Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.2.2-g-cos-^p-a+b-sin-^m-c+d-sin-^n

Nasser M. Abbasi

June 29, 2021 Compiled on June 29, 2021 at 9:46pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1563]. This is test number [74].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.81 (1560)	% 0.19 (3)
Mathematica	% 97.18 (1519)	% 2.82 (44)
Maple	% 88.29 (1380)	% 11.71 (183)
Maxima	% 62.96 (984)	% 37.04 (579)
Fricas	% 77.80 (1216)	% 22.20 (347)
Sympy	% 14.20 (222)	% 85.80 (1341)
Giac	% 71.08 (1111)	% 28.92 (452)
Mupad	% 72.36 (1131)	% 27.64 (432)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

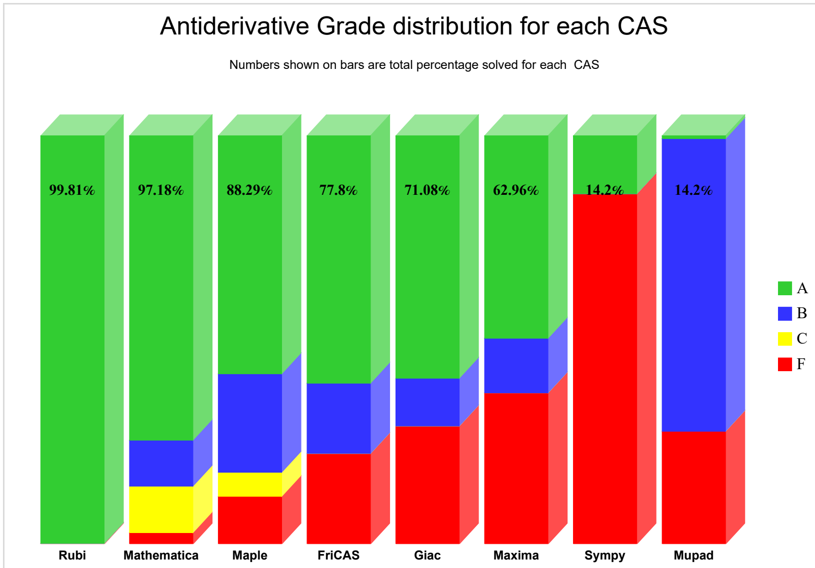
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

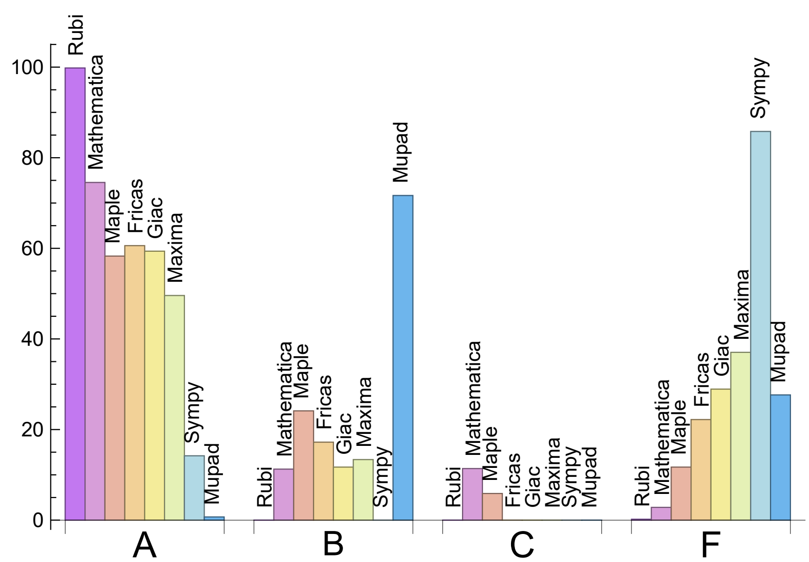
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.81	0.00	0.00	0.19
Mathematica	74.54	11.26	11.39	2.82
Maple	58.29	24.12	5.89	11.71
Maxima	49.58	13.37	0.00	37.04
Fricas	60.59	17.21	0.00	22.20
Sympy	14.20	0.00	0.00	85.80
Giac	59.37	11.71	0.00	28.92
Mupad	0.70	71.66	0.00	27.64

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	3	100.00 %	0.00 %	0.00 %
Mathematica	44	77.27 %	22.73 %	0.00 %
Maple	183	100.00 %	0.00 %	0.00 %
Maxima	579	73.58 %	7.08 %	19.34 %
Fricas	347	72.62 %	26.51 %	0.86 %
Sympy	1341	19.76 %	80.01 %	0.22 %
Giac	452	64.60 %	29.20 %	6.19 %
Mupad	432	98.61 %	1.39 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

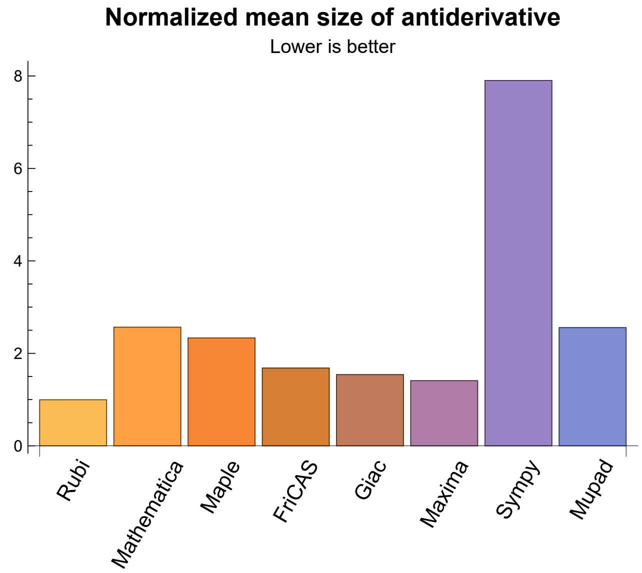
1.3 Performance

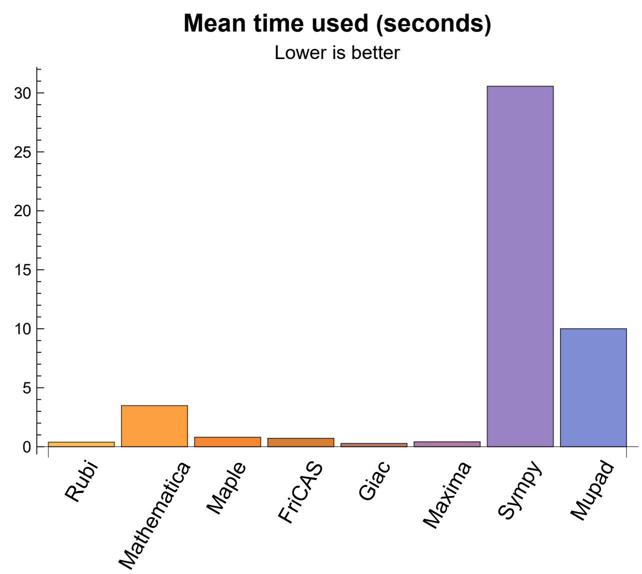
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.38	163.20	1.00	125.00	1.00
Mathematica	3.48	414.38	2.57	145.00	1.07
Maple	0.80	537.92	2.33	182.00	1.44
Maxima	0.41	163.26	1.41	119.00	1.05
Fricas	0.71	242.52	1.68	147.00	1.38
Sympy	30.56	836.31	7.90	335.00	2.80
Giac	0.27	202.05	1.54	152.00	1.40
Mupad	10.00	406.57	2.56	208.00	1.97

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{1191, 1192, 1193, 1194, 1519, 1520, 1521, 1522, 1526, 1527, 1560}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {65, 70, 77, 78, 81, 84, 85, 86, 87, 164, 165, 167, 168, 349, 478, 488, 818, 911, 937, 938, 942, 945, 1030, 1042, 1049, 1074, 1075, 1084, 1091, 1111, 1121, 1122, 1133, 1134, 1141, 1157, 1188, 1189, 1190, 1247, 1248, 1262, 1263, 1272, 1276, 1292, 1293, 1307, 1308, 1326, 1327, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1480, 1516, 1517, 1518, 1550, 1561, 1562, 1563}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

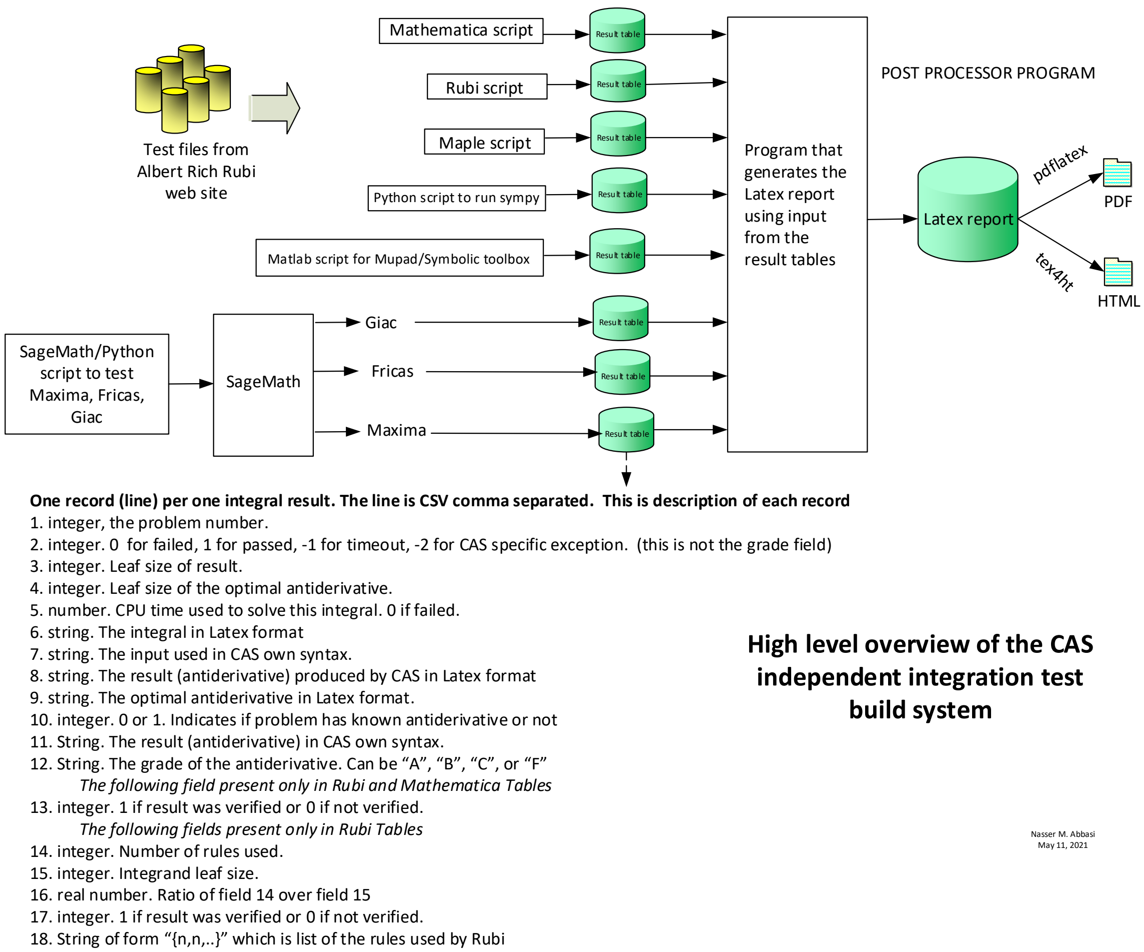
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563 }

B grade: { }

C grade: { }

F grade: { 1479, 1480, 1515 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 14, 15, 16, 18, 19, 20, 21, 22, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 69, 71, 72, 74, 75, 76, 79, 80, 81, 82, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 155, 156, 157, 158, 168, 173, 176, 177, 178, 179, 180, 181, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 299, 301, 303, 304, 305, 306, 307, 308, 309, 316, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 336, 337, 338, 339, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 374, 375, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 413, 414, 415, 417, 418, 419, 421, 425, 427, 428, 429, 430, 431, 432, 434, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 464, 465, 466, 467, 471, 472, 473, 474, 475, 480, 490, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 602, 603, 604, 605, 606, 607, 608, 609, 610, 612, 613, 614, 615, 616, 617, 619, 620, 621, 622, 623, 628, 629, 630, 631, 637, 638, 639, 640, 641, 642, 643, 647, 650, 651, 652, 653, 654, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 700, 701, 702, 703, 704, 710, 711, 712, 713, 714, 715, 721, 727, 728, 729, 730, 731, 733, 735, 736, 737, 738, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 759, 762, 763, 764, 765, 766, 768, 769, 770, 772, 773, 776, 778, 779, 780, 781, 782, 783, 784, 787, 788, 789, 790, 791, 792, 795, 796, 797, 798, 799, 800, 801, 802, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 819, 820, 821, 822, 823, 824, 825, 826, 829, 830, 831, 832, 833, 834, 835, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 938, 942, 945, 953, 954, 955, 956, 957, 961, 962, 963, 965, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1033, 1034, 1035, 1037, 1038, 1039, 1040, 1041, 1050, 1051, 1052, 1053, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1101, 1102, 1105, 1106, 1107, 1108, 1109, 1110, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1151, 1152, 1160, 1168, 1169, 1170, 1176, 1177, 1178, 1184, 1185, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1267, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1411, 1432, 1433, 1434, 1443, 1444, 1445, 1446, 1447, 1448, 1450, 1451, 1452, 1453, 1454, 1456, 1457, 1458, 1459, 1460, 1461, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1492, 1493, 1494, 1495, 1496, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1517, 1519, 1520, 1521, 1522, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560 }

B grade: { 13, 17, 23, 28, 35, 41, 42, 43, 44, 45, 77, 83, 167, 174, 188, 257, 282, 292, 293, 297, 298, 300, 302, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 326, 327, 328, 334, 335, 340, 341, 342, 376, 377, 388, 403, 410, 411, 412, 416, 420, 422, 423, 424, 426, 433, 435, 436, 437, 438, 439, 440, 441, 449, 463, 468, 469, 470, 476, 477, 478, 479, 492, 567, 585, 586, 587, 600, 601, 611, 618, 624, 625, 626, 627, 632, 633, 634, 635, 636, 644, 645, 646, 648, 649, 699, 705, 706, 707, 708, 709, 716, 717, 718, 719, 720, 722, 723, 724, 725, 726, 732, 734, 739, 740, 741, 757, 758, 760, 761, 767, 771, 774, 775, 777, 785, 786, 793, 794, 803, 818, 827, 828, 836, 837, 911, 964, 966, 967, 968, 1042, 1049, 1065, 1066, 1074, 1075, 1091, 1103, 1104, 1111, 1122, 1135, 1136, 1183, 1250, 1266, 1268, 1292, 1293, 1294, 1307, 1308, 1309, 1327, 1328, 1357, 1358, 1449, 1455, 1462, 1497, 1505, 1516, 1518, 1561, 1562, 1563 }

C grade: { 6, 7, 53, 61, 65, 70, 73, 78, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 108, 118, 129, 137, 146, 164, 165, 175, 184, 270, 271, 343, 344, 345, 346, 347, 348, 349, 372, 373, 481, 482, 483, 484, 485, 486, 487, 488, 489, 581, 582, 858, 859, 860, 936, 937, 958, 959, 960, 998, 1030, 1036, 1054, 1055, 1099, 1100, 1145, 1146, 1147, 1148, 1149, 1150, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1171, 1172, 1173, 1174, 1175, 1179, 1180, 1181, 1182, 1186, 1187, 1188, 1189, 1190, 1276, 1277, 1278, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1489, 1490, 1491, 1515 }

F grade: { 66, 67, 68, 152, 153, 154, 159, 160, 161, 162, 163, 166, 169, 170, 171, 172, 182, 183, 491, 655, 939, 940, 941, 943, 944, 946, 947, 948, 949, 950, 951, 952, 1031, 1032, 1043, 1044, 1045, 1046, 1047, 1048, 1514, 1523, 1524, 1525 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 6, 9, 10, 11, 12, 14, 15, 18, 19, 20, 21, 22, 24, 25, 26, 30, 31, 32, 33, 34, 36, 37, 38, 39, 43, 47, 49, 50, 51, 52, 53, 57, 58, 59, 60, 62, 64, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 301, 302, 303, 304, 305, 306, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 414, 417, 418, 425, 427, 429, 430, 431, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 631, 632, 639, 642, 648, 649, 650, 651, 652, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 668, 669, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 715, 731, 736, 745, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 766, 768, 769, 770, 771, 772, 773, 774, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 815, 816, 817, 818, 821, 822, 825, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 866, 867, 868, 869, 876, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 935, 936, 953, 954, 955, 956, 957, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 972, 977, 978, 979, 995, 996, 997, 1003, 1004, 1005, 1006, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1081, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1146, 1154, 1162, 1172, 1180, 1191, 1192, 1193, 1194, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1215, 1216, 1217, 1218, 1219, 1221, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1249, 1250, 1251, 1253, 1255, 1279, 1280, 1281, 1282, 1283, 1284, 1289, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1374, 1376, 1380, 1382, 1386, 1388, 1393, 1395, 1400, 1406, 1410, 1418, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1468, 1469, 1470, 1471, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1497, 1498, 1499, 1500, 1504, 1506, 1507, 1508, 1519, 1520, 1521, 1522, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1535, 1536, 1537, 1538, 1539, 1540, 1546, 1547, 1548, 1549, 1554, 1555, 1556, 1557, 1558, 1560 }

B grade: { 5, 7, 8, 13, 16, 17, 23, 27, 28, 29, 35, 40, 41, 42, 44, 45, 46, 48, 54, 55, 56, 61, 63, 297, 298, 299, 300, 307, 308, 309, 315, 385, 409, 410, 411, 412, 413, 415, 416, 419, 420, 421, 422, 423, 424, 426, 428, 432, 433, 434, 506, 507, 508, 509, 510, 521, 522, 597, 624, 625, 626, 627, 628, 629, 630, 633, 634, 635, 636, 637, 638, 640, 641, 643, 644, 645, 646, 647, 667, 670, 671, 672, 673, 674, 675, 676, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 732, 733, 734, 735, 737, 738, 739, 740, 741, 742, 743, 744, 746, 747, 748, 749, 750, 765, 767, 775, 811, 812, 813, 814, 819, 820, 823, 824, 826, 861, 862, 863, 864, 865, 870, 871, 872, 873, 874, 875, 877, 894, 937, 958, 970, 971, 973, 974, 975, 976, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 998, 999, 1000, 1001, 1002, 1007, 1008, 1015, 1078, 1079, 1080, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1210, 1211, 1212, 1213, 1214, 1220, 1222, 1248, 1252, 1254, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1285, 1286, 1287, 1288, 1290, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1355, 1408, 1409, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1467, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1496, 1501, 1502, 1503, 1505, 1515, 1533, 1534, 1541, 1542, 1543, 1544, 1545, 1550, 1551, 1552, 1553, 1559 }

C grade: { 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 174, 1370, 1371, 1372, 1373, 1375, 1377, 1378, 1379, 1381, 1383, 1384, 1385, 1387, 1389, 1390, 1391, 1392, 1394, 1396, 1397, 1398, 1399, 1401, 1402, 1403, 1404, 1405, 1407 }

F grade: { 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 258, 259, 260, 261, 262, 263, 264, 265, 490, 491, 492, 493, 565, 566, 567, 568, 569, 570, 571, 653, 654, 655, 697, 698, 699, 700, 701, 702, 703, 704, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1195, 1196, 1197, 1235, 1236, 1237, 1238, 1509, 1510, 1511, 1512, 1513, 1514, 1516, 1517, 1518, 1523, 1524, 1525, 1561, 1562, 1563 }

2.1.4 Maxima

A grade: { 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 311, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 435, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 774, 776, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 915, 916, 922, 923, 928, 929, 930, 931, 935, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 1000, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1023, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1191, 1192, 1193, 1194, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1279, 1280, 1281, 1282, 1283, 1284, 1295, 1296, 1297, 1298, 1299, 1300, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1332, 1333, 1334, 1335, 1336, 1337, 1346, 1347, 1348, 1349, 1350, 1351, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1519, 1520, 1521, 1522, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1560 }

B grade: { 5, 14, 25, 38, 46, 52, 59, 73, 74, 75, 175, 176, 177, 185, 252, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 442, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 772, 773, 775, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 913, 914, 920, 921, 999, 1001, 1002, 1020, 1021, 1022, 1559 }

C grade: { }

F grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 262, 263, 264, 265, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 570, 571, 653, 654, 655, 703, 704, 911, 912, 917, 918, 919, 924, 925, 926, 927, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 1019, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1195, 1196, 1197, 1237, 1238, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1523, 1524, 1525, 1561, 1562, 1563 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 29, 30, 31, 32, 33, 34, 43, 45, 46, 47, 49, 55, 56, 62, 63, 64, 73, 74, 75, 81, 82, 83, 174, 181, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 253, 254, 255, 256, 257, 260, 261, 266, 267, 268, 269, 275, 276, 277, 278, 279, 285, 286, 287, 288, 289, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 315, 316, 321, 322, 323, 324, 329, 330, 331, 336, 337, 338, 343, 344, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 377, 378, 379, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 443, 444, 453, 454, 455, 464, 465, 472, 473, 481, 482, 483, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 568, 569, 572, 573, 574, 575, 576, 577, 578, 579, 580, 588, 589, 590, 591, 592, 593, 594, 595, 596, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 700, 701, 702, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 759, 760, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 807, 819, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 857, 858, 861, 862, 863, 865, 866, 870, 871, 872, 873, 874, 875, 876, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 916, 922, 923, 928, 929, 930, 931, 935, 953, 954, 955, 956, 957, 959, 960, 961, 962, 963, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 977, 978, 979, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 995, 996, 997, 998, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1017, 1018, 1021, 1022, 1023, 1033, 1034, 1035, 1040, 1041, 1050, 1051, 1052, 1053, 1059, 1060, 1061, 1062, 1063, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1085, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1135, 1136, 1191, 1192, 1193, 1194, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1256, 1257, 1258, 1259, 1260, 1266, 1267, 1268, 1269, 1270, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1307, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1320, 1321, 1322, 1323, 1324, 1326, 1330, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1346, 1347, 1348, 1349, 1350, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1471, 1472, 1473, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1521, 1522, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1560 }

B grade: { 17, 23, 28, 35, 41, 42, 44, 76, 79, 80, 175, 176, 177, 213, 220, 248, 252, 258, 259, 270, 271, 272, 273, 274, 280, 281, 282, 283, 284, 290, 291, 292, 293, 302, 311, 312, 313, 314, 317, 318, 319, 320, 325, 326, 327, 328, 332, 333, 334, 335, 339, 340, 341, 342, 345, 346, 347, 348, 349, 372, 373, 374, 375, 376, 385, 386, 387, 401, 416, 435, 436, 437, 438, 439, 440, 441, 442, 445, 446, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 460, 461, 462, 463, 466, 467, 468, 469, 470, 471, 474, 475, 476, 477, 478, 479, 480, 484, 485, 486, 487, 488, 489, 563, 564, 565, 566, 567, 581, 582, 583, 584, 585, 586, 587, 597, 598, 599, 601, 603, 616, 633, 697, 698, 699, 717, 753, 754, 755, 756, 757, 758, 761, 762, 763, 764, 768, 769, 770, 771, 802, 803, 804, 805, 806, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 820, 856, 859, 860, 864, 867, 868, 869, 877, 913, 914, 915, 920, 921, 936, 937, 958, 964, 975, 976, 980, 993, 994, 999, 1015, 1016, 1020, 1054, 1055, 1056, 1057, 1058, 1064, 1081, 1082, 1083, 1084, 1086, 1087, 1088, 1089, 1090, 1099, 1100, 1101, 1102, 1103, 1132, 1133, 1134, 1137, 1138, 1139, 1140, 1141, 1142, 1231, 1232, 1233, 1234, 1235, 1236, 1250, 1251, 1252, 1253, 1254, 1255, 1261, 1262, 1263, 1264, 1265, 1271, 1272, 1273, 1274, 1275, 1293, 1306, 1308, 1309, 1318, 1319, 1325, 1327, 1328, 1329, 1331, 1344, 1345, 1351, 1369, 1468, 1469, 1470, 1474, 1475, 1476, 1477, 1556, 1557, 1558, 1559 }

C grade: { }

F grade: { 6, 7, 14, 15, 16, 24, 25, 26, 27, 36, 37, 38, 39, 40, 48, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 77, 78, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 178, 179, 180, 182, 183, 184, 189, 262, 263, 264, 265, 490, 491, 492, 493, 570, 571, 653, 654, 655, 703, 704, 911, 912, 917, 918, 919, 924, 925, 926, 927, 932, 933, 934, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 1019, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1036, 1037, 1038, 1039, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1091, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1195, 1196, 1197, 1237, 1238, 1276, 1277, 1278, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1478, 1479, 1480, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1523, 1524, 1525, 1561, 1562, 1563 }

2.1.6 Sympy

A grade: { 190, 191, 197, 198, 206, 207, 208, 217, 218, 219, 220, 224, 225, 226, 227, 232, 233, 234, 235, 240, 241, 242, 243, 244, 249, 250, 251, 252, 253, 258, 259, 260, 261, 266, 267, 268, 275, 276, 277, 285, 286, 294, 297, 298, 299, 300, 307, 308, 309, 310, 315, 316, 317, 321, 350, 351, 352, 353, 357, 358, 359, 365, 366, 367, 368, 378, 379, 380, 381, 392, 393, 394, 395, 407, 409, 410, 411, 412, 421, 422, 423, 424, 432, 433, 434, 494, 495, 496, 497, 498, 511, 512, 513, 521, 522, 533, 534, 535, 544, 545, 546, 555, 556, 572, 573, 574, 575, 588, 589, 590, 591, 605, 606, 607, 608, 625, 626, 627, 635, 636, 646, 656, 657, 658, 659, 660, 661, 680, 681, 682, 683, 708, 709, 726, 913, 914, 915, 916, 920, 921, 922, 923, 928, 929, 930, 931, 935, 953, 954, 955, 956, 961, 962, 963, 969, 970, 971, 972, 977, 978, 979, 986, 987, 988, 989, 995, 996, 997, 1003, 1004, 1005, 1006, 1011, 1012, 1013, 1014, 1022, 1023, 1050, 1051, 1052, 1059, 1060, 1061, 1069, 1070, 1092, 1093, 1094, 1095, 1105, 1106, 1107, 1116, 1117, 1191, 1198, 1199, 1200, 1201, 1202, 1215, 1216, 1236, 1239, 1240, 1241, 1242, 1243, 1279, 1280, 1281, 1528, 1529, 1530, 1531, 1536, 1537, 1538, 1539, 1547, 1555 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 192, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 209, 210, 211, 212, 213, 214, 215, 216, 221, 222, 223, 228, 229, 230, 231, 236, 237, 238, 239, 245, 246, 247, 248, 254, 255, 256, 257, 262, 263, 264, 265, 269, 270, 271, 272, 273, 274, 278, 279, 280, 281, 282, 283, 284, 287, 288, 289, 290, 291, 292, 293, 295, 296, 301, 302, 303, 304, 305, 306, 311, 312, 313, 314, 318, 319, 320, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 354, 355, 356, 360, 361, 362, 363, 364, 369, 370, 371, 372, 373, 374, 375, 376, 377, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 408, 413, 414, 415, 416, 417, 418, 419, 420, 425, 426, 427, 428, 429, 430, 431, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 514, 515, 516, 517, 518, 519, 520, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 536, 537, 538, 539, 540, 541, 542, 543, 547, 548, 549, 550, 551, 552, 553, 554, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 629, 630, 631, 632, 633, 634, 637, 638, 639, 640, 641, 642, 643, 644, 645, 647, 648, 649, 650, 651, 652, 653, 654, 655, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 917, 918, 919, 924, 925, 926, 927, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 957, 958, 959, 960, 964, 965, 966, 967, 968, 973, 974, 975, 976, 980, 981, 982, 983, 984, 985, 990, 991, 992, 993, 994, 998, 999, 1000, 1001, 1002, 1007, 1008, 1009, 1010, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1053, 1054, 1055, 1056, 1057, 1058, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1192, 1193, 1194, 1195, 1196, 1197, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1237, 1238, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1532, 1533, 1534, 1535, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563 }

2.1.7 Giac

A grade: { 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 278, 280, 281, 283, 285, 286, 287, 288, 290, 291, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 336, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 472, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 559, 560, 561, 562, 563, 564, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 598, 599, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 705, 706, 707, 708, 710, 711, 712, 713, 714, 715, 716, 719, 720, 722, 723, 724, 725, 726, 727, 728, 729, 731, 732, 733, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 748, 749, 750, 751, 752, 754, 755, 756, 757, 758, 759, 760, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 825, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 931, 935, 953, 954, 955, 956, 958, 959, 960, 961, 962, 963, 964, 969, 971, 972, 975, 976, 977, 978, 979, 981, 982, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1050, 1051, 1052, 1053, 1055, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1086, 1087, 1088, 1089, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1104, 1105, 1106, 1107, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1131, 1132, 1133, 1134, 1135, 1136, 1138, 1139, 1140, 1141, 1142, 1193, 1194, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1239, 1240, 1241, 1242, 1243, 1249, 1256, 1257, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1279, 1280, 1281, 1282, 1283, 1284, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1295, 1296, 1297, 1298, 1299, 1300, 1304, 1306, 1307, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1323, 1324, 1326, 1329, 1330, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1443, 1444, 1445, 1446, 1447, 1449, 1450, 1451, 1452, 1453, 1455, 1456, 1457, 1458, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1519, 1520, 1522, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1552, 1553, 1554, 1555, 1556, 1557, 1560 }

B grade: { 257, 258, 259, 260, 261, 270, 272, 279, 282, 284, 289, 292, 293, 302, 329, 330, 331, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 374, 375, 376, 385, 399, 408, 414, 416, 426, 453, 454, 464, 465, 466, 467, 468, 469, 470, 471, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489, 554, 555, 556, 557, 558, 583, 584, 585, 587, 597, 601, 603, 627, 633, 641, 648, 709, 717, 718, 721, 730, 734, 747, 753, 761, 824, 826, 870, 871, 872, 913, 914, 915, 916, 920, 921, 922, 923, 928, 929, 930, 957, 965, 966, 967, 968, 970, 973, 974, 980, 983, 984, 985, 986, 987, 988, 989, 990, 991, 1001, 1002, 1020, 1021, 1022, 1023, 1054, 1056, 1071, 1072, 1073, 1085, 1090, 1101, 1102, 1103, 1108, 1121, 1130, 1137, 1244, 1245, 1246, 1247, 1248, 1250, 1251, 1252, 1253, 1254, 1255, 1258, 1268, 1285, 1286, 1294, 1301, 1302, 1303, 1305, 1308, 1309, 1320, 1321, 1322, 1325, 1327, 1328, 1331, 1344, 1369, 1448, 1454, 1459, 1469, 1477, 1550, 1551, 1558, 1559 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 262, 263, 264, 265, 325, 326, 327, 328, 332, 333, 334, 335, 346, 445, 446, 447, 448, 449, 450, 451, 452, 455, 456, 457, 458, 459, 460, 461, 462, 463, 485, 490, 491, 492, 493, 565, 566, 567, 568, 569, 570, 571, 653, 654, 655, 697, 698, 699, 700, 701, 702, 703, 704, 911, 912, 917, 918, 919, 924, 925, 926, 927, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 1019, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1091, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1195, 1196, 1197, 1235, 1236, 1237, 1238, 1276, 1277, 1278, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1478, 1479, 1480, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1521, 1523, 1524, 1525, 1561, 1562, 1563 }

2.1.8 Mupad

A grade: { 1191, 1192, 1193, 1194, 1519, 1520, 1521, 1522, 1526, 1527, 1560 }

B grade: { 1, 2, 3, 4, 5, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 49, 62, 73, 74, 75, 76, 79, 80, 81, 82, 83, 175, 176, 177, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 913, 914, 915, 916, 920, 921, 922, 923, 928, 929, 930, 931, 935, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1020, 1021, 1022, 1023, 1033, 1034, 1035, 1040, 1041, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559 }

C grade: { }

F grade: { 6, 7, 8, 14, 15, 16, 17, 24, 25, 26, 27, 28, 36, 37, 38, 39, 40, 41, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 77, 78, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 189, 262, 263, 264, 265, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 570, 571, 653, 654, 655, 703, 704, 911, 912, 917, 918, 919, 924, 925, 926, 927, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 1019, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1036, 1037, 1038, 1039, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1091, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1195, 1196, 1197, 1237, 1238, 1276, 1277, 1278, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1478, 1479, 1480, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1523, 1524, 1525, 1561, 1562, 1563 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	104	133	0	110	0	0	121
normalized size	1	1.00	1.13	1.45	0.00	1.20	0.00	0.00	1.32
time (sec)	N/A	0.398	0.568	0.447	0.000	0.442	0.000	0.000	11.449

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	94	106	0	96	0	0	110
normalized size	1	1.00	1.02	1.15	0.00	1.04	0.00	0.00	1.20
time (sec)	N/A	0.391	0.476	0.420	0.000	0.446	0.000	0.000	10.453

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	83	90	0	75	0	0	97
normalized size	1	1.00	0.90	0.98	0.00	0.82	0.00	0.00	1.05
time (sec)	N/A	0.390	0.399	0.395	0.000	0.435	0.000	0.000	1.714

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	59	55	0	54	0	0	64
normalized size	1	1.00	0.64	0.60	0.00	0.59	0.00	0.00	0.70
time (sec)	N/A	0.372	0.170	0.383	0.000	0.422	0.000	0.000	0.895

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	62	94	387	59	0	0	73
normalized size	1	1.00	1.38	2.09	8.60	1.31	0.00	0.00	1.62
time (sec)	N/A	0.284	0.303	0.380	0.496	0.419	0.000	0.000	8.959

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	115	138	0	0	0	0	-1
normalized size	1	1.00	1.16	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.425	1.088	0.376	0.000	0.664	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	104	192	0	0	0	0	-1
normalized size	1	1.00	1.07	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.431	0.834	0.380	0.000	0.846	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	90	96	0	79	0	0	-1
normalized size	1	1.00	1.88	2.00	0.00	1.65	0.00	0.00	-0.02
time (sec)	N/A	0.319	0.394	0.381	0.000	0.440	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	166	133	0	116	0	0	319
normalized size	1	1.00	1.19	0.95	0.00	0.83	0.00	0.00	2.28
time (sec)	N/A	0.524	1.212	0.422	0.000	0.511	0.000	0.000	12.429

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	156	116	0	102	0	0	122
normalized size	1	1.00	1.11	0.83	0.00	0.73	0.00	0.00	0.87
time (sec)	N/A	0.529	0.967	0.363	0.000	0.476	0.000	0.000	11.444

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	82	67	0	73	0	0	79
normalized size	1	1.00	0.59	0.48	0.00	0.52	0.00	0.00	0.56
time (sec)	N/A	0.521	0.616	0.339	0.000	0.463	0.000	0.000	1.477

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	83	90	0	75	0	0	97
normalized size	1	1.00	0.90	0.98	0.00	0.82	0.00	0.00	1.05
time (sec)	N/A	0.390	0.392	0.390	0.000	0.422	0.000	0.000	9.813

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	111	141	0	77	0	0	87
normalized size	1	1.00	2.47	3.13	0.00	1.71	0.00	0.00	1.93
time (sec)	N/A	0.311	0.546	0.360	0.000	0.440	0.000	0.000	9.398

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	130	173	844	0	0	0	-1
normalized size	1	1.00	0.88	1.18	5.74	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.546	1.064	0.362	0.955	0.815	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	169	223	0	0	0	0	-1
normalized size	1	1.00	1.17	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.547	1.059	0.368	0.000	1.144	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	191	276	0	0	0	0	-1
normalized size	1	1.00	1.30	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.566	1.364	0.422	0.000	1.492	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	110	127	0	105	0	0	-1
normalized size	1	1.00	2.29	2.65	0.00	2.19	0.00	0.00	-0.02
time (sec)	N/A	0.337	1.424	0.354	0.000	0.446	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	118	152	0	127	0	0	236
normalized size	1	1.00	1.22	1.57	0.00	1.31	0.00	0.00	2.43
time (sec)	N/A	0.436	1.953	0.385	0.000	0.445	0.000	0.000	13.867

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	176	143	0	128	0	0	376
normalized size	1	1.00	0.94	0.76	0.00	0.68	0.00	0.00	2.00
time (sec)	N/A	0.624	3.110	0.444	0.000	0.486	0.000	0.000	12.623

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	87	77	0	101	0	0	179
normalized size	1	1.00	0.46	0.41	0.00	0.54	0.00	0.00	0.95
time (sec)	N/A	0.620	0.648	0.371	0.000	0.491	0.000	0.000	11.802

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	152	116	0	102	0	0	122
normalized size	1	1.00	1.09	0.83	0.00	0.73	0.00	0.00	0.87
time (sec)	N/A	0.517	0.767	0.388	0.000	0.463	0.000	0.000	11.517

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	106	0	96	0	0	108
normalized size	1	1.00	1.00	1.15	0.00	1.04	0.00	0.00	1.17
time (sec)	N/A	0.395	0.505	0.410	0.000	0.456	0.000	0.000	2.410

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	119	199	0	98	0	0	102
normalized size	1	1.00	2.64	4.42	0.00	2.18	0.00	0.00	2.27
time (sec)	N/A	0.307	0.922	0.362	0.000	0.467	0.000	0.000	1.875

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	140	218	0	0	0	0	-1
normalized size	1	1.00	0.73	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.642	2.609	0.355	0.000	0.595	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	181	273	1120	0	0	0	-1
normalized size	1	1.00	0.94	1.42	5.83	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.646	2.376	0.374	0.508	1.291	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	209	325	0	0	0	0	-1
normalized size	1	1.00	1.07	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.657	2.744	0.376	0.000	1.742	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	234	397	0	0	0	0	-1
normalized size	1	1.00	1.21	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.663	4.135	0.377	0.000	2.570	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	117	153	0	127	0	0	-1
normalized size	1	1.00	2.44	3.19	0.00	2.65	0.00	0.00	-0.02
time (sec)	N/A	0.339	4.288	0.342	0.000	0.488	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	130	187	0	163	0	0	317
normalized size	1	1.00	1.34	1.93	0.00	1.68	0.00	0.00	3.27
time (sec)	N/A	0.438	6.275	0.372	0.000	0.534	0.000	0.000	14.860

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	209	169	0	144	0	0	462
normalized size	1	1.00	0.89	0.72	0.00	0.61	0.00	0.00	1.96
time (sec)	N/A	0.725	5.626	0.512	0.000	1.225	0.000	0.000	13.533

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	97	87	0	117	0	0	247
normalized size	1	1.00	0.41	0.37	0.00	0.50	0.00	0.00	1.05
time (sec)	N/A	0.735	1.263	0.394	0.000	0.522	0.000	0.000	12.115

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	127	143	0	128	0	0	376
normalized size	1	1.00	0.68	0.76	0.00	0.68	0.00	0.00	2.00
time (sec)	N/A	0.620	2.414	0.411	0.000	0.494	0.000	0.000	12.548

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	115	133	0	115	0	0	319
normalized size	1	1.00	0.82	0.95	0.00	0.82	0.00	0.00	2.28
time (sec)	N/A	0.515	1.279	0.404	0.000	0.477	0.000	0.000	12.144

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	104	133	0	110	0	0	121
normalized size	1	1.00	1.13	1.45	0.00	1.20	0.00	0.00	1.32
time (sec)	N/A	0.386	0.538	0.402	0.000	0.436	0.000	0.000	11.485

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	142	245	0	111	0	0	113
normalized size	1	1.00	3.16	5.44	0.00	2.47	0.00	0.00	2.51
time (sec)	N/A	0.307	1.442	0.398	0.000	0.467	0.000	0.000	10.690

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	473	250	0	0	0	0	-1
normalized size	1	1.00	1.96	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.760	6.473	0.388	0.000	1.141	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	196	307	0	0	0	0	-1
normalized size	1	1.00	0.82	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.754	4.809	0.392	0.000	1.730	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	223	365	1396	0	0	0	-1
normalized size	1	1.00	0.93	1.53	5.84	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.764	6.553	0.418	0.536	2.277	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	442	435	0	0	0	0	-1
normalized size	1	1.00	1.83	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.781	6.612	0.408	0.000	3.391	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	437	490	0	0	0	0	-1
normalized size	1	1.00	1.80	2.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.785	6.649	0.398	0.000	4.658	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	412	188	0	150	0	0	-1
normalized size	1	1.00	8.58	3.92	0.00	3.12	0.00	0.00	-0.02
time (sec)	N/A	0.339	6.741	0.362	0.000	0.470	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	419	230	0	191	0	0	373
normalized size	1	1.00	4.32	2.37	0.00	1.97	0.00	0.00	3.85
time (sec)	N/A	0.444	6.791	0.398	0.000	0.501	0.000	0.000	14.413

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	419	240	0	205	0	0	764
normalized size	1	1.00	2.89	1.66	0.00	1.41	0.00	0.00	5.27
time (sec)	N/A	0.539	6.868	0.433	0.000	0.507	0.000	0.000	14.929

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	134	195	0	98	0	0	96
normalized size	1	1.00	2.98	4.33	0.00	2.18	0.00	0.00	2.13
time (sec)	N/A	0.306	0.897	0.381	0.000	0.449	0.000	0.000	2.003

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	120	147	0	78	0	0	83
normalized size	1	1.00	2.67	3.27	0.00	1.73	0.00	0.00	1.84
time (sec)	N/A	0.309	0.512	0.368	0.000	0.449	0.000	0.000	9.964

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	62	90	388	59	0	0	67
normalized size	1	1.00	1.38	2.00	8.62	1.31	0.00	0.00	1.49
time (sec)	N/A	0.284	0.285	0.370	1.632	0.443	0.000	0.000	0.931

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	44	42	0	49	0	0	52
normalized size	1	1.00	1.02	0.98	0.00	1.14	0.00	0.00	1.21
time (sec)	N/A	0.288	0.299	0.364	0.000	0.466	0.000	0.000	8.869

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	104	138	0	0	0	0	-1
normalized size	1	1.00	1.93	2.56	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.340	0.414	0.368	0.000	0.782	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	79	51	0	61	0	0	88
normalized size	1	1.00	1.88	1.21	0.00	1.45	0.00	0.00	2.10
time (sec)	N/A	0.321	0.494	0.365	0.000	0.451	0.000	0.000	9.771

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	471	252	0	0	0	0	-1
normalized size	1	1.00	1.97	1.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.745	6.469	0.395	0.000	1.146	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	138	214	0	0	0	0	-1
normalized size	1	1.00	0.73	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.639	2.370	0.364	0.000	0.590	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	134	175	844	0	0	0	-1
normalized size	1	1.00	0.92	1.21	5.82	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.529	1.115	0.351	1.056	0.795	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	113	137	0	0	0	0	-1
normalized size	1	1.00	1.18	1.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.412	1.057	0.371	0.000	0.684	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	102	137	0	0	0	0	-1
normalized size	1	1.00	2.00	2.69	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.333	0.430	0.389	0.000	0.774	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	103	174	0	160	0	0	-1
normalized size	1	1.00	1.98	3.35	0.00	3.08	0.00	0.00	-0.02
time (sec)	N/A	0.340	0.534	0.358	0.000	0.483	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	163	244	0	319	0	0	-1
normalized size	1	1.00	1.57	2.35	0.00	3.07	0.00	0.00	-0.01
time (sec)	N/A	0.436	0.775	0.352	0.000	0.521	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	553	349	0	0	0	0	-1
normalized size	1	1.00	1.94	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.865	6.642	0.365	0.000	2.262	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	179	306	0	0	0	0	-1
normalized size	1	1.00	0.76	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.752	5.309	0.409	0.000	1.706	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	164	270	1120	0	0	0	-1
normalized size	1	1.00	0.86	1.41	5.86	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.639	2.494	0.408	1.613	1.207	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	153	229	0	0	0	0	-1
normalized size	1	1.00	1.07	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.538	1.133	0.402	0.000	1.033	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	100	192	0	0	0	0	-1
normalized size	1	1.00	1.03	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.423	0.985	0.364	0.000	0.845	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	80	50	0	60	0	0	55
normalized size	1	1.00	1.86	1.16	0.00	1.40	0.00	0.00	1.28
time (sec)	N/A	0.321	0.521	0.363	0.000	0.438	0.000	0.000	9.261

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	163	246	0	313	0	0	-1
normalized size	1	1.00	1.57	2.37	0.00	3.01	0.00	0.00	-0.01
time (sec)	N/A	0.444	0.803	0.354	0.000	0.520	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	163	196	0	262	0	0	-1
normalized size	1	1.00	1.07	1.29	0.00	1.72	0.00	0.00	-0.01
time (sec)	N/A	0.540	0.878	0.359	0.000	0.513	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	3426	0	0	0	0	0	-1
normalized size	1	1.00	30.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.301	14.258	4.197	0.000	0.498	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	180.013	8.860	0.000	0.486	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	180.001	7.567	0.000	0.463	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.156	180.003	3.149	0.000	0.455	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.109	1.651	0.000	0.478	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	6442	0	0	0	0	0	-1
normalized size	1	1.00	83.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	19.684	4.023	0.000	0.455	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	88	0	0	0	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.207	3.326	0.000	0.474	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	91	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.147	3.017	0.000	0.459	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	695	0	558	395	0	0	1060
normalized size	1	1.00	2.85	0.00	2.29	1.62	0.00	0.00	4.34
time (sec)	N/A	0.615	6.559	0.796	0.661	0.493	0.000	0.000	14.970

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	149	0	423	243	0	0	528
normalized size	1	1.00	0.87	0.00	2.46	1.41	0.00	0.00	3.07
time (sec)	N/A	0.467	3.104	0.735	0.517	0.480	0.000	0.000	13.973

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	111	0	312	155	0	0	104
normalized size	1	1.00	1.04	0.00	2.92	1.45	0.00	0.00	0.97
time (sec)	N/A	0.354	0.570	0.707	0.528	0.492	0.000	0.000	1.720

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	85	0	0	108	0	0	68
normalized size	1	1.00	1.70	0.00	0.00	2.16	0.00	0.00	1.36
time (sec)	N/A	0.262	0.374	0.673	0.000	0.460	0.000	0.000	0.899

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	218	0	0	0	0	0	-1
normalized size	1	1.00	2.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.362	6.655	0.652	0.000	0.460	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	3174	0	0	0	0	0	-1
normalized size	1	1.00	40.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.372	6.669	0.681	0.000	0.476	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	85	0	0	108	0	0	68
normalized size	1	1.00	1.70	0.00	0.00	2.16	0.00	0.00	1.36
time (sec)	N/A	0.254	0.336	0.010	0.000	0.503	0.000	0.000	0.002

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	85	0	0	108	0	0	68
normalized size	1	1.00	1.70	0.00	0.00	2.16	0.00	0.00	1.36
time (sec)	N/A	0.254	0.356	0.864	0.000	0.475	0.000	0.000	9.110

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	176	0	0	105	0	0	334
normalized size	1	1.00	0.97	0.00	0.00	0.58	0.00	0.00	1.84
time (sec)	N/A	0.448	17.033	5.366	0.000	0.511	0.000	0.000	15.748

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	142	0	0	76	0	0	177
normalized size	1	1.00	1.25	0.00	0.00	0.67	0.00	0.00	1.55
time (sec)	N/A	0.335	12.609	5.396	0.000	0.457	0.000	0.000	10.599

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	109	0	0	48	0	0	101
normalized size	1	1.00	2.02	0.00	0.00	0.89	0.00	0.00	1.87
time (sec)	N/A	0.251	5.519	4.824	0.000	0.497	0.000	0.000	1.043

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	589	0	0	0	0	0	-1
normalized size	1	1.00	5.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.380	21.555	3.579	0.000	0.490	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	1045	0	0	0	0	0	-1
normalized size	1	1.00	9.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.371	29.645	1.353	0.000	0.542	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	1519	0	0	0	0	0	-1
normalized size	1	1.00	13.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.323	21.708	1.834	0.000	0.542	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	4270	0	0	0	0	0	-1
normalized size	1	1.00	36.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.368	25.901	2.962	0.000	0.514	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	311	425	0	0	0	0	-1
normalized size	1	1.00	0.91	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.743	8.015	0.760	0.000	0.546	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	281	392	0	0	0	0	-1
normalized size	1	1.00	0.97	1.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.428	1.932	0.642	0.000	0.536	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	255	372	0	0	0	0	-1
normalized size	1	1.00	1.09	1.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.129	1.717	0.621	0.000	0.521	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	249	346	0	0	0	0	-1
normalized size	1	1.00	1.40	1.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.803	2.391	0.671	0.000	0.523	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	197	362	0	0	0	0	-1
normalized size	1	1.00	1.61	2.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.570	1.921	0.581	0.000	0.491	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	211	2835	0	0	0	0	-1
normalized size	1	1.00	1.72	23.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.582	1.691	0.671	0.000	0.498	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	229	2040	0	0	0	0	-1
normalized size	1	1.00	1.26	11.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.879	2.077	0.559	0.000	0.553	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	256	966	0	0	0	0	-1
normalized size	1	1.00	1.08	4.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.165	2.180	0.558	0.000	0.529	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	291	1126	0	0	0	0	-1
normalized size	1	1.00	1.00	3.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.460	2.634	0.609	0.000	0.530	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	193	382	0	0	0	0	-1
normalized size	1	1.00	0.55	1.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.727	1.308	0.630	0.000	0.584	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	113	356	0	0	0	0	-1
normalized size	1	1.00	0.38	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.503	0.694	0.564	0.000	0.580	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	257	372	0	0	0	0	-1
normalized size	1	1.00	1.09	1.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.139	7.599	0.604	0.000	0.540	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	148	384	0	0	0	0	-1
normalized size	1	1.00	0.82	2.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.850	0.773	0.535	0.000	0.515	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	207	2894	0	0	0	0	-1
normalized size	1	1.00	1.14	15.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.876	1.746	0.568	0.000	0.631	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	191	3499	0	0	0	0	-1
normalized size	1	1.00	1.03	18.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.887	1.393	0.552	0.000	0.571	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	218	2684	0	0	0	0	-1
normalized size	1	1.00	0.90	11.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.194	2.337	0.565	0.000	0.543	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	464	1138	0	0	0	0	-1
normalized size	1	1.00	1.55	3.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.497	6.472	0.561	0.000	0.626	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	532	1298	0	0	0	0	-1
normalized size	1	1.00	1.49	3.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.808	6.494	0.621	0.000	0.590	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	120	366	0	0	0	0	-1
normalized size	1	1.00	0.30	0.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.089	1.164	0.589	0.000	0.548	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	189	382	0	0	0	0	-1
normalized size	1	1.00	0.54	1.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.757	1.280	0.677	0.000	0.514	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	281	394	0	0	0	0	-1
normalized size	1	1.00	0.97	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.420	2.044	0.647	0.000	0.578	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	158	415	0	0	0	0	-1
normalized size	1	1.00	0.68	1.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.142	1.771	0.596	0.000	0.544	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	240	2945	0	0	0	0	-1
normalized size	1	1.00	1.00	12.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.140	6.341	0.646	0.000	0.589	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	245	3549	0	0	0	0	-1
normalized size	1	1.00	1.01	14.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.175	2.435	0.660	0.000	0.607	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	246	4183	0	0	0	0	-1
normalized size	1	1.00	1.01	17.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.197	2.639	0.713	0.000	0.590	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	464	3455	0	0	0	0	-1
normalized size	1	1.00	1.55	11.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.483	6.538	0.712	0.000	0.590	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	532	1313	0	0	0	0	-1
normalized size	1	1.00	1.49	3.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.787	6.570	0.780	0.000	0.550	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	600	1473	0	0	0	0	-1
normalized size	1	1.00	1.45	3.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.080	6.679	0.750	0.000	0.591	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	226	392	0	0	0	0	-1
normalized size	1	1.00	0.49	0.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.390	3.523	0.855	0.000	0.560	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	212	404	0	0	0	0	-1
normalized size	1	1.00	0.52	0.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.042	2.921	0.729	0.000	0.564	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	360	425	0	0	0	0	-1
normalized size	1	1.00	1.05	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.720	4.322	0.717	0.000	0.555	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	181	436	0	0	0	0	-1
normalized size	1	1.00	0.63	1.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.437	3.714	0.564	0.000	0.528	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	284	2998	0	0	0	0	-1
normalized size	1	1.00	0.97	10.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.455	6.521	0.623	0.000	0.510	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	267	3601	0	0	0	0	-1
normalized size	1	1.00	0.90	12.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.488	4.638	0.620	0.000	0.514	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	406	4237	0	0	0	0	-1
normalized size	1	1.00	1.35	14.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.541	6.582	0.617	0.000	0.560	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	464	4829	0	0	0	0	-1
normalized size	1	1.00	1.55	16.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.534	6.632	0.606	0.000	0.556	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	532	3910	0	0	0	0	-1
normalized size	1	1.00	1.49	10.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.840	6.684	0.594	0.000	0.565	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	600	1484	0	0	0	0	-1
normalized size	1	1.00	1.45	3.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.171	6.786	0.624	0.000	0.580	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	668	1644	0	0	0	0	-1
normalized size	1	1.00	1.42	3.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.487	6.865	0.737	0.000	0.588	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	174	415	0	0	0	0	-1
normalized size	1	1.00	0.74	1.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.121	1.459	0.529	0.000	0.500	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	157	382	0	0	0	0	-1
normalized size	1	1.00	0.87	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.838	0.677	0.499	0.000	0.493	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	215	361	0	0	0	0	-1
normalized size	1	1.00	1.76	2.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.540	2.498	0.551	0.000	0.487	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	111	334	0	0	0	0	-1
normalized size	1	1.00	1.63	4.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.279	0.360	0.498	0.000	0.507	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	148	925	0	0	0	0	-1
normalized size	1	1.00	1.22	7.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.570	0.726	0.598	0.000	0.496	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	204	781	0	0	0	0	-1
normalized size	1	1.00	1.14	4.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.860	1.523	0.559	0.000	0.492	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	240	955	0	0	0	0	-1
normalized size	1	1.00	1.03	4.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.149	2.149	0.561	0.000	0.504	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	282	2994	0	0	0	0	-1
normalized size	1	1.00	0.96	10.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.420	6.532	0.620	0.000	0.503	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	238	2946	0	0	0	0	-1
normalized size	1	1.00	0.99	12.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.136	4.806	0.570	0.000	0.524	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	200	2890	0	0	0	0	-1
normalized size	1	1.00	1.10	15.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.844	1.552	0.556	0.000	0.494	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	213	2838	0	0	0	0	-1
normalized size	1	1.00	1.73	23.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.561	1.870	0.600	0.000	0.510	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	170	925	0	0	0	0	-1
normalized size	1	1.00	1.40	7.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.564	0.672	0.592	0.000	0.515	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	92	363	0	0	0	0	-1
normalized size	1	1.00	0.52	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.879	0.726	0.571	0.000	0.515	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	134	877	0	0	0	0	-1
normalized size	1	1.00	0.57	3.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.159	1.108	0.540	0.000	0.534	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	155	1177	0	0	0	0	-1
normalized size	1	1.00	0.53	4.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.474	1.449	0.598	0.000	0.511	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	356	3654	0	0	0	0	-1
normalized size	1	1.00	1.00	10.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.728	6.712	0.654	0.000	0.566	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	250	3598	0	0	0	0	-1
normalized size	1	1.00	0.84	12.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.424	4.681	0.596	0.000	0.526	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	230	3551	0	0	0	0	-1
normalized size	1	1.00	0.95	14.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.136	2.318	0.566	0.000	0.534	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	180	3497	0	0	0	0	-1
normalized size	1	1.00	0.97	18.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.852	1.160	0.558	0.000	0.508	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	230	2040	0	0	0	0	-1
normalized size	1	1.00	1.26	11.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.832	2.099	0.553	0.000	0.506	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	189	777	0	0	0	0	-1
normalized size	1	1.00	1.06	4.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.842	1.518	0.551	0.000	0.498	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	133	877	0	0	0	0	-1
normalized size	1	1.00	0.56	3.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.161	1.059	0.555	0.000	0.517	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	104	395	0	0	0	0	-1
normalized size	1	1.00	0.36	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.472	1.107	0.560	0.000	0.527	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	171	947	0	0	0	0	-1
normalized size	1	1.00	0.49	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.794	6.049	0.646	0.000	0.562	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	126	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.293	2.153	0.535	0.000	0.515	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	180.007	2.957	0.000	0.507	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.276	180.010	1.533	0.000	0.483	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	169.601	0.812	0.000	0.523	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.153	0.307	0.000	0.491	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.275	0.119	0.665	0.000	0.552	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	96	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.281	0.204	1.365	0.000	0.496	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	96	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.283	0.210	1.403	0.000	0.501	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.357	180.003	0.443	0.000	0.497	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.351	180.004	0.467	0.000	0.522	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.319	180.001	0.451	0.000	0.501	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.326	180.011	0.418	0.000	0.497	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.367	81.680	0.395	0.000	0.495	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	2320	0	0	0	0	0	-1
normalized size	1	1.00	20.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.357	53.658	0.392	0.000	0.502	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	832	0	0	0	0	0	-1
normalized size	1	1.00	7.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.311	13.592	0.008	0.000	0.506	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.313	180.013	0.469	0.000	0.508	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	382	0	0	0	0	0	-1
normalized size	1	1.00	3.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.370	18.003	1.389	0.000	0.502	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	202	0	0	0	0	0	-1
normalized size	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.367	6.196	1.109	0.000	0.494	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.359	104.976	0.707	0.000	0.527	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.311	16.296	0.467	0.000	0.508	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.341	176.640	0.668	0.000	0.509	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.353	84.495	1.002	0.000	0.492	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	133	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.287	41.732	9.720	0.000	0.527	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	132	8507	0	30	0	0	-1
normalized size	1	1.00	2.32	149.25	0.00	0.53	0.00	0.00	-0.02
time (sec)	N/A	0.228	84.635	13.417	0.000	0.498	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	1513	0	978	651	0	0	1149
normalized size	1	1.00	7.45	0.00	4.82	3.21	0.00	0.00	5.66
time (sec)	N/A	0.678	6.809	66.340	0.982	0.554	0.000	0.000	17.207

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	143	0	485	298	0	0	476
normalized size	1	1.00	1.13	0.00	3.82	2.35	0.00	0.00	3.75
time (sec)	N/A	0.408	1.376	26.100	2.800	0.516	0.000	0.000	18.128

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	96	0	207	129	0	0	74
normalized size	1	1.00	1.66	0.00	3.57	2.22	0.00	0.00	1.28
time (sec)	N/A	0.163	0.721	37.176	1.261	0.510	0.000	0.000	9.417

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	115	0	0	0	0	0	-1
normalized size	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	71.445	25.909	0.000	0.485	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	135	0	0	0	0	0	-1
normalized size	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.244	49.700	32.330	0.000	0.488	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	136	0	0	0	0	0	-1
normalized size	1	1.00	1.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.244	41.143	53.336	0.000	0.476	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	94	0	0	48	0	0	-1
normalized size	1	1.00	1.84	0.00	0.00	0.94	0.00	0.00	-0.02
time (sec)	N/A	0.172	1.003	0.964	0.000	0.521	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.382	149.712	1.770	0.000	0.564	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.374	142.841	1.474	0.000	0.507	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	207	0	0	0	0	0	-1
normalized size	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.369	27.102	95.905	0.000	0.505	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	145	83	0	0	53
normalized size	1	1.00	1.00	0.00	2.64	1.51	0.00	0.00	0.96
time (sec)	N/A	0.171	0.792	1.054	0.471	0.501	0.000	0.000	9.074

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	132	0	0	126	0	0	128
normalized size	1	1.00	1.06	0.00	0.00	1.01	0.00	0.00	1.02
time (sec)	N/A	0.411	27.822	1.326	0.000	0.486	0.000	0.000	10.157

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	183	0	0	183	0	0	887
normalized size	1	1.00	0.90	0.00	0.00	0.90	0.00	0.00	4.35
time (sec)	N/A	0.665	32.563	1.840	0.000	0.500	0.000	0.000	15.647

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	2681	0	0	264	0	0	1623
normalized size	1	1.00	9.24	0.00	0.00	0.91	0.00	0.00	5.60
time (sec)	N/A	0.937	38.701	2.977	0.000	0.527	0.000	0.000	17.649

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	139	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.460	42.535	10.491	0.000	0.517	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	28	50	42	28	24
normalized size	1	1.00	1.00	0.85	0.85	1.52	1.27	0.85	0.73
time (sec)	N/A	0.045	0.011	0.090	0.297	0.461	1.311	0.132	0.051

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	28	28	39	41	28	24
normalized size	1	1.00	0.91	0.85	0.85	1.18	1.24	0.85	0.73
time (sec)	N/A	0.036	0.090	0.046	0.306	0.483	0.565	0.152	0.040

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	25	22	24	0	23	38
normalized size	1	1.00	1.08	1.04	0.92	1.00	0.00	0.96	1.58
time (sec)	N/A	0.022	0.038	0.129	0.305	0.484	0.000	0.131	8.814

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	33	28	25	33	0	26	55
normalized size	1	1.00	1.32	1.12	1.00	1.32	0.00	1.04	2.20
time (sec)	N/A	0.036	0.044	0.084	0.304	0.474	0.000	0.132	8.580

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	27	24	29	0	24	25
normalized size	1	1.00	0.97	0.90	0.80	0.97	0.00	0.80	0.83
time (sec)	N/A	0.042	0.020	0.128	0.305	0.479	0.000	0.133	8.517

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	39	0	26	39
normalized size	1	1.00	1.00	0.82	0.79	1.18	0.00	0.79	1.18
time (sec)	N/A	0.043	0.025	0.086	0.302	0.436	0.000	0.149	8.602

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	40	0	26	26
normalized size	1	1.00	1.00	0.82	0.79	1.21	0.00	0.79	0.79
time (sec)	N/A	0.043	0.019	0.092	0.303	0.448	0.000	0.152	8.565

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	45	45	72	63	45	36
normalized size	1	1.00	0.96	0.82	0.82	1.31	1.15	0.82	0.65
time (sec)	N/A	0.067	0.337	0.092	0.297	0.454	2.603	0.164	8.492

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	45	45	58	63	45	36
normalized size	1	1.00	0.69	0.82	0.82	1.05	1.15	0.82	0.65
time (sec)	N/A	0.048	0.053	0.095	0.298	0.458	1.275	0.167	8.505

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	46	41	43	0	42	119
normalized size	1	1.00	1.00	0.98	0.87	0.91	0.00	0.89	2.53
time (sec)	N/A	0.039	0.025	0.175	0.337	0.481	0.000	0.168	8.988

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	38	46	40	46	0	41	111
normalized size	1	1.00	0.88	1.07	0.93	1.07	0.00	0.95	2.58
time (sec)	N/A	0.055	0.022	0.133	0.352	0.478	0.000	0.155	8.892

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	48	43	62	0	44	111
normalized size	1	1.00	0.89	1.02	0.91	1.32	0.00	0.94	2.36
time (sec)	N/A	0.065	0.018	0.185	0.351	0.476	0.000	0.150	8.866

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	39	41	56	0	41	41
normalized size	1	1.00	0.67	1.30	1.37	1.87	0.00	1.37	1.37
time (sec)	N/A	0.058	0.027	0.146	0.332	0.471	0.000	0.148	8.903

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	39	43	57	0	43	43
normalized size	1	1.00	1.00	0.71	0.78	1.04	0.00	0.78	0.78
time (sec)	N/A	0.066	0.029	0.138	0.309	0.447	0.000	0.155	8.853

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	39	43	65	0	43	43
normalized size	1	1.00	1.00	0.71	0.78	1.18	0.00	0.78	0.78
time (sec)	N/A	0.067	0.026	0.136	0.368	0.440	0.000	0.166	8.918

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	39	43	70	0	43	43
normalized size	1	1.00	1.00	0.71	0.78	1.27	0.00	0.78	0.78
time (sec)	N/A	0.066	0.033	0.155	0.340	0.482	0.000	0.159	8.907

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	80	58	58	98	80	58	57
normalized size	1	1.00	1.10	0.79	0.79	1.34	1.10	0.79	0.78
time (sec)	N/A	0.074	0.336	0.105	0.312	0.466	8.628	0.198	0.065

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	70	58	58	85	82	58	57
normalized size	1	1.00	0.96	0.79	0.79	1.16	1.12	0.79	0.78
time (sec)	N/A	0.071	0.282	0.095	0.328	0.457	5.139	0.185	0.062

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	30	57	58	71	76	58	56
normalized size	1	1.00	0.67	1.27	1.29	1.58	1.69	1.29	1.24
time (sec)	N/A	0.045	0.124	0.095	0.329	0.463	2.553	0.155	0.060

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	62	55	59	0	56	102
normalized size	1	1.00	1.00	0.95	0.85	0.91	0.00	0.86	1.57
time (sec)	N/A	0.044	0.028	0.183	0.334	0.479	0.000	0.183	8.655

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	63	54	78	0	55	156
normalized size	1	1.00	1.00	1.02	0.87	1.26	0.00	0.89	2.52
time (sec)	N/A	0.062	0.030	0.142	0.311	0.503	0.000	0.193	8.593

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	62	54	77	0	55	163
normalized size	1	1.00	0.87	1.02	0.89	1.26	0.00	0.90	2.67
time (sec)	N/A	0.069	0.022	0.193	0.322	0.486	0.000	0.177	8.573

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	57	64	58	91	0	59	147
normalized size	1	1.00	0.88	0.98	0.89	1.40	0.00	0.91	2.26
time (sec)	N/A	0.071	0.020	0.137	0.419	0.493	0.000	0.179	8.616

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	49	54	72	0	54	54
normalized size	1	1.00	0.67	1.63	1.80	2.40	0.00	1.80	1.80
time (sec)	N/A	0.057	0.023	0.144	0.499	0.451	0.000	0.178	8.614

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	71	49	56	81	0	56	56
normalized size	1	1.00	1.16	0.80	0.92	1.33	0.00	0.92	0.92
time (sec)	N/A	0.065	0.031	0.142	0.510	0.455	0.000	0.177	8.604

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	49	56	86	0	56	56
normalized size	1	1.00	1.00	0.67	0.77	1.18	0.00	0.77	0.77
time (sec)	N/A	0.072	0.030	0.161	0.343	0.454	0.000	0.198	8.603

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	49	56	93	0	56	56
normalized size	1	1.00	1.00	0.67	0.77	1.27	0.00	0.77	0.77
time (sec)	N/A	0.073	0.031	0.193	0.376	0.632	0.000	0.192	8.665

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	100	71	71	124	97	71	70
normalized size	1	1.00	1.10	0.78	0.78	1.36	1.07	0.78	0.77
time (sec)	N/A	0.085	0.844	0.106	0.315	0.904	19.923	0.247	8.455

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	70	71	111	95	71	69
normalized size	1	1.00	1.02	0.80	0.81	1.26	1.08	0.81	0.78
time (sec)	N/A	0.081	0.534	0.099	0.297	0.509	11.722	0.222	8.446

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	80	70	71	97	95	71	69
normalized size	1	1.00	1.19	1.04	1.06	1.45	1.42	1.06	1.03
time (sec)	N/A	0.075	0.337	0.097	0.311	0.493	7.240	0.191	8.414

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	30	71	71	85	97	71	70
normalized size	1	1.00	0.67	1.58	1.58	1.89	2.16	1.58	1.56
time (sec)	N/A	0.045	0.099	0.098	0.586	0.523	3.872	0.195	0.048

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	78	68	72	0	69	118
normalized size	1	1.00	1.00	0.96	0.84	0.89	0.00	0.85	1.46
time (sec)	N/A	0.047	0.038	0.184	0.430	0.526	0.000	0.199	8.620

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	79	67	91	0	68	235
normalized size	1	1.00	1.00	1.01	0.86	1.17	0.00	0.87	3.01
time (sec)	N/A	0.066	0.037	0.147	0.403	0.502	0.000	0.202	8.647

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	54	79	66	98	0	67	207
normalized size	1	1.00	0.68	0.99	0.82	1.22	0.00	0.84	2.59
time (sec)	N/A	0.076	0.077	0.185	0.373	0.507	0.000	0.202	8.618

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	60	80	63	58	80	76	68
normalized size	1	1.00	0.71	0.94	0.74	0.68	0.94	0.89	0.80
time (sec)	N/A	0.087	0.132	0.121	0.406	0.643	3.201	0.143	8.456

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	50	64	53	48	66	64	56
normalized size	1	1.00	0.75	0.96	0.79	0.72	0.99	0.96	0.84
time (sec)	N/A	0.078	0.109	0.125	0.305	0.534	1.960	0.159	0.056

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	38	48	41	36	53	45	35
normalized size	1	1.00	0.78	0.98	0.84	0.73	1.08	0.92	0.71
time (sec)	N/A	0.070	0.061	0.125	0.312	0.509	1.110	0.160	0.047

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	32	30	26	37	31	26
normalized size	1	1.00	0.81	1.03	0.97	0.84	1.19	1.00	0.84
time (sec)	N/A	0.046	0.021	0.072	0.315	0.497	0.684	0.140	8.459

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	33	31	28	0	33	32
normalized size	1	1.00	1.00	1.03	0.97	0.88	0.00	1.03	1.00
time (sec)	N/A	0.036	0.017	0.199	0.319	0.468	0.000	0.151	8.609

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	49	43	51	0	45	55
normalized size	1	1.00	1.00	1.07	0.93	1.11	0.00	0.98	1.20
time (sec)	N/A	0.062	0.035	0.184	0.306	0.500	0.000	0.196	8.599

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	64	55	72	0	57	106
normalized size	1	1.00	1.00	1.02	0.87	1.14	0.00	0.90	1.68
time (sec)	N/A	0.079	0.039	0.230	0.318	0.532	0.000	0.158	8.823

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	81	65	102	0	67	139
normalized size	1	1.00	1.00	0.99	0.79	1.24	0.00	0.82	1.70
time (sec)	N/A	0.082	0.047	0.202	0.313	0.573	0.000	0.159	8.591

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	73	83	70	81	201	107	72
normalized size	1	1.00	0.84	0.95	0.80	0.93	2.31	1.23	0.83
time (sec)	N/A	0.089	0.582	0.224	0.310	0.525	4.340	0.178	0.055

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	66	59	72	170	90	59
normalized size	1	1.00	1.01	0.94	0.84	1.03	2.43	1.29	0.84
time (sec)	N/A	0.080	0.184	0.218	0.320	0.553	2.808	0.173	8.471

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	55	50	47	57	126	70	45
normalized size	1	1.00	1.06	0.96	0.90	1.10	2.42	1.35	0.87
time (sec)	N/A	0.073	0.204	0.227	0.378	0.610	1.452	0.169	0.083

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	27	35	34	40	95	56	34
normalized size	1	1.00	0.73	0.95	0.92	1.08	2.57	1.51	0.92
time (sec)	N/A	0.048	0.029	0.191	0.341	0.559	1.044	0.158	0.048

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	50	46	59	0	45	87
normalized size	1	1.00	0.69	0.96	0.88	1.13	0.00	0.87	1.67
time (sec)	N/A	0.049	0.055	0.281	0.490	0.535	0.000	0.153	8.576

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	45	68	68	104	0	69	136
normalized size	1	1.00	0.66	1.00	1.00	1.53	0.00	1.01	2.00
time (sec)	N/A	0.074	0.160	0.261	0.316	0.500	0.000	0.159	8.609

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	61	83	80	147	0	87	168
normalized size	1	1.00	0.72	0.98	0.94	1.73	0.00	1.02	1.98
time (sec)	N/A	0.087	0.204	0.296	0.296	0.497	0.000	0.190	8.613

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	98	99	90	195	0	103	202
normalized size	1	1.00	0.97	0.98	0.89	1.93	0.00	1.02	2.00
time (sec)	N/A	0.099	2.458	0.292	0.323	0.521	0.000	0.180	8.628

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	106	101	95	117	394	89	108
normalized size	1	1.00	0.95	0.91	0.86	1.05	3.55	0.80	0.97
time (sec)	N/A	0.106	0.696	0.239	0.318	0.522	8.058	0.223	0.129

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	78	85	81	107	347	73	91
normalized size	1	1.00	0.84	0.91	0.87	1.15	3.73	0.78	0.98
time (sec)	N/A	0.096	2.083	0.236	0.311	0.484	5.468	0.205	8.490

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	70	68	71	95	303	56	59
normalized size	1	1.00	0.95	0.92	0.96	1.28	4.09	0.76	0.80
time (sec)	N/A	0.090	0.403	0.230	0.299	0.557	3.056	0.213	0.074

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	65	54	60	75	257	45	44
normalized size	1	1.00	1.08	0.90	1.00	1.25	4.28	0.75	0.73
time (sec)	N/A	0.081	0.650	0.225	0.408	0.528	1.933	0.184	0.056

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	44	46	99	28	37
normalized size	1	1.00	1.00	1.10	1.47	1.53	3.30	0.93	1.23
time (sec)	N/A	0.046	0.030	0.201	0.393	0.514	1.701	0.159	0.053

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	52	68	72	104	0	59	148
normalized size	1	1.00	0.70	0.92	0.97	1.41	0.00	0.80	2.00
time (sec)	N/A	0.058	0.180	0.286	0.433	0.520	0.000	0.178	8.761

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	61	86	91	152	0	77	193
normalized size	1	1.00	0.68	0.96	1.01	1.69	0.00	0.86	2.14
time (sec)	N/A	0.084	0.314	0.287	0.448	0.480	0.000	0.193	8.652

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	71	102	103	196	0	86	227
normalized size	1	1.00	0.66	0.94	0.95	1.81	0.00	0.80	2.10
time (sec)	N/A	0.102	0.574	0.311	0.313	0.513	0.000	0.191	8.684

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	81	118	113	242	0	97	260
normalized size	1	1.00	0.64	0.94	0.90	1.92	0.00	0.77	2.06
time (sec)	N/A	0.113	5.433	0.302	0.315	0.533	0.000	0.212	8.676

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	119	103	105	144	588	84	114
normalized size	1	1.00	1.03	0.89	0.91	1.24	5.07	0.72	0.98
time (sec)	N/A	0.104	0.883	0.227	0.348	0.519	10.054	0.217	8.532

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	127	86	94	132	527	66	69
normalized size	1	1.00	1.34	0.91	0.99	1.39	5.55	0.69	0.73
time (sec)	N/A	0.097	6.561	0.235	0.379	0.538	5.566	0.206	0.123

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	61	72	83	112	466	55	54
normalized size	1	1.00	0.73	0.87	1.00	1.35	5.61	0.66	0.65
time (sec)	N/A	0.093	0.368	0.215	0.305	0.507	3.476	0.207	0.063

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	53	43	67	72	192	38	54
normalized size	1	1.00	1.77	1.43	2.23	2.40	6.40	1.27	1.80
time (sec)	N/A	0.065	0.180	0.221	0.400	0.482	3.264	0.221	8.483

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	30	33	57	62	129	28	37
normalized size	1	1.00	0.65	0.72	1.24	1.35	2.80	0.61	0.80
time (sec)	N/A	0.052	0.033	0.200	0.312	0.476	3.495	0.172	8.462

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	62	86	95	152	0	69	206
normalized size	1	1.00	0.64	0.89	0.98	1.57	0.00	0.71	2.12
time (sec)	N/A	0.064	0.362	0.253	0.363	0.515	0.000	0.186	9.526

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	73	104	114	201	0	87	251
normalized size	1	1.00	0.66	0.94	1.03	1.81	0.00	0.78	2.26
time (sec)	N/A	0.094	1.006	0.262	0.390	0.543	0.000	0.201	8.722

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	85	120	126	242	0	97	286
normalized size	1	1.00	0.65	0.92	0.96	1.85	0.00	0.74	2.18
time (sec)	N/A	0.118	3.418	0.283	0.352	0.535	0.000	0.232	8.690

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	118	42	61	86	0	2182	43
normalized size	1	1.00	2.31	0.82	1.20	1.69	0.00	42.78	0.84
time (sec)	N/A	0.062	0.146	0.154	0.406	0.499	0.000	2.255	8.730

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	80	0	103	302	1833	593	370
normalized size	1	1.00	0.70	0.00	0.90	2.65	16.08	5.20	3.25
time (sec)	N/A	0.116	0.266	8.357	0.311	0.560	61.694	0.349	11.762

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	65	0	83	210	1061	379	242
normalized size	1	1.00	0.71	0.00	0.91	2.31	11.66	4.16	2.66
time (sec)	N/A	0.095	0.174	6.390	0.466	0.523	27.521	0.316	10.416

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	0	63	135	530	213	147
normalized size	1	1.00	0.74	0.00	0.93	1.99	7.79	3.13	2.16
time (sec)	N/A	0.086	0.218	6.525	0.316	0.505	12.064	0.243	9.641

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	0	39	62	190	86	67
normalized size	1	1.00	0.93	0.00	0.95	1.51	4.63	2.10	1.63
time (sec)	N/A	0.052	0.296	3.786	0.443	0.498	5.084	0.222	9.184

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.075	0.037	1.292	0.000	0.535	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.074	0.038	4.023	0.000	0.490	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.074	0.058	3.787	0.000	0.497	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.075	0.031	3.615	0.000	0.526	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	71	95	65	73	192	92	226
normalized size	1	1.00	0.68	0.90	0.62	0.70	1.83	0.88	2.15
time (sec)	N/A	0.166	0.200	0.132	0.335	0.494	4.337	0.166	12.131

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	54	77	52	62	144	62	198
normalized size	1	1.00	0.67	0.95	0.64	0.77	1.78	0.77	2.44
time (sec)	N/A	0.128	0.110	0.122	0.313	0.504	2.520	0.183	11.973

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	42	57	39	51	119	47	198
normalized size	1	1.00	0.65	0.88	0.60	0.78	1.83	0.72	3.05
time (sec)	N/A	0.093	0.103	0.081	0.310	0.527	1.228	0.157	12.332

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	74	63	57	60	0	87	160
normalized size	1	1.00	1.45	1.24	1.12	1.18	0.00	1.71	3.14
time (sec)	N/A	0.064	0.073	0.255	0.309	0.542	0.000	0.146	8.718

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	75	57	54	84	0	108	108
normalized size	1	1.00	1.83	1.39	1.32	2.05	0.00	2.63	2.63
time (sec)	N/A	0.055	0.043	0.190	0.462	0.528	0.000	0.156	8.800

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	109	81	66	114	0	95	145
normalized size	1	1.00	2.10	1.56	1.27	2.19	0.00	1.83	2.79
time (sec)	N/A	0.077	0.044	0.252	0.454	0.526	0.000	0.163	8.676

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	95	80	61	119	0	115	111
normalized size	1	1.00	1.83	1.54	1.17	2.29	0.00	2.21	2.13
time (sec)	N/A	0.108	0.038	0.221	0.314	0.504	0.000	0.167	8.577

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	135	102	80	137	0	116	112
normalized size	1	1.00	1.82	1.38	1.08	1.85	0.00	1.57	1.51
time (sec)	N/A	0.126	0.054	0.219	0.316	0.503	0.000	0.203	8.593

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	177	124	92	169	0	144	143
normalized size	1	1.00	1.97	1.38	1.02	1.88	0.00	1.60	1.59
time (sec)	N/A	0.129	0.073	0.217	0.319	0.539	0.000	0.177	8.648

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	86	151	105	98	275	123	331
normalized size	1	1.00	0.64	1.12	0.78	0.73	2.04	0.91	2.45
time (sec)	N/A	0.248	0.502	0.172	0.331	0.539	7.686	0.223	12.308

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	76	142	93	85	309	106	257
normalized size	1	1.00	0.74	1.38	0.90	0.83	3.00	1.03	2.50
time (sec)	N/A	0.168	0.448	0.171	0.331	0.512	4.788	0.192	12.163

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	105	57	95	69	72	172	72	262
normalized size	1	1.15	0.63	1.04	0.76	0.79	1.89	0.79	2.88
time (sec)	N/A	0.130	0.199	0.155	0.336	0.507	2.480	0.171	12.132

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	86	75	86	0	101	188
normalized size	1	1.00	1.00	1.21	1.06	1.21	0.00	1.42	2.65
time (sec)	N/A	0.120	0.381	0.337	0.349	0.533	0.000	0.173	9.043

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	94	89	79	105	0	143	201
normalized size	1	1.00	1.27	1.20	1.07	1.42	0.00	1.93	2.72
time (sec)	N/A	0.104	0.591	0.293	0.439	0.543	0.000	0.180	8.811

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	102	93	104	143	0	128	213
normalized size	1	1.00	1.40	1.27	1.42	1.96	0.00	1.75	2.92
time (sec)	N/A	0.130	0.673	0.352	0.583	0.529	0.000	0.192	8.777

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	140	117	83	166	0	141	193
normalized size	1	1.00	1.92	1.60	1.14	2.27	0.00	1.93	2.64
time (sec)	N/A	0.221	0.580	0.317	0.423	0.528	0.000	0.196	8.937

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	209	112	130	155	0	164	161
normalized size	1	1.00	2.55	1.37	1.59	1.89	0.00	2.00	1.96
time (sec)	N/A	0.201	0.112	0.312	0.345	0.512	0.000	0.240	8.622

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	189	136	109	189	0	164	160
normalized size	1	1.00	1.89	1.36	1.09	1.89	0.00	1.64	1.60
time (sec)	N/A	0.210	0.755	0.312	0.344	0.522	0.000	0.223	8.629

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	229	160	187	227	0	228	339
normalized size	1	1.00	1.85	1.29	1.51	1.83	0.00	1.84	2.73
time (sec)	N/A	0.256	0.716	0.340	0.353	0.513	0.000	0.228	9.658

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	86	194	129	98	379	123	331
normalized size	1	1.00	0.65	1.47	0.98	0.74	2.87	0.93	2.51
time (sec)	N/A	0.305	0.670	0.163	0.329	0.505	7.838	0.227	12.159

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	133	76	156	106	85	328	106	349
normalized size	1	1.14	0.65	1.33	0.91	0.73	2.80	0.91	2.98
time (sec)	N/A	0.181	0.379	0.168	0.311	0.487	4.876	0.202	10.718

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	82	111	99	101	0	144	244
normalized size	1	1.00	0.83	1.12	1.00	1.02	0.00	1.45	2.46
time (sec)	N/A	0.170	0.658	0.357	0.320	0.501	0.000	0.206	10.357

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	106	105	93	121	0	162	264
normalized size	1	1.00	1.15	1.14	1.01	1.32	0.00	1.76	2.87
time (sec)	N/A	0.138	1.130	0.317	0.426	0.526	0.000	0.216	8.766

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	112	113	124	159	0	184	259
normalized size	1	1.00	1.14	1.15	1.27	1.62	0.00	1.88	2.64
time (sec)	N/A	0.149	1.040	0.415	0.428	0.536	0.000	0.243	8.681

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	148	117	117	180	0	161	249
normalized size	1	1.00	1.63	1.29	1.29	1.98	0.00	1.77	2.74
time (sec)	N/A	0.171	0.427	0.337	0.431	0.521	0.000	0.242	8.675

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	133	141	147	190	0	174	237
normalized size	1	1.00	1.33	1.41	1.47	1.90	0.00	1.74	2.37
time (sec)	N/A	0.221	0.558	0.329	0.409	0.523	0.000	0.242	8.893

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	267	136	155	190	0	196	291
normalized size	1	1.00	2.67	1.36	1.55	1.90	0.00	1.96	2.91
time (sec)	N/A	0.240	0.127	0.325	0.330	0.488	0.000	0.265	9.304

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	252	160	200	227	0	228	339
normalized size	1	1.00	2.03	1.29	1.61	1.83	0.00	1.84	2.73
time (sec)	N/A	0.282	3.590	0.352	0.348	0.501	0.000	0.281	9.675

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	151	182	128	85	381	106	349
normalized size	1	1.00	1.10	1.33	0.93	0.62	2.78	0.77	2.55
time (sec)	N/A	0.157	0.443	0.321	0.319	0.490	5.091	0.213	10.728

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	95	135	125	115	0	181	295
normalized size	1	1.00	0.81	1.15	1.07	0.98	0.00	1.55	2.52
time (sec)	N/A	0.199	0.878	0.371	0.327	0.539	0.000	0.238	10.419

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	136	127	117	135	0	194	295
normalized size	1	1.00	1.17	1.09	1.01	1.16	0.00	1.67	2.54
time (sec)	N/A	0.168	1.626	0.316	0.425	0.522	0.000	0.249	8.834

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	281	245	258	68	1360	114	107
normalized size	1	1.00	2.70	2.36	2.48	0.65	13.08	1.10	1.03
time (sec)	N/A	0.136	5.238	0.304	0.423	0.481	33.858	0.170	11.913

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	271	245	237	58	1049	114	79
normalized size	1	1.00	3.11	2.82	2.72	0.67	12.06	1.31	0.91
time (sec)	N/A	0.130	1.722	0.286	0.433	0.496	19.757	0.147	8.605

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	46	141	156	45	563	75	66
normalized size	1	1.00	0.74	2.27	2.52	0.73	9.08	1.21	1.06
time (sec)	N/A	0.117	0.087	0.277	0.419	0.489	11.346	0.142	10.464

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	161	142	133	34	366	72	33
normalized size	1	1.00	3.58	3.16	2.96	0.76	8.13	1.60	0.73
time (sec)	N/A	0.072	0.588	0.195	0.414	0.523	6.175	0.137	8.686

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	37	37	52	37	0	31	79
normalized size	1	1.00	1.68	1.68	2.36	1.68	0.00	1.41	3.59
time (sec)	N/A	0.072	0.092	0.383	0.429	0.487	0.000	0.150	8.770

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	69	56	70	62	0	65	25
normalized size	1	1.00	2.38	1.93	2.41	2.14	0.00	2.24	0.86
time (sec)	N/A	0.055	0.239	0.376	0.330	0.488	0.000	0.158	8.599

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	94	94	115	88	0	94	87
normalized size	1	1.00	1.77	1.77	2.17	1.66	0.00	1.77	1.64
time (sec)	N/A	0.103	0.415	0.418	0.336	0.505	0.000	0.199	8.653

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	126	132	153	119	0	128	119
normalized size	1	1.00	1.75	1.83	2.12	1.65	0.00	1.78	1.65
time (sec)	N/A	0.123	0.598	0.449	0.318	0.495	0.000	0.171	8.621

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	125	170	195	143	0	157	151
normalized size	1	1.00	1.32	1.79	2.05	1.51	0.00	1.65	1.59
time (sec)	N/A	0.134	1.129	0.428	0.328	0.507	0.000	0.191	8.675

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	189	208	234	173	0	187	291
normalized size	1	1.00	1.66	1.82	2.05	1.52	0.00	1.64	2.55
time (sec)	N/A	0.138	0.658	0.463	0.338	0.500	0.000	0.190	9.250

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	209	300	398	144	3580	145	147
normalized size	1	1.00	1.88	2.70	3.59	1.30	32.25	1.31	1.32
time (sec)	N/A	0.245	1.489	0.464	0.444	0.512	61.076	0.198	12.340

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	165	198	312	119	2263	106	120
normalized size	1	1.00	1.99	2.39	3.76	1.43	27.27	1.28	1.45
time (sec)	N/A	0.222	0.970	0.460	0.431	0.500	37.331	0.195	12.378

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	163	226	100	1248	91	95
normalized size	1	1.00	1.00	2.36	3.28	1.45	18.09	1.32	1.38
time (sec)	N/A	0.272	0.152	0.438	0.433	0.485	21.855	0.184	10.792

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	117	64	139	77	479	78	68
normalized size	1	1.00	2.49	1.36	2.96	1.64	10.19	1.66	1.45
time (sec)	N/A	0.072	0.327	0.385	0.444	0.486	11.905	0.158	8.865

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	115	40	55	103	0	38	39
normalized size	1	1.00	2.88	1.00	1.38	2.58	0.00	0.95	0.98
time (sec)	N/A	0.157	0.158	0.594	0.324	0.507	0.000	0.164	8.675

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	216	77	116	160	0	90	87
normalized size	1	1.00	4.00	1.43	2.15	2.96	0.00	1.67	1.61
time (sec)	N/A	0.096	0.798	0.716	0.329	0.524	0.000	0.193	8.686

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	364	114	161	246	0	116	120
normalized size	1	1.00	4.67	1.46	2.06	3.15	0.00	1.49	1.54
time (sec)	N/A	0.220	0.785	0.762	0.340	0.501	0.000	0.193	8.658

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	472	153	199	302	0	146	153
normalized size	1	1.00	5.19	1.68	2.19	3.32	0.00	1.60	1.68
time (sec)	N/A	0.251	1.292	0.581	0.330	0.531	0.000	0.199	8.645

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	197	205	314	163	2264	117	121
normalized size	1	1.00	2.03	2.11	3.24	1.68	23.34	1.21	1.25
time (sec)	N/A	0.263	1.089	0.410	0.449	0.498	63.548	0.216	12.071

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	96	106	228	144	1246	80	94
normalized size	1	1.00	1.26	1.39	3.00	1.89	16.39	1.05	1.24
time (sec)	N/A	0.181	0.671	0.510	0.429	0.497	38.822	0.178	11.322

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	145	83	142	124	529	60	54
normalized size	1	1.00	2.38	1.36	2.33	2.03	8.67	0.98	0.89
time (sec)	N/A	0.109	0.417	0.418	0.441	0.514	22.211	0.219	8.774

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	185	82	143	194	0	66	64
normalized size	1	1.00	2.72	1.21	2.10	2.85	0.00	0.97	0.94
time (sec)	N/A	0.193	0.393	0.609	0.339	0.572	0.000	0.183	8.900

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	255	119	202	279	0	109	145
normalized size	1	1.00	3.11	1.45	2.46	3.40	0.00	1.33	1.77
time (sec)	N/A	0.208	1.564	0.595	0.345	0.598	0.000	0.212	8.807

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	308	157	247	365	0	143	178
normalized size	1	1.00	2.91	1.48	2.33	3.44	0.00	1.35	1.68
time (sec)	N/A	0.269	5.946	0.691	0.336	0.516	0.000	0.223	8.660

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	171	130	355	243	1501	120	207
normalized size	1	1.00	1.19	0.90	2.47	1.69	10.42	0.83	1.44
time (sec)	N/A	0.162	0.964	0.481	0.356	0.502	109.193	0.249	9.056

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	109	85	0	151	0	189	-1
normalized size	1	1.00	0.56	0.44	0.00	0.78	0.00	0.98	-0.01
time (sec)	N/A	0.568	1.230	1.042	0.000	0.534	0.000	0.263	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	99	75	0	130	0	99	-1
normalized size	1	1.00	0.80	0.60	0.00	1.05	0.00	0.80	-0.01
time (sec)	N/A	0.358	0.599	1.017	0.000	0.477	0.000	0.272	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	89	65	0	111	0	129	-1
normalized size	1	1.00	0.97	0.71	0.00	1.21	0.00	1.40	-0.01
time (sec)	N/A	0.188	0.404	1.057	0.000	0.472	0.000	0.218	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	143	103	0	250	0	0	-1
normalized size	1	1.00	1.54	1.11	0.00	2.69	0.00	0.00	-0.01
time (sec)	N/A	0.339	0.211	1.205	0.000	0.475	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	206	125	0	279	0	0	-1
normalized size	1	1.00	2.31	1.40	0.00	3.13	0.00	0.00	-0.01
time (sec)	N/A	0.199	1.003	1.095	0.000	0.522	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	249	126	0	319	0	0	-1
normalized size	1	1.00	2.47	1.25	0.00	3.16	0.00	0.00	-0.01
time (sec)	N/A	0.397	0.806	1.112	0.000	0.488	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	285	144	0	361	0	0	-1
normalized size	1	1.00	2.08	1.05	0.00	2.64	0.00	0.00	-0.01
time (sec)	N/A	0.480	1.419	1.136	0.000	0.496	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	120	97	0	189	0	412	-1
normalized size	1	1.00	0.52	0.42	0.00	0.81	0.00	1.77	-0.00
time (sec)	N/A	0.720	3.874	0.936	0.000	0.487	0.000	0.641	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	110	87	0	166	0	288	-1
normalized size	1	1.00	0.71	0.56	0.00	1.06	0.00	1.85	-0.01
time (sec)	N/A	0.432	1.919	0.859	0.000	0.485	0.000	0.497	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	100	77	0	145	0	226	-1
normalized size	1	1.00	0.81	0.62	0.00	1.17	0.00	1.82	-0.01
time (sec)	N/A	0.255	1.434	0.893	0.000	0.495	0.000	0.382	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	145	121	0	282	0	0	-1
normalized size	1	1.00	1.18	0.98	0.00	2.29	0.00	0.00	-0.01
time (sec)	N/A	0.459	0.262	1.079	0.000	0.522	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	233	144	0	315	0	0	-1
normalized size	1	1.00	1.93	1.19	0.00	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.308	0.792	1.178	0.000	0.495	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	271	151	0	359	0	0	-1
normalized size	1	1.00	2.07	1.15	0.00	2.74	0.00	0.00	-0.01
time (sec)	N/A	0.518	0.679	1.199	0.000	0.514	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	286	144	0	380	0	0	-1
normalized size	1	1.00	2.06	1.04	0.00	2.73	0.00	0.00	-0.01
time (sec)	N/A	0.573	0.920	1.238	0.000	0.512	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	97	74	0	136	0	224	-1
normalized size	1	1.00	0.61	0.47	0.00	0.86	0.00	1.42	-0.01
time (sec)	N/A	0.404	1.273	0.967	0.000	0.476	0.000	0.600	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	87	64	0	115	0	220	-1
normalized size	1	1.00	0.95	0.70	0.00	1.25	0.00	2.39	-0.01
time (sec)	N/A	0.343	0.352	1.510	0.000	0.495	0.000	0.563	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	77	54	0	96	0	155	-1
normalized size	1	1.00	1.28	0.90	0.00	1.60	0.00	2.58	-0.02
time (sec)	N/A	0.127	0.432	0.893	0.000	0.497	0.000	0.520	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	116	88	0	236	0	258	-1
normalized size	1	1.00	1.84	1.40	0.00	3.75	0.00	4.10	-0.02
time (sec)	N/A	0.216	0.131	1.036	0.000	0.530	0.000	0.636	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	138	103	0	263	0	360	-1
normalized size	1	1.00	2.23	1.66	0.00	4.24	0.00	5.81	-0.02
time (sec)	N/A	0.101	0.305	0.905	0.000	0.493	0.000	0.630	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	272	124	0	320	0	505	-1
normalized size	1	1.00	2.72	1.24	0.00	3.20	0.00	5.05	-0.01
time (sec)	N/A	0.317	1.828	1.235	0.000	0.500	0.000	0.691	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	292	144	0	367	0	546	-1
normalized size	1	1.00	2.16	1.07	0.00	2.72	0.00	4.04	-0.01
time (sec)	N/A	0.403	0.735	1.133	0.000	0.492	0.000	0.762	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	201	148	0	259	0	375	-1
normalized size	1	1.00	1.09	0.80	0.00	1.41	0.00	2.04	-0.01
time (sec)	N/A	0.589	1.878	1.302	0.000	0.512	0.000	0.700	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	150	112	0	236	0	303	-1
normalized size	1	1.00	1.07	0.80	0.00	1.69	0.00	2.16	-0.01
time (sec)	N/A	0.348	0.287	1.679	0.000	0.496	0.000	0.675	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	149	110	0	215	0	246	-1
normalized size	1	1.00	1.38	1.02	0.00	1.99	0.00	2.28	-0.01
time (sec)	N/A	0.158	0.895	1.174	0.000	0.504	0.000	0.626	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	130	97	0	291	0	0	-1
normalized size	1	1.00	1.53	1.14	0.00	3.42	0.00	0.00	-0.01
time (sec)	N/A	0.259	0.209	1.042	0.000	0.510	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	206	134	0	421	0	471	-1
normalized size	1	1.00	1.82	1.19	0.00	3.73	0.00	4.17	-0.01
time (sec)	N/A	0.224	2.148	1.115	0.000	0.525	0.000	0.798	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	309	164	0	508	0	620	-1
normalized size	1	1.00	2.02	1.07	0.00	3.32	0.00	4.05	-0.01
time (sec)	N/A	0.544	3.556	1.368	0.000	0.495	0.000	0.890	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	332	182	0	564	0	695	-1
normalized size	1	1.00	1.74	0.95	0.00	2.95	0.00	3.64	-0.01
time (sec)	N/A	0.714	2.352	1.492	0.000	0.525	0.000	0.940	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	92	50	72	90	50	49
normalized size	1	1.00	0.78	1.42	0.77	1.11	1.38	0.77	0.75
time (sec)	N/A	0.069	0.301	0.238	0.309	0.492	7.828	0.207	0.066

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	74	50	62	90	50	49
normalized size	1	1.00	0.78	1.14	0.77	0.95	1.38	0.77	0.75
time (sec)	N/A	0.072	0.221	0.218	0.315	0.536	4.306	0.210	0.062

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	58	54	50	51	66	50	49
normalized size	1	1.00	1.18	1.10	1.02	1.04	1.35	1.02	1.00
time (sec)	N/A	0.081	0.100	0.225	0.313	0.575	2.387	0.171	0.059

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	36	48	39	60	48	46
normalized size	1	1.00	0.98	0.80	1.07	0.87	1.33	1.07	1.02
time (sec)	N/A	0.033	0.016	0.283	0.310	0.524	1.215	0.146	0.055

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	60	47	51	0	48	92
normalized size	1	1.00	1.00	1.07	0.84	0.91	0.00	0.86	1.64
time (sec)	N/A	0.055	0.032	0.329	0.326	0.545	0.000	0.181	8.727

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	82	46	68	0	47	140
normalized size	1	1.00	1.00	1.55	0.87	1.28	0.00	0.89	2.64
time (sec)	N/A	0.058	0.035	0.283	0.311	0.527	0.000	0.183	8.696

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	60	83	45	69	0	46	146
normalized size	1	1.00	1.11	1.54	0.83	1.28	0.00	0.85	2.70
time (sec)	N/A	0.034	0.114	0.345	0.333	0.517	0.000	0.164	8.737

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	28	30	29	47	277	29	26
normalized size	1	1.00	0.76	0.81	0.78	1.27	7.49	0.78	0.70
time (sec)	N/A	0.097	0.133	0.150	0.325	0.482	20.003	0.148	0.045

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	28	30	29	37	224	29	26
normalized size	1	1.00	0.76	0.81	0.78	1.00	6.05	0.78	0.70
time (sec)	N/A	0.080	0.099	0.098	0.310	0.476	10.745	0.140	8.537

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	28	25	25	158	25	22
normalized size	1	1.00	0.75	0.88	0.78	0.78	4.94	0.78	0.69
time (sec)	N/A	0.042	0.042	0.187	0.310	0.479	5.868	0.153	0.041

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	33	27	25	0	28	71
normalized size	1	1.00	0.79	1.14	0.93	0.86	0.00	0.97	2.45
time (sec)	N/A	0.077	0.033	0.226	0.311	0.525	0.000	0.172	8.762

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	22	30	29	34	0	30	59
normalized size	1	1.00	0.73	1.00	0.97	1.13	0.00	1.00	1.97
time (sec)	N/A	0.079	0.038	0.181	0.681	0.545	0.000	0.156	8.697

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	25	26	30	0	26	23
normalized size	1	1.00	0.75	0.78	0.81	0.94	0.00	0.81	0.72
time (sec)	N/A	0.065	0.032	0.230	0.315	0.473	0.000	0.155	8.627

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	28	29	26	38	0	26	36
normalized size	1	1.00	0.76	0.78	0.70	1.03	0.00	0.70	0.97
time (sec)	N/A	0.081	0.056	0.203	0.313	0.475	0.000	0.172	8.757

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	28	29	26	41	0	26	25
normalized size	1	1.00	0.76	0.78	0.70	1.11	0.00	0.70	0.68
time (sec)	N/A	0.098	0.045	0.212	0.312	0.463	0.000	0.185	8.645

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	84	124	71	95	272	107	353
normalized size	1	1.00	0.59	0.87	0.50	0.66	1.90	0.75	2.47
time (sec)	N/A	0.169	0.278	0.247	0.331	0.504	19.382	0.242	12.343

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	71	106	61	84	248	92	320
normalized size	1	1.00	0.56	0.83	0.48	0.66	1.95	0.72	2.52
time (sec)	N/A	0.165	0.217	0.242	0.309	0.510	11.407	0.221	12.476

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	81	88	65	73	192	107	292
normalized size	1	1.00	0.79	0.85	0.63	0.71	1.86	1.04	2.83
time (sec)	N/A	0.133	0.208	0.260	0.321	0.507	6.938	0.190	12.274

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	71	68	52	62	167	92	292
normalized size	1	1.00	0.82	0.78	0.60	0.71	1.92	1.06	3.36
time (sec)	N/A	0.099	0.159	0.245	0.309	0.551	4.241	0.167	12.220

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	81	97	81	88	0	145	245
normalized size	1	1.00	0.91	1.09	0.91	0.99	0.00	1.63	2.75
time (sec)	N/A	0.087	0.149	0.360	0.313	0.525	0.000	0.191	10.189

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	77	119	90	107	0	142	244
normalized size	1	1.00	0.93	1.43	1.08	1.29	0.00	1.71	2.94
time (sec)	N/A	0.114	0.431	0.306	0.432	0.543	0.000	0.200	8.767

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	143	101	139	0	163	239
normalized size	1	1.00	1.00	1.52	1.07	1.48	0.00	1.73	2.54
time (sec)	N/A	0.109	0.798	0.368	0.452	0.522	0.000	0.216	8.708

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	125	106	92	160	0	141	228
normalized size	1	1.00	1.52	1.29	1.12	1.95	0.00	1.72	2.78
time (sec)	N/A	0.075	0.051	0.223	0.436	0.523	0.000	0.193	8.699

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	153	128	107	180	0	153	217
normalized size	1	1.00	1.74	1.45	1.22	2.05	0.00	1.74	2.47
time (sec)	N/A	0.099	0.049	0.224	0.416	0.530	0.000	0.224	8.924

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	135	116	86	160	0	173	289
normalized size	1	1.00	1.82	1.57	1.16	2.16	0.00	2.34	3.91
time (sec)	N/A	0.118	0.035	0.221	0.353	0.514	0.000	0.267	10.434

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	175	138	106	187	0	201	337
normalized size	1	1.00	1.79	1.41	1.08	1.91	0.00	2.05	3.44
time (sec)	N/A	0.152	0.044	0.250	0.334	0.495	0.000	0.218	9.506

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	239	160	118	221	0	229	385
normalized size	1	1.00	2.10	1.40	1.04	1.94	0.00	2.01	3.38
time (sec)	N/A	0.154	0.079	0.269	0.321	0.531	0.000	0.218	9.946

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	279	182	138	239	0	201	337
normalized size	1	1.00	2.05	1.34	1.01	1.76	0.00	1.48	2.48
time (sec)	N/A	0.171	0.081	0.265	0.318	0.577	0.000	0.240	10.217

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	116	218	123	124	554	174	469
normalized size	1	1.00	0.63	1.18	0.66	0.67	2.99	0.94	2.54
time (sec)	N/A	0.339	0.728	0.280	0.373	0.555	34.602	0.322	12.211

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	86	162	101	111	335	123	437
normalized size	1	1.00	0.54	1.02	0.64	0.70	2.11	0.77	2.75
time (sec)	N/A	0.255	0.708	0.280	0.342	0.600	22.161	0.311	12.153

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	96	164	102	98	420	140	363
normalized size	1	1.00	0.68	1.16	0.72	0.70	2.98	0.99	2.57
time (sec)	N/A	0.258	0.514	0.286	0.316	0.550	14.016	0.265	12.351

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	86	106	82	85	223	123	388
normalized size	1	1.00	0.67	0.82	0.64	0.66	1.73	0.95	3.01
time (sec)	N/A	0.139	0.325	0.273	0.355	0.494	8.007	0.260	10.747

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	96	127	98	115	0	181	293
normalized size	1	1.00	0.81	1.07	0.82	0.97	0.00	1.52	2.46
time (sec)	N/A	0.143	0.849	0.465	0.329	0.624	0.000	0.224	10.215

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	83	137	128	135	0	210	310
normalized size	1	1.00	0.72	1.18	1.10	1.16	0.00	1.81	2.67
time (sec)	N/A	0.197	0.491	0.415	0.448	0.688	0.000	0.226	8.827

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	158	161	151	172	0	178	303
normalized size	1	1.00	1.61	1.64	1.54	1.76	0.00	1.82	3.09
time (sec)	N/A	0.157	2.121	0.486	0.461	0.583	0.000	0.236	8.911

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	191	190	139	192	0	209	293
normalized size	1	1.00	1.95	1.94	1.42	1.96	0.00	2.13	2.99
time (sec)	N/A	0.155	5.322	0.440	0.450	0.603	0.000	0.252	8.810

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	215	149	167	219	0	162	265
normalized size	1	1.00	1.85	1.28	1.44	1.89	0.00	1.40	2.28
time (sec)	N/A	0.183	1.235	0.332	0.430	0.526	0.000	0.279	8.770

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	200	170	124	239	0	207	275
normalized size	1	1.00	1.69	1.44	1.05	2.03	0.00	1.75	2.33
time (sec)	N/A	0.191	0.533	0.333	0.446	0.524	0.000	0.254	9.159

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	267	152	181	211	0	229	339
normalized size	1	1.00	2.02	1.15	1.37	1.60	0.00	1.73	2.57
time (sec)	N/A	0.262	0.106	0.362	0.352	0.517	0.000	0.288	9.754

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	291	200	233	271	0	293	319
normalized size	1	1.00	1.65	1.14	1.32	1.54	0.00	1.66	1.81
time (sec)	N/A	0.320	0.955	0.386	0.351	0.516	0.000	0.304	9.117

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	313	224	177	304	0	261	387
normalized size	1	1.00	1.86	1.33	1.05	1.81	0.00	1.55	2.30
time (sec)	N/A	0.274	1.371	0.411	0.340	0.500	0.000	0.299	12.071

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	353	248	283	340	0	357	395
normalized size	1	1.00	1.62	1.14	1.30	1.56	0.00	1.64	1.81
time (sec)	N/A	0.353	1.223	0.408	0.333	0.528	0.000	0.322	9.523

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	126	288	169	137	648	191	506
normalized size	1	1.00	0.62	1.42	0.83	0.67	3.19	0.94	2.49
time (sec)	N/A	0.393	1.040	0.286	0.345	0.663	50.883	0.425	11.720

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	116	252	149	124	595	174	572
normalized size	1	1.00	0.64	1.38	0.82	0.68	3.27	0.96	3.14
time (sec)	N/A	0.382	1.082	0.288	0.316	0.498	33.513	0.396	10.812

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	106	216	138	111	486	157	437
normalized size	1	1.00	0.67	1.36	0.87	0.70	3.06	0.99	2.75
time (sec)	N/A	0.323	0.849	0.278	0.321	0.513	21.625	0.329	12.280

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	96	178	115	98	440	140	461
normalized size	1	1.00	0.61	1.13	0.73	0.62	2.80	0.89	2.94
time (sec)	N/A	0.194	0.452	0.273	0.319	0.469	13.194	0.285	10.761

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	102	149	135	128	0	229	355
normalized size	1	1.00	0.71	1.04	0.94	0.90	0.00	1.60	2.48
time (sec)	N/A	0.202	1.021	0.473	0.321	0.481	0.000	0.326	10.510

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	148	152	141	147	0	226	356
normalized size	1	1.00	1.13	1.16	1.08	1.12	0.00	1.73	2.72
time (sec)	N/A	0.196	2.054	0.418	0.447	0.483	0.000	0.272	8.811

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	164	161	183	185	0	241	347
normalized size	1	1.00	1.20	1.18	1.34	1.35	0.00	1.76	2.53
time (sec)	N/A	0.183	2.859	0.486	0.440	0.494	0.000	0.322	8.808

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	201	190	185	206	0	250	339
normalized size	1	1.00	1.50	1.42	1.38	1.54	0.00	1.87	2.53
time (sec)	N/A	0.178	6.132	0.446	0.441	0.480	0.000	0.288	8.781

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	215	215	209	231	0	241	329
normalized size	1	1.00	1.56	1.56	1.51	1.67	0.00	1.75	2.38
time (sec)	N/A	0.192	1.219	0.447	0.432	0.484	0.000	0.314	8.782

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	216	173	180	252	0	226	554
normalized size	1	1.00	1.64	1.31	1.36	1.91	0.00	1.71	4.20
time (sec)	N/A	0.216	0.511	0.346	0.529	0.470	0.000	0.323	10.111

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	217	194	215	290	0	239	313
normalized size	1	1.00	1.29	1.15	1.28	1.73	0.00	1.42	1.86
time (sec)	N/A	0.275	0.781	0.365	0.422	0.499	0.000	0.339	9.380

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	363	176	206	247	0	261	387
normalized size	1	1.00	2.42	1.17	1.37	1.65	0.00	1.74	2.58
time (sec)	N/A	0.284	0.128	0.378	0.347	0.511	0.000	0.338	10.320

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	313	200	246	271	0	293	319
normalized size	1	1.00	1.78	1.14	1.40	1.54	0.00	1.66	1.81
time (sec)	N/A	0.338	5.092	0.387	0.333	0.475	0.000	0.380	9.221

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	313	224	268	304	0	325	357
normalized size	1	1.00	1.61	1.15	1.38	1.57	0.00	1.68	1.84
time (sec)	N/A	0.351	1.333	0.408	0.354	0.478	0.000	0.371	9.315

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	366	248	308	340	0	357	395
normalized size	1	1.00	1.69	1.15	1.43	1.57	0.00	1.65	1.83
time (sec)	N/A	0.390	2.171	0.470	0.358	0.481	0.000	0.406	9.552

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	116	306	186	124	746	174	572
normalized size	1	1.00	0.62	1.64	0.99	0.66	3.99	0.93	3.06
time (sec)	N/A	0.234	1.188	0.279	0.352	0.485	34.276	0.417	10.836

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	209	190	218	219	0	274	384
normalized size	1	1.00	1.49	1.36	1.56	1.56	0.00	1.96	2.74
time (sec)	N/A	0.228	5.259	0.441	0.431	0.489	0.000	0.370	8.805

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	86	381	380	80	2635	166	159
normalized size	1	1.00	0.64	2.82	2.81	0.59	19.52	1.23	1.18
time (sec)	N/A	0.198	0.258	0.269	0.439	0.444	88.918	0.200	11.375

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	377	347	339	70	2067	153	147
normalized size	1	1.00	3.22	2.97	2.90	0.60	17.67	1.31	1.26
time (sec)	N/A	0.191	4.954	0.258	0.443	0.461	72.097	0.196	11.309

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	258	279	278	60	1464	127	120
normalized size	1	1.00	2.84	3.07	3.05	0.66	16.09	1.40	1.32
time (sec)	N/A	0.162	2.435	0.246	0.439	0.473	53.158	0.158	12.009

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	219	279	257	50	1134	127	43
normalized size	1	1.00	3.00	3.82	3.52	0.68	15.53	1.74	0.59
time (sec)	N/A	0.112	1.682	0.173	0.425	0.440	24.777	0.170	8.798

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	159	156	57	0	88	136
normalized size	1	1.00	1.02	2.69	2.64	0.97	0.00	1.49	2.31
time (sec)	N/A	0.098	0.181	0.359	0.427	0.465	0.000	0.158	8.853

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	93	97	154	80	0	113	147
normalized size	1	1.00	1.90	1.98	3.14	1.63	0.00	2.31	3.00
time (sec)	N/A	0.116	0.425	0.393	0.423	0.461	0.000	0.170	8.723

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	102	112	138	104	0	103	159
normalized size	1	1.00	1.76	1.93	2.38	1.79	0.00	1.78	2.74
time (sec)	N/A	0.106	0.464	0.433	0.424	0.464	0.000	0.224	8.748

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	124	132	155	111	0	127	115
normalized size	1	1.00	2.14	2.28	2.67	1.91	0.00	2.19	1.98
time (sec)	N/A	0.088	0.488	0.442	0.312	0.449	0.000	0.190	8.669

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	125	132	154	132	0	129	119
normalized size	1	1.00	1.52	1.61	1.88	1.61	0.00	1.57	1.45
time (sec)	N/A	0.148	0.971	0.432	0.382	0.445	0.000	0.230	8.683

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	189	170	195	161	0	157	151
normalized size	1	1.00	1.89	1.70	1.95	1.61	0.00	1.57	1.51
time (sec)	N/A	0.172	0.594	0.464	0.320	0.458	0.000	0.196	8.754

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	229	246	274	188	0	216	339
normalized size	1	1.00	1.85	1.98	2.21	1.52	0.00	1.74	2.73
time (sec)	N/A	0.180	0.564	0.477	0.358	0.457	0.000	0.222	9.680

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	418	381	396	88	0	166	160
normalized size	1	1.00	2.84	2.59	2.69	0.60	0.00	1.13	1.09
time (sec)	N/A	0.224	4.831	0.442	0.486	0.445	0.000	0.228	12.217

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	76	347	353	78	2271	153	146
normalized size	1	1.00	0.59	2.69	2.74	0.60	17.60	1.19	1.13
time (sec)	N/A	0.225	0.239	0.411	0.540	0.444	143.138	0.197	11.242

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	308	279	290	68	1608	127	89
normalized size	1	1.00	3.02	2.74	2.84	0.67	15.76	1.25	0.87
time (sec)	N/A	0.199	1.322	0.399	0.488	0.444	94.093	0.219	8.718

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	258	245	247	58	1153	114	79
normalized size	1	1.00	2.97	2.82	2.84	0.67	13.25	1.31	0.91
time (sec)	N/A	0.198	1.355	0.360	0.472	0.435	59.860	0.172	8.662

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	204	177	184	43	694	88	55
normalized size	1	1.00	2.91	2.53	2.63	0.61	9.91	1.26	0.79
time (sec)	N/A	0.110	0.808	0.348	0.444	0.435	36.059	0.163	8.637

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	46	60	82	45	0	52	97
normalized size	1	1.00	1.28	1.67	2.28	1.25	0.00	1.44	2.69
time (sec)	N/A	0.129	0.134	0.513	0.452	0.498	0.000	0.188	8.718

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	98	74	93	70	0	73	95
normalized size	1	1.00	2.80	2.11	2.66	2.00	0.00	2.09	2.71
time (sec)	N/A	0.149	0.367	0.538	0.446	0.462	0.000	0.192	8.832

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	86	93	115	93	0	98	84
normalized size	1	1.00	1.59	1.72	2.13	1.72	0.00	1.81	1.56
time (sec)	N/A	0.151	0.529	0.602	0.341	0.476	0.000	0.215	8.689

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	121	132	153	123	0	128	119
normalized size	1	1.00	1.83	2.00	2.32	1.86	0.00	1.94	1.80
time (sec)	N/A	0.126	0.912	0.612	0.340	0.448	0.000	0.219	8.665

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	116	170	195	149	0	157	151
normalized size	1	1.00	1.21	1.77	2.03	1.55	0.00	1.64	1.57
time (sec)	N/A	0.187	1.517	0.605	0.352	0.480	0.000	0.237	8.723

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	189	208	233	179	0	186	289
normalized size	1	1.00	1.69	1.86	2.08	1.60	0.00	1.66	2.58
time (sec)	N/A	0.241	0.747	0.639	0.369	0.452	0.000	0.230	9.304

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	229	246	275	204	0	215	339
normalized size	1	1.00	1.66	1.78	1.99	1.48	0.00	1.56	2.46
time (sec)	N/A	0.248	0.833	0.663	0.335	0.451	0.000	0.267	9.738

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	195	300	398	144	3578	145	146
normalized size	1	1.00	1.79	2.75	3.65	1.32	32.83	1.33	1.34
time (sec)	N/A	0.259	1.337	0.415	0.427	0.445	147.462	0.240	12.345

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	181	198	312	123	2264	106	121
normalized size	1	1.00	2.08	2.28	3.59	1.41	26.02	1.22	1.39
time (sec)	N/A	0.227	1.136	0.384	0.418	0.431	95.057	0.227	12.349

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	143	163	225	100	1244	91	94
normalized size	1	1.00	1.79	2.04	2.81	1.25	15.55	1.14	1.18
time (sec)	N/A	0.139	0.645	0.403	0.422	0.439	61.269	0.205	10.796

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	122	58	78	117	0	47	115
normalized size	1	1.00	2.71	1.29	1.73	2.60	0.00	1.04	2.56
time (sec)	N/A	0.183	0.284	0.577	0.416	0.460	0.000	0.207	8.729

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	156	77	116	165	0	90	87
normalized size	1	1.00	2.89	1.43	2.15	3.06	0.00	1.67	1.61
time (sec)	N/A	0.236	0.644	0.589	0.456	0.446	0.000	0.224	8.668

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	213	115	161	246	0	116	120
normalized size	1	1.00	2.73	1.47	2.06	3.15	0.00	1.49	1.54
time (sec)	N/A	0.249	5.901	0.678	0.332	0.447	0.000	0.249	8.693

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	251	153	199	302	0	146	153
normalized size	1	1.00	2.61	1.59	2.07	3.15	0.00	1.52	1.59
time (sec)	N/A	0.155	5.033	0.673	0.485	0.458	0.000	0.238	8.690

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	601	191	241	381	0	174	176
normalized size	1	1.00	5.14	1.63	2.06	3.26	0.00	1.49	1.50
time (sec)	N/A	0.300	6.163	0.683	0.379	0.482	0.000	0.278	9.131

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	143	100	269	195	0	92	157
normalized size	1	1.00	2.47	1.72	4.64	3.36	0.00	1.59	2.71
time (sec)	N/A	0.106	1.261	0.455	0.516	0.438	0.000	0.292	8.892

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	131	293	115	335	243	0	106	181
normalized size	1	1.47	3.29	1.29	3.76	2.73	0.00	1.19	2.03
time (sec)	N/A	0.459	2.609	0.455	0.543	0.455	0.000	0.380	9.051

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	195	130	401	291	0	120	205
normalized size	1	1.00	1.24	0.83	2.55	1.85	0.00	0.76	1.31
time (sec)	N/A	0.570	3.176	0.534	0.346	0.461	0.000	0.476	9.407

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	109	85	0	172	0	219	-1
normalized size	1	1.00	0.70	0.54	0.00	1.10	0.00	1.40	-0.01
time (sec)	N/A	0.424	3.907	0.985	0.000	0.440	0.000	0.285	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	99	75	0	151	0	189	-1
normalized size	1	1.00	0.80	0.60	0.00	1.22	0.00	1.52	-0.01
time (sec)	N/A	0.257	2.020	0.882	0.000	0.437	0.000	0.254	0.000

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	195	141	0	294	0	0	-1
normalized size	1	1.00	1.23	0.89	0.00	1.85	0.00	0.00	-0.01
time (sec)	N/A	0.492	0.328	1.334	0.000	0.459	0.000	0.000	0.000

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	258	162	0	320	0	0	-1
normalized size	1	1.00	1.74	1.09	0.00	2.16	0.00	0.00	-0.01
time (sec)	N/A	0.477	0.741	1.278	0.000	0.490	0.000	0.000	0.000

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	297	178	0	359	0	0	-1
normalized size	1	1.00	1.90	1.14	0.00	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.412	0.903	1.392	0.000	0.466	0.000	0.000	0.000

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	309	170	0	380	0	0	-1
normalized size	1	1.00	1.90	1.04	0.00	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.389	1.417	1.383	0.000	0.491	0.000	0.000	0.000

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	367	162	0	415	0	0	-1
normalized size	1	1.00	2.12	0.94	0.00	2.40	0.00	0.00	-0.01
time (sec)	N/A	0.538	2.737	1.412	0.000	0.460	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	403	180	0	461	0	0	-1
normalized size	1	1.00	1.93	0.86	0.00	2.21	0.00	0.00	-0.00
time (sec)	N/A	0.691	4.438	1.402	0.000	0.470	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	485	198	0	525	0	0	-1
normalized size	1	1.00	1.98	0.81	0.00	2.14	0.00	0.00	-0.00
time (sec)	N/A	0.812	7.685	1.323	0.000	0.489	0.000	0.000	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	191	216	0	567	0	0	-1
normalized size	1	1.00	0.68	0.77	0.00	2.02	0.00	0.00	-0.00
time (sec)	N/A	0.948	2.030	1.391	0.000	0.519	0.000	0.000	0.000

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	120	97	0	210	0	474	-1
normalized size	1	1.00	0.64	0.52	0.00	1.12	0.00	2.52	-0.01
time (sec)	N/A	0.515	9.080	1.112	0.000	0.458	0.000	1.429	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	110	87	0	189	0	412	-1
normalized size	1	1.00	0.71	0.56	0.00	1.21	0.00	2.64	-0.01
time (sec)	N/A	0.325	5.181	0.862	0.000	0.437	0.000	0.809	0.000

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	219	159	0	332	0	0	-1
normalized size	1	1.00	1.10	0.80	0.00	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.711	0.477	1.293	0.000	0.466	0.000	0.000	0.000

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	283	180	0	360	0	0	-1
normalized size	1	1.00	1.59	1.01	0.00	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.649	1.323	1.275	0.000	0.459	0.000	0.000	0.000

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	322	178	0	404	0	0	-1
normalized size	1	1.00	1.73	0.96	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.571	1.073	1.174	0.000	0.478	0.000	0.000	0.000

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	334	196	0	424	0	0	-1
normalized size	1	1.00	1.70	0.99	0.00	2.15	0.00	0.00	-0.01
time (sec)	N/A	0.500	1.276	1.500	0.000	0.503	0.000	0.000	0.000

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	392	188	0	460	0	0	-1
normalized size	1	1.00	1.91	0.92	0.00	2.24	0.00	0.00	-0.00
time (sec)	N/A	0.703	1.460	1.418	0.000	0.477	0.000	0.000	0.000

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	404	180	0	488	0	0	-1
normalized size	1	1.00	1.88	0.84	0.00	2.27	0.00	0.00	-0.00
time (sec)	N/A	0.808	1.641	1.480	0.000	0.495	0.000	0.000	0.000

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	486	198	0	557	0	0	-1
normalized size	1	1.00	1.92	0.78	0.00	2.20	0.00	0.00	-0.00
time (sec)	N/A	0.919	2.508	1.463	0.000	0.489	0.000	0.000	0.000

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	522	216	0	600	0	0	-1
normalized size	1	1.00	1.79	0.74	0.00	2.06	0.00	0.00	-0.00
time (sec)	N/A	1.061	4.938	1.660	0.000	0.503	0.000	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	2303	234	0	657	0	0	-1
normalized size	1	1.00	7.00	0.71	0.00	2.00	0.00	0.00	-0.00
time (sec)	N/A	1.187	6.258	1.877	0.000	0.513	0.000	0.000	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	143	74	0	155	0	342	-1
normalized size	1	1.00	1.15	0.60	0.00	1.25	0.00	2.76	-0.01
time (sec)	N/A	0.406	1.697	1.353	0.000	0.433	0.000	0.810	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	87	64	0	136	0	279	-1
normalized size	1	1.00	0.95	0.70	0.00	1.48	0.00	3.03	-0.01
time (sec)	N/A	0.193	1.595	1.405	0.000	0.438	0.000	0.602	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	169	123	0	279	0	386	-1
normalized size	1	1.00	1.30	0.95	0.00	2.15	0.00	2.97	-0.01
time (sec)	N/A	0.564	0.251	1.896	0.000	0.467	0.000	0.697	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	190	126	0	306	0	476	-1
normalized size	1	1.00	1.60	1.06	0.00	2.57	0.00	4.00	-0.01
time (sec)	N/A	0.503	0.381	2.039	0.000	0.499	0.000	0.799	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	296	150	0	346	0	554	-1
normalized size	1	1.00	2.37	1.20	0.00	2.77	0.00	4.43	-0.01
time (sec)	N/A	0.543	3.736	2.046	0.000	0.503	0.000	0.795	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	292	144	0	369	0	583	-1
normalized size	1	1.00	2.16	1.07	0.00	2.73	0.00	4.32	-0.01
time (sec)	N/A	0.597	0.604	2.099	0.000	0.485	0.000	0.804	0.000

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	374	162	0	426	0	736	-1
normalized size	1	1.00	2.20	0.95	0.00	2.51	0.00	4.33	-0.01
time (sec)	N/A	0.887	0.900	2.051	0.000	0.477	0.000	0.923	0.000

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	410	180	0	472	0	802	-1
normalized size	1	1.00	2.00	0.88	0.00	2.30	0.00	3.91	-0.00
time (sec)	N/A	1.143	1.006	2.096	0.000	0.484	0.000	0.952	0.000

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	102	77	0	161	0	286	-1
normalized size	1	1.00	0.50	0.38	0.00	0.79	0.00	1.40	-0.00
time (sec)	N/A	0.776	5.586	0.997	0.000	0.465	0.000	0.727	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	67	0	142	0	282	-1
normalized size	1	1.00	1.00	0.73	0.00	1.54	0.00	3.07	-0.01
time (sec)	N/A	0.370	3.717	1.017	0.000	0.445	0.000	0.719	0.000

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	82	57	0	121	0	216	-1
normalized size	1	1.00	1.37	0.95	0.00	2.02	0.00	3.60	-0.02
time (sec)	N/A	0.151	1.901	0.874	0.000	0.438	0.000	0.673	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	147	103	0	260	0	313	-1
normalized size	1	1.00	1.50	1.05	0.00	2.65	0.00	3.19	-0.01
time (sec)	N/A	0.362	0.268	1.096	0.000	0.491	0.000	0.835	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	220	123	0	291	0	424	-1
normalized size	1	1.00	2.34	1.31	0.00	3.10	0.00	4.51	-0.01
time (sec)	N/A	0.402	0.701	1.098	0.000	0.482	0.000	0.852	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	274	126	0	337	0	513	-1
normalized size	1	1.00	2.58	1.19	0.00	3.18	0.00	4.84	-0.01
time (sec)	N/A	0.488	1.819	1.281	0.000	0.510	0.000	0.887	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	294	144	0	383	0	589	-1
normalized size	1	1.00	2.04	1.00	0.00	2.66	0.00	4.09	-0.01
time (sec)	N/A	0.539	0.790	1.239	0.000	0.527	0.000	0.952	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	376	162	0	442	0	737	-1
normalized size	1	1.00	2.07	0.89	0.00	2.43	0.00	4.05	-0.01
time (sec)	N/A	0.727	0.958	1.294	0.000	0.491	0.000	1.107	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	412	180	0	492	0	808	-1
normalized size	1	1.00	1.87	0.82	0.00	2.24	0.00	3.67	-0.00
time (sec)	N/A	0.876	1.385	1.359	0.000	0.495	0.000	1.156	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	224	166	0	299	0	504	-1
normalized size	1	1.00	0.86	0.64	0.00	1.15	0.00	1.94	-0.00
time (sec)	N/A	1.359	1.075	1.234	0.000	0.513	0.000	0.956	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	225	166	0	280	0	434	-1
normalized size	1	1.00	1.01	0.75	0.00	1.26	0.00	1.95	-0.00
time (sec)	N/A	1.080	3.280	1.283	0.000	0.522	0.000	0.906	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	201	132	0	258	0	358	-1
normalized size	1	1.00	1.19	0.78	0.00	1.53	0.00	2.12	-0.01
time (sec)	N/A	0.428	2.507	1.095	0.000	0.489	0.000	0.894	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	177	130	0	239	0	297	-1
normalized size	1	1.00	1.29	0.95	0.00	1.74	0.00	2.17	-0.01
time (sec)	N/A	0.235	1.620	1.195	0.000	0.520	0.000	0.823	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	154	116	0	377	0	0	-1
normalized size	1	1.00	1.36	1.03	0.00	3.34	0.00	0.00	-0.01
time (sec)	N/A	0.391	0.400	1.005	0.000	0.521	0.000	0.000	0.000

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	170	132	0	421	0	472	-1
normalized size	1	1.00	1.50	1.17	0.00	3.73	0.00	4.18	-0.01
time (sec)	N/A	0.524	2.697	1.070	0.000	0.498	0.000	0.984	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	309	164	0	508	0	620	-1
normalized size	1	1.00	2.02	1.07	0.00	3.32	0.00	4.05	-0.01
time (sec)	N/A	0.733	3.558	1.291	0.000	0.493	0.000	1.495	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	332	182	0	564	0	695	-1
normalized size	1	1.00	1.74	0.95	0.00	2.95	0.00	3.64	-0.01
time (sec)	N/A	0.947	2.411	1.484	0.000	0.502	0.000	1.159	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	414	200	0	643	0	845	-1
normalized size	1	1.00	1.81	0.87	0.00	2.81	0.00	3.69	-0.00
time (sec)	N/A	1.314	5.121	1.396	0.000	0.520	0.000	1.294	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	164	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.292	11.093	0.000	0.505	0.000	0.000	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.268	6.119	0.000	0.462	0.000	0.000	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	441	0	0	0	0	0	-1
normalized size	1	1.00	3.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	11.479	4.783	0.000	0.474	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	312	0	0	0	0	0	-1
normalized size	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	4.921	6.952	0.000	0.482	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	138	72	106	136	133	71
normalized size	1	1.00	1.00	1.42	0.74	1.09	1.40	1.37	0.73
time (sec)	N/A	0.089	0.447	0.227	0.349	0.457	42.244	0.314	8.718

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	87	120	72	95	136	118	71
normalized size	1	1.00	0.90	1.24	0.74	0.98	1.40	1.22	0.73
time (sec)	N/A	0.084	0.326	0.244	0.503	0.556	27.036	0.258	8.699

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	97	102	72	84	114	133	71
normalized size	1	1.00	1.20	1.26	0.89	1.04	1.41	1.64	0.88
time (sec)	N/A	0.127	0.361	0.283	0.581	0.728	17.225	0.234	0.054

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	87	84	72	73	114	118	71
normalized size	1	1.00	1.07	1.04	0.89	0.90	1.41	1.46	0.88
time (sec)	N/A	0.126	0.285	0.241	0.858	0.791	10.441	0.213	8.694

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	78	64	72	62	90	103	71
normalized size	1	1.00	1.20	0.98	1.11	0.95	1.38	1.58	1.09
time (sec)	N/A	0.091	0.161	0.245	0.584	0.580	6.258	0.203	8.699

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	94	69	74	0	70	126
normalized size	1	1.00	1.00	1.09	0.80	0.86	0.00	0.81	1.47
time (sec)	N/A	0.067	0.037	0.374	0.542	0.745	0.000	0.173	8.871

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	116	69	91	0	79	250
normalized size	1	1.00	1.00	1.40	0.83	1.10	0.00	0.95	3.01
time (sec)	N/A	0.079	0.039	0.308	0.609	0.796	0.000	0.194	8.878

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	139	68	102	0	82	229
normalized size	1	1.00	0.90	1.62	0.79	1.19	0.00	0.95	2.66
time (sec)	N/A	0.079	0.112	0.385	0.596	0.693	0.000	0.198	8.861

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	159	69	117	0	81	218
normalized size	1	1.00	0.89	1.87	0.81	1.38	0.00	0.95	2.56
time (sec)	N/A	0.072	0.150	0.336	0.500	0.809	0.000	0.198	8.794

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	87	136	69	110	0	82	207
normalized size	1	1.00	1.07	1.68	0.85	1.36	0.00	1.01	2.56
time (sec)	N/A	0.044	0.201	0.324	0.786	0.830	0.000	0.213	9.033

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	92	160	72	124	0	84	193
normalized size	1	1.00	1.07	1.86	0.84	1.44	0.00	0.98	2.24
time (sec)	N/A	0.071	0.169	0.321	0.683	0.769	0.000	0.252	8.898

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	110	70	100	0	70	69
normalized size	1	1.00	1.00	1.80	1.15	1.64	0.00	1.15	1.13
time (sec)	N/A	0.104	0.025	0.337	0.712	0.647	0.000	0.245	8.884

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	128	70	106	0	70	70
normalized size	1	1.00	1.00	1.97	1.08	1.63	0.00	1.08	1.08
time (sec)	N/A	0.116	0.026	0.359	0.595	0.671	0.000	0.247	8.834

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	88	148	70	109	0	70	70
normalized size	1	1.00	1.09	1.83	0.86	1.35	0.00	0.86	0.86
time (sec)	N/A	0.121	0.160	0.364	0.512	0.694	0.000	0.241	8.844

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	88	166	70	115	0	70	70
normalized size	1	1.00	1.09	2.05	0.86	1.42	0.00	0.86	0.86
time (sec)	N/A	0.122	0.113	0.375	0.691	0.699	0.000	0.237	8.884

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	184	70	122	0	70	70
normalized size	1	1.00	0.91	1.90	0.72	1.26	0.00	0.72	0.72
time (sec)	N/A	0.082	0.174	0.373	0.450	0.820	0.000	0.262	8.859

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	202	70	128	0	70	70
normalized size	1	1.00	0.91	2.08	0.72	1.32	0.00	0.72	0.72
time (sec)	N/A	0.082	0.194	0.288	0.508	0.680	0.000	0.248	8.976

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	110	158	97	111	189	168	96
normalized size	1	1.00	0.87	1.24	0.76	0.87	1.49	1.32	0.76
time (sec)	N/A	0.125	0.791	0.171	0.511	0.823	26.151	0.327	8.738

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	99	156	97	98	190	151	96
normalized size	1	1.00	0.91	1.43	0.89	0.90	1.74	1.39	0.88
time (sec)	N/A	0.126	0.725	0.267	0.619	0.793	16.758	0.284	8.762

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	90	102	97	85	139	134	96
normalized size	1	1.00	1.01	1.15	1.09	0.96	1.56	1.51	1.08
time (sec)	N/A	0.084	0.319	0.237	0.484	0.727	9.877	0.247	8.812

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	78	122	94	99	0	95	132
normalized size	1	1.00	0.66	1.03	0.79	0.83	0.00	0.80	1.11
time (sec)	N/A	0.101	0.083	0.471	0.463	0.756	0.000	0.236	9.066

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	114	130	94	118	0	107	333
normalized size	1	1.00	1.00	1.14	0.82	1.04	0.00	0.94	2.92
time (sec)	N/A	0.118	0.053	0.408	0.692	0.773	0.000	0.234	8.946

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	76	155	93	131	0	109	297
normalized size	1	1.00	0.66	1.34	0.80	1.13	0.00	0.94	2.56
time (sec)	N/A	0.118	0.171	0.472	0.558	0.744	0.000	0.259	8.944

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	74	97	93	115	0	107	288
normalized size	1	1.00	0.67	0.88	0.85	1.05	0.00	0.97	2.62
time (sec)	N/A	0.101	0.182	0.424	0.407	0.851	0.000	0.273	8.918

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	76	211	94	152	0	108	276
normalized size	1	1.00	0.66	1.82	0.81	1.31	0.00	0.93	2.38
time (sec)	N/A	0.067	0.454	0.425	0.655	0.834	0.000	0.285	8.796

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	76	178	94	153	0	109	267
normalized size	1	1.00	0.68	1.59	0.84	1.37	0.00	0.97	2.38
time (sec)	N/A	0.102	0.129	0.425	0.561	0.825	0.000	0.267	8.788

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	102	202	97	167	0	111	217
normalized size	1	1.00	0.86	1.70	0.82	1.40	0.00	0.93	1.82
time (sec)	N/A	0.121	0.042	0.444	0.661	0.753	0.000	0.289	8.975

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	110	208	110	111	255	168	109
normalized size	1	1.00	0.99	1.87	0.99	1.00	2.30	1.51	0.98
time (sec)	N/A	0.128	0.856	0.265	0.657	0.979	30.001	0.390	8.715

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	100	170	110	98	202	151	108
normalized size	1	1.00	1.12	1.91	1.24	1.10	2.27	1.70	1.21
time (sec)	N/A	0.083	0.678	0.259	0.520	0.692	17.300	0.316	0.073

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	88	144	107	112	0	108	178
normalized size	1	1.00	0.64	1.05	0.78	0.82	0.00	0.79	1.30
time (sec)	N/A	0.106	0.108	0.471	0.515	0.640	0.000	0.276	8.989

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	86	147	107	131	0	120	371
normalized size	1	1.00	0.65	1.11	0.80	0.98	0.00	0.90	2.79
time (sec)	N/A	0.123	0.171	0.398	0.508	0.715	0.000	0.313	9.185

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	86	154	106	145	0	120	342
normalized size	1	1.00	0.65	1.16	0.80	1.09	0.00	0.90	2.57
time (sec)	N/A	0.128	0.152	0.462	0.613	0.779	0.000	0.321	9.014

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	86	179	108	159	0	122	333
normalized size	1	1.00	0.66	1.37	0.82	1.21	0.00	0.93	2.54
time (sec)	N/A	0.110	0.256	0.428	0.447	0.772	0.000	0.320	8.982

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	86	211	108	159	0	121	322
normalized size	1	1.00	0.66	1.61	0.82	1.21	0.00	0.92	2.46
time (sec)	N/A	0.070	0.498	0.428	0.519	0.787	0.000	0.358	8.940

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	86	234	107	179	0	122	311
normalized size	1	1.00	0.65	1.76	0.80	1.35	0.00	0.92	2.34
time (sec)	N/A	0.110	0.173	0.441	0.750	0.690	0.000	0.353	8.946

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	113	203	108	180	0	122	296
normalized size	1	1.00	0.85	1.53	0.81	1.35	0.00	0.92	2.23
time (sec)	N/A	0.126	0.043	0.519	0.525	0.854	0.000	0.407	9.843

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	96	179	119	172	0	135	378
normalized size	1	1.00	0.66	1.23	0.82	1.19	0.00	0.93	2.61
time (sec)	N/A	0.121	0.158	0.454	0.792	0.681	0.000	0.414	8.970

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	96	129	120	144	0	134	368
normalized size	1	1.00	0.65	0.87	0.81	0.97	0.00	0.91	2.49
time (sec)	N/A	0.079	0.154	0.441	0.717	0.970	0.000	0.456	9.073

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	96	235	120	192	0	134	357
normalized size	1	1.00	0.66	1.61	0.82	1.32	0.00	0.92	2.45
time (sec)	N/A	0.116	0.183	0.432	0.574	0.592	0.000	0.453	9.086

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	49	67	981	49	57
normalized size	1	1.00	0.66	0.67	0.67	0.92	13.44	0.67	0.78
time (sec)	N/A	0.113	0.355	0.253	0.675	0.694	81.792	0.169	0.068

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	49	59	862	49	57
normalized size	1	1.00	0.66	0.67	0.67	0.81	11.81	0.67	0.78
time (sec)	N/A	0.156	0.208	0.242	0.578	0.636	51.018	0.182	0.063

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	49	49	49	741	49	57
normalized size	1	1.00	0.87	0.89	0.89	0.89	13.47	0.89	1.04
time (sec)	N/A	0.107	0.156	0.189	0.550	0.634	30.744	0.181	0.063

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	62	51	48	0	61	102
normalized size	1	1.00	0.75	0.95	0.78	0.74	0.00	0.94	1.57
time (sec)	N/A	0.089	0.052	0.383	0.833	0.633	0.000	0.172	9.051

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	45	63	52	65	0	65	146
normalized size	1	1.00	0.73	1.02	0.84	1.05	0.00	1.05	2.35
time (sec)	N/A	0.108	0.061	0.426	0.320	0.758	0.000	0.193	8.801

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	45	61	52	61	0	63	150
normalized size	1	1.00	0.75	1.02	0.87	1.02	0.00	1.05	2.50
time (sec)	N/A	0.106	0.086	0.461	0.907	0.656	0.000	0.187	8.927

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	48	63	50	75	0	62	138
normalized size	1	1.00	0.75	0.98	0.78	1.17	0.00	0.97	2.16
time (sec)	N/A	0.092	0.079	0.457	0.766	0.776	0.000	0.197	8.948

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	30	49	46	63	0	46	45
normalized size	1	1.00	0.59	0.96	0.90	1.24	0.00	0.90	0.88
time (sec)	N/A	0.088	0.047	0.464	0.807	0.613	0.000	0.196	8.932

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	49	46	71	0	46	46
normalized size	1	1.00	0.87	0.89	0.84	1.29	0.00	0.84	0.84
time (sec)	N/A	0.136	0.113	0.483	0.775	0.656	0.000	0.229	8.958

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	46	76	0	46	46
normalized size	1	1.00	0.66	0.67	0.63	1.04	0.00	0.63	0.63
time (sec)	N/A	0.111	0.104	0.493	0.818	0.525	0.000	0.215	8.942

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	46	84	0	46	46
normalized size	1	1.00	0.66	0.67	0.63	1.15	0.00	0.63	0.63
time (sec)	N/A	0.109	0.106	0.539	1.004	0.550	0.000	0.230	8.963

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	39	67	682	39	36
normalized size	1	1.00	0.69	0.71	0.71	1.22	12.40	0.71	0.65
time (sec)	N/A	0.106	0.536	0.380	0.719	0.753	128.341	0.226	0.061

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	39	59	588	39	36
normalized size	1	1.00	0.69	0.71	0.71	1.07	10.69	0.71	0.65
time (sec)	N/A	0.104	0.663	0.362	0.570	0.613	82.057	0.178	0.054

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	39	47	493	39	36
normalized size	1	1.00	0.69	0.71	0.71	0.85	8.96	0.71	0.65
time (sec)	N/A	0.068	0.214	0.299	0.809	0.765	53.002	0.173	8.832

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	36	46	39	36	0	47	120
normalized size	1	1.00	0.77	0.98	0.83	0.77	0.00	1.00	2.55
time (sec)	N/A	0.082	0.039	0.455	0.510	0.514	0.000	0.176	9.108

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	32	46	41	42	0	53	110
normalized size	1	1.00	0.74	1.07	0.95	0.98	0.00	1.23	2.56
time (sec)	N/A	0.099	0.043	0.482	0.573	0.763	0.000	0.200	8.919

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	38	48	40	55	0	52	104
normalized size	1	1.00	0.81	1.02	0.85	1.17	0.00	1.11	2.21
time (sec)	N/A	0.101	0.047	0.526	0.849	0.693	0.000	0.223	8.887

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	37	36	52	0	36	34
normalized size	1	1.00	0.65	1.19	1.16	1.68	0.00	1.16	1.10
time (sec)	N/A	0.079	0.042	0.537	0.307	0.629	0.000	0.233	8.931

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	36	57	0	36	36
normalized size	1	1.00	0.69	0.71	0.65	1.04	0.00	0.65	0.65
time (sec)	N/A	0.046	0.065	0.532	0.533	0.706	0.000	0.261	8.923

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	36	65	0	36	36
normalized size	1	1.00	0.69	0.71	0.65	1.18	0.00	0.65	0.65
time (sec)	N/A	0.085	0.074	0.563	0.409	0.763	0.000	0.229	8.941

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	36	72	0	36	36
normalized size	1	1.00	0.69	0.71	0.65	1.31	0.00	0.65	0.65
time (sec)	N/A	0.103	0.076	0.585	0.963	0.701	0.000	0.273	8.951

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	71	97	73	70	0	193	83
normalized size	1	1.00	0.70	0.95	0.72	0.69	0.00	1.89	0.81
time (sec)	N/A	0.126	0.968	0.421	0.424	0.670	0.000	0.276	0.064

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	59	81	61	56	1698	167	69
normalized size	1	1.00	0.72	0.99	0.74	0.68	20.71	2.04	0.84
time (sec)	N/A	0.118	0.938	0.440	0.412	0.610	135.282	0.221	8.819

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	65	53	48	1102	141	57
normalized size	1	1.00	0.75	0.96	0.78	0.71	16.21	2.07	0.84
time (sec)	N/A	0.076	0.346	0.392	0.596	0.627	86.477	0.210	0.055

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	32	46	43	34	0	103	95
normalized size	1	1.00	0.71	1.02	0.96	0.76	0.00	2.29	2.11
time (sec)	N/A	0.086	0.040	0.553	0.488	0.828	0.000	0.211	8.919

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	35	50	44	52	0	101	71
normalized size	1	1.00	0.74	1.06	0.94	1.11	0.00	2.15	1.51
time (sec)	N/A	0.106	0.048	0.582	0.868	0.846	0.000	0.250	8.982

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	66	55	76	0	115	107
normalized size	1	1.00	0.75	1.02	0.85	1.17	0.00	1.77	1.65
time (sec)	N/A	0.113	0.080	0.668	0.798	0.767	0.000	0.248	8.879

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	59	82	65	106	0	145	139
normalized size	1	1.00	0.71	0.99	0.78	1.28	0.00	1.75	1.67
time (sec)	N/A	0.100	0.116	0.676	0.721	0.681	0.000	0.249	8.990

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	69	97	75	131	0	174	171
normalized size	1	1.00	0.72	1.01	0.78	1.36	0.00	1.81	1.78
time (sec)	N/A	0.068	0.312	0.680	0.673	0.808	0.000	0.265	8.877

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	79	114	85	161	0	204	203
normalized size	1	1.00	0.68	0.97	0.73	1.38	0.00	1.74	1.74
time (sec)	N/A	0.119	0.130	0.661	0.872	0.582	0.000	0.270	8.961

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	81	116	100	235	0	218	233
normalized size	1	1.00	0.68	0.97	0.83	1.96	0.00	1.82	1.94
time (sec)	N/A	0.083	0.714	0.684	0.543	0.831	0.000	0.301	8.886

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	91	131	110	283	0	248	266
normalized size	1	1.00	0.67	0.97	0.81	2.10	0.00	1.84	1.97
time (sec)	N/A	0.132	0.283	0.685	0.777	0.757	0.000	0.315	8.906

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	123	0	161	616	0	0	923
normalized size	1	1.00	0.68	0.00	0.89	3.40	0.00	0.00	5.10
time (sec)	N/A	0.181	0.585	21.576	0.927	0.773	0.000	0.000	15.448

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	110	0	143	473	0	0	819
normalized size	1	1.00	0.69	0.00	0.89	2.96	0.00	0.00	5.12
time (sec)	N/A	0.167	0.391	16.812	0.967	0.825	0.000	0.000	14.661

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	345	0	109	282	0	0	550
normalized size	1	1.00	2.80	0.00	0.89	2.29	0.00	0.00	4.47
time (sec)	N/A	0.119	1.384	9.863	0.595	0.729	0.000	0.000	13.561

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	74	0	124	134	0	0	228
normalized size	1	1.00	0.81	0.00	1.36	1.47	0.00	0.00	2.51
time (sec)	N/A	0.139	0.701	5.916	0.801	0.882	0.000	0.000	10.205

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	0	81	105	0	0	146
normalized size	1	1.00	0.74	0.00	1.19	1.54	0.00	0.00	2.15
time (sec)	N/A	0.127	0.119	13.917	0.920	0.763	0.000	0.000	9.540

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.105	6.211	0.000	0.700	0.000	0.000	0.000

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	72	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.105	12.282	0.000	0.854	0.000	0.000	0.000

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	121	134	86	106	318	167	447
normalized size	1	1.00	0.73	0.81	0.52	0.64	1.93	1.01	2.71
time (sec)	N/A	0.200	0.567	0.243	0.376	0.702	44.944	0.295	11.872

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	101	116	76	95	294	137	414
normalized size	1	1.00	0.68	0.78	0.51	0.64	1.97	0.92	2.78
time (sec)	N/A	0.206	0.348	0.246	0.422	0.776	29.349	0.263	12.186

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	91	98	76	84	248	122	386
normalized size	1	1.00	0.73	0.78	0.61	0.67	1.98	0.98	3.09
time (sec)	N/A	0.155	0.321	0.237	0.503	0.759	18.971	0.260	12.357

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	91	78	63	73	223	122	96
normalized size	1	1.00	0.83	0.72	0.58	0.67	2.05	1.12	0.88
time (sec)	N/A	0.120	0.263	0.247	0.350	0.741	11.233	0.192	9.391

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	100	131	106	110	0	201	327
normalized size	1	1.00	0.79	1.03	0.83	0.87	0.00	1.58	2.57
time (sec)	N/A	0.109	0.117	0.377	0.392	0.800	0.000	0.193	10.707

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	98	153	121	129	0	198	313
normalized size	1	1.00	0.81	1.26	1.00	1.07	0.00	1.64	2.59
time (sec)	N/A	0.134	0.265	0.291	0.468	0.830	0.000	0.199	8.923

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	117	177	131	162	0	214	321
normalized size	1	1.00	0.87	1.32	0.98	1.21	0.00	1.60	2.40
time (sec)	N/A	0.144	2.752	0.370	0.566	0.711	0.000	0.203	8.858

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	174	199	122	182	0	220	310
normalized size	1	1.00	1.34	1.53	0.94	1.40	0.00	1.69	2.38
time (sec)	N/A	0.140	6.096	0.321	0.598	0.835	0.000	0.214	8.970

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	138	221	136	202	0	213	300
normalized size	1	1.00	1.03	1.65	1.01	1.51	0.00	1.59	2.24
time (sec)	N/A	0.126	1.427	0.332	0.615	0.702	0.000	0.236	8.817

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	164	159	125	222	0	199	291
normalized size	1	1.00	1.34	1.30	1.02	1.82	0.00	1.63	2.39
time (sec)	N/A	0.099	0.070	0.226	0.437	0.618	0.000	0.222	8.867

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	193	181	137	254	0	208	285
normalized size	1	1.00	1.51	1.41	1.07	1.98	0.00	1.62	2.23
time (sec)	N/A	0.132	0.064	0.251	0.427	0.787	0.000	0.268	9.460

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	175	152	106	210	0	228	385
normalized size	1	1.00	1.82	1.58	1.10	2.19	0.00	2.38	4.01
time (sec)	N/A	0.140	0.048	0.270	0.403	0.875	0.000	0.236	9.942

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	215	174	126	225	0	256	285
normalized size	1	1.00	1.76	1.43	1.03	1.84	0.00	2.10	2.34
time (sec)	N/A	0.184	0.062	0.268	0.370	0.776	0.000	0.282	9.154

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	301	196	138	259	0	256	285
normalized size	1	1.00	2.18	1.42	1.00	1.88	0.00	1.86	2.07
time (sec)	N/A	0.199	0.086	0.287	0.352	0.604	0.000	0.270	9.186

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	341	218	158	289	0	284	319
normalized size	1	1.00	2.13	1.36	0.99	1.81	0.00	1.78	1.99
time (sec)	N/A	0.209	0.086	0.292	0.565	0.658	0.000	0.294	9.400

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	363	240	168	320	0	340	387
normalized size	1	1.00	2.06	1.36	0.95	1.82	0.00	1.93	2.20
time (sec)	N/A	0.218	0.101	0.294	0.333	0.912	0.000	0.291	9.980

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	136	238	138	137	656	208	518
normalized size	1	1.00	0.65	1.14	0.66	0.66	3.14	1.00	2.48
time (sec)	N/A	0.396	1.387	0.280	0.359	0.717	63.182	0.429	11.118

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	126	172	116	124	384	191	543
normalized size	1	1.00	0.69	0.94	0.63	0.68	2.10	1.04	2.97
time (sec)	N/A	0.266	0.990	0.276	0.409	0.804	40.491	0.369	12.055

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	106	184	128	111	529	157	469
normalized size	1	1.00	0.64	1.12	0.78	0.67	3.21	0.95	2.84
time (sec)	N/A	0.296	0.632	0.275	0.337	0.746	27.323	0.312	12.125

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	106	116	93	98	282	157	501
normalized size	1	1.00	0.69	0.76	0.61	0.64	1.84	1.03	3.27
time (sec)	N/A	0.154	0.629	0.247	0.979	0.757	16.873	0.259	10.853

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	112	165	123	141	0	245	384
normalized size	1	1.00	0.70	1.02	0.76	0.88	0.00	1.52	2.39
time (sec)	N/A	0.164	0.425	0.479	0.433	0.575	0.000	0.247	10.840

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	110	175	173	161	0	274	401
normalized size	1	1.00	0.70	1.11	1.09	1.02	0.00	1.73	2.54
time (sec)	N/A	0.226	0.312	0.423	0.778	0.803	0.000	0.266	8.973

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	174	199	191	199	0	244	377
normalized size	1	1.00	1.24	1.42	1.36	1.42	0.00	1.74	2.69
time (sec)	N/A	0.209	5.530	0.485	0.813	0.934	0.000	0.264	8.939

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	209	223	190	219	0	274	384
normalized size	1	1.00	1.37	1.46	1.24	1.43	0.00	1.79	2.51
time (sec)	N/A	0.215	3.368	0.431	0.757	0.754	0.000	0.277	8.927

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	227	247	206	245	0	259	373
normalized size	1	1.00	1.48	1.61	1.35	1.60	0.00	1.69	2.44
time (sec)	N/A	0.197	1.295	0.441	0.436	0.810	0.000	0.300	8.940

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	264	293	184	265	0	272	363
normalized size	1	1.00	1.90	2.11	1.32	1.91	0.00	1.96	2.61
time (sec)	N/A	0.249	1.410	0.449	0.647	0.813	0.000	0.294	8.898

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	270	205	220	303	0	259	657
normalized size	1	1.00	1.72	1.31	1.40	1.93	0.00	1.65	4.18
time (sec)	N/A	0.235	1.611	0.366	0.462	0.913	0.000	0.313	11.010

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	262	229	154	323	0	270	351
normalized size	1	1.00	1.62	1.41	0.95	1.99	0.00	1.67	2.17
time (sec)	N/A	0.228	1.086	0.382	0.428	0.836	0.000	0.313	9.839

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	401	192	221	255	0	260	387
normalized size	1	1.00	2.20	1.05	1.21	1.40	0.00	1.43	2.13
time (sec)	N/A	0.313	0.110	0.388	0.538	0.872	0.000	0.358	11.164

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	313	216	155	291	0	324	357
normalized size	1	1.00	2.06	1.42	1.02	1.91	0.00	2.13	2.35
time (sec)	N/A	0.285	1.628	0.416	0.365	0.768	0.000	0.340	9.406

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	353	240	273	327	0	324	357
normalized size	1	1.00	1.55	1.05	1.20	1.43	0.00	1.42	1.57
time (sec)	N/A	0.391	1.291	0.398	0.321	0.764	0.000	0.359	9.435

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	187	264	197	360	0	388	433
normalized size	1	1.00	0.96	1.36	1.02	1.86	0.00	2.00	2.23
time (sec)	N/A	0.302	3.050	0.389	0.328	0.936	0.000	0.368	9.974

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	197	288	323	384	0	420	471
normalized size	1	1.00	0.73	1.07	1.20	1.42	0.00	1.56	1.74
time (sec)	N/A	0.426	4.724	0.408	0.325	1.020	0.000	0.421	10.358

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	146	308	184	150	748	225	612
normalized size	1	1.00	0.65	1.38	0.82	0.67	3.34	1.00	2.73
time (sec)	N/A	0.423	2.304	0.287	0.340	1.069	92.942	0.552	12.192

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	136	272	164	137	699	208	683
normalized size	1	1.00	0.65	1.30	0.78	0.66	3.34	1.00	3.27
time (sec)	N/A	0.413	1.612	0.275	0.351	0.827	63.752	0.513	11.023

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	126	236	164	124	597	191	543
normalized size	1	1.00	0.69	1.29	0.90	0.68	3.26	1.04	2.97
time (sec)	N/A	0.334	1.139	0.286	0.323	0.785	41.840	0.418	12.123

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	116	198	141	111	542	174	572
normalized size	1	1.00	0.64	1.09	0.78	0.61	2.99	0.96	3.16
time (sec)	N/A	0.204	0.917	0.275	0.322	0.661	26.982	0.362	10.963

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	122	187	171	154	0	277	429
normalized size	1	1.00	0.66	1.01	0.92	0.83	0.00	1.50	2.32
time (sec)	N/A	0.236	0.699	0.491	0.451	0.838	0.000	0.313	10.854

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	168	190	186	173	0	290	429
normalized size	1	1.00	0.97	1.10	1.08	1.00	0.00	1.68	2.48
time (sec)	N/A	0.244	1.882	0.419	0.412	0.771	0.000	0.330	9.378

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	664	199	239	212	0	306	438
normalized size	1	1.00	3.67	1.10	1.32	1.17	0.00	1.69	2.42
time (sec)	N/A	0.253	6.374	0.501	0.489	0.822	0.000	0.370	8.916

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	219	223	246	231	0	292	429
normalized size	1	1.00	1.24	1.27	1.40	1.31	0.00	1.66	2.44
time (sec)	N/A	0.211	1.431	0.453	0.424	0.700	0.000	0.381	8.994

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	235	247	268	258	0	313	419
normalized size	1	1.00	1.32	1.39	1.51	1.45	0.00	1.76	2.35
time (sec)	N/A	0.219	0.856	0.428	0.411	0.809	0.000	0.397	8.956

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	271	293	250	278	0	276	408
normalized size	1	1.00	1.55	1.67	1.43	1.59	0.00	1.58	2.33
time (sec)	N/A	0.299	1.708	0.449	0.418	0.770	0.000	0.394	8.924

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	289	316	275	316	0	307	396
normalized size	1	1.00	1.59	1.74	1.51	1.74	0.00	1.69	2.18
time (sec)	N/A	0.248	1.862	0.477	0.410	0.662	0.000	0.427	8.940

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	292	228	233	336	0	291	388
normalized size	1	1.00	1.70	1.33	1.35	1.95	0.00	1.69	2.26
time (sec)	N/A	0.290	1.401	0.385	0.417	0.764	0.000	0.421	9.044

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	279	253	265	362	0	302	389
normalized size	1	1.00	1.17	1.06	1.11	1.52	0.00	1.27	1.63
time (sec)	N/A	0.355	1.212	0.450	0.413	0.751	0.000	0.468	10.302

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	459	216	246	291	0	324	357
normalized size	1	1.00	2.30	1.08	1.23	1.46	0.00	1.62	1.78
time (sec)	N/A	0.359	0.138	0.401	0.427	1.022	0.000	0.425	9.404

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	365	240	286	327	0	356	395
normalized size	1	1.00	1.60	1.05	1.25	1.43	0.00	1.56	1.73
time (sec)	N/A	0.427	2.064	0.403	0.339	0.738	0.000	0.469	9.647

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	187	264	308	360	0	388	433
normalized size	1	1.00	0.76	1.07	1.25	1.46	0.00	1.58	1.76
time (sec)	N/A	0.436	3.578	0.407	0.493	0.688	0.000	0.458	9.976

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	197	288	348	384	0	420	471
normalized size	1	1.00	0.73	1.07	1.29	1.42	0.00	1.56	1.74
time (sec)	N/A	0.464	4.715	0.408	0.398	1.012	0.000	0.544	10.395

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	283	312	368	417	0	452	509
normalized size	1	1.00	0.99	1.09	1.29	1.46	0.00	1.58	1.78
time (sec)	N/A	0.462	6.504	0.408	0.361	0.988	0.000	0.496	10.900

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	229	223	294	245	0	324	474
normalized size	1	1.00	1.29	1.25	1.65	1.38	0.00	1.82	2.66
time (sec)	N/A	0.281	1.660	0.444	0.410	0.888	0.000	0.485	8.979

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	429	517	502	90	0	218	211
normalized size	1	1.00	2.70	3.25	3.16	0.57	0.00	1.37	1.33
time (sec)	N/A	0.216	8.406	0.296	0.484	0.912	0.000	0.205	11.407

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	375	483	461	80	3580	205	199
normalized size	1	1.00	2.66	3.43	3.27	0.57	25.39	1.45	1.41
time (sec)	N/A	0.215	8.728	0.275	0.524	0.632	122.266	0.172	11.619

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	715	415	400	70	2773	179	172
normalized size	1	1.00	6.22	3.61	3.48	0.61	24.11	1.56	1.50
time (sec)	N/A	0.177	11.389	0.270	0.564	0.717	78.793	0.157	12.749

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	377	415	379	60	2307	179	173
normalized size	1	1.00	3.89	4.28	3.91	0.62	23.78	1.85	1.78
time (sec)	N/A	0.126	5.078	0.207	0.516	0.847	49.686	0.148	12.486

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	86	296	280	84	0	143	225
normalized size	1	1.00	0.85	2.93	2.77	0.83	0.00	1.42	2.23
time (sec)	N/A	0.123	0.371	0.406	0.540	0.691	0.000	0.161	10.434

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	122	230	277	104	0	147	229
normalized size	1	1.00	1.28	2.42	2.92	1.09	0.00	1.55	2.41
time (sec)	N/A	0.156	0.789	0.443	0.485	0.627	0.000	0.171	9.026

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	152	234	261	126	0	167	223
normalized size	1	1.00	1.43	2.21	2.46	1.19	0.00	1.58	2.10
time (sec)	N/A	0.161	0.485	0.473	0.576	0.815	0.000	0.183	8.976

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	138	173	240	148	0	157	212
normalized size	1	1.00	1.47	1.84	2.55	1.57	0.00	1.67	2.26
time (sec)	N/A	0.138	0.910	0.495	0.526	0.809	0.000	0.188	8.973

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	232	188	217	171	0	167	317
normalized size	1	1.00	2.27	1.84	2.13	1.68	0.00	1.64	3.11
time (sec)	N/A	0.133	0.672	0.483	0.485	0.915	0.000	0.204	9.419

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	189	208	234	155	0	187	183
normalized size	1	1.00	2.30	2.54	2.85	1.89	0.00	2.28	2.23
time (sec)	N/A	0.111	0.779	0.502	0.665	0.756	0.000	0.202	9.044

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	418	415	416	80	0	179	173
normalized size	1	1.00	3.10	3.07	3.08	0.59	0.00	1.33	1.28
time (sec)	N/A	0.347	3.151	0.337	0.429	0.712	0.000	0.213	12.661

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	362	347	353	70	2271	153	146
normalized size	1	1.00	3.48	3.34	3.39	0.67	21.84	1.47	1.40
time (sec)	N/A	0.268	2.065	0.330	0.479	0.824	139.914	0.180	11.703

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	262	313	310	60	1720	140	81
normalized size	1	1.00	2.62	3.13	3.10	0.60	17.20	1.40	0.81
time (sec)	N/A	0.126	1.133	0.303	0.421	0.533	87.470	0.191	8.999

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	69	160	188	71	0	91	167
normalized size	1	1.00	0.95	2.19	2.58	0.97	0.00	1.25	2.29
time (sec)	N/A	0.201	0.318	0.514	0.407	0.848	0.000	0.173	9.469

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	116	196	202	88	0	131	175
normalized size	1	1.00	1.57	2.65	2.73	1.19	0.00	1.77	2.36
time (sec)	N/A	0.203	0.534	0.533	0.410	0.822	0.000	0.210	9.063

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	134	134	204	118	0	128	186
normalized size	1	1.00	1.84	1.84	2.79	1.62	0.00	1.75	2.55
time (sec)	N/A	0.225	0.578	0.559	0.409	0.763	0.000	0.209	9.104

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	124	149	176	139	0	137	261
normalized size	1	1.00	1.70	2.04	2.41	1.90	0.00	1.88	3.58
time (sec)	N/A	0.315	1.284	0.569	0.446	0.485	0.000	0.241	9.297

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	116	170	194	138	0	158	151
normalized size	1	1.00	1.41	2.07	2.37	1.68	0.00	1.93	1.84
time (sec)	N/A	0.300	1.418	0.628	0.331	0.725	0.000	0.251	9.027

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	189	170	195	167	0	157	149
normalized size	1	1.00	1.89	1.70	1.95	1.67	0.00	1.57	1.49
time (sec)	N/A	0.147	0.560	0.656	0.316	0.749	0.000	0.254	9.024

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	229	246	274	196	0	216	339
normalized size	1	1.00	1.85	1.98	2.21	1.58	0.00	1.74	2.73
time (sec)	N/A	0.332	0.678	0.679	0.322	0.766	0.000	0.285	10.110

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	366	381	373	78	0	166	160
normalized size	1	1.00	2.84	2.95	2.89	0.60	0.00	1.29	1.24
time (sec)	N/A	0.242	2.187	0.470	0.426	0.939	0.000	0.261	11.556

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	310	279	290	68	0	127	95
normalized size	1	1.00	2.95	2.66	2.76	0.65	0.00	1.21	0.90
time (sec)	N/A	0.219	1.767	0.431	1.467	0.691	0.000	0.213	9.034

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	105	255	279	267	58	1246	127	78
normalized size	1	1.25	3.04	3.32	3.18	0.69	14.83	1.51	0.93
time (sec)	N/A	0.166	1.375	0.427	0.444	0.723	166.361	0.194	9.004

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	63	159	161	59	0	89	150
normalized size	1	1.00	1.05	2.65	2.68	0.98	0.00	1.48	2.50
time (sec)	N/A	0.147	0.214	0.619	0.433	0.658	0.000	0.203	9.437

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	106	97	158	82	0	111	151
normalized size	1	1.00	2.16	1.98	3.22	1.67	0.00	2.27	3.08
time (sec)	N/A	0.163	0.495	0.665	0.426	0.691	0.000	0.211	9.479

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	126	112	138	109	0	108	161
normalized size	1	1.00	2.10	1.87	2.30	1.82	0.00	1.80	2.68
time (sec)	N/A	0.175	0.468	0.682	0.408	0.797	0.000	0.252	9.347

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	115	132	153	123	0	128	119
normalized size	1	1.00	1.60	1.83	2.12	1.71	0.00	1.78	1.65
time (sec)	N/A	0.190	1.301	0.678	0.321	0.769	0.000	0.241	9.246

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	125	170	195	149	0	156	151
normalized size	1	1.00	1.34	1.83	2.10	1.60	0.00	1.68	1.62
time (sec)	N/A	0.205	2.104	0.680	0.323	0.716	0.000	0.261	9.290

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	189	208	234	179	0	187	291
normalized size	1	1.00	1.66	1.82	2.05	1.57	0.00	1.64	2.55
time (sec)	N/A	0.177	1.801	0.678	0.318	0.797	0.000	0.277	9.847

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	188	0	0	0	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.285	0.617	27.622	0.000	0.805	0.000	0.000	0.000

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	164	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.288	20.091	0.000	0.853	0.000	0.000	0.000

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.298	11.869	0.000	0.913	0.000	0.000	0.000

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	117	166	94	128	184	163	93
normalized size	1	1.00	0.91	1.29	0.73	0.99	1.43	1.26	0.72
time (sec)	N/A	0.101	1.029	0.238	0.316	0.735	117.154	0.406	0.095

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	137	148	94	117	160	193	93
normalized size	1	1.00	1.21	1.31	0.83	1.04	1.42	1.71	0.82
time (sec)	N/A	0.140	0.732	0.231	0.318	0.599	83.456	0.368	8.818

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	127	130	94	106	160	178	93
normalized size	1	1.00	1.12	1.15	0.83	0.94	1.42	1.58	0.82
time (sec)	N/A	0.137	0.555	0.236	0.310	0.856	56.204	0.323	0.076

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	117	112	94	95	138	163	93
normalized size	1	1.00	1.21	1.15	0.97	0.98	1.42	1.68	0.96
time (sec)	N/A	0.132	0.606	0.239	0.309	0.893	37.468	0.291	0.076

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	94	94	84	138	133	93
normalized size	1	1.00	1.00	0.97	0.97	0.87	1.42	1.37	0.96
time (sec)	N/A	0.128	0.453	0.236	0.314	0.831	24.119	0.258	8.954

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	60	74	94	73	114	118	93
normalized size	1	1.00	0.74	0.91	1.16	0.90	1.41	1.46	1.15
time (sec)	N/A	0.090	0.383	0.235	0.380	0.646	15.732	0.229	9.024

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	106	128	91	96	0	92	160
normalized size	1	1.00	0.90	1.08	0.77	0.81	0.00	0.78	1.36
time (sec)	N/A	0.077	0.134	0.339	0.343	0.815	0.000	0.269	9.164

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	102	150	91	113	0	101	340
normalized size	1	1.00	0.89	1.32	0.80	0.99	0.00	0.89	2.98
time (sec)	N/A	0.087	0.131	0.290	0.364	0.695	0.000	0.206	9.325

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	100	173	90	124	0	104	311
normalized size	1	1.00	0.87	1.50	0.78	1.08	0.00	0.90	2.70
time (sec)	N/A	0.087	0.122	0.352	0.357	0.883	0.000	0.237	9.173

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	103	195	92	139	0	104	300
normalized size	1	1.00	0.87	1.65	0.78	1.18	0.00	0.88	2.54
time (sec)	N/A	0.086	0.214	0.310	0.331	0.679	0.000	0.223	9.071

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	105	217	92	142	0	103	290
normalized size	1	1.00	0.89	1.84	0.78	1.20	0.00	0.87	2.46
time (sec)	N/A	0.089	0.543	0.296	0.351	0.728	0.000	0.245	9.010

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	102	239	91	157	0	103	281
normalized size	1	1.00	0.89	2.08	0.79	1.37	0.00	0.90	2.44
time (sec)	N/A	0.082	0.195	0.312	0.338	0.723	0.000	0.238	9.096

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	111	195	91	158	0	104	267
normalized size	1	1.00	0.97	1.70	0.79	1.37	0.00	0.90	2.32
time (sec)	N/A	0.053	0.387	0.339	0.316	0.918	0.000	0.277	10.118

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	115	217	94	172	0	106	270
normalized size	1	1.00	0.97	1.82	0.79	1.45	0.00	0.89	2.27
time (sec)	N/A	0.085	0.392	0.359	0.317	0.570	0.000	0.262	9.316

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	138	92	131	0	92	91
normalized size	1	1.00	1.00	1.86	1.24	1.77	0.00	1.24	1.23
time (sec)	N/A	0.107	0.032	0.363	0.326	0.640	0.000	0.275	9.392

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	156	92	139	0	92	92
normalized size	1	1.00	1.00	1.93	1.14	1.72	0.00	1.14	1.14
time (sec)	N/A	0.119	0.071	0.379	0.351	0.835	0.000	0.277	9.229

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	86	176	92	144	0	92	92
normalized size	1	1.00	0.89	1.81	0.95	1.48	0.00	0.95	0.95
time (sec)	N/A	0.125	0.185	0.377	0.370	1.067	0.000	0.288	9.214

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	86	194	92	152	0	92	92
normalized size	1	1.00	0.89	2.00	0.95	1.57	0.00	0.95	0.95
time (sec)	N/A	0.124	0.140	0.388	0.333	0.934	0.000	0.280	9.277

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	86	212	92	153	0	92	92
normalized size	1	1.00	0.76	1.88	0.81	1.35	0.00	0.81	0.81
time (sec)	N/A	0.133	0.225	0.375	0.352	1.033	0.000	0.298	9.195

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	86	230	92	161	0	92	92
normalized size	1	1.00	0.76	2.04	0.81	1.42	0.00	0.81	0.81
time (sec)	N/A	0.135	0.221	0.388	0.365	0.670	0.000	0.317	9.355

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	86	248	92	166	0	92	92
normalized size	1	1.00	0.67	1.92	0.71	1.29	0.00	0.71	0.71
time (sec)	N/A	0.099	0.236	0.376	0.347	0.573	0.000	0.340	9.239

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	69	109	0	69	83
normalized size	1	1.00	0.62	0.63	0.63	1.00	0.00	0.63	0.76
time (sec)	N/A	0.128	0.597	0.318	0.376	0.517	0.000	0.209	0.080

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	69	99	0	69	83
normalized size	1	1.00	0.62	0.63	0.63	0.91	0.00	0.63	0.76
time (sec)	N/A	0.127	0.917	0.306	0.418	0.471	0.000	0.226	8.986

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	69	89	0	69	83
normalized size	1	1.00	0.62	0.63	0.63	0.82	0.00	0.63	0.76
time (sec)	N/A	0.127	0.410	0.300	0.370	0.477	0.000	0.203	8.940

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	69	69	79	1906	69	83
normalized size	1	1.00	0.75	0.76	0.76	0.87	20.95	0.76	0.91
time (sec)	N/A	0.160	0.564	0.256	0.328	0.507	169.579	0.200	0.059

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	69	69	69	1719	69	83
normalized size	1	1.00	0.75	0.76	0.76	0.76	18.89	0.76	0.91
time (sec)	N/A	0.162	0.290	0.217	0.327	0.470	114.129	0.183	8.962

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	69	69	59	1530	69	83
normalized size	1	1.00	0.93	0.95	0.95	0.81	20.96	0.95	1.14
time (sec)	N/A	0.115	0.264	0.187	0.326	0.448	76.744	0.173	9.127

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	65	67	49	1096	67	80
normalized size	1	1.00	0.97	0.96	0.99	0.72	16.12	0.99	1.18
time (sec)	N/A	0.063	0.187	0.308	0.326	0.452	49.954	0.148	9.137

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	68	94	71	70	0	88	140
normalized size	1	1.00	0.69	0.95	0.72	0.71	0.00	0.89	1.41
time (sec)	N/A	0.100	0.063	0.368	0.366	0.463	0.000	0.193	9.286

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	66	94	74	85	0	95	272
normalized size	1	1.00	0.69	0.99	0.78	0.89	0.00	1.00	2.86
time (sec)	N/A	0.120	0.125	0.418	0.354	0.478	0.000	0.213	9.364

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	66	94	74	91	0	94	231
normalized size	1	1.00	0.68	0.97	0.76	0.94	0.00	0.97	2.38
time (sec)	N/A	0.118	0.110	0.468	0.363	0.509	0.000	0.214	9.225

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	66	94	73	107	0	87	221
normalized size	1	1.00	0.68	0.97	0.75	1.10	0.00	0.90	2.28
time (sec)	N/A	0.120	0.168	0.472	0.317	0.494	0.000	0.287	9.168

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	66	93	72	104	0	83	214
normalized size	1	1.00	0.70	0.99	0.77	1.11	0.00	0.88	2.28
time (sec)	N/A	0.119	0.295	0.459	0.329	0.482	0.000	0.236	9.343

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	68	97	70	118	0	82	204
normalized size	1	1.00	0.68	0.97	0.70	1.18	0.00	0.82	2.04
time (sec)	N/A	0.104	0.101	0.480	0.361	0.471	0.000	0.223	9.240

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	67	66	96	0	66	63
normalized size	1	1.00	0.90	0.99	0.97	1.41	0.00	0.97	0.93
time (sec)	N/A	0.090	0.134	0.470	0.342	0.465	0.000	0.220	9.040

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	69	66	104	0	66	66
normalized size	1	1.00	0.93	0.95	0.90	1.42	0.00	0.90	0.90
time (sec)	N/A	0.139	0.157	0.522	0.378	0.472	0.000	0.239	8.974

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	69	66	107	0	66	66
normalized size	1	1.00	0.75	0.76	0.73	1.18	0.00	0.73	0.73
time (sec)	N/A	0.160	0.145	0.530	0.327	0.476	0.000	0.246	9.035

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	69	66	115	0	66	65
normalized size	1	1.00	0.75	0.76	0.73	1.26	0.00	0.73	0.71
time (sec)	N/A	0.160	0.183	0.557	0.308	0.454	0.000	0.234	9.041

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	66	120	0	66	66
normalized size	1	1.00	0.62	0.63	0.61	1.10	0.00	0.61	0.61
time (sec)	N/A	0.121	0.120	0.577	0.320	0.458	0.000	0.260	9.075

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	66	128	0	66	65
normalized size	1	1.00	0.62	0.63	0.61	1.17	0.00	0.61	0.60
time (sec)	N/A	0.123	0.128	0.684	0.319	0.467	0.000	0.250	9.041

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	66	131	0	66	65
normalized size	1	1.00	0.62	0.63	0.61	1.20	0.00	0.61	0.60
time (sec)	N/A	0.122	0.126	0.658	0.316	0.499	0.000	0.270	9.140

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	126	0	165	697	0	0	1130
normalized size	1	1.00	0.68	0.00	0.90	3.79	0.00	0.00	6.14
time (sec)	N/A	0.178	0.955	41.963	0.354	0.585	0.000	0.000	17.091

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	126	0	165	628	0	0	1142
normalized size	1	1.00	0.68	0.00	0.90	3.41	0.00	0.00	6.21
time (sec)	N/A	0.176	0.780	28.834	0.322	0.571	0.000	0.000	16.449

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	659	0	148	445	0	0	901
normalized size	1	1.00	3.95	0.00	0.89	2.66	0.00	0.00	5.40
time (sec)	N/A	0.138	3.239	17.354	0.320	0.552	0.000	0.000	15.897

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	95	0	241	243	0	0	568
normalized size	1	1.00	0.69	0.00	1.76	1.77	0.00	0.00	4.15
time (sec)	N/A	0.164	0.325	9.335	0.368	0.495	0.000	0.000	14.291

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	117	0	126	169	0	0	280
normalized size	1	1.00	1.27	0.00	1.37	1.84	0.00	0.00	3.04
time (sec)	N/A	0.143	0.361	22.115	0.367	0.474	0.000	0.000	11.260

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	66	0	126	160	0	0	242
normalized size	1	1.00	0.72	0.00	1.37	1.74	0.00	0.00	2.63
time (sec)	N/A	0.137	0.197	5.837	0.368	0.496	0.000	0.000	10.615

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	104	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.267	9.074	0.000	0.479	0.000	0.000	0.000

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	108	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.216	2.804	0.000	0.484	0.000	0.000	0.000

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	518	755	705	110	0	309	303
normalized size	1	1.00	2.48	3.61	3.37	0.53	0.00	1.48	1.45
time (sec)	N/A	0.282	14.279	0.341	0.444	0.483	0.000	0.232	11.850

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	573	653	624	100	0	270	263
normalized size	1	1.00	3.13	3.57	3.41	0.55	0.00	1.48	1.44
time (sec)	N/A	0.240	12.161	0.322	0.439	0.472	0.000	0.222	11.605

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	533	619	583	90	0	257	251
normalized size	1	1.00	3.23	3.75	3.53	0.55	0.00	1.56	1.52
time (sec)	N/A	0.237	14.164	0.312	0.442	0.471	0.000	0.207	11.524

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	479	551	522	80	4490	231	224
normalized size	1	1.00	3.45	3.96	3.76	0.58	32.30	1.66	1.61
time (sec)	N/A	0.197	9.638	0.306	0.432	0.473	170.693	0.175	11.604

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	481	551	501	70	3888	231	225
normalized size	1	1.00	3.98	4.55	4.14	0.58	32.13	1.91	1.86
time (sec)	N/A	0.145	10.355	0.212	0.427	0.468	111.638	0.161	11.682

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	102	432	402	104	0	195	305
normalized size	1	1.00	0.71	3.02	2.81	0.73	0.00	1.36	2.13
time (sec)	N/A	0.146	0.265	0.436	0.425	0.500	0.000	0.173	11.830

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	146	367	379	124	0	199	296
normalized size	1	1.00	1.07	2.68	2.77	0.91	0.00	1.45	2.16
time (sec)	N/A	0.174	0.699	0.447	0.429	0.482	0.000	0.195	9.042

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	179	371	383	148	0	216	303
normalized size	1	1.00	1.19	2.47	2.55	0.99	0.00	1.44	2.02
time (sec)	N/A	0.189	0.532	0.562	0.425	0.491	0.000	0.213	9.019

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	197	306	362	168	0	228	290
normalized size	1	1.00	1.35	2.10	2.48	1.15	0.00	1.56	1.99
time (sec)	N/A	0.180	0.809	0.644	0.429	0.486	0.000	0.380	9.064

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	252	310	340	191	0	224	286
normalized size	1	1.00	1.68	2.07	2.27	1.27	0.00	1.49	1.91
time (sec)	N/A	0.177	0.749	0.546	0.417	0.459	0.000	0.220	9.034

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	264	249	319	211	0	217	279
normalized size	1	1.00	1.91	1.80	2.31	1.53	0.00	1.57	2.02
time (sec)	N/A	0.151	0.999	0.519	0.433	0.482	0.000	0.221	9.028

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	317	264	298	236	0	224	413
normalized size	1	1.00	2.23	1.86	2.10	1.66	0.00	1.58	2.91
time (sec)	N/A	0.179	1.022	0.553	0.454	0.491	0.000	0.243	10.387

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	284	284	315	198	0	244	387
normalized size	1	1.00	2.68	2.68	2.97	1.87	0.00	2.30	3.65
time (sec)	N/A	0.148	0.967	0.552	0.361	0.478	0.000	0.230	10.615

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	291	322	354	216	0	274	435
normalized size	1	1.00	2.17	2.40	2.64	1.61	0.00	2.04	3.25
time (sec)	N/A	0.218	1.004	0.559	0.326	0.473	0.000	0.263	11.388

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	313	322	355	249	0	273	435
normalized size	1	1.00	2.06	2.12	2.34	1.64	0.00	1.80	2.86
time (sec)	N/A	0.235	1.366	0.601	0.333	0.480	0.000	0.243	12.553

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	386	360	394	272	0	303	483
normalized size	1	1.00	2.19	2.05	2.24	1.55	0.00	1.72	2.74
time (sec)	N/A	0.248	1.546	0.619	0.357	0.493	0.000	0.285	13.995

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	187	436	475	302	0	360	579
normalized size	1	1.00	0.96	2.25	2.45	1.56	0.00	1.86	2.98
time (sec)	N/A	0.253	2.942	0.629	0.331	0.532	0.000	0.269	16.266

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	1453	653	648	110	0	270	264
normalized size	1	1.00	7.16	3.22	3.19	0.54	0.00	1.33	1.30
time (sec)	N/A	0.412	10.592	0.511	0.492	0.510	0.000	0.313	11.509

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	585	619	605	100	0	257	250
normalized size	1	1.00	3.16	3.35	3.27	0.54	0.00	1.39	1.35
time (sec)	N/A	0.458	8.382	0.456	0.440	0.462	0.000	0.258	11.629

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	430	551	542	90	0	231	225
normalized size	1	1.00	2.70	3.47	3.41	0.57	0.00	1.45	1.42
time (sec)	N/A	0.363	6.911	0.443	0.433	0.469	0.000	0.289	11.681

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	481	483	479	80	0	205	198
normalized size	1	1.00	3.41	3.43	3.40	0.57	0.00	1.45	1.40
time (sec)	N/A	0.374	3.885	0.379	0.441	0.488	0.000	0.235	11.703

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	418	449	436	70	3196	192	186
normalized size	1	1.00	3.37	3.62	3.52	0.56	25.77	1.55	1.50
time (sec)	N/A	0.138	4.858	0.309	0.459	0.459	172.040	0.192	12.652

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	93	329	333	94	0	156	262
normalized size	1	1.00	0.78	2.76	2.80	0.79	0.00	1.31	2.20
time (sec)	N/A	0.237	0.711	0.538	0.429	0.497	0.000	0.216	10.758

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	128	333	348	113	0	186	279
normalized size	1	1.00	1.10	2.87	3.00	0.97	0.00	1.60	2.41
time (sec)	N/A	0.302	1.521	0.541	0.436	0.503	0.000	0.217	9.212

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	158	234	330	140	0	168	270
normalized size	1	1.00	1.63	2.41	3.40	1.44	0.00	1.73	2.78
time (sec)	N/A	0.253	1.994	0.623	0.441	0.529	0.000	0.231	9.070

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	184	272	306	161	0	194	253
normalized size	1	1.00	1.90	2.80	3.15	1.66	0.00	2.00	2.61
time (sec)	N/A	0.243	2.451	0.651	0.445	0.480	0.000	0.233	9.156

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	219	173	263	187	0	159	232
normalized size	1	1.00	1.89	1.49	2.27	1.61	0.00	1.37	2.00
time (sec)	N/A	0.281	1.875	0.707	0.432	0.471	0.000	0.247	9.083

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	254	226	258	207	0	195	365
normalized size	1	1.00	2.15	1.92	2.19	1.75	0.00	1.65	3.09
time (sec)	N/A	0.321	1.121	0.744	0.434	0.479	0.000	0.264	9.851

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	145	246	275	183	0	215	339
normalized size	1	1.00	1.10	1.86	2.08	1.39	0.00	1.63	2.57
time (sec)	N/A	0.342	1.893	0.655	0.333	0.462	0.000	0.301	10.344

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	251	284	314	216	0	245	387
normalized size	1	1.00	2.02	2.29	2.53	1.74	0.00	1.98	3.12
time (sec)	N/A	0.260	1.111	0.737	0.333	0.476	0.000	0.279	10.841

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	291	322	355	239	0	273	435
normalized size	1	1.00	1.65	1.83	2.02	1.36	0.00	1.55	2.47
time (sec)	N/A	0.407	0.946	0.695	0.332	0.481	0.000	0.311	11.959

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	313	284	314	269	0	245	387
normalized size	1	1.00	1.86	1.69	1.87	1.60	0.00	1.46	2.30
time (sec)	N/A	0.386	1.871	0.712	0.340	0.491	0.000	0.300	12.558

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	353	398	435	294	0	331	531
normalized size	1	1.00	1.62	1.83	2.00	1.35	0.00	1.52	2.44
time (sec)	N/A	0.487	1.814	0.751	0.429	0.501	0.000	0.340	14.342

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	186	436	474	324	0	361	579
normalized size	1	1.00	0.89	2.08	2.26	1.54	0.00	1.72	2.76
time (sec)	N/A	0.432	4.226	0.765	0.332	0.522	0.000	0.336	16.548

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	482	517	499	90	0	218	212
normalized size	1	1.00	2.99	3.21	3.10	0.56	0.00	1.35	1.32
time (sec)	N/A	0.477	3.994	0.414	0.432	0.462	0.000	0.289	11.944

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	429	415	416	80	0	179	172
normalized size	1	1.00	3.23	3.12	3.13	0.60	0.00	1.35	1.29
time (sec)	N/A	0.401	8.922	0.395	0.466	0.455	0.000	0.241	12.639

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	366	415	393	70	0	179	173
normalized size	1	1.00	2.79	3.17	3.00	0.53	0.00	1.37	1.32
time (sec)	N/A	0.170	1.992	0.387	0.469	0.469	0.000	0.241	12.493

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	80	239	269	84	0	129	222
normalized size	1	1.00	0.81	2.41	2.72	0.85	0.00	1.30	2.24
time (sec)	N/A	0.243	0.409	0.569	0.600	0.483	0.000	0.247	10.995

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	126	230	285	104	0	147	231
normalized size	1	1.00	1.37	2.50	3.10	1.13	0.00	1.60	2.51
time (sec)	N/A	0.222	0.988	0.608	0.432	0.474	0.000	0.253	9.124

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	144	234	267	130	0	172	228
normalized size	1	1.00	1.47	2.39	2.72	1.33	0.00	1.76	2.33
time (sec)	N/A	0.247	0.894	0.675	0.571	0.483	0.000	0.262	9.242

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	132	173	242	150	0	157	219
normalized size	1	1.00	1.43	1.88	2.63	1.63	0.00	1.71	2.38
time (sec)	N/A	0.274	2.650	0.701	0.427	0.480	0.000	0.267	9.257

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	165	188	218	164	0	166	315
normalized size	1	1.00	1.70	1.94	2.25	1.69	0.00	1.71	3.25
time (sec)	N/A	0.318	2.454	0.710	0.503	0.485	0.000	0.312	9.526

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	189	208	235	169	0	186	291
normalized size	1	1.00	1.89	2.08	2.35	1.69	0.00	1.86	2.91
time (sec)	N/A	0.337	1.763	0.712	0.404	0.459	0.000	0.311	9.602

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	242	246	274	196	0	216	339
normalized size	1	1.00	1.95	1.98	2.21	1.58	0.00	1.74	2.73
time (sec)	N/A	0.364	1.013	0.715	0.326	0.450	0.000	0.331	10.182

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	251	284	315	226	0	244	387
normalized size	1	1.00	1.79	2.03	2.25	1.61	0.00	1.74	2.76
time (sec)	N/A	0.231	0.992	0.734	0.327	0.458	0.000	0.355	10.858

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	317	322	354	249	0	274	435
normalized size	1	1.00	1.91	1.94	2.13	1.50	0.00	1.65	2.62
time (sec)	N/A	0.406	5.507	0.758	0.334	0.498	0.000	0.362	11.501

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	104	75	130	0	105	257
normalized size	1	1.00	1.00	1.27	0.91	1.59	0.00	1.28	3.13
time (sec)	N/A	0.131	0.310	0.399	0.441	0.455	0.000	0.179	14.820

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	63	94	62	104	0	90	160
normalized size	1	1.00	0.97	1.45	0.95	1.60	0.00	1.38	2.46
time (sec)	N/A	0.102	0.106	0.389	0.418	0.447	0.000	0.168	11.418

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	47	59	39	80	0	81	99
normalized size	1	1.00	1.21	1.51	1.00	2.05	0.00	2.08	2.54
time (sec)	N/A	0.104	0.035	0.305	0.422	0.444	0.000	0.175	9.213

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	36	32	32	60	0	29	24
normalized size	1	1.00	1.33	1.19	1.19	2.22	0.00	1.07	0.89
time (sec)	N/A	0.047	0.020	0.143	0.414	0.450	0.000	0.160	8.977

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	56	47	48	108	0	34	35
normalized size	1	1.00	1.56	1.31	1.33	3.00	0.00	0.94	0.97
time (sec)	N/A	0.074	0.033	0.414	0.338	0.460	0.000	0.175	8.951

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	68	69	59	165	0	87	77
normalized size	1	1.00	1.42	1.44	1.23	3.44	0.00	1.81	1.60
time (sec)	N/A	0.109	0.078	0.379	0.450	0.460	0.000	0.205	8.967

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	172	93	84	261	0	102	113
normalized size	1	1.00	2.29	1.24	1.12	3.48	0.00	1.36	1.51
time (sec)	N/A	0.130	1.499	0.455	0.318	0.470	0.000	0.213	8.898

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	205	116	98	308	0	130	144
normalized size	1	1.00	2.25	1.27	1.08	3.38	0.00	1.43	1.58
time (sec)	N/A	0.131	4.722	0.442	0.312	0.448	0.000	0.213	8.990

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	161	148	98	152	0	119	288
normalized size	1	1.00	1.81	1.66	1.10	1.71	0.00	1.34	3.24
time (sec)	N/A	0.208	0.515	0.499	0.620	0.432	0.000	0.196	14.710

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	145	117	84	125	0	102	183
normalized size	1	1.00	2.04	1.65	1.18	1.76	0.00	1.44	2.58
time (sec)	N/A	0.086	0.394	0.420	0.525	0.470	0.000	0.201	11.386

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	90	76	57	101	0	89	117
normalized size	1	1.00	2.09	1.77	1.33	2.35	0.00	2.07	2.72
time (sec)	N/A	0.057	0.368	0.322	1.071	0.441	0.000	0.190	9.180

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	69	55	65	126	0	38	39
normalized size	1	1.00	1.57	1.25	1.48	2.86	0.00	0.86	0.89
time (sec)	N/A	0.126	0.114	0.551	0.307	0.459	0.000	0.202	8.889

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	96	92	72	192	0	98	86
normalized size	1	1.00	1.66	1.59	1.24	3.31	0.00	1.69	1.48
time (sec)	N/A	0.223	0.392	0.602	0.483	0.446	0.000	0.200	8.927

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	124	104	124	300	0	116	124
normalized size	1	1.00	1.44	1.21	1.44	3.49	0.00	1.35	1.44
time (sec)	N/A	0.210	1.096	0.599	0.330	0.451	0.000	0.227	8.956

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	125	212	162	178	0	167	363
normalized size	1	1.00	1.13	1.91	1.46	1.60	0.00	1.50	3.27
time (sec)	N/A	0.170	0.789	0.598	0.415	0.444	0.000	0.212	14.701

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	115	167	117	154	0	119	288
normalized size	1	1.00	1.29	1.88	1.31	1.73	0.00	1.34	3.24
time (sec)	N/A	0.124	0.483	0.509	0.422	0.448	0.000	0.215	14.759

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	145	130	97	125	0	102	183
normalized size	1	1.00	2.16	1.94	1.45	1.87	0.00	1.52	2.73
time (sec)	N/A	0.064	0.497	0.423	0.425	0.444	0.000	0.184	11.393

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	74	70	84	151	0	49	112
normalized size	1	1.00	1.54	1.46	1.75	3.15	0.00	1.02	2.33
time (sec)	N/A	0.104	0.138	0.562	0.431	0.459	0.000	0.193	8.925

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	96	93	88	194	0	98	86
normalized size	1	1.00	1.71	1.66	1.57	3.46	0.00	1.75	1.54
time (sec)	N/A	0.135	0.372	0.609	0.329	0.459	0.000	0.212	8.968

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	124	117	135	300	0	116	125
normalized size	1	1.00	1.55	1.46	1.69	3.75	0.00	1.45	1.56
time (sec)	N/A	0.156	1.093	0.699	0.341	0.474	0.000	0.228	8.948

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	211	128	160	354	0	148	160
normalized size	1	1.00	2.15	1.31	1.63	3.61	0.00	1.51	1.63
time (sec)	N/A	0.175	6.141	0.590	0.334	0.474	0.000	0.231	8.972

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	148	126	236	80	0	125	129
normalized size	1	1.00	1.78	1.52	2.84	0.96	0.00	1.51	1.55
time (sec)	N/A	0.142	0.393	0.365	0.427	0.456	0.000	0.193	13.516

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	111	104	154	70	0	77	79
normalized size	1	1.00	1.59	1.49	2.20	1.00	0.00	1.10	1.13
time (sec)	N/A	0.111	0.356	0.355	0.423	0.459	0.000	0.230	11.178

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	106	70	90	47	0	68	47
normalized size	1	1.00	2.12	1.40	1.80	0.94	0.00	1.36	0.94
time (sec)	N/A	0.090	0.137	0.318	0.318	0.466	0.000	0.202	8.898

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	104	70	110	47	0	57	60
normalized size	1	1.00	2.81	1.89	2.97	1.27	0.00	1.54	1.62
time (sec)	N/A	0.093	0.136	0.280	0.320	0.430	0.000	0.187	8.906

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	149	103	136	115	0	83	92
normalized size	1	1.00	1.89	1.30	1.72	1.46	0.00	1.05	1.16
time (sec)	N/A	0.120	0.610	0.423	0.324	0.475	0.000	0.177	9.899

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	245	139	215	162	0	133	150
normalized size	1	1.00	2.63	1.49	2.31	1.74	0.00	1.43	1.61
time (sec)	N/A	0.194	0.628	0.433	0.335	0.469	0.000	0.184	9.114

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	191	267	421	132	0	160	172
normalized size	1	1.00	1.28	1.79	2.83	0.89	0.00	1.07	1.15
time (sec)	N/A	0.306	0.586	0.474	0.431	0.456	0.000	0.233	17.673

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	148	169	335	122	0	151	156
normalized size	1	1.00	1.23	1.41	2.79	1.02	0.00	1.26	1.30
time (sec)	N/A	0.278	0.574	0.443	0.433	0.469	0.000	0.231	15.384

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	143	146	249	112	0	103	105
normalized size	1	1.00	1.35	1.38	2.35	1.06	0.00	0.97	0.99
time (sec)	N/A	0.282	0.558	0.432	0.428	0.444	0.000	0.219	14.089

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	84	100	164	76	0	94	111
normalized size	1	1.00	1.27	1.52	2.48	1.15	0.00	1.42	1.68
time (sec)	N/A	0.259	0.279	0.426	0.333	0.478	0.000	0.217	9.275

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	86	100	184	77	0	94	132
normalized size	1	1.00	1.18	1.37	2.52	1.05	0.00	1.29	1.81
time (sec)	N/A	0.190	0.258	0.409	0.334	0.433	0.000	0.212	9.307

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	82	100	204	80	0	94	159
normalized size	1	1.00	1.15	1.41	2.87	1.13	0.00	1.32	2.24
time (sec)	N/A	0.128	0.248	0.356	0.330	0.477	0.000	0.195	9.386

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	196	145	250	168	0	109	117
normalized size	1	1.00	1.70	1.26	2.17	1.46	0.00	0.95	1.02
time (sec)	N/A	0.266	0.519	0.556	0.334	0.452	0.000	0.187	10.782

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	289	182	309	218	0	161	216
normalized size	1	1.00	2.22	1.40	2.38	1.68	0.00	1.24	1.66
time (sec)	N/A	0.310	0.833	0.561	0.336	0.463	0.000	0.225	10.779

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	328	219	354	260	0	187	191
normalized size	1	1.00	2.08	1.39	2.24	1.65	0.00	1.18	1.21
time (sec)	N/A	0.346	0.730	0.615	0.340	0.470	0.000	0.232	10.086

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	224	211	421	159	0	177	182
normalized size	1	1.00	1.48	1.40	2.79	1.05	0.00	1.17	1.21
time (sec)	N/A	0.341	0.683	0.526	0.439	0.460	0.000	0.303	15.500

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	214	187	335	150	0	129	131
normalized size	1	1.00	1.51	1.32	2.36	1.06	0.00	0.91	0.92
time (sec)	N/A	0.340	0.800	0.500	0.429	0.454	0.000	0.299	15.649

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	104	130	230	104	0	120	135
normalized size	1	1.00	1.02	1.27	2.25	1.02	0.00	1.18	1.32
time (sec)	N/A	0.329	0.495	0.497	0.338	0.421	0.000	0.264	9.692

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	104	130	250	102	0	120	158
normalized size	1	1.00	1.18	1.48	2.84	1.16	0.00	1.36	1.80
time (sec)	N/A	0.315	0.382	0.488	0.335	0.429	0.000	0.254	9.735

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	104	130	270	104	0	120	183
normalized size	1	1.00	1.01	1.26	2.62	1.01	0.00	1.17	1.78
time (sec)	N/A	0.243	0.347	0.466	0.338	0.434	0.000	0.271	10.152

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	104	130	290	106	0	120	206
normalized size	1	1.00	1.05	1.31	2.93	1.07	0.00	1.21	2.08
time (sec)	N/A	0.142	0.379	0.453	0.341	0.451	0.000	0.269	10.012

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	341	187	336	218	0	135	143
normalized size	1	1.00	2.26	1.24	2.23	1.44	0.00	0.89	0.95
time (sec)	N/A	0.293	0.429	0.651	0.345	0.464	0.000	0.274	11.331

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	351	224	395	265	0	187	274
normalized size	1	1.00	2.17	1.38	2.44	1.64	0.00	1.15	1.69
time (sec)	N/A	0.347	0.885	0.647	0.341	0.489	0.000	0.221	10.842

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	84	164	96	108	0	182	306
normalized size	1	1.00	0.72	1.40	0.82	0.92	0.00	1.56	2.62
time (sec)	N/A	0.135	0.409	0.464	0.416	0.474	0.000	0.201	14.962

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	76	154	87	98	0	134	243
normalized size	1	1.00	0.75	1.52	0.86	0.97	0.00	1.33	2.41
time (sec)	N/A	0.130	0.254	0.456	0.422	0.465	0.000	0.201	14.619

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	81	98	65	88	0	124	185
normalized size	1	1.00	1.12	1.36	0.90	1.22	0.00	1.72	2.57
time (sec)	N/A	0.096	0.043	0.361	0.417	0.480	0.000	0.194	12.718

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	69	88	55	75	0	74	117
normalized size	1	1.00	1.15	1.47	0.92	1.25	0.00	1.23	1.95
time (sec)	N/A	0.098	0.042	0.369	0.420	0.456	0.000	0.182	10.138

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	82	39	49	0	67	74
normalized size	1	1.00	1.00	1.82	0.87	1.09	0.00	1.49	1.64
time (sec)	N/A	0.104	0.037	0.363	0.315	0.449	0.000	0.178	9.030

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	36	26	50	0	53	50
normalized size	1	1.00	1.00	1.09	0.79	1.52	0.00	1.61	1.52
time (sec)	N/A	0.084	0.022	0.204	0.320	0.443	0.000	0.169	9.028

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	85	82	73	126	0	81	90
normalized size	1	1.00	1.25	1.21	1.07	1.85	0.00	1.19	1.32
time (sec)	N/A	0.087	0.121	0.429	0.319	0.477	0.000	0.205	9.941

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	109	106	83	170	0	129	145
normalized size	1	1.00	1.35	1.31	1.02	2.10	0.00	1.59	1.79
time (sec)	N/A	0.124	0.057	0.462	0.321	0.484	0.000	0.211	9.099

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	359	138	106	222	0	148	180
normalized size	1	1.00	3.26	1.25	0.96	2.02	0.00	1.35	1.64
time (sec)	N/A	0.136	6.103	0.510	0.321	0.484	0.000	0.225	8.984

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	120	159	186	120	196	0	135	287
normalized size	1	1.19	1.57	1.84	1.19	1.94	0.00	1.34	2.84
time (sec)	N/A	0.205	1.264	0.487	0.425	0.496	0.000	0.210	14.686

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	131	162	95	168	0	86	182
normalized size	1	1.00	1.52	1.88	1.10	1.95	0.00	1.00	2.12
time (sec)	N/A	0.246	1.021	0.475	0.424	0.457	0.000	0.215	12.162

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	79	114	71	141	0	67	102
normalized size	1	1.00	1.25	1.81	1.13	2.24	0.00	1.06	1.62
time (sec)	N/A	0.207	0.052	0.402	0.416	0.455	0.000	0.205	9.290

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	72	99	56	98	0	38	34
normalized size	1	1.00	1.20	1.65	0.93	1.63	0.00	0.63	0.57
time (sec)	N/A	0.086	0.287	0.372	0.321	0.436	0.000	0.197	9.143

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	142	92	90	231	0	73	98
normalized size	1	1.00	1.95	1.26	1.23	3.16	0.00	1.00	1.34
time (sec)	N/A	0.187	0.509	0.605	0.323	0.463	0.000	0.199	9.497

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	135	156	107	329	0	118	144
normalized size	1	1.00	1.55	1.79	1.23	3.78	0.00	1.36	1.66
time (sec)	N/A	0.274	0.924	0.727	0.333	0.480	0.000	0.224	9.454

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	190	168	160	428	0	150	182
normalized size	1	1.00	1.52	1.34	1.28	3.42	0.00	1.20	1.46
time (sec)	N/A	0.308	1.988	0.710	0.331	0.505	0.000	0.247	9.072

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	177	266	165	220	0	187	317
normalized size	1	1.00	1.49	2.24	1.39	1.85	0.00	1.57	2.66
time (sec)	N/A	0.194	2.133	0.606	0.415	0.485	0.000	0.259	14.924

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	159	246	145	196	0	135	287
normalized size	1	1.00	1.57	2.44	1.44	1.94	0.00	1.34	2.84
time (sec)	N/A	0.162	1.480	0.585	0.426	0.458	0.000	0.262	14.962

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	133	184	107	169	0	87	182
normalized size	1	1.00	1.73	2.39	1.39	2.19	0.00	1.13	2.36
time (sec)	N/A	0.219	1.243	0.503	0.424	0.472	0.000	0.232	12.299

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	107	126	84	143	0	67	102
normalized size	1	1.00	1.67	1.97	1.31	2.23	0.00	1.05	1.59
time (sec)	N/A	0.139	0.724	0.399	0.421	0.451	0.000	0.201	9.261

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	144	115	103	231	0	73	98
normalized size	1	1.00	2.00	1.60	1.43	3.21	0.00	1.01	1.36
time (sec)	N/A	0.141	0.628	0.639	0.328	0.467	0.000	0.249	9.226

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	135	155	123	334	0	118	144
normalized size	1	1.00	1.57	1.80	1.43	3.88	0.00	1.37	1.67
time (sec)	N/A	0.170	0.957	0.717	0.324	0.489	0.000	0.250	9.206

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	190	202	182	428	0	150	183
normalized size	1	1.00	1.73	1.84	1.65	3.89	0.00	1.36	1.66
time (sec)	N/A	0.190	2.016	0.835	0.331	0.482	0.000	0.273	9.143

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	287	214	205	528	0	194	239
normalized size	1	1.00	2.24	1.67	1.60	4.12	0.00	1.52	1.87
time (sec)	N/A	0.206	6.178	0.695	0.336	0.502	0.000	0.307	11.409

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	252	360	238	247	0	200	437
normalized size	1	1.00	1.76	2.52	1.66	1.73	0.00	1.40	3.06
time (sec)	N/A	0.204	1.569	0.680	0.432	0.479	0.000	0.272	16.825

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	158	268	158	197	0	135	287
normalized size	1	1.00	1.56	2.65	1.56	1.95	0.00	1.34	2.84
time (sec)	N/A	0.163	1.796	0.573	0.422	0.457	0.000	0.240	14.851

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	224	210	400	108	0	149	172
normalized size	1	1.00	1.91	1.79	3.42	0.92	0.00	1.27	1.47
time (sec)	N/A	0.158	0.685	0.436	0.422	0.476	0.000	0.250	18.669

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	191	166	318	98	0	130	131
normalized size	1	1.00	1.82	1.58	3.03	0.93	0.00	1.24	1.25
time (sec)	N/A	0.129	0.606	0.429	0.427	0.452	0.000	0.232	16.139

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	106	130	214	75	0	120	73
normalized size	1	1.00	1.54	1.88	3.10	1.09	0.00	1.74	1.06
time (sec)	N/A	0.096	0.268	0.412	0.326	0.431	0.000	0.236	9.550

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	106	115	234	75	0	120	86
normalized size	1	1.00	1.93	2.09	4.25	1.36	0.00	2.18	1.56
time (sec)	N/A	0.138	0.293	0.408	0.337	0.443	0.000	0.213	9.858

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	106	130	254	73	0	109	99
normalized size	1	1.00	1.45	1.78	3.48	1.00	0.00	1.49	1.36
time (sec)	N/A	0.160	0.337	0.384	0.345	0.445	0.000	0.233	10.159

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	106	130	274	75	0	120	112
normalized size	1	1.00	1.93	2.36	4.98	1.36	0.00	2.18	2.04
time (sec)	N/A	0.108	0.264	0.345	0.333	0.421	0.000	0.244	10.852

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	267	187	320	149	0	136	143
normalized size	1	1.00	2.32	1.63	2.78	1.30	0.00	1.18	1.24
time (sec)	N/A	0.124	0.658	0.497	0.338	0.461	0.000	0.207	11.529

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	341	223	379	194	0	178	257
normalized size	1	1.00	2.71	1.77	3.01	1.54	0.00	1.41	2.04
time (sec)	N/A	0.166	0.603	0.500	0.335	0.469	0.000	0.221	10.822

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	267	253	507	150	0	175	198
normalized size	1	1.00	1.72	1.63	3.27	0.97	0.00	1.13	1.28
time (sec)	N/A	0.290	0.788	0.585	0.435	0.485	0.000	0.304	17.730

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	257	230	421	140	0	155	156
normalized size	1	1.00	1.84	1.64	3.01	1.00	0.00	1.11	1.11
time (sec)	N/A	0.300	0.571	0.560	0.428	0.476	0.000	0.289	16.890

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	126	145	296	104	0	146	160
normalized size	1	1.00	1.48	1.71	3.48	1.22	0.00	1.72	1.88
time (sec)	N/A	0.272	0.263	0.560	0.339	0.478	0.000	0.295	12.368

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	126	160	316	103	0	146	184
normalized size	1	1.00	1.38	1.76	3.47	1.13	0.00	1.60	2.02
time (sec)	N/A	0.157	0.259	0.566	0.340	0.466	0.000	0.273	12.384

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	126	130	336	104	0	120	207
normalized size	1	1.00	1.38	1.43	3.69	1.14	0.00	1.32	2.27
time (sec)	N/A	0.288	0.328	0.513	0.339	0.454	0.000	0.276	14.065

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	126	160	356	103	0	146	231
normalized size	1	1.00	1.38	1.76	3.91	1.13	0.00	1.60	2.54
time (sec)	N/A	0.306	0.374	0.495	0.340	0.438	0.000	0.263	14.343

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	134	160	376	104	0	146	254
normalized size	1	1.00	1.44	1.72	4.04	1.12	0.00	1.57	2.73
time (sec)	N/A	0.115	0.302	0.464	0.335	0.453	0.000	0.263	14.350

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	352	229	422	200	0	161	169
normalized size	1	1.00	2.36	1.54	2.83	1.34	0.00	1.08	1.13
time (sec)	N/A	0.253	0.608	0.765	0.338	0.461	0.000	0.216	12.343

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	442	266	481	250	0	204	331
normalized size	1	1.00	2.70	1.62	2.93	1.52	0.00	1.24	2.02
time (sec)	N/A	0.330	6.095	0.645	0.338	0.481	0.000	0.246	11.238

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	277	303	526	292	0	238	243
normalized size	1	1.00	1.43	1.56	2.71	1.51	0.00	1.23	1.25
time (sec)	N/A	0.366	0.574	0.697	0.343	0.481	0.000	0.266	10.766

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	273	272	487	177	0	181	169
normalized size	1	1.00	1.53	1.53	2.74	0.99	0.00	1.02	0.95
time (sec)	N/A	0.368	0.514	0.569	0.438	0.483	0.000	0.406	17.808

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	185	190	362	130	0	172	184
normalized size	1	1.00	1.53	1.57	2.99	1.07	0.00	1.42	1.52
time (sec)	N/A	0.347	0.378	0.571	0.343	0.468	0.000	0.388	13.173

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	185	190	382	128	0	172	208
normalized size	1	1.00	1.76	1.81	3.64	1.22	0.00	1.64	1.98
time (sec)	N/A	0.338	0.205	0.559	0.343	0.466	0.000	0.387	13.192

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	185	175	402	130	0	159	232
normalized size	1	1.00	1.46	1.38	3.17	1.02	0.00	1.25	1.83
time (sec)	N/A	0.223	0.293	0.547	0.335	0.455	0.000	0.352	14.330

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	185	190	422	130	0	161	255
normalized size	1	1.00	1.76	1.81	4.02	1.24	0.00	1.53	2.43
time (sec)	N/A	0.335	0.318	0.559	0.342	0.463	0.000	0.349	16.124

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	185	190	442	130	0	172	279
normalized size	1	1.00	1.46	1.50	3.48	1.02	0.00	1.35	2.20
time (sec)	N/A	0.362	0.326	0.543	0.343	0.450	0.000	0.335	14.871

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	185	190	442	130	0	172	279
normalized size	1	1.00	1.50	1.54	3.59	1.06	0.00	1.40	2.27
time (sec)	N/A	0.161	0.282	0.595	0.340	0.433	0.000	0.328	15.001

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	204	271	508	250	0	187	195
normalized size	1	1.00	1.09	1.45	2.72	1.34	0.00	1.00	1.04
time (sec)	N/A	0.360	1.377	0.789	0.343	0.479	0.000	0.237	12.556

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	230	308	567	297	0	230	390
normalized size	1	1.00	1.15	1.54	2.84	1.48	0.00	1.15	1.95
time (sec)	N/A	0.393	0.639	0.804	0.347	0.489	0.000	0.282	11.316

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	166	190	488	153	0	172	279
normalized size	1	1.00	1.14	1.31	3.37	1.06	0.00	1.19	1.92
time (sec)	N/A	0.313	0.453	0.721	0.351	0.437	0.000	0.461	16.845

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	166	220	508	154	0	198	303
normalized size	1	1.00	1.14	1.52	3.50	1.06	0.00	1.37	2.09
time (sec)	N/A	0.409	0.456	0.729	0.345	0.439	0.000	0.439	15.893

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	184	166	218	528	153	0	198	327
normalized size	1	1.29	1.16	1.52	3.69	1.07	0.00	1.38	2.29
time (sec)	N/A	0.356	0.491	0.726	0.347	0.451	0.000	0.400	15.996

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	133	205	106	172	0	113	286
normalized size	1	1.00	1.00	1.54	0.80	1.29	0.00	0.85	2.15
time (sec)	N/A	0.111	0.451	0.280	0.318	0.491	0.000	0.256	9.817

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	147	95	159	0	101	235
normalized size	1	1.00	1.07	1.28	0.83	1.38	0.00	0.88	2.04
time (sec)	N/A	0.068	0.271	0.270	0.312	0.466	0.000	0.254	9.289

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	106	133	86	136	0	93	205
normalized size	1	1.00	1.01	1.27	0.82	1.30	0.00	0.89	1.95
time (sec)	N/A	0.094	0.239	0.247	0.313	0.478	0.000	0.226	9.259

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	114	86	136	0	90	167
normalized size	1	1.00	1.00	1.36	1.02	1.62	0.00	1.07	1.99
time (sec)	N/A	0.102	0.208	0.240	0.311	0.458	0.000	0.223	14.457

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	74	100	84	135	0	91	167
normalized size	1	1.00	0.88	1.19	1.00	1.61	0.00	1.08	1.99
time (sec)	N/A	0.098	0.030	0.235	0.310	0.475	0.000	0.223	14.456

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	74	92	84	135	0	91	167
normalized size	1	1.00	1.21	1.51	1.38	2.21	0.00	1.49	2.74
time (sec)	N/A	0.067	0.029	0.179	0.312	0.466	0.000	0.205	14.586

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	99	100	95	175	0	104	99
normalized size	1	1.00	0.85	0.85	0.81	1.50	0.00	0.89	0.85
time (sec)	N/A	0.107	0.216	0.396	0.317	0.491	0.000	0.240	0.095

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	76	120	114	229	0	121	118
normalized size	1	1.00	0.59	0.93	0.88	1.78	0.00	0.94	0.91
time (sec)	N/A	0.121	0.194	0.349	0.347	0.468	0.000	0.285	0.096

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	86	151	127	294	0	125	134
normalized size	1	1.00	0.60	1.06	0.89	2.06	0.00	0.87	0.94
time (sec)	N/A	0.132	0.739	0.370	0.313	0.482	0.000	0.283	9.312

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	90	173	138	343	0	149	145
normalized size	1	1.00	0.56	1.07	0.85	2.12	0.00	0.92	0.90
time (sec)	N/A	0.141	1.163	0.398	0.366	0.472	0.000	0.288	0.114

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	75	261	96	168	0	102	283
normalized size	1	1.00	0.63	2.19	0.81	1.41	0.00	0.86	2.38
time (sec)	N/A	0.088	0.241	0.296	0.421	0.465	0.000	0.294	9.227

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	67	213	83	154	0	88	225
normalized size	1	1.00	0.66	2.11	0.82	1.52	0.00	0.87	2.23
time (sec)	N/A	0.131	0.122	0.289	0.314	0.464	0.000	0.292	9.270

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	91	173	72	125	0	78	166
normalized size	1	1.00	1.05	1.99	0.83	1.44	0.00	0.90	1.91
time (sec)	N/A	0.139	0.392	0.288	0.316	0.485	0.000	0.255	9.296

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	39	174	72	125	0	77	123
normalized size	1	1.00	0.61	2.72	1.12	1.95	0.00	1.20	1.92
time (sec)	N/A	0.118	0.108	0.271	0.310	0.464	0.000	0.231	11.552

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	126	64	120	0	95	106
normalized size	1	1.00	0.56	1.97	1.00	1.88	0.00	1.48	1.66
time (sec)	N/A	0.087	0.077	0.263	0.312	0.479	0.000	0.223	11.126

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	66	112	84	166	0	91	91
normalized size	1	1.00	0.65	1.11	0.83	1.64	0.00	0.90	0.90
time (sec)	N/A	0.115	0.291	0.662	0.480	0.480	0.000	0.276	0.085

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	74	176	104	240	0	115	110
normalized size	1	1.00	0.64	1.52	0.90	2.07	0.00	0.99	0.95
time (sec)	N/A	0.142	0.267	0.616	0.315	0.513	0.000	0.300	9.050

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	84	199	119	302	0	125	126
normalized size	1	1.00	0.63	1.49	0.89	2.25	0.00	0.93	0.94
time (sec)	N/A	0.148	1.131	0.566	0.310	0.472	0.000	0.298	0.098

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	133	215	133	370	0	142	140
normalized size	1	1.00	0.89	1.43	0.89	2.47	0.00	0.95	0.93
time (sec)	N/A	0.164	6.057	0.559	0.447	0.499	0.000	0.346	9.055

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	73	325	96	141	0	242	321
normalized size	1	1.00	0.64	2.85	0.84	1.24	0.00	2.12	2.82
time (sec)	N/A	0.084	0.307	0.309	0.430	0.498	0.000	0.286	11.559

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	61	309	82	128	0	209	263
normalized size	1	1.00	0.64	3.22	0.85	1.33	0.00	2.18	2.74
time (sec)	N/A	0.111	0.240	0.298	0.311	0.457	0.000	0.303	10.929

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	53	237	70	110	0	178	205
normalized size	1	1.00	0.68	3.04	0.90	1.41	0.00	2.28	2.63
time (sec)	N/A	0.119	0.228	0.316	0.354	0.475	0.000	0.274	10.129

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	45	220	59	86	0	125	313
normalized size	1	1.00	0.70	3.44	0.92	1.34	0.00	1.95	4.89
time (sec)	N/A	0.110	0.144	0.288	0.306	0.472	0.000	0.268	9.707

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	154	42	44	0	32	30
normalized size	1	1.00	0.97	4.97	1.35	1.42	0.00	1.03	0.97
time (sec)	N/A	0.059	0.030	0.273	0.330	0.436	0.000	0.233	9.294

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	54	172	70	126	0	123	61
normalized size	1	1.00	0.70	2.23	0.91	1.64	0.00	1.60	0.79
time (sec)	N/A	0.105	0.231	0.609	0.485	0.489	0.000	0.269	9.117

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	63	176	90	185	0	166	80
normalized size	1	1.00	0.68	1.89	0.97	1.99	0.00	1.78	0.86
time (sec)	N/A	0.158	0.232	0.623	0.318	0.458	0.000	0.315	0.085

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	73	241	103	235	0	198	97
normalized size	1	1.00	0.66	2.17	0.93	2.12	0.00	1.78	0.87
time (sec)	N/A	0.148	0.787	0.689	0.318	0.471	0.000	0.328	0.093

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	153	208	209	207	0	179	567
normalized size	1	1.00	0.65	0.88	0.89	0.88	0.00	0.76	2.40
time (sec)	N/A	0.252	6.126	0.477	0.327	0.544	0.000	0.360	10.637

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	143	192	197	197	0	161	512
normalized size	1	1.00	0.65	0.87	0.90	0.90	0.00	0.73	2.33
time (sec)	N/A	0.229	6.147	0.473	0.395	0.548	0.000	0.361	10.337

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	133	175	186	187	0	147	485
normalized size	1	1.00	0.67	0.88	0.93	0.94	0.00	0.74	2.44
time (sec)	N/A	0.212	6.136	0.460	0.334	0.515	0.000	0.413	10.144

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	117	162	175	167	0	136	432
normalized size	1	1.00	0.62	0.86	0.93	0.89	0.00	0.72	2.30
time (sec)	N/A	0.184	3.784	0.453	0.323	0.526	0.000	0.364	9.360

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	101	162	175	167	0	136	388
normalized size	1	1.00	0.78	1.25	1.35	1.28	0.00	1.05	2.98
time (sec)	N/A	0.172	0.923	0.401	0.322	0.502	0.000	0.337	17.052

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	101	162	175	167	0	136	388
normalized size	1	1.00	0.75	1.21	1.31	1.25	0.00	1.01	2.90
time (sec)	N/A	0.236	0.902	0.399	0.321	0.475	0.000	0.309	17.117

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	92	144	173	167	0	136	388
normalized size	1	1.00	0.61	0.95	1.14	1.10	0.00	0.89	2.55
time (sec)	N/A	0.245	1.116	0.395	0.318	0.482	0.000	0.330	17.207

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	101	162	175	167	0	136	388
normalized size	1	1.00	0.67	1.08	1.17	1.11	0.00	0.91	2.59
time (sec)	N/A	0.226	0.672	0.388	0.362	0.501	0.000	0.313	17.148

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	92	144	175	167	0	136	388
normalized size	1	1.00	0.61	0.96	1.17	1.11	0.00	0.91	2.59
time (sec)	N/A	0.225	0.853	0.392	0.320	0.484	0.000	0.310	17.024

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	92	144	175	167	0	136	388
normalized size	1	1.00	0.62	0.97	1.18	1.13	0.00	0.92	2.62
time (sec)	N/A	0.199	0.524	0.359	0.314	0.478	0.000	0.276	17.116

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	92	144	175	167	0	136	388
normalized size	1	1.00	0.71	1.11	1.35	1.28	0.00	1.05	2.98
time (sec)	N/A	0.148	0.953	0.327	0.313	0.490	0.000	0.290	17.038

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	145	162	175	167	0	136	158
normalized size	1	1.00	0.88	0.98	1.06	1.01	0.00	0.82	0.96
time (sec)	N/A	0.126	0.514	0.416	0.313	0.484	0.000	0.258	0.235

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	189	176	187	202	0	149	191
normalized size	1	1.00	0.94	0.87	0.93	1.00	0.00	0.74	0.95
time (sec)	N/A	0.201	6.135	0.450	0.330	0.524	0.000	0.275	0.163

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	201	193	205	258	0	170	212
normalized size	1	1.00	0.93	0.89	0.94	1.19	0.00	0.78	0.98
time (sec)	N/A	0.239	6.141	0.476	0.324	0.503	0.000	0.264	9.329

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	213	208	217	311	0	182	223
normalized size	1	1.00	0.92	0.90	0.94	1.34	0.00	0.78	0.96
time (sec)	N/A	0.246	6.188	0.532	0.336	0.529	0.000	0.299	9.244

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	231	225	227	360	0	187	233
normalized size	1	1.00	0.91	0.89	0.90	1.42	0.00	0.74	0.92
time (sec)	N/A	0.264	6.136	0.510	0.328	0.532	0.000	0.298	9.264

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	139	248	91	115	0	138	215
normalized size	1	1.00	1.53	2.73	1.00	1.26	0.00	1.52	2.36
time (sec)	N/A	0.214	1.017	0.484	0.319	0.436	0.000	0.242	13.518

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	169	227	236	217	0	181	648
normalized size	1	1.00	0.64	0.86	0.89	0.82	0.00	0.69	2.45
time (sec)	N/A	0.283	6.168	0.505	0.337	0.559	0.000	0.408	11.204

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	159	212	225	207	0	167	595
normalized size	1	1.00	0.64	0.86	0.91	0.84	0.00	0.68	2.41
time (sec)	N/A	0.261	6.188	0.480	0.381	0.552	0.000	0.399	10.845

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	137	198	214	187	0	156	539
normalized size	1	1.00	0.59	0.85	0.92	0.80	0.00	0.67	2.31
time (sec)	N/A	0.235	4.968	0.480	0.535	0.544	0.000	0.365	9.400

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	122	198	214	187	0	156	497
normalized size	1	1.00	0.79	1.29	1.39	1.21	0.00	1.01	3.23
time (sec)	N/A	0.193	2.419	0.464	0.525	0.528	0.000	0.375	17.054

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	121	198	214	187	0	156	496
normalized size	1	1.00	0.76	1.24	1.34	1.17	0.00	0.98	3.10
time (sec)	N/A	0.271	2.422	0.436	0.342	0.516	0.000	0.355	16.770

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	124	180	214	187	0	156	496
normalized size	1	1.00	0.70	1.01	1.20	1.05	0.00	0.88	2.79
time (sec)	N/A	0.280	1.677	0.401	0.407	0.519	0.000	0.352	16.704

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	122	198	214	187	0	156	496
normalized size	1	1.00	0.69	1.12	1.22	1.06	0.00	0.89	2.82
time (sec)	N/A	0.266	2.757	0.408	0.327	0.514	0.000	0.360	16.645

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	116	180	214	187	0	156	496
normalized size	1	1.00	0.60	0.93	1.10	0.96	0.00	0.80	2.56
time (sec)	N/A	0.273	5.485	0.395	0.483	0.513	0.000	0.325	16.618

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	116	180	214	187	0	156	496
normalized size	1	1.00	0.60	0.94	1.11	0.97	0.00	0.81	2.58
time (sec)	N/A	0.253	5.397	0.395	0.428	0.503	0.000	0.321	16.705

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	104	162	214	187	0	156	496
normalized size	1	1.00	0.60	0.93	1.23	1.07	0.00	0.90	2.85
time (sec)	N/A	0.242	2.809	0.382	0.336	0.512	0.000	0.306	17.339

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	122	198	214	187	0	156	496
normalized size	1	1.00	0.71	1.15	1.24	1.09	0.00	0.91	2.88
time (sec)	N/A	0.219	2.615	0.352	0.489	0.535	0.000	0.289	16.881

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	116	180	214	187	0	156	496
normalized size	1	1.00	0.75	1.17	1.39	1.21	0.00	1.01	3.22
time (sec)	N/A	0.165	5.526	0.319	0.338	0.511	0.000	0.305	16.754

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	165	198	214	187	0	156	199
normalized size	1	1.00	0.79	0.94	1.02	0.89	0.00	0.74	0.95
time (sec)	N/A	0.167	1.401	0.406	0.405	0.519	0.000	0.239	9.409

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	228	212	226	222	0	169	231
normalized size	1	1.00	0.92	0.86	0.91	0.90	0.00	0.68	0.94
time (sec)	N/A	0.244	6.205	0.457	0.494	0.571	0.000	0.241	0.232

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	240	229	245	278	0	190	252
normalized size	1	1.00	0.92	0.87	0.94	1.06	0.00	0.73	0.96
time (sec)	N/A	0.291	6.206	0.466	0.351	0.558	0.000	0.272	9.456

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	254	244	257	331	0	202	263
normalized size	1	1.00	0.91	0.87	0.92	1.19	0.00	0.72	0.94
time (sec)	N/A	0.302	6.233	0.516	0.326	0.544	0.000	0.286	9.637

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	347	0	0	0	0	0	-1
normalized size	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.349	3.321	19.459	0.000	0.479	0.000	0.000	0.000

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.142	0.615	0.000	0.513	0.000	0.000	0.000

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	130	0	486	902	11900	1840	863
normalized size	1	1.00	0.74	0.00	2.78	5.15	68.00	10.51	4.93
time (sec)	N/A	0.206	0.604	4.380	1.296	0.591	138.036	0.241	18.138

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	105	0	313	545	5596	1001	617
normalized size	1	1.00	0.76	0.00	2.25	3.92	40.26	7.20	4.44
time (sec)	N/A	0.165	0.339	4.436	0.661	0.543	54.770	0.201	13.584

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	78	0	183	294	2159	463	302
normalized size	1	1.00	0.77	0.00	1.81	2.91	21.38	4.58	2.99
time (sec)	N/A	0.150	0.365	3.248	0.711	0.506	20.587	0.191	11.259

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	52	0	87	116	586	156	121
normalized size	1	1.00	0.85	0.00	1.43	1.90	9.61	2.56	1.98
time (sec)	N/A	0.096	0.501	1.163	0.693	0.493	7.129	0.163	10.082

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.124	0.071	2.424	0.000	0.447	0.000	0.000	0.000

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	61	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.109	0.066	3.895	0.000	0.516	0.000	0.000	0.000

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.117	0.068	3.974	0.000	0.521	0.000	0.000	0.000

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	143	0	457	744	9238	1845	1656
normalized size	1	1.00	0.84	0.00	2.69	4.38	54.34	10.85	9.74
time (sec)	N/A	0.182	0.691	19.374	0.698	0.551	102.309	0.280	16.867

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	113	0	294	403	4310	1003	703
normalized size	1	1.00	0.85	0.00	2.21	3.03	32.41	7.54	5.29
time (sec)	N/A	0.140	0.394	8.888	0.751	0.512	41.430	0.242	13.442

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	83	0	171	189	1622	462	305
normalized size	1	1.00	0.86	0.00	1.78	1.97	16.90	4.81	3.18
time (sec)	N/A	0.112	0.413	6.351	0.669	0.476	15.951	0.162	11.480

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	51	0	83	70	428	156	99
normalized size	1	1.00	0.86	0.00	1.41	1.19	7.25	2.64	1.68
time (sec)	N/A	0.066	0.121	4.617	0.833	0.479	5.592	0.146	9.764

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.103	0.097	2.113	0.000	0.465	0.000	0.000	0.000

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.097	0.094	4.577	0.000	0.477	0.000	0.000	0.000

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.103	0.075	4.353	0.000	0.537	0.000	0.000	0.000

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	61	61	0	0	0	0	0	-1
normalized size	1	1.13	1.13	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.078	0.075	5.194	0.000	0.475	0.000	0.000	0.000

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	150	0	159	197	2747	402	349
normalized size	1	1.00	1.12	0.00	1.19	1.47	20.50	3.00	2.60
time (sec)	N/A	0.119	1.396	6.763	0.909	0.482	67.344	0.274	12.194

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	94	0	119	140	1508	274	224
normalized size	1	1.00	0.87	0.00	1.10	1.30	13.96	2.54	2.07
time (sec)	N/A	0.097	0.572	3.231	0.691	0.479	28.583	0.284	10.741

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	0	84	93	697	170	138
normalized size	1	1.00	0.96	0.00	1.05	1.16	8.71	2.12	1.72
time (sec)	N/A	0.084	0.188	4.776	0.637	0.452	11.847	0.264	9.919

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	0	56	54	248	92	62
normalized size	1	1.00	0.80	0.00	1.04	1.00	4.59	1.70	1.15
time (sec)	N/A	0.053	0.026	1.785	0.911	0.452	4.802	0.258	9.419

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.053	1.882	0.000	0.461	0.000	0.000	0.000

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.054	1.145	0.000	0.478	0.000	0.000	0.000

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.081	0.070	1.197	0.000	0.501	0.000	0.000	0.000

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	84	64	96	74	72	199	87	276
normalized size	1	1.06	0.81	1.22	0.94	0.91	2.52	1.10	3.49
time (sec)	N/A	0.092	0.597	0.358	0.336	0.440	0.989	0.169	10.240

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	220	160	0	667	0	0	-1
normalized size	1	1.00	1.79	1.30	0.00	5.42	0.00	0.00	-0.01
time (sec)	N/A	0.551	2.795	1.861	0.000	0.594	0.000	0.000	0.000

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	208404	4463	0	2071	0	0	-1
normalized size	1	1.00	1478.04	31.65	0.00	14.69	0.00	0.00	-0.01
time (sec)	N/A	0.672	33.311	0.604	0.000	1.107	0.000	0.000	0.000

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	158	0	0	0	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.245	0.743	1.112	0.000	0.545	0.000	0.000	0.000

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	116.952	3.483	0.000	0.611	0.000	0.000	0.000

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	77.427	3.280	0.000	0.550	0.000	0.000	0.000

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	16.120	1.350	0.000	0.490	0.000	0.000	0.000

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	229	0	0	0	0	0	-1
normalized size	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.229	1.009	1.327	0.000	0.477	0.000	0.000	0.000

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	8.830	5.727	0.000	0.519	0.000	0.000	0.000

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	20.687	4.015	0.000	0.504	0.000	0.000	0.000

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	160	0	0	0	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	1.375	1.239	0.000	0.611	0.000	0.000	0.000

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	1.634	3.581	0.000	0.665	0.000	0.000	0.000

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.746	1.559	0.000	0.569	0.000	0.000	0.000

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	30.710	1.293	0.000	0.655	0.000	0.000	0.000

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.245	17.612	5.531	0.000	0.783	0.000	0.000	0.000

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	19.803	3.888	0.000	0.802	0.000	0.000	0.000

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	25.132	6.306	0.000	0.687	0.000	0.000	0.000

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	2.428	4.479	0.000	0.762	0.000	0.000	0.000

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	194	128	134	97	228	182	134
normalized size	1	1.00	1.45	0.96	1.00	0.72	1.70	1.36	1.00
time (sec)	N/A	0.142	0.805	0.457	0.356	0.607	13.880	0.302	0.122

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	130	108	104	81	178	145	102
normalized size	1	1.00	1.27	1.06	1.02	0.79	1.75	1.42	1.00
time (sec)	N/A	0.108	0.662	0.451	0.346	0.717	5.332	0.237	0.081

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	88	72	65	128	100	72
normalized size	1	1.00	1.00	1.13	0.92	0.83	1.64	1.28	0.92
time (sec)	N/A	0.094	0.811	0.456	0.351	0.785	1.765	0.205	9.007

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	46	44	42	48	75	52	40
normalized size	1	1.00	0.94	0.90	0.86	0.98	1.53	1.06	0.82
time (sec)	N/A	0.063	0.391	0.230	0.346	0.705	0.438	0.140	0.069

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	68	47	29	31	0	114	35
normalized size	1	1.00	2.00	1.38	0.85	0.91	0.00	3.35	1.03
time (sec)	N/A	0.070	0.035	0.360	0.358	0.837	0.000	0.174	0.069

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	260	129	55	90	0	84	43
normalized size	1	1.00	5.53	2.74	1.17	1.91	0.00	1.79	0.91
time (sec)	N/A	0.086	0.632	0.569	0.344	0.714	0.000	0.231	9.121

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	357	173	115	182	0	152	98
normalized size	1	1.00	3.57	1.73	1.15	1.82	0.00	1.52	0.98
time (sec)	N/A	0.122	1.563	0.596	0.345	0.848	0.000	0.224	0.137

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	451	217	171	222	0	201	155
normalized size	1	1.00	2.87	1.38	1.09	1.41	0.00	1.28	0.99
time (sec)	N/A	0.168	1.755	0.598	0.352	0.730	0.000	0.230	9.257

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	164	138	124	97	416	176	504
normalized size	1	1.00	1.19	1.00	0.90	0.70	3.01	1.28	3.65
time (sec)	N/A	0.141	0.841	0.505	0.339	0.906	9.704	0.249	10.725

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	120	118	98	81	306	133	391
normalized size	1	1.00	1.08	1.06	0.88	0.73	2.76	1.20	3.52
time (sec)	N/A	0.113	0.591	0.493	0.403	0.806	3.504	0.215	10.506

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	64	96	74	65	199	83	276
normalized size	1	1.00	0.76	1.14	0.88	0.77	2.37	0.99	3.29
time (sec)	N/A	0.095	0.671	0.359	0.379	0.787	1.006	0.182	10.553

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	85	54	56	73	0	36	33
normalized size	1	1.00	2.93	1.86	1.93	2.52	0.00	1.24	1.14
time (sec)	N/A	0.049	0.354	0.461	0.430	0.524	0.000	0.180	9.166

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	97	72	59	69	0	94	107
normalized size	1	1.00	1.94	1.44	1.18	1.38	0.00	1.88	2.14
time (sec)	N/A	0.068	0.626	0.519	0.321	0.760	0.000	0.186	9.247

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	223	102	86	112	0	225	224
normalized size	1	1.00	3.05	1.40	1.18	1.53	0.00	3.08	3.07
time (sec)	N/A	0.073	1.272	0.547	0.366	0.741	0.000	0.207	11.137

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	315	130	107	149	0	345	320
normalized size	1	1.00	3.28	1.35	1.11	1.55	0.00	3.59	3.33
time (sec)	N/A	0.080	2.033	0.603	0.327	0.801	0.000	0.215	12.639

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	407	158	126	185	0	465	416
normalized size	1	1.00	3.42	1.33	1.06	1.55	0.00	3.91	3.50
time (sec)	N/A	0.087	4.340	0.632	0.328	0.643	0.000	0.233	13.295

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	86	231	168	127	440	239	168
normalized size	1	1.00	0.64	1.72	1.25	0.95	3.28	1.78	1.25
time (sec)	N/A	0.178	1.186	0.475	0.310	0.855	24.905	0.440	9.199

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	70	201	142	109	335	202	140
normalized size	1	1.00	0.67	1.91	1.35	1.04	3.19	1.92	1.33
time (sec)	N/A	0.149	0.358	0.473	0.314	0.768	10.169	0.329	0.123

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	171	96	91	228	116	96
normalized size	1	1.00	0.85	2.19	1.23	1.17	2.92	1.49	1.23
time (sec)	N/A	0.109	0.432	0.485	0.312	0.758	3.511	0.246	9.075

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	75	68	72	117	88	66
normalized size	1	1.00	0.96	1.47	1.33	1.41	2.29	1.73	1.29
time (sec)	N/A	0.070	0.095	0.240	0.312	0.878	0.987	0.161	9.095

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	127	52	54	0	220	63
normalized size	1	1.00	0.85	2.12	0.87	0.90	0.00	3.67	1.05
time (sec)	N/A	0.096	0.082	0.368	0.311	0.786	0.000	0.203	0.075

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	189	37	55	0	112	44
normalized size	1	1.00	0.95	4.40	0.86	1.28	0.00	2.60	1.02
time (sec)	N/A	0.090	0.081	0.600	0.350	0.807	0.000	0.213	0.065

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	75	281	87	161	0	130	73
normalized size	1	1.00	0.97	3.65	1.13	2.09	0.00	1.69	0.95
time (sec)	N/A	0.119	0.140	0.569	0.492	0.692	0.000	0.242	0.109

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	90	379	148	271	0	209	136
normalized size	1	1.00	0.68	2.87	1.12	2.05	0.00	1.58	1.03
time (sec)	N/A	0.157	0.706	0.734	0.433	0.775	0.000	0.268	9.216

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	216	245	208	135	719	235	622
normalized size	1	1.00	1.10	1.25	1.06	0.69	3.67	1.20	3.17
time (sec)	N/A	0.211	5.099	0.500	0.336	0.704	17.401	0.384	10.806

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	171	215	171	115	539	192	494
normalized size	1	1.00	1.04	1.30	1.04	0.70	3.27	1.16	2.99
time (sec)	N/A	0.189	1.731	0.490	0.452	0.742	6.699	0.301	10.998

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	133	182	134	95	371	130	367
normalized size	1	1.00	0.99	1.36	1.00	0.71	2.77	0.97	2.74
time (sec)	N/A	0.168	0.978	0.388	0.332	0.682	2.426	0.230	10.465

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	91	123	104	128	0	125	110
normalized size	1	1.00	1.65	2.24	1.89	2.33	0.00	2.27	2.00
time (sec)	N/A	0.093	0.245	0.611	0.498	0.682	0.000	0.201	9.297

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	121	162	108	120	0	78	77
normalized size	1	1.00	1.66	2.22	1.48	1.64	0.00	1.07	1.05
time (sec)	N/A	0.116	0.019	0.655	0.441	0.488	0.000	0.198	9.148

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	178	231	147	113	0	192	175
normalized size	1	1.00	1.71	2.22	1.41	1.09	0.00	1.85	1.68
time (sec)	N/A	0.125	0.020	0.639	0.483	0.676	0.000	0.213	9.361

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	130	295	178	157	0	325	274
normalized size	1	1.00	1.01	2.29	1.38	1.22	0.00	2.52	2.12
time (sec)	N/A	0.134	0.340	0.713	0.343	0.681	0.000	0.235	12.231

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	156	359	207	197	0	461	370
normalized size	1	1.00	1.01	2.33	1.34	1.28	0.00	2.99	2.40
time (sec)	N/A	0.141	0.452	0.712	0.471	0.686	0.000	0.255	12.954

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	181	423	238	237	0	597	466
normalized size	1	1.00	1.01	2.36	1.33	1.32	0.00	3.34	2.60
time (sec)	N/A	0.151	0.915	0.808	0.706	0.770	0.000	0.298	13.960

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	86	345	182	155	636	283	177
normalized size	1	1.00	0.64	2.57	1.36	1.16	4.75	2.11	1.32
time (sec)	N/A	0.182	1.527	0.489	0.509	0.746	38.223	0.894	0.200

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	70	305	158	129	471	230	156
normalized size	1	1.00	0.67	2.90	1.50	1.23	4.49	2.19	1.49
time (sec)	N/A	0.152	0.437	0.488	0.365	0.729	16.660	0.598	0.144

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	53	265	126	115	313	172	126
normalized size	1	1.00	0.68	3.40	1.62	1.47	4.01	2.21	1.62
time (sec)	N/A	0.134	0.243	0.484	0.443	0.733	6.341	0.345	9.142

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	36	98	84	94	151	116	81
normalized size	1	1.00	0.71	1.92	1.65	1.84	2.96	2.27	1.59
time (sec)	N/A	0.062	0.088	0.227	0.309	0.558	2.016	0.227	9.073

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	68	161	73	77	0	289	100
normalized size	1	1.00	0.84	1.99	0.90	0.95	0.00	3.57	1.23
time (sec)	N/A	0.095	0.130	0.363	0.353	0.693	0.000	0.211	0.088

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	48	290	52	89	0	228	63
normalized size	1	1.00	0.77	4.68	0.84	1.44	0.00	3.68	1.02
time (sec)	N/A	0.109	0.119	0.575	0.306	0.540	0.000	0.263	0.084

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	312	47	49	0	82	36
normalized size	1	1.00	0.86	7.26	1.09	1.14	0.00	1.91	0.84
time (sec)	N/A	0.082	0.047	0.583	0.305	0.748	0.000	0.262	9.154

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	95	521	123	242	0	158	112
normalized size	1	1.00	0.90	4.96	1.17	2.30	0.00	1.50	1.07
time (sec)	N/A	0.135	0.256	0.573	0.307	0.716	0.000	0.293	9.142

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	151	669	185	353	0	237	172
normalized size	1	1.00	0.93	4.13	1.14	2.18	0.00	1.46	1.06
time (sec)	N/A	0.186	0.660	0.648	0.318	0.746	0.000	0.315	9.184

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	344	363	284	155	1042	273	711
normalized size	1	1.00	1.49	1.57	1.23	0.67	4.51	1.18	3.08
time (sec)	N/A	0.268	6.053	0.464	0.336	0.929	27.746	0.745	10.832

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	183	323	232	135	823	217	584
normalized size	1	1.00	0.92	1.62	1.16	0.68	4.12	1.08	2.92
time (sec)	N/A	0.239	2.248	0.454	0.325	0.710	11.667	0.379	10.690

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	146	279	199	111	588	165	451
normalized size	1	1.00	0.92	1.75	1.25	0.70	3.70	1.04	2.84
time (sec)	N/A	0.217	1.419	0.348	0.319	0.615	4.381	0.294	10.729

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	219	167	173	0	147	234
normalized size	1	1.00	0.90	2.41	1.84	1.90	0.00	1.62	2.57
time (sec)	N/A	0.105	0.258	0.681	0.414	0.678	0.000	0.197	11.512

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	121	248	164	167	0	93	140
normalized size	1	1.00	1.75	3.59	2.38	2.42	0.00	1.35	2.03
time (sec)	N/A	0.154	1.111	0.670	0.421	0.701	0.000	0.250	9.813

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	94	333	188	188	0	146	113
normalized size	1	1.00	0.88	3.11	1.76	1.76	0.00	1.36	1.06
time (sec)	N/A	0.154	0.178	0.657	0.334	0.690	0.000	0.250	11.305

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	135	435	228	146	0	260	213
normalized size	1	1.00	1.17	3.78	1.98	1.27	0.00	2.26	1.85
time (sec)	N/A	0.146	0.495	0.674	0.370	0.859	0.000	0.286	11.177

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	176	535	270	188	0	393	322
normalized size	1	1.00	1.26	3.82	1.93	1.34	0.00	2.81	2.30
time (sec)	N/A	0.151	0.607	0.693	0.355	0.677	0.000	0.298	12.213

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	69	107	104	84	3363	139	124
normalized size	1	1.00	0.66	1.02	0.99	0.80	32.03	1.32	1.18
time (sec)	N/A	0.147	0.227	0.559	0.320	0.755	83.076	0.208	0.108

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	72	75	72	66	1703	95	82
normalized size	1	1.00	0.91	0.95	0.91	0.84	21.56	1.20	1.04
time (sec)	N/A	0.116	0.150	0.449	0.324	0.724	30.975	0.192	9.201

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	44	43	44	49	588	51	47
normalized size	1	1.00	0.77	0.75	0.77	0.86	10.32	0.89	0.82
time (sec)	N/A	0.095	0.100	0.436	0.334	0.775	9.861	0.184	0.076

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	31	51	34	31	60	35	36
normalized size	1	1.00	0.86	1.42	0.94	0.86	1.67	0.97	1.00
time (sec)	N/A	0.082	0.036	0.257	0.321	0.561	0.582	0.169	9.233

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	112	58	73	0	79	43
normalized size	1	1.00	0.98	2.49	1.29	1.62	0.00	1.76	0.96
time (sec)	N/A	0.094	0.064	0.456	0.321	0.666	0.000	0.194	0.103

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	75	169	113	161	0	147	96
normalized size	1	1.00	0.82	1.86	1.24	1.77	0.00	1.62	1.05
time (sec)	N/A	0.138	0.287	0.556	0.331	0.612	0.000	0.226	0.128

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	105	245	165	194	0	192	151
normalized size	1	1.00	0.72	1.68	1.13	1.33	0.00	1.32	1.03
time (sec)	N/A	0.189	0.566	0.530	0.346	0.722	0.000	0.265	9.184

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	142	321	220	224	0	236	206
normalized size	1	1.00	0.69	1.57	1.07	1.09	0.00	1.15	1.00
time (sec)	N/A	0.249	0.884	0.498	0.342	0.614	0.000	0.313	9.347

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	52	82	83	82	2705	95	98
normalized size	1	1.00	0.66	1.04	1.05	1.04	34.24	1.20	1.24
time (sec)	N/A	0.125	0.166	0.608	0.340	0.807	132.038	0.230	0.075

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	34	58	61	64	1182	73	68
normalized size	1	1.00	0.67	1.14	1.20	1.25	23.18	1.43	1.33
time (sec)	N/A	0.103	0.057	0.594	0.323	0.819	53.394	0.225	9.122

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	51	85	54	48	1096	92	61
normalized size	1	1.00	0.77	1.29	0.82	0.73	16.61	1.39	0.92
time (sec)	N/A	0.107	0.097	0.609	0.322	0.691	19.007	0.200	0.073

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	56	43	45	121	76	41
normalized size	1	1.00	0.93	1.27	0.98	1.02	2.75	1.73	0.93
time (sec)	N/A	0.068	0.073	0.448	0.399	0.634	0.837	0.175	0.056

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	150	84	134	0	104	71
normalized size	1	1.00	0.97	2.11	1.18	1.89	0.00	1.46	1.00
time (sec)	N/A	0.107	0.124	0.625	0.426	0.654	0.000	0.196	9.248

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	87	207	139	230	0	169	121
normalized size	1	1.00	0.71	1.68	1.13	1.87	0.00	1.37	0.98
time (sec)	N/A	0.155	0.741	0.757	0.419	0.863	0.000	0.321	0.151

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	123	283	207	260	0	214	193
normalized size	1	1.00	0.69	1.58	1.16	1.45	0.00	1.20	1.08
time (sec)	N/A	0.206	0.692	0.757	0.340	0.595	0.000	0.336	9.499

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	160	359	252	290	0	258	240
normalized size	1	1.00	0.68	1.52	1.07	1.23	0.00	1.09	1.02
time (sec)	N/A	0.279	1.481	0.765	0.353	0.766	0.000	0.393	9.597

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	154	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.269	0.465	7.633	0.000	0.750	0.000	0.000	0.000

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	132	0	1207	333	0	1402	783
normalized size	1	1.00	0.83	0.00	7.59	2.09	0.00	8.82	4.92
time (sec)	N/A	0.166	0.796	29.033	0.427	0.793	0.000	0.285	17.899

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	103	0	643	221	0	861	517
normalized size	1	1.00	0.84	0.00	5.23	1.80	0.00	7.00	4.20
time (sec)	N/A	0.135	0.401	13.371	0.421	0.661	0.000	0.225	15.820

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	0	285	135	5243	458	272
normalized size	1	1.00	1.00	0.00	3.06	1.45	56.38	4.92	2.92
time (sec)	N/A	0.118	0.314	6.831	0.400	0.710	54.478	0.185	11.741

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	51	0	83	70	428	156	99
normalized size	1	1.00	0.86	0.00	1.41	1.19	7.25	2.64	1.68
time (sec)	N/A	0.071	0.125	4.703	0.405	0.703	5.819	0.160	10.166

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.118	2.121	0.000	0.764	0.000	0.000	0.000

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	82	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.172	0.746	0.000	0.656	0.000	0.000	0.000

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	76	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.180	0.850	0.000	0.774	0.000	0.000	0.000

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	111	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.887	22.621	0.000	0.808	0.000	0.000	0.000

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	111	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.487	9.508	0.000	0.747	0.000	0.000	0.000

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	111	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.330	4.226	0.000	0.786	0.000	0.000	0.000

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	3925	0	0	0	0	0	-1
normalized size	1	1.00	31.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	6.546	0.761	0.000	0.744	0.000	0.000	0.000

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	1.766	0.822	0.000	0.846	0.000	0.000	0.000

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	3.913	0.937	0.000	0.731	0.000	0.000	0.000

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	160	0	0	197	0	0	441
normalized size	1	1.00	0.67	0.00	0.00	0.82	0.00	0.00	1.85
time (sec)	N/A	0.441	0.573	9.041	0.000	0.798	0.000	0.000	18.419

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	119	0	0	131	0	0	234
normalized size	1	1.00	0.71	0.00	0.00	0.78	0.00	0.00	1.39
time (sec)	N/A	0.306	0.296	8.198	0.000	0.755	0.000	0.000	11.940

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	83	0	0	84	0	0	129
normalized size	1	1.00	0.81	0.00	0.00	0.82	0.00	0.00	1.26
time (sec)	N/A	0.212	0.179	7.814	0.000	0.528	0.000	0.000	1.279

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	300	0	0	0	0	0	-1
normalized size	1	1.00	1.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.264	3.751	4.136	0.000	0.584	0.000	0.000	0.000

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	144	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.559	4.821	0.000	0.650	0.000	0.000	0.000

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	150	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.272	0.659	4.061	0.000	0.734	0.000	0.000	0.000

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	155	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.276	0.907	4.137	0.000	0.742	0.000	0.000	0.000

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	0	0	33	0	0	33
normalized size	1	1.00	1.03	0.00	0.00	1.03	0.00	0.00	1.03
time (sec)	N/A	0.117	0.168	10.905	0.000	0.627	0.000	0.000	9.705

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	0	0	35	0	0	35
normalized size	1	1.00	1.03	0.00	0.00	1.03	0.00	0.00	1.03
time (sec)	N/A	0.115	0.065	10.270	0.000	0.828	0.000	0.000	0.516

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	798	0	0	0	0	0	-1
normalized size	1	1.00	4.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.280	10.397	5.997	0.000	1.130	0.000	0.000	0.000

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	153	0	0	0	0	0	0	-1
normalized size	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	22.607	4.077	0.000	0.817	0.000	0.000	0.000

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	151	0	0	0	0	0	0	-1
normalized size	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	4.281	1.538	0.000	0.691	0.000	0.000	0.000

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	155	0	0	0	0	0	0	-1
normalized size	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	7.701	0.976	0.000	1.069	0.000	0.000	0.000

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	153	0	0	0	0	0	0	-1
normalized size	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	12.056	4.010	0.000	1.053	0.000	0.000	0.000

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	153	0	0	0	0	0	0	-1
normalized size	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	15.569	2.742	0.000	1.380	0.000	0.000	0.000

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	153	0	0	0	0	0	0	-1
normalized size	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.234	20.902	3.396	0.000	1.387	0.000	0.000	0.000

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	893	0	0	0	0	0	-1
normalized size	1	1.00	5.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.429	11.130	6.288	0.000	1.015	0.000	0.000	0.000

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	77	95	65	73	192	92	153
normalized size	1	1.00	0.73	0.90	0.62	0.70	1.83	0.88	1.46
time (sec)	N/A	0.164	0.177	0.128	0.303	0.536	3.568	0.204	12.983

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	59	77	52	62	144	62	125
normalized size	1	1.00	0.73	0.95	0.64	0.77	1.78	0.77	1.54
time (sec)	N/A	0.131	0.106	0.117	0.304	0.682	1.808	0.183	12.892

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	57	39	51	119	47	125
normalized size	1	1.00	0.94	0.88	0.60	0.78	1.83	0.72	1.92
time (sec)	N/A	0.094	0.116	0.091	0.370	0.690	0.903	0.146	12.661

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	74	63	57	60	0	87	157
normalized size	1	1.00	1.45	1.24	1.12	1.18	0.00	1.71	3.08
time (sec)	N/A	0.069	0.056	0.277	0.362	0.625	0.000	0.154	9.874

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	75	57	54	84	0	108	158
normalized size	1	1.00	1.83	1.39	1.32	2.05	0.00	2.63	3.85
time (sec)	N/A	0.056	0.037	0.240	0.495	0.840	0.000	0.171	9.545

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	109	81	66	114	0	95	151
normalized size	1	1.00	2.10	1.56	1.27	2.19	0.00	1.83	2.90
time (sec)	N/A	0.080	0.049	0.328	0.505	0.808	0.000	0.172	9.360

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	95	80	61	119	0	115	111
normalized size	1	1.00	1.83	1.54	1.17	2.29	0.00	2.21	2.13
time (sec)	N/A	0.109	0.036	0.404	0.384	0.674	0.000	0.186	9.309

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	135	102	80	137	0	116	112
normalized size	1	1.00	1.82	1.38	1.08	1.85	0.00	1.57	1.51
time (sec)	N/A	0.128	0.040	0.321	0.312	0.744	0.000	0.190	9.327

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	177	124	92	169	0	144	143
normalized size	1	1.00	1.97	1.38	1.02	1.88	0.00	1.60	1.59
time (sec)	N/A	0.133	0.069	0.322	0.406	0.745	0.000	0.180	9.333

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	132	150	104	104	275	141	233
normalized size	1	1.00	0.69	0.79	0.55	0.55	1.45	0.74	1.23
time (sec)	N/A	0.382	0.551	0.203	0.402	0.764	5.911	0.243	12.974

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	120	141	92	103	309	115	112
normalized size	1	1.00	0.74	0.87	0.56	0.63	1.90	0.71	0.69
time (sec)	N/A	0.382	0.217	0.215	0.386	0.636	3.569	0.236	9.615

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	77	94	68	73	172	82	180
normalized size	1	1.00	0.73	0.89	0.64	0.69	1.62	0.77	1.70
time (sec)	N/A	0.159	0.368	0.186	0.375	0.859	1.962	0.182	12.772

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	91	83	74	84	0	133	225
normalized size	1	1.00	1.01	0.92	0.82	0.93	0.00	1.48	2.50
time (sec)	N/A	0.253	0.216	0.399	0.308	0.766	0.000	0.200	9.868

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	116	102	79	118	0	148	277
normalized size	1	1.00	1.49	1.31	1.01	1.51	0.00	1.90	3.55
time (sec)	N/A	0.098	0.408	0.384	0.420	0.777	0.000	0.188	9.707

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	155	126	103	168	0	148	397
normalized size	1	1.00	1.74	1.42	1.16	1.89	0.00	1.66	4.46
time (sec)	N/A	0.306	0.890	0.510	0.416	0.646	0.000	0.197	10.198

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	538	114	82	167	0	167	231
normalized size	1	1.00	5.60	1.19	0.85	1.74	0.00	1.74	2.41
time (sec)	N/A	0.390	6.187	0.422	0.522	0.754	0.000	0.214	9.480

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	579	173	129	200	0	182	165
normalized size	1	1.00	4.71	1.41	1.05	1.63	0.00	1.48	1.34
time (sec)	N/A	0.365	6.170	0.461	0.407	0.790	0.000	0.218	9.328

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	236	156	108	195	0	222	187
normalized size	1	1.00	1.59	1.05	0.73	1.32	0.00	1.50	1.26
time (sec)	N/A	0.386	0.818	0.488	0.379	0.692	0.000	0.216	9.357

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	296	244	186	283	0	276	245
normalized size	1	1.00	1.74	1.44	1.09	1.66	0.00	1.62	1.44
time (sec)	N/A	0.396	0.801	0.470	0.330	0.801	0.000	0.232	9.444

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	157	196	131	141	394	166	455
normalized size	1	1.00	0.68	0.84	0.56	0.61	1.70	0.72	1.96
time (sec)	N/A	0.572	0.823	0.220	0.327	0.720	6.285	0.277	10.658

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	158	108	116	340	139	425
normalized size	1	1.00	0.85	0.97	0.66	0.71	2.09	0.85	2.61
time (sec)	N/A	0.295	0.761	0.207	0.319	0.695	3.609	0.253	10.746

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	129	150	101	116	0	293	567
normalized size	1	1.00	0.95	1.10	0.74	0.85	0.00	2.15	4.17
time (sec)	N/A	0.413	0.294	0.426	0.401	0.753	0.000	0.233	11.122

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	143	125	95	143	0	199	289
normalized size	1	1.00	1.40	1.23	0.93	1.40	0.00	1.95	2.83
time (sec)	N/A	0.142	1.319	0.402	0.414	0.713	0.000	0.237	9.474

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	192	171	128	216	0	272	585
normalized size	1	1.00	1.39	1.24	0.93	1.57	0.00	1.97	4.24
time (sec)	N/A	0.465	1.315	0.502	0.503	0.746	0.000	0.299	9.463

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	615	159	119	231	0	222	477
normalized size	1	1.00	4.46	1.15	0.86	1.67	0.00	1.61	3.46
time (sec)	N/A	0.483	6.189	0.462	0.419	0.735	0.000	0.249	10.709

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	690	207	149	265	0	234	348
normalized size	1	1.00	4.54	1.36	0.98	1.74	0.00	1.54	2.29
time (sec)	N/A	0.511	6.230	0.480	0.420	0.632	0.000	0.264	9.865

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	344	227	157	275	0	290	241
normalized size	1	1.00	1.88	1.24	0.86	1.50	0.00	1.58	1.32
time (sec)	N/A	0.569	1.292	0.487	0.334	0.718	0.000	0.266	9.439

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	369	276	202	310	0	354	292
normalized size	1	1.00	1.74	1.30	0.95	1.46	0.00	1.67	1.38
time (sec)	N/A	0.598	2.097	0.481	0.396	0.780	0.000	0.294	9.689

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	246	460	0	643	0	261	1688
normalized size	1	1.00	1.31	2.45	0.00	3.42	0.00	1.39	8.98
time (sec)	N/A	0.742	2.525	0.554	0.000	0.896	0.000	0.196	11.853

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	129	353	0	568	0	211	479
normalized size	1	1.00	0.84	2.31	0.00	3.71	0.00	1.38	3.13
time (sec)	N/A	0.492	0.422	0.509	0.000	0.693	0.000	0.195	10.400

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	130	229	0	479	0	191	269
normalized size	1	1.00	1.23	2.16	0.00	4.52	0.00	1.80	2.54
time (sec)	N/A	0.153	1.068	0.454	0.000	0.782	0.000	0.192	10.024

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	153	0	483	0	130	523
normalized size	1	1.00	1.05	1.66	0.00	5.25	0.00	1.41	5.68
time (sec)	N/A	0.242	0.223	0.682	0.000	1.021	0.000	0.210	10.162

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	139	245	0	768	0	218	1616
normalized size	1	1.00	1.21	2.13	0.00	6.68	0.00	1.90	14.05
time (sec)	N/A	0.435	0.760	0.752	0.000	0.981	0.000	0.199	11.414

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	196	307	0	1130	0	257	966
normalized size	1	1.00	1.25	1.96	0.00	7.20	0.00	1.64	6.15
time (sec)	N/A	0.768	3.091	0.811	0.000	1.291	0.000	0.215	9.895

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	385	390	0	1471	0	329	1089
normalized size	1	1.00	1.99	2.02	0.00	7.62	0.00	1.70	5.64
time (sec)	N/A	1.038	6.341	0.820	0.000	0.845	0.000	0.224	9.989

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	288	845	0	1058	0	535	4943
normalized size	1	1.00	1.08	3.18	0.00	3.98	0.00	2.01	18.58
time (sec)	N/A	0.878	5.878	0.582	0.000	1.146	0.000	0.243	17.722

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	159	711	0	919	0	302	3031
normalized size	1	1.00	0.88	3.95	0.00	5.11	0.00	1.68	16.84
time (sec)	N/A	0.562	1.221	0.538	0.000	0.755	0.000	0.220	15.788

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	289	576	0	793	0	256	2709
normalized size	1	1.00	1.73	3.45	0.00	4.75	0.00	1.53	16.22
time (sec)	N/A	0.279	2.058	0.506	0.000	0.785	0.000	0.231	13.930

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	632	0	996	0	277	1610
normalized size	1	1.00	1.00	4.10	0.00	6.47	0.00	1.80	10.45
time (sec)	N/A	0.477	1.179	0.721	0.000	0.931	0.000	0.257	12.439

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	195	729	0	1394	0	339	1762
normalized size	1	1.00	0.97	3.61	0.00	6.90	0.00	1.68	8.72
time (sec)	N/A	0.795	5.782	0.735	0.000	1.394	0.000	0.270	10.631

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	330	803	0	1922	0	526	1906
normalized size	1	1.00	1.23	2.99	0.00	7.14	0.00	1.96	7.09
time (sec)	N/A	1.151	6.334	0.931	0.000	1.624	0.000	0.285	11.006

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	3348	4675	0	0	0	0	-1
normalized size	1	1.00	9.65	13.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.774	23.009	0.660	0.000	0.641	0.000	0.000	0.000

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	92	124	71	95	272	107	223
normalized size	1	1.00	0.64	0.87	0.50	0.66	1.90	0.75	1.56
time (sec)	N/A	0.191	0.249	0.295	0.308	0.689	15.147	0.299	13.317

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	77	106	61	84	248	92	209
normalized size	1	1.00	0.61	0.83	0.48	0.66	1.95	0.72	1.65
time (sec)	N/A	0.187	0.190	0.279	0.376	0.753	9.163	0.224	13.166

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	88	88	65	73	192	107	181
normalized size	1	1.00	0.85	0.85	0.63	0.71	1.86	1.04	1.76
time (sec)	N/A	0.148	0.187	0.286	0.385	0.830	5.494	0.200	13.025

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	77	68	52	62	167	92	181
normalized size	1	1.00	0.89	0.78	0.60	0.71	1.92	1.06	2.08
time (sec)	N/A	0.111	0.167	0.277	0.304	0.719	3.241	0.174	12.860

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	109	97	81	88	0	145	242
normalized size	1	1.00	1.22	1.09	0.91	0.99	0.00	1.63	2.72
time (sec)	N/A	0.100	0.124	0.417	0.402	0.809	0.000	0.170	10.824

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	105	119	91	107	0	142	241
normalized size	1	1.00	1.27	1.43	1.10	1.29	0.00	1.71	2.90
time (sec)	N/A	0.134	0.330	0.390	0.504	0.764	0.000	0.209	9.388

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	132	143	101	139	0	163	236
normalized size	1	1.00	1.40	1.52	1.07	1.48	0.00	1.73	2.51
time (sec)	N/A	0.122	1.476	0.464	0.446	0.731	0.000	0.193	9.373

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	125	106	92	160	0	141	225
normalized size	1	1.00	1.52	1.29	1.12	1.95	0.00	1.72	2.74
time (sec)	N/A	0.085	0.042	0.307	0.483	0.685	0.000	0.195	9.446

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	153	128	107	180	0	153	221
normalized size	1	1.00	1.74	1.45	1.22	2.05	0.00	1.74	2.51
time (sec)	N/A	0.111	0.050	0.339	0.455	0.742	0.000	0.209	9.674

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	135	116	86	160	0	173	174
normalized size	1	1.00	1.82	1.57	1.16	2.16	0.00	2.34	2.35
time (sec)	N/A	0.133	0.036	0.339	0.315	0.797	0.000	0.225	9.559

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	175	138	106	187	0	201	205
normalized size	1	1.00	1.79	1.41	1.08	1.91	0.00	2.05	2.09
time (sec)	N/A	0.165	0.044	0.341	0.320	0.725	0.000	0.242	9.522

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	239	160	118	221	0	229	399
normalized size	1	1.00	2.10	1.40	1.04	1.94	0.00	2.01	3.50
time (sec)	N/A	0.173	0.077	0.343	0.391	0.894	0.000	0.230	10.995

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	279	182	138	239	0	201	205
normalized size	1	1.00	2.05	1.34	1.01	1.76	0.00	1.48	1.51
time (sec)	N/A	0.184	0.073	0.339	0.385	0.787	0.000	0.257	9.877

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	144	161	100	116	335	142	309
normalized size	1	1.00	0.48	0.53	0.33	0.39	1.11	0.47	1.03
time (sec)	N/A	0.654	0.961	0.333	0.318	0.909	15.575	0.372	12.924

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	141	163	101	128	420	150	139
normalized size	1	1.00	0.51	0.59	0.36	0.46	1.51	0.54	0.50
time (sec)	N/A	0.627	0.618	0.331	0.346	0.752	9.875	0.314	9.943

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	132	105	81	85	223	141	256
normalized size	1	1.00	1.02	0.81	0.63	0.66	1.73	1.09	1.98
time (sec)	N/A	0.181	0.426	0.328	0.384	0.742	5.768	0.216	13.013

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	190	125	123	97	112	0	213	319
normalized size	1	1.64	1.08	1.06	0.84	0.97	0.00	1.84	2.75
time (sec)	N/A	0.447	0.503	0.545	0.387	0.640	0.000	0.212	11.123

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	167	191	127	155	0	274	578
normalized size	1	1.00	0.92	1.06	0.70	0.86	0.00	1.51	3.19
time (sec)	N/A	0.521	0.699	0.536	0.494	0.803	0.000	0.222	9.545

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	191	208	150	210	0	252	397
normalized size	1	1.00	1.01	1.10	0.79	1.11	0.00	1.33	2.10
time (sec)	N/A	0.484	3.357	0.652	0.479	0.750	0.000	0.236	9.444

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	293	199	138	218	0	241	584
normalized size	1	1.00	2.20	1.50	1.04	1.64	0.00	1.81	4.39
time (sec)	N/A	0.162	6.186	0.449	0.557	0.565	0.000	0.259	11.747

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	270	223	166	260	0	244	825
normalized size	1	1.00	1.52	1.25	0.93	1.46	0.00	1.37	4.63
time (sec)	N/A	0.463	2.737	0.484	0.415	0.712	0.000	0.281	10.894

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	285	165	123	241	0	263	346
normalized size	1	1.00	1.36	0.79	0.59	1.15	0.00	1.26	1.66
time (sec)	N/A	0.516	1.537	0.503	0.414	0.676	0.000	0.254	10.103

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	319	253	180	274	0	309	262
normalized size	1	1.00	1.35	1.07	0.76	1.16	0.00	1.31	1.11
time (sec)	N/A	0.604	0.887	0.488	0.406	0.800	0.000	0.274	9.669

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	322	194	134	248	0	347	302
normalized size	1	1.00	1.23	0.74	0.51	0.95	0.00	1.33	1.16
time (sec)	N/A	0.629	1.326	0.472	0.327	0.613	0.000	0.290	9.921

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	204	218	140	164	505	204	578
normalized size	1	1.00	0.58	0.62	0.40	0.46	1.43	0.58	1.63
time (sec)	N/A	0.928	1.282	0.335	0.452	0.985	16.181	0.462	10.773

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	189	180	117	136	456	184	552
normalized size	1	1.00	0.97	0.93	0.60	0.70	2.35	0.95	2.85
time (sec)	N/A	0.327	0.871	0.332	0.399	0.865	10.059	0.358	10.921

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	191	211	137	150	0	427	690
normalized size	1	1.00	0.76	0.84	0.55	0.60	0.00	1.71	2.76
time (sec)	N/A	0.659	0.526	0.575	0.425	0.976	0.000	0.266	11.574

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	194	216	143	179	0	345	674
normalized size	1	1.00	0.85	0.94	0.62	0.78	0.00	1.51	2.94
time (sec)	N/A	0.677	3.037	0.538	0.511	0.778	0.000	0.297	9.592

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	252	279	186	260	0	400	718
normalized size	1	1.00	1.09	1.21	0.81	1.13	0.00	1.73	3.11
time (sec)	N/A	0.692	6.161	0.656	0.463	0.636	0.000	0.308	9.595

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	355	264	187	293	0	421	405
normalized size	1	1.00	1.83	1.36	0.96	1.51	0.00	2.17	2.09
time (sec)	N/A	0.223	6.250	0.598	0.471	0.836	0.000	0.323	9.649

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	381	316	212	331	0	343	699
normalized size	1	1.00	2.04	1.69	1.13	1.77	0.00	1.83	3.74
time (sec)	N/A	0.656	6.292	0.636	0.654	0.893	0.000	0.331	9.500

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	405	260	182	334	0	356	1007
normalized size	1	1.00	1.78	1.15	0.80	1.47	0.00	1.57	4.44
time (sec)	N/A	0.711	1.297	0.511	0.493	0.906	0.000	0.349	12.284

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	408	302	217	373	0	399	446
normalized size	1	1.00	1.48	1.10	0.79	1.36	0.00	1.45	1.62
time (sec)	N/A	0.756	1.865	0.510	0.422	0.855	0.000	0.360	9.934

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	324	309	208	347	0	456	359
normalized size	1	1.00	1.07	1.02	0.69	1.15	0.00	1.50	1.18
time (sec)	N/A	0.860	0.971	0.498	0.330	0.626	0.000	0.388	9.906

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	268	358	248	384	0	457	381
normalized size	1	1.00	0.80	1.07	0.74	1.15	0.00	1.37	1.14
time (sec)	N/A	0.907	1.602	0.501	0.329	0.678	0.000	0.389	9.951

Problem 1127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	378	1119	0	649	0	536	2390
normalized size	1	1.00	1.23	3.64	0.00	2.11	0.00	1.75	7.79
time (sec)	N/A	1.016	4.573	0.584	0.000	0.834	0.000	0.232	12.321

Problem 1128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	325	938	0	604	0	449	1003
normalized size	1	1.00	1.22	3.51	0.00	2.26	0.00	1.68	3.76
time (sec)	N/A	0.758	3.673	0.499	0.000	0.937	0.000	0.209	11.795

Problem 1129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	247	627	0	507	0	300	964
normalized size	1	1.00	1.52	3.85	0.00	3.11	0.00	1.84	5.91
time (sec)	N/A	0.306	2.514	0.474	0.000	0.690	0.000	0.198	11.526

Problem 1130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	161	380	0	516	0	286	2773
normalized size	1	1.00	1.18	2.77	0.00	3.77	0.00	2.09	20.24
time (sec)	N/A	0.274	0.712	0.712	0.000	0.924	0.000	0.219	11.153

Problem 1131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	182	396	0	675	0	260	6377
normalized size	1	1.00	1.18	2.57	0.00	4.38	0.00	1.69	41.41
time (sec)	N/A	0.304	1.786	0.751	0.000	0.860	0.000	0.236	12.091

Problem 1132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	180	191	339	0	804	0	275	675
normalized size	1	1.14	1.21	2.15	0.00	5.09	0.00	1.74	4.27
time (sec)	N/A	0.450	4.301	0.874	0.000	0.995	0.000	0.233	9.729

Problem 1133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	403	527	0	1149	0	356	973
normalized size	1	1.00	1.69	2.21	0.00	4.83	0.00	1.50	4.09
time (sec)	N/A	0.703	6.215	0.787	0.000	0.757	0.000	0.253	9.801

Problem 1134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	496	634	0	1578	0	461	1158
normalized size	1	1.00	1.70	2.17	0.00	5.40	0.00	1.58	3.97
time (sec)	N/A	1.047	6.294	0.841	0.000	1.146	0.000	0.282	9.829

Problem 1135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	1250	1193	0	1110	0	540	3581
normalized size	1	1.00	3.78	3.60	0.00	3.35	0.00	1.63	10.82
time (sec)	N/A	1.008	10.562	0.609	0.000	0.827	0.000	0.300	16.596

Problem 1136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	1030	880	0	976	0	393	2034
normalized size	1	1.00	3.63	3.10	0.00	3.44	0.00	1.38	7.16
time (sec)	N/A	0.738	6.444	0.534	0.000	0.847	0.000	0.272	14.338

Problem 1137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	274	639	0	837	0	429	1743
normalized size	1	1.00	1.58	3.69	0.00	4.84	0.00	2.48	10.08
time (sec)	N/A	0.275	3.464	0.544	0.000	0.799	0.000	0.241	12.408

Problem 1138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	176	600	0	1042	0	275	3001
normalized size	1	1.00	1.01	3.43	0.00	5.95	0.00	1.57	17.15
time (sec)	N/A	0.288	1.778	0.832	0.000	1.418	0.000	0.230	13.505

Problem 1139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	184	489	0	1064	0	273	956
normalized size	1	1.00	1.01	2.69	0.00	5.85	0.00	1.50	5.25
time (sec)	N/A	0.472	2.551	0.789	0.000	0.936	0.000	0.258	9.878

Problem 1140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	319	642	0	1560	0	395	1100
normalized size	1	1.00	1.46	2.94	0.00	7.16	0.00	1.81	5.05
time (sec)	N/A	0.772	6.184	0.907	0.000	1.200	0.000	0.287	9.904

Problem 1141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	459	780	0	2027	0	451	1261
normalized size	1	1.00	1.59	2.70	0.00	7.01	0.00	1.56	4.36
time (sec)	N/A	1.099	6.204	0.885	0.000	1.681	0.000	0.292	10.044

Problem 1142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	347	889	0	2592	0	550	1487
normalized size	1	1.00	1.02	2.61	0.00	7.62	0.00	1.62	4.37
time (sec)	N/A	1.459	4.765	0.892	0.000	1.882	0.000	0.348	10.215

Problem 1143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	327	1619	0	0	0	0	-1
normalized size	1	1.00	0.71	3.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.048	5.098	2.166	0.000	0.947	0.000	0.000	0.000

Problem 1144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	326	1356	0	0	0	0	-1
normalized size	1	1.00	0.98	4.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.634	4.116	1.944	0.000	0.757	0.000	0.000	0.000

Problem 1145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	435	1155	0	0	0	0	-1
normalized size	1	1.00	1.29	3.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.892	3.506	2.193	0.000	61.372	0.000	0.000	0.000

Problem 1146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	422	656	0	0	0	0	-1
normalized size	1	1.00	1.31	2.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.876	3.650	1.895	0.000	0.000	0.000	0.000	0.000

Problem 1147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	450	1364	0	0	0	0	-1
normalized size	1	1.00	1.30	3.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.898	3.576	2.073	0.000	0.000	0.000	0.000	0.000

Problem 1148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	473	1495	0	0	0	0	-1
normalized size	1	1.00	1.35	4.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.862	5.706	2.132	0.000	76.644	0.000	0.000	0.000

Problem 1149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	643	1761	0	0	0	0	-1
normalized size	1	1.00	1.56	4.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.235	6.622	2.456	0.000	0.000	0.000	0.000	0.000

Problem 1150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	702	2075	0	0	0	0	-1
normalized size	1	1.00	1.45	4.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.622	6.697	2.503	0.000	0.000	0.000	0.000	0.000

Problem 1151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	528	528	382	1801	0	0	0	0	-1
normalized size	1	1.00	0.72	3.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.208	15.662	2.032	0.000	1.353	0.000	0.000	0.000

Problem 1152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	382	1619	0	0	0	0	-1
normalized size	1	1.00	0.97	4.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.859	12.072	1.956	0.000	1.174	0.000	0.000	0.000

Problem 1153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	477	1405	0	0	0	0	-1
normalized size	1	1.00	1.22	3.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.159	3.822	1.918	0.000	3.064	0.000	0.000	0.000

Problem 1154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	452	726	0	0	0	0	-1
normalized size	1	1.00	1.21	1.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.165	4.235	1.774	0.000	0.000	0.000	0.000	0.000

Problem 1155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	434	1379	0	0	0	0	-1
normalized size	1	1.00	1.13	3.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.165	3.409	2.087	0.000	0.000	0.000	0.000	0.000

Problem 1156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	600	1511	0	0	0	0	-1
normalized size	1	1.00	1.55	3.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.100	6.620	2.159	0.000	146.735	0.000	0.000	0.000

Problem 1157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	641	1760	0	0	0	0	-1
normalized size	1	1.00	1.57	4.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.237	6.703	2.352	0.000	123.545	0.000	0.000	0.000

Problem 1158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	700	2075	0	0	0	0	-1
normalized size	1	1.00	1.45	4.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.652	6.756	2.401	0.000	0.000	0.000	0.000	0.000

Problem 1159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	771	2458	0	0	0	0	-1
normalized size	1	1.00	1.40	4.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.984	6.783	3.029	0.000	0.000	0.000	0.000	0.000

Problem 1160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	450	1801	0	0	0	0	-1
normalized size	1	1.00	1.00	3.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.075	21.645	1.865	0.000	1.238	0.000	0.000	0.000

Problem 1161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	521	1573	0	0	0	0	-1
normalized size	1	1.00	1.17	3.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.422	4.022	2.042	0.000	0.000	0.000	0.000	0.000

Problem 1162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	496	865	0	0	0	0	-1
normalized size	1	1.00	1.16	2.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.422	4.786	1.953	0.000	100.796	0.000	0.000	0.000

Problem 1163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	460	1520	0	0	0	0	-1
normalized size	1	1.00	1.07	3.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.423	5.538	2.238	0.000	0.000	0.000	0.000	0.000

Problem 1164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	466	1526	0	0	0	0	-1
normalized size	1	1.00	1.09	3.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.355	3.653	2.065	0.000	124.017	0.000	0.000	0.000

Problem 1165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	655	1777	0	0	0	0	-1
normalized size	1	1.00	1.46	3.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.511	6.697	2.086	0.000	0.000	0.000	0.000	0.000

Problem 1166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	700	2075	0	0	0	0	-1
normalized size	1	1.00	1.45	4.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.623	6.840	2.418	0.000	0.000	0.000	0.000	0.000

Problem 1167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	771	2458	0	0	0	0	-1
normalized size	1	1.00	1.40	4.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.971	6.813	2.848	0.000	0.000	0.000	0.000	0.000

Problem 1168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	382	1619	0	0	0	0	-1
normalized size	1	1.00	0.81	3.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.185	5.593	1.753	0.000	1.190	0.000	0.000	0.000

Problem 1169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	326	1356	0	0	0	0	-1
normalized size	1	1.00	0.80	3.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.886	4.357	1.705	0.000	1.471	0.000	0.000	0.000

Problem 1170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	275	1190	0	0	0	0	-1
normalized size	1	1.00	0.97	4.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	3.137	1.623	0.000	0.907	0.000	0.000	0.000

Problem 1171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	408	1018	0	0	0	0	-1
normalized size	1	1.00	1.42	3.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.655	3.752	1.860	0.000	3.328	0.000	0.000	0.000

Problem 1172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	416	704	0	0	0	0	-1
normalized size	1	1.00	1.46	2.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.673	3.639	3.085	0.000	0.000	0.000	0.000	0.000

Problem 1173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	443	913	0	0	0	0	-1
normalized size	1	1.00	1.44	2.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	3.348	3.310	0.000	3.808	0.000	0.000	0.000

Problem 1174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	475	1496	0	0	0	0	-1
normalized size	1	1.00	1.35	4.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.881	5.720	2.134	0.000	0.000	0.000	0.000	0.000

Problem 1175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	647	1761	0	0	0	0	-1
normalized size	1	1.00	1.57	4.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.244	6.701	2.342	0.000	0.000	0.000	0.000	0.000

Problem 1176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	466	326	1356	0	0	0	0	-1
normalized size	1	1.00	0.70	2.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.214	6.884	1.923	0.000	1.289	0.000	0.000	0.000

Problem 1177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	275	1190	0	0	0	0	-1
normalized size	1	1.00	0.69	2.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.904	5.302	1.854	0.000	1.490	0.000	0.000	0.000

Problem 1178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	222	943	0	0	0	0	-1
normalized size	1	1.00	0.85	3.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.424	4.157	1.651	0.000	2.181	0.000	0.000	0.000

Problem 1179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	419	1010	0	0	0	0	-1
normalized size	1	1.00	1.42	3.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.678	3.866	1.875	0.000	2.645	0.000	0.000	0.000

Problem 1180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	433	618	0	0	0	0	-1
normalized size	1	1.00	1.47	2.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.705	3.602	1.690	0.000	0.000	0.000	0.000	0.000

Problem 1181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	435	1349	0	0	0	0	-1
normalized size	1	1.00	1.19	3.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.928	5.034	2.016	0.000	0.000	0.000	0.000	0.000

Problem 1182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	468	1496	0	0	0	0	-1
normalized size	1	1.00	1.12	3.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.155	5.846	2.180	0.000	1.387	0.000	0.000	0.000

Problem 1183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	1044	2033	0	0	0	0	-1
normalized size	1	1.00	2.23	4.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.181	9.831	2.103	0.000	1.246	0.000	0.000	0.000

Problem 1184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	257	1642	0	0	0	0	-1
normalized size	1	1.00	0.63	4.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.918	7.964	1.903	0.000	1.199	0.000	0.000	0.000

Problem 1185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	211	1430	0	0	0	0	-1
normalized size	1	1.00	0.83	5.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.440	6.278	1.921	0.000	1.170	0.000	0.000	0.000

Problem 1186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	443	1375	0	0	0	0	-1
normalized size	1	1.00	1.42	4.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.681	5.181	6.696	0.000	73.817	0.000	0.000	0.000

Problem 1187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	445	2112	0	0	0	0	-1
normalized size	1	1.00	1.29	6.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.988	5.464	2.187	0.000	0.000	0.000	0.000	0.000

Problem 1188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	622	2617	0	0	0	0	-1
normalized size	1	1.00	1.53	6.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.265	6.708	2.429	0.000	0.000	0.000	0.000	0.000

Problem 1189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	680	2870	0	0	0	0	-1
normalized size	1	1.00	1.48	6.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.564	6.970	2.438	0.000	0.000	0.000	0.000	0.000

Problem 1190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	510	510	1670	24365	0	0	0	0	-1
normalized size	1	1.00	3.27	47.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.910	6.554	1.160	0.000	1.011	0.000	0.000	0.000

Problem 1191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.149	24.520	0.938	0.000	28.001	0.000	0.000	0.000

Problem 1192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.101	6.496	1.063	0.000	1.136	0.000	0.000	0.000

Problem 1193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.114	4.995	1.596	0.000	0.825	0.000	0.000	0.000

Problem 1194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.112	5.650	1.397	0.000	0.897	0.000	0.000	0.000

Problem 1195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	623	623	195	0	0	0	0	0	-1
normalized size	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.759	0.849	28.104	0.000	1.215	0.000	0.000	0.000

Problem 1196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	167	0	0	0	0	0	-1
normalized size	1	1.00	0.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.120	0.341	17.906	0.000	1.033	0.000	0.000	0.000

Problem 1197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	111	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.162	7.331	0.000	0.794	0.000	0.000	0.000

Problem 1198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	105	138	72	106	136	133	71
normalized size	1	1.00	1.08	1.42	0.74	1.09	1.40	1.37	0.73
time (sec)	N/A	0.130	0.396	0.265	0.316	0.908	32.617	0.319	0.083

Problem 1199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	94	120	72	95	136	118	71
normalized size	1	1.00	0.97	1.24	0.74	0.98	1.40	1.22	0.73
time (sec)	N/A	0.120	0.306	0.267	0.320	0.663	20.995	0.319	0.058

Problem 1200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	105	102	72	84	136	133	71
normalized size	1	1.00	1.30	1.26	0.89	1.04	1.68	1.64	0.88
time (sec)	N/A	0.136	0.289	0.265	0.314	0.831	13.677	0.252	0.057

Problem 1201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	94	84	72	73	136	118	71
normalized size	1	1.00	1.16	1.04	0.89	0.90	1.68	1.46	0.88
time (sec)	N/A	0.139	0.304	0.267	0.315	0.581	8.394	0.243	0.055

Problem 1202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	86	64	72	62	90	103	71
normalized size	1	1.00	1.32	0.98	1.11	0.95	1.38	1.58	1.09
time (sec)	N/A	0.098	0.221	0.271	0.312	0.582	5.068	0.212	11.594

Problem 1203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	94	69	74	0	70	126
normalized size	1	1.00	1.00	1.09	0.80	0.86	0.00	0.81	1.47
time (sec)	N/A	0.081	0.033	0.384	0.315	0.752	0.000	0.208	12.003

Problem 1204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	116	69	91	0	79	250
normalized size	1	1.00	1.00	1.40	0.83	1.10	0.00	0.95	3.01
time (sec)	N/A	0.093	0.032	0.344	0.313	0.813	0.000	0.187	11.927

Problem 1205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	139	68	102	0	82	229
normalized size	1	1.00	0.90	1.62	0.79	1.19	0.00	0.95	2.66
time (sec)	N/A	0.092	0.227	0.385	0.316	1.259	0.000	0.227	11.885

Problem 1206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	159	69	117	0	81	218
normalized size	1	1.00	0.89	1.87	0.81	1.38	0.00	0.95	2.56
time (sec)	N/A	0.085	0.151	0.385	0.329	0.669	0.000	0.206	11.911

Problem 1207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	87	136	69	110	0	82	207
normalized size	1	1.00	1.07	1.68	0.85	1.36	0.00	1.01	2.56
time (sec)	N/A	0.053	0.265	0.408	0.315	0.570	0.000	0.225	12.261

Problem 1208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	92	160	72	124	0	84	193
normalized size	1	1.00	1.07	1.86	0.84	1.44	0.00	0.98	2.24
time (sec)	N/A	0.084	0.185	0.404	0.316	0.797	0.000	0.220	11.803

Problem 1209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	110	70	100	0	70	69
normalized size	1	1.00	1.00	1.80	1.15	1.64	0.00	1.15	1.13
time (sec)	N/A	0.112	0.028	0.434	0.316	0.726	0.000	0.238	11.791

Problem 1210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	128	70	106	0	70	70
normalized size	1	1.00	1.00	1.97	1.08	1.63	0.00	1.08	1.08
time (sec)	N/A	0.122	0.028	0.451	0.339	0.930	0.000	0.226	11.606

Problem 1211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	88	148	70	109	0	70	70
normalized size	1	1.00	1.09	1.83	0.86	1.35	0.00	0.86	0.86
time (sec)	N/A	0.129	0.108	0.346	0.347	0.505	0.000	0.244	11.642

Problem 1212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	88	166	70	115	0	70	70
normalized size	1	1.00	1.09	2.05	0.86	1.42	0.00	0.86	0.86
time (sec)	N/A	0.131	0.117	0.460	0.319	0.616	0.000	0.273	11.669

Problem 1213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	184	70	122	0	70	70
normalized size	1	1.00	0.91	1.90	0.72	1.26	0.00	0.72	0.72
time (sec)	N/A	0.095	0.106	0.331	0.329	1.228	0.000	0.258	11.641

Problem 1214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	202	70	128	0	70	70
normalized size	1	1.00	0.91	2.08	0.72	1.32	0.00	0.72	0.72
time (sec)	N/A	0.095	0.107	0.439	0.339	0.696	0.000	0.244	11.686

Problem 1215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	169	155	108	121	214	173	108
normalized size	1	1.00	1.22	1.12	0.78	0.88	1.55	1.25	0.78
time (sec)	N/A	0.188	0.776	0.311	0.327	0.711	14.270	0.334	11.438

Problem 1216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	138	101	108	85	163	152	108
normalized size	1	1.00	1.00	0.73	0.78	0.62	1.18	1.10	0.78
time (sec)	N/A	0.128	0.684	0.311	0.314	1.047	8.696	0.267	11.468

Problem 1217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	105	119	105	96	0	118	153
normalized size	1	1.00	0.81	0.92	0.81	0.74	0.00	0.91	1.18
time (sec)	N/A	0.134	0.151	0.532	0.317	0.891	0.000	0.238	11.736

Problem 1218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	142	185	105	135	0	127	445
normalized size	1	1.00	1.14	1.48	0.84	1.08	0.00	1.02	3.56
time (sec)	N/A	0.159	0.052	0.451	0.330	0.657	0.000	0.242	12.016

Problem 1219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	103	197	104	160	0	140	331
normalized size	1	1.00	0.81	1.55	0.82	1.26	0.00	1.10	2.61
time (sec)	N/A	0.157	0.295	0.535	0.330	1.349	0.000	0.258	11.684

Problem 1220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	103	255	103	158	0	127	315
normalized size	1	1.00	0.86	2.12	0.86	1.32	0.00	1.06	2.62
time (sec)	N/A	0.141	0.277	0.503	0.331	0.994	0.000	0.279	11.712

Problem 1221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	107	220	105	177	0	138	310
normalized size	1	1.00	0.85	1.75	0.83	1.40	0.00	1.10	2.46
time (sec)	N/A	0.092	0.738	0.534	0.318	0.742	0.000	0.289	11.688

Problem 1222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	105	279	105	166	0	131	297
normalized size	1	1.00	0.85	2.25	0.85	1.34	0.00	1.06	2.40
time (sec)	N/A	0.140	0.187	0.541	0.320	0.640	0.000	0.268	11.713

Problem 1223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	107	196	108	183	0	134	274
normalized size	1	1.00	0.82	1.51	0.83	1.41	0.00	1.03	2.11
time (sec)	N/A	0.158	0.173	0.565	0.342	1.023	0.000	0.299	11.865

Problem 1224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	104	218	106	146	0	118	105
normalized size	1	1.00	0.81	1.69	0.82	1.13	0.00	0.91	0.81
time (sec)	N/A	0.159	0.200	0.557	0.329	0.858	0.000	0.285	11.794

Problem 1225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	108	173	106	148	0	118	107
normalized size	1	1.00	0.78	1.25	0.77	1.07	0.00	0.86	0.78
time (sec)	N/A	0.161	0.249	0.562	0.333	0.744	0.000	0.327	11.776

Problem 1226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	264	342	218	279	0	300	375
normalized size	1	1.00	1.12	1.46	0.93	1.19	0.00	1.28	1.60
time (sec)	N/A	0.281	2.192	0.546	0.320	0.863	0.000	0.227	0.135

Problem 1227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	225	285	184	246	0	249	254
normalized size	1	1.00	1.17	1.48	0.95	1.27	0.00	1.29	1.32
time (sec)	N/A	0.239	1.493	0.522	0.335	1.050	0.000	0.204	11.458

Problem 1228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	188	229	148	203	0	194	161
normalized size	1	1.00	1.20	1.46	0.94	1.29	0.00	1.24	1.03
time (sec)	N/A	0.155	1.036	0.550	0.332	1.037	0.000	0.193	0.088

Problem 1229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	111	169	118	189	0	154	338
normalized size	1	1.00	0.92	1.41	0.98	1.58	0.00	1.28	2.82
time (sec)	N/A	0.159	0.515	0.757	0.325	0.813	0.000	0.193	12.076

Problem 1230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	95	151	120	214	0	131	313
normalized size	1	1.00	0.87	1.39	1.10	1.96	0.00	1.20	2.87
time (sec)	N/A	0.170	0.621	0.796	0.333	0.802	0.000	0.218	12.101

Problem 1231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	116	189	147	335	0	190	280
normalized size	1	1.00	0.89	1.44	1.12	2.56	0.00	1.45	2.14
time (sec)	N/A	0.200	0.756	0.884	0.337	1.165	0.000	0.231	11.923

Problem 1232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	127	209	158	401	0	211	319
normalized size	1	1.00	0.86	1.42	1.07	2.73	0.00	1.44	2.17
time (sec)	N/A	0.206	1.899	0.839	0.321	0.846	0.000	0.250	11.717

Problem 1233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	187	282	189	542	0	278	439
normalized size	1	1.00	0.99	1.50	1.01	2.88	0.00	1.48	2.34
time (sec)	N/A	0.171	6.139	0.882	0.367	0.853	0.000	0.265	11.805

Problem 1234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	220	343	225	696	0	332	628
normalized size	1	1.00	0.97	1.52	1.00	3.08	0.00	1.47	2.78
time (sec)	N/A	0.261	3.170	0.903	0.352	0.997	0.000	0.243	11.902

Problem 1235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	139	0	178	572	0	0	887
normalized size	1	1.00	0.82	0.00	1.05	3.36	0.00	0.00	5.22
time (sec)	N/A	0.213	0.763	26.341	0.371	1.003	0.000	0.000	18.921

Problem 1236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	97	0	109	282	8675	0	550
normalized size	1	1.00	0.79	0.00	0.89	2.29	70.53	0.00	4.47
time (sec)	N/A	0.137	0.209	11.377	0.329	0.842	100.868	0.000	16.337

Problem 1237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	133	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.338	0.582	3.267	0.000	0.796	0.000	0.000	0.000

Problem 1238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	143	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.365	0.441	8.674	0.000	0.882	0.000	0.000	0.000

Problem 1239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	210	225	135	161	488	214	441
normalized size	1	1.00	0.88	0.95	0.57	0.68	2.05	0.90	1.85
time (sec)	N/A	0.372	2.232	0.326	0.339	0.889	77.817	0.787	15.014

Problem 1240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	202	237	137	182	656	226	207
normalized size	1	1.00	0.81	0.95	0.55	0.73	2.62	0.90	0.83
time (sec)	N/A	0.355	1.469	0.327	0.427	0.782	54.310	0.768	13.555

Problem 1241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	197	171	115	128	384	217	386
normalized size	1	1.00	1.05	0.91	0.61	0.68	2.05	1.16	2.06
time (sec)	N/A	0.315	1.133	0.326	0.338	0.782	35.401	0.566	14.944

Problem 1242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	193	183	127	149	529	189	237
normalized size	1	1.00	0.96	0.91	0.63	0.74	2.63	0.94	1.18
time (sec)	N/A	0.267	0.902	0.326	0.668	0.893	23.775	0.363	11.928

Problem 1243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	161	115	92	97	282	176	332
normalized size	1	1.00	1.06	0.76	0.61	0.64	1.86	1.16	2.18
time (sec)	N/A	0.212	1.020	0.325	0.994	0.733	14.944	0.281	15.045

Problem 1244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	166	160	122	137	0	291	415
normalized size	1	1.00	1.06	1.02	0.78	0.87	0.00	1.85	2.64
time (sec)	N/A	0.207	0.285	0.549	0.553	1.076	0.000	0.270	13.710

Problem 1245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	220	250	172	188	0	368	683
normalized size	1	1.00	1.24	1.40	0.97	1.06	0.00	2.07	3.84
time (sec)	N/A	0.463	0.401	0.460	0.765	0.756	0.000	0.267	11.894

Problem 1246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	250	261	190	244	0	346	484
normalized size	1	1.00	1.39	1.45	1.06	1.36	0.00	1.92	2.69
time (sec)	N/A	0.305	6.188	0.548	0.586	1.236	0.000	0.329	11.757

Problem 1247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	336	321	189	252	0	366	665
normalized size	1	1.00	1.90	1.81	1.07	1.42	0.00	2.07	3.76
time (sec)	N/A	0.440	6.283	0.516	0.583	0.678	0.000	0.308	12.096

Problem 1248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	337	334	205	309	0	346	479
normalized size	1	1.00	1.94	1.92	1.18	1.78	0.00	1.99	2.75
time (sec)	N/A	0.278	6.180	0.540	0.503	0.797	0.000	0.327	11.743

Problem 1249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	351	302	183	306	0	337	888
normalized size	1	1.00	1.74	1.50	0.91	1.51	0.00	1.67	4.40
time (sec)	N/A	0.188	1.121	0.558	0.556	0.819	0.000	0.308	16.040

Problem 1250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	384	318	219	360	0	337	985
normalized size	1	1.00	2.19	1.82	1.25	2.06	0.00	1.93	5.63
time (sec)	N/A	0.261	1.049	0.456	0.423	0.565	0.000	0.349	15.062

Problem 1251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	280	222	153	320	0	356	379
normalized size	1	1.00	1.77	1.41	0.97	2.03	0.00	2.25	2.40
time (sec)	N/A	0.414	1.397	0.459	0.418	1.034	0.000	0.335	11.871

Problem 1252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	282	333	220	338	0	402	343
normalized size	1	1.00	1.77	2.09	1.38	2.13	0.00	2.53	2.16
time (sec)	N/A	0.319	0.814	0.455	0.337	0.861	0.000	0.370	12.689

Problem 1253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	204	232	154	291	0	408	373
normalized size	1	1.00	1.35	1.54	1.02	1.93	0.00	2.70	2.47
time (sec)	N/A	0.405	1.161	0.465	0.332	0.734	0.000	0.339	11.864

Problem 1254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	244	404	272	455	0	468	394
normalized size	1	1.00	1.16	1.92	1.30	2.17	0.00	2.23	1.88
time (sec)	N/A	0.345	1.471	0.462	0.334	0.793	0.000	0.402	11.934

Problem 1255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	250	303	196	363	0	502	448
normalized size	1	1.00	1.26	1.53	0.99	1.83	0.00	2.54	2.26
time (sec)	N/A	0.457	1.672	0.456	0.341	0.816	0.000	0.360	16.332

Problem 1256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	531	2076	0	871	0	965	3724
normalized size	1	1.00	1.01	3.95	0.00	1.66	0.00	1.84	7.09
time (sec)	N/A	1.909	9.179	0.568	0.000	1.096	0.000	0.268	18.113

Problem 1257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	462	1817	0	814	0	835	4067
normalized size	1	1.00	0.98	3.86	0.00	1.73	0.00	1.77	8.63
time (sec)	N/A	1.550	8.166	0.542	0.000	1.012	0.000	0.242	16.382

Problem 1258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	371	1321	0	697	0	593	3135
normalized size	1	1.00	1.61	5.72	0.00	3.02	0.00	2.57	13.57
time (sec)	N/A	0.513	4.355	0.466	0.000	1.241	0.000	0.226	14.530

Problem 1259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	207	778	0	698	0	353	5197
normalized size	1	1.00	0.78	2.92	0.00	2.62	0.00	1.33	19.54
time (sec)	N/A	0.351	1.741	0.698	0.000	1.266	0.000	0.259	13.601

Problem 1260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	215	680	0	901	0	384	5214
normalized size	1	1.00	0.85	2.68	0.00	3.55	0.00	1.51	20.53
time (sec)	N/A	0.342	2.758	0.770	0.000	1.500	0.000	0.224	13.483

Problem 1261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	315	618	0	1210	0	463	4294
normalized size	1	1.00	1.25	2.46	0.00	4.82	0.00	1.84	17.11
time (sec)	N/A	0.336	6.194	0.809	0.000	1.744	0.000	0.278	12.902

Problem 1262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	428	678	0	1437	0	399	4740
normalized size	1	1.00	1.49	2.36	0.00	5.01	0.00	1.39	16.52
time (sec)	N/A	0.377	6.296	0.771	0.000	1.535	0.000	0.276	12.781

Problem 1263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	487	718	0	1576	0	475	1117
normalized size	1	1.00	1.61	2.37	0.00	5.20	0.00	1.57	3.69
time (sec)	N/A	1.211	6.227	0.808	0.000	1.537	0.000	0.288	12.032

Problem 1264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	361	897	0	2011	0	596	1424
normalized size	1	1.00	0.85	2.12	0.00	4.74	0.00	1.41	3.36
time (sec)	N/A	1.521	1.606	0.823	0.000	2.054	0.000	0.289	12.161

Problem 1265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	447	1048	0	2588	0	736	1810
normalized size	1	1.00	0.93	2.18	0.00	5.39	0.00	1.53	3.77
time (sec)	N/A	1.959	1.724	0.816	0.000	2.349	0.000	0.297	12.579

Problem 1266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	2015	2174	0	1128	0	968	4362
normalized size	1	1.00	3.76	4.06	0.00	2.10	0.00	1.81	8.14
time (sec)	N/A	2.199	15.307	0.586	0.000	1.304	0.000	0.336	48.182

Problem 1267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	517	1676	0	995	0	724	3700
normalized size	1	1.00	1.07	3.46	0.00	2.05	0.00	1.49	7.63
time (sec)	N/A	1.717	12.867	0.645	0.000	1.247	0.000	0.283	23.853

Problem 1268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	1250	1325	0	837	0	581	2529
normalized size	1	1.00	5.27	5.59	0.00	3.53	0.00	2.45	10.67
time (sec)	N/A	0.465	8.054	0.571	0.000	0.910	0.000	0.247	16.795

Problem 1269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	243	988	0	1007	0	635	5354
normalized size	1	1.00	0.61	2.48	0.00	2.52	0.00	1.59	13.42
time (sec)	N/A	0.517	1.950	0.802	0.000	1.442	0.000	0.255	14.551

Problem 1270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	332	903	0	1171	0	461	4223
normalized size	1	1.00	1.06	2.88	0.00	3.73	0.00	1.47	13.45
time (sec)	N/A	0.493	6.209	0.816	0.000	1.411	0.000	0.275	13.695

Problem 1271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	384	943	0	1658	0	512	4381
normalized size	1	1.00	0.97	2.39	0.00	4.20	0.00	1.30	11.09
time (sec)	N/A	0.523	6.260	0.870	0.000	1.719	0.000	0.292	13.019

Problem 1272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	490	873	0	1553	0	478	1082
normalized size	1	1.00	1.49	2.65	0.00	4.72	0.00	1.45	3.29
time (sec)	N/A	1.329	6.267	0.856	0.000	1.460	0.000	0.293	12.413

Problem 1273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	363	1070	0	2022	0	603	1275
normalized size	1	1.00	1.02	3.01	0.00	5.70	0.00	1.70	3.59
time (sec)	N/A	1.682	1.385	0.906	0.000	1.298	0.000	0.344	12.554

Problem 1274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	448	1252	0	2571	0	731	1614
normalized size	1	1.00	0.91	2.54	0.00	5.23	0.00	1.49	3.28
time (sec)	N/A	2.155	1.787	0.902	0.000	2.197	0.000	0.335	12.626

Problem 1275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	728	1576	0	3687	0	1018	2795
normalized size	1	1.00	1.21	2.63	0.00	6.14	0.00	1.70	4.66
time (sec)	N/A	3.220	3.146	0.923	0.000	2.656	0.000	0.391	13.286

Problem 1276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	712	712	1906	56846	0	0	0	0	-1
normalized size	1	1.00	2.68	79.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.645	6.975	2.755	0.000	1.122	0.000	0.000	0.000

Problem 1277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	161	127	371	0	0	0	0	-1
normalized size	1	1.01	0.80	2.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.428	0.450	0.926	0.000	0.684	0.000	0.000	0.000

Problem 1278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	105	615	0	0	0	0	-1
normalized size	1	1.00	0.54	3.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.474	0.658	0.928	0.000	0.854	0.000	0.000	0.000

Problem 1279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	66	73	67	71	105	68	64
normalized size	1	1.00	0.87	0.96	0.88	0.93	1.38	0.89	0.84
time (sec)	N/A	0.092	0.199	0.178	0.321	0.997	1.714	0.160	0.068

Problem 1280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	54	49	47	87	50	47
normalized size	1	1.00	0.89	0.98	0.89	0.85	1.58	0.91	0.85
time (sec)	N/A	0.080	0.108	0.176	0.314	0.807	1.051	0.160	0.065

Problem 1281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	35	33	31	66	34	31
normalized size	1	1.00	0.97	1.03	0.97	0.91	1.94	1.00	0.91
time (sec)	N/A	0.050	0.022	0.122	0.338	1.033	0.667	0.163	0.055

Problem 1282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	35	33	31	0	35	48
normalized size	1	1.00	1.00	1.03	0.97	0.91	0.00	1.03	1.41
time (sec)	N/A	0.041	0.017	0.220	0.313	0.923	0.000	0.147	11.826

Problem 1283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	35	47	56	0	49	89
normalized size	1	1.00	1.00	0.70	0.94	1.12	0.00	0.98	1.78
time (sec)	N/A	0.072	0.039	0.194	0.335	0.777	0.000	0.155	11.812

Problem 1284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	73	66	100	0	71	132
normalized size	1	1.00	1.00	1.01	0.92	1.39	0.00	0.99	1.83
time (sec)	N/A	0.091	0.049	0.250	0.332	0.908	0.000	0.157	11.785

Problem 1285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	177	871	0	427	0	467	376
normalized size	1	1.00	0.75	3.71	0.00	1.82	0.00	1.99	1.60
time (sec)	N/A	0.913	1.766	0.318	0.000	0.718	0.000	0.170	13.390

Problem 1286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	146	657	0	380	0	366	2616
normalized size	1	1.00	0.76	3.44	0.00	1.99	0.00	1.92	13.70
time (sec)	N/A	0.664	1.113	0.278	0.000	0.861	0.000	0.182	13.348

Problem 1287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	130	318	0	315	0	207	225
normalized size	1	1.00	0.88	2.15	0.00	2.13	0.00	1.40	1.52
time (sec)	N/A	0.459	0.263	0.274	0.000	0.908	0.000	0.158	12.478

Problem 1288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	104	214	0	275	0	159	190
normalized size	1	1.00	1.04	2.14	0.00	2.75	0.00	1.59	1.90
time (sec)	N/A	0.165	0.233	0.237	0.000	0.854	0.000	0.157	12.069

Problem 1289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	90	137	0	262	0	94	896
normalized size	1	1.00	1.20	1.83	0.00	3.49	0.00	1.25	11.95
time (sec)	N/A	0.185	0.098	0.426	0.000	1.069	0.000	0.166	11.975

Problem 1290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	108	155	0	314	0	129	204
normalized size	1	1.00	1.35	1.94	0.00	3.92	0.00	1.61	2.55
time (sec)	N/A	0.253	0.238	0.437	0.000	0.857	0.000	0.196	11.940

Problem 1291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	181	166	0	472	0	198	790
normalized size	1	1.00	1.59	1.46	0.00	4.14	0.00	1.74	6.93
time (sec)	N/A	0.456	0.848	0.480	0.000	1.045	0.000	0.191	12.453

Problem 1292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	351	250	0	591	0	270	749
normalized size	1	1.00	2.29	1.63	0.00	3.86	0.00	1.76	4.90
time (sec)	N/A	0.674	6.232	0.455	0.000	1.132	0.000	0.188	13.513

Problem 1293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	430	315	0	808	0	336	873
normalized size	1	1.00	2.22	1.62	0.00	4.16	0.00	1.73	4.50
time (sec)	N/A	0.953	6.265	0.490	0.000	1.016	0.000	0.203	12.069

Problem 1294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	506	439	0	959	0	444	1007
normalized size	1	1.00	2.13	1.84	0.00	4.03	0.00	1.87	4.23
time (sec)	N/A	1.227	1.851	0.495	0.000	0.949	0.000	0.199	12.081

Problem 1295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	127	182	131	127	0	149	133
normalized size	1	1.00	0.85	1.22	0.88	0.85	0.00	1.00	0.89
time (sec)	N/A	0.199	0.283	0.296	0.333	0.788	0.000	0.173	0.083

Problem 1296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	104	144	105	97	0	117	107
normalized size	1	1.00	0.87	1.21	0.88	0.82	0.00	0.98	0.90
time (sec)	N/A	0.172	0.356	0.294	0.314	0.534	0.000	0.155	11.623

Problem 1297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	79	106	79	78	0	85	78
normalized size	1	1.00	0.89	1.19	0.89	0.88	0.00	0.96	0.88
time (sec)	N/A	0.114	0.201	0.213	0.322	0.687	0.000	0.164	0.068

Problem 1298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	68	54	55	0	56	98
normalized size	1	1.00	0.90	1.15	0.92	0.93	0.00	0.95	1.66
time (sec)	N/A	0.117	0.067	0.397	0.323	0.713	0.000	0.179	11.861

Problem 1299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	72	57	69	0	59	118
normalized size	1	1.00	0.90	1.20	0.95	1.15	0.00	0.98	1.97
time (sec)	N/A	0.122	0.080	0.393	0.308	0.656	0.000	0.167	11.857

Problem 1300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	106	77	118	0	88	144
normalized size	1	1.00	0.77	1.26	0.92	1.40	0.00	1.05	1.71
time (sec)	N/A	0.092	0.156	0.454	0.310	0.786	0.000	0.176	11.758

Problem 1301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	274	1501	0	526	0	726	600
normalized size	1	1.00	0.97	5.32	0.00	1.87	0.00	2.57	2.13
time (sec)	N/A	1.001	2.207	0.314	0.000	1.029	0.000	0.213	14.637

Problem 1302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	186	941	0	457	0	458	511
normalized size	1	1.00	0.79	4.00	0.00	1.94	0.00	1.95	2.17
time (sec)	N/A	0.717	2.044	0.307	0.000	0.789	0.000	0.172	13.369

Problem 1303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	155	760	0	414	0	371	453
normalized size	1	1.00	0.97	4.78	0.00	2.60	0.00	2.33	2.85
time (sec)	N/A	0.325	1.079	0.247	0.000	0.709	0.000	0.168	12.950

Problem 1304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	143	334	0	350	0	183	1320
normalized size	1	1.00	1.15	2.69	0.00	2.82	0.00	1.48	10.65
time (sec)	N/A	0.288	0.263	0.441	0.000	0.867	0.000	0.191	14.031

Problem 1305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	146	249	0	396	0	221	1167
normalized size	1	1.00	1.40	2.39	0.00	3.81	0.00	2.12	11.22
time (sec)	N/A	0.270	0.778	0.438	0.000	1.059	0.000	0.201	13.781

Problem 1306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	204	286	0	572	0	217	2718
normalized size	1	1.00	1.66	2.33	0.00	4.65	0.00	1.76	22.10
time (sec)	N/A	0.301	1.686	0.485	0.000	0.959	0.000	0.204	12.676

Problem 1307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	350	348	0	633	0	273	654
normalized size	1	1.00	2.27	2.26	0.00	4.11	0.00	1.77	4.25
time (sec)	N/A	0.446	6.125	0.456	0.000	0.876	0.000	0.205	12.424

Problem 1308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	433	455	0	904	0	375	953
normalized size	1	1.00	2.19	2.30	0.00	4.57	0.00	1.89	4.81
time (sec)	N/A	0.761	6.211	0.486	0.000	1.351	0.000	0.214	12.156

Problem 1309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	507	583	0	1051	0	484	1082
normalized size	1	1.00	2.08	2.39	0.00	4.31	0.00	1.98	4.43
time (sec)	N/A	1.050	1.824	0.493	0.000	1.832	0.000	0.219	12.181

Problem 1310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	180	329	205	199	0	261	236
normalized size	1	1.00	0.85	1.55	0.97	0.94	0.00	1.23	1.11
time (sec)	N/A	0.238	1.267	0.296	0.308	0.680	0.000	0.202	0.132

Problem 1311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	153	273	172	164	0	213	191
normalized size	1	1.00	0.85	1.52	0.96	0.91	0.00	1.18	1.06
time (sec)	N/A	0.207	0.718	0.294	0.315	0.771	0.000	0.185	11.782

Problem 1312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	128	215	139	142	0	165	150
normalized size	1	1.00	0.86	1.45	0.94	0.96	0.00	1.11	1.01
time (sec)	N/A	0.141	0.638	0.223	0.310	0.899	0.000	0.177	0.075

Problem 1313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	101	140	99	104	0	106	254
normalized size	1	1.00	0.94	1.31	0.93	0.97	0.00	0.99	2.37
time (sec)	N/A	0.140	0.143	0.425	0.301	0.962	0.000	0.169	12.098

Problem 1314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	86	124	91	133	0	105	233
normalized size	1	1.00	0.90	1.29	0.95	1.39	0.00	1.09	2.43
time (sec)	N/A	0.155	0.176	0.423	0.301	0.723	0.000	0.202	12.029

Problem 1315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	97	140	99	174	0	130	238
normalized size	1	1.00	0.92	1.33	0.94	1.66	0.00	1.24	2.27
time (sec)	N/A	0.180	0.390	0.483	0.314	0.679	0.000	0.204	11.980

Problem 1316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	110	163	113	198	0	151	227
normalized size	1	1.00	0.92	1.36	0.94	1.65	0.00	1.26	1.89
time (sec)	N/A	0.174	0.285	0.447	0.305	0.894	0.000	0.218	11.928

Problem 1317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	115	216	139	271	0	201	281
normalized size	1	1.00	0.78	1.46	0.94	1.83	0.00	1.36	1.90
time (sec)	N/A	0.133	1.043	0.481	0.306	0.713	0.000	0.239	11.828

Problem 1318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	179	274	170	346	0	251	381
normalized size	1	1.00	1.00	1.53	0.95	1.93	0.00	1.40	2.13
time (sec)	N/A	0.205	6.121	0.487	0.307	1.002	0.000	0.210	11.917

Problem 1319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	165	330	206	464	0	301	514
normalized size	1	1.00	0.78	1.56	0.97	2.19	0.00	1.42	2.42
time (sec)	N/A	0.241	2.860	0.554	0.318	0.735	0.000	0.229	12.314

Problem 1320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	467	467	403	2587	0	706	0	1244	4505
normalized size	1	1.00	0.86	5.54	0.00	1.51	0.00	2.66	9.65
time (sec)	N/A	1.814	3.318	0.317	0.000	0.961	0.000	0.208	15.077

Problem 1321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	324	1808	0	619	0	863	3797
normalized size	1	1.00	0.79	4.43	0.00	1.52	0.00	2.12	9.31
time (sec)	N/A	1.458	3.069	0.301	0.000	0.911	0.000	0.190	14.463

Problem 1322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	275	1551	0	570	0	735	3683
normalized size	1	1.00	1.21	6.80	0.00	2.50	0.00	3.22	16.15
time (sec)	N/A	0.515	2.257	0.255	0.000	1.031	0.000	0.175	14.383

Problem 1323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	220	827	0	508	0	398	4866
normalized size	1	1.00	0.87	3.28	0.00	2.02	0.00	1.58	19.31
time (sec)	N/A	0.286	0.513	0.446	0.000	1.241	0.000	0.229	13.399

Problem 1324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	208	557	0	549	0	302	4892
normalized size	1	1.00	1.14	3.04	0.00	3.00	0.00	1.65	26.73
time (sec)	N/A	0.255	1.405	0.437	0.000	1.282	0.000	0.195	13.099

Problem 1325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	259	483	0	770	0	431	4898
normalized size	1	1.00	1.49	2.78	0.00	4.43	0.00	2.48	28.15
time (sec)	N/A	0.385	5.264	0.483	0.000	1.817	0.000	0.211	13.091

Problem 1326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	379	442	0	801	0	317	4712
normalized size	1	1.00	1.92	2.24	0.00	4.07	0.00	1.61	23.92
time (sec)	N/A	0.275	6.198	0.463	0.000	1.952	0.000	0.216	15.568

Problem 1327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	275	448	523	0	1034	0	396	4334
normalized size	1	1.41	2.30	2.68	0.00	5.30	0.00	2.03	22.23
time (sec)	N/A	0.290	6.191	0.490	0.000	1.510	0.000	0.240	13.398

Problem 1328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	307	504	629	0	1079	0	490	1099
normalized size	1	1.27	2.09	2.61	0.00	4.48	0.00	2.03	4.56
time (sec)	N/A	1.120	1.394	0.492	0.000	1.588	0.000	0.227	12.389

Problem 1329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	356	780	0	1462	0	627	1289
normalized size	1	1.00	0.98	2.15	0.00	4.03	0.00	1.73	3.55
time (sec)	N/A	1.484	1.518	0.506	0.000	2.155	0.000	0.257	12.337

Problem 1330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	442	952	0	1645	0	776	1513
normalized size	1	1.00	1.06	2.28	0.00	3.94	0.00	1.86	3.63
time (sec)	N/A	1.833	1.990	0.519	0.000	1.817	0.000	0.242	12.385

Problem 1331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	593	1143	0	2082	0	948	1861
normalized size	1	1.00	1.25	2.40	0.00	4.37	0.00	1.99	3.91
time (sec)	N/A	2.215	3.587	0.506	0.000	3.459	0.000	0.262	12.665

Problem 1332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	83	95	82	100	0	85	134
normalized size	1	1.00	0.89	1.02	0.88	1.08	0.00	0.91	1.44
time (sec)	N/A	0.181	0.208	0.367	0.311	0.602	0.000	0.211	12.272

Problem 1333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	72	81	68	74	0	71	117
normalized size	1	1.00	0.90	1.01	0.85	0.92	0.00	0.89	1.46
time (sec)	N/A	0.156	0.066	0.358	0.383	1.045	0.000	0.210	11.940

Problem 1334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	87	76	65	63	0	71	91
normalized size	1	1.00	1.18	1.03	0.88	0.85	0.00	0.96	1.23
time (sec)	N/A	0.064	0.081	0.293	0.314	0.685	0.000	0.205	12.020

Problem 1335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	84	95	80	93	0	86	87
normalized size	1	1.00	0.90	1.02	0.86	1.00	0.00	0.92	0.94
time (sec)	N/A	0.146	0.110	0.400	0.312	1.032	0.000	0.212	0.171

Problem 1336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	113	95	143	0	113	98
normalized size	1	1.00	0.88	1.03	0.86	1.30	0.00	1.03	0.89
time (sec)	N/A	0.187	0.243	0.416	0.321	0.946	0.000	0.205	11.919

Problem 1337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	144	114	224	0	148	125
normalized size	1	1.00	1.00	1.09	0.86	1.70	0.00	1.12	0.95
time (sec)	N/A	0.202	0.532	0.480	0.307	1.245	0.000	0.232	11.855

Problem 1338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	221	283	0	521	0	208	2098
normalized size	1	1.00	0.82	1.06	0.00	1.94	0.00	0.78	7.83
time (sec)	N/A	0.396	1.532	0.428	0.000	1.039	0.000	0.219	16.815

Problem 1339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	186	162	0	431	0	173	1656
normalized size	1	1.00	1.02	0.89	0.00	2.36	0.00	0.95	9.05
time (sec)	N/A	0.273	1.135	0.369	0.000	0.781	0.000	0.208	14.278

Problem 1340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	152	138	0	369	0	131	1538
normalized size	1	1.00	1.14	1.04	0.00	2.77	0.00	0.98	11.56
time (sec)	N/A	0.178	0.836	0.405	0.000	0.748	0.000	0.287	14.046

Problem 1341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	152	117	0	305	0	107	148
normalized size	1	1.00	1.58	1.22	0.00	3.18	0.00	1.11	1.54
time (sec)	N/A	0.107	0.188	0.378	0.000	0.796	0.000	0.253	11.964

Problem 1342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	151	116	0	308	0	106	151
normalized size	1	1.00	1.84	1.41	0.00	3.76	0.00	1.29	1.84
time (sec)	N/A	0.099	0.181	0.303	0.000	0.659	0.000	0.216	12.019

Problem 1343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	191	130	0	457	0	135	659
normalized size	1	1.00	1.62	1.10	0.00	3.87	0.00	1.14	5.58
time (sec)	N/A	0.230	0.377	0.439	0.000	0.997	0.000	0.207	12.883

Problem 1344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	150	205	169	0	582	0	259	778
normalized size	1	1.17	1.60	1.32	0.00	4.55	0.00	2.02	6.08
time (sec)	N/A	0.288	1.028	0.440	0.000	1.016	0.000	0.209	13.682

Problem 1345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	212	261	227	0	878	0	245	1570
normalized size	1	1.17	1.44	1.25	0.00	4.85	0.00	1.35	8.67
time (sec)	N/A	0.360	3.096	0.501	0.000	1.907	0.000	0.243	15.348

Problem 1346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	117	164	142	157	0	177	217
normalized size	1	1.00	0.93	1.30	1.13	1.25	0.00	1.40	1.72
time (sec)	N/A	0.202	0.552	0.407	0.320	0.888	0.000	0.241	12.291

Problem 1347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	108	123	132	154	0	168	206
normalized size	1	1.00	0.93	1.06	1.14	1.33	0.00	1.45	1.78
time (sec)	N/A	0.231	0.472	0.435	0.314	0.925	0.000	0.231	12.216

Problem 1348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	162	123	132	155	0	170	208
normalized size	1	1.00	1.38	1.05	1.13	1.32	0.00	1.45	1.78
time (sec)	N/A	0.165	0.405	0.355	0.317	0.856	0.000	0.234	12.114

Problem 1349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	151	181	156	213	0	210	170
normalized size	1	1.00	0.97	1.16	1.00	1.37	0.00	1.35	1.09
time (sec)	N/A	0.242	0.693	0.494	0.317	1.641	0.000	0.232	12.417

Problem 1350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	174	199	200	287	0	279	195
normalized size	1	1.00	1.02	1.16	1.17	1.68	0.00	1.63	1.14
time (sec)	N/A	0.270	0.778	0.494	0.322	1.850	0.000	0.237	12.403

Problem 1351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	168	231	244	440	0	275	240
normalized size	1	1.00	0.85	1.17	1.24	2.23	0.00	1.40	1.22
time (sec)	N/A	0.312	1.442	0.541	0.322	3.049	0.000	0.267	12.479

Problem 1352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	195	269	0	476	0	241	372
normalized size	1	1.00	1.10	1.52	0.00	2.69	0.00	1.36	2.10
time (sec)	N/A	0.229	1.417	0.392	0.000	0.740	0.000	0.272	17.428

Problem 1353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	184	222	0	465	0	227	370
normalized size	1	1.00	1.30	1.56	0.00	3.27	0.00	1.60	2.61
time (sec)	N/A	0.239	1.446	0.418	0.000	0.928	0.000	0.330	17.158

Problem 1354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	200	224	0	470	0	229	375
normalized size	1	1.00	1.21	1.36	0.00	2.85	0.00	1.39	2.27
time (sec)	N/A	0.238	1.283	0.413	0.000	0.849	0.000	0.318	17.402

Problem 1355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	203	272	0	469	0	240	378
normalized size	1	1.00	1.47	1.97	0.00	3.40	0.00	1.74	2.74
time (sec)	N/A	0.223	1.305	0.353	0.000	0.661	0.000	0.232	17.061

Problem 1356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	334	279	0	680	0	308	2162
normalized size	1	1.00	1.72	1.44	0.00	3.51	0.00	1.59	11.14
time (sec)	N/A	0.409	4.877	0.489	0.000	2.402	0.000	0.239	16.203

Problem 1357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	247	450	317	0	831	0	357	2317
normalized size	1	1.12	2.05	1.44	0.00	3.78	0.00	1.62	10.53
time (sec)	N/A	0.474	6.436	0.484	0.000	1.812	0.000	0.227	17.384

Problem 1358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	947	376	0	1182	0	417	5035
normalized size	1	1.00	2.85	1.13	0.00	3.56	0.00	1.26	15.17
time (sec)	N/A	0.548	6.266	0.552	0.000	3.095	0.000	0.240	18.007

Problem 1359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	212	338	316	429	0	403	806
normalized size	1	1.00	0.88	1.41	1.32	1.79	0.00	1.68	3.36
time (sec)	N/A	0.618	3.008	0.487	0.380	1.743	0.000	0.303	14.185

Problem 1360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	198	321	303	351	0	384	627
normalized size	1	1.00	0.90	1.45	1.37	1.59	0.00	1.74	2.84
time (sec)	N/A	0.539	2.496	0.487	0.326	1.620	0.000	0.313	14.068

Problem 1361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	187	308	289	303	0	371	549
normalized size	1	1.00	0.90	1.48	1.39	1.46	0.00	1.78	2.64
time (sec)	N/A	0.512	1.752	0.484	0.335	1.044	0.000	0.294	13.381

Problem 1362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	184	304	288	261	0	343	498
normalized size	1	1.00	0.90	1.49	1.41	1.28	0.00	1.68	2.44
time (sec)	N/A	0.351	1.405	0.451	0.326	0.945	0.000	0.309	12.542

Problem 1363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	169	260	276	261	0	333	507
normalized size	1	1.00	0.89	1.37	1.45	1.37	0.00	1.75	2.67
time (sec)	N/A	0.431	1.584	0.398	0.510	0.972	0.000	0.307	12.455

Problem 1364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	166	261	267	260	0	326	471
normalized size	1	1.00	0.91	1.43	1.47	1.43	0.00	1.79	2.59
time (sec)	N/A	0.373	1.246	0.415	0.321	1.139	0.000	0.291	12.448

Problem 1365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	163	262	265	258	0	325	498
normalized size	1	1.00	0.92	1.47	1.49	1.45	0.00	1.83	2.80
time (sec)	N/A	0.380	1.239	0.409	0.330	1.206	0.000	0.280	12.515

Problem 1366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	244	259	267	255	0	323	483
normalized size	1	1.00	1.38	1.46	1.51	1.44	0.00	1.82	2.73
time (sec)	N/A	0.243	0.939	0.350	0.323	0.998	0.000	0.266	12.402

Problem 1367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	220	321	299	344	0	391	346
normalized size	1	1.00	0.94	1.38	1.28	1.48	0.00	1.68	1.48
time (sec)	N/A	0.365	2.890	0.454	0.341	4.106	0.000	0.234	12.428

Problem 1368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	257	340	361	425	0	418	373
normalized size	1	1.00	1.03	1.36	1.44	1.70	0.00	1.67	1.49
time (sec)	N/A	0.405	6.212	0.475	0.347	4.810	0.000	0.246	12.473

Problem 1369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	281	371	422	640	0	589	412
normalized size	1	1.00	1.03	1.35	1.54	2.34	0.00	2.15	1.50
time (sec)	N/A	0.474	6.256	0.551	0.348	7.882	0.000	0.265	12.699

Problem 1370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	500	500	816	1674	0	0	0	0	-1
normalized size	1	1.00	1.63	3.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.237	26.854	6.597	0.000	0.000	0.000	0.000	0.000

Problem 1371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	789	2363	0	0	0	0	-1
normalized size	1	1.00	1.76	5.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.960	26.688	7.479	0.000	0.000	0.000	0.000	0.000

Problem 1372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	372	924	0	0	0	0	-1
normalized size	1	1.00	1.01	2.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.872	16.877	6.204	0.000	0.000	0.000	0.000	0.000

Problem 1373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	351	884	0	0	0	0	-1
normalized size	1	1.00	1.03	2.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.735	19.796	6.866	0.000	0.000	0.000	0.000	0.000

Problem 1374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	534	188	0	0	0	0	-1
normalized size	1	1.00	1.50	0.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.833	14.363	2.938	0.000	0.000	0.000	0.000	0.000

Problem 1375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	1550	1266	0	0	0	0	-1
normalized size	1	1.00	3.58	2.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.939	27.104	13.133	0.000	0.000	0.000	0.000	0.000

Problem 1376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	1582	307	0	0	0	0	-1
normalized size	1	1.00	2.91	0.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.057	29.185	3.251	0.000	0.000	0.000	0.000	0.000

Problem 1377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	1991	3600	0	0	0	0	-1
normalized size	1	1.00	3.21	5.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.541	27.839	10.475	0.000	0.000	0.000	0.000	0.000

Problem 1378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	1953	1778	0	0	0	0	-1
normalized size	1	1.00	3.80	3.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.214	27.429	8.438	0.000	0.000	0.000	0.000	0.000

Problem 1379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	1909	2432	0	0	0	0	-1
normalized size	1	1.00	4.48	5.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.983	26.955	7.276	0.000	0.000	0.000	0.000	0.000

Problem 1380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	484	216	0	0	0	0	-1
normalized size	1	1.00	1.10	0.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.138	6.332	2.939	0.000	0.000	0.000	0.000	0.000

Problem 1381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	2099	2324	0	0	0	0	-1
normalized size	1	1.00	4.48	4.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.257	27.110	13.642	0.000	0.000	0.000	0.000	0.000

Problem 1382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	2129	312	0	0	0	0	-1
normalized size	1	1.00	3.71	0.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.370	29.476	3.247	0.000	0.000	0.000	0.000	0.000

Problem 1383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	867	4548	0	0	0	0	-1
normalized size	1	1.00	1.42	7.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.359	27.201	8.449	0.000	0.000	0.000	0.000	0.000

Problem 1384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	824	1937	0	0	0	0	-1
normalized size	1	1.00	1.64	3.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.194	27.116	6.956	0.000	0.000	0.000	0.000	0.000

Problem 1385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	737	2612	0	0	0	0	-1
normalized size	1	1.00	1.78	6.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.972	24.809	7.894	0.000	0.000	0.000	0.000	0.000

Problem 1386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	484	259	0	0	0	0	-1
normalized size	1	1.00	1.14	0.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.149	24.460	3.168	0.000	0.000	0.000	0.000	0.000

Problem 1387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	462	462	1556	1987	0	0	0	0	-1
normalized size	1	1.00	3.37	4.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.261	27.278	14.199	0.000	0.000	0.000	0.000	0.000

Problem 1388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	1590	318	0	0	0	0	-1
normalized size	1	1.00	2.85	0.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.343	29.424	3.071	0.000	124.003	0.000	0.000	0.000

Problem 1389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	1953	1455	0	0	0	0	-1
normalized size	1	1.00	3.84	2.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.506	26.918	6.867	0.000	0.000	0.000	0.000	0.000

Problem 1390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	1915	2015	0	0	0	0	-1
normalized size	1	1.00	4.19	4.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.180	26.524	7.313	0.000	0.000	0.000	0.000	0.000

Problem 1391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	1326	855	0	0	0	0	-1
normalized size	1	1.00	3.49	2.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.931	25.414	6.547	0.000	0.000	0.000	0.000	0.000

Problem 1392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	546	1181	0	0	0	0	-1
normalized size	1	1.00	1.55	3.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.725	6.204	6.053	0.000	0.000	0.000	0.000	0.000

Problem 1393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	698	186	0	0	0	0	-1
normalized size	1	1.00	1.89	0.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.818	20.619	3.056	0.000	0.000	0.000	0.000	0.000

Problem 1394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	2093	1217	0	0	0	0	-1
normalized size	1	1.00	4.67	2.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.975	30.002	13.885	0.000	0.000	0.000	0.000	0.000

Problem 1395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	2129	315	0	0	0	0	-1
normalized size	1	1.00	3.82	0.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.032	30.846	3.456	0.000	0.000	0.000	0.000	0.000

Problem 1396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	584	584	820	1990	0	0	0	0	-1
normalized size	1	1.00	1.40	3.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.276	26.819	8.634	0.000	0.000	0.000	0.000	0.000

Problem 1397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	793	1613	0	0	0	0	-1
normalized size	1	1.00	1.56	3.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.067	26.860	11.944	0.000	0.000	0.000	0.000	0.000

Problem 1398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	785	1105	0	0	0	0	-1
normalized size	1	1.00	1.73	2.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.881	17.006	8.012	0.000	0.000	0.000	0.000	0.000

Problem 1399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	783	1938	0	0	0	0	-1
normalized size	1	1.00	1.90	4.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.953	16.533	9.045	0.000	0.000	0.000	0.000	0.000

Problem 1400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	507	507	1587	425	0	0	0	0	-1
normalized size	1	1.00	3.13	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.361	27.980	4.584	0.000	0.000	0.000	0.000	0.000

Problem 1401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	627	627	1635	3469	0	0	0	0	-1
normalized size	1	1.00	2.61	5.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.493	28.436	17.192	0.000	0.000	0.000	0.000	0.000

Problem 1402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	1958	1268	0	0	0	0	-1
normalized size	1	1.00	3.26	2.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.398	26.837	11.508	0.000	0.000	0.000	0.000	0.000

Problem 1403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	528	528	1193	2331	0	0	0	0	-1
normalized size	1	1.00	2.26	4.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.086	23.865	13.090	0.000	0.000	0.000	0.000	0.000

Problem 1404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	1184	1089	0	0	0	0	-1
normalized size	1	1.00	2.53	2.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.901	23.345	10.387	0.000	0.000	0.000	0.000	0.000

Problem 1405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	1183	2322	0	0	0	0	-1
normalized size	1	1.00	2.74	5.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.921	23.249	13.246	0.000	0.000	0.000	0.000	0.000

Problem 1406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	527	527	2136	627	0	0	0	0	-1
normalized size	1	1.00	4.05	1.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.314	30.573	5.241	0.000	0.000	0.000	0.000	0.000

Problem 1407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	651	651	2183	2312	0	0	0	0	-1
normalized size	1	1.00	3.35	3.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.464	29.961	19.530	0.000	0.000	0.000	0.000	0.000

Problem 1408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	926	926	1623	4649	0	0	0	0	-1
normalized size	1	1.00	1.75	5.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.842	27.587	1.004	0.000	0.000	0.000	0.000	0.000

Problem 1409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	578	578	176	3290	0	0	0	0	-1
normalized size	1	1.00	0.30	5.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.134	18.097	0.839	0.000	0.000	0.000	0.000	0.000

Problem 1410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	178	744	0	0	0	0	-1
normalized size	1	1.00	0.35	1.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.835	11.032	0.856	0.000	0.000	0.000	0.000	0.000

Problem 1411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	182	590	0	0	0	0	-1
normalized size	1	1.00	0.88	2.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.415	7.826	0.665	0.000	0.000	0.000	0.000	0.000

Problem 1412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	1619	2502	0	0	0	0	-1
normalized size	1	1.00	5.06	7.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.748	24.724	0.700	0.000	0.000	0.000	0.000	0.000

Problem 1413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	1645	2672	0	0	0	0	-1
normalized size	1	1.00	4.49	7.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.029	22.078	0.733	0.000	0.000	0.000	0.000	0.000

Problem 1414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	1726	6208	0	0	0	0	-1
normalized size	1	1.00	3.36	12.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.454	23.613	0.750	0.000	0.000	0.000	0.000	0.000

Problem 1415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	598	598	1768	6593	0	0	0	0	-1
normalized size	1	1.00	2.96	11.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.824	22.551	0.854	0.000	0.000	0.000	0.000	0.000

Problem 1416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	982	982	1898	2547	0	0	0	0	-1
normalized size	1	1.00	1.93	2.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.691	28.809	0.945	0.000	0.000	0.000	0.000	0.000

Problem 1417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	611	611	604	1926	0	0	0	0	-1
normalized size	1	1.00	0.99	3.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.030	21.405	0.809	0.000	0.000	0.000	0.000	0.000

Problem 1418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	178	944	0	0	0	0	-1
normalized size	1	1.00	0.31	1.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.972	11.747	0.632	0.000	0.000	0.000	0.000	0.000

Problem 1419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	1095	2587	0	0	0	0	-1
normalized size	1	1.00	3.41	8.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.702	20.498	0.656	0.000	0.000	0.000	0.000	0.000

Problem 1420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	1138	3014	0	0	0	0	-1
normalized size	1	1.00	2.62	6.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.016	21.506	0.646	0.000	0.000	0.000	0.000	0.000

Problem 1421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	1165	5828	0	0	0	0	-1
normalized size	1	1.00	2.22	11.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.353	21.557	0.717	0.000	0.000	0.000	0.000	0.000

Problem 1422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	688	688	1210	6707	0	0	0	0	-1
normalized size	1	1.00	1.76	9.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.779	21.220	0.812	0.000	0.000	0.000	0.000	0.000

Problem 1423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	936	936	1615	6311	0	0	0	0	-1
normalized size	1	1.00	1.73	6.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.600	27.035	0.963	0.000	0.000	0.000	0.000	0.000

Problem 1424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	572	572	1399	5224	0	0	0	0	-1
normalized size	1	1.00	2.45	9.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.023	25.806	0.686	0.000	0.000	0.000	0.000	0.000

Problem 1425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	616	616	1611	5212	0	0	0	0	-1
normalized size	1	1.00	2.62	8.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.136	26.649	0.739	0.000	0.000	0.000	0.000	0.000

Problem 1426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	1656	4668	0	0	0	0	-1
normalized size	1	1.00	4.61	13.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.812	26.354	0.677	0.000	0.000	0.000	0.000	0.000

Problem 1427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	1734	10138	0	0	0	0	-1
normalized size	1	1.00	3.34	19.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.210	24.614	0.758	0.000	0.000	0.000	0.000	0.000

Problem 1428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	612	612	1776	10704	0	0	0	0	-1
normalized size	1	1.00	2.90	17.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.584	23.746	0.836	0.000	0.000	0.000	0.000	0.000

Problem 1429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	822	822	1850	17102	0	0	0	0	-1
normalized size	1	1.00	2.25	20.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.141	25.018	0.982	0.000	0.000	0.000	0.000	0.000

Problem 1430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	616	616	1318	2344	0	0	0	0	-1
normalized size	1	1.00	2.14	3.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.141	27.075	0.891	0.000	0.000	0.000	0.000	0.000

Problem 1431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	518	941	0	0	0	0	-1
normalized size	1	1.00	1.02	1.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.772	16.452	0.867	0.000	0.000	0.000	0.000	0.000

Problem 1432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	166	527	0	0	0	0	-1
normalized size	1	1.00	0.79	2.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.354	4.442	0.800	0.000	0.000	0.000	0.000	0.000

Problem 1433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	209	631	0	0	0	0	-1
normalized size	1	1.00	0.77	2.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.586	10.098	0.681	0.000	0.000	0.000	0.000	0.000

Problem 1434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	225	2286	0	0	0	0	-1
normalized size	1	1.00	0.70	7.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.835	10.801	0.718	0.000	0.000	0.000	0.000	0.000

Problem 1435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	1140	2987	0	0	0	0	-1
normalized size	1	1.00	2.69	7.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.164	20.225	0.780	0.000	0.000	0.000	0.000	0.000

Problem 1436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1064	1064	1290	4619	0	0	0	0	-1
normalized size	1	1.00	1.21	4.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.624	80.658	0.891	0.000	0.000	0.000	0.000	0.000

Problem 1437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	1648	2540	0	0	0	0	-1
normalized size	1	1.00	4.35	6.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.827	23.875	0.885	0.000	0.000	0.000	0.000	0.000

Problem 1438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	1274	2536	0	0	0	0	-1
normalized size	1	1.00	3.41	6.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.911	23.120	0.892	0.000	0.000	0.000	0.000	0.000

Problem 1439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	1279	2559	0	0	0	0	-1
normalized size	1	1.00	3.37	6.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.920	21.806	0.716	0.000	0.000	0.000	0.000	0.000

Problem 1440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	1707	3104	0	0	0	0	-1
normalized size	1	1.00	3.01	5.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.413	25.007	0.721	0.000	0.000	0.000	0.000	0.000

Problem 1441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	1727	3315	0	0	0	0	-1
normalized size	1	1.00	2.57	4.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.811	24.369	0.778	0.000	0.000	0.000	0.000	0.000

Problem 1442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	717	3490	0	0	0	0	-1
normalized size	1	1.00	2.17	10.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.793	16.088	1.432	0.000	0.000	0.000	0.000	0.000

Problem 1443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	104	75	72	0	119	112
normalized size	1	1.00	1.00	1.27	0.91	0.88	0.00	1.45	1.37
time (sec)	N/A	0.137	0.416	0.413	0.439	0.711	0.000	0.214	18.223

Problem 1444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	63	94	62	61	0	104	98
normalized size	1	1.00	0.97	1.45	0.95	0.94	0.00	1.60	1.51
time (sec)	N/A	0.107	0.187	0.404	0.424	0.779	0.000	0.182	15.966

Problem 1445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	47	59	39	47	0	58	55
normalized size	1	1.00	1.24	1.55	1.03	1.24	0.00	1.53	1.45
time (sec)	N/A	0.064	0.032	0.309	0.434	0.532	0.000	0.174	12.480

Problem 1446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	36	32	32	36	0	43	41
normalized size	1	1.00	1.33	1.19	1.19	1.33	0.00	1.59	1.52
time (sec)	N/A	0.049	0.016	0.172	0.504	0.650	0.000	0.168	11.935

Problem 1447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	56	47	48	65	0	48	52
normalized size	1	1.00	1.56	1.31	1.33	1.81	0.00	1.33	1.44
time (sec)	N/A	0.080	0.032	0.415	0.349	0.426	0.000	0.201	11.936

Problem 1448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	68	69	59	96	0	103	92
normalized size	1	1.00	1.42	1.44	1.23	2.00	0.00	2.15	1.92
time (sec)	N/A	0.111	0.077	0.400	0.332	0.431	0.000	0.202	11.907

Problem 1449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	172	93	84	128	0	116	127
normalized size	1	1.00	2.29	1.24	1.12	1.71	0.00	1.55	1.69
time (sec)	N/A	0.137	0.343	0.457	0.329	0.436	0.000	0.203	11.929

Problem 1450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	104	147	97	91	0	172	149
normalized size	1	1.00	1.11	1.56	1.03	0.97	0.00	1.83	1.59
time (sec)	N/A	0.183	0.427	0.547	0.616	0.429	0.000	0.224	18.494

Problem 1451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	77	116	83	81	0	137	145
normalized size	1	1.00	0.82	1.23	0.88	0.86	0.00	1.46	1.54
time (sec)	N/A	0.139	0.426	0.452	0.428	0.422	0.000	0.214	17.039

Problem 1452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	66	75	56	59	0	82	81
normalized size	1	1.00	1.57	1.79	1.33	1.40	0.00	1.95	1.93
time (sec)	N/A	0.064	0.310	0.312	0.456	0.445	0.000	0.173	12.253

Problem 1453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	70	58	68	64	77	0	57	62
normalized size	1	1.52	1.26	1.48	1.39	1.67	0.00	1.24	1.35
time (sec)	N/A	0.168	0.197	0.546	0.362	0.451	0.000	0.212	11.849

Problem 1454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	102	90	71	113	0	128	108
normalized size	1	1.00	1.73	1.53	1.20	1.92	0.00	2.17	1.83
time (sec)	N/A	0.281	0.347	0.599	0.449	0.445	0.000	0.207	11.872

Problem 1455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	124	238	140	123	186	0	157	148
normalized size	1	1.24	2.38	1.40	1.23	1.86	0.00	1.57	1.48
time (sec)	N/A	0.251	0.485	0.562	0.453	0.462	0.000	0.229	11.854

Problem 1456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	196	162	123	192	0	204	194
normalized size	1	1.00	1.88	1.56	1.18	1.85	0.00	1.96	1.87
time (sec)	N/A	0.231	0.995	0.572	0.381	0.446	0.000	0.244	11.862

Problem 1457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	147	214	164	135	0	336	323
normalized size	1	1.00	0.75	1.09	0.83	0.69	0.00	1.71	1.64
time (sec)	N/A	0.261	0.757	0.640	0.476	0.443	0.000	0.235	16.023

Problem 1458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	113	169	119	116	0	207	249
normalized size	1	1.00	0.77	1.16	0.82	0.79	0.00	1.42	1.71
time (sec)	N/A	0.209	0.742	0.552	0.468	0.434	0.000	0.204	16.657

Problem 1459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	91	132	99	90	0	148	219
normalized size	1	1.00	1.21	1.76	1.32	1.20	0.00	1.97	2.92
time (sec)	N/A	0.072	0.550	0.452	0.478	0.416	0.000	0.198	16.288

Problem 1460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	83	100	86	99	0	86	154
normalized size	1	1.00	1.06	1.28	1.10	1.27	0.00	1.10	1.97
time (sec)	N/A	0.152	0.300	0.602	0.413	0.443	0.000	0.211	11.930

Problem 1461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	114	111	90	131	0	148	120
normalized size	1	1.00	1.31	1.28	1.03	1.51	0.00	1.70	1.38
time (sec)	N/A	0.195	0.396	0.651	0.376	0.429	0.000	0.220	11.906

Problem 1462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	267	161	137	196	0	179	166
normalized size	1	1.00	2.02	1.22	1.04	1.48	0.00	1.36	1.26
time (sec)	N/A	0.228	0.572	0.732	0.325	0.443	0.000	0.257	11.913

Problem 1463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	287	209	162	252	0	245	218
normalized size	1	1.00	1.75	1.27	0.99	1.54	0.00	1.49	1.33
time (sec)	N/A	0.262	1.359	0.629	0.370	0.432	0.000	0.275	11.888

Problem 1464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	236	303	0	724	0	264	4797
normalized size	1	1.00	1.06	1.36	0.00	3.26	0.00	1.19	21.61
time (sec)	N/A	0.364	1.977	0.546	0.000	0.511	0.000	0.269	19.769

Problem 1465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	162	219	0	534	0	222	276
normalized size	1	1.00	0.76	1.03	0.00	2.52	0.00	1.05	1.30
time (sec)	N/A	0.298	0.975	0.532	0.000	0.507	0.000	0.238	15.276

Problem 1466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	169	282	0	569	0	251	313
normalized size	1	1.00	0.84	1.41	0.00	2.84	0.00	1.26	1.56
time (sec)	N/A	0.308	0.911	0.510	0.000	0.504	0.000	0.235	15.961

Problem 1467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	169	280	0	557	0	243	310
normalized size	1	1.00	1.27	2.11	0.00	4.19	0.00	1.83	2.33
time (sec)	N/A	0.211	0.857	0.444	0.000	0.472	0.000	0.224	16.322

Problem 1468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	203	304	0	957	0	314	2076
normalized size	1	1.00	0.89	1.33	0.00	4.18	0.00	1.37	9.07
time (sec)	N/A	0.310	2.221	0.592	0.000	1.302	0.000	0.237	14.862

Problem 1469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	254	346	0	1355	0	523	2151
normalized size	1	1.00	1.02	1.40	0.00	5.46	0.00	2.11	8.67
time (sec)	N/A	0.372	3.247	0.618	0.000	1.273	0.000	0.256	13.332

Problem 1470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	356	404	0	1844	0	423	2302
normalized size	1	1.00	1.21	1.37	0.00	6.25	0.00	1.43	7.80
time (sec)	N/A	0.395	6.476	0.656	0.000	2.304	0.000	0.262	14.093

Problem 1471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	195	590	0	906	0	351	585
normalized size	1	1.00	0.50	1.52	0.00	2.34	0.00	0.90	1.51
time (sec)	N/A	0.580	3.188	0.543	0.000	0.523	0.000	0.351	18.233

Problem 1472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	204	702	0	917	0	377	583
normalized size	1	1.00	0.56	1.92	0.00	2.51	0.00	1.03	1.59
time (sec)	N/A	0.491	3.511	0.551	0.000	0.531	0.000	0.314	17.851

Problem 1473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	212	766	0	934	0	384	627
normalized size	1	1.00	0.61	2.19	0.00	2.67	0.00	1.10	1.79
time (sec)	N/A	0.561	3.165	0.559	0.000	0.513	0.000	0.312	18.207

Problem 1474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	206	643	0	895	0	365	624
normalized size	1	1.00	1.01	3.15	0.00	4.39	0.00	1.79	3.06
time (sec)	N/A	0.364	3.031	0.514	0.000	0.515	0.000	0.316	18.107

Problem 1475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	322	787	0	1623	0	411	2999
normalized size	1	1.00	0.80	1.96	0.00	4.04	0.00	1.02	7.46
time (sec)	N/A	0.510	6.592	0.663	0.000	3.024	0.000	0.297	17.794

Problem 1476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	379	829	0	2140	0	633	3122
normalized size	1	1.00	0.89	1.96	0.00	5.05	0.00	1.49	7.36
time (sec)	N/A	0.590	6.352	0.674	0.000	2.623	0.000	0.298	15.777

Problem 1477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	432	897	0	2672	0	900	3266
normalized size	1	1.00	0.92	1.91	0.00	5.69	0.00	1.91	6.95
time (sec)	N/A	0.608	6.779	0.727	0.000	4.571	0.000	0.371	16.824

Problem 1478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	198	666	0	0	0	0	-1
normalized size	1	1.00	1.25	4.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.265	6.226	0.723	0.000	0.435	0.000	0.000	0.000

Problem 1479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	F	F	F(-1)	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	0	602	2377	0	0	0	0	-1
normalized size	1	0.00	1.93	7.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.190	22.905	0.611	0.000	0.434	0.000	0.000	0.000

Problem 1480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	F	F	F(-1)	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	366	0	667	2490	0	0	0	0	-1
normalized size	1	0.00	1.82	6.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.391	22.946	0.716	0.000	0.448	0.000	0.000	0.000

Problem 1481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	156	219	132	149	0	135	346
normalized size	1	1.00	1.01	1.41	0.85	0.96	0.00	0.87	2.23
time (sec)	N/A	0.169	0.595	0.243	0.300	0.452	0.000	0.290	12.271

Problem 1482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	133	205	121	138	0	124	304
normalized size	1	1.00	0.99	1.52	0.90	1.02	0.00	0.92	2.25
time (sec)	N/A	0.133	0.361	0.238	0.307	0.452	0.000	0.281	12.157

Problem 1483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	123	147	108	114	0	108	261
normalized size	1	1.00	1.06	1.27	0.93	0.98	0.00	0.93	2.25
time (sec)	N/A	0.109	0.315	0.243	0.306	0.440	0.000	0.265	12.099

Problem 1484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	106	133	100	104	0	100	221
normalized size	1	1.00	1.03	1.29	0.97	1.01	0.00	0.97	2.15
time (sec)	N/A	0.120	0.314	0.235	0.407	0.447	0.000	0.290	12.030

Problem 1485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	84	114	89	93	0	81	144
normalized size	1	1.00	1.14	1.54	1.20	1.26	0.00	1.09	1.95
time (sec)	N/A	0.133	0.269	0.233	0.404	0.436	0.000	0.274	18.080

Problem 1486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	100	86	91	0	78	144
normalized size	1	1.00	1.00	1.35	1.16	1.23	0.00	1.05	1.95
time (sec)	N/A	0.139	0.024	0.230	0.350	0.439	0.000	0.373	18.069

Problem 1487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	92	75	80	0	67	158
normalized size	1	1.00	1.00	1.24	1.01	1.08	0.00	0.91	2.14
time (sec)	N/A	0.105	0.024	0.196	0.339	0.423	0.000	0.226	18.539

Problem 1488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	100	109	125	0	113	116
normalized size	1	1.00	1.00	1.01	1.10	1.26	0.00	1.14	1.17
time (sec)	N/A	0.105	1.263	0.431	0.375	0.444	0.000	0.242	11.874

Problem 1489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	76	120	126	159	0	134	130
normalized size	1	1.00	0.66	1.04	1.10	1.38	0.00	1.17	1.13
time (sec)	N/A	0.143	0.386	0.304	0.312	0.435	0.000	0.289	11.887

Problem 1490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	86	151	140	211	0	133	146
normalized size	1	1.00	0.64	1.12	1.04	1.56	0.00	0.99	1.08
time (sec)	N/A	0.157	0.608	0.355	0.325	0.459	0.000	0.321	0.111

Problem 1491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	90	173	151	248	0	160	157
normalized size	1	1.00	0.58	1.12	0.97	1.60	0.00	1.03	1.01
time (sec)	N/A	0.161	0.817	0.371	0.364	0.467	0.000	0.298	11.913

Problem 1492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	186	355	180	198	0	198	433
normalized size	1	1.00	0.98	1.88	0.95	1.05	0.00	1.05	2.29
time (sec)	N/A	0.359	1.558	0.320	0.305	0.504	0.000	0.306	12.333

Problem 1493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	164	270	157	178	0	175	377
normalized size	1	1.00	1.01	1.67	0.97	1.10	0.00	1.08	2.33
time (sec)	N/A	0.270	2.152	0.315	0.445	0.471	0.000	0.315	12.224

Problem 1494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	151	262	148	151	0	157	332
normalized size	1	1.00	1.01	1.75	0.99	1.01	0.00	1.05	2.21
time (sec)	N/A	0.285	1.021	0.316	0.329	0.465	0.000	0.290	12.211

Problem 1495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	129	168	123	127	0	130	247
normalized size	1	1.00	1.11	1.45	1.06	1.09	0.00	1.12	2.13
time (sec)	N/A	0.223	0.378	0.304	0.335	0.457	0.000	0.282	12.087

Problem 1496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	85	209	120	124	0	124	191
normalized size	1	1.00	0.91	2.25	1.29	1.33	0.00	1.33	2.05
time (sec)	N/A	0.194	0.769	0.297	0.308	0.438	0.000	0.265	17.052

Problem 1497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	215	122	97	104	0	89	183
normalized size	1	1.00	2.99	1.69	1.35	1.44	0.00	1.24	2.54
time (sec)	N/A	0.099	2.937	0.257	0.389	0.442	0.000	0.240	18.871

Problem 1498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	137	125	130	144	0	134	131
normalized size	1	1.00	1.09	0.99	1.03	1.14	0.00	1.06	1.04
time (sec)	N/A	0.217	0.925	0.606	0.333	0.459	0.000	0.252	0.134

Problem 1499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	162	195	163	202	0	186	169
normalized size	1	1.00	0.96	1.16	0.97	1.20	0.00	1.11	1.01
time (sec)	N/A	0.354	2.825	0.604	0.347	0.465	0.000	0.330	11.869

Problem 1500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	182	209	183	285	0	190	194
normalized size	1	1.00	0.98	1.13	0.99	1.54	0.00	1.03	1.05
time (sec)	N/A	0.384	3.663	0.545	0.407	0.460	0.000	0.335	0.135

Problem 1501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	199	420	217	238	0	251	512
normalized size	1	1.00	0.99	2.08	1.07	1.18	0.00	1.24	2.53
time (sec)	N/A	0.341	1.019	0.336	0.355	0.492	0.000	0.344	12.119

Problem 1502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	174	385	190	208	0	221	449
normalized size	1	1.00	0.98	2.18	1.07	1.18	0.00	1.25	2.54
time (sec)	N/A	0.368	0.567	0.331	0.325	0.487	0.000	0.316	12.109

Problem 1503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	147	297	173	176	0	188	356
normalized size	1	1.00	1.04	2.09	1.22	1.24	0.00	1.32	2.51
time (sec)	N/A	0.304	0.444	0.325	0.338	0.469	0.000	0.293	12.094

Problem 1504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	140	263	151	156	0	168	299
normalized size	1	1.00	0.97	1.83	1.05	1.08	0.00	1.17	2.08
time (sec)	N/A	0.253	0.389	0.317	0.333	0.470	0.000	0.273	12.136

Problem 1505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	370	231	140	143	0	142	228
normalized size	1	1.00	4.11	2.57	1.56	1.59	0.00	1.58	2.53
time (sec)	N/A	0.111	1.470	0.307	0.380	0.449	0.000	0.256	16.993

Problem 1506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	157	216	160	173	0	175	169
normalized size	1	1.00	0.95	1.31	0.97	1.05	0.00	1.06	1.02
time (sec)	N/A	0.247	0.568	0.684	0.331	0.463	0.000	0.352	11.906

Problem 1507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	161	221	188	226	0	210	182
normalized size	1	1.00	0.94	1.29	1.10	1.32	0.00	1.23	1.06
time (sec)	N/A	0.371	1.449	0.641	0.342	0.467	0.000	0.297	0.134

Problem 1508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	190	285	217	337	0	240	221
normalized size	1	1.00	0.86	1.29	0.98	1.52	0.00	1.09	1.00
time (sec)	N/A	0.434	3.097	0.739	0.343	0.476	0.000	0.346	11.949

Problem 1509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	164	0	0	0	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.530	0.199	5.405	0.000	0.435	0.000	0.000	0.000

Problem 1510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	158	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.296	0.162	5.232	0.000	0.456	0.000	0.000	0.000

Problem 1511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	158	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.258	0.223	4.755	0.000	0.457	0.000	0.000	0.000

Problem 1512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.116	1.700	0.000	0.442	0.000	0.000	0.000

Problem 1513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	241	0	0	0	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.541	0.461	1.908	0.000	0.448	0.000	0.000	0.000

Problem 1514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.566	15.014	2.260	0.000	1.422	0.000	0.000	0.000

Problem 1515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	B	F	F	F(-1)	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	502	0	1600	5578	0	0	0	0	-1
normalized size	1	0.00	3.19	11.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.370	9.987	1.104	0.000	0.505	0.000	0.000	0.000

Problem 1516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	3522	0	0	0	0	0	-1
normalized size	1	1.00	7.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.242	7.085	3.051	0.000	0.572	0.000	0.000	0.000

Problem 1517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	398	0	0	0	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.651	5.132	1.068	0.000	0.515	0.000	0.000	0.000

Problem 1518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	301	0	0	0	0	0	-1
normalized size	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	2.131	0.541	0.000	0.464	0.000	0.000	0.000

Problem 1519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.211	62.988	0.993	0.000	0.000	0.000	0.000	0.000

Problem 1520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.208	46.864	5.776	0.000	0.000	0.000	0.000	0.000

Problem 1521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.131	7.466	0.988	0.000	2.658	0.000	0.000	0.000

Problem 1522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.196	36.287	0.751	0.000	1.040	0.000	0.000	0.000

Problem 1523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.512	8.849	3.253	0.000	0.560	0.000	0.000	0.000

Problem 1524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	375	373	0	0	0	0	0	0	-1
normalized size	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.635	3.567	1.188	0.000	0.482	0.000	0.000	0.000

Problem 1525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.325	0.565	0.000	0.478	0.000	0.000	0.000

Problem 1526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.141	3.334	1.752	0.000	0.474	0.000	0.000	0.000

Problem 1527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.133	35.595	6.540	0.000	0.690	0.000	0.000	0.000

Problem 1528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	151	128	151	106	228	182	156
normalized size	1	1.00	0.80	0.68	0.80	0.56	1.21	0.97	0.83
time (sec)	N/A	0.242	0.796	0.462	0.348	0.478	15.627	0.315	0.127

Problem 1529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	116	108	116	88	178	145	118
normalized size	1	1.00	0.81	0.76	0.81	0.62	1.24	1.01	0.83
time (sec)	N/A	0.171	0.295	0.461	0.528	0.460	5.837	0.242	12.057

Problem 1530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	80	88	80	70	128	100	83
normalized size	1	1.00	0.82	0.91	0.82	0.72	1.32	1.03	0.86
time (sec)	N/A	0.115	0.258	0.458	1.129	0.446	1.898	0.202	11.991

Problem 1531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	45	44	45	51	75	52	44
normalized size	1	1.00	0.87	0.85	0.87	0.98	1.44	1.00	0.85
time (sec)	N/A	0.054	0.082	0.225	0.318	0.435	0.475	0.158	12.037

Problem 1532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	68	83	64	66	0	67	53
normalized size	1	1.00	1.06	1.30	1.00	1.03	0.00	1.05	0.83
time (sec)	N/A	0.108	0.029	0.358	0.314	0.469	0.000	0.172	0.121

Problem 1533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	129	78	92	0	84	63
normalized size	1	1.00	0.92	2.19	1.32	1.56	0.00	1.42	1.07
time (sec)	N/A	0.076	0.224	0.553	0.310	0.462	0.000	0.212	0.110

Problem 1534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	82	173	112	114	0	114	91
normalized size	1	1.00	0.93	1.97	1.27	1.30	0.00	1.30	1.03
time (sec)	N/A	0.090	0.585	0.530	0.316	0.478	0.000	0.225	0.141

Problem 1535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	104	217	143	135	0	139	120
normalized size	1	1.00	0.88	1.84	1.21	1.14	0.00	1.18	1.02
time (sec)	N/A	0.105	0.882	0.556	0.313	0.472	0.000	0.262	12.442

Problem 1536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	295	229	238	174	440	279	236
normalized size	1	1.00	0.85	0.66	0.68	0.50	1.26	0.80	0.68
time (sec)	N/A	0.392	1.501	0.479	0.311	0.506	25.923	0.501	0.180

Problem 1537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	227	199	184	147	335	231	180
normalized size	1	1.00	0.98	0.86	0.80	0.64	1.45	1.00	0.78
time (sec)	N/A	0.259	0.501	0.540	0.321	0.488	10.360	0.384	0.107

Problem 1538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	111	169	128	120	228	168	127
normalized size	1	1.00	0.84	1.28	0.97	0.91	1.73	1.27	0.96
time (sec)	N/A	0.170	0.248	0.480	0.333	0.465	3.677	0.266	12.298

Problem 1539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	41	73	74	92	117	86	71
normalized size	1	1.00	0.76	1.35	1.37	1.70	2.17	1.59	1.31
time (sec)	N/A	0.076	0.069	0.240	0.307	0.437	1.016	0.172	0.066

Problem 1540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	81	161	109	111	0	129	80
normalized size	1	1.00	0.86	1.71	1.16	1.18	0.00	1.37	0.85
time (sec)	N/A	0.174	0.207	0.375	0.335	0.485	0.000	0.196	12.370

Problem 1541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	174	231	122	136	0	146	118
normalized size	1	1.00	1.55	2.06	1.09	1.21	0.00	1.30	1.05
time (sec)	N/A	0.180	1.535	0.584	0.341	0.477	0.000	0.257	12.363

Problem 1542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	186	299	171	173	0	187	181
normalized size	1	1.00	1.52	2.45	1.40	1.42	0.00	1.53	1.48
time (sec)	N/A	0.156	1.784	0.556	0.438	0.468	0.000	0.274	12.378

Problem 1543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	242	396	211	203	0	229	220
normalized size	1	1.00	1.51	2.48	1.32	1.27	0.00	1.43	1.38
time (sec)	N/A	0.205	1.538	0.583	0.446	0.487	0.000	0.269	12.445

Problem 1544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	218	689	366	366	0	511	435
normalized size	1	1.00	0.69	2.19	1.16	1.16	0.00	1.62	1.38
time (sec)	N/A	0.356	0.866	0.399	0.440	0.550	0.000	0.243	12.366

Problem 1545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	148	397	220	222	0	286	253
normalized size	1	1.00	0.73	1.97	1.09	1.10	0.00	1.42	1.25
time (sec)	N/A	0.248	0.431	0.404	0.572	0.496	0.000	0.204	0.098

Problem 1546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	89	186	112	112	0	129	122
normalized size	1	1.00	0.80	1.68	1.01	1.01	0.00	1.16	1.10
time (sec)	N/A	0.163	0.376	0.395	0.505	0.466	0.000	0.201	0.078

Problem 1547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	56	40	38	104	41	41
normalized size	1	1.00	0.95	1.37	0.98	0.93	2.54	1.00	1.00
time (sec)	N/A	0.070	0.046	0.253	0.519	0.455	0.713	0.153	0.063

Problem 1548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	99	156	79	88	0	87	89
normalized size	1	1.00	1.10	1.73	0.88	0.98	0.00	0.97	0.99
time (sec)	N/A	0.148	0.185	0.445	0.560	0.524	0.000	0.203	0.313

Problem 1549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	197	297	175	234	0	260	197
normalized size	1	1.00	1.24	1.87	1.10	1.47	0.00	1.64	1.24
time (sec)	N/A	0.290	0.765	0.496	0.726	0.795	0.000	0.315	0.523

Problem 1550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	321	586	367	413	0	539	427
normalized size	1	1.00	1.22	2.23	1.40	1.57	0.00	2.05	1.62
time (sec)	N/A	0.448	1.312	0.506	0.384	1.772	0.000	0.403	12.955

Problem 1551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	565	990	632	643	0	907	729
normalized size	1	1.00	1.48	2.58	1.65	1.68	0.00	2.37	1.90
time (sec)	N/A	0.681	2.489	0.505	0.455	4.323	0.000	0.323	13.435

Problem 1552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	396	721	377	508	0	570	682
normalized size	1	1.00	1.22	2.23	1.16	1.57	0.00	1.76	2.10
time (sec)	N/A	0.399	1.625	0.695	0.467	0.553	0.000	0.266	12.149

Problem 1553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	234	422	229	322	0	328	290
normalized size	1	1.00	1.14	2.05	1.11	1.56	0.00	1.59	1.41
time (sec)	N/A	0.272	2.162	0.661	0.386	0.540	0.000	0.245	0.120

Problem 1554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	111	202	118	178	0	188	128
normalized size	1	1.00	0.98	1.79	1.04	1.58	0.00	1.66	1.13
time (sec)	N/A	0.169	0.537	0.662	0.310	0.493	0.000	0.228	12.099

Problem 1555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	42	63	48	54	178	80	48
normalized size	1	1.00	0.88	1.31	1.00	1.12	3.71	1.67	1.00
time (sec)	N/A	0.077	0.101	0.412	0.459	0.451	1.360	0.170	0.063

Problem 1556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	178	240	147	283	0	205	131
normalized size	1	1.00	1.32	1.78	1.09	2.10	0.00	1.52	0.97
time (sec)	N/A	0.194	1.367	0.686	0.490	0.713	0.000	0.230	0.417

Problem 1557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	246	388	346	598	0	335	327
normalized size	1	1.00	1.08	1.70	1.52	2.62	0.00	1.47	1.43
time (sec)	N/A	0.326	1.717	0.805	0.522	1.347	0.000	0.292	12.662

Problem 1558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	370	675	659	881	0	761	615
normalized size	1	1.00	0.99	1.81	1.77	2.37	0.00	2.05	1.65
time (sec)	N/A	0.564	4.279	0.816	0.399	3.301	0.000	0.342	13.462

Problem 1559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	766	1080	1083	1244	0	1185	1024
normalized size	1	1.00	1.39	1.96	1.97	2.26	0.00	2.15	1.86
time (sec)	N/A	0.928	6.206	0.859	0.398	8.221	0.000	0.429	14.385

Problem 1560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.097	5.168	2.558	0.000	0.544	0.000	0.000	0.000

Problem 1561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	5085	0	0	0	0	0	-1
normalized size	1	1.00	15.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.416	34.724	3.784	0.000	0.459	0.000	0.000	0.000

Problem 1562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	12568	0	0	0	0	0	-1
normalized size	1	1.00	24.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.524	55.253	3.411	0.000	0.468	0.000	0.000	0.000

Problem 1563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	5113	0	0	0	0	0	-1
normalized size	1	1.00	16.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.616	30.048	4.095	0.000	0.440	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [1382] had the largest ratio of [.5455]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	38	0.079
2	A	3	3	1.00	38	0.079
3	A	3	3	1.00	38	0.079
4	A	3	3	1.00	38	0.079
5	A	2	2	1.00	38	0.053
6	A	5	5	1.00	38	0.132
7	A	5	5	1.00	38	0.132
8	A	2	2	1.00	38	0.053
9	A	4	3	1.00	38	0.079
10	A	4	3	1.00	38	0.079
11	A	4	3	1.00	38	0.079
12	A	3	3	1.00	38	0.079
13	A	2	2	1.00	38	0.053
14	A	6	5	1.00	38	0.132
15	A	6	6	1.00	38	0.158

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	6	5	1.00	38	0.132
17	A	2	2	1.00	38	0.053
18	A	3	3	1.00	38	0.079
19	A	5	3	1.00	38	0.079
20	A	5	3	1.00	38	0.079
21	A	4	3	1.00	38	0.079
22	A	3	3	1.00	38	0.079
23	A	2	2	1.00	38	0.053
24	A	7	5	1.00	38	0.132
25	A	7	6	1.00	38	0.158
26	A	7	6	1.00	38	0.158
27	A	7	5	1.00	38	0.132
28	A	2	2	1.00	38	0.053
29	A	3	3	1.00	38	0.079
30	A	6	3	1.00	38	0.079
31	A	6	3	1.00	38	0.079
32	A	5	3	1.00	38	0.079
33	A	4	3	1.00	38	0.079
34	A	3	3	1.00	38	0.079
35	A	2	2	1.00	38	0.053
36	A	8	5	1.00	38	0.132
37	A	8	6	1.00	38	0.158
38	A	8	6	1.00	38	0.158
39	A	8	6	1.00	38	0.158
40	A	8	5	1.00	38	0.132
41	A	2	2	1.00	38	0.053

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	3	3	1.00	38	0.079
43	A	4	3	1.00	38	0.079
44	A	2	2	1.00	38	0.053
45	A	2	2	1.00	38	0.053
46	A	2	2	1.00	38	0.053
47	A	2	2	1.00	38	0.053
48	A	4	4	1.00	38	0.105
49	A	2	2	1.00	38	0.053
50	A	8	5	1.00	38	0.132
51	A	7	5	1.00	38	0.132
52	A	6	5	1.00	38	0.132
53	A	5	5	1.00	38	0.132
54	A	4	4	1.00	38	0.105
55	A	3	3	1.00	38	0.079
56	A	4	4	1.00	38	0.105
57	A	9	6	1.00	38	0.158
58	A	8	6	1.00	38	0.158
59	A	7	6	1.00	38	0.158
60	A	6	6	1.00	38	0.158
61	A	5	5	1.00	38	0.132
62	A	2	2	1.00	38	0.053
63	A	4	4	1.00	38	0.105
64	A	5	4	1.00	38	0.105
65	A	5	5	1.00	34	0.147
66	A	4	4	1.00	34	0.118
67	A	4	4	1.00	34	0.118

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
68	A	4	4	1.00	32	0.125
69	A	3	3	1.00	21	0.143
70	A	3	3	1.00	34	0.088
71	A	4	4	1.00	34	0.118
72	A	4	4	1.00	34	0.118
73	A	5	3	1.00	36	0.083
74	A	4	3	1.00	36	0.083
75	A	3	3	1.00	36	0.083
76	A	2	2	1.00	36	0.056
77	A	4	4	1.00	36	0.111
78	A	4	4	1.00	36	0.111
79	A	2	2	1.00	36	0.056
80	A	2	2	1.00	36	0.056
81	A	4	3	1.00	38	0.079
82	A	3	3	1.00	38	0.079
83	A	2	2	1.00	38	0.053
84	A	5	5	1.00	38	0.132
85	A	5	5	1.00	38	0.132
86	A	5	5	1.00	36	0.139
87	A	5	5	1.00	38	0.132
88	A	8	4	1.00	42	0.095
89	A	7	4	1.00	42	0.095
90	A	6	4	1.00	42	0.095
91	A	5	4	1.00	42	0.095
92	A	4	4	1.00	42	0.095
93	A	4	4	1.00	42	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	5	5	1.00	42	0.119
95	A	6	5	1.00	42	0.119
96	A	7	5	1.00	42	0.119
97	A	8	4	1.00	42	0.095
98	A	7	4	1.00	42	0.095
99	A	6	4	1.00	42	0.095
100	A	5	4	1.00	42	0.095
101	A	5	5	1.00	42	0.119
102	A	5	4	1.00	42	0.095
103	A	6	5	1.00	42	0.119
104	A	7	5	1.00	42	0.119
105	A	8	5	1.00	42	0.119
106	A	9	4	1.00	42	0.095
107	A	8	4	1.00	42	0.095
108	A	7	4	1.00	42	0.095
109	A	6	4	1.00	42	0.095
110	A	6	5	1.00	42	0.119
111	A	6	5	1.00	42	0.119
112	A	6	4	1.00	42	0.095
113	A	7	5	1.00	42	0.119
114	A	8	5	1.00	42	0.119
115	A	9	5	1.00	42	0.119
116	A	10	4	1.00	42	0.095
117	A	9	4	1.00	42	0.095
118	A	8	4	1.00	42	0.095
119	A	7	4	1.00	42	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
120	A	7	5	1.00	42	0.119
121	A	7	5	1.00	42	0.119
122	A	7	5	1.00	42	0.119
123	A	7	4	1.00	42	0.095
124	A	8	5	1.00	42	0.119
125	A	9	5	1.00	42	0.119
126	A	10	5	1.00	42	0.119
127	A	6	4	1.00	42	0.095
128	A	5	4	1.00	42	0.095
129	A	4	4	1.00	42	0.095
130	A	3	3	1.00	42	0.071
131	A	4	4	1.00	42	0.095
132	A	5	4	1.00	42	0.095
133	A	6	4	1.00	42	0.095
134	A	7	5	1.00	42	0.119
135	A	6	5	1.00	42	0.119
136	A	5	5	1.00	42	0.119
137	A	4	4	1.00	42	0.095
138	A	4	4	1.00	42	0.095
139	A	5	4	1.00	42	0.095
140	A	6	4	1.00	42	0.095
141	A	7	4	1.00	42	0.095
142	A	8	5	1.00	42	0.119
143	A	7	5	1.00	42	0.119
144	A	6	5	1.00	42	0.119
145	A	5	4	1.00	42	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
146	A	5	5	1.00	42	0.119
147	A	5	4	1.00	42	0.095
148	A	6	4	1.00	42	0.095
149	A	7	4	1.00	42	0.095
150	A	8	4	1.00	42	0.095
151	A	4	4	1.00	38	0.105
152	A	4	4	1.00	38	0.105
153	A	4	4	1.00	38	0.105
154	A	4	4	1.00	36	0.111
155	A	3	3	1.00	25	0.120
156	A	4	4	1.00	38	0.105
157	A	4	4	1.00	38	0.105
158	A	4	4	1.00	38	0.105
159	A	4	4	1.00	40	0.100
160	A	4	4	1.00	40	0.100
161	A	4	4	1.00	40	0.100
162	A	4	4	1.00	40	0.100
163	A	4	4	1.00	40	0.100
164	A	4	4	1.00	40	0.100
165	A	4	4	1.00	40	0.100
166	A	4	4	1.00	40	0.100
167	A	4	4	1.00	42	0.095
168	A	4	4	1.00	42	0.095
169	A	4	4	1.00	42	0.095
170	A	4	4	1.00	40	0.100
171	A	4	4	1.00	42	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
172	A	4	4	1.00	42	0.095
173	A	4	4	1.00	36	0.111
174	A	4	4	1.00	42	0.095
175	A	3	2	1.00	40	0.050
176	A	2	2	1.00	40	0.050
177	A	1	1	1.00	40	0.025
178	A	4	4	1.00	40	0.100
179	A	4	4	1.00	40	0.100
180	A	4	4	1.00	40	0.100
181	A	3	3	1.00	40	0.075
182	A	4	4	1.00	45	0.089
183	A	4	4	1.00	45	0.089
184	A	4	4	1.00	45	0.089
185	A	1	1	1.00	43	0.023
186	A	2	2	1.00	45	0.044
187	A	3	2	1.00	45	0.044
188	A	4	2	1.00	45	0.044
189	A	5	5	1.00	36	0.139
190	A	4	3	1.00	25	0.120
191	A	4	3	1.00	23	0.130
192	A	3	2	1.00	17	0.118
193	A	4	3	1.00	23	0.130
194	A	3	3	1.00	25	0.120
195	A	4	3	1.00	25	0.120
196	A	4	3	1.00	25	0.120
197	A	4	3	1.00	27	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
198	A	4	3	1.00	25	0.120
199	A	3	2	1.00	19	0.105
200	A	4	3	1.00	25	0.120
201	A	4	3	1.00	27	0.111
202	A	3	3	1.00	27	0.111
203	A	4	3	1.00	27	0.111
204	A	4	3	1.00	27	0.111
205	A	4	3	1.00	27	0.111
206	A	4	3	1.00	27	0.111
207	A	4	3	1.00	27	0.111
208	A	4	3	1.00	25	0.120
209	A	3	2	1.00	19	0.105
210	A	4	3	1.00	25	0.120
211	A	4	3	1.00	27	0.111
212	A	4	3	1.00	27	0.111
213	A	3	3	1.00	27	0.111
214	A	4	4	1.00	27	0.148
215	A	4	3	1.00	27	0.111
216	A	4	3	1.00	27	0.111
217	A	4	3	1.00	27	0.111
218	A	4	3	1.00	27	0.111
219	A	4	3	1.00	27	0.111
220	A	4	3	1.00	25	0.120
221	A	3	2	1.00	19	0.105
222	A	4	3	1.00	25	0.120
223	A	4	3	1.00	27	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	4	3	1.00	27	0.111
225	A	4	3	1.00	27	0.111
226	A	4	3	1.00	27	0.111
227	A	4	3	1.00	25	0.120
228	A	4	4	1.00	19	0.210
229	A	4	3	1.00	25	0.120
230	A	4	3	1.00	27	0.111
231	A	4	3	1.00	27	0.111
232	A	4	3	1.00	27	0.111
233	A	4	3	1.00	27	0.111
234	A	4	3	1.00	27	0.111
235	A	4	3	1.00	25	0.120
236	A	3	2	1.00	19	0.105
237	A	4	3	1.00	25	0.120
238	A	4	3	1.00	27	0.111
239	A	4	3	1.00	27	0.111
240	A	4	3	1.00	27	0.111
241	A	4	3	1.00	27	0.111
242	A	4	3	1.00	27	0.111
243	A	4	3	1.00	27	0.111
244	A	3	3	1.00	25	0.120
245	A	3	2	1.00	19	0.105
246	A	4	3	1.00	25	0.120
247	A	4	3	1.00	27	0.111
248	A	4	3	1.00	27	0.111
249	A	4	3	1.00	27	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
250	A	4	3	1.00	27	0.111
251	A	4	3	1.00	27	0.111
252	A	3	3	1.00	27	0.111
253	A	4	3	1.00	25	0.120
254	A	3	2	1.00	19	0.105
255	A	4	3	1.00	25	0.120
256	A	4	3	1.00	27	0.111
257	A	4	4	1.00	21	0.190
258	A	3	2	1.00	27	0.074
259	A	3	2	1.00	27	0.074
260	A	3	2	1.00	27	0.074
261	A	3	2	1.00	25	0.080
262	A	2	2	1.00	27	0.074
263	A	2	2	1.00	27	0.074
264	A	2	2	1.00	27	0.074
265	A	2	2	1.00	27	0.074
266	A	8	6	1.00	27	0.222
267	A	7	6	1.00	27	0.222
268	A	6	6	1.00	25	0.240
269	A	6	6	1.00	23	0.261
270	A	7	6	1.00	19	0.316
271	A	5	5	1.00	25	0.200
272	A	5	5	1.00	27	0.185
273	A	6	6	1.00	27	0.222
274	A	7	6	1.00	27	0.222
275	A	12	7	1.00	29	0.241

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
276	A	5	4	1.00	29	0.138
277	A	5	5	1.15	27	0.185
278	A	9	8	1.00	25	0.320
279	A	8	6	1.00	21	0.286
280	A	7	6	1.00	27	0.222
281	A	8	7	1.00	29	0.241
282	A	9	6	1.00	29	0.207
283	A	10	7	1.00	29	0.241
284	A	12	6	1.00	29	0.207
285	A	15	7	1.00	29	0.241
286	A	6	5	1.14	27	0.185
287	A	12	9	1.00	25	0.360
288	A	10	7	1.00	21	0.333
289	A	10	7	1.00	27	0.259
290	A	10	6	1.00	29	0.207
291	A	11	8	1.00	29	0.276
292	A	12	7	1.00	29	0.241
293	A	14	7	1.00	29	0.241
294	A	6	4	1.00	21	0.190
295	A	15	10	1.00	25	0.400
296	A	12	6	1.00	21	0.286
297	A	6	4	1.00	29	0.138
298	A	6	4	1.00	29	0.138
299	A	5	4	1.00	29	0.138
300	A	4	4	1.00	27	0.148
301	A	3	3	1.00	25	0.120

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
302	A	4	4	1.00	21	0.190
303	A	5	5	1.00	27	0.185
304	A	5	4	1.00	29	0.138
305	A	6	4	1.00	29	0.138
306	A	6	4	1.00	29	0.138
307	A	12	7	1.00	29	0.241
308	A	9	7	1.00	29	0.241
309	A	8	7	1.00	29	0.241
310	A	3	2	1.00	27	0.074
311	A	5	4	1.00	25	0.160
312	A	7	5	1.00	21	0.238
313	A	9	7	1.00	27	0.259
314	A	11	7	1.00	29	0.241
315	A	9	7	1.00	29	0.241
316	A	4	4	1.00	29	0.138
317	A	3	3	1.00	27	0.111
318	A	7	5	1.00	25	0.200
319	A	10	7	1.00	21	0.333
320	A	11	8	1.00	27	0.296
321	A	5	4	1.00	27	0.148
322	A	7	7	1.00	31	0.226
323	A	4	4	1.00	31	0.129
324	A	3	3	1.00	29	0.103
325	A	5	5	1.00	27	0.185
326	A	4	4	1.00	23	0.174
327	A	5	5	1.00	29	0.172

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
328	A	6	6	1.00	31	0.194
329	A	8	7	1.00	31	0.226
330	A	5	4	1.00	31	0.129
331	A	4	3	1.00	29	0.103
332	A	6	5	1.00	27	0.185
333	A	5	5	1.00	23	0.217
334	A	6	5	1.00	29	0.172
335	A	6	5	1.00	31	0.161
336	A	6	6	1.00	31	0.194
337	A	4	4	1.00	31	0.129
338	A	2	2	1.00	29	0.069
339	A	4	4	1.00	27	0.148
340	A	4	4	1.00	23	0.174
341	A	5	5	1.00	29	0.172
342	A	6	5	1.00	31	0.161
343	A	8	7	1.00	31	0.226
344	A	5	5	1.00	31	0.161
345	A	4	4	1.00	29	0.138
346	A	6	5	1.00	27	0.185
347	A	6	5	1.00	23	0.217
348	A	8	6	1.00	29	0.207
349	A	9	6	1.00	31	0.194
350	A	4	3	1.00	27	0.111
351	A	4	3	1.00	27	0.111
352	A	6	5	1.00	25	0.200
353	A	3	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
354	A	4	3	1.00	25	0.120
355	A	4	3	1.00	25	0.120
356	A	3	2	1.00	19	0.105
357	A	4	3	1.00	29	0.103
358	A	5	3	1.00	27	0.111
359	A	2	1	1.00	21	0.048
360	A	4	3	1.00	27	0.111
361	A	4	3	1.00	27	0.111
362	A	5	4	1.00	21	0.190
363	A	4	3	1.00	27	0.111
364	A	4	3	1.00	29	0.103
365	A	9	6	1.00	27	0.222
366	A	9	6	1.00	27	0.222
367	A	8	6	1.00	27	0.222
368	A	7	6	1.00	25	0.240
369	A	8	6	1.00	25	0.240
370	A	9	8	1.00	27	0.296
371	A	9	7	1.00	25	0.280
372	A	9	7	1.00	19	0.368
373	A	7	5	1.00	25	0.200
374	A	6	5	1.00	27	0.185
375	A	7	6	1.00	27	0.222
376	A	8	6	1.00	27	0.222
377	A	9	6	1.00	27	0.222
378	A	16	6	1.00	29	0.207
379	A	13	7	1.00	29	0.241

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
380	A	14	6	1.00	29	0.207
381	A	6	5	1.00	27	0.185
382	A	11	8	1.00	27	0.296
383	A	13	7	1.00	29	0.241
384	A	12	8	1.00	27	0.296
385	A	12	7	1.00	21	0.333
386	A	13	6	1.00	27	0.222
387	A	10	7	1.00	29	0.241
388	A	11	6	1.00	29	0.207
389	A	14	6	1.00	29	0.207
390	A	13	7	1.00	29	0.241
391	A	16	6	1.00	29	0.207
392	A	19	6	1.00	29	0.207
393	A	19	7	1.00	29	0.241
394	A	17	7	1.00	29	0.241
395	A	7	5	1.00	27	0.185
396	A	15	9	1.00	27	0.333
397	A	15	7	1.00	29	0.241
398	A	15	8	1.00	27	0.296
399	A	14	8	1.00	21	0.381
400	A	15	7	1.00	27	0.259
401	A	15	6	1.00	29	0.207
402	A	14	8	1.00	29	0.276
403	A	14	7	1.00	29	0.241
404	A	16	7	1.00	29	0.241
405	A	17	7	1.00	29	0.241

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	19	7	1.00	29	0.241
407	A	8	5	1.00	29	0.172
408	A	17	8	1.00	21	0.381
409	A	8	6	1.00	29	0.207
410	A	8	6	1.00	29	0.207
411	A	7	6	1.00	29	0.207
412	A	6	6	1.00	27	0.222
413	A	6	6	1.00	27	0.222
414	A	6	6	1.00	29	0.207
415	A	5	5	1.00	27	0.185
416	A	5	5	1.00	21	0.238
417	A	6	6	1.00	27	0.222
418	A	7	6	1.00	29	0.207
419	A	8	6	1.00	29	0.207
420	A	11	5	1.00	29	0.172
421	A	12	5	1.00	29	0.172
422	A	10	5	1.00	29	0.172
423	A	10	5	1.00	29	0.172
424	A	4	4	1.00	27	0.148
425	A	4	4	1.00	27	0.148
426	A	6	5	1.00	29	0.172
427	A	8	6	1.00	27	0.222
428	A	9	6	1.00	21	0.286
429	A	10	5	1.00	27	0.185
430	A	10	5	1.00	29	0.172
431	A	12	5	1.00	29	0.172

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
432	A	12	7	1.00	29	0.241
433	A	9	7	1.00	29	0.241
434	A	4	4	1.00	27	0.148
435	A	5	4	1.00	27	0.148
436	A	7	6	1.00	29	0.207
437	A	9	7	1.00	27	0.259
438	A	11	6	1.00	21	0.286
439	A	14	7	1.00	27	0.259
440	A	2	2	1.00	27	0.074
441	A	18	4	1.47	29	0.138
442	A	24	4	1.00	29	0.138
443	A	5	4	1.00	31	0.129
444	A	4	3	1.00	29	0.103
445	A	9	9	1.00	29	0.310
446	A	8	8	1.00	31	0.258
447	A	7	7	1.00	29	0.241
448	A	7	7	1.00	23	0.304
449	A	9	6	1.00	29	0.207
450	A	11	6	1.00	31	0.194
451	A	13	6	1.00	31	0.194
452	A	15	6	1.00	31	0.194
453	A	6	4	1.00	31	0.129
454	A	5	3	1.00	29	0.103
455	A	12	12	1.00	29	0.414
456	A	10	10	1.00	31	0.323
457	A	9	9	1.00	29	0.310

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
458	A	8	8	1.00	23	0.348
459	A	11	9	1.00	29	0.310
460	A	12	9	1.00	31	0.290
461	A	14	9	1.00	31	0.290
462	A	16	9	1.00	31	0.290
463	A	18	9	1.00	31	0.290
464	A	5	4	1.00	31	0.129
465	A	3	3	1.00	29	0.103
466	A	13	10	1.00	29	0.345
467	A	11	8	1.00	31	0.258
468	A	11	8	1.00	29	0.276
469	A	11	7	1.00	23	0.304
470	A	15	8	1.00	29	0.276
471	A	17	8	1.00	31	0.258
472	A	12	7	1.00	31	0.226
473	A	4	4	1.00	31	0.129
474	A	2	2	1.00	29	0.069
475	A	6	6	1.00	29	0.207
476	A	9	6	1.00	31	0.194
477	A	8	6	1.00	29	0.207
478	A	10	6	1.00	23	0.261
479	A	12	6	1.00	29	0.207
480	A	14	6	1.00	31	0.194
481	A	18	9	1.00	31	0.290
482	A	16	9	1.00	31	0.290
483	A	6	5	1.00	31	0.161

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
484	A	5	4	1.00	29	0.138
485	A	9	6	1.00	29	0.207
486	A	12	7	1.00	31	0.226
487	A	14	8	1.00	29	0.276
488	A	16	8	1.00	23	0.348
489	A	18	8	1.00	29	0.276
490	A	5	2	1.00	29	0.069
491	A	3	2	1.00	27	0.074
492	A	3	2	1.00	29	0.069
493	A	5	4	1.00	29	0.138
494	A	4	3	1.00	27	0.111
495	A	4	3	1.00	27	0.111
496	A	7	5	1.00	27	0.185
497	A	7	5	1.00	27	0.185
498	A	6	5	1.00	25	0.200
499	A	4	3	1.00	25	0.120
500	A	4	3	1.00	27	0.111
501	A	4	3	1.00	27	0.111
502	A	4	3	1.00	25	0.120
503	A	3	2	1.00	19	0.105
504	A	4	3	1.00	25	0.120
505	A	6	5	1.00	27	0.185
506	A	6	5	1.00	27	0.185
507	A	7	5	1.00	27	0.185
508	A	7	5	1.00	27	0.185
509	A	4	3	1.00	27	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
510	A	4	3	1.00	27	0.111
511	A	4	3	1.00	29	0.103
512	A	4	3	1.00	29	0.103
513	A	4	3	1.00	27	0.111
514	A	4	3	1.00	27	0.111
515	A	4	3	1.00	29	0.103
516	A	4	3	1.00	29	0.103
517	A	4	3	1.00	27	0.111
518	A	3	2	1.00	21	0.095
519	A	4	3	1.00	27	0.111
520	A	4	3	1.00	29	0.103
521	A	4	3	1.00	29	0.103
522	A	4	3	1.00	27	0.111
523	A	4	3	1.00	27	0.111
524	A	4	3	1.00	29	0.103
525	A	4	3	1.00	29	0.103
526	A	4	3	1.00	27	0.111
527	A	3	2	1.00	21	0.095
528	A	4	3	1.00	27	0.111
529	A	4	3	1.00	29	0.103
530	A	4	3	1.00	27	0.111
531	A	3	2	1.00	21	0.095
532	A	4	3	1.00	27	0.111
533	A	4	3	1.00	29	0.103
534	A	7	3	1.00	29	0.103
535	A	6	5	1.00	27	0.185

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
536	A	4	3	1.00	27	0.111
537	A	4	3	1.00	29	0.103
538	A	4	3	1.00	29	0.103
539	A	4	3	1.00	27	0.111
540	A	5	4	1.00	21	0.190
541	A	6	5	1.00	27	0.185
542	A	4	3	1.00	29	0.103
543	A	4	3	1.00	29	0.103
544	A	4	3	1.00	29	0.103
545	A	4	3	1.00	29	0.103
546	A	4	3	1.00	27	0.111
547	A	4	3	1.00	27	0.111
548	A	4	3	1.00	29	0.103
549	A	4	3	1.00	29	0.103
550	A	3	3	1.00	27	0.111
551	A	3	2	1.00	21	0.095
552	A	4	3	1.00	27	0.111
553	A	4	3	1.00	29	0.103
554	A	4	3	1.00	29	0.103
555	A	4	3	1.00	29	0.103
556	A	4	3	1.00	27	0.111
557	A	4	3	1.00	27	0.111
558	A	4	3	1.00	29	0.103
559	A	4	3	1.00	29	0.103
560	A	4	3	1.00	27	0.111
561	A	3	2	1.00	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
562	A	4	3	1.00	27	0.111
563	A	3	2	1.00	21	0.095
564	A	4	3	1.00	27	0.111
565	A	3	2	1.00	29	0.069
566	A	3	2	1.00	29	0.069
567	A	3	2	1.00	27	0.074
568	A	3	2	1.00	29	0.069
569	A	3	2	1.00	29	0.069
570	A	4	3	1.00	29	0.103
571	A	4	4	1.00	29	0.138
572	A	10	6	1.00	27	0.222
573	A	10	6	1.00	27	0.222
574	A	9	6	1.00	27	0.222
575	A	8	6	1.00	25	0.240
576	A	9	6	1.00	25	0.240
577	A	10	8	1.00	27	0.296
578	A	11	8	1.00	27	0.296
579	A	11	7	1.00	27	0.259
580	A	11	8	1.00	25	0.320
581	A	11	7	1.00	19	0.368
582	A	9	5	1.00	25	0.200
583	A	7	5	1.00	27	0.185
584	A	8	6	1.00	27	0.222
585	A	9	6	1.00	27	0.222
586	A	10	6	1.00	27	0.222
587	A	10	6	1.00	27	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
588	A	18	6	1.00	29	0.207
589	A	14	7	1.00	29	0.241
590	A	16	6	1.00	29	0.207
591	A	7	5	1.00	27	0.185
592	A	12	8	1.00	27	0.296
593	A	17	7	1.00	29	0.241
594	A	16	7	1.00	29	0.241
595	A	17	8	1.00	29	0.276
596	A	16	8	1.00	27	0.296
597	A	15	7	1.00	21	0.333
598	A	17	6	1.00	27	0.222
599	A	12	7	1.00	29	0.241
600	A	13	6	1.00	29	0.207
601	A	12	7	1.00	29	0.241
602	A	16	6	1.00	29	0.207
603	A	14	7	1.00	29	0.241
604	A	18	6	1.00	29	0.207
605	A	21	6	1.00	29	0.207
606	A	21	7	1.00	29	0.241
607	A	19	7	1.00	29	0.241
608	A	8	5	1.00	27	0.185
609	A	17	9	1.00	27	0.333
610	A	19	7	1.00	29	0.241
611	A	17	8	1.00	29	0.276
612	A	15	8	1.00	29	0.276
613	A	16	8	1.00	27	0.296

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
614	A	16	7	1.00	21	0.333
615	A	18	7	1.00	27	0.259
616	A	18	6	1.00	29	0.207
617	A	17	8	1.00	29	0.276
618	A	16	7	1.00	29	0.241
619	A	18	7	1.00	29	0.241
620	A	19	7	1.00	29	0.241
621	A	21	7	1.00	29	0.241
622	A	21	6	1.00	29	0.207
623	A	22	7	1.00	29	0.241
624	A	9	6	1.00	29	0.207
625	A	9	6	1.00	29	0.207
626	A	8	6	1.00	29	0.207
627	A	7	6	1.00	27	0.222
628	A	8	6	1.00	27	0.222
629	A	9	8	1.00	29	0.276
630	A	9	7	1.00	29	0.241
631	A	8	7	1.00	29	0.241
632	A	7	5	1.00	27	0.185
633	A	6	5	1.00	21	0.238
634	A	13	8	1.00	29	0.276
635	A	6	5	1.00	29	0.172
636	A	5	4	1.00	27	0.148
637	A	10	9	1.00	27	0.333
638	A	9	7	1.00	29	0.241
639	A	8	7	1.00	29	0.241

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
640	A	9	8	1.00	29	0.276
641	A	10	7	1.00	27	0.259
642	A	11	5	1.00	21	0.238
643	A	13	7	1.00	27	0.259
644	A	14	5	1.00	29	0.172
645	A	12	5	1.00	29	0.172
646	A	5	5	1.25	27	0.185
647	A	7	6	1.00	27	0.222
648	A	7	6	1.00	29	0.207
649	A	8	6	1.00	29	0.207
650	A	10	6	1.00	29	0.207
651	A	12	6	1.00	27	0.222
652	A	12	5	1.00	21	0.238
653	A	6	2	1.00	29	0.069
654	A	5	2	1.00	29	0.069
655	A	3	2	1.00	27	0.074
656	A	4	3	1.00	27	0.111
657	A	8	6	1.00	27	0.222
658	A	8	6	1.00	27	0.222
659	A	7	5	1.00	27	0.185
660	A	7	5	1.00	27	0.185
661	A	6	5	1.00	25	0.200
662	A	4	3	1.00	25	0.120
663	A	4	3	1.00	27	0.111
664	A	4	3	1.00	27	0.111
665	A	4	3	1.00	27	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
666	A	4	3	1.00	27	0.111
667	A	4	3	1.00	25	0.120
668	A	3	2	1.00	19	0.105
669	A	4	3	1.00	25	0.120
670	A	6	5	1.00	27	0.185
671	A	6	5	1.00	27	0.185
672	A	7	5	1.00	27	0.185
673	A	7	5	1.00	27	0.185
674	A	8	6	1.00	27	0.222
675	A	8	6	1.00	27	0.222
676	A	4	3	1.00	27	0.111
677	A	4	3	1.00	29	0.103
678	A	4	3	1.00	29	0.103
679	A	4	3	1.00	29	0.103
680	A	7	5	1.00	29	0.172
681	A	7	5	1.00	29	0.172
682	A	6	5	1.00	27	0.185
683	A	3	2	1.00	21	0.095
684	A	4	3	1.00	27	0.111
685	A	4	3	1.00	29	0.103
686	A	4	3	1.00	29	0.103
687	A	4	3	1.00	29	0.103
688	A	4	3	1.00	29	0.103
689	A	4	3	1.00	27	0.111
690	A	6	5	1.00	21	0.238
691	A	6	5	1.00	27	0.185

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
692	A	7	5	1.00	29	0.172
693	A	7	5	1.00	29	0.172
694	A	4	3	1.00	29	0.103
695	A	4	3	1.00	29	0.103
696	A	4	3	1.00	29	0.103
697	A	3	2	1.00	29	0.069
698	A	3	2	1.00	29	0.069
699	A	3	2	1.00	27	0.074
700	A	3	2	1.00	29	0.069
701	A	3	2	1.00	29	0.069
702	A	3	2	1.00	29	0.069
703	A	8	4	1.00	29	0.138
704	A	4	4	1.00	29	0.138
705	A	11	6	1.00	29	0.207
706	A	10	6	1.00	29	0.207
707	A	10	6	1.00	29	0.207
708	A	9	6	1.00	29	0.207
709	A	8	6	1.00	27	0.222
710	A	9	6	1.00	27	0.222
711	A	10	8	1.00	29	0.276
712	A	11	8	1.00	29	0.276
713	A	11	7	1.00	29	0.241
714	A	11	8	1.00	29	0.276
715	A	10	7	1.00	29	0.241
716	A	9	5	1.00	27	0.185
717	A	7	5	1.00	21	0.238

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
718	A	8	6	1.00	27	0.222
719	A	9	6	1.00	29	0.207
720	A	10	6	1.00	29	0.207
721	A	10	6	1.00	29	0.207
722	A	15	7	1.00	29	0.241
723	A	17	7	1.00	29	0.241
724	A	14	8	1.00	29	0.276
725	A	15	7	1.00	29	0.241
726	A	6	4	1.00	27	0.148
727	A	12	9	1.00	27	0.333
728	A	14	8	1.00	29	0.276
729	A	13	9	1.00	29	0.310
730	A	13	8	1.00	29	0.276
731	A	14	7	1.00	29	0.241
732	A	11	8	1.00	29	0.276
733	A	12	7	1.00	27	0.259
734	A	19	5	1.00	21	0.238
735	A	15	7	1.00	27	0.259
736	A	14	8	1.00	29	0.276
737	A	17	7	1.00	29	0.241
738	A	15	7	1.00	29	0.241
739	A	18	8	1.00	29	0.276
740	A	16	8	1.00	29	0.276
741	A	6	5	1.00	27	0.185
742	A	13	10	1.00	27	0.370
743	A	11	8	1.00	29	0.276

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
744	A	11	8	1.00	29	0.276
745	A	11	7	1.00	29	0.241
746	A	12	9	1.00	29	0.310
747	A	13	8	1.00	29	0.276
748	A	15	8	1.00	27	0.296
749	A	17	4	1.00	21	0.190
750	A	18	8	1.00	27	0.296
751	A	8	7	1.00	27	0.259
752	A	8	7	1.00	25	0.280
753	A	5	5	1.00	19	0.263
754	A	5	4	1.00	23	0.174
755	A	6	6	1.00	25	0.240
756	A	7	6	1.00	27	0.222
757	A	8	7	1.00	27	0.259
758	A	8	7	1.00	27	0.259
759	A	12	8	1.00	27	0.296
760	A	6	5	1.00	21	0.238
761	A	3	2	1.00	25	0.080
762	A	9	7	1.00	27	0.259
763	A	10	8	1.00	29	0.276
764	A	12	7	1.00	29	0.241
765	A	11	6	1.00	27	0.222
766	A	8	6	1.00	21	0.286
767	A	2	2	1.00	25	0.080
768	A	4	3	1.00	27	0.111
769	A	6	5	1.00	29	0.172

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
770	A	8	6	1.00	29	0.207
771	A	10	6	1.00	29	0.207
772	A	7	5	1.00	29	0.172
773	A	6	4	1.00	27	0.148
774	A	5	4	1.00	21	0.190
775	A	5	4	1.00	25	0.160
776	A	7	5	1.00	27	0.185
777	A	8	6	1.00	29	0.207
778	A	15	10	1.00	29	0.345
779	A	13	8	1.00	29	0.276
780	A	12	8	1.00	29	0.276
781	A	11	7	1.00	27	0.259
782	A	10	5	1.00	21	0.238
783	A	4	4	1.00	25	0.160
784	A	11	8	1.00	27	0.296
785	A	12	8	1.00	29	0.276
786	A	15	8	1.00	29	0.276
787	A	16	10	1.00	29	0.345
788	A	16	9	1.00	29	0.310
789	A	14	8	1.00	29	0.276
790	A	14	7	1.00	27	0.259
791	A	14	5	1.00	21	0.238
792	A	5	4	1.00	25	0.160
793	A	14	10	1.00	27	0.370
794	A	14	10	1.00	29	0.345
795	A	9	7	1.00	27	0.259

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
796	A	9	7	1.00	25	0.280
797	A	8	5	1.00	19	0.263
798	A	6	4	1.00	25	0.160
799	A	5	4	1.00	27	0.148
800	A	5	4	1.00	25	0.160
801	A	7	5	1.00	25	0.200
802	A	8	6	1.00	27	0.222
803	A	9	7	1.00	27	0.259
804	A	4	4	1.19	21	0.190
805	A	7	7	1.00	27	0.259
806	A	4	4	1.00	29	0.138
807	A	4	4	1.00	27	0.148
808	A	5	5	1.00	27	0.185
809	A	7	7	1.00	29	0.241
810	A	8	8	1.00	29	0.276
811	A	10	7	1.00	21	0.333
812	A	8	6	1.00	27	0.222
813	A	5	5	1.00	29	0.172
814	A	4	4	1.00	27	0.148
815	A	6	4	1.00	27	0.148
816	A	8	6	1.00	29	0.207
817	A	10	7	1.00	29	0.241
818	A	12	7	1.00	29	0.241
819	A	13	7	1.00	21	0.333
820	A	8	6	1.00	29	0.207
821	A	8	5	1.00	29	0.172

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
822	A	8	5	1.00	27	0.185
823	A	6	5	1.00	21	0.238
824	A	6	5	1.00	27	0.185
825	A	7	4	1.00	29	0.138
826	A	6	5	1.00	27	0.185
827	A	7	5	1.00	27	0.185
828	A	8	6	1.00	29	0.207
829	A	14	8	1.00	29	0.276
830	A	13	8	1.00	29	0.276
831	A	11	7	1.00	27	0.259
832	A	10	6	1.00	21	0.286
833	A	12	6	1.00	27	0.222
834	A	12	6	1.00	29	0.207
835	A	4	3	1.00	27	0.111
836	A	11	8	1.00	27	0.296
837	A	12	8	1.00	29	0.276
838	A	15	8	1.00	29	0.276
839	A	17	9	1.00	29	0.310
840	A	14	8	1.00	29	0.276
841	A	14	7	1.00	27	0.259
842	A	14	5	1.00	21	0.238
843	A	15	6	1.00	27	0.222
844	A	15	6	1.00	29	0.207
845	A	5	3	1.00	27	0.111
846	A	14	10	1.00	27	0.370
847	A	14	10	1.00	29	0.345

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
848	A	17	5	1.00	21	0.238
849	A	18	6	1.00	27	0.222
850	A	8	4	1.29	29	0.138
851	A	4	3	1.00	25	0.120
852	A	3	2	1.00	19	0.105
853	A	4	3	1.00	25	0.120
854	A	5	4	1.00	27	0.148
855	A	5	4	1.00	27	0.148
856	A	5	4	1.00	25	0.160
857	A	4	3	1.00	25	0.120
858	A	4	3	1.00	27	0.111
859	A	4	3	1.00	27	0.111
860	A	4	3	1.00	27	0.111
861	A	3	2	1.00	21	0.095
862	A	4	3	1.00	27	0.111
863	A	4	3	1.00	29	0.103
864	A	5	4	1.00	29	0.138
865	A	5	4	1.00	27	0.148
866	A	4	3	1.00	27	0.111
867	A	4	3	1.00	29	0.103
868	A	4	3	1.00	29	0.103
869	A	4	3	1.00	29	0.103
870	A	3	2	1.00	21	0.095
871	A	4	3	1.00	27	0.111
872	A	4	3	1.00	29	0.103
873	A	4	3	1.00	29	0.103

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
874	A	3	3	1.00	27	0.111
875	A	4	3	1.00	27	0.111
876	A	4	3	1.00	29	0.103
877	A	4	3	1.00	29	0.103
878	A	4	3	1.00	29	0.103
879	A	4	3	1.00	29	0.103
880	A	4	3	1.00	29	0.103
881	A	4	3	1.00	27	0.111
882	A	8	5	1.00	21	0.238
883	A	8	6	1.00	27	0.222
884	A	9	6	1.00	29	0.207
885	A	9	6	1.00	29	0.207
886	A	9	6	1.00	29	0.207
887	A	9	6	1.00	29	0.207
888	A	8	6	1.00	27	0.222
889	A	4	3	1.00	21	0.143
890	A	4	3	1.00	27	0.111
891	A	4	3	1.00	29	0.103
892	A	4	3	1.00	29	0.103
893	A	4	3	1.00	29	0.103
894	A	11	5	1.00	29	0.172
895	A	4	3	1.00	29	0.103
896	A	4	3	1.00	29	0.103
897	A	4	3	1.00	27	0.111
898	A	9	5	1.00	21	0.238
899	A	9	6	1.00	27	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
900	A	10	6	1.00	29	0.207
901	A	10	6	1.00	29	0.207
902	A	11	7	1.00	29	0.241
903	A	11	7	1.00	29	0.241
904	A	10	6	1.00	29	0.207
905	A	10	6	1.00	29	0.207
906	A	9	6	1.00	27	0.222
907	A	4	3	1.00	21	0.143
908	A	4	3	1.00	27	0.111
909	A	4	3	1.00	29	0.103
910	A	4	3	1.00	29	0.103
911	A	5	4	1.00	33	0.121
912	A	3	3	1.00	31	0.097
913	A	3	2	1.00	31	0.065
914	A	3	2	1.00	31	0.065
915	A	3	2	1.00	31	0.065
916	A	3	2	1.00	29	0.069
917	A	2	2	1.00	31	0.065
918	A	2	2	1.00	31	0.065
919	A	2	2	1.00	31	0.065
920	A	3	2	1.00	31	0.065
921	A	3	2	1.00	31	0.065
922	A	3	2	1.00	31	0.065
923	A	3	2	1.00	29	0.069
924	A	2	2	1.00	31	0.065
925	A	2	2	1.00	31	0.065

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
926	A	2	2	1.00	31	0.065
927	A	3	3	1.13	27	0.111
928	A	4	3	1.00	27	0.111
929	A	4	3	1.00	27	0.111
930	A	4	3	1.00	27	0.111
931	A	4	3	1.00	25	0.120
932	A	2	2	1.00	19	0.105
933	A	3	3	1.00	25	0.120
934	A	3	3	1.00	27	0.111
935	A	4	4	1.06	29	0.138
936	A	6	6	1.00	35	0.171
937	A	6	6	1.00	37	0.162
938	A	4	4	1.00	33	0.121
939	A	3	3	1.00	33	0.091
940	A	3	3	1.00	33	0.091
941	A	3	3	1.00	31	0.097
942	A	4	4	1.00	33	0.121
943	A	3	3	1.00	33	0.091
944	A	3	3	1.00	33	0.091
945	A	4	4	1.00	33	0.121
946	A	3	3	1.00	33	0.091
947	A	3	3	1.00	31	0.097
948	A	3	3	1.00	33	0.091
949	A	4	4	1.00	33	0.121
950	A	3	3	1.00	33	0.091
951	A	3	3	1.00	33	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
952	A	3	3	1.00	33	0.091
953	A	3	2	1.00	29	0.069
954	A	3	2	1.00	29	0.069
955	A	3	2	1.00	29	0.069
956	A	3	2	1.00	27	0.074
957	A	3	2	1.00	27	0.074
958	A	4	3	1.00	29	0.103
959	A	4	3	1.00	29	0.103
960	A	4	3	1.00	29	0.103
961	A	6	4	1.00	29	0.138
962	A	5	4	1.00	29	0.138
963	A	4	4	1.00	29	0.138
964	A	2	2	1.00	29	0.069
965	A	3	3	1.00	29	0.103
966	A	3	2	1.00	29	0.069
967	A	3	2	1.00	29	0.069
968	A	3	2	1.00	29	0.069
969	A	3	2	1.00	31	0.065
970	A	3	2	1.00	31	0.065
971	A	3	2	1.00	31	0.065
972	A	3	2	1.00	29	0.069
973	A	3	2	1.00	29	0.069
974	A	3	2	1.00	31	0.065
975	A	4	3	1.00	31	0.097
976	A	4	3	1.00	31	0.097
977	A	7	5	1.00	31	0.161

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
978	A	6	5	1.00	31	0.161
979	A	5	5	1.00	31	0.161
980	A	3	2	1.00	31	0.065
981	A	4	4	1.00	31	0.129
982	A	4	3	1.00	31	0.097
983	A	4	3	1.00	31	0.097
984	A	4	3	1.00	31	0.097
985	A	4	3	1.00	31	0.097
986	A	3	2	1.00	31	0.065
987	A	3	2	1.00	31	0.065
988	A	3	2	1.00	31	0.065
989	A	3	2	1.00	29	0.069
990	A	3	2	1.00	29	0.069
991	A	3	2	1.00	31	0.065
992	A	2	2	1.00	31	0.065
993	A	4	3	1.00	31	0.097
994	A	4	3	1.00	31	0.097
995	A	8	5	1.00	31	0.161
996	A	7	5	1.00	31	0.161
997	A	6	5	1.00	31	0.161
998	A	2	2	1.00	31	0.065
999	A	4	4	1.00	31	0.129
1000	A	4	4	1.00	31	0.129
1001	A	4	3	1.00	31	0.097
1002	A	4	3	1.00	31	0.097
1003	A	3	2	1.00	31	0.065

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1004	A	3	2	1.00	31	0.065
1005	A	3	2	1.00	31	0.065
1006	A	3	2	1.00	29	0.069
1007	A	4	3	1.00	29	0.103
1008	A	4	3	1.00	31	0.097
1009	A	4	3	1.00	31	0.097
1010	A	4	3	1.00	31	0.097
1011	A	3	2	1.00	31	0.065
1012	A	3	2	1.00	31	0.065
1013	A	3	2	1.00	31	0.065
1014	A	3	2	1.00	29	0.069
1015	A	4	3	1.00	29	0.103
1016	A	4	3	1.00	31	0.097
1017	A	4	3	1.00	31	0.097
1018	A	4	3	1.00	31	0.097
1019	A	4	4	1.00	33	0.121
1020	A	3	2	1.00	31	0.065
1021	A	3	2	1.00	31	0.065
1022	A	3	2	1.00	31	0.065
1023	A	3	2	1.00	29	0.069
1024	A	3	3	1.00	29	0.103
1025	A	3	3	1.00	31	0.097
1026	A	3	3	1.00	31	0.097
1027	A	4	4	1.00	31	0.129
1028	A	4	4	1.00	31	0.129
1029	A	4	4	1.00	31	0.129

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1030	A	4	4	1.00	31	0.129
1031	A	4	4	1.00	31	0.129
1032	A	4	4	1.00	31	0.129
1033	A	4	3	1.00	38	0.079
1034	A	3	3	1.00	38	0.079
1035	A	2	2	1.00	38	0.053
1036	A	4	4	1.00	38	0.105
1037	A	4	4	1.00	36	0.111
1038	A	4	4	1.00	38	0.105
1039	A	4	4	1.00	38	0.105
1040	A	1	1	1.00	40	0.025
1041	A	1	1	1.00	40	0.025
1042	A	4	4	1.00	35	0.114
1043	A	3	3	1.03	35	0.086
1044	A	3	3	1.04	33	0.091
1045	A	3	3	1.04	35	0.086
1046	A	3	3	1.03	35	0.086
1047	A	3	3	1.03	35	0.086
1048	A	3	3	1.03	35	0.086
1049	A	5	5	1.00	35	0.143
1050	A	8	6	1.00	27	0.222
1051	A	7	6	1.00	27	0.222
1052	A	6	6	1.00	25	0.240
1053	A	6	6	1.00	23	0.261
1054	A	7	6	1.00	19	0.316
1055	A	5	5	1.00	25	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1056	A	5	5	1.00	27	0.185
1057	A	6	6	1.00	27	0.222
1058	A	7	6	1.00	27	0.222
1059	A	10	8	1.00	29	0.276
1060	A	9	8	1.00	29	0.276
1061	A	5	4	1.00	27	0.148
1062	A	6	6	1.00	25	0.240
1063	A	9	7	1.00	21	0.333
1064	A	6	6	1.00	27	0.222
1065	A	6	6	1.00	29	0.207
1066	A	8	8	1.00	29	0.276
1067	A	9	9	1.00	29	0.310
1068	A	9	8	1.00	29	0.276
1069	A	10	9	1.00	29	0.310
1070	A	6	4	1.00	27	0.148
1071	A	7	7	1.00	25	0.280
1072	A	11	9	1.00	21	0.429
1073	A	7	7	1.00	27	0.259
1074	A	7	7	1.00	29	0.241
1075	A	7	7	1.00	29	0.241
1076	A	9	9	1.00	29	0.310
1077	A	10	10	1.00	29	0.345
1078	A	9	9	1.00	29	0.310
1079	A	8	8	1.00	29	0.276
1080	A	5	5	1.00	27	0.185
1081	A	8	8	1.00	25	0.320

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1082	A	8	7	1.00	21	0.333
1083	A	9	8	1.00	27	0.296
1084	A	10	8	1.00	29	0.276
1085	A	9	8	1.00	29	0.276
1086	A	8	8	1.00	29	0.276
1087	A	6	6	1.00	27	0.222
1088	A	8	7	1.00	25	0.280
1089	A	9	8	1.00	21	0.381
1090	A	10	8	1.00	27	0.296
1091	A	5	5	1.00	35	0.143
1092	A	9	6	1.00	27	0.222
1093	A	9	6	1.00	27	0.222
1094	A	8	6	1.00	27	0.222
1095	A	7	6	1.00	25	0.240
1096	A	8	6	1.00	25	0.240
1097	A	9	8	1.00	27	0.296
1098	A	9	7	1.00	25	0.280
1099	A	9	7	1.00	19	0.368
1100	A	7	5	1.00	25	0.200
1101	A	6	5	1.00	27	0.185
1102	A	7	6	1.00	27	0.222
1103	A	8	6	1.00	27	0.222
1104	A	9	6	1.00	27	0.222
1105	A	10	8	1.00	29	0.276
1106	A	9	8	1.00	29	0.276
1107	A	6	4	1.00	27	0.148

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1108	A	6	6	1.64	27	0.222
1109	A	6	6	1.00	29	0.207
1110	A	6	6	1.00	27	0.222
1111	A	13	9	1.00	21	0.429
1112	A	6	6	1.00	27	0.222
1113	A	6	6	1.00	29	0.207
1114	A	8	8	1.00	29	0.276
1115	A	9	9	1.00	29	0.310
1116	A	10	8	1.00	29	0.276
1117	A	7	4	1.00	27	0.148
1118	A	7	6	1.00	27	0.222
1119	A	7	6	1.00	29	0.207
1120	A	7	6	1.00	27	0.222
1121	A	17	10	1.00	21	0.476
1122	A	7	6	1.00	27	0.222
1123	A	7	6	1.00	29	0.207
1124	A	7	6	1.00	29	0.207
1125	A	9	8	1.00	29	0.276
1126	A	10	9	1.00	29	0.310
1127	A	9	7	1.00	29	0.241
1128	A	8	7	1.00	29	0.241
1129	A	6	6	1.00	27	0.222
1130	A	6	6	1.00	27	0.222
1131	A	6	6	1.00	29	0.207
1132	A	7	7	1.14	27	0.259
1133	A	8	7	1.00	21	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1134	A	9	7	1.00	27	0.259
1135	A	9	7	1.00	29	0.241
1136	A	8	7	1.00	29	0.241
1137	A	6	5	1.00	27	0.185
1138	A	6	6	1.00	27	0.222
1139	A	7	7	1.00	29	0.241
1140	A	8	7	1.00	27	0.259
1141	A	9	7	1.00	21	0.333
1142	A	10	7	1.00	27	0.259
1143	A	10	9	1.00	31	0.290
1144	A	8	7	1.00	29	0.241
1145	A	10	10	1.00	29	0.345
1146	A	10	10	1.00	31	0.323
1147	A	10	10	1.00	29	0.345
1148	A	10	10	1.00	23	0.435
1149	A	11	11	1.00	29	0.379
1150	A	12	11	1.00	31	0.355
1151	A	11	9	1.00	31	0.290
1152	A	9	7	1.00	29	0.241
1153	A	11	10	1.00	29	0.345
1154	A	11	10	1.00	31	0.323
1155	A	11	10	1.00	29	0.345
1156	A	11	11	1.00	23	0.478
1157	A	11	10	1.00	29	0.345
1158	A	12	11	1.00	31	0.355
1159	A	13	11	1.00	31	0.355

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1160	A	10	7	1.00	29	0.241
1161	A	12	10	1.00	29	0.345
1162	A	12	10	1.00	31	0.323
1163	A	12	10	1.00	29	0.345
1164	A	12	11	1.00	23	0.478
1165	A	12	11	1.00	29	0.379
1166	A	12	10	1.00	31	0.323
1167	A	13	11	1.00	31	0.355
1168	A	10	8	1.00	31	0.258
1169	A	9	8	1.00	31	0.258
1170	A	7	6	1.00	29	0.207
1171	A	9	9	1.00	29	0.310
1172	A	9	9	1.00	31	0.290
1173	A	9	9	1.00	29	0.310
1174	A	10	10	1.00	23	0.435
1175	A	11	10	1.00	29	0.345
1176	A	10	8	1.00	31	0.258
1177	A	9	8	1.00	31	0.258
1178	A	7	7	1.00	29	0.241
1179	A	9	9	1.00	29	0.310
1180	A	9	9	1.00	31	0.290
1181	A	10	10	1.00	29	0.345
1182	A	11	10	1.00	23	0.435
1183	A	10	8	1.00	31	0.258
1184	A	9	8	1.00	31	0.258
1185	A	7	6	1.00	29	0.207

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1186	A	9	9	1.00	29	0.310
1187	A	10	10	1.00	31	0.323
1188	A	11	10	1.00	29	0.345
1189	A	12	10	1.00	23	0.435
1190	A	8	7	1.00	35	0.200
1191	A	0	0	0.00	0	0.000
1192	A	0	0	0.00	0	0.000
1193	A	0	0	0.00	0	0.000
1194	A	0	0	0.00	0	0.000
1195	A	8	6	1.00	29	0.207
1196	A	7	6	1.00	29	0.207
1197	A	3	2	1.00	27	0.074
1198	A	4	3	1.00	27	0.111
1199	A	4	3	1.00	27	0.111
1200	A	7	5	1.00	27	0.185
1201	A	7	5	1.00	27	0.185
1202	A	6	5	1.00	25	0.200
1203	A	4	3	1.00	25	0.120
1204	A	4	3	1.00	27	0.111
1205	A	4	3	1.00	27	0.111
1206	A	4	3	1.00	25	0.120
1207	A	3	2	1.00	19	0.105
1208	A	4	3	1.00	25	0.120
1209	A	6	5	1.00	27	0.185
1210	A	6	5	1.00	27	0.185
1211	A	7	5	1.00	27	0.185

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1212	A	7	5	1.00	27	0.185
1213	A	4	3	1.00	27	0.111
1214	A	4	3	1.00	27	0.111
1215	A	4	3	1.00	29	0.103
1216	A	4	3	1.00	27	0.111
1217	A	4	3	1.00	27	0.111
1218	A	4	3	1.00	29	0.103
1219	A	4	3	1.00	29	0.103
1220	A	4	3	1.00	27	0.111
1221	A	3	2	1.00	21	0.095
1222	A	4	3	1.00	27	0.111
1223	A	4	3	1.00	29	0.103
1224	A	4	3	1.00	29	0.103
1225	A	4	3	1.00	29	0.103
1226	A	4	3	1.00	29	0.103
1227	A	4	3	1.00	29	0.103
1228	A	4	3	1.00	27	0.111
1229	A	4	3	1.00	27	0.111
1230	A	4	3	1.00	29	0.103
1231	A	4	3	1.00	29	0.103
1232	A	4	3	1.00	27	0.111
1233	A	3	2	1.00	21	0.095
1234	A	4	3	1.00	27	0.111
1235	A	3	2	1.00	29	0.069
1236	A	3	2	1.00	27	0.074
1237	A	5	4	1.00	29	0.138

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1238	A	5	4	1.00	29	0.138
1239	A	12	7	1.00	29	0.241
1240	A	12	9	1.00	29	0.310
1241	A	11	7	1.00	29	0.241
1242	A	11	8	1.00	29	0.276
1243	A	7	4	1.00	27	0.148
1244	A	9	5	1.00	27	0.185
1245	A	12	8	1.00	29	0.276
1246	A	11	8	1.00	29	0.276
1247	A	12	9	1.00	29	0.310
1248	A	12	9	1.00	27	0.333
1249	A	16	10	1.00	21	0.476
1250	A	11	9	1.00	27	0.333
1251	A	9	5	1.00	29	0.172
1252	A	9	9	1.00	29	0.310
1253	A	9	5	1.00	29	0.172
1254	A	11	10	1.00	29	0.345
1255	A	10	5	1.00	29	0.172
1256	A	11	8	1.00	29	0.276
1257	A	10	8	1.00	29	0.276
1258	A	7	6	1.00	27	0.222
1259	A	16	11	1.00	27	0.407
1260	A	16	11	1.00	29	0.379
1261	A	16	11	1.00	29	0.379
1262	A	17	10	1.00	29	0.345
1263	A	9	7	1.00	27	0.259

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1264	A	10	7	1.00	21	0.333
1265	A	11	7	1.00	27	0.259
1266	A	11	8	1.00	29	0.276
1267	A	10	8	1.00	29	0.276
1268	A	7	6	1.00	27	0.222
1269	A	20	11	1.00	27	0.407
1270	A	20	11	1.00	29	0.379
1271	A	21	11	1.00	29	0.379
1272	A	9	7	1.00	29	0.241
1273	A	10	7	1.00	27	0.259
1274	A	11	7	1.00	21	0.333
1275	A	13	7	1.00	29	0.241
1276	A	8	7	1.00	35	0.200
1277	A	9	9	1.01	37	0.243
1278	A	6	6	1.00	37	0.162
1279	A	4	3	1.00	27	0.111
1280	A	4	3	1.00	27	0.111
1281	A	4	3	1.00	25	0.120
1282	A	4	4	1.00	19	0.210
1283	A	4	3	1.00	25	0.120
1284	A	4	3	1.00	27	0.111
1285	A	10	8	1.00	29	0.276
1286	A	9	8	1.00	29	0.276
1287	A	8	8	1.00	29	0.276
1288	A	5	5	1.00	27	0.185
1289	A	6	6	1.00	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1290	A	7	7	1.00	21	0.333
1291	A	8	8	1.00	27	0.296
1292	A	9	8	1.00	29	0.276
1293	A	10	8	1.00	29	0.276
1294	A	11	8	1.00	29	0.276
1295	A	4	3	1.00	29	0.103
1296	A	4	3	1.00	29	0.103
1297	A	4	3	1.00	27	0.111
1298	A	4	3	1.00	27	0.111
1299	A	4	3	1.00	27	0.111
1300	A	3	2	1.00	21	0.095
1301	A	9	7	1.00	29	0.241
1302	A	8	7	1.00	29	0.241
1303	A	6	5	1.00	27	0.185
1304	A	6	6	1.00	27	0.222
1305	A	6	6	1.00	29	0.207
1306	A	6	6	1.00	27	0.222
1307	A	7	7	1.00	21	0.333
1308	A	8	7	1.00	27	0.259
1309	A	9	7	1.00	29	0.241
1310	A	4	3	1.00	29	0.103
1311	A	4	3	1.00	29	0.103
1312	A	4	3	1.00	27	0.111
1313	A	4	3	1.00	27	0.111
1314	A	4	3	1.00	29	0.103
1315	A	4	3	1.00	29	0.103

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1316	A	4	3	1.00	27	0.111
1317	A	3	2	1.00	21	0.095
1318	A	4	3	1.00	27	0.111
1319	A	4	3	1.00	29	0.103
1320	A	11	7	1.00	29	0.241
1321	A	10	7	1.00	29	0.241
1322	A	7	5	1.00	27	0.185
1323	A	14	9	1.00	27	0.333
1324	A	13	10	1.00	29	0.345
1325	A	6	6	1.00	29	0.207
1326	A	13	9	1.00	29	0.310
1327	A	15	8	1.41	27	0.296
1328	A	9	7	1.27	21	0.333
1329	A	10	7	1.00	27	0.259
1330	A	11	7	1.00	29	0.241
1331	A	12	7	1.00	29	0.241
1332	A	4	3	1.00	27	0.111
1333	A	4	3	1.00	25	0.120
1334	A	3	2	1.00	19	0.105
1335	A	4	3	1.00	25	0.120
1336	A	4	3	1.00	27	0.111
1337	A	4	3	1.00	27	0.111
1338	A	14	13	1.00	29	0.448
1339	A	12	11	1.00	29	0.379
1340	A	9	8	1.00	27	0.296
1341	A	8	7	1.00	21	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1342	A	5	5	1.00	25	0.200
1343	A	10	9	1.00	27	0.333
1344	A	13	11	1.17	29	0.379
1345	A	17	12	1.17	29	0.414
1346	A	4	3	1.00	21	0.143
1347	A	5	4	1.00	27	0.148
1348	A	5	4	1.00	27	0.148
1349	A	4	3	1.00	27	0.111
1350	A	4	3	1.00	29	0.103
1351	A	4	3	1.00	29	0.103
1352	A	13	9	1.00	21	0.429
1353	A	10	9	1.00	27	0.333
1354	A	10	9	1.00	29	0.310
1355	A	6	5	1.00	27	0.185
1356	A	12	10	1.00	27	0.370
1357	A	15	12	1.12	29	0.414
1358	A	20	13	1.00	29	0.448
1359	A	6	4	1.00	29	0.138
1360	A	6	4	1.00	29	0.138
1361	A	6	4	1.00	27	0.148
1362	A	5	3	1.00	21	0.143
1363	A	6	4	1.00	27	0.148
1364	A	6	5	1.00	29	0.172
1365	A	6	5	1.00	29	0.172
1366	A	6	4	1.00	27	0.148
1367	A	4	3	1.00	27	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1368	A	4	3	1.00	29	0.103
1369	A	4	3	1.00	29	0.103
1370	A	21	14	1.00	33	0.424
1371	A	18	13	1.00	33	0.394
1372	A	15	12	1.00	33	0.364
1373	A	12	10	1.00	31	0.323
1374	A	16	11	1.00	31	0.355
1375	A	19	14	1.00	33	0.424
1376	A	25	15	1.00	33	0.454
1377	A	24	16	1.00	33	0.485
1378	A	20	15	1.00	33	0.454
1379	A	13	11	1.00	31	0.355
1380	A	21	16	1.00	31	0.516
1381	A	24	17	1.00	33	0.515
1382	A	30	18	1.00	33	0.546
1383	A	24	16	1.00	33	0.485
1384	A	20	15	1.00	33	0.454
1385	A	13	11	1.00	31	0.355
1386	A	21	16	1.00	31	0.516
1387	A	24	17	1.00	33	0.515
1388	A	30	18	1.00	33	0.546
1389	A	23	15	1.00	33	0.454
1390	A	19	14	1.00	33	0.424
1391	A	15	13	1.00	33	0.394
1392	A	12	10	1.00	31	0.323
1393	A	16	11	1.00	31	0.355

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1394	A	19	14	1.00	33	0.424
1395	A	25	15	1.00	33	0.454
1396	A	22	15	1.00	33	0.454
1397	A	18	14	1.00	33	0.424
1398	A	15	13	1.00	33	0.394
1399	A	13	11	1.00	31	0.355
1400	A	21	16	1.00	31	0.516
1401	A	25	18	1.00	33	0.546
1402	A	22	16	1.00	33	0.485
1403	A	18	14	1.00	33	0.424
1404	A	15	13	1.00	33	0.394
1405	A	13	11	1.00	31	0.355
1406	A	21	16	1.00	31	0.516
1407	A	25	18	1.00	33	0.546
1408	A	31	15	1.00	37	0.405
1409	A	19	14	1.00	37	0.378
1410	A	16	12	1.00	37	0.324
1411	A	5	4	1.00	37	0.108
1412	A	9	8	1.00	37	0.216
1413	A	11	9	1.00	37	0.243
1414	A	16	9	1.00	37	0.243
1415	A	19	9	1.00	37	0.243
1416	A	31	16	1.00	37	0.432
1417	A	19	15	1.00	37	0.405
1418	A	18	14	1.00	37	0.378
1419	A	8	7	1.00	37	0.189

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1420	A	12	9	1.00	37	0.243
1421	A	15	9	1.00	37	0.243
1422	A	20	9	1.00	37	0.243
1423	A	31	17	1.00	37	0.460
1424	A	19	15	1.00	37	0.405
1425	A	20	16	1.00	37	0.432
1426	A	10	9	1.00	37	0.243
1427	A	15	10	1.00	37	0.270
1428	A	18	10	1.00	37	0.270
1429	A	24	10	1.00	37	0.270
1430	A	19	14	1.00	37	0.378
1431	A	15	11	1.00	37	0.297
1432	A	4	3	1.00	37	0.081
1433	A	7	6	1.00	37	0.162
1434	A	9	7	1.00	37	0.189
1435	A	13	8	1.00	37	0.216
1436	A	31	17	1.00	37	0.460
1437	A	10	9	1.00	37	0.243
1438	A	11	10	1.00	37	0.270
1439	A	11	10	1.00	37	0.270
1440	A	16	12	1.00	37	0.324
1441	A	19	12	1.00	37	0.324
1442	A	8	7	1.00	37	0.189
1443	A	8	7	1.00	27	0.259
1444	A	8	7	1.00	25	0.280
1445	A	7	5	1.00	19	0.263

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1446	A	5	4	1.00	23	0.174
1447	A	6	6	1.00	25	0.240
1448	A	7	6	1.00	27	0.222
1449	A	8	7	1.00	27	0.259
1450	A	8	7	1.00	27	0.259
1451	A	11	9	1.00	21	0.429
1452	A	4	3	1.00	25	0.120
1453	A	8	8	1.52	27	0.296
1454	A	7	6	1.00	29	0.207
1455	A	10	9	1.24	29	0.310
1456	A	8	7	1.00	29	0.241
1457	A	17	8	1.00	27	0.296
1458	A	14	10	1.00	21	0.476
1459	A	3	3	1.00	25	0.120
1460	A	11	8	1.00	27	0.296
1461	A	12	9	1.00	29	0.310
1462	A	14	9	1.00	29	0.310
1463	A	15	8	1.00	29	0.276
1464	A	12	7	1.00	29	0.241
1465	A	12	7	1.00	27	0.259
1466	A	12	7	1.00	21	0.333
1467	A	6	6	1.00	25	0.240
1468	A	13	8	1.00	27	0.296
1469	A	15	10	1.00	29	0.345
1470	A	17	11	1.00	29	0.379
1471	A	18	8	1.00	29	0.276

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1472	A	18	8	1.00	27	0.296
1473	A	18	8	1.00	21	0.381
1474	A	7	6	1.00	25	0.240
1475	A	19	9	1.00	27	0.333
1476	A	21	11	1.00	29	0.379
1477	A	23	12	1.00	29	0.414
1478	A	2	2	1.00	35	0.057
1479	F	0	0	N/A	0	N/A
1480	F	0	0	N/A	0	N/A
1481	A	11	8	1.00	27	0.296
1482	A	10	8	1.00	25	0.320
1483	A	7	5	1.00	19	0.263
1484	A	7	5	1.00	25	0.200
1485	A	6	5	1.00	27	0.185
1486	A	6	6	1.00	27	0.222
1487	A	6	6	1.00	25	0.240
1488	A	8	6	1.00	25	0.240
1489	A	10	8	1.00	27	0.296
1490	A	10	8	1.00	27	0.296
1491	A	11	8	1.00	27	0.296
1492	A	9	6	1.00	27	0.222
1493	A	8	5	1.00	21	0.238
1494	A	9	6	1.00	27	0.222
1495	A	7	5	1.00	29	0.172
1496	A	5	5	1.00	29	0.172

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1497	A	6	5	1.00	27	0.185
1498	A	6	5	1.00	27	0.185
1499	A	6	4	1.00	29	0.138
1500	A	6	4	1.00	29	0.138
1501	A	8	5	1.00	21	0.238
1502	A	9	6	1.00	27	0.222
1503	A	8	6	1.00	29	0.207
1504	A	7	6	1.00	29	0.207
1505	A	5	5	1.00	27	0.185
1506	A	6	5	1.00	27	0.185
1507	A	6	4	1.00	29	0.138
1508	A	6	4	1.00	29	0.138
1509	A	6	4	1.00	29	0.138
1510	A	5	4	1.00	29	0.138
1511	A	5	4	1.00	29	0.138
1512	A	4	3	1.00	27	0.111
1513	A	10	3	1.00	29	0.103
1514	A	17	5	1.00	29	0.172
1515	F	0	0	N/A	0	N/A
1516	A	11	8	1.00	35	0.229
1517	A	10	7	1.00	33	0.212
1518	A	2	2	1.00	23	0.087
1519	A	0	0	0.00	0	0.000
1520	A	0	0	0.00	0	0.000
1521	A	0	0	0.00	0	0.000
1522	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1523	A	11	8	1.00	33	0.242
1524	A	10	7	0.99	31	0.226
1525	A	2	2	1.00	21	0.095
1526	A	0	0	0.00	0	0.000
1527	A	0	0	0.00	0	0.000
1528	A	3	2	1.00	29	0.069
1529	A	3	2	1.00	29	0.069
1530	A	3	2	1.00	29	0.069
1531	A	3	2	1.00	27	0.074
1532	A	5	4	1.00	27	0.148
1533	A	3	3	1.00	29	0.103
1534	A	4	4	1.00	29	0.138
1535	A	5	4	1.00	29	0.138
1536	A	3	2	1.00	31	0.065
1537	A	3	2	1.00	31	0.065
1538	A	3	2	1.00	31	0.065
1539	A	3	2	1.00	29	0.069
1540	A	6	4	1.00	29	0.138
1541	A	5	4	1.00	31	0.129
1542	A	4	4	1.00	31	0.129
1543	A	5	5	1.00	31	0.161
1544	A	3	2	1.00	31	0.065
1545	A	3	2	1.00	31	0.065
1546	A	3	2	1.00	31	0.065
1547	A	3	2	1.00	29	0.069
1548	A	3	2	1.00	29	0.069

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1549	A	4	3	1.00	31	0.097
1550	A	5	3	1.00	31	0.097
1551	A	6	3	1.00	31	0.097
1552	A	3	2	1.00	31	0.065
1553	A	3	2	1.00	31	0.065
1554	A	3	2	1.00	31	0.065
1555	A	3	2	1.00	29	0.069
1556	A	3	2	1.00	29	0.069
1557	A	4	3	1.00	31	0.097
1558	A	5	3	1.00	31	0.097
1559	A	6	3	1.00	31	0.097
1560	A	0	0	0.00	0	0.000
1561	A	4	2	1.00	35	0.057
1562	A	5	2	1.00	35	0.057
1563	A	5	3	1.00	35	0.086

Chapter 3

Listing of integrals

3.1 $\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=92

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}{6cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{15cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/15*a*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/c/f/(a+a*sin(f*x+e))^(1/2)-1/6*co
s(f*x+e)*(c-c*sin(f*x+e))^(9/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A] time = 0.40, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}{6cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{15cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(15*c*f*Sqrt[a + a*Sin[e + f*x
]]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/2))/(6
*c*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
```

```
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx = \frac{\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9}}{6cf}$$

$$= -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{15cf\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)\sqrt{}}{}$$

Mathematica [A] time = 0.57, size = 104, normalized size = 1.13

$$\frac{c^3\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(1080\sin(e+fx)+20\sin(3(e+fx))-36\sin(5(e+fx))+}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] (c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(405*
Cos[2*(e + f*x)] + 90*Cos[4*(e + f*x)] - 5*Cos[6*(e + f*x)] + 1080*Sin[e +
f*x] + 20*Sin[3*(e + f*x)] - 36*Sin[5*(e + f*x)]))/(960*f)
```

fricas [A] time = 0.44, size = 110, normalized size = 1.20

$$-\frac{\left(5\,c^3\cos\left(fx+e\right)^6-30\,c^3\cos\left(fx+e\right)^4+25\,c^3+2\left(9\,c^3\cos\left(fx+e\right)^4-8\,c^3\cos\left(fx+e\right)^2-16\,c^3\right)\sin\left(fx+\right.\right.}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/30*(5*c^3*cos(f*x + e)^6 - 30*c^3*cos(f*x + e)^4 + 25*c^3 + 2*(9*c^3*cos
(f*x + e)^4 - 8*c^3*cos(f*x + e)^2 - 16*c^3)*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-144*c^3*f*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(16*f)^2
-96*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*sin(3*f*x+3*exp(1))/(96*f)^2+480*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(160*f)^2-448*c^
3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*c
os(2*f*x+2*exp(1))/(64*f)^2-3328*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(256*f)^2+384*c^3*f*s
```

```
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(6*
f*x+6*exp(1))/(384*f)^2-1664*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-128*f)^2+256*c^3*f*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-4*f
*x-4*exp(1))/(-256*f)^2)
```

maple [A] time = 0.45, size = 133, normalized size = 1.45

$$\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{7}{2}}\sin\left(fx+e\right)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\left(5\left(\cos^{8}\left(fx+e\right)\right)+3\left(\cos^{6}\left(fx+e\right)\right)\sin\left(fx+e\right)+\right.}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/30/f*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(5*cos
(f*x+e)^8+3*cos(f*x+e)^6*sin(f*x+e)+4*cos(f*x+e)^6+7*sin(f*x+e)*cos(f*x+e)^
4+7*cos(f*x+e)^2*sin(f*x+e)-7*cos(f*x+e)^2+28*sin(f*x+e)+28)/cos(f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(7/2)*cos(f*x + e)
^2, x)
```

mupad [B] time = 11.45, size = 121, normalized size = 1.32

$$\frac{c^3\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(405\cos\left(e+fx\right)+495\cos\left(3e+3fx\right)+85\cos\left(5e+5f\right.\right.}{960f\left(\cos\left(2e+2fx\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (c^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(405*cos(e
+ f*x) + 495*cos(3*e + 3*f*x) + 85*cos(5*e + 5*f*x) - 5*cos(7*e + 7*f*x) +
1100*sin(2*e + 2*f*x) - 16*sin(4*e + 4*f*x) - 36*sin(6*e + 6*f*x)))/(960*f*
(cos(2*e + 2*f*x) + 1))
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(7/2)*(a+a*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

3.2 $\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=92

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{5cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/10*a*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/c/f/(a+a*sin(f*x+e))^(1/2)-1/5*co
s(f*x+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A] time = 0.39, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{5cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(10*c*f*Sqrt[a + a*Sin[e + f*x
]]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))/(5
*c*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))}{5cf}$$
$$= -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10cf\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)}{}$$

Mathematica [A] time = 0.48, size = 94, normalized size = 1.02

$$\frac{c^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(70\sin(e+fx)+5\sin(3(e+fx))-\sin(5(e+fx))+20\cos}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] (c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(20*C
os[2*(e + f*x)] + 5*Cos[4*(e + f*x)] + 70*Sin[e + f*x] + 5*Sin[3*(e + f*x)]
 - Sin[5*(e + f*x)]))/(80*f)
```

fricas [A] time = 0.45, size = 96, normalized size = 1.04

$$\frac{\left(5\,c^2\cos\left(fx+e\right)^4-5\,c^2-2\left(c^2\cos\left(fx+e\right)^4-2\,c^2\cos\left(fx+e\right)^2-4\,c^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-}}{10\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/10*(5*c^2*cos(f*x + e)^4 - 5*c^2 - 2*(c^2*cos(f*x + e)^4 - 2*c^2*cos(f*x
+ e)^2 - 4*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sq
rt(2*a)*sqrt(2*c)*(-112*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(16*f)^2-288*c^2*f*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/
(96*f)^2+160*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp
(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(160*f)^2-16*c^2*f*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))/(16*f)^
2-32*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))*cos(4*f*x+4*exp(1))/(32*f)^2-16*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-16*f)^2)
```

maple [A] time = 0.42, size = 106, normalized size = 1.15

$$\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\ \left(2\left(\cos^{6}\left(fx+e\right)\right)+\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+2}{10f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/10/f*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(2*cos
(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4+2*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e)+
6*sin(f*x+e)+6)/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2)*cos(f*x + e)
^2, x)
```

mupad [B] time = 10.45, size = 110, normalized size = 1.20

$$\frac{c^2\sqrt{a\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\,\left(20\cos\left(e+f\,x\right)+25\cos\left(3\,e+3\,f\,x\right)+5\cos\left(5\,e+5\,f\,x\right)}{80\,f\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(20*cos(e +
 f*x) + 25*cos(3*e + 3*f*x) + 5*cos(5*e + 5*f*x) + 75*sin(2*e + 2*f*x) + 4*
sin(4*e + 4*f*x) - sin(6*e + 6*f*x)))/(80*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(5/2)*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.3 $\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=92

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{4cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/6*a*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/c/f/(a+a*sin(f*x+e))^(1/2)-1/4*cos
(f*x+e)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A] time = 0.39, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{4cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(6*c*f*Sqrt[a + a*Sin[e + f*x]
]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))/(4*
c*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx = \frac{\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))}{4cf}$$

$$= -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6cf\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)}{}$$

Mathematica [A] time = 0.40, size = 83, normalized size = 0.90

$$\frac{c\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(8(9\sin(e+fx)+\sin(3(e+fx)))+12\cos(2(e+fx))+3\,c}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] (c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(12*Cos
[2*(e + f*x)] + 3*Cos[4*(e + f*x)] + 8*(9*Sin[e + f*x] + Sin[3*(e + f*x)]))
)/(96*f)
```

fricas [A] time = 0.43, size = 75, normalized size = 0.82

$$\frac{\left(3\,c\cos\left(fx+e\right)^4+4\left(c\cos\left(fx+e\right)^2+2\,c\right)\sin\left(fx+e\right)-3\,c\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/12*(3*c*cos(f*x + e)^4 + 4*(c*cos(f*x + e)^2 + 2*c)*sin(f*x + e) - 3*c)*s
qrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(
2*c)*(-24*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*sin(f*x+exp(1))/(8*f)^2-24*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(24*f)^2-32*c*f*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x
+2*exp(1))/(32*f)^2-64*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*
(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(64*f)^2-32*c*f*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(
-32*f)^2)
```

maple [A] time = 0.40, size = 90, normalized size = 0.98

$$\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\;\left(3\left(\cos^{4}\left(fx+e\right)\right)+\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+4}{12f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/12/f*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(3*cos
(f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)+4*cos(f*x+e)^2+5*sin(f*x+e)+5)/cos(f*x+e)
^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)*cos(f*x + e)
^2, x)
```

mupad [B] time = 1.71, size = 97, normalized size = 1.05

$$\frac{c\sqrt{a\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\,\left(12\,\cos\left(e+f\,x\right)+15\,\cos\left(3\,e+3\,f\,x\right)+3\,\cos\left(5\,e+5\,f\,x\right)+}{96\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] (c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(12*cos(e + f
*x) + 15*cos(3*e + 3*f*x) + 3*cos(5*e + 5*f*x) + 80*sin(2*e + 2*f*x) + 8*si
n(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.4 $\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=92

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}{3cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/3*a*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/c/f/(a+a*sin(f*x+e))^(1/2)-1/3*cos
(f*x+e)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A] time = 0.37, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}{3cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*c*f*Sqrt[a + a*Sin[e + f*x]
]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))/(3*
c*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
```

```
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/}}{3cf}$$
$$= -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3cf\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)\sqrt{}}{}$$

Mathematica [A] time = 0.17, size = 59, normalized size = 0.64

$$\frac{(9\sin(e+fx)+\sin(3(e+fx)))\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(9*Sin[e
+ f*x] + Sin[3*(e + f*x)]))/(12*f)
```

fricas [A] time = 0.42, size = 54, normalized size = 0.59

$$\frac{\left(\cos\left(fx+e\right)^2+2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/3*(cos(f*x + e)^2 + 2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)
*sin(f*x + e)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-24*f*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(8*f)^2-24*f*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*
x+3*exp(1))/(24*f)^2)
```

maple [A] time = 0.38, size = 55, normalized size = 0.60

$$\frac{\left(\cos^2\left(fx+e\right)+2\right)\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\ \sin\left(fx+e\right)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}}{3f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*(cos(f*x+e)^2+2)*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e
)))^(1/2)/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\ \cos\left(fx+e\right)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2
, x)
```

mupad [B] time = 0.90, size = 64, normalized size = 0.70

$$\frac{\left(10\sin\left(2e+2fx\right)+\sin\left(4e+4fx\right)\right)\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}{12f\left(\cos\left(2e+2fx\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] ((10*sin(2*e + 2*f*x) + sin(4*e + 4*f*x))*(a*(sin(e + f*x) + 1))^(1/2)*(-c*
(sin(e + f*x) - 1))^(1/2))/(12*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\cos^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))*cos(e + f*x
)**2, x)
```

3.5 $\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int (a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2af\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.30, size = 62, normalized size = 1.38

$$-\frac{\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(\cos(2(e+fx))-4\sin(e+fx))}{4cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -1/4*(Sec[e + f*x]*(Cos[2*(e + f*x)] - 4*Sin[e + f*x])*Sqrt[a*(1 + Sin[e +
f*x])]*Sqrt[c - c*Sin[e + f*x]])/(c*f)
```

fricas [A] time = 0.42, size = 59, normalized size = 1.31

$$-\frac{\left(\cos\left(fx+e\right)^2-2\,\sin\left(fx+e\right)-1\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\,cf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/2*(cos(f*x + e)^2 - 2*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e) + a)*sqrt(-c
*sin(f*x + e) + c)/(c*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (8*pi/x/2)>(-8*pi/x/2)4*sqrt(2*a)*((sqrt(c*tan(1/2*exp(1))
```

^2+c)*(-324259173170675712*tan(1/2*exp(1))^7+1188950301625810944*tan(1/2*exp(1))^5-684547143360315392*tan(1/2*exp(1))^3+108086391056891904*tan(1/2*exp(1)))+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))^2*(40532396646334464*tan(1/2*exp(1))^8-81064793292668928*tan(1/2*exp(1))^7-378302368699121664*tan(1/2*exp(1))^6+297237575406452736*tan(1/2*exp(1))^5+567453553048682496*tan(1/2*exp(1))^4-171136785840078848*tan(1/2*exp(1))^3-162129586585337856*tan(1/2*exp(1))^2+27021597764222976*tan(1/2*exp(1))+4503599627370496)+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))^3*(1125899906842624*tan(1/2*exp(1))^9-6755399441055744*tan(1/2*exp(1))^8-40532396646334464*tan(1/2*exp(1))^7+103582791429521408*tan(1/2*exp(1))^6+141863388262170624*tan(1/2*exp(1))^5-135107988821114880*tan(1/2*exp(1))^4-94575592174780416*tan(1/2*exp(1))^3+40532396646334464*tan(1/2*exp(1))^2+10133099161583616*tan(1/2*exp(1))-2251799813685248)+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))*(-4503599627370496*tan(1/2*exp(1))^9-27021597764222976*tan(1/2*exp(1))^8+162129586585337856*tan(1/2*exp(1))^7+414331165718085632*tan(1/2*exp(1))^6-567453553048682496*tan(1/2*exp(1))^5-540431955284459520*tan(1/2*exp(1))^4+378302368699121664*tan(1/2*exp(1))^3+162129586585337856*tan(1/2*exp(1))^2-40532396646334464*tan(1/2*exp(1))-9007199254740992)+sqrt(c*tan(1/2*exp(1))^2+c)*(324259173170675712*tan(1/2*exp(1))^7-1188950301625810944*tan(1/2*exp(1))^5+684547143360315392*tan(1/2*exp(1))^3-108086391056891904*tan(1/2*exp(1)))*tan(1/4*exp(1))^6+sqrt(c*tan(1/2*exp(1))^2+c)*(-648518346341351424*tan(1/2*exp(1))^8+4107282860161892352*tan(1/2*exp(1))^6-7133701809754865664*tan(1/2*exp(1))^4+1945555039024054272*tan(1/2*exp(1))^2)*tan(1/4*exp(1))^5+sqrt(c*tan(1/2*exp(1))^2+c)*(2161727821137838080*tan(1/2*exp(1))^8-13690942867206307840*tan(1/2*exp(1))^6+23779006032516218880*tan(1/2*exp(1))^4-6485183463413514240*tan(1/2*exp(1))^2)*tan(1/4*exp(1))^3+sqrt(c*tan(1/2*exp(1))^2+c)*(4863887597560135680*tan(1/2*exp(1))^7-17834254524387164160*tan(1/2*exp(1))^5+10268207150404730880*tan(1/2*exp(1))^3-1621295865853378560*tan(1/2*exp(1)))*tan(1/4*exp(1))^2+sqrt(c*tan(1/2*exp(1))^2+c)*(-648518346341351424*tan(1/2*exp(1))^8+4107282860161892352*tan(1/2*exp(1))^6-7133701809754865664*tan(1/2*exp(1))^4+1945555039024054272*tan(1/2*exp(1))^2)*tan(1/4*exp(1))+sqrt(c*tan(1/2*exp(1))^2+c)*(-4863887597560135680*tan(1/2*exp(1))^7+17834254524387164160*tan(1/2*exp(1))^5-10268207150404730880*tan(1/2*exp(1))^3+1621295865853378560*tan(1/2*exp(1)))*tan(1/4*exp(1))^4+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))^2*(90071992547409920*tan(1/2*exp(1))^9+540431955284459520*tan(1/2*exp(1))^8-3242591731706757120*tan(1/2*exp(1))^7-3422735716801576960*tan(1/2*exp(1))^6+11349071060973649920*tan(1/2*exp(1))^5+5944751508129054720*tan(1/2*exp(1))^4-7566047373982433280*tan(1/2*exp(1))^3-1621295865853378560*tan(1/2*exp(1))^2+810647932926689280*tan(1/2*exp(1)))*tan(1/4*exp(1))^3+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))^2*(607985949695016960*tan(1/2*exp(1))^8-1215971899390033920*tan(1/2*exp(1))^7-5674535530486824960*tan(1/2*exp(1))^6+4458563631096791040*tan(1/2*exp(1))^5+8511803295730237440*tan(1/2*exp(1))^4-2567051787601182720*tan(1/2*e

xp(1))^3-2431943798780067840*tan(1/2*exp(1))^2+405323966463344640*tan(1/2*e
xp(1))+67553994410557440)*tan(1/4*exp(1))^4+sqrt(c*tan(1/2*exp(1))^2+c)*(ta
n(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))^2*(-27021597764222
976*tan(1/2*exp(1))^9-162129586585337856*tan(1/2*exp(1))^8+9727775195120271
36*tan(1/2*exp(1))^7+1026820715040473088*tan(1/2*exp(1))^6-3404721318292094
976*tan(1/2*exp(1))^5-1783425452438716416*tan(1/2*exp(1))^4+226981421219472
9984*tan(1/2*exp(1))^3+486388759756013568*tan(1/2*exp(1))^2-243194379878006
784*tan(1/2*exp(1)))*tan(1/4*exp(1))^5+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2
*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))^2*(-27021597764222976*t
an(1/2*exp(1))^9-162129586585337856*tan(1/2*exp(1))^8+972777519512027136*ta
n(1/2*exp(1))^7+1026820715040473088*tan(1/2*exp(1))^6-3404721318292094976*t
an(1/2*exp(1))^5-1783425452438716416*tan(1/2*exp(1))^4+2269814212194729984*
tan(1/2*exp(1))^3+486388759756013568*tan(1/2*exp(1))^2-243194379878006784*t
an(1/2*exp(1)))*tan(1/4*exp(1))+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f
*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))^2*(-40532396646334464*tan(1/2*
exp(1))^8+81064793292668928*tan(1/2*exp(1))^7+378302368699121664*tan(1/2*ex
p(1))^6-297237575406452736*tan(1/2*exp(1))^5-567453553048682496*tan(1/2*exp
(1))^4+171136785840078848*tan(1/2*exp(1))^3+162129586585337856*tan(1/2*exp(
1))^2-27021597764222976*tan(1/2*exp(1))-4503599627370496)*tan(1/4*exp(1))^6
+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*
x+2*exp(1))))^2*(-607985949695016960*tan(1/2*exp(1))^8+1215971899390033920*
tan(1/2*exp(1))^7+5674535530486824960*tan(1/2*exp(1))^6-4458563631096791040
*tan(1/2*exp(1))^5-8511803295730237440*tan(1/2*exp(1))^4+256705178760118272
0*tan(1/2*exp(1))^3+2431943798780067840*tan(1/2*exp(1))^2-40532396646334464
0*tan(1/2*exp(1))-67553994410557440)*tan(1/4*exp(1))^2+sqrt(c*tan(1/2*exp(1
))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))^3*(1688
8498602639360*tan(1/2*exp(1))^9-101330991615836160*tan(1/2*exp(1))^8-607985
949695016960*tan(1/2*exp(1))^7+1553741871442821120*tan(1/2*exp(1))^6+212795
0823932559360*tan(1/2*exp(1))^5-2026619832316723200*tan(1/2*exp(1))^4-14186
33882621706240*tan(1/2*exp(1))^3+607985949695016960*tan(1/2*exp(1))^2+15199
6487423754240*tan(1/2*exp(1))-33776997205278720)*tan(1/4*exp(1))^4+sqrt(c*t
an(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1
))))^3*(45035996273704960*tan(1/2*exp(1))^9-202661983231672320*tan(1/2*exp(
1))^8-810647932926689280*tan(1/2*exp(1))^7+1891511843495608320*tan(1/2*exp(
1))^6+2702159776422297600*tan(1/2*exp(1))^5-2837267765243412480*tan(1/2*exp
(1))^4-2071655828590428160*tan(1/2*exp(1))^3+810647932926689280*tan(1/2*exp
(1))^2+135107988821114880*tan(1/2*exp(1))-22517998136852480)*tan(1/4*exp(1)
)^3+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2
*f*x+2*exp(1))))^3*(-1125899906842624*tan(1/2*exp(1))^9+6755399441055744*ta
n(1/2*exp(1))^8+40532396646334464*tan(1/2*exp(1))^7-103582791429521408*tan(
1/2*exp(1))^6-141863388262170624*tan(1/2*exp(1))^5+135107988821114880*tan(1
/2*exp(1))^4+94575592174780416*tan(1/2*exp(1))^3-40532396646334464*tan(1/2*
exp(1))^2-10133099161583616*tan(1/2*exp(1))+2251799813685248)*tan(1/4*exp(1
))^6+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/
2*f*x+2*exp(1))))^3*(-13510798882111488*tan(1/2*exp(1))^9+60798594969501696

*tan(1/2*exp(1))^8+243194379878006784*tan(1/2*exp(1))^7-567453553048682496*
tan(1/2*exp(1))^6-810647932926689280*tan(1/2*exp(1))^5+851180329573023744*t
an(1/2*exp(1))^4+621496748577128448*tan(1/2*exp(1))^3-243194379878006784*ta
n(1/2*exp(1))^2-40532396646334464*tan(1/2*exp(1))+6755399441055744)*tan(1/4
*exp(1))^5+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1
/2*(1/2*f*x+2*exp(1))))^3*(-13510798882111488*tan(1/2*exp(1))^9+60798594969
501696*tan(1/2*exp(1))^8+243194379878006784*tan(1/2*exp(1))^7-5674535530486
82496*tan(1/2*exp(1))^6-810647932926689280*tan(1/2*exp(1))^5+85118032957302
3744*tan(1/2*exp(1))^4+621496748577128448*tan(1/2*exp(1))^3-243194379878006
784*tan(1/2*exp(1))^2-40532396646334464*tan(1/2*exp(1))+6755399441055744)*t
an(1/4*exp(1))+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/t
an(1/2*(1/2*f*x+2*exp(1))))^3*(-16888498602639360*tan(1/2*exp(1))^9+1013309
91615836160*tan(1/2*exp(1))^8+607985949695016960*tan(1/2*exp(1))^7-15537418
71442821120*tan(1/2*exp(1))^6-2127950823932559360*tan(1/2*exp(1))^5+2026619
832316723200*tan(1/2*exp(1))^4+1418633882621706240*tan(1/2*exp(1))^3-607985
949695016960*tan(1/2*exp(1))^2-151996487423754240*tan(1/2*exp(1))+337769972
05278720)*tan(1/4*exp(1))^2+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x+2
exp(1)))-1/tan(1/2(1/2*f*x+2*exp(1))))*(4503599627370496*tan(1/2*exp(1))^
9+27021597764222976*tan(1/2*exp(1))^8-162129586585337856*tan(1/2*exp(1))^7-
414331165718085632*tan(1/2*exp(1))^6+567453553048682496*tan(1/2*exp(1))^5+5
40431955284459520*tan(1/2*exp(1))^4-378302368699121664*tan(1/2*exp(1))^3-16
2129586585337856*tan(1/2*exp(1))^2+40532396646334464*tan(1/2*exp(1))+900719
9254740992)*tan(1/4*exp(1))^6+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x
+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))*(67553994410557440*tan(1/2*exp(1
))^9+405323966463344640*tan(1/2*exp(1))^8-2431943798780067840*tan(1/2*exp(1
))^7-6214967485771284480*tan(1/2*exp(1))^6+8511803295730237440*tan(1/2*exp(
1))^5+8106479329266892800*tan(1/2*exp(1))^4-5674535530486824960*tan(1/2*exp
(1))^3-2431943798780067840*tan(1/2*exp(1))^2+607985949695016960*tan(1/2*exp
(1))+135107988821114880)*tan(1/4*exp(1))^2+sqrt(c*tan(1/2*exp(1))^2+c)*(tan
(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))*(180143985094819840
*tan(1/2*exp(1))^9+810647932926689280*tan(1/2*exp(1))^8-3242591731706757120
*tan(1/2*exp(1))^7-7566047373982433280*tan(1/2*exp(1))^6+108086391056891904
00*tan(1/2*exp(1))^5+11349071060973649920*tan(1/2*exp(1))^4-828662331436171
2640*tan(1/2*exp(1))^3-3242591731706757120*tan(1/2*exp(1))^2+54043195528445
9520*tan(1/2*exp(1))+90071992547409920)*tan(1/4*exp(1))^3+sqrt(c*tan(1/2*ex
p(1))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))*(-54
043195528445952*tan(1/2*exp(1))^9-243194379878006784*tan(1/2*exp(1))^8+9727
77519512027136*tan(1/2*exp(1))^7+2269814212194729984*tan(1/2*exp(1))^6-3242
591731706757120*tan(1/2*exp(1))^5-3404721318292094976*tan(1/2*exp(1))^4+248
5986994308513792*tan(1/2*exp(1))^3+972777519512027136*tan(1/2*exp(1))^2-162
129586585337856*tan(1/2*exp(1))-27021597764222976)*tan(1/4*exp(1))^5+sqrt(c
*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp
(1))))*(-54043195528445952*tan(1/2*exp(1))^9-243194379878006784*tan(1/2*exp
(1))^8+972777519512027136*tan(1/2*exp(1))^7+2269814212194729984*tan(1/2*exp
(1))^6-3242591731706757120*tan(1/2*exp(1))^5-3404721318292094976*tan(1/2*ex

p(1))^4+248598699430851379 2*tan(1/2*exp(1))^3+972777519512027136*tan(1/2*ex
p(1))^2-162129586585337856*tan(1/2*exp(1))-27021597764222976)*tan(1/4*exp(1
))+sqrt(c*tan(1/2*exp(1))^2+c)*(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*
f*x+2*exp(1))))*(-67553994410557440*tan(1/2*exp(1))^9-405323966463344640*ta
n(1/2*exp(1))^8+243194379878006784 0*tan(1/2*exp(1))^7+6214967485771284480*t
an(1/2*exp(1))^6-8511803295730237440*tan(1/2*exp(1))^5-8106479329266892800*
tan(1/2*exp(1))^4+5674535530486824960*tan(1/2*exp(1))^3+2431943798780067840
*tan(1/2*exp(1))^2-607985949695016960*tan(1/2*exp(1))-135107988821114880)*t
an(1/4*exp(1))^4)/(-(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1
))))^2-4)^2/((2251799813685248*sqrt(2)*c*tan(1/2*exp(1))^10+112589990684262
40*sqrt(2)*c*tan(1/2*exp(1))^8+22517998136852480*sqrt(2)*c*tan(1/2*exp(1))^
6+22517998136852480*sqrt(2)*c*tan(1/2*exp(1))^4+11258999068426240*sqrt(2)*c
*tan(1/2*exp(1))^2+2251799813685248*sqrt(2)*c)*tan(1/4*exp(1))^6+(675539944
1055744*sqrt(2)*c*tan(1/2*exp(1))^10+33776997205278720*sqrt(2)*c*tan(1/2*ex
p(1))^8+67553994410557440*sqrt(2)*c*tan(1/2*exp(1))^6+67553994410557440*sqr
t(2)*c*tan(1/2*exp(1))^4+33776997205278720*sqrt(2)*c*tan(1/2*exp(1))^2+6755
399441055744*sqrt(2)*c)*tan(1/4*exp(1))^4+(6755399441055744*sqrt(2)*c*tan(1
/2*exp(1))^10+33776997205278720*sqrt(2)*c*tan(1/2*exp(1))^8+675539944105574
40*sqrt(2)*c*tan(1/2*exp(1))^6+67553994410557440*sqrt(2)*c*tan(1/2*exp(1))^
4+33776997205278720*sqrt(2)*c*tan(1/2*exp(1))^2+6755399441055744*sqrt(2)*c)
*tan(1/4*exp(1))^2+2251799813685248*sqrt(2)*c*tan(1/2*exp(1))^10+1125899906
8426240*sqrt(2)*c*tan(1/2*exp(1))^8+22517998136852480*sqrt(2)*c*tan(1/2*exp
(1))^6+22517998136852480*sqrt(2)*c*tan(1/2*exp(1))^4+11258999068426240*sqrt
(2)*c*tan(1/2*exp(1))^2+2251799813685248*sqrt(2)*c)+1/4*(1/2*pi*sign(tan(1/
2*(1/2*f*x+2*exp(1))))+atan(1/2*(tan(1/2*(1/2*f*x+2*exp(1)))^2-1)/tan(1/2*(
1/2*f*x+2*exp(1)))))*(sqrt(c*tan(1/2*exp(1))^2+c)*(1073741824*sqrt(2)*tan(1
/2*exp(1))^3-3221225472*sqrt(2)*tan(1/2*exp(1)))+sqrt(c*tan(1/2*exp(1))^2+c
)*(16106127360*sqrt(2)*tan(1/2*exp(1))^3-48318382080*sqrt(2)*tan(1/2*exp(1)
))*tan(1/4*exp(1))^4+sqrt(c*tan(1/2*exp(1))^2+c)*(-1073741824*sqrt(2)*tan(1
/2*exp(1))^3+3221225472*sqrt(2)*tan(1/2*exp(1)))*tan(1/4*exp(1))^6+sqrt(c*t
an(1/2*exp(1))^2+c)*(-19327352832*sqrt(2)*tan(1/2*exp(1))^2+6442450944*sqrt
(2))*tan(1/4*exp(1))^5+sqrt(c*tan(1/2*exp(1))^2+c)*(64424509440*sqrt(2)*tan
(1/2*exp(1))^2-21474836480*sqrt(2))*tan(1/4*exp(1))^3+sqrt(c*tan(1/2*exp(1)
)^2+c)*(-16106127360*sqrt(2)*tan(1/2*exp(1))^3+48318382080*sqrt(2)*tan(1/2*
exp(1)))*tan(1/4*exp(1))^2+sqrt(c*tan(1/2*exp(1))^2+c)*(-19327352832*sqrt(2
)*tan(1/2*exp(1))^2+6442450944*sqrt(2))*tan(1/4*exp(1)))/(-(2147483648*c*ta
n(1/2*exp(1))^4+4294967296*c*tan(1/2*exp(1))^2+2147483648*c)*tan(1/4*exp(1)
)^6-(6442450944*c*tan(1/2*exp(1))^4+12884901888*c*tan(1/2*exp(1))^2+6442450
944*c)*tan(1/4*exp(1))^4-(6442450944*c*tan(1/2*exp(1))^4+12884901888*c*tan(
1/2*exp(1))^2+6442450944*c)*tan(1/4*exp(1))^2-2147483648*c*tan(1/2*exp(1))^
4-4294967296*c*tan(1/2*exp(1))^2-2147483648*c))/f

maple [B] time = 0.38, size = 94, normalized size = 2.09

$$\frac{\sin\left(fx+e\right)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos^{2}\left(fx+e\right)\right)+\sin\left(fx+e\right)+2\cos\left(fx+e\right)-\right.}{2f\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)
^2+sin(f*x+e)+2*cos(f*x+e)-1)/(-c*(sin(f*x+e)-1))^(1/2)/(1-cos(f*x+e)+sin(f
*x+e))
```

maxima [B] time = 0.50, size = 387, normalized size = 8.60

$$-\frac{\frac{2\sqrt{a}\sqrt{c}+\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sqrt{a}\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}{c+\frac{2c\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{c\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}}-\frac{2\sqrt{a}\sqrt{c}-\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}{c+\frac{2c\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{c\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}}+\frac{2\left(\frac{\sqrt{a}\sqrt{c}\sin\left(f\right.}{\cos\left(fx+e\right)+}\right.}{c+}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/2*((2*sqrt(a)*sqrt(c) + sqrt(a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1)
+ 3*sqrt(a)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(a)*sqrt(c)*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3)/(c + 2*c*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - (2*sqrt(a)*sqrt(c) - sqrt(
a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(a)*sqrt(c)*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 - sqrt(a)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
)/(c + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + c*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4) + 2*(sqrt(a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(a)
*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(a)*sqrt(c)*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3)/(c + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + c*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

mupad [B] time = 8.96, size = 73, normalized size = 1.62

$$-\frac{\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(\cos\left(e+f\,x\right)+\cos\left(3\,e+3\,f\,x\right)-4\,\sin\left(2\,e+2\,f\,x\right)\right)}{4\,c\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(cos(e + f*x)
+ cos(3*e + 3*f*x) - 4*sin(2*e + 2*f*x)))/(4*c*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\cos^2\left(e+fx\right)}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*cos(e + f*x)**2/sqrt(-c*(sin(e + f*x) -
 1)), x)
```

3.6 $\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{2a\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2*a*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)-cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{2a\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[
c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{2\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{(2a\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,d}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+}}$$

$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{(2a\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{c+x}\,dx, x\right.}{cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c}}$$

$$= -\frac{2a\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)\sqrt{a+a\sin(e}}{cf\sqrt{c-c\sin(e+fx}}$$

Mathematica [C] time = 1.09, size = 115, normalized size = 1.16

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\left(4\log\left(i-e^{i(e+fx)}\right)+\sin(e+fx)-2ifx\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{f(c-c\sin(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -(((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*((-2*
I)*f*x + 4*Log[I - E^(I*(e + f*x))] + Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(3/2)))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^2}{c^2\cos\left(fx+e\right)^2+2c^2\sin\left(fx+e\right)-2c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2
/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.38, size = 138, normalized size = 1.39

$$\frac{\left(\sin\left(fx+e\right)+4\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-2\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)\right)\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos^2\left(fx+e\right)\right)-2\right.}{f\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\left(-c\left(\sin\left(fx+e\right)-1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(sin(f*x+e)+4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*ln(2/(cos(f*
x+e)+1)))*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a
*(1+sin(f*x+e)))^(1/2)/(1-cos(f*x+e)+sin(f*x+e))/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}\,\cos\left(fx+e\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(3/
2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(3/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\cos^2\left(e+fx\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*cos(e + f*x)**2/(-c*(sin(e + f*x) - 1))
**(3/2), x)
```

3.7 $\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(3/2)+a*cos(f*x+e)*l
n(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2739, 2737, 2667, 31}

$$\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*(c - c*Sin[e + f*x])^(3/2)) +
(a*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{(a\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{c\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+f}$$

$$= \frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{(a\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{c+x}\,dx, x, -}{c^2f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\,\mathrm{si}}$$

$$= \frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{a\cos(e+fx)\log(1-\sin(e+fx}{c^2f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e}$$

Mathematica [C] time = 0.83, size = 104, normalized size = 1.07

$$\frac{\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\left(2\log\left(i-e^{i(e+fx)}\right)+\left(ifx-2\log\left(i-e^{i(e+fx)}\right)\right)\sin(e+fx)-ifx+2\right)}{c^2f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(2 - I*f*x + 2*Log[I - E^(I*(e + f
*x))] + (I*f*x - 2*Log[I - E^(I*(e + f*x))])*Sin[e + f*x]))/(c^2*f*Sqrt[c -
 c*Sin[e + f*x]])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^{2}}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2
/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e))
, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.38, size = 192, normalized size = 1.98

$$\frac{\left(2\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\sin\left(fx+e\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)-2\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-2\sin\left(\right.}{f\left(1-\cos\left(fx+e\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-sin(f*x+e)*ln(
2/(cos(f*x+e)+1))-2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*sin(f*x+e)
```

```
+ln(2/(cos(f*x+e)+1)))*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos
(f*x+e)+2)*(a*(1+sin(f*x+e)))^(1/2)/(1-cos(f*x+e)+sin(f*x+e))/(-c*(sin(f*x+
e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(5/
2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(5/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\cos^{2}\left(e+fx\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*cos(e + f*x)**2/(-c*(sin(e + f*x) - 1))
**(5/2), x)
```

3.8 $\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=48

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4acf(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/4*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/c/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A] time = 0.32, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4acf(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*a*c*f*(c - c*Sin[e + f*x])^(5/
2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4acf(c-c\sin(e+fx))^{5/2}}$$

Mathematica [A] time = 0.39, size = 90, normalized size = 1.88

$$\frac{\sin(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}{c^4 f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (Sin[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(c^4*f*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
))
```

fricas [A] time = 0.44, size = 79, normalized size = 1.65

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{c^4 f\cos\left(fx+e\right)^3+2\,c^4 f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^4 f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(c^4*f*cos
(f*x + e)^3 + 2*c^4*f*cos(f*x + e)*sin(f*x + e) - 2*c^4*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.38, size = 96, normalized size = 2.00

$$-\frac{\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sin\left(fx+e\right)\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos^{2}\left(fx+e\right)\right)-2\sin\left(fx+e\right)-\cos\left(fx+e\right)\right.}{f\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{7}{2}}\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/f*(a*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^
2-2*sin(f*x+e)-cos(f*x+e)+2)/(-c*(sin(f*x+e)-1))^(7/2)/(1-cos(f*x+e)+sin(f*
x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}\,\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(7/
2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\cos\left(e+f\,x\right)}^2\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(7/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.9 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=140

$$-\frac{4a^2\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{105cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{7cf}-\frac{2a\cos(e+fx)\sqrt{a}}{}$$

```
[Out] -1/7*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/c/f-4/105*a^2
*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/c/f/(a+a*sin(f*x+e))^(1/2)-2/21*a*cos(f*
x+e)*(c-c*sin(f*x+e))^(9/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A] time = 0.52, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{4a^2\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{105cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{7cf}-\frac{2a\cos(e+fx)\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-4*a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(105*c*f*Sqrt[a + a*Sin[e
+ f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(9/2))/(21*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(9/2))/(7*c*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx = \frac{\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9/2}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+ }{7cf}$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+}{21cf}$$

$$= -\frac{4a^2\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{105cf\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(}{}$$

Mathematica [A] time = 1.21, size = 166, normalized size = 1.19

$$-\frac{c^3(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}\,(4725\sin(e+fx)+665\sin(3(e+fx))+21\sin(5}{6720f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -1/6720*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*
Sin[e + f*x]]*(1050*Cos[2*(e + f*x)] + 420*Cos[4*(e + f*x)] + 70*Cos[6*(e +
 f*x)] + 4725*Sin[e + f*x] + 665*Sin[3*(e + f*x)] + 21*Sin[5*(e + f*x)] - 1
5*Sin[7*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [A] time = 0.51, size = 116, normalized size = 0.83

$$\frac{\left(35\,ac^3\cos\left(fx+e\right)^6-35\,ac^3-\left(15\,ac^3\cos\left(fx+e\right)^6-24\,ac^3\cos\left(fx+e\right)^4-32\,ac^3\cos\left(fx+e\right)^2-64\,ac^3\right)\sin}{105\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 1/105*(35*a*c^3*cos(f*x + e)^6 - 35*a*c^3 - (15*a*c^3*cos(f*x + e)^6 - 24*a
*c^3*cos(f*x + e)^4 - 32*a*c^3*cos(f*x + e)^2 - 64*a*c^3)*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-5760*a*c^3*f*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+
exp(1))/(128*f)^2-7296*a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(384*f)^2-640*a*c^3*f*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*
exp(1))/(640*f)^2+896*a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
/2*(f*x+exp(1))-1/4*pi))*sin(7*f*x+7*exp(1))/(896*f)^2-32*a*c^3*f*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*ex
p(1))/(32*f)^2-384*a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*
(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(128*f)^2-192*a*c^3*f*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(6*f*x+6*exp(
1))/(192*f)^2-192*a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-64*f)^2-128*a*c^3*f*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-4*f*x-4*exp
(1))/(-128*f)^2)
```

maple [A] time = 0.42, size = 133, normalized size = 0.95

$$\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{7}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(15\left(\cos^{8}\left(fx+e\right)\right)+5\left(\cos^{6}\left(fx+e\right)\right)\sin\left(fx+e\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/105/f*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(15*c
os(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)+16*cos(f*x+e)^6+13*sin(f*x+e)*cos(f*x
+e)^4+16*cos(f*x+e)^4+29*cos(f*x+e)^2*sin(f*x+e)+58*sin(f*x+e)+58)/cos(f*x+
e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(7/2)*cos(f*x +
e)^2, x)
```

mupad [B] time = 12.43, size = 319, normalized size = 2.28

$$e^{-e\,7\mathrm{i}-f\,x\,7\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{5\,a\,c^3\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{16\,f}+\frac{a\,c^3\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{8\,f}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((5*a*c^3*exp(e*7i + f*x*7
i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(16*f) + (a*c^3*exp(e*7i +
f*x*7i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(8*f) + (a*c^3*exp(e*7
i + f*x*7i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(48*f) + (19*a*c^3
*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) + (
a*c^3*exp(e*7i + f*x*7i)*sin(5*e + 5*f*x)*(a + a*sin(e + f*x))^(1/2))/(160*
f) - (a*c^3*exp(e*7i + f*x*7i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2))
```

```
/(224*f) + (45*a*c^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(a + a*sin(e + f*x))^(
1/2))/(32*f)))/(2*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.10 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=140

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6cf}-\frac{2a\cos(e+fx)\sqrt{a}}{}$$

```
[Out] -1/6*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/c/f-1/15*a^2*
cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/c/f/(a+a*sin(f*x+e))^(1/2)-2/15*a*cos(f*x
+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A] time = 0.53, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6cf}-\frac{2a\cos(e+fx)\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(15*c*f*Sqrt[a + a*Sin[e + f
*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/
2))/(15*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x]
)^(7/2))/(6*c*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+f}{6cf}$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+}{15cf}$$

$$= -\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15cf\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+}{}$$

Mathematica [A] time = 0.97, size = 156, normalized size = 1.11

$$\frac{c^2(\sin(e+fx)-1)^2(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}\,(600\sin(e+fx)+100\sin(3(e+fx))+12\sin(5(e+}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\mathrm{co}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (c^2*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
f*x]]*(75*Cos[2*(e + f*x)] + 30*Cos[4*(e + f*x)] + 5*Cos[6*(e + f*x)] + 600
*Sin[e + f*x] + 100*Sin[3*(e + f*x)] + 12*Sin[5*(e + f*x)]))/(960*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [A] time = 0.48, size = 102, normalized size = 0.73

$$\frac{\left(5\,ac^2\cos\left(fx+e\right)^6-5\,ac^2+2\left(3\,ac^2\cos\left(fx+e\right)^4+4\,ac^2\cos\left(fx+e\right)^2+8\,ac^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/30*(5*a*c^2*cos(f*x + e)^6 - 5*a*c^2 + 2*(3*a*c^2*cos(f*x + e)^4 + 4*a*c^
2*cos(f*x + e)^2 + 8*a*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*
sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-80*a*c^2*f*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(16*f)^
2-480*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*sin(3*f*x+3*exp(1))/(96*f)^2-160*a*c^2*f*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(160*f)^2-
64*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))*cos(2*f*x+2*exp(1))/(64*f)^2-768*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/
4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(256*f)^2-384
*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))*cos(6*f*x+6*exp(1))/(384*f)^2-384*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-128*f)^2-25
6*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*cos(-4*f*x-4*exp(1))/(-256*f)^2)
```

maple [A] time = 0.36, size = 116, normalized size = 0.83

$$\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(5\left(\cos^{6}\left(fx+e\right)\right)+\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+\right.}{30f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/30/f*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(5*cos
(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4+6*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e)+
8*cos(f*x+e)^2+11*sin(f*x+e)+11)/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c \sin\left(fx+e\right)+c\right)^{\frac{5}{2}} \cos\left(fx+e\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(5/2)*cos(f*x +
e)^2, x)
```

mupad [B] time = 11.44, size = 122, normalized size = 0.87

$$\frac{a\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(75\,\cos\left(e+f\,x\right)+105\,\cos\left(3\,e+3\,f\,x\right)+35\,\cos\left(5\,e+5\,f\right.}{960\,f\,\left(\cos\left(2\,e+2\,f\,x\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (a*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(75*cos(e
 + f*x) + 105*cos(3*e + 3*f*x) + 35*cos(5*e + 5*f*x) + 5*cos(7*e + 7*f*x) +
 700*sin(2*e + 2*f*x) + 112*sin(4*e + 4*f*x) + 12*sin(6*e + 6*f*x)))/(960*f
*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.11 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=140

$$-\frac{2a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5cf}-\frac{a\cos(e+fx)\sqrt{a}}{}$$

```
[Out] -1/5*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/c/f-2/15*a^2*
cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/c/f/(a+a*sin(f*x+e))^(1/2)-1/5*a*cos(f*x+
e)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [**A**] time = 0.52, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{2a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5cf}-\frac{a\cos(e+fx)\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*c*f*Sqrt[a + a*Sin[e +
 f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2))/(5*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(5/2))/(5*c*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+f}{5cf}$$

$$= -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+f}{5cf}$$

$$= -\frac{2a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15cf\sqrt{a+a\sin(e+fx)}} - \frac{a\cos(e+}{}$$

Mathematica [A] time = 0.62, size = 82, normalized size = 0.59

$$-\frac{c(\sin(e+fx)-1)(150\sin(e+fx)+25\sin(3(e+fx))+3\sin(5(e+fx)))\sec^3(e+fx)(a(\sin(e+fx)+1))^{3/2}\sqrt{c}}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -1/240*(c*Sec[e + f*x]^3*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*S
qrt[c - c*Sin[e + f*x]]*(150*Sin[e + f*x] + 25*Sin[3*(e + f*x)] + 3*Sin[5*(
e + f*x)]))/f
```

fricas [A] time = 0.46, size = 73, normalized size = 0.52

$$\frac{\left(3\,ac\cos\left(fx+e\right)^4+4\,ac\cos\left(fx+e\right)^2+8\,ac\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/15*(3*a*c*cos(f*x + e)^4 + 4*a*c*cos(f*x + e)^2 + 8*a*c)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-80*a*c*f*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))
/(16*f)^2-480*a*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+ex
p(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(96*f)^2-160*a*c*f*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(160*f
)^2)
```

maple [A] time = 0.34, size = 67, normalized size = 0.48

$$\frac{\left(3\left(\cos^4\left(fx+e\right)\right)+4\left(\cos^2\left(fx+e\right)\right)+8\right)\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{15f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/15/f*(3*cos(f*x+e)^4+4*cos(f*x+e)^2+8)*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+
e)*(a*(1+sin(f*x+e)))^(3/2)/cos(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2)*cos(f*x +
e)^2, x)
```

mupad [B] time = 1.48, size = 79, normalized size = 0.56

$$\frac{a\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(175\,\sin\left(2\,e+2\,f\,x\right)+28\,\sin\left(4\,e+4\,f\,x\right)+3\,\sin\left(6\,e+6\,f\right.\right.}{240\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] (a*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(175*sin(2*
e + 2*f*x) + 28*sin(4*e + 4*f*x) + 3*sin(6*e + 6*f*x)))/(240*f*(cos(2*e + 2
*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.12 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=92

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/6*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f/(c-c*sin(f*x+e))^(1/2)+1/4*cos(
f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.39, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*a*f*Sqrt[c - c*Sin[e + f*x]]
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(4*a
*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
```

```
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{4af}$$
$$= \frac{c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6af\sqrt{c-c\sin(e+fx)}} + \frac{\cos(e+fx)(a}{}$$

Mathematica [A] time = 0.39, size = 83, normalized size = 0.90

$$\frac{a\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}\,(8(9\sin(e+fx)+\sin(3(e+fx)))-12\cos(2(e+fx))-3\text{co}}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] (a*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-12*Co
s[2*(e + f*x)] - 3*Cos[4*(e + f*x)] + 8*(9*Sin[e + f*x] + Sin[3*(e + f*x)])
))/(96*f)
```

fricas [A] time = 0.42, size = 75, normalized size = 0.82

$$-\frac{\left(3\,a\cos\left(fx+e\right)^4-4\left(a\cos\left(fx+e\right)^2+2\,a\right)\sin\left(fx+e\right)-3\,a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/12*(3*a*cos(f*x + e)^4 - 4*(a*cos(f*x + e)^2 + 2*a)*sin(f*x + e) - 3*a)*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(
2*c)*(-24*a*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*sin(f*x+exp(1))/(8*f)^2-24*a*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(24*f)^2+32*a*f*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x
+2*exp(1))/(32*f)^2+64*a*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*
(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(64*f)^2+32*a*f*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(
-32*f)^2)
```

maple [A] time = 0.39, size = 90, normalized size = 0.98

$$-\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(-3\left(\cos^{4}\left(fx+e\right)\right)+\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right.\right.}{12f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/12/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(-3*c
os(f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)-4*cos(f*x+e)^2+5*sin(f*x+e)-5)/cos(f*x+
e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)
^2, x)
```

mupad [B] time = 9.81, size = 97, normalized size = 1.05

$$-\frac{a\sqrt{a\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\,\left(12\,\cos\left(e+f\,x\right)+15\,\cos\left(3\,e+3\,f\,x\right)+3\,\cos\left(5\,e+5\,f\,x\right)-}{96\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -(a*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(12*cos(e +
f*x) + 15*cos(3*e + 3*f*x) + 3*cos(5*e + 5*f*x) - 80*sin(2*e + 2*f*x) - 8*s
in(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.13 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/3*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int (a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

Mathematica [B] time = 0.55, size = 111, normalized size = 2.47

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(15\sin(e+fx)-\sin(3(e+fx))-6\cos(2(e+fx)))}{12f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(-6*Cos
[2*(e + f*x)] + 15*Sin[e + f*x] - Sin[3*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.44, size = 77, normalized size = 1.71

$$-\frac{\left(3\,a\cos\left(fx+e\right)^2+\left(a\cos\left(fx+e\right)^2-4\,a\right)\sin\left(fx+e\right)-3\,a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\,cf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/3*(3*a*cos(f*x + e)^2 + (a*cos(f*x + e)^2 - 4*a)*sin(f*x + e) - 3*a)*sqr
t(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.36, size = 141, normalized size = 3.13

$$\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(\cos^{3}\left(fx+e\right)+\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+2\left(\cos^{2}\left(fx+e\right)\right)-3\sin\left(f\right.\right.}{3f\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\left(\cos^{2}\left(fx+e\right)+\sin\left(fx+e\right)\cos\left(fx+e\right)+\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(cos(f*x+e)^3+cos(f*x+e)^2*sin(f*
x+e)+2*cos(f*x+e)^2-3*sin(f*x+e)*cos(f*x+e)-4*cos(f*x+e)-sin(f*x+e)+1)/(-c*
(sin(f*x+e)-1))^(1/2)/(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(
f*x+e)-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) +
c), x)
```

mupad [B] time = 9.40, size = 87, normalized size = 1.93

$$-\frac{a\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(6\cos\left(e+fx\right)+6\cos\left(3e+3fx\right)-14\sin\left(2e+2fx\right)+\right.}{12cf\left(\cos\left(2e+2fx\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] -(a*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(6*cos(e + f
*x) + 6*cos(3*e + 3*f*x) - 14*sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(12*c*f
*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

3.14 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)-4*a^2*cos
(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-
2*a*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$-\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (-4*a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f
*Sqrt[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c
*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} + \frac{2\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/}}{2cf\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/}}{2cf\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/}}{2cf\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin}}{cf\sqrt{c-c\sin(e+f}}$$

Mathematica [A] time = 1.06, size = 130, normalized size = 0.88

$$-\frac{(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12\sin(e+fx)-\cos(2(e+fx))+32\log\left(\cos\left(\frac{1}{2}(e\right.\right.\right.}{4cf\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] -1/4*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(-
Cos[2*(e + f*x)] + 32*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 12*Sin[e +
 f*x]))/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x
]])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\cos\left(fx+e\right)^2\sin\left(fx+e\right)+a\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(a*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x +
e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.36, size = 173, normalized size = 1.18

$$-\frac{\left(-\left(\cos^2\left(fx+e\right)\right)+6\sin\left(fx+e\right)+16\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-8\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)+1\right)\left(\sin\left(fx+e\right)\cos\right.}{2f\left(\cos^2\left(fx+e\right)+\sin\left(fx+e\right)\cos\left(fx+e\right)+\cos\left(fx+e\right)-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(-cos(f*x+e)^2+6*sin(f*x+e)+16*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))-8*ln(2/(cos(f*x+e)+1))+1)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f
*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)^2+sin(f*x+e)*cos(f
*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [B] time = 0.96, size = 844, normalized size = 5.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/2*(16*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - 8*a^(3/2
)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + (10*a^(3/2) - 11*a
^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 15*a^(3/2)*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 - 20*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^(3/2)*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 7*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5)/(c^(3/2) - 2*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^(3/2)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2*c^(3/2)*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)
 - (10*a^(3/2) - 13*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 25*a^(3/2)*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 20*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 15*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 9*a^(3/2)*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)/(c^(3/2) - 2*c^(3/2)*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^(3/2)*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 - 2*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6) + 2*(5*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) - 5*a
^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*a^(3/2)*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 - 5*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^(3/2)
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(c^(3/2) - 2*c^(3/2)*sin(f*x + e)/(co
s(f*x + e) + 1) + 3*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^(3/2)
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 - 2*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.15 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(3/2)+4*a^2*cos(f*x+
e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2*a
*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(c*f*(c - c*Sin[e + f*x])^(3/2))
+ (4*a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) + (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(
c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{2\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{4a^2\cos(e+fx)\log(1-\sin(e+}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(}$$

Mathematica [A] time = 1.06, size = 169, normalized size = 1.17

$$\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos(2(e+fx))+16\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{2c^2f(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(7 +
Cos[2*(e + f*x)] + 16*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (2 - 16*Lo
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*c^2*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\cos\left(fx+e\right)^2\sin\left(fx+e\right)+a\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(a*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*c
os(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.37, size = 223, normalized size = 1.55

$$\frac{\left(8 \sin \left(f x+e\right) \ln \left(-\frac{-1+\cos \left(f x+e\right)+\sin \left(f x+e\right)}{\sin \left(f x+e\right)}\right)-\left(\cos ^{2}\left(f x+e\right)\right)-4 \sin \left(f x+e\right) \ln \left(\frac{2}{\cos \left(f x+e\right)+1}\right)-8 \ln \left(-\frac{-1+\cos \left(f x+\right.}{\sin \left(\right.}\right.}{f \left(\cos ^{2}\left(f x+e\right)+\sin \left(f x+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(8*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)^2-4
*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-8*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))-5*sin(f*x+e)+4*ln(2/(cos(f*x+e)+1))+1)*(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+
e)+cos(f*x+e)+2*sin(f*x+e)-2)*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)^2+sin(f*
x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sin \left(f x+e\right)+a\right)^{\frac{3}{2}} \cos \left(f x+e\right)^{2}}{\left(-c \sin \left(f x+e\right)+c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.16 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{a^2\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)-a*cos(f*x+
e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)-a^2*cos(f*x+e)*ln(1-
sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2739, 2737, 2667, 31}

$$-\frac{a^2\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e + f*x])^(5/2)
) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*(c - c*Sin[e + f*x])^(
3/2)) - (a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*Sin[e +
f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}}$$

Mathematica [A] time = 1.36, size = 191, normalized size = 1.30

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos(2(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{c^3f(\sin(e+fx)-1)^3\sqrt{c-c\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] -((a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(-2
 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e + f*x)]*Log[Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]] + 4*(1 + Log[Cos[(e + f*x)/2] - Sin[(e + f*x
)/2]])*Sin[e + f*x]))/(c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Si
n[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]))
```

fricas [F] time = 1.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a\cos\left(fx+e\right)^2\sin\left(fx+e\right)+a\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] integral((a*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e
)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.42, size = 276, normalized size = 1.88

$$-\frac{\left(\left(\cos^2\left(fx+e\right)\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)-2\left(\cos^2\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+2\sin\left(fx+e\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/f*(cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))+2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-4*sin(f*x+e)*ln(-(
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)^2-2*ln(2/(cos(f*x+e)+1))
+4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2)*(sin(f*x+e)*cos(f*x+e)-cos
(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)^2
+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}} \cos\left(fx+e\right)^{2}}{\left(-c \sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(7/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.17 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=48

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6acf(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/6*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/c/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A] time = 0.34, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6acf(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(9/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*a*c*f*(c - c*Sin[e + f*x])^(7/
2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6acf(c-c\sin(e+fx))^{7/2}}$$

Mathematica [B] time = 1.42, size = 110, normalized size = 2.29

$$-\frac{a(3\cos(2(e+fx))-5)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{6c^4f(\sin(e+fx)-1)^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] -1/6*(a*(-5 + 3*Cos[2*(e + f*x)])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*S
qrt[a*(1 + Sin[e + f*x])])/(c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1
 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.45, size = 105, normalized size = 2.19

$$\frac{\left(3\,a\cos\left(fx+e\right)^2-4\,a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(3\,c^5f\cos\left(fx+e\right)^3-4\,c^5f\cos\left(fx+e\right)-\left(c^5f\cos\left(fx+e\right)^3-4\,c^5f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] 1/3*(3*a*cos(f*x + e)^2 - 4*a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(3*c^5*f*cos(f*x + e)^3 - 4*c^5*f*cos(f*x + e) - (c^5*f*cos(f*x + e)
^3 - 4*c^5*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.35, size = 127, normalized size = 2.65

$$-\frac{\left(\cos^2\left(fx+e\right)-4\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos^2\left(fx+e\right)\right)-2\sin\left(f\right.\right.}{3f\left(\cos^2\left(fx+e\right)+\sin\left(fx+e\right)\cos\left(fx+e\right)+\cos\left(fx+e\right)-2\sin\left(fx+e\right)-2\right)\left(-c\left(\sin\left(fx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/3/f*(cos(f*x+e)^2-4)*(a*(1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(sin(f*x+e)*cos
(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(cos(f*x+e)^2+sin(f*x+e)*co
s(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(9/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(9/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

3.18 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48ac^2f(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8acf(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/8*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/c/f/(c-c*sin(f*x+e))^(9/2)+1/48*cos
(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/c^2/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A] time = 0.44, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48ac^2f(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8acf(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(11/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*a*c*f*(c - c*Sin[e + f*x])^(9/
2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*a*c^2*f*(c - c*Sin[e +
f*x])^(7/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8acf(c-c\sin(e+fx))^{9/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{8ac^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8acf(c-c\sin(e+fx))^{9/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{48ac^2f(c-c\sin(e+fx))^{7/2}}$$

Mathematica [A] time = 1.95, size = 118, normalized size = 1.22

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(4\sin(e+fx)-3\cos(2(e+fx))+5)}{12c^5f(\sin(e+fx)-1)^5\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] -1/12*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]
*(5 - 3*Cos[2*(e + f*x)] + 4*Sin[e + f*x]))/(c^5*f*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.44, size = 127, normalized size = 1.31

$$-\frac{\left(3\,a\cos\left(fx+e\right)^2-2\,a\sin\left(fx+e\right)-4\,a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(c^6f\cos\left(fx+e\right)^5-8\,c^6f\cos\left(fx+e\right)^3+8\,c^6f\cos\left(fx+e\right)+4\left(c^6f\cos\left(fx+e\right)^3-2\,c^6f\cos\left(fx+e\right)\right)\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="fricas")
```

```
[Out] -1/6*(3*a*cos(f*x + e)^2 - 2*a*sin(f*x + e) - 4*a)*sqrt(a*sin(f*x + e) + a)
*sqrt(-c*sin(f*x + e) + c)/(c^6*f*cos(f*x + e)^5 - 8*c^6*f*cos(f*x + e)^3 +
 8*c^6*f*cos(f*x + e) + 4*(c^6*f*cos(f*x + e)^3 - 2*c^6*f*cos(f*x + e))*sin
(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.38, size = 152, normalized size = 1.57

$$\frac{\left(\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-4\left(\cos^2\left(fx+e\right)\right)-4\sin\left(fx+e\right)+10\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\left(\sin\right.}{6f\left(\cos^2\left(fx+e\right)+\sin\left(fx+e\right)\cos\left(fx+e\right)+\cos\left(fx+e\right)-2\sin\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 1/6/f*(cos(f*x+e)^2*sin(f*x+e)-4*cos(f*x+e)^2-4*sin(f*x+e)+10)*(a*(1+sin(f*
x+e)))^(3/2)*sin(f*x+e)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-co
s(f*x+e)+2)/(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/
(-c*(sin(f*x+e)-1))^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
11/2), x)
```

mupad [B] time = 13.87, size = 236, normalized size = 2.43

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{40\,a\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^6\,f}-\frac{8\,a\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c^6\,f}+\frac{32\,a\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}}{}\right.}{84\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}-54\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)+2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)-96\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(2\,e+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(11/2)
,x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((40*a*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^
(1/2))/(3*c^6*f) - (8*a*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a*sin(e +
f*x))^(1/2))/(c^6*f) + (32*a*exp(e*5i + f*x*5i)*sin(e + f*x)*(a + a*sin(e +
 f*x))^(1/2))/(3*c^6*f)))/(84*cos(e + f*x)*exp(e*5i + f*x*5i) - 54*exp(e*5i
 + f*x*5i)*cos(3*e + 3*f*x) + 2*exp(e*5i + f*x*5i)*cos(5*e + 5*f*x) - 96*ex
p(e*5i + f*x*5i)*sin(2*e + 2*f*x) + 16*exp(e*5i + f*x*5i)*sin(4*e + 4*f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

3.19 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=188

$$-\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35cf\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}{14cf}-\frac{\cos(e+fx)(a\sin}{}$$

```
[Out] -3/28*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/c/f-1/8*co
s(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(9/2)/c/f-1/35*a^3*cos(f*x
+e)*(c-c*sin(f*x+e))^(9/2)/c/f/(a+a*sin(f*x+e))^(1/2)-1/14*a^2*cos(f*x+e)*(
c-c*sin(f*x+e))^(9/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A] time = 0.62, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}{14cf}-\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(35*c*f*Sqrt[a + a*Sin[e + f
*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/
2))/(14*c*f) - (3*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(9/2))/(28*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin
[e + f*x])^(9/2))/(8*c*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
```

```
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx = \frac{\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+f}{8cf}$$

$$= -\frac{3a\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+}{28cf}$$

$$= -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+f}{14cf}$$

$$= -\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35cf\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+}{}$$

Mathematica [A] time = 3.11, size = 176, normalized size = 0.94

$$-\frac{c^3(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}\,(19600\sin(e+fx)+3920\sin(3(e+fx))+784\sin}{35840f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -1/35840*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c
*Sin[e + f*x]]*(1960*Cos[2*(e + f*x)] + 980*Cos[4*(e + f*x)] + 280*Cos[6*(e
 + f*x)] + 35*Cos[8*(e + f*x)] + 19600*Sin[e + f*x] + 3920*Sin[3*(e + f*x)]
 + 784*Sin[5*(e + f*x)] + 80*Sin[7*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.49, size = 128, normalized size = 0.68

$$\frac{\left(35\,a^2c^3\cos\left(fx+e\right)^8-35\,a^2c^3+8\left(5\,a^2c^3\cos\left(fx+e\right)^6+6\,a^2c^3\cos\left(fx+e\right)^4+8\,a^2c^3\cos\left(fx+e\right)^2+16\,a^2c^3\right.\right.}{280\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 1/280*(35*a^2*c^3*cos(f*x + e)^8 - 35*a^2*c^3 + 8*(5*a^2*c^3*cos(f*x + e)^6
 + 6*a^2*c^3*cos(f*x + e)^4 + 8*a^2*c^3*cos(f*x + e)^2 + 16*a^2*c^3)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sq
rt(2*c)*(-4480*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(128*f)^2-8064*a^2*c^3*f*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))
/(384*f)^2-4480*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(640*f)^2-896*a^2*c^3*f*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(7*f*x+7*exp
(1))/(896*f)^2-2560*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))/(512*f)^2-9216*a^2*c^3*f*sign(
```

```
sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+
4*exp(1))/(1024*f)^2-7680*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(6*f*x+6*exp(1))/(1536*f)^2-2048*a^2*c^3*
f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos
(8*f*x+8*exp(1))/(2048*f)^2-4608*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-512*f)^2-5120*
a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*cos(-4*f*x-4*exp(1))/(-1024*f)^2-1536*a^2*c^3*f*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-6*f*x-6*exp(1))/(-1536
*f)^2)
```

maple [A] time = 0.44, size = 143, normalized size = 0.76

$$\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{7}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(35\left(\cos^{8}\left(fx+e\right)\right)+5\left(\cos^{6}\left(fx+e\right)\right)\sin\left(fx+e\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/280/f*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(35*c
os(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)+40*cos(f*x+e)^6+13*sin(f*x+e)*cos(f*x
+e)^4+48*cos(f*x+e)^4+29*cos(f*x+e)^2*sin(f*x+e)+64*cos(f*x+e)^2+93*sin(f*x
+e)+93)/cos(f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(7/2)*cos(f*x +
e)^2, x)
```

mupad [B] time = 12.62, size = 376, normalized size = 2.00

$$\frac{\mathrm{e}^{-e\,8\mathrm{i}-f\,x\,8\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{35\,a^2\,c^3\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}+\frac{7\,a^2\,c^3\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (exp(- e*8i - f*x*8i)*(c - c*sin(e + f*x))^(1/2)*((35*a^2*c^3*exp(e*8i + f*
x*8i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (7*a^2*c^3*exp(e*8i
 + f*x*8i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) + (7*a^2*c^3
*exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(128*f) +
(a^2*c^3*exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(6
4*f) + (a^2*c^3*exp(e*8i + f*x*8i)*cos(8*e + 8*f*x)*(a + a*sin(e + f*x))^(1
/2))/(512*f) + (7*a^2*c^3*exp(e*8i + f*x*8i)*sin(3*e + 3*f*x)*(a + a*sin(e
+ f*x))^(1/2))/(32*f) + (7*a^2*c^3*exp(e*8i + f*x*8i)*sin(5*e + 5*f*x)*(a +
 a*sin(e + f*x))^(1/2))/(160*f) + (a^2*c^3*exp(e*8i + f*x*8i)*sin(7*e + 7*f
*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.20 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=188

$$-\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35cf\sqrt{a\sin(e+fx)+a}}-\frac{4a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{35cf}-\frac{\cos(e+fx)(a\sin$$

```
[Out] -1/7*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/c/f-1/7*cos
(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/c/f-2/35*a^3*cos(f*x+
e)*(c-c*sin(f*x+e))^(7/2)/c/f/(a+a*sin(f*x+e))^(1/2)-4/35*a^2*cos(f*x+e)*(c
-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A] time = 0.62, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{4a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{35cf}-\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(35*c*f*Sqrt[a + a*Sin[e +
 f*x]]) - (4*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(7/2))/(35*c*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e
+ f*x])^(7/2))/(7*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Si
n[e + f*x])^(7/2))/(7*c*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
```

```
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+}{7cf}$$
$$= -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+}{7cf}$$
$$= -\frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e}{35cf}$$
$$= -\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35cf\sqrt{a+a\sin(e+fx)}} - \frac{4a^2\cos}{}$$

Mathematica [A] time = 0.65, size = 87, normalized size = 0.46

$$\frac{a^2c^2(1225\sin(e+fx)+245\sin(3(e+fx))+49\sin(5(e+fx))+5\sin(7(e+fx)))\sec(e+fx)\sqrt{a(\sin(e+fx)+}}{2240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (a^2*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
1225*Sin[e + f*x] + 245*Sin[3*(e + f*x)] + 49*Sin[5*(e + f*x)] + 5*Sin[7*(e
 + f*x)]))/(2240*f)
```

fricas [A] time = 0.49, size = 101, normalized size = 0.54

$$\frac{\left(5\,a^2c^2\cos\left(fx+e\right)^6+6\,a^2c^2\cos\left(fx+e\right)^4+8\,a^2c^2\cos\left(fx+e\right)^2+16\,a^2c^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right.}}{35\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/35*(5*a^2*c^2*cos(f*x + e)^6 + 6*a^2*c^2*cos(f*x + e)^4 + 8*a^2*c^2*cos(f
*x + e)^2 + 16*a^2*c^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*
sin(f*x + e)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-4480*a
^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))*sin(f*x+exp(1))/(128*f)^2-8064*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(384*f)^2-4480*
a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*sin(5*f*x+5*exp(1))/(640*f)^2-896*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(7*f*x+7*exp(1))/(896*f)^2)
```

maple [A] time = 0.37, size = 77, normalized size = 0.41

$$\frac{\left(5\left(\cos^6\left(fx+e\right)\right)+6\left(\cos^4\left(fx+e\right)\right)+8\left(\cos^2\left(fx+e\right)\right)+16\right)\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\right.\right.}{35f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/35/f*(5*cos(f*x+e)^6+6*cos(f*x+e)^4+8*cos(f*x+e)^2+16)*(-c*(sin(f*x+e)-1)
)^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(5/2)*cos(f*x +
e)^2, x)
```

mupad [B] time = 11.80, size = 179, normalized size = 0.95

$$\frac{\frac{1225\,a^2\,c^2\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{32}+\frac{245\,a^2\,c^2\,\sin\left(3\,e+3\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{32}+\frac{49\,a^2\,c^2\,\sin\left(5\,e+5\,f\,x\right.}{}}{70\,f\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] ((1225*a^2*c^2*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(1/2))/32 + (245*a^2*c^2*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2)*(c -
c*sin(e + f*x))^(1/2))/32 + (49*a^2*c^2*sin(5*e + 5*f*x)*(a + a*sin(e + f*x
))^(1/2)*(c - c*sin(e + f*x))^(1/2))/32 + (5*a^2*c^2*sin(7*e + 7*f*x)*(a +
a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2))/32)/(70*f*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.21 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=140

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{15af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6af}+\frac{2c\cos(e+fx)(a\sin}{}$$

```
[Out] 1/6*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/a/f+1/15*c^2*c
os(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/f/(c-c*sin(f*x+e))^(1/2)+2/15*c*cos(f*x+
e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.52, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{15af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6af}+\frac{2c\cos(e+fx)(a\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(15*a*f*Sqrt[c - c*Sin[e + f*
x]]) + (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]
])/(15*a*f) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])
^(3/2))/(6*a*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+f}{6af}$$
$$= \frac{2c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e}}{15af}$$
$$= \frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{15af\sqrt{c-c\sin(e+fx)}} + \frac{2c\cos(e+}{}$$

Mathematica [A] time = 0.77, size = 152, normalized size = 1.09

$$-\frac{c(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}\,(600\sin(e+fx)+100\sin(3(e+fx))+12\sin(5(e}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -1/960*(c*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e
 + f*x]]*(-75*Cos[2*(e + f*x)] - 30*Cos[4*(e + f*x)] - 5*Cos[6*(e + f*x)] +
 600*Sin[e + f*x] + 100*Sin[3*(e + f*x)] + 12*Sin[5*(e + f*x)]))/(f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.46, size = 102, normalized size = 0.73

$$-\frac{\left(5\,a^2c\cos\left(fx+e\right)^6-5\,a^2c-2\left(3\,a^2c\cos\left(fx+e\right)^4+4\,a^2c\cos\left(fx+e\right)^2+8\,a^2c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right.}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/30*(5*a^2*c*cos(f*x + e)^6 - 5*a^2*c - 2*(3*a^2*c*cos(f*x + e)^4 + 4*a^2
*c*cos(f*x + e)^2 + 8*a^2*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c
*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-80*a^2*c*f*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(16*f)^
2-480*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*sin(3*f*x+3*exp(1))/(96*f)^2-160*a^2*c*f*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(160*f)^2+
64*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))*cos(2*f*x+2*exp(1))/(64*f)^2+768*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/
4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(256*f)^2+384
*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))*cos(6*f*x+6*exp(1))/(384*f)^2+384*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-128*f)^2+25
6*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*cos(-4*f*x-4*exp(1))/(-256*f)^2)
```

maple [A] time = 0.39, size = 116, normalized size = 0.83

$$-\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(-5\left(\cos^{6}\left(fx+e\right)\right)+\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)\right.}{30f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/30/f*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(-5*c
os(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4-6*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e
)-8*cos(f*x+e)^2+11*sin(f*x+e)-11)/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(3/2)*cos(f*x +
e)^2, x)
```

mupad [B] time = 11.52, size = 122, normalized size = 0.87

$$-\frac{a^2\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(75\,\cos\left(e+f\,x\right)+105\,\cos\left(3\,e+3\,f\,x\right)+35\,\cos\left(5\,e+5\right.\right.}{960\,f\,\left(\cos\left(2\,e+2\,f\,x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] -(a^2*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(75*cos(
e + f*x) + 105*cos(3*e + 3*f*x) + 35*cos(5*e + 5*f*x) + 5*cos(7*e + 7*f*x)
- 700*sin(2*e + 2*f*x) - 112*sin(4*e + 4*f*x) - 12*sin(6*e + 6*f*x)))/(960*
f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.22 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=92

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/10*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/f/(c-c*sin(f*x+e))^(1/2)+1/5*cos
(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.40, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*a*f*Sqrt[c - c*Sin[e + f*x]
]) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(5*
a*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
```

```
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx}}{5af}$$
$$= \frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10af\sqrt{c-c\sin(e+fx)}} + \frac{\cos(e+fx)(}{}$$

Mathematica [A] time = 0.50, size = 92, normalized size = 1.00

$$-\frac{a^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}\,(-70\sin(e+fx)-5\sin(3(e+fx))+\sin(5(e+fx))+20}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -1/80*(a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]
*(20*Cos[2*(e + f*x)] + 5*Cos[4*(e + f*x)] - 70*Sin[e + f*x] - 5*Sin[3*(e +
 f*x)] + Sin[5*(e + f*x)]))/f
```

fricas [A] time = 0.46, size = 96, normalized size = 1.04

$$-\frac{\left(5\,a^2\cos\left(fx+e\right)^4-5\,a^2+2\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2-4\,a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{}}{10\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/10*(5*a^2*cos(f*x + e)^4 - 5*a^2 + 2*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x
 + e)^2 - 4*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sq
rt(2*a)*sqrt(2*c)*(-112*a^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(16*f)^2-288*a^2*f*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/
(96*f)^2+160*a^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp
(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(160*f)^2+16*a^2*f*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))/(16*f)^
2+32*a^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))*cos(4*f*x+4*exp(1))/(32*f)^2+16*a^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-16*f)^2)
```

maple [A] time = 0.41, size = 106, normalized size = 1.15

$$-\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(-2\left(\cos^{6}\left(fx+e\right)\right)+\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)\right.}{10f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/10/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(-2*c
os(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4-2*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e
)+6*sin(f*x+e)-6)/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)
^2, x)
```

mupad [B] time = 2.41, size = 108, normalized size = 1.17

$$-\frac{a^2\sqrt{a\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\,\left(20\,\cos\left(e+f\,x\right)+25\,\cos\left(3\,e+3\,f\,x\right)+5\,\cos\left(5\,e+5\,f\,x\right.\right.}{80\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -(a^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(20*cos(e
+ f*x) + 25*cos(3*e + 3*f*x) + 5*cos(5*e + 5*f*x) - 75*sin(2*e + 2*f*x) - 4
*sin(4*e + 4*f*x) + sin(6*e + 6*f*x)))/(80*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.23 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int (a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

Mathematica [B] time = 0.92, size = 119, normalized size = 2.64

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(56\sin(e+fx)-8\sin(3(e+fx))-28\cos(2(e+fx)}{32f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(-28*Co
s[2*(e + f*x)] + Cos[4*(e + f*x)] + 56*Sin[e + f*x] - 8*Sin[3*(e + f*x)]))/
(32*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.47, size = 98, normalized size = 2.18

$$\frac{\left(a^2\cos\left(fx+e\right)^4-8\,a^2\cos\left(fx+e\right)^2+7\,a^2-4\left(a^2\cos\left(fx+e\right)^2-2\,a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c}}{4\,cf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/4*(a^2*cos(f*x + e)^4 - 8*a^2*cos(f*x + e)^2 + 7*a^2 - 4*(a^2*cos(f*x + e
)^2 - 2*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
c)/(c*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.36, size = 199, normalized size = 4.42

$$\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right)\right)-\left(\cos^{4}\left(fx+e\right)\right)+3\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+\right.\right.}{4f\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\left(\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-\left(\cos^{3}\left(fx+e\right)\right)+2\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/4/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+
e)^4+3*cos(f*x+e)^2*sin(f*x+e)+4*cos(f*x+e)^3-7*sin(f*x+e)*cos(f*x+e)+4*cos
(f*x+e)^2-sin(f*x+e)-8*cos(f*x+e)+1)/(-c*(sin(f*x+e)-1))^(1/2)/(cos(f*x+e)^
2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+
e)+2*cos(f*x+e)-4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) +
c), x)
```

mupad [B] time = 1.88, size = 102, normalized size = 2.27

$$-\frac{a^{2}\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(28\cos\left(e+fx\right)+27\cos\left(3e+3fx\right)-\cos\left(5e+5fx\right)-\right.}{32cf\left(\cos\left(2e+2fx\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] -(a^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(28*cos(e
+ f*x) + 27*cos(3*e + 3*f*x) - cos(5*e + 5*f*x) - 48*sin(2*e + 2*f*x) + 8*s
in(4*e + 4*f*x)))/(32*c*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.24 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{8a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{4a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)-1/3*cos(f*x
+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(1/2)-8*a^3*cos(f*x+e)*ln(1
-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-4*a^2*cos(f*
x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$-\frac{4a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{8a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)}{cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (-8*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) - (4*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c
*f*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/
(c*f*Sqrt[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/
(3*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf\sqrt{c-c\sin(e+fx)}} + \frac{2\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e+fx))^{5}}{3cf\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin(e+fx}{cf\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin(e+fx}{cf\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin(e+fx}{cf\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{8a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4a^2\cos(e+fx)\sqrt{a+a\,\mathrm{si}}}{cf\sqrt{c-c\sin(e+}}$$

Mathematica [A] time = 2.61, size = 140, normalized size = 0.73

$$-\frac{(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(87\sin(e+fx)-\sin(3(e+fx))-12\cos(2(e+fx))\right.}{12cf\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] -1/12*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(
-12*Cos[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 87*Si
n[e + f*x] - Sin[3*(e + f*x)]))/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
5*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2\sin\left(fx+e\right)-2\,a^2\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin(}}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*co
s(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f
*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.36, size = 218, normalized size = 1.13

$$\frac{\left(\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+6\left(\cos^2\left(fx+e\right)\right)+24\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)-22\sin\left(fx+e\right)-48\ln\left(-\frac{-1+\cos\left(fx+e\right)+\text{s}}{\sin\left(fx+\right.}\right.\right.}{3f\left(\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-\left(\cos^3\left(fx+e\right)\right)+2\sin\left(fx+e\right)\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/3/f*(cos(f*x+e)^2*sin(f*x+e)+6*cos(f*x+e)^2+24*ln(2/(cos(f*x+e)+1))-22*si
n(f*x+e)-48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-6)*(sin(f*x+e)*cos(f
*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(5/2)/(cos
(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+3*cos(f*x+e)^2-4*
sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.25 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=192

$$\frac{12a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{3a\cos(e+fx)(a\sin(e+fx)+}{2c^2f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(3/2)+3/2*a*cos(f*x+
e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)+12*a^3*cos(f*x+e)*ln
(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+6*a^2*co
s(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$\frac{6a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{12a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3a\cos(e+fx)(a\sin(e+fx)+}{2c^2f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(c*f*(c - c*Sin[e + f*x])^(3/2))
+ (12*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]]) + (6*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]
)/(c^2*f*Sqrt[c - c*Sin[e + f*x]]) + (3*a*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(2*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{3\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{3a\cos(e+fx)(a+a\sin(e+fx))}{2c^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{6a^2\cos(e+fx)\sqrt{a+a\sin(e+f}}{c^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{6a^2\cos(e+fx)\sqrt{a+a\sin(e+f}}{c^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{6a^2\cos(e+fx)\sqrt{a+a\sin(e+f}}{c^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{12a^3\cos(e+fx)\log(1-\sin(e}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$

Mathematica [A] time = 2.38, size = 181, normalized size = 0.94

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin(3(e+fx))+18\cos(2(e+fx))+192\log\left(\cos\right.\right.}{8c^2f(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(44
 + 18*Cos[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (39
 - 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e +
f*x)]))/(8*c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^
2*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2\sin\left(fx+e\right)-2\,a^2\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(f\right.}}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*co
s(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos
(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.37, size = 273, normalized size = 1.42

$$-\frac{\left(-\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+48\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-24\sin\left(fx+e\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)-\right.}{2f\left(\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-\left(\mathrm{co}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*(-cos(f*x+e)^2*sin(f*x+e)+48*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))-24*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-9*cos(f*x+e)^2-48*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*ln(2/(cos(f*x+e)+1))-25*sin(f*x+e)+9
)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(
f*x+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e
)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(5/2)
```

maxima [B] time = 0.51, size = 1120, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] -1/6*(144*a^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) - 72*a^(
5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) + (46*a^(5/2) - 1
21*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 149*a^(5/2)*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 179*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 148*a
^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 43*a^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 33*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15*a^(5
/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(c^(5/2) - 4*c^(5/2)*sin(f*x + e)/
(cos(f*x + e) + 1) + 8*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 12*c^(
5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*c^(5/2)*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 12*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*c^(5/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 4*c^(5/2)*sin(f*x + e)^7/(cos(f*x +
e) + 1)^7 + c^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) - (46*a^(5/2) - 19
9*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 335*a^(5/2)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 509*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 496*a^
(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 373*a^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 219*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 63*a^(
5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(c^(5/2) - 4*c^(5/2)*sin(f*x + e)
/(cos(f*x + e) + 1) + 8*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 12*c^
(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*c^(5/2)*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 12*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*c^(5/2
)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 4*c^(5/2)*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7 + c^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) + 6*(13*a^(5/2)*s
in(f*x + e)/(cos(f*x + e) + 1) - 39*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 55*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 74*a^(5/2)*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 55*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 - 39*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 13*a^(5/2)*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7)/(c^(5/2) - 4*c^(5/2)*sin(f*x + e)/(cos(f*x + e) +
 1) + 8*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 12*c^(5/2)*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3 + 14*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 12*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*c^(5/2)*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - 4*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(
5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.26 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{6a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{3a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f\sqrt{c-c\sin(e+fx)}}-\frac{3a\cos(e+fx)(a\sin(e+fx)\cdots}{2c^2f(c-c\sin(e+fx))^{3/\cdots}}$$

```
[Out] 1/2*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(5/2)-3/2*a*cos(
f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)-6*a^3*cos(f*x+e)
*ln(1-sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-3*a^2
*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^3/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$-\frac{3a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f\sqrt{c-c\sin(e+fx)}}-\frac{6a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{3a\cos(e+fx)(a\sin(e+fx)\cdots}{2c^2f(c-c\sin(e+fx))^{3/\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*c*f*(c - c*Sin[e + f*x])^(5/2)
) - (3*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^2*f*(c - c*Sin[e + f
*x])^(3/2)) - (6*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (3*a^2*Cos[e + f*x]*Sqrt[a + a*Si
n[e + f*x]])/(c^3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{2c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e+fx)}{2c^2f(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e+fx)}{2c^2f(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e+fx)}{2c^2f(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e+fx)}{2c^2f(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e+fx)}{2c^2f(c-c\sin(e+fx))^{3/2}}$$

Mathematica [A] time = 2.74, size = 209, normalized size = 1.07

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin(3(e+fx))-72\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}\right.\right.\right.}{4c^3f(\sin(e+fx)-}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] -1/4*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])
]*(-28 - 72*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*Cos[2*(e + f*x)]*(
-1 + 6*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) + (41 + 96*Log[Cos[(e + f*
x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f*x)]))/(c^3*f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x
]])
```

fricas [F] time = 1.74, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2\sin\left(fx+e\right)-2\,a^2\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(\right.}}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*c
os(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(
f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*
sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.38, size = 325, normalized size = 1.67

$$\frac{\left(\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+12\left(\cos^2\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-6\left(\cos^2\left(fx+e\right)\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/f*(cos(f*x+e)^2*sin(f*x+e)+12*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))-6*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-10*cos(f*x+e)^2+24*sin(f*x
+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*sin(f*x+e)*ln(2/(cos(f*x+
e)+1))-6*sin(f*x+e)-24*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+12*ln(2/(
cos(f*x+e)+1))+10)*(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x
+e)-2)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos
(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(sin(f*x
+e)-1))^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(7/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.27 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=193

$$\frac{a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f(c-c\sin(e+fx))^{3/2}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/3*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)-1/2*a*cos(
f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)+a^2*cos(f*x+e)*(
a+a*sin(f*x+e))^(1/2)/c^3/f/(c-c*sin(f*x+e))^(3/2)+a^3*cos(f*x+e)*ln(1-sin(
f*x+e))/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2739, 2737, 2667, 31}

$$\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(9/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c*f*(c - c*Sin[e + f*x])^(7/2)
) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^2*f*(c - c*Sin[e + f*x
])^(5/2)) + (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^3*f*(c - c*Sin[e
 + f*x])^(3/2)) + (a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^4*f*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}$$

Mathematica [A] time = 4.13, size = 234, normalized size = 1.21

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(3\sin(3(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+f\right.\right.\right.}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(34
 + 30*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 18*Cos[2*(e + f*x)]*(1 + L
og[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) - 9*(4 + 5*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]])*Sin[e + f*x] + 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
]*Sin[3*(e + f*x)]))/(6*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + S
in[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 2.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2\sin\left(fx+e\right)-2\,a^2\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin}}{5\,c^5\cos\left(fx+e\right)^4-20\,c^5\cos\left(fx+e\right)^2+16\,c^5-\left(c^5\cos\left(fx+e\right)^4-12\,c^5\cos\left(fx+e\right)^2+16\,c^5\right)\mathrm{s}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*c
os(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^5*co
s(f*x + e)^4 - 20*c^5*cos(f*x + e)^2 + 16*c^5 - (c^5*cos(f*x + e)^4 - 12*c^
5*cos(f*x + e)^2 + 16*c^5)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.38, size = 397, normalized size = 2.06

$$\frac{\left(6\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-3\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)-18\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/3/f*(6*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
-3*sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-18*cos(f*x+e)^2*ln(-(-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))-8*cos(f*x+e)^2*sin(f*x+e)+9*cos(f*x+e)^2*ln
(2/(cos(f*x+e)+1))+6*cos(f*x+e)^2-24*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))+12*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+24*ln(-(-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))+14*sin(f*x+e)-12*ln(2/(cos(f*x+e)+1))-6)*(cos(f*x+e)^
2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)*(a*(1+sin(f*x+e)))^(5/2)
```

```
/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+
e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(sin(f*x+e)-1))^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(9/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(9/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

3.28 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=48

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8acf(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/8*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/c/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A] time = 0.34, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8acf(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(11/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*a*c*f*(c - c*Sin[e + f*x])^(9/
2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8acf(c-c\sin(e+fx))^{9/2}}$$

Mathematica [B] time = 4.29, size = 117, normalized size = 2.44

$$\frac{a^2(\sin(3(e+fx))-7\sin(e+fx))\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{4c^5f(\sin(e+fx)-1)^5\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(-7
*Sin[e + f*x] + Sin[3*(e + f*x)]))/(4*c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.49, size = 127, normalized size = 2.65

$$-\frac{\left(a^2\cos\left(fx+e\right)^2-2\,a^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{c^6f\cos\left(fx+e\right)^5-8\,c^6f\cos\left(fx+e\right)^3+8\,c^6f\cos\left(fx+e\right)+4\left(c^6f\cos\left(fx+e\right)^3-2\,c^6f\cos\left(fx+e\right)\right)\sin(fx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="fricas")
```

```
[Out] -(a^2*cos(f*x + e)^2 - 2*a^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*sin(f*x + e)/(c^6*f*cos(f*x + e)^5 - 8*c^6*f*cos(f*x + e)^3 + 8*c^6*f
*cos(f*x + e) + 4*(c^6*f*cos(f*x + e)^3 - 2*c^6*f*cos(f*x + e))*sin(f*x + e
))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.34, size = 153, normalized size = 3.19

$$-\frac{\left(\cos^{2}\left(fx+e\right)-2\right)\left(\cos^{2}\left(fx+e\right)-\sin\left(fx+e\right)\cos\left(fx+e\right)+\cos\left(fx+e\right)+2\sin\left(fx+e\right)-2}{f\left(\cos^{3}\left(fx+e\right)-\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-3\left(\cos^{2}\left(fx+e\right)\right)-2\sin\left(fx+e\right)\cos\left(fx+e\right)-2\cos\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] -1/f*(cos(f*x+e)^2-2)*(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(
f*x+e)-2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(f*x+e)^2*si
n(f*x+e)-3*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4
)/(-c*(sin(f*x+e)-1))^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)^{2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(11/2)
,x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(11/2)
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(11/2),x)
[Out] Timed out
```

3.29 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80ac^2f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10acf(c-c\sin(e+fx))^{11/2}}$$

```
[Out] 1/10*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/c/f/(c-c*sin(f*x+e))^(11/2)+1/80*c
os(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/c^2/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A] time = 0.44, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80ac^2f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10acf(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(13/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*a*c*f*(c - c*Sin[e + f*x])^(1
1/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*a*c^2*f*(c - c*Sin[e
+ f*x])^(9/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10acf(c-c\sin(e+fx))^{11/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{10ac^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10acf(c-c\sin(e+fx))^{11/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{80ac^2f(c-c\sin(e+fx))^{9/2}}$$

Mathematica [A] time = 6.27, size = 130, normalized size = 1.34

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(35\sin(e+fx)-5\sin(3(e+fx))-10\cos(2(e+fx)}{40c^6f(\sin(e+fx)-1)^6\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(13/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(14
 - 10*Cos[2*(e + f*x)] + 35*Sin[e + f*x] - 5*Sin[3*(e + f*x)]))/(40*c^6*f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^6*Sqrt[c - c*Sin[e
 + f*x]])
```

fricas [A] time = 0.53, size = 163, normalized size = 1.68

$$-\frac{\left(5\,a^2\cos\left(fx+e\right)^2-6\,a^2+5\left(a^2\cos\left(fx+e\right)^2-2\,a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c}}{10\left(5\,c^7f\cos\left(fx+e\right)^5-20\,c^7f\cos\left(fx+e\right)^3+16\,c^7f\cos\left(fx+e\right)-\left(c^7f\cos\left(fx+e\right)^5-12\,c^7f\cos\left(fx+e\right)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="fricas")

[Out] -1/10*(5*a^2*cos(f*x + e)^2 - 6*a^2 + 5*(a^2*cos(f*x + e)^2 - 2*a^2)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^7*f*cos(f*x
 + e)^5 - 20*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e) - (c^7*f*cos(f*x
+ e)^5 - 12*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="giac")

[Out] Timed out

maple [B] time = 0.37, size = 187, normalized size = 1.93

$$\frac{\left(\cos^4\left(fx+e\right)+5\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-17\left(\cos^2\left(fx+e\right)\right)-10\sin\left(fx+e\right)+26\right)\left(a\left(1+\sin\left(fx+e\right)\right.\right.}{10f\left(\cos^3\left(fx+e\right)-\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-3\left(\cos^2\left(fx+e\right)\right)-2\sin\left(fx+e\right)\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x)

[Out] 1/10/f*(cos(f*x+e)^4+5*cos(f*x+e)^2*sin(f*x+e)-17*cos(f*x+e)^2-10*sin(f*x+e
)+26)*(a*(1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+
e)+cos(f*x+e)+2*sin(f*x+e)-2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos(f
*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(sin(f*x+e
)-1))^(13/2)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="maxima")

[Out] Timed out

mupad [B] time = 14.86, size = 317, normalized size = 3.27

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,112{}\mathrm{i}}{5\,c^7\,f}+\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,56{}\mathrm{i}}{c^7\,f}-\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\mathrm{co}}{}\right.}{\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,264{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,220{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,20{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(2\,e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(13/2)
,x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((a^2*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(
1/2)*112i)/(5*c^7*f) + (a^2*exp(e*6i + f*x*6i)*sin(e + f*x)*(a + a*sin(e +
f*x))^(1/2)*56i)/(c^7*f) - (a^2*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a + a*
sin(e + f*x))^(1/2)*16i)/(c^7*f) - (a^2*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)
*(a + a*sin(e + f*x))^(1/2)*8i)/(c^7*f)))/(cos(e + f*x)*exp(e*6i + f*x*6i)*
264i - exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*220i + exp(e*6i + f*x*6i)*cos(5*
e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*
6i)*sin(4*e + 4*f*x)*88i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

3.30 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx$

Optimal. Leaf size=236

$$-\frac{4a^4\cos(e+fx)(c-c\sin(e+fx))^{11/2}}{315cf\sqrt{a\sin(e+fx)+a}}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{11/2}}{105cf}-\frac{a^2\cos(e+fx)}{}$$

```
[Out] -1/15*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(11/2)/c/f-4/4
5*a*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(11/2)/c/f-1/10*cos(
f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(11/2)/c/f-4/315*a^4*cos(f*x
+e)*(c-c*sin(f*x+e))^(11/2)/c/f/(a+a*sin(f*x+e))^(1/2)-4/105*a^3*cos(f*x+e)
*(c-c*sin(f*x+e))^(11/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A] time = 0.72, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{11/2}}{15cf}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))}{105cf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-4*a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(11/2))/(315*c*f*Sqrt[a + a*Sin[e
 + f*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(11/2))/(105*c*f) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*
Sin[e + f*x])^(11/2))/(15*c*f) - (4*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/
2)*(c - c*Sin[e + f*x])^(11/2))/(45*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x
])^(7/2)*(c - c*Sin[e + f*x])^(11/2))/(10*c*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
```

```
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{11/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+f}{10cf}$$
$$= -\frac{4a\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+}{45cf}$$
$$= -\frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+}{15cf}$$
$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+}{105cf}$$
$$= -\frac{4a^4\cos(e+fx)(c-c\sin(e+fx))^{11/2}}{315cf\sqrt{a+a\sin(e+fx)}} - \frac{4a^3\cos(}{}$$

Mathematica [A] time = 5.63, size = 209, normalized size = 0.89

$$\frac{a^3c^4(\sin(e+fx)-1)^4(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}\,(158760\sin(e+fx)+35280\sin(}{322}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9
/2),x]
```

```
[Out] (a^3*c^4*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x
])]*Sqrt[c - c*Sin[e + f*x]]*(13230*Cos[2*(e + f*x)] + 7560*Cos[4*(e + f*x)
] + 2835*Cos[6*(e + f*x)] + 630*Cos[8*(e + f*x)] + 63*Cos[10*(e + f*x)] + 1
58760*Sin[e + f*x] + 35280*Sin[3*(e + f*x)] + 9072*Sin[5*(e + f*x)] + 1620*
Sin[7*(e + f*x)] + 140*Sin[9*(e + f*x)]))/(322560*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

fricas [A] time = 1.23, size = 144, normalized size = 0.61

$$\frac{\left(63\,a^3c^4\cos\left(fx+e\right)^{10} - 63\,a^3c^4 + 2\left(35\,a^3c^4\cos\left(fx+e\right)^8 + 40\,a^3c^4\cos\left(fx+e\right)^6 + 48\,a^3c^4\cos\left(fx+e\right)^4 + 64\right.\right.}{630\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] 1/630*(63*a^3*c^4*cos(f*x + e)^10 - 63*a^3*c^4 + 2*(35*a^3*c^4*cos(f*x + e)
^8 + 40*a^3*c^4*cos(f*x + e)^6 + 48*a^3*c^4*cos(f*x + e)^4 + 64*a^3*c^4*cos
(f*x + e)^2 + 128*a^3*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*s
in(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
```

```
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-16128*a^3*c^4*f*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))
/(256*f)^2-8064*a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(384*f)^2-5760*a^3*c^4*f*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*ex
p(1))/(640*f)^2-32256*a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))*sin(7*f*x+7*exp(1))/(3584*f)^2-4608*a^3*c^4*f*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(9*f
*x+9*exp(1))/(4608*f)^2-7168*a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))/(1024*f)^2-7168*a^3*c
^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*
cos(4*f*x+4*exp(1))/(1024*f)^2-7680*a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(6*f*x+6*exp(1))/(1536*f)^2-573
44*a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1
/4*pi))*cos(8*f*x+8*exp(1))/(8192*f)^2-10240*a^3*c^4*f*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(10*f*x+10*exp(1))/(10
240*f)^2-3584*a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*
x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-512*f)^2-5120*a^3*c^4*f*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-4*f*x-4*e
xp(1))/(-1024*f)^2-43008*a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-6*f*x-6*exp(1))/(-6144*f)^2-8192*a^3*c^4
*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*co
s(-8*f*x-8*exp(1))/(-8192*f)^2)
```

maple [A] time = 0.51, size = 169, normalized size = 0.72

$$\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{9}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(63\left(\cos^{10}\left(fx+e\right)\right)+7\sin\left(fx+e\right)\left(\cos^{8}\left(fx+e\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/630/f*(-c*(sin(f*x+e)-1))^(9/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(63*c
os(f*x+e)^10+7*sin(f*x+e)*cos(f*x+e)^8+70*cos(f*x+e)^8+17*cos(f*x+e)^6*sin(
f*x+e)+80*cos(f*x+e)^6+33*sin(f*x+e)*cos(f*x+e)^4+96*cos(f*x+e)^4+65*cos(f*
x+e)^2*sin(f*x+e)+128*cos(f*x+e)^2+193*sin(f*x+e)+193)/cos(f*x+e)^9
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(9/2)*cos(f*x +
e)^2, x)
```

mupad [B] time = 13.53, size = 462, normalized size = 1.96

$$\frac{e^{-e\,10i-f\,x\,10i}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{63\,a^3\,c^4\,e^{e\,10i+f\,x\,10i}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}+\frac{21\,a^3\,c^4\,e^{e\,10i+f\,x\,10i}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+\right.}}{256\,f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(9/2),x)
```

```
[Out] (exp(- e*10i - f*x*10i)*(c - c*sin(e + f*x))^(1/2)*((63*a^3*c^4*exp(e*10i +
 f*x*10i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) + (21*a^3*c^4*exp
(e*10i + f*x*10i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(256*f) + (3
*a^3*c^4*exp(e*10i + f*x*10i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/
(64*f) + (9*a^3*c^4*exp(e*10i + f*x*10i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*
x))^(1/2))/(512*f) + (a^3*c^4*exp(e*10i + f*x*10i)*cos(8*e + 8*f*x)*(a + a*
sin(e + f*x))^(1/2))/(256*f) + (a^3*c^4*exp(e*10i + f*x*10i)*cos(10*e + 10*
f*x)*(a + a*sin(e + f*x))^(1/2))/(2560*f) + (7*a^3*c^4*exp(e*10i + f*x*10i)
*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (9*a^3*c^4*exp(e*10i
 + f*x*10i)*sin(5*e + 5*f*x)*(a + a*sin(e + f*x))^(1/2))/(160*f) + (9*a^3*c
^4*exp(e*10i + f*x*10i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2))/(896*f
) + (a^3*c^4*exp(e*10i + f*x*10i)*sin(9*e + 9*f*x)*(a + a*sin(e + f*x))^(1/
2))/(1152*f)))/(2*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

3.31 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=236

$$-\frac{8a^4\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315cf\sqrt{a\sin(e+fx)+a}}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}{63cf}-\frac{2a^2\cos(e+fx)(}{}$$

```
[Out] -2/21*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/c/f-1/9*
a*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(9/2)/c/f-1/9*cos(f*x+
e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2)/c/f-8/315*a^4*cos(f*x+e)*(
c-c*sin(f*x+e))^(9/2)/c/f/(a+a*sin(f*x+e))^(1/2)-4/63*a^3*cos(f*x+e)*(c-c*s
in(f*x+e))^(9/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A] time = 0.73, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{2a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{21cf}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx)}{63cf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-8*a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(315*c*f*Sqrt[a + a*Sin[e
+ f*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x]
)^(9/2))/(63*c*f) - (2*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*S
in[e + f*x])^(9/2))/(21*c*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(
c - c*Sin[e + f*x])^(9/2))/(9*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/
2)*(c - c*Sin[e + f*x])^(9/2))/(9*c*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
```

```
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{9/2}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+}{9cf}$$

$$= -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+}{9cf}$$

$$= -\frac{2a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(}{21cf}$$

$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e}{63cf}$$

$$= -\frac{8a^4\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315cf\sqrt{a+a\sin(e+fx)}} - \frac{4a^3\cos}{}$$

Mathematica [A] time = 1.26, size = 97, normalized size = 0.41

$$\frac{a^3c^3(39690\sin(e+fx)+8820\sin(3(e+fx))+2268\sin(5(e+fx))+405\sin(7(e+fx))+35\sin(9(e+fx)))\,\mathrm{s}}{80640f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (a^3*c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
39690*Sin[e + f*x] + 8820*Sin[3*(e + f*x)] + 2268*Sin[5*(e + f*x)] + 405*Si
n[7*(e + f*x)] + 35*Sin[9*(e + f*x)]))/(80640*f)
```

fricas [A] time = 0.52, size = 117, normalized size = 0.50

$$\frac{\left(35\,a^3c^3\cos\left(fx+e\right)^8+40\,a^3c^3\cos\left(fx+e\right)^6+48\,a^3c^3\cos\left(fx+e\right)^4+64\,a^3c^3\cos\left(fx+e\right)^2+128\,a^3c^3\right)\sqrt{a\sin}}{315\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 1/315*(35*a^3*c^3*cos(f*x + e)^8 + 40*a^3*c^3*cos(f*x + e)^6 + 48*a^3*c^3*c
os(f*x + e)^4 + 64*a^3*c^3*cos(f*x + e)^2 + 128*a^3*c^3)*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(
2*c)*(-16128*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))*sin(f*x+exp(1))/(256*f)^2-8064*a^3*c^3*f*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(
384*f)^2-5760*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*
x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(640*f)^2-32256*a^3*c^3*f*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(7*f*x+7*exp
(1))/(3584*f)^2-4608*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))*sin(9*f*x+9*exp(1))/(4608*f)^2)
```

maple [A] time = 0.39, size = 87, normalized size = 0.37

$$\frac{\left(35\left(\cos^{8}\left(fx+e\right)\right)+40\left(\cos^{6}\left(fx+e\right)\right)+48\left(\cos^{4}\left(fx+e\right)\right)+64\left(\cos^{2}\left(fx+e\right)\right)+128\right)\left(-c\left(\sin\left(fx+e\right)-1\right.\right.}{315f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/315/f*(35*cos(f*x+e)^8+40*cos(f*x+e)^6+48*cos(f*x+e)^4+64*cos(f*x+e)^2+12
8)*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/cos(f*x+e)
^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(7/2)*cos(f*x +
e)^2, x)
```

mupad [B] time = 12.12, size = 247, normalized size = 1.05

$$\frac{e^{-e\,9i-f\,x\,9i}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{63\,a^3\,c^3\,e^{e\,9i+f\,x\,9i}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}+\frac{7\,a^3\,c^3\,e^{e\,9i+f\,x\,9i}\,\sin\left(3\,e+3\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}+\right.}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (exp(- e*9i - f*x*9i)*(c - c*sin(e + f*x))^(1/2)*((63*a^3*c^3*exp(e*9i + f*
x*9i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) + (7*a^3*c^3*exp(e*9i
 + f*x*9i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (9*a^3*c^3
*exp(e*9i + f*x*9i)*sin(5*e + 5*f*x)*(a + a*sin(e + f*x))^(1/2))/(160*f) +
(9*a^3*c^3*exp(e*9i + f*x*9i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2))/
(896*f) + (a^3*c^3*exp(e*9i + f*x*9i)*sin(9*e + 9*f*x)*(a + a*sin(e + f*x))
^(1/2))/(1152*f)))/(2*cos(e + f*x))
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

3.32 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=188

$$\frac{c^3\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{35af\sqrt{c-c\sin(e+fx)}}+\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{9/2}\sqrt{c-c\sin(e+fx)}}{14af}+\frac{\cos(e+fx)(a\sin(}{}$$

```
[Out] 3/28*c*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)*(c-c*sin(f*x+e))^(3/2)/a/f+1/8*cos
(f*x+e)*(a+a*sin(f*x+e))^(9/2)*(c-c*sin(f*x+e))^(5/2)/a/f+1/35*c^3*cos(f*x+
e)*(a+a*sin(f*x+e))^(9/2)/a/f/(c-c*sin(f*x+e))^(1/2)+1/14*c^2*cos(f*x+e)*(a
+a*sin(f*x+e))^(9/2)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.62, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{9/2}\sqrt{c-c\sin(e+fx)}}{14af}+\frac{c^3\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{35af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(35*a*f*Sqrt[c - c*Sin[e + f*
x]]) + (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*Sqrt[c - c*Sin[e + f*x]
])/(14*a*f) + (3*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*(c - c*Sin[e + f
*x])^(3/2))/(28*a*f) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*(c - c*Sin[
e + f*x])^(5/2))/(8*a*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
```

```
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx)}{8af}$$

$$= \frac{3c\cos(e+fx)(a+a\sin(e+fx))^{9/2}(c-c\sin(e+}{28af}$$

$$= \frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e+}}{14af}$$

$$= \frac{c^3\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{35af\sqrt{c-c\sin(e+fx)}} + \frac{c^2\cos(e+}{}$$

Mathematica [A] time = 2.41, size = 127, normalized size = 0.68

$$\frac{a^3c^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}\,(19600\sin(e+fx)+3920\sin(3(e+fx))+784\sin(5(e+}{3584}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (a^3*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-1960*Cos[2*(e + f*x)] - 980*Cos[4*(e + f*x)] - 280*Cos[6*(e + f*x)] - 35*C
os[8*(e + f*x)] + 19600*Sin[e + f*x] + 3920*Sin[3*(e + f*x)] + 784*Sin[5*(e
 + f*x)] + 80*Sin[7*(e + f*x)]))/(35840*f)
```

fricas [A] time = 0.49, size = 128, normalized size = 0.68

$$-\frac{\left(35\,a^3c^2\cos\left(fx+e\right)^8-35\,a^3c^2-8\left(5\,a^3c^2\cos\left(fx+e\right)^6+6\,a^3c^2\cos\left(fx+e\right)^4+8\,a^3c^2\cos\left(fx+e\right)^2+16\,a^3\right.\right.}{280\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/280*(35*a^3*c^2*cos(f*x + e)^8 - 35*a^3*c^2 - 8*(5*a^3*c^2*cos(f*x + e)^
6 + 6*a^3*c^2*cos(f*x + e)^4 + 8*a^3*c^2*cos(f*x + e)^2 + 16*a^3*c^2)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e)
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sq
rt(2*c)*(-4480*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(128*f)^2-8064*a^3*c^2*f*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))
/(384*f)^2-4480*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(640*f)^2-896*a^3*c^2*f*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(7*f*x+7*exp
(1))/(896*f)^2+2560*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
```

```
/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))/(512*f)^2+9216*a^3*c^2*f*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+
4*exp(1))/(1024*f)^2+7680*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(6*f*x+6*exp(1))/(1536*f)^2+2048*a^3*c^2*
f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos
(8*f*x+8*exp(1))/(2048*f)^2+4608*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-512*f)^2+5120*
a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*cos(-4*f*x-4*exp(1))/(-1024*f)^2+1536*a^3*c^2*f*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-6*f*x-6*exp(1))/(-1536
*f)^2)
```

maple [A] time = 0.41, size = 143, normalized size = 0.76

$$-\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(-35\left(\cos^{8}\left(fx+e\right)\right)+5\left(\cos^{6}\left(fx+e\right)\right)\sin\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/280/f*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(-35
*cos(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)-40*cos(f*x+e)^6+13*sin(f*x+e)*cos(f
*x+e)^4-48*cos(f*x+e)^4+29*cos(f*x+e)^2*sin(f*x+e)-64*cos(f*x+e)^2+93*sin(f
*x+e)-93)/cos(f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(5/2)*cos(f*x +
e)^2, x)
```

mupad [B] time = 12.55, size = 376, normalized size = 2.00

$$\frac{e^{-e\,8i-f\,x\,8i}\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{35\,a^3\,c^2\,e^{e\,8i+f\,x\,8i}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}-\frac{7\,a^3\,c^2\,e^{e\,8i+f\,x\,8i}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (exp(- e*8i - f*x*8i)*(c - c*sin(e + f*x))^(1/2)*((35*a^3*c^2*exp(e*8i + f*
x*8i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) - (7*a^3*c^2*exp(e*8i
 + f*x*8i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) - (7*a^3*c^2
*exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(128*f) -
(a^3*c^2*exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(6
4*f) - (a^3*c^2*exp(e*8i + f*x*8i)*cos(8*e + 8*f*x)*(a + a*sin(e + f*x))^(1
/2))/(512*f) + (7*a^3*c^2*exp(e*8i + f*x*8i)*sin(3*e + 3*f*x)*(a + a*sin(e
+ f*x))^(1/2))/(32*f) + (7*a^3*c^2*exp(e*8i + f*x*8i)*sin(5*e + 5*f*x)*(a +
 a*sin(e + f*x))^(1/2))/(160*f) + (a^3*c^2*exp(e*8i + f*x*8i)*sin(7*e + 7*f
*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.33 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=140

$$\frac{4c^2\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{105af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}(c-c\sin(e+fx))^{3/2}}{7af}+\frac{2c\cos(e+fx)(a\sin}{}$$

```
[Out] 1/7*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)*(c-c*sin(f*x+e))^(3/2)/a/f+4/105*c^2*
cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/f/(c-c*sin(f*x+e))^(1/2)+2/21*c*cos(f*x
+e)*(a+a*sin(f*x+e))^(9/2)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.52, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$\frac{4c^2\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{105af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}(c-c\sin(e+fx))^{3/2}}{7af}+\frac{2c\cos(e+fx)(a\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (4*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(105*a*f*Sqrt[c - c*Sin[e +
 f*x]]) + (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*Sqrt[c - c*Sin[e + f
*x]])/(21*a*f) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*(c - c*Sin[e + f*
x])^(3/2))/(7*a*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}(c-c\sin(e+f}{7af}$$
$$= \frac{2c\cos(e+fx)(a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e}}{21af}$$
$$= \frac{4c^2\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{105af\sqrt{c-c\sin(e+fx)}} + \frac{2c\cos(e}{}$$

Mathematica [A] time = 1.28, size = 115, normalized size = 0.82

$$\frac{a^3c\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}\,(4725\sin(e+fx)+665\sin(3(e+fx))+21\sin(5(e+fx}{6720f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (a^3*c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-1
050*Cos[2*(e + f*x)] - 420*Cos[4*(e + f*x)] - 70*Cos[6*(e + f*x)] + 4725*Si
n[e + f*x] + 665*Sin[3*(e + f*x)] + 21*Sin[5*(e + f*x)] - 15*Sin[7*(e + f*x
)]))/(6720*f)
```

fricas [A] time = 0.48, size = 115, normalized size = 0.82

$$-\frac{\left(35\,a^3c\cos\left(fx+e\right)^6-35\,a^3c+\left(15\,a^3c\cos\left(fx+e\right)^6-24\,a^3c\cos\left(fx+e\right)^4-32\,a^3c\cos\left(fx+e\right)^2-64\,a^3c\right)s}{105\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/105*(35*a^3*c*cos(f*x + e)^6 - 35*a^3*c + (15*a^3*c*cos(f*x + e)^6 - 24*
a^3*c*cos(f*x + e)^4 - 32*a^3*c*cos(f*x + e)^2 - 64*a^3*c)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-5760*a^3*c*f*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+
exp(1))/(128*f)^2-7296*a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(384*f)^2-640*a^3*c*f*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*
exp(1))/(640*f)^2+896*a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
/2*(f*x+exp(1))-1/4*pi))*sin(7*f*x+7*exp(1))/(896*f)^2+32*a^3*c*f*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*ex
p(1))/(32*f)^2+384*a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*
(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(128*f)^2+192*a^3*c*f*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(6*f*x+6*exp(
1))/(192*f)^2+192*a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-64*f)^2+128*a^3*c*f*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-4*f*x-4*exp
(1))/(-128*f)^2)
```

maple [A] time = 0.40, size = 133, normalized size = 0.95

$$-\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(-15\left(\cos^{8}\left(fx+e\right)\right)+5\left(\cos^{6}\left(fx+e\right)\right)\sin\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/105/f*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(-15
*cos(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)-16*cos(f*x+e)^6+13*sin(f*x+e)*cos(f
*x+e)^4-16*cos(f*x+e)^4+29*cos(f*x+e)^2*sin(f*x+e)+58*sin(f*x+e)-58)/cos(f*
x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(3/2)*cos(f*x +
e)^2, x)
```

mupad [B] time = 12.14, size = 319, normalized size = 2.28

$$-\frac{e^{-e\,7i-f\,x\,7i}\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{5\,a^3\,c\,e^{e\,7i+f\,x\,7i}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{16\,f}+\frac{a^3\,c\,e^{e\,7i+f\,x\,7i}\,\cos\left(4\,e+4\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{8\,f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] -(exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((5*a^3*c*exp(e*7i + f*x*
7i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(16*f) + (a^3*c*exp(e*7i +
 f*x*7i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(8*f) + (a^3*c*exp(e*
7i + f*x*7i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(48*f) - (19*a^3*
c*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) -
(a^3*c*exp(e*7i + f*x*7i)*sin(5*e + 5*f*x)*(a + a*sin(e + f*x))^(1/2))/(160
*f) + (a^3*c*exp(e*7i + f*x*7i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2)
```

```
)/(224*f) - (45*a^3*c*exp(e*7i + f*x*7i)*sin(e + f*x)*(a + a*sin(e + f*x))^
(1/2))/(32*f)))/(2*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.34 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=92

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}\sqrt{c-c\sin(e+fx)}}{6af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{15af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/15*c*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/f/(c-c*sin(f*x+e))^(1/2)+1/6*cos
(f*x+e)*(a+a*sin(f*x+e))^(9/2)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.39, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}\sqrt{c-c\sin(e+fx)}}{6af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{15af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(15*a*f*Sqrt[c - c*Sin[e + f*x]
]) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*Sqrt[c - c*Sin[e + f*x]])/(6*
a*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
```

```
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{3/2}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e+fx)}}{6af}$$
$$= \frac{c\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{15af\sqrt{c-c\sin(e+fx)}} + \frac{\cos(e+fx)(a}{}$$

Mathematica [A] time = 0.54, size = 104, normalized size = 1.13

$$\frac{a^3\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}\,(1080\sin(e+fx)+20\sin(3(e+fx))-36\sin(5(e+fx))-}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] (a^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-405
*Cos[2*(e + f*x)] - 90*Cos[4*(e + f*x)] + 5*Cos[6*(e + f*x)] + 1080*Sin[e +
 f*x] + 20*Sin[3*(e + f*x)] - 36*Sin[5*(e + f*x)]))/(960*f)
```

fricas [A] time = 0.44, size = 110, normalized size = 1.20

$$\frac{\left(5\,a^3\cos\left(fx+e\right)^6-30\,a^3\cos\left(fx+e\right)^4+25\,a^3-2\left(9\,a^3\cos\left(fx+e\right)^4-8\,a^3\cos\left(fx+e\right)^2-16\,a^3\right)\sin\left(fx+e\right)}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/30*(5*a^3*cos(f*x + e)^6 - 30*a^3*cos(f*x + e)^4 + 25*a^3 - 2*(9*a^3*cos(
f*x + e)^4 - 8*a^3*cos(f*x + e)^2 - 16*a^3)*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-144*a^3*f*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(16*f)^2
-96*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*sin(3*f*x+3*exp(1))/(96*f)^2+480*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(160*f)^2+448*a^
3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*c
os(2*f*x+2*exp(1))/(64*f)^2+3328*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(256*f)^2-384*a^3*f*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(6*
f*x+6*exp(1))/(384*f)^2+1664*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-128*f)^2-256*a^3*f*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-4*f
*x-4*exp(1))/(-256*f)^2)
```

maple [A] time = 0.40, size = 133, normalized size = 1.45

$$-\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\ \sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(-5\left(\cos^{8}\left(fx+e\right)\right)+3\left(\cos^{6}\left(fx+e\right)\right)\sin\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/30/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(-5*c
os(f*x+e)^8+3*cos(f*x+e)^6*sin(f*x+e)-4*cos(f*x+e)^6+7*sin(f*x+e)*cos(f*x+e
)^4+7*cos(f*x+e)^2*sin(f*x+e)+7*cos(f*x+e)^2+28*sin(f*x+e)-28)/cos(f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)
^2, x)
```

mupad [B] time = 11.49, size = 121, normalized size = 1.32

$$-\frac{a^3\sqrt{a\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\,\left(405\cos\left(e+f\,x\right)+495\cos\left(3\,e+3\,f\,x\right)+85\cos\left(5\,e+5\right.\right.}{960\,f\,\left(\cos\left(2\,e+2\,f\,x\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -(a^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(405*cos(e
 + f*x) + 495*cos(3*e + 3*f*x) + 85*cos(5*e + 5*f*x) - 5*cos(7*e + 7*f*x) -
 1100*sin(2*e + 2*f*x) + 16*sin(4*e + 4*f*x) + 36*sin(6*e + 6*f*x)))/(960*f
*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.35 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/5*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(5*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e+fx)}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

Mathematica [B] time = 1.44, size = 142, normalized size = 3.16

$$\frac{a^3(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(210\sin(e+fx)-45\sin(3(e+fx))}{80f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*Sqrt[a*(1 +
 Sin[e + f*x])]*(-120*Cos[2*(e + f*x)] + 10*Cos[4*(e + f*x)] + 210*Sin[e +
f*x] - 45*Sin[3*(e + f*x)] + Sin[5*(e + f*x)]))/(80*f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.47, size = 111, normalized size = 2.47

$$\frac{\left(5\,a^3\cos\left(fx+e\right)^4-20\,a^3\cos\left(fx+e\right)^2+15\,a^3+\left(a^3\cos\left(fx+e\right)^4-12\,a^3\cos\left(fx+e\right)^2+16\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{}}{5\,cf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/5*(5*a^3*cos(f*x + e)^4 - 20*a^3*cos(f*x + e)^2 + 15*a^3 + (a^3*cos(f*x +
 e)^4 - 12*a^3*cos(f*x + e)^2 + 16*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c)/(c*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.40, size = 245, normalized size = 5.44

$$\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(\cos^{5}\left(fx+e\right)+\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+4\left(\cos^{4}\left(fx+e\right)\right)-5\sin\left(f\right.\right.}{5f\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\ \left(\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right)\right)+\cos^{4}\left(fx+e\right)-4\left(\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/5/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(cos(f*x+e)^5+sin(f*x+e)*cos(f*x+
e)^4+4*cos(f*x+e)^4-5*sin(f*x+e)*cos(f*x+e)^3-12*cos(f*x+e)^3-7*cos(f*x+e)^
2*sin(f*x+e)-8*cos(f*x+e)^2+15*sin(f*x+e)*cos(f*x+e)+16*cos(f*x+e)+sin(f*x+
e)-1)/(-c*(sin(f*x+e)-1))^(1/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos
(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+
8*sin(f*x+e)-4*cos(f*x+e)+8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) +
c), x)
```

mupad [B] time = 10.69, size = 113, normalized size = 2.51

$$-\frac{a^{3}\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(120\,\cos\left(e+f\,x\right)+110\,\cos\left(3\,e+3\,f\,x\right)-10\,\cos\left(5\,e+5\right.\right.}{80\,c\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] -(a^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(120*cos(e
 + f*x) + 110*cos(3*e + 3*f*x) - 10*cos(5*e + 5*f*x) - 165*sin(2*e + 2*f*x)
 + 44*sin(4*e + 4*f*x) - sin(6*e + 6*f*x)))/(80*c*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.36 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{16a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{8a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{2a^2\cos(e+fx)(a\sin(e+fx)-}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)-2/3*a*c
os(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(1/2)-1/4*cos(f*x+e)*
(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(1/2)-16*a^4*cos(f*x+e)*ln(1-si
n(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-8*a^3*cos(f*x+e
)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$-\frac{8a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{2a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{16a^4\cos(e+fx)\log(1-\sin(e+}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (-16*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) - (8*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(
c*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3
/2))/(c*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x]
)^(5/2))/(3*c*f*Sqrt[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*(a + a*Sin[e + f*
x])^(7/2))/(4*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf\sqrt{c-c\sin(e+fx)}} + \frac{2\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= -\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e+fx)}{4cf\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{2a\cos(e+fx)(a+a\sin(e}{3cf\sqrt{c-c\sin(e+f}}$$

$$= -\frac{8a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{2a^2\cos(e+fx)(a+a\sin(e+}{cf\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{8a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{2a^2\cos(e+fx)(a+a\sin(e+}{cf\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{8a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{2a^2\cos(e+fx)(a+a\sin(e+}{cf\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{16a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{8a^3\cos(e+fx)\sqrt{a+a}}{cf\sqrt{c-c\sin(e+}}$$

Mathematica [A] time = 6.47, size = 473, normalized size = 1.96

$$\frac{5\sin(3(e+fx))(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{12f(c-c\sin(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} - \frac{65\sin(e+fx)(a(\sin(e+fx)+1))}{4f(c-c\sin(e+fx))^{3/2}\left(\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (23*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e
+ f*x]))^(7/2))/(8*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
 f*x])^(3/2)) - (Cos[4*(e + f*x)]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(
```

```
a*(1 + Sin[e + f*x]))^(7/2))/(32*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*
(c - c*Sin[e + f*x])^(3/2)) - (32*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(3/2)) - (65*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sin[e + f*x]*(a*(1 + Sin[e + f*x]))^(7
/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(3/2
)) + (5*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2
)*Sin[3*(e + f*x)])/(12*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Si
n[e + f*x])^(3/2))
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2+\left(a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(\right.}}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^4
 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.39, size = 250, normalized size = 1.04

$$-\frac{3\left(\cos^4\left(fx+e\right)\right)-20\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-72\left(\cos^2\left(fx+e\right)\right)+384\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+20}{12f\left(\sin\left(fx+e\right)\left(\cos^3\left(fx+e\right)\right)+\cos^4\left(fx+e\right)-4\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+3\left(\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/12/f*(3*cos(f*x+e)^4-20*cos(f*x+e)^2*sin(f*x+e)-72*cos(f*x+e)^2+384*ln(-
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+200*sin(f*x+e)-192*ln(2/(cos(f*x+e)+
1))+69)*(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)*(a*(
1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*s
in(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e
)-4*cos(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.37 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=238

$$\frac{32a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{16a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{4a^2\cos(e+fx)(a\sin(e+fx)+}{c^2f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(3/2)+4*a^2*cos(f*x+
e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)+4/3*a*cos(f*x+e)*(a+
a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)+32*a^4*cos(f*x+e)*ln(1-sin
(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+16*a^3*cos(f*x
+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$\frac{16a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{4a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{32a^4\cos(e+fx)\log(1-\sin(e+f}{c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e-}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(c*f*(c - c*Sin[e + f*x])^(3/2))
+ (32*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]]) + (16*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]
])/(c^2*f*Sqrt[c - c*Sin[e + f*x]]) + (4*a^2*Cos[e + f*x]*(a + a*Sin[e + f*
x])^(3/2))/(c^2*f*Sqrt[c - c*Sin[e + f*x]]) + (4*a*Cos[e + f*x]*(a + a*Sin[
e + f*x])^(5/2))/(3*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{4\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{4a\cos(e+fx)(a+a\sin(e+fx))^{5}}{3c^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{4a^2\cos(e+fx)(a+a\sin(e+fx))}{c^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{16a^3\cos(e+fx)\sqrt{a+a\sin(e+f}}{c^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{16a^3\cos(e+fx)\sqrt{a+a\sin(e+f}}{c^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{16a^3\cos(e+fx)\sqrt{a+a\sin(e+f}}{c^2f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{32a^4\cos(e+fx)\log(1-\sin(e+}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(}}$$

<u>**Mathematica**</u> [A] time = 4.81, size = 196, normalized size = 0.82

$$-\frac{a^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(-396\sin(e+fx)-16\sin(3(e+fx))-172\cos(2(e}{24c^2f(\sin(e+fx)-1)^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -1/24*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x]
)]*(-177 - 172*Cos[2*(e + f*x)] + Cos[4*(e + f*x)] - 1536*Log[Cos[(e + f*x)
/2] - Sin[(e + f*x)/2]] - 396*Sin[e + f*x] + 1536*Log[Cos[(e + f*x)/2] - Si
```

```
n[(e + f*x)/2]]*Sin[e + f*x] - 16*Sin[3*(e + f*x)]))/(c^2*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 1.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2+\left(a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin}}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^4
 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)
*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.39, size = 307, normalized size = 1.29

$$-\frac{\left(-\left(\cos^4\left(fx+e\right)\right)+8\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-192\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+96\sin\left(fx\right.\right.}{3f\left(\sin\left(fx+e\right)\left(\cos^3\left(fx+e\right)\right)+\cos^4\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/3/f*(-cos(f*x+e)^4+8*cos(f*x+e)^2*sin(f*x+e)-192*sin(f*x+e)*ln(-(-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))+96*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+44*cos(f*
x+e)^2+91*sin(f*x+e)+192*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-96*ln(2
/(cos(f*x+e)+1))-43)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f
*x+e)+2)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*c
```

```
os(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^
2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.38 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{24a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{12a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f\sqrt{c-c\sin(e+fx)}}-\frac{3a^2\cos(e+fx)(a\sin(e+fx}{c^3f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(5/2)-2*a*cos(f*
x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)-3*a^2*cos(f*x+e)*(
a+a*sin(f*x+e))^(3/2)/c^3/f/(c-c*sin(f*x+e))^(1/2)-24*a^4*cos(f*x+e)*ln(1-s
in(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-12*a^3*cos(f
*x+e)*(a+a*sin(f*x+e))^(1/2)/c^3/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$-\frac{12a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f\sqrt{c-c\sin(e+fx)}}-\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{c^3f\sqrt{c-c\sin(e+fx)}}-\frac{24a^4\cos(e+fx)\log(1-\sin(e}{c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2*c*f*(c - c*Sin[e + f*x])^(5/2)
) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(c^2*f*(c - c*Sin[e + f*x
])^(3/2)) - (24*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (12*a^3*Cos[e + f*x]*Sqrt[a + a*Si
n[e + f*x]])/(c^3*f*Sqrt[c - c*Sin[e + f*x]]) - (3*a^2*Cos[e + f*x]*(a + a*
Sin[e + f*x])^(3/2))/(c^3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx)}{c^2f(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx)}{c^2f(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx)}{c^2f(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx)}{c^2f(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx)}{c^2f(c-c\sin(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx)}{c^2f(c-c\sin(e+fx))^{3/2}}$$

Mathematica [A] time = 6.55, size = 223, normalized size = 0.93

$$\frac{a^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(-320\sin(e+fx)-24\sin(3(e+fx))+\cos(4(e+\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(27
3 + Cos[4*(e + f*x)] + Cos[2*(e + f*x)]*(106 - 384*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]]) + 1152*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 320*Sin
[e + f*x] - 1536*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] - 24
```

```
*Sin[3*(e + f*x)]))/(16*c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + S
in[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 2.28, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2+\left(a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin}}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^
4 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4
*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.42, size = 365, normalized size = 1.53

$$-\frac{\left(-\left(\cos^4\left(fx+e\right)\right)+12\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+96\left(\cos^2\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-48\left(\cos^2\right.\right.}{2f\left(\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/2/f*(-cos(f*x+e)^4+12*cos(f*x+e)^2*sin(f*x+e)+96*cos(f*x+e)^2*ln(-(-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))-48*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+192*s
in(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-96*sin(f*x+e)*ln(2/(co
s(f*x+e)+1))-73*cos(f*x+e)^2-58*sin(f*x+e)-192*ln(-(-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))+96*ln(2/(cos(f*x+e)+1))+74)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e
```

```
)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x
+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*co
s(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(7
/2)
```

maxima [B] time = 0.54, size = 1396, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] 1/30*(1440*a^(7/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 720*a
^(7/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + (334*a^(7/2)
- 1449*a^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 2693*a^(7/2)*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 3278*a^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 3199*a^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2014*a^(7/2)*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 315*a^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 + 10*a^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 525*a^(7/2)*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8 + 75*a^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)/
(c^(7/2) - 6*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 17*c^(7/2)*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - 32*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 46*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 52*c^(7/2)*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5 + 46*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
32*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 17*c^(7/2)*sin(f*x + e)^8/
(cos(f*x + e) + 1)^8 - 6*c^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + c^(7
/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10) - (334*a^(7/2) - 2079*a^(7/2)*si
n(f*x + e)/(cos(f*x + e) + 1) + 6203*a^(7/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 10698*a^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15049*a^(7/2)*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 15354*a^(7/2)*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 12165*a^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 7410*a^(7
/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 2985*a^(7/2)*sin(f*x + e)^8/(cos(
f*x + e) + 1)^8 - 555*a^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)/(c^(7/2)
 - 6*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 17*c^(7/2)*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 32*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 46*c^
(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 52*c^(7/2)*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 46*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 32*c^(7/
2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 17*c^(7/2)*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 - 6*c^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + c^(7/2)*sin(
f*x + e)^10/(cos(f*x + e) + 1)^10) + 10*(75*a^(7/2)*sin(f*x + e)/(cos(f*x +
 e) + 1) - 375*a^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 854*a^(7/2)*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1257*a^(7/2)*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + 1534*a^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1257*a^(7/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 854*a^(7/2)*sin(f*x + e)^7/(cos(f*x
```

```
+ e) + 1)^7 - 375*a^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 75*a^(7/2)*
sin(f*x + e)^9/(cos(f*x + e) + 1)^9)/(c^(7/2) - 6*c^(7/2)*sin(f*x + e)/(cos
(f*x + e) + 1) + 17*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 32*c^(7/2
)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 46*c^(7/2)*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 - 52*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 46*c^(7/2)*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - 32*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 + 17*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 6*c^(7/2)*sin(f
*x + e)^9/(cos(f*x + e) + 1)^9 + c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)
^10))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(7/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.39 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=241

$$\frac{8a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^4f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2\cos(e+fx)(a\sin(e+fx)+}{c^3f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/3*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(7/2)-2/3*a*cos(
f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)+2*a^2*cos(f*x+e)
*(a+a*sin(f*x+e))^(3/2)/c^3/f/(c-c*sin(f*x+e))^(3/2)+8*a^4*cos(f*x+e)*ln(1-
sin(f*x+e))/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+4*a^3*cos(f
*x+e)*(a+a*sin(f*x+e))^(1/2)/c^4/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^4f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{c^3f(c-c\sin(e+fx))^{3/2}}+\frac{8a^4\cos(e+fx)\log(1-\sin(e+f}{c^4f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(9/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(3*c*f*(c - c*Sin[e + f*x])^(7/2)
) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c^2*f*(c - c*Sin[e + f
*x])^(5/2)) + (2*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(c^3*f*(c - c
*Sin[e + f*x])^(3/2)) + (8*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^4*f*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (4*a^3*Cos[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(c^4*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{4\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{3c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx)}{3c^2f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx)}{3c^2f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx)}{3c^2f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx)}{3c^2f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx)}{3c^2f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx)}{3c^2f(c-c\sin(e+fx))^{5/2}}$$

Mathematica [A] time = 6.61, size = 442, normalized size = 1.83

$$\frac{\sin(e+fx)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}{f(c-c\sin(e+fx))^{9/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} + \frac{24(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+f\right.\right.}{f(c-c\sin(e+fx))^{9/2}\left(\sin\left(\frac{1}{2}(e+fx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] (16*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(
3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(9/2)) - (
16*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(9/2)) + (24*
```

```
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(9/2)) + (16*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^9*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7
*(c - c*Sin[e + f*x])^(9/2)) + ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*Sin
[e + f*x]*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^7*(c - c*Sin[e + f*x])^(9/2))
```

fricas [F] time = 3.39, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2+\left(a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin}}{5\,c^5\cos\left(fx+e\right)^4-20\,c^5\cos\left(fx+e\right)^2+16\,c^5-\left(c^5\cos\left(fx+e\right)^4-12\,c^5\cos\left(fx+e\right)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^
4 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c)/(5*c^5*cos(f*x + e)^4 - 20*c^5*cos(f*x + e)^2 + 16*c^5 - (c
^5*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 + 16*c^5)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.41, size = 435, normalized size = 1.80

$$-\frac{\left(48\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-24\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/3/f*(48*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))-24*sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-3*cos(f*x+e)^4-49*cos(f*
x+e)^2*sin(f*x+e)-144*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))+72*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-192*sin(f*x+e)*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))+96*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+63*cos(f*x+e)^2+7
6*sin(f*x+e)+192*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-96*ln(2/(cos(f*
x+e)+1))-60)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)
*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e
)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(
f*x+e)-4*cos(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(9/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(9/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

3.40 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=243

$$-\frac{a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^4f(c-c\sin(e+fx))^{3/2}}+\frac{a^2\cos(e+fx)(a\sin(e+fx)+a}{2c^3f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/4*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(9/2)-1/3*a*cos(
f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(7/2)+1/2*a^2*cos(f*x+
e)*(a+a*sin(f*x+e))^(3/2)/c^3/f/(c-c*sin(f*x+e))^(5/2)-a^3*cos(f*x+e)*(a+a*
sin(f*x+e))^(1/2)/c^4/f/(c-c*sin(f*x+e))^(3/2)-a^4*cos(f*x+e)*ln(1-sin(f*x+
e))/c^5/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2739, 2737, 2667, 31}

$$-\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^4f(c-c\sin(e+fx))^{3/2}}+\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^3f(c-c\sin(e+fx))^{5/2}}-\frac{a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(11/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*c*f*(c - c*Sin[e + f*x])^(9/2)
) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c^2*f*(c - c*Sin[e + f*x
])^(7/2)) + (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^3*f*(c - c*S
in[e + f*x])^(5/2)) - (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^4*f*(c
 - c*Sin[e + f*x])^(3/2)) - (a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^5*f
*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{7/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{7/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{7/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{7/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{7/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{7/2}}$$

Mathematica [A] time = 6.65, size = 437, normalized size = 1.80

$$-\frac{8(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}{f(c-c\sin(e+fx))^{11/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}+\frac{12(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\right.}{f(c-c\sin(e+fx))^{11/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] (4*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) - (32
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) + (12
```

```
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) - (8*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) - (2*Log[C
os[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^1
1*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*
(c - c*Sin[e + f*x])^(11/2))
```

fricas [F] time = 4.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(3\,a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2+\left(a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin}}{c^6\cos\left(fx+e\right)^6-18\,c^6\cos\left(fx+e\right)^4+48\,c^6\cos\left(fx+e\right)^2-32\,c^6+2\left(3\,c^6\cos\left(fx+e\right)^4-16\,c^6\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="fricas")
```

```
[Out] integral((3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^4
 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(c^6*cos(f*x + e)^6 - 18*c^6*cos(f*x + e)^4 + 48*c^6*cos(f*x
 + e)^2 - 32*c^6 + 2*(3*c^6*cos(f*x + e)^4 - 16*c^6*cos(f*x + e)^2 + 16*c^6
)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.40, size = 490, normalized size = 2.02

$$-\frac{\left(6\left(\cos^4\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-3\left(\cos^4\left(fx+e\right)\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)+24\sin\left(fx+e\right)\left(\cos^2\left(fx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] -1/3/f*(6*cos(f*x+e)^4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*cos(f*x
+e)^4*ln(2/(cos(f*x+e)+1))+24*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))-8*cos(f*x+e)^4-12*sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f
*x+e)+1))-48*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-8*cos(
f*x+e)^2*sin(f*x+e)+24*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-48*sin(f*x+e)*ln(-
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+28*cos(f*x+e)^2+24*sin(f*x+e)*ln(2/(
cos(f*x+e)+1))+48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+8*sin(f*x+e)-2
4*ln(2/(cos(f*x+e)+1))-20)*(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2
*sin(f*x+e)-2)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)
^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f
*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(11/2)
,x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(11/2)
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

3.41 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=48

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{10acf(c-c\sin(e+fx))^{11/2}}$$

```
[Out] 1/10*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/c/f/(c-c*sin(f*x+e))^(11/2)
```

Rubi [A] time = 0.34, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{10acf(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(13/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(10*a*c*f*(c - c*Sin[e + f*x])^(1
1/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{10acf(c-c\sin(e+fx))^{11/2}}$$

Mathematica [B] time = 6.74, size = 412, normalized size = 8.58

$$\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}{f(c-c\sin(e+fx))^{13/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^{7}}-\frac{4(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\right.\right.}{f(c-c\sin(e+fx))^{13/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(13/2),x]
```

```
[Out] (16*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(
5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) -
(8*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) + (8*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) - (4*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) + ((Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2))
```

fricas [B] time = 0.47, size = 150, normalized size = 3.12

$$\frac{\left(5\,a^3\cos\left(fx+e\right)^4-20\,a^3\cos\left(fx+e\right)^2+16\,a^3\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right.}}{5\left(5\,c^7f\cos\left(fx+e\right)^5-20\,c^7f\cos\left(fx+e\right)^3+16\,c^7f\cos\left(fx+e\right)-\left(c^7f\cos\left(fx+e\right)^5-12\,c^7f\cos\left(fx+e\right)^3+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="fricas")
```

```
[Out] 1/5*(5*a^3*cos(f*x + e)^4 - 20*a^3*cos(f*x + e)^2 + 16*a^3)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^7*f*cos(f*x + e)^5 - 20*c^7*f*cos(
```

```
f*x + e)^3 + 16*c^7*f*cos(f*x + e) - (c^7*f*cos(f*x + e)^5 - 12*c^7*f*cos(f
*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.36, size = 188, normalized size = 3.92

$$\frac{\left(\cos^{4}\left(fx+e\right)-12\left(\cos^{2}\left(fx+e\right)\right)+16\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\sin\left(fx+e\right)\left(\cos^{2}\right.}{5f\left(\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right)\right)+\cos^{4}\left(fx+e\right)-4\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+3\left(\cos^{3}\left(fx+e\right)\right)-4\sin\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x)
```

```
[Out] 1/5/f*(cos(f*x+e)^4-12*cos(f*x+e)^2+16)*(a*(1+sin(f*x+e)))^(7/2)*sin(f*x+e)
*(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(sin(f*x+e)
*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f
*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(sin(f*x+e
)-1))^(13/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{13/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(13/2)
,x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(13/2)
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

3.42 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{120ac^2f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{12acf(c-c\sin(e+fx))^{13/2}}$$

```
[Out] 1/12*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/c/f/(c-c*sin(f*x+e))^(13/2)+1/120*
cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/c^2/f/(c-c*sin(f*x+e))^(11/2)
```

Rubi [A] time = 0.44, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{120ac^2f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{12acf(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(15/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(12*a*c*f*(c - c*Sin[e + f*x])^(1
3/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(120*a*c^2*f*(c - c*Sin[e
 + f*x])^(11/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{13/2}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{12acf(c-c\sin(e+fx))^{13/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{12ac^2}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{12acf(c-c\sin(e+fx))^{13/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{120ac^2f(c-c\sin(e+fx))^{11/2}}$$

Mathematica [B] time = 6.79, size = 419, normalized size = 4.32

$$\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}{2f(c-c\sin(e+fx))^{15/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} - \frac{8(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}\right.\right.}{3f(c-c\sin(e+fx))^{15/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(15/2),x]
```

```
[Out] (8*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(3
*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) - (
32*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(5
*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + (
6*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) - (8*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + ((Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2))
```

fricas [B] time = 0.50, size = 191, normalized size = 1.97

$$-\frac{\left(15\,a^{3}\cos\left(fx+e\right)^{4}-60\,a^{3}\cos\left(fx+e\right)^{2}+48\,a^{3}-4\left(5\,a^{3}\cos\left(fx+e\right)^{2}-8\,a^{3}\right)\sin\left(fx+e\right.}{30\left(c^{8}f\cos\left(fx+e\right)^{7}-18\,c^{8}f\cos\left(fx+e\right)^{5}+48\,c^{8}f\cos\left(fx+e\right)^{3}-32\,c^{8}f\cos\left(fx+e\right)+2\left(3\,c^{8}f\cos\left(fx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, al
gorithm="fricas")
```

```
[Out] -1/30*(15*a^3*cos(f*x + e)^4 - 60*a^3*cos(f*x + e)^2 + 48*a^3 - 4*(5*a^3*co
s(f*x + e)^2 - 8*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*
x + e) + c)/(c^8*f*cos(f*x + e)^7 - 18*c^8*f*cos(f*x + e)^5 + 48*c^8*f*cos(
f*x + e)^3 - 32*c^8*f*cos(f*x + e) + 2*(3*c^8*f*cos(f*x + e)^5 - 16*c^8*f*c
os(f*x + e)^3 + 16*c^8*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, al
gorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.40, size = 230, normalized size = 2.37

$$-\frac{\left(3\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)-18\left(\cos^{4}\left(fx+e\right)\right)-36\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+116\left(\cos^{2}\left(fx+e\right)\right)+\right.}{30f\left(\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right)\right)+\cos^{4}\left(fx+e\right)-4\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+3\left(\cos^{3}\left(\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x)
```

```
[Out] -1/30/f*(3*sin(f*x+e)*cos(f*x+e)^4-18*cos(f*x+e)^4-36*cos(f*x+e)^2*sin(f*x+
e)+116*cos(f*x+e)^2+48*sin(f*x+e)-128)*(a*(1+sin(f*x+e)))^(7/2)*sin(f*x+e)*
(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(sin(f*x+e)*
cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*
x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(sin(f*x+e)
-1))^(15/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, al
gorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 14.41, size = 373, normalized size = 3.85

$$-\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{504\,a^3\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{5\,c^8\,f}+\frac{576\,a^3\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{5\,c^8\,f}-\frac{96\,a^3\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\mathrm{co}}{}\right.}{-858\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}+858\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)-130\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)+2\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\mathrm{cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(15/2)
,x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((504*a^3*exp(e*7i + f*x*7i)*(a + a*sin(e + f*
x))^(1/2))/(5*c^8*f) + (576*a^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(a + a*sin(
e + f*x))^(1/2))/(5*c^8*f) - (96*a^3*exp(e*7i + f*x*7i)*cos(2*e + 2*f*x)*(a
 + a*sin(e + f*x))^(1/2))/(c^8*f) + (8*a^3*exp(e*7i + f*x*7i)*cos(4*e + 4*f
*x)*(a + a*sin(e + f*x))^(1/2))/(c^8*f) - (64*a^3*exp(e*7i + f*x*7i)*sin(3*
e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(3*c^8*f)))/(858*exp(e*7i + f*x*7i)*
cos(3*e + 3*f*x) - 858*cos(e + f*x)*exp(e*7i + f*x*7i) - 130*exp(e*7i + f*x
*7i)*cos(5*e + 5*f*x) + 2*exp(e*7i + f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*
7i + f*x*7i)*sin(2*e + 2*f*x) - 416*exp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 2
4*exp(e*7i + f*x*7i)*sin(6*e + 6*f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(15/2),x)
```

```
[Out] Timed out
```

3.43 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{17/2}}\,dx$

Optimal. Leaf size=145

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{840ac^3f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{84ac^2f(c-c\sin(e+fx))^{13/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}}$$

```
[Out] 1/14*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/c/f/(c-c*sin(f*x+e))^(15/2)+1/84*c
os(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/c^2/f/(c-c*sin(f*x+e))^(13/2)+1/840*cos(
f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/c^3/f/(c-c*sin(f*x+e))^(11/2)
```

Rubi [A] time = 0.54, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{840ac^3f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{84ac^2f(c-c\sin(e+fx))^{13/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(17/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(14*a*c*f*(c - c*Sin[e + f*x])^(1
5/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(84*a*c^2*f*(c - c*Sin[e
+ f*x])^(13/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(840*a*c^3*f*(c
 - c*Sin[e + f*x])^(11/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{15/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{13/2}}\,dx}{7ac^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{84ac^2f(c-c\sin(e+fx))^{13/2}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{84ac^2f(c-c\sin(e+fx))^{13/2}}$$

Mathematica [B] time = 6.87, size = 419, normalized size = 2.89

$$\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}{3f(c-c\sin(e+fx))^{17/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^{7}} - \frac{2(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}\right.\right.}{f(c-c\sin(e+fx))^{17/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(17/2),x]
```

```
[Out] (16*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(
7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) -
(16*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(
3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) +
(24*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(
5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) -
```

```
(2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) + ((C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2))
```

fricas [A] time = 0.51, size = 205, normalized size = 1.41

$$-\frac{\left(35\,a^3\cos\left(fx+e\right)^4-154\,a^3\cos\left(fx+e\right)^2+128\,a^3-14\left(5\,a^3\cos\left(fx+e\right)^2-8\,a \cdots\right.\right.}{105\left(7\,c^9 f\cos\left(fx+e\right)^7-56\,c^9 f\cos\left(fx+e\right)^5+112\,c^9 f\cos\left(fx+e\right)^3-64\,c^9 f\cos\left(fx+e\right)-\left(c^9 f\cos\left(fx+\cdots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, al
gorithm="fricas")
```

```
[Out] -1/105*(35*a^3*cos(f*x + e)^4 - 154*a^3*cos(f*x + e)^2 + 128*a^3 - 14*(5*a^
3*cos(f*x + e)^2 - 8*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c)/(7*c^9*f*cos(f*x + e)^7 - 56*c^9*f*cos(f*x + e)^5 + 112*c^9
*f*cos(f*x + e)^3 - 64*c^9*f*cos(f*x + e) - (c^9*f*cos(f*x + e)^7 - 24*c^9*
f*cos(f*x + e)^5 + 80*c^9*f*cos(f*x + e)^3 - 64*c^9*f*cos(f*x + e))*sin(f*x
 + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, al
gorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.43, size = 240, normalized size = 1.66

$$-\frac{\left(9\left(\cos^6\left(fx+e\right)\right)+63\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)-216\left(\cos^4\left(fx+e\right)\right)-406\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+\cdots\right.}{105 f\left(\sin\left(fx+e\right)\left(\cos^3\left(fx+e\right)\right)+\cos^4\left(fx+e\right)-4\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x)
```

```
[Out] -1/105/f*(9*cos(f*x+e)^6+63*sin(f*x+e)*cos(f*x+e)^4-216*cos(f*x+e)^4-406*co
s(f*x+e)^2*sin(f*x+e)+790*cos(f*x+e)^2+448*sin(f*x+e)-688)*(a*(1+sin(f*x+e)
```

```
))^(7/2)*sin(f*x+e)*(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*
x+e)-2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*c
os(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e
)+8)/(-c*(sin(f*x+e)-1))^(17/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, al
gorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 14.93, size = 764, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(17/2)
,x)
```

```
[Out] -((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(
(a^3*exp(e*4i + f*x*4i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2))^(1/2)*16i)/(3*c^9*f) + (64*a^3*exp(e*5i + f*x*5i)*(a + a*((e
xp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(3*c^9*f) -
(a^3*exp(e*6i + f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2))^(1/2)*1088i)/(15*c^9*f) - (576*a^3*exp(e*7i + f*x*7i)*(a + a
*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(5*c^9*f
) + (a^3*exp(e*8i + f*x*8i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
 + f*x*1i)*1i)/2))^(1/2)*5472i)/(35*c^9*f) + (576*a^3*exp(e*9i + f*x*9i)*(a
 + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(5*c
^9*f) - (a^3*exp(e*10i + f*x*10i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (ex
p(e*1i + f*x*1i)*1i)/2))^(1/2)*1088i)/(15*c^9*f) - (64*a^3*exp(e*11i + f*x*
11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2
))/(3*c^9*f) + (a^3*exp(e*12i + f*x*12i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/
2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*16i)/(3*c^9*f)))/(exp(e*1i + f*x*1i)*
14i - 90*exp(e*2i + f*x*2i) - exp(e*3i + f*x*3i)*350i + 910*exp(e*4i + f*x*
4i) + exp(e*5i + f*x*5i)*1638i - 2002*exp(e*6i + f*x*6i) - exp(e*7i + f*x*7
i)*1430i - exp(e*9i + f*x*9i)*1430i + 2002*exp(e*10i + f*x*10i) + exp(e*11i
 + f*x*11i)*1638i - 910*exp(e*12i + f*x*12i) - exp(e*13i + f*x*13i)*350i +
90*exp(e*14i + f*x*14i) + exp(e*15i + f*x*15i)*14i - exp(e*16i + f*x*16i) +
 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(17/2),x)
[Out] Timed out
```

3.44 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/4*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*c*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 0.90, size = 134, normalized size = 2.98

$$\frac{c^2(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(56\sin(e+fx)-8\sin(3(e+fx))+}{32f\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c
*Sin[e + f*x]]*(28*Cos[2*(e + f*x)] - Cos[4*(e + f*x)] + 56*Sin[e + f*x] -
8*Sin[3*(e + f*x)]))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*Sqrt[a*(
1 + Sin[e + f*x])])
```

fricas [B] time = 0.45, size = 98, normalized size = 2.18

$$-\frac{\left(c^2\cos\left(fx+e\right)^4-8\,c^2\cos\left(fx+e\right)^2+7\,c^2+4\left(c^2\cos\left(fx+e\right)^2-2\,c^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-}}{4\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/4*(c^2*cos(f*x + e)^4 - 8*c^2*cos(f*x + e)^2 + 7*c^2 + 4*(c^2*cos(f*x +
e)^2 - 2*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)/(a*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.38, size = 195, normalized size = 4.33

$$\frac{\sin\left(fx+e\right)\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\left(\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right)\right)+\cos^{4}\left(fx+e\right)+3\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)\right.}{4f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\left(\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+\cos^{3}\left(fx+e\right)+2\sin\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/4/f*sin(f*x+e)*(-c*(sin(f*x+e)-1))^(5/2)*(sin(f*x+e)*cos(f*x+e)^3+cos(f*x
+e)^4+3*cos(f*x+e)^2*sin(f*x+e)-4*cos(f*x+e)^3-7*sin(f*x+e)*cos(f*x+e)-4*co
s(f*x+e)^2-sin(f*x+e)+8*cos(f*x+e)-1)/(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^
2*sin(f*x+e)+cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+
e)-2*cos(f*x+e)+4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)*cos(f*x + e)^2/sqrt(a*sin(f*x + e) +
a), x)
```

mupad [B] time = 2.00, size = 96, normalized size = 2.13

$$-\frac{c^{2}\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(28\,\cos\left(e+f\,x\right)+27\,\cos\left(3\,e+3\,f\,x\right)-\cos\left(5\,e+5\,f\,x\right)+48\,\sin\left(2\,e+2\,f\,x\right)-8\right.}{64\,f\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(\sin\left(e+f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(1/2),
x)
```

```
[Out] -(c^2*(-c*(sin(e + f*x) - 1))^(1/2)*(28*cos(e + f*x) + 27*cos(3*e + 3*f*x)
- cos(5*e + 5*f*x) + 48*sin(2*e + 2*f*x) - 8*sin(4*e + 4*f*x)))/(64*f*(a*(s
in(e + f*x) + 1))^(1/2)*(sin(e + f*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.45 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/3*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*c*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a+a\sin(e+fx)}}$$

Mathematica [**B**] time = 0.51, size = 120, normalized size = 2.67

$$-\frac{c(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(15\sin(e+fx)-\sin(3(e+fx))+6\cos(2(e+fx)))}{12f\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] -1/12*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c -
 c*Sin[e + f*x]]*(6*Cos[2*(e + f*x)] + 15*Sin[e + f*x] - Sin[3*(e + f*x)]))
/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [**A**] time = 0.45, size = 78, normalized size = 1.73

$$\frac{\left(3\,c\cos\left(fx+e\right)^2-\left(c\cos\left(fx+e\right)^2-4\,c\right)\sin\left(fx+e\right)-3\,c\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/3*(3*c*cos(f*x + e)^2 - (c*cos(f*x + e)^2 - 4*c)*sin(f*x + e) - 3*c)*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.37, size = 147, normalized size = 3.27

$$\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\left(\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-\left(\cos^{3}\left(fx+e\right)\right)-3\sin\left(fx+e\right)\cos\left(fx+e\right)\right.}{3f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos^{2}\left(fx+e\right)\right)-2\sin\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(cos(f*x+e)^2*sin(f*x+e)-cos(f*x
+e)^3-3*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)^2-sin(f*x+e)+4*cos(f*x+e)-1)/(a*
(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(
f*x+e)+2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)*cos(f*x + e)^2/sqrt(a*sin(f*x + e) +
a), x)
```

mupad [B] time = 9.96, size = 83, normalized size = 1.84

$$-\frac{c\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(6\cos\left(e+fx\right)+6\cos\left(3e+3fx\right)+14\sin\left(2e+2fx\right)-\sin\left(4e+4fx\right)\right)}{24f\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(\sin\left(e+fx\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(1/2),
x)
```

```
[Out] -(c*(-c*(sin(e + f*x) - 1))^(1/2)*(6*cos(e + f*x) + 6*cos(3*e + 3*f*x) + 14
*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(24*f*(a*(sin(e + f*x) + 1))^(1/2)*(
sin(e + f*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\cos^{2}\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*cos(e + f*x)**2/sqrt(a*(sin(e + f*x
) + 1)), x)
```

3.46 $\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.29, size = 62, normalized size = 1.38

$$\frac{\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(4\sin(e+fx)+\cos(2(e+fx)))}{4af}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]
],x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(Cos[2*(e + f*x)] + 4*Sin[e + f*x]
)*Sqrt[c - c*Sin[e + f*x]])/(4*a*f)
```

fricas [A] time = 0.44, size = 59, normalized size = 1.31

$$\frac{\left(\cos\left(fx+e\right)^2+2\,\sin\left(fx+e\right)-1\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/2*(cos(f*x + e)^2 + 2*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*
sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8
```

```
*pi/x/2)-32*sqrt(2*c)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+1
/4*(2*exp(1)-pi)))^4/sqrt(2)/sqrt(a)/(tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^
2+1)^4/f/sign(tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^4-1)
```

maple [B] time = 0.37, size = 90, normalized size = 2.00

$$\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\ \sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)+\sin\left(fx+e\right)\cos\left(fx+e\right)-2\cos\left(fx+e\right)+\sin\left(fx+e\right)+\right.}{2f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\ \left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(cos(f*x+e)^2+sin(f*x+e)*cos(f*x
+e)-2*cos(f*x+e)+sin(f*x+e)+1)/(a*(1+sin(f*x+e)))^(1/2)/(-1+cos(f*x+e)+sin(
f*x+e))
```

maxima [B] time = 1.63, size = 388, normalized size = 8.62

$$-\frac{\frac{2\sqrt{a}\sqrt{c}+\frac{\sqrt{a}\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sqrt{a}\sqrt{c}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{\sqrt{a}\sqrt{c}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}{a+\frac{2a\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}}-\frac{2\sqrt{a}\sqrt{c}-\frac{\sqrt{a}\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sqrt{a}\sqrt{c}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{\sqrt{a}\sqrt{c}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}{a+\frac{2a\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}}+\frac{2\left(\frac{\sqrt{a}\sqrt{c}\sin(f}{\cos(fx+e)+}\right.}{a+}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/2*((2*sqrt(a)*sqrt(c) + sqrt(a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1)
+ sqrt(a)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(a)*sqrt(c)*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - (2*sqrt(a)*sqrt(c) - sqrt(a)
*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sqrt(a)*sqrt(c)*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 - sqrt(a)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
)/(a + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4) + 2*(sqrt(a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) - sqrt(a)
*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(a)*sqrt(c)*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3)/(a + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

mupad [B] time = 0.93, size = 67, normalized size = 1.49

$$-\frac{\sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\,\left(\cos\left(e+f\,x\right)+\cos\left(3\,e+3\,f\,x\right)+4\,\sin\left(2\,e+2\,f\,x\right)\right)}{8\,f\,\sqrt{a\left(\sin\left(e+f\,x\right)+1\right)}\,\left(\sin\left(e+f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(1/2),
x)
```

```
[Out] -((-c*(sin(e + f*x) - 1))^(1/2)*(cos(e + f*x) + cos(3*e + 3*f*x) + 4*sin(2*
e + 2*f*x)))/(8*f*(a*(sin(e + f*x) + 1))^(1/2)*(sin(e + f*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\cos^{2}\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*cos(e + f*x)**2/sqrt(a*(sin(e + f*x) +
 1)), x)
```

3.47 $\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$-\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.30, size = 44, normalized size = 1.02

$$\frac{\sin(2(e+fx))}{2f\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
),x]
```

```
[Out] Sin[2*(e + f*x)]/(2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.47, size = 49, normalized size = 1.14

$$\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{acf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(a*c*f*cos(
f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^2}{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)), x)
```

maple [A] time = 0.36, size = 42, normalized size = 0.98

$$\frac{\cos\left(fx+e\right)\sin\left(fx+e\right)}{f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/f*cos(f*x+e)*sin(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)), x)
```

mupad [B] time = 8.87, size = 52, normalized size = 1.21

$$-\frac{\sin\left(2\,e+2\,f\,x\right)\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}}{2\,c\,f\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(\sin\left(e+f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2)),
x)
```

```
[Out] -(sin(2*e + 2*f*x)*(-c*(sin(e + f*x) - 1))^(1/2))/(2*c*f*(a*(sin(e + f*x) +
 1))^(1/2)*(sin(e + f*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x)
- 1))), x)
```

3.48 $\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=54

$$-\frac{\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)
```

Rubi [A] time = 0.34, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2841, 2737, 2667, 31}

$$-\frac{\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -((Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$
$$= -\frac{\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.41, size = 104, normalized size = 1.93

$$-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f\sqrt{a(\sin(e+fx)+1)}\,(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/
2)),x]
```

```
[Out] (-2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(f*Sqrt[a*(1 + Sin[e + f*x
])]*(c - c*Sin[e + f*x])^(3/2))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{ac^2\sin\left(fx+e\right)-ac^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c^2*sin(f*x
 + e) - a*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3
/2)), x)
```

maple [B] time = 0.37, size = 138, normalized size = 2.56

$$-\frac{\left(2\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)\right)\left(-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)\right)\left(\cos^{2}\left(fx+e\right)-\sin\left(fx+\right.\right.}{2f\left(-1+\cos\left(fx+e\right)\right)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\left(-c\left(\sin\left(fx+e\right)-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln(2/(cos(f*x+e)+1)))*
(-1+cos(f*x+e)-sin(f*x+e))*(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2
*sin(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))
^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3
/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)^2}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3/2)),
x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1)
)**(3/2)), x)
```

3.49 $\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\cos(e+fx)}{cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}$$

```
[Out] cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$\frac{\cos(e+fx)}{cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] Cos[e + f*x]/(c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)}{cf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}$$

Mathematica [A] time = 0.49, size = 79, normalized size = 1.88

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f\sqrt{a(\sin(e+fx)+1)}\,(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2]))/(f*Sqrt[a*(1 + Sin[e + f*x])]*(c - c*Sin[e + f*x])^(5/2))
```

fricas [A] time = 0.45, size = 61, normalized size = 1.45

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{ac^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^3f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c^3*f*cos(f*x + e)*s
in(f*x + e) - a*c^3*f*cos(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^2}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5
/2)), x)
```

maple [A] time = 0.36, size = 51, normalized size = 1.21

$$-\frac{\left(\sin\left(fx+e\right)-1\right)\cos\left(fx+e\right)\sin\left(fx+e\right)}{f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(sin(f*x+e)-1)*cos(f*x+e)*sin(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin
(f*x+e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(fx+e\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5
/2)), x)
```

mupad [B] time = 9.77, size = 88, normalized size = 2.10

$$-\frac{2\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(4\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+\sin\left(2\,e+2\,f\,x\right)-2\right)}{c^3\,f\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(12\sin\left(e+f\,x\right)^{2}-15\,\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5/2)),
x)
```

```
[Out] -(2*(-c*(sin(e + f*x) - 1))^(1/2)*(sin(2*e + 2*f*x) + 4*sin(e/2 + (f*x)/2)^
2 - 2))/(c^3*f*(a*(sin(e + f*x) + 1))^(1/2)*(sin(3*e + 3*f*x) - 15*sin(e +
f*x) + 12*sin(e + f*x)^2 + 4))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

3.50 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=239

$$\frac{16c^4\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{3}}{af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] 2*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)+2/3*c*co
s(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(1/2)+1/4*cos(f*x+e)*(
c-c*sin(f*x+e))^(7/2)/a/f/(a+a*sin(f*x+e))^(1/2)+16*c^4*cos(f*x+e)*ln(1+sin
(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+8*c^3*cos(f*x+e)
*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$\frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a\sin(e+fx)+a}}+\frac{16c^4\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (16*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) + (8*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a
*f*Sqrt[a + a*Sin[e + f*x]]) + (2*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/
2))/(a*f*Sqrt[a + a*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*(c - c*Sin[e + f*x])
^(5/2))/(3*a*f*Sqrt[a + a*Sin[e + f*x]]) + (Cos[e + f*x]*(c - c*Sin[e + f*x
])^(7/2))/(4*a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{9/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= \frac{2c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c\cos(e+fx)(c-c\sin(e+fx)}{3af\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{16c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{8c^3\cos(e+fx)\sqrt{c-c\sin(}{af\sqrt{a+a\sin(e+fx}$$

Mathematica [A] time = 6.47, size = 471, normalized size = 1.97

$$\frac{5\sin(3(e+fx))(c-c\sin(e+fx))^{7/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{12f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7} - \frac{23\cos(2(e+fx))(c-c\sin(e+fx))^{7/}}{8f(a(\sin(e+fx)+1))^{3/2}\left(\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-23*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e
+ f*x])^(7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e +
 f*x]))^(3/2)) + (Cos[4*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*
```

```
(c - c*Sin[e + f*x])^(7/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(
a*(1 + Sin[e + f*x]))^(3/2)) + (32*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(7/2))/(f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(3/2)) - (65*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[e + f*x]*(c - c*Sin[e + f*x])^(7/
2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(3/
2)) + (5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(7/2)
*Sin[3*(e + f*x)])/(12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Si
n[e + f*x]))^(3/2))
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,c^{3}\cos\left(fx+e\right)^{4}-4\,c^{3}\cos\left(fx+e\right)^{2}-\left(c^{3}\cos\left(fx+e\right)^{4}-4\,c^{3}\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(\right.}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((3*c^3*cos(f*x + e)^4 - 4*c^3*cos(f*x + e)^2 - (c^3*cos(f*x + e)^4
 - 4*c^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.40, size = 252, normalized size = 1.05

$$\frac{\left(3\left(\cos^{4}\left(fx+e\right)\right)+20\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-72\left(\cos^{2}\left(fx+e\right)\right)-192\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)-200\sin\left(fx+\right.\right.}{12f\left(\cos^{4}\left(fx+e\right)-\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right)\right)+3\left(\cos^{3}\left(fx+e\right)\right)+4\left(\cos^{2}\left(fx+e\right)\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/12/f*(3*cos(f*x+e)^4+20*cos(f*x+e)^2*sin(f*x+e)-72*cos(f*x+e)^2-192*ln(2/
(cos(f*x+e)+1))-200*sin(f*x+e)+384*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))+69)*(-c*(sin(f*x+e)-1))^(7/2)*(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*
x+e)-2*sin(f*x+e)-2)/(cos(f*x+e)^4-sin(f*x+e)*cos(f*x+e)^3+3*cos(f*x+e)^3+4
*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2+4*sin(f*x+e)*cos(f*x+e)-4*cos(f*x+e
)-8*sin(f*x+e)+8)/(a*(1+sin(f*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.51 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{8c^3\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/}}{af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)+1/3*cos(f*x+
e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(1/2)+8*c^3*cos(f*x+e)*ln(1+
sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+4*c^2*cos(f*x
+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$\frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{8c^3\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/}}{af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (8*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]]) + (4*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*
f*Sqrt[a + a*Sin[e + f*x]]) + (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(
a*f*Sqrt[a + a*Sin[e + f*x]]) + (Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(
3*a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3af\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3af\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{8c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{4c^2\cos(e+fx)\sqrt{c-c\sin}}{af\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 2.37, size = 138, normalized size = 0.73

$$\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(-87\sin(e+fx)+\sin(3(e+fx))-12\cos(2(e+fx)\right.}{12f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
(3/2),x]

[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(-12*
Cos[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 87*Sin[e
+ f*x] + Sin[3*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*
(1 + Sin[e + f*x]))^(3/2))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2\cos\left(fx+e\right)^4+2\,c^2\cos\left(fx+e\right)^2\sin\left(fx+e\right)-2\,c^2\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx\right.}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((c^2*cos(f*x + e)^4 + 2*c^2*cos(f*x + e)^2*sin(f*x + e) - 2*c^2*co
s(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f
*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.36, size = 214, normalized size = 1.13

$$\frac{\left(\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-6\left(\cos^2\left(fx+e\right)\right)-22\sin\left(fx+e\right)-24\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)+48\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin}{\sin\left(fx+e\right)}\right.\right.}{3f\left(\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+\cos^3\left(fx+e\right)+2\sin\left(fx+e\right)\cos\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/3/f*(cos(f*x+e)^2*sin(f*x+e)-6*cos(f*x+e)^2-22*sin(f*x+e)-24*ln(2/(cos(f*
x+e)+1))+48*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+6)*(-c*(sin(f*x+e)-1
))^(5/2)*(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(co
s(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)-3*cos(f*x+e)^2-4
*sin(f*x+e)-2*cos(f*x+e)+4)/(a*(1+sin(f*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.52 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{4c^2\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] 1/2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)+4*c^2*cos(
f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2
*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$\frac{4c^2\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (4*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*
Sqrt[a + a*Sin[e + f*x]]) + (Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*a*
f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= \frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{4c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2c\cos(e+fx)\sqrt{c-c\sin(}}{af\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 1.12, size = 134, normalized size = 0.92

$$\frac{c(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(12\sin(e+fx)+\cos(2(e+fx))-32\mathrm{l}\right.}{4f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-1 + Sin[e + f*x])*(Cos[2*(e +
f*x)] - 32*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 12*Sin[e + f*x])*Sqrt
[c - c*Sin[e + f*x]])/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 +
Sin[e + f*x]))^(3/2))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c\cos\left(fx+e\right)^2\sin\left(fx+e\right)-c\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((c*cos(f*x + e)^2*sin(f*x + e) - c*cos(f*x + e)^2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e
) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.35, size = 175, normalized size = 1.21

$$\frac{\left(-\left(\cos^2\left(fx+e\right)\right)+16\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-6\sin\left(fx+e\right)-8\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)+1\right)\left(-c\left(\sin\left(fx+e\right)-1\right.\right.}{2f\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos^2\left(fx+e\right)\right)-2\sin\left(fx+e\right)-\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(-cos(f*x+e)^2+16*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-6*sin(f*
x+e)-8*ln(2/(cos(f*x+e)+1))+1)*(-c*(sin(f*x+e)-1))^(3/2)*(cos(f*x+e)^2+sin(
f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(sin(f*x+e)*cos(f*x+e)-cos(f*x
+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(a*(1+sin(f*x+e)))^(3/2)
```

maxima [B] time = 1.06, size = 844, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] -1/2*(16*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(3/2) - 8*c^(3/
2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - (10*c^(3/2) + 13*
c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 25*c^(3/2)*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 20*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*c^(3/2)
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9*c^(3/2)*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5)/(a^(3/2) + 2*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^(3/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^(3/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 2*a^(3/2)*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + a^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6) + (10*c^(3/2) + 11*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 15*c^(3/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20*c^(3/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7*c^(3/2)*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5)/(a^(3/2) + 2*a^(3/2)*sin(f*x + e)/(cos(f*x
+ e) + 1) + 3*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^(3/2)*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 + 2*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + a^(3/2)*sin(f*x + e)^
6/(cos(f*x + e) + 1)^6) - 2*(5*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5*
c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*c^(3/2)*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 5*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*c^(3/2
)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^(3/2) + 2*a^(3/2)*sin(f*x + e)/(c
os(f*x + e) + 1) + 3*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^(3/2
)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^(3/2)*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 2*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + a^(3/2)*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(3/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\cos^{2}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*cos(e + f*x)**2/(a*(sin(e + f*x) +
1))**(3/2), x)
```

3.53 $\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{2c\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*c*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))
^(1/2)+cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{2c\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*Sqrt[a
 + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx &= \frac{\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}\\
&= \frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}}+\frac{2\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}\\
&= \frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}}+\frac{(2c\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\\
&= \frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}}+\frac{(2c\cos(e+fx))\,\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,a\,\text{s}\ldots\right.}{af\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e\ldots}}\\
&= \frac{2c\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)\sqrt{c-c\sin(e+f\ldots}}{af\sqrt{a+a\sin(e+fx)}}
\end{aligned}
$$

Mathematica [C] time = 1.06, size = 113, normalized size = 1.18

$$-\frac{\sqrt{c-c\sin(e+fx)}\left(-4\log\left(e^{i(e+fx)}+i\right)+\sin(e+fx)+2ifx\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -((((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*((2*I)*f*x - 4*Log[I + E^(I*(e +
 f*x))] + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^2}{a^2\cos\left(fx+e\right)^2-2a^2\sin\left(fx+e\right)-2a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2
/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8
*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)4*sqrt(2*c)*(1/2*(-tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi))
)^2-6-1/tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2)/(tan(1/2*(1/2*f*x+1/4*(2*ex
p(1)-pi)))^2+2+1/tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2)/sign(-tan(1/2*(1/2
*f*x+1/4*(2*exp(1)-pi)))^2+1)+1/2*ln(tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2
+2+1/tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2)/sign(-tan(1/2*(1/2*f*x+1/4*(2*
```

```
exp(1)-pi)))^2+1)-1/2*ln(tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2-2+1/tan(1/2
*(1/2*f*x+1/4*(2*exp(1)-pi)))^2)/sign(-tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))
^2+1))*sign(sin(1/2*(f*x+exp(1))-1/4*pi))/sqrt(2)/sqrt(a)/a/f
```

maple [A] time = 0.37, size = 137, normalized size = 1.43

$$-\frac{\left(4\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-2\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)-\sin\left(fx+e\right)\right)\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\ \left(\cos^{2}\left(fx+e\right)+\sin\left(\right.\right.}{f\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(4*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*ln(2/(cos(f*x+e)+1))-s
in(f*x+e))*(-c*(sin(f*x+e)-1))^(1/2)*(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+co
s(f*x+e)-2*sin(f*x+e)-2)/(-1+cos(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+e)))^(3/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}\,\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(3/
2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(3/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\cos^{2}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*cos(e + f*x)**2/(a*(sin(e + f*x) + 1))
**(3/2), x)
```

3.54 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/
2)
```

Rubi [A] time = 0.33, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2841, 2737, 2667, 31}

$$\frac{\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] (Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.43, size = 102, normalized size = 2.00

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx\right.\right.}{f(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] (2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(f*(a*(1 + Sin[e + f*x]))^(
3/2)*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{a^2c\sin\left(fx+e\right)+a^2c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*c*sin(f*x
+ e) + a^2*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) +
 c)), x)
```

maple [B] time = 0.39, size = 137, normalized size = 2.69

$$\frac{\left(-\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)+2\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\right)\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\left(\cos^2\left(fx+e\right)+\sin\left(fx+\right.\right.}{2f\left(-1+\cos\left(fx+e\right)\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(fx+e\right)\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(-ln(2/(cos(f*x+e)+1))+2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)))*
(-1+cos(f*x+e)+sin(f*x+e))*(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2
*sin(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) +
 c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)^2}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*
x) - 1))), x)
```

3.55 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{acf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] arctanh(sin(f*x+e))*cos(f*x+e)/a/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{acf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)),
x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(a*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)\int \sec(e+fx)\,dx}{ac\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{acf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.53, size = 103, normalized size = 1.98

$$\frac{\cos^3(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{cf(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] (Cos[e + f*x]^3*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]]))/(c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))
^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.48, size = 160, normalized size = 3.08

$$\left[\frac{\sqrt{ac}\log\left(-\frac{ac\cos(fx+e)^3-2\,ac\cos(fx+e)-2\,\sqrt{ac}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}\,\sin(fx+e)}{\cos(fx+e)^3}\right)}{2\,a^2c^2f}, -\frac{\sqrt{-ac}\,\arctan\left(\frac{\sqrt{-ac}\,\sqrt{a\sin(fx+e)+}}{ac\cos(fx+e}\right.}{a^2c^2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/2*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x +
e)^3)/(a^2*c^2*f), -sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*s
qrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))/(a^2*c^2*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^
(3/2)), x)
```

maple [B] time = 0.36, size = 174, normalized size = 3.35

$$\frac{\left(\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\right)\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos^{2}\left(fx+e\right)\right)-2\sin\left(f\right.\right.}{2f\left(-1+\cos\left(fx+e\right)\right)\left(a\left(1+\sin\left(fx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e)))*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x
+e)+2)*(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(-1+c
os(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^
(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\cos\left(e+f\,x\right)}^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2)),
x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/((a*(sin(e + f*x) + 1))**(3/2)*(-c*(sin(e + f*x) -
 1))**(3/2)), x)
```

3.56 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=104

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2ac^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2acf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/2*cos(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*arct
anh(sin(f*x+e))*cos(f*x+e)/a/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^
(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2841, 2743, 2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2ac^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2acf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)),
x]
```

```
[Out] Cos[e + f*x]/(2*a*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
+ (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*c^2*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)}{2acf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+}}$$

$$= \frac{\cos(e+fx)}{2acf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{\cos(e}{2ac^2\sqrt{a+a\,s}}$$

$$= \frac{\cos(e+fx)}{2acf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{\tanh^{-1}}{2ac^2f\sqrt{a+a}}$$

Mathematica [A] time = 0.77, size = 163, normalized size = 1.57

$$\frac{\cos^3(e+fx)\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+\sin(e+fx\right.}{2c^2f(\sin(e+fx)-1)^2(a(\sin(e+fx)+1))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] (Cos[e + f*x]^3*(1 - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2]] + (Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] -
 Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*c^2*f*(-1 + Si
n[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.52, size = 319, normalized size = 3.07

$$\left[\frac{\sqrt{ac}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)\right)\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin}{\cos\left(fx+e\right)^{3}}\right.}{4\left(a^{2}c^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-a^{2}c^{3}f\cos\left(fx+e\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*(cos(f*x + e)*sin(f*x + e) - cos(f*x + e))*log(-(a*c*cos(f*
x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)*sin(f*x + e) - a^2*c
^3*f*cos(f*x + e)), -1/2*(sqrt(-a*c)*(cos(f*x + e)*sin(f*x + e) - cos(f*x +
 e))*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(
a*c*cos(f*x + e)*sin(f*x + e))) + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c))/(a^2*c^3*f*cos(f*x + e)*sin(f*x + e) - a^2*c^3*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^
(5/2)), x)
```

maple [B] time = 0.35, size = 244, normalized size = 2.35

$$\frac{\left(\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\sin\left(fx+e\right)-\ln\left(-\frac{-1+\cos(f}{\mathrm{si}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/4/f*(sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-sin(f*x+e)*ln(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+sin(f*x+e)-ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))+ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)))*(cos(f*x+e)^
2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)*(cos(f*x+e)^2+sin(f*x+e)
*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(
3/2)/(-c*(sin(f*x+e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^
(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2)),
x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.57 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=285

$$-\frac{80c^5\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{10c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/a/f/(a+a*sin(f*x+e))^(3/2)-10*c^3*cos(f*
x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)-10/3*c^2*cos(f*x+e
)*(c-c*sin(f*x+e))^(5/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)-5/4*c*cos(f*x+e)*(c-c
*sin(f*x+e))^(7/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)-80*c^5*cos(f*x+e)*ln(1+sin(
f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-40*c^4*cos(f*x+
e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$-\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{10c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{10c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (-80*c^5*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) - (40*c^4*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]]
)/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (10*c^3*Cos[e + f*x]*(c - c*Sin[e + f*
x])^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (10*c^2*Cos[e + f*x]*(c - c*S
in[e + f*x])^(5/2))/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (5*c*Cos[e + f*x]*
(c - c*Sin[e + f*x])^(7/2))/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f
*x]*(c - c*Sin[e + f*x])^(9/2))/(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{11/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{5\int \frac{(c-c\sin(e+fx))^{9/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{5c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e+fx))^{9/}}{af(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{10c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} - \frac{5c\cos(e+fx)(c-c\sin(e+}{4a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{10c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{10c^2\cos(e+fx)(c-c\sin(e-}{3a^2f\sqrt{a+a\sin(e+fx}}$$

$$= -\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{10c^3\cos(e+fx)(c-c\sin(e+}{a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{10c^3\cos(e+fx)(c-c\sin(e+}{a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{10c^3\cos(e+fx)(c-c\sin(e+}{a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{80c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{40c^4\cos(e+fx)\sqrt{c-c\,s}}{a^2f\sqrt{a+a\sin(e+}}$$

Mathematica [A] time = 6.64, size = 553, normalized size = 1.94

$$\frac{203\sin(e+fx)(c-c\sin(e+fx))^{9/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{4f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9} + \frac{47\cos(2(e+fx))(c-c\sin(e+fx))^{9/2}}{8f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (-32*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(9/2))/(f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) + (4
7*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f
*x])^(9/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*
x]))^(5/2)) - (Cos[4*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c
- c*Sin[e + f*x])^(9/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(
1 + Sin[e + f*x]))^(5/2)) - (160*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2))/(f*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) + (203*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sin[e + f*x]*(c - c*Sin[e + f*x])^(9/2
))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2
)) - (7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2)*
Sin[3*(e + f*x)])/(12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin
[e + f*x]))^(5/2))
```

fricas [F] time = 2.26, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c^4\cos\left(fx+e\right)^6 - 8\,c^4\cos\left(fx+e\right)^4 + 8\,c^4\cos\left(fx+e\right)^2 + 4\left(c^4\cos\left(fx+e\right)^4 - 2\,c^4\cos\left(fx+e\right)^2\right)\right.}{3\,a^3\cos\left(fx+e\right)^2 - 4\,a^3 + \left(a^3\cos\left(fx+e\right)^2 - 4\,a^3\right)\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(c^4*cos(f*x + e)^6 - 8*c^4*cos(f*x + e)^4 + 8*c^4*cos(f*x + e)^2
 + 4*(c^4*cos(f*x + e)^4 - 2*c^4*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3
*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.36, size = 349, normalized size = 1.22

$$\frac{\left(3\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right) - 25\left(\cos^4\left(fx+e\right)\right) - 116\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right) + 500\left(\cos^2\left(fx+e\right)\right) - \right.}{12f\left(\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right) + \cos^5\left(fx+e\right) + 4\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/12/f*(3*sin(f*x+e)*cos(f*x+e)^4-25*cos(f*x+e)^4-116*cos(f*x+e)^2*sin(f*x+
e)+500*cos(f*x+e)^2-960*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+1920*sin(f*x+e)*ln(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-859*sin(f*x+e)-960*ln(2/(cos(f*x+e)
+1))+1920*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-475)*(-c*(sin(f*x+e)-1
))^(9/2)*(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(si
n(f*x+e)*cos(f*x+e)^4+cos(f*x+e)^5+4*sin(f*x+e)*cos(f*x+e)^3-5*cos(f*x+e)^4
-12*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^3-8*sin(f*x+e)*cos(f*x+e)+20*cos(f
*x+e)^2+16*sin(f*x+e)+8*cos(f*x+e)-16)/(a*(1+sin(f*x+e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(9/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.58 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{32c^4\cos(e+fx)\log(\sin(e+fx)+1)}{a^2 f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2 f\sqrt{a\sin(e+fx)+a}}-\frac{4c^2\cos(e+fx)(c-c\sin(e+f}{a^2 f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/a/f/(a+a*sin(f*x+e))^(3/2)-4*c^2*cos(f*x
+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)-4/3*c*cos(f*x+e)*(c
-c*sin(f*x+e))^(5/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)-32*c^4*cos(f*x+e)*ln(1+si
n(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-16*c^3*cos(f*
x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$-\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2 f\sqrt{a\sin(e+fx)+a}}-\frac{4c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2 f\sqrt{a\sin(e+fx)+a}}-\frac{32c^4\cos(e+fx)\log(\sin(e+fx)+}{a^2 f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e-}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (-32*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) - (16*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]]
)/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (4*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x
])^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (4*c*Cos[e + f*x]*(c - c*Sin[e
 + f*x])^(5/2))/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f*x]*(c - c*S
in[e + f*x])^(7/2))/(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{4\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{4c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e+fx))^{7/}}{af(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{4c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{4c\cos(e+fx)(c-c\sin(e+f}{3a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{4c^2\cos(e+fx)(c-c\sin(e+f}{a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{4c^2\cos(e+fx)(c-c\sin(e+f}{a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{4c^2\cos(e+fx)(c-c\sin(e+f}{a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{32c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{16c^3\cos(e+fx)\sqrt{c-c\,s}}{a^2f\sqrt{a+a\sin(e+}}$$

Mathematica [A] time = 5.31, size = 179, normalized size = 0.76

$$\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(396\sin(e+fx)+16\sin(3(e+fx))-172\cos(2(e+f\right.}{24f(a(\sin(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(-177
 - 172*Cos[2*(e + f*x)] + Cos[4*(e + f*x)] - 1536*Log[Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]] + 396*Sin[e + f*x] - 1536*Log[Cos[(e + f*x)/2] + Sin[(e + f
```

```
*x)/2]]*Sin[e + f*x] + 16*Sin[3*(e + f*x)]))/(24*f*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [F] time = 1.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,c^3\cos\left(fx+e\right)^4-4\,c^3\cos\left(fx+e\right)^2-\left(c^3\cos\left(fx+e\right)^4-4\,c^3\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(\right.}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((3*c^3*cos(f*x + e)^4 - 4*c^3*cos(f*x + e)^2 - (c^3*cos(f*x + e)^4
 - 4*c^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)
*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.41, size = 306, normalized size = 1.29

$$\frac{\left(\cos^4\left(fx+e\right)+8\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-44\left(\cos^2\left(fx+e\right)\right)-192\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right.\right.}{3f\left(\cos^4\left(fx+e\right)-\sin\left(fx+e\right)\left(\cos^3\left(fx+e\right)\right)+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/3/f*(cos(f*x+e)^4+8*cos(f*x+e)^2*sin(f*x+e)-44*cos(f*x+e)^2-192*sin(f*x+e
)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+96*sin(f*x+e)*ln(2/(cos(f*x+e)
+1))+91*sin(f*x+e)-192*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+96*ln(2/(
cos(f*x+e)+1))+43)*(-c*(sin(f*x+e)-1))^(7/2)*(cos(f*x+e)^2+sin(f*x+e)*cos(f
*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(cos(f*x+e)^4-sin(f*x+e)*cos(f*x+e)^3+3*co
```

```
s(f*x+e)^3+4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2+4*sin(f*x+e)*cos(f*x+e)
-4*cos(f*x+e)-8*sin(f*x+e)+8)/(a*(1+sin(f*x+e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.59 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{12c^3\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{3c\cos(e+fx)(c-c\sin(e+f}{2a^2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(3/2)-3/2*c*cos(f*x
+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)-12*c^3*cos(f*x+e)*l
n(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-6*c^2*c
os(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$-\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{12c^3\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{3c\cos(e+fx)(c-c\sin(e+f}{2a^2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (-12*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) - (6*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])
/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (3*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^
(3/2))/(2*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f*x]*(c - c*Sin[e + f*
x])^(5/2))/(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{3\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{3c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e+fx))}{af(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{3c\cos(e+fx)(c-c\sin(e+f}{2a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{3c\cos(e+fx)(c-c\sin(e+f}{2a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{3c\cos(e+fx)(c-c\sin(e+f}{2a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{12c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{6c^2\cos(e+fx)\sqrt{c-c}}{a^2f\sqrt{a+a\sin(e}}$$

Mathematica [A] time = 2.49, size = 164, normalized size = 0.86

$$\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin(3(e+fx))-18\cos(2(e+fx))-192\log\left(\sin\left(\frac{1}{2}\right.\right.\right.}{8f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(-44
- 18*Cos[2*(e + f*x)] - 192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (39
- 192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f
*x)]))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5
/2))
```

fricas [F] time = 1.21, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^{2}\cos\left(fx+e\right)^{4}+2\,c^{2}\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-2\,c^{2}\cos\left(fx+e\right)^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(f\right.}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((c^2*cos(f*x + e)^4 + 2*c^2*cos(f*x + e)^2*sin(f*x + e) - 2*c^2*co
s(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos
(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.41, size = 270, normalized size = 1.41

$$\frac{\left(\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-9\left(\cos^{2}\left(fx+e\right)\right)+24\sin\left(fx+e\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)-48\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(\right.}{\mathrm{s}}\right.\right.}{2f\left(\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+\cos^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/2/f*(cos(f*x+e)^2*sin(f*x+e)-9*cos(f*x+e)^2+24*sin(f*x+e)*ln(2/(cos(f*x+e
)+1))-48*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+24*ln(2/(cos
(f*x+e)+1))+25*sin(f*x+e)-48*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+9)*
(-c*(sin(f*x+e)-1))^(5/2)*(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*
sin(f*x+e)-2)/(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)
-3*cos(f*x+e)^2-4*sin(f*x+e)-2*cos(f*x+e)+4)/(a*(1+sin(f*x+e)))^(5/2)
```

maxima [B] time = 1.61, size = 1120, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] 1/6*(144*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) - 72*c^(5
/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2) - (46*c^(5/2) + 19
9*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 335*c^(5/2)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 509*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 496*c^
(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 373*c^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 219*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*c^(
5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(a^(5/2) + 4*a^(5/2)*sin(f*x + e)
/(cos(f*x + e) + 1) + 8*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*a^
(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a^(5/2)*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 12*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*a^(5/2
)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*a^(5/2)*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7 + a^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) + (46*c^(5/2) + 1
21*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 149*c^(5/2)*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 179*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 148*c
^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 43*c^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 33*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15*c^(5
/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(a^(5/2) + 4*a^(5/2)*sin(f*x + e)/
(cos(f*x + e) + 1) + 8*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*a^(
5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a^(5/2)*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 12*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*a^(5/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*a^(5/2)*sin(f*x + e)^7/(cos(f*x +
e) + 1)^7 + a^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) - 6*(13*c^(5/2)*si
n(f*x + e)/(cos(f*x + e) + 1) + 39*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 55*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 74*c^(5/2)*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 55*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 + 39*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 13*c^(5/2)*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7)/(a^(5/2) + 4*a^(5/2)*sin(f*x + e)/(cos(f*x + e) +
1) + 8*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*a^(5/2)*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 14*a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 12*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*a^(5/2)*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 + 4*a^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + a^(5
/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.60 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{4c^2\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(3/2)-4*c^2*cos(f*x
+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-2*
c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$-\frac{4c^2\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (-4*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) - (2*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a
^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/
(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{2\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/}}{af(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/}}{af(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/}}{af(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{4c^2\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2c\cos(e+fx)\sqrt{c-c\,\mathrm{s}}}{a^2f\sqrt{a+a\sin(e-}}$$

Mathematica [A] time = 1.13, size = 153, normalized size = 1.07

$$-\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos(2(e+fx))+16\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.\right.}{2f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\mathrm{si}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -1/2*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(7
 + Cos[2*(e + f*x)] + 16*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*(-1 +
 8*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(c\cos\left(fx+e\right)^2\sin\left(fx+e\right)-c\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((c*cos(f*x + e)^2*sin(f*x + e) - c*cos(f*x + e)^2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*co
s(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.40, size = 229, normalized size = 1.60

$$-\frac{\left(8\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-4\sin\left(fx+e\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)+\cos^{2}\left(fx+e\right)-5\sin\left(fx+e\right)+8}{f\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/f*(8*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-4*sin(f*x+e)*
ln(2/(cos(f*x+e)+1))+cos(f*x+e)^2-5*sin(f*x+e)+8*ln(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))-4*ln(2/(cos(f*x+e)+1))-1)*(-c*(sin(f*x+e)-1))^(3/2)*(cos(f
*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(sin(f*x+e)*cos(f*
x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(a*(1+sin(f*x+e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.61 $\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{c\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -c*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)-cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.42, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2739, 2737, 2667, 31}

$$-\frac{c\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]])) - (Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*(a
+ a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(c\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{a\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+}$$

$$= -\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(c\cos(e+fx))\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}$$

$$= -\frac{c\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)\sqrt{c-c\sin(e}{af(a+a\sin(e+fx))}$$

Mathematica [C] time = 0.98, size = 100, normalized size = 1.03

$$\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}\left(-2\log\left(e^{i(e+fx)}+i\right)+\left(ifx-2\log\left(e^{i(e+fx)}+i\right)\right)\sin(e+fx)+ifx-2\right)}{a^2f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]]*(-2 + I*f*x - 2*Log[I + E^(I*(e + f*
x))] + (I*f*x - 2*Log[I + E^(I*(e + f*x))])*Sin[e + f*x]))/(a^2*f*Sqrt[a*(1
 + Sin[e + f*x])])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^{2}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2
/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e))
, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.36, size = 192, normalized size = 1.98

$$\frac{\left(2\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\sin\left(fx+e\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)+2\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-2\sin\left(\right.}{f\left(-1+\cos\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(2*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-sin(f*x+e)*ln(
2/(cos(f*x+e)+1))+2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*sin(f*x+e)
```

```
-ln(2/(cos(f*x+e)+1)))*(-c*(sin(f*x+e)-1))^(1/2)*(cos(f*x+e)^2+sin(f*x+e)*c
os(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(-1+cos(f*x+e)+sin(f*x+e))/(a*(1+sin(f
*x+e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(5/
2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(5/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\cos^{2}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*cos(e + f*x)**2/(a*(sin(e + f*x) + 1))
**(5/2), x)
```

3.62 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$-\frac{\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -(Cos[e + f*x]/(a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.52, size = 80, normalized size = 1.86

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{f(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] -(((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^3)/(f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x]]))
```

fricas [A] time = 0.44, size = 60, normalized size = 1.40

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{a^3cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^3cf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^3*c*f*cos(f*x + e)*s
in(f*x + e) + a^3*c*f*cos(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) +
 c)), x)
```

maple [A] time = 0.36, size = 50, normalized size = 1.16

$$\frac{\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sin\left(fx+e\right)}{f\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(1+sin(f*x+e))*cos(f*x+e)*sin(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(sin(
f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) +
 c)), x)
```

mupad [B] time = 9.26, size = 55, normalized size = 1.28

$$-\frac{2\cos\left(e+f\,x\right)\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}}{a^2\,c\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1/2)),
x)
```

```
[Out] -(2*cos(e + f*x)*(-c*(sin(e + f*x) - 1))^(1/2))/(a^2*c*f*(cos(2*e + 2*f*x)
+ 1)*(a*(sin(e + f*x) + 1))^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

3.63 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=104

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2a^2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{2acf(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*cos(f*x+e)/a/c/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/2*arc
tanh(sin(f*x+e))*cos(f*x+e)/a^2/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))
^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2841, 2743, 2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2a^2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{2acf(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)),
x]
```

```
[Out] -Cos[e + f*x]/(2*a*c*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])
 + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{\int \frac{}{\sqrt{a+a\sin(e}}}{}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{\cos(}{2a^2c\sqrt{a+}}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{\tanh}{2a^2cf\sqrt{a+}}$$

Mathematica [A] time = 0.80, size = 163, normalized size = 1.57

$$\frac{\cos^3(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+\sin(e+fx)\left(\right.\right.}{2cf(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] (Cos[e + f*x]^3*(1 + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2]] + (Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] -
 Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*c*f*(-1 + Sin[
e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.52, size = 313, normalized size = 3.01

$$\left[\frac{\sqrt{ac}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)+\cos\left(fx+e\right)\right)\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin}{\cos\left(fx+e\right)^{3}}\right.}{4\left(a^{3}c^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{3}c^{2}f\cos\left(fx+e\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))*log(-(a*c*cos(f*
x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)*sin(f*x + e) + a^3*c
^2*f*cos(f*x + e)), -1/2*(sqrt(-a*c)*(cos(f*x + e)*sin(f*x + e) + cos(f*x +
 e))*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(
a*c*cos(f*x + e)*sin(f*x + e))) + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c))/(a^3*c^2*f*cos(f*x + e)*sin(f*x + e) + a^3*c^2*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^
(3/2)), x)
```

maple [B] time = 0.35, size = 246, normalized size = 2.37

$$-\frac{\left(\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\sin\left(fx+e\right)+\ln\left(-\frac{-1+\cos}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/f*(sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-sin(f*x+e)*ln
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-sin(f*x+e)+ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)))*(cos(f*x+e)
^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)*(cos(f*x+e)^2+sin(f*x+e
)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^
(5/2)/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^
(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2)),
x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.64 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=152

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2a^2c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2a^2cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{c}{2acf(a\sin(e+fx}$$

```
[Out] -1/2*cos(f*x+e)/a/c/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+1/2*cos
(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*arctanh(s
in(f*x+e))*cos(f*x+e)/a^2/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/
2)
```

Rubi [A] time = 0.54, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2841, 2743, 2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2a^2c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2a^2cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{c}{2acf(a\sin(e+fx}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)),
x]
```

```
[Out] -Cos[e + f*x]/(2*a*c*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2
)) + Cos[e + f*x]/(2*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(3/2)) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a^2*c^2*f*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{\int \frac{}{\sqrt{a+a\sin}}}{}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{2a^2cf\sqrt{a}}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{2a^2cf\sqrt{a}}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{2a^2cf\sqrt{a}}$$

Mathematica [A] time = 0.88, size = 163, normalized size = 1.07

$$\frac{\sec(e+fx)\left(2\sin(e+fx)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{4a^2c^2f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] (Sec[e + f*x]*(-Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x
)/2] + Sin[(e + f*x)/2]] + Cos[2*(e + f*x)]*(-Log[Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*Sin[e + f*x]))/
(4*a^2*c^2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.51, size = 262, normalized size = 1.72

$$\left[\frac{\sqrt{ac}\cos\left(fx+e\right)^{3}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)+2\,\sqrt{a\sin\left(fx+e\right)-}}{4\,a^{3}c^{3}f\cos\left(fx+e\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*cos(f*x + e)^3*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e
) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x
+ e))/cos(f*x + e)^3) + 2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)*sin(f*x + e))/(a^3*c^3*f*cos(f*x + e)^3), -1/2*(sqrt(-a*c)*arctan(sqrt(-a
*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*si
n(f*x + e)))*cos(f*x + e)^3 - sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*sin(f*x + e))/(a^3*c^3*f*cos(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^
(5/2)), x)
```

maple [A] time = 0.36, size = 196, normalized size = 1.29

$$-\frac{\left(\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\sin\left(fx+e\right)\right)\left(\cos^{2}\right.}{4f\left(-1+\cos\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/4/f*(cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)^
2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-sin(f*x+e))*(cos(f*x+e)^2-sin(
f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)*(cos(f*x+e)^2+sin(f*x+e)*cos(f
*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(5/2)/(
-c*(sin(f*x+e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^
(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(e+f\,x\right)}^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2)),
x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.65 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=114

$$\frac{c^2 2^{n+\frac{3}{2}}\cos^3(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2}(2m+3),\frac{1}{2}(-2n-1);\frac{1}{2}(\ldots\right.}{f(2m+3)}$$

```
[Out] 2^(3/2+n)*c^2*cos(f*x+e)^3*hypergeom([-1/2-n, 3/2+m],[5/2+m],1/2+1/2*sin(f*
x+e))*(1-sin(f*x+e))^(1/2-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n)/f/(
3+2*m)
```

Rubi [A] time = 0.30, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2841, 2745, 2689, 70, 69}

$$\frac{c^2 2^{n+\frac{3}{2}}\cos^3(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2}(2m+3),\frac{1}{2}(-2n-1);\frac{1}{2}(\ldots\right.}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(3/2 + n)*c^2*Cos[e + f*x]^3*Hypergeometric2F1[(3 + 2*m)/2, (-1 - 2*n)/2
, (5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 - n)*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 + n))/(f*(3 + 2*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{1+n}\,dx}{ac}$$

$$= \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+\right.$$

$$= \frac{\left(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(c-\right.}{}$$

$$= \frac{\left(2^{\frac{1}{2}+n}c^3\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))\right.}{}$$

$$= \frac{2^{\frac{3}{2}+n}c^2\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(3+2m),\frac{1}{2}(-1-2n);\frac{1}{2}\right.}{}$$

Mathematica [C] time = 14.26, size = 3426, normalized size = 30.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (-64*(AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)
/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*AppellF1[1/2 + n, -2*m, 2*(2 + m +
n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*A
ppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2,
-Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2 + n, -2*m, 5 + 2*(m + n), 3/2
 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + P
i/2 - f*x)/2]^(2 + 2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*(m + n))*Sin[(-e +
Pi/2 - f*x)/2]^(2 + 2*n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*Tan[
(-e + Pi/2 - f*x)/4])/(f*(1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(
(16*m*(AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x
)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*AppellF1[1/2 + n, -2*m, 2*(2 + m +
 n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*
AppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2,
 -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2 + n, -2*m, 5 + 2*(m + n), 3/
2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e +
Pi/2 - f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*(m + n))*Sin[(-e +
 Pi/2 - f*x)/2]^(2*n)*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)
/4]^2)^(-1 - 2*m))/(1 + 2*n) + (4*(AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3
/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*Appell
F1[1/2 + n, -2*m, 2*(2 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[
(-e + Pi/2 - f*x)/4]^2] - 5*AppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n,
 Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2
+ n, -2*m, 5 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + P
i/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x)/4]^
2)^(1 + 2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(2*n))/((1 + 2*n)*(1 - Tan[(-e
+ Pi/2 - f*x)/4]^2)^(2*m)) + (16*n*(AppellF1[1/2 + n, -2*m, 2*(1 + m + n),
3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*Appel
lF1[1/2 + n, -2*m, 2*(2 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2] - 5*AppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n
, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2
 + n, -2*m, 5 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*(Sec[(-e + Pi/2 - f*x
)/4]^2)^(2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(-1 + 2*n)*Tan[(-e + Pi/2 - f*
x)/4])/((1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (16*m*(AppellF1
[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2] + 8*AppellF1[1/2 + n, -2*m, 2*(2 + m + n), 3/2 + n, T
an[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*AppellF1[1/2 +
```

```
n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2] - 4*AppellF1[1/2 + n, -2*m, 5 + 2*(m + n), 3/2 + n, Tan[(-e
+ Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^
(-1 + 2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*(m + n))*Sin[(-e + Pi/2 - f*x)/2
]^(1 + 2*n)*Tan[(-e + Pi/2 - f*x)/4])/((1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)
/4]^2)^(2*m)) + (16*(m + n)*(AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n
, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*AppellF1[1/2
 + n, -2*m, 2*(2 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2] - 5*AppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(
-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2 + n, -
2*m, 5 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*
(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(2*n)*Tan[(-e + Pi/2 - f*x)/4]^2)/((1 + 2
*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (16*Cos[(-e + Pi/2 - f*x)/2]^
(2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(2*
n)*Tan[(-e + Pi/2 - f*x)/4]*(-((m*(1/2 + n)*AppellF1[3/2 + n, 1 - 2*m, 2*(1
 + m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2
]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 + n)) - ((1/2 +
 n)*(1 + m + n)*AppellF1[3/2 + n, -2*m, 1 + 2*(1 + m + n), 5/2 + n, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^
2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 + n) - 5*(-((m*(1/2 + n)*AppellF1[3/2 + n,
 1 - 2*m, 3 + 2*(m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi
/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 +
 n)) - ((1/2 + n)*(3 + 2*(m + n))*AppellF1[3/2 + n, -2*m, 4 + 2*(m + n), 5/
2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + P
i/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2 + n))) - 4*(-((m*(1/2 + n
)*AppellF1[3/2 + n, 1 - 2*m, 5 + 2*(m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/
4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/
2 - f*x)/4])/(3/2 + n)) - ((1/2 + n)*(5 + 2*(m + n))*AppellF1[3/2 + n, -2*m
, 6 + 2*(m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x
)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2 + n)))
 + 8*(-((m*(1/2 + n)*AppellF1[3/2 + n, 1 - 2*m, 2*(2 + m + n), 5/2 + n, Tan
[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)
/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 + n)) - ((1/2 + n)*(2 + m + n)*AppellF
1[3/2 + n, -2*m, 1 + 2*(2 + m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -T
an[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)
/4])/(3/2 + n))))/((1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="
```

```
fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin \left(fx + e\right) + a\right)^{m} \left(-c \sin \left(fx + e\right) + c\right)^{n} \cos \left(fx + e\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

maple [F] time = 4.20, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx + e\right)\right) \left(a + a \sin \left(fx + e\right)\right)^{m} \left(c - c \sin \left(fx + e\right)\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin \left(fx + e\right) + a\right)^{m} \left(-c \sin \left(fx + e\right) + c\right)^{n} \cos \left(fx + e\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos \left(e + f\,x\right)^{2} \left(a + a\,\sin \left(e + f\,x\right)\right)^{m} \left(c - c\,\sin \left(e + f\,x\right)\right)^{n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.66 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=86

$$-\frac{a^4c^3 2^{m+\frac{3}{2}}\cos^9(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{9}{2},-m-\frac{1}{2};\frac{11}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{9f}$$

```
[Out] -1/9*2^(3/2+m)*a^4*c^3*cos(f*x+e)^9*hypergeom([9/2, -1/2-m],[11/2],1/2-1/2*
sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-4+m)/f
```

Rubi [A] time = 0.21, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2840, 2689, 70, 69}

$$-\frac{a^4c^3 2^{m+\frac{3}{2}}\cos^9(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{9}{2},-m-\frac{1}{2};\frac{11}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -(2^(3/2 + m)*a^4*c^3*Cos[e + f*x]^9*Hypergeometric2F1[9/2, -1/2 - m, 11/2,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(
-4 + m))/(9*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx = \left(a^3c^3\right)\int \cos^8(e+fx)(a+a\sin(e+fx))^{-3+m}\,dx$$

$$= \frac{\left(a^5c^3\cos^9(e+fx)\right)\text{Subst}\left(\int (a-ax)^{7/2}(a+ax)^{\frac{1}{2}+}\right.}{f(a-a\sin(e+fx))^{9/2}(a+a\sin(e+}$$

$$= \frac{\left(2^{\frac{1}{2}+m}a^5c^3\cos^9(e+fx)(a+a\sin(e+fx))^{-4+m}\left(\frac{a+}{}\right.\right.}{f(}$$

$$= -\frac{2^{\frac{3}{2}+m}a^4c^3\cos^9(e+fx)\,{}_2F_1\left(\frac{9}{2},-\frac{1}{2}-m;\frac{11}{2};\frac{1}{2}(1-\text{s}\right.}{}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]
[Out] $Aborted
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,c^3\cos\left(fx+e\right)^4-4\,c^3\cos\left(fx+e\right)^2-\left(c^3\cos\left(fx+e\right)^4-4\,c^3\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)\right)\left(a\sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="
fricas")
```

```
[Out] integral(-(3*c^3*cos(f*x + e)^4 - 4*c^3*cos(f*x + e)^2 - (c^3*cos(f*x + e)^
4 - 4*c^3*cos(f*x + e)^2)*sin(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(c \sin\left(fx+e\right)-c\right)^{3}\left(a \sin\left(fx+e\right)+a\right)^{m} \cos\left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="
giac")
```

```
[Out] integrate(-(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

maple [F] time = 8.86, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(a+a \sin\left(fx+e\right)\right)^{m}\left(c-c \sin\left(fx+e\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^{2}\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}\left(c-c\,\sin\left(e+f\,x\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3,x)
```

```
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.67 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=86

$$-\frac{a^3c^2 2^{m+\frac{3}{2}}\cos^7(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{7}{2},-m-\frac{1}{2};\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{7f}$$

```
[Out] -1/7*2^(3/2+m)*a^3*c^2*cos(f*x+e)^7*hypergeom([7/2, -1/2-m],[9/2],1/2-1/2*s
in(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-3+m)/f
```

Rubi [A] time = 0.21, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2840, 2689, 70, 69}

$$-\frac{a^3c^2 2^{m+\frac{3}{2}}\cos^7(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{7}{2},-m-\frac{1}{2};\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -(2^(3/2 + m)*a^3*c^2*Cos[e + f*x]^7*Hypergeometric2F1[7/2, -1/2 - m, 9/2,
(1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(-
3 + m))/(7*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^6(e+fx)(a+a\sin(e+fx))^{-2+m}\,dx$$

$$= \frac{\left(a^4c^2\cos^7(e+fx)\right)\text{Subst}\left(\int (a-ax)^{5/2}(a+ax)^{\frac{1}{2}+m}\right.}{f(a-a\sin(e+fx))^{7/2}(a+a\sin(e+}$$

$$= \frac{\left(2^{\frac{1}{2}+m}a^4c^2\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}\left(\frac{a+}{}\right.\right.}{f(}$$

$$= -\frac{2^{\frac{3}{2}+m}a^3c^2\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2},-\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\text{si}\right.}{}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
[Out] $Aborted
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c^2\cos\left(fx+e\right)^4+2\,c^2\cos\left(fx+e\right)^2\sin\left(fx+e\right)-2\,c^2\cos\left(fx+e\right)^2\right)\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] integral(-(c^2*cos(f*x + e)^4 + 2*c^2*cos(f*x + e)^2*sin(f*x + e) - 2*c^2*c
os(f*x + e)^2)*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(fx+e)-c)^2(a\sin(fx+e)+a)^m\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

maple [F] time = 7.57, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+a\sin(fx+e))^m(c-c\sin(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(fx+e)-c)^2(a\sin(fx+e)+a)^m\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] integrate((c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+fx)^2\,(a+a\,\sin(e+fx))^m\,(c-c\,\sin(e+fx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2,x)
```

```
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \left(a\sin\left(e+fx\right)+a\right)^m\cos^2\left(e+fx\right)dx+\int\left(-2\left(a\sin\left(e+fx\right)+a\right)^m\sin\left(e+fx\right)\cos^2\left(e+fx\right)\right)dx+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**2,x)
```

```
[Out] c**2*(Integral((a*sin(e + f*x) + a)**m*cos(e + f*x)**2, x) + Integral(-2*(a
*sin(e + f*x) + a)**m*sin(e + f*x)*cos(e + f*x)**2, x) + Integral((a*sin(e
+ f*x) + a)**m*sin(e + f*x)**2*cos(e + f*x)**2, x))
```

3.68 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=84

$$-\frac{a^2c2^{m+\frac{3}{2}}\cos^5(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{1}{2};\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f}$$

```
[Out] -1/5*2^(3/2+m)*a^2*c*cos(f*x+e)^5*hypergeom([5/2, -1/2-m],[7/2],1/2-1/2*sin
(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-2+m)/f
```

Rubi [A] time = 0.16, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2840, 2689, 70, 69}

$$-\frac{a^2c2^{m+\frac{3}{2}}\cos^5(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{1}{2};\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
```

```
[Out] -(2^(3/2 + m)*a^2*c*Cos[e + f*x]^5*Hypergeometric2F1[5/2, -1/2 - m, 7/2, (1
 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(-2
+ m))/(5*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx = (ac)\int \cos^4(e+fx)(a+a\sin(e+fx))^{-1+m}\,dx$$

$$= \frac{\left(a^3c\cos^5(e+fx)\right)\text{Subst}\left(\int (a-ax)^{3/2}(a+ax)^{\frac{1}{2}+m}\,a\right.}{f(a-a\sin(e+fx))^{5/2}(a+a\sin(e+fx$$

$$= \frac{\left(2^{\frac{1}{2}+m}a^3c\cos^5(e+fx)(a+a\sin(e+fx))^{-2+m}\left(\frac{a+a\,s}{}\right.\right.}{f(a}$$

$$= -\frac{2^{\frac{3}{2}+m}a^2c\cos^5(e+fx)\,{}_2F_1\left(\frac{5}{2},-\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(\right.}{}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
[Out] $Aborted
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c\cos\left(fx+e\right)^2\sin\left(fx+e\right)-c\cos\left(fx+e\right)^2\right)\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral(-(c*cos(f*x + e)^2*sin(f*x + e) - c*cos(f*x + e)^2)*(a*sin(f*x + e
) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(-(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

maple [F] time = 3.15, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -integrate((c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(e+f\,x\right)}^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)),x)
```

```
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\left(-\left(a\sin\left(e+fx\right)+a\right)^{m}\cos^{2}\left(e+fx\right)\right)dx+\int\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)\cos^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e)),x)
```

```
[Out] -c*(Integral(-(a*sin(e + f*x) + a)**m*cos(e + f*x)**2, x) + Integral((a*sin
(e + f*x) + a)**m*sin(e + f*x)*cos(e + f*x)**2, x))
```

3.69 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=81

$$-\frac{a2^{m+\frac{3}{2}}\cos^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f}$$

```
[Out] -1/3*2^(3/2+m)*a*cos(f*x+e)^3*hypergeom([3/2, -1/2-m],[5/2],1/2-1/2*sin(f*x
+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-1+m)/f
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2689, 70, 69}

$$-\frac{a2^{m+\frac{3}{2}}\cos^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(2^(3/2 + m)*a*Cos[e + f*x]^3*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 - S
in[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(-1 + m)
)/(3*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
```

```
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos^3(e+fx)\right)\operatorname{Subst}\left(\int \sqrt{a-ax}\,(a+ax)^{\frac{1}{2}+m}\,dx,x,\sin(e+fx)\right)}{f(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}$$

$$= \frac{\left(2^{\frac{1}{2}+m}a^2\cos^3(e+fx)(a+a\sin(e+fx))^{-1+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{-\frac{1}{2}-m}\right)\operatorname{Subs}}{f(a-a\sin(e+fx))^{3/2}}$$

$$= -\frac{2^{\frac{3}{2}+m}a\cos^3(e+fx)\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e}{3f}$$

Mathematica [A] time = 0.11, size = 78, normalized size = 0.96

$$-\frac{2^{m+\frac{3}{2}}\cos^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{3}{2}}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -1/3*(2^(3/2 + m)*Cos[e + f*x]^3*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 -
 Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-3/2 - m)*(a*(1 + Sin[e + f*x]))^m)/f
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

maple [F] time = 1.65, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^m\cos^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cos(e + f*x)**2, x)
```

3.70 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=77

$$-\frac{2^{m+\frac{3}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m-\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

```
[Out] -2^(3/2+m)*cos(f*x+e)*hypergeom([1/2, -1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+
sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/c/f
```

Rubi [A] time = 0.16, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2840, 2652, 2651}

$$-\frac{2^{m+\frac{3}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m-\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x]),x]
```

```
[Out] -((2^(3/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sin[
e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(c*f))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
```

```
m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}\,dx}{ac}$$
$$= \frac{\left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (1+\sin(e+fx))^{1+m}\,dx}{c}$$
$$= -\frac{2^{\frac{3}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+f$$
$$cf$$

Mathematica [C] time = 19.68, size = 6442, normalized size = 83.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x]),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c), x)
```

maple [F] time = 4.02, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}}{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^{2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)),x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a\sin\left(e+fx\right)+a\right)^{m}\cos^{2}\left(e+fx\right)}{\sin\left(e+fx\right)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e)),x)
[Out] -Integral((a*sin(e + f*x) + a)**m*cos(e + f*x)**2/(sin(e + f*x) - 1), x)/c
```

3.71 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=81

$$\frac{2^{m+\frac{3}{2}}\sec(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{1}{2},-m-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{ac^2f}$$

```
[Out] 2^(3/2+m)*hypergeom([-1/2, -1/2-m],[1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)*(1+
sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(1+m)/a/c^2/f
```

Rubi [A] time = 0.21, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2840, 2689, 70, 69}

$$\frac{2^{m+\frac{3}{2}}\sec(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{1}{2},-m-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{ac^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(3/2 + m)*Hypergeometric2F1[-1/2, -1/2 - m, 1/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(1 + m))/(a*
c^2*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx = \frac{\int \sec^2(e+fx)(a+a\sin(e+fx))^{2+m}\,dx}{a^2c^2}$$

$$= \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\mathrm{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}}{(a-ax)^{3/2}}\right.}{c^2f}$$

$$= \frac{\left(2^{\frac{1}{2}+m}\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(a+a\sin(e+fx))^{1+m}\left(\frac{a+a\sin(e+f\right.\right.}{a}}{c^2f}$$

$$= \frac{2^{\frac{3}{2}+m}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e+f}{ac^2f}$$

Mathematica [A] time = 0.21, size = 88, normalized size = 1.09

$$\frac{2^{m+\frac{3}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(-\frac{1}{2},-m-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{c^2f(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(3/2 + m)*Cos[e + f*x]*Hypergeometric2F1[-1/2, -1/2 - m, 1/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a*(1 + Sin[e + f*x]))^m)/(c^2*f*
(1 - Sin[e + f*x]))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c^2*cos(f*x + e)^2 + 2*c^2
*sin(f*x + e) - 2*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(c\sin\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c)^2, x)
```

maple [F] time = 3.33, size = 0, normalized size = 0.00

$$\int\frac{\left(\cos^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c-c\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(c\sin\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2,x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(a\sin\left(e+fx\right)+a\right)^m\cos^2\left(e+fx\right)}{\sin^2\left(e+fx\right)-2\sin\left(e+fx\right)+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**m*cos(e + f*x)**2/(sin(e + f*x)**2 - 2*sin(e
 + f*x) + 1), x)/c**2
```

3.72 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=86

$$\frac{2^{m+\frac{3}{2}}\sec^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{3}{2},-m-\frac{1}{2};-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3a^2c^3f}$$

```
[Out] 1/3*2^(3/2+m)*hypergeom([-3/2, -1/2-m],[-1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e
)^3*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(2+m)/a^2/c^3/f
```

Rubi [A] time = 0.21, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2840, 2689, 70, 69}

$$\frac{2^{m+\frac{3}{2}}\sec^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{3}{2},-m-\frac{1}{2};-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3a^2c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(3/2 + m)*Hypergeometric2F1[-3/2, -1/2 - m, -1/2, (1 - Sin[e + f*x])/2]*
Sec[e + f*x]^3*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(2 + m))/
(3*a^2*c^3*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx = \frac{\int \sec^4(e+fx)(a+a\sin(e+fx))^{3+m}\,dx}{a^3c^3}$$

$$= \frac{\left(\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+ax}{(a-a}\right.}{ac^3f}$$

$$= \frac{\left(2^{\frac{1}{2}+m}\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{2+m}\left(\frac{a+a\sin(e}{a}\right.\right.}{ac^3f}$$

$$= \frac{2^{\frac{3}{2}+m}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^3(e+fx)(1+\sin(e+}{3a^2c^3f}$$

Mathematica [A] time = 0.15, size = 91, normalized size = 1.06

$$\frac{2^{m+\frac{3}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(-\frac{3}{2},-m-\frac{1}{2};-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3c^3f(1-\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(3/2 + m)*Cos[e + f*x]*Hypergeometric2F1[-3/2, -1/2 - m, -1/2, (1 - Sin[
e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a*(1 + Sin[e + f*x]))^m)/(3*c^3
*f*(1 - Sin[e + f*x])^2)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="
fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(3*c^3*cos(f*x + e)^2 - 4*c
^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(c\sin\left(fx+e\right)-c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="
giac")
```

```
[Out] integrate(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c)^3, x)
```

maple [F] time = 3.02, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c-c\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(c\sin\left(fx+e\right)-c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="
maxima")
```

```
[Out] -integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3,x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.73 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=244

$$\frac{768c^3\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+7)(2m+9)\left(4m^2+16m+15\right)\sqrt{c-c\sin(e+fx)}}+\frac{192c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)}{af(2m+9)\left(4m^2+24m+35\right)}$$

```
[Out] 24*c*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(3/2)/a/f/(4*m^2+32
*m+63)+2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(5/2)/a/f/(9+2*
m)+768*c^3*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(4*m^2+16*m+15)/(4*m^2+32*
m+63)/(c-c*sin(f*x+e))^(1/2)+192*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c
*sin(f*x+e))^(1/2)/a/f/(8*m^3+84*m^2+286*m+315)
```

Rubi [A] time = 0.62, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2841, 2740, 2738}

$$\frac{192c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^{m+1}}{af(2m+9)\left(4m^2+24m+35\right)}+\frac{768c^3\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+7)(2m+9)\left(4m^2+16m+15\right)\sqrt{c-c\sin(}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (768*c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(7 + 2*m)*(9 + 2*m
)*(15 + 16*m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) + (192*c^2*Cos[e + f*x]*(a
+ a*Sin[e + f*x])^(1 + m)*Sqrt[c - c*Sin[e + f*x]])/(a*f*(9 + 2*m)*(35 + 24
*m + 4*m^2)) + (24*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e
 + f*x])^(3/2))/(a*f*(63 + 32*m + 4*m^2)) + (2*Cos[e + f*x]*(a + a*Sin[e +
f*x])^(1 + m)*(c - c*Sin[e + f*x])^(5/2))/(a*f*(9 + 2*m))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
```

```
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$
$$= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+f}{af(9+2m)}$$
$$= \frac{24c\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+}{af\left(63+32m+4m^2\right)}$$
$$= \frac{192c^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(}}{af(5+2m)\left(63+32m+4m^2\right)}$$
$$= \frac{768c^3\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)(5+2m)\left(63+32m+4m^2\right)\sqrt{c-c\sin(}}$$

Mathematica [C] time = 6.56, size = 695, normalized size = 2.85

$$\frac{(c-c\sin(e+fx))^{5/2}(a(\sin(e+fx)+1))^m\left(\frac{(8m^3+108m^2+590m+2205)\left(\left(\frac{3}{8}-\frac{3i}{8}\right)\sin\left(\frac{1}{2}(e+fx)\right)+\left(\frac{3}{8}+\frac{3i}{8}\right)\cos\left(\frac{1}{2}(e+fx)\right)\right)}{(2m+3)(2m+5)(2m+7)(2m+9)}+\frac{(8m^3+108m^2}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^(5/2)*(((2205 + 590*m + 108*
m^2 + 8*m^3)*((3/8 + (3*I)/8)*Cos[(e + f*x)/2] + (3/8 - (3*I)/8)*Sin[(e + f
*x)/2]))/((3 + 2*m)*(5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((2205 + 590*m + 108*m
^2 + 8*m^3)*((3/8 - (3*I)/8)*Cos[(e + f*x)/2] + (3/8 + (3*I)/8)*Sin[(e + f*
x)/2]))/((3 + 2*m)*(5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((191*m + 48*m^2 + 4*m^
3)*((1 - I)*Cos[(3*(e + f*x))/2] - (1 + I)*Sin[(3*(e + f*x))/2]))/((3 + 2*m
)*(5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((191*m + 48*m^2 + 4*m^3)*((1 + I)*Cos[(
3*(e + f*x))/2] - (1 - I)*Sin[(3*(e + f*x))/2]))/((3 + 2*m)*(5 + 2*m)*(7 +
2*m)*(9 + 2*m)) + ((21 + 2*m)*((3/2 + (3*I)/2)*Cos[(5*(e + f*x))/2] + (3/2
- (3*I)/2)*Sin[(5*(e + f*x))/2]))/((5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((21 +
2*m)*((3/2 - (3*I)/2)*Cos[(5*(e + f*x))/2] + (3/2 + (3*I)/2)*Sin[(5*(e + f*
x))/2]))/((5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((15 + 2*m)*((3/16 - (3*I)/16)*C
os[(7*(e + f*x))/2] - (3/16 + (3*I)/16)*Sin[(7*(e + f*x))/2]))/((7 + 2*m)*(
9 + 2*m)) + ((15 + 2*m)*((3/16 + (3*I)/16)*Cos[(7*(e + f*x))/2] - (3/16 - (
3*I)/16)*Sin[(7*(e + f*x))/2]))/((7 + 2*m)*(9 + 2*m)) + ((-1/16 + I/16)*Cos
[(9*(e + f*x))/2] - (1/16 + I/16)*Sin[(9*(e + f*x))/2])/(9 + 2*m) + ((-1/16
 - I/16)*Cos[(9*(e + f*x))/2] - (1/16 - I/16)*Sin[(9*(e + f*x))/2])/(9 + 2*
m)))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.49, size = 395, normalized size = 1.62

$$-\frac{2\left(\left(8\,c^2m^3+60\,c^2m^2+142\,c^2m+105\,c^2\right)\cos\left(fx+e\right)^5-\left(8\,c^2m^3+108\,c^2m^2+334\,c^2m+285\,c^2\right)\cos\left(fx+e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorit
hm="fricas")
```

```
[Out] -2*((8*c^2*m^3 + 60*c^2*m^2 + 142*c^2*m + 105*c^2)*cos(f*x + e)^5 - (8*c^2*
m^3 + 108*c^2*m^2 + 334*c^2*m + 285*c^2)*cos(f*x + e)^4 - 2*(8*c^2*m^3 + 84
*c^2*m^2 + 334*c^2*m + 339*c^2)*cos(f*x + e)^3 - 384*c^2*cos(f*x + e) - 96*
(2*c^2*m - c^2)*cos(f*x + e)^2 - 768*c^2 + ((8*c^2*m^3 + 60*c^2*m^2 + 142*c
^2*m + 105*c^2)*cos(f*x + e)^4 + 2*(8*c^2*m^3 + 84*c^2*m^2 + 238*c^2*m + 19
5*c^2)*cos(f*x + e)^3 - 384*c^2*cos(f*x + e) - 96*(2*c^2*m + 3*c^2)*cos(f*x
 + e)^2 - 768*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e)
+ a)^m/(16*f*m^4 + 192*f*m^3 + 824*f*m^2 + 1488*f*m + (16*f*m^4 + 192*f*m^3
 + 824*f*m^2 + 1488*f*m + 945*f)*cos(f*x + e) - (16*f*m^4 + 192*f*m^3 + 824
*f*m^2 + 1488*f*m + 945*f)*sin(f*x + e) + 945*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2
, x)
```

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

maxima [B] time = 0.66, size = 558, normalized size = 2.29

$$-\frac{2\left(\left(8\,m^3+108\,m^2+526\,m+957\right)a^m c^{\frac{5}{2}}-\frac{3\left(8\,m^3+76\,m^2+142\,m-315\right)a^m c^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{24\left(4\,m^2+16\,m-81\right)a^m c^{\frac{5}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorit
hm="maxima")
```

```
[Out] -2*((8*m^3 + 108*m^2 + 526*m + 957)*a^m*c^(5/2) - 3*(8*m^3 + 76*m^2 + 142*m
 - 315)*a^m*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) - 24*(4*m^2 + 16*m - 81
)*a^m*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 16*(4*m^3 + 36*m^2 + 95
*m + 315)*a^m*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*(8*m^3 + 60*m
^2 + 206*m - 567)*a^m*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 6*(8*m^
3 + 60*m^2 + 206*m - 567)*a^m*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 16*(4*m^3 + 36*m^2 + 95*m + 315)*a^m*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 - 24*(4*m^2 + 16*m - 81)*a^m*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 3*(8*m^3 + 76*m^2 + 142*m - 315)*a^m*c^(5/2)*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 + (8*m^3 + 108*m^2 + 526*m + 957)*a^m*c^(5/2)*sin(f*x + e)^9/(
cos(f*x + e) + 1)^9)*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((16*m^4 + 192*m^3 + 824*m^2 +
1488*m + 2*(16*m^4 + 192*m^3 + 824*m^2 + 1488*m + 945)*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + (16*m^4 + 192*m^3 + 824*m^2 + 1488*m + 945)*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 945)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(
5/2))
```

mupad [B] time = 14.97, size = 1060, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
3*c^2*exp(e*7i + f*x*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^m*(48*m + 4*m^2 + 63))/(f*(m*1488i + m^2*824i + m^3*192i +
m^4*16i + 945i)) - (c^2*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2))^m*(m*142i + m^2*60i + m^3*8i + 105i))/(8*f*(m*1488i + m^2*82
4i + m^3*192i + m^4*16i + 945i)) + (3*c^2*exp(e*2i + f*x*2i)*(a + a*((exp(-
 e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*48i + m^2*4i + 63i
))/(f*(m*1488i + m^2*824i + m^3*192i + m^4*16i + 945i)) - (c^2*exp(e*9i + f
*x*9i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*
(142*m + 60*m^2 + 8*m^3 + 105))/(8*f*(m*1488i + m^2*824i + m^3*192i + m^4*1
6i + 945i)) + (3*c^2*exp(e*1i + f*x*1i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2
 - (exp(e*1i + f*x*1i)*1i)/2))^m*(270*m + 92*m^2 + 8*m^3 + 225))/(8*f*(m*14
88i + m^2*824i + m^3*192i + m^4*16i + 945i)) + (3*c^2*exp(e*8i + f*x*8i)*(a
 + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*270i +
 m^2*92i + m^3*8i + 225i))/(8*f*(m*1488i + m^2*824i + m^3*192i + m^4*16i +
945i)) + (3*c^2*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (e
xp(e*1i + f*x*1i)*1i)/2))^m*(590*m + 108*m^2 + 8*m^3 + 2205))/(4*f*(m*1488i
 + m^2*824i + m^3*192i + m^4*16i + 945i)) + (3*c^2*exp(e*4i + f*x*4i)*(a +
a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*590i + m^
2*108i + m^3*8i + 2205i))/(4*f*(m*1488i + m^2*824i + m^3*192i + m^4*16i + 9
45i)) + (2*c^2*m*exp(e*3i + f*x*3i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (
exp(e*1i + f*x*1i)*1i)/2))^m*(48*m + 4*m^2 + 191))/(f*(m*1488i + m^2*824i +
 m^3*192i + m^4*16i + 945i)) + (2*c^2*m*exp(e*6i + f*x*6i)*(a + a*((exp(- e
*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*48i + m^2*4i + 191i)
)/(f*(m*1488i + m^2*824i + m^3*192i + m^4*16i + 945i))))/(exp(e*5i + f*x*5i
) + (exp(e*4i + f*x*4i)*(1488*m + 824*m^2 + 192*m^3 + 16*m^4 + 945))/(m*148
8i + m^2*824i + m^3*192i + m^4*16i + 945i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.74 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{64c^2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+7)\left(4m^2+16m+15\right)\sqrt{c-c\sin(e+fx)}}+\frac{16c\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^{m+1}}{af\left(4m^2+24m+35\right)}+\frac{2c}{}$$

```
[Out] 2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(3/2)/a/f/(7+2*m)+64*c
^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(8*m^3+60*m^2+142*m+105)/(c-c*sin(
f*x+e))^(1/2)+16*c*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(1/2)
/a/f/(4*m^2+24*m+35)
```

Rubi [A] time = 0.47, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2841, 2740, 2738}

$$\frac{64c^2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+7)\left(4m^2+16m+15\right)\sqrt{c-c\sin(e+fx)}}+\frac{16c\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^{m+1}}{af\left(4m^2+24m+35\right)}+\frac{2c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (64*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(7 + 2*m)*(15 + 16*
m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) + (16*c*Cos[e + f*x]*(a + a*Sin[e + f*
x])^(1 + m)*Sqrt[c - c*Sin[e + f*x]])/(a*f*(35 + 24*m + 4*m^2)) + (2*Cos[e
+ f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(3/2))/(a*f*(7 + 2
*m))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
```

```
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$

$$= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+}{af(7+2m)}$$

$$= \frac{16c\cos(e+fx)(a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e}}{af\left(35+24m+4m^2\right)}$$

$$= \frac{64c^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\left(35+24m+4m^2\right)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 3.10, size = 149, normalized size = 0.87

$$-\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(a(\sin(e+fx)+1))^m\left(4\left(4m^2+24m+27\right)\sin(e+f\right.}{f(2m+3)(2m+5)(2m+7)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2),
x]

[Out] -((c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^m*Sqrt[
c - c*Sin[e + f*x]]*(-157 - 80*m - 12*m^2 + (15 + 16*m + 4*m^2)*Cos[2*(e +
f*x)] + 4*(27 + 24*m + 4*m^2)*Sin[e + f*x]))/(f*(3 + 2*m)*(5 + 2*m)*(7 + 2*
m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))
```

fricas [A] time = 0.48, size = 243, normalized size = 1.41

$$\frac{2\left(\left(4\,cm^2+16\,cm+15\,c\right)\cos\left(fx+e\right)^4+\left(4\,cm^2+32\,cm+39\,c\right)\cos\left(fx+e\right)^3+8\,(2\,cm-c)\cos\left(fx+e\right)^2+32}{8\,fm^3+60\,fm^2+142\,fm+\left(8\,fm^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorit
hm="fricas")
```

```
[Out] 2*((4*c*m^2 + 16*c*m + 15*c)*cos(f*x + e)^4 + (4*c*m^2 + 32*c*m + 39*c)*cos
(f*x + e)^3 + 8*(2*c*m - c)*cos(f*x + e)^2 + 32*c*cos(f*x + e) - ((4*c*m^2
+ 16*c*m + 15*c)*cos(f*x + e)^3 - 8*(2*c*m + 3*c)*cos(f*x + e)^2 - 32*c*cos
(f*x + e) - 64*c)*sin(f*x + e) + 64*c)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x
 + e) + a)^m/(8*f*m^3 + 60*f*m^2 + 142*f*m + (8*f*m^3 + 60*f*m^2 + 142*f*m
+ 105*f)*cos(f*x + e) - (8*f*m^3 + 60*f*m^2 + 142*f*m + 105*f)*sin(f*x + e)
 + 105*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2
, x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int\left(\cos^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

maxima [B] time = 0.52, size = 423, normalized size = 2.46

$$-\frac{2\left(\left(4\,m^2+32\,m+71\right)a^m c^{\frac{3}{2}}-\frac{\left(4\,m^2-105\right)a^m c^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\left(12\,m^2+64\,m-91\right)a^m c^{\frac{3}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{\left(12\,m^2+32\,m+245\right)a^m c^{\frac{3}{2}}\sin\left(fx+e\right.}{\left(\cos\left(fx+e\right)+1\right)^3}}{\left(8\,m^3+60\,m^2+142\,m+\frac{2\left(8\,m^3+60\right.}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorit
hm="maxima")
```

```
[Out] -2*((4*m^2 + 32*m + 71)*a^m*c^(3/2) - (4*m^2 - 105)*a^m*c^(3/2)*sin(f*x + e
)/(cos(f*x + e) + 1) - (12*m^2 + 64*m - 91)*a^m*c^(3/2)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + (12*m^2 + 32*m + 245)*a^m*c^(3/2)*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + (12*m^2 + 32*m + 245)*a^m*c^(3/2)*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - (12*m^2 + 64*m - 91)*a^m*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 - (4*m^2 - 105)*a^m*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + (4
*m^2 + 32*m + 71)*a^m*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)*e^(2*m*l
og(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 1))/((8*m^3 + 60*m^2 + 142*m + 2*(8*m^3 + 60*m^2 + 142*m + 105)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + (8*m^3 + 60*m^2 + 142*m + 105)*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 105)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 1)^(3/2))
```

mupad [B] time = 13.97, size = 528, normalized size = 3.07

$$-\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{c\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(m^2\,4{}\mathrm{i}+m\,16{}\mathrm{i}+15{}\mathrm{i}\right)}{4\,f\,\left(8\,m^3+60\,m^2+142\,m+105\right)}-\frac{c\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(4\,m^2+16\,m+15\right)}{4\,f\,\left(8\,m^3+60\,m^2+142\,m+105\right)}-\frac{c\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,\left(a+a\,\sin\right.}{4\,f\,\left(8\,m^3+6\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((c*(a + a*sin(e + f*x))^m*(m*16i + m^2*4i + 1
5i))/(4*f*(142*m + 60*m^2 + 8*m^3 + 105)) - (c*exp(e*7i + f*x*7i)*(a + a*si
n(e + f*x))^m*(16*m + 4*m^2 + 15))/(4*f*(142*m + 60*m^2 + 8*m^3 + 105)) - (
c*exp(e*1i + f*x*1i)*(a + a*sin(e + f*x))^m*(48*m + 4*m^2 + 63))/(4*f*(142*
m + 60*m^2 + 8*m^3 + 105)) - (c*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^m*(
80*m + 12*m^2 - 35))/(4*f*(142*m + 60*m^2 + 8*m^3 + 105)) + (c*exp(e*6i + f
*x*6i)*(a + a*sin(e + f*x))^m*(m*48i + m^2*4i + 63i))/(4*f*(142*m + 60*m^2
```

```
+ 8*m^3 + 105)) + (c*exp(e*2i + f*x*2i)*(a + a*sin(e + f*x))^m*(m*80i + m^2
*12i - 35i))/(4*f*(142*m + 60*m^2 + 8*m^3 + 105)) - (c*exp(e*3i + f*x*3i)*(
a + a*sin(e + f*x))^m*(112*m + 12*m^2 + 525))/(4*f*(142*m + 60*m^2 + 8*m^3
+ 105)) + (c*exp(e*4i + f*x*4i)*(a + a*sin(e + f*x))^m*(m*112i + m^2*12i +
525i))/(4*f*(142*m + 60*m^2 + 8*m^3 + 105))))/(exp(e*4i + f*x*4i) - (exp(e*
3i + f*x*3i)*(m*142i + m^2*60i + m^3*8i + 105i))/(142*m + 60*m^2 + 8*m^3 +
105))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.75 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=107

$$\frac{8c\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af\left(4m^2+16m+15\right)\sqrt{c-c\sin(e+fx)}}+\frac{2\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^{m+1}}{af(2m+5)}$$

```
[Out] 8*c*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(4*m^2+16*m+15)/(c-c*sin(f*x+e))^
(1/2)+2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(1/2)/a/f/(5+2*m
)
```

Rubi [A] time = 0.35, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2841, 2740, 2738}

$$\frac{8c\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af\left(4m^2+16m+15\right)\sqrt{c-c\sin(e+fx)}}+\frac{2\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^{m+1}}{af(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (8*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(15 + 16*m + 4*m^2)*Sq
rt[c - c*Sin[e + f*x]]) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt
[c - c*Sin[e + f*x]])/(a*f*(5 + 2*m))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{3/2}\,dx}{ac}$$
$$= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+fx)}}{af(5+2m)}$$
$$= \frac{8c\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af\left(15+16m+4m^2\right)\sqrt{c-c\sin(e+fx)}} + \frac{2\cos(e+}{}$$

Mathematica [A] time = 0.57, size = 111, normalized size = 1.04

$$-\frac{2\sqrt{c-c\sin(e+fx)}\,((2m+3)\sin(e+fx)-2m-7)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(a(\sin(e+fx)+1))^m}{f(2m+3)(2m+5)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^m*Sqrt[c
 - c*Sin[e + f*x]]*(-7 - 2*m + (3 + 2*m)*Sin[e + f*x]))/(f*(3 + 2*m)*(5 + 2
*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [A] time = 0.49, size = 155, normalized size = 1.45

$$\frac{2\left((2\,m+3)\cos\left(fx+e\right)^3+(2\,m-1)\cos\left(fx+e\right)^2+\left((2\,m+3)\cos\left(fx+e\right)^2+4\,\cos\left(fx+e\right)+8\right)\sin\left(fx+e\right)}{4\,fm^2+16\,fm+\left(4\,fm^2+16\,fm+15\,f\right)\cos\left(fx+e\right)-\left(4\,fm^2+16\,fm\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 2*((2*m + 3)*cos(f*x + e)^3 + (2*m - 1)*cos(f*x + e)^2 + ((2*m + 3)*cos(f*x
 + e)^2 + 4*cos(f*x + e) + 8)*sin(f*x + e) + 4*cos(f*x + e) + 8)*sqrt(-c*si
n(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(4*f*m^2 + 16*f*m + (4*f*m^2 + 16*f*
m + 15*f)*cos(f*x + e) - (4*f*m^2 + 16*f*m + 15*f)*sin(f*x + e) + 15*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2,
x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\sqrt{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

maxima [B] time = 0.53, size = 312, normalized size = 2.92

$$-\frac{2\left(a^{m}\sqrt{c}\,(2\,m+7)+\frac{a^{m}\sqrt{c}\,(2\,m+15)\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{2\,a^{m}\sqrt{c}\,(2\,m-5)\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{2\,a^{m}\sqrt{c}\,(2\,m-5)\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{a^{m}\sqrt{c}\,(2\,m+15)\sin\left(fx+e\right.}{\left(\cos\left(fx+e\right)+1\right)^{4}}\right.}{\left(4\,m^{2}+16\,m+\frac{2\left(4\,m^{2}+16\,m+15\right)\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{\left(4\,m^{2}+16\,m+15\right)\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorit
hm="maxima")
```

```
[Out] -2*(a^m*sqrt(c)*(2*m + 7) + a^m*sqrt(c)*(2*m + 15)*sin(f*x + e)/(cos(f*x +
e) + 1) - 2*a^m*sqrt(c)*(2*m - 5)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a
^m*sqrt(c)*(2*m - 5)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a^m*sqrt(c)*(2*m
 + 15)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^m*sqrt(c)*(2*m + 7)*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5)*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1
```

```
) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((4*m^2 + 16*m + 2*(4*m
^2 + 16*m + 15)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + (4*m^2 + 16*m + 15)*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15)*f*sqrt(sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 1))
```

mupad [B] time = 1.72, size = 104, normalized size = 0.97

$$-\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(25\,\cos\left(e+f\,x\right)+3\,\cos\left(3\,e+3\,f\,x\right)+8\,\sin\left(2\,e+2\,f\,x\right)+6$$
$$2\,f\,\left(\sin\left(e+f\,x\right)-1\right)\,\left(4\,m^2+16\,m+15\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(25*cos(e + f*x) +
 3*cos(3*e + 3*f*x) + 8*sin(2*e + 2*f*x) + 6*m*cos(e + f*x) + 2*m*cos(3*e +
 3*f*x)))/(2*f*(sin(e + f*x) - 1)*(16*m + 4*m^2 + 15))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(\sin\left(e+fx\right)+1\right)\right)^m\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\cos^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*sqrt(-c*(sin(e + f*x) - 1))*cos(e + f*x)
**2, x)
```

3.76 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=50

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(3+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2841, 2738}

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3 + 2*m)*Sqrt[c - c*Sin
[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+fx)}\,dx}{ac}$$
$$= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.37, size = 85, normalized size = 1.70

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(a(\sin(e+fx)+1))^m}{f(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^3*(a*(1 + Sin[e + f*x]))^m)/(f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.46, size = 108, normalized size = 2.16

$$-\frac{2\left(\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m}{2cfm+3cf+\left(2cfm+3cf\right)\cos\left(fx+e\right)-\left(2cfm+3cf\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="fricas")
```

```
[Out] -2*(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)*sq
rt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(2*c*f*m + 3*c*f + (2*c*f*m
+ 3*c*f)*cos(f*x + e) - (2*c*f*m + 3*c*f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) + c),
x)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}}{\sqrt{c-c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 0.90, size = 68, normalized size = 1.36

$$-\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^{m}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(2\,\cos\left(e+f\,x\right)+\sin\left(2\,e+2\,f\,x\right)\right)}{c\,f\,\left(2\,m+3\right)\,\left(\sin\left(e+f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(2*cos(e + f*x) +
sin(2*e + 2*f*x)))/(c*f*(2*m + 3)*(sin(e + f*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\cos^{2}\left(e+fx\right)}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cos(e + f*x)**2/sqrt(-c*(sin(e + f*x) -
1)), x)
```

3.77 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+\frac{3}{2};m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{acf(2m+3)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*hypergeom([1, 3/2+m],[5/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e)
)^(1+m)/a/c/f/(3+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2841, 2745, 2667, 68}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+\frac{3}{2};m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{acf(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^(1 + m))/(a*c*f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
```

```
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{1+m}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)\int \sec(e+fx)(a+a\sin(e+fx))^{\frac{3}{2}+m}\,dx}{ac\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\,\mathrm{Subst}\left(\int \frac{(a+x)^{\frac{1}{2}+m}}{a-x}\,dx, x, a\sin(e+fx)\right)}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{3}{2}+m;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx)}{acf(3+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [B] time = 6.66, size = 218, normalized size = 2.87

$$-\frac{2^{-2m-\frac{5}{2}}\cos^2\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a\sin(e+fx)+a)^m\left(\sec^4\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -((2^(-5/2 - 2*m)*Cos[(-e + Pi/2 - f*x)/2]^2*(-(4^(1 + m)*Hypergeometric2F1
[1, 2 + 2*m, 3 + 2*m, Cos[(-e + Pi/2 - f*x)/2]]) + Hypergeometric2F1[2 + 2*
```

```
m, 2 + 2*m, 3 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2]*Sec[(-e + Pi/2 - f
*x)/4]^4*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^3*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(c - c*Sin[e + f*x])^(3/2)))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(
c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(3/2)
, x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c-c\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(3/2)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^m\cos^2\left(e+fx\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cos(e + f*x)**2/(-c*(sin(e + f*x) - 1))*
*(3/2), x)
```

3.78 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(2,m+\frac{3}{2};m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{2ac^2f(2m+3)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*cos(f*x+e)*hypergeom([2, 3/2+m],[5/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*
x+e))^(1+m)/a/c^2/f/(3+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2841, 2745, 2667, 68}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(2,m+\frac{3}{2};m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{2ac^2f(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[2, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^(1 + m))/(2*a*c^2*f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]]
)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
```

```
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{1+m}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)\int \sec^3(e+fx)(a+a\sin(e+fx))^{\frac{5}{2}+m}\,dx}{a^2c^2\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\,\text{Subst}\left(\int \frac{(a+x)^{\frac{1}{2}+m}}{(a-x)^2}\,dx, x, a\sin(e+fx)\right)}{c^2f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(2,\frac{3}{2}+m;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx)}{2ac^2f(3+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] time = 6.67, size = 3174, normalized size = 40.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] (2^(-3/2 - 2*m)*(-(4^m*Hypergeometric2F1[1, 2*m, 1 + 2*m, Cos[(-e + Pi/2 -
f*x)/2]]) + Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)
/4]^2)/2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^5*(a + a*Sin[e + f*x])^m)/(f*m*(c - c*Sin[e + f*x])^(5/2)) - ((Cos
```

```
[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a +
 a*Sin[e + f*x])^m*(AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -
Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi
/2 - f*x)/4]^2 - (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Co
t[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)
/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 -
f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan
[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*(-1 + Tan[(-e +
 Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m)))/(4*S
qrt[2]*f*(c - c*Sin[e + f*x])^(5/2)*(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[P
i/4 + (e - Pi/2 + f*x)/2])^3*(-1/8*(m*Cos[(-e + Pi/2 - f*x)/4]*(Cos[(-e + P
i/2 - f*x)/4]^2)^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/4]*(AppellF1[1, -2*m, 2*m
, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi
/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2 - (AppellF1[1, -2*m, 2*m,
2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2
- f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4
]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*AppellF1[
1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e +
 Pi/2 - f*x)/4]^2]*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 -
f*x)/4]^4)^(2*m))/(1 + 2*m)))/Sqrt[2] + ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)
*((AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2 - f*x)/4])
/2 + m*AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^
3 + (Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2*(-1/2*(m*
AppellF1[2, 1 - 2*m, 2*m, 3, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) - (m*Appell
F1[2, -2*m, 1 + 2*m, 3, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/
4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/2) + (m*AppellF1
[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*
Cot[(-e + Pi/2 - f*x)/4]^3*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e
+ Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + m*Appel
lF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^
2]*Cot[(-e + Pi/2 - f*x)/4]^3*(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(-1 - 2*m)*(
Csc[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m
) + (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2
- f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m
)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(2*(1 - Cot[(-e + Pi/2 - f*x)/4]^
2)^(2*m)) - (Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*
((m*AppellF1[2, 1 - 2*m, 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/
2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 - f*x)/4]^2)/2 + (m*
AppellF1[2, -2*m, 1 + 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 -
 f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 - f*x)/4]^2)/2)*(1 - Ta
n[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (
m*AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f
```

```
*x)/4]^2]*Csc[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Sec[(
-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m))/(1 - Cot[(
-e + Pi/2 - f*x)/4]^2)^(2*m) + (AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan
[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2
- f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m)
)/(2^(2*m)*(1 + 2*m)) + (2^(1 - 2*m)*(-1/2*((1 + 2*m)*AppellF1[2 + 2*m, 2*m
, 2, 3 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)
/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2 + 2*m) - (m*
(1 + 2*m)*AppellF1[2 + 2*m, 1 + 2*m, 1, 3 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)
/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-
e + Pi/2 - f*x)/4])/(2*(2 + 2*m)))*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - T
an[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m) - (2^(2 - 2*m)*m*AppellF1[1 + 2
*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2
 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^3*(-1 + T
an[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(-1 + 2*m))/(1
+ 2*m)))/(8*Sqrt[2])))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(
3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(5/2)
, x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c-c\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(5/2)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^{2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.79 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=50

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(3+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2841, 2738}

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3 + 2*m)*Sqrt[c - c*Sin
[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+fx)}\,dx}{ac}$$
$$= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.34, size = 85, normalized size = 1.70

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(a(\sin(e+fx)+1))^m}{f(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^3*(a*(1 + Sin[e + f*x]))^m)/(f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.50, size = 108, normalized size = 2.16

$$-\frac{2\left(\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right.}{2\,cfm+3\,cf+\left(2\,cfm+3\,cf\right)\cos\left(fx+e\right)-\left(2\,cfm+3\,cf\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="fricas")
```

```
[Out] -2*(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)*sq
rt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(2*c*f*m + 3*c*f + (2*c*f*m
+ 3*c*f)*cos(f*x + e) - (2*c*f*m + 3*c*f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) + c),
x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m}{\sqrt{c-c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 0.00, size = 68, normalized size = 1.36

$$-\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(2\,\cos\left(e+f\,x\right)+\sin\left(2\,e+2\,f\,x\right)\right)}{c\,f\,\left(2\,m+3\right)\,\left(\sin\left(e+f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(2*cos(e + f*x) +
sin(2*e + 2*f*x)))/(c*f*(2*m + 3)*(sin(e + f*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^m\cos^2\left(e+fx\right)}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cos(e + f*x)**2/sqrt(-c*(sin(e + f*x) -
1)), x)
```

3.80 $\int \frac{\cos^2(e+fx)(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

Optimal. Leaf size=50

$$\frac{2\cos(e+fx)(c\sin(e+fx)+c)^{m+1}}{cf(2m+3)\sqrt{a-a\sin(e+fx)}}$$

```
[Out] 2*cos(f*x+e)*(c+c*sin(f*x+e))^(1+m)/c/f/(3+2*m)/(a-a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2841, 2738}

$$\frac{2\cos(e+fx)(c\sin(e+fx)+c)^{m+1}}{cf(2m+3)\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]],x]
```

```
[Out] (2*Cos[e + f*x]*(c + c*Sin[e + f*x])^(1 + m))/(c*f*(3 + 2*m)*Sqrt[a - a*Sin
[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \frac{\int \sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{1+m}\,dx}{ac}$$
$$= \frac{2\cos(e+fx)(c+c\sin(e+fx))^{1+m}}{cf(3+2m)\sqrt{a-a\sin(e+fx)}}$$

Mathematica [A] time = 0.36, size = 85, normalized size = 1.70

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(c(\sin(e+fx)+1))^m}{f(2m+3)\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]],
x]
```

```
[Out] (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^3*(c*(1 + Sin[e + f*x]))^m)/(f*(3 + 2*m)*Sqrt[a - a*Sin[e + f*x]])
```

fricas [B] time = 0.48, size = 108, normalized size = 2.16

$$-\frac{2\left(\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2\right)\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right.}{2\,afm+3\,af+\left(2\,afm+3\,af\right)\cos\left(fx+e\right)-\left(2\,afm+3\,af\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorit
hm="fricas")
```

```
[Out] -2*(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)*sq
rt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^m/(2*a*f*m + 3*a*f + (2*a*f*m
+ 3*a*f)*cos(f*x + e) - (2*a*f*m + 3*a*f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(fx+e\right)+c\right)^m\cos\left(fx+e\right)^2}{\sqrt{-a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((c*sin(f*x + e) + c)^m*cos(f*x + e)^2/sqrt(-a*sin(f*x + e) + a),
x)
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2\left(fx+e\right)\right)\left(c+c\sin\left(fx+e\right)\right)^m}{\sqrt{a-a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 9.11, size = 68, normalized size = 1.36

$$-\frac{\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(c\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(2\,\cos\left(e+f\,x\right)+\sin\left(2\,e+2\,f\,x\right)\right)}{a\,f\,\left(2\,m+3\right)\,\left(\sin\left(e+f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1/2),x)
```

```
[Out] -((-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^m*(2*cos(e + f*x) +
sin(2*e + 2*f*x)))/(a*f*(2*m + 3)*(sin(e + f*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^m\cos^2\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*(sin(e + f*x) + 1))**m*cos(e + f*x)**2/sqrt(-a*(sin(e + f*x) -
1)), x)
```

3.81 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-5-m}\,dx$

Optimal. Leaf size=182

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-2}}{ac^3f(2m+7)\left(4m^2+16m+15\right)}+\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx}{ac^2f\left(4m^2+24m+35\right)}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(-4-m)/a/c/f/(7+2*m)+2*c
os(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(-3-m)/a/c^2/f/(4*m^2+24*
m+35)+2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(-2-m)/a/c^3/f/(
8*m^3+60*m^2+142*m+105)
```

Rubi [A] time = 0.45, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2743, 2742}

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-3}}{ac^2f\left(4m^2+24m+35\right)}+\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx}{ac^3f(2m+7)\left(4m^2+16m+15\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-5 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-4 - m))/(
a*c*f*(7 + 2*m)) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[
e + f*x])^(-3 - m))/(a*c^2*f*(35 + 24*m + 4*m^2)) + (2*Cos[e + f*x]*(a + a*
Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-2 - m))/(a*c^3*f*(7 + 2*m)*(15
 + 16*m + 4*m^2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-5-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-4-m}}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+}{acf(7+2m)}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+}{acf(7+2m)}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+}{acf(7+2m)}$$

Mathematica [A] time = 17.03, size = 176, normalized size = 0.97

$$\frac{2^{-m-2}\cos^3\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m-7}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-5}\left(\cos\left(\frac{1}{2}(e\right.\right.}{f(2m+3)(2m+5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-5 -
m),x]
```

```
[Out] (2^(-2 - m)*Cos[(-e + Pi/2 - f*x)/2]^3*Sin[(-e + Pi/2 - f*x)/2]^(-7 - 2*m)*
(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-5 - m)*(4*(6 + 5*m + m^2) + C
os[2*(-e + Pi/2 - f*x)] - 2*(5 + 2*m)*Sin[e + f*x]))/(f*(3 + 2*m)*(5 + 2*m)
*(7 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-5 - m)))
```

fricas [A] time = 0.51, size = 105, normalized size = 0.58

$$-\frac{\left(2\,\cos\left(fx+e\right)^{5}+2\,(2\,m+5)\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-\left(4\,m^{2}+20\,m+25\right)\cos\left(fx+e\right)^{3}\right)\left(a\sin\left(fx+e\right)+\right.}{8\,fm^{3}+60\,fm^{2}+142\,fm+105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x, algori
thm="fricas")
```

```
[Out] -(2*cos(f*x + e)^5 + 2*(2*m + 5)*cos(f*x + e)^3*sin(f*x + e) - (4*m^2 + 20*
m + 25)*cos(f*x + e)^3)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m -
5)/(8*f*m^3 + 60*f*m^2 + 142*f*m + 105*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-5}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x, algori
thm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 5)*cos(f*x + e
)^2, x)
```

maple [F] time = 5.37, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-5-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-5}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x, algori
thm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 5)*cos(f*x + e
)^2, x)
```

mupad [B] time = 15.75, size = 334, normalized size = 1.84

$$\frac{\cos\left(e+f\,x\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(24\,m^2+120\,m+140\right)}{8\,f\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{m+5}\,\left(8\,m^3+60\,m^2+142\,m+105\right)}-\frac{\cos\left(5\,e+5\,f\,x\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{8\,f\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{m+5}\,\left(8\,m^3+60\,m^2+142\,m\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 5),x)
```

```
[Out] (cos(e + f*x)*(a + a*sin(e + f*x))^m*(120*m + 24*m^2 + 140))/(8*f*(c - c*si
n(e + f*x))^(m + 5)*(142*m + 60*m^2 + 8*m^3 + 105)) - (cos(5*e + 5*f*x)*(a
+ a*sin(e + f*x))^m)/(8*f*(c - c*sin(e + f*x))^(m + 5)*(142*m + 60*m^2 + 8*
m^3 + 105)) + (sin(4*e + 4*f*x)*(m*4i + 10i)*(a + a*sin(e + f*x))^m*1i)/(8*
f*(c - c*sin(e + f*x))^(m + 5)*(142*m + 60*m^2 + 8*m^3 + 105)) + (sin(2*e +
 2*f*x)*(m*8i + 20i)*(a + a*sin(e + f*x))^m*1i)/(8*f*(c - c*sin(e + f*x))^(
m + 5)*(142*m + 60*m^2 + 8*m^3 + 105)) + (cos(3*e + 3*f*x)*(a + a*sin(e + f
*x))^m*(40*m + 8*m^2 + 45))/(8*f*(c - c*sin(e + f*x))^(m + 5)*(142*m + 60*m
^2 + 8*m^3 + 105))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-5-m),x)
```

```
[Out] Timed out
```

3.82 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}\,dx$

Optimal. Leaf size=114

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-2}}{ac^2f\left(4m^2+16m+15\right)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m}}{acf(2m+5)}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(-3-m)/a/c/f/(5+2*m)+cos
(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(-2-m)/a/c^2/f/(4*m^2+16*m+
15)
```

Rubi [A] time = 0.33, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-2}}{ac^2f\left(4m^2+16m+15\right)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m}}{acf(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-3 - m))/(
a*c*f*(5 + 2*m)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e
+ f*x])^(-2 - m))/(a*c^2*f*(15 + 16*m + 4*m^2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-3-m}}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+}{acf(5+2m)}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+}{acf(5+2m)}$$

Mathematica [A] time = 12.61, size = 142, normalized size = 1.25

$$-\frac{2^{-m-1}\cos^3\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m-5}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(\sin(e+fx)-2(m+2))(a\sin(e+fx)+a)^m(c-c\,\mathrm{si}}{f(2m+3)(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 -
m),x]
```

```
[Out] -((2^(-1 - m)*Cos[(-e + Pi/2 - f*x)/2]^3*Sin[(-e + Pi/2 - f*x)/2]^(-5 - 2*m
)*(-2*(2 + m) + Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(
-4 - m))/(f*(3 + 2*m)*(5 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-
4 - m))))
```

fricas [A] time = 0.46, size = 76, normalized size = 0.67

$$\frac{\left(2\,(m+2)\cos\left(fx+e\right)^3-\cos\left(fx+e\right)^3\sin\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^{-m-4}}{4\,fm^2+16\,fm+15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x, algori
thm="fricas")
```

```
[Out] (2*(m + 2)*cos(f*x + e)^3 - cos(f*x + e)^3*sin(f*x + e))*(a*sin(f*x + e) +
a)^m*(-c*sin(f*x + e) + c)^(-m - 4)/(4*f*m^2 + 16*f*m + 15*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-4}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x, algori
thm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 4)*cos(f*x + e
)^2, x)
```

maple [F] time = 5.40, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-4-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-4}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x, algori
thm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 4)*cos(f*x + e
)^2, x)
```

mupad [B] time = 10.60, size = 177, normalized size = 1.55

$$-\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^{m}\,\left(2\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)+48\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+16\,\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^{2}+12\,m}{c^{4}\,f\,\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)^{m}\,\left(4\,m^{2}+16\,m+15\right)\,\left(56\,\sin\left(e+f\,x\right)^{2}-56\,\sin\left(e+f\,x\right)-2\,\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 4),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(2*sin(2*e + 2*f*x) + sin(4*e + 4*f*x) + 48*sin(
e/2 + (f*x)/2)^2 + 16*sin((3*e)/2 + (3*f*x)/2)^2 + 12*m*(2*sin(e/2 + (f*x)/
2)^2 - 1) + 4*m*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1) - 32))/(c^4*f*(-c*(sin(e
 + f*x) - 1))^m*(16*m + 4*m^2 + 15)*(8*sin(3*e + 3*f*x) - 56*sin(e + f*x) -
 2*sin(2*e + 2*f*x)^2 + 56*sin(e + f*x)^2 + 8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-4-m),x)
```

```
[Out] Timed out
```

3.83 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx$

Optimal. Leaf size=54

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-2}}{acf(2m+3)}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(-2-m)/a/c/f/(3+2*m)
```

Rubi [A] time = 0.25, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-2}}{acf(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-2 - m))/(
a*c*f*(3 + 2*m))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-2-m}}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+}{acf(3+2m)}$$

Mathematica [B] time = 5.52, size = 109, normalized size = 2.02

$$\frac{2^{-m}\sin^3\left(\frac{1}{4}(2e+2fx+\pi)\right)\cos^{-2m-3}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^{-m}\left(\cos\left(\frac{1}{2}(e+\right.\right.}{c^3f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 -
m),x]
```

```
[Out] (Cos[(2*e + Pi + 2*f*x)/4]^(-3 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^(2*m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4]^3)/(2^m*c^3*f*(3
+ 2*m)*(c - c*Sin[e + f*x])^m)
```

fricas [A] time = 0.50, size = 48, normalized size = 0.89

$$\frac{\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^{-m-3}\cos\left(fx+e\right)^3}{2fm+3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algori
thm="fricas")
```

```
[Out] (a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)*cos(f*x + e)^3/(2*f*m
 + 3*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^{-m-3}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algori
thm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)*cos(f*x + e
)^2, x)
```

maple [F] time = 4.82, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-3-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-3}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algori
thm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)*cos(f*x + e
)^2, x)
```

mupad [B] time = 1.04, size = 101, normalized size = 1.87

$$-\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(6\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^2-4\right)}{c^3\,f\,\left(2\,m+3\right)\,\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)^m\,\left(12\sin\left(e+f\,x\right)^2-15\,\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 3),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(6*sin(e/2 + (f*x)/2)^2 + 2*sin((3*e)/2 + (3*f*x
)/2)^2 - 4))/(c^3*f*(2*m + 3)*(-c*(sin(e + f*x) - 1))^m*(sin(3*e + 3*f*x) -
 15*sin(e + f*x) + 12*sin(e + f*x)^2 + 4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-3-m),x)
```

```
[Out] Timed out
```

3.84 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx$

Optimal. Leaf size=113

$$\frac{2^{-m-\frac{1}{2}}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m+3),\frac{1}{2}(2m+3);\right.}{f(2m+3)}$$

```
[Out] 2^(-1/2-m)*cos(f*x+e)^3*hypergeom([3/2+m, 3/2+m],[5/2+m],1/2+1/2*sin(f*x+e)
)*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(3+2*
m)
```

Rubi [A] time = 0.38, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2745, 2689, 70, 69}

$$\frac{2^{-m-\frac{1}{2}}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m+3),\frac{1}{2}(2m+3);\right.}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m),x]
```

```
[Out] (2^(-1/2 - m)*Cos[e + f*x]^3*Hypergeometric2F1[(3 + 2*m)/2, (3 + 2*m)/2, (5
 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[e
+ f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-1-m}\,d}{ac}$$

$$= \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+\right.$$

$$= \frac{\left(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(c\right.}{}$$

$$= \frac{\left(2^{-\frac{3}{2}-m}c\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx\right.}{}$$

$$= \frac{2^{-\frac{1}{2}-m}\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(3+2m),\frac{1}{2}(3+2m);\frac{1}{2}\right.}{}$$

Mathematica [C] time = 21.56, size = 589, normalized size = 5.21

$$-\frac{2^{1-m}(2m-3)\cos^2\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\cot\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)\csc^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right.}{f\left(4m^2-1\right)\left(8(m+1)F_1\left(\frac{3}{2}-m;-2m-1,1;\frac{5}{2}-m;\tan^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right),-\tan^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)\right)+4F_1\left(\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 -
m),x]
```

```
[Out] -((2^(1 - m)*(-3 + 2*m)*Cos[(-e + Pi/2 - f*x)/2]^2*Cot[(-e + Pi/2 - f*x)/4]
*Csc[(-e + Pi/2 - f*x)/4]^2*(-((1 + 2*m)*AppellF1[1/2 - m, -2*(1 + m), 1, 3
/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]) + (-1 + 2
*m)*Cot[(-e + Pi/2 - f*x)/4]^2*Hypergeometric2F1[-1/2 - m, -2*(1 + m), 1/2
- m, Tan[(-e + Pi/2 - f*x)/4]^2])*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(-2 - m))/(f*(-1 + 4*m^2)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^(2*(-2 - m))*(8*(1 + m)*AppellF1[3/2 - m, -1 - 2*m,
1, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 4*Ap
pellF1[3/2 - m, -2*(1 + m), 2, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2] + (-3 + 2*m)*(2*AppellF1[1/2 - m, -2*(1 + m), 1, 3/2
- m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/
2 - f*x)/4]^2 - Csc[(-e + Pi/2 - f*x)/4]^4*(1 + Sin[e + f*x])*(1 - Tan[(-e
+ Pi/2 - f*x)/4]^2)^(2*m)))))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}\cos\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algori
thm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)*cos(f*x + e)
^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algori
thm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)*cos(f*x + e
)^2, x)
```

maple [F] time = 3.58, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{-2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algori
thm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)*cos(f*x + e
)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{m+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 2),x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-2-m),x)
```

```
[Out] Timed out
```

3.85 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=114

$$\frac{c2^{\frac{1}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2m+3);\right.}{f(2m+3)}$$

```
[Out] 2^(1/2-m)*c*cos(f*x+e)^3*hypergeom([1/2+m, 3/2+m],[5/2+m],1/2+1/2*sin(f*x+e
))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(3+2
*m)
```

Rubi [A] time = 0.37, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2745, 2689, 70, 69}

$$\frac{c2^{\frac{1}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2m+3);\right.}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (2^(1/2 - m)*c*Cos[e + f*x]^3*Hypergeometric2F1[(1 + 2*m)/2, (3 + 2*m)/2, (
5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[e
 + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-m}\,dx}{ac}$$

$$= \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+\right.$$

$$= \frac{\left(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(c\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}-m}c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+f\right.}{}$$

$$= \frac{2^{\frac{1}{2}-m}c\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(3+2m);\frac{1}{2}\right.}{}$$

Mathematica [C] time = 29.65, size = 1045, normalized size = 9.17

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 -
m),x]
```

```
[Out] (2^(3 - m)*(-3 + 2*m)*(AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 -
 f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2 - m, -2*m, 2, 3/2
 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 4*AppellF1
[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f
*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/4]^3*Cos[(-e + Pi/2 - f*x)/2]^2*Sin[(-e +
Pi/2 - f*x)/4]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(-1
 + 2*m)*((-3 + 2*m)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 - 4*(-3 +
2*m)*AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 + 4*(-3 + 2*m)*AppellF1[1
/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x
)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 + 8*AppellF1[3/2 - m, -2*m, 3, 5/2 - m,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(-1 + Cos[(-e + Pi
/2 - f*x)/2]) + 2*(2*m*AppellF1[3/2 - m, 1 - 2*m, 1, 5/2 - m, Tan[(-e + Pi/
2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*m*AppellF1[3/2 - m, 1 - 2*m
, 2, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*
m*AppellF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2 - m, -2*m, 2, 5/2 - m, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 12*AppellF1[3/2 - m, -2*m, 4,
5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Sin[(-e
+ Pi/2 - f*x)/4]^2)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^(2*(-1 - m)))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algori
thm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1)*cos(f*x + e)
^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algori
thm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1)*cos(f*x + e
)^2, x)
```

maple [F] time = 1.35, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algori
thm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1)*cos(f*x + e
)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 1),x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 1), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

3.86 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx$

Optimal. Leaf size=116

$$\frac{c^2 2^{\frac{3}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m-1),\frac{1}{2}(2m+3);\frac{1}{2}\right.}{f(2m+3)}$$

```
[Out] 2^(3/2-m)*c^2*cos(f*x+e)^3*hypergeom([3/2+m, -1/2+m],[5/2+m],1/2+1/2*sin(f*
x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(
3+2*m)
```

Rubi [A] time = 0.32, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2841, 2745, 2689, 70, 69}

$$\frac{c^2 2^{\frac{3}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m-1),\frac{1}{2}(2m+3);\frac{1}{2}\right.}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(3/2 - m)*c^2*Cos[e + f*x]^3*Hypergeometric2F1[(-1 + 2*m)/2, (3 + 2*m)/2
, (5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{1-m}\,dx}{ac}$$

$$= \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+\right.$$

$$= \frac{\left(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(c\right.}{}$$

$$= \frac{\left(2^{\frac{1}{2}-m}c^3\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx\right.}{}$$

$$= \frac{2^{\frac{3}{2}-m}c^2\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(-1+2m),\frac{1}{2}(3+2m);\right.}{}$$

Mathematica [C] time = 21.71, size = 1519, normalized size = 13.09

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(6 - m)*(-3 + 2*m)*(AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 -
 f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*AppellF1[1/2 - m, -2*m, 3, 3/2
 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*AppellF1
[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f
*x)/4]^2] - 4*AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^
2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/4]^5*Cos[(-e + Pi/2
- f*x)/2]^2*Sin[(-e + Pi/2 - f*x)/4]^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^(2*m)*(a + a*Sin[e + f*x])^m)/(f*(-1 + 2*m)*((-3 + 2*m)*AppellF1[1/2 - m,
 -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]
*Cos[(-e + Pi/2 - f*x)/4]^2 - 5*(-3 + 2*m)*AppellF1[1/2 - m, -2*m, 3, 3/2 -
 m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2
 - f*x)/4]^2 + 8*(-3 + 2*m)*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + P
i/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 +
12*AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 - 8*m*AppellF1[1/2 - m, -2*
m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos
[(-e + Pi/2 - f*x)/4]^2 + 4*m*AppellF1[3/2 - m, 1 - 2*m, 2, 5/2 - m, Tan[(-
e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]
^2 - 20*m*AppellF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2
, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2 + 32*m*AppellF1[3
/2 - m, 1 - 2*m, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2 - 16*m*AppellF1[3/2 - m, 1 - 2*m, 5,
5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e +
 Pi/2 - f*x)/4]^2 + 4*AppellF1[3/2 - m, -2*m, 3, 5/2 - m, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2 - 30*App
ellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2 + 64*AppellF1[3/2 - m, -2*m, 5, 5
/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e +
Pi/2 - f*x)/4]^2 - 40*AppellF1[3/2 - m, -2*m, 6, 5/2 - m, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2)*Sin[(-e
 + Pi/2 - f*x)/2]^(2*m)*(c - c*Sin[e + f*x])^m)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{m}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm
="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm
="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^m, x)
```

maple [F] time = 1.83, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm
="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^{2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m,x)
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/((c-c*sin(f*x+e))**m),x)
[Out] Timed out
```

3.87 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx$

Optimal. Leaf size=116

$$\frac{c^3 2^{\frac{5}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m-3),\frac{1}{2}(2m+3);\right.}{f(2m+3)}$$

```
[Out] 2^(5/2-m)*c^3*cos(f*x+e)^3*hypergeom([3/2+m, -3/2+m],[5/2+m],1/2+1/2*sin(f*
x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(
3+2*m)
```

Rubi [A] time = 0.37, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2745, 2689, 70, 69}

$$\frac{c^3 2^{\frac{5}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m-3),\frac{1}{2}(2m+3);\right.}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m),x]
```

```
[Out] (2^(5/2 - m)*c^3*Cos[e + f*x]^3*Hypergeometric2F1[(-3 + 2*m)/2, (3 + 2*m)/2
, (5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{2-m}\,dx}{ac}$$

$$= \Big(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+$$

$$= \frac{\Big(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(c-}{}$$

$$= \frac{\Big(2^{\frac{3}{2}-m}c^4\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx)}{}$$

$$= \frac{2^{\frac{5}{2}-m}c^3\cos^3(e+fx)\,{}_2F_1\Big(\frac{1}{2}(-3+2m),\frac{1}{2}(3+2m);}{}$$

Mathematica [C] time = 25.90, size = 4270, normalized size = 36.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m
),x]
```

```
[Out] (2^(9 - m)*(AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2,
 -Tan[(-e + Pi/2 - f*x)/4]^2] - 6*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(
-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 13*AppellF1[1/2 - m,
-2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]
- 12*AppellF1[1/2 - m, -2*m, 6, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2] + 4*AppellF1[1/2 - m, -2*m, 7, 3/2 - m, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*(a + a*Sin[e + f*x])^m*(c - c
*Sin[e + f*x])^(1 - m)*((Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Cos[Pi/4 + (e - Pi/
2 + f*x)/2]^6)/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x
)/2])^(2*m) - (2*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Cos[Pi/4 + (e - Pi/2 + f*x)
/2]^5*Sin[Pi/4 + (e - Pi/2 + f*x)/2])/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin
[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) - (Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Cos[Pi
/4 + (e - Pi/2 + f*x)/2]^4*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^2)/(Cos[Pi/4 + (e
 - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + (4*Cos[(-e + Pi
/2 - f*x)/2]^(2*m)*Cos[Pi/4 + (e - Pi/2 + f*x)/2]^3*Sin[Pi/4 + (e - Pi/2 +
f*x)/2]^3)/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2]
)^(2*m) - (Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Cos[Pi/4 + (e - Pi/2 + f*x)/2]^2*
Sin[Pi/4 + (e - Pi/2 + f*x)/2]^4)/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/
4 + (e - Pi/2 + f*x)/2])^(2*m) - (2*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Cos[Pi/4
 + (e - Pi/2 + f*x)/2]*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^5)/(Cos[Pi/4 + (e - P
i/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + (Cos[(-e + Pi/2 - f
*x)/2]^(2*m)*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^6)/(Cos[Pi/4 + (e - Pi/2 + f*x)
/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m))*Tan[(-e + Pi/2 - f*x)/4])/(f*(
-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^(2*(1 - m))*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(-((2^(9 - m)*m*(App
ellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2] - 6*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x
)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 13*AppellF1[1/2 - m, -2*m, 5, 3/2 -
m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 12*AppellF1[1
/2 - m, -2*m, 6, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x
)/4]^2] + 4*AppellF1[1/2 - m, -2*m, 7, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2,
 -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/
2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(
-1 - 2*m))/((-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m))) - (2^(7 - m)*(Appel
lF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
- f*x)/4]^2] - 6*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/
```

4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 13*AppellF1[1/2 - m, -2*m, 5, 3/2 - m,
 Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 12*AppellF1[1/2
 - m, -2*m, 6, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/
4]^2] + 4*AppellF1[1/2 - m, -2*m, 7, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -
Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/2
- f*x)/4]^2)/((-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(1 - Tan[(-e + Pi/2
 - f*x)/4]^2)^(2*m)) + (2^(9 - m)*m*(AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Ta
n[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 6*AppellF1[1/2 - m
, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2
] + 13*AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2] - 12*AppellF1[1/2 - m, -2*m, 6, 3/2 - m, Tan[(-e +
 Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 4*AppellF1[1/2 - m, -2*m,
 7, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[
(-e + Pi/2 - f*x)/2]^(1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(-1 - 2*m)*Tan[(-e
+ Pi/2 - f*x)/4])/((-1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (2^
(9 - m)*m*(AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2,
-Tan[(-e + Pi/2 - f*x)/4]^2] - 6*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-
e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 13*AppellF1[1/2 - m, -
2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] -
 12*AppellF1[1/2 - m, -2*m, 6, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2] + 4*AppellF1[1/2 - m, -2*m, 7, 3/2 - m, Tan[(-e + Pi/
2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(-1 +
 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(1 - 2*m)*Tan[(-e + Pi/2 - f*x)/4])/((-1 + 2
m)(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (2^(9 - m)*Cos[(-e + Pi/2 - f
*x)/2]^(2*m)*Tan[(-e + Pi/2 - f*x)/4]*(-(((1/2 - m)*m*AppellF1[3/2 - m, 1 -
 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*
Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)) - (3*(1/2 -
 m)*AppellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/
(2*(3/2 - m)) - 6*(-(((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 4, 5/2 - m, Ta
n[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x
)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)) - (2*(1/2 - m)*AppellF1[3/2 - m
, -2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2
]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)) + 13*(-((
(1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]
^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2
- f*x)/4])/(3/2 - m)) - (5*(1/2 - m)*AppellF1[3/2 - m, -2*m, 6, 5/2 - m, Ta
n[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x
)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2 - m))) - 12*(-(((1/2 - m)*m*Appell
F1[3/2 - m, 1 - 2*m, 6, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 -
m)) - (3*(1/2 - m)*AppellF1[3/2 - m, -2*m, 7, 5/2 - m, Tan[(-e + Pi/2 - f*x
)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + P
i/2 - f*x)/4])/(3/2 - m)) + 4*(-(((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 7,
 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e

```
+ Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)) - (7*(1/2 - m)*Appe
llF1[3/2 - m, -2*m, 8, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2
- m)))))/((-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(1 - Tan[(-e + Pi/2 - f
*x)/4]^2)^(2*m))))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a \sin\left(fx+e\right)+a\right)^{m}\left(-c \sin\left(fx+e\right)+c\right)^{-m+1} \cos\left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorit
hm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1)*cos(f*x + e)
^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{m}\left(-c \sin\left(fx+e\right)+c\right)^{-m+1} \cos\left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorit
hm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1)*cos(f*x + e
)^2, x)
```

maple [F] time = 2.96, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(a+a \sin\left(fx+e\right)\right)^{m}\left(c-c \sin\left(fx+e\right)\right)^{1-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{m}\left(-c \sin\left(fx+e\right)+c\right)^{-m+1} \cos\left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorit
hm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1)*cos(f*x + e
)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m),x)
```

```
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1-m),x)
```

```
[Out] Timed out
```

3.88 $\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=343

$$\frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2ac^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2ac^3\sqrt{c-c\,\mathrm{si}}}{7fg}$$

```
[Out] 10/77*a*c^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e)
)^(1/2)+2/33*a*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f/g/(a+a*sin(f
*x+e))^(1/2)-2/11*a*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f/g/(a+a*si
n(f*x+e))^(1/2)+2/3*a*c^4*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(
c-c*sin(f*x+e))^(1/2)+2*a*c^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/
2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^
(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2/7*a*c^3*(g*cos(f*x+
e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.74, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2ac^3\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{7fg\sqrt{a\sin(e+fx)+a}}+\frac{10ac^2(c-c\sin(e+fx))^{3}}{77fg\sqrt{a\sin(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (2*a*c^4*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (2*a*c^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
+ (2*a*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(7*f*g*Sqrt[a +
 a*Sin[e + f*x]]) + (10*a*c^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2))/(77*f*g*Sqrt[a + a*Sin[e + f*x]]) + (2*a*c*(g*Cos[e + f*x])^(5/2)*(c
- c*Sin[e + f*x])^(5/2))/(33*f*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(g*Cos[e
+ f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2))/(11*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{11fg\sqrt{a+a\sin(e+fx)}} +$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{33fg\sqrt{a+a\sin(e+fx)}} -$$

$$= \frac{10ac^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{77fg\sqrt{a+a\sin(e+fx)}} +$$

$$= \frac{2ac^3(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{7fg\sqrt{a+a\sin(e+fx)}} +$$

$$= \frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} +$$

$$= \frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} +$$

$$= \frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} +$$

$$= \frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} +$$

Mathematica [C] time = 8.02, size = 311, normalized size = 0.91

$$\frac{c^4ge^{-5i(e+fx)}\left(e^{i(e+fx)}-i\right)\left(4928e^{7i(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)+\sqrt{1+e^{2i(e+fx)}}\left(154e^{i(e+fx)}+423ie^{2i(e+fx)}-308e\right.\right.}{369}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] (c^4*(-I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*I)*(e +
f*x))]*(-21*I + 154*E^(I*(e + f*x)) + (423*I)*E^((2*I)*(e + f*x)) - 308*E^(
(3*I)*(e + f*x)) + (1374*I)*E^((4*I)*(e + f*x)) - 7392*E^((5*I)*(e + f*x))
+ (1374*I)*E^((6*I)*(e + f*x)) + 308*E^((7*I)*(e + f*x)) + (423*I)*E^((8*I)
*(e + f*x)) - 154*E^((9*I)*(e + f*x)) - (21*I)*E^((10*I)*(e + f*x))) + 4928
*E^((7*I)*(e + f*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))]
)*Sqrt[a*(1 + Sin[e + f*x])])/(3696*E^((5*I)*(e + f*x))*(I + E^(I*(e + f*x)
))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,c^{3}g\cos\left(fx+e\right)^{3}-4\,c^{3}g\cos\left(fx+e\right)-\left(c^{3}g\cos\left(fx+e\right)^{3}-4\,c^{3}g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(3*c^3*g*cos(f*x + e)^3 - 4*c^3*g*cos(f*x + e) - (c^3*g*cos(f*x +
 e)^3 - 4*c^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

maple [C] time = 0.76, size = 425, normalized size = 1.24

$$\frac{2\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{7}{2}}\left(-21\left(\cos^{6}\left(fx+e\right)\right)\sin\left(fx+e\right)+231i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos\left(fx+e\right)\sin\left(f\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/231/f*(-c*(sin(f*x+e)-1))^(7/2)*(-21*cos(f*x+e)^6*sin(f*x+e)+231*I*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)+77*cos(f*x+e)^6+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+132*sin(f*x+e)*cos(f*x+e)^4
```

```
-154*cos(f*x+e)^4-154*cos(f*x+e)^2+231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*
(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+
4)/cos(f*x+e)^3/sin(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(7/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2)*(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.89 $\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=290

$$\frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22ac^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22ac^2\sqrt{c-c}}{105}$$

```
[Out] 2/21*a*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e))^(
1/2)-2/9*a*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))
^(1/2)+22/45*a*c^3*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin
(f*x+e))^(1/2)+22/15*a*c^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e
)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/
2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+22/105*a*c^2*(g*cos(f*x+
e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.43, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22ac^2\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{105fg\sqrt{a\sin(e+fx)+a}}+\frac{22ac^3g\sqrt{\cos(e+fx)}\,E}{15f\sqrt{a\sin(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (22*a*c^3*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]]) + (22*a*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ell
ipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f
*x]]) + (22*a*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(105*f*g
*Sqrt[a + a*Sin[e + f*x]]) + (2*a*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(3/2))/(21*f*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)
*(c - c*Sin[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}} + \frac{1}{3}
$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{21fg\sqrt{a+a\sin(e+fx)}} - 2a$$

$$= \frac{22ac^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{105fg\sqrt{a+a\sin(e+fx)}} + 2a$$

$$= \frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + $$

$$= \frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + $$

$$= \frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + $$

$$= \frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + $$

Mathematica [C] time = 1.93, size = 281, normalized size = 0.97

$$-\frac{c^3ge^{-4i(e+fx)}\left(e^{i(e+fx)}-i\right)\left(\sqrt{1+e^{2i(e+fx)}}\left(-180ie^{i(e+fx)}+238e^{2i(e+fx)}-540ie^{3i(e+fx)}+3696e^{4i(e+fx)}-540ie^{5i(e+fx)}\right.\right.}{2520f\left(e^{i(e+fx)}+i\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] -1/2520*(c^3*(-I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*
I)*(e + f*x))]*(-35 - (180*I)*E^(I*(e + f*x)) + 238*E^((2*I)*(e + f*x)) - (
540*I)*E^((3*I)*(e + f*x)) + 3696*E^((4*I)*(e + f*x)) - (540*I)*E^((5*I)*(e
 + f*x)) - 238*E^((6*I)*(e + f*x)) - (180*I)*E^((7*I)*(e + f*x)) + 35*E^((8
*I)*(e + f*x))) - 2464*E^((6*I)*(e + f*x))*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(E^((4*I)*(e + f*x))*(I
 + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[c - c*Sin[e + f*x]
])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c^2g\cos\left(fx+e\right)^3+2\,c^2g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^2g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*g*cos(f*x + e)^3 + 2*c^2*g*cos(f*x + e)*sin(f*x + e) - 2*c^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(
f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

maple [C] time = 0.64, size = 392, normalized size = 1.35

$$-\frac{2\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\left(231i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\cos\left(fx+e\right)\sin\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/315/f*(-c*(sin(f*x+e)-1))^(5/2)*(231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*cos(f*x+e)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+35*cos
(f*x+e)^6+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)+90*sin(f*x+e)*cos(f*x+e)^4-112*cos(f*x+e)^4-154*cos(f*
x+e)^2+231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f
*x+e)^2+2*sin(f*x+e)-2)/sin(f*x+e)/cos(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2)*(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.90 $\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=235

$$\frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6ac^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a(c-c\sin(e}{7fg}$$

```
[Out] -2/7*a*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e))^(1/
2)+2/5*a*c^2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)+6/5*a*c^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipt
icE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+
a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+6/35*a*c*(g*cos(f*x+e))^(5/2)*(c
-c*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.13, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6ac^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a(c-c\sin(e}{7fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (2*a*c^2*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (6*a*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
) + (6*a*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(35*f*g*Sqrt[a
+ a*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)
)/(7*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{7fg\sqrt{a+a\sin(e+fx)}}+\frac{1}{7}($$

$$= \frac{6ac(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{35fg\sqrt{a+a\sin(e+fx)}}-\frac{2a(g}{}$$

$$= \frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{6a}{}$$

$$= \frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{6a}{}$$

$$= \frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{6a}{}$$

$$= \frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{6a}{}$$

Mathematica [C] time = 1.72, size = 255, normalized size = 1.09

$$\frac{c^2 g e^{-3i(e+fx)} \left(e^{i(e+fx)} - i\right) \left(112 e^{5i(e+fx)} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(e+fx)}\right) + \sqrt{1+e^{2i(e+fx)}} \left(-14 e^{i(e+fx)} + 15 i e^{2i(e+fx)} - 168 e^{3i(e+fx)} \cdots\right.\right.}{140 f \left(e^{i(e+fx)} + i\right) \sqrt{1+e^{2i(e+fx)}} \sqrt{c - c \cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] (c^2*(-I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*I)*(e +
f*x))]*(5*I - 14*E^(I*(e + f*x)) + (15*I)*E^((2*I)*(e + f*x)) - 168*E^((3*I
)*(e + f*x)) + (15*I)*E^((4*I)*(e + f*x)) + 14*E^((5*I)*(e + f*x)) + (5*I)*
E^((6*I)*(e + f*x))) + 112*E^((5*I)*(e + f*x))*Hypergeometric2F1[1/2, 3/4,
7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(140*E^((3*I)*(e +
f*x))*(I + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[c - c*Sin[
e + f*x]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(cg\cos\left(fx+e\right)\sin\left(fx+e\right) - cg\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right) \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(c*g*cos(f*x + e)*sin(f*x + e) - c*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}} \sqrt{a\sin\left(fx+e\right)+a} \left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

maple [C] time = 0.62, size = 372, normalized size = 1.58

$$-\frac{2\left(21i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos\left(fx+e\right)\sin\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)-21i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos}{\cos}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/35/f*(21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*si
n(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*El
lipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+5*sin(f*x+e)*cos(f*x+e)^4+21*I*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-
7*cos(f*x+e)^4-14*cos(f*x+e)^2+21*cos(f*x+e))*(-c*(sin(f*x+e)-1))^(3/2)*(g*
cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/sin(f*x+e)/cos(f*
x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.91 $\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}$

Optimal. Leaf size=178

$$-\frac{2a\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}}+\frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{6acg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e\right.}{5f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] 2/5*a*c*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)+6/5*a*c*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin
(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f
*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-2/5*a*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x
+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{2a\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}}+\frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{6acg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e\right.}{5f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] (2*a*c*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]]) + (6*a*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[
(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) -
(2*a*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*f*g*Sqrt[a + a*Sin
[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5fg\sqrt{a+a\sin(e+fx)}}+\frac{1}{5}(3a$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} - \frac{2a}{}$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} - \frac{2a}{}$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} - \frac{2a}{}$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} + \frac{6a}{}$$

Mathematica [C] time = 2.39, size = 249, normalized size = 1.40

$$\frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\sec^3(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{3/2}\Big(12(\cos(fx)-i\sin(fx))\sqrt{i}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Csc[e/2]*Sec[e/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sin[e
+ f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-11*Cos[f*x] - 13*Cos[2*e + f*x] + Cos[2
*e + 3*f*x] - Cos[4*e + 3*f*x] + 12*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^(
(2*I)*f*x)*(Cos[e] + I*Sin[e])^2)]*(Cos[f*x] - I*Sin[f*x])*Sqrt[1 + Cos[2*(
e + f*x)] + I*Sin[2*(e + f*x)]] + 4*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((
2*I)*f*x)*(Cos[e] + I*Sin[e])^2)]*(Cos[f*x] + I*Sin[f*x])*Sqrt[1 + Cos[2*(e
 + f*x)] + I*Sin[2*(e + f*x)]]))/(40*f)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c), x)
```

maple [C] time = 0.67, size = 346, normalized size = 1.94

$$\frac{2\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\left(3i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\cos\left(fx+e\right)\sin\left(fx+e\right)\text{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/5/f*(-c*(sin(f*x+e)-1))^(1/2)*(3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(
f*x+e),I)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*si
n(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF(I*(-
1+cos(f*x+e))/sin(f*x+e),I)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-cos(f*
x+e)^4-2*cos(f*x+e)^2+3*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))
^(1/2)/sin(f*x+e)/cos(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.92 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=122

$$\frac{2ag\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2/3*a*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/
2)+2*a*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*
f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f*x+e)
)^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{2ag\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-2*a*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Si
n[e + f*x]]) + (2*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e
+ f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
```

```
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx &= -\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+a\int\frac{(g\,\mathrm{c}}{\sqrt{a+a\sin(e+}}\\ &= -\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{(ag\cos(e+fx))\int}{\sqrt{a+a\sin(e+fx)}}\\ &= -\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{\left(ag\sqrt{\cos(e+fx)}\right.}{\sqrt{a+a\sin}}\\ &= -\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{2ag\sqrt{\cos(e+fx)}}{f\sqrt{a+a\sin(}}\end{aligned}$$

Mathematica [C] time = 1.92, size = 197, normalized size = 1.61

$$-\frac{ig\sqrt{ice^{-i(e+fx)}\left(e^{i(e+fx)}-i\right)^2}\sqrt{ge^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)}\left(4e^{3i(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)-i\sqrt{1+e^{2i(e+fx)}}\left(-6ie^{i}\right.\right.}{3cf\left(1+e^{2i(e+fx)}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] ((-1/3*I)*Sqrt[(I*c*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*g*Sqrt[((1 +
 E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*((-I)*Sqrt[1 + E^((2*I)*(e + f*x)
```

```
)]*(1 - (6*I)*E^(I*(e + f*x)) + E^((2*I)*(e + f*x))) + 4*E^((3*I)*(e + f*x)
)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e
 + f*x])])/(c*(1 + E^((2*I)*(e + f*x)))^(3/2)*f)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g\cos\left(fx+e\right)}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g*cos(f*x + e)/(c*sin(f*x + e) - c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.58, size = 362, normalized size = 2.97

$$\frac{2\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\left(3i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\cos\left(fx+e\right)\sin\left(fx+e\right)\text{EllipticF}\left(\frac{i(-1}{s}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/3/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(1/2)*(3*I*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*
(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*(1/(cos(f*x+e)+1))^(1/2
```

```
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2+3*cos(f*x+e))/(1+sin(f
*x+e))/sin(f*x+e)/cos(f*x+e)/(-c*(sin(f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/sqrt(-c*sin(f*x +
 e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(1/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.93 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{6ag\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 4*a*(g*cos(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-
6*a*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x
+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c/f/(a+a*sin(f*x+e))
^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2850, 2842, 2640, 2639}

$$\frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{6ag\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(3/2)) - (6*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
 f*x)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
```

```
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{(3a)\int \frac{(g\cos(e+}{\sqrt{a+a\sin(e+fx)}}}{c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{(3ag\cos(e+fx))\int}{c\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{\left(3ag\sqrt{\cos(e+fx)}\right.}{c\sqrt{a+a\sin}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{6ag\sqrt{\cos(e+fx)}\sqrt{}}{cf\sqrt{a+a\sin(e}}$$

Mathematica [C] time = 1.69, size = 211, normalized size = 1.72

$$-\frac{2g\sqrt{ge^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)}\left(2e^{2i(e+fx)}\left(e^{i(e+fx)}-i\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)+\sqrt{1+e^{2i(e+fx)}}\left(i-5e^{i(e+fx)}\right)\right)\sqrt{a(}}{cf\left(e^{i(e+fx)}+i\right)\sqrt{1+e^{2i(e+fx)}}\sqrt{ice^{-i(e+fx)}\left(e^{i(e+fx)}-i\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (-2*g*Sqrt[((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*((I - 5*E^(I*(e +
 f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))] + 2*E^((2*I)*(e + f*x))*(-I + E^(I*(e
 + f*x)))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1
 + Sin[e + f*x]))])/(c*Sqrt[(I*c*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*
(I + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g\cos\left(fx+e\right)}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g*cos(f*x + e)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.67, size = 2835, normalized size = 23.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e))*(-10*cos(f*x+e)^2-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*sin(f*x+e)+6*I*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*I*s
in(f*x+e)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*cos(f*x+e)^3-cos(f*x+e)^
4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)
```

+1)^2)^(1/2)-1)/sin(f*x+e)^2)-4*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+4*
cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-6*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x
+e)^2)+6*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e
)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+ln(-2*(2*cos(f*x+e)^2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
*sin(f*x+e)-ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-4*cos(f*x+e)*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)
/sin(f*x+e)^2)+4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+cos(f*x+e)^4*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)+6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+2*cos
(f*x+e)^2*sin(f*x+e)+sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-si
n(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e
)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*sin(f*x+e)*cos(f*x+e)^2*l
n(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(3/2)-3*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)+3*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-3*sin(f*x+e)*cos(f*
x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-12*I*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+
12*I*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*I*cos(f*x+e)^3*EllipticE(I*(-

```
1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)+6*I*cos(f*x+e)^3*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*I*cos(f*x+e)*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)+6*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)
/sin(f*x+e)^2)+(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*
x+e)))^(1/2)/(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)
/(-c*(sin(f*x+e)-1))^(3/2)/sin(f*x+e)/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

3.94 $\int \frac{(g \cos(e+fx))^{3/2} \sqrt{a+a \sin(e+fx)}}{(c-c \sin(e+fx))^{5/2}} \, dx$

Optimal. Leaf size=182

$$\frac{6ag\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{6a(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{4a}{5fg\sqrt{a\sin(e+}}$$

```
[Out] 4/5*a*(g*cos(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
)-6/5*a*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^
(1/2)+6/5*a*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin
(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^2/f/(a+a*s
in(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.88, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{6ag\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{6a(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{4a}{5fg\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(5/2)) - (6*a*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*Sqrt[a + a*Sin[e + f*
x]]*(c - c*Sin[e + f*x])^(3/2)) + (6*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e +
f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} - \frac{(3a)\int \frac{(g\cos(e\cdots}{\sqrt{a+a\sin(e+fx)}\,(\cdots}}{5c\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} - \frac{6a(g\,\mathrm{co}\cdots}{5cfg\sqrt{a+a\sin(e+\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} - \frac{6a(g\,\mathrm{co}\cdots}{5cfg\sqrt{a+a\sin(e+\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} - \frac{6a(g\,\mathrm{co}\cdots}{5cfg\sqrt{a+a\sin(e+\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} - \frac{6a(g\,\mathrm{co}\cdots}{5cfg\sqrt{a+a\sin(e+\cdots}}$$

Mathematica [C] time = 2.08, size = 229, normalized size = 1.26

$$\frac{4ig\sqrt{ge^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)}\left(e^{i(e+fx)}\left(e^{i(e+fx)}-i\right)^3 {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)+\left(4ie^{i(e+fx)}-3e^{2i(e+fx)}+5\right)\sqrt{1+e^{2i(e\cdots}}\cdots}{5cf\left(e^{i(e+fx)}+i\right)\sqrt{1+e^{2i(e+fx)}}\left(ice^{-i(e+fx)}\left(e^{i(e+fx)}-i\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (((4*I)/5)*g*Sqrt[((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*((5 + (4*I
)*E^(I*(e + f*x)) - 3*E^((2*I)*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))] +
E^(I*(e + f*x))*(-I + E^(I*(e + f*x)))^3*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(c*((I*c*(-I + E^(I*(e +
f*x)))^2)/E^(I*(e + f*x)))^(3/2)*(I + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e
 + f*x))]*f)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g\cos\left(fx+e\right)}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g*cos(f*x + e)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 -
 4*c^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.56, size = 2040, normalized size = 11.21

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/10/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(1/2)*(-1+cos(f*x+e))^3*(si
n(f*x+e)-1)*(-12*sin(f*x+e)*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)+12*I*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f
*x+e),I)-12*I*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))
/sin(f*x+e),I)+8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-8*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-5*cos(f*x+e)^3*ln(-2*(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+5*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-20*cos(f*x+e)^3*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)+12*I*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+24*I*sin(f*x+e)*cos(f*x+e)*EllipticF(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-24*I*sin(f*x+e)*cos(f*x+
e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-12*I*sin(
f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
```

```
)^(1/2)-4*sin(f*x+e)*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-5*cos(
f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*si
n(f*x+e)+5*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)*sin(f*x+e)-12*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)+12*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)+5*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)-5*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/
sin(f*x+e)^2)+8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+20*cos(f*x+
e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+24*I*cos(f*x+e)^3*EllipticF(I*(-1+c
os(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-24*I*cos(f*x+e)^3*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-24*I*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)+24*I*EllipticE(I*(-
1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+12*I*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-12*I*Ellipt
icE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e))/(1+sin
(f*x+e))/sin(f*x+e)^7/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/(-c*(sin(f*x+e)-
1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\cos\left(e+f\,x\right)\right)^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

3.95 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=237

$$\frac{2ag\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{}{15cfg\sqrt{a\sin}}$$

```
[Out] 4/9*a*(g*cos(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2
)-2/15*a*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))
^(1/2)-2/15*a*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(
f*x+e))^(1/2)+2/15*a*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Elli
pticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^3
/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.16, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$-\frac{2a(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{2ag\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{}{15cfg\sqrt{a\,s}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(7/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(15*c*f*g*Sqrt[a + a*Sin[e + f
*x]]*(c - c*Sin[e + f*x])^(5/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(15*c^2*f*g
*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (2*a*g*Sqrt[Cos[e +
 f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3*f*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

x]

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} - \frac{a\int \frac{(g\cos(e+fx}{\sqrt{a+a\sin(e+fx)}\,(c-c}}{3c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} - \frac{2a(g\,\mathrm{cc}}{15cfg\sqrt{a+a\sin(e-}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} - \frac{2a(g\,\mathrm{cc}}{15cfg\sqrt{a+a\sin(e-}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} - \frac{2a(g\,\mathrm{cc}}{15cfg\sqrt{a+a\sin(e-}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} - \frac{2a(g\,\mathrm{cc}}{15cfg\sqrt{a+a\sin(e-}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} - \frac{2a(g\,\mathrm{cc}}{15cfg\sqrt{a+a\sin(e-}}$$

Mathematica [C] time = 2.18, size = 256, normalized size = 1.08

$$\frac{4e^{3i(e+fx)}\left(ge^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\right)^{3/2}\left(\left(e^{i(e+fx)}-i\right)^5\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)+\sqrt{1+e^{2i(e+fx)}}\left(e^{i(e+fx)}+15ie^{2i(e+fx)}\right.\right.}{45c^3f\left(e^{i(e+fx)}-i\right)^4\left(e^{i(e+fx)}+i\right)\left(1+e^{2i(e+fx)}\right)^{3/2}\sqrt{ice^{-i(e+fx)}\left(e^{i(e+fx)}-i\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e +
f*x])^(7/2),x]
```

```
[Out] (4*E^((3*I)*(e + f*x))*(((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x)))^(3/2
)*(Sqrt[1 + E^((2*I)*(e + f*x))]*(-29*I + E^(I*(e + f*x)) + (15*I)*E^((2*I)
*(e + f*x)) - 3*E^((3*I)*(e + f*x))) + (-I + E^(I*(e + f*x)))^5*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(4
5*c^3*(-I + E^(I*(e + f*x)))^4*Sqrt[(I*c*(-I + E^(I*(e + f*x)))^2)/E^(I*(e
+ f*x))]*(I + E^(I*(e + f*x)))*(1 + E^((2*I)*(e + f*x)))^(3/2)*f)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g\cos\left(fx+e\right)}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g*cos(f*x + e)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4
*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.56, size = 966, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/45/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*cos(f*x+e)
-sin(f*x+e)-cos(f*x+e)+1)*(-9*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*
x+e)^2-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-3*I*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*sin(f*x+e)*cos(f*x+e)^4+9*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x
+e)^2+6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-12*I*cos(
f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)
*cos(f*x+e)^4-6*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+12*I*cos(f*x+e)^2*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)+3*cos(f*x+e)^4+6*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f
```

```
*x+e),I)+6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+6*cos(f*x+e)^2*sin(f*x+e)
+10*cos(f*x+e)^3-16*sin(f*x+e)*cos(f*x+e)-19*cos(f*x+e)^2+10*sin(f*x+e)-4*c
os(f*x+e)+10)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(1+sin(f*x+e))/(-c*(sin(f*x+e)-
1))^(7/2)/sin(f*x+e)^5/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(7/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

3.96 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=292

$$\frac{2ag\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{65c^4f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a(g\cos(e+fx))^{5/2}}{65c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{}{65c^2fg\sqrt{a}}$$

```
[Out] 4/13*a*(g*cos(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(1/
2)-2/39*a*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)
)^(1/2)-2/65*a*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin
(f*x+e))^(1/2)-2/65*a*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))^(3/2)/(
a+a*sin(f*x+e))^(1/2)+2/65*a*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2
*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(
1/2)/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.46, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$-\frac{2a(g\cos(e+fx))^{5/2}}{65c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{2a(g\cos(e+fx))^{5/2}}{65c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}+\frac{2ag\sqrt{\cos(e}}{65c^4f\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(13*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(9/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(39*c*f*g*Sqrt[a + a*Sin[e +
f*x]]*(c - c*Sin[e + f*x])^(7/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(65*c^2*f*
g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) - (2*a*(g*Cos[e + f*
x])^(5/2))/(65*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
 + (2*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
)/(65*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}} - \frac{(3a)\int \frac{(g\,\mathrm{co}}{\sqrt{a+a\sin(e+f}}}{1}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}} - \frac{2a(g}{39cfg\sqrt{a+a\sin(}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}} - \frac{2a(g}{39cfg\sqrt{a+a\sin(}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}} - \frac{2a(g}{39cfg\sqrt{a+a\sin(}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}} - \frac{2a(g}{39cfg\sqrt{a+a\sin(}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}} - \frac{2a(g}{39cfg\sqrt{a+a\sin(}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}} - \frac{2a(g}{39cfg\sqrt{a+a\sin(}}$$

Mathematica [C] time = 2.63, size = 291, normalized size = 1.00

$$\frac{4e^{3i(e+fx)}\left(ge^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\right)^{3/2}\left(\sqrt{1+e^{2i(e+fx)}}\left(149ie^{i(e+fx)}+44e^{2i(e+fx)}-64ie^{3i(e+fx)}+21e^{4i(e+fx)}+3ie^{5i(}\right.\right.}{195c^4f\left(1-ie^{i(e+fx)}\right)\left(e^{i(e+fx)}-i\right)^6\left(1+e^{2i(e+fx)}\right)^{3/2}\sqrt{ice^{-i(}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e +
f*x])^(9/2),x]
```

```
[Out] (4*E^((3*I)*(e + f*x))*(((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x)))^(3/2
)*(Sqrt[1 + E^((2*I)*(e + f*x))]*(-1 + (149*I)*E^(I*(e + f*x)) + 44*E^((2*I
)*(e + f*x)) - (64*I)*E^((3*I)*(e + f*x)) + 21*E^((4*I)*(e + f*x)) + (3*I)*
E^((5*I)*(e + f*x))) - I*(-I + E^(I*(e + f*x)))^7*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(195*c^4*(1 - I*
E^(I*(e + f*x)))*(-I + E^(I*(e + f*x)))^6*Sqrt[(I*c*(-I + E^(I*(e + f*x)))^
2)/E^(I*(e + f*x))]*(1 + E^((2*I)*(e + f*x)))^(3/2)*f)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g\cos\left(fx+e\right)}{5\,c^5\cos\left(fx+e\right)^4-20\,c^5\cos\left(fx+e\right)^2+16\,c^5-\left(c^5\cos\left(fx+e\right)^4-12\,c^5\cos\left(fx+e\right)^2+16\,c^5\right)\sin\left(f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g*cos(f*x + e)/(5*c^5*cos(f*x + e)^4 - 20*c^5*cos(f*x + e)^2 + 16*c^5
 - (c^5*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 + 16*c^5)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.61, size = 1126, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 2/195/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*cos(f*x+e
)-sin(f*x+e)-cos(f*x+e)+1)*(9*I*sin(f*x+e)*cos(f*x+e)^4*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)-3*I*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*cos(f*x+e)^6*Elliptic
F(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)+21*I*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-18
*I*cos(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+12*I*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-12
*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
```

```
f*x+e)/(cos(f*x+e)+1))^(1/2)-9*I*sin(f*x+e)*cos(f*x+e)^4*EllipticE(I*(-1+co
s(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)-12*I*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-21*I*sin(f*x+e)*cos(f*x+e
)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*sin(f*x+e)*cos(f*x+e)^4+27*I*cos(f*x+e)^2*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)-27*I*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5*sin(
f*x+e)*cos(f*x+e)^3+9*cos(f*x+e)^4+18*I*cos(f*x+e)^4*EllipticE(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)+12*I*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+10*cos(f*x+e)^2*sin(f*x+e)+24
*cos(f*x+e)^3-42*sin(f*x+e)*cos(f*x+e)-45*cos(f*x+e)^2+30*sin(f*x+e)-18*cos
(f*x+e)+30)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(1+sin(f*x+e))/(-c*(sin(f*x+e)-1)
)^(9/2)/sin(f*x+e)^5/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(9/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
9/2),x)
```

```
[Out] Timed out
```

3.97 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=352

$$\frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14a^2c^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2a^2c^2\sqrt{c-}}{1}$$

```
[Out] 2/33*a^2*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e))
^(1/2)-14/99*a^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f/g/(a+a*sin(f
*x+e))^(1/2)-2/11*a*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*
x+e))^(1/2)/f/g+14/45*a^2*c^3*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/
2)/(c-c*sin(f*x+e))^(1/2)+14/15*a^2*c^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(
1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*co
s(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2/15*a^2*c^
2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.73, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2a^2c^2\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}}+\frac{14a^2c^3g\sqrt{\cos(e+fx)}}{15f\sqrt{a\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (14*a^2*c^3*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (14*a^2*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]
*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]) + (2*a^2*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(15
*f*g*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin
[e + f*x])^(3/2))/(33*f*g*Sqrt[a + a*Sin[e + f*x]]) - (14*a^2*(g*Cos[e + f*
x])^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(99*f*g*Sqrt[a + a*Sin[e + f*x]]) - (
2*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5
/2))/(11*f*g)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}\,(c-}{11fg}$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{99fg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2a^2c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{33fg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2a^2c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{15fg\sqrt{a+a\sin(e+fx)}} +$$

$$= \frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 1.31, size = 193, normalized size = 0.55

$$\frac{c^2(\sin(e+fx)-1)^2(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{3/2}\left(3696E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(e}\right.}{3960f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (c^2*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3
/2)*Sqrt[c - c*Sin[e + f*x]]*(3696*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e +
 f*x]]*(450*Cos[e + f*x] + 225*Cos[3*(e + f*x)] + 45*Cos[5*(e + f*x)] + 836
*Sin[2*(e + f*x)] + 110*Sin[4*(e + f*x)])))/(3960*f*Cos[e + f*x]^(3/2)*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3
)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(ac^2g\cos\left(fx+e\right)^3\sin\left(fx+e\right)-ac^2g\cos\left(fx+e\right)^3\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*g*cos(f*x + e)^3*sin(f*x + e) - a*c^2*g*cos(f*x + e)^3)*sq
rt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.63, size = 382, normalized size = 1.09

$$-\frac{2\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\left(45\left(\cos^6\left(fx+e\right)\right)\sin\left(fx+e\right)+231i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\cos\left(fx+e\right)\sin\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/495/f*(-c*(sin(f*x+e)-1))^(5/2)*(45*cos(f*x+e)^6*sin(f*x+e)+231*I*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)-55*cos(f*x+e)^6+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-22*cos(f*x+e)^4-154*cos(f*x
+e)^2+231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(3/2)/(sin(f*
x+e)-1)/sin(f*x+e)/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e
) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

3.98 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=295

$$\frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14a^2c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a^2(c-c\sin\ldots}{9f\ldots}$$

```
[Out] -2/9*a^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e))^(
1/2)-2/9*a*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/
2)/f/g+14/45*a^2*c^2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)+14/15*a^2*c^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))
^(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2/15*a^2*c*(g*cos(f*
x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.50, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14a^2c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a^2(c-c\sin\ldots}{9f\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (14*a^2*c^2*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (14*a^2*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]
*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]) + (2*a^2*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(15*f
*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e +
 f*x])^(3/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2
)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))/(9*f*g)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}\,(c-}{9fg}$$

$$= -\frac{2a^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{9fg\sqrt{a+a\sin(e+fx)}} -$$

$$= \frac{2a^2c(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{15fg\sqrt{a+a\sin(e+fx)}} - \frac{2a}{}$$

$$= \frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} +$$

$$= \frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} +$$

$$= \frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} +$$

$$= \frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} +$$

Mathematica [A] time = 0.69, size = 113, normalized size = 0.38

$$-\frac{c(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{3/2}\left(168E\left(\frac{1}{2}(e+fx)\middle|2\right)+(38\sin(2(e\right.}{180f\cos^{\frac{9}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/180*(c*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))
^(3/2)*Sqrt[c - c*Sin[e + f*x]]*(168*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e
 + f*x]]*(38*Sin[2*(e + f*x)] + 5*Sin[4*(e + f*x)])))/(f*Cos[e + f*x]^(9/2)
)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,acg\cos\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*a*c*g*cos(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

maple [C] time = 0.56, size = 356, normalized size = 1.21

$$-\frac{2\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\left(21i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\cos\left(fx+e\right)\sin\left(fx+e\right)\mathrm{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/45/f*(-c*(sin(f*x+e)-1))^(3/2)*(21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)-21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(
f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5*cos(f*x
+e)^6+21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-21*I*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)+2*cos(f*x+e)^4+14*cos(f*x+e)^2-21*cos(f*x+e))*(g*cos(f*x+e))
^(3/2)*(a*(1+sin(f*x+e)))^(3/2)/sin(f*x+e)/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

3.99 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}$

Optimal. Leaf size=235

$$-\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6a^2cg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c(a\sin(e+}{7f}$$

```
[Out] 2/7*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(1/2
)-2/5*a^2*c*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)+6/5*a^2*c*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipti
cE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a
*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-6/35*a*c*(g*cos(f*x+e))^(5/2)*(a+
a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.14, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6a^2cg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c(a\sin(e+}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-2*a^2*c*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (6*a^2*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]
]) - (6*a*c*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(35*f*g*Sqrt[c
 - c*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(7*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{7fg\sqrt{c-c\sin(e+fx)}} + \frac{1}{7}(3$$

$$= -\frac{6ac(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{35fg\sqrt{c-c\sin(e+fx)}} + \frac{2c}{}$$

$$= -\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{6}{}$$

$$= -\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{6}{}$$

$$= -\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{6}{}$$

$$= -\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{6}{}$$

Mathematica [C] time = 7.60, size = 257, normalized size = 1.09

$$\frac{ia^2ge^{-3i(e+fx)}\left(e^{i(e+fx)}+i\right)\left(112ie^{5i(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)+\sqrt{1+e^{2i(e+fx)}}\left(-14ie^{i(e+fx)}+15e^{2i(e+fx)}-168ie\right.\right.}{140f\left(e^{i(e+fx)}-i\right)\sqrt{1+e^{2i(e+fx)}}\sqrt{a(\sin}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] ((I/140)*a^2*(I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*I
)*(e + f*x))]*(5 - (14*I)*E^(I*(e + f*x)) + 15*E^((2*I)*(e + f*x)) - (168*I
)*E^((3*I)*(e + f*x)) + 15*E^((4*I)*(e + f*x)) + (14*I)*E^((5*I)*(e + f*x))
 + 5*E^((6*I)*(e + f*x))) + (112*I)*E^((5*I)*(e + f*x))*Hypergeometric2F1[1
/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[c - c*Sin[e + f*x]])/(E^((3*I)*(e
 + f*x))*(-I + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[a*(1 +
 Sin[e + f*x])])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ag\cos\left(fx+e\right)\sin\left(fx+e\right)+ag\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral((a*g*cos(f*x + e)*sin(f*x + e) + a*g*cos(f*x + e))*sqrt(g*cos(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

maple [C] time = 0.60, size = 372, normalized size = 1.58

$$-\frac{2\left(-21i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\cos\left(fx+e\right)\sin\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)+21i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{}{c}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/35/f*(-21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x
+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+21*I*Ellipti
cE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5*sin(f*x+e)*cos(f*x+e)^4-21*I*Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
+7*cos(f*x+e)^4+14*cos(f*x+e)^2-21*cos(f*x+e))*(-c*(sin(f*x+e)-1))^(1/2)*(g
*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(3/2)/(1+sin(f*x+e))/sin(f*x+e)/cos(f
*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(1/2), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.100 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=180

$$-\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{14a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a\sqrt{a\sin(e\cdots}}{5fg\cdots}$$

```
[Out] -14/15*a^2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))
^(1/2)+14/5*a^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE
(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*s
in(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-2/5*a*(g*cos(f*x+e))^(5/2)*(a+a*sin
(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{14a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a\sqrt{a\sin(e\cdots}}{5fg\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (-14*a^2*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
) - (2*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(5*f*g*Sqrt[c - c
*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg\sqrt{c-c\sin(e+fx)}}+\frac{1}{5}(7a)\int \frac{(g\cos(e+$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{2a(g\cos(e+fx}{5fg\sqrt{c}}$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{2a(g\cos(e+fx}{5fg\sqrt{c}}$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{2a(g\cos(e+fx}{5fg\sqrt{c}}$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{14a^2g\sqrt{\cos(e+}}{5f\sqrt{a+a}}$$

Mathematica [A] time = 0.77, size = 148, normalized size = 0.82

$$-\frac{(a(\sin(e+fx)+1))^{3/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{\cos(e+fx)}\,(3\sin(2(e+fx))+\right.}{15f\cos^{\frac{3}{2}}(e+fx)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/Sqrt[c - c*Si
n[e + f*x]],x]
```

```
[Out] -1/15*((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 +
 Sin[e + f*x]))^(3/2)*(-42*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(
20*Cos[e + f*x] + 3*Sin[2*(e + f*x)])))/(f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(ag\cos\left(fx+e\right)\sin\left(fx+e\right)+ag\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)}}{c\sin\left(fx+e\right)-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a*g*cos(f*x + e)*sin(f*x + e) + a*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) -
 c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.54, size = 384, normalized size = 2.13

$$-\frac{2\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(21i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\cos\left(fx+e\right)\sin\left(fx+e\right)\text{EllipticE}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/15/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(3/2)*(21*I*EllipticE(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(
```

```
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)+21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-21*I*EllipticF(I*(-1+c
os(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)-3*cos(f*x+e)^4+10*cos(f*x+e)^2*sin(f*x+e)+24*cos(f*x+e)
^2-21*cos(f*x+e))/(2*sin(f*x+e)-cos(f*x+e)^2+2)/sin(f*x+e)/cos(f*x+e)/(-c*(
sin(f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x
 + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(1/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.101 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=182

$$\frac{14a^2(g\cos(e+fx))^{5/2}}{3cfg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{14a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{4a\sqrt{a\sin(e+}}{fg(c-}$$

```
[Out] 4*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(3/2)+
14/3*a^2*(g*cos(f*x+e))^(5/2)/c/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))
^(1/2)-14*a^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(s
in(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c/f/(a+a*s
in(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.88, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$\frac{14a^2(g\cos(e+fx))^{5/2}}{3cfg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{14a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{4a\sqrt{a\sin(e+}}{fg(c-}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(f*g*(c - c*Sin[e + f
*x])^(3/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(3*c*f*g*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e
+ f*x]]*EllipticE[(e + f*x)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} - \frac{(7a)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+}}{\sqrt{c-c\sin(e+}}}{c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{14a^2(g\cos(e}{3cfg\sqrt{a+a\sin(e+fx}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{14a^2(g\cos(e}{3cfg\sqrt{a+a\sin(e+fx}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{14a^2(g\cos(e}{3cfg\sqrt{a+a\sin(e+fx}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{14a^2(g\cos(e}{3cfg\sqrt{a+a\sin(e+fx}}$$

Mathematica [A] time = 1.75, size = 207, normalized size = 1.14

$$-\frac{2(a(\sin(e+fx)+1))^{3/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.\right.}{3cf(\sin(e+fx)-1)\cos^{\frac{3}{2}}(e+fx)\sqrt{c-c\sin}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(-21*Ell
ipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + Sqrt[Cos[e +
 f*x]]*(Cos[(e + f*x)/2]*(12 + Cos[e + f*x]) - (-12 + Cos[e + f*x])*Sin[(e
+ f*x)/2]))*(a*(1 + Sin[e + f*x]))^(3/2))/(3*c*f*Cos[e + f*x]^(3/2)*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^3*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(ag\cos\left(fx+e\right)\sin\left(fx+e\right)+ag\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right.}}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a*g*cos(f*x + e)*sin(f*x + e) + a*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)
^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.57, size = 2894, normalized size = 15.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/3/f*(-1+cos(f*x+e))*(-33*cos(f*x+e)^2-3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-1)/sin(f*x+e)^2)-3*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-21*I*cos
(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)+21*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-42*I*cos(f*x+e)^2*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)+42*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*cos(f*x+e)
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1
+cos(f*x+e))/sin(f*x+e),I)+8*cos(f*x+e)^3+21*I*sin(f*x+e)*cos(f*x+e)^2*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)+21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*
x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-
```

cos(f*x+e)^4-12*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+12*cos(f*x+e)^3*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-1)/sin(f*x+e)^2)-18*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2
)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+18*cos(
f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+9*cos(f*x+e)^2*sin(f*x+e)+3*ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(3/2)*sin(f*x+e)-3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-12*cos(f*
x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+12*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*
cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+sin(f*x+e)*cos(f*x+e)^3+3*sin(f*x+e)
*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*sin(f*x+e)*cos(f*x+e)^3*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-1)/sin(f*x+e)^2)+9*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-9*
sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/s
in(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+9*sin(f*x+e)*cos(f*x+e)*l
n(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(3/2)-9*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2
)-21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-21*I*sin(f*x+e)*cos(
f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))
^(3/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+3*cos(

```
f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(3/2)/sin(f*x+e)/
cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

3.102 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{42a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{28a^2(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{4a\sqrt{a\sin(e}{5fg(}$$

```
[Out] -28/5*a^2*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)
)^(1/2)+4/5*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+
e))^(5/2)+42/5*a^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipt
icE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^2/f
/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2850, 2842, 2640, 2639}

$$\frac{42a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{28a^2(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{4a\sqrt{a\sin(e}{5fg(}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(5*f*g*(c - c*Sin[e +
 f*x])^(5/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*Sqrt[a + a*Sin[e +
 f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (42*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Co
s[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{(7a)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a}}{(c-c\sin(e+}}{5c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{28a^2(g\cos}{5cfg\sqrt{a+a\sin(e+f}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{28a^2(g\cos}{5cfg\sqrt{a+a\sin(e+f}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{28a^2(g\cos}{5cfg\sqrt{a+a\sin(e+f}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{28a^2(g\cos}{5cfg\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 1.39, size = 191, normalized size = 1.03

$$-\frac{a\sqrt{\cos(e+fx)}\sqrt{a(\sin(e+fx)+1)}\,(g\cos(e+fx))^{3/2}\left(8\sqrt{\cos(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)-2\sin\left(\frac{3}{2}(e+fx)\right)+c\right.\right.}{5c^2f(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] -1/5*(a*Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*Sqrt[a*(1 + Sin[e + f*x])
]*(-42*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 +
8*Sqrt[Cos[e + f*x]]*(Cos[(e + f*x)/2] + 2*Cos[(3*(e + f*x))/2] + Sin[(e +
f*x)/2] - 2*Sin[(3*(e + f*x))/2])))/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^3*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(ag\cos\left(fx+e\right)\sin\left(fx+e\right)+ag\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)}}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a*g*cos(f*x + e)*sin(f*x + e) + a*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x +
e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.55, size = 3499, normalized size = 18.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/5/f*(-1+cos(f*x+e))*(-38*cos(f*x+e)^2-10*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
```

```
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+10*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-1)/sin(f*x+e)^2)+10*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-21*I*s
in(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+5*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)
^2)*sin(f*x+e)*cos(f*x+e)^4-5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*c
os(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+
e)^4+21*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+9*cos(f*x+e)^3+
5*cos(f*x+e)^4-10*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(
2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+
e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+10*cos(f*x+e)^3*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)-40*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+40*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+5*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-co
s(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e
)^2)-5*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+46*cos(f*x+e)^2*sin(f*x+e)+63
*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-63*I*cos(f*x+e)^2*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)-21*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*cos(f*x+e)^4*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+co
s(f*x+e))/sin(f*x+e),I)+42*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-42*I*cos(
f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE
(I*(-1+cos(f*x+e))/sin(f*x+e),I)+10*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-
10*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*
```

```
x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-35*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x
+e)^2)+35*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-10*cos(f*x+e)^4*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/
sin(f*x+e)^2)-5*sin(f*x+e)*cos(f*x+e)^3+25*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-1)/sin(f*x+e)^2)-25*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+45
*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-45*sin(f*x+e)*cos(f*x+
e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+35*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(3/2)-35*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-42*I*cos(f*x+e)
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+42*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*sin(f*x+e)-63*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+63*I
*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(g*cos(f*x+e))^(3/2)*(a
*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*c
os(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(
5/2)/sin(f*x+e)/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
```

```
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

3.103 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=243

$$-\frac{14a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{}{45cfg\sqrt{}}$$

```
[Out] -28/45*a^2*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e
))^(1/2)+14/15*a^2*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a
*sin(f*x+e))^(1/2)+4/9*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c
-c*sin(f*x+e))^(7/2)-14/15*a^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))
^(1/2)/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.19, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{14a^2(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{14a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{}{45cfg\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(9*f*g*(c - c*Sin[e +
 f*x])^(7/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(45*c*f*g*Sqrt[a + a*Sin[e
+ f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(15*c
^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (14*a^2*g*Sqr
t[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3*f*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

x]

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{(7a)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+}}{(c-c\sin(e+fx}}{9c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{28a^2(g\cos}{45cfg\sqrt{a+a\sin(e+f}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{28a^2(g\cos}{45cfg\sqrt{a+a\sin(e+f}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{28a^2(g\cos}{45cfg\sqrt{a+a\sin(e+f}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{28a^2(g\cos}{45cfg\sqrt{a+a\sin(e+f}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{28a^2(g\cos}{45cfg\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 2.34, size = 218, normalized size = 0.90

$$\frac{a\sqrt{\cos(e+fx)}\sqrt{a(\sin(e+fx)+1)}(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}\left(-74\sin\left(\frac{1}{2}(e+fx)\right)+15\sin\left(\frac{3}{2}(e+fx)\right)+\right.\right.}{90c^3f(\sin(e+fx)-1)^3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(7/2),x]
```

```
[Out] (a*Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*Sqrt[a*(1 + Sin[e + f*x])]*(84
*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + Sqrt[C
os[e + f*x]]*(-74*Cos[(e + f*x)/2] - 15*Cos[(3*(e + f*x))/2] + 21*Cos[(5*(e
 + f*x))/2] - 74*Sin[(e + f*x)/2] + 15*Sin[(3*(e + f*x))/2] + 21*Sin[(5*(e
+ f*x))/2])))/(90*c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-1 + Sin[e
 + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ag\cos\left(fx+e\right)\sin\left(fx+e\right)+ag\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)}}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] integral((a*g*cos(f*x + e)*sin(f*x + e) + a*g*cos(f*x + e))*sqrt(g*cos(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x + e)^
4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x +
 e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.56, size = 2684, normalized size = 11.05

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/90/f*(cos(f*x+e)+1)*(-1+cos(f*x+e))^4*(-12*sin(f*x+e)*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+168*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+90*cos(f*x+e)^3*ln(-2*(2*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-90*cos(f*x+e)^3*ln(-(2*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+88*cos(f*x+e)^3*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-90*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+90*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+84*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)+80*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-164*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-180*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)+45*sin(f*x+e)*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-45*sin(f*x+e)*cos(f*x+e)^3*ln(-2*(2*
```

```
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+90*cos(f*x+e)*ln(-
2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)-90
*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)
*sin(f*x+e)+248*sin(f*x+e)*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+
336*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)-168*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
*sin(f*x+e)+168*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)*sin(f*x+e)-168*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)+336*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*cos(f*x+e)-336*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-336*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+80*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)*sin(f*x+e)-88*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+168*I*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+c
os(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-168*I*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+84*I*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)*cos(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-84
*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)^4*EllipticE(I*(-1+cos(f*x+e))
/sin(f*x+e),I)+168*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-168*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-84*I*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+84*I*si
n(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)-336*I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)+336*I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*
EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(sin(f*x+e)-1)*(a*(1+sin(f*x+e))
```

```
)^(3/2)*(g*cos(f*x+e))^(3/2)/sin(f*x+e)^9/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)/(-c*(sin(f*x+e)-1))^(7/2)/(2*sin(f*x+e)-cos(f*x+e)^2+2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(7/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

3.104 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{14a^2 g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{195c^4 f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{195c^3 fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{\cdots}{195c^2 f\cdots}$$

```
[Out] -28/117*a^2*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+
e))^(1/2)+14/195*a^2*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(5/2)/(a
+a*sin(f*x+e))^(1/2)+14/195*a^2*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e
))^(3/2)/(a+a*sin(f*x+e))^(1/2)+4/13*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e)
)^(1/2)/f/g/(c-c*sin(f*x+e))^(9/2)-14/195*a^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2
)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)
*(g*cos(f*x+e))^(1/2)/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.50, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{14a^2(g\cos(e+fx))^{5/2}}{195c^3 fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{195c^2 fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}-\frac{14a^2 g\sqrt{\cos(\cdots}}{195c^4 f\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(9/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(13*f*g*(c - c*Sin[e
+ f*x])^(9/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(117*c*f*g*Sqrt[a + a*Sin[
e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(19
5*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (14*a^2*(g
*Cos[e + f*x])^(5/2))/(195*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
f*x])^(3/2)) - (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[
(e + f*x)/2, 2])/(195*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x
]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{(7a)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+}}{(c-c\sin(e+fx}}{13c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{28a^2(g\cos}{117cfg\sqrt{a+a\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{28a^2(g\cos}{117cfg\sqrt{a+a\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{28a^2(g\cos}{117cfg\sqrt{a+a\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{28a^2(g\cos}{117cfg\sqrt{a+a\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{28a^2(g\cos}{117cfg\sqrt{a+a\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{28a^2(g\cos}{117cfg\sqrt{a+a\sin(e+}}$$

Mathematica [A] time = 6.47, size = 464, normalized size = 1.55

$$\frac{\sec(e+fx)(a(\sin(e+fx)+1))^{3/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9\left(\frac{28\sin\left(\frac{1}{2}(e+fx)\right)}{195\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+f\right.\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(9/2),x]
```

```
[Out] (-14*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(3/2))/(195*f*Cos[e + f*x]^(3/2)*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(9/2)) + ((g*C
os[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(14
/195 + 8/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6) - 64/(117*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^4) + 14/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
```

```
])^2) + (16*Sin[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
- (128*Sin[(e + f*x)/2])/(117*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) + (2
8*Sin[(e + f*x)/2])/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) + (28*Sin
[(e + f*x)/2])/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e +
 f*x]))^(3/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*
x])^(9/2))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ag\cos\left(fx+e\right)\sin\left(fx+e\right)+ag\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+\right.}}{5\,c^5\cos\left(fx+e\right)^4-20\,c^5\cos\left(fx+e\right)^2+16\,c^5-\left(c^5\cos\left(fx+e\right)^4-12\,c^5\cos\left(fx+e\right)^2+16\,c^5\right)\sin\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="fricas")
```

```
[Out] integral((a*g*cos(f*x + e)*sin(f*x + e) + a*g*cos(f*x + e))*sqrt(g*cos(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^5*cos(f*x + e
)^4 - 20*c^5*cos(f*x + e)^2 + 16*c^5 - (c^5*cos(f*x + e)^4 - 12*c^5*cos(f*x
 + e)^2 + 16*c^5)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.56, size = 1138, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -2/585/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(3/2)*(sin(f*x+e)*cos(f*x+
e)-sin(f*x+e)-cos(f*x+e)+1)*(-147*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin
(f*x+e),I)+84*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-84*I*(1/(cos(f*x+e)+1)
```

```
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f
*x+e),I)-126*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c
os(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+84*I*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)+126*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
cos(f*x+e)^4*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-84*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)+189*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+63*I*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+
e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-189*I*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e
))/sin(f*x+e),I)+147*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21
*I*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*sin(f*x+e)*cos(f*x+e)^4-63*I*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*
x+e)^4*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*cos(f*x+e)^6*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+
e))/sin(f*x+e),I)-160*sin(f*x+e)*cos(f*x+e)^3+63*cos(f*x+e)^4+265*cos(f*x+e
)^2*sin(f*x+e)-222*cos(f*x+e)^3+96*sin(f*x+e)*cos(f*x+e)+75*cos(f*x+e)^2-18
0*sin(f*x+e)+264*cos(f*x+e)-180)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(2*sin(f*x+e
)-cos(f*x+e)^2+2)/(-c*(sin(f*x+e)-1))^(9/2)/sin(f*x+e)^5/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(9/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
9/2),x)
```

```
[Out] Timed out
```

3.105 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=357

$$-\frac{14a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{1105c^5f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{1105c^4fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{}{1105c}$$

```
[Out] -28/221*a^2*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+
e))^(1/2)+14/663*a^2*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(7/2)/(a
+a*sin(f*x+e))^(1/2)+14/1105*a^2*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+
e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+14/1105*a^2*(g*cos(f*x+e))^(5/2)/c^4/f/g/(
c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+4/17*a*(g*cos(f*x+e))^(5/2)*(a
+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(11/2)-14/1105*a^2*g*(cos(1/2*f*x
+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*c
os(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^5/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(
f*x+e))^(1/2)
```

Rubi [A] time = 1.81, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{14a^2(g\cos(e+fx))^{5/2}}{1105c^4fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{1105c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}+\frac{}{663c^2fg\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(11/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(17*f*g*(c - c*Sin[e
+ f*x])^(11/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(221*c*f*g*Sqrt[a + a*Sin
[e + f*x]]*(c - c*Sin[e + f*x])^(9/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(6
63*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (14*a^2*(
g*Cos[e + f*x])^(5/2))/(1105*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(5/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(1105*c^4*f*g*Sqrt[a + a*S
in[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqr
t[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(1105*c^5*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{(7a)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+}}{(c-c\sin(e+fx}}{17c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{28a^2(g\cos}{221cfg\sqrt{a+a\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{28a^2(g\cos}{221cfg\sqrt{a+a\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{28a^2(g\cos}{221cfg\sqrt{a+a\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{28a^2(g\cos}{221cfg\sqrt{a+a\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{28a^2(g\cos}{221cfg\sqrt{a+a\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{28a^2(g\cos}{221cfg\sqrt{a+a\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{28a^2(g\cos}{221cfg\sqrt{a+a\sin(e+}}$$

Mathematica [A] time = 6.49, size = 532, normalized size = 1.49

$$\frac{\sec(e+fx)(a(\sin(e+fx)+1))^{3/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}\left(\frac{28\sin\left(\frac{1}{2}(e+fx)\right)}{1105\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\right.\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(11/2),x]
```

```
[Out] (-14*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(3/2))/(1105*f*Cos[e + f*x]^(3/2
```

```
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(11/2)) + ((
g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11
*(14/1105 + 8/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8) - 80/(221*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^6) + 14/(663*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^4) + 14/(1105*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (16*Sin[(e
 + f*x)/2])/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9) - (160*Sin[(e + f*
x)/2])/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7) + (28*Sin[(e + f*x)/2]
)/(663*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) + (28*Sin[(e + f*x)/2])/(11
05*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) + (28*Sin[(e + f*x)/2])/(1105*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(3/2))/(f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(11/2))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(ag\cos\left(fx+e\right)\sin\left(fx+e\right)+ag\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{}}{c^6\cos\left(fx+e\right)^6-18\,c^6\cos\left(fx+e\right)^4+48\,c^6\cos\left(fx+e\right)^2-32\,c^6+2\left(3\,c^6\cos\left(fx+e\right)^4-16\,c^6\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="fricas")
```

```
[Out] integral(-(a*g*cos(f*x + e)*sin(f*x + e) + a*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^6*cos(f*x + e)
^6 - 18*c^6*cos(f*x + e)^4 + 48*c^6*cos(f*x + e)^2 - 32*c^6 + 2*(3*c^6*cos(
f*x + e)^4 - 16*c^6*cos(f*x + e)^2 + 16*c^6)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.62, size = 1298, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x)
```

[Out] -2/3315/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(3/2)*(sin(f*x+e)*cos(f*x+e)-sin(f*x+e)-cos(f*x+e)+1)*(-780-780*sin(f*x+e)+948*cos(f*x+e)+605*cos(f*x+e)^2+612*sin(f*x+e)*cos(f*x+e)+35*cos(f*x+e)^5-941*cos(f*x+e)^3+21*I*sin(f*x+e)*cos(f*x+e)^6*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+154*cos(f*x+e)^4-21*cos(f*x+e)^6+168*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-168*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+775*cos(f*x+e)^2*sin(f*x+e)-523*sin(f*x+e)*cos(f*x+e)^3-84*sin(f*x+e)*cos(f*x+e)^4-21*I*sin(f*x+e)*cos(f*x+e)^6*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+336*I*cos(f*x+e)^4*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-336*I*cos(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-420*I*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+420*I*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-168*I*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+168*I*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-84*I*cos(f*x+e)^6*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+84*I*cos(f*x+e)^6*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-189*I*sin(f*x+e)*cos(f*x+e)^4*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+189*I*sin(f*x+e)*cos(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+336*I*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-336*I*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(2*sin(f*x+e)-cos(f*x+e)^2+2)/(-c*(sin(f*x+e)-1))^(11/2)/sin(f*x+e)^5/cos(f*x+e)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxima")

[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(11/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(11/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
11/2),x)
```

```
[Out] Timed out
```

3.106 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=406

$$\frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{154a^3c^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{195f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22a^3c^2\sqrt{}}{}$$

```
[Out] -2/13*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/
f/g+2/39*a^3*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x
+e))^(1/2)-14/117*a^3*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f/g/(a+a*
sin(f*x+e))^(1/2)-2/13*a^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)*(a+a
*sin(f*x+e))^(1/2)/f/g+154/585*a^3*c^3*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*
x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+154/195*a^3*c^3*g*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^
(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+
22/195*a^3*c^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x
+e))^(1/2)
```

Rubi [A] time = 2.09, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{22a^3c^2\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{195fg\sqrt{a\sin(e+fx)+a}}+\frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{154a^3c^3g\sqrt{\cos(e+fx}}{195f\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (154*a^3*c^3*(g*Cos[e + f*x])^(5/2))/(585*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (154*a^3*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*
x]]*EllipticE[(e + f*x)/2, 2])/(195*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]]) + (22*a^3*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]
)/(195*f*g*Sqrt[a + a*Sin[e + f*x]]) + (2*a^3*c*(g*Cos[e + f*x])^(5/2)*(c -
 c*Sin[e + f*x])^(3/2))/(39*f*g*Sqrt[a + a*Sin[e + f*x]]) - (14*a^3*(g*Cos[
e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(117*f*g*Sqrt[a + a*Sin[e + f*x
]]) - (2*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(5/2))/(13*f*g) - (2*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(
3/2)*(c - c*Sin[e + f*x])^(5/2))/(13*f*g)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}(c-}{13fg}$$

$$= -\frac{2a^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}\,(c-}{13fg}$$

$$= -\frac{14a^3(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{117fg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2a^3c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{39fg\sqrt{a+a\sin(e+fx)}} -$$

$$= \frac{22a^3c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{195fg\sqrt{a+a\sin(e+fx)}} +$$

$$= \frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 1.16, size = 120, normalized size = 0.30

$$\frac{a^2c^2\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{3/2}\left(7392E\left(\frac{1}{2}(e+fx)\middle|2\right)+(1897\sin(2(e+fx))+400\right.}{9360f\cos^{\frac{5}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (a^2*c^2*(g*Cos[e + f*x])^(3/2)*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e
 + f*x]]*(7392*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(1897*Sin[2*(
e + f*x)] + 400*Sin[4*(e + f*x)] + 45*Sin[6*(e + f*x)])))/(9360*f*Cos[e + f
*x]^(5/2))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,a^2c^2g\cos\left(fx+e\right)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*a^2*c^2*g*cos(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e
) + c)^(5/2), x)
```

maple [C] time = 0.59, size = 366, normalized size = 0.90

$$\frac{2\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\left(-45\left(\cos^8\left(fx+e\right)\right)+231i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\cos\left(fx+e\right)\sin\left(fx+e\right)\mathrm{Elli}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/585/f*(-c*(sin(f*x+e)-1))^(5/2)*(-45*cos(f*x+e)^8+231*I*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticF(I
*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)-10*cos(f*x+e)^6+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-22*cos(f*x+e)^4-154*cos(f*x+e)^2+231*c
os(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)/cos(f*x+e)^7/sin(f
*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e
) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

3.107 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=352

$$-\frac{14a^3c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14a^3c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a^2c^2\sqrt{a}}{}$$

```
[Out] -2/33*a*c^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e)
)^(1/2)+14/99*c^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(
f*x+e))^(1/2)-14/45*a^3*c^2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)
/(c-c*sin(f*x+e))^(1/2)+14/15*a^3*c^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/
2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(
f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-2/15*a^2*c^2*
(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(1/2)+2/11
*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f/g
```

Rubi [A] time = 1.76, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{14a^3c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a^2c^2\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{15fg\sqrt{c-c\sin(e+fx)}}+\frac{14a^3c^2g\sqrt{\cos(e+fx)}}{15f\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-14*a^3*c^2*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]]) + (14*a^3*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]
]*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) - (2*a^2*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(1
5*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*c^2*(g*Cos[e + f*x])^(5/2)*(a + a*Si
n[e + f*x])^(3/2))/(33*f*g*Sqrt[c - c*Sin[e + f*x]]) + (14*c^2*(g*Cos[e + f
*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(99*f*g*Sqrt[c - c*Sin[e + f*x]]) +
(2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f
*x]])/(11*f*g)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}\sqrt{c}}{11fg}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{99fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2ac^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{33fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^2c^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{15fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{14a^3c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{14a^3c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{14a^3c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{14a^3c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 1.28, size = 189, normalized size = 0.54

$$\frac{c(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}\,(-836\sin(2(e+fx)\right.}{3960f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (c*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*
Sqrt[c - c*Sin[e + f*x]]*(-3696*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*
x]]*(450*Cos[e + f*x] + 225*Cos[3*(e + f*x)] + 45*Cos[5*(e + f*x)] - 836*Si
n[2*(e + f*x)] - 110*Sin[4*(e + f*x)])))/(3960*f*Cos[e + f*x]^(3/2)*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(a^2cg\cos\left(fx+e\right)^3\sin\left(fx+e\right)+a^2cg\cos\left(fx+e\right)^3\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(f\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*g*cos(f*x + e)^3*sin(f*x + e) + a^2*c*g*cos(f*x + e)^3)*sqr
t(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

maple [C] time = 0.68, size = 382, normalized size = 1.09

$$-\frac{2\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\left(45\left(\cos^{6}\left(fx+e\right)\right)\sin\left(fx+e\right)+231i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\cos\left(fx+e\right)\sin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/495/f*(-c*(sin(f*x+e)-1))^(3/2)*(45*cos(f*x+e)^6*sin(f*x+e)+231*I*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*
EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)+55*cos(f*x+e)^6+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e
),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+22*cos(f*x+e)^4+154*cos(f*x
+e)^2-231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)/(1+sin(
f*x+e))/sin(f*x+e)/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

3.108 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}$

Optimal. Leaf size=290

$$-\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22a^3cg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{22a^2c\sqrt{a\,\mathrm{si}}}{10}$$

```
[Out] -2/21*a*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^
(1/2)+2/9*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e)
)^(1/2)-22/45*a^3*c*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*si
n(f*x+e))^(1/2)+22/15*a^3*c*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*
e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1
/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-22/105*a^2*c*(g*cos(f*x
+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.42, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{22a^2c\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{105fg\sqrt{c-c\sin(e+fx)}}+\frac{22a^3cg\sqrt{\cos(e+fx)}}{15f\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-22*a^3*c*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
- c*Sin[e + f*x]]) + (22*a^3*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*El
lipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
f*x]]) - (22*a^2*c*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(105*f*
g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e +
f*x])^(3/2))/(21*f*g*Sqrt[c - c*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2
)*(a + a*Sin[e + f*x])^(5/2))/(9*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\begin{aligned}
\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx &= \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg\sqrt{c-c\sin(e+fx)}}+\frac{1}{3}\\
&= -\frac{2ac(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg\sqrt{c-c\sin(e+fx)}}+\\
&= -\frac{22a^2c(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{105fg\sqrt{c-c\sin(e+fx)}}-\\
&= -\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 2.04, size = 281, normalized size = 0.97

$$\frac{a^3ge^{-4i(e+fx)}\left(e^{i(e+fx)}+i\right)\left(\sqrt{1+e^{2i(e+fx)}}\left(180ie^{i(e+fx)}+238e^{2i(e+fx)}+540ie^{3i(e+fx)}+3696e^{4i(e+fx)}+540ie^{5i(e+fx)}-\right.\right.}{2520f\left(e^{i(e+fx)}-i\right)\sqrt{1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] (a^3*(I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*I)*(e + f
*x))]*(-35 + (180*I)*E^(I*(e + f*x)) + 238*E^((2*I)*(e + f*x)) + (540*I)*E^
((3*I)*(e + f*x)) + 3696*E^((4*I)*(e + f*x)) + (540*I)*E^((5*I)*(e + f*x))
- 238*E^((6*I)*(e + f*x)) + (180*I)*E^((7*I)*(e + f*x)) + 35*E^((8*I)*(e +
f*x))) - 2464*E^((6*I)*(e + f*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I
)*(e + f*x))])*Sqrt[c - c*Sin[e + f*x]])/(2520*E^((4*I)*(e + f*x))*(-I + E^
(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2g\cos\left(fx+e\right)^3-2\,a^2g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^2g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(
f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

maple [C] time = 0.65, size = 394, normalized size = 1.36

$$-\frac{2\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\left(231i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\cos\left(fx+e\right)\sin\left(fx+e\right)\text{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/315/f*(-c*(sin(f*x+e)-1))^(1/2)*(231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))
/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-35*cos
(f*x+e)^6+90*sin(f*x+e)*cos(f*x+e)^4+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e
),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+112*cos(f*x+e)^4+154*cos(f*
x+e)^2-231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)/(2*sin
(f*x+e)-cos(f*x+e)^2+2)/sin(f*x+e)/cos(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.109 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=234

$$-\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{22a^2\sqrt{a\sin}}{35}$$

```
[Out] -2/7*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(1/
2)-22/15*a^3*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)+22/5*a^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipti
cE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a
*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-22/35*a^2*(g*cos(f*x+e))^(5/2)*(a
+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.14, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{22a^2\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{35fg\sqrt{c-c\sin(e+fx)}}+\frac{22a^3g\sqrt{\cos(e+fx)}\,E}{5f\sqrt{a\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (-22*a^3*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (22*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
) - (22*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(35*f*g*Sqrt[c
 - c*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(7*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{7fg\sqrt{c-c\sin(e+fx)}} + \frac{1}{7}(11a)\int \frac{(g\cos(e-$$

$$= -\frac{22a^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{35fg\sqrt{c-c\sin(e+fx)}} - \frac{2a(g\cos(e+fx))^{5/}}{7fg\sqrt{c-c}}$$

$$= -\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22a^2(g\cos(e+f}{35fg\sqrt{c}}$$

$$= -\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22a^2(g\cos(e+f}{35fg\sqrt{c}}$$

$$= -\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22a^2(g\cos(e+f}{35fg\sqrt{c}}$$

$$= -\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22a^3g\sqrt{\cos(e+f}}{5f\sqrt{a+a\sin}}$$

Mathematica [A] time = 1.77, size = 158, normalized size = 0.68

$$-\frac{(a(\sin(e+fx)+1))^{5/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{\cos(e+fx)}\,(126\sin(2(e+fx)\right.}{210f\cos^{\frac{3}{2}}(e+fx)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/Sqrt[c - c*Si
n[e + f*x]],x]
```

```
[Out] -1/210*((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1
+ Sin[e + f*x]))^(5/2)*(-924*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*x]]
*(515*Cos[e + f*x] - 15*Cos[3*(e + f*x)] + 126*Sin[2*(e + f*x)])))/(f*Cos[e
 + f*x]^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c - c*Sin[e + f*
x]])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2g\cos\left(fx+e\right)^3-2\,a^2g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^2g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx\right.}}{c\sin\left(fx+e\right)-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral((a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2*
g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(c*sin(f*x + e) - c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.60, size = 415, normalized size = 1.77

$$\frac{2\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(231i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\cos\left(fx+e\right)\sin\left(fx+e\right)\operatorname{EllipticE}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/105/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)*(231*I*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*Elliptic
E(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/s
in(f*x+e),I)+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)-15*sin(f*x+e)*cos(f*x+e)^4-63*cos(f*x+e)^4+140*cos(
f*x+e)^2*sin(f*x+e)+294*cos(f*x+e)^2-231*cos(f*x+e))/(cos(f*x+e)^2*sin(f*x+
e)+3*cos(f*x+e)^2-4*sin(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(1/2)/sin(f*x+e)/cos(
f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x
 + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(1/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.110 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=241

$$\frac{154a^3(g\cos(e+fx))^{5/2}}{15cfg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{154a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22a^2\sqrt{a\sin}}{5c}$$

```
[Out] 4*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(3/2)+
154/15*a^3*(g*cos(f*x+e))^(5/2)/c/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)-154/5*a^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipt
icE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c/f/(
a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+22/5*a^2*(g*cos(f*x+e))^(5/2)*
(a+a*sin(f*x+e))^(1/2)/c/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.14, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$\frac{154a^3(g\cos(e+fx))^{5/2}}{15cfg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22a^2\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{5cfg\sqrt{c-c\sin(e+fx)}}-\frac{154a^3g\sqrt{\cos(e+fx)}\,E}{5cf\sqrt{a\sin(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(f*g*(c - c*Sin[e +
 f*x])^(3/2)) + (154*a^3*(g*Cos[e + f*x])^(5/2))/(15*c*f*g*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (154*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*C
os[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) + (22*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e +
f*x]])/(5*c*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} - \frac{(11a)\int \frac{(g\cos(e+fx))^{3/2}(a}{\sqrt{c-c\sin}}}{c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22a^2(g\cos(e+fx))^{5}}{5cfg\sqrt{c-c}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{154a^3(g\cos}{15cfg\sqrt{a+a\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{154a^3(g\cos}{15cfg\sqrt{a+a\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{154a^3(g\cos}{15cfg\sqrt{a+a\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{154a^3(g\cos}{15cfg\sqrt{a+a\sin(e+}}$$

Mathematica [A] time = 6.34, size = 240, normalized size = 1.00

$$-\frac{(a(\sin(e+fx)+1))^{5/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(520\sin\left(\frac{1}{2}(e+fx\right.\right.\right.}{30cf(\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] -1/30*((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(a*(1
 + Sin[e + f*x]))^(5/2)*(-924*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) + Sqrt[Cos[e + f*x]]*(520*Cos[(e + f*x)/2] + 37*Cos[(3*(
e + f*x))/2] + 3*Cos[(5*(e + f*x))/2] + 520*Sin[(e + f*x)/2] - 37*Sin[(3*(e
 + f*x))/2] + 3*Sin[(5*(e + f*x))/2])))/(c*f*Cos[e + f*x]^(3/2)*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^5*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2 g\cos\left(fx+e\right)^3 - 2a^2 g\cos\left(fx+e\right)\sin\left(fx+e\right) - 2a^2 g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+\right.}}{c^2\cos\left(fx+e\right)^2 + 2c^2\sin\left(fx+e\right) - 2c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral((a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2*
g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.65, size = 2945, normalized size = 12.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/15/f*(-1+cos(f*x+e))*(351*cos(f*x+e)^2+30*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-30
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)+30*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*co
s(f*x+e)^5-94*cos(f*x+e)^3+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)+231*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+20*cos(f*x+e)
^4+120*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e
)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-120*cos(f*x+e)^3*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)+180*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2
*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
```

```
x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-180*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)-111*cos(f*x+e)^2*sin(f*x+e)-30*ln(-2*(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)*sin(f*x+e)+30*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+120*cos(f*x
+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-120*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-30
*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-17*sin(f*x+e)*cos(f*x+e)^3-30*sin(f
*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+30*sin(f*x+e)*cos(f*x+e)^3*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-1)/sin(f*x+e)^2)-90*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)+90*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-90*sin(f*x+e)*cos
(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+90*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(3/2)+3*sin(f*x+e)*cos(f*x+e)^4-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f
*x+e)^3-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)+231*I*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*cos(f*x+e)+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3-462*I*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*cos(f*x+e)^2+462*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e
)^2-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2-231*I*(1/(cos(f
```

```
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*Ell
ipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+
e)))^(5/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+
3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*
x+e)+8)/cos(f*x+e)/(-c*(sin(f*x+e)-1))^(3/2)/sin(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

3.111 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=243

$$-\frac{154a^3(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{154a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{44a^2\sqrt{a\,s}}{5}$$

```
[Out] 4/5*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(5/2
)-44/5*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/c/f/g/(c-c*sin(f*x+e
))^(3/2)-154/15*a^3*(g*cos(f*x+e))^(5/2)/c^2/f/g/(a+a*sin(f*x+e))^(1/2)/(c-
c*sin(f*x+e))^(1/2)+154/5*a^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/
2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^
(1/2)/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.17, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$-\frac{154a^3(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{154a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{44a^2\sqrt{a\,s}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(5*f*g*(c - c*Sin[e
 + f*x])^(5/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/
(5*c*f*g*(c - c*Sin[e + f*x])^(3/2)) - (154*a^3*(g*Cos[e + f*x])^(5/2))/(15
*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (154*a^3*g*Sq
rt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{(11a)\int \frac{(g\cos(e+fx))^{3/2}(}{(c-c\sin(}}{5c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{5cfg(c-c\sin}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{5cfg(c-c\sin}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{5cfg(c-c\sin}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{5cfg(c-c\sin}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{5cfg(c-c\sin}$$

Mathematica [A] time = 2.44, size = 245, normalized size = 1.01

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\,(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(226\sin\left(\frac{1}{2}(e+fx\right.\right.\right.}{30c^2f(\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] -1/30*(a^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*S
qrt[a*(1 + Sin[e + f*x])]*(-924*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(226*Cos[(e + f*x)/2] + 327*Cos
[(3*(e + f*x))/2] - 5*Cos[(5*(e + f*x))/2] + 226*Sin[(e + f*x)/2] - 327*Sin
[(3*(e + f*x))/2] - 5*Sin[(5*(e + f*x))/2])))/(c^2*f*Cos[e + f*x]^(3/2)*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e +
 f*x]])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^{2} g \cos\left(fx+e\right)^{3} - 2\, a^{2} g \cos\left(fx+e\right) \sin\left(fx+e\right) - 2\, a^{2} g \cos\left(fx+e\right)\right) \sqrt{g \cos\left(fx+e\right)} \sqrt{a \sin\left(fx\right.}}{3\, c^{3} \cos\left(fx+e\right)^{2} - 4\, c^{3} - \left(c^{3} \cos\left(fx+e\right)^{2} - 4\, c^{3}\right) \sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral((a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2*
g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*s
in(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.66, size = 3549, normalized size = 14.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/15/f*(-1+cos(f*x+e))*(438*cos(f*x+e)^2+90*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-90*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)-90*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+231*I
*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-5*cos(f*x+e)^5-89*cos(f*
x+e)^3-70*cos(f*x+e)^4-45*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
```

ln(-2(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+45*cos(f
x+e)^5(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-45*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*l
n(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)
*cos(f*x+e)^4+45*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^4+90*cos(f
x+e)^3(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-90*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(
f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^
2)+360*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e
)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-360*cos(f*x+e)^2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)-486*cos(f*x+e)^2*sin(f*x+e)-90*ln(-2*(2*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*
sin(f*x+e)+90*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(
f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^
2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+315*cos(f*x+e)*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-1)/sin(f*x+e)^2)-315*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-
(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+90*cos(f*x+e)^4
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)+65*sin(f*x+e)*cos(f*x+e)^3-225*sin(f*x+e)*cos(f*
x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+225*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/
sin(f*x+e)^2)-405*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+405*s
in(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-315*sin(f*x+e)*cos(f*x+e)*

```
ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(3/2)+315*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)-5*sin(f*x+e)*cos(f*x+e)^4-462*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*s
in(f*x+e)+693*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-693*I*sin
(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*sin(f*x+e)*cos(f*x+e)^
3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-
1+cos(f*x+e))/sin(f*x+e),I)+462*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(
f*x+e)+231*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*cos(f*x+e)^4*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)-693*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+693*I*co
s(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-462*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)+462*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(g*cos(f*x+e))^(3/2)*(a*(1
+sin(f*x+e)))^(5/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*si
n(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)
-4*cos(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(5/2)/sin(f*x+e)/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\cos\left(e+fx\right)\right)^{3/2}\left(a+a\sin\left(e+fx\right)\right)^{5/2}}{\left(c-c\sin\left(e+fx\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

3.112 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=243

$$-\frac{154a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{308a^3(g\cos(e+fx))^{5/2}}{45c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{44a^2\cdots}{\cdots}$$

```
[Out] 4/9*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(7/2
)+308/45*a^3*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f
*x+e))^(1/2)-44/45*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/c/f/g/(c
-c*sin(f*x+e))^(5/2)-154/15*a^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+
1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e)
)^(1/2)/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.20, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2850, 2842, 2640, 2639}

$$\frac{308a^3(g\cos(e+fx))^{5/2}}{45c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{154a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{44a^2\sqrt{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(9*f*g*(c - c*Sin[e
 + f*x])^(7/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/
(45*c*f*g*(c - c*Sin[e + f*x])^(5/2)) + (308*a^3*(g*Cos[e + f*x])^(5/2))/(4
5*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (154*a^3*g
*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3
*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{(11a)\int \frac{(g\cos(e+fx))^{3/2}(a}{(c-c\sin(e}}{9c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{45cfg(c-c\,\mathrm{si}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{45cfg(c-c\,\mathrm{si}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{45cfg(c-c\,\mathrm{si}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{45cfg(c-c\,\mathrm{si}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{45cfg(c-c\,\mathrm{si}}$$

Mathematica [A] time = 2.64, size = 246, normalized size = 1.01

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\,(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(2\sqrt{\cos(e+fx)}\left(182\sin\left(\frac{1}{2}(e+\right.\right.\right.}{90c^3f(\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(7/2),x]
```

```
[Out] -1/90*(a^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*S
qrt[a*(1 + Sin[e + f*x])]*(-924*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^5 + 2*Sqrt[Cos[e + f*x]]*(182*Cos[(e + f*x)/2] + 195*C
os[(3*(e + f*x))/2] - 93*Cos[(5*(e + f*x))/2] + 182*Sin[(e + f*x)/2] - 195*
Sin[(3*(e + f*x))/2] - 93*Sin[(5*(e + f*x))/2])))/(c^3*f*Cos[e + f*x]^(3/2)
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*Sqrt[c - c*Sin
[e + f*x]])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2g\cos\left(fx+e\right)^3-2\,a^2g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^2g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(f\right.}}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(
f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*c
os(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.71, size = 4183, normalized size = 17.21

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/45/f*(-1+cos(f*x+e))*(1928*cos(f*x+e)^2+540*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x
+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-
540*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-1)/sin(f*x+e)^2)-1350*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)
+90*cos(f*x+e)^5-268*cos(f*x+e)^3-1848*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*sin(f*x+e)-1182*cos(f*x+e)^4-810*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)+810*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+135*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)*sin(f*x+e)*cos(f*x+e)^4-135*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*c
os(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+
e)^4+2025*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-2025*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-1)/sin(f*x+e)^2)-1768*cos(f*x+e)^2*sin(f*x+e)-540*ln(-2*(2*cos(f*x+e)^2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)*sin(f*x+e)+540*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(
f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+1890*cos(f*x+e)*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)-1890*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+1350*c
```

```
os(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+348*sin(f*x+e)*cos(f*x+e)^3-945*sin(f
*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+945*sin(f*x+e)*cos(f*x+e)^3*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)-2295*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(3/2)+2295*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-1890*sin(f*x
+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-c
os(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+
e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+1890*sin(f*x+e)*cos(f*x+e)*ln(-(
2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+
e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)+90*sin(f*x+e)*cos(f*x+e)^4-135*cos(f*x+e)^6*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)+135*cos(f*x+e)^6*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(
-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-462*I*cos(f*x+
e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)+462*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)
+1848*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-1848*I*cos(f*x+e)*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/
sin(f*x+e),I)+462*I*cos(f*x+e)^5*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-1848*I*cos(f*x+e
)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*
(-1+cos(f*x+e))/sin(f*x+e),I)+1848*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)
+2772*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+462*I*sin(f*x+e)*cos(f*x+e)^
4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-
1+cos(f*x+e))/sin(f*x+e),I)-462*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)+462*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+1848*I*cos
(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+135*sin(f*x+e)*cos(f*x+e)^5*(-
```

```
cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)-135*sin(f*x+e)*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-co
s(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e
)^2)-2772*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-462*I*cos(f*x+e)^5*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)-462*I*sin(f*x+e)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)-2772*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+2772*I*sin(f*x+e
)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*
x+e)))^(5/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e
)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(
f*x+e)+8)/(-c*(sin(f*x+e)-1))^(7/2)/sin(f*x+e)/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(7/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

3.113 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=300

$$\frac{154a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{195c^4f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{154a^3(g\cos(e+fx))^{5/2}}{195c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{}{585c^2f}$$

```
[Out] 4/13*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(9/
2)+308/585*a^3*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin
(f*x+e))^(1/2)-154/195*a^3*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))^(3
/2)/(a+a*sin(f*x+e))^(1/2)-44/117*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))
^(1/2)/c/f/g/(c-c*sin(f*x+e))^(7/2)+154/195*a^3*g*(cos(1/2*f*x+1/2*e)^2)^(1
/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/
2)*(g*cos(f*x+e))^(1/2)/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.48, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$-\frac{154a^3(g\cos(e+fx))^{5/2}}{195c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{308a^3(g\cos(e+fx))^{5/2}}{585c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}+\frac{154a^3g\sqrt{\cos}}{195c^4f}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(9/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(13*f*g*(c - c*Sin[
e + f*x])^(9/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])
/(117*c*f*g*(c - c*Sin[e + f*x])^(7/2)) + (308*a^3*(g*Cos[e + f*x])^(5/2))/
(585*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) - (154*a^
3*(g*Cos[e + f*x])^(5/2))/(195*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[
e + f*x])^(3/2)) + (154*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(195*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
+ f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{(11a)\int \frac{(g\cos(e+fx))^{3/2}(a}{(c-c\sin(e}}{13c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{117cfg(c-c\,\mathrm{s}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{117cfg(c-c\,\mathrm{s}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{117cfg(c-c\,\mathrm{s}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{117cfg(c-c\,\mathrm{s}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{117cfg(c-c\,\mathrm{s}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e+fx))^{5/}}{117cfg(c-c\,\mathrm{s}}$$

Mathematica [A] time = 6.54, size = 464, normalized size = 1.55

$$\frac{154E\left(\frac{1}{2}(e+fx)\middle|2\right)(a(\sin(e+fx)+1))^{5/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}{195f\cos^{\frac{3}{2}}(e+fx)(c-c\sin(e+fx))^{9/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5} + \frac{\sec(e+fx)(a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(9/2),x]
```

```
[Out] (154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))/(195*f*Cos[e + f*x]^(3/2)*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2)) + ((g*C
os[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(-1
54/195 + 16/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6) - 232/(117*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^4) + 236/(195*(Cos[(e + f*x)/2] - Sin[(e + f
```

```
*x)/2])^2) + (32*Sin[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^7) - (464*Sin[(e + f*x)/2])/(117*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5)
 + (472*Sin[(e + f*x)/2])/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) - (
308*Sin[(e + f*x)/2])/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 +
Sin[e + f*x]))^(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin
[e + f*x])^(9/2))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2 g \cos\left(fx+e\right)^3 - 2\,a^2 g \cos\left(fx+e\right)\sin\left(fx+e\right) - 2\,a^2 g \cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(f\right.}}{5\,c^5\cos\left(fx+e\right)^4 - 20\,c^5\cos\left(fx+e\right)^2 + 16\,c^5 - \left(c^5\cos\left(fx+e\right)^4 - 12\,c^5\cos\left(fx+e\right)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(
f*x + e) + c)/(5*c^5*cos(f*x + e)^4 - 20*c^5*cos(f*x + e)^2 + 16*c^5 - (c^5
*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 + 16*c^5)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.71, size = 3455, normalized size = 11.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/1170/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)*(-1+cos(f*x+e))^4*(s
in(f*x+e)-1)*(cos(f*x+e)+1)*(3696*I*(1/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)-3696*I*(1/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x
```

+e))/sin(f*x+e),I)-2340*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^5+5
85*cos(f*x+e)^5*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-co
s(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e
)^2)-585*cos(f*x+e)^5*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/s
in(f*x+e)^2)+2925*cos(f*x+e)^3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-1)/sin(f*x+e)^2)-2772*I*(1/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e
))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^4+5544*I*(1/(cos(f*x+e)+1))^(1/2)*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^3-5544*I*(1/(cos(
f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e
)^3-924*I*(1/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*si
n(f*x+e)*cos(f*x+e)^2+924*I*(1/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2-7392*I*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+7392*I*(1/(cos(f*x
+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-9
24*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x
+e)^6+1848*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*cos(f*x+e)^5-1848*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*cos(f*x+e)^5-3696*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f
x+e)+1))^(1/2)(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e
))/sin(f*x+e),I)*cos(f*x+e)^4+3696*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+c
os(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4-9240*I*(1/(cos(f*x+e)+1))^(1/2)*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3+9240*I*(1/(cos(f*x+e)+1))^(1/
2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3-924*I*(1/(cos(f*x+e)+
1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2+924*I*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2+3696*I*
(1/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+7
392*I*(1/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*

```
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*
x+e)-7392*I*(1/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
cos(f*x+e)+2752*sin(f*x+e)*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-924*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)*cos(f*x+e)^4-1868*sin
(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+1755*sin(f*x+e)*c
os(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)
-1755*sin(f*x+e)*cos(f*x+e)^3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-1)/sin(f*x+e)^2)+2340*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)-2340*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)-2925*cos(f*x+e)^3*ln
(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+7476*cos(f*x+
e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-2340*cos(f*x+e)*ln(-2*(2*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+2340*cos(f*x+e)*ln(-(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+432*cos(f*x+e)^4*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1440*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+
1008*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-3696*I*(1/(cos(f*x+e
)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+2256*sin(f*x+e
)*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+924*I*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6-1440*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-5136*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)+2772*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*sin(f*x+e)*cos(f*x+e)^4)/(cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-4*
sin(f*x+e)-4)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/(-c*(sin(f*x+e)-1))^(9/2
)/sin(f*x+e)^9
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2
```

```
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(9/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
9/2),x)
```

```
[Out] Timed out
```

3.114 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=357

$$\frac{154a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{3315c^5f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{154a^3(g\cos(e+fx))^{5/2}}{3315c^4fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{}{3315}$$

```
[Out] 4/17*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(11
/2)+308/1989*a^3*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(7/2)/(a+a*s
in(f*x+e))^(1/2)-154/3315*a^3*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))
^(5/2)/(a+a*sin(f*x+e))^(1/2)-154/3315*a^3*(g*cos(f*x+e))^(5/2)/c^4/f/g/(c-
c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-44/221*a^2*(g*cos(f*x+e))^(5/2)*
(a+a*sin(f*x+e))^(1/2)/c/f/g/(c-c*sin(f*x+e))^(9/2)+154/3315*a^3*g*(cos(1/2
*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2
))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^5/f/(a+a*sin(f*x+e))^(1/2)/(c-c*
sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.79, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$-\frac{154a^3(g\cos(e+fx))^{5/2}}{3315c^4fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{154a^3(g\cos(e+fx))^{5/2}}{3315c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}+\frac{}{1989c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(11/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(17*f*g*(c - c*Sin[
e + f*x])^(11/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]
)/(221*c*f*g*(c - c*Sin[e + f*x])^(9/2)) + (308*a^3*(g*Cos[e + f*x])^(5/2))
/(1989*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) - (154*
a^3*(g*Cos[e + f*x])^(5/2))/(3315*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(5/2)) - (154*a^3*(g*Cos[e + f*x])^(5/2))/(3315*c^4*f*g*Sqrt[a
 + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (154*a^3*g*Sqrt[Cos[e + f*
x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(3315*c^5*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{(11a)\int \frac{(g\cos(e+fx))^{3/2}}{(c-c\sin}}{17}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+fx))}{221cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+fx))}{221cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+fx))}{221cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+fx))}{221cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+fx))}{221cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+fx))}{221cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+fx))}{221cfg(c-c}$$

Mathematica [A] time = 6.57, size = 532, normalized size = 1.49

$$\frac{154E\left(\frac{1}{2}(e+fx)\middle|2\right)(a(\sin(e+fx)+1))^{5/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}{3315f\cos^{\frac{3}{2}}(e+fx)(c-c\sin(e+fx))^{11/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^{5}} + \frac{\sec(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(11/2),x]
```

```
[Out] (154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(5/2))/(3315*f*Cos[e + f*x]^(3/2
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(11/2)) + ((
```

```
g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11
*(-154/3315 + 16/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8) - 296/(221*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^6) + 1172/(1989*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^4) - 154/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (3
2*Sin[(e + f*x)/2])/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9) - (592*Sin
[(e + f*x)/2])/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7) + (2344*Sin[(e
 + f*x)/2])/(1989*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) - (308*Sin[(e +
f*x)/2])/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) - (308*Sin[(e + f*x
)/2])/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^
(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(11/
2))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2 g \cos\left(fx+e\right)^3 - 2\,a^2 g \cos\left(fx+e\right)\sin\left(fx+e\right) - 2\,a^2 g \cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+\right.}}{c^6\cos\left(fx+e\right)^6 - 18\,c^6\cos\left(fx+e\right)^4 + 48\,c^6\cos\left(fx+e\right)^2 - 32\,c^6 + 2\left(3\,c^6\cos\left(fx+e\right)^4 - 16\,c^6\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="fricas")
```

```
[Out] integral((a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2*
g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(c^6*cos(f*x + e)^6 - 18*c^6*cos(f*x + e)^4 + 48*c^6*cos(f*x +
 e)^2 - 32*c^6 + 2*(3*c^6*cos(f*x + e)^4 - 16*c^6*cos(f*x + e)^2 + 16*c^6)*
sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.78, size = 1313, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 2/9945/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)*(sin(f*x+e)*cos(f*x+
e)-sin(f*x+e)-cos(f*x+e)+1)*(-4680-4680*sin(f*x+e)+2832*cos(f*x+e)+9920*cos
(f*x+e)^2+6528*sin(f*x+e)*cos(f*x+e)+1848*I*EllipticF(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-1848*
I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)+2930*cos(f*x+e)^5-6224*cos(f*x+e)^3+2079*I*sin(
f*x+e)*cos(f*x+e)^4*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-5009*cos(f*x+e)^4+231*cos(f*x
+e)^6+1420*cos(f*x+e)^2*sin(f*x+e)-4192*sin(f*x+e)*cos(f*x+e)^3+924*sin(f*x
+e)*cos(f*x+e)^4+231*I*sin(f*x+e)*cos(f*x+e)^6*EllipticF(I*(-1+cos(f*x+e))/
sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-23
1*I*sin(f*x+e)*cos(f*x+e)^6*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-2079*I*sin(f*x+e)*cos
(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3696*I*sin(f*x+e)*cos(f*x+e)^2*Elliptic
F(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)-3696*I*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-
1848*I*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+1848*I*sin(f*x+e)*EllipticE(I*(
-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)-924*I*cos(f*x+e)^6*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+924*I*cos(f*x+e)^
6*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)+3696*I*cos(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3
696*I*cos(f*x+e)^4*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-4620*I*cos(f*x+e)^2*EllipticF(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)+4620*I*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(cos(f*x+e)^
2+2*cos(f*x+e)+1)/(cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)-4)/(
-c*(sin(f*x+e)-1))^(11/2)/sin(f*x+e)^5/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(11/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
11/2),x)
```

```
[Out] Timed out
```

3.115 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=414

$$\frac{22a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{3315c^6f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{22a^3(g\cos(e+fx))^{5/2}}{3315c^5fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{}{3315c}$$

```
[Out] 4/21*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(13
/2)+44/663*a^3*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(9/2)/(a+a*sin
(f*x+e))^(1/2)-22/1989*a^3*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))^(7
/2)/(a+a*sin(f*x+e))^(1/2)-22/3315*a^3*(g*cos(f*x+e))^(5/2)/c^4/f/g/(c-c*si
n(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)-22/3315*a^3*(g*cos(f*x+e))^(5/2)/c^5
/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-44/357*a^2*(g*cos(f*x+e)
)^(5/2)*(a+a*sin(f*x+e))^(1/2)/c/f/g/(c-c*sin(f*x+e))^(11/2)+22/3315*a^3*g*
(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e
),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^6/f/(a+a*sin(f*x+e))^(1/
2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 2.08, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$-\frac{22a^3(g\cos(e+fx))^{5/2}}{3315c^5fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{22a^3(g\cos(e+fx))^{5/2}}{3315c^4fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}-\frac{}{1989c^3}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(13/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(21*f*g*(c - c*Sin[
e + f*x])^(13/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]
)/(357*c*f*g*(c - c*Sin[e + f*x])^(11/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2))
/(663*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/2)) - (22*a^
3*(g*Cos[e + f*x])^(5/2))/(1989*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin
[e + f*x])^(7/2)) - (22*a^3*(g*Cos[e + f*x])^(5/2))/(3315*c^4*f*g*Sqrt[a +
a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) - (22*a^3*(g*Cos[e + f*x])^(5/2
))/(3315*c^5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (22
*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(
3315*c^6*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{(11a)\int \frac{(g\cos(e+fx))^{3/2}}{(c-c\sin}}{21}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))}{357cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))}{357cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))}{357cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))}{357cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))}{357cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))}{357cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))}{357cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))}{357cfg(c-c}$$

Mathematica [A] time = 6.68, size = 600, normalized size = 1.45

$$\frac{22E\left(\frac{1}{2}(e+fx)\middle|2\right)(a(\sin(e+fx)+1))^{5/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{13}}{3315f\cos^{\frac{3}{2}}(e+fx)(c-c\sin(e+fx))^{13/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^{5}} + \frac{\sec(e+fx)(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(13/2),x]
```

```
[Out] (22*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])^13*(a*(1 + Sin[e + f*x]))^(5/2))/(3315*f*Cos[e + f*x]^(3/2)
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(13/2)) + ((g
*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^13*
(-22/3315 + 16/(21*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10) - 120/(119*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^8) + 84/(221*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])^6) - 22/(1989*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4) - 22/(331
5*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (32*Sin[(e + f*x)/2])/(21*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^11) - (240*Sin[(e + f*x)/2])/(119*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^9) + (168*Sin[(e + f*x)/2])/(221*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^7) - (44*Sin[(e + f*x)/2])/(1989*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^5) - (44*Sin[(e + f*x)/2])/(3315*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^3) - (44*Sin[(e + f*x)/2])/(3315*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^5*(c - c*Sin[e + f*x])^(13/2))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2 g \cos\left(fx+e\right)^3 - 2\,a^2 g \cos\left(fx+e\right)\sin\left(fx+e\right) - 2\,a^2 g \cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a}}{7\,c^7\cos\left(fx+e\right)^6 - 56\,c^7\cos\left(fx+e\right)^4 + 112\,c^7\cos\left(fx+e\right)^2 - 64\,c^7 - \left(c^7\cos\left(fx+e\right)^6 - 24\,c^7\cos\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="fricas")
```

```
[Out] integral((a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2*
g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(7*c^7*cos(f*x + e)^6 - 56*c^7*cos(f*x + e)^4 + 112*c^7*cos(f*
x + e)^2 - 64*c^7 - (c^7*cos(f*x + e)^6 - 24*c^7*cos(f*x + e)^4 + 80*c^7*co
s(f*x + e)^2 - 64*c^7)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.75, size = 1473, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x)
```

```
[Out] -2/69615/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)*(sin(f*x+e)*cos(f*
x+e)-sin(f*x+e)-cos(f*x+e)+1)*(26520+26520*sin(f*x+e)-22824*cos(f*x+e)-4360
0*cos(f*x+e)^2-30216*sin(f*x+e)*cos(f*x+e)-385*sin(f*x+e)*cos(f*x+e)^5-1199
8*cos(f*x+e)^5+35284*cos(f*x+e)^3+1155*I*sin(f*x+e)*cos(f*x+e)^6*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)+17773*cos(f*x+e)^4-1155*cos(f*x+e)^6+3696*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)-3696*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-18020*cos(f*x+e)^2*sin(f*x+e)+2
4488*sin(f*x+e)*cos(f*x+e)^3-2618*sin(f*x+e)*cos(f*x+e)^4+231*cos(f*x+e)^6*
sin(f*x+e)-1155*I*sin(f*x+e)*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*c
os(f*x+e)^8*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*cos(f*x+e)^8*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)-3234*I*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3234*I*cos(f*x+e)^6*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+c
os(f*x+e))/sin(f*x+e),I)+9471*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-9471
*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-10164*I*cos(f*x+e)^2*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)+10164*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3696*I*sin(f*x
+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*
(-1+cos(f*x+e))/sin(f*x+e),I)+3696*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-5
775*I*sin(f*x+e)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+5775*I*sin(f*x+e)*c
os(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+8316*I*sin(f*x+e)*cos(f*x+e)^2*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*
x+e))/sin(f*x+e),I)-8316*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
))*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-4*
sin(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(13/2)/sin(f*x+e)^5/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(13/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(13/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(13/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
13/2),x)
```

```
[Out] Timed out
```

3.116 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=463

$$-\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{154a^4c^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{195f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^3c}{}$$

```
[Out] 2/15*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/f
/g-2/39*a^2*c^3*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*
x+e))^(1/2)-14/585*a*c^3*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c
-c*sin(f*x+e))^(1/2)+14/195*c^3*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(7/2)
/f/g/(c-c*sin(f*x+e))^(1/2)-154/585*a^4*c^3*(g*cos(f*x+e))^(5/2)/f/g/(a+a*s
in(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+154/195*a^4*c^3*g*(cos(1/2*f*x+1/2*
e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*
x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(
1/2)-22/195*a^3*c^3*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*si
n(f*x+e))^(1/2)+22/195*c^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(7/2)*(c-c
*sin(f*x+e))^(1/2)/f/g
```

Rubi [A] time = 2.39, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^3c^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{195fg\sqrt{c-c\sin(e+fx)}}-\frac{2a^2c^3(a\sin(e+fx)}{39fg\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (-154*a^4*c^3*(g*Cos[e + f*x])^(5/2))/(585*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) + (154*a^4*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f
*x]]*EllipticE[(e + f*x)/2, 2])/(195*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*
Sin[e + f*x]]) - (22*a^3*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]
])/(195*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a^2*c^3*(g*Cos[e + f*x])^(5/2)*(
a + a*Sin[e + f*x])^(3/2))/(39*f*g*Sqrt[c - c*Sin[e + f*x]]) - (14*a*c^3*(g
*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(585*f*g*Sqrt[c - c*Sin[e
+ f*x]]) + (14*c^3*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(7/2))/(195*
f*g*Sqrt[c - c*Sin[e + f*x]]) + (22*c^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e
 + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(195*f*g) + (2*c*(g*Cos[e + f*x])^
(5/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2))/(15*f*g)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{7/2}(c-}{15fg}$$

$$= \frac{22c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{7/2}}{195fg}$$

$$= \frac{14c^3(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{7/2}}{195fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{14ac^3(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5}}{585fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^2c^3(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3}}{39fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{22a^3c^3(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{195fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+f}}$$

$$= -\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+f}}$$

$$= -\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+f}}$$

$$= -\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+f}}$$

Mathematica [A] time = 3.52, size = 226, normalized size = 0.49

$$-\frac{a^3c^2(\sin(e+fx)-1)^2(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+f}\right.}{18720f\cos^{\frac{3}{2}}(e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] -1/18720*(a^3*c^2*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])^2*(1 + Sin[e +
 f*x])^3*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-14784*Ellipt
```

```
icE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(1365*Cos[e + f*x] + 819*Cos[3*(e
+ f*x)] + 273*Cos[5*(e + f*x)] + 39*Cos[7*(e + f*x)] - 3794*Sin[2*(e + f*x)
] - 800*Sin[4*(e + f*x)] - 90*Sin[6*(e + f*x)])))/(f*Cos[e + f*x]^(3/2)*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3c^2g\cos\left(fx+e\right)^5\sin\left(fx+e\right)+a^3c^2g\cos\left(fx+e\right)^5\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral((a^3*c^2*g*cos(f*x + e)^5*sin(f*x + e) + a^3*c^2*g*cos(f*x + e)^5)
*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.86, size = 392, normalized size = 0.85

$$-\frac{2\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\left(39\left(\cos^8\left(fx+e\right)\right)\sin\left(fx+e\right)+45\left(\cos^8\left(fx+e\right)\right)+10\left(\cos^6\left(fx+e\right)\right)-231i\sqrt{\frac{}{\text{co}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/585/f*(-c*(sin(f*x+e)-1))^(5/2)*(39*cos(f*x+e)^8*sin(f*x+e)+45*cos(f*x+e
)^8+10*cos(f*x+e)^6-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+2
31*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*
cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1
```

```
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+22*cos(f*x+
e)^4+154*cos(f*x+e)^2-231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)
))^(7/2)/(1+sin(f*x+e))/cos(f*x+e)^7/sin(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e
) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^
(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

3.117 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=409

$$-\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14a^4c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{13f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a^3c^2\sqrt{a\,\mathrm{s}}}{1}$$

```
[Out] -10/143*a^2*c^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*
x+e))^(1/2)-14/429*a*c^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c
-c*sin(f*x+e))^(1/2)+14/143*c^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(7/2)
/f/g/(c-c*sin(f*x+e))^(1/2)-14/39*a^4*c^2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin
(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+14/13*a^4*c^2*g*(cos(1/2*f*x+1/2*e)^2
)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)
^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
-2/13*a^3*c^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+
e))^(1/2)+2/13*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e
))^(1/2)/f/g
```

Rubi [A] time = 2.04, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a^3c^2\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{13fg\sqrt{c-c\sin(e+fx)}}-\frac{10a^2c^2(a\sin(e+fx)+}{143fg\sqrt{c-}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-14*a^4*c^2*(g*Cos[e + f*x])^(5/2))/(39*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]]) + (14*a^4*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]
]*EllipticE[(e + f*x)/2, 2])/(13*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) - (2*a^3*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(1
3*f*g*Sqrt[c - c*Sin[e + f*x]]) - (10*a^2*c^2*(g*Cos[e + f*x])^(5/2)*(a + a
*Sin[e + f*x])^(3/2))/(143*f*g*Sqrt[c - c*Sin[e + f*x]]) - (14*a*c^2*(g*Cos
[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(429*f*g*Sqrt[c - c*Sin[e + f*
x]]) + (14*c^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(7/2))/(143*f*g*
Sqrt[c - c*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x]
)^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(13*f*g)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{7/2}\sqrt{c-}}{13fg}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{7/2}}{143fg\sqrt{c-c\sin(e+fx)}} +$$

$$= -\frac{14ac^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{429fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{10a^2c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/}}{143fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^3c^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg\sqrt{c-c\sin(e+fx)}} -$$

$$= -\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 2.92, size = 212, normalized size = 0.52

$$\frac{a^3c(\sin(e+fx)-1)(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}(-1\right.}{6864f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (a^3*c*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^3*Sqrt
[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-7392*EllipticE[(e + f*x)/
2, 2] + Sqrt[Cos[e + f*x]]*(1560*Cos[e + f*x] + 780*Cos[3*(e + f*x)] + 156*
Cos[5*(e + f*x)] - 1507*Sin[2*(e + f*x)] - 88*Sin[4*(e + f*x)] + 33*Sin[6*(
```

```
e + f*x)])))/(6864*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^3cg\cos\left(fx+e\right)^5-2\,a^3cg\cos\left(fx+e\right)^3\sin\left(fx+e\right)-2\,a^3cg\cos\left(fx+e\right)^3\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a^3*c*g*cos(f*x + e)^5 - 2*a^3*c*g*cos(f*x + e)^3*sin(f*x + e) -
 2*a^3*c*g*cos(f*x + e)^3)*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(-c*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

maple [C] time = 0.73, size = 404, normalized size = 0.99

$$-\frac{2\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\left(-33\left(\cos^8\left(fx+e\right)\right)+78\left(\cos^6\left(fx+e\right)\right)\sin\left(fx+e\right)+231i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(f\right.}{\cos\left(fx\right.}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/429/f*(-c*(sin(f*x+e)-1))^(3/2)*(-33*cos(f*x+e)^8+78*cos(f*x+e)^6*sin(f*
x+e)+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+88*cos(f*x+e)^6+231*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(
```

```
f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+22*cos(f*x
+e)^4+154*cos(f*x+e)^2-231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e
)))^(7/2)/(2*sin(f*x+e)-cos(f*x+e)^2+2)/sin(f*x+e)/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^
(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

3.118 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx}$

Optimal. Leaf size=343

$$-\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2a^4cg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^3c\sqrt{a\sin(}}{7f}$$

[Out] -10/77*a^2*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(1/2)-2/33*a*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(1/2)+2/11*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(7/2)/f/g/(c-c*sin(f*x+e))^(1/2)-2/3*a^4*c*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2*a^4*c*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-2/7*a^3*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 1.72, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^3c\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{7fg\sqrt{c-c\sin(e+fx)}}-\frac{10a^2c(a\sin(e+fx)+a}{77fg\sqrt{c-c\,s}}$$

Antiderivative was successfully verified.

[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]],x]

[Out] (-2*a^4*c*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*a^4*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*a^3*c*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(7*f*g*Sqrt[c - c*Sin[e + f*x]]) - (10*a^2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(77*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(33*f*g*Sqrt[c - c*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(7/2))/(11*f*g*Sqrt[c - c*Sin[e + f*x]])

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{7/2}}{11fg\sqrt{c-c\sin(e+fx)}} + \frac{1}{11}$$

$$= -\frac{2ac(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{33fg\sqrt{c-c\sin(e+fx)}} +$$

$$= -\frac{10a^2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{77fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^3c(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{7fg\sqrt{c-c\sin(e+fx)}} -$$

$$= -\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} -$$

$$= -\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} -$$

$$= -\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} -$$

$$= -\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} +$$

Mathematica [C] time = 4.32, size = 360, normalized size = 1.05

$$\frac{\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{3/2}\left(-\frac{a^3\sqrt{\cos(e+fx)}\sqrt{a(\sin(e+fx)+1)}\,(1374\cos(e+fx)+423\cos(3(e+fx))-7(44\sin(2(e+fx))-22\sin(4}{1848f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}\right.}{\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]*(((2 + 2*I)*a^4*E^((I/2)*(
e + f*x))*(I + E^(I*(e + f*x)))*((1 + E^((2*I)*(e + f*x)))/E^(I*(e + f*x)))
^(3/2)*(Sqrt[1 + E^((2*I)*(e + f*x))] + (-1 + E^((2*I)*e))*Hypergeometric2F
1[-1/4, 1/2, 3/4, -E^((2*I)*(e + f*x))]))/((-1 + E^((2*I)*e))*Sqrt[((-I)*a*
(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(1 + E^((2*I)*(e + f*x)))^(3/2)*f
) - (a^3*Sqrt[Cos[e + f*x]]*Sqrt[a*(1 + Sin[e + f*x])]*(1374*Cos[e + f*x] +
```

```
423*Cos[3*(e + f*x)] - 7*(3*Cos[5*(e + f*x)] - 528*Cot[e] + 44*Sin[2*(e +
f*x)] - 22*Sin[4*(e + f*x)])))/(1848*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
))))/(Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,a^3g\cos\left(fx+e\right)^3-4\,a^3g\cos\left(fx+e\right)+\left(a^3g\cos\left(fx+e\right)^3-4\,a^3g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
 e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

maple [C] time = 0.72, size = 425, normalized size = 1.24

$$-\frac{2\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\left(21\left(\cos^{6}\left(fx+e\right)\right)\sin\left(fx+e\right)+231i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos\left(fx+e\right)\sin\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/231/f*(-c*(sin(f*x+e)-1))^(1/2)*(21*cos(f*x+e)^6*sin(f*x+e)+231*I*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)+77*cos(f*x+e)^6-132*sin(f*x+e)*cos(f*x+e)^4+231*I*(1/(
```

```
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
-154*cos(f*x+e)^4-154*cos(f*x+e)^2+231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*
(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)-
4)/sin(f*x+e)/cos(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^
(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.119 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=288

$$-\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{22a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^3\sqrt{a\sin(e}{21fg}$$

```
[Out] -10/21*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))
^(1/2)-2/9*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e
))^(1/2)-22/9*a^4*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(
f*x+e))^(1/2)+22/3*a^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*El
lipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f
/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-22/21*a^3*(g*cos(f*x+e))^(5/
2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.44, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{21fg\sqrt{c-c\sin(e+fx)}}-\frac{10a^2(a\sin(e+fx)+a)^3}{21fg\sqrt{c-c\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (-22*a^4*(g*Cos[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (22*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 - (22*a^3*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(21*f*g*Sqrt[c
- c*Sin[e + f*x]]) - (10*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3
/2))/(21*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)*(a + a
*Sin[e + f*x])^(5/2))/(9*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg\sqrt{c-c\sin(e+fx)}} + \frac{1}{3}(5a)\int \frac{(g\cos(e}{}$$

$$= -\frac{10a^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg\sqrt{c-c\sin(e+fx)}} - \frac{2a(g\cos(e+fx}{9fg\sqrt{c}}$$

$$= -\frac{22a^3(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{21fg\sqrt{c-c\sin(e+fx)}} - \frac{10a^2(g\cos(e+fx}{21fg\sqrt{c}}$$

$$= -\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22a^3(g\cos(e+f}{21fg\sqrt{c}}$$

$$= -\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22a^3(g\cos(e+f}{21fg\sqrt{c}}$$

$$= -\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22a^3(g\cos(e+f}{21fg\sqrt{c}}$$

$$= -\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22a^4g\sqrt{\cos(e+f}}{3f\sqrt{a+a\,s}}$$

Mathematica [A] time = 3.71, size = 181, normalized size = 0.63

$$-\frac{a^3(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\,(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{\cos(e+fx)}\,(3\right.}{252f\cos^{\frac{3}{2}}(e+fx)\sqrt{c-c\sin(e+fx)}\,\left(\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/Sqrt[c - c*Si
n[e + f*x]],x]
```

```
[Out] -1/252*(a^3*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1
 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x])]*(-1848*EllipticE[(e + f*x)/2,
 2] + Sqrt[Cos[e + f*x]]*(1128*Cos[e + f*x] - 72*Cos[3*(e + f*x)] + 350*Sin
[2*(e + f*x)] - 7*Sin[4*(e + f*x)])))/(f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,a^3g\cos\left(fx+e\right)^3-4\,a^3g\cos\left(fx+e\right)+\left(a^3g\cos\left(fx+e\right)^3-4\,a^3g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos}}{c\sin\left(fx+e\right)-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral((3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.56, size = 436, normalized size = 1.51

$$-\frac{2\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(231i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\cos\left(fx+e\right)\sin\left(fx+e\right)\mathrm{Elliptic}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/63/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)*(231*I*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*Elliptic
F(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/s
in(f*x+e),I)-7*cos(f*x+e)^6+36*sin(f*x+e)*cos(f*x+e)^4+231*I*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+98*cos(f*
x+e)^4-168*cos(f*x+e)^2*sin(f*x+e)-322*cos(f*x+e)^2+231*cos(f*x+e))/(-cos(f
*x+e)^4+4*cos(f*x+e)^2*sin(f*x+e)+8*cos(f*x+e)^2-8*sin(f*x+e)-8)/sin(f*x+e)
/cos(f*x+e)/(-c*(sin(f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x
 + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(1/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.120 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=294

$$\frac{22a^4(g\cos(e+fx))^{5/2}}{cfg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{66a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{66a^3\sqrt{a\sin(e}{7cfg}$$

```
[Out] 4*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(3/2)+
30/7*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/c/f/g/(c-c*sin(f*x+e))
^(1/2)+22*a^4*(g*cos(f*x+e))^(5/2)/c/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*
x+e))^(1/2)-66*a^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipt
icE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c/f/(
a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+66/7*a^3*(g*cos(f*x+e))^(5/2)*
(a+a*sin(f*x+e))^(1/2)/c/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.46, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$\frac{22a^4(g\cos(e+fx))^{5/2}}{cfg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{66a^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{7cfg\sqrt{c-c\sin(e+fx)}}+\frac{30a^2(a\sin(e+fx)+a)^3}{7cfg\sqrt{c-c\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(f*g*(c - c*Sin[e +
 f*x])^(3/2)) + (22*a^4*(g*Cos[e + f*x])^(5/2))/(c*f*g*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (66*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e
+ f*x]]*EllipticE[(e + f*x)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (66*a^3*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/
(7*c*f*g*Sqrt[c - c*Sin[e + f*x]]) + (30*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*
Sin[e + f*x])^(3/2))/(7*c*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} - \frac{(15a)\int \frac{(g\cos(e+fx))^{3/\cdots}}{\sqrt{c-c\cdots}}}{c\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{30a^2(g\cos(e+fx)\cdots}{7cfg\sqrt{c-\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{66a^3(g\cos(e+fx)\cdots}{7cfg\sqrt{c-\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22a^4(g\,\mathrm{co}\cdots}{cfg\sqrt{a+a\sin(e+\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22a^4(g\,\mathrm{co}\cdots}{cfg\sqrt{a+a\sin(e+\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22a^4(g\,\mathrm{co}\cdots}{cfg\sqrt{a+a\sin(e+\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22a^4(g\,\mathrm{co}\cdots}{cfg\sqrt{a+a\sin(e+\cdots}}$$

Mathematica [A] time = 6.52, size = 284, normalized size = 0.97

$$\frac{\sec(e+fx)(a(\sin(e+fx)+1))^{7/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin(2(e+fx))+\frac{109}{14}\mathrm{c}\cdots\right.}{f(c-c\sin(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+f\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] (-66*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(f*Cos[e + f*x]^(3/2)*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(3/2)) + ((g*Cos[e
 + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 +
 Sin[e + f*x]))^(7/2)*(32 + (109*Cos[e + f*x])/14 - Cos[3*(e + f*x)]/14 + (
64*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + Sin[2*(e + f*x
)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(3/2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,a^3 g\cos\left(fx+e\right)^3-4\,a^3 g\cos\left(fx+e\right)+\left(a^3 g\cos\left(fx+e\right)^3-4\,a^3 g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\,\mathrm{co}}}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral((3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x
 + e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.62, size = 2998, normalized size = 10.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/7/f*(-1+cos(f*x+e))*(343*cos(f*x+e)^2+28*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-28*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)+28*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+sin(f
*x+e)*cos(f*x+e)^5+6*cos(f*x+e)^5-98*cos(f*x+e)^3+231*I*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticF(I*(
-1+cos(f*x+e))/sin(f*x+e),I)+231*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1
```

```
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(
f*x+e),I)+28*cos(f*x+e)^4-cos(f*x+e)^6+112*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)-112*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f
*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+168*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-1)/sin(f*x+e)^2)-168*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-119*cos(f*
x+e)^2*sin(f*x+e)-28*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+28*ln(-(2*cos(f
*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(3/2)*sin(f*x+e)+112*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2
)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-112*cos
(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-28*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)
-21*sin(f*x+e)*cos(f*x+e)^3-28*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*
x+e)^2)+28*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(
-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-84*sin(f*x+e)*
cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)
^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+84*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(3/2)-84*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+84*
sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+7*sin(f*x+e)*cos(f*x+e)^4-2
31*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I
*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3-231*I*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)
```

```
*cos(f*x+e)+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)+231*I*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*cos(f*x+e)^3-462*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2+462*I
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x
+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2)
*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^4-cos
(f*x+e)^5+4*sin(f*x+e)*cos(f*x+e)^3+5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x+
e)+8*cos(f*x+e)^3-8*sin(f*x+e)*cos(f*x+e)-20*cos(f*x+e)^2+16*sin(f*x+e)-8*c
os(f*x+e)+16)/(-c*(sin(f*x+e)-1))^(3/2)/sin(f*x+e)/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

3.121 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=298

$$-\frac{154a^4(g\cos(e+fx))^{5/2}}{5c^2fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{462a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{66a^3\sqrt{a\sin}}{5}$$

```
[Out] 4/5*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(5/2
)-12*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/c/f/g/(c-c*sin(f*x+e))
^(3/2)-154/5*a^4*(g*cos(f*x+e))^(5/2)/c^2/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)+462/5*a^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e
)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/
2)/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-66/5*a^3*(g*cos(f*x+
e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/c^2/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.49, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$-\frac{154a^4(g\cos(e+fx))^{5/2}}{5c^2fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{66a^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{5c^2fg\sqrt{c-c\sin(e+fx)}}+\frac{462a^4g\sqrt{\cos(e+fx)}\,}{5c^2f\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(5*f*g*(c - c*Sin[e
 + f*x])^(5/2)) - (12*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2)
)/(c*f*g*(c - c*Sin[e + f*x])^(3/2)) - (154*a^4*(g*Cos[e + f*x])^(5/2))/(5*
c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (462*a^4*g*Sqr
t[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sq
rt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (66*a^3*(g*Cos[e + f*x])
^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(5*c^2*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{(3a)\int \frac{(g\cos(e+fx))^{3/2}(a\cdots}{(c-c\sin(e\cdots}}{c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e+fx))^{5/\cdots}}{cfg(c-c\,\mathrm{si}\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e+fx))^{5/\cdots}}{cfg(c-c\,\mathrm{si}\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e+fx))^{5/\cdots}}{cfg(c-c\,\mathrm{si}\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e+fx))^{5/\cdots}}{cfg(c-c\,\mathrm{si}\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e+fx))^{5/\cdots}}{cfg(c-c\,\mathrm{si}\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e+fx))^{5/\cdots}}{cfg(c-c\,\mathrm{si}\cdots}$$

Mathematica [A] time = 4.64, size = 267, normalized size = 0.90

$$-\frac{a^3\sqrt{a(\sin(e+fx)+1)}\,(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(487\sin\left(\frac{1}{2}(e+fx\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] -1/20*(a^3*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*S
qrt[a*(1 + Sin[e + f*x])]*(-1848*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(487*Cos[(e + f*x)/2] + 633*Co
s[(3*(e + f*x))/2] - 17*Cos[(5*(e + f*x))/2] + Cos[(7*(e + f*x))/2] + 487*S
in[(e + f*x)/2] - 633*Sin[(3*(e + f*x))/2] - 17*Sin[(5*(e + f*x))/2] - Sin[
(7*(e + f*x))/2])))/(c^2*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,a^3 g\cos\left(fx+e\right)^3-4\,a^3 g\cos\left(fx+e\right)+\left(a^3 g\cos\left(fx+e\right)^3-4\,a^3 g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos}}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral((3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^
3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.62, size = 3601, normalized size = 12.08

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/5/f*(-1+cos(f*x+e))*(446*cos(f*x+e)^2+80*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-80*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-1)/sin(f*x+e)^2)-80*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+231*I*
sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-sin(f*x+e)*cos(f*x+e)^5-8
*cos(f*x+e)^5-85*cos(f*x+e)^3-78*cos(f*x+e)^4+cos(f*x+e)^6-40*cos(f*x+e)^5*
```

(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-1)/sin(f*x+e)^2)+40*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-40*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^4+40*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)
^2)*sin(f*x+e)*cos(f*x+e)^4+80*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-80*
cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+320*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)-320*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f
x+e)^2(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-478*cos(f*x+e)^2*sin(f*
x+e)-80*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+80*ln(-(2*cos(f*x+e)^2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
*sin(f*x+e)+280*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-280*cos(f*x+e)*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-1)/sin(f*x+e)^2)+80*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*l
n(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(
f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+69*sin(f*x+e
)*cos(f*x+e)^3-200*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+200*
sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-360*sin(f*x+e)*cos(f*x+e)
^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+360*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e
)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)

```
^2)^(3/2)-280*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+280*sin(f*x
+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)
^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-9*sin(f*x+e)*cos(f*x+e)^4-462*I*co
s(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+693*I*sin(f*x+e)*cos(f*x+e)^2
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1
+cos(f*x+e))/sin(f*x+e),I)-693*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*
x+e),I)-231*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+462*I*cos(f
*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+231*I*cos(f*x+e)^4*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/s
in(f*x+e),I)-231*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-693*I*cos(f*x+e)^
2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-
1+cos(f*x+e))/sin(f*x+e),I)+693*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-46
2*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+462*I*cos(f*x+e)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x
+e)^4-cos(f*x+e)^5+4*sin(f*x+e)*cos(f*x+e)^3+5*cos(f*x+e)^4-12*cos(f*x+e)^2
*sin(f*x+e)+8*cos(f*x+e)^3-8*sin(f*x+e)*cos(f*x+e)-20*cos(f*x+e)^2+16*sin(f
*x+e)-8*cos(f*x+e)+16)/cos(f*x+e)/(-c*(sin(f*x+e)-1))^(5/2)/sin(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\cos\left(e+fx\right)\right)^{3/2}\left(a+a\sin\left(e+fx\right)\right)^{7/2}}{\left(c-c\sin\left(e+fx\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

3.122 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=300

$$\frac{154a^4(g\cos(e+fx))^{5/2}}{9c^3fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{154a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{3c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{44a^3\sqrt{a\,\mathrm{si}}}{3c}$$

```
[Out] 4/9*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(7/2
)-4/3*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/c/f/g/(c-c*sin(f*x+e)
)^(5/2)+44/3*a^3*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/c^2/f/g/(c-c*s
in(f*x+e))^(3/2)+154/9*a^4*(g*cos(f*x+e))^(5/2)/c^3/f/g/(a+a*sin(f*x+e))^(1
/2)/(c-c*sin(f*x+e))^(1/2)-154/3*a^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2
*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f
*x+e))^(1/2)/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.54, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$\frac{154a^4(g\cos(e+fx))^{5/2}}{9c^3fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{44a^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{3c^2fg(c-c\sin(e+fx))^{3/2}}-\frac{154a^4g\sqrt{\cos(e+fx)}\,E}{3c^3f\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(9*f*g*(c - c*Sin[e
 + f*x])^(7/2)) - (4*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))
/(3*c*f*g*(c - c*Sin[e + f*x])^(5/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2)*Sqrt
[a + a*Sin[e + f*x]])/(3*c^2*f*g*(c - c*Sin[e + f*x])^(3/2)) + (154*a^4*(g*
Cos[e + f*x])^(5/2))/(9*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]) - (154*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
 f*x)/2, 2])/(3*c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

x]

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{(5a)\int \frac{(g\cos(e+fx))^{3/2}(}{(c-c\sin(}}{3c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{3cfg(c-c\,s}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{3cfg(c-c\,s}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{3cfg(c-c\,s}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{3cfg(c-c\,s}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{3cfg(c-c\,s}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{3cfg(c-c\,s}$$

Mathematica [A] time = 6.58, size = 406, normalized size = 1.35

$$\frac{\sec(e+fx)(a(\sin(e+fx)+1))^{7/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\frac{2}{3}\cos(e+fx)+\frac{}{3(\cos(}\right.}{f(c-c\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(7/2),x]
```

```
[Out] (-154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*Cos[e + f*x]^(3/2)*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(7/2)) + ((g*Co
s[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(112
/3 + (2*Cos[e + f*x])/3 + 32/(9*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4) -
32/(3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (64*Sin[(e + f*x)/2])/(9*(
```

```
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) - (64*Sin[(e + f*x)/2])/(3*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^3) + (224*Sin[(e + f*x)/2])/(3*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(7/2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,a^3 g\cos\left(fx+e\right)^3-4\,a^3 g\cos\left(fx+e\right)+\left(a^3 g\cos\left(fx+e\right)^3-4\,a^3 g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\ldots}}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
 e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*
x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.62, size = 4237, normalized size = 14.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/9/f*(-1+cos(f*x+e))*(-940*cos(f*x+e)^2-216*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+21
6*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-1)/sin(f*x+e)^2)+540*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
```

)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*
sin(f*x+e)*cos(f*x+e)^5-54*cos(f*x+e)^5+146*cos(f*x+e)^3-231*I*sin(f*x+e)*c
os(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+567*cos(f*x+e)^4+3*cos(f*x+e)^6+324*c
os(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-324*cos(f*x+e)^5*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*
x+e)^2)-54*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^4+54*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-
1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^4+54*cos(f*x+e)^6*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(
f*x+e)^2)-54*cos(f*x+e)^6*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f
x+e)^2(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-810*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-1)/sin(f*x+e)^2)+810*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+908*cos(f*
x+e)^2*sin(f*x+e)+216*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/s
in(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-216*ln(-(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(3/2)*sin(f*x+e)-756*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+756*c
os(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-540*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)
^2)-162*sin(f*x+e)*cos(f*x+e)^3+378*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/s
in(f*x+e)^2)-378*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+918*sin(
f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1

```
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-918*sin(f*x+e)*cos(f*x+e)^2
*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(3/2)+756*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)-756*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-57*sin(f*x+e)*cos
(f*x+e)^4-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3+231*I*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*cos(f*x+e)^3+231*I*sin(f*x+e)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)-54*sin(f*x+e)*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2
*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+54*sin(f*x+e)*
cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-924*I*(1/(cos(f*x+e)+1))^(1/2)*cos(f
*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)+924*I*(1/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos(f*x+e)^5*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*
x+e))/sin(f*x+e),I)-231*I*cos(f*x+e)^5*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+924*I*cos(
f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-924*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)-1386*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+1386*I*(1/(cos(f*x+e)+1)
)^(1/2)*cos(f*x+e)^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)+231*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)-231*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+1386*I*sin(f*x+e
)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-1386*I*sin(f*x+e)*cos(f*x+e)^2*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)+924*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e
)-924*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e))*(g*cos(f*x+e))^(3/
2)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^4-cos(f*x+e)^5+4*sin(f*x
```

```
+e)*cos(f*x+e)^3+5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x+e)+8*cos(f*x+e)^3-8
*sin(f*x+e)*cos(f*x+e)-20*cos(f*x+e)^2+16*sin(f*x+e)-8*cos(f*x+e)+16)/(-c*(
sin(f*x+e)-1))^(7/2)/sin(f*x+e)/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(7/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

3.123 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=300

$$\frac{154a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{13c^4f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{308a^4(g\cos(e+fx))^{5/2}}{39c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{44a^3\sqrt{a\ ...}}{39 ...}$$

```
[Out] 4/13*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(9/
2)-20/39*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/c/f/g/(c-c*sin(f*x
+e))^(7/2)-308/39*a^4*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))^(3/2)/(
a+a*sin(f*x+e))^(1/2)+44/39*a^3*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)
/c^2/f/g/(c-c*sin(f*x+e))^(5/2)+154/13*a^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/c
os(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g
*cos(f*x+e))^(1/2)/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.53, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2850, 2842, 2640, 2639}

$$-\frac{308a^4(g\cos(e+fx))^{5/2}}{39c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{44a^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{39c^2fg(c-c\sin(e+fx))^{5/2}}+\frac{154a^4g\sqrt{\cos(e+f ...}}{13c^4f\sqrt{a\sin(...}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(9/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(13*f*g*(c - c*Sin[
e + f*x])^(9/2)) - (20*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(39*c*f*g*(c - c*Sin[e + f*x])^(7/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2)*S
qrt[a + a*Sin[e + f*x]])/(39*c^2*f*g*(c - c*Sin[e + f*x])^(5/2)) - (308*a^4
*(g*Cos[e + f*x])^(5/2))/(39*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(3/2)) + (154*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(13*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f
*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{(15a)\int \frac{(g\cos(e+fx))^{3/2}(}{(c-c\sin(}}{13c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e+fx))^{5/}}{39cfg(c-c\,s}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e+fx))^{5/}}{39cfg(c-c\,s}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e+fx))^{5/}}{39cfg(c-c\,s}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e+fx))^{5/}}{39cfg(c-c\,s}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e+fx))^{5/}}{39cfg(c-c\,s}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e+fx))^{5/}}{39cfg(c-c\,s}$$

Mathematica [A] time = 6.63, size = 464, normalized size = 1.55

$$\frac{154E\left(\frac{1}{2}(e+fx)\middle|2\right)(a(\sin(e+fx)+1))^{7/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}{13f\cos^{\frac{3}{2}}(e+fx)(c-c\sin(e+fx))^{9/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} + \frac{\sec(e+fx)(a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(9/2),x]
```

```
[Out] (154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(13*f*Cos[e + f*x]^(3/2)*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(9/2)) + ((g*Co
s[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(-12
8/13 + 32/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6) - 224/(39*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^4) + 80/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
```

```
])^2) + (64*Sin[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
- (448*Sin[(e + f*x)/2])/(39*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) + (16
0*Sin[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) - (256*Sin
[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e +
f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x
])^(9/2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,a^3 g\cos\left(fx+e\right)^3-4\,a^3 g\cos\left(fx+e\right)+\left(a^3 g\cos\left(fx+e\right)^3-4\,a^3 g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g}}{5\,c^5\cos\left(fx+e\right)^4-20\,c^5\cos\left(fx+e\right)^2+16\,c^5-\left(c^5\cos\left(fx+e\right)^4-12\,c^5\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
 e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^5*cos(f*x + e)^4 - 20*c^5*cos
(f*x + e)^2 + 16*c^5 - (c^5*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 + 16*c^5
)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.61, size = 4829, normalized size = 16.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 2/39/f*(-1+cos(f*x+e))*(1800*cos(f*x+e)^2+624*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-6
24*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
```

os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+1848*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-1848*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+462*I*cos(f*x+e)^5*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+2772*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-2772*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-462*I*cos(f*x+e)^5*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+2541*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-2541*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+924*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-924*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-2184*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-78*cos(f*x+e)^7*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-39*sin(f*x+e)*cos(f*x+e)^5+78*cos(f*x+e)^7*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+237*cos(f*x+e)^5-332*cos(f*x+e)^3-1848*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-1472*cos(f*x+e)^4+39*cos(f*x+e)^6-1092*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+1092*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+858*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^4-858*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^4-546*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+546*cos(f*x+e)^3*(-cos(f*x+

e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)+2184*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-
2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f
x+e)-2(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-2184*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-1896*cos(f*x+e)^2*sin(f*x+e)-624*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(3/2)*sin(f*x+e)+624*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+2184*c
os(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-2184*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-co
s(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e
)^2)+2184*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+284*sin(f*x+e)*cos(f*x+e)^
3-546*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+546*sin(f*x+e)*co
s(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-2496*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(3/2)+2496*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-2
184*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-
1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+2184*sin(f*x+e)*cos(f
*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+660*sin(f*x+e)*cos(f*x+e)^4+1848*I*cos(f*x+
e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(
-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+78*sin(f*x+e)*cos(f*x+e)^6*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-1)/sin(f*x+e)^2)-78*sin(f*x+e)*cos(f*x+e)^6*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+54

```
6*sin(f*x+e)*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-546*sin(f*x+e)*cos(f*
x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-2772*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))
/sin(f*x+e),I)+2772*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231
*I*sin(f*x+e)*cos(f*x+e)^5*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-1155*I*sin(f*x+e)*cos(
f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+1155*I*sin(f*x+e)*cos(f*x+e)^4*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e
))/sin(f*x+e),I)-231*I*sin(f*x+e)*cos(f*x+e)^5*(1/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(
g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^4-cos(f
*x+e)^5+4*sin(f*x+e)*cos(f*x+e)^3+5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x+e)
+8*cos(f*x+e)^3-8*sin(f*x+e)*cos(f*x+e)-20*cos(f*x+e)^2+16*sin(f*x+e)-8*cos
(f*x+e)+16)/(-c*(sin(f*x+e)-1))^(9/2)/sin(f*x+e)/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(9/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
9/2),x)
```

```
[Out] Timed out
```

3.124 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=357

$$-\frac{154a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{221c^5f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{154a^4(g\cos(e+fx))^{5/2}}{221c^4fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{}{663c^3}$$

```
[Out] 4/17*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(11
/2)-60/221*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/c/f/g/(c-c*sin(f
*x+e))^(9/2)-308/663*a^4*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))^(5/2
)/(a+a*sin(f*x+e))^(1/2)+154/221*a^4*(g*cos(f*x+e))^(5/2)/c^4/f/g/(c-c*sin(
f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+220/663*a^3*(g*cos(f*x+e))^(5/2)*(a+a*
sin(f*x+e))^(1/2)/c^2/f/g/(c-c*sin(f*x+e))^(7/2)-154/221*a^4*g*(cos(1/2*f*x
+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*c
os(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^5/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(
f*x+e))^(1/2)
```

Rubi [A] time = 1.84, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{154a^4(g\cos(e+fx))^{5/2}}{221c^4fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{308a^4(g\cos(e+fx))^{5/2}}{663c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}+\frac{220a^3\sqrt{a\sin}}{663c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(11/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(17*f*g*(c - c*Sin[
e + f*x])^(11/2)) - (60*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/
2))/(221*c*f*g*(c - c*Sin[e + f*x])^(9/2)) + (220*a^3*(g*Cos[e + f*x])^(5/2
)*Sqrt[a + a*Sin[e + f*x]])/(663*c^2*f*g*(c - c*Sin[e + f*x])^(7/2)) - (308
*a^4*(g*Cos[e + f*x])^(5/2))/(663*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(5/2)) + (154*a^4*(g*Cos[e + f*x])^(5/2))/(221*c^4*f*g*Sqrt[a
+ a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (154*a^4*g*Sqrt[Cos[e + f*x
]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(221*c^5*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{(15a)\int \frac{(g\cos(e+fx))^{3/2}(}{(c-c\sin(}}{17c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+fx))^{5/}}{221cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+fx))^{5/}}{221cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+fx))^{5/}}{221cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+fx))^{5/}}{221cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+fx))^{5/}}{221cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+fx))^{5/}}{221cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+fx))^{5/}}{221cfg(c-c}$$

Mathematica [A] time = 6.68, size = 532, normalized size = 1.49

$$\sec(e+fx)(a(\sin(e+fx)+1))^{7/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}\left(\frac{308\sin\left(\frac{1}{2}(e+fx)\right)}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(11/2),x]
```

```
[Out] (-154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(7/2))/(221*f*Cos[e + f*x]^(3/2
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) + ((
```

```
g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11
*(154/221 + 32/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8) - 864/(221*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^6) + 2096/(663*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^4) - 288/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (64*Si
n[(e + f*x)/2])/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9) - (1728*Sin[(e
 + f*x)/2])/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7) + (4192*Sin[(e +
f*x)/2])/(663*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) - (576*Sin[(e + f*x)
/2])/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) + (308*Sin[(e + f*x)/2])
/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(7/2))
/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,a^3 g\cos\left(fx+e\right)^3 - 4\,a^3 g\cos\left(fx+e\right) + \left(a^3 g\cos\left(fx+e\right)^3 - 4\,a^3 g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\,c}}{c^6\cos\left(fx+e\right)^6 - 18\,c^6\cos\left(fx+e\right)^4 + 48\,c^6\cos\left(fx+e\right)^2 - 32\,c^6 + 2\left(3\,c^6\cos\left(fx+e\right)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="fricas")
```

```
[Out] integral((3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^6*cos(f*x + e)^6 - 18*c^6*cos(f*
x + e)^4 + 48*c^6*cos(f*x + e)^2 - 32*c^6 + 2*(3*c^6*cos(f*x + e)^4 - 16*c^
6*cos(f*x + e)^2 + 16*c^6)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.59, size = 3910, normalized size = 10.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] -1/1326/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)*(-1+cos(f*x+e))^4*(
sin(f*x+e)-1)*(cos(f*x+e)+1)*(-2960*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*co
s(f*x+e)^5+2652*cos(f*x+e)^5*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)-2652*cos(f*x+e)^5*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)+7956*cos(f*x+e)^3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+663*sin(f*x+e)*cos(f*x+e)^5*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-752*sin(f*x+e)*cos(f
*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1044*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)*sin(f*x+e)*cos(f*x+e)^4-11840*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)+5304*sin(f*x+e)*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-5304*sin(f*x+e)*cos(f*x+e)^3*
ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+5304*cos(
f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*si
n(f*x+e)-5304*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/
sin(f*x+e)^2)*sin(f*x+e)-7956*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+7856*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-5304*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-1)/sin(f*x+e)^2)+5304*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-1)/sin(f*x+e)^2)+5356*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-663*sin(f*x+e)*cos(f*x+e)^5*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-1)/sin(f*x+e)^2)+2652*sin(f*x+e)*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)+2496*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-6928*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-3696*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^6*
EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+9888*sin(f*x+e)*cos(f*x+e)*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3696*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^6*Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-924*cos(f*x+e)^6*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)+2496*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-4896
*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-7392*I*(1/(cos(f*x+e)+1))^
(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+7392*I*(1/(cos(f*x+e)+1))^(1/2)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
```

```
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+14784*I*sin(f*x+e)*cos(f*x+e)^3*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+14784*I*sin(f*x+
e)*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I
)-14784*I*sin(f*x+e)*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)+924*I*sin(f*x+e)*cos(f*x+e)^6*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-924*I*sin(f*x+e)*cos(f*x+e)^6*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+1848*I*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*sin(f*x+e)*cos(f*x+e)^5*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-
1848*I*sin(f*x+e)*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)-6468*I*sin(f*x+e)*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+6468*I*sin(f*x+e)*cos(f*x+e)^4*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-14784*I*sin(f*x+e)
*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I
)-7392*I*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)+7392*I*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+
e))/sin(f*x+e),I)+7392*I*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)-7392*I*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(-1+cos(f*x+e))/sin(f*x+e),I)+22176*I*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-22176*I*cos(f*x+e)^3*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3696*I*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3696*I*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+7392*I*
(1/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-7
392*I*(1/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*
x+e)-14784*I*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)
```

```
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/si
n(f*x+e),I)+14784*I*(1/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*cos(f*x+e))/(-cos(f*x+e)^4+4*cos(f*x+e)^2*sin(f*x+e)+8*cos(f*x+e)^2
-8*sin(f*x+e)-8)/sin(f*x+e)^9/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/(-c*(sin
(f*x+e)-1))^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(11/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
11/2),x)
```

```
[Out] Timed out
```

3.125 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=414

$$-\frac{22a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{663c^6f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22a^4(g\cos(e+fx))^{5/2}}{663c^5fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{}{663c}$$

```
[Out] 4/21*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(13
/2)-20/119*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/c/f/g/(c-c*sin(f
*x+e))^(11/2)-220/1989*a^4*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))^(7
/2)/(a+a*sin(f*x+e))^(1/2)+22/663*a^4*(g*cos(f*x+e))^(5/2)/c^4/f/g/(c-c*sin
(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+22/663*a^4*(g*cos(f*x+e))^(5/2)/c^5/f
/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+220/1547*a^3*(g*cos(f*x+e)
)^(5/2)*(a+a*sin(f*x+e))^(1/2)/c^2/f/g/(c-c*sin(f*x+e))^(9/2)-22/663*a^4*g*
(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e
),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^6/f/(a+a*sin(f*x+e))^(1/
2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 2.17, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{22a^4(g\cos(e+fx))^{5/2}}{663c^5fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{22a^4(g\cos(e+fx))^{5/2}}{663c^4fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}-\frac{}{1989c^3fg}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(13/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(21*f*g*(c - c*Sin[
e + f*x])^(13/2)) - (20*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/
2))/(119*c*f*g*(c - c*Sin[e + f*x])^(11/2)) + (220*a^3*(g*Cos[e + f*x])^(5/
2)*Sqrt[a + a*Sin[e + f*x]])/(1547*c^2*f*g*(c - c*Sin[e + f*x])^(9/2)) - (2
20*a^4*(g*Cos[e + f*x])^(5/2))/(1989*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(7/2)) + (22*a^4*(g*Cos[e + f*x])^(5/2))/(663*c^4*f*g*Sqrt[
a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (22*a^4*(g*Cos[e + f*x])^
(5/2))/(663*c^5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) -
(22*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
)/(663*c^6*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{(5a)\int \frac{(g\cos(e+fx))^{3/2}(\cdots}{(c-c\sin(\cdots}}{7c\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))\cdots}{119cfg(c-c\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))\cdots}{119cfg(c-c\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))\cdots}{119cfg(c-c\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))\cdots}{119cfg(c-c\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))\cdots}{119cfg(c-c\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))\cdots}{119cfg(c-c\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))\cdots}{119cfg(c-c\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))\cdots}{119cfg(c-c\cdots}$$

Mathematica [A] time = 6.79, size = 600, normalized size = 1.45

$$\frac{\sec(e+fx)(a(\sin(e+fx)+1))^{7/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{13}\left(\frac{44\sin\left(\frac{1}{2}(e+fx)\right)}{663\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(\cdots\right.\right.}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(13/2),x]
```

```
[Out] (-22*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^13*(a*(1 + Sin[e + f*x]))^(7/2))/(663*f*Cos[e + f*x]^(3/2)
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) + ((g
*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^13*
(22/663 + 32/(21*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10) - 352/(119*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])^8) + 464/(221*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^6) - 1216/(1989*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4) + 22/(66
3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (64*Sin[(e + f*x)/2])/(21*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^11) - (704*Sin[(e + f*x)/2])/(119*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^9) + (928*Sin[(e + f*x)/2])/(221*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^7) - (2432*Sin[(e + f*x)/2])/(1989*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^5) + (44*Sin[(e + f*x)/2])/(663*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^3) + (44*Sin[(e + f*x)/2])/(663*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,a^3 g\cos\left(fx+e\right)^3-4\,a^3 g\cos\left(fx+e\right)+\left(a^3 g\cos\left(fx+e\right)^3-4\,a^3 g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos}}{7\,c^7\cos\left(fx+e\right)^6-56\,c^7\cos\left(fx+e\right)^4+112\,c^7\cos\left(fx+e\right)^2-64\,c^7-\left(c^7\cos\left(fx+e\right)^6-24\,c^7\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="fricas")
```

```
[Out] integral((3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(7*c^7*cos(f*x + e)^6 - 56*c^7*cos(
f*x + e)^4 + 112*c^7*cos(f*x + e)^2 - 64*c^7 - (c^7*cos(f*x + e)^6 - 24*c^7
*cos(f*x + e)^4 + 80*c^7*cos(f*x + e)^2 - 64*c^7)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 0.62, size = 1484, normalized size = 3.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x)

[Out] 2/13923/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)*(sin(f*x+e)*cos(f*x+e)-sin(f*x+e)-cos(f*x+e)+1)*(-10608-10608*sin(f*x+e)+14304*cos(f*x+e)+12092*cos(f*x+e)^2+6912*sin(f*x+e)*cos(f*x+e)+4256*sin(f*x+e)*cos(f*x+e)^5+6566*cos(f*x+e)^5-20408*cos(f*x+e)^3+1155*I*sin(f*x+e)*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-791*cos(f*x+e)^4-1155*cos(f*x+e)^6+3696*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3696*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+19108*cos(f*x+e)^2*sin(f*x+e)-12640*sin(f*x+e)*cos(f*x+e)^3-7259*sin(f*x+e)*cos(f*x+e)^4+231*cos(f*x+e)^6*sin(f*x+e)-1155*I*sin(f*x+e)*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos(f*x+e)^8*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*cos(f*x+e)^8*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3234*I*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3234*I*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+9471*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-9471*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-10164*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+10164*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3696*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3696*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-5775*I*sin(f*x+e)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+5775*I*sin(f*x+e)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+8316*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-8316*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(-cos(f*x+e)^4+4*cos(f*x+e)^2*sin(f*x+e)+8*cos(f*x+e)^2-8*sin(f*x+e)-8)/(-c*(sin(f*x+e)-1))^(13/2)/sin(f*x+e)^5/cos(f*x+e)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(13/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(13/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(13/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
13/2),x)
```

```
[Out] Timed out
```

3.126 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx$

Optimal. Leaf size=471

$$-\frac{22a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5525c^7f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22a^4(g\cos(e+fx))^{5/2}}{5525c^6fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{}{552}$$

```
[Out] 4/25*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(15
/2)-4/35*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/c/f/g/(c-c*sin(f*x
+e))^(13/2)-44/1105*a^4*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))^(9/2)
/(a+a*sin(f*x+e))^(1/2)+22/3315*a^4*(g*cos(f*x+e))^(5/2)/c^4/f/g/(c-c*sin(f
*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2)+22/5525*a^4*(g*cos(f*x+e))^(5/2)/c^5/f/
g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+22/5525*a^4*(g*cos(f*x+e))^
(5/2)/c^6/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+44/595*a^3*(g*c
os(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/c^2/f/g/(c-c*sin(f*x+e))^(11/2)-22/
5525*a^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/
2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^7/f/(a+a*sin(
f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 2.49, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{22a^4(g\cos(e+fx))^{5/2}}{5525c^6fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{22a^4(g\cos(e+fx))^{5/2}}{5525c^5fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}+\frac{}{3315c^4f}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(15/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(25*f*g*(c - c*Sin[
e + f*x])^(15/2)) - (4*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(35*c*f*g*(c - c*Sin[e + f*x])^(13/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2)*
Sqrt[a + a*Sin[e + f*x]])/(595*c^2*f*g*(c - c*Sin[e + f*x])^(11/2)) - (44*a
^4*(g*Cos[e + f*x])^(5/2))/(1105*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Si
n[e + f*x])^(9/2)) + (22*a^4*(g*Cos[e + f*x])^(5/2))/(3315*c^4*f*g*Sqrt[a +
 a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (22*a^4*(g*Cos[e + f*x])^(5/
2))/(5525*c^5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (2
2*a^4*(g*Cos[e + f*x])^(5/2))/(5525*c^6*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c
*Sin[e + f*x])^(3/2)) - (22*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
```

```
llipticE[(e + f*x)/2, 2])/(5525*c^7*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{(3a)\int \frac{(g\cos(e+fx))^{3/2}(}{(c-c\sin(}}{5c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{35cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{35cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{35cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{35cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{35cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{35cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{35cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{35cfg(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5}}{35cfg(c-c}$$

Mathematica [A] time = 6.87, size = 668, normalized size = 1.42

$$\frac{\sec(e+fx)(a(\sin(e+fx)+1))^{7/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{15}\left(\frac{44\sin\left(\frac{1}{2}(e+fx)\right)}{5525\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}\right.\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(15/2),x]
```

```
[Out] (-22*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^15*(a*(1 + Sin[e + f*x]))^(7/2))/(5525*f*Cos[e + f*x]^(3/2
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + ((
g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^15
*(22/5525 + 32/(25*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^12) - 416/(175*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^10) + 4656/(2975*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^8) - 2144/(5525*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6) + 2
2/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4) + 22/(5525*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2) + (64*Sin[(e + f*x)/2])/(25*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^13) - (832*Sin[(e + f*x)/2])/(175*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^11) + (9312*Sin[(e + f*x)/2])/(2975*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^9) - (4288*Sin[(e + f*x)/2])/(5525*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])^7) + (44*Sin[(e + f*x)/2])/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^5) + (44*Sin[(e + f*x)/2])/(5525*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^3) + (44*Sin[(e + f*x)/2])/(5525*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))
*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(
c - c*Sin[e + f*x])^(15/2))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,a^3 g\cos\left(fx+e\right)^3-4\,a^3 g\cos\left(fx+e\right)+\left(a^3 g\cos\left(fx+e\right)^3-4\,a^3 g\cos\left(fx+e\right)\right)\sin\left(fx\right.}{c^8\cos\left(fx+e\right)^8-32\,c^8\cos\left(fx+e\right)^6+160\,c^8\cos\left(fx+e\right)^4-256\,c^8\cos\left(fx+e\right)^2+128\,c^8+8\left(c^8\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/
2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
 e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^8*cos(f*x + e)^8 - 32*c^8*cos(f
*x + e)^6 + 160*c^8*cos(f*x + e)^4 - 256*c^8*cos(f*x + e)^2 + 128*c^8 + 8*(
c^8*cos(f*x + e)^6 - 10*c^8*cos(f*x + e)^4 + 24*c^8*cos(f*x + e)^2 - 16*c^8
)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.74, size = 1644, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x)
```

```
[Out] -2/116025/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)*(sin(f*x+e)*cos(f
*x+e)-sin(f*x+e)-cos(f*x+e)+1)*(74256+74256*sin(f*x+e)-81648*cos(f*x+e)-112
324*cos(f*x+e)^2-66864*sin(f*x+e)*cos(f*x+e)-20895*sin(f*x+e)*cos(f*x+e)^5-
231*cos(f*x+e)^8-50003*cos(f*x+e)^5+130804*cos(f*x+e)^3+231*I*EllipticE(I*(
-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^8*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+34757*cos(f*x+e)^4+4004*cos(f*x+e)^6-9
9836*cos(f*x+e)^2*sin(f*x+e)+85052*sin(f*x+e)*cos(f*x+e)^3+385*cos(f*x+e)^7
+29673*sin(f*x+e)*cos(f*x+e)^4-7392*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+7392*I*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)+15246*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin
(f*x+e)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)-15246*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e
)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-18480*I*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+18480*I*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)-1386*cos(f*x+e)^6*sin(f*x+e)-231*I*EllipticF(I
*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^8*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-1386*I*EllipticE(I*(-1+cos(f*x+e))/s
in(f*x+e),I)*cos(f*x+e)^8*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)+1386*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^8*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+10164*I*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)-10164*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-
23562*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+23562*I*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)+22176*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*
x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-22176*I*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+7392*I*EllipticE(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)-7392*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-4389*I*EllipticE(I*(-1+c
```

```
os(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+4389*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*sin(f*x+e)*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2))*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(-cos(f*x+e)^4+4*cos(f*x+e)^2
*sin(f*x+e)+8*cos(f*x+e)^2-8*sin(f*x+e)-8)/(-c*(sin(f*x+e)-1))^(15/2)/sin(f
*x+e)^5/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(15/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(15/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(15/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
15/2),x)
```

```
[Out] Timed out
```

3.127 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=234

$$\frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22c^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22c^2\sqrt{c-c\text{s}}}{35f}$$

```
[Out] 2/7*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e))^(1/2
)+22/15*c^3*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)+22/5*c^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Elliptic
E(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*
sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+22/35*c^2*(g*cos(f*x+e))^(5/2)*(c-
c*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.12, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22c^2\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{35fg\sqrt{a\sin(e+fx)+a}}+\frac{22c^3g\sqrt{\cos(e+fx)}\,E\Big(}{5f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] (22*c^3*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (22*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 + (22*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(35*f*g*Sqrt[a
+ a*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)
)/(7*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{7fg\sqrt{a+a\sin(e+fx)}} + \frac{1}{7}(11c)\int \frac{(g\cos(e+f}{\sqrt{a}}$$

$$= \frac{22c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{35fg\sqrt{a+a\sin(e+fx)}} + \frac{2c(g\cos(e+fx))^{5/2}(c}{7fg\sqrt{a+a\,\mathrm{si}}}$$

$$= \frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22c^2(g\cos(e+fx))}{35fg\sqrt{a+}}$$

$$= \frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22c^2(g\cos(e+fx))}{35fg\sqrt{a+}}$$

$$= \frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22c^2(g\cos(e+fx))}{35fg\sqrt{a+}}$$

$$= \frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22c^3g\sqrt{\cos(e+fx)}}{5f\sqrt{a+a\sin(}}$$

Mathematica [A] time = 1.46, size = 174, normalized size = 0.74

$$\frac{c^2(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(924E\left(\frac{1}{2}(e+fx)\right|\right.}{210f\cos^{\frac{3}{2}}(e+fx)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Si
n[e + f*x]],x]
```

```
[Out] (c^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin
[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]*(924*EllipticE[(e + f*x)/2, 2] + Sqrt
[Cos[e + f*x]]*(515*Cos[e + f*x] - 3*(5*Cos[3*(e + f*x)] + 42*Sin[2*(e + f*
x)]))))/(210*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*S
qrt[a*(1 + Sin[e + f*x])])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c^2g\cos\left(fx+e\right)^3+2\,c^2g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^2g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{-c\sin\left(f\right.}}{\sqrt{a\sin\left(fx+e\right)+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*g*cos(f*x + e)^3 + 2*c^2*g*cos(f*x + e)*sin(f*x + e) - 2*c^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(
f*x + e) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.53, size = 415, normalized size = 1.77

$$-\frac{2\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(231i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\cos\left(fx+e\right)\sin\left(fx+e\right)\mathrm{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)-231\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/105/f*(g*cos(f*x+e))^(3/2)*(231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+
15*sin(f*x+e)*cos(f*x+e)^4-63*cos(f*x+e)^4-140*cos(f*x+e)^2*sin(f*x+e)+294*
cos(f*x+e)^2-231*cos(f*x+e))*(-c*(sin(f*x+e)-1))^(5/2)/(cos(f*x+e)^2*sin(f*
x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+4)/sin(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/cos
(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x
 + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)
)^(1/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)
)^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.128 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=180

$$\frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2c\sqrt{c-c\sin(\ldots}}{5fg\sqrt{\ldots}}$$

```
[Out] 14/15*c^2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^
(1/2)+14/5*c^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(
sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*si
n(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2/5*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(
f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2c\sqrt{c-c\sin(\ldots}}{5fg\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] (14*c^2*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (14*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 + (2*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*f*g*Sqrt[a + a*
Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{2c(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5fg\sqrt{a+a\sin(e+fx)}} + \frac{1}{5}(7c)\int \frac{(g\cos(e+fx}{\sqrt{a+}}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2c(g\cos(e+fx))^{5}}{5fg\sqrt{a+}}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2c(g\cos(e+fx))^{5}}{5fg\sqrt{a+}}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2c(g\cos(e+fx))^{5}}{5fg\sqrt{a+}}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{14c^2g\sqrt{\cos(e+f}}{5f\sqrt{a+a\sin}}$$

Mathematica [A] time = 0.68, size = 157, normalized size = 0.87

$$-\frac{c(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(42E\left(\frac{1}{2}(e+fx)\middle|2\right)\right.}{15f\cos^{\frac{3}{2}}(e+fx)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Si
n[e + f*x]],x]
```

```
[Out] -1/15*(c*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 +
 Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]*(42*EllipticE[(e + f*x)/2, 2] + Sqr
t[Cos[e + f*x]]*(20*Cos[e + f*x] - 3*Sin[2*(e + f*x)])))/(f*Cos[e + f*x]^(3
/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(cg\cos\left(fx+e\right)\sin\left(fx+e\right)-cg\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(c*g*cos(f*x + e)*sin(f*x + e) - c*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.50, size = 382, normalized size = 2.12

$$-\frac{2\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(21i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\cos\left(fx+e\right)\sin\left(fx+e\right)\text{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)-21i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/15/f*(g*cos(f*x+e))^(3/2)*(21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin
```

```
(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF(I*(-
1+cos(f*x+e))/sin(f*x+e),I)-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*cos
(f*x+e)^4+10*cos(f*x+e)^2*sin(f*x+e)-24*cos(f*x+e)^2+21*cos(f*x+e))*(-c*(si
n(f*x+e)-1))^(3/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/sin(f*x+e)/cos(f*x+e)/(a*(
1+sin(f*x+e)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x
 + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)
)^(1/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)
)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.129 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=122

$$\frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2cg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2/3*c*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
)+2*c*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f
*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))
^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2cg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] (2*c*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin
[e + f*x]]) + (2*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
 f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
```

```
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} + c\int \frac{(g\cos}{\sqrt{a+a\sin(e+f}}$$

$$= \frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} + \frac{(cg\cos(e+fx))\int\sqrt{}}{\sqrt{a+a\sin(e+fx)}\,\sqrt{}}$$

$$= \frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} + \frac{\left(cg\sqrt{\cos(e+fx)}\,\sqrt{g}\right.}{\sqrt{a+a\sin(e}}$$

$$= \frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} + \frac{2cg\sqrt{\cos(e+fx)}\,\sqrt{g}}{f\sqrt{a+a\sin(e+}}$$

Mathematica [C] time = 2.50, size = 215, normalized size = 1.76

$$\frac{\sqrt{e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)}\sqrt{-iae^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}\left(12ie^{i(e+fx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(e+fx)}\right)+\sqrt{1+e^{2i(e+fx)}}\left(-6ie^{i(e}\right.\right.}{3af\left(1+e^{2i(e+fx)}\right)^{3/2}\cos^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[
e + f*x]],x]
```

```
[Out] (Sqrt[((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*Sqrt[(1 + E^((2*I)*
(e + f*x)))/E^(I*(e + f*x))]*(g*Cos[e + f*x])^(3/2)*(Sqrt[1 + E^((2*I)*(e +
```

```
f*x))]*(1 - (6*I)*E^(I*(e + f*x)) + E^((2*I)*(e + f*x))) + (12*I)*E^(I*(e
+ f*x))*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(e + f*x))])*Sqrt[c - c
*Sin[e + f*x]])/(3*a*(1 + E^((2*I)*(e + f*x)))^(3/2)*f*Cos[e + f*x]^(3/2))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}\,g\cos\left(fx+e\right)}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*g*cos(f*x + e)/sqrt
(a*sin(f*x + e) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.55, size = 361, normalized size = 2.96

$$-\frac{2\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\left(3i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\cos\left(fx+e\right)\sin\left(fx+e\right)\text{EllipticF}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/3/f*(g*cos(f*x+e))^(3/2)*(-c*(sin(f*x+e)-1))^(1/2)*(3*I*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(
I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin
(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*(1/(cos(f*x+e)+1))^(1
```

```
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)+cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2+3*cos(f*x+e))/(sin(f
*x+e)-1)/(a*(1+sin(f*x+e)))^(1/2)/sin(f*x+e)/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x +
 e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)
)^(1/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)
)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.130 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=68

$$\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1
/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/
2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2842, 2640, 2639}

$$\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(f*
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
```

```
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(g\cos(e+fx))\int\sqrt{g\cos(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{\left(g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\right)\int\sqrt{\cos(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{2g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.36, size = 111, normalized size = 1.63

$$\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f\cos^{\frac{3}{2}}(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(f*Cos[e + f*x]^(3/2)
*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g}{ac\cos\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g/(a*c*cos(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

maple [C] time = 0.50, size = 334, normalized size = 4.91

$$2\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\cos\left(fx+e\right)\sin\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)-i\sqrt{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/f*(g*cos(f*x+e))^(3/2)*(I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
-I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+I*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)-I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)^2+cos(f*x+
e))/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)/sin(f*x+e)/cos(f*x+e
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(1/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(e+fx\right)\right)^{\frac{3}{2}}}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Integral((g*cos(e + f*x))**(3/2)/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e
 + f*x) - 1))), x)
```

3.131 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=121

$$\frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*(g*cos(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-2*
g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2
*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c/f/(a+a*sin(f*x+e))^(1/
2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$\frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x
)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
```

```
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx = \frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{\int \frac{(g\cos(e+f}{\sqrt{a+a\sin(e+fx)}\,\sqrt{}}}{c}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{(g\cos(e+fx))}{c\sqrt{a+a\sin(e+}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{\left(g\sqrt{\cos(e+fx)}\right.}{c\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{2g\sqrt{\cos(e+fx)}}{cf\sqrt{a+a\sin}}$$

Mathematica [A] time = 0.73, size = 148, normalized size = 1.22

$$\frac{2(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{\cos(e+fx)}\,\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+E\right.}{cf\cos^{\frac{3}{2}}(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(EllipticE[
(e + f*x)/2, 2]*(-Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + Sqrt[Cos[e + f*x]]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(c*f*Cos[e + f*x]^(3/2)*Sqrt[a*(1
+ Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g}{ac^2\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^2\cos\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g/(a*c^2*cos(f*x + e)*sin(f*x + e) - a*c^2*cos(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

maple [C] time = 0.60, size = 925, normalized size = 7.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(sin(f*x+e)-1)*(4*I*sin(f*x+e
)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),
I)-4*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticE(I*(-1+cos(f*x
```

```
+e))/sin(f*x+e),I)+8*I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)-8*I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+4*I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-4*I*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*
EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-4*sin(f*x+e)*cos(f*x+e
)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)+cos(f*x+e)*ln(-(2*cos(f
*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)+4*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)*sin(f*x+e)-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))/(cos(f*x+e)+1)/(a*(1
+sin(f*x+e)))^(1/2)/sin(f*x+e)^5/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/(-c*(
sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)+a}\,(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(3/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(3/2)), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.132 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=179

$$-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{}{5fg\sqrt{a\sin(e}}$$

```
[Out] 2/5*(g*cos(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+
2/5*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
)-2/5*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f
*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^2/f/(a+a*sin(f*x
+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{}{5fg\sqrt{a\sin(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(5/2)) + (2*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*Sqrt[a + a*Sin[e + f*x]]*
(c - c*Sin[e + f*x])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*
EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin
[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx = \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{\int \frac{(g\cos(e+\ldots}{\sqrt{a+a\sin(e+fx)}\,(c\ldots}}{5c\ldots}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{2(g\ldots}{5cfg\sqrt{a+a\sin(\ldots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{2(g\ldots}{5cfg\sqrt{a+a\sin(\ldots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{2(g\ldots}{5cfg\sqrt{a+a\sin(\ldots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{2(g\ldots}{5cfg\sqrt{a+a\sin(\ldots}}$$

Mathematica [A] time = 1.52, size = 204, normalized size = 1.14

$$\frac{(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{\cos(e+fx)}\left(4\sin\ldots\right.\right.}{5c^2f(\sin(e+fx)-1)^2\cos^{\frac{3}{2}}(e+fx)\sqrt{a\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(5/2)),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*(-2*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(3*Cos[(e + f*x)/2] + Cos[(3*(e
+ f*x))/2] + 4*Sin[(e + f*x)/2]^3)))/(5*c^2*f*Cos[e + f*x]^(3/2)*(-1 + Sin[
e + f*x])^2*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g}{ac^3\cos\left(fx+e\right)^3+2\,ac^3\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,ac^3\cos\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g/(a*c^3*cos(f*x + e)^3 + 2*a*c^3*cos(f*x + e)*sin(f*x + e) - 2*a*c^
3*cos(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

maple [C] time = 0.56, size = 781, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/5/f*(g*cos(f*x+e))^(3/2)*(sin(f*x+e)*cos(f*x+e)-sin(f*x+e)-cos(f*x+e)+1)
*(I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4-I*EllipticF(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(
f*x+e)^4+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-I*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-2*I*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x
+e)^2+2*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2-I*EllipticE(I*(-1+cos(f*x+
e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)+I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+I*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e
),I)-I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)+cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^2-sin(f
*x+e)+2*cos(f*x+e)-1)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(a*(1+sin(f*x+e)))^(1/2
)/(-c*(sin(f*x+e)-1))^(5/2)/sin(f*x+e)^5/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(5/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.133 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=233

$$-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{}{15cfg\sqrt{a\sin}}$$

```
[Out] 2/9*(g*cos(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2)+
2/15*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/
2)+2/15*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)
)^(1/2)-2/15*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(si
n(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^3/f/(a+a*
sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.15, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$\frac{2(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{}{15cfg\sqrt{a\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
7/2)),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(7/2)) + (2*(g*Cos[e + f*x])^(5/2))/(15*c*f*g*Sqrt[a + a*Sin[e + f*x]]
*(c - c*Sin[e + f*x])^(5/2)) + (2*(g*Cos[e + f*x])^(5/2))/(15*c^2*f*g*Sqrt[
a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*S
qrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}\,dx = \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} + \frac{\int \frac{(g\cos(e\cdots}{\sqrt{a+a\sin(e+fx)}\cdots}}{3\cdots}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} + \frac{2\cdots}{15cfg\sqrt{a+a\sin\cdots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} + \frac{2\cdots}{15cfg\sqrt{a+a\sin\cdots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} + \frac{2\cdots}{15cfg\sqrt{a+a\sin\cdots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} + \frac{2\cdots}{15cfg\sqrt{a+a\sin\cdots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} + \frac{2\cdots}{15cfg\sqrt{a+a\sin\cdots}}$$

Mathematica [A] time = 2.15, size = 240, normalized size = 1.03

$$\frac{(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{\cos(e+fx)}\left(-32\sin\ldots\right.\right.}{90c^3f\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(7/2)),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*(12*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^5 + Sqrt[Cos[e + f*x]]*(-32*Cos[(e + f*x)/2] - 15*Cos[(
3*(e + f*x))/2] + 3*Cos[(5*(e + f*x))/2] - 32*Sin[(e + f*x)/2] + 15*Sin[(3*
(e + f*x))/2] + 3*Sin[(5*(e + f*x))/2])))/(90*c^3*f*Cos[e + f*x]^(3/2)*(-1
+ Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g}{3\,ac^4\cos\left(fx+e\right)^3-4\,ac^4\cos\left(fx+e\right)-\left(ac^4\cos\left(fx+e\right)^3-4\,ac^4\cos\left(fx+e\right)\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g/(3*a*c^4*cos(f*x + e)^3 - 4*a*c^4*cos(f*x + e) - (a*c^4*cos(f*x +
e)^3 - 4*a*c^4*cos(f*x + e))*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(7/2)), x)
```

maple [C] time = 0.56, size = 955, normalized size = 4.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/45/f*(g*cos(f*x+e))^(3/2)*(sin(f*x+e)*cos(f*x+e)-sin(f*x+e)-cos(f*x+e)+1)
*(9*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-9*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*Ellipti
cE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+6*I*cos(f*
x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*sin(f*x+e)*cos(f*x+e)^4*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/si
n(f*x+e),I)+6*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*sin(f*x+e)*cos(f*x
+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)-12*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)
-6*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4-6*I*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)-3*cos(f*x+e)^4+12*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+6*I*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)-6*cos(f*x+e)^2*sin(f*x+e)+5*cos(f*x+e)^3+sin(f*x+e)
*cos(f*x+e)+4*cos(f*x+e)^2+5*sin(f*x+e)-11*cos(f*x+e)+5)*(cos(f*x+e)^2+2*co
s(f*x+e)+1)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(7/2)/sin(f*x+e)^5
/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(7/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.134 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=294

$$-\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{66c^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{66c^3\sqrt{c-c\,\mathrm{s}}}{7af}$$

```
[Out] -4*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(3/2)
-30/7*c^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/a/f/g/(a+a*sin(f*x+e)
)^(1/2)-22*c^4*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f
*x+e))^(1/2)-66*c^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellip
ticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/f/
(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-66/7*c^3*(g*cos(f*x+e))^(5/2)
*(c-c*sin(f*x+e))^(1/2)/a/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.42, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$-\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{66c^3\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{7afg\sqrt{a\sin(e+fx)+a}}-\frac{30c^2(c-c\sin(e+fx))^{3}}{7afg\sqrt{a\sin(e}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (-22*c^4*(g*Cos[e + f*x])^(5/2))/(a*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) - (66*c^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 - (66*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(7*a*f*g*Sqrt[a
 + a*Sin[e + f*x]]) - (30*c^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2))/(7*a*f*g*Sqrt[a + a*Sin[e + f*x]]) - (4*c*(g*Cos[e + f*x])^(5/2)*(c -
 c*Sin[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}} - \frac{(15c)\int \frac{(g\cos(e+fx))^{3/}}{\sqrt{a+a}}}{a}$$

$$= -\frac{30c^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{7afg\sqrt{a+a\sin(e+fx)}} - \frac{4c(g\cos(e+fx))}{fg(a+a}$$

$$= -\frac{66c^3(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{7afg\sqrt{a+a\sin(e+fx)}} - \frac{30c^2(g\cos(e+fx)}{7afg\sqrt{a+}}$$

$$= -\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{66c^3(g\cos(e+fx}{7afg\sqrt{a+}}$$

$$= -\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{66c^3(g\cos(e+fx}{7afg\sqrt{a+}}$$

$$= -\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{66c^3(g\cos(e+fx}{7afg\sqrt{a+}}$$

$$= -\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{66c^4g\sqrt{\cos(e+f}}{af\sqrt{a+a\,\mathrm{si}}}$$

Mathematica [A] time = 6.53, size = 282, normalized size = 0.96

$$\frac{\sec(e+fx)(c-c\sin(e+fx))^{7/2}(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin(2(e+fx))-\frac{109}{14}\cos\right.}{f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+f\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] (-66*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(7/2))/(f*Cos[e + f*x]^(3/2)*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(3/2)) + ((g*Cos[e
 + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*
Sin[e + f*x])^(7/2)*(-32 - (109*Cos[e + f*x])/14 + Cos[3*(e + f*x)]/14 + (6
4*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + Sin[2*(e + f*x)
```

```
]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(3/2)
)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,c^{3}g\cos\left(fx+e\right)^{3}-4\,c^{3}g\cos\left(fx+e\right)-\left(c^{3}g\cos\left(fx+e\right)^{3}-4\,c^{3}g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral((3*c^3*g*cos(f*x + e)^3 - 4*c^3*g*cos(f*x + e) - (c^3*g*cos(f*x +
e)^3 - 4*c^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x
 + e) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.62, size = 2994, normalized size = 10.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/7/f*(-1+cos(f*x+e))*(-343*cos(f*x+e)^2-28*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+28
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)-28*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+231*
I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos
```

(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+sin(f*x+e)*cos(f*x+e)^5-6
*cos(f*x+e)^5+98*cos(f*x+e)^3-28*cos(f*x+e)^4+cos(f*x+e)^6-112*cos(f*x+e)^3
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)+112*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-168
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+168*cos(f*x+e)^2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(
f*x+e)^2)-119*cos(f*x+e)^2*sin(f*x+e)-28*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*
x+e)+28*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-112*cos(f*x+e)*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/s
in(f*x+e)^2)+112*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+28*cos(f*x+e)^4*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-1)/sin(f*x+e)^2)-21*sin(f*x+e)*cos(f*x+e)^3-28*sin(f*x+e)*cos(f*x+e)^3*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-1)/sin(f*x+e)^2)+28*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-c
os(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+
e)^2)-84*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+84*sin(f*x+e)*
cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-84*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(3/2)+84*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+231*I*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*
x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+7*sin(f*x+e)*cos(f*x+e)^4-
231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(

```
I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3+231*I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*cos(f*x+e)^3-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)-231*I*sin(f*x+e)*
cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+
e))/sin(f*x+e),I)+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)-462*I*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+
e))/sin(f*x+e),I)*cos(f*x+e)^2+462*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2
)*(g*cos(f*x+e))^(3/2)*(-c*(sin(f*x+e)-1))^(7/2)/(sin(f*x+e)*cos(f*x+e)^4+c
os(f*x+e)^5+4*sin(f*x+e)*cos(f*x+e)^3-5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*
x+e)-8*cos(f*x+e)^3-8*sin(f*x+e)*cos(f*x+e)+20*cos(f*x+e)^2+16*sin(f*x+e)+8
*cos(f*x+e)-16)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e
) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)
)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

3.135 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{154c^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{22c^2\sqrt{c-}}{5a}$$

```
[Out] -4*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e))^(3/2)
-154/15*c^3*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+
e))^(1/2)-154/5*c^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellip
ticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/f/
(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-22/5*c^2*(g*cos(f*x+e))^(5/2)
*(c-c*sin(f*x+e))^(1/2)/a/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.14, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$-\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{22c^2\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{5afg\sqrt{a\sin(e+fx)+a}}-\frac{154c^3g\sqrt{\cos(e+fx)}\,E}{5af\sqrt{a\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (-154*c^3*(g*Cos[e + f*x])^(5/2))/(15*a*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) - (154*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*El
lipticE[(e + f*x)/2, 2])/(5*a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]) - (22*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*a*f*g
*Sqrt[a + a*Sin[e + f*x]]) - (4*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x
])^(3/2))/(f*g*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{fg(a+a\sin(e+fx))^{3/2}} - \frac{(11c)\int \frac{(g\cos(e+fx))^{3/2}(}{\sqrt{a+a\sin}}}{a}$$

$$= -\frac{22c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5afg\sqrt{a+a\sin(e+fx)}} - \frac{4c(g\cos(e+fx))^{5/2}(}{fg(a+a\sin}$$

$$= -\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22c^2(g\cos(e+f}{5afg\sqrt{a}}$$

$$= -\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22c^2(g\cos(e+f}{5afg\sqrt{a}}$$

$$= -\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22c^2(g\cos(e+f}{5afg\sqrt{a}}$$

$$= -\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{154c^3g\sqrt{\cos(e+}}{5af\sqrt{a+a}}$$

Mathematica [A] time = 4.81, size = 238, normalized size = 0.99

$$-\frac{c^2(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\,\left(-\right.\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] -1/30*(c^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(
-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]*(924*EllipticE[(e + f*x)/2, 2
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + Sqrt[Cos[e + f*x]]*(520*Cos[(e +
f*x)/2] + 37*Cos[(3*(e + f*x))/2] + 3*Cos[(5*(e + f*x))/2] - 520*Sin[(e + f
*x)/2] + 37*Sin[(3*(e + f*x))/2] - 3*Sin[(5*(e + f*x))/2])))/(f*Cos[e + f*x
]^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(3/2
))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2 g \cos\left(fx+e\right)^3 + 2\,c^2 g \cos\left(fx+e\right)\sin\left(fx+e\right) - 2\,c^2 g \cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx\right.}}{a^2\cos\left(fx+e\right)^2 - 2\,a^2\sin\left(fx+e\right) - 2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral((c^2*g*cos(f*x + e)^3 + 2*c^2*g*cos(f*x + e)*sin(f*x + e) - 2*c^2*
g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.57, size = 2946, normalized size = 12.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/15/f*(-1+cos(f*x+e))*(351*cos(f*x+e)^2+30*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-30
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)+30*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*co
s(f*x+e)^5-94*cos(f*x+e)^3+231*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*co
s(f*x+e)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+20*cos(f*x+e)
```

^4+120*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e
)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-120*cos(f*x+e)^3*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)+180*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2
*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-180*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)+111*cos(f*x+e)^2*sin(f*x+e)+30*ln(-2*(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)*sin(f*x+e)-30*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+120*cos(f*x
+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-120*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-30
*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+17*sin(f*x+e)*cos(f*x+e)^3+30*sin(f
*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-30*sin(f*x+e)*cos(f*x+e)^3*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-1)/sin(f*x+e)^2)+90*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)-90*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+90*sin(f*x+e)*cos
(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-90*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(3/2)-3*sin(f*x+e)*cos(f*x+e)^4-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e
))/sin(f*x+e),I)+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)-231*I*(1/(cos(f
x+e)+1))^(1/2)(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e

```
))/sin(f*x+e),I)*cos(f*x+e)-231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(
f*x+e)^2+462*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2-462*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*cos(f*x+e)^2+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3-231*I*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3)*(-c*(sin(f*x+e)-1))^(5/2)*(g*cos(f*
x+e))^(3/2)/(cos(f*x+e)^4-sin(f*x+e)*cos(f*x+e)^3+3*cos(f*x+e)^3+4*cos(f*x+
e)^2*sin(f*x+e)-8*cos(f*x+e)^2+4*sin(f*x+e)*cos(f*x+e)-4*cos(f*x+e)-8*sin(f
*x+e)+8)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e
) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)
)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

3.136 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{14c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{4c\sqrt{c-c\,\mathrm{si}}}{fg(}$$

```
[Out] -14/3*c^2*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)-14*c^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(
sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/f/(a+a*
sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-4*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(
f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.84, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$-\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{14c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{4c\sqrt{c-c\,\mathrm{si}}}{fg(}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (-14*c^2*(g*Cos[e + f*x])^(5/2))/(3*a*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]]) - (14*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]
]) - (4*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(f*g*(a + a*Sin[
e + f*x])^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{fg(a+a\sin(e+fx))^{3/2}} - \frac{(7c)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c}}{\sqrt{a+a\sin(e}}}{a}$$

$$= -\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos(e+fx)}{fg(a+a}$$

$$= -\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos(e+fx)}{fg(a+a}$$

$$= -\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos(e+fx)}{fg(a+a}$$

$$= -\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{14c^2g\sqrt{\cos(e+}}{af\sqrt{a+a\,s}}$$

Mathematica [A] time = 1.55, size = 200, normalized size = 1.10

$$\frac{2c(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\,(\mathrm{co}\right.}{3f\cos^{\frac{3}{2}}(e+fx)(a(\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] (2*c*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(21*Ell
ipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + Sqrt[Cos[e +
 f*x]]*(Cos[(e + f*x)/2]*(12 + Cos[e + f*x]) + (-12 + Cos[e + f*x])*Sin[(e
+ f*x)/2]))*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*f*Cos[e + f*x]
^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(3/2)
)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(cg\cos\left(fx+e\right)\sin\left(fx+e\right)-cg\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral((c*g*cos(f*x + e)*sin(f*x + e) - c*g*cos(f*x + e))*sqrt(g*cos(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^
2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.56, size = 2890, normalized size = 15.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/3/f*(-1+cos(f*x+e))*(33*cos(f*x+e)^2+3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-1)/sin(f*x+e)^2)+3*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*
ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-8*cos(f*x
+e)^3+21*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+21*I*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+cos(f*x+e)^4+12*cos(f*x+e)^3*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-1)/sin(f*x+e)^2)-12*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*l
n(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(
f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+18*cos(f*x+e
)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-18*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+
```

e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+9
*cos(f*x+e)^2*sin(f*x+e)+3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-3*ln(-(2*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(3/2)*sin(f*x+e)+12*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-12*
cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(
f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^
2)+sin(f*x+e)*cos(f*x+e)^3+3*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-c
os(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+
e)^2)-3*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+9*sin(f*x+e)*cos(
f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x
+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-9*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f
x+e)^2(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(3/2)+9*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-9*sin(f*x
+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)
^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-21*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)+21*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/si
n(f*x+e),I)+21*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-42*I*cos(f*x+e)^2*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos
(f*x+e))/sin(f*x+e),I)+42*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*cos
(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
E(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(-c*(sin(f*x+e)-1))^(3/2)*(g*cos(f*x+e))
^(3/2)/(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)-3*cos(

```
f*x+e)^2-4*sin(f*x+e)-2*cos(f*x+e)+4)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e
)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e
) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)
)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

3.137 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{4c(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{6cg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -4*c*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)
-6*c*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*
x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e)
)^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2850, 2842, 2640, 2639}

$$-\frac{4c(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{6cg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-4*c*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Si
n[e + f*x]]) - (6*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e
+ f*x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
```

```
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{(3c)\int \frac{(g\cos(e+}{\sqrt{a+a\sin(e+fx)}\,\sqrt{}}}{a}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{(3cg\cos(e+fx))\int}{a\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{\left(3cg\sqrt{\cos(e+fx)}\,\sqrt{}\right.}{a\sqrt{a+a\sin(}}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{6cg\sqrt{\cos(e+fx)}\,\sqrt{g}}{af\sqrt{a+a\sin(e}}$$

Mathematica [C] time = 1.87, size = 213, normalized size = 1.73

$$\frac{2g\sqrt{ge^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)}\left(2e^{2i(e+fx)}\left(e^{i(e+fx)}+i\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)-\left(5e^{i(e+fx)}+i\right)\sqrt{1+e^{2i(e+fx)}}\right)\sqrt{c-c}}{af\left(e^{i(e+fx)}-i\right)\sqrt{1+e^{2i(e+fx)}}\sqrt{-iae^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (2*g*Sqrt[((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*(-((I + 5*E^(I*(e
+ f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]) + 2*E^((2*I)*(e + f*x))*(I + E^(I*(
e + f*x)))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[c -
 c*Sin[e + f*x]])/(a*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*a*(I + E^(I*(e + f*x
)))^2)/E^(I*(e + f*x))]*Sqrt[1 + E^((2*I)*(e + f*x))]*f)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,g\cos\left(fx+e\right)}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g*cos(f*x + e)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.60, size = 2838, normalized size = 23.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(-1+cos(f*x+e))*(-10*cos(f*x+e)^2-(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*
ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-
1)/sin(f*x+e)^2)-cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+2*cos(f*x+e)^3-6*
```

```
I*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-4*cos(f*x+e)^3*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-1)/sin(f*x+e)^2)+4*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(
-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-6*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)+6*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-2*cos
(f*x+e)^2*sin(f*x+e)-ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+ln(-(2*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(3/2)*sin(f*x+e)-4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(
-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(
f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+4*cos(f*x+e)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)+cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2
)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-sin(f*x+e
)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+sin(f*x+e)*cos(f*x+e)^3*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)-3*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+3*s
in(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-3*sin(f*x+e)*cos(f*x+e)*ln
(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos
(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(3/2)+3*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
+6*I*cos(f*x+e)^3*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*I*cos(f*x+e)^3*EllipticF(I*(-
1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)+6*I*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(co
```

```
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*I*cos(f*x+e)*Ellipti
cF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)+6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+12*I
*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-12*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f
*x+e),I)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos
(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+6*I*sin(f*x+e)
*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(g*cos(f*x+e))^(3/2)*(-c*(sin(f*x+
e)-1))^(1/2)/(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)
/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)
)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

3.138 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=121

$$-\frac{2(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)-2
*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/
2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(1
/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$-\frac{2(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f
*x]]),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[
e + f*x]]) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*
x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
```

```
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{\int \frac{(g\cos(e+f}{\sqrt{a+a\sin(e+fx)}\sqrt{}}}{a}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{(g\cos(e+fx))}{a\sqrt{a+a\sin(e+}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{\left(g\sqrt{\cos(e+fx)}\right.}{a\sqrt{a+a\,\mathrm{s}}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{2g\sqrt{\cos(e+fx)}}{af\sqrt{a+a\sin}}$$

Mathematica [A] time = 0.67, size = 170, normalized size = 1.40

$$-\frac{2(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\,\left(\mathrm{co}\right.\right.}{f\cos^{\frac{3}{2}}(e+fx)(a(\sin(e+fx)+1))^{3/2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin
[e + f*x]]),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^2*(Sqrt[Cos[e + f*x]]*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2]) + EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])))/(f*Cos[e + f*x]^(3/2)*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
 f*x]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g}{a^{2}c\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}c\cos\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g/(a^2*c*cos(f*x + e)*sin(f*x + e) + a^2*c*cos(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*
x + e) + c)), x)
```

maple [C] time = 0.59, size = 925, normalized size = 7.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(1+sin(f*x+e))*(4*I*sin(f*x+e)
*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)-4*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
```

```
+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticE(I*(-1+cos(f*x+
e))/sin(f*x+e),I)+8*I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticF(
I*(-1+cos(f*x+e))/sin(f*x+e),I)-8*I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+4*I*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-4*I*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+4*sin(f*x+e)*cos(f*x+e)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)+cos(f*x+e)*ln(-2*(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)+4*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*sin(f*x+e)-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))/(cos(f*x+e)+1)/sin(f*
x+e)^5/(a*(1+sin(f*x+e)))^(3/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/(-c*(s
in(f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*
x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(1/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(1/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.139 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$\frac{2(g\cos(e+fx))^{5/2}}{afg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{2(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}\right.}{acf\sqrt{a\sin(e+fx}}$$

```
[Out] -2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+2
*(g*cos(f*x+e))^(5/2)/a/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-2
*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/
2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/c/f/(a+a*sin(f*x+e))^
(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.88, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$\frac{2(g\cos(e+fx))^{5/2}}{afg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{2(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}\right.}{acf\sqrt{a\sin(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(3/2)),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(3/2)) + (2*(g*Cos[e + f*x])^(5/2))/(a*f*g*Sqrt[a + a*Sin[e + f*x]]*(
c - c*Sin[e + f*x])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(a*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
+ f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{\int \frac{(g}{\sqrt{a+a\sin(e}}}{}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{afg\sqrt{a+a}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{afg\sqrt{a+a}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{afg\sqrt{a+a}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{afg\sqrt{a+a}}$$

Mathematica [A] time = 0.73, size = 92, normalized size = 0.52

$$-\frac{2(g\cos(e+fx))^{5/2}\left(\sin(e+fx)-\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{cfg(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)),x]

[Out] (-2*(g*Cos[e + f*x])^(5/2)*(-(Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]) + Sin[e + f*x]))/(c*f*g*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x]])

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g}{a^2c^2\cos\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*g/(a^2*c^2*cos(f*x + e)^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2)), x)

maple [C] time = 0.57, size = 363, normalized size = 2.06

$$-\frac{2\left(\cos\left(fx+e\right)+1\right)^2\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-1+\cos\left(fx+e\right)\right)^2\left(1+\sin\left(fx+e\right)\right)\left(\sin\left(fx+e\right)-1\right)\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)

```
[Out] -2/f*(cos(f*x+e)+1)^2*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(1+sin(f*x+e))
*(sin(f*x+e)-1)*(I*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-I*Ellipt
icF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos
(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+I*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)-I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/(a*(1+sin(f*x+e)
))^(3/2)/(-c*(sin(f*x+e)-1))^(3/2)/sin(f*x+e)^5/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x +
e) + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(3/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

3.140 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{6g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5ac^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{6(g\cos(e+fx))^{5/2}}{5acfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{}{5afg\sqrt{a\sin(}}$$

```
[Out] -2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+6
/5*(g*cos(f*x+e))^(5/2)/a/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)
+6/5*(g*cos(f*x+e))^(5/2)/a/c/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(
1/2)-6/5*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/
2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/c^2/f/(a+a*si
n(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.16, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$-\frac{6g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5ac^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{6(g\cos(e+fx))^{5/2}}{5acfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{}{5afg\sqrt{a\sin(}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(5/2)),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*Sqrt[a + a*Sin[e + f*x]]
*(c - c*Sin[e + f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2))/(5*a*c*f*g*Sqrt[a
 + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (6*g*Sqrt[Cos[e + f*x]]*Sq
rt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a*c^2*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{3\int \frac{\cdots}{\sqrt{a+a\sin\cdots}}}{\cdots}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{\cdots}{5afg\sqrt{a+\cdots}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{\cdots}{5afg\sqrt{a+\cdots}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{\cdots}{5afg\sqrt{a+\cdots}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{\cdots}{5afg\sqrt{a+\cdots}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{\cdots}{5afg\sqrt{a+\cdots}}$$

Mathematica [A] time = 1.11, size = 134, normalized size = 0.57

$$-\frac{\sqrt{\cos(e+fx)}\,(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}\,(-6\sin(e+fx)-3\cos(2(e+fx))+1)+E\left(\frac{1}{2}(e+fx)\middle|2\right)(6\cos$$
$$\overline{5c^2f(\sin(e+fx)-1)^2(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
 f*x])^(5/2)),x]
```

```
[Out] -1/5*(Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*(Sqrt[Cos[e + f*x]]*(1 - 3*
Cos[2*(e + f*x)] - 6*Sin[e + f*x]) + EllipticE[(e + f*x)/2, 2]*(6*Cos[e + f
*x] - 3*Sin[2*(e + f*x)])))/(c^2*f*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*
x]))^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g}{a^2c^3\cos\left(fx+e\right)^3\sin\left(fx+e\right)-a^2c^3\cos\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g/(a^2*c^3*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*cos(f*x + e)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x +
e) + c)^(5/2)), x)
```

maple [C] time = 0.54, size = 877, normalized size = 3.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/5/f*(cos(f*x+e)+1)^2*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(1+sin(f*x+e
))*(sin(f*x+e)-1)*(-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)+3*I*sin(f*x+e)
*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*cos(f*x+e)^2*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)-3*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)+3*I*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)+3*I*cos(f*x+e)^3*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*Ellipt
icE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)-3*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3
*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(
-1+cos(f*x+e))/sin(f*x+e),I)-3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*sin
(f*x+e)*cos(f*x+e)-2*sin(f*x+e)-3*cos(f*x+e)+3)/(a*(1+sin(f*x+e)))^(3/2)/(-
c*(sin(f*x+e)-1))^(5/2)/cos(f*x+e)/sin(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x +
e) + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(5/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

3.141 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=294

$$-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{3ac^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2(g\cos(e+fx))^{5/2}}{3ac^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{}{3acfg\sqrt{a}}$$

```
[Out] -2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)+1
0/9*(g*cos(f*x+e))^(5/2)/a/f/g/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2
)+2/3*(g*cos(f*x+e))^(5/2)/a/c/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^
(1/2)+2/3*(g*cos(f*x+e))^(5/2)/a/c^2/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*
x+e))^(1/2)-2/3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE
(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/c^3/f/
(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.47, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$\frac{2(g\cos(e+fx))^{5/2}}{3ac^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{3ac^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{}{3acfg\sqrt{a\,\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(7/2)),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(7/2)) + (10*(g*Cos[e + f*x])^(5/2))/(9*a*f*g*Sqrt[a + a*Sin[e + f*x]
]*(c - c*Sin[e + f*x])^(7/2)) + (2*(g*Cos[e + f*x])^(5/2))/(3*a*c*f*g*Sqrt[
a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (2*(g*Cos[e + f*x])^(5/2)
)/(3*a*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (2*g*
Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(3*a*c^3
*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{5\int \frac{\cdots}{\sqrt{a+a\sin\cdots}}}{\cdots}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{\cdots}{9afg\sqrt{a+\cdots}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{\cdots}{9afg\sqrt{a+\cdots}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{\cdots}{9afg\sqrt{a+\cdots}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{\cdots}{9afg\sqrt{a+\cdots}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{\cdots}{9afg\sqrt{a+\cdots}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{\cdots}{9afg\sqrt{a+\cdots}}$$

<u>Mathematica</u> [A] time = 1.45, size = 155, normalized size = 0.53

$$\frac{\sqrt{\cos(e+fx)}\,(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}\,(-17\sin(e+fx)+3\sin(3(e+fx))-12\cos(2(e+fx))+4)+3\cdots\right.}{18c^3f(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\,\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
 f*x])^(7/2)),x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*(3*EllipticE[(e + f*x)/2, 2]*(5*
Cos[e + f*x] - Cos[3*(e + f*x)] - 4*Sin[2*(e + f*x)]) + Sqrt[Cos[e + f*x]]*
(4 - 12*Cos[2*(e + f*x)] - 17*Sin[e + f*x] + 3*Sin[3*(e + f*x)])))/(18*c^3*
f*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x
]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g}{a^2c^4\cos\left(fx+e\right)^5+2\,a^2c^4\cos\left(fx+e\right)^3\sin\left(fx+e\right)-2\,a^2c^4\cos\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g/(a^2*c^4*cos(f*x + e)^5 + 2*a^2*c^4*cos(f*x + e)^3*sin(f*x + e) -
2*a^2*c^4*cos(f*x + e)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x +
e) + c)^(7/2)), x)
```

maple [C] time = 0.60, size = 1177, normalized size = 4.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -2/9/f*(cos(f*x+e)+1)^2*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(1+sin(f*x+e
))*(sin(f*x+e)-1)*(-6*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-6*I*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+
e))/sin(f*x+e),I)-3*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+6*I
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1
+cos(f*x+e))/sin(f*x+e),I)+3*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
```

```
e),I)-6*I*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/s
in(f*x+e),I)*sin(f*x+e)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)
+3*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+6*I*cos(f*x+e)*Ellip
ticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)+6*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-6*I*cos(f*x
+e)^3*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*
I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos
(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+6*I*sin(f*x+e)*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+
e))/sin(f*x+e),I)+6*I*cos(f*x+e)^3*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*cos(f*x+e)^3
+6*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)^2-4*sin(f*x+e)-6*cos(f*x+e)+5)/(a*(1+
sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(7/2)/cos(f*x+e)/sin(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x +
e) + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(7/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

3.142 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=357

$$\frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{1254c^5g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{1254c^4\sqrt{}}{3}$$

```
[Out] 76/5*c^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))
^(3/2)-4/5*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f/g/(a+a*sin(f*x+e
))^(5/2)+114/7*c^3*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/a^2/f/g/(a+a
*sin(f*x+e))^(1/2)+418/5*c^5*(g*cos(f*x+e))^(5/2)/a^2/f/g/(a+a*sin(f*x+e))^
(1/2)/(c-c*sin(f*x+e))^(1/2)+1254/5*c^5*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(
1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*co
s(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+1254/35
*c^4*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/a^2/f/g/(a+a*sin(f*x+e))^(
1/2)
```

Rubi [A] time = 1.73, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$\frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{1254c^4\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{35a^2fg\sqrt{a\sin(e+fx)+a}}+\frac{114c^3(c-c\sin(e+}{7a^2fg\sqrt{a\,s}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (418*c^5*(g*Cos[e + f*x])^(5/2))/(5*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (1254*c^5*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) + (1254*c^4*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(35
*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]) + (114*c^3*(g*Cos[e + f*x])^(5/2)*(c - c
*Sin[e + f*x])^(3/2))/(7*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]) + (76*c^2*(g*Cos
[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^
(3/2)) - (4*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2))/(5*f*g*(a
+ a*Sin[e + f*x])^(5/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{5fg(a+a\sin(e+fx))^{5/2}} - \frac{(19c)\int \frac{(g\cos(e+fx))^{3/}}{(a+a\sin}}{5a}$$

$$= \frac{76c^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}} - \frac{4c(g\cos(e+fx))^{5/}}{5fg(a+a\,s}$$

$$= \frac{114c^3(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{7a^2fg\sqrt{a+a\sin(e+fx)}} + \frac{76c^2(g\cos(e+fx}{5afg(a+}$$

$$= \frac{1254c^4(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{35a^2fg\sqrt{a+a\sin(e+fx)}} + \frac{114c^3(g\cos(e+f}{7a^2fg\sqrt{a}}$$

$$= \frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{1254c^4(g\cos(e+}{35a^2fg}$$

$$= \frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{1254c^4(g\cos(e+}{35a^2fg}$$

$$= \frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{1254c^4(g\cos(e+}{35a^2fg}$$

$$= \frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{1254c^5g\sqrt{\cos(e}}{5a^2f\sqrt{a+}}$$

Mathematica [A] time = 6.71, size = 356, normalized size = 1.00

$$\frac{1254E\left(\frac{1}{2}(e+fx)\middle|2\right)(c-c\sin(e+fx))^{9/2}(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{5f\cos^{\frac{3}{2}}(e+fx)(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9} + \frac{\sec(e+fx)(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] (1254*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2))/(5*f*Cos[e + f*x]^(3/2)*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) + ((g*Co
```

```
s[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c -
 c*Sin[e + f*x])^(9/2)*(736/5 + (221*Cos[e + f*x])/14 - Cos[3*(e + f*x)]/14
 + (128*Sin[(e + f*x)/2])/(5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) - 64/
(5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) - (1472*Sin[(e + f*x)/2])/(5*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])) - (7*Sin[2*(e + f*x)])/5))/(f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c^4 g\cos\left(fx+e\right)^5-8\,c^4 g\cos\left(fx+e\right)^3+8\,c^4 g\cos\left(fx+e\right)+4\left(c^4 g\cos\left(fx+e\right)^3-2\,c^4 g\cos\left(fx+\right.\right.\right.}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(-(c^4*g*cos(f*x + e)^5 - 8*c^4*g*cos(f*x + e)^3 + 8*c^4*g*cos(f*x
+ e) + 4*(c^4*g*cos(f*x + e)^3 - 2*c^4*g*cos(f*x + e))*sin(f*x + e))*sqrt(g
*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*co
s(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.65, size = 3654, normalized size = 10.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/35/f*(-1+cos(f*x+e))*(8554*cos(f*x+e)^2+1400*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)
-1400*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+
```

e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-1400*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+39*sin(f*x+e)*cos(f*x+e)^5-4389*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-192*cos(f*x+e)^5-1575*cos(f*x+e)^3-1582*cos(f*x+e)^4+44*cos(f*x+e)^6+5*cos(f*x+e)^7-700*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+700*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+700*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^4-700*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^4+1400*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-1400*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+5600*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-5600*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+9002*cos(f*x+e)^2*sin(f*x+e)+1400*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-1400*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+4900*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-4900*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+1400*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-1351*sin(f*x+e)*cos(f*x+e)^3+3500*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3500*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x

```
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+6300*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(3/2)-6300*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+490
0*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)
/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-4900*sin(f*x+e)*cos(f*x
+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(3/2)+231*sin(f*x+e)*cos(f*x+e)^4-5*cos(f*x+e)^6*si
n(f*x+e)-8778*I*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+8778*I*cos(
f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+4389*I*cos(f*x+e)^3*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*sin(f*x+e)-13167*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*si
n(f*x+e)+13167*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+4389*I*c
os(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-8778*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)+8778*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-4389*I*cos(f*x+e)^4*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)-13167*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+13167*
I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(g*cos(f*x+e))^(3/2)*(-c*(sin(f*x
+e)-1))^(9/2)/(cos(f*x+e)^6-sin(f*x+e)*cos(f*x+e)^5+5*cos(f*x+e)^5+6*sin(f*
x+e)*cos(f*x+e)^4-18*cos(f*x+e)^4+12*sin(f*x+e)*cos(f*x+e)^3-20*cos(f*x+e)^
3-32*cos(f*x+e)^2*sin(f*x+e)+48*cos(f*x+e)^2-16*sin(f*x+e)*cos(f*x+e)+16*co
s(f*x+e)+32*sin(f*x+e)-32)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\cos\left(e+fx\right)\right)^{3/2}\left(c-c\sin\left(e+fx\right)\right)^{9/2}}{\left(a+a\sin\left(e+fx\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x)
)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

3.143 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=298

$$\frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{462c^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{66c^3\sqrt{c-c}}{5a^2}$$

```
[Out] 12*c^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/a/f/g/(a+a*sin(f*x+e))^(
3/2)-4/5*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))
^(5/2)+154/5*c^4*(g*cos(f*x+e))^(5/2)/a^2/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)+462/5*c^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e
)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/
2)/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+66/5*c^3*(g*cos(f*x+
e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/a^2/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.42, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$\frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{66c^3\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}}+\frac{462c^4g\sqrt{\cos(e+fx)}\,E\left(\right.}{5a^2f\sqrt{a\sin(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (154*c^4*(g*Cos[e + f*x])^(5/2))/(5*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (462*c^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*El
lipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]) + (66*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*a^2
*f*g*Sqrt[a + a*Sin[e + f*x]]) + (12*c^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[
e + f*x])^(3/2))/(a*f*g*(a + a*Sin[e + f*x])^(3/2)) - (4*c*(g*Cos[e + f*x])
^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

x]

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}} - \frac{(3c)\int \frac{(g\cos(e+fx))^{3/2}(c-}{(a+a\sin(e-}}{a}$$

$$= \frac{12c^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{afg(a+a\sin(e+fx))^{3/2}} - \frac{4c(g\cos(e+fx))^{5/2}(}{5fg(a+a\sin}$$

$$= \frac{66c^3(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5a^2fg\sqrt{a+a\sin(e+fx)}} + \frac{12c^2(g\cos(e+fx))^{5/2}}{afg(a+a\sin}$$

$$= \frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{66c^3(g\cos(e+fx)}{5a^2fg\sqrt{a-}}$$

$$= \frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{66c^3(g\cos(e+fx)}{5a^2fg\sqrt{a-}}$$

$$= \frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{66c^3(g\cos(e+fx)}{5a^2fg\sqrt{a-}}$$

$$= \frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{462c^4g\sqrt{\cos(e+f}}{5a^2f\sqrt{a+a\,\mathrm{s}}}$$

Mathematica [A] time = 4.68, size = 250, normalized size = 0.84

$$\frac{c^3\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(-487\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] (c^3*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[c
- c*Sin[e + f*x]]*(1848*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(487*Cos[(e + f*x)/2] + 633*Cos[(3*(e +
 f*x))/2] - 17*Cos[(5*(e + f*x))/2] + Cos[(7*(e + f*x))/2] - 487*Sin[(e + f
*x)/2] + 633*Sin[(3*(e + f*x))/2] + 17*Sin[(5*(e + f*x))/2] + Sin[(7*(e + f
*x))/2])))/(20*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(
a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,c^3 g\cos\left(fx+e\right)^3-4\,c^3 g\cos\left(fx+e\right)-\left(c^3 g\cos\left(fx+e\right)^3-4\,c^3 g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral((3*c^3*g*cos(f*x + e)^3 - 4*c^3*g*cos(f*x + e) - (c^3*g*cos(f*x +
e)^3 - 4*c^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^
3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.60, size = 3598, normalized size = 12.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/5/f*(-1+cos(f*x+e))*(446*cos(f*x+e)^2+80*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-80*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)-80*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+sin(f
*x+e)*cos(f*x+e)^5+231*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-
8*cos(f*x+e)^5-85*cos(f*x+e)^3-78*cos(f*x+e)^4+cos(f*x+e)^6-40*cos(f*x+e)^5
```

```
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)+40*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+40*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^4-40*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-co
s(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e
)^2)*sin(f*x+e)*cos(f*x+e)^4+80*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-80
*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+320*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(
f*x+e)^2)-320*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(
f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+478*cos(f*x+e)^2*sin(f
*x+e)+80*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x
+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-80*ln(-(2*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2
)*sin(f*x+e)+280*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*c
os(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-280*cos(f*x+e)*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-1)/sin(f*x+e)^2)+80*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*
ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos
(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-69*sin(f*x+
e)*cos(f*x+e)^3+200*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-200
*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+360*sin(f*x+e)*cos(f*x+e
)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-360*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1
```

```
)^2)^(3/2)+280*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-280*sin(f*
x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-co
s(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e
)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+9*sin(f*x+e)*cos(f*x+e)^4-462*I*c
os(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+231*I*cos(f*x+e)^4*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e
))/sin(f*x+e),I)-231*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-693*I*cos(f*x
+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)+693*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)-462*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+462*I*cos(f*x+e)*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/s
in(f*x+e),I)+693*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-693*I*
sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*sin(f*x+e)*cos(f*x+
e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)+462*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*s
in(f*x+e))*(g*cos(f*x+e))^(3/2)*(-c*(sin(f*x+e)-1))^(7/2)/(sin(f*x+e)*cos(f
*x+e)^4+cos(f*x+e)^5+4*sin(f*x+e)*cos(f*x+e)^3-5*cos(f*x+e)^4-12*cos(f*x+e)
^2*sin(f*x+e)-8*cos(f*x+e)^3-8*sin(f*x+e)*cos(f*x+e)+20*cos(f*x+e)^2+16*sin
(f*x+e)+8*cos(f*x+e)-16)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\cos\left(e+fx\right)\right)^{3/2}\left(c-c\sin\left(e+fx\right)\right)^{7/2}}{\left(a+a\sin\left(e+fx\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)
)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

3.144 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=243

$$\frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{154c^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{44c^2\sqrt{c-}}{5}$$

```
[Out] -4/5*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e))^(5/
2)+154/15*c^3*(g*cos(f*x+e))^(5/2)/a^2/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(
f*x+e))^(1/2)+154/5*c^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*E
llipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/
a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+44/5*c^2*(g*cos(f*x+e))
^(5/2)*(c-c*sin(f*x+e))^(1/2)/a/f/g/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A] time = 1.14, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$\frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{154c^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{44c^2\sqrt{c-}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (154*c^3*(g*Cos[e + f*x])^(5/2))/(15*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]]) + (154*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) + (44*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*a*
f*g*(a + a*Sin[e + f*x])^(3/2)) - (4*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e
+ f*x])^(3/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{5fg(a+a\sin(e+fx))^{5/2}} - \frac{(11c)\int \frac{(g\cos(e+fx))^{3/}}{(a+a\sin}}{5a}$$

$$= \frac{44c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5afg(a+a\sin(e+fx))^{3/2}} - \frac{4c(g\cos(e+fx))^{5/2}(}{5fg(a+a\sin}$$

$$= \frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{44c^2(g\cos(e+}{5afg(a+}$$

$$= \frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{44c^2(g\cos(e+}{5afg(a+}$$

$$= \frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{44c^2(g\cos(e+}{5afg(a+}$$

$$= \frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{154c^3g\sqrt{\cos(e}}{5a^2f\sqrt{a+}}$$

Mathematica [A] time = 2.32, size = 230, normalized size = 0.95

$$\frac{c^2\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(-226\sin\left(\frac{1}{2}(e+fx\right.\right.\right.}{30f\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] (c^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[c
- c*Sin[e + f*x]]*(924*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(226*Cos[(e + f*x)/2] + 327*Cos[(3*(e +
f*x))/2] - 5*Cos[(5*(e + f*x))/2] - 226*Sin[(e + f*x)/2] + 327*Sin[(3*(e +
f*x))/2] + 5*Sin[(5*(e + f*x))/2])))/(30*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2g\cos\left(fx+e\right)^3+2\,c^2g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^2g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx\right.}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral((c^2*g*cos(f*x + e)^3 + 2*c^2*g*cos(f*x + e)*sin(f*x + e) - 2*c^2*
g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*s
in(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.57, size = 3551, normalized size = 14.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/15/f*(-1+cos(f*x+e))*(-438*cos(f*x+e)^2-90*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+90
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)+90*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-693*
I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-
1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2+5*cos(f*x+e)^5+89*cos(f
*x+e)^3+70*cos(f*x+e)^4+45*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2
)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-45*cos(
f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-45*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*
ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e
```

```
)*cos(f*x+e)^4+45*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^4-90*cos(
f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+90*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)
^2)-360*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+360*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-
1)/sin(f*x+e)^2)-486*cos(f*x+e)^2*sin(f*x+e)-90*ln(-2*(2*cos(f*x+e)^2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
*sin(f*x+e)+90*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)
^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-315*cos(f*x+e)*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-1)/sin(f*x+e)^2)+315*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(
-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-90*cos(f*x+e)^
4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-1)/sin(f*x+e)^2)+65*sin(f*x+e)*cos(f*x+e)^3-225*sin(f*x+e)*cos(f
*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+225*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)
/sin(f*x+e)^2)-405*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+405*
sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/s
in(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-315*sin(f*x+e)*cos(f*x+e)
*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)+315*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)-5*sin(f*x+e)*cos(f*x+e)^4-462*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
```

```
sin(f*x+e)+462*I*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+693*I*cos(
f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+231*I*cos(f*x+e)^4*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)-231*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-693*I*cos(f*x+
e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I
*(-1+cos(f*x+e))/sin(f*x+e),I)+693*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
-462*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+462*I*cos(f*x+e)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/si
n(f*x+e),I)+231*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*s
in(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(-c*(sin(f*x+e)-1))^(5/2)
*(g*cos(f*x+e))^(3/2)/(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4-4*cos(f*x+e)^2*
sin(f*x+e)-3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)+8*cos(f*x+e)^2+8*sin(f*x+
e)+4*cos(f*x+e)-8)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)
)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

3.145 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{42c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{4c\sqrt{c-c\sin}}{5fg(}$$

```
[Out] 28/5*c^2*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))
^(1/2)+42/5*c^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE
(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a^2/f/(a
+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-4/5*c*(g*cos(f*x+e))^(5/2)*(c-c
*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(5/2)
```

Rubi [A] time = 0.85, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2850, 2842, 2640, 2639}

$$\frac{42c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{4c\sqrt{c-c\sin}}{5fg(}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (28*c^2*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c
- c*Sin[e + f*x]]) + (42*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Elli
pticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]) - (4*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*f*g*(a +
 a*Sin[e + f*x])^(5/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5fg(a+a\sin(e+fx))^{5/2}} - \frac{(7c)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c}}{(a+a\sin(e+}}{5a}$$

$$= \frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos(e+fx)}{5fg(a+a}$$

$$= \frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos(e+fx)}{5fg(a+a}$$

$$= \frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos(e+fx)}{5fg(a+a}$$

$$= \frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{42c^2g\sqrt{\cos(e+}}{5a^2f\sqrt{a+a}}$$

Mathematica [A] time = 1.16, size = 180, normalized size = 0.97

$$\frac{c\sqrt{\cos(e+fx)}\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(8\sqrt{\cos(e+fx)}\left(-\sin\left(\frac{1}{2}(e+fx)\right)+2\sin\left(\frac{3}{2}(e+fx)\right)+\mathrm{co}\right.\right.}{5f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] (c*Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]*(42*E
llipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 8*Sqrt[C
os[e + f*x]]*(Cos[(e + f*x)/2] + 2*Cos[(3*(e + f*x))/2] - Sin[(e + f*x)/2]
+ 2*Sin[(3*(e + f*x))/2])))/(5*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a
*(1 + Sin[e + f*x]))^(5/2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cg\cos\left(fx+e\right)\sin\left(fx+e\right)-cg\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral((c*g*cos(f*x + e)*sin(f*x + e) - c*g*cos(f*x + e))*sqrt(g*cos(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e
)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.56, size = 3497, normalized size = 18.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/5/f*(-1+cos(f*x+e))*(38*cos(f*x+e)^2+10*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
```

```
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-10*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-1)/sin(f*x+e)^2)-10*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2
)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-21*I*co
s(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-9*cos(f*x+e)^3-5*cos(f*x+e)
^4-5*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+5*cos(f*x+e)^5*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)+5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^4-5*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^4+10*cos(f*x+e)^3*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)-10*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+40*cos(f*x+e)^2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-1)/sin(f*x+e)^2)-40*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+46*cos(f*x
+e)^2*sin(f*x+e)+10*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-10*ln(-(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)*sin(f*x+e)+35*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*
ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-35*cos(f*
x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+10*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-5*
sin(f*x+e)*cos(f*x+e)^3+25*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)
```

```
^2)-25*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+45*sin(f*x+e)*cos(
f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x
+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-45*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(
f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(3/2)+35*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-35*sin(
f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x
+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+63*I*cos(f*x+e)^2*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/s
in(f*x+e),I)-63*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*cos(f*x+e)^4*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+
cos(f*x+e))/sin(f*x+e),I)+21*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)+21*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+42*I*cos(f*x+e)*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e
))/sin(f*x+e),I)-42*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-42*I*sin(f*x+e)*
cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+42*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*sin(f*x+e)-63*I*cos(f*x+e)^2*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+63*I*
cos(f*x+e)^2*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(g*cos(f*x+e))^(3/2)*(-c
*(sin(f*x+e)-1))^(3/2)/(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*sin(f*x+e)*c
os(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)-2*cos(f*x+e)+4)/sin(f*x+e)/cos(f*x+e)
/(a*(1+sin(f*x+e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2
```

```
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\cos\left(e+fx\right)\right)^{3/2}\left(c-c\sin\left(e+fx\right)\right)^{3/2}}{\left(a+a\sin\left(e+fx\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)
)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

3.146 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=182

$$\frac{6cg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{6c(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{4c\cdots}{5fg(a\sin(e+\cdots}$$

```
[Out] -4/5*c*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/
2)+6/5*c*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))
^(1/2)+6/5*c*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(si
n(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a^2/f/(a+a*
sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{6cg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{6c(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{4c\cdots}{5fg(a\sin(e+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (-4*c*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*
Sin[e + f*x]]) + (6*c*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])
^(3/2)*Sqrt[c - c*Sin[e + f*x]]) + (6*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e +
 f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
- c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(3c)\int \frac{(g\cos(}{(a+a\sin(e+fx))}}{5a}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{6c(g\,c}{5afg(a+a\sin(e+}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{6c(g\,c}{5afg(a+a\sin(e+}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{6c(g\,c}{5afg(a+a\sin(e+}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{6c(g\,c}{5afg(a+a\sin(e+}$$

Mathematica [C] time = 2.10, size = 230, normalized size = 1.26

$$\frac{4ig\sqrt{ge^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)}\left(e^{i(e+fx)}\left(e^{i(e+fx)}+i\right)^3 {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)+\left(-4ie^{i(e+fx)}-3e^{2i(e+fx)}+5\right)\sqrt{1+e^{2i}}\right.}{5af\left(e^{i(e+fx)}-i\right)\sqrt{1+e^{2i(e+fx)}}\left(-iae^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (((4*I)/5)*g*Sqrt[((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*((5 - (4*I
)*E^(I*(e + f*x)) - 3*E^((2*I)*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))] +
E^(I*(e + f*x))*(I + E^(I*(e + f*x)))^3*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(e + f*x))])*Sqrt[c - c*Sin[e + f*x]])/(a*(-I + E^(I*(e + f*x)))*((
(-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x)))^(3/2)*Sqrt[1 + E^((2*I)*(e
 + f*x))]*f)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g\cos\left(fx+e\right)}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g*cos(f*x + e)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 -
 4*a^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.55, size = 2040, normalized size = 11.21

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/10/f*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^3*(1+sin(f*x+e))*(-c*(sin(f*x+
e)-1))^(1/2)*(-5*cos(f*x+e)^3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-1)/sin(f*x+e)^2)+12*sin(f*x+e)*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)+5*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/
sin(f*x+e)^2)*sin(f*x+e)-5*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)+5*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-20*cos(f*x+e)^3*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)+5*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-1)/sin(f*x+e)^2)-5*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)+8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-8*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-12*I*cos(f*x+e)^4*EllipticF(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+12*I*(1/(cos(f*x+e)+1))^(1/
2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-12*I*(1/(cos(f*x+e)+1))^(1/2)*(-co
```

```
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
E(I*(-1+cos(f*x+e))/sin(f*x+e),I)+4*sin(f*x+e)*cos(f*x+e)*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)+12*I*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)-8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)
+20*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+24*I*sin(f*x+e)*cos(f*x
+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-24*I*sin
(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)+12*I*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)-12*I*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-24*I*cos(f*x+e)^3*EllipticF(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+24*I*cos(f*x+e)^3*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+12*I*(1/(cos(f*x+e)+1)
)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-12*I*(1/(cos(f*x+e
)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+24*I*(1/(cos(f
*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)-24*I*(1/(c
os(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e))/(sin(
f*x+e)-1)/sin(f*x+e)^7/(a*(1+sin(f*x+e)))^(5/2)/(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)
)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

3.147 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=179

$$-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{2}{5fg(a\sin(e+}$$

```
[Out] -2/5*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)
-2/5*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/
2)-2/5*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*
f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*
x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{2}{5fg(a\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f
*x]]),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Si
n[e + f*x]]) - (2*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^(3/
2)*Sqrt[c - c*Sin[e + f*x]]) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]
*EllipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Si
n[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{\int \frac{(g\cos}{(a+a\sin(e+fx)}}{}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{2}{5afg(a+a\sin}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{2}{5afg(a+a\sin}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{2}{5afg(a+a\sin}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{2}{5afg(a+a\sin}$$

Mathematica [A] time = 1.52, size = 189, normalized size = 1.06

$$-\frac{(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\,\left(-4\right.\right.}{5f\cos^{\frac{3}{2}}(e+fx)(a(\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin
[e + f*x]]),x]
```

```
[Out] -1/5*((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^2*(2*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(3*Cos[(e + f*x)/2] + Cos[(3
*(e + f*x))/2] - 4*Sin[(e + f*x)/2]^3)))/(f*Cos[e + f*x]^(3/2)*(a*(1 + Sin[
e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g}{a^3c\cos\left(fx+e\right)^3-2\,a^3c\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3c\cos\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g/(a^3*c*cos(f*x + e)^3 - 2*a^3*c*cos(f*x + e)*sin(f*x + e) - 2*a^3*
c*cos(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*
x + e) + c)), x)
```

maple [C] time = 0.55, size = 777, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/5/f*(g*cos(f*x+e))^(3/2)*(sin(f*x+e)*cos(f*x+e)-sin(f*x+e)+cos(f*x+e)-1)
*(-I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4+I*EllipticE(I*(-1+cos(f*x+e))/s
in(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos
(f*x+e)^4+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2-I*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2+2*I*EllipticF(I*(-1+cos(f*x+e))/sin(
f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*
x+e)^2-2*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2-I*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)+I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-I*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
E(I*(-1+cos(f*x+e))/sin(f*x+e),I)+cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^2-sin(
f*x+e)-2*cos(f*x+e)+1)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(a*(1+sin(f*x+e)))^(5/
2)/(-c*(sin(f*x+e)-1))^(1/2)/sin(f*x+e)^5/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*
x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))
^(1/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))
^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

3.148 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{6g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{6(g\cos(e+fx))^{5/2}}{5acfg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{\ldots}{5cfg(a\sin(\ldots}$$

```
[Out] 2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2)-6/
5*(g*cos(f*x+e))^(5/2)/c/f/g/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)-
6/5*(g*cos(f*x+e))^(5/2)/a/c/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1
/2)-6/5*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2
*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a^2/c/f/(a+a*sin
(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.16, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$-\frac{6g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{6(g\cos(e+fx))^{5/2}}{5acfg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{\ldots}{5cfg(a\sin(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])
^(3/2)),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(3/2)) - (6*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*(a + a*Sin[e + f*x])^(5/2
)*Sqrt[c - c*Sin[e + f*x]]) - (6*(g*Cos[e + f*x])^(5/2))/(5*a*c*f*g*(a + a*
Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]) - (6*g*Sqrt[Cos[e + f*x]]*Sqr
t[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a^2*c*f*Sqrt[a + a*Sin[e +
f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} + \frac{3\int \frac{(g\,\text{co}}{(a+a\sin(e+fx}}{}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{6}{5cfg(a+a\sin}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{6}{5cfg(a+a\sin}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{6}{5cfg(a+a\sin}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{6}{5cfg(a+a\sin}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{6}{5cfg(a+a\sin}$$

Mathematica [A] time = 1.06, size = 133, normalized size = 0.56

$$\frac{\sqrt{\cos(e+fx)}\,(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}\,(-6\sin(e+fx)+3\cos(2(e+fx))-1)+3E\left(\frac{1}{2}(e+fx)\middle|2\right)(\sin(\right.}{5cf(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
 f*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*(Sqrt[Cos[e + f*x]]*(-1 + 3*Cos[
2*(e + f*x)] - 6*Sin[e + f*x]) + 3*EllipticE[(e + f*x)/2, 2]*(2*Cos[e + f*x
] + Sin[2*(e + f*x)])))/(5*c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(
5/2)*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g}{a^3c^2\cos\left(fx+e\right)^3\sin\left(fx+e\right)+a^3c^2\cos\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g/(a^3*c^2*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*cos(f*x + e)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(3/2)), x)
```

maple [C] time = 0.56, size = 877, normalized size = 3.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/5/f*(cos(f*x+e)+1)^2*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(1+sin(f*x+e
))*(sin(f*x+e)-1)*(3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+3*I*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*EllipticF(I*(-
1+cos(f*x+e))/sin(f*x+e),I)-3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*s
in(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)
*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*cos(f*x+e)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
cos(f*x+e)^3*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*cos(f*x+e)+3*I*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*Ell
ipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*EllipticF(I*(-1+cos(f*x+e))/sin(
f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*sin(
f*x+e)*cos(f*x+e)+2*sin(f*x+e)-3*cos(f*x+e)+3)/(a*(1+sin(f*x+e)))^(5/2)/(-c
*(sin(f*x+e)-1))^(3/2)/cos(f*x+e)/sin(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2)),x)

[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),x)

[Out] Timed out

3.149 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{6g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{6(g\cos(e+fx))^{5/2}}{5a^2cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{}{5a^2fg\sqrt{a\,\mathrm{si}}}$$

```
[Out] -2/5*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2)
-2*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)
+6/5*(g*cos(f*x+e))^(5/2)/a^2/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(
1/2)+6/5*(g*cos(f*x+e))^(5/2)/a^2/c/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x
+e))^(1/2)-6/5*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(
sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a^2/c^2/f
/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.47, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$-\frac{6g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{6(g\cos(e+fx))^{5/2}}{5a^2cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{}{5a^2fg\sqrt{a\,\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])
^(5/2)),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e
+ f*x])^(5/2)) - (2*(g*Cos[e + f*x])^(5/2))/(a*f*g*(a + a*Sin[e + f*x])^(3/
2)*(c - c*Sin[e + f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2))/(5*a^2*f*g*Sqrt
[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2
))/(5*a^2*c*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (6*g
*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a^2*
c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

x]

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} + \frac{\int \frac{(g}{(a+a\sin(e+}}{}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{afg(a+a\,\mathrm{si}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{afg(a+a\,\mathrm{si}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{afg(a+a\,\mathrm{si}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{afg(a+a\,\mathrm{si}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{afg(a+a\,\mathrm{si}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{afg(a+a\,\mathrm{si}}$$

Mathematica [A] time = 1.11, size = 104, normalized size = 0.36

$$\frac{\sec^3(e+fx)(g\cos(e+fx))^{3/2}\left(7\sin(e+fx)+3\sin(3(e+fx))-12\cos^{\frac{5}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{10a^2c^2f\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
 f*x])^(5/2)),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Sec[e + f*x]^3*(-12*Cos[e + f*x]^(5/2)*EllipticE[(e
 + f*x)/2, 2] + 7*Sin[e + f*x] + 3*Sin[3*(e + f*x)]))/(10*a^2*c^2*f*Sqrt[a*
(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g}{a^3c^3\cos\left(fx+e\right)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g/(a^3*c^3*cos(f*x + e)^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(5/2)), x)
```

maple [C] time = 0.56, size = 395, normalized size = 1.36

$$-\frac{2\left(\cos\left(fx+e\right)+1\right)^{2}\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-1+\cos\left(fx+e\right)\right)^{2}\left(1+\sin\left(fx+e\right)\right)\left(\sin\left(fx+e\right)-1\right)\left(3i\sin\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/5/f*(cos(f*x+e)+1)^2*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(1+sin(f*x+e
))*(sin(f*x+e)-1)*(3*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*
I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*sin(f*x+e)*cos(f*x+
e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos
(f*x+e)^2-3*cos(f*x+e)^3+2*cos(f*x+e)^2+1)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(si
n(f*x+e)-1))^(5/2)/sin(f*x+e)^5/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))
^(5/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))
^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

3.150 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=350

$$-\frac{14g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15a^2c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14(g\cos(e+fx))^{5/2}}{15a^2c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{}{15a^2cf}$$

```
[Out] -2/5*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2)
-14/5*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7
/2)+14/9*(g*cos(f*x+e))^(5/2)/a^2/f/g/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e
))^(1/2)+14/15*(g*cos(f*x+e))^(5/2)/a^2/c/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*s
in(f*x+e))^(1/2)+14/15*(g*cos(f*x+e))^(5/2)/a^2/c^2/f/g/(c-c*sin(f*x+e))^(3
/2)/(a+a*sin(f*x+e))^(1/2)-14/15*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x
+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e
))^(1/2)/a^2/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.79, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$\frac{14(g\cos(e+fx))^{5/2}}{15a^2c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{14g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15a^2c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{}{15a^2cfg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])
^(7/2)),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e
+ f*x])^(7/2)) - (14*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^
(3/2)*(c - c*Sin[e + f*x])^(7/2)) + (14*(g*Cos[e + f*x])^(5/2))/(9*a^2*f*g*
Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (14*(g*Cos[e + f*x])
^(5/2))/(15*a^2*c*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))
+ (14*(g*Cos[e + f*x])^(5/2))/(15*a^2*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c -
 c*Sin[e + f*x])^(3/2)) - (14*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ell
ipticE[(e + f*x)/2, 2])/(15*a^2*c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} + \frac{7\int \frac{}{(a+a\sin}}{}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{}{5afg(a+}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{}{5afg(a+}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{}{5afg(a+}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{}{5afg(a+}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{}{5afg(a+}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{}{5afg(a+}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{}{5afg(a+}$$

Mathematica [A] time = 6.05, size = 171, normalized size = 0.49

$$-\frac{\sqrt{\cos(e+fx)}\,(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}\,(98\sin(e+fx)+42\sin(3(e+fx))+28\cos(2(e+fx))+21\cos\right.}{180c^3f(\sin(e+fx)-1)^3(a(\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
 f*x])^(7/2)),x]
```

```
[Out] -1/180*(Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*(42*EllipticE[(e + f*x)/2
, 2]*(-3*Cos[e + f*x] - Cos[3*(e + f*x)] + 4*Cos[e + f*x]^3*Sin[e + f*x]) +
 Sqrt[Cos[e + f*x]]*(-9 + 28*Cos[2*(e + f*x)] + 21*Cos[4*(e + f*x)] + 98*Si
n[e + f*x] + 42*Sin[3*(e + f*x)])))/(c^3*f*(-1 + Sin[e + f*x])^3*(a*(1 + Si
n[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g}{a^{3}c^{4}\cos\left(fx+e\right)^{5}\sin\left(fx+e\right)-a^{3}c^{4}\cos\left(fx+e\right)^{5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g/(a^3*c^4*cos(f*x + e)^5*sin(f*x + e) - a^3*c^4*cos(f*x + e)^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(7/2)), x)
```

maple [C] time = 0.65, size = 947, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/45/f*(cos(f*x+e)+1)^2*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(1+sin(f*x+e
))*(sin(f*x+e)-1)*(-21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4+21*I*(1/(co
s(f*x+e)+1))^(1/2)*cos(f*x+e)^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE
(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/s
in(f*x+e),I)+21*I*(1/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^5*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*(1/(cos(f*x+e
)+1))^(1/2)*cos(f*x+e)^4*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+
cos(f*x+e))/sin(f*x+e),I)-21*I*(1/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^5*(cos(f
```

```
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*
(1/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*sin(f*x+e)-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-2
1*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*(1/(cos(f*x+e)+1
))^(1/2)*cos(f*x+e)^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)-21*I*(1/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*sin(f
*x+e)*cos(f*x+e)^3+14*cos(f*x+e)^2*sin(f*x+e)+21*cos(f*x+e)^3-14*cos(f*x+e)
^2+2*sin(f*x+e)-7)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(sin(f*x+e)-1))^(7/2)/cos(f
*x+e)/sin(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos(e+f\,x))^{3/2}}{(a+a\,\sin(e+f\,x))^{5/2}\,(c-c\,\sin(e+f\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))
^(7/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))
^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

3.151 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=119

$$\frac{c2^{n+\frac{9}{4}}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{-n-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(-4n-\right.}{fg(4m+5)}$$

```
[Out] 2^(9/4+n)*c*(g*cos(f*x+e))^(5/2)*hypergeom([5/4+m, -1/4-n],[9/4+m],1/2+1/2*
sin(f*x+e))*(1-sin(f*x+e))^(-1/4-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1
+n)/f/g/(5+4*m)
```

Rubi [A] time = 0.29, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2853, 2689, 70, 69}

$$\frac{c2^{n+\frac{9}{4}}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{-n-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(-4n-\right.}{fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(9/4 + n)*c*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(5 + 4*m)/4, (-1 -
4*n)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 - n)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(f*g*(5 + 4*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin$$

$$= \frac{\Big(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-c\,\mathrm{s}}{}$$

$$= \frac{\Big(2^{\frac{1}{4}+n}c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c}{}$$

$$= \frac{2^{\frac{9}{4}+n}c(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(5+4m),\frac{1}{4}(-1-}{}$$

Mathematica [A] time = 2.15, size = 126, normalized size = 1.06

$$-\frac{8g\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right)\sqrt{g\cos(e+fx)}\,(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n\csc^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{m+n+}}{f(4n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^n,x]
```

```
[Out] (-8*g*Sqrt[g*Cos[e + f*x]]*Cos[(2*e + Pi + 2*f*x)/4]^2*(Csc[(2*e + Pi + 2*f
*x)/4]^2)^(3/2 + m + n)*Hypergeometric2F1[5/4 + n, 5/2 + m + n, 9/4 + n, -T
an[(2*e - Pi + 2*f*x)/4]^2]*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n
)/(f*(5 + 4*n))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, alg
orithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
n*g*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^n, x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.152 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=93

$$-\frac{a^4c^3 2^{m+\frac{9}{4}}(g\cos(e+fx))^{17/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{17}{4},-m-\frac{1}{4};\frac{21}{4};\frac{1}{2}(1-\sin(e+fx)\right.}{17fg^7}$$

```
[Out] -1/17*2^(9/4+m)*a^4*c^3*(g*cos(f*x+e))^(17/2)*hypergeom([17/4, -1/4-m],[21/
4],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-4+m)/f/g^
7
```

Rubi [A] time = 0.28, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2840, 2689, 70, 69}

$$-\frac{a^4c^3 2^{m+\frac{9}{4}}(g\cos(e+fx))^{17/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{17}{4},-m-\frac{1}{4};\frac{21}{4};\frac{1}{2}(1-\sin(e+fx)\right.}{17fg^7}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -(2^(9/4 + m)*a^4*c^3*(g*Cos[e + f*x])^(17/2)*Hypergeometric2F1[17/4, -1/4
- m, 21/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e
 + f*x])^(-4 + m))/(17*f*g^7)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx = \frac{\left(a^3c^3\right)\int (g\cos(e+fx))^{15/2}(a+a\sin(e+fx))^{-3}}{g^6}$$

$$= \frac{\left(a^5c^3(g\cos(e+fx))^{17/2}\right)\text{Subst}\left(\int (a-ax)^{13/4}(a\right.}{fg^7(a-a\sin(e+fx))^{17/4}(a+a\sin}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^5c^3(g\cos(e+fx))^{17/2}(a+a\sin(e+fx))\right.}{}$$

$$= -\frac{2^{\frac{9}{4}+m}a^4c^3(g\cos(e+fx))^{17/2}\,{}_2F_1\left(\frac{17}{4},-\frac{1}{4}-m;\frac{2}{4}\right.}{}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^3,x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,c^3g\cos\left(fx+e\right)^3-4\,c^3g\cos\left(fx+e\right)-\left(c^3g\cos\left(fx+e\right)^3-4\,c^3g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] integral(-(3*c^3*g*cos(f*x + e)^3 - 4*c^3*g*cos(f*x + e) - (c^3*g*cos(f*x +
 e)^3 - 4*c^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*(a*sin(f*x
 + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c\sin\left(fx+e\right)-c\right)^3\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] integrate(-(g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) +
a)^m, x)
```

maple [F] time = 2.96, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c\sin\left(fx+e\right)-c\right)^3\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] -integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) +
a)^m, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3,x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3, x
)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.153 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=93

$$-\frac{a^3c^2 2^{m+\frac{9}{4}}(g\cos(e+fx))^{13/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{13}{4},-m-\frac{1}{4};\frac{17}{4};\frac{1}{2}(1-\sin(e+fx)\right.}{13fg^5}$$

```
[Out] -1/13*2^(9/4+m)*a^3*c^2*(g*cos(f*x+e))^(13/2)*hypergeom([13/4, -1/4-m],[17/
4],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-3+m)/f/g^
5
```

Rubi [A] time = 0.28, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2840, 2689, 70, 69}

$$-\frac{a^3c^2 2^{m+\frac{9}{4}}(g\cos(e+fx))^{13/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{13}{4},-m-\frac{1}{4};\frac{17}{4};\frac{1}{2}(1-\sin(e+fx)\right.}{13fg^5}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -(2^(9/4 + m)*a^3*c^2*(g*Cos[e + f*x])^(13/2)*Hypergeometric2F1[13/4, -1/4
- m, 17/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e
 + f*x])^(-3 + m))/(13*f*g^5)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx = \frac{\left(a^2c^2\right)\int (g\cos(e+fx))^{11/2}(a+a\sin(e+fx))^{-2\cdots}}{g^4}$$

$$= \frac{\left(a^4c^2(g\cos(e+fx))^{13/2}\right)\text{Subst}\left(\int (a-ax)^{9/4}(a\cdots\right.}{fg^5(a-a\sin(e+fx))^{13/4}(a+a\,\text{si}\cdots}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^4c^2(g\cos(e+fx))^{13/2}(a+a\sin(e+fx))\cdots\right.}{\cdots}$$

$$= -\frac{2^{\frac{9}{4}+m}a^3c^2(g\cos(e+fx))^{13/2}\,{}_2F_1\left(\frac{13}{4},-\frac{1}{4}-m;\frac{1}{4}\cdots\right.}{\cdots}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^2,x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c^2 g\cos\left(fx+e\right)^3 + 2\,c^2 g\cos\left(fx+e\right)\sin\left(fx+e\right) - 2\,c^2 g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] integral(-(c^2*g*cos(f*x + e)^3 + 2*c^2*g*cos(f*x + e)*sin(f*x + e) - 2*c^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c\sin\left(fx+e\right)-c\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a
)^m, x)
```

maple [F] time = 1.53, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c\sin\left(fx+e\right)-c\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a
)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left (g\,\cos \left (e+f\,x\right)\right)}^{3/2}\,{\left (a+a\,\sin \left (e+f\,x\right)\right)}^m\,{\left (c-c\,\sin \left (e+f\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2,x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.154 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=91

$$-\frac{a^2c2^{m+\frac{9}{4}}(g\cos(e+fx))^{9/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{9}{4},-m-\frac{1}{4};\frac{13}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{9fg^3}$$

```
[Out] -1/9*2^(9/4+m)*a^2*c*(g*cos(f*x+e))^(9/2)*hypergeom([9/4, -1/4-m],[13/4],1/
2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-2+m)/f/g^3
```

Rubi [A] time = 0.21, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2840, 2689, 70, 69}

$$-\frac{a^2c2^{m+\frac{9}{4}}(g\cos(e+fx))^{9/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{9}{4},-m-\frac{1}{4};\frac{13}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{9fg^3}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
```

```
[Out] -(2^(9/4 + m)*a^2*c*(g*Cos[e + f*x])^(9/2)*Hypergeometric2F1[9/4, -1/4 - m,
 13/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f
*x])^(-2 + m))/(9*f*g^3)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx = \frac{(ac)\int (g\cos(e+fx))^{7/2}(a+a\sin(e+fx))^{-1+m}\,d}{g^2}$$

$$= \frac{\left(a^3c(g\cos(e+fx))^{9/2}\right)\text{Subst}\left(\int (a-ax)^{5/4}(a+a\right.}{fg^3(a-a\sin(e+fx))^{9/4}(a+a\sin(}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^3c(g\cos(e+fx))^{9/2}(a+a\sin(e+fx))^{-2+\right.}{}$$

$$= -\frac{2^{\frac{9}{4}+m}a^2c(g\cos(e+fx))^{9/2}\,{}_2F_1\left(\frac{9}{4},-\frac{1}{4}-m;\frac{13}{4};\frac{1}{2}\right.}{}$$

Mathematica [F] time = 169.60, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
), x]
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(cg\cos\left(fx+e\right)\sin\left(fx+e\right)-cg\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral(-(c*g*cos(f*x + e)*sin(f*x + e) - c*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] integrate(-(g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)
^m, x)
```

maple [F] time = 0.81, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] -integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)
^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)),x)
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e)),x)
[Out] Timed out
```

3.155 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=88

$$-\frac{a2^{m+\frac{9}{4}}(g\cos(e+fx))^{5/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{5}{4},-m-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{5fg}$$

```
[Out] -1/5*2^(9/4+m)*a*(g*cos(f*x+e))^(5/2)*hypergeom([5/4, -1/4-m],[9/4],1/2-1/2
*sin(f*x+e))*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-1+m)/f/g
```

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2689, 70, 69}

$$-\frac{a2^{m+\frac{9}{4}}(g\cos(e+fx))^{5/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{5}{4},-m-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{5fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(2^(9/4 + m)*a*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[5/4, -1/4 - m, 9/4
, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^
(-1 + m))/(5*f*g)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
```

```
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2(g\cos(e+fx))^{5/2}\right)\text{Subst}\left(\int \sqrt[4]{a-ax}\,(a+ax)^{\frac{1}{4}+m}\,dx, x, \sin(e+\right.}{fg(a-a\sin(e+fx))^{5/4}(a+a\sin(e+fx))^{5/4}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{-1+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{-\frac{1}{4}-}\right.}{fg(a-a\sin(e+fx}$$

$$= -\frac{2^{\frac{9}{4}+m}a(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},-\frac{1}{4}-m;\frac{9}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1-}{5fg}$$

Mathematica [A] time = 0.15, size = 85, normalized size = 0.97

$$-\frac{2^{m+\frac{9}{4}}(g\cos(e+fx))^{5/2}(\sin(e+fx)+1)^{-m-\frac{5}{4}}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{5}{4},-m-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{5fg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -1/5*(2^(9/4 + m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[5/4, -1/4 - m, 9
/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-5/4 - m)*(a*(1 + Sin[e + f*x
]))^m)/(f*g)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^m g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x)
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m,x)
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

3.156 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=84

$$-\frac{g2^{m+\frac{9}{4}}\sqrt{g\cos(e+fx)}\,(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{4},-m-\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

```
[Out] -2^(9/4+m)*g*hypergeom([1/4, -1/4-m],[5/4],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e
))^(-1/4-m)*(a+a*sin(f*x+e))^m*(g*cos(f*x+e))^(1/2)/c/f
```

Rubi [A] time = 0.27, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2840, 2689, 70, 69}

$$-\frac{g2^{m+\frac{9}{4}}\sqrt{g\cos(e+fx)}\,(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{4},-m-\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x]),x]
```

```
[Out] -((2^(9/4 + m)*g*Sqrt[g*Cos[e + f*x]]*Hypergeometric2F1[1/4, -1/4 - m, 5/4,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^m
)/(c*f))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx = \frac{g^2\int \frac{(a+a\sin(e+fx))^{1+m}}{\sqrt{g\cos(e+fx)}}\,dx}{ac}$$

$$= \frac{\left(ag\sqrt{g\cos(e+fx)}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{4}+m}}{(a-ax)^{3/4}}\,dx, x, \sin(e+fx)\right)}{cf\sqrt[4]{a-a\sin(e+fx)}\,\sqrt[4]{a+a\sin(e+fx)}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}ag\sqrt{g\cos(e+fx)}\,(a+a\sin(e+fx))^m\left(\frac{a+a\sin(e+fx)}{a}\right)^{-\frac{1}{4}-m}\right)}{cf\sqrt[4]{a-a\sin(e+fx)}}$$

$$= -\frac{2^{\frac{9}{4}+m}g\sqrt{g\cos(e+fx)}\,{}_2F_1\left(\frac{1}{4},-\frac{1}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1}{cf}$$

Mathematica [A] time = 0.12, size = 84, normalized size = 1.00

$$-\frac{g2^{m+\frac{9}{4}}\sqrt{g\cos(e+fx)}\,(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{1}{4},-m-\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x]),x]
```

```
[Out] -((2^(9/4 + m)*g*Sqrt[g*Cos[e + f*x]]*Hypergeometric2F1[1/4, -1/4 - m, 5/4,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a*(1 + Sin[e + f*x]))
^m)/(c*f))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e)/(c*sin
(f*x + e) - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] integrate(-(g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) -
c), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}}{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] -integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) -
c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{c-c\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.157 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=93

$$\frac{g^3 2^{m+\frac{9}{4}}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{4},-m-\frac{1}{4};\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{3ac^2f(g\cos(e+fx))^{3/2}}$$

```
[Out] 1/3*2^(9/4+m)*g^3*hypergeom([-3/4, -1/4-m],[1/4],1/2-1/2*sin(f*x+e))*(1+sin
(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(1+m)/a/c^2/f/(g*cos(f*x+e))^(3/2)
```

Rubi [A] time = 0.28, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2840, 2689, 70, 69}

$$\frac{g^3 2^{m+\frac{9}{4}}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{4},-m-\frac{1}{4};\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{3ac^2f(g\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^2,
x]
```

```
[Out] (2^(9/4 + m)*g^3*Hypergeometric2F1[-3/4, -1/4 - m, 1/4, (1 - Sin[e + f*x])/
2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^(1 + m))/(3*a*c^2*f*(
g*Cos[e + f*x])^(3/2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx = \frac{g^4\int \frac{(a+a\sin(e+fx))^{2+m}}{(g\cos(e+fx))^{5/2}}\,dx}{a^2c^2}$$

$$= \frac{\left(g^3(a-a\sin(e+fx))^{3/4}(a+a\sin(e+fx))^{3/4}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{4}+m}}{(a-ax)^{7/4}}\right.}{c^2f(g\cos(e+fx))^{3/2}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}g^3(a-a\sin(e+fx))^{3/4}(a+a\sin(e+fx))^{1+m}\left(\frac{a+a\sin(e+fx)}{a}\right.\right.}{c^2f(g\cos(e+fx))^{3/2}}$$

$$= \frac{2^{\frac{9}{4}+m}g^3\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))}{3ac^2f(g\cos(e+fx))^{3/2}}$$

Mathematica [A] time = 0.20, size = 96, normalized size = 1.03

$$-\frac{g2^{m+\frac{9}{4}}\sqrt{g\cos(e+fx)}\,(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(-\frac{3}{4},-m-\frac{1}{4};\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{3c^2f(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^2,x]
```

```
[Out] -1/3*(2^(9/4 + m)*g*Sqrt[g*Cos[e + f*x]]*Hypergeometric2F1[-3/4, -1/4 - m,
1/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a*(1 + Sin[e + f*
x]))^m)/(c^2*f*(-1 + Sin[e + f*x]))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e)/(c^2*c
os(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c
)^2, x)
```

maple [F] time = 1.36, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c-c\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c
)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2,
x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2,
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.158 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3} dx$

Optimal. Leaf size=93

$$\frac{g^5 2^{m+\frac{9}{4}}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{7}{4},-m-\frac{1}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{7a^2c^3f(g\cos(e+fx))^{7/2}}$$

```
[Out] 1/7*2^(9/4+m)*g^5*hypergeom([-7/4, -1/4-m],[-3/4],1/2-1/2*sin(f*x+e))*(1+si
n(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(2+m)/a^2/c^3/f/(g*cos(f*x+e))^(7/2)
```

Rubi [A] time = 0.28, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2840, 2689, 70, 69}

$$\frac{g^5 2^{m+\frac{9}{4}}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{7}{4},-m-\frac{1}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{7a^2c^3f(g\cos(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^3,
x]
```

```
[Out] (2^(9/4 + m)*g^5*Hypergeometric2F1[-7/4, -1/4 - m, -3/4, (1 - Sin[e + f*x])
/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^(2 + m))/(7*a^2*c^3*
f*(g*Cos[e + f*x])^(7/2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx = \frac{g^6\int \frac{(a+a\sin(e+fx))^{3+m}}{(g\cos(e+fx))^{9/2}}\,dx}{a^3c^3}$$

$$= \frac{\left(g^5(a-a\sin(e+fx))^{7/4}(a+a\sin(e+fx))^{7/4}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{4}+m}}{(a-ax)^{11/4}} \cdots\right.}{ac^3f(g\cos(e+fx))^{7/2}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}g^5(a-a\sin(e+fx))^{7/4}(a+a\sin(e+fx))^{2+m}\left(\frac{a+a\sin(e+fx)}{a}\cdots\right.\right.}{ac^3f(g\cos(e+fx))^{7\cdots}}$$

$$= \frac{2^{\frac{9}{4}+m}g^5\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{4}-m;-\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx\cdots}{7a^2c^3f(g\cos(e+fx))^{7/2}}$$

Mathematica [A] time = 0.21, size = 96, normalized size = 1.03

$$\frac{g2^{m+\frac{9}{4}}\sqrt{g\cos(e+fx)}\,(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(-\frac{7}{4},-m-\frac{1}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{7c^3f(\sin(e+fx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^3,x]
```

```
[Out] (2^(9/4 + m)*g*Sqrt[g*Cos[e + f*x]]*Hypergeometric2F1[-7/4, -1/4 - m, -3/4,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a*(1 + Sin[e + f*x]))
^m)/(7*c^3*f*(-1 + Sin[e + f*x])^2)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e)/(3*c^3
*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] integrate(-(g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) -
c)^3, x)
```

maple [F] time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c-c\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] -integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) -
c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3,
x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3,
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.159 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=114

$$-\frac{a^3c^2 2^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{15/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{15}{4},\right.}{15fg^6}$$

```
[Out] -1/15*2^(9/4+m)*a^3*c^2*(g*cos(f*x+e))^(15/2)*hypergeom([15/4, -1/4-m],[19/
4],1/2-1/2*sin(f*x+e))*sec(f*x+e)*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^
(-3+m)*(c-c*sin(f*x+e))^(1/2)/f/g^6
```

Rubi [A] time = 0.36, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2853, 2689, 70, 69}

$$-\frac{a^3c^2 2^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{15/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{15}{4},\right.}{15fg^6}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -(2^(9/4 + m)*a^3*c^2*(g*Cos[e + f*x])^(15/2)*Hypergeometric2F1[15/4, -1/4
- m, 19/4, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(-1/4 - m)
*(a + a*Sin[e + f*x])^(-3 + m)*Sqrt[c - c*Sin[e + f*x]])/(15*f*g^6)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx = \frac{\left(a^2c^2\sec(e+fx)\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin\ldots}\right.}{\ldots}$$

$$= \frac{\left(a^4c^2(g\cos(e+fx))^{15/2}\sec(e+fx)\sqrt{c-c\sin\ldots}\right.}{fg^6(a-a\sin(e\ldots}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^4c^2(g\cos(e+fx))^{15/2}\sec(e+fx)(a+\ldots\right.}{\ldots}$$

$$= -\frac{2^{\frac{9}{4}+m}a^3c^2(g\cos(e+fx))^{15/2}\,{}_2F_1\left(\frac{15}{4},-\frac{1}{4}-m\ldots\right.}{\ldots}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(5/2),x]
```

[Out] $Aborted

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c^2 g\cos\left(fx+e\right)^3+2\,c^2 g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^2 g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{-c\sin\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(c^2*g*cos(f*x + e)^3 + 2*c^2*g*cos(f*x + e)*sin(f*x + e) - 2*c^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x
+ e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e
) + a)^m, x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e
) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(5/2
),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(5/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(5/2)
,x)
```

```
[Out] Timed out
```

3.160 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=112

$$-\frac{a^2c2^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{11/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{11}{4},-\right.}{11fg^4}$$

```
[Out] -1/11*2^(9/4+m)*a^2*c*(g*cos(f*x+e))^(11/2)*hypergeom([11/4, -1/4-m],[15/4]
,1/2-1/2*sin(f*x+e))*sec(f*x+e)*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-
2+m)*(c-c*sin(f*x+e))^(1/2)/f/g^4
```

Rubi [A] time = 0.35, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2853, 2689, 70, 69}

$$-\frac{a^2c2^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{11/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{11}{4},-\right.}{11fg^4}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -(2^(9/4 + m)*a^2*c*(g*Cos[e + f*x])^(11/2)*Hypergeometric2F1[11/4, -1/4 -
m, 15/4, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(-1/4 - m)*(
a + a*Sin[e + f*x])^(-2 + m)*Sqrt[c - c*Sin[e + f*x]])/(11*f*g^4)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx = \frac{\left(ac\sec(e+fx)\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin\right.}{}$$

$$= \frac{\left(a^3c(g\cos(e+fx))^{11/2}\sec(e+fx)\sqrt{c-c\sin\right.}{fg^4(a-a\sin(e}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^3c(g\cos(e+fx))^{11/2}\sec(e+fx)(a+\right.}{}$$

$$= -\frac{2^{\frac{9}{4}+m}a^2c(g\cos(e+fx))^{11/2}\,{}_2F_1\left(\frac{11}{4},-\frac{1}{4}-m\right.}{}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(cg\cos\left(fx+e\right)\sin\left(fx+e\right)-cg\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(c*g*cos(f*x + e)*sin(f*x + e) - c*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e
) + a)^m, x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e
) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(g\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(3/2
),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(3/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(3/2)
,x)
```

```
[Out] Timed out
```

3.161 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}$

Optimal. Leaf size=109

$$-\frac{a2^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{7/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{7}{4},-m-\right.}{7fg^2}$$

```
[Out] -1/7*2^(9/4+m)*a*(g*cos(f*x+e))^(7/2)*hypergeom([7/4, -1/4-m],[11/4],1/2-1/
2*sin(f*x+e))*sec(f*x+e)*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-1+m)*(c
-c*sin(f*x+e))^(1/2)/f/g^2
```

Rubi [A] time = 0.32, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2853, 2689, 70, 69}

$$-\frac{a2^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{7/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{7}{4},-m-\right.}{7fg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] -(2^(9/4 + m)*a*(g*Cos[e + f*x])^(7/2)*Hypergeometric2F1[7/4, -1/4 - m, 11/
4, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*
Sin[e + f*x])^(-1 + m)*Sqrt[c - c*Sin[e + f*x]])/(7*f*g^2)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx = \frac{\left(\sec(e+fx)\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+\right.}{}$$

$$= \frac{\left(a^2(g\cos(e+fx))^{7/2}\sec(e+fx)\sqrt{c-c\sin(e+\right.}{fg^2(a-a\sin(e+}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2(g\cos(e+fx))^{7/2}\sec(e+fx)(a+a\sin\right.}{}$$

$$= -\frac{2^{\frac{9}{4}+m}a(g\cos(e+fx))^{7/2}\,{}_2F_1\left(\frac{7}{4},-\frac{1}{4}-m;\frac{11}{4};\frac{1}{2}\right.}{}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m*g*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e)
+ a)^m, x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\sqrt{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e)
+ a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1/2
),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1/2)
,x)
```

```
[Out] Timed out
```

3.162 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{a2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e+\right.}{3f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/3*2^(9/4+m)*a*cos(f*x+e)*(g*cos(f*x+e))^(3/2)*hypergeom([3/4, -1/4-m],[7
/4],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-1+m)/f/(
c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2853, 2689, 70, 69}

$$-\frac{a2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e+\right.}{3f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -(2^(9/4 + m)*a*Cos[e + f*x]*(g*Cos[e + f*x])^(3/2)*Hypergeometric2F1[3/4,
-1/4 - m, 7/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*S
in[e + f*x])^(-1 + m))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(g\cos(e+fx))\int \sqrt{g\cos(e+fx)}\,(a+a\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)(g\cos(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{4}+m}}{\sqrt[4]{a-ax}}\,dx, x, \sin(e\right.}{f(a-a\sin(e+fx))^{3/4}(a+a\sin(e+fx))^{5/4}\sqrt{c-c\sin(e+f}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2\cos(e+fx)(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{-1+m}\left(\frac{a-}{}\right.\right.}{f(a-a\sin(e+fx))^{3/4}\sqrt{c}}$$

$$= -\frac{2^{\frac{9}{4}+m}a\cos(e+fx)(g\cos(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},-\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin\right.}{3f\sqrt{c-c\sin(}}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m*g*cos(f*x + e)/(c*sin(f*x + e) - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e
) + c), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}}{\sqrt{c-c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e
) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1
/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2)
,x)
```

```
[Out] Timed out
```

3.163 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{g^2 2^{m+\frac{9}{4}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{4},-m-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{cf\sqrt{c-c\sin(e+fx)}\sqrt{g\cos(e+fx)}}$$

```
[Out] 2^(9/4+m)*g^2*cos(f*x+e)*hypergeom([-1/4, -1/4-m],[3/4],1/2-1/2*sin(f*x+e))
*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^m/c/f/(g*cos(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2853, 2689, 70, 69}

$$\frac{g^2 2^{m+\frac{9}{4}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{4},-m-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{cf\sqrt{c-c\sin(e+fx)}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (2^(9/4 + m)*g^2*Cos[e + f*x]*Hypergeometric2F1[-1/4, -1/4 - m, 3/4, (1 - S
in[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^m)/(c*f*
Sqrt[g*Cos[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(g^3\cos(e+fx)\right)\int \frac{(a+a\sin(e+fx))^{\frac{3}{2}+m}}{(g\cos(e+fx))^{3/2}}\,dx}{ac\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(ag^2\cos(e+fx)\sqrt[4]{a-a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{4}+m}}{(a-ax)^{5/4}}\,dx, x, \sin\ldots\right.}{cf\sqrt{g\cos(e+fx)}\,\sqrt[4]{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}\ldots}$$

$$= \frac{\left(2^{\frac{1}{4}+m}ag^2\cos(e+fx)\sqrt[4]{a-a\sin(e+fx)}\,(a+a\sin(e+fx))^m\left(\frac{a+\ldots}{\ldots}\right.\right.}{cf\sqrt{g\cos(e+fx)}\,\sqrt{c-\ldots}}$$

$$= \frac{2^{\frac{9}{4}+m}g^2\cos(e+fx)\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\ldots}{cf\sqrt{g\cos(e+fx)}\,\sqrt{c-c\sin(e+\ldots}}$$

Mathematica [F] time = 81.68, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^(3/2), x]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m*g*cos(f*x + e)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^(3/2), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c-c\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3
/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3
/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(3/2)
,x)
```

```
[Out] Timed out
```

3.164 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=114

$$\frac{g^4 2^{m+\frac{9}{4}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{5}{4},-m-\frac{1}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{5ac^2f\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}$$

```
[Out] 1/5*2^(9/4+m)*g^4*cos(f*x+e)*hypergeom([-5/4, -1/4-m],[-1/4],1/2-1/2*sin(f*
x+e))*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(1+m)/a/c^2/f/(g*cos(f*x+e))
^(5/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2853, 2689, 70, 69}

$$\frac{g^4 2^{m+\frac{9}{4}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{5}{4},-m-\frac{1}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{5ac^2f\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (2^(9/4 + m)*g^4*Cos[e + f*x]*Hypergeometric2F1[-5/4, -1/4 - m, -1/4, (1 -
Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^(1 + m)
)/(5*a*c^2*f*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(g^5\cos(e+fx)\right)\int \frac{(a+a\sin(e+fx))^{\frac{5}{2}+m}}{(g\cos(e+fx))^{7/2}}\,dx}{a^2c^2\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(g^4\cos(e+fx)(a-a\sin(e+fx))^{5/4}(a+a\sin(e+fx))^{3/4}\right)\text{Subst}}{c^2f(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}g^4\cos(e+fx)(a-a\sin(e+fx))^{5/4}(a+a\sin(e+fx))^{1+m}\right.}{c^2f(g\cos(e+fx))^{5/2}\sqrt{c}}$$

$$= \frac{2^{\frac{9}{4}+m}g^4\cos(e+fx)\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{4}-m;-\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+}{5ac^2f(g\cos(e+fx))^{5/2}\sqrt{c-c\sin}}$$

Mathematica [C] time = 53.66, size = 2320, normalized size = 20.35

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^(5/2),x]

[Out] ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Cos[(-e + Pi/2 - f*x)/2]*(g*Cos[e + f*x
])^(3/2)*(5*AppellF1[3/4, -1/2 - 2*m, 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2,
 -Tan[(-e + Pi/2 - f*x)/4]^2] + 3*AppellF1[-5/4, -1/2 - 2*m, 2*m, -1/4, Tan
[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)
/4]^4)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^5*(a + a*Sin[e + f*x])^m*Tan[(-e + Pi/2 - f*x)/4])/(60*Sqrt[2]*f*S
qrt[Cos[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)*(Cos[Pi/4 + (e - Pi/2 + f*x)/2
] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^4*Sqrt[2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2
]*(-1/240*((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Sqrt[Cos[e + f*x]]*(5*AppellF
1[3/4, -1/2 - 2*m, 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2] + 3*AppellF1[-5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e + Pi/2 - f*x)/
4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^4)*(Sec[(-e + P
i/2 - f*x)/4]^2)^(2 + 2*m)*Tan[(-e + Pi/2 - f*x)/4]^2)/(2 - 2*Tan[(-e + Pi/
2 - f*x)/4]^2)^(3/2) - ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Sqrt[Cos[e + f*x
]]*(5*AppellF1[3/4, -1/2 - 2*m, 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[
(-e + Pi/2 - f*x)/4]^2] + 3*AppellF1[-5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e +
 Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^4)
*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2 + 2*m))/(480*Sqrt[2 - 2*Tan[(-e + Pi/2 - f
x)/4]^2]) + (m(Cos[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m)*Sqrt[Cos[e + f*x]]*
(5*AppellF1[3/4, -1/2 - 2*m, 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2] + 3*AppellF1[-5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^4)*(S
ec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Sin[(-e + Pi/2 - f*x)/4]^2)/(120*Sqrt[
2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2]) - ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(5
*AppellF1[3/4, -1/2 - 2*m, 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2] + 3*AppellF1[-5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^4)*(Sec
[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Sin[e + f*x]*Tan[(-e + Pi/2 - f*x)/4])/(
240*Sqrt[Cos[e + f*x]]*Sqrt[2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2]) - ((1 + 2*m)
*(Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Sqrt[Cos[e + f*x]]*(5*AppellF1[3/4, -1/
2 - 2*m, 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]
 + 3*AppellF1[-5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^4)*(Sec[(-e + Pi/2 - f*x)
/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2 - f*x)/4]^2)/(240*Sqrt[2 - 2*Tan[(-e + Pi/2
 - f*x)/4]^2]) - ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Sqrt[Cos[e + f*x]]*(Se
c[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2 - f*x)/4]*(-3*AppellF1[-
5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f
*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^3*Csc[(-e + Pi/2 - f*x)/4]^2 + 3*Cot[(-e
 + Pi/2 - f*x)/4]^4*(-5*m*AppellF1[-1/4, -1/2 - 2*m, 1 + 2*m, 3/4, Tan[(-e
+ Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2
*Tan[(-e + Pi/2 - f*x)/4] + (5*(-1/2 - 2*m)*AppellF1[-1/4, 1/2 - 2*m, 2*m,
3/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/

```
2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/2) + 5*((-3*m*AppellF1[7/4, -1/2 -
2*m, 1 + 2*m, 11/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2
]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/7 + (3*(-1/2 - 2*m)*
AppellF1[7/4, 1/2 - 2*m, 2*m, 11/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/14))
)/(120*Sqrt[2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2])))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m*g*cos(f*x + e)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4
*c^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^(5/2), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c-c\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/2),x)

[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(5/2),x)

[Out] Timed out

3.165 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{a2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e\right.}{3f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/3*2^(9/4+m)*a*cos(f*x+e)*(g*cos(f*x+e))^(3/2)*hypergeom([3/4, -1/4-m],[7
/4],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-1+m)/f/(
c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2853, 2689, 70, 69}

$$-\frac{a2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e\right.}{3f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -(2^(9/4 + m)*a*Cos[e + f*x]*(g*Cos[e + f*x])^(3/2)*Hypergeometric2F1[3/4,
-1/4 - m, 7/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*S
in[e + f*x])^(-1 + m))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(g\cos(e+fx))\int\sqrt{g\cos(e+fx)}\,(a+a\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)(g\cos(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{(a+ax)^{\frac{1}{4}+m}}{\sqrt[4]{a-ax}}\,dx, x, \sin(e+\right.}{f(a-a\sin(e+fx))^{3/4}(a+a\sin(e+fx))^{5/4}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2\cos(e+fx)(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{-1+m}\left(\frac{a+a}{}\right.\right.}{f(a-a\sin(e+fx))^{3/4}\sqrt{c-}}$$

$$= -\frac{2^{\frac{9}{4}+m}a\cos(e+fx)(g\cos(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},-\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(\right.}{3f\sqrt{c-c\sin(e}}$$

Mathematica [C] time = 13.59, size = 832, normalized size = 7.85

$$-\frac{}{3f\left(616F_1\left(\frac{3}{4};-2m-\frac{1}{2},2m+3;\frac{7}{4};\tan^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right),-\tan^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)\right)\cos^4\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)-\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] (-88*Sqrt[2]*Cos[(-e + Pi/2 - f*x)/4]^6*Cos[e + f*x]*(g*Cos[e + f*x])^(3/2)
*Csc[(-e + Pi/2 - f*x)/2]*Sec[(-e + Pi/2 - f*x)/2]^2*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])*(a + a*Sin[e + f*x])^m*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(-
7*AppellF1[3/4, -1/2 - 2*m, 3 + 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[
(-e + Pi/2 - f*x)/4]^2] + 3*AppellF1[7/4, -1/2 - 2*m, 3 + 2*m, 11/4, Tan[(-
e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]
^2))/(3*f*(616*AppellF1[3/4, -1/2 - 2*m, 3 + 2*m, 7/4, Tan[(-e + Pi/2 - f*x
)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^4 - 4*Sin[(-e
 + Pi/2 - f*x)/4]^2*(88*(3 + 2*m)*AppellF1[7/4, -1/2 - 2*m, 4 + 2*m, 11/4,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f
*x)/4]^2 + 44*(1 + 4*m)*AppellF1[7/4, 1/2 - 2*m, 3 + 2*m, 11/4, Tan[(-e + P
i/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 +
77*AppellF1[7/4, -1/2 - 2*m, 3 + 2*m, 11/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Ta
n[(-e + Pi/2 - f*x)/4]^2]*(1 + Cos[(-e + Pi/2 - f*x)/2]) - 28*((6 + 4*m)*Ap
pellF1[11/4, -1/2 - 2*m, 4 + 2*m, 15/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2] + (1 + 4*m)*AppellF1[11/4, 1/2 - 2*m, 3 + 2*m, 15/4,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Sin[(-e + Pi/2 -
f*x)/4]^2))*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m*g*cos(f*x + e)/(c*sin(f*x + e) - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e
) + c), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}}{\sqrt{c-c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e
) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{\sqrt{c-c\,\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1
/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2)
,x)
```

```
[Out] Timed out
```

3.166 $\int \frac{(g\cos(e+fx))^{3/2}(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{c2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(c\sin(e+fx)+c)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e+\right.}{3f\sqrt{a-a\sin(e+fx)}}$$

```
[Out] -1/3*2^(9/4+m)*c*cos(f*x+e)*(g*cos(f*x+e))^(3/2)*hypergeom([3/4, -1/4-m],[7
/4],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/4-m)*(c+c*sin(f*x+e))^(-1+m)/f/(
a-a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2853, 2689, 70, 69}

$$-\frac{c2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(c\sin(e+fx)+c)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e+\right.}{3f\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]
],x]
```

```
[Out] -(2^(9/4 + m)*c*Cos[e + f*x]*(g*Cos[e + f*x])^(3/2)*Hypergeometric2F1[3/4,
-1/4 - m, 7/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(c + c*S
in[e + f*x])^(-1 + m))/(3*f*Sqrt[a - a*Sin[e + f*x]])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \frac{(g\cos(e+fx))\int\sqrt{g\cos(e+fx)}\,(c+c\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\left(c^2\cos(e+fx)(g\cos(e+fx))^{3/2}\right)\operatorname{Subst}\left(\int\frac{(c+cx)^{\frac{1}{4}+m}}{\sqrt[4]{c-cx}}\,dx,x,\sin(e\right.}{f\sqrt{a-a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/4}(c+c\sin(e+fx))^{5}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}c^2\cos(e+fx)(g\cos(e+fx))^{3/2}(c+c\sin(e+fx))^{-1+m}\left(\frac{c+}{}\right.\right.}{f\sqrt{a-a\sin(e+fx)}\,(c-c}$$

$$= -\frac{2^{\frac{9}{4}+m}c\cos(e+fx)(g\cos(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},-\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin\right.}{3f\sqrt{a-a\sin(}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]],x]

[Out] $Aborted

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{m}g\cos\left(fx+e\right)}{a\sin\left(fx+e\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^m*g*cos(f*x + e)/(a*sin(f*x + e) - a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c\sin\left(fx+e\right)+c\right)^{m}}{\sqrt{-a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e) + a), x)

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c+c\sin\left(fx+e\right)\right)^{m}}{\sqrt{a-a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)

[Out] int((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c\sin\left(fx+e\right)+c\right)^{m}}{\sqrt{-a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e
) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(c+c\,\sin\left(e+f\,x\right)\right)^{m}}{\sqrt{a-a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1
/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1
/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2)
,x)
```

```
[Out] Timed out
```

3.167 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx$

Optimal. Leaf size=123

$$\frac{2^{-m-\frac{3}{4}}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(4m+1\right.}{c^2fg(4m+5)}$$

```
[Out] 2^(-3/4-m)*(g*cos(f*x+e))^(5/2)*hypergeom([5/4+m, 11/4+m],[9/4+m],1/2+1/2*s
in(f*x+e))*(1-sin(f*x+e))^(-1/4+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-
m)/c^2/f/g/(5+4*m)
```

Rubi [A] time = 0.37, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2853, 2689, 70, 69}

$$\frac{2^{-m-\frac{3}{4}}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(4m+1\right.}{c^2fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3
- m),x]
```

```
[Out] (2^(-3/4 - m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(5 + 4*m)/4, (11 + 4
*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a
+ a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c^2*f*g*(5 + 4*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-$$

$$= \frac{\Big(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c}{}$$

$$= \frac{\Big(2^{-\frac{11}{4}-m}(g\cos(e+fx))^{5/2}(a+a\sin(e+fx}{}$$

$$= \frac{2^{-\frac{3}{4}-m}(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(5+4m),\frac{1}{4}(\right.}{}$$

Mathematica [B] time = 18.00, size = 382, normalized size = 3.11

$$\frac{2^{-m-4}\sec^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)\sec(e+fx)(g\cos(e+fx))^{3/2}\sin^{-2m}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\left(1-\tan^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(-3 - m),x]
```

```
[Out] (2^(-4 - m)*(g*Cos[e + f*x])^(3/2)*((-3 + 8*m + 16*m^2)*Cot[(-e + Pi/2 - f*
x)/4]^4*Hypergeometric2F1[-3/2 - 2*m, -7/4 - m, -3/4 - m, Tan[(-e + Pi/2 -
f*x)/4]^2] + (7 + 4*m)*(2*(-1 + 4*m)*Cot[(-e + Pi/2 - f*x)/4]^2*Hypergeomet
ric2F1[-3/2 - 2*m, -3/4 - m, 1/4 - m, Tan[(-e + Pi/2 - f*x)/4]^2] + (3 + 4*
m)*Hypergeometric2F1[-3/2 - 2*m, 1/4 - m, 5/4 - m, Tan[(-e + Pi/2 - f*x)/4]
^2]))*Sec[(-e + Pi/2 - f*x)/4]^2*Sec[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c
*Sin[e + f*x])^(-3 - m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1/2 - 2*m))/(f*(
-1 + 4*m)*(3 + 4*m)*(7 + 4*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^(2*(-3 - m)))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-3}g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m - 3)*g*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x
, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 3), x)
```

maple [F] time = 1.39, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-3-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{\frac{3}{2}}(a\sin(fx+e)+a)^{m}(-c\sin(fx+e)+c)^{-m-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x
, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{m}}{(c-c\sin(e+fx))^{m+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
 + 3),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
 + 3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-3-m
),x)
```

```
[Out] Timed out
```

3.168 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx$

Optimal. Leaf size=123

$$\frac{2^{\frac{1}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(4m+7)\right.}{cfg(4m+5)}$$

```
[Out] 2^(1/4-m)*(g*cos(f*x+e))^(5/2)*hypergeom([5/4+m, 7/4+m],[9/4+m],1/2+1/2*sin
(f*x+e))*(1-sin(f*x+e))^(-1/4+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)
/c/f/g/(5+4*m)
```

Rubi [A] time = 0.37, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2853, 2689, 70, 69}

$$\frac{2^{\frac{1}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(4m+7)\right.}{cfg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2
- m),x]
```

```
[Out] (2^(1/4 - m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(5 + 4*m)/4, (7 + 4*m
)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a +
a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c*f*g*(5 + 4*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-$$

$$= \frac{\Big(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c}{}$$

$$= \frac{\Big(2^{-\frac{7}{4}-m}(g\cos(e+fx))^{5/2}(a+a\sin(e+fx)}{}$$

$$= \frac{2^{\frac{1}{4}-m}(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(5+4m),\frac{1}{4}(7\right.}{}$$

Mathematica [A] time = 6.20, size = 202, normalized size = 1.64

$$\frac{g2^{-m-1}\csc^2\left(\frac{1}{8}(-2e-2fx+\pi)\right)\sqrt{g\cos(e+fx)}\,\cos^{-2m}\left(\frac{1}{4}(2e+2fx+\pi)\right)\left(1-\tan^2\left(\frac{1}{8}(2e+2fx-\pi)\right)\right)^{-2m-\frac{1}{2}}(}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(-2 - m),x]
```

```
[Out] (2^(-1 - m)*g*Sqrt[g*Cos[e + f*x]]*Csc[(-2*e + Pi - 2*f*x)/8]^2*Hypergeomet
ric2F1[-3/2 - 2*m, -3/4 - m, 1/4 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2]*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^(2*(2 + m))*(a*(1 + Sin[e + f*x]))^m*(1 - Ta
n[(2*e - Pi + 2*f*x)/8]^2)^(-1/2 - 2*m))/(c^2*f*(3 + 4*m)*Cos[(2*e + Pi + 2
*f*x)/4]^(2*m)*(-1 + Sin[e + f*x])^2*(c - c*Sin[e + f*x])^m)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m - 2)*g*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x
, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 2), x)
```

maple [F] time = 1.11, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x
, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\cos\left(e+fx\right)\right)^{3/2}\left(a+a\sin\left(e+fx\right)\right)^m}{\left(c-c\sin\left(e+fx\right)\right)^{m+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
 + 2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
 + 2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-2-m
),x)
```

```
[Out] Timed out
```

3.169 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=120

$$\frac{2^{\frac{5}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+3),\frac{1}{4}(4m+5)\right.}{fg(4m+5)}$$

```
[Out] 2^(5/4-m)*(g*cos(f*x+e))^(5/2)*hypergeom([5/4+m, 3/4+m],[9/4+m],1/2+1/2*sin
(f*x+e))*(1-sin(f*x+e))^(-1/4+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)
/f/g/(5+4*m)
```

Rubi [A] time = 0.36, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2853, 2689, 70, 69}

$$\frac{2^{\frac{5}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+3),\frac{1}{4}(4m+5)\right.}{fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1
- m),x]
```

```
[Out] (2^(5/4 - m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(3 + 4*m)/4, (5 + 4*m
)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a +
a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-$$

$$= \frac{\Big(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c}{}$$

$$= \frac{\Big(2^{-\frac{3}{4}-m}c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx}{}$$

$$= \frac{2^{\frac{5}{4}-m}(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(3+4m),\frac{1}{4}(5}{}$$

Mathematica [F] time = 104.98, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(-1 - m),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(-1 - m), x]
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m - 1)*g*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x
, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 1), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x
, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\cos\left(e+fx\right)\right)^{3/2}\left(a+a\sin\left(e+fx\right)\right)^m}{\left(c-c\sin\left(e+fx\right)\right)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
 + 1),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
 + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-1-m
),x)
```

```
[Out] Timed out
```

3.170 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx$

Optimal. Leaf size=121

$$\frac{c2^{\frac{9}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-1),\frac{1}{4}(4m+5\right.}{fg(4m+5)}$$

```
[Out] 2^(9/4-m)*c*(g*cos(f*x+e))^(5/2)*hypergeom([-1/4+m, 5/4+m],[9/4+m],1/2+1/2*
sin(f*x+e))*(1-sin(f*x+e))^(-1/4+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1
-m)/f/g/(5+4*m)
```

Rubi [A] time = 0.31, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2853, 2689, 70, 69}

$$\frac{c2^{\frac{9}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-1),\frac{1}{4}(4m+5\right.}{fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^m,
x]
```

```
[Out] (2^(9/4 - m)*c*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(-1 + 4*m)/4, (5 +
4*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c$$

$$= \frac{\Big(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-}{}$$

$$= \frac{\Big(2^{\frac{1}{4}-m}c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))}{}$$

$$= \frac{2^{\frac{9}{4}-m}c(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(-1+4m),\frac{1}{4}(5\right.}{}$$

Mathematica [F] time = 16.30, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^m,x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^m, x]
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{\left(-c\sin\left(fx+e\right)+c\right)^{m}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, a
lgorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e)/(-c*sin
(f*x + e) + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, a
lgorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^m, x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m,
x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m,
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/((c-c*sin(f*x+e))**m),x
)
```

```
[Out] Timed out
```

3.171 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx$

Optimal. Leaf size=123

$$\frac{c^2 2^{\frac{13}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-5),\frac{1}{4}(4m+$$
$$\overline{\qquad fg(4m+5)\qquad}$$

```
[Out] 2^(13/4-m)*c^2*(g*cos(f*x+e))^(5/2)*hypergeom([5/4+m, -5/4+m],[9/4+m],1/2+1
/2*sin(f*x+e))*(1-sin(f*x+e))^(-1/4+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^
(-1-m)/f/g/(5+4*m)
```

Rubi [A] time = 0.34, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2853, 2689, 70, 69}

$$\frac{c^2 2^{\frac{13}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-5),\frac{1}{4}(4m+$$
$$\overline{\qquad fg(4m+5)\qquad}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 -
 m),x]
```

```
[Out] (2^(13/4 - m)*c^2*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(-5 + 4*m)/4, (5
 + 4*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)
*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-$$

$$= \frac{\Big(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-}{}$$

$$= \frac{\Big(2^{\frac{5}{4}-m}c^3(g\cos(e+fx))^{5/2}(a+a\sin(e+fx)}{}$$

$$= \frac{2^{\frac{13}{4}-m}c^2(g\cos(e+fx))^{5/2}\,{}_2F_1\Big(\frac{1}{4}(-5+4m),}{}$$

Mathematica [F] time = 176.64, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(1 - m),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(1 - m), x]
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1}g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x,
 algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m + 1)*g*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m + 1), x)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 -
 m),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 -
 m), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1-m)
,x)
```

```
[Out] Timed out
```

3.172 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx$

Optimal. Leaf size=123

$$\frac{c^3 2^{\frac{17}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-9),\frac{1}{4}(4m+\right.}{fg(4m+5)}$$

```
[Out] 2^(17/4-m)*c^3*(g*cos(f*x+e))^(5/2)*hypergeom([5/4+m, -9/4+m],[9/4+m],1/2+1
/2*sin(f*x+e))*(1-sin(f*x+e))^(-1/4+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^
(-1-m)/f/g/(5+4*m)
```

Rubi [A] time = 0.35, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2853, 2689, 70, 69}

$$\frac{c^3 2^{\frac{17}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-9),\frac{1}{4}(4m+\right.}{fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 -
 m),x]
```

```
[Out] (2^(17/4 - m)*c^3*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(-9 + 4*m)/4, (5
 + 4*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)
*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-$$

$$= \frac{\Big(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-}{}$$

$$= \frac{\Big(2^{\frac{9}{4}-m}c^4(g\cos(e+fx))^{5/2}(a+a\sin(e+fx)}{}$$

$$= \frac{2^{\frac{17}{4}-m}c^3(g\cos(e+fx))^{5/2}\,{}_2F_1\Big(\frac{1}{4}(-9+4m),}{}$$

Mathematica [F] time = 84.50, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(2 - m),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(2 - m), x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+2}g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x,
 algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m + 2)*g*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m + 2), x)
```

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(g\cos\left(e+fx\right)\right)^{3/2}\left(a+a\sin\left(e+fx\right)\right)^{m}\left(c-c\sin\left(e+fx\right)\right)^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 -
 m),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 -
 m), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(2-m)
,x)
```

```
[Out] Timed out
```

3.173 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=135

$$\frac{c2^{n+\frac{p}{2}+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{p+1}(1-\sin(e+fx))^{\frac{1}{2}(-2n-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2n-p+\right.}{fg(2m+p+1)}$$

```
[Out] 2^(1/2+n+1/2*p)*c*(g*cos(f*x+e))^(1+p)*hypergeom([1/2+m+1/2*p, 1/2-n-1/2*p]
,[3/2+m+1/2*p],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2-n-1/2*p)*(a+a*sin(f*
x+e))^m*(c-c*sin(f*x+e))^(-1+n)/f/g/(1+2*m+p)
```

Rubi [A] time = 0.29, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2853, 2689, 70, 69}

$$\frac{c2^{n+\frac{p}{2}+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{p+1}(1-\sin(e+fx))^{\frac{1}{2}(-2n-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2n-p+\right.}{fg(2m+p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n + p/2)*c*(g*Cos[e + f*x])^(1 + p)*Hypergeometric2F1[(1 - 2*n -
p)/2, (1 + 2*m + p)/2, (3 + 2*m + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e +
f*x])^((1 - 2*n - p)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n
))/(f*g*(1 + 2*m + p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin$$

$$= \frac{\Big(c^2(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))^m(c-c}{}$$

$$= \frac{\Big(2^{-\frac{1}{2}+n+\frac{p}{2}}c^2(g\cos(e+fx))^{1+p}(a+a\sin(e+fx)}{}$$

$$= \frac{2^{\frac{1}{2}+n+\frac{p}{2}}c(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(1-2n-p),\frac{1}{2}\right.}{}$$

Mathematica [A] time = 41.73, size = 133, normalized size = 0.99

$$\frac{2\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n(g\cos(e+fx))^p\sec^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m+n+p}}{f(2n+p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,
x]
```

```
[Out] (2*(g*Cos[e + f*x])^p*Hypergeometric2F1[1 + m + n + p, (1 + 2*n + p)/2, (3
+ 2*n + p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(
m + n + p)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n*Tan[(2*e - Pi +
2*f*x)/4])/(f*(1 + 2*n + p))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n
, x)
```

maple [F] time = 9.72, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{p}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(g\cos\left(e+f\,x\right)\right)^p\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
```

```
[Out] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.174 $\int (g\cos(e+fx))^{1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1+m}\,dx$

Optimal. Leaf size=57

$$-\frac{g\log(1-\sin(e+fx))(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m(g\cos(e+fx))^{-2m}}{cf}$$

```
[Out] -g*ln(1-sin(f*x+e))*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m/c/f/((g*cos(f*x+e
))^(2*m))
```

Rubi [A] time = 0.23, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2843, 12, 2667, 31}

$$-\frac{g\log(1-\sin(e+fx))(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m(g\cos(e+fx))^{-2m}}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
(-1 + m),x]
```

```
[Out] -((g*Log[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^m)/(
c*f*(g*Cos[e + f*x])^(2*m)))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2843

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*
m + p - 1, 0] && EqQ[m - n - 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^{1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1+m}\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(\ldots$$

$$= \Big(g(g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m\ldots$$

$$= -\frac{\Big(g(g\cos(e+fx))^{-2m}(a+a\sin(e+fx)\ldots}{\ldots}$$

$$= -\frac{g(g\cos(e+fx))^{-2m}\log(1-\sin(e+fx\ldots}{\ldots}$$

Mathematica [B] time = 84.63, size = 132, normalized size = 2.32

$$\frac{g2^{m+1}\cos^{2m}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^m(g\cos(e+fx))^{-2m}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\ldots\right.}{cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^(-1 + m),x]
```

```
[Out] (2^(1 + m)*g*Cos[(2*e + Pi + 2*f*x)/4]^(2*m)*(Log[Csc[(2*e + 3*Pi + 2*f*x)/
8]^2] - Log[Tan[(-2*e + Pi - 2*f*x)/8]])*(a*(1 + Sin[e + f*x]))^m*(c - c*Si
n[e + f*x])^m)/(c*f*(g*Cos[e + f*x])^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])^(2*m))
```

fricas [A] time = 0.50, size = 30, normalized size = 0.53

$$-\frac{a\left(\frac{ac}{g^2}\right)^{m-1}\log\left(-\sin\left(fx+e\right)+1\right)}{fg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+m)
,x, algorithm="fricas")
```

```
[Out] -a*(a*c/g^2)^(m - 1)*log(-sin(f*x + e) + 1)/(f*g)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+m)
,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)(4*pi*exp(m*ln(abs(a))+m*ln(abs(c))-2*m*ln(abs(g))-ln(abs(c))+ln(ab
s(g)))*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)
*tan((4*m*pi*floor(-(sign(a)-2)/4)+4*m*pi*floor(-(sign(c)-4)/4)+m*pi*sign(a
)+m*pi*sign(c)-2*m*pi*sign(g)-4*pi*floor(-(sign(c)-4)/4)-pi*sign(c)+pi*sign
(g)+pi)/4)^2-4*pi*exp(m*ln(abs(a))+m*ln(abs(c))-2*m*ln(abs(g))-ln(abs(c))+l
n(abs(g)))*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4
/pi)+4*pi*exp(m*ln(abs(a))+m*ln(abs(c))-2*m*ln(abs(g))-ln(abs(c))+ln(abs(g)
))*floor((f*x+pi+exp(1))*1/2/pi)*tan((4*m*pi*floor(-(sign(a)-2)/4)+4*m*pi*f
loor(-(sign(c)-4)/4)+m*pi*sign(a)+m*pi*sign(c)-2*m*pi*sign(g)-4*pi*floor(-(
sign(c)-4)/4)-pi*sign(c)+pi*sign(g)+pi)/4)^2-4*pi*exp(m*ln(abs(a))+m*ln(abs
(c))-2*m*ln(abs(g))-ln(abs(c))+ln(abs(g)))*floor((f*x+pi+exp(1))*1/2/pi)+2*
pi*exp(m*ln(abs(a))+m*ln(abs(c))-2*m*ln(abs(g))-ln(abs(c))+ln(abs(g)))*sign
(tan((f*x+exp(1))/2)^2-1)*tan((4*m*pi*floor(-(sign(a)-2)/4)+4*m*pi*floor(-(
sign(c)-4)/4)+m*pi*sign(a)+m*pi*sign(c)-2*m*pi*sign(g)-4*pi*floor(-(sign(c)
-4)/4)-pi*sign(c)+pi*sign(g)+pi)/4)^2-2*pi*exp(m*ln(abs(a))+m*ln(abs(c))-2*
m*ln(abs(g))-ln(abs(c))+ln(abs(g)))*sign(tan((f*x+exp(1))/2)^2-1)+3*pi*exp(
m*ln(abs(a))+m*ln(abs(c))-2*m*ln(abs(g))-ln(abs(c))+ln(abs(g)))*tan((4*m*pi
*floor(-(sign(a)-2)/4)+4*m*pi*floor(-(sign(c)-4)/4)+m*pi*sign(a)+m*pi*sign(
c)-2*m*pi*sign(g)-4*pi*floor(-(sign(c)-4)/4)-pi*sign(c)+pi*sign(g)+pi)/4)^2
-3*pi*exp(m*ln(abs(a))+m*ln(abs(c))-2*m*ln(abs(g))-ln(abs(c))+ln(abs(g)))-2
*exp(1)*exp(m*ln(abs(a))+m*ln(abs(c))-2*m*ln(abs(g))-ln(abs(c))+ln(abs(g)))
*tan((4*m*pi*floor(-(sign(a)-2)/4)+4*m*pi*floor(-(sign(c)-4)/4)+m*pi*sign(a
)+m*pi*sign(c)-2*m*pi*sign(g)-4*pi*floor(-(sign(c)-4)/4)-pi*sign(c)+pi*sign
(g)+pi)/4)^2+2*exp(1)*exp(m*ln(abs(a))+m*ln(abs(c))-2*m*ln(abs(g))-ln(abs(c
))+ln(abs(g)))-4*exp(m*ln(abs(a))+m*ln(abs(c))-2*m*ln(abs(g))-ln(abs(c))+ln
(abs(g)))*ln((2*tan((f*x+exp(1))/2)^2-4*tan((f*x+exp(1))/2)+2)/(tan((f*x+ex
p(1))/2)^2+1))*tan((4*m*pi*floor(-(sign(a)-2)/4)+4*m*pi*floor(-(sign(c)-4)/
4)+m*pi*sign(a)+m*pi*sign(c)-2*m*pi*sign(g)-4*pi*floor(-(sign(c)-4)/4)-pi*s
```

```
ign(c)+pi*sign(g)+pi)/4))/(2*f*tan((4*m*pi*floor(-(sign(a)-2)/4)+4*m*pi*flo
or(-(sign(c)-4)/4)+m*pi*sign(a)+m*pi*sign(c)-2*m*pi*sign(g)-4*pi*floor(-(si
gn(c)-4)/4)-pi*sign(c)+pi*sign(g)+pi)/4)^2+2*f)
```

maple [C] time = 13.42, size = 8507, normalized size = 149.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+m),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{-2\,m+1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+m)
,x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m + 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^(m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{1-2\,m}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1 - 2*m)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^
(m - 1),x)
```

```
[Out] int((g*cos(e + f*x))^(1 - 2*m)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^
(m - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-1
+m),x)
```

```
[Out] Timed out
```

3.175 $\int (g\cos(e+fx))^{5-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=203

$$-\frac{8a^3(a\sin(e+fx)+a)^{m-3}(c-c\sin(e+fx))^n(g\cos(e+fx))^{6-2m}}{fg(-m+n+3)(-m+n+4)(-m+n+5)}-\frac{4a^2(a\sin(e+fx)+a)^{m-2}(c-c\sin(e+fx))^n(}{fg(-m+n+4)(-m+n+5}$$

```
[Out] -8*a^3*(g*cos(f*x+e))^(6-2*m)*(a+a*sin(f*x+e))^(-3+m)*(c-c*sin(f*x+e))^n/f/
g/(3-m+n)/(4-m+n)/(5-m+n)-4*a^2*(g*cos(f*x+e))^(6-2*m)*(a+a*sin(f*x+e))^(-2
+m)*(c-c*sin(f*x+e))^n/f/g/(4-m+n)/(5-m+n)-a*(g*cos(f*x+e))^(6-2*m)*(a+a*si
n(f*x+e))^(-1+m)*(c-c*sin(f*x+e))^n/f/g/(5-m+n)
```

Rubi [A] time = 0.68, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2846, 2844}

$$-\frac{8a^3(a\sin(e+fx)+a)^{m-3}(c-c\sin(e+fx))^n(g\cos(e+fx))^{6-2m}}{fg(-m+n+3)(-m+n+4)(-m+n+5)}-\frac{4a^2(a\sin(e+fx)+a)^{m-2}(c-c\sin(e+fx))^n(}{fg(-m+n+4)(-m+n+5}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
n,x]
```

```
[Out] (-8*a^3*(g*Cos[e + f*x])^(6 - 2*m)*(a + a*Sin[e + f*x])^(-3 + m)*(c - c*Sin
[e + f*x])^n)/(f*g*(3 - m + n)*(4 - m + n)*(5 - m + n)) - (4*a^2*(g*Cos[e +
 f*x])^(6 - 2*m)*(a + a*Sin[e + f*x])^(-2 + m)*(c - c*Sin[e + f*x])^n)/(f*g
*(4 - m + n)*(5 - m + n)) - (a*(g*Cos[e + f*x])^(6 - 2*m)*(a + a*Sin[e + f*
x])^(-1 + m)*(c - c*Sin[e + f*x])^n)/(f*g*(5 - m + n))
```

Rule 2844

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])
^n)/(f*g*(m - n - 1)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m - n -
1, 0]
```

Rule 2846

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
```

```
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[Simplify[m + p/2 - 1/2], 0] &&  !LtQ[n, -1] &&  !(IGtQ[Simpli
fy[n + p/2 - 1/2], 0] && GtQ[m - n, 0]) &&  !(ILtQ[Simplify[m + n + p], 0]
&& GtQ[Simplify[2*m + n + (3*p)/2 + 1], 0])
```

Rubi steps

$$\int (g\cos(e+fx))^{5-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = -\frac{a(g\cos(e+fx))^{6-2m}(a+a\sin(e+fx))^{-1+}}{fg(5-m+n)}$$
$$= -\frac{4a^2(g\cos(e+fx))^{6-2m}(a+a\sin(e+fx))^{-}}{fg(4-m+n)(5-m+}$$
$$= -\frac{8a^3(g\cos(e+fx))^{6-2m}(a+a\sin(e+fx))^{-}}{fg(3-m+n)(4-m+n)(5}$$

Mathematica [C] time = 6.81, size = 1513, normalized size = 7.45

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(5 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^n,x]
```

```
[Out] (Cos[e + f*x]^(-5 + 2*n)*(g*Cos[e + f*x])^(5 - 2*m)*(a*(1 + Sin[e + f*x]))^
m*(c - c*Sin[e + f*x])^(n - (n*(Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e
 + f*x]]))/Log[c - c*Sin[e + f*x]])*((E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1
 + Sin[e + f*x])] + Log[c - c*Sin[e + f*x]]))*(256 - 41*m + 3*m^2 + 41*n -
6*m*n + 3*n^2))/(8*(-5 + m - n)*(-4 + m - n)*(-3 + m - n)) + ((300 - 23*m +
 m^2 + 23*n - 2*m*n + n^2)*((-1/16*I)*E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1
 + Sin[e + f*x])] + Log[c - c*Sin[e + f*x]]))*Cos[e + f*x] - (E^(n*(-2*Log[
Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e + f*x]]))*Sin[e
 + f*x])/16))/((-5 + m - n)*(-4 + m - n)*(-3 + m - n)) + ((300 - 23*m + m^2
 + 23*n - 2*m*n + n^2)*((I/16)*E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[
e + f*x])] + Log[c - c*Sin[e + f*x]]))*Cos[e + f*x] - (E^(n*(-2*Log[Cos[e +
 f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e + f*x]]))*Sin[e + f*x]
)/16))/((-5 + m - n)*(-4 + m - n)*(-3 + m - n)) + ((-11*m + m^2 + 11*n - 2*
m*n + n^2)*((E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c
 - c*Sin[e + f*x]]))*Cos[2*(e + f*x)])/4 - (I/4)*E^(n*(-2*Log[Cos[e + f*x]]
 + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e + f*x]]))*Sin[2*(e + f*x)]))
```

```
/((-5 + m - n)*(-4 + m - n)*(-3 + m - n)) + ((-11*m + m^2 + 11*n - 2*m*n +
n^2)*((E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*S
in[e + f*x]]))*Cos[2*(e + f*x)])/4 + (I/4)*E^(n*(-2*Log[Cos[e + f*x]] + Log
[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e + f*x]]))*Sin[2*(e + f*x)]))/((-5
+ m - n)*(-4 + m - n)*(-3 + m - n)) + ((100 - 53*m + 3*m^2 + 53*n - 6*m*n +
 3*n^2)*((-1/32*I)*E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] +
 Log[c - c*Sin[e + f*x]]))*Cos[3*(e + f*x)] - (E^(n*(-2*Log[Cos[e + f*x]] +
 Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e + f*x]]))*Sin[3*(e + f*x)])/32
))/((-5 + m - n)*(-4 + m - n)*(-3 + m - n)) + ((100 - 53*m + 3*m^2 + 53*n -
 6*m*n + 3*n^2)*((I/32)*E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x
])] + Log[c - c*Sin[e + f*x]]))*Cos[3*(e + f*x)] - (E^(n*(-2*Log[Cos[e + f*
x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e + f*x]]))*Sin[3*(e + f*x)
])/32))/((-5 + m - n)*(-4 + m - n)*(-3 + m - n)) + ((m - n)*((E^(n*(-2*Log[
Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e + f*x]]))*Cos[4
*(e + f*x)])/16 - (I/16)*E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*
x])] + Log[c - c*Sin[e + f*x]]))*Sin[4*(e + f*x)]))/((-5 + m - n)*(-4 + m -
 n)) + ((m - n)*((E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] +
Log[c - c*Sin[e + f*x]]))*Cos[4*(e + f*x)])/16 + (I/16)*E^(n*(-2*Log[Cos[e
+ f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e + f*x]]))*Sin[4*(e +
f*x)]))/((-5 + m - n)*(-4 + m - n)) + ((-1/32*I)*E^(n*(-2*Log[Cos[e + f*x]]
 + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e + f*x]]))*Cos[5*(e + f*x)] -
 (E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e
+ f*x]]))*Sin[5*(e + f*x)])/32)/(-5 + m - n) + ((I/32)*E^(n*(-2*Log[Cos[e +
 f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e + f*x]]))*Cos[5*(e + f
*x)] - (E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*
Sin[e + f*x]]))*Sin[5*(e + f*x)])/32)/(-5 + m - n)))/f
```

fricas [B] time = 0.55, size = 651, normalized size = 3.21

$$-\frac{}{4\,fm^3-4\,fn^3-\left(fm^3-fn^3-12\,fm^2+3\left(fm-4\,f\right)n^2+47\,fm-\left(3\,fm^2-24\,fm+47\,f\right)n-60\,f\right)\cos\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="fricas")
```

```
[Out] -((m^2 - (2*m - 7)*n + n^2 - 7*m + 12)*cos(f*x + e)^3 - (m^2 - (2*m - 11)*n
 + n^2 - 11*m + 24)*cos(f*x + e)^2 - 2*(m^2 - (2*m - 9)*n + n^2 - 9*m + 22)
*cos(f*x + e) - ((m^2 - (2*m - 7)*n + n^2 - 7*m + 12)*cos(f*x + e)^2 + 2*(m
^2 - (2*m - 9)*n + n^2 - 9*m + 18)*cos(f*x + e) - 8)*sin(f*x + e) - 8)*(g*c
os(f*x + e))^(-2*m + 5)*(a*sin(f*x + e) + a)^m*e^(2*n*log(g*cos(f*x + e)) -
 n*log(a*sin(f*x + e) + a) + n*log(a*c/g^2))/(4*f*m^3 - 4*f*n^3 - (f*m^3 -
f*n^3 - 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*f*m + 47*f)*n
```

```
 - 60*f)*cos(f*x + e)^3 - 48*f*m^2 + 12*(f*m - 4*f)*n^2 - 3*(f*m^3 - f*n^3
- 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*f*m + 47*f)*n - 60*
f)*cos(f*x + e)^2 + 188*f*m - 4*(3*f*m^2 - 24*f*m + 47*f)*n + 2*(f*m^3 - f*
n^3 - 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*f*m + 47*f)*n -
 60*f)*cos(f*x + e) + (4*f*m^3 - 4*f*n^3 - 48*f*m^2 + 12*(f*m - 4*f)*n^2 -
(f*m^3 - f*n^3 - 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*f*m
+ 47*f)*n - 60*f)*cos(f*x + e)^2 + 188*f*m - 4*(3*f*m^2 - 24*f*m + 47*f)*n
+ 2*(f*m^3 - f*n^3 - 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*
f*m + 47*f)*n - 60*f)*cos(f*x + e) - 240*f)*sin(f*x + e) - 240*f)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 66.34, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{5-2m}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

maxima [B] time = 0.98, size = 978, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="maxima")
```

```
[Out] ((m^2 - m*(2*n + 11) + n^2 + 11*n + 32)*a^m*c^n*g^5 - 2*(m^2 - m*(2*n + 15)
 + n^2 + 15*n + 60)*a^m*c^n*g^5*sin(f*x + e)/(cos(f*x + e) + 1) - (3*m^2 -
m*(6*n + 1) + 3*n^2 + n - 160)*a^m*c^n*g^5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 8*(m^2 - m*(2*n + 7) + n^2 + 7*n - 20)*a^m*c^n*g^5*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 2*(m^2 - m*(2*n - 5) + n^2 - 5*n + 160)*a^m*c^n*g^5*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*(3*m^2 - m*(6*n + 13) + 3*n^2 + 13*n +
 116)*a^m*c^n*g^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*(m^2 - m*(2*n - 5
```

```
) + n^2 - 5*n + 160)*a^m*c^n*g^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 8*(m
^2 - m*(2*n + 7) + n^2 + 7*n - 20)*a^m*c^n*g^5*sin(f*x + e)^7/(cos(f*x + e)
 + 1)^7 - (3*m^2 - m*(6*n + 1) + 3*n^2 + n - 160)*a^m*c^n*g^5*sin(f*x + e)^
8/(cos(f*x + e) + 1)^8 - 2*(m^2 - m*(2*n + 15) + n^2 + 15*n + 60)*a^m*c^n*g
^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + (m^2 - m*(2*n + 11) + n^2 + 11*n +
 32)*a^m*c^n*g^5*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*e^(2*n*log(sin(f*x
+ e)/(cos(f*x + e) + 1) - 1) - 2*m*log(-sin(f*x + e)/(cos(f*x + e) + 1) + 1
) + m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1) - n*log(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1))/(((m^3 - 3*m^2*(n + 4) - n^3 + (3*n^2 + 24*n + 47
)*m - 12*n^2 - 47*n - 60)*g^(2*m) + 5*(m^3 - 3*m^2*(n + 4) - n^3 + (3*n^2 +
 24*n + 47)*m - 12*n^2 - 47*n - 60)*g^(2*m)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 10*(m^3 - 3*m^2*(n + 4) - n^3 + (3*n^2 + 24*n + 47)*m - 12*n^2 - 47*
n - 60)*g^(2*m)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*(m^3 - 3*m^2*(n +
4) - n^3 + (3*n^2 + 24*n + 47)*m - 12*n^2 - 47*n - 60)*g^(2*m)*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 5*(m^3 - 3*m^2*(n + 4) - n^3 + (3*n^2 + 24*n + 47
)*m - 12*n^2 - 47*n - 60)*g^(2*m)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + (m^
3 - 3*m^2*(n + 4) - n^3 + (3*n^2 + 24*n + 47)*m - 12*n^2 - 47*n - 60)*g^(2*
m)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*f)
```

mupad [B] time = 17.21, size = 1149, normalized size = 5.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5 - 2*m)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^
n,x)
```

```
[Out] ((c - c*sin(e + f*x))^n*((exp(e*2i + f*x*2i)*(g*(exp(- e*1i - f*x*1i)/2 + e
xp(e*1i + f*x*1i)/2))^(5 - 2*m)*(a + a*sin(e + f*x))^m*(22*n - 22*m - 4*m*n
 + 2*m^2 + 2*n^2 + 80))/(f*(47*n - 47*m - 24*m*n - 3*m*n^2 + 3*m^2*n + 12*m
^2 - m^3 + 12*n^2 + n^3 + 60)) - (exp(e*5i + f*x*5i)*(g*(exp(- e*1i - f*x*1
i)/2 + exp(e*1i + f*x*1i)/2))^(5 - 2*m)*(a + a*sin(e + f*x))^m*(n*7i - m*7i
 - m*n*2i + m^2*1i + n^2*1i + 12i))/(f*(47*n - 47*m - 24*m*n - 3*m*n^2 + 3*
m^2*n + 12*m^2 - m^3 + 12*n^2 + n^3 + 60)) - ((g*(exp(- e*1i - f*x*1i)/2 +
exp(e*1i + f*x*1i)/2))^(5 - 2*m)*(a + a*sin(e + f*x))^m*(7*n - 7*m - 2*m*n
+ m^2 + n^2 + 12))/(f*(47*n - 47*m - 24*m*n - 3*m*n^2 + 3*m^2*n + 12*m^2 -
m^3 + 12*n^2 + n^3 + 60)) + (exp(e*4i + f*x*4i)*(g*(exp(- e*1i - f*x*1i)/2
+ exp(e*1i + f*x*1i)/2))^(5 - 2*m)*(a + a*sin(e + f*x))^m*(29*n - 29*m - 6*
m*n + 3*m^2 + 3*n^2 + 60))/(f*(47*n - 47*m - 24*m*n - 3*m*n^2 + 3*m^2*n + 1
2*m^2 - m^3 + 12*n^2 + n^3 + 60)) + (exp(e*1i + f*x*1i)*(g*(exp(- e*1i - f*
x*1i)/2 + exp(e*1i + f*x*1i)/2))^(5 - 2*m)*(a + a*sin(e + f*x))^m*(n*29i -
m*29i - m*n*6i + m^2*3i + n^2*3i + 60i))/(f*(47*n - 47*m - 24*m*n - 3*m*n^2
 + 3*m^2*n + 12*m^2 - m^3 + 12*n^2 + n^3 + 60)) + (exp(e*3i + f*x*3i)*(g*(e
xp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(5 - 2*m)*(a + a*sin(e + f*x
))^m*(n*22i - m*22i - m*n*4i + m^2*2i + n^2*2i + 80i))/(f*(47*n - 47*m - 24
```

```
*m*n - 3*m*n^2 + 3*m^2*n + 12*m^2 - m^3 + 12*n^2 + n^3 + 60))))/(5*exp(e*1i
 + f*x*1i) - 10*exp(e*3i + f*x*3i) + exp(e*5i + f*x*5i) + (n*47i - m*47i -
m*n*24i - m*n^2*3i + m^2*n*3i + m^2*12i - m^3*1i + n^2*12i + n^3*1i + 60i)/
(47*n - 47*m - 24*m*n - 3*m*n^2 + 3*m^2*n + 12*m^2 - m^3 + 12*n^2 + n^3 + 6
0) - (10*exp(e*2i + f*x*2i)*(n*47i - m*47i - m*n*24i - m*n^2*3i + m^2*n*3i
+ m^2*12i - m^3*1i + n^2*12i + n^3*1i + 60i))/(47*n - 47*m - 24*m*n - 3*m*n
^2 + 3*m^2*n + 12*m^2 - m^3 + 12*n^2 + n^3 + 60) + (5*exp(e*4i + f*x*4i)*(n
*47i - m*47i - m*n*24i - m*n^2*3i + m^2*n*3i + m^2*12i - m^3*1i + n^2*12i +
 n^3*1i + 60i))/(47*n - 47*m - 24*m*n - 3*m*n^2 + 3*m^2*n + 12*m^2 - m^3 +
12*n^2 + n^3 + 60))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x
)
```

```
[Out] Timed out
```

3.176 $\int (g\cos(e+fx))^{3-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=127

$$-\frac{2a^2(a\sin(e+fx)+a)^{m-2}(c-c\sin(e+fx))^n(g\cos(e+fx))^{4-2m}}{fg(-m+n+2)(-m+n+3)}-\frac{a(a\sin(e+fx)+a)^{m-1}(c-c\sin(e+fx))^n(g}{fg(-m+n+3)}$$

```
[Out] -2*a^2*(g*cos(f*x+e))^(4-2*m)*(a+a*sin(f*x+e))^(-2+m)*(c-c*sin(f*x+e))^n/f/
g/(2-m+n)/(3-m+n)-a*(g*cos(f*x+e))^(4-2*m)*(a+a*sin(f*x+e))^(-1+m)*(c-c*sin
(f*x+e))^n/f/g/(3-m+n)
```

Rubi [A] time = 0.41, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2846, 2844}

$$-\frac{2a^2(a\sin(e+fx)+a)^{m-2}(c-c\sin(e+fx))^n(g\cos(e+fx))^{4-2m}}{fg(-m+n+2)(-m+n+3)}-\frac{a(a\sin(e+fx)+a)^{m-1}(c-c\sin(e+fx))^n(g}{fg(-m+n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
n,x]
```

```
[Out] (-2*a^2*(g*Cos[e + f*x])^(4 - 2*m)*(a + a*Sin[e + f*x])^(-2 + m)*(c - c*Sin
[e + f*x])^n)/(f*g*(2 - m + n)*(3 - m + n)) - (a*(g*Cos[e + f*x])^(4 - 2*m)
*(a + a*Sin[e + f*x])^(-1 + m)*(c - c*Sin[e + f*x])^n)/(f*g*(3 - m + n))
```

Rule 2844

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])
^n)/(f*g*(m - n - 1)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m - n -
1, 0]
```

Rule 2846

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
```

```
2, 0] && IGtQ[Simplify[m + p/2 - 1/2], 0] &&  !LtQ[n, -1] &&  !(IGtQ[Simpli
fy[n + p/2 - 1/2], 0] && GtQ[m - n, 0]) &&  !(ILtQ[Simplify[m + n + p], 0]
&& GtQ[Simplify[2*m + n + (3*p)/2 + 1], 0])
```

Rubi steps

$$\int (g\cos(e+fx))^{3-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = -\frac{a(g\cos(e+fx))^{4-2m}(a+a\sin(e+fx))^{-1+}}{fg(3-m+n)}$$
$$= -\frac{2a^2(g\cos(e+fx))^{4-2m}(a+a\sin(e+fx))^{-}}{fg(2-m+n)(3-m+}$$

Mathematica [A] time = 1.38, size = 143, normalized size = 1.13

$$-\frac{g^3\cos^{2n}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(g\cos(e+fx))^{-2m}((-m+n+2)\sin(e+fx)-m+n+}{f(-m+n+2)(-}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^n,x]
```

```
[Out] -((E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e
 + f*x]]))*g^3*Cos[e + f*x]^(2*n)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(
a*(1 + Sin[e + f*x]))^(m - n)*(4 - m + n + (2 - m + n)*Sin[e + f*x]))/(f*(2
 - m + n)*(3 - m + n)*(g*Cos[e + f*x])^(2*m)))
```

fricas [B] time = 0.52, size = 298, normalized size = 2.35

$$\frac{\left((m-n-2)\cos\left(fx+e\right)^2+(m-n-4)\cos\left(fx+e\right)+\left((m-\right.\right.}{2fm^2+2fn^2-\left(fm^2+fn^2-5fm-\left(2fm-5f\right)n+6f\right)\cos\left(fx+e\right)^2-10fm-2\left(2fm-5f\right)n+\left(fm^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="fricas")
```

```
[Out] ((m - n - 2)*cos(f*x + e)^2 + (m - n - 4)*cos(f*x + e) + ((m - n - 2)*cos(f
*x + e) + 2)*sin(f*x + e) - 2)*(g*cos(f*x + e))^(-2*m + 3)*(a*sin(f*x + e)
+ a)^m*e^(2*n*log(g*cos(f*x + e)) - n*log(a*sin(f*x + e) + a) + n*log(a*c/g
```

```
^2))/(2*f*m^2 + 2*f*n^2 - (f*m^2 + f*n^2 - 5*f*m - (2*f*m - 5*f)*n + 6*f)*c
os(f*x + e)^2 - 10*f*m - 2*(2*f*m - 5*f)*n + (f*m^2 + f*n^2 - 5*f*m - (2*f*
m - 5*f)*n + 6*f)*cos(f*x + e) + (2*f*m^2 + 2*f*n^2 - 10*f*m - 2*(2*f*m - 5
*f)*n + (f*m^2 + f*n^2 - 5*f*m - (2*f*m - 5*f)*n + 6*f)*cos(f*x + e) + 12*f
)*sin(f*x + e) + 12*f)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 26.10, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{3-2m}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

maxima [B] time = 2.80, size = 485, normalized size = 3.82

$$\frac{\left(a^{m}c^{n}g^{3}(m-n-4)-\frac{2\,a^{m}c^{n}g^{3}(m-n-6)\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{a^{m}c^{n}g^{3}(m-n+12)\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{4\,a^{m}c^{n}g^{3}(m-n+2)\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{a^{m}c^{n}g^{3}(m-n+12)\,\mathrm{si}}{\left(\cos\left(fx+e\right)+\right.}}{\left(\left(m^{2}-m(2\,n+5)+n^{2}+5\,n+6\right)g^{2\,m}+\frac{3\left(m^{2}-m(2\,n+5)+n^{2}+5\,n+6\right.}{\left(\cos\left(fx+e\right)+1\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="maxima")
```

```
[Out] (a^m*c^n*g^3*(m - n - 4) - 2*a^m*c^n*g^3*(m - n - 6)*sin(f*x + e)/(cos(f*x
+ e) + 1) - a^m*c^n*g^3*(m - n + 12)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
4*a^m*c^n*g^3*(m - n + 2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^m*c^n*g^3
*(m - n + 12)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2*a^m*c^n*g^3*(m - n -
6)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + a^m*c^n*g^3*(m - n - 4)*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6)*e^(2*n*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)
```

```
- 2*m*log(-sin(f*x + e)/(cos(f*x + e) + 1) + 1) + m*log(sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 1) - n*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/(((
m^2 - m*(2*n + 5) + n^2 + 5*n + 6)*g^(2*m) + 3*(m^2 - m*(2*n + 5) + n^2 + 5
*n + 6)*g^(2*m)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*(m^2 - m*(2*n + 5)
+ n^2 + 5*n + 6)*g^(2*m)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + (m^2 - m*(2*
n + 5) + n^2 + 5*n + 6)*g^(2*m)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*f)
```

mupad [B]   time = 18.13, size = 476, normalized size = 3.75

$$-\frac{\left(c-c\,\sin\left(e+f\,x\right)\right)^n\,\left(\frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3-2\,m}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(n-m+2\right)}{f\,\left(m^2-2\,m\,n-5\,m+n^2+5\,n+6\right)}-\frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3-2\,m}\,\left(\cos\left(3\,e+3\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\,1{}\mathrm{i}\right)\,\left(a+a\,\mathrm{si}\right.}{f\,\left(m^2-2\,m\,n-5\,m+n^2+5\,n+6\right)}\right.}{3\,\cos\left(e+f\,x\right)+\sin\left(e+f\,x\right)\,3{}\mathrm{i}-\cos\left(3\,e+3\,f\,x\right)-\mathrm{sin}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3 - 2*m)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^
n,x)
```

```
[Out] -((c - c*sin(e + f*x))^n*(((g*cos(e + f*x))^(3 - 2*m)*(a + a*sin(e + f*x))^
m*(n - m + 2))/(f*(5*n - 5*m - 2*m*n + m^2 + n^2 + 6)) - ((g*cos(e + f*x))^
(3 - 2*m)*(cos(3*e + 3*f*x) + sin(3*e + 3*f*x)*1i)*(a + a*sin(e + f*x))^m*(
n*1i - m*1i + 2i))/(f*(5*n - 5*m - 2*m*n + m^2 + n^2 + 6)) - ((g*cos(e + f*
x))^(3 - 2*m)*(cos(e + f*x) + sin(e + f*x)*1i)*(a + a*sin(e + f*x))^m*(n*1i
 - m*1i + 6i))/(f*(5*n - 5*m - 2*m*n + m^2 + n^2 + 6)) + ((g*cos(e + f*x))^
(3 - 2*m)*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i)*(a + a*sin(e + f*x))^m*(
n - m + 6))/(f*(5*n - 5*m - 2*m*n + m^2 + n^2 + 6))))/(3*cos(e + f*x) + sin
(e + f*x)*3i - cos(3*e + 3*f*x) - sin(3*e + 3*f*x)*1i + (n*5i - m*5i - m*n*
2i + m^2*1i + n^2*1i + 6i)/(5*n - 5*m - 2*m*n + m^2 + n^2 + 6) - (3*(cos(2*
e + 2*f*x) + sin(2*e + 2*f*x)*1i)*(n*5i - m*5i - m*n*2i + m^2*1i + n^2*1i +
 6i))/(5*n - 5*m - 2*m*n + m^2 + n^2 + 6))
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x
)
```

```
[Out] Timed out
```

3.177 $\int (g\cos(e+fx))^{1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=58

$$-\frac{a(a\sin(e+fx)+a)^{m-1}(c-c\sin(e+fx))^n(g\cos(e+fx))^{2-2m}}{fg(-m+n+1)}$$

```
[Out] -a*(g*cos(f*x+e))^(2-2*m)*(a+a*sin(f*x+e))^(-1+m)*(c-c*sin(f*x+e))^n/f/g/(1
-m+n)
```

Rubi [A] time = 0.16, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2844}

$$-\frac{a(a\sin(e+fx)+a)^{m-1}(c-c\sin(e+fx))^n(g\cos(e+fx))^{2-2m}}{fg(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
n,x]
```

```
[Out] -((a*(g*Cos[e + f*x])^(2 - 2*m)*(a + a*Sin[e + f*x])^(-1 + m)*(c - c*Sin[e
+ f*x])^n)/(f*g*(1 - m + n)))
```

Rule 2844

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])
^n)/(f*g*(m - n - 1)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m - n -
1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^{1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx=-\frac{a(g\cos(e+fx))^{2-2m}(a+a\sin(e+fx))^{-1+m}}{fg(1-m+n)}$$

Mathematica [A] time = 0.72, size = 96, normalized size = 1.66

$$\frac{g(\sin(e+fx)-1)\cos^{2n}(e+fx)(g\cos(e+fx))^{-2m}(a(\sin(e+fx)+1))^{m-n}\exp(n(\log(a(\sin(e+fx)+1))+\log(c}{f(-m+n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(g*Cos[e + f*x])^(1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]

[Out] (E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e + f*x]]))*g*Cos[e + f*x]^(2*n)*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(m - n))/(f*(1 - m + n)*(g*Cos[e + f*x])^(2*m))

fricas [B] time = 0.51, size = 129, normalized size = 2.22

$$\frac{\left(g\cos\left(fx+e\right)\right)^{-2m+1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)e^{\left(2n\log\left(g\cos\left(fx+e\right)\right)-n\log\left(a\sin\left(fx+e\right)+a\right)\right.}}{fm-fn+\left(fm-fn-f\right)\cos\left(fx+e\right)+\left(fm-fn-f\right)\sin\left(fx+e\right)-f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="fricas")

[Out] (g*cos(f*x + e))^(-2*m + 1)*(a*sin(f*x + e) + a)^m*(cos(f*x + e) - sin(f*x + e) + 1)*e^(2*n*log(g*cos(f*x + e)) - n*log(a*sin(f*x + e) + a) + n*log(a*c/g^2))/(f*m - f*n + (f*m - f*n - f)*cos(f*x + e) + (f*m - f*n - f)*sin(f*x + e) - f)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*

```
pi/x/2)>(-8*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unab
le to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi
/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable
 to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*
pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8
*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to c
heck sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/
2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)(-exp(m*ln(2)-2*m*ln
(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+m*ln(a
bs(a))-2*m*ln(abs(g))-n*ln(2)+2*n*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan
((2*f*x-pi+2*exp(1))/8)^2+1))+n*ln(abs(c))-ln(2)+ln(4*abs(tan((2*f*x-pi+2*e
xp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+ln(abs(g)))*tan((2*f*x-8*m*pi*
```

```
floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)-8*m*pi
*floor((f*x+pi+exp(1))*1/2/pi)+8*m*pi*floor(-(sign(a)-2)/4)+2*m*pi*sign(a)-
4*m*pi*sign(g)-4*m*pi*sign(tan((f*x+exp(1))/2)^2-1)-2*m*pi+8*n*pi*floor((2*
f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+8*n*pi*floor((f
*x+pi+exp(1))*1/2/pi)+8*n*pi*floor(-(sign(c)-4)/4)+2*n*pi*sign(c)+4*n*pi*si
gn(tan((f*x+exp(1))/2)^2-1)+2*n*pi+4*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp
(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+4*pi*floor((f*x+pi+exp(1))*1/2/pi)+2*pi*s
ign(g)+2*pi*sign(tan((f*x+exp(1))/2)^2-1)+pi+2*exp(1))/8)^2*tan((f*x+exp(1)
)/2)^2+2*exp(m*ln(2)-2*m*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-p
i+2*exp(1))/8)^2+1))+m*ln(abs(a))-2*m*ln(abs(g))-n*ln(2)+2*n*ln(4*abs(tan((
2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+n*ln(abs(c))-ln(2)
+ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+ln(
abs(g)))*tan((2*f*x-8*m*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+
pi+2*exp(1))*1/4/pi)-8*m*pi*floor((f*x+pi+exp(1))*1/2/pi)+8*m*pi*floor(-(si
gn(a)-2)/4)+2*m*pi*sign(a)-4*m*pi*sign(g)-4*m*pi*sign(tan((f*x+exp(1))/2)^2
-1)-2*m*pi+8*n*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(
1))*1/4/pi)+8*n*pi*floor((f*x+pi+exp(1))*1/2/pi)+8*n*pi*floor(-(sign(c)-4)/
4)+2*n*pi*sign(c)+4*n*pi*sign(tan((f*x+exp(1))/2)^2-1)+2*n*pi+4*pi*floor((2
*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+4*pi*floor((f*
x+pi+exp(1))*1/2/pi)+2*pi*sign(g)+2*pi*sign(tan((f*x+exp(1))/2)^2-1)+pi+2*e
xp(1))/8)^2*tan((f*x+exp(1))/2)-exp(m*ln(2)-2*m*ln(4*abs(tan((2*f*x-pi+2*ex
p(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+m*ln(abs(a))-2*m*ln(abs(g))-n*l
n(2)+2*n*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2
+1))+n*ln(abs(c))-ln(2)+ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi
+2*exp(1))/8)^2+1))+ln(abs(g)))*tan((2*f*x-8*m*pi*floor((2*f*x-4*pi*floor((
f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)-8*m*pi*floor((f*x+pi+exp(1))*1/
2/pi)+8*m*pi*floor(-(sign(a)-2)/4)+2*m*pi*sign(a)-4*m*pi*sign(g)-4*m*pi*sig
n(tan((f*x+exp(1))/2)^2-1)-2*m*pi+8*n*pi*floor((2*f*x-4*pi*floor((f*x+pi+ex
p(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+8*n*pi*floor((f*x+pi+exp(1))*1/2/pi)+8*n
*pi*floor(-(sign(c)-4)/4)+2*n*pi*sign(c)+4*n*pi*sign(tan((f*x+exp(1))/2)^2-
1)+2*n*pi+4*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))
*1/4/pi)+4*pi*floor((f*x+pi+exp(1))*1/2/pi)+2*pi*sign(g)+2*pi*sign(tan((f*x
+exp(1))/2)^2-1)+pi+2*exp(1))/8)^2-2*exp(m*ln(2)-2*m*ln(4*abs(tan((2*f*x-pi
+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+m*ln(abs(a))-2*m*ln(abs(g)
)-n*ln(2)+2*n*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))
/8)^2+1))+n*ln(abs(c))-ln(2)+ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f
*x-pi+2*exp(1))/8)^2+1))+ln(abs(g)))*tan((2*f*x-8*m*pi*floor((2*f*x-4*pi*fl
oor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)-8*m*pi*floor((f*x+pi+exp(1
))*1/2/pi)+8*m*pi*floor(-(sign(a)-2)/4)+2*m*pi*sign(a)-4*m*pi*sign(g)-4*m*p
i*sign(tan((f*x+exp(1))/2)^2-1)-2*m*pi+8*n*pi*floor((2*f*x-4*pi*floor((f*x+
pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+8*n*pi*floor((f*x+pi+exp(1))*1/2/pi
)+8*n*pi*floor(-(sign(c)-4)/4)+2*n*pi*sign(c)+4*n*pi*sign(tan((f*x+exp(1))/
2)^2-1)+2*n*pi+4*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*ex
p(1))*1/4/pi)+4*pi*floor((f*x+pi+exp(1))*1/2/pi)+2*pi*sign(g)+2*pi*sign(tan
((f*x+exp(1))/2)^2-1)+pi+2*exp(1))/8)*tan((f*x+exp(1))/2)^2+2*exp(m*ln(2)-2
```

```
*m*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+m
*ln(abs(a))-2*m*ln(abs(g))-n*ln(2)+2*n*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))
/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+n*ln(abs(c))-ln(2)+ln(4*abs(tan((2*f*x-p
i+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+ln(abs(g)))*tan((2*f*x-8*
m*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)-8
*m*pi*floor((f*x+pi+exp(1))*1/2/pi)+8*m*pi*floor(-(sign(a)-2)/4)+2*m*pi*sig
n(a)-4*m*pi*sign(g)-4*m*pi*sign(tan((f*x+exp(1))/2)^2-1)-2*m*pi+8*n*pi*floo
r((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+8*n*pi*flo
or((f*x+pi+exp(1))*1/2/pi)+8*n*pi*floor(-(sign(c)-4)/4)+2*n*pi*sign(c)+4*n*
pi*sign(tan((f*x+exp(1))/2)^2-1)+2*n*pi+4*pi*floor((2*f*x-4*pi*floor((f*x+p
i+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+4*pi*floor((f*x+pi+exp(1))*1/2/pi)+2
*pi*sign(g)+2*pi*sign(tan((f*x+exp(1))/2)^2-1)+pi+2*exp(1))/8)+exp(m*ln(2)-
2*m*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+
m*ln(abs(a))-2*m*ln(abs(g))-n*ln(2)+2*n*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8)
)/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+n*ln(abs(c))-ln(2)+ln(4*abs(tan((2*f*x-
pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+ln(abs(g)))*tan((f*x+exp
(1))/2)^2-2*exp(m*ln(2)-2*m*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*
x-pi+2*exp(1))/8)^2+1))+m*ln(abs(a))-2*m*ln(abs(g))-n*ln(2)+2*n*ln(4*abs(ta
n((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+n*ln(abs(c))-ln
(2)+ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+
ln(abs(g)))*tan((f*x+exp(1))/2)+exp(m*ln(2)-2*m*ln(4*abs(tan((2*f*x-pi+2*ex
p(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+m*ln(abs(a))-2*m*ln(abs(g))-n*l
n(2)+2*n*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2
+1))+n*ln(abs(c))-ln(2)+ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi
+2*exp(1))/8)^2+1))+ln(abs(g))))/(f*m*tan((2*f*x-8*m*pi*floor((2*f*x-4*pi*f
loor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)-8*m*pi*floor((f*x+pi+exp(
1))*1/2/pi)+8*m*pi*floor(-(sign(a)-2)/4)+2*m*pi*sign(a)-4*m*pi*sign(g)-4*m*
pi*sign(tan((f*x+exp(1))/2)^2-1)-2*m*pi+8*n*pi*floor((2*f*x-4*pi*floor((f*x
+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+8*n*pi*floor((f*x+pi+exp(1))*1/2/p
i)+8*n*pi*floor(-(sign(c)-4)/4)+2*n*pi*sign(c)+4*n*pi*sign(tan((f*x+exp(1))
/2)^2-1)+2*n*pi+4*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*e
xp(1))*1/4/pi)+4*pi*floor((f*x+pi+exp(1))*1/2/pi)+2*pi*sign(g)+2*pi*sign(ta
n((f*x+exp(1))/2)^2-1)+pi+2*exp(1))/8)^2*tan((f*x+exp(1))/2)^2+f*m*tan((2*f
*x-8*m*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/
pi)-8*m*pi*floor((f*x+pi+exp(1))*1/2/pi)+8*m*pi*floor(-(sign(a)-2)/4)+2*m*p
i*sign(a)-4*m*pi*sign(g)-4*m*pi*sign(tan((f*x+exp(1))/2)^2-1)-2*m*pi+8*n*pi
*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+8*n*p
i*floor((f*x+pi+exp(1))*1/2/pi)+8*n*pi*floor(-(sign(c)-4)/4)+2*n*pi*sign(c)
+4*n*pi*sign(tan((f*x+exp(1))/2)^2-1)+2*n*pi+4*pi*floor((2*f*x-4*pi*floor((
f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+4*pi*floor((f*x+pi+exp(1))*1/2/
pi)+2*pi*sign(g)+2*pi*sign(tan((f*x+exp(1))/2)^2-1)+pi+2*exp(1))/8)^2+f*m*t
an((f*x+exp(1))/2)^2+f*m-f*n*tan((2*f*x-8*m*pi*floor((2*f*x-4*pi*floor((f*x
+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)-8*m*pi*floor((f*x+pi+exp(1))*1/2/p
i)+8*m*pi*floor(-(sign(a)-2)/4)+2*m*pi*sign(a)-4*m*pi*sign(g)-4*m*pi*sign(t
an((f*x+exp(1))/2)^2-1)-2*m*pi+8*n*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1
```

```
))*1/2/pi)+pi+2*exp(1))*1/4/pi)+8*n*pi*floor((f*x+pi+exp(1))*1/2/pi)+8*n*pi
*floor(-(sign(c)-4)/4)+2*n*pi*sign(c)+4*n*pi*sign(tan((f*x+exp(1))/2)^2-1)+
2*n*pi+4*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/
4/pi)+4*pi*floor((f*x+pi+exp(1))*1/2/pi)+2*pi*sign(g)+2*pi*sign(tan((f*x+ex
p(1))/2)^2-1)+pi+2*exp(1))/8)^2*tan((f*x+exp(1))/2)^2-f*n*tan((2*f*x-8*m*pi
*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)-8*m*p
i*floor((f*x+pi+exp(1))*1/2/pi)+8*m*pi*floor(-(sign(a)-2)/4)+2*m*pi*sign(a)
-4*m*pi*sign(g)-4*m*pi*sign(tan((f*x+exp(1))/2)^2-1)-2*m*pi+8*n*pi*floor((2
*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+8*n*pi*floor((
f*x+pi+exp(1))*1/2/pi)+8*n*pi*floor(-(sign(c)-4)/4)+2*n*pi*sign(c)+4*n*pi*s
ign(tan((f*x+exp(1))/2)^2-1)+2*n*pi+4*pi*floor((2*f*x-4*pi*floor((f*x+pi+ex
p(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+4*pi*floor((f*x+pi+exp(1))*1/2/pi)+2*pi*
sign(g)+2*pi*sign(tan((f*x+exp(1))/2)^2-1)+pi+2*exp(1))/8)^2-f*n*tan((f*x+e
xp(1))/2)^2-f*n-f*tan((2*f*x-8*m*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))
*1/2/pi)+pi+2*exp(1))*1/4/pi)-8*m*pi*floor((f*x+pi+exp(1))*1/2/pi)+8*m*pi*f
loor(-(sign(a)-2)/4)+2*m*pi*sign(a)-4*m*pi*sign(g)-4*m*pi*sign(tan((f*x+exp
(1))/2)^2-1)-2*m*pi+8*n*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+
pi+2*exp(1))*1/4/pi)+8*n*pi*floor((f*x+pi+exp(1))*1/2/pi)+8*n*pi*floor(-(si
gn(c)-4)/4)+2*n*pi*sign(c)+4*n*pi*sign(tan((f*x+exp(1))/2)^2-1)+2*n*pi+4*pi
*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+4*pi*
floor((f*x+pi+exp(1))*1/2/pi)+2*pi*sign(g)+2*pi*sign(tan((f*x+exp(1))/2)^2-
1)+pi+2*exp(1))/8)^2*tan((f*x+exp(1))/2)^2-f*tan((2*f*x-8*m*pi*floor((2*f*x
-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)-8*m*pi*floor((f*x+
pi+exp(1))*1/2/pi)+8*m*pi*floor(-(sign(a)-2)/4)+2*m*pi*sign(a)-4*m*pi*sign(
g)-4*m*pi*sign(tan((f*x+exp(1))/2)^2-1)-2*m*pi+8*n*pi*floor((2*f*x-4*pi*flo
or((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+8*n*pi*floor((f*x+pi+exp(1)
)*1/2/pi)+8*n*pi*floor(-(sign(c)-4)/4)+2*n*pi*sign(c)+4*n*pi*sign(tan((f*x+
exp(1))/2)^2-1)+2*n*pi+4*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)
+pi+2*exp(1))*1/4/pi)+4*pi*floor((f*x+pi+exp(1))*1/2/pi)+2*pi*sign(g)+2*pi*
sign(tan((f*x+exp(1))/2)^2-1)+pi+2*exp(1))/8)^2-f*tan((f*x+exp(1))/2)^2-f)
```

maple [F] time = 37.18, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{1-2m}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

maxima [B] time = 1.26, size = 207, normalized size = 3.57

$$\frac{\left(a^{m} c^{n} g - \frac{2 \, a^{m} c^{n} g \sin\left(f x + e\right)}{\cos\left(f x + e\right) + 1} + \frac{a^{m} c^{n} g \sin\left(f x + e\right)^{2}}{{\left(\cos\left(f x + e\right) + 1\right)}^{2}}\right)} e^{\left(2 \, n \log\left(\frac{\sin\left(f x + e\right)}{\cos\left(f x + e\right) + 1} - 1\right) - 2 \, m \log\left(-\frac{\sin\left(f x + e\right)}{\cos\left(f x + e\right) + 1} + 1\right) + m \log\left(\frac{\sin\left(f x + e\right)^{2}}{{\left(\cos\left(f x + e\right) + 1\right)}^{2}} + 1\right) - n \log\left(\frac{\sin\left(f x + e\right)^{2}}{{\left(\cos\left(f x + e\right) + 1\right)}^{2}} + 1\right)\right)}}{{\left(g^{2 \, m} {\left(m - n - 1\right)} + \frac{g^{2 \, m} {\left(m - n - 1\right)} \sin\left(f x + e\right)^{2}}{{\left(\cos\left(f x + e\right) + 1\right)}^{2}}\right)} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="maxima")
```

```
[Out] (a^m*c^n*g - 2*a^m*c^n*g*sin(f*x + e)/(cos(f*x + e) + 1) + a^m*c^n*g*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2)*e^(2*n*log(sin(f*x + e)/(cos(f*x + e) + 1) -
 1) - 2*m*log(-sin(f*x + e)/(cos(f*x + e) + 1) + 1) + m*log(sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 1) - n*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
/((g^(2*m)*(m - n - 1) + g^(2*m)*(m - n - 1)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2)*f)
```

mupad [B] time = 9.42, size = 74, normalized size = 1.28

$$-\frac{g\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m\,{\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^n}{2\,f\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{2\,m}\,\left(\sin\left(e+f\,x\right)+1\right)\,\left(n-m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1 - 2*m)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^
n,x)
```

```
[Out] -(g*(cos(2*e + 2*f*x) + 1)*(a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))
^n)/(2*f*(g*cos(e + f*x))^(2*m)*(sin(e + f*x) + 1)*(n - m + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x
)
```

```
[Out] Timed out
```

3.178 $\int (g\cos(e+fx))^{-1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=81

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-2m}\,{}_2F_1\left(1,n-m;-m+n+1;\frac{1}{2}(1-\sin(e+fx))\right)}{2fg(m-n)}$$

```
[Out] 1/2*hypergeom([1, -m+n],[1-m+n],1/2-1/2*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c-c
*sin(f*x+e))^n/f/g/(m-n)/((g*cos(f*x+e))^(2*m))
```

Rubi [A] time = 0.23, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2853, 12, 2667, 68}

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-2m}\,{}_2F_1\left(1,n-m;-m+n+1;\frac{1}{2}(1-\sin(e+fx))\right)}{2fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])
^n,x]
```

```
[Out] (Hypergeometric2F1[1, -m + n, 1 - m + n, (1 - Sin[e + f*x])/2]*(a + a*Sin[e
 + f*x])^m*(c - c*Sin[e + f*x])^n)/(2*f*g*(m - n)*(g*Cos[e + f*x])^(2*m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c$$

$$= \frac{\Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c}{}$$

$$= -\frac{\Big(c(g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c}{}$$

$$= \frac{(g\cos(e+fx))^{-2m}\,{}_2F_1\Big(1,-m+n;1-m+n}{}$$

Mathematica [A] time = 71.44, size = 115, normalized size = 1.42

$$\frac{(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-2m}\sec^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{n-m}\,{}_2F_1\Big(n-m,n-m;-m+n}{2fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
 f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[-m + n, -m + n, 1 - m + n, -Tan[(2*e - Pi + 2*f*x)/4]^2]
*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(-m + n)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin
[e + f*x])^n)/(2*f*g*(m - n)*(g*Cos[e + f*x])^(2*m))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g\cos\left(fx+e\right)\right)^{-2\,m-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-2*m - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e
) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-2\,m-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

maple [F] time = 25.91, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-1-2m}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-2\,m-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\sin\left(e+f\,x\right)\right)^m\,\left(c-c\sin\left(e+f\,x\right)\right)^n}{\left(g\cos\left(e+f\,x\right)\right)^{2\,m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n)/(g*cos(e + f*x))^(2*m +
 1),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n)/(g*cos(e + f*x))^(2*m +
 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,
x)
```

```
[Out] Timed out
```

3.179 $\int (g\cos(e+fx))^{-3-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=85

$$\frac{c(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-2m}\,{}_2F_1\left(2,-m+n-1;n-m;\frac{1}{2}(1-\sin(e+fx))\right)}{4fg^3(m-n+1)}$$

```
[Out] 1/4*c*hypergeom([2, -1-m+n],[-m+n],1/2-1/2*sin(f*x+e))*(a+a*sin(f*x+e))^m*(
c-c*sin(f*x+e))^(-1+n)/f/g^3/(1+m-n)/((g*cos(f*x+e))^(2*m))
```

Rubi [A] time = 0.24, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2853, 12, 2667, 68}

$$\frac{c(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-2m}\,{}_2F_1\left(2,-m+n-1;n-m;\frac{1}{2}(1-\sin(e+fx))\right)}{4fg^3(m-n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-3 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])
^n,x]
```

```
[Out] (c*Hypergeometric2F1[2, -1 - m + n, -m + n, (1 - Sin[e + f*x])/2]*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(4*f*g^3*(1 + m - n)*(g*Cos[e
+ f*x])^(2*m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{-3-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c$$

$$= \frac{\Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c}{}$$

$$= -\frac{\Big(c^3(g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c}{}$$

$$= \frac{c(g\cos(e+fx))^{-2m}\,{}_2F_1\Big(2,-1-m+n;-m+}{}$$

Mathematica [A] time = 49.70, size = 135, normalized size = 1.59

$$\frac{\cot^2\left(\frac{1}{4}(2e+2fx-\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-2m}\sec^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{n-m}\,{}_2F}{8fg^3(m-n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-3 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
 f*x])^n,x]
```

```
[Out] (Cot[(2*e - Pi + 2*f*x)/4]^2*Hypergeometric2F1[-2 - m + n, -1 - m + n, -m +
 n, -Tan[(2*e - Pi + 2*f*x)/4]^2]*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(-m + n)*(a
*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/(8*f*g^3*(1 + m - n)*(g*Cos[
e + f*x])^(2*m))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g\cos\left(fx+e\right)\right)^{-2\,m-3}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-2*m - 3)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e
) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-2\,m-3}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 3)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

maple [F] time = 32.33, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-3-2m}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-2\,m-3}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 3)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^n}{\left(g\,\cos\left(e+f\,x\right)\right)^{2\,m+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n)/(g*cos(e + f*x))^(2*m +
 3),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n)/(g*cos(e + f*x))^(2*m +
 3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-3-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,
x)
```

```
[Out] Timed out
```

3.180 $\int (g\cos(e+fx))^{-5-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=88

$$\frac{c^2(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}(g\cos(e+fx))^{-2m}\,{}_2F_1\left(3,-m+n-2;-m+n-1;\frac{1}{2}(1-\sin(e+fx\right.}{8fg^5(m-n+2)}$$

```
[Out] 1/8*c^2*hypergeom([3, -2-m+n],[-1-m+n],1/2-1/2*sin(f*x+e))*(a+a*sin(f*x+e))
^m*(c-c*sin(f*x+e))^(-2+n)/f/g^5/(2+m-n)/((g*cos(f*x+e))^(2*m))
```

Rubi [A] time = 0.24, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2853, 12, 2667, 68}

$$\frac{c^2(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}(g\cos(e+fx))^{-2m}\,{}_2F_1\left(3,-m+n-2;-m+n-1;\frac{1}{2}(1-\sin(e+fx\right.}{8fg^5(m-n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-5 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])
^n,x]
```

```
[Out] (c^2*Hypergeometric2F1[3, -2 - m + n, -1 - m + n, (1 - Sin[e + f*x])/2]*(a
+ a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 + n))/(8*f*g^5*(2 + m - n)*(g*
Cos[e + f*x])^(2*m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{-5-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c$$

$$= \frac{\Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c}{}$$

$$= -\frac{\Big(c^5(g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c}{}$$

$$= \frac{c^2(g\cos(e+fx))^{-2m}\,{}_2F_1\Big(3,-2-m+n;-1-}{}$$

Mathematica [A] time = 41.14, size = 136, normalized size = 1.55

$$\frac{\cot^4\left(\frac{1}{4}(2e+2fx-\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-2m}\sec^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{n-m}\,{}_2F}{32fg^5(m-n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-5 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
 f*x])^n,x]
```

```
[Out] (Cot[(2*e - Pi + 2*f*x)/4]^4*Hypergeometric2F1[-4 - m + n, -2 - m + n, -1 -
 m + n, -Tan[(2*e - Pi + 2*f*x)/4]^2]*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(-m + n
)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/(32*f*g^5*(2 + m - n)*(g
*Cos[e + f*x])^(2*m))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g\cos\left(fx+e\right)\right)^{-2m-5}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-2*m - 5)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e
) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-2m-5}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 5)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

maple [F] time = 53.34, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-5-2m}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-2m-5}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 5)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^n}{\left(g\,\cos\left(e+f\,x\right)\right)^{2\,m+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n)/(g*cos(e + f*x))^(2*m +
 5),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n)/(g*cos(e + f*x))^(2*m +
 5), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-5-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,
x)
```

```
[Out] Timed out
```

3.181 $\int (g\cos(e+fx))^{-1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\,dx$

Optimal. Leaf size=51

$$\frac{\tanh^{-1}(\sin(e+fx))(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m(g\cos(e+fx))^{-2m}}{fg}$$

```
[Out] arctanh(sin(f*x+e))*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m/f/g/((g*cos(f*x+e
))^(2*m))
```

Rubi [A] time = 0.17, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2847, 12, 3770}

$$\frac{\tanh^{-1}(\sin(e+fx))(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m(g\cos(e+fx))^{-2m}}{fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])
^m,x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^m)/(f*g*
(g*Cos[e + f*x])^(2*m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2847

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
&& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p + 1, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-\right.$$
$$= \frac{\left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-\right.}{g}$$
$$= \frac{\tanh^{-1}(\sin(e+fx))(g\cos(e+fx))^{-2m}(a+a}{fg}$$

Mathematica [A] time = 1.00, size = 94, normalized size = 1.84

$$\frac{\sqrt{-\tan^2(e+fx)}\csc(e+fx)\cos^{2(m+1)}(e+fx)\sin^{-1}(\sec(e+fx))(g\cos(e+fx))^{-2m-1}\exp(m(\log(a(\sin(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
 f*x])^m,x]
```

```
[Out] (E^(m*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e +
 f*x]]))*ArcSin[Sec[e + f*x]]*Cos[e + f*x]^(2*(1 + m))*(g*Cos[e + f*x])^(-1
 - 2*m)*Csc[e + f*x]*Sqrt[-Tan[e + f*x]^2])/f
```

fricas [A] time = 0.52, size = 48, normalized size = 0.94

$$\frac{\left(\frac{ac}{g^2}\right)^m\log\left(\sin\left(fx+e\right)+1\right)-\left(\frac{ac}{g^2}\right)^m\log\left(-\sin\left(fx+e\right)+1\right)}{2\,fg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x,
algorithm="fricas")
```

```
[Out] 1/2*((a*c/g^2)^m*log(sin(f*x + e) + 1) - (a*c/g^2)^m*log(-sin(f*x + e) + 1)
)/(f*g)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-2\,m-1}\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x,
algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^m, x)
```

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-2m}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x)
```

```
[Out] int((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{-2\,m-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x,
algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{m}}{\left(g\,\cos\left(e+f\,x\right)\right)^{2\,m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^m)/(g*cos(e + f*x))^(2*m +
 1),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^m)/(g*cos(e + f*x))^(2*m +
 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**m,
x)
```

```
[Out] Timed out
```

3.182 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3+n}\,dx$

Optimal. Leaf size=134

$$\frac{c^3 2^{-\frac{m}{2}+\frac{n}{2}+3}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{1}{2}(m-n-4),\frac{m-}{2}\right.}{fg(m-n)}$$

```
[Out] 2^(3-1/2*m+1/2*n)*c^3*(g*cos(f*x+e))^(-n-m)*hypergeom([1/2*m-1/2*n, -2+1/2*
m-1/2*n],[1+1/2*m-1/2*n],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2*m-1/2*n)*(
a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/f/g/(m-n)
```

Rubi [A] time = 0.38, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2853, 2689, 70, 69}

$$\frac{c^3 2^{-\frac{m}{2}+\frac{n}{2}+3}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{1}{2}(m-n-4),\frac{m-}{2}\right.}{fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(3 + n),x]
```

```
[Out] (2^(3 - m/2 + n/2)*c^3*(g*Cos[e + f*x])^(-m - n)*Hypergeometric2F1[(-4 + m
- n)/2, (m - n)/2, (2 + m - n)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^
((m - n)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(f*g*(m - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3+n}\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c$$

$$= \frac{\Big(c^2(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))}{}$$

$$= \frac{\Big(2^{2-\frac{m}{2}+\frac{n}{2}}c^4(g\cos(e+fx))^{-m-n}(a+a\sin(}{}$$

$$= \frac{2^{3-\frac{m}{2}+\frac{n}{2}}c^3(g\cos(e+fx))^{-m-n}\,{}_2F_1\Big(\frac{1}{2}(-4+}{}$$

Mathematica [F] time = 149.71, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3+n}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(3 + n),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(3 + n), x]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n)
,x, algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
 e) + c)^(n + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n)
,x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 3), x)
```

maple [F] time = 1.77, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-1-m-n}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{3+n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n)
,x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{n+3}}{\left(g\,\cos\left(e+f\,x\right)\right)^{m+n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n + 3))/(g*cos(e + f*x))^
(m + n + 1),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n + 3))/(g*cos(e + f*x))^
(m + n + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(3
+n),x)
```

```
[Out] Timed out
```

3.183 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2+n}\,dx$

Optimal. Leaf size=134

$$\frac{c^2 2^{-\frac{m}{2}+\frac{n}{2}+2}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{1}{2}(m-n-2),\frac{m-}{2}\right.}{fg(m-n)}$$

[Out] 2^(2-1/2*m+1/2*n)*c^2*(g*cos(f*x+e))^(-n-m)*hypergeom([1/2*m-1/2*n, -1+1/2*m-1/2*n],[1+1/2*m-1/2*n],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2*m-1/2*n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/f/g/(m-n)

Rubi [A] time = 0.37, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2853, 2689, 70, 69}

$$\frac{c^2 2^{-\frac{m}{2}+\frac{n}{2}+2}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{1}{2}(m-n-2),\frac{m-}{2}\right.}{fg(m-n)}$$

Antiderivative was successfully verified.

[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 + n),x]

[Out] (2^(2 - m/2 + n/2)*c^2*(g*Cos[e + f*x])^(-m - n)*Hypergeometric2F1[(-2 + m - n)/2, (m - n)/2, (2 + m - n)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^((m - n)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(f*g*(m - n))

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2+n}\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c$$

$$= \frac{\Big(c^2(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))}{}$$

$$= \frac{\Big(2^{1-\frac{m}{2}+\frac{n}{2}}c^3(g\cos(e+fx))^{-m-n}(a+a\sin(}{}$$

$$= \frac{2^{2-\frac{m}{2}+\frac{n}{2}}c^2(g\cos(e+fx))^{-m-n}\,{}_2F_1\Big(\frac{1}{2}(-2+}{}$$

Mathematica [F] time = 142.84, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2+n}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(2 + n),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(2 + n), x]
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n)
,x, algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
 e) + c)^(n + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n+2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n)
,x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 2), x)
```

maple [F] time = 1.47, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-1-m-n}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{2+n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n+2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n)
,x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\sin\left(e+f\,x\right)\right)^m\,\left(c-c\sin\left(e+f\,x\right)\right)^{n+2}}{\left(g\cos\left(e+f\,x\right)\right)^{m+n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n + 2))/(g*cos(e + f*x))^
(m + n + 1),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n + 2))/(g*cos(e + f*x))^
(m + n + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(2
+n),x)
```

```
[Out] Timed out
```

3.184 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1+n}\,dx$

Optimal. Leaf size=131

$$\frac{c2^{-\frac{m}{2}+\frac{n}{2}+1}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{m-n}{2},\frac{m-n}{2};\frac{1}{2}(m-\right.}{fg(m-n)}$$

```
[Out] 2^(1-1/2*m+1/2*n)*c*(g*cos(f*x+e))^(-n-m)*hypergeom([1/2*m-1/2*n, 1/2*m-1/2
*n],[1+1/2*m-1/2*n],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2*m-1/2*n)*(a+a*s
in(f*x+e))^m*(c-c*sin(f*x+e))^n/f/g/(m-n)
```

Rubi [A] time = 0.37, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2853, 2689, 70, 69}

$$\frac{c2^{-\frac{m}{2}+\frac{n}{2}+1}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{m-n}{2},\frac{m-n}{2};\frac{1}{2}(m-\right.}{fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(1 + n),x]
```

```
[Out] (2^(1 - m/2 + n/2)*c*(g*Cos[e + f*x])^(-m - n)*Hypergeometric2F1[(m - n)/2,
 (m - n)/2, (2 + m - n)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^((m - n
)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(f*g*(m - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1+n}\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c$$

$$= \frac{\Big(c^2(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))}{}$$

$$= \frac{\Big(2^{-\frac{m}{2}+\frac{n}{2}}c^2(g\cos(e+fx))^{-m-n}(a+a\sin(e}{}$$

$$= \frac{2^{1-\frac{m}{2}+\frac{n}{2}}c(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{m-n}{2},\frac{m-}{2}\right.}{}$$

Mathematica [C] time = 27.10, size = 207, normalized size = 1.58

$$\frac{ic(\sin(e+fx)-1)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}\left({}_2F_1\left(1,-m+n+1;-m+n+2;\right.\right.}{fg(m-n-1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(1 + n),x]
```

```
[Out] (I*c*(g*Cos[e + f*x])^(-m - n)*(Hypergeometric2F1[1, 1 - m + n, 2 - m + n,
((-I)*(-1 + Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])] - Hypergeometric2F1[
1, 1 - m + n, 2 - m + n, (I*(-1 + Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])
])*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/(f*
g*(-1 + m - n)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(g \cos\left(fx+e\right)\right)^{-m-n-1} \left(a \sin\left(fx+e\right)+a\right)^{m} \left(-c \sin\left(fx+e\right)+c\right)^{n+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n)
,x, algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
 e) + c)^(n + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g \cos\left(fx+e\right)\right)^{-m-n-1} \left(a \sin\left(fx+e\right)+a\right)^{m} \left(-c \sin\left(fx+e\right)+c\right)^{n+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n)
,x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 1), x)
```

maple [F] time = 95.90, size = 0, normalized size = 0.00

$$\int \left(g \cos\left(fx+e\right)\right)^{-1-m-n} \left(a+a \sin\left(fx+e\right)\right)^{m} \left(c-c \sin\left(fx+e\right)\right)^{1+n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g \cos\left(fx+e\right)\right)^{-m-n-1} \left(a \sin\left(fx+e\right)+a\right)^{m} \left(-c \sin\left(fx+e\right)+c\right)^{n+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n)
,x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{n+1}}{\left(g\,\cos\left(e+f\,x\right)\right)^{m+n+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n + 1))/(g*cos(e + f*x))^
(m + n + 1),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n + 1))/(g*cos(e + f*x))^
(m + n + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1
+n),x)
```

```
[Out] Timed out
```

3.185 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=55

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{fg(m-n)}$$

```
[Out] (g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/f/g/(m-n)
```

Rubi [A] time = 0.17, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2848}

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^n,x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(
f*g*(m - n))
```

Rule 2848

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(m - n)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + p + 1, 0] && NeQ[m, n]
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx=\frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c}{fg(m-n)}$$

Mathematica [A] time = 0.79, size = 55, normalized size = 1.00

$$\frac{(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^n,x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)
/(f*g*(m - n))
```

fricas [A] time = 0.50, size = 83, normalized size = 1.51

$$\frac{\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)e^{\left(2\,n\log\left(g\cos\left(fx+e\right)\right)-n\log\left(a\sin\left(fx+e\right)+a\right)+n\log\left(\frac{ac}{g^{2}}\right)\right)}}{fm-fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="fricas")
```

```
[Out] (g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*cos(f*x + e)*e^(2*n*lo
g(g*cos(f*x + e)) - n*log(a*sin(f*x + e) + a) + n*log(a*c/g^2))/(f*m - f*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^n, x)
```

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{-1-m-n}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

maxima [B] time = 0.47, size = 145, normalized size = 2.64

$$\frac{a^{m}c^{n}g^{-m-n-1}e^{\left(m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-n\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)+2\,n\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)-m\log\left(-\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-n\log\left(-\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)\right)}}{f(m-n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="maxima")
```

```
[Out] a^m*c^n*g^(-m - n - 1)*e^(m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - n*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) + 1) + 2*n*log(sin(f*x + e)/(cos(f*x + e)
 + 1) - 1) - m*log(-sin(f*x + e)/(cos(f*x + e) + 1) + 1) - n*log(-sin(f*x +
 e)/(cos(f*x + e) + 1) + 1))/(f*(m - n))
```

mupad [B] time = 9.07, size = 53, normalized size = 0.96

$$\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)^n}{f\,g\,\left(g\,\cos\left(e+f\,x\right)\right)^{m+n}\,\left(m-n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n)/(g*cos(e + f*x))^(m + n
 + 1),x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^n)/(f*g*(g*cos(e + f*x))^
(m + n)*(m - n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,
x)
```

```
[Out] Timed out
```

3.186 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1+n}\,dx$

Optimal. Leaf size=125

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-m-n}}{fg(m-n+2)}+\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+}{cfg(m-n)(m-n+2)}$$

```
[Out] (g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n)/f/g/(2+m-n
)+(g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/c/f/g/(m-n)/(
2+m-n)
```

Rubi [A] time = 0.41, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2849, 2848}

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-m-n}}{fg(m-n+2)}+\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+}{cfg(m-n)(m-n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(-1 + n),x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1
+ n))/(f*g*(2 + m - n)) + ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m
*(c - c*Sin[e + f*x])^n)/(c*f*g*(m - n)*(2 + m - n))
```

Rule 2848

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(m - n)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + p + 1, 0] && NeQ[m, n]
```

Rule 2849

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2
- b^2, 0] && ILtQ[Simplify[m + n + p + 1], 0] && NeQ[2*m + p + 1, 0] && (Su
```

```
mSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1+n}\,dx = \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^{\cdots}}{fg(2+m-n)\cdots}$$
$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^{\cdots}}{fg(2+m-n)\cdots}$$

Mathematica [A] time = 27.82, size = 132, normalized size = 1.06

$$-\frac{2^{n-1}\cos^{2(n-1)}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^{n-1}(\sin(e+fx)-m+n-1)\left(\cos\left(\frac{1}{2}(\cdots\right.\right.}{fg(m-n)(m-n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(-1 + n),x]
```

```
[Out] -((2^(-1 + n)*(g*Cos[e + f*x])^(-m - n)*Cos[(2*e + Pi + 2*f*x)/4]^(2*(-1 +
n))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2 - 2*n)*(a*(1 + Sin[e + f*x]))^
m*(-1 - m + n + Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 + n))/(f*g*(m - n)*(
2 + m - n)))
```

fricas [A] time = 0.49, size = 126, normalized size = 1.01

$$\frac{\left((m-n+1)\cos\left(fx+e\right)-\cos\left(fx+e\right)\sin\left(fx+e\right)\right)\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}e^{\left(2\,(n-1)\log\left(g\cdots\right.\right.}}{fm^2+fn^2+2\,fm-2\left(fm+f\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n
),x, algorithm="fricas")
```

```
[Out] ((m - n + 1)*cos(f*x + e) - cos(f*x + e)*sin(f*x + e))*(g*cos(f*x + e))^(-m
 - n - 1)*(a*sin(f*x + e) + a)^m*e^(2*(n - 1)*log(g*cos(f*x + e)) - (n - 1)
*log(a*sin(f*x + e) + a) + (n - 1)*log(a*c/g^2))/(f*m^2 + f*n^2 + 2*f*m - 2
*(f*m + f)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n - 1), x)
```

maple [F] time = 1.33, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-m-n}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-1+n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n
),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 10.16, size = 128, normalized size = 1.02

$$\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)^n\,\left(2\,\cos\left(e+f\,x\right)-\sin\left(2\,e+2\,f\,x\right)+2\,m\,\cos\left(e+f\,x\right)-2\,n\,\mathrm{co}\right.}{c\,f\,g\,\left(g\,\cos\left(e+f\,x\right)\right)^{m+n}\,\left(2\,\cos\left(e+f\,x\right)-\sin\left(2\,e+2\,f\,x\right)\right)\,\left(m^2-2\,m\,n+2\,m+n^2-2\,n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n - 1))/(g*cos(e + f*x))^
(m + n + 1),x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^n*(2*cos(e + f*x) - sin(2
*e + 2*f*x) + 2*m*cos(e + f*x) - 2*n*cos(e + f*x)))/(c*f*g*(g*cos(e + f*x))
```

```
^(m + n)*(2*cos(e + f*x) - sin(2*e + 2*f*x))*(2*m - 2*n - 2*m*n + m^2 + n^2
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-
1+n),x)
```

```
[Out] Timed out
```

3.187 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2+n}\,dx$

Optimal. Leaf size=204

$$\frac{2(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{c^2fg(m-n)(m-n+2)(m-n+4)}+\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}(g\cos(e-$$

```
[Out] (g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n)/f/g/(4+m-n
)+2*(g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n)/c/f/g/
(2+m-n)/(4+m-n)+2*(g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))
^n/c^2/f/g/(m-n)/(2+m-n)/(4+m-n)
```

Rubi [A] time = 0.67, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2849, 2848}

$$\frac{2(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{c^2fg(m-n)(m-n+2)(m-n+4)}+\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}(g\cos(e-$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(-2 + n),x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2
+ n))/(f*g*(4 + m - n)) + (2*(g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])
^m*(c - c*Sin[e + f*x])^(-1 + n))/(c*f*g*(2 + m - n)*(4 + m - n)) + (2*(g*C
os[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(c^2*f
*g*(m - n)*(2 + m - n)*(4 + m - n))
```

Rule 2848

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(m - n)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + p + 1, 0] && NeQ[m, n]
```

Rule 2849

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
```

```
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2
- b^2, 0] && ILtQ[Simplify[m + n + p + 1], 0] && NeQ[2*m + p + 1, 0] && (Su
mSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2+n}\,dx = \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^{}}{fg(4+m-n)}$$

$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^{}}{fg(4+m-n)}$$

$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^{}}{fg(4+m-n)}$$

Mathematica [A] time = 32.56, size = 183, normalized size = 0.90

$$\frac{2^{n-2}\cos(e+fx)\sin^{2n-4}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(\right.\right.}{f(m-n)(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(-2 + n),x]
```

```
[Out] (2^(-2 + n)*Cos[e + f*x]*(g*Cos[e + f*x])^(-1 - m - n)*Sin[(-e + Pi/2 - f*x
)/2]^(-4 + 2*n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 + n)*(3 + 4
*m + m^2 - 4*n - 2*m*n + n^2 + Cos[2*(-e + Pi/2 - f*x)] - 2*(2 + m - n)*Sin
[e + f*x]))/(f*(m - n)*(2 + m - n)*(4 + m - n)*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^(2*(-2 + n)))
```

fricas [A] time = 0.50, size = 183, normalized size = 0.90

$$-\frac{\left(2\cos\left(fx+e\right)^3+2\left(m-n+2\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(m^2-2\left(m+2\right)n+n^2+4m+4\right)\cos\left(fx+e\right)\right)\left(\right.}{fm^3-fn^3+6fm^2+3\left(fm+2f\right)n^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n
),x, algorithm="fricas")
```

```
[Out] -(2*cos(f*x + e)^3 + 2*(m - n + 2)*cos(f*x + e)*sin(f*x + e) - (m^2 - 2*(m
+ 2)*n + n^2 + 4*m + 4)*cos(f*x + e))*(g*cos(f*x + e))^(-m - n - 1)*(a*sin(
f*x + e) + a)^m*e^(2*(n - 2)*log(g*cos(f*x + e)) - (n - 2)*log(a*sin(f*x +
e) + a) + (n - 2)*log(a*c/g^2))/(f*m^3 - f*n^3 + 6*f*m^2 + 3*(f*m + 2*f)*n^
2 + 8*f*m - (3*f*m^2 + 12*f*m + 8*f)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n - 2), x)
```

maple [F] time = 1.84, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-m-n}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-2+n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n
),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 15.65, size = 887, normalized size = 4.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n - 2))/(g*cos(e + f*x))^
(m + n + 1),x)
```

```
[Out] -exp(- e*3i - f*x*3i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2))^(n - 2)*((a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2))^m/(4*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(m
+ n + 1)*(8*m - 8*n - 12*m*n + 3*m*n^2 - 3*m^2*n + 6*m^2 + m^3 + 6*n^2 - n^
3)) + (exp(e*6i + f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
 f*x*1i)*1i)/2))^m)/(4*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)
)^(m + n + 1)*(8*m - 8*n - 12*m*n + 3*m*n^2 - 3*m^2*n + 6*m^2 + m^3 + 6*n^2
 - n^3)) - (exp(e*2i + f*x*2i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e
*1i + f*x*1i)*1i)/2))^m*(8*m - 8*n - 4*m*n + 2*m^2 + 2*n^2 + 5))/(4*f*(g*(e
xp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(m + n + 1)*(8*m - 8*n - 12*
m*n + 3*m*n^2 - 3*m^2*n + 6*m^2 + m^3 + 6*n^2 - n^3)) - (exp(e*4i + f*x*4i)
*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(8*m -
 8*n - 4*m*n + 2*m^2 + 2*n^2 + 5))/(4*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*
1i + f*x*1i)/2))^(m + n + 1)*(8*m - 8*n - 12*m*n + 3*m*n^2 - 3*m^2*n + 6*m^
2 + m^3 + 6*n^2 - n^3)) + (exp(e*1i + f*x*1i)*(a + a*((exp(- e*1i - f*x*1i)
*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*2i - n*2i + 4i))/(4*f*(g*(exp(- e
*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(m + n + 1)*(8*m - 8*n - 12*m*n +
3*m*n^2 - 3*m^2*n + 6*m^2 + m^3 + 6*n^2 - n^3)) - (exp(e*5i + f*x*5i)*(a +
a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*2i - n*2i
 + 4i))/(4*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(m + n + 1
)*(8*m - 8*n - 12*m*n + 3*m*n^2 - 3*m^2*n + 6*m^2 + m^3 + 6*n^2 - n^3)))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-
2+n),x)
```

```
[Out] Timed out
```

3.188 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3+n}\,dx$

Optimal. Leaf size=290

$$\frac{6(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{c^3fg(m-n)(m-n+2)(m-n+4)(m-n+6)}+\frac{6(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e}{c^2fg(m-n+2)(m-n+4)(m-n+6}$$

```
[Out] (g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n)/f/g/(6+m-n
)+3*(g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n)/c/f/g/
(4+m-n)/(6+m-n)+6*(g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))
^(-1+n)/c^2/f/g/(2+m-n)/(4+m-n)/(6+m-n)+6*(g*cos(f*x+e))^(-n-m)*(a+a*sin(f*
x+e))^m*(c-c*sin(f*x+e))^n/c^3/f/g/(m-n)/(2+m-n)/(4+m-n)/(6+m-n)
```

Rubi [A] time = 0.94, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2849, 2848}

$$\frac{6(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-m-n}}{c^2fg(m-n+2)(m-n+4)(m-n+6)}+\frac{6(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e}{c^3fg(m-n)(m-n+2)(m-n+4)(m-n}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(-3 + n),x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3
+ n))/(f*g*(6 + m - n)) + (3*(g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])
^m*(c - c*Sin[e + f*x])^(-2 + n))/(c*f*g*(4 + m - n)*(6 + m - n)) + (6*(g*C
os[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))
/(c^2*f*g*(2 + m - n)*(4 + m - n)*(6 + m - n)) + (6*(g*Cos[e + f*x])^(-m -
n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(c^3*f*g*(m - n)*(2 + m -
 n)*(4 + m - n)*(6 + m - n))
```

Rule 2848

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(m - n)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + p + 1, 0] && NeQ[m, n]
```

Rule 2849

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
```

```
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2
- b^2, 0] && ILtQ[Simplify[m + n + p + 1], 0] && NeQ[2*m + p + 1, 0] && (Su
mSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3+n}\,dx = \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^{}}{fg(6+m-n)}$$

$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^{}}{fg(6+m-n)}$$

$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^{}}{fg(6+m-n)}$$

$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^{}}{fg(6+m-n)}$$

Mathematica [B] time = 38.70, size = 2681, normalized size = 9.24

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(-3 + n),x]
```

```
[Out] -((2^(-4 - m + 2*n)*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Cos[e + f*x]*(g*Cos[e +
f*x])^(-1 - m - n)*Csc[(-e + Pi/2 - f*x)/2]^6*(Cos[(-e + Pi/2 - f*x)/8]*(-S
in[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8]))^(2*n)*(Cos[(-e + P
i/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8] - S
in[(5*(-e + Pi/2 - f*x))/8] + Sin[(7*(-e + Pi/2 - f*x))/8]))^(-m - n)*(a +
a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 + n)*(-30 - 46*m - 18*m^2 - 2*m^
3 + 46*n + 36*m*n + 6*m^2*n - 18*n^2 - 6*m*n^2 + 2*n^3 - 6*(3 + m - n)*Cos[
2*(-e + Pi/2 - f*x)] + 3*Cos[3*(-e + Pi/2 - f*x)] + 3*(15 + 2*m^2 - 4*m*(-3
 + n) - 12*n + 2*n^2)*Sin[e + f*x])*(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[P
i/4 + (e - Pi/2 + f*x)/2])^(-7 + 2*n))/(f*(m - n)*(2 + m - n)*(4 + m - n)*(
6 + m - n)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-3 + n))*((2^(-4 - m +
 2*n)*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Csc[(-e + Pi/2 - f*x)/2]^6*(Cos[(-e +
Pi/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8]))^
```

(2*n)*(Cos[(-e + Pi/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + P
i/2 - f*x))/8] - Sin[(5*(-e + Pi/2 - f*x))/8] + Sin[(7*(-e + Pi/2 - f*x))/8
]))^(-m - n)*(-3*(15 + 2*m^2 - 4*m*(-3 + n) - 12*n + 2*n^2)*Cos[e + f*x] +
12*(3 + m - n)*Sin[2*(-e + Pi/2 - f*x)] - 9*Sin[3*(-e + Pi/2 - f*x)]))/((m
- n)*(2 + m - n)*(4 + m - n)*(6 + m - n)) - (2^(-4 - m + 2*n)*m*Cos[(-e + P
i/2 - f*x)/2]^(-1 + 2*m)*Csc[(-e + Pi/2 - f*x)/2]^5*(Cos[(-e + Pi/2 - f*x)/
8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8]))^(2*n)*(Cos[(
-e + Pi/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/
8] - Sin[(5*(-e + Pi/2 - f*x))/8] + Sin[(7*(-e + Pi/2 - f*x))/8]))^(-m - n)
*(-30 - 46*m - 18*m^2 - 2*m^3 + 46*n + 36*m*n + 6*m^2*n - 18*n^2 - 6*m*n^2
+ 2*n^3 - 6*(3 + m - n)*Cos[2*(-e + Pi/2 - f*x)] + 3*Cos[3*(-e + Pi/2 - f*x
)] + 3*(15 + 2*m^2 - 4*m*(-3 + n) - 12*n + 2*n^2)*Sin[e + f*x]))/((m - n)*(
2 + m - n)*(4 + m - n)*(6 + m - n)) - (3*2^(-4 - m + 2*n)*Cos[(-e + Pi/2 -
f*x)/2]^(1 + 2*m)*Csc[(-e + Pi/2 - f*x)/2]^7*(Cos[(-e + Pi/2 - f*x)/8]*(-Si
n[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8]))^(2*n)*(Cos[(-e + Pi
/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8] - Si
n[(5*(-e + Pi/2 - f*x))/8] + Sin[(7*(-e + Pi/2 - f*x))/8]))^(-m - n)*(-30 -
 46*m - 18*m^2 - 2*m^3 + 46*n + 36*m*n + 6*m^2*n - 18*n^2 - 6*m*n^2 + 2*n^3
 - 6*(3 + m - n)*Cos[2*(-e + Pi/2 - f*x)] + 3*Cos[3*(-e + Pi/2 - f*x)] + 3*
(15 + 2*m^2 - 4*m*(-3 + n) - 12*n + 2*n^2)*Sin[e + f*x]))/((m - n)*(2 + m -
 n)*(4 + m - n)*(6 + m - n)) + (2^(-3 - m + 2*n)*n*Cos[(-e + Pi/2 - f*x)/2]
^(2*m)*Csc[(-e + Pi/2 - f*x)/2]^6*(Cos[(-e + Pi/2 - f*x)/8]*(-Sin[(-e + Pi/
2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8]))^(-1 + 2*n)*(Cos[(-e + Pi/2 - f
x)/8](-1/8*Cos[(-e + Pi/2 - f*x)/8] + (3*Cos[(3*(-e + Pi/2 - f*x))/8])/8)
 - (Sin[(-e + Pi/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2
 - f*x))/8]))/8)*(Cos[(-e + Pi/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin
[(3*(-e + Pi/2 - f*x))/8] - Sin[(5*(-e + Pi/2 - f*x))/8] + Sin[(7*(-e + Pi/
2 - f*x))/8]))^(-m - n)*(-30 - 46*m - 18*m^2 - 2*m^3 + 46*n + 36*m*n + 6*m^
2*n - 18*n^2 - 6*m*n^2 + 2*n^3 - 6*(3 + m - n)*Cos[2*(-e + Pi/2 - f*x)] + 3
Cos[3(-e + Pi/2 - f*x)] + 3*(15 + 2*m^2 - 4*m*(-3 + n) - 12*n + 2*n^2)*Si
n[e + f*x]))/((m - n)*(2 + m - n)*(4 + m - n)*(6 + m - n)) + (2^(-4 - m + 2
n)(-m - n)*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Csc[(-e + Pi/2 - f*x)/2]^6*(Cos
[(-e + Pi/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x)
)/8]))^(2*n)*(Cos[(-e + Pi/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*
(-e + Pi/2 - f*x))/8] - Sin[(5*(-e + Pi/2 - f*x))/8] + Sin[(7*(-e + Pi/2 -
f*x))/8]))^(-1 - m - n)*(Cos[(-e + Pi/2 - f*x)/8]*(-1/8*Cos[(-e + Pi/2 - f*
x)/8] + (3*Cos[(3*(-e + Pi/2 - f*x))/8])/8 - (5*Cos[(5*(-e + Pi/2 - f*x))/8
])/8 + (7*Cos[(7*(-e + Pi/2 - f*x))/8])/8) - (Sin[(-e + Pi/2 - f*x)/8]*(-Si
n[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8] - Sin[(5*(-e + Pi/2 -
 f*x))/8] + Sin[(7*(-e + Pi/2 - f*x))/8]))/8)*(-30 - 46*m - 18*m^2 - 2*m^3
+ 46*n + 36*m*n + 6*m^2*n - 18*n^2 - 6*m*n^2 + 2*n^3 - 6*(3 + m - n)*Cos[2*
(-e + Pi/2 - f*x)] + 3*Cos[3*(-e + Pi/2 - f*x)] + 3*(15 + 2*m^2 - 4*m*(-3 +
 n) - 12*n + 2*n^2)*Sin[e + f*x]))/((m - n)*(2 + m - n)*(4 + m - n)*(6 + m
- n)))*(Cos[Pi/4 + (e - Pi/2 + f*x)/2] + Sin[Pi/4 + (e - Pi/2 + f*x)/2]))

fricas [A] time = 0.53, size = 264, normalized size = 0.91

$$-\frac{\left(6\,(m-n+3)\cos\left(fx+e\right)^{3}-\left(m^{3}+3\,(m+3)n^{2}-n^{3}+9\,m^{2}-\left(3\,m^{2}+18\,m+26\right)n+26\,m+24\right)\cos\left(fx+\right.}{fm^{4}+fn^{4}+12\,fm^{3}-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n
),x, algorithm="fricas")
```

```
[Out] -(6*(m - n + 3)*cos(f*x + e)^3 - (m^3 + 3*(m + 3)*n^2 - n^3 + 9*m^2 - (3*m^
2 + 18*m + 26)*n + 26*m + 24)*cos(f*x + e) - 3*(2*cos(f*x + e)^3 - (m^2 - 2
*(m + 3)*n + n^2 + 6*m + 8)*cos(f*x + e))*sin(f*x + e))*(g*cos(f*x + e))^(-
m - n - 1)*(a*sin(f*x + e) + a)^m*e^(2*(n - 3)*log(g*cos(f*x + e)) - (n - 3
)*log(a*sin(f*x + e) + a) + (n - 3)*log(a*c/g^2))/(f*m^4 + f*n^4 + 12*f*m^3
 - 4*(f*m + 3*f)*n^3 + 44*f*m^2 + 2*(3*f*m^2 + 18*f*m + 22*f)*n^2 + 48*f*m
- 4*(f*m^3 + 9*f*m^2 + 22*f*m + 12*f)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n - 3), x)
```

maple [F] time = 2.98, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-m-n}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-3+n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n
),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 17.65, size = 1623, normalized size = 5.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n - 3))/(g*cos(e + f*x))^
(m + n + 1),x)
```

```
[Out] -exp(- e*4i - f*x*4i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2))^(n - 3)*((3*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^m)/(8*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))
^(m + n + 1)*(m*48i - n*48i - m*n*88i + m*n^2*36i - m^2*n*36i - m*n^3*4i -
m^3*n*4i + m^2*44i + m^3*12i + m^4*1i + n^2*44i - n^3*12i + n^4*1i + m^2*n^
2*6i)) - (3*exp(e*8i + f*x*8i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e
*1i + f*x*1i)*1i)/2))^m)/(8*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(m + n + 1)*(m*48i - n*48i - m*n*88i + m*n^2*36i - m^2*n*36i - m*n^3
*4i - m^3*n*4i + m^2*44i + m^3*12i + m^4*1i + n^2*44i - n^3*12i + n^4*1i +
m^2*n^2*6i)) - (exp(e*2i + f*x*2i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (e
xp(e*1i + f*x*1i)*1i)/2))^m*(36*m - 36*n - 12*m*n + 6*m^2 + 6*n^2 + 42))/(8
*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(m + n + 1)*(m*48i -
 n*48i - m*n*88i + m*n^2*36i - m^2*n*36i - m*n^3*4i - m^3*n*4i + m^2*44i +
m^3*12i + m^4*1i + n^2*44i - n^3*12i + n^4*1i + m^2*n^2*6i)) + (exp(e*6i +
f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m
*(36*m - 36*n - 12*m*n + 6*m^2 + 6*n^2 + 42))/(8*f*(g*(exp(- e*1i - f*x*1i)
/2 + exp(e*1i + f*x*1i)/2))^(m + n + 1)*(m*48i - n*48i - m*n*88i + m*n^2*36
i - m^2*n*36i - m*n^3*4i - m^3*n*4i + m^2*44i + m^3*12i + m^4*1i + n^2*44i
- n^3*12i + n^4*1i + m^2*n^2*6i)) + (exp(e*1i + f*x*1i)*(a + a*((exp(- e*1i
 - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*6i - n*6i + 18i))/(8*f*
(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(m + n + 1)*(m*48i - n*
48i - m*n*88i + m*n^2*36i - m^2*n*36i - m*n^3*4i - m^3*n*4i + m^2*44i + m^3
*12i + m^4*1i + n^2*44i - n^3*12i + n^4*1i + m^2*n^2*6i)) + (exp(e*7i + f*x
*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m
*6i - n*6i + 18i))/(8*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))
^(m + n + 1)*(m*48i - n*48i - m*n*88i + m*n^2*36i - m^2*n*36i - m*n^3*4i -
m^3*n*4i + m^2*44i + m^3*12i + m^4*1i + n^2*44i - n^3*12i + n^4*1i + m^2*n^
2*6i)) - (exp(e*3i + f*x*3i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1
i + f*x*1i)*1i)/2))^m*(m*86i - n*86i - m*n*72i + m*n^2*12i - m^2*n*12i + m^
2*36i + m^3*4i + n^2*36i - n^3*4i + 42i))/(8*f*(g*(exp(- e*1i - f*x*1i)/2 +
 exp(e*1i + f*x*1i)/2))^(m + n + 1)*(m*48i - n*48i - m*n*88i + m*n^2*36i -
```

```
m^2*n*36i - m*n^3*4i - m^3*n*4i + m^2*44i + m^3*12i + m^4*1i + n^2*44i - n^
3*12i + n^4*1i + m^2*n^2*6i)) - (exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f
*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*86i - n*86i - m*n*72i + m*n
^2*12i - m^2*n*12i + m^2*36i + m^3*4i + n^2*36i - n^3*4i + 42i))/(8*f*(g*(e
xp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(m + n + 1)*(m*48i - n*48i -
 m*n*88i + m*n^2*36i - m^2*n*36i - m*n^3*4i - m^3*n*4i + m^2*44i + m^3*12i
+ m^4*1i + n^2*44i - n^3*12i + n^4*1i + m^2*n^2*6i)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-
3+n),x)
```

```
[Out] Timed out
```

3.189 $\int (g\sec(e+fx))^p(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=138

$$\frac{c2^{n-\frac{p}{2}+\frac{1}{2}}\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\sec(e+fx))^p(1-\sin(e+fx))^{\frac{1}{2}(-2n+p+1)}\,{}_2F_1\left(\frac{1}{2}(2\right.}{f(2m-p+1)}$$

[Out] 2^(1/2+n-1/2*p)*c*cos(f*x+e)*hypergeom([1/2+m-1/2*p, 1/2-n+1/2*p],[3/2+m-1/2*p],1/2+1/2*sin(f*x+e))*(g*sec(f*x+e))^p*(1-sin(f*x+e))^(1/2-n+1/2*p)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n)/f/(1+2*m-p)

Rubi [A] time = 0.46, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2926, 2853, 2689, 70, 69}

$$\frac{c2^{n-\frac{p}{2}+\frac{1}{2}}\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\sec(e+fx))^p(1-\sin(e+fx))^{\frac{1}{2}(-2n+p+1)}\,{}_2F_1\left(\frac{1}{2}(2\right.}{f(2m-p+1)}$$

Antiderivative was successfully verified.

[In] Int[(g*Sec[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]

[Out] (2^(1/2 + n - p/2)*c*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m - p)/2, (1 - 2*n + p)/2, (3 + 2*m - p)/2, (1 + Sin[e + f*x])/2]*(g*Sec[e + f*x])^p*(1 - Sin[e + f*x])^((1 - 2*n + p)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(f*(1 + 2*m - p))

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2926

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]*(g*Sec[e + f*x])^FracPart[p
], Int[((a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(g*Cos[e + f*x])^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int (g\sec(e+fx))^p(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \left((g\cos(e+fx))^p(g\sec(e+fx))^p\right)\int (g\cos(e+$$

$$= \left((g\cos(e+fx))^{-2m+p}(g\sec(e+fx))^p(a+a\sin($$

$$= \frac{\left(c^2\cos(e+fx)(g\sec(e+fx))^p(a+a\sin(e+fx\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}+n-\frac{p}{2}}c^2\cos(e+fx)(g\sec(e+fx))^p(a+a\sin\right.}{}$$

$$= \frac{2^{\frac{1}{2}+n-\frac{p}{2}}c\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m-p),\frac{1}{2}(1-2\right.}{}$$

Mathematica [A] time = 42.54, size = 139, normalized size = 1.01

$$\frac{2\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n(g\sec(e+fx))^p\sec^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m+n-p}\ {}_2F}{f(2n-p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,
x]
```

```
[Out] (2*Hypergeometric2F1[1 + m + n - p, 1/2 + n - p/2, 3/2 + n - p/2, -Tan[(2*e
 - Pi + 2*f*x)/4]^2]*(g*Sec[e + f*x])^p*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(m +
n - p)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n*Tan[(2*e - Pi + 2*f*
x)/4])/(f*(1 + 2*n - p))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g\sec\left(fx+e\right)\right)^p\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\sec\left(fx+e\right)\right)^p\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n
, x)
```

maple [F] time = 10.49, size = 0, normalized size = 0.00

$$\int\left(g\sec\left(fx+e\right)\right)^p\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g \sec \left(fx+e\right)\right)^{p} \left(a \sin \left(fx+e\right)+a\right)^{m} \left(-c \sin \left(fx+e\right)+c\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac {g}{\cos \left(e+f\,x\right)}\right)}^p\,{\left(a+a\,\sin \left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin \left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
```

```
[Out] int((g/cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.190 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=33

$$\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] 1/3*a*sin(d*x+c)^3/d+1/4*a*sin(d*x+c)^4/d
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 43}

$$\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+x)}{a^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^2(a+x)\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(ax^2+x^3\right)\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{a\sin^3(c+dx)}{3d} + \frac{a\sin^4(c+dx)}{4d}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{a\sin^4(c+dx)}{4d} + \frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
[Out] (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d)
```

fricas [A] time = 0.46, size = 50, normalized size = 1.52

$$\frac{3\,a\cos\left(dx+c\right)^4 - 6\,a\cos\left(dx+c\right)^2 - 4\left(a\cos\left(dx+c\right)^2 - a\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/12*(3*a*cos(d*x + c)^4 - 6*a*cos(d*x + c)^2 - 4*(a*cos(d*x + c)^2 - a)*si
n(d*x + c))/d
```

giac [A] time = 0.13, size = 28, normalized size = 0.85

$$\frac{3\,a\sin\left(dx+c\right)^4 + 4\,a\sin\left(dx+c\right)^3}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3)/d
```

maple [A] time = 0.09, size = 28, normalized size = 0.85

$$\frac{\frac{\left(\sin^4(dx+c)\right)a}{4}+\frac{\left(\sin^3(dx+c)\right)a}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/4*sin(d*x+c)^4*a+1/3*sin(d*x+c)^3*a)
```

maxima [A] time = 0.30, size = 28, normalized size = 0.85

$$\frac{3\,a\sin\left(dx+c\right)^4+4\,a\sin\left(dx+c\right)^3}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3)/d
```

mupad [B] time = 0.05, size = 24, normalized size = 0.73

$$\frac{a\sin\left(c+d\,x\right)^3\,\left(3\,\sin\left(c+d\,x\right)+4\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x)^3*(3*sin(c + d*x) + 4))/(12*d)
```

sympy [A] time = 1.31, size = 42, normalized size = 1.27

$$\begin{cases} \frac{a\sin^4\left(c+dx\right)}{4d}+\frac{a\sin^3\left(c+dx\right)}{3d} & \text{for}\: d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)\sin^2\left(c\right)\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)**4/(4*d) + a*sin(c + d*x)**3/(3*d), Ne(d, 0)), (x
*(a*sin(c) + a)*sin(c)**2*cos(c), True))
```

3.191 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=33

$$\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^2(c+dx)}{2d}$$

```
[Out] 1/2*a*sin(d*x+c)^2/d+1/3*a*sin(d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2833, 12, 43}

$$\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)}{a}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}(\int x(a+x)\,dx, x, a\sin(c+dx))}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(ax+x^2\right)\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{a\sin^2(c+dx)}{2d} + \frac{a\sin^3(c+dx)}{3d}$$

Mathematica [A] time = 0.09, size = 30, normalized size = 0.91

$$\frac{4a\sin^3(c+dx) - 3a\cos(2(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
[Out] (-3*a*Cos[2*(c + d*x)] + 4*a*Sin[c + d*x]^3)/(12*d)
```

fricas [A] time = 0.48, size = 39, normalized size = 1.18

$$-\frac{3\,a\cos\left(dx+c\right)^2 + 2\left(a\cos\left(dx+c\right)^2 - a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/6*(3*a*cos(d*x + c)^2 + 2*(a*cos(d*x + c)^2 - a)*sin(d*x + c))/d
```

giac [A] time = 0.15, size = 28, normalized size = 0.85

$$\frac{2\,a\sin\left(dx+c\right)^3 + 3\,a\sin\left(dx+c\right)^2}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2)/d
```

maple [A] time = 0.05, size = 28, normalized size = 0.85

$$\frac{\frac{\left(\sin^3(dx+c)\right)a}{3}+\frac{\left(\sin^2(dx+c)\right)a}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*sin(d*x+c)^3*a+1/2*sin(d*x+c)^2*a)
```

maxima [A] time = 0.31, size = 28, normalized size = 0.85

$$\frac{2\,a\sin\left(dx+c\right)^3+3\,a\sin\left(dx+c\right)^2}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2)/d
```

mupad [B] time = 0.04, size = 24, normalized size = 0.73

$$\frac{a\sin\left(c+d\,x\right)^2\,\left(2\,\sin\left(c+d\,x\right)+3\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x)^2*(2*sin(c + d*x) + 3))/(6*d)
```

sympy [A] time = 0.56, size = 41, normalized size = 1.24

$$\begin{cases}\frac{a\sin^3\left(c+dx\right)}{3d}+\frac{a\sin^2\left(c+dx\right)}{2d} & \text{for } d\neq 0\\ x\left(a\sin\left(c\right)+a\right)\sin\left(c\right)\cos\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)**2/(2*d), Ne(d, 0)), (x
*(a*sin(c) + a)*sin(c)*cos(c), True))
```

3.192 $\int \cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2707, 43}

$$\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sin(c+dx))\,dx&=\frac{\text{Subst}\left(\int \frac{a+x}{x}\,dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int \left(1+\frac{a}{x}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 26, normalized size = 1.08

$$\frac{a(\sin(c+dx)+\log(\tan(c+dx))+\log(\cos(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
[Out] (a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]] + Sin[c + d*x]))/d
```

fricas [A] time = 0.48, size = 24, normalized size = 1.00

$$\frac{a\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] (a*log(1/2*sin(d*x + c)) + a*sin(d*x + c))/d
```

giac [A] time = 0.13, size = 23, normalized size = 0.96

$$\frac{a\log\left(|\sin\left(dx+c\right)|\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] (a*log(abs(sin(d*x + c))) + a*sin(d*x + c))/d
```

maple [A] time = 0.13, size = 25, normalized size = 1.04

$$\frac{a\ln\left(\sin\left(dx+c\right)\right)}{d}+\frac{a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x)
[Out] a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d
```

maxima [A] time = 0.30, size = 22, normalized size = 0.92

$$\frac{a\log\left(\sin\left(dx+c\right)\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sin(d*x + c)) + a*sin(d*x + c))/d
```

mupad [B] time = 8.81, size = 38, normalized size = 1.58

$$\frac{a\left(\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\sin\left(c+d\,x\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] (a*(log(tan(c/2 + (d*x)/2)) + sin(c + d*x) - log(tan(c/2 + (d*x)/2)^2 + 1))
)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos\left(c+dx\right)\csc\left(c+dx\right)dx+\int\sin\left(c+dx\right)\cos\left(c+dx\right)\csc\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)*csc(c + d*x), x) + Integral(sin(c + d*x)*cos(c + d
*x)*csc(c + d*x), x))
```

3.193 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=25

$$\frac{a\log(\sin(c+dx))}{d}-\frac{a\csc(c+dx)}{d}$$

```
[Out] -a*csc(d*x+c)/d+a*ln(sin(d*x+c))/d
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2833, 12, 43}

$$\frac{a\log(\sin(c+dx))}{d}-\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a+x)}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{a+x}{x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\csc(c+dx)}{d} + \frac{a\log(\sin(c+dx))}{d}$$

Mathematica [A] time = 0.04, size = 33, normalized size = 1.32

$$\frac{a(\log(\tan(c+dx)) + \log(\cos(c+dx)))}{d} - \frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
[Out] -((a*Csc[c + d*x])/d) + (a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d
```

fricas [A] time = 0.47, size = 33, normalized size = 1.32

$$\frac{a\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c) - a}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] (a*log(1/2*sin(d*x + c))*sin(d*x + c) - a)/(d*sin(d*x + c))
```

giac [A] time = 0.13, size = 26, normalized size = 1.04

$$\frac{a\log(|\sin(dx+c)|) - \frac{a}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] (a*log(abs(sin(d*x + c))) - a/sin(d*x + c))/d
```

maple [A] time = 0.08, size = 28, normalized size = 1.12

$$-\frac{a}{d\sin\left(dx+c\right)}+\frac{a\ln\left(\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] -a/d/sin(d*x+c)+a*ln(sin(d*x+c))/d
```

maxima [A] time = 0.30, size = 25, normalized size = 1.00

$$\frac{a\log\left(\sin\left(dx+c\right)\right)-\frac{a}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sin(d*x + c)) - a/sin(d*x + c))/d
```

mupad [B] time = 8.58, size = 55, normalized size = 2.20

$$-\frac{a\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+2\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)+\frac{1}{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] -(a*(tan(c/2 + (d*x)/2) - 2*log(tan(c/2 + (d*x)/2)) + 2*log(tan(c/2 + (d*x)
/2)^2 + 1) + 1/tan(c/2 + (d*x)/2)))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos\left(c+dx\right)\csc^2\left(c+dx\right)dx+\int\sin\left(c+dx\right)\cos\left(c+dx\right)\csc^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)*csc(c + d*x)**2, x) + Integral(sin(c + d*x)*cos(c
+ d*x)*csc(c + d*x)**2, x))
```

3.194 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=30

$$-\frac{\csc^2(c+dx)(a\sin(c+dx)+a)^2}{2ad}$$

```
[Out] -1/2*csc(d*x+c)^2*(a+a*sin(d*x+c))^2/a/d
```

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 37}

$$-\frac{\csc^2(c+dx)(a\sin(c+dx)+a)^2}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2)/(2*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a+x)}{x^3}\,dx,x,a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{a+x}{x^3}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^2(c+dx)(a+a\sin(c+dx))^2}{2ad}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.97

$$-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d)
```

fricas [A] time = 0.48, size = 29, normalized size = 0.97

$$\frac{2\,a\sin\left(dx+c\right)+a}{2\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*sin(d*x + c) + a)/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.13, size = 24, normalized size = 0.80

$$-\frac{2\,a\sin\left(dx+c\right)+a}{2\,d\sin\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*sin(d*x + c) + a)/(d*sin(d*x + c)^2)
```

maple [A] time = 0.13, size = 27, normalized size = 0.90

$$\frac{a\left(-\frac{1}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] a/d*(-1/sin(d*x+c)-1/2/sin(d*x+c)^2)
```

maxima [A] time = 0.30, size = 24, normalized size = 0.80

$$-\frac{2\,a\sin\left(dx+c\right)+a}{2\,d\sin\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*sin(d*x + c) + a)/(d*sin(d*x + c)^2)
```

mupad [B] time = 8.52, size = 25, normalized size = 0.83

$$-\frac{\frac{a}{2}+a\,\sin\left(c+d\,x\right)}{d\sin\left(c+d\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] -(a/2 + a*sin(c + d*x))/(d*sin(c + d*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos\left(c+dx\right)\csc^3\left(c+dx\right)dx+\int\sin\left(c+dx\right)\cos\left(c+dx\right)\csc^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)*csc(c + d*x)**3, x) + Integral(sin(c + d*x)*cos(c
+ d*x)*csc(c + d*x)**3, x))
```

3.195 $\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=33

$$-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}$$

```
[Out] -1/2*a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 43}

$$-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a+x)}{x^4}\,dx,x,a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{a+x}{x^4}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{a}{x^4}+\frac{1}{x^3}\right)dx,x,a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\csc^2(c+dx)}{2d} - \frac{a\csc^3(c+dx)}{3d}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{a\csc^3(c+dx)}{3d} - \frac{a\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
[Out] -1/2*(a*Csc[c + d*x]^2)/d - (a*Csc[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.44, size = 39, normalized size = 1.18

$$\frac{3\,a\sin(dx+c)+2\,a}{6\left(d\cos(dx+c)^2-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/6*(3*a*sin(d*x + c) + 2*a)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.15, size = 26, normalized size = 0.79

$$-\frac{3\,a\sin(dx+c)+2\,a}{6\,d\sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/6*(3*a*sin(d*x + c) + 2*a)/(d*sin(d*x + c)^3)
```

maple [A] time = 0.09, size = 27, normalized size = 0.82

$$\frac{a\left(-\frac{1}{2\sin(dx+c)^2}-\frac{1}{3\sin(dx+c)^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] a/d*(-1/2/sin(d*x+c)^2-1/3/sin(d*x+c)^3)
```

maxima [A] time = 0.30, size = 26, normalized size = 0.79

$$-\frac{3\,a\sin\left(dx+c\right)+2\,a}{6\,d\sin\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*a*sin(d*x + c) + 2*a)/(d*sin(d*x + c)^3)
```

mupad [B] time = 8.60, size = 39, normalized size = 1.18

$$-\frac{\frac{5\,a\,\sin(c+d\,x)}{16}+\frac{a\left(\frac{3\,\sin(3\,c+3\,d\,x)}{16}+1\right)}{3}}{d\sin\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] -((5*a*sin(c + d*x))/16 + (a*((3*sin(3*c + 3*d*x))/16 + 1))/3)/(d*sin(c + d
*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos\left(c+dx\right)\csc^4\left(c+dx\right)dx+\int\sin\left(c+dx\right)\cos\left(c+dx\right)\csc^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)*csc(c + d*x)**4, x) + Integral(sin(c + d*x)*cos(c
+ d*x)*csc(c + d*x)**4, x))
```

3.196 $\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=33

$$-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}$$

```
[Out] -1/3*a*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/d
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 43}

$$-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^5(a+x)}{x^5}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^4\,\text{Subst}\left(\int \frac{a+x}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \left(\frac{a}{x^5}+\frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\csc^3(c+dx)}{3d} - \frac{a\csc^4(c+dx)}{4d}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{a\csc^4(c+dx)}{4d} - \frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
[Out] -1/3*(a*Csc[c + d*x]^3)/d - (a*Csc[c + d*x]^4)/(4*d)
```

fricas [A] time = 0.45, size = 40, normalized size = 1.21

$$-\frac{4\,a\sin\left(dx+c\right)+3\,a}{12\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/12*(4*a*sin(d*x + c) + 3*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.15, size = 26, normalized size = 0.79

$$-\frac{4\,a\sin\left(dx+c\right)+3\,a}{12\,d\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/12*(4*a*sin(d*x + c) + 3*a)/(d*sin(d*x + c)^4)
```

maple [A] time = 0.09, size = 27, normalized size = 0.82

$$\frac{a\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] a/d*(-1/4/sin(d*x+c)^4-1/3/sin(d*x+c)^3)
```

maxima [A] time = 0.30, size = 26, normalized size = 0.79

$$-\frac{4\,a\sin\left(dx+c\right)+3\,a}{12\,d\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*a*sin(d*x + c) + 3*a)/(d*sin(d*x + c)^4)
```

mupad [B] time = 8.56, size = 26, normalized size = 0.79

$$-\frac{\frac{a}{4}+\frac{a\,\sin(c+d\,x)}{3}}{d\sin\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] -(a/4 + (a*sin(c + d*x))/3)/(d*sin(c + d*x)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.197 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin^4(c+dx)}{2d}+\frac{a^2\sin^3(c+dx)}{3d}$$

```
[Out] 1/3*a^2*sin(d*x+c)^3/d+1/2*a^2*sin(d*x+c)^4/d+1/5*a^2*sin(d*x+c)^5/d
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin^4(c+dx)}{2d}+\frac{a^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^4)/(2*d) + (a^2*Sin[c + d*x]
^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+x)^2}{a^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^2(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2x^2+2ax^3+x^4\right)\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^4(c+dx)}{2d}+\frac{a^2\sin^5(c+dx)}{5d}$$

Mathematica [**A**] time = 0.34, size = 53, normalized size = 0.96

$$\frac{a^2\left(104\sin^3(c+dx)+15\cos(4(c+dx))-12\left(2\sin^3(c+dx)+5\right)\cos(2(c+dx))\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(15*Cos[4*(c + d*x)] + 104*Sin[c + d*x]^3 - 12*Cos[2*(c + d*x)]*(5 + 2
*Sin[c + d*x]^3)))/(240*d)
```

fricas [**A**] time = 0.45, size = 72, normalized size = 1.31

$$\frac{15\,a^2\cos\left(dx+c\right)^4-30\,a^2\cos\left(dx+c\right)^2+2\left(3\,a^2\cos\left(dx+c\right)^4-11\,a^2\cos\left(dx+c\right)^2+8\,a^2\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(15*a^2*cos(d*x + c)^4 - 30*a^2*cos(d*x + c)^2 + 2*(3*a^2*cos(d*x + c)
^4 - 11*a^2*cos(d*x + c)^2 + 8*a^2)*sin(d*x + c))/d
```

giac [**A**] time = 0.16, size = 45, normalized size = 0.82

$$\frac{6\,a^2\sin\left(dx+c\right)^5+15\,a^2\sin\left(dx+c\right)^4+10\,a^2\sin\left(dx+c\right)^3}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*a^2*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4 + 10*a^2*sin(d*x + c)^3)
/d
```

maple [A] time = 0.09, size = 45, normalized size = 0.82

$$\frac{\frac{\left(\sin^{5}(dx+c)\right)a^{2}}{5}+\frac{a^{2}\left(\sin^{4}(dx+c)\right)}{2}+\frac{a^{2}\left(\sin^{3}(dx+c)\right)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(1/5*sin(d*x+c)^5*a^2+1/2*a^2*sin(d*x+c)^4+1/3*a^2*sin(d*x+c)^3)
```

maxima [A] time = 0.30, size = 45, normalized size = 0.82

$$\frac{6\,a^{2}\sin\left(dx+c\right)^{5}+15\,a^{2}\sin\left(dx+c\right)^{4}+10\,a^{2}\sin\left(dx+c\right)^{3}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(6*a^2*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4 + 10*a^2*sin(d*x + c)^3)
/d
```

mupad [B] time = 8.49, size = 36, normalized size = 0.65

$$\frac{a^{2}\sin\left(c+d\,x\right)^{3}\,\left(6\sin\left(c+d\,x\right)^{2}+15\,\sin\left(c+d\,x\right)+10\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x)^3*(15*sin(c + d*x) + 6*sin(c + d*x)^2 + 10))/(30*d)
```

sympy [A] time = 2.60, size = 63, normalized size = 1.15

$$\begin{cases} \frac{a^{2}\sin^{5}\left(c+dx\right)}{5d}+\frac{a^{2}\sin^{4}\left(c+dx\right)}{2d}+\frac{a^{2}\sin^{3}\left(c+dx\right)}{3d} & \text{for}\ d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)^{2}\sin^{2}\left(c\right)\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sin(c + d*x)**5/(5*d) + a**2*sin(c + d*x)**4/(2*d) + a**2*s
in(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**2*cos(c), Tru
e))
```

3.198 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^2(c+dx)}{2d}$$

```
[Out] 1/2*a^2*sin(d*x+c)^2/d+2/3*a^2*sin(d*x+c)^3/d+1/4*a^2*sin(d*x+c)^4/d
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 43}

$$\frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^2)/(2*d) + (2*a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*
x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^2}{a}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2x+2ax^2+x^3\right)\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{a^2\sin^2(c+dx)}{2d} + \frac{2a^2\sin^3(c+dx)}{3d} + \frac{a^2\sin^4(c+dx)}{4d}$$

Mathematica [A] time = 0.05, size = 38, normalized size = 0.69

$$\frac{a^2\sin^2(c+dx)\left(3\sin^2(c+dx)+8\sin(c+dx)+6\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^2*(6 + 8*Sin[c + d*x] + 3*Sin[c + d*x]^2))/(12*d)
```

fricas [A] time = 0.46, size = 58, normalized size = 1.05

$$\frac{3\,a^2\cos\left(dx+c\right)^4 - 12\,a^2\cos\left(dx+c\right)^2 - 8\left(a^2\cos\left(dx+c\right)^2 - a^2\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*cos(d*x + c)^4 - 12*a^2*cos(d*x + c)^2 - 8*(a^2*cos(d*x + c)^2
- a^2)*sin(d*x + c))/d
```

giac [A] time = 0.17, size = 45, normalized size = 0.82

$$\frac{3\,a^2\sin\left(dx+c\right)^4 + 8\,a^2\sin\left(dx+c\right)^3 + 6\,a^2\sin\left(dx+c\right)^2}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^2*sin(d*x + c)^4 + 8*a^2*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)^2)/d
```

maple [A] time = 0.10, size = 45, normalized size = 0.82

$$\frac{\frac{a^2\left(\sin^4(dx+c)\right)}{4}+\frac{2a^2\left(\sin^3(dx+c)\right)}{3}+\frac{\left(\sin^2(dx+c)\right)a^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(1/4*a^2*sin(d*x+c)^4+2/3*a^2*sin(d*x+c)^3+1/2*sin(d*x+c)^2*a^2)
```

maxima [A] time = 0.30, size = 45, normalized size = 0.82

$$\frac{3\,a^2\sin\left(dx+c\right)^4+8\,a^2\sin\left(dx+c\right)^3+6\,a^2\sin\left(dx+c\right)^2}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^2*sin(d*x + c)^4 + 8*a^2*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)^2)/d
```

mupad [B] time = 8.51, size = 36, normalized size = 0.65

$$\frac{a^2\sin\left(c+d\,x\right)^2\,\left(3\sin\left(c+d\,x\right)^2+8\,\sin\left(c+d\,x\right)+6\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x)^2*(8*sin(c + d*x) + 3*sin(c + d*x)^2 + 6))/(12*d)
```

sympy [A] time = 1.28, size = 63, normalized size = 1.15

$$\begin{cases} \frac{a^2\sin^4\left(c+dx\right)}{4d}+\frac{2a^2\sin^3\left(c+dx\right)}{3d}+\frac{a^2\sin^2\left(c+dx\right)}{2d} & \text{for}\ d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)^2\sin\left(c\right)\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sin(c + d*x)**4/(4*d) + 2*a**2*sin(c + d*x)**3/(3*d) + a**2
*sin(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)*cos(c), True
))
```

3.199 $\int \cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=47

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

```
[Out] a^2*ln(sin(d*x+c))/d+2*a^2*sin(d*x+c)/d+1/2*a^2*sin(d*x+c)^2/d
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 43}

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)/(
2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(2a+\frac{a^2}{x}+x\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\log(\sin(c+dx))}{d} + \frac{2a^2\sin(c+dx)}{d} + \frac{a^2\sin^2(c+dx)}{2d}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.00

$$\frac{a^2\sin^2(c+dx)}{2d} + \frac{2a^2\sin(c+dx)}{d} + \frac{a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)/(
2*d)
```

fricas [A] time = 0.48, size = 43, normalized size = 0.91

$$-\frac{a^2\cos\left(dx+c\right)^2 - 2\,a^2\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right) - 4\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*cos(d*x + c)^2 - 2*a^2*log(1/2*sin(d*x + c)) - 4*a^2*sin(d*x + c)
)/d
```

giac [A] time = 0.17, size = 42, normalized size = 0.89

$$\frac{a^2\sin\left(dx+c\right)^2 + 2\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right) + 4\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*sin(d*x + c)^2 + 2*a^2*log(abs(sin(d*x + c))) + 4*a^2*sin(d*x + c)
)/d
```

maple [A] time = 0.18, size = 46, normalized size = 0.98

$$\frac{a^2 \ln (\sin (dx+c))}{d}+\frac{2a^2 \sin (dx+c)}{d}+\frac{a^2 \left(\sin^2 (dx+c)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] a^2*ln(sin(d*x+c))/d+2*a^2*sin(d*x+c)/d+1/2*a^2*sin(d*x+c)^2/d
```

maxima [A] time = 0.34, size = 41, normalized size = 0.87

$$\frac{a^2 \sin (dx+c)^2+2\,a^2 \log (\sin (dx+c))+4\,a^2 \sin (dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*sin(d*x + c)^2 + 2*a^2*log(sin(d*x + c)) + 4*a^2*sin(d*x + c))/d
```

mupad [B] time = 8.99, size = 119, normalized size = 2.53

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{4\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (2*a^2*tan(c/2 + (d*x)/2)^2 + 4*a^2*tan(c
/2 + (d*x)/2)^3 + 4*a^2*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 + ta
n(c/2 + (d*x)/2)^4 + 1)) - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int \cos (c+dx) \csc (c+dx)\, dx+\int 2 \sin (c+dx) \cos (c+dx) \csc (c+dx)\, dx+\int \sin^2 (c+dx) \cos (c+dx) \text{cs}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(cos(c + d*x)*csc(c + d*x), x) + Integral(2*sin(c + d*x)*cos(
c + d*x)*csc(c + d*x), x) + Integral(sin(c + d*x)**2*cos(c + d*x)*csc(c + d
*x), x))
```

3.200 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=43

$$\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}$$

```
[Out] -a^2*csc(d*x+c)/d+2*a^2*ln(sin(d*x+c))/d+a^2*sin(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 43}

$$\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a^2*Log[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/
d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a+x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a+x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(1+\frac{a^2}{x^2}+\frac{2a}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^2\csc(c+dx)}{d} + \frac{2a^2\log(\sin(c+dx))}{d} + \frac{a^2\sin(c+dx)}{d}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.88

$$a^2\left(\frac{\sin(c+dx)}{d} - \frac{\csc(c+dx)}{d} + \frac{2\log(\sin(c+dx))}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*(-(Csc[c + d*x]/d) + (2*Log[Sin[c + d*x]])/d + Sin[c + d*x]/d)
```

fricas [A] time = 0.48, size = 46, normalized size = 1.07

$$-\frac{a^2\cos\left(dx+c\right)^2 - 2\,a^2\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)}{d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*cos(d*x + c)^2 - 2*a^2*log(1/2*sin(d*x + c))*sin(d*x + c))/(d*sin(d*x
 + c))
```

giac [A] time = 0.15, size = 41, normalized size = 0.95

$$\frac{2\,a^2\log\left(|\sin\left(dx+c\right)|\right) + a^2\sin\left(dx+c\right) - \frac{a^2}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a^2*log(abs(sin(d*x + c))) + a^2*sin(d*x + c) - a^2/sin(d*x + c))/d
```

maple [A] time = 0.13, size = 46, normalized size = 1.07

$$\frac{a^2 \sin(dx+c)}{d} - \frac{a^2}{d \sin(dx+c)} + \frac{2a^2 \ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] a^2*sin(d*x+c)/d-a^2/d/sin(d*x+c)+2*a^2*ln(sin(d*x+c))/d
```

maxima [A] time = 0.35, size = 40, normalized size = 0.93

$$\frac{2\,a^2 \log(\sin(dx+c)) + a^2 \sin(dx+c) - \frac{a^2}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*log(sin(d*x + c)) + a^2*sin(d*x + c) - a^2/sin(d*x + c))/d
```

mupad [B] time = 8.89, size = 111, normalized size = 2.58

$$\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^2}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (2*a^2*log(tan(c/2 + (d*x)/2)))/d + (3*a^2*tan(c/2 + (d*x)/2)^2 - a^2)/(d*(
2*tan(c/2 + (d*x)/2) + 2*tan(c/2 + (d*x)/2)^3)) - (a^2*tan(c/2 + (d*x)/2))/
(2*d) - (2*a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \cos(c+dx)\csc^2(c+dx)\,dx + \int 2\sin(c+dx)\cos(c+dx)\csc^2(c+dx)\,dx + \int \sin^2(c+dx)\cos(c+dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(cos(c + d*x)*csc(c + d*x)**2, x) + Integral(2*sin(c + d*x)*c
os(c + d*x)*csc(c + d*x)**2, x) + Integral(sin(c + d*x)**2*cos(c + d*x)*csc
(c + d*x)**2, x))
```

3.201 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=47

$$-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

```
[Out] -2*a^2*csc(d*x+c)/d-1/2*a^2*csc(d*x+c)^2/d+a^2*ln(sin(d*x+c))/d
```

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) + (a^2*Log[Sin[c + d*x
]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a+x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(a+x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{a^2}{x^3}+\frac{2a}{x^2}+\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{2a^2\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{2d} + \frac{a^2\log(\sin(c+dx))}{d}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.89

$$a^2\left(-\frac{\csc^2(c+dx)}{2d} - \frac{2\csc(c+dx)}{d} + \frac{\log(\sin(c+dx))}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
[Out] a^2*((-2*Csc[c + d*x])/d - Csc[c + d*x]^2/(2*d) + Log[Sin[c + d*x]]/d)
```

fricas [A] time = 0.48, size = 62, normalized size = 1.32

$$\frac{4\,a^2\sin(dx+c) + a^2 + 2\left(a^2\cos(dx+c)^2 - a^2\right)\log\left(\frac{1}{2}\,\sin(dx+c)\right)}{2\left(d\cos(dx+c)^2 - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] 1/2*(4*a^2*sin(d*x + c) + a^2 + 2*(a^2*cos(d*x + c)^2 - a^2)*log(1/2*sin(d*
x + c)))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.15, size = 44, normalized size = 0.94

$$\frac{2\,a^2\log(|\sin(dx+c)|) - \frac{4\,a^2\sin(dx+c)+a^2}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(sin(d*x + c))) - (4*a^2*sin(d*x + c) + a^2)/sin(d*x + c)
^2)/d
```

maple [A] time = 0.18, size = 48, normalized size = 1.02

$$-\frac{2a^2}{d\sin(dx+c)} + \frac{a^2\ln(\sin(dx+c))}{d} - \frac{a^2}{2d\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2*a^2/d/sin(d*x+c)+a^2*ln(sin(d*x+c))/d-1/2*a^2/d/sin(d*x+c)^2
```

maxima [A] time = 0.35, size = 43, normalized size = 0.91

$$\frac{2\,a^2\log(\sin(dx+c)) - \frac{4\,a^2\sin(dx+c)+a^2}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*log(sin(d*x + c)) - (4*a^2*sin(d*x + c) + a^2)/sin(d*x + c)^2)/d
```

mupad [B] time = 8.87, size = 111, normalized size = 2.36

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d} - \frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{8}+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d} - \frac{a^2\,\ln\left(\tan\left(\frac{c}{2}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) - (cot(c
/2 + (d*x)/2)^2*(a^2/8 + a^2*tan(c/2 + (d*x)/2)))/d - (a^2*tan(c/2 + (d*x)/
2))/d - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\cos(c+dx)\csc^3(c+dx)\,dx + \int 2\sin(c+dx)\cos(c+dx)\csc^3(c+dx)\,dx + \int\sin^2(c+dx)\cos(c+dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(cos(c + d*x)*csc(c + d*x)**3, x) + Integral(2*sin(c + d*x)*c
os(c + d*x)*csc(c + d*x)**3, x) + Integral(sin(c + d*x)**2*cos(c + d*x)*csc
(c + d*x)**3, x))
```

3.202 $\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=30

$$-\frac{\csc^3(c+dx)(a\sin(c+dx)+a)^3}{3ad}$$

[Out] -1/3*csc(d*x+c)^3*(a+a*sin(d*x+c))^3/a/d

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 37}

$$-\frac{\csc^3(c+dx)(a\sin(c+dx)+a)^3}{3ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]

[Out] -(Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*a*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a+x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{(a+x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^3(c+dx)(a+a\sin(c+dx))^3}{3ad}$$

Mathematica [A] time = 0.03, size = 20, normalized size = 0.67

$$-\frac{a^2(\csc(c+dx)+1)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/3*(a^2*(1 + Csc[c + d*x])^3)/d
```

fricas [A] time = 0.47, size = 56, normalized size = 1.87

$$-\frac{3\,a^2\cos\left(dx+c\right)^2 - 3\,a^2\sin\left(dx+c\right) - 4\,a^2}{3\left(d\cos\left(dx+c\right)^2 - d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*cos(d*x + c)^2 - 3*a^2*sin(d*x + c) - 4*a^2)/((d*cos(d*x + c)^2
 - d)*sin(d*x + c))
```

giac [A] time = 0.15, size = 41, normalized size = 1.37

$$-\frac{3\,a^2\sin\left(dx+c\right)^2 + 3\,a^2\sin\left(dx+c\right) + a^2}{3\,d\sin\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*a^2*sin(d*x + c)^2 + 3*a^2*sin(d*x + c) + a^2)/(d*sin(d*x + c)^3)
```

maple [A] time = 0.15, size = 39, normalized size = 1.30

$$\frac{a^{2} \left(-\frac{1}{\sin(dx+c)}-\frac{1}{\sin(dx+c)^{2}}-\frac{1}{3\sin(dx+c)^{3}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
[Out] a^2/d*(-1/sin(d*x+c)-1/sin(d*x+c)^2-1/3/sin(d*x+c)^3)
```

maxima [A] time = 0.33, size = 41, normalized size = 1.37

$$-\frac{3\,a^{2}\sin\left(dx+c\right)^{2}+3\,a^{2}\sin\left(dx+c\right)+a^{2}}{3\,d\sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] -1/3*(3*a^2*sin(d*x + c)^2 + 3*a^2*sin(d*x + c) + a^2)/(d*sin(d*x + c)^3)
```

mupad [B] time = 8.90, size = 41, normalized size = 1.37

$$-\frac{a^{2}\sin\left(c+d\,x\right)^{2}+a^{2}\,\sin\left(c+d\,x\right)+\frac{a^{2}}{3}}{d\sin\left(c+d\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^2)/sin(c + d*x)^4,x)
[Out] -(a^2*sin(c + d*x) + a^2/3 + a^2*sin(c + d*x)^2)/(d*sin(c + d*x)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
[Out] Timed out
```

3.203 $\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$-\frac{a^2\csc^4(c+dx)}{4d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{2d}$$

```
[Out] -1/2*a^2*csc(d*x+c)^2/d-2/3*a^2*csc(d*x+c)^3/d-1/4*a^2*csc(d*x+c)^4/d
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{a^2\csc^4(c+dx)}{4d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^2)/(2*d) - (2*a^2*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d
*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^5(a+x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^4\,\text{Subst}\left(\int \frac{(a+x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \left(\frac{a^2}{x^5}+\frac{2a}{x^4}+\frac{1}{x^3}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^2\csc^2(c+dx)}{2d} - \frac{2a^2\csc^3(c+dx)}{3d} - \frac{a^2\csc^4(c+dx)}{4d}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 1.00

$$-\frac{a^2\csc^4(c+dx)}{4d} - \frac{2a^2\csc^3(c+dx)}{3d} - \frac{a^2\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(a^2*Csc[c + d*x]^2)/d - (2*a^2*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d
*x]^4)/(4*d)
```

fricas [A] time = 0.45, size = 57, normalized size = 1.04

$$\frac{6\,a^2\cos(dx+c)^2 - 8\,a^2\sin(dx+c) - 9\,a^2}{12\left(d\cos(dx+c)^4 - 2\,d\cos(dx+c)^2 + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a^2*cos(d*x + c)^2 - 8*a^2*sin(d*x + c) - 9*a^2)/(d*cos(d*x + c)^4
- 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.16, size = 43, normalized size = 0.78

$$-\frac{6\,a^2\sin(dx+c)^2 + 8\,a^2\sin(dx+c) + 3\,a^2}{12\,d\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/12*(6*a^2*sin(d*x + c)^2 + 8*a^2*sin(d*x + c) + 3*a^2)/(d*sin(d*x + c)^4
)
```

maple [A] time = 0.14, size = 39, normalized size = 0.71

$$\frac{a^{2}\left(-\frac{1}{2\sin(dx+c)^{2}}-\frac{1}{4\sin(dx+c)^{4}}-\frac{2}{3\sin(dx+c)^{3}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] a^2/d*(-1/2/sin(d*x+c)^2-1/4/sin(d*x+c)^4-2/3/sin(d*x+c)^3)
```

maxima [A] time = 0.31, size = 43, normalized size = 0.78

$$-\frac{6\,a^{2}\sin\left(dx+c\right)^{2}+8\,a^{2}\sin\left(dx+c\right)+3\,a^{2}}{12\,d\sin\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(6*a^2*sin(d*x + c)^2 + 8*a^2*sin(d*x + c) + 3*a^2)/(d*sin(d*x + c)^4
)
```

mupad [B] time = 8.85, size = 43, normalized size = 0.78

$$-\frac{\frac{a^{2}\sin(c+d\,x)^{2}}{2}+\frac{2\,a^{2}\,\sin(c+d\,x)}{3}+\frac{a^{2}}{4}}{d\sin\left(c+d\,x\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] -((2*a^2*sin(c + d*x))/3 + a^2/4 + (a^2*sin(c + d*x)^2)/2)/(d*sin(c + d*x)^
4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.204 $\int \cot(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$-\frac{a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{2d}-\frac{a^2\csc^3(c+dx)}{3d}$$

```
[Out] -1/3*a^2*csc(d*x+c)^3/d-1/2*a^2*csc(d*x+c)^4/d-1/5*a^2*csc(d*x+c)^5/d
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{2d}-\frac{a^2\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x
]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a+x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^5\,\text{Subst}\left(\int \frac{(a+x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{a^2}{x^6}+\frac{2a}{x^5}+\frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^2\csc^3(c+dx)}{3d} - \frac{a^2\csc^4(c+dx)}{2d} - \frac{a^2\csc^5(c+dx)}{5d}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 1.00

$$-\frac{a^2\csc^5(c+dx)}{5d} - \frac{a^2\csc^4(c+dx)}{2d} - \frac{a^2\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/3*(a^2*Csc[c + d*x]^3)/d - (a^2*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x
]^5)/(5*d)
```

fricas [A] time = 0.44, size = 65, normalized size = 1.18

$$\frac{10\,a^2\cos(dx+c)^2 - 15\,a^2\sin(dx+c) - 16\,a^2}{30\left(d\cos(dx+c)^4 - 2\,d\cos(dx+c)^2 + d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(10*a^2*cos(d*x + c)^2 - 15*a^2*sin(d*x + c) - 16*a^2)/((d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.17, size = 43, normalized size = 0.78

$$-\frac{10\,a^2\sin(dx+c)^2 + 15\,a^2\sin(dx+c) + 6\,a^2}{30\,d\sin(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/30*(10*a^2*sin(d*x + c)^2 + 15*a^2*sin(d*x + c) + 6*a^2)/(d*sin(d*x + c)^5)

maple [A] time = 0.14, size = 39, normalized size = 0.71

$$\frac{a^2\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{2\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x)

[Out] a^2/d*(-1/5/sin(d*x+c)^5-1/2/sin(d*x+c)^4-1/3/sin(d*x+c)^3)

maxima [A] time = 0.37, size = 43, normalized size = 0.78

$$-\frac{10\,a^2\sin\left(dx+c\right)^2+15\,a^2\sin\left(dx+c\right)+6\,a^2}{30\,d\sin\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/30*(10*a^2*sin(d*x + c)^2 + 15*a^2*sin(d*x + c) + 6*a^2)/(d*sin(d*x + c)^5)

mupad [B] time = 8.92, size = 43, normalized size = 0.78

$$-\frac{\frac{a^2\sin(c+d\,x)^2}{3}+\frac{a^2\,\sin(c+d\,x)}{2}+\frac{a^2}{5}}{d\sin\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^2)/sin(c + d*x)^6,x)

[Out] -((a^2*sin(c + d*x))/2 + a^2/5 + (a^2*sin(c + d*x)^2)/3)/(d*sin(c + d*x)^5)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)

[Out] Timed out

3.205 $\int \cot(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{4d}$$

```
[Out] -1/4*a^2*csc(d*x+c)^4/d-2/5*a^2*csc(d*x+c)^5/d-1/6*a^2*csc(d*x+c)^6/d
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^4)/(4*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2*Csc[c + d
*x]^6)/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a+x)^2}{x^7}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{(a+x)^2}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(\frac{a^2}{x^7}+\frac{2a}{x^6}+\frac{1}{x^5}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^2\csc^4(c+dx)}{4d} - \frac{2a^2\csc^5(c+dx)}{5d} - \frac{a^2\csc^6(c+dx)}{6d}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 1.00

$$-\frac{a^2\csc^6(c+dx)}{6d} - \frac{2a^2\csc^5(c+dx)}{5d} - \frac{a^2\csc^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*(a^2*Csc[c + d*x]^4)/d - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2*Csc[c + d
*x]^6)/(6*d)
```

fricas [A] time = 0.48, size = 70, normalized size = 1.27

$$-\frac{15\,a^2\cos(dx+c)^2 - 24\,a^2\sin(dx+c) - 25\,a^2}{60\left(d\cos(dx+c)^6 - 3\,d\cos(dx+c)^4 + 3\,d\cos(dx+c)^2 - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(15*a^2*cos(d*x + c)^2 - 24*a^2*sin(d*x + c) - 25*a^2)/(d*cos(d*x + c
)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.16, size = 43, normalized size = 0.78

$$-\frac{15\,a^2\sin(dx+c)^2 + 24\,a^2\sin(dx+c) + 10\,a^2}{60\,d\sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(15*a^2*sin(d*x + c)^2 + 24*a^2*sin(d*x + c) + 10*a^2)/(d*sin(d*x + c
)^6)
```

maple [A] time = 0.16, size = 39, normalized size = 0.71

$$\frac{a^{2} \left(-\frac{1}{6 \sin \left(d x +c \right)^{6}}-\frac{2}{5 \sin \left(d x +c \right)^{5}}-\frac{1}{4 \sin \left(d x +c \right)^{4}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x)
```

```
[Out] a^2/d*(-1/6/sin(d*x+c)^6-2/5/sin(d*x+c)^5-1/4/sin(d*x+c)^4)
```

maxima [A] time = 0.34, size = 43, normalized size = 0.78

$$-\frac{15 \, a^{2} \sin \left(d x + c\right)^{2} + 24 \, a^{2} \sin \left(d x + c\right) + 10 \, a^{2}}{60 \, d \sin \left(d x + c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*(15*a^2*sin(d*x + c)^2 + 24*a^2*sin(d*x + c) + 10*a^2)/(d*sin(d*x + c
)^6)
```

mupad [B] time = 8.91, size = 43, normalized size = 0.78

$$-\frac{\frac{a^2\,{\sin\left(c+d\,x\right)}^2}{4}+\frac{2\,a^2\,\sin\left(c+d\,x\right)}{5}+\frac{a^2}{6}}{d\,{\sin\left(c+d\,x\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] -((2*a^2*sin(c + d*x))/5 + a^2/6 + (a^2*sin(c + d*x)^2)/4)/(d*sin(c + d*x)^
6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.206 $\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=73

$$\frac{a^3\sin^7(c+dx)}{7d}+\frac{a^3\sin^6(c+dx)}{2d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^4(c+dx)}{4d}$$

```
[Out] 1/4*a^3*sin(d*x+c)^4/d+3/5*a^3*sin(d*x+c)^5/d+1/2*a^3*sin(d*x+c)^6/d+1/7*a^
3*sin(d*x+c)^7/d
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^3\sin^7(c+dx)}{7d}+\frac{a^3\sin^6(c+dx)}{2d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^4)/(4*d) + (3*a^3*Sin[c + d*x]^5)/(5*d) + (a^3*Sin[c + d*
x]^6)/(2*d) + (a^3*Sin[c + d*x]^7)/(7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^3}{a^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^3(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3x^3+3a^2x^4+3ax^5+x^6\right)dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{a^3\sin^4(c+dx)}{4d} + \frac{3a^3\sin^5(c+dx)}{5d} + \frac{a^3\sin^6(c+dx)}{2d} + \frac{a^3\sin}{}$$

Mathematica [A] time = 0.34, size = 80, normalized size = 1.10

$$-\frac{a^3(-1015\sin(c+dx)+525\sin(3(c+dx))-119\sin(5(c+dx))+5\sin(7(c+dx))+805\cos(2(c+dx))-280\cos}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/2240*(a^3*(-350 + 805*Cos[2*(c + d*x)] - 280*Cos[4*(c + d*x)] + 35*Cos[6
*(c + d*x)] - 1015*Sin[c + d*x] + 525*Sin[3*(c + d*x)] - 119*Sin[5*(c + d*x
)] + 5*Sin[7*(c + d*x)]))/d
```

fricas [A] time = 0.47, size = 98, normalized size = 1.34

$$-\frac{70\,a^3\cos(dx+c)^6-245\,a^3\cos(dx+c)^4+280\,a^3\cos(dx+c)^2+4\left(5\,a^3\cos(dx+c)^6-36\,a^3\cos(dx+c)^4+5\right.}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/140*(70*a^3*cos(d*x + c)^6 - 245*a^3*cos(d*x + c)^4 + 280*a^3*cos(d*x +
c)^2 + 4*(5*a^3*cos(d*x + c)^6 - 36*a^3*cos(d*x + c)^4 + 57*a^3*cos(d*x + c
)^2 - 26*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.20, size = 58, normalized size = 0.79

$$\frac{20\,a^3\sin(dx+c)^7+70\,a^3\sin(dx+c)^6+84\,a^3\sin(dx+c)^5+35\,a^3\sin(dx+c)^4}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/140*(20*a^3*sin(d*x + c)^7 + 70*a^3*sin(d*x + c)^6 + 84*a^3*sin(d*x + c)^
5 + 35*a^3*sin(d*x + c)^4)/d
```

maple [A] time = 0.10, size = 58, normalized size = 0.79

$$\frac{\frac{a^3\left(\sin^7(dx+c)\right)}{7}+\frac{a^3\left(\sin^6(dx+c)\right)}{2}+\frac{3a^3\left(\sin^5(dx+c)\right)}{5}+\frac{a^3\left(\sin^4(dx+c)\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(1/7*a^3*sin(d*x+c)^7+1/2*a^3*sin(d*x+c)^6+3/5*a^3*sin(d*x+c)^5+1/4*a^3
*sin(d*x+c)^4)
```

maxima [A] time = 0.31, size = 58, normalized size = 0.79

$$\frac{20\,a^3\sin\left(dx+c\right)^7+70\,a^3\sin\left(dx+c\right)^6+84\,a^3\sin\left(dx+c\right)^5+35\,a^3\sin\left(dx+c\right)^4}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/140*(20*a^3*sin(d*x + c)^7 + 70*a^3*sin(d*x + c)^6 + 84*a^3*sin(d*x + c)^
5 + 35*a^3*sin(d*x + c)^4)/d
```

mupad [B] time = 0.07, size = 57, normalized size = 0.78

$$\frac{\frac{a^3\sin(c+d\,x)^7}{7}+\frac{a^3\sin(c+d\,x)^6}{2}+\frac{3\,a^3\sin(c+d\,x)^5}{5}+\frac{a^3\sin(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^3*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^4)/4 + (3*a^3*sin(c + d*x)^5)/5 + (a^3*sin(c + d*x)^6)/2
 + (a^3*sin(c + d*x)^7)/7)/d
```

sympy [A] time = 8.63, size = 80, normalized size = 1.10

$$\begin{cases} \frac{a^3\sin^7\left(c+dx\right)}{7d}+\frac{a^3\sin^6\left(c+dx\right)}{2d}+\frac{3a^3\sin^5\left(c+dx\right)}{5d}+\frac{a^3\sin^4\left(c+dx\right)}{4d} & \text{for } d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)^3\sin^3\left(c\right)\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**7/(7*d) + a**3*sin(c + d*x)**6/(2*d) + 3*a**3
*sin(c + d*x)**5/(5*d) + a**3*sin(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c
) + a)**3*sin(c)**3*cos(c), True))
```

3.207 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=73

$$\frac{a^3\sin^6(c+dx)}{6d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{3a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{3d}$$

```
[Out] 1/3*a^3*sin(d*x+c)^3/d+3/4*a^3*sin(d*x+c)^4/d+3/5*a^3*sin(d*x+c)^5/d+1/6*a^
3*sin(d*x+c)^6/d
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^3\sin^6(c+dx)}{6d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{3a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^4)/(4*d) + (3*a^3*Sin[c +
d*x]^5)/(5*d) + (a^3*Sin[c + d*x]^6)/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+x)^3}{a^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^2(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3x^2+3a^2x^3+3ax^4+x^5\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{a^3\sin^3(c+dx)}{3d} + \frac{3a^3\sin^4(c+dx)}{4d} + \frac{3a^3\sin^5(c+dx)}{5d} + \frac{a^3 s}{}$$

Mathematica [**A**] time = 0.28, size = 70, normalized size = 0.96

$$-\frac{a^3(-1200\sin(c+dx)+520\sin(3(c+dx))-72\sin(5(c+dx))+870\cos(2(c+dx))-240\cos(4(c+dx))+10\,c}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/1920*(a^3*(-45 + 870*Cos[2*(c + d*x)] - 240*Cos[4*(c + d*x)] + 10*Cos[6*
(c + d*x)] - 1200*Sin[c + d*x] + 520*Sin[3*(c + d*x)] - 72*Sin[5*(c + d*x)]
))/d
```

fricas [**A**] time = 0.46, size = 85, normalized size = 1.16

$$-\frac{10\,a^3\cos(dx+c)^6 - 75\,a^3\cos(dx+c)^4 + 120\,a^3\cos(dx+c)^2 - 4\left(9\,a^3\cos(dx+c)^4 - 23\,a^3\cos(dx+c)^2 + 14\,a\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(10*a^3*cos(d*x + c)^6 - 75*a^3*cos(d*x + c)^4 + 120*a^3*cos(d*x + c)
^2 - 4*(9*a^3*cos(d*x + c)^4 - 23*a^3*cos(d*x + c)^2 + 14*a^3)*sin(d*x + c)
)/d
```

giac [**A**] time = 0.19, size = 58, normalized size = 0.79

$$\frac{10\,a^3\sin(dx+c)^6 + 36\,a^3\sin(dx+c)^5 + 45\,a^3\sin(dx+c)^4 + 20\,a^3\sin(dx+c)^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(10*a^3*sin(d*x + c)^6 + 36*a^3*sin(d*x + c)^5 + 45*a^3*sin(d*x + c)^4
 + 20*a^3*sin(d*x + c)^3)/d
```

maple [A] time = 0.10, size = 58, normalized size = 0.79

$$\frac{\frac{a^3\left(\sin^6(dx+c)\right)}{6}+\frac{3a^3\left(\sin^5(dx+c)\right)}{5}+\frac{3a^3\left(\sin^4(dx+c)\right)}{4}+\frac{a^3\left(\sin^3(dx+c)\right)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(1/6*a^3*sin(d*x+c)^6+3/5*a^3*sin(d*x+c)^5+3/4*a^3*sin(d*x+c)^4+1/3*a^3
*sin(d*x+c)^3)
```

maxima [A] time = 0.33, size = 58, normalized size = 0.79

$$\frac{10\,a^3\sin\left(dx+c\right)^6+36\,a^3\sin\left(dx+c\right)^5+45\,a^3\sin\left(dx+c\right)^4+20\,a^3\sin\left(dx+c\right)^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(10*a^3*sin(d*x + c)^6 + 36*a^3*sin(d*x + c)^5 + 45*a^3*sin(d*x + c)^4
 + 20*a^3*sin(d*x + c)^3)/d
```

mupad [B] time = 0.06, size = 57, normalized size = 0.78

$$\frac{\frac{a^3\sin(c+d\,x)^6}{6}+\frac{3\,a^3\sin(c+d\,x)^5}{5}+\frac{3\,a^3\sin(c+d\,x)^4}{4}+\frac{a^3\sin(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^3)/3 + (3*a^3*sin(c + d*x)^4)/4 + (3*a^3*sin(c + d*x)^5)
/5 + (a^3*sin(c + d*x)^6)/6)/d
```

sympy [A] time = 5.14, size = 82, normalized size = 1.12

$$\begin{cases} \frac{a^3\sin^6\left(c+dx\right)}{6d}+\frac{3a^3\sin^5\left(c+dx\right)}{5d}+\frac{3a^3\sin^4\left(c+dx\right)}{4d}+\frac{a^3\sin^3\left(c+dx\right)}{3d} & \text{for } d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)^3\sin^2\left(c\right)\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**6/(6*d) + 3*a**3*sin(c + d*x)**5/(5*d) + 3*a*
*3*sin(c + d*x)**4/(4*d) + a**3*sin(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin
(c) + a)**3*sin(c)**2*cos(c), True))
```

3.208 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=45

$$\frac{(a\sin(c+dx)+a)^5}{5a^2d}-\frac{(a\sin(c+dx)+a)^4}{4ad}$$

```
[Out] -1/4*(a+a*sin(d*x+c))^4/a/d+1/5*(a+a*sin(d*x+c))^5/a^2/d
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 43}

$$\frac{(a\sin(c+dx)+a)^5}{5a^2d}-\frac{(a\sin(c+dx)+a)^4}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a + a*Sin[c + d*x])^4/(4*a*d) + (a + a*Sin[c + d*x])^5/(5*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^3}{a}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-a(a+x)^3+(a+x)^4\right)\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= -\frac{(a+a\sin(c+dx))^4}{4ad} + \frac{(a+a\sin(c+dx))^5}{5a^2d}$$

Mathematica [A] time = 0.12, size = 30, normalized size = 0.67

$$\frac{a^3(\sin(c+dx)+1)^4(4\sin(c+dx)-1)}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^4*(-1 + 4*Sin[c + d*x]))/(20*d)
```

fricas [A] time = 0.46, size = 71, normalized size = 1.58

$$\frac{15\,a^3\cos\left(dx+c\right)^4 - 40\,a^3\cos\left(dx+c\right)^2 + 4\left(a^3\cos\left(dx+c\right)^4 - 7\,a^3\cos\left(dx+c\right)^2 + 6\,a^3\right)\sin\left(dx+c\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/20*(15*a^3*cos(d*x + c)^4 - 40*a^3*cos(d*x + c)^2 + 4*(a^3*cos(d*x + c)^4
 - 7*a^3*cos(d*x + c)^2 + 6*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.16, size = 58, normalized size = 1.29

$$\frac{4\,a^3\sin\left(dx+c\right)^5 + 15\,a^3\sin\left(dx+c\right)^4 + 20\,a^3\sin\left(dx+c\right)^3 + 10\,a^3\sin\left(dx+c\right)^2}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/20*(4*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3
+ 10*a^3*sin(d*x + c)^2)/d
```

maple [A] time = 0.10, size = 57, normalized size = 1.27

$$\frac{\frac{a^3\left(\sin^5(dx+c)\right)}{5}+\frac{3a^3\left(\sin^4(dx+c)\right)}{4}+a^3\left(\sin^3(dx+c)\right)+\frac{a^3\left(\sin^2(dx+c)\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(1/5*a^3*sin(d*x+c)^5+3/4*a^3*sin(d*x+c)^4+a^3*sin(d*x+c)^3+1/2*a^3*sin
(d*x+c)^2)
```

maxima [A] time = 0.33, size = 58, normalized size = 1.29

$$\frac{4\,a^3\sin(dx+c)^5+15\,a^3\sin(dx+c)^4+20\,a^3\sin(dx+c)^3+10\,a^3\sin(dx+c)^2}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(4*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3
+ 10*a^3*sin(d*x + c)^2)/d
```

mupad [B] time = 0.06, size = 56, normalized size = 1.24

$$\frac{\frac{a^3\sin(c+d\,x)^5}{5}+\frac{3\,a^3\sin(c+d\,x)^4}{4}+a^3\sin(c+d\,x)^3+\frac{a^3\sin(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^2)/2 + a^3*sin(c + d*x)^3 + (3*a^3*sin(c + d*x)^4)/4 + (
a^3*sin(c + d*x)^5)/5)/d
```

sympy [A] time = 2.55, size = 76, normalized size = 1.69

$$\begin{cases}\frac{a^3\sin^5(c+dx)}{5d}+\frac{3a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{d}+\frac{a^3\sin^2(c+dx)}{2d} & \text{for } d\neq 0\\ x\,(a\sin(c)+a)^3\sin(c)\cos(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**5/(5*d) + 3*a**3*sin(c + d*x)**4/(4*d) + a**3
*sin(c + d*x)**3/d + a**3*sin(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*sin(c) +
a)**3*sin(c)*cos(c), True))
```

3.209 $\int \cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=65

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

```
[Out] a^3*ln(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d+3/2*a^3*sin(d*x+c)^2/d+1/3*a^3*sin(
d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 43}

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)
/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^3}{x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(3a^2+\frac{a^3}{x}+3ax+x^2\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin^3(c+}{3d}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.00

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)
/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.48, size = 59, normalized size = 0.91

$$-\frac{9\,a^3\cos\left(dx+c\right)^2-6\,a^3\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+2\left(a^3\cos\left(dx+c\right)^2-10\,a^3\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(9*a^3*cos(d*x + c)^2 - 6*a^3*log(1/2*sin(d*x + c)) + 2*(a^3*cos(d*x +
 c)^2 - 10*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.18, size = 56, normalized size = 0.86

$$\frac{2\,a^3\sin\left(dx+c\right)^3+9\,a^3\sin\left(dx+c\right)^2+6\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)+18\,a^3\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 6*a^3*log(abs(sin(d*x +
c))) + 18*a^3*sin(d*x + c))/d
```

maple [A] time = 0.18, size = 62, normalized size = 0.95

$$\frac{a^3 \ln(\sin(dx+c))}{d} + \frac{3a^3 \sin(dx+c)}{d} + \frac{3a^3 \left(\sin^2(dx+c)\right)}{2d} + \frac{a^3 \left(\sin^3(dx+c)\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] a^3*ln(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d+3/2*a^3*sin(d*x+c)^2/d+1/3*a^3*sin(
d*x+c)^3/d
```

maxima [A] time = 0.33, size = 55, normalized size = 0.85

$$\frac{2\,a^3 \sin(dx+c)^3 + 9\,a^3 \sin(dx+c)^2 + 6\,a^3 \log(\sin(dx+c)) + 18\,a^3 \sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 6*a^3*log(sin(d*x + c))
+ 18*a^3*sin(d*x + c))/d
```

mupad [B] time = 8.65, size = 102, normalized size = 1.57

$$\frac{10\,a^3\,\sin(c+d\,x)}{3\,d} - \frac{a^3\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d} + \frac{a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{3\,a^3\cos(c+d\,x)^2}{2\,d} - \frac{a^3\cos(c+d\,x)^2\,\sin(c+d\,x)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] (10*a^3*sin(c + d*x))/(3*d) - (a^3*log(1/cos(c/2 + (d*x)/2)^2))/d + (a^3*lo
g(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (3*a^3*cos(c + d*x)^2)/(2*d)
- (a^3*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int \cos(c+dx)\csc(c+dx)\,dx + \int 3\sin(c+dx)\cos(c+dx)\csc(c+dx)\,dx + \int 3\sin^2(c+dx)\cos(c+dx)\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(cos(c + d*x)*csc(c + d*x), x) + Integral(3*sin(c + d*x)*cos(
c + d*x)*csc(c + d*x), x) + Integral(3*sin(c + d*x)**2*cos(c + d*x)*csc(c +
 d*x), x) + Integral(sin(c + d*x)**3*cos(c + d*x)*csc(c + d*x), x))
```

3.210 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=62

$$\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

```
[Out] -a^3*csc(d*x+c)/d+3*a^3*ln(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d+1/2*a^3*sin(d*x
+c)^2/d
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 43}

$$\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) + (3*a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x]
)/d + (a^3*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a+x)^3}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a+x)^3}{x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(3a+\frac{a^3}{x^2}+\frac{3a^2}{x}+x\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a}{}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 1.00

$$\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) + (3*a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x]
)/d + (a^3*Sin[c + d*x]^2)/(2*d)
```

fricas [A] time = 0.50, size = 78, normalized size = 1.26

$$-\frac{12\,a^3\cos(dx+c)^2-12\,a^3\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-8\,a^3+\left(2\,a^3\cos(dx+c)^2-a^3\right)\sin(dx+c)}{4\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(12*a^3*cos(d*x + c)^2 - 12*a^3*log(1/2*sin(d*x + c))*sin(d*x + c) - 8
*a^3 + (2*a^3*cos(d*x + c)^2 - a^3)*sin(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.19, size = 55, normalized size = 0.89

$$\frac{a^3\sin(dx+c)^2+6\,a^3\log(|\sin(dx+c)|)+6\,a^3\sin(dx+c)-\frac{2\,a^3}{\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(a^3*sin(d*x + c)^2 + 6*a^3*log(abs(sin(d*x + c))) + 6*a^3*sin(d*x + c)
 - 2*a^3/sin(d*x + c))/d
```

maple [A] time = 0.14, size = 63, normalized size = 1.02

$$\frac{a^3 \left(\sin^2(dx+c)\right)}{2d}+\frac{3a^3 \sin(dx+c)}{d}-\frac{a^3}{d\sin(dx+c)}+\frac{3a^3\ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sin(d*x+c)^2/d+3*a^3*sin(d*x+c)/d-a^3/d/sin(d*x+c)+3*a^3*ln(sin(d*x
+c))/d
```

maxima [A] time = 0.31, size = 54, normalized size = 0.87

$$\frac{a^3 \sin(dx+c)^2+6\,a^3\log(\sin(dx+c))+6\,a^3\sin(dx+c)-\frac{2\,a^3}{\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a^3*sin(d*x + c)^2 + 6*a^3*log(sin(d*x + c)) + 6*a^3*sin(d*x + c) - 2*
a^3/sin(d*x + c))/d
```

mupad [B] time = 8.59, size = 156, normalized size = 2.52

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{11\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^3}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d + (10*a^3*tan(c/2 + (d*x)/2)^2 + 4*a^3*ta
n(c/2 + (d*x)/2)^3 + 11*a^3*tan(c/2 + (d*x)/2)^4 - a^3)/(d*(2*tan(c/2 + (d*
x)/2) + 4*tan(c/2 + (d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^5)) - (a^3*tan(c/2 +
(d*x)/2))/(2*d) - (3*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\cos(c+dx)\csc^2(c+dx)\,dx+\int 3\sin(c+dx)\cos(c+dx)\csc^2(c+dx)\,dx+\int 3\sin^2(c+dx)\cos(c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(cos(c + d*x)*csc(c + d*x)**2, x) + Integral(3*sin(c + d*x)*c
os(c + d*x)*csc(c + d*x)**2, x) + Integral(3*sin(c + d*x)**2*cos(c + d*x)*c
sc(c + d*x)**2, x) + Integral(sin(c + d*x)**3*cos(c + d*x)*csc(c + d*x)**2,
 x))
```

3.211 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=61

$$\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

```
[Out] -3*a^3*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d+3*a^3*ln(sin(d*x+c))/d+a^3*sin(d
*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) + (3*a^3*Log[Sin[c + d
*x]])/d + (a^3*Sin[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a+x)^3}{x^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(a+x)^3}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(1+\frac{a^3}{x^3}+\frac{3a^2}{x^2}+\frac{3a}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{3a^3\csc(c+dx)}{d} - \frac{a^3\csc^2(c+dx)}{2d} + \frac{3a^3\log(\sin(c+dx))}{d} +$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.87

$$a^3\left(\frac{\sin(c+dx)}{d} - \frac{\csc^2(c+dx)}{2d} - \frac{3\csc(c+dx)}{d} + \frac{3\log(\sin(c+dx))}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((-3*Csc[c + d*x])/d - Csc[c + d*x]^2/(2*d) + (3*Log[Sin[c + d*x]])/d +
 Sin[c + d*x]/d)
```

fricas [A] time = 0.49, size = 77, normalized size = 1.26

$$\frac{a^3 + 6\left(a^3\cos(dx+c)^2 - a^3\right)\log\left(\frac{1}{2}\sin(dx+c)\right) + 2\left(a^3\cos(dx+c)^2 + 2a^3\right)\sin(dx+c)}{2\left(d\cos(dx+c)^2 - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3 + 6*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*sin(d*x + c)) + 2*(a^3*cos(
d*x + c)^2 + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.18, size = 55, normalized size = 0.90

$$\frac{6a^3\log(|\sin(dx+c)|) + 2a^3\sin(dx+c) - \frac{6a^3\sin(dx+c)+a^3}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^3*log(abs(sin(d*x + c))) + 2*a^3*sin(d*x + c) - (6*a^3*sin(d*x + c
) + a^3)/sin(d*x + c)^2)/d
```

maple [A] time = 0.19, size = 62, normalized size = 1.02

$$\frac{a^3 \sin (dx+c)}{d}-\frac{3a^3}{d \sin (dx+c)}+\frac{3a^3 \ln (\sin (dx+c))}{d}-\frac{a^3}{2d \sin (dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] a^3*sin(d*x+c)/d-3*a^3/d/sin(d*x+c)+3*a^3*ln(sin(d*x+c))/d-1/2*a^3/d/sin(d*
x+c)^2
```

maxima [A] time = 0.32, size = 54, normalized size = 0.89

$$\frac{6\,a^3 \log (\sin (dx+c))+2\,a^3 \sin (dx+c)-\frac{6\,a^3 \sin (dx+c)+a^3}{\sin (dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*a^3*log(sin(d*x + c)) + 2*a^3*sin(d*x + c) - (6*a^3*sin(d*x + c) + a
^3)/sin(d*x + c)^2)/d
```

mupad [B] time = 8.57, size = 163, normalized size = 2.67

$$\frac{3\,a^3\,\ln \left(\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-2\,a^3\,\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{a^3\,\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+6\,a^3\,\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3}{2}}{d\,\left(4\,\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{a^3\,\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{3\,a^3\,\mathrm{tan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d - ((a^3*tan(c/2 + (d*x)/2)^2)/2 - 2*a^3*t
an(c/2 + (d*x)/2)^3 + a^3/2 + 6*a^3*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*
x)/2)^2 + 4*tan(c/2 + (d*x)/2)^4)) - (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*
a^3*tan(c/2 + (d*x)/2))/(2*d) - (3*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.212 $\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=65

$$-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

```
[Out] -3*a^3*csc(d*x+c)/d-3/2*a^3*csc(d*x+c)^2/d-1/3*a^3*csc(d*x+c)^3/d+a^3*ln(si
n(d*x+c))/d
```

Rubi [A] time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (3*a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^
3)/(3*d) + (a^3*Log[Sin[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a+x)^3}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{(a+x)^3}{x^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{3a^2}{x^3}+\frac{3a}{x^2}+\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{3a^3\csc(c+dx)}{d} - \frac{3a^3\csc^2(c+dx)}{2d} - \frac{a^3\csc^3(c+dx)}{3d} + \frac{a^3\,1}{}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.88

$$a^3\left(-\frac{\csc^3(c+dx)}{3d} - \frac{3\csc^2(c+dx)}{2d} - \frac{3\csc(c+dx)}{d} + \frac{\log(\sin(c+dx))}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((-3*Csc[c + d*x])/d - (3*Csc[c + d*x]^2)/(2*d) - Csc[c + d*x]^3/(3*d)
+ Log[Sin[c + d*x]]/d)
```

fricas [A] time = 0.49, size = 91, normalized size = 1.40

$$-\frac{18\,a^3\cos(dx+c)^2 - 9\,a^3\sin(dx+c) - 20\,a^3 - 6\left(a^3\cos(dx+c)^2 - a^3\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)}{6\left(d\cos(dx+c)^2 - d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(18*a^3*cos(d*x + c)^2 - 9*a^3*sin(d*x + c) - 20*a^3 - 6*(a^3*cos(d*x
+ c)^2 - a^3)*log(1/2*sin(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*s
in(d*x + c))
```

giac [A] time = 0.18, size = 59, normalized size = 0.91

$$\frac{6\,a^3\log\left(|\sin(dx+c)|\right) - \frac{18\,a^3\sin(dx+c)^2 + 9\,a^3\sin(dx+c) + 2\,a^3}{\sin(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(6*a^3*log(abs(sin(d*x + c))) - (18*a^3*sin(d*x + c)^2 + 9*a^3*sin(d*x
+ c) + 2*a^3)/sin(d*x + c)^3)/d
```

maple [A] time = 0.14, size = 64, normalized size = 0.98

$$-\frac{3a^{3}}{d\sin\left(dx+c\right)}+\frac{a^{3}\ln\left(\sin\left(dx+c\right)\right)}{d}-\frac{3a^{3}}{2d\sin\left(dx+c\right)^{2}}-\frac{a^{3}}{3d\sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -3*a^3/d/sin(d*x+c)+a^3*ln(sin(d*x+c))/d-3/2*a^3/d/sin(d*x+c)^2-1/3*a^3/d/s
in(d*x+c)^3
```

maxima [A] time = 0.42, size = 58, normalized size = 0.89

$$\frac{6\,a^{3}\log\left(\sin\left(dx+c\right)\right)-\frac{18\,a^{3}\sin\left(dx+c\right)^{2}+9\,a^{3}\sin\left(dx+c\right)+2\,a^{3}}{\sin\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^3*log(sin(d*x + c)) - (18*a^3*sin(d*x + c)^2 + 9*a^3*sin(d*x + c)
+ 2*a^3)/sin(d*x + c)^3)/d
```

mupad [B] time = 8.62, size = 147, normalized size = 2.26

$$\frac{a^{3}\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{3\,a^{3}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{8\,d}-\frac{a^{3}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\,\left(13\,a^{3}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+3\,a^{3}\,\tan\left(\frac{c}{2}+\right.\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d - (3*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - (a^3
*tan(c/2 + (d*x)/2)^3)/(24*d) - (cot(c/2 + (d*x)/2)^3*(13*a^3*tan(c/2 + (d*
x)/2)^2 + a^3/3 + 3*a^3*tan(c/2 + (d*x)/2)))/(8*d) - (13*a^3*tan(c/2 + (d*x
)/2))/(8*d) - (a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.213 $\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=30

$$-\frac{\csc^4(c+dx)(a\sin(c+dx)+a)^4}{4ad}$$

[Out] -1/4*csc(d*x+c)^4*(a+a*sin(d*x+c))^4/a/d

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 37}

$$-\frac{\csc^4(c+dx)(a\sin(c+dx)+a)^4}{4ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]

[Out] -(Csc[c + d*x]^4*(a + a*Sin[c + d*x])^4)/(4*a*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^5(a+x)^3}{x^5}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^4\,\text{Subst}\left(\int \frac{(a+x)^3}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^4(c+dx)(a+a\sin(c+dx))^4}{4ad}$$

Mathematica [**A**] time = 0.02, size = 20, normalized size = 0.67

$$-\frac{a^3(\csc(c+dx)+1)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/4*(a^3*(1 + Csc[c + d*x])^4)/d
```

fricas [**B**] time = 0.45, size = 72, normalized size = 2.40

$$\frac{6\,a^3\cos(dx+c)^2 - 7\,a^3 + 4\left(a^3\cos(dx+c)^2 - 2\,a^3\right)\sin(dx+c)}{4\left(d\cos(dx+c)^4 - 2\,d\cos(dx+c)^2 + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*a^3*cos(d*x + c)^2 - 7*a^3 + 4*(a^3*cos(d*x + c)^2 - 2*a^3)*sin(d*x
+ c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [**A**] time = 0.18, size = 54, normalized size = 1.80

$$-\frac{4\,a^3\sin(dx+c)^3 + 6\,a^3\sin(dx+c)^2 + 4\,a^3\sin(dx+c) + a^3}{4\,d\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*a^3*sin(d*x + c)^3 + 6*a^3*sin(d*x + c)^2 + 4*a^3*sin(d*x + c) + a^
3)/(d*sin(d*x + c)^4)
```

maple [A] time = 0.14, size = 49, normalized size = 1.63

$$\frac{a^3\left(-\frac{1}{\sin(dx+c)}-\frac{3}{2\sin(dx+c)^2}-\frac{1}{4\sin(dx+c)^4}-\frac{1}{\sin(dx+c)^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] a^3/d*(-1/sin(d*x+c)-3/2/sin(d*x+c)^2-1/4/sin(d*x+c)^4-1/sin(d*x+c)^3)
```

maxima [A] time = 0.50, size = 54, normalized size = 1.80

$$-\frac{4\,a^3\sin\left(dx+c\right)^3+6\,a^3\sin\left(dx+c\right)^2+4\,a^3\sin\left(dx+c\right)+a^3}{4\,d\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^3*sin(d*x + c)^3 + 6*a^3*sin(d*x + c)^2 + 4*a^3*sin(d*x + c) + a^
3)/(d*sin(d*x + c)^4)
```

mupad [B] time = 8.61, size = 54, normalized size = 1.80

$$-\frac{4\,a^3\sin\left(c+d\,x\right)^3+6\,a^3\sin\left(c+d\,x\right)^2+4\,a^3\,\sin\left(c+d\,x\right)+a^3}{4\,d\sin\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] -(4*a^3*sin(c + d*x) + a^3 + 6*a^3*sin(c + d*x)^2 + 4*a^3*sin(c + d*x)^3)/(
4*d*sin(c + d*x)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.214 $\int \cot(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=61

$$\frac{\csc^4(c+dx)(a\sin(c+dx)+a)^4}{20ad}-\frac{\csc^5(c+dx)(a\sin(c+dx)+a)^4}{5ad}$$

```
[Out] 1/20*csc(d*x+c)^4*(a+a*sin(d*x+c))^4/a/d-1/5*csc(d*x+c)^5*(a+a*sin(d*x+c))^
4/a/d
```

Rubi **[A]** time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2833, 12, 45, 37}

$$\frac{\csc^4(c+dx)(a\sin(c+dx)+a)^4}{20ad}-\frac{\csc^5(c+dx)(a\sin(c+dx)+a)^4}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^4*(a + a*Sin[c + d*x])^4)/(20*a*d) - (Csc[c + d*x]^5*(a + a*S
in[c + d*x])^4)/(5*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a+x)^3}{x^6}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^5\,\text{Subst}\left(\int \frac{(a+x)^3}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^5(c+dx)(a+a\sin(c+dx))^4}{5ad} - \frac{a^4\,\text{Subst}\left(\int \frac{(a+x)^3}{x^5}\,dx, x\right.}{5d}$$
$$= \frac{\csc^4(c+dx)(a+a\sin(c+dx))^4}{20ad} - \frac{\csc^5(c+dx)(a+a\sin(c}{5ad}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 1.16

$$-\frac{a^3\csc^5(c+dx)}{5d} - \frac{3a^3\csc^4(c+dx)}{4d} - \frac{a^3\csc^3(c+dx)}{d} - \frac{a^3\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*(a^3*Csc[c + d*x]^2)/d - (a^3*Csc[c + d*x]^3)/d - (3*a^3*Csc[c + d*x]^
4)/(4*d) - (a^3*Csc[c + d*x]^5)/(5*d)
```

fricas [A] time = 0.45, size = 81, normalized size = 1.33

$$\frac{20\,a^3\cos(dx+c)^2 - 24\,a^3 + 5\left(2\,a^3\cos(dx+c)^2 - 5\,a^3\right)\sin(dx+c)}{20\left(d\cos(dx+c)^4 - 2\,d\cos(dx+c)^2 + d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/20*(20*a^3*cos(d*x + c)^2 - 24*a^3 + 5*(2*a^3*cos(d*x + c)^2 - 5*a^3)*sin
(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.18, size = 56, normalized size = 0.92

$$-\frac{10\,a^{3}\sin\left(dx+c\right)^{3}+20\,a^{3}\sin\left(dx+c\right)^{2}+15\,a^{3}\sin\left(dx+c\right)+4\,a^{3}}{20\,d\sin\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(10*a^3*sin(d*x + c)^3 + 20*a^3*sin(d*x + c)^2 + 15*a^3*sin(d*x + c)
+ 4*a^3)/(d*sin(d*x + c)^5)
```

maple [A] time = 0.14, size = 49, normalized size = 0.80

$$\frac{a^{3}\left(-\frac{1}{5\sin\left(dx+c\right)^{5}}-\frac{1}{2\sin\left(dx+c\right)^{2}}-\frac{3}{4\sin\left(dx+c\right)^{4}}-\frac{1}{\sin\left(dx+c\right)^{3}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x)
```

```
[Out] a^3/d*(-1/5/sin(d*x+c)^5-1/2/sin(d*x+c)^2-3/4/sin(d*x+c)^4-1/sin(d*x+c)^3)
```

maxima [A] time = 0.51, size = 56, normalized size = 0.92

$$-\frac{10\,a^{3}\sin\left(dx+c\right)^{3}+20\,a^{3}\sin\left(dx+c\right)^{2}+15\,a^{3}\sin\left(dx+c\right)+4\,a^{3}}{20\,d\sin\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/20*(10*a^3*sin(d*x + c)^3 + 20*a^3*sin(d*x + c)^2 + 15*a^3*sin(d*x + c)
+ 4*a^3)/(d*sin(d*x + c)^5)
```

mupad [B] time = 8.60, size = 56, normalized size = 0.92

$$-\frac{10\,a^{3}\sin\left(c+d\,x\right)^{3}+20\,a^{3}\sin\left(c+d\,x\right)^{2}+15\,a^{3}\,\sin\left(c+d\,x\right)+4\,a^{3}}{20\,d\sin\left(c+d\,x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] -(15*a^3*sin(c + d*x) + 4*a^3 + 20*a^3*sin(c + d*x)^2 + 10*a^3*sin(c + d*x)
^3)/(20*d*sin(c + d*x)^5)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.215 $\int \cot(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=73

$$-\frac{a^3\csc^6(c+dx)}{6d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{3d}$$

```
[Out] -1/3*a^3*csc(d*x+c)^3/d-3/4*a^3*csc(d*x+c)^4/d-3/5*a^3*csc(d*x+c)^5/d-1/6*a
^3*csc(d*x+c)^6/d
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{a^3\csc^6(c+dx)}{6d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Csc[c + d*x]^3)/(3*d) - (3*a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c +
 d*x]^5)/(5*d) - (a^3*Csc[c + d*x]^6)/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a+x)^3}{x^7}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{(a+x)^3}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(\frac{a^3}{x^7}+\frac{3a^2}{x^6}+\frac{3a}{x^5}+\frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a}{}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.00

$$-\frac{a^3\csc^6(c+dx)}{6d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/3*(a^3*Csc[c + d*x]^3)/d - (3*a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c +
 d*x]^5)/(5*d) - (a^3*Csc[c + d*x]^6)/(6*d)
```

fricas [A] time = 0.45, size = 86, normalized size = 1.18

$$-\frac{45\,a^3\cos(dx+c)^2-55\,a^3+4\left(5\,a^3\cos(dx+c)^2-14\,a^3\right)\sin(dx+c)}{60\left(d\cos(dx+c)^6-3\,d\cos(dx+c)^4+3\,d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(45*a^3*cos(d*x + c)^2 - 55*a^3 + 4*(5*a^3*cos(d*x + c)^2 - 14*a^3)*s
in(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
d)
```

giac [A] time = 0.20, size = 56, normalized size = 0.77

$$-\frac{20\,a^3\sin(dx+c)^3+45\,a^3\sin(dx+c)^2+36\,a^3\sin(dx+c)+10\,a^3}{60\,d\sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/60*(20*a^3*sin(d*x + c)^3 + 45*a^3*sin(d*x + c)^2 + 36*a^3*sin(d*x + c)
+ 10*a^3)/(d*sin(d*x + c)^6)
```

maple [A] time = 0.16, size = 49, normalized size = 0.67

$$\frac{a^3\left(-\frac{1}{6\sin(dx+c)^6}-\frac{3}{5\sin(dx+c)^5}-\frac{3}{4\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x)
```

```
[Out] a^3/d*(-1/6/sin(d*x+c)^6-3/5/sin(d*x+c)^5-3/4/sin(d*x+c)^4-1/3/sin(d*x+c)^3
)
```

maxima [A] time = 0.34, size = 56, normalized size = 0.77

$$-\frac{20\,a^3\sin\left(dx+c\right)^3+45\,a^3\sin\left(dx+c\right)^2+36\,a^3\sin\left(dx+c\right)+10\,a^3}{60\,d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(20*a^3*sin(d*x + c)^3 + 45*a^3*sin(d*x + c)^2 + 36*a^3*sin(d*x + c)
+ 10*a^3)/(d*sin(d*x + c)^6)
```

mupad [B] time = 8.60, size = 56, normalized size = 0.77

$$-\frac{a^3\,\left(-20\sin\left(c+d\,x\right)^6+20\sin\left(c+d\,x\right)^3+45\sin\left(c+d\,x\right)^2+36\,\sin\left(c+d\,x\right)+10\right)}{60\,d\sin\left(c+d\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] -(a^3*(36*sin(c + d*x) + 45*sin(c + d*x)^2 + 20*sin(c + d*x)^3 - 20*sin(c +
 d*x)^6 + 10))/(60*d*sin(c + d*x)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.216 $\int \cot(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=73

$$-\frac{a^3\csc^7(c+dx)}{7d}-\frac{a^3\csc^6(c+dx)}{2d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^4(c+dx)}{4d}$$

```
[Out] -1/4*a^3*csc(d*x+c)^4/d-3/5*a^3*csc(d*x+c)^5/d-1/2*a^3*csc(d*x+c)^6/d-1/7*a
^3*csc(d*x+c)^7/d
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{a^3\csc^7(c+dx)}{7d}-\frac{a^3\csc^6(c+dx)}{2d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c + d*x]^5)/(5*d) - (a^3*Csc[c + d
*x]^6)/(2*d) - (a^3*Csc[c + d*x]^7)/(7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^8(a+x)^3}{x^8}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^7\,\text{Subst}\left(\int \frac{(a+x)^3}{x^8}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{a^3}{x^8}+\frac{3a^2}{x^7}+\frac{3a}{x^6}+\frac{1}{x^5}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^3\csc^4(c+dx)}{4d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^6(c+dx)}{2d}-\frac{a^3\,\text{c}}{}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.00

$$-\frac{a^3\csc^7(c+dx)}{7d}-\frac{a^3\csc^6(c+dx)}{2d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/4*(a^3*Csc[c + d*x]^4)/d - (3*a^3*Csc[c + d*x]^5)/(5*d) - (a^3*Csc[c + d
*x]^6)/(2*d) - (a^3*Csc[c + d*x]^7)/(7*d)
```

fricas [A] time = 0.63, size = 93, normalized size = 1.27

$$-\frac{84\,a^3\cos(dx+c)^2-104\,a^3+35\left(a^3\cos(dx+c)^2-3\,a^3\right)\sin(dx+c)}{140\left(d\cos(dx+c)^6-3\,d\cos(dx+c)^4+3\,d\cos(dx+c)^2-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/140*(84*a^3*cos(d*x + c)^2 - 104*a^3 + 35*(a^3*cos(d*x + c)^2 - 3*a^3)*s
in(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
 d)*sin(d*x + c))
```

giac [A] time = 0.19, size = 56, normalized size = 0.77

$$-\frac{35\,a^3\sin(dx+c)^3+84\,a^3\sin(dx+c)^2+70\,a^3\sin(dx+c)+20\,a^3}{140\,d\sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/140*(35*a^3*sin(d*x + c)^3 + 84*a^3*sin(d*x + c)^2 + 70*a^3*sin(d*x + c)
 + 20*a^3)/(d*sin(d*x + c)^7)
```

maple [A] time = 0.19, size = 49, normalized size = 0.67

$$\frac{a^3\left(-\frac{1}{2\sin(dx+c)^6}-\frac{3}{5\sin(dx+c)^5}-\frac{1}{7\sin(dx+c)^7}-\frac{1}{4\sin(dx+c)^4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x)
```

```
[Out] a^3/d*(-1/2/sin(d*x+c)^6-3/5/sin(d*x+c)^5-1/7/sin(d*x+c)^7-1/4/sin(d*x+c)^4
)
```

maxima [A] time = 0.38, size = 56, normalized size = 0.77

$$-\frac{35\,a^3\sin\left(dx+c\right)^3+84\,a^3\sin\left(dx+c\right)^2+70\,a^3\sin\left(dx+c\right)+20\,a^3}{140\,d\sin\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/140*(35*a^3*sin(d*x + c)^3 + 84*a^3*sin(d*x + c)^2 + 70*a^3*sin(d*x + c)
 + 20*a^3)/(d*sin(d*x + c)^7)
```

mupad [B] time = 8.66, size = 56, normalized size = 0.77

$$-\frac{35\,a^3\sin\left(c+d\,x\right)^3+84\,a^3\sin\left(c+d\,x\right)^2+70\,a^3\,\sin\left(c+d\,x\right)+20\,a^3}{140\,d\sin\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x)^8,x)
```

```
[Out] -(70*a^3*sin(c + d*x) + 20*a^3 + 84*a^3*sin(c + d*x)^2 + 35*a^3*sin(c + d*x
)^3)/(140*d*sin(c + d*x)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**8*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.217 $\int \cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=91

$$\frac{a^4\sin^9(c+dx)}{9d}+\frac{a^4\sin^8(c+dx)}{2d}+\frac{6a^4\sin^7(c+dx)}{7d}+\frac{2a^4\sin^6(c+dx)}{3d}+\frac{a^4\sin^5(c+dx)}{5d}$$

```
[Out] 1/5*a^4*sin(d*x+c)^5/d+2/3*a^4*sin(d*x+c)^6/d+6/7*a^4*sin(d*x+c)^7/d+1/2*a^
4*sin(d*x+c)^8/d+1/9*a^4*sin(d*x+c)^9/d
```

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^4\sin^9(c+dx)}{9d}+\frac{a^4\sin^8(c+dx)}{2d}+\frac{6a^4\sin^7(c+dx)}{7d}+\frac{2a^4\sin^6(c+dx)}{3d}+\frac{a^4\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Sin[c + d*x]^5)/(5*d) + (2*a^4*Sin[c + d*x]^6)/(3*d) + (6*a^4*Sin[c +
d*x]^7)/(7*d) + (a^4*Sin[c + d*x]^8)/(2*d) + (a^4*Sin[c + d*x]^9)/(9*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+x)^4}{a^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^4(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4x^4+4a^3x^5+6a^2x^6+4ax^7+x^8\right)dx, x, a\sin(c$$
$$= \frac{a^4\sin^5(c+dx)}{5d} + \frac{2a^4\sin^6(c+dx)}{3d} + \frac{6a^4\sin^7(c+dx)}{7d} + \frac{a^4$$

Mathematica [A] time = 0.84, size = 100, normalized size = 1.10

$$\frac{a^4(52290\sin(c+dx) - 30660\sin(3(c+dx)) + 9828\sin(5(c+dx)) - 1395\sin(7(c+dx)) + 35\sin(9(c+dx)) -}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(4095 - 42840*Cos[2*(c + d*x)] + 18900*Cos[4*(c + d*x)] - 4200*Cos[6*(
c + d*x)] + 315*Cos[8*(c + d*x)] + 52290*Sin[c + d*x] - 30660*Sin[3*(c + d*
x)] + 9828*Sin[5*(c + d*x)] - 1395*Sin[7*(c + d*x)] + 35*Sin[9*(c + d*x)]))
/(80640*d)
```

fricas [A] time = 0.90, size = 124, normalized size = 1.36

$$\frac{315\,a^4\cos(dx+c)^8 - 1680\,a^4\cos(dx+c)^6 + 3150\,a^4\cos(dx+c)^4 - 2520\,a^4\cos(dx+c)^2 + 2\left(35\,a^4\cos(dx+\right.}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/630*(315*a^4*cos(d*x + c)^8 - 1680*a^4*cos(d*x + c)^6 + 3150*a^4*cos(d*x
+ c)^4 - 2520*a^4*cos(d*x + c)^2 + 2*(35*a^4*cos(d*x + c)^8 - 410*a^4*cos(d
*x + c)^6 + 1083*a^4*cos(d*x + c)^4 - 1076*a^4*cos(d*x + c)^2 + 368*a^4)*si
n(d*x + c))/d
```

giac [A] time = 0.25, size = 71, normalized size = 0.78

$$\frac{70\,a^4\sin(dx+c)^9 + 315\,a^4\sin(dx+c)^8 + 540\,a^4\sin(dx+c)^7 + 420\,a^4\sin(dx+c)^6 + 126\,a^4\sin(dx+c)^5}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/630*(70*a^4*sin(d*x + c)^9 + 315*a^4*sin(d*x + c)^8 + 540*a^4*sin(d*x + c
)^7 + 420*a^4*sin(d*x + c)^6 + 126*a^4*sin(d*x + c)^5)/d
```

maple [A] time = 0.11, size = 71, normalized size = 0.78

$$\frac{\frac{a^4\left(\sin^9(dx+c)\right)}{9}+\frac{a^4\left(\sin^8(dx+c)\right)}{2}+\frac{6a^4\left(\sin^7(dx+c)\right)}{7}+\frac{2a^4\left(\sin^6(dx+c)\right)}{3}+\frac{a^4\left(\sin^5(dx+c)\right)}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(1/9*a^4*sin(d*x+c)^9+1/2*a^4*sin(d*x+c)^8+6/7*a^4*sin(d*x+c)^7+2/3*a^4
*sin(d*x+c)^6+1/5*a^4*sin(d*x+c)^5)
```

maxima [A] time = 0.32, size = 71, normalized size = 0.78

$$\frac{70\,a^4\sin\left(dx+c\right)^9+315\,a^4\sin\left(dx+c\right)^8+540\,a^4\sin\left(dx+c\right)^7+420\,a^4\sin\left(dx+c\right)^6+126\,a^4\sin\left(dx+c\right)^5}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/630*(70*a^4*sin(d*x + c)^9 + 315*a^4*sin(d*x + c)^8 + 540*a^4*sin(d*x + c
)^7 + 420*a^4*sin(d*x + c)^6 + 126*a^4*sin(d*x + c)^5)/d
```

mupad [B] time = 8.46, size = 70, normalized size = 0.77

$$\frac{\frac{a^4\sin(c+d\,x)^9}{9}+\frac{a^4\sin(c+d\,x)^8}{2}+\frac{6\,a^4\sin(c+d\,x)^7}{7}+\frac{2\,a^4\sin(c+d\,x)^6}{3}+\frac{a^4\sin(c+d\,x)^5}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^4*(a + a*sin(c + d*x))^4,x)
```

```
[Out] ((a^4*sin(c + d*x)^5)/5 + (2*a^4*sin(c + d*x)^6)/3 + (6*a^4*sin(c + d*x)^7)
/7 + (a^4*sin(c + d*x)^8)/2 + (a^4*sin(c + d*x)^9)/9)/d
```

sympy [A] time = 19.92, size = 97, normalized size = 1.07

$$\begin{cases} \frac{a^4\sin^9(c+dx)}{9d}+\frac{a^4\sin^8(c+dx)}{2d}+\frac{6a^4\sin^7(c+dx)}{7d}+\frac{2a^4\sin^6(c+dx)}{3d}+\frac{a^4\sin^5(c+dx)}{5d} & \text{for } d\neq 0 \\ x\left(a\sin(c)+a\right)^4\sin^4(c)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**9/(9*d) + a**4*sin(c + d*x)**8/(2*d) + 6*a**4
*sin(c + d*x)**7/(7*d) + 2*a**4*sin(c + d*x)**6/(3*d) + a**4*sin(c + d*x)**
5/(5*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*sin(c)**4*cos(c), True))
```

3.218 $\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=88

$$\frac{a^4\sin^8(c+dx)}{8d}+\frac{4a^4\sin^7(c+dx)}{7d}+\frac{a^4\sin^6(c+dx)}{d}+\frac{4a^4\sin^5(c+dx)}{5d}+\frac{a^4\sin^4(c+dx)}{4d}$$

```
[Out] 1/4*a^4*sin(d*x+c)^4/d+4/5*a^4*sin(d*x+c)^5/d+a^4*sin(d*x+c)^6/d+4/7*a^4*si
n(d*x+c)^7/d+1/8*a^4*sin(d*x+c)^8/d
```

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^4\sin^8(c+dx)}{8d}+\frac{4a^4\sin^7(c+dx)}{7d}+\frac{a^4\sin^6(c+dx)}{d}+\frac{4a^4\sin^5(c+dx)}{5d}+\frac{a^4\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Sin[c + d*x]^4)/(4*d) + (4*a^4*Sin[c + d*x]^5)/(5*d) + (a^4*Sin[c + d*
x]^6)/d + (4*a^4*Sin[c + d*x]^7)/(7*d) + (a^4*Sin[c + d*x]^8)/(8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^4}{a^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^3(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4x^3+4a^3x^4+6a^2x^5+4ax^6+x^7\right)dx, x, a\sin(c\right.}{a^4 d}$$
$$= \frac{a^4\sin^4(c+dx)}{4d} + \frac{4a^4\sin^5(c+dx)}{5d} + \frac{a^4\sin^6(c+dx)}{d} + \frac{4a^4}{}$$

Mathematica [A] time = 0.53, size = 90, normalized size = 1.02

$$\frac{a^4(87360\sin(c+dx) - 47040\sin(3(c+dx)) + 12096\sin(5(c+dx)) - 960\sin(7(c+dx)) - 69720\cos(2(c+dx)}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(36400 - 69720*Cos[2*(c + d*x)] + 26460*Cos[4*(c + d*x)] - 4200*Cos[6*
(c + d*x)] + 105*Cos[8*(c + d*x)] + 87360*Sin[c + d*x] - 47040*Sin[3*(c + d
*x)] + 12096*Sin[5*(c + d*x)] - 960*Sin[7*(c + d*x)]))/(107520*d)
```

fricas [A] time = 0.51, size = 111, normalized size = 1.26

$$\frac{35\,a^4\cos(dx+c)^8 - 420\,a^4\cos(dx+c)^6 + 1120\,a^4\cos(dx+c)^4 - 1120\,a^4\cos(dx+c)^2 - 32\left(5\,a^4\cos(dx+c)^6\right.}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/280*(35*a^4*cos(d*x + c)^8 - 420*a^4*cos(d*x + c)^6 + 1120*a^4*cos(d*x +
c)^4 - 1120*a^4*cos(d*x + c)^2 - 32*(5*a^4*cos(d*x + c)^6 - 22*a^4*cos(d*x
+ c)^4 + 29*a^4*cos(d*x + c)^2 - 12*a^4)*sin(d*x + c))/d
```

giac [A] time = 0.22, size = 71, normalized size = 0.81

$$\frac{35\,a^4\sin(dx+c)^8 + 160\,a^4\sin(dx+c)^7 + 280\,a^4\sin(dx+c)^6 + 224\,a^4\sin(dx+c)^5 + 70\,a^4\sin(dx+c)^4}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(35*a^4*sin(d*x + c)^8 + 160*a^4*sin(d*x + c)^7 + 280*a^4*sin(d*x + c
)^6 + 224*a^4*sin(d*x + c)^5 + 70*a^4*sin(d*x + c)^4)/d
```

maple [A] time = 0.10, size = 70, normalized size = 0.80

$$\frac{\frac{a^4\left(\sin^8(dx+c)\right)}{8} + \frac{4a^4\left(\sin^7(dx+c)\right)}{7} + a^4\left(\sin^6(dx+c)\right) + \frac{4a^4\left(\sin^5(dx+c)\right)}{5} + \frac{a^4\left(\sin^4(dx+c)\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(1/8*a^4*sin(d*x+c)^8+4/7*a^4*sin(d*x+c)^7+a^4*sin(d*x+c)^6+4/5*a^4*sin
(d*x+c)^5+1/4*a^4*sin(d*x+c)^4)
```

maxima [A] time = 0.30, size = 71, normalized size = 0.81

$$\frac{35\,a^4\sin(dx+c)^8 + 160\,a^4\sin(dx+c)^7 + 280\,a^4\sin(dx+c)^6 + 224\,a^4\sin(dx+c)^5 + 70\,a^4\sin(dx+c)^4}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(35*a^4*sin(d*x + c)^8 + 160*a^4*sin(d*x + c)^7 + 280*a^4*sin(d*x + c
)^6 + 224*a^4*sin(d*x + c)^5 + 70*a^4*sin(d*x + c)^4)/d
```

mupad [B] time = 8.45, size = 69, normalized size = 0.78

$$\frac{\frac{a^4\sin(c+d\,x)^8}{8} + \frac{4\,a^4\sin(c+d\,x)^7}{7} + a^4\sin(c+d\,x)^6 + \frac{4\,a^4\sin(c+d\,x)^5}{5} + \frac{a^4\sin(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^3*(a + a*sin(c + d*x))^4,x)
```

```
[Out] ((a^4*sin(c + d*x)^4)/4 + (4*a^4*sin(c + d*x)^5)/5 + a^4*sin(c + d*x)^6 + (
4*a^4*sin(c + d*x)^7)/7 + (a^4*sin(c + d*x)^8)/8)/d
```

sympy [A] time = 11.72, size = 95, normalized size = 1.08

$$\begin{cases} \frac{a^4\sin^8(c+dx)}{8d} + \frac{4a^4\sin^7(c+dx)}{7d} + \frac{a^4\sin^6(c+dx)}{d} + \frac{4a^4\sin^5(c+dx)}{5d} + \frac{a^4\sin^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x\left(a\sin(c)+a\right)^4\sin^3(c)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**8/(8*d) + 4*a**4*sin(c + d*x)**7/(7*d) + a**4
*sin(c + d*x)**6/d + 4*a**4*sin(c + d*x)**5/(5*d) + a**4*sin(c + d*x)**4/(4
*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*sin(c)**3*cos(c), True))
```

3.219 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=67

$$\frac{(a\sin(c+dx)+a)^7}{7a^3d}-\frac{(a\sin(c+dx)+a)^6}{3a^2d}+\frac{(a\sin(c+dx)+a)^5}{5ad}$$

```
[Out] 1/5*(a+a*sin(d*x+c))^5/a/d-1/3*(a+a*sin(d*x+c))^6/a^2/d+1/7*(a+a*sin(d*x+c)
)^7/a^3/d
```

Rubi [A] time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{(a\sin(c+dx)+a)^7}{7a^3d}-\frac{(a\sin(c+dx)+a)^6}{3a^2d}+\frac{(a\sin(c+dx)+a)^5}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a + a*Sin[c + d*x])^5/(5*a*d) - (a + a*Sin[c + d*x])^6/(3*a^2*d) + (a + a*
Sin[c + d*x])^7/(7*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+x)^4}{a^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^2(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2(a+x)^4 - 2a(a+x)^5 + (a+x)^6\right)\,dx, x, a\sin(c$$
$$= \frac{(a+a\sin(c+dx))^5}{5ad} - \frac{(a+a\sin(c+dx))^6}{3a^2 d} + \frac{(a+a\sin(c+$$

Mathematica [A] time = 0.34, size = 80, normalized size = 1.19

$$-\frac{a^4(-7245\sin(c+dx) + 3395\sin(3(c+dx)) - 609\sin(5(c+dx)) + 15\sin(7(c+dx)) + 5460\cos(2(c+dx)) - 1}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/6720*(a^4*(-630 + 5460*Cos[2*(c + d*x)] - 1680*Cos[4*(c + d*x)] + 140*Co
s[6*(c + d*x)] - 7245*Sin[c + d*x] + 3395*Sin[3*(c + d*x)] - 609*Sin[5*(c +
 d*x)] + 15*Sin[7*(c + d*x)]))/d
```

fricas [A] time = 0.49, size = 97, normalized size = 1.45

$$-\frac{70\,a^4\cos(dx+c)^6 - 315\,a^4\cos(dx+c)^4 + 420\,a^4\cos(dx+c)^2 + \left(15\,a^4\cos(dx+c)^6 - 171\,a^4\cos(dx+c)^4 +}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(70*a^4*cos(d*x + c)^6 - 315*a^4*cos(d*x + c)^4 + 420*a^4*cos(d*x +
c)^2 + (15*a^4*cos(d*x + c)^6 - 171*a^4*cos(d*x + c)^4 + 332*a^4*cos(d*x +
c)^2 - 176*a^4)*sin(d*x + c))/d
```

giac [A] time = 0.19, size = 71, normalized size = 1.06

$$\frac{15\,a^4\sin(dx+c)^7 + 70\,a^4\sin(dx+c)^6 + 126\,a^4\sin(dx+c)^5 + 105\,a^4\sin(dx+c)^4 + 35\,a^4\sin(dx+c)^3}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*a^4*sin(d*x + c)^7 + 70*a^4*sin(d*x + c)^6 + 126*a^4*sin(d*x + c)
^5 + 105*a^4*sin(d*x + c)^4 + 35*a^4*sin(d*x + c)^3)/d
```

maple [A] time = 0.10, size = 70, normalized size = 1.04

$$\frac{\frac{a^4\left(\sin^7(dx+c)\right)}{7}+\frac{2a^4\left(\sin^6(dx+c)\right)}{3}+\frac{6a^4\left(\sin^5(dx+c)\right)}{5}+a^4\left(\sin^4(dx+c)\right)+\frac{a^4\left(\sin^3(dx+c)\right)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(1/7*a^4*sin(d*x+c)^7+2/3*a^4*sin(d*x+c)^6+6/5*a^4*sin(d*x+c)^5+a^4*sin
(d*x+c)^4+1/3*a^4*sin(d*x+c)^3)
```

maxima [A] time = 0.31, size = 71, normalized size = 1.06

$$\frac{15\,a^4\sin(dx+c)^7+70\,a^4\sin(dx+c)^6+126\,a^4\sin(dx+c)^5+105\,a^4\sin(dx+c)^4+35\,a^4\sin(dx+c)^3}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/105*(15*a^4*sin(d*x + c)^7 + 70*a^4*sin(d*x + c)^6 + 126*a^4*sin(d*x + c)
^5 + 105*a^4*sin(d*x + c)^4 + 35*a^4*sin(d*x + c)^3)/d
```

mupad [B] time = 8.41, size = 69, normalized size = 1.03

$$\frac{\frac{a^4\sin(c+d\,x)^7}{7}+\frac{2\,a^4\sin(c+d\,x)^6}{3}+\frac{6\,a^4\sin(c+d\,x)^5}{5}+a^4\sin(c+d\,x)^4+\frac{a^4\sin(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^2*(a + a*sin(c + d*x))^4,x)
```

```
[Out] ((a^4*sin(c + d*x)^3)/3 + a^4*sin(c + d*x)^4 + (6*a^4*sin(c + d*x)^5)/5 + (
2*a^4*sin(c + d*x)^6)/3 + (a^4*sin(c + d*x)^7)/7)/d
```

sympy [A] time = 7.24, size = 95, normalized size = 1.42

$$\begin{cases} \frac{a^4\sin^7(c+dx)}{7d}+\frac{2a^4\sin^6(c+dx)}{3d}+\frac{6a^4\sin^5(c+dx)}{5d}+\frac{a^4\sin^4(c+dx)}{d}+\frac{a^4\sin^3(c+dx)}{3d} & \text{for } d\neq 0 \\ x\,(a\sin(c)+a)^4\sin^2(c)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**7/(7*d) + 2*a**4*sin(c + d*x)**6/(3*d) + 6*a*
*4*sin(c + d*x)**5/(5*d) + a**4*sin(c + d*x)**4/d + a**4*sin(c + d*x)**3/(3
*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*sin(c)**2*cos(c), True))
```

3.220 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=45

$$\frac{(a\sin(c+dx)+a)^6}{6a^2d}-\frac{(a\sin(c+dx)+a)^5}{5ad}$$

```
[Out] -1/5*(a+a*sin(d*x+c))^5/a/d+1/6*(a+a*sin(d*x+c))^6/a^2/d
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 43}

$$\frac{(a\sin(c+dx)+a)^6}{6a^2d}-\frac{(a\sin(c+dx)+a)^5}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(a + a*Sin[c + d*x])^5/(5*a*d) + (a + a*Sin[c + d*x])^6/(6*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^4}{a}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-a(a+x)^4+(a+x)^5\right)\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= -\frac{(a+a\sin(c+dx))^5}{5ad} + \frac{(a+a\sin(c+dx))^6}{6a^2d}$$

Mathematica [A] time = 0.10, size = 30, normalized size = 0.67

$$\frac{a^4(\sin(c+dx)+1)^5(5\sin(c+dx)-1)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Sin[c + d*x])^5*(-1 + 5*Sin[c + d*x]))/(30*d)
```

fricas [B] time = 0.52, size = 85, normalized size = 1.89

$$-\frac{5\,a^4\cos\left(dx+c\right)^6 - 60\,a^4\cos\left(dx+c\right)^4 + 120\,a^4\cos\left(dx+c\right)^2 - 8\left(3\,a^4\cos\left(dx+c\right)^4 - 11\,a^4\cos\left(dx+c\right)^2 + 8\,a^4\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/30*(5*a^4*cos(d*x + c)^6 - 60*a^4*cos(d*x + c)^4 + 120*a^4*cos(d*x + c)^
2 - 8*(3*a^4*cos(d*x + c)^4 - 11*a^4*cos(d*x + c)^2 + 8*a^4)*sin(d*x + c))/
d
```

giac [A] time = 0.19, size = 71, normalized size = 1.58

$$\frac{5\,a^4\sin\left(dx+c\right)^6 + 24\,a^4\sin\left(dx+c\right)^5 + 45\,a^4\sin\left(dx+c\right)^4 + 40\,a^4\sin\left(dx+c\right)^3 + 15\,a^4\sin\left(dx+c\right)^2}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(5*a^4*sin(d*x + c)^6 + 24*a^4*sin(d*x + c)^5 + 45*a^4*sin(d*x + c)^4
+ 40*a^4*sin(d*x + c)^3 + 15*a^4*sin(d*x + c)^2)/d
```

maple [A] time = 0.10, size = 71, normalized size = 1.58

$$\frac{\frac{a^4\left(\sin^6(dx+c)\right)}{6}+\frac{4a^4\left(\sin^5(dx+c)\right)}{5}+\frac{3a^4\left(\sin^4(dx+c)\right)}{2}+\frac{4a^4\left(\sin^3(dx+c)\right)}{3}+\frac{a^4\left(\sin^2(dx+c)\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(1/6*a^4*sin(d*x+c)^6+4/5*a^4*sin(d*x+c)^5+3/2*a^4*sin(d*x+c)^4+4/3*a^4
*sin(d*x+c)^3+1/2*a^4*sin(d*x+c)^2)
```

maxima [A] time = 0.59, size = 71, normalized size = 1.58

$$\frac{5\,a^4\sin\left(dx+c\right)^6+24\,a^4\sin\left(dx+c\right)^5+45\,a^4\sin\left(dx+c\right)^4+40\,a^4\sin\left(dx+c\right)^3+15\,a^4\sin\left(dx+c\right)^2}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/30*(5*a^4*sin(d*x + c)^6 + 24*a^4*sin(d*x + c)^5 + 45*a^4*sin(d*x + c)^4
+ 40*a^4*sin(d*x + c)^3 + 15*a^4*sin(d*x + c)^2)/d
```

mupad [B] time = 0.05, size = 70, normalized size = 1.56

$$\frac{\frac{a^4\sin(c+d\,x)^6}{6}+\frac{4\,a^4\sin(c+d\,x)^5}{5}+\frac{3\,a^4\sin(c+d\,x)^4}{2}+\frac{4\,a^4\sin(c+d\,x)^3}{3}+\frac{a^4\sin(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)*(a + a*sin(c + d*x))^4,x)
```

```
[Out] ((a^4*sin(c + d*x)^2)/2 + (4*a^4*sin(c + d*x)^3)/3 + (3*a^4*sin(c + d*x)^4)
/2 + (4*a^4*sin(c + d*x)^5)/5 + (a^4*sin(c + d*x)^6)/6)/d
```

sympy [A] time = 3.87, size = 97, normalized size = 2.16

$$\begin{cases} \frac{a^4\sin^6(c+dx)}{6d}+\frac{4a^4\sin^5(c+dx)}{5d}+\frac{3a^4\sin^4(c+dx)}{2d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin^2(c+dx)}{2d} & \text{for } d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)^4\sin\left(c\right)\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**6/(6*d) + 4*a**4*sin(c + d*x)**5/(5*d) + 3*a*
*4*sin(c + d*x)**4/(2*d) + 4*a**4*sin(c + d*x)**3/(3*d) + a**4*sin(c + d*x)
**2/(2*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*sin(c)*cos(c), True))
```

3.221 $\int \cot(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=81

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{3a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\log(\sin(c+dx))}{d}$$

```
[Out] a^4*ln(sin(d*x+c))/d+4*a^4*sin(d*x+c)/d+3*a^4*sin(d*x+c)^2/d+4/3*a^4*sin(d*
x+c)^3/d+1/4*a^4*sin(d*x+c)^4/d
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 43}

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{3a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Log[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (3*a^4*Sin[c + d*x]^2)
/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/(4*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^3+\frac{a^4}{x}+6a^2x+4ax^2+x^3\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\log(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{3a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin^3(c+}{3d}$$

Mathematica [A] time = 0.04, size = 81, normalized size = 1.00

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{3a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Log[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (3*a^4*Sin[c + d*x]^2)
/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/(4*d)
```

fricas [A] time = 0.53, size = 72, normalized size = 0.89

$$\frac{3\,a^4\cos\left(dx+c\right)^4-42\,a^4\cos\left(dx+c\right)^2+12\,a^4\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-16\,\left(a^4\cos\left(dx+c\right)^2-4\,a^4\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^4*cos(d*x + c)^4 - 42*a^4*cos(d*x + c)^2 + 12*a^4*log(1/2*sin(d*x
 + c)) - 16*(a^4*cos(d*x + c)^2 - 4*a^4)*sin(d*x + c))/d
```

giac [A] time = 0.20, size = 69, normalized size = 0.85

$$\frac{3\,a^4\sin\left(dx+c\right)^4+16\,a^4\sin\left(dx+c\right)^3+36\,a^4\sin\left(dx+c\right)^2+12\,a^4\log\left(\left|\sin\left(dx+c\right)\right|\right)+48\,a^4\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 36*a^4*sin(d*x + c)^2
+ 12*a^4*log(abs(sin(d*x + c))) + 48*a^4*sin(d*x + c))/d
```

maple [A] time = 0.18, size = 78, normalized size = 0.96

$$\frac{a^4\ln(\sin(dx+c))}{d}+\frac{4a^4\sin(dx+c)}{d}+\frac{3a^4\left(\sin^2(dx+c)\right)}{d}+\frac{4a^4\left(\sin^3(dx+c)\right)}{3d}+\frac{a^4\left(\sin^4(dx+c)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^4,x)
```

```
[Out] a^4*ln(sin(d*x+c))/d+4*a^4*sin(d*x+c)/d+3*a^4*sin(d*x+c)^2/d+4/3*a^4*sin(d*
x+c)^3/d+1/4*a^4*sin(d*x+c)^4/d
```

maxima [A] time = 0.43, size = 68, normalized size = 0.84

$$\frac{3\,a^4\sin(dx+c)^4+16\,a^4\sin(dx+c)^3+36\,a^4\sin(dx+c)^2+12\,a^4\log(\sin(dx+c))+48\,a^4\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 36*a^4*sin(d*x + c)^2
+ 12*a^4*log(sin(d*x + c)) + 48*a^4*sin(d*x + c))/d
```

mupad [B] time = 8.62, size = 118, normalized size = 1.46

$$\frac{16\,a^4\,\sin(c+d\,x)}{3\,d}-\frac{a^4\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}+\frac{a^4\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{7\,a^4\cos(c+d\,x)^2}{2\,d}+\frac{a^4\cos(c+d\,x)^4}{4\,d}-\frac{4\,a^4\cos(c+d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^4)/sin(c + d*x),x)
```

```
[Out] (16*a^4*sin(c + d*x))/(3*d) - (a^4*log(1/cos(c/2 + (d*x)/2)^2))/d + (a^4*lo
g(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (7*a^4*cos(c + d*x)^2)/(2*d)
+ (a^4*cos(c + d*x)^4)/(4*d) - (4*a^4*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int\cos(c+dx)\csc(c+dx)\,dx+\int4\sin(c+dx)\cos(c+dx)\csc(c+dx)\,dx+\int6\sin^2(c+dx)\cos(c+dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(cos(c + d*x)*csc(c + d*x), x) + Integral(4*sin(c + d*x)*cos(
c + d*x)*csc(c + d*x), x) + Integral(6*sin(c + d*x)**2*cos(c + d*x)*csc(c +
 d*x), x) + Integral(4*sin(c + d*x)**3*cos(c + d*x)*csc(c + d*x), x) + Inte
gral(sin(c + d*x)**4*cos(c + d*x)*csc(c + d*x), x))
```

3.222 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=78

$$\frac{a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}+\frac{6a^4\sin(c+dx)}{d}-\frac{a^4\csc(c+dx)}{d}+\frac{4a^4\log(\sin(c+dx))}{d}$$

```
[Out] -a^4*csc(d*x+c)/d+4*a^4*ln(sin(d*x+c))/d+6*a^4*sin(d*x+c)/d+2*a^4*sin(d*x+c
)^2/d+1/3*a^4*sin(d*x+c)^3/d
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 43}

$$\frac{a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}+\frac{6a^4\sin(c+dx)}{d}-\frac{a^4\csc(c+dx)}{d}+\frac{4a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -((a^4*Csc[c + d*x])/d) + (4*a^4*Log[Sin[c + d*x]])/d + (6*a^4*Sin[c + d*x]
)/d + (2*a^4*Sin[c + d*x]^2)/d + (a^4*Sin[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(6a^2+\frac{a^4}{x^2}+\frac{4a^3}{x}+4ax+x^2\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^4\csc(c+dx)}{d}+\frac{4a^4\log(\sin(c+dx))}{d}+\frac{6a^4\sin(c+dx)}{d}+$$

Mathematica [A] time = 0.04, size = 78, normalized size = 1.00

$$\frac{a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}+\frac{6a^4\sin(c+dx)}{d}-\frac{a^4\csc(c+dx)}{d}+\frac{4a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -((a^4*Csc[c + d*x])/d) + (4*a^4*Log[Sin[c + d*x]])/d + (6*a^4*Sin[c + d*x]
)/d + (2*a^4*Sin[c + d*x]^2)/d + (a^4*Sin[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.50, size = 91, normalized size = 1.17

$$\frac{a^4\cos(dx+c)^4-20\,a^4\cos(dx+c)^2+12\,a^4\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+16\,a^4-3\left(2\,a^4\cos(dx+c)^2-a^4\right.}{3\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(a^4*cos(d*x + c)^4 - 20*a^4*cos(d*x + c)^2 + 12*a^4*log(1/2*sin(d*x +
c))*sin(d*x + c) + 16*a^4 - 3*(2*a^4*cos(d*x + c)^2 - a^4)*sin(d*x + c))/(d
*sin(d*x + c))
```

giac [A] time = 0.20, size = 68, normalized size = 0.87

$$\frac{a^4\sin(dx+c)^3+6\,a^4\sin(dx+c)^2+12\,a^4\log(|\sin(dx+c)|)+18\,a^4\sin(dx+c)-\frac{3\,a^4}{\sin(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(a^4*sin(d*x + c)^3 + 6*a^4*sin(d*x + c)^2 + 12*a^4*log(abs(sin(d*x + c
))) + 18*a^4*sin(d*x + c) - 3*a^4/sin(d*x + c))/d
```

maple [A] time = 0.15, size = 79, normalized size = 1.01

$$\frac{a^4\left(\sin^3(dx+c)\right)}{3d}+\frac{2a^4\left(\sin^2(dx+c)\right)}{d}+\frac{6a^4\sin(dx+c)}{d}-\frac{a^4}{d\sin(dx+c)}+\frac{4a^4\ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/3*a^4*sin(d*x+c)^3/d+2*a^4*sin(d*x+c)^2/d+6*a^4*sin(d*x+c)/d-a^4/d/sin(d*
x+c)+4*a^4*ln(sin(d*x+c))/d
```

maxima [A] time = 0.40, size = 67, normalized size = 0.86

$$\frac{a^4\sin(dx+c)^3+6\,a^4\sin(dx+c)^2+12\,a^4\log(\sin(dx+c))+18\,a^4\sin(dx+c)-\frac{3\,a^4}{\sin(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(a^4*sin(d*x + c)^3 + 6*a^4*sin(d*x + c)^2 + 12*a^4*log(sin(d*x + c)) +
 18*a^4*sin(d*x + c) - 3*a^4/sin(d*x + c))/d
```

mupad [B] time = 8.65, size = 235, normalized size = 3.01

$$\frac{8\,a^4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d}-\frac{8\,a^4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}-\frac{4\,a^4\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}+\frac{4\,a^4\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{28\,a^4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{16\,a^4\cos}{3\,d\,\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^4)/sin(c + d*x)^2,x)
```

```
[Out] (8*a^4*cos(c/2 + (d*x)/2)^2)/d - (8*a^4*cos(c/2 + (d*x)/2)^4)/d - (4*a^4*lo
g(1/cos(c/2 + (d*x)/2)^2))/d + (4*a^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)))/d - (28*a^4*cos(c/2 + (d*x)/2)^3)/(3*d*sin(c/2 + (d*x)/2)) - (16*a^4
*cos(c/2 + (d*x)/2)^5)/(3*d*sin(c/2 + (d*x)/2)) + (8*a^4*cos(c/2 + (d*x)/2)
^7)/(3*d*sin(c/2 + (d*x)/2)) + (23*a^4*cos(c/2 + (d*x)/2))/(2*d*sin(c/2 + (
d*x)/2)) - (a^4*sin(c/2 + (d*x)/2))/(2*d*cos(c/2 + (d*x)/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
[Out] Timed out
```

3.223 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=80

$$\frac{a^4\sin^2(c+dx)}{2d}+\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^2(c+dx)}{2d}-\frac{4a^4\csc(c+dx)}{d}+\frac{6a^4\log(\sin(c+dx))}{d}$$

```
[Out] -4*a^4*csc(d*x+c)/d-1/2*a^4*csc(d*x+c)^2/d+6*a^4*ln(sin(d*x+c))/d+4*a^4*sin
(d*x+c)/d+1/2*a^4*sin(d*x+c)^2/d
```

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^4\sin^2(c+dx)}{2d}+\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^2(c+dx)}{2d}-\frac{4a^4\csc(c+dx)}{d}+\frac{6a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a^4*Csc[c + d*x])/d - (a^4*Csc[c + d*x]^2)/(2*d) + (6*a^4*Log[Sin[c + d
*x]])/d + (4*a^4*Sin[c + d*x])/d + (a^4*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(4a+\frac{a^4}{x^3}+\frac{4a^3}{x^2}+\frac{6a^2}{x}+x\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{4a^4\csc(c+dx)}{d}-\frac{a^4\csc^2(c+dx)}{2d}+\frac{6a^4\log(\sin(c+dx))}{d}$$

Mathematica [A] time = 0.08, size = 54, normalized size = 0.68

$$-\frac{a^4\left(-\sin^2(c+dx)-8\sin(c+dx)+\csc^2(c+dx)+8\csc(c+dx)-12\log(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/2*(a^4*(8*Csc[c + d*x] + Csc[c + d*x]^2 - 12*Log[Sin[c + d*x]] - 8*Sin[c
 + d*x] - Sin[c + d*x]^2))/d
```

fricas [A] time = 0.51, size = 98, normalized size = 1.22

$$-\frac{2\,a^4\cos(dx+c)^4-16\,a^4\cos(dx+c)^2\sin(dx+c)-3\,a^4\cos(dx+c)^2-a^4-24\left(a^4\cos(dx+c)^2-a^4\right)\log\left(\frac{1}{2}\right.}{4\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^4*cos(d*x + c)^4 - 16*a^4*cos(d*x + c)^2*sin(d*x + c) - 3*a^4*cos
(d*x + c)^2 - a^4 - 24*(a^4*cos(d*x + c)^2 - a^4)*log(1/2*sin(d*x + c)))/(d
*cos(d*x + c)^2 - d)
```

giac [A] time = 0.20, size = 67, normalized size = 0.84

$$\frac{a^4\sin(dx+c)^2+12\,a^4\log(|\sin(dx+c)|)+8\,a^4\sin(dx+c)-\frac{8\,a^4\sin(dx+c)+a^4}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(a^4*sin(d*x + c)^2 + 12*a^4*log(abs(sin(d*x + c))) + 8*a^4*sin(d*x + c
) - (8*a^4*sin(d*x + c) + a^4)/sin(d*x + c)^2)/d
```

maple [A] time = 0.18, size = 79, normalized size = 0.99

$$\frac{a^4\left(\sin^2(dx+c)\right)}{2d}+\frac{4a^4\sin(dx+c)}{d}-\frac{4a^4}{d\sin(dx+c)}+\frac{6a^4\ln(\sin(dx+c))}{d}-\frac{a^4}{2d\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/2*a^4*sin(d*x+c)^2/d+4*a^4*sin(d*x+c)/d-4*a^4/d/sin(d*x+c)+6*a^4*ln(sin(d
*x+c))/d-1/2*a^4/d/sin(d*x+c)^2
```

maxima [A] time = 0.37, size = 66, normalized size = 0.82

$$\frac{a^4\sin(dx+c)^2+12\,a^4\log(\sin(dx+c))+8\,a^4\sin(dx+c)-\frac{8\,a^4\sin(dx+c)+a^4}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*(a^4*sin(d*x + c)^2 + 12*a^4*log(sin(d*x + c)) + 8*a^4*sin(d*x + c) - (
8*a^4*sin(d*x + c) + a^4)/sin(d*x + c)^2)/d
```

mupad [B] time = 8.62, size = 207, normalized size = 2.59

$$\frac{6\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{-24\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{15\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}-16\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+a^4\tan\left(\right.}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^4)/sin(c + d*x)^3,x)
```

```
[Out] (6*a^4*log(tan(c/2 + (d*x)/2)))/d - (a^4*tan(c/2 + (d*x)/2)^2)/(8*d) - (a^4
*tan(c/2 + (d*x)/2)^2 - 16*a^4*tan(c/2 + (d*x)/2)^3 - (15*a^4*tan(c/2 + (d*
x)/2)^4)/2 - 24*a^4*tan(c/2 + (d*x)/2)^5 + a^4/2 + 8*a^4*tan(c/2 + (d*x)/2)
)/(d*(4*tan(c/2 + (d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2
```

```
)^6)) - (2*a^4*tan(c/2 + (d*x)/2))/d - (6*a^4*log(tan(c/2 + (d*x)/2)^2 + 1)
)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.224 $\int \frac{\cos(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{\sin^4(c+dx)}{4ad}-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] ln(1+sin(d*x+c))/a/d-sin(d*x+c)/a/d+1/2*sin(d*x+c)^2/a/d-1/3*sin(d*x+c)^3/a
/d+1/4*sin(d*x+c)^4/a/d
```

Rubi [A] time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^4(c+dx)}{4ad}-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) + Sin[c + d*x]^2/(2*a*d) -
 Sin[c + d*x]^3/(3*a*d) + Sin[c + d*x]^4/(4*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^3+a^2x-ax^2+x^3+\frac{a^4}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\log(1+\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{ad} + \frac{\sin^2(c+dx)}{2ad} - \frac{\sin^3(c+dx)}{3ad} + \frac{\sin^4(c+dx)}{4ad}$$

Mathematica [A] time = 0.13, size = 60, normalized size = 0.71

$$\frac{3\sin^4(c+dx)-4\sin^3(c+dx)+6\sin^2(c+dx)-12\sin(c+dx)+12\log(\sin(c+dx)+1)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (12*Log[1 + Sin[c + d*x]] - 12*Sin[c + d*x] + 6*Sin[c + d*x]^2 - 4*Sin[c +
d*x]^3 + 3*Sin[c + d*x]^4)/(12*a*d)
```

fricas [A] time = 0.64, size = 58, normalized size = 0.68

$$\frac{3\cos(dx+c)^4-12\cos(dx+c)^2+4\left(\cos(dx+c)^2-4\right)\sin(dx+c)+12\log(\sin(dx+c)+1)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(d*x + c)^4 - 12*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 4)*sin(d*x
 + c) + 12*log(sin(d*x + c) + 1))/(a*d)
```

giac [A] time = 0.14, size = 76, normalized size = 0.89

$$\frac{\frac{12\log(|\sin(dx+c)+1|)}{a}+\frac{3a^3\sin(dx+c)^4-4a^3\sin(dx+c)^3+6a^3\sin(dx+c)^2-12a^3\sin(dx+c)}{a^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*log(abs(sin(d*x + c) + 1))/a + (3*a^3*sin(d*x + c)^4 - 4*a^3*sin(d
*x + c)^3 + 6*a^3*sin(d*x + c)^2 - 12*a^3*sin(d*x + c))/a^4)/d
```

maple [A] time = 0.12, size = 80, normalized size = 0.94

$$\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{ad}-\frac{\sin\left(dx+c\right)}{ad}+\frac{\sin^{2}\left(dx+c\right)}{2ad}-\frac{\sin^{3}\left(dx+c\right)}{3da}+\frac{\sin^{4}\left(dx+c\right)}{4da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] ln(1+sin(d*x+c))/a/d-sin(d*x+c)/a/d+1/2*sin(d*x+c)^2/a/d-1/3*sin(d*x+c)^3/d
/a+1/4*sin(d*x+c)^4/d/a
```

maxima [A] time = 0.41, size = 63, normalized size = 0.74

$$\frac{\frac{3\,\sin(dx+c)^{4}-4\,\sin(dx+c)^{3}+6\,\sin(dx+c)^{2}-12\,\sin(dx+c)}{a}+\frac{12\,\log(\sin(dx+c)+1)}{a}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*sin(d*x + c)^4 - 4*sin(d*x + c)^3 + 6*sin(d*x + c)^2 - 12*sin(d*x
+ c))/a + 12*log(sin(d*x + c) + 1)/a)/d
```

mupad [B] time = 8.46, size = 68, normalized size = 0.80

$$\frac{\frac{\ln(\sin(c+d\,x)+1)}{a}-\frac{\sin(c+d\,x)}{a}+\frac{\sin(c+d\,x)^{2}}{2\,a}-\frac{\sin(c+d\,x)^{3}}{3\,a}+\frac{\sin(c+d\,x)^{4}}{4\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^4)/(a + a*sin(c + d*x)),x)
```

```
[Out] (log(sin(c + d*x) + 1)/a - sin(c + d*x)/a + sin(c + d*x)^2/(2*a) - sin(c +
d*x)^3/(3*a) + sin(c + d*x)^4/(4*a))/d
```

sympy [A] time = 3.20, size = 80, normalized size = 0.94

$$\begin{cases} \frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad}+\frac{\sin^{4}\left(c+dx\right)}{4ad}-\frac{\sin^{3}\left(c+dx\right)}{3ad}+\frac{\sin^{2}\left(c+dx\right)}{2ad}-\frac{\sin\left(c+dx\right)}{ad} & \text{for}\: d\neq 0 \\ \frac{x\sin^{4}\left(c\right)\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)**4/(4*a*d) - sin(c +
d*x)**3/(3*a*d) + sin(c + d*x)**2/(2*a*d) - sin(c + d*x)/(a*d), Ne(d, 0)),
(x*sin(c)**4*cos(c)/(a*sin(c) + a), True))
```

3.225 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] -ln(1+sin(d*x+c))/a/d+sin(d*x+c)/a/d-1/2*sin(d*x+c)^2/a/d+1/3*sin(d*x+c)^3/
a/d
```

Rubi [A] time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Log[1 + Sin[c + d*x]]/(a*d)) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d
) + Sin[c + d*x]^3/(3*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{a^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2-ax+x^2-\frac{a^3}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= -\frac{\log(1+\sin(c+dx))}{ad} + \frac{\sin(c+dx)}{ad} - \frac{\sin^2(c+dx)}{2ad} + \frac{\sin^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.11, size = 50, normalized size = 0.75

$$\frac{2\sin^3(c+dx)-3\sin^2(c+dx)+6\sin(c+dx)-6\log(\sin(c+dx)+1)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-6*Log[1 + Sin[c + d*x]] + 6*Sin[c + d*x] - 3*Sin[c + d*x]^2 + 2*Sin[c + d
*x]^3)/(6*a*d)
```

fricas [A] time = 0.53, size = 48, normalized size = 0.72

$$\frac{3\cos(dx+c)^2-2\left(\cos(dx+c)^2-4\right)\sin(dx+c)-6\log(\sin(dx+c)+1)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 4)*sin(d*x + c) - 6*log(sin(d*x
 + c) + 1))/(a*d)
```

giac [A] time = 0.16, size = 64, normalized size = 0.96

$$-\frac{\frac{6\log(|\sin(dx+c)+1|)}{a}-\frac{2a^2\sin(dx+c)^3-3a^2\sin(dx+c)^2+6a^2\sin(dx+c)}{a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/6*(6*log(abs(sin(d*x + c) + 1))/a - (2*a^2*sin(d*x + c)^3 - 3*a^2*sin(d*x + c)^2 + 6*a^2*sin(d*x + c))/a^3)/d

maple [A] time = 0.12, size = 64, normalized size = 0.96

$$-\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{ad}+\frac{\sin\left(dx+c\right)}{ad}-\frac{\sin^{2}\left(dx+c\right)}{2ad}+\frac{\sin^{3}\left(dx+c\right)}{3da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)

[Out] -ln(1+sin(d*x+c))/a/d+sin(d*x+c)/a/d-1/2*sin(d*x+c)^2/a/d+1/3*sin(d*x+c)^3/d/a

maxima [A] time = 0.31, size = 53, normalized size = 0.79

$$\frac{\frac{2\sin(dx+c)^{3}-3\sin(dx+c)^{2}+6\sin(dx+c)}{a}-\frac{6\log(\sin(dx+c)+1)}{a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/6*((2*sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 6*sin(d*x + c))/a - 6*log(sin(d*x + c) + 1)/a)/d

mupad [B] time = 0.06, size = 56, normalized size = 0.84

$$-\frac{\frac{\ln(\sin(c+d\,x)+1)}{a}-\frac{\sin(c+d\,x)}{a}+\frac{\sin(c+d\,x)^{2}}{2\,a}-\frac{\sin(c+d\,x)^{3}}{3\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*sin(c + d*x)^3)/(a + a*sin(c + d*x)),x)

[Out] -(log(sin(c + d*x) + 1)/a - sin(c + d*x)/a + sin(c + d*x)^2/(2*a) - sin(c + d*x)^3/(3*a))/d

sympy [A] time = 1.96, size = 66, normalized size = 0.99

$$\begin{cases} -\frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad}+\frac{\sin^{3}\left(c+dx\right)}{3ad}-\frac{\sin^{2}\left(c+dx\right)}{2ad}+\frac{\sin\left(c+dx\right)}{ad} & \text{for}\: d\neq 0 \\ \frac{x\sin^{3}\left(c\right)\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)**3/(3*a*d) - sin(c +
 d*x)**2/(2*a*d) + sin(c + d*x)/(a*d), Ne(d, 0)), (x*sin(c)**3*cos(c)/(a*si
n(c) + a), True))
```

3.226 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{\sin^2(c+dx)}{2ad} - \frac{\sin(c+dx)}{ad} + \frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] ln(1+sin(d*x+c))/a/d-sin(d*x+c)/a/d+1/2*sin(d*x+c)^2/a/d
```

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^2(c+dx)}{2ad} - \frac{\sin(c+dx)}{ad} + \frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) + Sin[c + d*x]^2/(2*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(-a+x+\frac{a^2}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\log(1+\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{ad} + \frac{\sin^2(c+dx)}{2ad}$$

Mathematica [A] time = 0.06, size = 38, normalized size = 0.78

$$\frac{\sin^2(c+dx) - 2\sin(c+dx) + 2\log(\sin(c+dx)+1)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] (2*Log[1 + Sin[c + d*x]] - 2*Sin[c + d*x] + Sin[c + d*x]^2)/(2*a*d)
```

fricas [A] time = 0.51, size = 36, normalized size = 0.73

$$-\frac{\cos(dx+c)^2 - 2\log(\sin(dx+c)+1) + 2\sin(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(cos(d*x + c)^2 - 2*log(sin(d*x + c) + 1) + 2*sin(d*x + c))/(a*d)
```

giac [A] time = 0.16, size = 45, normalized size = 0.92

$$\frac{\frac{2\log(|\sin(dx+c)+1|)}{a} + \frac{a\sin(dx+c)^2 - 2\,a\sin(dx+c)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(sin(d*x + c) + 1))/a + (a*sin(d*x + c)^2 - 2*a*sin(d*x + c))
/a^2)/d
```

maple [A] time = 0.12, size = 48, normalized size = 0.98

$$\frac{\ln(1+\sin(dx+c))}{ad}-\frac{\sin(dx+c)}{ad}+\frac{\sin^2(dx+c)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] ln(1+sin(d*x+c))/a/d-sin(d*x+c)/a/d+1/2*sin(d*x+c)^2/a/d
```

maxima [A] time = 0.31, size = 41, normalized size = 0.84

$$\frac{\frac{\sin(dx+c)^2-2\sin(dx+c)}{a}+\frac{2\log(\sin(dx+c)+1)}{a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((sin(d*x + c)^2 - 2*sin(d*x + c))/a + 2*log(sin(d*x + c) + 1)/a)/d
```

mupad [B] time = 0.05, size = 35, normalized size = 0.71

$$\frac{\ln(\sin(c+d\,x)+1)-\sin(c+d\,x)+\frac{\sin(c+d\,x)^2}{2}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] (log(sin(c + d*x) + 1) - sin(c + d*x) + sin(c + d*x)^2/2)/(a*d)
```

sympy [A] time = 1.11, size = 53, normalized size = 1.08

$$\begin{cases}\frac{\log(\sin(c+dx)+1)}{ad}+\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad} & \text{for } d\neq 0\\ \frac{x\sin^2(c)\cos(c)}{a\sin(c)+a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)**2/(2*a*d) - sin(c +
d*x)/(a*d), Ne(d, 0)), (x*sin(c)**2*cos(c)/(a*sin(c) + a), True))
```

3.227 $\int \frac{\cos(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{\sin(c+dx)}{ad} - \frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] -ln(1+sin(d*x+c))/a/d+sin(d*x+c)/a/d
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 43}

$$\frac{\sin(c+dx)}{ad} - \frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Log[1 + Sin[c + d*x]]/(a*d)) + Sin[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{a(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{a}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= -\frac{\log(1+\sin(c+dx))}{ad} + \frac{\sin(c+dx)}{ad}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.81

$$\frac{\sin(c+dx)-\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] (-Log[1 + Sin[c + d*x]] + Sin[c + d*x])/(a*d)
```

fricas [A] time = 0.50, size = 26, normalized size = 0.84

$$-\frac{\log\left(\sin\left(dx+c\right)+1\right)-\sin\left(dx+c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -(log(sin(d*x + c) + 1) - sin(d*x + c))/(a*d)
```

giac [A] time = 0.14, size = 31, normalized size = 1.00

$$-\frac{\frac{\log(|\sin(dx+c)+1|)}{a}-\frac{\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -(log(abs(sin(d*x + c) + 1))/a - sin(d*x + c)/a)/d
```

maple [A] time = 0.07, size = 32, normalized size = 1.03

$$-\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{ad}+\frac{\sin\left(dx+c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -ln(1+sin(d*x+c))/a/d+sin(d*x+c)/a/d
```

maxima [A] time = 0.32, size = 30, normalized size = 0.97

$$-\frac{\frac{\log(\sin(dx+c)+1)}{a}-\frac{\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c) + 1)/a - sin(d*x + c)/a)/d
```

mupad [B] time = 8.46, size = 26, normalized size = 0.84

$$-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)-\sin\left(c+d\,x\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] -(log(sin(c + d*x) + 1) - sin(c + d*x))/(a*d)
```

sympy [A] time = 0.68, size = 37, normalized size = 1.19

$$\begin{cases} -\frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad}+\frac{\sin\left(c+dx\right)}{ad} & \text{for}\ d\neq 0\\ \frac{x\sin\left(c\right)\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)/(a*d), Ne(d, 0)), (x
*sin(c)*cos(c)/(a*sin(c) + a), True))
```

3.228 $\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] ln(sin(d*x+c))/a/d-ln(1+sin(d*x+c))/a/d
```

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2707, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a\sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\log(1+\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

fricas [A] time = 0.47, size = 28, normalized size = 0.88

$$\frac{\log\left(\frac{1}{2}\sin\left(dx+c\right)\right) - \log\left(\sin\left(dx+c\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (log(1/2*sin(d*x + c)) - log(sin(d*x + c) + 1))/(a*d)
```

giac [A] time = 0.15, size = 33, normalized size = 1.03

$$-\frac{\frac{\log(|\sin(dx+c)+1|)}{a} - \frac{\log(|\sin(dx+c)|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c)))/a)/d
```

maple [A] time = 0.20, size = 33, normalized size = 1.03

$$\frac{\ln\left(\sin\left(dx+c\right)\right)}{ad} - \frac{\ln\left(1+\sin\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] ln(sin(d*x+c))/a/d-ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.32, size = 31, normalized size = 0.97

$$-\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c))}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c) + 1)/a - log(sin(d*x + c))/a)/d
```

mupad [B] time = 8.61, size = 32, normalized size = 1.00

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)) - 2*log(tan(c/2 + (d*x)/2) + 1))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

3.229 $\int \frac{\cot(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=46

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] -csc(d*x+c)/a/d-ln(sin(d*x+c))/a/d+ln(1+sin(d*x+c))/a/d
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 44}

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) + Log[1 + Sin[c + d*x]]/(a*
d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^2}{x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(1+\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) + Log[1 + Sin[c + d*x]]/(a*
d)
```

fricas [A] time = 0.50, size = 51, normalized size = 1.11

$$-\frac{\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-\log(\sin(dx+c)+1)\sin(dx+c)+1}{ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -(log(1/2*sin(d*x + c))*sin(d*x + c) - log(sin(d*x + c) + 1)*sin(d*x + c) +
 1)/(a*d*sin(d*x + c))
```

giac [A] time = 0.20, size = 45, normalized size = 0.98

$$\frac{\frac{\log(|\sin(dx+c)+1|)}{a}-\frac{\log(|\sin(dx+c)|)}{a}-\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c)))/a - 1/(a*sin(d*x + c
)))/d
```

maple [A] time = 0.18, size = 49, normalized size = 1.07

$$-\frac{1}{da\sin(dx+c)}-\frac{\ln(\sin(dx+c))}{ad}+\frac{\ln(1+\sin(dx+c))}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d/a/sin(d*x+c)-ln(sin(d*x+c))/a/d+ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.31, size = 43, normalized size = 0.93

$$\frac{\frac{\log(\sin(dx+c)+1)}{a}-\frac{\log(\sin(dx+c))}{a}-\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c) + 1)/a - log(sin(d*x + c))/a - 1/(a*sin(d*x + c)))/d
```

mupad [B] time = 8.60, size = 55, normalized size = 1.20

$$-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{1}{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] -(2*log(tan(c/2 + (d*x)/2)) - 4*log(tan(c/2 + (d*x)/2) + 1) + tan(c/2 + (d*
x)/2) + 1/tan(c/2 + (d*x)/2))/(2*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos(c+dx)\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

3.230 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=63

$$-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d+ln(sin(d*x+c))/a/d-ln(1+sin(d*x+c))/a/d
```

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) + Log[Sin[c + d*x]]/(a*d) - Log
[1 + Sin[c + d*x]]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^3}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{1}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{1}{a^2x^2}+\frac{1}{a^3x}-\frac{1}{a^3(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{\log(1+\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 1.00

$$-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) + Log[Sin[c + d*x]]/(a*d) - Log
[1 + Sin[c + d*x]]/(a*d)
```

fricas [A] time = 0.53, size = 72, normalized size = 1.14

$$\frac{2\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-2\left(\cos(dx+c)^2-1\right)\log(\sin(dx+c)+1)-2\sin(dx+c)+1}{2\left(ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 2*(cos(d*x + c)^2 - 1)*
log(sin(d*x + c) + 1) - 2*sin(d*x + c) + 1)/(a*d*cos(d*x + c)^2 - a*d)
```

giac [A] time = 0.16, size = 57, normalized size = 0.90

$$-\frac{\frac{2\log(|\sin(dx+c)+1|)}{a}-\frac{2\log(|\sin(dx+c)|)}{a}-\frac{2\sin(dx+c)-1}{a\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(sin(d*x + c) + 1))/a - 2*log(abs(sin(d*x + c)))/a - (2*sin(
d*x + c) - 1)/(a*sin(d*x + c)^2))/d
```

maple [A] time = 0.23, size = 64, normalized size = 1.02

$$-\frac{1}{2ad\sin\left(dx+c\right)^{2}}+\frac{\ln\left(\sin\left(dx+c\right)\right)}{ad}+\frac{1}{da\sin\left(dx+c\right)}-\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/a/d/sin(d*x+c)^2+ln(sin(d*x+c))/a/d+1/d/a/sin(d*x+c)-ln(1+sin(d*x+c))/
a/d
```

maxima [A] time = 0.32, size = 55, normalized size = 0.87

$$-\frac{\frac{2\log\left(\sin\left(dx+c\right)+1\right)}{a}-\frac{2\log\left(\sin\left(dx+c\right)\right)}{a}-\frac{2\sin\left(dx+c\right)-1}{a\sin\left(dx+c\right)^{2}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*log(sin(d*x + c) + 1)/a - 2*log(sin(d*x + c))/a - (2*sin(d*x + c) -
 1)/(a*sin(d*x + c)^2))/d
```

mupad [B] time = 8.82, size = 106, normalized size = 1.68

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,a\,d}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}+\frac{{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{2}\right)}{4\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - (2*log(tan(c
/2 + (d*x)/2) + 1))/(a*d) + tan(c/2 + (d*x)/2)/(2*a*d) + (cot(c/2 + (d*x)/2
)^2*(2*tan(c/2 + (d*x)/2) - 1/2))/(4*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos\left(c+dx\right)\csc^{3}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

3.231 $\int \frac{\cot(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] -csc(d*x+c)/a/d+1/2*csc(d*x+c)^2/a/d-1/3*csc(d*x+c)^3/a/d-ln(sin(d*x+c))/a/
d+ln(1+sin(d*x+c))/a/d
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) - L
og[Sin[c + d*x]]/(a*d) + Log[1 + Sin[c + d*x]]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^4}{x^4(a+x)}\,dx,x,a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{1}{x^4(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{1}{a^2x^3}+\frac{1}{a^3x^2}-\frac{1}{a^4x}+\frac{1}{a^4(a+x)}\right)dx,x,a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(1+\sin}{ad}$$

Mathematica [A] time = 0.05, size = 82, normalized size = 1.00

$$-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) - L
og[Sin[c + d*x]]/(a*d) + Log[1 + Sin[c + d*x]]/(a*d)
```

fricas [A] time = 0.57, size = 102, normalized size = 1.24

$$-\frac{6\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-6\left(\cos(dx+c)^2-1\right)\log(\sin(dx+c)+1)\sin(dx+c)}{6\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(6*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c))*sin(d*x + c) - 6*(cos(d*
x + c)^2 - 1)*log(sin(d*x + c) + 1)*sin(d*x + c) + 6*cos(d*x + c)^2 + 3*sin
(d*x + c) - 8)/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

giac [A] time = 0.16, size = 67, normalized size = 0.82

$$\frac{\frac{6\log(|\sin(dx+c)+1|)}{a}-\frac{6\log(|\sin(dx+c)|)}{a}-\frac{6\sin(dx+c)^2-3\sin(dx+c)+2}{a\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c) + 1))/a - 6*log(abs(sin(d*x + c)))/a - (6*sin(d
*x + c)^2 - 3*sin(d*x + c) + 2)/(a*sin(d*x + c)^3))/d
```

maple [A] time = 0.20, size = 81, normalized size = 0.99

$$-\frac{1}{3ad\sin\left(dx+c\right)^{3}}-\frac{1}{da\sin\left(dx+c\right)}+\frac{1}{2ad\sin\left(dx+c\right)^{2}}-\frac{\ln\left(\sin\left(dx+c\right)\right)}{ad}+\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3/a/d/sin(d*x+c)^3-1/d/a/sin(d*x+c)+1/2/a/d/sin(d*x+c)^2-ln(sin(d*x+c))/
a/d+ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.31, size = 65, normalized size = 0.79

$$\frac{\frac{6\log\left(\sin\left(dx+c\right)+1\right)}{a}-\frac{6\log\left(\sin\left(dx+c\right)\right)}{a}-\frac{6\sin\left(dx+c\right)^{2}-3\sin\left(dx+c\right)+2}{a\sin\left(dx+c\right)^{3}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*log(sin(d*x + c) + 1)/a - 6*log(sin(d*x + c))/a - (6*sin(d*x + c)^2
- 3*sin(d*x + c) + 2)/(a*sin(d*x + c)^3))/d
```

mupad [B] time = 8.59, size = 139, normalized size = 1.70

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a\,d}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(5\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - tan(c/2 + (d*x)/2)^3/(24*a*d) - log(tan(c/2
+ (d*x)/2))/(a*d) + (2*log(tan(c/2 + (d*x)/2) + 1))/(a*d) - (5*tan(c/2 + (d
*x)/2))/(8*a*d) - (cot(c/2 + (d*x)/2)^3*(5*tan(c/2 + (d*x)/2)^2 - tan(c/2 +
 (d*x)/2) + 1/3))/(8*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
[Out] Integral(cos(c + d*x)*csc(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

3.232 $\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=87

$$\frac{\sin^3(c+dx)}{3a^2d}-\frac{\sin^2(c+dx)}{a^2d}+\frac{3\sin(c+dx)}{a^2d}-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{4\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] -4*ln(1+sin(d*x+c))/a^2/d+3*sin(d*x+c)/a^2/d-sin(d*x+c)^2/a^2/d+1/3*sin(d*x
+c)^3/a^2/d-1/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^3(c+dx)}{3a^2d}-\frac{\sin^2(c+dx)}{a^2d}+\frac{3\sin(c+dx)}{a^2d}-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{4\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^2*d) + (3*Sin[c + d*x])/(a^2*d) - Sin[c + d*x
]^2/(a^2*d) + Sin[c + d*x]^3/(3*a^2*d) - 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(3a^2-2ax+x^2+\frac{a^4}{(a+x)^2}-\frac{4a^3}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= -\frac{4\log(1+\sin(c+dx))}{a^2 d}+\frac{3\sin(c+dx)}{a^2 d}-\frac{\sin^2(c+dx)}{a^2 d}+\frac{\sin^3(c+dx)}{3a^2 d}-\frac{}{d\left(a^2\right.}$$

Mathematica [A] time = 0.58, size = 73, normalized size = 0.84

$$\frac{\sin^3(c+dx)-3\sin^2(c+dx)+9\sin(c+dx)-12\log(\sin(c+dx)+1)-\frac{3}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}}{3a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-12*Log[1 + Sin[c + d*x]] - 3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 9*
Sin[c + d*x] - 3*Sin[c + d*x]^2 + Sin[c + d*x]^3)/(3*a^2*d)
```

fricas [A] time = 0.52, size = 81, normalized size = 0.93

$$\frac{2\cos(dx+c)^4-16\cos(dx+c)^2-24(\sin(dx+c)+1)\log(\sin(dx+c)+1)+\left(4\cos(dx+c)^2+17\right)\sin(dx+}{6\left(a^2 d\sin(dx+c)+a^2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(d*x + c)^4 - 16*cos(d*x + c)^2 - 24*(sin(d*x + c) + 1)*log(sin(d
*x + c) + 1) + (4*cos(d*x + c)^2 + 17)*sin(d*x + c) + 11)/(a^2*d*sin(d*x +
c) + a^2*d)
```

giac [A] time = 0.18, size = 107, normalized size = 1.23

$$-\frac{\frac{(a\sin(dx+c)+a)^3\left(\frac{6a}{a\sin(dx+c)+a}-\frac{18a^2}{(a\sin(dx+c)+a)^2}-1\right)}{a^5}-\frac{12\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a^2}+\frac{3}{(a\sin(dx+c)+a)a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*((a*sin(d*x + c) + a)^3*(6*a/(a*sin(d*x + c) + a) - 18*a^2/(a*sin(d*x
+ c) + a)^2 - 1)/a^5 - 12*log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)
^2*abs(a)))/a^2 + 3/((a*sin(d*x + c) + a)*a))/d
```

maple [A] time = 0.22, size = 83, normalized size = 0.95

$$\frac{\sin^3(dx+c)}{3a^2d}-\frac{\sin^2(dx+c)}{a^2d}+\frac{3\sin(dx+c)}{a^2d}-\frac{4\ln(1+\sin(dx+c))}{a^2d}-\frac{1}{d\,a^2\,(1+\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/3*sin(d*x+c)^3/a^2/d-sin(d*x+c)^2/a^2/d+3*sin(d*x+c)/a^2/d-4*ln(1+sin(d*x
+c))/a^2/d-1/d/a^2/(1+sin(d*x+c))
```

maxima [A] time = 0.31, size = 70, normalized size = 0.80

$$-\frac{\frac{3}{a^2\sin(dx+c)+a^2}-\frac{\sin(dx+c)^3-3\,\sin(dx+c)^2+9\,\sin(dx+c)}{a^2}+\frac{12\,\log(\sin(dx+c)+1)}{a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3/(a^2*sin(d*x + c) + a^2) - (sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 9*s
in(d*x + c))/a^2 + 12*log(sin(d*x + c) + 1)/a^2)/d
```

mupad [B] time = 0.05, size = 72, normalized size = 0.83

$$-\frac{\frac{1}{a^2\,\sin(c+d\,x)+a^2}+\frac{4\,\ln(\sin(c+d\,x)+1)}{a^2}-\frac{3\,\sin(c+d\,x)}{a^2}+\frac{\sin(c+d\,x)^2}{a^2}-\frac{\sin(c+d\,x)^3}{3\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^4)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] -(1/(a^2*sin(c + d*x) + a^2) + (4*log(sin(c + d*x) + 1))/a^2 - (3*sin(c + d
*x))/a^2 + sin(c + d*x)^2/a^2 - sin(c + d*x)^3/(3*a^2))/d
```

sympy [A] time = 4.34, size = 201, normalized size = 2.31

$$\begin{cases} -\frac{12\log(\sin(c+dx)+1)\sin(c+dx)}{3a^2d\sin(c+dx)+3a^2d} - \frac{12\log(\sin(c+dx)+1)}{3a^2d\sin(c+dx)+3a^2d} + \frac{\sin^4(c+dx)}{3a^2d\sin(c+dx)+3a^2d} - \frac{2\sin^3(c+dx)}{3a^2d\sin(c+dx)+3a^2d} + \frac{6\sin^2(c+dx)}{3a^2d\sin(c+dx)+3a^2d} - \frac{}{3a^2d} \\ \frac{x\sin^4(c)\cos(c)}{(a\sin(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-12*log(sin(c + d*x) + 1)*sin(c + d*x)/(3*a**2*d*sin(c + d*x) +
3*a**2*d) - 12*log(sin(c + d*x) + 1)/(3*a**2*d*sin(c + d*x) + 3*a**2*d) + s
in(c + d*x)**4/(3*a**2*d*sin(c + d*x) + 3*a**2*d) - 2*sin(c + d*x)**3/(3*a*
*2*d*sin(c + d*x) + 3*a**2*d) + 6*sin(c + d*x)**2/(3*a**2*d*sin(c + d*x) +
3*a**2*d) - 12/(3*a**2*d*sin(c + d*x) + 3*a**2*d), Ne(d, 0)), (x*sin(c)**4*
cos(c)/(a*sin(c) + a)**2, True))
```

3.233 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$\frac{\sin^2(c+dx)}{2a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{3\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] 3*ln(1+sin(d*x+c))/a^2/d-2*sin(d*x+c)/a^2/d+1/2*sin(d*x+c)^2/a^2/d+1/d/(a^2
+a^2*sin(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^2(c+dx)}{2a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{3\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*Log[1 + Sin[c + d*x]])/(a^2*d) - (2*Sin[c + d*x])/(a^2*d) + Sin[c + d*x]
^2/(2*a^2*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{a^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+x-\frac{a^3}{(a+x)^2}+\frac{3a^2}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{3\log(1+\sin(c+dx))}{a^2d} - \frac{2\sin(c+dx)}{a^2d} + \frac{\sin^2(c+dx)}{2a^2d} + \frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.18, size = 71, normalized size = 1.01

$$\frac{\sin^3(c+dx)-3\sin^2(c+dx)+\sin(c+dx)(6\log(\sin(c+dx)+1)-4)+6\log(\sin(c+dx)+1)+2}{2a^2d(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2 + 6*Log[1 + Sin[c + d*x]] + (-4 + 6*Log[1 + Sin[c + d*x]])*Sin[c + d*x]
- 3*Sin[c + d*x]^2 + Sin[c + d*x]^3)/(2*a^2*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.55, size = 72, normalized size = 1.03

$$\frac{6\cos(dx+c)^2+12(\sin(dx+c)+1)\log(\sin(dx+c)+1)-\left(2\cos(dx+c)^2+7\right)\sin(dx+c)-3}{4\left(a^2d\sin(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(d*x + c)^2 + 12*(sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (2*co
s(d*x + c)^2 + 7)*sin(d*x + c) - 3)/(a^2*d*sin(d*x + c) + a^2*d)
```

giac [A] time = 0.17, size = 90, normalized size = 1.29

$$-\frac{\frac{(a\sin(dx+c)+a)^2\left(\frac{6a}{a\sin(dx+c)+a}-1\right)}{a^4}+\frac{6\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a^2}-\frac{2}{(a\sin(dx+c)+a)a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((a*sin(d*x + c) + a)^2*(6*a/(a*sin(d*x + c) + a) - 1)/a^4 + 6*log(abs
(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a^2 - 2/((a*sin(d*x +
 c) + a)*a))/d
```

maple [A] time = 0.22, size = 66, normalized size = 0.94

$$\frac{\sin^2(dx+c)}{2a^2d}-\frac{2\sin(dx+c)}{a^2d}+\frac{3\ln(1+\sin(dx+c))}{a^2d}+\frac{1}{d\,a^2\,(1+\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2*sin(d*x+c)^2/a^2/d-2*sin(d*x+c)/a^2/d+3*ln(1+sin(d*x+c))/a^2/d+1/d/a^2/
(1+sin(d*x+c))
```

maxima [A] time = 0.32, size = 59, normalized size = 0.84

$$\frac{\frac{2}{a^2\sin(dx+c)+a^2}+\frac{\sin(dx+c)^2-4\sin(dx+c)}{a^2}+\frac{6\log(\sin(dx+c)+1)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2/(a^2*sin(d*x + c) + a^2) + (sin(d*x + c)^2 - 4*sin(d*x + c))/a^2 + 6
*log(sin(d*x + c) + 1)/a^2)/d
```

mupad [B] time = 8.47, size = 59, normalized size = 0.84

$$\frac{\frac{1}{a^2\,\sin(c+d\,x)+a^2}+\frac{3\,\ln(\sin(c+d\,x)+1)}{a^2}-\frac{2\,\sin(c+d\,x)}{a^2}+\frac{\sin(c+d\,x)^2}{2\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^3)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (1/(a^2*sin(c + d*x) + a^2) + (3*log(sin(c + d*x) + 1))/a^2 - (2*sin(c + d*
x))/a^2 + sin(c + d*x)^2/(2*a^2))/d
```

sympy [A] time = 2.81, size = 170, normalized size = 2.43

$$\begin{cases} \frac{6\log(\sin(c+dx)+1)\sin(c+dx)}{2a^2d\sin(c+dx)+2a^2d} + \frac{6\log(\sin(c+dx)+1)}{2a^2d\sin(c+dx)+2a^2d} + \frac{\sin^3(c+dx)}{2a^2d\sin(c+dx)+2a^2d} - \frac{3\sin^2(c+dx)}{2a^2d\sin(c+dx)+2a^2d} + \frac{6}{2a^2d\sin(c+dx)+2a^2d} & \text{for } d \neq 0 \\ \frac{x\sin^3(c)\cos(c)}{(a\sin(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((6*log(sin(c + d*x) + 1)*sin(c + d*x)/(2*a**2*d*sin(c + d*x) + 2*
a**2*d) + 6*log(sin(c + d*x) + 1)/(2*a**2*d*sin(c + d*x) + 2*a**2*d) + sin(
c + d*x)**3/(2*a**2*d*sin(c + d*x) + 2*a**2*d) - 3*sin(c + d*x)**2/(2*a**2*
d*sin(c + d*x) + 2*a**2*d) + 6/(2*a**2*d*sin(c + d*x) + 2*a**2*d), Ne(d, 0)
), (x*sin(c)**3*cos(c)/(a*sin(c) + a)**2, True))
```

3.234 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=52

$$\frac{\sin(c+dx)}{a^2d}-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] -2*ln(1+sin(d*x+c))/a^2/d+sin(d*x+c)/a^2/d-1/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin(c+dx)}{a^2d}-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*Log[1 + Sin[c + d*x]])/(a^2*d) + Sin[c + d*x]/(a^2*d) - 1/(d*(a^2 + a^2
*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^2}{(a+x)^2}-\frac{2a}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= -\frac{2\log(1+\sin(c+dx))}{a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.20, size = 55, normalized size = 1.06

$$\frac{4\sin(c+dx)-8\log(\sin(c+dx)+1)-\frac{4}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-8*Log[1 + Sin[c + d*x]] - 4/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 4*S
in[c + d*x])/(4*a^2*d)
```

fricas [A] time = 0.61, size = 57, normalized size = 1.10

$$-\frac{\cos\left(dx+c\right)^2+2\left(\sin\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-\sin\left(dx+c\right)}{a^2d\sin\left(dx+c\right)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c)^2 + 2*(sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - sin(d*x + c
))/(a^2*d*sin(d*x + c) + a^2*d)
```

giac [A] time = 0.17, size = 70, normalized size = 1.35

$$\frac{\frac{2\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a^2}+\frac{a\sin(dx+c)+a}{a^3}-\frac{1}{(a\sin(dx+c)+a)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a^2 + (a*si
n(d*x + c) + a)/a^3 - 1/((a*sin(d*x + c) + a)*a))/d
```

maple [A] time = 0.23, size = 50, normalized size = 0.96

$$\frac{\sin(dx+c)}{a^2 d}-\frac{2\ln(1+\sin(dx+c))}{a^2 d}-\frac{1}{d\,a^2\,(1+\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] sin(d*x+c)/a^2/d-2*ln(1+sin(d*x+c))/a^2/d-1/d/a^2/(1+sin(d*x+c))
```

maxima [A] time = 0.38, size = 47, normalized size = 0.90

$$-\frac{\frac{1}{a^2\sin(dx+c)+a^2}+\frac{2\log(\sin(dx+c)+1)}{a^2}-\frac{\sin(dx+c)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(1/(a^2*sin(d*x + c) + a^2) + 2*log(sin(d*x + c) + 1)/a^2 - sin(d*x + c)/a
^2)/d
```

mupad [B] time = 0.08, size = 45, normalized size = 0.87

$$\frac{\sin(c+d\,x)^2-2}{a^2\,d\,\left(\sin(c+d\,x)+1\right)}-\frac{2\,\ln\left(\sin(c+d\,x)+1\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^2 - 2)/(a^2*d*(sin(c + d*x) + 1)) - (2*log(sin(c + d*x) + 1))
/(a^2*d)
```

sympy [A] time = 1.45, size = 126, normalized size = 2.42

$$\begin{cases} -\frac{2\log(\sin(c+dx)+1)\sin(c+dx)}{a^2d\sin(c+dx)+a^2d}-\frac{2\log(\sin(c+dx)+1)}{a^2d\sin(c+dx)+a^2d}+\frac{\sin^2(c+dx)}{a^2d\sin(c+dx)+a^2d}-\frac{2}{a^2d\sin(c+dx)+a^2d} & \text{for } d\neq 0 \\ \frac{x\sin^2(c)\cos(c)}{(a\sin(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-2*log(sin(c + d*x) + 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**
2*d) - 2*log(sin(c + d*x) + 1)/(a**2*d*sin(c + d*x) + a**2*d) + sin(c + d*x
)**2/(a**2*d*sin(c + d*x) + a**2*d) - 2/(a**2*d*sin(c + d*x) + a**2*d), Ne(
d, 0)), (x*sin(c)**2*cos(c)/(a*sin(c) + a)**2, True))
```

3.235 $\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=37

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] ln(1+sin(d*x+c))/a^2/d+1/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 43}

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a^2*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{a(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{(a+x)^2}+\frac{1}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\log(1+\sin(c+dx))}{a^2d} + \frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 0.73

$$\frac{\frac{1}{\sin(c+dx)+1} + \log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
[Out] (Log[1 + Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(a^2*d)
```

fricas [A] time = 0.56, size = 40, normalized size = 1.08

$$\frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1)+1}{a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] ((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) + 1)/(a^2*d*sin(d*x + c) + a^2*d)
```

giac [A] time = 0.16, size = 56, normalized size = 1.51

$$-\frac{\frac{\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a} - \frac{1}{a\sin(dx+c)+a}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a - 1/(a*sin
(d*x + c) + a))/(a*d)
```

maple [A] time = 0.19, size = 35, normalized size = 0.95

$$\frac{\ln(1+\sin(dx+c))}{a^2d}+\frac{1}{d\,a^2\,(1+\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] ln(1+sin(d*x+c))/a^2/d+1/d/a^2/(1+sin(d*x+c))
```

maxima [A] time = 0.34, size = 34, normalized size = 0.92

$$\frac{\frac{1}{a^2\sin(dx+c)+a^2}+\frac{\log(\sin(dx+c)+1)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a^2*sin(d*x + c) + a^2) + log(sin(d*x + c) + 1)/a^2)/d
```

mupad [B] time = 0.05, size = 34, normalized size = 0.92

$$\frac{1}{a^2\,d\,\left(\sin\left(c+d\,x\right)+1\right)}+\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] 1/(a^2*d*(sin(c + d*x) + 1)) + log(sin(c + d*x) + 1)/(a^2*d)
```

sympy [A] time = 1.04, size = 95, normalized size = 2.57

$$\begin{cases} \frac{\log(\sin(c+dx)+1)\sin(c+dx)}{a^2d\sin(c+dx)+a^2d}+\frac{\log(\sin(c+dx)+1)}{a^2d\sin(c+dx)+a^2d}+\frac{1}{a^2d\sin(c+dx)+a^2d} & \text{for}\ d\neq 0 \\ \frac{x\sin(c)\cos(c)}{(a\sin(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((log(sin(c + d*x) + 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**2*d
) + log(sin(c + d*x) + 1)/(a**2*d*sin(c + d*x) + a**2*d) + 1/(a**2*d*sin(c
+ d*x) + a**2*d), Ne(d, 0)), (x*sin(c)*cos(c)/(a*sin(c) + a)**2, True))
```

3.236 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2} \, dx$

Optimal. Leaf size=52

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] ln(sin(d*x+c))/a^2/d-ln(1+sin(d*x+c))/a^2/d+1/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 44}

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[1 + Sin[c + d*x]]/(a^2*d) + 1/(d*(a^2 + a^2
*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{1}{a(a+x)^2} - \frac{1}{a^2(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^2d} - \frac{\log(1+\sin(c+dx))}{a^2d} + \frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.06, size = 36, normalized size = 0.69

$$\frac{\frac{1}{\sin(c+dx)+1} + \log(\sin(c+dx)) - \log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Log[Sin[c + d*x]] - Log[1 + Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(a^2*
d)
```

fricas [A] time = 0.54, size = 59, normalized size = 1.13

$$\frac{(\sin(dx+c)+1)\log\left(\frac{1}{2}\sin(dx+c)\right) - (\sin(dx+c)+1)\log(\sin(dx+c)+1) + 1}{a^2d\sin(dx+c) + a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((sin(d*x + c) + 1)*log(1/2*sin(d*x + c)) - (sin(d*x + c) + 1)*log(sin(d*x
+ c) + 1) + 1)/(a^2*d*sin(d*x + c) + a^2*d)
```

giac [A] time = 0.15, size = 45, normalized size = 0.87

$$\frac{a\left(\frac{\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^3} + \frac{1}{(a\sin(dx+c)+a)a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] a*(log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^3 + 1/((a*sin(d*x + c) + a)*a^2)
)/d
```

maple [A] time = 0.28, size = 50, normalized size = 0.96

$$\frac{\ln(\sin(dx+c))}{a^2 d} + \frac{1}{d\,a^2\,(1+\sin(dx+c))} - \frac{\ln(1+\sin(dx+c))}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] ln(sin(d*x+c))/a^2/d+1/d/a^2/(1+sin(d*x+c))-ln(1+sin(d*x+c))/a^2/d
```

maxima [A] time = 0.49, size = 46, normalized size = 0.88

$$\frac{\frac{1}{a^2\sin(dx+c)+a^2} - \frac{\log(\sin(dx+c)+1)}{a^2} + \frac{\log(\sin(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a^2*sin(d*x + c) + a^2) - log(sin(d*x + c) + 1)/a^2 + log(sin(d*x + c))
/a^2)/d
```

mupad [B] time = 8.58, size = 87, normalized size = 1.67

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} - \frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^2\,d} - \frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - (2*log(tan(c/2 + (d*x)/2) + 1))/(a^2*d) -
 (2*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 + a^2 + 2*a^2*tan(c/2
+ (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1),
x)/a**2
```

3.237 $\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc(c+dx)}{a^2d}-\frac{2\log(\sin(c+dx))}{a^2d}+\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] -csc(d*x+c)/a^2/d-2*ln(sin(d*x+c))/a^2/d+2*ln(1+sin(d*x+c))/a^2/d-1/d/(a^2+
a^2*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 44}

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc(c+dx)}{a^2d}-\frac{2\log(\sin(c+dx))}{a^2d}+\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]/(a^2*d)) - (2*Log[Sin[c + d*x]])/(a^2*d) + (2*Log[1 + Sin[c
+ d*x]])/(a^2*d) - 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^2}{x^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2}{a^3x}+\frac{1}{a^2(a+x)^2}+\frac{2}{a^3(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc(c+dx)}{a^2d}-\frac{2\log(\sin(c+dx))}{a^2d}+\frac{2\log(1+\sin(c+dx))}{a^2d}-\frac{1}{d\left(a^2+a^2\sin(c+\right.}$$

Mathematica [A] time = 0.16, size = 45, normalized size = 0.66

$$-\frac{\frac{1}{\sin(c+dx)+1}+\csc(c+dx)+2\log(\sin(c+dx))-2\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((Csc[c + d*x] + 2*Log[Sin[c + d*x]] - 2*Log[1 + Sin[c + d*x]] + (1 + Sin[
c + d*x])^(-1))/(a^2*d))
```

fricas [A] time = 0.50, size = 104, normalized size = 1.53

$$-\frac{2\left(\cos(dx+c)^2-\sin(dx+c)-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-2\left(\cos(dx+c)^2-\sin(dx+c)-1\right)\log(\sin(dx+c)+}{a^2d\cos(dx+c)^2-a^2d\sin(dx+c)-a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*(cos(d*x + c)^2 - sin(d*x + c) - 1)*log(1/2*sin(d*x + c)) - 2*(cos(d*x
+ c)^2 - sin(d*x + c) - 1)*log(sin(d*x + c) + 1) - 2*sin(d*x + c) - 1)/(a^2
*d*cos(d*x + c)^2 - a^2*d*sin(d*x + c) - a^2*d)
```

giac [A] time = 0.16, size = 69, normalized size = 1.01

$$-\frac{\frac{2\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^2}+\frac{1}{(a\sin(dx+c)+a)a}-\frac{1}{a^2\left(\frac{a}{a\sin(dx+c)+a}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^2 + 1/((a*sin(d*x + c) + a)*a)
- 1/(a^2*(a/(a*sin(d*x + c) + a) - 1)))/d
```

maple [A] time = 0.26, size = 68, normalized size = 1.00

$$-\frac{1}{a^2 d \sin(dx+c)} - \frac{2\ln(\sin(dx+c))}{a^2 d} - \frac{1}{d\,a^2(1+\sin(dx+c))} + \frac{2\ln(1+\sin(dx+c))}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/a^2/d/sin(d*x+c)-2*ln(sin(d*x+c))/a^2/d-1/d/a^2/(1+sin(d*x+c))+2*ln(1+si
n(d*x+c))/a^2/d
```

maxima [A] time = 0.32, size = 68, normalized size = 1.00

$$-\frac{\frac{2\,\sin(dx+c)+1}{a^2\sin(dx+c)^2+a^2\sin(dx+c)} - \frac{2\,\log(\sin(dx+c)+1)}{a^2} + \frac{2\,\log(\sin(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((2*sin(d*x + c) + 1)/(a^2*sin(d*x + c)^2 + a^2*sin(d*x + c)) - 2*log(sin(
d*x + c) + 1)/a^2 + 2*log(sin(d*x + c))/a^2)/d
```

mupad [B] time = 8.61, size = 136, normalized size = 2.00

$$\frac{4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^2\,d} - \frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} - \frac{-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1}{d\left(2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (4*log(tan(c/2 + (d*x)/2) + 1))/(a^2*d) - (2*log(tan(c/2 + (d*x)/2)))/(a^2*
d) - (2*tan(c/2 + (d*x)/2) - 3*tan(c/2 + (d*x)/2)^2 + 1)/(d*(4*a^2*tan(c/2
+ (d*x)/2)^2 + 2*a^2*tan(c/2 + (d*x)/2)^3 + 2*a^2*tan(c/2 + (d*x)/2))) - ta
n(c/2 + (d*x)/2)/(2*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

3.238 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{3\log(\sin(c+dx))}{a^2d}-\frac{3\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] 2*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a^2/d+3*ln(sin(d*x+c))/a^2/d-3*ln(1+sin
(d*x+c))/a^2/d+1/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{3\log(\sin(c+dx))}{a^2d}-\frac{3\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a^2*d) + (3*Log[Sin[c + d*x]])
/(a^2*d) - (3*Log[1 + Sin[c + d*x]])/(a^2*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^3}{x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{1}{x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{a^2x^3} - \frac{2}{a^3x^2} + \frac{3}{a^4x} - \frac{1}{a^3(a+x)^2} - \frac{3}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{2\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2a^2d} + \frac{3\log(\sin(c+dx))}{a^2d} - \frac{3\log(1+\sin(c+dx))}{a^2d} + \frac{}{d}$$

Mathematica [A] time = 0.20, size = 61, normalized size = 0.72

$$\frac{\frac{2}{\sin(c+dx)+1} - \csc^2(c+dx) + 4\csc(c+dx) + 6\log(\sin(c+dx)) - 6\log(\sin(c+dx)+1)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*Csc[c + d*x] - Csc[c + d*x]^2 + 6*Log[Sin[c + d*x]] - 6*Log[1 + Sin[c +
d*x]] + 2/(1 + Sin[c + d*x]))/(2*a^2*d)
```

fricas [A] time = 0.50, size = 147, normalized size = 1.73

$$\frac{6\cos(dx+c)^2 + 6\left(\cos(dx+c)^2 + \left(\cos(dx+c)^2 - 1\right)\sin(dx+c) - 1\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 6\left(\cos(dx+c)^2 + \right.}{2\left(a^2d\cos(dx+c)^2 - a^2d + \left(a^2d\cos(dx+c)^2 - a^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*cos(d*x + c)^2 + 6*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x +
c) - 1)*log(1/2*sin(d*x + c)) - 6*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*si
n(d*x + c) - 1)*log(sin(d*x + c) + 1) - 3*sin(d*x + c) - 5)/(a^2*d*cos(d*x
+ c)^2 - a^2*d + (a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

giac [A] time = 0.19, size = 87, normalized size = 1.02

$$\frac{\frac{6\,\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^2}+\frac{2}{(a\sin(dx+c)+a)a}-\frac{\frac{6\,a}{a\sin(dx+c)+a}-5}{a^2\left(\frac{a}{a\sin(dx+c)+a}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(6*log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^2 + 2/((a*sin(d*x + c) + a)*
a) - (6*a/(a*sin(d*x + c) + a) - 5)/(a^2*(a/(a*sin(d*x + c) + a) - 1)^2))/d
```

maple [A] time = 0.30, size = 83, normalized size = 0.98

$$-\frac{1}{2a^2d\sin(dx+c)^2}+\frac{2}{a^2d\sin(dx+c)}+\frac{3\ln(\sin(dx+c))}{a^2d}+\frac{1}{d\,a^2\,(1+\sin(dx+c))}-\frac{3\ln(1+\sin(dx+c))}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/2/a^2/d/sin(d*x+c)^2+2/a^2/d/sin(d*x+c)+3*ln(sin(d*x+c))/a^2/d+1/d/a^2/(
1+sin(d*x+c))-3*ln(1+sin(d*x+c))/a^2/d
```

maxima [A] time = 0.30, size = 80, normalized size = 0.94

$$\frac{\frac{6\,\sin(dx+c)^2+3\,\sin(dx+c)-1}{a^2\sin(dx+c)^3+a^2\sin(dx+c)^2}-\frac{6\,\log(\sin(dx+c)+1)}{a^2}+\frac{6\,\log(\sin(dx+c))}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((6*sin(d*x + c)^2 + 3*sin(d*x + c) - 1)/(a^2*sin(d*x + c)^3 + a^2*sin(
d*x + c)^2) - 6*log(sin(d*x + c) + 1)/a^2 + 6*log(sin(d*x + c))/a^2)/d
```

mupad [B] time = 8.61, size = 168, normalized size = 1.98

$$\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}-\frac{6\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^2\,d}+\frac{-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+3\tan\left(\frac{c}{2} \cdots\right.}{d\left(4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (3*log(tan(c/2 + (d*x)/2)))/(a^2*d) - tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (6*l
og(tan(c/2 + (d*x)/2) + 1))/(a^2*d) + (3*tan(c/2 + (d*x)/2) + (15*tan(c/2 +
 (d*x)/2)^2)/2 - 4*tan(c/2 + (d*x)/2)^3 - 1/2)/(d*(4*a^2*tan(c/2 + (d*x)/2)
^2 + 8*a^2*tan(c/2 + (d*x)/2)^3 + 4*a^2*tan(c/2 + (d*x)/2)^4)) + tan(c/2 +
(d*x)/2)/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

3.239 $\int \frac{\cot(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc^3(c+dx)}{3a^2d}+\frac{\csc^2(c+dx)}{a^2d}-\frac{3\csc(c+dx)}{a^2d}-\frac{4\log(\sin(c+dx))}{a^2d}+\frac{4\log(\sin(c+dx)+1)}{a^2d}$$

[Out] -3*csc(d*x+c)/a^2/d+csc(d*x+c)^2/a^2/d-1/3*csc(d*x+c)^3/a^2/d-4*ln(sin(d*x+c))/a^2/d+4*ln(1+sin(d*x+c))/a^2/d-1/d/(a^2+a^2*sin(d*x+c))

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc^3(c+dx)}{3a^2d}+\frac{\csc^2(c+dx)}{a^2d}-\frac{3\csc(c+dx)}{a^2d}-\frac{4\log(\sin(c+dx))}{a^2d}+\frac{4\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]

[Out] (-3*Csc[c + d*x])/(a^2*d) + Csc[c + d*x]^2/(a^2*d) - Csc[c + d*x]^3/(3*a^2*d) - (4*Log[Sin[c + d*x]])/(a^2*d) + (4*Log[1 + Sin[c + d*x]])/(a^2*d) - 1/(d*(a^2 + a^2*Sin[c + d*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^4}{x^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{1}{x^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{a^2x^4} - \frac{2}{a^3x^3} + \frac{3}{a^4x^2} - \frac{4}{a^5x} + \frac{1}{a^4(a+x)^2} + \frac{4}{a^5(a+x)}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{3\csc(c+dx)}{a^2d} + \frac{\csc^2(c+dx)}{a^2d} - \frac{\csc^3(c+dx)}{3a^2d} - \frac{4\log(\sin(c+dx))}{a^2d} + \frac{4\log(1+$$

Mathematica [A] time = 2.46, size = 98, normalized size = 0.97

$$-\frac{1}{a^2d(\sin(c+dx)+1)} - \frac{\csc^3(c+dx)}{3a^2d} + \frac{\csc^2(c+dx)}{a^2d} - \frac{3\csc(c+dx)}{a^2d} - \frac{4\log(\sin(c+dx))}{a^2d} + \frac{4\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*Csc[c + d*x])/(a^2*d) + Csc[c + d*x]^2/(a^2*d) - Csc[c + d*x]^3/(3*a^2*
d) - (4*Log[Sin[c + d*x]])/(a^2*d) + (4*Log[1 + Sin[c + d*x]])/(a^2*d) - 1/
(a^2*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.52, size = 195, normalized size = 1.93

$$\frac{6\cos(dx+c)^2 - 12\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 - \left(\cos(dx+c)^2 - 1\right)\sin(dx+c) + 1\right)\log\left(\frac{1}{2}\sin(dx+c)\right) + }{3\left(a^2d\cos(dx+c)^4 - 2a^2d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*cos(d*x + c)^2 - 12*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x +
c)^2 - 1)*sin(d*x + c) + 1)*log(1/2*sin(d*x + c)) + 12*(cos(d*x + c)^4 - 2*
cos(d*x + c)^2 - (cos(d*x + c)^2 - 1)*sin(d*x + c) + 1)*log(sin(d*x + c) +
1) + 2*(6*cos(d*x + c)^2 - 5)*sin(d*x + c) - 7)/(a^2*d*cos(d*x + c)^4 - 2*a
^2*d*cos(d*x + c)^2 + a^2*d - (a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

giac [A] time = 0.18, size = 103, normalized size = 1.02

$$-\frac{\frac{12\,\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^2}+\frac{3}{(a\sin(dx+c)+a)a}+\frac{\frac{30\,a}{a\sin(dx+c)+a}-\frac{18\,a^2}{(a\sin(dx+c)+a)^2}-13}{a^2\left(\frac{a}{a\sin(dx+c)+a}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(12*log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^2 + 3/((a*sin(d*x + c) + a
)*a) + (30*a/(a*sin(d*x + c) + a) - 18*a^2/(a*sin(d*x + c) + a)^2 - 13)/(a^
2*(a/(a*sin(d*x + c) + a) - 1)^3))/d
```

maple [A] time = 0.29, size = 99, normalized size = 0.98

$$-\frac{1}{3a^2d\sin(dx+c)^3}+\frac{1}{a^2d\sin(dx+c)^2}-\frac{3}{a^2d\sin(dx+c)}-\frac{4\ln(\sin(dx+c))}{a^2d}-\frac{1}{d\,a^2\,(1+\sin(dx+c))}+\frac{4\ln(1+\mathrm{si}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/3/a^2/d/sin(d*x+c)^3+1/a^2/d/sin(d*x+c)^2-3/a^2/d/sin(d*x+c)-4*ln(sin(d*
x+c))/a^2/d-1/d/a^2/(1+sin(d*x+c))+4*ln(1+sin(d*x+c))/a^2/d
```

maxima [A] time = 0.32, size = 90, normalized size = 0.89

$$-\frac{\frac{12\,\sin(dx+c)^3+6\,\sin(dx+c)^2-2\,\sin(dx+c)+1}{a^2\,\sin(dx+c)^4+a^2\,\sin(dx+c)^3}-\frac{12\,\log(\sin(dx+c)+1)}{a^2}+\frac{12\,\log(\sin(dx+c))}{a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*((12*sin(d*x + c)^3 + 6*sin(d*x + c)^2 - 2*sin(d*x + c) + 1)/(a^2*sin(
d*x + c)^4 + a^2*sin(d*x + c)^3) - 12*log(sin(d*x + c) + 1)/a^2 + 12*log(si
n(d*x + c))/a^2)/d
```

mupad [B] time = 8.63, size = 202, normalized size = 2.00

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^2\,d}-\frac{-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{1}{3}}{d\left(8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+16\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{4\,\ln\left(\mathrm{tan}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(4*a^2*d) - tan(c/2 + (d*x)/2)^3/(24*a^2*d) - ((28*tan
(c/2 + (d*x)/2)^2)/3 - (4*tan(c/2 + (d*x)/2))/3 + 24*tan(c/2 + (d*x)/2)^3 -
 3*tan(c/2 + (d*x)/2)^4 + 1/3)/(d*(8*a^2*tan(c/2 + (d*x)/2)^3 + 16*a^2*tan(
c/2 + (d*x)/2)^4 + 8*a^2*tan(c/2 + (d*x)/2)^5)) - (4*log(tan(c/2 + (d*x)/2)
))/(a^2*d) + (8*log(tan(c/2 + (d*x)/2) + 1))/(a^2*d) - (13*tan(c/2 + (d*x)/
2))/(8*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc^4(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**4/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

3.240 $\int \frac{\cos(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=111

$$\frac{\sin^3(c+dx)}{3a^3d}-\frac{3\sin^2(c+dx)}{2a^3d}+\frac{6\sin(c+dx)}{a^3d}-\frac{5}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{10\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx}$$

```
[Out] -10*ln(1+sin(d*x+c))/a^3/d+6*sin(d*x+c)/a^3/d-3/2*sin(d*x+c)^2/a^3/d+1/3*si
n(d*x+c)^3/a^3/d+1/2/a/d/(a+a*sin(d*x+c))^2-5/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^3(c+dx)}{3a^3d}-\frac{3\sin^2(c+dx)}{2a^3d}+\frac{6\sin(c+dx)}{a^3d}-\frac{5}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{10\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-10*Log[1 + Sin[c + d*x]])/(a^3*d) + (6*Sin[c + d*x])/(a^3*d) - (3*Sin[c +
 d*x]^2)/(2*a^3*d) + Sin[c + d*x]^3/(3*a^3*d) + 1/(2*a*d*(a + a*Sin[c + d*x
])^2) - 5/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{a^5(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(6a^2-3ax+x^2-\frac{a^5}{(a+x)^3}+\frac{5a^4}{(a+x)^2}-\frac{10a^3}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= -\frac{10\log(1+\sin(c+dx))}{a^3 d}+\frac{6\sin(c+dx)}{a^3 d}-\frac{3\sin^2(c+dx)}{2a^3 d}+\frac{\sin^3(c+dx)}{3a^3 d}+\frac{}{2a\ldots}$$

Mathematica [A] time = 0.70, size = 106, normalized size = 0.95

$$\frac{32\sin^5(c+dx)-80\sin^4(c+dx)+320\sin^3(c+dx)+\sin^2(c+dx)(1023-960\log(\sin(c+dx)+1))-6\sin(c+\ldots}{96a^3 d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-417 - 960*Log[1 + Sin[c + d*x]] - 6*(-21 + 320*Log[1 + Sin[c + d*x]])*Sin
[c + d*x] + (1023 - 960*Log[1 + Sin[c + d*x]])*Sin[c + d*x]^2 + 320*Sin[c +
 d*x]^3 - 80*Sin[c + d*x]^4 + 32*Sin[c + d*x]^5)/(96*a^3*d*(1 + Sin[c + d*x
])^2)
```

fricas [A] time = 0.52, size = 117, normalized size = 1.05

$$\frac{10\cos(dx+c)^4+115\cos(dx+c)^2-120\left(\cos(dx+c)^2-2\sin(dx+c)-2\right)\log(\sin(dx+c)+1)-2\left(2\cos(d\ldots\right.}{12\left(a^3 d\cos(dx+c)^2-2a^3 d\sin(dx+c)-2a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(10*cos(d*x + c)^4 + 115*cos(d*x + c)^2 - 120*(cos(d*x + c)^2 - 2*sin(
d*x + c) - 2)*log(sin(d*x + c) + 1) - 2*(2*cos(d*x + c)^4 - 24*cos(d*x + c)
^2 + 37)*sin(d*x + c) - 80)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) -
2*a^3*d)
```

giac [A] time = 0.22, size = 89, normalized size = 0.80

$$-\frac{\frac{60\log(|\sin(dx+c)+1|)}{a^3}+\frac{3(10\sin(dx+c)+9)}{a^3(\sin(dx+c)+1)^2}-\frac{2a^6\sin(dx+c)^3-9a^6\sin(dx+c)^2+36a^6\sin(dx+c)}{a^9}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(60*log(abs(sin(d*x + c) + 1))/a^3 + 3*(10*sin(d*x + c) + 9)/(a^3*(sin
(d*x + c) + 1)^2) - (2*a^6*sin(d*x + c)^3 - 9*a^6*sin(d*x + c)^2 + 36*a^6*s
in(d*x + c))/a^9)/d
```

maple [A] time = 0.24, size = 101, normalized size = 0.91

$$\frac{\sin^3(dx+c)}{3a^3d}-\frac{3\left(\sin^2(dx+c)\right)}{2a^3d}+\frac{6\sin(dx+c)}{a^3d}+\frac{1}{2d\,a^3\,(1+\sin(dx+c))^2}-\frac{10\ln(1+\sin(dx+c))}{a^3d}-\frac{5}{d\,a^3\,(1+\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/3*sin(d*x+c)^3/a^3/d-3/2*sin(d*x+c)^2/a^3/d+6*sin(d*x+c)/a^3/d+1/2/d/a^3/
(1+sin(d*x+c))^2-10*ln(1+sin(d*x+c))/a^3/d-5/d/a^3/(1+sin(d*x+c))
```

maxima [A] time = 0.32, size = 95, normalized size = 0.86

$$-\frac{\frac{3(10\sin(dx+c)+9)}{a^3\sin(dx+c)^2+2a^3\sin(dx+c)+a^3}-\frac{2\sin(dx+c)^3-9\sin(dx+c)^2+36\sin(dx+c)}{a^3}+\frac{60\log(\sin(dx+c)+1)}{a^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(10*sin(d*x + c) + 9)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^
3) - (2*sin(d*x + c)^3 - 9*sin(d*x + c)^2 + 36*sin(d*x + c))/a^3 + 60*log(s
in(d*x + c) + 1)/a^3)/d
```

mupad [B] time = 0.13, size = 108, normalized size = 0.97

$$\frac{6\sin(c+d\,x)}{a^3\,d}-\frac{5\sin(c+d\,x)+\frac{9}{2}}{d\left(a^3\sin(c+d\,x)^2+2\,a^3\,\sin(c+d\,x)+a^3\right)}-\frac{10\ln(\sin(c+d\,x)+1)}{a^3\,d}-\frac{3\sin(c+d\,x)^2}{2\,a^3\,d}+\frac{\sin(c}{3\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^5)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (6*sin(c + d*x))/(a^3*d) - (5*sin(c + d*x) + 9/2)/(d*(2*a^3*sin(c + d*x) +
a^3 + a^3*sin(c + d*x)^2)) - (10*log(sin(c + d*x) + 1))/(a^3*d) - (3*sin(c
+ d*x)^2)/(2*a^3*d) + sin(c + d*x)^3/(3*a^3*d)
```

sympy [A] time = 8.06, size = 394, normalized size = 3.55

$$\begin{cases} -\frac{60\log(\sin(c+dx)+1)\sin^2(c+dx)}{6a^3d\sin^2(c+dx)+12a^3d\sin(c+dx)+6a^3d} - \frac{120\log(\sin(c+dx)+1)\sin(c+dx)}{6a^3d\sin^2(c+dx)+12a^3d\sin(c+dx)+6a^3d} - \frac{60\log(\sin(c+dx)+1)}{6a^3d\sin^2(c+dx)+12a^3d\sin(c+dx)+6a^3d} + \frac{}{6a^3d\sin^2(c} \\ \frac{x\sin^5(c)\cos(c)}{(a\sin(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-60*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(6*a**3*d*sin(c + d*x)
**2 + 12*a**3*d*sin(c + d*x) + 6*a**3*d) - 120*log(sin(c + d*x) + 1)*sin(c
+ d*x)/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d*sin(c + d*x) + 6*a**3*d) - 60*
log(sin(c + d*x) + 1)/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d*sin(c + d*x) +
6*a**3*d) + 2*sin(c + d*x)**5/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d*sin(c +
 d*x) + 6*a**3*d) - 5*sin(c + d*x)**4/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d
*sin(c + d*x) + 6*a**3*d) + 20*sin(c + d*x)**3/(6*a**3*d*sin(c + d*x)**2 +
12*a**3*d*sin(c + d*x) + 6*a**3*d) - 120*sin(c + d*x)/(6*a**3*d*sin(c + d*x
)**2 + 12*a**3*d*sin(c + d*x) + 6*a**3*d) - 90/(6*a**3*d*sin(c + d*x)**2 +
12*a**3*d*sin(c + d*x) + 6*a**3*d), Ne(d, 0)), (x*sin(c)**5*cos(c)/(a*sin(c
) + a)**3, True))
```

3.241 $\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=93

$$\frac{\sin^2(c+dx)}{2a^3d}-\frac{3\sin(c+dx)}{a^3d}+\frac{4}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{6\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] 6*ln(1+sin(d*x+c))/a^3/d-3*sin(d*x+c)/a^3/d+1/2*sin(d*x+c)^2/a^3/d-1/2/a/d/
(a+a*sin(d*x+c))^2+4/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^2(c+dx)}{2a^3d}-\frac{3\sin(c+dx)}{a^3d}+\frac{4}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{6\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*Log[1 + Sin[c + d*x]])/(a^3*d) - (3*Sin[c + d*x])/(a^3*d) + Sin[c + d*x]
^2/(2*a^3*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) + 4/(d*(a^3 + a^3*Sin[c + d
*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{a^4(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(-3a+x+\frac{a^4}{(a+x)^3}-\frac{4a^3}{(a+x)^2}+\frac{6a^2}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{6\log(1+\sin(c+dx))}{a^3d} - \frac{3\sin(c+dx)}{a^3d} + \frac{\sin^2(c+dx)}{2a^3d} - \frac{1}{2ad(a+a\sin(c+dx))^2}$$

Mathematica [A] time = 2.08, size = 78, normalized size = 0.84

$$\frac{8\sin^2(c+dx)+\left(\frac{64}{(\sin(c+dx)+1)^2}-48\right)\sin(c+dx)+96\log(\sin(c+dx)+1)+\frac{56}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}}{16a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (96*Log[1 + Sin[c + d*x]] + 56/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 8*
Sin[c + d*x]^2 + Sin[c + d*x]*(-48 + 64/(1 + Sin[c + d*x])^2))/(16*a^3*d)
```

fricas [A] time = 0.48, size = 107, normalized size = 1.15

$$-\frac{2\cos(dx+c)^4+19\cos(dx+c)^2-24\left(\cos(dx+c)^2-2\sin(dx+c)-2\right)\log(\sin(dx+c)+1)+2\left(4\cos(dx\right.}{4\left(a^3d\cos(dx+c)^2-2a^3d\sin(dx+c)-2a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(d*x + c)^4 + 19*cos(d*x + c)^2 - 24*(cos(d*x + c)^2 - 2*sin(d*x
 + c) - 2)*log(sin(d*x + c) + 1) + 2*(4*cos(d*x + c)^2 - 3)*sin(d*x + c) -
8)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

giac [A] time = 0.21, size = 73, normalized size = 0.78

$$\frac{\frac{12\log(|\sin(dx+c)+1|)}{a^3}+\frac{8\sin(dx+c)+7}{a^3(\sin(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^2-6a^3\sin(dx+c)}{a^6}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(12*log(abs(sin(d*x + c) + 1))/a^3 + (8*sin(d*x + c) + 7)/(a^3*(sin(d*x
 + c) + 1)^2) + (a^3*sin(d*x + c)^2 - 6*a^3*sin(d*x + c))/a^6)/d
```

maple [A] time = 0.24, size = 85, normalized size = 0.91

$$\frac{\sin^2(dx+c)}{2a^3d}-\frac{3\sin(dx+c)}{a^3d}-\frac{1}{2d\,a^3\,(1+\sin(dx+c))^2}+\frac{6\ln(1+\sin(dx+c))}{a^3d}+\frac{4}{d\,a^3\,(1+\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2*sin(d*x+c)^2/a^3/d-3*sin(d*x+c)/a^3/d-1/2/d/a^3/(1+sin(d*x+c))^2+6*ln(1
+sin(d*x+c))/a^3/d+4/d/a^3/(1+sin(d*x+c))
```

maxima [A] time = 0.31, size = 81, normalized size = 0.87

$$\frac{\frac{8\sin(dx+c)+7}{a^3\sin(dx+c)^2+2a^3\sin(dx+c)+a^3}+\frac{\sin(dx+c)^2-6\sin(dx+c)}{a^3}+\frac{12\log(\sin(dx+c)+1)}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((8*sin(d*x + c) + 7)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3) +
 (sin(d*x + c)^2 - 6*sin(d*x + c))/a^3 + 12*log(sin(d*x + c) + 1)/a^3)/d
```

mupad [B] time = 8.49, size = 91, normalized size = 0.98

$$\frac{6\ln(\sin(c+d\,x)+1)}{a^3\,d}+\frac{4\sin(c+d\,x)+\frac{7}{2}}{d\left(a^3\sin(c+d\,x)^2+2\,a^3\sin(c+d\,x)+a^3\right)}-\frac{3\sin(c+d\,x)}{a^3\,d}+\frac{\sin(c+d\,x)^2}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^4)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (6*log(sin(c + d*x) + 1))/(a^3*d) + (4*sin(c + d*x) + 7/2)/(d*(2*a^3*sin(c
+ d*x) + a^3 + a^3*sin(c + d*x)^2)) - (3*sin(c + d*x))/(a^3*d) + sin(c + d*
x)^2/(2*a^3*d)
```

sympy [A] time = 5.47, size = 347, normalized size = 3.73

$$\begin{cases} \frac{12\log\left(\sin\left(c+dx\right)+1\right)\sin^{2}\left(c+dx\right)}{2a^{3}d\sin^{2}\left(c+dx\right)+4a^{3}d\sin\left(c+dx\right)+2a^{3}d}+\frac{24\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{2a^{3}d\sin^{2}\left(c+dx\right)+4a^{3}d\sin\left(c+dx\right)+2a^{3}d}+\frac{12\log\left(\sin\left(c+dx\right)+1\right)}{2a^{3}d\sin^{2}\left(c+dx\right)+4a^{3}d\sin\left(c+dx\right)+2a^{3}d}+\frac{\sin}{2a^{3}d\sin^{2}\left(c+dx\right)} \\ \frac{x\sin^{4}\left(c\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{3}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((12*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)*
*2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 24*log(sin(c + d*x) + 1)*sin(c + d
*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 12*log(
sin(c + d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**
3*d) + sin(c + d*x)**4/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) +
2*a**3*d) - 4*sin(c + d*x)**3/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c +
d*x) + 2*a**3*d) + 24*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin
(c + d*x) + 2*a**3*d) + 18/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x
) + 2*a**3*d), Ne(d, 0)), (x*sin(c)**4*cos(c)/(a*sin(c) + a)**3, True))
```

3.242 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=74

$$\frac{\sin(c+dx)}{a^3d}-\frac{3}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{3\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] -3*ln(1+sin(d*x+c))/a^3/d+sin(d*x+c)/a^3/d+1/2/a/d/(a+a*sin(d*x+c))^2-3/d/(
a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin(c+dx)}{a^3d}-\frac{3}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{3\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*Log[1 + Sin[c + d*x]])/(a^3*d) + Sin[c + d*x]/(a^3*d) + 1/(2*a*d*(a + a
*Sin[c + d*x])^2) - 3/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{a^3(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{a^3}{(a+x)^3}+\frac{3a^2}{(a+x)^2}-\frac{3a}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= -\frac{3\log(1+\sin(c+dx))}{a^3 d}+\frac{\sin(c+dx)}{a^3 d}+\frac{1}{2ad(a+a\sin(c+dx))^2}-\frac{3}{d\left(a^3+a^3\sin\right.}$$

Mathematica [A] time = 0.40, size = 70, normalized size = 0.95

$$\frac{\frac{\sin^2(c+dx)}{(\sin(c+dx)+1)^2}+4\sin(c+dx)+\frac{-10\sin(c+dx)-9}{(\sin(c+dx)+1)^2}-12\log(\sin(c+dx)+1)}{4a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-12*Log[1 + Sin[c + d*x]] + 4*Sin[c + d*x] + (-9 - 10*Sin[c + d*x])/(1 + S
in[c + d*x])^2 + Sin[c + d*x]^2/(1 + Sin[c + d*x])^2)/(4*a^3*d)
```

fricas [A] time = 0.56, size = 95, normalized size = 1.28

$$\frac{4\cos(dx+c)^2-6\left(\cos(dx+c)^2-2\sin(dx+c)-2\right)\log(\sin(dx+c)+1)+2\left(\cos(dx+c)^2+1\right)\sin(dx+c)+}{2\left(a^3 d\cos(dx+c)^2-2a^3 d\sin(dx+c)-2a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*cos(d*x + c)^2 - 6*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x
 + c) + 1) + 2*(cos(d*x + c)^2 + 1)*sin(d*x + c) + 1)/(a^3*d*cos(d*x + c)^2
 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

giac [A] time = 0.21, size = 56, normalized size = 0.76

$$-\frac{\frac{6\log(|\sin(dx+c)+1|)}{a^3}-\frac{2\sin(dx+c)}{a^3}+\frac{6\sin(dx+c)+5}{a^3(\sin(dx+c)+1)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*log(abs(sin(d*x + c) + 1))/a^3 - 2*sin(d*x + c)/a^3 + (6*sin(d*x +
c) + 5)/(a^3*(sin(d*x + c) + 1)^2))/d
```

maple [A] time = 0.23, size = 68, normalized size = 0.92

$$\frac{\sin\left(dx+c\right)}{a^{3}d}+\frac{1}{2d\,a^{3}\left(1+\sin\left(dx+c\right)\right)^{2}}-\frac{3\ln\left(1+\sin\left(dx+c\right)\right)}{a^{3}d}-\frac{3}{d\,a^{3}\left(1+\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] sin(d*x+c)/a^3/d+1/2/d/a^3/(1+sin(d*x+c))^2-3*ln(1+sin(d*x+c))/a^3/d-3/d/a^
3/(1+sin(d*x+c))
```

maxima [A] time = 0.30, size = 71, normalized size = 0.96

$$-\frac{\frac{6\,\sin(dx+c)+5}{a^{3}\sin(dx+c)^{2}+2\,a^{3}\sin(dx+c)+a^{3}}+\frac{6\,\log(\sin(dx+c)+1)}{a^{3}}-\frac{2\,\sin(dx+c)}{a^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((6*sin(d*x + c) + 5)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3)
+ 6*log(sin(d*x + c) + 1)/a^3 - 2*sin(d*x + c)/a^3)/d
```

mupad [B] time = 0.07, size = 59, normalized size = 0.80

$$\frac{\sin\left(c+d\,x\right)}{a^{3}\,d}-\frac{3\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^{3}\,d}-\frac{3\,\sin\left(c+d\,x\right)+\frac{5}{2}}{a^{3}\,d\,{\left(\sin\left(c+d\,x\right)+1\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^3)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] sin(c + d*x)/(a^3*d) - (3*log(sin(c + d*x) + 1))/(a^3*d) - (3*sin(c + d*x)
+ 5/2)/(a^3*d*(sin(c + d*x) + 1)^2)
```

sympy [A] time = 3.06, size = 303, normalized size = 4.09

$$\begin{cases} -\frac{6\log\left(\sin\left(c+dx\right)+1\right)\sin^{2}\left(c+dx\right)}{2a^{3}d\sin^{2}\left(c+dx\right)+4a^{3}d\sin\left(c+dx\right)+2a^{3}d}-\frac{12\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{2a^{3}d\sin^{2}\left(c+dx\right)+4a^{3}d\sin\left(c+dx\right)+2a^{3}d}-\frac{6\log\left(\sin\left(c+dx\right)+1\right)}{2a^{3}d\sin^{2}\left(c+dx\right)+4a^{3}d\sin\left(c+dx\right)+2a^{3}d}+\frac{}{2a^{3}d\sin^{2}\left(c+\right.} \\ \frac{x\sin^{3}\left(c\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{3}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-6*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)*
*2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 12*log(sin(c + d*x) + 1)*sin(c + d
*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 6*log(s
in(c + d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3
*d) + 2*sin(c + d*x)**3/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) +
 2*a**3*d) - 12*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d
*x) + 2*a**3*d) - 9/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a
**3*d), Ne(d, 0)), (x*sin(c)**3*cos(c)/(a*sin(c) + a)**3, True))
```

3.243 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=60

$$\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] ln(1+sin(d*x+c))/a^3/d-1/2/a/d/(a+a*sin(d*x+c))^2+2/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a^3*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) + 2/(d*(a^
3 + a^3*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{(a+x)^3} - \frac{2a}{(a+x)^2} + \frac{1}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\log(1+\sin(c+dx))}{a^3d} - \frac{1}{2ad(a+a\sin(c+dx))^2} + \frac{2}{d\left(a^3+a^3\sin(c+dx)\right)}$$

Mathematica [A] time = 0.65, size = 65, normalized size = 1.08

$$\frac{\frac{16\sin(c+dx)}{(\sin(c+dx)+1)^2} + 8\log(\sin(c+dx)+1) + \frac{12}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (8*Log[1 + Sin[c + d*x]] + 12/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + (16
*Sin[c + d*x])/(1 + Sin[c + d*x])^2)/(8*a^3*d)
```

fricas [A] time = 0.53, size = 75, normalized size = 1.25

$$\frac{2\left(\cos(dx+c)^2 - 2\sin(dx+c) - 2\right)\log(\sin(dx+c)+1) - 4\sin(dx+c) - 3}{2\left(a^3d\cos(dx+c)^2 - 2a^3d\sin(dx+c) - 2a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - 4*sin(
d*x + c) - 3)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

giac [A] time = 0.18, size = 45, normalized size = 0.75

$$\frac{\frac{2\log(|\sin(dx+c)+1|)}{a^3} + \frac{4\sin(dx+c)+3}{a^3(\sin(dx+c)+1)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(sin(d*x + c) + 1))/a^3 + (4*sin(d*x + c) + 3)/(a^3*(sin(d*x
+ c) + 1)^2))/d
```

maple [A] time = 0.22, size = 54, normalized size = 0.90

$$-\frac{1}{2d\,a^{3}\left(1+\sin\left(dx+c\right)\right)^{2}}+\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{a^{3}d}+\frac{2}{d\,a^{3}\left(1+\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d/a^3/(1+sin(d*x+c))^2+ln(1+sin(d*x+c))/a^3/d+2/d/a^3/(1+sin(d*x+c))
```

maxima [A] time = 0.41, size = 60, normalized size = 1.00

$$\frac{\frac{4\,\sin\left(dx+c\right)+3}{a^{3}\sin\left(dx+c\right)^{2}+2\,a^{3}\sin\left(dx+c\right)+a^{3}}+\frac{2\,\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((4*sin(d*x + c) + 3)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3) +
 2*log(sin(d*x + c) + 1)/a^3)/d
```

mupad [B] time = 0.06, size = 44, normalized size = 0.73

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^{3}\,d}+\frac{2\,\sin\left(c+d\,x\right)+\frac{3}{2}}{a^{3}\,d\,\left(\sin\left(c+d\,x\right)+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] log(sin(c + d*x) + 1)/(a^3*d) + (2*sin(c + d*x) + 3/2)/(a^3*d*(sin(c + d*x)
 + 1)^2)
```

sympy [A] time = 1.93, size = 257, normalized size = 4.28

$$\begin{cases} \frac{2\log\left(\sin\left(c+dx\right)+1\right)\sin^{2}\left(c+dx\right)}{2a^{3}d\sin^{2}\left(c+dx\right)+4a^{3}d\sin\left(c+dx\right)+2a^{3}d}+\frac{4\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{2a^{3}d\sin^{2}\left(c+dx\right)+4a^{3}d\sin\left(c+dx\right)+2a^{3}d}+\frac{2\log\left(\sin\left(c+dx\right)+1\right)}{2a^{3}d\sin^{2}\left(c+dx\right)+4a^{3}d\sin\left(c+dx\right)+2a^{3}d}+\frac{4}{2a^{3}d\sin^{2}\left(c+d\right.} \\ \frac{x\sin^{2}\left(c\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{3}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)**
2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 4*log(sin(c + d*x) + 1)*sin(c + d*x
)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 2*log(sin
(c + d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d
) + 4*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a*
*3*d) + 3/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d), Ne
(d, 0)), (x*sin(c)**2*cos(c)/(a*sin(c) + a)**3, True))
```

3.244 $\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=30

$$\frac{\sin^2(c+dx)}{2ad(a\sin(c+dx)+a)^2}$$

[Out] 1/2*sin(d*x+c)^2/a/d/(a+a*sin(d*x+c))^2

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 37}

$$\frac{\sin^2(c+dx)}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]

[Out] Sin[c + d*x]^2/(2*a*d*(a + a*Sin[c + d*x])^2)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{a(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\sin^2(c+dx)}{2ad(a+a\sin(c+dx))^2}$$

Mathematica [**A**] time = 0.03, size = 30, normalized size = 1.00

$$\frac{\sin^2(c+dx)}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sin[c + d*x]^2/(2*a*d*(a + a*Sin[c + d*x])^2)
```

fricas [**A**] time = 0.51, size = 46, normalized size = 1.53

$$\frac{2\sin(dx+c)+1}{2\left(a^3d\cos(dx+c)^2-2a^3d\sin(dx+c)-2a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*sin(d*x + c) + 1)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a
^3*d)
```

giac [**A**] time = 0.16, size = 28, normalized size = 0.93

$$-\frac{2\sin(dx+c)+1}{2a^3d(\sin(dx+c)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*sin(d*x + c) + 1)/(a^3*d*(sin(d*x + c) + 1)^2)
```

maple [A] time = 0.20, size = 33, normalized size = 1.10

$$\frac{\frac{1}{2(1+\sin(dx+c))^2}-\frac{1}{1+\sin(dx+c)}}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(1/2/(1+sin(d*x+c))^2-1/(1+sin(d*x+c)))
```

maxima [A] time = 0.39, size = 44, normalized size = 1.47

$$-\frac{2\,\sin\left(dx+c\right)+1}{2\left(a^3\sin\left(dx+c\right)^2+2\,a^3\sin\left(dx+c\right)+a^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*sin(d*x + c) + 1)/((a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3)*
d)
```

mupad [B] time = 0.05, size = 37, normalized size = 1.23

$$\frac{1}{2\,a^3\,d\,{\left(\sin\left(c+d\,x\right)+1\right)}^2}-\frac{1}{a^3\,d\,\left(\sin\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] 1/(2*a^3*d*(sin(c + d*x) + 1)^2) - 1/(a^3*d*(sin(c + d*x) + 1))
```

sympy [A] time = 1.70, size = 99, normalized size = 3.30

$$\begin{cases}-\frac{2\sin\left(c+dx\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}-\frac{1}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d} & \text{for } d\neq 0\\ \frac{x\sin\left(c\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-2*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x
) + 2*a**3*d) - 1/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**
3*d), Ne(d, 0)), (x*sin(c)*cos(c)/(a*sin(c) + a)**3, True))
```

3.245 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=74

$$\frac{1}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] ln(sin(d*x+c))/a^3/d-ln(1+sin(d*x+c))/a^3/d+1/2/a/d/(a+a*sin(d*x+c))^2+1/d/
(a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 44}

$$\frac{1}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[1 + Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a + a
*Sin[c + d*x])^2) + 1/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{1}{a(a+x)^3} - \frac{1}{a^2(a+x)^2} - \frac{1}{a^3(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^3d} - \frac{\log(1+\sin(c+dx))}{a^3d} + \frac{1}{2ad(a+a\sin(c+dx))^2} + \frac{1}{d\left(a^3+a^3\sin\right.}$$

Mathematica [A] time = 0.18, size = 52, normalized size = 0.70

$$\frac{\frac{2\sin(c+dx)+3}{(\sin(c+dx)+1)^2} + 2\log(\sin(c+dx)) - 2\log(\sin(c+dx)+1)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*Log[Sin[c + d*x]] - 2*Log[1 + Sin[c + d*x]] + (3 + 2*Sin[c + d*x])/(1 +
Sin[c + d*x])^2)/(2*a^3*d)
```

fricas [A] time = 0.52, size = 104, normalized size = 1.41

$$\frac{2\left(\cos(dx+c)^2 - 2\sin(dx+c) - 2\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 2\left(\cos(dx+c)^2 - 2\sin(dx+c) - 2\right)\log(\sin(dx+}{2\left(a^3d\cos(dx+c)^2 - 2a^3d\sin(dx+c) - 2a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(1/2*sin(d*x + c)) - 2*(cos
(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - 2*sin(d*x + c) -
3)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

giac [A] time = 0.18, size = 59, normalized size = 0.80

$$-\frac{\frac{2\log(|\sin(dx+c)+1|)}{a^3} - \frac{2\log(|\sin(dx+c)|)}{a^3} - \frac{2\sin(dx+c)+3}{a^3(\sin(dx+c)+1)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(sin(d*x + c) + 1))/a^3 - 2*log(abs(sin(d*x + c)))/a^3 - (2*
sin(d*x + c) + 3)/(a^3*(sin(d*x + c) + 1)^2))/d
```

maple [A] time = 0.29, size = 68, normalized size = 0.92

$$\frac{\ln\left(\sin\left(dx+c\right)\right)}{a^{3}d}+\frac{1}{2d\,a^{3}\left(1+\sin\left(dx+c\right)\right)^{2}}+\frac{1}{d\,a^{3}\left(1+\sin\left(dx+c\right)\right)}-\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] ln(sin(d*x+c))/a^3/d+1/2/d/a^3/(1+sin(d*x+c))^2+1/d/a^3/(1+sin(d*x+c))-ln(1
+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.43, size = 72, normalized size = 0.97

$$\frac{\frac{2\,\sin\left(dx+c\right)+3}{a^{3}\,\sin\left(dx+c\right)^{2}+2\,a^{3}\,\sin\left(dx+c\right)+a^{3}}-\frac{2\,\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}}+\frac{2\,\log\left(\sin\left(dx+c\right)\right)}{a^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((2*sin(d*x + c) + 3)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3) -
 2*log(sin(d*x + c) + 1)/a^3 + 2*log(sin(d*x + c))/a^3)/d
```

mupad [B] time = 8.76, size = 148, normalized size = 2.00

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^{3}\,d}-\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^{3}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+4\,a^{3}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+6\,a^{3}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+4\,a^{3}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^{3}\right)}-\frac{2\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) - (4*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)
/2)^2 + 4*tan(c/2 + (d*x)/2)^3)/(d*(6*a^3*tan(c/2 + (d*x)/2)^2 + 4*a^3*tan(
c/2 + (d*x)/2)^3 + a^3*tan(c/2 + (d*x)/2)^4 + a^3 + 4*a^3*tan(c/2 + (d*x)/2
))) - (2*log(tan(c/2 + (d*x)/2) + 1))/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos\left(c+dx\right)\csc\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3
*sin(c + d*x) + 1), x)/a**3
```

3.246 $\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=90

$$-\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc(c+dx)}{a^3d}-\frac{3\log(\sin(c+dx))}{a^3d}+\frac{3\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] -csc(d*x+c)/a^3/d-3*ln(sin(d*x+c))/a^3/d+3*ln(1+sin(d*x+c))/a^3/d-1/2/a/d/(
a+a*sin(d*x+c))^2-2/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 44}

$$-\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc(c+dx)}{a^3d}-\frac{3\log(\sin(c+dx))}{a^3d}+\frac{3\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Csc[c + d*x]/(a^3*d)) - (3*Log[Sin[c + d*x]])/(a^3*d) + (3*Log[1 + Sin[c
+ d*x]])/(a^3*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) - 2/(d*(a^3 + a^3*Sin[c
 + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^2}{x^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{a^3x^2} - \frac{3}{a^4x} + \frac{1}{a^2(a+x)^3} + \frac{2}{a^3(a+x)^2} + \frac{3}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc(c+dx)}{a^3d} - \frac{3\log(\sin(c+dx))}{a^3d} + \frac{3\log(1+\sin(c+dx))}{a^3d} - \frac{1}{2ad(a+a\sin(c+$$

Mathematica [A] time = 0.31, size = 61, normalized size = 0.68

$$-\frac{\frac{4}{\sin(c+dx)+1} + \frac{1}{(\sin(c+dx)+1)^2} + 2\csc(c+dx) + 6\log(\sin(c+dx)) - 6\log(\sin(c+dx)+1)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*(2*Csc[c + d*x] + 6*Log[Sin[c + d*x]] - 6*Log[1 + Sin[c + d*x]] + (1 +
 Sin[c + d*x])^(-2) + 4/(1 + Sin[c + d*x]))/(a^3*d)
```

fricas [A] time = 0.48, size = 152, normalized size = 1.69

$$-\frac{6\cos(dx+c)^2 + 6\left(2\cos(dx+c)^2 + \left(\cos(dx+c)^2 - 2\right)\sin(dx+c) - 2\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 6\left(2\cos(dx\right.}{2\left(2a^3d\cos(dx+c)^2 - 2a^3d + \left(a^3d\cos(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*cos(d*x + c)^2 + 6*(2*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2)*sin(d*x
 + c) - 2)*log(1/2*sin(d*x + c)) - 6*(2*cos(d*x + c)^2 + (cos(d*x + c)^2 -
2)*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - 9*sin(d*x + c) - 8)/(2*a^3*d*c
os(d*x + c)^2 - 2*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.19, size = 77, normalized size = 0.86

$$\frac{\frac{6\log(|\sin(dx+c)+1|)}{a^3} - \frac{6\log(|\sin(dx+c)|)}{a^3} - \frac{6\sin(dx+c)^2+9\sin(dx+c)+2}{a^3(\sin(dx+c)+1)^2\sin(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*log(abs(sin(d*x + c) + 1))/a^3 - 6*log(abs(sin(d*x + c)))/a^3 - (6*s
in(d*x + c)^2 + 9*sin(d*x + c) + 2)/(a^3*(sin(d*x + c) + 1)^2*sin(d*x + c))
)/d
```

maple [A] time = 0.29, size = 86, normalized size = 0.96

$$-\frac{1}{a^{3} d \sin \left(d x+c\right)}-\frac{3 \ln \left(\sin \left(d x+c\right)\right)}{a^{3} d}-\frac{1}{2 d\, a^{3} \left(1+\sin \left(d x+c\right)\right)^{2}}-\frac{2}{d\, a^{3} \left(1+\sin \left(d x+c\right)\right)}+\frac{3 \ln \left(1+\sin \left(d x+c\right)\right)}{a^{3} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/a^3/d/sin(d*x+c)-3*ln(sin(d*x+c))/a^3/d-1/2/d/a^3/(1+sin(d*x+c))^2-2/d/a
^3/(1+sin(d*x+c))+3*ln(1+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.45, size = 91, normalized size = 1.01

$$-\frac{\frac{6 \, \sin\left(d x + c\right)^{2} + 9 \, \sin\left(d x + c\right) + 2}{a^{3} \sin\left(d x + c\right)^{3} + 2 \, a^{3} \sin\left(d x + c\right)^{2} + a^{3} \sin\left(d x + c\right)} - \frac{6 \, \log\left(\sin\left(d x + c\right) + 1\right)}{a^{3}} + \frac{6 \, \log\left(\sin\left(d x + c\right)\right)}{a^{3}}}{2 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((6*sin(d*x + c)^2 + 9*sin(d*x + c) + 2)/(a^3*sin(d*x + c)^3 + 2*a^3*s
in(d*x + c)^2 + a^3*sin(d*x + c)) - 6*log(sin(d*x + c) + 1)/a^3 + 6*log(sin
(d*x + c))/a^3)/d
```

mupad [B] time = 8.65, size = 193, normalized size = 2.14

$$\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1}{d\,\left(2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+12\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+8\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{3\,\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (6*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) + 16*tan(c/2 + (d*x)/2)^3 +
11*tan(c/2 + (d*x)/2)^4 - 1)/(d*(8*a^3*tan(c/2 + (d*x)/2)^2 + 12*a^3*tan(c/
```

```
2 + (d*x)/2)^3 + 8*a^3*tan(c/2 + (d*x)/2)^4 + 2*a^3*tan(c/2 + (d*x)/2)^5 +
2*a^3*tan(c/2 + (d*x)/2))) - (3*log(tan(c/2 + (d*x)/2)))/(a^3*d) + (6*log(t
an(c/2 + (d*x)/2) + 1))/(a^3*d) - tan(c/2 + (d*x)/2)/(2*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos{(c+dx)}\csc^2{(c+dx)}}{\sin^3{(c+dx)}+3\sin^2{(c+dx)}+3\sin{(c+dx)}+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

3.247 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=108

$$\frac{3}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{6\log(\sin(c+dx))}{a^3d}-\frac{6\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] 3*csc(d*x+c)/a^3/d-1/2*csc(d*x+c)^2/a^3/d+6*ln(sin(d*x+c))/a^3/d-6*ln(1+sin
(d*x+c))/a^3/d+1/2/a/d/(a+a*sin(d*x+c))^2+3/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$\frac{3}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{6\log(\sin(c+dx))}{a^3d}-\frac{6\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^3*d) + (6*Log[Sin[c + d*x]])
/(a^3*d) - (6*Log[1 + Sin[c + d*x]])/(a^3*d) + 1/(2*a*d*(a + a*Sin[c + d*x]
)^2) + 3/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^3}{x^3(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{1}{x^3(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{a^3x^3} - \frac{3}{a^4x^2} + \frac{6}{a^5x} - \frac{1}{a^3(a+x)^3} - \frac{3}{a^4(a+x)^2} - \frac{6}{a^5(a+x)}\right) dx, x, a\sin(c+dx\right.}{d}$$
$$= \frac{3\csc(c+dx)}{a^3d} - \frac{\csc^2(c+dx)}{2a^3d} + \frac{6\log(\sin(c+dx))}{a^3d} - \frac{6\log(1+\sin(c+dx))}{a^3d} + $$

Mathematica [A] time = 0.57, size = 71, normalized size = 0.66

$$\frac{\frac{6}{\sin(c+dx)+1} + \frac{1}{(\sin(c+dx)+1)^2} - \csc^2(c+dx) + 6\csc(c+dx) + 12\log(\sin(c+dx)) - 12\log(\sin(c+dx)+1)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*Csc[c + d*x] - Csc[c + d*x]^2 + 12*Log[Sin[c + d*x]] - 12*Log[1 + Sin[c
+ d*x]] + (1 + Sin[c + d*x])^(-2) + 6/(1 + Sin[c + d*x]))/(2*a^3*d)
```

fricas [A] time = 0.51, size = 196, normalized size = 1.81

$$-\frac{18\cos(dx+c)^2 - 12\left(\cos(dx+c)^4 - 3\cos(dx+c)^2 - 2\left(\cos(dx+c)^2 - 1\right)\sin(dx+c) + 2\right)\log\left(\frac{1}{2}\sin(dx+\right.}{2\left(a^3d\cos(dx+c)^4 - 3a^3d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(18*cos(d*x + c)^2 - 12*(cos(d*x + c)^4 - 3*cos(d*x + c)^2 - 2*(cos(d*
x + c)^2 - 1)*sin(d*x + c) + 2)*log(1/2*sin(d*x + c)) + 12*(cos(d*x + c)^4
- 3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 1)*sin(d*x + c) + 2)*log(sin(d*x +
 c) + 1) + 4*(3*cos(d*x + c)^2 - 4)*sin(d*x + c) - 17)/(a^3*d*cos(d*x + c)^
4 - 3*a^3*d*cos(d*x + c)^2 + 2*a^3*d - 2*(a^3*d*cos(d*x + c)^2 - a^3*d)*sin
(d*x + c))
```

giac [A] time = 0.19, size = 86, normalized size = 0.80

$$-\frac{\frac{12\,\log\left(|\sin\left(dx+c\right)+1|\right)}{a^3}-\frac{12\,\log\left(|\sin\left(dx+c\right)|\right)}{a^3}-\frac{12\,\sin\left(dx+c\right)^3+18\,\sin\left(dx+c\right)^2+4\,\sin\left(dx+c\right)-1}{\left(\sin\left(dx+c\right)^2+\sin\left(dx+c\right)\right)^2a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(12*log(abs(sin(d*x + c) + 1))/a^3 - 12*log(abs(sin(d*x + c)))/a^3 - (
12*sin(d*x + c)^3 + 18*sin(d*x + c)^2 + 4*sin(d*x + c) - 1)/((sin(d*x + c)^
2 + sin(d*x + c))^2*a^3))/d
```

maple [A] time = 0.31, size = 102, normalized size = 0.94

$$-\frac{1}{2a^3d\sin\left(dx+c\right)^2}+\frac{3}{a^3d\sin\left(dx+c\right)}+\frac{6\ln\left(\sin\left(dx+c\right)\right)}{a^3d}+\frac{1}{2d\,a^3\left(1+\sin\left(dx+c\right)\right)^2}+\frac{3}{d\,a^3\left(1+\sin\left(dx+c\right)\right)}-\frac{6\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2/a^3/d/sin(d*x+c)^2+3/a^3/d/sin(d*x+c)+6*ln(sin(d*x+c))/a^3/d+1/2/d/a^3
/(1+sin(d*x+c))^2+3/d/a^3/(1+sin(d*x+c))-6*ln(1+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.31, size = 103, normalized size = 0.95

$$\frac{\frac{12\,\sin\left(dx+c\right)^3+18\,\sin\left(dx+c\right)^2+4\,\sin\left(dx+c\right)-1}{a^3\,\sin\left(dx+c\right)^4+2\,a^3\,\sin\left(dx+c\right)^3+a^3\,\sin\left(dx+c\right)^2}-\frac{12\,\log\left(\sin\left(dx+c\right)+1\right)}{a^3}+\frac{12\,\log\left(\sin\left(dx+c\right)\right)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((12*sin(d*x + c)^3 + 18*sin(d*x + c)^2 + 4*sin(d*x + c) - 1)/(a^3*sin(
d*x + c)^4 + 2*a^3*sin(d*x + c)^3 + a^3*sin(d*x + c)^2) - 12*log(sin(d*x +
c) + 1)/a^3 + 12*log(sin(d*x + c))/a^3)/d
```

mupad [B] time = 8.68, size = 227, normalized size = 2.10

$$\frac{6\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}+\frac{-26\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{65\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+21\tan\left(\frac{c}{2}+\right.}{d\left(4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+24\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+16\,a^3\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (6*log(tan(c/2 + (d*x)/2)))/(a^3*d) - tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (4*t
an(c/2 + (d*x)/2) + 21*tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^3 - (65*
tan(c/2 + (d*x)/2)^4)/2 - 26*tan(c/2 + (d*x)/2)^5 - 1/2)/(d*(4*a^3*tan(c/2
+ (d*x)/2)^2 + 16*a^3*tan(c/2 + (d*x)/2)^3 + 24*a^3*tan(c/2 + (d*x)/2)^4 +
16*a^3*tan(c/2 + (d*x)/2)^5 + 4*a^3*tan(c/2 + (d*x)/2)^6)) - (12*log(tan(c/
2 + (d*x)/2) + 1))/(a^3*d) + (3*tan(c/2 + (d*x)/2))/(2*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

3.248 $\int \frac{\cot(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=126

$$-\frac{4}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^2(c+dx)}{2a^3d}-\frac{6\csc(c+dx)}{a^3d}-\frac{10\log(\sin(c+dx))}{a^3d}+\frac{10\log(\sin(c+dx)+}{a^3d}$$

```
[Out] -6*csc(d*x+c)/a^3/d+3/2*csc(d*x+c)^2/a^3/d-1/3*csc(d*x+c)^3/a^3/d-10*ln(sin
(d*x+c))/a^3/d+10*ln(1+sin(d*x+c))/a^3/d-1/2/a/d/(a+a*sin(d*x+c))^2-4/d/(a^
3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$-\frac{4}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^2(c+dx)}{2a^3d}-\frac{6\csc(c+dx)}{a^3d}-\frac{10\log(\sin(c+dx))}{a^3d}+\frac{10\log(\sin(c+dx)+}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-6*Csc[c + d*x])/(a^3*d) + (3*Csc[c + d*x]^2)/(2*a^3*d) - Csc[c + d*x]^3/(
3*a^3*d) - (10*Log[Sin[c + d*x]])/(a^3*d) + (10*Log[1 + Sin[c + d*x]])/(a^3
*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) - 4/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^4}{x^4(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{1}{x^4(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{a^3x^4} - \frac{3}{a^4x^3} + \frac{6}{a^5x^2} - \frac{10}{a^6x} + \frac{1}{a^4(a+x)^3} + \frac{4}{a^5(a+x)^2} + \frac{10}{a^6(a+x)}\right)dx, x, a\sin\right.}{d}$$
$$= -\frac{6\csc(c+dx)}{a^3d} + \frac{3\csc^2(c+dx)}{2a^3d} - \frac{\csc^3(c+dx)}{3a^3d} - \frac{10\log(\sin(c+dx))}{a^3d} + \frac{10\log}{}$$

Mathematica [A] time = 5.43, size = 81, normalized size = 0.64

$$-\frac{\frac{3(8\sin(c+dx)+9)}{(\sin(c+dx)+1)^2} + 2\csc^3(c+dx) - 9\csc^2(c+dx) + 36\csc(c+dx) + 60\log(\sin(c+dx)) - 60\log(\sin(c+dx)+1}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/6*(36*Csc[c + d*x] - 9*Csc[c + d*x]^2 + 2*Csc[c + d*x]^3 + 60*Log[Sin[c
+ d*x]] - 60*Log[1 + Sin[c + d*x]] + (3*(9 + 8*Sin[c + d*x]))/(1 + Sin[c +
d*x])^2)/(a^3*d)
```

fricas [B] time = 0.53, size = 242, normalized size = 1.92

$$-\frac{60\cos(dx+c)^4 - 140\cos(dx+c)^2 + 60\left(2\cos(dx+c)^4 - 4\cos(dx+c)^2 + \left(\cos(dx+c)^4 - 3\cos(dx+c)^2\right.\right.}{6\left(2a^3d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(60*cos(d*x + c)^4 - 140*cos(d*x + c)^2 + 60*(2*cos(d*x + c)^4 - 4*cos
(d*x + c)^2 + (cos(d*x + c)^4 - 3*cos(d*x + c)^2 + 2)*sin(d*x + c) + 2)*log
(1/2*sin(d*x + c)) - 60*(2*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + (cos(d*x + c
)^4 - 3*cos(d*x + c)^2 + 2)*sin(d*x + c) + 2)*log(sin(d*x + c) + 1) - 5*(18
*cos(d*x + c)^2 - 17)*sin(d*x + c) + 82)/(2*a^3*d*cos(d*x + c)^4 - 4*a^3*d*
```

```
cos(d*x + c)^2 + 2*a^3*d + (a^3*d*cos(d*x + c)^4 - 3*a^3*d*cos(d*x + c)^2 +
 2*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.21, size = 97, normalized size = 0.77

$$\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a^3} - \frac{60 \log(|\sin(dx+c)|)}{a^3} - \frac{60 \sin(dx+c)^4+90 \sin(dx+c)^3+20 \sin(dx+c)^2-5 \sin(dx+c)+2}{a^3(\sin(dx+c)+1)^2 \sin(dx+c)^3}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(60*log(abs(sin(d*x + c) + 1))/a^3 - 60*log(abs(sin(d*x + c)))/a^3 - (6
0*sin(d*x + c)^4 + 90*sin(d*x + c)^3 + 20*sin(d*x + c)^2 - 5*sin(d*x + c) +
 2)/(a^3*(sin(d*x + c) + 1)^2*sin(d*x + c)^3))/d
```

maple [A] time = 0.30, size = 118, normalized size = 0.94

$$-\frac{1}{3a^3 d \sin(dx+c)^3} + \frac{3}{2a^3 d \sin(dx+c)^2} - \frac{6}{a^3 d \sin(dx+c)} - \frac{10 \ln(\sin(dx+c))}{a^3 d} - \frac{1}{2d\, a^3 (1+\sin(dx+c))^2} - \frac{}{d\, a^3 (1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/3/a^3/d/sin(d*x+c)^3+3/2/a^3/d/sin(d*x+c)^2-6/a^3/d/sin(d*x+c)-10*ln(sin
(d*x+c))/a^3/d-1/2/d/a^3/(1+sin(d*x+c))^2-4/d/a^3/(1+sin(d*x+c))+10*ln(1+si
n(d*x+c))/a^3/d
```

maxima [A] time = 0.31, size = 113, normalized size = 0.90

$$-\frac{\frac{60 \sin(dx+c)^4+90 \sin(dx+c)^3+20 \sin(dx+c)^2-5 \sin(dx+c)+2}{a^3 \sin(dx+c)^5+2 a^3 \sin(dx+c)^4+a^3 \sin(dx+c)^3} - \frac{60 \log(\sin(dx+c)+1)}{a^3} + \frac{60 \log(\sin(dx+c))}{a^3}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*((60*sin(d*x + c)^4 + 90*sin(d*x + c)^3 + 20*sin(d*x + c)^2 - 5*sin(d*
x + c) + 2)/(a^3*sin(d*x + c)^5 + 2*a^3*sin(d*x + c)^4 + a^3*sin(d*x + c)^3
) - 60*log(sin(d*x + c) + 1)/a^3 + 60*log(sin(d*x + c))/a^3)/d
```

mupad [B] time = 8.68, size = 260, normalized size = 2.06

$$\frac{3 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^3\,d} - \frac{10\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d} + \frac{20\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^3\,d} - \frac{25 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,d} - \frac{-55 \tan(}{d\,\left(8\,a^3 \tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (3*tan(c/2 + (d*x)/2)^2)/(8*a^3*d) - tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (10*
log(tan(c/2 + (d*x)/2)))/(a^3*d) + (20*log(tan(c/2 + (d*x)/2) + 1))/(a^3*d)
 - (25*tan(c/2 + (d*x)/2))/(8*a^3*d) - (15*tan(c/2 + (d*x)/2)^2 - (5*tan(c/
2 + (d*x)/2))/3 + (250*tan(c/2 + (d*x)/2)^3)/3 + (175*tan(c/2 + (d*x)/2)^4)
/3 - 47*tan(c/2 + (d*x)/2)^5 - 55*tan(c/2 + (d*x)/2)^6 + 1/3)/(d*(8*a^3*tan
(c/2 + (d*x)/2)^3 + 32*a^3*tan(c/2 + (d*x)/2)^4 + 48*a^3*tan(c/2 + (d*x)/2)
^5 + 32*a^3*tan(c/2 + (d*x)/2)^6 + 8*a^3*tan(c/2 + (d*x)/2)^7))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

3.249 $\int \frac{\cos(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=116

$$\frac{\sin^2(c+dx)}{2a^4d}-\frac{4\sin(c+dx)}{a^4d}+\frac{10}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{10\log(\sin(c+dx)+1)}{a^4d}-\frac{5}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{}{3ad(a\,\mathrm{s}}$$

```
[Out] 10*ln(1+sin(d*x+c))/a^4/d-4*sin(d*x+c)/a^4/d+1/2*sin(d*x+c)^2/a^4/d+1/3/a/d
/(a+a*sin(d*x+c))^3-5/2/d/(a^2+a^2*sin(d*x+c))^2+10/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^2(c+dx)}{2a^4d}-\frac{4\sin(c+dx)}{a^4d}+\frac{10}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{5}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{10\log(\sin(c+dx)+1)}{a^4d}+\frac{}{3ad(a\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (10*Log[1 + Sin[c + d*x]])/(a^4*d) - (4*Sin[c + d*x])/(a^4*d) + Sin[c + d*x
]^2/(2*a^4*d) + 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 5/(2*d*(a^2 + a^2*Sin[c
+ d*x])^2) + 10/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{a^5(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{a^6 d}$$

$$= \frac{\text{Subst}\left(\int \left(-4a+x-\frac{a^5}{(a+x)^4}+\frac{5a^4}{(a+x)^3}-\frac{10a^3}{(a+x)^2}+\frac{10a^2}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^6 d}$$

$$= \frac{10\log(1+\sin(c+dx))}{a^4 d} - \frac{4\sin(c+dx)}{a^4 d} + \frac{\sin^2(c+dx)}{2a^4 d} + \frac{1}{3ad(a+a\sin(c+dx$$

Mathematica [A] time = 0.88, size = 119, normalized size = 1.03

$$\frac{3\sin^5(c+dx)-15\sin^4(c+dx)+\sin^3(c+dx)(60\log(\sin(c+dx)+1)-63)+9\sin^2(c+dx)(20\log(\sin(c+dx}{6a^4 d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (47 + 60*Log[1 + Sin[c + d*x]] + 9*(9 + 20*Log[1 + Sin[c + d*x]])*Sin[c + d
*x] + 9*(-1 + 20*Log[1 + Sin[c + d*x]])*Sin[c + d*x]^2 + (-63 + 60*Log[1 +
Sin[c + d*x]])*Sin[c + d*x]^3 - 15*Sin[c + d*x]^4 + 3*Sin[c + d*x]^5)/(6*a^
4*d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.52, size = 144, normalized size = 1.24

$$\frac{30\cos(dx+c)^4-87\cos(dx+c)^2+120\left(3\cos(dx+c)^2+\left(\cos(dx+c)^2-4\right)\sin(dx+c)-4\right)\log(\sin(dx+}{12\left(3a^4 d\cos(dx+c)^2-4a^4 d+\left(a^4 d\cos(dx+c)^2-4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(30*cos(d*x + c)^4 - 87*cos(d*x + c)^2 + 120*(3*cos(d*x + c)^2 + (cos(
d*x + c)^2 - 4)*sin(d*x + c) - 4)*log(sin(d*x + c) + 1) - 3*(2*cos(d*x + c)
^4 + 39*cos(d*x + c)^2 + 10)*sin(d*x + c) - 34)/(3*a^4*d*cos(d*x + c)^2 - 4
*a^4*d + (a^4*d*cos(d*x + c)^2 - 4*a^4*d)*sin(d*x + c))
```

giac [A] time = 0.22, size = 84, normalized size = 0.72

$$\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a^4} + \frac{60 \sin(dx+c)^2+105 \sin(dx+c)+47}{a^4(\sin(dx+c)+1)^3} + \frac{3\left(a^4 \sin(dx+c)^2-8 a^4 \sin(dx+c)\right)}{a^8}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(60*log(abs(sin(d*x + c) + 1))/a^4 + (60*sin(d*x + c)^2 + 105*sin(d*x +
 c) + 47)/(a^4*(sin(d*x + c) + 1)^3) + 3*(a^4*sin(d*x + c)^2 - 8*a^4*sin(d*
x + c))/a^8)/d
```

maple [A] time = 0.23, size = 103, normalized size = 0.89

$$\frac{\sin^2(dx+c)}{2a^4d} - \frac{4\sin(dx+c)}{a^4d} + \frac{1}{3d\,a^4(1+\sin(dx+c))^3} - \frac{5}{2d\,a^4(1+\sin(dx+c))^2} + \frac{10\ln(1+\sin(dx+c))}{a^4d} + \frac{}{d\,a^4(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/2*sin(d*x+c)^2/a^4/d-4*sin(d*x+c)/a^4/d+1/3/d/a^4/(1+sin(d*x+c))^3-5/2/d/
a^4/(1+sin(d*x+c))^2+10*ln(1+sin(d*x+c))/a^4/d+10/d/a^4/(1+sin(d*x+c))
```

maxima [A] time = 0.35, size = 105, normalized size = 0.91

$$\frac{\frac{60 \sin(dx+c)^2+105 \sin(dx+c)+47}{a^4 \sin(dx+c)^3+3 a^4 \sin(dx+c)^2+3 a^4 \sin(dx+c)+a^4} + \frac{3\left(\sin(dx+c)^2-8 \sin(dx+c)\right)}{a^4} + \frac{60 \log(\sin(dx+c)+1)}{a^4}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*((60*sin(d*x + c)^2 + 105*sin(d*x + c) + 47)/(a^4*sin(d*x + c)^3 + 3*a^
4*sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4) + 3*(sin(d*x + c)^2 - 8*sin(d*
x + c))/a^4 + 60*log(sin(d*x + c) + 1)/a^4)/d
```

mupad [B] time = 8.53, size = 114, normalized size = 0.98

$$\frac{10\ \ln(\sin(c+d\,x)+1)}{a^4\,d} - \frac{4\ \sin(c+d\,x)}{a^4\,d} + \frac{10\sin(c+d\,x)^2 + \frac{35\ \sin(c+d\,x)}{2} + \frac{47}{6}}{d\,\left(a^4\sin(c+d\,x)^3 + 3\,a^4\sin(c+d\,x)^2 + 3\,a^4\ \sin(c+d\,x) + a^4\right)} + \frac{\sin(}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^5)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] (10*log(sin(c + d*x) + 1))/(a^4*d) - (4*sin(c + d*x))/(a^4*d) + ((35*sin(c
+ d*x))/2 + 10*sin(c + d*x)^2 + 47/6)/(d*(3*a^4*sin(c + d*x) + a^4 + 3*a^4*
sin(c + d*x)^2 + a^4*sin(c + d*x)^3)) + sin(c + d*x)^2/(2*a^4*d)
```

sympy [A] time = 10.05, size = 588, normalized size = 5.07

$$\begin{cases} \frac{60 \log{(\sin{(c+dx)}+1)} \sin^{3}{(c+dx)}}{6a^{4}d \sin^{3}{(c+dx)}+18a^{4}d \sin^{2}{(c+dx)}+18a^{4}d \sin{(c+dx)}+6a^{4}d} + \frac{180 \log{(\sin{(c+dx)}+1)} \sin^{2}{(c+dx)}}{6a^{4}d \sin^{3}{(c+dx)}+18a^{4}d \sin^{2}{(c+dx)}+18a^{4}d \sin{(c+dx)}+6a^{4}d} + \frac{180}{6a^{4}d \sin^{3}{(c+dx)}+} \\ \frac{x \sin^{5}{(c)} \cos{(c)}}{(a \sin{(c)}+a)^{4}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**5/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((60*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(6*a**4*d*sin(c + d*x)*
*3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 180*l
og(sin(c + d*x) + 1)*sin(c + d*x)**2/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*
sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 180*log(sin(c + d*x)
 + 1)*sin(c + d*x)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 +
18*a**4*d*sin(c + d*x) + 6*a**4*d) + 60*log(sin(c + d*x) + 1)/(6*a**4*d*sin
(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*
d) + 3*sin(c + d*x)**5/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**
2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) - 15*sin(c + d*x)**4/(6*a**4*d*sin(c
 + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d)
 + 180*sin(c + d*x)**2/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**
2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 270*sin(c + d*x)/(6*a**4*d*sin(c +
 d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) +
 110/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(
c + d*x) + 6*a**4*d), Ne(d, 0)), (x*sin(c)**5*cos(c)/(a*sin(c) + a)**4, Tru
e))
```

3.250 $\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=95

$$\frac{\sin(c+dx)}{a^4d}-\frac{6}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{4\log(\sin(c+dx)+1)}{a^4d}+\frac{2}{d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

```
[Out] -4*ln(1+sin(d*x+c))/a^4/d+sin(d*x+c)/a^4/d-1/3/a/d/(a+a*sin(d*x+c))^3+2/d/(
a^2+a^2*sin(d*x+c))^2-6/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin(c+dx)}{a^4d}-\frac{6}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{2}{d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{4\log(\sin(c+dx)+1)}{a^4d}-\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^4*d) + Sin[c + d*x]/(a^4*d) - 1/(3*a*d*(a + a
*Sin[c + d*x])^3) + 2/(d*(a^2 + a^2*Sin[c + d*x])^2) - 6/(d*(a^4 + a^4*Sin[
c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a^4(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^4}{(a+x)^4}-\frac{4a^3}{(a+x)^3}+\frac{6a^2}{(a+x)^2}-\frac{4a}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= -\frac{4\log(1+\sin(c+dx))}{a^4d}+\frac{\sin(c+dx)}{a^4d}-\frac{1}{3ad(a+a\sin(c+dx))^3}+\frac{}{d\left(a^2+a^2\right.}$$

Mathematica [A] time = 6.56, size = 127, normalized size = 1.34

$$-\frac{3(2\sin(c+dx)+1)^2}{16a^4d(\sin(c+dx)+1)^3}-\frac{\frac{252\sin^2(c+dx)+444\sin(c+dx)+197}{(\sin(c+dx)+1)^3}-48\sin(c+dx)+192\log(\sin(c+dx)+1)}{48a^4d}-\frac{}{24a^4d\left(\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/24*1/(a^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) - (3*(1 + 2*Sin[c +
 d*x])^2)/(16*a^4*d*(1 + Sin[c + d*x])^3) - (192*Log[1 + Sin[c + d*x]] - 48
*Sin[c + d*x] + (197 + 444*Sin[c + d*x] + 252*Sin[c + d*x]^2)/(1 + Sin[c +
d*x])^3)/(48*a^4*d)
```

fricas [A] time = 0.54, size = 132, normalized size = 1.39

$$-\frac{3\cos(dx+c)^4+3\cos(dx+c)^2+12\left(3\cos(dx+c)^2+\left(\cos(dx+c)^2-4\right)\sin(dx+c)-4\right)\log(\sin(dx+c)}{3\left(3a^4d\cos(dx+c)^2-4a^4d+\left(a^4d\cos(dx+c)^2-4a^4d\right)\sin(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 + 12*(3*cos(d*x + c)^2 + (cos(d*x
 + c)^2 - 4)*sin(d*x + c) - 4)*log(sin(d*x + c) + 1) - 9*(cos(d*x + c)^2 +
2)*sin(d*x + c) - 19)/(3*a^4*d*cos(d*x + c)^2 - 4*a^4*d + (a^4*d*cos(d*x +
c)^2 - 4*a^4*d)*sin(d*x + c))
```

giac [A] time = 0.21, size = 66, normalized size = 0.69

$$-\frac{\frac{12 \log(|\sin(dx+c)+1|)}{a^4} - \frac{3 \sin(dx+c)}{a^4} + \frac{18 \sin(dx+c)^2+30 \sin(dx+c)+13}{a^4(\sin(dx+c)+1)^3}}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*log(abs(sin(d*x + c) + 1))/a^4 - 3*sin(d*x + c)/a^4 + (18*sin(d*x
+ c)^2 + 30*sin(d*x + c) + 13)/(a^4*(sin(d*x + c) + 1)^3))/d
```

maple [A] time = 0.24, size = 86, normalized size = 0.91

$$\frac{\sin(dx+c)}{a^4 d} - \frac{1}{3d\,a^4\,(1+\sin(dx+c))^3} - \frac{4\ln(1+\sin(dx+c))}{a^4 d} - \frac{6}{d\,a^4\,(1+\sin(dx+c))} + \frac{2}{d\,a^4\,(1+\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x)
```

```
[Out] sin(d*x+c)/a^4/d-1/3/d/a^4/(1+sin(d*x+c))^3-4*ln(1+sin(d*x+c))/a^4/d-6/d/a^
4/(1+sin(d*x+c))+2/d/a^4/(1+sin(d*x+c))^2
```

maxima [A] time = 0.38, size = 94, normalized size = 0.99

$$-\frac{\frac{18 \sin(dx+c)^2+30 \sin(dx+c)+13}{a^4 \sin(dx+c)^3+3\,a^4 \sin(dx+c)^2+3\,a^4 \sin(dx+c)+a^4} + \frac{12 \log(\sin(dx+c)+1)}{a^4} - \frac{3 \sin(dx+c)}{a^4}}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((18*sin(d*x + c)^2 + 30*sin(d*x + c) + 13)/(a^4*sin(d*x + c)^3 + 3*a^
4*sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4) + 12*log(sin(d*x + c) + 1)/a^4
 - 3*sin(d*x + c)/a^4)/d
```

mupad [B] time = 0.12, size = 69, normalized size = 0.73

$$\frac{\sin(c+d\,x)}{a^4\,d} - \frac{4\,\ln(\sin(c+d\,x)+1)}{a^4\,d} - \frac{6\sin(c+d\,x)^2+10\,\sin(c+d\,x)+\frac{13}{3}}{a^4\,d\,(\sin(c+d\,x)+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^4)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] sin(c + d*x)/(a^4*d) - (4*log(sin(c + d*x) + 1))/(a^4*d) - (10*sin(c + d*x)
 + 6*sin(c + d*x)^2 + 13/3)/(a^4*d*(sin(c + d*x) + 1)^3)
```

sympy [A] time = 5.57, size = 527, normalized size = 5.55

$$\begin{cases} -\frac{12\log(\sin(c+dx)+1)\sin^3(c+dx)}{3a^4d\sin^3(c+dx)+9a^4d\sin^2(c+dx)+9a^4d\sin(c+dx)+3a^4d} - \frac{36\log(\sin(c+dx)+1)\sin^2(c+dx)}{3a^4d\sin^3(c+dx)+9a^4d\sin^2(c+dx)+9a^4d\sin(c+dx)+3a^4d} - \frac{36\log(s}{3a^4d\sin^3(c+dx)+9a^4} \\ \frac{x\sin^4(c)\cos(c)}{(a\sin(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((-12*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(3*a**4*d*sin(c + d*x)
**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 36*log
(sin(c + d*x) + 1)*sin(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin
(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 36*log(sin(c + d*x) + 1)
*sin(c + d*x)/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4
*d*sin(c + d*x) + 3*a**4*d) - 12*log(sin(c + d*x) + 1)/(3*a**4*d*sin(c + d*
x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) + 3*si
n(c + d*x)**4/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4
*d*sin(c + d*x) + 3*a**4*d) - 36*sin(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3
+ 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 54*sin(c +
 d*x)/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c
 + d*x) + 3*a**4*d) - 22/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)*
*2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d), Ne(d, 0)), (x*sin(c)**4*cos(c)/(a*s
in(c) + a)**4, True))
```

3.251 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=83

$$\frac{3}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{\log(\sin(c+dx)+1)}{a^4d}-\frac{3}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

```
[Out] ln(1+sin(d*x+c))/a^4/d+1/3/a/d/(a+a*sin(d*x+c))^3-3/2/d/(a^2+a^2*sin(d*x+c)
)^2+3/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{3}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{3}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{\log(\sin(c+dx)+1)}{a^4d}+\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a^4*d) + 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 3/(2*d*(
a^2 + a^2*Sin[c + d*x])^2) + 3/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{a^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a^3}{(a+x)^4} + \frac{3a^2}{(a+x)^3} - \frac{3a}{(a+x)^2} + \frac{1}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\log(1+\sin(c+dx))}{a^4 d} + \frac{1}{3ad(a+a\sin(c+dx))^3} - \frac{3}{2d\left(a^2+a^2\sin(c+dx)\right)^2} +$$

Mathematica [A] time = 0.37, size = 61, normalized size = 0.73

$$\frac{18\sin^2(c+dx) + 27\sin(c+dx) + 6(\sin(c+dx)+1)^3\log(\sin(c+dx)+1) + 11}{6a^4 d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (11 + 27*Sin[c + d*x] + 18*Sin[c + d*x]^2 + 6*Log[1 + Sin[c + d*x]]*(1 + Si
n[c + d*x])^3)/(6*a^4*d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.51, size = 112, normalized size = 1.35

$$\frac{18\cos(dx+c)^2 + 6\left(3\cos(dx+c)^2 + \left(\cos(dx+c)^2 - 4\right)\sin(dx+c) - 4\right)\log(\sin(dx+c)+1) - 27\sin(dx+}{6\left(3a^4 d\cos(dx+c)^2 - 4a^4 d + \left(a^4 d\cos(dx+c)^2 - 4a^4 d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(18*cos(d*x + c)^2 + 6*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*x
 + c) - 4)*log(sin(d*x + c) + 1) - 27*sin(d*x + c) - 29)/(3*a^4*d*cos(d*x +
 c)^2 - 4*a^4*d + (a^4*d*cos(d*x + c)^2 - 4*a^4*d)*sin(d*x + c))
```

giac [A] time = 0.21, size = 55, normalized size = 0.66

$$\frac{\frac{6\log(|\sin(dx+c)+1|)}{a^4} + \frac{18\sin(dx+c)^2 + 27\sin(dx+c) + 11}{a^4(\sin(dx+c)+1)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c) + 1))/a^4 + (18*sin(d*x + c)^2 + 27*sin(d*x + c
) + 11)/(a^4*(sin(d*x + c) + 1)^3))/d
```

maple [A] time = 0.22, size = 72, normalized size = 0.87

$$\frac{1}{3d\,a^4\,(1+\sin(dx+c))^3}-\frac{3}{2d\,a^4\,(1+\sin(dx+c))^2}+\frac{\ln(1+\sin(dx+c))}{a^4d}+\frac{3}{d\,a^4\,(1+\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/3/d/a^4/(1+sin(d*x+c))^3-3/2/d/a^4/(1+sin(d*x+c))^2+ln(1+sin(d*x+c))/a^4/
d+3/d/a^4/(1+sin(d*x+c))
```

maxima [A] time = 0.30, size = 83, normalized size = 1.00

$$\frac{\frac{18\,\sin(dx+c)^2+27\,\sin(dx+c)+11}{a^4\sin(dx+c)^3+3\,a^4\sin(dx+c)^2+3\,a^4\sin(dx+c)+a^4}+\frac{6\,\log(\sin(dx+c)+1)}{a^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*((18*sin(d*x + c)^2 + 27*sin(d*x + c) + 11)/(a^4*sin(d*x + c)^3 + 3*a^4
*sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4) + 6*log(sin(d*x + c) + 1)/a^4)/
d
```

mupad [B] time = 0.06, size = 54, normalized size = 0.65

$$\frac{\ln(\sin(c+d\,x)+1)}{a^4\,d}+\frac{3\sin(c+d\,x)^2+\frac{9\,\sin(c+d\,x)}{2}+\frac{11}{6}}{a^4\,d\,(\sin(c+d\,x)+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^3)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] log(sin(c + d*x) + 1)/(a^4*d) + ((9*sin(c + d*x))/2 + 3*sin(c + d*x)^2 + 11
/6)/(a^4*d*(sin(c + d*x) + 1)^3)
```

sympy [A] time = 3.48, size = 466, normalized size = 5.61

$$\begin{cases} \frac{6\log(\sin(c+dx)+1)\sin^3(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d} + \frac{18\log(\sin(c+dx)+1)\sin^2(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d} + \frac{18\,\mathrm{l}}{6a^4d\sin^3(c+dx)+} \\ \frac{x\sin^3(c)\cos(c)}{(a\sin(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((6*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(6*a**4*d*sin(c + d*x)**
3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 18*log
(sin(c + d*x) + 1)*sin(c + d*x)**2/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*si
n(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 18*log(sin(c + d*x) +
1)*sin(c + d*x)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*
a**4*d*sin(c + d*x) + 6*a**4*d) + 6*log(sin(c + d*x) + 1)/(6*a**4*d*sin(c +
 d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) +
 18*sin(c + d*x)**2/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 +
 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 27*sin(c + d*x)/(6*a**4*d*sin(c + d*x
)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 11/
(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d
*x) + 6*a**4*d), Ne(d, 0)), (x*sin(c)**3*cos(c)/(a*sin(c) + a)**4, True))
```

3.252 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=30

$$\frac{\sin^3(c+dx)}{3ad(a\sin(c+dx)+a)^3}$$

```
[Out] 1/3*sin(d*x+c)^3/a/d/(a+a*sin(d*x+c))^3
```

Rubi [A] time = 0.07, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 37}

$$\frac{\sin^3(c+dx)}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] Sin[c + d*x]^3/(3*a*d*(a + a*Sin[c + d*x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\sin^3(c+dx)}{3ad(a+a\sin(c+dx))^3}$$

Mathematica [A] time = 0.18, size = 53, normalized size = 1.77

$$\frac{-6\sin(c+dx)+3\cos(2(c+dx))-5}{6a^4d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-5 + 3*Cos[2*(c + d*x)] - 6*Sin[c + d*x])/(6*a^4*d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^6)
```

fricas [B] time = 0.48, size = 72, normalized size = 2.40

$$-\frac{3\,\cos\left(dx+c\right)^2-3\,\sin\left(dx+c\right)-4}{3\left(3\,a^4d\cos\left(dx+c\right)^2-4\,a^4d+\left(a^4d\cos\left(dx+c\right)^2-4\,a^4d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(d*x + c)^2 - 3*sin(d*x + c) - 4)/(3*a^4*d*cos(d*x + c)^2 - 4*a^
4*d + (a^4*d*cos(d*x + c)^2 - 4*a^4*d)*sin(d*x + c))
```

giac [A] time = 0.22, size = 38, normalized size = 1.27

$$-\frac{3\,\sin\left(dx+c\right)^2+3\,\sin\left(dx+c\right)+1}{3\,a^4d(\sin\left(dx+c\right)+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*sin(d*x + c)^2 + 3*sin(d*x + c) + 1)/(a^4*d*(sin(d*x + c) + 1)^3)
```

maple [A] time = 0.22, size = 43, normalized size = 1.43

$$\frac{-\frac{1}{3(1+\sin(dx+c))^3}+\frac{1}{(1+\sin(dx+c))^2}-\frac{1}{1+\sin(dx+c)}}{d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d/a^4*(-1/3/(1+sin(d*x+c))^3+1/(1+sin(d*x+c))^2-1/(1+sin(d*x+c)))
```

maxima [B] time = 0.40, size = 67, normalized size = 2.23

$$-\frac{3\sin(dx+c)^2+3\sin(dx+c)+1}{3\left(a^4\sin(dx+c)^3+3a^4\sin(dx+c)^2+3a^4\sin(dx+c)+a^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*sin(d*x + c)^2 + 3*sin(d*x + c) + 1)/((a^4*sin(d*x + c)^3 + 3*a^4*s
in(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4)*d)
```

mupad [B] time = 8.48, size = 54, normalized size = 1.80

$$\frac{1}{a^4\,d\,(\sin(c+d\,x)+1)^2}-\frac{1}{a^4\,d\,\left(\sin(c+d\,x)+1\right)}-\frac{1}{3\,a^4\,d\,(\sin(c+d\,x)+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^2)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] 1/(a^4*d*(sin(c + d*x) + 1)^2) - 1/(a^4*d*(sin(c + d*x) + 1)) - 1/(3*a^4*d*
(sin(c + d*x) + 1)^3)
```

sympy [A] time = 3.26, size = 192, normalized size = 6.40

$$\begin{cases} -\frac{3\sin^2(c+dx)}{3a^4d\sin^3(c+dx)+9a^4d\sin^2(c+dx)+9a^4d\sin(c+dx)+3a^4d}-\frac{3\sin(c+dx)}{3a^4d\sin^3(c+dx)+9a^4d\sin^2(c+dx)+9a^4d\sin(c+dx)+3a^4d}-\frac{}{3a^4d\sin^3(c+dx)+9a^4d} \\ \frac{x\sin^2(c)\cos(c)}{(a\sin(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((-3*sin(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c +
d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 3*sin(c + d*x)/(3*a**4*d*sin(
c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d)
- 1/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c +
 d*x) + 3*a**4*d), Ne(d, 0)), (x*sin(c)**2*cos(c)/(a*sin(c) + a)**4, True))
```

3.253 $\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=46

$$\frac{1}{3ad(a\sin(c+dx)+a)^3}-\frac{1}{2d\left(a^2\sin(c+dx)+a^2\right)^2}$$

```
[Out] 1/3/a/d/(a+a*sin(d*x+c))^3-1/2/d/(a^2+a^2*sin(d*x+c))^2
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 43}

$$\frac{1}{3ad(a\sin(c+dx)+a)^3}-\frac{1}{2d\left(a^2\sin(c+dx)+a^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 1/(2*d*(a^2 + a^2*Sin[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x}{a(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{(a+x)^4}+\frac{1}{(a+x)^3}\right)dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{1}{3ad(a+a\sin(c+dx))^3} - \frac{1}{2d\left(a^2+a^2\sin(c+dx)\right)^2}$$

Mathematica [A] time = 0.03, size = 30, normalized size = 0.65

$$-\frac{3\sin(c+dx)+1}{6a^4d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/6*(1 + 3*Sin[c + d*x])/(a^4*d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.48, size = 62, normalized size = 1.35

$$\frac{3\,\sin\left(dx+c\right)+1}{6\left(3\,a^4d\cos\left(dx+c\right)^2-4\,a^4d+\left(a^4d\cos\left(dx+c\right)^2-4\,a^4d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*sin(d*x + c) + 1)/(3*a^4*d*cos(d*x + c)^2 - 4*a^4*d + (a^4*d*cos(d*x
 + c)^2 - 4*a^4*d)*sin(d*x + c))
```

giac [A] time = 0.17, size = 28, normalized size = 0.61

$$-\frac{3\,\sin\left(dx+c\right)+1}{6\,a^4d\left(\sin\left(dx+c\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*sin(d*x + c) + 1)/(a^4*d*(sin(d*x + c) + 1)^3)
```

maple [A] time = 0.20, size = 33, normalized size = 0.72

$$\frac{-\frac{1}{2(1+\sin(dx+c))^2}+\frac{1}{3(1+\sin(dx+c))^3}}{d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^4,x)
[Out] 1/d/a^4*(-1/2/(1+sin(d*x+c))^2+1/3/(1+sin(d*x+c))^3)
```

maxima [A] time = 0.31, size = 57, normalized size = 1.24

$$-\frac{3\,\sin\left(dx+c\right)+1}{6\left(a^4\sin\left(dx+c\right)^3+3\,a^4\sin\left(dx+c\right)^2+3\,a^4\sin\left(dx+c\right)+a^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
[Out] -1/6*(3*sin(d*x + c) + 1)/((a^4*sin(d*x + c)^3 + 3*a^4*sin(d*x + c)^2 + 3*a
^4*sin(d*x + c) + a^4)*d)
```

mupad [B] time = 8.46, size = 37, normalized size = 0.80

$$\frac{1}{3\,a^4\,d\,\left(\sin\left(c+d\,x\right)+1\right)^3}-\frac{1}{2\,a^4\,d\,\left(\sin\left(c+d\,x\right)+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x))/(a + a*sin(c + d*x))^4,x)
[Out] 1/(3*a^4*d*(sin(c + d*x) + 1)^3) - 1/(2*a^4*d*(sin(c + d*x) + 1)^2)
```

sympy [A] time = 3.49, size = 129, normalized size = 2.80

$$\begin{cases} -\frac{3\sin\left(c+dx\right)}{6a^4d\sin^3\left(c+dx\right)+18a^4d\sin^2\left(c+dx\right)+18a^4d\sin\left(c+dx\right)+6a^4d}-\frac{1}{6a^4d\sin^3\left(c+dx\right)+18a^4d\sin^2\left(c+dx\right)+18a^4d\sin\left(c+dx\right)+6a^4d} & \text{for}\ d\neq 0 \\ \frac{x\sin\left(c\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((-3*sin(c + d*x)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*
x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) - 1/(6*a**4*d*sin(c + d*x)**3 +
18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d), Ne(d, 0)),
(x*sin(c)*cos(c)/(a*sin(c) + a)**4, True))
```

3.254 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=97

$$\frac{1}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{\log(\sin(c+dx))}{a^4d}-\frac{\log(\sin(c+dx)+1)}{a^4d}+\frac{1}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{1}{3ad(a\sin(c+dx)+a}$$

[Out] ln(sin(d*x+c))/a^4/d-ln(1+sin(d*x+c))/a^4/d+1/3/a/d/(a+a*sin(d*x+c))^3+1/2/d/(a^2+a^2*sin(d*x+c))^2+1/d/(a^4+a^4*sin(d*x+c))

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 44}

$$\frac{1}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{1}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{\log(\sin(c+dx))}{a^4d}-\frac{\log(\sin(c+dx)+1)}{a^4d}+\frac{1}{3ad(a\sin(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^4*d) - Log[1 + Sin[c + d*x]]/(a^4*d) + 1/(3*a*d*(a + a*Sin[c + d*x])^3) + 1/(2*d*(a^2 + a^2*Sin[c + d*x])^2) + 1/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^4x} - \frac{1}{a(a+x)^4} - \frac{1}{a^2(a+x)^3} - \frac{1}{a^3(a+x)^2} - \frac{1}{a^4(a+x)}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^4d} - \frac{\log(1+\sin(c+dx))}{a^4d} + \frac{1}{3ad(a+a\sin(c+dx))^3} + \frac{1}{2d\left(a^2+a^2\sin\right.}$$

Mathematica [A] time = 0.36, size = 62, normalized size = 0.64

$$\frac{\frac{6\sin^2(c+dx)+15\sin(c+dx)+11}{(\sin(c+dx)+1)^3} + 6\log(\sin(c+dx)) - 6\log(\sin(c+dx)+1)}{6a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (6*Log[Sin[c + d*x]] - 6*Log[1 + Sin[c + d*x]] + (11 + 15*Sin[c + d*x] + 6*
Sin[c + d*x]^2)/(1 + Sin[c + d*x])^3)/(6*a^4*d)
```

fricas [A] time = 0.51, size = 152, normalized size = 1.57

$$\frac{6\cos(dx+c)^2 + 6\left(3\cos(dx+c)^2 + \left(\cos(dx+c)^2-4\right)\sin(dx+c) - 4\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 6\left(3\cos(dx+\right.}{6\left(3a^4d\cos(dx+c)^2 - 4a^4d + \left(a^4d\cos(dx+c)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*cos(d*x + c)^2 + 6*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*x
+ c) - 4)*log(1/2*sin(d*x + c)) - 6*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4
)*sin(d*x + c) - 4)*log(sin(d*x + c) + 1) - 15*sin(d*x + c) - 17)/(3*a^4*d*
cos(d*x + c)^2 - 4*a^4*d + (a^4*d*cos(d*x + c)^2 - 4*a^4*d)*sin(d*x + c))
```

giac [A] time = 0.19, size = 69, normalized size = 0.71

$$-\frac{\frac{6\log(|\sin(dx+c)+1|)}{a^4} - \frac{6\log(|\sin(dx+c)|)}{a^4} - \frac{6\sin(dx+c)^2+15\sin(dx+c)+11}{a^4(\sin(dx+c)+1)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(6*log(abs(sin(d*x + c) + 1))/a^4 - 6*log(abs(sin(d*x + c)))/a^4 - (6*
sin(d*x + c)^2 + 15*sin(d*x + c) + 11)/(a^4*(sin(d*x + c) + 1)^3))/d
```

maple [A] time = 0.25, size = 86, normalized size = 0.89

$$\frac{\ln(\sin(dx+c))}{a^4 d}+\frac{1}{3d\,a^4\,(1+\sin(dx+c))^3}+\frac{1}{2d\,a^4\,(1+\sin(dx+c))^2}+\frac{1}{d\,a^4\,(1+\sin(dx+c))}-\frac{\ln(1+\sin(dx+c))}{a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] ln(sin(d*x+c))/a^4/d+1/3/d/a^4/(1+sin(d*x+c))^3+1/2/d/a^4/(1+sin(d*x+c))^2+
1/d/a^4/(1+sin(d*x+c))-ln(1+sin(d*x+c))/a^4/d
```

maxima [A] time = 0.36, size = 95, normalized size = 0.98

$$\frac{\frac{6\,\sin(dx+c)^2+15\,\sin(dx+c)+11}{a^4\,\sin(dx+c)^3+3\,a^4\,\sin(dx+c)^2+3\,a^4\,\sin(dx+c)+a^4}-\frac{6\,\log(\sin(dx+c)+1)}{a^4}+\frac{6\,\log(\sin(dx+c))}{a^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*((6*sin(d*x + c)^2 + 15*sin(d*x + c) + 11)/(a^4*sin(d*x + c)^3 + 3*a^4*
sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4) - 6*log(sin(d*x + c) + 1)/a^4 +
6*log(sin(d*x + c))/a^4)/d
```

mupad [B] time = 9.53, size = 206, normalized size = 2.12

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^4\,d}-\frac{6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+18\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{80\,\tan\left(\frac{c}{2}\right.}{3}}{d\,\left(a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+15\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+20\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)*(a + a*sin(c + d*x))^4),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^4*d) - (2*log(tan(c/2 + (d*x)/2) + 1))/(a^4*d) -
 (6*tan(c/2 + (d*x)/2) + 18*tan(c/2 + (d*x)/2)^2 + (80*tan(c/2 + (d*x)/2)^3
)/3 + 18*tan(c/2 + (d*x)/2)^4 + 6*tan(c/2 + (d*x)/2)^5)/(d*(15*a^4*tan(c/2
+ (d*x)/2)^2 + 20*a^4*tan(c/2 + (d*x)/2)^3 + 15*a^4*tan(c/2 + (d*x)/2)^4 +
```

```
6*a^4*tan(c/2 + (d*x)/2)^5 + a^4*tan(c/2 + (d*x)/2)^6 + a^4 + 6*a^4*tan(c/2
 + (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6
*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.255 $\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=111

$$-\frac{3}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\csc(c+dx)}{a^4d}-\frac{4\log(\sin(c+dx))}{a^4d}+\frac{4\log(\sin(c+dx)+1)}{a^4d}-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{1}{3ad\ldots}$$

```
[Out] -csc(d*x+c)/a^4/d-4*ln(sin(d*x+c))/a^4/d+4*ln(1+sin(d*x+c))/a^4/d-1/3/a/d/(
a+a*sin(d*x+c))^3-1/d/(a^2+a^2*sin(d*x+c))^2-3/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 44}

$$-\frac{3}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{\csc(c+dx)}{a^4d}-\frac{4\log(\sin(c+dx))}{a^4d}+\frac{4\log(\sin(c+dx)+1)}{a^4d}-\frac{1}{3ad\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(Csc[c + d*x]/(a^4*d)) - (4*Log[Sin[c + d*x]])/(a^4*d) + (4*Log[1 + Sin[c
+ d*x]])/(a^4*d) - 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 1/(d*(a^2 + a^2*Sin[c
 + d*x])^2) - 3/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{a^2}{x^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{a^4x^2} - \frac{4}{a^5x} + \frac{1}{a^2(a+x)^4} + \frac{2}{a^3(a+x)^3} + \frac{3}{a^4(a+x)^2} + \frac{4}{a^5(a+x)}\right) dx, x, a\sin(c+\right.}{d}$$
$$= -\frac{\csc(c+dx)}{a^4d} - \frac{4\log(\sin(c+dx))}{a^4d} + \frac{4\log(1+\sin(c+dx))}{a^4d} - \frac{1}{3ad(a+a\sin(c+}$$

Mathematica [A] time = 1.01, size = 73, normalized size = 0.66

$$-\frac{\frac{9}{\sin(c+dx)+1} + \frac{3}{(\sin(c+dx)+1)^2} + \frac{1}{(\sin(c+dx)+1)^3} + 3\csc(c+dx) + 12\log(\sin(c+dx)) - 12\log(\sin(c+dx)+1)}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/3*(3*Csc[c + d*x] + 12*Log[Sin[c + d*x]] - 12*Log[1 + Sin[c + d*x]] + (1
 + Sin[c + d*x])^(-3) + 3/(1 + Sin[c + d*x])^2 + 9/(1 + Sin[c + d*x]))/(a^4
*d)
```

fricas [A] time = 0.54, size = 201, normalized size = 1.81

$$\frac{30\cos(dx+c)^2 - 12\left(\cos(dx+c)^4 - 5\cos(dx+c)^2 - \left(3\cos(dx+c)^2 - 4\right)\sin(dx+c) + 4\right)\log\left(\frac{1}{2}\sin(dx+\right.}{3\left(a^4d\cos(dx+c)^4 - 5a^4d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(30*cos(d*x + c)^2 - 12*(cos(d*x + c)^4 - 5*cos(d*x + c)^2 - (3*cos(d*x
 + c)^2 - 4)*sin(d*x + c) + 4)*log(1/2*sin(d*x + c)) + 12*(cos(d*x + c)^4 -
 5*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - 4)*sin(d*x + c) + 4)*log(sin(d*x +
c) + 1) + 2*(6*cos(d*x + c)^2 - 17)*sin(d*x + c) - 33)/(a^4*d*cos(d*x + c)^
4 - 5*a^4*d*cos(d*x + c)^2 + 4*a^4*d - (3*a^4*d*cos(d*x + c)^2 - 4*a^4*d)*s
in(d*x + c))
```

giac [A] time = 0.20, size = 87, normalized size = 0.78

$$\frac{\frac{12\log(|\sin(dx+c)+1|)}{a^4}-\frac{12\log(|\sin(dx+c)|)}{a^4}-\frac{12\sin(dx+c)^3+30\sin(dx+c)^2+22\sin(dx+c)+3}{a^4(\sin(dx+c)+1)^3\sin(dx+c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(12*log(abs(sin(d*x + c) + 1))/a^4 - 12*log(abs(sin(d*x + c)))/a^4 - (1
2*sin(d*x + c)^3 + 30*sin(d*x + c)^2 + 22*sin(d*x + c) + 3)/(a^4*(sin(d*x +
 c) + 1)^3*sin(d*x + c)))/d
```

maple [A] time = 0.26, size = 104, normalized size = 0.94

$$-\frac{1}{a^4 d\sin(dx+c)}-\frac{4\ln(\sin(dx+c))}{a^4 d}-\frac{1}{3d\,a^4(1+\sin(dx+c))^3}-\frac{1}{d\,a^4(1+\sin(dx+c))^2}-\frac{3}{d\,a^4(1+\sin(dx+c))}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/a^4/d/sin(d*x+c)-4*ln(sin(d*x+c))/a^4/d-1/3/d/a^4/(1+sin(d*x+c))^3-1/d/a
^4/(1+sin(d*x+c))^2-3/d/a^4/(1+sin(d*x+c))+4*ln(1+sin(d*x+c))/a^4/d
```

maxima [A] time = 0.39, size = 114, normalized size = 1.03

$$-\frac{\frac{12\sin(dx+c)^3+30\sin(dx+c)^2+22\sin(dx+c)+3}{a^4\sin(dx+c)^4+3a^4\sin(dx+c)^3+3a^4\sin(dx+c)^2+a^4\sin(dx+c)}-\frac{12\log(\sin(dx+c)+1)}{a^4}+\frac{12\log(\sin(dx+c))}{a^4}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((12*sin(d*x + c)^3 + 30*sin(d*x + c)^2 + 22*sin(d*x + c) + 3)/(a^4*si
n(d*x + c)^4 + 3*a^4*sin(d*x + c)^3 + 3*a^4*sin(d*x + c)^2 + a^4*sin(d*x +
c)) - 12*log(sin(d*x + c) + 1)/a^4 + 12*log(sin(d*x + c))/a^4)/d
```

mupad [B] time = 8.72, size = 251, normalized size = 2.26

$$\frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+74\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{307\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}+60\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{d\left(2a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+12a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+30a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+40a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+30a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^2*(a + a*sin(c + d*x))^4),x)
```

```
[Out] (9*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2) + 60*tan(c/2 + (d*x)/2)^3 +
(307*tan(c/2 + (d*x)/2)^4)/3 + 74*tan(c/2 + (d*x)/2)^5 + 23*tan(c/2 + (d*x)
/2)^6 - 1)/(d*(12*a^4*tan(c/2 + (d*x)/2)^2 + 30*a^4*tan(c/2 + (d*x)/2)^3 +
40*a^4*tan(c/2 + (d*x)/2)^4 + 30*a^4*tan(c/2 + (d*x)/2)^5 + 12*a^4*tan(c/2
+ (d*x)/2)^6 + 2*a^4*tan(c/2 + (d*x)/2)^7 + 2*a^4*tan(c/2 + (d*x)/2))) - (4
*log(tan(c/2 + (d*x)/2)))/(a^4*d) + (8*log(tan(c/2 + (d*x)/2) + 1))/(a^4*d)
 - tan(c/2 + (d*x)/2)/(2*a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc^2(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**4 + 4*sin(c + d*x)**3
+ 6*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.256 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=131

$$\frac{6}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\csc^2(c+dx)}{2a^4d}+\frac{4\csc(c+dx)}{a^4d}+\frac{10\log(\sin(c+dx))}{a^4d}-\frac{10\log(\sin(c+dx)+1)}{a^4d}+\frac{3}{2d\left(a^2\sin(c+\right.}$$

```
[Out] 4*csc(d*x+c)/a^4/d-1/2*csc(d*x+c)^2/a^4/d+10*ln(sin(d*x+c))/a^4/d-10*ln(1+s
in(d*x+c))/a^4/d+1/3/a/d/(a+a*sin(d*x+c))^3+3/2/d/(a^2+a^2*sin(d*x+c))^2+6/
d/(a^4+a^4*sin(d*x+c))
```

Rubi [A] time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$\frac{6}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{3}{2d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{\csc^2(c+dx)}{2a^4d}+\frac{4\csc(c+dx)}{a^4d}+\frac{10\log(\sin(c+dx))}{a^4d}-\frac{10\log(\sin(}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (4*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^4*d) + (10*Log[Sin[c + d*x]]
)/(a^4*d) - (10*Log[1 + Sin[c + d*x]])/(a^4*d) + 1/(3*a*d*(a + a*Sin[c + d*
x])^3) + 3/(2*d*(a^2 + a^2*Sin[c + d*x])^2) + 6/(d*(a^4 + a^4*Sin[c + d*x])
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{a^3}{x^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{1}{x^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{a^4x^3} - \frac{4}{a^5x^2} + \frac{10}{a^6x} - \frac{1}{a^3(a+x)^4} - \frac{3}{a^4(a+x)^3} - \frac{6}{a^5(a+x)^2} - \frac{10}{a^6(a+x)}\right)dx, x, a \text{s}$$
$$= \frac{4\csc(c+dx)}{a^4d} - \frac{\csc^2(c+dx)}{2a^4d} + \frac{10\log(\sin(c+dx))}{a^4d} - \frac{10\log(1+\sin(c+dx))}{a^4d}$$

Mathematica [A] time = 3.42, size = 85, normalized size = 0.65

$$\frac{\frac{36}{\sin(c+dx)+1} + \frac{9}{(\sin(c+dx)+1)^2} + \frac{2}{(\sin(c+dx)+1)^3} - 3\csc^2(c+dx) + 24\csc(c+dx) + 60\log(\sin(c+dx)) - 60\log(\sin(c}{6a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (24*Csc[c + d*x] - 3*Csc[c + d*x]^2 + 60*Log[Sin[c + d*x]] - 60*Log[1 + Sin
[c + d*x]] + 2/(1 + Sin[c + d*x])^3 + 9/(1 + Sin[c + d*x])^2 + 36/(1 + Sin[
c + d*x]))/(6*a^4*d)
```

fricas [A] time = 0.54, size = 242, normalized size = 1.85

$$\frac{60\cos(dx+c)^4 - 230\cos(dx+c)^2 + 60\left(3\cos(dx+c)^4 - 7\cos(dx+c)^2 + \left(\cos(dx+c)^4 - 5\cos(dx+c)^2\right.\right.}{6\left(3a^4d\,\text{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*cos(d*x + c)^4 - 230*cos(d*x + c)^2 + 60*(3*cos(d*x + c)^4 - 7*cos(
d*x + c)^2 + (cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 4)*sin(d*x + c) + 4)*log(
1/2*sin(d*x + c)) - 60*(3*cos(d*x + c)^4 - 7*cos(d*x + c)^2 + (cos(d*x + c)
```

```
^4 - 5*cos(d*x + c)^2 + 4)*sin(d*x + c) + 4)*log(sin(d*x + c) + 1) - 15*(10
*cos(d*x + c)^2 - 11)*sin(d*x + c) + 167)/(3*a^4*d*cos(d*x + c)^4 - 7*a^4*d
*cos(d*x + c)^2 + 4*a^4*d + (a^4*d*cos(d*x + c)^4 - 5*a^4*d*cos(d*x + c)^2
+ 4*a^4*d)*sin(d*x + c))
```

giac [A] time = 0.23, size = 97, normalized size = 0.74

$$-\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a^4} - \frac{60 \log(|\sin(dx+c)|)}{a^4} - \frac{60 \sin(dx+c)^4+150 \sin(dx+c)^3+110 \sin(dx+c)^2+15 \sin(dx+c)-3}{a^4(\sin(dx+c)+1)^3 \sin(dx+c)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(60*log(abs(sin(d*x + c) + 1))/a^4 - 60*log(abs(sin(d*x + c)))/a^4 - (
60*sin(d*x + c)^4 + 150*sin(d*x + c)^3 + 110*sin(d*x + c)^2 + 15*sin(d*x +
c) - 3)/(a^4*(sin(d*x + c) + 1)^3*sin(d*x + c)^2))/d
```

maple [A] time = 0.28, size = 120, normalized size = 0.92

$$-\frac{1}{2a^4 d \sin(dx+c)^2}+\frac{4}{a^4 d \sin(dx+c)}+\frac{10 \ln(\sin(dx+c))}{a^4 d}+\frac{1}{3d\, a^4 (1+\sin(dx+c))^3}+\frac{3}{2d\, a^4 (1+\sin(dx+c))^2}+\frac{}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/2/a^4/d/sin(d*x+c)^2+4/a^4/d/sin(d*x+c)+10*ln(sin(d*x+c))/a^4/d+1/3/d/a^
4/(1+sin(d*x+c))^3+3/2/d/a^4/(1+sin(d*x+c))^2+6/d/a^4/(1+sin(d*x+c))-10*ln(
1+sin(d*x+c))/a^4/d
```

maxima [A] time = 0.35, size = 126, normalized size = 0.96

$$\frac{\frac{60 \sin(dx+c)^4+150 \sin(dx+c)^3+110 \sin(dx+c)^2+15 \sin(dx+c)-3}{a^4 \sin(dx+c)^5+3 a^4 \sin(dx+c)^4+3 a^4 \sin(dx+c)^3+a^4 \sin(dx+c)^2} - \frac{60 \log(\sin(dx+c)+1)}{a^4} + \frac{60 \log(\sin(dx+c))}{a^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*((60*sin(d*x + c)^4 + 150*sin(d*x + c)^3 + 110*sin(d*x + c)^2 + 15*sin(
d*x + c) - 3)/(a^4*sin(d*x + c)^5 + 3*a^4*sin(d*x + c)^4 + 3*a^4*sin(d*x +
c)^3 + a^4*sin(d*x + c)^2) - 60*log(sin(d*x + c) + 1)/a^4 + 60*log(sin(d*x
+ c))/a^4)/d
```

mupad [B] time = 8.69, size = 286, normalized size = 2.18

$$\frac{10\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^4\,d}-\frac{72\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{465\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+\frac{881\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}+\frac{255\,\mathrm{t}}{}}{d\,\left(4\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+24\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+60\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+80\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^3*(a + a*sin(c + d*x))^4),x)
```

```
[Out] (10*log(tan(c/2 + (d*x)/2)))/(a^4*d) - tan(c/2 + (d*x)/2)^2/(8*a^4*d) - ((2
55*tan(c/2 + (d*x)/2)^4)/2 - (81*tan(c/2 + (d*x)/2)^2)/2 - 30*tan(c/2 + (d*
x)/2)^3 - 5*tan(c/2 + (d*x)/2) + (881*tan(c/2 + (d*x)/2)^5)/3 + (465*tan(c/
2 + (d*x)/2)^6)/2 + 72*tan(c/2 + (d*x)/2)^7 + 1/2)/(d*(4*a^4*tan(c/2 + (d*x
)/2)^2 + 24*a^4*tan(c/2 + (d*x)/2)^3 + 60*a^4*tan(c/2 + (d*x)/2)^4 + 80*a^4
*tan(c/2 + (d*x)/2)^5 + 60*a^4*tan(c/2 + (d*x)/2)^6 + 24*a^4*tan(c/2 + (d*x
)/2)^7 + 4*a^4*tan(c/2 + (d*x)/2)^8)) - (20*log(tan(c/2 + (d*x)/2) + 1))/(a
^4*d) + (2*tan(c/2 + (d*x)/2))/(a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc^3(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**4 + 4*sin(c + d*x)**3
+ 6*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.257 $\int \cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{a\sin(c+dx)+a}}{d}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

```
[Out] -2*arctanh((a+a*sin(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+a*sin(d*x+c))^(1/
2)/d
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2707, 50, 63, 207}

$$\frac{2\sqrt{a\sin(c+dx)+a}}{d}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + a*Si
n[c + d*x]])/d
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{2\sqrt{a+a\sin(c+dx)}}{d} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{2\sqrt{a+a\sin(c+dx)}}{d} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+a\sin(c+dx)}\right)}{d}\\ &= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+a\sin(c+dx)}}{d}\end{aligned}$$

Mathematica [B] time = 0.15, size = 118, normalized size = 2.31

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)+\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)+1\right)-\text{lo}\right.}{d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((2*Cos[(c + d*x)/2] + Log[1 - Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1
 + Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 2*Sin[(c + d*x)/2])*Sqrt[a*(1 + S
in[c + d*x]])])/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.50, size = 86, normalized size = 1.69

$$\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^2+4\sqrt{a\sin(dx+c)+a}\sqrt{a}(\sin(dx+c)+2)-8a\sin(dx+c)-9a}{\cos(dx+c)^2-1}\right)+4\sqrt{a\sin(dx+c)+a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(sqrt(a)*log((a*cos(d*x + c)^2 + 4*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(si
n(d*x + c) + 2) - 8*a*sin(d*x + c) - 9*a)/(cos(d*x + c)^2 - 1)) + 4*sqrt(a*
sin(d*x + c) + a))/d
```

giac [B] time = 2.26, size = 2182, normalized size = 42.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*sqrt(a)*((sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*
c)^3*tan(1/4*c)^6 - 6*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c
)^3*tan(1/4*c)^5 + 3*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)
^2*tan(1/4*c)^6 - 15*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)
^3*tan(1/4*c)^4 + 18*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)
^2*tan(1/4*c)^5 - 3*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*
tan(1/4*c)^6 + 20*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*
tan(1/4*c)^3 - 45*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*
tan(1/4*c)^4 + 18*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*ta
n(1/4*c)^5 - sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^6 + 15*
sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^2 - 60*
sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^3 + 45*
sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^4 - 6*sqr
t(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^5 - 6*sqrt(2)*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c) + 45*sqrt(2)*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^2 - 60*sqrt(2)*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^3 + 15*sqrt(2)*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^4 - sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))*tan(1/2*c)^3 + 18*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/
2*c)^2*tan(1/4*c) - 45*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*
c)*tan(1/4*c)^2 + 20*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)
^3 - 3*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2 + 18*sqrt(2
)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c) - 15*sqrt(2)*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 + 3*sqrt(2)*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))*tan(1/2*c) - 6*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*tan(1/4*c) + sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))*log(abs(2*tan
(1/4*d*x + c)*tan(1/2*c)^3 - 6*tan(1/4*d*x + c)*tan(1/2*c) + 6*tan(1/2*c)^2
 - 2*(tan(1/2*c)^2 + 1)^(3/2) - 2)/abs(2*tan(1/4*d*x + c)*tan(1/2*c)^3 - 6*
tan(1/4*d*x + c)*tan(1/2*c) + 6*tan(1/2*c)^2 + 2*(tan(1/2*c)^2 + 1)^(3/2) -
```

```
 2))/((tan(1/4*c)^6 + 3*tan(1/4*c)^4 + 3*tan(1/4*c)^2 + 1)*(tan(1/2*c)^2 +
1)^(3/2)) - (sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1
/4*c)^6 + 6*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/
4*c)^5 - 3*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4
*c)^6 - 15*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4
*c)^4 + 18*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4
*c)^5 - 3*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)
^6 - 20*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)
^3 + 45*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)
^4 - 18*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^5
 + sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^6 + 15*sqrt(2)*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^2 - 60*sqrt(2)*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^3 + 45*sqrt(2)*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^4 - 6*sqrt(2)*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^5 + 6*sqrt(2)*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c) - 45*sqrt(2)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^2 + 60*sqrt(2)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^3 - 15*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))*tan(1/4*c)^4 - sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan
(1/2*c)^3 + 18*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan
(1/4*c) - 45*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4
*c)^2 + 20*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^3 + 3*sqr
t(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2 - 18*sqrt(2)*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c) + 15*sqrt(2)*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 + 3*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))*tan(1/2*c) - 6*sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/
4*c) - sqrt(2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))*log(abs(6*tan(1/4*d*x +
 c)*tan(1/2*c)^2 - 2*tan(1/2*c)^3 - 2*(tan(1/2*c)^2 + 1)^(3/2) - 2*tan(1/4*
d*x + c) + 6*tan(1/2*c))/abs(6*tan(1/4*d*x + c)*tan(1/2*c)^2 - 2*tan(1/2*c)
^3 + 2*(tan(1/2*c)^2 + 1)^(3/2) - 2*tan(1/4*d*x + c) + 6*tan(1/2*c)))/((tan
(1/4*c)^6 + 3*tan(1/4*c)^4 + 3*tan(1/4*c)^2 + 1)*(tan(1/2*c)^2 + 1)^(3/2))
+ 8*(sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^6 - 6*
sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^5 + sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^6 - 15*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^4 + 6*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*tan(1/4*c)^5 + 20*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*t
an(1/4*c)^3 - 15*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^4 + 15*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^2 - 20*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^3 - 6*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*tan(1/4*d*x + c)*tan(1/4*c) + 15*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
tan(1/4*c)^2 - sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c) + 6*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) - sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c)))/((sqrt(2)*tan(1/4*c)^6 + 3*sqrt(2)*tan(1/4*c)^4 + 3*sqrt(2)*tan(1/4
*c)^2 + sqrt(2))*(tan(1/4*d*x + c)^2 + 1)))/d
```

maple [A] time = 0.15, size = 42, normalized size = 0.82

$$\frac{2\sqrt{a+a\sin(dx+c)}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}}{\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(2*(a+a*sin(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+a*sin(d*x+c))^(1/2)/a^(1
/2)))
```

maxima [A] time = 0.41, size = 61, normalized size = 1.20

$$\frac{\sqrt{a}\log\left(\frac{\sqrt{a\sin(dx+c)+a}-\sqrt{a}}{\sqrt{a\sin(dx+c)+a}+\sqrt{a}}\right)+2\sqrt{a\sin(dx+c)+a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima
")
```

```
[Out] (sqrt(a)*log((sqrt(a*sin(d*x + c) + a) - sqrt(a))/(sqrt(a*sin(d*x + c) + a)
 + sqrt(a))) + 2*sqrt(a*sin(d*x + c) + a))/d
```

mupad [B] time = 8.73, size = 43, normalized size = 0.84

$$\frac{2\sqrt{a+a\ \sin(c+d\,x)}}{d}-\frac{2\sqrt{a}\ \operatorname{atanh}\left(\frac{\sqrt{a+a\ \sin(c+d\,x)}}{\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x),x)
```

```
[Out] (2*(a + a*sin(c + d*x))^(1/2))/d - (2*a^(1/2)*atanh((a + a*sin(c + d*x))^(1
/2)/a^(1/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a(\sin(c+dx)+1)}\cos(c+dx)\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)*csc(c + d*x), x)
```

3.258 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=114

$$\frac{a^4\sin^{n+1}(c+dx)}{d(n+1)}+\frac{4a^4\sin^{n+2}(c+dx)}{d(n+2)}+\frac{6a^4\sin^{n+3}(c+dx)}{d(n+3)}+\frac{4a^4\sin^{n+4}(c+dx)}{d(n+4)}+\frac{a^4\sin^{n+5}(c+dx)}{d(n+5)}$$

```
[Out] a^4*sin(d*x+c)^(1+n)/d/(1+n)+4*a^4*sin(d*x+c)^(2+n)/d/(2+n)+6*a^4*sin(d*x+c
)^(3+n)/d/(3+n)+4*a^4*sin(d*x+c)^(4+n)/d/(4+n)+a^4*sin(d*x+c)^(5+n)/d/(5+n)
```

Rubi [A] time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 43}

$$\frac{a^4\sin^{n+1}(c+dx)}{d(n+1)}+\frac{4a^4\sin^{n+2}(c+dx)}{d(n+2)}+\frac{6a^4\sin^{n+3}(c+dx)}{d(n+3)}+\frac{4a^4\sin^{n+4}(c+dx)}{d(n+4)}+\frac{a^4\sin^{n+5}(c+dx)}{d(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (4*a^4*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (6*a^4*Sin[c + d*x]^(3 + n))/(d*(3 + n)) + (4*a^4*Sin[c + d*x]^(4
+ n))/(d*(4 + n)) + (a^4*Sin[c + d*x]^(5 + n))/(d*(5 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (a+x)^4\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(a^4\left(\frac{x}{a}\right)^n + 4a^4\left(\frac{x}{a}\right)^{1+n} + 6a^4\left(\frac{x}{a}\right)^{2+n} + 4a^4\left(\frac{x}{a}\right)^{3+n} + \right.\right.}{ad}$$
$$= \frac{a^4\sin^{1+n}(c+dx)}{d(1+n)} + \frac{4a^4\sin^{2+n}(c+dx)}{d(2+n)} + \frac{6a^4\sin^{3+n}(c+dx)}{d(3+n)}$$

Mathematica [A] time = 0.27, size = 80, normalized size = 0.70

$$\frac{a^4\sin^{n+1}(c+dx)\left(\frac{\sin^4(c+dx)}{n+5} + \frac{4\sin^3(c+dx)}{n+4} + \frac{6\sin^2(c+dx)}{n+3} + \frac{4\sin(c+dx)}{n+2} + \frac{1}{n+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (4*Sin[c + d*x])/(2 + n) + (6*Sin
[c + d*x]^2)/(3 + n) + (4*Sin[c + d*x]^3)/(4 + n) + Sin[c + d*x]^4/(5 + n))
)/d
```

fricas [B] time = 0.56, size = 302, normalized size = 2.65

$$\frac{\left(8\,a^4n^4 + 96\,a^4n^3 + 400\,a^4n^2 + 672\,a^4n + 4\left(a^4n^4 + 11\,a^4n^3 + 41\,a^4n^2 + 61\,a^4n + 30\,a^4\right)\cos(dx+c)^4 + 360\,a^4 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] (8*a^4*n^4 + 96*a^4*n^3 + 400*a^4*n^2 + 672*a^4*n + 4*(a^4*n^4 + 11*a^4*n^3
 + 41*a^4*n^2 + 61*a^4*n + 30*a^4)*cos(d*x + c)^4 + 360*a^4 - 4*(3*a^4*n^4
+ 35*a^4*n^3 + 141*a^4*n^2 + 229*a^4*n + 120*a^4)*cos(d*x + c)^2 + (8*a^4*n
^4 + 96*a^4*n^3 + 400*a^4*n^2 + 672*a^4*n + (a^4*n^4 + 10*a^4*n^3 + 35*a^4*
n^2 + 50*a^4*n + 24*a^4)*cos(d*x + c)^4 + 384*a^4 - 4*(2*a^4*n^4 + 23*a^4*n
^3 + 91*a^4*n^2 + 142*a^4*n + 72*a^4)*cos(d*x + c)^2)*sin(d*x + c))*sin(d*x
 + c)^n/(d*n^5 + 15*d*n^4 + 85*d*n^3 + 225*d*n^2 + 274*d*n + 120*d)
```

giac [B] time = 0.35, size = 593, normalized size = 5.20

$$\frac{a^4n^4\sin(dx+c)^n\sin(dx+c)^5 + 4\,a^4n^4\sin(dx+c)^n\sin(dx+c)^4 + 10\,a^4n^3\sin(dx+c)^n\sin(dx+c)^5 + 6\,a^4n^4\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] (a^4*n^4*sin(d*x + c)^n*sin(d*x + c)^5 + 4*a^4*n^4*sin(d*x + c)^n*sin(d*x +
 c)^4 + 10*a^4*n^3*sin(d*x + c)^n*sin(d*x + c)^5 + 6*a^4*n^4*sin(d*x + c)^n
*sin(d*x + c)^3 + 44*a^4*n^3*sin(d*x + c)^n*sin(d*x + c)^4 + 35*a^4*n^2*sin
(d*x + c)^n*sin(d*x + c)^5 + 4*a^4*n^4*sin(d*x + c)^n*sin(d*x + c)^2 + 72*a
^4*n^3*sin(d*x + c)^n*sin(d*x + c)^3 + 164*a^4*n^2*sin(d*x + c)^n*sin(d*x +
 c)^4 + 50*a^4*n*sin(d*x + c)^n*sin(d*x + c)^5 + a^4*n^4*sin(d*x + c)^n*sin
(d*x + c) + 52*a^4*n^3*sin(d*x + c)^n*sin(d*x + c)^2 + 294*a^4*n^2*sin(d*x
+ c)^n*sin(d*x + c)^3 + 244*a^4*n*sin(d*x + c)^n*sin(d*x + c)^4 + 24*a^4*si
n(d*x + c)^n*sin(d*x + c)^5 + 14*a^4*n^3*sin(d*x + c)^n*sin(d*x + c) + 236*
a^4*n^2*sin(d*x + c)^n*sin(d*x + c)^2 + 468*a^4*n*sin(d*x + c)^n*sin(d*x +
c)^3 + 120*a^4*sin(d*x + c)^n*sin(d*x + c)^4 + 71*a^4*n^2*sin(d*x + c)^n*si
n(d*x + c) + 428*a^4*n*sin(d*x + c)^n*sin(d*x + c)^2 + 240*a^4*sin(d*x + c)
^n*sin(d*x + c)^3 + 154*a^4*n*sin(d*x + c)^n*sin(d*x + c) + 240*a^4*sin(d*x
 + c)^n*sin(d*x + c)^2 + 120*a^4*sin(d*x + c)^n*sin(d*x + c))/((n^5 + 15*n^
4 + 85*n^3 + 225*n^2 + 274*n + 120)*d)
```

maple [F] time = 8.36, size = 0, normalized size = 0.00

$$\int \cos\left(dx+c\right)\left(\sin^{n}\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x)
```

maxima [A] time = 0.31, size = 103, normalized size = 0.90

$$\frac{\frac{a^{4}\sin(dx+c)^{n+5}}{n+5}+\frac{4\,a^{4}\sin(dx+c)^{n+4}}{n+4}+\frac{6\,a^{4}\sin(dx+c)^{n+3}}{n+3}+\frac{4\,a^{4}\sin(dx+c)^{n+2}}{n+2}+\frac{a^{4}\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] (a^4*sin(d*x + c)^(n + 5)/(n + 5) + 4*a^4*sin(d*x + c)^(n + 4)/(n + 4) + 6*
a^4*sin(d*x + c)^(n + 3)/(n + 3) + 4*a^4*sin(d*x + c)^(n + 2)/(n + 2) + a^4
*sin(d*x + c)^(n + 1)/(n + 1))/d
```

mupad [B] time = 11.76, size = 370, normalized size = 3.25

$$\frac{a^{4}\sin\left(c+d\,x\right)^{n}\,\left(4888\,n+5040\,\sin\left(c+d\,x\right)-2880\,\cos\left(2\,c+2\,d\,x\right)+240\,\cos\left(4\,c+4\,d\,x\right)-1080\,\sin\left(3\,c+3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^n*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (a^4*sin(c + d*x)^n*(4888*n + 5040*sin(c + d*x) - 2880*cos(2*c + 2*d*x) + 2
40*cos(4*c + 4*d*x) - 1080*sin(3*c + 3*d*x) + 24*sin(5*c + 5*d*x) + 8580*n*
sin(c + d*x) - 5376*n*cos(2*c + 2*d*x) + 488*n*cos(4*c + 4*d*x) - 2122*n*si
n(3*c + 3*d*x) + 50*n*sin(5*c + 5*d*x) + 5014*n^2*sin(c + d*x) + 1188*n^3*s
in(c + d*x) + 98*n^4*sin(c + d*x) + 2872*n^2 + 680*n^3 + 56*n^4 - 3200*n^2*
cos(2*c + 2*d*x) - 768*n^3*cos(2*c + 2*d*x) - 64*n^4*cos(2*c + 2*d*x) + 328
*n^2*cos(4*c + 4*d*x) + 88*n^3*cos(4*c + 4*d*x) + 8*n^4*cos(4*c + 4*d*x) -
1351*n^2*sin(3*c + 3*d*x) - 338*n^3*sin(3*c + 3*d*x) - 29*n^4*sin(3*c + 3*d
*x) + 35*n^2*sin(5*c + 5*d*x) + 10*n^3*sin(5*c + 5*d*x) + n^4*sin(5*c + 5*d
*x) + 2640))/(16*d*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
```

sympy [A] time = 61.69, size = 1833, normalized size = 16.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**4*sin(c)**n*cos(c), Eq(d, 0)), (a**4*log(sin(c
 + d*x))/d - 4*a**4/(d*sin(c + d*x)) - 3*a**4/(d*sin(c + d*x)**2) - 4*a**4/
(3*d*sin(c + d*x)**3) - a**4/(4*d*sin(c + d*x)**4), Eq(n, -5)), (4*a**4*log
(sin(c + d*x))/d + a**4*sin(c + d*x)/d - 6*a**4/(d*sin(c + d*x)) - 2*a**4/(
d*sin(c + d*x)**2) - a**4/(3*d*sin(c + d*x)**3), Eq(n, -4)), (6*a**4*log(si
n(c + d*x))/d + a**4*sin(c + d*x)**2/(2*d) + 4*a**4*sin(c + d*x)/d - 4*a**4
/(d*sin(c + d*x)) - a**4/(2*d*sin(c + d*x)**2), Eq(n, -3)), (4*a**4*log(sin
(c + d*x))/d + a**4*sin(c + d*x)**3/(3*d) + 2*a**4*sin(c + d*x)**2/d + 6*a*
*4*sin(c + d*x)/d - a**4/(d*sin(c + d*x)), Eq(n, -2)), (a**4*log(sin(c + d*
x))/d + a**4*sin(c + d*x)**4/(4*d) + 4*a**4*sin(c + d*x)**3/(3*d) + 3*a**4*
sin(c + d*x)**2/d + 4*a**4*sin(c + d*x)/d, Eq(n, -1)), (a**4*n**4*sin(c + d
*x)**5*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d
*n + 120*d) + 4*a**4*n**4*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**5 + 15*d*n*
*4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 6*a**4*n**4*sin(c + d*x)**
3*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n +
120*d) + 4*a**4*n**4*sin(c + d*x)**2*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 +
85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + a**4*n**4*sin(c + d*x)*sin(c +
d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 1
0*a**4*n**3*sin(c + d*x)**5*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3
 + 225*d*n**2 + 274*d*n + 120*d) + 44*a**4*n**3*sin(c + d*x)**4*sin(c + d*x
)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 72*a
**4*n**3*sin(c + d*x)**3*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 +
225*d*n**2 + 274*d*n + 120*d) + 52*a**4*n**3*sin(c + d*x)**2*sin(c + d*x)**
```

```
n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 14*a**4
*n**3*sin(c + d*x)*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*
n**2 + 274*d*n + 120*d) + 35*a**4*n**2*sin(c + d*x)**5*sin(c + d*x)**n/(d*n
**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 164*a**4*n**2
*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n*
*2 + 274*d*n + 120*d) + 294*a**4*n**2*sin(c + d*x)**3*sin(c + d*x)**n/(d*n*
*5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 236*a**4*n**2*
sin(c + d*x)**2*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**
2 + 274*d*n + 120*d) + 71*a**4*n**2*sin(c + d*x)*sin(c + d*x)**n/(d*n**5 +
15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 50*a**4*n*sin(c + d
*x)**5*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d
*n + 120*d) + 244*a**4*n*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**5 + 15*d*n**
4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 468*a**4*n*sin(c + d*x)**3*
sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 12
0*d) + 428*a**4*n*sin(c + d*x)**2*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*
d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 154*a**4*n*sin(c + d*x)*sin(c + d*
x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 24*
a**4*sin(c + d*x)**5*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*
d*n**2 + 274*d*n + 120*d) + 120*a**4*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**
5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 240*a**4*sin(c
+ d*x)**3*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 27
4*d*n + 120*d) + 240*a**4*sin(c + d*x)**2*sin(c + d*x)**n/(d*n**5 + 15*d*n*
*4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 120*a**4*sin(c + d*x)*sin(
c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d)
, True))
```

3.259 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}+\frac{3a^3\sin^{n+3}(c+dx)}{d(n+3)}+\frac{a^3\sin^{n+4}(c+dx)}{d(n+4)}$$

```
[Out] a^3*sin(d*x+c)^(1+n)/d/(1+n)+3*a^3*sin(d*x+c)^(2+n)/d/(2+n)+3*a^3*sin(d*x+c
)^(3+n)/d/(3+n)+a^3*sin(d*x+c)^(4+n)/d/(4+n)
```

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 43}

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}+\frac{3a^3\sin^{n+3}(c+dx)}{d(n+3)}+\frac{a^3\sin^{n+4}(c+dx)}{d(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (3*a^3*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (3*a^3*Sin[c + d*x]^(3 + n))/(d*(3 + n)) + (a^3*Sin[c + d*x]^(4 +
n))/(d*(4 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (a+x)^3\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(a^3\left(\frac{x}{a}\right)^n + 3a^3\left(\frac{x}{a}\right)^{1+n} + 3a^3\left(\frac{x}{a}\right)^{2+n} + a^3\left(\frac{x}{a}\right)^{3+n}\right)\right.}{ad}$$
$$= \frac{a^3\sin^{1+n}(c+dx)}{d(1+n)} + \frac{3a^3\sin^{2+n}(c+dx)}{d(2+n)} + \frac{3a^3\sin^{3+n}(c+dx}{d(3+n)}$$

Mathematica [A] time = 0.17, size = 65, normalized size = 0.71

$$\frac{a^3\sin^{n+1}(c+dx)\left(\frac{\sin^3(c+dx)}{n+4} + \frac{3\sin^2(c+dx)}{n+3} + \frac{3\sin(c+dx)}{n+2} + \frac{1}{n+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (3*Sin[c + d*x])/(2 + n) + (3*Sin
[c + d*x]^2)/(3 + n) + Sin[c + d*x]^3/(4 + n)))/d
```

fricas [B] time = 0.52, size = 210, normalized size = 2.31

$$\frac{\left(4\,a^3n^3 + 30\,a^3n^2 + \left(a^3n^3 + 6\,a^3n^2 + 11\,a^3n + 6\,a^3\right)\cos\left(dx+c\right)^4 + 68\,a^3n + 42\,a^3 - \left(5\,a^3n^3 + 36\,a^3n^2 + 79\,a^3n\right.}{dn^4 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (4*a^3*n^3 + 30*a^3*n^2 + (a^3*n^3 + 6*a^3*n^2 + 11*a^3*n + 6*a^3)*cos(d*x
+ c)^4 + 68*a^3*n + 42*a^3 - (5*a^3*n^3 + 36*a^3*n^2 + 79*a^3*n + 48*a^3)*c
os(d*x + c)^2 + (4*a^3*n^3 + 30*a^3*n^2 + 68*a^3*n + 48*a^3 - 3*(a^3*n^3 +
7*a^3*n^2 + 14*a^3*n + 8*a^3)*cos(d*x + c)^2)*sin(d*x + c))*sin(d*x + c)^n/
(d*n^4 + 10*d*n^3 + 35*d*n^2 + 50*d*n + 24*d)
```

giac [B] time = 0.32, size = 379, normalized size = 4.16

$$\frac{a^3n^3\sin\left(dx+c\right)^n\sin\left(dx+c\right)^4 + 3\,a^3n^3\sin\left(dx+c\right)^n\sin\left(dx+c\right)^3 + 6\,a^3n^2\sin\left(dx+c\right)^n\sin\left(dx+c\right)^4 + 3\,a^3n^3\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (a^3*n^3*sin(d*x + c)^n*sin(d*x + c)^4 + 3*a^3*n^3*sin(d*x + c)^n*sin(d*x +
 c)^3 + 6*a^3*n^2*sin(d*x + c)^n*sin(d*x + c)^4 + 3*a^3*n^3*sin(d*x + c)^n*
sin(d*x + c)^2 + 21*a^3*n^2*sin(d*x + c)^n*sin(d*x + c)^3 + 11*a^3*n*sin(d*
x + c)^n*sin(d*x + c)^4 + a^3*n^3*sin(d*x + c)^n*sin(d*x + c) + 24*a^3*n^2*
sin(d*x + c)^n*sin(d*x + c)^2 + 42*a^3*n*sin(d*x + c)^n*sin(d*x + c)^3 + 6*
a^3*sin(d*x + c)^n*sin(d*x + c)^4 + 9*a^3*n^2*sin(d*x + c)^n*sin(d*x + c) +
 57*a^3*n*sin(d*x + c)^n*sin(d*x + c)^2 + 24*a^3*sin(d*x + c)^n*sin(d*x + c
)^3 + 26*a^3*n*sin(d*x + c)^n*sin(d*x + c) + 36*a^3*sin(d*x + c)^n*sin(d*x
+ c)^2 + 24*a^3*sin(d*x + c)^n*sin(d*x + c))/((n^4 + 10*n^3 + 35*n^2 + 50*n
 + 24)*d)
```

maple [F] time = 6.39, size = 0, normalized size = 0.00

$$\int \cos \left(dx+c\right) \left(\sin^{n}\left(dx+c\right)\right) \left(a+a\sin \left(dx+c\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

maxima [A] time = 0.47, size = 83, normalized size = 0.91

$$\frac{\frac{a^3 \sin(dx+c)^{n+4}}{n+4} + \frac{3\,a^3 \sin(dx+c)^{n+3}}{n+3} + \frac{3\,a^3 \sin(dx+c)^{n+2}}{n+2} + \frac{a^3 \sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (a^3*sin(d*x + c)^(n + 4)/(n + 4) + 3*a^3*sin(d*x + c)^(n + 3)/(n + 3) + 3*
a^3*sin(d*x + c)^(n + 2)/(n + 2) + a^3*sin(d*x + c)^(n + 1)/(n + 1))/d
```

mupad [B] time = 10.42, size = 242, normalized size = 2.66

$$\frac{a^3 \sin\left(c+d\,x\right)^n\,\left(261\,n+336\,\sin\left(c+d\,x\right)-168\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)-48\,\sin\left(3\,c+3\,d\,x\right)+46\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^n*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (a^3*sin(c + d*x)^n*(261*n + 336*sin(c + d*x) - 168*cos(2*c + 2*d*x) + 6*co
s(4*c + 4*d*x) - 48*sin(3*c + 3*d*x) + 460*n*sin(c + d*x) - 272*n*cos(2*c +
```

```
 2*d*x) + 11*n*cos(4*c + 4*d*x) - 84*n*sin(3*c + 3*d*x) + 198*n^2*sin(c + d
*x) + 26*n^3*sin(c + d*x) + 114*n^2 + 15*n^3 - 120*n^2*cos(2*c + 2*d*x) - 1
6*n^3*cos(2*c + 2*d*x) + 6*n^2*cos(4*c + 4*d*x) + n^3*cos(4*c + 4*d*x) - 42
*n^2*sin(3*c + 3*d*x) - 6*n^3*sin(3*c + 3*d*x) + 162))/(8*d*(50*n + 35*n^2
+ 10*n^3 + n^4 + 24))
```

sympy [A] time = 27.52, size = 1061, normalized size = 11.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**3*sin(c)**n*cos(c), Eq(d, 0)), (a**3*log(sin(c
 + d*x))/d - 3*a**3/(d*sin(c + d*x)) - 3*a**3/(2*d*sin(c + d*x)**2) - a**3/
(3*d*sin(c + d*x)**3), Eq(n, -4)), (3*a**3*log(sin(c + d*x))/d + a**3*sin(c
 + d*x)/d - 3*a**3/(d*sin(c + d*x)) - a**3/(2*d*sin(c + d*x)**2), Eq(n, -3)
), (3*a**3*log(sin(c + d*x))/d + a**3*sin(c + d*x)**2/(2*d) + 3*a**3*sin(c
+ d*x)/d - a**3/(d*sin(c + d*x)), Eq(n, -2)), (a**3*log(sin(c + d*x))/d + a
**3*sin(c + d*x)**3/(3*d) + 3*a**3*sin(c + d*x)**2/(2*d) + 3*a**3*sin(c + d
*x)/d, Eq(n, -1)), (a**3*n**3*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**4 + 10*
d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 3*a**3*n**3*sin(c + d*x)**3*sin(c + d
*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 3*a**3*n**3*sin(c
 + d*x)**2*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d)
 + a**3*n**3*sin(c + d*x)*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 +
 50*d*n + 24*d) + 6*a**3*n**2*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**4 + 10*
d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 21*a**3*n**2*sin(c + d*x)**3*sin(c +
d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 24*a**3*n**2*sin
(c + d*x)**2*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*
d) + 9*a**3*n**2*sin(c + d*x)*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n*
*2 + 50*d*n + 24*d) + 11*a**3*n*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**4 + 1
0*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 42*a**3*n*sin(c + d*x)**3*sin(c + d
*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 57*a**3*n*sin(c +
 d*x)**2*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) +
 26*a**3*n*sin(c + d*x)*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 5
0*d*n + 24*d) + 6*a**3*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**4 + 10*d*n**3
+ 35*d*n**2 + 50*d*n + 24*d) + 24*a**3*sin(c + d*x)**3*sin(c + d*x)**n/(d*n
**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 36*a**3*sin(c + d*x)**2*sin(
c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 24*a**3*sin(
c + d*x)*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d),
True))
```

3.260 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=68

$$\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2\sin^{n+2}(c+dx)}{d(n+2)}+\frac{a^2\sin^{n+3}(c+dx)}{d(n+3)}$$

```
[Out] a^2*sin(d*x+c)^(1+n)/d/(1+n)+2*a^2*sin(d*x+c)^(2+n)/d/(2+n)+a^2*sin(d*x+c)^
(3+n)/d/(3+n)
```

Rubi [A] time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 43}

$$\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2\sin^{n+2}(c+dx)}{d(n+2)}+\frac{a^2\sin^{n+3}(c+dx)}{d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a^2*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (a^2*Sin[c + d*x]^(3 + n))/(d*(3 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (a+x)^2\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(\frac{x}{a}\right)^n + 2a^2\left(\frac{x}{a}\right)^{1+n} + a^2\left(\frac{x}{a}\right)^{2+n}\right)dx, x, a\sin(c+\right.}{ad}$$
$$= \frac{a^2\sin^{1+n}(c+dx)}{d(1+n)} + \frac{2a^2\sin^{2+n}(c+dx)}{d(2+n)} + \frac{a^2\sin^{3+n}(c+dx)}{d(3+n)}$$

Mathematica [A] time = 0.22, size = 50, normalized size = 0.74

$$\frac{a^2\sin^{n+1}(c+dx)\left(\frac{\sin^2(c+dx)}{n+3} + \frac{2\sin(c+dx)}{n+2} + \frac{1}{n+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (2*Sin[c + d*x])/(2 + n) + Sin[c
+ d*x]^2/(3 + n)))/d
```

fricas [A] time = 0.50, size = 135, normalized size = 1.99

$$\frac{\left(2\,a^2n^2 + 8\,a^2n - 2\left(a^2n^2 + 4\,a^2n + 3\,a^2\right)\cos(dx+c)^2 + 6\,a^2 + \left(2\,a^2n^2 + 8\,a^2n - \left(a^2n^2 + 3\,a^2n + 2\,a^2\right)\cos(dx\right.\right.}{dn^3 + 6\,dn^2 + 11\,dn + 6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*n^2 + 8*a^2*n - 2*(a^2*n^2 + 4*a^2*n + 3*a^2)*cos(d*x + c)^2 + 6*a^2
 + (2*a^2*n^2 + 8*a^2*n - (a^2*n^2 + 3*a^2*n + 2*a^2)*cos(d*x + c)^2 + 8*a^
2)*sin(d*x + c))*sin(d*x + c)^n/(d*n^3 + 6*d*n^2 + 11*d*n + 6*d)
```

giac [B] time = 0.24, size = 213, normalized size = 3.13

$$\frac{a^2n^2\sin(dx+c)^n\sin(dx+c)^3 + 2\,a^2n^2\sin(dx+c)^n\sin(dx+c)^2 + 3\,a^2n\sin(dx+c)^n\sin(dx+c)^3 + a^2n^2\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a^2*n^2*sin(d*x + c)^n*sin(d*x + c)^3 + 2*a^2*n^2*sin(d*x + c)^n*sin(d*x +
 c)^2 + 3*a^2*n*sin(d*x + c)^n*sin(d*x + c)^3 + a^2*n^2*sin(d*x + c)^n*sin(
d*x + c) + 8*a^2*n*sin(d*x + c)^n*sin(d*x + c)^2 + 2*a^2*sin(d*x + c)^n*sin
(d*x + c)^3 + 5*a^2*n*sin(d*x + c)^n*sin(d*x + c) + 6*a^2*sin(d*x + c)^n*si
n(d*x + c)^2 + 6*a^2*sin(d*x + c)^n*sin(d*x + c))/((n^3 + 6*n^2 + 11*n + 6)
*d)
```

maple [F] time = 6.52, size = 0, normalized size = 0.00

$$\int \cos \left(dx+c\right) \left(\sin^{n}\left(dx+c\right)\right) \left(a+a \sin \left(dx+c\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

maxima [A] time = 0.32, size = 63, normalized size = 0.93

$$\frac{\frac{a^{2} \sin(dx+c)^{n+3}}{n+3} + \frac{2\, a^{2} \sin(dx+c)^{n+2}}{n+2} + \frac{a^{2} \sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (a^2*sin(d*x + c)^(n + 3)/(n + 3) + 2*a^2*sin(d*x + c)^(n + 2)/(n + 2) + a^
2*sin(d*x + c)^(n + 1)/(n + 1))/d
```

mupad [B] time = 9.64, size = 147, normalized size = 2.16

$$\frac{a^{2} \sin\left(c+d\,x\right)^{n}\,\left(16\,n+30\,\sin\left(c+d\,x\right)-2\,\sin\left(3\,c+3\,d\,x\right)+29\,n\,\sin\left(c+d\,x\right)+16\,n\,\left(2\sin\left(c+d\,x\right)^{2}-1\right)-3\,n}{4\,d\,\left(n^{3}+6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^n*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x)^n*(16*n + 30*sin(c + d*x) - 2*sin(3*c + 3*d*x) + 29*n*sin
(c + d*x) + 16*n*(2*sin(c + d*x)^2 - 1) - 3*n*sin(3*c + 3*d*x) + 7*n^2*sin(
c + d*x) + 4*n^2*(2*sin(c + d*x)^2 - 1) + 4*n^2 + 24*sin(c + d*x)^2 - n^2*s
in(3*c + 3*d*x)))/(4*d*(11*n + 6*n^2 + n^3 + 6))
```

sympy [A] time = 12.06, size = 530, normalized size = 7.79

$$\begin{cases} x(a\sin(c)+a)^2\sin^n(c)\cos(c) \\ \frac{a^2\log(\sin(c+dx))}{d}-\frac{2a^2}{d\sin(c+dx)}-\frac{a^2}{2d\sin^2(c+dx)} \\ \frac{2a^2\log(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2}{d\sin(c+dx)} \\ \frac{a^2\log(\sin(c+dx))}{d}+\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d} \\ \frac{a^2n^2\sin^3(c+dx)\sin^n(c+dx)}{dn^3+6dn^2+11dn+6d}+\frac{2a^2n^2\sin^2(c+dx)\sin^n(c+dx)}{dn^3+6dn^2+11dn+6d}+\frac{a^2n^2\sin(c+dx)\sin^n(c+dx)}{dn^3+6dn^2+11dn+6d}+\frac{3a^2n\sin^3(c+dx)\sin^n(c+dx)}{dn^3+6dn^2+11dn+6d}+\frac{8a^2n\sin^2(c+dx}{dn^3+6dn^2+} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**2*sin(c)**n*cos(c), Eq(d, 0)), (a**2*log(sin(c
 + d*x))/d - 2*a**2/(d*sin(c + d*x)) - a**2/(2*d*sin(c + d*x)**2), Eq(n, -3
)), (2*a**2*log(sin(c + d*x))/d + a**2*sin(c + d*x)/d - a**2/(d*sin(c + d*x
)), Eq(n, -2)), (a**2*log(sin(c + d*x))/d + a**2*sin(c + d*x)**2/(2*d) + 2*
a**2*sin(c + d*x)/d, Eq(n, -1)), (a**2*n**2*sin(c + d*x)**3*sin(c + d*x)**n
/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 2*a**2*n**2*sin(c + d*x)**2*sin(c + d
*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + a**2*n**2*sin(c + d*x)*sin(c +
d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 3*a**2*n*sin(c + d*x)**3*sin(c
 + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 8*a**2*n*sin(c + d*x)**2*si
n(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 5*a**2*n*sin(c + d*x)*si
n(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 2*a**2*sin(c + d*x)**3*s
in(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 6*a**2*sin(c + d*x)**2*
sin(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 6*a**2*sin(c + d*x)*si
n(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d), True))
```

3.261 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}$$

```
[Out] a*sin(d*x+c)^(1+n)/d/(1+n)+a*sin(d*x+c)^(2+n)/d/(2+n)
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2833, 43}

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (a*Sin[c + d*x]^(2 + n))/(d*(2 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (a+x)\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(a\left(\frac{x}{a}\right)^n + a\left(\frac{x}{a}\right)^{1+n}\right) dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\sin^{1+n}(c+dx)}{d(1+n)} + \frac{a\sin^{2+n}(c+dx)}{d(2+n)}$$

Mathematica [A] time = 0.30, size = 38, normalized size = 0.93

$$\frac{a\sin^{n+1}(c+dx)((n+1)\sin(c+dx)+n+2)}{d(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n)*(2 + n + (1 + n)*Sin[c + d*x]))/(d*(1 + n)*(2 + n))
```

fricas [A] time = 0.50, size = 62, normalized size = 1.51

$$-\frac{\left((an+a)\cos\left(dx+c\right)^2 - an - (an+2\,a)\sin\left(dx+c\right) - a\right)\sin\left(dx+c\right)^n}{dn^2+3\,dn+2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((a*n + a)*cos(d*x + c)^2 - a*n - (a*n + 2*a)*sin(d*x + c) - a)*sin(d*x +
c)^n/(d*n^2 + 3*d*n + 2*d)
```

giac [B] time = 0.22, size = 86, normalized size = 2.10

$$\frac{an\sin\left(dx+c\right)^n\sin\left(dx+c\right)^2 + an\sin\left(dx+c\right)^n\sin\left(dx+c\right) + a\sin\left(dx+c\right)^n\sin\left(dx+c\right)^2 + 2\,a\sin\left(dx+c\right)^n \sin}{\left(n^2+3\,n+2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*n*sin(d*x + c)^n*sin(d*x + c)^2 + a*n*sin(d*x + c)^n*sin(d*x + c) + a*si
n(d*x + c)^n*sin(d*x + c)^2 + 2*a*sin(d*x + c)^n*sin(d*x + c))/((n^2 + 3*n
+ 2)*d)
```

maple [F] time = 3.79, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\left(\sin^n(dx+c)\right)\left(a+a\sin(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

maxima [A] time = 0.44, size = 39, normalized size = 0.95

$$\frac{\frac{a\sin(dx+c)^{n+2}}{n+2}+\frac{a\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*sin(d*x + c)^(n + 2)/(n + 2) + a*sin(d*x + c)^(n + 1)/(n + 1))/d
```

mupad [B] time = 9.18, size = 67, normalized size = 1.63

$$\frac{a\sin(c+d\,x)^n\,\left(n+4\,\sin(c+d\,x)+2\,n\,\sin(c+d\,x)+n\,\left(2\sin(c+d\,x)^2-1\right)+2\sin(c+d\,x)^2\right)}{2\,d\,\left(n^2+3\,n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^n*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x)^n*(n + 4*sin(c + d*x) + 2*n*sin(c + d*x) + n*(2*sin(c + d*x
)^2 - 1) + 2*sin(c + d*x)^2))/(2*d*(3*n + n^2 + 2))
```

sympy [A] time = 5.08, size = 190, normalized size = 4.63

$$\begin{cases} x\left(a\sin(c)+a\right)\sin^n(c)\cos(c) & \text{for } d=0 \\ \frac{a\log(\sin(c+dx))}{d}-\frac{a}{d\sin(c+dx)} & \text{for } n=-2 \\ \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d} & \text{for } n=-1 \\ \frac{an\sin^2(c+dx)\sin^n(c+dx)}{dn^2+3dn+2d}+\frac{an\sin(c+dx)\sin^n(c+dx)}{dn^2+3dn+2d}+\frac{a\sin^2(c+dx)\sin^n(c+dx)}{dn^2+3dn+2d}+\frac{2a\sin(c+dx)\sin^n(c+dx)}{dn^2+3dn+2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)*sin(c)**n*cos(c), Eq(d, 0)), (a*log(sin(c + d*x
))/d - a/(d*sin(c + d*x)), Eq(n, -2)), (a*log(sin(c + d*x))/d + a*sin(c + d
*x)/d, Eq(n, -1)), (a*n*sin(c + d*x)**2*sin(c + d*x)**n/(d*n**2 + 3*d*n + 2
*d) + a*n*sin(c + d*x)*sin(c + d*x)**n/(d*n**2 + 3*d*n + 2*d) + a*sin(c + d
*x)**2*sin(c + d*x)**n/(d*n**2 + 3*d*n + 2*d) + 2*a*sin(c + d*x)*sin(c + d*
x)**n/(d*n**2 + 3*d*n + 2*d), True))
```

3.262 $\int \frac{\cos(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{ad(n+1)}$$

```
[Out] hypergeom([1, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(1+n)/a/d/(1+n)
```

Rubi [A] time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 64}

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
*d*(1 + n))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{ad(1+n)}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 1.00

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
*d*(1 + n))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin(dx+c)^n\cos(dx+c)}{a\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n\cos(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

maple [F] time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)\left(\sin^n(dx+c)\right)}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)}{a\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)\,{\sin\left(c+d\,x\right)}^{n}}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.263 $\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=38

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(2,n+1;n+2;-\sin(c+dx))}{a^2d(n+1)}$$

```
[Out] hypergeom([2, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(1+n)/a^2/d/(1+n)
```

Rubi [A] time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 64}

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(2,n+1;n+2;-\sin(c+dx))}{a^2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^2*d*(1 + n))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{{}_2F_1(2,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^2d(1+n)}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 1.00

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(2,n+1;n+2;-\sin(c+dx))}{a^2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^2*d*(1 + n))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sin(dx+c)^n\cos(dx+c)}{a^2\cos(dx+c)^2 - 2a^2\sin(dx+c) - 2a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x +
 c) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n\cos(dx+c)}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^2, x)
```

maple [F] time = 4.02, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)\left(\sin^n(dx+c)\right)}{(a+a\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n\cos(dx+c)}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(c+d\,x)\,\sin(c+d\,x)^n}{\left(a+a\,\sin(c+d\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.264 $\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=38

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(3,n+1;n+2;-\sin(c+dx))}{a^3d(n+1)}$$

```
[Out] hypergeom([3, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(1+n)/a^3/d/(1+n)
```

Rubi [A] time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 64}

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(3,n+1;n+2;-\sin(c+dx))}{a^3d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^3*d*(1 + n))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{{}_2F_1(3,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^3d(1+n)}$$

Mathematica [A] time = 0.06, size = 38, normalized size = 1.00

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(3,n+1;n+2;-\sin(c+dx))}{a^3d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^3*d*(1 + n))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sin(dx+c)^n\cos(dx+c)}{3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)/(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*
cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n\cos(dx+c)}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^3, x)
```

maple [F] time = 3.79, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)\left(\sin^n(dx+c)\right)}{(a+a\sin(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n \cos(dx+c)}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(c+d\,x)\,\sin(c+d\,x)^n}{(a+a\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.265 $\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=38

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(4,n+1;n+2;-\sin(c+dx))}{a^4d(n+1)}$$

```
[Out] hypergeom([4, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(1+n)/a^4/d/(1+n)
```

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 64}

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(4,n+1;n+2;-\sin(c+dx))}{a^4d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Hypergeometric2F1[4, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^4*d*(1 + n))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{{}_2F_1(4,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4d(1+n)}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 1.00

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(4,n+1;n+2;-\sin(c+dx))}{a^4d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Hypergeometric2F1[4, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^4*d*(1 + n))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin(dx+c)^n\cos(dx+c)}{a^4\cos(dx+c)^4 - 8\,a^4\cos(dx+c)^2 + 8\,a^4 - 4\left(a^4\cos(dx+c)^2 - 2\,a^4\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)/(a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x +
c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n\cos(dx+c)}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^4, x)
```

maple [F] time = 3.62, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)\left(\sin^n(dx+c)\right)}{(a+a\sin(dx+c))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)}{\left(a\sin\left(dx+c\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)\,{\sin\left(c+d\,x\right)}^{n}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.266 $\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{a\sin(c+dx)\cos(c+dx}{16d}$$

```
[Out] 1/16*a*x-1/3*a*cos(d*x+c)^3/d+1/5*a*cos(d*x+c)^5/d+1/16*a*cos(d*x+c)*sin(d*
x+c)/d-1/8*a*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a*cos(d*x+c)^3*sin(d*x+c)^3/d
```

Rubi [A] time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2565, 14, 2568, 2635, 8}

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{a\sin(c+dx)\cos(c+dx}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/16 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) - (a*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (a*Cos
[c + d*x]^3*Sin[c + d*x]^3)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
```

```
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^2(c+dx)\sin^3(c+dx)\,dx + a\int \cos^2(c+dx)\sin^4(c \\ &= -\frac{a\cos^3(c+dx)\sin^3(c+dx)}{6d} + \frac{1}{2}a\int \cos^2(c+dx)\sin^2(c+ \\ &= -\frac{a\cos^3(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+dx)\sin^3(c+dx)}{6d} + \\ &= -\frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c+d}{16d} \\ &= \frac{ax}{16} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c}{16d}\end{aligned}$$

Mathematica [A] time = 0.20, size = 71, normalized size = 0.68

$$\frac{a(-15\sin(2(c+dx)) - 15\sin(4(c+dx)) + 5\sin(6(c+dx)) - 120\cos(c+dx) - 20\cos(3(c+dx)) + 12\cos(5(c+}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(60*d*x - 120*Cos[c + d*x] - 20*Cos[3*(c + d*x)] + 12*Cos[5*(c + d*x)] -
 15*Sin[2*(c + d*x)] - 15*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(960*d)
```

fricas [A] time = 0.49, size = 73, normalized size = 0.70

$$\frac{48\,a\cos\left(dx+c\right)^{5}-80\,a\cos\left(dx+c\right)^{3}+15\,adx+5\left(8\,a\cos\left(dx+c\right)^{5}-14\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 15*a*d*x + 5*(8*a*cos(d*
x + c)^5 - 14*a*cos(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.17, size = 92, normalized size = 0.88

$$\frac{1}{16}\,ax+\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}+\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\sin\left(4\,dx+4\,c\right)}{64\,d}-\frac{a\sin\left(2\,dx+2\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*a*x + 1/80*a*cos(5*d*x + 5*c)/d - 1/48*a*cos(3*d*x + 3*c)/d - 1/8*a*co
s(d*x + c)/d + 1/192*a*sin(6*d*x + 6*c)/d - 1/64*a*sin(4*d*x + 4*c)/d - 1/6
4*a*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.13, size = 95, normalized size = 0.90

$$\frac{a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{3}(dx+c)\right)}{6}-\frac{\left(\cos^{3}(dx+c)\right)\sin(dx+c)}{8}+\frac{\cos(dx+c)\sin(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{3}(dx+c)\right)}{5}-\frac{2\left(\cos^{3}(dx+\right.}{15}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*cos
(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+a*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*
cos(d*x+c)^3))
```

maxima [A] time = 0.33, size = 65, normalized size = 0.62

$$\frac{64\left(3\,\cos\left(dx+c\right)^{5}-5\,\cos\left(dx+c\right)^{3}\right)a-5\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-12\,dx-12\,c+3\,\sin\left(4\,dx+4\,c\right)\right)a}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*(64*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a - 5*(4*sin(2*d*x + 2*c)^3
 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a)/d
```

mupad [B] time = 12.13, size = 226, normalized size = 2.15

$$\frac{a\,x}{16}+\frac{\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{17\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\left(\frac{a\,(225\,c+225\,d\,x-960)}{240}-\frac{15\,a\,(c+d\,x)}{16}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-\frac{19\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\left(\frac{a\,(300\,c+300}{240}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*x)/16 + ((a*(15*c + 15*d*x - 64))/240 - (a*tan(c/2 + (d*x)/2))/8 - (a*(c
 + d*x))/16 + tan(c/2 + (d*x)/2)^2*((a*(90*c + 90*d*x - 384))/240 - (3*a*(c
 + d*x))/8) + tan(c/2 + (d*x)/2)^6*((a*(300*c + 300*d*x - 640))/240 - (5*a*
(c + d*x))/4) + tan(c/2 + (d*x)/2)^8*((a*(225*c + 225*d*x - 960))/240 - (15
*a*(c + d*x))/16) - (17*a*tan(c/2 + (d*x)/2)^3)/24 + (19*a*tan(c/2 + (d*x)/
2)^5)/4 - (19*a*tan(c/2 + (d*x)/2)^7)/4 + (17*a*tan(c/2 + (d*x)/2)^9)/24 +
(a*tan(c/2 + (d*x)/2)^11)/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [A] time = 4.34, size = 192, normalized size = 1.83

$$\begin{cases} \frac{ax\sin^{6}(c+dx)}{16}+\frac{3ax\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{3ax\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{ax\cos^{6}(c+dx)}{16}+\frac{a\sin^{5}(c+dx)\cos(c+dx)}{16d}-\frac{a\sin^{3}(c+dx)\cos^{3}}{6d} \\ x\left(a\sin(c)+a\right)\sin^{3}(c)\cos^{2}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**6/16 + 3*a*x*sin(c + d*x)**4*cos(c + d*x)**2/1
6 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a*x*cos(c + d*x)**6/16 + a*s
in(c + d*x)**5*cos(c + d*x)/(16*d) - a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - a*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - a*sin(c + d*x)*cos(c + d*x)**
5/(16*d) - 2*a*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*
*3*cos(c)**2, True))
```

3.267 $\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}$$

```
[Out] 1/8*a*x-1/3*a*cos(d*x+c)^3/d+1/5*a*cos(d*x+c)^5/d+1/8*a*cos(d*x+c)*sin(d*x+
c)/d-1/4*a*cos(d*x+c)^3*sin(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 14}

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c +
d*x]*Sin[c + d*x])/(8*d) - (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
```

```
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^2(c+dx)\sin^2(c+dx)\,dx + a\int \cos^2(c+dx)\sin^3(c$$

$$= -\frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}a\int \cos^2(c+dx)\,dx - \frac{a\,\mathrm{Su}}{}$$

$$= \frac{a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{8}a$$

$$= \frac{ax}{8} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c}{8d}$$

Mathematica [A] time = 0.11, size = 54, normalized size = 0.67

$$\frac{a(-15\sin(4(c+dx)) - 60\cos(c+dx) - 10\cos(3(c+dx)) + 6\cos(5(c+dx)) + 60c + 60dx)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(60*c + 60*d*x - 60*Cos[c + d*x] - 10*Cos[3*(c + d*x)] + 6*Cos[5*(c + d*
x)] - 15*Sin[4*(c + d*x)]))/(480*d)
```

fricas [A] time = 0.50, size = 62, normalized size = 0.77

$$\frac{24\,a\cos(dx+c)^5 - 40\,a\cos(dx+c)^3 + 15\,adx - 15\left(2\,a\cos(dx+c)^3 - a\cos(dx+c)\right)\sin(dx+c)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/120*(24*a*cos(d*x + c)^5 - 40*a*cos(d*x + c)^3 + 15*a*d*x - 15*(2*a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*x + c))/d

giac [A] time = 0.18, size = 62, normalized size = 0.77

$$\frac{1}{8}\,ax + \frac{a\cos\left(5\,dx + 5\,c\right)}{80\,d} - \frac{a\cos\left(3\,dx + 3\,c\right)}{48\,d} - \frac{a\cos\left(dx + c\right)}{8\,d} - \frac{a\sin\left(4\,dx + 4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/8*a*x + 1/80*a*cos(5*d*x + 5*c)/d - 1/48*a*cos(3*d*x + 3*c)/d - 1/8*a*cos(d*x + c)/d - 1/32*a*sin(4*d*x + 4*c)/d

maple [A] time = 0.12, size = 77, normalized size = 0.95

$$\frac{a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{3}(dx+c)\right)}{5}-\frac{2\left(\cos^{3}(dx+c)\right)}{15}\right)+a\left(-\frac{\left(\cos^{3}(dx+c)\right)\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)

[Out] 1/d*(a*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+a*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c))

maxima [A] time = 0.31, size = 52, normalized size = 0.64

$$\frac{32\left(3\,\cos\left(dx + c\right)^{5} - 5\,\cos\left(dx + c\right)^{3}\right)a + 15\left(4\,dx + 4\,c - \sin\left(4\,dx + 4\,c\right)\right)a}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/480*(32*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a + 15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a)/d

mupad [B] time = 11.97, size = 198, normalized size = 2.44

$$\frac{a\,x}{8}+\frac{\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+\left(\frac{a\,(150\,c+150\,d\,x-480)}{120}-\frac{5\,a\,(c+d\,x)}{4}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{a\,(150\,c+150\,d\,x+160)}{120}-\frac{5\,a\,(c+d\,x)}{4}\right)}{d\left(\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*x)/8 + ((a*(15*c + 15*d*x - 32))/120 - (a*tan(c/2 + (d*x)/2))/4 - (a*(c
+ d*x))/8 + tan(c/2 + (d*x)/2)^2*((a*(75*c + 75*d*x - 160))/120 - (5*a*(c +
 d*x))/8) + tan(c/2 + (d*x)/2)^4*((a*(150*c + 150*d*x + 160))/120 - (5*a*(c
 + d*x))/4) + tan(c/2 + (d*x)/2)^6*((a*(150*c + 150*d*x - 480))/120 - (5*a*
(c + d*x))/4) + (3*a*tan(c/2 + (d*x)/2)^3)/2 - (3*a*tan(c/2 + (d*x)/2)^7)/2
 + (a*tan(c/2 + (d*x)/2)^9)/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

sympy [A] time = 2.52, size = 144, normalized size = 1.78

$$\begin{cases} \frac{ax\sin^4(c+dx)}{8} + \frac{ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{ax\cos^4(c+dx)}{8} + \frac{a\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{a\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{a\sin(c+dx)\cos^3(c+d}{8d} \\ x(a\sin(c)+a)\sin^2(c)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**4/8 + a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
a*x*cos(c + d*x)**4/8 + a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a*sin(c + d*
x)**2*cos(c + d*x)**3/(3*d) - a*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*a*co
s(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**2*cos(c)**2, Tru
e))
```

3.268 $\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}$$

```
[Out] 1/8*a*x-1/3*a*cos(d*x+c)^3/d+1/8*a*cos(d*x+c)*sin(d*x+c)/d-1/4*a*cos(d*x+c)
^3*sin(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2838, 2565, 30, 2568, 2635, 8}

$$-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(8*d) -
(a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^2(c+dx)\sin(c+dx)\,dx + a\int \cos^2(c+dx)\sin^2(c+\\
&= -\frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}a\int \cos^2(c+dx)\,dx - \frac{a\,\mathrm{Sub}}{}\\
&= -\frac{a\cos^3(c+dx)}{3d} + \frac{a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+dx}{4}\\
&= \frac{ax}{8} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c}{}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 42, normalized size = 0.65

$$
-\frac{a(3(\sin(4(c+dx))-4dx)+24\cos(c+dx)+8\cos(3(c+dx)))}{96d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/96*(a*(24*Cos[c + d*x] + 8*Cos[3*(c + d*x)] + 3*(-4*d*x + Sin[4*(c + d*x
)])))/d
```

fricas [A] time = 0.53, size = 51, normalized size = 0.78

$$
-\frac{8\,a\cos\left(dx+c\right)^3-3\,adx+3\left(2\,a\cos\left(dx+c\right)^3-a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*a*cos(d*x + c)^3 - 3*a*d*x + 3*(2*a*cos(d*x + c)^3 - a*cos(d*x + c
))*sin(d*x + c))/d
```

giac [A] time = 0.16, size = 47, normalized size = 0.72

$$\frac{1}{8}\,ax - \frac{a\cos\left(3\,dx + 3\,c\right)}{12\,d} - \frac{a\cos\left(dx + c\right)}{4\,d} - \frac{a\sin\left(4\,dx + 4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*a*x - 1/12*a*cos(3*d*x + 3*c)/d - 1/4*a*cos(d*x + c)/d - 1/32*a*sin(4*d
*x + 4*c)/d
```

maple [A] time = 0.08, size = 57, normalized size = 0.88

$$\frac{a\left(-\frac{\left(\cos^{3}(dx+c)\right)\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{\left(\cos^{3}(dx+c)\right)a}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*
c)-1/3*cos(d*x+c)^3*a)
```

maxima [A] time = 0.31, size = 39, normalized size = 0.60

$$-\frac{32\,a\cos\left(dx + c\right)^{3} - 3\,\left(4\,dx + 4\,c - \sin\left(4\,dx + 4\,c\right)\right)a}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*a*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a)/d
```

mupad [B] time = 12.33, size = 198, normalized size = 3.05

$$\frac{a\,x}{8}+\frac{\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\left(\frac{a\,(12\,c+12\,d\,x-48)}{24}-\frac{a\,(c+d\,x)}{2}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+\left(\frac{a\,(18\,c+18\,d\,x-48)}{24}-\frac{3\,a\,(c+d\,x)}{4}\right)\tan\left(\frac{c}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*x)/8 + ((a*(3*c + 3*d*x - 16))/24 - (a*tan(c/2 + (d*x)/2))/4 - (a*(c + d
*x))/8 + tan(c/2 + (d*x)/2)^2*((a*(12*c + 12*d*x - 16))/24 - (a*(c + d*x))/
2) + tan(c/2 + (d*x)/2)^6*((a*(12*c + 12*d*x - 48))/24 - (a*(c + d*x))/2) +
 tan(c/2 + (d*x)/2)^4*((a*(18*c + 18*d*x - 48))/24 - (3*a*(c + d*x))/4) + (
7*a*tan(c/2 + (d*x)/2)^3)/4 - (7*a*tan(c/2 + (d*x)/2)^5)/4 + (a*tan(c/2 + (
d*x)/2)^7)/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [A] time = 1.23, size = 119, normalized size = 1.83

$$\begin{cases} \frac{ax\sin^4(c+dx)}{8} + \frac{ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{ax\cos^4(c+dx)}{8} + \frac{a\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{a\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{a\cos^3(c+dx)}{3d} & \text{for } a \\ x(a\sin(c)+a)\sin(c)\cos^2(c) & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**4/8 + a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
a*x*cos(c + d*x)**4/8 + a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a*sin(c + d*
x)*cos(c + d*x)**3/(8*d) - a*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c)
 + a)*sin(c)*cos(c)**2, True))
```

3.269 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=51

$$\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{ax}{2}$$

```
[Out] 1/2*a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/
d
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2838, 2592, 321, 206, 2635, 8}

$$\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/2 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^2(c+dx)\,dx + a\int \cos(c+dx)\cot(c+dx)\,dx \\
&= \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx - \frac{a\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,\right.}{d} \\
&= \frac{ax}{2} + \frac{a\cos(c+dx)}{d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d} - \frac{a\,\text{Subst}\left(\int\right.}{} \\
&= \frac{ax}{2} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 74, normalized size = 1.45

$$
\frac{a(c+dx)}{2d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\cos(c+dx)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) + (a*Cos[c + d*x])/d - (a*Log[Cos[(c + d*x)/2]])/d + (a
*Log[Sin[(c + d*x)/2]])/d + (a*Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.54, size = 60, normalized size = 1.18

$$\frac{adx + a\cos\left(dx + c\right)\sin\left(dx + c\right) + 2\,a\cos\left(dx + c\right) - a\log\left(\frac{1}{2}\,\cos\left(dx + c\right) + \frac{1}{2}\right) + a\log\left(-\frac{1}{2}\,\cos\left(dx + c\right) + \frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + a*cos(d*x + c)*sin(d*x + c) + 2*a*cos(d*x + c) - a*log(1/2*cos
(d*x + c) + 1/2) + a*log(-1/2*cos(d*x + c) + 1/2))/d
```

giac [A] time = 0.15, size = 87, normalized size = 1.71

$$\frac{(dx + c)a + 2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} - 2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2} - a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a + 2*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(a*tan(1/2*d*x +
1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.26, size = 63, normalized size = 1.24

$$\frac{a\cos\left(dx + c\right)\sin\left(dx + c\right)}{2d} + \frac{ax}{2} + \frac{ca}{2d} + \frac{a\cos\left(dx + c\right)}{d} + \frac{a\ln\left(\csc\left(dx + c\right) - \cot\left(dx + c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2*a*cos(d*x+c)*sin(d*x+c)/d+1/2*a*x+1/2/d*c*a+a*cos(d*x+c)/d+1/d*a*ln(csc
(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.31, size = 57, normalized size = 1.12

$$\frac{\left(2\,dx + 2\,c + \sin\left(2\,dx + 2\,c\right)\right)a + 2\,a\left(2\,\cos\left(dx + c\right) - \log\left(\cos\left(dx + c\right) + 1\right) + \log\left(\cos\left(dx + c\right) - 1\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 2*a*(2*cos(d*x + c) - log(cos(d*x
 + c) + 1) + log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.72, size = 160, normalized size = 3.14

$$\frac{-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,a}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{a\,\mathrm{atan}\left(\frac{a^2}{2\,a^2-a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{}{2}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] (2*a + a*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^2 - a*tan(c/2 + (d*x)/
2)^3)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 1)) + (a*log(tan(
c/2 + (d*x)/2)))/d + (a*atan(a^2/(2*a^2 - a^2*tan(c/2 + (d*x)/2)) + (2*a^2*
tan(c/2 + (d*x)/2))/(2*a^2 - a^2*tan(c/2 + (d*x)/2))))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^2(c+dx)\csc(c+dx)\,dx+\int\sin(c+dx)\cos^2(c+dx)\csc(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**2*csc(c + d*x), x) + Integral(sin(c + d*x)*cos(c
+ d*x)**2*csc(c + d*x), x))
```

3.270 $\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-ax$$

```
[Out] -a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d-a*cot(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2710, 2592, 321, 206, 3473, 8}

$$\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx &= \int \left(a\cos(c+dx)\cot(c+dx)+a\cot^2(c+dx)\right)\,dx \\
&= a\int \cos(c+dx)\cot(c+dx)\,dx + a\int \cot^2(c+dx)\,dx \\
&= -\frac{a\cot(c+dx)}{d} - a\int 1\,dx - \frac{a\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -ax + \frac{a\cos(c+dx)}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(c+dx\right.}{d} \\
&= -ax - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} - \frac{a\cot(c+dx)}{d}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 75, normalized size = 1.83

$$
-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}+\frac{a\cos(c+dx)}{d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c
 + d*x]^2])/d - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/2]])/d
```

fricas [B] time = 0.53, size = 84, normalized size = 2.05

$$-\frac{a\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\,a\cos\left(dx+c\right)+2\left(adx-a\right.}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - a*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + 2*a*cos(d*x + c) + 2*(a*d*x - a*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

giac [B] time = 0.16, size = 108, normalized size = 2.63

$$-\frac{6\left(dx+c\right)a-6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a - 6*a*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + (2*a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 10*a*
tan(1/2*d*x + 1/2*c) + 3*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))
)/d
```

maple [A] time = 0.19, size = 57, normalized size = 1.39

$$-ax-\frac{a\cot\left(dx+c\right)}{d}+\frac{a\cos\left(dx+c\right)}{d}+\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}-\frac{ca}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] -a*x-a*cot(d*x+c)/d+a*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/d*c*a
```

maxima [A] time = 0.46, size = 54, normalized size = 1.32

$$-\frac{2\left(dx+c+\frac{1}{\tan\left(dx+c\right)}\right)a-a\left(2\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a - a*(2*cos(d*x + c) - log(cos(d*x + c)
 + 1) + log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.80, size = 108, normalized size = 2.63

$$\frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)}\right)+a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\cos\left(c+d\,x\right)-\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{2}}{d\,\sin\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] (2*a*atan((2^(1/2)*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)))/(2*cos(c/2 -
pi/4 + (d*x)/2))) + a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*co
s(c + d*x) - (a*sin(2*c + 2*d*x))/2)/(d*sin(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^{2}\left(c+dx\right)\csc^{2}\left(c+dx\right)dx+\int\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)\csc^{2}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**2*csc(c + d*x)**2, x) + Integral(sin(c + d*x)*cos
(c + d*x)**2*csc(c + d*x)**2, x))
```

3.271 $\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$-\frac{a\cot(c+dx)}{d}+\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-ax$$

```
[Out] -a*x+1/2*a*arctanh(cos(d*x+c))/d-a*cot(d*x+c)/d-1/2*a*cot(d*x+c)*csc(d*x+c)
/d
```

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2838, 2611, 3770, 3473, 8}

$$-\frac{a\cot(c+dx)}{d}+\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) + (a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x])/d - (a*Cot[c +
d*x]*Csc[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^2(c+dx)\,dx + a\int \cot^2(c+dx)\csc(c+dx)\,dx$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d} - \frac{1}{2}a\int \csc(c+d$$
$$= -ax + \frac{a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot(c+dx)}{2}$$

Mathematica [C] time = 0.04, size = 109, normalized size = 2.10

$$-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + $$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1,
1/2, -Tan[c + d*x]^2])/d + (a*Log[Cos[(c + d*x)/2]])/(2*d) - (a*Log[Sin[(c
+ d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

fricas [B] time = 0.53, size = 114, normalized size = 2.19

$$-\frac{4\,adx\cos(dx+c)^2 - 4\,adx - 4\,a\cos(dx+c)\sin(dx+c) - 2\,a\cos(dx+c) - \left(a\cos(dx+c)^2 - a\right)\log\left(\frac{1}{2}\cos(\right.}{4\left(d\cos(dx+c)^2 - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*d*x*cos(d*x + c)^2 - 4*a*d*x - 4*a*cos(d*x + c)*sin(d*x + c) - 2*
a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + (a*co
s(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.16, size = 95, normalized size = 1.83

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,(dx+c)a-4\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,a\tan\left(\frac{1}{2}\,dx+\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 - 8*(d*x + c)*a - 4*a*log(abs(tan(1/2*d*x + 1
/2*c))) + 4*a*tan(1/2*d*x + 1/2*c) + (6*a*tan(1/2*d*x + 1/2*c)^2 - 4*a*tan(
1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^2)/d
```

maple [A] time = 0.25, size = 81, normalized size = 1.56

$$-ax-\frac{a\cot(dx+c)}{d}-\frac{ca}{d}-\frac{a\left(\cos^3(dx+c)\right)}{2d\sin(dx+c)^2}-\frac{a\cos(dx+c)}{2d}-\frac{a\ln(\csc(dx+c)-\cot(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] -a*x-a*cot(d*x+c)/d-1/d*c*a-1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^3-1/2*a*cos(d*x
+c)/d-1/2/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.45, size = 66, normalized size = 1.27

$$-\frac{4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a-a\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c + 1/tan(d*x + c))*a - a*(2*cos(d*x + c)/(cos(d*x + c)^2 -
1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.68, size = 145, normalized size = 2.79

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{2\,a\,\mathrm{atan}\left(\frac{2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a\tan\left(\frac{c}{2}+\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) - (a*cot(c/2 + (d*x)/2))/(2*d) - (2*a*atan((2*
cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2) - 2*sin(c/2 +
(d*x)/2))))/d - (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*d) - (a*c
ot(c/2 + (d*x)/2)^2)/(8*d) + (a*tan(c/2 + (d*x)/2)^2)/(8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \cos^2(c+dx)\csc^3(c+dx)\,dx+\int \sin(c+dx)\cos^2(c+dx)\csc^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**2*csc(c + d*x)**3, x) + Integral(sin(c + d*x)*cos
(c + d*x)**2*csc(c + d*x)**3, x))
```

3.272 $\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] 1/2*a*arctanh(cos(d*x+c))/d-1/3*a*cot(d*x+c)^3/d-1/2*a*cot(d*x+c)*csc(d*x+c
)/d
```

Rubi [A] time = 0.11, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2838, 2607, 30, 2611, 3770}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x]^3)/(3*d) - (a*Cot[c + d*x
]*Csc[c + d*x])/(2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc(c+dx)\,dx + a\int \cot^2(c+dx)\csc^2(c+$$

$$= -\frac{a\cot(c+dx)\csc(c+dx)}{2d} - \frac{1}{2}a\int \csc(c+dx)\,dx + \frac{a\,\mathrm{Subs}}{}$$

$$= \frac{a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot(c+dx)\csc(}{2d}$$

Mathematica [A] time = 0.04, size = 95, normalized size = 1.83

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(a*Cot[c + d*x]^3)/d - (a*Csc[(c + d*x)/2]^2)/(8*d) + (a*Log[Cos[(c +
d*x)/2]])/(2*d) - (a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/
(8*d)
```

fricas [B] time = 0.50, size = 119, normalized size = 2.29

$$\frac{4\,a\cos\left(dx+c\right)^3+6\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+3\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3}{12\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*cos(d*x + c)^3 + 6*a*cos(d*x + c)*sin(d*x + c) + 3*(a*cos(d*x + c
)^2 - a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(a*cos(d*x + c)^2 - a
)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*
x + c))
```

giac [B] time = 0.17, size = 115, normalized size = 2.21

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{22\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}\cdots}{\cdots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 12*a*log(abs(
tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x + 1/2*c) + (22*a*tan(1/2*d*x + 1/2
*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*
d*x + 1/2*c)^3)/d
```

maple [A] time = 0.22, size = 80, normalized size = 1.54

$$-\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{2d\sin\left(dx+c\right)^{2}}-\frac{a\cos\left(dx+c\right)}{2d}-\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{2d}-\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^3-1/2*a*cos(d*x+c)/d-1/2/d*a*ln(csc(d*x+c)
-cot(d*x+c))-1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^3
```

maxima [A] time = 0.31, size = 61, normalized size = 1.17

$$\frac{3\,a\left(\frac{2\,\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{2}-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{4\,a}{\tan\left(dx+c\right)^{3}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - lo
g(cos(d*x + c) - 1)) - 4*a/tan(d*x + c)^3)/d
```

mupad [B] time = 8.58, size = 111, normalized size = 2.13

$$\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d}-\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24d}-\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\,\mathrm{t}\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a*tan(c/2 + (d*x)/2))/(8*d) + (a*tan(c/2
+ (d*x)/2)^3)/(24*d) - (a*log(tan(c/2 + (d*x)/2)))/(2*d) - (cot(c/2 + (d*x)
/2)^3*(a/3 + a*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.273 $\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] 1/8*a*arctanh(cos(d*x+c))/d-1/3*a*cot(d*x+c)^3/d+1/8*a*cot(d*x+c)*csc(d*x+c
)/d-1/4*a*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A] time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 30}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^3)/(3*d) + (a*Cot[c + d*x
]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^2(c+dx)\,dx + a\int \cot^2(c+dx)\csc^3(c$$

$$= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}a\int \csc^3(c+dx)\,dx + \frac{a\,\mathrm{Su}}{}$$

$$= -\frac{a\cot^3(c+dx)}{3d} + \frac{a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)}{4}$$

$$= \frac{a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^3(c+dx)}{3d} + \frac{a\cot(c+dx)\csc}{8d}$$

Mathematica [A] time = 0.05, size = 135, normalized size = 1.82

$$-\frac{a\cot^3(c+dx)}{3d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(a*Cot[c + d*x]^3)/d + (a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x
)/2]^4)/(64*d) + (a*Log[Cos[(c + d*x)/2]])/(8*d) - (a*Log[Sin[(c + d*x)/2]]
)/(8*d) - (a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

fricas [B] time = 0.50, size = 137, normalized size = 1.85

$$-\frac{16\,a\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+6\,a\cos\left(dx+c\right)^{3}+6\,a\cos\left(dx+c\right)-3\,\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\ldots}{48\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(16*a*cos(d*x + c)^3*sin(d*x + c) + 6*a*cos(d*x + c)^3 + 6*a*cos(d*x
+ c) - 3*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) +
 1/2) + 3*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c)
 + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.20, size = 116, normalized size = 1.57

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{50\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\ldots\right.}{\ldots}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*a*tan(1/2*d*x + 1/2*c)^3 - 24*a*log(a
bs(tan(1/2*d*x + 1/2*c))) - 24*a*tan(1/2*d*x + 1/2*c) + (50*a*tan(1/2*d*x +
 1/2*c)^4 + 24*a*tan(1/2*d*x + 1/2*c)^3 - 8*a*tan(1/2*d*x + 1/2*c) - 3*a)/t
an(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.22, size = 102, normalized size = 1.38

$$-\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)^{3}}-\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{4d\sin\left(dx+c\right)^{4}}-\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^{2}}-\frac{a\cos\left(dx+c\right)}{8d}-\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^3-1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^3-1/8/d*
a/sin(d*x+c)^2*cos(d*x+c)^3-1/8*a*cos(d*x+c)/d-1/8/d*a*ln(csc(d*x+c)-cot(d*
x+c))
```

maxima [A] time = 0.32, size = 80, normalized size = 1.08

$$-\frac{3\,a\left(\frac{2\,\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\,a}{\tan\left(dx+c\right)^{3}}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/48*(3*a*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x +
 c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 16*a/tan(d*x
+ c)^3)/d
```

mupad [B] time = 8.59, size = 112, normalized size = 1.51

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\cdots\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2))/(8*d) + (a*tan(c/2
 + (d*x)/2)^4)/(64*d) - (a*log(tan(c/2 + (d*x)/2)))/(8*d) - (cot(c/2 + (d*x
)/2)^4*(a/4 + (2*a*tan(c/2 + (d*x)/2))/3 - 2*a*tan(c/2 + (d*x)/2)^3))/(16*d
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.274 $\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=90

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] 1/8*a*arctanh(cos(d*x+c))/d-1/3*a*cot(d*x+c)^3/d-1/5*a*cot(d*x+c)^5/d+1/8*a
*cot(d*x+c)*csc(d*x+c)/d-1/4*a*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A] time = 0.13, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2607, 14, 2611, 3768, 3770}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^3)/(3*d) - (a*Cot[c + d*x
]^5)/(5*d) + (a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c +
d*x]^3)/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^3(c+dx)\,dx + a\int \cot^2(c+dx)\csc^4(c$$

$$= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}a\int \csc^3(c+dx)\,dx + \frac{a\,\mathrm{Su}}{}$$

$$= \frac{a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{8}a$$

$$= \frac{a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} +$$

Mathematica [A] time = 0.07, size = 177, normalized size = 1.97

$$\frac{2a\cot(c+dx)}{15d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Cot[c + d*x])/(15*d) + (a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x
)/2]^4)/(64*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*Cot[c + d*x]*C
```

```
sc[c + d*x]^4)/(5*d) + (a*Log[Cos[(c + d*x)/2]])/(8*d) - (a*Log[Sin[(c + d*
x)/2]])/(8*d) - (a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*
d)
```

fricas [B] time = 0.54, size = 169, normalized size = 1.88

$$\frac{32\,a\cos\left(dx+c\right)^{5}-80\,a\cos\left(dx+c\right)^{3}+15\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(\right.}{240\left(d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(32*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 15*(a*cos(d*x + c)^4 - 2
*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(a*cos
(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 30*(a*cos(d*x + c)^3 + a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.18, size = 144, normalized size = 1.60

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-120\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-60\,a\tan\left(\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*a*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(
1/2*d*x + 1/2*c)^3 - 120*a*log(abs(tan(1/2*d*x + 1/2*c))) - 60*a*tan(1/2*d*
x + 1/2*c) + (274*a*tan(1/2*d*x + 1/2*c)^5 + 60*a*tan(1/2*d*x + 1/2*c)^4 -
10*a*tan(1/2*d*x + 1/2*c)^2 - 15*a*tan(1/2*d*x + 1/2*c) - 6*a)/tan(1/2*d*x
+ 1/2*c)^5)/d
```

maple [A] time = 0.22, size = 124, normalized size = 1.38

$$-\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{4d\sin\left(dx+c\right)^{4}}-\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^{2}}-\frac{a\cos\left(dx+c\right)}{8d}-\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{8d}-\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)^{5}}-\frac{2a\left(\mathrm{co}\right.}{15d\,\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^3-1/8/d*a/sin(d*x+c)^2*cos(d*x+c)^3-1/8*a*
cos(d*x+c)/d-1/8/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/5/d*a/sin(d*x+c)^5*cos(d*x
+c)^3-2/15/d*a/sin(d*x+c)^3*cos(d*x+c)^3
```

maxima [A] time = 0.32, size = 92, normalized size = 1.02

$$-\frac{15\,a\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\left(5\,\tan(dx+c)^2+3\right)a}{\tan(dx+c)^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(15*a*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x
 + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 16*(5*tan(d
*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

mupad [B] time = 8.65, size = 143, normalized size = 1.59

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(-2\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^3)/(96*d) - (a*tan(c/2 + (d*x)/2))/(16*d) + (a*tan(c/
2 + (d*x)/2)^4)/(64*d) + (a*tan(c/2 + (d*x)/2)^5)/(160*d) - (a*log(tan(c/2
+ (d*x)/2)))/(8*d) - (cot(c/2 + (d*x)/2)^5*(a/5 + (a*tan(c/2 + (d*x)/2))/2
+ (a*tan(c/2 + (d*x)/2)^2)/3 - 2*a*tan(c/2 + (d*x)/2)^4))/(32*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.275 $\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=135

$$-\frac{a^2\cos^7(c+dx)}{7d}+\frac{3a^2\cos^5(c+dx)}{5d}-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{a^2\sin^3(c+dx)\cos^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

```
[Out] 1/8*a^2*x-2/3*a^2*cos(d*x+c)^3/d+3/5*a^2*cos(d*x+c)^5/d-1/7*a^2*cos(d*x+c)^
7/d+1/8*a^2*cos(d*x+c)*sin(d*x+c)/d-1/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-1/3*a
^2*cos(d*x+c)^3*sin(d*x+c)^3/d
```

Rubi [A] time = 0.25, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{a^2\cos^7(c+dx)}{7d}+\frac{3a^2\cos^5(c+dx)}{5d}-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{a^2\sin^3(c+dx)\cos^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*x)/8 - (2*a^2*Cos[c + d*x]^3)/(3*d) + (3*a^2*Cos[c + d*x]^5)/(5*d) - (
a^2*Cos[c + d*x]^7)/(7*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^2*Co
s[c + d*x]^3*Sin[c + d*x])/(4*d) - (a^2*Cos[c + d*x]^3*Sin[c + d*x]^3)/(3*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^2(c+dx)\sin^3(c+dx)+2a^2\cos^2(c+dx)\sin^4(c\right.$$

$$= a^2\int \cos^2(c+dx)\sin^3(c+dx)\,dx + a^2\int \cos^2(c+dx)\sin$$

$$= -\frac{a^2\cos^3(c+dx)\sin^3(c+dx)}{3d} + a^2\int \cos^2(c+dx)\sin^2(c$$

$$= -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{a^2\cos^3(c+dx)\sin^3(c+dx}{3d}$$

$$= -\frac{2a^2\cos^3(c+dx)}{3d} + \frac{3a^2\cos^5(c+dx)}{5d} - \frac{a^2\cos^7(c+dx)}{7d} +$$

$$= \frac{a^2x}{8} - \frac{2a^2\cos^3(c+dx)}{3d} + \frac{3a^2\cos^5(c+dx)}{5d} - \frac{a^2\cos^7(c+}{7d}$$

Mathematica [A] time = 0.50, size = 86, normalized size = 0.64

$$\frac{a^2(-210\sin(2(c+dx))-210\sin(4(c+dx))+70\sin(6(c+dx))-1365\cos(c+dx)-175\cos(3(c+dx))+147\text{co}}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(840*c + 840*d*x - 1365*Cos[c + d*x] - 175*Cos[3*(c + d*x)] + 147*Cos[
5*(c + d*x)] - 15*Cos[7*(c + d*x)] - 210*Sin[2*(c + d*x)] - 210*Sin[4*(c +
d*x)] + 70*Sin[6*(c + d*x)]))/(6720*d)
```

fricas [A] time = 0.54, size = 98, normalized size = 0.73

$$-\frac{120\,a^2\cos\left(dx+c\right)^7-504\,a^2\cos\left(dx+c\right)^5+560\,a^2\cos\left(dx+c\right)^3-105\,a^2dx-35\left(8\,a^2\cos\left(dx+c\right)^5-14\,a^2\cos\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/840*(120*a^2*cos(d*x + c)^7 - 504*a^2*cos(d*x + c)^5 + 560*a^2*cos(d*x +
 c)^3 - 105*a^2*d*x - 35*(8*a^2*cos(d*x + c)^5 - 14*a^2*cos(d*x + c)^3 + 3*
a^2*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.22, size = 123, normalized size = 0.91

$$\frac{1}{8}\,a^2x-\frac{a^2\cos\left(7\,dx+7\,c\right)}{448\,d}+\frac{7\,a^2\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{5\,a^2\cos\left(3\,dx+3\,c\right)}{192\,d}-\frac{13\,a^2\cos\left(dx+c\right)}{64\,d}+\frac{a^2\sin\left(6\,dx+6\,c\right)}{96\,d}-\frac{a^2\,\text{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*x - 1/448*a^2*cos(7*d*x + 7*c)/d + 7/320*a^2*cos(5*d*x + 5*c)/d - 5
/192*a^2*cos(3*d*x + 3*c)/d - 13/64*a^2*cos(d*x + c)/d + 1/96*a^2*sin(6*d*x
 + 6*c)/d - 1/32*a^2*sin(4*d*x + 4*c)/d - 1/32*a^2*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.17, size = 151, normalized size = 1.12

$$\frac{a^2\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^3(dx+c)\right)}{7}-\frac{4\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{35}-\frac{8\left(\cos^3(dx+c)\right)}{105}\right)+2a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c}{8}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*cos(d*x+c)^3-8/1
05*cos(d*x+c)^3)+2*a^2*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin
(d*x+c)+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+a^2*(-1/5*sin(d*x+c)^2*
cos(d*x+c)^3-2/15*cos(d*x+c)^3))
```

maxima [A] time = 0.33, size = 105, normalized size = 0.78

$$-\frac{32\left(15\cos\left(dx+c\right)^{7}-42\cos\left(dx+c\right)^{5}+35\cos\left(dx+c\right)^{3}\right)a^{2}-224\left(3\cos\left(dx+c\right)^{5}-5\cos\left(dx+c\right)^{3}\right)a^{2}+35}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3360*(32*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^2
 - 224*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2 + 35*(4*sin(2*d*x + 2*c)^3
 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^2)/d
```

mupad [B] time = 12.31, size = 331, normalized size = 2.45

$$\frac{a^2\,x}{8}-\frac{\frac{a^2\,\left(c+d\,x\right)}{8}+\frac{5\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-\frac{97\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{97\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{12}-\frac{5\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3}-\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{4}-\frac{a^2\,\left(105\,c+105\,d\,x\right.}{840}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*x)/8 - ((a^2*(c + d*x))/8 + (5*a^2*tan(c/2 + (d*x)/2)^3)/3 - (97*a^2*t
an(c/2 + (d*x)/2)^5)/12 + (97*a^2*tan(c/2 + (d*x)/2)^9)/12 - (5*a^2*tan(c/2
 + (d*x)/2)^11)/3 - (a^2*tan(c/2 + (d*x)/2)^13)/4 - (a^2*(105*c + 105*d*x -
 352))/840 + tan(c/2 + (d*x)/2)^2*((7*a^2*(c + d*x))/8 - (a^2*(735*c + 735*
d*x - 2464))/840) + tan(c/2 + (d*x)/2)^10*((21*a^2*(c + d*x))/8 - (a^2*(220
5*c + 2205*d*x - 3360))/840) + tan(c/2 + (d*x)/2)^4*((21*a^2*(c + d*x))/8 -
 (a^2*(2205*c + 2205*d*x - 4032))/840) + tan(c/2 + (d*x)/2)^6*((35*a^2*(c +
 d*x))/8 - (a^2*(3675*c + 3675*d*x + 2240))/840) + tan(c/2 + (d*x)/2)^8*((3
5*a^2*(c + d*x))/8 - (a^2*(3675*c + 3675*d*x - 14560))/840) + (a^2*tan(c/2
+ (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [A] time = 7.69, size = 275, normalized size = 2.04

$$\begin{cases} \frac{a^2x\sin^6(c+dx)}{8} + \frac{3a^2x\sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{3a^2x\sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{a^2x\cos^6(c+dx)}{8} + \frac{a^2\sin^5(c+dx)\cos(c+dx)}{8d} - \frac{a^2\sin^4(c+dx)}{3d} \\ x(a\sin(c)+a)^2\sin^3(c)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/8 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x)
**2/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + a**2*x*cos(c + d*x)**6
/8 + a**2*sin(c + d*x)**5*cos(c + d*x)/(8*d) - a**2*sin(c + d*x)**4*cos(c +
 d*x)**3/(3*d) - a**2*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 4*a**2*sin(c
+ d*x)**2*cos(c + d*x)**5/(15*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*
d) - a**2*sin(c + d*x)*cos(c + d*x)**5/(8*d) - 8*a**2*cos(c + d*x)**7/(105*
d) - 2*a**2*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)*
*3*cos(c)**2, True))
```

3.276 $\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=103

$$-\frac{a^2\cos^5(c+dx)}{10d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{3a^2x}{16}-\frac{\cos^3(c+dx)(a\sin(c+dx)+$$

[Out] 3/16*a^2*x-1/10*a^2*cos(d*x+c)^5/d+3/16*a^2*cos(d*x+c)*sin(d*x+c)/d+1/8*a^2*cos(d*x+c)^3*sin(d*x+c)/d-1/6*cos(d*x+c)^3*(a+a*sin(d*x+c))^3/a/d

Rubi [A] time = 0.17, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2870, 2669, 2635, 8}

$$-\frac{a^2\cos^5(c+dx)}{10d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{3a^2x}{16}-\frac{\cos^3(c+dx)(a\sin(c+dx)+$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]

[Out] (3*a^2*x)/16 - (a^2*Cos[c + d*x]^5)/(10*d) + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(6*a*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2870

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
```

```
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(2*b*f*g*(m + 1)), x] + Dist[a/(2*g^2)
, Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{\cos^3(c+dx)(a+a\sin(c+dx))^3}{6ad}+\frac{1}{2}a\int \cos^4(c+dx)(a+$$
$$= -\frac{a^2\cos^5(c+dx)}{10d}-\frac{\cos^3(c+dx)(a+a\sin(c+dx))^3}{6ad}+\frac{1}{2}a^2$$
$$= -\frac{a^2\cos^5(c+dx)}{10d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{8d}-\frac{\cos^3(c+$$
$$= -\frac{a^2\cos^5(c+dx)}{10d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^2\cos^3(c$$
$$= \frac{3a^2x}{16}-\frac{a^2\cos^5(c+dx)}{10d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^2}{}$$

Mathematica [A] time = 0.45, size = 76, normalized size = 0.74

$$\frac{a^2(-15\sin(2(c+dx))-45\sin(4(c+dx))+5\sin(6(c+dx))-240\cos(c+dx)-40\cos(3(c+dx))+24\cos(5(c+}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(180*c + 180*d*x - 240*Cos[c + d*x] - 40*Cos[3*(c + d*x)] + 24*Cos[5*(
c + d*x)] - 15*Sin[2*(c + d*x)] - 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)])
)/(960*d)
```

fricas [A] time = 0.51, size = 85, normalized size = 0.83

$$\frac{96\,a^2\cos(dx+c)^5-160\,a^2\cos(dx+c)^3+45\,a^2dx+5\left(8\,a^2\cos(dx+c)^5-26\,a^2\cos(dx+c)^3+9\,a^2\cos(dx+c\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/240*(96*a^2*cos(d*x + c)^5 - 160*a^2*cos(d*x + c)^3 + 45*a^2*d*x + 5*(8*a
^2*cos(d*x + c)^5 - 26*a^2*cos(d*x + c)^3 + 9*a^2*cos(d*x + c))*sin(d*x + c
))/d
```

giac [A] time = 0.19, size = 106, normalized size = 1.03

$$\frac{3}{16}\,a^2x+\frac{a^2\cos\left(5\,dx+5\,c\right)}{40\,d}-\frac{a^2\cos\left(3\,dx+3\,c\right)}{24\,d}-\frac{a^2\cos\left(dx+c\right)}{4\,d}+\frac{a^2\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{3\,a^2\sin\left(4\,dx+4\,c\right)}{64\,d}-\frac{a^2\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/16*a^2*x + 1/40*a^2*cos(5*d*x + 5*c)/d - 1/24*a^2*cos(3*d*x + 3*c)/d - 1/
4*a^2*cos(d*x + c)/d + 1/192*a^2*sin(6*d*x + 6*c)/d - 3/64*a^2*sin(4*d*x +
4*c)/d - 1/64*a^2*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.17, size = 142, normalized size = 1.38

$$\frac{a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{8}+\frac{\cos(dx+c)\sin(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+2a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2(\mathrm{co}}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*c
os(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+2*a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3
-2/15*cos(d*x+c)^3)+a^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*
x+c)+1/8*d*x+1/8*c))
```

maxima [A] time = 0.33, size = 93, normalized size = 0.90

$$\frac{128\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)a^2-5\left(4\,\sin\left(2\,dx+2\,c\right)^3-12\,dx-12\,c+3\,\sin\left(4\,dx+4\,c\right)\right)a^2+30\left(4\,d\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/960*(128*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2 - 5*(4*sin(2*d*x + 2*c
)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^2 + 30*(4*d*x + 4*c - sin(4*d*x
 + 4*c))*a^2)/d
```

mupad [B] time = 12.16, size = 257, normalized size = 2.50

$$\frac{3\,a^2\,x}{16}-\frac{\frac{3\,a^2\,(c+d\,x)}{16}-\frac{13\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24}-\frac{25\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{25\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{13\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{24}-\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\frac{a^2\,(45\,c+4}{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/16 - ((3*a^2*(c + d*x))/16 - (13*a^2*tan(c/2 + (d*x)/2)^3)/24 - (
25*a^2*tan(c/2 + (d*x)/2)^5)/4 + (25*a^2*tan(c/2 + (d*x)/2)^7)/4 + (13*a^2*
tan(c/2 + (d*x)/2)^9)/24 - (3*a^2*tan(c/2 + (d*x)/2)^11)/8 - (a^2*(45*c + 4
5*d*x - 128))/240 + tan(c/2 + (d*x)/2)^2*((9*a^2*(c + d*x))/8 - (a^2*(270*c
 + 270*d*x - 768))/240) + tan(c/2 + (d*x)/2)^6*((15*a^2*(c + d*x))/4 - (a^2
*(900*c + 900*d*x - 1280))/240) + tan(c/2 + (d*x)/2)^8*((45*a^2*(c + d*x))/
16 - (a^2*(675*c + 675*d*x - 1920))/240) + (3*a^2*tan(c/2 + (d*x)/2))/8)/(d
*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [A] time = 4.79, size = 309, normalized size = 3.00

$$\begin{cases} \frac{a^2 x \sin^6(c+dx)}{16} + \frac{3a^2 x \sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{a^2 x \sin^4(c+dx)}{8} + \frac{3a^2 x \sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{a^2 x \sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{a^2 x \cos^6(c+ \\ x(a\sin(c)+a)^2 \sin^2(c)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/16 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)*
*4/16 + a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + a**2*x*cos(c + d*x)**6/1
6 + a**2*x*cos(c + d*x)**4/8 + a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) - a
**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + a**2*sin(c + d*x)**3*cos(c + d*
x)/(8*d) - 2*a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - a**2*sin(c + d*x)
*cos(c + d*x)**5/(16*d) - a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 4*a**2*
cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**2*cos(c)**2
, True))
```

3.277 $\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=91

$$\frac{a^2\cos^5(c+dx)}{5d}-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{a^2x}{4}$$

```
[Out] 1/4*a^2*x-2/3*a^2*cos(d*x+c)^3/d+1/5*a^2*cos(d*x+c)^5/d+1/4*a^2*cos(d*x+c)*
sin(d*x+c)/d-1/2*a^2*cos(d*x+c)^3*sin(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{a^2\cos^3(c+dx)}{6d}-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{10d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{a^2x}{4}-\frac{\cos^3(c+dx)(a\sin(c+a}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*x)/4 - (a^2*Cos[c + d*x]^3)/(6*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(4
*d) - (Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2)/(5*d) - (Cos[c + d*x]^3*(a^2
+ a^2*Sin[c + d*x]))/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}+\frac{2}{5}\int \cos^2(c+dx)(a+a$$

$$= -\frac{\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}-\frac{\cos^3(c+dx)\left(a^2+a^2\sin\right.}{10d}$$

$$= -\frac{a^2\cos^3(c+dx)}{6d}-\frac{\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}-\frac{\cos^3(}{}$$

$$= -\frac{a^2\cos^3(c+dx)}{6d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{4d}-\frac{\cos^3(c+dx}{}$$

$$= \frac{a^2x}{4}-\frac{a^2\cos^3(c+dx)}{6d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{4d}-\frac{\cos^3(c}{}$$

Mathematica [A] time = 0.20, size = 57, normalized size = 0.63

$$\frac{a^2(-90\cos(c+dx)-25\cos(3(c+dx))+3(-5\sin(4(c+dx))+\cos(5(c+dx))+20c+20dx))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-90*Cos[c + d*x] - 25*Cos[3*(c + d*x)] + 3*(20*c + 20*d*x + Cos[5*(c
+ d*x)] - 5*Sin[4*(c + d*x)])))/(240*d)
```

fricas [A] time = 0.51, size = 72, normalized size = 0.79

$$\frac{12\,a^{2}\cos\left(dx+c\right)^{5}-40\,a^{2}\cos\left(dx+c\right)^{3}+15\,a^{2}dx-15\left(2\,a^{2}\cos\left(dx+c\right)^{3}-a^{2}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*a^2*cos(d*x + c)^5 - 40*a^2*cos(d*x + c)^3 + 15*a^2*d*x - 15*(2*a^
2*cos(d*x + c)^3 - a^2*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.17, size = 72, normalized size = 0.79

$$\frac{1}{4}\,a^{2}x+\frac{a^{2}\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{5\,a^{2}\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{3\,a^{2}\cos\left(dx+c\right)}{8\,d}-\frac{a^{2}\sin\left(4\,dx+4\,c\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*a^2*x + 1/80*a^2*cos(5*d*x + 5*c)/d - 5/48*a^2*cos(3*d*x + 3*c)/d - 3/8
*a^2*cos(d*x + c)/d - 1/16*a^2*sin(4*d*x + 4*c)/d
```

maple [A] time = 0.16, size = 95, normalized size = 1.04

$$\frac{a^{2}\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{3}(dx+c)\right)}{5}-\frac{2\left(\cos^{3}(dx+c)\right)}{15}\right)+2a^{2}\left(-\frac{\left(\cos^{3}(dx+c)\right)\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{a^{2}\left(\cos^{3}(dx+c)\right)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+2*a^2*(-1/4*cos
(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-1/3*a^2*cos(d
*x+c)^3)
```

maxima [A] time = 0.34, size = 69, normalized size = 0.76

$$-\frac{80\,a^{2}\cos\left(dx+c\right)^{3}-16\left(3\,\cos\left(dx+c\right)^{5}-5\,\cos\left(dx+c\right)^{3}\right)a^{2}-15\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^{2}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(80*a^2*cos(d*x + c)^3 - 16*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^
2 - 15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2)/d
```

mupad [B] time = 12.13, size = 262, normalized size = 2.88

$$\frac{a^2 x}{4} - \frac{\frac{a^2 (c+d x)}{4} - 3 a^2 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^3 + 3 a^2 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^7 - \frac{a^2 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^9}{2} - \frac{a^2 (15 c+15 d x-56)}{60} + \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^8 \left(\frac{5 a^2 (c+\cdots}{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*x)/4 - ((a^2*(c + d*x))/4 - 3*a^2*tan(c/2 + (d*x)/2)^3 + 3*a^2*tan(c/2
 + (d*x)/2)^7 - (a^2*tan(c/2 + (d*x)/2)^9)/2 - (a^2*(15*c + 15*d*x - 56))/6
0 + tan(c/2 + (d*x)/2)^8*((5*a^2*(c + d*x))/4 - (a^2*(75*c + 75*d*x - 120))
/60) + tan(c/2 + (d*x)/2)^2*((5*a^2*(c + d*x))/4 - (a^2*(75*c + 75*d*x - 16
0))/60) + tan(c/2 + (d*x)/2)^4*((5*a^2*(c + d*x))/2 - (a^2*(150*c + 150*d*x
 - 80))/60) + tan(c/2 + (d*x)/2)^6*((5*a^2*(c + d*x))/2 - (a^2*(150*c + 150
*d*x - 480))/60) + (a^2*tan(c/2 + (d*x)/2))/2)/(d*(tan(c/2 + (d*x)/2)^2 + 1
)^5)
```

sympy [A] time = 2.48, size = 172, normalized size = 1.89

$$\begin{cases} \frac{a^2 x \sin^4(c+dx)}{4} + \frac{a^2 x \sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{a^2 x \cos^4(c+dx)}{4} + \frac{a^2 \sin^3(c+dx)\cos(c+dx)}{4d} - \frac{a^2 \sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{a^2 \sin(c+dx)\cos^3\cdots}{4d} \\ x (a \sin(c) + a)^2 \sin(c) \cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**4/4 + a**2*x*sin(c + d*x)**2*cos(c + d*x)**
2/2 + a**2*x*cos(c + d*x)**4/4 + a**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) -
a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - a**2*sin(c + d*x)*cos(c + d*x)
**3/(4*d) - 2*a**2*cos(c + d*x)**5/(15*d) - a**2*cos(c + d*x)**3/(3*d), Ne(
d, 0)), (x*(a*sin(c) + a)**2*sin(c)*cos(c)**2, True))
```

3.278 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=71

$$-\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+a^2x$$

```
[Out] a^2*x-a^2*arctanh(cos(d*x+c))/d+a^2*cos(d*x+c)/d-1/3*a^2*cos(d*x+c)^3/d+a^2
*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2873, 2635, 8, 2592, 321, 206, 2565, 30}

$$-\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*x - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d - (a^2*Cos[c +
 d*x]^3)/(3*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(2a^2\cos^2(c+dx) + a^2\cos(c+dx)\cot(c+dx) + a^2\cos^2(\right.$$

$$= a^2\int \cos(c+dx)\cot(c+dx)\,dx + a^2\int \cos^2(c+dx)\sin(c+$$

$$= \frac{a^2\cos(c+dx)\sin(c+dx)}{d} + a^2\int 1\,dx - \frac{a^2\,\mathrm{Subst}\left(\int x^2\,dx,\right.}{d}$$

$$= a^2x + \frac{a^2\cos(c+dx)}{d} - \frac{a^2\cos^3(c+dx)}{3d} + \frac{a^2\cos(c+dx)\sin}{d}$$

$$= a^2x - \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^2\cos(c+dx)}{d} - \frac{a^2\cos^3(c}{3d}$$

Mathematica [A] time = 0.38, size = 71, normalized size = 1.00

$$\frac{a^2\left(9\cos(c+dx) - \cos(3(c+dx)) + 6\left(\sin(2(c+dx)) + 2\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + c + \right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(9*Cos[c + d*x] - Cos[3*(c + d*x)] + 6*(2*(c + d*x - Log[Cos[(c + d*x)
/2]] + Log[Sin[(c + d*x)/2]]) + Sin[2*(c + d*x)])))/(12*d)
```

fricas [A] time = 0.53, size = 86, normalized size = 1.21

$$-\frac{2\,a^2\cos(dx+c)^3 - 6\,a^2dx - 6\,a^2\cos(dx+c)\sin(dx+c) - 6\,a^2\cos(dx+c) + 3\,a^2\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) -}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a^2*cos(d*x + c)^3 - 6*a^2*d*x - 6*a^2*cos(d*x + c)*sin(d*x + c) -
6*a^2*cos(d*x + c) + 3*a^2*log(1/2*cos(d*x + c) + 1/2) - 3*a^2*log(-1/2*cos
(d*x + c) + 1/2))/d
```

giac [A] time = 0.17, size = 101, normalized size = 1.42

$$\frac{3\,(dx+c)a^2 + 3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - \frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^2 + 3*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(3*a^2*tan(
1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2
*c) - 2*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.34, size = 86, normalized size = 1.21

$$-\frac{a^2 \left(\cos^3 (dx+c)\right)}{3d}+\frac{a^2 \cos (dx+c) \sin (dx+c)}{d}+a^2x+\frac{a^2c}{d}+\frac{a^2 \ln (\csc (dx+c)-\cot (dx+c))}{d}+\frac{a^2 \cos (dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/3*a^2*cos(d*x+c)^3/d+a^2*cos(d*x+c)*sin(d*x+c)/d+a^2*x+1/d*a^2*c+1/d*a^2
*ln(csc(d*x+c)-cot(d*x+c))+a^2*cos(d*x+c)/d
```

maxima [A] time = 0.35, size = 75, normalized size = 1.06

$$-\frac{2\,a^2 \cos (dx+c)^3 - 3\,(2\,dx+2\,c+\sin (2\,dx+2\,c))a^2 - 3\,a^2\Big(2\,\cos (dx+c) - \log (\cos (dx+c)+1) + \log (\cos (d}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^2*cos(d*x + c)^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 3*a^2
*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.04, size = 188, normalized size = 2.65

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{-2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{4\,a^2}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{2\,a^2\,\mathrm{atan}\left(\frac{4\,a^4}{4\,a^4-4\,a^4\,\tan(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (4*a^2*tan(c/2 + (d*x)/2)^2 - 2*a^2*tan(c
/2 + (d*x)/2)^5 + (4*a^2)/3 + 2*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*
x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (2*a^2*atan
```

```
((4*a^4)/(4*a^4 - 4*a^4*tan(c/2 + (d*x)/2)) + (4*a^4*tan(c/2 + (d*x)/2))/(4
*a^4 - 4*a^4*tan(c/2 + (d*x)/2))))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int \cos^2(c+dx)\csc(c+dx)\,dx + \int 2\sin(c+dx)\cos^2(c+dx)\csc(c+dx)\,dx + \int \sin^2(c+dx)\cos^2(c+d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(cos(c + d*x)**2*csc(c + d*x), x) + Integral(2*sin(c + d*x)*c
os(c + d*x)**2*csc(c + d*x), x) + Integral(sin(c + d*x)**2*cos(c + d*x)**2*
csc(c + d*x), x))
```

3.279 $\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=74

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2x}{2}$$

```
[Out] -1/2*a^2*x-2*a^2*arctanh(cos(d*x+c))/d+2*a^2*cos(d*x+c)/d-a^2*cot(d*x+c)/d+
1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 3770, 3767, 8, 2638, 2635}

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x)/2 - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d - (a^
2*Cot[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
```

```
p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(2a^4\csc(c+dx)+a^4\csc^2(c+dx)-2a^4\sin(c+dx)-a^4\sin^2(c+dx\right.}{a^2}$$
$$= a^2\int \csc^2(c+dx)\,dx - a^2\int \sin^2(c+dx)\,dx + \left(2a^2\right)\int \csc(c+dx)\,dx -$$
$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+}{2d}$$
$$= -\frac{a^2x}{2} - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} - \frac{a^2\cot(c+dx)}{d} +$$

Mathematica [A] time = 0.59, size = 94, normalized size = 1.27

$$-\frac{a^2\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(7\cos(c+dx)+\cos(3(c+dx))+4\sin(c+dx)\left(-4\cos(c+dx)-4\log\left(\sin\right.\right.\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/16*(a^2*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(7*Cos[c + d*x] + Cos[3*(c + d
*x)] + 4*(c + d*x - 4*Cos[c + d*x] + 4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c
 + d*x)/2]])*Sin[c + d*x]))/d
```

fricas [A] time = 0.54, size = 105, normalized size = 1.42

$$-\frac{a^2\cos(dx+c)^3 + 2a^2\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 2a^2\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) + a^2c}{2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(a^2*cos(d*x + c)^3 + 2*a^2*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 2*a^2*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + a^2*cos(d*x + c) + (a^2*
d*x - 4*a^2*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

giac [B] time = 0.18, size = 143, normalized size = 1.93

$$-\frac{(dx+c)a^2 - 4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} + \frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 4\,a^2\tan\left(\frac{1}{2}\,d\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)*a^2 - 4*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - a^2*tan(1/2*d*
x + 1/2*c) + (4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c) + 2*(a
^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*x
+ 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.29, size = 89, normalized size = 1.20

$$\frac{a^2\cos(dx+c)\sin(dx+c)}{2d} - \frac{a^2x}{2} - \frac{a^2c}{2d} + \frac{2a^2\cos(dx+c)}{d} + \frac{2a^2\ln(\csc(dx+c)-\cot(dx+c))}{d} - \frac{a^2\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*cos(d*x+c)*sin(d*x+c)/d-1/2*a^2*x-1/2/d*a^2*c+2*a^2*cos(d*x+c)/d+2/
d*a^2*ln(csc(d*x+c)-cot(d*x+c))-a^2*cot(d*x+c)/d
```

maxima [A] time = 0.44, size = 79, normalized size = 1.07

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2 - 4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2 + 4\,a^2\left(2\,\cos(dx+c) - \log(\cos(dx+c)+1) + \log(\mathrm{co}\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 4*(d*x + c + 1/tan(d*x + c))*a^
2 + 4*a^2*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))
/d
```

mupad [B] time = 8.81, size = 201, normalized size = 2.72

$$\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{-3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+8\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a^2}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{a^2\,\mathrm{atan}\left(\frac{a^4}{4\,a^4+a^4\tan\left(\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (2*a^2*log(tan(c/2 + (d*x)/2)))/d + (8*a^2*tan(c/2 + (d*x)/2)^3 - 3*a^2*tan
(c/2 + (d*x)/2)^4 - a^2 + 8*a^2*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2
) + 4*tan(c/2 + (d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^5)) + (a^2*atan(a^4/(4*a^
4 + a^4*tan(c/2 + (d*x)/2)) - (4*a^4*tan(c/2 + (d*x)/2))/(4*a^4 + a^4*tan(c
/2 + (d*x)/2))))/d + (a^2*tan(c/2 + (d*x)/2))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\cos^2(c+dx)\csc^2(c+dx)\,dx+\int 2\sin(c+dx)\cos^2(c+dx)\csc^2(c+dx)\,dx+\int\sin^2(c+dx)\cos^2(c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(cos(c + d*x)**2*csc(c + d*x)**2, x) + Integral(2*sin(c + d*x
)*cos(c + d*x)**2*csc(c + d*x)**2, x) + Integral(sin(c + d*x)**2*cos(c + d*
x)**2*csc(c + d*x)**2, x))
```

3.280 $\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=73

$$\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-2a^2x$$

```
[Out] -2*a^2*x-1/2*a^2*arctanh(cos(d*x+c))/d+a^2*cos(d*x+c)/d-2*a^2*cot(d*x+c)/d-
1/2*a^2*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2872, 3767, 8, 3768, 3770, 2638}

$$\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -2*a^2*x - (a^2*ArcTanh[Cos[c + d*x]])/(2*d) + (a^2*Cos[c + d*x])/d - (2*a^
2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-2a^4+2a^4\csc^2(c+dx)+a^4\csc^3(c+dx)-a^4\sin(c+d\right.}{a^2}$$
$$= -2a^2x+a^2\int \csc^3(c+dx)\,dx-a^2\int \sin(c+dx)\,dx+\left(2a^2\right.$$
$$= -2a^2x+\frac{a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{1}{2}a^2$$
$$= -2a^2x-\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}+\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot}{}$$

Mathematica [A] time = 0.67, size = 102, normalized size = 1.40

$$\frac{a^2\left(8\cos(c+dx)+8\tan\left(\frac{1}{2}(c+dx)\right)-8\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)+4\log\left(\sin\left(\frac{1}{2}(c\right.\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-16*c - 16*d*x + 8*Cos[c + d*x] - 8*Cot[(c + d*x)/2] - Csc[(c + d*x)/
2]^2 - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]
^2 + 8*Tan[(c + d*x)/2]))/(8*d)
```

fricas [B] time = 0.53, size = 143, normalized size = 1.96

$$-\frac{8a^2dx\cos(dx+c)^2-4a^2\cos(dx+c)^3-8a^2dx-8a^2\cos(dx+c)\sin(dx+c)+2a^2\cos(dx+c)+\left(a^2\cos(\right.}{4\left(d\cos(dx+c)^2-d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/4*(8*a^2*d*x*cos(d*x + c)^2 - 4*a^2*cos(d*x + c)^3 - 8*a^2*d*x - 8*a^2*c
os(d*x + c)*sin(d*x + c) + 2*a^2*cos(d*x + c) + (a^2*cos(d*x + c)^2 - a^2)*
log(1/2*cos(d*x + c) + 1/2) - (a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x +
 c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.19, size = 128, normalized size = 1.75

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,(dx+c)a^2+4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{16\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\frac{6\,a^2\,\mathrm{t}}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 - 16*(d*x + c)*a^2 + 4*a^2*log(abs(tan(1/2*
d*x + 1/2*c))) + 8*a^2*tan(1/2*d*x + 1/2*c) + 16*a^2/(tan(1/2*d*x + 1/2*c)^
2 + 1) - (6*a^2*tan(1/2*d*x + 1/2*c)^2 + 8*a^2*tan(1/2*d*x + 1/2*c) + a^2)/
tan(1/2*d*x + 1/2*c)^2)/d
```

maple [A] time = 0.35, size = 93, normalized size = 1.27

$$\frac{a^2\cos(dx+c)}{2d}+\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d}-2a^2x-\frac{2a^2\cot(dx+c)}{d}-\frac{2a^2c}{d}-\frac{a^2\left(\cos^3(dx+c)\right)}{2d\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*cos(d*x+c)/d+1/2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2*a^2*x-2*a^2*cot(
d*x+c)/d-2/d*a^2*c-1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3
```

maxima [A] time = 0.58, size = 104, normalized size = 1.42

$$-\frac{8\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2-a^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)-2\,a^2\left(2\cos(dx+c)\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

[Out] -1/4*(8*(d*x + c + 1/tan(d*x + c))*a^2 - a^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) - 2*a^2*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d

mupad [B] time = 8.78, size = 213, normalized size = 2.92

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d}-\frac{4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\frac{15a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}+4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{a^2}{2}}{d\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}+\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}+\frac{4a^2\,\mathrm{ata}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^2)/sin(c + d*x)^3,x)

[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) - (4*a^2*tan(c/2 + (d*x)/2)^3 - (15*a^2*tan(c/2 + (d*x)/2)^2)/2 + a^2/2 + 4*a^2*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 4*tan(c/2 + (d*x)/2)^4)) + (a^2*log(tan(c/2 + (d*x)/2)))/(2*d) + (4*a^2*atan((16*a^4)/(4*a^4 + 16*a^4*tan(c/2 + (d*x)/2)) - (4*a^4*tan(c/2 + (d*x)/2))/(4*a^4 + 16*a^4*tan(c/2 + (d*x)/2))))/d + (a^2*tan(c/2 + (d*x)/2))/d

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)

[Out] Timed out

3.281 $\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=73

$$-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}-a^2x$$

```
[Out] -a^2*x+a^2*arctanh(cos(d*x+c))/d-a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d-a^
2*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A] time = 0.22, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 3473, 8, 2611, 3770, 2607, 30}

$$-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (a^2*ArcTanh[Cos[c + d*x]])/d - (a^2*Cot[c + d*x])/d - (a^2*Cot[
c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(a^2\cot^2(c+dx)+2a^2\cot^2(c+dx)\csc(c+dx)+a^2\cot\right.\\ &= a^2\int \cot^2(c+dx)\,dx+a^2\int \cot^2(c+dx)\csc^2(c+dx)\,dx+\\ &= -\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}-a^2\int 1\,dx-\\ &= -a^2x+\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(}{3}\end{aligned}$$

Mathematica [A] time = 0.58, size = 140, normalized size = 1.92

$$-\frac{a^2\left(-8\tan\left(\frac{1}{2}(c+dx)\right)+8\cot\left(\frac{1}{2}(c+dx)\right)+6\csc^2\left(\frac{1}{2}(c+dx)\right)-6\sec^2\left(\frac{1}{2}(c+dx)\right)+24\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/24*(a^2*(24*c + 24*d*x + 8*Cot[(c + d*x)/2] + 6*Csc[(c + d*x)/2]^2 - 24*
Log[Cos[(c + d*x)/2]] + 24*Log[Sin[(c + d*x)/2]] - 6*Sec[(c + d*x)/2]^2 - 8
```

```
*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + (Csc[(c + d*x)/2]^4*Sin[c + d*x])/2 -
8*Tan[(c + d*x)/2]))/d
```

fricas [B] time = 0.53, size = 166, normalized size = 2.27

$$-\frac{4\,a^2\cos\left(dx+c\right)^3-6\,a^2\cos\left(dx+c\right)-3\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\left(a^2\cos\right.}{6\left(d\cos\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/6*(4*a^2*cos(d*x + c)^3 - 6*a^2*cos(d*x + c) - 3*(a^2*cos(d*x + c)^2 - a
^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(a^2*cos(d*x + c)^2 - a^2)
*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*(a^2*d*x*cos(d*x + c)^2 - a^
2*d*x - a^2*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c
))
```

giac [A] time = 0.20, size = 141, normalized size = 1.93

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\left(dx+c\right)a^2-24\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*(d*x +
 c)*a^2 - 24*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 9*a^2*tan(1/2*d*x + 1/2*c
) + (44*a^2*tan(1/2*d*x + 1/2*c)^3 - 9*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*t
an(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.32, size = 117, normalized size = 1.60

$$-a^2x-\frac{a^2\cot\left(dx+c\right)}{d}-\frac{a^2c}{d}-\frac{a^2\left(\cos^3\left(dx+c\right)\right)}{d\sin\left(dx+c\right)^2}-\frac{a^2\cos\left(dx+c\right)}{d}-\frac{a^2\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}-\frac{a^2\left(\cos^3\left(dx+\right.\right.}{3d\sin\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -a^2*x-a^2*cot(d*x+c)/d-1/d*a^2*c-1/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3-a^2*cos
(d*x+c)/d-1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*a^2/sin(d*x+c)^3*cos(d*x+
c)^3
```

maxima [A] time = 0.42, size = 83, normalized size = 1.14

$$-\frac{6\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2-3\,a^2\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{2\,a^2}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/6*(6*(d*x + c + 1/tan(d*x + c))*a^2 - 3*a^2*(2*cos(d*x + c)/(cos(d*x + c
)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 2*a^2/tan(d*x +
 c)^3)/d
```

mupad [B] time = 8.94, size = 193, normalized size = 2.64

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{2\,a^2\,\mathrm{atan}\left(\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a^2\,\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(4*d) - (a^2*cot(c/2 + (d*x)/2)^3)/(24*d) - (a^2
*cot(c/2 + (d*x)/2)^2)/(4*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (2*a^2*a
tan((cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2) - sin(c/2
 + (d*x)/2))))/d - (a^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (3*
a^2*cot(c/2 + (d*x)/2))/(8*d) + (3*a^2*tan(c/2 + (d*x)/2))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.282 $\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=82

$$-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] 5/8*a^2*arctanh(cos(d*x+c))/d-2/3*a^2*cot(d*x+c)^3/d-3/8*a^2*cot(d*x+c)*csc
(d*x+c)/d-1/4*a^2*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A] time = 0.20, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3770, 2607, 30, 3768}

$$-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (2*a^2*Cot[c + d*x]^3)/(3*d) - (3*a^2
*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^2(c+dx)\csc(c+dx)+2a^2\cot^2(c+dx)\csc^2(c+\right.$$
$$= a^2\int \cot^2(c+dx)\csc(c+dx)\,dx + a^2\int \cot^2(c+dx)\csc^3($$
$$= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d} - \frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}$$
$$= \frac{a^2\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a^2\cot^3(c+dx)}{3d} - \frac{3a^2\cot(c+d}{8}$$
$$= \frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{2a^2\cot^3(c+dx)}{3d} - \frac{3a^2\cot(c+}{}$$

Mathematica [B] time = 0.11, size = 209, normalized size = 2.55

$$a^2\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{3d}+\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{3d}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{3\sec^2\left(\frac{1}{2}\right.}{32}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*(Cot[(c + d*x)/2]/(3*d) - (3*Csc[(c + d*x)/2]^2)/(32*d) - (Cot[(c + d*x
)/2]*Csc[(c + d*x)/2]^2)/(12*d) - Csc[(c + d*x)/2]^4/(64*d) + (5*Log[Cos[(c
```

```
 + d*x)/2]])/(8*d) - (5*Log[Sin[(c + d*x)/2]])/(8*d) + (3*Sec[(c + d*x)/2]^
2)/(32*d) + Sec[(c + d*x)/2]^4/(64*d) - Tan[(c + d*x)/2]/(3*d) + (Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/(12*d))
```

fricas [B] time = 0.51, size = 155, normalized size = 1.89

$$-\frac{32\,a^2\cos\left(dx+c\right)^3\sin\left(dx+c\right)-18\,a^2\cos\left(dx+c\right)^3+30\,a^2\cos\left(dx+c\right)-15\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right.\right.}{48\left(d\cos\left(dx+c\right)^4-2\,\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(32*a^2*cos(d*x + c)^3*sin(d*x + c) - 18*a^2*cos(d*x + c)^3 + 30*a^2*
cos(d*x + c) - 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2
*cos(d*x + c) + 1/2) + 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)
*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [B] time = 0.24, size = 164, normalized size = 2.00

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-48\,a^2}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*a^
2*tan(1/2*d*x + 1/2*c)^2 - 120*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 48*a^2*
tan(1/2*d*x + 1/2*c) + (250*a^2*tan(1/2*d*x + 1/2*c)^4 + 48*a^2*tan(1/2*d*x
 + 1/2*c)^3 - 24*a^2*tan(1/2*d*x + 1/2*c)^2 - 16*a^2*tan(1/2*d*x + 1/2*c) -
 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.31, size = 112, normalized size = 1.37

$$-\frac{5a^2\left(\cos^3\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^2}-\frac{5a^2\cos\left(dx+c\right)}{8d}-\frac{5a^2\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{8d}-\frac{2a^2\left(\cos^3\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)^3}-\frac{a^2\left(\cos^3\left(dx+c\right.\right.}{4d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -5/8/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3-5/8*a^2*cos(d*x+c)/d-5/8/d*a^2*ln(csc(
d*x+c)-cot(d*x+c))-2/3/d*a^2/sin(d*x+c)^3*cos(d*x+c)^3-1/4/d*a^2/sin(d*x+c)
^4*cos(d*x+c)^3
```

maxima [A] time = 0.34, size = 130, normalized size = 1.59

$$-\frac{3\,a^2\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-12\,a^2\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx\right.\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/48*(3*a^2*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x
 + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 12*a^2*(2*c
os(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c)
 - 1)) + 32*a^2/tan(d*x + c)^3)/d
```

mupad [B] time = 8.62, size = 161, normalized size = 1.96

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{5\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(-4\,a^2\tan\left(\frac{c}{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(12*d) + (a^2
*tan(c/2 + (d*x)/2)^4)/(64*d) - (5*a^2*log(tan(c/2 + (d*x)/2)))/(8*d) - (co
t(c/2 + (d*x)/2)^4*(2*a^2*tan(c/2 + (d*x)/2)^2 - 4*a^2*tan(c/2 + (d*x)/2)^3
 + a^2/4 + (4*a^2*tan(c/2 + (d*x)/2))/3))/(16*d) - (a^2*tan(c/2 + (d*x)/2))
/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.283 $\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=100

$$-\frac{a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{2d}+\frac{a^2\cot(c+dx)\csc(c+}{4d}$$

```
[Out] 1/4*a^2*arctanh(cos(d*x+c))/d-2/3*a^2*cot(d*x+c)^3/d-1/5*a^2*cot(d*x+c)^5/d
+1/4*a^2*cot(d*x+c)*csc(d*x+c)/d-1/2*a^2*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A] time = 0.21, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2607, 30, 2611, 3768, 3770, 14}

$$-\frac{a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{2d}+\frac{a^2\cot(c+dx)\csc(c+}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Cos[c + d*x]])/(4*d) - (2*a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot
[c + d*x]^5)/(5*d) + (a^2*Cot[c + d*x]*Csc[c + d*x])/(4*d) - (a^2*Cot[c + d
*x]*Csc[c + d*x]^3)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx = \int \Big(a^2\cot^2(c+dx)\csc^2(c+dx)+2a^2\cot^2(c+dx)\csc^3(c+$$
$$= a^2\int \cot^2(c+dx)\csc^2(c+dx)\,dx + a^2\int \cot^2(c+dx)\csc^4$$
$$= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{2d} - \frac{1}{2}a^2\int \csc^3(c+dx)\,dx + \frac{a^2}{}$$
$$= -\frac{a^2\cot^3(c+dx)}{3d} + \frac{a^2\cot(c+dx)\csc(c+dx)}{4d} - \frac{a^2\cot(c+}{}$$
$$= \frac{a^2\tanh^{-1}(\cos(c+dx))}{4d} - \frac{2a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+d}{5d}$$

Mathematica [A] time = 0.75, size = 189, normalized size = 1.89

$$-\frac{a^2\csc^5(c+dx)\Big(180\sin(2(c+dx))+30\sin(4(c+dx))+200\cos(c+dx)+20\cos(3(c+dx))-28\cos(5(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/960*(a^2*Csc[c + d*x]^5*(200*Cos[c + d*x] + 20*Cos[3*(c + d*x)] - 28*Cos
[5*(c + d*x)] - 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 150*Log[Sin[(c + d
*x)/2]]*Sin[c + d*x] + 180*Sin[2*(c + d*x)] + 75*Log[Cos[(c + d*x)/2]]*Sin[
3*(c + d*x)] - 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 30*Sin[4*(c + d*
x)] - 15*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*Log[Sin[(c + d*x)/2]]*
Sin[5*(c + d*x)]))/d
```

fricas [B] time = 0.52, size = 189, normalized size = 1.89

$$\frac{56\,a^2\cos\left(dx+c\right)^5-80\,a^2\cos\left(dx+c\right)^3+15\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{120\left(d\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(56*a^2*cos(d*x + c)^5 - 80*a^2*cos(d*x + c)^3 + 15*(a^2*cos(d*x + c)
^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) - 30*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c))*sin(d*x +
 c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.22, size = 164, normalized size = 1.64

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+25\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-90\,a^2}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a^2*tan(1/2*d*x + 1/2*c)^4 + 25*a^
2*tan(1/2*d*x + 1/2*c)^3 - 120*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 90*a^2*
tan(1/2*d*x + 1/2*c) + (274*a^2*tan(1/2*d*x + 1/2*c)^5 + 90*a^2*tan(1/2*d*x
 + 1/2*c)^4 - 25*a^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c) -
 3*a^2)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.31, size = 136, normalized size = 1.36

$$-\frac{7a^2\left(\cos^3\left(dx+c\right)\right)}{15d\sin\left(dx+c\right)^3}-\frac{a^2\left(\cos^3\left(dx+c\right)\right)}{2d\sin\left(dx+c\right)^4}-\frac{a^2\left(\cos^3\left(dx+c\right)\right)}{4d\sin\left(dx+c\right)^2}-\frac{a^2\cos\left(dx+c\right)}{4d}-\frac{a^2\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{4d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -7/15/d*a^2/sin(d*x+c)^3*cos(d*x+c)^3-1/2/d*a^2/sin(d*x+c)^4*cos(d*x+c)^3-1
/4/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3-1/4*a^2*cos(d*x+c)/d-1/4/d*a^2*ln(csc(d*
x+c)-cot(d*x+c))-1/5/d*a^2/sin(d*x+c)^5*cos(d*x+c)^3
```

maxima [A] time = 0.34, size = 109, normalized size = 1.09

$$-\frac{15\,a^{2}\left(\frac{2\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{40\,a^{2}}{\tan\left(dx+c\right)^{3}}+\frac{8\left(5\,\tan\left(dx+c\right)^{2}+3\right)a^{2}}{\tan\left(dx+c\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*(15*a^2*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 40*a^2/ta
n(d*x + c)^3 + 8*(5*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

mupad [B] time = 8.63, size = 160, normalized size = 1.60

$$\frac{5\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{96\,d}+\frac{a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{32\,d}+\frac{a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{160\,d}-\frac{a^{2}\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}\left(-6\,a^{2}\tan\left(\frac{c}{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] (5*a^2*tan(c/2 + (d*x)/2)^3)/(96*d) + (a^2*tan(c/2 + (d*x)/2)^4)/(32*d) + (
a^2*tan(c/2 + (d*x)/2)^5)/(160*d) - (a^2*log(tan(c/2 + (d*x)/2)))/(4*d) - (
cot(c/2 + (d*x)/2)^5*((5*a^2*tan(c/2 + (d*x)/2)^2)/3 - 6*a^2*tan(c/2 + (d*x
)/2)^4 + a^2/5 + a^2*tan(c/2 + (d*x)/2)))/(32*d) - (3*a^2*tan(c/2 + (d*x)/2
))/(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.284 $\int \cot^2(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=124

$$-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^2\cot(c+dx)\csc^3(}{24d}$$

```
[Out] 3/16*a^2*arctanh(cos(d*x+c))/d-2/3*a^2*cot(d*x+c)^3/d-2/5*a^2*cot(d*x+c)^5/
d+3/16*a^2*cot(d*x+c)*csc(d*x+c)/d-5/24*a^2*cot(d*x+c)*csc(d*x+c)^3/d-1/6*a
^2*cot(d*x+c)*csc(d*x+c)^5/d
```

Rubi [A] time = 0.26, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3768, 3770, 2607, 14}

$$-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^2\cot(c+dx)\csc^3(}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^2*Cot[c + d*x]^3)/(3*d) - (2*a^
2*Cot[c + d*x]^5)/(5*d) + (3*a^2*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (5*a^2
*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(6
*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^2(c+dx)\csc^3(c+dx) + 2a^2\cot^2(c+dx)\csc^4(c+\right.$$

$$= a^2\int \cot^2(c+dx)\csc^3(c+dx)\,dx + a^2\int \cot^2(c+dx)\csc^5$$

$$= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{a^2\cot(c+dx)\csc^5(c+dx)}{6d}$$

$$= \frac{a^2\cot(c+dx)\csc(c+dx)}{8d} - \frac{5a^2\cot(c+dx)\csc^3(c+dx)}{24d} -$$

$$= \frac{a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{2a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+}{5d}$$

$$= \frac{3a^2\tanh^{-1}(\cos(c+dx))}{16d} - \frac{2a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+}{5d}$$

Mathematica [A] time = 0.72, size = 229, normalized size = 1.85

$$\frac{a^2\csc^6(c+dx)\Big(-960\sin(2(c+dx)) - 384\sin(4(c+dx)) + 64\sin(6(c+dx)) - 1500\cos(c+dx) + 130\cos(3(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c + d*x]^6*(-1500*Cos[c + d*x] + 130*Cos[3*(c + d*x)] + 90*Cos[5*(
c + d*x)] + 450*Log[Cos[(c + d*x)/2]] - 675*Cos[2*(c + d*x)]*Log[Cos[(c + d
*x)/2]] + 270*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 45*Cos[6*(c + d*x)]*
Log[Cos[(c + d*x)/2]] - 450*Log[Sin[(c + d*x)/2]] + 675*Cos[2*(c + d*x)]*Lo
g[Sin[(c + d*x)/2]] - 270*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 45*Cos[6
*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 960*Sin[2*(c + d*x)] - 384*Sin[4*(c + d
*x)] + 64*Sin[6*(c + d*x)]))/(7680*d)
```

fricas [B] time = 0.51, size = 227, normalized size = 1.83

$$-\frac{90\,a^2\cos\left(dx+c\right)^5-80\,a^2\cos\left(dx+c\right)^3-90\,a^2\cos\left(dx+c\right)-45\left(a^2\cos\left(dx+c\right)^6-3\,a^2\cos\left(dx+c\right)^4+3\,a^2\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(90*a^2*cos(d*x + c)^5 - 80*a^2*cos(d*x + c)^3 - 90*a^2*cos(d*x + c)
 - 45*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a
^2)*log(1/2*cos(d*x + c) + 1/2) + 45*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x +
c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2) + 64*(2*a^2
*cos(d*x + c)^5 - 5*a^2*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^6 - 3
*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [B] time = 0.23, size = 228, normalized size = 1.84

$$\frac{5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+40\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^2\tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*tan(1/2*d*x + 1/2*c)^5 + 45*a
^2*tan(1/2*d*x + 1/2*c)^4 + 40*a^2*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*
d*x + 1/2*c)^2 - 360*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 240*a^2*tan(1/2*d
*x + 1/2*c) + (882*a^2*tan(1/2*d*x + 1/2*c)^6 + 240*a^2*tan(1/2*d*x + 1/2*c
)^5 + 15*a^2*tan(1/2*d*x + 1/2*c)^4 - 40*a^2*tan(1/2*d*x + 1/2*c)^3 - 45*a^
```

```
2*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*tan(1/2*d*x + 1/2*c) - 5*a^2)/tan(1/2*d*x
 + 1/2*c)^6)/d
```

maple [A] time = 0.34, size = 160, normalized size = 1.29

$$-\frac{3a^2\left(\cos^3(dx+c)\right)}{8d\sin(dx+c)^4}-\frac{3a^2\left(\cos^3(dx+c)\right)}{16d\sin(dx+c)^2}-\frac{3a^2\cos(dx+c)}{16d}-\frac{3a^2\ln(\csc(dx+c)-\cot(dx+c))}{16d}-\frac{2a^2\left(\cos^3(dx+\right.}{5d\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/8/d*a^2/sin(d*x+c)^4*cos(d*x+c)^3-3/16/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3-3
/16*a^2*cos(d*x+c)/d-3/16/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/5/d*a^2/sin(d*x
+c)^5*cos(d*x+c)^3-4/15/d*a^2/sin(d*x+c)^3*cos(d*x+c)^3-1/6/d*a^2/sin(d*x+c
)^6*cos(d*x+c)^3
```

maxima [A] time = 0.35, size = 187, normalized size = 1.51

$$-\frac{5\,a^2\left(\frac{2\left(3\,\cos(dx+c)^5-8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log(\cos(dx+c)+1)+3\,\log(\cos(dx+c)-1)\right)+30\,a^2\left(\frac{2\left(\cos(dx\right.}{\cos(dx+c)^4}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*(5*a^2*(2*(3*cos(d*x + c)^5 - 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(co
s(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c
) + 1) + 3*log(cos(d*x + c) - 1)) + 30*a^2*(2*(cos(d*x + c)^3 + cos(d*x + c
))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(co
s(d*x + c) - 1)) + 64*(5*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

mupad [B] time = 9.66, size = 339, normalized size = 2.73

$$-\frac{a^2\left(5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-24\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] -(a^2*(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 - 24*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^11 + 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) -
 45*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 40*cos(c/2 + (d*x)/2)^3*si
n(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 240*cos
(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 240*cos(c/2 + (d*x)/2)^7*sin(c/2 +
 (d*x)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 40*cos(c/2 + (
d*x)/2)^9*sin(c/2 + (d*x)/2)^3 + 45*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2
)^2 + 360*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*s
in(c/2 + (d*x)/2)^6))/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.285 $\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=132

$$-\frac{a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^5(c+dx)}{d}-\frac{4a^3\cos^3(c+dx)}{3d}-\frac{a^3\sin^3(c+dx)\cos^3(c+dx)}{2d}-\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{8d}$$

```
[Out] 5/16*a^3*x-4/3*a^3*cos(d*x+c)^3/d+a^3*cos(d*x+c)^5/d-1/7*a^3*cos(d*x+c)^7/d
+5/16*a^3*cos(d*x+c)*sin(d*x+c)/d-5/8*a^3*cos(d*x+c)^3*sin(d*x+c)/d-1/2*a^3
*cos(d*x+c)^3*sin(d*x+c)^3/d
```

Rubi [A] time = 0.30, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2873, 2568, 2635, 8, 2565, 14, 270}

$$-\frac{a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^5(c+dx)}{d}-\frac{4a^3\cos^3(c+dx)}{3d}-\frac{a^3\sin^3(c+dx)\cos^3(c+dx)}{2d}-\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/16 - (4*a^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cos[c + d*x]^5)/d - (a^3
*Cos[c + d*x]^7)/(7*d) + (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (5*a^3*
Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (a^3*Cos[c + d*x]^3*Sin[c + d*x]^3)/(2
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^2(c+dx)\sin^2(c+dx)+3a^3\cos^2(c+dx)\sin^3(c+\right.$$

$$= a^3\int \cos^2(c+dx)\sin^2(c+dx)\,dx + a^3\int \cos^2(c+dx)\sin^5($$

$$= -\frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{a^3\cos^3(c+dx)\sin^3(c+dx)}{2d}$$

$$= \frac{a^3\cos(c+dx)\sin(c+dx)}{8d} - \frac{5a^3\cos^3(c+dx)\sin(c+dx)}{8d} -$$

$$= \frac{a^3x}{8} - \frac{4a^3\cos^3(c+dx)}{3d} + \frac{a^3\cos^5(c+dx)}{d} - \frac{a^3\cos^7(c+dx)}{7d}$$

$$= \frac{5a^3x}{16} - \frac{4a^3\cos^3(c+dx)}{3d} + \frac{a^3\cos^5(c+dx)}{d} - \frac{a^3\cos^7(c+dx)}{7d}$$

Mathematica [A] time = 0.67, size = 86, normalized size = 0.65

$$\frac{a^3(-63\sin(2(c+dx))-105\sin(4(c+dx))+21\sin(6(c+dx))-609\cos(c+dx)-91\cos(3(c+dx))+63\cos(5}{1344d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(420*c + 420*d*x - 609*Cos[c + d*x] - 91*Cos[3*(c + d*x)] + 63*Cos[5*(
c + d*x)] - 3*Cos[7*(c + d*x)] - 63*Sin[2*(c + d*x)] - 105*Sin[4*(c + d*x)]
 + 21*Sin[6*(c + d*x)]))/(1344*d)
```

fricas [A] time = 0.51, size = 98, normalized size = 0.74

$$-\frac{48\,a^3\cos\left(dx+c\right)^7-336\,a^3\cos\left(dx+c\right)^5+448\,a^3\cos\left(dx+c\right)^3-105\,a^3dx-21\left(8\,a^3\cos\left(dx+c\right)^5-18\,a^3\cos\right.}{336\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/336*(48*a^3*cos(d*x + c)^7 - 336*a^3*cos(d*x + c)^5 + 448*a^3*cos(d*x +
c)^3 - 105*a^3*d*x - 21*(8*a^3*cos(d*x + c)^5 - 18*a^3*cos(d*x + c)^3 + 5*a
^3*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.23, size = 123, normalized size = 0.93

$$\frac{5}{16}\,a^3x-\frac{a^3\cos\left(7\,dx+7\,c\right)}{448\,d}+\frac{3\,a^3\cos\left(5\,dx+5\,c\right)}{64\,d}-\frac{13\,a^3\cos\left(3\,dx+3\,c\right)}{192\,d}-\frac{29\,a^3\cos\left(dx+c\right)}{64\,d}+\frac{a^3\sin\left(6\,dx+6\,c\right)}{64\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 5/16*a^3*x - 1/448*a^3*cos(7*d*x + 7*c)/d + 3/64*a^3*cos(5*d*x + 5*c)/d - 1
3/192*a^3*cos(3*d*x + 3*c)/d - 29/64*a^3*cos(d*x + c)/d + 1/64*a^3*sin(6*d*
x + 6*c)/d - 5/64*a^3*sin(4*d*x + 4*c)/d - 3/64*a^3*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.16, size = 194, normalized size = 1.47

$$\frac{a^3\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^3(dx+c)\right)}{7}-\frac{4\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{35}-\frac{8\left(\cos^3(dx+c)\right)}{105}\right)+3a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\left(\cos^3(dx+c)\right)\sin(dx\right.}{8}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*cos(d*x+c)^3-8/1
05*cos(d*x+c)^3)+3*a^3*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin
(d*x+c)+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+3*a^3*(-1/5*sin(d*x+c)^
2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+a^3*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos
(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c))
```

maxima [A] time = 0.33, size = 129, normalized size = 0.98

$$-\frac{64\left(15\cos\left(dx+c\right)^{7}-42\cos\left(dx+c\right)^{5}+35\cos\left(dx+c\right)^{3}\right)a^{3}-1344\left(3\cos\left(dx+c\right)^{5}-5\cos\left(dx+c\right)^{3}\right)a^{3}+105}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6720*(64*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^3
 - 1344*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^3 + 105*(4*sin(2*d*x + 2*c)
^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^3 - 210*(4*d*x + 4*c - sin(4*d*x
 + 4*c))*a^3)/d
```

mupad [B] time = 12.16, size = 331, normalized size = 2.51

$$\frac{5\,a^3\,x}{16}-\frac{\frac{5\,a^3\,(c+d\,x)}{16}+\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{119\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8}+\frac{119\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{8}-\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{2}-\frac{5\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}-\frac{a^3\,(105\,c+}{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (5*a^3*x)/16 - ((5*a^3*(c + d*x))/16 + (3*a^3*tan(c/2 + (d*x)/2)^3)/2 - (11
9*a^3*tan(c/2 + (d*x)/2)^5)/8 + (119*a^3*tan(c/2 + (d*x)/2)^9)/8 - (3*a^3*t
an(c/2 + (d*x)/2)^11)/2 - (5*a^3*tan(c/2 + (d*x)/2)^13)/8 - (a^3*(105*c + 1
05*d*x - 320))/336 + tan(c/2 + (d*x)/2)^2*((35*a^3*(c + d*x))/16 - (a^3*(73
5*c + 735*d*x - 2240))/336) + tan(c/2 + (d*x)/2)^4*((105*a^3*(c + d*x))/16
- (a^3*(2205*c + 2205*d*x - 2688))/336) + tan(c/2 + (d*x)/2)^6*((175*a^3*(c
 + d*x))/16 - (a^3*(3675*c + 3675*d*x - 896))/336) + tan(c/2 + (d*x)/2)^10*
((105*a^3*(c + d*x))/16 - (a^3*(2205*c + 2205*d*x - 4032))/336) + tan(c/2 +
 (d*x)/2)^8*((175*a^3*(c + d*x))/16 - (a^3*(3675*c + 3675*d*x - 10304))/336
) + (5*a^3*tan(c/2 + (d*x)/2))/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [A] time = 7.84, size = 379, normalized size = 2.87

$$\begin{cases} \frac{3a^3x\sin^6{(c+dx)}}{16} + \frac{9a^3x\sin^4{(c+dx)}\cos^2{(c+dx)}}{16} + \frac{a^3x\sin^4{(c+dx)}}{8} + \frac{9a^3x\sin^2{(c+dx)}\cos^4{(c+dx)}}{16} + \frac{a^3x\sin^2{(c+dx)}\cos^2{(c+dx)}}{4} + \frac{3a^3x\cos^6 \\ x\left(a\sin{(c)}+a\right)^3\sin^2{(c)}\cos^2{(c)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**6/16 + 9*a**3*x*sin(c + d*x)**4*cos(c + d
*x)**2/16 + a**3*x*sin(c + d*x)**4/8 + 9*a**3*x*sin(c + d*x)**2*cos(c + d*x
)**4/16 + a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**3*x*cos(c + d*x)*
*6/16 + a**3*x*cos(c + d*x)**4/8 + 3*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*
d) - a**3*sin(c + d*x)**4*cos(c + d*x)**3/(3*d) - a**3*sin(c + d*x)**3*cos(
c + d*x)**3/(2*d) + a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 4*a**3*sin(c
+ d*x)**2*cos(c + d*x)**5/(15*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**3/d -
 3*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - a**3*sin(c + d*x)*cos(c + d*x
)**3/(8*d) - 8*a**3*cos(c + d*x)**7/(105*d) - 2*a**3*cos(c + d*x)**5/(5*d),
 Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)**2*cos(c)**2, True))
```

3.286 $\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=117

$$\frac{3a^3\cos^5(c+dx)}{5d}-\frac{4a^3\cos^3(c+dx)}{3d}-\frac{a^3\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{7a^3\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{7a^3\sin(c+dx)\,c}{16d}$$

```
[Out] 7/16*a^3*x-4/3*a^3*cos(d*x+c)^3/d+3/5*a^3*cos(d*x+c)^5/d+7/16*a^3*cos(d*x+c
)*sin(d*x+c)/d-7/8*a^3*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a^3*cos(d*x+c)^3*sin(d
*x+c)^3/d
```

Rubi [A] time = 0.18, antiderivative size = 133, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{7a^3\cos^3(c+dx)}{24d}-\frac{7\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{40d}+\frac{7a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{7a^3x}{16}-\frac{a\cos^3(c+dx)(a\sin}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*a^3*x)/16 - (7*a^3*Cos[c + d*x]^3)/(24*d) + (7*a^3*Cos[c + d*x]*Sin[c +
d*x])/(16*d) - (a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2)/(10*d) - (Cos[c +
d*x]^3*(a + a*Sin[c + d*x])^3)/(6*d) - (7*Cos[c + d*x]^3*(a^3 + a^3*Sin[c +
 d*x]))/(40*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx &= -\frac{\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d}+\frac{1}{2}\int \cos^2(c+dx)(a+\\ &= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{10d}-\frac{\cos^3(c+dx)(a+a\,\mathrm{si}}{6d}\\ &= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{10d}-\frac{\cos^3(c+dx)(a+a\,\mathrm{si}}{6d}\\ &= -\frac{7a^3\cos^3(c+dx)}{24d}-\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{10d}-\frac{\mathrm{c}}{}\\ &= -\frac{7a^3\cos^3(c+dx)}{24d}+\frac{7a^3\cos(c+dx)\sin(c+dx)}{16d}-\frac{a\cos^3(}{}\\ &= \frac{7a^3x}{16}-\frac{7a^3\cos^3(c+dx)}{24d}+\frac{7a^3\cos(c+dx)\sin(c+dx)}{16d}-\frac{a}{}\end{aligned}$$

Mathematica [A] time = 0.38, size = 76, normalized size = 0.65

$$\frac{a^3(-15\sin(2(c+dx))-105\sin(4(c+dx))+5\sin(6(c+dx))-600\cos(c+dx)-140\cos(3(c+dx))+36\cos(5}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(450*c + 420*d*x - 600*Cos[c + d*x] - 140*Cos[3*(c + d*x)] + 36*Cos[5*
(c + d*x)] - 15*Sin[2*(c + d*x)] - 105*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)
]))/(960*d)
```

fricas [A] time = 0.49, size = 85, normalized size = 0.73

$$\frac{144\,a^3\cos\left(dx+c\right)^5-320\,a^3\cos\left(dx+c\right)^3+105\,a^3dx+5\left(8\,a^3\cos\left(dx+c\right)^5-50\,a^3\cos\left(dx+c\right)^3+21\,a^3\cos\left(dx\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(144*a^3*cos(d*x + c)^5 - 320*a^3*cos(d*x + c)^3 + 105*a^3*d*x + 5*(8
*a^3*cos(d*x + c)^5 - 50*a^3*cos(d*x + c)^3 + 21*a^3*cos(d*x + c))*sin(d*x
+ c))/d
```

giac [A] time = 0.20, size = 106, normalized size = 0.91

$$\frac{7}{16}\,a^3x+\frac{3\,a^3\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{7\,a^3\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{5\,a^3\cos\left(dx+c\right)}{8\,d}+\frac{a^3\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{7\,a^3\sin\left(4\,dx+4\,c\right)}{64\,d}-\frac{a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 7/16*a^3*x + 3/80*a^3*cos(5*d*x + 5*c)/d - 7/48*a^3*cos(3*d*x + 3*c)/d - 5/
8*a^3*cos(d*x + c)/d + 1/192*a^3*sin(6*d*x + 6*c)/d - 7/64*a^3*sin(4*d*x +
4*c)/d - 1/64*a^3*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.17, size = 156, normalized size = 1.33

$$\frac{a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{8}+\frac{\cos(dx+c)\sin(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3\right.}{1}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*c
os(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+3*a^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3
-2/15*cos(d*x+c)^3)+3*a^3*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(
d*x+c)+1/8*d*x+1/8*c)-1/3*a^3*cos(d*x+c)^3)
```

maxima [A] time = 0.31, size = 106, normalized size = 0.91

$$-\frac{320\,a^3\cos\left(dx+c\right)^3-192\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)a^3+5\left(4\,\sin\left(2\,dx+2\,c\right)^3-12\,dx-12\,c+3\,\sin\left(4\right.\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(320*a^3*cos(d*x + c)^3 - 192*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*
a^3 + 5*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^3 - 9
0*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^3)/d
```

mupad [B] time = 10.72, size = 349, normalized size = 2.98

$$\frac{7a^3x}{16} - \frac{\frac{37a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4} - \frac{37a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4} - \frac{73a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24} + \frac{73a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{24} - \frac{7a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{8} + \frac{a^3(105c+105dx)}{240} - \frac{a^3 \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (7*a^3*x)/16 - ((37*a^3*tan(c/2 + (d*x)/2)^7)/4 - (37*a^3*tan(c/2 + (d*x)/2
)^5)/4 - (73*a^3*tan(c/2 + (d*x)/2)^3)/24 + (73*a^3*tan(c/2 + (d*x)/2)^9)/2
4 - (7*a^3*tan(c/2 + (d*x)/2)^11)/8 + (a^3*(105*c + 105*d*x))/240 - (a^3*(1
05*c + 105*d*x - 352))/240 + tan(c/2 + (d*x)/2)^10*((a^3*(105*c + 105*d*x))
/40 - (a^3*(630*c + 630*d*x - 480))/240) + tan(c/2 + (d*x)/2)^2*((a^3*(105*
c + 105*d*x))/40 - (a^3*(630*c + 630*d*x - 1632))/240) + tan(c/2 + (d*x)/2)
^4*((a^3*(105*c + 105*d*x))/16 - (a^3*(1575*c + 1575*d*x - 960))/240) + tan
(c/2 + (d*x)/2)^8*((a^3*(105*c + 105*d*x))/16 - (a^3*(1575*c + 1575*d*x - 4
320))/240) + tan(c/2 + (d*x)/2)^6*((a^3*(105*c + 105*d*x))/12 - (a^3*(2100*
c + 2100*d*x - 3520))/240) + (7*a^3*tan(c/2 + (d*x)/2))/8)/(d*(tan(c/2 + (d
*x)/2)^2 + 1)^6)
```

sympy [A] time = 4.88, size = 328, normalized size = 2.80

$$\begin{cases} \frac{a^3x\sin^6(c+dx)}{16} + \frac{3a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3a^3x\sin^4(c+dx)}{8} + \frac{3a^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{a^3x\cos^{\cdots}}{1\cdots} \\ x(a\sin(c)+a)^3\sin(c)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(c + d*x)**6/16 + 3*a**3*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x
)**4/16 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + a**3*x*cos(c + d*x)*
*6/16 + 3*a**3*x*cos(c + d*x)**4/8 + a**3*sin(c + d*x)**5*cos(c + d*x)/(16*
```

```
d) - a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 3*a**3*sin(c + d*x)**3*co
s(c + d*x)/(8*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**3/d - a**3*sin(c + d*
x)*cos(c + d*x)**5/(16*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*a
**3*cos(c + d*x)**5/(5*d) - a**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*si
n(c) + a)**3*sin(c)*cos(c)**2, True))
```

3.287 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=99

$$-\frac{a^3\cos^3(c+dx)}{d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}-\frac{a^3\tanh^{-1}(\cos(c+$$

```
[Out] 13/8*a^3*x-a^3*arctanh(cos(d*x+c))/d+a^3*cos(d*x+c)/d-a^3*cos(d*x+c)^3/d+13
/8*a^3*cos(d*x+c)*sin(d*x+c)/d-1/4*a^3*cos(d*x+c)^3*sin(d*x+c)/d
```

Rubi [A] time = 0.17, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2873, 2635, 8, 2592, 321, 206, 2565, 30, 2568}

$$-\frac{a^3\cos^3(c+dx)}{d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}-\frac{a^3\tanh^{-1}(\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (13*a^3*x)/8 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Cos[c + d*x])/d - (a^3*
Cos[c + d*x]^3)/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^3*Cos[c +
 d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(3a^3\cos^2(c+dx) + a^3\cos(c+dx)\cot(c+dx) + 3a^3\cos^2\right.$$
$$= a^3\int \cos(c+dx)\cot(c+dx)\,dx + a^3\int \cos^2(c+dx)\sin^2(c$$
$$= \frac{3a^3\cos(c+dx)\sin(c+dx)}{2d} - \frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d} +$$
$$= \frac{3a^3x}{2} + \frac{a^3\cos(c+dx)}{d} - \frac{a^3\cos^3(c+dx)}{d} + \frac{13a^3\cos(c+dx)}{8d}$$
$$= \frac{13a^3x}{8} - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} - \frac{a^3\cos^3}{d}$$

Mathematica [A] time = 0.66, size = 82, normalized size = 0.83

$$\frac{a^3\left(24\sin(2(c+dx)) - \sin(4(c+dx)) + 8\cos(c+dx) - 8\cos(3(c+dx)) + 32\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 32\log\left(\cos\right.\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(52*c + 52*d*x + 8*Cos[c + d*x] - 8*Cos[3*(c + d*x)] - 32*Log[Cos[(c +
 d*x)/2]] + 32*Log[Sin[(c + d*x)/2]] + 24*Sin[2*(c + d*x)] - Sin[4*(c + d*x
)]))/(32*d)
```

fricas [A] time = 0.50, size = 101, normalized size = 1.02

$$-\frac{8\,a^3\cos(dx+c)^3 - 13\,a^3dx - 8\,a^3\cos(dx+c) + 4\,a^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - 4\,a^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a^3*cos(d*x + c)^3 - 13*a^3*d*x - 8*a^3*cos(d*x + c) + 4*a^3*log(1/
2*cos(d*x + c) + 1/2) - 4*a^3*log(-1/2*cos(d*x + c) + 1/2) + (2*a^3*cos(d*x
 + c)^3 - 13*a^3*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.21, size = 144, normalized size = 1.45

$$\frac{13\,(dx+c)a^3 + 8\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - \frac{2\left(11\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 16\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 + 19\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 19\,a^3\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(13*(d*x + c)*a^3 + 8*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(11*a^3*ta
n(1/2*d*x + 1/2*c)^7 + 16*a^3*tan(1/2*d*x + 1/2*c)^6 + 19*a^3*tan(1/2*d*x +
 1/2*c)^5 - 19*a^3*tan(1/2*d*x + 1/2*c)^3 - 16*a^3*tan(1/2*d*x + 1/2*c)^2 -
 11*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.36, size = 111, normalized size = 1.12

$$-\frac{a^3 \left(\cos^3(dx+c)\right) \sin(dx+c)}{4d}+\frac{13a^3 \cos(dx+c) \sin(dx+c)}{8d}+\frac{13a^3x}{8}+\frac{13a^3c}{8d}-\frac{a^3 \left(\cos^3(dx+c)\right)}{d}+\frac{a^3 \cos(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/4*a^3*cos(d*x+c)^3*sin(d*x+c)/d+13/8*a^3*cos(d*x+c)*sin(d*x+c)/d+13/8*a^
3*x+13/8/d*a^3*c-a^3*cos(d*x+c)^3/d+a^3*cos(d*x+c)/d+1/d*a^3*ln(csc(d*x+c)-
cot(d*x+c))
```

maxima [A] time = 0.32, size = 99, normalized size = 1.00

$$-\frac{32\,a^3 \cos(dx+c)^3 - (4\,dx+4\,c-\sin(4\,dx+4\,c))a^3 - 24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3 - 16\,a^3\big(2\,\cos(dx+c)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(32*a^3*cos(d*x + c)^3 - (4*d*x + 4*c - sin(4*d*x + 4*c))*a^3 - 24*(2
*d*x + 2*c + sin(2*d*x + 2*c))*a^3 - 16*a^3*(2*cos(d*x + c) - log(cos(d*x +
 c) + 1) + log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 10.36, size = 244, normalized size = 2.46

$$\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{13\,a^3\,\mathrm{atan}\left(\frac{169\,a^6}{16\left(\frac{13\,a^6}{2}-\frac{169\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{13\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{13\,a^6}{2}-\frac{169\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}+\frac{-\frac{11\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-4\,a^3\,\tan\left(\frac{c}{2}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d + (13*a^3*atan((169*a^6)/(16*((13*a^6)/2 -
(169*a^6*tan(c/2 + (d*x)/2))/16)) + (13*a^6*tan(c/2 + (d*x)/2))/(2*((13*a^6
)/2 - (169*a^6*tan(c/2 + (d*x)/2))/16))))/(4*d) + (4*a^3*tan(c/2 + (d*x)/2)
^2 + (19*a^3*tan(c/2 + (d*x)/2)^3)/4 - (19*a^3*tan(c/2 + (d*x)/2)^5)/4 - 4*
a^3*tan(c/2 + (d*x)/2)^6 - (11*a^3*tan(c/2 + (d*x)/2)^7)/4 + (11*a^3*tan(c/
2 + (d*x)/2))/4)/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*ta
n(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int \cos^2(c+dx) \csc(c+dx)\, dx + \int 3 \sin(c+dx) \cos^2(c+dx) \csc(c+dx)\, dx + \int 3 \sin^2(c+dx) \cos^2(c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(cos(c + d*x)**2*csc(c + d*x), x) + Integral(3*sin(c + d*x)*c
os(c + d*x)**2*csc(c + d*x), x) + Integral(3*sin(c + d*x)**2*cos(c + d*x)**
2*csc(c + d*x), x) + Integral(sin(c + d*x)**3*cos(c + d*x)**2*csc(c + d*x),
 x))
```

3.288 $\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=92

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3x}{2}$$

```
[Out] 1/2*a^3*x-3*a^3*arctanh(cos(d*x+c))/d+3*a^3*cos(d*x+c)/d-1/3*a^3*cos(d*x+c)
^3/d-a^3*cot(d*x+c)/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.14, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 3770, 3767, 8, 2638, 2635, 2633}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*x)/2 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d - (a^3
*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c +
 d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(2a^5+3a^5\csc(c+dx)+a^5\csc^2(c+dx)-2a^5\sin(c+dx)-3a^5\sin^2$$
$$= 2a^3x+a^3\int\csc^2(c+dx)\,dx-a^3\int\sin^3(c+dx)\,dx-\left(2a^3\right)\int\sin(c+$$
$$= 2a^3x-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3\cos(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin}{2d}$$
$$= \frac{a^3x}{2}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}-$$

Mathematica [A] time = 1.13, size = 106, normalized size = 1.15

$$-\frac{a^3\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left((15-66\sin(c+dx))\cos(c+dx)+(2\sin(c+dx)+9)\cos(3(c+dx))-12\,s}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/48*(a^3*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x]*(15 - 66*Sin[c +
 d*x]) - 12*(c + d*x - 6*Log[Cos[(c + d*x)/2]] + 6*Log[Sin[(c + d*x)/2]])*S
in[c + d*x] + Cos[3*(c + d*x)]*(9 + 2*Sin[c + d*x])))/d
```

fricas [A] time = 0.53, size = 121, normalized size = 1.32

$$-\frac{9\,a^3\cos\left(dx+c\right)^3+9\,a^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,a^3\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\,a^3}{6\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(9*a^3*cos(d*x + c)^3 + 9*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
 - 9*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*a^3*cos(d*x + c) + (
2*a^3*cos(d*x + c)^3 - 3*a^3*d*x - 18*a^3*cos(d*x + c))*sin(d*x + c))/(d*si
n(d*x + c))
```

giac [A] time = 0.22, size = 162, normalized size = 1.76

$$\frac{3\,(dx+c)a^3+18\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*a^3 + 18*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/
2*d*x + 1/2*c) - 3*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)
- 2*(9*a^3*tan(1/2*d*x + 1/2*c)^5 - 12*a^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^3*
tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) - 16*a^3)/(tan(1/2*d*x
+ 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.32, size = 105, normalized size = 1.14

$$-\frac{a^3\left(\cos^3\left(dx+c\right)\right)}{3d}+\frac{3a^3\cos\left(dx+c\right)\sin\left(dx+c\right)}{2d}+\frac{a^3x}{2}+\frac{a^3c}{2d}+\frac{3a^3\cos\left(dx+c\right)}{d}+\frac{3a^3\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/3*a^3*cos(d*x+c)^3/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d+1/2*a^3*x+1/2/d*a^3
*c+3*a^3*cos(d*x+c)/d+3/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-a^3*cot(d*x+c)/d
```

maxima [A] time = 0.43, size = 93, normalized size = 1.01

$$-\frac{4\,a^3\cos\left(dx+c\right)^3-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^3+12\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3-18\,a^3\left(2\,\cos\left(dx+c\right)-\log\left(\right.\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/12*(4*a^3*cos(d*x + c)^3 - 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(
d*x + c + 1/tan(d*x + c))*a^3 - 18*a^3*(2*cos(d*x + c) - log(cos(d*x + c) +
 1) + log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.77, size = 264, normalized size = 2.87

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{a^3\,\mathrm{atan}\left(\frac{a^6}{6\,a^6-a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{6\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6\,a^6-a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{-7\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d + (a^3*atan(a^6/(6*a^6 - a^6*tan(c/2 + (d
*x)/2)) + (6*a^6*tan(c/2 + (d*x)/2))/(6*a^6 - a^6*tan(c/2 + (d*x)/2))))/d +
 (a^3*tan(c/2 + (d*x)/2))/(2*d) + (3*a^3*tan(c/2 + (d*x)/2)^2 + 24*a^3*tan(
c/2 + (d*x)/2)^3 - 3*a^3*tan(c/2 + (d*x)/2)^4 + 8*a^3*tan(c/2 + (d*x)/2)^5
- 7*a^3*tan(c/2 + (d*x)/2)^6 - a^3 + (32*a^3*tan(c/2 + (d*x)/2))/3)/(d*(2*t
an(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3 + 6*tan(c/2 + (d*x)/2)^5 + 2*tan
(c/2 + (d*x)/2)^7))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.289 $\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=98

$$\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{5a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -5/2*a^3*x-5/2*a^3*arctanh(cos(d*x+c))/d+3*a^3*cos(d*x+c)/d-3*a^3*cot(d*x+c
)/d-1/2*a^3*cot(d*x+c)*csc(d*x+c)/d+1/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2872, 3770, 3767, 8, 3768, 2638, 2635}

$$\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{5a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-5*a^3*x)/2 - (5*a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (3*a^3*Cos[c + d*x])/d
 - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (a^3*Co
s[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
```

```
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(-2a^5+2a^5\csc(c+dx)+3a^5\csc^2(c+dx)+a^5\csc^3(c+\right.}{a^2}$$

$$= -2a^3x+a^3\int\csc^3(c+dx)\,dx-a^3\int\sin^2(c+dx)\,dx+\left(2a\right.$$

$$= -2a^3x-\frac{2a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\,\mathrm{co}}{}$$

$$= -\frac{5a^3x}{2}-\frac{5a^3\tanh^{-1}(\cos(c+dx))}{2d}+\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3}{}$$

Mathematica [A] time = 1.04, size = 112, normalized size = 1.14

$$\frac{a^3\left(2\sin(2(c+dx))+24\cos(c+dx)+12\tan\left(\frac{1}{2}(c+dx)\right)-12\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^2\left(\frac{1}{2}(c+\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-20*c - 20*d*x + 24*Cos[c + d*x] - 12*Cot[(c + d*x)/2] - Csc[(c + d*x
)/2]^2 - 20*Log[Cos[(c + d*x)/2]] + 20*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x
)/2]^2 + 2*Sin[2*(c + d*x)] + 12*Tan[(c + d*x)/2]))/(8*d)
```

fricas [A] time = 0.54, size = 159, normalized size = 1.62

$$-\frac{10\,a^3dx\cos\left(dx+c\right)^2-12\,a^3\cos\left(dx+c\right)^3-10\,a^3dx+10\,a^3\cos\left(dx+c\right)+5\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\cos\left(\right.\right.}{4\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/4*(10*a^3*d*x*cos(d*x + c)^2 - 12*a^3*cos(d*x + c)^3 - 10*a^3*d*x + 10*a
^3*cos(d*x + c) + 5*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2)
- 5*(a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*cos(d*
x + c)^3 + 5*a^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

giac [B] time = 0.24, size = 184, normalized size = 1.88

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-20\,(dx+c)a^3+20\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{10\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+20}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 - 20*(d*x + c)*a^3 + 20*a^3*log(abs(tan(1/2
*d*x + 1/2*c))) + 12*a^3*tan(1/2*d*x + 1/2*c) - (10*a^3*tan(1/2*d*x + 1/2*c
)^6 + 20*a^3*tan(1/2*d*x + 1/2*c)^5 - 27*a^3*tan(1/2*d*x + 1/2*c)^4 + 16*a^
3*tan(1/2*d*x + 1/2*c)^3 - 36*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d
*x + 1/2*c) + a^3)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2)/d
```

maple [A] time = 0.42, size = 113, normalized size = 1.15

$$\frac{a^3\cos\left(dx+c\right)\sin\left(dx+c\right)}{2d}-\frac{5a^3x}{2}-\frac{5a^3c}{2d}+\frac{5a^3\cos\left(dx+c\right)}{2d}+\frac{5a^3\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{2d}-\frac{3a^3\cot\left(dx+c\right)}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*cos(d*x+c)*sin(d*x+c)/d-5/2*a^3*x-5/2/d*a^3*c+5/2*a^3*cos(d*x+c)/d+
5/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3*a^3*cot(d*x+c)/d-1/2/d*a^3/sin(d*x+c)
^2*cos(d*x+c)^3
```

maxima [A] time = 0.43, size = 124, normalized size = 1.27

$$\frac{(2\,dx + 2\,c + \sin(2\,dx + 2\,c))a^3 - 12\left(dx + c + \frac{1}{\tan(dx+c)}\right)a^3 + a^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1} + \log(\cos(dx + c) + 1) - \log(\cos(\ldots}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 - 12*(d*x + c + 1/tan(d*x + c))*a
^3 + a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log
(cos(d*x + c) - 1)) + 6*a^3*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(c
os(d*x + c) - 1)))/d
```

mupad [B] time = 8.68, size = 259, normalized size = 2.64

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{5\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{47\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-23\,a^3\tan\ldots}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (5*a^3*log(tan(c/2 + (d*x)/2)))/(2*d) -
(8*a^3*tan(c/2 + (d*x)/2)^3 - 23*a^3*tan(c/2 + (d*x)/2)^2 - (47*a^3*tan(c/2
 + (d*x)/2)^4)/2 + 10*a^3*tan(c/2 + (d*x)/2)^5 + a^3/2 + 6*a^3*tan(c/2 + (d
*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (
d*x)/2)^6)) + (5*a^3*atan((25*a^6)/(25*a^6 + 25*a^6*tan(c/2 + (d*x)/2)) - (
25*a^6*tan(c/2 + (d*x)/2))/(25*a^6 + 25*a^6*tan(c/2 + (d*x)/2))))/d + (3*a^
3*tan(c/2 + (d*x)/2))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.290 $\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}-3a^3x$$

```
[Out] -3*a^3*x+1/2*a^3*arctanh(cos(d*x+c))/d+a^3*cos(d*x+c)/d-3*a^3*cot(d*x+c)/d-
1/3*a^3*cot(d*x+c)^3/d-3/2*a^3*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A] time = 0.17, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2872, 3770, 3767, 8, 3768, 2638}

$$\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}-3a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -3*a^3*x + (a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*Cos[c + d*x])/d - (3*a^
3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c
+ d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(-3a^5-2a^5\csc(c+dx)+2a^5\csc^2(c+dx)+3a^5\csc^3(c\right.}{a^2}$$

$$= -3a^3x + a^3\int \csc^4(c+dx)\,dx - a^3\int \sin(c+dx)\,dx - \left(2a\right.$$

$$= -3a^3x + \frac{2a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} - \frac{3a^3 c}{}$$

$$= -3a^3x + \frac{a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{a^3\cos(c+dx)}{d} - \frac{3a^3 co}{}$$

Mathematica [A] time = 0.43, size = 148, normalized size = 1.63

$$\frac{a^3\left(24\cos(c+dx)+32\tan\left(\frac{1}{2}(c+dx)\right)-32\cot\left(\frac{1}{2}(c+dx)\right)-9\csc^2\left(\frac{1}{2}(c+dx)\right)+9\sec^2\left(\frac{1}{2}(c+dx)\right)-12\log\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-72*c - 72*d*x + 24*Cos[c + d*x] - 32*Cot[(c + d*x)/2] - 9*Csc[(c + d
*x)/2]^2 + 12*Log[Cos[(c + d*x)/2]] - 12*Log[Sin[(c + d*x)/2]] + 9*Sec[(c +
 d*x)/2]^2 + 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (Csc[(c + d*x)/2]^4*Sin[
c + d*x])/2 + 32*Tan[(c + d*x)/2]))/(24*d)
```

fricas [B] time = 0.52, size = 180, normalized size = 1.98

$$-\frac{32\,a^3\cos(dx+c)^3-36\,a^3\cos(dx+c)-3\left(a^3\cos(dx+c)^2-a^3\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+3\left(a^3\,\mathrm{c}\right.}{12\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(32*a^3*cos(d*x + c)^3 - 36*a^3*cos(d*x + c) - 3*(a^3*cos(d*x + c)^2
- a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(a^3*cos(d*x + c)^2 - a
^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*(6*a^3*d*x*cos(d*x + c)^2
 - 2*a^3*cos(d*x + c)^3 - 6*a^3*d*x - a^3*cos(d*x + c))*sin(d*x + c))/((d*c
os(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.24, size = 161, normalized size = 1.77

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,(dx+c)a^3-12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+33\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^3*tan(1/2*d*x + 1/2*c)^2 - 72*(d*x +
 c)*a^3 - 12*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 33*a^3*tan(1/2*d*x + 1/2*
c) + 48*a^3/(tan(1/2*d*x + 1/2*c)^2 + 1) + (22*a^3*tan(1/2*d*x + 1/2*c)^3 -
 33*a^3*tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) - a^3)/tan(1/2*
d*x + 1/2*c)^3)/d
```

maple [A] time = 0.34, size = 117, normalized size = 1.29

$$-\frac{a^3\cos(dx+c)}{2d}-\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d}-3a^3x-\frac{3a^3\cot(dx+c)}{d}-\frac{3a^3c}{d}-\frac{3a^3\left(\cos^3(dx+c)\right)}{2d\sin(dx+c)^2}-\frac{a^3\left(\cos^3\right.}{3d\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2*a^3*cos(d*x+c)/d-1/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3*a^3*x-3*a^3*cot
(d*x+c)/d-3/d*a^3*c-3/2/d*a^3/sin(d*x+c)^2*cos(d*x+c)^3-1/3/d*a^3/sin(d*x+c
)^3*cos(d*x+c)^3
```

maxima [A] time = 0.43, size = 117, normalized size = 1.29

$$-\frac{36\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3-9\,a^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)-6\,a^3\left(2\,\cos\left(dx\right.\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/12*(36*(d*x + c + 1/tan(d*x + c))*a^3 - 9*a^3*(2*cos(d*x + c)/(cos(d*x +
 c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) - 6*a^3*(2*cos(
d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 4*a^3/tan(d*x +
 c)^3)/d
```

mupad [B] time = 8.68, size = 249, normalized size = 2.74

$$\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{6\,a^3\,\mathrm{atan}\left(\frac{36\,a^6}{6\,a^6-36\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{6\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6\,a^6-36\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] (3*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^3*tan(c/2 + (d*x)/2)^3)/(24*d) - (a
^3*log(tan(c/2 + (d*x)/2)))/(2*d) - (6*a^3*atan((36*a^6)/(6*a^6 - 36*a^6*ta
n(c/2 + (d*x)/2)) + (6*a^6*tan(c/2 + (d*x)/2))/(6*a^6 - 36*a^6*tan(c/2 + (d
*x)/2))))/d - ((34*a^3*tan(c/2 + (d*x)/2)^2)/3 - 13*a^3*tan(c/2 + (d*x)/2)^
3 + 11*a^3*tan(c/2 + (d*x)/2)^4 + a^3/3 + 3*a^3*tan(c/2 + (d*x)/2))/(d*(8*t
an(c/2 + (d*x)/2)^3 + 8*tan(c/2 + (d*x)/2)^5)) + (11*a^3*tan(c/2 + (d*x)/2)
)/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.291 $\int \cot^2(c+dx) \csc^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=100

$$-\frac{a^3\cot^3(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{13a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{11a^3\cot(c+dx)\csc(c+\cdots}{8d}$$

```
[Out] -a^3*x+13/8*a^3*arctanh(cos(d*x+c))/d-a^3*cot(d*x+c)/d-a^3*cot(d*x+c)^3/d-1
1/8*a^3*cot(d*x+c)*csc(d*x+c)/d-1/4*a^3*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A] time = 0.22, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2873, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$-\frac{a^3\cot^3(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{13a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{11a^3\cot(c+dx)\csc(c+\cdots}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (13*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (a^3*Cot[c + d*x])/d - (a
^3*Cot[c + d*x]^3)/d - (11*a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*Cot[
c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^2(c+dx) + 3a^3\cot^2(c+dx)\csc(c+dx) + 3a^3\,\mathrm{co}\right.$$

$$= a^3\int \cot^2(c+dx)\,dx + a^3\int \cot^2(c+dx)\csc^3(c+dx)\,dx +$$

$$= -\frac{a^3\cot(c+dx)}{d} - \frac{3a^3\cot(c+dx)\csc(c+dx)}{2d} - \frac{a^3\cot(c+}{}$$

$$= -a^3x + \frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot}{}$$

$$= -a^3x + \frac{13a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^3\cot(c+dx)}{d} - \frac{a^3\,\mathrm{co}}{}$$

Mathematica [A] time = 0.56, size = 133, normalized size = 1.33

$$\frac{a^3\left(-22\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)+22\sec^2\left(\frac{1}{2}(c+dx)\right)-\left((4\sin(c+dx)+1)\csc^4\left(\frac{1}{2}(c+dx)\right)\right)-8\left(13\right.\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-22*Csc[(c + d*x)/2]^2 + 22*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^4 -
 8*(8*c + 8*d*x - 13*Log[Cos[(c + d*x)/2]] + 13*Log[Sin[(c + d*x)/2]] - 8*C
sc[c + d*x]^3*Sin[(c + d*x)/2]^4) - Csc[(c + d*x)/2]^4*(1 + 4*Sin[c + d*x])
))/(64*d)
```

fricas [B] time = 0.52, size = 190, normalized size = 1.90

$$-\frac{16\,a^3dx\cos\left(dx+c\right)^4-32\,a^3dx\cos\left(dx+c\right)^2-22\,a^3\cos\left(dx+c\right)^3+16\,a^3dx+16\,a^3\cos\left(dx+c\right)\sin\left(dx+c\right)+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*(16*a^3*d*x*cos(d*x + c)^4 - 32*a^3*d*x*cos(d*x + c)^2 - 22*a^3*cos(d
*x + c)^3 + 16*a^3*d*x + 16*a^3*cos(d*x + c)*sin(d*x + c) + 26*a^3*cos(d*x
+ c) - 13*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x
 + c) + 1/2) + 13*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(-1/
2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.24, size = 174, normalized size = 1.74

$$\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+72\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-192\,(dx+c)a^3-312\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx\right.\right.\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^3*tan(1/2*d*x + 1/2*c)^4 + 24*a^3*tan(1/2*d*x + 1/2*c)^3 + 72*a^
3*tan(1/2*d*x + 1/2*c)^2 - 192*(d*x + c)*a^3 - 312*a^3*log(abs(tan(1/2*d*x
+ 1/2*c))) + 24*a^3*tan(1/2*d*x + 1/2*c) + (650*a^3*tan(1/2*d*x + 1/2*c)^4
```

```
- 24*a^3*tan(1/2*d*x + 1/2*c)^3 - 72*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*a^3*ta
n(1/2*d*x + 1/2*c) - 3*a^3)/tan(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.33, size = 141, normalized size = 1.41

$$-a^3x-\frac{a^3\cot(dx+c)}{d}-\frac{a^3c}{d}-\frac{13a^3\left(\cos^3(dx+c)\right)}{8d\sin(dx+c)^2}-\frac{13a^3\cos(dx+c)}{8d}-\frac{13a^3\ln(\csc(dx+c)-\cot(dx+c))}{8d}-\frac{a^3\left(\mathrm{co}\right.}{d\,\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -a^3*x-a^3*cot(d*x+c)/d-1/d*a^3*c-13/8/d*a^3/sin(d*x+c)^2*cos(d*x+c)^3-13/8
*a^3*cos(d*x+c)/d-13/8/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-1/d*a^3/sin(d*x+c)^3
*cos(d*x+c)^3-1/4/d*a^3/sin(d*x+c)^4*cos(d*x+c)^3
```

maxima [A] time = 0.41, size = 147, normalized size = 1.47

$$-\frac{16\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3+a^3\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-12\,a^3\left(\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(16*(d*x + c + 1/tan(d*x + c))*a^3 + a^3*(2*(cos(d*x + c)^3 + cos(d*x
 + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + lo
g(cos(d*x + c) - 1)) - 12*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(co
s(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 16*a^3/tan(d*x + c)^3)/d
```

mupad [B] time = 8.89, size = 237, normalized size = 2.37

$$\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{3\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{\cdot}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] (3*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - (a^3*cot(c/2 + (d*x)/2)^3)/(8*d) - (a^
3*cot(c/2 + (d*x)/2)^4)/(64*d) - (3*a^3*cot(c/2 + (d*x)/2)^2)/(8*d) + (a^3*
tan(c/2 + (d*x)/2)^3)/(8*d) + (a^3*tan(c/2 + (d*x)/2)^4)/(64*d) - (2*a^3*at
an((8*cos(c/2 + (d*x)/2) + 13*sin(c/2 + (d*x)/2))/(13*cos(c/2 + (d*x)/2) -
```

```
8*sin(c/2 + (d*x)/2))))/d - (13*a^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/(8*d) - (a^3*cot(c/2 + (d*x)/2))/(8*d) + (a^3*tan(c/2 + (d*x)/2))/(8*d
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.292 $\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=100

$$-\frac{a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{7a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] 7/8*a^3*arctanh(cos(d*x+c))/d-4/3*a^3*cot(d*x+c)^3/d-1/5*a^3*cot(d*x+c)^5/d
-1/8*a^3*cot(d*x+c)*csc(d*x+c)/d-3/4*a^3*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A] time = 0.24, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2611, 3770, 2607, 30, 3768, 14}

$$-\frac{a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{7a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (4*a^3*Cot[c + d*x]^3)/(3*d) - (a^3*C
ot[c + d*x]^5)/(5*d) - (a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (3*a^3*Cot[c
 + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^2(c+dx)\csc(c+dx)+3a^3\cot^2(c+dx)\csc^2(c+d\right.$$

$$= a^3\int \cot^2(c+dx)\csc(c+dx)\,dx + a^3\int \cot^2(c+dx)\csc^4(c$$

$$= -\frac{a^3\cot(c+dx)\csc(c+dx)}{2d} - \frac{3a^3\cot(c+dx)\csc^3(c+dx)}{4d}$$

$$= \frac{a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^3\cot^3(c+dx)}{d} - \frac{a^3\cot(c+dx)\,c}{8d}$$

$$= \frac{7a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{4a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+d}{5d}$$

Mathematica [B] time = 0.13, size = 267, normalized size = 2.67

$$a^3\left(-\frac{17\tan\left(\frac{1}{2}(c+dx)\right)}{30d}+\frac{17\cot\left(\frac{1}{2}(c+dx)\right)}{30d}-\frac{3\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{3\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\mathrm{se}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((17*Cot[(c + d*x)/2])/(30*d) - Csc[(c + d*x)/2]^2/(32*d) - (59*Cot[(c
+ d*x)/2]*Csc[(c + d*x)/2]^2)/(480*d) - (3*Csc[(c + d*x)/2]^4)/(64*d) - (Co
t[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(160*d) + (7*Log[Cos[(c + d*x)/2]])/(8*d
) - (7*Log[Sin[(c + d*x)/2]])/(8*d) + Sec[(c + d*x)/2]^2/(32*d) + (3*Sec[(c
 + d*x)/2]^4)/(64*d) - (17*Tan[(c + d*x)/2])/(30*d) + (59*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2])/(480*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(160*d)
)
```

fricas [B] time = 0.49, size = 190, normalized size = 1.90

$$\frac{272\,a^3\cos\left(dx+c\right)^5-320\,a^3\cos\left(dx+c\right)^3+105\left(a^3\cos\left(dx+c\right)^4-2\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)\right.}{240}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/240*(272*a^3*cos(d*x + c)^5 - 320*a^3*cos(d*x + c)^3 + 105*(a^3*cos(d*x +
 c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) - 105*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) + 30*(a^3*cos(d*x + c)^3 - 7*a^3*cos(d*x + c))*sin
(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [B] time = 0.27, size = 196, normalized size = 1.96

$$\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+130\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-840\,a^3\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*tan(1/2*d*x + 1/2*c)^4 + 130*a
^3*tan(1/2*d*x + 1/2*c)^3 + 120*a^3*tan(1/2*d*x + 1/2*c)^2 - 840*a^3*log(ab
s(tan(1/2*d*x + 1/2*c))) - 420*a^3*tan(1/2*d*x + 1/2*c) + (1918*a^3*tan(1/2
*d*x + 1/2*c)^5 + 420*a^3*tan(1/2*d*x + 1/2*c)^4 - 120*a^3*tan(1/2*d*x + 1/
2*c)^3 - 130*a^3*tan(1/2*d*x + 1/2*c)^2 - 45*a^3*tan(1/2*d*x + 1/2*c) - 6*a
^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.32, size = 136, normalized size = 1.36

$$-\frac{7a^3\left(\cos^3(dx+c)\right)}{8d\sin(dx+c)^2}-\frac{7a^3\cos(dx+c)}{8d}-\frac{7a^3\ln(\csc(dx+c)-\cot(dx+c))}{8d}-\frac{17a^3\left(\cos^3(dx+c)\right)}{15d\sin(dx+c)^3}-\frac{3a^3\left(\cos^3(dx+\right.}{4d\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -7/8/d*a^3/sin(d*x+c)^2*cos(d*x+c)^3-7/8*a^3*cos(d*x+c)/d-7/8/d*a^3*ln(csc(
d*x+c)-cot(d*x+c))-17/15/d*a^3/sin(d*x+c)^3*cos(d*x+c)^3-3/4/d*a^3/sin(d*x+
c)^4*cos(d*x+c)^3-1/5/d*a^3/sin(d*x+c)^5*cos(d*x+c)^3
```

maxima [A] time = 0.33, size = 155, normalized size = 1.55

$$-\frac{45\,a^3\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-60\,a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(d\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/240*(45*a^3*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 60*a^3*(2
*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x +
c) - 1)) + 240*a^3/tan(d*x + c)^3 + 16*(5*tan(d*x + c)^2 + 3)*a^3/tan(d*x +
 c)^5)/d
```

mupad [B] time = 9.30, size = 291, normalized size = 2.91

$$-\frac{a^3\left(6\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-45\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+45\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-130\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] -(a^3*(6*cos(c/2 + (d*x)/2)^10 - 6*sin(c/2 + (d*x)/2)^10 - 45*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^9 + 45*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) - 1
30*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 - 120*cos(c/2 + (d*x)/2)^3*sin
(c/2 + (d*x)/2)^7 + 420*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 - 420*cos
(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 120*cos(c/2 + (d*x)/2)^7*sin(c/2 +
 (d*x)/2)^3 + 130*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 840*log(sin(c
/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5
))/(960*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.293 $\int \cot^2(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=124

$$-\frac{3a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{7a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{17a^3\cot(c+dx)\csc^3}{24d}$$

```
[Out] 7/16*a^3*arctanh(cos(d*x+c))/d-4/3*a^3*cot(d*x+c)^3/d-3/5*a^3*cot(d*x+c)^5/
d+7/16*a^3*cot(d*x+c)*csc(d*x+c)/d-17/24*a^3*cot(d*x+c)*csc(d*x+c)^3/d-1/6*
a^3*cot(d*x+c)*csc(d*x+c)^5/d
```

Rubi [A] time = 0.28, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2607, 30, 2611, 3768, 3770, 14}

$$-\frac{3a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{7a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{17a^3\cot(c+dx)\csc^3}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*a^3*ArcTanh[Cos[c + d*x]])/(16*d) - (4*a^3*Cot[c + d*x]^3)/(3*d) - (3*a^
3*Cot[c + d*x]^5)/(5*d) + (7*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (17*a^
3*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(
6*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^2(c+dx)\csc^2(c+dx)+3a^3\cot^2(c+dx)\csc^3(c+\right.$$

$$= a^3\int \cot^2(c+dx)\csc^2(c+dx)\,dx + a^3\int \cot^2(c+dx)\csc^5$$

$$= -\frac{3a^3\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{a^3\cot(c+dx)\csc^5(c+dx}{6d}$$

$$= -\frac{a^3\cot^3(c+dx)}{3d} + \frac{3a^3\cot(c+dx)\csc(c+dx)}{8d} - \frac{17a^3\cot}{}$$

$$= \frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{4a^3\cot^3(c+dx)}{3d} - \frac{3a^3\cot^5(c+}{5d}$$

$$= \frac{7a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{4a^3\cot^3(c+dx)}{3d} - \frac{3a^3\cot^5(c+}{5d}$$

Mathematica [B] time = 3.59, size = 252, normalized size = 2.03

$$-\frac{a^3\sin(c+dx)(\sin(c+dx)+1)^3\left(\csc^6\left(\frac{1}{2}(c+dx)\right)(5\csc(c+dx)+18)+\csc^4\left(\frac{1}{2}(c+dx)\right)(90\csc(c+dx)+34)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/1920*(a^3*(Csc[(c + d*x)/2]^6*(18 + 5*Csc[c + d*x]) + Csc[(c + d*x)/2]^4
*(34 + 90*Csc[c + d*x]) - 2*Csc[(c + d*x)/2]^2*(176 + 105*Csc[c + d*x]) - 8
40*Csc[c + d*x]*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + (97 + 159
*Cos[c + d*x] + 44*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^6 + 840*Csc[c + d*x]^
3*Sin[(c + d*x)/2]^2 - 1440*Csc[c + d*x]^5*Sin[(c + d*x)/2]^4 - 320*Csc[c +
 d*x]^7*Sin[(c + d*x)/2]^6)*Sin[c + d*x]*(1 + Sin[c + d*x])^3)/(d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [B] time = 0.50, size = 227, normalized size = 1.83

$$-\frac{210\,a^3\cos(dx+c)^5-80\,a^3\cos(dx+c)^3-210\,a^3\cos(dx+c)-105\left(a^3\cos(dx+c)^6-3\,a^3\cos(dx+c)^4+3\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/480*(210*a^3*cos(d*x + c)^5 - 80*a^3*cos(d*x + c)^3 - 210*a^3*cos(d*x +
c) - 105*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2
- a^3)*log(1/2*cos(d*x + c) + 1/2) + 105*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*
x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) + 32*(1
1*a^3*cos(d*x + c)^5 - 20*a^3*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)
^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [B] time = 0.28, size = 228, normalized size = 1.84

$$\frac{5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+140\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^3\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^3*tan(1/2*d*x + 1/2*c)^5 + 105*
a^3*tan(1/2*d*x + 1/2*c)^4 + 140*a^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/
2*d*x + 1/2*c)^2 - 840*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 600*a^3*tan(1/2
*d*x + 1/2*c) + (2058*a^3*tan(1/2*d*x + 1/2*c)^6 + 600*a^3*tan(1/2*d*x + 1/
2*c)^5 + 15*a^3*tan(1/2*d*x + 1/2*c)^4 - 140*a^3*tan(1/2*d*x + 1/2*c)^3 - 1
05*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*a^3*tan(1/2*d*x + 1/2*c) - 5*a^3)/tan(1/
2*d*x + 1/2*c)^6)/d
```

maple [A] time = 0.35, size = 160, normalized size = 1.29

$$-\frac{11a^3\left(\cos^3(dx+c)\right)}{15d\sin(dx+c)^3}-\frac{7a^3\left(\cos^3(dx+c)\right)}{8d\sin(dx+c)^4}-\frac{7a^3\left(\cos^3(dx+c)\right)}{16d\sin(dx+c)^2}-\frac{7a^3\cos(dx+c)}{16d}-\frac{7a^3\ln(\csc(dx+c)-\cot(dx}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -11/15/d*a^3/sin(d*x+c)^3*cos(d*x+c)^3-7/8/d*a^3/sin(d*x+c)^4*cos(d*x+c)^3-
7/16/d*a^3/sin(d*x+c)^2*cos(d*x+c)^3-7/16*a^3*cos(d*x+c)/d-7/16/d*a^3*ln(cs
c(d*x+c)-cot(d*x+c))-3/5/d*a^3/sin(d*x+c)^5*cos(d*x+c)^3-1/6/d*a^3/sin(d*x+
c)^6*cos(d*x+c)^3
```

maxima [A] time = 0.35, size = 200, normalized size = 1.61

$$-\frac{5\,a^3\left(\frac{2\left(3\,\cos(dx+c)^5-8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos(dx+c)+1\right)+3\,\log\left(\cos(dx+c)-1\right)\right)+90\,a^3\left(\frac{2\left(\cos(dx\right.}{\cos(dx+c)^4}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/480*(5*a^3*(2*(3*cos(d*x + c)^5 - 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(co
s(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c
) + 1) + 3*log(cos(d*x + c) - 1)) + 90*a^3*(2*(cos(d*x + c)^3 + cos(d*x + c
))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(co
s(d*x + c) - 1)) + 160*a^3/tan(d*x + c)^3 + 96*(5*tan(d*x + c)^2 + 3)*a^3/t
an(d*x + c)^5)/d
```

mupad [B] time = 9.67, size = 339, normalized size = 2.73

$$-\frac{a^3\left(5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-36\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+36\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] -(a^3*(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 - 36*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^11 + 36*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) -
 105*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 140*cos(c/2 + (d*x)/2)^3*
sin(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 600*c
os(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 600*cos(c/2 + (d*x)/2)^7*sin(c/2
 + (d*x)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 140*cos(c/2
+ (d*x)/2)^9*sin(c/2 + (d*x)/2)^3 + 105*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*
x)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)
^6*sin(c/2 + (d*x)/2)^6))/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6
)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.294 $\int \cos^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=137

$$-\frac{7a^4\cos^3(c+dx)}{8d}-\frac{21\cos^3(c+dx)\left(a^4\sin(c+dx)+a^4\right)}{40d}+\frac{21a^4\sin(c+dx)\cos(c+dx)}{16d}+\frac{21a^4x}{16}-\frac{3\cos^3(c+dx)}{}$$

```
[Out] 21/16*a^4*x-7/8*a^4*cos(d*x+c)^3/d+21/16*a^4*cos(d*x+c)*sin(d*x+c)/d-1/6*a*
cos(d*x+c)^3*(a+a*sin(d*x+c))^3/d-3/10*cos(d*x+c)^3*(a^2+a^2*sin(d*x+c))^2/
d-21/40*cos(d*x+c)^3*(a^4+a^4*sin(d*x+c))/d
```

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2678, 2669, 2635, 8}

$$-\frac{7a^4\cos^3(c+dx)}{8d}-\frac{3\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)^2}{10d}-\frac{21\cos^3(c+dx)\left(a^4\sin(c+dx)+a^4\right)}{40d}+\frac{21a^4\sin(c+d}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (21*a^4*x)/16 - (7*a^4*Cos[c + d*x]^3)/(8*d) + (21*a^4*Cos[c + d*x]*Sin[c +
 d*x])/(16*d) - (a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(6*d) - (3*Cos[c
+ d*x]^3*(a^2 + a^2*Sin[c + d*x])^2)/(10*d) - (21*Cos[c + d*x]^3*(a^4 + a^4
*Sin[c + d*x]))/(40*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^4\,dx = -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d} + \frac{1}{2}(3a)\int \cos^2(c+dx)(a+a\sin(c$$

$$= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d} - \frac{3\cos^3(c+dx)\left(a^2+a^2\sin(c+dx\right.}{10d}$$

$$= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d} - \frac{3\cos^3(c+dx)\left(a^2+a^2\sin(c+dx\right.}{10d}$$

$$= -\frac{7a^4\cos^3(c+dx)}{8d} - \frac{a\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d} - \frac{3\cos^3(c+dx)}{}$$

$$= -\frac{7a^4\cos^3(c+dx)}{8d} + \frac{21a^4\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^3(c+dx)(a+a}{6d}$$

$$= \frac{21a^4x}{16} - \frac{7a^4\cos^3(c+dx)}{8d} + \frac{21a^4\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^3(c+d}{}$$

Mathematica [A] time = 0.44, size = 151, normalized size = 1.10

$$-\frac{a^4\left(630\sqrt{1-\sin(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\,\left(40\sin^6(c+dx)+152\sin^5(c+dx)+158\sin^4\right.\right.}{240d(\sin(c+dx)-1)^2(\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/240*(a^4*Cos[c + d*x]^3*(630*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt
[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(448 - 373*Sin[c + d*x] - 331*S
in[c + d*x]^2 - 94*Sin[c + d*x]^3 + 158*Sin[c + d*x]^4 + 152*Sin[c + d*x]^5
 + 40*Sin[c + d*x]^6)))/(d*(-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^(3/2))
```

fricas [A] time = 0.49, size = 85, normalized size = 0.62

$$\frac{192\,a^4\cos(dx+c)^5 - 640\,a^4\cos(dx+c)^3 + 315\,a^4dx + 5\left(8\,a^4\cos(dx+c)^5 - 86\,a^4\cos(dx+c)^3 + 63\,a^4\cos(d\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(192*a^4*cos(d*x + c)^5 - 640*a^4*cos(d*x + c)^3 + 315*a^4*d*x + 5*(8
*a^4*cos(d*x + c)^5 - 86*a^4*cos(d*x + c)^3 + 63*a^4*cos(d*x + c))*sin(d*x
+ c))/d
```

giac [A] time = 0.21, size = 106, normalized size = 0.77

$$\frac{21}{16}\,a^4x + \frac{a^4\cos(5\,dx+5\,c)}{20\,d} - \frac{5\,a^4\cos(3\,dx+3\,c)}{12\,d} - \frac{3\,a^4\cos(dx+c)}{2\,d} + \frac{a^4\sin(6\,dx+6\,c)}{192\,d} - \frac{13\,a^4\sin(4\,dx+4\,c)}{64\,d} + \frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 21/16*a^4*x + 1/20*a^4*cos(5*d*x + 5*c)/d - 5/12*a^4*cos(3*d*x + 3*c)/d - 3
/2*a^4*cos(d*x + c)/d + 1/192*a^4*sin(6*d*x + 6*c)/d - 13/64*a^4*sin(4*d*x
+ 4*c)/d + 15/64*a^4*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.32, size = 182, normalized size = 1.33

$$\frac{a^4\left(-\frac{(\sin^3(dx+c))(\cos^3(dx+c))}{6} - \frac{(\cos^3(dx+c))\sin(dx+c)}{8} + \frac{\cos(dx+c)\sin(dx+c)}{16} + \frac{dx}{16} + \frac{c}{16}\right) + 4a^4\left(-\frac{(\sin^2(dx+c))(\cos^3(dx+c))}{5} - \frac{2(\mathrm{co}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*c
os(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+4*a^4*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3
-2/15*cos(d*x+c)^3)+6*a^4*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(
d*x+c)+1/8*d*x+1/8*c)-4/3*a^4*cos(d*x+c)^3+a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c))
```

maxima [A] time = 0.32, size = 128, normalized size = 0.93

$$-\frac{1280\,a^4\cos(dx+c)^3 - 256\left(3\,\cos(dx+c)^5 - 5\,\cos(dx+c)^3\right)a^4 + 5\left(4\,\sin(2\,dx+2\,c)^3 - 12\,dx - 12\,c + 3\,\mathrm{sin}\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/960*(1280*a^4*cos(d*x + c)^3 - 256*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)
*a^4 + 5*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^4 -
180*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^4 - 240*(2*d*x + 2*c + sin(2*d*x + 2
*c))*a^4)/d
```

mupad [B] time = 10.73, size = 349, normalized size = 2.55

$$\frac{21\,a^4\,x}{16}-\frac{\frac{63\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{63\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{235\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24}+\frac{235\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{24}-\frac{5\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{a^4\,(315\,c+315\,d\,x)}{240}-a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (21*a^4*x)/16 - ((63*a^4*tan(c/2 + (d*x)/2)^7)/4 - (63*a^4*tan(c/2 + (d*x)/
2)^5)/4 - (235*a^4*tan(c/2 + (d*x)/2)^3)/24 + (235*a^4*tan(c/2 + (d*x)/2)^9
)/24 - (5*a^4*tan(c/2 + (d*x)/2)^11)/8 + (a^4*(315*c + 315*d*x))/240 - (a^4
*(315*c + 315*d*x - 896))/240 + tan(c/2 + (d*x)/2)^10*((a^4*(315*c + 315*d*
x))/40 - (a^4*(1890*c + 1890*d*x - 1920))/240) + tan(c/2 + (d*x)/2)^2*((a^4
*(315*c + 315*d*x))/40 - (a^4*(1890*c + 1890*d*x - 3456))/240) + tan(c/2 +
(d*x)/2)^4*((a^4*(315*c + 315*d*x))/16 - (a^4*(4725*c + 4725*d*x - 3840))/2
40) + tan(c/2 + (d*x)/2)^8*((a^4*(315*c + 315*d*x))/16 - (a^4*(4725*c + 472
5*d*x - 9600))/240) + tan(c/2 + (d*x)/2)^6*((a^4*(315*c + 315*d*x))/12 - (a
^4*(6300*c + 6300*d*x - 8960))/240) + (5*a^4*tan(c/2 + (d*x)/2))/8)/(d*(tan
(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [A] time = 5.09, size = 381, normalized size = 2.78

$$\begin{cases} \frac{a^4x\sin^6(c+dx)}{16}+\frac{3a^4x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3a^4x\sin^4(c+dx)}{4}+\frac{3a^4x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{2}+\frac{a^4x\sin^2(}{2} \\ x\,(a\sin(c)+a)^4\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x*sin(c + d*x)**6/16 + 3*a**4*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**4*x*sin(c + d*x)**4/4 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*x
)**4/16 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + a**4*x*sin(c + d*x)*
```

```
*2/2 + a**4*x*cos(c + d*x)**6/16 + 3*a**4*x*cos(c + d*x)**4/4 + a**4*x*cos(
c + d*x)**2/2 + a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) - a**4*sin(c + d*x
)**3*cos(c + d*x)**3/(6*d) + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/(4*d) - 4*
a**4*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - a**4*sin(c + d*x)*cos(c + d*x)
**5/(16*d) - 3*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + a**4*sin(c + d*x)*
cos(c + d*x)/(2*d) - 8*a**4*cos(c + d*x)**5/(15*d) - 4*a**4*cos(c + d*x)**3
/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*cos(c)**2, True))
```

3.295 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=117

$$\frac{a^4\cos^5(c+dx)}{5d}-\frac{7a^4\cos^3(c+dx)}{3d}+\frac{a^4\cos(c+dx)}{d}-\frac{a^4\sin(c+dx)\cos^3(c+dx)}{d}+\frac{5a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^4 \text{t}}{}$$

```
[Out] 5/2*a^4*x-a^4*arctanh(cos(d*x+c))/d+a^4*cos(d*x+c)/d-7/3*a^4*cos(d*x+c)^3/d
+1/5*a^4*cos(d*x+c)^5/d+5/2*a^4*cos(d*x+c)*sin(d*x+c)/d-a^4*cos(d*x+c)^3*si
n(d*x+c)/d
```

Rubi [A] time = 0.20, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2873, 2635, 8, 2592, 321, 206, 2565, 30, 2568, 14}

$$\frac{a^4\cos^5(c+dx)}{5d}-\frac{7a^4\cos^3(c+dx)}{3d}+\frac{a^4\cos(c+dx)}{d}-\frac{a^4\sin(c+dx)\cos^3(c+dx)}{d}+\frac{5a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^4 \text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (5*a^4*x)/2 - (a^4*ArcTanh[Cos[c + d*x]])/d + (a^4*Cos[c + d*x])/d - (7*a^4
*Cos[c + d*x]^3)/(3*d) + (a^4*Cos[c + d*x]^5)/(5*d) + (5*a^4*Cos[c + d*x]*S
in[c + d*x])/(2*d) - (a^4*Cos[c + d*x]^3*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
```

```
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^4\,dx = \int \left(4a^4\cos^2(c+dx) + a^4\cos(c+dx)\cot(c+dx) + 6a^4\cos^2(c\right.$$
$$= a^4\int \cos(c+dx)\cot(c+dx)\,dx + a^4\int \cos^2(c+dx)\sin^3(c+$$
$$= \frac{2a^4\cos(c+dx)\sin(c+dx)}{d} - \frac{a^4\cos^3(c+dx)\sin(c+dx)}{d} + a$$
$$= 2a^4x + \frac{a^4\cos(c+dx)}{d} - \frac{2a^4\cos^3(c+dx)}{d} + \frac{5a^4\cos(c+dx)\,\mathrm{s}}{2d}$$
$$= \frac{5a^4x}{2} - \frac{a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^4\cos(c+dx)}{d} - \frac{7a^4\cos^3(c}{3d}$$

Mathematica [A] time = 0.88, size = 95, normalized size = 0.81

$$\frac{a^4\left(-150\cos(c+dx) - 125\cos(3(c+dx)) + 3\cos(5(c+dx)) + 30\left(8\sin(2(c+dx)) - \sin(4(c+dx)) + 8\log\left(\sin\right.\right.\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(-150*Cos[c + d*x] - 125*Cos[3*(c + d*x)] + 3*Cos[5*(c + d*x)] + 30*(2
0*c + 20*d*x - 8*Log[Cos[(c + d*x)/2]] + 8*Log[Sin[(c + d*x)/2]] + 8*Sin[2*
(c + d*x)] - Sin[4*(c + d*x)])))/(240*d)
```

fricas [A] time = 0.54, size = 115, normalized size = 0.98

$$\frac{6\,a^4\cos\left(dx+c\right)^5 - 70\,a^4\cos\left(dx+c\right)^3 + 75\,a^4dx + 30\,a^4\cos\left(dx+c\right) - 15\,a^4\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right) + 15\,a^4\log}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(6*a^4*cos(d*x + c)^5 - 70*a^4*cos(d*x + c)^3 + 75*a^4*d*x + 30*a^4*co
s(d*x + c) - 15*a^4*log(1/2*cos(d*x + c) + 1/2) + 15*a^4*log(-1/2*cos(d*x +
 c) + 1/2) - 15*(2*a^4*cos(d*x + c)^3 - 5*a^4*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.24, size = 181, normalized size = 1.55

$$\frac{75\,(dx+c)a^4+30\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(45\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+150\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+210\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+300\,a^4\tan(}{}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(75*(d*x + c)*a^4 + 30*a^4*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(45*a^4*
tan(1/2*d*x + 1/2*c)^9 + 150*a^4*tan(1/2*d*x + 1/2*c)^8 + 210*a^4*tan(1/2*d
*x + 1/2*c)^7 + 300*a^4*tan(1/2*d*x + 1/2*c)^6 + 40*a^4*tan(1/2*d*x + 1/2*c
)^4 - 210*a^4*tan(1/2*d*x + 1/2*c)^3 + 20*a^4*tan(1/2*d*x + 1/2*c)^2 - 45*a
^4*tan(1/2*d*x + 1/2*c) + 34*a^4)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

maple [A] time = 0.37, size = 135, normalized size = 1.15

$$-\frac{a^4\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5d}-\frac{32a^4\left(\cos^3(dx+c)\right)}{15d}-\frac{a^4\left(\cos^3(dx+c)\right)\sin(dx+c)}{d}+\frac{5a^4\cos(dx+c)\sin(}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/5/d*a^4*cos(d*x+c)^3*sin(d*x+c)^2-32/15*a^4*cos(d*x+c)^3/d-a^4*cos(d*x+c
)^3*sin(d*x+c)/d+5/2*a^4*cos(d*x+c)*sin(d*x+c)/d+5/2*a^4*x+5/2/d*a^4*c+a^4*
cos(d*x+c)/d+1/d*a^4*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.33, size = 125, normalized size = 1.07

$$-\frac{240\,a^4\cos(dx+c)^3-8\left(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3\right)a^4-15\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^4-120\,(2\,dx}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/120*(240*a^4*cos(d*x + c)^3 - 8*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^
4 - 15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^4 - 120*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a^4 - 60*a^4*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x
+ c) - 1)))/d
```

mupad [B] time = 10.42, size = 295, normalized size = 2.52

$$\frac{a^4\ \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{5\,a^4\ \mathrm{atan}\left(\frac{25\,a^8}{10\,a^8-25\,a^8\ \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{10\,a^8\ \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{10\,a^8-25\,a^8\ \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{3\,a^4\ \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+10\,a^4\ \tan\left(\frac{c}{2}+\frac{d}{}\right.}{d\,\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^4)/sin(c + d*x),x)
```

```
[Out] (a^4*log(tan(c/2 + (d*x)/2)))/d + (5*a^4*atan((25*a^8)/(10*a^8 - 25*a^8*tan
(c/2 + (d*x)/2)) + (10*a^8*tan(c/2 + (d*x)/2))/(10*a^8 - 25*a^8*tan(c/2 + (
d*x)/2))))/d - ((4*a^4*tan(c/2 + (d*x)/2)^2)/3 - 14*a^4*tan(c/2 + (d*x)/2)^
3 + (8*a^4*tan(c/2 + (d*x)/2)^4)/3 + 20*a^4*tan(c/2 + (d*x)/2)^6 + 14*a^4*t
an(c/2 + (d*x)/2)^7 + 10*a^4*tan(c/2 + (d*x)/2)^8 + 3*a^4*tan(c/2 + (d*x)/2
)^9 + (34*a^4)/15 - 3*a^4*tan(c/2 + (d*x)/2))/(d*(5*tan(c/2 + (d*x)/2)^2 +
10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8
+ tan(c/2 + (d*x)/2)^10 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.296 $\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=116

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{23a^4\sin(c+dx)\cos(c+dx)}{8d}$$

```
[Out] 17/8*a^4*x-4*a^4*arctanh(cos(d*x+c))/d+4*a^4*cos(d*x+c)/d-4/3*a^4*cos(d*x+c
)^3/d-a^4*cot(d*x+c)/d+23/8*a^4*cos(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos(d*x+c)*
sin(d*x+c)^3/d
```

Rubi [A] time = 0.17, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2709, 3770, 3767, 8, 2635, 2633}

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{23a^4\sin(c+dx)\cos(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (17*a^4*x)/8 - (4*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x])/d - (
4*a^4*Cos[c + d*x]^3)/(3*d) - (a^4*Cot[c + d*x])/d + (23*a^4*Cos[c + d*x]*S
in[c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(5a^6+4a^6\csc(c+dx)+a^6\csc^2(c+dx)-5a^6\sin^2(c+dx)-4a^6\sin^3(\right.}{a^2}$$
$$= 5a^4x+a^4\int \csc^2(c+dx)\,dx-a^4\int \sin^4(c+dx)\,dx+\left(4a^4\right)\int \csc(c+d$$
$$= 5a^4x-\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{5a^4\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^4\cos(}{}$$
$$= \frac{5a^4x}{2}-\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}-$$
$$= \frac{17a^4x}{8}-\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}$$

Mathematica [A] time = 1.63, size = 136, normalized size = 1.17

$$\frac{a^4\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(408c\sin(c+dx)+408dx\sin(c+dx)+320\sin(2(c+dx))-32\sin(4(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-48*Cos[c + d*x] - 147*Cos[3*(c + d
*x)] + 3*Cos[5*(c + d*x)] + 408*c*Sin[c + d*x] + 408*d*x*Sin[c + d*x] - 768
```

```
*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 768*Log[Sin[(c + d*x)/2]]*Sin[c + d*x
] + 320*Sin[2*(c + d*x)] - 32*Sin[4*(c + d*x)]))/(384*d)
```

fricas [A] time = 0.52, size = 135, normalized size = 1.16

$$\frac{6\,a^4\cos(dx+c)^5 - 81\,a^4\cos(dx+c)^3 - 48\,a^4\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) + 48\,a^4\log\left(-\frac{1}{2}\cos(dx+c\right.}{24\,d\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/24*(6*a^4*cos(d*x + c)^5 - 81*a^4*cos(d*x + c)^3 - 48*a^4*log(1/2*cos(d*x
 + c) + 1/2)*sin(d*x + c) + 48*a^4*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) + 51*a^4*cos(d*x + c) - (32*a^4*cos(d*x + c)^3 - 51*a^4*d*x - 96*a^4*cos(
d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.25, size = 194, normalized size = 1.67

$$\frac{51\,(dx+c)a^4 + 96\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) + 12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - \frac{12\left(8\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^4\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} - \frac{2\left(69\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(51*(d*x + c)*a^4 + 96*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 12*a^4*tan
(1/2*d*x + 1/2*c) - 12*(8*a^4*tan(1/2*d*x + 1/2*c) + a^4)/tan(1/2*d*x + 1/2
*c) - 2*(69*a^4*tan(1/2*d*x + 1/2*c)^7 + 93*a^4*tan(1/2*d*x + 1/2*c)^5 - 19
2*a^4*tan(1/2*d*x + 1/2*c)^4 - 93*a^4*tan(1/2*d*x + 1/2*c)^3 - 256*a^4*tan(
1/2*d*x + 1/2*c)^2 - 69*a^4*tan(1/2*d*x + 1/2*c) - 64*a^4)/(tan(1/2*d*x + 1
/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.32, size = 127, normalized size = 1.09

$$-\frac{a^4\left(\cos^3(dx+c)\right)\sin(dx+c)}{4d} + \frac{25a^4\cos(dx+c)\sin(dx+c)}{8d} + \frac{17a^4x}{8} + \frac{17a^4c}{8d} - \frac{4a^4\left(\cos^3(dx+c)\right)}{3d} + \frac{4a^4\cos(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

[Out] -1/4*a^4*cos(d*x+c)^3*sin(d*x+c)/d+25/8*a^4*cos(d*x+c)*sin(d*x+c)/d+17/8*a^4*x+17/8/d*a^4*c-4/3*a^4*cos(d*x+c)^3/d+4*a^4*cos(d*x+c)/d+4/d*a^4*ln(csc(d*x+c)-cot(d*x+c))-a^4*cot(d*x+c)/d

maxima [A] time = 0.43, size = 117, normalized size = 1.01

$$-\frac{128\,a^4\cos\left(dx+c\right)^3-3\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^4-144\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^4+96\left(dx+c+\frac{1}{\tan}\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")

[Out] -1/96*(128*a^4*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^4 - 144*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 96*(d*x + c + 1/tan(d*x + c))*a^4 - 192*a^4*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d

mupad [B] time = 8.83, size = 295, normalized size = 2.54

$$\frac{4\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{17\,a^4\,\mathrm{atan}\left(\frac{289\,a^8}{16\left(34\,a^8-\frac{289\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{34\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{34\,a^8-\frac{289\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}}\right)}{4\,d}+\frac{-\frac{25\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}-\frac{39\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{2}}{d\left(2\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^4)/sin(c + d*x)^2,x)

[Out] (4*a^4*log(tan(c/2 + (d*x)/2)))/d + (17*a^4*atan((289*a^8)/(16*(34*a^8 - (289*a^8*tan(c/2 + (d*x)/2))/16)) + (34*a^8*tan(c/2 + (d*x)/2))/(34*a^8 - (289*a^8*tan(c/2 + (d*x)/2))/16)))/(4*d) + ((15*a^4*tan(c/2 + (d*x)/2)^2)/2 + (128*a^4*tan(c/2 + (d*x)/2)^3)/3 + (19*a^4*tan(c/2 + (d*x)/2)^4)/2 + 32*a^4*tan(c/2 + (d*x)/2)^5 - (39*a^4*tan(c/2 + (d*x)/2)^6)/2 - (25*a^4*tan(c/2 + (d*x)/2)^8)/2 - a^4 + (32*a^4*tan(c/2 + (d*x)/2))/3)/(d*(2*tan(c/2 + (d*x)/2) + 8*tan(c/2 + (d*x)/2)^3 + 12*tan(c/2 + (d*x)/2)^5 + 8*tan(c/2 + (d*x)/2)^7 + 2*tan(c/2 + (d*x)/2)^9)) + (a^4*tan(c/2 + (d*x)/2))/(2*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.297 $\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=104

$$\frac{\cos^5(c+dx)}{5ad}-\frac{2\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin^3(c+dx)\cos(c+dx)}{4ad}-\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

```
[Out] 3/8*x/a+cos(d*x+c)/a/d-2/3*cos(d*x+c)^3/a/d+1/5*cos(d*x+c)^5/a/d-3/8*cos(d*
x+c)*sin(d*x+c)/a/d-1/4*cos(d*x+c)*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.14, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2839, 2635, 8, 2633}

$$\frac{\cos^5(c+dx)}{5ad}-\frac{2\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin^3(c+dx)\cos(c+dx)}{4ad}-\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(8*a) + Cos[c + d*x]/(a*d) - (2*Cos[c + d*x]^3)/(3*a*d) + Cos[c + d*x
]^5/(5*a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) - (Cos[c + d*x]*Sin[c +
 d*x]^3)/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
```

```
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sin^4(c+dx)\,dx}{a} - \frac{\int \sin^5(c+dx)\,dx}{a}$$
$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{4ad} + \frac{3\int \sin^2(c+dx)\,dx}{4a} + \frac{\mathrm{Subst}\left(\int \left(1-2x^2+x^4\right) d\right.}{ad}$$
$$= \frac{\cos(c+dx)}{ad} - \frac{2\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{3\cos(c+dx)\sin(c+dx)}{8ad} - \frac{c}{}$$
$$= \frac{3x}{8a} + \frac{\cos(c+dx)}{ad} - \frac{2\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{3\cos(c+dx)\sin(c+dx}{8ad}$$

Mathematica [B] time = 5.24, size = 281, normalized size = 2.70

$$\frac{1}{480}\left(\frac{60\sin^2\left(\frac{1}{2}(c+dx)\right)}{d(a\sin(c+dx)+a)} - \frac{300\sin(c)\sin(dx)}{ad} + \frac{50\sin(3c)\sin(3dx)}{ad} - \frac{6\sin(5c)\sin(5dx)}{ad} + \frac{30\sin(c+dx)}{ad(\sin(c+dx)+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((180*x)/a + (300*Cos[c]*Cos[d*x])/(a*d) - (50*Cos[3*c]*Cos[3*d*x])/(a*d) +
 (6*Cos[5*c]*Cos[5*d*x])/(a*d) - (120*Cos[2*d*x]*Sin[2*c])/(a*d) + (15*Cos[
4*d*x]*Sin[4*c])/(a*d) - (300*Sin[c]*Sin[d*x])/(a*d) - (120*Cos[2*c]*Sin[2*
d*x])/(a*d) + (50*Sin[3*c]*Sin[3*d*x])/(a*d) + (15*Cos[4*c]*Sin[4*d*x])/(a*
d) - (6*Sin[5*c]*Sin[5*d*x])/(a*d) - (60*Sin[(d*x)/2])/(a*d*(Cos[c/2] + Sin
[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (30*Sin[c + d*x])/(a*d*(1 +
 Sin[c + d*x])) + (60*Sin[(c + d*x)/2]^2)/(d*(a + a*Sin[c + d*x])))/480
```

fricas [A] time = 0.48, size = 68, normalized size = 0.65

$$\frac{24\cos(dx+c)^5 - 80\cos(dx+c)^3 + 45\,dx + 15\left(2\cos(dx+c)^3 - 5\cos(dx+c)\right)\sin(dx+c) + 120\cos(dx+}{120\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*cos(d*x + c)^5 - 80*cos(d*x + c)^3 + 45*d*x + 15*(2*cos(d*x + c)^
3 - 5*cos(d*x + c))*sin(d*x + c) + 120*cos(d*x + c))/(a*d)
```

giac [A] time = 0.17, size = 114, normalized size = 1.10

$$\frac{\frac{45\,(dx+c)}{a}+\frac{2\left(45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+210\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+640\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-210\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+320\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+64\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5}a}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(45*(d*x + c)/a + 2*(45*tan(1/2*d*x + 1/2*c)^9 + 210*tan(1/2*d*x + 1/
2*c)^7 + 640*tan(1/2*d*x + 1/2*c)^4 - 210*tan(1/2*d*x + 1/2*c)^3 + 320*tan(
1/2*d*x + 1/2*c)^2 - 45*tan(1/2*d*x + 1/2*c) + 64)/((tan(1/2*d*x + 1/2*c)^2
 + 1)^5*a))/d
```

maple [B] time = 0.30, size = 245, normalized size = 2.36

$$\frac{3\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{7\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{32\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{7\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{16}{3ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 3/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9+7/2/a/d/(1+tan(1/2*
d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7+32/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*ta
n(1/2*d*x+1/2*c)^4-7/2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3+
16/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2-3/4/a/d/(1+tan(1/2
*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)+16/15/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5+3/
4/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.42, size = 258, normalized size = 2.48

$$-\frac{\frac{\frac{45\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{320\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{210\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{640\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{210\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{45\,\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-64}{a+\frac{5\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10\,a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{10\,a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{5\,a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{45\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((45*sin(d*x + c)/(cos(d*x + c) + 1) - 320*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 210*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 640*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 210*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 45*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9 - 64)/(a + 5*a*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 10*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a*sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 + 5*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a*sin(d*x + c)^10
/(cos(d*x + c) + 1)^10) - 45*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 11.91, size = 107, normalized size = 1.03

$$\frac{3\,x}{8\,a}+\frac{\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+\frac{32\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{16}{15}}{a\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^4)/(a + a*sin(c + d*x)),x)
```

```
[Out] (3*x)/(8*a) + ((16*tan(c/2 + (d*x)/2)^2)/3 - (3*tan(c/2 + (d*x)/2))/4 - (7*
tan(c/2 + (d*x)/2)^3)/2 + (32*tan(c/2 + (d*x)/2)^4)/3 + (7*tan(c/2 + (d*x)/
2)^7)/2 + (3*tan(c/2 + (d*x)/2)^9)/4 + 16/15)/(a*d*(tan(c/2 + (d*x)/2)^2 +
1)^5)
```

sympy [A] time = 33.86, size = 1360, normalized size = 13.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((45*d*x*tan(c/2 + d*x/2)**10/(120*a*d*tan(c/2 + d*x/2)**10 + 600*
a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 +
 d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 225*d*x*tan(c/2 + d*x
/2)**8/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a
*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d
*x/2)**2 + 120*a*d) + 450*d*x*tan(c/2 + d*x/2)**6/(120*a*d*tan(c/2 + d*x/2)
**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*
d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 450*d*x*ta
n(c/2 + d*x/2)**4/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)*
*8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*
tan(c/2 + d*x/2)**2 + 120*a*d) + 225*d*x*tan(c/2 + d*x/2)**2/(120*a*d*tan(c
/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**
```

```
6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) +
 45*d*x/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*
a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 +
d*x/2)**2 + 120*a*d) + 90*tan(c/2 + d*x/2)**9/(120*a*d*tan(c/2 + d*x/2)**10
 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*ta
n(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 420*tan(c/2 +
d*x/2)**7/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 120
0*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2
+ d*x/2)**2 + 120*a*d) + 1280*tan(c/2 + d*x/2)**4/(120*a*d*tan(c/2 + d*x/2)
**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*
d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 420*tan(c/
2 + d*x/2)**3/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 +
 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(
c/2 + d*x/2)**2 + 120*a*d) + 640*tan(c/2 + d*x/2)**2/(120*a*d*tan(c/2 + d*x
/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200
*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 90*tan(
c/2 + d*x/2)/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 +
1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c
/2 + d*x/2)**2 + 120*a*d) + 128/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan
(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)
**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d), Ne(d, 0)), (x*sin(c)**4*cos(c
)**2/(a*sin(c) + a), True))
```

3.298 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=87

$$\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}+\frac{\sin^3(c+dx)\cos(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}-\frac{3x}{8a}$$

```
[Out] -3/8*x/a-cos(d*x+c)/a/d+1/3*cos(d*x+c)^3/a/d+3/8*cos(d*x+c)*sin(d*x+c)/a/d+
1/4*cos(d*x+c)*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.13, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2839, 2633, 2635, 8}

$$\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}+\frac{\sin^3(c+dx)\cos(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}-\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(8*a) - Cos[c + d*x]/(a*d) + Cos[c + d*x]^3/(3*a*d) + (3*Cos[c + d*x
]*Sin[c + d*x])/(8*a*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
```

```
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sin^3(c+dx)\,dx}{a} - \frac{\int \sin^4(c+dx)\,dx}{a}$$
$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4ad} - \frac{3\int \sin^2(c+dx)\,dx}{4a} - \frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(\right.}{ad}$$
$$= -\frac{\cos(c+dx)}{ad} + \frac{\cos^3(c+dx)}{3ad} + \frac{3\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos(c+dx)\sin^3(c}{4ad}$$
$$= -\frac{3x}{8a} - \frac{\cos(c+dx)}{ad} + \frac{\cos^3(c+dx)}{3ad} + \frac{3\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos(c+dx)\,\mathrm{s}}{4a}$$

Mathematica [B] time = 1.72, size = 271, normalized size = 3.11

$$\frac{-72dx\sin\left(\frac{c}{2}\right)+72\sin\left(\frac{c}{2}+dx\right)-72\sin\left(\frac{3c}{2}+dx\right)+24\sin\left(\frac{3c}{2}+2dx\right)+24\sin\left(\frac{5c}{2}+2dx\right)-8\sin\left(\frac{5c}{2}+3dx\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (24*(c - 3*d*x)*Cos[c/2] - 72*Cos[c/2 + d*x] - 72*Cos[(3*c)/2 + d*x] + 24*C
os[(3*c)/2 + 2*d*x] - 24*Cos[(5*c)/2 + 2*d*x] + 8*Cos[(5*c)/2 + 3*d*x] + 8*
Cos[(7*c)/2 + 3*d*x] - 3*Cos[(7*c)/2 + 4*d*x] + 3*Cos[(9*c)/2 + 4*d*x] - 48
*Sin[c/2] + 24*c*Sin[c/2] - 72*d*x*Sin[c/2] + 72*Sin[c/2 + d*x] - 72*Sin[(3
*c)/2 + d*x] + 24*Sin[(3*c)/2 + 2*d*x] + 24*Sin[(5*c)/2 + 2*d*x] - 8*Sin[(5
*c)/2 + 3*d*x] + 8*Sin[(7*c)/2 + 3*d*x] - 3*Sin[(7*c)/2 + 4*d*x] - 3*Sin[(9
*c)/2 + 4*d*x])/(192*a*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.50, size = 58, normalized size = 0.67

$$\frac{8\cos\left(dx+c\right)^3 - 9\,dx - 3\left(2\cos\left(dx+c\right)^3 - 5\cos\left(dx+c\right)\right)\sin\left(dx+c\right) - 24\cos\left(dx+c\right)}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*cos(d*x + c)^3 - 9*d*x - 3*(2*cos(d*x + c)^3 - 5*cos(d*x + c))*sin(
d*x + c) - 24*cos(d*x + c))/(a*d)
```

giac [A] time = 0.15, size = 114, normalized size = 1.31

$$-\frac{\frac{9\,(dx+c)}{a}+\frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+64\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+16\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(9*(d*x + c)/a + 2*(9*tan(1/2*d*x + 1/2*c)^7 + 33*tan(1/2*d*x + 1/2*c
)^5 + 48*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*d*x + 1/2*c)^3 + 64*tan(1/2*d*
x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 16)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4
*a))/d
```

maple [B] time = 0.29, size = 245, normalized size = 2.82

$$-\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{11\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{4\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{11\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{1}{3ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -3/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-11/4/a/d/(1+tan(1/
2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan
(1/2*d*x+1/2*c)^4+11/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-
16/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2+3/4/a/d/(1+tan(1/2
*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-4/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4-3/4/
a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 237, normalized size = 2.72

$$\frac{\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{64\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{33\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{48\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{33\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{9\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-16}{a+\frac{4\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\,a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4\,a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}-\frac{9\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 64*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 48*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 9*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 - 16)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*
a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 9*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a)/d
```

mupad [B] time = 8.61, size = 79, normalized size = 0.91

$$\frac{\cos(c+d\,x)^3}{3\,a\,d}-\frac{\cos(c+d\,x)}{a\,d}-\frac{3\,x}{8\,a}-\frac{\cos(c+d\,x)^3\,\sin(c+d\,x)}{4\,a\,d}+\frac{5\,\cos(c+d\,x)\,\sin(c+d\,x)}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] cos(c + d*x)^3/(3*a*d) - cos(c + d*x)/(a*d) - (3*x)/(8*a) - (cos(c + d*x)^3
*sin(c + d*x))/(4*a*d) + (5*cos(c + d*x)*sin(c + d*x))/(8*a*d)
```

sympy [A] time = 19.76, size = 1049, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-9*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*
tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)
**2 + 24*a*d) - 36*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 96
*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d
*x/2)**2 + 24*a*d) - 54*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8
 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/
2 + d*x/2)**2 + 24*a*d) - 36*d*x*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*x/
2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*t
an(c/2 + d*x/2)**2 + 24*a*d) - 9*d*x/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*t
an(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)*
*2 + 24*a*d) - 18*tan(c/2 + d*x/2)**7/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*
tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)
**2 + 24*a*d) - 66*tan(c/2 + d*x/2)**5/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d
*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2
)**2 + 24*a*d) - 96*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*
d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/
2)**2 + 24*a*d) + 66*tan(c/2 + d*x/2)**3/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a
```

```
*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x
/2)**2 + 24*a*d) - 128*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*x/2)**8 + 96
*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d
*x/2)**2 + 24*a*d) + 18*tan(c/2 + d*x/2)/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a
*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x
/2)**2 + 24*a*d) - 32/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)
**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d), N
e(d, 0)), (x*sin(c)**3*cos(c)**2/(a*sin(c) + a), True))
```

3.299 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=62

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

```
[Out] 1/2*x/a+cos(d*x+c)/a/d-1/3*cos(d*x+c)^3/a/d-1/2*cos(d*x+c)*sin(d*x+c)/a/d
```

Rubi [A] time = 0.12, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2839, 2635, 8, 2633}

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(2*a) + Cos[c + d*x]/(a*d) - Cos[c + d*x]^3/(3*a*d) - (Cos[c + d*x]*Sin[c
 + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
```

```
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sin^2(c+dx)\,dx}{a}-\frac{\int \sin^3(c+dx)\,dx}{a}\\ &= -\frac{\cos(c+dx)\sin(c+dx)}{2ad}+\frac{\int 1\,dx}{2a}+\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(c+dx)\right)}{ad}\\ &= \frac{x}{2a}+\frac{\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)\sin(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.09, size = 46, normalized size = 0.74

$$\frac{-3\sin(2(c+dx))+9\cos(c+dx)-\cos(3(c+dx))+6c+6dx}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6*c + 6*d*x + 9*Cos[c + d*x] - Cos[3*(c + d*x)] - 3*Sin[2*(c + d*x)])/(12*
a*d)
```

fricas [A] time = 0.49, size = 45, normalized size = 0.73

$$-\frac{2\cos\left(dx+c\right)^3-3\,dx+3\cos\left(dx+c\right)\sin\left(dx+c\right)-6\cos\left(dx+c\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*cos(d*x + c)^3 - 3*d*x + 3*cos(d*x + c)*sin(d*x + c) - 6*cos(d*x +
c))/(a*d)
```

giac [A] time = 0.14, size = 75, normalized size = 1.21

$$\frac{\frac{3\,(dx+c)}{a}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^5 + 12*tan(1/2*d*x + 1/2*c)^
2 - 3*tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

maple [B] time = 0.28, size = 141, normalized size = 2.27

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4}{3ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{\arctan\left(t\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+4/a/d/(1+tan(1/2*d*x+
1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d
*x+1/2*c)+4/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3+1/a/d*arctan(tan(1/2*d*x+1/2*c
))
```

maxima [B] time = 0.42, size = 156, normalized size = 2.52

$$-\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-4}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*((3*sin(d*x + c)/(cos(d*x + c) + 1) - 12*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4)/(a + 3*a*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
)/d
```

mupad [B] time = 10.46, size = 66, normalized size = 1.06

$$\frac{x}{2a}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{4}{3}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] x/(2*a) + (4*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)
^5 + 4/3)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

sympy [A] time = 11.35, size = 563, normalized size = 9.08

$$\begin{cases} \frac{3dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad} + \frac{9dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad} + \frac{}{6ad\tan^6\left(\frac{c}{2}+\right.} \\ \frac{x\sin^2(c)\cos^2(c)}{a\sin(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*ta
n(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 9*d*x*tan(c/2 + d
*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*t
an(c/2 + d*x/2)**2 + 6*a*d) + 9*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*
x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d)
+ 3*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*ta
n(c/2 + d*x/2)**2 + 6*a*d) + 6*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)*
*6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 24*
tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4
 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 6*tan(c/2 + d*x/2)/(6*a*d*tan(c/2
+ d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a
*d) + 8/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*ta
n(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**2/(a*sin(c) + a
), True))
```

3.300 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=45

$$-\frac{\cos(c+dx)}{ad}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x}{2a}$$

```
[Out] -1/2*x/a-cos(d*x+c)/a/d+1/2*cos(d*x+c)*sin(d*x+c)/a/d
```

Rubi [A] time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2839, 2638, 2635, 8}

$$-\frac{\cos(c+dx)}{ad}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -x/(2*a) - Cos[c + d*x]/(a*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sin(c+dx)\,dx}{a} - \frac{\int \sin^2(c+dx)\,dx}{a} \\ &= -\frac{\cos(c+dx)}{ad} + \frac{\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\int 1\,dx}{2a} \\ &= -\frac{x}{2a} - \frac{\cos(c+dx)}{ad} + \frac{\cos(c+dx)\sin(c+dx)}{2ad}\end{aligned}$$

Mathematica [B] time = 0.59, size = 161, normalized size = 3.58

$$\frac{-4dx\sin\left(\frac{c}{2}\right)+4\sin\left(\frac{c}{2}+dx\right)-4\sin\left(\frac{3c}{2}+dx\right)+\sin\left(\frac{3c}{2}+2dx\right)+\sin\left(\frac{5c}{2}+2dx\right)+2\cos\left(\frac{c}{2}\right)(c-2dx)-4\cos}{8ad\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*(c - 2*d*x)*Cos[c/2] - 4*Cos[c/2 + d*x] - 4*Cos[(3*c)/2 + d*x] + Cos[(3*
c)/2 + 2*d*x] - Cos[(5*c)/2 + 2*d*x] - 4*Sin[c/2] + 2*c*Sin[c/2] - 4*d*x*Si
n[c/2] + 4*Sin[c/2 + d*x] - 4*Sin[(3*c)/2 + d*x] + Sin[(3*c)/2 + 2*d*x] + S
in[(5*c)/2 + 2*d*x])/(8*a*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.52, size = 34, normalized size = 0.76

$$-\frac{dx-\cos(dx+c)\sin(dx+c)+2\cos(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d*x - cos(d*x + c)*sin(d*x + c) + 2*cos(d*x + c))/(a*d)
```

giac [A] time = 0.14, size = 72, normalized size = 1.60

$$-\frac{\frac{dx+c}{a}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/a + 2*(tan(1/2*d*x + 1/2*c)^3 + 2*tan(1/2*d*x + 1/2*c)^2 -
tan(1/2*d*x + 1/2*c) + 2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

maple [B] time = 0.20, size = 142, normalized size = 3.16

$$-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{2}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{\arctan\left(\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-2/a/d/(1+tan(1/2*d*x
+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*
d*x+1/2*c)-2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2-1/a/d*arctan(tan(1/2*d*x+1/2*c)
)
```

maxima [B] time = 0.41, size = 133, normalized size = 2.96

$$\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-2}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
 sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2)/(a + 2*a*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 8.69, size = 33, normalized size = 0.73

$$-\frac{x}{2\,a}-\frac{\cos\left(c+d\,x\right)-\frac{\sin(2\,c+2\,d\,x)}{4}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] - x/(2*a) - (cos(c + d*x) - sin(2*c + 2*d*x)/4)/(a*d)
```

sympy [A] time = 6.18, size = 366, normalized size = 8.13

$$\begin{cases} -\frac{dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} - \frac{2dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} - \frac{dx}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} - \frac{}{2ad\tan^4\left(\right.} \\ \frac{x\sin(c)\cos^2(c)}{a\sin(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(
c/2 + d*x/2)**2 + 2*a*d) - 2*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*x/2
)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - d*x/(2*a*d*tan(c/2 + d*x/2)**4
+ 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*tan(c/2 + d*x/2)**3/(2*a*d*tan(c/2
 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 4*tan(c/2 + d*x/2)**2/(
2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 2*tan(c/2
+ d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) -
4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d), Ne(d, 0)
), (x*sin(c)*cos(c)**2/(a*sin(c) + a), True))
```

3.301 $\int \frac{\cos(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=22

$$-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{a}$$

```
[Out] -x/a-arctanh(cos(d*x+c))/a/d
```

Rubi [A] time = 0.07, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2839, 3770, 8}

$$-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int 1\,dx}{a} + \frac{\int \csc(c+dx)\,dx}{a}$$
$$= -\frac{x}{a} - \frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Mathematica [A] time = 0.09, size = 37, normalized size = 1.68

$$-\frac{-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+c+dx}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] -((c + d*x + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])/(a*d))
```

fricas [A] time = 0.49, size = 37, normalized size = 1.68

$$-\frac{2\,dx+\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(2*d*x + log(1/2*cos(d*x + c) + 1/2) - log(-1/2*cos(d*x + c) + 1/2))/(
a*d)
```

giac [A] time = 0.15, size = 31, normalized size = 1.41

$$-\frac{\frac{dx+c}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -((d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c)))/a)/d
```

maple [A] time = 0.38, size = 37, normalized size = 1.68

$$-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/a/d*arctan(tan(1/2*d*x+1/2*c))+1/a/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 52, normalized size = 2.36

$$-\frac{\frac{2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - log(sin(d*x + c)/(cos(d*x +
 c) + 1))/a)/d
```

mupad [B] time = 8.77, size = 79, normalized size = 3.59

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)}\right)}{a\,d}+\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] (2*atan((2^(1/2)*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)))/(2*cos(c/2 - pi
/4 + (d*x)/2))))/(a*d) + log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^{2}{\left(c+dx \right)} \csc{\left(c+dx \right)}}{\sin{\left(c+dx \right)}+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

3.302 $\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}$$

```
[Out] arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2706, 3767, 8, 3770}

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \csc(c+dx)\,dx}{a}+\frac{\int \csc^2(c+dx)\,dx}{a}$$
$$=\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{ad}$$
$$=\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}$$

Mathematica [B] time = 0.24, size = 69, normalized size = 2.38

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos(c+dx)+\sin(c+dx)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x] + (-Log[Cos[(c + d*x)
/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x]))/(a*d)
```

fricas [B] time = 0.49, size = 62, normalized size = 2.14

$$\frac{\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)}{2\,ad\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - log(-1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) - 2*cos(d*x + c))/(a*d*sin(d*x + c))
```

giac [B] time = 0.16, size = 65, normalized size = 2.24

$$-\frac{\frac{2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

[Out] -1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a - tan(1/2*d*x + 1/2*c)/a - (2*tan(
1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)))/d

maple [A] time = 0.38, size = 56, normalized size = 1.93

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{1}{2ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)

[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)-1/2/a/d/tan(1/2*d*x+1/2*c)-1/a/d*ln(tan(1/2*d*x+
1/2*c))

maxima [B] time = 0.33, size = 70, normalized size = 2.41

$$-\frac{\frac{2\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\cos(dx+c)+1}{a\sin(dx+c)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/2*(2*log(sin(d*x + c)/(cos(d*x + c) + 1))/a + (cos(d*x + c) + 1)/(a*sin(
d*x + c)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d

mupad [B] time = 8.60, size = 25, normalized size = 0.86

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\cot\left(c+d\,x\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)

[Out] -(log(tan(c/2 + (d*x)/2)) + cot(c + d*x))/(a*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^{2}\left(c+dx\right)\csc^{2}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

3.303 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{\cot(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d+cot(d*x+c)/a/d-1/2*cot(d*x+c)*csc(d*x+c)/a/d
```

Rubi [A] time = 0.10, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 3768, 3770, 3767, 8}

$$\frac{\cot(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) + Cot[c + d*x]/(a*d) - (Cot[c + d*x]*Csc[c +
 d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \csc^2(c+dx)\,dx}{a}+\frac{\int \csc^3(c+dx)\,dx}{a}\\ &= -\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \csc(c+dx)\,dx}{2a}+\frac{\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{ad}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)}{ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.42, size = 94, normalized size = 1.77

$$\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\sin(2(c+dx))-\cos(c+dx)+\sin^2(c+dx)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(c\right.\right.\right.}{8ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(-Cos[c + d*x] + (-Log[Cos[(c + d*
x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^2 + Sin[2*(c + d*x)]))/(8*a*d*
(1 + Sin[c + d*x]))
```

fricas [A] time = 0.50, size = 88, normalized size = 1.66

$$-\frac{\left(\cos\left(dx+c\right)^2-1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(\cos\left(dx+c\right)^2-1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+4\cos\left(dx+c\right)\sin}{4\left(ad\cos\left(dx+c\right)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c)^2 -
1)*log(-1/2*cos(d*x + c) + 1/2) + 4*cos(d*x + c)*sin(d*x + c) - 2*cos(d*x +
 c))/(a*d*cos(d*x + c)^2 - a*d)
```

giac [A] time = 0.20, size = 94, normalized size = 1.77

$$\frac{\frac{4\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}-\frac{6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(4*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a*tan(1/2*d*x + 1/2*c)^2 - 4*a*t
an(1/2*d*x + 1/2*c))/a^2 - (6*tan(1/2*d*x + 1/2*c)^2 - 4*tan(1/2*d*x + 1/2*
c) + 1)/(a*tan(1/2*d*x + 1/2*c)^2))/d
```

maple [A] time = 0.42, size = 94, normalized size = 1.77

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{1}{2ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{1}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/8/a/d*tan(1/2*d*x+1/2*c)^2-1/2/a/d*tan(1/2*d*x+1/2*c)+1/2/a/d/tan(1/2*d*x
+1/2*c)+1/2/a/d*ln(tan(1/2*d*x+1/2*c))-1/8/a/d/tan(1/2*d*x+1/2*c)^2
```

maxima [B] time = 0.34, size = 115, normalized size = 2.17

$$-\frac{\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a}-\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)^2}{a\sin(dx+c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)/a - 4*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (4*sin(d*x + c)/(cos(d*
x + c) + 1) - 1)*(cos(d*x + c) + 1)^2/(a*sin(d*x + c)^2))/d
```

mupad [B] time = 8.65, size = 87, normalized size = 1.64

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d}+\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{1}{2}\right)}{4\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) + log(tan(c/2 + (d*x)/2))/(2*a*d) - tan(c/2 +
(d*x)/2)/(2*a*d) + (cot(c/2 + (d*x)/2)^2*(2*tan(c/2 + (d*x)/2) - 1/2))/(4*a
*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

3.304 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=72

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] 1/2*arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d-1/3*cot(d*x+c)^3/a/d+1/2*cot(d*x
+c)*csc(d*x+c)/a/d
```

Rubi [A] time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2839, 3767, 3768, 3770}

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(2*a*d) - Cot[c + d*x]/(a*d) - Cot[c + d*x]^3/(3*a*d)
 + (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \csc^3(c+dx)\,dx}{a} + \frac{\int \csc^4(c+dx)\,dx}{a}$$
$$= \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\int \csc(c+dx)\,dx}{2a} - \frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, \cot(c+a$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot(c+dx)}{ad} - \frac{\cot^3(c+dx)}{3ad} + \frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Mathematica [A] time = 0.60, size = 126, normalized size = 1.75

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-12(\sin(c+dx)-1)\cos(c+dx)-4\left(\cos(\right.\right.}{192ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/192*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Csc[(c + d*x)/2] + Sec[(c + d*x)
/2])^2*(-12*Cos[c + d*x]*(-1 + Sin[c + d*x]) - 4*(Cos[3*(c + d*x)] + 3*(Log
[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3)))/(a*d*(1 + Sin
[c + d*x]))
```

fricas [A] time = 0.50, size = 119, normalized size = 1.65

$$-\frac{8\cos(dx+c)^3 - 3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) + 3\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(\right.}{12\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(8*cos(d*x + c)^3 - 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*
x + c) + 6*cos(d*x + c)*sin(d*x + c) - 12*cos(d*x + c))/((a*d*cos(d*x + c)^
2 - a*d)*sin(d*x + c))
```

giac [A] time = 0.17, size = 128, normalized size = 1.78

$$-\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+9a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}-\frac{22\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\tan\left(\frac{1}{2}dx+\right.}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a - (a^2*tan(1/2*d*x + 1/2*c)^3 -
3*a^2*tan(1/2*d*x + 1/2*c)^2 + 9*a^2*tan(1/2*d*x + 1/2*c))/a^3 - (22*tan(1/
2*d*x + 1/2*c)^3 - 9*tan(1/2*d*x + 1/2*c)^2 + 3*tan(1/2*d*x + 1/2*c) - 1)/(
a*tan(1/2*d*x + 1/2*c)^3))/d
```

maple [A] time = 0.45, size = 132, normalized size = 1.83

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{3}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{1}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{}{24ad\,\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/24/a/d*tan(1/2*d*x+1/2*c)^3-1/8/a/d*tan(1/2*d*x+1/2*c)^2+3/8/a/d*tan(1/2*
d*x+1/2*c)-3/8/a/d/tan(1/2*d*x+1/2*c)-1/2/a/d*ln(tan(1/2*d*x+1/2*c))+1/8/a/
d/tan(1/2*d*x+1/2*c)^2-1/24/a/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.32, size = 153, normalized size = 2.12

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/24*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - 12*log(sin(d*x + c)/(cos(d
*x + c) + 1))/a + (3*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

mupad [B] time = 8.62, size = 119, normalized size = 1.65

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - log(tan(c/2
+ (d*x)/2))/(2*a*d) + (3*tan(c/2 + (d*x)/2))/(8*a*d) - (cot(c/2 + (d*x)/2)^
3*(3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2) + 1/3))/(8*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^2(c+dx)\csc^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

3.305 $\int \frac{\cot^2(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

```
[Out] -3/8*arctanh(cos(d*x+c))/a/d+cot(d*x+c)/a/d+1/3*cot(d*x+c)^3/a/d-3/8*cot(d*
x+c)*csc(d*x+c)/a/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d
```

Rubi [A] time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2839, 3768, 3770, 3767}

$$\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(8*a*d) + Cot[c + d*x]/(a*d) + Cot[c + d*x]^3/(3
*a*d) - (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x]^
3)/(4*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \csc^4(c+dx)\,dx}{a} + \frac{\int \csc^5(c+dx)\,dx}{a}$$
$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{3\int \csc^3(c+dx)\,dx}{4a} + \frac{\text{Subst}\left(\int\left(1+x^2\right)\,dx, x, \cot\right.}{ad}$$
$$= \frac{\cot(c+dx)}{ad} + \frac{\cot^3(c+dx)}{3ad} - \frac{3\cot(c+dx)\csc(c+dx)}{8ad} - \frac{\cot(c+dx)\csc^3(c+}{4ad}$$
$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{8ad} + \frac{\cot(c+dx)}{ad} + \frac{\cot^3(c+dx)}{3ad} - \frac{3\cot(c+dx)\csc(c+}{8ad}$$

Mathematica [A] time = 1.13, size = 125, normalized size = 1.32

$$-\frac{\csc^4(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-48\sin(2(c+dx))+66\cos(c+dx)+2(16\sin(c+dx)-9)\right.}{192ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/192*(Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(66*Cos[c +
d*x] + 72*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 +
2*Cos[3*(c + d*x)]*(-9 + 16*Sin[c + d*x]) - 48*Sin[2*(c + d*x)]))/(a*d*(1 +
 Sin[c + d*x]))
```

fricas [A] time = 0.51, size = 143, normalized size = 1.51

$$\frac{18\cos(dx+c)^3 - 9\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) + 9\left(\cos(dx+c)^4 - 2\cos(dx\right.}{48\left(ad\cos(dx+c)^4 - 2ad\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(18*cos(d*x + c)^3 - 9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2
*cos(d*x + c) + 1/2) + 9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*c
os(d*x + c) + 1/2) - 16*(2*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c) -
30*cos(d*x + c))/(a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

giac [A] time = 0.19, size = 157, normalized size = 1.65

$$\frac{\frac{72\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-8a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+24a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-72a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}-\frac{150\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-72\tan}{}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(72*log(abs(tan(1/2*d*x + 1/2*c)))/a + (3*a^3*tan(1/2*d*x + 1/2*c)^4
- 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 24*a^3*tan(1/2*d*x + 1/2*c)^2 - 72*a^3*tan
(1/2*d*x + 1/2*c))/a^4 - (150*tan(1/2*d*x + 1/2*c)^4 - 72*tan(1/2*d*x + 1/2
*c)^3 + 24*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*
d*x + 1/2*c)^4))/d
```

maple [A] time = 0.43, size = 170, normalized size = 1.79

$$\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{3}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{}{8ad\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/64/a/d*tan(1/2*d*x+1/2*c)^4-1/24/a/d*tan(1/2*d*x+1/2*c)^3+1/8/a/d*tan(1/2
*d*x+1/2*c)^2-3/8/a/d*tan(1/2*d*x+1/2*c)+3/8/a/d/tan(1/2*d*x+1/2*c)+3/8/a/d
*ln(tan(1/2*d*x+1/2*c))-1/8/a/d/tan(1/2*d*x+1/2*c)^2-1/64/a/d/tan(1/2*d*x+1
/2*c)^4+1/24/a/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.33, size = 195, normalized size = 2.05

$$-\frac{\frac{\frac{72\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a}-\frac{72\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{72\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-3\right)(\cos(dx+c)+1}{a\sin(dx+c)^4}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/192*((72*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4)/a - 72*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (8*sin(d*x
+ c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 72*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 - 3)*(cos(d*x + c) + 1)^4/(a*sin(d*x + c)^4)
)/d
```

mupad [B] time = 8.68, size = 151, normalized size = 1.59

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}+\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}+\frac{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(6\,\tan\left(\frac{c}{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^5*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - tan(c/2 + (d*x)/2)^3/(24*a*d) + tan(c/2 + (d
*x)/2)^4/(64*a*d) + (3*log(tan(c/2 + (d*x)/2)))/(8*a*d) - (3*tan(c/2 + (d*x
)/2))/(8*a*d) + (cot(c/2 + (d*x)/2)^4*((2*tan(c/2 + (d*x)/2))/3 - 2*tan(c/2
 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^3 - 1/4))/(16*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)\csc^5(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**5/(sin(c + d*x) + 1), x)/a
```

3.306 $\int \frac{\cot^2(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=114

$$-\frac{\cot^5(c+dx)}{5ad}-\frac{2\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

```
[Out] 3/8*arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d-2/3*cot(d*x+c)^3/a/d-1/5*cot(d*x
+c)^5/a/d+3/8*cot(d*x+c)*csc(d*x+c)/a/d+1/4*cot(d*x+c)*csc(d*x+c)^3/a/d
```

Rubi [A] time = 0.14, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2839, 3767, 3768, 3770}

$$-\frac{\cot^5(c+dx)}{5ad}-\frac{2\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]/(a*d) - (2*Cot[c + d*x]^3)
/(3*a*d) - Cot[c + d*x]^5/(5*a*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) +
 (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \csc^5(c+dx)\,dx}{a} + \frac{\int \csc^6(c+dx)\,dx}{a}$$
$$= \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} - \frac{3\int \csc^3(c+dx)\,dx}{4a} - \frac{\mathrm{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x$$
$$= -\frac{\cot(c+dx)}{ad} - \frac{2\cot^3(c+dx)}{3ad} - \frac{\cot^5(c+dx)}{5ad} + \frac{3\cot(c+dx)\csc(c+dx)}{8ad} + \frac{\mathrm{co}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{8ad} - \frac{\cot(c+dx)}{ad} - \frac{2\cot^3(c+dx)}{3ad} - \frac{\cot^5(c+dx)}{5ad} + \frac{3\cot(c$$

Mathematica [A] time = 0.66, size = 189, normalized size = 1.66

$$\frac{\csc^5(c+dx)\left(420\sin(2(c+dx)) - 90\sin(4(c+dx)) - 640\cos(c+dx) + 320\cos(3(c+dx)) - 64\cos(5(c+dx)) -$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^5*(-640*Cos[c + d*x] + 320*Cos[3*(c + d*x)] - 64*Cos[5*(c + d
*x)] + 450*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 450*Log[Sin[(c + d*x)/2]]*S
in[c + d*x] + 420*Sin[2*(c + d*x)] - 225*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d
*x)] + 225*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 90*Sin[4*(c + d*x)] + 4
5*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 45*Log[Sin[(c + d*x)/2]]*Sin[5*(
c + d*x)]))/(1920*a*d)
```

fricas [A] time = 0.50, size = 173, normalized size = 1.52

$$-\frac{128\cos(dx+c)^5 - 320\cos(dx+c)^3 - 45\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx}{240\left(ad$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/240*(128*cos(d*x + c)^5 - 320*cos(d*x + c)^3 - 45*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 45*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*(3*cos(d*x + c)^3 - 5*cos(d*x + c))*sin(d*x + c) + 240*cos(d*x + c))/((a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))

giac [A] time = 0.19, size = 187, normalized size = 1.64

$$-\frac{\frac{360 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a} - \frac{6 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5 - 15 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4 + 50 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3 - 120 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 + 300 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^5} - \cdots}{960 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/960*(360*log(abs(tan(1/2*d*x + 1/2*c)))/a - (6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^4*tan(1/2*d*x + 1/2*c)^4 + 50*a^4*tan(1/2*d*x + 1/2*c)^3 - 120*a^4*tan(1/2*d*x + 1/2*c)^2 + 300*a^4*tan(1/2*d*x + 1/2*c))/a^5 - (822*tan(1/2*d*x + 1/2*c)^5 - 300*tan(1/2*d*x + 1/2*c)^4 + 120*tan(1/2*d*x + 1/2*c)^3 - 50*tan(1/2*d*x + 1/2*c)^2 + 15*tan(1/2*d*x + 1/2*c) - 6)/(a*tan(1/2*d*x + 1/2*c)^5))/d

maple [A] time = 0.46, size = 208, normalized size = 1.82

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad} - \frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad} + \frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96ad} - \frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad} + \frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad} - \frac{5}{16ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{3\ln\left(\tan \cdots\right.}{8 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*csc(d*x+c)^6/(a+a*sin(d*x+c)),x)

[Out] 1/160/a/d*tan(1/2*d*x+1/2*c)^5-1/64/a/d*tan(1/2*d*x+1/2*c)^4+5/96/a/d*tan(1/2*d*x+1/2*c)^3-1/8/a/d*tan(1/2*d*x+1/2*c)^2+5/16/a/d*tan(1/2*d*x+1/2*c)-5/16/a/d/tan(1/2*d*x+1/2*c)-3/8/a/d*ln(tan(1/2*d*x+1/2*c))-1/160/a/d/tan(1/2*d*x+1/2*c)^5+1/8/a/d/tan(1/2*d*x+1/2*c)^2+1/64/a/d/tan(1/2*d*x+1/2*c)^4-5/96/a/d/tan(1/2*d*x+1/2*c)^3

maxima [B] time = 0.34, size = 234, normalized size = 2.05

$$\frac{\frac{\frac{300 \sin(dx+c)}{\cos(dx+c)+1} - \frac{120 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{50 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{15 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{6 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a} - \frac{360 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{\left(\frac{15 \sin(dx+c)}{\cos(dx+c)+1} - \frac{50 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{120 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \cdots\right.}{a \sin(dx+c)^5 \cdots}}{960 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*((300*sin(d*x + c)/(cos(d*x + c) + 1) - 120*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 360*log(sin(
d*x + c)/(cos(d*x + c) + 1))/a + (15*sin(d*x + c)/(cos(d*x + c) + 1) - 50*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 - 300*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6)*(cos(d*x + c) + 1)^5/(a*si
n(d*x + c)^5))/d
```

mupad [B] time = 9.25, size = 291, normalized size = 2.55

$$-\frac{6\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-15\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-50\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^6*(a + a*sin(c + d*x))),x)
```

```
[Out] -(6*cos(c/2 + (d*x)/2)^10 - 6*sin(c/2 + (d*x)/2)^10 + 15*cos(c/2 + (d*x)/2)
*sin(c/2 + (d*x)/2)^9 - 15*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) - 50*cos
(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 120*cos(c/2 + (d*x)/2)^3*sin(c/2 +
 (d*x)/2)^7 - 300*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 + 300*cos(c/2 +
 (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 - 120*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)
/2)^3 + 50*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 360*log(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)/(960*
a*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.307 $\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=111

$$\frac{2\cos^3(c+dx)}{3a^2d}-\frac{4\cos(c+dx)}{a^2d}+\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}+\frac{11\sin(c+dx)\cos(c+dx)}{8a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{27x}{8a^2}$$

```
[Out] -27/8*x/a^2-4*cos(d*x+c)/a^2/d+2/3*cos(d*x+c)^3/a^2/d+11/8*cos(d*x+c)*sin(d
*x+c)/a^2/d+1/4*cos(d*x+c)*sin(d*x+c)^3/a^2/d-2*cos(d*x+c)/a^2/d/(1+sin(d*x
+c))
```

Rubi [A] time = 0.25, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2874, 2966, 2638, 2635, 8, 2633, 2648}

$$\frac{2\cos^3(c+dx)}{3a^2d}-\frac{4\cos(c+dx)}{a^2d}+\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}+\frac{11\sin(c+dx)\cos(c+dx)}{8a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{27x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-27*x)/(8*a^2) - (4*Cos[c + d*x])/(a^2*d) + (2*Cos[c + d*x]^3)/(3*a^2*d) +
 (11*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(
4*a^2*d) - (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\sin^4(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}$$

$$= \frac{\int \left(-2+2\sin(c+dx)-2\sin^2(c+dx)+2\sin^3(c+dx)-\sin^4(c+dx)+\frac{}{1+\sin}\right.}{a^2}$$

$$= -\frac{2x}{a^2} - \frac{\int \sin^4(c+dx)\,dx}{a^2} + \frac{2\int \sin(c+dx)\,dx}{a^2} - \frac{2\int \sin^2(c+dx)\,dx}{a^2} + \frac{2\int \mathrm{si}}{}$$

$$= -\frac{2x}{a^2} - \frac{2\cos(c+dx)}{a^2d} + \frac{\cos(c+dx)\sin(c+dx)}{a^2d} + \frac{\cos(c+dx)\sin^3(c+dx)}{4a^2d} -$$

$$= -\frac{3x}{a^2} - \frac{4\cos(c+dx)}{a^2d} + \frac{2\cos^3(c+dx)}{3a^2d} + \frac{11\cos(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos(c-}{}$$

$$= -\frac{27x}{8a^2} - \frac{4\cos(c+dx)}{a^2d} + \frac{2\cos^3(c+dx)}{3a^2d} + \frac{11\cos(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos(c}{}$$

Mathematica [A] time = 1.49, size = 209, normalized size = 1.88

$$\frac{-648dx\sin\left(c+\frac{dx}{2}\right)+4\sin\left(c+\frac{dx}{2}\right)-264\sin\left(2c+\frac{3dx}{2}\right)+56\sin\left(2c+\frac{5dx}{2}\right)+13\sin\left(4c+\frac{7dx}{2}\right)-3\sin\left(4c+\frac{9}{}\right.}{192a^2d\left(\sin\left(\frac{c}{2}\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((4 - 648*d*x)*Cos[(d*x)/2] - 340*Cos[c + (d*x)/2] - 264*Cos[c + (3*d*x)/2]
 - 56*Cos[3*c + (5*d*x)/2] + 13*Cos[3*c + (7*d*x)/2] + 3*Cos[5*c + (9*d*x)/
2] + 1100*Sin[(d*x)/2] + 4*Sin[c + (d*x)/2] - 648*d*x*Sin[c + (d*x)/2] - 26
4*Sin[2*c + (3*d*x)/2] + 56*Sin[2*c + (5*d*x)/2] + 13*Sin[4*c + (7*d*x)/2]
- 3*Sin[4*c + (9*d*x)/2])/(192*a^2*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.51, size = 144, normalized size = 1.30

$$\frac{6\cos(dx+c)^5+16\cos(dx+c)^4-29\cos(dx+c)^3-81\,dx-3\,(27\,dx+35)\cos(dx+c)-96\cos(dx+c)^2-}{24\left(a^2d\cos(dx+c)+a^2d\,\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(6*cos(d*x + c)^5 + 16*cos(d*x + c)^4 - 29*cos(d*x + c)^3 - 81*d*x - 3
*(27*d*x + 35)*cos(d*x + c) - 96*cos(d*x + c)^2 - (6*cos(d*x + c)^4 - 10*co
s(d*x + c)^3 + 81*d*x - 39*cos(d*x + c)^2 + 57*cos(d*x + c) - 48)*sin(d*x +
 c) - 48)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

giac [A] time = 0.20, size = 145, normalized size = 1.31

$$-\frac{\frac{81\,(dx+c)}{a^2}+\frac{96}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{2\left(33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+57\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-57\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4 a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(81*(d*x + c)/a^2 + 96/(a^2*(tan(1/2*d*x + 1/2*c) + 1)) + 2*(33*tan(1
/2*d*x + 1/2*c)^7 + 48*tan(1/2*d*x + 1/2*c)^6 + 57*tan(1/2*d*x + 1/2*c)^5 +
 240*tan(1/2*d*x + 1/2*c)^4 - 57*tan(1/2*d*x + 1/2*c)^3 + 272*tan(1/2*d*x +
 1/2*c)^2 - 33*tan(1/2*d*x + 1/2*c) + 80)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a
^2))/d
```

maple [B] time = 0.46, size = 300, normalized size = 2.70

$$-\frac{11\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{19\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{20\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -11/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-4/d/a^2/(1+tan(
1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6-19/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^4*tan(1/2*d*x+1/2*c)^5-20/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/
2*c)^4+19/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-68/3/d/a^
2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2+11/4/d/a^2/(1+tan(1/2*d*x
+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-20/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4-27/4/
d/a^2*arctan(tan(1/2*d*x+1/2*c))-4/d/a^2/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.44, size = 398, normalized size = 3.59

$$-\frac{\frac{\frac{47\sin(dx+c)}{\cos(dx+c)+1}+\frac{431\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{215\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{471\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{297\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{297\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{81\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{81\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+128}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{4a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}+\frac{81\,\mathrm{ar}}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*((47*sin(d*x + c)/(cos(d*x + c) + 1) + 431*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 215*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 471*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 297*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 297*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 + 81*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 81*s
in(d*x + c)^8/(cos(d*x + c) + 1)^8 + 128)/(a^2 + a^2*sin(d*x + c)/(cos(d*x
+ c) + 1) + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^2*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 + 6*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 6*a^2*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4*a^2*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + 4*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a^2*sin(d*x + c)^8/(cos(
d*x + c) + 1)^8 + a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9) + 81*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 12.34, size = 147, normalized size = 1.32

$$-\frac{27\,x}{8\,a^2}-\frac{\frac{27\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{4}+\frac{27\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{99\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{4}+\frac{99\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{157\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{4}+\frac{215\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{431\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{12}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^4)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - (27*x)/(8*a^2) - ((47*tan(c/2 + (d*x)/2))/12 + (431*tan(c/2 + (d*x)/2)^2)
/12 + (215*tan(c/2 + (d*x)/2)^3)/12 + (157*tan(c/2 + (d*x)/2)^4)/4 + (99*ta
n(c/2 + (d*x)/2)^5)/4 + (99*tan(c/2 + (d*x)/2)^6)/4 + (27*tan(c/2 + (d*x)/2
)^7)/4 + (27*tan(c/2 + (d*x)/2)^8)/4 + 32/3)/(a^2*d*(tan(c/2 + (d*x)/2) + 1
)*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [A] time = 61.08, size = 3580, normalized size = 32.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-81*d*x*tan(c/2 + d*x/2)**9/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*
a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(
c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x
/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24
```

*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 81*d*x*tan(c/2 + d*x/2)**8/(24*a**2
*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2
+ d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**
5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**
2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 324*d*x
*tan(c/2 + d*x/2)**7/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d
*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 +
144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d
*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 +
d*x/2) + 24*a**2*d) - 324*d*x*tan(c/2 + d*x/2)**6/(24*a**2*d*tan(c/2 + d*x/
2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*
a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*ta
n(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x
/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 486*d*x*tan(c/2 + d*x/2)
5/(24*a2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**
2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/
2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)
3 + 96*a2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*
d) - 486*d*x*tan(c/2 + d*x/2)**4/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d
*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 +
d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4
 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*
d*tan(c/2 + d*x/2) + 24*a**2*d) - 324*d*x*tan(c/2 + d*x/2)**3/(24*a**2*d*ta
n(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x
/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 1
44*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*t
an(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 324*d*x*tan(
c/2 + d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)
8 + 96*a2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a
2*d*tan(c/2 + d*x/2)5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(
c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2
) + 24*a**2*d) - 81*d*x*tan(c/2 + d*x/2)/(24*a**2*d*tan(c/2 + d*x/2)**9 + 2
4*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*ta
n(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d
*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 +
24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 81*d*x/(24*a**2*d*tan(c/2 + d*x/2
)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a
2*d*tan(c/2 + d*x/2)6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan
(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/
2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 162*tan(c/2 + d*x/2)**8/(
24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*t
an(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d
*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 +
 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) -
162*tan(c/2 + d*x/2)**7/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2

```
+ d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6
 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**
2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2
 + d*x/2) + 24*a**2*d) - 594*tan(c/2 + d*x/2)**6/(24*a**2*d*tan(c/2 + d*x/2
)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a
**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan
(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/
2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 594*tan(c/2 + d*x/2)**5/(
24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*t
an(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d
*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 +
 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) -
942*tan(c/2 + d*x/2)**4/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2
+ d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6
 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**
2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2
 + d*x/2) + 24*a**2*d) - 430*tan(c/2 + d*x/2)**3/(24*a**2*d*tan(c/2 + d*x/2
)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a
**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan
(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/
2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 862*tan(c/2 + d*x/2)**2/(
24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*t
an(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d
*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 +
 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) -
94*tan(c/2 + d*x/2)/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*
x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 1
44*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*
tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d
*x/2) + 24*a**2*d) - 256/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2
 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**
6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a*
*2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/
2 + d*x/2) + 24*a**2*d), Ne(d, 0)), (x*sin(c)**4*cos(c)**2/(a*sin(c) + a)**
2, True))
```

3.308 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=83

$$-\frac{\cos^3(c+dx)}{3a^2d}+\frac{3\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}+\frac{3x}{a^2}$$

```
[Out] 3*x/a^2+3*cos(d*x+c)/a^2/d-1/3*cos(d*x+c)^3/a^2/d-cos(d*x+c)*sin(d*x+c)/a^2
/d+2*cos(d*x+c)/a^2/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.22, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2874, 2966, 2638, 2635, 8, 2633, 2648}

$$-\frac{\cos^3(c+dx)}{3a^2d}+\frac{3\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}+\frac{3x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*x)/a^2 + (3*Cos[c + d*x])/(a^2*d) - Cos[c + d*x]^3/(3*a^2*d) - (Cos[c +
d*x]*Sin[c + d*x])/(a^2*d) + (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\sin^3(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}$$
$$= \frac{\int \left(2-2\sin(c+dx)+2\sin^2(c+dx)-\sin^3(c+dx)-\frac{2}{1+\sin(c+dx)}\right)dx}{a^2}$$
$$= \frac{2x}{a^2}-\frac{\int \sin^3(c+dx)\,dx}{a^2}-\frac{2\int \sin(c+dx)\,dx}{a^2}+\frac{2\int \sin^2(c+dx)\,dx}{a^2}-\frac{2\int \frac{}{1+\text{si}}}{}$$
$$= \frac{2x}{a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}+\frac{\int 1\,d}{a^2}$$
$$= \frac{3x}{a^2}+\frac{3\cos(c+dx)}{a^2d}-\frac{\cos^3(c+dx)}{3a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{a^2d}+\frac{2\cos(c+}{a^2d(1+\sin(}$$

Mathematica [A] time = 0.97, size = 165, normalized size = 1.99

$$-\frac{-72dx\sin\left(c+\frac{dx}{2}\right)-27\sin\left(2c+\frac{3dx}{2}\right)+5\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(4c+\frac{7dx}{2}\right)-31\cos\left(c+\frac{dx}{2}\right)-27\cos\left(c+\frac{3dx}{2}\right)}{24a^2d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/24*(-72*d*x*Cos[(d*x)/2] - 31*Cos[c + (d*x)/2] - 27*Cos[c + (3*d*x)/2] -
 5*Cos[3*c + (5*d*x)/2] + Cos[3*c + (7*d*x)/2] + 131*Sin[(d*x)/2] - 72*d*x*
Sin[c + (d*x)/2] - 27*Sin[2*c + (3*d*x)/2] + 5*Sin[2*c + (5*d*x)/2] + Sin[4
*c + (7*d*x)/2])/(a^2*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]))
```

fricas [A] time = 0.50, size = 119, normalized size = 1.43

$$-\frac{\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{3}-9\,dx-3\,(3\,dx+4)\cos\left(dx+c\right)-9\,\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{3}-9\,dx+3\,\mathrm{c}\right.}{3\left(a^{2}d\cos\left(dx+c\right)+a^{2}d\sin\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 9*d*x - 3*(3*d*x + 4)*cos(d*x + c
) - 9*cos(d*x + c)^2 + (cos(d*x + c)^3 - 9*d*x + 3*cos(d*x + c)^2 - 6*cos(d
*x + c) + 6)*sin(d*x + c) - 6)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a
^2*d)
```

giac [A] time = 0.20, size = 106, normalized size = 1.28

$$\frac{\frac{9\,(dx+c)}{a^{2}}+\frac{12}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a^{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(9*(d*x + c)/a^2 + 12/(a^2*(tan(1/2*d*x + 1/2*c) + 1)) + 2*(3*tan(1/2*d
*x + 1/2*c)^5 + 6*tan(1/2*d*x + 1/2*c)^4 + 18*tan(1/2*d*x + 1/2*c)^2 - 3*ta
n(1/2*d*x + 1/2*c) + 8)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2))/d
```

maple [B] time = 0.46, size = 198, normalized size = 2.39

$$\frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{4\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{12\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{}{3d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+4/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4+12/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*ta
n(1/2*d*x+1/2*c)^2-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)+16
/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3+6/d/a^2*arctan(tan(1/2*d*x+1/2*c))+4/d/
a^2/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.43, size = 312, normalized size = 3.76

$$\frac{2\left(\frac{\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{33\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{24\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{9\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+14}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 2/3*((5*sin(d*x + c)/(cos(d*x + c) + 1) + 33*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 18*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 24*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 9*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 9*sin(d*x + c)^6/(co
s(d*x + c) + 1)^6 + 14)/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*a^2*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 + a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7) + 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2
)/d
```

mupad [B] time = 12.38, size = 120, normalized size = 1.45

$$\frac{3x}{a^2}+\frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+22\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (3*x)/a^2 + ((10*tan(c/2 + (d*x)/2))/3 + 22*tan(c/2 + (d*x)/2)^2 + 12*tan(c
/2 + (d*x)/2)^3 + 16*tan(c/2 + (d*x)/2)^4 + 6*tan(c/2 + (d*x)/2)^5 + 6*tan(
```

```
c/2 + (d*x)/2)^6 + 28/3)/(a^2*d*(tan(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2
)^2 + 1)^3)
```

sympy [A] time = 37.33, size = 2263, normalized size = 27.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((9*d*x*tan(c/2 + d*x/2)**7/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2
*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 +
d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3
*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 9*d*x*tan(c/2 + d*x/2)**6/(3*a**2*d*
tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x
/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a*
*2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 27*d*x*t
an(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2
)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2
*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 +
d*x/2) + 3*a**2*d) + 27*d*x*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)*
*7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d
*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*
x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 27*d*x*tan(c/2 + d*x/2)**
3/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*t
an(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/
2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d
) + 27*d*x*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan
(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)
**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*
d*tan(c/2 + d*x/2) + 3*a**2*d) + 9*d*x*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 +
 d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 +
9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(
c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 9*d*x/(3*a**2*d*t
an(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/
2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**
2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 18*tan(c/
2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6
+ 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*ta
n(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2
) + 3*a**2*d) + 18*tan(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a*
*2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2
+ d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 +
 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 48*tan(c/2 + d*x/2)**4/(3*a**2*d*t
an(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/
```

```
2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**
2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 36*tan(c/
2 + d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6
+ 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*ta
n(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2
) + 3*a**2*d) + 66*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a*
*2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2
+ d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 +
 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 10*tan(c/2 + d*x/2)/(3*a**2*d*tan(
c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)*
*5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d
*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 28/(3*a**2*d
*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*
x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a
**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d), Ne(d, 0)
), (x*sin(c)**3*cos(c)**2/(a*sin(c) + a)**2, True))
```

3.309 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=69

$$-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{5x}{2a^2}$$

```
[Out] -5/2*x/a^2-2*cos(d*x+c)/a^2/d+1/2*cos(d*x+c)*sin(d*x+c)/a^2/d-2*cos(d*x+c)/
a^2/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.27, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2874, 2950, 2709, 2638, 2635, 8, 2648}

$$-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{5x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*x)/(2*a^2) - (2*Cos[c + d*x])/(a^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*
a^2*d) - (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2950

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\sin^2(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}$$
$$= \frac{\int (a-a\sin(c+dx))^2\tan^2(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(-2+2\sin(c+dx)-\sin^2(c+dx)+\frac{2}{1+\sin(c+dx)}\right)dx}{a^2}$$
$$= -\frac{2x}{a^2}-\frac{\int \sin^2(c+dx)\,dx}{a^2}+\frac{2\int \sin(c+dx)\,dx}{a^2}+\frac{2\int \frac{1}{1+\sin(c+dx)}\,dx}{a^2}$$
$$= -\frac{2x}{a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}-\frac{\int 1\,d}{2a^2}$$
$$= -\frac{5x}{2a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}$$

Mathematica [A] time = 0.15, size = 69, normalized size = 1.00

$$\frac{-10(c+dx)+\sin(2(c+dx))-8\cos(c+dx)+\frac{16\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-10*(c + d*x) - 8*Cos[c + d*x] + (16*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]) + Sin[2*(c + d*x)])/(4*a^2*d)
```

fricas [A] time = 0.48, size = 100, normalized size = 1.45

$$-\frac{\cos(dx+c)^3+5\,dx+(5\,dx+7)\cos(dx+c)+4\,\cos(dx+c)^2+\left(5\,dx-\cos(dx+c)^2+3\,\cos(dx+c)-4\right)\sin}{2\left(a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(cos(d*x + c)^3 + 5*d*x + (5*d*x + 7)*cos(d*x + c) + 4*cos(d*x + c)^2
+ (5*d*x - cos(d*x + c)^2 + 3*cos(d*x + c) - 4)*sin(d*x + c) + 4)/(a^2*d*co
s(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

giac [A] time = 0.18, size = 91, normalized size = 1.32

$$-\frac{\frac{5\,(dx+c)}{a^2}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}+\frac{8}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*(d*x + c)/a^2 + 2*(tan(1/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*c)^
2 - tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + 8/(a^2
*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [B] time = 0.44, size = 163, normalized size = 2.36

$$-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{4}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-5\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-4/d/a^2/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*ta
n(1/2*d*x+1/2*c)-4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2-5/d/a^2*arctan(tan(1/2*
d*x+1/2*c))-4/d/a^2/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.43, size = 226, normalized size = 3.28

$$-\frac{\frac{\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+8}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2\,a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{5\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -((3*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 + 8)/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 2*a^2*sin(d*x + c
```

```
)^2/(cos(d*x + c) + 1)^2 + 2*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^2*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5) + 5*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 10.79, size = 95, normalized size = 1.38

$$-\frac{5x}{2a^2}-\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+8}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - (5*x)/(2*a^2) - (3*tan(c/2 + (d*x)/2) + 11*tan(c/2 + (d*x)/2)^2 + 5*tan(c
/2 + (d*x)/2)^3 + 5*tan(c/2 + (d*x)/2)^4 + 8)/(a^2*d*(tan(c/2 + (d*x)/2) +
1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

sympy [A] time = 21.85, size = 1248, normalized size = 18.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-5*d*x*tan(c/2 + d*x/2)**5/(2*a**2*d*tan(c/2 + d*x/2)**5 + 2*a**
2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**3 + 4*a**2*d*tan(c/2 +
 d*x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) + 2*a**2*d) - 5*d*x*tan(c/2 + d*x/2)
**4/(2*a**2*d*tan(c/2 + d*x/2)**5 + 2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d
*tan(c/2 + d*x/2)**3 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d*tan(c/2 + d*
x/2) + 2*a**2*d) - 10*d*x*tan(c/2 + d*x/2)**3/(2*a**2*d*tan(c/2 + d*x/2)**5
 + 2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**3 + 4*a**2*d*t
an(c/2 + d*x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) + 2*a**2*d) - 10*d*x*tan(c/2
 + d*x/2)**2/(2*a**2*d*tan(c/2 + d*x/2)**5 + 2*a**2*d*tan(c/2 + d*x/2)**4 +
 4*a**2*d*tan(c/2 + d*x/2)**3 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d*tan
(c/2 + d*x/2) + 2*a**2*d) - 5*d*x*tan(c/2 + d*x/2)/(2*a**2*d*tan(c/2 + d*x/
2)**5 + 2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**3 + 4*a**
2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) + 2*a**2*d) - 5*d*x/(2*
a**2*d*tan(c/2 + d*x/2)**5 + 2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/
2 + d*x/2)**3 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) +
2*a**2*d) - 10*tan(c/2 + d*x/2)**4/(2*a**2*d*tan(c/2 + d*x/2)**5 + 2*a**2*d
*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**3 + 4*a**2*d*tan(c/2 + d*
x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) + 2*a**2*d) - 10*tan(c/2 + d*x/2)**3/(2
*a**2*d*tan(c/2 + d*x/2)**5 + 2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c
```

```
/2 + d*x/2)**3 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) +
 2*a**2*d) - 22*tan(c/2 + d*x/2)**2/(2*a**2*d*tan(c/2 + d*x/2)**5 + 2*a**2*
d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**3 + 4*a**2*d*tan(c/2 + d
*x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) + 2*a**2*d) - 6*tan(c/2 + d*x/2)/(2*a*
*2*d*tan(c/2 + d*x/2)**5 + 2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2
+ d*x/2)**3 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) + 2*
a**2*d) - 16/(2*a**2*d*tan(c/2 + d*x/2)**5 + 2*a**2*d*tan(c/2 + d*x/2)**4 +
 4*a**2*d*tan(c/2 + d*x/2)**3 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d*tan
(c/2 + d*x/2) + 2*a**2*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**2/(a*sin(c) + a)
**2, True))
```

3.310 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$\frac{\cos(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{2x}{a^2}$$

```
[Out] 2*x/a^2+cos(d*x+c)/a^2/d+2*cos(d*x+c)/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2857, 2638}

$$\frac{\cos(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*x)/a^2 + Cos[c + d*x]/(a^2*d) + (2*Cos[c + d*x])/(d*(a^2 + a^2*Sin[c + d
*x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{2\cos(c+dx)}{d\left(a^2+a^2\sin(c+dx)\right)} - \frac{\int(-2a+a\sin(c+dx))\,dx}{a^3}$$
$$= \frac{2x}{a^2} + \frac{2\cos(c+dx)}{d\left(a^2+a^2\sin(c+dx)\right)} - \frac{\int \sin(c+dx)\,dx}{a^2}$$
$$= \frac{2x}{a^2} + \frac{\cos(c+dx)}{a^2 d} + \frac{2\cos(c+dx)}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [B] time = 0.33, size = 117, normalized size = 2.49

$$\frac{12dx\sin\left(c+\frac{dx}{2}\right)+3\sin\left(2c+\frac{3dx}{2}\right)+2\cos\left(c+\frac{dx}{2}\right)+3\cos\left(c+\frac{3dx}{2}\right)-28\sin\left(\frac{dx}{2}\right)+12dx\cos\left(\frac{dx}{2}\right)}{6a^2d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (12*d*x*Cos[(d*x)/2] + 2*Cos[c + (d*x)/2] + 3*Cos[c + (3*d*x)/2] - 28*Sin[(
d*x)/2] + 12*d*x*Sin[c + (d*x)/2] + 3*Sin[2*c + (3*d*x)/2])/(6*a^2*d*(Cos[c
/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.49, size = 77, normalized size = 1.64

$$\frac{2\,dx+(2\,dx+3)\cos(dx+c)+\cos(dx+c)^2+(2\,dx+\cos(dx+c)-2)\sin(dx+c)+2}{a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (2*d*x + (2*d*x + 3)*cos(d*x + c) + cos(d*x + c)^2 + (2*d*x + cos(d*x + c)
- 2)*sin(d*x + c) + 2)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

giac [A] time = 0.16, size = 78, normalized size = 1.66

$$\frac{2\left(\frac{dx+c}{a^2}+\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((d*x + c)/a^2 + (2*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 3)/((
tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 1)
*a^2))/d
```

maple [A] time = 0.38, size = 64, normalized size = 1.36

$$\frac{2}{d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{2}}+\frac{4}{d\,a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)+4/d/a^2*arctan(tan(1/2*d*x+1/2*c))+4/d/a^2
/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.44, size = 139, normalized size = 2.96

$$\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+3}{a^{2}+\frac{a^{2}\sin(dx+c)}{\cos(dx+c)+1}+\frac{a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{2}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*((sin(d*x + c)/(cos(d*x + c) + 1) + 2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 3)/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1) + a^2*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 2*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 8.86, size = 68, normalized size = 1.45

$$\frac{2\,x}{a^{2}}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6}{a^{2}\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (2*x)/a^2 + (2*tan(c/2 + (d*x)/2) + 4*tan(c/2 + (d*x)/2)^2 + 6)/(a^2*d*(tan
(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [A] time = 11.90, size = 479, normalized size = 10.19

$$\begin{cases} \frac{2dx\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{2dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{2}{a^2d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} \\ \frac{x\sin(c)\cos^2(c)}{(a\sin(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*d*x*tan(c/2 + d*x/2)**3/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*t
an(c/2 + d*x/2)**2 + a**2*d*tan(c/2 + d*x/2) + a**2*d) + 2*d*x*tan(c/2 + d*
x/2)**2/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*tan(c/2 + d*x/2)**2 + a**2*d*t
an(c/2 + d*x/2) + a**2*d) + 2*d*x*tan(c/2 + d*x/2)/(a**2*d*tan(c/2 + d*x/2)
**3 + a**2*d*tan(c/2 + d*x/2)**2 + a**2*d*tan(c/2 + d*x/2) + a**2*d) + 2*d*
x/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*tan(c/2 + d*x/2)**2 + a**2*d*tan(c/2
 + d*x/2) + a**2*d) + 4*tan(c/2 + d*x/2)**2/(a**2*d*tan(c/2 + d*x/2)**3 + a
**2*d*tan(c/2 + d*x/2)**2 + a**2*d*tan(c/2 + d*x/2) + a**2*d) + 2*tan(c/2 +
 d*x/2)/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*tan(c/2 + d*x/2)**2 + a**2*d*t
an(c/2 + d*x/2) + a**2*d) + 6/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*tan(c/2
+ d*x/2)**2 + a**2*d*tan(c/2 + d*x/2) + a**2*d), Ne(d, 0)), (x*sin(c)*cos(c
)**2/(a*sin(c) + a)**2, True))
```

3.311 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=40

$$\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

```
[Out] -arctanh(cos(d*x+c))/a^2/d+2*cos(d*x+c)/a^2/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.16, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2874, 2966, 3770, 2648}

$$\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x
]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \frac{\csc(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}\\ &= \frac{\int \left(\csc(c+dx)-\frac{2}{1+\sin(c+dx)}\right)dx}{a^2}\\ &= \frac{\int \csc(c+dx)\,dx}{a^2}-\frac{2\int \frac{1}{1+\sin(c+dx)}\,dx}{a^2}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.16, size = 115, normalized size = 2.88

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sin\left(\frac{1}{2}\right.\right.}{a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2]*(Log[Cos[(c +
d*x)/2]] - Log[Sin[(c + d*x)/2]]) + (4 + Log[Cos[(c + d*x)/2]] - Log[Sin[(c
 + d*x)/2]])*Sin[(c + d*x)/2]))/(a^2*d*(1 + Sin[c + d*x])^2))
```

fricas [B] time = 0.51, size = 103, normalized size = 2.58

$$-\frac{(\cos(dx+c)+\sin(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(\cos(dx+c)+\sin(dx+c)+1)\log\left(-\frac{1}{2}\cos(dx+\right.}{2\left(a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((cos(d*x + c) + sin(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(
d*x + c) + sin(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) - 4*cos(d*x + c)
+ 4*sin(d*x + c) - 4)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

giac [A] time = 0.16, size = 38, normalized size = 0.95

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{4}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 4/(a^2*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.59, size = 40, normalized size = 1.00

$$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}+\frac{4}{d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*ln(tan(1/2*d*x+1/2*c))+4/d/a^2/(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.32, size = 55, normalized size = 1.38

$$\frac{\frac{4}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (4/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1)) + log(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^2)/d
```

mupad [B] time = 8.68, size = 39, normalized size = 0.98

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{4}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) + 4/(a^2*d*(tan(c/2 + (d*x)/2) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)\csc(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

3.312 $\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=54

$$-\frac{\cot(c+dx)}{a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot(c+dx)}{a^2d(\csc(c+dx)+1)}$$

```
[Out] 2*arctanh(cos(d*x+c))/a^2/d-3*cot(d*x+c)/a^2/d+2*cot(d*x+c)/a^2/d/(1+sin(d*
x+c))
```

Rubi [A] time = 0.10, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2709, 3770, 3767, 8, 3777}

$$-\frac{\cot(c+dx)}{a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot(c+dx)}{a^2d(\csc(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Cot[c + d*x])
/(a^2*d*(1 + Csc[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \left(2-2\csc(c+dx)+\csc^2(c+dx)-\frac{2}{1+\csc(c+dx)}\right)dx}{a^2}$$
$$= \frac{2x}{a^2}+\frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\,dx}{a^2}-\frac{2\int \frac{1}{1+\csc(c+dx)}\,dx}{a^2}$$
$$= \frac{2x}{a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot(c+dx)}{a^2d(1+\csc(c+dx))}+\frac{2\int -1\,dx}{a^2}-\frac{\text{Subst}(\int 1\,dx, x}{a^2d}$$
$$= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\cot(c+dx)}{a^2d(1+\csc(c+dx))}$$

Mathematica [B] time = 0.80, size = 216, normalized size = 4.00

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{3}{2}(c+dx)\right)\left(-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3*(Cos[(3*(c + d*x))/2]*(5 + 2*Log[Cos[(c + d*x)/2]] - 2*Log[Sin[(c + d*
x)/2]]) + Cos[(c + d*x)/2]*(-3 - 2*Log[Cos[(c + d*x)/2]] + 2*Log[Sin[(c + d
*x)/2]]) + 2*(-2*Log[Cos[(c + d*x)/2]] + 2*Log[Sin[(c + d*x)/2]] + Cos[c +
d*x]*(1 - 2*Log[Cos[(c + d*x)/2]] + 2*Log[Sin[(c + d*x)/2]]))*Sin[(c + d*x)
/2]))/(a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [B] time = 0.52, size = 160, normalized size = 2.96

$$\frac{3\cos(dx+c)^2+\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left(\cos(dx+c)^2-\right.}{a^2d\cos(dx+c)^2-a^2d-\left(a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (3*cos(d*x + c)^2 + (cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*
log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x
+ c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + (3*cos(d*x + c) + 2)*sin(d*x + c)
+ cos(d*x + c) - 2)/(a^2*d*cos(d*x + c)^2 - a^2*d - (a^2*d*cos(d*x + c) + a
^2*d)*sin(d*x + c))
```

giac [A] time = 0.19, size = 90, normalized size = 1.67

$$-\frac{\frac{4\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - tan(1/2*d*x + 1/2*c)/a^2 - (2*
tan(1/2*d*x + 1/2*c)^2 - 7*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c)
^2 + tan(1/2*d*x + 1/2*c))*a^2))/d
```

maple [A] time = 0.72, size = 77, normalized size = 1.43

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{1}{2d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}-\frac{4}{d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/2/d/a^2/tan(1/2*d*x+1/2*c)-2/d/a^2*ln(tan(1/
2*d*x+1/2*c))-4/d/a^2/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.33, size = 116, normalized size = 2.15

$$-\frac{\frac{\frac{9 \sin(dx+c)}{\cos(dx+c)+1}+1}{\frac{a^2 \sin(dx+c)}{\cos(dx+c)+1}+\frac{a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{4 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/(a^2*sin(d*x + c)/(cos(d*x +
c) + 1) + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 4*log(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2 - sin(d*x + c)/(a^2*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 8.69, size = 87, normalized size = 1.61

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1}{d\,\left(2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^2*d) - (9*tan(c/2 + (d*x)/2) + 1)/(d*(2*a^2*tan(c/2
 + (d*x)/2)^2 + 2*a^2*tan(c/2 + (d*x)/2))) - (2*log(tan(c/2 + (d*x)/2)))/(a
^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

3.313 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=78

$$\frac{2\cot(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}$$

```
[Out] -5/2*arctanh(cos(d*x+c))/a^2/d+2*cot(d*x+c)/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)
/a^2/d+2*cos(d*x+c)/a^2/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.22, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2874, 2966, 3770, 3767, 8, 3768, 2648}

$$\frac{2\cot(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) - (Cot[c +
d*x]*Csc[c + d*x])/(2*a^2*d) + (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}$$

$$= \frac{\int \left(2\csc(c+dx)-2\csc^2(c+dx)+\csc^3(c+dx)-\frac{2}{1+\sin(c+dx)}\right)dx}{a^2}$$

$$= \frac{\int \csc^3(c+dx)\,dx}{a^2}+\frac{2\int \csc(c+dx)\,dx}{a^2}-\frac{2\int \csc^2(c+dx)\,dx}{a^2}-\frac{2\int \frac{1}{1+\sin(c+dx)}}{a^2}$$

$$= -\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}+\int$$

$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2\cos(}{a^2d(1+s}$$

Mathematica [B] time = 0.78, size = 364, normalized size = 4.67

$$-\frac{4\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}{d(a\sin(c+dx)+a)^2}-\frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx\right.\right.}{2d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-4*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(d*(a + a*Sin
[c + d*x])^2) + (Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/
(d*(a + a*Sin[c + d*x])^2) - (Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^4)/(8*d*(a + a*Sin[c + d*x])^2) - (5*Log[Cos[(c + d*x)/2]]*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(2*d*(a + a*Sin[c + d*x])^2) + (5*Log[
Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(2*d*(a + a*Sin[
c + d*x])^2) + (Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
/(8*d*(a + a*Sin[c + d*x])^2) - ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Ta
n[(c + d*x)/2])/(d*(a + a*Sin[c + d*x])^2)
```

fricas [B] time = 0.50, size = 246, normalized size = 3.15

$$\frac{16\cos(dx+c)^3+10\cos(dx+c)^2-5\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4*(16*cos(d*x + c)^3 + 10*cos(d*x + c)^2 - 5*(cos(d*x + c)^3 + cos(d*x +
c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)*log(1/2*cos(d*
x + c) + 1/2) + 5*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*s
in(d*x + c) - cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) - 2*(8*cos(d*x
 + c)^2 + 3*cos(d*x + c) - 4)*sin(d*x + c) - 14*cos(d*x + c) - 8)/(a^2*d*co
s(d*x + c)^3 + a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d + (a^2*d*c
os(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

giac [A] time = 0.19, size = 116, normalized size = 1.49

$$\frac{\frac{20\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}+\frac{32}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{30\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(20*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (a^2*tan(1/2*d*x + 1/2*c)^2 -
8*a^2*tan(1/2*d*x + 1/2*c))/a^4 + 32/(a^2*(tan(1/2*d*x + 1/2*c) + 1)) - (30
*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d*x + 1/
2*c)^2))/d
```

maple [A] time = 0.76, size = 114, normalized size = 1.46

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2}-\frac{1}{8d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{1}{d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}+\frac{4}{d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^2*tan(1/2*d*x+1/2*c)-1/8/d/a^2/tan(1/2
*d*x+1/2*c)^2+1/d/a^2/tan(1/2*d*x+1/2*c)+5/2/d/a^2*ln(tan(1/2*d*x+1/2*c))+4
/d/a^2/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.34, size = 161, normalized size = 2.06

$$\frac{\frac{\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}{\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}-\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^2}+\frac{20\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*((7*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 1)/(a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3) - (8*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(
cos(d*x + c) + 1)^2)/a^2 + 20*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 8.66, size = 120, normalized size = 1.54

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}+\frac{5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^2\,d}+\frac{20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\frac{1}{2}}{d\left(4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) + (5*log(tan(c/2 + (d*x)/2)))/(2*a^2*d) + ((
7*tan(c/2 + (d*x)/2))/2 + 20*tan(c/2 + (d*x)/2)^2 - 1/2)/(d*(4*a^2*tan(c/2
+ (d*x)/2)^2 + 4*a^2*tan(c/2 + (d*x)/2)^3)) - tan(c/2 + (d*x)/2)/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)\csc^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

3.314 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)}{a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

```
[Out] 3*arctanh(cos(d*x+c))/a^2/d-3*cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d+cot(d
*x+c)*csc(d*x+c)/a^2/d-2*cos(d*x+c)/a^2/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.25, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2874, 2966, 3770, 3767, 8, 3768, 2648}

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)}{a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^2*d) - (3*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]
^3/(3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d) - (2*Cos[c + d*x])/(a^2*
d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \frac{\csc^4(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2} \\
&= \frac{\int \left(-2\csc(c+dx)+2\csc^2(c+dx)-2\csc^3(c+dx)+\csc^4(c+dx)+\frac{2}{1+\sin(c+dx)}\right.}{a^2} \\
&= \frac{\int \csc^4(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\,dx}{a^2}+\frac{2\int \csc^2(c+dx)\,dx}{a^2}-\frac{2\int \csc^3(c+dx}{a^2} \\
&= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}-\frac{\int \mathrm{cs}}{} \\
&= \frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{3\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)\csc(c+dx}{a^2d}
\end{aligned}
$$

Mathematica [B] time = 1.29, size = 472, normalized size = 5.19

$$\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^3\left(12\sin\left(\frac{1}{2}(c+dx)\right)-6\sin\left(\frac{3}{2}(c+dx)\right)-2\sin\left(\frac{5}{2}(c+dx)\right)+8\sin\left(\frac{7}{2}(c+d\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^3*(-10*Cos[(5*(c + d*x))/2] + 20*Cos
[(7*(c + d*x))/2] - 9*Cos[(5*(c + d*x))/2]*Log[Cos[(c + d*x)/2]] + 9*Cos[(7
*(c + d*x))/2]*Log[Cos[(c + d*x)/2]] + 3*Cos[(c + d*x)/2]*(8 + 9*Log[Cos[(c
 + d*x)/2]] - 9*Log[Sin[(c + d*x)/2]]) - 3*Cos[(3*(c + d*x))/2]*(14 + 9*Log
[Cos[(c + d*x)/2]] - 9*Log[Sin[(c + d*x)/2]]) + 9*Cos[(5*(c + d*x))/2]*Log[
Sin[(c + d*x)/2]] - 9*Cos[(7*(c + d*x))/2]*Log[Sin[(c + d*x)/2]] + 12*Sin[(
c + d*x)/2] + 27*Log[Cos[(c + d*x)/2]]*Sin[(c + d*x)/2] - 27*Log[Sin[(c + d
*x)/2]]*Sin[(c + d*x)/2] - 6*Sin[(3*(c + d*x))/2] + 27*Log[Cos[(c + d*x)/2]
]*Sin[(3*(c + d*x))/2] - 27*Log[Sin[(c + d*x)/2]]*Sin[(3*(c + d*x))/2] - 2*
Sin[(5*(c + d*x))/2] - 9*Log[Cos[(c + d*x)/2]]*Sin[(5*(c + d*x))/2] + 9*Log
[Sin[(c + d*x)/2]]*Sin[(5*(c + d*x))/2] + 8*Sin[(7*(c + d*x))/2] - 9*Log[Co
s[(c + d*x)/2]]*Sin[(7*(c + d*x))/2] + 9*Log[Sin[(c + d*x)/2]]*Sin[(7*(c +
d*x))/2]))/(192*a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [B] time = 0.53, size = 302, normalized size = 3.32

$$28\cos(dx+c)^4+10\cos(dx+c)^3-42\cos(dx+c)^2+9\left(\cos(dx+c)^4-2\cos(dx+c)^2-\left(\cos(dx+c)^3+\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(28*cos(d*x + c)^4 + 10*cos(d*x + c)^3 - 42*cos(d*x + c)^2 + 9*(cos(d*x
 + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c
) - 1)*sin(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 9*(cos(d*x + c)^4 -
2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sin
(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(14*cos(d*x + c)^3 + 9*cos(
d*x + c)^2 - 12*cos(d*x + c) - 6)*sin(d*x + c) - 12*cos(d*x + c) + 12)/(a^2
*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d - (a^2*d*cos(d*x + c)^3
+ a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

giac [A] time = 0.20, size = 146, normalized size = 1.60

$$-\frac{\frac{72\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{96}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{132\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}-\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^4\tan}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(72*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 96/(a^2*(tan(1/2*d*x + 1/2*c
) + 1)) - (132*tan(1/2*d*x + 1/2*c)^3 - 33*tan(1/2*d*x + 1/2*c)^2 + 6*tan(1
/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)^3) - (a^4*tan(1/2*d*x + 1/2*
c)^3 - 6*a^4*tan(1/2*d*x + 1/2*c)^2 + 33*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.58, size = 153, normalized size = 1.68

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^2}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^2}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^2}-\frac{1}{24d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{1}{4d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{11}{8d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^2*tan(1/2*d*x+1/2*c)^2+11/8/d/a^2*t
an(1/2*d*x+1/2*c)-1/24/d/a^2/tan(1/2*d*x+1/2*c)^3+1/4/d/a^2/tan(1/2*d*x+1/2
*c)^2-11/8/d/a^2/tan(1/2*d*x+1/2*c)-3/d/a^2*ln(tan(1/2*d*x+1/2*c))-4/d/a^2/
(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.33, size = 199, normalized size = 2.19

$$\frac{\frac{\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{27\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{129\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1}{\frac{a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{33\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{72\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((5*sin(d*x + c)/(cos(d*x + c) + 1) - 27*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 129*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a^2*sin(d*x + c)^3/(
```

```
cos(d*x + c) + 1)^3 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (33*sin(d*
x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 72*log(sin(d*x + c)/(cos(d*x + c) + 1))
/a^2)/d
```

mupad [B] time = 8.65, size = 153, normalized size = 1.68

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a^2\,d}-\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{43\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{1}{3}}{d\,\left(8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+11\,\mathrm{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^2*d) - tan(c/2 + (d*x)/2)^2/(4*a^2*d) - (3*log(t
an(c/2 + (d*x)/2)))/(a^2*d) - (9*tan(c/2 + (d*x)/2)^2 - (5*tan(c/2 + (d*x)/
2))/3 + 43*tan(c/2 + (d*x)/2)^3 + 1/3)/(d*(8*a^2*tan(c/2 + (d*x)/2)^3 + 8*a
^2*tan(c/2 + (d*x)/2)^4)) + (11*tan(c/2 + (d*x)/2))/(8*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)\csc^4(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**4/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

3.315 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=97

$$-\frac{3\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{19\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{11x}{2a^3}$$

```
[Out] -11/2*x/a^3-3*cos(d*x+c)/a^3/d+1/2*cos(d*x+c)*sin(d*x+c)/a^3/d+2/3*cos(d*x+
c)/a^3/d/(1+sin(d*x+c))^2-19/3*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.26, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2874, 2966, 2638, 2635, 8, 2650, 2648}

$$-\frac{3\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{19\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{11x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*x)/(2*a^3) - (3*Cos[c + d*x])/(a^3*d) + (Cos[c + d*x]*Sin[c + d*x])/(2
*a^3*d) + (2*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x])^2) - (19*Cos[c + d*x
])/(3*a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \frac{\sin^3(c+dx)(a-a\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx}{a^2}$$

$$= \frac{\int \left(-\frac{5}{a}+\frac{3\sin(c+dx)}{a}-\frac{\sin^2(c+dx)}{a}-\frac{2}{a(1+\sin(c+dx))^2}+\frac{7}{a(1+\sin(c+dx))}\right)dx}{a^2}$$

$$= -\frac{5x}{a^3}-\frac{\int \sin^2(c+dx)\,dx}{a^3}-\frac{2\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3}+\frac{3\int \sin(c+dx)\,dx}{a^3}+\frac{7\int \frac{}{1+}}{}$$

$$= -\frac{5x}{a^3}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}+\frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}-\frac{}{a}$$

$$= -\frac{11x}{2a^3}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}+\frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}-$$

Mathematica [B] time = 1.09, size = 197, normalized size = 2.03

$$-\frac{1980dx\sin\left(c+\frac{dx}{2}\right)+660dx\sin\left(c+\frac{3dx}{2}\right)+498\sin\left(2c+\frac{3dx}{2}\right)+135\sin\left(2c+\frac{5dx}{2}\right)+15\sin\left(4c+\frac{7dx}{2}\right)-1326}{240a^3d\left(\sin\left(\frac{c}{2}\right)+\mathrm{co}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/240*(1980*d*x*Cos[(d*x)/2] - 1326*Cos[c + (d*x)/2] + 2012*Cos[c + (3*d*x
)/2] - 660*d*x*Cos[2*c + (3*d*x)/2] - 135*Cos[3*c + (5*d*x)/2] + 15*Cos[3*c
 + (7*d*x)/2] - 3216*Sin[(d*x)/2] + 1980*d*x*Sin[c + (d*x)/2] + 660*d*x*Sin
[c + (3*d*x)/2] + 498*Sin[2*c + (3*d*x)/2] + 135*Sin[2*c + (5*d*x)/2] + 15*
Sin[4*c + (7*d*x)/2])/(a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^3)
```

fricas [A] time = 0.50, size = 163, normalized size = 1.68

$$\frac{3\cos\left(dx+c\right)^4-\left(33\,dx-53\right)\cos\left(dx+c\right)^2-12\cos\left(dx+c\right)^3+66\,dx+\left(33\,dx+64\right)\cos\left(dx+c\right)+\left(3\cos\left(dx+\right.\right.}{6\left(a^3d\cos\left(dx+c\right)^2-a^3d\cos\left(dx+c\right)-2\,a^3d-\left(a^3d\cos\left(dx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*cos(d*x + c)^4 - (33*d*x - 53)*cos(d*x + c)^2 - 12*cos(d*x + c)^3 +
66*d*x + (33*d*x + 64)*cos(d*x + c) + (3*cos(d*x + c)^3 + 66*d*x + (33*d*x
+ 68)*cos(d*x + c) + 15*cos(d*x + c)^2 + 4)*sin(d*x + c) - 4)/(a^3*d*cos(d*
x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (a^3*d*cos(d*x + c) + 2*a^3*d)*si
n(d*x + c))
```

giac [A] time = 0.22, size = 117, normalized size = 1.21

$$-\frac{\frac{33\,(dx+c)}{a^3}+\frac{6\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}+\frac{4\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(33*(d*x + c)/a^3 + 6*(tan(1/2*d*x + 1/2*c)^3 + 6*tan(1/2*d*x + 1/2*c)
^2 - tan(1/2*d*x + 1/2*c) + 6)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) + 4*(15
```

```
*tan(1/2*d*x + 1/2*c)^2 + 36*tan(1/2*d*x + 1/2*c) + 17)/(a^3*(tan(1/2*d*x +
 1/2*c) + 1)^3))/d
```

maple [B] time = 0.41, size = 205, normalized size = 2.11

$$-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{6}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-11$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-6/d/a^3/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*ta
n(1/2*d*x+1/2*c)-6/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2-11/d/a^3*arctan(tan(1/2
*d*x+1/2*c))+8/3/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3-4/d/a^3/(tan(1/2*d*x+1/2*c)
+1)^2-10/d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.45, size = 314, normalized size = 3.24

$$-\frac{\frac{\frac{123\sin(dx+c)}{\cos(dx+c)+1}+\frac{161\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{210\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{154\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{99\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{33\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+52}{a^3+\frac{3a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{7a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{7a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{33\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/3*((123*sin(d*x + c)/(cos(d*x + c) + 1) + 161*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 210*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 154*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 99*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 33*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 52)/(a^3 + 3*a^3*sin(d*x + c)/(cos(d*x + c) + 1
) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 7*a^3*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 7*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 5*a^3*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 + 3*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a
^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 33*arctan(sin(d*x + c)/(cos(d*x +
 c) + 1))/a^3)/d
```

mupad [B] time = 12.07, size = 121, normalized size = 1.25

$$-\frac{11\,x}{2\,a^3}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+33\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{154\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+70\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{161\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+41\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (11*x)/(2*a^3) - (41*tan(c/2 + (d*x)/2) + (161*tan(c/2 + (d*x)/2)^2)/3 +
70*tan(c/2 + (d*x)/2)^3 + (154*tan(c/2 + (d*x)/2)^4)/3 + 33*tan(c/2 + (d*x)
/2)^5 + 11*tan(c/2 + (d*x)/2)^6 + 52/3)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^3*(
tan(c/2 + (d*x)/2)^2 + 1)^2)
```

sympy [A] time = 63.55, size = 2264, normalized size = 23.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-33*d*x*tan(c/2 + d*x/2)**7/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a
**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c
/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)
**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 99*d*x*tan(c/2 + d*x/2)**6/(
6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*ta
n(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x
/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**
3*d) - 165*d*x*tan(c/2 + d*x/2)**5/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*
d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 +
 d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2
+ 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 231*d*x*tan(c/2 + d*x/2)**4/(6*a
**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c
/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)
**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d
) - 231*d*x*tan(c/2 + d*x/2)**3/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*t
an(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*
x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 1
8*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 165*d*x*tan(c/2 + d*x/2)**2/(6*a**3
*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2
+ d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3
 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) -
 99*d*x*tan(c/2 + d*x/2)/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2
+ d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4
 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*
d*tan(c/2 + d*x/2) + 6*a**3*d) - 33*d*x/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*
a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(
c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2
)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 66*tan(c/2 + d*x/2)**6/(6*a
**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c
```

```
/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)
**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d
) - 198*tan(c/2 + d*x/2)**5/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c
/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)
**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a*
*3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 308*tan(c/2 + d*x/2)**4/(6*a**3*d*tan(c
/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)
**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a*
*3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 420*tan
(c/2 + d*x/2)**3/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)
**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a*
*3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/
2 + d*x/2) + 6*a**3*d) - 322*tan(c/2 + d*x/2)**2/(6*a**3*d*tan(c/2 + d*x/2)
**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a*
*3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/
2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 246*tan(c/2 + d*x/
2)/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*
d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 +
 d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6
*a**3*d) - 104/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**
6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3
*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2
+ d*x/2) + 6*a**3*d), Ne(d, 0)), (x*sin(c)**3*cos(c)**2/(a*sin(c) + a)**3,
True))
```

3.316 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=76

$$\frac{3\cos(c+dx)}{a^3d}+\frac{3x}{a^3}+\frac{2\cos^3(c+dx)}{ad(a\sin(c+dx)+a)^2}-\frac{\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^3}$$

```
[Out] 3*x/a^3+3*cos(d*x+c)/a^3/d-1/3*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^3+2*cos(d*x+
c)^3/a/d/(a+a*sin(d*x+c))^2
```

Rubi [A] time = 0.18, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2871, 2680, 2682, 8}

$$\frac{3\cos(c+dx)}{a^3d}+\frac{3x}{a^3}+\frac{2\cos^3(c+dx)}{ad(a\sin(c+dx)+a)^2}-\frac{\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*x)/a^3 + (3*Cos[c + d*x])/(a^3*d) - Cos[c + d*x]^3/(3*d*(a + a*Sin[c + d
*x])^3) + (2*Cos[c + d*x]^3)/(a*d*(a + a*Sin[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2871

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*m), x] - Dist[1/g^2, Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && EqQ[a^2 - b^2, 0] && EqQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= -\frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3}-\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx\\ &= -\frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3}+\frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2}+\frac{3\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx}{a^2}\\ &= \frac{3\cos(c+dx)}{a^3d}-\frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3}+\frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2}+\frac{3\int 1\,dx}{a^3}\\ &= \frac{3x}{a^3}+\frac{3\cos(c+dx)}{a^3d}-\frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3}+\frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2}\end{aligned}$$

Mathematica [A] time = 0.67, size = 96, normalized size = 1.26

$$\frac{3\cos(c+dx)-\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(13\sin(c+dx)+11)}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}-\frac{2}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}+9c+9dx}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (9*c + 9*d*x + 3*Cos[c + d*x] - 2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 -
 (2*Sin[(c + d*x)/2]*(11 + 13*Sin[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^3)/(3*a^3*d)
```

fricas [A] time = 0.50, size = 144, normalized size = 1.89

$$\frac{(9\,dx-16)\cos(dx+c)^2+3\cos(dx+c)^3-18\,dx-(9\,dx+17)\cos(dx+c)-\left(18\,dx+(9\,dx+19)\cos(dx+c)\right.}{3\left(a^3d\cos(dx+c)^2-a^3d\cos(dx+c)-2\,a^3d-\left(a^3d\cos(dx+c)+2\,a^3d\right)\sin(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/3*((9*d*x - 16)*cos(d*x + c)^2 + 3*cos(d*x + c)^3 - 18*d*x - (9*d*x + 17)
*cos(d*x + c) - (18*d*x + (9*d*x + 19)*cos(d*x + c) + 3*cos(d*x + c)^2 + 2)
*sin(d*x + c) + 2)/(a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (
a^3*d*cos(d*x + c) + 2*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.18, size = 80, normalized size = 1.05

$$\frac{\frac{9\,(dx+c)}{a^3} + \frac{6}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3} + \frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(9*(d*x + c)/a^3 + 6/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3) + 2*(9*tan(1/2*
d*x + 1/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) + 11)/(a^3*(tan(1/2*d*x + 1/2*c) +
 1)^3))/d
```

maple [A] time = 0.51, size = 106, normalized size = 1.39

$$\frac{2}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}-\frac{8}{3d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{4}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{d\,a^3\left(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)+6/d/a^3*arctan(tan(1/2*d*x+1/2*c))-8/3/d/a
^3/(tan(1/2*d*x+1/2*c)+1)^3+4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+6/d/a^3/(tan(1
/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.43, size = 228, normalized size = 3.00

$$\frac{2\left(\frac{\frac{33\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{29\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{27\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+14}{a^3+\frac{3\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{4\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{9\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 2/3*((33*sin(d*x + c)/(cos(d*x + c) + 1) + 29*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 27*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 14)/(a^3 + 3*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 4*a^3*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5) + 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 11.32, size = 94, normalized size = 1.24

$$\frac{3\,x}{a^3}+\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{58\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+22\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{28}{3}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (3*x)/a^3 + (22*tan(c/2 + (d*x)/2) + (58*tan(c/2 + (d*x)/2)^2)/3 + 18*tan(c
/2 + (d*x)/2)^3 + 6*tan(c/2 + (d*x)/2)^4 + 28/3)/(a^3*d*(tan(c/2 + (d*x)/2)
 + 1)^3*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [A] time = 38.82, size = 1246, normalized size = 16.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((9*d*x*tan(c/2 + d*x/2)**5/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3
*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2
+ d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 27*d*x*tan(c/2 + d*x/
2)**4/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**
3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2
+ d*x/2) + 3*a**3*d) + 36*d*x*tan(c/2 + d*x/2)**3/(3*a**3*d*tan(c/2 + d*x/2
)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a*
*3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 36*d*x*t
an(c/2 + d*x/2)**2/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2
)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a*
*3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 27*d*x*tan(c/2 + d*x/2)/(3*a**3*d*tan(c
/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)*
*3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d)
+ 9*d*x/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a
**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/
2 + d*x/2) + 3*a**3*d) + 18*tan(c/2 + d*x/2)**4/(3*a**3*d*tan(c/2 + d*x/2)*
```

```
*5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3
*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 54*tan(c/2
 + d*x/2)**3/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 +
 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*t
an(c/2 + d*x/2) + 3*a**3*d) + 58*tan(c/2 + d*x/2)**2/(3*a**3*d*tan(c/2 + d*
x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12
*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 66*ta
n(c/2 + d*x/2)/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4
 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d
*tan(c/2 + d*x/2) + 3*a**3*d) + 28/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d
*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 +
d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d), Ne(d, 0)), (x*sin(c)**2*
cos(c)**2/(a*sin(c) + a)**3, True))
```

3.317 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=61

$$-\frac{7\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}-\frac{x}{a^3}+\frac{2\cos(c+dx)}{3ad(a\sin(c+dx)+a)^2}$$

```
[Out] -x/a^3-7/3*cos(d*x+c)/a^3/d/(1+sin(d*x+c))+2/3*cos(d*x+c)/a/d/(a+a*sin(d*x+
c))^2
```

Rubi [A] time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2857, 2735, 2648}

$$-\frac{7\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}-\frac{x}{a^3}+\frac{2\cos(c+dx)}{3ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - (7*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x])) + (2*Cos[c + d*x])
/(3*a*d*(a + a*Sin[c + d*x])^2)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{2\cos(c+dx)}{3ad(a+a\sin(c+dx))^2} - \frac{\int \frac{-4a+3a\sin(c+dx)}{a+a\sin(c+dx)}\,dx}{3a^3}$$
$$= -\frac{x}{a^3} + \frac{2\cos(c+dx)}{3ad(a+a\sin(c+dx))^2} + \frac{7\int \frac{1}{a+a\sin(c+dx)}\,dx}{3a^2}$$
$$= -\frac{x}{a^3} + \frac{2\cos(c+dx)}{3ad(a+a\sin(c+dx))^2} - \frac{7\cos(c+dx)}{3d\left(a^3+a^3\sin(c+dx)\right)}$$

Mathematica [B] time = 0.42, size = 145, normalized size = 2.38

$$-\frac{180dx\sin\left(c+\frac{dx}{2}\right)+60dx\sin\left(c+\frac{3dx}{2}\right)+3\sin\left(2c+\frac{3dx}{2}\right)-351\cos\left(c+\frac{dx}{2}\right)+277\cos\left(c+\frac{3dx}{2}\right)-60dx\cos\left(2\right.}{120a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/120*(180*d*x*Cos[(d*x)/2] - 351*Cos[c + (d*x)/2] + 277*Cos[c + (3*d*x)/2
] - 60*d*x*Cos[2*c + (3*d*x)/2] - 471*Sin[(d*x)/2] + 180*d*x*Sin[c + (d*x)/
2] + 60*d*x*Sin[c + (3*d*x)/2] + 3*Sin[2*c + (3*d*x)/2])/(a^3*d*(Cos[c/2] +
 Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

fricas [B] time = 0.51, size = 124, normalized size = 2.03

$$-\frac{(3\,dx-7)\cos(dx+c)^2-6\,dx-(3\,dx+5)\cos(dx+c)-(6\,dx+(3\,dx+7)\cos(dx+c)+2)\sin(dx+c)+2}{3\left(a^3d\cos(dx+c)^2-a^3d\cos(dx+c)-2\,a^3d-\left(a^3d\cos(dx+c)+2\,a^3d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*((3*d*x - 7)*cos(d*x + c)^2 - 6*d*x - (3*d*x + 5)*cos(d*x + c) - (6*d*
x + (3*d*x + 7)*cos(d*x + c) + 2)*sin(d*x + c) + 2)/(a^3*d*cos(d*x + c)^2 -
 a^3*d*cos(d*x + c) - 2*a^3*d - (a^3*d*cos(d*x + c) + 2*a^3*d)*sin(d*x + c)
)
```

giac [A] time = 0.22, size = 60, normalized size = 0.98

$$-\frac{\frac{3\,(dx+c)}{a^3}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^3 + 2*(3*tan(1/2*d*x + 1/2*c)^2 + 12*tan(1/2*d*x + 1/2*
c) + 5)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

maple [A] time = 0.42, size = 83, normalized size = 1.36

$$-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}+\frac{8}{3d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{4}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/d/a^3*arctan(tan(1/2*d*x+1/2*c))+8/3/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3-4/d/
a^3/(tan(1/2*d*x+1/2*c)+1)^2-2/d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.44, size = 142, normalized size = 2.33

$$-\frac{2\left(\frac{\frac{12\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+5}{a^3+\frac{3\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{3\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/3*((12*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 5)/(a^3 + 3*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^3*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 3*ar
ctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 8.77, size = 54, normalized size = 0.89

$$-\frac{x}{a^3}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{10}{3}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - x/a^3 - (8*tan(c/2 + (d*x)/2) + 2*tan(c/2 + (d*x)/2)^2 + 10/3)/(a^3*d*(ta
n(c/2 + (d*x)/2) + 1)^3)
```

sympy [A] time = 22.21, size = 529, normalized size = 8.67

$$\begin{cases} -\frac{3dx\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^3d} - \frac{9dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^3d} - \frac{\quad}{3a^3d\tan^3\left(\frac{c}{2}\right.} \\ \frac{x\sin(c)\cos^2(c)}{(a\sin(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-3*d*x*tan(c/2 + d*x/2)**3/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**
3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) - 9*d*x*tan
(c/2 + d*x/2)**2/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*tan(c/2 + d*x/2)*
*2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) - 9*d*x*tan(c/2 + d*x/2)/(3*a**3
*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 +
d*x/2) + 3*a**3*d) - 3*d*x/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*tan(c/2
 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) - 6*tan(c/2 + d*x/2)**
2/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*t
an(c/2 + d*x/2) + 3*a**3*d) - 24*tan(c/2 + d*x/2)/(3*a**3*d*tan(c/2 + d*x/2
)**3 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d)
 - 10/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3
*d*tan(c/2 + d*x/2) + 3*a**3*d), Ne(d, 0)), (x*sin(c)*cos(c)**2/(a*sin(c) +
 a)**3, True))
```

3.318 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=68

$$\frac{5\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}$$

```
[Out] -arctanh(cos(d*x+c))/a^3/d+2/3*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+5/3*cos(d*
x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.19, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2874, 2966, 3770, 2650, 2648}

$$\frac{5\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^3*d)) + (2*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d
*x])^2) + (5*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \frac{\csc(c+dx)(a-a\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx}{a^2}\\ &= \frac{\int \left(\frac{\csc(c+dx)}{a}-\frac{2}{a(1+\sin(c+dx))^2}-\frac{1}{a(1+\sin(c+dx))}\right)dx}{a^2}\\ &= \frac{\int \csc(c+dx)\,dx}{a^3}-\frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3}-\frac{2\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}+\frac{\cos(c+dx)}{a^3d(1+\sin(c+dx))}-\frac{2\int \frac{}{1+}\end{aligned}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}+\frac{5\cos(c+dx)}{3a^3d(1+\sin(c+dx))}$$

Mathematica [B] time = 0.39, size = 185, normalized size = 2.72

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-4\sin\left(\frac{1}{2}(c+dx)\right)-10\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-4*Sin[(c + d*x)/2] + 2*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]) - 10*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 - 3*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)
```

```
/2])^3 + 3*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3))/
(3*d*(a + a*Sin[c + d*x])^3)
```

fricas [B] time = 0.57, size = 194, normalized size = 2.85

$$-\frac{10\cos(dx+c)^2 + 3\left(\cos(dx+c)^2 - (\cos(dx+c)+2)\sin(dx+c) - \cos(dx+c) - 2\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right.}{6\left(a^3 d\cos(dx+c)^2 - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(10*cos(d*x + c)^2 + 3*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x +
c) - cos(d*x + c) - 2)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^2 - (c
os(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)*log(-1/2*cos(d*x + c) + 1
/2) + 2*(5*cos(d*x + c) - 2)*sin(d*x + c) + 14*cos(d*x + c) + 4)/(a^3*d*cos
(d*x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (a^3*d*cos(d*x + c) + 2*a^3*d)
*sin(d*x + c))
```

giac [A] time = 0.18, size = 66, normalized size = 0.97

$$\frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3} + \frac{2\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 2*(9*tan(1/2*d*x + 1/2*c)^2 + 1
2*tan(1/2*d*x + 1/2*c) + 7)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

maple [A] time = 0.61, size = 82, normalized size = 1.21

$$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}+\frac{8}{3d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{4}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{6}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*ln(tan(1/2*d*x+1/2*c))+8/3/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3-4/d/a^3/(
tan(1/2*d*x+1/2*c)+1)^2+6/d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.34, size = 143, normalized size = 2.10

$$\frac{\frac{2\left(\frac{12\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+7\right)}{a^3+\frac{3\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(2*(12*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 7)/(a^3 + 3*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^3*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 3*l
og(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 8.90, size = 64, normalized size = 0.94

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}+\frac{6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{14}{3}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) + (8*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)
/2)^2 + 14/3)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)\csc(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

3.319 $\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$-\frac{\cot(c+dx)}{a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{13\cot(c+dx)}{3a^3d(\csc(c+dx)+1)}+\frac{2\cot(c+dx)}{3a^3d(\csc(c+dx)+1)^2}$$

```
[Out] 3*arctanh(cos(d*x+c))/a^3/d-14/3*cot(d*x+c)/a^3/d+2/3*cot(d*x+c)/a^3/d/(1+s
in(d*x+c))^2+3*cot(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.21, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 3770, 3767, 8, 3777, 3919, 3794}

$$-\frac{\cot(c+dx)}{a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{13\cot(c+dx)}{3a^3d(\csc(c+dx)+1)}+\frac{2\cot(c+dx)}{3a^3d(\csc(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) + (2*Cot[c + d*x])
/(3*a^3*d*(1 + Csc[c + d*x])^2) - (13*Cot[c + d*x])/(3*a^3*d*(1 + Csc[c + d
*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \left(\frac{5}{a}-\frac{3\csc(c+dx)}{a}+\frac{\csc^2(c+dx)}{a}+\frac{2}{a(1+\csc(c+dx))^2}-\frac{7}{a(1+\csc(c+dx))}\right)dx}{a^2}$$

$$= \frac{5x}{a^3}+\frac{\int \csc^2(c+dx)\,dx}{a^3}+\frac{2\int \frac{1}{(1+\csc(c+dx))^2}\,dx}{a^3}-\frac{3\int \csc(c+dx)\,dx}{a^3}-\frac{7\int \frac{1}{1+\csc(c+dx)}}{a^3}$$

$$= \frac{5x}{a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{2\cot(c+dx)}{3a^3d(1+\csc(c+dx))^2}-\frac{7\cot(c+dx)}{a^3d(1+\csc(c+dx))}-\frac{2\int}{}$$

$$= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{2\cot(c+dx)}{3a^3d(1+\csc(c+dx))^2}-\frac{7\cot(c+dx)}{a^3d(1+\csc(c+dx}$$

$$= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{2\cot(c+dx)}{3a^3d(1+\csc(c+dx))^2}-\frac{13\cot(c+dx)}{3a^3d(1+\csc(c+d}$$

Mathematica [B] time = 1.56, size = 255, normalized size = 3.11

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(8\sin\left(\frac{1}{2}(c+dx)\right)+44\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(8*Sin[(c + d*x)/2] - 4*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]) + 44*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2 - 3*Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3
 + 18*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 18*Lo
g[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 3*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^3*Tan[(c + d*x)/2]))/(6*d*(a + a*Sin[c + d*x])^3
)
```

fricas [B] time = 0.60, size = 279, normalized size = 3.40

$$-\frac{28\cos(dx+c)^3-10\cos(dx+c)^2-9\left(\cos(dx+c)^3+2\cos(dx+c)^2+\left(\cos(dx+c)^2-\cos(dx+c)-2\right)\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(28*cos(d*x + c)^3 - 10*cos(d*x + c)^2 - 9*(cos(d*x + c)^3 + 2*cos(d*x
 + c)^2 + (cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) - cos(d*x + c) -
 2)*log(1/2*cos(d*x + c) + 1/2) + 9*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + (c
os(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) - cos(d*x + c) - 2)*log(-1/2
*cos(d*x + c) + 1/2) - 2*(14*cos(d*x + c)^2 + 19*cos(d*x + c) + 2)*sin(d*x
+ c) - 34*cos(d*x + c) + 4)/(a^3*d*cos(d*x + c)^3 + 2*a^3*d*cos(d*x + c)^2
- a^3*d*cos(d*x + c) - 2*a^3*d + (a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c)
 - 2*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.21, size = 109, normalized size = 1.33

$$-\frac{\frac{18\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}-\frac{3\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\frac{4\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+13\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(18*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*tan(1/2*d*x + 1/2*c)/a^3 -
3*(6*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)) + 4*(15*tan(1/2*d
*x + 1/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) + 13)/(a^3*(tan(1/2*d*x + 1/2*c) +
1)^3))/d
```

maple [A] time = 0.60, size = 119, normalized size = 1.45

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}-\frac{1}{2d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}-\frac{8}{3d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{4}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^3*ln(tan(1/
2*d*x+1/2*c))-8/3/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3+4/d/a^3/(tan(1/2*d*x+1/2*c
)+1)^2-10/d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.34, size = 202, normalized size = 2.46

$$-\frac{\frac{\frac{61\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{105\,\sin\left(dx+c\right)^2}{\left(\cos\left(dx+c\right)+1\right)^2}+\frac{63\,\sin\left(dx+c\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}+3}{\frac{a^3\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{3\,a^3\sin\left(dx+c\right)^2}{\left(\cos\left(dx+c\right)+1\right)^2}+\frac{3\,a^3\sin\left(dx+c\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}+\frac{a^3\sin\left(dx+c\right)^4}{\left(\cos\left(dx+c\right)+1\right)^4}}+\frac{18\,\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a^3}-\frac{3\,\sin\left(dx+c\right)}{a^3\left(\cos\left(dx+c\right)+1\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*((61*sin(d*x + c)/(cos(d*x + c) + 1) + 105*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 63*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3)/(a^3*sin(d*x + c)/(c
os(d*x + c) + 1) + 3*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) +
18*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 3*sin(d*x + c)/(a^3*(cos(d*x
+ c) + 1)))/d
```

mupad [B] time = 8.81, size = 145, normalized size = 1.77

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}-\frac{21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{61\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+1}{d\,\left(2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+6\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{3\,\ln\left(\tan\left(\right.\right.}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - ((61*tan(c/2 + (d*x)/2))/3 + 35*tan(c/2 + (d
*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^3 + 1)/(d*(6*a^3*tan(c/2 + (d*x)/2)^2 + 6*
a^3*tan(c/2 + (d*x)/2)^3 + 2*a^3*tan(c/2 + (d*x)/2)^4 + 2*a^3*tan(c/2 + (d*
x)/2))) - (3*log(tan(c/2 + (d*x)/2)))/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)*
*2 + 3*sin(c + d*x) + 1), x)/a**3
```

3.320 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=106

$$\frac{3\cot(c+dx)}{a^3d}+\frac{17\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}$$

```
[Out] -11/2*arctanh(cos(d*x+c))/a^3/d+3*cot(d*x+c)/a^3/d-1/2*cot(d*x+c)*csc(d*x+c
)/a^3/d+2/3*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+17/3*cos(d*x+c)/a^3/d/(1+sin(
d*x+c))
```

Rubi [A] time = 0.27, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2874, 2966, 3770, 3767, 8, 3768, 2650, 2648}

$$\frac{3\cot(c+dx)}{a^3d}+\frac{17\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cot[c + d*x])/(a^3*d) - (Cot[c +
 d*x]*Csc[c + d*x])/(2*a^3*d) + (2*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x]
)^2) + (17*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx}{a^2}$$
$$= \frac{\int \left(\frac{5\csc(c+dx)}{a} - \frac{3\csc^2(c+dx)}{a} + \frac{\csc^3(c+dx)}{a} - \frac{2}{a(1+\sin(c+dx))^2} - \frac{5}{a(1+\sin(c+dx))}\right)dx}{a^2}$$
$$= \frac{\int \csc^3(c+dx)\,dx}{a^3} - \frac{2\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3} - \frac{3\int \csc^2(c+dx)\,dx}{a^3} + \frac{5\int \csc(c+dx)}{a^3}$$
$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2} + -$$
$$= -\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{3\cot(c+dx)}{a^3d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{2\cos}{3a^3d(1+}$$

Mathematica [B] time = 5.95, size = 308, normalized size = 2.91

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-32\sin\left(\frac{1}{2}(c+dx)\right)-272\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-32*Sin[(c + d*x)/2] - 3*(1 + Cot
[(c + d*x)/2])^3*Sin[(c + d*x)/2] + 16*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
) - 272*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 36*Cot[(
c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 132*Log[Cos[(c + d*x)
/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 132*Log[Sin[(c + d*x)/2]]*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 36*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^3*Tan[(c + d*x)/2] + 3*Cos[(c + d*x)/2]*(1 + Tan[(c + d*x)/2])^3))/(2
4*a^3*d*(1 + Sin[c + d*x])^3)
```

fricas [B] time = 0.52, size = 365, normalized size = 3.44

$$-\frac{104\cos(dx+c)^4+142\cos(dx+c)^3-90\cos(dx+c)^2+33\left(\cos(dx+c)^4-\cos(dx+c)^3-3\cos(dx+c)^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(104*cos(d*x + c)^4 + 142*cos(d*x + c)^3 - 90*cos(d*x + c)^2 + 33*(co
s(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 - (cos(d*x + c)^3 + 2*cos(
d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) + cos(d*x + c) + 2)*log(1/2*cos
(d*x + c) + 1/2) - 33*(cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 -
 (cos(d*x + c)^3 + 2*cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) + cos(
d*x + c) + 2)*log(-1/2*cos(d*x + c) + 1/2) + 2*(52*cos(d*x + c)^3 - 19*cos(
d*x + c)^2 - 64*cos(d*x + c) + 4)*sin(d*x + c) - 136*cos(d*x + c) - 8)/(a^3
*d*cos(d*x + c)^4 - a^3*d*cos(d*x + c)^3 - 3*a^3*d*cos(d*x + c)^2 + a^3*d*c
os(d*x + c) + 2*a^3*d - (a^3*d*cos(d*x + c)^3 + 2*a^3*d*cos(d*x + c)^2 - a^
3*d*cos(d*x + c) - 2*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.22, size = 143, normalized size = 1.35

$$\frac{\frac{132 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} - \frac{3\left(66\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2} + \frac{3\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^6} + \frac{16\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{a^3\left(\tan\left(\right.\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(132*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*(66*tan(1/2*d*x + 1/2*c)^2
 - 12*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c)^2) + 3*(a^3*tan(1
/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x + 1/2*c))/a^6 + 16*(21*tan(1/2*d*x +
 1/2*c)^2 + 36*tan(1/2*d*x + 1/2*c) + 19)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^3
))/d
```

maple [A] time = 0.69, size = 157, normalized size = 1.48

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}-\frac{1}{8d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{3}{2d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{11\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^3}+\frac{8}{3d\,a^3\left(\tan\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^3*tan(1/2*d*x+1/2*c)-1/8/d/a^3/tan(1
/2*d*x+1/2*c)^2+3/2/d/a^3/tan(1/2*d*x+1/2*c)+11/2/d/a^3*ln(tan(1/2*d*x+1/2*
c))+8/3/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3-4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+14/
d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.34, size = 247, normalized size = 2.33

$$\frac{\frac{\frac{27 \sin(dx+c)}{\cos(dx+c)+1} + \frac{403 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{681 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{372 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - 3}{\frac{a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3 a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}} - \frac{3\left(\frac{12 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^3} + \frac{132 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/24*((27*sin(d*x + c)/(cos(d*x + c) + 1) + 403*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 681*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 372*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 3)/(a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) - 3*(12*sin(d*x + c)/(cos(d*x
+ c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^3 + 132*log(sin(d*x + c)
/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 8.66, size = 178, normalized size = 1.68

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}+\frac{11\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^3\,d}+\frac{62\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{227\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{403\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6}+\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\left(4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+12\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+12\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a^3\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (11*log(tan(c/2 + (d*x)/2)))/(2*a^3*d) + (
(9*tan(c/2 + (d*x)/2))/2 + (403*tan(c/2 + (d*x)/2)^2)/6 + (227*tan(c/2 + (d
*x)/2)^3)/2 + 62*tan(c/2 + (d*x)/2)^4 - 1/2)/(d*(4*a^3*tan(c/2 + (d*x)/2)^2
 + 12*a^3*tan(c/2 + (d*x)/2)^3 + 12*a^3*tan(c/2 + (d*x)/2)^4 + 4*a^3*tan(c/
2 + (d*x)/2)^5)) - (3*tan(c/2 + (d*x)/2))/(2*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^{2}(c+dx)\csc^{3}(c+dx)}{\sin^{3}(c+dx)+3\sin^{2}(c+dx)+3\sin(c+dx)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)*
*2 + 3*sin(c + d*x) + 1), x)/a**3
```

3.321 $\int \frac{\cos^2(e+fx)\sin(e+fx)}{(a+a\sin(e+fx))^6}\,dx$

Optimal. Leaf size=144

$$\frac{4\cos(e+fx)}{315f\left(a^6\sin(e+fx)+a^6\right)}+\frac{4\cos(e+fx)}{315f\left(a^3\sin(e+fx)+a^3\right)^2}+\frac{2\cos(e+fx)}{105f\left(a^2\sin(e+fx)+a^2\right)^3}-\frac{19\cos(e+fx)}{63a^2f(a\sin(e+fx)+a)^4}$$

```
[Out] 2/9*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^5-19/63*cos(f*x+e)/a^2/f/(a+a*sin(f*x+e
))^4+2/105*cos(f*x+e)/f/(a^2+a^2*sin(f*x+e))^3+4/315*cos(f*x+e)/f/(a^3+a^3*
sin(f*x+e))^2+4/315*cos(f*x+e)/f/(a^6+a^6*sin(f*x+e))
```

Rubi [A] time = 0.16, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2857, 2750, 2650, 2648}

$$\frac{4\cos(e+fx)}{315f\left(a^6\sin(e+fx)+a^6\right)}+\frac{4\cos(e+fx)}{315f\left(a^3\sin(e+fx)+a^3\right)^2}+\frac{2\cos(e+fx)}{105f\left(a^2\sin(e+fx)+a^2\right)^3}-\frac{19\cos(e+fx)}{63a^2f(a\sin(e+fx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sin[e + f*x])/(a + a*Sin[e + f*x])^6,x]
```

```
[Out] (2*Cos[e + f*x])/(9*a*f*(a + a*Sin[e + f*x])^5) - (19*Cos[e + f*x])/(63*a^2
*f*(a + a*Sin[e + f*x])^4) + (2*Cos[e + f*x])/(105*f*(a^2 + a^2*Sin[e + f*x
])^3) + (4*Cos[e + f*x])/(315*f*(a^3 + a^3*Sin[e + f*x])^2) + (4*Cos[e + f*
x])/(315*f*(a^6 + a^6*Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)\sin(e+fx)}{(a+a\sin(e+fx))^6}\,dx = \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5} - \frac{\int \frac{-10a+9a\sin(e+fx)}{(a+a\sin(e+fx))^4}\,dx}{9a^3}$$

$$= \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5} - \frac{19\cos(e+fx)}{63a^2f(a+a\sin(e+fx))^4} - \frac{2\int \frac{1}{(a+a\sin(e+fx))^3}\,dx}{21a^3}$$

$$= \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5} - \frac{19\cos(e+fx)}{63a^2f(a+a\sin(e+fx))^4} + \frac{2\cos(e+fx)}{105f\left(a^2+a^2\sin(e+\right.}$$

$$= \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5} - \frac{19\cos(e+fx)}{63a^2f(a+a\sin(e+fx))^4} + \frac{2\cos(e+fx)}{105f\left(a^2+a^2\sin(e+\right.}$$

$$= \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5} - \frac{19\cos(e+fx)}{63a^2f(a+a\sin(e+fx))^4} + \frac{2\cos(e+fx)}{105f\left(a^2+a^2\sin(e+\right.}$$

Mathematica [A] time = 0.96, size = 171, normalized size = 1.19

$$-\frac{2562\sin\left(2e+\frac{3fx}{2}\right)-900\sin\left(2e+\frac{5fx}{2}\right)-27\sin\left(4e+\frac{7fx}{2}\right)+25\sin\left(4e+\frac{9fx}{2}\right)+378\cos\left(e+\frac{fx}{2}\right)+210\cos}{13860a^6f\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sin[e + f*x])/(a + a*Sin[e + f*x])^6,x]
```

```
[Out] -1/13860*(378*Cos[e + (f*x)/2] + 210*Cos[e + (3*f*x)/2] - 108*Cos[3*e + (5*
f*x)/2] + 225*Cos[3*e + (7*f*x)/2] + 3*Cos[5*e + (9*f*x)/2] + 3150*Sin[(f*x
)/2] + 2562*Sin[2*e + (3*f*x)/2] - 900*Sin[2*e + (5*f*x)/2] - 27*Sin[4*e +
(7*f*x)/2] + 25*Sin[4*e + (9*f*x)/2])/(a^6*f*(Cos[e/2] + Sin[e/2])*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^9)
```

fricas [A] time = 0.50, size = 243, normalized size = 1.69

$$\frac{4\cos\left(fx+e\right)^{5}-16\cos\left(fx+e\right)^{4}-50\cos\left(fx+e\right)^{3}-65\cos\left(fx+e\right)^{2}-\left(4\cos\left(fx+e\right)^{4}+\right.}{315\left(a^{6}f\cos\left(fx+e\right)^{5}+5\,a^{6}f\cos\left(fx+e\right)^{4}-8\,a^{6}f\cos\left(fx+e\right)^{3}-20\,a^{6}f\cos\left(fx+e\right)^{2}+8\,a^{6}f\cos\left(fx+e\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] 1/315*(4*cos(f*x + e)^5 - 16*cos(f*x + e)^4 - 50*cos(f*x + e)^3 - 65*cos(f*
x + e)^2 - (4*cos(f*x + e)^4 + 20*cos(f*x + e)^3 - 30*cos(f*x + e)^2 + 35*c
os(f*x + e) + 70)*sin(f*x + e) + 35*cos(f*x + e) + 70)/(a^6*f*cos(f*x + e)^
5 + 5*a^6*f*cos(f*x + e)^4 - 8*a^6*f*cos(f*x + e)^3 - 20*a^6*f*cos(f*x + e)
^2 + 8*a^6*f*cos(f*x + e) + 16*a^6*f + (a^6*f*cos(f*x + e)^4 - 4*a^6*f*cos(
f*x + e)^3 - 12*a^6*f*cos(f*x + e)^2 + 8*a^6*f*cos(f*x + e) + 16*a^6*f)*sin
(f*x + e))
```

giac [A] time = 0.25, size = 120, normalized size = 0.83

$$-\frac{2\left(315\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+315\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}+945\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+441\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+609\tan\left(\right.\right.}{315\,a^{6}f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -2/315*(315*tan(1/2*f*x + 1/2*e)^7 + 315*tan(1/2*f*x + 1/2*e)^6 + 945*tan(1
/2*f*x + 1/2*e)^5 + 441*tan(1/2*f*x + 1/2*e)^4 + 609*tan(1/2*f*x + 1/2*e)^3
 + 81*tan(1/2*f*x + 1/2*e)^2 + 99*tan(1/2*f*x + 1/2*e) + 11)/(a^6*f*(tan(1/
2*f*x + 1/2*e) + 1)^9)
```

maple [A] time = 0.48, size = 130, normalized size = 0.90

$$\frac{-\frac{248}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{6}}+\frac{64}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{9}}+\frac{336}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{8}}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}+\frac{12}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{36}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}}{f\,a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*sin(f*x+e)/(a+a*sin(f*x+e))^6,x)
```

```
[Out] 4/f/a^6*(-62/3/(tan(1/2*f*x+1/2*e)+1)^6+16/9/(tan(1/2*f*x+1/2*e)+1)^9+84/5/
(tan(1/2*f*x+1/2*e)+1)^5-8/(tan(1/2*f*x+1/2*e)+1)^8-1/2/(tan(1/2*f*x+1/2*e)
+1)^2+3/(tan(1/2*f*x+1/2*e)+1)^3-9/(tan(1/2*f*x+1/2*e)+1)^4+116/7/(tan(1/2*
f*x+1/2*e)+1)^7)
```

maxima [B] time = 0.36, size = 355, normalized size = 2.47

$$-\frac{2\left(\frac{99\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{81\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{609\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{441\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{945\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}+\frac{315\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)}\cdots\right.}{315\left(a^6+\frac{9a^6\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{36a^6\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{84a^6\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{126a^6\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{126a^6\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}+\frac{84a^6\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}+\frac{36a^6}{(\cos\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -2/315*(99*sin(f*x + e)/(cos(f*x + e) + 1) + 81*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 609*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 441*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 945*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 315*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 + 315*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 11)/
((a^6 + 9*a^6*sin(f*x + e)/(cos(f*x + e) + 1) + 36*a^6*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 84*a^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*a^6*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 126*a^6*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 84*a^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 36*a^6*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7 + 9*a^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^6*sin(f*x
 + e)^9/(cos(f*x + e) + 1)^9)*f)
```

mupad [B] time = 9.06, size = 207, normalized size = 1.44

$$-\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(11\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+99\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+81\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+609\cos\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*sin(e + f*x))/(a + a*sin(e + f*x))^6,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)^2*(11*cos(e/2 + (f*x)/2)^7 + 315*sin(e/2 + (f*x)/2)^
7 + 315*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^6 + 99*cos(e/2 + (f*x)/2)^6*s
in(e/2 + (f*x)/2) + 945*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^5 + 441*cos
(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^4 + 609*cos(e/2 + (f*x)/2)^4*sin(e/2 +
```

```
(f*x)/2)^3 + 81*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^2))/(315*a^6*f*(co
s(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2))^9)
```

sympy [A] time = 109.19, size = 1501, normalized size = 10.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*sin(f*x+e)/(a+a*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-630*tan(e/2 + f*x/2)**7/(315*a**6*f*tan(e/2 + f*x/2)**9 + 2835*
a**6*f*tan(e/2 + f*x/2)**8 + 11340*a**6*f*tan(e/2 + f*x/2)**7 + 26460*a**6*
f*tan(e/2 + f*x/2)**6 + 39690*a**6*f*tan(e/2 + f*x/2)**5 + 39690*a**6*f*tan
(e/2 + f*x/2)**4 + 26460*a**6*f*tan(e/2 + f*x/2)**3 + 11340*a**6*f*tan(e/2
+ f*x/2)**2 + 2835*a**6*f*tan(e/2 + f*x/2) + 315*a**6*f) - 630*tan(e/2 + f*
x/2)**6/(315*a**6*f*tan(e/2 + f*x/2)**9 + 2835*a**6*f*tan(e/2 + f*x/2)**8 +
 11340*a**6*f*tan(e/2 + f*x/2)**7 + 26460*a**6*f*tan(e/2 + f*x/2)**6 + 3969
0*a**6*f*tan(e/2 + f*x/2)**5 + 39690*a**6*f*tan(e/2 + f*x/2)**4 + 26460*a**
6*f*tan(e/2 + f*x/2)**3 + 11340*a**6*f*tan(e/2 + f*x/2)**2 + 2835*a**6*f*ta
n(e/2 + f*x/2) + 315*a**6*f) - 1890*tan(e/2 + f*x/2)**5/(315*a**6*f*tan(e/2
 + f*x/2)**9 + 2835*a**6*f*tan(e/2 + f*x/2)**8 + 11340*a**6*f*tan(e/2 + f*x
/2)**7 + 26460*a**6*f*tan(e/2 + f*x/2)**6 + 39690*a**6*f*tan(e/2 + f*x/2)**
5 + 39690*a**6*f*tan(e/2 + f*x/2)**4 + 26460*a**6*f*tan(e/2 + f*x/2)**3 + 1
1340*a**6*f*tan(e/2 + f*x/2)**2 + 2835*a**6*f*tan(e/2 + f*x/2) + 315*a**6*f
) - 882*tan(e/2 + f*x/2)**4/(315*a**6*f*tan(e/2 + f*x/2)**9 + 2835*a**6*f*t
an(e/2 + f*x/2)**8 + 11340*a**6*f*tan(e/2 + f*x/2)**7 + 26460*a**6*f*tan(e/
2 + f*x/2)**6 + 39690*a**6*f*tan(e/2 + f*x/2)**5 + 39690*a**6*f*tan(e/2 + f
*x/2)**4 + 26460*a**6*f*tan(e/2 + f*x/2)**3 + 11340*a**6*f*tan(e/2 + f*x/2)
**2 + 2835*a**6*f*tan(e/2 + f*x/2) + 315*a**6*f) - 1218*tan(e/2 + f*x/2)**3
/(315*a**6*f*tan(e/2 + f*x/2)**9 + 2835*a**6*f*tan(e/2 + f*x/2)**8 + 11340*
a**6*f*tan(e/2 + f*x/2)**7 + 26460*a**6*f*tan(e/2 + f*x/2)**6 + 39690*a**6*
f*tan(e/2 + f*x/2)**5 + 39690*a**6*f*tan(e/2 + f*x/2)**4 + 26460*a**6*f*tan
(e/2 + f*x/2)**3 + 11340*a**6*f*tan(e/2 + f*x/2)**2 + 2835*a**6*f*tan(e/2 +
 f*x/2) + 315*a**6*f) - 162*tan(e/2 + f*x/2)**2/(315*a**6*f*tan(e/2 + f*x/2
)**9 + 2835*a**6*f*tan(e/2 + f*x/2)**8 + 11340*a**6*f*tan(e/2 + f*x/2)**7 +
 26460*a**6*f*tan(e/2 + f*x/2)**6 + 39690*a**6*f*tan(e/2 + f*x/2)**5 + 3969
0*a**6*f*tan(e/2 + f*x/2)**4 + 26460*a**6*f*tan(e/2 + f*x/2)**3 + 11340*a**
6*f*tan(e/2 + f*x/2)**2 + 2835*a**6*f*tan(e/2 + f*x/2) + 315*a**6*f) - 198*
tan(e/2 + f*x/2)/(315*a**6*f*tan(e/2 + f*x/2)**9 + 2835*a**6*f*tan(e/2 + f*
x/2)**8 + 11340*a**6*f*tan(e/2 + f*x/2)**7 + 26460*a**6*f*tan(e/2 + f*x/2)*
*6 + 39690*a**6*f*tan(e/2 + f*x/2)**5 + 39690*a**6*f*tan(e/2 + f*x/2)**4 +
26460*a**6*f*tan(e/2 + f*x/2)**3 + 11340*a**6*f*tan(e/2 + f*x/2)**2 + 2835*
a**6*f*tan(e/2 + f*x/2) + 315*a**6*f) - 22/(315*a**6*f*tan(e/2 + f*x/2)**9
+ 2835*a**6*f*tan(e/2 + f*x/2)**8 + 11340*a**6*f*tan(e/2 + f*x/2)**7 + 2646
0*a**6*f*tan(e/2 + f*x/2)**6 + 39690*a**6*f*tan(e/2 + f*x/2)**5 + 39690*a**
```

```
6*f*tan(e/2 + f*x/2)**4 + 26460*a**6*f*tan(e/2 + f*x/2)**3 + 11340*a**6*f*t
an(e/2 + f*x/2)**2 + 2835*a**6*f*tan(e/2 + f*x/2) + 315*a**6*f), Ne(f, 0)),
 (x*sin(e)*cos(e)**2/(a*sin(e) + a)**6, True))
```

3.322 $\int \cos^2(c+dx)\sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=193

$$\frac{2\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}+\frac{2a\sin^4(c+dx)\cos(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{38a\sin^3(c+dx)\cos(c+dx)}{693d\sqrt{a\sin(c+dx)+a}}-\frac{76\cos}{}$$

```
[Out] -76/1155*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a/d-76/495*a*cos(d*x+c)/d/(a+a*s
in(d*x+c))^(1/2)-38/693*a*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+
2/99*a*cos(d*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+152/3465*cos(d*x+c)
*(a+a*sin(d*x+c))^(1/2)/d+2/11*cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^(1/
2)/d
```

Rubi [A] time = 0.57, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2879, 2976, 2981, 2770, 2759, 2751, 2646}

$$\frac{2\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}+\frac{2a\sin^4(c+dx)\cos(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{38a\sin^3(c+dx)\cos(c+dx)}{693d\sqrt{a\sin(c+dx)+a}}-\frac{76\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-76*a*Cos[c + d*x])/(495*d*Sqrt[a + a*Sin[c + d*x]]) - (38*a*Cos[c + d*x]*
Sin[c + d*x]^3)/(693*d*Sqrt[a + a*Sin[c + d*x]]) + (2*a*Cos[c + d*x]*Sin[c
+ d*x]^4)/(99*d*Sqrt[a + a*Sin[c + d*x]]) + (152*Cos[c + d*x]*Sqrt[a + a*Si
n[c + d*x]])/(3465*d) + (2*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[a + a*Sin[c + d
*x]])/(11*d) - (76*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(1155*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2879

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{3/2}\,dx}{a^2}$$

$$= \frac{2\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\sin(c+dx)}}{11d} + \frac{2\int \sin^3(c+$$

$$= \frac{2a\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^4(c+dx)\sqrt{a}}{11d}$$

$$= -\frac{38a\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{38a\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{38a\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{76a\cos(c+dx)}{495d\sqrt{a+a\sin(c+dx)}} - \frac{38a\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} +$$

Mathematica [A] time = 1.23, size = 109, normalized size = 0.56

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(7638\sin(c+dx)-1330\sin(3(c+dx))-3540\cos(2(}{13860d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/13860*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])
]*(5657 - 3540*Cos[2*(c + d*x)] + 315*Cos[4*(c + d*x)] + 7638*Sin[c + d*x]
- 1330*Sin[3*(c + d*x)]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.53, size = 151, normalized size = 0.78

$$\frac{2\left(315\cos(dx+c)^6+350\cos(dx+c)^5-500\cos(dx+c)^4-586\cos(dx+c)^3+17\cos(dx+c)^2+\left(315\cos(\right.\right.}{346}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2/3465*(315*cos(d*x + c)^6 + 350*cos(d*x + c)^5 - 500*cos(d*x + c)^4 - 586*
cos(d*x + c)^3 + 17*cos(d*x + c)^2 + (315*cos(d*x + c)^5 - 35*cos(d*x + c)^
4 - 535*cos(d*x + c)^3 + 51*cos(d*x + c)^2 + 68*cos(d*x + c) + 136)*sin(d*x
 + c) - 68*cos(d*x + c) - 136)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d
*sin(d*x + c) + d)
```

giac [A] time = 0.26, size = 189, normalized size = 0.98

$$\frac{1}{55440}\sqrt{2}\sqrt{a}\left(\frac{385\cos\left(\frac{1}{4}\pi+\frac{9}{2}dx+\frac{9}{2}c\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}-\frac{693\cos\left(\frac{1}{4}\pi+\frac{5}{2}dx+\frac{5}{2}c\right)\operatorname{sgn}\left(\cos\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/55440*sqrt(2)*sqrt(a)*(385*cos(1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))/d - 693*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))/d - 6930*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))/d + 315*cos(-1/4*pi + 11/2*d*x + 11/2*c)*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))/d - 495*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))/d - 2310*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c))/d)
```

maple [A] time = 1.04, size = 85, normalized size = 0.44

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)a\left(\sin\left(dx+c\right)-1\right)^{2}\left(315\left(\sin^{4}\left(dx+c\right)\right)+665\left(\sin^{3}\left(dx+c\right)\right)+570\left(\sin^{2}\left(dx+c\right)\right)+456\,\mathrm{s}\right.}{3465\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/3465*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^2*(315*sin(d*x+c)^4+665*sin(d*x+c)^
3+570*sin(d*x+c)^2+456*sin(d*x+c)+304)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*sin(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)^3\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.323 $\int \cos^2(c+dx)\sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{8a^2\cos^3(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{9ad}+\frac{4\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos^3}{21d\sqrt{a\sin}}$$

```
[Out] -8/63*a^2*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-2/9*cos(d*x+c)^3*(a+a*sin(d
*x+c))^(3/2)/a/d-2/21*a*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+4/21*cos(d*x+
c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.36, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2878, 2856, 2674, 2673}

$$-\frac{8a^2\cos^3(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{9ad}+\frac{4\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos^3}{21d\sqrt{a\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^3)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*
x]^3)/(21*d*Sqrt[a + a*Sin[c + d*x]]) + (4*Cos[c + d*x]^3*Sqrt[a + a*Sin[c
+ d*x]])/(21*d) - (2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(9*a*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
```

```
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{9ad}+\frac{2\int \cos^2(c+dx)\left(\frac{3a}{2}\right.}{}$$

$$= \frac{4\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}-\frac{2\cos^3(c+dx)(a+a\sin}{9ad}$$

$$= -\frac{2a\cos^3(c+dx)}{21d\sqrt{a+a\sin(c+dx)}}+\frac{4\cos^3(c+dx)\sqrt{a+a\sin(c+dx}}{21d}$$

$$= -\frac{8a^2\cos^3(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^3(c+dx)}{21d\sqrt{a+a\sin(c+dx)}}+\frac{4\cos}{}$$

Mathematica [A] time = 0.60, size = 99, normalized size = 0.80

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(-69\sin(c+dx)+7\sin(3(c+dx))+30\cos(2(c+dx))}{126d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*(-62 +
30*Cos[2*(c + d*x)] - 69*Sin[c + d*x] + 7*Sin[3*(c + d*x)]))/(126*d*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.48, size = 130, normalized size = 1.05

$$\frac{2\left(7\cos(dx+c)^5-\cos(dx+c)^4-11\cos(dx+c)^3+\cos(dx+c)^2-\left(7\cos(dx+c)^4+8\cos(dx+c)^3-3\cos\right.\right.}{63\left(d\cos(dx+c)+d\sin(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2/63*(7*cos(d*x + c)^5 - cos(d*x + c)^4 - 11*cos(d*x + c)^3 + cos(d*x + c)^
2 - (7*cos(d*x + c)^4 + 8*cos(d*x + c)^3 - 3*cos(d*x + c)^2 - 4*cos(d*x + c
) - 8)*sin(d*x + c) - 4*cos(d*x + c) - 8)*sqrt(a*sin(d*x + c) + a)/(d*cos(d
*x + c) + d*sin(d*x + c) + d)
```

giac [A] time = 0.27, size = 99, normalized size = 0.80

$$-\frac{1}{504}\sqrt{2}\sqrt{a}\left(\frac{9\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{4}\pi+\frac{7}{2}dx+\frac{7}{2}c\right)}{d}+\frac{7\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/504*sqrt(2)*sqrt(a)*(9*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi +
7/2*d*x + 7/2*c)/d + 7*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 9/
2*d*x + 9/2*c)/d - 126*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/
2*d*x + 1/2*c)/d)
```

maple [A] time = 1.02, size = 75, normalized size = 0.60

$$-\frac{2\left(1+\sin(dx+c)\right)a\left(\sin(dx+c)-1\right)^2\left(7\left(\sin^3(dx+c)\right)+15\left(\sin^2(dx+c)\right)+12\sin(dx+c)+8\right)}{63\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/63*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^2*(7*sin(d*x+c)^3+15*sin(d*x+c)^2+12*
sin(d*x+c)+8)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin(dx+c)+a}\,\cos(dx+c)^2\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*sin(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x)**2*cos(c + d*x)**2, x)
```

3.324 $\int \cos^2(c+dx)\sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=92

$$-\frac{8a^2\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}-\frac{2a\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -8/105*a^2*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-2/35*a*cos(d*x+c)^3/d/(a+a
*sin(d*x+c))^(1/2)-2/7*cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2856, 2674, 2673}

$$-\frac{8a^2\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}-\frac{2a\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^3)/(105*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d
*x]^3)/(35*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + a*Sin[c
 + d*x]])/(7*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
```

```
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7d} + \frac{1}{7}\int \cos^2(c+dx)\sqrt{a+$$

$$= -\frac{2a\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7d}$$

$$= -\frac{8a^2\cos^3(c+dx)}{105d(a+a\sin(c+dx))^{3/2}} - \frac{2a\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos$$

Mathematica [A] time = 0.40, size = 89, normalized size = 0.97

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(66\sin(c+dx)-15\cos(2(c+dx))+59)}{105d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/105*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*
(59 - 15*Cos[2*(c + d*x)] + 66*Sin[c + d*x]))/(d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]))
```

fricas [A] time = 0.47, size = 111, normalized size = 1.21

$$-\frac{2\left(15\cos(dx+c)^4+18\cos(dx+c)^3-\cos(dx+c)^2+\left(15\cos(dx+c)^3-3\cos(dx+c)^2-4\cos(dx+c)-8\right.}{105\left(d\cos(dx+c)+d\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/105*(15*cos(d*x + c)^4 + 18*cos(d*x + c)^3 - cos(d*x + c)^2 + (15*cos(d*
x + c)^3 - 3*cos(d*x + c)^2 - 4*cos(d*x + c) - 8)*sin(d*x + c) + 4*cos(d*x
+ c) + 8)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [A] time = 0.22, size = 129, normalized size = 1.40

$$-\frac{1}{420}\sqrt{2}\sqrt{a}\left(\frac{21\cos\left(\frac{1}{4}\pi+\frac{5}{2}dx+\frac{5}{2}c\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}+\frac{105\cos\left(\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\operatorname{sgn}\left(\cos\left(\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] -1/420*sqrt(2)*sqrt(a)*(21*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))/d + 105*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))/d + 15*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))/d + 35*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d)
```

maple [A] time = 1.06, size = 65, normalized size = 0.71

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)a\left(\sin\left(dx+c\right)-1\right)^{2}\left(15\left(\sin^{2}\left(dx+c\right)\right)+33\sin\left(dx+c\right)+22\right)}{105\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/105*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^2*(15*sin(d*x+c)^2+33*sin(d*x+c)+22)
/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(c+d\,x\right)^{2}\,\sin\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x)*cos(c + d*x)**2, x)
```

3.325 $\int \cos(c+dx)\cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}+\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d+2/3*a*cos(d
*x+c)/d/(a+a*sin(d*x+c))^(1/2)+2/3*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.34, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2874, 2976, 2981, 2773, 206}

$$\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}+\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
2*a*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
```

```
, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\int \csc(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{3/2}\,dx}{a^2}$$

$$= \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \frac{2\int \csc(c+dx)\sqrt{a+a\sin}$$

$$= \frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \int$$

$$= \frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{(2}{}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos}{}$$

Mathematica [A] time = 0.21, size = 143, normalized size = 1.54

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(-3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)+3\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)-3\log\left(-\sin\right.\right.}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(3*Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2] - 3*
Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Log[1 - Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] - 3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3*d*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [B] time = 0.48, size = 250, normalized size = 2.69

$$3\sqrt{a}\left(\cos\left(dx+c\right)+\sin\left(dx+c\right)+1\right)\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4
*(cos(d*x + c)^2 + (cos(d*x + c) - 1)*sin(d*x + c) + 2*cos(d*x + c) + 1)*sq
rt(a*sin(d*x + c) + a))/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [A] time = 1.20, size = 103, normalized size = 1.11

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(3a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)+\left(a-a\sin\left(dx+c\right)\right)^{\frac{3}{2}}-3a\sqrt{a-a\sin(}\right.}{3a\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*a^(3/2)*arctanh((a-a*sin(d
*x+c))^(1/2)/a^(1/2))+(a-a*sin(d*x+c))^(3/2)-3*a*(a-a*sin(d*x+c))^(1/2))/a/
cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{2}\csc\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x),x)
```

```
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\cos^{2}\left(c+dx\right)\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**2*csc(c + d*x), x)
```

3.326 $\int \cot^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=89

$$\frac{3a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

```
[Out] -arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d+3*a*cos(d*x+c
)/d/(a+a*sin(d*x+c))^(1/2)-cot(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.20, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2716, 2981, 2773, 206}

$$\frac{3a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d) + (
3*a*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Sqrt[a + a*S
in[c + d*x]])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2716

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] &&  !LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} + \frac{\int \csc(c+dx)\left(\frac{a}{2}-\frac{3}{2}a\sin(c+dx)\right)\sqrt{a}}{a}$$

$$= \frac{3a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} + \frac{1}{2}\int \csc(c+dx$$

$$= \frac{3a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\right.}{}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{3a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a}}{d}$$

Mathematica [B] time = 1.00, size = 206, normalized size = 2.31

$$\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{3}{2}(c+dx)\right)-4\cos\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{3}{2}(c+dx\right.\right.}{d\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)-\mathrm{se}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(-4*Cos[(c + d*x)/2] + 2*Cos
[(3*(c + d*x))/2] + 4*Sin[(c + d*x)/2] - Log[1 + Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]]*Sin[c + d*x] + Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[
c + d*x] + 2*Sin[(3*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x
)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + Sec[(c + d*x)/4]))
```

fricas [B] time = 0.52, size = 279, normalized size = 3.13

$$\frac{\left(\cos (dx+c)^2-(\cos (dx+c)+1)\sin (dx+c)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+\right.}{\cos(dx+c)^3+\cos(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*((cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*
cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3
)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a
*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)
/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos
(d*x + c) - 1)) - 4*(2*cos(d*x + c)^2 + (2*cos(d*x + c) + 3)*sin(d*x + c) -
 cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2 - (d*cos(d*x
 + c) + d)*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.10, size = 125, normalized size = 1.40

$$\frac{(1+\sin (dx+c))\sqrt{-a(\sin (dx+c)-1)}\,\left(\sin (dx+c)\left(2\sqrt{a-a\sin (dx+c)}\,a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)a^2\right)-\right.}{\sin (dx+c)\,a^{\frac{3}{2}}\cos (dx+c)\sqrt{a+a\sin (dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] (1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(sin(d*x+c)*(2*(a-a*sin(d*x+c))^(1
/2)*a^(3/2)-arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^2)-(a-a*sin(d*x+c))^(
1/2)*a^(3/2))/sin(d*x+c)/a^(3/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin (dx+c)+a}\,\cos (dx+c)^2\csc (dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^2,x)
```

```
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\cos^{2}\left(c+dx\right)\csc^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**2*csc(c + d*x)**2, x)
```

3.327 $\int \cot^2(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=101

$$-\frac{a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{5\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

```
[Out] 5/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-1/4*a*cot(
d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^
(1/2)/d
```

Rubi [A] time = 0.40, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2874, 2975, 2980, 2773, 206}

$$-\frac{a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{5\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (5*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
- (a*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d
*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
```

```
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{3/2}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} + \frac{\int \csc^2(c+dx)}{}$$

$$= -\frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(}}{2d}$$

$$= -\frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(}}{2d}$$

$$= \frac{5\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c}{}$$

Mathematica [B] time = 0.81, size = 249, normalized size = 2.47

$$-\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-2\sin\left(\frac{1}{2}(c+dx)\right)+6\sin\left(\frac{3}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)+6\cos\left(\frac{3}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/4*(Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(2*Cos[(c + d*x)/2] + 6
*Cos[(3*(c + d*x))/2] - 5*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 5*
Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 5*Log[1 - C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] - 5*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] - 2*Sin[(c + d*x)/2] + 6*Sin[(3*(c + d*x))/2]))/
(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^2)
```

fricas [B] time = 0.49, size = 319, normalized size = 3.16

$$\frac{5\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(5*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 +
4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*s
qrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (c
os(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(3*cos(d*x + c)^2
+ (3*cos(d*x + c) + 1)*sin(d*x + c) + 2*cos(d*x + c) - 1)*sqrt(a*sin(d*x +
c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x
 + c)^2 - d)*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

[Out] Timed out

maple [A] time = 1.11, size = 126, normalized size = 1.25

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left(5\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)\left(\sin^2(dx+c)\right)a^2+3\left(-a(\sin(dx+c)-1)\right)\right.}{4a^{\frac{3}{2}}\sin(dx+c)^2\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(3/2)*(5*arctanh((-a*(sin(d*
x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+3*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2
)-5*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2))/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*x
+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin(dx+c)+a}\,\cos(dx+c)^2\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*csc(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2\,\sqrt{a+a\,\sin(c+d\,x)}}{\sin(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^3,x)
```

```
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sin(c+dx)+1)}\,\cos^2(c+dx)\csc^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**2*csc(c + d*x)**3, x)
```

3.328 $\int \cot^2(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$\frac{3a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{a\cot(c+dx)\cdots}{12d\sqrt{a\sin(\cdots}}$$

```
[Out] 3/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d+3/8*a*cot(
d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/12*a*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x
+c))^(1/2)-1/3*cot(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.48, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2874, 2975, 2980, 2772, 2773, 206}

$$\frac{3a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{a\cot(c+dx)\cdots}{12d\sqrt{a\sin(\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (3*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
+ (3*a*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c
 + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2*Sqr
t[a + a*Sin[c + d*x]])/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{3/2}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \frac{\int \csc^3(c+$$

$$= -\frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+}}{3d}$$

$$= \frac{3a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c}{}$$

$$= \frac{3a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c}{}$$

$$= \frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} + \frac{3a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a\,c}{12}$$

Mathematica [B] time = 1.42, size = 285, normalized size = 2.08

$$-\frac{\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(12\sin\left(\frac{1}{2}(c+dx)\right)+58\sin\left(\frac{3}{2}(c+dx)\right)-18\sin\left(\frac{5}{2}(c+dx)\right)-12\cos\left(\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/24*(Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-12*Cos[(c + d*x)/2]
 + 58*Cos[(3*(c + d*x))/2] + 18*Cos[(5*(c + d*x))/2] + 12*Sin[(c + d*x)/2]
- 27*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 27*Log[1 -
 Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 58*Sin[(3*(c + d*x))/2
] - 18*Sin[(5*(c + d*x))/2] + 9*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
]*Sin[3*(c + d*x)] - 9*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(
c + d*x)]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4
]^2)^3)
```

fricas [B] time = 0.50, size = 361, normalized size = 2.64

$$\frac{9\left(\cos(dx+c)^4-2\cos(dx+c)^2-\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sin(dx+c)+1\right)\sqrt{a}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)
^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4
*(9*cos(d*x + c)^3 + 19*cos(d*x + c)^2 - (9*cos(d*x + c)^2 - 10*cos(d*x + c
) - 11)*sin(d*x + c) - cos(d*x + c) - 11)*sqrt(a*sin(d*x + c) + a))/(d*cos(
d*x + c)^4 - 2*d*cos(d*x + c)^2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*
cos(d*x + c) - d)*sin(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.14, size = 144, normalized size = 1.05

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left(9(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{3}{2}}+9\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^4\left(\sin^3(dx\right.\right.}{24a^{\frac{7}{2}}\sin(dx+c)^3\cos(dx+c)\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(9*(-a*(sin(d*x+c)-1)
)^(5/2)*a^(3/2)+9*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^4*sin(d*x+c)
^3-8*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)-9*(-a*(sin(d*x+c)-1))^(1/2)*a^(7/2))
/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin(dx+c)+a}\,\cos(dx+c)^2\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*csc(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^4,x)
```

```
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.329 $\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=233

$$-\frac{38a^2\sin^4(c+dx)\cos(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{862a^2\sin^3(c+dx)\cos(c+dx)}{9009d\sqrt{a\sin(c+dx)+a}}-\frac{1724a^2\cos(c+dx)}{6435d\sqrt{a\sin(c+dx)+a}}+\frac{2\sin^4(c+dx)\cos(c$$

```
[Out] -1724/15015*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d+2/13*cos(d*x+c)*sin(d*x+c)^
4*(a+a*sin(d*x+c))^(3/2)/d-1724/6435*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2
)-862/9009*a^2*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-38/1287*a^2
*cos(d*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+3448/45045*a*cos(d*x+c)*(
a+a*sin(d*x+c))^(1/2)/d+6/143*a*cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^(1
/2)/d
```

Rubi [A] time = 0.72, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2879, 2976, 2981, 2770, 2759, 2751, 2646}

$$-\frac{38a^2\sin^4(c+dx)\cos(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{862a^2\sin^3(c+dx)\cos(c+dx)}{9009d\sqrt{a\sin(c+dx)+a}}-\frac{1724a^2\cos(c+dx)}{6435d\sqrt{a\sin(c+dx)+a}}+\frac{2\sin^4(c+dx)\cos(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-1724*a^2*Cos[c + d*x])/(6435*d*Sqrt[a + a*Sin[c + d*x]]) - (862*a^2*Cos[c
 + d*x]*Sin[c + d*x]^3)/(9009*d*Sqrt[a + a*Sin[c + d*x]]) - (38*a^2*Cos[c +
 d*x]*Sin[c + d*x]^4)/(1287*d*Sqrt[a + a*Sin[c + d*x]]) + (3448*a*Cos[c + d
*x]*Sqrt[a + a*Sin[c + d*x]])/(45045*d) + (6*a*Cos[c + d*x]*Sin[c + d*x]^4*
Sqrt[a + a*Sin[c + d*x]])/(143*d) - (1724*Cos[c + d*x]*(a + a*Sin[c + d*x])
^(3/2))/(15015*d) + (2*Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^(3/
2))/(13*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2879

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{5/2}\,dx}{a^2}$$

$$= \frac{2\cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^{3/2}}{13d} + \frac{2\int \sin^3}{}$$

$$= \frac{6a\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\sin(c+dx)}}{143d} + \frac{2\cos(c+}{}$$

$$= -\frac{38a^2\cos(c+dx)\sin^4(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}} + \frac{6a\cos(c+dx)\sin^4(c+}{143}$$

$$= -\frac{862a^2\cos(c+dx)\sin^3(c+dx)}{9009d\sqrt{a+a\sin(c+dx)}} - \frac{38a^2\cos(c+dx)\sin^4(c}{1287d\sqrt{a+a\sin(c+}}$$

$$= -\frac{862a^2\cos(c+dx)\sin^3(c+dx)}{9009d\sqrt{a+a\sin(c+dx)}} - \frac{38a^2\cos(c+dx)\sin^4(c}{1287d\sqrt{a+a\sin(c+}}$$

$$= -\frac{862a^2\cos(c+dx)\sin^3(c+dx)}{9009d\sqrt{a+a\sin(c+dx)}} - \frac{38a^2\cos(c+dx)\sin^4(c}{1287d\sqrt{a+a\sin(c+}}$$

$$= -\frac{1724a^2\cos(c+dx)}{6435d\sqrt{a+a\sin(c+dx)}} - \frac{862a^2\cos(c+dx)\sin^3(c+d}{9009d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 3.87, size = 120, normalized size = 0.52

$$-\frac{a\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(381174\sin(c+dx)-77665\sin(3(c+dx))+3465\,\mathrm{si}}{360360d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/360360*(a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*
x])]*(281816 - 194160*Cos[2*(c + d*x)] + 22680*Cos[4*(c + d*x)] + 381174*Si
n[c + d*x] - 77665*Sin[3*(c + d*x)] + 3465*Sin[5*(c + d*x)]))/(d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.49, size = 189, normalized size = 0.81

$$-\frac{2\left(3465\,a\cos\left(dx+c\right)^{7}-4410\,a\cos\left(dx+c\right)^{6}-14140\,a\cos\left(dx+c\right)^{5}+7330\,a\cos\left(dx+c\right)^{4}+15299\,a\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] -2/45045*(3465*a*cos(d*x + c)^7 - 4410*a*cos(d*x + c)^6 - 14140*a*cos(d*x +
 c)^5 + 7330*a*cos(d*x + c)^4 + 15299*a*cos(d*x + c)^3 - 568*a*cos(d*x + c)
^2 + 2272*a*cos(d*x + c) - (3465*a*cos(d*x + c)^6 + 7875*a*cos(d*x + c)^5 -
 6265*a*cos(d*x + c)^4 - 13595*a*cos(d*x + c)^3 + 1704*a*cos(d*x + c)^2 + 2
272*a*cos(d*x + c) + 4544*a)*sin(d*x + c) + 4544*a)*sqrt(a*sin(d*x + c) + a
)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [B] time = 0.64, size = 412, normalized size = 1.77

$$\frac{1}{1441440}\sqrt{2}\left(\frac{10010\,a\cos\left(\frac{1}{4}\,\pi+\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{18018\,a\cos\left(\frac{1}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\mathrm{sg}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/1441440*sqrt(2)*(10010*a*cos(1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))/d - 18018*a*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))/d - 180180*a*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))/d + 8190*a*cos(-1/4*pi + 11/2*d*x + 11/2*c)*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))/d - 12870*a*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 60060*a*cos(-1/4*pi + 3/2*d*x + 3/2*c
)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 4095*a*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))*sin(1/4*pi + 11/2*d*x + 11/2*c)/d - 12870*a*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))*sin(1/4*pi + 7/2*d*x + 7/2*c)/d - 15015*a*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))*sin(1/4*pi + 3/2*d*x + 3/2*c)/d + 3465*a*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))*sin(-1/4*pi + 13/2*d*x + 13/2*c)/d - 10010*a*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 9/2*d*x + 9/2*c)/d - 9009*a*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 5/2*d*x + 5/2*c)/d + 180180*a*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.94, size = 97, normalized size = 0.42

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)a^{2}\left(\sin\left(dx+c\right)-1\right)^{2}\left(3465\left(\sin^{5}\left(dx+c\right)\right)+11340\left(\sin^{4}\left(dx+c\right)\right)+15085\left(\sin^{3}\left(dx+c\right)\right)\right.}{45045\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/45045*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^2*(3465*sin(d*x+c)^5+11340*sin(d
*x+c)^4+15085*sin(d*x+c)^3+12930*sin(d*x+c)^2+10344*sin(d*x+c)+6896)/cos(d*
x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*sin(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2\sin(c+d\,x)^3\,(a+a\,\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.330 $\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=156

$$-\frac{64a^3\cos^3(c+dx)}{385d(a\sin(c+dx)+a)^{3/2}}-\frac{48a^2\cos^3(c+dx)}{385d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{5/2}}{11ad}+\frac{4\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{33d}$$

```
[Out] -64/385*a^3*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)+4/33*cos(d*x+c)^3*(a+a*si
n(d*x+c))^(3/2)/d-2/11*cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2)/a/d-48/385*a^2*c
os(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-6/77*a*cos(d*x+c)^3*(a+a*sin(d*x+c))^(
1/2)/d
```

Rubi [A] time = 0.43, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2878, 2856, 2674, 2673}

$$-\frac{48a^2\cos^3(c+dx)}{385d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^3(c+dx)}{385d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{5/2}}{11ad}+\frac{4\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{33d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^3)/(385*d*(a + a*Sin[c + d*x])^(3/2)) - (48*a^2*Cos[c
 + d*x]^3)/(385*d*Sqrt[a + a*Sin[c + d*x]]) - (6*a*Cos[c + d*x]^3*Sqrt[a +
a*Sin[c + d*x]])/(77*d) + (4*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(33
*d) - (2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2))/(11*a*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos^3(c+dx)(a+a\sin(c+dx))^{5/2}}{11ad} + \frac{2\int \cos^2(c+dx)\Big(}{}$$
$$= \frac{4\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{33d} - \frac{2\cos^3(c+dx)(a+a}{11ad}$$
$$= -\frac{6a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{77d} + \frac{4\cos^3(c+dx)(a+}{33d}$$
$$= -\frac{48a^2\cos^3(c+dx)}{385d\sqrt{a+a\sin(c+dx)}} - \frac{6a\cos^3(c+dx)\sqrt{a+a\sin(c}}{77d}$$
$$= -\frac{64a^3\cos^3(c+dx)}{385d(a+a\sin(c+dx))^{3/2}} - \frac{48a^2\cos^3(c+dx)}{385d\sqrt{a+a\sin(c+dx)}} -$$

Mathematica [A] time = 1.92, size = 110, normalized size = 0.71

$$-\frac{a\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(5076\sin(c+dx)-700\sin(3(c+dx))-2280\cos(2(}{4620d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/4620*(a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x]
)]*(4159 - 2280*Cos[2*(c + d*x)] + 105*Cos[4*(c + d*x)] + 5076*Sin[c + d*x]
 - 700*Sin[3*(c + d*x)]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.49, size = 166, normalized size = 1.06

$$\frac{2\left(105\,a\cos\left(dx+c\right)^{6}+245\,a\cos\left(dx+c\right)^{5}-185\,a\cos\left(dx+c\right)^{4}-397\,a\cos\left(dx+c\right)^{3}+24\,a\cos\left(dx+c\right)^{2}-96\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 2/1155*(105*a*cos(d*x + c)^6 + 245*a*cos(d*x + c)^5 - 185*a*cos(d*x + c)^4
- 397*a*cos(d*x + c)^3 + 24*a*cos(d*x + c)^2 - 96*a*cos(d*x + c) + (105*a*c
os(d*x + c)^5 - 140*a*cos(d*x + c)^4 - 325*a*cos(d*x + c)^3 + 72*a*cos(d*x
+ c)^2 + 96*a*cos(d*x + c) + 192*a)*sin(d*x + c) - 192*a)*sqrt(a*sin(d*x +
c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [B] time = 0.50, size = 288, normalized size = 1.85

$$\frac{1}{55440}\sqrt{2}\left(\frac{385\,a\cos\left(\frac{1}{4}\,\pi+\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{693\,a\cos\left(\frac{1}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\mathrm{sgn}\left(\cos\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/55440*sqrt(2)*(385*a*cos(1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))/d - 693*a*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d - 6930*a*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))/d + 315*a*cos(-1/4*pi + 11/2*d*x + 11/2*c)*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))/d - 495*a*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))/d - 2310*a*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))/d - 990*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin
(1/4*pi + 7/2*d*x + 7/2*c)/d - 770*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*si
n(-1/4*pi + 9/2*d*x + 9/2*c)/d + 13860*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*sin(-1/4*pi + 1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.86, size = 87, normalized size = 0.56

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)a^{2}\left(\sin\left(dx+c\right)-1\right)^{2}\left(105\left(\sin^{4}\left(dx+c\right)\right)+350\left(\sin^{3}\left(dx+c\right)\right)+465\left(\sin^{2}\left(dx+c\right)\right)+372\right.}{1155\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/1155*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^2*(105*sin(d*x+c)^4+350*sin(d*x+c
)^3+465*sin(d*x+c)^2+372*sin(d*x+c)+248)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/
d
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*sin(d*x + c)^2, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\,\sin(c+d\,x)^2\,\left(a+a\,\sin(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.331 $\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=124

$$-\frac{64a^3\cos^3(c+dx)}{315d(a\sin(c+dx)+a)^{3/2}}-\frac{16a^2\cos^3(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{9d}-\frac{2a\cos^3(c+dx)\sqrt{a}}{21d}$$

```
[Out] -64/315*a^3*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-2/9*cos(d*x+c)^3*(a+a*sin
(d*x+c))^(3/2)/d-16/105*a^2*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-2/21*a*co
s(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.26, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2856, 2674, 2673}

$$-\frac{16a^2\cos^3(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^3(c+dx)}{315d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{9d}-\frac{2a\cos^3(c+dx)\sqrt{a}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^3)/(315*d*(a + a*Sin[c + d*x])^(3/2)) - (16*a^2*Cos[c
 + d*x]^3)/(105*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^3*Sqrt[a +
a*Sin[c + d*x]])/(21*d) - (2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(9*
d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{9d} + \frac{1}{3}\int \cos^2(c+dx)(a$$

$$= -\frac{2a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d} - \frac{2\cos^3(c+dx)(a+a}{9d}$$

$$= -\frac{16a^2\cos^3(c+dx)}{105d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos^3(c+dx)\sqrt{a+a\sin(c+}}{21d}$$

$$= -\frac{64a^3\cos^3(c+dx)}{315d(a+a\sin(c+dx))^{3/2}} - \frac{16a^2\cos^3(c+dx)}{105d\sqrt{a+a\sin(c+dx)}} - 2$$

Mathematica [A] time = 1.43, size = 100, normalized size = 0.81

$$\frac{a\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(-741\sin(c+dx)+35\sin(3(c+dx))+240\cos(2(c+}{630d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*(-664
 + 240*Cos[2*(c + d*x)] - 741*Sin[c + d*x] + 35*Sin[3*(c + d*x)]))/(630*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.49, size = 145, normalized size = 1.17

$$\frac{2\left(35\,a\cos(dx+c)^5-50\,a\cos(dx+c)^4-109\,a\cos(dx+c)^3+8\,a\cos(dx+c)^2-32\,a\cos(dx+c)-\left(35\,a\cos(\right.\right.}{315\,(d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 2/315*(35*a*cos(d*x + c)^5 - 50*a*cos(d*x + c)^4 - 109*a*cos(d*x + c)^3 + 8
*a*cos(d*x + c)^2 - 32*a*cos(d*x + c) - (35*a*cos(d*x + c)^4 + 85*a*cos(d*x
 + c)^3 - 24*a*cos(d*x + c)^2 - 32*a*cos(d*x + c) - 64*a)*sin(d*x + c) - 64
*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [B] time = 0.38, size = 226, normalized size = 1.82

$$-\frac{1}{2520}\sqrt{2}\left(\frac{126\,a\cos\left(\frac{1}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{630\,a\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\operatorname{sgn}\left(\cos\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] -1/2520*sqrt(2)*(126*a*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))/d + 630*a*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d + 90*a*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))/d + 210*a*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))/d + 45*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi +
 7/2*d*x + 7/2*c)/d + 35*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
+ 9/2*d*x + 9/2*c)/d - 630*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*p
i + 1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.89, size = 77, normalized size = 0.62

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)a^{2}\left(\sin\left(dx+c\right)-1\right)^{2}\left(35\left(\sin^{3}\left(dx+c\right)\right)+120\left(\sin^{2}\left(dx+c\right)\right)+159\sin\left(dx+c\right)+106\right)}{315\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/315*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^2*(35*sin(d*x+c)^3+120*sin(d*x+c)^
2+159*sin(d*x+c)+106)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^2 \sin(c+dx) \left(a+a\sin(c+dx)\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin(c+dx)+1\right)\right)^{\frac{3}{2}} \sin(c+dx)\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*sin(c + d*x)*cos(c + d*x)**2, x)
```

3.332 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=123

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{2a^2\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}+\frac{2\cos(c+dx)(a\sin(c$$

```
[Out] -2*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d+2/5*cos(d*x
+c)*(a+a*sin(d*x+c))^(3/2)/d-2/5*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+2/
5*a*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.46, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2874, 2976, 2981, 2773, 206}

$$-\frac{2a^2\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}+\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}+\frac{2\cos(c+dx)(a\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
2*a^2*Cos[c + d*x])/(5*d*Sqrt[a + a*Sin[c + d*x]]) + (2*a*Cos[c + d*x]*Sqrt
[a + a*Sin[c + d*x]])/(5*d) + (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(
5*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
```

```
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\int \csc(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{5/2}\,dx}{a^2}$$

$$= \frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d} + \frac{2\int \csc(c+dx)(a+a$$

$$= \frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} + \frac{2\cos(c+dx)(a+a\sin}{5d}$$

$$= -\frac{2a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx}}{5d}$$

$$= -\frac{2a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx}}{5d}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{2a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2}{}$$

Mathematica [A] time = 0.26, size = 145, normalized size = 1.18

$$\frac{(a(\sin(c+dx)+1))^{3/2}\left(5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)+5\cos\left(\frac{3}{2}(c+dx)\right)-\cos\left(\frac{5}{2}(c+dx)\right)-10\log\left(-\sin\right.\right.}{10d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(3/2)*(5*Cos[(3*(c + d*x))/2] - Cos[(5*(c + d*x))/2
] - 10*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 10*Log[1 - Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] + 5*Sin[(3*(c + d*x))/2] + Sin[(5*(c + d*x))/2])
)/(10*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

fricas [B] time = 0.52, size = 282, normalized size = 2.29

$$\frac{5\left(a\cos(dx+c)+a\sin(dx+c)+a\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/10*(5*(a*cos(d*x + c) + a*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3
 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c)
 - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c)
+ (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)
^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)
) - 4*(a*cos(d*x + c)^3 - 2*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) - (a*cos(d*
x + c)^2 + 3*a*cos(d*x + c) + a)*sin(d*x + c) + a)*sqrt(a*sin(d*x + c) + a)
)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [A] time = 1.08, size = 121, normalized size = 0.98

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(-5a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)+\left(a-a\sin\left(dx+c\right)\right)^{\frac{5}{2}}-5\left(a-a\sin\left(dx\right.\right.}{5a\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/5*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-5*a^(5/2)*arctanh((a-a*sin(d
*x+c))^(1/2)/a^(1/2))+(a-a*sin(d*x+c))^(5/2)-5*(a-a*sin(d*x+c))^(3/2)*a+5*a
^2*(a-a*sin(d*x+c))^(1/2))/a/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\csc\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\cos\left(c+d\,x\right)^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x),x)
```

```
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.333 $\int \cot^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=121

$$-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}+\frac{11a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{5a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{\cot(c+dx)(a\sin(c+a}{d}$$

```
[Out] -3*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-cot(d*x+c)*
(a+a*sin(d*x+c))^(3/2)/d+11/3*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+5/3*a
*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.31, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2716, 2976, 2981, 2773, 206}

$$\frac{11a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}+\frac{5a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{\cot(c+dx)(a\sin(c+dx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
11*a^2*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) + (5*a*Cos[c + d*x]*Sqr
t[a + a*Sin[c + d*x]])/(3*d) - (Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2716

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] &&  !LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{\cot(c+dx)(a+a\sin(c+dx))^{3/2}}{d}+\frac{\int \csc(c+dx)\left(\frac{3a}{2}-\frac{5}{2}a\sin(c+d\cdots\right.}{a}$$

$$=\frac{5a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}-\frac{\cot(c+dx)(a+a\sin(c+dx))^{3/2}}{d}$$

$$=\frac{11a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{5a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}-\frac{\cot(c+\cdots}{}$$

$$=\frac{11a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{5a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}-\frac{\cot(c+\cdots}{}$$

$$=-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}+\frac{11a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{5a\cos(c+\cdots}{}$$

Mathematica [A] time = 0.79, size = 233, normalized size = 1.93

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-14\sin\left(\frac{1}{2}(c+dx)\right)-9\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{5}{2}(c+dx)\right)+14\cos\left(\frac{1}{2}(c\right.\right.}{3d\left(\cot\left(\frac{1}{2}(c+dx)\right)+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/3*(a*Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(14*Cos[(c + d*x)/2]
- 9*Cos[(3*(c + d*x))/2] + Cos[(5*(c + d*x))/2] - 14*Sin[(c + d*x)/2] + 9*L
og[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 9*Log[1 - Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 9*Sin[(3*(c + d*x))/2] - Sin[
(5*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4] - Sec[(c + d
*x)/4])*(Csc[(c + d*x)/4] + Sec[(c + d*x)/4]))
```

fricas [B] time = 0.49, size = 315, normalized size = 2.60

$$\frac{9\left(a\cos\left(dx+c\right)^2-\left(a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)-a\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin\right.}{\cos(dx+c)^3+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(9*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(d*x + c) - a)*sqrt(a)*
log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x +
 c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a
) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x +
c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)) + 4*(2*a*cos(d*x + c)^3 - 8*a*cos(d*x + c)^2 + a*cos
(d*x + c) - (2*a*cos(d*x + c)^2 + 10*a*cos(d*x + c) + 11*a)*sin(d*x + c) +
11*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2 - (d*cos(d*x + c) + d)*si
n(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.18, size = 144, normalized size = 1.19

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)\left(2(a-a\sin(dx+c))^{\frac{3}{2}}\sqrt{a}-12\sqrt{a-a\sin(dx+c)}\,a^{\frac{3}{2}}+\right.\right.}{3\sin(dx+c)\sqrt{a}\,\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(sin(d*x+c)*(2*(a-a*sin(d*x+c
))^(3/2)*a^(1/2)-12*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+9*arctanh((a-a*sin(d*x+c
))^(1/2)/a^(1/2))*a^2)+3*(a-a*sin(d*x+c))^(1/2)*a^(3/2))/sin(d*x+c)/a^(1/2)
/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{3/2}}{\sin(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^2,x)
```

```
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.334 $\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=131

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}+\frac{13a^2\cos(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{3a\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}-\frac{\cot(c+dx)\csc(c+dx)(a}{2d}$$

```
[Out] 1/4*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/2*cot(d*
x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d+13/4*a^2*cos(d*x+c)/d/(a+a*sin(d*x
+c))^(1/2)-3/4*a*cot(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.52, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2874, 2975, 2981, 2773, 206}

$$\frac{13a^2\cos(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{3a\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}-\frac{\cot(c+dx)\csc(c+dx)(a}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d) +
(13*a^2*Cos[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a*Cot[c + d*x]*Sq
rt[a + a*Sin[c + d*x]])/(4*d) - (Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d
*x])^(3/2))/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
```

```
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{5/2}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}}{2d} + \frac{\int \csc^2(c+$$

$$= -\frac{3a\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} - \frac{\cot(c+dx)\csc(c+dx}{2}$$

$$= \frac{13a^2\cos(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{3a\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}$$

$$= \frac{13a^2\cos(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{3a\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}$$

$$= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} + \frac{13a^2\cos(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{3a\,\mathrm{co}}{}$$

Mathematica [B] time = 0.68, size = 271, normalized size = 2.07

$$-\frac{a\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(22\sin\left(\frac{1}{2}(c+dx)\right)+22\sin\left(\frac{3}{2}(c+dx)\right)-8\sin\left(\frac{5}{2}(c+dx)\right)-22\cos\left(\frac{1}{2}(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/4*(a*Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(-22*Cos[(c + d*x)/2]
 + 22*Cos[(3*(c + d*x))/2] + 8*Cos[(5*(c + d*x))/2] - Log[1 + Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]] + Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] + Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - Cos[2*(c + d*
x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 22*Sin[(c + d*x)/2] + 22
*Sin[(3*(c + d*x))/2] - 8*Sin[(5*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2])*
(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^2)
```

fricas [B] time = 0.51, size = 359, normalized size = 2.74

$$\frac{\left(a\cos(dx+c)^3 + a\cos(dx+c)^2 - a\cos(dx+c) + \left(a\cos(dx+c)^2 - a\right)\sin(dx+c) - a\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3 - 7\,a\,\mathrm{co}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*((a*cos(d*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x +
c)^2 - a)*sin(d*x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c
)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c)
- 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^
2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^
2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(8*a*cos(d*x
 + c)^3 + 15*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - (8*a*cos(d*x + c)^2 - 7*
a*cos(d*x + c) - 13*a)*sin(d*x + c) - 13*a)*sqrt(a*sin(d*x + c) + a))/(d*co
s(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c)^2 - d)*s
in(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.20, size = 151, normalized size = 1.15

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(8\sqrt{-a(\sin(dx+c)-1)}\left(\sin^2(dx+c)\right)a^{\frac{3}{2}}+\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right.\right.}{4\sin(dx+c)^2\sqrt{a}\,\cos(dx+c)\sqrt{a+a\sin}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(8*(-a*(sin(d*x+c)-1))^(1/2)*s
in(d*x+c)^2*a^(3/2)+arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2
*a^2+7*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)-9*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2
))/sin(d*x+c)^2/a^(1/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*csc(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{3/2}}{\sin(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^3,x)
```

```
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.335 $\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=139

$$\frac{13a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}+\frac{5a^2\cot(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}-\frac{a\cot(c+$$

```
[Out] 13/8*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/3*cot(d
*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)/d+5/24*a^2*cot(d*x+c)/d/(a+a*sin(
d*x+c))^(1/2)-1/4*a*cot(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.57, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2874, 2975, 2980, 2773, 206}

$$\frac{5a^2\cot(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}+\frac{13a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{\cot(c+dx)\csc^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}-\frac{a\cot(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (13*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 + (5*a^2*Cot[c + d*x])/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*C
sc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(4*d) - (Cot[c + d*x]*Csc[c + d*x]^2*
(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
```

```
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{5/2}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}}{3d} + \frac{\int \csc^3(}{}$$

$$= -\frac{a\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} - \frac{\cot(c+}{}$$

$$= \frac{5a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)\sqrt{a+}}{4d}$$

$$= \frac{5a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)\sqrt{a+}}{4d}$$

$$= \frac{13a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} + \frac{5a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [B] time = 0.92, size = 286, normalized size = 2.06

$$-\frac{a\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-12\sin\left(\frac{1}{2}(c+dx)\right)+70\sin\left(\frac{3}{2}(c+dx)\right)+18\sin\left(\frac{5}{2}(c+dx)\right)+12\,\mathrm{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/24*(a*Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(12*Cos[(c + d*x)/2
] + 70*Cos[(3*(c + d*x))/2] - 18*Cos[(5*(c + d*x))/2] - 12*Sin[(c + d*x)/2]
 - 117*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 117*Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 70*Sin[(3*(c + d*x)
)/2] + 18*Sin[(5*(c + d*x))/2] + 39*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]]*Sin[3*(c + d*x)] - 39*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Si
n[3*(c + d*x)]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d
*x)/4]^2)^3)
```

fricas [B] time = 0.51, size = 380, normalized size = 2.73

$$\frac{39\left(a\cos(dx+c)^4-2\,a\cos(dx+c)^2-\left(a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a\right)\sin(dx+c)+a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(39*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a*cos
(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x
+ c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*
x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x
 + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*
x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c
) - 1)) + 4*(9*a*cos(d*x + c)^3 - 13*a*cos(d*x + c)^2 - 17*a*cos(d*x + c) -
 (9*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 5*a)*sin(d*x + c) + 5*a)*sqrt(a*
sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 - (d*cos(d*x + c)
^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.24, size = 144, normalized size = 1.04

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left(-39\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^3\left(\sin^3(dx+c)\right)+9\left(-a\left(\sin(dx+c)\right.\right.\right.}{24a^{\frac{3}{2}}\sin(dx+c)^3\cos(dx+c)\sqrt{a+a\sin(dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(3/2)*(-39*arctanh((-a*(si
n(d*x+c)-1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^3+9*(-a*(sin(d*x+c)-1))^(5/2)*a^
(1/2)-40*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)+39*(-a*(sin(d*x+c)-1))^(1/2)*a^(
5/2))/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*csc(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{\sin\left(c+d\,x\right)^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^4,x)
```

```
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.336 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=158

$$-\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{105a^2d}+\frac{2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}+\frac{8\cos(c+dx)\sqrt{a\sin}}{315ad}$$

```
[Out] -4/105*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a^2/d-4/45*cos(d*x+c)/d/(a+a*sin(d
*x+c))^(1/2)-2/63*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+2/9*cos(
d*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+8/315*cos(d*x+c)*(a+a*sin(d*x+
c))^(1/2)/a/d
```

Rubi [A] time = 0.40, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2879, 2981, 2770, 2759, 2751, 2646}

$$-\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{105a^2d}+\frac{2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}+\frac{8\cos(c+dx)\sqrt{a\sin}}{315ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-4*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sin[c +
 d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sin[c + d*x]^4)/
(9*d*Sqrt[a + a*Sin[c + d*x]]) + (8*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/
(315*a*d) - (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(105*a^2*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2879

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))\sqrt{a+a\sin(c+dx)}\,dx}{a^2}$$
$$= \frac{2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{\int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{9a}$$
$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{2\int \sin^2(c+dx)\sqrt{a}}{2}$$
$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} - \frac{4\cos(c+dx)(a+a}{105a^2}$$
$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{8\cos(c+dx)\sqrt{a+}}{315ad}$$
$$= -\frac{4\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^4(c+}{9d\sqrt{a+a\sin(c+dx}}$$

Mathematica [A] time = 1.27, size = 97, normalized size = 0.61

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(-201\sin(c+dx)+35\sin(3(c+dx))+}{630d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])*(-124 + 60*Cos[2*(c + d*x)] - 201*Sin[c + d*x] + 35*Sin[3*(c + d*x)]))/
(630*d*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [A] time = 0.48, size = 136, normalized size = 0.86

$$\frac{2\left(35\cos(dx+c)^5+40\cos(dx+c)^4-64\cos(dx+c)^3-82\cos(dx+c)^2-\left(35\cos(dx+c)^4-5\cos(dx+c)^3\right.\right.}{315\left(ad\cos(dx+c)+ad\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2/315*(35*cos(d*x + c)^5 + 40*cos(d*x + c)^4 - 64*cos(d*x + c)^3 - 82*cos(d
*x + c)^2 - (35*cos(d*x + c)^4 - 5*cos(d*x + c)^3 - 69*cos(d*x + c)^2 + 13*
```

```
cos(d*x + c) + 26)*sin(d*x + c) + 13*cos(d*x + c) + 26)*sqrt(a*sin(d*x + c)
 + a)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

giac [A] time = 0.60, size = 224, normalized size = 1.42

$$-\frac{4\left(\frac{13\sqrt{2}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{a}}+\frac{4\left(\frac{2a^4}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\left(\frac{9a^4}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{63a^4}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{63a^4}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{2a^4\tan\left(\frac{1}{2}dx\cdots\right.}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\cdots\right.\right.}\cdots\right.\right.\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\cdots\right.}\cdots\right.}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -4/315*(13*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(a) + 4*(2*a^4/sgn(tan
(1/2*d*x + 1/2*c) + 1) + (9*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) - (63*a^4/sgn
(tan(1/2*d*x + 1/2*c) + 1) - (63*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) - (2*a^4
*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c) + 1) + 9*a^4/sgn(tan(1/2*d
*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x
 + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(9/2))/
d
```

maple [A] time = 0.97, size = 74, normalized size = 0.47

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\left(\sin\left(dx+c\right)-1\right)^{2}\left(35\left(\sin^{3}\left(dx+c\right)\right)+30\left(\sin^{2}\left(dx+c\right)\right)+24\sin\left(dx+c\right)+16\right)}{315\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/315*(1+sin(d*x+c))*(sin(d*x+c)-1)^2*(35*sin(d*x+c)^3+30*sin(d*x+c)^2+24*
sin(d*x+c)+16)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)^{3}}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^2 \sin(c+dx)^3}{\sqrt{a+a\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.337 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=92

$$-\frac{2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7ad}+\frac{12\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{22a\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -22/105*a*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)+12/35*cos(d*x+c)^3/d/(a+a*s
in(d*x+c))^(1/2)-2/7*cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.34, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2877, 2856, 2674, 2673}

$$-\frac{2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7ad}+\frac{12\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{22a\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-22*a*Cos[c + d*x]^3)/(105*d*(a + a*Sin[c + d*x])^(3/2)) + (12*Cos[c + d*x
]^3)/(35*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + a*Sin[c +
 d*x]])/(7*a*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
```

```
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2877

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\cos^3(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \cos^2(c+dx)\left(-\frac{a}{2}-2a\sin(c+dx)\right)\sqrt{a+a\sin(c+d}}{2a^2}$$

$$= \frac{\cos^3(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7ad} + \frac{11\int \cos^2(c+dx)}{}$$

$$= \frac{12\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7ad} + \frac{11}{35}\int \frac{\cos^2(c}{\sqrt{a+a\sin}}$$

$$= -\frac{22a\cos^3(c+dx)}{105d(a+a\sin(c+dx))^{3/2}} + \frac{12\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)\sqrt{a+a}}{7ad}$$

Mathematica [A] time = 0.35, size = 87, normalized size = 0.95

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(24\sin(c+dx)-15\cos(2(c+dx))+3}{105d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/105*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])*(31 - 15*Cos[2*(c + d*x)] + 24*Sin[c + d*x]))/(d*Sqrt[a*(1 + Sin
[c + d*x])])
```

fricas [A] time = 0.49, size = 115, normalized size = 1.25

$$-\frac{2\left(15\cos\left(dx+c\right)^{4}-3\cos\left(dx+c\right)^{3}-29\cos\left(dx+c\right)^{2}+\left(15\cos\left(dx+c\right)^{3}+18\cos\left(dx+c\right)^{2}-11\cos\left(dx+\right.\right.\right.}{105\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] -2/105*(15*cos(d*x + c)^4 - 3*cos(d*x + c)^3 - 29*cos(d*x + c)^2 + (15*cos(
d*x + c)^3 + 18*cos(d*x + c)^2 - 11*cos(d*x + c) - 22)*sin(d*x + c) + 11*co
s(d*x + c) + 22)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c) + a*d*sin(d*x +
 c) + a*d)
```

giac [B] time = 0.56, size = 220, normalized size = 2.39

$$\frac{4\left(\frac{11\sqrt{2}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{a}}-\frac{2\left(\left(\left(\frac{7a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{35a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{35a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{2a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{7a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}\right.\right.\right.\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+\right.}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 4/105*(11*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(a) - 2*((7*a^3/sgn(tan
(1/2*d*x + 1/2*c) + 1) - (35*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (35*a^3/sg
n(tan(1/2*d*x + 1/2*c) + 1) - (2*a^3*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x
 + 1/2*c) + 1) + 7*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c))
*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^2 + 2*a^3
/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2))/d
```

maple [A] time = 1.51, size = 64, normalized size = 0.70

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\left(\sin\left(dx+c\right)-1\right)^{2}\left(15\left(\sin^{2}\left(dx+c\right)\right)+12\sin\left(dx+c\right)+8\right)}{105d\cos\left(dx+c\right)\sqrt{a\left(1+\sin\left(dx+c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d*(1+sin(d*x+c))*(sin(d*x+c)-1)^2*(15*sin(d*x+c)^2+12*sin(d*x+c)+8)/
cos(d*x+c)/(a*(1+sin(d*x+c)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2 \sin(dx+c)^2}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2 \sin(c+d\,x)^2}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)\cos^2(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)**2*cos(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.338 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{2a\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] 2/15*a*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-2/5*cos(d*x+c)^3/d/(a+a*sin(d*
x+c))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2856, 2673}

$$\frac{2a\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*a*Cos[c + d*x]^3)/(15*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^3)
/(5*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\cos^3(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{1}{5}\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= \frac{2a\cos^3(c+dx)}{15d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^3(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}$$

<u>Mathematica</u> [A] time = 0.43, size = 77, normalized size = 1.28

$$-\frac{2(3\sin(c+dx)+2)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{15d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])*(2 + 3*Sin[c + d*x]))/(15*d*Sqrt[a*(1 + Sin[c + d*x])])
```

<u>fricas</u> [A] time = 0.50, size = 96, normalized size = 1.60

$$-\frac{2\left(3\cos\left(dx+c\right)^3+4\cos\left(dx+c\right)^2-\left(3\cos\left(dx+c\right)^2-\cos\left(dx+c\right)-2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a\sin}}{15\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/15*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - cos(d*x +
c) - 2)*sin(d*x + c) - cos(d*x + c) - 2)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(
d*x + c) + a*d*sin(d*x + c) + a*d)
```

<u>giac</u> [B] time = 0.52, size = 155, normalized size = 2.58

$$-\frac{4\left(\frac{\sqrt{2}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}}-\frac{\left(\left(\left(\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{5\,a^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{5\,a^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\frac{a^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{5}{2}}}\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] -4/15*(sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(a) - (((a^2*tan(1/2*d*x +
 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c) + 1) - 5*a^2/sgn(tan(1/2*d*x + 1/2*c) +
1))*tan(1/2*d*x + 1/2*c) + 5*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x
 + 1/2*c)^2 - a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*x + 1/2*c)^2
+ a)^(5/2))/d
```

maple [A] time = 0.89, size = 54, normalized size = 0.90

$$-\frac{2 \left(1+\sin \left(d x+c\right)\right) \left(\sin \left(d x+c\right)-1\right)^{2} \left(3 \sin \left(d x+c\right)+2\right)}{15 \cos \left(d x+c\right) \sqrt{a+a \sin \left(d x+c\right)}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/15*(1+sin(d*x+c))*(sin(d*x+c)-1)^2*(3*sin(d*x+c)+2)/cos(d*x+c)/(a+a*sin(
d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos \left(d x+c\right)^{2} \sin \left(d x+c\right)}{\sqrt{a \sin \left(d x+c\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\cos \left(c+d\,x\right)}^2\,\sin \left(c+d\,x\right)}{\sqrt{a+a\,\sin \left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*sin(c + d*x))/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin {\left (c + d x \right)} \cos^{2}{\left (c + d x \right)}}{\sqrt{a \left(\sin {\left (c + d x \right)} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)*cos(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.339 $\int \frac{\cos(c+dx)\cot(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=63

$$\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+2*cos(d*x+c
)/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2874, 2981, 2773, 206}

$$\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(Sqrt[a]*d) +
 (2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
```

```
, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\cot(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\int \csc(c+dx)(a-a\sin(c+dx))\sqrt{a+a\sin(c+dx)}\,dx}{a^2}\\
&= \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{a}\\
&= \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 116, normalized size = 1.84

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-2\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)-\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.\right.}{d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((2*Cos[(c + d*x)/2] - Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[1
 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*Sin[(c + d*x)/2])*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [B] time = 0.53, size = 236, normalized size = 3.75

$$\frac{\sqrt{a}\left(\cos\left(dx+c\right)+\sin\left(dx+c\right)+1\right)\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin\ldots}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\ldots}\right.}{2\left(ad\cos\left(dx+c\right)+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*co
s(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*co
s(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + co
s(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*s
qrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a*d*cos(d*x + c
) + a*d*sin(d*x + c) + a*d)
```

giac [B] time = 0.64, size = 258, normalized size = 4.10

$$-\frac{\frac{2\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{1}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}+\frac{\left(2\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-\sqrt{-a}\,\sqrt{a}\,\log\left(\sqrt{2}\,\sqrt{a}+\sqrt{a}\right)+2\,\sqrt{2}\,\sqrt{-a}\,\sqrt{a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{-a}\,a}-\ldots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] -(2*(tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 1/sgn(tan(1/2*d*x
 + 1/2*c) + 1))/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + (2*a*arctan((sqrt(2)*s
qrt(a) + sqrt(a))/sqrt(-a)) - sqrt(-a)*sqrt(a)*log(sqrt(2)*sqrt(a) + sqrt(a
)) + 2*sqrt(2)*sqrt(-a)*sqrt(a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a)
 - 2*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + log(abs(-sqrt(a)
*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(a)*sgn(t
an(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 1.04, size = 88, normalized size = 1.40

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(\sqrt{a}\;\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)-\sqrt{a-a\sin\left(dx+c\right)}\right)}{a\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(a^(1/2)*arctanh((a-a*sin(d*x+c
))^(1/2)/a^(1/2))-(a-a*sin(d*x+c))^(1/2))/a/cos(d*x+c)/(a+a*sin(d*x+c))^(1/
2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{(dx+c)}^2 \csc{(dx+c)}}{\sqrt{a\sin{(dx+c)}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos{(c+d\,x)}^2}{\sin{(c+d\,x)}\,\sqrt{a+a\,\sin{(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2{(c+dx)}\csc{(c+dx)}}{\sqrt{a\,(\sin{(c+dx)}+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.340 $\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=62

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)-cot(d*x+c)/d/(
a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2716, 21, 2773, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(Sqrt[a]*d) - Cot[
c + d*x]/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2716

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
```

```
2] &&  !LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{\int \frac{\csc(c+dx)\left(-\frac{a}{2}-\frac{1}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a}\\
&= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{2a}\\
&= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}
\end{aligned}
$$

Mathematica [B] time = 0.30, size = 138, normalized size = 2.23

$$\frac{\left(\tan\left(\frac{1}{2}(c+dx)\right)+1\right)\csc\left(\frac{1}{4}(c+dx)\right)\sec\left(\frac{1}{4}(c+dx)\right)\left(2\sin\left(\frac{1}{2}(c+dx)\right)-2\cos\left(\frac{1}{2}(c+dx)\right)+\sin(c+dx)\left(\log\left(-\right.\right.\right.}{8d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/4]*Sec[(c + d*x)/4]*(-2*Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2
] + (Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]])*Sin[c + d*x])*(1 + Tan[(c + d*x)/2]))/(8*d*Sqrt[a*(1
 + Sin[c + d*x])])
```

fricas [B] time = 0.49, size = 263, normalized size = 4.24

$$\frac{\left(\cos\left(dx+c\right)^2-\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)\right.}{\cos(dx+c)^3+\cos(d\right.}}{4\left(ad\cos\left(dx+c\right)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*((cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*
cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3
)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a
*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)
/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos
(d*x + c) - 1)) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) +
 1))/(a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

giac [B] time = 0.63, size = 360, normalized size = 5.81

$$\frac{\left(2\,\sqrt{2}\,\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-\sqrt{2}\,\sqrt{-a}\,\log\left(\sqrt{2}\,\sqrt{a}+\sqrt{a}\right)+2\,\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-\sqrt{-a}\,\log\left(\sqrt{2}\,\sqrt{a}+\sqrt{a}\right)-\sqrt{2}\,\sqrt{-a}-3\,\sqrt{-a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\sqrt{2}\,\sqrt{-a}\,\sqrt{a}+\sqrt{-a}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/2*((2*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - sqrt
(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 2*sqrt(a)*arctan((sqrt(2)*sqr
t(a) + sqrt(a))/sqrt(-a)) - sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - sqrt(
2)*sqrt(-a) - 3*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(2)*sqrt(-a)*s
qrt(a) + sqrt(-a)*sqrt(a)) - 2*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c
) + 1)) + log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a)/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*sqrt(a)/(((sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*sgn(tan(1/2*d
*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.90, size = 103, normalized size = 1.66

$$-\frac{\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}}{\sqrt{a}}\right)a\sin\left(dx+c\right)+\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{a}\right)}{a^{\frac{3}{2}}\sin\left(dx+c\right)\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-arctanh((a-a*sin(d*x+c))^(1/2)/
a^(1/2))*a*sin(d*x+c)+(a-a*sin(d*x+c))^(1/2)*a^(1/2))/a^(3/2)/sin(d*x+c)/co
s(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2 \csc(dx+c)^2}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+d\,x)^2}{\sin(c+d\,x)^2\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.341 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=100

$$\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] 1/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+1/4*cot(d*
x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^
(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2874, 2980, 2772, 2773, 206}

$$\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(4*Sqrt[a]*d) + Co
t[c + d*x]/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x])/(2*
d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))\sqrt{a+a\sin(c+dx)}\,dx}{a^2}\\
&= -\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{4a}\\
&= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+}{8a}\\
&= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+}{\sqrt{a+a\sin(c}}\right.}{4d}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{a}\,d} + \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}
\end{aligned}
$$

Mathematica [B] time = 1.83, size = 272, normalized size = 2.72

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(4\tan\left(\frac{1}{4}(c+dx)\right)+4\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)+\sec^2\left(\frac{1}{4}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8 + 4*Cot[(c + d*x)/4] - Csc[(c +
d*x)/4]^2 + 4*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*Log[1 - Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Cos[(c + d*x)/4]
 - Sin[(c + d*x)/4])^2 - (8*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] - Sin[(c +
d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (8*Sin[(c + d*x)/4])
/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 4*Tan[(c + d*x)/4]))/(32*d*Sqrt[a*
(1 + Sin[c + d*x])])
```

fricas [B] time = 0.50, size = 320, normalized size = 3.20

$$\frac{\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^2-1\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c}{}\right.}{16\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*((cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c)
- cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*
(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqr
t(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*
cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos
(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(cos(d*x + c)^2 + (c
os(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a
))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d + (a*d
*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

giac [B] time = 0.69, size = 505, normalized size = 5.05

$$\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{2}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)+\frac{\left(4\,\sqrt{2}\,\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-2\,\sqrt{2}\,\sqrt{-a}\,\log\left(\sqrt{2}\,\sqrt{a}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/8*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(tan(1/2*d*x + 1/2*c)/(a*sgn(tan(1/
2*d*x + 1/2*c) + 1)) - 2/(a*sgn(tan(1/2*d*x + 1/2*c) + 1))) + (4*sqrt(2)*sq
rt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 2*sqrt(2)*sqrt(-a)*log
(sqrt(2)*sqrt(a) + sqrt(a)) + 6*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/
sqrt(-a)) - 3*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 14*sqrt(2)*sqrt(-a)
 + 18*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(2*sqrt(2)*sqrt(-a)*sqrt(a) +
 3*sqrt(-a)*sqrt(a)) - 2*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)
) + log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*((sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3 - 2*(sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) + (sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a + 2*a^(3/2))/(((sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^2*sgn(tan(
1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 1.24, size = 124, normalized size = 1.24

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left((-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{3}{2}}-\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^3\left(\sin^2(dx+\right.\right.}{4a^{\frac{7}{2}}\sin(dx+c)^2\cos(dx+c)\sqrt{a+a\sin(dx+c)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*((-a*(sin(d*x+c)-1))^
(3/2)*a^(3/2)-arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^2+(
-a*(sin(d*x+c)-1))^(1/2)*a^(5/2))/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{2}\csc\left(dx+c\right)^{3}}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{2}}{\sin\left(c+d\,x\right)^{3}\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(c+dx\right)\csc^{3}\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.342 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{a}\,d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] 1/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+1/8*cot(d*
x+c)/d/(a+a*sin(d*x+c))^(1/2)+1/12*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))
^(1/2)-1/3*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2874, 2980, 2772, 2773, 206}

$$\frac{\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{a}\,d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(8*Sqrt[a]*d) + Co
t[c + d*x]/(8*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c + d*x])/(12
*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*d*Sqrt[a +
a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))\sqrt{a+a\sin(c+dx)}\,dx}{a^2}$$
$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{6a}$$
$$= \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc^2(c+dx)\sqrt{a+a\sin(}}{8a}$$
$$= \frac{\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} -$$
$$= \frac{\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} +$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{a}\,d} + \frac{\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} -$$

Mathematica [B] time = 0.74, size = 292, normalized size = 2.16

$$\frac{\csc^9\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(60\sin\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{3}{2}(c+dx)\right)+6\sin\left(\frac{5}{2}(c+dx)\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^9*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-60*Cos[(c + d*x
)/2] + 2*Cos[(3*(c + d*x))/2] - 6*Cos[(5*(c + d*x))/2] + 60*Sin[(c + d*x)/2
] + 9*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 9*Log[1 -
 Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 2*Sin[(3*(c + d*x))/2]
 + 6*Sin[(5*(c + d*x))/2] - 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*
Sin[3*(c + d*x)] + 3*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c
+ d*x)]))/(24*d*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3*Sqrt[a*(1 + Sin
[c + d*x])])
```

fricas [B] time = 0.49, size = 367, normalized size = 2.72

$$\frac{3\left(\cos(dx+c)^4-2\cos(dx+c)^2-\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sin(dx+c)+1\right)\sqrt{a}\log\left(\frac{a\,c}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/96*(3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(3*cos(d*x + c)^3 + cos(d*x + c)^2 - (3*cos(d*x + c)^2 + 2*cos(d*x + c) + 7)*sin(d*x + c) + 5*cos(d*x + c) + 7)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d - (a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))

giac [B] time = 0.76, size = 546, normalized size = 4.04

$$\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\left(\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{3}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")

[Out] 1/48*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*tan(1/2*d*x + 1/2*c)/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 3/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c) + 2/(a*sgn(tan(1/2*d*x + 1/2*c) + 1))) + (30*sqrt(2)*a^(3/2)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 15*sqrt(2)*sqrt(-a)*a*log(sqrt(2)*sqrt(a) + sqrt(a)) + 42*a^(3/2)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 21*sqrt(-a)*a*log(sqrt(2)*sqrt(a) + sqrt(a)) - 88*sqrt(2)*sqrt(-a)*a - 126*sqrt(-a)*a)*sgn(tan(1/2*d*x + 1/2*c) + 1)/(5*sqrt(2)*sqrt(-a)*a^(3/2) + 7*sqrt(-a)*a^(3/2)) - 6*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 2*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(a) - 3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2 - 2*a^(5/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d

maple [A] time = 1.13, size = 144, normalized size = 1.07

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left(3(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{5}{2}}-8(-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{7}{2}}+3\,\mathrm{arctanh}\right.}{24a^{\frac{11}{2}}\sin(dx+c)^{3}\cos(dx+c)\sqrt{a+a\sin(dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(11/2)*(3*(-a*(sin(d*x+c)-1
))^(5/2)*a^(5/2)-8*(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2)+3*arctanh((-a*(sin(d*x
+c)-1))^(1/2)/a^(1/2))*a^5*sin(d*x+c)^3-3*(-a*(sin(d*x+c)-1))^(1/2)*a^(9/2)
)/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{2}\csc(dx+c)^{4}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)^4/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{2}}{\sin(c+d\,x)^{4}\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.343 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=184

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{76\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{105a^2d}+\frac{2\sin^3(c+dx)\cos(c+dx)}{7ad\sqrt{a\sin(c+dx)+a}}-\frac{16\sin^2(c+dx)}{35ad\sqrt{a\sin(c}$$

```
[Out] 2*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*
2^(1/2)-344/105*cos(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-16/35*cos(d*x+c)*sin(
d*x+c)^2/a/d/(a+a*sin(d*x+c))^(1/2)+2/7*cos(d*x+c)*sin(d*x+c)^3/a/d/(a+a*si
n(d*x+c))^(1/2)+76/105*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.59, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2879, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{76\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{105a^2d}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2\sin^3(c+dx)\cos(c+dx)}{7ad\sqrt{a\sin(c+dx)+a}}-\frac{16\sin^2(c+dx)}{35ad\sqrt{a\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(3/2)*d) - (344*Cos[c + d*x])/(105*a*d*Sqrt[a + a*Sin[c + d*x]]) - (
16*Cos[c + d*x]*Sin[c + d*x]^2)/(35*a*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[
c + d*x]*Sin[c + d*x]^3)/(7*a*d*Sqrt[a + a*Sin[c + d*x]]) + (76*Cos[c + d*x
]*Sqrt[a + a*Sin[c + d*x]])/(105*a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2879

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \frac{\sin^3(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{2\cos(c+dx)\sin^3(c+dx)}{7ad\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\sin^2(c+dx)\left(-3a^2+4a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{7a^3}$$

$$= -\frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^3(c+dx)}{7ad\sqrt{a+a\sin(c+dx)}} + \frac{4\int \frac{\sin(c+dx)\left(8a^3-\right.}{\sqrt{a+a\,\mathrm{si}}}}{35}$$

$$= -\frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^3(c+dx)}{7ad\sqrt{a+a\sin(c+dx)}} + \frac{4\int \frac{8a^3\sin(c+dx)-}{\sqrt{a+a\,\mathrm{si}}}}{35a}$$

$$= -\frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^3(c+dx)}{7ad\sqrt{a+a\sin(c+dx)}} + \frac{76\cos(c+dx)\sqrt{}}{105}$$

$$= -\frac{344\cos(c+dx)}{105ad\sqrt{a+a\sin(c+dx)}} - \frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin}{7ad\sqrt{a+a\sin(}}$$

$$= -\frac{344\cos(c+dx)}{105ad\sqrt{a+a\sin(c+dx)}} - \frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin}{7ad\sqrt{a+a\sin(}}$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{344\cos(c+dx)}{105ad\sqrt{a+a\sin(c+dx)}} - \frac{16\cos(c+dx)}{35ad\sqrt{a+a\,\mathrm{s}}}$$

Mathematica [C] time = 1.88, size = 201, normalized size = 1.09

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(1365\sin\left(\frac{1}{2}(c+dx)\right)+245\sin\left(\frac{3}{2}(c+dx)\right)-63\sin\left(\frac{5}{2}(c+dx)\right)-15\sin\left(\frac{7}{2}(c+dx)\right)-136\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]

[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((1680 + 1680*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)
*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 1365*
Cos[(c + d*x)/2] + 245*Cos[(3*(c + d*x))/2] + 63*Cos[(5*(c + d*x))/2] - 15*
Cos[(7*(c + d*x))/2] + 1365*Sin[(c + d*x)/2] + 245*Sin[(3*(c + d*x))/2] - 6
```

```
3*Sin[(5*(c + d*x))/2] - 15*Sin[(7*(c + d*x))/2]))/(420*a^2*d*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.51, size = 259, normalized size = 1.41

$$\frac{\frac{105\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-2\left(15\cos(d\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/105*(105*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)
^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(
cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c
)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 2*(15*
cos(d*x + c)^4 - 24*cos(d*x + c)^3 - 92*cos(d*x + c)^2 + (15*cos(d*x + c)^3
 + 39*cos(d*x + c)^2 - 53*cos(d*x + c) - 211)*sin(d*x + c) + 158*cos(d*x +
c) + 211)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c
) + a^2*d)
```

giac [B] time = 0.70, size = 375, normalized size = 2.04

$$\frac{2\left(\frac{\sqrt{2}\left(210\,a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+211\sqrt{-a}\sqrt{a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{-a}\,a^2}-\frac{210\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{2\left(\left(\left(\left(\left(\left(\cdots\right.\right.\right.\right.\right.\right.}{\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 2/105*(sqrt(2)*(210*a*arctan(sqrt(a)/sqrt(-a)) + 211*sqrt(-a)*sqrt(a))*sgn(
tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^2) - 210*sqrt(2)*arctan(-1/2*sqrt(2)*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a
))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*((((((((67*a^2*t
an(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 105*a^2/sgn(tan(1/2*d*x
```

```
 + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 287*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1
))*tan(1/2*d*x + 1/2*c) - 385*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*
x + 1/2*c) + 385*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) -
287*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 105*a^2/sgn(t
an(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 67*a^2/sgn(tan(1/2*d*x + 1
/2*c) + 1))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2))/d
```

maple [A] time = 1.30, size = 148, normalized size = 0.80

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(105a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-15\left(a-a\sin\left(dx+c\right)\right)^{\frac{7}{2}}+21}{105a^{5}\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/105*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(105*a^(7/2)*2^(1/2)*arctanh
(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-15*(a-a*sin(d*x+c))^(7/2)+21*(
a-a*sin(d*x+c))^(5/2)*a-35*(a-a*sin(d*x+c))^(3/2)*a^2-105*a^3*(a-a*sin(d*x+
c))^(1/2))/a^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)^{3}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)^3/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{2}\,\sin\left(c+d\,x\right)^{3}}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.344 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=140

$$-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5a^2d}-\frac{2\cos^3(c+dx)}{5ad\sqrt{a\sin(c+dx)+a}}+\frac{18\cos(c+dx)}{5ad\sqrt{a\sin(c+dx)}}$$

```
[Out] -2*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d
*2^(1/2)+18/5*cos(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-2/5*cos(d*x+c)^3/a/d/(a
+a*sin(d*x+c))^(1/2)-4/5*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.35, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2878, 2858, 2751, 2649, 206}

$$-\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5a^2d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\cos^3(c+dx)}{5ad\sqrt{a\sin(c+dx)+a}}+\frac{18\cos(c+dx)}{5ad\sqrt{a\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
]])])/(a^(3/2)*d) + (18*Cos[c + d*x])/(5*a*d*Sqrt[a + a*Sin[c + d*x]]) - (2*
Cos[c + d*x]^3)/(5*a*d*Sqrt[a + a*Sin[c + d*x]]) - (4*Cos[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/(5*a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2858

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(d*Cos[e + f*x]*(
a + b*Sin[e + f*x])^(m + 2))/(b^2*f*(m + 3)), x] - Dist[1/(b^2*(m + 3)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(b*d*(m + 2) - a*c*(m + 3) + (b*c*(m + 3) -
a*d*(m + 4))*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 - b^2, 0] && GeQ[m, -3/2] && LtQ[m, 0]
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2\cos^3(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\cos^2(c+dx)\left(-\frac{a}{2}-3a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{5a}$$

$$= -\frac{2\cos^3(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5a^2d} - \frac{4\int \frac{-\frac{3a^2}{4}+\frac{27}{4}a^2\sin}{\sqrt{a+a\sin(c+}}}{15a^3}$$

$$= \frac{18\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+a\sin(c}}{5a^2d}$$

$$= \frac{18\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+a\sin(c}}{5a^2d}$$

$$= -\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{18\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+d}{5ad\sqrt{a+a\sin(c}}$$

Mathematica [C] time = 0.29, size = 150, normalized size = 1.07

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-30\sin\left(\frac{1}{2}(c+dx)\right)-5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)+30\cos\left(\frac{1}{2}(c+\cdots\right.\right.}{10d(a(\sin(c+\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*((40 + 40*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 30*Cos[(c + d*x)/2] - 5*Cos
[(3*(c + d*x))/2] - Cos[(5*(c + d*x))/2] - 30*Sin[(c + d*x)/2] - 5*Sin[(3*(
c + d*x))/2] + Sin[(5*(c + d*x))/2]))/(10*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [A] time = 0.50, size = 236, normalized size = 1.69

$$\frac{\frac{5\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-2\left(\cos\left(dx+\cdots\right.\right.}{5\left(a^2d\cos\left(dx+c\right)+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/5*(5*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 -
 (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(
d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
- (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 2*(cos(d*x
 + c)^3 + 3*cos(d*x + c)^2 - (cos(d*x + c)^2 - 2*cos(d*x + c) - 9)*sin(d*x
+ c) - 7*cos(d*x + c) - 9)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c) +
a^2*d*sin(d*x + c) + a^2*d)
```

giac [B] time = 0.67, size = 303, normalized size = 2.16

$$-\frac{2\left(\frac{\sqrt{2}\left(10\,a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+9\sqrt{-a}\sqrt{a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}\,a^2}-\frac{10\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{2\left(\left(\left(\left(\left(\frac{3\cdots}{\mathrm{sgn}(\cdots}\right.\right.\right.\right.\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -2/5*(sqrt(2)*(10*a*arctan(sqrt(a)/sqrt(-a)) + 9*sqrt(-a)*sqrt(a))*sgn(tan(
1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^2) - 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sq
rt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(((((3*a*tan(1/2*d*x
 + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 5*a/sgn(tan(1/2*d*x + 1/2*c) + 1)
)*tan(1/2*d*x + 1/2*c) + 10*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x +
1/2*c) - 10*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 5*a/sgn
(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 3*a/sgn(tan(1/2*d*x + 1/
2*c) + 1))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2))/d
```

maple [A] time = 1.68, size = 112, normalized size = 0.80

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(-5a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)+\left(a-a\sin\left(dx+c\right)\right)^{\frac{5}{2}}+5a^{2}\sqrt{a}\right.}{5d\,a^{4}\cos\left(dx+c\right)\sqrt{a\left(1+\sin\left(dx+c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/5/d/a^4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-5*a^(5/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+(a-a*sin(d*x+c))^(5/2)+5*a^
2*(a-a*sin(d*x+c))^(1/2))/cos(d*x+c)/(a*(1+sin(d*x+c)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)^{2}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{2}\sin\left(c+d\,x\right)^{2}}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)\cos^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sin(c + d*x)**2*cos(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.345 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}-\frac{10\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] 2*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*
2^(1/2)-10/3*cos(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)+2/3*cos(d*x+c)*(a+a*sin(
d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.16, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2858, 2751, 2649, 206}

$$\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{10\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(3/2)*d) - (10*Cos[c + d*x])/(3*a*d*Sqrt[a + a*Sin[c + d*x]]) + (2*C
os[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2858

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(d*Cos[e + f*x]*(
a + b*Sin[e + f*x])^(m + 2))/(b^2*f*(m + 3)), x] - Dist[1/(b^2*(m + 3)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(b*d*(m + 2) - a*c*(m + 3) + (b*c*(m + 3) -
a*d*(m + 4))*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 - b^2, 0] && GeQ[m, -3/2] && LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} - \frac{2\int \frac{\frac{a}{2}-\frac{5}{2}a\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a^2}$$

$$= -\frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} - \frac{2\int \frac{1}{\sqrt{a+a\sin(c+d}}}{a}$$

$$= -\frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} + \frac{4\,\mathrm{Subst}\left(\int \frac{1}{2a-}\right.}{}$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+}}{3a^2d}$$

Mathematica [C] time = 0.90, size = 149, normalized size = 1.38

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(9\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)-9\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)+(12+12i)(-1)\right.}{3a^2d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((12 + 12*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1
)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 9*Cos[(c
+ d*x)/2] + Cos[(3*(c + d*x))/2] + 9*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2
]))/(3*a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [B] time = 0.50, size = 215, normalized size = 1.99

$$\frac{\frac{3\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+2\left(\cos\left(dx+c\right.\right.}{3\left(a^2d\cos\left(dx+c\right)+a^2d\sin\left(dx+c\right)+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/3*(3*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 -
 (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(
d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
- (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 2*(cos(d*x
 + c)^2 + (cos(d*x + c) + 5)*sin(d*x + c) - 4*cos(d*x + c) - 5)*sqrt(a*sin(
d*x + c) + a))/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

giac [B] time = 0.63, size = 246, normalized size = 2.28

$$\frac{2\left(\frac{2\left(\left(\left(\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}+\frac{\left(6\sqrt{2}\,a\arctan\left(\right.\right.}{}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] 2/3*(2*(((2*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 3/sgn(tan(
1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 3/sgn(tan(1/2*d*x + 1/2*c) +
1))*tan(1/2*d*x + 1/2*c) - 2/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*x
+ 1/2*c)^2 + a)^(3/2) + (6*sqrt(2)*a*arctan(sqrt(a)/sqrt(-a)) + 5*sqrt(2)*s
qrt(-a)*sqrt(a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^2) - 6*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))
/d
```

maple [A] time = 1.17, size = 110, normalized size = 1.02

$$-\frac{2\left(1+\sin \left(dx+c\right)\right) \sqrt{-a\left(\sin \left(dx+c\right)-1\right)}\, \left(-3a^{\frac{3}{2}}\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\, \sqrt{2}}{2\sqrt{a}}\right)+\left(a-a\sin \left(dx+c\right)\right)^{\frac{3}{2}}+3a\sqrt{}\right.}{3a^{3}\cos \left(dx+c\right) \sqrt{a+a\sin \left(dx+c\right)}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/3/a^3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-3*a^(3/2)*2^(1/2)*arcta
nh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+(a-a*sin(d*x+c))^(3/2)+3*a*(
a-a*sin(d*x+c))^(1/2))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos \left(dx+c\right)^{2} \sin \left(dx+c\right)}{\left(a\sin \left(dx+c\right)+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos \left(c+d\,x\right)}^2\,\sin \left(c+d\,x\right)}{{\left(a+a\,\sin \left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*sin(c + d*x))/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin \left(c+dx\right) \cos^{2}\left(c+dx\right)}{\left(a\left(\sin \left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sin(c + d*x)*cos(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.346 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d+2*arctanh(1
/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2874, 2985, 2649, 206, 2773}

$$\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) +
 (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(3/2)*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\int \frac{\csc(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}\\
&= \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{a^2}-\frac{2\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{a}\\
&= -\frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad}+\frac{4\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}\end{aligned}$$

Mathematica [C] time = 0.21, size = 130, normalized size = 1.53

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left((4+4i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(c+dx)\right)-1\right)\right)+\log\left(-\text{s}\right.\right.}{d(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((((4 + 4*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)
/4])] + Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Cos[(c + d*x
```

```
)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(d*(a*(1
 + Sin[c + d*x]))^(3/2)))
```

fricas [B] time = 0.51, size = 291, normalized size = 3.42

$$\frac{2\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\cdots}{\cdots}\right)}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(2*sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x +
c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/s
qrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x +
 c) - cos(d*x + c) - 2)) + sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)
^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)))/(a^2*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos((d*t_nostep+c)/2-pi/4))]Discontinuities at zeroe
s of cos((d*t_nostep+c)/2-pi/4) were not checkedWarning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep+1)]Evaluation time: 0.47Error: Bad Argument Type
```

maple [A] time = 1.04, size = 97, normalized size = 1.14

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)\right)}{a^{\frac{3}{2}}\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(d*
x+c))^(1/2)*2^(1/2)/a^(1/2))-arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))/a^(3/
2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2 \csc(dx+c)}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{\sin(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.347 $\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] 3*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d-2*arctanh(1/
2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-cot(
d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2715, 2985, 2649, 206, 2773}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) -
(2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(3/2)*d) - Cot[c + d*x]/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2715

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(a*f*Tan[e + f*x]), x] + Di
st[1/b^2, Int[((a + b*Sin[e + f*x])^(m + 1)*(b*m - a*(m + 1)*Sin[e + f*x]))
```

```
/Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(-\frac{3a}{2}+\frac{1}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{3\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{2a^2} + \frac{2\int \frac{1}{\sqrt{a+a\sin(c+a}}$$

$$= -\frac{\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{3\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad} - \frac{4\,\text{Subst}\left(\int \frac{1}{2a-x^2}\right.}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx}}$$

Mathematica [C] time = 2.15, size = 206, normalized size = 1.82

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-\cot\left(\frac{1}{4}(c+dx)\right)+(16+16i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(c+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*((16 + 16*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - Cot[(c + d*x)/4] + 2*(3*Log
[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[1 - Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + Sec[(c + d*x)/2] + Csc[c + d*x]*Sin[(c + d*x)/4]^2 - Csc[
c + d*x]*Sin[(c + d*x)/4]*Sin[(3*(c + d*x))/4])))/(4*d*(a*(1 + Sin[c + d*x]
))^(3/2))
```

fricas [B] time = 0.53, size = 421, normalized size = 3.73

$$\frac{3\left(\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}+4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+\right.}{\cos(dx+c)^{3}+\cos}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((
a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) +
 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9
*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) -
a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - c
os(d*x + c) - 1)) + 4*sqrt(2)*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(
d*x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) +
3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos
(d*x + c) - 2))/sqrt(a) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*
x + c) + 1))/(a^2*d*cos(d*x + c)^2 - a^2*d - (a^2*d*cos(d*x + c) + a^2*d)*s
in(d*x + c))
```

giac [B] time = 0.80, size = 471, normalized size = 4.17

$$\frac{\frac{\left(6\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-8\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-3\sqrt{2}\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)+6\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-16\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-3\sqrt{-a}\log\left(\sqrt{2}\right.\right.}{\sqrt{2}\sqrt{-a}\,a^{\frac{3}{2}}+\sqrt{-a}\,a^{\frac{3}{2}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/2*((6*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 8*sq
rt(2)*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - 3*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqr
t(a) + sqrt(a)) + 6*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) -
16*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - 3*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt
(a)) - sqrt(2)*sqrt(-a) - 3*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(2
)*sqrt(-a)*a^(3/2) + sqrt(-a)*a^(3/2)) + 8*sqrt(2)*arctan(-1/2*sqrt(2)*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/s
qrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 6*arctan(-(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a
)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c
) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c)
+ 1)) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^2*sgn(tan(1/2*d*x + 1/2*c) +
1)) + 2/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^2 - a)*sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 1.12, size = 134, normalized size = 1.19

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)\,a^2\left(2\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)-3\operatorname{arctanh}\left(\frac{\sqrt{a-a\,s}}{}\right.\right.\right.}{a^{\frac{7}{2}}\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin(dx+c)}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/a^(7/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(sin(d*x+c)*a^2*(2*2^(1
/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-3*arctanh((a-a*sin(
d*x+c))^(1/2)/a^(1/2)))+(a-a*sin(d*x+c))^(1/2)*a^(3/2))/sin(d*x+c)/cos(d*x+
c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2\csc(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{\sin(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.348 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{5\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -11/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d+2*arctan
h(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+
5/4*cot(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)/a/d/(a+
a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2874, 2984, 2985, 2649, 206, 2773}

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{5\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(3/2)*d
) + (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d
*x]])])/(a^(3/2)*d) + (5*Cot[c + d*x])/(4*a*d*Sqrt[a + a*Sin[c + d*x]]) - (
Cot[c + d*x]*Csc[c + d*x])/(2*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(-\frac{5a^2}{2}+\frac{3}{2}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^3}$$

$$= \frac{5\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(\frac{11a^3}{4}-\frac{5}{4}a^3\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}}{2a^4}$$

$$= \frac{5\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{11\int \csc(c+dx)\sqrt{a+a\sin}}{8a^2}$$

$$= \frac{5\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} - \frac{11\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a}{\sqrt{a+}}\right.}{4ad}$$

$$= -\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{5\cot(c+}{4ad\sqrt{a+a\sin}}$$

Mathematica [C] time = 3.56, size = 309, normalized size = 2.02

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(12\tan\left(\frac{1}{4}(c+dx)\right)+12\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)+\sec^2\left(\frac{1}{4}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-24 - (128 + 128*I)*(-1)^(3/4)*Ar
cTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 12*Cot[(c + d*x)/4]
 - Csc[(c + d*x)/4]^2 - 44*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4
4*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Co
s[(c + d*x)/4] - Sin[(c + d*x)/4])^2 - (24*Sin[(c + d*x)/4])/(Cos[(c + d*x)
/4] - Sin[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (24*S
in[(c + d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 12*Tan[(c + d*x)/4
]))/(32*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.50, size = 508, normalized size = 3.32

$$11\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(11*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x +
c) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8
*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (
cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 16*sqrt(2)*(a*cos(d
*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x + c)^2 - a)*sin(
d*x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) +
3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos
(d*x + c) - 2))/sqrt(a) - 4*(5*cos(d*x + c)^2 + (5*cos(d*x + c) + 7)*sin(d*
x + c) - 2*cos(d*x + c) - 7)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^
3 + a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d + (a^2*d*cos(d*x + c)
^2 - a^2*d)*sin(d*x + c))
```

giac [B] time = 0.89, size = 620, normalized size = 4.05

$$\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{6}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)-\frac{\left(44\,\sqrt{2}\,\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-96\,\sqrt{2}\,\sqrt{a}\,\arctan\left(\frac{-\sqrt{}}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/8*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(tan(1/2*d*x + 1/2*c)/(a^2*sgn(tan(
1/2*d*x + 1/2*c) + 1)) - 6/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (44*sqrt(
```

```
2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 96*sqrt(2)*sqrt(a
)*arctan(sqrt(a)/sqrt(-a)) - 22*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt
(a)) + 66*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 128*sqrt(a
)*arctan(sqrt(a)/sqrt(-a)) - 33*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 3
0*sqrt(2)*sqrt(-a) - 38*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(2*sqrt(2)*
sqrt(-a)*a^(3/2) + 3*sqrt(-a)*a^(3/2)) - 32*sqrt(2)*arctan(-1/2*sqrt(2)*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/
sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 22*arctan(-(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(
-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 11*log(abs(-sqrt(a)*tan(1/2*d*x + 1/
2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*
c) + 1)) + 2*((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^3 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))^2*sqrt(a) + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))*a + 6*a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 - a)^2*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 1.37, size = 164, normalized size = 1.07

$$-\frac{\left(1+\sin \left(dx+c\right)\right) \sqrt{-a \left(\sin \left(dx+c\right)-1\right)}\, \left(11 a^{4} \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right) \left(\sin^{2}\left(dx+c\right)\right)+5 \left(-a \left(\sin \left(dx+c\right)-\right.\right.}{4 a^{\frac{11}{2}} \sin \left(dx+c\right)^{2} \cos \left(dx+c\right) \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/4/a^(11/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(11*a^4*arctanh((-a*
(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2+5*(-a*(sin(d*x+c)-1))^(3/2)*a^(
5/2)-8*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*s
in(d*x+c)^2-3*(-a*(sin(d*x+c)-1))^(1/2)*a^(7/2))/sin(d*x+c)^2/cos(d*x+c)/(a
+a*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{\sin\left(c+d\,x\right)^3\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^3(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.349 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=191

$$\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{3/2}d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{9\cot(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}+$$

```
[Out] 23/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d-2*arctanh
(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-9
/8*cot(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)+7/12*cot(d*x+c)*csc(d*x+c)/a/d/(a+
a*sin(d*x+c))^(1/2)-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2874, 2984, 2985, 2649, 206, 2773}

$$\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{3/2}d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{9\cot(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (23*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*a^(3/2)*d)
 - (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*
x]])])/(a^(3/2)*d) - (9*Cot[c + d*x])/(8*a*d*Sqrt[a + a*Sin[c + d*x]]) + (7
*Cot[c + d*x]*Csc[c + d*x])/(12*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*
x]*Csc[c + d*x]^2)/(3*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \frac{\csc^4(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^3(c+dx)\left(-\frac{7a^2}{2}+\frac{5}{2}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a^3}$$

$$= \frac{7\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(\frac{27a^3}{4}-\frac{21}{4}a^3\sin\right.}{\sqrt{a+a\sin(c+dx)}}}{6a^4}$$

$$= -\frac{9\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{7\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+d}{3ad\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{9\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{7\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+d}{3ad\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{9\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{7\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+d}{3ad\sqrt{a+a\sin(c+dx}}$$

$$= \frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8a^{3/2}d} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{9\cot(c+}{8ad\sqrt{a+a\sin(}}$$

Mathematica [C] time = 2.35, size = 332, normalized size = 1.74

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-\frac{8\csc^9\left(\frac{1}{2}(c+dx)\right)\left(-228\sin\left(\frac{1}{2}(c+dx)\right)-110\sin\left(\frac{3}{2}(c+dx)\right)+54\sin\left(\frac{5}{2}(c+dx)\right)+228\cos\left(\frac{1}{2}(c+dx)\right)-1\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*((768 + 768*I)*(-1)^(3/4)*ArcTanh[
(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - (8*Csc[(c + d*x)/2]^9*(22
8*Cos[(c + d*x)/2] - 110*Cos[(3*(c + d*x))/2] - 54*Cos[(5*(c + d*x))/2] - 2
28*Sin[(c + d*x)/2] - 207*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[
c + d*x] + 207*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] -
110*Sin[(3*(c + d*x))/2] + 54*Sin[(5*(c + d*x))/2] + 69*Log[1 + Cos[(c + d*
```

```
x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 69*Log[1 - Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]*Sin[3*(c + d*x)]))/(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]
^2)^3))/(192*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.52, size = 564, normalized size = 2.95

$$\frac{69\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)+1\right)\sqrt{a}\,\log\left(\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(69*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c
)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2
*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a
*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
96*sqrt(2)*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a*c
os(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*log(-(cos(d*x + c)^2
- (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos
(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 4*(27*cos
(d*x + c)^3 + 41*cos(d*x + c)^2 - (27*cos(d*x + c)^2 - 14*cos(d*x + c) - 49
)*sin(d*x + c) - 35*cos(d*x + c) - 49)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos
(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d - (a^2*d*cos(d*x + c)^3 + a^2*
d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

giac [B] time = 0.94, size = 695, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/48*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*tan(1/2*d*x + 1/2*c)/(a^2*sgn(
tan(1/2*d*x + 1/2*c) + 1)) - 9/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2
*d*x + 1/2*c) + 38/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))) + (690*sqrt(2)*sqrt
(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 1344*sqrt(2)*sqrt(a)*arc
```

```
tan(sqrt(a)/sqrt(-a)) - 345*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a))
 + 966*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 1920*sqrt(a)*
arctan(sqrt(a)/sqrt(-a)) - 483*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 49
6*sqrt(2)*sqrt(-a) - 714*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(5*sqrt(2)
*sqrt(-a)*a^(3/2) + 7*sqrt(-a)*a^(3/2)) + 192*sqrt(2)*arctan(-1/2*sqrt(2)*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a)
)/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 138*arctan(-(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sq
rt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 69*log(abs(-sqrt(a)*tan(1/2*d*x +
 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1
/2*c) + 1)) - 2*(9*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^5 - 42*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^4*sqrt(a) + 72*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^2*a^(3/2) - 9*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))*a^2 - 38*a^(5/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3*a*sgn(tan(1/2*d*x + 1/2*c)
 + 1)))/d
```

maple [A] time = 1.49, size = 182, normalized size = 0.95

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(-69a^6\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)\left(\sin^3(dx+c)\right)+27(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{7}{2}}-40(-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{9}{2}}+48\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}\sqrt{2}}{2\sqrt{a}}\right)a^6\sin^3(dx+c)+21\sqrt{-a(\sin(dx+c)-1)}a^{\frac{11}{2}}\right)}{24a^{\frac{15}{2}}\sin^3(dx+c)\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/24/a^(15/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-69*a^6*arctanh((-
a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^3+27*(-a*(sin(d*x+c)-1))^(5/2)*
a^(7/2)-40*(-a*(sin(d*x+c)-1))^(3/2)*a^(9/2)+48*2^(1/2)*arctanh(1/2*(-a*(si
n(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a^6*sin(d*x+c)^3+21*(-a*(sin(d*x+c)-1))
^(1/2)*a^(11/2))/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + d\,x)^2}{\sin(c + d\,x)^4\,(a + a\,\sin(c + d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.350 $\int \cos^3(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^6(c+dx)}{6d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}$$

```
[Out] 1/4*a*sin(d*x+c)^4/d+1/5*a*sin(d*x+c)^5/d-1/6*a*sin(d*x+c)^6/d-1/7*a*sin(d*
x+c)^7/d
```

Rubi [A] time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 75}

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^6(c+dx)}{6d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^4)/(4*d) + (a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^6)/(6
*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)x^3(a+x)^2}{a^3}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int (a-x)x^3(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3x^3+a^2x^4-ax^5-x^6\right)dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{a\sin^4(c+dx)}{4d} + \frac{a\sin^5(c+dx)}{5d} - \frac{a\sin^6(c+dx)}{6d} - \frac{a\sin^7(c+$$

Mathematica [A] time = 0.30, size = 51, normalized size = 0.78

$$\frac{a\left(-315\cos(2(c+dx))+35\cos(6(c+dx))+96\sin^5(c+dx)(5\cos(2(c+dx))+9)\right)}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-315*Cos[2*(c + d*x)] + 35*Cos[6*(c + d*x)] + 96*(9 + 5*Cos[2*(c + d*x)
])*Sin[c + d*x]^5))/(6720*d)
```

fricas [A] time = 0.49, size = 72, normalized size = 1.11

$$\frac{70\,a\cos\left(dx+c\right)^6-105\,a\cos\left(dx+c\right)^4+12\left(5\,a\cos\left(dx+c\right)^6-8\,a\cos\left(dx+c\right)^4+a\cos\left(dx+c\right)^2+2\,a\right)\sin\left(dx-\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(70*a*cos(d*x + c)^6 - 105*a*cos(d*x + c)^4 + 12*(5*a*cos(d*x + c)^6
- 8*a*cos(d*x + c)^4 + a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

giac [A] time = 0.21, size = 50, normalized size = 0.77

$$-\frac{60\,a\sin\left(dx+c\right)^7+70\,a\sin\left(dx+c\right)^6-84\,a\sin\left(dx+c\right)^5-105\,a\sin\left(dx+c\right)^4}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/420*(60*a*sin(d*x + c)^7 + 70*a*sin(d*x + c)^6 - 84*a*sin(d*x + c)^5 - 1
05*a*sin(d*x + c)^4)/d
```

maple [A] time = 0.24, size = 92, normalized size = 1.42

$$\frac{a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{4}(dx+c)\right)}{7}-\frac{3\left(\cos^{4}(dx+c)\right)\sin(dx+c)}{35}+\frac{\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{35}\right)+a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{4}(dx+c)\right)}{6}-\frac{\left(\cos^{4}(dx+c)\right)}{12}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*cos(d*x+c)^4*sin(d*x+c)+1/35*(2
+cos(d*x+c)^2)*sin(d*x+c))+a*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c
)^4))
```

maxima [A] time = 0.31, size = 50, normalized size = 0.77

$$-\frac{60\,a\sin\left(dx+c\right)^{7}+70\,a\sin\left(dx+c\right)^{6}-84\,a\sin\left(dx+c\right)^{5}-105\,a\sin\left(dx+c\right)^{4}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(60*a*sin(d*x + c)^7 + 70*a*sin(d*x + c)^6 - 84*a*sin(d*x + c)^5 - 1
05*a*sin(d*x + c)^4)/d
```

mupad [B] time = 0.07, size = 49, normalized size = 0.75

$$\frac{-\frac{a\sin(c+d\,x)^{7}}{7}-\frac{a\sin(c+d\,x)^{6}}{6}+\frac{a\sin(c+d\,x)^{5}}{5}+\frac{a\sin(c+d\,x)^{4}}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*sin(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^4)/4 + (a*sin(c + d*x)^5)/5 - (a*sin(c + d*x)^6)/6 - (a*si
n(c + d*x)^7)/7)/d
```

sympy [A] time = 7.83, size = 90, normalized size = 1.38

$$\begin{cases} \frac{2a\sin^{7}\left(c+dx\right)}{35d}+\frac{a\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{5d}-\frac{a\sin^{2}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{4d}-\frac{a\cos^{6}\left(c+dx\right)}{12d} & \text{for}\: d\neq 0\\ x\left(a\sin\left(c\right)+a\right)\sin^{3}\left(c\right)\cos^{3}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**7/(35*d) + a*sin(c + d*x)**5*cos(c + d*x)**2/(
5*d) - a*sin(c + d*x)**2*cos(c + d*x)**4/(4*d) - a*cos(c + d*x)**6/(12*d),
Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**3*cos(c)**3, True))
```

3.351 $\int \cos^3(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\sin^6(c+dx)}{6d}-\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] 1/3*a*sin(d*x+c)^3/d+1/4*a*sin(d*x+c)^4/d-1/5*a*sin(d*x+c)^5/d-1/6*a*sin(d*
x+c)^6/d
```

Rubi [A] time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 75}

$$-\frac{a\sin^6(c+dx)}{6d}-\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) - (a*Sin[c + d*x]^5)/(5
*d) - (a*Sin[c + d*x]^6)/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)x^2(a+x)^2}{a^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int (a-x)x^2(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3x^2+a^2x^3-ax^4-x^5\right)dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{a\sin^3(c+dx)}{3d} + \frac{a\sin^4(c+dx)}{4d} - \frac{a\sin^5(c+dx)}{5d} - \frac{a\sin^6(c+}{6d}$$

Mathematica [A] time = 0.22, size = 51, normalized size = 0.78

$$\frac{a\left(-45\cos(2(c+dx))+5\cos(6(c+dx))+32\sin^3(c+dx)(3\cos(2(c+dx))+7)\right)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-45*Cos[2*(c + d*x)] + 5*Cos[6*(c + d*x)] + 32*(7 + 3*Cos[2*(c + d*x)])
*Sin[c + d*x]^3))/(960*d)
```

fricas [A] time = 0.54, size = 62, normalized size = 0.95

$$\frac{10\,a\cos\left(dx+c\right)^6 - 15\,a\cos\left(dx+c\right)^4 - 4\left(3\,a\cos\left(dx+c\right)^4 - a\cos\left(dx+c\right)^2 - 2\,a\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(10*a*cos(d*x + c)^6 - 15*a*cos(d*x + c)^4 - 4*(3*a*cos(d*x + c)^4 - a
*cos(d*x + c)^2 - 2*a)*sin(d*x + c))/d
```

giac [A] time = 0.21, size = 50, normalized size = 0.77

$$-\frac{10\,a\sin\left(dx+c\right)^6 + 12\,a\sin\left(dx+c\right)^5 - 15\,a\sin\left(dx+c\right)^4 - 20\,a\sin\left(dx+c\right)^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(10*a*sin(d*x + c)^6 + 12*a*sin(d*x + c)^5 - 15*a*sin(d*x + c)^4 - 20
*a*sin(d*x + c)^3)/d
```

maple [A] time = 0.22, size = 74, normalized size = 1.14

$$\frac{a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{4}(dx+c)\right)}{6}-\frac{\left(\cos^{4}(dx+c)\right)}{12}\right)+a\left(-\frac{\left(\cos^{4}(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{15}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+a*(-1/5*cos(d*x+c
)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.31, size = 50, normalized size = 0.77

$$-\frac{10\,a\sin\left(dx+c\right)^{6}+12\,a\sin\left(dx+c\right)^{5}-15\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(10*a*sin(d*x + c)^6 + 12*a*sin(d*x + c)^5 - 15*a*sin(d*x + c)^4 - 20
*a*sin(d*x + c)^3)/d
```

mupad [B] time = 0.06, size = 49, normalized size = 0.75

$$\frac{-\frac{a\sin(c+d\,x)^{6}}{6}-\frac{a\sin(c+d\,x)^{5}}{5}+\frac{a\sin(c+d\,x)^{4}}{4}+\frac{a\sin(c+d\,x)^{3}}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*sin(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^3)/3 + (a*sin(c + d*x)^4)/4 - (a*sin(c + d*x)^5)/5 - (a*si
n(c + d*x)^6)/6)/d
```

sympy [A] time = 4.31, size = 90, normalized size = 1.38

$$\begin{cases} \frac{2a\sin^{5}(c+dx)}{15d}+\frac{a\sin^{3}(c+dx)\cos^{2}(c+dx)}{3d}-\frac{a\sin^{2}(c+dx)\cos^{4}(c+dx)}{4d}-\frac{a\cos^{6}(c+dx)}{12d} & \text{for } d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)\sin^{2}\left(c\right)\cos^{3}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**5/(15*d) + a*sin(c + d*x)**3*cos(c + d*x)**2/(
3*d) - a*sin(c + d*x)**2*cos(c + d*x)**4/(4*d) - a*cos(c + d*x)**6/(12*d),
Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**2*cos(c)**3, True))
```

3.352 $\int \cos^3(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=49

$$-\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^4(c+dx)}{4d}$$

```
[Out] -1/4*a*cos(d*x+c)^4/d+1/3*a*sin(d*x+c)^3/d-1/5*a*sin(d*x+c)^5/d
```

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2834, 2565, 30, 2564, 14}

$$-\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^4)/(4*d) + (a*Sin[c + d*x]^3)/(3*d) - (a*Sin[c + d*x]^5)/(
5*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^3(c+dx)\sin(c+dx)\,dx + a\int \cos^3(c+dx)\sin^2(c+a\\ &= -\frac{a\,\mathrm{Subst}\left(\int x^3\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\mathrm{Subst}\left(\int x^2\left(1-x^2\right)\,dz\right.}{d}\\ &= -\frac{a\cos^4(c+dx)}{4d} + \frac{a\,\mathrm{Subst}\left(\int\left(x^2-x^4\right)\,dx, x, \sin(c+dx)\right)}{d}\\ &= -\frac{a\cos^4(c+dx)}{4d} + \frac{a\sin^3(c+dx)}{3d} - \frac{a\sin^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.10, size = 58, normalized size = 1.18

$$-\frac{a(-60\sin(c+dx)+10\sin(3(c+dx))+6\sin(5(c+dx))+60\cos(2(c+dx))+15\cos(4(c+dx))+45)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/480*(a*(45 + 60*Cos[2*(c + d*x)] + 15*Cos[4*(c + d*x)] - 60*Sin[c + d*x]
 + 10*Sin[3*(c + d*x)] + 6*Sin[5*(c + d*x)]))/d
```

fricas [A] time = 0.57, size = 51, normalized size = 1.04

$$-\frac{15\,a\cos\left(dx+c\right)^4+4\left(3\,a\cos\left(dx+c\right)^4-a\cos\left(dx+c\right)^2-2\,a\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(15*a*cos(d*x + c)^4 + 4*(3*a*cos(d*x + c)^4 - a*cos(d*x + c)^2 - 2*a
)*sin(d*x + c))/d
```

giac [A] time = 0.17, size = 50, normalized size = 1.02

$$-\frac{12\,a\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}-30\,a\sin\left(dx+c\right)^{2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(12*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 30
*a*sin(d*x + c)^2)/d
```

maple [A] time = 0.22, size = 54, normalized size = 1.10

$$\frac{a\left(-\frac{\left(\cos^{4}(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{\left(\cos^{4}(dx+c)\right)a}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/5*cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-1/4*
cos(d*x+c)^4*a)
```

maxima [A] time = 0.31, size = 50, normalized size = 1.02

$$-\frac{12\,a\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}-30\,a\sin\left(dx+c\right)^{2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(12*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 30
*a*sin(d*x + c)^2)/d
```

mupad [B] time = 0.06, size = 49, normalized size = 1.00

$$\frac{-\frac{a\sin(c+d\,x)^5}{5}-\frac{a\sin(c+d\,x)^4}{4}+\frac{a\sin(c+d\,x)^3}{3}+\frac{a\sin(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*sin(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^2)/2 + (a*sin(c + d*x)^3)/3 - (a*sin(c + d*x)^4)/4 - (a*si
n(c + d*x)^5)/5)/d
```

sympy [A] time = 2.39, size = 66, normalized size = 1.35

$$\begin{cases} \frac{2a\sin^5(c+dx)}{15d} + \frac{a\sin^3(c+dx)\cos^2(c+dx)}{3d} - \frac{a\cos^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)\sin(c)\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**5/(15*d) + a*sin(c + d*x)**3*cos(c + d*x)**2/(
3*d) - a*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*cos(c)*
*3, True))
```

3.353 $\int \cos^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=45

$$\frac{2(a\sin(c+dx)+a)^3}{3a^2d}-\frac{(a\sin(c+dx)+a)^4}{4a^3d}$$

```
[Out] 2/3*(a+a*sin(d*x+c))^3/a^2/d-1/4*(a+a*sin(d*x+c))^4/a^3/d
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^3}{3a^2d}-\frac{(a\sin(c+dx)+a)^4}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^3)/(3*a^2*d) - (a + a*Sin[c + d*x])^4/(4*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(2a(a+x)^2-(a+x)^3\right)\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{2(a+a\sin(c+dx))^3}{3a^2d} - \frac{(a+a\sin(c+dx))^4}{4a^3d}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.98

$$-\frac{a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} - \frac{a\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x]),x]
[Out] -1/4*(a*Cos[c + d*x]^4)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.52, size = 39, normalized size = 0.87

$$-\frac{3\,a\cos\left(dx+c\right)^4 - 4\left(a\cos\left(dx+c\right)^2 + 2\,a\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/12*(3*a*cos(d*x + c)^4 - 4*(a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

giac [A] time = 0.15, size = 48, normalized size = 1.07

$$-\frac{3\,a\sin\left(dx+c\right)^4 + 4\,a\sin\left(dx+c\right)^3 - 6\,a\sin\left(dx+c\right)^2 - 12\,a\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*a*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

maple [A] time = 0.28, size = 36, normalized size = 0.80

$$\frac{-\frac{\left(\cos^4(dx+c)\right)a}{4} + \frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/4*cos(d*x+c)^4*a+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.31, size = 48, normalized size = 1.07

$$-\frac{3\,a\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}-6\,a\sin\left(dx+c\right)^{2}-12\,a\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*a*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

mupad [B] time = 0.06, size = 46, normalized size = 1.02

$$\frac{-\frac{a\sin(c+d\,x)^4}{4}-\frac{a\sin(c+d\,x)^3}{3}+\frac{a\sin(c+d\,x)^2}{2}+a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x) + (a*sin(c + d*x)^2)/2 - (a*sin(c + d*x)^3)/3 - (a*sin(c +
d*x)^4)/4)/d
```

sympy [A] time = 1.22, size = 60, normalized size = 1.33

$$\begin{cases} \frac{2a\sin^{3}\left(c+dx\right)}{3d}+\frac{a\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}-\frac{a\cos^{4}\left(c+dx\right)}{4d} & \text{for}\ d\neq 0\\ x\left(a\sin\left(c\right)+a\right)\cos^{3}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d - a
*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)*cos(c)**3, True))
```

3.354 $\int \cos^2(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=56

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d-1/2*a*sin(d*x+c)^2/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2836, 12, 75}

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d) - (
a*Sin[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)(a+x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2+\frac{a^3}{x}-ax-x^2\right)dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin^3}{}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.00

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d) - (
a*Sin[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.54, size = 51, normalized size = 0.91

$$\frac{3\,a\cos\left(dx+c\right)^2+6\,a\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+2\left(a\cos\left(dx+c\right)^2+2\,a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*cos(d*x + c)^2 + 6*a*log(1/2*sin(d*x + c)) + 2*(a*cos(d*x + c)^2 +
 2*a)*sin(d*x + c))/d
```

giac [A] time = 0.18, size = 48, normalized size = 0.86

$$-\frac{2\,a\sin\left(dx+c\right)^3+3\,a\sin\left(dx+c\right)^2-6\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-6\,a\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 6*a*log(abs(sin(d*x + c)))
- 6*a*sin(d*x + c))/d
```

maple [A] time = 0.33, size = 60, normalized size = 1.07

$$\frac{\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)a}{3d}+\frac{2a\sin\left(dx+c\right)}{3d}+\frac{a\left(\cos^{2}\left(dx+c\right)\right)}{2d}+\frac{a\ln\left(\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/3/d*cos(d*x+c)^2*sin(d*x+c)*a+2/3*a*sin(d*x+c)/d+1/2*a*cos(d*x+c)^2/d+a*l
n(sin(d*x+c))/d
```

maxima [A] time = 0.33, size = 47, normalized size = 0.84

$$-\frac{2\,a\sin\left(dx+c\right)^{3}+3\,a\sin\left(dx+c\right)^{2}-6\,a\log\left(\sin\left(dx+c\right)\right)-6\,a\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 6*a*log(sin(d*x + c)) - 6*a
*sin(d*x + c))/d
```

mupad [B] time = 8.73, size = 92, normalized size = 1.64

$$\frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}+\frac{a\cos\left(c+d\,x\right)^2}{2\,d}+\frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(a + a*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*log(1/cos(c/2 + (d*x)
/2)^2))/d + (a*cos(c + d*x)^2)/(2*d) + (2*a*sin(c + d*x))/(3*d) + (a*cos(c
+ d*x)^2*sin(c + d*x))/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^{3}\left(c+dx\right)\csc\left(c+dx\right)dx+\int\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)\csc\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**3*csc(c + d*x), x) + Integral(sin(c + d*x)*cos(c
+ d*x)**3*csc(c + d*x), x))
```

3.355 $\int \cos(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=53

$$-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] -a*csc(d*x+c)/d+a*ln(sin(d*x+c))/d-a*sin(d*x+c)/d-1/2*a*sin(d*x+c)^2/d
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2836, 12, 75}

$$-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a*S
in[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)(a+x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-a+\frac{a^3}{x^2}+\frac{a^2}{x}-x\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-\frac{a\sin^2}{}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 1.00

$$-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a*S
in[c + d*x]^2)/(2*d)
```

fricas [A] time = 0.53, size = 68, normalized size = 1.28

$$\frac{4\,a\cos\left(dx+c\right)^2+4\,a\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+\left(2\,a\cos\left(dx+c\right)^2-a\right)\sin\left(dx+c\right)-8\,a}{4\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*cos(d*x + c)^2 + 4*a*log(1/2*sin(d*x + c))*sin(d*x + c) + (2*a*cos
(d*x + c)^2 - a)*sin(d*x + c) - 8*a)/(d*sin(d*x + c))
```

giac [A] time = 0.18, size = 47, normalized size = 0.89

$$-\frac{a\sin\left(dx+c\right)^2-2\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+2\,a\sin\left(dx+c\right)+\frac{2\,a}{\sin\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(a*sin(d*x + c)^2 - 2*a*log(abs(sin(d*x + c))) + 2*a*sin(d*x + c) + 2*
a/sin(d*x + c))/d
```

maple [A] time = 0.28, size = 82, normalized size = 1.55

$$\frac{a\left(\cos^{2}\left(dx+c\right)\right)}{2d}+\frac{a\ln\left(\sin\left(dx+c\right)\right)}{d}-\frac{a\left(\cos^{4}\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}-\frac{\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)a}{d}-\frac{2a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2*a*cos(d*x+c)^2/d+a*ln(sin(d*x+c))/d-1/d*a/sin(d*x+c)*cos(d*x+c)^4-1/d*c
os(d*x+c)^2*sin(d*x+c)*a-2*a*sin(d*x+c)/d
```

maxima [A] time = 0.31, size = 46, normalized size = 0.87

$$-\frac{a\sin\left(dx+c\right)^{2}-2\,a\log\left(\sin\left(dx+c\right)\right)+2\,a\sin\left(dx+c\right)+\frac{2\,a}{\sin\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(a*sin(d*x + c)^2 - 2*a*log(sin(d*x + c)) + 2*a*sin(d*x + c) + 2*a/sin
(d*x + c))/d
```

mupad [B] time = 8.70, size = 140, normalized size = 2.64

$$\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{5\,a\tan{\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,a\tan{\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+6\,a\tan\left(\right.}{d\,\left(2\tan{\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+4\tan{\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+2\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(a + a*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a*ta
n(c/2 + (d*x)/2))/(2*d) - (a + 6*a*tan(c/2 + (d*x)/2)^2 + 4*a*tan(c/2 + (d*
x)/2)^3 + 5*a*tan(c/2 + (d*x)/2)^4)/(d*(2*tan(c/2 + (d*x)/2) + 4*tan(c/2 +
(d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^{3}\left(c+dx\right)\csc^{2}\left(c+dx\right)dx+\int\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)\csc^{2}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**3*csc(c + d*x)**2, x) + Integral(sin(c + d*x)*cos
(c + d*x)**3*csc(c + d*x)**2, x))
```

3.356 $\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] -a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-a*ln(sin(d*x+c))/d-a*sin(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 75}

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (a*Log[Sin[c + d*x]])/d
- (a*Sin[c + d*x])/d
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^2}{x^3}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^3}{x^3}+\frac{a^2}{x^2}-\frac{a}{x}\right)dx,x,a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}$$

Mathematica [A] time = 0.11, size = 60, normalized size = 1.11

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}-\frac{a\left(\cot^2(c+dx)+2\log(\tan(c+dx))+2\log(\cos(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Ta
n[c + d*x]]))/(2*d) - (a*Sin[c + d*x])/d
```

fricas [A] time = 0.52, size = 69, normalized size = 1.28

$$-\frac{2\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+2\left(a\cos\left(dx+c\right)^2-2a\right)\sin\left(dx+c\right)-a}{2\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 2*(a*cos(d*x + c)^2
- 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.16, size = 46, normalized size = 0.85

$$-\frac{2\,a\log\left(|\sin\left(dx+c\right)|\right)+2\,a\sin\left(dx+c\right)+\frac{2\,a\sin(dx+c)+a}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*log(abs(sin(d*x + c))) + 2*a*sin(d*x + c) + (2*a*sin(d*x + c) + a
)/sin(d*x + c)^2)/d
```

maple [A] time = 0.34, size = 83, normalized size = 1.54

$$-\frac{a\left(\cos^4(dx+c)\right)}{d\sin(dx+c)}-\frac{\left(\cos^2(dx+c)\right)\sin(dx+c)\,a}{d}-\frac{2a\sin(dx+c)}{d}-\frac{a\left(\cot^2(dx+c)\right)}{2d}-\frac{a\ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^4-1/d*cos(d*x+c)^2*sin(d*x+c)*a-2*a*sin(d*x+c)
/d-1/2*a*cot(d*x+c)^2/d-a*ln(sin(d*x+c))/d
```

maxima [A] time = 0.33, size = 45, normalized size = 0.83

$$-\frac{2\,a\log(\sin(dx+c))+2\,a\sin(dx+c)+\frac{2\,a\sin(dx+c)+a}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*log(sin(d*x + c)) + 2*a*sin(d*x + c) + (2*a*sin(d*x + c) + a)/sin
(d*x + c)^2)/d
```

mupad [B] time = 8.74, size = 146, normalized size = 2.70

$$\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{a\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(a + a*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a*tan(c/2 + (d*x)/2))/(2*d) - (a/2 +
 2*a*tan(c/2 + (d*x)/2) + (a*tan(c/2 + (d*x)/2)^2)/2 + 10*a*tan(c/2 + (d*x)
/2)^3)/(d*(4*tan(c/2 + (d*x)/2)^2 + 4*tan(c/2 + (d*x)/2)^4)) - (a*tan(c/2 +
 (d*x)/2)^2)/(8*d) - (a*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.357 $\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^4(c+dx)}{4ad}$$

```
[Out] 1/3*sin(d*x+c)^3/a/d-1/4*sin(d*x+c)^4/a/d
```

Rubi [A] time = 0.10, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^3/(3*a*d) - Sin[c + d*x]^4/(4*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)x^2}{a^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int (a-x)x^2\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(ax^2-x^3\right)\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\sin^3(c+dx)}{3ad} - \frac{\sin^4(c+dx)}{4ad}$$

Mathematica [A] time = 0.13, size = 28, normalized size = 0.76

$$\frac{(4-3\sin(c+dx))\sin^3(c+dx)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((4 - 3*Sin[c + d*x])*Sin[c + d*x]^3)/(12*a*d)
```

fricas [A] time = 0.48, size = 47, normalized size = 1.27

$$-\frac{3\,\cos\left(dx+c\right)^4 - 6\,\cos\left(dx+c\right)^2 + 4\left(\cos\left(dx+c\right)^2 - 1\right)\sin\left(dx+c\right)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*cos(d*x + c)^4 - 6*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 1)*sin(d*x
 + c))/(a*d)
```

giac [A] time = 0.15, size = 29, normalized size = 0.78

$$-\frac{3\,\sin\left(dx+c\right)^4 - 4\,\sin\left(dx+c\right)^3}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*sin(d*x + c)^4 - 4*sin(d*x + c)^3)/(a*d)
```

maple [A] time = 0.15, size = 30, normalized size = 0.81

$$-\frac{\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^3(dx+c)\right)}{3}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/a/d*(1/4*sin(d*x+c)^4-1/3*sin(d*x+c)^3)
```

maxima [A] time = 0.32, size = 29, normalized size = 0.78

$$-\frac{3\,\sin\left(dx+c\right)^4-4\,\sin\left(dx+c\right)^3}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*sin(d*x + c)^4 - 4*sin(d*x + c)^3)/(a*d)
```

mupad [B] time = 0.05, size = 26, normalized size = 0.70

$$-\frac{\sin\left(c+d\,x\right)^3\,\left(3\,\sin\left(c+d\,x\right)-4\right)}{12\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)^3*(3*sin(c + d*x) - 4))/(12*a*d)
```

sympy [A] time = 20.00, size = 277, normalized size = 7.49

$$\begin{cases} \frac{8\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad}-\frac{12\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad} \\ \frac{x\sin^2(c)\cos^3(c)}{a\sin(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*tan(c/2 + d*x/2)**5/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/
2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3
```

```
*a*d) - 12*tan(c/2 + d*x/2)**4/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2
+ d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a
*d) + 8*tan(c/2 + d*x/2)**3/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 + d
*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*d)
, Ne(d, 0)), (x*sin(c)**2*cos(c)**3/(a*sin(c) + a), True))
```

3.358 $\int \frac{\cos^3(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\sin^2(c+dx)}{2ad}-\frac{\sin^3(c+dx)}{3ad}$$

```
[Out] 1/2*sin(d*x+c)^2/a/d-1/3*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.08, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2835, 2564, 30}

$$\frac{\sin^2(c+dx)}{2ad}-\frac{\sin^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^2/(2*a*d) - Sin[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos(c+dx)\sin(c+dx)\,dx}{a} - \frac{\int \cos(c+dx)\sin^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}(\int x\,dx, x, \sin(c+dx))}{ad} - \frac{\text{Subst}\left(\int x^2\,dx, x, \sin(c+dx)\right)}{ad}$$
$$= \frac{\sin^2(c+dx)}{2ad} - \frac{\sin^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.10, size = 28, normalized size = 0.76

$$\frac{(3-2\sin(c+dx))\sin^2(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] ((3 - 2*Sin[c + d*x])*Sin[c + d*x]^2)/(6*a*d)
```

fricas [A] time = 0.48, size = 37, normalized size = 1.00

$$-\frac{3\cos(dx+c)^2 - 2\left(\cos(dx+c)^2 - 1\right)\sin(dx+c)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/6*(3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 1)*sin(d*x + c))/(a*d)
```

giac [A] time = 0.14, size = 29, normalized size = 0.78

$$-\frac{2\sin(dx+c)^3 - 3\sin(dx+c)^2}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/6*(2*sin(d*x + c)^3 - 3*sin(d*x + c)^2)/(a*d)
```

maple [A] time = 0.10, size = 30, normalized size = 0.81

$$-\frac{\frac{\left(\sin^3(dx+c)\right)}{3} - \frac{\left(\sin^2(dx+c)\right)}{2}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/a/d*(1/3*sin(d*x+c)^3-1/2*sin(d*x+c)^2)
```

maxima [A] time = 0.31, size = 29, normalized size = 0.78

$$-\frac{2\sin(dx+c)^{3}-3\sin(dx+c)^{2}}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sin(d*x + c)^3 - 3*sin(d*x + c)^2)/(a*d)
```

mupad [B] time = 8.54, size = 26, normalized size = 0.70

$$-\frac{\sin(c+d\,x)^{2}\,\left(2\sin(c+d\,x)-3\right)}{6\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*sin(c + d*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)^2*(2*sin(c + d*x) - 3))/(6*a*d)
```

sympy [A] time = 10.74, size = 224, normalized size = 6.05

$$\begin{cases} \frac{6\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad}-\frac{8\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad}+\frac{}{3ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a} \\ \frac{x\sin(c)\cos^{3}(c)}{a\sin(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((6*tan(c/2 + d*x/2)**4/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2
 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) - 8*tan(c/2 + d*x/2)**3/(
3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x
/2)**2 + 3*a*d) + 6*tan(c/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*
tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d), Ne(d, 0)), (x*sin
(c)*cos(c)**3/(a*sin(c) + a), True))
```

3.359 $\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

```
[Out] sin(d*x+c)/a/d-1/2*sin(d*x+c)^2/a/d
```

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2667}

$$\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.04, size = 24, normalized size = 0.75

$$-\frac{(\sin(c+dx)-2)\sin(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
[Out] -1/2*((-2 + Sin[c + d*x])*Sin[c + d*x])/(a*d)
```

fricas [A] time = 0.48, size = 25, normalized size = 0.78

$$\frac{\cos(dx+c)^2 + 2\sin(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(cos(d*x + c)^2 + 2*sin(d*x + c))/(a*d)
```

giac [A] time = 0.15, size = 25, normalized size = 0.78

$$-\frac{\sin(dx+c)^2 - 2\sin(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

maple [A] time = 0.19, size = 28, normalized size = 0.88

$$-\frac{\frac{\left(\sin^2(dx+c)\right)}{2} - \sin(dx+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
[Out] -1/a/d*(1/2*sin(d*x+c)^2-sin(d*x+c))
```

maxima [A] time = 0.31, size = 25, normalized size = 0.78

$$-\frac{\sin(dx+c)^2 - 2\sin(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

mupad [B] time = 0.04, size = 22, normalized size = 0.69

$$-\frac{\sin\left(c+d\,x\right)\,\left(\sin\left(c+d\,x\right)-2\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)*(sin(c + d*x) - 2))/(2*a*d)
```

sympy [A] time = 5.87, size = 158, normalized size = 4.94

$$\begin{cases} \frac{2\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{2\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for}\ d \neq 0 \\ \frac{x\cos^{3}\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*tan(c/2 +
 d*x/2)**2 + a*d) - 2*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*
tan(c/2 + d*x/2)**2 + a*d) + 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**4 +
2*a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) + a), T
rue))
```

3.360 $\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}$$

```
[Out] ln(sin(d*x+c))/a/d-sin(d*x+c)/a/d
```

Rubi [A] time = 0.08, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 43}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)}{x}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{a-x}{x}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{ad}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.79

$$\frac{\log(\sin(c+dx)) - \sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] (Log[Sin[c + d*x]] - Sin[c + d*x])/(a*d)
```

fricas [A] time = 0.52, size = 25, normalized size = 0.86

$$\frac{\log\left(\frac{1}{2}\sin\left(dx+c\right)\right) - \sin\left(dx+c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] (log(1/2*sin(d*x + c)) - sin(d*x + c))/(a*d)
```

giac [A] time = 0.17, size = 28, normalized size = 0.97

$$\frac{\frac{\log(|\sin(dx+c)|)}{a} - \frac{\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] (log(abs(sin(d*x + c)))/a - sin(d*x + c)/a)/d
```

maple [A] time = 0.23, size = 33, normalized size = 1.14

$$-\frac{1}{ad\csc\left(dx+c\right)}-\frac{\ln\left(\csc\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/a/d/csc(d*x+c)-1/a/d*ln(csc(d*x+c))
```

maxima [A] time = 0.31, size = 27, normalized size = 0.93

$$\frac{\frac{\log(\sin(dx+c))}{a}-\frac{\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c))/a - sin(d*x + c)/a)/d
```

mupad [B] time = 8.76, size = 71, normalized size = 2.45

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) - (2*tan(c/2 + (d*x)/2))/(d*(a + a*tan(c/2 +
(d*x)/2)^2)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.361 $\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=30

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] -csc(d*x+c)/a/d-ln(sin(d*x+c))/a/d
```

Rubi [A] time = 0.08, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 43}

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{a-x}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{\csc(c+dx)}{ad} - \frac{\log(\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.73

$$-\frac{\csc(c+dx)+\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] -((Csc[c + d*x] + Log[Sin[c + d*x]])/(a*d))
```

fricas [A] time = 0.54, size = 34, normalized size = 1.13

$$-\frac{\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+1}{ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -(log(1/2*sin(d*x + c))*sin(d*x + c) + 1)/(a*d*sin(d*x + c))
```

giac [A] time = 0.16, size = 30, normalized size = 1.00

$$-\frac{\frac{\log(|\sin(dx+c)|)}{a}+\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -(log(abs(sin(d*x + c)))/a + 1/(a*sin(d*x + c)))/d
```

maple [A] time = 0.18, size = 30, normalized size = 1.00

$$-\frac{\csc(dx+c)}{ad}+\frac{\ln(\csc(dx+c))}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -csc(d*x+c)/a/d+1/a/d*ln(csc(d*x+c))
```

maxima [A] time = 0.68, size = 29, normalized size = 0.97

$$-\frac{\frac{\log(\sin(dx+c))}{a}+\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c))/a + 1/(a*sin(d*x + c)))/d
```

mupad [B] time = 8.70, size = 59, normalized size = 1.97

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)+\frac{1}{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] -(log(tan(c/2 + (d*x)/2)) + tan(c/2 + (d*x)/2)/2 - log(tan(c/2 + (d*x)/2)^2
 + 1) + 1/(2*tan(c/2 + (d*x)/2)))/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.362 $\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad}$$

```
[Out] csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d
```

Rubi [A] time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2706, 2606, 30, 8}

$$\frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot(c+dx)\csc(c+dx)\,dx}{a} + \frac{\int \cot(c+dx)\csc^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}(\int 1\,dx, x, \csc(c+dx))}{ad} - \frac{\text{Subst}(\int x\,dx, x, \csc(c+dx))}{ad}$$
$$= \frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad}$$

Mathematica [A] time = 0.03, size = 24, normalized size = 0.75

$$-\frac{(\csc(c+dx)-2)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*((-2 + Csc[c + d*x])*Csc[c + d*x])/(a*d)
```

fricas [A] time = 0.47, size = 30, normalized size = 0.94

$$-\frac{2\sin(dx+c)-1}{2\left(ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sin(d*x + c) - 1)/(a*d*cos(d*x + c)^2 - a*d)
```

giac [A] time = 0.16, size = 26, normalized size = 0.81

$$\frac{2\sin(dx+c)-1}{2\,ad\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)
```

maple [A] time = 0.23, size = 25, normalized size = 0.78

$$\frac{-\frac{\left(\csc^2(dx+c)\right)}{2}+\csc(dx+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/a/d*(-1/2*csc(d*x+c)^2+csc(d*x+c))
```

maxima [A] time = 0.32, size = 26, normalized size = 0.81

$$\frac{2 \sin(dx+c)-1}{2 \, ad \sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)
```

mupad [B] time = 8.63, size = 23, normalized size = 0.72

$$\frac{\sin(c+d\,x)-\frac{1}{2}}{a\,d\sin(c+d\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x) - 1/2)/(a*d*sin(c + d*x)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.363 $\int \frac{\cot^3(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] 1/2*csc(d*x+c)^2/a/d-1/3*csc(d*x+c)^3/a/d
```

Rubi [A] time = 0.08, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 43}

$$\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^3*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{a-x}{x^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{a}{x^4}-\frac{1}{x^3}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^2(c+dx)}{2ad} - \frac{\csc^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.06, size = 28, normalized size = 0.76

$$\frac{(3\sin(c+dx)-2)\csc^3(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^3*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] (Csc[c + d*x]^3*(-2 + 3*Sin[c + d*x]))/(6*a*d)
```

fricas [A] time = 0.48, size = 38, normalized size = 1.03

$$-\frac{3\sin(dx+c)-2}{6\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/6*(3*sin(d*x + c) - 2)/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

giac [A] time = 0.17, size = 26, normalized size = 0.70

$$\frac{3\sin(dx+c)-2}{6\,ad\sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/6*(3*sin(d*x + c) - 2)/(a*d*sin(d*x + c)^3)
```

maple [A] time = 0.20, size = 29, normalized size = 0.78

$$\frac{-\frac{\left(\csc^{3}(dx+c)\right)}{3}+\frac{\left(\csc^{2}(dx+c)\right)}{2}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
[Out] 1/a/d*(-1/3*csc(d*x+c)^3+1/2*csc(d*x+c)^2)
```

maxima [A] time = 0.31, size = 26, normalized size = 0.70

$$\frac{3\sin\left(dx+c\right)-2}{6\,ad\sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/6*(3*sin(d*x + c) - 2)/(a*d*sin(d*x + c)^3)
```

mupad [B] time = 8.76, size = 36, normalized size = 0.97

$$\frac{\frac{5\,\sin(c+d\,x)}{16}+\frac{\sin(3\,c+3\,d\,x)}{16}-\frac{1}{3}}{a\,d\,\sin\left(c+d\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
[Out] ((5*sin(c + d*x))/16 + sin(3*c + 3*d*x)/16 - 1/3)/(a*d*sin(c + d*x)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.364 $\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^4(c+dx)}{4ad}$$

```
[Out] 1/3*csc(d*x+c)^3/a/d-1/4*csc(d*x+c)^4/a/d
```

Rubi [A] time = 0.10, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^3*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^3/(3*a*d) - Csc[c + d*x]^4/(4*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^5(a-x)}{x^5}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{a-x}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{a}{x^5}-\frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^3(c+dx)}{3ad} - \frac{\csc^4(c+dx)}{4ad}$$

Mathematica [A] time = 0.04, size = 28, normalized size = 0.76

$$\frac{(4\sin(c+dx)-3)\csc^4(c+dx)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^3*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] (Csc[c + d*x]^4*(-3 + 4*Sin[c + d*x]))/(12*a*d)
```

fricas [A] time = 0.46, size = 41, normalized size = 1.11

$$\frac{4\,\sin\left(dx+c\right)-3}{12\left(ad\cos\left(dx+c\right)^{4}-2\,ad\cos\left(dx+c\right)^{2}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/12*(4*sin(d*x + c) - 3)/(a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

giac [A] time = 0.19, size = 26, normalized size = 0.70

$$\frac{4\,\sin\left(dx+c\right)-3}{12\,ad\sin\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/12*(4*sin(d*x + c) - 3)/(a*d*sin(d*x + c)^4)
```

maple [A] time = 0.21, size = 29, normalized size = 0.78

$$\frac{-\frac{\left(\csc^4(dx+c)\right)}{4}+\frac{\left(\csc^3(dx+c)\right)}{3}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^5/(a+a*sin(d*x+c)),x)
[Out] 1/a/d*(-1/4*csc(d*x+c)^4+1/3*csc(d*x+c)^3)
```

maxima [A] time = 0.31, size = 26, normalized size = 0.70

$$\frac{4\sin\left(dx+c\right)-3}{12\,ad\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/12*(4*sin(d*x + c) - 3)/(a*d*sin(d*x + c)^4)
```

mupad [B] time = 8.64, size = 25, normalized size = 0.68

$$\frac{\frac{\sin(c+d\,x)}{3}-\frac{1}{4}}{a\,d\sin\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)^5*(a + a*sin(c + d*x))),x)
[Out] (sin(c + d*x)/3 - 1/4)/(a*d*sin(c + d*x)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.365 $\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=143

$$-\frac{a\cos^9(c+dx)}{9d}+\frac{2a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\,\mathrm{si}}{}$$

[Out] 3/128*a*x-1/5*a*cos(d*x+c)^5/d+2/7*a*cos(d*x+c)^7/d-1/9*a*cos(d*x+c)^9/d+3/128*a*cos(d*x+c)*sin(d*x+c)/d+1/64*a*cos(d*x+c)^3*sin(d*x+c)/d-1/16*a*cos(d*x+c)^5*sin(d*x+c)/d-1/8*a*cos(d*x+c)^5*sin(d*x+c)^3/d

Rubi [A] time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 270}

$$-\frac{a\cos^9(c+dx)}{9d}+\frac{2a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]

[Out] (3*a*x)/128 - (a*Cos[c + d*x]^5)/(5*d) + (2*a*Cos[c + d*x]^7)/(7*d) - (a*Cos[c + d*x]^9)/(9*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) - (a*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (a*Cos[c + d*x]^5*Sin[c + d*x]^3)/(8*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^4(c+dx)\,dx + a\int \cos^4(c+dx)\sin^5(c+$$

$$= -\frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{8}(3a)\int \cos^4(c+dx)\sin^2(c$$

$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} +$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{2a\cos^7(c+dx)}{7d} - \frac{a\cos^9(c+dx)}{9d} + \frac{a\cos$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{2a\cos^7(c+dx)}{7d} - \frac{a\cos^9(c+dx)}{9d} + \frac{3a\cos$$

$$= \frac{3ax}{128} - \frac{a\cos^5(c+dx)}{5d} + \frac{2a\cos^7(c+dx)}{7d} - \frac{a\cos^9(c+dx)}{9d} +$$

Mathematica [A] time = 0.28, size = 84, normalized size = 0.59

$$\frac{a(-2520\sin(4(c+dx)) + 315\sin(8(c+dx)) - 7560\cos(c+dx) - 1680\cos(3(c+dx)) + 1008\cos(5(c+dx)) + 18}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(7560*c + 7560*d*x - 7560*Cos[c + d*x] - 1680*Cos[3*(c + d*x)] + 1008*Co
s[5*(c + d*x)] + 180*Cos[7*(c + d*x)] - 140*Cos[9*(c + d*x)] - 2520*Sin[4*(
c + d*x)] + 315*Sin[8*(c + d*x)]))/(322560*d)
```

fricas [A] time = 0.50, size = 95, normalized size = 0.66

$$-\frac{4480\,a\cos\left(dx+c\right)^{9}-11520\,a\cos\left(dx+c\right)^{7}+8064\,a\cos\left(dx+c\right)^{5}-945\,adx-315\left(16\,a\cos\left(dx+c\right)^{7}-24\,a\,c\right.}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/40320*(4480*a*cos(d*x + c)^9 - 11520*a*cos(d*x + c)^7 + 8064*a*cos(d*x +
 c)^5 - 945*a*d*x - 315*(16*a*cos(d*x + c)^7 - 24*a*cos(d*x + c)^5 + 2*a*co
s(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.24, size = 107, normalized size = 0.75

$$\frac{3}{128}\,ax-\frac{a\cos\left(9\,dx+9\,c\right)}{2304\,d}+\frac{a\cos\left(7\,dx+7\,c\right)}{1792\,d}+\frac{a\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{192\,d}-\frac{3\,a\cos\left(dx+c\right)}{128\,d}+\frac{a\sin\left(8\,dx\right.}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/128*a*x - 1/2304*a*cos(9*d*x + 9*c)/d + 1/1792*a*cos(7*d*x + 7*c)/d + 1/3
20*a*cos(5*d*x + 5*c)/d - 1/192*a*cos(3*d*x + 3*c)/d - 3/128*a*cos(d*x + c)
/d + 1/1024*a*sin(8*d*x + 8*c)/d - 1/128*a*sin(4*d*x + 4*c)/d
```

maple [A] time = 0.25, size = 124, normalized size = 0.87

$$\frac{a\left(-\frac{\left(\sin^{4}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{9}-\frac{4\left(\sin^{2}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{63}-\frac{8\left(\cos^{5}(dx+c)\right)}{315}\right)+a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^{5}(dx+c)\right)}{16}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315
*cos(d*x+c)^5)+a*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)
^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c))
```

maxima [A] time = 0.33, size = 71, normalized size = 0.50

$$-\frac{1024\left(35\cos\left(dx+c\right)^9-90\cos\left(dx+c\right)^7+63\cos\left(dx+c\right)^5\right)a-315\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\sin\left(4\right.\right.}{322560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/322560*(1024*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a - 315*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a)/d
```

mupad [B] time = 12.34, size = 353, normalized size = 2.47

$$\frac{3\,a\,x}{128}+\frac{\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{64}+\frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{32}-\frac{155\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32}+\left(\frac{a\,(79380\,c+79380\,d\,x-430080)}{40320}-\frac{63\,a\,(c+d\,x)}{32}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^4*(a + a*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/128 + ((a*(945*c + 945*d*x - 2048))/40320 - (3*a*tan(c/2 + (d*x)/2)
)/64 - (3*a*(c + d*x))/128 + tan(c/2 + (d*x)/2)^2*((a*(8505*c + 8505*d*x -
18432))/40320 - (27*a*(c + d*x))/128) + tan(c/2 + (d*x)/2)^4*((a*(34020*c +
 34020*d*x - 73728))/40320 - (27*a*(c + d*x))/32) + tan(c/2 + (d*x)/2)^6*((
a*(79380*c + 79380*d*x + 258048))/40320 - (63*a*(c + d*x))/32) + tan(c/2 +
(d*x)/2)^12*((a*(79380*c + 79380*d*x - 430080))/40320 - (63*a*(c + d*x))/32
) + tan(c/2 + (d*x)/2)^10*((a*(119070*c + 119070*d*x + 645120))/40320 - (18
9*a*(c + d*x))/64) + tan(c/2 + (d*x)/2)^8*((a*(119070*c + 119070*d*x - 9031
68))/40320 - (189*a*(c + d*x))/64) - (13*a*tan(c/2 + (d*x)/2)^3)/32 + (155*
a*tan(c/2 + (d*x)/2)^5)/32 - (169*a*tan(c/2 + (d*x)/2)^7)/32 + (169*a*tan(c
/2 + (d*x)/2)^11)/32 - (155*a*tan(c/2 + (d*x)/2)^13)/32 + (13*a*tan(c/2 + (
d*x)/2)^15)/32 + (3*a*tan(c/2 + (d*x)/2)^17)/64)/(d*(tan(c/2 + (d*x)/2)^2 +
 1)^9)
```

sympy [A] time = 19.38, size = 272, normalized size = 1.90

$$\begin{cases}\frac{3ax\sin^8(c+dx)}{128}+\frac{3ax\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{9ax\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{3ax\sin^2(c+dx)\cos^6(c+dx)}{32}+\frac{3ax\cos^8(c+dx)}{128}+\frac{3a\sin^7(c+dx)}{128}\\ x\left(a\sin\left(c\right)+a\right)\sin^4\left(c\right)\cos^4\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**8/128 + 3*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 9*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*a*x*sin(c + d*x)**2*cos
(c + d*x)**6/32 + 3*a*x*cos(c + d*x)**8/128 + 3*a*sin(c + d*x)**7*cos(c + d
*x)/(128*d) + 11*a*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - a*sin(c + d*x)
**4*cos(c + d*x)**5/(5*d) - 11*a*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) -
4*a*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - 3*a*sin(c + d*x)*cos(c + d*x)*
*7/(128*d) - 8*a*cos(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(
c)**4*cos(c)**4, True))
```

3.366 $\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=127

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\sin(c+dx)\cos^3(c+d...}{64d}$$

```
[Out] 3/128*a*x-1/5*a*cos(d*x+c)^5/d+1/7*a*cos(d*x+c)^7/d+3/128*a*cos(d*x+c)*sin(
d*x+c)/d+1/64*a*cos(d*x+c)^3*sin(d*x+c)/d-1/16*a*cos(d*x+c)^5*sin(d*x+c)/d-
1/8*a*cos(d*x+c)^5*sin(d*x+c)^3/d
```

Rubi [A] time = 0.16, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2565, 14, 2568, 2635, 8}

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\sin(c+dx)\cos^3(c+d...}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/128 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^7)/(7*d) + (3*a*Co
s[c + d*x]*Sin[c + d*x])/(128*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) -
 (a*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (a*Cos[c + d*x]^5*Sin[c + d*x]^3)
/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^3(c+dx)\,dx + a\int \cos^4(c+dx)\sin^4(c$$

$$= -\frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{8}(3a)\int \cos^4(c+dx)\sin^2($$

$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} +$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^3(c+dx)\sin(c+}{64d}$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{3a\cos(c+dx)\sin(c+}{128d}$$

$$= \frac{3ax}{128} - \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{3a\cos(c+dx)\sin}{128d}$$

Mathematica [A] time = 0.22, size = 71, normalized size = 0.56

$$\frac{a(-280\sin(4(c+dx)) + 35\sin(8(c+dx)) - 1680\cos(c+dx) - 560\cos(3(c+dx)) + 112\cos(5(c+dx)) + 80\cos}{35840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(840*d*x - 1680*Cos[c + d*x] - 560*Cos[3*(c + d*x)] + 112*Cos[5*(c + d*x
)] + 80*Cos[7*(c + d*x)] - 280*Sin[4*(c + d*x)] + 35*Sin[8*(c + d*x)]))/(35
840*d)
```

fricas [A] time = 0.51, size = 84, normalized size = 0.66

$$\frac{640\,a\cos\left(dx+c\right)^{7}-896\,a\cos\left(dx+c\right)^{5}+105\,adx+35\left(16\,a\cos\left(dx+c\right)^{7}-24\,a\cos\left(dx+c\right)^{5}+2\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4480*(640*a*cos(d*x + c)^7 - 896*a*cos(d*x + c)^5 + 105*a*d*x + 35*(16*a*
cos(d*x + c)^7 - 24*a*cos(d*x + c)^5 + 2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c
))*sin(d*x + c))/d
```

giac [A] time = 0.22, size = 92, normalized size = 0.72

$$\frac{3}{128}\,ax+\frac{a\cos\left(7\,dx+7\,c\right)}{448\,d}+\frac{a\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{3\,a\cos\left(dx+c\right)}{64\,d}+\frac{a\sin\left(8\,dx+8\,c\right)}{1024\,d}-\frac{a\sin\left(4\,dx+4\,c\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/128*a*x + 1/448*a*cos(7*d*x + 7*c)/d + 1/320*a*cos(5*d*x + 5*c)/d - 1/64*
a*cos(3*d*x + 3*c)/d - 3/64*a*cos(d*x + c)/d + 1/1024*a*sin(8*d*x + 8*c)/d
- 1/128*a*sin(4*d*x + 4*c)/d
```

maple [A] time = 0.24, size = 106, normalized size = 0.83

$$\frac{a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^{5}(dx+c)\right)}{16}+\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{64}+\frac{3dx}{128}+\frac{3c}{128}\right)+a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{7}-\frac{2\left(\cos^{5}(dx+c)\right)}{35}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+a*(-1/7*sin(d*x+c
)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5))
```

maxima [A] time = 0.31, size = 61, normalized size = 0.48

$$\frac{1024\left(5\cos\left(dx+c\right)^{7}-7\cos\left(dx+c\right)^{5}\right)a+35\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\sin\left(4\,dx+4\,c\right)\right)a}{35840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/35840*(1024*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a + 35*(24*d*x + 24*c +
 sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a)/d
```

mupad [B] time = 12.48, size = 320, normalized size = 2.52

$$\frac{3\,a\,x}{128}+\frac{\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+\frac{23\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+\left(\frac{a\,(2940\,c+2940\,d\,x-17920)}{4480}-\frac{21\,a\,(c+d\,x)}{32}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-\frac{333\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/128 + ((a*(105*c + 105*d*x - 512))/4480 - (3*a*tan(c/2 + (d*x)/2))/
64 - (3*a*(c + d*x))/128 + tan(c/2 + (d*x)/2)^2*((a*(840*c + 840*d*x - 4096
))/4480 - (3*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^4*((a*(2940*c + 2940*d*x
 + 3584))/4480 - (21*a*(c + d*x))/32) + tan(c/2 + (d*x)/2)^12*((a*(2940*c +
 2940*d*x - 17920))/4480 - (21*a*(c + d*x))/32) + tan(c/2 + (d*x)/2)^8*((a*
(7350*c + 7350*d*x - 17920))/4480 - (105*a*(c + d*x))/64) + tan(c/2 + (d*x)
/2)^6*((a*(5880*c + 5880*d*x - 28672))/4480 - (21*a*(c + d*x))/16) - (23*a*
tan(c/2 + (d*x)/2)^3)/64 + (333*a*tan(c/2 + (d*x)/2)^5)/64 - (671*a*tan(c/2
 + (d*x)/2)^7)/64 + (671*a*tan(c/2 + (d*x)/2)^9)/64 - (333*a*tan(c/2 + (d*x
)/2)^11)/64 + (23*a*tan(c/2 + (d*x)/2)^13)/64 + (3*a*tan(c/2 + (d*x)/2)^15)
/64)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^8)
```

sympy [A] time = 11.41, size = 248, normalized size = 1.95

$$\begin{cases}\frac{3ax\sin^{8}(c+dx)}{128}+\frac{3ax\sin^{6}(c+dx)\cos^{2}(c+dx)}{32}+\frac{9ax\sin^{4}(c+dx)\cos^{4}(c+dx)}{64}+\frac{3ax\sin^{2}(c+dx)\cos^{6}(c+dx)}{32}+\frac{3ax\cos^{8}(c+dx)}{128}+\frac{3a\sin^{7}(c+\cdots}{\cdots} & \\ x\left(a\sin(c)+a\right)\sin^{3}(c)\cos^{4}(c) & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**8/128 + 3*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 9*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*a*x*sin(c + d*x)**2*cos
(c + d*x)**6/32 + 3*a*x*cos(c + d*x)**8/128 + 3*a*sin(c + d*x)**7*cos(c + d
*x)/(128*d) + 11*a*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*a*sin(c + d
*x)**3*cos(c + d*x)**5/(128*d) - a*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) -
3*a*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 2*a*cos(c + d*x)**7/(35*d), Ne(d
, 0)), (x*(a*sin(c) + a)*sin(c)**3*cos(c)**4, True))
```

3.367 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=103

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx}{16d}$$

```
[Out] 1/16*a*x-1/5*a*cos(d*x+c)^5/d+1/7*a*cos(d*x+c)^7/d+1/16*a*cos(d*x+c)*sin(d*
x+c)/d+1/24*a*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 14}

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/16 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^7)/(7*d) + (a*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a*Co
s[c + d*x]^5*Sin[c + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
```

```
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^2(c+dx)\,dx + a\int \cos^4(c+dx)\sin^3(c+$$

$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}a\int \cos^4(c+dx)\,dx - \frac{a\,\mathrm{Sub}}{}$$

$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{8}a$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos(c+dx)\sin(c+dx}{16d}$$

$$= \frac{ax}{16} - \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos(c+dx)\sin(c-}{16d}$$

Mathematica [A] time = 0.21, size = 81, normalized size = 0.79

$$\frac{a(105\sin(2(c+dx)) - 105\sin(4(c+dx)) - 35\sin(6(c+dx)) - 315\cos(c+dx) - 105\cos(3(c+dx)) + 21\cos(5(c}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(420*d*x - 315*Cos[c + d*x] - 105*Cos[3*(c + d*x)] + 21*Cos[5*(c + d*x)]
 + 15*Cos[7*(c + d*x)] + 105*Sin[2*(c + d*x)] - 105*Sin[4*(c + d*x)] - 35*S
in[6*(c + d*x)]))/(6720*d)
```

fricas [A] time = 0.51, size = 73, normalized size = 0.71

$$\frac{240\, a \cos\left(dx+c\right)^{7} - 336\, a \cos\left(dx+c\right)^{5} + 105\, adx - 35\left(8\, a \cos\left(dx+c\right)^{5} - 2\, a \cos\left(dx+c\right)^{3} - 3\, a \cos\left(dx+c\right)\right)}{1680\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*cos(d*x + c)^7 - 336*a*cos(d*x + c)^5 + 105*a*d*x - 35*(8*a*c
os(d*x + c)^5 - 2*a*cos(d*x + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.19, size = 107, normalized size = 1.04

$$\frac{1}{16}\, ax + \frac{a \cos\left(7\, dx + 7\, c\right)}{448\, d} + \frac{a \cos\left(5\, dx + 5\, c\right)}{320\, d} - \frac{a \cos\left(3\, dx + 3\, c\right)}{64\, d} - \frac{3\, a \cos\left(dx + c\right)}{64\, d} - \frac{a \sin\left(6\, dx + 6\, c\right)}{192\, d} - \frac{a \sin\left(4\, dx +\right.}{64\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*a*x + 1/448*a*cos(7*d*x + 7*c)/d + 1/320*a*cos(5*d*x + 5*c)/d - 1/64*a
*cos(3*d*x + 3*c)/d - 3/64*a*cos(d*x + c)/d - 1/192*a*sin(6*d*x + 6*c)/d -
1/64*a*sin(4*d*x + 4*c)/d + 1/64*a*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.26, size = 88, normalized size = 0.85

$$\frac{a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{7} - \frac{2\left(\cos^{5}(dx+c)\right)}{35}\right) + a\left(-\frac{\sin(dx+c)\left(\cos^{5}(dx+c)\right)}{6} + \frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24} + \frac{dx}{16} + \frac{c}{16}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+a*(-1/6*sin(d*x+c
)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*
c))
```

maxima [A] time = 0.32, size = 65, normalized size = 0.63

$$\frac{192\left(5\, \cos\left(dx+c\right)^{7} - 7\, \cos\left(dx+c\right)^{5}\right)a + 35\left(4\, \sin\left(2\, dx + 2\, c\right)^{3} + 12\, dx + 12\, c - 3\, \sin\left(4\, dx + 4\, c\right)\right)a}{6720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

[Out] 1/6720*(192*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a + 35*(4*sin(2*d*x + 2*c
)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a)/d

mupad [B] time = 12.27, size = 292, normalized size = 2.83

$$\frac{a\,x}{16}+\frac{\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}-\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{6}+\left(\frac{a\,(2205\,c+2205\,d\,x-6720)}{1680}-\frac{21\,a\,(c+d\,x)}{16}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{31\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\left(\frac{a\,(3675\,c\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x)),x)

[Out] (a*x)/16 + ((a*(105*c + 105*d*x - 192))/1680 - (a*tan(c/2 + (d*x)/2))/8 - (
a*(c + d*x))/16 + tan(c/2 + (d*x)/2)^2*((a*(735*c + 735*d*x - 1344))/1680 -
 (7*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^4*((a*(2205*c + 2205*d*x + 2688))
/1680 - (21*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^10*((a*(2205*c + 2205*d*x
 - 6720))/1680 - (21*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^8*((a*(3675*c +
3675*d*x + 6720))/1680 - (35*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^6*((a*(3
675*c + 3675*d*x - 13440))/1680 - (35*a*(c + d*x))/16) + (11*a*tan(c/2 + (d
*x)/2)^3)/6 - (31*a*tan(c/2 + (d*x)/2)^5)/24 + (31*a*tan(c/2 + (d*x)/2)^9)/
24 - (11*a*tan(c/2 + (d*x)/2)^11)/6 + (a*tan(c/2 + (d*x)/2)^13)/8)/(d*(tan(
c/2 + (d*x)/2)^2 + 1)^7)

sympy [A] time = 6.94, size = 192, normalized size = 1.86

$$\begin{cases} \frac{ax\sin^{6}(c+dx)}{16}+\frac{3ax\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{3ax\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{ax\cos^{6}(c+dx)}{16}+\frac{a\sin^{5}(c+dx)\cos(c+dx)}{16d}+\frac{a\sin^{3}(c+dx)\cos^{3}(c}{6d} \\ x\left(a\sin(c)+a\right)\sin^{2}(c)\cos^{4}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)

[Out] Piecewise((a*x*sin(c + d*x)**6/16 + 3*a*x*sin(c + d*x)**4*cos(c + d*x)**2/1
6 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a*x*cos(c + d*x)**6/16 + a*s
in(c + d*x)**5*cos(c + d*x)/(16*d) + a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - a*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - a*sin(c + d*x)*cos(c + d*x)**
5/(16*d) - 2*a*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*
*2*cos(c)**4, True))

3.368 $\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}+\frac{ax}{16}$$

```
[Out] 1/16*a*x-1/5*a*cos(d*x+c)^5/d+1/16*a*cos(d*x+c)*sin(d*x+c)/d+1/24*a*cos(d*x
+c)^3*sin(d*x+c)/d-1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2838, 2565, 30, 2568, 2635, 8}

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}+\frac{ax}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/16 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(16*d)
+ (a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
```

```
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin(c+dx)\,dx + a\int \cos^4(c+dx)\sin^2(c+d$$

$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}a\int \cos^4(c+dx)\,dx - \frac{a\,\text{Subs}}{}$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a\cos^5(c+dx}{6d}$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c+dx)}{16d} + \frac{a\cos^3(c+dx)}{24d}$$

$$= \frac{ax}{16} - \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c+dx)}{16d} + \frac{a\cos^3(c+}{}$$

Mathematica [A] time = 0.16, size = 71, normalized size = 0.82

$$-\frac{a(-15\sin(2(c+dx))+15\sin(4(c+dx))+5\sin(6(c+dx))+120\cos(c+dx)+60\cos(3(c+dx))+12\cos(5(c}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/960*(a*(-60*d*x + 120*Cos[c + d*x] + 60*Cos[3*(c + d*x)] + 12*Cos[5*(c +
 d*x)] - 15*Sin[2*(c + d*x)] + 15*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)]))/d
```

fricas [A] time = 0.55, size = 62, normalized size = 0.71

$$-\frac{48\,a\cos\left(dx+c\right)^{5}-15\,adx+5\,\left(8\,a\cos\left(dx+c\right)^{5}-2\,a\cos\left(dx+c\right)^{3}-3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(48*a*cos(d*x + c)^5 - 15*a*d*x + 5*(8*a*cos(d*x + c)^5 - 2*a*cos(d*
x + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.17, size = 92, normalized size = 1.06

$$\frac{1}{16}\,ax-\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}-\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{a\sin\left(2\,dx+2\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*a*x - 1/80*a*cos(5*d*x + 5*c)/d - 1/16*a*cos(3*d*x + 3*c)/d - 1/8*a*co
s(d*x + c)/d - 1/192*a*sin(6*d*x + 6*c)/d - 1/64*a*sin(4*d*x + 4*c)/d + 1/6
4*a*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.24, size = 68, normalized size = 0.78

$$\frac{a\left(-\frac{\sin\left(dx+c\right)\left(\cos^{5}\left(dx+c\right)\right)}{6}+\frac{\left(\cos^{3}\left(dx+c\right)+\frac{3\cos\left(dx+c\right)}{2}\right)\sin\left(dx+c\right)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin
(d*x+c)+1/16*d*x+1/16*c)-1/5*a*cos(d*x+c)^5)
```

maxima [A] time = 0.31, size = 52, normalized size = 0.60

$$-\frac{192\,a\cos\left(dx+c\right)^{5}-5\,\left(4\,\sin\left(2\,dx+2\,c\right)^{3}+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/960*(192*a*cos(d*x + c)^5 - 5*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*
sin(4*d*x + 4*c))*a)/d
```

mupad [B] time = 12.22, size = 292, normalized size = 3.36

$$\frac{a\,x}{16}+\frac{\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\left(\frac{a\,(90\,c+90\,d\,x-480)}{240}-\frac{3\,a\,(c+d\,x)}{8}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{47\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\left(\frac{a\,(225\,c+225\,d\,x-480)}{240}-\frac{15\,a\,(c+d\,x)}{16}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*x)/16 + ((a*(15*c + 15*d*x - 96))/240 - (a*tan(c/2 + (d*x)/2))/8 - (a*(c
 + d*x))/16 + tan(c/2 + (d*x)/2)^2*((a*(90*c + 90*d*x - 96))/240 - (3*a*(c
+ d*x))/8) + tan(c/2 + (d*x)/2)^10*((a*(90*c + 90*d*x - 480))/240 - (3*a*(c
 + d*x))/8) + tan(c/2 + (d*x)/2)^8*((a*(225*c + 225*d*x - 480))/240 - (15*a
*(c + d*x))/16) + tan(c/2 + (d*x)/2)^4*((a*(225*c + 225*d*x - 960))/240 - (
15*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^6*((a*(300*c + 300*d*x - 960))/240
 - (5*a*(c + d*x))/4) + (47*a*tan(c/2 + (d*x)/2)^3)/24 - (13*a*tan(c/2 + (d
*x)/2)^5)/4 + (13*a*tan(c/2 + (d*x)/2)^7)/4 - (47*a*tan(c/2 + (d*x)/2)^9)/2
4 + (a*tan(c/2 + (d*x)/2)^11)/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [A] time = 4.24, size = 167, normalized size = 1.92

$$\begin{cases} \frac{ax\sin^{6}(c+dx)}{16}+\frac{3ax\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{3ax\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{ax\cos^{6}(c+dx)}{16}+\frac{a\sin^{5}(c+dx)\cos(c+dx)}{16d}+\frac{a\sin^{3}(c+dx)\cos^{3}(c+dx)}{6d} \\ x\left(a\sin(c)+a\right)\sin(c)\cos^{4}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**6/16 + 3*a*x*sin(c + d*x)**4*cos(c + d*x)**2/1
6 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a*x*cos(c + d*x)**6/16 + a*s
in(c + d*x)**5*cos(c + d*x)/(16*d) + a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - a*cos(c + d*x)**5/(5*d), Ne(d,
0)), (x*(a*sin(c) + a)*sin(c)*cos(c)**4, True))
```

3.369 $\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=89

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+$$

```
[Out] 3/8*a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d+3/8*a*c
os(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*sin(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2838, 2592, 302, 206, 2635, 8}

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c + d
*x]^3)/(3*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*Cos[c + d*x]^3*Si
n[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\,dx + a\int \cos^3(c+dx)\cot(c+dx)\,dx$$

$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx - \frac{a\,\mathrm{Sub}}{}$$

$$= \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{8}(3$$

$$= \frac{3ax}{8} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{3a\cos(c+dx)\sin(c}{8d}$$

$$= \frac{3ax}{8} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx}{3d}$$

Mathematica [A] time = 0.15, size = 81, normalized size = 0.91

$$\frac{a\left(120\cos(c+dx) + 8\cos(3(c+dx)) + 3\left(8\sin(2(c+dx)) + \sin(4(c+dx)) + 32\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 32\log\left(\right.\right.\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(120*Cos[c + d*x] + 8*Cos[3*(c + d*x)] + 3*(12*c + 12*d*x - 32*Log[Cos[(
c + d*x)/2]] + 32*Log[Sin[(c + d*x)/2]] + 8*Sin[2*(c + d*x)] + Sin[4*(c + d
*x)]))))/(96*d)
```

fricas [A] time = 0.53, size = 88, normalized size = 0.99

$$\frac{8\,a\cos\left(dx+c\right)^{3}+9\,adx+24\,a\cos\left(dx+c\right)-12\,a\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+12\,a\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+3}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*a*cos(d*x + c)^3 + 9*a*d*x + 24*a*cos(d*x + c) - 12*a*log(1/2*cos(d
*x + c) + 1/2) + 12*a*log(-1/2*cos(d*x + c) + 1/2) + 3*(2*a*cos(d*x + c)^3
+ 3*a*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.19, size = 145, normalized size = 1.63

$$\frac{9\left(dx+c\right)a+24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-96\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+9}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a + 24*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(15*a*tan(1/2
*d*x + 1/2*c)^7 - 48*a*tan(1/2*d*x + 1/2*c)^6 - 9*a*tan(1/2*d*x + 1/2*c)^5
- 96*a*tan(1/2*d*x + 1/2*c)^4 + 9*a*tan(1/2*d*x + 1/2*c)^3 - 80*a*tan(1/2*d
*x + 1/2*c)^2 - 15*a*tan(1/2*d*x + 1/2*c) - 32*a)/(tan(1/2*d*x + 1/2*c)^2 +
 1)^4)/d
```

maple [A] time = 0.36, size = 97, normalized size = 1.09

$$\frac{a\left(\cos^{3}\left(dx+c\right)\right)\sin\left(dx+c\right)}{4d}+\frac{3a\cos\left(dx+c\right)\sin\left(dx+c\right)}{8d}+\frac{3ax}{8}+\frac{3ca}{8d}+\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{3d}+\frac{a\cos\left(dx+c\right)}{d}+\frac{a\ln(c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4*a*cos(d*x+c)^3*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+3/8*a*x+3/8/d
*c*a+1/3*a*cos(d*x+c)^3/d+a*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.31, size = 81, normalized size = 0.91

$$\frac{16\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)a+3(12\,dx+12\,c+\sin}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(16*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*a)/d
```

mupad [B] time = 10.19, size = 245, normalized size = 2.75

$$\frac{-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{20\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{5\,a\tan\left(\frac{c}{2}+\cdots\right.}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] ((8*a)/3 + (5*a*tan(c/2 + (d*x)/2))/4 + (20*a*tan(c/2 + (d*x)/2)^2)/3 - (3*
a*tan(c/2 + (d*x)/2)^3)/4 + 8*a*tan(c/2 + (d*x)/2)^4 + (3*a*tan(c/2 + (d*x)
/2)^5)/4 + 4*a*tan(c/2 + (d*x)/2)^6 - (5*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(4*t
an(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan
(c/2 + (d*x)/2)^8 + 1)) + (a*log(tan(c/2 + (d*x)/2)))/d + (3*a*atan((9*a^2)
/(16*((3*a^2)/2 - (9*a^2*tan(c/2 + (d*x)/2))/16)) + (3*a^2*tan(c/2 + (d*x)/
2))/(2*((3*a^2)/2 - (9*a^2*tan(c/2 + (d*x)/2))/16))))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^4(c+dx)\csc(c+dx)\,dx+\int\sin(c+dx)\cos^4(c+dx)\csc(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**4*csc(c + d*x), x) + Integral(sin(c + d*x)*cos(c
+ d*x)**4*csc(c + d*x), x))
```

3.370 $\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=83

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{3ax}{2}$$

```
[Out] -3/2*a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d-3/2*a*
cot(d*x+c)/d+1/2*a*cos(d*x+c)^2*cot(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2838, 2591, 288, 321, 203, 2592, 302, 206}

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{3ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*x)/2 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c +
d*x]^3)/(3*d) - (3*a*Cot[c + d*x])/(2*d) + (a*Cos[c + d*x]^2*Cot[c + d*x])/
(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^3(c+dx)\cot(c+dx)\,dx + a\int \cos^2(c+dx)\cot^2(c$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,d\right.}{d}$$

$$= \frac{a\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{a\,\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,d\right.}{d}$$

$$= \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{3a\cot(c+dx)}{2d} + \frac{a\cos^2(c}{}$$

$$= -\frac{3ax}{2} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c}{3d}$$

Mathematica [A] time = 0.43, size = 77, normalized size = 0.93

$$\frac{a\left(15\cos(c+dx)+\cos(3(c+dx))-3\left(\sin(2(c+dx))+4\cot(c+dx)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(15*Cos[c + d*x] + Cos[3*(c + d*x)] - 3*(6*c + 6*d*x + 4*Cot[c + d*x] +
4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c + d*x)/2]] + Sin[2*(c + d*x)])))/(12
*d)
```

fricas [A] time = 0.54, size = 107, normalized size = 1.29

$$\frac{3\,a\cos\left(dx+c\right)^3-3\,a\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\,a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,a\cos}{6\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*cos(d*x + c)^3 - 3*a*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*
a*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*a*cos(d*x + c) + (2*a*cos(d
*x + c)^3 - 9*a*d*x + 6*a*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.20, size = 142, normalized size = 1.71

$$-\frac{9\,(dx+c)a-6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+\cdots\right)}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)*a - 6*a*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + 3*(2*a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c) - 2*(3*a*t
an(1/2*d*x + 1/2*c)^5 + 12*a*tan(1/2*d*x + 1/2*c)^4 + 12*a*tan(1/2*d*x + 1/
2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) + 8*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.31, size = 119, normalized size = 1.43

$$\frac{a\left(\cos^{3}(dx+c)\right)}{3d}+\frac{a\cos(dx+c)}{d}+\frac{a\ln(\csc(dx+c)-\cot(dx+c))}{d}-\frac{a\left(\cos^{5}(dx+c)\right)}{d\sin(dx+c)}-\frac{a\left(\cos^{3}(dx+c)\right)\sin(dx+\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/3*a*cos(d*x+c)^3/d+a*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/d*a/s
in(d*x+c)*cos(d*x+c)^5-a*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a*cos(d*x+c)*sin(d*x
+c)/d-3/2*a*x-3/2/d*c*a
```

maxima [A] time = 0.43, size = 90, normalized size = 1.08

$$\frac{\left(2\,\cos(dx+c)^{3}+6\,\cos(dx+c)-3\,\log(\cos(dx+c)+1)+3\,\log(\cos(dx+c)-1)\right)a-3\left(3\,dx+3\,c+\frac{3\,\tan(\cdots}{\tan(dx+c)\cdots}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*log(c
os(d*x + c) - 1))*a - 3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)
^3 + tan(d*x + c)))*a)/d
```

mupad [B] time = 8.77, size = 244, normalized size = 2.94

$$\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+8\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+8\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{16\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}}{d\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] ((16*a*tan(c/2 + (d*x)/2))/3 - a - 5*a*tan(c/2 + (d*x)/2)^2 + 8*a*tan(c/2 +
 (d*x)/2)^3 - 3*a*tan(c/2 + (d*x)/2)^4 + 8*a*tan(c/2 + (d*x)/2)^5 + a*tan(c
/2 + (d*x)/2)^6)/(d*(2*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3 + 6*tan(
c/2 + (d*x)/2)^5 + 2*tan(c/2 + (d*x)/2)^7)) + (a*tan(c/2 + (d*x)/2))/(2*d)
+ (a*log(tan(c/2 + (d*x)/2)))/d + (3*a*atan((9*a^2)/(6*a^2 + 9*a^2*tan(c/2
+ (d*x)/2)) - (6*a^2*tan(c/2 + (d*x)/2))/(6*a^2 + 9*a^2*tan(c/2 + (d*x)/2))
))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.371 $\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=94

$$-\frac{3a\cos(c+dx)}{2d}-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}$$

```
[Out] -3/2*a*x+3/2*a*arctanh(cos(d*x+c))/d-3/2*a*cos(d*x+c)/d-3/2*a*cot(d*x+c)/d+
1/2*a*cos(d*x+c)^2*cot(d*x+c)/d-1/2*a*cos(d*x+c)*cot(d*x+c)^2/d
```

Rubi [A] time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2838, 2592, 288, 321, 206, 2591, 203}

$$-\frac{3a\cos(c+dx)}{2d}-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*x)/2 + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) -
 (3*a*Cot[c + d*x])/(2*d) + (a*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (a*Cos[
c + d*x]*Cot[c + d*x]^2)/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^2(c+dx)\cot^2(c+dx)\,dx + a\int \cos(c+dx)\cot^3(c+$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\right.}{d}$$

$$= \frac{a\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d} + \frac{(3}{}$$

$$= -\frac{3a\cos(c+dx)}{2d} - \frac{3a\cot(c+dx)}{2d} + \frac{a\cos^2(c+dx)\cot(c+d}{2d}$$

$$= -\frac{3ax}{2} + \frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3a\cos(c+dx)}{2d} - \frac{3a\cot(c}{2d}$$

Mathematica [A] time = 0.80, size = 94, normalized size = 1.00

$$-\frac{a\left(2\sin(2(c+dx))+8\cos(c+dx)+8\cot(c+dx)+\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2\left(\frac{1}{2}(c+dx)\right)+12\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*(12*c + 12*d*x + 8*Cos[c + d*x] + 8*Cot[c + d*x] + Csc[(c + d*x)/2]
^2 - 12*Log[Cos[(c + d*x)/2]] + 12*Log[Sin[(c + d*x)/2]] - Sec[(c + d*x)/2]
^2 + 2*Sin[2*(c + d*x)]))/d
```

fricas [A] time = 0.52, size = 139, normalized size = 1.48

$$-\frac{6\,adx\cos\left(dx+c\right)^2+4\,a\cos\left(dx+c\right)^3-6\,adx-6\,a\cos\left(dx+c\right)-3\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right.}{4\left(d\cos\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*d*x*cos(d*x + c)^2 + 4*a*cos(d*x + c)^3 - 6*a*d*x - 6*a*cos(d*x +
 c) - 3*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 3*(a*cos(d*x +
 c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 2*(a*cos(d*x + c)^3 - 3*a*cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.22, size = 163, normalized size = 1.73

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,(dx+c)a-12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+4\,a\tan\left(\frac{1}{2}\,dx\right.}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 - 12*(d*x + c)*a - 12*a*log(abs(tan(1/2*d*x +
 1/2*c))) + 4*a*tan(1/2*d*x + 1/2*c) + (6*a*tan(1/2*d*x + 1/2*c)^6 + 4*a*ta
n(1/2*d*x + 1/2*c)^5 - 5*a*tan(1/2*d*x + 1/2*c)^4 - 16*a*tan(1/2*d*x + 1/2*
c)^3 - 12*a*tan(1/2*d*x + 1/2*c)^2 - 4*a*tan(1/2*d*x + 1/2*c) - a)/(tan(1/2
*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2)/d
```

maple [A] time = 0.37, size = 143, normalized size = 1.52

$$-\frac{a\left(\cos^5(dx+c)\right)}{d\sin(dx+c)}-\frac{a\left(\cos^3(dx+c)\right)\sin(dx+c)}{d}-\frac{3a\cos(dx+c)\sin(dx+c)}{2d}-\frac{3ax}{2}-\frac{3ca}{2d}-\frac{a\left(\cos^5(dx+c)\right)}{2d\sin(dx+c)^2}-\frac{a(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^5-a*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a*cos(d*x+c)
*sin(d*x+c)/d-3/2*a*x-3/2/d*c*a-1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^5-1/2*a*cos
(d*x+c)^3/d-3/2*a*cos(d*x+c)/d-3/2/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.45, size = 101, normalized size = 1.07

$$-\frac{2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a-a\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos(dx+c)+3\,\log(\cos(dx+c)+1)-3\,\log(\cos(dx\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*a - a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos
(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.71, size = 239, normalized size = 2.54

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{17\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) - (a/2 + 2*a*tan(c/2 + (d*x)/2) + 9*a*tan(c/2
+ (d*x)/2)^2 + 8*a*tan(c/2 + (d*x)/2)^3 + (17*a*tan(c/2 + (d*x)/2)^4)/2 - 2
*a*tan(c/2 + (d*x)/2)^5)/(d*(4*tan(c/2 + (d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^
4 + 4*tan(c/2 + (d*x)/2)^6)) + (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*a*log(ta
n(c/2 + (d*x)/2)))/(2*d) - (3*a*atan((9*a^2)/(9*a^2 - 9*a^2*tan(c/2 + (d*x)
/2)) + (9*a^2*tan(c/2 + (d*x)/2))/(9*a^2 - 9*a^2*tan(c/2 + (d*x)/2))))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.372 $\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=82

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}+ax$$

```
[Out] a*x+3/2*a*arctanh(cos(d*x+c))/d-3/2*a*cos(d*x+c)/d+a*cot(d*x+c)/d-1/2*a*cos
(d*x+c)*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d
```

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2710, 2592, 288, 321, 206, 3473, 8}

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] a*x + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (a*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx = \int \left(a\cos(c+dx)\cot^3(c+dx)+a\cot^4(c+dx)\right)\,dx$$

$$= a\int \cos(c+dx)\cot^3(c+dx)\,dx + a\int \cot^4(c+dx)\,dx$$

$$= -\frac{a\cot^3(c+dx)}{3d} - a\int \cot^2(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx, x, \cos(c+\right.}{d}$$

$$= \frac{a\cot(c+dx)}{d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} + a\int 1\,dx +$$

$$= ax - \frac{3a\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot}{}$$

$$= ax + \frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3a\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{a\cos(c}{}$$

Mathematica [C] time = 0.05, size = 125, normalized size = 1.52

$$-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} - \frac{a\cos(c+dx)}{d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{3a\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*Hy
pergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*a*Log[Cos[(c +
d*x)/2]])/(2*d) - (3*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2
)/(8*d)
```

fricas [B] time = 0.52, size = 160, normalized size = 1.95

$$\frac{16\,a\cos\left(dx+c\right)^3 + 9\left(a\cos\left(dx+c\right)^2 - a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right) - 9\left(a\cos\left(dx+c\right)^2 - a\right)\log\left(-\frac{1}{2}\right.}{12\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(16*a*cos(d*x + c)^3 + 9*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) +
 1/2)*sin(d*x + c) - 9*(a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2)*
```

```
sin(d*x + c) - 12*a*cos(d*x + c) + 6*(2*a*d*x*cos(d*x + c)^2 - 2*a*cos(d*x
+ c)^3 - 2*a*d*x + 3*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*
sin(d*x + c))
```

giac [A] time = 0.19, size = 141, normalized size = 1.72

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,(dx+c)a-36\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*
a - 36*a*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*a/
(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*a*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*
d*x + 1/2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.22, size = 106, normalized size = 1.29

$$-\frac{a\left(\cos^{5}(dx+c)\right)}{2d\sin(dx+c)^{2}}-\frac{a\left(\cos^{3}(dx+c)\right)}{2d}-\frac{3a\cos(dx+c)}{2d}-\frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{a\left(\cot^{3}(dx+c)\right)}{3d}+\frac{a\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^5-1/2*a*cos(d*x+c)^3/d-3/2*a*cos(d*x+c)/d-
3/2/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/3*a*cot(d*x+c)^3/d+a*cot(d*x+c)/d+a*x+1
/d*c*a
```

maxima [A] time = 0.44, size = 92, normalized size = 1.12

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^{2}-1}{\tan(dx+c)^{3}}\right)a+3\,a\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^{2}-1}-4\,\cos(dx+c)+3\,\log(\cos(dx+c)+1)-3\,\log(\cos(dx+c)-\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + 3*a*(2*co
s(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
- 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.70, size = 228, normalized size = 2.78

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+17\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{3}}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a/3 + a*tan(c/2 + (d*x)/2) - (14*a*tan(c/
2 + (d*x)/2)^2)/3 + 17*a*tan(c/2 + (d*x)/2)^3 - 5*a*tan(c/2 + (d*x)/2)^4)/(
d*(8*tan(c/2 + (d*x)/2)^3 + 8*tan(c/2 + (d*x)/2)^5)) - (5*a*tan(c/2 + (d*x)
/2))/(8*d) + (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (3*a*log(tan(c/2 + (d*x)/2))
)/(2*d) - (2*a*atan((4*a^2)/(6*a^2 + 4*a^2*tan(c/2 + (d*x)/2)) - (6*a^2*tan
(c/2 + (d*x)/2))/(6*a^2 + 4*a^2*tan(c/2 + (d*x)/2))))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.373 $\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=88

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}+$$

```
[Out] a*x-3/8*a*arctanh(cos(d*x+c))/d+a*cot(d*x+c)/d-1/3*a*cot(d*x+c)^3/d+3/8*a*c
ot(d*x+c)*csc(d*x+c)/d-1/4*a*cot(d*x+c)^3*csc(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2838, 2611, 3770, 3473, 8}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] a*x - (3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (a*Cot[c + d*x])/d - (a*Cot[c + d
*x]^3)/(3*d) + (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]^3*Cs
c[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^4(c+dx)\,dx + a\int \cot^4(c+dx)\csc(c+dx)\,dx$$
$$= -\frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{1}{4}(3a)\int \cot^2$$
$$= \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} + \frac{3a\cot(c+dx)\csc(c+dx)}{8d}$$
$$= ax - \frac{3a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx}{3d}$$

Mathematica [C] time = 0.05, size = 153, normalized size = 1.74

$$-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (a*Cot[c
+ d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) - (3*a*Lo
g[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) - (5*a*Sec[(
c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

fricas [B] time = 0.53, size = 180, normalized size = 2.05

$$\frac{48\,adx\cos(dx+c)^4 - 96\,adx\cos(dx+c)^2 - 30\,a\cos(dx+c)^3 + 48\,adx + 18\,a\cos(dx+c) - 9\left(a\cos(dx+c)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(48*a*d*x*cos(d*x + c)^4 - 96*a*d*x*cos(d*x + c)^2 - 30*a*cos(d*x + c)
^3 + 48*a*d*x + 18*a*cos(d*x + c) - 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^
2 + a)*log(1/2*cos(d*x + c) + 1/2) + 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)
^2 + a)*log(-1/2*cos(d*x + c) + 1/2) - 16*(4*a*cos(d*x + c)^3 - 3*a*cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.22, size = 153, normalized size = 1.74

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+8\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+192\,(dx+c)a+72\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*a*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1
/2*d*x + 1/2*c)^2 + 192*(d*x + c)*a + 72*a*log(abs(tan(1/2*d*x + 1/2*c))) -
 120*a*tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 - 120*a*tan(1/2
*d*x + 1/2*c)^3 - 24*a*tan(1/2*d*x + 1/2*c)^2 + 8*a*tan(1/2*d*x + 1/2*c) +
3*a)/tan(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.22, size = 128, normalized size = 1.45

$$-\frac{a\left(\cot^3(dx+c)\right)}{3d}+\frac{a\cot(dx+c)}{d}+ax+\frac{ca}{d}-\frac{a\left(\cos^5(dx+c)\right)}{4d\sin(dx+c)^4}+\frac{a\left(\cos^5(dx+c)\right)}{8d\sin(dx+c)^2}+\frac{a\left(\cos^3(dx+c)\right)}{8d}+\frac{3a\cos(dx+}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3*a*cot(d*x+c)^3/d+a*cot(d*x+c)/d+a*x+1/d*c*a-1/4/d*a/sin(d*x+c)^4*cos(d
*x+c)^5+1/8/d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/8*a*cos(d*x+c)^3/d+3/8*a*cos(d*
x+c)/d+3/8/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.42, size = 107, normalized size = 1.22

$$\frac{16\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a-3\,a\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a - 3*a*(2*(
5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)
+ 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.92, size = 217, normalized size = 2.47

$$\frac{2\,a\,\mathrm{atan}\left(\frac{8\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-8\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{8\,d}+\frac{5\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{a\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (2*a*atan((8*cos(c/2 + (d*x)/2) + 3*sin(c/2 + (d*x)/2))/(3*cos(c/2 + (d*x)/
2) - 8*sin(c/2 + (d*x)/2))))/d + (3*a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)))/(8*d) + (5*a*cot(c/2 + (d*x)/2))/(8*d) - (5*a*tan(c/2 + (d*x)/2))/(8
*d) + (a*cot(c/2 + (d*x)/2)^2)/(8*d) - (a*cot(c/2 + (d*x)/2)^3)/(24*d) - (a
*cot(c/2 + (d*x)/2)^4)/(64*d) - (a*tan(c/2 + (d*x)/2)^2)/(8*d) + (a*tan(c/2
 + (d*x)/2)^3)/(24*d) + (a*tan(c/2 + (d*x)/2)^4)/(64*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.374 $\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] -3/8*a*arctanh(cos(d*x+c))/d-1/5*a*cot(d*x+c)^5/d+3/8*a*cot(d*x+c)*csc(d*x+
c)/d-1/4*a*cot(d*x+c)^3*csc(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2838, 2607, 30, 2611, 3770}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^5)/(5*d) + (3*a*Cot[c
+ d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]^3*Csc[c + d*x])/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc(c+dx)\,dx + a\int \cot^4(c+dx)\csc^2(c+$$

$$= -\frac{a\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{1}{4}(3a)\int \cot^2(c+dx)\csc(c+$$

$$= -\frac{a\cot^5(c+dx)}{5d} + \frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot^3(c+d}{}$$

$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^5(c+dx)}{5d} + \frac{3a\cot(c+dx)}{8d}$$

Mathematica [A] time = 0.04, size = 135, normalized size = 1.82

$$-\frac{a\cot^5(c+dx)}{5d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3a\log\left(\sin\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/5*(a*Cot[c + d*x]^5)/d + (5*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d
*x)/2]^4)/(64*d) - (3*a*Log[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*
x)/2]])/(8*d) - (5*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(6
4*d)
```

fricas [B] time = 0.51, size = 160, normalized size = 2.16

$$-\frac{16\,a\cos(dx+c)^5 + 15\left(a\cos(dx+c)^4 - 2\,a\cos(dx+c)^2 + a\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - 15\left(a\cos\right.}{80\left(d\cos(dx+c)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/80*(16*a*cos(d*x + c)^5 + 15*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 10*(5*a*cos(d*x + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))

giac [B] time = 0.27, size = 173, normalized size = 2.34

$$\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-40\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+120\,a\log\left(\left|\tan\left(\frac{1}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/320*(2*a*tan(1/2*d*x + 1/2*c)^5 + 5*a*tan(1/2*d*x + 1/2*c)^4 - 10*a*tan(1/2*d*x + 1/2*c)^3 - 40*a*tan(1/2*d*x + 1/2*c)^2 + 120*a*log(abs(tan(1/2*d*x + 1/2*c))) + 20*a*tan(1/2*d*x + 1/2*c) - (274*a*tan(1/2*d*x + 1/2*c)^5 + 20*a*tan(1/2*d*x + 1/2*c)^4 - 40*a*tan(1/2*d*x + 1/2*c)^3 - 10*a*tan(1/2*d*x + 1/2*c)^2 + 5*a*tan(1/2*d*x + 1/2*c) + 2*a)/tan(1/2*d*x + 1/2*c)^5)/d

maple [A] time = 0.22, size = 116, normalized size = 1.57

$$-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{4d\sin\left(dx+c\right)^{4}}+\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^{2}}+\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{8d}+\frac{3a\cos\left(dx+c\right)}{8d}+\frac{3a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{8d}-\frac{a\left(\right.}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c)),x)

[Out] -1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^5+1/8/d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/8*a*cos(d*x+c)^3/d+3/8*a*cos(d*x+c)/d+3/8/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^5

maxima [A] time = 0.35, size = 86, normalized size = 1.16

$$-\frac{5\,a\left(\frac{2\left(5\,\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\,a}{\tan\left(dx+c\right)^{5}}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")

```
[Out] -1/80*(5*a*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 16*a/
tan(d*x + c)^5)/d
```

mupad [B] time = 10.43, size = 289, normalized size = 3.91

$$\frac{a\left(2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-10\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (a*(2*sin(c/2 + (d*x)/2)^10 - 2*cos(c/2 + (d*x)/2)^10 + 5*cos(c/2 + (d*x)/2
)*sin(c/2 + (d*x)/2)^9 - 5*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) - 10*cos
(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 - 40*cos(c/2 + (d*x)/2)^3*sin(c/2 +
(d*x)/2)^7 + 20*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 - 20*cos(c/2 + (d
*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 40*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^
3 + 10*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 120*log(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5))/(320*d*c
os(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.375 $\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=98

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{a\cot(c+dx)\text{cs}}{16d}$$

```
[Out] -1/16*a*arctanh(cos(d*x+c))/d-1/5*a*cot(d*x+c)^5/d-1/16*a*cot(d*x+c)*csc(d*
x+c)/d+1/8*a*cot(d*x+c)*csc(d*x+c)^3/d-1/6*a*cot(d*x+c)^3*csc(d*x+c)^3/d
```

Rubi [A] time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 30}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{a\cot(c+dx)\text{cs}}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]*Csc[c + d*x])/(16*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^3)/(6*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^2(c+dx)\,dx + a\int \cot^4(c+dx)\csc^3(c$$

$$= -\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{1}{2}a\int \cot^2(c+dx)\csc^3(c+$$

$$= -\frac{a\cot^5(c+dx)}{5d} + \frac{a\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a\cot^3(c+d}{}$$

$$= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot(c+dx)\csc(c+dx)}{16d} + \frac{a\cot(c+dx)}{8}$$

$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\cot(c+dx)\csc}{16d}$$

Mathematica [A] time = 0.04, size = 175, normalized size = 1.79

$$-\frac{a\cot^5(c+dx)}{5d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\sec^4\left(\frac{1}{2}(c+\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/5*(a*Cot[c + d*x]^5)/d - (a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c + d*x
)/2]^4)/(64*d) - (a*Csc[(c + d*x)/2]^6)/(384*d) - (a*Log[Cos[(c + d*x)/2]])
```

```
/(16*d) + (a*Log[Sin[(c + d*x)/2]])/(16*d) + (a*Sec[(c + d*x)/2]^2)/(64*d)
- (a*Sec[(c + d*x)/2]^4)/(64*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

fricas [B] time = 0.50, size = 187, normalized size = 1.91

$$\frac{96\,a\cos\left(dx+c\right)^{5}\sin\left(dx+c\right)+30\,a\cos\left(dx+c\right)^{5}+80\,a\cos\left(dx+c\right)^{3}-30\,a\cos\left(dx+c\right)-15\left(a\cos\left(dx+c\right)^{6}-\right.}{480\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/480*(96*a*cos(d*x + c)^5*sin(d*x + c) + 30*a*cos(d*x + c)^5 + 80*a*cos(d*
x + c)^3 - 30*a*cos(d*x + c) - 15*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 +
3*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 15*(a*cos(d*x + c)^6
- 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2)
)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [B] time = 0.22, size = 201, normalized size = 2.05

$$\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-60\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a\tan\left(\frac{1}{2}\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*a*tan(1/2*d*x + 1/2*c)^6 + 12*a*tan(1/2*d*x + 1/2*c)^5 - 15*a*tan
(1/2*d*x + 1/2*c)^4 - 60*a*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2*
c)^2 + 120*a*log(abs(tan(1/2*d*x + 1/2*c))) + 120*a*tan(1/2*d*x + 1/2*c) -
(294*a*tan(1/2*d*x + 1/2*c)^6 + 120*a*tan(1/2*d*x + 1/2*c)^5 - 15*a*tan(1/2
*d*x + 1/2*c)^4 - 60*a*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2*c)^2
 + 12*a*tan(1/2*d*x + 1/2*c) + 5*a)/tan(1/2*d*x + 1/2*c)^6)/d
```

maple [A] time = 0.25, size = 138, normalized size = 1.41

$$-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)^{5}}-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{6d\sin\left(dx+c\right)^{6}}-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{24d\sin\left(dx+c\right)^{4}}+\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{48d\sin\left(dx+c\right)^{2}}+\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{48d}+\frac{a\cos\left(dx+c\right)}{16d}+\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^5-1/6/d*a/sin(d*x+c)^6*cos(d*x+c)^5-1/24/d
*a/sin(d*x+c)^4*cos(d*x+c)^5+1/48/d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/48*a*cos(
d*x+c)^3/d+1/16*a*cos(d*x+c)/d+1/16/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.33, size = 106, normalized size = 1.08

$$\frac{5\,a\left(\frac{2\left(3\,\cos\left(dx+c\right)^{5}+8\,\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{6}-3\,\cos\left(dx+c\right)^{4}+3\,\cos\left(dx+c\right)^{2}-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{96\,a}{\tan\left(dx+c\right)^{5}}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(5*a*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d
*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) +
 1) + 3*log(cos(d*x + c) - 1)) - 96*a/tan(d*x + c)^5)/d
```

mupad [B] time = 9.51, size = 337, normalized size = 3.44

$$\frac{a\left(5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+12\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-15\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^7,x)
```

```
[Out] (a*(5*sin(c/2 + (d*x)/2)^12 - 5*cos(c/2 + (d*x)/2)^12 + 12*cos(c/2 + (d*x)/
2)*sin(c/2 + (d*x)/2)^11 - 12*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) - 15
*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 60*cos(c/2 + (d*x)/2)^3*sin(c
/2 + (d*x)/2)^9 - 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 120*cos(c/
2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 120*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d
*x)/2)^5 + 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 60*cos(c/2 + (d*x
)/2)^9*sin(c/2 + (d*x)/2)^3 + 15*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2
 + 120*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*sin(
c/2 + (d*x)/2)^6))/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.376 $\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=114

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-$$

```
[Out] -1/16*a*arctanh(cos(d*x+c))/d-1/5*a*cot(d*x+c)^5/d-1/7*a*cot(d*x+c)^7/d-1/1
6*a*cot(d*x+c)*csc(d*x+c)/d+1/8*a*cot(d*x+c)*csc(d*x+c)^3/d-1/6*a*cot(d*x+c
)^3*csc(d*x+c)^3/d
```

Rubi [A] time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2607, 14, 2611, 3768, 3770}

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]^7)/(7*d) - (a*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (a*Cot[c + d*x]*Csc[c
 + d*x]^3)/(8*d) - (a*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^3(c+dx)\,dx + a\int \cot^4(c+dx)\csc^4(c+$$

$$= -\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{1}{2}a\int \cot^2(c+dx)\csc^3(c+$$

$$= \frac{a\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d} + \frac{1}{8}$$

$$= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\cot(c+dx)\csc(c+dx}{16d}$$

$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} -$$

Mathematica [B] time = 0.08, size = 239, normalized size = 2.10

$$-\frac{2a\cot(c+dx)}{35d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\sec^4\left(\frac{1}{2}(c+\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(35*d) - (a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c + d*
x)/2]^4)/(64*d) - (a*Csc[(c + d*x)/2]^6)/(384*d) - (a*Cot[c + d*x]*Csc[c +
d*x]^2)/(35*d) + (8*a*Cot[c + d*x]*Csc[c + d*x]^4)/(35*d) - (a*Cot[c + d*x]
*Csc[c + d*x]^6)/(7*d) - (a*Log[Cos[(c + d*x)/2]])/(16*d) + (a*Log[Sin[(c +
 d*x)/2]])/(16*d) + (a*Sec[(c + d*x)/2]^2)/(64*d) - (a*Sec[(c + d*x)/2]^4)/
(64*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

fricas [B] time = 0.53, size = 221, normalized size = 1.94

$$-\frac{192\,a\cos\left(dx+c\right)^{7}-672\,a\cos\left(dx+c\right)^{5}+105\left(a\cos\left(dx+c\right)^{6}-3\,a\cos\left(dx+c\right)^{4}+3\,a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3360*(192*a*cos(d*x + c)^7 - 672*a*cos(d*x + c)^5 + 105*(a*cos(d*x + c)^
6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) - 105*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x +
 c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 70*(3*a*cos(d*x + c)
^5 + 8*a*cos(d*x + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^
6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [B] time = 0.22, size = 229, normalized size = 2.01

$$\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-105\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-105\,a\tan\left(\frac{1}{2}\,dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*a*tan(1/2*d*x + 1/2*c)^7 + 35*a*tan(1/2*d*x + 1/2*c)^6 - 21*a*t
an(1/2*d*x + 1/2*c)^5 - 105*a*tan(1/2*d*x + 1/2*c)^4 - 105*a*tan(1/2*d*x +
1/2*c)^3 - 105*a*tan(1/2*d*x + 1/2*c)^2 + 840*a*log(abs(tan(1/2*d*x + 1/2*c
))) + 315*a*tan(1/2*d*x + 1/2*c) - (2178*a*tan(1/2*d*x + 1/2*c)^7 + 315*a*t
an(1/2*d*x + 1/2*c)^6 - 105*a*tan(1/2*d*x + 1/2*c)^5 - 105*a*tan(1/2*d*x +
1/2*c)^4 - 105*a*tan(1/2*d*x + 1/2*c)^3 - 21*a*tan(1/2*d*x + 1/2*c)^2 + 35*
a*tan(1/2*d*x + 1/2*c) + 15*a)/tan(1/2*d*x + 1/2*c)^7)/d
```

maple [A] time = 0.27, size = 160, normalized size = 1.40

$$-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{6d\sin\left(dx+c\right)^{6}}-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{24d\sin\left(dx+c\right)^{4}}+\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{48d\sin\left(dx+c\right)^{2}}+\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{48d}+\frac{a\cos\left(dx+c\right)}{16d}+\frac{a\ln\left(\csc\left(dx+c\right)-\mathrm{c}\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/6/d*a/sin(d*x+c)^6*cos(d*x+c)^5-1/24/d*a/sin(d*x+c)^4*cos(d*x+c)^5+1/48/
d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/48*a*cos(d*x+c)^3/d+1/16*a*cos(d*x+c)/d+1/1
6/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/7/d*a/sin(d*x+c)^7*cos(d*x+c)^5-2/35/d*a/
sin(d*x+c)^5*cos(d*x+c)^5
```

maxima [A] time = 0.32, size = 118, normalized size = 1.04

$$\frac{35\,a\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{96\left(7\,\tan(dx+c)^2+5\right)}{\tan(dx+c)^7}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3360*(35*a*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
 + 1) + 3*log(cos(d*x + c) - 1)) - 96*(7*tan(d*x + c)^2 + 5)*a/tan(d*x + c)
^7)/d
```

mupad [B] time = 9.95, size = 385, normalized size = 3.38

$$\frac{a\left(15\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-15\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+35\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-35\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^8,x)
```

```
[Out] (a*(15*sin(c/2 + (d*x)/2)^14 - 15*cos(c/2 + (d*x)/2)^14 + 35*cos(c/2 + (d*x
)/2)*sin(c/2 + (d*x)/2)^13 - 35*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) -
21*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 105*cos(c/2 + (d*x)/2)^3*si
n(c/2 + (d*x)/2)^11 - 105*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 - 105*
cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 + 315*cos(c/2 + (d*x)/2)^6*sin(c/
2 + (d*x)/2)^8 - 315*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 105*cos(c/
2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5 + 105*cos(c/2 + (d*x)/2)^10*sin(c/2 + (
d*x)/2)^4 + 105*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 21*cos(c/2 + (
d*x)/2)^12*sin(c/2 + (d*x)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2))*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7))/(13440*d*cos(c/2 + (d*x)/
2)^7*sin(c/2 + (d*x)/2)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.377 $\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=136

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d}+\frac{a\cot(c+dx)\csc^5(c+dx}{16d}$$

```
[Out] -3/128*a*arctanh(cos(d*x+c))/d-1/5*a*cot(d*x+c)^5/d-1/7*a*cot(d*x+c)^7/d-3/
128*a*cot(d*x+c)*csc(d*x+c)/d-1/64*a*cot(d*x+c)*csc(d*x+c)^3/d+1/16*a*cot(d
*x+c)*csc(d*x+c)^5/d-1/8*a*cot(d*x+c)^3*csc(d*x+c)^5/d
```

Rubi [A] time = 0.17, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 14}

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d}+\frac{a\cot(c+dx)\csc^5(c+dx}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(128*d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c
+ d*x]^7)/(7*d) - (3*a*Cot[c + d*x]*Csc[c + d*x])/(128*d) - (a*Cot[c + d*x]
*Csc[c + d*x]^3)/(64*d) + (a*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^5)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^4(c+dx)\,dx + a\int \cot^4(c+dx)\csc^5(c+$$

$$= -\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{1}{8}(3a)\int \cot^2(c+dx)\csc^5(c+$$

$$= \frac{a\cot(c+dx)\csc^5(c+dx)}{16d} - \frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d} + \frac{1}{16}$$

$$= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{64d}$$

$$= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{3a\cot(c+dx)\csc(c+dx)}{128d}$$

$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{128d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} -$$

Mathematica [B] time = 0.08, size = 279, normalized size = 2.05

$$-\frac{2a\cot(c+dx)}{35d} - \frac{a\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d} + \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{512d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d} - \frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{512d} + \frac{a\sec^8\left(\frac{1}{2}(c+a\right.}{2048d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(35*d) - (3*a*Csc[(c + d*x)/2]^2)/(512*d) + (a*Csc[(c +
 d*x)/2]^4)/(1024*d) + (a*Csc[(c + d*x)/2]^6)/(512*d) - (a*Csc[(c + d*x)/2]
^8)/(2048*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(35*d) + (8*a*Cot[c + d*x]*C
sc[c + d*x]^4)/(35*d) - (a*Cot[c + d*x]*Csc[c + d*x]^6)/(7*d) - (3*a*Log[Co
s[(c + d*x)/2]])/(128*d) + (3*a*Log[Sin[(c + d*x)/2]])/(128*d) + (3*a*Sec[(
c + d*x)/2]^2)/(512*d) - (a*Sec[(c + d*x)/2]^4)/(1024*d) - (a*Sec[(c + d*x)
/2]^6)/(512*d) + (a*Sec[(c + d*x)/2]^8)/(2048*d)
```

fricas [A] time = 0.58, size = 239, normalized size = 1.76

$$\frac{210\,a\cos\left(dx+c\right)^{7} - 770\,a\cos\left(dx+c\right)^{5} - 770\,a\cos\left(dx+c\right)^{3} + 210\,a\cos\left(dx+c\right) - 105\,\left(a\cos\left(dx+c\right)^{8} - 4\,a\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8960*(210*a*cos(d*x + c)^7 - 770*a*cos(d*x + c)^5 - 770*a*cos(d*x + c)^3
+ 210*a*cos(d*x + c) - 105*(a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos
(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 105*(a*
cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)
^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 256*(2*a*cos(d*x + c)^7 - 7*a*cos(d*
x + c)^5)*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*
x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.24, size = 201, normalized size = 1.48

$$\frac{35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8} + 80\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} - 112\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} - 280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4} - 560\,a\tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/71680*(35*a*tan(1/2*d*x + 1/2*c)^8 + 80*a*tan(1/2*d*x + 1/2*c)^7 - 112*a*
tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^4 - 560*a*tan(1/2*d*x +
 1/2*c)^3 + 1680*a*log(abs(tan(1/2*d*x + 1/2*c))) + 1680*a*tan(1/2*d*x + 1/
2*c) - (4566*a*tan(1/2*d*x + 1/2*c)^8 + 1680*a*tan(1/2*d*x + 1/2*c)^7 - 560
*a*tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^4 - 112*a*tan(1/2*d*
x + 1/2*c)^3 + 80*a*tan(1/2*d*x + 1/2*c) + 35*a)/tan(1/2*d*x + 1/2*c)^8)/d
```

maple [A] time = 0.26, size = 182, normalized size = 1.34

$$-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{7d\sin\left(dx+c\right)^{7}}-\frac{2a\left(\cos^{5}\left(dx+c\right)\right)}{35d\sin\left(dx+c\right)^{5}}-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^{8}}-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{16d\sin\left(dx+c\right)^{6}}-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{64d\sin\left(dx+c\right)^{4}}+\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{128d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/7/d*a/sin(d*x+c)^7*cos(d*x+c)^5-2/35/d*a/sin(d*x+c)^5*cos(d*x+c)^5-1/8/d
*a/sin(d*x+c)^8*cos(d*x+c)^5-1/16/d*a/sin(d*x+c)^6*cos(d*x+c)^5-1/64/d*a/si
n(d*x+c)^4*cos(d*x+c)^5+1/128/d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/128*a*cos(d*x
+c)^3/d+3/128*a*cos(d*x+c)/d+3/128/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.32, size = 138, normalized size = 1.01

$$\frac{35\,a\left(\frac{2\left(3\,\cos(dx+c)^{7}-11\,\cos(dx+c)^{5}-11\,\cos(dx+c)^{3}+3\,\cos(dx+c)\right)}{\cos(dx+c)^{8}-4\,\cos(dx+c)^{6}+6\,\cos(dx+c)^{4}-4\,\cos(dx+c)^{2}+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{256\,(7}{}}{8960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8960*(35*a*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3 +
 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*
cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) -
256*(7*tan(d*x + c)^2 + 5)*a/tan(d*x + c)^7)/d
```

mupad [B] time = 10.22, size = 337, normalized size = 2.48

$$\frac{a\left(35\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-35\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+80\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}-80\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^9,x)
```

```
[Out] (a*(35*sin(c/2 + (d*x)/2)^16 - 35*cos(c/2 + (d*x)/2)^16 + 80*cos(c/2 + (d*x
)/2)*sin(c/2 + (d*x)/2)^15 - 80*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2) -
112*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^13 - 280*cos(c/2 + (d*x)/2)^4*s
in(c/2 + (d*x)/2)^12 - 560*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^11 + 168
0*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^9 - 1680*cos(c/2 + (d*x)/2)^9*sin
```

```
(c/2 + (d*x)/2)^7 + 560*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^5 + 280*co
s(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 + 112*cos(c/2 + (d*x)/2)^13*sin(c/
2 + (d*x)/2)^3 + 1680*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 +
(d*x)/2)^8*sin(c/2 + (d*x)/2)^8))/(71680*d*cos(c/2 + (d*x)/2)^8*sin(c/2 + (
d*x)/2)^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.378 $\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=185

$$-\frac{2a^2\cos^9(c+dx)}{9d}+\frac{4a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{3a^2\sin^3(c+dx)\cos^5(c+dx)}{16d}$$

```
[Out] 9/256*a^2*x-2/5*a^2*cos(d*x+c)^5/d+4/7*a^2*cos(d*x+c)^7/d-2/9*a^2*cos(d*x+c
)^9/d+9/256*a^2*cos(d*x+c)*sin(d*x+c)/d+3/128*a^2*cos(d*x+c)^3*sin(d*x+c)/d
-3/32*a^2*cos(d*x+c)^5*sin(d*x+c)/d-3/16*a^2*cos(d*x+c)^5*sin(d*x+c)^3/d-1/
10*a^2*cos(d*x+c)^5*sin(d*x+c)^5/d
```

Rubi [A] time = 0.34, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2568, 2635, 8, 2565, 270}

$$-\frac{2a^2\cos^9(c+dx)}{9d}+\frac{4a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{3a^2\sin^3(c+dx)\cos^5(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (9*a^2*x)/256 - (2*a^2*Cos[c + d*x]^5)/(5*d) + (4*a^2*Cos[c + d*x]^7)/(7*d)
 - (2*a^2*Cos[c + d*x]^9)/(9*d) + (9*a^2*Cos[c + d*x]*Sin[c + d*x])/(256*d)
 + (3*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) - (3*a^2*Cos[c + d*x]^5*Sin[
c + d*x])/(32*d) - (3*a^2*Cos[c + d*x]^5*Sin[c + d*x]^3)/(16*d) - (a^2*Cos[
c + d*x]^5*Sin[c + d*x]^5)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)\sin^4(c+dx)+2a^2\cos^4(c+dx)\sin^5(c\right.$$

$$= a^2\int \cos^4(c+dx)\sin^4(c+dx)\,dx + a^2\int \cos^4(c+dx)\sin$$

$$= -\frac{a^2\cos^5(c+dx)\sin^3(c+dx)}{8d} - \frac{a^2\cos^5(c+dx)\sin^5(c+d}{10d}$$

$$= -\frac{a^2\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{3a^2\cos^5(c+dx)\sin^3(c+d}{16d}$$

$$= -\frac{2a^2\cos^5(c+dx)}{5d} + \frac{4a^2\cos^7(c+dx)}{7d} - \frac{2a^2\cos^9(c+dx)}{9d}$$

$$= -\frac{2a^2\cos^5(c+dx)}{5d} + \frac{4a^2\cos^7(c+dx)}{7d} - \frac{2a^2\cos^9(c+dx)}{9d}$$

$$= \frac{3a^2x}{128} - \frac{2a^2\cos^5(c+dx)}{5d} + \frac{4a^2\cos^7(c+dx)}{7d} - \frac{2a^2\cos^9(c}{9d}$$

$$= \frac{9a^2x}{256} - \frac{2a^2\cos^5(c+dx)}{5d} + \frac{4a^2\cos^7(c+dx)}{7d} - \frac{2a^2\cos^9(c}{9d}$$

Mathematica [A] time = 0.73, size = 116, normalized size = 0.63

$$\frac{a^2(-1260\sin(2(c+dx))-7560\sin(4(c+dx))+630\sin(6(c+dx))+945\sin(8(c+dx))-126\sin(10(c+dx))-\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(22680*c + 22680*d*x - 30240*Cos[c + d*x] - 6720*Cos[3*(c + d*x)] + 40
32*Cos[5*(c + d*x)] + 720*Cos[7*(c + d*x)] - 560*Cos[9*(c + d*x)] - 1260*Si
n[2*(c + d*x)] - 7560*Sin[4*(c + d*x)] + 630*Sin[6*(c + d*x)] + 945*Sin[8*(
c + d*x)] - 126*Sin[10*(c + d*x)]))/(645120*d)
```

fricas [A] time = 0.56, size = 124, normalized size = 0.67

$$-\frac{17920\,a^2\cos(dx+c)^9-46080\,a^2\cos(dx+c)^7+32256\,a^2\cos(dx+c)^5-2835\,a^2dx+63\left(128\,a^2\cos(dx+c)^9\ldots\right.}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/80640*(17920*a^2*cos(d*x + c)^9 - 46080*a^2*cos(d*x + c)^7 + 32256*a^2*c
os(d*x + c)^5 - 2835*a^2*d*x + 63*(128*a^2*cos(d*x + c)^9 - 496*a^2*cos(d*x
 + c)^7 + 488*a^2*cos(d*x + c)^5 - 30*a^2*cos(d*x + c)^3 - 45*a^2*cos(d*x +
 c))*sin(d*x + c))/d
```

giac [A] time = 0.32, size = 174, normalized size = 0.94

$$\frac{9}{256}\,a^2x-\frac{a^2\cos(9\,dx+9\,c)}{1152\,d}+\frac{a^2\cos(7\,dx+7\,c)}{896\,d}+\frac{a^2\cos(5\,dx+5\,c)}{160\,d}-\frac{a^2\cos(3\,dx+3\,c)}{96\,d}-\frac{3\,a^2\cos(dx+c)}{64\,d}-\frac{a^2\sin\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 9/256*a^2*x - 1/1152*a^2*cos(9*d*x + 9*c)/d + 1/896*a^2*cos(7*d*x + 7*c)/d
+ 1/160*a^2*cos(5*d*x + 5*c)/d - 1/96*a^2*cos(3*d*x + 3*c)/d - 3/64*a^2*cos
(d*x + c)/d - 1/5120*a^2*sin(10*d*x + 10*c)/d + 3/2048*a^2*sin(8*d*x + 8*c)
/d + 1/1024*a^2*sin(6*d*x + 6*c)/d - 3/256*a^2*sin(4*d*x + 4*c)/d - 1/512*a
^2*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.28, size = 218, normalized size = 1.18

$$\frac{a^2\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^5(dx+c)\right)}{10}-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{16}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{32}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{128}+\frac{3dx}{256}+\frac{3c}{256}\right)+\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1/
32*sin(d*x+c)*cos(d*x+c)^5+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3
/256*d*x+3/256*c)+2*a^2*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*c
os(d*x+c)^5-8/315*cos(d*x+c)^5)+a^2*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*si
n(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d
*x+3/128*c))
```

maxima [A] time = 0.37, size = 123, normalized size = 0.66

$$-\frac{4096\left(35\cos(dx+c)^9-90\cos(dx+c)^7+63\cos(dx+c)^5\right)a^2+63\left(32\sin(2dx+2c)^5-120dx-120c-5\right.}{645120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/645120*(4096*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a^2 + 63*(32*sin(2*d*x + 2*c)^5 - 120*d*x - 120*c - 5*sin(8*d*x + 8*c) + 4
0*sin(4*d*x + 4*c))*a^2 - 630*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d
*x + 4*c))*a^2)/d
```

mupad [B] time = 12.21, size = 469, normalized size = 2.54

$$\frac{9a^2x}{256}-\frac{\frac{9a^2(c+dx)}{256}+\frac{87a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{128}-\frac{553a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160}-\frac{491a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{32}+\frac{2555a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{64}-\frac{2555a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{64}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^4*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (9*a^2*x)/256 - ((9*a^2*(c + d*x))/256 + (87*a^2*tan(c/2 + (d*x)/2)^3)/128
- (553*a^2*tan(c/2 + (d*x)/2)^5)/160 - (491*a^2*tan(c/2 + (d*x)/2)^7)/32 +
(2555*a^2*tan(c/2 + (d*x)/2)^9)/64 - (2555*a^2*tan(c/2 + (d*x)/2)^11)/64 +
(491*a^2*tan(c/2 + (d*x)/2)^13)/32 + (553*a^2*tan(c/2 + (d*x)/2)^15)/160 -
(87*a^2*tan(c/2 + (d*x)/2)^17)/128 - (9*a^2*tan(c/2 + (d*x)/2)^19)/128 - (a
^2*(2835*c + 2835*d*x - 8192))/80640 + tan(c/2 + (d*x)/2)^2*((45*a^2*(c + d
*x))/128 - (a^2*(28350*c + 28350*d*x - 81920))/80640) + tan(c/2 + (d*x)/2)^
4*((405*a^2*(c + d*x))/256 - (a^2*(127575*c + 127575*d*x - 368640))/80640)
+ tan(c/2 + (d*x)/2)^6*((135*a^2*(c + d*x))/32 - (a^2*(340200*c + 340200*d*
```

```
x + 737280))/80640) + tan(c/2 + (d*x)/2)^12*((945*a^2*(c + d*x))/128 - (a^2
*(595350*c + 595350*d*x + 860160))/80640) + tan(c/2 + (d*x)/2)^14*((135*a^2
*(c + d*x))/32 - (a^2*(340200*c + 340200*d*x - 1720320))/80640) + tan(c/2 +
 (d*x)/2)^10*((567*a^2*(c + d*x))/64 - (a^2*(714420*c + 714420*d*x - 103219
2))/80640) + tan(c/2 + (d*x)/2)^8*((945*a^2*(c + d*x))/128 - (a^2*(595350*c
 + 595350*d*x - 2580480))/80640) + (9*a^2*tan(c/2 + (d*x)/2))/128)/(d*(tan(
c/2 + (d*x)/2)^2 + 1)^10)
```

sympy [A] time = 34.60, size = 554, normalized size = 2.99

$$\begin{cases} \frac{3a^2x\sin^{10}(c+dx)}{256} + \frac{15a^2x\sin^8(c+dx)\cos^2(c+dx)}{256} + \frac{3a^2x\sin^8(c+dx)}{128} + \frac{15a^2x\sin^6(c+dx)\cos^4(c+dx)}{128} + \frac{3a^2x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{15a^2 \\ x(a\sin(c)+a)^2\sin^4(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**10/256 + 15*a**2*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 3*a**2*x*sin(c + d*x)**8/128 + 15*a**2*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 3*a**2*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 15*a**2*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 9*a**2*x*sin(c + d*x)**4*cos(c + d*x
)**4/64 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 3*a**2*x*sin(c +
d*x)**2*cos(c + d*x)**6/32 + 3*a**2*x*cos(c + d*x)**10/256 + 3*a**2*x*cos(c
 + d*x)**8/128 + 3*a**2*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**2*sin(c
 + d*x)**7*cos(c + d*x)**3/(128*d) + 3*a**2*sin(c + d*x)**7*cos(c + d*x)/(1
28*d) - a**2*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 11*a**2*sin(c + d*x)*
*5*cos(c + d*x)**3/(128*d) - 2*a**2*sin(c + d*x)**4*cos(c + d*x)**5/(5*d) -
 7*a**2*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) - 11*a**2*sin(c + d*x)**3*c
os(c + d*x)**5/(128*d) - 8*a**2*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - 3*
a**2*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 3*a**2*sin(c + d*x)*cos(c + d*x
)**7/(128*d) - 16*a**2*cos(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a*sin(c) + a
)**2*sin(c)**4*cos(c)**4, True))
```

3.379 $\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=159

$$-\frac{a^2\cos^9(c+dx)}{9d}+\frac{3a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)\cos^5(c+dx)}{4d}-\frac{a^2\sin(c+dx)\cos^5(c+dx)}{8d}$$

```
[Out] 3/64*a^2*x-2/5*a^2*cos(d*x+c)^5/d+3/7*a^2*cos(d*x+c)^7/d-1/9*a^2*cos(d*x+c)
^9/d+3/64*a^2*cos(d*x+c)*sin(d*x+c)/d+1/32*a^2*cos(d*x+c)^3*sin(d*x+c)/d-1/
8*a^2*cos(d*x+c)^5*sin(d*x+c)/d-1/4*a^2*cos(d*x+c)^5*sin(d*x+c)^3/d
```

Rubi [A] time = 0.25, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{a^2\cos^9(c+dx)}{9d}+\frac{3a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)\cos^5(c+dx)}{4d}-\frac{a^2\sin(c+dx)\cos^5(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/64 - (2*a^2*Cos[c + d*x]^5)/(5*d) + (3*a^2*Cos[c + d*x]^7)/(7*d)
- (a^2*Cos[c + d*x]^9)/(9*d) + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(64*d) + (
a^2*Cos[c + d*x]^3*Sin[c + d*x])/(32*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x])
/(8*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)\sin^3(c+dx)+2a^2\cos^4(c+dx)\sin^4(c+\right.$$

$$= a^2\int \cos^4(c+dx)\sin^3(c+dx)\,dx + a^2\int \cos^4(c+dx)\sin^5($$

$$= -\frac{a^2\cos^5(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{4}\left(3a^2\right)\int \cos^4(c+dx)\sin$$

$$= -\frac{a^2\cos^5(c+dx)\sin(c+dx)}{8d}-\frac{a^2\cos^5(c+dx)\sin^3(c+dx)}{4d}$$

$$= -\frac{2a^2\cos^5(c+dx)}{5d}+\frac{3a^2\cos^7(c+dx)}{7d}-\frac{a^2\cos^9(c+dx)}{9d}+$$

$$= -\frac{2a^2\cos^5(c+dx)}{5d}+\frac{3a^2\cos^7(c+dx)}{7d}-\frac{a^2\cos^9(c+dx)}{9d}+$$

$$= \frac{3a^2x}{64}-\frac{2a^2\cos^5(c+dx)}{5d}+\frac{3a^2\cos^7(c+dx)}{7d}-\frac{a^2\cos^9(c+}{9d}$$

Mathematica [A] time = 0.71, size = 86, normalized size = 0.54

$$\frac{a^2(-2520\sin(4(c+dx))+315\sin(8(c+dx))-11340\cos(c+dx)-3360\cos(3(c+dx))+1008\cos(5(c+dx))}{161280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(7560*c + 7560*d*x - 11340*Cos[c + d*x] - 3360*Cos[3*(c + d*x)] + 1008
*Cos[5*(c + d*x)] + 450*Cos[7*(c + d*x)] - 70*Cos[9*(c + d*x)] - 2520*Sin[4
*(c + d*x)] + 315*Sin[8*(c + d*x)]))/(161280*d)
```

fricas [A] time = 0.60, size = 111, normalized size = 0.70

$$-\frac{2240\,a^2\cos\left(dx+c\right)^9-8640\,a^2\cos\left(dx+c\right)^7+8064\,a^2\cos\left(dx+c\right)^5-945\,a^2dx-315\left(16\,a^2\cos\left(dx+c\right)^7-2\right.}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20160*(2240*a^2*cos(d*x + c)^9 - 8640*a^2*cos(d*x + c)^7 + 8064*a^2*cos(
d*x + c)^5 - 945*a^2*d*x - 315*(16*a^2*cos(d*x + c)^7 - 24*a^2*cos(d*x + c)
^5 + 2*a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.31, size = 123, normalized size = 0.77

$$\frac{3}{64}\,a^2x-\frac{a^2\cos\left(9\,dx+9\,c\right)}{2304\,d}+\frac{5\,a^2\cos\left(7\,dx+7\,c\right)}{1792\,d}+\frac{a^2\cos\left(5\,dx+5\,c\right)}{160\,d}-\frac{a^2\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{9\,a^2\cos\left(dx+c\right)}{128\,d}+\frac{a^2\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/64*a^2*x - 1/2304*a^2*cos(9*d*x + 9*c)/d + 5/1792*a^2*cos(7*d*x + 7*c)/d
+ 1/160*a^2*cos(5*d*x + 5*c)/d - 1/48*a^2*cos(3*d*x + 3*c)/d - 9/128*a^2*co
s(d*x + c)/d + 1/512*a^2*sin(8*d*x + 8*c)/d - 1/64*a^2*sin(4*d*x + 4*c)/d
```

maple [A] time = 0.28, size = 162, normalized size = 1.02

$$\frac{a^2\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^5(dx+c)\right)}{9}-\frac{4\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{63}-\frac{8\left(\cos^5(dx+c)\right)}{315}\right)+2a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c\right.}{16}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/3
15*cos(d*x+c)^5)+2*a^2*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(
d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+a
^2*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5))
```

maxima [A] time = 0.34, size = 101, normalized size = 0.64

$$-\frac{512\left(35\cos\left(dx+c\right)^{9}-90\cos\left(dx+c\right)^{7}+63\cos\left(dx+c\right)^{5}\right)a^{2}-4608\left(5\cos\left(dx+c\right)^{7}-7\cos\left(dx+c\right)^{5}\right)a^{2}-3}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/161280*(512*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*
a^2 - 4608*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^2 - 315*(24*d*x + 24*c +
 sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^2)/d
```

mupad [B] time = 12.15, size = 437, normalized size = 2.75

$$\frac{3\,a^2\,x}{64}-\frac{\frac{3\,a^2\,(c+d\,x)}{64}+\frac{13\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{16}-\frac{155\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{16}+\frac{169\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{16}-\frac{169\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{16}+\frac{155\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{16}-\frac{13\,a}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/64 - ((3*a^2*(c + d*x))/64 + (13*a^2*tan(c/2 + (d*x)/2)^3)/16 - (
155*a^2*tan(c/2 + (d*x)/2)^5)/16 + (169*a^2*tan(c/2 + (d*x)/2)^7)/16 - (169
*a^2*tan(c/2 + (d*x)/2)^11)/16 + (155*a^2*tan(c/2 + (d*x)/2)^13)/16 - (13*a
^2*tan(c/2 + (d*x)/2)^15)/16 - (3*a^2*tan(c/2 + (d*x)/2)^17)/32 - (a^2*(945
*c + 945*d*x - 3328))/20160 + tan(c/2 + (d*x)/2)^2*((27*a^2*(c + d*x))/64 -
 (a^2*(8505*c + 8505*d*x - 29952))/20160) + tan(c/2 + (d*x)/2)^4*((27*a^2*(
c + d*x))/16 - (a^2*(34020*c + 34020*d*x - 39168))/20160) + tan(c/2 + (d*x)
/2)^14*((27*a^2*(c + d*x))/16 - (a^2*(34020*c + 34020*d*x - 80640))/20160)
+ tan(c/2 + (d*x)/2)^6*((63*a^2*(c + d*x))/16 - (a^2*(79380*c + 79380*d*x +
 16128))/20160) + tan(c/2 + (d*x)/2)^12*((63*a^2*(c + d*x))/16 - (a^2*(7938
0*c + 79380*d*x - 295680))/20160) + tan(c/2 + (d*x)/2)^10*((189*a^2*(c + d*
x))/32 - (a^2*(119070*c + 119070*d*x + 241920))/20160) + tan(c/2 + (d*x)/2)
^8*((189*a^2*(c + d*x))/32 - (a^2*(119070*c + 119070*d*x - 661248))/20160)
+ (3*a^2*tan(c/2 + (d*x)/2))/32)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

sympy [A] time = 22.16, size = 335, normalized size = 2.11

$$\begin{cases} \frac{3a^2x\sin^8(c+dx)}{64} + \frac{3a^2x\sin^6(c+dx)\cos^2(c+dx)}{16} + \frac{9a^2x\sin^4(c+dx)\cos^4(c+dx)}{32} + \frac{3a^2x\sin^2(c+dx)\cos^6(c+dx)}{16} + \frac{3a^2x\cos^8(c+dx)}{64} + \frac{3a^2\sin}{} \\ x(a\sin(c)+a)^2\sin^3(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**8/64 + 3*a**2*x*sin(c + d*x)**6*cos(c + d
*x)**2/16 + 9*a**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 3*a**2*x*sin(c +
d*x)**2*cos(c + d*x)**6/16 + 3*a**2*x*cos(c + d*x)**8/64 + 3*a**2*sin(c + d
*x)**7*cos(c + d*x)/(64*d) + 11*a**2*sin(c + d*x)**5*cos(c + d*x)**3/(64*d)
 - a**2*sin(c + d*x)**4*cos(c + d*x)**5/(5*d) - 11*a**2*sin(c + d*x)**3*cos
(c + d*x)**5/(64*d) - 4*a**2*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - a**2*
sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**7
/(64*d) - 8*a**2*cos(c + d*x)**9/(315*d) - 2*a**2*cos(c + d*x)**7/(35*d), N
e(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**3*cos(c)**4, True))
```

3.380 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=141

$$\frac{2a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{11a^2\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{11a^2\sin(c+dx}{192}$$

[Out] 11/128*a^2*x-2/5*a^2*cos(d*x+c)^5/d+2/7*a^2*cos(d*x+c)^7/d+11/128*a^2*cos(d*x+c)*sin(d*x+c)/d+11/192*a^2*cos(d*x+c)^3*sin(d*x+c)/d-11/48*a^2*cos(d*x+c)^5*sin(d*x+c)/d-1/8*a^2*cos(d*x+c)^5*sin(d*x+c)^3/d

Rubi [A] time = 0.26, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2873, 2568, 2635, 8, 2565, 14}

$$\frac{2a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{11a^2\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{11a^2\sin(c+dx}{192}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]

[Out] (11*a^2*x)/128 - (2*a^2*Cos[c + d*x]^5)/(5*d) + (2*a^2*Cos[c + d*x]^7)/(7*d) + (11*a^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (11*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) - (11*a^2*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x]^3)/(8*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)\sin^2(c+dx)+2a^2\cos^4(c+dx)\sin^3(c\right.$$
$$= a^2\int \cos^4(c+dx)\sin^2(c+dx)\,dx + a^2\int \cos^4(c+dx)\sin$$
$$= -\frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{a^2\cos^5(c+dx)\sin^3(c+dx}{8d}$$
$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{11a^2\cos^5(c+dx)\sin(c+dx}{48d}$$
$$= -\frac{2a^2\cos^5(c+dx)}{5d} + \frac{2a^2\cos^7(c+dx)}{7d} + \frac{a^2\cos(c+dx)\sin}{16d}$$
$$= \frac{a^2x}{16} - \frac{2a^2\cos^5(c+dx)}{5d} + \frac{2a^2\cos^7(c+dx)}{7d} + \frac{11a^2\cos(c+}{1}$$
$$= \frac{11a^2x}{128} - \frac{2a^2\cos^5(c+dx)}{5d} + \frac{2a^2\cos^7(c+dx)}{7d} + \frac{11a^2\cos(c}{}$$

Mathematica [A] time = 0.51, size = 96, normalized size = 0.68

$$\frac{a^2(1680\sin(2(c+dx)) - 2520\sin(4(c+dx)) - 560\sin(6(c+dx)) + 105\sin(8(c+dx)) - 10080\cos(c+dx) - 33}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(3360*c + 9240*d*x - 10080*Cos[c + d*x] - 3360*Cos[3*(c + d*x)] + 672*
Cos[5*(c + d*x)] + 480*Cos[7*(c + d*x)] + 1680*Sin[2*(c + d*x)] - 2520*Sin[
4*(c + d*x)] - 560*Sin[6*(c + d*x)] + 105*Sin[8*(c + d*x)]))/(107520*d)
```

fricas [A] time = 0.55, size = 98, normalized size = 0.70

$$\frac{3840\,a^2\cos\left(dx+c\right)^7-5376\,a^2\cos\left(dx+c\right)^5+1155\,a^2dx+35\left(48\,a^2\cos\left(dx+c\right)^7-136\,a^2\cos\left(dx+c\right)^5+22\,a^2}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/13440*(3840*a^2*cos(d*x + c)^7 - 5376*a^2*cos(d*x + c)^5 + 1155*a^2*d*x +
 35*(48*a^2*cos(d*x + c)^7 - 136*a^2*cos(d*x + c)^5 + 22*a^2*cos(d*x + c)^3
 + 33*a^2*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.27, size = 140, normalized size = 0.99

$$\frac{11}{128}\,a^2x+\frac{a^2\cos\left(7\,dx+7\,c\right)}{224\,d}+\frac{a^2\cos\left(5\,dx+5\,c\right)}{160\,d}-\frac{a^2\cos\left(3\,dx+3\,c\right)}{32\,d}-\frac{3\,a^2\cos\left(dx+c\right)}{32\,d}+\frac{a^2\sin\left(8\,dx+8\,c\right)}{1024\,d}-\frac{a^2\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 11/128*a^2*x + 1/224*a^2*cos(7*d*x + 7*c)/d + 1/160*a^2*cos(5*d*x + 5*c)/d
- 1/32*a^2*cos(3*d*x + 3*c)/d - 3/32*a^2*cos(d*x + c)/d + 1/1024*a^2*sin(8*
d*x + 8*c)/d - 1/192*a^2*sin(6*d*x + 6*c)/d - 3/128*a^2*sin(4*d*x + 4*c)/d
+ 1/64*a^2*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.29, size = 164, normalized size = 1.16

$$\frac{a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{64}+\frac{3dx}{128}+\frac{3c}{128}\right)+2a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(\right.}{7}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+2*a^2*(-1/7*sin
```

```
(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+a^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5
+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c))
```

maxima [A] time = 0.32, size = 102, normalized size = 0.72

$$\frac{6144\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^2+560\left(4\sin(2\,dx+2\,c)^3+12\,dx+12\,c-3\sin(4\,dx+4\,c)\right)a^2+105}{107520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/107520*(6144*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^2 + 560*(4*sin(2*d*x
 + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^2 + 105*(24*d*x + 24*c +
sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^2)/d
```

mupad [B] time = 12.35, size = 363, normalized size = 2.57

$$\frac{11\,a^2\,x}{128}-\frac{\frac{11\,a^2\,(c+d\,x)}{128}-\frac{259\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{192}-\frac{1103\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{192}+\frac{2261\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{192}-\frac{2261\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{192}+\frac{1103\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{1}}{192}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (11*a^2*x)/128 - ((11*a^2*(c + d*x))/128 - (259*a^2*tan(c/2 + (d*x)/2)^3)/1
92 - (1103*a^2*tan(c/2 + (d*x)/2)^5)/192 + (2261*a^2*tan(c/2 + (d*x)/2)^7)/
192 - (2261*a^2*tan(c/2 + (d*x)/2)^9)/192 + (1103*a^2*tan(c/2 + (d*x)/2)^11
)/192 + (259*a^2*tan(c/2 + (d*x)/2)^13)/192 - (11*a^2*tan(c/2 + (d*x)/2)^15
)/64 - (a^2*(1155*c + 1155*d*x - 3072))/13440 + tan(c/2 + (d*x)/2)^2*((11*a
^2*(c + d*x))/16 - (a^2*(9240*c + 9240*d*x - 24576))/13440) + tan(c/2 + (d*
x)/2)^4*((77*a^2*(c + d*x))/32 - (a^2*(32340*c + 32340*d*x + 21504))/13440)
 + tan(c/2 + (d*x)/2)^12*((77*a^2*(c + d*x))/32 - (a^2*(32340*c + 32340*d*x
 - 107520))/13440) + tan(c/2 + (d*x)/2)^8*((385*a^2*(c + d*x))/64 - (a^2*(8
0850*c + 80850*d*x - 107520))/13440) + tan(c/2 + (d*x)/2)^6*((77*a^2*(c + d
*x))/16 - (a^2*(64680*c + 64680*d*x - 172032))/13440) + (11*a^2*tan(c/2 + (
d*x)/2))/64)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^8)
```

sympy [A] time = 14.02, size = 420, normalized size = 2.98

$$\begin{cases} \frac{3a^2x\sin^8(c+dx)}{128}+\frac{3a^2x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{a^2x\sin^6(c+dx)}{16}+\frac{9a^2x\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{3a^2x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3a^2x\sin}{} \\ x\,(a\sin(c)+a)^2\sin^2(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)

[Out] Piecewise((3*a**2*x*sin(c + d*x)**8/128 + 3*a**2*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + a**2*x*sin(c + d*x)**6/16 + 9*a**2*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3*a**2*x*cos(c + d*x)**8/128 + a**2*x*cos(c + d*x)**6/16 + 3*a**2*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 11*a**2*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) + a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 11*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) + a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - 2*a**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**7/(128*d) - a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 4*a**2*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**2*cos(c)**4, True))

3.381 $\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=129

$$-\frac{a^2\cos^5(c+dx)}{15d}-\frac{\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{21d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{12d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{8d}+$$

```
[Out] 1/8*a^2*x-1/15*a^2*cos(d*x+c)^5/d+1/8*a^2*cos(d*x+c)*sin(d*x+c)/d+1/12*a^2*
cos(d*x+c)^3*sin(d*x+c)/d-1/7*cos(d*x+c)^5*(a+a*sin(d*x+c))^2/d-1/21*cos(d*
x+c)^5*(a^2+a^2*sin(d*x+c))/d
```

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{a^2\cos^5(c+dx)}{15d}-\frac{\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{21d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{12d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{8d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*x)/8 - (a^2*Cos[c + d*x]^5)/(15*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(12*d) - (Cos[c + d*x]^5*(a + a*Si
n[c + d*x])^2)/(7*d) - (Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x]))/(21*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{2}{7}\int \cos^4(c+dx)(a+a$$

$$= -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}-\frac{\cos^5(c+dx)\left(a^2+a^2\sin\right.}{21d}$$

$$= -\frac{a^2\cos^5(c+dx)}{15d}-\frac{\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}-\frac{\cos^5(}{}$$

$$= -\frac{a^2\cos^5(c+dx)}{15d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{12d}-\frac{\cos^5(c+d}{}$$

$$= -\frac{a^2\cos^5(c+dx)}{15d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+}{}$$

$$= \frac{a^2x}{8}-\frac{a^2\cos^5(c+dx)}{15d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos}{}$$

Mathematica [A] time = 0.33, size = 86, normalized size = 0.67

$$\frac{a^2(210\sin(2(c+dx))-210\sin(4(c+dx))-70\sin(6(c+dx))-1155\cos(c+dx)-525\cos(3(c+dx))-63\cos(5}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(840*c + 840*d*x - 1155*Cos[c + d*x] - 525*Cos[3*(c + d*x)] - 63*Cos[5
*(c + d*x)] + 15*Cos[7*(c + d*x)] + 210*Sin[2*(c + d*x)] - 210*Sin[4*(c + d
*x)] - 70*Sin[6*(c + d*x)]))/(6720*d)
```

fricas [A] time = 0.49, size = 85, normalized size = 0.66

$$\frac{120\,a^2\cos\left(dx+c\right)^7-336\,a^2\cos\left(dx+c\right)^5+105\,a^2dx-35\left(8\,a^2\cos\left(dx+c\right)^5-2\,a^2\cos\left(dx+c\right)^3-3\,a^2\cos\left(dx\right.\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*a^2*cos(d*x + c)^7 - 336*a^2*cos(d*x + c)^5 + 105*a^2*d*x - 35*(
8*a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x + c)^3 - 3*a^2*cos(d*x + c))*sin(d*x +
 c))/d
```

giac [A] time = 0.26, size = 123, normalized size = 0.95

$$\frac{1}{8}\,a^2x+\frac{a^2\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{3\,a^2\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{5\,a^2\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{11\,a^2\cos\left(dx+c\right)}{64\,d}-\frac{a^2\sin\left(6\,dx+6\,c\right)}{96\,d}-\frac{a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*x + 1/448*a^2*cos(7*d*x + 7*c)/d - 3/320*a^2*cos(5*d*x + 5*c)/d - 5
/64*a^2*cos(3*d*x + 3*c)/d - 11/64*a^2*cos(d*x + c)/d - 1/96*a^2*sin(6*d*x
+ 6*c)/d - 1/32*a^2*sin(4*d*x + 4*c)/d + 1/32*a^2*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.27, size = 106, normalized size = 0.82

$$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+2a^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+2*a^2*(-1/6*sin
(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x
+1/16*c)-1/5*a^2*cos(d*x+c)^5)
```

maxima [A] time = 0.35, size = 82, normalized size = 0.64

$$-\frac{672\,a^2\cos\left(dx+c\right)^5-96\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)a^2-35\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\right.}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*(672*a^2*cos(d*x + c)^5 - 96*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*
a^2 - 35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^2)/d
```

mupad [B] time = 10.75, size = 388, normalized size = 3.01

$$\frac{a^2 x}{8} - \frac{\frac{31 a^2 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^5}{12} - \frac{11 a^2 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^3}{3} - \frac{31 a^2 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^9}{12} + \frac{11 a^2 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{11}}{3} - \frac{a^2 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{13}}{4} + a^2 \left(\frac{c}{8} + \frac{d x}{8}\right) - a^2 \left(\frac{c}{8} + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*x)/8 - ((31*a^2*tan(c/2 + (d*x)/2)^5)/12 - (11*a^2*tan(c/2 + (d*x)/2)^
3)/3 - (31*a^2*tan(c/2 + (d*x)/2)^9)/12 + (11*a^2*tan(c/2 + (d*x)/2)^11)/3
- (a^2*tan(c/2 + (d*x)/2)^13)/4 + a^2*(c/8 + (d*x)/8) - a^2*(c/8 + (d*x)/8
- 18/35) + tan(c/2 + (d*x)/2)^12*(7*a^2*(c/8 + (d*x)/8) - a^2*((7*c)/8 + (7
*d*x)/8 - 2)) + tan(c/2 + (d*x)/2)^2*(7*a^2*(c/8 + (d*x)/8) - a^2*((7*c)/8
+ (7*d*x)/8 - 8/5)) + tan(c/2 + (d*x)/2)^10*(21*a^2*(c/8 + (d*x)/8) - a^2*(
(21*c)/8 + (21*d*x)/8 - 8)) + tan(c/2 + (d*x)/2)^4*(21*a^2*(c/8 + (d*x)/8)
- a^2*((21*c)/8 + (21*d*x)/8 - 14/5)) + tan(c/2 + (d*x)/2)^8*(35*a^2*(c/8 +
 (d*x)/8) - a^2*((35*c)/8 + (35*d*x)/8 - 2)) + tan(c/2 + (d*x)/2)^6*(35*a^2
*(c/8 + (d*x)/8) - a^2*((35*c)/8 + (35*d*x)/8 - 16)) + (a^2*tan(c/2 + (d*x)
/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [A] time = 8.01, size = 223, normalized size = 1.73

$$\begin{cases} \frac{a^2 x \sin^6(c+dx)}{8} + \frac{3a^2 x \sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{3a^2 x \sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{a^2 x \cos^6(c+dx)}{8} + \frac{a^2 \sin^5(c+dx)\cos(c+dx)}{8d} + \frac{a^2 \sin^3(c+dx)}{3d} \\ x\left(a \sin(c) + a\right)^2 \sin(c) \cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/8 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x)
**2/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + a**2*x*cos(c + d*x)**6
/8 + a**2*sin(c + d*x)**5*cos(c + d*x)/(8*d) + a**2*sin(c + d*x)**3*cos(c +
 d*x)**3/(3*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - a**2*sin(c +
d*x)*cos(c + d*x)**5/(8*d) - 2*a**2*cos(c + d*x)**7/(35*d) - a**2*cos(c + d
*x)**5/(5*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)*cos(c)**4, True))
```

3.382 $\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=119

$$-\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}-\frac{a}{}$$

```
[Out] 3/4*a^2*x-a^2*arctanh(cos(d*x+c))/d+a^2*cos(d*x+c)/d+1/3*a^2*cos(d*x+c)^3/d
-1/5*a^2*cos(d*x+c)^5/d+3/4*a^2*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)^
3*sin(d*x+c)/d
```

Rubi [A] time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2873, 2635, 8, 2592, 302, 206, 2565, 30}

$$-\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}-\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/4 - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d + (a^2*C
os[c + d*x]^3)/(3*d) - (a^2*Cos[c + d*x]^5)/(5*d) + (3*a^2*Cos[c + d*x]*Sin
[c + d*x])/(4*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(2a^2\cos^4(c+dx) + a^2\cos^3(c+dx)\cot(c+dx) + a^2\cos\right.$$

$$= a^2\int \cos^3(c+dx)\cot(c+dx)\,dx + a^2\int \cos^4(c+dx)\sin(c$$

$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}\left(3a^2\right)\int \cos^2(c+dx)\,dx -$$

$$= -\frac{a^2\cos^5(c+dx)}{5d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{4d} + \frac{a^2\cos^3(c}{}$$

$$= \frac{3a^2x}{4} + \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} - \frac{a^2\cos^5(c+dx)}{5d}$$

$$= \frac{3a^2x}{4} - \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3}{3}$$

Mathematica [A] time = 0.85, size = 96, normalized size = 0.81

$$\frac{a^2\left(270\cos(c+dx) + 5\cos(3(c+dx)) - 3\cos(5(c+dx)) + 15\left(8\sin(2(c+dx)) + \sin(4(c+dx)) + 4\left(4\log\left(\sin\right.\right.\right.\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(270*Cos[c + d*x] + 5*Cos[3*(c + d*x)] - 3*Cos[5*(c + d*x)] + 15*(4*(3
*c + 3*d*x - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]]) + 8*Sin[2*(
c + d*x)] + Sin[4*(c + d*x)])))/(240*d)
```

fricas [A] time = 0.62, size = 115, normalized size = 0.97

$$-\frac{12\,a^2\cos\left(dx+c\right)^5 - 20\,a^2\cos\left(dx+c\right)^3 - 45\,a^2dx - 60\,a^2\cos\left(dx+c\right) + 30\,a^2\log\left(\frac{1}{2}\cos\left(dx+c\right) + \frac{1}{2}\right) - 30\,a^2}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(12*a^2*cos(d*x + c)^5 - 20*a^2*cos(d*x + c)^3 - 45*a^2*d*x - 60*a^2*
cos(d*x + c) + 30*a^2*log(1/2*cos(d*x + c) + 1/2) - 30*a^2*log(-1/2*cos(d*x
 + c) + 1/2) - 15*(2*a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c))*sin(d*x + c))
/d
```

giac [A] time = 0.22, size = 181, normalized size = 1.52

$$\frac{45\,(dx+c)a^2+60\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(75\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+30\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-360\,a^2\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\right.}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(45*(d*x + c)*a^2 + 60*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(75*a^2*
tan(1/2*d*x + 1/2*c)^9 - 60*a^2*tan(1/2*d*x + 1/2*c)^8 + 30*a^2*tan(1/2*d*x
 + 1/2*c)^7 - 360*a^2*tan(1/2*d*x + 1/2*c)^6 - 320*a^2*tan(1/2*d*x + 1/2*c)
^4 - 30*a^2*tan(1/2*d*x + 1/2*c)^3 - 280*a^2*tan(1/2*d*x + 1/2*c)^2 - 75*a^
2*tan(1/2*d*x + 1/2*c) - 68*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

maple [A] time = 0.46, size = 127, normalized size = 1.07

$$-\frac{a^2\left(\cos^5(dx+c)\right)}{5d}+\frac{a^2\left(\cos^3(dx+c)\right)\sin(dx+c)}{2d}+\frac{3a^2\cos(dx+c)\sin(dx+c)}{4d}+\frac{3a^2x}{4}+\frac{3a^2c}{4d}+\frac{a^2\left(\cos^3(dx+c)\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/5*a^2*cos(d*x+c)^5/d+1/2*a^2*cos(d*x+c)^3*sin(d*x+c)/d+3/4*a^2*cos(d*x+c
)*sin(d*x+c)/d+3/4*a^2*x+3/4/d*a^2*c+1/3*a^2*cos(d*x+c)^3/d+a^2*cos(d*x+c)/
d+1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.33, size = 98, normalized size = 0.82

$$-\frac{48\,a^2\cos(dx+c)^5-40\left(2\,\cos(dx+c)^3+6\,\cos(dx+c)-3\,\log(\cos(dx+c)+1)+3\,\log(\cos(dx+c)-1)\right)a^{}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(48*a^2*cos(d*x + c)^5 - 40*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*l
og(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^2 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2)/d
```

mupad [B] time = 10.22, size = 293, normalized size = 2.46

$$\frac{a^2\ \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{3\,a^2\,\mathrm{atan}\left(\frac{9\,a^4}{4\left(3\,a^4-\frac{9\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)}+\frac{3\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3\,a^4-\frac{9\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}\right)}{2\,d}+\frac{-\frac{5\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2}+2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-}{d\,\left(\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (3*a^2*atan((9*a^4)/(4*(3*a^4 - (9*a^4*ta
n(c/2 + (d*x)/2))/4)) + (3*a^4*tan(c/2 + (d*x)/2))/(3*a^4 - (9*a^4*tan(c/2
+ (d*x)/2))/4)))/(2*d) + ((28*a^2*tan(c/2 + (d*x)/2)^2)/3 + a^2*tan(c/2 + (
d*x)/2)^3 + (32*a^2*tan(c/2 + (d*x)/2)^4)/3 + 12*a^2*tan(c/2 + (d*x)/2)^6 -
 a^2*tan(c/2 + (d*x)/2)^7 + 2*a^2*tan(c/2 + (d*x)/2)^8 - (5*a^2*tan(c/2 + (
d*x)/2)^9)/2 + (34*a^2)/15 + (5*a^2*tan(c/2 + (d*x)/2))/2)/(d*(5*tan(c/2 +
(d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2
+ (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.383 $\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{2a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}-\frac{a^2\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{8d}-\frac{2a^2\ t}{}$$

```
[Out] -9/8*a^2*x-2*a^2*arctanh(cos(d*x+c))/d+2*a^2*cos(d*x+c)/d+2/3*a^2*cos(d*x+c
)^3/d-a^2*cot(d*x+c)/d+1/8*a^2*cos(d*x+c)*sin(d*x+c)/d-1/4*a^2*cos(d*x+c)*s
in(d*x+c)^3/d
```

Rubi [A] time = 0.20, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2872, 3770, 3767, 8, 2638, 2635, 2633}

$$\frac{2a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}-\frac{a^2\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{8d}-\frac{2a^2\ t}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-9*a^2*x)/8 - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d + (
2*a^2*Cos[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[
c + d*x])/(8*d) - (a^2*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-a^6+2a^6\csc(c+dx)+a^6\csc^2(c+dx)-4a^6\sin(c+d\right.}{}$$

$$= -a^2x + a^2\int \csc^2(c+dx)\,dx - a^2\int \sin^2(c+dx)\,dx + a^2$$

$$= -a^2x - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^2\cos(c+dx)}{d} + \frac{a^2 c}{}$$

$$= -\frac{3a^2x}{2} - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{2a}{}$$

$$= -\frac{9a^2x}{8} - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{2a}{}$$

Mathematica [A] time = 0.49, size = 83, normalized size = 0.72

$$\frac{a^2\left(240\cos(c+dx)+16\cos(3(c+dx))-3\left(-\sin(4(c+dx))+32\cot(c+dx)-64\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+64\,\mathrm{lo}\right.\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(240*Cos[c + d*x] + 16*Cos[3*(c + d*x)] - 3*(36*c + 36*d*x + 32*Cot[c
+ d*x] + 64*Log[Cos[(c + d*x)/2]] - 64*Log[Sin[(c + d*x)/2]] - Sin[4*(c + d
*x)])))/(96*d)
```

fricas [A] time = 0.69, size = 135, normalized size = 1.16

$$-\frac{6\,a^{2}\cos\left(dx+c\right)^{5}-9\,a^{2}\cos\left(dx+c\right)^{3}+24\,a^{2}\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-24\,a^{2}\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)\right.}{24\,d\sin\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*a^2*cos(d*x + c)^5 - 9*a^2*cos(d*x + c)^3 + 24*a^2*log(1/2*cos(d*x
 + c) + 1/2)*sin(d*x + c) - 24*a^2*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) + 27*a^2*cos(d*x + c) - (16*a^2*cos(d*x + c)^3 - 27*a^2*d*x + 48*a^2*cos(
d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.23, size = 210, normalized size = 1.81

$$-\frac{27\,(dx+c)a^{2}-48\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-12\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{12\left(4\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{2}\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(27*(d*x + c)*a^2 - 48*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 12*a^2*ta
n(1/2*d*x + 1/2*c) + 12*(4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/
2*c) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^7 - 96*a^2*tan(1/2*d*x + 1/2*c)^6 - 21
*a^2*tan(1/2*d*x + 1/2*c)^5 - 192*a^2*tan(1/2*d*x + 1/2*c)^4 + 21*a^2*tan(1
/2*d*x + 1/2*c)^3 - 160*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/
2*c) - 64*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.42, size = 137, normalized size = 1.18

$$-\frac{3a^{2}\left(\cos^{3}\left(dx+c\right)\right)\sin\left(dx+c\right)}{4d}-\frac{9a^{2}\cos\left(dx+c\right)\sin\left(dx+c\right)}{8d}-\frac{9a^{2}x}{8}-\frac{9a^{2}c}{8d}+\frac{2a^{2}\left(\cos^{3}\left(dx+c\right)\right)}{3d}+\frac{2a^{2}\cos\left(dx+c\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-9/8*a^2*cos(d*x+c)*sin(d*x+c)/d-9/8*a^2*
x-9/8/d*a^2*c+2/3*a^2*cos(d*x+c)^3/d+2*a^2*cos(d*x+c)/d+2/d*a^2*ln(csc(d*x+
c)-cot(d*x+c))-1/d*a^2/sin(d*x+c)*cos(d*x+c)^5
```

maxima [A] time = 0.45, size = 128, normalized size = 1.10

$$\frac{32\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)a^2+3(12\,dx+12\,c+s}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/96*(32*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a^2 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d
*x + 2*c))*a^2 - 48*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 +
 tan(d*x + c)))*a^2)/d
```

mupad [B] time = 8.83, size = 310, normalized size = 2.67

$$\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{9\,a^2\,\mathrm{atan}\left(\frac{81\,a^4}{16\left(9\,a^4+\frac{81\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}-\frac{9\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{9\,a^4+\frac{81\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}}\right)}{4\,d}-\frac{\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}-16\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{d\left(2\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (2*a^2*log(tan(c/2 + (d*x)/2)))/d + (9*a^2*atan((81*a^4)/(16*(9*a^4 + (81*a
^4*tan(c/2 + (d*x)/2))/16)) - (9*a^4*tan(c/2 + (d*x)/2))/(9*a^4 + (81*a^4*t
an(c/2 + (d*x)/2))/16)))/(4*d) - ((7*a^2*tan(c/2 + (d*x)/2)^2)/2 - (80*a^2*
tan(c/2 + (d*x)/2)^3)/3 + (19*a^2*tan(c/2 + (d*x)/2)^4)/2 - 32*a^2*tan(c/2
+ (d*x)/2)^5 + (a^2*tan(c/2 + (d*x)/2)^6)/2 - 16*a^2*tan(c/2 + (d*x)/2)^7 +
 (3*a^2*tan(c/2 + (d*x)/2)^8)/2 + a^2 - (32*a^2*tan(c/2 + (d*x)/2))/3)/(d*(
2*tan(c/2 + (d*x)/2) + 8*tan(c/2 + (d*x)/2)^3 + 12*tan(c/2 + (d*x)/2)^5 + 8
*tan(c/2 + (d*x)/2)^7 + 2*tan(c/2 + (d*x)/2)^9)) + (a^2*tan(c/2 + (d*x)/2))
/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
[Out] Timed out
```

3.384 $\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=98

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{2a^2\cot(c+dx)}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -3*a^2*x+1/2*a^2*arctanh(cos(d*x+c))/d+1/3*a^2*cos(d*x+c)^3/d-2*a^2*cot(d*x
+c)/d-1/2*a^2*cot(d*x+c)*csc(d*x+c)/d-a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.16, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2872, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{2a^2\cot(c+dx)}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -3*a^2*x + (a^2*ArcTanh[Cos[c + d*x]])/(2*d) + (a^2*Cos[c + d*x]^3)/(3*d) -
 (2*a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]*Csc[c + d*x])/(2*d) - (a^2*Cos[
c + d*x]*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int\left(-4a^6 - a^6\csc(c+dx) + 2a^6\csc^2(c+dx) + a^6\csc^3(c+dx\right.}{a^4}$$

$$= -4a^2x - a^2\int\csc(c+dx)\,dx + a^2\int\csc^3(c+dx)\,dx - a^2\int s$$

$$= -4a^2x + \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^2\cos(c+dx)}{d} - \frac{a^2\cot(c}{}$$

$$= -3a^2x + \frac{a^2\tanh^{-1}(\cos(c+dx))}{2d} + \frac{a^2\cos^3(c+dx)}{3d} - \frac{2a^2\cot}{}$$

Mathematica [A] time = 2.12, size = 158, normalized size = 1.61

$$\frac{a^2(\sin(c+dx)+1)^2\left(6\cos(c+dx)+2\cos(3(c+dx))+3\left(-4\sin(2(c+dx))+8\tan\left(\frac{1}{2}(c+dx)\right)-8\cot\left(\frac{1}{2}(c+\right.\right.\right.}{24d\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(6*Cos[c + d*x] + 2*Cos[3*(c + d*x)] + 3*(-24*c -
 24*d*x - 8*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 + 4*Log[Cos[(c + d*x)/2]]
 - 4*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 - 4*Sin[2*(c + d*x)] + 8*Ta
n[(c + d*x)/2])))/(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

fricas [A] time = 0.58, size = 172, normalized size = 1.76

$$\frac{4\,a^2\cos\left(dx+c\right)^5-36\,a^2dx\cos\left(dx+c\right)^2-4\,a^2\cos\left(dx+c\right)^3+36\,a^2dx+6\,a^2\cos\left(dx+c\right)+3\left(a^2\cos\left(dx+c\right)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(4*a^2*cos(d*x + c)^5 - 36*a^2*d*x*cos(d*x + c)^2 - 4*a^2*cos(d*x + c)
^3 + 36*a^2*d*x + 6*a^2*cos(d*x + c) + 3*(a^2*cos(d*x + c)^2 - a^2)*log(1/2
*cos(d*x + c) + 1/2) - 3*(a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) +
 1/2) - 12*(a^2*cos(d*x + c)^3 - 3*a^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2 - d)
```

giac [A] time = 0.24, size = 178, normalized size = 1.82

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,(dx+c)a^2-12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{\tan\left(\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*tan(1/2*d*x + 1/2*c)^2 - 72*(d*x + c)*a^2 - 12*a^2*log(abs(tan(
1/2*d*x + 1/2*c))) + 24*a^2*tan(1/2*d*x + 1/2*c) + 3*(6*a^2*tan(1/2*d*x + 1
```

```
/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2 + 16*(3*
a^2*tan(1/2*d*x + 1/2*c)^5 + 3*a^2*tan(1/2*d*x + 1/2*c)^4 - 3*a^2*tan(1/2*d
*x + 1/2*c) + a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.49, size = 161, normalized size = 1.64

$$-\frac{a^2\left(\cos^3(dx+c)\right)}{6d}-\frac{a^2\cos(dx+c)}{2d}-\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{2a^2\left(\cos^5(dx+c)\right)}{d\sin(dx+c)}-\frac{2a^2\left(\cos^3(dx+c)\right)\mathrm{s}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/6*a^2*cos(d*x+c)^3/d-1/2*a^2*cos(d*x+c)/d-1/2/d*a^2*ln(csc(d*x+c)-cot(d*
x+c))-2/d*a^2/sin(d*x+c)*cos(d*x+c)^5-2*a^2*cos(d*x+c)^3*sin(d*x+c)/d-3*a^2
*cos(d*x+c)*sin(d*x+c)/d-3*a^2*x-3/d*a^2*c-1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c
)^5
```

maxima [A] time = 0.46, size = 151, normalized size = 1.54

$$\frac{2\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)a^2-12\left(3dx+3c+\frac{3}{\tan(a}\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*lo
g(cos(d*x + c) - 1))*a^2 - 12*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*
x + c)^3 + tan(d*x + c)))*a^2 + 3*a^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1)
- 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.91, size = 303, normalized size = 3.09

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d}-\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}-\frac{6a^2\,\mathrm{atan}\left(\frac{36a^4}{6a^4-36a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{6a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^4-36a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}-\frac{-4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) - (a^2*log(tan(c/2 + (d*x)/2)))/(2*d) - (6
*a^2*atan((36*a^4)/(6*a^4 - 36*a^4*tan(c/2 + (d*x)/2)) + (6*a^4*tan(c/2 + (
```

```
d*x)/2))/(6*a^4 - 36*a^4*tan(c/2 + (d*x)/2))))/d - (20*a^2*tan(c/2 + (d*x)/
2)^3 - (7*a^2*tan(c/2 + (d*x)/2)^2)/6 + (3*a^2*tan(c/2 + (d*x)/2)^4)/2 + 12
*a^2*tan(c/2 + (d*x)/2)^5 - (15*a^2*tan(c/2 + (d*x)/2)^6)/2 - 4*a^2*tan(c/2
 + (d*x)/2)^7 + a^2/2 + 4*a^2*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^
2 + 12*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 + 4*tan(c/2 + (d*x)/2
)^8)) + (a^2*tan(c/2 + (d*x)/2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.385 $\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=98

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+a}{d}$$

```
[Out] -1/2*a^2*x+3*a^2*arctanh(cos(d*x+c))/d-2*a^2*cos(d*x+c)/d-1/3*a^2*cot(d*x+c
)^3/d-a^2*cot(d*x+c)*csc(d*x+c)/d-1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.16, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 3770, 3767, 8, 3768, 2638, 2635}

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+a}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x)/2 + (3*a^2*ArcTanh[Cos[c + d*x]])/d - (2*a^2*Cos[c + d*x])/d - (a^
2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d - (a^2*Cos[c +
d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
```

```
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-a^6-4a^6\csc(c+dx)-a^6\csc^2(c+dx)+2a^6\csc^3(c+dx)+a^6\csc^4\right.}{a^4}$$
$$= -a^2x-a^2\int \csc^2(c+dx)\,dx+a^2\int \csc^4(c+dx)\,dx+a^2\int \sin^2(c+dx$$
$$= -a^2x+\frac{4a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc}{d}$$
$$= -\frac{a^2x}{2}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-$$

Mathematica [A] time = 5.32, size = 191, normalized size = 1.95

$$\frac{a^2(\sin(c+dx)+1)^2\left(-12(c+dx)-6\sin(2(c+dx))-48\cos(c+dx)-4\tan\left(\frac{1}{2}(c+dx)\right)+4\cot\left(\frac{1}{2}(c+dx)\right)-6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(-12*(c + d*x) - 48*Cos[c + d*x] + 4*Cot[(c + d*x
)/2] - 6*Csc[(c + d*x)/2]^2 + 72*Log[Cos[(c + d*x)/2]] - 72*Log[Sin[(c + d*
x)/2]] + 6*Sec[(c + d*x)/2]^2 + 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (Csc[
(c + d*x)/2]^4*Sin[c + d*x])/2 - 6*Sin[2*(c + d*x)] - 4*Tan[(c + d*x)/2]))/
(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

fricas [B] time = 0.60, size = 192, normalized size = 1.96

$$3\,a^2\cos(dx+c)^5 - 4\,a^2\cos(dx+c)^3 + 3\,a^2\cos(dx+c) + 9\left(a^2\cos(dx+c)^2 - a^2\right)\log\left(\frac{1}{2}\,\cos(dx+c) + \frac{1}{2}\right)\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^5 - 4*a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c) + 9*(
a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a^2
*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(a^2*d
*x*cos(d*x + c)^2 + 4*a^2*cos(d*x + c)^3 - a^2*d*x - 6*a^2*cos(d*x + c))*si
n(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [B] time = 0.25, size = 209, normalized size = 2.13

$$\frac{a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 6\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 12\,(dx+c)a^2 - 72\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - 3\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right.}{24\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*(d*x +
 c)*a^2 - 72*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a^2*tan(1/2*d*x + 1/2*c
) + 24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan
(1/2*d*x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a^2*tan(1/
2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*tan(1/2*d*x + 1/2*c
) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [B] time = 0.44, size = 190, normalized size = 1.94

$$-\frac{a^2\left(\cos^5(dx+c)\right)}{d\sin(dx+c)} - \frac{a^2\left(\cos^3(dx+c)\right)\sin(dx+c)}{d} - \frac{3a^2\cos(dx+c)\sin(dx+c)}{2d} - \frac{a^2x}{2} - \frac{a^2c}{2d} - \frac{a^2\left(\cos^5(dx+c)\right)}{d\sin(dx+c)^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/sin(d*x+c)*cos(d*x+c)^5-a^2*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a^2*cos(
d*x+c)*sin(d*x+c)/d-1/2*a^2*x-1/2/d*a^2*c-1/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5
-a^2*cos(d*x+c)^3/d-3*a^2*cos(d*x+c)/d-3/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/
3*a^2*cot(d*x+c)^3/d+a^2*cot(d*x+c)/d
```

maxima [A] time = 0.45, size = 139, normalized size = 1.42

$$-\frac{3\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^2-2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-3\,a^2\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos(dx+c)+3\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/6*(3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*a^2 - 2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*a
^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x +
c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.81, size = 293, normalized size = 2.99

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,d}-\frac{3\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a^2\,\mathrm{atan}\left(\frac{a^4}{6\,a^4-a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{6\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6\,a^4-a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}-\frac{-9\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(4*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (3*a
^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*atan(a^4/(6*a^4 - a^4*tan(c/2 + (d*x)/
2)) + (6*a^4*tan(c/2 + (d*x)/2))/(6*a^4 - a^4*tan(c/2 + (d*x)/2))))/d - (36
*a^2*tan(c/2 + (d*x)/2)^3 - (a^2*tan(c/2 + (d*x)/2)^2)/3 + (19*a^2*tan(c/2
+ (d*x)/2)^4)/3 + 34*a^2*tan(c/2 + (d*x)/2)^5 - 9*a^2*tan(c/2 + (d*x)/2)^6
+ a^2/3 + 2*a^2*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 16*tan(c/2
 + (d*x)/2)^5 + 8*tan(c/2 + (d*x)/2)^7)) - (a^2*tan(c/2 + (d*x)/2))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.386 $\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{2a^2\cot(c+dx)}{d}+\frac{9a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a^2 c}{}$$

```
[Out] 2*a^2*x+9/8*a^2*arctanh(cos(d*x+c))/d-a^2*cos(d*x+c)/d+2*a^2*cot(d*x+c)/d-2
/3*a^2*cot(d*x+c)^3/d+1/8*a^2*cot(d*x+c)*csc(d*x+c)/d-1/4*a^2*cot(d*x+c)*cs
c(d*x+c)^3/d
```

Rubi [A] time = 0.18, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2872, 3770, 3767, 8, 3768, 2638}

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{2a^2\cot(c+dx)}{d}+\frac{9a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a^2 c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] 2*a^2*x + (9*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (a^2*Cos[c + d*x])/d + (2*a
^2*Cot[c + d*x])/d - (2*a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*x]*Csc[c
 + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(2a^6 - a^6\csc(c+dx) - 4a^6\csc^2(c+dx) - a^6\csc^3(c+dx) \cdots\right.}{a^4}$$
$$= 2a^2x - a^2\int \csc(c+dx)\,dx - a^2\int \csc^3(c+dx)\,dx + a^2\int \mathrm{cs}\cdots$$
$$= 2a^2x + \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cot(c+\cdots}{\cdots}$$
$$= 2a^2x + \frac{3a^2\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^2\cos(c+dx)}{d} + \frac{2a^2\cot(\cdots}{d}$$
$$= 2a^2x + \frac{9a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^2\cos(c+dx)}{d} + \frac{2a^2\cot(\cdots}{d}$$

Mathematica [A] time = 1.23, size = 215, normalized size = 1.85

$$-\frac{a^2\sin(c+dx)(\sin(c+dx)+1)^2\left(192\cot(c+dx)+\csc^4\left(\frac{1}{2}(c+dx)\right)(3\csc(c+dx)+8)-2\csc^2\left(\frac{1}{2}(c+dx)\right)(3\,\mathrm{cs}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/192*(a^2*(192*Cot[c + d*x] + Csc[(c + d*x)/2]^4*(8 + 3*Csc[c + d*x]) - 2
*Csc[(c + d*x)/2]^2*(64 + 3*Csc[c + d*x]) - 24*Csc[c + d*x]*(16*(c + d*x) +
```

```
9*Log[Cos[(c + d*x)/2]] - 9*Log[Sin[(c + d*x)/2]]) + 8*(7 + 8*Cos[c + d*x]
)*Sec[(c + d*x)/2]^4 + 24*Csc[c + d*x]^3*Sin[(c + d*x)/2]^2 - 48*Csc[c + d*
x]^5*Sin[(c + d*x)/2]^4)*Sin[c + d*x]*(1 + Sin[c + d*x])^2)/(d*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^4)
```

fricas [B] time = 0.53, size = 219, normalized size = 1.89

$$\frac{96\,a^2dx\cos\left(dx+c\right)^4-48\,a^2\cos\left(dx+c\right)^5-192\,a^2dx\cos\left(dx+c\right)^2+90\,a^2\cos\left(dx+c\right)^3+96\,a^2dx-54\,a^2\cos(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/48*(96*a^2*d*x*cos(d*x + c)^4 - 48*a^2*cos(d*x + c)^5 - 192*a^2*d*x*cos(d
*x + c)^2 + 90*a^2*cos(d*x + c)^3 + 96*a^2*d*x - 54*a^2*cos(d*x + c) + 27*(
a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2
) - 27*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x +
 c) + 1/2) - 32*(4*a^2*cos(d*x + c)^3 - 3*a^2*cos(d*x + c))*sin(d*x + c))/(
d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.28, size = 162, normalized size = 1.40

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+384\,(dx+c)a^2-216\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-240\,a^2\tan\left(\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a^2*tan(1/2*d*x + 1/2*c)^3 + 384*(
d*x + c)*a^2 - 216*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 240*a^2*tan(1/2*d*x
 + 1/2*c) - 384*a^2/(tan(1/2*d*x + 1/2*c)^2 + 1) + (450*a^2*tan(1/2*d*x + 1
/2*c)^4 + 240*a^2*tan(1/2*d*x + 1/2*c)^3 - 16*a^2*tan(1/2*d*x + 1/2*c) - 3*
a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.33, size = 149, normalized size = 1.28

$$-\frac{3a^2\left(\cos^5\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^2}-\frac{3a^2\left(\cos^3\left(dx+c\right)\right)}{8d}-\frac{9a^2\cos\left(dx+c\right)}{8d}-\frac{9a^2\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{8d}-\frac{2a^2\left(\cot^3\left(dx+\right.\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/8/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5-3/8*a^2*cos(d*x+c)^3/d-9/8*a^2*cos(d*x
+c)/d-9/8/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/3*a^2*cot(d*x+c)^3/d+2*a^2*cot(
d*x+c)/d+2*a^2*x+2/d*a^2*c-1/4/d*a^2/sin(d*x+c)^4*cos(d*x+c)^5
```

maxima [A] time = 0.43, size = 167, normalized size = 1.44

$$\frac{32\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-3\,a^2\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/48*(32*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*a^2*
(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 +
 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 12*a^2*(2*cos(d*
x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*
log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.77, size = 265, normalized size = 2.28

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{9\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{4\,a^2\,\mathrm{atan}\left(\frac{16\,a^4}{9\,a^4+16\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{9\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{9\,a^4+16\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^3)/(12*d) + (a^2*tan(c/2 + (d*x)/2)^4)/(64*d) - (9*
a^2*log(tan(c/2 + (d*x)/2)))/(8*d) - (4*a^2*atan((16*a^4)/(9*a^4 + 16*a^4*t
an(c/2 + (d*x)/2)) - (9*a^4*tan(c/2 + (d*x)/2))/(9*a^4 + 16*a^4*tan(c/2 + (
d*x)/2))))/d - (5*a^2*tan(c/2 + (d*x)/2))/(4*d) - ((a^2*tan(c/2 + (d*x)/2)^
2)/4 - (56*a^2*tan(c/2 + (d*x)/2)^3)/3 + 32*a^2*tan(c/2 + (d*x)/2)^4 - 20*a
^2*tan(c/2 + (d*x)/2)^5 + a^2/4 + (4*a^2*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(
c/2 + (d*x)/2)^4 + 16*tan(c/2 + (d*x)/2)^6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.387 $\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=118

$$-\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot^3(c+dx)\csc(c+dx)}{2d}+\frac{3a^2\cot$$

```
[Out] a^2*x-3/4*a^2*arctanh(cos(d*x+c))/d+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d
-1/5*a^2*cot(d*x+c)^5/d+3/4*a^2*cot(d*x+c)*csc(d*x+c)/d-1/2*a^2*cot(d*x+c)^
3*csc(d*x+c)/d
```

Rubi [A] time = 0.19, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 3473, 8, 2611, 3770, 2607, 30}

$$-\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot^3(c+dx)\csc(c+dx)}{2d}+\frac{3a^2\cot$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*x - (3*a^2*ArcTanh[Cos[c + d*x]])/(4*d) + (a^2*Cot[c + d*x])/d - (a^2*C
ot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]^5)/(5*d) + (3*a^2*Cot[c + d*x]*Csc
[c + d*x])/(4*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
```

```
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(a^2\cot^4(c+dx)+2a^2\cot^4(c+dx)\csc(c+dx)+a^2\cot^4\right.\\
&= a^2\int \cot^4(c+dx)\,dx+a^2\int \cot^4(c+dx)\csc^2(c+dx)\,dx+\\
&= -\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^3(c+dx)\csc(c+dx)}{2d}-a^2\int \cot^2\\
&= \frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{3a^2\,c}{}\\
&= a^2x-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{4d}+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(}{3}\end{aligned}$$

Mathematica [A] time = 0.53, size = 200, normalized size = 1.69

$$\frac{a^2\left(-272\tan\left(\frac{1}{2}(c+dx)\right)+272\cot\left(\frac{1}{2}(c+dx)\right)+150\csc^2\left(\frac{1}{2}(c+dx)\right)+15\sec^4\left(\frac{1}{2}(c+dx)\right)-150\sec^2\left(\frac{1}{2}(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(480*c + 480*d*x + 272*Cot[(c + d*x)/2] + 150*Csc[(c + d*x)/2]^2 - 360
*Log[Cos[(c + d*x)/2]] + 360*Log[Sin[(c + d*x)/2]] - 150*Sec[(c + d*x)/2]^2
 + 15*Sec[(c + d*x)/2]^4 - 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 96*Csc[c +
 d*x]^5*Sin[(c + d*x)/2]^6 + (Csc[(c + d*x)/2]^4*(-30 + Sin[c + d*x]))/2 -
(3*Csc[(c + d*x)/2]^6*Sin[c + d*x])/2 - 272*Tan[(c + d*x)/2]))/(480*d)
```

fricas [B] time = 0.52, size = 239, normalized size = 2.03

$$136\,a^2\cos(dx+c)^5 - 280\,a^2\cos(dx+c)^3 + 120\,a^2\cos(dx+c) - 45\left(a^2\cos(dx+c)^4 - 2\,a^2\cos(dx+c)^2 + a^2\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(136*a^2*cos(d*x + c)^5 - 280*a^2*cos(d*x + c)^3 + 120*a^2*cos(d*x +
c) - 45*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) + 45*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a
^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*(4*a^2*d*x*cos(d*x + c)^
4 - 8*a^2*d*x*cos(d*x + c)^2 - 5*a^2*cos(d*x + c)^3 + 4*a^2*d*x + 3*a^2*cos
(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d
*x + c))
```

giac [A] time = 0.25, size = 207, normalized size = 1.75

$$3\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 + 15\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^4 + 5\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 120\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 480\,(dx+c)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a^2*tan(1/2*d*x + 1/2*c)^4 + 5*a^2
*tan(1/2*d*x + 1/2*c)^3 - 120*a^2*tan(1/2*d*x + 1/2*c)^2 + 480*(d*x + c)*a^
2 + 360*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 270*a^2*tan(1/2*d*x + 1/2*c) -
 (822*a^2*tan(1/2*d*x + 1/2*c)^5 - 270*a^2*tan(1/2*d*x + 1/2*c)^4 - 120*a^2
*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^2*tan(1/2*d*x
 + 1/2*c) + 3*a^2)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.33, size = 170, normalized size = 1.44

$$-\frac{a^2\left(\cot^3(dx+c)\right)}{3d} + \frac{a^2\cot(dx+c)}{d} + a^2x + \frac{a^2c}{d} - \frac{a^2\left(\cos^5(dx+c)\right)}{2d\sin(dx+c)^4} + \frac{a^2\left(\cos^5(dx+c)\right)}{4d\sin(dx+c)^2} + \frac{a^2\left(\cos^3(dx+c)\right)}{4d} + \frac{3a^2\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/3*a^2*cot(d*x+c)^3/d+a^2*cot(d*x+c)/d+a^2*x+1/d*a^2*c-1/2/d*a^2/sin(d*x+
c)^4*cos(d*x+c)^5+1/4/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5+1/4*a^2*cos(d*x+c)^3/
d+3/4*a^2*cos(d*x+c)/d+3/4/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/5/d*a^2/sin(d*
x+c)^5*cos(d*x+c)^5
```

maxima [A] time = 0.45, size = 124, normalized size = 1.05

$$\frac{40\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-15\,a^2\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right.\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/120*(40*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 15*a^
2*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2
 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) - 24*a^2/tan(d*x
 + c)^5)/d
```

mupad [B] time = 9.16, size = 275, normalized size = 2.33

$$\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] (a^2*cot(c/2 + (d*x)/2)^2)/(4*d) - (a^2*cot(c/2 + (d*x)/2)^3)/(96*d) - (a^2
*cot(c/2 + (d*x)/2)^4)/(32*d) - (a^2*cot(c/2 + (d*x)/2)^5)/(160*d) - (a^2*t
an(c/2 + (d*x)/2)^2)/(4*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(96*d) + (a^2*tan(c
/2 + (d*x)/2)^4)/(32*d) + (a^2*tan(c/2 + (d*x)/2)^5)/(160*d) + (2*a^2*atan(
(4*cos(c/2 + (d*x)/2) + 3*sin(c/2 + (d*x)/2))/(3*cos(c/2 + (d*x)/2) - 4*sin
(c/2 + (d*x)/2))))/d + (3*a^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(
4*d) + (9*a^2*cot(c/2 + (d*x)/2))/(16*d) - (9*a^2*tan(c/2 + (d*x)/2))/(16*d
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.388 $\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=132

$$-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{7a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot^3(c+dx)\csc^3(c+dx)}{6d}-\frac{a^2\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{a^2\cot(c$$

```
[Out] -7/16*a^2*arctanh(cos(d*x+c))/d-2/5*a^2*cot(d*x+c)^5/d+5/16*a^2*cot(d*x+c)*
csc(d*x+c)/d-1/4*a^2*cot(d*x+c)^3*csc(d*x+c)/d+1/8*a^2*cot(d*x+c)*csc(d*x+c
)^3/d-1/6*a^2*cot(d*x+c)^3*csc(d*x+c)^3/d
```

Rubi [A] time = 0.26, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2873, 2611, 3770, 2607, 30, 3768}

$$-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{7a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot^3(c+dx)\csc^3(c+dx)}{6d}-\frac{a^2\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{a^2\cot(c$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*a^2*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) + (5*a
^2*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x])/(4
*d) + (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a^2*Cot[c + d*x]^3*Csc[c +
 d*x]^3)/(6*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)\csc(c+dx)+2a^2\cot^4(c+dx)\csc^2(c+d\right.$$
$$= a^2\int \cot^4(c+dx)\csc(c+dx)\,dx + a^2\int \cot^4(c+dx)\csc^3(c$$
$$= -\frac{a^2\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{a^2\cot^3(c+dx)\csc^3(c+dx)}{6d}$$
$$= -\frac{2a^2\cot^5(c+dx)}{5d} + \frac{3a^2\cot(c+dx)\csc(c+dx)}{8d} - \frac{a^2\cot^3(c}{}$$
$$= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{2a^2\cot^5(c+dx)}{5d} + \frac{5a^2\cot(c+}{}$$
$$= -\frac{7a^2\tanh^{-1}(\cos(c+dx))}{16d} - \frac{2a^2\cot^5(c+dx)}{5d} + \frac{5a^2\cot(c+}{}$$

Mathematica [B] time = 0.11, size = 267, normalized size = 2.02

$$a^2\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{5d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{5d} - \frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{9\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{9\sec^2\left(\frac{1}{2}(c-\right.}{64d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*(-1/5*Cot[(c + d*x)/2]/d + (9*Csc[(c + d*x)/2]^2)/(64*d) + (7*Cot[(c +
d*x)/2]*Csc[(c + d*x)/2]^2)/(80*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/
(80*d) - Csc[(c + d*x)/2]^6/(384*d) - (7*Log[Cos[(c + d*x)/2]])/(16*d) + (7
*Log[Sin[(c + d*x)/2]])/(16*d) - (9*Sec[(c + d*x)/2]^2)/(64*d) + Sec[(c + d
*x)/2]^6/(384*d) + Tan[(c + d*x)/2]/(5*d) - (7*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/(80*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(80*d))
```

fricas [A] time = 0.52, size = 211, normalized size = 1.60

$$192\, a^2 \cos\left(dx+c\right)^5 \sin\left(dx+c\right) - 270\, a^2 \cos\left(dx+c\right)^5 + 560\, a^2 \cos\left(dx+c\right)^3 - 210\, a^2 \cos\left(dx+c\right) - 105\left(a^2 \cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/480*(192*a^2*cos(d*x + c)^5*sin(d*x + c) - 270*a^2*cos(d*x + c)^5 + 560*a
^2*cos(d*x + c)^3 - 210*a^2*cos(d*x + c) - 105*(a^2*cos(d*x + c)^6 - 3*a^2*
cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2) +
105*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2
)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*
d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.29, size = 229, normalized size = 1.73

$$5\, a^2 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^6 + 24\, a^2 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^5 + 15\, a^2 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^4 - 120\, a^2 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^3 - 255\, a^2 \tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a
^2*tan(1/2*d*x + 1/2*c)^4 - 120*a^2*tan(1/2*d*x + 1/2*c)^3 - 255*a^2*tan(1/
2*d*x + 1/2*c)^2 + 840*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 240*a^2*tan(1/2
*d*x + 1/2*c) - (2058*a^2*tan(1/2*d*x + 1/2*c)^6 + 240*a^2*tan(1/2*d*x + 1/
2*c)^5 - 255*a^2*tan(1/2*d*x + 1/2*c)^4 - 120*a^2*tan(1/2*d*x + 1/2*c)^3 +
15*a^2*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*tan(1/2*d*x + 1/2*c) + 5*a^2)/tan(1/
2*d*x + 1/2*c)^6)/d
```

maple [A] time = 0.36, size = 152, normalized size = 1.15

$$-\frac{7a^2\left(\cos^5(dx+c)\right)}{24d\sin(dx+c)^4}+\frac{7a^2\left(\cos^5(dx+c)\right)}{48d\sin(dx+c)^2}+\frac{7a^2\left(\cos^3(dx+c)\right)}{48d}+\frac{7a^2\cos(dx+c)}{16d}+\frac{7a^2\ln(\csc(dx+c)-\cot(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -7/24/d*a^2/sin(d*x+c)^4*cos(d*x+c)^5+7/48/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5+
7/48*a^2*cos(d*x+c)^3/d+7/16*a^2*cos(d*x+c)/d+7/16/d*a^2*ln(csc(d*x+c)-cot(
d*x+c))-2/5/d*a^2/sin(d*x+c)^5*cos(d*x+c)^5-1/6/d*a^2/sin(d*x+c)^6*cos(d*x+
c)^5
```

maxima [A] time = 0.35, size = 181, normalized size = 1.37

$$\frac{5\,a^2\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log(\cos(dx+c)+1)+3\,\log(\cos(dx+c)-1)\right)-30\,a^2\left(\frac{2\left(5\,\cos(dx+c)\right.}{\cos(dx+c)^4-2}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/480*(5*a^2*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
 + 1) + 3*log(cos(d*x + c) - 1)) - 30*a^2*(2*(5*cos(d*x + c)^3 - 3*cos(d*x
+ c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3
*log(cos(d*x + c) - 1)) - 192*a^2/tan(d*x + c)^5)/d
```

mupad [B] time = 9.75, size = 339, normalized size = 2.57

$$a^2\left(5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+24\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+15\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] (a^2*(5*sin(c/2 + (d*x)/2)^12 - 5*cos(c/2 + (d*x)/2)^12 + 24*cos(c/2 + (d*x
)/2)*sin(c/2 + (d*x)/2)^11 - 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) +
15*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 120*cos(c/2 + (d*x)/2)^3*si
```

```
n(c/2 + (d*x)/2)^9 - 255*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 240*co
s(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 240*cos(c/2 + (d*x)/2)^7*sin(c/2
+ (d*x)/2)^5 + 255*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 120*cos(c/2
+ (d*x)/2)^9*sin(c/2 + (d*x)/2)^3 - 15*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x
)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^
6*sin(c/2 + (d*x)/2)^6))/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.389 $\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=176

$$-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{11a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{a^2\cot^3(c+dx)\csc}{6d}$$

```
[Out] -11/128*a^2*arctanh(cos(d*x+c))/d-2/5*a^2*cot(d*x+c)^5/d-2/7*a^2*cot(d*x+c)
^7/d-11/128*a^2*cot(d*x+c)*csc(d*x+c)/d+7/64*a^2*cot(d*x+c)*csc(d*x+c)^3/d-
1/6*a^2*cot(d*x+c)^3*csc(d*x+c)^3/d+1/16*a^2*cot(d*x+c)*csc(d*x+c)^5/d-1/8*
a^2*cot(d*x+c)^3*csc(d*x+c)^5/d
```

Rubi [A] time = 0.32, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2873, 2611, 3768, 3770, 2607, 14}

$$-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{11a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{a^2\cot^3(c+dx)\csc}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-11*a^2*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (2
*a^2*Cot[c + d*x]^7)/(7*d) - (11*a^2*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (
7*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x
]^3)/(6*d) + (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a^2*Cot[c + d*x]^3
*Csc[c + d*x]^5)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
```

```
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)\csc^3(c+dx)+2a^2\cot^4(c+dx)\csc^4(c+\right.$$

$$= a^2\int \cot^4(c+dx)\csc^3(c+dx)\,dx + a^2\int \cot^4(c+dx)\csc^5$$

$$= -\frac{a^2\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{a^2\cot^3(c+dx)\csc^5(c+dx}{8d}$$

$$= \frac{a^2\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a^2\cot^3(c+dx)\csc^3(c+dx)}{6d}$$

$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot(c+dx)\csc(}{16d}$$

$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{16d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c-}{7d}$$

$$= -\frac{11a^2\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(}{7d}$$

Mathematica [A] time = 0.96, size = 291, normalized size = 1.65

$$-\frac{a^2 \csc^8(c+dx)\Big(86016\sin(2(c+dx))+64512\sin(4(c+dx))+12288\sin(6(c+dx))-1536\sin(8(c+dx))+158$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/1720320*(a^2*Csc[c + d*x]^8*(158270*Cos[c + d*x] + 77210*Cos[3*(c + d*x)
] - 18130*Cos[5*(c + d*x)] - 2310*Cos[7*(c + d*x)] + 40425*Log[Cos[(c + d*x
)/2]] - 64680*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 32340*Cos[4*(c + d*x
)]*Log[Cos[(c + d*x)/2]] - 9240*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 11
55*Cos[8*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 40425*Log[Sin[(c + d*x)/2]] + 6
4680*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 32340*Cos[4*(c + d*x)]*Log[Si
n[(c + d*x)/2]] + 9240*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 1155*Cos[8*
(c + d*x)]*Log[Sin[(c + d*x)/2]] + 86016*Sin[2*(c + d*x)] + 64512*Sin[4*(c
+ d*x)] + 12288*Sin[6*(c + d*x)] - 1536*Sin[8*(c + d*x)]))/d
```

fricas [A] time = 0.52, size = 271, normalized size = 1.54

$$\frac{2310\,a^2\cos(dx+c)^7+490\,a^2\cos(dx+c)^5-8470\,a^2\cos(dx+c)^3+2310\,a^2\cos(dx+c)-1155\Big(a^2\cos(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/26880*(2310*a^2*cos(d*x + c)^7 + 490*a^2*cos(d*x + c)^5 - 8470*a^2*cos(d*
x + c)^3 + 2310*a^2*cos(d*x + c) - 1155*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x
 + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*
x + c) + 1/2) + 1155*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos
(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x + c) + 1/2) + 15
36*(2*a^2*cos(d*x + c)^7 - 7*a^2*cos(d*x + c)^5)*sin(d*x + c))/(d*cos(d*x +
 c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.30, size = 293, normalized size = 1.66

$$\frac{105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+480\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+560\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-672\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2520\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/215040*(105*a^2*tan(1/2*d*x + 1/2*c)^8 + 480*a^2*tan(1/2*d*x + 1/2*c)^7 +
 560*a^2*tan(1/2*d*x + 1/2*c)^6 - 672*a^2*tan(1/2*d*x + 1/2*c)^5 - 2520*a^2
*tan(1/2*d*x + 1/2*c)^4 - 3360*a^2*tan(1/2*d*x + 1/2*c)^3 - 1680*a^2*tan(1/
2*d*x + 1/2*c)^2 + 18480*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 10080*a^2*tan
(1/2*d*x + 1/2*c) - (50226*a^2*tan(1/2*d*x + 1/2*c)^8 + 10080*a^2*tan(1/2*d
*x + 1/2*c)^7 - 1680*a^2*tan(1/2*d*x + 1/2*c)^6 - 3360*a^2*tan(1/2*d*x + 1/
2*c)^5 - 2520*a^2*tan(1/2*d*x + 1/2*c)^4 - 672*a^2*tan(1/2*d*x + 1/2*c)^3 +
 560*a^2*tan(1/2*d*x + 1/2*c)^2 + 480*a^2*tan(1/2*d*x + 1/2*c) + 105*a^2)/t
an(1/2*d*x + 1/2*c)^8)/d
```

maple [A] time = 0.39, size = 200, normalized size = 1.14

$$-\frac{11a^2\left(\cos^5(dx+c)\right)}{48d\sin(dx+c)^6}-\frac{11a^2\left(\cos^5(dx+c)\right)}{192d\sin(dx+c)^4}+\frac{11a^2\left(\cos^5(dx+c)\right)}{384d\sin(dx+c)^2}+\frac{11a^2\left(\cos^3(dx+c)\right)}{384d}+\frac{11a^2\cos(dx+c)}{128d}+\frac{11a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -11/48/d*a^2/sin(d*x+c)^6*cos(d*x+c)^5-11/192/d*a^2/sin(d*x+c)^4*cos(d*x+c)
^5+11/384/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5+11/384*a^2*cos(d*x+c)^3/d+11/128*
a^2*cos(d*x+c)/d+11/128/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/7/d*a^2/sin(d*x+c
)^7*cos(d*x+c)^5-4/35/d*a^2/sin(d*x+c)^5*cos(d*x+c)^5-1/8/d*a^2/sin(d*x+c)^
8*cos(d*x+c)^5
```

maxima [A] time = 0.35, size = 233, normalized size = 1.32

$$\frac{105\,a^2\left(\frac{2\left(3\,\cos(dx+c)^7-11\,\cos(dx+c)^5-11\,\cos(dx+c)^3+3\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-3\,\log\left(\cos(dx+c)+1\right)+3\,\log\left(\cos(dx+c)-1\right)\right)+2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/26880*(105*a^2*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)
^3 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)
) + 280*a^2*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(
d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
+ 1) + 3*log(cos(d*x + c) - 1)) - 1536*(7*tan(d*x + c)^2 + 5)*a^2/tan(d*x +
 c)^7)/d
```

mupad [B] time = 9.12, size = 319, normalized size = 1.81

$$\frac{a^2 \cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{a^2 \cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{64\,d}+\frac{3\,a^2 \cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}+\frac{a^2 \cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{320\,d}-\frac{a^2 \cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}-\frac{a^2 \cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{448\,d}-- a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^9,x)
```

```
[Out] (a^2*cot(c/2 + (d*x)/2)^2)/(128*d) + (a^2*cot(c/2 + (d*x)/2)^3)/(64*d) + (3
*a^2*cot(c/2 + (d*x)/2)^4)/(256*d) + (a^2*cot(c/2 + (d*x)/2)^5)/(320*d) - (
a^2*cot(c/2 + (d*x)/2)^6)/(384*d) - (a^2*cot(c/2 + (d*x)/2)^7)/(448*d) - (a
^2*cot(c/2 + (d*x)/2)^8)/(2048*d) - (a^2*tan(c/2 + (d*x)/2)^2)/(128*d) - (a
^2*tan(c/2 + (d*x)/2)^3)/(64*d) - (3*a^2*tan(c/2 + (d*x)/2)^4)/(256*d) - (a
^2*tan(c/2 + (d*x)/2)^5)/(320*d) + (a^2*tan(c/2 + (d*x)/2)^6)/(384*d) + (a^
2*tan(c/2 + (d*x)/2)^7)/(448*d) + (a^2*tan(c/2 + (d*x)/2)^8)/(2048*d) + (11
*a^2*log(tan(c/2 + (d*x)/2)))/(128*d) - (3*a^2*cot(c/2 + (d*x)/2))/(64*d) +
 (3*a^2*tan(c/2 + (d*x)/2))/(64*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.390 $\int \cot^4(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=168

$$-\frac{a^2\cot^9(c+dx)}{9d}-\frac{3a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{64d}-\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{4d}+$$

```
[Out] -3/64*a^2*arctanh(cos(d*x+c))/d-2/5*a^2*cot(d*x+c)^5/d-3/7*a^2*cot(d*x+c)^7
/d-1/9*a^2*cot(d*x+c)^9/d-3/64*a^2*cot(d*x+c)*csc(d*x+c)/d-1/32*a^2*cot(d*x
+c)*csc(d*x+c)^3/d+1/8*a^2*cot(d*x+c)*csc(d*x+c)^5/d-1/4*a^2*cot(d*x+c)^3*c
sc(d*x+c)^5/d
```

Rubi [A] time = 0.27, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2607, 14, 2611, 3768, 3770, 270}

$$-\frac{a^2\cot^9(c+dx)}{9d}-\frac{3a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{64d}-\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*a^2*ArcTanh[Cos[c + d*x]])/(64*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (3*a
^2*Cot[c + d*x]^7)/(7*d) - (a^2*Cot[c + d*x]^9)/(9*d) - (3*a^2*Cot[c + d*x]
*Csc[c + d*x])/(64*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(32*d) + (a^2*Cot
[c + d*x]*Csc[c + d*x]^5)/(8*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x]^5)/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)\csc^4(c+dx) + 2a^2\cot^4(c+dx)\csc^5(c+\right.$$

$$= a^2\int \cot^4(c+dx)\csc^4(c+dx)\,dx + a^2\int \cot^4(c+dx)\csc^6$$

$$= -\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{4d} - \frac{1}{4}\left(3a^2\right)\int \cot^2(c+dx)\csc$$

$$= \frac{a^2\cot(c+dx)\csc^5(c+dx)}{8d} - \frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{4d}$$

$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{3a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} -$$

$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{3a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} -$$

$$= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{64d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{3a^2\cot^7(c}{7d}$$

Mathematica [A] time = 1.37, size = 313, normalized size = 1.86

$$-\frac{a^2\csc^9(c+dx)\left(212940\sin(2(c+dx)) + 195300\sin(4(c+dx)) + 16380\sin(6(c+dx)) - 1890\sin(8(c+dx)) +\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/5160960*(a^2*Csc[c + d*x]^9*(451584*Cos[c + d*x] + 155904*Cos[3*(c + d*x
)] - 20736*Cos[5*(c + d*x)] - 14976*Cos[7*(c + d*x)] + 1664*Cos[9*(c + d*x)
] + 119070*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 119070*Log[Sin[(c + d*x)/2]
]*Sin[c + d*x] + 212940*Sin[2*(c + d*x)] - 79380*Log[Cos[(c + d*x)/2]]*Sin[
3*(c + d*x)] + 79380*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 195300*Sin[4*
(c + d*x)] + 34020*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 34020*Log[Sin[(
c + d*x)/2]]*Sin[5*(c + d*x)] + 16380*Sin[6*(c + d*x)] - 8505*Log[Cos[(c +
d*x)/2]]*Sin[7*(c + d*x)] + 8505*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] - 1
890*Sin[8*(c + d*x)] + 945*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)] - 945*Log
[Sin[(c + d*x)/2]]*Sin[9*(c + d*x)]))/d
```

fricas [A] time = 0.50, size = 304, normalized size = 1.81

$$-\frac{3328\,a^2\cos(dx+c)^9 - 14976\,a^2\cos(dx+c)^7 + 16128\,a^2\cos(dx+c)^5 + 945\left(a^2\cos(dx+c)^8 - 4\,a^2\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/40320*(3328*a^2*cos(d*x + c)^9 - 14976*a^2*cos(d*x + c)^7 + 16128*a^2*co
s(d*x + c)^5 + 945*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d
*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) - 945*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)
^4 - 4*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
- 630*(3*a^2*cos(d*x + c)^7 - 11*a^2*cos(d*x + c)^5 - 11*a^2*cos(d*x + c)^3
 + 3*a^2*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^
6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.30, size = 261, normalized size = 1.55

$$\frac{70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+315\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+450\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-1008\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2520\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/322560*(70*a^2*tan(1/2*d*x + 1/2*c)^9 + 315*a^2*tan(1/2*d*x + 1/2*c)^8 +
450*a^2*tan(1/2*d*x + 1/2*c)^7 - 1008*a^2*tan(1/2*d*x + 1/2*c)^5 - 2520*a^2
*tan(1/2*d*x + 1/2*c)^4 - 3360*a^2*tan(1/2*d*x + 1/2*c)^3 + 15120*a^2*log(a
bs(tan(1/2*d*x + 1/2*c))) + 11340*a^2*tan(1/2*d*x + 1/2*c) - (42774*a^2*tan
(1/2*d*x + 1/2*c)^9 + 11340*a^2*tan(1/2*d*x + 1/2*c)^8 - 3360*a^2*tan(1/2*d
*x + 1/2*c)^6 - 2520*a^2*tan(1/2*d*x + 1/2*c)^5 - 1008*a^2*tan(1/2*d*x + 1/
2*c)^4 + 450*a^2*tan(1/2*d*x + 1/2*c)^2 + 315*a^2*tan(1/2*d*x + 1/2*c) + 70
*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

maple [A] time = 0.41, size = 224, normalized size = 1.33

$$-\frac{13a^2\left(\cos^5(dx+c)\right)}{63d\sin(dx+c)^7}-\frac{26a^2\left(\cos^5(dx+c)\right)}{315d\sin(dx+c)^5}-\frac{a^2\left(\cos^5(dx+c)\right)}{4d\sin(dx+c)^8}-\frac{a^2\left(\cos^5(dx+c)\right)}{8d\sin(dx+c)^6}-\frac{a^2\left(\cos^5(dx+c)\right)}{32d\sin(dx+c)^4}+\frac{a^2\left(\cos^5\right.}{64d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -13/63/d*a^2/sin(d*x+c)^7*cos(d*x+c)^5-26/315/d*a^2/sin(d*x+c)^5*cos(d*x+c)
^5-1/4/d*a^2/sin(d*x+c)^8*cos(d*x+c)^5-1/8/d*a^2/sin(d*x+c)^6*cos(d*x+c)^5-
```

```
1/32/d*a^2/sin(d*x+c)^4*cos(d*x+c)^5+1/64/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5+1
/64*a^2*cos(d*x+c)^3/d+3/64*a^2*cos(d*x+c)/d+3/64/d*a^2*ln(csc(d*x+c)-cot(d
*x+c))-1/9/d*a^2/sin(d*x+c)^9*cos(d*x+c)^5
```

maxima [A] time = 0.34, size = 177, normalized size = 1.05

$$\frac{315\,a^2\left(\frac{2\left(3\,\cos(dx+c)^7-11\,\cos(dx+c)^5-11\,\cos(dx+c)^3+3\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] 1/40320*(315*a^2*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)
^3 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)
) - 1152*(7*tan(d*x + c)^2 + 5)*a^2/tan(d*x + c)^7 - 128*(63*tan(d*x + c)^4
 + 90*tan(d*x + c)^2 + 35)*a^2/tan(d*x + c)^9)/d
```

mupad [B] time = 12.07, size = 387, normalized size = 2.30

$$a^2\left(70\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-70\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+315\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}-315\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^10,x)
```

```
[Out] (a^2*(70*sin(c/2 + (d*x)/2)^18 - 70*cos(c/2 + (d*x)/2)^18 + 315*cos(c/2 + (
d*x)/2)*sin(c/2 + (d*x)/2)^17 - 315*cos(c/2 + (d*x)/2)^17*sin(c/2 + (d*x)/2
) + 450*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^16 - 1008*cos(c/2 + (d*x)/2
)^4*sin(c/2 + (d*x)/2)^14 - 2520*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^13
 - 3360*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^12 + 11340*cos(c/2 + (d*x)/
2)^8*sin(c/2 + (d*x)/2)^10 - 11340*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)
^8 + 3360*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^6 + 2520*cos(c/2 + (d*x)
/2)^13*sin(c/2 + (d*x)/2)^5 + 1008*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)
^4 - 450*cos(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^2 + 15120*log(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^9))/(32
2560*d*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**10*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.391 $\int \cot^4(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=218

$$-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{4a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{9a^2\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^2\cot^3(c+dx)\csc^7(c+dx)}{10d}-$$

```
[Out] -9/256*a^2*arctanh(cos(d*x+c))/d-2/5*a^2*cot(d*x+c)^5/d-4/7*a^2*cot(d*x+c)^
7/d-2/9*a^2*cot(d*x+c)^9/d-9/256*a^2*cot(d*x+c)*csc(d*x+c)/d-3/128*a^2*cot(
d*x+c)*csc(d*x+c)^3/d+9/160*a^2*cot(d*x+c)*csc(d*x+c)^5/d-1/8*a^2*cot(d*x+c
)^3*csc(d*x+c)^5/d+3/80*a^2*cot(d*x+c)*csc(d*x+c)^7/d-1/10*a^2*cot(d*x+c)^3
*csc(d*x+c)^7/d
```

Rubi [A] time = 0.35, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3768, 3770, 2607, 270}

$$-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{4a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{9a^2\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^2\cot^3(c+dx)\csc^7(c+dx)}{10d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-9*a^2*ArcTanh[Cos[c + d*x]])/(256*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (4*
a^2*Cot[c + d*x]^7)/(7*d) - (2*a^2*Cot[c + d*x]^9)/(9*d) - (9*a^2*Cot[c + d
*x]*Csc[c + d*x])/(256*d) - (3*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (
9*a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(160*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*
x]^5)/(8*d) + (3*a^2*Cot[c + d*x]*Csc[c + d*x]^7)/(80*d) - (a^2*Cot[c + d*x
]^3*Csc[c + d*x]^7)/(10*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)\csc^5(c+dx) + 2a^2\cot^4(c+dx)\csc^6(c+\right.$$
$$= a^2\int \cot^4(c+dx)\csc^5(c+dx)\,dx + a^2\int \cot^4(c+dx)\csc^7$$
$$= -\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{a^2\cot^3(c+dx)\csc^7(c+dx)}{10d}$$
$$= \frac{a^2\cot(c+dx)\csc^5(c+dx)}{16d} - \frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d}$$
$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{4a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} -$$
$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{4a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} -$$
$$= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{4a^2\cot^7(c}{7d}$$
$$= -\frac{9a^2\tanh^{-1}(\cos(c+dx))}{256d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{4a^2\cot^7(c}{7d}$$

Mathematica [A] time = 1.22, size = 353, normalized size = 1.62

$$-\frac{a^2\csc^{10}(c+dx)\left(1720320\sin(2(c+dx)) + 1228800\sin(4(c+dx)) + 184320\sin(6(c+dx)) - 40960\sin(8(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/41287680*(a^2*Csc[c + d*x]^10*(3219300*Cos[c + d*x] + 1237320*Cos[3*(c +
 d*x)] - 278712*Cos[5*(c + d*x)] - 54810*Cos[7*(c + d*x)] + 5670*Cos[9*(c +
 d*x)] + 357210*Log[Cos[(c + d*x)/2]] - 595350*Cos[2*(c + d*x)]*Log[Cos[(c
+ d*x)/2]] + 340200*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 127575*Cos[6*(
c + d*x)]*Log[Cos[(c + d*x)/2]] + 28350*Cos[8*(c + d*x)]*Log[Cos[(c + d*x)/
2]] - 2835*Cos[10*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 357210*Log[Sin[(c + d*
x)/2]] + 595350*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 340200*Cos[4*(c +
d*x)]*Log[Sin[(c + d*x)/2]] + 127575*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]]
 - 28350*Cos[8*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 2835*Cos[10*(c + d*x)]*Lo
g[Sin[(c + d*x)/2]] + 1720320*Sin[2*(c + d*x)] + 1228800*Sin[4*(c + d*x)] +
 184320*Sin[6*(c + d*x)] - 40960*Sin[8*(c + d*x)] + 4096*Sin[10*(c + d*x)])
)/d
```

fricas [A] time = 0.53, size = 340, normalized size = 1.56

$$5670\,a^2\cos\left(dx+c\right)^9 - 26460\,a^2\cos\left(dx+c\right)^7 + 16128\,a^2\cos\left(dx+c\right)^5 + 26460\,a^2\cos\left(dx+c\right)^3 - 5670\,a^2\cos\left(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/161280*(5670*a^2*cos(d*x + c)^9 - 26460*a^2*cos(d*x + c)^7 + 16128*a^2*co
s(d*x + c)^5 + 26460*a^2*cos(d*x + c)^3 - 5670*a^2*cos(d*x + c) - 2835*(a^2
*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 10*a^2*co
s(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2) + 28
35*(a^2*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 10
*a^2*cos(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1
/2) + 1024*(8*a^2*cos(d*x + c)^9 - 36*a^2*cos(d*x + c)^7 + 63*a^2*cos(d*x +
 c)^5)*sin(d*x + c))/(d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x
 + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.32, size = 357, normalized size = 1.64

$$126\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10} + 560\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 945\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8 + 720\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 630\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/1290240*(126*a^2*tan(1/2*d*x + 1/2*c)^10 + 560*a^2*tan(1/2*d*x + 1/2*c)^9
 + 945*a^2*tan(1/2*d*x + 1/2*c)^8 + 720*a^2*tan(1/2*d*x + 1/2*c)^7 - 630*a^
2*tan(1/2*d*x + 1/2*c)^6 - 4032*a^2*tan(1/2*d*x + 1/2*c)^5 - 7560*a^2*tan(1
/2*d*x + 1/2*c)^4 - 6720*a^2*tan(1/2*d*x + 1/2*c)^3 + 1260*a^2*tan(1/2*d*x
+ 1/2*c)^2 + 45360*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 30240*a^2*tan(1/2*d
*x + 1/2*c) - (132858*a^2*tan(1/2*d*x + 1/2*c)^10 + 30240*a^2*tan(1/2*d*x +
 1/2*c)^9 + 1260*a^2*tan(1/2*d*x + 1/2*c)^8 - 6720*a^2*tan(1/2*d*x + 1/2*c)
^7 - 7560*a^2*tan(1/2*d*x + 1/2*c)^6 - 4032*a^2*tan(1/2*d*x + 1/2*c)^5 - 63
0*a^2*tan(1/2*d*x + 1/2*c)^4 + 720*a^2*tan(1/2*d*x + 1/2*c)^3 + 945*a^2*tan
(1/2*d*x + 1/2*c)^2 + 560*a^2*tan(1/2*d*x + 1/2*c) + 126*a^2)/tan(1/2*d*x +
 1/2*c)^10)/d
```

maple [A] time = 0.41, size = 248, normalized size = 1.14

$$-\frac{3a^2\left(\cos^5(dx+c)\right)}{16d\sin(dx+c)^8}-\frac{3a^2\left(\cos^5(dx+c)\right)}{32d\sin(dx+c)^6}-\frac{3a^2\left(\cos^5(dx+c)\right)}{128d\sin(dx+c)^4}+\frac{3a^2\left(\cos^5(dx+c)\right)}{256d\sin(dx+c)^2}+\frac{3a^2\left(\cos^3(dx+c)\right)}{256d}+\frac{9a^2 c\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/16/d*a^2/sin(d*x+c)^8*cos(d*x+c)^5-3/32/d*a^2/sin(d*x+c)^6*cos(d*x+c)^5-
3/128/d*a^2/sin(d*x+c)^4*cos(d*x+c)^5+3/256/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5
+3/256*a^2*cos(d*x+c)^3/d+9/256*a^2*cos(d*x+c)/d+9/256/d*a^2*ln(csc(d*x+c)-
cot(d*x+c))-2/9/d*a^2/sin(d*x+c)^9*cos(d*x+c)^5-8/63/d*a^2/sin(d*x+c)^7*cos
(d*x+c)^5-16/315/d*a^2/sin(d*x+c)^5*cos(d*x+c)^5-1/10/d*a^2/sin(d*x+c)^10*c
os(d*x+c)^5
```

maxima [A] time = 0.33, size = 283, normalized size = 1.30

$$\frac{63\,a^2\left(\frac{2\left(15\,\cos(dx+c)^9-70\,\cos(dx+c)^7+128\,\cos(dx+c)^5+70\,\cos(dx+c)^3-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^8+10\,\cos(dx+c)^6-10\,\cos(dx+c)^4+5\,\cos(dx+c)^2-1}-15\,\log\left(\cos(dx+c)+1\right)+15\,\log\left(\cos(\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] 1/161280*(63*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 + 128*cos(d*x +
c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x +
c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*l
og(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 630*a^2*(2*(3*cos(d*x +
c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3 + 3*cos(d*x + c))/(cos(d*x + c
)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 3*log(c
os(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 1024*(63*tan(d*x + c)^4 + 90*
tan(d*x + c)^2 + 35)*a^2/tan(d*x + c)^9)/d
```

mupad [B] time = 9.52, size = 395, normalized size = 1.81

$$\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{192\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}+\frac{3\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{512\,d}+\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{320\,d}+\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2048\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{1792\,d}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^11,x)
```

```
[Out] (a^2*cot(c/2 + (d*x)/2)^3)/(192*d) - (a^2*cot(c/2 + (d*x)/2)^2)/(1024*d) +
(3*a^2*cot(c/2 + (d*x)/2)^4)/(512*d) + (a^2*cot(c/2 + (d*x)/2)^5)/(320*d) +
 (a^2*cot(c/2 + (d*x)/2)^6)/(2048*d) - (a^2*cot(c/2 + (d*x)/2)^7)/(1792*d)
- (3*a^2*cot(c/2 + (d*x)/2)^8)/(4096*d) - (a^2*cot(c/2 + (d*x)/2)^9)/(2304*
d) - (a^2*cot(c/2 + (d*x)/2)^10)/(10240*d) + (a^2*tan(c/2 + (d*x)/2)^2)/(10
24*d) - (a^2*tan(c/2 + (d*x)/2)^3)/(192*d) - (3*a^2*tan(c/2 + (d*x)/2)^4)/(
512*d) - (a^2*tan(c/2 + (d*x)/2)^5)/(320*d) - (a^2*tan(c/2 + (d*x)/2)^6)/(2
048*d) + (a^2*tan(c/2 + (d*x)/2)^7)/(1792*d) + (3*a^2*tan(c/2 + (d*x)/2)^8)
/(4096*d) + (a^2*tan(c/2 + (d*x)/2)^9)/(2304*d) + (a^2*tan(c/2 + (d*x)/2)^1
0)/(10240*d) + (9*a^2*log(tan(c/2 + (d*x)/2)))/(256*d) - (3*a^2*cot(c/2 + (
d*x)/2))/(128*d) + (3*a^2*tan(c/2 + (d*x)/2))/(128*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**11*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.392 $\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=203

$$\frac{a^3\cos^{11}(c+dx)}{11d}-\frac{2a^3\cos^9(c+dx)}{3d}+\frac{9a^3\cos^7(c+dx)}{7d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{3a^3\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{5a^3\sin}{}$$

```
[Out] 15/256*a^3*x-4/5*a^3*cos(d*x+c)^5/d+9/7*a^3*cos(d*x+c)^7/d-2/3*a^3*cos(d*x+
c)^9/d+1/11*a^3*cos(d*x+c)^11/d+15/256*a^3*cos(d*x+c)*sin(d*x+c)/d+5/128*a^
3*cos(d*x+c)^3*sin(d*x+c)/d-5/32*a^3*cos(d*x+c)^5*sin(d*x+c)/d-5/16*a^3*cos
(d*x+c)^5*sin(d*x+c)^3/d-3/10*a^3*cos(d*x+c)^5*sin(d*x+c)^5/d
```

Rubi [A] time = 0.39, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2568, 2635, 8, 2565, 270}

$$\frac{a^3\cos^{11}(c+dx)}{11d}-\frac{2a^3\cos^9(c+dx)}{3d}+\frac{9a^3\cos^7(c+dx)}{7d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{3a^3\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{5a^3\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (15*a^3*x)/256 - (4*a^3*Cos[c + d*x]^5)/(5*d) + (9*a^3*Cos[c + d*x]^7)/(7*d
) - (2*a^3*Cos[c + d*x]^9)/(3*d) + (a^3*Cos[c + d*x]^11)/(11*d) + (15*a^3*C
os[c + d*x]*Sin[c + d*x])/(256*d) + (5*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(12
8*d) - (5*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(32*d) - (5*a^3*Cos[c + d*x]^5*S
in[c + d*x]^3)/(16*d) - (3*a^3*Cos[c + d*x]^5*Sin[c + d*x]^5)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^4(c+dx)\sin^4(c+dx)+3a^3\cos^4(c+dx)\sin^5(c+\right.$$

$$= a^3\int \cos^4(c+dx)\sin^4(c+dx)\,dx + a^3\int \cos^4(c+dx)\sin^7($$

$$= -\frac{a^3\cos^5(c+dx)\sin^3(c+dx)}{8d} - \frac{3a^3\cos^5(c+dx)\sin^5(c+d}{10d}$$

$$= -\frac{a^3\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{5a^3\cos^5(c+dx)\sin^3(c+dx}{16d}$$

$$= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos^7(c+dx)}{7d} - \frac{2a^3\cos^9(c+dx)}{3d} +$$

$$= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos^7(c+dx)}{7d} - \frac{2a^3\cos^9(c+dx)}{3d} +$$

$$= \frac{3a^3x}{128} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos^7(c+dx)}{7d} - \frac{2a^3\cos^9(c+}{3d}$$

$$= \frac{15a^3x}{256} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos^7(c+dx)}{7d} - \frac{2a^3\cos^9(c}{3d}$$

Mathematica [A] time = 1.04, size = 126, normalized size = 0.62

$$a^3(-13860\sin(2(c+dx)) - 46200\sin(4(c+dx)) + 6930\sin(6(c+dx)) + 5775\sin(8(c+dx)) - 1386\sin(10(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(138600*c + 138600*d*x - 198660*Cos[c + d*x] - 41580*Cos[3*(c + d*x)]
+ 27258*Cos[5*(c + d*x)] + 3630*Cos[7*(c + d*x)] - 3850*Cos[9*(c + d*x)] +
210*Cos[11*(c + d*x)] - 13860*Sin[2*(c + d*x)] - 46200*Sin[4*(c + d*x)] + 6
930*Sin[6*(c + d*x)] + 5775*Sin[8*(c + d*x)] - 1386*Sin[10*(c + d*x)]))/(23
65440*d)
```

fricas [A] time = 0.66, size = 137, normalized size = 0.67

$$26880\,a^3\cos(dx+c)^{11} - 197120\,a^3\cos(dx+c)^9 + 380160\,a^3\cos(dx+c)^7 - 236544\,a^3\cos(dx+c)^5 + 17325\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/295680*(26880*a^3*cos(d*x + c)^11 - 197120*a^3*cos(d*x + c)^9 + 380160*a^
3*cos(d*x + c)^7 - 236544*a^3*cos(d*x + c)^5 + 17325*a^3*d*x - 231*(384*a^3
*cos(d*x + c)^9 - 1168*a^3*cos(d*x + c)^7 + 984*a^3*cos(d*x + c)^5 - 50*a^3
*cos(d*x + c)^3 - 75*a^3*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.42, size = 191, normalized size = 0.94

$$\frac{15}{256}\,a^3x + \frac{a^3\cos(11\,dx+11\,c)}{11264\,d} - \frac{5\,a^3\cos(9\,dx+9\,c)}{3072\,d} + \frac{11\,a^3\cos(7\,dx+7\,c)}{7168\,d} + \frac{59\,a^3\cos(5\,dx+5\,c)}{5120\,d} - \frac{9\,a^3\cos(3\,dx}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 15/256*a^3*x + 1/11264*a^3*cos(11*d*x + 11*c)/d - 5/3072*a^3*cos(9*d*x + 9*
c)/d + 11/7168*a^3*cos(7*d*x + 7*c)/d + 59/5120*a^3*cos(5*d*x + 5*c)/d - 9/
512*a^3*cos(3*d*x + 3*c)/d - 43/512*a^3*cos(d*x + c)/d - 3/5120*a^3*sin(10*
d*x + 10*c)/d + 5/2048*a^3*sin(8*d*x + 8*c)/d + 3/1024*a^3*sin(6*d*x + 6*c)
/d - 5/256*a^3*sin(4*d*x + 4*c)/d - 3/512*a^3*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.29, size = 288, normalized size = 1.42

$$a^3\left(-\frac{(\sin^6(dx+c))(\cos^5(dx+c))}{11} - \frac{2(\sin^4(dx+c))(\cos^5(dx+c))}{33} - \frac{8(\sin^2(dx+c))(\cos^5(dx+c))}{231} - \frac{16(\cos^5(dx+c))}{1155}\right) + 3a^3\left(-\frac{(\sin^5(dx+c))(\cos^5}{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/11*sin(d*x+c)^6*cos(d*x+c)^5-2/33*sin(d*x+c)^4*cos(d*x+c)^5-8/
231*sin(d*x+c)^2*cos(d*x+c)^5-16/1155*cos(d*x+c)^5)+3*a^3*(-1/10*sin(d*x+c)
^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1/32*sin(d*x+c)*cos(d*x+c)^5
+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)+3*a^3*(-
1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+
c)^5)+a^3*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64
*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c))
```

maxima [A] time = 0.34, size = 169, normalized size = 0.83

$$\frac{2048\left(105\cos(dx+c)^{11}-385\cos(dx+c)^{9}+495\cos(dx+c)^{7}-231\cos(dx+c)^{5}\right)a^{3}-22528\left(35\cos(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/2365440*(2048*(105*cos(d*x + c)^11 - 385*cos(d*x + c)^9 + 495*cos(d*x + c
)^7 - 231*cos(d*x + c)^5)*a^3 - 22528*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^
7 + 63*cos(d*x + c)^5)*a^3 - 693*(32*sin(2*d*x + 2*c)^5 - 120*d*x - 120*c -
 5*sin(8*d*x + 8*c) + 40*sin(4*d*x + 4*c))*a^3 + 2310*(24*d*x + 24*c + sin(
8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^3)/d
```

mupad [B] time = 11.72, size = 506, normalized size = 2.49

$$\frac{15\,a^3\,x}{256}-\frac{\frac{15\,a^3\,(c+d\,x)}{256}+\frac{5\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}-\frac{231\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640}-\frac{242\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{5}+\frac{3987\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{64}-\frac{3987\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^4*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (15*a^3*x)/256 - ((15*a^3*(c + d*x))/256 + (5*a^3*tan(c/2 + (d*x)/2)^3)/4 -
 (231*a^3*tan(c/2 + (d*x)/2)^5)/640 - (242*a^3*tan(c/2 + (d*x)/2)^7)/5 + (3
987*a^3*tan(c/2 + (d*x)/2)^9)/64 - (3987*a^3*tan(c/2 + (d*x)/2)^13)/64 + (2
42*a^3*tan(c/2 + (d*x)/2)^15)/5 + (231*a^3*tan(c/2 + (d*x)/2)^17)/640 - (5*
a^3*tan(c/2 + (d*x)/2)^19)/4 - (15*a^3*tan(c/2 + (d*x)/2)^21)/128 - (a^3*(1
7325*c + 17325*d*x - 53248))/295680 + tan(c/2 + (d*x)/2)^2*((165*a^3*(c + d
*x))/256 - (a^3*(190575*c + 190575*d*x - 585728))/295680) + tan(c/2 + (d*x)
```

```
/2)^4*((825*a^3*(c + d*x))/256 - (a^3*(952875*c + 952875*d*x - 2928640))/29
5680) + tan(c/2 + (d*x)/2)^6*((2475*a^3*(c + d*x))/256 - (a^3*(2858625*c +
2858625*d*x + 675840))/295680) + tan(c/2 + (d*x)/2)^8*((2475*a^3*(c + d*x))
/128 - (a^3*(5717250*c + 5717250*d*x - 3379200))/295680) + tan(c/2 + (d*x)/
2)^16*((2475*a^3*(c + d*x))/256 - (a^3*(2858625*c + 2858625*d*x - 9461760))
/295680) + tan(c/2 + (d*x)/2)^14*((2475*a^3*(c + d*x))/128 - (a^3*(5717250*
c + 5717250*d*x - 14192640))/295680) + tan(c/2 + (d*x)/2)^12*((3465*a^3*(c
+ d*x))/128 - (a^3*(8004150*c + 8004150*d*x + 16084992))/295680) + tan(c/2
+ (d*x)/2)^10*((3465*a^3*(c + d*x))/128 - (a^3*(8004150*c + 8004150*d*x - 4
0685568))/295680) + (15*a^3*tan(c/2 + (d*x)/2))/128)/(d*(tan(c/2 + (d*x)/2)
^2 + 1)^11)
```

sympy [A] time = 50.88, size = 648, normalized size = 3.19

$$\begin{cases} \frac{9a^3x\sin^{10}(c+dx)}{256} + \frac{45a^3x\sin^8(c+dx)\cos^2(c+dx)}{256} + \frac{3a^3x\sin^8(c+dx)}{128} + \frac{45a^3x\sin^6(c+dx)\cos^4(c+dx)}{128} + \frac{3a^3x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{45a}{} \\ x(a\sin(c)+a)^3\sin^4(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((9*a**3*x*sin(c + d*x)**10/256 + 45*a**3*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 3*a**3*x*sin(c + d*x)**8/128 + 45*a**3*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 3*a**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 45*a**3*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 9*a**3*x*sin(c + d*x)**4*cos(c + d*x
)**4/64 + 45*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 3*a**3*x*sin(c +
d*x)**2*cos(c + d*x)**6/32 + 9*a**3*x*cos(c + d*x)**10/256 + 3*a**3*x*cos(c
 + d*x)**8/128 + 9*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 21*a**3*sin(
c + d*x)**7*cos(c + d*x)**3/(128*d) + 3*a**3*sin(c + d*x)**7*cos(c + d*x)/(
128*d) - a**3*sin(c + d*x)**6*cos(c + d*x)**5/(5*d) - 3*a**3*sin(c + d*x)**
5*cos(c + d*x)**5/(10*d) + 11*a**3*sin(c + d*x)**5*cos(c + d*x)**3/(128*d)
- 6*a**3*sin(c + d*x)**4*cos(c + d*x)**7/(35*d) - 3*a**3*sin(c + d*x)**4*co
s(c + d*x)**5/(5*d) - 21*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) - 11*
a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 8*a**3*sin(c + d*x)**2*cos(c
 + d*x)**9/(105*d) - 12*a**3*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - 9*a**
3*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**
7/(128*d) - 16*a**3*cos(c + d*x)**11/(1155*d) - 8*a**3*cos(c + d*x)**9/(105
*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)**4*cos(c)**4, True))
```

3.393 $\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=182

$$-\frac{a^3\cos^9(c+dx)}{3d}+\frac{a^3\cos^7(c+dx)}{d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{a^3\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{7a^3\sin^3(c+dx)\cos^5(c+dx)}{16d}$$

```
[Out] 21/256*a^3*x-4/5*a^3*cos(d*x+c)^5/d+a^3*cos(d*x+c)^7/d-1/3*a^3*cos(d*x+c)^9
/d+21/256*a^3*cos(d*x+c)*sin(d*x+c)/d+7/128*a^3*cos(d*x+c)^3*sin(d*x+c)/d-7
/32*a^3*cos(d*x+c)^5*sin(d*x+c)/d-7/16*a^3*cos(d*x+c)^5*sin(d*x+c)^3/d-1/10
*a^3*cos(d*x+c)^5*sin(d*x+c)^5/d
```

Rubi [A] time = 0.38, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{a^3\cos^9(c+dx)}{3d}+\frac{a^3\cos^7(c+dx)}{d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{a^3\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{7a^3\sin^3(c+dx)\cos^5(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (21*a^3*x)/256 - (4*a^3*Cos[c + d*x]^5)/(5*d) + (a^3*Cos[c + d*x]^7)/d - (a
^3*Cos[c + d*x]^9)/(3*d) + (21*a^3*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (7*
a^3*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) - (7*a^3*Cos[c + d*x]^5*Sin[c + d*
x])/(32*d) - (7*a^3*Cos[c + d*x]^5*Sin[c + d*x]^3)/(16*d) - (a^3*Cos[c + d*
x]^5*Sin[c + d*x]^5)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^4(c+dx)\sin^3(c+dx)+3a^3\cos^4(c+dx)\sin^4(c+\right.$$
$$= a^3\int \cos^4(c+dx)\sin^3(c+dx)\,dx + a^3\int \cos^4(c+dx)\sin^6($$
$$= -\frac{3a^3\cos^5(c+dx)\sin^3(c+dx)}{8d} - \frac{a^3\cos^5(c+dx)\sin^5(c+d}{10d}$$
$$= -\frac{3a^3\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{7a^3\cos^5(c+dx)\sin^3(c+d}{16d}$$
$$= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{a^3\cos^7(c+dx)}{d} - \frac{a^3\cos^9(c+dx)}{3d} + \frac{3}{}$$
$$= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{a^3\cos^7(c+dx)}{d} - \frac{a^3\cos^9(c+dx)}{3d} + \frac{9}{}$$
$$= \frac{9a^3x}{128} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{a^3\cos^7(c+dx)}{d} - \frac{a^3\cos^9(c+dx}{3d}$$
$$= \frac{21a^3x}{256} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{a^3\cos^7(c+dx)}{d} - \frac{a^3\cos^9(c+d}{3d}$$

Mathematica [A] time = 1.08, size = 116, normalized size = 0.64

$$\frac{a^3(-60\sin(2(c+dx)) - 840\sin(4(c+dx)) + 30\sin(6(c+dx)) + 105\sin(8(c+dx)) - 6\sin(10(c+dx)) - 3600\,\mathrm{co}}{307}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(2700*c + 2520*d*x - 3600*Cos[c + d*x] - 960*Cos[3*(c + d*x)] + 384*Co
s[5*(c + d*x)] + 120*Cos[7*(c + d*x)] - 40*Cos[9*(c + d*x)] - 60*Sin[2*(c +
 d*x)] - 840*Sin[4*(c + d*x)] + 30*Sin[6*(c + d*x)] + 105*Sin[8*(c + d*x)]
- 6*Sin[10*(c + d*x)]))/(30720*d)
```

fricas [A] time = 0.50, size = 124, normalized size = 0.68

$$-\frac{1280\,a^3\cos(dx+c)^9 - 3840\,a^3\cos(dx+c)^7 + 3072\,a^3\cos(dx+c)^5 - 315\,a^3dx + 3\left(128\,a^3\cos(dx+c)^9 - 816\right.}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/3840*(1280*a^3*cos(d*x + c)^9 - 3840*a^3*cos(d*x + c)^7 + 3072*a^3*cos(d
*x + c)^5 - 315*a^3*d*x + 3*(128*a^3*cos(d*x + c)^9 - 816*a^3*cos(d*x + c)^
7 + 968*a^3*cos(d*x + c)^5 - 70*a^3*cos(d*x + c)^3 - 105*a^3*cos(d*x + c))*
sin(d*x + c))/d
```

giac [A] time = 0.40, size = 174, normalized size = 0.96

$$\frac{21}{256}a^3x - \frac{a^3\cos(9\,dx+9\,c)}{768\,d} + \frac{a^3\cos(7\,dx+7\,c)}{256\,d} + \frac{a^3\cos(5\,dx+5\,c)}{80\,d} - \frac{a^3\cos(3\,dx+3\,c)}{32\,d} - \frac{15\,a^3\cos(dx+c)}{128\,d} - \frac{a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 21/256*a^3*x - 1/768*a^3*cos(9*d*x + 9*c)/d + 1/256*a^3*cos(7*d*x + 7*c)/d
+ 1/80*a^3*cos(5*d*x + 5*c)/d - 1/32*a^3*cos(3*d*x + 3*c)/d - 15/128*a^3*co
s(d*x + c)/d - 1/5120*a^3*sin(10*d*x + 10*c)/d + 7/2048*a^3*sin(8*d*x + 8*c
)/d + 1/1024*a^3*sin(6*d*x + 6*c)/d - 7/256*a^3*sin(4*d*x + 4*c)/d - 1/512*
a^3*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.29, size = 252, normalized size = 1.38

$$\frac{a^3\left(-\frac{(\sin^5(dx+c))(\cos^5(dx+c))}{10} - \frac{(\sin^3(dx+c))(\cos^5(dx+c))}{16} - \frac{\sin(dx+c)(\cos^5(dx+c))}{32} + \frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{128} + \frac{3dx}{256} + \frac{3c}{256}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1/
32*sin(d*x+c)*cos(d*x+c)^5+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3
/256*d*x+3/256*c)+3*a^3*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*c
os(d*x+c)^5-8/315*cos(d*x+c)^5)+3*a^3*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*
sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128
*d*x+3/128*c)+a^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5))
```

maxima [A] time = 0.32, size = 149, normalized size = 0.82

$$-\frac{2048\left(35\cos(dx+c)^9 - 90\cos(dx+c)^7 + 63\cos(dx+c)^5\right)a^3 - 6144\left(5\cos(dx+c)^7 - 7\cos(dx+c)^5\right)a^3 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/215040*(2048*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a^3 - 6144*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^3 + 21*(32*sin(2*d*x +
2*c)^5 - 120*d*x - 120*c - 5*sin(8*d*x + 8*c) + 40*sin(4*d*x + 4*c))*a^3 -
630*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^3)/d
```

mupad [B] time = 10.81, size = 572, normalized size = 3.14

$$\frac{21\,a^3\,x}{256}-\frac{\frac{203\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{128}-\frac{1973\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160}-\frac{463\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{32}+\frac{3231\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{64}-\frac{3231\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64}+\frac{463\,a^3\,\tan\left(\frac{c}{2}+\right.}{32}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^3*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (21*a^3*x)/256 - ((203*a^3*tan(c/2 + (d*x)/2)^3)/128 - (1973*a^3*tan(c/2 +
(d*x)/2)^5)/160 - (463*a^3*tan(c/2 + (d*x)/2)^7)/32 + (3231*a^3*tan(c/2 + (
d*x)/2)^9)/64 - (3231*a^3*tan(c/2 + (d*x)/2)^11)/64 + (463*a^3*tan(c/2 + (d
*x)/2)^13)/32 + (1973*a^3*tan(c/2 + (d*x)/2)^15)/160 - (203*a^3*tan(c/2 + (
d*x)/2)^17)/128 - (21*a^3*tan(c/2 + (d*x)/2)^19)/128 + (a^3*(315*c + 315*d*
x))/3840 - (a^3*(315*c + 315*d*x - 1024))/3840 + tan(c/2 + (d*x)/2)^18*((a^
3*(315*c + 315*d*x))/384 - (a^3*(3150*c + 3150*d*x))/3840) + tan(c/2 + (d*x
)/2)^2*((a^3*(315*c + 315*d*x))/384 - (a^3*(3150*c + 3150*d*x - 10240))/384
0) + tan(c/2 + (d*x)/2)^16*((3*a^3*(315*c + 315*d*x))/256 - (a^3*(14175*c +
 14175*d*x - 15360))/3840) + tan(c/2 + (d*x)/2)^4*((3*a^3*(315*c + 315*d*x)
)/256 - (a^3*(14175*c + 14175*d*x - 30720))/3840) + tan(c/2 + (d*x)/2)^6*((
a^3*(315*c + 315*d*x))/32 - (a^3*(37800*c + 37800*d*x + 30720))/3840) + tan
(c/2 + (d*x)/2)^12*((7*a^3*(315*c + 315*d*x))/128 - (a^3*(66150*c + 66150*d
*x + 30720))/3840) + tan(c/2 + (d*x)/2)^14*((a^3*(315*c + 315*d*x))/32 - (a
^3*(37800*c + 37800*d*x - 153600))/3840) + tan(c/2 + (d*x)/2)^10*((21*a^3*(
315*c + 315*d*x))/320 - (a^3*(79380*c + 79380*d*x - 129024))/3840) + tan(c/
2 + (d*x)/2)^8*((7*a^3*(315*c + 315*d*x))/128 - (a^3*(66150*c + 66150*d*x -
 245760))/3840) + (21*a^3*tan(c/2 + (d*x)/2))/128)/(d*(tan(c/2 + (d*x)/2)^2
 + 1)^10)
```

sympy [A] time = 33.51, size = 595, normalized size = 3.27

$$\begin{cases}\frac{3a^3x\sin^{10}(c+dx)}{256}+\frac{15a^3x\sin^8(c+dx)\cos^2(c+dx)}{256}+\frac{9a^3x\sin^8(c+dx)}{128}+\frac{15a^3x\sin^6(c+dx)\cos^4(c+dx)}{128}+\frac{9a^3x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{15a^3}{} \\ x\,(a\sin(c)+a)^3\sin^3(c)\cos^4(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**10/256 + 15*a**3*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 9*a**3*x*sin(c + d*x)**8/128 + 15*a**3*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 9*a**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 15*a**3*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 27*a**3*x*sin(c + d*x)**4*cos(c + d*
x)**4/64 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 9*a**3*x*sin(c +
 d*x)**2*cos(c + d*x)**6/32 + 3*a**3*x*cos(c + d*x)**10/256 + 9*a**3*x*cos(
c + d*x)**8/128 + 3*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**3*sin(
c + d*x)**7*cos(c + d*x)**3/(128*d) + 9*a**3*sin(c + d*x)**7*cos(c + d*x)/(
128*d) - a**3*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 33*a**3*sin(c + d*x)
**5*cos(c + d*x)**3/(128*d) - 3*a**3*sin(c + d*x)**4*cos(c + d*x)**5/(5*d)
- 7*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) - 33*a**3*sin(c + d*x)**3*
cos(c + d*x)**5/(128*d) - 12*a**3*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) -
a**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**3*sin(c + d*x)*cos(c + d*
x)**9/(256*d) - 9*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 8*a**3*cos(c
+ d*x)**9/(105*d) - 2*a**3*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c)
+ a)**3*sin(c)**3*cos(c)**4, True))
```

3.394 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=159

$$-\frac{a^3\cos^9(c+dx)}{9d}+\frac{5a^3\cos^7(c+dx)}{7d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{3a^3\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{17a^3\sin(c+dx)\cos^5(c+dx)}{48d}$$

```
[Out] 17/128*a^3*x-4/5*a^3*cos(d*x+c)^5/d+5/7*a^3*cos(d*x+c)^7/d-1/9*a^3*cos(d*x+
c)^9/d+17/128*a^3*cos(d*x+c)*sin(d*x+c)/d+17/192*a^3*cos(d*x+c)^3*sin(d*x+c
)/d-17/48*a^3*cos(d*x+c)^5*sin(d*x+c)/d-3/8*a^3*cos(d*x+c)^5*sin(d*x+c)^3/d
```

Rubi [A] time = 0.32, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2873, 2568, 2635, 8, 2565, 14, 270}

$$-\frac{a^3\cos^9(c+dx)}{9d}+\frac{5a^3\cos^7(c+dx)}{7d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{3a^3\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{17a^3\sin(c+dx)\cos^5(c+dx)}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (17*a^3*x)/128 - (4*a^3*Cos[c + d*x]^5)/(5*d) + (5*a^3*Cos[c + d*x]^7)/(7*d
) - (a^3*Cos[c + d*x]^9)/(9*d) + (17*a^3*Cos[c + d*x]*Sin[c + d*x])/(128*d)
 + (17*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) - (17*a^3*Cos[c + d*x]^5*Si
n[c + d*x])/(48*d) - (3*a^3*Cos[c + d*x]^5*Sin[c + d*x]^3)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^4(c+dx)\sin^2(c+dx)+3a^3\cos^4(c+dx)\sin^3(c\right.$$

$$= a^3\int \cos^4(c+dx)\sin^2(c+dx)\,dx + a^3\int \cos^4(c+dx)\sin$$

$$= -\frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{3a^3\cos^5(c+dx)\sin^3(c+a}{8d}$$

$$= \frac{a^3\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{17a^3\cos^5(c+dx)\sin(c+dx}{48d}$$

$$= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{5a^3\cos^7(c+dx)}{7d} - \frac{a^3\cos^9(c+dx)}{9d} +$$

$$= \frac{a^3x}{16} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{5a^3\cos^7(c+dx)}{7d} - \frac{a^3\cos^9(c+}{9d}$$

$$= \frac{17a^3x}{128} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{5a^3\cos^7(c+dx)}{7d} - \frac{a^3\cos^9(c}{9d}$$

Mathematica [A] time = 0.85, size = 106, normalized size = 0.67

$$\frac{a^3(5040\sin(2(c+dx))-12600\sin(4(c+dx))-1680\sin(6(c+dx))+945\sin(8(c+dx))-52920\cos(c+dx)-1}{32256}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(30240*c + 42840*d*x - 52920*Cos[c + d*x] - 16800*Cos[3*(c + d*x)] + 4
032*Cos[5*(c + d*x)] + 2340*Cos[7*(c + d*x)] - 140*Cos[9*(c + d*x)] + 5040*
Sin[2*(c + d*x)] - 12600*Sin[4*(c + d*x)] - 1680*Sin[6*(c + d*x)] + 945*Sin
[8*(c + d*x)]))/(322560*d)
```

fricas [A] time = 0.51, size = 111, normalized size = 0.70

$$-\frac{4480\,a^3\cos(dx+c)^9-28800\,a^3\cos(dx+c)^7+32256\,a^3\cos(dx+c)^5-5355\,a^3dx-105\left(144\,a^3\cos(dx+c)^7\right.}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/40320*(4480*a^3*cos(d*x + c)^9 - 28800*a^3*cos(d*x + c)^7 + 32256*a^3*co
s(d*x + c)^5 - 5355*a^3*d*x - 105*(144*a^3*cos(d*x + c)^7 - 280*a^3*cos(d*x
 + c)^5 + 34*a^3*cos(d*x + c)^3 + 51*a^3*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.33, size = 157, normalized size = 0.99

$$\frac{17}{128}\,a^3x-\frac{a^3\cos(9\,dx+9\,c)}{2304\,d}+\frac{13\,a^3\cos(7\,dx+7\,c)}{1792\,d}+\frac{a^3\cos(5\,dx+5\,c)}{80\,d}-\frac{5\,a^3\cos(3\,dx+3\,c)}{96\,d}-\frac{21\,a^3\cos(dx+c)}{128\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 17/128*a^3*x - 1/2304*a^3*cos(9*d*x + 9*c)/d + 13/1792*a^3*cos(7*d*x + 7*c)
/d + 1/80*a^3*cos(5*d*x + 5*c)/d - 5/96*a^3*cos(3*d*x + 3*c)/d - 21/128*a^3
*cos(d*x + c)/d + 3/1024*a^3*sin(8*d*x + 8*c)/d - 1/192*a^3*sin(6*d*x + 6*c
)/d - 5/128*a^3*sin(4*d*x + 4*c)/d + 1/64*a^3*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.28, size = 216, normalized size = 1.36

$$a^3\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^5(dx+c)\right)}{9}-\frac{4\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{63}-\frac{8\left(\cos^5(dx+c)\right)}{315}\right)+3a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/3
15*cos(d*x+c)^5)+3*a^3*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(
d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+3
*a^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+a^3*(-1/6*sin(d*x+c
)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*
c))
```

maxima [A] time = 0.32, size = 138, normalized size = 0.87

$$-\frac{1024\left(35\cos\left(dx+c\right)^{9}-90\cos\left(dx+c\right)^{7}+63\cos\left(dx+c\right)^{5}\right)a^{3}-27648\left(5\cos\left(dx+c\right)^{7}-7\cos\left(dx+c\right)^{5}\right)a^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/322560*(1024*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a^3 - 27648*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^3 - 1680*(4*sin(2*d*x
+ 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^3 - 945*(24*d*x + 24*c + s
in(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^3)/d
```

mupad [B] time = 12.28, size = 437, normalized size = 2.75

$$\frac{17\,a^3\,x}{128}-\frac{\frac{17\,a^3\,(c+d\,x)}{128}-\frac{35\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96}-\frac{537\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{32}+\frac{531\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{32}-\frac{531\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{32}+\frac{537\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (17*a^3*x)/128 - ((17*a^3*(c + d*x))/128 - (35*a^3*tan(c/2 + (d*x)/2)^3)/96
 - (537*a^3*tan(c/2 + (d*x)/2)^5)/32 + (531*a^3*tan(c/2 + (d*x)/2)^7)/32 -
(531*a^3*tan(c/2 + (d*x)/2)^11)/32 + (537*a^3*tan(c/2 + (d*x)/2)^13)/32 + (
35*a^3*tan(c/2 + (d*x)/2)^15)/96 - (17*a^3*tan(c/2 + (d*x)/2)^17)/64 - (a^3
*(5355*c + 5355*d*x - 15872))/40320 + tan(c/2 + (d*x)/2)^2*((153*a^3*(c + d
*x))/128 - (a^3*(48195*c + 48195*d*x - 142848))/40320) + tan(c/2 + (d*x)/2)
^4*((153*a^3*(c + d*x))/32 - (a^3*(192780*c + 192780*d*x - 87552))/40320) +
 tan(c/2 + (d*x)/2)^14*((153*a^3*(c + d*x))/32 - (a^3*(192780*c + 192780*d*
x - 483840))/40320) + tan(c/2 + (d*x)/2)^6*((357*a^3*(c + d*x))/32 - (a^3*(
```

```
449820*c + 449820*d*x - 419328))/40320) + tan(c/2 + (d*x)/2)^10*((1071*a^3*
(c + d*x))/64 - (a^3*(674730*c + 674730*d*x + 161280))/40320) + tan(c/2 + (
d*x)/2)^12*((357*a^3*(c + d*x))/32 - (a^3*(449820*c + 449820*d*x - 913920))
/40320) + tan(c/2 + (d*x)/2)^8*((1071*a^3*(c + d*x))/64 - (a^3*(674730*c +
674730*d*x - 2161152))/40320) + (17*a^3*tan(c/2 + (d*x)/2))/64)/(d*(tan(c/2
 + (d*x)/2)^2 + 1)^9)
```

sympy [A] time = 21.62, size = 486, normalized size = 3.06

$$\begin{cases} \frac{9a^3x\sin^8(c+dx)}{128} + \frac{9a^3x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{a^3x\sin^6(c+dx)}{16} + \frac{27a^3x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{3a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{9a^3x\sin \\ x\left(a\sin(c)+a\right)^3\sin^2(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((9*a**3*x*sin(c + d*x)**8/128 + 9*a**3*x*sin(c + d*x)**6*cos(c +
d*x)**2/32 + a**3*x*sin(c + d*x)**6/16 + 27*a**3*x*sin(c + d*x)**4*cos(c +
d*x)**4/64 + 3*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a**3*x*sin(c +
 d*x)**2*cos(c + d*x)**6/32 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 +
 9*a**3*x*cos(c + d*x)**8/128 + a**3*x*cos(c + d*x)**6/16 + 9*a**3*sin(c +
d*x)**7*cos(c + d*x)/(128*d) + 33*a**3*sin(c + d*x)**5*cos(c + d*x)**3/(128
*d) + a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) - a**3*sin(c + d*x)**4*cos(c
 + d*x)**5/(5*d) - 33*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) + a**3*s
in(c + d*x)**3*cos(c + d*x)**3/(6*d) - 4*a**3*sin(c + d*x)**2*cos(c + d*x)*
*7/(35*d) - 3*a**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 9*a**3*sin(c + d
*x)*cos(c + d*x)**7/(128*d) - a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 8*
a**3*cos(c + d*x)**9/(315*d) - 6*a**3*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x
*(a*sin(c) + a)**3*sin(c)**2*cos(c)**4, True))
```

3.395 $\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=157

$$-\frac{9a^3\cos^5(c+dx)}{80d}-\frac{9\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{112d}+\frac{9a^3\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{27a^3\sin(c+dx)\cos(c+$$

```
[Out] 27/128*a^3*x-9/80*a^3*cos(d*x+c)^5/d+27/128*a^3*cos(d*x+c)*sin(d*x+c)/d+9/6
4*a^3*cos(d*x+c)^3*sin(d*x+c)/d-3/56*a*cos(d*x+c)^5*(a+a*sin(d*x+c))^2/d-1/
8*cos(d*x+c)^5*(a+a*sin(d*x+c))^3/d-9/112*cos(d*x+c)^5*(a^3+a^3*sin(d*x+c))
/d
```

Rubi [A] time = 0.19, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{9a^3\cos^5(c+dx)}{80d}-\frac{9\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{112d}+\frac{9a^3\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{27a^3\sin(c+dx)\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (27*a^3*x)/128 - (9*a^3*Cos[c + d*x]^5)/(80*d) + (27*a^3*Cos[c + d*x]*Sin[c
 + d*x])/(128*d) + (9*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) - (3*a*Cos[c
+ d*x]^5*(a + a*Sin[c + d*x])^2)/(56*d) - (Cos[c + d*x]^5*(a + a*Sin[c + d*
x])^3)/(8*d) - (9*Cos[c + d*x]^5*(a^3 + a^3*Sin[c + d*x]))/(112*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^3}{8d}+\frac{3}{8}\int \cos^4(c+dx)(a+a$$

$$= -\frac{3a\cos^5(c+dx)(a+a\sin(c+dx))^2}{56d}-\frac{\cos^5(c+dx)(a+a\,\mathrm{si}}{8d}$$

$$= -\frac{3a\cos^5(c+dx)(a+a\sin(c+dx))^2}{56d}-\frac{\cos^5(c+dx)(a+a\,\mathrm{si}}{8d}$$

$$= -\frac{9a^3\cos^5(c+dx)}{80d}-\frac{3a\cos^5(c+dx)(a+a\sin(c+dx))^2}{56d}-$$

$$= -\frac{9a^3\cos^5(c+dx)}{80d}+\frac{9a^3\cos^3(c+dx)\sin(c+dx)}{64d}-\frac{3a\cos^5(}{}$$

$$= -\frac{9a^3\cos^5(c+dx)}{80d}+\frac{27a^3\cos(c+dx)\sin(c+dx)}{128d}+\frac{9a^3\cos}{}$$

$$= \frac{27a^3x}{128}-\frac{9a^3\cos^5(c+dx)}{80d}+\frac{27a^3\cos(c+dx)\sin(c+dx)}{128d}+$$

Mathematica [A] time = 0.45, size = 96, normalized size = 0.61

$$\frac{a^3(1680\sin(2(c+dx))-1960\sin(4(c+dx))-560\sin(6(c+dx))+35\sin(8(c+dx))-9520\cos(c+dx)-3920}{35840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(8400*c + 7560*d*x - 9520*Cos[c + d*x] - 3920*Cos[3*(c + d*x)] - 112*C
os[5*(c + d*x)] + 240*Cos[7*(c + d*x)] + 1680*Sin[2*(c + d*x)] - 1960*Sin[4
*(c + d*x)] - 560*Sin[6*(c + d*x)] + 35*Sin[8*(c + d*x)]))/(35840*d)
```

fricas [A] time = 0.47, size = 98, normalized size = 0.62

$$\frac{1920\,a^3\cos\left(dx+c\right)^7-3584\,a^3\cos\left(dx+c\right)^5+945\,a^3dx+35\left(16\,a^3\cos\left(dx+c\right)^7-88\,a^3\cos\left(dx+c\right)^5+18\,a^3\cos\right.}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4480*(1920*a^3*cos(d*x + c)^7 - 3584*a^3*cos(d*x + c)^5 + 945*a^3*d*x + 3
5*(16*a^3*cos(d*x + c)^7 - 88*a^3*cos(d*x + c)^5 + 18*a^3*cos(d*x + c)^3 +
27*a^3*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.29, size = 140, normalized size = 0.89

$$\frac{27}{128}\,a^3x+\frac{3\,a^3\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{a^3\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{7\,a^3\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{17\,a^3\cos\left(dx+c\right)}{64\,d}+\frac{a^3\sin\left(8\,dx+8\,c\right)}{1024\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 27/128*a^3*x + 3/448*a^3*cos(7*d*x + 7*c)/d - 1/320*a^3*cos(5*d*x + 5*c)/d
- 7/64*a^3*cos(3*d*x + 3*c)/d - 17/64*a^3*cos(d*x + c)/d + 1/1024*a^3*sin(8
*d*x + 8*c)/d - 1/64*a^3*sin(6*d*x + 6*c)/d - 7/128*a^3*sin(4*d*x + 4*c)/d
+ 3/64*a^3*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.27, size = 178, normalized size = 1.13

$$\frac{a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{64}+\frac{3dx}{128}+\frac{3c}{128}\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5\right.}{7}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+3*a^3*(-1/7*sin
(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+3*a^3*(-1/6*sin(d*x+c)*cos(d*x+c)
```

```
^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)-1/5*a^3*c
os(d*x+c)^5)
```

maxima [A] time = 0.32, size = 115, normalized size = 0.73

$$-\frac{7168\,a^3\cos\left(dx+c\right)^5-3072\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)a^3-560\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\mathrm{s}\right.}{35840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/35840*(7168*a^3*cos(d*x + c)^5 - 3072*(5*cos(d*x + c)^7 - 7*cos(d*x + c)
^5)*a^3 - 560*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a
^3 - 35*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^3)/d
```

mupad [B] time = 10.76, size = 461, normalized size = 2.94

$$\frac{27\,a^3\,x}{128}-\frac{\frac{919\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{64}-\frac{437\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{64}-\frac{305\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{64}-\frac{919\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{64}+\frac{437\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64}+\frac{305\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{64}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (27*a^3*x)/128 - ((919*a^3*tan(c/2 + (d*x)/2)^7)/64 - (437*a^3*tan(c/2 + (d
*x)/2)^5)/64 - (305*a^3*tan(c/2 + (d*x)/2)^3)/64 - (919*a^3*tan(c/2 + (d*x)
/2)^9)/64 + (437*a^3*tan(c/2 + (d*x)/2)^11)/64 + (305*a^3*tan(c/2 + (d*x)/2
)^13)/64 - (27*a^3*tan(c/2 + (d*x)/2)^15)/64 + (a^3*(945*c + 945*d*x))/4480
 - (a^3*(945*c + 945*d*x - 3328))/4480 + tan(c/2 + (d*x)/2)^14*((a^3*(945*c
 + 945*d*x))/560 - (a^3*(7560*c + 7560*d*x - 8960))/4480) + tan(c/2 + (d*x)
/2)^2*((a^3*(945*c + 945*d*x))/560 - (a^3*(7560*c + 7560*d*x - 17664))/4480
) + tan(c/2 + (d*x)/2)^4*((a^3*(945*c + 945*d*x))/160 - (a^3*(26460*c + 264
60*d*x - 12544))/4480) + tan(c/2 + (d*x)/2)^12*((a^3*(945*c + 945*d*x))/160
 - (a^3*(26460*c + 26460*d*x - 80640))/4480) + tan(c/2 + (d*x)/2)^10*((a^3*
(945*c + 945*d*x))/80 - (a^3*(52920*c + 52920*d*x - 44800))/4480) + tan(c/2
 + (d*x)/2)^6*((a^3*(945*c + 945*d*x))/80 - (a^3*(52920*c + 52920*d*x - 141
568))/4480) + tan(c/2 + (d*x)/2)^8*((a^3*(945*c + 945*d*x))/64 - (a^3*(6615
0*c + 66150*d*x - 116480))/4480) + (27*a^3*tan(c/2 + (d*x)/2))/64)/(d*(tan(
c/2 + (d*x)/2)^2 + 1)^8)
```

sympy [A] time = 13.19, size = 440, normalized size = 2.80

$$\begin{cases} \frac{3a^3x\sin^8(c+dx)}{128} + \frac{3a^3x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{3a^3x\sin^6(c+dx)}{16} + \frac{9a^3x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{9a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3a^3x\,\mathrm{si}}{} \\ x(a\sin(c)+a)^3\sin(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**8/128 + 3*a**3*x*sin(c + d*x)**6*cos(c +
d*x)**2/32 + 3*a**3*x*sin(c + d*x)**6/16 + 9*a**3*x*sin(c + d*x)**4*cos(c +
 d*x)**4/64 + 9*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*a**3*x*sin(c
+ d*x)**2*cos(c + d*x)**6/32 + 9*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16
+ 3*a**3*x*cos(c + d*x)**8/128 + 3*a**3*x*cos(c + d*x)**6/16 + 3*a**3*sin(c
 + d*x)**7*cos(c + d*x)/(128*d) + 11*a**3*sin(c + d*x)**5*cos(c + d*x)**3/(
128*d) + 3*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 11*a**3*sin(c + d*x)*
*3*cos(c + d*x)**5/(128*d) + a**3*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3
*a**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**3*sin(c + d*x)*cos(c + d
*x)**7/(128*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 6*a**3*cos(c
+ d*x)**7/(35*d) - a**3*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x*(a*sin(c) + a)
**3*sin(c)*cos(c)**4, True))
```

3.396 $\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=143

$$-\frac{3a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{19a^3\sin(c+dx)\cos^3(c+dx)}{24d}+$$

```
[Out] 19/16*a^3*x-a^3*arctanh(cos(d*x+c))/d+a^3*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3
/d-3/5*a^3*cos(d*x+c)^5/d+19/16*a^3*cos(d*x+c)*sin(d*x+c)/d+19/24*a^3*cos(d
*x+c)^3*sin(d*x+c)/d-1/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2873, 2635, 8, 2592, 302, 206, 2565, 30, 2568}

$$-\frac{3a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{19a^3\sin(c+dx)\cos^3(c+dx)}{24d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (19*a^3*x)/16 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Cos[c + d*x])/d + (a^3
*Cos[c + d*x]^3)/(3*d) - (3*a^3*Cos[c + d*x]^5)/(5*d) + (19*a^3*Cos[c + d*x
]*Sin[c + d*x])/(16*d) + (19*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a^3
*Cos[c + d*x]^5*Sin[c + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(3a^3\cos^4(c+dx)+a^3\cos^3(c+dx)\cot(c+dx)+3a^3\cos^4\right.$$

$$= a^3\int \cos^3(c+dx)\cot(c+dx)\,dx + a^3\int \cos^4(c+dx)\sin^2(c$$

$$= \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} +$$

$$= -\frac{3a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{19a^3\cos^3}{}$$

$$= \frac{9a^3x}{8} + \frac{a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} - \frac{3a^3\cos^5(c+dx)}{5d}$$

$$= \frac{19a^3x}{16} - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(}{3}$$

Mathematica [A] time = 1.02, size = 102, normalized size = 0.71

$$\frac{a^3\left(735\sin(2(c+dx))+75\sin(4(c+dx))-5\sin(6(c+dx))+840\cos(c+dx)-100\cos(3(c+dx))-36\cos(5(c\right.}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1140*c + 1140*d*x + 840*Cos[c + d*x] - 100*Cos[3*(c + d*x)] - 36*Cos[
5*(c + d*x)] - 960*Log[Cos[(c + d*x)/2]] + 960*Log[Sin[(c + d*x)/2]] + 735*
Sin[2*(c + d*x)] + 75*Sin[4*(c + d*x)] - 5*Sin[6*(c + d*x)]))/(960*d)
```

fricas [A] time = 0.48, size = 128, normalized size = 0.90

$$-\frac{144\,a^3\cos(dx+c)^5-80\,a^3\cos(dx+c)^3-285\,a^3dx-240\,a^3\cos(dx+c)+120\,a^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-1}{240}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*a^3*cos(d*x + c)^5 - 80*a^3*cos(d*x + c)^3 - 285*a^3*d*x - 240*
a^3*cos(d*x + c) + 120*a^3*log(1/2*cos(d*x + c) + 1/2) - 120*a^3*log(-1/2*c
os(d*x + c) + 1/2) + 5*(8*a^3*cos(d*x + c)^5 - 38*a^3*cos(d*x + c)^3 - 57*a
^3*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.33, size = 229, normalized size = 1.60

$$285\,(dx+c)a^3+240\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(435\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+240\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+865\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1200}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(285*(d*x + c)*a^3 + 240*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(435*
a^3*tan(1/2*d*x + 1/2*c)^11 + 240*a^3*tan(1/2*d*x + 1/2*c)^10 + 865*a^3*tan
(1/2*d*x + 1/2*c)^9 - 1200*a^3*tan(1/2*d*x + 1/2*c)^8 - 210*a^3*tan(1/2*d*x
 + 1/2*c)^7 - 1760*a^3*tan(1/2*d*x + 1/2*c)^6 + 210*a^3*tan(1/2*d*x + 1/2*c
)^5 - 1440*a^3*tan(1/2*d*x + 1/2*c)^4 - 865*a^3*tan(1/2*d*x + 1/2*c)^3 - 12
96*a^3*tan(1/2*d*x + 1/2*c)^2 - 435*a^3*tan(1/2*d*x + 1/2*c) - 176*a^3)/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 0.47, size = 149, normalized size = 1.04

$$-\frac{a^3\left(\cos^5(dx+c)\right)\sin(dx+c)}{6d}+\frac{19a^3\left(\cos^3(dx+c)\right)\sin(dx+c)}{24d}+\frac{19a^3\cos(dx+c)\sin(dx+c)}{16d}+\frac{19a^3x}{16}+\frac{19a^3}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d+19/24*a^3*cos(d*x+c)^3*sin(d*x+c)/d+19/1
6*a^3*cos(d*x+c)*sin(d*x+c)/d+19/16*a^3*x+19/16/d*a^3*c-3/5*a^3*cos(d*x+c)^
5/d+1/3*a^3*cos(d*x+c)^3/d+a^3*cos(d*x+c)/d+1/d*a^3*ln(csc(d*x+c)-cot(d*x+c
))
```

maxima [A] time = 0.32, size = 135, normalized size = 0.94

$$-\frac{576\,a^3\cos(dx+c)^5-160\left(2\,\cos(dx+c)^3+6\,\cos(dx+c)-3\,\log(\cos(dx+c)+1)+3\,\log(\cos(dx+c)-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(576*a^3*cos(d*x + c)^5 - 160*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3
*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^3 - 5*(4*sin(2*d*x + 2*
c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^3 - 90*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d
```

mupad [B] time = 10.51, size = 355, normalized size = 2.48

$$\frac{a^3 \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{19\,a^3\,\mathrm{atan}\left(\frac{361\,a^6}{64\left(\frac{19\,a^6}{4}-\frac{361\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}+\frac{19\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\left(\frac{19\,a^6}{4}-\frac{361\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}\right)}{8\,d}+\frac{-\frac{29\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-2\,a^3\tan\left(\frac{c}{2}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d + (19*a^3*atan((361*a^6)/(64*((19*a^6)/4 -
(361*a^6*tan(c/2 + (d*x)/2))/64)) + (19*a^6*tan(c/2 + (d*x)/2))/(4*((19*a^6
)/4 - (361*a^6*tan(c/2 + (d*x)/2))/64))))/(8*d) + ((54*a^3*tan(c/2 + (d*x)/
2)^2)/5 + (173*a^3*tan(c/2 + (d*x)/2)^3)/24 + 12*a^3*tan(c/2 + (d*x)/2)^4 -
 (7*a^3*tan(c/2 + (d*x)/2)^5)/4 + (44*a^3*tan(c/2 + (d*x)/2)^6)/3 + (7*a^3*
tan(c/2 + (d*x)/2)^7)/4 + 10*a^3*tan(c/2 + (d*x)/2)^8 - (173*a^3*tan(c/2 +
(d*x)/2)^9)/24 - 2*a^3*tan(c/2 + (d*x)/2)^10 - (29*a^3*tan(c/2 + (d*x)/2)^1
1)/8 + (22*a^3)/15 + (29*a^3*tan(c/2 + (d*x)/2))/8)/(d*(6*tan(c/2 + (d*x)/2
)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x
)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.397 $\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=131

$$-\frac{a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}-\frac{3a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{11a^3\sin(c+\ldots}{}$$

```
[Out] -3/8*a^3*x-3*a^3*arctanh(cos(d*x+c))/d+3*a^3*cos(d*x+c)/d+a^3*cos(d*x+c)^3/
d-1/5*a^3*cos(d*x+c)^5/d-a^3*cot(d*x+c)/d+11/8*a^3*cos(d*x+c)*sin(d*x+c)/d-
3/4*a^3*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.20, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2872, 3770, 3767, 8, 2638, 2635, 2633}

$$-\frac{a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}-\frac{3a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{11a^3\sin(c+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*x)/8 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d + (
a^3*Cos[c + d*x]^3)/d - (a^3*Cos[c + d*x]^5)/(5*d) - (a^3*Cot[c + d*x])/d +
 (11*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (3*a^3*Cos[c + d*x]*Sin[c + d*x
]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(a^7+3a^7\csc(c+dx)+a^7\csc^2(c+dx)-5a^7\sin(c+dx)\right.}{}$$

$$= a^3x + a^3\int \csc^2(c+dx)\,dx + a^3\int \sin^3(c+dx)\,dx + a^3\int s$$

$$= a^3x - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{5a^3\cos(c+dx)}{d} + \frac{5a^3\cos}{}$$

$$= -\frac{3a^3x}{2} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} + \frac{a^3c}{}$$

$$= -\frac{3a^3x}{8} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} + \frac{a^3c}{}$$

Mathematica [A] time = 2.05, size = 148, normalized size = 1.13

$$\frac{(a\sin(c+dx)+a)^3\left(-60(c+dx)+80\sin(2(c+dx))+15\sin(4(c+dx))+580\cos(c+dx)+30\cos(3(c+dx))-\right.}{160d\left(\sin\left(\frac{1}{2}(c+dx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-60*(c + d*x) + 580*Cos[c + d*x] + 30*Cos[3*(c + d
*x)] - 2*Cos[5*(c + d*x)] - 80*Cot[(c + d*x)/2] - 480*Log[Cos[(c + d*x)/2]]
 + 480*Log[Sin[(c + d*x)/2]] + 80*Sin[2*(c + d*x)] + 15*Sin[4*(c + d*x)] +
80*Tan[(c + d*x)/2]))/(160*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.48, size = 147, normalized size = 1.12

$$-\frac{30\,a^3\cos\left(dx+c\right)^5-5\,a^3\cos\left(dx+c\right)^3+60\,a^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-60\,a^3\log\left(-\frac{1}{2}\,\cos\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/40*(30*a^3*cos(d*x + c)^5 - 5*a^3*cos(d*x + c)^3 + 60*a^3*log(1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) - 60*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
c) + 15*a^3*cos(d*x + c) + (8*a^3*cos(d*x + c)^5 - 40*a^3*cos(d*x + c)^3 +
15*a^3*d*x - 120*a^3*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.27, size = 226, normalized size = 1.73

$$-\frac{15\left(dx+c\right)a^3-120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{20\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(55\,a^3\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/40*(15*(d*x + c)*a^3 - 120*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 20*a^3*t
an(1/2*d*x + 1/2*c) + 20*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1
/2*c) + 2*(55*a^3*tan(1/2*d*x + 1/2*c)^9 - 200*a^3*tan(1/2*d*x + 1/2*c)^8 -
 10*a^3*tan(1/2*d*x + 1/2*c)^7 - 720*a^3*tan(1/2*d*x + 1/2*c)^6 - 800*a^3*t
an(1/2*d*x + 1/2*c)^4 + 10*a^3*tan(1/2*d*x + 1/2*c)^3 - 560*a^3*tan(1/2*d*x
 + 1/2*c)^2 - 55*a^3*tan(1/2*d*x + 1/2*c) - 152*a^3)/(tan(1/2*d*x + 1/2*c)^
2 + 1)^5)/d
```

maple [A] time = 0.42, size = 152, normalized size = 1.16

$$-\frac{a^3\left(\cos^5\left(dx+c\right)\right)}{5d}-\frac{a^3\left(\cos^3\left(dx+c\right)\right)\sin\left(dx+c\right)}{4d}-\frac{3a^3\cos\left(dx+c\right)\sin\left(dx+c\right)}{8d}-\frac{3a^3x}{8}-\frac{3a^3c}{8d}+\frac{a^3\left(\cos^3\left(dx+c\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/5*a^3*cos(d*x+c)^5/d-1/4*a^3*cos(d*x+c)^3*sin(d*x+c)/d-3/8*a^3*cos(d*x+c
)*sin(d*x+c)/d-3/8*a^3*x-3/8/d*a^3*c+a^3*cos(d*x+c)^3/d+3*a^3*cos(d*x+c)/d+
3/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-1/d*a^3/sin(d*x+c)*cos(d*x+c)^5
```

maxima [A] time = 0.45, size = 141, normalized size = 1.08

$$-\frac{32\,a^3\cos\left(dx+c\right)^5-80\left(2\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)a}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/160*(32*a^3*cos(d*x + c)^5 - 80*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*l
og(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^3 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 + 80*(3*d*x + 3*c + (3*tan(d*x +
c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*a^3)/d
```

mupad [B] time = 8.81, size = 356, normalized size = 2.72

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{-\frac{13\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{2}+20\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+72\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-10\,a}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d + ((a^3*tan(c/2 + (d*x)/2)^2)/2 + 56*a^3*
tan(c/2 + (d*x)/2)^3 - 11*a^3*tan(c/2 + (d*x)/2)^4 + 80*a^3*tan(c/2 + (d*x)
/2)^5 - 10*a^3*tan(c/2 + (d*x)/2)^6 + 72*a^3*tan(c/2 + (d*x)/2)^7 - 4*a^3*t
an(c/2 + (d*x)/2)^8 + 20*a^3*tan(c/2 + (d*x)/2)^9 - (13*a^3*tan(c/2 + (d*x)
/2)^10)/2 - a^3 + (76*a^3*tan(c/2 + (d*x)/2))/5)/(d*(2*tan(c/2 + (d*x)/2) +
 10*tan(c/2 + (d*x)/2)^3 + 20*tan(c/2 + (d*x)/2)^5 + 20*tan(c/2 + (d*x)/2)^
7 + 10*tan(c/2 + (d*x)/2)^9 + 2*tan(c/2 + (d*x)/2)^11)) + (3*a^3*atan((9*a^
6)/(16*((9*a^6)/2 + (9*a^6*tan(c/2 + (d*x)/2))/16)) - (9*a^6*tan(c/2 + (d*x
)/2))/(2*((9*a^6)/2 + (9*a^6*tan(c/2 + (d*x)/2))/16))))/(4*d) + (a^3*tan(c/
2 + (d*x)/2))/(2*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.398 $\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=137

$$\frac{a^3\cos^3(c+dx)}{d}+\frac{2a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{7a^3\sin(c+dx)\cos(c+dx)}{8d}-3a^3$$

```
[Out] -33/8*a^3*x-3/2*a^3*arctanh(cos(d*x+c))/d+2*a^3*cos(d*x+c)/d+a^3*cos(d*x+c)
^3/d-3*a^3*cot(d*x+c)/d-1/2*a^3*cot(d*x+c)*csc(d*x+c)/d-7/8*a^3*cos(d*x+c)*
sin(d*x+c)/d-1/4*a^3*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.18, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2872, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$\frac{a^3\cos^3(c+dx)}{d}+\frac{2a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{7a^3\sin(c+dx)\cos(c+dx)}{8d}-3a^3$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-33*a^3*x)/8 - (3*a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (2*a^3*Cos[c + d*x])/
d + (a^3*Cos[c + d*x]^3)/d - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]*Csc
[c + d*x])/(2*d) - (7*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^3*Cos[c + d
*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(-5a^7+a^7\csc(c+dx)+3a^7\csc^2(c+dx)+a^7\csc^3(c+dx\right.}{}$$

$$= -5a^3x + a^3\int \csc(c+dx)\,dx + a^3\int \csc^3(c+dx)\,dx + a^3\int$$

$$= -5a^3x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{5a^3\cos(c+dx)}{d} - \frac{a^3\cot(}{}$$

$$= -\frac{9a^3x}{2} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{2a^3\cos(c+dx)}{d} + \frac{a^3\cos}{}$$

$$= -\frac{33a^3x}{8} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{2a^3\cos(c+dx)}{d} + \frac{a^3\,c}{}$$

Mathematica [A] time = 2.86, size = 164, normalized size = 1.20

$$\frac{(a\sin(c+dx)+a)^3\left(-132(c+dx)-16\sin(2(c+dx))+\sin(4(c+dx))+88\cos(c+dx)+8\cos(3(c+dx))+48\,t\right.}{32d\left(\sin\left(\frac{1}{2}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-132*(c + d*x) + 88*Cos[c + d*x] + 8*Cos[3*(c + d*
x)] - 48*Cot[(c + d*x)/2] - 4*Csc[(c + d*x)/2]^2 - 48*Log[Cos[(c + d*x)/2]]
 + 48*Log[Sin[(c + d*x)/2]] + 4*Sec[(c + d*x)/2]^2 - 16*Sin[2*(c + d*x)] +
Sin[4*(c + d*x)] + 48*Tan[(c + d*x)/2]))/(32*d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^6)
```

fricas [A] time = 0.49, size = 185, normalized size = 1.35

$$\frac{8\,a^3\cos(dx+c)^5 - 33\,a^3dx\cos(dx+c)^2 + 8\,a^3\cos(dx+c)^3 + 33\,a^3dx - 12\,a^3\cos(dx+c) - 6\left(a^3\cos(dx+c)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/8*(8*a^3*cos(d*x + c)^5 - 33*a^3*d*x*cos(d*x + c)^2 + 8*a^3*cos(d*x + c)^
3 + 33*a^3*d*x - 12*a^3*cos(d*x + c) - 6*(a^3*cos(d*x + c)^2 - a^3)*log(1/2
*cos(d*x + c) + 1/2) + 6*(a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) +
```

```
 1/2) + (2*a^3*cos(d*x + c)^5 - 11*a^3*cos(d*x + c)^3 + 33*a^3*cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.32, size = 241, normalized size = 1.76

$$a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 33\,(dx + c)a^3 + 12\,a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) + 12\,a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - \frac{18\,a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 +}{\tan\left(\frac{1}{2}\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 - 33*(d*x + c)*a^3 + 12*a^3*log(abs(tan(1/2
*d*x + 1/2*c))) + 12*a^3*tan(1/2*d*x + 1/2*c) - (18*a^3*tan(1/2*d*x + 1/2*c
)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^2 + 2*(7*a^3*
tan(1/2*d*x + 1/2*c)^7 + 40*a^3*tan(1/2*d*x + 1/2*c)^6 + 15*a^3*tan(1/2*d*x
 + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2*c)^4 - 15*a^3*tan(1/2*d*x + 1/2*c)^3
 + 56*a^3*tan(1/2*d*x + 1/2*c)^2 - 7*a^3*tan(1/2*d*x + 1/2*c) + 24*a^3)/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.49, size = 161, normalized size = 1.18

$$-\frac{11a^3\left(\cos^3(dx+c)\right)\sin(dx+c)}{4d} - \frac{33a^3\cos(dx+c)\sin(dx+c)}{8d} - \frac{33a^3x}{8} - \frac{33a^3c}{8d} + \frac{a^3\left(\cos^3(dx+c)\right)}{2d} + \frac{3a^3\cos(\ldots}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -11/4*a^3*cos(d*x+c)^3*sin(d*x+c)/d-33/8*a^3*cos(d*x+c)*sin(d*x+c)/d-33/8*a
^3*x-33/8/d*a^3*c+1/2*a^3*cos(d*x+c)^3/d+3/2*a^3*cos(d*x+c)/d+3/2/d*a^3*ln(
csc(d*x+c)-cot(d*x+c))-3/d*a^3/sin(d*x+c)*cos(d*x+c)^5-1/2/d*a^3/sin(d*x+c)
^2*cos(d*x+c)^5
```

maxima [A] time = 0.44, size = 183, normalized size = 1.34

$$\frac{16\left(2\cos(dx+c)^3 + 6\cos(dx+c) - 3\log(\cos(dx+c)+1) + 3\log(\cos(dx+c)-1)\right)a^3 + (12\,dx + 12\,c + \mathrm{si}\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/32*(16*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a^3 + (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*a^3 - 48*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + t
an(d*x + c)))*a^3 + 8*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x
+ c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.81, size = 347, normalized size = 2.53

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}+\frac{33\,a^3\,\mathrm{atan}\left(\frac{1089\,a^6}{16\left(\frac{99\,a^6}{4}+\frac{1089\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}-\frac{99\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\left(\frac{99\,a^6}{4}+\frac{1089\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}+\frac{a^3\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (3*a^3*log(tan(c/2 + (d*x)/2)))/(2*d) +
(33*a^3*atan((1089*a^6)/(16*((99*a^6)/4 + (1089*a^6*tan(c/2 + (d*x)/2))/16)
) - (99*a^6*tan(c/2 + (d*x)/2))/(4*((99*a^6)/4 + (1089*a^6*tan(c/2 + (d*x)/
2))/16))))/(4*d) + (22*a^3*tan(c/2 + (d*x)/2)^2 - 31*a^3*tan(c/2 + (d*x)/2)
^3 + 53*a^3*tan(c/2 + (d*x)/2)^4 - 51*a^3*tan(c/2 + (d*x)/2)^5 + 70*a^3*tan
(c/2 + (d*x)/2)^6 - 9*a^3*tan(c/2 + (d*x)/2)^7 + (79*a^3*tan(c/2 + (d*x)/2)
^8)/2 + a^3*tan(c/2 + (d*x)/2)^9 - a^3/2 - 6*a^3*tan(c/2 + (d*x)/2))/(d*(4*
tan(c/2 + (d*x)/2)^2 + 16*tan(c/2 + (d*x)/2)^4 + 24*tan(c/2 + (d*x)/2)^6 +
16*tan(c/2 + (d*x)/2)^8 + 4*tan(c/2 + (d*x)/2)^10)) + (3*a^3*tan(c/2 + (d*x
)/2))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.399 $\int \cot^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=134

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{2a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{2a^3\cot(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3\tanh^{-1}(\mathrm{co}}{2d}$$

[Out] -7/2*a^3*x+7/2*a^3*arctanh(cos(d*x+c))/d-2*a^3*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3/d-2*a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d-3/2*a^3*cot(d*x+c)*csc(d*x+c)/d-3/2*a^3*cos(d*x+c)*sin(d*x+c)/d

Rubi [A] time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2709, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{2a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{2a^3\cot(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3\tanh^{-1}(\mathrm{co}}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]

[Out] (-7*a^3*x)/2 + (7*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (2*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) - (2*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) - (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(-5a^7 - 5a^7\csc(c+dx) + a^7\csc^2(c+dx) + 3a^7\csc^3(c+dx) + a^7\csc^4\right.}{a^4}$$
$$= -5a^3x + a^3\int \csc^2(c+dx)\,dx + a^3\int \csc^4(c+dx)\,dx + a^3\int \sin(c+dx)$$
$$= -5a^3x + \frac{5a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^3\cos(c+dx)}{d} - \frac{3a^3\cot(c+dx)\csc}{2d}$$
$$= -\frac{7a^3x}{2} + \frac{7a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} -$$

Mathematica [A] time = 6.13, size = 201, normalized size = 1.50

$$a^3(\sin(c+dx)+1)^3\left(-84(c+dx)-18\sin(2(c+dx))-42\cos(c+dx)+2\cos(3(c+dx))+20\tan\left(\frac{1}{2}(c+dx)\right)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-84*(c + d*x) - 42*Cos[c + d*x] + 2*Cos[3*(c + d
*x)] - 20*Cot[(c + d*x)/2] - 9*Csc[(c + d*x)/2]^2 + 84*Log[Cos[(c + d*x)/2]
] - 84*Log[Sin[(c + d*x)/2]] + 9*Sec[(c + d*x)/2]^2 + 8*Csc[c + d*x]^3*Sin[
(c + d*x)/2]^4 - (Csc[(c + d*x)/2]^4*Sin[c + d*x])/2 - 18*Sin[2*(c + d*x)]
+ 20*Tan[(c + d*x)/2]))/(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.48, size = 206, normalized size = 1.54

$$18\,a^3\cos\left(dx+c\right)^5-56\,a^3\cos\left(dx+c\right)^3+42\,a^3\cos\left(dx+c\right)+21\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(18*a^3*cos(d*x + c)^5 - 56*a^3*cos(d*x + c)^3 + 42*a^3*cos(d*x + c) +
 21*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2
1*(a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 2*
(2*a^3*cos(d*x + c)^5 - 21*a^3*d*x*cos(d*x + c)^2 - 14*a^3*cos(d*x + c)^3 +
 21*a^3*d*x + 21*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*si
n(d*x + c))
```

giac [B] time = 0.29, size = 250, normalized size = 1.87

$$3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+27\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-252\left(dx+c\right)a^3-252\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+63\,a^3\tan\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/72*(3*a^3*tan(1/2*d*x + 1/2*c)^3 + 27*a^3*tan(1/2*d*x + 1/2*c)^2 - 252*(d
*x + c)*a^3 - 252*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 63*a^3*tan(1/2*d*x +
 1/2*c) + (154*a^3*tan(1/2*d*x + 1/2*c)^9 + 153*a^3*tan(1/2*d*x + 1/2*c)^8
+ 291*a^3*tan(1/2*d*x + 1/2*c)^7 - 192*a^3*tan(1/2*d*x + 1/2*c)^6 - 195*a^3
*tan(1/2*d*x + 1/2*c)^5 - 414*a^3*tan(1/2*d*x + 1/2*c)^4 - 167*a^3*tan(1/2*
d*x + 1/2*c)^3 - 72*a^3*tan(1/2*d*x + 1/2*c)^2 - 27*a^3*tan(1/2*d*x + 1/2*c
) - 3*a^3)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^3)/d
```

maple [A] time = 0.45, size = 190, normalized size = 1.42

$$-\frac{7a^3\left(\cos^3\left(dx+c\right)\right)}{6d}-\frac{7a^3\cos\left(dx+c\right)}{2d}-\frac{7a^3\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{2d}-\frac{3a^3\left(\cos^5\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}-\frac{3a^3\left(\cos^3\left(dx+\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -7/6*a^3*cos(d*x+c)^3/d-7/2*a^3*cos(d*x+c)/d-7/2/d*a^3*ln(csc(d*x+c)-cot(d*
x+c))-3/d*a^3/sin(d*x+c)*cos(d*x+c)^5-3*a^3*cos(d*x+c)^3*sin(d*x+c)/d-9/2*a
^3*cos(d*x+c)*sin(d*x+c)/d-7/2*a^3*x-7/2/d*a^3*c-3/2/d*a^3/sin(d*x+c)^2*cos
(d*x+c)^5-1/3*a^3*cot(d*x+c)^3/d+a^3*cot(d*x+c)/d
```

maxima [A] time = 0.44, size = 185, normalized size = 1.38

$$\frac{2\left(2\cos\left(dx+c\right)^3+6\cos\left(dx+c\right)-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)a^3-18\left(3\,dx+3\,c+\frac{3}{\tan(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*lo
g(cos(d*x + c) - 1))*a^3 - 18*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*
x + c)^3 + tan(d*x + c)))*a^3 + 4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan
(d*x + c)^3)*a^3 + 9*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x +
 c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.78, size = 339, normalized size = 2.53

$$\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{7\,a^3\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{7\,a^3\,\mathrm{atan}\left(\frac{49\,a^6}{49\,a^6-49\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{49\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{49\,a^6-49\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] (3*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^3*tan(c/2 + (d*x)/2)^3)/(24*d) - (7
*a^3*log(tan(c/2 + (d*x)/2)))/(2*d) - (7*a^3*atan((49*a^6)/(49*a^6 - 49*a^6
*tan(c/2 + (d*x)/2)) + (49*a^6*tan(c/2 + (d*x)/2))/(49*a^6 - 49*a^6*tan(c/2
 + (d*x)/2))))/d + (7*a^3*tan(c/2 + (d*x)/2))/(8*d) - (8*a^3*tan(c/2 + (d*x
)/2)^2 + (107*a^3*tan(c/2 + (d*x)/2)^3)/3 + 46*a^3*tan(c/2 + (d*x)/2)^4 + 7
3*a^3*tan(c/2 + (d*x)/2)^5 + (64*a^3*tan(c/2 + (d*x)/2)^6)/3 + 19*a^3*tan(c
/2 + (d*x)/2)^7 - 17*a^3*tan(c/2 + (d*x)/2)^8 + a^3/3 + 3*a^3*tan(c/2 + (d*
x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 24*tan(c/2 + (d*x)/2)^5 + 24*tan(c/2 +
(d*x)/2)^7 + 8*tan(c/2 + (d*x)/2)^9))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.400 $\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=138

$$-\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{2a^3\cot(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{33a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3 \mathrm{co}}{}$$

```
[Out] 3/2*a^3*x+33/8*a^3*arctanh(cos(d*x+c))/d-3*a^3*cos(d*x+c)/d+2*a^3*cot(d*x+c
)/d-a^3*cot(d*x+c)^3/d-7/8*a^3*cot(d*x+c)*csc(d*x+c)/d-1/4*a^3*cot(d*x+c)*c
sc(d*x+c)^3/d-1/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2872, 3770, 3767, 8, 3768, 2638, 2635}

$$-\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{2a^3\cot(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{33a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3 \mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*a^3*x)/2 + (33*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (3*a^3*Cos[c + d*x])/d
 + (2*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/d - (7*a^3*Cot[c + d*x]*Cs
c[c + d*x])/(8*d) - (a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) - (a^3*Cos[c +
d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(a^7-5a^7\csc(c+dx)-5a^7\csc^2(c+dx)+a^7\csc^3(c+dx)\right.}{}$$

$$= a^3x+a^3\int \csc^3(c+dx)\,dx+a^3\int \csc^5(c+dx)\,dx+a^3\int$$

$$= a^3x+\frac{5a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(}{}$$

$$= \frac{3a^3x}{2}+\frac{9a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cos(c+dx)}{d}+\frac{2a^3c}{}$$

$$= \frac{3a^3x}{2}+\frac{33a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^3\cos(c+dx)}{d}+\frac{2a^3}{}$$

Mathematica [A] time = 1.22, size = 215, normalized size = 1.56

$$\frac{a^3(\sin(c+dx)+1)^3\left(96(c+dx)-16\sin(2(c+dx))-192\cos(c+dx)-96\tan\left(\frac{1}{2}(c+dx)\right)+96\cot\left(\frac{1}{2}(c+dx)\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(96*(c + d*x) - 192*Cos[c + d*x] + 96*Cot[(c + d*
x)/2] - 14*Csc[(c + d*x)/2]^2 - Csc[(c + d*x)/2]^4 + 264*Log[Cos[(c + d*x)/
2]] - 264*Log[Sin[(c + d*x)/2]] + 14*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^
4 + 64*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 4*Csc[(c + d*x)/2]^4*Sin[c + d*x
] - 16*Sin[2*(c + d*x)] - 96*Tan[(c + d*x)/2]))/(64*d*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^6)
```

fricas [A] time = 0.48, size = 231, normalized size = 1.67

$$\frac{24\,a^3dx\cos\left(dx+c\right)^4-48\,a^3\cos\left(dx+c\right)^5-48\,a^3dx\cos\left(dx+c\right)^2+110\,a^3\cos\left(dx+c\right)^3+24\,a^3dx-66\,a^3\cos\left(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(24*a^3*d*x*cos(d*x + c)^4 - 48*a^3*cos(d*x + c)^5 - 48*a^3*d*x*cos(d*
x + c)^2 + 110*a^3*cos(d*x + c)^3 + 24*a^3*d*x - 66*a^3*cos(d*x + c) + 33*(
a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2
) - 33*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x +
 c) + 1/2) - 8*(a^3*cos(d*x + c)^5 + 4*a^3*cos(d*x + c)^3 - 3*a^3*cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.31, size = 241, normalized size = 1.75

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+8\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+16\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+96\,(dx+c)a^3-264\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*tan(1/2*d*x + 1/2*c)^4 + 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 16*a^3*ta
n(1/2*d*x + 1/2*c)^2 + 96*(d*x + c)*a^3 - 264*a^3*log(abs(tan(1/2*d*x + 1/2
*c))) - 88*a^3*tan(1/2*d*x + 1/2*c) + 64*(a^3*tan(1/2*d*x + 1/2*c)^3 - 6*a^
3*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^3)/(tan(1/2*d*x +
 1/2*c)^2 + 1)^2 + (550*a^3*tan(1/2*d*x + 1/2*c)^4 + 88*a^3*tan(1/2*d*x + 1
/2*c)^3 - 16*a^3*tan(1/2*d*x + 1/2*c)^2 - 8*a^3*tan(1/2*d*x + 1/2*c) - a^3)
/tan(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.45, size = 215, normalized size = 1.56

$$-\frac{a^3\left(\cos^5(dx+c)\right)}{d\sin(dx+c)}-\frac{a^3\left(\cos^3(dx+c)\right)\sin(dx+c)}{d}-\frac{3a^3\cos(dx+c)\sin(dx+c)}{2d}+\frac{3a^3x}{2}+\frac{3a^3c}{2d}-\frac{11a^3\left(\cos^5(dx+\right.}{8d\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/d*a^3/sin(d*x+c)*cos(d*x+c)^5-a^3*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a^3*cos(
d*x+c)*sin(d*x+c)/d+3/2*a^3*x+3/2/d*a^3*c-11/8/d*a^3/sin(d*x+c)^2*cos(d*x+c
)^5-11/8*a^3*cos(d*x+c)^3/d-33/8*a^3*cos(d*x+c)/d-33/8/d*a^3*ln(csc(d*x+c)-
cot(d*x+c))-a^3*cot(d*x+c)^3/d+3*a^3*cot(d*x+c)/d-1/4/d*a^3/sin(d*x+c)^4*co
s(d*x+c)^5
```

maxima [A] time = 0.43, size = 209, normalized size = 1.51

$$-\frac{8\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^3-16\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3+a^3\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(8*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x +
c)))*a^3 - 16*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 + a
^3*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^
2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) - 12*a^3*(2*cos
(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) -
 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.78, size = 329, normalized size = 2.38

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{33\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{3\,a^3\,\mathrm{atan}\left(\frac{9\,a^6}{\frac{99\,a^6}{4}+9\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(4*d) + (a^3*tan(c/2 + (d*x)/2)^3)/(8*d) + (a^3*
tan(c/2 + (d*x)/2)^4)/(64*d) - (33*a^3*log(tan(c/2 + (d*x)/2)))/(8*d) - (3*
a^3*atan((9*a^6)/((99*a^6)/4 + 9*a^6*tan(c/2 + (d*x)/2)) - (99*a^6*tan(c/2
+ (d*x)/2))/(4*((99*a^6)/4 + 9*a^6*tan(c/2 + (d*x)/2)))))/d - (11*a^3*tan(c
/2 + (d*x)/2))/(8*d) - ((9*a^3*tan(c/2 + (d*x)/2)^2)/2 - 18*a^3*tan(c/2 + (
d*x)/2)^3 + (417*a^3*tan(c/2 + (d*x)/2)^4)/4 - 26*a^3*tan(c/2 + (d*x)/2)^5
+ 100*a^3*tan(c/2 + (d*x)/2)^6 - 38*a^3*tan(c/2 + (d*x)/2)^7 + a^3/4 + 2*a^
3*tan(c/2 + (d*x)/2))/(d*(16*tan(c/2 + (d*x)/2)^4 + 32*tan(c/2 + (d*x)/2)^6
 + 16*tan(c/2 + (d*x)/2)^8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.401 $\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=132

$$-\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{3a^3\cot(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^3\cot(c+dx)\,c}{4d}$$

```
[Out] 3*a^3*x+3/8*a^3*arctanh(cos(d*x+c))/d-a^3*cos(d*x+c)/d+3*a^3*cot(d*x+c)/d-a
^3*cot(d*x+c)^3/d-1/5*a^3*cot(d*x+c)^5/d+11/8*a^3*cot(d*x+c)*csc(d*x+c)/d-3
/4*a^3*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A] time = 0.22, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2872, 3770, 3767, 8, 3768, 2638}

$$-\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{3a^3\cot(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^3\cot(c+dx)\,c}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] 3*a^3*x + (3*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (a^3*Cos[c + d*x])/d + (3*a
^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/d - (a^3*Cot[c + d*x]^5)/(5*d) +
(11*a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]
^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(3a^7 + a^7\csc(c+dx) - 5a^7\csc^2(c+dx) - 5a^7\csc^3(c+dx\right.}{}$$

$$= 3a^3x + a^3\int \csc(c+dx)\,dx + a^3\int \csc^4(c+dx)\,dx + a^3\int c$$

$$= 3a^3x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^3\cos(c+dx)}{d} + \frac{5a^3\cot(c}{}$$

$$= 3a^3x + \frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^3\cos(c+dx)}{d} + \frac{3a^3\cot}{}$$

$$= 3a^3x + \frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^3\cos(c+dx)}{d} + \frac{3a^3\cot}{}$$

Mathematica [A] time = 0.51, size = 216, normalized size = 1.64

$$\frac{a^3\left(-320\cos(c+dx) - 608\tan\left(\frac{1}{2}(c+dx)\right) + 608\cot\left(\frac{1}{2}(c+dx)\right) - 15\csc^4\left(\frac{1}{2}(c+dx)\right) + 110\csc^2\left(\frac{1}{2}(c+dx)\right) + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(960*c + 960*d*x - 320*Cos[c + d*x] + 608*Cot[(c + d*x)/2] + 110*Csc[(
c + d*x)/2]^2 - 15*Csc[(c + d*x)/2]^4 + 120*Log[Cos[(c + d*x)/2]] - 120*Log
```

```
[Sin[(c + d*x)/2]] - 110*Sec[(c + d*x)/2]^2 + 15*Sec[(c + d*x)/2]^4 + 208*C
sc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 64*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 - 1
3*Csc[(c + d*x)/2]^4*Sin[c + d*x] - Csc[(c + d*x)/2]^6*Sin[c + d*x] - 608*T
an[(c + d*x)/2]))/(320*d)
```

fricas [B] time = 0.47, size = 252, normalized size = 1.91

$$304\,a^3\cos\left(dx+c\right)^5 - 560\,a^3\cos\left(dx+c\right)^3 + 240\,a^3\cos\left(dx+c\right) + 15\left(a^3\cos\left(dx+c\right)^4 - 2\,a^3\cos\left(dx+c\right)^2 + a^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/80*(304*a^3*cos(d*x + c)^5 - 560*a^3*cos(d*x + c)^3 + 240*a^3*cos(d*x + c
) + 15*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x +
c) + 1/2)*sin(d*x + c) - 15*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^
3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 10*(24*a^3*d*x*cos(d*x + c)^
4 - 8*a^3*cos(d*x + c)^5 - 48*a^3*d*x*cos(d*x + c)^2 + 5*a^3*cos(d*x + c)^3
 + 24*a^3*d*x - 3*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*
cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.32, size = 226, normalized size = 1.71

$$2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 30\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 80\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 960\,(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/320*(2*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*a^3*tan(1/2*d*x + 1/2*c)^4 + 30*a^
3*tan(1/2*d*x + 1/2*c)^3 - 80*a^3*tan(1/2*d*x + 1/2*c)^2 + 960*(d*x + c)*a^
3 - 120*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 580*a^3*tan(1/2*d*x + 1/2*c) -
 640*a^3/(tan(1/2*d*x + 1/2*c)^2 + 1) + (274*a^3*tan(1/2*d*x + 1/2*c)^5 + 5
80*a^3*tan(1/2*d*x + 1/2*c)^4 + 80*a^3*tan(1/2*d*x + 1/2*c)^3 - 30*a^3*tan(
1/2*d*x + 1/2*c)^2 - 15*a^3*tan(1/2*d*x + 1/2*c) - 2*a^3)/tan(1/2*d*x + 1/2
*c)^5)/d
```

maple [A] time = 0.35, size = 173, normalized size = 1.31

$$-\frac{a^3\left(\cos^5\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^2} - \frac{a^3\left(\cos^3\left(dx+c\right)\right)}{8d} - \frac{3a^3\cos\left(dx+c\right)}{8d} - \frac{3a^3\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{8d} - \frac{a^3\left(\cot^3\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/8/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5-1/8*a^3*cos(d*x+c)^3/d-3/8*a^3*cos(d*x
+c)/d-3/8/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-a^3*cot(d*x+c)^3/d+3*a^3*x+3*a^3*
cot(d*x+c)/d+3/d*a^3*c-3/4/d*a^3/sin(d*x+c)^4*cos(d*x+c)^5-1/5/d*a^3/sin(d*
x+c)^5*cos(d*x+c)^5
```

maxima [A] time = 0.53, size = 180, normalized size = 1.36

$$\frac{80\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3-15\,a^3\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)\cdots\right.\right.}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/80*(80*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 - 15*a^3
*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2
+ 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 20*a^3*(2*cos(d
*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3
*log(cos(d*x + c) - 1)) - 16*a^3/tan(d*x + c)^5)/d
```

mupad [B] time = 10.11, size = 554, normalized size = 4.20

$$-\frac{a^3\left(2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-15\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+15\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] -(a^3*(2*cos(c/2 + (d*x)/2)^12 - 2*sin(c/2 + (d*x)/2)^12 - 15*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^11 + 15*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) -
 32*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 65*cos(c/2 + (d*x)/2)^3*si
n(c/2 + (d*x)/2)^9 + 550*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 80*cos
(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 560*cos(c/2 + (d*x)/2)^7*sin(c/2 +
 (d*x)/2)^5 - 550*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 65*cos(c/2 +
(d*x)/2)^9*sin(c/2 + (d*x)/2)^3 + 32*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/
2)^2 + 120*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*
sin(c/2 + (d*x)/2)^7 + 120*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c
```

```
/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5 + 1920*atan((8*cos(c/2 + (d*x)/2) - si
n(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2) + 8*sin(c/2 + (d*x)/2)))*cos(c/2 + (d
*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 1920*atan((8*cos(c/2 + (d*x)/2) - sin(c/2 +
 (d*x)/2))/(cos(c/2 + (d*x)/2) + 8*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^
7*sin(c/2 + (d*x)/2)^5))/(320*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5*(
cos(c/2 + (d*x)/2)^2 + sin(c/2 + (d*x)/2)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.402 $\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=168

$$-\frac{3a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{19a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{6d}-\frac{3a^3}{}$$

```
[Out] a^3*x-19/16*a^3*arctanh(cos(d*x+c))/d+a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3
/d-3/5*a^3*cot(d*x+c)^5/d+17/16*a^3*cot(d*x+c)*csc(d*x+c)/d-3/4*a^3*cot(d*x
+c)^3*csc(d*x+c)/d+1/8*a^3*cot(d*x+c)*csc(d*x+c)^3/d-1/6*a^3*cot(d*x+c)^3*c
sc(d*x+c)^3/d
```

Rubi [A] time = 0.27, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2873, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$-\frac{3a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{19a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{6d}-\frac{3a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*x - (19*a^3*ArcTanh[Cos[c + d*x]])/(16*d) + (a^3*Cot[c + d*x])/d - (a^3
*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]^5)/(5*d) + (17*a^3*Cot[c + d*x
]*Csc[c + d*x])/(16*d) - (3*a^3*Cot[c + d*x]^3*Csc[c + d*x])/(4*d) + (a^3*C
ot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)+3a^3\cot^4(c+dx)\csc(c+dx)+3a^3\cot^4\right.$$
$$= a^3\int \cot^4(c+dx)\,dx + a^3\int \cot^4(c+dx)\csc^3(c+dx)\,dx +$$
$$= -\frac{a^3\cot^3(c+dx)}{3d} - \frac{3a^3\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{a^3\cot^3(c}{}$$
$$= \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} - \frac{3a^3\cot^5(c+dx)}{5d} + \frac{9a^3\,c}{}$$
$$= a^3x - \frac{9a^3\tanh^{-1}(\cos(c+dx))}{8d} + \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c}{3d}$$
$$= a^3x - \frac{19a^3\tanh^{-1}(\cos(c+dx))}{16d} + \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(}{3a}$$

Mathematica [A] time = 0.78, size = 217, normalized size = 1.29

$$\frac{a^3\left(-704\tan\left(\frac{1}{2}(c+dx)\right)+704\cot\left(\frac{1}{2}(c+dx)\right)+870\csc^2\left(\frac{1}{2}(c+dx)\right)+5\sec^6\left(\frac{1}{2}(c+dx)\right)+60\sec^4\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1920*c + 1920*d*x + 704*Cot[(c + d*x)/2] + 870*Csc[(c + d*x)/2]^2 - 2
280*Log[Cos[(c + d*x)/2]] + 2280*Log[Sin[(c + d*x)/2]] - 870*Sec[(c + d*x)/
2]^2 + 60*Sec[(c + d*x)/2]^4 + 5*Sec[(c + d*x)/2]^6 - 1376*Csc[c + d*x]^3*S
in[(c + d*x)/2]^4 - Csc[(c + d*x)/2]^6*(5 + 18*Sin[c + d*x]) + Csc[(c + d*x
)/2]^4*(-60 + 86*Sin[c + d*x]) - 704*Tan[(c + d*x)/2] + 36*Sec[(c + d*x)/2]
^4*Tan[(c + d*x)/2]))/(1920*d)
```

fricas [A] time = 0.50, size = 290, normalized size = 1.73

$$\frac{480\,a^3dx\cos(dx+c)^6 - 1440\,a^3dx\cos(dx+c)^4 - 870\,a^3\cos(dx+c)^5 + 1440\,a^3dx\cos(dx+c)^2 + 1520\,a^3\cos(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/480*(480*a^3*d*x*cos(d*x + c)^6 - 1440*a^3*d*x*cos(d*x + c)^4 - 870*a^3*c
os(d*x + c)^5 + 1440*a^3*d*x*cos(d*x + c)^2 + 1520*a^3*cos(d*x + c)^3 - 480
```

```
*a^3*d*x - 570*a^3*cos(d*x + c) - 285*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x +
 c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2) + 285*(a^3*
cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(-1/
2*cos(d*x + c) + 1/2) - 32*(11*a^3*cos(d*x + c)^5 - 35*a^3*cos(d*x + c)^3 +
 15*a^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4
+ 3*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.34, size = 239, normalized size = 1.42

$$5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+75\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-100\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-735\,a^3\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^3*tan(1/2*d*x + 1/2*c)^5 + 75*a
^3*tan(1/2*d*x + 1/2*c)^4 - 100*a^3*tan(1/2*d*x + 1/2*c)^3 - 735*a^3*tan(1/
2*d*x + 1/2*c)^2 + 1920*(d*x + c)*a^3 + 2280*a^3*log(abs(tan(1/2*d*x + 1/2*
c))) - 840*a^3*tan(1/2*d*x + 1/2*c) - (5586*a^3*tan(1/2*d*x + 1/2*c)^6 - 84
0*a^3*tan(1/2*d*x + 1/2*c)^5 - 735*a^3*tan(1/2*d*x + 1/2*c)^4 - 100*a^3*tan
(1/2*d*x + 1/2*c)^3 + 75*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a^3*tan(1/2*d*x +
1/2*c) + 5*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

maple [A] time = 0.36, size = 194, normalized size = 1.15

$$-\frac{a^3\left(\cot^3(dx+c)\right)}{3d}+\frac{a^3\cot(dx+c)}{d}+a^3x+\frac{a^3c}{d}-\frac{19a^3\left(\cos^5(dx+c)\right)}{24d\sin(dx+c)^4}+\frac{19a^3\left(\cos^5(dx+c)\right)}{48d\sin(dx+c)^2}+\frac{19a^3\left(\cos^3(dx+c)\right)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/3*a^3*cot(d*x+c)^3/d+a^3*cot(d*x+c)/d+a^3*x+1/d*a^3*c-19/24/d*a^3/sin(d*
x+c)^4*cos(d*x+c)^5+19/48/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5+19/48*a^3*cos(d*x
+c)^3/d+19/16*a^3*cos(d*x+c)/d+19/16/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3/5/d*
a^3/sin(d*x+c)^5*cos(d*x+c)^5-1/6/d*a^3/sin(d*x+c)^6*cos(d*x+c)^5
```

maxima [A] time = 0.42, size = 215, normalized size = 1.28

$$\frac{160\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3+5\,a^3\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos(dx+c)+1\right)+3\,\log\left(\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/480*(160*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 + 5*a^
3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^6
 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1)) - 90*a^3*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(
d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*
x + c) - 1)) - 288*a^3/tan(d*x + c)^5)/d
```

mupad [B] time = 9.38, size = 313, normalized size = 1.86

$$\frac{49\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{5\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{5\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{128\,d}-\frac{3\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}-\frac{49\,a^3\tan\left(\frac{c}{2}-\right.}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] (49*a^3*cot(c/2 + (d*x)/2)^2)/(128*d) + (5*a^3*cot(c/2 + (d*x)/2)^3)/(96*d)
 - (5*a^3*cot(c/2 + (d*x)/2)^4)/(128*d) - (3*a^3*cot(c/2 + (d*x)/2)^5)/(160
*d) - (a^3*cot(c/2 + (d*x)/2)^6)/(384*d) - (49*a^3*tan(c/2 + (d*x)/2)^2)/(1
28*d) - (5*a^3*tan(c/2 + (d*x)/2)^3)/(96*d) + (5*a^3*tan(c/2 + (d*x)/2)^4)/
(128*d) + (3*a^3*tan(c/2 + (d*x)/2)^5)/(160*d) + (a^3*tan(c/2 + (d*x)/2)^6)
/(384*d) + (2*a^3*atan((16*cos(c/2 + (d*x)/2) + 19*sin(c/2 + (d*x)/2))/(19*
cos(c/2 + (d*x)/2) - 16*sin(c/2 + (d*x)/2))))/d + (19*a^3*log(sin(c/2 + (d*
x)/2)/cos(c/2 + (d*x)/2)))/(16*d) + (7*a^3*cot(c/2 + (d*x)/2))/(16*d) - (7*
a^3*tan(c/2 + (d*x)/2))/(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.403 $\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=150

$$-\frac{a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{9a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)\csc(}{4d}$$

```
[Out] -9/16*a^3*arctanh(cos(d*x+c))/d-4/5*a^3*cot(d*x+c)^5/d-1/7*a^3*cot(d*x+c)^7
/d+3/16*a^3*cot(d*x+c)*csc(d*x+c)/d-1/4*a^3*cot(d*x+c)^3*csc(d*x+c)/d+3/8*a
^3*cot(d*x+c)*csc(d*x+c)^3/d-1/2*a^3*cot(d*x+c)^3*csc(d*x+c)^3/d
```

Rubi [A] time = 0.28, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2611, 3770, 2607, 30, 3768, 14}

$$-\frac{a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{9a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)\csc(}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-9*a^3*ArcTanh[Cos[c + d*x]])/(16*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (a^3
*Cot[c + d*x]^7)/(7*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (a^3*Co
t[c + d*x]^3*Csc[c + d*x])/(4*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d
) - (a^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)\csc(c+dx)+3a^3\cot^4(c+dx)\csc^2(c+d\right.$$
$$= a^3\int \cot^4(c+dx)\csc(c+dx)\,dx + a^3\int \cot^4(c+dx)\csc^4(c$$
$$= -\frac{a^3\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{2d}$$
$$= -\frac{3a^3\cot^5(c+dx)}{5d} + \frac{3a^3\cot(c+dx)\csc(c+dx)}{8d} - \frac{a^3\cot^3(c}{}$$
$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+}{7d}$$
$$= -\frac{9a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+}{7d}$$

Mathematica [B] time = 0.13, size = 363, normalized size = 2.42

$$a^3\left(\frac{23\tan\left(\frac{1}{2}(c+dx)\right)}{70d}-\frac{23\cot\left(\frac{1}{2}(c+dx)\right)}{70d}-\frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{128d}+\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{7\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\sec^6}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((-23*Cot[(c + d*x)/2])/(70*d) + (7*Csc[(c + d*x)/2]^2)/(64*d) + (297*C
ot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(2240*d) + Csc[(c + d*x)/2]^4/(32*d) -
(31*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(2240*d) - Csc[(c + d*x)/2]^6/(128
*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^6)/(896*d) - (9*Log[Cos[(c + d*x)/
2]])/(16*d) + (9*Log[Sin[(c + d*x)/2]])/(16*d) - (7*Sec[(c + d*x)/2]^2)/(64
*d) - Sec[(c + d*x)/2]^4/(32*d) + Sec[(c + d*x)/2]^6/(128*d) + (23*Tan[(c +
 d*x)/2])/(70*d) - (297*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(2240*d) + (31
*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(2240*d) + (Sec[(c + d*x)/2]^6*Tan[(c
 + d*x)/2])/(896*d))
```

fricas [A] time = 0.51, size = 247, normalized size = 1.65

$$-\frac{736\,a^3\cos\left(dx+c\right)^7-896\,a^3\cos\left(dx+c\right)^5+315\left(a^3\cos\left(dx+c\right)^6-3\,a^3\cos\left(dx+c\right)^4+3\,a^3\cos\left(dx+c\right)^2-a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/1120*(736*a^3*cos(d*x + c)^7 - 896*a^3*cos(d*x + c)^5 + 315*(a^3*cos(d*x
 + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) - 315*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4
 + 3*a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
70*(7*a^3*cos(d*x + c)^5 - 24*a^3*cos(d*x + c)^3 + 9*a^3*cos(d*x + c))*sin(
d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
*sin(d*x + c))
```

giac [A] time = 0.34, size = 261, normalized size = 1.74

$$\frac{5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+77\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-455\,a^3\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5*a^3*tan(1/2*d*x + 1/2*c)^7 + 35*a^3*tan(1/2*d*x + 1/2*c)^6 + 77*a
^3*tan(1/2*d*x + 1/2*c)^5 - 35*a^3*tan(1/2*d*x + 1/2*c)^4 - 455*a^3*tan(1/2
*d*x + 1/2*c)^3 - 665*a^3*tan(1/2*d*x + 1/2*c)^2 + 2520*a^3*log(abs(tan(1/2
*d*x + 1/2*c))) + 945*a^3*tan(1/2*d*x + 1/2*c) - (6534*a^3*tan(1/2*d*x + 1/
2*c)^7 + 945*a^3*tan(1/2*d*x + 1/2*c)^6 - 665*a^3*tan(1/2*d*x + 1/2*c)^5 -
455*a^3*tan(1/2*d*x + 1/2*c)^4 - 35*a^3*tan(1/2*d*x + 1/2*c)^3 + 77*a^3*tan
(1/2*d*x + 1/2*c)^2 + 35*a^3*tan(1/2*d*x + 1/2*c) + 5*a^3)/tan(1/2*d*x + 1/
2*c)^7)/d
```

maple [A] time = 0.38, size = 176, normalized size = 1.17

$$-\frac{3a^3\left(\cos^5\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^4}+\frac{3a^3\left(\cos^5\left(dx+c\right)\right)}{16d\sin\left(dx+c\right)^2}+\frac{3a^3\left(\cos^3\left(dx+c\right)\right)}{16d}+\frac{9a^3\cos\left(dx+c\right)}{16d}+\frac{9a^3\ln\left(\csc\left(dx+c\right)-\cot\left(dx+\right.\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -3/8/d*a^3/sin(d*x+c)^4*cos(d*x+c)^5+3/16/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5+3
/16*a^3*cos(d*x+c)^3/d+9/16*a^3*cos(d*x+c)/d+9/16/d*a^3*ln(csc(d*x+c)-cot(d
*x+c))-23/35/d*a^3/sin(d*x+c)^5*cos(d*x+c)^5-1/2/d*a^3/sin(d*x+c)^6*cos(d*x
+c)^5-1/7/d*a^3/sin(d*x+c)^7*cos(d*x+c)^5
```

maxima [A] time = 0.35, size = 206, normalized size = 1.37

$$\frac{35\,a^3\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-70\,a^3\left(\frac{2\left(5\,\cos(dx+\right.}{\cos(dx+c)^4-}\right.}{1120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/1120*(35*a^3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(c
os(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x +
c) + 1) + 3*log(cos(d*x + c) - 1)) - 70*a^3*(2*(5*cos(d*x + c)^3 - 3*cos(d*
x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) -
 3*log(cos(d*x + c) - 1)) - 672*a^3/tan(d*x + c)^5 - 32*(7*tan(d*x + c)^2 +
 5)*a^3/tan(d*x + c)^7)/d
```

mupad [B] time = 10.32, size = 387, normalized size = 2.58

$$a^3\left(5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+35\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-35\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^8,x)
```

```
[Out] (a^3*(5*sin(c/2 + (d*x)/2)^14 - 5*cos(c/2 + (d*x)/2)^14 + 35*cos(c/2 + (d*x
)/2)*sin(c/2 + (d*x)/2)^13 - 35*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) +
77*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 35*cos(c/2 + (d*x)/2)^3*sin
(c/2 + (d*x)/2)^11 - 455*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 - 665*c
os(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 + 945*cos(c/2 + (d*x)/2)^6*sin(c/2
 + (d*x)/2)^8 - 945*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 665*cos(c/2
 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5 + 455*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d
*x)/2)^4 + 35*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 - 77*cos(c/2 + (d*
x)/2)^12*sin(c/2 + (d*x)/2)^2 + 2520*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2))*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7))/(4480*d*cos(c/2 + (d*x)/2)
^7*sin(c/2 + (d*x)/2)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.404 $\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=176

$$-\frac{3a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{27a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^3\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{a^3\cot^3(c+dx)\csc}{2d}$$

```
[Out] -27/128*a^3*arctanh(cos(d*x+c))/d-4/5*a^3*cot(d*x+c)^5/d-3/7*a^3*cot(d*x+c)
^7/d-27/128*a^3*cot(d*x+c)*csc(d*x+c)/d+23/64*a^3*cot(d*x+c)*csc(d*x+c)^3/d
-1/2*a^3*cot(d*x+c)^3*csc(d*x+c)^3/d+1/16*a^3*cot(d*x+c)*csc(d*x+c)^5/d-1/8
*a^3*cot(d*x+c)^3*csc(d*x+c)^5/d
```

Rubi [A] time = 0.34, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2873, 2607, 30, 2611, 3768, 3770, 14}

$$-\frac{3a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{27a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^3\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{a^3\cot^3(c+dx)\csc}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-27*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (3
*a^3*Cot[c + d*x]^7)/(7*d) - (27*a^3*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (
23*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a^3*Cot[c + d*x]^3*Csc[c + d*
x]^3)/(2*d) + (a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a^3*Cot[c + d*x]^
3*Csc[c + d*x]^5)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)\csc^2(c+dx)+3a^3\cot^4(c+dx)\csc^3(c+\right.$$

$$= a^3\int \cot^4(c+dx)\csc^2(c+dx)\,dx + a^3\int \cot^4(c+dx)\csc^5($$

$$= -\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{2d} - \frac{a^3\cot^3(c+dx)\csc^5(c+dx)}{8d}$$

$$= -\frac{a^3\cot^5(c+dx)}{5d} + \frac{3a^3\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a^3\cot^3(c}{}$$

$$= -\frac{4a^3\cot^5(c+dx)}{5d} - \frac{3a^3\cot^7(c+dx)}{7d} - \frac{3a^3\cot(c+dx)\csc(}{16d}$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{3a^3\cot^7(c+}{7d}$$

$$= -\frac{27a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{3a^3\cot^7(c}{7d}$$

Mathematica [A] time = 5.09, size = 313, normalized size = 1.78

$$-\frac{a^3\sin(c+dx)(\sin(c+dx)+1)^3\left(10(7\csc(c+dx)+24)\csc^8\left(\frac{1}{2}(c+dx)\right)+8(105\csc(c+dx)-76)\csc^6\left(\frac{1}{2}(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/143360*(a^3*(10*Csc[(c + d*x)/2]^8*(24 + 7*Csc[c + d*x]) + 8*Csc[(c + d*
x)/2]^6*(-76 + 105*Csc[c + d*x]) + 8*Csc[(c + d*x)/2]^2*(1664 + 945*Csc[c +
 d*x]) - 4*Csc[(c + d*x)/2]^4*(856 + 1715*Csc[c + d*x]) - 4*(-7560*Csc[c +
d*x]*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + (703 + 1056*Cos[c +
d*x] + 517*Cos[2*(c + d*x)] + 104*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^8 + 75
60*Csc[c + d*x]^3*Sin[(c + d*x)/2]^2 - 27440*Csc[c + d*x]^5*Sin[(c + d*x)/2
]^4 + 13440*Csc[c + d*x]^7*Sin[(c + d*x)/2]^6 + 4480*Csc[c + d*x]^9*Sin[(c
+ d*x)/2]^8))*Sin[c + d*x]*(1 + Sin[c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^6)
```

fricas [A] time = 0.47, size = 271, normalized size = 1.54

$$\frac{1890\,a^3\cos(dx+c)^7+2030\,a^3\cos(dx+c)^5-6930\,a^3\cos(dx+c)^3+1890\,a^3\cos(dx+c)-945\left(a^3\cos(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/8960*(1890*a^3*cos(d*x + c)^7 + 2030*a^3*cos(d*x + c)^5 - 6930*a^3*cos(d*
x + c)^3 + 1890*a^3*cos(d*x + c) - 945*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x
+ c)^6 + 6*a^3*cos(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x
 + c) + 1/2) + 945*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos(d
*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2) + 256*
(13*a^3*cos(d*x + c)^7 - 28*a^3*cos(d*x + c)^5)*sin(d*x + c))/(d*cos(d*x +
c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.38, size = 293, normalized size = 1.66

$$\frac{35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+240\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+560\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+112\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1960\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/71680*(35*a^3*tan(1/2*d*x + 1/2*c)^8 + 240*a^3*tan(1/2*d*x + 1/2*c)^7 + 5
60*a^3*tan(1/2*d*x + 1/2*c)^6 + 112*a^3*tan(1/2*d*x + 1/2*c)^5 - 1960*a^3*t
an(1/2*d*x + 1/2*c)^4 - 3920*a^3*tan(1/2*d*x + 1/2*c)^3 - 1680*a^3*tan(1/2*
d*x + 1/2*c)^2 + 15120*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 9520*a^3*tan(1/
2*d*x + 1/2*c) - (41094*a^3*tan(1/2*d*x + 1/2*c)^8 + 9520*a^3*tan(1/2*d*x +
 1/2*c)^7 - 1680*a^3*tan(1/2*d*x + 1/2*c)^6 - 3920*a^3*tan(1/2*d*x + 1/2*c)
^5 - 1960*a^3*tan(1/2*d*x + 1/2*c)^4 + 112*a^3*tan(1/2*d*x + 1/2*c)^3 + 560
*a^3*tan(1/2*d*x + 1/2*c)^2 + 240*a^3*tan(1/2*d*x + 1/2*c) + 35*a^3)/tan(1/
2*d*x + 1/2*c)^8)/d
```

maple [A] time = 0.39, size = 200, normalized size = 1.14

$$-\frac{13a^3\left(\cos^5\left(dx+c\right)\right)}{35d\sin\left(dx+c\right)^5}-\frac{9a^3\left(\cos^5\left(dx+c\right)\right)}{16d\sin\left(dx+c\right)^6}-\frac{9a^3\left(\cos^5\left(dx+c\right)\right)}{64d\sin\left(dx+c\right)^4}+\frac{9a^3\left(\cos^5\left(dx+c\right)\right)}{128d\sin\left(dx+c\right)^2}+\frac{9a^3\left(\cos^3\left(dx+c\right)\right)}{128d}+\frac{27a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -13/35/d*a^3/sin(d*x+c)^5*cos(d*x+c)^5-9/16/d*a^3/sin(d*x+c)^6*cos(d*x+c)^5
-9/64/d*a^3/sin(d*x+c)^4*cos(d*x+c)^5+9/128/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5
+9/128*a^3*cos(d*x+c)^3/d+27/128*a^3*cos(d*x+c)/d+27/128/d*a^3*ln(csc(d*x+c
```

```
)-cot(d*x+c))-3/7/d*a^3/sin(d*x+c)^7*cos(d*x+c)^5-1/8/d*a^3/sin(d*x+c)^8*co
s(d*x+c)^5
```

maxima [A] time = 0.33, size = 246, normalized size = 1.40

$$35\,a^3\left(\frac{2\left(3\,\cos(dx+c)^7-11\,\cos(dx+c)^5-11\,\cos(dx+c)^3+3\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+280$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8960*(35*a^3*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3
 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 -
4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))
+ 280*a^3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*
x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) +
1) + 3*log(cos(d*x + c) - 1)) - 1792*a^3/tan(d*x + c)^5 - 768*(7*tan(d*x +
c)^2 + 5)*a^3/tan(d*x + c)^7)/d
```

mupad [B] time = 9.22, size = 319, normalized size = 1.81

$$\frac{3\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{7\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{128\,d}+\frac{7\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}-\frac{a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640\,d}-\frac{a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{128\,d}-\frac{3\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{896\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^9,x)
```

```
[Out] (3*a^3*cot(c/2 + (d*x)/2)^2)/(128*d) + (7*a^3*cot(c/2 + (d*x)/2)^3)/(128*d)
 + (7*a^3*cot(c/2 + (d*x)/2)^4)/(256*d) - (a^3*cot(c/2 + (d*x)/2)^5)/(640*d
) - (a^3*cot(c/2 + (d*x)/2)^6)/(128*d) - (3*a^3*cot(c/2 + (d*x)/2)^7)/(896*
d) - (a^3*cot(c/2 + (d*x)/2)^8)/(2048*d) - (3*a^3*tan(c/2 + (d*x)/2)^2)/(12
8*d) - (7*a^3*tan(c/2 + (d*x)/2)^3)/(128*d) - (7*a^3*tan(c/2 + (d*x)/2)^4)/
(256*d) + (a^3*tan(c/2 + (d*x)/2)^5)/(640*d) + (a^3*tan(c/2 + (d*x)/2)^6)/(
128*d) + (3*a^3*tan(c/2 + (d*x)/2)^7)/(896*d) + (a^3*tan(c/2 + (d*x)/2)^8)/
(2048*d) + (27*a^3*log(tan(c/2 + (d*x)/2)))/(128*d) - (17*a^3*cot(c/2 + (d*
x)/2))/(128*d) + (17*a^3*tan(c/2 + (d*x)/2))/(128*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.405 $\int \cot^4(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=194

$$-\frac{a^3\cot^9(c+dx)}{9d}-\frac{5a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{17a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{8d}-$$

```
[Out] -17/128*a^3*arctanh(cos(d*x+c))/d-4/5*a^3*cot(d*x+c)^5/d-5/7*a^3*cot(d*x+c)
^7/d-1/9*a^3*cot(d*x+c)^9/d-17/128*a^3*cot(d*x+c)*csc(d*x+c)/d+5/64*a^3*cot
(d*x+c)*csc(d*x+c)^3/d-1/6*a^3*cot(d*x+c)^3*csc(d*x+c)^3/d+3/16*a^3*cot(d*x
+c)*csc(d*x+c)^5/d-3/8*a^3*cot(d*x+c)^3*csc(d*x+c)^5/d
```

Rubi [A] time = 0.35, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2873, 2611, 3768, 3770, 2607, 14, 270}

$$-\frac{a^3\cot^9(c+dx)}{9d}-\frac{5a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{17a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{8d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-17*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (5
*a^3*Cot[c + d*x]^7)/(7*d) - (a^3*Cot[c + d*x]^9)/(9*d) - (17*a^3*Cot[c + d
*x]*Csc[c + d*x])/(128*d) + (5*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a
^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x]^
5)/(16*d) - (3*a^3*Cot[c + d*x]^3*Csc[c + d*x]^5)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)\csc^3(c+dx) + 3a^3\cot^4(c+dx)\csc^4(c+\right.$$

$$= a^3\int \cot^4(c+dx)\csc^3(c+dx)\,dx + a^3\int \cot^4(c+dx)\csc^6($$

$$= -\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{3a^3\cot^3(c+dx)\csc^5(c+dx}{8d}$$

$$= \frac{a^3\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{6d} +$$

$$= -\frac{4a^3\cot^5(c+dx)}{5d} - \frac{5a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{9d} - \frac{a^3}{}$$

$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{5a^3\cot^7(c+}{7d}$$

$$= -\frac{17a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{5a^3\cot^7(c}{7d}$$

Mathematica [A] time = 1.33, size = 313, normalized size = 1.61

$$-\frac{a^3\csc^9(c+dx)\left(669060\sin(2(c+dx)) + 676620\sin(4(c+dx)) - 14700\sin(6(c+dx)) - 10710\sin(8(c+dx)) +\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/10321920*(a^3*Csc[c + d*x]^9*(1161216*Cos[c + d*x] + 247296*Cos[3*(c + d
*x)] - 198144*Cos[5*(c + d*x)] - 71424*Cos[7*(c + d*x)] + 7936*Cos[9*(c + d
*x)] + 674730*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 674730*Log[Sin[(c + d*x)
/2]]*Sin[c + d*x] + 669060*Sin[2*(c + d*x)] - 449820*Log[Cos[(c + d*x)/2]]*
Sin[3*(c + d*x)] + 449820*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 676620*S
in[4*(c + d*x)] + 192780*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 192780*Lo
g[Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 14700*Sin[6*(c + d*x)] - 48195*Log[C
os[(c + d*x)/2]]*Sin[7*(c + d*x)] + 48195*Log[Sin[(c + d*x)/2]]*Sin[7*(c +
d*x)] - 10710*Sin[8*(c + d*x)] + 5355*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)
] - 5355*Log[Sin[(c + d*x)/2]]*Sin[9*(c + d*x)]))/d
```

fricas [A] time = 0.48, size = 304, normalized size = 1.57

$$-\frac{15872\,a^3\cos(dx+c)^9 - 71424\,a^3\cos(dx+c)^7 + 64512\,a^3\cos(dx+c)^5 + 5355\left(a^3\cos(dx+c)^8 - 4\,a^3\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] -1/80640*(15872*a^3*cos(d*x + c)^9 - 71424*a^3*cos(d*x + c)^7 + 64512*a^3*c
os(d*x + c)^5 + 5355*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos
(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d
*x + c) - 5355*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x +
 c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
c) - 210*(51*a^3*cos(d*x + c)^7 - 59*a^3*cos(d*x + c)^5 - 187*a^3*cos(d*x +
 c)^3 + 51*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x
 + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.37, size = 325, normalized size = 1.68

$$\frac{140\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+945\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+2340\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1680\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-403}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/645120*(140*a^3*tan(1/2*d*x + 1/2*c)^9 + 945*a^3*tan(1/2*d*x + 1/2*c)^8 +
 2340*a^3*tan(1/2*d*x + 1/2*c)^7 + 1680*a^3*tan(1/2*d*x + 1/2*c)^6 - 4032*a
^3*tan(1/2*d*x + 1/2*c)^5 - 12600*a^3*tan(1/2*d*x + 1/2*c)^4 - 16800*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 5040*a^3*tan(1/2*d*x + 1/2*c)^2 + 85680*a^3*log(abs(
tan(1/2*d*x + 1/2*c))) + 52920*a^3*tan(1/2*d*x + 1/2*c) - (242386*a^3*tan(1
/2*d*x + 1/2*c)^9 + 52920*a^3*tan(1/2*d*x + 1/2*c)^8 - 5040*a^3*tan(1/2*d*x
 + 1/2*c)^7 - 16800*a^3*tan(1/2*d*x + 1/2*c)^6 - 12600*a^3*tan(1/2*d*x + 1/
2*c)^5 - 4032*a^3*tan(1/2*d*x + 1/2*c)^4 + 1680*a^3*tan(1/2*d*x + 1/2*c)^3
+ 2340*a^3*tan(1/2*d*x + 1/2*c)^2 + 945*a^3*tan(1/2*d*x + 1/2*c) + 140*a^3)
/tan(1/2*d*x + 1/2*c)^9)/d
```

maple [A] time = 0.41, size = 224, normalized size = 1.15

$$-\frac{17a^3\left(\cos^5(dx+c)\right)}{48d\sin(dx+c)^6}-\frac{17a^3\left(\cos^5(dx+c)\right)}{192d\sin(dx+c)^4}+\frac{17a^3\left(\cos^5(dx+c)\right)}{384d\sin(dx+c)^2}+\frac{17a^3\left(\cos^3(dx+c)\right)}{384d}+\frac{17a^3\cos(dx+c)}{128d}+17$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -17/48/d*a^3/sin(d*x+c)^6*cos(d*x+c)^5-17/192/d*a^3/sin(d*x+c)^4*cos(d*x+c)
^5+17/384/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5+17/384*a^3*cos(d*x+c)^3/d+17/128*
a^3*cos(d*x+c)/d+17/128/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-31/63/d*a^3/sin(d*x
+c)^7*cos(d*x+c)^5-62/315/d*a^3/sin(d*x+c)^5*cos(d*x+c)^5-3/8/d*a^3/sin(d*x
+c)^8*cos(d*x+c)^5-1/9/d*a^3/sin(d*x+c)^9*cos(d*x+c)^5
```

maxima [A] time = 0.35, size = 268, normalized size = 1.38

$$945\,a^3\left(\frac{2\left(3\,\cos(dx+c)^7-11\,\cos(dx+c)^5-11\,\cos(dx+c)^3+3\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+84$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] 1/80640*(945*a^3*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)
^3 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)
) + 840*a^3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(
d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
+ 1) + 3*log(cos(d*x + c) - 1)) - 6912*(7*tan(d*x + c)^2 + 5)*a^3/tan(d*x +
 c)^7 - 256*(63*tan(d*x + c)^4 + 90*tan(d*x + c)^2 + 35)*a^3/tan(d*x + c)^9
)/d
```

mupad [B] time = 9.32, size = 357, normalized size = 1.84

$$\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{5\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{192\,d}+\frac{5\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}+\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}-\frac{13\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3584\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^10,x)
```

```
[Out] (a^3*cot(c/2 + (d*x)/2)^2)/(128*d) + (5*a^3*cot(c/2 + (d*x)/2)^3)/(192*d) +
 (5*a^3*cot(c/2 + (d*x)/2)^4)/(256*d) + (a^3*cot(c/2 + (d*x)/2)^5)/(160*d)
- (a^3*cot(c/2 + (d*x)/2)^6)/(384*d) - (13*a^3*cot(c/2 + (d*x)/2)^7)/(3584*
d) - (3*a^3*cot(c/2 + (d*x)/2)^8)/(2048*d) - (a^3*cot(c/2 + (d*x)/2)^9)/(46
08*d) - (a^3*tan(c/2 + (d*x)/2)^2)/(128*d) - (5*a^3*tan(c/2 + (d*x)/2)^3)/(
192*d) - (5*a^3*tan(c/2 + (d*x)/2)^4)/(256*d) - (a^3*tan(c/2 + (d*x)/2)^5)/
(160*d) + (a^3*tan(c/2 + (d*x)/2)^6)/(384*d) + (13*a^3*tan(c/2 + (d*x)/2)^7
)/(3584*d) + (3*a^3*tan(c/2 + (d*x)/2)^8)/(2048*d) + (a^3*tan(c/2 + (d*x)/2
)^9)/(4608*d) + (17*a^3*log(tan(c/2 + (d*x)/2)))/(128*d) - (21*a^3*cot(c/2
+ (d*x)/2))/(256*d) + (21*a^3*tan(c/2 + (d*x)/2))/(256*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**10*(a+a*sin(d*x+c))**3,x)
[Out] Timed out
```

3.406 $\int \cot^4(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=216

$$-\frac{a^3\cot^9(c+dx)}{3d}-\frac{a^3\cot^7(c+dx)}{d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{21a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^3\cot^3(c+dx)\csc^7(c+dx)}{10d}-\frac{3a}{}$$

```
[Out] -21/256*a^3*arctanh(cos(d*x+c))/d-4/5*a^3*cot(d*x+c)^5/d-a^3*cot(d*x+c)^7/d
-1/3*a^3*cot(d*x+c)^9/d-21/256*a^3*cot(d*x+c)*csc(d*x+c)/d-7/128*a^3*cot(d*
x+c)*csc(d*x+c)^3/d+29/160*a^3*cot(d*x+c)*csc(d*x+c)^5/d-3/8*a^3*cot(d*x+c)
^3*csc(d*x+c)^5/d+3/80*a^3*cot(d*x+c)*csc(d*x+c)^7/d-1/10*a^3*cot(d*x+c)^3*
csc(d*x+c)^7/d
```

Rubi [A] time = 0.39, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2873, 2607, 14, 2611, 3768, 3770, 270}

$$-\frac{a^3\cot^9(c+dx)}{3d}-\frac{a^3\cot^7(c+dx)}{d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{21a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^3\cot^3(c+dx)\csc^7(c+dx)}{10d}-\frac{3a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-21*a^3*ArcTanh[Cos[c + d*x]])/(256*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (a
^3*Cot[c + d*x]^7)/d - (a^3*Cot[c + d*x]^9)/(3*d) - (21*a^3*Cot[c + d*x]*Cs
c[c + d*x])/(256*d) - (7*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (29*a^3
*Cot[c + d*x]*Csc[c + d*x]^5)/(160*d) - (3*a^3*Cot[c + d*x]^3*Csc[c + d*x]^
5)/(8*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x]^7)/(80*d) - (a^3*Cot[c + d*x]^3
*Csc[c + d*x]^7)/(10*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)\csc^4(c+dx) + 3a^3\cot^4(c+dx)\csc^5(c+\right.$$
$$= a^3\int \cot^4(c+dx)\csc^4(c+dx)\,dx + a^3\int \cot^4(c+dx)\csc^7($$
$$= -\frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{a^3\cot^3(c+dx)\csc^7(c+dx}{10d}$$
$$= \frac{3a^3\cot(c+dx)\csc^5(c+dx)}{16d} - \frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{8d}$$
$$= -\frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{d} - \frac{a^3\cot^9(c+dx)}{3d} - \frac{3a^3}{}$$
$$= -\frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{d} - \frac{a^3\cot^9(c+dx)}{3d} - \frac{9a^3}{}$$
$$= -\frac{9a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+}{d}$$
$$= -\frac{21a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+}{d}$$

Mathematica [A] time = 2.17, size = 366, normalized size = 1.69

$$\frac{a^3(\sin(c+dx)+1)^3\left(4096\tan\left(\frac{1}{2}(c+dx)\right) - 4096\cot\left(\frac{1}{2}(c+dx)\right) - 1260\csc^2\left(\frac{1}{2}(c+dx)\right) + 6\sec^{10}\left(\frac{1}{2}(c+dx)\right) +\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-4096*Cot[(c + d*x)/2] - 1260*Csc[(c + d*x)/2]^2
 - 5040*Log[Cos[(c + d*x)/2]] + 5040*Log[Sin[(c + d*x)/2]] + 1260*Sec[(c +
d*x)/2]^2 - 180*Sec[(c + d*x)/2]^4 - 390*Sec[(c + d*x)/2]^6 + 75*Sec[(c + d
*x)/2]^8 + 6*Sec[(c + d*x)/2]^10 + 64*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 4
*Csc[(c + d*x)/2]^4*(-45 + Sin[c + d*x]) + 5*Csc[(c + d*x)/2]^8*(-15 + 4*Si
n[c + d*x]) - 2*Csc[(c + d*x)/2]^10*(3 + 10*Sin[c + d*x]) + 6*Csc[(c + d*x)
/2]^6*(65 + 42*Sin[c + d*x]) + 4096*Tan[(c + d*x)/2] - 504*Sec[(c + d*x)/2]
^4*Tan[(c + d*x)/2] - 40*Sec[(c + d*x)/2]^6*Tan[(c + d*x)/2] + 40*Sec[(c +
d*x)/2]^8*Tan[(c + d*x)/2]))/(61440*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^6)
```

fricas [A] time = 0.48, size = 340, normalized size = 1.57

$$630\,a^3\cos\left(dx+c\right)^9-2940\,a^3\cos\left(dx+c\right)^7+768\,a^3\cos\left(dx+c\right)^5+2940\,a^3\cos\left(dx+c\right)^3-630\,a^3\cos\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] 1/7680*(630*a^3*cos(d*x + c)^9 - 2940*a^3*cos(d*x + c)^7 + 768*a^3*cos(d*x
+ c)^5 + 2940*a^3*cos(d*x + c)^3 - 630*a^3*cos(d*x + c) - 315*(a^3*cos(d*x
+ c)^10 - 5*a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10*a^3*cos(d*x + c
)^4 + 5*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2) + 315*(a^3*co
s(d*x + c)^10 - 5*a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10*a^3*cos(d
*x + c)^4 + 5*a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) + 512*
(2*a^3*cos(d*x + c)^9 - 9*a^3*cos(d*x + c)^7 + 12*a^3*cos(d*x + c)^5)*sin(d
*x + c))/(d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10
*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.41, size = 357, normalized size = 1.65

$$6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-30\,a^3\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/61440*(6*a^3*tan(1/2*d*x + 1/2*c)^10 + 40*a^3*tan(1/2*d*x + 1/2*c)^9 + 10
5*a^3*tan(1/2*d*x + 1/2*c)^8 + 120*a^3*tan(1/2*d*x + 1/2*c)^7 - 30*a^3*tan(
1/2*d*x + 1/2*c)^6 - 384*a^3*tan(1/2*d*x + 1/2*c)^5 - 840*a^3*tan(1/2*d*x +
 1/2*c)^4 - 960*a^3*tan(1/2*d*x + 1/2*c)^3 + 60*a^3*tan(1/2*d*x + 1/2*c)^2
+ 5040*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 3600*a^3*tan(1/2*d*x + 1/2*c) -
 (14762*a^3*tan(1/2*d*x + 1/2*c)^10 + 3600*a^3*tan(1/2*d*x + 1/2*c)^9 + 60*
a^3*tan(1/2*d*x + 1/2*c)^8 - 960*a^3*tan(1/2*d*x + 1/2*c)^7 - 840*a^3*tan(1
/2*d*x + 1/2*c)^6 - 384*a^3*tan(1/2*d*x + 1/2*c)^5 - 30*a^3*tan(1/2*d*x + 1
/2*c)^4 + 120*a^3*tan(1/2*d*x + 1/2*c)^3 + 105*a^3*tan(1/2*d*x + 1/2*c)^2 +
 40*a^3*tan(1/2*d*x + 1/2*c) + 6*a^3)/tan(1/2*d*x + 1/2*c)^10)/d
```

maple [A] time = 0.47, size = 248, normalized size = 1.15

$$-\frac{a^3\left(\cos^5\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)^7}-\frac{2a^3\left(\cos^5\left(dx+c\right)\right)}{15d\sin\left(dx+c\right)^5}-\frac{7a^3\left(\cos^5\left(dx+c\right)\right)}{16d\sin\left(dx+c\right)^8}-\frac{7a^3\left(\cos^5\left(dx+c\right)\right)}{32d\sin\left(dx+c\right)^6}-\frac{7a^3\left(\cos^5\left(dx+c\right)\right)}{128d\sin\left(dx+c\right)^4}+\frac{7a^3\left(\mathrm{co}\right.}{256d\,\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/3/d*a^3/sin(d*x+c)^7*cos(d*x+c)^5-2/15/d*a^3/sin(d*x+c)^5*cos(d*x+c)^5-7
/16/d*a^3/sin(d*x+c)^8*cos(d*x+c)^5-7/32/d*a^3/sin(d*x+c)^6*cos(d*x+c)^5-7/
128/d*a^3/sin(d*x+c)^4*cos(d*x+c)^5+7/256/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5+7
/256*a^3*cos(d*x+c)^3/d+21/256*a^3*cos(d*x+c)/d+21/256/d*a^3*ln(csc(d*x+c)-
cot(d*x+c))-1/3/d*a^3/sin(d*x+c)^9*cos(d*x+c)^5-1/10/d*a^3/sin(d*x+c)^10*co
s(d*x+c)^5
```

maxima [A] time = 0.36, size = 308, normalized size = 1.43

$$21\,a^{3}\left(\frac{2\left(15\,\cos(dx+c)^{9}-70\,\cos(dx+c)^{7}+128\,\cos(dx+c)^{5}+70\,\cos(dx+c)^{3}-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^{8}+10\,\cos(dx+c)^{6}-10\,\cos(dx+c)^{4}+5\,\cos(dx+c)^{2}-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] 1/53760*(21*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 + 128*cos(d*x + c
)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x + c
)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*lo
g(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 630*a^3*(2*(3*cos(d*x + c
)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3 + 3*cos(d*x + c))/(cos(d*x + c)
^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 3*log(co
s(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 1536*(7*tan(d*x + c)^2 + 5)*a^
3/tan(d*x + c)^7 - 512*(63*tan(d*x + c)^4 + 90*tan(d*x + c)^2 + 35)*a^3/tan
(d*x + c)^9)/d
```

mupad [B] time = 9.55, size = 395, normalized size = 1.83

$$\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{64\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}+\frac{7\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{512\,d}+\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2048\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{512\,d}-\frac{7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^11,x)
```

```
[Out] (a^3*cot(c/2 + (d*x)/2)^3)/(64*d) - (a^3*cot(c/2 + (d*x)/2)^2)/(1024*d) + (
7*a^3*cot(c/2 + (d*x)/2)^4)/(512*d) + (a^3*cot(c/2 + (d*x)/2)^5)/(160*d) +
(a^3*cot(c/2 + (d*x)/2)^6)/(2048*d) - (a^3*cot(c/2 + (d*x)/2)^7)/(512*d) -
(7*a^3*cot(c/2 + (d*x)/2)^8)/(4096*d) - (a^3*cot(c/2 + (d*x)/2)^9)/(1536*d)
 - (a^3*cot(c/2 + (d*x)/2)^10)/(10240*d) + (a^3*tan(c/2 + (d*x)/2)^2)/(1024
```

```
*d) - (a^3*tan(c/2 + (d*x)/2)^3)/(64*d) - (7*a^3*tan(c/2 + (d*x)/2)^4)/(512
*d) - (a^3*tan(c/2 + (d*x)/2)^5)/(160*d) - (a^3*tan(c/2 + (d*x)/2)^6)/(2048
*d) + (a^3*tan(c/2 + (d*x)/2)^7)/(512*d) + (7*a^3*tan(c/2 + (d*x)/2)^8)/(40
96*d) + (a^3*tan(c/2 + (d*x)/2)^9)/(1536*d) + (a^3*tan(c/2 + (d*x)/2)^10)/(
10240*d) + (21*a^3*log(tan(c/2 + (d*x)/2)))/(256*d) - (15*a^3*cot(c/2 + (d*
x)/2))/(256*d) + (15*a^3*tan(c/2 + (d*x)/2))/(256*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**11*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.407 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=187

$$-\frac{11a^4\cos^7(c+dx)}{112d}-\frac{11\cos^7(c+dx)\left(a^4\sin(c+dx)+a^4\right)}{144d}+\frac{11a^4\sin(c+dx)\cos^5(c+dx)}{96d}+\frac{55a^4\sin(c+dx)\cos^3(}{384d}$$

```
[Out] 55/256*a^4*x-11/112*a^4*cos(d*x+c)^7/d+55/256*a^4*cos(d*x+c)*sin(d*x+c)/d+5
5/384*a^4*cos(d*x+c)^3*sin(d*x+c)/d+11/96*a^4*cos(d*x+c)^5*sin(d*x+c)/d-1/1
0*cos(d*x+c)^5*(a+a*sin(d*x+c))^5/a/d-1/18*cos(d*x+c)^7*(a^2+a^2*sin(d*x+c)
)^2/d-11/144*cos(d*x+c)^7*(a^4+a^4*sin(d*x+c))/d
```

Rubi [A] time = 0.23, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2870, 2678, 2669, 2635, 8}

$$-\frac{11a^4\cos^7(c+dx)}{112d}-\frac{\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)^2}{18d}-\frac{11\cos^7(c+dx)\left(a^4\sin(c+dx)+a^4\right)}{144d}+\frac{11a^4\sin(c+dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (55*a^4*x)/256 - (11*a^4*Cos[c + d*x]^7)/(112*d) + (55*a^4*Cos[c + d*x]*Sin
[c + d*x])/(256*d) + (55*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + (11*a^4
*Cos[c + d*x]^5*Sin[c + d*x])/(96*d) - (Cos[c + d*x]^5*(a + a*Sin[c + d*x])
^5)/(10*a*d) - (Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x])^2)/(18*d) - (11*Cos
[c + d*x]^7*(a^4 + a^4*Sin[c + d*x]))/(144*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2870

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(2*b*f*g*(m + 1)), x] + Dist[a/(2*g^2)
, Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^4\,dx = -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^5}{10ad}+\frac{1}{2}a\int \cos^6(c+dx)(a$$

$$= -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^5}{10ad}-\frac{\cos^7(c+dx)\left(a^2+a^2\right.}{18d}$$

$$= -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^5}{10ad}-\frac{\cos^7(c+dx)\left(a^2+a^2\right.}{18d}$$

$$= -\frac{11a^4\cos^7(c+dx)}{112d}-\frac{\cos^5(c+dx)(a+a\sin(c+dx))^5}{10ad}-$$

$$= -\frac{11a^4\cos^7(c+dx)}{112d}+\frac{11a^4\cos^5(c+dx)\sin(c+dx)}{96d}-\cos$$

$$= -\frac{11a^4\cos^7(c+dx)}{112d}+\frac{55a^4\cos^3(c+dx)\sin(c+dx)}{384d}+11a$$

$$= -\frac{11a^4\cos^7(c+dx)}{112d}+\frac{55a^4\cos(c+dx)\sin(c+dx)}{256d}+55a$$

$$= \frac{55a^4x}{256}-\frac{11a^4\cos^7(c+dx)}{112d}+\frac{55a^4\cos(c+dx)\sin(c+dx)}{256d}$$

Mathematica [A] time = 1.19, size = 116, normalized size = 0.62

$$\frac{a^4(8820\sin(2(c+dx))-42840\sin(4(c+dx))-2730\sin(6(c+dx))+4095\sin(8(c+dx))-126\sin(10(c+dx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(136080*c + 138600*d*x - 181440*Cos[c + d*x] - 53760*Cos[3*(c + d*x)]
+ 16128*Cos[5*(c + d*x)] + 7200*Cos[7*(c + d*x)] - 1120*Cos[9*(c + d*x)] +
8820*Sin[2*(c + d*x)] - 42840*Sin[4*(c + d*x)] - 2730*Sin[6*(c + d*x)] + 40
95*Sin[8*(c + d*x)] - 126*Sin[10*(c + d*x)]))/(645120*d)
```

fricas [A] time = 0.49, size = 124, normalized size = 0.66

$$-\frac{35840\,a^4\cos(dx+c)^9-138240\,a^4\cos(dx+c)^7+129024\,a^4\cos(dx+c)^5-17325\,a^4dx+21\left(384\,a^4\cos(dx+\right.}{80640}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/80640*(35840*a^4*cos(d*x + c)^9 - 138240*a^4*cos(d*x + c)^7 + 129024*a^4
*cos(d*x + c)^5 - 17325*a^4*d*x + 21*(384*a^4*cos(d*x + c)^9 - 3888*a^4*cos
(d*x + c)^7 + 5704*a^4*cos(d*x + c)^5 - 550*a^4*cos(d*x + c)^3 - 825*a^4*co
s(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.42, size = 174, normalized size = 0.93

$$\frac{55}{256}\,a^4x-\frac{a^4\cos(9\,dx+9\,c)}{576\,d}+\frac{5\,a^4\cos(7\,dx+7\,c)}{448\,d}+\frac{a^4\cos(5\,dx+5\,c)}{40\,d}-\frac{a^4\cos(3\,dx+3\,c)}{12\,d}-\frac{9\,a^4\cos(dx+c)}{32\,d}-\frac{a^4\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 55/256*a^4*x - 1/576*a^4*cos(9*d*x + 9*c)/d + 5/448*a^4*cos(7*d*x + 7*c)/d
+ 1/40*a^4*cos(5*d*x + 5*c)/d - 1/12*a^4*cos(3*d*x + 3*c)/d - 9/32*a^4*cos(
d*x + c)/d - 1/5120*a^4*sin(10*d*x + 10*c)/d + 13/2048*a^4*sin(8*d*x + 8*c)
/d - 13/3072*a^4*sin(6*d*x + 6*c)/d - 17/256*a^4*sin(4*d*x + 4*c)/d + 7/512
*a^4*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.28, size = 306, normalized size = 1.64

$$\frac{a^4\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^5(dx+c)\right)}{10}-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{16}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{32}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{128}+\frac{3dx}{256}+\frac{3c}{256}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1/
32*sin(d*x+c)*cos(d*x+c)^5+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3
/256*d*x+3/256*c)+4*a^4*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*c
os(d*x+c)^5-8/315*cos(d*x+c)^5)+6*a^4*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*
sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128
*d*x+3/128*c)+4*a^4*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+a^4*
(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
+1/16*d*x+1/16*c))
```

maxima [A] time = 0.35, size = 186, normalized size = 0.99

$$-\frac{8192\left(35\cos\left(dx+c\right)^{9}-90\cos\left(dx+c\right)^{7}+63\cos\left(dx+c\right)^{5}\right)a^{4}-73728\left(5\cos\left(dx+c\right)^{7}-7\cos\left(dx+c\right)^{5}\right)a^{4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] -1/645120*(8192*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a^4 - 73728*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^4 + 63*(32*sin(2*d*x +
 2*c)^5 - 120*d*x - 120*c - 5*sin(8*d*x + 8*c) + 40*sin(4*d*x + 4*c))*a^4 -
 3360*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^4 - 378
0*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^4)/d
```

mupad [B] time = 10.84, size = 572, normalized size = 3.06

$$\frac{55\,a^4\,x}{256}-\frac{\frac{571\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{384}-\frac{14149\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{480}-\frac{469\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{32}+\frac{4293\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{64}-\frac{4293\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64}+\frac{469\,a^4\,\tan(}{32}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (55*a^4*x)/256 - ((571*a^4*tan(c/2 + (d*x)/2)^3)/384 - (14149*a^4*tan(c/2 +
 (d*x)/2)^5)/480 - (469*a^4*tan(c/2 + (d*x)/2)^7)/32 + (4293*a^4*tan(c/2 +
(d*x)/2)^9)/64 - (4293*a^4*tan(c/2 + (d*x)/2)^11)/64 + (469*a^4*tan(c/2 + (
d*x)/2)^13)/32 + (14149*a^4*tan(c/2 + (d*x)/2)^15)/480 - (571*a^4*tan(c/2 +
 (d*x)/2)^17)/384 - (55*a^4*tan(c/2 + (d*x)/2)^19)/128 + (a^4*(17325*c + 17
325*d*x))/80640 - (a^4*(17325*c + 17325*d*x - 53248))/80640 + tan(c/2 + (d*
```

```
x)/2)^18*((a^4*(17325*c + 17325*d*x))/8064 - (a^4*(173250*c + 173250*d*x))/
80640) + tan(c/2 + (d*x)/2)^2*((a^4*(17325*c + 17325*d*x))/8064 - (a^4*(173
250*c + 173250*d*x - 532480))/80640) + tan(c/2 + (d*x)/2)^4*((a^4*(17325*c
+ 17325*d*x))/1792 - (a^4*(779625*c + 779625*d*x - 1105920))/80640) + tan(c
/2 + (d*x)/2)^16*((a^4*(17325*c + 17325*d*x))/1792 - (a^4*(779625*c + 77962
5*d*x - 1290240))/80640) + tan(c/2 + (d*x)/2)^6*((a^4*(17325*c + 17325*d*x)
)/672 - (a^4*(2079000*c + 2079000*d*x - 368640))/80640) + tan(c/2 + (d*x)/2
)^12*((a^4*(17325*c + 17325*d*x))/384 - (a^4*(3638250*c + 3638250*d*x - 860
160))/80640) + tan(c/2 + (d*x)/2)^14*((a^4*(17325*c + 17325*d*x))/672 - (a^
4*(2079000*c + 2079000*d*x - 6021120))/80640) + tan(c/2 + (d*x)/2)^10*((a^4
*(17325*c + 17325*d*x))/320 - (a^4*(4365900*c + 4365900*d*x - 6709248))/806
40) + tan(c/2 + (d*x)/2)^8*((a^4*(17325*c + 17325*d*x))/384 - (a^4*(3638250
*c + 3638250*d*x - 10321920))/80640) + (55*a^4*tan(c/2 + (d*x)/2))/128)/(d*
(tan(c/2 + (d*x)/2)^2 + 1)^10)
```

sympy [A] time = 34.28, size = 746, normalized size = 3.99

$$\begin{cases} \frac{3a^4x\sin^{10}(c+dx)}{256} + \frac{15a^4x\sin^8(c+dx)\cos^2(c+dx)}{256} + \frac{9a^4x\sin^8(c+dx)}{64} + \frac{15a^4x\sin^6(c+dx)\cos^4(c+dx)}{128} + \frac{9a^4x\sin^6(c+dx)\cos^2(c+dx)}{16} + \frac{a^4x\sin \\ x(a\sin(c)+a)^4\sin^2(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**10/256 + 15*a**4*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 9*a**4*x*sin(c + d*x)**8/64 + 15*a**4*x*sin(c + d*x)**6*cos
(c + d*x)**4/128 + 9*a**4*x*sin(c + d*x)**6*cos(c + d*x)**2/16 + a**4*x*sin
(c + d*x)**6/16 + 15*a**4*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 27*a**4*x
*sin(c + d*x)**4*cos(c + d*x)**4/32 + 3*a**4*x*sin(c + d*x)**4*cos(c + d*x)
**2/16 + 15*a**4*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 9*a**4*x*sin(c + d
*x)**2*cos(c + d*x)**6/16 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3
*a**4*x*cos(c + d*x)**10/256 + 9*a**4*x*cos(c + d*x)**8/64 + a**4*x*cos(c +
 d*x)**6/16 + 3*a**4*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**4*sin(c +
d*x)**7*cos(c + d*x)**3/(128*d) + 9*a**4*sin(c + d*x)**7*cos(c + d*x)/(64*d
) - a**4*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 33*a**4*sin(c + d*x)**5*c
os(c + d*x)**3/(64*d) + a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 4*a**4*s
in(c + d*x)**4*cos(c + d*x)**5/(5*d) - 7*a**4*sin(c + d*x)**3*cos(c + d*x)*
*7/(128*d) - 33*a**4*sin(c + d*x)**3*cos(c + d*x)**5/(64*d) + a**4*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) - 16*a**4*sin(c + d*x)**2*cos(c + d*x)**7/(35
*d) - 4*a**4*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**4*sin(c + d*x)*co
s(c + d*x)**9/(256*d) - 9*a**4*sin(c + d*x)*cos(c + d*x)**7/(64*d) - a**4*s
in(c + d*x)*cos(c + d*x)**5/(16*d) - 32*a**4*cos(c + d*x)**9/(315*d) - 8*a*
*4*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*sin(c)**2*cos(c)
**4, True))
```

3.408 $\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=140

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{19a^4\sin(c+dx)\cos(c+dx)}{8d}+$$

```
[Out] -61/8*a^4*x+2*a^4*arctanh(cos(d*x+c))/d+4/3*a^4*cos(d*x+c)^3/d-5*a^4*cot(d*
x+c)/d-1/3*a^4*cot(d*x+c)^3/d-2*a^4*cot(d*x+c)*csc(d*x+c)/d-19/8*a^4*cos(d*
x+c)*sin(d*x+c)/d-1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.23, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2709, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{19a^4\sin(c+dx)\cos(c+dx)}{8d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-61*a^4*x)/8 + (2*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x]^3)/(3
*d) - (5*a^4*Cot[c + d*x])/d - (a^4*Cot[c + d*x]^3)/(3*d) - (2*a^4*Cot[c +
d*x]*Csc[c + d*x])/d - (19*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[
c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(-10a^8-4a^8\csc(c+dx)+4a^8\csc^2(c+dx)+4a^8\csc^3(c+dx)+a^8\text{cs}\right.}{}$$

$$= -10a^4x + a^4\int \csc^4(c+dx)\,dx + a^4\int \sin^4(c+dx)\,dx - \left(4a^4\right)\int \csc(c+$$

$$= -10a^4x + \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos(c+dx)}{d} - \frac{2a^4\cot(c+dx)\text{c}}{d}$$

$$= -8a^4x + \frac{2a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos^3(c+dx)}{3d} - \frac{5a^4\cot(c+dx)}{d}$$

$$= -\frac{61a^4x}{8} + \frac{2a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos^3(c+dx)}{3d} - \frac{5a^4\cot(c+dx}{d}$$

Mathematica [A] time = 5.26, size = 209, normalized size = 1.49

$$\frac{a^4(\sin(c+dx)+1)^4\left(-732(c+dx)-120\sin(2(c+dx))+3\sin(4(c+dx))+96\cos(c+dx)+32\cos(3(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Sin[c + d*x])^4*(-732*(c + d*x) + 96*Cos[c + d*x] + 32*Cos[3*(c +
 d*x)] - 224*Cot[(c + d*x)/2] - 48*Csc[(c + d*x)/2]^2 + 192*Log[Cos[(c + d*
x)/2]] - 192*Log[Sin[(c + d*x)/2]] + 48*Sec[(c + d*x)/2]^2 + 32*Csc[c + d*x
]^3*Sin[(c + d*x)/2]^4 - 2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 120*Sin[2*(c +
 d*x)] + 3*Sin[4*(c + d*x)] + 224*Tan[(c + d*x)/2]))/(96*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^8)
```

fricas [A] time = 0.49, size = 219, normalized size = 1.56

$$-\frac{6\,a^4\cos\left(dx+c\right)^7-75\,a^4\cos\left(dx+c\right)^5+244\,a^4\cos\left(dx+c\right)^3-183\,a^4\cos\left(dx+c\right)-24\left(a^4\cos\left(dx+c\right)^2-a^4\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^7 - 75*a^4*cos(d*x + c)^5 + 244*a^4*cos(d*x + c)^
3 - 183*a^4*cos(d*x + c) - 24*(a^4*cos(d*x + c)^2 - a^4)*log(1/2*cos(d*x +
c) + 1/2)*sin(d*x + c) + 24*(a^4*cos(d*x + c)^2 - a^4)*log(-1/2*cos(d*x + c
) + 1/2)*sin(d*x + c) - (32*a^4*cos(d*x + c)^5 - 183*a^4*d*x*cos(d*x + c)^2
 - 32*a^4*cos(d*x + c)^3 + 183*a^4*d*x + 48*a^4*cos(d*x + c))*sin(d*x + c))
/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [B] time = 0.37, size = 274, normalized size = 1.96

$$\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-183\,(dx+c)a^4-48\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+57\,a^4\tan\left(\frac{1}{2}\,dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*tan(1/2*d*x + 1/2*c)^2 - 183*(d*x
 + c)*a^4 - 48*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 57*a^4*tan(1/2*d*x + 1/
2*c) + (88*a^4*tan(1/2*d*x + 1/2*c)^3 - 57*a^4*tan(1/2*d*x + 1/2*c)^2 - 12*
a^4*tan(1/2*d*x + 1/2*c) - a^4)/tan(1/2*d*x + 1/2*c)^3 + 2*(57*a^4*tan(1/2*
d*x + 1/2*c)^7 + 96*a^4*tan(1/2*d*x + 1/2*c)^6 + 81*a^4*tan(1/2*d*x + 1/2*c
)^5 + 96*a^4*tan(1/2*d*x + 1/2*c)^4 - 81*a^4*tan(1/2*d*x + 1/2*c)^3 + 32*a^
4*tan(1/2*d*x + 1/2*c)^2 - 57*a^4*tan(1/2*d*x + 1/2*c) + 32*a^4)/(tan(1/2*d
*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.44, size = 190, normalized size = 1.36

$$-\frac{23a^4\left(\cos^3(dx+c)\right)\sin(dx+c)}{4d}-\frac{69a^4\cos(dx+c)\sin(dx+c)}{8d}-\frac{61a^4x}{8}-\frac{61a^4c}{8d}-\frac{2a^4\left(\cos^3(dx+c)\right)}{3d}-\frac{2a^4\cos(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -23/4*a^4*cos(d*x+c)^3*sin(d*x+c)/d-69/8*a^4*cos(d*x+c)*sin(d*x+c)/d-61/8*a
^4*x-61/8/d*a^4*c-2/3*a^4*cos(d*x+c)^3/d-2*a^4*cos(d*x+c)/d-2/d*a^4*ln(csc(
d*x+c)-cot(d*x+c))-6/d*a^4/sin(d*x+c)*cos(d*x+c)^5-2/d*a^4/sin(d*x+c)^2*cos
(d*x+c)^5-1/3*a^4*cot(d*x+c)^3/d+a^4*cot(d*x+c)/d
```

maxima [A] time = 0.43, size = 218, normalized size = 1.56

$$\frac{64\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)a^4+3(12dx+12c+\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/96*(64*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d
*x + 2*c))*a^4 - 288*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3
+ tan(d*x + c)))*a^4 + 32*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c
)^3)*a^4 + 96*a^4*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3
*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.81, size = 384, normalized size = 2.74

$$\frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,d}+\frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{2\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{61\,a^4\,\mathrm{atan}\left(\frac{3721\,a^8}{16\left(61\,a^8-\frac{3721\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{61\,a^8\tan\left(\frac{c}{2}+\right.}{61\,a^8-\frac{3721\,a^8\tan}{16}}\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^4)/sin(c + d*x)^4,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^2)/(2*d) + (a^4*tan(c/2 + (d*x)/2)^3)/(24*d) - (2*a
^4*log(tan(c/2 + (d*x)/2)))/d - (61*a^4*atan((3721*a^8)/(16*(61*a^8 - (3721
*a^8*tan(c/2 + (d*x)/2))/16)) + (61*a^8*tan(c/2 + (d*x)/2))/(61*a^8 - (3721
*a^8*tan(c/2 + (d*x)/2))/16)))/(4*d) + (19*a^4*tan(c/2 + (d*x)/2))/(8*d) -
((61*a^4*tan(c/2 + (d*x)/2)^2)/3 - (16*a^4*tan(c/2 + (d*x)/2)^3)/3 + 116*a^
4*tan(c/2 + (d*x)/2)^4 + (8*a^4*tan(c/2 + (d*x)/2)^5)/3 + (508*a^4*tan(c/2
+ (d*x)/2)^6)/3 - 48*a^4*tan(c/2 + (d*x)/2)^7 + (67*a^4*tan(c/2 + (d*x)/2)^
8)/3 - 60*a^4*tan(c/2 + (d*x)/2)^9 - 19*a^4*tan(c/2 + (d*x)/2)^10 + a^4/3 +
 4*a^4*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 32*tan(c/2 + (d*x)/
2)^5 + 48*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9 + 8*tan(c/2 + (d*x
)/2)^11))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.409 $\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{\cos^7(c+dx)}{7ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}+\frac{\sin(c+dx)\cos(c+dx)}{16ad}$$

```
[Out] 1/16*x/a+1/3*cos(d*x+c)^3/a/d-2/5*cos(d*x+c)^5/a/d+1/7*cos(d*x+c)^7/a/d+1/1
6*cos(d*x+c)*sin(d*x+c)/a/d-1/8*cos(d*x+c)^3*sin(d*x+c)/a/d-1/6*cos(d*x+c)^
3*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.20, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2568, 2635, 8, 2565, 270}

$$\frac{\cos^7(c+dx)}{7ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}+\frac{\sin(c+dx)\cos(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(16*a) + Cos[c + d*x]^3/(3*a*d) - (2*Cos[c + d*x]^5)/(5*a*d) + Cos[c + d*
x]^7/(7*a*d) + (Cos[c + d*x]*Sin[c + d*x])/(16*a*d) - (Cos[c + d*x]^3*Sin[c
 + d*x])/(8*a*d) - (Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^5(c+dx)\,dx}{a}$$

$$= -\frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{2a} + \frac{\text{Subst}\left(\int x^2 \left(\right.\right.}{}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{8ad} - \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\int \cos^2(c+dx)\,dx}{8a} +$$

$$= \frac{\cos^3(c+dx)}{3ad} - \frac{2\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} + \frac{\cos(c+dx)\sin(c+dx)}{16ad} - \frac{\text{co}}{}$$

$$= \frac{x}{16a} + \frac{\cos^3(c+dx)}{3ad} - \frac{2\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} + \frac{\cos(c+dx)\sin(c+d}{16ad}$$

Mathematica [A] time = 0.26, size = 86, normalized size = 0.64

$$\frac{-105\sin(2(c+dx)) - 105\sin(4(c+dx)) + 35\sin(6(c+dx)) + 525\cos(c+dx) + 35\cos(3(c+dx)) - 63\cos(5(c}{6720ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (420*c + 420*d*x + 525*Cos[c + d*x] + 35*Cos[3*(c + d*x)] - 63*Cos[5*(c + d
*x)] + 15*Cos[7*(c + d*x)] - 105*Sin[2*(c + d*x)] - 105*Sin[4*(c + d*x)] +
35*Sin[6*(c + d*x)])/(6720*a*d)
```

fricas [A] time = 0.44, size = 80, normalized size = 0.59

$$\frac{240\cos(dx+c)^7 - 672\cos(dx+c)^5 + 560\cos(dx+c)^3 + 105\,dx + 35\left(8\cos(dx+c)^5 - 14\cos(dx+c)^3 + 3\right.}{1680\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*cos(d*x + c)^7 - 672*cos(d*x + c)^5 + 560*cos(d*x + c)^3 + 105*
d*x + 35*(8*cos(d*x + c)^5 - 14*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x +
c))/(a*d)
```

giac [A] time = 0.20, size = 166, normalized size = 1.23

$$\frac{\frac{105\,(dx+c)}{a} + \frac{2\left(105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13} + 700\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11} - 3395\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9} + 8960\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8} - 4480\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6} + 3395\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^7 a}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1680*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^13 + 700*tan(1/2*d*x
+ 1/2*c)^11 - 3395*tan(1/2*d*x + 1/2*c)^9 + 8960*tan(1/2*d*x + 1/2*c)^8 - 4
480*tan(1/2*d*x + 1/2*c)^6 + 3395*tan(1/2*d*x + 1/2*c)^5 + 2688*tan(1/2*d*x
 + 1/2*c)^4 - 700*tan(1/2*d*x + 1/2*c)^3 + 896*tan(1/2*d*x + 1/2*c)^2 - 105
*tan(1/2*d*x + 1/2*c) + 128)/((tan(1/2*d*x + 1/2*c)^2 + 1)^7*a))/d
```

maple [B] time = 0.27, size = 381, normalized size = 2.82

$$\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7} + \frac{5\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7} - \frac{97\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7} + \frac{32\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7} - \frac{1}{3ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/8/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13+5/6/a/d/(1+tan(1/2
*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11-97/24/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7
*tan(1/2*d*x+1/2*c)^9+32/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c
)^8-16/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6+97/24/a/d/(1+t
an(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^5+16/5/a/d/(1+tan(1/2*d*x+1/2*c)^
2)^7*tan(1/2*d*x+1/2*c)^4-5/6/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/
2*c)^3+16/15/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2-1/8/a/d/(1
+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)+16/105/a/d/(1+tan(1/2*d*x+1/2*c
)^2)^7+1/8/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.44, size = 380, normalized size = 2.81

$$-\frac{\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1} - \frac{896 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{700 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{2688 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{3395 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{4480 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{8960 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{3395 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{700 \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}} - \frac{105 \sin}{(\cos(d}}{a + \frac{7a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{21a\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{35a\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{35a\sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{21a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{7a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} + \frac{a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}}{840\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 896*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 700*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2688*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 3395*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4480*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 8960*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 + 3395*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 700*sin(d*x + c)^11/(cos(d*x
 + c) + 1)^11 - 105*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 128)/(a + 7*a*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 21*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 35*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 35*a*sin(d*x + c)^8/(cos(d*x
 + c) + 1)^8 + 21*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 7*a*sin(d*x + c
)^12/(cos(d*x + c) + 1)^12 + a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14) - 105
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 11.38, size = 159, normalized size = 1.18

$$\frac{x}{16\,a} + \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8} + \frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{6} - \frac{97\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} + \frac{32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3} - \frac{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3} + \frac{97\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{24} + \frac{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{5} - \frac{5}{}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^4)/(a + a*sin(c + d*x)),x)
```

```
[Out] x/(16*a) + ((16*tan(c/2 + (d*x)/2)^2)/15 - tan(c/2 + (d*x)/2)/8 - (5*tan(c/
2 + (d*x)/2)^3)/6 + (16*tan(c/2 + (d*x)/2)^4)/5 + (97*tan(c/2 + (d*x)/2)^5)
```

```
/24 - (16*tan(c/2 + (d*x)/2)^6)/3 + (32*tan(c/2 + (d*x)/2)^8)/3 - (97*tan(c
/2 + (d*x)/2)^9)/24 + (5*tan(c/2 + (d*x)/2)^11)/6 + tan(c/2 + (d*x)/2)^13/8
 + 16/105)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [A] time = 88.92, size = 2635, normalized size = 19.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((105*d*x*tan(c/2 + d*x/2)**14/(1680*a*d*tan(c/2 + d*x/2)**14 + 11
760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*t
an(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*
x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 735*d*x*tan(c/2 + d*x
/2)**12/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 3
5280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*t
an(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*
x/2)**2 + 1680*a*d) + 2205*d*x*tan(c/2 + d*x/2)**10/(1680*a*d*tan(c/2 + d*x
/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 +
 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*
tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 3675*d*x*
tan(c/2 + d*x/2)**8/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*
x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 +
 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*
tan(c/2 + d*x/2)**2 + 1680*a*d) + 3675*d*x*tan(c/2 + d*x/2)**6/(1680*a*d*ta
n(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d
*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 +
 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d)
+ 2205*d*x*tan(c/2 + d*x/2)**4/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*t
an(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 +
d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 +
 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 735*d*x*tan(c/2 + d*x/2)**2/(1
680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*t
an(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d
*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 +
1680*a*d) + 105*d*x/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*
x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 +
 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*
tan(c/2 + d*x/2)**2 + 1680*a*d) + 210*tan(c/2 + d*x/2)**13/(1680*a*d*tan(c/
2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2
)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 352
80*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 14
00*tan(c/2 + d*x/2)**11/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2
+ d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)*
```

```
*8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*
a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 6790*tan(c/2 + d*x/2)**9/(1680*a*d*ta
n(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d
*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 +
 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d)
+ 17920*tan(c/2 + d*x/2)**8/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(
c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x
/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11
760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 8960*tan(c/2 + d*x/2)**6/(1680*a*
d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2
 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)*
*6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a
*d) + 6790*tan(c/2 + d*x/2)**5/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*t
an(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 +
d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 +
 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 5376*tan(c/2 + d*x/2)**4/(1680
*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(
c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/
2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 168
0*a*d) - 1400*tan(c/2 + d*x/2)**3/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*
d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2
 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**
4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 1792*tan(c/2 + d*x/2)**2/(1
680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*t
an(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d
*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 +
1680*a*d) - 210*tan(c/2 + d*x/2)/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d
*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2
+ d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4
 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 256/(1680*a*d*tan(c/2 + d*x/
2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 +
58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*t
an(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d), Ne(d, 0)),
(x*sin(c)**4*cos(c)**4/(a*sin(c) + a), True))
```

3.410 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{3ad}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

```
[Out] -1/16*x/a-1/3*cos(d*x+c)^3/a/d+1/5*cos(d*x+c)^5/a/d-1/16*cos(d*x+c)*sin(d*x
+c)/a/d+1/8*cos(d*x+c)^3*sin(d*x+c)/a/d+1/6*cos(d*x+c)^3*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2565, 14, 2568, 2635, 8}

$$\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{3ad}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -x/(16*a) - Cos[c + d*x]^3/(3*a*d) + Cos[c + d*x]^5/(5*a*d) - (Cos[c + d*x]
*Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(8*a*d) + (Cos[c +
d*x]^3*Sin[c + d*x]^3)/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
```

```
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{a}\\ &= \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{2a} - \frac{\text{Subst}\left(\int x^2\left(1\right.\right.}{}\\ &= \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} - \frac{\int \cos^2(c+dx)\,dx}{8a} -\\ &= -\frac{\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(}{8ad}\\ &= -\frac{x}{16a} - \frac{\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+d}{8}\end{aligned}$$

Mathematica [B] time = 4.95, size = 377, normalized size = 3.22

$$\frac{-120dx\sin\left(\frac{c}{2}\right)+120\sin\left(\frac{c}{2}+dx\right)-120\sin\left(\frac{3c}{2}+dx\right)+15\sin\left(\frac{3c}{2}+2dx\right)+15\sin\left(\frac{5c}{2}+2dx\right)+20\sin\left(\frac{5c}{2}+3d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (30*(3*c - 4*d*x)*Cos[c/2] - 120*Cos[c/2 + d*x] - 120*Cos[(3*c)/2 + d*x] +
15*Cos[(3*c)/2 + 2*d*x] - 15*Cos[(5*c)/2 + 2*d*x] - 20*Cos[(5*c)/2 + 3*d*x]
 - 20*Cos[(7*c)/2 + 3*d*x] + 15*Cos[(7*c)/2 + 4*d*x] - 15*Cos[(9*c)/2 + 4*d
*x] + 12*Cos[(9*c)/2 + 5*d*x] + 12*Cos[(11*c)/2 + 5*d*x] - 5*Cos[(11*c)/2 +
 6*d*x] + 5*Cos[(13*c)/2 + 6*d*x] - 180*Sin[c/2] + 90*c*Sin[c/2] - 120*d*x*
Sin[c/2] + 120*Sin[c/2 + d*x] - 120*Sin[(3*c)/2 + d*x] + 15*Sin[(3*c)/2 + 2
*d*x] + 15*Sin[(5*c)/2 + 2*d*x] + 20*Sin[(5*c)/2 + 3*d*x] - 20*Sin[(7*c)/2
+ 3*d*x] + 15*Sin[(7*c)/2 + 4*d*x] + 15*Sin[(9*c)/2 + 4*d*x] - 12*Sin[(9*c)
/2 + 5*d*x] + 12*Sin[(11*c)/2 + 5*d*x] - 5*Sin[(11*c)/2 + 6*d*x] - 5*Sin[(1
3*c)/2 + 6*d*x])/(1920*a*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.46, size = 70, normalized size = 0.60

$$\frac{48\cos(dx+c)^5-80\cos(dx+c)^3-15\,dx-5\left(8\cos(dx+c)^5-14\cos(dx+c)^3+3\cos(dx+c)\right)\sin(dx+c)}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 15*d*x - 5*(8*cos(d*x + c)^5
 - 14*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x + c))/(a*d)
```

giac [A] time = 0.20, size = 153, normalized size = 1.31

$$-\frac{\frac{15\,(dx+c)}{a}+\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+85\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+480\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-570\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+320\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+570\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-85\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)^{6}a}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(15*(d*x + c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^11 + 85*tan(1/2*d*x + 1
/2*c)^9 + 480*tan(1/2*d*x + 1/2*c)^8 - 570*tan(1/2*d*x + 1/2*c)^7 + 320*tan
(1/2*d*x + 1/2*c)^6 + 570*tan(1/2*d*x + 1/2*c)^5 - 85*tan(1/2*d*x + 1/2*c)^
3 + 192*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 32)/((tan(1/2*d*
x + 1/2*c)^2 + 1)^6*a))/d
```

maple [B] time = 0.26, size = 347, normalized size = 2.97

$$-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{17\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{4\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{19\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{8}{3ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/8/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11-17/24/a/d/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9-4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*t
an(1/2*d*x+1/2*c)^8+19/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^
7-8/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6-19/4/a/d/(1+tan(1
/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5+17/24/a/d/(1+tan(1/2*d*x+1/2*c)^2)^
6*tan(1/2*d*x+1/2*c)^3-8/5/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)^2+1/8/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-4/15/a/d/(1+tan(1
/2*d*x+1/2*c)^2)^6-1/8/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.44, size = 339, normalized size = 2.90

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{192\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{85\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{570\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{320\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{570\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{480\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{85\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{15\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-32}{a+\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}} - \frac{15\,\mathrm{arc}}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 192*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 85*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 570*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 320*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 570*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 - 480*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 85*s
in(d*x + c)^9/(cos(d*x + c) + 1)^9 - 15*sin(d*x + c)^11/(cos(d*x + c) + 1)^
11 - 32)/(a + 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^1
0 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 15*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a)/d
```

mupad [B] time = 11.31, size = 147, normalized size = 1.26

$$-\frac{x}{16\,a}-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{24}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\frac{19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] - x/(16*a) - ((8*tan(c/2 + (d*x)/2)^2)/5 - tan(c/2 + (d*x)/2)/8 - (17*tan(c
/2 + (d*x)/2)^3)/24 + (19*tan(c/2 + (d*x)/2)^5)/4 + (8*tan(c/2 + (d*x)/2)^6
)/3 - (19*tan(c/2 + (d*x)/2)^7)/4 + 4*tan(c/2 + (d*x)/2)^8 + (17*tan(c/2 +
(d*x)/2)^9)/24 + tan(c/2 + (d*x)/2)^11/8 + 4/15)/(a*d*(tan(c/2 + (d*x)/2)^2
 + 1)^6)
```

sympy [A] time = 72.10, size = 2067, normalized size = 17.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-15*d*x*tan(c/2 + d*x/2)**12/(240*a*d*tan(c/2 + d*x/2)**12 + 144
0*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/
2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2
 + 240*a*d) - 90*d*x*tan(c/2 + d*x/2)**10/(240*a*d*tan(c/2 + d*x/2)**12 + 1
440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(
c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)*
*2 + 240*a*d) - 225*d*x*tan(c/2 + d*x/2)**8/(240*a*d*tan(c/2 + d*x/2)**12 +
 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*ta
n(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2
)**2 + 240*a*d) - 300*d*x*tan(c/2 + d*x/2)**6/(240*a*d*tan(c/2 + d*x/2)**12
 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*
tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x
/2)**2 + 240*a*d) - 225*d*x*tan(c/2 + d*x/2)**4/(240*a*d*tan(c/2 + d*x/2)**
12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*
d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d
*x/2)**2 + 240*a*d) - 90*d*x*tan(c/2 + d*x/2)**2/(240*a*d*tan(c/2 + d*x/2)*
*12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a
*d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 +
d*x/2)**2 + 240*a*d) - 15*d*x/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(
c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)
**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d
) - 30*tan(c/2 + d*x/2)**11/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/
2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**
6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d)
- 170*tan(c/2 + d*x/2)**9/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2
+ d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6
+ 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) -
960*tan(c/2 + d*x/2)**8/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 +
d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 +
3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) + 11
40*tan(c/2 + d*x/2)**7/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d
*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3
```

```
600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 640
*tan(c/2 + d*x/2)**6/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x
/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 360
0*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 1140*
tan(c/2 + d*x/2)**5/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/
2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600
*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) + 170*ta
n(c/2 + d*x/2)**3/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)
**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a
*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 384*tan(
c/2 + d*x/2)**2/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**
10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d
*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) + 30*tan(c/2
 + d*x/2)/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3
600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c
/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 64/(240*a*d*tan(
c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2
)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a
*d*tan(c/2 + d*x/2)**2 + 240*a*d), Ne(d, 0)), (x*sin(c)**3*cos(c)**4/(a*sin
(c) + a), True))
```

3.411 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$-\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{\sin(c+dx)\cos(c+dx)}{8ad}+\frac{x}{8a}$$

```
[Out] 1/8*x/a+1/3*cos(d*x+c)^3/a/d-1/5*cos(d*x+c)^5/a/d+1/8*cos(d*x+c)*sin(d*x+c)
/a/d-1/4*cos(d*x+c)^3*sin(d*x+c)/a/d
```

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2568, 2635, 8, 2565, 14}

$$-\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{\sin(c+dx)\cos(c+dx)}{8ad}+\frac{x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(8*a) + Cos[c + d*x]^3/(3*a*d) - Cos[c + d*x]^5/(5*a*d) + (Cos[c + d*x]*S
in[c + d*x])/(8*a*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
```

```
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a}\\ &= -\frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\int \cos^2(c+dx)\,dx}{4a} + \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx, x,\right.}{ad}\\ &= \frac{\cos(c+dx)\sin(c+dx)}{8ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\int 1\,dx}{8a} + \frac{\text{Subst}\left(\int \left(x^2\right.\right.}{}\\ &= \frac{x}{8a} + \frac{\cos^3(c+dx)}{3ad} - \frac{\cos^5(c+dx)}{5ad} + \frac{\cos(c+dx)\sin(c+dx)}{8ad} - \frac{\cos^3(c+dx)}{4a}\end{aligned}$$

Mathematica [B] time = 2.44, size = 258, normalized size = 2.84

$$\frac{120dx\sin\left(\frac{c}{2}\right)-60\sin\left(\frac{c}{2}+dx\right)+60\sin\left(\frac{3c}{2}+dx\right)-10\sin\left(\frac{5c}{2}+3dx\right)+10\sin\left(\frac{7c}{2}+3dx\right)-15\sin\left(\frac{7c}{2}+4dx\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (120*d*x*Cos[c/2] + 60*Cos[c/2 + d*x] + 60*Cos[(3*c)/2 + d*x] + 10*Cos[(5*c
)/2 + 3*d*x] + 10*Cos[(7*c)/2 + 3*d*x] - 15*Cos[(7*c)/2 + 4*d*x] + 15*Cos[(
9*c)/2 + 4*d*x] - 6*Cos[(9*c)/2 + 5*d*x] - 6*Cos[(11*c)/2 + 5*d*x] + 120*Si
n[c/2] + 120*d*x*Sin[c/2] - 60*Sin[c/2 + d*x] + 60*Sin[(3*c)/2 + d*x] - 10*
Sin[(5*c)/2 + 3*d*x] + 10*Sin[(7*c)/2 + 3*d*x] - 15*Sin[(7*c)/2 + 4*d*x] -
15*Sin[(9*c)/2 + 4*d*x] + 6*Sin[(9*c)/2 + 5*d*x] - 6*Sin[(11*c)/2 + 5*d*x])
/(960*a*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.47, size = 60, normalized size = 0.66

$$-\frac{24\cos(dx+c)^5-40\cos(dx+c)^3-15\,dx+15\left(2\cos(dx+c)^3-\cos(dx+c)\right)\sin(dx+c)}{120\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/120*(24*cos(d*x + c)^5 - 40*cos(d*x + c)^3 - 15*d*x + 15*(2*cos(d*x + c)
^3 - cos(d*x + c))*sin(d*x + c))/(a*d)
```

giac [A] time = 0.16, size = 127, normalized size = 1.40

$$\frac{\frac{15\,(dx+c)}{a}+\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-90\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+240\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-80\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+90\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+80\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^5a}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(15*(d*x + c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^9 - 90*tan(1/2*d*x + 1/2
*c)^7 + 240*tan(1/2*d*x + 1/2*c)^6 - 80*tan(1/2*d*x + 1/2*c)^4 + 90*tan(1/2
*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 16)
/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a))/d
```

maple [B] time = 0.25, size = 279, normalized size = 3.07

$$\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{3\left(\right.}{2ad\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9-3/2/a/d/(1+tan(1/2*
d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7+4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1
/2*d*x+1/2*c)^6-4/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4+3/2
/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3+4/3/a/d/(1+tan(1/2*d*x
+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2-1/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/
2*d*x+1/2*c)+4/15/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5+1/4/a/d*arctan(tan(1/2*d*x
+1/2*c))
```

maxima [B] time = 0.44, size = 278, normalized size = 3.05

$$-\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{80\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{90\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{80\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{240\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{90\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{15\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-16}{a+\frac{5a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 80*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 90*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 80*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 240*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 90*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 - 15*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 16)/(a +
 5*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + 10*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8 + a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 15*arctan(s
in(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 12.01, size = 120, normalized size = 1.32

$$\frac{x}{8a}+\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{4}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{2}+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}+\frac{4}{15}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] x/(8*a) + ((4*tan(c/2 + (d*x)/2)^2)/3 - tan(c/2 + (d*x)/2)/4 + (3*tan(c/2 +
 (d*x)/2)^3)/2 - (4*tan(c/2 + (d*x)/2)^4)/3 + 4*tan(c/2 + (d*x)/2)^6 - (3*t
an(c/2 + (d*x)/2)^7)/2 + tan(c/2 + (d*x)/2)^9/4 + 4/15)/(a*d*(tan(c/2 + (d*
x)/2)^2 + 1)^5)
```

sympy [A] time = 53.16, size = 1464, normalized size = 16.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((15*d*x*tan(c/2 + d*x/2)**10/(120*a*d*tan(c/2 + d*x/2)**10 + 600*
a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 +
 d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 75*d*x*tan(c/2 + d*x/
2)**8/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*
d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*
x/2)**2 + 120*a*d) + 150*d*x*tan(c/2 + d*x/2)**6/(120*a*d*tan(c/2 + d*x/2)*
*10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d
*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 150*d*x*tan
(c/2 + d*x/2)**4/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**
8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*t
an(c/2 + d*x/2)**2 + 120*a*d) + 75*d*x*tan(c/2 + d*x/2)**2/(120*a*d*tan(c/2
 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6
+ 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 1
5*d*x/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*
d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*
x/2)**2 + 120*a*d) + 30*tan(c/2 + d*x/2)**9/(120*a*d*tan(c/2 + d*x/2)**10 +
 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(
c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 180*tan(c/2 + d*
x/2)**7/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*
a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 +
d*x/2)**2 + 120*a*d) + 480*tan(c/2 + d*x/2)**6/(120*a*d*tan(c/2 + d*x/2)**1
0 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*t
an(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 160*tan(c/2 +
 d*x/2)**4/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 12
00*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2
 + d*x/2)**2 + 120*a*d) + 180*tan(c/2 + d*x/2)**3/(120*a*d*tan(c/2 + d*x/2)
**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*
d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 160*tan(c/
2 + d*x/2)**2/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 +
 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(
c/2 + d*x/2)**2 + 120*a*d) - 30*tan(c/2 + d*x/2)/(120*a*d*tan(c/2 + d*x/2)*
*10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d
*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 32/(120*a*d
*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*
x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*
a*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**4/(a*sin(c) + a), True))
```

3.412 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\sin(c+dx)\cos(c+dx)}{8ad}-\frac{x}{8a}$$

```
[Out] -1/8*x/a-1/3*cos(d*x+c)^3/a/d-1/8*cos(d*x+c)*sin(d*x+c)/a/d+1/4*cos(d*x+c)^
3*sin(d*x+c)/a/d
```

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2565, 30, 2568, 2635, 8}

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\sin(c+dx)\cos(c+dx)}{8ad}-\frac{x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -x/(8*a) - Cos[c + d*x]^3/(3*a*d) - (Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + (
Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
```

```
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^2(c+dx)\sin(c+dx)\,dx}{a}-\frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a}\\ &= \frac{\cos^3(c+dx)\sin(c+dx)}{4ad}-\frac{\int \cos^2(c+dx)\,dx}{4a}-\frac{\text{Subst}\left(\int x^2\,dx,x,\cos(c+dx)\right)}{ad}\\ &= -\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}-\frac{\int 1\,dx}{8a}\\ &= -\frac{x}{8a}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}\end{aligned}$$

Mathematica [B] time = 1.68, size = 219, normalized size = 3.00

$$-\frac{24dx\sin\left(\frac{c}{2}\right)-24\sin\left(\frac{c}{2}+dx\right)+24\sin\left(\frac{3c}{2}+dx\right)-8\sin\left(\frac{5c}{2}+3dx\right)+8\sin\left(\frac{7c}{2}+3dx\right)-3\sin\left(\frac{7c}{2}+4dx\right)-3\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/192*(-24*(c - d*x)*Cos[c/2] + 24*Cos[c/2 + d*x] + 24*Cos[(3*c)/2 + d*x]
+ 8*Cos[(5*c)/2 + 3*d*x] + 8*Cos[(7*c)/2 + 3*d*x] - 3*Cos[(7*c)/2 + 4*d*x]
```

```
+ 3*Cos[(9*c)/2 + 4*d*x] + 48*Sin[c/2] - 24*c*Sin[c/2] + 24*d*x*Sin[c/2] -
24*Sin[c/2 + d*x] + 24*Sin[(3*c)/2 + d*x] - 8*Sin[(5*c)/2 + 3*d*x] + 8*Sin[
(7*c)/2 + 3*d*x] - 3*Sin[(7*c)/2 + 4*d*x] - 3*Sin[(9*c)/2 + 4*d*x])/(a*d*(C
os[c/2] + Sin[c/2]))
```

fricas [A] time = 0.44, size = 50, normalized size = 0.68

$$-\frac{8\cos(dx+c)^3+3\,dx-3\left(2\cos(dx+c)^3-\cos(dx+c)\right)\sin(dx+c)}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*cos(d*x + c)^3 + 3*d*x - 3*(2*cos(d*x + c)^3 - cos(d*x + c))*sin(d
*x + c))/(a*d)
```

giac [A] time = 0.17, size = 127, normalized size = 1.74

$$-\frac{\frac{3\,(dx+c)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+24\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+24\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\tan\left(\frac{1}{2}\,d\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(3*(d*x + c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^7 + 24*tan(1/2*d*x + 1/2*c
)^6 - 21*tan(1/2*d*x + 1/2*c)^5 + 24*tan(1/2*d*x + 1/2*c)^4 + 21*tan(1/2*d*
x + 1/2*c)^3 + 8*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 8)/((tan
(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

maple [B] time = 0.17, size = 279, normalized size = 3.82

$$-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{7}{4ad\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-2/a/d/(1+tan(1/2*d
*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6+7/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(
1/2*d*x+1/2*c)^5-2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4-7/4/
```

```
a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-2/3/a/d/(1+tan(1/2*d*x+
1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2+1/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2
*d*x+1/2*c)-2/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4-1/4/a/d*arctan(tan(1/2*d*x+1
/2*c))
```

maxima [B] time = 0.42, size = 257, normalized size = 3.52

$$\frac{\frac{\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{8\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{21\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{24\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{24\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-8}{a+\frac{4\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{3\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*sin(d*x + c)/(cos(d*x + c) + 1) - 8*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 21*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 24*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 24*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 8)/(a + 4*a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin(d*x + c)^8/(cos(d*x + c
) + 1)^8) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 8.80, size = 43, normalized size = 0.59

$$-\frac{6\,\cos\left(c+d\,x\right)+2\,\cos\left(3\,c+3\,d\,x\right)-\frac{3\,\sin(4\,c+4\,d\,x)}{4}+3\,d\,x}{24\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] -(6*cos(c + d*x) + 2*cos(3*c + 3*d*x) - (3*sin(4*c + 4*d*x))/4 + 3*d*x)/(24
*a*d)
```

sympy [A] time = 24.78, size = 1134, normalized size = 15.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-3*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*
tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)
**2 + 24*a*d) - 12*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 96
```

```
*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d
*x/2)**2 + 24*a*d) - 18*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8
 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/
2 + d*x/2)**2 + 24*a*d) - 12*d*x*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*x/
2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*t
an(c/2 + d*x/2)**2 + 24*a*d) - 3*d*x/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*t
an(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)*
*2 + 24*a*d) - 6*tan(c/2 + d*x/2)**7/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*t
an(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)*
*2 + 24*a*d) - 48*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*
tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)
**2 + 24*a*d) + 42*tan(c/2 + d*x/2)**5/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d
*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2
)**2 + 24*a*d) - 48*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*
d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/
2)**2 + 24*a*d) - 42*tan(c/2 + d*x/2)**3/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a
*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x
/2)**2 + 24*a*d) - 16*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*x/2)**8 + 96*
a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*
x/2)**2 + 24*a*d) + 6*tan(c/2 + d*x/2)/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d
*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2
)**2 + 24*a*d) - 16/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**
6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d), Ne(
d, 0)), (x*sin(c)*cos(c)**4/(a*sin(c) + a), True))
```

3.413 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{2a}$$

```
[Out] -1/2*x/a-arctanh(cos(d*x+c))/a/d+cos(d*x+c)/a/d-1/2*cos(d*x+c)*sin(d*x+c)/a
/d
```

Rubi [A] time = 0.10, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2592, 321, 206, 2635, 8}

$$\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -x/(2*a) - ArcTanh[Cos[c + d*x]]/(a*d) + Cos[c + d*x]/(a*d) - (Cos[c + d*x]
*Sin[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cos^2(c+dx)\,dx}{a}+\frac{\int \cos(c+dx)\cot(c+dx)\,dx}{a}\\ &= -\frac{\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\int 1\,dx}{2a}-\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(c+dx)\right)}{ad}\\ &= -\frac{x}{2a}+\frac{\cos(c+dx)}{ad}-\frac{\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(c+d\right.}{ad}\\ &= -\frac{x}{2a}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\cos(c+dx)}{ad}-\frac{\cos(c+dx)\sin(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.18, size = 60, normalized size = 1.02

$$-\frac{\sin(2(c+dx))-4\cos(c+dx)+2\left(-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+c+dx\right)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/4*(-4*Cos[c + d*x] + 2*(c + d*x + 2*Log[Cos[(c + d*x)/2]] - 2*Log[Sin[(c
 + d*x)/2]]) + Sin[2*(c + d*x)])/(a*d)
```

fricas [A] time = 0.47, size = 57, normalized size = 0.97

$$-\frac{dx + \cos(dx + c)\sin(dx + c) - 2\cos(dx + c) + \log\left(\frac{1}{2}\cos(dx + c) + \frac{1}{2}\right) - \log\left(-\frac{1}{2}\cos(dx + c) + \frac{1}{2}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d*x + cos(d*x + c)*sin(d*x + c) - 2*cos(d*x + c) + log(1/2*cos(d*x +
c) + 1/2) - log(-1/2*cos(d*x + c) + 1/2))/(a*d)
```

giac [A] time = 0.16, size = 88, normalized size = 1.49

$$-\frac{\frac{dx+c}{a} - \frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a} - \frac{2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2 a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/a - 2*log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(tan(1/2*d*x + 1
/2*c)^3 + 2*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 2)/((tan(1/2*d*
x + 1/2*c)^2 + 1)^2*a))/d
```

maple [B] time = 0.36, size = 159, normalized size = 2.69

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{\arctan\left(\tan\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+2/a/d/(1+tan(1/2*d*x+
1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d
*x+1/2*c)+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2-1/a/d*arctan(tan(1/2*d*x+1/2*c))
+1/a/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 156, normalized size = 2.64

$$-\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-2}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -((sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - log(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d

mupad [B] time = 8.85, size = 136, normalized size = 2.31

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}+\frac{\mathrm{atan}\left(\frac{1}{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))),x)

[Out] log(tan(c/2 + (d*x)/2))/(a*d) + (2*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^3 + 2)/(d*(a + 2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4)) + atan(1/(tan(c/2 + (d*x)/2) + 2) - (2*tan(c/2 + (d*x)/2))/(tan(c/2 + (d*x)/2) + 2))/(a*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^4(c+dx)\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c)),x)

[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(sin(c + d*x) + 1), x)/a

3.414 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=49

$$-\frac{\cos(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{a}$$

```
[Out] -x/a+arctanh(cos(d*x+c))/a/d-cos(d*x+c)/a/d-cot(d*x+c)/a/d
```

Rubi [A] time = 0.12, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 3473, 8, 2592, 321, 206}

$$-\frac{\cos(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) + ArcTanh[Cos[c + d*x]]/(a*d) - Cos[c + d*x]/(a*d) - Cot[c + d*x]/(a
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cos(c+dx)\cot(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\,dx}{a}\\
&= -\frac{\cot(c+dx)}{ad}-\frac{\int 1\,dx}{a}+\frac{\operatorname{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(c+dx)\right)}{ad}\\
&= -\frac{x}{a}-\frac{\cos(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}+\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(c+dx)\right)}{ad}\\
&= -\frac{x}{a}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cos(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.42, size = 93, normalized size = 1.90

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)^2\left(\cos(c+dx)+\sin(c+dx)\left(\cos(c+dx)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\right.\right.\right.}{2ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*((1 + Cot[(c + d*x)/2])^2*(Cos[c + d*x] + (c + d*x + Cos[c + d*x] - Lo
g[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x])*Tan[(c + d*x)/2]
)/(a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.46, size = 80, normalized size = 1.63

$$-\frac{2\,(dx+\cos(dx+c))\sin(dx+c)-\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c}{2\,ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(d*x + cos(d*x + c))*sin(d*x + c) - log(1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 2*cos(d*x + c))/(a*
d*sin(d*x + c))
```

giac [B] time = 0.17, size = 113, normalized size = 2.31

$$-\frac{\frac{6\,(dx+c)}{a}+\frac{6\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)/a + 6*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3*tan(1/2*d*x +
1/2*c)/a - (2*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c)^2 - 10*tan(1/
2*d*x + 1/2*c) - 3)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a))/d
```

maple [A] time = 0.39, size = 97, normalized size = 1.98

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{2}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{2ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)-2/a/d/(1+tan(1/2*d*x+1/2*c)^2)-2/a/d*arctan(tan(
1/2*d*x+1/2*c))-1/2/a/d/tan(1/2*d*x+1/2*c)-1/a/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.42, size = 154, normalized size = 3.14

$$-\frac{\frac{\frac{4\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}+\frac{4\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{2\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/2*((4*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/(a*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 2*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d

mupad [B] time = 8.72, size = 147, normalized size = 3.00

$$\frac{2\,\mathrm{atan}\left(\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4}+\frac{4}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4}\right)}{a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1}{d\left(2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{\tan\left(\frac{c}{2}+\cdots\right.}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)

[Out] (2*atan((4*tan(c/2 + (d*x)/2))/(4*tan(c/2 + (d*x)/2) - 4) + 4/(4*tan(c/2 + (d*x)/2) - 4)))/(a*d) - log(tan(c/2 + (d*x)/2))/(a*d) - (4*tan(c/2 + (d*x)/ 2) + tan(c/2 + (d*x)/2)^2 + 1)/(d*(2*a*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^3)) + tan(c/2 + (d*x)/2)/(2*a*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^4(c+dx)\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)

[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**2/(sin(c + d*x) + 1), x)/a

3.415 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{x}{a}$$

```
[Out] x/a+1/2*arctanh(cos(d*x+c))/a/d+cot(d*x+c)/a/d-1/2*cot(d*x+c)*csc(d*x+c)/a/
d
```

Rubi [A] time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2611, 3770, 3473, 8}

$$\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + ArcTanh[Cos[c + d*x]]/(2*a*d) + Cot[c + d*x]/(a*d) - (Cot[c + d*x]*Cs
c[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^2(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a}\\ &= \frac{\cot(c+dx)}{ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}-\frac{\int \csc(c+dx)\,dx}{2a}+\frac{\int 1\,dx}{a}\\ &= \frac{x}{a}+\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)}{ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.46, size = 102, normalized size = 1.76

$$\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left((2\sin(c+dx)-1)\cos(c+dx)+\sin^2(c+dx)\left(-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+1\right.\right.}{8ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*((2*c + 2*d*x + Log[Cos[(c + d*x)/
2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^2 + Cos[c + d*x]*(-1 + 2*Sin[c +
d*x])))/(8*a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.46, size = 104, normalized size = 1.79

$$\frac{4\,dx\cos\left(dx+c\right)^2-4\,dx+\left(\cos\left(dx+c\right)^2-1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(\cos\left(dx+c\right)^2-1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right.\right.}{4\left(ad\cos\left(dx+c\right)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*x*cos(d*x + c)^2 - 4*d*x + (cos(d*x + c)^2 - 1)*log(1/2*cos(d*x +
c) + 1/2) - (cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) - 4*cos(d*x +
 c)*sin(d*x + c) + 2*cos(d*x + c))/(a*d*cos(d*x + c)^2 - a*d)
```

giac [A] time = 0.22, size = 103, normalized size = 1.78

$$\frac{\frac{8\,(dx+c)}{a} - \frac{4\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} + \frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2} + \frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 1}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*(d*x + c)/a - 4*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a*tan(1/2*d*x +
1/2*c)^2 - 4*a*tan(1/2*d*x + 1/2*c))/a^2 + (6*tan(1/2*d*x + 1/2*c)^2 + 4*ta
n(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)^2))/d
```

maple [B] time = 0.43, size = 112, normalized size = 1.93

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad} - \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad} + \frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad} - \frac{1}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2} + \frac{1}{2ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/8/a/d*tan(1/2*d*x+1/2*c)^2-1/2/a/d*tan(1/2*d*x+1/2*c)+2/a/d*arctan(tan(1/
2*d*x+1/2*c))-1/8/a/d/tan(1/2*d*x+1/2*c)^2+1/2/a/d/tan(1/2*d*x+1/2*c)-1/2/a
/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.42, size = 138, normalized size = 2.38

$$-\frac{\frac{\frac{4\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a} - \frac{16\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{4\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\left(\frac{4\,\sin(dx+c)}{\cos(dx+c)+1} - 1\right)(\cos(dx+c)+1)^2}{a\sin(dx+c)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)/a - 16*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 4*log(sin(d*x + c)/
```

```
(cos(d*x + c) + 1))/a - (4*sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(cos(d*x +
c) + 1)^2/(a*sin(d*x + c)^2))/d
```

mupad [B] time = 8.75, size = 159, normalized size = 2.74

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8ad}-\frac{2\,\mathrm{atan}\left(\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{ad}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8ad}-\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2ad}+\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - (2*atan((2*cos(c/2 + (d*x)/2) - sin(c/2 + (d
*x)/2))/(cos(c/2 + (d*x)/2) + 2*sin(c/2 + (d*x)/2))))/(a*d) - cot(c/2 + (d*
x)/2)^2/(8*a*d) - log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(2*a*d) + cot(
c/2 + (d*x)/2)/(2*a*d) - tan(c/2 + (d*x)/2)/(2*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^4(c+dx)\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

3.416 $\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d-1/3*cot(d*x+c)^3/a/d+1/2*cot(d*x+c)*csc(d*x+c)
/a/d
```

Rubi [A] time = 0.09, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) - Cot[c + d*x]^3/(3*a*d) + (Cot[c + d*x]*Csc
[c + d*x])/(2*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= \frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \csc(c+dx)\,dx}{2a}+\frac{\mathrm{Subst}\left(\int x^2\,dx,x,-\cot(c+dx)\right)}{ad}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}\end{aligned}$$

Mathematica [B] time = 0.49, size = 124, normalized size = 2.14

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\cos(3(c+dx))+(3-6\sin(c+dx))\cos(\right.}{96ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/96*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/
2])^2*(Cos[3*(c + d*x)] + Cos[c + d*x]*(3 - 6*Sin[c + d*x]) + 6*(Log[Cos[(c
 + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3))/(a*d*(1 + Sin[c + d*x
]))
```

fricas [B] time = 0.45, size = 111, normalized size = 1.91

$$\frac{4\,\cos\left(dx+c\right)^3-3\left(\cos\left(dx+c\right)^2-1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\left(\cos\left(dx+c\right)^2-1\right)\log\left(-\frac{1}{2}\,\cos\right.}{12\left(ad\cos\left(dx+c\right)^2-ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(d*x + c)^3 - 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)
*sin(d*x + c) + 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) - 6*cos(d*x + c)*sin(d*x + c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x +
c))
```

giac [B] time = 0.19, size = 127, normalized size = 2.19

$$\frac{\frac{12 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{22 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a^2*tan(1/2*d*x + 1/2*c)^3 - 3
*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c))/a^3 - (22*tan(1/2
*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 1)/(a
*tan(1/2*d*x + 1/2*c)^3))/d
```

maple [B] time = 0.44, size = 132, normalized size = 2.28

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{1}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{1}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{}{24ad\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/24/a/d*tan(1/2*d*x+1/2*c)^3-1/8/a/d*tan(1/2*d*x+1/2*c)^2-1/8/a/d*tan(1/2*
d*x+1/2*c)+1/8/a/d/tan(1/2*d*x+1/2*c)+1/2/a/d*ln(tan(1/2*d*x+1/2*c))+1/8/a/
d/tan(1/2*d*x+1/2*c)^2-1/24/a/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.31, size = 155, normalized size = 2.67

$$-\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/24*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - 12*log(sin(d*x + c)/(cos(
d*x + c) + 1))/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

mupad [B] time = 8.67, size = 115, normalized size = 1.98

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) + log(tan(c/2
+ (d*x)/2))/(2*a*d) - tan(c/2 + (d*x)/2)/(8*a*d) + (cot(c/2 + (d*x)/2)^3*(t
an(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2 - 1/3))/(8*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.417 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{\cot^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

```
[Out] 1/8*arctanh(cos(d*x+c))/a/d+1/3*cot(d*x+c)^3/a/d+1/8*cot(d*x+c)*csc(d*x+c)/
a/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d
```

Rubi [A] time = 0.15, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2611, 3768, 3770, 2607, 30}

$$\frac{\cot^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(8*a*d) + Cot[c + d*x]^3/(3*a*d) + (Cot[c + d*x]*Csc[
c + d*x])/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a} + \frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a}$$
$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad} - \frac{\int \csc^3(c+dx)\,dx}{4a} - \frac{\text{Subst}\left(\int x^2\,dx, x, -\cot(c+d\right.}{ad}$$
$$= \frac{\cot^3(c+dx)}{3ad} + \frac{\cot(c+dx)\csc(c+dx)}{8ad} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} - \frac{\int \csc(c+}{8a}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{8ad} + \frac{\cot^3(c+dx)}{3ad} + \frac{\cot(c+dx)\csc(c+dx)}{8ad} - \frac{\cot(c+dx)\,c}{4a}$$

Mathematica [A] time = 0.97, size = 125, normalized size = 1.52

$$\frac{\csc^4(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-42\cos(c+dx)+2(8\sin(c+dx)-3)\cos(3(c+dx))+24\left(\right.\right.}{192ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(-42*Cos[c + d*x] +
 2*Cos[3*(c + d*x)]*(-3 + 8*Sin[c + d*x]) + 24*((Log[Cos[(c + d*x)/2]] - Lo
```

```
g[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 + Sin[2*(c + d*x)])))/(192*a*d*(1 + Sin
[c + d*x]))
```

fricas [A] time = 0.45, size = 132, normalized size = 1.61

$$\frac{16 \cos(dx+c)^3 \sin(dx+c) - 6 \cos(dx+c)^3 + 3\left(\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) -}{48\left(ad\cos(dx+c)^4 - 2\,ad\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(16*cos(d*x + c)^3*sin(d*x + c) - 6*cos(d*x + c)^3 + 3*(cos(d*x + c)^4
 - 2*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^4 -
2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2) - 6*cos(d*x + c))/(a*d*c
os(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

giac [A] time = 0.23, size = 129, normalized size = 1.57

$$-\frac{\frac{24 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a} - \frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 8a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 24a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4} - \frac{50\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 8\tan\left(\frac{1}{2}dx\right.}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(24*log(abs(tan(1/2*d*x + 1/2*c)))/a - (3*a^3*tan(1/2*d*x + 1/2*c)^4
 - 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 24*a^3*tan(1/2*d*x + 1/2*c))/a^4 - (50*ta
n(1/2*d*x + 1/2*c)^4 - 24*tan(1/2*d*x + 1/2*c)^3 + 8*tan(1/2*d*x + 1/2*c) -
 3)/(a*tan(1/2*d*x + 1/2*c)^4))/d
```

maple [A] time = 0.43, size = 132, normalized size = 1.61

$$\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad} - \frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad} + \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad} - \frac{1}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad} - \frac{1}{64ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4} + \frac{}{24ad\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/64/a/d*tan(1/2*d*x+1/2*c)^4-1/24/a/d*tan(1/2*d*x+1/2*c)^3+1/8/a/d*tan(1/2
*d*x+1/2*c)-1/8/a/d/tan(1/2*d*x+1/2*c)-1/8/a/d*ln(tan(1/2*d*x+1/2*c))-1/64/
a/d/tan(1/2*d*x+1/2*c)^4+1/24/a/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.38, size = 154, normalized size = 1.88

$$\frac{\frac{\frac{24\sin(dx+c)}{\cos(dx+c)+1}-\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a}-\frac{24\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-3\right)(\cos(dx+c)+1)^4}{a\sin(dx+c)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/192*((24*sin(d*x + c)/(cos(d*x + c) + 1) - 8*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/a - 24*log(sin(d*x + c)/(c
os(d*x + c) + 1))/a + (8*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 - 3)*(cos(d*x + c) + 1)^4/(a*sin(d*x + c)^4))/d
```

mupad [B] time = 8.68, size = 119, normalized size = 1.45

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}\right.}{16\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^4/(64*a*d) - tan(c/2 + (d*x)/2)^3/(24*a*d) - log(tan(c/2
 + (d*x)/2))/(8*a*d) + tan(c/2 + (d*x)/2)/(8*a*d) - (cot(c/2 + (d*x)/2)^4*(
2*tan(c/2 + (d*x)/2)^3 - (2*tan(c/2 + (d*x)/2))/3 + 1/4))/(16*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.418 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=100

$$-\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

```
[Out] -1/8*arctanh(cos(d*x+c))/a/d-1/3*cot(d*x+c)^3/a/d-1/5*cot(d*x+c)^5/a/d-1/8*
cot(d*x+c)*csc(d*x+c)/a/d+1/4*cot(d*x+c)*csc(d*x+c)^3/a/d
```

Rubi [A] time = 0.17, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2607, 14, 2611, 3768, 3770}

$$-\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(8*a*d) - Cot[c + d*x]^3/(3*a*d) - Cot[c + d*x]^5/(5
*a*d) - (Cot[c + d*x]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]*Csc[c + d*x]^3)
/(4*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a}$$
$$= \frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{\int \csc^3(c+dx)\,dx}{4a}+\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,-\right.}{ad}$$
$$= -\frac{\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{\int \csc(c+dx)\,dx}{8a}+\text{Su}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot^5(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx}{8ad}$$

Mathematica [A] time = 0.59, size = 189, normalized size = 1.89

$$-\frac{\csc^5(c+dx)\left(-180\sin(2(c+dx))-30\sin(4(c+dx))+320\cos(c+dx)+80\cos(3(c+dx))-16\cos(5(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/1920*(Csc[c + d*x]^5*(320*Cos[c + d*x] + 80*Cos[3*(c + d*x)] - 16*Cos[5*
(c + d*x)] + 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 150*Log[Sin[(c + d*x)
/2]]*Sin[c + d*x] - 180*Sin[2*(c + d*x)] - 75*Log[Cos[(c + d*x)/2]]*Sin[3*(
c + d*x)] + 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 30*Sin[4*(c + d*x)]
 + 15*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin
[5*(c + d*x)]))/(a*d)
```

fricas [A] time = 0.46, size = 161, normalized size = 1.61

$$\frac{32\,\cos\left(dx+c\right)^5-80\,\cos\left(dx+c\right)^3-15\left(\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right.}{240\left(ad\cos\left(dx+c\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(32*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 15*(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)^
4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*(c
os(d*x + c)^3 + cos(d*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^4 - 2*a*d*co
s(d*x + c)^2 + a*d)*sin(d*x + c))
```

giac [A] time = 0.20, size = 157, normalized size = 1.57

$$\frac{\frac{120\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-60\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^5}-\frac{274\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-60\,\tan}{}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a + (6*a^4*tan(1/2*d*x + 1/2*c)^5
 - 15*a^4*tan(1/2*d*x + 1/2*c)^4 + 10*a^4*tan(1/2*d*x + 1/2*c)^3 - 60*a^4*t
an(1/2*d*x + 1/2*c))/a^5 - (274*tan(1/2*d*x + 1/2*c)^5 - 60*tan(1/2*d*x + 1
/2*c)^4 + 10*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 6)/(a*tan(1
/2*d*x + 1/2*c)^5))/d
```

maple [A] time = 0.46, size = 170, normalized size = 1.70

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{96ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{1}{16ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{1}{160ad\tan\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/160/a/d*tan(1/2*d*x+1/2*c)^5-1/64/a/d*tan(1/2*d*x+1/2*c)^4+1/96/a/d*tan(1
/2*d*x+1/2*c)^3-1/16/a/d*tan(1/2*d*x+1/2*c)+1/16/a/d/tan(1/2*d*x+1/2*c)+1/8
/a/d*ln(tan(1/2*d*x+1/2*c))-1/160/a/d/tan(1/2*d*x+1/2*c)^5+1/64/a/d/tan(1/2
*d*x+1/2*c)^4-1/96/a/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.32, size = 195, normalized size = 1.95

$$-\frac{\frac{\frac{60\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{60\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-6\right)(\cos(dx+c)+}{a\sin(dx+c)^5}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/960*((60*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5)/a - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (15*sin(d
*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 60*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6)*(cos(d*x + c) + 1)^5/(a*sin(d*x + c)
^5))/d
```

mupad [B] time = 8.75, size = 151, normalized size = 1.51

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^6*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(96*a*d) - tan(c/2 + (d*x)/2)^4/(64*a*d) + tan(c/2 + (
d*x)/2)^5/(160*a*d) + log(tan(c/2 + (d*x)/2))/(8*a*d) - tan(c/2 + (d*x)/2)/
(16*a*d) + (cot(c/2 + (d*x)/2)^5*(tan(c/2 + (d*x)/2)/2 - tan(c/2 + (d*x)/2)
^2/3 + 2*tan(c/2 + (d*x)/2)^4 - 1/5))/(32*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.419 $\int \frac{\cot^4(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}+\frac{\cot(c+dx)\csc(c+dx)}{16ad}$$

```
[Out] 1/16*arctanh(cos(d*x+c))/a/d+1/3*cot(d*x+c)^3/a/d+1/5*cot(d*x+c)^5/a/d+1/16
*cot(d*x+c)*csc(d*x+c)/a/d+1/24*cot(d*x+c)*csc(d*x+c)^3/a/d-1/6*cot(d*x+c)*
csc(d*x+c)^5/a/d
```

Rubi [A] time = 0.18, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2611, 3768, 3770, 2607, 14}

$$\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}+\frac{\cot(c+dx)\csc(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(16*a*d) + Cot[c + d*x]^3/(3*a*d) + Cot[c + d*x]^5/(5
*a*d) + (Cot[c + d*x]*Csc[c + d*x])/(16*a*d) + (Cot[c + d*x]*Csc[c + d*x]^3
)/(24*a*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a} + \frac{\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{a}$$

$$= -\frac{\cot(c+dx)\csc^5(c+dx)}{6ad} - \frac{\int \csc^5(c+dx)\,dx}{6a} - \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx, x, -c\right.}{ad}$$

$$= \frac{\cot(c+dx)\csc^3(c+dx)}{24ad} - \frac{\cot(c+dx)\csc^5(c+dx)}{6ad} - \frac{\int \csc^3(c+dx)\,dx}{8a} - \text{Sub}$$

$$= \frac{\cot^3(c+dx)}{3ad} + \frac{\cot^5(c+dx)}{5ad} + \frac{\cot(c+dx)\csc(c+dx)}{16ad} + \frac{\cot(c+dx)\csc^3(c+}{24ad}$$

$$= \frac{\tanh^{-1}(\cos(c+dx))}{16ad} + \frac{\cot^3(c+dx)}{3ad} + \frac{\cot^5(c+dx)}{5ad} + \frac{\cot(c+dx)\csc(c+dx)}{16ad}$$

Mathematica [A] time = 0.56, size = 229, normalized size = 1.85

$$-\frac{\csc^6(c+dx)\left(-480\sin(2(c+dx)) - 192\sin(4(c+dx)) + 32\sin(6(c+dx)) + 1140\cos(c+dx) + 170\cos(3(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7680*(Csc[c + d*x]^6*(1140*Cos[c + d*x] + 170*Cos[3*(c + d*x)] - 30*Cos[
5*(c + d*x)] - 150*Log[Cos[(c + d*x)/2]] + 225*Cos[2*(c + d*x)]*Log[Cos[(c
+ d*x)/2]] - 90*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 15*Cos[6*(c + d*x)
]*Log[Cos[(c + d*x)/2]] + 150*Log[Sin[(c + d*x)/2]] - 225*Cos[2*(c + d*x)]*
Log[Sin[(c + d*x)/2]] + 90*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 15*Cos[
6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 480*Sin[2*(c + d*x)] - 192*Sin[4*(c +
d*x)] + 32*Sin[6*(c + d*x)]))/(a*d)
```

fricas [A] time = 0.46, size = 188, normalized size = 1.52

$$-\frac{30 \cos(dx+c)^5 - 80 \cos(dx+c)^3 - 15\left(\cos(dx+c)^6 - 3\cos(dx+c)^4 + 3\cos(dx+c)^2 - 1\right)\log\left(\frac{1}{2}\cos(dx \cdots}{480\,(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/480*(30*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 15*(cos(d*x + c)^6 - 3*cos(
d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*
x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) +
 1/2) - 32*(2*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*sin(d*x + c) - 30*cos(d*x
+ c))/(a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a
*d)
```

giac [A] time = 0.22, size = 216, normalized size = 1.74

$$-\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a} - \frac{5a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 - 12a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 15a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 20a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 15a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 120 \cdots}{a^6}}{19 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1920*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - (5*a^5*tan(1/2*d*x + 1/2*c)
^6 - 12*a^5*tan(1/2*d*x + 1/2*c)^5 + 15*a^5*tan(1/2*d*x + 1/2*c)^4 - 20*a^5
*tan(1/2*d*x + 1/2*c)^3 - 15*a^5*tan(1/2*d*x + 1/2*c)^2 + 120*a^5*tan(1/2*d
*x + 1/2*c))/a^6 - (294*tan(1/2*d*x + 1/2*c)^6 - 120*tan(1/2*d*x + 1/2*c)^5
 + 15*tan(1/2*d*x + 1/2*c)^4 + 20*tan(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x +
 1/2*c)^2 + 12*tan(1/2*d*x + 1/2*c) - 5)/(a*tan(1/2*d*x + 1/2*c)^6))/d
```

maple [B] time = 0.48, size = 246, normalized size = 1.98

$$\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{384ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad}+\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{128ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{96ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{128ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{1}{384ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/384/a/d*tan(1/2*d*x+1/2*c)^6-1/160/a/d*tan(1/2*d*x+1/2*c)^5+1/128/a/d*tan
(1/2*d*x+1/2*c)^4-1/96/a/d*tan(1/2*d*x+1/2*c)^3-1/128/a/d*tan(1/2*d*x+1/2*c
)^2+1/16/a/d*tan(1/2*d*x+1/2*c)-1/384/a/d/tan(1/2*d*x+1/2*c)^6-1/16/a/d/tan
(1/2*d*x+1/2*c)-1/16/a/d*ln(tan(1/2*d*x+1/2*c))+1/160/a/d/tan(1/2*d*x+1/2*c
)^5+1/128/a/d/tan(1/2*d*x+1/2*c)^2-1/128/a/d/tan(1/2*d*x+1/2*c)^4+1/96/a/d/
tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.36, size = 274, normalized size = 2.21

$$\frac{\frac{\frac{120\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{12\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{20}{(\cos}\right.}{}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/1920*((120*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 12*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6)/a - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a + (
12*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 120*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x + c) + 1)^6
/(a*sin(d*x + c)^6))/d
```

mupad [B] time = 9.68, size = 339, normalized size = 2.73

$$-\frac{5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+12\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-15\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] -(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 + 12*cos(c/2 + (d*x)/2)
*sin(c/2 + (d*x)/2)^11 - 12*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) - 15*c
os(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 20*cos(c/2 + (d*x)/2)^3*sin(c/2
 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 - 120*cos(c/2
+ (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 120*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x
)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 20*cos(c/2 + (d*x)/
2)^9*sin(c/2 + (d*x)/2)^3 + 15*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2 +
 120*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*sin(c/
2 + (d*x)/2)^6)/(1920*a*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.420 $\int \frac{\cos^4(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=147

$$\frac{\cos^7(c+dx)}{7a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin^5(c+dx)\cos(c+dx)}{3a^2d}+\frac{5\sin^3(c+dx)\cos(c+dx)}{12a^2d}$$

```
[Out] -5/8*x/a^2-2*cos(d*x+c)/a^2/d+5/3*cos(d*x+c)^3/a^2/d-4/5*cos(d*x+c)^5/a^2/d
+1/7*cos(d*x+c)^7/a^2/d+5/8*cos(d*x+c)*sin(d*x+c)/a^2/d+5/12*cos(d*x+c)*sin
(d*x+c)^3/a^2/d+1/3*cos(d*x+c)*sin(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.22, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 2633, 2635, 8}

$$\frac{\cos^7(c+dx)}{7a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin^5(c+dx)\cos(c+dx)}{3a^2d}+\frac{5\sin^3(c+dx)\cos(c+dx)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*x)/(8*a^2) - (2*Cos[c + d*x])/(a^2*d) + (5*Cos[c + d*x]^3)/(3*a^2*d) -
(4*Cos[c + d*x]^5)/(5*a^2*d) + Cos[c + d*x]^7/(7*a^2*d) + (5*Cos[c + d*x]*S
in[c + d*x])/(8*a^2*d) + (5*Cos[c + d*x]*Sin[c + d*x]^3)/(12*a^2*d) + (Cos[
c + d*x]*Sin[c + d*x]^5)/(3*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sin^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sin^5(c+dx)-2a^2\sin^6(c+dx)+a^2\sin^7(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \sin^5(c+dx)\,dx}{a^2}+\frac{\int \sin^7(c+dx)\,dx}{a^2}-\frac{2\int \sin^6(c+dx)\,dx}{a^2}$$
$$= \frac{\cos(c+dx)\sin^5(c+dx)}{3a^2d}-\frac{5\int \sin^4(c+dx)\,dx}{3a^2}-\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx\right.}{a^2d}$$
$$= -\frac{2\cos(c+dx)}{a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{\cos^7(c+dx)}{7a^2d}+\frac{5\cos(c}{}$$
$$= -\frac{2\cos(c+dx)}{a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{\cos^7(c+dx)}{7a^2d}+\frac{5\cos(c}{}$$
$$= -\frac{5x}{8a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{\cos^7(c+dx)}{7a^2d}+\frac{5}{}$$

Mathematica [B] time = 4.83, size = 418, normalized size = 2.84

$$\frac{-8400dx\sin\left(\frac{c}{2}\right)+7875\sin\left(\frac{c}{2}+dx\right)-7875\sin\left(\frac{3c}{2}+dx\right)+3150\sin\left(\frac{3c}{2}+2dx\right)+3150\sin\left(\frac{5c}{2}+2dx\right)-1435}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-210*(1 + 40*d*x)*Cos[c/2] - 7875*Cos[c/2 + d*x] - 7875*Cos[(3*c)/2 + d*x]
 + 3150*Cos[(3*c)/2 + 2*d*x] - 3150*Cos[(5*c)/2 + 2*d*x] + 1435*Cos[(5*c)/2
 + 3*d*x] + 1435*Cos[(7*c)/2 + 3*d*x] - 630*Cos[(7*c)/2 + 4*d*x] + 630*Cos[
(9*c)/2 + 4*d*x] - 231*Cos[(9*c)/2 + 5*d*x] - 231*Cos[(11*c)/2 + 5*d*x] + 7
0*Cos[(11*c)/2 + 6*d*x] - 70*Cos[(13*c)/2 + 6*d*x] + 15*Cos[(13*c)/2 + 7*d*
x] + 15*Cos[(15*c)/2 + 7*d*x] + 210*Sin[c/2] - 8400*d*x*Sin[c/2] + 7875*Sin
[c/2 + d*x] - 7875*Sin[(3*c)/2 + d*x] + 3150*Sin[(3*c)/2 + 2*d*x] + 3150*Si
n[(5*c)/2 + 2*d*x] - 1435*Sin[(5*c)/2 + 3*d*x] + 1435*Sin[(7*c)/2 + 3*d*x]
- 630*Sin[(7*c)/2 + 4*d*x] - 630*Sin[(9*c)/2 + 4*d*x] + 231*Sin[(9*c)/2 + 5
*d*x] - 231*Sin[(11*c)/2 + 5*d*x] + 70*Sin[(11*c)/2 + 6*d*x] + 70*Sin[(13*c
)/2 + 6*d*x] - 15*Sin[(13*c)/2 + 7*d*x] + 15*Sin[(15*c)/2 + 7*d*x])/(13440*
a^2*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.45, size = 88, normalized size = 0.60

$$\frac{120\,\cos\left(dx+c\right)^{7}-672\,\cos\left(dx+c\right)^{5}+1400\,\cos\left(dx+c\right)^{3}-525\,dx+35\left(8\,\cos\left(dx+c\right)^{5}-26\,\cos\left(dx+c\right)^{3}+3\right.}{840\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/840*(120*cos(d*x + c)^7 - 672*cos(d*x + c)^5 + 1400*cos(d*x + c)^3 - 525*
d*x + 35*(8*cos(d*x + c)^5 - 26*cos(d*x + c)^3 + 33*cos(d*x + c))*sin(d*x +
 c) - 1680*cos(d*x + c))/(a^2*d)
```

giac [A] time = 0.23, size = 166, normalized size = 1.13

$$-\frac{\frac{525\,(dx+c)}{a^{2}}+\frac{2\left(525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+3500\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+9905\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+4480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+24640\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-9905\,\tan\left(\frac{1}{2}\,c\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{7}a^{2}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(525*(d*x + c)/a^2 + 2*(525*tan(1/2*d*x + 1/2*c)^13 + 3500*tan(1/2*d
*x + 1/2*c)^11 + 9905*tan(1/2*d*x + 1/2*c)^9 + 4480*tan(1/2*d*x + 1/2*c)^8
+ 24640*tan(1/2*d*x + 1/2*c)^6 - 9905*tan(1/2*d*x + 1/2*c)^5 + 17472*tan(1/
2*d*x + 1/2*c)^4 - 3500*tan(1/2*d*x + 1/2*c)^3 + 5824*tan(1/2*d*x + 1/2*c)^
2 - 525*tan(1/2*d*x + 1/2*c) + 832)/((tan(1/2*d*x + 1/2*c)^2 + 1)^7*a^2))/d
```

maple [B] time = 0.44, size = 381, normalized size = 2.59

$$-\frac{5\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{25\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{283\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{32\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -5/4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13-25/3/a^2/d/(1+t
an(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11-283/12/a^2/d/(1+tan(1/2*d*x+1/
2*c)^2)^7*tan(1/2*d*x+1/2*c)^9-32/3/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/
2*d*x+1/2*c)^8-176/3/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6+
283/12/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^5-208/5/a^2/d/(1
+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4+25/3/a^2/d/(1+tan(1/2*d*x+1/2
*c)^2)^7*tan(1/2*d*x+1/2*c)^3-208/15/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1
/2*d*x+1/2*c)^2+5/4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)-208
/105/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^7-5/4/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.49, size = 396, normalized size = 2.69

$$\frac{\frac{\frac{525\sin(dx+c)}{\cos(dx+c)+1}-\frac{5824\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3500\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{17472\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9905\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{24640\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{4480\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{9905\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{3500\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{525}{(\cos}}{a^2+\frac{7a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{21a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{21a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{7a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/420*((525*sin(d*x + c)/(cos(d*x + c) + 1) - 5824*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 3500*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 17472*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 9905*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 24640*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 4480*sin(d*x + c)^8/(cos(d*x + c) + 1
)^8 - 9905*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 3500*sin(d*x + c)^11/(cos(
d*x + c) + 1)^11 - 525*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 832)/(a^2 +
7*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 21*a^2*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 35*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 35*a^2*sin(d*x +
 c)^8/(cos(d*x + c) + 1)^8 + 21*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 7*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + a^2*sin(d*x + c)^14/(cos(d*x
 + c) + 1)^14) - 525*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 12.22, size = 160, normalized size = 1.09

$$-\frac{5\,x}{8\,a^2}-\frac{\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{4}+\frac{25\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3}+\frac{283\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{12}+\frac{32\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3}+\frac{176\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}-\frac{283\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{12}+\frac{208\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{5}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^5)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - (5*x)/(8*a^2) - ((208*tan(c/2 + (d*x)/2)^2)/15 - (5*tan(c/2 + (d*x)/2))/4
 - (25*tan(c/2 + (d*x)/2)^3)/3 + (208*tan(c/2 + (d*x)/2)^4)/5 - (283*tan(c/
2 + (d*x)/2)^5)/12 + (176*tan(c/2 + (d*x)/2)^6)/3 + (32*tan(c/2 + (d*x)/2)^
8)/3 + (283*tan(c/2 + (d*x)/2)^9)/12 + (25*tan(c/2 + (d*x)/2)^11)/3 + (5*ta
n(c/2 + (d*x)/2)^13)/4 + 208/105)/(a^2*d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.421 $\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=129

$$\frac{2\cos^5(c+dx)}{5a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{\sin^5(c+dx)\cos(c+dx)}{6a^2d}-\frac{11\sin^3(c+dx)\cos(c+dx)}{24a^2d}-\frac{11\sin(c+}{}$$

[Out] 11/16*x/a^2+2*cos(d*x+c)/a^2/d-4/3*cos(d*x+c)^3/a^2/d+2/5*cos(d*x+c)^5/a^2/d-11/16*cos(d*x+c)*sin(d*x+c)/a^2/d-11/24*cos(d*x+c)*sin(d*x+c)^3/a^2/d-1/6*cos(d*x+c)*sin(d*x+c)^5/a^2/d

Rubi [A] time = 0.23, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 2635, 8, 2633}

$$\frac{2\cos^5(c+dx)}{5a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{\sin^5(c+dx)\cos(c+dx)}{6a^2d}-\frac{11\sin^3(c+dx)\cos(c+dx)}{24a^2d}-\frac{11\sin(c+}{}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]

[Out] (11*x)/(16*a^2) + (2*Cos[c + d*x])/(a^2*d) - (4*Cos[c + d*x]^3)/(3*a^2*d) + (2*Cos[c + d*x]^5)/(5*a^2*d) - (11*Cos[c + d*x]*Sin[c + d*x])/(16*a^2*d) - (11*Cos[c + d*x]*Sin[c + d*x]^3)/(24*a^2*d) - (Cos[c + d*x]*Sin[c + d*x]^5)/(6*a^2*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sin^4(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\
&= \frac{\int \left(a^2\sin^4(c+dx)-2a^2\sin^5(c+dx)+a^2\sin^6(c+dx)\right)dx}{a^4}\\
&= \frac{\int \sin^4(c+dx)\,dx}{a^2}+\frac{\int \sin^6(c+dx)\,dx}{a^2}-\frac{2\int \sin^5(c+dx)\,dx}{a^2}\\
&= -\frac{\cos(c+dx)\sin^3(c+dx)}{4a^2d}-\frac{\cos(c+dx)\sin^5(c+dx)}{6a^2d}+\frac{3\int \sin^2(c+dx)\,dx}{4a^2}+\\
&= \frac{2\cos(c+dx)}{a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{3\cos(c+dx)\sin(c+dx)}{8a^2d}-\\
&= \frac{3x}{8a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{11\cos(c+dx)\sin(c}{16a^2d}\\
&= \frac{11x}{16a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{11\cos(c+dx)\sin(c}{16a^2d}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 76, normalized size = 0.59

$$\frac{-465\sin(2(c+dx))+75\sin(4(c+dx))-5\sin(6(c+dx))+1200\cos(c+dx)-200\cos(3(c+dx))+24\cos(5(c}{960a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (660*c + 660*d*x + 1200*Cos[c + d*x] - 200*Cos[3*(c + d*x)] + 24*Cos[5*(c +
 d*x)] - 465*Sin[2*(c + d*x)] + 75*Sin[4*(c + d*x)] - 5*Sin[6*(c + d*x)])/(
960*a^2*d)
```

fricas [A] time = 0.44, size = 78, normalized size = 0.60

$$\frac{96\cos(dx+c)^5-320\cos(dx+c)^3+165\,dx-5\left(8\cos(dx+c)^5-38\cos(dx+c)^3+63\cos(dx+c)\right)\sin(dx}{240\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/240*(96*cos(d*x + c)^5 - 320*cos(d*x + c)^3 + 165*d*x - 5*(8*cos(d*x + c)
^5 - 38*cos(d*x + c)^3 + 63*cos(d*x + c))*sin(d*x + c) + 480*cos(d*x + c))/
(a^2*d)
```

giac [A] time = 0.20, size = 153, normalized size = 1.19

$$\frac{\frac{165\,(dx+c)}{a^2}+\frac{2\left(165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+935\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+1410\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+2560\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-1410\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+3840\tan\left(\frac{1}{2}dx+\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^6a^2}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(165*(d*x + c)/a^2 + 2*(165*tan(1/2*d*x + 1/2*c)^11 + 935*tan(1/2*d*x
 + 1/2*c)^9 + 1410*tan(1/2*d*x + 1/2*c)^7 + 2560*tan(1/2*d*x + 1/2*c)^6 - 1
410*tan(1/2*d*x + 1/2*c)^5 + 3840*tan(1/2*d*x + 1/2*c)^4 - 935*tan(1/2*d*x
+ 1/2*c)^3 + 1536*tan(1/2*d*x + 1/2*c)^2 - 165*tan(1/2*d*x + 1/2*c) + 256)/
((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a^2))/d
```

maple [B] time = 0.41, size = 347, normalized size = 2.69

$$\frac{11\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{187\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{47\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{64\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 11/8/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11+187/24/a^2/d/(1
+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9+47/4/a^2/d/(1+tan(1/2*d*x+1/2
*c)^2)^6*tan(1/2*d*x+1/2*c)^7+64/3/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2
*d*x+1/2*c)^6-47/4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5+32
```

```
/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4-187/24/a^2/d/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+64/5/a^2/d/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)^2-11/8/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+
1/2*c)+32/15/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^6+11/8/d/a^2*arctan(tan(1/2*d*x
+1/2*c))
```

maxima [B] time = 0.54, size = 353, normalized size = 2.74

$$-\frac{\frac{\frac{165\sin(dx+c)}{\cos(dx+c)+1}-\frac{1536\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{935\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3840\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1410\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2560\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{1410\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{935\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{165\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-256}{a^2+\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}} -}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*((165*sin(d*x + c)/(cos(d*x + c) + 1) - 1536*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 935*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3840*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 1410*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2560*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1410*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7 - 935*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 165*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 - 256)/(a^2 + 6*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a
^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^2*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^2*sin(d*x + c)^
10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 165
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 11.24, size = 146, normalized size = 1.13

$$\frac{11\,x}{16\,a^2}+\frac{\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{187\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{47\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}-\frac{47\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{187\tan\left(\frac{c}{2}+\right.}{24}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^4)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (11*x)/(16*a^2) + ((64*tan(c/2 + (d*x)/2)^2)/5 - (11*tan(c/2 + (d*x)/2))/8
- (187*tan(c/2 + (d*x)/2)^3)/24 + 32*tan(c/2 + (d*x)/2)^4 - (47*tan(c/2 + (
d*x)/2)^5)/4 + (64*tan(c/2 + (d*x)/2)^6)/3 + (47*tan(c/2 + (d*x)/2)^7)/4 +
(187*tan(c/2 + (d*x)/2)^9)/24 + (11*tan(c/2 + (d*x)/2)^11)/8 + 32/15)/(a^2*
d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [A] time = 143.14, size = 2271, normalized size = 17.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((165*d*x*tan(c/2 + d*x/2)**12/(240*a**2*d*tan(c/2 + d*x/2)**12 +
1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a
**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*t
an(c/2 + d*x/2)**2 + 240*a**2*d) + 990*d*x*tan(c/2 + d*x/2)**10/(240*a**2*d
*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(
c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d
*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 2475*d*x*tan(c/2
 + d*x/2)**8/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2
)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 +
 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a*
*2*d) + 3300*d*x*tan(c/2 + d*x/2)**6/(240*a**2*d*tan(c/2 + d*x/2)**12 + 144
0*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2
*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(
c/2 + d*x/2)**2 + 240*a**2*d) + 2475*d*x*tan(c/2 + d*x/2)**4/(240*a**2*d*ta
n(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2
 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/
2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 990*d*x*tan(c/2 + d
*x/2)**2/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**1
0 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 360
0*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d
) + 165*d*x/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)
**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 +
3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**
2*d) + 330*tan(c/2 + d*x/2)**11/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**
2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*ta
n(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 +
 d*x/2)**2 + 240*a**2*d) + 1870*tan(c/2 + d*x/2)**9/(240*a**2*d*tan(c/2 + d
*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)
**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1
440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 2820*tan(c/2 + d*x/2)**7/(24
0*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**
2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan
(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 5120*tan
(c/2 + d*x/2)**6/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d
*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)*
*6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 24
0*a**2*d) - 2820*tan(c/2 + d*x/2)**5/(240*a**2*d*tan(c/2 + d*x/2)**12 + 144
```

```
0*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2
*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(
c/2 + d*x/2)**2 + 240*a**2*d) + 7680*tan(c/2 + d*x/2)**4/(240*a**2*d*tan(c/
2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d
*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**
4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) - 1870*tan(c/2 + d*x/2)**
3/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 360
0*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*
d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 307
2*tan(c/2 + d*x/2)**2/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/
2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*
x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2
 + 240*a**2*d) - 330*tan(c/2 + d*x/2)/(240*a**2*d*tan(c/2 + d*x/2)**12 + 14
40*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**
2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan
(c/2 + d*x/2)**2 + 240*a**2*d) + 512/(240*a**2*d*tan(c/2 + d*x/2)**12 + 144
0*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2
*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(
c/2 + d*x/2)**2 + 240*a**2*d), Ne(d, 0)), (x*sin(c)**4*cos(c)**4/(a*sin(c)
+ a)**2, True))
```

3.422 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=102

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^3(c+dx)}{a^2d}-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin^3(c+dx)\cos(c+dx)}{2a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{3x}{4a^2}$$

```
[Out] -3/4*x/a^2-2*cos(d*x+c)/a^2/d+cos(d*x+c)^3/a^2/d-1/5*cos(d*x+c)^5/a^2/d+3/4
*cos(d*x+c)*sin(d*x+c)/a^2/d+1/2*cos(d*x+c)*sin(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.20, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2869, 2757, 2633, 2635, 8}

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^3(c+dx)}{a^2d}-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin^3(c+dx)\cos(c+dx)}{2a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{3x}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*x)/(4*a^2) - (2*Cos[c + d*x])/(a^2*d) + Cos[c + d*x]^3/(a^2*d) - Cos[c
+ d*x]^5/(5*a^2*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(4*a^2*d) + (Cos[c + d*x
]*Sin[c + d*x]^3)/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
```

```
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sin^3(c+dx)-2a^2\sin^4(c+dx)+a^2\sin^5(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \sin^3(c+dx)\,dx}{a^2}+\frac{\int \sin^5(c+dx)\,dx}{a^2}-\frac{2\int \sin^4(c+dx)\,dx}{a^2}$$
$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2a^2d}-\frac{3\int \sin^2(c+dx)\,dx}{2a^2}-\frac{\mathrm{Subst}\left(\int \left(1-x^2\right)dx,x,\cos(c\right.}{a^2d}$$
$$= -\frac{2\cos(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{a^2d}-\frac{\cos^5(c+dx)}{5a^2d}+\frac{3\cos(c+dx)\sin(c+dx)}{4a^2d}+$$
$$= -\frac{3x}{4a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{a^2d}-\frac{\cos^5(c+dx)}{5a^2d}+\frac{3\cos(c+dx)\sin(c+a}{4a^2d}$$

Mathematica [B] time = 1.32, size = 308, normalized size = 3.02

$$-\frac{120dx\sin\left(\frac{c}{2}\right)-110\sin\left(\frac{c}{2}+dx\right)+110\sin\left(\frac{3c}{2}+dx\right)-40\sin\left(\frac{3c}{2}+2dx\right)-40\sin\left(\frac{5c}{2}+2dx\right)+15\sin\left(\frac{5c}{2}+3dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/160*(5*(1 + 24*d*x)*Cos[c/2] + 110*Cos[c/2 + d*x] + 110*Cos[(3*c)/2 + d*
x] - 40*Cos[(3*c)/2 + 2*d*x] + 40*Cos[(5*c)/2 + 2*d*x] - 15*Cos[(5*c)/2 + 3
*d*x] - 15*Cos[(7*c)/2 + 3*d*x] + 5*Cos[(7*c)/2 + 4*d*x] - 5*Cos[(9*c)/2 +
4*d*x] + Cos[(9*c)/2 + 5*d*x] + Cos[(11*c)/2 + 5*d*x] - 5*Sin[c/2] + 120*d*
x*Sin[c/2] - 110*Sin[c/2 + d*x] + 110*Sin[(3*c)/2 + d*x] - 40*Sin[(3*c)/2 +
```

```
 2*d*x] - 40*Sin[(5*c)/2 + 2*d*x] + 15*Sin[(5*c)/2 + 3*d*x] - 15*Sin[(7*c)/
2 + 3*d*x] + 5*Sin[(7*c)/2 + 4*d*x] + 5*Sin[(9*c)/2 + 4*d*x] - Sin[(9*c)/2
+ 5*d*x] + Sin[(11*c)/2 + 5*d*x])/(a^2*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.44, size = 68, normalized size = 0.67

$$-\frac{4\cos(dx+c)^5-20\cos(dx+c)^3+15dx+5\left(2\cos(dx+c)^3-5\cos(dx+c)\right)\sin(dx+c)+40\cos(dx+c}{20a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20*(4*cos(d*x + c)^5 - 20*cos(d*x + c)^3 + 15*d*x + 5*(2*cos(d*x + c)^3
- 5*cos(d*x + c))*sin(d*x + c) + 40*cos(d*x + c))/(a^2*d)
```

giac [A] time = 0.22, size = 127, normalized size = 1.25

$$-\frac{\frac{15(dx+c)}{a^2}+\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+70\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+200\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-70\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^5a^2}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(15*(d*x + c)/a^2 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 70*tan(1/2*d*x + 1
/2*c)^7 + 40*tan(1/2*d*x + 1/2*c)^6 + 200*tan(1/2*d*x + 1/2*c)^4 - 70*tan(1
/2*d*x + 1/2*c)^3 + 120*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) +
24)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^2))/d
```

maple [B] time = 0.40, size = 279, normalized size = 2.74

$$-\frac{3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{7\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{20\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/2/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9-7/a^2/d/(1+tan(1
/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7-4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^5*
tan(1/2*d*x+1/2*c)^6-20/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)
```

```
^4+7/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3-12/a^2/d/(1+tan(
1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2+3/2/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)
^5*tan(1/2*d*x+1/2*c)-12/5/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^5-3/2/d/a^2*arcta
n(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.49, size = 290, normalized size = 2.84

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{120\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{70\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{200\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{40\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{70\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{15\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-24}{a^2+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/10*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 120*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 70*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 200*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 40*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 70*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 - 15*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 24)/(a^2
 + 5*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 10*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*x
 + c)^8/(cos(d*x + c) + 1)^8 + a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) -
 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 8.72, size = 89, normalized size = 0.87

$$\frac{3\cos(3c+3dx)}{16a^2d}-\frac{11\cos(c+dx)}{8a^2d}-\frac{3x}{4a^2}-\frac{\cos(5c+5dx)}{80a^2d}+\frac{\sin(2c+2dx)}{2a^2d}-\frac{\sin(4c+4dx)}{16a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (3*cos(3*c + 3*d*x))/(16*a^2*d) - (11*cos(c + d*x))/(8*a^2*d) - (3*x)/(4*a^
2) - cos(5*c + 5*d*x)/(80*a^2*d) + sin(2*c + 2*d*x)/(2*a^2*d) - sin(4*c + 4
*d*x)/(16*a^2*d)
```

sympy [A] time = 94.09, size = 1608, normalized size = 15.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-15*d*x*tan(c/2 + d*x/2)**10/(20*a**2*d*tan(c/2 + d*x/2)**10 + 1
00*a**2*d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d
*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a**2*d) - 75*d*x
*tan(c/2 + d*x/2)**8/(20*a**2*d*tan(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 +
 d*x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**
4 + 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a**2*d) - 150*d*x*tan(c/2 + d*x/2)*
*6/(20*a**2*d*tan(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 + d*x/2)**8 + 200*a
**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan
(c/2 + d*x/2)**2 + 20*a**2*d) - 150*d*x*tan(c/2 + d*x/2)**4/(20*a**2*d*tan(
c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c/2 + d*
x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2 +
 20*a**2*d) - 75*d*x*tan(c/2 + d*x/2)**2/(20*a**2*d*tan(c/2 + d*x/2)**10 +
100*a**2*d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*
d*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a**2*d) - 15*d*
x/(20*a**2*d*tan(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 + d*x/2)**8 + 200*a*
*2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(
c/2 + d*x/2)**2 + 20*a**2*d) - 30*tan(c/2 + d*x/2)**9/(20*a**2*d*tan(c/2 +
d*x/2)**10 + 100*a**2*d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**
6 + 200*a**2*d*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a*
*2*d) - 140*tan(c/2 + d*x/2)**7/(20*a**2*d*tan(c/2 + d*x/2)**10 + 100*a**2*
d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d*tan(c/2
 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a**2*d) - 80*tan(c/2 + d
*x/2)**6/(20*a**2*d*tan(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 + d*x/2)**8 +
 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**4 + 100*a**2
*d*tan(c/2 + d*x/2)**2 + 20*a**2*d) - 400*tan(c/2 + d*x/2)**4/(20*a**2*d*ta
n(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c/2 +
d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2
 + 20*a**2*d) + 140*tan(c/2 + d*x/2)**3/(20*a**2*d*tan(c/2 + d*x/2)**10 + 1
00*a**2*d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d
*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a**2*d) - 240*ta
n(c/2 + d*x/2)**2/(20*a**2*d*tan(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 + d*
x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**4 +
 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a**2*d) + 30*tan(c/2 + d*x/2)/(20*a**2
*d*tan(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c
/2 + d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/
2)**2 + 20*a**2*d) - 48/(20*a**2*d*tan(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/
2 + d*x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2
)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a**2*d), Ne(d, 0)), (x*sin(c)**3
*cos(c)**4/(a*sin(c) + a)**2, True))
```

3.423 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=87

$$-\frac{2\cos^3(c+dx)}{3a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}-\frac{7\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{7x}{8a^2}$$

```
[Out] 7/8*x/a^2+2*cos(d*x+c)/a^2/d-2/3*cos(d*x+c)^3/a^2/d-7/8*cos(d*x+c)*sin(d*x+
c)/a^2/d-1/4*cos(d*x+c)*sin(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.20, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 2635, 8, 2633}

$$-\frac{2\cos^3(c+dx)}{3a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}-\frac{7\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{7x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (7*x)/(8*a^2) + (2*Cos[c + d*x])/(a^2*d) - (2*Cos[c + d*x]^3)/(3*a^2*d) - (
7*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a
^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
```

```
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sin^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\sin^2(c+dx) - 2a^2\sin^3(c+dx) + a^2\sin^4(c+dx)\right)dx}{a^4}$$

$$= \frac{\int \sin^2(c+dx)\,dx}{a^2} + \frac{\int \sin^4(c+dx)\,dx}{a^2} - \frac{2\int \sin^3(c+dx)\,dx}{a^2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{4a^2d} + \frac{\int 1\,dx}{2a^2} + \frac{3\int \sin^2(c+}{4a^2}$$

$$= \frac{x}{2a^2} + \frac{2\cos(c+dx)}{a^2d} - \frac{2\cos^3(c+dx)}{3a^2d} - \frac{7\cos(c+dx)\sin(c+dx)}{8a^2d} - \frac{\cos(c+}{}$$

$$= \frac{7x}{8a^2} + \frac{2\cos(c+dx)}{a^2d} - \frac{2\cos^3(c+dx)}{3a^2d} - \frac{7\cos(c+dx)\sin(c+dx)}{8a^2d} - \frac{\cos(c+}{}$$

Mathematica [B] time = 1.35, size = 258, normalized size = 2.97

$$\frac{168dx\sin\left(\frac{c}{2}\right) - 144\sin\left(\frac{c}{2}+dx\right) + 144\sin\left(\frac{3c}{2}+dx\right) - 48\sin\left(\frac{3c}{2}+2dx\right) - 48\sin\left(\frac{5c}{2}+2dx\right) + 16\sin\left(\frac{5c}{2}+3dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (168*d*x*Cos[c/2] + 144*Cos[c/2 + d*x] + 144*Cos[(3*c)/2 + d*x] - 48*Cos[(3
*c)/2 + 2*d*x] + 48*Cos[(5*c)/2 + 2*d*x] - 16*Cos[(5*c)/2 + 3*d*x] - 16*Cos
[(7*c)/2 + 3*d*x] + 3*Cos[(7*c)/2 + 4*d*x] - 3*Cos[(9*c)/2 + 4*d*x] + 8*Sin
[c/2] + 168*d*x*Sin[c/2] - 144*Sin[c/2 + d*x] + 144*Sin[(3*c)/2 + d*x] - 48
*Sin[(3*c)/2 + 2*d*x] - 48*Sin[(5*c)/2 + 2*d*x] + 16*Sin[(5*c)/2 + 3*d*x] -
```

```
16*Sin[(7*c)/2 + 3*d*x] + 3*Sin[(7*c)/2 + 4*d*x] + 3*Sin[(9*c)/2 + 4*d*x])
/(192*a^2*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.44, size = 58, normalized size = 0.67

$$-\frac{16\cos\left(dx+c\right)^{3}-21\,dx-3\left(2\cos\left(dx+c\right)^{3}-9\cos\left(dx+c\right)\right)\sin\left(dx+c\right)-48\cos\left(dx+c\right)}{24\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/24*(16*cos(d*x + c)^3 - 21*d*x - 3*(2*cos(d*x + c)^3 - 9*cos(d*x + c))*s
in(d*x + c) - 48*cos(d*x + c))/(a^2*d)
```

giac [A] time = 0.17, size = 114, normalized size = 1.31

$$\frac{\frac{21\,(dx+c)}{a^{2}}+\frac{2\left(21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+45\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+96\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-45\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+128\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+32\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}a^{2}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(21*(d*x + c)/a^2 + 2*(21*tan(1/2*d*x + 1/2*c)^7 + 45*tan(1/2*d*x + 1/
2*c)^5 + 96*tan(1/2*d*x + 1/2*c)^4 - 45*tan(1/2*d*x + 1/2*c)^3 + 128*tan(1/
2*d*x + 1/2*c)^2 - 21*tan(1/2*d*x + 1/2*c) + 32)/((tan(1/2*d*x + 1/2*c)^2 +
 1)^4*a^2))/d
```

maple [B] time = 0.36, size = 245, normalized size = 2.82

$$\frac{7\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{2}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{15\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{2}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{8\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{15\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{2}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 7/4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+15/4/a^2/d/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5+8/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^
4*tan(1/2*d*x+1/2*c)^4-15/4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/
2*c)^3+32/3/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2-7/4/a^2/d
```

```
/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)+8/3/a^2/d/(1+tan(1/2*d*x+1/2
*c)^2)^4+7/4/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.47, size = 247, normalized size = 2.84

$$-\frac{\frac{\frac{21\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{128\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{45\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{96\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{45\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{21\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-32}{a^2+\frac{4\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\,a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{21\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) - 128*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 45*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 96*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 45*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 21*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 - 32)/(a^2 + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 6*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^2*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 21*arctan(sin
(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 8.66, size = 79, normalized size = 0.91

$$\frac{7\,x}{8\,a^2}+\frac{2\,\cos\left(c+d\,x\right)}{a^2\,d}-\frac{2\cos\left(c+d\,x\right)^3}{3\,a^2\,d}+\frac{\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,a^2\,d}-\frac{9\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (7*x)/(8*a^2) + (2*cos(c + d*x))/(a^2*d) - (2*cos(c + d*x)^3)/(3*a^2*d) + (
cos(c + d*x)^3*sin(c + d*x))/(4*a^2*d) - (9*cos(c + d*x)*sin(c + d*x))/(8*a
^2*d)
```

sympy [A] time = 59.86, size = 1153, normalized size = 13.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((21*d*x*tan(c/2 + d*x/2)**8/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a
**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(
c/2 + d*x/2)**2 + 24*a**2*d) + 84*d*x*tan(c/2 + d*x/2)**6/(24*a**2*d*tan(c/
```

```
2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)
**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 126*d*x*tan(c/2 + d*x/2)
**4/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a*
*2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 84*
d*x*tan(c/2 + d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2
+ d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**
2 + 24*a**2*d) + 21*d*x/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2
+ d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**
2 + 24*a**2*d) + 42*tan(c/2 + d*x/2)**7/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96
*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*ta
n(c/2 + d*x/2)**2 + 24*a**2*d) + 90*tan(c/2 + d*x/2)**5/(24*a**2*d*tan(c/2
+ d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**
4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 192*tan(c/2 + d*x/2)**4/(2
4*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*t
an(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) - 90*tan(c/
2 + d*x/2)**3/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**
6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**
2*d) + 256*tan(c/2 + d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*t
an(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d
*x/2)**2 + 24*a**2*d) - 42*tan(c/2 + d*x/2)/(24*a**2*d*tan(c/2 + d*x/2)**8
+ 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*
d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 64/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96
*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*ta
n(c/2 + d*x/2)**2 + 24*a**2*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**4/(a*sin(c)
 + a)**2, True))
```

3.424 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{x}{a^2}-\frac{\cos^5(c+dx)}{d(a\sin(c+dx)+a)^2}$$

```
[Out] -x/a^2-2/3*cos(d*x+c)^3/a^2/d-cos(d*x+c)*sin(d*x+c)/a^2/d-cos(d*x+c)^5/d/(a
+a*sin(d*x+c))^2
```

Rubi [A] time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2859, 2682, 2635, 8}

$$-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{x}{a^2}-\frac{\cos^5(c+dx)}{d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*Cos[c + d*x]^3)/(3*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(a^2*
d) - Cos[c + d*x]^5/(d*(a + a*Sin[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= -\frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^2}-\frac{2\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx}{a}\\ &= -\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^2}-\frac{2\int \cos^2(c+dx)\,dx}{a^2}\\ &= -\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{a^2d}-\frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^2}-\frac{\int 1\,dx}{a^2}\\ &= -\frac{x}{a^2}-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{a^2d}-\frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^2}\end{aligned}$$

Mathematica [B] time = 0.81, size = 204, normalized size = 2.91

$$\frac{-24dx\sin\left(\frac{c}{2}\right)+21\sin\left(\frac{c}{2}+dx\right)-21\sin\left(\frac{3c}{2}+dx\right)+6\sin\left(\frac{3c}{2}+2dx\right)+6\sin\left(\frac{5c}{2}+2dx\right)-\sin\left(\frac{5c}{2}+3dx\right)+\sin\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*(1 + 12*d*x)*Cos[c/2] - 21*Cos[c/2 + d*x] - 21*Cos[(3*c)/2 + d*x] + 6*C
os[(3*c)/2 + 2*d*x] - 6*Cos[(5*c)/2 + 2*d*x] + Cos[(5*c)/2 + 3*d*x] + Cos[(
7*c)/2 + 3*d*x] + 2*Sin[c/2] - 24*d*x*Sin[c/2] + 21*Sin[c/2 + d*x] - 21*Sin
[(3*c)/2 + d*x] + 6*Sin[(3*c)/2 + 2*d*x] + 6*Sin[(5*c)/2 + 2*d*x] - Sin[(5*
c)/2 + 3*d*x] + Sin[(7*c)/2 + 3*d*x])/(24*a^2*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.44, size = 43, normalized size = 0.61

$$\frac{\cos\left(dx+c\right)^{3}-3\,dx+3\,\cos\left(dx+c\right)\sin\left(dx+c\right)-6\,\cos\left(dx+c\right)}{3\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 - 3*d*x + 3*cos(d*x + c)*sin(d*x + c) - 6*cos(d*x + c))
/(a^2*d)
```

giac [A] time = 0.16, size = 88, normalized size = 1.26

$$-\frac{\frac{3\,(dx+c)}{a^2}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^2 + 2*(3*tan(1/2*d*x + 1/2*c)^5 + 3*tan(1/2*d*x + 1/2*c
)^4 + 12*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 5)/((tan(1/2*d*x
 + 1/2*c)^2 + 1)^3*a^2))/d
```

maple [B] time = 0.35, size = 177, normalized size = 2.53

$$-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5-2/d/a^2/(1+tan(1/2
*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4-8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*ta
n(1/2*d*x+1/2*c)^2+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)-10
/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.44, size = 184, normalized size = 2.63

$$\frac{2\left(\frac{\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{12\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-5}{a^2+\frac{3\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{3\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/3*((3*sin(d*x + c)/(cos(d*x + c) + 1) - 12*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3*sin(d*x + c)^5/(cos(d*x +
```

```
 c) + 1)^5 - 5)/(a^2 + 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 8.64, size = 55, normalized size = 0.79

$$\frac{\cos(3c+3dx)}{12a^2d}-\frac{7\cos(c+dx)}{4a^2d}-\frac{x}{a^2}+\frac{\sin(2c+2dx)}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] cos(3*c + 3*d*x)/(12*a^2*d) - (7*cos(c + d*x))/(4*a^2*d) - x/a^2 + sin(2*c
+ 2*d*x)/(2*a^2*d)
```

sympy [A] time = 36.06, size = 694, normalized size = 9.91

$$\begin{cases} -\frac{3dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^2d} - \frac{9dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^2d} - \frac{}{3a^2d\tan^6(} \\ \frac{x\sin(c)\cos^4(c)}{(a\sin(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-3*d*x*tan(c/2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**
2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 9*d*x*
tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/
2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 9*d*x*tan(c/2 + d*x/2)**
2/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*t
an(c/2 + d*x/2)**2 + 3*a**2*d) - 3*d*x/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a*
*2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 6*tan
(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)*
*4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 6*tan(c/2 + d*x/2)**4/(3*a*
*2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2
+ d*x/2)**2 + 3*a**2*d) - 24*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)
**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*
d) + 6*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 +
d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 10/(3*a**2*d*tan(c/2
 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2
+ 3*a**2*d), Ne(d, 0)), (x*sin(c)*cos(c)**4/(a*sin(c) + a)**2, True))
```

3.425 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=36

$$-\frac{\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2x}{a^2}$$

```
[Out] -2*x/a^2-arctanh(cos(d*x+c))/a^2/d-cos(d*x+c)/a^2/d
```

Rubi [A] time = 0.13, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2869, 2746, 2735, 3770}

$$-\frac{\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*x)/a^2 - ArcTanh[Cos[c + d*x]]/(a^2*d) - Cos[c + d*x]/(a^2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\ &= -\frac{\cos(c+dx)}{a^2d}+\frac{\int \csc(c+dx)\left(a^2-2a^2\sin(c+dx)\right)\,dx}{a^4}\\ &= -\frac{2x}{a^2}-\frac{\cos(c+dx)}{a^2d}+\frac{\int \csc(c+dx)\,dx}{a^2}\\ &= -\frac{2x}{a^2}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cos(c+dx)}{a^2d}\end{aligned}$$

Mathematica [A] time = 0.13, size = 46, normalized size = 1.28

$$-\frac{\cos(c+dx)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2c+2dx}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2*c + 2*d*x + Cos[c + d*x] + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2
]])/(a^2*d))
```

fricas [A] time = 0.50, size = 45, normalized size = 1.25

$$-\frac{4\,dx+2\,\cos\left(dx+c\right)+\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*d*x + 2*cos(d*x + c) + log(1/2*cos(d*x + c) + 1/2) - log(-1/2*cos(d
*x + c) + 1/2))/(a^2*d)
```

giac [A] time = 0.19, size = 52, normalized size = 1.44

$$-\frac{\frac{2\,(dx+c)}{a^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*(d*x + c)/a^2 - log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 2/((tan(1/2*d*x +
1/2*c)^2 + 1)*a^2))/d
```

maple [A] time = 0.51, size = 60, normalized size = 1.67

$$-\frac{2}{a^{2}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{2}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^2*arctan(tan(1/2*d*x+1/2*c))+1/d/a^
2*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.45, size = 82, normalized size = 2.28

$$-\frac{\frac{2}{a^{2}+\frac{a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}}+\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2/(a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 4*arctan(sin(d*x + c)
/(cos(d*x + c) + 1))/a^2 - log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 8.72, size = 97, normalized size = 2.69

$$\frac{4\,\mathrm{atan}\left(\frac{16}{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8}-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8}\right)}{a^{2}\,d}-\frac{2}{d\,\left(a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+a^{2}\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (4*atan(16/(16*tan(c/2 + (d*x)/2) + 8) - (8*tan(c/2 + (d*x)/2))/(16*tan(c/2
 + (d*x)/2) + 8)))/(a^2*d) - 2/(d*(a^2*tan(c/2 + (d*x)/2)^2 + a^2)) + log(t
an(c/2 + (d*x)/2))/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^4(c+dx)\csc(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

3.426 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=35

$$-\frac{\cot(c+dx)}{a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{x}{a^2}$$

```
[Out] x/a^2+2*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a^2/d
```

Rubi [A] time = 0.15, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 3770, 3767, 8}

$$-\frac{\cot(c+dx)}{a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] x/a^2 + (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\ &= \frac{\int \left(a^2-2a^2\csc(c+dx)+a^2\csc^2(c+dx)\right)\,dx}{a^4}\\ &= \frac{x}{a^2}+\frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\,dx}{a^2}\\ &= \frac{x}{a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\mathrm{Subst}(\int 1\,dx,x,\cot(c+dx))}{a^2d}\\ &= \frac{x}{a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}\end{aligned}$$

Mathematica [B] time = 0.37, size = 98, normalized size = 2.80

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(2(c+dx)+\tan\left(\frac{1}{2}(c+dx)\right)-\cot\left(\frac{1}{2}(c+dx)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\right.}{2d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(2*(c + d*x) - Cot[(c + d*x)/2] +
4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c + d*x)/2]] + Tan[(c + d*x)/2]))/(2*d
*(a + a*Sin[c + d*x])^2)
```

fricas [A] time = 0.46, size = 70, normalized size = 2.00

$$\frac{dx\sin(dx+c)+\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-\cos(dx+c)}{a^2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (d*x*sin(d*x + c) + log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - log(-1/2*cos
(d*x + c) + 1/2)*sin(d*x + c) - cos(d*x + c))/(a^2*d*sin(d*x + c))
```

giac [B] time = 0.19, size = 73, normalized size = 2.09

$$\frac{\frac{2\,(dx+c)}{a^2} - \frac{4\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} + \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2} + \frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/a^2 - 4*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + tan(1/2*d*x +
 1/2*c)/a^2 + (4*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)))/d
```

maple [B] time = 0.54, size = 74, normalized size = 2.11

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2} + \frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2} - \frac{1}{2d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/2/d/a^2/t
an(1/2*d*x+1/2*c)-2/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.45, size = 93, normalized size = 2.66

$$\frac{\frac{4\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} - \frac{4\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} - \frac{\cos(dx+c)+1}{a^2\sin(dx+c)} + \frac{\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/2*(4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 4*log(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2 - (cos(d*x + c) + 1)/(a^2*sin(d*x + c)) + sin(d*x + c)
/(a^2*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 8.83, size = 95, normalized size = 2.71

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{5}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{5\,\cos\left(\frac{c}{2}-\mathrm{atan}\left(\frac{1}{2}\right)+\frac{d\,x}{2}\right)}\right)}{a^2\,d}-\frac{\cot\left(c+d\,x\right)}{a^2\,d}-\frac{2\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] - (2*atan((5^(1/2)*(cos(c/2 + (d*x)/2) - 2*sin(c/2 + (d*x)/2)))/(5*cos(c/2
- atan(1/2) + (d*x)/2))))/(a^2*d) - cot(c + d*x)/(a^2*d) - (2*log(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^{4}{\left(c+dx \right)} \csc^{2}{\left(c+dx \right)}}{\sin^{2}{\left(c+dx \right)}+2\sin{\left(c+dx \right)}+1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

3.427 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=54

$$\frac{2\cot(c+dx)}{a^2d}-\frac{3\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}$$

```
[Out] -3/2*arctanh(cos(d*x+c))/a^2/d+2*cot(d*x+c)/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)
/a^2/d
```

Rubi [A] time = 0.15, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2869, 2757, 3770, 3767, 8, 3768}

$$\frac{2\cot(c+dx)}{a^2d}-\frac{3\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) - (Cot[c +
d*x]*Csc[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\ &= \frac{\int \left(a^2\csc(c+dx)-2a^2\csc^2(c+dx)+a^2\csc^3(c+dx)\right)dx}{a^4}\\ &= \frac{\int \csc(c+dx)\,dx}{a^2}+\frac{\int \csc^3(c+dx)\,dx}{a^2}-\frac{2\int \csc^2(c+dx)\,dx}{a^2}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{\int \csc(c+dx)\,dx}{2a^2}+\frac{2\,\text{Subst}(\\ &= -\frac{3\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}\end{aligned}$$

Mathematica [A] time = 0.53, size = 86, normalized size = 1.59

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(\cot(c+dx)(\csc(c+dx)-4)+3\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+\right.\right.\right.\right.}{2a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*((Cot[c + d*x]*(-4 + Csc[c + d*x]) + 3*(Log[Cos[(c + d*x)/2]] - Log[Si
n[(c + d*x)/2]]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(a^2*d*(1 + Sin[
c + d*x])^2)
```

fricas [A] time = 0.48, size = 93, normalized size = 1.72

$$-\frac{3\left(\cos\left(dx+c\right)^2-1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(\cos\left(dx+c\right)^2-1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+8\cos\left(dx+c\right.}{4\left(a^2d\cos\left(dx+c\right)^2-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/4*(3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^
2 - 1)*log(-1/2*cos(d*x + c) + 1/2) + 8*cos(d*x + c)*sin(d*x + c) - 2*cos(d
*x + c))/(a^2*d*cos(d*x + c)^2 - a^2*d)
```

giac [A] time = 0.22, size = 98, normalized size = 1.81

$$\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}-\frac{18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (a^2*tan(1/2*d*x + 1/2*c)^2 -
8*a^2*tan(1/2*d*x + 1/2*c))/a^4 - (18*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*
x + 1/2*c) + 1)/(a^2*tan(1/2*d*x + 1/2*c)^2))/d
```

maple [A] time = 0.60, size = 93, normalized size = 1.72

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2d}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2}+\frac{1}{d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}-\frac{1}{8a^2d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/8/a^2/d*tan(1/2*d*x+1/2*c)^2-1/d/a^2*tan(1/2*d*x+1/2*c)+1/d/a^2/tan(1/2*d
*x+1/2*c)+3/2/d/a^2*ln(tan(1/2*d*x+1/2*c))-1/8/a^2/d/tan(1/2*d*x+1/2*c)^2
```

maxima [B] time = 0.34, size = 115, normalized size = 2.13

$$-\frac{\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)^2}{a^2\sin(dx+c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*((8*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - (8*sin(d*x + c)/(c
os(d*x + c) + 1) - 1)*(cos(d*x + c) + 1)^2/(a^2*sin(d*x + c)^2))/d
```

mupad [B] time = 8.69, size = 84, normalized size = 1.56

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}+\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{8}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) + (3*log(tan(c/2 + (d*x)/2)))/(2*a^2*d) - ta
n(c/2 + (d*x)/2)/(a^2*d) + (cot(c/2 + (d*x)/2)^2*(tan(c/2 + (d*x)/2) - 1/8)
)/(a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.428 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

```
[Out] arctanh(cos(d*x+c))/a^2/d-2*cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d+cot(d*x
+c)*csc(d*x+c)/a^2/d
```

Rubi [A] time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2708, 2757, 3767, 8, 3768, 3770}

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a^2*d) - (2*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(
3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\
&= \frac{\int \left(a^2\csc^2(c+dx)-2a^2\csc^3(c+dx)+a^2\csc^4(c+dx)\right)\,dx}{a^4}\\
&= \frac{\int \csc^2(c+dx)\,dx}{a^2}+\frac{\int \csc^4(c+dx)\,dx}{a^2}-\frac{2\int \csc^3(c+dx)\,dx}{a^2}\\
&= \frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{\int \csc(c+dx)\,dx}{a^2}-\frac{\operatorname{Subst}(\int 1\,dx,x,\cot(c+dx))}{a^2d}-\frac{\operatorname{Subst}(
\end{aligned}$$

$$= \frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

Mathematica [A] time = 0.91, size = 121, normalized size = 1.83

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)^4\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-9\cos(c+dx)+5\cos(3(c+dx))+6\left(\sin(2(c+dx))+2\right.\right.}{96a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((1 + Cot[(c + d*x)/2])^4*Sec[(c + d*x)/2]^2*(-9*Cos[c + d*x] + 5*Cos[3*(c
+ d*x)] + 6*(2*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]
^3 + Sin[2*(c + d*x)]))*Tan[(c + d*x)/2])/(96*a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.45, size = 123, normalized size = 1.86

$$-\frac{10\,\cos\left(dx+c\right)^{3}-3\,\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\,\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(-\frac{1}{2}\,\mathrm{co}\right.}{6\,\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\right)\sin\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/6*(10*cos(d*x + c)^3 - 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*
x + c) + 6*cos(d*x + c)*sin(d*x + c) - 12*cos(d*x + c))/((a^2*d*cos(d*x + c
)^2 - a^2*d)*sin(d*x + c))
```

giac [A] time = 0.22, size = 128, normalized size = 1.94

$$-\frac{\frac{24\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}}-\frac{44\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}-\frac{a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+21\,a^{4}\tan\left(\right.}{a^{6}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(24*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (44*tan(1/2*d*x + 1/2*c)^3 -
 21*tan(1/2*d*x + 1/2*c)^2 + 6*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x +
 1/2*c)^3) - (a^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^4*tan(1/2*d*x + 1/2*c)^2 + 2
1*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [B] time = 0.61, size = 132, normalized size = 2.00

$$\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^{2}}-\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^{2}d}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{2}}-\frac{7}{8d\,a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{2}}+\frac{1}{4a^{2}d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-\frac{}{24a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/24/a^2/d*tan(1/2*d*x+1/2*c)^3-1/4/a^2/d*tan(1/2*d*x+1/2*c)^2+7/8/d/a^2*ta
n(1/2*d*x+1/2*c)-7/8/d/a^2/tan(1/2*d*x+1/2*c)-1/d/a^2*ln(tan(1/2*d*x+1/2*c)
)+1/4/a^2/d/tan(1/2*d*x+1/2*c)^2-1/24/a^2/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.34, size = 153, normalized size = 2.32

$$\frac{\frac{\frac{21 \sin(dx+c)}{\cos(dx+c)+1} - \frac{6 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{24 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} + \frac{\left(\frac{6 \sin(dx+c)}{\cos(dx+c)+1} - \frac{21 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1\right)(\cos(dx+c)+1)^3}{a^2 \sin(dx+c)^3}}{24\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((21*sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 24*log(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2 + (6*sin(d*x + c)/(cos(d*x + c) + 1) - 21*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a^2*sin(d*x + c)^3))/d
```

mupad [B] time = 8.67, size = 119, normalized size = 1.80

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^2\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\tan\left(\frac{c}{2}+\right.\right.}{8\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^2*d) - tan(c/2 + (d*x)/2)^2/(4*a^2*d) - log(tan(
c/2 + (d*x)/2))/(a^2*d) + (7*tan(c/2 + (d*x)/2))/(8*a^2*d) - (cot(c/2 + (d*
x)/2)^3*(7*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2) + 1/3))/(8*a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.429 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=96

$$\frac{2\cot^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{7\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{7\cot(c+dx)\csc(c+dx)}{8a^2d}$$

```
[Out] -7/8*arctanh(cos(d*x+c))/a^2/d+2*cot(d*x+c)/a^2/d+2/3*cot(d*x+c)^3/a^2/d-7/
8*cot(d*x+c)*csc(d*x+c)/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.19, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2869, 2757, 3768, 3770, 3767}

$$\frac{2\cot^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{7\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{7\cot(c+dx)\csc(c+dx)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*ArcTanh[Cos[c + d*x]])/(8*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (2*Cot[c
+ d*x]^3)/(3*a^2*d) - (7*Cot[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^2*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(a^2\csc^3(c+dx)-2a^2\csc^4(c+dx)+a^2\csc^5(c+dx)\right)dx}{a^4} \\
&= \frac{\int \csc^3(c+dx)\,dx}{a^2}+\frac{\int \csc^5(c+dx)\,dx}{a^2}-\frac{2\int \csc^4(c+dx)\,dx}{a^2} \\
&= -\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{\int \csc(c+dx)\,dx}{2a^2}+\frac{3\int \mathrm{cs}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{7\cot(c+dx)\csc(c+}{8a^2d} \\
&= -\frac{7\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{7\cot(c+dx)\csc(c-}{8a^2d}
\end{aligned}$$

Mathematica [A] time = 1.52, size = 116, normalized size = 1.21

$$-\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^4\left(-48\sin(2(c+dx))+45\cos(c+dx)+(32\sin(c+dx)-21)\cos(3(c+dx)\right.}{1536a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/1536*((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^4*(45*Cos[c + d*x] + 84*(Log
[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 + Cos[3*(c + d*x
)]*(-21 + 32*Sin[c + d*x]) - 48*Sin[2*(c + d*x)]))/(a^2*d*(1 + Sin[c + d*x]
)^2)
```

fricas [A] time = 0.48, size = 149, normalized size = 1.55

$$\frac{42\,\cos\left(dx+c\right)^{3}-21\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+21\left(\cos\left(dx+c\right)^{4}-2\,\cos\right.}{48\left(a^{2}d\cos\left(dx+c\right)^{4}-2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/48*(42*cos(d*x + c)^3 - 21*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/
2*cos(d*x + c) + 1/2) + 21*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2
*cos(d*x + c) + 1/2) - 32*(2*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c)
- 54*cos(d*x + c))/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)
```

giac [A] time = 0.24, size = 157, normalized size = 1.64

$$\frac{\frac{168\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}}-\frac{350\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-144\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}}+\frac{3\,a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-16\,a^{6}\,\mathrm{ta}}{}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(168*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (350*tan(1/2*d*x + 1/2*c)^4
 - 144*tan(1/2*d*x + 1/2*c)^3 + 48*tan(1/2*d*x + 1/2*c)^2 - 16*tan(1/2*d*x
+ 1/2*c) + 3)/(a^2*tan(1/2*d*x + 1/2*c)^4) + (3*a^6*tan(1/2*d*x + 1/2*c)^4
- 16*a^6*tan(1/2*d*x + 1/2*c)^3 + 48*a^6*tan(1/2*d*x + 1/2*c)^2 - 144*a^6*t
an(1/2*d*x + 1/2*c))/a^8)/d
```

maple [A] time = 0.60, size = 170, normalized size = 1.77

$$\frac{\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a^{2}d}-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{12d\,a^{2}}+\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^{2}d}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{2}}+\frac{3}{4d\,a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{2}}-\frac{}{4a^{2}d\,\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/64/a^2/d*tan(1/2*d*x+1/2*c)^4-1/12/a^2/d*tan(1/2*d*x+1/2*c)^3+1/4/a^2/d*t
an(1/2*d*x+1/2*c)^2-3/4/d/a^2*tan(1/2*d*x+1/2*c)+3/4/d/a^2/tan(1/2*d*x+1/2*
```

```
c)+7/8/d/a^2*ln(tan(1/2*d*x+1/2*c))-1/4/a^2/d/tan(1/2*d*x+1/2*c)^2-1/64/a^2
/d/tan(1/2*d*x+1/2*c)^4+1/12/a^2/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.35, size = 195, normalized size = 2.03

$$-\frac{\frac{\frac{144\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{16\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^2}-\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{16\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{144\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-3\right)(\cos(dx+c)+1}{a^2\sin(dx+c)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/192*((144*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4)/a^2 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - (16*
sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
144*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3)*(cos(d*x + c) + 1)^4/(a^2*sin(
d*x + c)^4))/d
```

mupad [B] time = 8.72, size = 151, normalized size = 1.57

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^2\,d}+\frac{7\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^2\,d}-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^2\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(12\tan\left(\frac{c}{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(4*a^2*d) - tan(c/2 + (d*x)/2)^3/(12*a^2*d) + tan(c/2
+ (d*x)/2)^4/(64*a^2*d) + (7*log(tan(c/2 + (d*x)/2)))/(8*a^2*d) - (3*tan(c/
2 + (d*x)/2))/(4*a^2*d) + (cot(c/2 + (d*x)/2)^4*((4*tan(c/2 + (d*x)/2))/3 -
 4*tan(c/2 + (d*x)/2)^2 + 12*tan(c/2 + (d*x)/2)^3 - 1/4))/(16*a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.430 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=112

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}+\frac{3\cot(c+dx)\csc}{4a^2d}$$

```
[Out] 3/4*arctanh(cos(d*x+c))/a^2/d-2*cot(d*x+c)/a^2/d-cot(d*x+c)^3/a^2/d-1/5*cot
(d*x+c)^5/a^2/d+3/4*cot(d*x+c)*csc(d*x+c)/a^2/d+1/2*cot(d*x+c)*csc(d*x+c)^3
/a^2/d
```

Rubi [A] time = 0.24, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2869, 2757, 3767, 3768, 3770}

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}+\frac{3\cot(c+dx)\csc}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(4*a^2*d) - (2*Cot[c + d*x])/(a^2*d) - Cot[c + d*
x]^3/(a^2*d) - Cot[c + d*x]^5/(5*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(4*
a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(2*a^2*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\csc^4(c+dx)-2a^2\csc^5(c+dx)+a^2\csc^6(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \csc^4(c+dx)\,dx}{a^2}+\frac{\int \csc^6(c+dx)\,dx}{a^2}-\frac{2\int \csc^5(c+dx)\,dx}{a^2}$$
$$= \frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}-\frac{3\int \csc^3(c+dx)\,dx}{2a^2}-\frac{\text{Subst}\left(\int \left(1+x^2\right)dx,x,\cot(c$$
$$= -\frac{2\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{3\cot(c+dx)\csc(c+dx)}{4a^2d}+\text{co}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}-\frac{2\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{3\cot(c$$

Mathematica [A] time = 0.75, size = 189, normalized size = 1.69

$$\frac{\csc^5(c+dx)\left(140\sin(2(c+dx))-30\sin(4(c+dx))-160\cos(c+dx)+120\cos(3(c+dx))-24\cos(5(c+dx))-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^5*(-160*Cos[c + d*x] + 120*Cos[3*(c + d*x)] - 24*Cos[5*(c + d
*x)] + 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 150*Log[Sin[(c + d*x)/2]]*S
```

```
in[c + d*x] + 140*Sin[2*(c + d*x)] - 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*
x)] + 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 30*Sin[4*(c + d*x)] + 15*
Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin[5*(c
+ d*x)]))/(320*a^2*d)
```

fricas [A] time = 0.45, size = 179, normalized size = 1.60

$$-\frac{48\cos(dx+c)^5-120\cos(dx+c)^3-15\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx}{40\left(a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/40*(48*cos(d*x + c)^5 - 120*cos(d*x + c)^3 - 15*(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)
^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 10*(
3*cos(d*x + c)^3 - 5*cos(d*x + c))*sin(d*x + c) + 80*cos(d*x + c))/((a^2*d*
cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

giac [A] time = 0.23, size = 186, normalized size = 1.66

$$-\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{274\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-110\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}-\frac{a^8\tan(}{}}{160d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/160*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (274*tan(1/2*d*x + 1/2*c)^
5 - 110*tan(1/2*d*x + 1/2*c)^4 + 40*tan(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x
 + 1/2*c)^2 + 5*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)^5) - (a
^8*tan(1/2*d*x + 1/2*c)^5 - 5*a^8*tan(1/2*d*x + 1/2*c)^4 + 15*a^8*tan(1/2*d
*x + 1/2*c)^3 - 40*a^8*tan(1/2*d*x + 1/2*c)^2 + 110*a^8*tan(1/2*d*x + 1/2*c
))/a^10)/d
```

maple [A] time = 0.64, size = 208, normalized size = 1.86

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{160a^2d}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{32a^2d}+\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d\,a^2}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2d}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^2}-\frac{11}{16d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{3\ln\left(\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x)

[Out] 1/160/a^2/d*tan(1/2*d*x+1/2*c)^5-1/32/a^2/d*tan(1/2*d*x+1/2*c)^4+3/32/a^2/d*tan(1/2*d*x+1/2*c)^3-1/4/a^2/d*tan(1/2*d*x+1/2*c)^2+11/16/d/a^2*tan(1/2*d*x+1/2*c)-11/16/d/a^2/tan(1/2*d*x+1/2*c)-3/4/d/a^2*ln(tan(1/2*d*x+1/2*c))-1/160/a^2/d/tan(1/2*d*x+1/2*c)^5+1/4/a^2/d/tan(1/2*d*x+1/2*c)^2+1/32/a^2/d/tan(1/2*d*x+1/2*c)^4-3/32/a^2/d/tan(1/2*d*x+1/2*c)^3

maxima [B] time = 0.37, size = 233, normalized size = 2.08

$$\frac{\frac{\frac{110 \sin(dx+c)}{\cos(dx+c)+1}-\frac{40 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}-\frac{120 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{5 \sin(dx+c)}{\cos(dx+c)+1}-\frac{15 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{40 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{11}{(c}\right.}{a^2 \sin(dx+c)^5}}{160\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/160*((110*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + (5*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 110*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1)*(cos(d*x + c) + 1)^5/(a^2*sin(d*x + c)^5))/d

mupad [B] time = 9.30, size = 289, normalized size = 2.58

$$-\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}-5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-15\cos\left(\frac{c}{2}+\frac{d}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(sin(c + d*x)^6*(a + a*sin(c + d*x))^2),x)

[Out] -(cos(c/2 + (d*x)/2)^10 - sin(c/2 + (d*x)/2)^10 + 5*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^9 - 5*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) - 15*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 40*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^7 - 110*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 + 110*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 - 40*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3 + 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 120*log(sin(c/2 + (d*x)/2)/c

```
os(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)/(160*a^2*d*co
s(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.431 $\int \frac{\cot^4(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{2\cot^5(c+dx)}{5a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{11\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}-\frac{11\cot(c+dx)\,c}{24a^2}$$

```
[Out] -11/16*arctanh(cos(d*x+c))/a^2/d+2*cot(d*x+c)/a^2/d+4/3*cot(d*x+c)^3/a^2/d+
2/5*cot(d*x+c)^5/a^2/d-11/16*cot(d*x+c)*csc(d*x+c)/a^2/d-11/24*cot(d*x+c)*c
sc(d*x+c)^3/a^2/d-1/6*cot(d*x+c)*csc(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.25, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 3768, 3770, 3767}

$$\frac{2\cot^5(c+dx)}{5a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{11\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}-\frac{11\cot(c+dx)\,c}{24a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(16*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (4*Cot[
c + d*x]^3)/(3*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) - (11*Cot[c + d*x]*Csc
[c + d*x])/(16*a^2*d) - (11*Cot[c + d*x]*Csc[c + d*x]^3)/(24*a^2*d) - (Cot[
c + d*x]*Csc[c + d*x]^5)/(6*a^2*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc^7(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(a^2\csc^5(c+dx)-2a^2\csc^6(c+dx)+a^2\csc^7(c+dx)\right)\,dx}{a^4} \\
&= \frac{\int \csc^5(c+dx)\,dx}{a^2}+\frac{\int \csc^7(c+dx)\,dx}{a^2}-\frac{2\int \csc^6(c+dx)\,dx}{a^2} \\
&= -\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}+\frac{3\int \csc^3(c+dx)\,dx}{4a^2}+ \\
&= \frac{2\cot(c+dx)}{a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8a^2d}- \\
&= -\frac{3\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}- \\
&= -\frac{11\tanh^{-1}(\cos(c+dx))}{16a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-
\end{aligned}$$

Mathematica [A] time = 0.83, size = 229, normalized size = 1.66

$$\frac{\csc^6(c+dx)\left(3840\sin(2(c+dx))-1536\sin(4(c+dx))+256\sin(6(c+dx))-2820\cos(c+dx)+1870\cos(3(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^6*(-2820*Cos[c + d*x] + 1870*Cos[3*(c + d*x)] - 330*Cos[5*(c
+ d*x)] - 1650*Log[Cos[(c + d*x)/2]] + 2475*Cos[2*(c + d*x)]*Log[Cos[(c + d
*x)/2]] - 990*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 165*Cos[6*(c + d*x)]
*Log[Cos[(c + d*x)/2]] + 1650*Log[Sin[(c + d*x)/2]] - 2475*Cos[2*(c + d*x)]
*Log[Sin[(c + d*x)/2]] + 990*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 165*C
os[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 3840*Sin[2*(c + d*x)] - 1536*Sin[4*
(c + d*x)] + 256*Sin[6*(c + d*x)]))/(7680*a^2*d)
```

fricas [A] time = 0.45, size = 204, normalized size = 1.48

$$\frac{330\,\cos\left(dx+c\right)^{5}-880\,\cos\left(dx+c\right)^{3}-165\left(\cos\left(dx+c\right)^{6}-3\,\cos\left(dx+c\right)^{4}+3\,\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\,\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/480*(330*cos(d*x + c)^5 - 880*cos(d*x + c)^3 - 165*(cos(d*x + c)^6 - 3*co
s(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 165*(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c
) + 1/2) - 64*(8*cos(d*x + c)^5 - 20*cos(d*x + c)^3 + 15*cos(d*x + c))*sin(
d*x + c) + 630*cos(d*x + c))/(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4
 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)
```

giac [A] time = 0.27, size = 215, normalized size = 1.56

$$\frac{\frac{1320\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}}-\frac{3234\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-1200\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+465\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-200\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+75\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-24\,\tan\left(\right.}{a^{2}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(1320*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (3234*tan(1/2*d*x + 1/2*c
)^6 - 1200*tan(1/2*d*x + 1/2*c)^5 + 465*tan(1/2*d*x + 1/2*c)^4 - 200*tan(1/
2*d*x + 1/2*c)^3 + 75*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 5)
/(a^2*tan(1/2*d*x + 1/2*c)^6) + (5*a^10*tan(1/2*d*x + 1/2*c)^6 - 24*a^10*ta
n(1/2*d*x + 1/2*c)^5 + 75*a^10*tan(1/2*d*x + 1/2*c)^4 - 200*a^10*tan(1/2*d*
x + 1/2*c)^3 + 465*a^10*tan(1/2*d*x + 1/2*c)^2 - 1200*a^10*tan(1/2*d*x + 1/
2*c))/a^12)/d
```

maple [A] time = 0.66, size = 246, normalized size = 1.78

$$\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{384a^2d}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{80a^2d}+\frac{5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128a^2d}-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48d\,a^2}+\frac{31\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128a^2d}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^2}-\frac{}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/384/a^2/d*tan(1/2*d*x+1/2*c)^6-1/80/a^2/d*tan(1/2*d*x+1/2*c)^5+5/128/a^2/
d*tan(1/2*d*x+1/2*c)^4-5/48/a^2/d*tan(1/2*d*x+1/2*c)^3+31/128/a^2/d*tan(1/2
*d*x+1/2*c)^2-5/8/d/a^2*tan(1/2*d*x+1/2*c)-1/384/a^2/d/tan(1/2*d*x+1/2*c)^6
+5/8/d/a^2/tan(1/2*d*x+1/2*c)+11/16/d/a^2*ln(tan(1/2*d*x+1/2*c))+1/80/a^2/d
/tan(1/2*d*x+1/2*c)^5-31/128/a^2/d/tan(1/2*d*x+1/2*c)^2-5/128/a^2/d/tan(1/2
*d*x+1/2*c)^4+5/48/a^2/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.34, size = 275, normalized size = 1.99

$$-\frac{\frac{\frac{1200\sin(dx+c)}{\cos(dx+c)+1}-\frac{465\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{200\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{75\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{24\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a^2}-\frac{1320\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\left(\frac{24\sin(dx+c)}{\cos(dx+c)+1}-\frac{75\sin(dx+c)}{(\cos(dx+c)+1}\right.}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/1920*((1200*sin(d*x + c)/(cos(d*x + c) + 1) - 465*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 75*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6)/a^2 - 1320*log(sin(d*x + c)/(cos(d*x + c) + 1)
)/a^2 - (24*sin(d*x + c)/(cos(d*x + c) + 1) - 75*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 465*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 1200*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x
 + c) + 1)^6/(a^2*sin(d*x + c)^6))/d
```

mupad [B] time = 9.74, size = 339, normalized size = 2.46

$$\frac{5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+75\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^7*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (5*sin(c/2 + (d*x)/2)^12 - 5*cos(c/2 + (d*x)/2)^12 - 24*cos(c/2 + (d*x)/2)*
sin(c/2 + (d*x)/2)^11 + 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) + 75*co
s(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 200*cos(c/2 + (d*x)/2)^3*sin(c/2
 + (d*x)/2)^9 + 465*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 - 1200*cos(c/
2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 1200*cos(c/2 + (d*x)/2)^7*sin(c/2 + (
d*x)/2)^5 - 465*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 200*cos(c/2 + (
d*x)/2)^9*sin(c/2 + (d*x)/2)^3 - 75*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2
)^2 + 1320*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*
sin(c/2 + (d*x)/2)^6)/(1920*a^2*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6
)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.432 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=109

$$-\frac{\cos^3(c+dx)}{a^3d}+\frac{7\cos(c+dx)}{a^3d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^3d}-\frac{19\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}+\frac{51x}{8a^3}$$

```
[Out] 51/8*x/a^3+7*cos(d*x+c)/a^3/d-cos(d*x+c)^3/a^3/d-19/8*cos(d*x+c)*sin(d*x+c)
/a^3/d-1/4*cos(d*x+c)*sin(d*x+c)^3/a^3/d+4*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.26, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2875, 2872, 2638, 2635, 8, 2633, 2648}

$$-\frac{\cos^3(c+dx)}{a^3d}+\frac{7\cos(c+dx)}{a^3d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^3d}-\frac{19\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}+\frac{51x}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (51*x)/(8*a^3) + (7*Cos[c + d*x])/(a^3*d) - Cos[c + d*x]^3/(a^3*d) - (19*Co
s[c + d*x]*Sin[c + d*x])/(8*a^3*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a^3*d
) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sin(c+dx)(a-a\sin(c+dx))^3\tan^2(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(4a-4a\sin(c+dx)+4a\sin^2(c+dx)-3a\sin^3(c+dx)+a\sin^4(c+dx)-\right.}{a^4}$$
$$= \frac{4x}{a^3}+\frac{\int \sin^4(c+dx)\,dx}{a^3}-\frac{3\int \sin^3(c+dx)\,dx}{a^3}-\frac{4\int \sin(c+dx)\,dx}{a^3}+\frac{4\int \sin^{}}{}$$
$$= \frac{4x}{a^3}+\frac{4\cos(c+dx)}{a^3d}-\frac{2\cos(c+dx)\sin(c+dx)}{a^3d}-\frac{\cos(c+dx)\sin^3(c+dx)}{4a^3d}+$$
$$= \frac{6x}{a^3}+\frac{7\cos(c+dx)}{a^3d}-\frac{\cos^3(c+dx)}{a^3d}-\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}-\frac{\cos(c+dx}{}$$
$$= \frac{51x}{8a^3}+\frac{7\cos(c+dx)}{a^3d}-\frac{\cos^3(c+dx)}{a^3d}-\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}-\frac{\cos(c+}{}$$

Mathematica [A] time = 1.34, size = 195, normalized size = 1.79

$$\frac{2040dx\sin\left(c+\frac{dx}{2}\right)+800\sin\left(2c+\frac{3dx}{2}\right)-160\sin\left(2c+\frac{5dx}{2}\right)-35\sin\left(4c+\frac{7dx}{2}\right)+5\sin\left(4c+\frac{9dx}{2}\right)+997\cos\left(\right.}{320a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2040*d*x*Cos[(d*x)/2] + 997*Cos[c + (d*x)/2] + 800*Cos[c + (3*d*x)/2] + 16
0*Cos[3*c + (5*d*x)/2] - 35*Cos[3*c + (7*d*x)/2] - 5*Cos[5*c + (9*d*x)/2] -
 3563*Sin[(d*x)/2] + 2040*d*x*Sin[c + (d*x)/2] + 800*Sin[2*c + (3*d*x)/2] -
 160*Sin[2*c + (5*d*x)/2] - 35*Sin[4*c + (7*d*x)/2] + 5*Sin[4*c + (9*d*x)/2
])/(320*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.44, size = 144, normalized size = 1.32

$$-\frac{2\cos(dx+c)^5+8\cos(dx+c)^4-15\cos(dx+c)^3-51\,dx-(51\,dx+67)\cos(dx+c)-56\cos(dx+c)^2-\left(2\right.}{8\left(a^3d\cos(dx+c)+a^3d\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/8*(2*cos(d*x + c)^5 + 8*cos(d*x + c)^4 - 15*cos(d*x + c)^3 - 51*d*x - (5
1*d*x + 67)*cos(d*x + c) - 56*cos(d*x + c)^2 - (2*cos(d*x + c)^4 - 6*cos(d*
x + c)^3 + 51*d*x - 21*cos(d*x + c)^2 + 35*cos(d*x + c) - 32)*sin(d*x + c)
- 32)/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

giac [A] time = 0.24, size = 145, normalized size = 1.33

$$\frac{\frac{51\,(dx+c)}{a^3} + \frac{64}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)} + \frac{2\left(19\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+32\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+27\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+144\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-27\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+16\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4 a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(51*(d*x + c)/a^3 + 64/(a^3*(tan(1/2*d*x + 1/2*c) + 1)) + 2*(19*tan(1/2
*d*x + 1/2*c)^7 + 32*tan(1/2*d*x + 1/2*c)^6 + 27*tan(1/2*d*x + 1/2*c)^5 + 1
44*tan(1/2*d*x + 1/2*c)^4 - 27*tan(1/2*d*x + 1/2*c)^3 + 160*tan(1/2*d*x + 1
/2*c)^2 - 19*tan(1/2*d*x + 1/2*c) + 48)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3
))/d
```

maple [B] time = 0.42, size = 300, normalized size = 2.75

$$\frac{19\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{8\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{27\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{36\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 19/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+8/d/a^3/(1+tan(1
/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6+27/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)
^4*tan(1/2*d*x+1/2*c)^5+36/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2
*c)^4-27/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3+40/d/a^3/(
1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2-19/4/d/a^3/(1+tan(1/2*d*x+1/
2*c)^2)^4*tan(1/2*d*x+1/2*c)+12/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4+51/4/d/a^3
*arctan(tan(1/2*d*x+1/2*c))+8/d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.43, size = 398, normalized size = 3.65

$$\frac{\frac{\frac{29\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{269\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{133\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{309\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{171\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{187\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{51\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{51\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+80}{a^3+\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{4\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6\,a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{4\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{4\,a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^3\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}+\frac{51\,\mathrm{arcta}\cdots}{\cdots}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*((29*sin(d*x + c)/(cos(d*x + c) + 1) + 269*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 133*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 309*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 171*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 187*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 51*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 51*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 + 80)/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c
) + 1) + 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^3*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 6*a^3*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + 4*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a^3*sin(d*x + c)^8/(cos(d*x
 + c) + 1)^8 + a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9) + 51*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 12.35, size = 146, normalized size = 1.34

$$\frac{51\,x}{8\,a^3}+\frac{\frac{51\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{4}+\frac{51\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{187\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{4}+\frac{171\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{309\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{4}+\frac{133\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{269\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}+\cdots}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (51*x)/(8*a^3) + ((29*tan(c/2 + (d*x)/2))/4 + (269*tan(c/2 + (d*x)/2)^2)/4
+ (133*tan(c/2 + (d*x)/2)^3)/4 + (309*tan(c/2 + (d*x)/2)^4)/4 + (171*tan(c/
2 + (d*x)/2)^5)/4 + (187*tan(c/2 + (d*x)/2)^6)/4 + (51*tan(c/2 + (d*x)/2)^7
)/4 + (51*tan(c/2 + (d*x)/2)^8)/4 + 20)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)*(ta
n(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [A] time = 147.46, size = 3578, normalized size = 32.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((51*d*x*tan(c/2 + d*x/2)**9/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**
3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2
 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**
4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*
```

d*tan(c/2 + d*x/2) + 8*a**3*d) + 51*d*x*tan(c/2 + d*x/2)**8/(8*a**3*d*tan(c
/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)*
*7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**
3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2
 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 204*d*x*tan(c/2 + d*
x/2)**7/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a
3*d*tan(c/2 + d*x/2)7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c
/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)
3 + 32*a3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d)
 + 204*d*x*tan(c/2 + d*x/2)**6/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan
(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/
2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*
a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c
/2 + d*x/2) + 8*a**3*d) + 306*d*x*tan(c/2 + d*x/2)**5/(8*a**3*d*tan(c/2 + d
*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 3
2*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*ta
n(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x
/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 306*d*x*tan(c/2 + d*x/2)**
4/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*
tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d
*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 +
32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 204
*d*x*tan(c/2 + d*x/2)**3/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 +
 d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6
+ 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d
*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d
*x/2) + 8*a**3*d) + 204*d*x*tan(c/2 + d*x/2)**2/(8*a**3*d*tan(c/2 + d*x/2)*
*9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3
*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2
+ d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2
 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 51*d*x*tan(c/2 + d*x/2)/(8*a**3*
d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 +
d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 +
 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*
tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 51*d*x/(8*a**
3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2
+ d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5
 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*
d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 102*tan(c/2
 + d*x/2)**8/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 +
 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*
tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d
*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a*
*3*d) + 102*tan(c/2 + d*x/2)**7/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*ta
n(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x

```
/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32
*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(
c/2 + d*x/2) + 8*a**3*d) + 374*tan(c/2 + d*x/2)**6/(8*a**3*d*tan(c/2 + d*x/
2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a
**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c
/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)
**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 342*tan(c/2 + d*x/2)**5/(8*a*
*3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2
 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**
5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3
*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 618*tan(c/
2 + d*x/2)**4/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8
+ 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d
*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 +
d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a
**3*d) + 266*tan(c/2 + d*x/2)**3/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*t
an(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*
x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 3
2*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan
(c/2 + d*x/2) + 8*a**3*d) + 538*tan(c/2 + d*x/2)**2/(8*a**3*d*tan(c/2 + d*x
/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*
a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(
c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2
)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 58*tan(c/2 + d*x/2)/(8*a**3*
d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 +
d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 +
 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*
tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 160/(8*a**3*d
*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d
*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 +
48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*t
an(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d), Ne(d, 0)), (x*s
in(c)**3*cos(c)**4/(a*sin(c) + a)**3, True))
```

3.433 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=87

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{5\cos(c+dx)}{a^3d}+\frac{3\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{11x}{2a^3}$$

```
[Out] -11/2*x/a^3-5*cos(d*x+c)/a^3/d+1/3*cos(d*x+c)^3/a^3/d+3/2*cos(d*x+c)*sin(d*
x+c)/a^3/d-4*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.23, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2709, 2638, 2635, 8, 2633, 2648}

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{5\cos(c+dx)}{a^3d}+\frac{3\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{11x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*x)/(2*a^3) - (5*Cos[c + d*x])/(a^3*d) + Cos[c + d*x]^3/(3*a^3*d) + (3*
Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c +
 d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int (a-a\sin(c+dx))^3\tan^2(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(-4a+4a\sin(c+dx)-3a\sin^2(c+dx)+a\sin^3(c+dx)+\frac{4a}{1+\sin(c+dx)}\right)dx}{a^4}$$

$$= -\frac{4x}{a^3}+\frac{\int \sin^3(c+dx)\,dx}{a^3}-\frac{3\int \sin^2(c+dx)\,dx}{a^3}+\frac{4\int \sin(c+dx)\,dx}{a^3}+\frac{4\int \frac{1}{1+\cdots}}{\cdots}$$

$$= -\frac{4x}{a^3}-\frac{4\cos(c+dx)}{a^3d}+\frac{3\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}-\frac{3\cdots}{\cdots}$$

$$= -\frac{11x}{2a^3}-\frac{5\cos(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3a^3d}+\frac{3\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{4\cos\cdots}{a^3d(1+\cdots}$$

Mathematica [B] time = 1.14, size = 181, normalized size = 2.08

$$\frac{-660dx\sin\left(c+\frac{dx}{2}\right)+\sin\left(c+\frac{dx}{2}\right)-240\sin\left(2c+\frac{3dx}{2}\right)+40\sin\left(2c+\frac{5dx}{2}\right)+5\sin\left(4c+\frac{7dx}{2}\right)-286\cos\left(c+\frac{dx}{2}\right)}{120a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((1 - 660*d*x)*Cos[(d*x)/2] - 286*Cos[c + (d*x)/2] - 240*Cos[c + (3*d*x)/2]
 - 40*Cos[3*c + (5*d*x)/2] + 5*Cos[3*c + (7*d*x)/2] + 1244*Sin[(d*x)/2] + S
in[c + (d*x)/2] - 660*d*x*Sin[c + (d*x)/2] - 240*Sin[2*c + (3*d*x)/2] + 40*
Sin[2*c + (5*d*x)/2] + 5*Sin[4*c + (7*d*x)/2])/(120*a^3*d*(Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.43, size = 123, normalized size = 1.41

$$\frac{2\cos(dx+c)^4-7\cos(dx+c)^3-33dx-3(11dx+15)\cos(dx+c)-30\cos(dx+c)^2+\left(2\cos(dx+c)^3-33\right.}{6\left(a^3d\cos(dx+c)+a^3d\sin(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*cos(d*x + c)^4 - 7*cos(d*x + c)^3 - 33*d*x - 3*(11*d*x + 15)*cos(d*x
 + c) - 30*cos(d*x + c)^2 + (2*cos(d*x + c)^3 - 33*d*x + 9*cos(d*x + c)^2 -
 21*cos(d*x + c) + 24)*sin(d*x + c) - 24)/(a^3*d*cos(d*x + c) + a^3*d*sin(d
*x + c) + a^3*d)
```

giac [A] time = 0.23, size = 106, normalized size = 1.22

$$-\frac{\frac{33(dx+c)}{a^3}+\frac{48}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{2\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+60\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+28\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3a^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(33*(d*x + c)/a^3 + 48/(a^3*(tan(1/2*d*x + 1/2*c) + 1)) + 2*(9*tan(1/2
*d*x + 1/2*c)^5 + 24*tan(1/2*d*x + 1/2*c)^4 + 60*tan(1/2*d*x + 1/2*c)^2 - 9
*tan(1/2*d*x + 1/2*c) + 28)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^3))/d
```

maple [B] time = 0.38, size = 198, normalized size = 2.28

$$-\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{8\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{20\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5-8/d/a^3/(1+tan(1/2
*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4-20/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*t
an(1/2*d*x+1/2*c)^2+3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)-2
8/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3-11/d/a^3*arctan(tan(1/2*d*x+1/2*c))-8/
d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.42, size = 312, normalized size = 3.59

$$-\frac{\frac{\frac{19\sin(dx+c)}{\cos(dx+c)+1}+\frac{123\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{60\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{96\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{33\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{33\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+52}{a^{3}+\frac{a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{3a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}+\frac{33\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/3*((19*sin(d*x + c)/(cos(d*x + c) + 1) + 123*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 60*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 96*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 + 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 33*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 + 52)/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 3
*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*a^3*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 + a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(
d*x + c)^7/(cos(d*x + c) + 1)^7) + 33*arctan(sin(d*x + c)/(cos(d*x + c) + 1
))/a^3)/d
```

mupad [B] time = 12.35, size = 121, normalized size = 1.39

$$-\frac{11\,x}{2\,a^{3}}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+41\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+\frac{19\tan\left(\frac{c}{2}\right.}{3}}{a^{3}\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (11*x)/(2*a^3) - ((19*tan(c/2 + (d*x)/2))/3 + 41*tan(c/2 + (d*x)/2)^2 + 2
0*tan(c/2 + (d*x)/2)^3 + 32*tan(c/2 + (d*x)/2)^4 + 11*tan(c/2 + (d*x)/2)^5
+ 11*tan(c/2 + (d*x)/2)^6 + 52/3)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)*(tan(c/2
+ (d*x)/2)^2 + 1)^3)
```

sympy [A] time = 95.06, size = 2264, normalized size = 26.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-33*d*x*tan(c/2 + d*x/2)**7/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a*
*3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/
2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)*
*2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 33*d*x*tan(c/2 + d*x/2)**6/(6*
a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c
/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)
**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d)
 - 99*d*x*tan(c/2 + d*x/2)**5/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(
c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2
)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a*
*3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 99*d*x*tan(c/2 + d*x/2)**4/(6*a**3*d*ta
n(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/
2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*
a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 99*d*x
*tan(c/2 + d*x/2)**3/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x
/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18
*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(
c/2 + d*x/2) + 6*a**3*d) - 99*d*x*tan(c/2 + d*x/2)**2/(6*a**3*d*tan(c/2 + d
*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 1
8*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*ta
n(c/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 33*d*x*tan(c/2
+ d*x/2)/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*
a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(
c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2)
 + 6*a**3*d) - 33*d*x/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*
x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 1
8*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan
(c/2 + d*x/2) + 6*a**3*d) - 66*tan(c/2 + d*x/2)**6/(6*a**3*d*tan(c/2 + d*x/
2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a
```

```
**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c
/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 66*tan(c/2 + d*x/2
)**5/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3
*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2
+ d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6
*a**3*d) - 192*tan(c/2 + d*x/2)**4/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d
*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 +
d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 +
 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 120*tan(c/2 + d*x/2)**3/(6*a**3*d*
tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*
x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 1
8*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 246*
tan(c/2 + d*x/2)**2/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/
2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*
a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(c
/2 + d*x/2) + 6*a**3*d) - 38*tan(c/2 + d*x/2)/(6*a**3*d*tan(c/2 + d*x/2)**7
 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d
*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 +
d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 104/(6*a**3*d*tan(c/2 +
 d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 +
 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*
tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d), Ne(d, 0)), (x*
sin(c)**2*cos(c)**4/(a*sin(c) + a)**3, True))
```

3.434 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=80

$$\frac{9\cos(c+dx)}{2a^3d}+\frac{3\cos^3(c+dx)}{2d\left(a^3\sin(c+dx)+a^3\right)}+\frac{9x}{2a^3}+\frac{\cos^5(c+dx)}{d(a\sin(c+dx)+a)^3}$$

```
[Out] 9/2*x/a^3+9/2*cos(d*x+c)/a^3/d+cos(d*x+c)^5/d/(a+a*sin(d*x+c))^3+3/2*cos(d*
x+c)^3/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2859, 2679, 2682, 8}

$$\frac{9\cos(c+dx)}{2a^3d}+\frac{3\cos^3(c+dx)}{2d\left(a^3\sin(c+dx)+a^3\right)}+\frac{9x}{2a^3}+\frac{\cos^5(c+dx)}{d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (9*x)/(2*a^3) + (9*Cos[c + d*x])/(2*a^3*d) + Cos[c + d*x]^5/(d*(a + a*Sin[c
 + d*x])^3) + (3*Cos[c + d*x]^3)/(2*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^3}+\frac{3\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{a}\\
&= \frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^3}+\frac{3\cos^3(c+dx)}{2d\left(a^3+a^3\sin(c+dx)\right)}+\frac{9\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx}{2a^2}\\
&= \frac{9\cos(c+dx)}{2a^3d}+\frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^3}+\frac{3\cos^3(c+dx)}{2d\left(a^3+a^3\sin(c+dx)\right)}+\frac{9\int 1\,dx}{2a^3}\\
&= \frac{9x}{2a^3}+\frac{9\cos(c+dx)}{2a^3d}+\frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^3}+\frac{3\cos^3(c+dx)}{2d\left(a^3+a^3\sin(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.64, size = 143, normalized size = 1.79

$$\frac{180dx\sin\left(c+\frac{dx}{2}\right)+55\sin\left(2c+\frac{3dx}{2}\right)-5\sin\left(2c+\frac{5dx}{2}\right)+59\cos\left(c+\frac{dx}{2}\right)+55\cos\left(c+\frac{3dx}{2}\right)+5\cos\left(3c+\frac{5dx}{2}\right.}{40a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (180*d*x*Cos[(d*x)/2] + 59*Cos[c + (d*x)/2] + 55*Cos[c + (3*d*x)/2] + 5*Cos
[3*c + (5*d*x)/2] - 381*Sin[(d*x)/2] + 180*d*x*Sin[c + (d*x)/2] + 55*Sin[2*
c + (3*d*x)/2] - 5*Sin[2*c + (5*d*x)/2])/(40*a^3*d*(Cos[c/2] + Sin[c/2])*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.44, size = 100, normalized size = 1.25

$$\frac{\cos(dx+c)^3+9\,dx+(9\,dx+13)\cos(dx+c)+6\,\cos(dx+c)^2+\left(9\,dx-\cos(dx+c)^2+5\,\cos(dx+c)-8\right)\mathrm{si}}{2\left(a^3d\cos(dx+c)+a^3d\sin(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(d*x + c)^3 + 9*d*x + (9*d*x + 13)*cos(d*x + c) + 6*cos(d*x + c)^2
+ (9*d*x - cos(d*x + c)^2 + 5*cos(d*x + c) - 8)*sin(d*x + c) + 8)/(a^3*d*co
s(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

giac [A] time = 0.20, size = 91, normalized size = 1.14

$$\frac{\frac{9\,(dx+c)}{a^3} + \frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^3} + \frac{16}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(9*(d*x + c)/a^3 + 2*(tan(1/2*d*x + 1/2*c)^3 + 6*tan(1/2*d*x + 1/2*c)^2
 - tan(1/2*d*x + 1/2*c) + 6)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) + 16/(a^3
*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [B] time = 0.40, size = 163, normalized size = 2.04

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} + \frac{6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} - \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} + \frac{6}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} + \frac{9\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+6/d/a^3/(1+tan(1/2*
d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2-1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan
(1/2*d*x+1/2*c)+6/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2+9/d/a^3*arctan(tan(1/2*d
*x+1/2*c))+8/d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.42, size = 225, normalized size = 2.81

$$\frac{\frac{\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{21\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{7\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+14}{a^3+\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}} + \frac{9\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] ((5*sin(d*x + c)/(cos(d*x + c) + 1) + 21*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 14)/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 2*a^3*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^3*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5) + 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 10.80, size = 94, normalized size = 1.18

$$\frac{9\,x}{2\,a^3}+\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+14}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (9*x)/(2*a^3) + (5*tan(c/2 + (d*x)/2) + 21*tan(c/2 + (d*x)/2)^2 + 7*tan(c/2
 + (d*x)/2)^3 + 9*tan(c/2 + (d*x)/2)^4 + 14)/(a^3*d*(tan(c/2 + (d*x)/2) + 1
)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

sympy [A] time = 61.27, size = 1244, normalized size = 15.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((9*d*x*tan(c/2 + d*x/2)**5/(2*a**3*d*tan(c/2 + d*x/2)**5 + 2*a**3
*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**3 + 4*a**3*d*tan(c/2 +
d*x/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) + 2*a**3*d) + 9*d*x*tan(c/2 + d*x/2)*
*4/(2*a**3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*
tan(c/2 + d*x/2)**3 + 4*a**3*d*tan(c/2 + d*x/2)**2 + 2*a**3*d*tan(c/2 + d*x
/2) + 2*a**3*d) + 18*d*x*tan(c/2 + d*x/2)**3/(2*a**3*d*tan(c/2 + d*x/2)**5
+ 2*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**3 + 4*a**3*d*ta
n(c/2 + d*x/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) + 2*a**3*d) + 18*d*x*tan(c/2
+ d*x/2)**2/(2*a**3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d*tan(c/2 + d*x/2)**4 +
4*a**3*d*tan(c/2 + d*x/2)**3 + 4*a**3*d*tan(c/2 + d*x/2)**2 + 2*a**3*d*tan(
c/2 + d*x/2) + 2*a**3*d) + 9*d*x*tan(c/2 + d*x/2)/(2*a**3*d*tan(c/2 + d*x/2
)**5 + 2*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**3 + 4*a**3
*d*tan(c/2 + d*x/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) + 2*a**3*d) + 9*d*x/(2*a
**3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2
```

```
 + d*x/2)**3 + 4*a**3*d*tan(c/2 + d*x/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) + 2
*a**3*d) + 18*tan(c/2 + d*x/2)**4/(2*a**3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d*
tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**3 + 4*a**3*d*tan(c/2 + d*x
/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) + 2*a**3*d) + 14*tan(c/2 + d*x/2)**3/(2*
a**3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/
2 + d*x/2)**3 + 4*a**3*d*tan(c/2 + d*x/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) +
2*a**3*d) + 42*tan(c/2 + d*x/2)**2/(2*a**3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d
*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**3 + 4*a**3*d*tan(c/2 + d*
x/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) + 2*a**3*d) + 10*tan(c/2 + d*x/2)/(2*a*
*3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2
+ d*x/2)**3 + 4*a**3*d*tan(c/2 + d*x/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) + 2*
a**3*d) + 28/(2*a**3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d*tan(c/2 + d*x/2)**4 +
 4*a**3*d*tan(c/2 + d*x/2)**3 + 4*a**3*d*tan(c/2 + d*x/2)**2 + 2*a**3*d*tan
(c/2 + d*x/2) + 2*a**3*d), Ne(d, 0)), (x*sin(c)*cos(c)**4/(a*sin(c) + a)**3
, True))
```

3.435 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=45

$$\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{x}{a^3}$$

```
[Out] x/a^3-arctanh(cos(d*x+c))/a^3/d+4*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.18, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2875, 2872, 3770, 2648}

$$\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] x/a^3 - ArcTanh[Cos[c + d*x]]/(a^3*d) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c
+ d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
```

```
LtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc(c+dx)\sec^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\
&= \frac{\int \left(a+a\csc(c+dx)-\frac{4a}{1+\sin(c+dx)}\right)dx}{a^4}\\
&= \frac{x}{a^3}+\frac{\int \csc(c+dx)\,dx}{a^3}-\frac{4\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3}\\
&= \frac{x}{a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}
\end{aligned}$$

Mathematica [B] time = 0.28, size = 122, normalized size = 2.71

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+c+dx\right)+\right.}{a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(Cos[(c + d*x)/2]*(c + d*x - Log[C
os[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]) + (-8 + c + d*x - Log[Cos[(c + d*
x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[(c + d*x)/2]))/(a^3*d*(1 + Sin[c + d*x]
)^3)
```

fricas [B] time = 0.46, size = 117, normalized size = 2.60

$$\frac{2\,dx+2\,(dx+4)\cos\left(dx+c\right)-\left(\cos\left(dx+c\right)+\sin\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\cos\left(dx+c\right)+\sin\left(d\right.\right.}{2\left(a^3d\cos\left(dx+c\right)+a^3d\sin\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*x + 2*(d*x + 4)*cos(d*x + c) - (cos(d*x + c) + sin(d*x + c) + 1)*l
og(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c) + sin(d*x + c) + 1)*log(-1/2*cos
(d*x + c) + 1/2) + 2*(d*x - 4)*sin(d*x + c) + 8)/(a^3*d*cos(d*x + c) + a^3*
d*sin(d*x + c) + a^3*d)
```

giac [A] time = 0.21, size = 47, normalized size = 1.04

$$\frac{\frac{dx+c}{a^3} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} + \frac{8}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^3 + log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 8/(a^3*(tan(1/2*d*x +
 1/2*c) + 1)))/d
```

maple [A] time = 0.58, size = 58, normalized size = 1.29

$$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3} + \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3} + \frac{8}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*arctan(tan(1/2*d*x+1/2*c))+1/d/a^3*ln(tan(1/2*d*x+1/2*c))+8/d/a^3/(
tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.42, size = 78, normalized size = 1.73

$$\frac{\frac{8}{a^3+\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}} + \frac{2\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} + \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (8/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c) + 1)) + 2*arctan(sin(d*x + c)/(cos
(d*x + c) + 1))/a^3 + log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 8.73, size = 115, normalized size = 2.56

$$\frac{8}{d\left(a^3+a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3d}+\frac{2\,\mathrm{atan}\left(\frac{4a^3}{4a^3-4a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{4a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3-4a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] 8/(d*(a^3 + a^3*tan(c/2 + (d*x)/2))) + log(tan(c/2 + (d*x)/2))/(a^3*d) + (2
*atan((4*a^3)/(4*a^3 - 4*a^3*tan(c/2 + (d*x)/2)) + (4*a^3*tan(c/2 + (d*x)/2
))/(4*a^3 - 4*a^3*tan(c/2 + (d*x)/2))))/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^4(c+dx)\csc(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

3.436 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=54

$$-\frac{\cot(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}$$

```
[Out] 3*arctanh(cos(d*x+c))/a^3/d-cot(d*x+c)/a^3/d-4*cos(d*x+c)/a^3/d/(1+sin(d*x+
c))
```

Rubi [A] time = 0.24, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2875, 2872, 3770, 3767, 8, 2648}

$$-\frac{\cot(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) - (4*Cos[c + d*x])
/(a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc^2(c+dx)\sec^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\
&= \frac{\int \left(-3a\csc(c+dx)+a\csc^2(c+dx)+\frac{4a}{1+\sin(c+dx)}\right)dx}{a^4}\\
&= \frac{\int \csc^2(c+dx)\,dx}{a^3}-\frac{3\int \csc(c+dx)\,dx}{a^3}+\frac{4\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3}\\
&= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}-\frac{\text{Subst}\left(\int 1\,dx,x,\cot(c+dx)\right)}{a^3d}\\
&= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}
\end{aligned}$$

Mathematica [B] time = 0.64, size = 156, normalized size = 2.89

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(\cot^2\left(\frac{1}{2}(c+dx)\right)+6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*((Cos[(c + d*x)/2]*(-17 + Cot[(c + d*x)/2]^2 - 6*Log[Cos[(c + d*x)/2]]
 + 6*Log[Sin[(c + d*x)/2]] + Cot[(c + d*x)/2]*(1 - 6*Log[Cos[(c + d*x)/2]]
+ 6*Log[Sin[(c + d*x)/2]])) - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^5*Tan[(c + d*x)/2])/(a^3*d*(1 + Sin[c + d*x])^3)
```

fricas [B] time = 0.45, size = 165, normalized size = 3.06

$$\frac{10\cos(dx+c)^2+3\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\left(\cos(dx+\right.}{2\left(a^3d\cos(dx+c)^2-a^3d-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/2*(10*cos(d*x + c)^2 + 3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c
) - 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)
*sin(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(5*cos(d*x + c) + 4)*si
n(d*x + c) + 2*cos(d*x + c) - 8)/(a^3*d*cos(d*x + c)^2 - a^3*d - (a^3*d*cos
(d*x + c) + a^3*d)*sin(d*x + c))
```

giac [A] time = 0.22, size = 90, normalized size = 1.67

$$-\frac{\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-14\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - tan(1/2*d*x + 1/2*c)/a^3 - (3*
tan(1/2*d*x + 1/2*c)^2 - 14*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c
)^2 + tan(1/2*d*x + 1/2*c))*a^3))/d
```

maple [A] time = 0.59, size = 77, normalized size = 1.43

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}-\frac{1}{2d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}-\frac{8}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^3*ln(tan(1/2*d*x+1/2*c))-8/d/a^3/(tan(1/2*d*x+1/2*c)+1)

maxima [B] time = 0.46, size = 116, normalized size = 2.15

$$-\frac{\frac{\frac{17\,\sin(dx+c)}{\cos(dx+c)+1}+1}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -1/2*((17*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 6*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d

mupad [B] time = 8.67, size = 87, normalized size = 1.61

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}-\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1}{d\,\left(2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)

[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - (17*tan(c/2 + (d*x)/2) + 1)/(d*(2*a^3*tan(c/2 + (d*x)/2)^2 + 2*a^3*tan(c/2 + (d*x)/2))) - (3*log(tan(c/2 + (d*x)/2)))/(a^3*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^4(c+dx)\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)

[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x)/a**3

3.437 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=78

$$\frac{3\cot(c+dx)}{a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{9\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}$$

```
[Out] -9/2*arctanh(cos(d*x+c))/a^3/d+3*cot(d*x+c)/a^3/d-1/2*cot(d*x+c)*csc(d*x+c)
/a^3/d+4*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.25, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2872, 3770, 3767, 8, 3768, 2648}

$$\frac{3\cot(c+dx)}{a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{9\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-9*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cot[c + d*x])/(a^3*d) - (Cot[c +
d*x]*Csc[c + d*x])/(2*a^3*d) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^3(c+dx)\sec^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(4a\csc(c+dx)-3a\csc^2(c+dx)+a\csc^3(c+dx)-\frac{4a}{1+\sin(c+dx)}\right)dx}{a^4}$$

$$= \frac{\int \csc^3(c+dx)\,dx}{a^3}-\frac{3\int \csc^2(c+dx)\,dx}{a^3}+\frac{4\int \csc(c+dx)\,dx}{a^3}-\frac{4\int \frac{1}{1+\sin(c+dx)}\,d}{a^3}$$

$$= -\frac{4\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}+\frac{\int c}{}$$

$$= -\frac{9\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{4\cos(c}{a^3d(1+\sin}$$

Mathematica [B] time = 5.90, size = 213, normalized size = 2.73

$$-\frac{\sin^8\left(\frac{1}{2}(c+dx)\right)\sin^7(c+dx)\left(\csc^2\left(\frac{1}{2}(c+dx)\right)+2\csc(c+dx)\right)^5\left((\csc(c+dx)-6)\csc^6\left(\frac{1}{2}(c+dx)\right)-8(\csc(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/512*((Csc[(c + d*x)/2]^2 + 2*Csc[c + d*x])^5*(Csc[(c + d*x)/2]^6*(-6 + C
sc[c + d*x]) - 8*(-6 + Csc[c + d*x])*Csc[c + d*x]^3 + 2*Csc[(c + d*x)/2]^4*
Csc[c + d*x]*(-6 + Csc[c + d*x] + 18*Log[Cos[(c + d*x)/2]] - 18*Log[Sin[(c
+ d*x)/2]]) - 4*Csc[(c + d*x)/2]^2*Csc[c + d*x]^2*(-38 + Csc[c + d*x] - 18*
Log[Cos[(c + d*x)/2]] + 18*Log[Sin[(c + d*x)/2]]))*Sin[(c + d*x)/2]^8*Sin[c
 + d*x]^7)/(a^3*d*(1 + Sin[c + d*x])^3)
```

fricas [B] time = 0.45, size = 246, normalized size = 3.15

$$\frac{28\cos(dx+c)^3+18\cos(dx+c)^2-9\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(28*cos(d*x + c)^3 + 18*cos(d*x + c)^2 - 9*(cos(d*x + c)^3 + cos(d*x +
c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)*log(1/2*cos(d*
x + c) + 1/2) + 9*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*s
in(d*x + c) - cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) - 2*(14*cos(d*
x + c)^2 + 5*cos(d*x + c) - 8)*sin(d*x + c) - 26*cos(d*x + c) - 16)/(a^3*d*
cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - a^3*d + (a^3*d
*cos(d*x + c)^2 - a^3*d)*sin(d*x + c))
```

giac [A] time = 0.25, size = 116, normalized size = 1.49

$$\frac{\frac{36\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{64}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{54\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(36*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 64/(a^3*(tan(1/2*d*x + 1/2*c)
+ 1)) - (54*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(
1/2*d*x + 1/2*c)^2) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x + 1/
2*c))/a^6)/d
```

maple [A] time = 0.68, size = 115, normalized size = 1.47

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}-\frac{1}{8d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{3}{2d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{9\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^3}+\frac{8}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^3*tan(1/2*d*x+1/2*c)-1/8/d/a^3/tan(1
/2*d*x+1/2*c)^2+3/2/d/a^3/tan(1/2*d*x+1/2*c)+9/2/d/a^3*ln(tan(1/2*d*x+1/2*c
))+8/d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.33, size = 161, normalized size = 2.06

$$\frac{\frac{\frac{11\sin(dx+c)}{\cos(dx+c)+1}+\frac{76\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}{\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}-\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^3}+\frac{36\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8*((11*sin(d*x + c)/(cos(d*x + c) + 1) + 76*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 1)/(a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3) - (12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2
/(cos(d*x + c) + 1)^2)/a^3 + 36*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 8.69, size = 120, normalized size = 1.54

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}+\frac{9\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^3\,d}+\frac{38\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\frac{1}{2}}{d\left(4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (9*log(tan(c/2 + (d*x)/2)))/(2*a^3*d) + ((
11*tan(c/2 + (d*x)/2))/2 + 38*tan(c/2 + (d*x)/2)^2 - 1/2)/(d*(4*a^3*tan(c/2
 + (d*x)/2)^2 + 4*a^3*tan(c/2 + (d*x)/2)^3)) - (3*tan(c/2 + (d*x)/2))/(2*a^
3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.438 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=96

$$-\frac{\cot^3(c+dx)}{3a^3d}-\frac{5\cot(c+dx)}{a^3d}+\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{4\cot(c+dx)}{a^3d(\csc(c+dx)+1)}$$

```
[Out] 11/2*arctanh(cos(d*x+c))/a^3/d-13*cot(d*x+c)/a^3/d-13/3*cot(d*x+c)^3/a^3/d+
11/2*cot(d*x+c)*csc(d*x+c)/a^3/d+4*cot(d*x+c)*csc(d*x+c)^2/a^3/d/(1+sin(d*x
+c))
```

Rubi [A] time = 0.15, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 3770, 3767, 8, 3768, 3777}

$$-\frac{\cot^3(c+dx)}{3a^3d}-\frac{5\cot(c+dx)}{a^3d}+\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{4\cot(c+dx)}{a^3d(\csc(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (11*ArcTanh[Cos[c + d*x]])/(2*a^3*d) - (5*Cot[c + d*x])/(a^3*d) - Cot[c + d
*x]^3/(3*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d) - (4*Cot[c + d*x]
)/(a^3*d*(1 + Csc[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \left(4a-4a\csc(c+dx)+4a\csc^2(c+dx)-3a\csc^3(c+dx)+a\csc^4(c+dx)-\frac{4}{1+\csc(\ldots}\right.}{a^4}$$

$$= \frac{4x}{a^3}+\frac{\int \csc^4(c+dx)\,dx}{a^3}-\frac{3\int \csc^3(c+dx)\,dx}{a^3}-\frac{4\int \csc(c+dx)\,dx}{a^3}+\frac{4\int \csc^2(c+\ldots}{a^3}$$

$$= \frac{4x}{a^3}+\frac{4\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{4\cot(c+dx)}{a^3d(1+\csc(c+dx))}-\ldots$$

$$= \frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{5\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}\ldots$$

Mathematica [B] time = 5.03, size = 251, normalized size = 2.61

$$-\frac{\sin^2\left(\frac{1}{2}(c+dx)\right)\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)^5\csc^3(c+dx)\left(-4\sin^8\left(\frac{1}{2}(c+dx)\right)-8\sin(c+dx)(7\sin(c+dx)-2)\sin\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/12*((1 + Cot[(c + d*x)/2])^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^2*(-4*Sin[(
c + d*x)/2]^8 - 8*Sin[(c + d*x)/2]^6*Sin[c + d*x]*(-2 + 7*Sin[c + d*x]) + (
Sin[c + d*x]^4*(-8 + Cot[(c + d*x)/2] + 28*Sin[c + d*x]))/4 - (Sin[(c + d*x
)/2]^2*Sin[c + d*x]^3*(9 + (-28 + 66*Log[Cos[(c + d*x)/2]] - 66*Log[Sin[(c
+ d*x)/2]])*Sin[c + d*x]))/2 + Sin[(c + d*x)/2]^4*Sin[c + d*x]^2*(9 - 2*(62
 + 33*Log[Cos[(c + d*x)/2]] - 33*Log[Sin[(c + d*x)/2]])*Sin[c + d*x])))/(a^
3*d*(1 + Sin[c + d*x])^3)
```

fricas [B] time = 0.46, size = 302, normalized size = 3.15

$$\frac{104\,\cos\left(dx+c\right)^{4}+38\,\cos\left(dx+c\right)^{3}-156\,\cos\left(dx+c\right)^{2}+33\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(104*cos(d*x + c)^4 + 38*cos(d*x + c)^3 - 156*cos(d*x + c)^2 + 33*(cos
(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x
 + c) - 1)*sin(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 33*(cos(d*x + c)
^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1
)*sin(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(52*cos(d*x + c)^3 + 3
3*cos(d*x + c)^2 - 45*cos(d*x + c) - 24)*sin(d*x + c) - 42*cos(d*x + c) + 4
8)/(a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d - (a^3*d*cos(d*x
+ c)^3 + a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

giac [A] time = 0.24, size = 146, normalized size = 1.52

$$-\frac{\frac{132\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{3}}+\frac{192}{a^{3}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{242\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-57\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}-\frac{a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-9\,a^{6}\,\cdots}{\cdots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/24*(132*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 192/(a^3*(tan(1/2*d*x + 1/2
*c) + 1)) - (242*tan(1/2*d*x + 1/2*c)^3 - 57*tan(1/2*d*x + 1/2*c)^2 + 9*tan
(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)^3) - (a^6*tan(1/2*d*x + 1/
2*c)^3 - 9*a^6*tan(1/2*d*x + 1/2*c)^2 + 57*a^6*tan(1/2*d*x + 1/2*c))/a^9)/d
```

maple [A] time = 0.67, size = 153, normalized size = 1.59

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^3}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^3}+\frac{19\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^3}-\frac{1}{24d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{3}{8d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{19}{8d\,a^3\tan\left(\frac{dx}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d/a^3*tan(1/2*d*x+1/2*c)^3-3/8/d/a^3*tan(1/2*d*x+1/2*c)^2+19/8/d/a^3*t
an(1/2*d*x+1/2*c)-1/24/d/a^3/tan(1/2*d*x+1/2*c)^3+3/8/d/a^3/tan(1/2*d*x+1/2
*c)^2-19/8/d/a^3/tan(1/2*d*x+1/2*c)-11/2/d/a^3*ln(tan(1/2*d*x+1/2*c))-8/d/a
^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.48, size = 199, normalized size = 2.07

$$\frac{\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{249\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1}{\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{57\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{132\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/24*((8*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 249*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a^3*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (57*sin(d*
x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3)/a^3 - 132*log(sin(d*x + c)/(cos(d*x + c) + 1)
)/a^3)/d
```

mupad [B] time = 8.69, size = 153, normalized size = 1.59

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^3\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}-\frac{11\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^3\,d}-\frac{83\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{1}{3}}{d\left(8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (3*tan(c/2 + (d*x)/2)^2)/(8*a^3*d) - (11*
log(tan(c/2 + (d*x)/2)))/(2*a^3*d) - (16*tan(c/2 + (d*x)/2)^2 - (8*tan(c/2
+ (d*x)/2))/3 + 83*tan(c/2 + (d*x)/2)^3 + 1/3)/(d*(8*a^3*tan(c/2 + (d*x)/2)
^3 + 8*a^3*tan(c/2 + (d*x)/2)^4)) + (19*tan(c/2 + (d*x)/2))/(8*a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.439 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=117

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{7\cot(c+dx)}{a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{51\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{19\cot(c$$

```
[Out] -51/8*arctanh(cos(d*x+c))/a^3/d+7*cot(d*x+c)/a^3/d+cot(d*x+c)^3/a^3/d-19/8*
cot(d*x+c)*csc(d*x+c)/a^3/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a^3/d+4*cos(d*x+c)/
a^3/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.30, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2872, 3770, 3767, 8, 3768, 2648}

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{7\cot(c+dx)}{a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{51\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{19\cot(c$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-51*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + (7*Cot[c + d*x])/(a^3*d) + Cot[c +
d*x]^3/(a^3*d) - (19*Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) - (Cot[c + d*x]*C
sc[c + d*x]^3)/(4*a^3*d) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^5(c+dx)\sec^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \Big(4a\csc(c+dx) - 4a\csc^2(c+dx) + 4a\csc^3(c+dx) - 3a\csc^4(c+dx) + a\csc$$

$$a^4$$

$$= \frac{\int \csc^5(c+dx)\,dx}{a^3} - \frac{3\int \csc^4(c+dx)\,dx}{a^3} + \frac{4\int \csc(c+dx)\,dx}{a^3} - \frac{4\int \csc^2(c+d}{a^3}$$

$$= -\frac{4\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}$$

$$= -\frac{6\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{7\cot(c+dx)}{a^3d} + \frac{\cot^3(c+dx)}{a^3d} - \frac{19\cot(c+dx)\csc(c}{8a^3d}$$

$$= -\frac{51\tanh^{-1}(\cos(c+dx))}{8a^3d} + \frac{7\cot(c+dx)}{a^3d} + \frac{\cot^3(c+dx)}{a^3d} - \frac{19\cot(c+dx)\csc(}{8a^3d}$$

Mathematica [B] time = 6.16, size = 601, normalized size = 5.14

$$-\frac{8\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}{d(a\sin(c+dx)+a)^3} - \frac{51\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}{8d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-8*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/(d*(a + a*Sin
[c + d*x])^3) + (3*Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6
)/(d*(a + a*Sin[c + d*x])^3) - (19*Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^6)/(32*d*(a + a*Sin[c + d*x])^3) + (Cot[(c + d*x)/2]*Csc[(
c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(8*d*(a + a*Sin[c +
d*x])^3) - (Csc[(c + d*x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(64
*d*(a + a*Sin[c + d*x])^3) - (51*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^6)/(8*d*(a + a*Sin[c + d*x])^3) + (51*Log[Sin[(c + d*x)/2
]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(8*d*(a + a*Sin[c + d*x])^3) +
(19*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(32*d*(a +
a*Sin[c + d*x])^3) + (Sec[(c + d*x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^6)/(64*d*(a + a*Sin[c + d*x])^3) - (3*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^6*Tan[(c + d*x)/2])/(d*(a + a*Sin[c + d*x])^3) - (Sec[(c + d*x)/2]^2*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*Tan[(c + d*x)/2])/(8*d*(a + a*Sin[c
+ d*x])^3)
```

fricas [B] time = 0.48, size = 381, normalized size = 3.26

$$\frac{160\cos(dx+c)^5+102\cos(dx+c)^4-298\cos(dx+c)^3-170\cos(dx+c)^2-51\left(\cos(dx+c)^5+\cos(dx+c)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(160*cos(d*x + c)^5 + 102*cos(d*x + c)^4 - 298*cos(d*x + c)^3 - 170*co
s(d*x + c)^2 - 51*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*c
os(d*x + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(
d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 51*(cos(d*x + c)^5 + cos(d*x +
c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x +
c)^2 + 1)*sin(d*x + c) + cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) - 2
*(80*cos(d*x + c)^4 + 29*cos(d*x + c)^3 - 120*cos(d*x + c)^2 - 35*cos(d*x +
 c) + 32)*sin(d*x + c) + 134*cos(d*x + c) + 64)/(a^3*d*cos(d*x + c)^5 + a^3
*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 + a^3*d
*cos(d*x + c) + a^3*d + (a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^
3*d)*sin(d*x + c))
```

giac [A] time = 0.28, size = 174, normalized size = 1.49

$$\frac{\frac{408\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{512}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{850\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-200\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}+\frac{a^9\tan}{}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(408*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 512/(a^3*(tan(1/2*d*x + 1/2*
c) + 1)) - (850*tan(1/2*d*x + 1/2*c)^4 - 200*tan(1/2*d*x + 1/2*c)^3 + 40*ta
n(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c
)^4) + (a^9*tan(1/2*d*x + 1/2*c)^4 - 8*a^9*tan(1/2*d*x + 1/2*c)^3 + 40*a^9*
tan(1/2*d*x + 1/2*c)^2 - 200*a^9*tan(1/2*d*x + 1/2*c))/a^12)/d
```

maple [A] time = 0.68, size = 191, normalized size = 1.63

$$\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d\,a^3}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^3}+\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^3}-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^3}-\frac{1}{64d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{1}{8d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/64/d/a^3*tan(1/2*d*x+1/2*c)^4-1/8/d/a^3*tan(1/2*d*x+1/2*c)^3+5/8/d/a^3*ta
n(1/2*d*x+1/2*c)^2-25/8/d/a^3*tan(1/2*d*x+1/2*c)-1/64/d/a^3/tan(1/2*d*x+1/2
*c)^4+1/8/d/a^3/tan(1/2*d*x+1/2*c)^3-5/8/d/a^3/tan(1/2*d*x+1/2*c)^2+25/8/d/
a^3/tan(1/2*d*x+1/2*c)+51/8/d/a^3*ln(tan(1/2*d*x+1/2*c))+8/d/a^3/(tan(1/2*d
*x+1/2*c)+1)
```

maxima [B] time = 0.38, size = 241, normalized size = 2.06

$$\frac{\frac{\frac{7\sin(dx+c)}{\cos(dx+c)+1}-\frac{32\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{160\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{712\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-1}{\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}-\frac{\frac{200\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^3}+\frac{408\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/64*((7*sin(d*x + c)/(cos(d*x + c) + 1) - 32*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 160*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 712*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 1)/(a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5) - (200*sin(d*x + c)/(cos(d*x + c) + 1) - 40*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
- sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/a^3 + 408*log(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^3)/d
```

mupad [B] time = 9.13, size = 176, normalized size = 1.50

$$\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,a^3\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^3\,d}+\frac{51\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^3\,d}-\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{89\tan\ldots}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (5*tan(c/2 + (d*x)/2)^2)/(8*a^3*d) - tan(c/2 + (d*x)/2)^3/(8*a^3*d) + tan(c
/2 + (d*x)/2)^4/(64*a^3*d) + (51*log(tan(c/2 + (d*x)/2)))/(8*a^3*d) - (25*t
an(c/2 + (d*x)/2))/(8*a^3*d) + (cot(c/2 + (d*x)/2)^4*((7*tan(c/2 + (d*x)/2)
)/64 - tan(c/2 + (d*x)/2)^2/2 + (5*tan(c/2 + (d*x)/2)^3)/2 + (89*tan(c/2 +
(d*x)/2)^4)/8 - 1/64))/(a^3*d*(tan(c/2 + (d*x)/2) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.440 $\int \frac{\cos^4(e+fx)\sin(e+fx)}{(a+a\sin(e+fx))^6}\,dx$

Optimal. Leaf size=58

$$\frac{\cos^5(e+fx)}{7f(a\sin(e+fx)+a)^6}-\frac{6\cos^5(e+fx)}{35af(a\sin(e+fx)+a)^5}$$

```
[Out] 1/7*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^6-6/35*cos(f*x+e)^5/a/f/(a+a*sin(f*x+e)
)^5
```

Rubi [A] time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2859, 2671}

$$\frac{\cos^5(e+fx)}{7f(a\sin(e+fx)+a)^6}-\frac{6\cos^5(e+fx)}{35af(a\sin(e+fx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*Sin[e + f*x])/(a + a*Sin[e + f*x])^6,x]
```

```
[Out] Cos[e + f*x]^5/(7*f*(a + a*Sin[e + f*x])^6) - (6*Cos[e + f*x]^5)/(35*a*f*(a
 + a*Sin[e + f*x])^5)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)\sin(e+fx)}{(a+a\sin(e+fx))^6}\,dx = \frac{\cos^5(e+fx)}{7f(a+a\sin(e+fx))^6} + \frac{6\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^5}\,dx}{7a}$$
$$= \frac{\cos^5(e+fx)}{7f(a+a\sin(e+fx))^6} - \frac{6\cos^5(e+fx)}{35af(a+a\sin(e+fx))^5}$$

Mathematica [B] time = 1.26, size = 143, normalized size = 2.47

$$\frac{1134\sin\left(2e+\frac{3fx}{2}\right)-224\sin\left(2e+\frac{5fx}{2}\right)+\sin\left(4e+\frac{7fx}{2}\right)+4585\cos\left(e+\frac{fx}{2}\right)-2982\cos\left(e+\frac{3fx}{2}\right)-1148\cos\left(3\right.}{4620a^6f\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^4*Sin[e + f*x])/(a + a*Sin[e + f*x])^6,x]
```

```
[Out] (4585*Cos[e + (f*x)/2] - 2982*Cos[e + (3*f*x)/2] - 1148*Cos[3*e + (5*f*x)/2
] + 197*Cos[3*e + (7*f*x)/2] + 2275*Sin[(f*x)/2] + 1134*Sin[2*e + (3*f*x)/2
] - 224*Sin[2*e + (5*f*x)/2] + Sin[4*e + (7*f*x)/2])/(4620*a^6*f*(Cos[e/2]
+ Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

fricas [B] time = 0.44, size = 195, normalized size = 3.36

$$\frac{6\cos\left(fx+e\right)^4-11\cos\left(fx+e\right)^3-27\cos\left(fx+e\right)^2+\left(6\cos\left(fx+e\right)^3+17\cos\left(fx+e\right)^2-10\,\mathrm{c}\right.}{35\left(a^6f\cos\left(fx+e\right)^4-3a^6f\cos\left(fx+e\right)^3-8a^6f\cos\left(fx+e\right)^2+4a^6f\cos\left(fx+e\right)+8a^6f-\left(a^6f\cos\left(fx+e\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] 1/35*(6*cos(f*x + e)^4 - 11*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + (6*cos(f*x
 + e)^3 + 17*cos(f*x + e)^2 - 10*cos(f*x + e) - 20)*sin(f*x + e) + 10*cos(f
*x + e) + 20)/(a^6*f*cos(f*x + e)^4 - 3*a^6*f*cos(f*x + e)^3 - 8*a^6*f*cos(
f*x + e)^2 + 4*a^6*f*cos(f*x + e) + 8*a^6*f - (a^6*f*cos(f*x + e)^3 + 4*a^6
*f*cos(f*x + e)^2 - 4*a^6*f*cos(f*x + e) - 8*a^6*f)*sin(f*x + e))
```

giac [A] time = 0.29, size = 92, normalized size = 1.59

$$-\frac{2\left(35\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-35\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+70\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-14\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+7\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.\right.}{35a^6f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -2/35*(35*tan(1/2*f*x + 1/2*e)^5 - 35*tan(1/2*f*x + 1/2*e)^4 + 70*tan(1/2*f
*x + 1/2*e)^3 - 14*tan(1/2*f*x + 1/2*e)^2 + 7*tan(1/2*f*x + 1/2*e) + 1)/(a^
6*f*(tan(1/2*f*x + 1/2*e) + 1)^7)
```

maple [A] time = 0.46, size = 100, normalized size = 1.72

$$\frac{\frac{224}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}+\frac{64}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{7}}-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{6}}-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}+\frac{12}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}}{f\,a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*sin(f*x+e)/(a+a*sin(f*x+e))^6,x)
```

```
[Out] 4/f/a^6*(56/5/(tan(1/2*f*x+1/2*e)+1)^5-1/2/(tan(1/2*f*x+1/2*e)+1)^2+16/7/(t
an(1/2*f*x+1/2*e)+1)^7-8/(tan(1/2*f*x+1/2*e)+1)^6-8/(tan(1/2*f*x+1/2*e)+1)^
4+3/(tan(1/2*f*x+1/2*e)+1)^3)
```

maxima [B] time = 0.52, size = 269, normalized size = 4.64

$$-\frac{2\left(\frac{7\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{14\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{70\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{35\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{35\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+1\right)}{35\left(a^{6}+\frac{7a^{6}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{21a^{6}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{35a^{6}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{35a^{6}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{21a^{6}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{7a^{6}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{a^{6}\sin\left(f\right.}{\left(\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -2/35*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 14*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 70*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 35*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1)/((a^6 + 7*a^6*s
in(f*x + e)/(cos(f*x + e) + 1) + 21*a^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 35*a^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*a^6*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 21*a^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*a^6*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + a^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)*f)
```

mupad [B] time = 8.89, size = 157, normalized size = 2.71

$$-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}+7\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)-14\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+70\cos\left(\frac{e}{2}\right.\right.}{35\,a^{6}\,f\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*sin(e + f*x))/(a + a*sin(e + f*x))^6,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)^2*(cos(e/2 + (f*x)/2)^5 + 35*sin(e/2 + (f*x)/2)^5 -
35*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^4 + 7*cos(e/2 + (f*x)/2)^4*sin(e/2
 + (f*x)/2) + 70*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^3 - 14*cos(e/2 + (
f*x)/2)^3*sin(e/2 + (f*x)/2)^2))/(35*a^6*f*(cos(e/2 + (f*x)/2) + sin(e/2 +
(f*x)/2))^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*sin(f*x+e)/(a+a*sin(f*x+e))**6,x)
```

```
[Out] Timed out
```

3.441 $\int \frac{\cos^4(e+fx)\sin^2(e+fx)}{(a+a\sin(e+fx))^7}\,dx$

Optimal. Leaf size=89

$$-\frac{47\cos^5(e+fx)}{315a^2f(a\sin(e+fx)+a)^5}-\frac{a\cos^7(e+fx)}{18f(a\sin(e+fx)+a)^8}+\frac{25\cos^5(e+fx)}{126af(a\sin(e+fx)+a)^6}$$

```
[Out] -1/18*a*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^8+25/126*cos(f*x+e)^5/a/f/(a+a*sin(
f*x+e))^6-47/315*cos(f*x+e)^5/a^2/f/(a+a*sin(f*x+e))^5
```

Rubi [A] time = 0.46, antiderivative size = 131, normalized size of antiderivative = 1.47, number of steps used = 18, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2875, 2872, 2650, 2648}

$$-\frac{47\cos(e+fx)}{315a^7f(\sin(e+fx)+1)}+\frac{268\cos(e+fx)}{315a^7f(\sin(e+fx)+1)^2}-\frac{181\cos(e+fx)}{105a^7f(\sin(e+fx)+1)^3}+\frac{92\cos(e+fx)}{63a^7f(\sin(e+fx)+1)^4}-\frac{}{9a^7}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*Sin[e + f*x]^2)/(a + a*Sin[e + f*x])^7,x]
```

```
[Out] (-4*Cos[e + f*x])/(9*a^7*f*(1 + Sin[e + f*x])^5) + (92*Cos[e + f*x])/(63*a^
7*f*(1 + Sin[e + f*x])^4) - (181*Cos[e + f*x])/(105*a^7*f*(1 + Sin[e + f*x]
)^3) + (268*Cos[e + f*x])/(315*a^7*f*(1 + Sin[e + f*x])^2) - (47*Cos[e + f*
x])/(315*a^7*f*(1 + Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
```

```
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)\sin^2(e+fx)}{(a+a\sin(e+fx))^7}\,dx = \frac{\int \sec^8(e+fx)(a-a\sin(e+fx))^7\tan^2(e+fx)\,dx}{a^{14}}$$

$$= \frac{\int \left(\frac{4}{a^3(1+\sin(e+fx))^5} - \frac{12}{a^3(1+\sin(e+fx))^4} + \frac{13}{a^3(1+\sin(e+fx))^3} - \frac{6}{a^3(1+\sin(e+fx))^2} + \frac{1}{a^3(1+\sin(e\cdots}\right.}{a^4}$$

$$= \frac{\int \frac{1}{1+\sin(e+fx)}\,dx}{a^7} + \frac{4\int \frac{1}{(1+\sin(e+fx))^5}\,dx}{a^7} - \frac{6\int \frac{1}{(1+\sin(e+fx))^2}\,dx}{a^7} - \frac{12\int \frac{1}{(1+\sin(e+fx)\cdots}}{a^7}$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{12\cos(e+fx)}{7a^7f(1+\sin(e+fx))^4} - \frac{13\cos(e+fx)}{5a^7f(1+\sin(e+fx))^3} + \cdots$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{92\cos(e+fx)}{63a^7f(1+\sin(e+fx))^4} - \frac{11\cos(e+fx)}{7a^7f(1+\sin(e+fx))^3} \cdots$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{92\cos(e+fx)}{63a^7f(1+\sin(e+fx))^4} - \frac{181\cos(e+fx)}{105a^7f(1+\sin(e+fx)\cdots}$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{92\cos(e+fx)}{63a^7f(1+\sin(e+fx))^4} - \frac{181\cos(e+fx)}{105a^7f(1+\sin(e+fx)\cdots}$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{92\cos(e+fx)}{63a^7f(1+\sin(e+fx))^4} - \frac{181\cos(e+fx)}{105a^7f(1+\sin(e+fx)\cdots}$$

Mathematica [B] time = 2.61, size = 293, normalized size = 3.29

$$\frac{1890\sin\left(e+\frac{fx}{2}\right) + 1260\sin\left(e+\frac{3fx}{2}\right) + 659400\sin\left(2e+\frac{3fx}{2}\right) - 303192\sin\left(2e+\frac{5fx}{2}\right) - 540\sin\left(3e+\frac{5fx}{2}\right) - 13\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^4*Sin[e + f*x]^2)/(a + a*Sin[e + f*x])^7,x]
```

```
[Out] (1890*Cos[(f*x)/2] + 718830*Cos[e + (f*x)/2] - 467208*Cos[e + (3*f*x)/2] -
1260*Cos[2*e + (3*f*x)/2] - 540*Cos[2*e + (5*f*x)/2] - 179640*Cos[3*e + (5*
f*x)/2] + 30753*Cos[3*e + (7*f*x)/2] + 135*Cos[4*e + (7*f*x)/2] + 15*Cos[4*
e + (9*f*x)/2] - 15*Cos[5*e + (9*f*x)/2] + 971082*Sin[(f*x)/2] + 1890*Sin[e
 + (f*x)/2] + 1260*Sin[e + (3*f*x)/2] + 659400*Sin[2*e + (3*f*x)/2] - 30319
2*Sin[2*e + (5*f*x)/2] - 540*Sin[3*e + (5*f*x)/2] - 135*Sin[3*e + (7*f*x)/2
] - 89955*Sin[4*e + (7*f*x)/2] + 13427*Sin[4*e + (9*f*x)/2] + 15*Sin[5*e +
(9*f*x)/2])/(720720*a^7*f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^9)
```

fricas [B] time = 0.46, size = 243, normalized size = 2.73

$$-\frac{47\cos\left(fx+e\right)^{5}+127\cos\left(fx+e\right)^{4}-115\cos\left(fx+e\right)^{3}-265\cos\left(fx+e\right)^{2}-\left(47\cos\left(fx+e\right.\right.}{315\left(a^{7}f\cos\left(fx+e\right)^{5}+5a^{7}f\cos\left(fx+e\right)^{4}-8a^{7}f\cos\left(fx+e\right)^{3}-20a^{7}f\cos\left(fx+e\right)^{2}+8a^{7}f\cos\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^2/(a+a*sin(f*x+e))^7,x, algorithm="fricas
")
```

```
[Out] -1/315*(47*cos(f*x + e)^5 + 127*cos(f*x + e)^4 - 115*cos(f*x + e)^3 - 265*c
os(f*x + e)^2 - (47*cos(f*x + e)^4 - 80*cos(f*x + e)^3 - 195*cos(f*x + e)^2
 + 70*cos(f*x + e) + 140)*sin(f*x + e) + 70*cos(f*x + e) + 140)/(a^7*f*cos(
f*x + e)^5 + 5*a^7*f*cos(f*x + e)^4 - 8*a^7*f*cos(f*x + e)^3 - 20*a^7*f*cos
(f*x + e)^2 + 8*a^7*f*cos(f*x + e) + 16*a^7*f + (a^7*f*cos(f*x + e)^4 - 4*a
^7*f*cos(f*x + e)^3 - 12*a^7*f*cos(f*x + e)^2 + 8*a^7*f*cos(f*x + e) + 16*a
^7*f)*sin(f*x + e))
```

giac [A] time = 0.38, size = 106, normalized size = 1.19

$$-\frac{4\left(210\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-315\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+441\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-126\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+36\tan\left(\frac{1}{2}\right.\right.}{315a^{7}f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^2/(a+a*sin(f*x+e))^7,x, algorithm="giac")
```

```
[Out] -4/315*(210*tan(1/2*f*x + 1/2*e)^6 - 315*tan(1/2*f*x + 1/2*e)^5 + 441*tan(1
/2*f*x + 1/2*e)^4 - 126*tan(1/2*f*x + 1/2*e)^3 + 36*tan(1/2*f*x + 1/2*e)^2
+ 9*tan(1/2*f*x + 1/2*e) + 1)/(a^7*f*(tan(1/2*f*x + 1/2*e) + 1)^9)
```

maple [A] time = 0.46, size = 115, normalized size = 1.29

$$\frac{\frac{352}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{6}}-\frac{832}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{7}}+\frac{20}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{8}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}+\frac{64}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{8}}-\frac{328}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{128}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}}{f\,a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*sin(f*x+e)^2/(a+a*sin(f*x+e))^7,x)
```

```
[Out] 8/f/a^7*(44/3/(tan(1/2*f*x+1/2*e)+1)^6-104/7/(tan(1/2*f*x+1/2*e)+1)^7+5/2/(
tan(1/2*f*x+1/2*e)+1)^4-1/3/(tan(1/2*f*x+1/2*e)+1)^3+8/(tan(1/2*f*x+1/2*e)+
1)^8-41/5/(tan(1/2*f*x+1/2*e)+1)^5-16/9/(tan(1/2*f*x+1/2*e)+1)^9)
```

maxima [B] time = 0.54, size = 335, normalized size = 3.76

$$-\frac{4\left(\frac{9\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{36\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{126\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{441\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{315\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{210\,\mathrm{s}}{(\cos(}\right.}{315\left(a^{7}+\frac{9a^{7}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{36a^{7}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{84a^{7}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{126a^{7}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{126a^{7}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{84a^{7}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{36a^{7}\,\mathrm{si}}{(\cos(f}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^2/(a+a*sin(f*x+e))^7,x, algorithm="maxima
")
```

```
[Out] -4/315*(9*sin(f*x + e)/(cos(f*x + e) + 1) + 36*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 441*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 210*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 + 1)/((a^7 + 9*a^7*sin(f*x + e)/(cos(f*x + e) + 1
) + 36*a^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*a^7*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 126*a^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 126*a^7*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*a^7*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 + 36*a^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*a^7*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8 + a^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)*f)
```

mupad [B] time = 9.05, size = 181, normalized size = 2.03

$$-\frac{4\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}+9\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)+36\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}-126\cos\left(\frac{e}{2}\right.\right.}{315\,a^{7}\,f\left(\cos\left(\frac{e}{2}+\frac{f}{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*sin(e + f*x)^2)/(a + a*sin(e + f*x))^7,x)
```

```
[Out] -(4*cos(e/2 + (f*x)/2)^3*(cos(e/2 + (f*x)/2)^6 + 210*sin(e/2 + (f*x)/2)^6 -
 315*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^5 + 9*cos(e/2 + (f*x)/2)^5*sin(e
/2 + (f*x)/2) + 441*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 - 126*cos(e/2
 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^3 + 36*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x
)/2)^2))/(315*a^7*f*(cos(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2))^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*sin(f*x+e)**2/(a+a*sin(f*x+e))**7,x)
```

```
[Out] Timed out
```

3.442 $\int \frac{\cos^4(e+fx)\sin^3(e+fx)}{(a+a\sin(e+fx))^8}\,dx$

Optimal. Leaf size=157

$$-\frac{152\cos(e+fx)}{1155a^8f(\sin(e+fx)+1)}+\frac{1003\cos(e+fx)}{1155a^8f(\sin(e+fx)+1)^2}-\frac{846\cos(e+fx)}{385a^8f(\sin(e+fx)+1)^3}+\frac{617\cos(e+fx)}{231a^8f(\sin(e+fx)+1)^4}-3$$

```
[Out] 4/11*cos(f*x+e)/a^8/f/(1+sin(f*x+e))^6-52/33*cos(f*x+e)/a^8/f/(1+sin(f*x+e)
)^5+617/231*cos(f*x+e)/a^8/f/(1+sin(f*x+e))^4-846/385*cos(f*x+e)/a^8/f/(1+s
in(f*x+e))^3+1003/1155*cos(f*x+e)/a^8/f/(1+sin(f*x+e))^2-152/1155*cos(f*x+e
)/a^8/f/(1+sin(f*x+e))
```

Rubi [A] time = 0.57, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2875, 2872, 2650, 2648}

$$-\frac{152\cos(e+fx)}{1155a^8f(\sin(e+fx)+1)}+\frac{1003\cos(e+fx)}{1155a^8f(\sin(e+fx)+1)^2}-\frac{846\cos(e+fx)}{385a^8f(\sin(e+fx)+1)^3}+\frac{617\cos(e+fx)}{231a^8f(\sin(e+fx)+1)^4}-3$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*Sin[e + f*x]^3)/(a + a*Sin[e + f*x])^8,x]
```

```
[Out] (4*Cos[e + f*x])/(11*a^8*f*(1 + Sin[e + f*x])^6) - (52*Cos[e + f*x])/(33*a^
8*f*(1 + Sin[e + f*x])^5) + (617*Cos[e + f*x])/(231*a^8*f*(1 + Sin[e + f*x]
)^4) - (846*Cos[e + f*x])/(385*a^8*f*(1 + Sin[e + f*x])^3) + (1003*Cos[e +
f*x])/(1155*a^8*f*(1 + Sin[e + f*x])^2) - (152*Cos[e + f*x])/(1155*a^8*f*(1
 + Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
```

```
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)\sin^3(e+fx)}{(a+a\sin(e+fx))^8}\,dx = \frac{\int \sec^9(e+fx)(a-a\sin(e+fx))^8\tan^3(e+fx)\,dx}{a^{16}}$$

$$= \frac{\int \Big(-\frac{4}{a^4(1+\sin(e+fx))^6} + \frac{16}{a^4(1+\sin(e+fx))^5} - \frac{25}{a^4(1+\sin(e+fx))^4} + \frac{19}{a^4(1+\sin(e+fx))^3} - \frac{}{a^4(1+\mathrm{s}}}{a^4}$$

$$= \frac{\int \frac{1}{1+\sin(e+fx)}\,dx}{a^8} - \frac{4\int \frac{1}{(1+\sin(e+fx))^6}\,dx}{a^8} - \frac{7\int \frac{1}{(1+\sin(e+fx))^2}\,dx}{a^8} + \frac{16\int \frac{1}{(1+\sin(e+f}}{a^8}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6} - \frac{16\cos(e+fx)}{9a^8f(1+\sin(e+fx))^5} + \frac{25\cos(e+fx)}{7a^8f(1+\sin(e+fx))^4}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6} - \frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5} + \frac{23\cos(e+fx)}{9a^8f(1+\sin(e+fx))^4}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6} - \frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5} + \frac{617\cos(e+fx)}{231a^8f(1+\sin(e+fx}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6} - \frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5} + \frac{617\cos(e+fx)}{231a^8f(1+\sin(e+fx}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6} - \frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5} + \frac{617\cos(e+fx)}{231a^8f(1+\sin(e+fx}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6} - \frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5} + \frac{617\cos(e+fx)}{231a^8f(1+\sin(e+fx}$$

Mathematica [A] time = 3.18, size = 195, normalized size = 1.24

$$-\frac{-299970\sin\left(2e+\frac{3fx}{2}\right)+145695\sin\left(2e+\frac{5fx}{2}\right)+44990\sin\left(4e+\frac{7fx}{2}\right)-6710\sin\left(4e+\frac{9fx}{2}\right)+\sin\left(6e+\frac{11fx}{2}\right)}{240240a^{8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^4*Sin[e + f*x]^3)/(a + a*Sin[e + f*x])^8,x]
```

```
[Out] -1/240240*(-486024*Cos[e + (f*x)/2] + 351450*Cos[e + (3*f*x)/2] + 180015*Co
s[3*e + (5*f*x)/2] - 63580*Cos[3*e + (7*f*x)/2] - 15004*Cos[5*e + (9*f*x)/2
] + 1975*Cos[5*e + (11*f*x)/2] - 425964*Sin[(f*x)/2] - 299970*Sin[2*e + (3*
f*x)/2] + 145695*Sin[2*e + (5*f*x)/2] + 44990*Sin[4*e + (7*f*x)/2] - 6710*S
in[4*e + (9*f*x)/2] + Sin[6*e + (11*f*x)/2])/(a^8*f*(Cos[e/2] + Sin[e/2])*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^11)
```

fricas [B] time = 0.46, size = 291, normalized size = 1.85

$$\frac{152\cos\left(fx+e\right)^{6}-243\cos\left(fx+e\right)^{5}-745\cos\left(fx+e\right)^{4}+455\cos\left(fx+e\right)^{3}+1015\cos\left(fx+\right.}{1155\left(a^{8}f\cos\left(fx+e\right)^{6}-5a^{8}f\cos\left(fx+e\right)^{5}-18a^{8}f\cos\left(fx+e\right)^{4}+20a^{8}f\cos\left(fx+e\right)^{3}+48a^{8}f\cos\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^3/(a+a*sin(f*x+e))^8,x, algorithm="fricas
")
```

```
[Out] 1/1155*(152*cos(f*x + e)^6 - 243*cos(f*x + e)^5 - 745*cos(f*x + e)^4 + 455*
cos(f*x + e)^3 + 1015*cos(f*x + e)^2 + (152*cos(f*x + e)^5 + 395*cos(f*x +
e)^4 - 350*cos(f*x + e)^3 - 805*cos(f*x + e)^2 + 210*cos(f*x + e) + 420)*si
n(f*x + e) - 210*cos(f*x + e) - 420)/(a^8*f*cos(f*x + e)^6 - 5*a^8*f*cos(f*
x + e)^5 - 18*a^8*f*cos(f*x + e)^4 + 20*a^8*f*cos(f*x + e)^3 + 48*a^8*f*cos
(f*x + e)^2 - 16*a^8*f*cos(f*x + e) - 32*a^8*f - (a^8*f*cos(f*x + e)^5 + 6*
a^8*f*cos(f*x + e)^4 - 12*a^8*f*cos(f*x + e)^3 - 32*a^8*f*cos(f*x + e)^2 +
16*a^8*f*cos(f*x + e) + 32*a^8*f)*sin(f*x + e))
```

giac [A] time = 0.48, size = 120, normalized size = 0.76

$$-\frac{4\left(1155\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}-2079\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}+2541\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-825\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+165\tan\right.}{1155a^{8}f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^3/(a+a*sin(f*x+e))^8,x, algorithm="giac")
```

```
[Out] -4/1155*(1155*tan(1/2*f*x + 1/2*e)^7 - 2079*tan(1/2*f*x + 1/2*e)^6 + 2541*t
an(1/2*f*x + 1/2*e)^5 - 825*tan(1/2*f*x + 1/2*e)^4 + 165*tan(1/2*f*x + 1/2*
e)^3 + 55*tan(1/2*f*x + 1/2*e)^2 + 11*tan(1/2*f*x + 1/2*e) + 1)/(a^8*f*(tan
(1/2*f*x + 1/2*e) + 1)^11)
```

maple [A] time = 0.53, size = 130, normalized size = 0.83

$$\frac{\frac{256}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{11}}+\frac{2064}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{7}}-\frac{136}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{6}}-\frac{384}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{8}}-\frac{128}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{10}}+\frac{176}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}+\frac{89}{3\left(\tan\left(\frac{fx}{2}\right.\right.}}{f\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*sin(f*x+e)^3/(a+a*sin(f*x+e))^8,x)
```

```
[Out] 16/f/a^8*(16/11/(tan(1/2*f*x+1/2*e)+1)^11+129/7/(tan(1/2*f*x+1/2*e)+1)^7-17
/2/(tan(1/2*f*x+1/2*e)+1)^6-24/(tan(1/2*f*x+1/2*e)+1)^8-8/(tan(1/2*f*x+1/2*
e)+1)^10+11/5/(tan(1/2*f*x+1/2*e)+1)^5+56/3/(tan(1/2*f*x+1/2*e)+1)^9-1/4/(t
an(1/2*f*x+1/2*e)+1)^4)
```

maxima [B] time = 0.35, size = 401, normalized size = 2.55

$$-\frac{4\left(\frac{11\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{55\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{165\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{825\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{2541\sin\left(\right.}{\left(\cos\left(fx+\right.\right.}\right.}{1155\left(a^8+\frac{11a^8\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{55a^8\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{165a^8\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{330a^8\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{462a^8\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{462a^8\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^3/(a+a*sin(f*x+e))^8,x, algorithm="maxima
")
```

```
[Out] -4/1155*(11*sin(f*x + e)/(cos(f*x + e) + 1) + 55*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 165*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 825*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 2541*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2079*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
1)/((a^8 + 11*a^8*sin(f*x + e)/(cos(f*x + e) + 1) + 55*a^8*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 165*a^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*a^8
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 462*a^8*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 + 462*a^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 330*a^8*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7 + 165*a^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 55
*a^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*a^8*sin(f*x + e)^10/(cos(f*x
+ e) + 1)^10 + a^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11)*f)
```

mupad [B] time = 9.41, size = 205, normalized size = 1.31

$$-\frac{4\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+11\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+55\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+165\cos\left(\frac{e}{2}\right.\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*sin(e + f*x)^3)/(a + a*sin(e + f*x))^8,x)
```

```
[Out] -(4*cos(e/2 + (f*x)/2)^4*(cos(e/2 + (f*x)/2)^7 + 1155*sin(e/2 + (f*x)/2)^7
- 2079*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^6 + 11*cos(e/2 + (f*x)/2)^6*si
n(e/2 + (f*x)/2) + 2541*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^5 - 825*cos
(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^4 + 165*cos(e/2 + (f*x)/2)^4*sin(e/2 +
 (f*x)/2)^3 + 55*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^2))/(1155*a^8*f*(c
os(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2))^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*sin(f*x+e)**3/(a+a*sin(f*x+e))**8,x)
```

```
[Out] Timed out
```

3.443 $\int \cos^4(c+dx)\sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=156

$$-\frac{1472a^3\cos^5(c+dx)}{45045d(a\sin(c+dx)+a)^{5/2}}-\frac{368a^2\cos^5(c+dx)}{9009d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{13ad}+\frac{20\cos^5(c+d}{}$$

```
[Out] -1472/45045*a^3*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-368/9009*a^2*cos(d*x+
c)^5/d/(a+a*sin(d*x+c))^(3/2)-2/13*cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2)/a/d-
46/1287*a*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)+20/143*cos(d*x+c)^5*(a+a*si
n(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.42, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2878, 2856, 2674, 2673}

$$-\frac{368a^2\cos^5(c+dx)}{9009d(a\sin(c+dx)+a)^{3/2}}-\frac{1472a^3\cos^5(c+dx)}{45045d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{13ad}+\frac{20\cos^5(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-1472*a^3*Cos[c + d*x]^5)/(45045*d*(a + a*Sin[c + d*x])^(5/2)) - (368*a^2*
Cos[c + d*x]^5)/(9009*d*(a + a*Sin[c + d*x])^(3/2)) - (46*a*Cos[c + d*x]^5)
/(1287*d*Sqrt[a + a*Sin[c + d*x]]) + (20*Cos[c + d*x]^5*Sqrt[a + a*Sin[c +
d*x]])/(143*d) - (2*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(13*a*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)\sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= -\frac{2\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{13ad}+\frac{2\int\cos^4(c+dx)\left(\frac{3a}{2}\right.}{}\\ &= \frac{20\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{143d}-\frac{2\cos^5(c+dx)(a+a\,\mathrm{si}}{13ad}\\ &= -\frac{46a\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}}+\frac{20\cos^5(c+dx)\sqrt{a+a\sin(c+}}{143d}\\ &= -\frac{368a^2\cos^5(c+dx)}{9009d(a+a\sin(c+dx))^{3/2}}-\frac{46a\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}}+\\ &= -\frac{1472a^3\cos^5(c+dx)}{45045d(a+a\sin(c+dx))^{5/2}}-\frac{368a^2\cos^5(c+dx)}{9009d(a+a\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 3.91, size = 109, normalized size = 0.70

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5(119780\sin(c+dx)-21420\sin(3(c+dx))-62440\,\mathrm{co}}{180180d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/180180*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x]
)]*(81183 - 62440*Cos[2*(c + d*x)] + 3465*Cos[4*(c + d*x)] + 119780*Sin[c +
 d*x] - 21420*Sin[3*(c + d*x)]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.44, size = 172, normalized size = 1.10

$$\frac{2\left(3465\cos\left(dx+c\right)^{7}-315\cos\left(dx+c\right)^{6}-4585\cos\left(dx+c\right)^{5}+115\cos\left(dx+c\right)^{4}-184\cos\left(dx+c\right)^{3}+368\,\mathrm{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2/45045*(3465*cos(d*x + c)^7 - 315*cos(d*x + c)^6 - 4585*cos(d*x + c)^5 + 1
15*cos(d*x + c)^4 - 184*cos(d*x + c)^3 + 368*cos(d*x + c)^2 - (3465*cos(d*x
 + c)^6 + 3780*cos(d*x + c)^5 - 805*cos(d*x + c)^4 - 920*cos(d*x + c)^3 - 1
104*cos(d*x + c)^2 - 1472*cos(d*x + c) - 2944)*sin(d*x + c) - 1472*cos(d*x
+ c) - 2944)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [A] time = 0.29, size = 219, normalized size = 1.40

$$-\frac{1}{1441440}\sqrt{2}\sqrt{a}\left(\frac{4095\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{4}\pi+\frac{11}{2}dx+\frac{11}{2}c\right)}{d}+\frac{12870\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/1441440*sqrt(2)*sqrt(a)*(4095*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/
4*pi + 11/2*d*x + 11/2*c)/d + 12870*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin
(1/4*pi + 7/2*d*x + 7/2*c)/d - 15015*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*si
n(1/4*pi + 3/2*d*x + 3/2*c)/d + 3465*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*si
n(-1/4*pi + 13/2*d*x + 13/2*c)/d + 10010*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*sin(-1/4*pi + 9/2*d*x + 9/2*c)/d - 9009*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*sin(-1/4*pi + 5/2*d*x + 5/2*c)/d - 180180*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)/d)
```

maple [A] time = 0.98, size = 85, normalized size = 0.54

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a\left(\sin\left(dx+c\right)-1\right)^{3}\left(3465\left(\sin^{4}\left(dx+c\right)\right)+10710\left(\sin^{3}\left(dx+c\right)\right)+12145\left(\sin^{2}\left(dx+c\right)\right)+6\right.}{45045\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/45045*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^3*(3465*sin(d*x+c)^4+10710*sin(d*x+
c)^3+12145*sin(d*x+c)^2+6940*sin(d*x+c)+2776)/cos(d*x+c)/(a+a*sin(d*x+c))^(
1/2)/d
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sin(dx+c)+a} \cos(dx+c)^4 \sin(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^4 \sin(c+dx)^2 \sqrt{a+a\sin(c+dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sin(c+dx)+1)} \sin^2(c+dx) \cos^4(c+dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x)**2*cos(c + d*x)**4, x)
```

3.444 $\int \cos^4(c+dx)\sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{64a^3\cos^5(c+dx)}{3465d(a\sin(c+dx)+a)^{5/2}}-\frac{16a^2\cos^5(c+dx)}{693d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}-\frac{2a\cos^5(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -64/3465*a^3*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-16/693*a^2*cos(d*x+c)^5/
d/(a+a*sin(d*x+c))^(3/2)-2/99*a*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)-2/11*
cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.26, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2856, 2674, 2673}

$$-\frac{16a^2\cos^5(c+dx)}{693d(a\sin(c+dx)+a)^{3/2}}-\frac{64a^3\cos^5(c+dx)}{3465d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}-\frac{2a\cos^5(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^5)/(3465*d*(a + a*Sin[c + d*x])^(5/2)) - (16*a^2*Cos[
c + d*x]^5)/(693*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^5)/(99*d
*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(1
1*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11d}+\frac{1}{11}\int \cos^4(c+dx)\sqrt{a\ldots}$$

$$= -\frac{2a\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11d}\ldots$$

$$= -\frac{16a^2\cos^5(c+dx)}{693d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos\ldots}{\ldots}$$

$$= -\frac{64a^3\cos^5(c+dx)}{3465d(a+a\sin(c+dx))^{5/2}}-\frac{16a^2\cos^5(c+dx)}{693d(a+a\sin(c+dx))^{3/2}}-\ldots$$

Mathematica [A] time = 2.02, size = 99, normalized size = 0.80

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5(-5165\sin(c+dx)+315\sin(3(c+dx))+1960\cos(2(c\ldots}{6930d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*(-3648
+ 1960*Cos[2*(c + d*x)] - 5165*Sin[c + d*x] + 315*Sin[3*(c + d*x)]))/(6930*
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.44, size = 151, normalized size = 1.22

$$-\frac{2\left(315\cos(dx+c)^6+350\cos(dx+c)^5-5\cos(dx+c)^4+8\cos(dx+c)^3-16\cos(dx+c)^2+\left(315\cos(dx\ldots\right.\right.}{3465\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/3465*(315*cos(d*x + c)^6 + 350*cos(d*x + c)^5 - 5*cos(d*x + c)^4 + 8*cos
(d*x + c)^3 - 16*cos(d*x + c)^2 + (315*cos(d*x + c)^5 - 35*cos(d*x + c)^4 -
 40*cos(d*x + c)^3 - 48*cos(d*x + c)^2 - 64*cos(d*x + c) - 128)*sin(d*x + c
) + 64*cos(d*x + c) + 128)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin
(d*x + c) + d)
```

giac [A] time = 0.25, size = 189, normalized size = 1.52

$$-\frac{1}{55440}\sqrt{2}\sqrt{a}\left(\frac{385\cos\left(\frac{1}{4}\pi+\frac{9}{2}dx+\frac{9}{2}c\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}+\frac{2079\cos\left(\frac{1}{4}\pi+\frac{5}{2}dx+\frac{5}{2}c\right)\operatorname{sgn}\left(\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] -1/55440*sqrt(2)*sqrt(a)*(385*cos(1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))/d + 2079*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))/d + 6930*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))/d + 315*cos(-1/4*pi + 11/2*d*x + 11/2*c)*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))/d + 1485*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))/d + 2310*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))/d)
```

maple [A] time = 0.88, size = 75, normalized size = 0.60

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a\left(\sin\left(dx+c\right)-1\right)^{3}\left(315\left(\sin^{3}\left(dx+c\right)\right)+980\left(\sin^{2}\left(dx+c\right)\right)+1055\sin\left(dx+c\right)+422\right)}{3465\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3465*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^3*(315*sin(d*x+c)^3+980*sin(d*x+c)^2
+1055*sin(d*x+c)+422)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^4 \sin(c+dx) \sqrt{a+a\sin(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.445 $\int \cos^3(c+dx)\cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=159

$$-\frac{2a\sin^3(c+dx)\cos(c+dx)}{7d\sqrt{a\sin(c+dx)+a}}-\frac{12\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35ad}+\frac{164\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{105d}+\frac{8a}{15d\sqrt{a}}$$

```
[Out] -12/35*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a/d-2*arctanh(cos(d*x+c)*a^(1/2)/(
a+a*sin(d*x+c))^(1/2))*a^(1/2)/d+8/15*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)
-2/7*a*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+164/105*cos(d*x+c)*
(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.49, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2881, 2770, 2759, 2751, 2646, 3046, 2981, 2773, 206}

$$-\frac{2a\sin^3(c+dx)\cos(c+dx)}{7d\sqrt{a\sin(c+dx)+a}}-\frac{12\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35ad}+\frac{164\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{105d}+\frac{8a}{15d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
8*a*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sin[c
 + d*x]^3)/(7*d*Sqrt[a + a*Sin[c + d*x]]) + (164*Cos[c + d*x]*Sqrt[a + a*Si
n[c + d*x]])/(105*d) - (12*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(35*a*d
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
```

```
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc(c+dx)\sqrt{a+$$

$$= -\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)\sqrt{a+a\sin(c+}{3d}$$

$$= \frac{4a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}} + \frac{4\,\mathrm{co}}{}$$

$$= \frac{4a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}} + \frac{164}{}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{8a\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - $$

Mathematica [A] time = 0.33, size = 195, normalized size = 1.23

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(-525\sin\left(\frac{1}{2}(c+dx)\right)+175\sin\left(\frac{3}{2}(c+dx)\right)-21\sin\left(\frac{5}{2}(c+dx)\right)+15\sin\left(\frac{7}{2}(c+dx)\right)+52}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(525*Cos[(c + d*x)/2] + 175*Cos[(3*(c + d*x))/2
] + 21*Cos[(5*(c + d*x))/2] + 15*Cos[(7*(c + d*x))/2] - 420*Log[1 + Cos[(c
```

```
+ d*x)/2] - Sin[(c + d*x)/2]] + 420*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] - 525*Sin[(c + d*x)/2] + 175*Sin[(3*(c + d*x))/2] - 21*Sin[(5*(c + d*
x))/2] + 15*Sin[(7*(c + d*x))/2]))/(420*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]))
```

fricas [B] time = 0.46, size = 294, normalized size = 1.85

$$\frac{105\sqrt{a}\left(\cos\left(dx+c\right)+\sin\left(dx+c\right)+1\right)\log\left(\frac{a\cos\left(dx+c\right)^{3}-7\,a\cos\left(dx+c\right)^{2}-4\left(\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)+3\right)\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)-3\right)\sqrt{a}}{\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/210*(105*sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3
- 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c)
- 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) +
 (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^
3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1))
 + 4*(15*cos(d*x + c)^4 + 18*cos(d*x + c)^3 + 34*cos(d*x + c)^2 + (15*cos(d
*x + c)^3 - 3*cos(d*x + c)^2 + 31*cos(d*x + c) - 43)*sin(d*x + c) + 74*cos(
d*x + c) + 43)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c) + d*sin(d*x + c) +
 d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [A] time = 1.33, size = 141, normalized size = 0.89

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(105a^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)-15\left(a-a\sin\left(dx+c\right)\right)^{\frac{7}{2}}+63\left(a-a\right.\right.}{105a^{3}\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/105*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(105*a^(7/2)*arctanh((a-a*s
in(d*x+c))^(1/2)/a^(1/2))-15*(a-a*sin(d*x+c))^(7/2)+63*(a-a*sin(d*x+c))^(5/
2)*a-35*(a-a*sin(d*x+c))^(3/2)*a^2-105*a^3*(a-a*sin(d*x+c))^(1/2))/a^3/cos(
d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{4}\csc\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x),x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\cos^{4}\left(c+dx\right)\csc\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**4*csc(c + d*x), x)
```

3.446 $\int \cos^2(c+dx)\cot^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5ad}+\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}+\frac{61a\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{d}$$

```
[Out] -2/5*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a/d-arctanh(cos(d*x+c)*a^(1/2)/(a+a*
sin(d*x+c))^(1/2))*a^(1/2)/d+61/15*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+4/
15*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d-cot(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.48, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2881, 2759, 2751, 2646, 3044, 2981, 2773, 206}

$$-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5ad}+\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}+\frac{61a\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d) + (
61*a*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) + (4*Cos[c + d*x]*Sqrt[a
 + a*Sin[c + d*x]])/(15*d) - (Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d - (2
*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(5*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^2(c+dx)\sqrt{a+$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} - \frac{2\cos(c+dx)(a+a\sin(c}{5ad}$$

$$= \frac{5a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d} -$$

$$= \frac{61a\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{61a\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} + \frac{4c}{}$$

Mathematica [A] time = 0.74, size = 258, normalized size = 1.74

$$\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(155\sin\left(\frac{1}{2}(c+dx)\right)+87\sin\left(\frac{3}{2}(c+dx)\right)-5\sin\left(\frac{5}{2}(c+dx)\right)+3\sin\left(\frac{7}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(-155*Cos[(c + d*x)/2] + 87*
Cos[(3*(c + d*x))/2] + 5*Cos[(5*(c + d*x))/2] + 3*Cos[(7*(c + d*x))/2] + 15
5*Sin[(c + d*x)/2] - 30*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c
+ d*x] + 30*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 87*
Sin[(3*(c + d*x))/2] - 5*Sin[(5*(c + d*x))/2] + 3*Sin[(7*(c + d*x))/2]))/(3
0*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c +
d*x)/4] + Sec[(c + d*x)/4]))
```

fricas [B] time = 0.49, size = 320, normalized size = 2.16

$$\frac{15\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx\right.}{\cos(dx+c)^3+\mathrm{co}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log
((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c)
 + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) -
 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c)
- a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) -
 cos(d*x + c) - 1)) - 4*(6*cos(d*x + c)^4 + 8*cos(d*x + c)^3 + 40*cos(d*x +
 c)^2 + (6*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + 38*cos(d*x + c) + 61)*sin(d*
x + c) - 23*cos(d*x + c) - 61)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2
- (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.28, size = 162, normalized size = 1.09

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)\left(6(a-a\sin(dx+c))^{\frac{5}{2}}a^{\frac{3}{2}}-20(a-a\sin(dx+c))^{\frac{3}{2}}a^{\frac{5}{2}}-\right.\right.}{15a^{\frac{7}{2}}\sin(dx+c)\cos(dx+c)\sqrt{a+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/15*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(sin(d*x+c)*(6*(a-a*
sin(d*x+c))^(5/2)*a^(3/2)-20*(a-a*sin(d*x+c))^(3/2)*a^(5/2)-30*(a-a*sin(d*x
+c))^(1/2)*a^(7/2)+15*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^4)+15*(a-a*
sin(d*x+c))^(1/2)*a^(7/2))/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin(dx+c)+a}\,\cos(dx+c)^4\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4\,\sqrt{a+a\,\sin(c+d\,x)}}{\sin(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^2,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.447 $\int \cos(c+dx)\cot^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=156

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{13\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}$$

```
[Out] 13/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-2/3*a*cos
(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/4*a*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-
2/3*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+a*sin(
d*x+c))^(1/2)/d
```

Rubi [A] time = 0.41, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2881, 2751, 2646, 3044, 2980, 2773, 206}

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{13\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (13*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
 - (2*a*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x])/(4*
d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*
d) - (Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^3(c+dx)\sqrt{a+$$

$$= -\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{\cot(c+dx)\csc(c+dx}{2d}$$

$$= -\frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c}{}$$

$$= -\frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c}{}$$

$$= \frac{13\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{}{4d}$$

Mathematica [A] time = 0.90, size = 297, normalized size = 1.90

$$\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(26\sin\left(\frac{1}{2}(c+dx)\right)-14\sin\left(\frac{3}{2}(c+dx)\right)-12\sin\left(\frac{5}{2}(c+dx)\right)+4\sin\left(\frac{7}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(-26*Cos[(c + d*x)/2] - 14*C
os[(3*(c + d*x))/2] + 12*Cos[(5*(c + d*x))/2] + 4*Cos[(7*(c + d*x))/2] + 39
*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 39*Cos[2*(c + d*x)]*Log[1 +
 Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 39*Log[1 - Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] + 39*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]] + 26*Sin[(c + d*x)/2] - 14*Sin[(3*(c + d*x))/2] - 12*Sin[(5*(c + d*x))
/2] + 4*Sin[(7*(c + d*x))/2]))/(12*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/
4]^2 - Sec[(c + d*x)/4]^2)^2)
```

fricas [B] time = 0.47, size = 359, normalized size = 2.30

$$\frac{39\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(39*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x +
c) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 +
 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8
*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (
cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(8*cos(d*x + c)^4
 + 16*cos(d*x + c)^3 - 9*cos(d*x + c)^2 + (8*cos(d*x + c)^3 - 8*cos(d*x + c
)^2 - 17*cos(d*x + c) + 5)*sin(d*x + c) - 22*cos(d*x + c) - 5)*sqrt(a*sin(d
*x + c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*co
s(d*x + c)^2 - d)*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.39, size = 178, normalized size = 1.14

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(24a^{\frac{3}{2}}\sqrt{-a(\sin(dx+c)-1)}\left(\sin^{2}(dx+c)\right)-8(-a(\sin(dx+c)-1))\right.}{12a^{\frac{3}{2}}\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/12*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(24*a^(3/2)*(-a*(sin(d*x+c)-
1))^(1/2)*sin(d*x+c)^2-8*(-a*(sin(d*x+c)-1))^(3/2)*sin(d*x+c)^2*a^(1/2)-39*
arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+15*(-a*(sin(d*x
+c)-1))^(1/2)*a^(3/2)-9*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2))/a^(3/2)/sin(d*x+
c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin(dx+c)+a}\,\cos(dx+c)^{4}\csc(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^3,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.448 $\int \cot^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=163

$$-\frac{2a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{11a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+}}{3d}$$

```
[Out] 11/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-2*a*cos(d
*x+c)/d/(a+a*sin(d*x+c))^(1/2)+11/8*a*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1
/12*a*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/3*cot(d*x+c)*csc(d*x
+c)^2*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.39, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2718, 2646, 3044, 2980, 2772, 2773, 206}

$$-\frac{2a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{11a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 - (2*a*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) + (11*a*Cot[c + d*x])/(8
*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x])/(12*d*Sqrt[a +
 a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]])/
(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2718

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
```

```
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \sqrt{a+a\sin(c+dx)}\,dx + \int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,\left(1-2\sin^2\right.$$

$$= -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \ldots$$

$$= -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+\ldots}{}$$

$$= -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{11a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+d\ldots}{12d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{11a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+d\ldots}{12d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{11\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{11a\cot(c+d\ldots}{8d\sqrt{a+a\sin(c+\ldots}}$$

Mathematica [A] time = 1.42, size = 309, normalized size = 1.90

$$\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-252\sin\left(\frac{1}{2}(c+dx)\right)-250\sin\left(\frac{3}{2}(c+dx)\right)+114\sin\left(\frac{5}{2}(c+dx)\right)+48\sin\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(252*Cos[(c + d*x)/2] - 250
*Cos[(3*(c + d*x))/2] - 114*Cos[(5*(c + d*x))/2] + 48*Cos[(7*(c + d*x))/2]
- 252*Sin[(c + d*x)/2] + 99*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Si
n[c + d*x] - 99*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] -
 250*Sin[(3*(c + d*x))/2] + 114*Sin[(5*(c + d*x))/2] - 33*Log[1 + Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 33*Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 48*Sin[(7*(c + d*x))/2]))/(24*d*(1 +
 Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3)
```

fricas [B] time = 0.49, size = 380, normalized size = 2.33

$$33\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 - \left(\cos(dx+c)^3 + \cos(dx+c)^2 - \cos(dx+c) - 1\right)\sin(dx+c) + 1\right)\sqrt{a}\,\log\left(\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(33*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c
)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2
*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a
*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
4*(48*cos(d*x + c)^4 - 33*cos(d*x + c)^3 - 139*cos(d*x + c)^2 + (48*cos(d*x
 + c)^3 + 81*cos(d*x + c)^2 - 58*cos(d*x + c) - 83)*sin(d*x + c) + 25*cos(d
*x + c) + 83)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c
)^2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x +
c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.38, size = 170, normalized size = 1.04

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(48\sqrt{-a(\sin(dx+c)-1)}\,a^{\frac{7}{2}}\left(\sin^3(dx+c)\right)-15\sqrt{-a(\sin(dx+c)-}\right.}{24a^{\frac{7}{2}}\sin(dx+c)^3\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(48*(-a*(sin(d*x+c)-1))^(1/2
)*a^(7/2)*sin(d*x+c)^3-15*(-a*(sin(d*x+c)-1))^(1/2)*a^(7/2)+56*(-a*(sin(d*x
+c)-1))^(3/2)*a^(5/2)-33*(-a*(sin(d*x+c)-1))^(5/2)*a^(3/2)-33*arctanh((-a*(
sin(d*x+c)-1))^(1/2)/a^(1/2))*a^4*sin(d*x+c)^3)/a^(7/2)/sin(d*x+c)^3/cos(d*
x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin(dx+c)+a}\,\cos(dx+c)^4\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^4,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.449 $\int \cot^4(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=173

$$\frac{61a\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{67\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}-\frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}-\frac{a\cot(c+d}{24d\sqrt{a\sin}}$$

```
[Out] -67/64*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d+61/64*a
*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+61/96*a*cot(d*x+c)*csc(d*x+c)/d/(a+a*s
in(d*x+c))^(1/2)-1/24*a*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)-1/
4*cot(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.54, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2881, 2773, 206, 3044, 2980, 2772}

$$\frac{61a\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{67\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}-\frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}-\frac{a\cot(c+d}{24d\sqrt{a\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-67*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*
d) + (61*a*Cot[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (61*a*Cot[c + d*
x]*Csc[c + d*x])/(96*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c +
d*x]^2)/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^3*Sqrt
[a + a*Sin[c + d*x]])/(4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^5(c+dx)\sqrt{a+
$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} + \frac{\int \csc^4(c+
$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a+a\sin(c+dx)}}
$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{61a\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}}
$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{61a\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} +
$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{61a\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} +
$$

$$= -\frac{67\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} + \frac{61a\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} +
$$

Mathematica [B] time = 2.74, size = 367, normalized size = 2.12

$$-\frac{\csc^{13}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-442\sin\left(\frac{1}{2}(c+dx)\right)-162\sin\left(\frac{3}{2}(c+dx)\right)-122\sin\left(\frac{5}{2}(c+dx)\right)+366
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/192*(Csc[(c + d*x)/2]^13*Sqrt[a*(1 + Sin[c + d*x])]*(442*Cos[(c + d*x)/2
] - 162*Cos[(3*(c + d*x))/2] + 122*Cos[(5*(c + d*x))/2] + 366*Cos[(7*(c + d
*x))/2] + 603*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 804*Cos[2*(c +
 d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 201*Cos[4*(c + d*x)]*
Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 603*Log[1 - Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] + 804*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] - 201*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]] - 442*Sin[(c + d*x)/2] - 162*Sin[(3*(c + d*x))/2] - 122*Sin[(5*(c +
d*x))/2] + 366*Sin[(7*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d
*x)/4]^2 - Sec[(c + d*x)/4]^2)^4)
```

fricas [B] time = 0.46, size = 415, normalized size = 2.40

$$\frac{201\left(\cos\left(dx+c\right)^{5}+\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{3}-2\,\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\sin(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/768*(201*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x
+ c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(d*x + c
) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)
^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x +
 c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c)
- a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 -
 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(183*cos(d*x + c)^4 + 122*cos(d*x
 + c)^3 - 188*cos(d*x + c)^2 + (183*cos(d*x + c)^3 + 61*cos(d*x + c)^2 - 12
7*cos(d*x + c) - 53)*sin(d*x + c) - 74*cos(d*x + c) + 53)*sqrt(a*sin(d*x +
c) + a))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 - 2*d*co
s(d*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
*sin(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.41, size = 162, normalized size = 0.94

$$\frac{\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(-201\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^{4}\left(\sin^{4}\left(dx+c\right)\right)+201\sqrt{-a\left(\sin\left(dx+\cdots\right.\right.}\right.}{192a^{\frac{7}{2}}\sin\left(dx+c\right)^{4}\cos\left(dx+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/192*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-201*arctanh((-a*(sin(d*x+c
)-1))^(1/2)/a^(1/2))*a^4*sin(d*x+c)^4+201*(-a*(sin(d*x+c)-1))^(1/2)*a^(7/2)
```

```
-737*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)+671*(-a*(sin(d*x+c)-1))^(5/2)*a^(3/2
)-183*(-a*(sin(d*x+c)-1))^(7/2)*a^(1/2))/a^(7/2)/sin(d*x+c)^4/cos(d*x+c)/(a
+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sin (dx + c) + a} \cos (dx + c)^4 \csc (dx + c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c + d\,x)^4 \sqrt{a + a\,\sin (c + d\,x)}}{\sin (c + d\,x)^5} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^5,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.450 $\int \cot^4(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=209

$$-\frac{31a\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}-\frac{31\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128d}-\frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{a\cot(c+}{40d\sqrt{a\text{s}}}$$

```
[Out] -31/128*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-31/128
*a*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+97/192*a*cot(d*x+c)*csc(d*x+c)/d/(a+
a*sin(d*x+c))^(1/2)+97/240*a*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/
2)-1/40*a*cot(d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-1/5*cot(d*x+c)*c
sc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.69, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2881, 2772, 2773, 206, 3044, 2980}

$$-\frac{31a\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}-\frac{31\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128d}-\frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{a\cot(c+}{40d\sqrt{a\text{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-31*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(128
*d) - (31*a*Cot[c + d*x])/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (97*a*Cot[c +
d*x]*Csc[c + d*x])/(192*d*Sqrt[a + a*Sin[c + d*x]]) + (97*a*Cot[c + d*x]*Cs
c[c + d*x]^2)/(240*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*
x]^3)/(40*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^4*Sqrt[a
 + a*Sin[c + d*x]])/(5*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^6(c+dx)\sqrt{a+$$

$$= -\frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a+a\sin}}{5d}$$

$$= -\frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^3(c+dx)}{40d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c}{}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{97a\,c}{240}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{97a\,c}{192}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{31a\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} + \frac{97}{}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{31a\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} + \frac{97}{}$$

$$= -\frac{31\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128d} - \frac{31a\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} +$$

Mathematica [**A**] time = 4.44, size = 403, normalized size = 1.93

$$-\frac{\csc^{16}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-10180\sin\left(\frac{1}{2}(c+dx)\right)-2240\sin\left(\frac{3}{2}(c+dx)\right)+1392\sin\left(\frac{5}{2}(c+dx)\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/1920*(Csc[(c + d*x)/2]^16*Sqrt[a*(1 + Sin[c + d*x])]*(10180*Cos[(c + d*x
)/2] - 2240*Cos[(3*(c + d*x))/2] - 1392*Cos[(5*(c + d*x))/2] + 4810*Cos[(7*
(c + d*x))/2] + 930*Cos[(9*(c + d*x))/2] - 10180*Sin[(c + d*x)/2] + 4650*Lo
g[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 4650*Log[1 - Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 2240*Sin[(3*(c + d*x))/2] +
 1392*Sin[(5*(c + d*x))/2] - 2325*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/
```

```
2]]*Sin[3*(c + d*x)] + 2325*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Si
n[3*(c + d*x)] + 4810*Sin[(7*(c + d*x))/2] - 930*Sin[(9*(c + d*x))/2] + 465
*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 465*Log[1
- Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(d*(1 + Cot[(c +
d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^5)
```

fricas [B] time = 0.47, size = 461, normalized size = 2.21

$$\frac{465\left(\cos(dx+c)^6 - 3\cos(dx+c)^4 + 3\cos(dx+c)^2 - \left(\cos(dx+c)^5 + \cos(dx+c)^4 - 2\cos(dx+c)^3 - 2\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/7680*(465*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos(d*
x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x +
 c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)
^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(465*cos(d*x
+ c)^5 + 1435*cos(d*x + c)^4 - 154*cos(d*x + c)^3 - 1662*cos(d*x + c)^2 - (
465*cos(d*x + c)^4 - 970*cos(d*x + c)^3 - 1124*cos(d*x + c)^2 + 538*cos(d*x
 + c) + 611)*sin(d*x + c) + 73*cos(d*x + c) + 611)*sqrt(a*sin(d*x + c) + a)
)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - (d*cos(d*x
+ c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 + d*cos
(d*x + c) + d)*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.40, size = 180, normalized size = 0.86

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(465(-a(\sin(dx+c)-1))^{\frac{9}{2}}a^{\frac{3}{2}} + 465\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^6\left(\mathrm{s}\right.\right.}{1920a^{\frac{11}{2}}\,\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/1920*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(11/2)*(465*(-a*(sin(d*x
+c)-1))^(9/2)*a^(3/2)+465*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^6*si
n(d*x+c)^5-890*(-a*(sin(d*x+c)-1))^(7/2)*a^(5/2)-896*(-a*(sin(d*x+c)-1))^(5
/2)*a^(7/2)+2170*(-a*(sin(d*x+c)-1))^(3/2)*a^(9/2)-465*(-a*(sin(d*x+c)-1))^
(1/2)*a^(11/2))/sin(d*x+c)^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sin(dx+c)+a}\cos(dx+c)^4\csc(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\,\sqrt{a+a\,\sin(c+d\,x)}}{\sin(c+d\,x)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^6,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.451 $\int \cot^4(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=245

$$-\frac{55a\cot(c+dx)}{512d\sqrt{a\sin(c+dx)+a}}-\frac{55\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{512d}-\frac{\cot(c+dx)\csc^5(c+dx)\sqrt{a\sin(c+dx)+a}}{6d}-\frac{a\cot(c}{60d\sqrt{a}}$$

```
[Out] -55/512*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-55/512
*a*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-55/768*a*cot(d*x+c)*csc(d*x+c)/d/(a+
a*sin(d*x+c))^(1/2)+329/960*a*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1
/2)+47/160*a*cot(d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-1/60*a*cot(d*
x+c)*csc(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)-1/6*cot(d*x+c)*csc(d*x+c)^5*(a+a
*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.81, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2881, 2772, 2773, 206, 3044, 2980}

$$-\frac{55a\cot(c+dx)}{512d\sqrt{a\sin(c+dx)+a}}-\frac{55\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{512d}-\frac{\cot(c+dx)\csc^5(c+dx)\sqrt{a\sin(c+dx)+a}}{6d}-\frac{a\cot(c}{60d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-55*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(512
*d) - (55*a*Cot[c + d*x])/(512*d*Sqrt[a + a*Sin[c + d*x]]) - (55*a*Cot[c +
d*x]*Csc[c + d*x])/(768*d*Sqrt[a + a*Sin[c + d*x]]) + (329*a*Cot[c + d*x]*C
sc[c + d*x]^2)/(960*d*Sqrt[a + a*Sin[c + d*x]]) + (47*a*Cot[c + d*x]*Csc[c
+ d*x]^3)/(160*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^4
)/(60*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^5*Sqrt[a + a
*Sin[c + d*x]])/(6*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
```

```
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^7(c+dx)\sqrt{a$$

$$= -\frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^5(c+dx)\sqrt{a+}}{6d}$$

$$= -\frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot}{60a}$$

$$= -\frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{47a}{16}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a}{}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{5}{}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{55a\cot(c+dx)}{512d\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{55a\cot(c+dx)}{512d\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{55\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{512d} - \frac{55a\cot(c+dx)}{512d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 7.69, size = 485, normalized size = 1.98

$$\frac{\csc^{19}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(24540\sin\left(\frac{1}{2}(c+dx)\right)-25684\sin\left(\frac{3}{2}(c+dx)\right)+14490\sin\left(\frac{5}{2}(c+dx)\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^19*Sqrt[a*(1 + Sin[c + d*x])]*(-24540*Cos[(c + d*x)/2] -
25684*Cos[(3*(c + d*x))/2] - 14490*Cos[(5*(c + d*x))/2] - 15006*Cos[(7*(c +
 d*x))/2] - 550*Cos[(9*(c + d*x))/2] - 1650*Cos[(11*(c + d*x))/2] - 8250*Lo
g[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12375*Cos[2*(c + d*x)]*Log[1 +
```

```
 Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4950*Cos[4*(c + d*x)]*Log[1 + Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 825*Cos[6*(c + d*x)]*Log[1 + Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]] + 8250*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
] - 12375*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4
950*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 825*Cos
[6*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24540*Sin[(c +
 d*x)/2] - 25684*Sin[(3*(c + d*x))/2] + 14490*Sin[(5*(c + d*x))/2] - 15006*
Sin[(7*(c + d*x))/2] + 550*Sin[(9*(c + d*x))/2] - 1650*Sin[(11*(c + d*x))/2
]))/(7680*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2
)^6)
```

fricas [B] time = 0.49, size = 525, normalized size = 2.14

$$\frac{825\left(\cos\left(dx+c\right)^{7}+\cos\left(dx+c\right)^{6}-3\,\cos\left(dx+c\right)^{5}-3\,\cos\left(dx+c\right)^{4}+3\,\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+\left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/30720*(825*(cos(d*x + c)^7 + cos(d*x + c)^6 - 3*cos(d*x + c)^5 - 3*cos(d*
x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^6 - 3*cos(d*
x + c)^4 + 3*cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x +
c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
 - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c
) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c)
 - cos(d*x + c) - 1)) + 4*(825*cos(d*x + c)^6 + 550*cos(d*x + c)^5 + 707*co
s(d*x + c)^4 + 1156*cos(d*x + c)^3 - 225*cos(d*x + c)^2 + (825*cos(d*x + c)
^5 + 275*cos(d*x + c)^4 + 982*cos(d*x + c)^3 - 174*cos(d*x + c)^2 - 399*cos
(d*x + c) + 27)*sin(d*x + c) - 426*cos(d*x + c) - 27)*sqrt(a*sin(d*x + c) +
 a))/(d*cos(d*x + c)^7 + d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^5 - 3*d*cos(d*
x + c)^4 + 3*d*cos(d*x + c)^3 + 3*d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*co
s(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c) -
d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.32, size = 198, normalized size = 0.81

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(-825\sqrt{-a(\sin(dx+c)-1)}\,a^{\frac{15}{2}}+4675(-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{13}{2}}+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/7680*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-825*(-a*(sin(d*x+c)-1))^
(1/2)*a^(15/2)+4675*(-a*(sin(d*x+c)-1))^(3/2)*a^(13/2)+1398*(-a*(sin(d*x+c)
-1))^(5/2)*a^(11/2)-7818*(-a*(sin(d*x+c)-1))^(7/2)*a^(9/2)+4675*(-a*(sin(d*
x+c)-1))^(9/2)*a^(7/2)-825*(-a*(sin(d*x+c)-1))^(11/2)*a^(5/2)+825*arctanh((
-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^8*sin(d*x+c)^6)/a^(15/2)/sin(d*x+c)^6/c
os(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin(dx+c)+a}\,\cos(dx+c)^4\csc(dx+c)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\sqrt{a+a\,\sin(c+d\,x)}}{\sin(c+d\,x)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^7,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^7, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.452 $\int \cot^4(c+dx)\csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=281

$$-\frac{61a\cot(c+dx)}{1024d\sqrt{a\sin(c+dx)+a}}-\frac{61\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{1024d}-\frac{\cot(c+dx)\csc^6(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}-\frac{a\cot(c+}{84d\sqrt{a}}$$

```
[Out] -61/1024*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-61/10
24*a*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-61/1536*a*cot(d*x+c)*csc(d*x+c)/d/
(a+a*sin(d*x+c))^(1/2)-61/1920*a*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))
^(1/2)+579/2240*a*cot(d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+193/840*
a*cot(d*x+c)*csc(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)-1/84*a*cot(d*x+c)*csc(d*
x+c)^5/d/(a+a*sin(d*x+c))^(1/2)-1/7*cot(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c)
)^(1/2)/d
```

Rubi [A] time = 0.95, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2881, 2772, 2773, 206, 3044, 2980}

$$-\frac{61a\cot(c+dx)}{1024d\sqrt{a\sin(c+dx)+a}}-\frac{61\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{1024d}-\frac{\cot(c+dx)\csc^6(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}-\frac{a\cot(c+}{84d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-61*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(102
4*d) - (61*a*Cot[c + d*x])/(1024*d*Sqrt[a + a*Sin[c + d*x]]) - (61*a*Cot[c
+ d*x]*Csc[c + d*x])/(1536*d*Sqrt[a + a*Sin[c + d*x]]) - (61*a*Cot[c + d*x]
*Csc[c + d*x]^2)/(1920*d*Sqrt[a + a*Sin[c + d*x]]) + (579*a*Cot[c + d*x]*Cs
c[c + d*x]^3)/(2240*d*Sqrt[a + a*Sin[c + d*x]]) + (193*a*Cot[c + d*x]*Csc[c
 + d*x]^4)/(840*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^
5)/(84*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^6*Sqrt[a +
a*Sin[c + d*x]])/(7*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^8(c+dx)\sqrt{a+$$

$$= -\frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^6(c+dx)\sqrt{a+}}{7d}$$

$$= -\frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot}{3d}$$

$$= -\frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot}{3d}$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{5a}{1}$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{61}{15}$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{61a\cot(c+dx)}{1024d\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{61a\cot(c+dx)}{1024d\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{61\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{1024d} - \frac{61a\cot(c+dx)}{1024d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 2.03, size = 191, normalized size = 0.68

$$\sqrt{a(\sin(c+dx)+1)}\left(-102480\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+1\right)+102480\log\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(-102480*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 102480*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Csc[c + d*x]
```

```
^7*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(-201298 - 244533*Cos[2*(c + d*x)]
 - 52094*Cos[4*(c + d*x)] + 6405*Cos[6*(c + d*x)] + 49128*Sin[c + d*x] - 17
9636*Sin[3*(c + d*x)] - 8540*Sin[5*(c + d*x)])))/(3440640*d*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]))
```

fricas [B] time = 0.52, size = 567, normalized size = 2.02

$$\frac{6405\left(\cos\left(dx+c\right)^{8}-4\,\cos\left(dx+c\right)^{6}+6\,\cos\left(dx+c\right)^{4}-4\,\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{7}+\cos\left(dx+c\right)^{6}-3\,\mathrm{co}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/430080*(6405*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*co
s(d*x + c)^2 - (cos(d*x + c)^7 + cos(d*x + c)^6 - 3*cos(d*x + c)^5 - 3*cos(
d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 - cos(d*x + c) - 1)*sin(d*
x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x
 + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(
d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x
+ c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c
)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(6405*cos(d*x + c)^7 + 2135*
cos(d*x + c)^6 - 22631*cos(d*x + c)^5 - 37613*cos(d*x + c)^4 + 1343*cos(d*x
 + c)^3 + 27477*cos(d*x + c)^2 - (6405*cos(d*x + c)^6 + 4270*cos(d*x + c)^5
 - 18361*cos(d*x + c)^4 + 19252*cos(d*x + c)^3 + 20595*cos(d*x + c)^2 - 688
2*cos(d*x + c) - 7359)*sin(d*x + c) - 477*cos(d*x + c) - 7359)*sqrt(a*sin(d
*x + c) + a))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 -
 4*d*cos(d*x + c)^2 - (d*cos(d*x + c)^7 + d*cos(d*x + c)^6 - 3*d*cos(d*x +
c)^5 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^3 + 3*d*cos(d*x + c)^2 - d*cos
(d*x + c) - d)*sin(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.39, size = 216, normalized size = 0.77

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left(6405(-a(\sin(dx+c)-1))^{\frac{13}{2}}a^{\frac{7}{2}}-42700(-a(\sin(dx+c)-1))^{\frac{11}{2}}a^{\frac{9}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/107520*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(19/2)*(6405*(-a*(sin(
d*x+c)-1))^(13/2)*a^(7/2)-42700*(-a*(sin(d*x+c)-1))^(11/2)*a^(9/2)+120841*(
-a*(sin(d*x+c)-1))^(9/2)*a^(11/2)+6405*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^
(1/2))*a^10*sin(d*x+c)^7-156672*(-a*(sin(d*x+c)-1))^(7/2)*a^(13/2)+51191*(-
a*(sin(d*x+c)-1))^(5/2)*a^(15/2)+42700*(-a*(sin(d*x+c)-1))^(3/2)*a^(17/2)-6
405*(-a*(sin(d*x+c)-1))^(1/2)*a^(19/2))/sin(d*x+c)^7/cos(d*x+c)/(a+a*sin(d*
x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin(dx+c)+a}\,\cos(dx+c)^4\csc(dx+c)^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\,\sqrt{a+a\,\sin(c+d\,x)}}{\sin(c+d\,x)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^8,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^8, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.453 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=188

$$-\frac{256a^4\cos^5(c+dx)}{6435d(a\sin(c+dx)+a)^{5/2}}-\frac{64a^3\cos^5(c+dx)}{1287d(a\sin(c+dx)+a)^{3/2}}-\frac{56a^2\cos^5(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos^5(c+dx)(a\sin(c+}{15ad}$$

```
[Out] -256/6435*a^4*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-64/1287*a^3*cos(d*x+c)^
5/d/(a+a*sin(d*x+c))^(3/2)+4/39*cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2)/d-2/15*
cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2)/a/d-56/1287*a^2*cos(d*x+c)^5/d/(a+a*sin
(d*x+c))^(1/2)-14/429*a*cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.51, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2878, 2856, 2674, 2673}

$$-\frac{56a^2\cos^5(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^5(c+dx)}{1287d(a\sin(c+dx)+a)^{3/2}}-\frac{256a^4\cos^5(c+dx)}{6435d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)(a\sin(c+}{15ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^5)/(6435*d*(a + a*Sin[c + d*x])^(5/2)) - (64*a^3*Cos
[c + d*x]^5)/(1287*d*(a + a*Sin[c + d*x])^(3/2)) - (56*a^2*Cos[c + d*x]^5)/
(1287*d*Sqrt[a + a*Sin[c + d*x]]) - (14*a*Cos[c + d*x]^5*Sqrt[a + a*Sin[c +
 d*x]])/(429*d) + (4*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(39*d) - (2
*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2))/(15*a*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos^5(c+dx)(a+a\sin(c+dx))^{5/2}}{15ad} + \frac{2\int \cos^4(c+dx)\Big(}{}$$

$$= \frac{4\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{39d} - \frac{2\cos^5(c+dx)(a+a}{15ad}$$

$$= -\frac{14a\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{429d} + \frac{4\cos^5(c+dx)(a-}{39}$$

$$= -\frac{56a^2\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}} - \frac{14a\cos^5(c+dx)\sqrt{a+a\sin(}}{429d}$$

$$= -\frac{64a^3\cos^5(c+dx)}{1287d(a+a\sin(c+dx))^{3/2}} - \frac{56a^2\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{256a^4\cos^5(c+dx)}{6435d(a+a\sin(c+dx))^{5/2}} - \frac{64a^3\cos^5(c+dx)}{1287d(a+a\sin(c+dx))^{3/}}$$

Mathematica [A] time = 9.08, size = 120, normalized size = 0.64

$$-\frac{a\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5(66470\sin(c+dx)-14445\sin(3(c+dx))+429\sin(}{51480d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/51480*(a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x
])]*(43122 - 36640*Cos[2*(c + d*x)] + 3630*Cos[4*(c + d*x)] + 66470*Sin[c +
 d*x] - 14445*Sin[3*(c + d*x)] + 429*Sin[5*(c + d*x)]))/(d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.46, size = 210, normalized size = 1.12

$$\frac{2\left(429\,a\cos\left(dx+c\right)^{8}+957\,a\cos\left(dx+c\right)^{7}-633\,a\cos\left(dx+c\right)^{6}-1301\,a\cos\left(dx+c\right)^{5}+20\,a\cos\left(dx+c\right)^{4}-32\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 2/6435*(429*a*cos(d*x + c)^8 + 957*a*cos(d*x + c)^7 - 633*a*cos(d*x + c)^6
- 1301*a*cos(d*x + c)^5 + 20*a*cos(d*x + c)^4 - 32*a*cos(d*x + c)^3 + 64*a*
cos(d*x + c)^2 - 256*a*cos(d*x + c) + (429*a*cos(d*x + c)^7 - 528*a*cos(d*x
 + c)^6 - 1161*a*cos(d*x + c)^5 + 140*a*cos(d*x + c)^4 + 160*a*cos(d*x + c)
^3 + 192*a*cos(d*x + c)^2 + 256*a*cos(d*x + c) + 512*a)*sin(d*x + c) - 512*
a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [B] time = 1.43, size = 474, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/2882880*sqrt(2)*(3465*a*cos(1/4*pi + 13/2*d*x + 13/2*c)*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))/d + 5005*a*cos(1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))/d - 27027*a*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))/d - 135135*a*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))/d + 3003*a*cos(-1/4*pi + 15/2*d*x + 15/2*c)*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 4095*a*cos(-1/4*pi + 11/2*d*x + 11/2*c
)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 19305*a*cos(-1/4*pi + 7/2*d*x + 7
/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 45045*a*cos(-1/4*pi + 3/2*d*x
 + 3/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 8190*a*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))*sin(1/4*pi + 11/2*d*x + 11/2*c)/d - 25740*a*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 7/2*d*x + 7/2*c)/d + 30030*a*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 3/2*d*x + 3/2*c)/d - 6930*a*sgn(cos(
```

```
-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 13/2*d*x + 13/2*c)/d - 20020*a*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 9/2*d*x + 9/2*c)/d + 18018*
a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 5/2*d*x + 5/2*c)/d + 36
0360*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)/d
)*sqrt(a)
```

maple [A] time = 1.11, size = 97, normalized size = 0.52

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a^{2}\left(\sin\left(dx+c\right)-1\right)^{3}\left(429\left(\sin^{5}\left(dx+c\right)\right)+1815\left(\sin^{4}\left(dx+c\right)\right)+3075\left(\sin^{3}\left(dx+c\right)\right)+2765\right.}{6435\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/6435*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^3*(429*sin(d*x+c)^5+1815*sin(d*x+c
)^4+3075*sin(d*x+c)^3+2765*sin(d*x+c)^2+1580*sin(d*x+c)+632)/cos(d*x+c)/(a+
a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.454 $\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=156

$$-\frac{256a^4\cos^5(c+dx)}{5005d(a\sin(c+dx)+a)^{5/2}}-\frac{64a^3\cos^5(c+dx)}{1001d(a\sin(c+dx)+a)^{3/2}}-\frac{8a^2\cos^5(c+dx)}{143d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos^5(c+dx)(a\sin(c+d}{13d}$$

```
[Out] -256/5005*a^4*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-64/1001*a^3*cos(d*x+c)^
5/d/(a+a*sin(d*x+c))^(3/2)-2/13*cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2)/d-8/143
*a^2*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)-6/143*a*cos(d*x+c)^5*(a+a*sin(d*
x+c))^(1/2)/d
```

Rubi [A] time = 0.32, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2856, 2674, 2673}

$$-\frac{8a^2\cos^5(c+dx)}{143d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^5(c+dx)}{1001d(a\sin(c+dx)+a)^{3/2}}-\frac{256a^4\cos^5(c+dx)}{5005d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)(a\sin(c+d}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^5)/(5005*d*(a + a*Sin[c + d*x])^(5/2)) - (64*a^3*Cos
[c + d*x]^5)/(1001*d*(a + a*Sin[c + d*x])^(3/2)) - (8*a^2*Cos[c + d*x]^5)/(
143*d*Sqrt[a + a*Sin[c + d*x]]) - (6*a*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*
x]])/(143*d) - (2*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(13*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{13d}+\frac{3}{13}\int \cos^4(c+dx)($$
$$= -\frac{6a\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{143d}-\frac{2\cos^5(c+dx)(a+a}{13d}$$
$$= -\frac{8a^2\cos^5(c+dx)}{143d\sqrt{a+a\sin(c+dx)}}-\frac{6a\cos^5(c+dx)\sqrt{a+a\sin(c+}}{143d}$$
$$= -\frac{64a^3\cos^5(c+dx)}{1001d(a+a\sin(c+dx))^{3/2}}-\frac{8a^2\cos^5(c+dx)}{143d\sqrt{a+a\sin(c+dx)}}-$$
$$= -\frac{256a^4\cos^5(c+dx)}{5005d(a+a\sin(c+dx))^{5/2}}-\frac{64a^3\cos^5(c+dx)}{1001d(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 5.18, size = 110, normalized size = 0.71

$$-\frac{a\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5(28230\sin(c+dx)-3290\sin(3(c+dx))-12600\cos}{20020d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/20020*(a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x
])]*(19559 - 12600*Cos[2*(c + d*x)] + 385*Cos[4*(c + d*x)] + 28230*Sin[c +
d*x] - 3290*Sin[3*(c + d*x)]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.44, size = 189, normalized size = 1.21

$$\frac{2\left(385\,a\cos(dx+c)^7-490\,a\cos(dx+c)^6-1015\,a\cos(dx+c)^5+20\,a\cos(dx+c)^4-32\,a\cos(dx+c)^3+64\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 2/5005*(385*a*cos(d*x + c)^7 - 490*a*cos(d*x + c)^6 - 1015*a*cos(d*x + c)^5
 + 20*a*cos(d*x + c)^4 - 32*a*cos(d*x + c)^3 + 64*a*cos(d*x + c)^2 - 256*a*
cos(d*x + c) - (385*a*cos(d*x + c)^6 + 875*a*cos(d*x + c)^5 - 140*a*cos(d*x
 + c)^4 - 160*a*cos(d*x + c)^3 - 192*a*cos(d*x + c)^2 - 256*a*cos(d*x + c)
- 512*a)*sin(d*x + c) - 512*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d
*sin(d*x + c) + d)
```

giac [B] time = 0.81, size = 412, normalized size = 2.64

$$-\frac{1}{1441440}\sqrt{2}\left(\frac{10010\,a\cos\left(\frac{1}{4}\pi+\frac{9}{2}dx+\frac{9}{2}c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}+\frac{54054\,a\cos\left(\frac{1}{4}\pi+\frac{5}{2}dx+\frac{5}{2}c\right)\mathrm{s}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] -1/1441440*sqrt(2)*(10010*a*cos(1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))/d + 54054*a*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c))/d + 180180*a*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))/d + 8190*a*cos(-1/4*pi + 11/2*d*x + 11/2*c)*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))/d + 38610*a*cos(-1/4*pi + 7/2*d*x + 7/2*c)*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 60060*a*cos(-1/4*pi + 3/2*d*x + 3/2*
c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 4095*a*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c))*sin(1/4*pi + 11/2*d*x + 11/2*c)/d + 12870*a*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))*sin(1/4*pi + 7/2*d*x + 7/2*c)/d - 15015*a*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))*sin(1/4*pi + 3/2*d*x + 3/2*c)/d + 3465*a*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 13/2*d*x + 13/2*c)/d + 10010*a*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 9/2*d*x + 9/2*c)/d - 9009*a*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 5/2*d*x + 5/2*c)/d - 180180*a*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)/d)*sqrt(a
)
```

maple [A] time = 0.86, size = 87, normalized size = 0.56

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a^{2}\left(\sin\left(dx+c\right)-1\right)^{3}\left(385\left(\sin^{4}\left(dx+c\right)\right)+1645\left(\sin^{3}\left(dx+c\right)\right)+2765\left(\sin^{2}\left(dx+c\right)\right)+229\right.}{5005\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/5005*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^3*(385*sin(d*x+c)^4+1645*sin(d*x+c
)^3+2765*sin(d*x+c)^2+2295*sin(d*x+c)+918)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin (dx+c)+a)^{\frac{3}{2}}\cos (dx+c)^4\sin (dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos (c+d\,x)^4\,\sin (c+d\,x)\,\left(a+a\,\sin (c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.455 $\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=199

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{2a^2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{34a^2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{14a^2\cos(c+dx)}{45d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -2*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d+16/105*cos(
d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-14/45*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/
2)-34/63*a^2*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-2/9*a^2*cos(d
*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+388/315*a*cos(d*x+c)*(a+a*sin(d
*x+c))^(1/2)/d
```

Rubi [A] time = 0.71, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2881, 2763, 21, 2770, 2759, 2751, 2646, 3046, 2976, 2981, 2773, 206}

$$-\frac{2a^2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{34a^2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{14a^2\cos(c+dx)}{45d\sqrt{a\sin(c+dx)+a}}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
14*a^2*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (34*a^2*Cos[c + d*x]
*Sin[c + d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a^2*Cos[c + d*x]*Sin[
c + d*x]^4)/(9*d*Sqrt[a + a*Sin[c + d*x]]) + (388*a*Cos[c + d*x]*Sqrt[a + a
*Sin[c + d*x]])/(315*d) + (16*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(105
*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
```

```
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc(c+dx)(a$$

$$= -\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)(a+a\sin(c}{5d}$$

$$= -\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{4a\cos(c+dx)\sqrt{a+a\sin(}}{5d}$$

$$= \frac{6a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a}{}$$

$$= \frac{6a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a}{}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{6a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{3}{}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{14a^2\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}} - $$

Mathematica [A] time = 0.48, size = 219, normalized size = 1.10

$$\frac{(a(\sin(c+dx)+1))^{3/2}\left(-1260\sin\left(\frac{1}{2}(c+dx)\right)+1470\sin\left(\frac{3}{2}(c+dx)\right)+126\sin\left(\frac{5}{2}(c+dx)\right)+135\sin\left(\frac{7}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(3/2)*(1260*Cos[(c + d*x)/2] + 1470*Cos[(3*(c + d*x
))/2] - 126*Cos[(5*(c + d*x))/2] + 135*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c
+ d*x))/2] - 2520*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2520*Log[1
 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 1260*Sin[(c + d*x)/2] + 1470*Sin[
(3*(c + d*x))/2] + 126*Sin[(5*(c + d*x))/2] + 135*Sin[(7*(c + d*x))/2] + 35
*Sin[(9*(c + d*x))/2]))/(2520*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

fricas [A] time = 0.47, size = 332, normalized size = 1.67

$$\frac{315\,(a\cos(dx+c)+a\sin(dx+c)+a)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)\right.}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(d\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/630*(315*(a*cos(d*x + c) + a*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c
)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x +
 c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x +
c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x +
 c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) -
 1)) - 4*(35*a*cos(d*x + c)^5 - 50*a*cos(d*x + c)^4 - 46*a*cos(d*x + c)^3 -
 118*a*cos(d*x + c)^2 - 158*a*cos(d*x + c) - (35*a*cos(d*x + c)^4 + 85*a*co
s(d*x + c)^3 + 39*a*cos(d*x + c)^2 + 157*a*cos(d*x + c) - a)*sin(d*x + c) -
 a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [A] time = 1.29, size = 159, normalized size = 0.80

$$-\frac{2\,(1+\sin(dx+c))\sqrt{-a\,(\sin(dx+c)-1)}\,\left(315a^{\frac{9}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)+35\,(a-a\sin(dx+c))^{\frac{9}{2}}-225a\,(a\right.}{315a^3\cos(dx+c)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/315*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(315*a^(9/2)*arctanh((a-a*s
in(d*x+c))^(1/2)/a^(1/2))+35*(a-a*sin(d*x+c))^(9/2)-225*a*(a-a*sin(d*x+c))^
(7/2)+441*(a-a*sin(d*x+c))^(5/2)*a^2-105*(a-a*sin(d*x+c))^(3/2)*a^3-315*a^4
*(a-a*sin(d*x+c))^(1/2))/a^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^{3/2}}{\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x),x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.456 $\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=178

$$-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}+\frac{171a^2\cos(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7ad}+\frac{4\cos(c+dx)(a\sin}{35d}$$

```
[Out] -3*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d+4/35*cos(d*
x+c)*(a+a*sin(d*x+c))^(3/2)/d-cot(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-2/7*cos(d
*x+c)*(a+a*sin(d*x+c))^(5/2)/a/d+171/35*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(
1/2)+69/35*a*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.65, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2881, 2759, 2751, 2647, 2646, 3044, 2976, 2981, 2773, 206}

$$\frac{171a^2\cos(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7ad}+\frac{4\cos(c+dx)(a\sin(c}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
171*a^2*Cos[c + d*x])/(35*d*Sqrt[a + a*Sin[c + d*x]]) + (69*a*Cos[c + d*x]*
Sqrt[a + a*Sin[c + d*x]])/(35*d) + (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/
2))/(35*d) - (Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/d - (2*Cos[c + d*x]*
(a + a*Sin[c + d*x])^(5/2))/(7*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

```
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^2(c+dx)(a$$

$$= -\frac{\cot(c+dx)(a+a\sin(c+dx))^{3/2}}{d} - \frac{2\cos(c+dx)(a+a\sin}{7ad}$$

$$= \frac{7a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \frac{4\cos(c+dx)(a+a\sin}{35d}$$

$$= \frac{19a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{69a\cos(c+dx)\sqrt{a+a\sin(c+dx}}{35d}$$

$$= \frac{171a^2\cos(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} + \frac{69a\cos(c+dx)\sqrt{a+a\sin(c+a}}{35d}$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{171a^2\cos(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} +$$

Mathematica [A] time = 1.32, size = 283, normalized size = 1.59

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-840\sin\left(\frac{1}{2}(c+dx)\right)-574\sin\left(\frac{3}{2}(c+dx)\right)-30\sin\left(\frac{5}{2}(c+dx)\right)-21\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/140*(a*Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(840*Cos[(c + d*x)/
2] - 574*Cos[(3*(c + d*x))/2] + 30*Cos[(5*(c + d*x))/2] - 21*Cos[(7*(c + d*
x))/2] + 5*Cos[(9*(c + d*x))/2] - 840*Sin[(c + d*x)/2] + 420*Log[1 + Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 420*Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]*Sin[c + d*x] - 574*Sin[(3*(c + d*x))/2] - 30*Sin[(5*(c
+ d*x))/2] - 21*Sin[(7*(c + d*x))/2] - 5*Sin[(9*(c + d*x))/2]))/(d*(1 + Cot
[(c + d*x)/2])*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + Se
c[(c + d*x)/4]))
```

fricas [B] time = 0.46, size = 360, normalized size = 2.02

$$\frac{105\left(a\cos(dx+c)^2-(a\cos(dx+c)+a)\sin(dx+c)-a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\right.}{\cos(dx+c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/140*(105*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(d*x + c) - a)*sqrt(
a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*
x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqr
t(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x
 + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x
+ c) - cos(d*x + c) - 1)) + 4*(10*a*cos(d*x + c)^5 - 16*a*cos(d*x + c)^4 -
8*a*cos(d*x + c)^3 - 120*a*cos(d*x + c)^2 + 33*a*cos(d*x + c) - (10*a*cos(d
*x + c)^4 + 26*a*cos(d*x + c)^3 + 18*a*cos(d*x + c)^2 + 138*a*cos(d*x + c)
+ 171*a)*sin(d*x + c) + 171*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2
- (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.28, size = 180, normalized size = 1.01

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)\left(140\sqrt{a-a\sin(dx+c)}\,a^{\frac{7}{2}}+70(a-a\sin(dx+c))^{\frac{3}{2}}a^{\frac{5}{2}}-\right.\right.}{35a^{\frac{5}{2}}\sin(dx+c)\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/35*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(5/2)*(sin(d*x+c)*(140*(a-a
*sin(d*x+c))^(1/2)*a^(7/2)+70*(a-a*sin(d*x+c))^(3/2)*a^(5/2)-56*(a-a*sin(d*
x+c))^(5/2)*a^(3/2)+10*a^(1/2)*(a-a*sin(d*x+c))^(7/2)-105*arctanh((a-a*sin(
d*x+c))^(1/2)/a^(1/2))*a^4)-35*(a-a*sin(d*x+c))^(1/2)*a^(7/2))/sin(d*x+c)/c
os(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{{\sin\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^2,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.457 $\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=186

$$\frac{9a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}+\frac{73a^2\cos(c+dx)}{20d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{2\cos(c+dx)(a\sin(c}{5d}$$

```
[Out] 9/4*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-2/5*cos(d*
x+c)*(a+a*sin(d*x+c))^(3/2)/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(3
/2)/d+73/20*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/5*a*cos(d*x+c)*(a+a*s
in(d*x+c))^(1/2)/d-3/4*a*cot(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.57, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2881, 2751, 2647, 2646, 3044, 2975, 2981, 2773, 206}

$$\frac{73a^2\cos(c+dx)}{20d\sqrt{a\sin(c+dx)+a}}+\frac{9a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{2\cos(c+dx)(a\sin(c}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (9*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
+ (73*a^2*Cos[c + d*x])/(20*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]
*Sqrt[a + a*Sin[c + d*x]])/(5*d) - (3*a*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x
]])/(4*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(5*d) - (Cot[c + d*
x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
```

```
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^3(c+dx)(a$$

$$= -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d} - \frac{\cot(c+dx)\csc(c+}{}$$

$$= -\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} - \frac{3a\cot(c+dx)\sqrt{a+}}{4d}$$

$$= \frac{73a^2\cos(c+dx)}{20d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sqrt{a+a\sin(c+d}}{5d}$$

$$= \frac{73a^2\cos(c+dx)}{20d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sqrt{a+a\sin(c+d}}{5d}$$

$$= \frac{9a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} + \frac{73a^2\cos(c+dx)}{20d\sqrt{a+a\sin(c+dx)}} -$$

Mathematica [A] time = 1.07, size = 322, normalized size = 1.73

$$-\frac{a\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(118\sin\left(\frac{1}{2}(c+dx)\right)+130\sin\left(\frac{3}{2}(c+dx)\right)-36\sin\left(\frac{5}{2}(c+dx)\right)-10\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/20*(a*Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(-118*Cos[(c + d*x)/
2] + 130*Cos[(3*(c + d*x))/2] + 36*Cos[(5*(c + d*x))/2] - 10*Cos[(7*(c + d*
x))/2] + 2*Cos[(9*(c + d*x))/2] - 45*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 45*Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 45*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 45*Cos[2*(c + d*x)]*Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 118*Sin[(c + d*x)/2] + 130*Sin[(
3*(c + d*x))/2] - 36*Sin[(5*(c + d*x))/2] - 10*Sin[(7*(c + d*x))/2] - 2*Sin
[(9*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c
+ d*x)/4]^2)^2)
```

fricas [B] time = 0.48, size = 404, normalized size = 2.17

$$\frac{45\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)^{2}-a\right)\sin\left(dx+c\right)-a\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/80*(45*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x
 + c)^2 - a)*sin(d*x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x
+ c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x +
c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x +
c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x +
c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(8*a*cos(
d*x + c)^5 - 16*a*cos(d*x + c)^4 + 16*a*cos(d*x + c)^3 + 99*a*cos(d*x + c)^
2 - 14*a*cos(d*x + c) - (8*a*cos(d*x + c)^4 + 24*a*cos(d*x + c)^3 + 40*a*co
s(d*x + c)^2 - 59*a*cos(d*x + c) - 73*a)*sin(d*x + c) - 73*a)*sqrt(a*sin(d*
x + c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos
(d*x + c)^2 - d)*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.17, size = 178, normalized size = 0.96

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left(40(-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{3}{2}}\left(\sin^2(dx+c)\right)-8(-a(\sin(dx+c)-1)\right.}{20a^{\frac{3}{2}}\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/20*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(40*(-a*(sin(d*x+c)-1))^(3/2)
*a^(3/2)*sin(d*x+c)^2-8*(-a*(sin(d*x+c)-1))^(5/2)*sin(d*x+c)^2*a^(1/2)+45*a
rctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^2-45*(-a*(sin(d*x+
c)-1))^(1/2)*a^(5/2)+35*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2))/a^(3/2)/sin(d*x+
c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^{3/2}}{\sin(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^3,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.458 $\int \cot^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=197

$$\frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{8a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{29a^2\cot(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}$$

```
[Out] 37/8*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/3*cot(d
*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)/d-8/3*a^2*cos(d*x+c)/d/(a+a*sin(d
*x+c))^(1/2)+29/24*a^2*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/3*a*cos(d*x+c)
*(a+a*sin(d*x+c))^(1/2)/d-1/4*a*cot(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2
)/d
```

Rubi [A] time = 0.50, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2718, 2647, 2646, 3044, 2975, 2980, 2773, 206}

$$-\frac{8a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{29a^2\cot(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}+\frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (37*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 - (8*a^2*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) + (29*a^2*Cot[c + d*
x])/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sqrt[a + a*Sin[c +
d*x]])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(4*d)
 - (Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2718

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int (a+a\sin(c+dx))^{3/2}\,dx + \int \csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\left(1-2\right.$$

$$= -\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{\cot(c+dx)\csc^2(c+dx)(a+a\sin}{3d}$$

$$= -\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{a\cot(c}{}$$

$$= -\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{29a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sqrt{a}}{3d}$$

$$= -\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{29a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sqrt{a}}{3d}$$

$$= \frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{29a^2\cot(c}{24d\sqrt{a+a\sin}}$$

Mathematica [A] time = 1.28, size = 334, normalized size = 1.70

$$-\frac{a\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(276\sin\left(\frac{1}{2}(c+dx)\right)+326\sin\left(\frac{3}{2}(c+dx)\right)-78\sin\left(\frac{5}{2}(c+dx)\right)-72\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/24*(a*Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-276*Cos[(c + d*x)
/2] + 326*Cos[(3*(c + d*x))/2] + 78*Cos[(5*(c + d*x))/2] - 72*Cos[(7*(c + d
*x))/2] + 8*Cos[(9*(c + d*x))/2] + 276*Sin[(c + d*x)/2] - 333*Log[1 + Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 333*Log[1 - Cos[(c + d*x)/2]
```

```
 + Sin[(c + d*x)/2]]*Sin[c + d*x] + 326*Sin[(3*(c + d*x))/2] - 78*Sin[(5*(c
 + d*x))/2] + 111*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d
*x)] - 111*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] -
72*Sin[(7*(c + d*x))/2] - 8*Sin[(9*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2]
)*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3)
```

fricas [B] time = 0.50, size = 424, normalized size = 2.15

$$\frac{111 \left(a \cos(dx+c)^4 - 2\, a \cos(dx+c)^2 - \left(a \cos(dx+c)^3 + a \cos(dx+c)^2 - a \cos(dx+c) - a\right) \sin(dx+c) + a\right) \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(111*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a*co
s(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x
 + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d
*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*
x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d
*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x +
c) - 1)) - 4*(16*a*cos(d*x + c)^5 - 64*a*cos(d*x + c)^4 - 17*a*cos(d*x + c)
^3 + 165*a*cos(d*x + c)^2 + 9*a*cos(d*x + c) - (16*a*cos(d*x + c)^4 + 80*a*
cos(d*x + c)^3 + 63*a*cos(d*x + c)^2 - 102*a*cos(d*x + c) - 93*a)*sin(d*x +
 c) - 93*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^
2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c)
 + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.50, size = 196, normalized size = 0.99

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(96a^{\frac{5}{2}}\sqrt{-a(\sin(dx+c)-1)}\left(\sin^3(dx+c)\right) - 16(-a(\sin(dx+c)-1 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(96*a^(5/2)*(-a*(sin(d*x+c)-
1))^(1/2)*sin(d*x+c)^3-16*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)*sin(d*x+c)^3-11
1*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^3+15*(-a*(sin(d
*x+c)-1))^(1/2)*a^(5/2)+8*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)-15*(-a*(sin(d*x
+c)-1))^(5/2)*a^(1/2))/a^(3/2)/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/
2)/d
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^4, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^{3/2}}{\sin(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^4,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^4, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.459 $\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=205

$$\frac{21a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}-\frac{2a^2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{149a^2\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}+\frac{19a^2\cot(c+dx)\csc(c+dx)}{32d\sqrt{a\sin(c+dx)+a}}-\text{cot}$$

```
[Out] 21/64*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/4*cot(
d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2)/d-2*a^2*cos(d*x+c)/d/(a+a*sin(d*
x+c))^(1/2)+149/64*a^2*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+19/32*a^2*cot(d*
x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/8*a*cot(d*x+c)*csc(d*x+c)^2*(a+a
*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.70, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2881, 2763, 21, 2773, 206, 3044, 2975, 2980, 2772}

$$-\frac{2a^2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{149a^2\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}+\frac{21a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}+\frac{19a^2\cot(c+dx)\csc(c+dx)}{32d\sqrt{a\sin(c+dx)+a}}-\text{c}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (21*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*d
) - (2*a^2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) + (149*a^2*Cot[c + d*
x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (19*a^2*Cot[c + d*x]*Csc[c + d*x])/(3
2*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a + a*S
in[c + d*x]])/(8*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/
2))/(4*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
```

```
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^5(c+dx)(a+$$

$$= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+dx)(a+a\sin(}{4d}$$

$$= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\,\mathrm{s}}}{8d}$$

$$= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{19a^2\cot(c+dx)\csc(c+dx)}{32d\sqrt{a+a\sin(c+dx)}} - \frac{a\,\mathrm{c}}{}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{1}{64a}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{1}{64a}$$

$$= \frac{21a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{1}{64a}$$

Mathematica [A] time = 1.46, size = 392, normalized size = 1.91

$$-\frac{a\csc^{13}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-1486\sin\left(\frac{1}{2}(c+dx)\right)-1030\sin\left(\frac{3}{2}(c+dx)\right)+754\sin\left(\frac{5}{2}(c+dx)\right)+4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/64*(a*Csc[(c + d*x)/2]^13*Sqrt[a*(1 + Sin[c + d*x])]*(1486*Cos[(c + d*x)
/2] - 1030*Cos[(3*(c + d*x))/2] - 754*Cos[(5*(c + d*x))/2] + 426*Cos[(7*(c
+ d*x))/2] + 128*Cos[(9*(c + d*x))/2] - 63*Log[1 + Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] + 84*Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - 21*Cos[4*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6
3*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 84*Cos[2*(c + d*x)]*Log[1
- Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 21*Cos[4*(c + d*x)]*Log[1 - Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] - 1486*Sin[(c + d*x)/2] - 1030*Sin[(3*(c + d
*x))/2] + 754*Sin[(5*(c + d*x))/2] + 426*Sin[(7*(c + d*x))/2] - 128*Sin[(9*
```

```
(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*
x)/4]^2)^4)
```

fricas [B] time = 0.48, size = 460, normalized size = 2.24

$$\frac{21\left(a\cos\left(dx+c\right)^{5}+a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{3}-2\,a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)^{4}-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/256*(21*(a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 - 2*a*c
os(d*x + c)^2 + a*cos(d*x + c) + (a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a
)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*
(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqr
t(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*
cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos
(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(128*a*cos(d*x + c)^
5 + 277*a*cos(d*x + c)^4 - 242*a*cos(d*x + c)^3 - 500*a*cos(d*x + c)^2 + 13
0*a*cos(d*x + c) - (128*a*cos(d*x + c)^4 - 149*a*cos(d*x + c)^3 - 391*a*cos
(d*x + c)^2 + 109*a*cos(d*x + c) + 239*a)*sin(d*x + c) + 239*a)*sqrt(a*sin(
d*x + c) + a))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 -
2*d*cos(d*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^
2 + d)*sin(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.42, size = 188, normalized size = 0.92

$$-\frac{\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(128\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,a^{\frac{7}{2}}\left(\sin^{4}\left(dx+c\right)\right)-21\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(d}}{\sqrt{a}}\right.\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/64*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(5/2)*(128*(-a*(sin(d*x+c)
-1))^(1/2)*a^(7/2)*sin(d*x+c)^4-21*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2
))*a^4*sin(d*x+c)^4+149*(-a*(sin(d*x+c)-1))^(7/2)*a^(1/2)-461*(-a*(sin(d*x+
c)-1))^(5/2)*a^(3/2)+435*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)-107*(-a*(sin(d*x
+c)-1))^(1/2)*a^(7/2))/sin(d*x+c)^4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^{3/2}}{\sin(c+d\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^5,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.460 $\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=215

$$-\frac{165a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128d}+\frac{91a^2\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}+\frac{31a^2\cot(c+dx)\csc^2(c+dx)}{80d\sqrt{a\sin(c+dx)+a}}+\frac{73a^2\cot(c+dx)\csc}{64d\sqrt{a\sin(c+dx}}$$

```
[Out] -165/128*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/5*c
ot(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2)/d+91/128*a^2*cot(d*x+c)/d/(a+
a*sin(d*x+c))^(1/2)+73/64*a^2*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^(1/2
)+31/80*a^2*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)-3/40*a*cot(d*x
+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.81, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2881, 2762, 21, 2773, 206, 3044, 2975, 2980, 2772}

$$\frac{91a^2\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}-\frac{165a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128d}+\frac{31a^2\cot(c+dx)\csc^2(c+dx)}{80d\sqrt{a\sin(c+dx)+a}}+\frac{73a^2\cot(c+dx)\csc(c}{64d\sqrt{a\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-165*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(12
8*d) + (91*a^2*Cot[c + d*x])/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (73*a^2*Cot
[c + d*x]*Csc[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (31*a^2*Cot[c + d
*x]*Csc[c + d*x]^2)/(80*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a*Cot[c + d*x]*Csc
[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]])/(40*d) - (Cot[c + d*x]*Csc[c + d*x]^4
*(a + a*Sin[c + d*x])^(3/2))/(5*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
```

```
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^6(c+dx)(a$$

$$= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)(a+a\sin}{5d}$$

$$= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{3a\cot(c+dx)\csc^3(c+dx)\sqrt{a+a}}{40d}$$

$$= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{31a^2\cot(c+dx)\csc^2(c+dx)}{80d\sqrt{a+a\sin(c+dx)}} - 3$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + 73$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{91a^2\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{91a^2\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{165a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128d} + \frac{91a^2\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 1.64, size = 404, normalized size = 1.88

$$-\frac{a\csc^{16}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-1380\sin\left(\frac{1}{2}(c+dx)\right)+320\sin\left(\frac{3}{2}(c+dx)\right)-1296\sin\left(\frac{5}{2}(c+dx)\right)+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/640*(a*Csc[(c + d*x)/2]^16*Sqrt[a*(1 + Sin[c + d*x])]*(1380*Cos[(c + d*x
)/2] + 320*Cos[(3*(c + d*x))/2] + 1296*Cos[(5*(c + d*x))/2] + 2010*Cos[(7*(
c + d*x))/2] - 910*Cos[(9*(c + d*x))/2] - 1380*Sin[(c + d*x)/2] + 8250*Log[
1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 8250*Log[1 - Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 320*Sin[(3*(c + d*x))/2] - 12
96*Sin[(5*(c + d*x))/2] - 4125*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
```

```
*Sin[3*(c + d*x)] + 4125*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3
*(c + d*x)] + 2010*Sin[(7*(c + d*x))/2] + 910*Sin[(9*(c + d*x))/2] + 825*Lo
g[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 825*Log[1 - C
os[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(d*(1 + Cot[(c + d*x
)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^5)
```

fricas [B] time = 0.49, size = 488, normalized size = 2.27

$$\frac{825\left(a\cos\left(dx+c\right)^{6}-3\,a\cos\left(dx+c\right)^{4}+3\,a\cos\left(dx+c\right)^{2}-\left(a\cos\left(dx+c\right)^{5}+a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{3}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/2560*(825*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - (
a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 - 2*a*cos(d*x + c)
^2 + a*cos(d*x + c) + a)*sin(d*x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) -
 4*(455*a*cos(d*x + c)^5 - 275*a*cos(d*x + c)^4 - 982*a*cos(d*x + c)^3 + 17
4*a*cos(d*x + c)^2 + 399*a*cos(d*x + c) - (455*a*cos(d*x + c)^4 + 730*a*cos
(d*x + c)^3 - 252*a*cos(d*x + c)^2 - 426*a*cos(d*x + c) - 27*a)*sin(d*x + c
) - 27*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4
+ 3*d*cos(d*x + c)^2 - (d*cos(d*x + c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x +
 c)^3 - 2*d*cos(d*x + c)^2 + d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.48, size = 180, normalized size = 0.84

$$\frac{\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(-825\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^{5}\left(\sin^{5}\left(dx+c\right)\right)+455\left(-a\left(\sin\left(dx+\right.\right.\right.\right.}{640a^{\frac{7}{2}}\,\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/640*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(-825*arctanh((-a*(s
in(d*x+c)-1))^(1/2)/a^(1/2))*a^5*sin(d*x+c)^5+455*(-a*(sin(d*x+c)-1))^(9/2)
*a^(1/2)-2550*(-a*(sin(d*x+c)-1))^(7/2)*a^(3/2)+4992*(-a*(sin(d*x+c)-1))^(5
/2)*a^(5/2)-3850*(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2)+825*(-a*(sin(d*x+c)-1))^
(1/2)*a^(9/2))/sin(d*x+c)^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^{3/2}}{\sin(c+d\,x)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^6,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.461 $\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=253

$$-\frac{179a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{512d}-\frac{179a^2\cot(c+dx)}{512d\sqrt{a\sin(c+dx)+a}}+\frac{137a^2\cot(c+dx)\csc^3(c+dx)}{480d\sqrt{a\sin(c+dx)+a}}+\frac{239a^2\cot(c+dx)\,c}{320d\sqrt{a\sin(c+}}$$

```
[Out] -179/512*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/6*c
ot(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2)/d-179/512*a^2*cot(d*x+c)/d/(a
+a*sin(d*x+c))^(1/2)+111/256*a^2*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^(
1/2)+239/320*a^2*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)+137/480*a
^2*cot(d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-1/20*a*cot(d*x+c)*csc(d
*x+c)^4*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.92, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2881, 2762, 21, 2772, 2773, 206, 3044, 2975, 2980}

$$-\frac{179a^2\cot(c+dx)}{512d\sqrt{a\sin(c+dx)+a}}-\frac{179a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{512d}+\frac{137a^2\cot(c+dx)\csc^3(c+dx)}{480d\sqrt{a\sin(c+dx)+a}}+\frac{239a^2\cot(c+dx)\,c}{320d\sqrt{a\sin(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-179*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(51
2*d) - (179*a^2*Cot[c + d*x])/(512*d*Sqrt[a + a*Sin[c + d*x]]) + (111*a^2*C
ot[c + d*x]*Csc[c + d*x])/(256*d*Sqrt[a + a*Sin[c + d*x]]) + (239*a^2*Cot[c
 + d*x]*Csc[c + d*x]^2)/(320*d*Sqrt[a + a*Sin[c + d*x]]) + (137*a^2*Cot[c +
 d*x]*Csc[c + d*x]^3)/(480*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Cs
c[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]])/(20*d) - (Cot[c + d*x]*Csc[c + d*x]^
5*(a + a*Sin[c + d*x])^(3/2))/(6*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
```

```
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^7(c+dx)(a$$

$$= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^5(c+dx)(a+}{6d}$$

$$= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^4(c+dx)\sqrt{a}}{20d}$$

$$= -\frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{137}{}$$

$$= -\frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{239}{}$$

$$= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} + \frac{1}{}$$

$$= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{179a^2\cot(c+dx)}{512d\sqrt{a+a\sin(c+dx)}} +$$

$$= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{179a^2\cot(c+dx)}{512d\sqrt{a+a\sin(c+dx)}} +$$

$$= -\frac{179a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{512d} - \frac{179a^2\cot(c+dx)}{512d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 2.51, size = 486, normalized size = 1.92

$$\frac{a\csc^{19}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-25140\sin\left(\frac{1}{2}(c+dx)\right)-71972\sin\left(\frac{3}{2}(c+dx)\right)+42690\sin\left(\frac{5}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*Csc[(c + d*x)/2]^19*Sqrt[a*(1 + Sin[c + d*x])]*(25140*Cos[(c + d*x)/2] -
 71972*Cos[(3*(c + d*x))/2] - 42690*Cos[(5*(c + d*x))/2] - 5718*Cos[(7*(c +
 d*x))/2] + 18690*Cos[(9*(c + d*x))/2] - 5370*Cos[(11*(c + d*x))/2] - 26850
*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 40275*Cos[2*(c + d*x)]*Log[
```

```
1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 16110*Cos[4*(c + d*x)]*Log[1 + C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2685*Cos[6*(c + d*x)]*Log[1 + Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] + 26850*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]] - 40275*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] + 16110*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] -
2685*Cos[6*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 25140*
Sin[(c + d*x)/2] - 71972*Sin[(3*(c + d*x))/2] + 42690*Sin[(5*(c + d*x))/2]
- 5718*Sin[(7*(c + d*x))/2] - 18690*Sin[(9*(c + d*x))/2] - 5370*Sin[(11*(c
+ d*x))/2]))/(7680*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c +
d*x)/4]^2)^6)
```

fricas [B] time = 0.49, size = 557, normalized size = 2.20

$$\frac{2685 \left(a \cos \left(dx+c\right)^{7}+a \cos \left(dx+c\right)^{6}-3 a \cos \left(dx+c\right)^{5}-3 a \cos \left(dx+c\right)^{4}+3 a \cos \left(dx+c\right)^{3}+3 a \cos \left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/30720*(2685*(a*cos(d*x + c)^7 + a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^5 - 3
*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - a*cos(d*x + c
) + (a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*sin(d*
x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x
 + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(
d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x
+ c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c
)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(2685*a*cos(d*x + c)^6 - 333
0*a*cos(d*x + c)^5 - 5649*a*cos(d*x + c)^4 + 7188*a*cos(d*x + c)^3 + 6715*a
*cos(d*x + c)^2 - 2578*a*cos(d*x + c) + (2685*a*cos(d*x + c)^5 + 6015*a*cos
(d*x + c)^4 + 366*a*cos(d*x + c)^3 - 6822*a*cos(d*x + c)^2 - 107*a*cos(d*x
+ c) + 2471*a)*sin(d*x + c) - 2471*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x
+ c)^7 + d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4 + 3*d*c
os(d*x + c)^3 + 3*d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c)^6 - 3
*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.46, size = 198, normalized size = 0.78

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(2685(-a(\sin(dx+c)-1))^{\frac{11}{2}}a^{\frac{3}{2}}-2685\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^{7}(s\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/7680*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(11/2)*(2685*(-a*(sin(d*x
+c)-1))^(11/2)*a^(3/2)-2685*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^7*
sin(d*x+c)^6-10095*(-a*(sin(d*x+c)-1))^(9/2)*a^(5/2)+7794*(-a*(sin(d*x+c)-1
))^(7/2)*a^(7/2)+10866*(-a*(sin(d*x+c)-1))^(5/2)*a^(9/2)-15215*(-a*(sin(d*x
+c)-1))^(3/2)*a^(11/2)+2685*(-a*(sin(d*x+c)-1))^(1/2)*a^(13/2))/sin(d*x+c)^
6/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{4}\csc(dx+c)^{7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^{3/2}}{\sin(c+d\,x)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^7,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^7, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.462 $\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=291

$$-\frac{171a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{1024d}-\frac{171a^2\cot(c+dx)}{1024d\sqrt{a\sin(c+dx)+a}}+\frac{9a^2\cot(c+dx)\csc^4(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}+\frac{1237a^2\cot(c+dx)\csc}{2240d\sqrt{a\sin(c+}}$$

```
[Out] -171/1024*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/7*
cot(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2)/d-171/1024*a^2*cot(d*x+c)/d/
(a+a*sin(d*x+c))^(1/2)-57/512*a^2*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^
(1/2)+199/640*a^2*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)+1237/224
0*a^2*cot(d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+9/40*a^2*cot(d*x+c)*
csc(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)-1/28*a*cot(d*x+c)*csc(d*x+c)^5*(a+a*s
in(d*x+c))^(1/2)/d
```

Rubi [A] time = 1.06, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2881, 2762, 21, 2772, 2773, 206, 3044, 2975, 2980}

$$-\frac{171a^2\cot(c+dx)}{1024d\sqrt{a\sin(c+dx)+a}}-\frac{171a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{1024d}+\frac{9a^2\cot(c+dx)\csc^4(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}+\frac{1237a^2\cot(c+dx)\csc}{2240d\sqrt{a\sin(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-171*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(10
24*d) - (171*a^2*Cot[c + d*x])/(1024*d*Sqrt[a + a*Sin[c + d*x]]) - (57*a^2*
Cot[c + d*x]*Csc[c + d*x])/(512*d*Sqrt[a + a*Sin[c + d*x]]) + (199*a^2*Cot[
c + d*x]*Csc[c + d*x]^2)/(640*d*Sqrt[a + a*Sin[c + d*x]]) + (1237*a^2*Cot[c
 + d*x]*Csc[c + d*x]^3)/(2240*d*Sqrt[a + a*Sin[c + d*x]]) + (9*a^2*Cot[c +
d*x]*Csc[c + d*x]^4)/(40*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[
c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(28*d) - (Cot[c + d*x]*Csc[c + d*x]^6*
(a + a*Sin[c + d*x])^(3/2))/(7*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^8(c+dx)(a$$

$$= -\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^6(c+dx)(a}{7d}$$

$$= -\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^5(c+dx)\sqrt{}}{28d}$$

$$= -\frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^2(c+dx}{3d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a}{}$$

$$= -\frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} + \frac{1}{}$$

$$= -\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{171a^2\cot(c+dx)}{1024d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{171a^2\cot(c+dx)}{1024d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{171a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{1024d} - \frac{171a^2\cot(c+dx)}{1024d\sqrt{a+a\sin(c+dx}}$$

Mathematica [A] time = 4.94, size = 522, normalized size = 1.79

$$\frac{a\csc^{22}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(306488\sin\left(\frac{1}{2}(c+dx)\right)-177170\sin\left(\frac{3}{2}(c+dx)\right)-6566\sin\left(\frac{5}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]

[Out] (a*Csc[(c + d*x)/2]^22*Sqrt[a*(1 + Sin[c + d*x])]*(-306488*Cos[(c + d*x)/2]
 - 177170*Cos[(3*(c + d*x))/2] + 6566*Cos[(5*(c + d*x))/2] - 219540*Cos[(7*
```

```
(c + d*x))/2] + 33292*Cos[(9*(c + d*x))/2] - 3990*Cos[(11*(c + d*x))/2] + 1
1970*Cos[(13*(c + d*x))/2] + 306488*Sin[(c + d*x)/2] - 209475*Log[1 + Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 209475*Log[1 - Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 177170*Sin[(3*(c + d*x))/2] - 6566*S
in[(5*(c + d*x))/2] + 125685*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*S
in[3*(c + d*x)] - 125685*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3
*(c + d*x)] - 219540*Sin[(7*(c + d*x))/2] - 33292*Sin[(9*(c + d*x))/2] - 41
895*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 41895*L
og[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 3990*Sin[(11
*(c + d*x))/2] - 11970*Sin[(13*(c + d*x))/2] + 5985*Log[1 + Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] - 5985*Log[1 - Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]*Sin[7*(c + d*x)]))/(35840*d*(1 + Cot[(c + d*x)/2])*(Csc[(c
+ d*x)/4]^2 - Sec[(c + d*x)/4]^2)^7)
```

fricas [B] time = 0.50, size = 600, normalized size = 2.06

$$\frac{5985\left(a\cos\left(dx+c\right)^{8}-4\,a\cos\left(dx+c\right)^{6}+6\,a\cos\left(dx+c\right)^{4}-4\,a\cos\left(dx+c\right)^{2}-\left(a\cos\left(dx+c\right)^{7}+a\cos\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/143360*(5985*(a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4
- 4*a*cos(d*x + c)^2 - (a*cos(d*x + c)^7 + a*cos(d*x + c)^6 - 3*a*cos(d*x +
 c)^5 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - a*co
s(d*x + c) - a)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x
 + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x
 + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x
 + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(5985*
a*cos(d*x + c)^7 + 1995*a*cos(d*x + c)^6 - 6811*a*cos(d*x + c)^5 - 14633*a*
cos(d*x + c)^4 - 5997*a*cos(d*x + c)^3 + 10097*a*cos(d*x + c)^2 + 1703*a*co
s(d*x + c) - (5985*a*cos(d*x + c)^6 + 3990*a*cos(d*x + c)^5 - 2821*a*cos(d*
x + c)^4 + 11812*a*cos(d*x + c)^3 + 5815*a*cos(d*x + c)^2 - 4282*a*cos(d*x
+ c) - 2579*a)*sin(d*x + c) - 2579*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x
+ c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 - (d*
cos(d*x + c)^7 + d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4
 + 3*d*cos(d*x + c)^3 + 3*d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x +
c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.66, size = 216, normalized size = 0.74

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(5985\sqrt{-a(\sin(dx+c)-1)}\,a^{\frac{17}{2}}-39900(-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{15}{2}}-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/35840*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(5985*(-a*(sin(d*x+c)-1))^
(1/2)*a^(17/2)-39900*(-a*(sin(d*x+c)-1))^(3/2)*a^(15/2)-1771*(-a*(sin(d*x+c
)-1))^(5/2)*a^(13/2)+95232*(-a*(sin(d*x+c)-1))^(7/2)*a^(11/2)-98581*(-a*(si
n(d*x+c)-1))^(9/2)*a^(9/2)+39900*(-a*(sin(d*x+c)-1))^(11/2)*a^(7/2)-5985*(-
a*(sin(d*x+c)-1))^(13/2)*a^(5/2)-5985*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(
1/2))*a^9*sin(d*x+c)^7)/a^(15/2)/sin(d*x+c)^7/cos(d*x+c)/(a+a*sin(d*x+c))^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^{3/2}}{\sin(c+d\,x)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^8,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^8, x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.463 $\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=329

$$-\frac{1587a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{16384d}-\frac{1587a^2\cot(c+dx)}{16384d\sqrt{a\sin(c+dx)+a}}+\frac{83a^2\cot(c+dx)\csc^5(c+dx)}{448d\sqrt{a\sin(c+dx)+a}}+\frac{1957a^2\cot(c+dx}{4480d\sqrt{a\sin(c}$$

```
[Out] -1587/16384*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/
8*cot(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2)/d-1587/16384*a^2*cot(d*x+c
)/d/(a+a*sin(d*x+c))^(1/2)-529/8192*a^2*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*
x+c))^(1/2)-529/10240*a^2*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)+
8653/35840*a^2*cot(d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+1957/4480*a
^2*cot(d*x+c)*csc(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+83/448*a^2*cot(d*x+c)*c
sc(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)-3/112*a*cot(d*x+c)*csc(d*x+c)^6*(a+a*s
in(d*x+c))^(1/2)/d
```

Rubi [A] time = 1.19, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2881, 2762, 21, 2772, 2773, 206, 3044, 2975, 2980}

$$-\frac{1587a^2\cot(c+dx)}{16384d\sqrt{a\sin(c+dx)+a}}-\frac{1587a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{16384d}+\frac{83a^2\cot(c+dx)\csc^5(c+dx)}{448d\sqrt{a\sin(c+dx)+a}}+\frac{1957a^2\cot(c+dx}{4480d\sqrt{a\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-1587*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(1
6384*d) - (1587*a^2*Cot[c + d*x])/(16384*d*Sqrt[a + a*Sin[c + d*x]]) - (529
*a^2*Cot[c + d*x]*Csc[c + d*x])/(8192*d*Sqrt[a + a*Sin[c + d*x]]) - (529*a^
2*Cot[c + d*x]*Csc[c + d*x]^2)/(10240*d*Sqrt[a + a*Sin[c + d*x]]) + (8653*a
^2*Cot[c + d*x]*Csc[c + d*x]^3)/(35840*d*Sqrt[a + a*Sin[c + d*x]]) + (1957*
a^2*Cot[c + d*x]*Csc[c + d*x]^4)/(4480*d*Sqrt[a + a*Sin[c + d*x]]) + (83*a^
2*Cot[c + d*x]*Csc[c + d*x]^5)/(448*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a*Cot[
c + d*x]*Csc[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]])/(112*d) - (Cot[c + d*x]*C
sc[c + d*x]^7*(a + a*Sin[c + d*x])^(3/2))/(8*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^9(c+dx)$$

$$= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^7(c+dx)(a}{8d}$$

$$= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{3a\cot(c+dx)\csc^6(c+dx}{112d}$$

$$= -\frac{5a^2\cot(c+dx)\csc^2(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^3(c+d}{4d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{25a^2\cot(c+dx)\csc(c+dx)}{32d\sqrt{a+a\sin(c+dx)}} - \frac{5a^2\cot(c+dx)\csc^2(c+}{8d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{75a^2\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{25a^2\cot(c+dx)\csc(c+dx)}{32d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{75a^2\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{25a^2\cot(c+dx)\csc(c+dx)}{32d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{75a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{75a^2\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{75a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{1587a^2\cot(c+dx)}{16384d\sqrt{a+a\sin(c+}}$$

$$= -\frac{75a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{1587a^2\cot(c+dx)}{16384d\sqrt{a+a\sin(c+}}$$

$$= -\frac{1587a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{16384d} - \frac{1587a^2\cot(c+d}{16384d\sqrt{a+a\sin(c}}$$

Mathematica [B] time = 6.26, size = 2303, normalized size = 7.00

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (6053*(a*(1 + Sin[c + d*x]))^(3/2))/(143360*d*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^3) - (6053*Cot[(c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(286720*
```

d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (179*Csc[(c + d*x)/4]^2*(a*(1 + Sin[c + d*x]))^(3/2))/(131072*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (107*Cot[(c + d*x)/4]*Csc[(c + d*x)/4]^2*(a*(1 + Sin[c + d*x]))^(3/2))/(573440*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (113*Csc[(c + d*x)/4]^4*(a*(1 + Sin[c + d*x]))^(3/2))/(262144*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (31*Cot[(c + d*x)/4]*Csc[(c + d*x)/4]^4*(a*(1 + Sin[c + d*x]))^(3/2))/(143360*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (Csc[(c + d*x)/4]^6*(a*(1 + Sin[c + d*x]))^(3/2))/(131072*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (3*Cot[(c + d*x)/4]*Csc[(c + d*x)/4]^6*(a*(1 + Sin[c + d*x]))^(3/2))/(229376*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (Csc[(c + d*x)/4]^8*(a*(1 + Sin[c + d*x]))^(3/2))/(524288*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (1587*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*(1 + Sin[c + d*x]))^(3/2))/(32768*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (1587*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*(1 + Sin[c + d*x]))^(3/2))/(32768*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (179*Sec[(c + d*x)/4]^2*(a*(1 + Sin[c + d*x]))^(3/2))/(131072*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (113*Sec[(c + d*x)/4]^4*(a*(1 + Sin[c + d*x]))^(3/2))/(262144*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (Sec[(c + d*x)/4]^6*(a*(1 + Sin[c + d*x]))^(3/2))/(131072*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (Sec[(c + d*x)/4]^8*(a*(1 + Sin[c + d*x]))^(3/2))/(524288*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (a*(1 + Sin[c + d*x]))^(3/2)/(32768*d*(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^8*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (5*(a*(1 + Sin[c + d*x]))^(3/2))/(114688*d*(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^6*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (5939*(a*(1 + Sin[c + d*x]))^(3/2))/(2293760*d*(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (5409*(a*(1 + Sin[c + d*x]))^(3/2))/(2293760*d*(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (3*Sin[(c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(14336*d*(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^7*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (31*Sin[(c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(17920*d*(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (107*Sin[(c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(143360*d*(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (6053*Sin[(c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(143360*d*(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (a*(1 + Sin[c + d*x]))^(3/2)/(32768*d*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^8*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (3*Sin[(c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(14336*d*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^7*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (19*(a*(1 + Sin[c + d*x]))^(3/2))/(114688*d*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^6*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (31*Sin[(c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(17920*d*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (1971*(a*(1 + Sin[c + d*x]))^(3/2))/(2293760*d*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (107*Sin[(c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(143360*d*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^3*(Cos[(c +

```
d*x)/2] + Sin[(c + d*x)/2])^3) - (7121*(a*(1 + Sin[c + d*x]))^(3/2))/(2293
760*d*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^3) - (6053*Sin[(c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(143360*
d*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3) - (6053*(a*(1 + Sin[c + d*x]))^(3/2)*Tan[(c + d*x)/4])/(286720*d*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (107*Sec[(c + d*x)/4]^2*(a*(1 + Sin[
c + d*x]))^(3/2)*Tan[(c + d*x)/4])/(573440*d*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^3) + (31*Sec[(c + d*x)/4]^4*(a*(1 + Sin[c + d*x]))^(3/2)*Tan[(c + d
*x)/4])/(143360*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (3*Sec[(c + d*
x)/4]^6*(a*(1 + Sin[c + d*x]))^(3/2)*Tan[(c + d*x)/4])/(229376*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^3)
```

fricas [B] time = 0.51, size = 657, normalized size = 2.00

$$\frac{55545\left(a\cos\left(dx+c\right)^{9}+a\cos\left(dx+c\right)^{8}-4\,a\cos\left(dx+c\right)^{7}-4\,a\cos\left(dx+c\right)^{6}+6\,a\cos\left(dx+c\right)^{5}+6\,a\cos\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/2293760*(55545*(a*cos(d*x + c)^9 + a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^7
- 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^5 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*
x + c)^3 - 4*a*cos(d*x + c)^2 + a*cos(d*x + c) + (a*cos(d*x + c)^8 - 4*a*co
s(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*sin(d*x + c) +
a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 +
 (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c)
+ a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)
*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*
sin(d*x + c) - cos(d*x + c) - 1)) + 4*(55545*a*cos(d*x + c)^8 + 37030*a*cos
(d*x + c)^7 - 214774*a*cos(d*x + c)^6 + 27358*a*cos(d*x + c)^5 + 199004*a*c
os(d*x + c)^4 - 185006*a*cos(d*x + c)^3 - 153786*a*cos(d*x + c)^2 + 48938*a
*cos(d*x + c) + (55545*a*cos(d*x + c)^7 + 18515*a*cos(d*x + c)^6 - 196259*a
*cos(d*x + c)^5 - 223617*a*cos(d*x + c)^4 - 24613*a*cos(d*x + c)^3 + 160393
*a*cos(d*x + c)^2 + 6607*a*cos(d*x + c) - 42331*a)*sin(d*x + c) + 42331*a)*
sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^9 + d*cos(d*x + c)^8 - 4*d*cos(d*
x + c)^7 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^5 + 6*d*cos(d*x + c)^4 - 4
*d*cos(d*x + c)^3 - 4*d*cos(d*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^8
 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*
x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.88, size = 234, normalized size = 0.71

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(55545(-a(\sin(dx+c)-1))^{\frac{15}{2}}a^{\frac{7}{2}}-425845(-a(\sin(dx+c)-1))^{\frac{13}{2}}a^{\frac{9}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/573440*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(19/2)*(55545*(-a*(sin(
d*x+c)-1))^(15/2)*a^(7/2)-425845*(-a*(sin(d*x+c)-1))^(13/2)*a^(9/2)+1418249
*(-a*(sin(d*x+c)-1))^(11/2)*a^(11/2)-55545*arctanh((-a*(sin(d*x+c)-1))^(1/2
)/a^(1/2))*a^11*sin(d*x+c)^8-2509197*(-a*(sin(d*x+c)-1))^(9/2)*a^(13/2)+217
6627*(-a*(sin(d*x+c)-1))^(7/2)*a^(15/2)-416759*(-a*(sin(d*x+c)-1))^(5/2)*a^
(17/2)-425845*(-a*(sin(d*x+c)-1))^(3/2)*a^(19/2)+55545*(-a*(sin(d*x+c)-1))^
(1/2)*a^(21/2))/sin(d*x+c)^8/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)^9\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^{3/2}}{\sin(c+d\,x)^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^9,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^9, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.464 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=124

$$-\frac{152a^2\cos^5(c+dx)}{3465d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{11ad}+\frac{20\cos^5(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{38a\cos^5(c+dx)}{693d(a\sin(c+dx)+a}$$

```
[Out] -152/3465*a^2*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-38/693*a*cos(d*x+c)^5/d
/(a+a*sin(d*x+c))^(3/2)+20/99*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)-2/11*co
s(d*x+c)^5*(a+a*sin(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.41, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2877, 2856, 2674, 2673}

$$-\frac{152a^2\cos^5(c+dx)}{3465d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{11ad}+\frac{20\cos^5(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{38a\cos^5(c+dx)}{693d(a\sin(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-152*a^2*Cos[c + d*x]^5)/(3465*d*(a + a*Sin[c + d*x])^(5/2)) - (38*a*Cos[c
 + d*x]^5)/(693*d*(a + a*Sin[c + d*x])^(3/2)) + (20*Cos[c + d*x]^5)/(99*d*S
qrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(11*
a*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2877

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\cos^5(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\int \cos^4(c+dx)\left(-\frac{a}{2}-4a\sin(c+dx)\right)\sqrt{a+a\sin(c+ \cdots}{4a^2}$$

$$= \frac{\cos^5(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11ad} + \frac{19\int \cos^4(c+d \cdots}{\cdots}$$

$$= \frac{20\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11ad} + \frac{19}{99}\int \frac{\cos^4 \cdots}{\sqrt{a+a\,\mathrm{s} \cdots}}$$

$$= -\frac{38a\cos^5(c+dx)}{693d(a+a\sin(c+dx))^{3/2}} + \frac{20\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^5(c+dx)\sqrt{a+ \cdots}}{11ad}$$

$$= -\frac{152a^2\cos^5(c+dx)}{3465d(a+a\sin(c+dx))^{5/2}} - \frac{38a\cos^5(c+dx)}{693d(a+a\sin(c+dx))^{3/2}} + \frac{20\cos^5(c+ \cdots}{99d\sqrt{a+a\sin(\cdots}}$$

Mathematica [A] time = 1.70, size = 143, normalized size = 1.15

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left(5773\sin\left(\frac{1}{2}(c+dx)\right)+3495\sin\left(\frac{3}{2}(c+dx)\right)-1505\sin\left(\frac{5}{2}(c+dx)\right)-31 \cdots\right.}{13860d\sqrt{a(\mathrm{si} \cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/13860*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(5773*Cos[(c + d*x)/2] -
3495*Cos[(3*(c + d*x))/2] - 1505*Cos[(5*(c + d*x))/2] + 315*Cos[(7*(c + d*x
))/2] + 5773*Sin[(c + d*x)/2] + 3495*Sin[(3*(c + d*x))/2] - 1505*Sin[(5*(c
+ d*x))/2] - 315*Sin[(7*(c + d*x))/2]))/(d*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [A] time = 0.43, size = 155, normalized size = 1.25

$$-\frac{2\left(315\cos(dx+c)^{6}-35\cos(dx+c)^{5}-445\cos(dx+c)^{4}+19\cos(dx+c)^{3}-38\cos(dx+c)^{2}+\left(315\cos(d\right.\right.}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] -2/3465*(315*cos(d*x + c)^6 - 35*cos(d*x + c)^5 - 445*cos(d*x + c)^4 + 19*c
os(d*x + c)^3 - 38*cos(d*x + c)^2 + (315*cos(d*x + c)^5 + 350*cos(d*x + c)^
4 - 95*cos(d*x + c)^3 - 114*cos(d*x + c)^2 - 152*cos(d*x + c) - 304)*sin(d*
x + c) + 152*cos(d*x + c) + 304)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c)
 + a*d*sin(d*x + c) + a*d)
```

giac [B] time = 0.81, size = 342, normalized size = 2.76

$$8\left(\frac{76\sqrt{2}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{a}}-\frac{\frac{34a^{5}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\left(\frac{187a^{5}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{1155a^{5}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{1287a^{5}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\left(\frac{231a^{5}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right.\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 8/3465*(76*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(a) - (34*a^5/sgn(tan(
1/2*d*x + 1/2*c) + 1) + (187*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) - (1155*a^5/
sgn(tan(1/2*d*x + 1/2*c) + 1) - (1287*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) + (
231*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) - (231*a^5/sgn(tan(1/2*d*x + 1/2*c) +
 1) + (1287*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) - (1155*a^5/sgn(tan(1/2*d*x +
 1/2*c) + 1) - 17*(2*a^5*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c) +
1) + 11*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*
x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c
))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^2)/(a*t
an(1/2*d*x + 1/2*c)^2 + a)^(11/2))/d
```

maple [A] time = 1.35, size = 74, normalized size = 0.60

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\left(\sin\left(dx+c\right)-1\right)^{3}\left(315\left(\sin^{3}\left(dx+c\right)\right)+595\left(\sin^{2}\left(dx+c\right)\right)+340\sin\left(dx+c\right)+136\right)}{3465\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3465*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(315*sin(d*x+c)^3+595*sin(d*x+c)^2+3
40*sin(d*x+c)+136)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{2}}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\cos\left(c+d\,x\right)^{4}\sin\left(c+d\,x\right)^{2}}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin^{2}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)**2*cos(c + d*x)**4/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.465 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{8a^2\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}+\frac{2a\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] 8/315*a^2*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)+2/63*a*cos(d*x+c)^5/d/(a+a*
sin(d*x+c))^(3/2)-2/9*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2856, 2674, 2673}

$$\frac{8a^2\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}+\frac{2a\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (8*a^2*Cos[c + d*x]^5)/(315*d*(a + a*Sin[c + d*x])^(5/2)) + (2*a*Cos[c + d*
x]^5)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^5)/(9*d*Sqrt[a +
a*Sin[c + d*x]])
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
```

```
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} - \frac{1}{9}\int \frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= \frac{2a\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} - \frac{1}{63}(4a)\int \frac{\cos^4(c+d}{(a+a\sin(c+}$$
$$= \frac{8a^2\cos^5(c+dx)}{315d(a+a\sin(c+dx))^{5/2}} + \frac{2a\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 1.59, size = 87, normalized size = 0.95

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(130\sin(c+dx)-35\cos(2(c+dx))\cdots}{315d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/315*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])*(87 - 35*Cos[2*(c + d*x)] + 130*Sin[c + d*x]))/(d*Sqrt[a*(1 + Si
n[c + d*x])])
```

fricas [A] time = 0.44, size = 136, normalized size = 1.48

$$-\frac{2\left(35\cos(dx+c)^5+40\cos(dx+c)^4-\cos(dx+c)^3+2\cos(dx+c)^2-\left(35\cos(dx+c)^4-5\cos(dx+c)^3\cdots\right.\right.}{315\,(ad\cos(dx+c)+ad\sin(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/315*(35*cos(d*x + c)^5 + 40*cos(d*x + c)^4 - cos(d*x + c)^3 + 2*cos(d*x
+ c)^2 - (35*cos(d*x + c)^4 - 5*cos(d*x + c)^3 - 6*cos(d*x + c)^2 - 8*cos(d
*x + c) - 16)*sin(d*x + c) - 8*cos(d*x + c) - 16)*sqrt(a*sin(d*x + c) + a)/
(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

giac [B] time = 0.60, size = 279, normalized size = 3.03

$$-\frac{4\left(\frac{8\sqrt{2}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}}+\frac{\frac{13\,a^4}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{99\,a^4}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{105\,a^4}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\frac{63\,a^4}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{63\,a^4}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right.}\right.\right.\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] -4/315*(8*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(a) + (13*a^4/sgn(tan(1
/2*d*x + 1/2*c) + 1) - (99*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) - (105*a^4/sgn
(tan(1/2*d*x + 1/2*c) + 1) + (63*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) - (63*a^
4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (105*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) +
(13*a^4*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c) + 1) - 99*a^4/sgn(t
an(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1
/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x +
1/2*c)^2)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(9/2))/d
```

maple [A] time = 1.40, size = 64, normalized size = 0.70

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\left(\sin\left(dx+c\right)-1\right)^{3}\left(35\left(\sin^{2}\left(dx+c\right)\right)+65\sin\left(dx+c\right)+26\right)}{315\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/315*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(35*sin(d*x+c)^2+65*sin(d*x+c)+26)/co
s(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^4 \sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.466 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=130

$$-\frac{2\sin^2(c+dx)\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15ad}+\frac{32\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+32/15*cos(d
*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/5*cos(d*x+c)*sin(d*x+c)^2/d/(a+a*sin(d*x+c
))^(1/2)+2/15*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.56, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2881, 2778, 2968, 3023, 2751, 2649, 206, 3046, 2985, 2773}

$$-\frac{2\sin^2(c+dx)\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15ad}+\frac{32\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(Sqrt[a]*d) +
 (32*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sin[c
+ d*x]^2)/(5*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a + a*Sin[c
 + d*x]])/(15*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= \frac{4\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\sin(c+dx)(-4a+a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,d}{5a}$$

$$= \frac{4\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{-4a\sin(c+dx)+a\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,d}{5a}$$

$$= \frac{4\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c}}{15ad}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{32\cos(c+d}{15d\sqrt{a+a\sin(c}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{32\cos(c+d}{15d\sqrt{a+a\sin(c}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{32\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+}{5d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.25, size = 169, normalized size = 1.30

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-60\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)-3\sin\left(\frac{5}{2}(c+dx)\right)+60\cos\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(60*Cos[(c + d*x)/2] + 5*Cos[(3*(c +
 d*x))/2] + 3*Cos[(5*(c + d*x))/2] - 30*Log[1 + Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] + 30*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 60*Sin[(c + d
*x)/2] + 5*Sin[(3*(c + d*x))/2] - 3*Sin[(5*(c + d*x))/2]))/(30*d*Sqrt[a*(1
+ Sin[c + d*x])])
```

fricas [B] time = 0.47, size = 279, normalized size = 2.15

$$\frac{15\sqrt{a}\left(\cos(dx+c)+\sin(dx+c)+1\right)\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/30*(15*sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
 4*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - cos(d*x + c)
+ 13)*sin(d*x + c) + 14*cos(d*x + c) + 13)*sqrt(a*sin(d*x + c) + a))/(a*d*c
os(d*x + c) + a*d*sin(d*x + c) + a*d)
```

giac [B] time = 0.70, size = 386, normalized size = 2.97

$$-\frac{\frac{\left(30\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-15\,\sqrt{-a}\,\sqrt{a}\,\log\left(\sqrt{2}\,\sqrt{a}+\sqrt{a}\right)+26\,\sqrt{2}\,\sqrt{-a}\,\sqrt{a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{-a}\,a}-\frac{30\arctan\left(-\frac{\sqrt{a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\sqrt{-a}}\right.}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] -1/15*((30*a*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 15*sqrt(-a)*sqr
t(a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 26*sqrt(2)*sqrt(-a)*sqrt(a))*sgn(tan(
1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a) - 30*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*
x + 1/2*c) + 1)) + 15*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(((((17
*a^2*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 15*a^2/sgn(tan(1/
2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 20*a^2/sgn(tan(1/2*d*x + 1/2*c)
 + 1))*tan(1/2*d*x + 1/2*c) - 20*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2
*d*x + 1/2*c) + 15*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c)
- 17*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2
))/d
```

maple [A] time = 1.90, size = 123, normalized size = 0.95

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(15a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)+3\left(a-a\sin\left(dx+c\right)\right)^{\frac{5}{2}}-5\left(a-a\sin\right.\right.}{15a^{3}\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/15*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(15*a^(5/2)*arctanh((a-a*sin
(d*x+c))^(1/2)/a^(1/2))+3*(a-a*sin(d*x+c))^(5/2)-5*(a-a*sin(d*x+c))^(3/2)*a
-15*a^2*(a-a*sin(d*x+c))^(1/2))/a^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4 \csc(dx+c)}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.467 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=119

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3ad}+\frac{4\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+4/3*cos(d*x+c)
/d/(a+a*sin(d*x+c))^(1/2)-cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/3*cos(d*x+c
)*(a+a*sin(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.50, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2881, 2759, 2751, 2649, 206, 3044, 2985, 2773}

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3ad}+\frac{4\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(Sqrt[a]*d) + (4*C
os[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - Cot[c + d*x]/(d*Sqrt[a + a*Si
n[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ
[m, -2^(-1)]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
```

```
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx=\int \frac{\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx+\int \frac{\csc^2(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$=-\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad}+\frac{2\int \frac{\frac{a}{2}-a\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,d}{3a}$$

$$=\frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sqrt{a+a\sin(c+d}}{3ad}$$

$$=\frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sqrt{a+a\sin(c+d}}{3ad}$$

$$=\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{2\,\mathrm{co}}{}$$

Mathematica [A] time = 0.38, size = 190, normalized size = 1.60

$$\left(\tan\left(\frac{1}{2}(c+dx)\right)+1\right)\csc\left(\frac{1}{4}(c+dx)\right)\sec\left(\frac{1}{4}(c+dx)\right)\left(10\sin\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{5}{2}(c+dx)\right)-1\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/4]*Sec[(c + d*x)/4]*(-10*Cos[(c + d*x)/2] + 3*Cos[(3*(c + d*
x))/2] + Cos[(5*(c + d*x))/2] + 10*Sin[(c + d*x)/2] + 3*Log[1 + Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 3*Log[1 - Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]]*Sin[c + d*x] + 3*Sin[(3*(c + d*x))/2] - Sin[(5*(c + d*x))/2])*
(1 + Tan[(c + d*x)/2]))/(24*d*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [B] time = 0.50, size = 306, normalized size = 2.57

$$3\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+\right.}{\cos(dx+c)^3+\cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log(
(a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c)
+ 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) -
9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) -
 a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) -
cos(d*x + c) - 1)) - 4*(2*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - (2*cos(d*x +
c)^2 - 2*cos(d*x + c) - 7)*sin(d*x + c) - 5*cos(d*x + c) - 7)*sqrt(a*sin(d*
x + c) + a))/(a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d*x + c) + a*d)*sin(d*x +
 c))
```

giac [B] time = 0.80, size = 476, normalized size = 4.00

$$\frac{\left(6\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-3\sqrt{2}\,\sqrt{-a}\,\sqrt{a}\,\log\left(\sqrt{2}\,\sqrt{a}+\sqrt{a}\right)+6\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-3\sqrt{-a}\,\sqrt{a}\,\log\left(\sqrt{2}\,\sqrt{a}+\sqrt{a}\right)-11\sqrt{2}\,\sqrt{-a}\,\sqrt{a}-25\sqrt{-a}\,\sqrt{a}\right)s}{\sqrt{2}\,\sqrt{-a}\,a+\sqrt{-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/6*((6*sqrt(2)*a*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 3*sqrt(2)*
sqrt(-a)*sqrt(a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 6*a*arctan((sqrt(2)*sqrt(
a) + sqrt(a))/sqrt(-a)) - 3*sqrt(-a)*sqrt(a)*log(sqrt(2)*sqrt(a) + sqrt(a))
 - 11*sqrt(2)*sqrt(-a)*sqrt(a) - 25*sqrt(-a)*sqrt(a))*sgn(tan(1/2*d*x + 1/2
*c) + 1)/(sqrt(2)*sqrt(-a)*a + sqrt(-a)*a) + ((((3*a*tan(1/2*d*x + 1/2*c)/s
gn(tan(1/2*d*x + 1/2*c) + 1) - 4*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d
*x + 1/2*c) + 18*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 12
*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 7*a/sgn(tan(1/2*d*
x + 1/2*c) + 1))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 6*arctan(-(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(
-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c
) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c)
+ 1)) + 6*sqrt(a)/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2 - a)*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 2.04, size = 126, normalized size = 1.06

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left(\sin(dx+c)\left(2(a-a\sin(dx+c))^{\frac{3}{2}}\sqrt{a}+3\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)a^2\right.\right.}{3a^{\frac{5}{2}}\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin(dx+c)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(5/2)*(sin(d*x+c)*(2*(a-a*si
n(d*x+c))^(3/2)*a^(1/2)+3*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^2)-3*(a
-a*sin(d*x+c))^(1/2)*a^(3/2))/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/
d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos (dx+c)^4 \csc (dx+c)^2}{\sqrt{a\sin (dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c+d\,x)^4}{\sin (c+d\,x)^2\,\sqrt{a+a\,\sin (c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.468 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=125

$$-\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] 9/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)-2*cos(d*x+
c)/d/(a+a*sin(d*x+c))^(1/2)+1/4*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/2*cot
(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2881, 2751, 2649, 206, 3044, 2984, 2985, 2773}

$$-\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (9*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*Sqrt[a]*d)
- (2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) + Cot[c + d*x]/(4*d*Sqrt[a
+ a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*
x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2773

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2881

Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IGtQ[m, 0]

Rule 2984

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rule 2985

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3044

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc^3(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(-\frac{a}{2}-\frac{5}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,d}{2a}$$

$$= -\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{\int}{}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{a}\,d} - \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\text{co}}{2}$$

Mathematica [B] time = 3.74, size = 296, normalized size = 2.37

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(64\sin\left(\frac{1}{2}(c+dx)\right)-64\cos\left(\frac{1}{2}(c+dx)\right)+4\tan\left(\frac{1}{4}(c+dx)\right)+4\cot\left(\frac{1}{4}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8 - 64*Cos[(c + d*x)/2] + 4*Cot[(c
 + d*x)/4] - Csc[(c + d*x)/4]^2 + 36*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*
```

```
x)/2]] - 36*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]
^2 + 2/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^2 - (8*Sin[(c + d*x)/4])/(Cos[
(c + d*x)/4] - Sin[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^
2 + (8*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 64*Sin[(c
+ d*x)/2] + 4*Tan[(c + d*x)/4]))/(32*d*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [B] time = 0.50, size = 346, normalized size = 2.77

$$9\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(9*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 +
4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*s
qrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (c
os(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(8*cos(d*x + c)^3
+ 9*cos(d*x + c)^2 - (8*cos(d*x + c)^2 - cos(d*x + c) - 11)*sin(d*x + c) -
10*cos(d*x + c) - 11)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x + c)^3 + a*d*c
os(d*x + c)^2 - a*d*cos(d*x + c) - a*d + (a*d*cos(d*x + c)^2 - a*d)*sin(d*x
 + c))
```

giac [B] time = 0.79, size = 554, normalized size = 4.43

$$\frac{\frac{\left(36\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-18\sqrt{2}\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)+54\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-27\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)+62\sqrt{2}\sqrt{-a}+82\sqrt{-a}\right)\mathrm{sgn}\left(\tan\cdots\right.}{2\sqrt{2}\sqrt{-a}\sqrt{a}+3\sqrt{-a}\sqrt{a}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/8*((36*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 18*
sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 54*sqrt(a)*arctan((sqrt(2
)*sqrt(a) + sqrt(a))/sqrt(-a)) - 27*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a))
```

```
 + 62*sqrt(2)*sqrt(-a) + 82*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(2*sqrt
(2)*sqrt(-a)*sqrt(a) + 3*sqrt(-a)*sqrt(a)) + (((tan(1/2*d*x + 1/2*c)/sgn(ta
n(1/2*d*x + 1/2*c) + 1) - 2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/
2*c) + 17/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 18/sgn(tan(
1/2*d*x + 1/2*c) + 1))/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) - 18*arctan(-(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(
sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 9*log(abs(-sqrt(a)*tan(1/2*d*x +
1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/
2*c) + 1)) + 2*((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^3 - 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^2*sqrt(a) + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))*a + 2*a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 - a)^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 2.05, size = 150, normalized size = 1.20

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(8a^{\frac{3}{2}}\sqrt{-a(\sin(dx+c)-1)}\left(\sin^2(dx+c)\right)-9\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)}}{\sqrt{a}}\right.\right.}{4a^{\frac{5}{2}}\sin(dx+c)^2\cos(dx+c)\sqrt{a+a\sin}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(5/2)*(8*a^(3/2)*(-a*(sin(d
*x+c)-1))^(1/2)*sin(d*x+c)^2-9*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*s
in(d*x+c)^2*a^2+(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)+(-a*(sin(d*x+c)-1))^(1/2)
*a^(3/2))/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4\csc(dx+c)^3}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)^3\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)),x)
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.469 $\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{9\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{a}\,d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -7/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+9/8*cot(d
*x+c)/d/(a+a*sin(d*x+c))^(1/2)+1/12*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c)
)^(1/2)-1/3*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2718, 2649, 206, 3044, 2984, 2985, 2773}

$$\frac{9\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{a}\,d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*Sqrt[a]*d)
 + (9*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c +
d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*d*
Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2718

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
```

```
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc^4(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^3(c+dx)\left(-\frac{a}{2}-\frac{7}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a} - \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x,\right.}{d}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx}{3d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{9\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{9\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{9\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{a}\,d} + \frac{9\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot}{3a}$$

Mathematica [B] time = 0.60, size = 292, normalized size = 2.16

$$\frac{\csc^9\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-36\sin\left(\frac{1}{2}(c+dx)\right)-46\sin\left(\frac{3}{2}(c+dx)\right)+54\sin\left(\frac{5}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^9*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(36*Cos[(c + d*x)
/2] - 46*Cos[(3*(c + d*x))/2] - 54*Cos[(5*(c + d*x))/2] - 36*Sin[(c + d*x)/
2] - 63*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 63*Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 46*Sin[(3*(c + d*x)
)/2] + 54*Sin[(5*(c + d*x))/2] + 21*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]]*Sin[3*(c + d*x)] - 21*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Si
```

```
n[3*(c + d*x)]))/(24*d*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3*Sqrt[a*(
1 + Sin[c + d*x])])
```

fricas [B] time = 0.48, size = 369, normalized size = 2.73

$$21\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)+1\right)\sqrt{a}\,\log\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(21*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c
)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2
*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a
*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) -
4*(27*cos(d*x + c)^3 + 25*cos(d*x + c)^2 - (27*cos(d*x + c)^2 + 2*cos(d*x +
 c) - 17)*sin(d*x + c) - 19*cos(d*x + c) - 17)*sqrt(a*sin(d*x + c) + a))/(a
*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d - (a*d*cos(d*x + c)^3 + a*d*
cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))
```

giac [B] time = 0.80, size = 583, normalized size = 4.32

$$\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\left(\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{3}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{22}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)-\frac{\left(2\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/48*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*tan(1/2*d*x + 1/2*c)/(a*sgn(ta
n(1/2*d*x + 1/2*c) + 1)) - 3/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x
 + 1/2*c) - 22/(a*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (210*sqrt(2)*sqrt(a)*ar
ctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 105*sqrt(2)*sqrt(-a)*log(sqrt(
2)*sqrt(a) + sqrt(a)) + 294*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt
(-a)) - 147*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 128*sqrt(2)*sqrt(-a)
```

```
- 186*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(5*sqrt(2)*sqrt(-a)*sqrt(a) +
 7*sqrt(-a)*sqrt(a)) + 42*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1
)) - 21*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 2*(3*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5 + 18*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(a) - 48*(sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(3/2) - 3*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2 + 22*a
^(5/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^2 - a)^3*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 2.10, size = 144, normalized size = 1.07

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(21\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^3\left(\sin^3(dx+c)\right)-21\sqrt{-a(\sin(dx+c}\right.}{24a^{\frac{7}{2}}\sin(dx+c)^3\cos(dx+c)\sqrt{a+a\sin(d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(21*arctanh((-a*(sin(d*x+c)-
1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^3-21*(-a*(sin(d*x+c)-1))^(1/2)*a^(5/2)+56
*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)-27*(-a*(sin(d*x+c)-1))^(5/2)*a^(1/2))/a^
(7/2)/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4\csc(dx+c)^4}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)^4/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)^4\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(1/2)),x)
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.470 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=170

$$-\frac{11\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64\sqrt{a}\,d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}+\frac{53\,\mathrm{co}}{96d}$$

```
[Out] -11/64*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)-11/64*c
ot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+53/96*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d
*x+c))^(1/2)+1/24*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)-1/4*cot(
d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2881, 2780, 2649, 206, 2773, 3044, 2984, 2985}

$$-\frac{11\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64\sqrt{a}\,d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}+\frac{53\,\mathrm{co}}{96d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*Sqrt[a]*
d) - (11*Cot[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (53*Cot[c + d*x]*C
sc[c + d*x])/(96*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c + d*x]^2
)/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*d*Sqrt
[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2780

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e +
f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
```

```
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc^5(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^4(c+dx)\left(-\frac{a}{2}-\frac{9}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a} + \int \csc(c+dx)\sqrt{a\,\cdots}$$

$$= \frac{\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^3(c+dx)\left(-\frac{53a^2}{4}-\frac{5}{4}a^2\sin(\cdots\right)}{\sqrt{a+a\sin(c+dx)}}}{12a^2}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{53\cot(c+dx)\,\mathrm{cs}\cdots}{96d\sqrt{a+a\sin(\cdots}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{11\cot(c+d\cdots}{64d\sqrt{a+a\sin(c\cdots}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{11\cot(c+d\cdots}{64d\sqrt{a+a\sin(c\cdots}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{11\cot(c+d\cdots}{64d\sqrt{a+a\sin(c\cdots}}$$

$$= -\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64\sqrt{a}\,d} - \frac{11\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} + \frac{53\cot(c+dx)\csc(c+\cdots}{96d\sqrt{a+a\sin(c+d\cdots}}$$

Mathematica [B] time = 0.90, size = 374, normalized size = 2.20

$$\csc^{12}\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-214\sin\left(\frac{1}{2}(c+dx)\right)-558\sin\left(\frac{3}{2}(c+dx)\right)+490\sin\left(\frac{5}{2}(c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^12*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(214*Cos[(c + d*
x)/2] - 558*Cos[(3*(c + d*x))/2] - 490*Cos[(5*(c + d*x))/2] + 66*Cos[(7*(c
+ d*x))/2] - 99*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 132*Cos[2*(c
 + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 33*Cos[4*(c + d*x)]
*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 99*Log[1 - Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] - 132*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] + 33*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] - 214*Sin[(c + d*x)/2] - 558*Sin[(3*(c + d*x))/2] + 490*Sin[(5*(c + d
*x))/2] + 66*Sin[(7*(c + d*x))/2]))/(192*d*(Csc[(c + d*x)/4]^2 - Sec[(c + d
*x)/4]^2)^4*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [B] time = 0.48, size = 426, normalized size = 2.51

$$33\left(\cos\left(dx+c\right)^{5}+\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{3}-2\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1\right)\sin\left(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/768*(33*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x +
 c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(d*x + c)
 + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^
2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x +
c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) -
 a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 -
1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(33*cos(d*x + c)^4 - 106*cos(d*x +
 c)^3 - 164*cos(d*x + c)^2 + (33*cos(d*x + c)^3 + 139*cos(d*x + c)^2 - 25*c
os(d*x + c) - 83)*sin(d*x + c) + 58*cos(d*x + c) + 83)*sqrt(a*sin(d*x + c)
+ a))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a
*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d + (a*d*cos(d*x + c)^4 - 2*a*d*co
s(d*x + c)^2 + a*d)*sin(d*x + c))
```

giac [B] time = 0.92, size = 736, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/384*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*(3*tan(1/2*d*x + 1/2*c)/(a*sg
n(tan(1/2*d*x + 1/2*c) + 1)) - 4/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2
*d*x + 1/2*c) - 33/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c)
+ 64/(a*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (792*sqrt(2)*a^(3/2)*arctan((sqrt
(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 396*sqrt(2)*sqrt(-a)*a*log(sqrt(2)*sqrt(
a) + sqrt(a)) + 1122*a^(3/2)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) -
 561*sqrt(-a)*a*log(sqrt(2)*sqrt(a) + sqrt(a)) + 2054*sqrt(2)*sqrt(-a)*a +
2896*sqrt(-a)*a)*sgn(tan(1/2*d*x + 1/2*c) + 1)/(12*sqrt(2)*sqrt(-a)*a^(3/2)
 + 17*sqrt(-a)*a^(3/2)) + 66*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)
+ 1)) - 33*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 2*(33*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7 - 48*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) - 57*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a + 192*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2)
- 57*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*
a^2 - 208*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^2*a^(5/2) + 33*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))*a^3 + 64*a^(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2 - a)^4*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 2.05, size = 162, normalized size = 0.95

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left(33(-a(\sin(dx+c)-1))^{\frac{7}{2}}a^{\frac{3}{2}}-33\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^{5}\left(\sin^{4}\right.\right.}{192a^{\frac{11}{2}}\sin(dx+c)^{4}\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/192*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(11/2)*(33*(-a*(sin(d*x+c)
-1))^(7/2)*a^(3/2)-33*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^5*sin(d*
x+c)^4+7*(-a*(sin(d*x+c)-1))^(5/2)*a^(5/2)-121*(-a*(sin(d*x+c)-1))^(3/2)*a^
(7/2)+33*(-a*(sin(d*x+c)-1))^(1/2)*a^(9/2))/sin(d*x+c)^4/cos(d*x+c)/(a+a*si
n(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4 \csc(dx+c)^5}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)^5/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)^5\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.471 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=205

$$-\frac{9\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128\sqrt{a}\,d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc^3(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}+\frac{29\,\mathrm{co}}{80d}$$

```
[Out] -9/128*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)-9/128*c
ot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-3/64*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*
x+c))^(1/2)+29/80*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)+1/40*cot
(d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-1/5*cot(d*x+c)*csc(d*x+c)^4/d
/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.14, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2881, 2779, 2985, 2649, 206, 2773, 3044, 2984}

$$-\frac{9\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128\sqrt{a}\,d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc^3(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}+\frac{29\,\mathrm{co}}{80d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-9*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(128*Sqrt[a]*
d) - (9*Cot[c + d*x])/(128*d*Sqrt[a + a*Sin[c + d*x]]) - (3*Cot[c + d*x]*Cs
c[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (29*Cot[c + d*x]*Csc[c + d*x]
^2)/(80*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c + d*x]^3)/(40*d*S
qrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*d*Sqrt[a + a*Si
n[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc^6(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^5(c+dx)\left(-\frac{a}{2}-\frac{11}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}}{5a}$$

$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc^3(c+dx)}{40d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} +$$

$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{29\cot(c+dx)\csc^2(c+dx)}{80d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc^3(c+dx}{40d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{9\cot(c+dx)}{128d\sqrt{a+a\sin(c+}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{9\cot(c+dx)}{128d\sqrt{a+a\sin(c+}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{9\cot(c+dx)}{128d\sqrt{a+a\sin(c+}}$$

$$= -\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128\sqrt{a}\,d} - \frac{9\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} - \frac{3\cot(c+dx)\csc(c+d}{64d\sqrt{a+a\sin(c+dx}}$$

<u>**Mathematica**</u> **[A]** time = 1.01, size = 410, normalized size = 2.00

$$-\frac{\csc^{15}\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-820\sin\left(\frac{1}{2}(c+dx)\right)+1600\sin\left(\frac{3}{2}(c+dx)\right)-1616\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/640*(Csc[(c + d*x)/2]^15*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(820*Cos[
(c + d*x)/2] + 1600*Cos[(3*(c + d*x))/2] + 1616*Cos[(5*(c + d*x))/2] - 30*C
os[(7*(c + d*x))/2] + 90*Cos[(9*(c + d*x))/2] - 820*Sin[(c + d*x)/2] + 450*
Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 450*Log[1 - Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 1600*Sin[(3*(c + d*x))/2]
- 1616*Sin[(5*(c + d*x))/2] - 225*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]]*Sin[3*(c + d*x)] + 225*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin
[3*(c + d*x)] - 30*Sin[(7*(c + d*x))/2] - 90*Sin[(9*(c + d*x))/2] + 45*Log[
1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 45*Log[1 - Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(d*(Csc[(c + d*x)/4]^2
- Sec[(c + d*x)/4]^2)^5*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [B] time = 0.48, size = 472, normalized size = 2.30

$$\frac{45\left(\cos\left(dx+c\right)^{6}-3\cos\left(dx+c\right)^{4}+3\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{5}+\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{3}-2\cos\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/2560*(45*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos(d*x
 + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x +
c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
+ 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
+ (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(45*cos(d*x +
c)^5 + 15*cos(d*x + c)^4 + 142*cos(d*x + c)^3 + 186*cos(d*x + c)^2 - (45*co
s(d*x + c)^4 + 30*cos(d*x + c)^3 + 172*cos(d*x + c)^2 - 14*cos(d*x + c) - 7
3)*sin(d*x + c) - 59*cos(d*x + c) - 73)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(
d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d
*x + c)^5 + a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^
2 + a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

giac [B] time = 0.95, size = 802, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/1280*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*((4*tan(1/2*d*x + 1/2*c)/(a*
sgn(tan(1/2*d*x + 1/2*c) + 1)) - 5/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1
/2*d*x + 1/2*c) - 12/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c
) + 35/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c) - 32/(a*sgn(
tan(1/2*d*x + 1/2*c) + 1))) - (2610*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a)
 + sqrt(a))/sqrt(-a)) - 1305*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)
) + 3690*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 1845*sqrt(-
a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 5058*sqrt(2)*sqrt(-a) - 7156*sqrt(-a))*
sgn(tan(1/2*d*x + 1/2*c) + 1)/(29*sqrt(2)*sqrt(-a)*sqrt(a) + 41*sqrt(-a)*sq
rt(a)) + 90*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 45*log(ab
s(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqr
t(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(35*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^9 - 80*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(a) - 110*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7*a + 240*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(3/2) - 80*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(5/2) + 110*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^3 + 8
0*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(
7/2) - 35*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))*a^4 - 32*a^(9/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 - a)^5*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 2.10, size = 180, normalized size = 0.88

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(45\sqrt{-a(\sin(dx+c)-1)}\,a^{\frac{13}{2}}-210(-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{11}{2}}-128(-\right.}{640a^{\frac{15}{2}}\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/640*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(45*(-a*(sin(d*x+c)-1))^(1/2
)*a^(13/2)-210*(-a*(sin(d*x+c)-1))^(3/2)*a^(11/2)-128*(-a*(sin(d*x+c)-1))^(
5/2)*a^(9/2)+210*(-a*(sin(d*x+c)-1))^(7/2)*a^(7/2)-45*(-a*(sin(d*x+c)-1))^(
9/2)*a^(5/2)-45*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^7*sin(d*x+c)^5
)/a^(15/2)/sin(d*x+c)^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sin\left(c+d\,x\right)^6\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^6*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^6*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.472 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=205

$$-\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{385a^3d}-\frac{2\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{11a^2d}+\frac{8\cos(c+dx)\sqrt{a\sin(c+dx)}}{1155a^2d}$$

```
[Out] -4/385*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a^3/d-4/165*cos(d*x+c)/a/d/(a+a*si
n(d*x+c))^(1/2)-2/231*cos(d*x+c)*sin(d*x+c)^3/a/d/(a+a*sin(d*x+c))^(1/2)+14
/33*cos(d*x+c)*sin(d*x+c)^4/a/d/(a+a*sin(d*x+c))^(1/2)+8/1155*cos(d*x+c)*(a
+a*sin(d*x+c))^(1/2)/a^2/d-2/11*cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^(1
/2)/a^2/d
```

Rubi [A] time = 0.78, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2880, 2770, 2759, 2751, 2646, 3046, 2981}

$$-\frac{2\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{11a^2d}-\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{385a^3d}+\frac{8\cos(c+dx)\sqrt{a\sin(c+dx)}}{1155a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-4*Cos[c + d*x])/(165*a*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sin[
c + d*x]^3)/(231*a*d*Sqrt[a + a*Sin[c + d*x]]) + (14*Cos[c + d*x]*Sin[c + d
*x]^4)/(33*a*d*Sqrt[a + a*Sin[c + d*x]]) + (8*Cos[c + d*x]*Sqrt[a + a*Sin[c
 + d*x]])/(1155*a^2*d) - (2*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[a + a*Sin[c +
d*x]])/(11*a^2*d) - (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(385*a^3*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,\left(1+\sin^2(c+dx)\right)\,dx}{a^2} - \frac{2\int \sin^4(c+dx)\sqrt{a}}{a^2}$$

$$= \frac{4\cos(c+dx)\sin^4(c+dx)}{9ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\sin(c+dx)}}{11a^2d} + $$

$$= \frac{32\cos(c+dx)\sin^3(c+dx)}{63ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+dx)\sin^4(c+dx)}{33ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4}{}$$

$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{231ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+dx)\sin^4(c+dx)}{33ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4}{}$$

$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{231ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+dx)\sin^4(c+dx)}{33ad\sqrt{a+a\sin(c+dx)}} - \frac{128\cos(c+dx)\sqrt{}}{315}$$

$$= \frac{64\cos(c+dx)}{45ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^3(c+dx)}{231ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+dx)\sin^4(c+}{33ad\sqrt{a+a\sin(c+}}$$

$$= -\frac{4\cos(c+dx)}{165ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^3(c+dx)}{231ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+dx)\sin^4(}{33ad\sqrt{a+a\sin(c}}$$

Mathematica [A] time = 5.59, size = 102, normalized size = 0.50

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5(-475\sin(c+dx)+105\sin(3(c+dx))+140\cos(2(c+}{2310a^2d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*(-204 +
 140*Cos[2*(c + d*x)] - 475*Sin[c + d*x] + 105*Sin[3*(c + d*x)]))/(2310*a^2
*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.47, size = 161, normalized size = 0.79

$$-\frac{2\left(105\cos(dx+c)^6-140\cos(dx+c)^5-460\cos(dx+c)^4+274\cos(dx+c)^3+607\cos(dx+c)^2+\left(105\cos\right.\right.}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr icas")

[Out] -2/1155*(105*cos(d*x + c)^6 - 140*cos(d*x + c)^5 - 460*cos(d*x + c)^4 + 274 *cos(d*x + c)^3 + 607*cos(d*x + c)^2 + (105*cos(d*x + c)^5 + 245*cos(d*x + c)^4 - 215*cos(d*x + c)^3 - 489*cos(d*x + c)^2 + 118*cos(d*x + c) + 236)*si n(d*x + c) - 118*cos(d*x + c) - 236)*sqrt(a*sin(d*x + c) + a)/(a^2*d*cos(d* x + c) + a^2*d*sin(d*x + c) + a^2*d)

giac [A] time = 0.73, size = 286, normalized size = 1.40

$$-\frac{8\left(\frac{59\sqrt{2}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}}+\frac{2\left(\frac{2a^4}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\left(\frac{11a^4}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{264a^4}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{693a^4}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{693a^4}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+ \right.\right.}\right.\right.\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi ac")

[Out] -8/1155*(59*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(3/2) + 2*(2*a^4/sgn(ta n(1/2*d*x + 1/2*c) + 1) + (11*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) - (264*a^4/ sgn(tan(1/2*d*x + 1/2*c) + 1) - (693*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) - (6 93*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) - (264*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) - (2*a^4*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c) + 1) + 11*a^4/s gn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)) *tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^2)*tan(1/ 2*d*x + 1/2*c)^2)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(11/2))/d

maple [A] time = 1.00, size = 77, normalized size = 0.38

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\left(\sin\left(dx+c\right)-1\right)^{3}\left(105\left(\sin^{3}\left(dx+c\right)\right)+70\left(\sin^{2}\left(dx+c\right)\right)+40\sin\left(dx+c\right)+16\right)}{1155a\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)

[Out] 2/1155/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(105*sin(d*x+c)^3+70*sin(d*x+c)^2+ 40*sin(d*x+c)+16)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{3}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{4}\,\sin\left(c+d\,x\right)^{3}}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.473 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=92

$$-\frac{2\cos^5(c+dx)}{9ad\sqrt{a\sin(c+dx)+a}}+\frac{20\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{46a\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] -46/315*a*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)+20/63*cos(d*x+c)^5/d/(a+a*s
in(d*x+c))^(3/2)-2/9*cos(d*x+c)^5/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2877, 2856, 2674, 2673}

$$-\frac{2\cos^5(c+dx)}{9ad\sqrt{a\sin(c+dx)+a}}+\frac{20\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{46a\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-46*a*Cos[c + d*x]^5)/(315*d*(a + a*Sin[c + d*x])^(5/2)) + (20*Cos[c + d*x
]^5)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^5)/(9*a*d*Sqrt[a +
 a*Sin[c + d*x]])
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
```

```
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2877

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\cos^5(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{\int \frac{\cos^4(c+dx)\left(-\frac{3a}{2}-2a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}\\ &= \frac{\cos^5(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{2\cos^5(c+dx)}{9ad\sqrt{a+a\sin(c+dx)}}+\frac{23\int \frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{36a}\\ &= \frac{20\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2\cos^5(c+dx)}{9ad\sqrt{a+a\sin(c+dx)}}+\frac{23}{63}\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))}\\ &= -\frac{46a\cos^5(c+dx)}{315d(a+a\sin(c+dx))^{5/2}}+\frac{20\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2\cos^5(c+dx)}{9ad\sqrt{a+a\sin(c+a}}\end{aligned}$$

Mathematica [A] time = 3.72, size = 92, normalized size = 1.00

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5(40\sin(c+dx)-35\cos(2(c+dx))+51)}{315a^2d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/315*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*
(51 - 35*Cos[2*(c + d*x)] + 40*Sin[c + d*x]))/(a^2*d*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]))
```

fricas [A] time = 0.44, size = 142, normalized size = 1.54

$$-\frac{2\left(35\,\cos\left(dx+c\right)^{5}+85\,\cos\left(dx+c\right)^{4}-73\,\cos\left(dx+c\right)^{3}-169\,\cos\left(dx+c\right)^{2}-\left(35\,\cos\left(dx+c\right)^{4}-50\,\cos\left(dx\right.\right.\right.}{315\left(a^{2}d\cos\left(dx+c\right)+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] -2/315*(35*cos(d*x + c)^5 + 85*cos(d*x + c)^4 - 73*cos(d*x + c)^3 - 169*cos
(d*x + c)^2 - (35*cos(d*x + c)^4 - 50*cos(d*x + c)^3 - 123*cos(d*x + c)^2 +
 46*cos(d*x + c) + 92)*sin(d*x + c) + 46*cos(d*x + c) + 92)*sqrt(a*sin(d*x
+ c) + a)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

giac [B] time = 0.72, size = 282, normalized size = 3.07

$$8\left(\frac{23\,\sqrt{2}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{\frac{3}{2}}}-\frac{\left(\frac{9\,a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{105\,a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{252\,a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{252\,a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{105\,a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}\right.\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 8/315*(23*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(3/2) - ((9*a^3/sgn(tan(1
/2*d*x + 1/2*c) + 1) - (105*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (252*a^3/sg
n(tan(1/2*d*x + 1/2*c) + 1) - (252*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (105
*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (2*a^3*tan(1/2*d*x + 1/2*c)^2/sgn(tan(
1/2*d*x + 1/2*c) + 1) + 9*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x +
1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*t
an(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^2 + 2*a^3/sgn(tan(1/2*d*x + 1/2*c
) + 1))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(9/2))/d
```

maple [A] time = 1.02, size = 67, normalized size = 0.73

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\left(\sin\left(dx+c\right)-1\right)^{3}\left(35\left(\sin^{2}\left(dx+c\right)\right)+20\sin\left(dx+c\right)+8\right)}{315a\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/315/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(35*sin(d*x+c)^2+20*sin(d*x+c)+8)/c
os(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4 \sin(c+d\,x)^2}{(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.474 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{6a\cos^5(c+dx)}{35d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] 6/35*a*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-2/7*cos(d*x+c)^5/d/(a+a*sin(d*
x+c))^(3/2)
```

Rubi [A] time = 0.15, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2856, 2673}

$$\frac{6a\cos^5(c+dx)}{35d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (6*a*Cos[c + d*x]^5)/(35*d*(a + a*Sin[c + d*x])^(5/2)) - (2*Cos[c + d*x]^5)
/(7*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} - \frac{3}{7}\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$
$$= \frac{6a\cos^5(c+dx)}{35d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 1.90, size = 82, normalized size = 1.37

$$-\frac{2(5\sin(c+dx)+2)\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}{35a^2d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*(2 +
 5*Sin[c + d*x]))/(35*a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [B] time = 0.44, size = 121, normalized size = 2.02

$$\frac{2\left(5\cos(dx+c)^4-8\cos(dx+c)^3-19\cos(dx+c)^2+\left(5\cos(dx+c)^3+13\cos(dx+c)^2-6\cos(dx+c)-12\right.\right.}{35\left(a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 2/35*(5*cos(d*x + c)^4 - 8*cos(d*x + c)^3 - 19*cos(d*x + c)^2 + (5*cos(d*x
+ c)^3 + 13*cos(d*x + c)^2 - 6*cos(d*x + c) - 12)*sin(d*x + c) + 6*cos(d*x
+ c) + 12)*sqrt(a*sin(d*x + c) + a)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c
) + a^2*d)
```

giac [B] time = 0.67, size = 216, normalized size = 3.60

$$-\frac{4\left(\frac{6\sqrt{2}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}}-\frac{\left(\left(\left(\left(\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{14a^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{35a^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\right.\right.}}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{7}{2}}}\right.}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] -4/35*(6*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(3/2) - (((((a^2*tan(1/2*d
*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c) + 1) - 14*a^2/sgn(tan(1/2*d*x + 1/2*
c) + 1))*tan(1/2*d*x + 1/2*c) + 35*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1
/2*d*x + 1/2*c) - 35*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c
) + 14*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c)^2 - a^2/sgn(
tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2))/d
```

maple [A] time = 0.87, size = 57, normalized size = 0.95

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\left(\sin\left(dx+c\right)-1\right)^{3}\left(5\sin\left(dx+c\right)+2\right)}{35a\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/35/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(5*sin(d*x+c)+2)/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{{\cos\left(c+d\,x\right)}^4\,\sin\left(c+d\,x\right)}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.475 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}+\frac{10\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d+10/3*cos(d*
x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-2/3*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.36, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2880, 2646, 3046, 2981, 2773, 206}

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{10\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) +
 (10*Cos[c + d*x])/(3*a*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[
a + a*Sin[c + d*x]])/(3*a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\left(1+\sin^2(c+dx)\right)dx}{a^2} - \frac{2\int \sqrt{a+a\sin(c+d}}{a^2}$$

$$= \frac{4\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} + \frac{2\int \csc(c+dx)\left(\frac{3}{2}\right.}{}$$

$$= \frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} + \frac{\int \csc(c+dx)\sqrt{a}}{}$$

$$= \frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} - \frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\right.}{}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin}}{3a^2d}$$

Mathematica [A] time = 0.27, size = 147, normalized size = 1.50

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-9\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)+9\cos\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{3}{2}(c+dx)\right.\right.}{3d(a(\sin(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(9*Cos[(c + d*x)/2] - Cos[(3*(c +
d*x))/2] - 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 9*Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]
))/(3*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.49, size = 260, normalized size = 2.65

$$\frac{3\sqrt{a}\,(\cos(dx+c)+\sin(dx+c)+1)\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-\right.}\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
```

```
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4
*(cos(d*x + c)^2 + (cos(d*x + c) + 5)*sin(d*x + c) - 4*cos(d*x + c) - 5)*sq
rt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

giac [B] time = 0.84, size = 313, normalized size = 3.19

$$-\frac{\frac{\left(6\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-3\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)+10\sqrt{2}\sqrt{-a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}\,a^{\frac{3}{2}}}+\frac{4\left(\left(\left(\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\cdots\right.\right.\right.}{\left(a\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] -1/3*((6*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 3*sqrt(-a)*
log(sqrt(2)*sqrt(a) + sqrt(a)) + 10*sqrt(2)*sqrt(-a))*sgn(tan(1/2*d*x + 1/2
*c) + 1)/(sqrt(-a)*a^(3/2)) + 4*(((2*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x +
 1/2*c) + 1) - 3/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 3/sg
n(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 2/sgn(tan(1/2*d*x + 1/2
*c) + 1))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 6*arctan(-(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a*s
gn(tan(1/2*d*x + 1/2*c) + 1)) + 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1))
)/d
```

maple [A] time = 1.10, size = 103, normalized size = 1.05

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(-3a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)+\left(a-a\sin\left(dx+c\right)\right)^{\frac{3}{2}}+3a\sqrt{a-a\sin\left(\cdots\right.}\right.}{3a^{3}\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/3/a^3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-3*a^(3/2)*arctanh((a-a*s
in(d*x+c))^(1/2)/a^(1/2))+(a-a*sin(d*x+c))^(3/2)+3*a*(a-a*sin(d*x+c))^(1/2)
)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4 \csc(dx+c)}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.476 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{a^2d}-\frac{\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] 3*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d-cos(d*x+c)/a
/d/(a+a*sin(d*x+c))^(1/2)-cot(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.40, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2880, 2773, 206, 3044, 21, 2763}

$$-\frac{\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{a^2d}+\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) -
Cos[c + d*x]/(a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Sqrt[a + a*Sin[
c + d*x]])/(a^2*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
```

```
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\left(1+\sin^2(c+dx)\right)dx}{a^2} - \frac{2\int \csc(c+dx)\sqrt{a+}}{a^2}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{a^2d} + \frac{\int \csc(c+dx)\left(\frac{a}{2}+\frac{1}{2}a\sin(c+dx)\right)\sqrt{a+a\,\mathrm{s}}}{a^3}$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{a^2d} + \frac{\int \csc(c+dx)(a+}{2}$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+}}{a^2d}$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+}}{a^2d}$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+}}{a^2d}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+}}{a^2d}$$

Mathematica [B] time = 0.70, size = 220, normalized size = 2.34

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(8\sin\left(\frac{1}{2}(c+dx)\right)-8\cos\left(\frac{1}{2}(c+dx)\right)-\tan\left(\frac{1}{4}(c+dx)\right)-\cot\left(\frac{1}{4}(c+dx)\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(2 - 8*Cos[(c + d*x)/2] - Cot[(c +
 d*x)/4] + 6*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 6*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] + (2*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] -
Sin[(c + d*x)/4]) - (2*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/
4]) + 8*Sin[(c + d*x)/2] - Tan[(c + d*x)/4]))/(4*d*(a*(1 + Sin[c + d*x]))^(
3/2))
```

fricas [B] time = 0.48, size = 291, normalized size = 3.10

$$\frac{3\left(\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}+4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(d\cdots\right.}{\cos(dx+c)^{3}+\mathrm{co}\cdots}\right.}{4\,(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((
a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) +
 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9
*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) -
a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - c
os(d*x + c) - 1)) + 4*(2*cos(d*x + c)^2 + (2*cos(d*x + c) + 1)*sin(d*x + c)
 + cos(d*x + c) - 1)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^2 - a^2*
d - (a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

giac [B] time = 0.85, size = 424, normalized size = 4.51

$$\frac{\frac{\left(6\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-3\sqrt{2}\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)+6\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-3\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)+3\sqrt{2}\sqrt{-a}+5\sqrt{-a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}d\cdots\right.\right.}{\sqrt{2}\sqrt{-a}\,a^{\frac{3}{2}}+\sqrt{-a}\,a^{\frac{3}{2}}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/2*((6*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 3*sq
rt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 6*sqrt(a)*arctan((sqrt(2)*s
qrt(a) + sqrt(a))/sqrt(-a)) - 3*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 3
*sqrt(2)*sqrt(-a) + 5*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(2)*sqrt
(-a)*a^(3/2) + sqrt(-a)*a^(3/2)) + ((tan(1/2*d*x + 1/2*c)/(a*sgn(tan(1/2*d*
x + 1/2*c) + 1)) + 4/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c
) - 3/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
 - 6*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 3*log(abs(-sqr
t(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(3/2)*s
gn(tan(1/2*d*x + 1/2*c) + 1)) + 2/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
```

```
tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)))
/d
```

maple [A] time = 1.10, size = 123, normalized size = 1.31

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left(\sin(dx+c)\left(2\sqrt{a-a\sin(dx+c)}\,\sqrt{a}-3\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)a\right)\cdots}{a^{\frac{5}{2}}\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/a^(5/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(sin(d*x+c)*(2*(a-a*sin
(d*x+c))^(1/2)*a^(1/2)-3*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a)+(a-a*si
n(d*x+c))^(1/2)*a^(1/2))/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)^2\,{\left(a+a\,\sin(c+d\,x)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.477 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2a^2d}+\frac{7\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -3/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d+7/4*cot(d
*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))
^(1/2)/a^2/d
```

Rubi [A] time = 0.49, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2880, 2772, 2773, 206, 3044, 2980}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2a^2d}+\frac{7\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(3/2)*d)
 + (7*Cot[c + d*x])/(4*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c
+ d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\left(1+\sin^2(c+dx)\right)dx}{a^2} - \frac{2\int \csc^2(c+dx)\sqrt{a}}{a}$$

$$= \frac{2\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2a^2d} + \frac{\int \csc^2(c}{}$$

$$= \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2a^2d} + \frac{11\int \mathrm{cs}}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{}}{2a^2d}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2d}$$

Mathematica [B] time = 1.82, size = 274, normalized size = 2.58

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(12\tan\left(\frac{1}{4}(c+dx)\right)+12\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)+\sec^2\left(\frac{1}{4}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-24 + 12*Cot[(c + d*x)/4] - Csc[(
c + d*x)/4]^2 - 12*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*Log[1
- Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Cos[(c + d
*x)/4] - Sin[(c + d*x)/4])^2 - (24*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] - Si
n[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (24*Sin[(c +
d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 12*Tan[(c + d*x)/4]))/(32*
d*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.51, size = 337, normalized size = 3.18

$$\frac{3\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx}{}\right.}{16\left(a^{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(3*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*s
qrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (c
os(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(5*cos(d*x + c)^2
+ (5*cos(d*x + c) + 7)*sin(d*x + c) - 2*cos(d*x + c) - 7)*sqrt(a*sin(d*x +
c) + a))/(a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c)
- a^2*d + (a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

giac [B] time = 0.89, size = 513, normalized size = 4.84

$$\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{6}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)-\frac{\left(12\,\sqrt{2}\,\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-6\,\sqrt{2}\,\sqrt{-a}\,\log\left(\sqrt{2}\,\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/8*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(tan(1/2*d*x + 1/2*c)/(a^2*sgn(tan(
1/2*d*x + 1/2*c) + 1)) - 6/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (12*sqrt(
2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 6*sqrt(2)*sqrt(-a
)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 18*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqr
t(a))/sqrt(-a)) - 9*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 30*sqrt(2)*sq
rt(-a) - 38*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(2*sqrt(2)*sqrt(-a)*a^(
3/2) + 3*sqrt(-a)*a^(3/2)) + 6*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2
*c) + 1)) - 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3 - 6*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) + (sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a + 6*a^(3/2))/((
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^
2*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 1.28, size = 126, normalized size = 1.19

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left(-3\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)\left(\sin^2(dx+c)\right)a^2+3\sqrt{-a(\sin(dx+c)-}\right.}{4a^{\frac{7}{2}}\sin(dx+c)^2\cos(dx+c)\sqrt{a+a\sin(dx+c)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-3*arctanh((-a*(sin(d*x+c)-1)
)^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+3*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)-5*(-a
*(sin(d*x+c)-1))^(3/2)*a^(1/2))/a^(7/2)/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*
x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)^3\,\left(a+a\,\sin(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.478 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{3/2}d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}-\frac{\cot(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}+\frac{11\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -1/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d-1/8*cot(d
*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)+11/12*cot(d*x+c)*csc(d*x+c)/a/d/(a+a*sin(d
*x+c))^(1/2)-1/3*cot(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.54, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2717, 2772, 2773, 206, 3044, 2980}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{3/2}d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}-\frac{\cot(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}+\frac{11\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(8*a^(3/2)*d) - C
ot[c + d*x]/(8*a*d*Sqrt[a + a*Sin[c + d*x]]) + (11*Cot[c + d*x]*Csc[c + d*x
])/(12*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a
+ a*Sin[c + d*x]])/(3*a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Dist[-2/(a*b), Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^3
, x], x] + Dist[1/a^2, Int[((a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^
2))/Sin[e + f*x]^4, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] &
& IntegerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\left(1+\sin^2(c+dx)\right)dx}{a^2} - \frac{2\int \csc^3(c+dx)\sqrt{a+a\,\mathrm{s}}}{a^2}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} + \frac{\int \csc^3(c+}{}$$

$$= \frac{3\cot(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{11\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a}}{3a^2d}$$

$$= -\frac{\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{11\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a}}{3a^2d}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{2a^{3/2}d} - \frac{\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{11\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8a^{3/2}d} - \frac{\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{11\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} -$$

Mathematica [B] time = 0.79, size = 294, normalized size = 2.04

$$\csc^9\left(\tfrac{1}{2}(c+dx)\right)\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\right)^3\left(132\sin\left(\tfrac{1}{2}(c+dx)\right)+62\sin\left(\tfrac{3}{2}(c+dx)\right)-6\sin\left(\tfrac{5}{2}(c+dx)\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Csc[(c + d*x)/2]^9*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-132*Cos[(c +
d*x)/2] + 62*Cos[(3*(c + d*x))/2] + 6*Cos[(5*(c + d*x))/2] + 132*Sin[(c + d
*x)/2] - 9*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 9*Lo
g[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 62*Sin[(3*(c + d*
x))/2] - 6*Sin[(5*(c + d*x))/2] + 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]]*Sin[3*(c + d*x)] - 3*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin
[3*(c + d*x)]))/(24*d*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3*(a*(1 + S
in[c + d*x]))^(3/2))
```

fricas [B] time = 0.53, size = 383, normalized size = 2.66

$$3\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 - \left(\cos(dx+c)^3 + \cos(dx+c)^2 - \cos(dx+c) - 1\right)\sin(dx+c) + 1\right)\sqrt{a}\,\log\left(\frac{a\,\mathrm{c}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)
^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4
*(3*cos(d*x + c)^3 + 17*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - 14*cos(d*x + c
) - 25)*sin(d*x + c) - 11*cos(d*x + c) - 25)*sqrt(a*sin(d*x + c) + a))/(a^2
*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d - (a^2*d*cos(d*x + c)^3
+ a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

giac [B] time = 0.95, size = 589, normalized size = 4.09

$$\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\left(\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{9}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{14}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/48*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*tan(1/2*d*x + 1/2*c)/(a^2*sgn(
tan(1/2*d*x + 1/2*c) + 1)) - 9/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2
*d*x + 1/2*c) + 14/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (30*sqrt(2)*sqrt(
a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 15*sqrt(2)*sqrt(-a)*log(s
qrt(2)*sqrt(a) + sqrt(a)) + 42*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/s
qrt(-a)) - 21*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 280*sqrt(2)*sqrt(-a
) + 402*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(5*sqrt(2)*sqrt(-a)*a^(3/2)
 + 7*sqrt(-a)*a^(3/2)) + 6*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)
+ 1)) - 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 2*(9*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5 - 18*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(a) + 24*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(3/2) - 9*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2 - 14
```

```
*a^(5/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2 - a)^3*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 1.24, size = 144, normalized size = 1.00

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(3(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{3}{2}}+3\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^4\left(\sin^3(dx\right.\right.}{24a^{\frac{11}{2}}\sin(dx+c)^3\cos(dx+c)\sqrt{a+a\sin(dx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/24/a^(11/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*(-a*(sin(d*x+c)-
1))^(5/2)*a^(3/2)+3*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^4*sin(d*x+
c)^3+8*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)-3*(-a*(sin(d*x+c)-1))^(1/2)*a^(7/2
))/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)^4\,\left(a+a\,\sin(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.479 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64a^{3/2}d}-\frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a\sin(c+dx)+a}}{4a^2d}-\frac{3\cot(c+dx)}{64ad\sqrt{a\sin(c+dx)+a}}+\frac{5\cot(c+dx)\cdots}{8ad\sqrt{a\sin(\cdots}}$$

```
[Out] -3/64*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d-3/64*cot
(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-1/32*cot(d*x+c)*csc(d*x+c)/a/d/(a+a*sin(
d*x+c))^(1/2)+5/8*cot(d*x+c)*csc(d*x+c)^2/a/d/(a+a*sin(d*x+c))^(1/2)-1/4*co
t(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.73, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2880, 2772, 2773, 206, 3044, 2980}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64a^{3/2}d}-\frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a\sin(c+dx)+a}}{4a^2d}-\frac{3\cot(c+dx)}{64ad\sqrt{a\sin(c+dx)+a}}+\frac{5\cot(c+dx\cdots}{8ad\sqrt{a\sin(\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*a^(3/2)*d
) - (3*Cot[c + d*x])/(64*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[
c + d*x])/(32*a*d*Sqrt[a + a*Sin[c + d*x]]) + (5*Cot[c + d*x]*Csc[c + d*x]^
2)/(8*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^3*Sqrt[a +
 a*Sin[c + d*x]])/(4*a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^5(c+dx)\sqrt{a+a\sin(c+dx)}\,\left(1+\sin^2(c+dx)\right)\,dx}{a^2} - \frac{2\int \csc^4(c+dx)\sqrt{a}}{a^2}$$

$$= \frac{2\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}}{4a^2d} + \frac{\int c}{}$$

$$= \frac{5\cot(c+dx)\csc(c+dx)}{6ad\sqrt{a+a\sin(c+dx)}} + \frac{5\cot(c+dx)\csc^2(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+d}{4a}$$

$$= \frac{5\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{32ad\sqrt{a+a\sin(c+dx)}} + \frac{5\cot(c+dx)\csc^2(c+dx}{8ad\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{3\cot(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{32ad\sqrt{a+a\sin(c+dx)}} + \frac{5\cot(c+dx)\csc^2(c+}{8ad\sqrt{a+a\sin(c+d}}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d} - \frac{3\cot(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{32ad\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64a^{3/2}d} - \frac{3\cot(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+d}{32ad\sqrt{a+a\sin(c+d}}$$

Mathematica [B] time = 0.96, size = 376, normalized size = 2.07

$$-\frac{\csc^{12}\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-446\sin\left(\frac{1}{2}(c+dx)\right)-182\sin\left(\frac{3}{2}(c+dx)\right)+2\sin\left(\frac{5}{2}(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/64*(Csc[(c + d*x)/2]^12*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(446*Cos
[(c + d*x)/2] - 182*Cos[(3*(c + d*x))/2] - 2*Cos[(5*(c + d*x))/2] - 6*Cos[(
7*(c + d*x))/2] + 9*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 12*Cos[2
*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Cos[4*(c + d*x
)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 9*Log[1 - Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]] + 12*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] - 3*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]] - 446*Sin[(c + d*x)/2] - 182*Sin[(3*(c + d*x))/2] + 2*Sin[(5*(c + d*x)
)/2] - 6*Sin[(7*(c + d*x))/2]))/(d*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2
)^4*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.49, size = 442, normalized size = 2.43

$$\frac{3\left(\cos\left(dx+c\right)^5+\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^3-2\,\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2+1\right)\sin\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/256*(3*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x +
c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(d*x + c)
+ 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c
) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) -
a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1
)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(3*cos(d*x + c)^4 + 2*cos(d*x + c)^
3 + 20*cos(d*x + c)^2 + (3*cos(d*x + c)^3 + cos(d*x + c)^2 + 21*cos(d*x + c
) + 39)*sin(d*x + c) - 18*cos(d*x + c) - 39)*sqrt(a*sin(d*x + c) + a))/(a^2
*d*cos(d*x + c)^5 + a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d
*cos(d*x + c)^2 + a^2*d*cos(d*x + c) + a^2*d + (a^2*d*cos(d*x + c)^4 - 2*a^
2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

giac [B] time = 1.11, size = 737, normalized size = 4.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/128*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*(tan(1/2*d*x + 1/2*c)/(a^2*sg
n(tan(1/2*d*x + 1/2*c) + 1)) - 4/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1
/2*d*x + 1/2*c) + 13/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2
*c) - 16/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (72*sqrt(2)*sqrt(a)*arctan(
(sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 36*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqr
t(a) + sqrt(a)) + 102*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a))
- 51*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 1134*sqrt(2)*sqrt(-a) - 1600
*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(12*sqrt(2)*sqrt(-a)*a^(3/2) + 17*
sqrt(-a)*a^(3/2)) + 6*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1))
 - 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(13*(sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7 - 32*(sqrt(a)*tan(1/2*d*x
```

```
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) - 5*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a + 48*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) - 5*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2 - 32*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/
2) + 13*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
*a^3 + 16*a^(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 - a)^4*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 1.29, size = 162, normalized size = 0.89

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left(-3(-a(\sin(dx+c)-1))^{\frac{7}{2}}a^{\frac{5}{2}}+11(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{7}{2}}+11(-\right.}{64a^{\frac{15}{2}}\sin(dx+c)^4\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/64*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-3*(-a*(sin(d*x+c)-1))^(7/2
)*a^(5/2)+11*(-a*(sin(d*x+c)-1))^(5/2)*a^(7/2)+11*(-a*(sin(d*x+c)-1))^(3/2)
*a^(9/2)-3*(-a*(sin(d*x+c)-1))^(1/2)*a^(11/2)+3*arctanh((-a*(sin(d*x+c)-1))
^(1/2)/a^(1/2))*a^6*sin(d*x+c)^4)/a^(15/2)/sin(d*x+c)^4/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)^5\,(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.480 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=220

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128a^{3/2}d}-\frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a\sin(c+dx)+a}}{5a^2d}-\frac{3\cot(c+dx)}{128ad\sqrt{a\sin(c+dx)+a}}+\frac{19\cot(c+ \cdots}{40ad\sqrt{a\,\mathrm{si}\cdots}}$$

```
[Out] -3/128*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d-3/128*c
ot(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-1/64*cot(d*x+c)*csc(d*x+c)/a/d/(a+a*si
n(d*x+c))^(1/2)-1/80*cot(d*x+c)*csc(d*x+c)^2/a/d/(a+a*sin(d*x+c))^(1/2)+19/
40*cot(d*x+c)*csc(d*x+c)^3/a/d/(a+a*sin(d*x+c))^(1/2)-1/5*cot(d*x+c)*csc(d*
x+c)^4*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.88, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2880, 2772, 2773, 206, 3044, 2980}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128a^{3/2}d}-\frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a\sin(c+dx)+a}}{5a^2d}-\frac{3\cot(c+dx)}{128ad\sqrt{a\sin(c+dx)+a}}+\frac{19\cot(c+ \cdots}{40ad\sqrt{a\,\mathrm{si}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(128*a^(3/2)*
d) - (3*Cot[c + d*x])/(128*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Cs
c[c + d*x])/(64*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^
2)/(80*a*d*Sqrt[a + a*Sin[c + d*x]]) + (19*Cot[c + d*x]*Csc[c + d*x]^3)/(40
*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^4*Sqrt[a + a*Si
n[c + d*x]])/(5*a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
```

```
f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^6(c+dx)\sqrt{a+a\sin(c+dx)}\left(1+\sin^2(c+dx)\right)dx}{a^2} - \frac{2\int \csc^5(c+dx)\sqrt{a}}{a}$$

$$= \frac{\cot(c+dx)\csc^3(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a+a\sin(c+dx)}}{5a^2d} + \frac{\int \mathrm{cs}}{}$$

$$= \frac{7\cot(c+dx)\csc^2(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} + \frac{19\cot(c+dx)\csc^3(c+dx)}{40ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c}{}$$

$$= \frac{35\cot(c+dx)\csc(c+dx)}{48ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{80ad\sqrt{a+a\sin(c+dx)}} + \frac{19\cot(c+dx)\csc^3(c}{40ad\sqrt{a+a\sin(c+}}$$

$$= \frac{35\cot(c+dx)}{32ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx}{80ad\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{3\cot(c+dx)}{128ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+}{80ad\sqrt{a+a\sin(c+}}$$

$$= \frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{32a^{3/2}d} - \frac{3\cot(c+dx)}{128ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+}{64ad\sqrt{a+a\sin(c+}}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128a^{3/2}d} - \frac{3\cot(c+dx)}{128ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+}{64ad\sqrt{a+a\sin(c+}}$$

Mathematica [A] time = 1.38, size = 412, normalized size = 1.87

$$-\frac{\csc^{15}\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-7100\sin\left(\frac{1}{2}(c+dx)\right)-2880\sin\left(\frac{3}{2}(c+dx)\right)+144\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/640*(Csc[(c + d*x)/2]^15*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(7100*C
os[(c + d*x)/2] - 2880*Cos[(3*(c + d*x))/2] - 144*Cos[(5*(c + d*x))/2] - 10
*Cos[(7*(c + d*x))/2] + 30*Cos[(9*(c + d*x))/2] - 7100*Sin[(c + d*x)/2] + 1
50*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 150*Log[1 -
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 2880*Sin[(3*(c + d*x))/
2] + 144*Sin[(5*(c + d*x))/2] - 75*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]]*Sin[3*(c + d*x)] + 75*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin
```

```
[3*(c + d*x)] - 10*Sin[(7*(c + d*x))/2] - 30*Sin[(9*(c + d*x))/2] + 15*Log[
1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*Log[1 - Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(d*(Csc[(c + d*x)/4]^2
- Sec[(c + d*x)/4]^2)^5*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.49, size = 492, normalized size = 2.24

$$\frac{15\left(\cos\left(dx+c\right)^{6}-3\,\cos\left(dx+c\right)^{4}+3\,\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{5}+\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{3}-2\,\cos\left(d\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/2560*(15*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos(d*x
 + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x +
c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
+ 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
+ (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(15*cos(d*x +
c)^5 + 5*cos(d*x + c)^4 - 38*cos(d*x + c)^3 - 194*cos(d*x + c)^2 - (15*cos(
d*x + c)^4 + 10*cos(d*x + c)^3 - 28*cos(d*x + c)^2 + 166*cos(d*x + c) + 317
)*sin(d*x + c) + 151*cos(d*x + c) + 317)*sqrt(a*sin(d*x + c) + a))/(a^2*d*c
os(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d - (
a^2*d*cos(d*x + c)^5 + a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^3 - 2*a^
2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

giac [B] time = 1.16, size = 808, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/1280*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*((4*tan(1/2*d*x + 1/2*c)/(a^
2*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 15/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))*
tan(1/2*d*x + 1/2*c) + 28/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x
+ 1/2*c) - 95/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c) + 1
28/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (870*sqrt(2)*sqrt(a)*arctan((sqrt
(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 435*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a)
 + sqrt(a)) + 1230*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 6
```

```
15*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 22282*sqrt(2)*sqrt(-a) + 31524
*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(29*sqrt(2)*sqrt(-a)*a^(3/2) + 41*
sqrt(-a)*a^(3/2)) + 30*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)
) - 15*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 2*(95*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^9 - 240*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(a) - 70*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7*a + 560*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(3/2) - 7
20*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^
(5/2) + 70*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^3*a^3 + 400*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*a^(7/2) - 95*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))*a^4 - 128*a^(9/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 1.36, size = 180, normalized size = 0.82

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(15(-a(\sin(dx+c)-1))^{\frac{9}{2}}a^{\frac{7}{2}}-70(-a(\sin(dx+c)-1))^{\frac{7}{2}}a^{\frac{9}{2}}+128(\right.}{640a^{\frac{19}{2}}\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/640/a^(19/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(15*(-a*(sin(d*x+c
)-1))^(9/2)*a^(7/2)-70*(-a*(sin(d*x+c)-1))^(7/2)*a^(9/2)+128*(-a*(sin(d*x+c
)-1))^(5/2)*a^(11/2)+15*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^8*sin(
d*x+c)^5+70*(-a*(sin(d*x+c)-1))^(3/2)*a^(13/2)-15*(-a*(sin(d*x+c)-1))^(1/2)
*a^(15/2))/sin(d*x+c)^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)^6\,(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^6*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^6*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.481 $\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=260

$$-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{1048\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{693a^3d}-\frac{2\sin^5(c+dx)\cos(c+dx)}{11a^2d\sqrt{a\sin(c+dx)+a}}+\frac{46\sin^4(c+}{99a^2d\sqrt{a\,s}}$$

```
[Out] -4*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d
*2^(1/2)+4496/693*cos(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)+200/231*cos(d*x+c
)*sin(d*x+c)^2/a^2/d/(a+a*sin(d*x+c))^(1/2)-424/693*cos(d*x+c)*sin(d*x+c)^3
/a^2/d/(a+a*sin(d*x+c))^(1/2)+46/99*cos(d*x+c)*sin(d*x+c)^4/a^2/d/(a+a*sin(
d*x+c))^(1/2)-2/11*cos(d*x+c)*sin(d*x+c)^5/a^2/d/(a+a*sin(d*x+c))^(1/2)-104
8/693*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 1.36, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2880, 2778, 2983, 2968, 3023, 2751, 2649, 206, 3046}

$$-\frac{2\sin^5(c+dx)\cos(c+dx)}{11a^2d\sqrt{a\sin(c+dx)+a}}+\frac{46\sin^4(c+dx)\cos(c+dx)}{99a^2d\sqrt{a\sin(c+dx)+a}}-\frac{424\sin^3(c+dx)\cos(c+dx)}{693a^2d\sqrt{a\sin(c+dx)+a}}+\frac{200\sin^2(c+dx)\cos(c}{231a^2d\sqrt{a\sin(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(5/2)*d) + (4496*Cos[c + d*x])/(693*a^2*d*Sqrt[a + a*Sin[c + d*x]])
 + (200*Cos[c + d*x]*Sin[c + d*x]^2)/(231*a^2*d*Sqrt[a + a*Sin[c + d*x]]) -
 (424*Cos[c + d*x]*Sin[c + d*x]^3)/(693*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (
46*Cos[c + d*x]*Sin[c + d*x]^4)/(99*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Co
s[c + d*x]*Sin[c + d*x]^5)/(11*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (1048*Cos[
c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(693*a^3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
```

```
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\sin^4(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\sin^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{4\cos(c+dx)\sin^4(c+dx)}{9a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^5(c+dx)}{11a^2d\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\sin^4(c+dx)\left(\frac{21a}{2}-\frac{1}{2}a\,\mathrm{si}\right.}{\sqrt{a+a\sin(c+d}}}{11a^3}$$

$$= -\frac{4\cos(c+dx)\sin^3(c+dx)}{63a^2d\sqrt{a+a\sin(c+dx)}} + \frac{46\cos(c+dx)\sin^4(c+dx)}{99a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^5}{11a^2d\sqrt{a+a\sin(}}$$

$$= \frac{76\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+dx)\sin^3(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{46\cos(c+dx)\,\mathrm{si}}{99a^2d\sqrt{a+a\,\mathrm{si}}}$$

$$= \frac{200\cos(c+dx)\sin^2(c+dx)}{231a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+dx)\sin^3(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{46\cos(c+dx)\,\mathrm{s}}{99a^2d\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{200\cos(c+dx)\sin^2(c+dx)}{231a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+dx)\sin^3(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{46\cos(c+dx)\,\mathrm{s}}{99a^2d\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{1144\cos(c+dx)}{315a^2d\sqrt{a+a\sin(c+dx)}} + \frac{200\cos(c+dx)\sin^2(c+dx)}{231a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+dx)\,\mathrm{s}}{693a^2d\sqrt{a+a\,\mathrm{si}}}$$

$$= \frac{4496\cos(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{200\cos(c+dx)\sin^2(c+dx)}{231a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+dx)\,\mathrm{s}}{693a^2d\sqrt{a+a\,\mathrm{si}}}$$

$$= -\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{4496\cos(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{200\cos(c+d}{231a^2d\sqrt{a+}}$$

$$= -\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{4496\cos(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{200\cos(c+d}{231a^2d\sqrt{a+}}$$

Mathematica [C] time = 1.08, size = 224, normalized size = 0.86

$$\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\right)^5\left(-73458\sin\left(\tfrac{1}{2}(c+dx)\right)-15246\sin\left(\tfrac{3}{2}(c+dx)\right)+4851\sin\left(\tfrac{5}{2}(c+dx)\right)+1\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*((88704 + 88704*I)*(-1)^(3/4)*ArcT
anh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 73458*Cos[(c + d*x)/2
] - 15246*Cos[(3*(c + d*x))/2] - 4851*Cos[(5*(c + d*x))/2] + 1485*Cos[(7*(c
 + d*x))/2] + 385*Cos[(9*(c + d*x))/2] - 63*Cos[(11*(c + d*x))/2] - 73458*S
in[(c + d*x)/2] - 15246*Sin[(3*(c + d*x))/2] + 4851*Sin[(5*(c + d*x))/2] +
1485*Sin[(7*(c + d*x))/2] - 385*Sin[(9*(c + d*x))/2] - 63*Sin[(11*(c + d*x)
)/2]))/(11088*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

fricas [A] time = 0.51, size = 299, normalized size = 1.15

$$2\left(\frac{693\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}} - \left(63\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 2/693*(693*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)
^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(
cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c
)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - (63*co
s(d*x + c)^6 - 161*cos(d*x + c)^5 - 562*cos(d*x + c)^4 + 622*cos(d*x + c)^3
 + 1759*cos(d*x + c)^2 + (63*cos(d*x + c)^5 + 224*cos(d*x + c)^4 - 338*cos(
d*x + c)^3 - 960*cos(d*x + c)^2 + 799*cos(d*x + c) + 2984)*sin(d*x + c) - 2
185*cos(d*x + c) - 2984)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c) + a^
3*d*sin(d*x + c) + a^3*d)
```

giac [B] time = 0.96, size = 504, normalized size = 1.94

$$-\frac{8\left(\frac{\sqrt{2}\left(693\,a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+746\sqrt{-a}\sqrt{a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}\,a^3} - \frac{693\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)} - \frac{\cdots}{\mathrm{sgn}(\tan\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] -8/693*(sqrt(2)*(693*a*arctan(sqrt(a)/sqrt(-a)) + 746*sqrt(-a)*sqrt(a))*sgn
(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^3) - 693*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(
a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) - (431*a^3/sgn(t
an(1/2*d*x + 1/2*c) + 1) - (693*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (2717*a
^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (3927*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1)
- (7326*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (8778*a^3/sgn(tan(1/2*d*x + 1/2
*c) + 1) - (8778*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (7326*a^3/sgn(tan(1/2*
d*x + 1/2*c) + 1) - (3927*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (2717*a^3/sgn
(tan(1/2*d*x + 1/2*c) + 1) + (431*a^3*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x
+ 1/2*c) + 1) - 693*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c)
)*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2
*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/
2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))/(a*tan(1/2*d*x + 1/2*c)^2
 + a)^(11/2))/d
```

maple [A] time = 1.23, size = 166, normalized size = 0.64

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(-1386a^{\frac{11}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)+63\left(a-a\sin\left(dx+c\right)\right)^{\frac{11}{2}}-\right.}{693a^{8}\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/693/a^8*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-1386*a^(11/2)*2^(1/2)*
arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+63*(a-a*sin(d*x+c))^(11
/2)-154*a*(a-a*sin(d*x+c))^(9/2)+198*(a-a*sin(d*x+c))^(7/2)*a^2+231*a^4*(a-
a*sin(d*x+c))^(3/2)+1386*a^5*(a-a*sin(d*x+c))^(1/2))/cos(d*x+c)/(a+a*sin(d*
x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{4}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^4/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (c+d x)^4 \sin (c+d x)^4}{(a+a \sin (c+d x))^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^4)/(a + a*sin(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^4*sin(c + d*x)^4)/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.482 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=222

$$\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{472\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{315a^3d}-\frac{2\sin^4(c+dx)\cos(c+dx)}{9a^2d\sqrt{a\sin(c+dx)+a}}+\frac{38\sin^3(c+dx)}{63a^2d\sqrt{a\sin(c}}$$

```
[Out] 4*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d*
2^(1/2)-2048/315*cos(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)-92/105*cos(d*x+c)*
sin(d*x+c)^2/a^2/d/(a+a*sin(d*x+c))^(1/2)+38/63*cos(d*x+c)*sin(d*x+c)^3/a^2
/d/(a+a*sin(d*x+c))^(1/2)-2/9*cos(d*x+c)*sin(d*x+c)^4/a^2/d/(a+a*sin(d*x+c)
)^(1/2)+472/315*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 1.08, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2880, 2778, 2983, 2968, 3023, 2751, 2649, 206, 3046}

$$-\frac{2\sin^4(c+dx)\cos(c+dx)}{9a^2d\sqrt{a\sin(c+dx)+a}}+\frac{38\sin^3(c+dx)\cos(c+dx)}{63a^2d\sqrt{a\sin(c+dx)+a}}-\frac{92\sin^2(c+dx)\cos(c+dx)}{105a^2d\sqrt{a\sin(c+dx)+a}}+\frac{472\cos(c+dx)\sqrt{a\sin(c}}{315a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(5/2)*d) - (2048*Cos[c + d*x])/(315*a^2*d*Sqrt[a + a*Sin[c + d*x]])
- (92*Cos[c + d*x]*Sin[c + d*x]^2)/(105*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (
38*Cos[c + d*x]*Sin[c + d*x]^3)/(63*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Co
s[c + d*x]*Sin[c + d*x]^4)/(9*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (472*Cos[c
+ d*x]*Sqrt[a + a*Sin[c + d*x]])/(315*a^3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\sin^3(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\sin^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{4\cos(c+dx)\sin^3(c+dx)}{7a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4(c+dx)}{9a^2d\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\sin^3(c+dx)\left(\frac{17a}{2}-\frac{1}{2}a\right.}{\sqrt{a+a\sin(c+}}}{9a^3}$$

$$= -\frac{4\cos(c+dx)\sin^2(c+dx)}{35a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c+dx)\sin^3(c+dx)}{63a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin}{9a^2d\sqrt{a+a\sin}}$$

$$= -\frac{92\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c+dx)\sin^3(c+dx)}{63a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\,\mathrm{s}}{9a^2d\sqrt{a+a\,\mathrm{si}}}$$

$$= -\frac{92\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c+dx)\sin^3(c+dx)}{63a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\,\mathrm{s}}{9a^2d\sqrt{a+a\,\mathrm{si}}}$$

$$= -\frac{296\cos(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} - \frac{92\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c+dx)\,\mathrm{s}}{63a^2d\sqrt{a+a\,\mathrm{s}}}$$

$$= -\frac{2048\cos(c+dx)}{315a^2d\sqrt{a+a\sin(c+dx)}} - \frac{92\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c+dx)\,\mathrm{s}}{63a^2d\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2048\cos(c+dx)}{315a^2d\sqrt{a+a\sin(c+dx)}} - \frac{92\cos(c+dx)}{105a^2d\sqrt{a+a}}$$

$$= \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2048\cos(c+dx)}{315a^2d\sqrt{a+a\sin(c+dx)}} - \frac{92\cos(c+dx)}{105a^2d\sqrt{a+a}}$$

Mathematica [C] time = 3.28, size = 225, normalized size = 1.01

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(16380\sin\left(\frac{1}{2}(c+dx)\right)+3150\sin\left(\frac{3}{2}(c+dx)\right)-882\sin\left(\frac{5}{2}(c+dx)\right)-225\sin\left(\frac{7}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((20160 + 20160*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/
2)*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 163
80*Cos[(c + d*x)/2] + 3150*Cos[(3*(c + d*x))/2] + 882*Cos[(5*(c + d*x))/2]
- 225*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c + d*x))/2] + 16380*Sin[(c + d*x)/
2] + 3150*Sin[(3*(c + d*x))/2] - 882*Sin[(5*(c + d*x))/2] - 225*Sin[(7*(c +
 d*x))/2] + 35*Sin[(9*(c + d*x))/2]))/(2520*a^3*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]))
```

fricas [A] time = 0.52, size = 280, normalized size = 1.26

$$2\left(\frac{315\,\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\,\sqrt{2}\,\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\,\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-\left(35\,\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 2/315*(315*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)
^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(
cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c
)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - (35*co
s(d*x + c)^5 + 130*cos(d*x + c)^4 - 208*cos(d*x + c)^3 - 634*cos(d*x + c)^2
 - (35*cos(d*x + c)^4 - 95*cos(d*x + c)^3 - 303*cos(d*x + c)^2 + 331*cos(d*
x + c) + 1292)*sin(d*x + c) + 961*cos(d*x + c) + 1292)*sqrt(a*sin(d*x + c)
+ a))/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

giac [B] time = 0.91, size = 434, normalized size = 1.95

$$8\left(\frac{\sqrt{2}\left(315\,a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+323\,\sqrt{-a}\,\sqrt{a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{-a}\,a^3}-\frac{315\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}+\sqrt{a}\right)}{2\,\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] 8/315*(sqrt(2)*(315*a*arctan(sqrt(a)/sqrt(-a)) + 323*sqrt(-a)*sqrt(a))*sgn(
tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^3) - 315*sqrt(2)*arctan(-1/2*sqrt(2)*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a
))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + ((((((((197*a^
2*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 315*a^2/sgn(tan(1/2*
d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 1044*a^2/sgn(tan(1/2*d*x + 1/2*c)
 + 1))*tan(1/2*d*x + 1/2*c) - 1470*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1
/2*d*x + 1/2*c) + 2142*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2
*c) - 2142*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 1470*a
^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 1044*a^2/sgn(tan(1
/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 315*a^2/sgn(tan(1/2*d*x + 1/2*
c) + 1))*tan(1/2*d*x + 1/2*c) - 197*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*t
an(1/2*d*x + 1/2*c)^2 + a)^(9/2))/d
```

maple [A] time = 1.28, size = 166, normalized size = 0.75

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(630a^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-35\left(a-a\sin\left(dx+c\right)\right)^{\frac{9}{2}}+45}{315a^{7}\cos\left(dx+c\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/315*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(630*a^(9/2)*2^(1/2)*arctanh
(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-35*(a-a*sin(d*x+c))^(9/2)+45*a
*(a-a*sin(d*x+c))^(7/2)-63*(a-a*sin(d*x+c))^(5/2)*a^2-105*(a-a*sin(d*x+c))^
(3/2)*a^3-630*a^4*(a-a*sin(d*x+c))^(1/2))/a^7/cos(d*x+c)/(a+a*sin(d*x+c))^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{3}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^4 \sin(c+dx)^3}{(a+a\sin(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.483 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos^5(c+dx)}{7ad(a\sin(c+dx)+a)^{3/2}}+\frac{4\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{5/2}}+$$

```
[Out] 4/7*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)+2/3*cos(d*x+c)^3/a/d/(a+a*sin(d*x
+c))^(3/2)-2/7*cos(d*x+c)^5/a/d/(a+a*sin(d*x+c))^(3/2)-4*arctanh(1/2*cos(d*
x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+4*cos(d*x+c)
/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2878, 2860, 2679, 2649, 206}

$$\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{2\cos^5(c+dx)}{7ad(a\sin(c+dx)+a)^{3/2}}+\frac{4\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{5/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(5/2)*d) + (4*Cos[c + d*x]^5)/(7*d*(a + a*Sin[c + d*x])^(5/2)) + (2
*Cos[c + d*x]^3)/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^5)/(7
*a*d*(a + a*Sin[c + d*x])^(3/2)) + (4*Cos[c + d*x])/(a^2*d*Sqrt[a + a*Sin[c
 + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos^5(c+dx)}{7ad(a+a\sin(c+dx))^{3/2}} + \frac{2\int \frac{\cos^4(c+dx)\left(-\frac{3a}{2}-5a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{5/2}}\,dx}{7a}$$
$$= \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^5(c+dx)}{7ad(a+a\sin(c+dx))^{3/2}} + \int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{5/}}$$
$$= \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} + \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+dx)}{7ad(a+a\sin(c+dx))^{}}$$
$$= \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} + \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+dx)}{7ad(a+a\sin(c+dx))^{}}$$
$$= \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} + \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+dx)}{7ad(a+a\sin(c+dx))^{}}$$
$$= -\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} + \frac{2\cos^3(c+}{3ad(a+a\sin(}$$

Mathematica [C] time = 2.51, size = 201, normalized size = 1.19

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(525\sin\left(\frac{1}{2}(c+dx)\right)+91\sin\left(\frac{3}{2}(c+dx)\right)-21\sin\left(\frac{5}{2}(c+dx)\right)-3\sin\left(\frac{7}{2}(c+dx)\right)-525\,\mathrm{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/84*(Sqrt[a*(1 + Sin[c + d*x])]*((672 + 672*I)*(-1)^(3/4)*ArcTanh[(1/2 +
I/2)*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 5
25*Cos[(c + d*x)/2] + 91*Cos[(3*(c + d*x))/2] + 21*Cos[(5*(c + d*x))/2] - 3
*Cos[(7*(c + d*x))/2] + 525*Sin[(c + d*x)/2] + 91*Sin[(3*(c + d*x))/2] - 21
*Sin[(5*(c + d*x))/2] - 3*Sin[(7*(c + d*x))/2]))/(a^3*d*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]))
```

fricas [A] time = 0.49, size = 258, normalized size = 1.53

$$\frac{2\left(\frac{21\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+\left(3\cos(dx \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 2/21*(21*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2
 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(co
s(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^
2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + (3*cos(d
*x + c)^4 - 9*cos(d*x + c)^3 - 31*cos(d*x + c)^2 + (3*cos(d*x + c)^3 + 12*c
os(d*x + c)^2 - 19*cos(d*x + c) - 80)*sin(d*x + c) + 61*cos(d*x + c) + 80)*
sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

giac [B] time = 0.89, size = 358, normalized size = 2.12

$$-\frac{8\left(\frac{\sqrt{2}\left(21\,a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+20\sqrt{-a}\sqrt{a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}\,a^3}-\frac{21\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{\left(\left(\left(\left(\left(\left(\frac{13a \cdots}{\mathrm{sgn}(\cdots}\right.\right.\right.\right.\right.\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] -8/21*(sqrt(2)*(21*a*arctan(sqrt(a)/sqrt(-a)) + 20*sqrt(-a)*sqrt(a))*sgn(ta
n(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^3) - 21*sqrt(2)*arctan(-1/2*sqrt(2)*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/
sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + (((((((13*a*tan(1/
2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 21*a/sgn(tan(1/2*d*x + 1/2*c
) + 1))*tan(1/2*d*x + 1/2*c) + 56*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*
```

```
d*x + 1/2*c) - 70*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 7
0*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 56*a/sgn(tan(1/2*
d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 21*a/sgn(tan(1/2*d*x + 1/2*c) + 1
))*tan(1/2*d*x + 1/2*c) - 13*a/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*
x + 1/2*c)^2 + a)^(7/2))/d
```

maple [A] time = 1.10, size = 132, normalized size = 0.78

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(42a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-3\left(a-a\sin\left(dx+c\right)\right)^{\frac{7}{2}}-7\left(a\right.\right.}{21a^{6}\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/21*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(42*a^(7/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-3*(a-a*sin(d*x+c))^(7/2)-7*(a-a
*sin(d*x+c))^(3/2)*a^2-42*a^3*(a-a*sin(d*x+c))^(1/2))/a^6/cos(d*x+c)/(a+a*s
in(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{2}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^4\,{\sin\left(c+d\,x\right)}^2}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.484 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos^5(c+dx)}{5d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^3(c+dx)}{3ad(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -2/5*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-2/3*cos(d*x+c)^3/a/d/(a+a*sin(d*
x+c))^(3/2)+4*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2)
)/a^(5/2)/d*2^(1/2)-4*cos(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2860, 2679, 2649, 206}

$$-\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{2\cos^5(c+dx)}{5d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^3(c+dx)}{3ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(5/2)*d) - (2*Cos[c + d*x]^5)/(5*d*(a + a*Sin[c + d*x])^(5/2)) - (2*
Cos[c + d*x]^3)/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) - (4*Cos[c + d*x])/(a^2*
d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
```

```
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$$
$$= -\frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{a}$$
$$= -\frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}}$$
$$= -\frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}}$$
$$= \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+d}$$

Mathematica [C] time = 1.62, size = 177, normalized size = 1.29

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(180\sin\left(\frac{1}{2}(c+dx)\right)+25\sin\left(\frac{3}{2}(c+dx)\right)-3\sin\left(\frac{5}{2}(c+dx)\right)-180\cos\left(\frac{1}{2}(c+dx)\right)+25\cos}{30a^3d\left(\sin\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((240 + 240*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(
-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 180*Cos
[(c + d*x)/2] + 25*Cos[(3*(c + d*x))/2] + 3*Cos[(5*(c + d*x))/2] + 180*Sin[
(c + d*x)/2] + 25*Sin[(3*(c + d*x))/2] - 3*Sin[(5*(c + d*x))/2]))/(30*a^3*d
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [B] time = 0.52, size = 239, normalized size = 1.74

$$\frac{2\left(\frac{15\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+\left(3\cos(\right.\right.}{15\left(a^3d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] 2/15*(15*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2
 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(co
s(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^
2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + (3*cos(d
*x + c)^3 + 14*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - 11*cos(d*x + c) - 52)*s
in(d*x + c) - 41*cos(d*x + c) - 52)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*
x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

giac [B] time = 0.82, size = 297, normalized size = 2.17

$$4\left(\frac{2\sqrt{2}\left(15\,a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+13\sqrt{-a}\sqrt{a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}\,a^3}+\frac{\left(\left(\left(\left(\frac{19\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{30}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{55}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\right.\right.}\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] 4/15*(2*sqrt(2)*(15*a*arctan(sqrt(a)/sqrt(-a)) + 13*sqrt(-a)*sqrt(a))*sgn(t
an(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^3) + (((((19*tan(1/2*d*x + 1/2*c)/sgn(
tan(1/2*d*x + 1/2*c) + 1) - 30/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x +
 1/2*c) + 55/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 55/sgn(t
an(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 30/sgn(tan(1/2*d*x + 1/2*c
) + 1))*tan(1/2*d*x + 1/2*c) - 19/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2
*d*x + 1/2*c)^2 + a)^(5/2) - 30*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(s
qrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 1.20, size = 130, normalized size = 0.95

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(30a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-3\left(a-a\sin\left(dx+c\right)\right)^{\frac{5}{2}}-5\left(a-\right.}{15a^{5}\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/15*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(30*a^(5/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-3*(a-a*sin(d*x+c))^(5/2)-5*(a-a*
sin(d*x+c))^(3/2)*a-30*a^2*(a-a*sin(d*x+c))^(1/2))/a^5/cos(d*x+c)/(a+a*sin(
d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^4\,\sin\left(c+d\,x\right)}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.485 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{2\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d+4*arctanh(1
/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-2*c
os(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2880, 2649, 206, 3046, 2985, 2773}

$$-\frac{2\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) +
 (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
]])])/(a^(5/2)*d) - (2*Cos[c + d*x])/(a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\csc(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{2\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\csc(c+dx)\left(\frac{a}{2}-\frac{1}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^3} + \frac{4\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,\right.}{a^2d}$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \csc(c+dx)\sqrt{a+}}{a^3}$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,\right.}{a^2d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2\cos(c+dx}{a^2d\sqrt{a+a\sin(c}}$$

Mathematica [C] time = 0.40, size = 154, normalized size = 1.36

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5\left(-2\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)+(8+8i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\right.\right.}{d(a(s}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((((8 + 8*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)
/4])] + 2*Cos[(c + d*x)/2] + Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*Sin[(c + d*x)/2])*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^5)/(d*(a*(1 + Sin[c + d*x]))^(5/2)))
```

fricas [B] time = 0.52, size = 377, normalized size = 3.34

$$\sqrt{a}\left(\cos(dx+c)+\sin(dx+c)+1\right)\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*co
s(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*co
s(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + co
s(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*s
qrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 - (cos(d*
x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c)
 - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d
*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 4*sqrt(a*sin(d*x +
 c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a^3*d*cos(d*x + c) + a^3*d*sin
(d*x + c) + a^3*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos((d*t_nostep+c)/2-pi/4))]Discontinuities at zeroe
s of cos((d*t_nostep+c)/2-pi/4) were not checkedWarning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep+1)]Evaluation time: 0.62Error: Bad Argument Type
```

maple [A] time = 1.00, size = 116, normalized size = 1.03

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(\sqrt{a-a\sin\left(dx+c\right)}+\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)-2\sqrt{a}\,\sqrt{2}\,\operatorname{arct}\right.}{a^{3}\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/a^3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*((a-a*sin(d*x+c))^(1/2)+a^(
1/2)*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))-2*a^(1/2)*2^(1/2)*arctanh(1/2*
```

```
(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/
d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4 \csc(dx+c)}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.486 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\cot(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] 5*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d-4*arctanh(1/
2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-cot(
d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2880, 2780, 2649, 206, 2773, 3044, 2985}

$$-\frac{\cot(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) -
(4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(5/2)*d) - Cot[c + d*x]/(a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2780

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e +
f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\csc^2(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(-\frac{a}{2}+\frac{3}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^3} - \frac{2\int \csc(c+dx)\sqrt{a}}{a}$$

$$= -\frac{\cot(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{2a^3} + \frac{2\int \frac{1}{\sqrt{a+a\sin}}}{a^2}$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{\cot(c+dx}{a^2d\sqrt{a+a\sin(c}}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{\cot(c+dx}{a^2d\sqrt{a+a\sin(c}}$$

Mathematica [C] time = 2.70, size = 170, normalized size = 1.50

$$\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5\left(-\tan\left(\frac{1}{4}(c+dx)\right)-\cot\left(\frac{1}{4}(c+dx)\right)+2\sec\left(\frac{1}{2}(c+dx)\right)+(32+32i)(-1)^{3/}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*((32 + 32*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - Cot[(c + d*x)/4] + 10*Log[1
 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 10*Log[1 - Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] + 2*Sec[(c + d*x)/2] - Tan[(c + d*x)/4]))/(4*d*(a*(1 + Sin[c
 + d*x]))^(5/2))
```

fricas [B] time = 0.50, size = 421, normalized size = 3.73

$$5\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(d\right.}{\cos(dx+c)^3+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(5*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((
a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) +
 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9
*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) -
a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - c
os(d*x + c) - 1)) + 8*sqrt(2)*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(
d*x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) +
3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos
(d*x + c) - 2))/sqrt(a) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*
x + c) + 1))/(a^3*d*cos(d*x + c)^2 - a^3*d - (a^3*d*cos(d*x + c) + a^3*d)*s
in(d*x + c))
```

giac [B] time = 0.98, size = 472, normalized size = 4.18

$$\frac{\left(10\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-32\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-5\sqrt{2}\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)+20\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-32\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-10\sqrt{-a}\log\right.}{\sqrt{2}\sqrt{-a}\,a^{\frac{5}{2}}+2\sqrt{-a}\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] 1/2*((10*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 32*
sqrt(2)*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - 5*sqrt(2)*sqrt(-a)*log(sqrt(2)*s
qrt(a) + sqrt(a)) + 20*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a))
 - 32*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - 10*sqrt(-a)*log(sqrt(2)*sqrt(a) +
sqrt(a)) - 3*sqrt(2)*sqrt(-a) - 2*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(
sqrt(2)*sqrt(-a)*a^(5/2) + 2*sqrt(-a)*a^(5/2)) + 16*sqrt(2)*arctan(-1/2*sqr
t(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + s
qrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 10*arctan(
-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-
a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 5*log(abs(-sqrt(a)*tan(1
/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(5/2)*sgn(tan(1/2
*d*x + 1/2*c) + 1)) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^3*sgn(tan(1/2*d
*x + 1/2*c) + 1)) + 2/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2 - a)*a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 1.07, size = 132, normalized size = 1.17

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left(\sin(dx+c)\,a\left(4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-5\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)\right)+\sqrt{a-a\sin(dx+c)}\,\sqrt{a}\right)}{a^{\frac{7}{2}}\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/a^(7/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(sin(d*x+c)*a*(4*2^(1/2
)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-5*arctanh((a-a*sin(d*
x+c))^(1/2)/a^(1/2)))+(a-a*sin(d*x+c))^(1/2)*a^(1/2))/sin(d*x+c)/cos(d*x+c)
/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.487 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{9\cot(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -23/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d+4*arctan
h(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+
9/4*cot(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)/a^2/d
/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2880, 2779, 2985, 2649, 206, 2773, 3044, 2984}

$$\frac{9\cot(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}-\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-23*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(5/2)*d
) + (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d
*x]])])/(a^(5/2)*d) + (9*Cot[c + d*x])/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]]) -
 (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
```

```
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\csc^3(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{2\cot(c+dx)}{a^2 d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2 d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(-\frac{a}{2}+\frac{7}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,d}{2a^3}$$

$$= \frac{9\cot(c+dx)}{4a^2 d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2 d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(\frac{15a^2}{4}-\frac{1}{4}a^2\sin(c+dx\right.}{\sqrt{a+a\sin(c+dx)}}}{2a^4}$$

$$= \frac{9\cot(c+dx)}{4a^2 d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2 d\sqrt{a+a\sin(c+dx)}} + \frac{15\int \csc(c+dx)\sqrt{a+a\,\mathrm{si}}}{8a^3}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{9\cot(c+d}{4a^2 d\sqrt{a+a\sin(}}$$

$$= -\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{5/2}d} + \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{9\cot(c+}{4a^2 d\sqrt{a+a\sin}}$$

Mathematica [C] time = 3.56, size = 309, normalized size = 2.02

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5\left(20\tan\left(\frac{1}{4}(c+dx)\right)+20\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)+\sec^2\left(\frac{1}{4}(c+dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(-40 - (256 + 256*I)*(-1)^(3/4)*Ar
cTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 20*Cot[(c + d*x)/4]
 - Csc[(c + d*x)/4]^2 - 92*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9
2*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Co
s[(c + d*x)/4] - Sin[(c + d*x)/4])^2 - (40*Sin[(c + d*x)/4])/(Cos[(c + d*x)
/4] - Sin[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (40*S
in[(c + d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 20*Tan[(c + d*x)/4
]))/(32*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

fricas [B] time = 0.49, size = 508, normalized size = 3.32

$$23\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(23*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x +
c) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8
*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (
cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 32*sqrt(2)*(a*cos(d
*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x + c)^2 - a)*sin(
d*x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) +
3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos
(d*x + c) - 2))/sqrt(a) - 4*(9*cos(d*x + c)^2 + (9*cos(d*x + c) + 11)*sin(d
*x + c) - 2*cos(d*x + c) - 11)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c
)^3 + a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - a^3*d + (a^3*d*cos(d*x +
c)^2 - a^3*d)*sin(d*x + c))
```

giac [B] time = 1.50, size = 620, normalized size = 4.05

$$\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{10}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)-\frac{\left(138\,\sqrt{2}\,\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-256\,\sqrt{2}\,\sqrt{a}\,\arctan\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] 1/8*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(tan(1/2*d*x + 1/2*c)/(a^3*sgn(tan(
1/2*d*x + 1/2*c) + 1)) - 10/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (138*sqr
t(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 256*sqrt(2)*sqr
t(a)*arctan(sqrt(a)/sqrt(-a)) - 69*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + s
qrt(a)) + 184*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 384*sq
rt(a)*arctan(sqrt(a)/sqrt(-a)) - 92*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a))
 - 58*sqrt(2)*sqrt(-a) - 92*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(3*sqrt
(2)*sqrt(-a)*a^(5/2) + 4*sqrt(-a)*a^(5/2)) - 64*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(
a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 46*arctan(-(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/
(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 23*log(abs(-sqrt(a)*tan(1/2*
d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(5/2)*sgn(tan(1/2*d*
x + 1/2*c) + 1)) + 2*((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^3 - 10*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^2*sqrt(a) + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))*a + 10*a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^2*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))
/d
```

maple [A] time = 1.29, size = 164, normalized size = 1.07

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\ (16\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{3}\left(\sin^{2}(dx+c)\right)+7\sqrt{-a(\sin(dx}{4a^{\frac{11}{2}}\sin(dx+c)^{2}\cos(dx+c)\sqrt{a+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(16*2^(1/2)*arctanh(1/2*(-a*(s
in(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a^3*sin(d*x+c)^2+7*(-a*(sin(d*x+c)-1))
^(1/2)*a^(5/2)-9*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)-23*arctanh((-a*(sin(d*x+
c)-1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^2)/a^(11/2)/sin(d*x+c)^2/cos(d*x+c)/(a
+a*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^4}{\sin(c+dx)^3 \left(a+a \sin(c+dx)\right)^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.488 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$\frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{5/2}d}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{19\cot(c+dx)}{8a^2d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a\sin(c+dx)+a}}+\frac{1}{1}$$

```
[Out] 45/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d-4*arctanh
(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1
9/8*cot(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)+13/12*cot(d*x+c)*csc(d*x+c)/a^2
/d/(a+a*sin(d*x+c))^(1/2)-1/3*cot(d*x+c)*csc(d*x+c)^2/a^2/d/(a+a*sin(d*x+c)
)^(1/2)
```

Rubi [A] time = 0.95, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2717, 2779, 2984, 2985, 2649, 206, 2773, 3044}

$$-\frac{19\cot(c+dx)}{8a^2d\sqrt{a\sin(c+dx)+a}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{5/2}d}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a\sin(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (45*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*a^(5/2)*d)
 - (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*
x]])])/(a^(5/2)*d) - (19*Cot[c + d*x])/(8*a^2*d*Sqrt[a + a*Sin[c + d*x]]) +
 (13*Cot[c + d*x]*Csc[c + d*x])/(12*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(3*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2717

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Dist[-2/(a*b), Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^3
, x], x] + Dist[1/a^2, Int[((a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^
2))/Sin[e + f*x]^4, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] &
& IntegerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\csc^4(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\csc^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^3(c+dx)\left(-\frac{a}{2}+\frac{11}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a^3}$$

$$= -\frac{\cot(c+dx)}{2a^2d\sqrt{a+a\sin(c+dx)}} + \frac{13\cot(c+dx)\csc(c+dx)}{12a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{19\cot(c+dx)}{8a^2d\sqrt{a+a\sin(c+dx)}} + \frac{13\cot(c+dx)\csc(c+dx)}{12a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{19\cot(c+dx)}{8a^2d\sqrt{a+a\sin(c+dx)}} + \frac{13\cot(c+dx)\csc(c+dx)}{12a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{2a^{5/2}d} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{19\cot(c+dx)}{8a^2d\sqrt{a+a\sin(c+d}}$$

$$= \frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8a^{5/2}d} - \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{19\cot(c+dx)}{8a^2d\sqrt{a+a\sin(c+}}$$

Mathematica [C] time = 2.41, size = 332, normalized size = 1.74

$$\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5\left(-\frac{8\csc^9\left(\frac{1}{2}(c+dx)\right)\left(-396\sin\left(\frac{1}{2}(c+dx)\right)-218\sin\left(\frac{3}{2}(c+dx)\right)+114\sin\left(\frac{5}{2}(c+dx)\right)+396\cos\left(\frac{1}{2}(c+dx)\right)-\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*((1536 + 1536*I)*(-1)^(3/4)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - (8*Csc[(c + d*x)/2]^9*(
396*Cos[(c + d*x)/2] - 218*Cos[(3*(c + d*x))/2] - 114*Cos[(5*(c + d*x))/2]
- 396*Sin[(c + d*x)/2] - 405*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*S
in[c + d*x] + 405*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x]
 - 218*Sin[(3*(c + d*x))/2] + 114*Sin[(5*(c + d*x))/2] + 135*Log[1 + Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 135*Log[1 - Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)]))/(Csc[(c + d*x)/4]^2 - Sec[(c + d
*x)/4]^2)^3))/(192*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

fricas [B] time = 0.50, size = 564, normalized size = 2.95

$$135\left(\cos(dx+c)^4-2\cos(dx+c)^2-\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sin(dx+c)+1\right)\sqrt{a}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(135*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x +
c)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
 192*sqrt(2)*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a
*cos(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*log(-(cos(d*x + c)^
2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(c
os(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)
^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 4*(57*c
os(d*x + c)^3 + 83*cos(d*x + c)^2 - (57*cos(d*x + c)^2 - 26*cos(d*x + c) -
```

```
91)*sin(d*x + c) - 65*cos(d*x + c) - 91)*sqrt(a*sin(d*x + c) + a))/(a^3*d*c
os(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d - (a^3*d*cos(d*x + c)^3 + a^
3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

giac [B] time = 1.16, size = 695, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] 1/48*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*tan(1/2*d*x + 1/2*c)/(a^3*sgn(
tan(1/2*d*x + 1/2*c) + 1)) - 15/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/
2*d*x + 1/2*c) + 74/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1))) + (1890*sqrt(2)*sq
rt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 3840*sqrt(2)*sqrt(a)*a
rctan(sqrt(a)/sqrt(-a)) - 945*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a
)) + 2700*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 5376*sqrt(
a)*arctan(sqrt(a)/sqrt(-a)) - 1350*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a))
- 1302*sqrt(2)*sqrt(-a) - 1808*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(7*s
qrt(2)*sqrt(-a)*a^(5/2) + 10*sqrt(-a)*a^(5/2)) + 384*sqrt(2)*arctan(-1/2*sq
rt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) +
sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 270*arcta
n(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt
(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 135*log(abs(-sqrt(a)*t
an(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(5/2)*sgn(tan
(1/2*d*x + 1/2*c) + 1)) - 2*(15*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^5 - 78*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^4*sqrt(a) + 144*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(3/2) - 15*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2 - 74*a^(5/2))/(((sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3*a^2*sgn(tan
(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 1.48, size = 182, normalized size = 0.95

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(-135\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^5\left(\sin^3(dx+c)\right)+57\,(-a\,(\sin(dx+\cdots\right.}{24a^{\frac{15}{2}}\sin(dx+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/24/a^(15/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-135*a^5*arctanh((
-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^3+57*(-a*(sin(d*x+c)-1))^(5/2)
```

```
*a^(5/2)+96*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*
a^5*sin(d*x+c)^3-88*(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2)+39*(-a*(sin(d*x+c)-1)
)^(1/2)*a^(9/2))/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sin\left(c+d\,x\right)^4\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(5/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.489 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{363\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{149\cot(c+dx)}{64a^2d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -363/64*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d+4*arct
anh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2
)+149/64*cot(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)-107/96*cot(d*x+c)*csc(d*x+
c)/a^2/d/(a+a*sin(d*x+c))^(1/2)+17/24*cot(d*x+c)*csc(d*x+c)^2/a^2/d/(a+a*si
n(d*x+c))^(1/2)-1/4*cot(d*x+c)*csc(d*x+c)^3/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.31, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2880, 2779, 2984, 2985, 2649, 206, 2773, 3044}

$$\frac{149\cot(c+dx)}{64a^2d\sqrt{a\sin(c+dx)+a}}-\frac{363\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-363*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*a^(5/2)
*d) + (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c +
 d*x]])])/(a^(5/2)*d) + (149*Cot[c + d*x])/(64*a^2*d*Sqrt[a + a*Sin[c + d*x
]]) - (107*Cot[c + d*x]*Csc[c + d*x])/(96*a^2*d*Sqrt[a + a*Sin[c + d*x]]) +
 (17*Cot[c + d*x]*Csc[c + d*x]^2)/(24*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (Co
t[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\csc^5(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\csc^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{2\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^4(c+dx)\left(-\frac{a}{2}+\frac{15}{2}a\sin(c\cdots\right.}{\sqrt{a+a\sin(c+dx)}}}{4a^3}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{6a^2d\sqrt{a+a\sin(c+dx)}} + \frac{17\cot(c+dx)\csc^2(c+dx)}{24a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+\cdots}{4a^2d\sqrt{a+a\sin(c+\cdots}}$$

$$= \frac{7\cot(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}} - \frac{107\cot(c+dx)\csc(c+dx)}{96a^2d\sqrt{a+a\sin(c+dx)}} + \frac{17\cot(c+dx)\csc^2(c\cdots}{24a^2d\sqrt{a+a\sin(c+\cdots}}$$

$$= \frac{149\cot(c+dx)}{64a^2d\sqrt{a+a\sin(c+dx)}} - \frac{107\cot(c+dx)\csc(c+dx)}{96a^2d\sqrt{a+a\sin(c+dx)}} + \frac{17\cot(c+dx)\csc^2(\cdots}{24a^2d\sqrt{a+a\sin(c\cdots}}$$

$$= \frac{149\cot(c+dx)}{64a^2d\sqrt{a+a\sin(c+dx)}} - \frac{107\cot(c+dx)\csc(c+dx)}{96a^2d\sqrt{a+a\sin(c+dx)}} + \frac{17\cot(c+dx)\csc^2(\cdots}{24a^2d\sqrt{a+a\sin(c\cdots}}$$

$$= -\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{5/2}d} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{149\cot(c\cdots}{64a^2d\sqrt{a+a\,\mathrm{si}\cdots}}$$

$$= -\frac{363\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64a^{5/2}d} + \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{149\cot(\cdots}{64a^2d\sqrt{a+a\cdots}}$$

Mathematica [C] time = 5.12, size = 414, normalized size = 1.81

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5\left(-\frac{16\csc^{12}\left(\frac{1}{2}(c+dx)\right)\left(-6250\sin\left(\frac{1}{2}(c+dx)\right)-4626\sin\left(\frac{3}{2}(c+dx)\right)+1750\sin\left(\frac{5}{2}(c+dx)\right)+894\sin\left(\frac{7}{2}(c+\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*((-24576 - 24576*I)*(-1)^(3/4)*Arc
Tanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - (16*Csc[(c + d*x)/2]
```

```
^12*(6250*Cos[(c + d*x)/2] - 4626*Cos[(3*(c + d*x))/2] - 1750*Cos[(5*(c + d
*x))/2] + 894*Cos[(7*(c + d*x))/2] + 3267*Log[1 + Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - 4356*Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 1089*Cos[4*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- 3267*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4356*Cos[2*(c + d*x)]
*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 1089*Cos[4*(c + d*x)]*Log[1
 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 6250*Sin[(c + d*x)/2] - 4626*Sin[
(3*(c + d*x))/2] + 1750*Sin[(5*(c + d*x))/2] + 894*Sin[(7*(c + d*x))/2]))/(
Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^4))/(3072*d*(a*(1 + Sin[c + d*x]))
^(5/2))
```

fricas [B] time = 0.52, size = 643, normalized size = 2.81

$$\frac{1089\left(\cos\left(dx+c\right)^{5}+\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{3}-2\,\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 1/768*(1089*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x
 + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(d*x +
c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c
)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x
+ c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c)
 - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2
- 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 1536*sqrt(2)*(a*cos(d*x + c)^5 + a
*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 - 2*a*cos(d*x + c)^2 + a*cos(d*x + c)
+ (a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*sin(d*x + c) + a)*log(-(cos(d
*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c)
 + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(
d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) -
 4*(447*cos(d*x + c)^4 - 214*cos(d*x + c)^3 - 1244*cos(d*x + c)^2 + (447*co
s(d*x + c)^3 + 661*cos(d*x + c)^2 - 583*cos(d*x + c) - 845)*sin(d*x + c) +
262*cos(d*x + c) + 845)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^5 + a
^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 + a^3
*d*cos(d*x + c) + a^3*d + (a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 +
a^3*d)*sin(d*x + c))
```

giac [B] time = 1.29, size = 845, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] 1/384*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*(3*tan(1/2*d*x + 1/2*c)/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 20/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c) + 159/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c) - 640/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (37026*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 73728*sqrt(2)*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - 18513*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 52272*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 104448*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - 26136*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 29680*sqrt(2)*sqrt(-a) - 42100*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(17*sqrt(2)*sqrt(-a)*a^(5/2) + 24*sqrt(-a)*a^(5/2)) - 3072*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2178*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 1089*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(5/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(159*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7 - 720*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) - 135*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a + 1920*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) - 135*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2 - 1840*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) + 159*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^3 + 640*a^(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^4*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d

maple [A] time = 1.40, size = 200, normalized size = 0.87

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(321\sqrt{-a(\sin(dx+c)-1)}\,a^{\frac{13}{2}}-1049(-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{11}{2}}+11\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x)

[Out] 1/192*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(321*(-a*(sin(d*x+c)-1))^(1/2)*a^(13/2)-1049*(-a*(sin(d*x+c)-1))^(3/2)*a^(11/2)+1127*(-a*(sin(d*x+c)-1))^(5/2)*a^(9/2)-447*(-a*(sin(d*x+c)-1))^(7/2)*a^(7/2)+768*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a^7*sin(d*x+c)^4-1089*a^7*arc

```
tanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^4)/a^(19/2)/sin(d*x+c)^4
/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sin\left(c+d\,x\right)^5\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.490 $\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=200

$$\frac{a^2\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{2a^2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] a^2*cos(d*x+c)*hypergeom([-3/2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*
x+c)^(1+n)/d/(1+n)/(cos(d*x+c)^2)^(1/2)+2*a^2*cos(d*x+c)*hypergeom([-3/2, 1
+1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)/(cos(d*x+c)^2)^(1/
2)+a^2*cos(d*x+c)*hypergeom([-3/2, 3/2+1/2*n],[5/2+1/2*n],sin(d*x+c)^2)*sin
(d*x+c)^(3+n)/d/(3+n)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2873, 2577}

$$\frac{a^2\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{2a^2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (2*a^2*Cos[c
+ d*x]*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2]) + (a^2*Cos[c + d*x]*Hyperg
eometric2F1[-3/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3 + n
))/(d*(3 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
```

```
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)\sin^n(c+dx)+2a^2\cos^4(c+dx)\sin^{1+n}(c\right.$$
$$= a^2\int \cos^4(c+dx)\sin^n(c+dx)\,dx + a^2\int \cos^4(c+dx)\sin^{2}$$
$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+d}{d(1+n)\sqrt{\cos^2(c+dx)}}$$

Mathematica [A] time = 0.29, size = 164, normalized size = 0.82

$$\frac{a^2\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\sin^{n+1}(c+dx)\left(\left(n^2+5n+6\right)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+(n+1)\sin(c+dx)\right.}{d(n+1)(n+2)(n+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*((6 + 5*n + n^2
)*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2] + (1 + n)*S
in[c + d*x]*(2*(3 + n)*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c
+ d*x]^2] + (2 + n)*Hypergeometric2F1[-3/2, (3 + n)/2, (5 + n)/2, Sin[c + d
*x]^2]*Sin[c + d*x])))/(d*(1 + n)*(2 + n)*(3 + n))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos(dx+c)^6 - 2\,a^2\cos(dx+c)^4\sin(dx+c) - 2\,a^2\cos(dx+c)^4\right)\sin(dx+c)^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral(-(a^2*cos(d*x + c)^6 - 2*a^2*cos(d*x + c)^4*sin(d*x + c) - 2*a^2*c
os(d*x + c)^4)*sin(d*x + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^2\sin(dx+c)^n\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

maple [F] time = 11.09, size = 0, normalized size = 0.00

$$\int \left(\cos^{4}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin\left(dx+c\right)+a)^{2}\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^n\,\left(a+a\,\sin\left(c+d\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + a*sin(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + a*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.491 $\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{a\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] a*cos(d*x+c)*hypergeom([-3/2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+
c)^(1+n)/d/(1+n)/(cos(d*x+c)^2)^(1/2)+a*cos(d*x+c)*hypergeom([-3/2, 1+1/2*n
],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2838, 2577}

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{a\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^
2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (a*Cos[c + d*x]
*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^n(c+dx)\,dx + a\int \cos^4(c+dx)\sin^{1+n}$$

$$= \frac{a\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx}{d(1+n)\sqrt{\cos^2(c+dx)}}$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
[Out] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x]), x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\cos(dx+c)^4\sin(dx+c)+a\cos(dx+c)^4\right)\sin(dx+c)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((a*cos(d*x + c)^4*sin(d*x + c) + a*cos(d*x + c)^4)*sin(d*x + c)^n,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)\sin(dx+c)^n\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

maple [F] time = 6.12, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)(\sin^n(dx+c))(a+a\sin(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)\sin(dx+c)^n\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^4\sin(c+d\,x)^n\,\left(a+a\,\sin(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + a*sin(c + d*x)),x)
[Out] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + a*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.492 $\int \frac{\cos^4(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{ad(n+1)\sqrt{\cos^2(c+dx)}}-\frac{\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c\right.}{ad(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] cos(d*x+c)*hypergeom([-1/2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+c)
^(1+n)/a/d/(1+n)/(cos(d*x+c)^2)^(1/2)-cos(d*x+c)*hypergeom([-1/2, 1+1/2*n],
[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/a/d/(2+n)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2839, 2577}

$$\frac{\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{ad(n+1)\sqrt{\cos^2(c+dx)}}-\frac{\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c\right.}{ad(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*Hypergeometric2F1[-1/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]
*Sin[c + d*x]^(1 + n))/(a*d*(1 + n)*Sqrt[Cos[c + d*x]^2]) - (Cos[c + d*x]*H
ypergeometric2F1[-1/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(
2 + n))/(a*d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^2(c+dx)\sin^n(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^{1+n}(c+dx)\,dx}{a}$$
$$= \frac{\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{ad(1+n)\sqrt{\cos^2(c+dx)}} - \frac{\cos(c+dx)\,{}_2F_1\left(\right.}{}$$

Mathematica [B] time = 11.48, size = 441, normalized size = 3.29

$$\frac{2^{n+1}\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^n\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^n\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\frac{{}_2F_1\left(\frac{n+1}{2},n+4;\frac{n}{}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2^(1 + n)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*Tan[(c + d*x)/2]*(Tan[(c
 + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^n*(1 + Tan[(c + d*x)/2]^2)^n*(Hypergeo
metric2F1[(1 + n)/2, 4 + n, (3 + n)/2, -Tan[(c + d*x)/2]^2]/(1 + n) + Tan[(
c + d*x)/2]*((-2*Hypergeometric2F1[(2 + n)/2, 4 + n, (4 + n)/2, -Tan[(c + d
*x)/2]^2])/(2 + n) + Tan[(c + d*x)/2]*(-(Hypergeometric2F1[(3 + n)/2, 4 + n
, (5 + n)/2, -Tan[(c + d*x)/2]^2]/(3 + n)) + (4*Hypergeometric2F1[(4 + n)/2
, 4 + n, (6 + n)/2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2])/(4 + n) - (Hyper
geometric2F1[4 + n, (5 + n)/2, (7 + n)/2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x
)/2]^2)/(5 + n) - (2*Hypergeometric2F1[4 + n, (6 + n)/2, (8 + n)/2, -Tan[(c
 + d*x)/2]^2]*Tan[(c + d*x)/2]^3)/(6 + n) + (Hypergeometric2F1[4 + n, (7 +
n)/2, (9 + n)/2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^4)/(7 + n)))))/(d*(a
 + a*Sin[c + d*x]))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin(dx+c)^n\cos(dx+c)^4}{a\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n \cos(dx+c)^4}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

maple [F] time = 4.78, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^4(dx+c)\right)(\sin^n(dx+c))}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n \cos(dx+c)^4}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4 \sin(c+d\,x)^n}{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^n)/(a + a*sin(c + d*x)),x)
[Out] int((cos(c + d*x)^4*sin(c + d*x)^n)/(a + a*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.493 $\int \frac{\cos^4(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=173

$$\frac{(2n+3)\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{a^2d(n+1)(n+2)\sqrt{\cos^2(c+dx)}}-\frac{2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2}\right.}{a^2d(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] -cos(d*x+c)*sin(d*x+c)^(1+n)/a^2/d/(2+n)+(3+2*n)*cos(d*x+c)*hypergeom([1/2,
 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^(1+n)/a^2/d/(1+n)/(2+n)/(c
os(d*x+c)^2)^(1/2)-2*cos(d*x+c)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(d*x+
c)^2)*sin(d*x+c)^(2+n)/a^2/d/(2+n)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2869, 2763, 2748, 2643}

$$\frac{(2n+3)\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{a^2d(n+1)(n+2)\sqrt{\cos^2(c+dx)}}-\frac{2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2}\right.}{a^2d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(1 + n))/(a^2*d*(2 + n))) + ((3 + 2*n)*Cos[c +
 d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c +
d*x]^(1 + n))/(a^2*d*(1 + n)*(2 + n)*Sqrt[Cos[c + d*x]^2]) - (2*Cos[c + d*x
]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(a^2*d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sin^n(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= -\frac{\cos(c+dx)\sin^{1+n}(c+dx)}{a^2d(2+n)} + \frac{\int \sin^n(c+dx)\left(a^2(3+2n)-2a^2(2+n)\sin(c+\right.}{a^4(2+n)}$$

$$= -\frac{\cos(c+dx)\sin^{1+n}(c+dx)}{a^2d(2+n)} - \frac{2\int \sin^{1+n}(c+dx)\,dx}{a^2} + \frac{(3+2n)\int \sin^n(c+dx}{a^2(2+n)}$$

$$= -\frac{\cos(c+dx)\sin^{1+n}(c+dx)}{a^2d(2+n)} + \frac{(3+2n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+d\right.}{a^2d(1+n)(2+n)\sqrt{\cos^2(c+dx}}$$

Mathematica [A] time = 4.92, size = 312, normalized size = 1.80

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sin^n(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)^n\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(\frac{{}_2F_1\left(\frac{n+1}{2},n+3;\frac{n+3}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right.\right.}{n+1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*(Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Sin[c + d
*x]^n*Tan[(c + d*x)/2]*(Hypergeometric2F1[(1 + n)/2, 3 + n, (3 + n)/2, -Tan
```

```
[(c + d*x)/2]^2]/(1 + n) + Tan[(c + d*x)/2]*((-4*Hypergeometric2F1[(2 + n)/
2, 3 + n, (4 + n)/2, -Tan[(c + d*x)/2]^2])/(2 + n) + Tan[(c + d*x)/2]*((6*H
ypergeometric2F1[(3 + n)/2, 3 + n, (5 + n)/2, -Tan[(c + d*x)/2]^2])/(3 + n)
 - (4*Hypergeometric2F1[3 + n, (4 + n)/2, (6 + n)/2, -Tan[(c + d*x)/2]^2]*T
an[(c + d*x)/2])/(4 + n) + (Hypergeometric2F1[3 + n, (5 + n)/2, (7 + n)/2,
-Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(5 + n)))))/(d*(a + a*Sin[c + d*x]
)^2)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)^{4}}{a^{2}\cos\left(dx+c\right)^{2}-2\,a^{2}\sin\left(dx+c\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)^4/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x
 + c) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)^{4}}{\left(a\sin\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a)^2, x)
```

maple [F] time = 6.95, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{4}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)}{\left(a+a\sin\left(dx+c\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)^{4}}{\left(a\sin\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^4 \sin(c+dx)^n}{(a+a\,\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^n)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^n)/(a + a*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.494 $\int \cos^5(c+dx)\sin^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^{10}(c+dx)}{10d}-\frac{2a\sin^9(c+dx)}{9d}-\frac{a\sin^8(c+dx)}{4d}+\frac{a\sin^7(c+dx)}{7d}+\frac{a\sin^6(c+dx)}{6d}$$

```
[Out] 1/6*a*sin(d*x+c)^6/d+1/7*a*sin(d*x+c)^7/d-1/4*a*sin(d*x+c)^8/d-2/9*a*sin(d*
x+c)^9/d+1/10*a*sin(d*x+c)^10/d+1/11*a*sin(d*x+c)^11/d
```

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^{10}(c+dx)}{10d}-\frac{2a\sin^9(c+dx)}{9d}-\frac{a\sin^8(c+dx)}{4d}+\frac{a\sin^7(c+dx)}{7d}+\frac{a\sin^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^6)/(6*d) + (a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^8)/(4
*d) - (2*a*Sin[c + d*x]^9)/(9*d) + (a*Sin[c + d*x]^10)/(10*d) + (a*Sin[c +
d*x]^11)/(11*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^5(a+x)^3}{a^5}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^5(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^{10}d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5x^5+a^4x^6-2a^3x^7-2a^2x^8+ax^9+x^{10}\right)dx, x, a$$
$$= \frac{a\sin^6(c+dx)}{6d}+\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^8(c+dx)}{4d}-\frac{2a\sin^9(c}{9d}$$

Mathematica [A] time = 0.45, size = 97, normalized size = 1.00

$$-\frac{a(-34650\sin(c+dx)+11550\sin(3(c+dx))+3465\sin(5(c+dx))-2475\sin(7(c+dx))-385\sin(9(c+dx))}{3548160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3548160*(a*(34650*Cos[2*(c + d*x)] - 5775*Cos[6*(c + d*x)] + 693*Cos[10*
(c + d*x)] - 34650*Sin[c + d*x] + 11550*Sin[3*(c + d*x)] + 3465*Sin[5*(c +
d*x)] - 2475*Sin[7*(c + d*x)] - 385*Sin[9*(c + d*x)] + 315*Sin[11*(c + d*x)
]))/d
```

fricas [A] time = 0.46, size = 106, normalized size = 1.09

$$-\frac{1386\,a\cos(dx+c)^{10}-3465\,a\cos(dx+c)^{8}+2310\,a\cos(dx+c)^{6}+20\left(63\,a\cos(dx+c)^{10}-161\,a\cos(dx+c)^{8}\right.}{13860\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/13860*(1386*a*cos(d*x + c)^10 - 3465*a*cos(d*x + c)^8 + 2310*a*cos(d*x +
 c)^6 + 20*(63*a*cos(d*x + c)^10 - 161*a*cos(d*x + c)^8 + 113*a*cos(d*x + c
)^6 - 3*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 - 8*a)*sin(d*x + c))/d
```

giac [A] time = 0.31, size = 133, normalized size = 1.37

$$-\frac{a\cos(10\,dx+10\,c)}{5120\,d}+\frac{5\,a\cos(6\,dx+6\,c)}{3072\,d}-\frac{5\,a\cos(2\,dx+2\,c)}{512\,d}-\frac{a\sin(11\,dx+11\,c)}{11264\,d}+\frac{a\sin(9\,dx+9\,c)}{9216\,d}+\frac{5\,a\sin(7}{716}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/5120*a*cos(10*d*x + 10*c)/d + 5/3072*a*cos(6*d*x + 6*c)/d - 5/512*a*cos(
2*d*x + 2*c)/d - 1/11264*a*sin(11*d*x + 11*c)/d + 1/9216*a*sin(9*d*x + 9*c)
/d + 5/7168*a*sin(7*d*x + 7*c)/d - 1/1024*a*sin(5*d*x + 5*c)/d - 5/1536*a*s
in(3*d*x + 3*c)/d + 5/512*a*sin(d*x + c)/d
```

maple [A] time = 0.23, size = 138, normalized size = 1.42

$$\frac{a\left(-\frac{\left(\sin^{5}(dx+c)\right)\left(\cos^{6}(dx+c)\right)}{11}-\frac{5\left(\sin^{3}(dx+c)\right)\left(\cos^{6}(dx+c)\right)}{99}-\frac{5\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{231}+\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{231}\right)+a\left(-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^3*cos(d*x+c)^6-5/23
1*sin(d*x+c)*cos(d*x+c)^6+1/231*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x
+c))+a*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/60
*cos(d*x+c)^6))
```

maxima [A] time = 0.35, size = 72, normalized size = 0.74

$$\frac{1260\,a\sin(dx+c)^{11}+1386\,a\sin(dx+c)^{10}-3080\,a\sin(dx+c)^{9}-3465\,a\sin(dx+c)^{8}+1980\,a\sin(dx+c)^{7}}{13860\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/13860*(1260*a*sin(d*x + c)^11 + 1386*a*sin(d*x + c)^10 - 3080*a*sin(d*x +
 c)^9 - 3465*a*sin(d*x + c)^8 + 1980*a*sin(d*x + c)^7 + 2310*a*sin(d*x + c)
^6)/d
```

mupad [B] time = 8.72, size = 71, normalized size = 0.73

$$\frac{\frac{a\sin(c+d\,x)^{11}}{11}+\frac{a\sin(c+d\,x)^{10}}{10}-\frac{2\,a\sin(c+d\,x)^{9}}{9}-\frac{a\sin(c+d\,x)^{8}}{4}+\frac{a\sin(c+d\,x)^{7}}{7}+\frac{a\sin(c+d\,x)^{6}}{6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^5*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^6)/6 + (a*sin(c + d*x)^7)/7 - (a*sin(c + d*x)^8)/4 - (2*a*
sin(c + d*x)^9)/9 + (a*sin(c + d*x)^10)/10 + (a*sin(c + d*x)^11)/11)/d
```

sympy [A] time = 42.24, size = 136, normalized size = 1.40

$$\begin{cases} \frac{8a\sin^{11}(c+dx)}{693d} + \frac{4a\sin^{9}(c+dx)\cos^{2}(c+dx)}{63d} + \frac{a\sin^{7}(c+dx)\cos^{4}(c+dx)}{7d} - \frac{a\sin^{4}(c+dx)\cos^{6}(c+dx)}{6d} - \frac{a\sin^{2}(c+dx)\cos^{8}(c+dx)}{12d} - \frac{a\cos^{10}(c+ \\ x\left(a\sin(c)+a\right)\sin^{5}(c)\cos^{5}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**11/(693*d) + 4*a*sin(c + d*x)**9*cos(c + d*x)*
*2/(63*d) + a*sin(c + d*x)**7*cos(c + d*x)**4/(7*d) - a*sin(c + d*x)**4*cos
(c + d*x)**6/(6*d) - a*sin(c + d*x)**2*cos(c + d*x)**8/(12*d) - a*cos(c + d
*x)**10/(60*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**5*cos(c)**5, True))
```

3.495 $\int \cos^5(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{a\sin^{10}(c+dx)}{10d}+\frac{a\sin^9(c+dx)}{9d}-\frac{a\sin^8(c+dx)}{4d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^6(c+dx)}{6d}+\frac{a\sin^5(c+dx)}{5d}$$

```
[Out] 1/5*a*sin(d*x+c)^5/d+1/6*a*sin(d*x+c)^6/d-2/7*a*sin(d*x+c)^7/d-1/4*a*sin(d*
x+c)^8/d+1/9*a*sin(d*x+c)^9/d+1/10*a*sin(d*x+c)^10/d
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a\sin^{10}(c+dx)}{10d}+\frac{a\sin^9(c+dx)}{9d}-\frac{a\sin^8(c+dx)}{4d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^6(c+dx)}{6d}+\frac{a\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^5)/(5*d) + (a*Sin[c + d*x]^6)/(6*d) - (2*a*Sin[c + d*x]^7)/
(7*d) - (a*Sin[c + d*x]^8)/(4*d) + (a*Sin[c + d*x]^9)/(9*d) + (a*Sin[c + d*
x]^10)/(10*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^4(a+x)^3}{a^4}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^4(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^9 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5x^4+a^4x^5-2a^3x^6-2a^2x^7+ax^8+x^9\right)dx, x, a\,\text{s}}{a^9 d}$$
$$= \frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^6(c+dx)}{6d}-\frac{2a\sin^7(c+dx)}{7d}-\frac{a\sin^8(c}{4d}$$

Mathematica [A] time = 0.33, size = 87, normalized size = 0.90

$$-\frac{a(-7560\sin(c+dx)+1680\sin(3(c+dx))+1008\sin(5(c+dx))-180\sin(7(c+dx))-140\sin(9(c+dx))+31}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/322560*(a*(3150*Cos[2*(c + d*x)] - 525*Cos[6*(c + d*x)] + 63*Cos[10*(c +
 d*x)] - 7560*Sin[c + d*x] + 1680*Sin[3*(c + d*x)] + 1008*Sin[5*(c + d*x)]
- 180*Sin[7*(c + d*x)] - 140*Sin[9*(c + d*x)]))/d
```

fricas [A] time = 0.56, size = 95, normalized size = 0.98

$$-\frac{126\,a\cos(dx+c)^{10}-315\,a\cos(dx+c)^{8}+210\,a\cos(dx+c)^{6}-4\left(35\,a\cos(dx+c)^{8}-50\,a\cos(dx+c)^{6}+3\,a\,c}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/1260*(126*a*cos(d*x + c)^10 - 315*a*cos(d*x + c)^8 + 210*a*cos(d*x + c)^
6 - 4*(35*a*cos(d*x + c)^8 - 50*a*cos(d*x + c)^6 + 3*a*cos(d*x + c)^4 + 4*a
*cos(d*x + c)^2 + 8*a)*sin(d*x + c))/d
```

giac [A] time = 0.26, size = 118, normalized size = 1.22

$$-\frac{a\cos(10\,dx+10\,c)}{5120\,d}+\frac{5\,a\cos(6\,dx+6\,c)}{3072\,d}-\frac{5\,a\cos(2\,dx+2\,c)}{512\,d}+\frac{a\sin(9\,dx+9\,c)}{2304\,d}+\frac{a\sin(7\,dx+7\,c)}{1792\,d}-\frac{a\sin(5\,dx}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/5120*a*cos(10*d*x + 10*c)/d + 5/3072*a*cos(6*d*x + 6*c)/d - 5/512*a*cos(2*d*x + 2*c)/d + 1/2304*a*sin(9*d*x + 9*c)/d + 1/1792*a*sin(7*d*x + 7*c)/d - 1/320*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x + 3*c)/d + 3/128*a*sin(d*x + c)/d

maple [A] time = 0.24, size = 120, normalized size = 1.24

$$\frac{a\left(-\frac{\left(\sin^{4}(dx+c)\right)\left(\cos^{6}(dx+c)\right)}{10}-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{6}(dx+c)\right)}{20}-\frac{\left(\cos^{6}(dx+c)\right)}{60}\right)+a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{6}(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{21}+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x)

[Out] 1/d*(a*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/60*cos(d*x+c)^6)+a*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))

maxima [A] time = 0.50, size = 72, normalized size = 0.74

$$\frac{126\,a\sin\left(dx+c\right)^{10}+140\,a\sin\left(dx+c\right)^{9}-315\,a\sin\left(dx+c\right)^{8}-360\,a\sin\left(dx+c\right)^{7}+210\,a\sin\left(dx+c\right)^{6}+252\,a}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/1260*(126*a*sin(d*x + c)^10 + 140*a*sin(d*x + c)^9 - 315*a*sin(d*x + c)^8 - 360*a*sin(d*x + c)^7 + 210*a*sin(d*x + c)^6 + 252*a*sin(d*x + c)^5)/d

mupad [B] time = 8.70, size = 71, normalized size = 0.73

$$\frac{\frac{a\sin(c+d\,x)^{10}}{10}+\frac{a\sin(c+d\,x)^{9}}{9}-\frac{a\sin(c+d\,x)^{8}}{4}-\frac{2\,a\sin(c+d\,x)^{7}}{7}+\frac{a\sin(c+d\,x)^{6}}{6}+\frac{a\sin(c+d\,x)^{5}}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^5*sin(c + d*x)^4*(a + a*sin(c + d*x)),x)

[Out] ((a*sin(c + d*x)^5)/5 + (a*sin(c + d*x)^6)/6 - (2*a*sin(c + d*x)^7)/7 - (a*sin(c + d*x)^8)/4 + (a*sin(c + d*x)^9)/9 + (a*sin(c + d*x)^10)/10)/d

sympy [A] time = 27.04, size = 136, normalized size = 1.40

$$\begin{cases} \frac{8a\sin^9(c+dx)}{315d} + \frac{4a\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{a\sin^5(c+dx)\cos^4(c+dx)}{5d} - \frac{a\sin^4(c+dx)\cos^6(c+dx)}{6d} - \frac{a\sin^2(c+dx)\cos^8(c+dx)}{12d} - \frac{a\cos^{10}(c+d}{60d} \\ x(a\sin(c)+a)\sin^4(c)\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**9/(315*d) + 4*a*sin(c + d*x)**7*cos(c + d*x)**
2/(35*d) + a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) - a*sin(c + d*x)**4*cos(
c + d*x)**6/(6*d) - a*sin(c + d*x)**2*cos(c + d*x)**8/(12*d) - a*cos(c + d*
x)**10/(60*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**4*cos(c)**5, True))
```

3.496 $\int \cos^5(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\sin^9(c+dx)}{9d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}$$

```
[Out] -1/6*a*cos(d*x+c)^6/d+1/8*a*cos(d*x+c)^8/d+1/5*a*sin(d*x+c)^5/d-2/7*a*sin(d
*x+c)^7/d+1/9*a*sin(d*x+c)^9/d
```

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2565, 14, 2564, 270}

$$\frac{a\sin^9(c+dx)}{9d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (a*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^5)/(
5*d) - (2*a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^5(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin^3(c+dx)\,dx + a\int \cos^5(c+dx)\sin^4(c+$$

$$= -\frac{a\,\text{Subst}\left(\int x^5\left(1-x^2\right)dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^4\left(\right.\right.}{}$$

$$= -\frac{a\,\text{Subst}\left(\int \left(x^5-x^7\right)dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \left(x^4-\right.\right.}{}$$

$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\cos^8(c+dx)}{8d} + \frac{a\sin^5(c+dx)}{5d} - \frac{2a\sin^7}{7}$$

Mathematica [A] time = 0.36, size = 97, normalized size = 1.20

$$\frac{a(7560\sin(c+dx) - 1680\sin(3(c+dx)) - 1008\sin(5(c+dx)) + 180\sin(7(c+dx)) + 140\sin(9(c+dx)) - 7560}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-7560*Cos[2*(c + d*x)] - 1260*Cos[4*(c + d*x)] + 840*Cos[6*(c + d*x)] +
 315*Cos[8*(c + d*x)] + 7560*Sin[c + d*x] - 1680*Sin[3*(c + d*x)] - 1008*Si
n[5*(c + d*x)] + 180*Sin[7*(c + d*x)] + 140*Sin[9*(c + d*x)]))/(322560*d)
```

fricas [A] time = 0.73, size = 84, normalized size = 1.04

$$\frac{315\,a\cos(dx+c)^8 - 420\,a\cos(dx+c)^6 + 8\left(35\,a\cos(dx+c)^8 - 50\,a\cos(dx+c)^6 + 3\,a\cos(dx+c)^4 + 4\,a\cos(\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/2520*(315*a*cos(d*x + c)^8 - 420*a*cos(d*x + c)^6 + 8*(35*a*cos(d*x + c)^8 - 50*a*cos(d*x + c)^6 + 3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*a)*sin(d*x + c))/d

giac [A] time = 0.23, size = 133, normalized size = 1.64

$$\frac{a\cos\left(8\,dx+8\,c\right)}{1024\,d}+\frac{a\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{a\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{3\,a\cos\left(2\,dx+2\,c\right)}{128\,d}+\frac{a\sin\left(9\,dx+9\,c\right)}{2304\,d}+\frac{a\sin\left(7\,dx+7\,c\right)}{1792\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/1024*a*cos(8*d*x + 8*c)/d + 1/384*a*cos(6*d*x + 6*c)/d - 1/256*a*cos(4*d*x + 4*c)/d - 3/128*a*cos(2*d*x + 2*c)/d + 1/2304*a*sin(9*d*x + 9*c)/d + 1/1792*a*sin(7*d*x + 7*c)/d - 1/320*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x + 3*c)/d + 3/128*a*sin(d*x + c)/d

maple [A] time = 0.28, size = 102, normalized size = 1.26

$$\frac{a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{6}(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{105}\right)+a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{6}(dx+c)\right)}{8}-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)

[Out] 1/d*(a*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6))

maxima [A] time = 0.58, size = 72, normalized size = 0.89

$$\frac{280\,a\sin\left(dx+c\right)^{9}+315\,a\sin\left(dx+c\right)^{8}-720\,a\sin\left(dx+c\right)^{7}-840\,a\sin\left(dx+c\right)^{6}+504\,a\sin\left(dx+c\right)^{5}+630\,a}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/2520*(280*a*sin(d*x + c)^9 + 315*a*sin(d*x + c)^8 - 720*a*sin(d*x + c)^7 - 840*a*sin(d*x + c)^6 + 504*a*sin(d*x + c)^5 + 630*a*sin(d*x + c)^4)/d

mupad [B] time = 0.05, size = 71, normalized size = 0.88

$$\frac{\frac{a\sin(c+d\,x)^9}{9}+\frac{a\sin(c+d\,x)^8}{8}-\frac{2\,a\sin(c+d\,x)^7}{7}-\frac{a\sin(c+d\,x)^6}{3}+\frac{a\sin(c+d\,x)^5}{5}+\frac{a\sin(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^4)/4 + (a*sin(c + d*x)^5)/5 - (a*sin(c + d*x)^6)/3 - (2*a*
sin(c + d*x)^7)/7 + (a*sin(c + d*x)^8)/8 + (a*sin(c + d*x)^9)/9)/d
```

sympy [A] time = 17.23, size = 114, normalized size = 1.41

$$\begin{cases} \frac{8a\sin^{9}(c+dx)}{315d}+\frac{4a\sin^{7}(c+dx)\cos^{2}(c+dx)}{35d}+\frac{a\sin^{5}(c+dx)\cos^{4}(c+dx)}{5d}-\frac{a\sin^{2}(c+dx)\cos^{6}(c+dx)}{6d}-\frac{a\cos^{8}(c+dx)}{24d} & \text{for}\ d\neq 0\\ x\left(a\sin(c)+a\right)\sin^{3}(c)\cos^{5}(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**9/(315*d) + 4*a*sin(c + d*x)**7*cos(c + d*x)**
2/(35*d) + a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) - a*sin(c + d*x)**2*cos(
c + d*x)**6/(6*d) - a*cos(c + d*x)**8/(24*d), Ne(d, 0)), (x*(a*sin(c) + a)*
sin(c)**3*cos(c)**5, True))
```

3.497 $\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}$$

```
[Out] -1/6*a*cos(d*x+c)^6/d+1/8*a*cos(d*x+c)^8/d+1/3*a*sin(d*x+c)^3/d-2/5*a*sin(d
*x+c)^5/d+1/7*a*sin(d*x+c)^7/d
```

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2564, 270, 2565, 14}

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (a*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^3)/(
3*d) - (2*a*Sin[c + d*x]^5)/(5*d) + (a*Sin[c + d*x]^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin^2(c+dx)\,dx + a\int \cos^5(c+dx)\sin^3(c+$$

$$= -\frac{a\,\text{Subst}\left(\int x^5\left(1-x^2\right)dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^2\right.}{}$$

$$= \frac{a\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \sin(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\cos^8(c+dx)}{8d} + \frac{a\sin^3(c+dx)}{3d} - \frac{2a\sin^5}{5}$$

Mathematica [A] time = 0.28, size = 87, normalized size = 1.07

$$-\frac{a(-8400\sin(c+dx)+560\sin(3(c+dx))+1008\sin(5(c+dx))+240\sin(7(c+dx))+2520\cos(2(c+dx))+42}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/107520*(a*(2520*Cos[2*(c + d*x)] + 420*Cos[4*(c + d*x)] - 280*Cos[6*(c +
 d*x)] - 105*Cos[8*(c + d*x)] - 8400*Sin[c + d*x] + 560*Sin[3*(c + d*x)] +
1008*Sin[5*(c + d*x)] + 240*Sin[7*(c + d*x)]))/d
```

fricas [A] time = 0.79, size = 73, normalized size = 0.90

$$\frac{105\,a\cos\left(dx+c\right)^8-140\,a\cos\left(dx+c\right)^6-8\left(15\,a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4-4\,a\cos\left(dx+c\right)^2-8\,a\right)\sin\left(d\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/840*(105*a*cos(d*x + c)^8 - 140*a*cos(d*x + c)^6 - 8*(15*a*cos(d*x + c)^6
 - 3*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 - 8*a)*sin(d*x + c))/d
```

giac [A] time = 0.21, size = 118, normalized size = 1.46

$$\frac{a\cos(8\,dx+8\,c)}{1024\,d}+\frac{a\cos(6\,dx+6\,c)}{384\,d}-\frac{a\cos(4\,dx+4\,c)}{256\,d}-\frac{3\,a\cos(2\,dx+2\,c)}{128\,d}-\frac{a\sin(7\,dx+7\,c)}{448\,d}-\frac{3\,a\sin(5\,dx+5}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1024*a*cos(8*d*x + 8*c)/d + 1/384*a*cos(6*d*x + 6*c)/d - 1/256*a*cos(4*d*
x + 4*c)/d - 3/128*a*cos(2*d*x + 2*c)/d - 1/448*a*sin(7*d*x + 7*c)/d - 3/32
0*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x + 3*c)/d + 5/64*a*sin(d*x + c)/d
```

maple [A] time = 0.24, size = 84, normalized size = 1.04

$$\frac{a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{6}(dx+c)\right)}{8}-\frac{\left(\cos^{6}(dx+c)\right)}{24}\right)+a\left(-\frac{\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+a*(-1/7*sin(d*x+c
)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.86, size = 72, normalized size = 0.89

$$\frac{105\,a\sin(dx+c)^{8}+120\,a\sin(dx+c)^{7}-280\,a\sin(dx+c)^{6}-336\,a\sin(dx+c)^{5}+210\,a\sin(dx+c)^{4}+280\,a\,s}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/840*(105*a*sin(d*x + c)^8 + 120*a*sin(d*x + c)^7 - 280*a*sin(d*x + c)^6 -
 336*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 + 280*a*sin(d*x + c)^3)/d
```

mupad [B] time = 8.69, size = 71, normalized size = 0.88

$$\frac{\frac{a\sin(c+d\,x)^{8}}{8}+\frac{a\sin(c+d\,x)^{7}}{7}-\frac{a\sin(c+d\,x)^{6}}{3}-\frac{2\,a\sin(c+d\,x)^{5}}{5}+\frac{a\sin(c+d\,x)^{4}}{4}+\frac{a\sin(c+d\,x)^{3}}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^3)/3 + (a*sin(c + d*x)^4)/4 - (2*a*sin(c + d*x)^5)/5 - (a*
sin(c + d*x)^6)/3 + (a*sin(c + d*x)^7)/7 + (a*sin(c + d*x)^8)/8)/d
```

sympy [A] time = 10.44, size = 114, normalized size = 1.41

$$\begin{cases} \frac{8a\sin^{7}(c+dx)}{105d} + \frac{4a\sin^{5}(c+dx)\cos^{2}(c+dx)}{15d} + \frac{a\sin^{3}(c+dx)\cos^{4}(c+dx)}{3d} - \frac{a\sin^{2}(c+dx)\cos^{6}(c+dx)}{6d} - \frac{a\cos^{8}(c+dx)}{24d} & \text{for } d \neq 0 \\ x\left(a\sin(c)+a\right)\sin^{2}(c)\cos^{5}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**7/(105*d) + 4*a*sin(c + d*x)**5*cos(c + d*x)**
2/(15*d) + a*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - a*sin(c + d*x)**2*cos(
c + d*x)**6/(6*d) - a*cos(c + d*x)**8/(24*d), Ne(d, 0)), (x*(a*sin(c) + a)*
sin(c)**2*cos(c)**5, True))
```

3.498 $\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^6(c+dx)}{6d}$$

```
[Out] -1/6*a*cos(d*x+c)^6/d+1/3*a*sin(d*x+c)^3/d-2/5*a*sin(d*x+c)^5/d+1/7*a*sin(d
*x+c)^7/d
```

Rubi [A] time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2834, 2565, 30, 2564, 270}

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (a*Sin[c + d*x]^3)/(3*d) - (2*a*Sin[c + d*x]^5)
/(5*d) + (a*Sin[c + d*x]^7)/(7*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^5(c+dx)\sin(c+dx)\,dx + a\int \cos^5(c+dx)\sin^2(c+d\\ &= -\frac{a\,\text{Subst}\left(\int x^5\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^2\left(1-x^2\right)^2 \right.}{d}\\ &= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \sin(c+d\right.}{d}\\ &= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\sin^3(c+dx)}{3d} - \frac{2a\sin^5(c+dx)}{5d} + \frac{a\sin^7(}{7}\end{aligned}$$

Mathematica [A] time = 0.16, size = 78, normalized size = 1.20

$$-\frac{a(-525\sin(c+dx)+35\sin(3(c+dx))+63\sin(5(c+dx))+15\sin(7(c+dx))+525\cos(2(c+dx))+210\cos(}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6720*(a*(350 + 525*Cos[2*(c + d*x)] + 210*Cos[4*(c + d*x)] + 35*Cos[6*(c
 + d*x)] - 525*Sin[c + d*x] + 35*Sin[3*(c + d*x)] + 63*Sin[5*(c + d*x)] + 1
5*Sin[7*(c + d*x)]))/d
```

fricas [A] time = 0.58, size = 62, normalized size = 0.95

$$-\frac{35\,a\cos\left(dx+c\right)^6+2\left(15\,a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4-4\,a\cos\left(dx+c\right)^2-8\,a\right)\sin\left(dx+c\right)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/210*(35*a*cos(d*x + c)^6 + 2*(15*a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 - 8*a)*sin(d*x + c))/d

giac [A] time = 0.20, size = 103, normalized size = 1.58

$$-\frac{a\cos(6\,dx+6\,c)}{192\,d}-\frac{a\cos(4\,dx+4\,c)}{32\,d}-\frac{5\,a\cos(2\,dx+2\,c)}{64\,d}-\frac{a\sin(7\,dx+7\,c)}{448\,d}-\frac{3\,a\sin(5\,dx+5\,c)}{320\,d}-\frac{a\sin(3\,dx+}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/192*a*cos(6*d*x + 6*c)/d - 1/32*a*cos(4*d*x + 4*c)/d - 5/64*a*cos(2*d*x + 2*c)/d - 1/448*a*sin(7*d*x + 7*c)/d - 3/320*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x + 3*c)/d + 5/64*a*sin(d*x + c)/d

maple [A] time = 0.24, size = 64, normalized size = 0.98

$$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{a\left(\cos^6(dx+c)\right)}{6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x)

[Out] 1/d*(a*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/6*a*cos(d*x+c)^6)

maxima [A] time = 0.58, size = 72, normalized size = 1.11

$$\frac{30\,a\sin(dx+c)^7+35\,a\sin(dx+c)^6-84\,a\sin(dx+c)^5-105\,a\sin(dx+c)^4+70\,a\sin(dx+c)^3+105\,a\sin(}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/210*(30*a*sin(d*x + c)^7 + 35*a*sin(d*x + c)^6 - 84*a*sin(d*x + c)^5 - 105*a*sin(d*x + c)^4 + 70*a*sin(d*x + c)^3 + 105*a*sin(d*x + c)^2)/d

mupad [B] time = 8.70, size = 71, normalized size = 1.09

$$\frac{\frac{a\sin(c+d\,x)^7}{7}+\frac{a\sin(c+d\,x)^6}{6}-\frac{2\,a\sin(c+d\,x)^5}{5}-\frac{a\sin(c+d\,x)^4}{2}+\frac{a\sin(c+d\,x)^3}{3}+\frac{a\sin(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^2)/2 + (a*sin(c + d*x)^3)/3 - (a*sin(c + d*x)^4)/2 - (2*a*
sin(c + d*x)^5)/5 + (a*sin(c + d*x)^6)/6 + (a*sin(c + d*x)^7)/7)/d
```

sympy [A] time = 6.26, size = 90, normalized size = 1.38

$$\begin{cases} \frac{8a\sin^7(c+dx)}{105d} + \frac{4a\sin^5(c+dx)\cos^2(c+dx)}{15d} + \frac{a\sin^3(c+dx)\cos^4(c+dx)}{3d} - \frac{a\cos^6(c+dx)}{6d} & \text{for } d \neq 0 \\ x\,(a\sin(c)+a)\sin(c)\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**7/(105*d) + 4*a*sin(c + d*x)**5*cos(c + d*x)**
2/(15*d) + a*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - a*cos(c + d*x)**6/(6*d
), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*cos(c)**5, True))
```

3.499 $\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}-\frac{2a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d-a*sin(d*x+c)^2/d-2/3*a*sin(d*x+c)^3/d+1/4
*a*sin(d*x+c)^4/d+1/5*a*sin(d*x+c)^5/d
```

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2836, 12, 88}

$$\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}-\frac{2a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/d - (2*a*
Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2(a+x)^3}{x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4+\frac{a^5}{x}-2a^3x-2a^2x^2+ax^3+x^4\right)dx, x, a\sin(c+\right.}{a^4 d}$$
$$= \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{d}-\frac{2a\sin^3}{3}$$

Mathematica [A] time = 0.04, size = 86, normalized size = 1.00

$$\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}-\frac{2a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/d - (2*a*
Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) + (a*Sin[c + d*x]^5)/(5*d)
```

fricas [A] time = 0.75, size = 74, normalized size = 0.86

$$\frac{15\,a\cos(dx+c)^4+30\,a\cos(dx+c)^2+60\,a\log\left(\frac{1}{2}\sin(dx+c)\right)+4\left(3\,a\cos(dx+c)^4+4\,a\cos(dx+c)^2+8\,a\right)s}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*a*cos(d*x + c)^4 + 30*a*cos(d*x + c)^2 + 60*a*log(1/2*sin(d*x + c)
) + 4*(3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*a)*sin(d*x + c))/d
```

giac [A] time = 0.17, size = 70, normalized size = 0.81

$$\frac{12\,a\sin(dx+c)^5+15\,a\sin(dx+c)^4-40\,a\sin(dx+c)^3-60\,a\sin(dx+c)^2+60\,a\log(|\sin(dx+c)|)+60\,a\sin}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/60*(12*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 40*a*sin(d*x + c)^3 - 60*a*sin(d*x + c)^2 + 60*a*log(abs(sin(d*x + c))) + 60*a*sin(d*x + c))/d

maple [A] time = 0.37, size = 94, normalized size = 1.09

$$\frac{8a\sin(dx+c)}{15d}+\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)\,a}{5d}+\frac{4\left(\cos^2(dx+c)\right)\sin(dx+c)\,a}{15d}+\frac{a\left(\cos^4(dx+c)\right)}{4d}+\frac{a\left(\cos^2(dx+\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c)),x)

[Out] 8/15*a*sin(d*x+c)/d+1/5/d*cos(d*x+c)^4*sin(d*x+c)*a+4/15/d*a*sin(d*x+c)*cos(d*x+c)^2+1/4*a*cos(d*x+c)^4/d+1/2*a*cos(d*x+c)^2/d+a*ln(sin(d*x+c))/d

maxima [A] time = 0.54, size = 69, normalized size = 0.80

$$\frac{12\,a\sin(dx+c)^5+15\,a\sin(dx+c)^4-40\,a\sin(dx+c)^3-60\,a\sin(dx+c)^2+60\,a\log(\sin(dx+c))+60\,a\sin}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/60*(12*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 40*a*sin(d*x + c)^3 - 60*a*sin(d*x + c)^2 + 60*a*log(sin(d*x + c)) + 60*a*sin(d*x + c))/d

mupad [B] time = 8.87, size = 126, normalized size = 1.47

$$\frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}+\frac{a\cos\left(c+d\,x\right)^2}{2\,d}+\frac{a\cos\left(c+d\,x\right)^4}{4\,d}+\frac{8\,a\,\sin\left(c+d\,x\right)}{15\,d}+\frac{4\,a\cos\left(c+d\,x\right)^2\,\sin(}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x),x)

[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*log(1/cos(c/2 + (d*x)/2)^2))/d + (a*cos(c + d*x)^2)/(2*d) + (a*cos(c + d*x)^4)/(4*d) + (8*a*sin(c + d*x))/(15*d) + (4*a*cos(c + d*x)^2*sin(c + d*x))/(15*d) + (a*cos(c + d*x)^4*sin(c + d*x))/(5*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^5(c+dx)\csc(c+dx)\,dx+\int\sin(c+dx)\cos^5(c+dx)\csc(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**5*csc(c + d*x), x) + Integral(sin(c + d*x)*cos(c
+ d*x)**5*csc(c + d*x), x))
```

3.500 $\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=83

$$\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] -a*csc(d*x+c)/d+a*ln(sin(d*x+c))/d-2*a*sin(d*x+c)/d-a*sin(d*x+c)^2/d+1/3*a*
sin(d*x+c)^3/d+1/4*a*sin(d*x+c)^4/d
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (2*a*Sin[c + d*x])/d - (a
*Sin[c + d*x]^2)/d + (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2(a+x)^3}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^3+\frac{a^5}{x^2}+\frac{a^4}{x}-2a^2x+ax^2+x^3\right)dx, x, a\sin(c+\right.}{a^3 d}$$
$$= -\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}-\frac{2a\sin(c+dx)}{d}-\frac{a\sin}{}$$

Mathematica [A] time = 0.04, size = 83, normalized size = 1.00

$$\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (2*a*Sin[c + d*x])/d - (a
*Sin[c + d*x]^2)/d + (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d)
```

fricas [A] time = 0.80, size = 91, normalized size = 1.10

$$\frac{32\,a\cos(dx+c)^4+128\,a\cos(dx+c)^2+96\,a\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+3\left(8\,a\cos(dx+c)^4+16\,a\cos(d\right.}{96\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(32*a*cos(d*x + c)^4 + 128*a*cos(d*x + c)^2 + 96*a*log(1/2*sin(d*x + c
))*sin(d*x + c) + 3*(8*a*cos(d*x + c)^4 + 16*a*cos(d*x + c)^2 - 11*a)*sin(d
*x + c) - 256*a)/(d*sin(d*x + c))
```

giac [A] time = 0.19, size = 79, normalized size = 0.95

$$\frac{3\,a\sin(dx+c)^4+4\,a\sin(dx+c)^3-12\,a\sin(dx+c)^2+12\,a\log(|\sin(dx+c)|)-24\,a\sin(dx+c)-\frac{12\,(a\sin(dx+c)}{\sin(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 12*a*log(abs(sin(d*x + c))) - 24*a*sin(d*x + c) - 12*(a*sin(d*x + c) + a)/sin(d*x + c))/d

maple [A] time = 0.31, size = 116, normalized size = 1.40

$$\frac{a\left(\cos^4(dx+c)\right)}{4d}+\frac{a\left(\cos^2(dx+c)\right)}{2d}+\frac{a\ln(\sin(dx+c))}{d}-\frac{a\left(\cos^6(dx+c)\right)}{d\sin(dx+c)}-\frac{8a\sin(dx+c)}{3d}-\frac{\left(\cos^4(dx+c)\right)\sin(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c)),x)

[Out] 1/4*a*cos(d*x+c)^4/d+1/2*a*cos(d*x+c)^2/d+a*ln(sin(d*x+c))/d-1/d*a/sin(d*x+c)*cos(d*x+c)^6-8/3*a*sin(d*x+c)/d-1/d*cos(d*x+c)^4*sin(d*x+c)*a-4/3/d*a*sin(d*x+c)*cos(d*x+c)^2

maxima [A] time = 0.61, size = 69, normalized size = 0.83

$$\frac{3\,a\sin(dx+c)^4+4\,a\sin(dx+c)^3-12\,a\sin(dx+c)^2+12\,a\log(\sin(dx+c))-24\,a\sin(dx+c)-\frac{12\,a}{\sin(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 12*a*log(sin(d*x + c)) - 24*a*sin(d*x + c) - 12*a/sin(d*x + c))/d

mupad [B] time = 8.88, size = 250, normalized size = 3.01

$$\frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}-\frac{4\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d}+\frac{8\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}-\frac{8\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{d}+\frac{4\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^2,x)

[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*log(1/cos(c/2 + (d*x)/2)^2))/d - (4*a*cos(c/2 + (d*x)/2)^2)/d + (8*a*cos(c/2 + (d*x)/2)^4)/d - (8*a*cos(c/2 + (d*x)/2)^6)/d + (4*a*cos(c/2 + (d*x)/2)^8)/d - (9*a*cos(c/2 + (d*x)/2))/(2*d*sin(c/2 + (d*x)/2)) - (a*sin(c/2 + (d*x)/2))/(2*d*cos(c/2 +

```
 (d*x)/2)) + (20*a*cos(c/2 + (d*x)/2)^3)/(3*d*sin(c/2 + (d*x)/2)) - (16*a*c
os(c/2 + (d*x)/2)^5)/(3*d*sin(c/2 + (d*x)/2)) + (8*a*cos(c/2 + (d*x)/2)^7)/
(3*d*sin(c/2 + (d*x)/2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.501 $\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^2(c+dx)}{2d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{2a\log(\sin(c+dx))}{d}$$

```
[Out] -a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-2*a*ln(sin(d*x+c))/d-2*a*sin(d*x+c)/d+
1/2*a*sin(d*x+c)^2/d+1/3*a*sin(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^2(c+dx)}{2d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{2a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (2*a*Log[Sin[c + d*x]])/
d - (2*a*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/(3
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2(a+x)^3}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^2+\frac{a^5}{x^3}+\frac{a^4}{x^2}-\frac{2a^3}{x}+ax+x^2\right)dx, x, a\sin(c+d$$
$$= -\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{2a\log(\sin(c+dx))}{d}-\frac{2a\,\text{s}$$

Mathematica [A] time = 0.11, size = 77, normalized size = 0.90

$$\frac{a\sin^3(c+dx)}{3d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}-\frac{a\left(-\sin^2(c+dx)+\csc^2(c+dx)+4\log(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (2*a*Sin[c + d*x])/d + (a*Sin[c + d*x]^3)/(3*d) - (
a*(Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - Sin[c + d*x]^2))/(2*d)
```

fricas [A] time = 0.69, size = 102, normalized size = 1.19

$$-\frac{6\,a\cos\left(dx+c\right)^4-9\,a\cos\left(dx+c\right)^2+24\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+4\left(a\cos\left(dx+c\right)^4+4\,a\cos}{12\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*cos(d*x + c)^4 - 9*a*cos(d*x + c)^2 + 24*(a*cos(d*x + c)^2 - a)*
log(1/2*sin(d*x + c)) + 4*(a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 - 8*a)*sin
(d*x + c) - 3*a)/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.20, size = 82, normalized size = 0.95

$$\frac{2\,a\sin\left(dx+c\right)^3+3\,a\sin\left(dx+c\right)^2-12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-12\,a\sin\left(dx+c\right)+\frac{3\left(6\,a\sin(dx+c)^2-2\,a\sin(dx+c)-a\right)}{\sin(dx+c)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 12*a*log(abs(sin(d*x + c)))
- 12*a*sin(d*x + c) + 3*(6*a*sin(d*x + c)^2 - 2*a*sin(d*x + c) - a)/sin(d*x
 + c)^2)/d
```

maple [A] time = 0.38, size = 139, normalized size = 1.62

$$-\frac{a\left(\cos^6(dx+c)\right)}{d\sin(dx+c)}-\frac{8a\sin(dx+c)}{3d}-\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)\,a}{d}-\frac{4\left(\cos^2(dx+c)\right)\sin(dx+c)\,a}{3d}-\frac{a\left(\cos^6(dx+c)\right)}{2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^6-8/3*a*sin(d*x+c)/d-1/d*cos(d*x+c)^4*sin(d*x+
c)*a-4/3/d*a*sin(d*x+c)*cos(d*x+c)^2-1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^6-1/2*
a*cos(d*x+c)^4/d-a*cos(d*x+c)^2/d-2*a*ln(sin(d*x+c))/d
```

maxima [A] time = 0.60, size = 68, normalized size = 0.79

$$\frac{2\,a\sin(dx+c)^3+3\,a\sin(dx+c)^2-12\,a\log(\sin(dx+c))-12\,a\sin(dx+c)-\frac{3\,(2\,a\sin(dx+c)+a)}{\sin(dx+c)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 12*a*log(sin(d*x + c)) - 12*
a*sin(d*x + c) - 3*(2*a*sin(d*x + c) + a)/sin(d*x + c)^2)/d
```

mupad [B] time = 8.86, size = 229, normalized size = 2.66

$$\frac{2\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{18\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+\frac{82\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-\frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+22\,a\tan\left(\frac{c}{2}+\right.}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (2*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a/2 + 2*a*tan(c/2 + (d*x)/2) + (3*
a*tan(c/2 + (d*x)/2)^2)/2 + 22*a*tan(c/2 + (d*x)/2)^3 - (13*a*tan(c/2 + (d*
```

```
x)/2)^4)/2 + (82*a*tan(c/2 + (d*x)/2)^5)/3 - (15*a*tan(c/2 + (d*x)/2)^6)/2
+ 18*a*tan(c/2 + (d*x)/2)^7)/(d*(4*tan(c/2 + (d*x)/2)^2 + 12*tan(c/2 + (d*x
)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 + 4*tan(c/2 + (d*x)/2)^8)) - (a*tan(c/2 +
(d*x)/2))/(2*d) - (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (2*a*log(tan(c/2 + (d*x)
/2)))/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.502 $\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=85

$$\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}+\frac{2a\csc(c+dx)}{d}-\frac{2a\log(\sin(c+dx))}{d}$$

```
[Out] 2*a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-2*a*ln(sin(d*x+c
))/d+a*sin(d*x+c)/d+1/2*a*sin(d*x+c)^2/d
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2836, 12, 88}

$$\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}+\frac{2a\csc(c+dx)}{d}-\frac{2a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
- (2*a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)^3}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(a+\frac{a^5}{x^4}+\frac{a^4}{x^3}-\frac{2a^3}{x^2}-\frac{2a^2}{x}+x\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{2a\csc(c+dx)}{d} - \frac{a\csc^2(c+dx)}{2d} - \frac{a\csc^3(c+dx)}{3d} - \frac{2a\log(\sin}{d}$$

Mathematica [A] time = 0.15, size = 76, normalized size = 0.89

$$\frac{a\sin(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} + \frac{2a\csc(c+dx)}{d} - \frac{a\left(-\sin^2(c+dx)+\csc^2(c+dx)+4\log(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (a*Sin[c + d*x])/d - (a*(
Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - Sin[c + d*x]^2))/(2*d)
```

fricas [A] time = 0.81, size = 117, normalized size = 1.38

$$-\frac{12\,a\cos(dx+c)^4 - 48\,a\cos(dx+c)^2 + 24\left(a\cos(dx+c)^2 - a\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c) + 3\left(2\,a\cos(dx\right.}{12\left(d\cos(dx+c)^2 - d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^4 - 48*a*cos(d*x + c)^2 + 24*(a*cos(d*x + c)^2 - a
)*log(1/2*sin(d*x + c))*sin(d*x + c) + 3*(2*a*cos(d*x + c)^4 - 3*a*cos(d*x
+ c)^2 - a)*sin(d*x + c) + 32*a)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.20, size = 81, normalized size = 0.95

$$\frac{3\,a\sin(dx+c)^2 - 12\,a\log(|\sin(dx+c)|) + 6\,a\sin(dx+c) + \frac{22\,a\sin(dx+c)^3 + 12\,a\sin(dx+c)^2 - 3\,a\sin(dx+c) - 2\,a}{\sin(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*sin(d*x + c)^2 - 12*a*log(abs(sin(d*x + c))) + 6*a*sin(d*x + c) +
(22*a*sin(d*x + c)^3 + 12*a*sin(d*x + c)^2 - 3*a*sin(d*x + c) - 2*a)/sin(d*
x + c)^3)/d
```

maple [A] time = 0.34, size = 159, normalized size = 1.87

$$-\frac{a\left(\cos^6(dx+c)\right)}{2d\sin(dx+c)^2}-\frac{a\left(\cos^4(dx+c)\right)}{2d}-\frac{a\left(\cos^2(dx+c)\right)}{d}-\frac{2a\ln(\sin(dx+c))}{d}-\frac{a\left(\cos^6(dx+c)\right)}{3d\sin(dx+c)^3}+\frac{a\left(\cos^6(dx+c\right.}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^6-1/2*a*cos(d*x+c)^4/d-a*cos(d*x+c)^2/d-2*
a*ln(sin(d*x+c))/d-1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^6+1/d*a/sin(d*x+c)*cos(d
*x+c)^6+8/3*a*sin(d*x+c)/d+1/d*cos(d*x+c)^4*sin(d*x+c)*a+4/3/d*a*sin(d*x+c)
*cos(d*x+c)^2
```

maxima [A] time = 0.50, size = 69, normalized size = 0.81

$$\frac{3\,a\sin(dx+c)^2-12\,a\log(\sin(dx+c))+6\,a\sin(dx+c)+\frac{12\,a\sin(dx+c)^2-3\,a\sin(dx+c)-2\,a}{\sin(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*sin(d*x + c)^2 - 12*a*log(sin(d*x + c)) + 6*a*sin(d*x + c) + (12*a
*sin(d*x + c)^2 - 3*a*sin(d*x + c) - 2*a)/sin(d*x + c)^3)/d
```

mupad [B] time = 8.79, size = 218, normalized size = 2.56

$$\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{2\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{2\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{23\,a\,\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] (7*a*tan(c/2 + (d*x)/2))/(8*d) + (2*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a
*tan(c/2 + (d*x)/2)^2)/(8*d) - (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (2*a*log(t
an(c/2 + (d*x)/2)))/d + ((19*a*tan(c/2 + (d*x)/2)^2)/3 - a*tan(c/2 + (d*x)/
2) - a/3 - 2*a*tan(c/2 + (d*x)/2)^3 + (89*a*tan(c/2 + (d*x)/2)^4)/3 + 15*a*
tan(c/2 + (d*x)/2)^5 + 23*a*tan(c/2 + (d*x)/2)^6)/(d*(8*tan(c/2 + (d*x)/2)^
3 + 16*tan(c/2 + (d*x)/2)^5 + 8*tan(c/2 + (d*x)/2)^7))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.503 $\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}+\frac{2a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] 2*a*csc(d*x+c)/d+a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/d
+a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 88}

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}+\frac{2a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (a*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^5}{x^5}+\frac{a^4}{x^4}-\frac{2a^3}{x^3}-\frac{2a^2}{x^2}+\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{2a\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\log}{}$$

Mathematica [A] time = 0.20, size = 87, normalized size = 1.07

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc(c+dx)}{d}+\frac{a\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\text{co}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (a*(2*Cot[c + d*x]^2 - Co
t[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*d) + (a*Sin[c
 + d*x])/d
```

fricas [A] time = 0.83, size = 110, normalized size = 1.36

$$-\frac{12\,a\cos\left(dx+c\right)^2-12\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\left(3\,a\cos\left(dx+c\right)^4-12\,a\,\text{c}\right.}{12\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)
*log(1/2*sin(d*x + c)) - 4*(3*a*cos(d*x + c)^4 - 12*a*cos(d*x + c)^2 + 8*a)
*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.21, size = 82, normalized size = 1.01

$$\frac{12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+12\,a\sin\left(dx+c\right)-\frac{25\,a\sin\left(dx+c\right)^4-24\,a\sin\left(dx+c\right)^3-12\,a\sin\left(dx+c\right)^2+4\,a\sin\left(dx+c\right)+3\,a}{\sin\left(dx+c\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(sin(d*x + c))) + 12*a*sin(d*x + c) - (25*a*sin(d*x + c)^
4 - 24*a*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*a*sin(d*x + c) + 3*a)/sin
(d*x + c)^4)/d
```

maple [A] time = 0.32, size = 136, normalized size = 1.68

$$-\frac{a\left(\cos^{6}\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)^{3}}+\frac{a\left(\cos^{6}\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}+\frac{8a\sin\left(dx+c\right)}{3d}+\frac{\left(\cos^{4}\left(dx+c\right)\right)\sin\left(dx+c\right)a}{d}+\frac{4\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^6+1/d*a/sin(d*x+c)*cos(d*x+c)^6+8/3*a*sin(
d*x+c)/d+1/d*cos(d*x+c)^4*sin(d*x+c)*a+4/3/d*a*sin(d*x+c)*cos(d*x+c)^2-1/4/
d*a*cot(d*x+c)^4+1/2*a*cot(d*x+c)^2/d+a*ln(sin(d*x+c))/d
```

maxima [A] time = 0.79, size = 69, normalized size = 0.85

$$\frac{12\,a\log\left(\sin\left(dx+c\right)\right)+12\,a\sin\left(dx+c\right)+\frac{24\,a\sin\left(dx+c\right)^{3}+12\,a\sin\left(dx+c\right)^{2}-4\,a\sin\left(dx+c\right)-3\,a}{\sin\left(dx+c\right)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*a*log(sin(d*x + c)) + 12*a*sin(d*x + c) + (24*a*sin(d*x + c)^3 + 1
2*a*sin(d*x + c)^2 - 4*a*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d
```

mupad [B] time = 9.03, size = 207, normalized size = 2.56

$$\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{46\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\frac{40\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{3}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{4}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{a}{4}}{d\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\right)}-\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (7*a*tan(c/2 + (d*x)/2))/(8*d) + ((11*a*tan(c/2 + (d*x)/2)^2)/4 - (2*a*tan(
c/2 + (d*x)/2))/3 - a/4 + (40*a*tan(c/2 + (d*x)/2)^3)/3 + 3*a*tan(c/2 + (d*
x)/2)^4 + 46*a*tan(c/2 + (d*x)/2)^5)/(d*(16*tan(c/2 + (d*x)/2)^4 + 16*tan(c
/2 + (d*x)/2)^6)) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (3*a*tan(c/2 + (d
```

```
*x)/2)^2)/(16*d) - (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2)^
4)/(64*d) + (a*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.504 $\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^4(c+dx)}{4d}+\frac{2a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] -a*csc(d*x+c)/d+a*csc(d*x+c)^2/d+2/3*a*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/d-
1/5*a*csc(d*x+c)^5/d+a*ln(sin(d*x+c))/d
```

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2836, 12, 88}

$$-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^4(c+dx)}{4d}+\frac{2a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Csc[c + d*x]^2)/d + (2*a*Csc[c + d*x]^3)/(3*d) -
 (a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) + (a*Log[Sin[c + d*x]]
)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2(a+x)^3}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{a^4}{x^5}-\frac{2a^3}{x^4}-\frac{2a^2}{x^3}+\frac{a}{x^2}+\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c}{4d}$$

Mathematica [A] time = 0.17, size = 92, normalized size = 1.07

$$-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{a\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (2*a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*
d) + (a*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Ta
n[c + d*x]]))/(4*d)
```

fricas [A] time = 0.77, size = 124, normalized size = 1.44

$$-\frac{60\,a\cos(dx+c)^4-80\,a\cos(dx+c)^2-60\left(a\cos(dx+c)^4-2\,a\cos(dx+c)^2+a\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx}{60\left(d\cos(dx+c)^4-2\,d\cos(dx+c)^2+d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(60*a*cos(d*x + c)^4 - 80*a*cos(d*x + c)^2 - 60*(a*cos(d*x + c)^4 - 2
*a*cos(d*x + c)^2 + a)*log(1/2*sin(d*x + c))*sin(d*x + c) + 15*(4*a*cos(d*x
 + c)^2 - 3*a)*sin(d*x + c) + 32*a)/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2
 + d)*sin(d*x + c))
```

giac [A] time = 0.25, size = 84, normalized size = 0.98

$$\frac{60\,a\log(|\sin(dx+c)|)-\frac{137\,a\sin(dx+c)^5+60\,a\sin(dx+c)^4-60\,a\sin(dx+c)^3-40\,a\sin(dx+c)^2+15\,a\sin(dx+c)+12\,a}{\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(sin(d*x + c))) - (137*a*sin(d*x + c)^5 + 60*a*sin(d*x +
c)^4 - 60*a*sin(d*x + c)^3 - 40*a*sin(d*x + c)^2 + 15*a*sin(d*x + c) + 12*a
)/sin(d*x + c)^5)/d
```

maple [A] time = 0.32, size = 160, normalized size = 1.86

$$-\frac{a\left(\cot^4(dx+c)\right)}{4d}+\frac{a\left(\cot^2(dx+c)\right)}{2d}+\frac{a\ln(\sin(dx+c))}{d}-\frac{a\left(\cos^6(dx+c)\right)}{5d\sin(dx+c)^5}+\frac{a\left(\cos^6(dx+c)\right)}{15d\sin(dx+c)^3}-\frac{a\left(\cos^6(dx+c)\right)}{5d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a*cot(d*x+c)^4+1/2*a*cot(d*x+c)^2/d+a*ln(sin(d*x+c))/d-1/5/d*a/sin(d
*x+c)^5*cos(d*x+c)^6+1/15/d*a/sin(d*x+c)^3*cos(d*x+c)^6-1/5/d*a/sin(d*x+c)*
cos(d*x+c)^6-8/15*a*sin(d*x+c)/d-1/5/d*cos(d*x+c)^4*sin(d*x+c)*a-4/15/d*a*s
in(d*x+c)*cos(d*x+c)^2
```

maxima [A] time = 0.68, size = 72, normalized size = 0.84

$$\frac{60\,a\log(\sin(dx+c))-\frac{60\,a\sin(dx+c)^4-60\,a\sin(dx+c)^3-40\,a\sin(dx+c)^2+15\,a\sin(dx+c)+12\,a}{\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*a*log(sin(d*x + c)) - (60*a*sin(d*x + c)^4 - 60*a*sin(d*x + c)^3 -
 40*a*sin(d*x + c)^2 + 15*a*sin(d*x + c) + 12*a)/sin(d*x + c)^5)/d
```

mupad [B] time = 8.90, size = 193, normalized size = 2.24

$$\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{a\tan\left(\frac{c}{2}+\right.}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (3*a*tan(c/2 + (d*x)/2)^2)/(16*d) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (
5*a*tan(c/2 + (d*x)/2))/(16*d) + (5*a*tan(c/2 + (d*x)/2)^3)/(96*d) - (a*tan
```

```
(c/2 + (d*x)/2)^4)/(64*d) - (a*tan(c/2 + (d*x)/2)^5)/(160*d) + (a*log(tan(c
/2 + (d*x)/2)))/d - (cot(c/2 + (d*x)/2)^5*(a/5 + (a*tan(c/2 + (d*x)/2))/2 -
 (5*a*tan(c/2 + (d*x)/2)^2)/3 - 6*a*tan(c/2 + (d*x)/2)^3 + 10*a*tan(c/2 + (
d*x)/2)^4))/(32*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.505 $\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=61

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}$$

```
[Out] -1/6*a*cot(d*x+c)^6/d-a*csc(d*x+c)/d+2/3*a*csc(d*x+c)^3/d-1/5*a*csc(d*x+c)^
5/d
```

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2607, 30, 2606, 194}

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Csc[c + d*x])/d + (2*a*Csc[c + d*x]^3)/(3*d)
 - (a*Csc[c + d*x]^5)/(5*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc(c+dx)\,dx + a\int \cot^5(c+dx)\csc^2(c+a$$

$$= -\frac{a\,\text{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(-1+x^2\right)^2}{d}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(c+dx}{d}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\csc(c+dx)}{d} + \frac{2a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c}{5a}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.00

$$-\frac{a\cot^6(c+dx)}{6d} - \frac{a\csc^5(c+dx)}{5d} + \frac{2a\csc^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cot[c + d*x]^6)/d - (a*Csc[c + d*x])/d + (2*a*Csc[c + d*x]^3)/(3*d)
 - (a*Csc[c + d*x]^5)/(5*d)
```

fricas [A] time = 0.65, size = 100, normalized size = 1.64

$$\frac{15\,a\cos\left(dx+c\right)^4 - 15\,a\cos\left(dx+c\right)^2 + 2\left(15\,a\cos\left(dx+c\right)^4 - 20\,a\cos\left(dx+c\right)^2 + 8\,a\right)\sin\left(dx+c\right) + 5\,a}{30\left(d\cos\left(dx+c\right)^6 - 3\,d\cos\left(dx+c\right)^4 + 3\,d\cos\left(dx+c\right)^2 - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*a*cos(d*x + c)^4 - 15*a*cos(d*x + c)^2 + 2*(15*a*cos(d*x + c)^4 -
20*a*cos(d*x + c)^2 + 8*a)*sin(d*x + c) + 5*a)/(d*cos(d*x + c)^6 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.25, size = 70, normalized size = 1.15

$$-\frac{30\,a\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}-15\,a\sin\left(dx+c\right)^{2}+6\,a\sin\left(dx+c\right)+5\,a}{30\,d\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(30*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 15
*a*sin(d*x + c)^2 + 6*a*sin(d*x + c) + 5*a)/(d*sin(d*x + c)^6)
```

maple [A] time = 0.34, size = 110, normalized size = 1.80

$$\frac{a\left(-\frac{\cos^{6}(dx+c)}{5\sin(dx+c)^{5}}+\frac{\cos^{6}(dx+c)}{15\sin(dx+c)^{3}}-\frac{\cos^{6}(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)-\frac{a\left(\cos^{6}(dx+c)\right)}{6\sin(dx+c)^{6}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/s
in(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-
1/6*a/sin(d*x+c)^6*cos(d*x+c)^6)
```

maxima [A] time = 0.71, size = 70, normalized size = 1.15

$$-\frac{30\,a\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}-15\,a\sin\left(dx+c\right)^{2}+6\,a\sin\left(dx+c\right)+5\,a}{30\,d\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(30*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 15
*a*sin(d*x + c)^2 + 6*a*sin(d*x + c) + 5*a)/(d*sin(d*x + c)^6)
```

mupad [B] time = 8.88, size = 69, normalized size = 1.13

$$-\frac{a\sin\left(c+d\,x\right)^{5}+\frac{a\sin(c+d\,x)^{4}}{2}-\frac{2\,a\sin(c+d\,x)^{3}}{3}-\frac{a\sin(c+d\,x)^{2}}{2}+\frac{a\,\sin(c+d\,x)}{5}+\frac{a}{6}}{d\sin\left(c+d\,x\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^7,x)
```

```
[Out] -(a/6 + (a*sin(c + d*x))/5 - (a*sin(c + d*x)^2)/2 - (2*a*sin(c + d*x)^3)/3
+ (a*sin(c + d*x)^4)/2 + a*sin(c + d*x)^5)/(d*sin(c + d*x)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.506 $\int \cot^5(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}$$

```
[Out] -1/6*a*cot(d*x+c)^6/d-1/3*a*csc(d*x+c)^3/d+2/5*a*csc(d*x+c)^5/d-1/7*a*csc(d
*x+c)^7/d
```

Rubi [A] time = 0.12, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2606, 270, 2607, 30}

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)
/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^2(c+dx)\,dx + a\int \cot^5(c+dx)\csc^3(c+$$

$$= -\frac{a\,\mathrm{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{d} - \frac{a\,\mathrm{Subst}\left(\int x^2\left(-1+x^2\right.\right.}{d}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\,\mathrm{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \csc(c+d\right.}{d}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\csc^3(c+dx)}{3d} + \frac{2a\csc^5(c+dx)}{5d} - \frac{a\csc^7(}{7}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.00

$$-\frac{a\cot^6(c+dx)}{6d} - \frac{a\csc^7(c+dx)}{7d} + \frac{2a\csc^5(c+dx)}{5d} - \frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cot[c + d*x]^6)/d - (a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)
/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

fricas [A] time = 0.67, size = 106, normalized size = 1.63

$$\frac{70\,a\cos(dx+c)^4 - 56\,a\cos(dx+c)^2 + 35\left(3\,a\cos(dx+c)^4 - 3\,a\cos(dx+c)^2 + a\right)\sin(dx+c) + 16\,a}{210\left(d\cos(dx+c)^6 - 3\,d\cos(dx+c)^4 + 3\,d\cos(dx+c)^2 - d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/210*(70*a*cos(d*x + c)^4 - 56*a*cos(d*x + c)^2 + 35*(3*a*cos(d*x + c)^4 -
 3*a*cos(d*x + c)^2 + a)*sin(d*x + c) + 16*a)/((d*cos(d*x + c)^6 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.25, size = 70, normalized size = 1.08

$$-\frac{105\, a \sin\left(dx + c\right)^{5} + 70\, a \sin\left(dx + c\right)^{4} - 105\, a \sin\left(dx + c\right)^{3} - 84\, a \sin\left(dx + c\right)^{2} + 35\, a \sin\left(dx + c\right) + 30\, a}{210\, d \sin\left(dx + c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/210*(105*a*sin(d*x + c)^5 + 70*a*sin(d*x + c)^4 - 105*a*sin(d*x + c)^3 -
 84*a*sin(d*x + c)^2 + 35*a*sin(d*x + c) + 30*a)/(d*sin(d*x + c)^7)
```

maple [B] time = 0.36, size = 128, normalized size = 1.97

$$\frac{-\frac{a\left(\cos^{6}(dx+c)\right)}{6\sin(dx+c)^{6}} + a\left(-\frac{\cos^{6}(dx+c)}{7\sin(dx+c)^{7}} - \frac{\cos^{6}(dx+c)}{35\sin(dx+c)^{5}} + \frac{\cos^{6}(dx+c)}{105\sin(dx+c)^{3}} - \frac{\cos^{6}(dx+c)}{35\sin(dx+c)} - \frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^8*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/6*a/sin(d*x+c)^6*cos(d*x+c)^6+a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/3
5/sin(d*x+c)^5*cos(d*x+c)^6+1/105/sin(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)
*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.60, size = 70, normalized size = 1.08

$$-\frac{105\, a \sin\left(dx + c\right)^{5} + 70\, a \sin\left(dx + c\right)^{4} - 105\, a \sin\left(dx + c\right)^{3} - 84\, a \sin\left(dx + c\right)^{2} + 35\, a \sin\left(dx + c\right) + 30\, a}{210\, d \sin\left(dx + c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/210*(105*a*sin(d*x + c)^5 + 70*a*sin(d*x + c)^4 - 105*a*sin(d*x + c)^3 -
 84*a*sin(d*x + c)^2 + 35*a*sin(d*x + c) + 30*a)/(d*sin(d*x + c)^7)
```

mupad [B] time = 8.83, size = 70, normalized size = 1.08

$$-\frac{105\, a \sin\left(c + d\, x\right)^{5} + 70\, a \sin\left(c + d\, x\right)^{4} - 105\, a \sin\left(c + d\, x\right)^{3} - 84\, a \sin\left(c + d\, x\right)^{2} + 35\, a\, \sin\left(c + d\, x\right) + 30\, a}{210\, d \sin\left(c + d\, x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^8,x)
```

```
[Out] -(30*a + 35*a*sin(c + d*x) - 84*a*sin(c + d*x)^2 - 105*a*sin(c + d*x)^3 + 7
0*a*sin(c + d*x)^4 + 105*a*sin(c + d*x)^5)/(210*d*sin(c + d*x)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**8*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.507 $\int \cot^5(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}$$

```
[Out] -1/6*a*cot(d*x+c)^6/d-1/8*a*cot(d*x+c)^8/d-1/3*a*csc(d*x+c)^3/d+2/5*a*csc(d
*x+c)^5/d-1/7*a*csc(d*x+c)^7/d
```

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2607, 14, 2606, 270}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Cot[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^3)/(
3*d) + (2*a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^3(c+dx)\,dx + a\int \cot^5(c+dx)\csc^4(c+$$

$$= -\frac{a\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int x\right.}{}$$

$$= -\frac{a\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\cot^8(c+dx)}{8d} - \frac{a\csc^3(c+dx)}{3d} + \frac{2a\csc^5(}{5a}$$

Mathematica [A] time = 0.16, size = 88, normalized size = 1.09

$$-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\left(3\csc^8(c+dx)-8\csc^6(c+dx)+6\csc^4(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(a*Csc[c + d*x]^3)/d + (2*a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)
/(7*d) - (a*(6*Csc[c + d*x]^4 - 8*Csc[c + d*x]^6 + 3*Csc[c + d*x]^8))/(24*d
)
```

fricas [A] time = 0.69, size = 109, normalized size = 1.35

$$-\frac{210\,a\cos\left(dx+c\right)^4-140\,a\cos\left(dx+c\right)^2+8\left(35\,a\cos\left(dx+c\right)^4-28\,a\cos\left(dx+c\right)^2+8\,a\right)\sin\left(dx+c\right)+35\,a}{840\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/840*(210*a*cos(d*x + c)^4 - 140*a*cos(d*x + c)^2 + 8*(35*a*cos(d*x + c)^
4 - 28*a*cos(d*x + c)^2 + 8*a)*sin(d*x + c) + 35*a)/(d*cos(d*x + c)^8 - 4*d
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.24, size = 70, normalized size = 0.86

$$-\frac{280\,a\sin(dx+c)^5+210\,a\sin(dx+c)^4-336\,a\sin(dx+c)^3-280\,a\sin(dx+c)^2+120\,a\sin(dx+c)+105\,a}{840\,d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/840*(280*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 336*a*sin(d*x + c)^3
- 280*a*sin(d*x + c)^2 + 120*a*sin(d*x + c) + 105*a)/(d*sin(d*x + c)^8)
```

maple [B] time = 0.36, size = 148, normalized size = 1.83

$$\frac{a\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{35}\right)+a\left(-\frac{\cos^6(dx+c)}{8\sin(dx+c)^8}-\frac{c}{24}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^9*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/105
/sin(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c
)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/8/sin(d*x+c)^8*cos(d*x+c)^6-1/24/si
n(d*x+c)^6*cos(d*x+c)^6))
```

maxima [A] time = 0.51, size = 70, normalized size = 0.86

$$-\frac{280\,a\sin(dx+c)^5+210\,a\sin(dx+c)^4-336\,a\sin(dx+c)^3-280\,a\sin(dx+c)^2+120\,a\sin(dx+c)+105\,a}{840\,d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*(280*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 336*a*sin(d*x + c)^3
- 280*a*sin(d*x + c)^2 + 120*a*sin(d*x + c) + 105*a)/(d*sin(d*x + c)^8)
```

mupad [B] time = 8.84, size = 70, normalized size = 0.86

$$-\frac{280\,a\sin\left(c+d\,x\right)^5+210\,a\sin\left(c+d\,x\right)^4-336\,a\sin\left(c+d\,x\right)^3-280\,a\sin\left(c+d\,x\right)^2+120\,a\,\sin\left(c+d\,x\right)+105\,a}{840\,d\sin\left(c+d\,x\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^9,x)
```

```
[Out] -(105*a + 120*a*sin(c + d*x) - 280*a*sin(c + d*x)^2 - 336*a*sin(c + d*x)^3
+ 210*a*sin(c + d*x)^4 + 280*a*sin(c + d*x)^5)/(840*d*sin(c + d*x)^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**9*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.508 $\int \cot^5(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^9(c+dx)}{9d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}$$

```
[Out] -1/6*a*cot(d*x+c)^6/d-1/8*a*cot(d*x+c)^8/d-1/5*a*csc(d*x+c)^5/d+2/7*a*csc(d
*x+c)^7/d-1/9*a*csc(d*x+c)^9/d
```

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2606, 270, 2607, 14}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^9(c+dx)}{9d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Cot[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^5)/(
5*d) + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^4(c+dx)\,dx + a\int \cot^5(c+dx)\csc^5(c+$$

$$= -\frac{a\,\text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int x\right.}{}$$

$$= -\frac{a\,\text{Subst}\left(\int \left(x^5+x^7\right)dx, x, -\cot(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(x^4\right.\right.}{}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\cot^8(c+dx)}{8d} - \frac{a\csc^5(c+dx)}{5d} + \frac{2a\csc^7(}{7d}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 1.09

$$-\frac{a\csc^9(c+dx)}{9d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\left(3\csc^8(c+dx)-8\csc^6(c+dx)+6\csc^4(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/5*(a*Csc[c + d*x]^5)/d + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)
/(9*d) - (a*(6*Csc[c + d*x]^4 - 8*Csc[c + d*x]^6 + 3*Csc[c + d*x]^8))/(24*d
)
```

fricas [A] time = 0.70, size = 115, normalized size = 1.42

$$-\frac{504\,a\cos\left(dx+c\right)^4-288\,a\cos\left(dx+c\right)^2+105\left(6\,a\cos\left(dx+c\right)^4-4\,a\cos\left(dx+c\right)^2+a\right)\sin\left(dx+c\right)+64\,a}{2520\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2520*(504*a*cos(d*x + c)^4 - 288*a*cos(d*x + c)^2 + 105*(6*a*cos(d*x + c
)^4 - 4*a*cos(d*x + c)^2 + a)*sin(d*x + c) + 64*a)/((d*cos(d*x + c)^8 - 4*d
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c)
)
```

giac [A] time = 0.24, size = 70, normalized size = 0.86

$$-\frac{630\,a\sin(dx+c)^5+504\,a\sin(dx+c)^4-840\,a\sin(dx+c)^3-720\,a\sin(dx+c)^2+315\,a\sin(dx+c)+280\,a}{2520\,d\sin(dx+c)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2520*(630*a*sin(d*x + c)^5 + 504*a*sin(d*x + c)^4 - 840*a*sin(d*x + c)^3
 - 720*a*sin(d*x + c)^2 + 315*a*sin(d*x + c) + 280*a)/(d*sin(d*x + c)^9)
```

maple [B] time = 0.38, size = 166, normalized size = 2.05

$$\frac{a\left(-\frac{\cos^6(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^6(dx+c)}{24\sin(dx+c)^6}\right)+a\left(-\frac{\cos^6(dx+c)}{9\sin(dx+c)^9}-\frac{\cos^6(dx+c)}{21\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{105\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{315\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{105\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^10*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/8/sin(d*x+c)^8*cos(d*x+c)^6-1/24/sin(d*x+c)^6*cos(d*x+c)^6)+a*(-
1/9/sin(d*x+c)^9*cos(d*x+c)^6-1/21/sin(d*x+c)^7*cos(d*x+c)^6-1/105/sin(d*x+
c)^5*cos(d*x+c)^6+1/315/sin(d*x+c)^3*cos(d*x+c)^6-1/105/sin(d*x+c)*cos(d*x+
c)^6-1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.69, size = 70, normalized size = 0.86

$$-\frac{630\,a\sin(dx+c)^5+504\,a\sin(dx+c)^4-840\,a\sin(dx+c)^3-720\,a\sin(dx+c)^2+315\,a\sin(dx+c)+280\,a}{2520\,d\sin(dx+c)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/2520*(630*a*sin(d*x + c)^5 + 504*a*sin(d*x + c)^4 - 840*a*sin(d*x + c)^3
 - 720*a*sin(d*x + c)^2 + 315*a*sin(d*x + c) + 280*a)/(d*sin(d*x + c)^9)
```

mupad [B] time = 8.88, size = 70, normalized size = 0.86

$$-\frac{\frac{a\sin(c+d\,x)^5}{4}+\frac{a\sin(c+d\,x)^4}{5}-\frac{a\sin(c+d\,x)^3}{3}-\frac{2\,a\sin(c+d\,x)^2}{7}+\frac{a\,\sin(c+d\,x)}{8}+\frac{a}{9}}{d\sin\left(c+d\,x\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^10,x)
```

```
[Out] -(a/9 + (a*sin(c + d*x))/8 - (2*a*sin(c + d*x)^2)/7 - (a*sin(c + d*x)^3)/3
+ (a*sin(c + d*x)^4)/5 + (a*sin(c + d*x)^5)/4)/(d*sin(c + d*x)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**10*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.509 $\int \cot^5(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\csc^{10}(c+dx)}{10d}-\frac{a\csc^9(c+dx)}{9d}+\frac{a\csc^8(c+dx)}{4d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}$$

```
[Out] -1/5*a*csc(d*x+c)^5/d-1/6*a*csc(d*x+c)^6/d+2/7*a*csc(d*x+c)^7/d+1/4*a*csc(d
*x+c)^8/d-1/9*a*csc(d*x+c)^9/d-1/10*a*csc(d*x+c)^10/d
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\csc^{10}(c+dx)}{10d}-\frac{a\csc^9(c+dx)}{9d}+\frac{a\csc^8(c+dx)}{4d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)/(6*d) + (2*a*Csc[c + d*x]^7)
/(7*d) + (a*Csc[c + d*x]^8)/(4*d) - (a*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d
*x]^10)/(10*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^{11}(a-x)^2(a+x)^3}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(\frac{a^5}{x^{11}}+\frac{a^4}{x^{10}}-\frac{2a^3}{x^9}-\frac{2a^2}{x^8}+\frac{a}{x^7}+\frac{1}{x^6}\right)dx, x, a\sin(c+\right.}{d}$$
$$= -\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^6(c+dx)}{6d}+\frac{2a\csc^7(c+dx)}{7d}+\frac{a\csc^8(}{4}$$

Mathematica [A] time = 0.17, size = 88, normalized size = 0.91

$$-\frac{a\csc^9(c+dx)}{9d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\left(6\csc^{10}(c+dx)-15\csc^8(c+dx)+10\csc^6(c+dx)\right)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/5*(a*Csc[c + d*x]^5)/d + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)
/(9*d) - (a*(10*Csc[c + d*x]^6 - 15*Csc[c + d*x]^8 + 6*Csc[c + d*x]^10))/(6
0*d)
```

fricas [A] time = 0.82, size = 122, normalized size = 1.26

$$\frac{210\,a\cos(dx+c)^4-105\,a\cos(dx+c)^2+4\left(63\,a\cos(dx+c)^4-36\,a\cos(dx+c)^2+8\,a\right)\sin(dx+c)+21\,a}{1260\left(d\cos(dx+c)^{10}-5\,d\cos(dx+c)^8+10\,d\cos(dx+c)^6-10\,d\cos(dx+c)^4+5\,d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1260*(210*a*cos(d*x + c)^4 - 105*a*cos(d*x + c)^2 + 4*(63*a*cos(d*x + c)^
4 - 36*a*cos(d*x + c)^2 + 8*a)*sin(d*x + c) + 21*a)/(d*cos(d*x + c)^10 - 5*
d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x
+ c)^2 - d)
```

giac [A] time = 0.26, size = 70, normalized size = 0.72

$$-\frac{252\,a\sin(dx+c)^5+210\,a\sin(dx+c)^4-360\,a\sin(dx+c)^3-315\,a\sin(dx+c)^2+140\,a\sin(dx+c)+126\,a}{1260\,d\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1260*(252*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 360*a*sin(d*x + c)^3
 - 315*a*sin(d*x + c)^2 + 140*a*sin(d*x + c) + 126*a)/(d*sin(d*x + c)^10)
```

maple [B] time = 0.37, size = 184, normalized size = 1.90

$$\frac{a\left(-\frac{\cos^6(dx+c)}{9\sin(dx+c)^9}-\frac{\cos^6(dx+c)}{21\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{105\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{315\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{105\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{105}\right)+a\left(-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^11*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^6-1/21/sin(d*x+c)^7*cos(d*x+c)^6-1/105
/sin(d*x+c)^5*cos(d*x+c)^6+1/315/sin(d*x+c)^3*cos(d*x+c)^6-1/105/sin(d*x+c)
*cos(d*x+c)^6-1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/1
0/sin(d*x+c)^10*cos(d*x+c)^6-1/20/sin(d*x+c)^8*cos(d*x+c)^6-1/60/sin(d*x+c)
^6*cos(d*x+c)^6))
```

maxima [A] time = 0.45, size = 70, normalized size = 0.72

$$-\frac{252\,a\sin(dx+c)^5+210\,a\sin(dx+c)^4-360\,a\sin(dx+c)^3-315\,a\sin(dx+c)^2+140\,a\sin(dx+c)+126\,a}{1260\,d\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/1260*(252*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 360*a*sin(d*x + c)^3
 - 315*a*sin(d*x + c)^2 + 140*a*sin(d*x + c) + 126*a)/(d*sin(d*x + c)^10)
```

mupad [B] time = 8.86, size = 70, normalized size = 0.72

$$-\frac{252\,a\sin(c+d\,x)^5+210\,a\sin(c+d\,x)^4-360\,a\sin(c+d\,x)^3-315\,a\sin(c+d\,x)^2+140\,a\,\sin(c+d\,x)+126}{1260\,d\sin(c+d\,x)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^11,x)
```

```
[Out] -(126*a + 140*a*sin(c + d*x) - 315*a*sin(c + d*x)^2 - 360*a*sin(c + d*x)^3
+ 210*a*sin(c + d*x)^4 + 252*a*sin(c + d*x)^5)/(1260*d*sin(c + d*x)^10)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**11*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.510 $\int \cot^5(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\csc^{11}(c+dx)}{11d}-\frac{a\csc^{10}(c+dx)}{10d}+\frac{2a\csc^9(c+dx)}{9d}+\frac{a\csc^8(c+dx)}{4d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}$$

```
[Out] -1/6*a*csc(d*x+c)^6/d-1/7*a*csc(d*x+c)^7/d+1/4*a*csc(d*x+c)^8/d+2/9*a*csc(d
*x+c)^9/d-1/10*a*csc(d*x+c)^10/d-1/11*a*csc(d*x+c)^11/d
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\csc^{11}(c+dx)}{11d}-\frac{a\csc^{10}(c+dx)}{10d}+\frac{2a\csc^9(c+dx)}{9d}+\frac{a\csc^8(c+dx)}{4d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^6)/(6*d) - (a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c + d*x]^8)/(
4*d) + (2*a*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d*x]^10)/(10*d) - (a*Csc[c +
 d*x]^11)/(11*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^{12}(a-x)^2(a+x)^3}{x^{12}}\,dx,x,a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^{12}}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{a^5}{x^{12}}+\frac{a^4}{x^{11}}-\frac{2a^3}{x^{10}}-\frac{2a^2}{x^9}+\frac{a}{x^8}+\frac{1}{x^7}\right)dx,x,a\sin(c+\right.}{d}$$
$$= -\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{a\csc^8(c+dx)}{4d}+\frac{2a\csc^9(}{9}$$

Mathematica [A] time = 0.19, size = 88, normalized size = 0.91

$$-\frac{a\csc^{11}(c+dx)}{11d}+\frac{2a\csc^9(c+dx)}{9d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a\left(6\csc^{10}(c+dx)-15\csc^8(c+dx)+10\csc^6(c+dx)\right)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7*(a*Csc[c + d*x]^7)/d + (2*a*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d*x]^11
)/(11*d) - (a*(10*Csc[c + d*x]^6 - 15*Csc[c + d*x]^8 + 6*Csc[c + d*x]^10))/
(60*d)
```

fricas [A] time = 0.68, size = 128, normalized size = 1.32

$$\frac{1980\,a\cos(dx+c)^4-880\,a\cos(dx+c)^2+231\left(10\,a\cos(dx+c)^4-5\,a\cos(dx+c)^2+a\right)\sin(dx+c)+160}{13860\left(d\cos(dx+c)^{10}-5\,d\cos(dx+c)^8+10\,d\cos(dx+c)^6-10\,d\cos(dx+c)^4+5\,d\cos(dx+c)^2-d\right)\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/13860*(1980*a*cos(d*x + c)^4 - 880*a*cos(d*x + c)^2 + 231*(10*a*cos(d*x +
 c)^4 - 5*a*cos(d*x + c)^2 + a)*sin(d*x + c) + 160*a)/((d*cos(d*x + c)^10 -
 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d
*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.25, size = 70, normalized size = 0.72

$$-\frac{2310\,a\sin(dx+c)^5+1980\,a\sin(dx+c)^4-3465\,a\sin(dx+c)^3-3080\,a\sin(dx+c)^2+1386\,a\sin(dx+c)+1}{13860\,d\sin(dx+c)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/13860*(2310*a*sin(d*x + c)^5 + 1980*a*sin(d*x + c)^4 - 3465*a*sin(d*x +
c)^3 - 3080*a*sin(d*x + c)^2 + 1386*a*sin(d*x + c) + 1260*a)/(d*sin(d*x + c
)^11)
```

maple [B] time = 0.29, size = 202, normalized size = 2.08

$$\frac{a\left(-\frac{\cos^6(dx+c)}{10\sin(dx+c)^{10}}-\frac{\cos^6(dx+c)}{20\sin(dx+c)^{8}}-\frac{\cos^6(dx+c)}{60\sin(dx+c)^{6}}\right)+a\left(-\frac{\cos^6(dx+c)}{11\sin(dx+c)^{11}}-\frac{5\left(\cos^6(dx+c)\right)}{99\sin(dx+c)^{9}}-\frac{5\left(\cos^6(dx+c)\right)}{231\sin(dx+c)^{7}}-\frac{\cos^6(dx+c)}{231\sin(dx+c)^{5}}+\frac{\cos}{693\,\mathrm{s}}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^12*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/10/sin(d*x+c)^10*cos(d*x+c)^6-1/20/sin(d*x+c)^8*cos(d*x+c)^6-1/6
0/sin(d*x+c)^6*cos(d*x+c)^6)+a*(-1/11/sin(d*x+c)^11*cos(d*x+c)^6-5/99/sin(d
*x+c)^9*cos(d*x+c)^6-5/231/sin(d*x+c)^7*cos(d*x+c)^6-1/231/sin(d*x+c)^5*cos
(d*x+c)^6+1/693/sin(d*x+c)^3*cos(d*x+c)^6-1/231/sin(d*x+c)*cos(d*x+c)^6-1/2
31*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.51, size = 70, normalized size = 0.72

$$-\frac{2310\,a\sin\left(dx+c\right)^{5}+1980\,a\sin\left(dx+c\right)^{4}-3465\,a\sin\left(dx+c\right)^{3}-3080\,a\sin\left(dx+c\right)^{2}+1386\,a\sin\left(dx+c\right)+}{13860\,d\sin\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/13860*(2310*a*sin(d*x + c)^5 + 1980*a*sin(d*x + c)^4 - 3465*a*sin(d*x +
c)^3 - 3080*a*sin(d*x + c)^2 + 1386*a*sin(d*x + c) + 1260*a)/(d*sin(d*x + c
)^11)
```

mupad [B] time = 8.98, size = 70, normalized size = 0.72

$$-\frac{\frac{a\sin(c+d\,x)^5}{6}+\frac{a\sin(c+d\,x)^4}{7}-\frac{a\sin(c+d\,x)^3}{4}-\frac{2\,a\sin(c+d\,x)^2}{9}+\frac{a\,\sin(c+d\,x)}{10}+\frac{a}{11}}{d\sin\left(c+d\,x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^12,x)
```

```
[Out] -(a/11 + (a*sin(c + d*x))/10 - (2*a*sin(c + d*x)^2)/9 - (a*sin(c + d*x)^3)/
4 + (a*sin(c + d*x)^4)/7 + (a*sin(c + d*x)^5)/6)/(d*sin(c + d*x)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**12*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.511 $\int \cos^5(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=127

$$\frac{a^2\sin^{10}(c+dx)}{10d}+\frac{2a^2\sin^9(c+dx)}{9d}-\frac{a^2\sin^8(c+dx)}{8d}-\frac{4a^2\sin^7(c+dx)}{7d}-\frac{a^2\sin^6(c+dx)}{6d}+\frac{2a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin}{}$$

```
[Out] 1/4*a^2*sin(d*x+c)^4/d+2/5*a^2*sin(d*x+c)^5/d-1/6*a^2*sin(d*x+c)^6/d-4/7*a^
2*sin(d*x+c)^7/d-1/8*a^2*sin(d*x+c)^8/d+2/9*a^2*sin(d*x+c)^9/d+1/10*a^2*sin
(d*x+c)^10/d
```

Rubi [A] time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^2\sin^{10}(c+dx)}{10d}+\frac{2a^2\sin^9(c+dx)}{9d}-\frac{a^2\sin^8(c+dx)}{8d}-\frac{4a^2\sin^7(c+dx)}{7d}-\frac{a^2\sin^6(c+dx)}{6d}+\frac{2a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^4)/(4*d) + (2*a^2*Sin[c + d*x]^5)/(5*d) - (a^2*Sin[c + d*
x]^6)/(6*d) - (4*a^2*Sin[c + d*x]^7)/(7*d) - (a^2*Sin[c + d*x]^8)/(8*d) + (
2*a^2*Sin[c + d*x]^9)/(9*d) + (a^2*Sin[c + d*x]^10)/(10*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^3(a+x)^4}{a^3}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^3(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= \frac{\text{Subst}\left(\int \left(a^6x^3+2a^5x^4-a^4x^5-4a^3x^6-a^2x^7+2ax^8+x^9\right)\right.}{a^8d}$$
$$= \frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^6(c+dx)}{6d}-\frac{4a^2}{}$$

Mathematica [A] time = 0.79, size = 110, normalized size = 0.87

$$-\frac{a^2(-15120\sin(c+dx)+3360\sin(3(c+dx))+2016\sin(5(c+dx))-360\sin(7(c+dx))-280\sin(9(c+dx))+}{32}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/322560*(a^2*(-2625 + 10710*Cos[2*(c + d*x)] + 1260*Cos[4*(c + d*x)] - 13
65*Cos[6*(c + d*x)] - 315*Cos[8*(c + d*x)] + 63*Cos[10*(c + d*x)] - 15120*S
in[c + d*x] + 3360*Sin[3*(c + d*x)] + 2016*Sin[5*(c + d*x)] - 360*Sin[7*(c
+ d*x)] - 280*Sin[9*(c + d*x)]))/d
```

fricas [A] time = 0.82, size = 111, normalized size = 0.87

$$-\frac{252\,a^2\cos(dx+c)^{10}-945\,a^2\cos(dx+c)^8+840\,a^2\cos(dx+c)^6-16\left(35\,a^2\cos(dx+c)^8-50\,a^2\cos(dx+c)^6\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2520*(252*a^2*cos(d*x + c)^10 - 945*a^2*cos(d*x + c)^8 + 840*a^2*cos(d*x
 + c)^6 - 16*(35*a^2*cos(d*x + c)^8 - 50*a^2*cos(d*x + c)^6 + 3*a^2*cos(d*x
 + c)^4 + 4*a^2*cos(d*x + c)^2 + 8*a^2)*sin(d*x + c))/d
```

giac [A] time = 0.33, size = 168, normalized size = 1.32

$$-\frac{a^2\cos(10\,dx+10\,c)}{5120\,d}+\frac{a^2\cos(8\,dx+8\,c)}{1024\,d}+\frac{13\,a^2\cos(6\,dx+6\,c)}{3072\,d}-\frac{a^2\cos(4\,dx+4\,c)}{256\,d}-\frac{17\,a^2\cos(2\,dx+2\,c)}{512\,d}+\frac{a^2\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/5120*a^2*cos(10*d*x + 10*c)/d + 1/1024*a^2*cos(8*d*x + 8*c)/d + 13/3072*
a^2*cos(6*d*x + 6*c)/d - 1/256*a^2*cos(4*d*x + 4*c)/d - 17/512*a^2*cos(2*d*
x + 2*c)/d + 1/1152*a^2*sin(9*d*x + 9*c)/d + 1/896*a^2*sin(7*d*x + 7*c)/d -
 1/160*a^2*sin(5*d*x + 5*c)/d - 1/96*a^2*sin(3*d*x + 3*c)/d + 3/64*a^2*sin(
d*x + c)/d
```

maple [A] time = 0.17, size = 158, normalized size = 1.24

$$\frac{a^2\left(-\frac{(\sin^4(dx+c))(\cos^6(dx+c))}{10}-\frac{(\sin^2(dx+c))(\cos^6(dx+c))}{20}-\frac{(\cos^6(dx+c))}{60}\right)+2a^2\left(-\frac{(\sin^3(dx+c))(\cos^6(dx+c))}{9}-\frac{\sin(dx+c)(\cos^6(dx+c))}{21}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/
60*cos(d*x+c)^6)+2*a^2*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(
d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a^2*(-1/8*si
n(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6))
```

maxima [A] time = 0.51, size = 97, normalized size = 0.76

$$\frac{252\,a^2\sin(dx+c)^{10}+560\,a^2\sin(dx+c)^9-315\,a^2\sin(dx+c)^8-1440\,a^2\sin(dx+c)^7-420\,a^2\sin(dx+c)^6+}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/2520*(252*a^2*sin(d*x + c)^10 + 560*a^2*sin(d*x + c)^9 - 315*a^2*sin(d*x
+ c)^8 - 1440*a^2*sin(d*x + c)^7 - 420*a^2*sin(d*x + c)^6 + 1008*a^2*sin(d*
x + c)^5 + 630*a^2*sin(d*x + c)^4)/d
```

mupad [B] time = 8.74, size = 96, normalized size = 0.76

$$\frac{\frac{a^2\sin(c+d\,x)^{10}}{10}+\frac{2\,a^2\sin(c+d\,x)^9}{9}-\frac{a^2\sin(c+d\,x)^8}{8}-\frac{4\,a^2\sin(c+d\,x)^7}{7}-\frac{a^2\sin(c+d\,x)^6}{6}+\frac{2\,a^2\sin(c+d\,x)^5}{5}+\frac{a^2\sin(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((a^2*sin(c + d*x)^4)/4 + (2*a^2*sin(c + d*x)^5)/5 - (a^2*sin(c + d*x)^6)/6
 - (4*a^2*sin(c + d*x)^7)/7 - (a^2*sin(c + d*x)^8)/8 + (2*a^2*sin(c + d*x)^
9)/9 + (a^2*sin(c + d*x)^10)/10)/d
```

sympy [A] time = 26.15, size = 189, normalized size = 1.49

$$\begin{cases} \frac{16a^2\sin^9(c+dx)}{315d} + \frac{8a^2\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{2a^2\sin^5(c+dx)\cos^4(c+dx)}{5d} - \frac{a^2\sin^4(c+dx)\cos^6(c+dx)}{6d} - \frac{a^2\sin^2(c+dx)\cos^8(c+dx)}{12d} - \frac{a^2\sin \\ x(a\sin(c)+a)^2\sin^3(c)\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((16*a**2*sin(c + d*x)**9/(315*d) + 8*a**2*sin(c + d*x)**7*cos(c +
 d*x)**2/(35*d) + 2*a**2*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) - a**2*sin(c
 + d*x)**4*cos(c + d*x)**6/(6*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**8/(12
*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**6/(6*d) - a**2*cos(c + d*x)**10/(6
0*d) - a**2*cos(c + d*x)**8/(24*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)*
*3*cos(c)**5, True))
```

3.512 $\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=109

$$\frac{(a\sin(c+dx)+a)^9}{9a^7d}-\frac{3(a\sin(c+dx)+a)^8}{4a^6d}+\frac{13(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(a\sin(c+dx)+a)^6}{a^4d}+\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

```
[Out] 4/5*(a+a*sin(d*x+c))^5/a^3/d-2*(a+a*sin(d*x+c))^6/a^4/d+13/7*(a+a*sin(d*x+c
))^7/a^5/d-3/4*(a+a*sin(d*x+c))^8/a^6/d+1/9*(a+a*sin(d*x+c))^9/a^7/d
```

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{(a\sin(c+dx)+a)^9}{9a^7d}-\frac{3(a\sin(c+dx)+a)^8}{4a^6d}+\frac{13(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(a\sin(c+dx)+a)^6}{a^4d}+\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^5)/(5*a^3*d) - (2*(a + a*Sin[c + d*x])^6)/(a^4*d) +
 (13*(a + a*Sin[c + d*x])^7)/(7*a^5*d) - (3*(a + a*Sin[c + d*x])^8)/(4*a^6*
d) + (a + a*Sin[c + d*x])^9/(9*a^7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2(a+x)^4}{a^2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^2(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^4(a+x)^4-12a^3(a+x)^5+13a^2(a+x)^6-6a(a\right.\right.}{a^7d}$$
$$= \frac{4(a+a\sin(c+dx))^5}{5a^3d} - \frac{2(a+a\sin(c+dx))^6}{a^4d} + \frac{13(a+a\sin}{7a^5}$$

Mathematica [A] time = 0.72, size = 99, normalized size = 0.91

$$-\frac{a^2(-16380\sin(c+dx)+1680\sin(3(c+dx))+2016\sin(5(c+dx))+270\sin(7(c+dx))-70\sin(9(c+dx))+7}{161280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/161280*(a^2*(7560*Cos[2*(c + d*x)] + 1260*Cos[4*(c + d*x)] - 840*Cos[6*(
c + d*x)] - 315*Cos[8*(c + d*x)] - 16380*Sin[c + d*x] + 1680*Sin[3*(c + d*x
)] + 2016*Sin[5*(c + d*x)] + 270*Sin[7*(c + d*x)] - 70*Sin[9*(c + d*x)]))/d
```

fricas [A] time = 0.79, size = 98, normalized size = 0.90

$$\frac{315\,a^2\cos(dx+c)^8-420\,a^2\cos(dx+c)^6+4\left(35\,a^2\cos(dx+c)^8-95\,a^2\cos(dx+c)^6+12\,a^2\cos(dx+c)^4+16\right.}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/1260*(315*a^2*cos(d*x + c)^8 - 420*a^2*cos(d*x + c)^6 + 4*(35*a^2*cos(d*x
 + c)^8 - 95*a^2*cos(d*x + c)^6 + 12*a^2*cos(d*x + c)^4 + 16*a^2*cos(d*x +
c)^2 + 32*a^2)*sin(d*x + c))/d
```

giac [A] time = 0.28, size = 151, normalized size = 1.39

$$\frac{a^2\cos(8\,dx+8\,c)}{512\,d}+\frac{a^2\cos(6\,dx+6\,c)}{192\,d}-\frac{a^2\cos(4\,dx+4\,c)}{128\,d}-\frac{3\,a^2\cos(2\,dx+2\,c)}{64\,d}+\frac{a^2\sin(9\,dx+9\,c)}{2304\,d}-\frac{3\,a^2\sin(7\,d}{1792}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/512*a^2*cos(8*d*x + 8*c)/d + 1/192*a^2*cos(6*d*x + 6*c)/d - 1/128*a^2*cos
(4*d*x + 4*c)/d - 3/64*a^2*cos(2*d*x + 2*c)/d + 1/2304*a^2*sin(9*d*x + 9*c)
/d - 3/1792*a^2*sin(7*d*x + 7*c)/d - 1/80*a^2*sin(5*d*x + 5*c)/d - 1/96*a^2
*sin(3*d*x + 3*c)/d + 13/128*a^2*sin(d*x + c)/d
```

maple [A] time = 0.27, size = 156, normalized size = 1.43

$$\frac{a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{105}\right)+2a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+2*a^2*(-1/8*sin(d*x+c)^2*c
os(d*x+c)^6-1/24*cos(d*x+c)^6)+a^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+
cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.62, size = 97, normalized size = 0.89

$$\frac{140\,a^2\sin(dx+c)^9+315\,a^2\sin(dx+c)^8-180\,a^2\sin(dx+c)^7-840\,a^2\sin(dx+c)^6-252\,a^2\sin(dx+c)^5+6}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/1260*(140*a^2*sin(d*x + c)^9 + 315*a^2*sin(d*x + c)^8 - 180*a^2*sin(d*x +
 c)^7 - 840*a^2*sin(d*x + c)^6 - 252*a^2*sin(d*x + c)^5 + 630*a^2*sin(d*x +
 c)^4 + 420*a^2*sin(d*x + c)^3)/d
```

mupad [B] time = 8.76, size = 96, normalized size = 0.88

$$\frac{\frac{a^2\sin(c+d\,x)^9}{9}+\frac{a^2\sin(c+d\,x)^8}{4}-\frac{a^2\sin(c+d\,x)^7}{7}-\frac{2\,a^2\sin(c+d\,x)^6}{3}-\frac{a^2\sin(c+d\,x)^5}{5}+\frac{a^2\sin(c+d\,x)^4}{2}+\frac{a^2\sin(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((a^2*sin(c + d*x)^3)/3 + (a^2*sin(c + d*x)^4)/2 - (a^2*sin(c + d*x)^5)/5 -
 (2*a^2*sin(c + d*x)^6)/3 - (a^2*sin(c + d*x)^7)/7 + (a^2*sin(c + d*x)^8)/4
 + (a^2*sin(c + d*x)^9)/9)/d
```

sympy [A] time = 16.76, size = 190, normalized size = 1.74

$$\begin{cases} \frac{8a^2\sin^9(c+dx)}{315d} + \frac{4a^2\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{8a^2\sin^7(c+dx)}{105d} + \frac{a^2\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{4a^2\sin^5(c+dx)\cos^2(c+dx)}{15d} + \frac{a^2\sin^3(c+dx)\cos^4(c+dx)}{3d} \\ x(a\sin(c)+a)^2\sin^2(c)\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((8*a**2*sin(c + d*x)**9/(315*d) + 4*a**2*sin(c + d*x)**7*cos(c +
d*x)**2/(35*d) + 8*a**2*sin(c + d*x)**7/(105*d) + a**2*sin(c + d*x)**5*cos(
c + d*x)**4/(5*d) + 4*a**2*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) + a**2*si
n(c + d*x)**3*cos(c + d*x)**4/(3*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**6/
(3*d) - a**2*cos(c + d*x)**8/(12*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)
**2*cos(c)**5, True))
```

3.513 $\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=89

$$\frac{(a\sin(c+dx)+a)^8}{8a^6d}-\frac{5(a\sin(c+dx)+a)^7}{7a^5d}+\frac{4(a\sin(c+dx)+a)^6}{3a^4d}-\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

```
[Out] -4/5*(a+a*sin(d*x+c))^5/a^3/d+4/3*(a+a*sin(d*x+c))^6/a^4/d-5/7*(a+a*sin(d*x
+c))^7/a^5/d+1/8*(a+a*sin(d*x+c))^8/a^6/d
```

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 77}

$$\frac{(a\sin(c+dx)+a)^8}{8a^6d}-\frac{5(a\sin(c+dx)+a)^7}{7a^5d}+\frac{4(a\sin(c+dx)+a)^6}{3a^4d}-\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-4*(a + a*Sin[c + d*x])^5)/(5*a^3*d) + (4*(a + a*Sin[c + d*x])^6)/(3*a^4*d
) - (5*(a + a*Sin[c + d*x])^7)/(7*a^5*d) + (a + a*Sin[c + d*x])^8/(8*a^6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x(a+x)^4}{a}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^3(a+x)^4+8a^2(a+x)^5-5a(a+x)^6+(a+x)\right.\right.}{a^6d}$$
$$= -\frac{4(a+a\sin(c+dx))^5}{5a^3d}+\frac{4(a+a\sin(c+dx))^6}{3a^4d}-\frac{5(a+a\sin}{7a^5}$$

Mathematica [A] time = 0.32, size = 90, normalized size = 1.01

$$-\frac{a^2(-16800\sin(c+dx)+1120\sin(3(c+dx))+2016\sin(5(c+dx))+480\sin(7(c+dx))+10920\cos(2(c+dx))}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/107520*(a^2*(-2590 + 10920*Cos[2*(c + d*x)] + 3780*Cos[4*(c + d*x)] + 28
0*Cos[6*(c + d*x)] - 105*Cos[8*(c + d*x)] - 16800*Sin[c + d*x] + 1120*Sin[3
*(c + d*x)] + 2016*Sin[5*(c + d*x)] + 480*Sin[7*(c + d*x)]))/d
```

fricas [A] time = 0.73, size = 85, normalized size = 0.96

$$\frac{105\,a^2\cos(dx+c)^8-280\,a^2\cos(dx+c)^6-16\left(15\,a^2\cos(dx+c)^6-3\,a^2\cos(dx+c)^4-4\,a^2\cos(dx+c)^2-8\,a^2\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*a^2*cos(d*x + c)^8 - 280*a^2*cos(d*x + c)^6 - 16*(15*a^2*cos(d*x
 + c)^6 - 3*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 - 8*a^2)*sin(d*x + c)
)/d
```

giac [A] time = 0.25, size = 134, normalized size = 1.51

$$\frac{a^2\cos(8\,dx+8\,c)}{1024\,d}-\frac{a^2\cos(6\,dx+6\,c)}{384\,d}-\frac{9\,a^2\cos(4\,dx+4\,c)}{256\,d}-\frac{13\,a^2\cos(2\,dx+2\,c)}{128\,d}-\frac{a^2\sin(7\,dx+7\,c)}{224\,d}-\frac{3\,a^2\sin(5}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1024*a^2*cos(8*d*x + 8*c)/d - 1/384*a^2*cos(6*d*x + 6*c)/d - 9/256*a^2*co
s(4*d*x + 4*c)/d - 13/128*a^2*cos(2*d*x + 2*c)/d - 1/224*a^2*sin(7*d*x + 7*
c)/d - 3/160*a^2*sin(5*d*x + 5*c)/d - 1/96*a^2*sin(3*d*x + 3*c)/d + 5/32*a^
2*sin(d*x + c)/d
```

maple [A] time = 0.24, size = 102, normalized size = 1.15

$$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+2a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{a^2\left(\cos^6(\right.}{6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+2*a^2*(-1/7*sin
(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1
/6*a^2*cos(d*x+c)^6)
```

maxima [A] time = 0.48, size = 97, normalized size = 1.09

$$\frac{105\,a^2\sin\left(dx+c\right)^8+240\,a^2\sin\left(dx+c\right)^7-140\,a^2\sin\left(dx+c\right)^6-672\,a^2\sin\left(dx+c\right)^5-210\,a^2\sin\left(dx+c\right)^4+5}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/840*(105*a^2*sin(d*x + c)^8 + 240*a^2*sin(d*x + c)^7 - 140*a^2*sin(d*x +
c)^6 - 672*a^2*sin(d*x + c)^5 - 210*a^2*sin(d*x + c)^4 + 560*a^2*sin(d*x +
c)^3 + 420*a^2*sin(d*x + c)^2)/d
```

mupad [B] time = 8.81, size = 96, normalized size = 1.08

$$\frac{\frac{a^2\sin(c+d\,x)^8}{8}+\frac{2\,a^2\sin(c+d\,x)^7}{7}-\frac{a^2\sin(c+d\,x)^6}{6}-\frac{4\,a^2\sin(c+d\,x)^5}{5}-\frac{a^2\sin(c+d\,x)^4}{4}+\frac{2\,a^2\sin(c+d\,x)^3}{3}+\frac{a^2\sin(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((a^2*sin(c + d*x)^2)/2 + (2*a^2*sin(c + d*x)^3)/3 - (a^2*sin(c + d*x)^4)/4
 - (4*a^2*sin(c + d*x)^5)/5 - (a^2*sin(c + d*x)^6)/6 + (2*a^2*sin(c + d*x)^
7)/7 + (a^2*sin(c + d*x)^8)/8)/d
```

sympy [A] time = 9.88, size = 139, normalized size = 1.56

$$\begin{cases} \frac{16a^2\sin^7(c+dx)}{105d} + \frac{8a^2\sin^5(c+dx)\cos^2(c+dx)}{15d} + \frac{2a^2\sin^3(c+dx)\cos^4(c+dx)}{3d} - \frac{a^2\sin^2(c+dx)\cos^6(c+dx)}{6d} - \frac{a^2\cos^8(c+dx)}{24d} - \frac{a^2\cos^6(c+dx)}{6d} \\ x\,(a\sin(c)+a)^2\sin(c)\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((16*a**2*sin(c + d*x)**7/(105*d) + 8*a**2*sin(c + d*x)**5*cos(c +
 d*x)**2/(15*d) + 2*a**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - a**2*sin(c
 + d*x)**2*cos(c + d*x)**6/(6*d) - a**2*cos(c + d*x)**8/(24*d) - a**2*cos(c
 + d*x)**6/(6*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)*cos(c)**5, True))
```

3.514 $\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=119

$$\frac{a^2\sin^6(c+dx)}{6d}+\frac{2a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^4(c+dx)}{4d}-\frac{4a^2\sin^3(c+dx)}{3d}-\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\log($$

```
[Out] a^2*ln(sin(d*x+c))/d+2*a^2*sin(d*x+c)/d-1/2*a^2*sin(d*x+c)^2/d-4/3*a^2*sin(
d*x+c)^3/d-1/4*a^2*sin(d*x+c)^4/d+2/5*a^2*sin(d*x+c)^5/d+1/6*a^2*sin(d*x+c)
^6/d
```

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^2\sin^6(c+dx)}{6d}+\frac{2a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^4(c+dx)}{4d}-\frac{4a^2\sin^3(c+dx)}{3d}-\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\log($$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(
2*d) - (4*a^2*Sin[c + d*x]^3)/(3*d) - (a^2*Sin[c + d*x]^4)/(4*d) + (2*a^2*S
in[c + d*x]^5)/(5*d) + (a^2*Sin[c + d*x]^6)/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(2a^5+\frac{a^6}{x}-a^4x-4a^3x^2-a^2x^3+2ax^4+x^5\right)dx, x,$$
$$= \frac{a^2\log(\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\sin^2(c+dx)}{2d}-\frac{4a$$

Mathematica [A] time = 0.08, size = 78, normalized size = 0.66

$$\frac{a^2\left(10\sin^6(c+dx)+24\sin^5(c+dx)-15\sin^4(c+dx)-80\sin^3(c+dx)-30\sin^2(c+dx)+120\sin(c+dx)+60}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(60*Log[Sin[c + d*x]] + 120*Sin[c + d*x] - 30*Sin[c + d*x]^2 - 80*Sin[
c + d*x]^3 - 15*Sin[c + d*x]^4 + 24*Sin[c + d*x]^5 + 10*Sin[c + d*x]^6))/(6
0*d)
```

fricas [A] time = 0.76, size = 99, normalized size = 0.83

$$-\frac{10\,a^2\cos(dx+c)^6-15\,a^2\cos(dx+c)^4-30\,a^2\cos(dx+c)^2-60\,a^2\log\left(\frac{1}{2}\sin(dx+c)\right)-8\left(3\,a^2\cos(dx+c)^4}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(10*a^2*cos(d*x + c)^6 - 15*a^2*cos(d*x + c)^4 - 30*a^2*cos(d*x + c)^
2 - 60*a^2*log(1/2*sin(d*x + c)) - 8*(3*a^2*cos(d*x + c)^4 + 4*a^2*cos(d*x
+ c)^2 + 8*a^2)*sin(d*x + c))/d
```

giac [A] time = 0.24, size = 95, normalized size = 0.80

$$\frac{10\,a^2\sin(dx+c)^6+24\,a^2\sin(dx+c)^5-15\,a^2\sin(dx+c)^4-80\,a^2\sin(dx+c)^3-30\,a^2\sin(dx+c)^2+60\,a^2\log}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*a^2*sin(d*x + c)^6 + 24*a^2*sin(d*x + c)^5 - 15*a^2*sin(d*x + c)^4
 - 80*a^2*sin(d*x + c)^3 - 30*a^2*sin(d*x + c)^2 + 60*a^2*log(abs(sin(d*x +
 c))) + 120*a^2*sin(d*x + c))/d
```

maple [A] time = 0.47, size = 122, normalized size = 1.03

$$-\frac{a^{2}\left(\cos^{6}\left(dx+c\right)\right)}{6d}+\frac{16a^{2}\sin\left(dx+c\right)}{15d}+\frac{2\sin\left(dx+c\right)a^{2}\left(\cos^{4}\left(dx+c\right)\right)}{5d}+\frac{8\sin\left(dx+c\right)a^{2}\left(\cos^{2}\left(dx+c\right)\right)}{15d}+\frac{\left(\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/6/d*a^2*cos(d*x+c)^6+16/15*a^2*sin(d*x+c)/d+2/5/d*sin(d*x+c)*a^2*cos(d*x
+c)^4+8/15/d*sin(d*x+c)*a^2*cos(d*x+c)^2+1/4/d*cos(d*x+c)^4*a^2+1/2/d*a^2*c
os(d*x+c)^2+a^2*ln(sin(d*x+c))/d
```

maxima [A] time = 0.46, size = 94, normalized size = 0.79

$$\frac{10\,a^{2}\sin\left(dx+c\right)^{6}+24\,a^{2}\sin\left(dx+c\right)^{5}-15\,a^{2}\sin\left(dx+c\right)^{4}-80\,a^{2}\sin\left(dx+c\right)^{3}-30\,a^{2}\sin\left(dx+c\right)^{2}+60\,a^{2}\,\mathrm{lo}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/60*(10*a^2*sin(d*x + c)^6 + 24*a^2*sin(d*x + c)^5 - 15*a^2*sin(d*x + c)^4
 - 80*a^2*sin(d*x + c)^3 - 30*a^2*sin(d*x + c)^2 + 60*a^2*log(sin(d*x + c))
 + 120*a^2*sin(d*x + c))/d
```

mupad [B] time = 9.07, size = 132, normalized size = 1.11

$$\frac{5\,a^{2}\,\sin\left(c+d\,x\right)}{4\,d}-\frac{a^{2}\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}}\right)}{d}+\frac{a^{2}\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{19\,a^{2}\,\cos\left(2\,c+2\,d\,x\right)}{64\,d}-\frac{a^{2}\,\cos\left(6\,c+6\,d\,x\right)}{192\,d}+\frac{5\,a^{2}\,\sin\left(\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (5*a^2*sin(c + d*x))/(4*d) - (a^2*log(1/cos(c/2 + (d*x)/2)^2))/d + (a^2*log
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (19*a^2*cos(2*c + 2*d*x))/(64*
```

```
d) - (a^2*cos(6*c + 6*d*x))/(192*d) + (5*a^2*sin(3*c + 3*d*x))/(24*d) + (a^
2*sin(5*c + 5*d*x))/(40*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.515 $\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=114

$$\frac{a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin^4(c+dx)}{2d}-\frac{a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin^2(c+dx)}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin}{d}$$

```
[Out] -a^2*csc(d*x+c)/d+2*a^2*ln(sin(d*x+c))/d-a^2*sin(d*x+c)/d-2*a^2*sin(d*x+c)^
2/d-1/3*a^2*sin(d*x+c)^3/d+1/2*a^2*sin(d*x+c)^4/d+1/5*a^2*sin(d*x+c)^5/d
```

Rubi [A] time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin^4(c+dx)}{2d}-\frac{a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin^2(c+dx)}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a^2*Log[Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/
d - (2*a^2*Sin[c + d*x]^2)/d - (a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*
x]^4)/(2*d) + (a^2*Sin[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$

$$= \frac{\text{Subst}\left(\int \left(-a^4+\frac{a^6}{x^2}+\frac{2a^5}{x}-4a^3x-a^2x^2+2ax^3+x^4\right)dx, x\right.}{a^3 d}$$

$$= -\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)}{d}-$$

Mathematica [A] time = 0.05, size = 114, normalized size = 1.00

$$\frac{a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin^4(c+dx)}{2d}-\frac{a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin^2(c+dx)}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a^2*Log[Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/
d - (2*a^2*Sin[c + d*x]^2)/d - (a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*
x]^4)/(2*d) + (a^2*Sin[c + d*x]^5)/(5*d)
```

fricas [A] time = 0.77, size = 118, normalized size = 1.04

$$-\frac{48\,a^2\cos(dx+c)^6-64\,a^2\cos(dx+c)^4-256\,a^2\cos(dx+c)^2-480\,a^2\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+512\,a}{240\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(48*a^2*cos(d*x + c)^6 - 64*a^2*cos(d*x + c)^4 - 256*a^2*cos(d*x + c
)^2 - 480*a^2*log(1/2*sin(d*x + c))*sin(d*x + c) + 512*a^2 - 15*(8*a^2*cos(
d*x + c)^4 + 16*a^2*cos(d*x + c)^2 - 11*a^2)*sin(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.23, size = 107, normalized size = 0.94

$$\frac{6\,a^2\sin(dx+c)^5+15\,a^2\sin(dx+c)^4-10\,a^2\sin(dx+c)^3-60\,a^2\sin(dx+c)^2+60\,a^2\log(|\sin(dx+c)|)-30\,a^2}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*a^2*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4 - 10*a^2*sin(d*x + c)^3
- 60*a^2*sin(d*x + c)^2 + 60*a^2*log(abs(sin(d*x + c))) - 30*a^2*sin(d*x +
c) - 30*(2*a^2*sin(d*x + c) + a^2)/sin(d*x + c))/d
```

maple [A] time = 0.41, size = 130, normalized size = 1.14

$$-\frac{32a^2\sin(dx+c)}{15d}-\frac{4\sin(dx+c)\,a^2\left(\cos^4(dx+c)\right)}{5d}-\frac{16\sin(dx+c)\,a^2\left(\cos^2(dx+c)\right)}{15d}+\frac{\left(\cos^4(dx+c)\right)a^2}{2d}+\frac{a^2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -32/15*a^2*sin(d*x+c)/d-4/5/d*sin(d*x+c)*a^2*cos(d*x+c)^4-16/15/d*sin(d*x+c
)*a^2*cos(d*x+c)^2+1/2/d*cos(d*x+c)^4*a^2+1/d*a^2*cos(d*x+c)^2+2*a^2*ln(sin
(d*x+c))/d-1/d*a^2/sin(d*x+c)*cos(d*x+c)^6
```

maxima [A] time = 0.69, size = 94, normalized size = 0.82

$$\frac{6\,a^2\sin(dx+c)^5+15\,a^2\sin(dx+c)^4-10\,a^2\sin(dx+c)^3-60\,a^2\sin(dx+c)^2+60\,a^2\log(\sin(dx+c))-30\,a}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*(6*a^2*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4 - 10*a^2*sin(d*x + c)^3
- 60*a^2*sin(d*x + c)^2 + 60*a^2*log(sin(d*x + c)) - 30*a^2*sin(d*x + c) -
30*a^2/sin(d*x + c))/d
```

mupad [B] time = 8.95, size = 333, normalized size = 2.92

$$\frac{16\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}-\frac{8\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d}-\frac{16\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{d}+\frac{8\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{d}-\frac{2\,a^2\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}+\frac{2\,a^2\,\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (16*a^2*cos(c/2 + (d*x)/2)^4)/d - (8*a^2*cos(c/2 + (d*x)/2)^2)/d - (16*a^2*
cos(c/2 + (d*x)/2)^6)/d + (8*a^2*cos(c/2 + (d*x)/2)^8)/d - (2*a^2*log(1/cos
(c/2 + (d*x)/2)^2))/d + (2*a^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d - (2*a^2*cos(c/2 + (d*x)/2)^3)/(3*d*sin(c/2 + (d*x)/2)) + (176*a^2*cos(c/
2 + (d*x)/2)^5)/(15*d*sin(c/2 + (d*x)/2)) - (328*a^2*cos(c/2 + (d*x)/2)^7)/
(15*d*sin(c/2 + (d*x)/2)) + (96*a^2*cos(c/2 + (d*x)/2)^9)/(5*d*sin(c/2 + (d
*x)/2)) - (32*a^2*cos(c/2 + (d*x)/2)^11)/(5*d*sin(c/2 + (d*x)/2)) - (5*a^2*
cos(c/2 + (d*x)/2))/(2*d*sin(c/2 + (d*x)/2)) - (a^2*sin(c/2 + (d*x)/2))/(2*
d*cos(c/2 + (d*x)/2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.516 $\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^3(c+dx)}{3d}-\frac{a^2\sin^2(c+dx)}{2d}-\frac{4a^2\sin(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}-\frac{a^2\log(s$$

```
[Out] -2*a^2*csc(d*x+c)/d-1/2*a^2*csc(d*x+c)^2/d-a^2*ln(sin(d*x+c))/d-4*a^2*sin(d
*x+c)/d-1/2*a^2*sin(d*x+c)^2/d+2/3*a^2*sin(d*x+c)^3/d+1/4*a^2*sin(d*x+c)^4/
d
```

Rubi [A] time = 0.12, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^3(c+dx)}{3d}-\frac{a^2\sin^2(c+dx)}{2d}-\frac{4a^2\sin(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}-\frac{a^2\log(s$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - (a^2*Log[Sin[c + d*x
]])/d - (4*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d) + (2*a^2*Sin[c
+ d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$

$$= \frac{\text{Subst}\left(\int \left(-4a^3+\frac{a^6}{x^3}+\frac{2a^5}{x^2}-\frac{a^4}{x}-a^2x+2ax^2+x^3\right)dx, x, a$$

$$= -\frac{2a^2\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{a^2\log(\sin(c+dx))}{d}-$$

Mathematica [A] time = 0.17, size = 76, normalized size = 0.66

$$-\frac{a^2\left(-3\sin^4(c+dx)-8\sin^3(c+dx)+6\sin^2(c+dx)+48\sin(c+dx)+6\csc^2(c+dx)+24\csc(c+dx)+12\log\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/12*(a^2*(24*Csc[c + d*x] + 6*Csc[c + d*x]^2 + 12*Log[Sin[c + d*x]] + 48*
Sin[c + d*x] + 6*Sin[c + d*x]^2 - 8*Sin[c + d*x]^3 - 3*Sin[c + d*x]^4))/d
```

fricas [A] time = 0.74, size = 131, normalized size = 1.13

$$\frac{24\,a^2\cos(dx+c)^6-24\,a^2\cos(dx+c)^4-9\,a^2\cos(dx+c)^2+57\,a^2-96\left(a^2\cos(dx+c)^2-a^2\right)\log\left(\frac{1}{2}\sin(dx+\right.}{96\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/96*(24*a^2*cos(d*x + c)^6 - 24*a^2*cos(d*x + c)^4 - 9*a^2*cos(d*x + c)^2
+ 57*a^2 - 96*(a^2*cos(d*x + c)^2 - a^2)*log(1/2*sin(d*x + c)) - 64*(a^2*co
s(d*x + c)^4 + 4*a^2*cos(d*x + c)^2 - 8*a^2)*sin(d*x + c))/(d*cos(d*x + c)^
2 - d)
```

giac [A] time = 0.26, size = 109, normalized size = 0.94

$$\frac{3\,a^2\sin(dx+c)^4+8\,a^2\sin(dx+c)^3-6\,a^2\sin(dx+c)^2-12\,a^2\log(|\sin(dx+c)|)-48\,a^2\sin(dx+c)+\frac{6\left(3\,a^2\sin\right.}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^2*sin(d*x + c)^4 + 8*a^2*sin(d*x + c)^3 - 6*a^2*sin(d*x + c)^2 -
12*a^2*log(abs(sin(d*x + c))) - 48*a^2*sin(d*x + c) + 6*(3*a^2*sin(d*x + c)
^2 - 4*a^2*sin(d*x + c) - a^2)/sin(d*x + c)^2)/d
```

maple [A] time = 0.47, size = 155, normalized size = 1.34

$$-\frac{\left(\cos^4(dx+c)\right)a^2}{4d}-\frac{a^2\left(\cos^2(dx+c)\right)}{2d}-\frac{a^2\ln(\sin(dx+c))}{d}-\frac{2a^2\left(\cos^6(dx+c)\right)}{d\sin(dx+c)}-\frac{16a^2\sin(dx+c)}{3d}-\frac{2\sin(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*cos(d*x+c)^4*a^2-1/2/d*a^2*cos(d*x+c)^2-a^2*ln(sin(d*x+c))/d-2/d*a^2
/sin(d*x+c)*cos(d*x+c)^6-16/3*a^2*sin(d*x+c)/d-2/d*sin(d*x+c)*a^2*cos(d*x+c
)^4-8/3/d*sin(d*x+c)*a^2*cos(d*x+c)^2-1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^6
```

maxima [A] time = 0.56, size = 93, normalized size = 0.80

$$\frac{3a^2\sin(dx+c)^4+8a^2\sin(dx+c)^3-6a^2\sin(dx+c)^2-12a^2\log(\sin(dx+c))-48a^2\sin(dx+c)-\frac{6\left(4a^2\sin\right.}{\sin(}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(3*a^2*sin(d*x + c)^4 + 8*a^2*sin(d*x + c)^3 - 6*a^2*sin(d*x + c)^2 -
12*a^2*log(sin(d*x + c)) - 48*a^2*sin(d*x + c) - 6*(4*a^2*sin(d*x + c) + a^
2)/sin(d*x + c)^2)/d
```

mupad [B] time = 8.94, size = 297, normalized size = 2.56

$$\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}-\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{36a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\frac{17a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{2}+\frac{272a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3}+2a^2}{d\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a^2*log(tan(c/2 + (d*x)/2)))/d - (
2*a^2*tan(c/2 + (d*x)/2)^2 + 48*a^2*tan(c/2 + (d*x)/2)^3 + 11*a^2*tan(c/2 +
 (d*x)/2)^4 + (296*a^2*tan(c/2 + (d*x)/2)^5)/3 + 2*a^2*tan(c/2 + (d*x)/2)^6
 + (272*a^2*tan(c/2 + (d*x)/2)^7)/3 + (17*a^2*tan(c/2 + (d*x)/2)^8)/2 + 36*
a^2*tan(c/2 + (d*x)/2)^9 + a^2/2 + 4*a^2*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2
+ (d*x)/2)^2 + 16*tan(c/2 + (d*x)/2)^4 + 24*tan(c/2 + (d*x)/2)^6 + 16*tan(c
/2 + (d*x)/2)^8 + 4*tan(c/2 + (d*x)/2)^10)) - (a^2*tan(c/2 + (d*x)/2))/d -
(a^2*tan(c/2 + (d*x)/2)^2)/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.517 $\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=110

$$\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^2(c+dx)}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{d}+\frac{a^2\csc(c+dx)}{d}-\frac{4a^2\log(\sin}{d}$$

```
[Out] a^2*csc(d*x+c)/d-a^2*csc(d*x+c)^2/d-1/3*a^2*csc(d*x+c)^3/d-4*a^2*ln(sin(d*x
+c))/d-a^2*sin(d*x+c)/d+a^2*sin(d*x+c)^2/d+1/3*a^2*sin(d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^2(c+dx)}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{d}+\frac{a^2\csc(c+dx)}{d}-\frac{4a^2\log(\sin}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/d - (a^2*Csc[c + d*x]^3)/(3*d)
- (4*a^2*Log[Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)
/d + (a^2*Sin[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)^4}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2+\frac{a^6}{x^4}+\frac{2a^5}{x^3}-\frac{a^4}{x^2}-\frac{4a^3}{x}+2ax+x^2\right)dx, x, a\sin$$
$$= \frac{a^2\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{4a^2\log$$

Mathematica [A] time = 0.18, size = 74, normalized size = 0.67

$$\frac{a^2\left(\sin^3(c+dx)+3\sin^2(c+dx)-3\sin(c+dx)-\csc^3(c+dx)-3\csc^2(c+dx)+3\csc(c+dx)-12\log(\sin(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(3*Csc[c + d*x] - 3*Csc[c + d*x]^2 - Csc[c + d*x]^3 - 12*Log[Sin[c + d
*x]] - 3*Sin[c + d*x] + 3*Sin[c + d*x]^2 + Sin[c + d*x]^3))/(3*d)
```

fricas [A] time = 0.85, size = 115, normalized size = 1.05

$$\frac{2\,a^2\cos(dx+c)^6-24\left(a^2\cos(dx+c)^2-a^2\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-3\left(2\,a^2\cos(dx+c)^4-3\,a^2\cos(d}{6\left(d\cos(dx+c)^2-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*a^2*cos(d*x + c)^6 - 24*(a^2*cos(d*x + c)^2 - a^2)*log(1/2*sin(d*x +
 c))*sin(d*x + c) - 3*(2*a^2*cos(d*x + c)^4 - 3*a^2*cos(d*x + c)^2 - a^2)*s
in(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.27, size = 107, normalized size = 0.97

$$\frac{a^2\sin(dx+c)^3+3\,a^2\sin(dx+c)^2-12\,a^2\log(|\sin(dx+c)|)-3\,a^2\sin(dx+c)+\frac{22\,a^2\sin(dx+c)^3+3\,a^2\sin(dx+c)^2-3\,a^2 s}{\sin(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(a^2*sin(d*x + c)^3 + 3*a^2*sin(d*x + c)^2 - 12*a^2*log(abs(sin(d*x + c
))) - 3*a^2*sin(d*x + c) + (22*a^2*sin(d*x + c)^3 + 3*a^2*sin(d*x + c)^2 -
3*a^2*sin(d*x + c) - a^2)/sin(d*x + c)^3)/d
```

maple [A] time = 0.42, size = 97, normalized size = 0.88

$$-\frac{a^{2}\left(\cos^{6}\left(dx+c\right)\right)}{d\sin\left(dx+c\right)^{2}}-\frac{\left(\cos^{4}\left(dx+c\right)\right)a^{2}}{d}-\frac{2a^{2}\left(\cos^{2}\left(dx+c\right)\right)}{d}-\frac{4a^{2}\ln\left(\sin\left(dx+c\right)\right)}{d}-\frac{a^{2}\left(\cos^{6}\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/sin(d*x+c)^2*cos(d*x+c)^6-1/d*cos(d*x+c)^4*a^2-2/d*a^2*cos(d*x+c)^
2-4*a^2*ln(sin(d*x+c))/d-1/3/d*a^2/sin(d*x+c)^3*cos(d*x+c)^6
```

maxima [A] time = 0.41, size = 93, normalized size = 0.85

$$\frac{a^{2}\sin\left(dx+c\right)^{3}+3\,a^{2}\sin\left(dx+c\right)^{2}-12\,a^{2}\log\left(\sin\left(dx+c\right)\right)-3\,a^{2}\sin\left(dx+c\right)+\frac{3\,a^{2}\sin(dx+c)^{2}-3\,a^{2}\sin(dx+c)-a^{2}}{\sin(dx+c)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(a^2*sin(d*x + c)^3 + 3*a^2*sin(d*x + c)^2 - 12*a^2*log(sin(d*x + c)) -
 3*a^2*sin(d*x + c) + (3*a^2*sin(d*x + c)^2 - 3*a^2*sin(d*x + c) - a^2)/sin
(d*x + c)^3)/d
```

mupad [B] time = 8.92, size = 288, normalized size = 2.62

$$\frac{3\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24\,d}-\frac{4\,a^{2}\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{4\,d}-\frac{13\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-30\,a^{2}\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] (3*a^2*tan(c/2 + (d*x)/2))/(8*d) - (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (4*a
^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*tan(c/2 + (d*x)/2)^2)/(4*d) - (6*a^2*t
```

```
an(c/2 + (d*x)/2)^3 - 2*a^2*tan(c/2 + (d*x)/2)^2 + 8*a^2*tan(c/2 + (d*x)/2)
^4 - 26*a^2*tan(c/2 + (d*x)/2)^5 + 2*a^2*tan(c/2 + (d*x)/2)^6 - 30*a^2*tan(
c/2 + (d*x)/2)^7 + 13*a^2*tan(c/2 + (d*x)/2)^8 + a^2/3 + 2*a^2*tan(c/2 + (d
*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 24*tan(c/2 + (d*x)/2)^5 + 24*tan(c/2 +
 (d*x)/2)^7 + 8*tan(c/2 + (d*x)/2)^9)) + (4*a^2*log(tan(c/2 + (d*x)/2)^2 +
1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.518 $\int \cot^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\csc^4(c+dx)}{4d}-\frac{2a^2\csc^3(c+dx)}{3d}+\frac{a^2\csc^2(c+dx)}{2d}+\frac{4a^2\csc(c+dx)}{d}-\frac{a^2\log(s$$

```
[Out] 4*a^2*csc(d*x+c)/d+1/2*a^2*csc(d*x+c)^2/d-2/3*a^2*csc(d*x+c)^3/d-1/4*a^2*cs
c(d*x+c)^4/d-a^2*ln(sin(d*x+c))/d+2*a^2*sin(d*x+c)/d+1/2*a^2*sin(d*x+c)^2/d
```

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\csc^4(c+dx)}{4d}-\frac{2a^2\csc^3(c+dx)}{3d}+\frac{a^2\csc^2(c+dx)}{2d}+\frac{4a^2\csc(c+dx)}{d}-\frac{a^2\log(s$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*a^2*Csc[c + d*x])/d + (a^2*Csc[c + d*x]^2)/(2*d) - (2*a^2*Csc[c + d*x]^3
)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) - (a^2*Log[Sin[c + d*x]])/d + (2*a^2*S
in[c + d*x])/d + (a^2*Sin[c + d*x]^2)/(2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(2a+\frac{a^6}{x^5}+\frac{2a^5}{x^4}-\frac{a^4}{x^3}-\frac{4a^3}{x^2}-\frac{a^2}{x}+x\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{4a^2\csc(c+dx)}{d}+\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{4d}-$$

Mathematica [A] time = 0.45, size = 76, normalized size = 0.66

$$\frac{a^2\left(6\sin^2(c+dx)+24\sin(c+dx)-3\csc^4(c+dx)-8\csc^3(c+dx)+6\csc^2(c+dx)+48\csc(c+dx)-12\log(\sin\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(48*Csc[c + d*x] + 6*Csc[c + d*x]^2 - 8*Csc[c + d*x]^3 - 3*Csc[c + d*x
]^4 - 12*Log[Sin[c + d*x]] + 24*Sin[c + d*x] + 6*Sin[c + d*x]^2))/(12*d)
```

fricas [A] time = 0.83, size = 152, normalized size = 1.31

$$-\frac{6\,a^2\cos(dx+c)^6-15\,a^2\cos(dx+c)^4+18\,a^2\cos(dx+c)^2-6\,a^2+12\left(a^2\cos(dx+c)^4-2\,a^2\cos(dx+c)^2+a\right.}{12\left(d\cos(dx+c)^4-2\,d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/12*(6*a^2*cos(d*x + c)^6 - 15*a^2*cos(d*x + c)^4 + 18*a^2*cos(d*x + c)^2
 - 6*a^2 + 12*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*sin
(d*x + c)) - 8*(3*a^2*cos(d*x + c)^4 - 12*a^2*cos(d*x + c)^2 + 8*a^2)*sin(d
*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.28, size = 108, normalized size = 0.93

$$\frac{6\,a^2\sin(dx+c)^2-12\,a^2\log(|\sin(dx+c)|)+24\,a^2\sin(dx+c)+\frac{25\,a^2\sin(dx+c)^4+48\,a^2\sin(dx+c)^3+6\,a^2\sin(dx+c)^2-8\,a^2\sin(d}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(6*a^2*sin(d*x + c)^2 - 12*a^2*log(abs(sin(d*x + c))) + 24*a^2*sin(d*x
 + c) + (25*a^2*sin(d*x + c)^4 + 48*a^2*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)
^2 - 8*a^2*sin(d*x + c) - 3*a^2)/sin(d*x + c)^4)/d
```

maple [A] time = 0.42, size = 211, normalized size = 1.82

$$-\frac{a^{2}\left(\cos^{6}\left(dx+c\right)\right)}{2d\sin\left(dx+c\right)^{2}}-\frac{\left(\cos^{4}\left(dx+c\right)\right)a^{2}}{2d}-\frac{a^{2}\left(\cos^{2}\left(dx+c\right)\right)}{d}-\frac{a^{2}\ln\left(\sin\left(dx+c\right)\right)}{d}-\frac{2a^{2}\left(\cos^{6}\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)^{3}}+\frac{2a^{2}\left(\cos^{6}\right.}{d\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^6-1/2/d*cos(d*x+c)^4*a^2-1/d*a^2*cos(d*x
+c)^2-a^2*ln(sin(d*x+c))/d-2/3/d*a^2/sin(d*x+c)^3*cos(d*x+c)^6+2/d*a^2/sin(
d*x+c)*cos(d*x+c)^6+16/3*a^2*sin(d*x+c)/d+2/d*sin(d*x+c)*a^2*cos(d*x+c)^4+8
/3/d*sin(d*x+c)*a^2*cos(d*x+c)^2-1/4/d*a^2*cot(d*x+c)^4+1/2/d*a^2*cot(d*x+c
)^2
```

maxima [A] time = 0.65, size = 94, normalized size = 0.81

$$\frac{6\,a^{2}\sin\left(dx+c\right)^{2}-12\,a^{2}\log\left(\sin\left(dx+c\right)\right)+24\,a^{2}\sin\left(dx+c\right)+\frac{48\,a^{2}\sin(dx+c)^{3}+6\,a^{2}\sin(dx+c)^{2}-8\,a^{2}\sin(dx+c)-3\,a^{2}}{\sin(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(6*a^2*sin(d*x + c)^2 - 12*a^2*log(sin(d*x + c)) + 24*a^2*sin(d*x + c)
 + (48*a^2*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)^2 - 8*a^2*sin(d*x + c) - 3*a
^2)/sin(d*x + c)^4)/d
```

mupad [B] time = 8.80, size = 276, normalized size = 2.38

$$\frac{a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{16\,d}-\frac{a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{12\,d}-\frac{a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{64\,d}-\frac{a^{2}\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{7\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d}+\frac{a^{2}\,\ln\left(\tan\left(\frac{c}{2}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(16*d) - (a^2*tan(c/2 + (d*x)/2)^3)/(12*d) - (a^
2*tan(c/2 + (d*x)/2)^4)/(64*d) - (a^2*log(tan(c/2 + (d*x)/2)))/d + (7*a^2*t
an(c/2 + (d*x)/2))/(4*d) + (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d + ((a^2*ta
n(c/2 + (d*x)/2)^2)/2 + (76*a^2*tan(c/2 + (d*x)/2)^3)/3 + (7*a^2*tan(c/2 +
(d*x)/2)^4)/4 + (356*a^2*tan(c/2 + (d*x)/2)^5)/3 + 33*a^2*tan(c/2 + (d*x)/2
)^6 + 92*a^2*tan(c/2 + (d*x)/2)^7 - a^2/4 - (4*a^2*tan(c/2 + (d*x)/2))/3)/(
d*(16*tan(c/2 + (d*x)/2)^4 + 32*tan(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2
)^8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.519 $\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=112

$$\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{2d}+\frac{a^2\csc^3(c+dx)}{3d}+\frac{2a^2\csc^2(c+dx)}{d}+\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\text{si}}{}$$

```
[Out] a^2*csc(d*x+c)/d+2*a^2*csc(d*x+c)^2/d+1/3*a^2*csc(d*x+c)^3/d-1/2*a^2*csc(d*
x+c)^4/d-1/5*a^2*csc(d*x+c)^5/d+2*a^2*ln(sin(d*x+c))/d+a^2*sin(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{2d}+\frac{a^2\csc^3(c+dx)}{3d}+\frac{2a^2\csc^2(c+dx)}{d}+\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c + d*x])/d + (2*a^2*Csc[c + d*x]^2)/d + (a^2*Csc[c + d*x]^3)/(3*d
) - (a^2*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x]^5)/(5*d) + (2*a^2*Log[Si
n[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2(a+x)^4}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(1+\frac{a^6}{x^6}+\frac{2a^5}{x^5}-\frac{a^4}{x^4}-\frac{4a^3}{x^3}-\frac{a^2}{x^2}+\frac{2a}{x}\right)dx, x, a\sin(c-$$
$$= \frac{a^2\csc(c+dx)}{d}+\frac{2a^2\csc^2(c+dx)}{d}+\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc$$

Mathematica [A] time = 0.13, size = 76, normalized size = 0.68

$$\frac{a^2\left(30\sin(c+dx)-6\csc^5(c+dx)-15\csc^4(c+dx)+10\csc^3(c+dx)+60\csc^2(c+dx)+30\csc(c+dx)+60\log\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(30*Csc[c + d*x] + 60*Csc[c + d*x]^2 + 10*Csc[c + d*x]^3 - 15*Csc[c +
d*x]^4 - 6*Csc[c + d*x]^5 + 60*Log[Sin[c + d*x]] + 30*Sin[c + d*x]))/(30*d)
```

fricas [A] time = 0.83, size = 153, normalized size = 1.37

$$-\frac{30\,a^2\cos(dx+c)^6-120\,a^2\cos(dx+c)^4+160\,a^2\cos(dx+c)^2-60\left(a^2\cos(dx+c)^4-2\,a^2\cos(dx+c)^2+a^2\right)}{30\left(d\cos(dx+c)^4-2\,d\cos(dx+c)^2+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/30*(30*a^2*cos(d*x + c)^6 - 120*a^2*cos(d*x + c)^4 + 160*a^2*cos(d*x + c
)^2 - 60*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*sin(d*x
+ c))*sin(d*x + c) - 64*a^2 + 15*(4*a^2*cos(d*x + c)^2 - 3*a^2)*sin(d*x + c
))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.27, size = 109, normalized size = 0.97

$$\frac{60\,a^2\log(|\sin(dx+c)|)+30\,a^2\sin(dx+c)-\frac{137\,a^2\sin(dx+c)^5-30\,a^2\sin(dx+c)^4-60\,a^2\sin(dx+c)^3-10\,a^2\sin(dx+c)^2+15\,a^2\sin(dx+c)}{\sin(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(60*a^2*log(abs(sin(d*x + c))) + 30*a^2*sin(d*x + c) - (137*a^2*sin(d*
x + c)^5 - 30*a^2*sin(d*x + c)^4 - 60*a^2*sin(d*x + c)^3 - 10*a^2*sin(d*x +
 c)^2 + 15*a^2*sin(d*x + c) + 6*a^2)/sin(d*x + c)^5)/d
```

maple [A] time = 0.42, size = 178, normalized size = 1.59

$$-\frac{4a^2\left(\cos^6(dx+c)\right)}{15d\sin(dx+c)^3}+\frac{4a^2\left(\cos^6(dx+c)\right)}{5d\sin(dx+c)}+\frac{32a^2\sin(dx+c)}{15d}+\frac{4\sin(dx+c)a^2\left(\cos^4(dx+c)\right)}{5d}+\frac{16\sin(dx+c)a}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -4/15/d*a^2/sin(d*x+c)^3*cos(d*x+c)^6+4/5/d*a^2/sin(d*x+c)*cos(d*x+c)^6+32/
15*a^2*sin(d*x+c)/d+4/5/d*sin(d*x+c)*a^2*cos(d*x+c)^4+16/15/d*sin(d*x+c)*a^
2*cos(d*x+c)^2-1/2/d*a^2*cot(d*x+c)^4+1/d*a^2*cot(d*x+c)^2+2*a^2*ln(sin(d*x
+c))/d-1/5/d*a^2/sin(d*x+c)^5*cos(d*x+c)^6
```

maxima [A] time = 0.56, size = 94, normalized size = 0.84

$$\frac{60\,a^2\log(\sin(dx+c))+30\,a^2\sin(dx+c)+\frac{30\,a^2\sin(dx+c)^4+60\,a^2\sin(dx+c)^3+10\,a^2\sin(dx+c)^2-15\,a^2\sin(dx+c)-6\,a^2}{\sin(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*(60*a^2*log(sin(d*x + c)) + 30*a^2*sin(d*x + c) + (30*a^2*sin(d*x + c)
^4 + 60*a^2*sin(d*x + c)^3 + 10*a^2*sin(d*x + c)^2 - 15*a^2*sin(d*x + c) -
6*a^2)/sin(d*x + c)^5)/d
```

mupad [B] time = 8.79, size = 267, normalized size = 2.38

$$\frac{82\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+12\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{55\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}-a^2\tan\left(\frac{c}{2}+\right.}{d\left(32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] ((2*a^2*tan(c/2 + (d*x)/2)^2)/15 + 11*a^2*tan(c/2 + (d*x)/2)^3 + (55*a^2*ta
n(c/2 + (d*x)/2)^4)/3 + 12*a^2*tan(c/2 + (d*x)/2)^5 + 82*a^2*tan(c/2 + (d*x
)/2)^6 - a^2/5 - a^2*tan(c/2 + (d*x)/2))/(d*(32*tan(c/2 + (d*x)/2)^5 + 32*t
an(c/2 + (d*x)/2)^7)) + (3*a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^2*tan(c/2 +
 (d*x)/2)^3)/(96*d) - (a^2*tan(c/2 + (d*x)/2)^4)/(32*d) - (a^2*tan(c/2 + (d
*x)/2)^5)/(160*d) + (2*a^2*log(tan(c/2 + (d*x)/2)))/d + (9*a^2*tan(c/2 + (d
*x)/2))/(16*d) - (2*a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.520 $\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=119

$$-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{a^2\csc^4(c+dx)}{4d}+\frac{4a^2\csc^3(c+dx)}{3d}+\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\log}{}$$

```
[Out] -2*a^2*csc(d*x+c)/d+1/2*a^2*csc(d*x+c)^2/d+4/3*a^2*csc(d*x+c)^3/d+1/4*a^2*c
sc(d*x+c)^4/d-2/5*a^2*csc(d*x+c)^5/d-1/6*a^2*csc(d*x+c)^6/d+a^2*ln(sin(d*x+
c))/d
```

Rubi [A] time = 0.12, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{a^2\csc^4(c+dx)}{4d}+\frac{4a^2\csc^3(c+dx)}{3d}+\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Csc[c + d*x])/d + (a^2*Csc[c + d*x]^2)/(2*d) + (4*a^2*Csc[c + d*x]^
3)/(3*d) + (a^2*Csc[c + d*x]^4)/(4*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2
*Csc[c + d*x]^6)/(6*d) + (a^2*Log[Sin[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a-x)^2(a+x)^4}{x^7}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{a^6}{x^7}+\frac{2a^5}{x^6}-\frac{a^4}{x^5}-\frac{4a^3}{x^4}-\frac{a^2}{x^3}+\frac{2a}{x^2}+\frac{1}{x}\right)dx, x, a\sin(\right.}{d}$$
$$= -\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\csc^2(c+dx)}{2d}+\frac{4a^2\csc^3(c+dx)}{3d}+\frac{a^2}{}$$

Mathematica [A] time = 0.04, size = 102, normalized size = 0.86

$$a^2\left(-\frac{\csc^6(c+dx)}{6d}-\frac{2\csc^5(c+dx)}{5d}+\frac{\csc^4(c+dx)}{4d}+\frac{4\csc^3(c+dx)}{3d}+\frac{\csc^2(c+dx)}{2d}-\frac{2\csc(c+dx)}{d}+\frac{\log(\sin(c}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*((-2*Csc[c + d*x])/d + Csc[c + d*x]^2/(2*d) + (4*Csc[c + d*x]^3)/(3*d)
+ Csc[c + d*x]^4/(4*d) - (2*Csc[c + d*x]^5)/(5*d) - Csc[c + d*x]^6/(6*d) +
Log[Sin[c + d*x]]/d)
```

fricas [A] time = 0.75, size = 167, normalized size = 1.40

$$-\frac{30\,a^2\cos(dx+c)^4-75\,a^2\cos(dx+c)^2+35\,a^2-60\left(a^2\cos(dx+c)^6-3\,a^2\cos(dx+c)^4+3\,a^2\cos(dx+c)^2-\right.}{60\left(d\cos(dx+c)^6-3\,d\cos(dx+c)^4+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/60*(30*a^2*cos(d*x + c)^4 - 75*a^2*cos(d*x + c)^2 + 35*a^2 - 60*(a^2*cos
(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*si
n(d*x + c)) - 8*(15*a^2*cos(d*x + c)^4 - 20*a^2*cos(d*x + c)^2 + 8*a^2)*sin
(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.29, size = 111, normalized size = 0.93

$$\frac{60\,a^2\log\left(|\sin\left(dx+c\right)|\right)-\frac{147\,a^2\sin(dx+c)^6+120\,a^2\sin(dx+c)^5-30\,a^2\sin(dx+c)^4-80\,a^2\sin(dx+c)^3-15\,a^2\sin(dx+c)^2+24\,a^2\sin(dx+c)+10\,a}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^2*log(abs(sin(d*x + c))) - (147*a^2*sin(d*x + c)^6 + 120*a^2*sin
(d*x + c)^5 - 30*a^2*sin(d*x + c)^4 - 80*a^2*sin(d*x + c)^3 - 15*a^2*sin(d*
x + c)^2 + 24*a^2*sin(d*x + c) + 10*a^2)/sin(d*x + c)^6)/d
```

maple [A] time = 0.44, size = 202, normalized size = 1.70

$$-\frac{a^2\left(\cot^4\left(dx+c\right)\right)}{4d}+\frac{a^2\left(\cot^2\left(dx+c\right)\right)}{2d}+\frac{a^2\ln\left(\sin\left(dx+c\right)\right)}{d}-\frac{2a^2\left(\cos^6\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)^5}+\frac{2a^2\left(\cos^6\left(dx+c\right)\right)}{15d\sin\left(dx+c\right)^3}-\frac{2a^2\left(\cos\right.}{5d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2*cot(d*x+c)^4+1/2/d*a^2*cot(d*x+c)^2+a^2*ln(sin(d*x+c))/d-2/5/d*a
^2/sin(d*x+c)^5*cos(d*x+c)^6+2/15/d*a^2/sin(d*x+c)^3*cos(d*x+c)^6-2/5/d*a^2
/sin(d*x+c)*cos(d*x+c)^6-16/15*a^2*sin(d*x+c)/d-2/5/d*sin(d*x+c)*a^2*cos(d*
x+c)^4-8/15/d*sin(d*x+c)*a^2*cos(d*x+c)^2-1/6/d*a^2/sin(d*x+c)^6*cos(d*x+c)
^6
```

maxima [A] time = 0.66, size = 97, normalized size = 0.82

$$\frac{60\,a^2\log\left(\sin\left(dx+c\right)\right)-\frac{120\,a^2\sin(dx+c)^5-30\,a^2\sin(dx+c)^4-80\,a^2\sin(dx+c)^3-15\,a^2\sin(dx+c)^2+24\,a^2\sin(dx+c)+10\,a^2}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/60*(60*a^2*log(sin(d*x + c)) - (120*a^2*sin(d*x + c)^5 - 30*a^2*sin(d*x +
 c)^4 - 80*a^2*sin(d*x + c)^3 - 15*a^2*sin(d*x + c)^2 + 24*a^2*sin(d*x + c)
 + 10*a^2)/sin(d*x + c)^6)/d
```

mupad [B] time = 8.98, size = 217, normalized size = 1.82

$$\frac{19\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{48\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{80\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}+\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\cot\left(\frac{c}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] (19*a^2*tan(c/2 + (d*x)/2)^2)/(128*d) + (5*a^2*tan(c/2 + (d*x)/2)^3)/(48*d)
 - (a^2*tan(c/2 + (d*x)/2)^5)/(80*d) - (a^2*tan(c/2 + (d*x)/2)^6)/(384*d) +
 (a^2*log(tan(c/2 + (d*x)/2)))/d - (cot(c/2 + (d*x)/2)^6*(40*a^2*tan(c/2 +
(d*x)/2)^5 - (19*a^2*tan(c/2 + (d*x)/2)^4)/2 - (20*a^2*tan(c/2 + (d*x)/2)^3
)/3 + a^2/6 + (4*a^2*tan(c/2 + (d*x)/2))/5))/(64*d) - (5*a^2*tan(c/2 + (d*x
)/2))/(8*d) - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.521 $\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=111

$$\frac{(a\sin(c+dx)+a)^{10}}{10a^7d}-\frac{2(a\sin(c+dx)+a)^9}{3a^6d}+\frac{13(a\sin(c+dx)+a)^8}{8a^5d}-\frac{12(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(a\sin(c+dx)+a}{3a^3d}$$

```
[Out] 2/3*(a+a*sin(d*x+c))^6/a^3/d-12/7*(a+a*sin(d*x+c))^7/a^4/d+13/8*(a+a*sin(d*
x+c))^8/a^5/d-2/3*(a+a*sin(d*x+c))^9/a^6/d+1/10*(a+a*sin(d*x+c))^10/a^7/d
```

Rubi [A] time = 0.13, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{(a\sin(c+dx)+a)^{10}}{10a^7d}-\frac{2(a\sin(c+dx)+a)^9}{3a^6d}+\frac{13(a\sin(c+dx)+a)^8}{8a^5d}-\frac{12(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(a\sin(c+dx)+a}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^6)/(3*a^3*d) - (12*(a + a*Sin[c + d*x])^7)/(7*a^4*d
) + (13*(a + a*Sin[c + d*x])^8)/(8*a^5*d) - (2*(a + a*Sin[c + d*x])^9)/(3*a
^6*d) + (a + a*Sin[c + d*x])^10/(10*a^7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2(a+x)^5}{a^2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^2(a+x)^5\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^4(a+x)^5 - 12a^3(a+x)^6 + 13a^2(a+x)^7 - 6a(a$$
$$a^7d$$
$$= \frac{2(a+a\sin(c+dx))^6}{3a^3d} - \frac{12(a+a\sin(c+dx))^7}{7a^4d} + \frac{13(a+a\,\text{si}}{8a}$$

Mathematica [A] time = 0.86, size = 110, normalized size = 0.99

$$-\frac{a^3(-63840\sin(c+dx) + 8960\sin(3(c+dx)) + 8064\sin(5(c+dx)) + 240\sin(7(c+dx)) - 560\sin(9(c+dx)) +}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/430080*(a^3*(-2835 + 34440*Cos[2*(c + d*x)] + 5040*Cos[4*(c + d*x)] - 40
60*Cos[6*(c + d*x)] - 1260*Cos[8*(c + d*x)] + 84*Cos[10*(c + d*x)] - 63840*
Sin[c + d*x] + 8960*Sin[3*(c + d*x)] + 8064*Sin[5*(c + d*x)] + 240*Sin[7*(c
 + d*x)] - 560*Sin[9*(c + d*x)]))/d
```

fricas [A] time = 0.98, size = 111, normalized size = 1.00

$$-\frac{84\,a^3\cos(dx+c)^{10} - 525\,a^3\cos(dx+c)^8 + 560\,a^3\cos(dx+c)^6 - 8\left(35\,a^3\cos(dx+c)^8 - 65\,a^3\cos(dx+c)^6 +\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/840*(84*a^3*cos(d*x + c)^10 - 525*a^3*cos(d*x + c)^8 + 560*a^3*cos(d*x +
 c)^6 - 8*(35*a^3*cos(d*x + c)^8 - 65*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x +
c)^4 + 8*a^3*cos(d*x + c)^2 + 16*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.39, size = 168, normalized size = 1.51

$$-\frac{a^3\cos(10\,dx+10\,c)}{5120\,d} + \frac{3\,a^3\cos(8\,dx+8\,c)}{1024\,d} + \frac{29\,a^3\cos(6\,dx+6\,c)}{3072\,d} - \frac{3\,a^3\cos(4\,dx+4\,c)}{256\,d} - \frac{41\,a^3\cos(2\,dx+2\,c)}{512\,d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/5120*a^3*cos(10*d*x + 10*c)/d + 3/1024*a^3*cos(8*d*x + 8*c)/d + 29/3072*
a^3*cos(6*d*x + 6*c)/d - 3/256*a^3*cos(4*d*x + 4*c)/d - 41/512*a^3*cos(2*d*
x + 2*c)/d + 1/768*a^3*sin(9*d*x + 9*c)/d - 1/1792*a^3*sin(7*d*x + 7*c)/d -
 3/160*a^3*sin(5*d*x + 5*c)/d - 1/48*a^3*sin(3*d*x + 3*c)/d + 19/128*a^3*si
n(d*x + c)/d
```

maple [B] time = 0.26, size = 208, normalized size = 1.87

$$\frac{a^3\left(-\frac{(\sin^4(dx+c))(\cos^6(dx+c))}{10}-\frac{(\sin^2(dx+c))(\cos^6(dx+c))}{20}-\frac{(\cos^6(dx+c))}{60}\right)+3a^3\left(-\frac{(\sin^3(dx+c))(\cos^6(dx+c))}{9}-\frac{\sin(dx+c)(\cos^6(dx+c))}{21}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/
60*cos(d*x+c)^6)+3*a^3*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(
d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+3*a^3*(-1/8*
sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+a^3*(-1/7*sin(d*x+c)*cos(d*x+c
)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.66, size = 110, normalized size = 0.99

$$\frac{84\,a^3\sin(dx+c)^{10}+280\,a^3\sin(dx+c)^9+105\,a^3\sin(dx+c)^8-600\,a^3\sin(dx+c)^7-700\,a^3\sin(dx+c)^6+1}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/840*(84*a^3*sin(d*x + c)^10 + 280*a^3*sin(d*x + c)^9 + 105*a^3*sin(d*x +
c)^8 - 600*a^3*sin(d*x + c)^7 - 700*a^3*sin(d*x + c)^6 + 168*a^3*sin(d*x +
c)^5 + 630*a^3*sin(d*x + c)^4 + 280*a^3*sin(d*x + c)^3)/d
```

mupad [B] time = 8.71, size = 109, normalized size = 0.98

$$\frac{\frac{a^3\sin(c+d\,x)^{10}}{10}+\frac{a^3\sin(c+d\,x)^9}{3}+\frac{a^3\sin(c+d\,x)^8}{8}-\frac{5\,a^3\sin(c+d\,x)^7}{7}-\frac{5\,a^3\sin(c+d\,x)^6}{6}+\frac{a^3\sin(c+d\,x)^5}{5}+\frac{3\,a^3\sin(c+d\,x)^4}{4}+\frac{a^3\sin(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^3)/3 + (3*a^3*sin(c + d*x)^4)/4 + (a^3*sin(c + d*x)^5)/5
 - (5*a^3*sin(c + d*x)^6)/6 - (5*a^3*sin(c + d*x)^7)/7 + (a^3*sin(c + d*x)^
8)/8 + (a^3*sin(c + d*x)^9)/3 + (a^3*sin(c + d*x)^10)/10)/d
```

sympy [A] time = 30.00, size = 255, normalized size = 2.30

$$\begin{cases} \frac{8a^3\sin^9(c+dx)}{105d} + \frac{12a^3\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{8a^3\sin^7(c+dx)}{105d} + \frac{3a^3\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{4a^3\sin^5(c+dx)\cos^2(c+dx)}{15d} - \frac{a^3\sin^4(c+dx)}{6a} \\ x\left(a\sin(c)+a\right)^3\sin^2(c)\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*sin(c + d*x)**9/(105*d) + 12*a**3*sin(c + d*x)**7*cos(c +
 d*x)**2/(35*d) + 8*a**3*sin(c + d*x)**7/(105*d) + 3*a**3*sin(c + d*x)**5*c
os(c + d*x)**4/(5*d) + 4*a**3*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) - a**3
*sin(c + d*x)**4*cos(c + d*x)**6/(6*d) + a**3*sin(c + d*x)**3*cos(c + d*x)*
*4/(3*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**8/(12*d) - a**3*sin(c + d*x)*
*2*cos(c + d*x)**6/(2*d) - a**3*cos(c + d*x)**10/(60*d) - a**3*cos(c + d*x)
**8/(8*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)**2*cos(c)**5, True))
```

3.522 $\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=89

$$\frac{(a\sin(c+dx)+a)^9}{9a^6d}-\frac{5(a\sin(c+dx)+a)^8}{8a^5d}+\frac{8(a\sin(c+dx)+a)^7}{7a^4d}-\frac{2(a\sin(c+dx)+a)^6}{3a^3d}$$

```
[Out] -2/3*(a+a*sin(d*x+c))^6/a^3/d+8/7*(a+a*sin(d*x+c))^7/a^4/d-5/8*(a+a*sin(d*x
+c))^8/a^5/d+1/9*(a+a*sin(d*x+c))^9/a^6/d
```

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 77}

$$\frac{(a\sin(c+dx)+a)^9}{9a^6d}-\frac{5(a\sin(c+dx)+a)^8}{8a^5d}+\frac{8(a\sin(c+dx)+a)^7}{7a^4d}-\frac{2(a\sin(c+dx)+a)^6}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*(a + a*Sin[c + d*x])^6)/(3*a^3*d) + (8*(a + a*Sin[c + d*x])^7)/(7*a^4*d
) - (5*(a + a*Sin[c + d*x])^8)/(8*a^5*d) + (a + a*Sin[c + d*x])^9/(9*a^6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x(a+x)^5}{a}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x(a+x)^5\,dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^3(a+x)^5+8a^2(a+x)^6-5a(a+x)^7+(a+x)\right.\right.}{a^6d}$$
$$= -\frac{2(a+a\sin(c+dx))^6}{3a^3d}+\frac{8(a+a\sin(c+dx))^7}{7a^4d}-\frac{5(a+a\sin}{8a^5}$$

Mathematica [A] time = 0.68, size = 100, normalized size = 1.12

$$\frac{a^3(16632\sin(c+dx)-1344\sin(3(c+dx))-2016\sin(5(c+dx))-396\sin(7(c+dx))+28\sin(9(c+dx))-9576}{64512d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(4662 - 9576*Cos[2*(c + d*x)] - 2772*Cos[4*(c + d*x)] + 168*Cos[6*(c +
 d*x)] + 189*Cos[8*(c + d*x)] + 16632*Sin[c + d*x] - 1344*Sin[3*(c + d*x)]
- 2016*Sin[5*(c + d*x)] - 396*Sin[7*(c + d*x)] + 28*Sin[9*(c + d*x)]))/(645
12*d)
```

fricas [A] time = 0.69, size = 98, normalized size = 1.10

$$\frac{189\,a^3\cos(dx+c)^8-336\,a^3\cos(dx+c)^6+8\left(7\,a^3\cos(dx+c)^8-37\,a^3\cos(dx+c)^6+6\,a^3\cos(dx+c)^4+8\,a^3\right.}{504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/504*(189*a^3*cos(d*x + c)^8 - 336*a^3*cos(d*x + c)^6 + 8*(7*a^3*cos(d*x +
 c)^8 - 37*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x + c)^4 + 8*a^3*cos(d*x + c)^2
 + 16*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.32, size = 151, normalized size = 1.70

$$\frac{3\,a^3\cos(8\,dx+8\,c)}{1024\,d}+\frac{a^3\cos(6\,dx+6\,c)}{384\,d}-\frac{11\,a^3\cos(4\,dx+4\,c)}{256\,d}-\frac{19\,a^3\cos(2\,dx+2\,c)}{128\,d}+\frac{a^3\sin(9\,dx+9\,c)}{2304\,d}-\frac{11\,a^3\,\text{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 3/1024*a^3*cos(8*d*x + 8*c)/d + 1/384*a^3*cos(6*d*x + 6*c)/d - 11/256*a^3*c
os(4*d*x + 4*c)/d - 19/128*a^3*cos(2*d*x + 2*c)/d + 1/2304*a^3*sin(9*d*x +
9*c)/d - 11/1792*a^3*sin(7*d*x + 7*c)/d - 1/32*a^3*sin(5*d*x + 5*c)/d - 1/4
8*a^3*sin(3*d*x + 3*c)/d + 33/128*a^3*sin(d*x + c)/d
```

maple [B] time = 0.26, size = 170, normalized size = 1.91

$$\frac{a^3\left(-\frac{(\sin^3(dx+c))(\cos^6(dx+c))}{9}-\frac{\sin(dx+c)(\cos^6(dx+c))}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{105}\right)+3a^3\left(-\frac{(\sin^2(dx+c))(\cos^6(dx+c))}{8}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+3*a^3*(-1/8*sin(d*x+c)^2*c
os(d*x+c)^6-1/24*cos(d*x+c)^6)+3*a^3*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/
3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/6*a^3*cos(d*x+c)^6)
```

maxima [A] time = 0.52, size = 110, normalized size = 1.24

$$\frac{56\,a^3\sin(dx+c)^9+189\,a^3\sin(dx+c)^8+72\,a^3\sin(dx+c)^7-420\,a^3\sin(dx+c)^6-504\,a^3\sin(dx+c)^5+126}{504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/504*(56*a^3*sin(d*x + c)^9 + 189*a^3*sin(d*x + c)^8 + 72*a^3*sin(d*x + c)
^7 - 420*a^3*sin(d*x + c)^6 - 504*a^3*sin(d*x + c)^5 + 126*a^3*sin(d*x + c)
^4 + 504*a^3*sin(d*x + c)^3 + 252*a^3*sin(d*x + c)^2)/d
```

mupad [B] time = 0.07, size = 108, normalized size = 1.21

$$\frac{\frac{a^3\sin(c+d\,x)^9}{9}+\frac{3\,a^3\sin(c+d\,x)^8}{8}+\frac{a^3\sin(c+d\,x)^7}{7}-\frac{5\,a^3\sin(c+d\,x)^6}{6}-a^3\sin(c+d\,x)^5+\frac{a^3\sin(c+d\,x)^4}{4}+a^3\sin(c+d\,x)^3+\frac{a^3\,\mathrm{si}}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^2)/2 + a^3*sin(c + d*x)^3 + (a^3*sin(c + d*x)^4)/4 - a^3
*sin(c + d*x)^5 - (5*a^3*sin(c + d*x)^6)/6 + (a^3*sin(c + d*x)^7)/7 + (3*a^
3*sin(c + d*x)^8)/8 + (a^3*sin(c + d*x)^9)/9)/d
```

sympy [A] time = 17.30, size = 202, normalized size = 2.27

$$\begin{cases} \frac{8a^3 \sin^9(c+dx)}{315d} + \frac{4a^3 \sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{8a^3 \sin^7(c+dx)}{35d} + \frac{a^3 \sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{4a^3 \sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{a^3 \sin^3(c+dx)\cos^4(c+dx)}{d} \\ x\left(a\sin(c)+a\right)^3 \sin(c)\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*sin(c + d*x)**9/(315*d) + 4*a**3*sin(c + d*x)**7*cos(c +
d*x)**2/(35*d) + 8*a**3*sin(c + d*x)**7/(35*d) + a**3*sin(c + d*x)**5*cos(c
 + d*x)**4/(5*d) + 4*a**3*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + a**3*sin(
c + d*x)**3*cos(c + d*x)**4/d - a**3*sin(c + d*x)**2*cos(c + d*x)**6/(2*d)
- a**3*cos(c + d*x)**8/(8*d) - a**3*cos(c + d*x)**6/(6*d), Ne(d, 0)), (x*(a
*sin(c) + a)**3*sin(c)*cos(c)**5, True))
```

3.523 $\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=137

$$\frac{a^3\sin^7(c+dx)}{7d}+\frac{a^3\sin^6(c+dx)}{2d}+\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^4(c+dx)}{4d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin}{}$$

```
[Out] a^3*ln(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d+1/2*a^3*sin(d*x+c)^2/d-5/3*a^3*sin(
d*x+c)^3/d-5/4*a^3*sin(d*x+c)^4/d+1/5*a^3*sin(d*x+c)^5/d+1/2*a^3*sin(d*x+c)
^6/d+1/7*a^3*sin(d*x+c)^7/d
```

Rubi [A] time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^3\sin^7(c+dx)}{7d}+\frac{a^3\sin^6(c+dx)}{2d}+\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^4(c+dx)}{4d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3*Sin[c + d*x]^2)/(
2*d) - (5*a^3*Sin[c + d*x]^3)/(3*d) - (5*a^3*Sin[c + d*x]^4)/(4*d) + (a^3*S
in[c + d*x]^5)/(5*d) + (a^3*Sin[c + d*x]^6)/(2*d) + (a^3*Sin[c + d*x]^7)/(7
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2(a+x)^5}{x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(3a^6+\frac{a^7}{x}+a^5x-5a^4x^2-5a^3x^3+a^2x^4+3ax^5+x\right.\right.}{a^4 d}$$
$$= \frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\sin^2(c+dx)}{2d}-\frac{5a}{}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 0.64

$$\frac{a^3\left(60\sin^7(c+dx)+210\sin^6(c+dx)+84\sin^5(c+dx)-525\sin^4(c+dx)-700\sin^3(c+dx)+210\sin^2(c+dx)\right.}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(420*Log[Sin[c + d*x]] + 1260*Sin[c + d*x] + 210*Sin[c + d*x]^2 - 700*
Sin[c + d*x]^3 - 525*Sin[c + d*x]^4 + 84*Sin[c + d*x]^5 + 210*Sin[c + d*x]^
6 + 60*Sin[c + d*x]^7))/(420*d)
```

fricas [A] time = 0.64, size = 112, normalized size = 0.82

$$-\frac{210\,a^3\cos(dx+c)^6-105\,a^3\cos(dx+c)^4-210\,a^3\cos(dx+c)^2-420\,a^3\log\left(\frac{1}{2}\sin(dx+c)\right)+4\left(15\,a^3\cos(dx\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/420*(210*a^3*cos(d*x + c)^6 - 105*a^3*cos(d*x + c)^4 - 210*a^3*cos(d*x +
 c)^2 - 420*a^3*log(1/2*sin(d*x + c)) + 4*(15*a^3*cos(d*x + c)^6 - 66*a^3*c
os(d*x + c)^4 - 88*a^3*cos(d*x + c)^2 - 176*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.28, size = 108, normalized size = 0.79

$$\frac{60\,a^{3}\sin\left(dx+c\right)^{7}+210\,a^{3}\sin\left(dx+c\right)^{6}+84\,a^{3}\sin\left(dx+c\right)^{5}-525\,a^{3}\sin\left(dx+c\right)^{4}-700\,a^{3}\sin\left(dx+c\right)^{3}+210}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/420*(60*a^3*sin(d*x + c)^7 + 210*a^3*sin(d*x + c)^6 + 84*a^3*sin(d*x + c)
^5 - 525*a^3*sin(d*x + c)^4 - 700*a^3*sin(d*x + c)^3 + 210*a^3*sin(d*x + c)
^2 + 420*a^3*log(abs(sin(d*x + c))) + 1260*a^3*sin(d*x + c))/d
```

maple [A] time = 0.47, size = 144, normalized size = 1.05

$$-\frac{a^{3}\left(\cos^{6}\left(dx+c\right)\right)\sin\left(dx+c\right)}{7d}+\frac{176a^{3}\sin\left(dx+c\right)}{105d}+\frac{22a^{3}\left(\cos^{4}\left(dx+c\right)\right)\sin\left(dx+c\right)}{35d}+\frac{88a^{3}\left(\cos^{2}\left(dx+c\right)\right)\sin}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/7/d*a^3*cos(d*x+c)^6*sin(d*x+c)+176/105*a^3*sin(d*x+c)/d+22/35/d*a^3*cos
(d*x+c)^4*sin(d*x+c)+88/105/d*a^3*cos(d*x+c)^2*sin(d*x+c)-1/2/d*a^3*cos(d*x
+c)^6+1/4/d*cos(d*x+c)^4*a^3+1/2/d*a^3*cos(d*x+c)^2+a^3*ln(sin(d*x+c))/d
```

maxima [A] time = 0.51, size = 107, normalized size = 0.78

$$\frac{60\,a^{3}\sin\left(dx+c\right)^{7}+210\,a^{3}\sin\left(dx+c\right)^{6}+84\,a^{3}\sin\left(dx+c\right)^{5}-525\,a^{3}\sin\left(dx+c\right)^{4}-700\,a^{3}\sin\left(dx+c\right)^{3}+210}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/420*(60*a^3*sin(d*x + c)^7 + 210*a^3*sin(d*x + c)^6 + 84*a^3*sin(d*x + c)
^5 - 525*a^3*sin(d*x + c)^4 - 700*a^3*sin(d*x + c)^3 + 210*a^3*sin(d*x + c)
^2 + 420*a^3*log(sin(d*x + c)) + 1260*a^3*sin(d*x + c))/d
```

mupad [B] time = 8.99, size = 178, normalized size = 1.30

$$\frac{176\,a^{3}\,\sin\left(c+d\,x\right)}{105\,d}-\frac{a^{3}\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}}\right)}{d}+\frac{a^{3}\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^{3}\cos\left(c+d\,x\right)^{2}}{2\,d}+\frac{a^{3}\cos\left(c+d\,x\right)^{4}}{4\,d}-\frac{a^{3}\cos\left(c+d\,x\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] (176*a^3*sin(c + d*x))/(105*d) - (a^3*log(1/cos(c/2 + (d*x)/2)^2))/d + (a^3
*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^3*cos(c + d*x)^2)/(2*d)
 + (a^3*cos(c + d*x)^4)/(4*d) - (a^3*cos(c + d*x)^6)/(2*d) + (88*a^3*cos(c
+ d*x)^2*sin(c + d*x))/(105*d) + (22*a^3*cos(c + d*x)^4*sin(c + d*x))/(35*d
) - (a^3*cos(c + d*x)^6*sin(c + d*x))/(7*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.524 $\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=133

$$\frac{a^3\sin^6(c+dx)}{6d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^4(c+dx)}{4d}-\frac{5a^3\sin^3(c+dx)}{3d}-\frac{5a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc(c}{d}$$

```
[Out] -a^3*csc(d*x+c)/d+3*a^3*ln(sin(d*x+c))/d+a^3*sin(d*x+c)/d-5/2*a^3*sin(d*x+c
)^2/d-5/3*a^3*sin(d*x+c)^3/d+1/4*a^3*sin(d*x+c)^4/d+3/5*a^3*sin(d*x+c)^5/d+
1/6*a^3*sin(d*x+c)^6/d
```

Rubi [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^3\sin^6(c+dx)}{6d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^4(c+dx)}{4d}-\frac{5a^3\sin^3(c+dx)}{3d}-\frac{5a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) + (3*a^3*Log[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])/
d - (5*a^3*Sin[c + d*x]^2)/(2*d) - (5*a^3*Sin[c + d*x]^3)/(3*d) + (a^3*Sin[
c + d*x]^4)/(4*d) + (3*a^3*Sin[c + d*x]^5)/(5*d) + (a^3*Sin[c + d*x]^6)/(6*
d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2(a+x)^5}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5+\frac{a^7}{x^2}+\frac{3a^6}{x}-5a^4x-5a^3x^2+a^2x^3+3ax^4+x^5\right.\right.}{a^3 d}$$
$$= -\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}-$$

Mathematica [A] time = 0.17, size = 86, normalized size = 0.65

$$-\frac{a^3\left(-10\sin^6(c+dx)-36\sin^5(c+dx)-15\sin^4(c+dx)+100\sin^3(c+dx)+150\sin^2(c+dx)-60\sin(c+dx)\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/60*(a^3*(60*Csc[c + d*x] - 180*Log[Sin[c + d*x]] - 60*Sin[c + d*x] + 150
*Sin[c + d*x]^2 + 100*Sin[c + d*x]^3 - 15*Sin[c + d*x]^4 - 36*Sin[c + d*x]^
5 - 10*Sin[c + d*x]^6))/d
```

fricas [A] time = 0.71, size = 131, normalized size = 0.98

$$-\frac{144\,a^3\cos(dx+c)^6-32\,a^3\cos(dx+c)^4-128\,a^3\cos(dx+c)^2-720\,a^3\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+256}{240\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/240*(144*a^3*cos(d*x + c)^6 - 32*a^3*cos(d*x + c)^4 - 128*a^3*cos(d*x +
c)^2 - 720*a^3*log(1/2*sin(d*x + c))*sin(d*x + c) + 256*a^3 + 5*(8*a^3*cos(
d*x + c)^6 - 36*a^3*cos(d*x + c)^4 - 72*a^3*cos(d*x + c)^2 + 47*a^3)*sin(d*
x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.31, size = 120, normalized size = 0.90

$$\frac{10\,a^3\sin(dx+c)^6+36\,a^3\sin(dx+c)^5+15\,a^3\sin(dx+c)^4-100\,a^3\sin(dx+c)^3-150\,a^3\sin(dx+c)^2+180\,a}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(10*a^3*sin(d*x + c)^6 + 36*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4
 - 100*a^3*sin(d*x + c)^3 - 150*a^3*sin(d*x + c)^2 + 180*a^3*log(abs(sin(d*
x + c))) + 60*a^3*sin(d*x + c) - 60*(3*a^3*sin(d*x + c) + a^3)/sin(d*x + c)
)/d
```

maple [A] time = 0.40, size = 147, normalized size = 1.11

$$-\frac{a^3\left(\cos^6(dx+c)\right)}{6d}-\frac{16a^3\sin(dx+c)}{15d}-\frac{2a^3\left(\cos^4(dx+c)\right)\sin(dx+c)}{5d}-\frac{8a^3\left(\cos^2(dx+c)\right)\sin(dx+c)}{15d}+\frac{3\left(\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/6/d*a^3*cos(d*x+c)^6-16/15*a^3*sin(d*x+c)/d-2/5/d*a^3*cos(d*x+c)^4*sin(d
*x+c)-8/15/d*a^3*cos(d*x+c)^2*sin(d*x+c)+3/4/d*cos(d*x+c)^4*a^3+3/2/d*a^3*c
os(d*x+c)^2+3*a^3*ln(sin(d*x+c))/d-1/d*a^3/sin(d*x+c)*cos(d*x+c)^6
```

maxima [A] time = 0.51, size = 107, normalized size = 0.80

$$\frac{10\,a^3\sin(dx+c)^6+36\,a^3\sin(dx+c)^5+15\,a^3\sin(dx+c)^4-100\,a^3\sin(dx+c)^3-150\,a^3\sin(dx+c)^2+180\,a}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(10*a^3*sin(d*x + c)^6 + 36*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4
 - 100*a^3*sin(d*x + c)^3 - 150*a^3*sin(d*x + c)^2 + 180*a^3*log(sin(d*x +
c)) + 60*a^3*sin(d*x + c) - 60*a^3/sin(d*x + c))/d
```

mupad [B] time = 9.18, size = 371, normalized size = 2.79

$$\frac{14\,a^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}-\frac{10\,a^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d}+\frac{8\,a^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3\,d}-\frac{28\,a^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{d}+\frac{32\,a^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{d}-\frac{32\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (14*a^3*cos(c/2 + (d*x)/2)^4)/d - (10*a^3*cos(c/2 + (d*x)/2)^2)/d + (8*a^3*
cos(c/2 + (d*x)/2)^6)/(3*d) - (28*a^3*cos(c/2 + (d*x)/2)^8)/d + (32*a^3*cos
(c/2 + (d*x)/2)^10)/d - (32*a^3*cos(c/2 + (d*x)/2)^12)/(3*d) - (3*a^3*log(1
/cos(c/2 + (d*x)/2)^2))/d + (3*a^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d - (46*a^3*cos(c/2 + (d*x)/2)^3)/(3*d*sin(c/2 + (d*x)/2)) + (688*a^3*c
os(c/2 + (d*x)/2)^5)/(15*d*sin(c/2 + (d*x)/2)) - (1064*a^3*cos(c/2 + (d*x)/
2)^7)/(15*d*sin(c/2 + (d*x)/2)) + (288*a^3*cos(c/2 + (d*x)/2)^9)/(5*d*sin(c
/2 + (d*x)/2)) - (96*a^3*cos(c/2 + (d*x)/2)^11)/(5*d*sin(c/2 + (d*x)/2)) +
(3*a^3*cos(c/2 + (d*x)/2))/(2*d*sin(c/2 + (d*x)/2)) - (a^3*sin(c/2 + (d*x)/
2))/(2*d*cos(c/2 + (d*x)/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.525 $\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=133

$$\frac{a^3\sin^5(c+dx)}{5d}+\frac{3a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{3d}-\frac{5a^3\sin^2(c+dx)}{2d}-\frac{5a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc}{}$$

```
[Out] -3*a^3*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d+a^3*ln(sin(d*x+c))/d-5*a^3*sin(d
*x+c)/d-5/2*a^3*sin(d*x+c)^2/d+1/3*a^3*sin(d*x+c)^3/d+3/4*a^3*sin(d*x+c)^4/
d+1/5*a^3*sin(d*x+c)^5/d
```

Rubi [A] time = 0.13, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^3\sin^5(c+dx)}{5d}+\frac{3a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{3d}-\frac{5a^3\sin^2(c+dx)}{2d}-\frac{5a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) + (a^3*Log[Sin[c + d*x
]])/d - (5*a^3*Sin[c + d*x])/d - (5*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c
+ d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^4)/(4*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2(a+x)^5}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-5a^4+\frac{a^7}{x^3}+\frac{3a^6}{x^2}+\frac{a^5}{x}-5a^3x+a^2x^2+3ax^3+x^4\right)$$
$$= -\frac{3a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{a^3\log(\sin(c+dx))}{d}-$$

Mathematica [A] time = 0.15, size = 86, normalized size = 0.65

$$-\frac{a^3\left(-12\sin^5(c+dx)-45\sin^4(c+dx)-20\sin^3(c+dx)+150\sin^2(c+dx)+300\sin(c+dx)+30\csc^2(c+dx)\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/60*(a^3*(180*Csc[c + d*x] + 30*Csc[c + d*x]^2 - 60*Log[Sin[c + d*x]] + 3
00*Sin[c + d*x] + 150*Sin[c + d*x]^2 - 20*Sin[c + d*x]^3 - 45*Sin[c + d*x]^
4 - 12*Sin[c + d*x]^5))/d
```

fricas [A] time = 0.78, size = 145, normalized size = 1.09

$$\frac{360\,a^3\cos(dx+c)^6+120\,a^3\cos(dx+c)^4-855\,a^3\cos(dx+c)^2+615\,a^3+480\left(a^3\cos(dx+c)^2-a^3\right)\log\left(\frac{1}{2}\sin\right.}{480\left(d\cos(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/480*(360*a^3*cos(d*x + c)^6 + 120*a^3*cos(d*x + c)^4 - 855*a^3*cos(d*x +
c)^2 + 615*a^3 + 480*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*sin(d*x + c)) + 32*
(3*a^3*cos(d*x + c)^6 - 14*a^3*cos(d*x + c)^4 - 56*a^3*cos(d*x + c)^2 + 112
*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.32, size = 120, normalized size = 0.90

$$\frac{12\,a^3\sin(dx+c)^5+45\,a^3\sin(dx+c)^4+20\,a^3\sin(dx+c)^3-150\,a^3\sin(dx+c)^2+60\,a^3\log(|\sin(dx+c)|)-3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(12*a^3*sin(d*x + c)^5 + 45*a^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3
 - 150*a^3*sin(d*x + c)^2 + 60*a^3*log(abs(sin(d*x + c))) - 300*a^3*sin(d*x
 + c) - 30*(3*a^3*sin(d*x + c)^2 + 6*a^3*sin(d*x + c) + a^3)/sin(d*x + c)^2
)/d
```

maple [A] time = 0.46, size = 154, normalized size = 1.16

$$-\frac{112a^3\sin(dx+c)}{15d}-\frac{14a^3\left(\cos^4(dx+c)\right)\sin(dx+c)}{5d}-\frac{56a^3\left(\cos^2(dx+c)\right)\sin(dx+c)}{15d}+\frac{\left(\cos^4(dx+c)\right)a^3}{4d}+\frac{a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -112/15*a^3*sin(d*x+c)/d-14/5/d*a^3*cos(d*x+c)^4*sin(d*x+c)-56/15/d*a^3*cos
(d*x+c)^2*sin(d*x+c)+1/4/d*cos(d*x+c)^4*a^3+1/2/d*a^3*cos(d*x+c)^2+a^3*ln(s
in(d*x+c))/d-3/d*a^3/sin(d*x+c)*cos(d*x+c)^6-1/2/d*a^3/sin(d*x+c)^2*cos(d*x
+c)^6
```

maxima [A] time = 0.61, size = 106, normalized size = 0.80

$$\frac{12\,a^3\sin(dx+c)^5+45\,a^3\sin(dx+c)^4+20\,a^3\sin(dx+c)^3-150\,a^3\sin(dx+c)^2+60\,a^3\log(\sin(dx+c))-3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(12*a^3*sin(d*x + c)^5 + 45*a^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3
 - 150*a^3*sin(d*x + c)^2 + 60*a^3*log(sin(d*x + c)) - 300*a^3*sin(d*x + c)
 - 30*(6*a^3*sin(d*x + c) + a^3)/sin(d*x + c)^2)/d
```

mupad [B] time = 9.01, size = 342, normalized size = 2.57

$$\frac{a^3\ \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{46\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\frac{81\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{2}+\frac{538\,a^3\tan\left(\frac{c}{2}+\right.}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d - (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*a^3
*tan(c/2 + (d*x)/2))/(2*d) - ((5*a^3*tan(c/2 + (d*x)/2)^2)/2 + 70*a^3*tan(c
/2 + (d*x)/2)^3 + 45*a^3*tan(c/2 + (d*x)/2)^4 + (628*a^3*tan(c/2 + (d*x)/2)
^5)/3 + 77*a^3*tan(c/2 + (d*x)/2)^6 + (3796*a^3*tan(c/2 + (d*x)/2)^7)/15 +
(149*a^3*tan(c/2 + (d*x)/2)^8)/2 + (538*a^3*tan(c/2 + (d*x)/2)^9)/3 + (81*a
^3*tan(c/2 + (d*x)/2)^10)/2 + 46*a^3*tan(c/2 + (d*x)/2)^11 + a^3/2 + 6*a^3*
tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 20*tan(c/2 + (d*x)/2)^4 +
40*tan(c/2 + (d*x)/2)^6 + 40*tan(c/2 + (d*x)/2)^8 + 20*tan(c/2 + (d*x)/2)^1
0 + 4*tan(c/2 + (d*x)/2)^12)) - (a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.526 $\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=131

$$\frac{a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{d}+\frac{a^3\sin^2(c+dx)}{2d}-\frac{5a^3\sin(c+dx)}{d}-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc(c}{d}$$

```
[Out] -a^3*csc(d*x+c)/d-3/2*a^3*csc(d*x+c)^2/d-1/3*a^3*csc(d*x+c)^3/d-5*a^3*ln(si
n(d*x+c))/d-5*a^3*sin(d*x+c)/d+1/2*a^3*sin(d*x+c)^2/d+a^3*sin(d*x+c)^3/d+1/
4*a^3*sin(d*x+c)^4/d
```

Rubi [A] time = 0.11, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{d}+\frac{a^3\sin^2(c+dx)}{2d}-\frac{5a^3\sin(c+dx)}{d}-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) - (3*a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^
3)/(3*d) - (5*a^3*Log[Sin[c + d*x]])/d - (5*a^3*Sin[c + d*x])/d + (a^3*Sin[
c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/d + (a^3*Sin[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)^5}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-5a^3+\frac{a^7}{x^4}+\frac{3a^6}{x^3}+\frac{a^5}{x^2}-\frac{5a^4}{x}+a^2x+3ax^2+x^3\right) dx$$
$$= -\frac{a^3\csc(c+dx)}{d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{3d}-\frac{5a^3 1$$

Mathematica [A] time = 0.26, size = 86, normalized size = 0.66

$$-\frac{a^3\left(-3\sin^4(c+dx)-12\sin^3(c+dx)-6\sin^2(c+dx)+60\sin(c+dx)+4\csc^3(c+dx)+18\csc^2(c+dx)+12\,\text{c}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/12*(a^3*(12*Csc[c + d*x] + 18*Csc[c + d*x]^2 + 4*Csc[c + d*x]^3 + 60*Log
[Sin[c + d*x]] + 60*Sin[c + d*x] - 6*Sin[c + d*x]^2 - 12*Sin[c + d*x]^3 - 3
*Sin[c + d*x]^4))/d
```

fricas [A] time = 0.77, size = 159, normalized size = 1.21

$$\frac{96\,a^3\cos(dx+c)^6+192\,a^3\cos(dx+c)^4-768\,a^3\cos(dx+c)^2+512\,a^3-480\left(a^3\cos(dx+c)^2-a^3\right)\log\left(\frac{1}{2}\,\sin}{96\left(d\cos(dx+c)^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/96*(96*a^3*cos(d*x + c)^6 + 192*a^3*cos(d*x + c)^4 - 768*a^3*cos(d*x + c)
^2 + 512*a^3 - 480*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*sin(d*x + c))*sin(d*x
 + c) + 3*(8*a^3*cos(d*x + c)^6 - 40*a^3*cos(d*x + c)^4 + 45*a^3*cos(d*x +
c)^2 + 35*a^3)*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.32, size = 122, normalized size = 0.93

$$\frac{3\,a^3\sin(dx+c)^4+12\,a^3\sin(dx+c)^3+6\,a^3\sin(dx+c)^2-60\,a^3\log(|\sin(dx+c)|)-60\,a^3\sin(dx+c)+\frac{2(55\,a}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^3*sin(d*x + c)^4 + 12*a^3*sin(d*x + c)^3 + 6*a^3*sin(d*x + c)^2 -
 60*a^3*log(abs(sin(d*x + c))) - 60*a^3*sin(d*x + c) + 2*(55*a^3*sin(d*x +
c)^3 - 6*a^3*sin(d*x + c)^2 - 9*a^3*sin(d*x + c) - 2*a^3)/sin(d*x + c)^3)/d
```

maple [A] time = 0.43, size = 179, normalized size = 1.37

$$-\frac{5\left(\cos^4(dx+c)\right)a^3}{4d}-\frac{5a^3\left(\cos^2(dx+c)\right)}{2d}-\frac{5a^3\ln(\sin(dx+c))}{d}-\frac{2a^3\left(\cos^6(dx+c)\right)}{d\sin(dx+c)}-\frac{16a^3\sin(dx+c)}{3d}-\frac{2a^3\left(\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -5/4/d*cos(d*x+c)^4*a^3-5/2/d*a^3*cos(d*x+c)^2-5*a^3*ln(sin(d*x+c))/d-2/d*a
^3/sin(d*x+c)*cos(d*x+c)^6-16/3*a^3*sin(d*x+c)/d-2/d*a^3*cos(d*x+c)^4*sin(d
*x+c)-8/3/d*a^3*cos(d*x+c)^2*sin(d*x+c)-3/2/d*a^3/sin(d*x+c)^2*cos(d*x+c)^6
-1/3/d*a^3/sin(d*x+c)^3*cos(d*x+c)^6
```

maxima [A] time = 0.45, size = 108, normalized size = 0.82

$$\frac{3\,a^3\sin(dx+c)^4+12\,a^3\sin(dx+c)^3+6\,a^3\sin(dx+c)^2-60\,a^3\log(\sin(dx+c))-60\,a^3\sin(dx+c)-\frac{2(6\,a^3\mathrm{s}}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(3*a^3*sin(d*x + c)^4 + 12*a^3*sin(d*x + c)^3 + 6*a^3*sin(d*x + c)^2 -
 60*a^3*log(sin(d*x + c)) - 60*a^3*sin(d*x + c) - 2*(6*a^3*sin(d*x + c)^2 +
 9*a^3*sin(d*x + c) + 2*a^3)/sin(d*x + c)^3)/d
```

mupad [B] time = 8.98, size = 333, normalized size = 2.54

$$\frac{5\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{5\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] (5*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a^3*tan(c/2 + (d*x)/2)^3)/(24*d)
 - (5*a^3*log(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2))/(8*d) - (
3*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - ((19*a^3*tan(c/2 + (d*x)/2)^2)/3 + 12*a
^3*tan(c/2 + (d*x)/2)^3 + 102*a^3*tan(c/2 + (d*x)/2)^4 + 2*a^3*tan(c/2 + (d
*x)/2)^5 + (622*a^3*tan(c/2 + (d*x)/2)^6)/3 - 52*a^3*tan(c/2 + (d*x)/2)^7 +
 (589*a^3*tan(c/2 + (d*x)/2)^8)/3 - 13*a^3*tan(c/2 + (d*x)/2)^9 + 85*a^3*ta
n(c/2 + (d*x)/2)^10 + a^3/3 + 3*a^3*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*
x)/2)^3 + 32*tan(c/2 + (d*x)/2)^5 + 48*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 +
(d*x)/2)^9 + 8*tan(c/2 + (d*x)/2)^11))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.527 $\int \cot^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=131

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{5a^3\csc(c}{d}$$

```
[Out] 5*a^3*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d-a^3*csc(d*x+c)^3/d-1/4*a^3*csc(d*
x+c)^4/d-5*a^3*ln(sin(d*x+c))/d+a^3*sin(d*x+c)/d+3/2*a^3*sin(d*x+c)^2/d+1/3
*a^3*sin(d*x+c)^3/d
```

Rubi [A] time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{5a^3\csc(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^3)/
d - (a^3*Csc[c + d*x]^4)/(4*d) - (5*a^3*Log[Sin[c + d*x]])/d + (a^3*Sin[c +
 d*x])/d + (3*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2+\frac{a^7}{x^5}+\frac{3a^6}{x^4}+\frac{a^5}{x^3}-\frac{5a^4}{x^2}-\frac{5a^3}{x}+3ax+x^2\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{5a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{d}-\frac{a^3\csc^4(c+dx)}{4d}-$$

Mathematica [A] time = 0.50, size = 86, normalized size = 0.66

$$\frac{a^3\left(4\sin^3(c+dx)+18\sin^2(c+dx)+12\sin(c+dx)-3\csc^4(c+dx)-12\csc^3(c+dx)-6\csc^2(c+dx)+60\csc(\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(60*Csc[c + d*x] - 6*Csc[c + d*x]^2 - 12*Csc[c + d*x]^3 - 3*Csc[c + d*
x]^4 - 60*Log[Sin[c + d*x]] + 12*Sin[c + d*x] + 18*Sin[c + d*x]^2 + 4*Sin[c
 + d*x]^3))/(12*d)
```

fricas [A] time = 0.79, size = 159, normalized size = 1.21

$$-\frac{18\,a^3\cos(dx+c)^6-45\,a^3\cos(dx+c)^4+30\,a^3\cos(dx+c)^2+60\left(a^3\cos(dx+c)^4-2\,a^3\cos(dx+c)^2+a^3\right)\text{lo}}{12\left(d\cos(dx+c)^4-2\,d\,\text{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(18*a^3*cos(d*x + c)^6 - 45*a^3*cos(d*x + c)^4 + 30*a^3*cos(d*x + c)^
2 + 60*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*sin(d*x +
c)) + 4*(a^3*cos(d*x + c)^6 - 6*a^3*cos(d*x + c)^4 + 24*a^3*cos(d*x + c)^2
- 16*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.36, size = 121, normalized size = 0.92

$$\frac{4\,a^3\sin(dx+c)^3+18\,a^3\sin(dx+c)^2-60\,a^3\log(|\sin(dx+c)|)+12\,a^3\sin(dx+c)+\frac{125\,a^3\sin(dx+c)^4+60\,a^3\sin(dx+c)}{\text{sin}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*(4*a^3*sin(d*x + c)^3 + 18*a^3*sin(d*x + c)^2 - 60*a^3*log(abs(sin(d*x
 + c))) + 12*a^3*sin(d*x + c) + (125*a^3*sin(d*x + c)^4 + 60*a^3*sin(d*x +
c)^3 - 6*a^3*sin(d*x + c)^2 - 12*a^3*sin(d*x + c) - 3*a^3)/sin(d*x + c)^4)/
d
```

maple [A] time = 0.43, size = 211, normalized size = 1.61

$$\frac{2a^3\left(\cos^6(dx+c)\right)}{d\sin(dx+c)}+\frac{16a^3\sin(dx+c)}{3d}+\frac{2a^3\left(\cos^4(dx+c)\right)\sin(dx+c)}{d}+\frac{8a^3\left(\cos^2(dx+c)\right)\sin(dx+c)}{3d}-\frac{3a^3\left(c\right.}{2d\,\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d*a^3/sin(d*x+c)*cos(d*x+c)^6+16/3*a^3*sin(d*x+c)/d+2/d*a^3*cos(d*x+c)^4*
sin(d*x+c)+8/3/d*a^3*cos(d*x+c)^2*sin(d*x+c)-3/2/d*a^3/sin(d*x+c)^2*cos(d*x
+c)^6-3/2/d*cos(d*x+c)^4*a^3-3/d*a^3*cos(d*x+c)^2-5*a^3*ln(sin(d*x+c))/d-1/
d*a^3/sin(d*x+c)^3*cos(d*x+c)^6-1/4/d*a^3*cot(d*x+c)^4+1/2/d*a^3*cot(d*x+c)
^2
```

maxima [A] time = 0.52, size = 108, normalized size = 0.82

$$\frac{4\,a^3\sin(dx+c)^3+18\,a^3\sin(dx+c)^2-60\,a^3\log(\sin(dx+c))+12\,a^3\sin(dx+c)+\frac{3\left(20\,a^3\sin(dx+c)^3-2\,a^3\sin(dx+c\right.}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(4*a^3*sin(d*x + c)^3 + 18*a^3*sin(d*x + c)^2 - 60*a^3*log(sin(d*x + c
)) + 12*a^3*sin(d*x + c) + 3*(20*a^3*sin(d*x + c)^3 - 2*a^3*sin(d*x + c)^2
- 4*a^3*sin(d*x + c) - a^3)/sin(d*x + c)^4)/d
```

mupad [B] time = 8.94, size = 322, normalized size = 2.46

$$\frac{66\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+93\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{620\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+\frac{347\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{4}+128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{39\,a^3\tan\left(\right.}{4}}{d\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] (28*a^3*tan(c/2 + (d*x)/2)^3 - (15*a^3*tan(c/2 + (d*x)/2)^2)/4 - (39*a^3*ta
n(c/2 + (d*x)/2)^4)/4 + 128*a^3*tan(c/2 + (d*x)/2)^5 + (347*a^3*tan(c/2 + (
d*x)/2)^6)/4 + (620*a^3*tan(c/2 + (d*x)/2)^7)/3 + 93*a^3*tan(c/2 + (d*x)/2)
^8 + 66*a^3*tan(c/2 + (d*x)/2)^9 - a^3/4 - 2*a^3*tan(c/2 + (d*x)/2))/(d*(16
*tan(c/2 + (d*x)/2)^4 + 48*tan(c/2 + (d*x)/2)^6 + 48*tan(c/2 + (d*x)/2)^8 +
 16*tan(c/2 + (d*x)/2)^10)) - (a^3*tan(c/2 + (d*x)/2)^3)/(8*d) - (a^3*tan(c
/2 + (d*x)/2)^4)/(64*d) - (3*a^3*tan(c/2 + (d*x)/2)^2)/(16*d) - (5*a^3*log(
tan(c/2 + (d*x)/2)))/d + (17*a^3*tan(c/2 + (d*x)/2))/(8*d) + (5*a^3*log(tan
(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.528 $\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=133

$$\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\csc^5(c+dx)}{5d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{3d}+\frac{5a^3\csc^2(c+dx)}{2d}+\frac{5a^3\csc}{}$$

```
[Out] 5*a^3*csc(d*x+c)/d+5/2*a^3*csc(d*x+c)^2/d-1/3*a^3*csc(d*x+c)^3/d-3/4*a^3*cs
c(d*x+c)^4/d-1/5*a^3*csc(d*x+c)^5/d+a^3*ln(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d
+1/2*a^3*sin(d*x+c)^2/d
```

Rubi [A] time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\csc^5(c+dx)}{5d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{3d}+\frac{5a^3\csc^2(c+dx)}{2d}+\frac{5a^3\csc}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*Csc[c + d*x])/d + (5*a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^3
)/(3*d) - (3*a^3*Csc[c + d*x]^4)/(4*d) - (a^3*Csc[c + d*x]^5)/(5*d) + (a^3*
Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2(a+x)^5}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(3a+\frac{a^7}{x^6}+\frac{3a^6}{x^5}+\frac{a^5}{x^4}-\frac{5a^4}{x^3}-\frac{5a^3}{x^2}+\frac{a^2}{x}+x\right)dx, x, a\right.}{d}$$
$$= \frac{5a^3\csc(c+dx)}{d}+\frac{5a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\,\text{c}}{}$$

Mathematica [A] time = 0.17, size = 86, normalized size = 0.65

$$\frac{a^3\left(30\sin^2(c+dx)+180\sin(c+dx)-12\csc^5(c+dx)-45\csc^4(c+dx)-20\csc^3(c+dx)+150\csc^2(c+dx)+3\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(300*Csc[c + d*x] + 150*Csc[c + d*x]^2 - 20*Csc[c + d*x]^3 - 45*Csc[c
+ d*x]^4 - 12*Csc[c + d*x]^5 + 60*Log[Sin[c + d*x]] + 180*Sin[c + d*x] + 30
*Sin[c + d*x]^2))/(60*d)
```

fricas [A] time = 0.69, size = 179, normalized size = 1.35

$$-\frac{180\,a^3\cos(dx+c)^6-840\,a^3\cos(dx+c)^4+1120\,a^3\cos(dx+c)^2-448\,a^3-60\left(a^3\cos(dx+c)^4-2\,a^3\cos(dx\right.}{60\left(d\cos(dx+c)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/60*(180*a^3*cos(d*x + c)^6 - 840*a^3*cos(d*x + c)^4 + 1120*a^3*cos(d*x +
 c)^2 - 448*a^3 - 60*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(
1/2*sin(d*x + c))*sin(d*x + c) + 15*(2*a^3*cos(d*x + c)^6 - 5*a^3*cos(d*x +
 c)^4 + 14*a^3*cos(d*x + c)^2 - 8*a^3)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2
*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.35, size = 122, normalized size = 0.92

$$\frac{30\,a^3\sin\left(dx+c\right)^2+60\,a^3\log\left(|\sin\left(dx+c\right)|\right)+180\,a^3\sin\left(dx+c\right)-\frac{137\,a^3\sin(dx+c)^5-300\,a^3\sin(dx+c)^4-150\,a^3\sin(dx+c)^3+}{\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(30*a^3*sin(d*x + c)^2 + 60*a^3*log(abs(sin(d*x + c))) + 180*a^3*sin(d
*x + c) - (137*a^3*sin(d*x + c)^5 - 300*a^3*sin(d*x + c)^4 - 150*a^3*sin(d*
x + c)^3 + 20*a^3*sin(d*x + c)^2 + 45*a^3*sin(d*x + c) + 12*a^3)/sin(d*x +
c)^5)/d
```

maple [A] time = 0.44, size = 234, normalized size = 1.76

$$-\frac{a^3\left(\cos^6\left(dx+c\right)\right)}{2d\sin\left(dx+c\right)^2}-\frac{\left(\cos^4\left(dx+c\right)\right)a^3}{2d}-\frac{a^3\left(\cos^2\left(dx+c\right)\right)}{d}+\frac{a^3\ln\left(\sin\left(dx+c\right)\right)}{d}-\frac{14a^3\left(\cos^6\left(dx+c\right)\right)}{15d\sin\left(dx+c\right)^3}+\frac{14a^3\left(\mathrm{co}\right.}{5d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3/sin(d*x+c)^2*cos(d*x+c)^6-1/2/d*cos(d*x+c)^4*a^3-1/d*a^3*cos(d*x
+c)^2+a^3*ln(sin(d*x+c))/d-14/15/d*a^3/sin(d*x+c)^3*cos(d*x+c)^6+14/5/d*a^3
/sin(d*x+c)*cos(d*x+c)^6+112/15*a^3*sin(d*x+c)/d+14/5/d*a^3*cos(d*x+c)^4*si
n(d*x+c)+56/15/d*a^3*cos(d*x+c)^2*sin(d*x+c)-3/4/d*a^3*cot(d*x+c)^4+3/2/d*a
^3*cot(d*x+c)^2-1/5/d*a^3/sin(d*x+c)^5*cos(d*x+c)^6
```

maxima [A] time = 0.75, size = 107, normalized size = 0.80

$$\frac{30\,a^3\sin\left(dx+c\right)^2+60\,a^3\log\left(\sin\left(dx+c\right)\right)+180\,a^3\sin\left(dx+c\right)+\frac{300\,a^3\sin(dx+c)^4+150\,a^3\sin(dx+c)^3-20\,a^3\sin(dx+c)^2-45}{\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(30*a^3*sin(d*x + c)^2 + 60*a^3*log(sin(d*x + c)) + 180*a^3*sin(d*x +
c) + (300*a^3*sin(d*x + c)^4 + 150*a^3*sin(d*x + c)^3 - 20*a^3*sin(d*x + c)
^2 - 45*a^3*sin(d*x + c) - 12*a^3)/sin(d*x + c)^5)/d
```

mupad [B] time = 8.95, size = 311, normalized size = 2.34

$$\frac{7a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{16d}-\frac{7a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{96d}-\frac{3a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160d}+\frac{a^3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{266a^3\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] (7*a^3*tan(c/2 + (d*x)/2)^2)/(16*d) - (7*a^3*tan(c/2 + (d*x)/2)^3)/(96*d) -
 (3*a^3*tan(c/2 + (d*x)/2)^4)/(64*d) - (a^3*tan(c/2 + (d*x)/2)^5)/(160*d) +
 (a^3*log(tan(c/2 + (d*x)/2)))/d + (11*a^3*tan(c/2 + (d*x)/2)^3 - (41*a^3*t
an(c/2 + (d*x)/2)^2)/15 + (1037*a^3*tan(c/2 + (d*x)/2)^4)/15 + (53*a^3*tan(
c/2 + (d*x)/2)^5)/2 + (1013*a^3*tan(c/2 + (d*x)/2)^6)/3 + 78*a^3*tan(c/2 +
(d*x)/2)^7 + 266*a^3*tan(c/2 + (d*x)/2)^8 - a^3/5 - (3*a^3*tan(c/2 + (d*x)/
2))/2)/(d*(32*tan(c/2 + (d*x)/2)^5 + 64*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 +
 (d*x)/2)^9)) + (37*a^3*tan(c/2 + (d*x)/2))/(16*d) - (a^3*log(tan(c/2 + (d*
x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.529 $\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=133

$$\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^6(c+dx)}{6d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^4(c+dx)}{4d}+\frac{5a^3\csc^3(c+dx)}{3d}+\frac{5a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc(}{d}$$

```
[Out] -a^3*csc(d*x+c)/d+5/2*a^3*csc(d*x+c)^2/d+5/3*a^3*csc(d*x+c)^3/d-1/4*a^3*csc
(d*x+c)^4/d-3/5*a^3*csc(d*x+c)^5/d-1/6*a^3*csc(d*x+c)^6/d+3*a^3*ln(sin(d*x+
c))/d+a^3*sin(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^6(c+dx)}{6d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^4(c+dx)}{4d}+\frac{5a^3\csc^3(c+dx)}{3d}+\frac{5a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) + (5*a^3*Csc[c + d*x]^2)/(2*d) + (5*a^3*Csc[c + d*x
]^3)/(3*d) - (a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c + d*x]^5)/(5*d) - (a
^3*Csc[c + d*x]^6)/(6*d) + (3*a^3*Log[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])
/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a-x)^2(a+x)^5}{x^7}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(1+\frac{a^7}{x^7}+\frac{3a^6}{x^6}+\frac{a^5}{x^5}-\frac{5a^4}{x^4}-\frac{5a^3}{x^3}+\frac{a^2}{x^2}+\frac{3a}{x}\right)dx, x,\right.}{d}$$
$$= -\frac{a^3\csc(c+dx)}{d}+\frac{5a^3\csc^2(c+dx)}{2d}+\frac{5a^3\csc^3(c+dx)}{3d}-\frac{a^3}{}$$

Mathematica [A] time = 0.04, size = 113, normalized size = 0.85

$$a^3\left(\frac{\sin(c+dx)}{d}-\frac{\csc^6(c+dx)}{6d}-\frac{3\csc^5(c+dx)}{5d}-\frac{\csc^4(c+dx)}{4d}+\frac{5\csc^3(c+dx)}{3d}+\frac{5\csc^2(c+dx)}{2d}-\frac{\csc(c+dx)}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*(-(Csc[c + d*x]/d) + (5*Csc[c + d*x]^2)/(2*d) + (5*Csc[c + d*x]^3)/(3*d
) - Csc[c + d*x]^4/(4*d) - (3*Csc[c + d*x]^5)/(5*d) - Csc[c + d*x]^6/(6*d)
+ (3*Log[Sin[c + d*x]])/d + Sin[c + d*x]/d)
```

fricas [A] time = 0.85, size = 180, normalized size = 1.35

$$-\frac{150\,a^3\cos(dx+c)^4-285\,a^3\cos(dx+c)^2+125\,a^3-180\left(a^3\cos(dx+c)^6-3\,a^3\cos(dx+c)^4+3\,a^3\cos(dx+\right.}{60\left(d\cos(dx+c)^6-3\,d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/60*(150*a^3*cos(d*x + c)^4 - 285*a^3*cos(d*x + c)^2 + 125*a^3 - 180*(a^3
*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/
2*sin(d*x + c)) - 4*(15*a^3*cos(d*x + c)^6 - 30*a^3*cos(d*x + c)^4 + 40*a^3
*cos(d*x + c)^2 - 16*a^3)*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c
)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.41, size = 122, normalized size = 0.92

$$\frac{180\,a^3\log\left(|\sin\left(dx+c\right)|\right)+60\,a^3\sin\left(dx+c\right)-\frac{441\,a^3\sin(dx+c)^6+60\,a^3\sin(dx+c)^5-150\,a^3\sin(dx+c)^4-100\,a^3\sin(dx+c)^3+15\,a^3\sin(}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(180*a^3*log(abs(sin(d*x + c))) + 60*a^3*sin(d*x + c) - (441*a^3*sin(d
*x + c)^6 + 60*a^3*sin(d*x + c)^5 - 150*a^3*sin(d*x + c)^4 - 100*a^3*sin(d*
x + c)^3 + 15*a^3*sin(d*x + c)^2 + 36*a^3*sin(d*x + c) + 10*a^3)/sin(d*x +
c)^6)/d
```

maple [A] time = 0.52, size = 203, normalized size = 1.53

$$-\frac{2a^3\left(\cos^6\left(dx+c\right)\right)}{15d\sin\left(dx+c\right)^3}+\frac{2a^3\left(\cos^6\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)}+\frac{16a^3\sin\left(dx+c\right)}{15d}+\frac{2a^3\left(\cos^4\left(dx+c\right)\right)\sin\left(dx+c\right)}{5d}+\frac{8a^3\left(\cos^2\left(dx+\right.\right.}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/15/d*a^3/sin(d*x+c)^3*cos(d*x+c)^6+2/5/d*a^3/sin(d*x+c)*cos(d*x+c)^6+16/
15*a^3*sin(d*x+c)/d+2/5/d*a^3*cos(d*x+c)^4*sin(d*x+c)+8/15/d*a^3*cos(d*x+c)
^2*sin(d*x+c)-3/4/d*a^3*cot(d*x+c)^4+3/2/d*a^3*cot(d*x+c)^2+3*a^3*ln(sin(d*
x+c))/d-3/5/d*a^3/sin(d*x+c)^5*cos(d*x+c)^6-1/6/d*a^3/sin(d*x+c)^6*cos(d*x+
c)^6
```

maxima [A] time = 0.53, size = 108, normalized size = 0.81

$$\frac{180\,a^3\log\left(\sin\left(dx+c\right)\right)+60\,a^3\sin\left(dx+c\right)-\frac{60\,a^3\sin(dx+c)^5-150\,a^3\sin(dx+c)^4-100\,a^3\sin(dx+c)^3+15\,a^3\sin(dx+c)^2+36\,a^3\sin(dx}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(180*a^3*log(sin(d*x + c)) + 60*a^3*sin(d*x + c) - (60*a^3*sin(d*x + c
)^5 - 150*a^3*sin(d*x + c)^4 - 100*a^3*sin(d*x + c)^3 + 15*a^3*sin(d*x + c)
^2 + 36*a^3*sin(d*x + c) + 10*a^3)/sin(d*x + c)^6)/d
```

mupad [B] time = 9.84, size = 296, normalized size = 2.23

$$\frac{67\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{11\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32\,d}-\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}+\frac{a^3\left(5760\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] (67*a^3*tan(c/2 + (d*x)/2)^2)/(128*d) + (11*a^3*tan(c/2 + (d*x)/2)^3)/(96*d
) - (a^3*tan(c/2 + (d*x)/2)^4)/(32*d) - (3*a^3*tan(c/2 + (d*x)/2)^5)/(160*d
) - (a^3*tan(c/2 + (d*x)/2)^6)/(384*d) + (a^3*(5760*log(tan(c/2 + (d*x)/2))
 - 5760*log(tan(c/2 + (d*x)/2)^2 + 1)))/(1920*d) - (a^3*tan(c/2 + (d*x)/2))
/(16*d) + (cot(c/2 + (d*x)/2)^6*((23*a^3*tan(c/2 + (d*x)/2)^3)/240 - (13*a^
3*tan(c/2 + (d*x)/2)^2)/384 + (63*a^3*tan(c/2 + (d*x)/2)^4)/128 + (5*a^3*ta
n(c/2 + (d*x)/2)^5)/96 + (67*a^3*tan(c/2 + (d*x)/2)^6)/128 + (31*a^3*tan(c/
2 + (d*x)/2)^7)/16 - a^3/384 - (3*a^3*tan(c/2 + (d*x)/2))/160))/(d*(tan(c/2
 + (d*x)/2)^2 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.530 $\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=145

$$\frac{a^4\sin^5(c+dx)}{5d}+\frac{a^4\sin^4(c+dx)}{d}+\frac{4a^4\sin^3(c+dx)}{3d}-\frac{2a^4\sin^2(c+dx)}{d}-\frac{10a^4\sin(c+dx)}{d}-\frac{a^4\csc^3(c+dx)}{3d}-\frac{2a^4\mathrm{cs}}{}$$

```
[Out] -4*a^4*csc(d*x+c)/d-2*a^4*csc(d*x+c)^2/d-1/3*a^4*csc(d*x+c)^3/d-4*a^4*ln(si
n(d*x+c))/d-10*a^4*sin(d*x+c)/d-2*a^4*sin(d*x+c)^2/d+4/3*a^4*sin(d*x+c)^3/d
+a^4*sin(d*x+c)^4/d+1/5*a^4*sin(d*x+c)^5/d
```

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^4\sin^5(c+dx)}{5d}+\frac{a^4\sin^4(c+dx)}{d}+\frac{4a^4\sin^3(c+dx)}{3d}-\frac{2a^4\sin^2(c+dx)}{d}-\frac{10a^4\sin(c+dx)}{d}-\frac{a^4\csc^3(c+dx)}{3d}-\frac{2a^4\mathrm{cs}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a^4*Csc[c + d*x])/d - (2*a^4*Csc[c + d*x]^2)/d - (a^4*Csc[c + d*x]^3)/(
3*d) - (4*a^4*Log[Sin[c + d*x]])/d - (10*a^4*Sin[c + d*x])/d - (2*a^4*Sin[c
 + d*x]^2)/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/d + (a^4
*Sin[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)^6}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^6}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-10a^4+\frac{a^8}{x^4}+\frac{4a^7}{x^3}+\frac{4a^6}{x^2}-\frac{4a^5}{x}-4a^3x+4a^2x^2+4a\right.\right.}{ad}$$
$$= -\frac{4a^4\csc(c+dx)}{d}-\frac{2a^4\csc^2(c+dx)}{d}-\frac{a^4\csc^3(c+dx)}{3d}-\frac{4a^4}{}$$

Mathematica [A] time = 0.16, size = 96, normalized size = 0.66

$$-\frac{a^4\left(-3\sin^5(c+dx)-15\sin^4(c+dx)-20\sin^3(c+dx)+30\sin^2(c+dx)+150\sin(c+dx)+5\csc^3(c+dx)+30\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/15*(a^4*(60*Csc[c + d*x] + 30*Csc[c + d*x]^2 + 5*Csc[c + d*x]^3 + 60*Log
[Sin[c + d*x]] + 150*Sin[c + d*x] + 30*Sin[c + d*x]^2 - 20*Sin[c + d*x]^3 -
 15*Sin[c + d*x]^4 - 3*Sin[c + d*x]^5))/d
```

fricas [A] time = 0.68, size = 172, normalized size = 1.19

$$-\frac{24\,a^4\cos(dx+c)^8-256\,a^4\cos(dx+c)^6-576\,a^4\cos(dx+c)^4+2304\,a^4\cos(dx+c)^2-1536\,a^4+480\left(a^4\cos\right.}{120\left(d\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/120*(24*a^4*cos(d*x + c)^8 - 256*a^4*cos(d*x + c)^6 - 576*a^4*cos(d*x +
c)^4 + 2304*a^4*cos(d*x + c)^2 - 1536*a^4 + 480*(a^4*cos(d*x + c)^2 - a^4)*
log(1/2*sin(d*x + c))*sin(d*x + c) - 15*(8*a^4*cos(d*x + c)^6 - 8*a^4*cos(d
```

```
*x + c)^4 - 3*a^4*cos(d*x + c)^2 + 19*a^4)*sin(d*x + c))/((d*cos(d*x + c)^2
 - d)*sin(d*x + c))
```

giac [A] time = 0.41, size = 135, normalized size = 0.93

$$\frac{3\,a^4\sin\left(dx+c\right)^5+15\,a^4\sin\left(dx+c\right)^4+20\,a^4\sin\left(dx+c\right)^3-30\,a^4\sin\left(dx+c\right)^2-60\,a^4\log\left(\left|\sin\left(dx+c\right)\right|\right)-150}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/15*(3*a^4*sin(d*x + c)^5 + 15*a^4*sin(d*x + c)^4 + 20*a^4*sin(d*x + c)^3
- 30*a^4*sin(d*x + c)^2 - 60*a^4*log(abs(sin(d*x + c))) - 150*a^4*sin(d*x +
 c) + 5*(22*a^4*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 - 6*a^4*sin(d*x + c)
 - a^4)/sin(d*x + c)^3)/d
```

maple [A] time = 0.45, size = 179, normalized size = 1.23

$$-\frac{64a^4\sin\left(dx+c\right)}{5d}-\frac{24a^4\sin\left(dx+c\right)\left(\cos^4\left(dx+c\right)\right)}{5d}-\frac{32a^4\left(\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)}{5d}-\frac{a^4\left(\cos^4\left(dx+c\right)\right)}{d}-\frac{2a^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -64/5*a^4*sin(d*x+c)/d-24/5/d*a^4*sin(d*x+c)*cos(d*x+c)^4-32/5/d*a^4*cos(d*
x+c)^2*sin(d*x+c)-1/d*a^4*cos(d*x+c)^4-2/d*a^4*cos(d*x+c)^2-4*a^4*ln(sin(d*
x+c))/d-5/d*a^4/sin(d*x+c)*cos(d*x+c)^6-2/d*a^4/sin(d*x+c)^2*cos(d*x+c)^6-1
/3/d*a^4/sin(d*x+c)^3*cos(d*x+c)^6
```

maxima [A] time = 0.79, size = 119, normalized size = 0.82

$$\frac{3\,a^4\sin\left(dx+c\right)^5+15\,a^4\sin\left(dx+c\right)^4+20\,a^4\sin\left(dx+c\right)^3-30\,a^4\sin\left(dx+c\right)^2-60\,a^4\log\left(\sin\left(dx+c\right)\right)-150}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/15*(3*a^4*sin(d*x + c)^5 + 15*a^4*sin(d*x + c)^4 + 20*a^4*sin(d*x + c)^3
- 30*a^4*sin(d*x + c)^2 - 60*a^4*log(sin(d*x + c)) - 150*a^4*sin(d*x + c) -
 5*(12*a^4*sin(d*x + c)^2 + 6*a^4*sin(d*x + c) + a^4)/sin(d*x + c)^3)/d
```

mupad [B] time = 8.97, size = 378, normalized size = 2.61

$$\frac{4\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{4\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{177\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+68\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^4)/sin(c + d*x)^4,x)
```

```
[Out] (4*a^4*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a^4*tan(c/2 + (d*x)/2)^3)/(24*d)
 - (4*a^4*log(tan(c/2 + (d*x)/2)))/d - ((56*a^4*tan(c/2 + (d*x)/2)^2)/3 + 2
0*a^4*tan(c/2 + (d*x)/2)^3 + (745*a^4*tan(c/2 + (d*x)/2)^4)/3 + 104*a^4*tan
(c/2 + (d*x)/2)^5 + 728*a^4*tan(c/2 + (d*x)/2)^6 + 104*a^4*tan(c/2 + (d*x)/
2)^7 + (4549*a^4*tan(c/2 + (d*x)/2)^8)/5 + 84*a^4*tan(c/2 + (d*x)/2)^9 + 64
0*a^4*tan(c/2 + (d*x)/2)^10 + 68*a^4*tan(c/2 + (d*x)/2)^11 + 177*a^4*tan(c/
2 + (d*x)/2)^12 + a^4/3 + 4*a^4*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2
)^3 + 40*tan(c/2 + (d*x)/2)^5 + 80*tan(c/2 + (d*x)/2)^7 + 80*tan(c/2 + (d*x
)/2)^9 + 40*tan(c/2 + (d*x)/2)^11 + 8*tan(c/2 + (d*x)/2)^13)) - (17*a^4*tan
(c/2 + (d*x)/2))/(8*d) - (a^4*tan(c/2 + (d*x)/2)^2)/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.531 $\int \cot^5(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=148

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}-\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^4(c+dx)}{4d}-\frac{4a^4\csc^3(c+dx)}{3d}-\frac{2a^4\mathrm{cs}}{}$$

```
[Out] 4*a^4*csc(d*x+c)/d-2*a^4*csc(d*x+c)^2/d-4/3*a^4*csc(d*x+c)^3/d-1/4*a^4*csc(
d*x+c)^4/d-10*a^4*ln(sin(d*x+c))/d-4*a^4*sin(d*x+c)/d+2*a^4*sin(d*x+c)^2/d+
4/3*a^4*sin(d*x+c)^3/d+1/4*a^4*sin(d*x+c)^4/d
```

Rubi [A] time = 0.08, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}-\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^4(c+dx)}{4d}-\frac{4a^4\csc^3(c+dx)}{3d}-\frac{2a^4\mathrm{cs}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (4*a^4*Csc[c + d*x])/d - (2*a^4*Csc[c + d*x]^2)/d - (4*a^4*Csc[c + d*x]^3)/
(3*d) - (a^4*Csc[c + d*x]^4)/(4*d) - (10*a^4*Log[Sin[c + d*x]])/d - (4*a^4*
Sin[c + d*x])/d + (2*a^4*Sin[c + d*x]^2)/d + (4*a^4*Sin[c + d*x]^3)/(3*d) +
 (a^4*Sin[c + d*x]^4)/(4*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^6}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^3+\frac{a^8}{x^5}+\frac{4a^7}{x^4}+\frac{4a^6}{x^3}-\frac{4a^5}{x^2}-\frac{10a^4}{x}+4a^2x+4ax^2+x^3\right)dx, x,\right.}{d}$$
$$= \frac{4a^4\csc(c+dx)}{d}-\frac{2a^4\csc^2(c+dx)}{d}-\frac{4a^4\csc^3(c+dx)}{3d}-\frac{a^4\csc^4(c+dx)}{4d}$$

Mathematica [A] time = 0.15, size = 96, normalized size = 0.65

$$\frac{a^4\left(3\sin^4(c+dx)+16\sin^3(c+dx)+24\sin^2(c+dx)-48\sin(c+dx)-3\csc^4(c+dx)-16\csc^3(c+dx)-24\csc\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(48*Csc[c + d*x] - 24*Csc[c + d*x]^2 - 16*Csc[c + d*x]^3 - 3*Csc[c + d
*x]^4 - 120*Log[Sin[c + d*x]] - 48*Sin[c + d*x] + 24*Sin[c + d*x]^2 + 16*Si
n[c + d*x]^3 + 3*Sin[c + d*x]^4))/(12*d)
```

fricas [A] time = 0.97, size = 144, normalized size = 0.97

$$\frac{24\,a^4\cos(dx+c)^8-128\,a^4\cos(dx+c)^6\sin(dx+c)-288\,a^4\cos(dx+c)^6+615\,a^4\cos(dx+c)^4-270\,a^4\cos(d}{96\left(d\cos(dx+c)^4-2\,d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/96*(24*a^4*cos(d*x + c)^8 - 128*a^4*cos(d*x + c)^6*sin(d*x + c) - 288*a^4
*cos(d*x + c)^6 + 615*a^4*cos(d*x + c)^4 - 270*a^4*cos(d*x + c)^2 - 105*a^4
 - 960*(a^4*cos(d*x + c)^4 - 2*a^4*cos(d*x + c)^2 + a^4)*log(1/2*sin(d*x +
c)))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.46, size = 134, normalized size = 0.91

$$\frac{3\,a^4\sin(dx+c)^4+16\,a^4\sin(dx+c)^3+24\,a^4\sin(dx+c)^2-120\,a^4\log(|\sin(dx+c)|)-48\,a^4\sin(dx+c)+\frac{250\,a}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 24*a^4*sin(d*x + c)^2
- 120*a^4*log(abs(sin(d*x + c))) - 48*a^4*sin(d*x + c) + (250*a^4*sin(d*x +
 c)^4 + 48*a^4*sin(d*x + c)^3 - 24*a^4*sin(d*x + c)^2 - 16*a^4*sin(d*x + c)
 - 3*a^4)/sin(d*x + c)^4)/d
```

maple [A] time = 0.44, size = 129, normalized size = 0.87

$$-\frac{11a^4\left(\cos^4(dx+c)\right)}{4d}-\frac{11a^4\left(\cos^2(dx+c)\right)}{2d}-\frac{10a^4\ln(\sin(dx+c))}{d}-\frac{3a^4\left(\cos^6(dx+c)\right)}{d\sin(dx+c)^2}-\frac{4a^4\left(\cos^6(dx+c)\right)}{3d\sin(dx+c)^3}-\frac{a^4}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -11/4/d*a^4*cos(d*x+c)^4-11/2/d*a^4*cos(d*x+c)^2-10*a^4*ln(sin(d*x+c))/d-3/
d*a^4/sin(d*x+c)^2*cos(d*x+c)^6-4/3/d*a^4/sin(d*x+c)^3*cos(d*x+c)^6-1/4/d*a
^4*cot(d*x+c)^4+1/2/d*a^4*cot(d*x+c)^2
```

maxima [A] time = 0.72, size = 120, normalized size = 0.81

$$\frac{3\,a^4\sin(dx+c)^4+16\,a^4\sin(dx+c)^3+24\,a^4\sin(dx+c)^2-120\,a^4\log(\sin(dx+c))-48\,a^4\sin(dx+c)+\frac{48\,a^4\cdots}{\cdots}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 24*a^4*sin(d*x + c)^2
- 120*a^4*log(sin(d*x + c)) - 48*a^4*sin(d*x + c) + (48*a^4*sin(d*x + c)^3
- 24*a^4*sin(d*x + c)^2 - 16*a^4*sin(d*x + c) - 3*a^4)/sin(d*x + c)^4)/d
```

mupad [B] time = 9.07, size = 368, normalized size = 2.49

$$\frac{3\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,d}-\frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{10\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{104\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-119\,a^4\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^4)/sin(c + d*x)^5,x)
```

```
[Out] (3*a^4*tan(c/2 + (d*x)/2))/(2*d) - (a^4*tan(c/2 + (d*x)/2)^3)/(6*d) - (a^4*
tan(c/2 + (d*x)/2)^4)/(64*d) - (10*a^4*log(tan(c/2 + (d*x)/2)))/d - (10*a^4
*tan(c/2 + (d*x)/2)^2 - (40*a^4*tan(c/2 + (d*x)/2)^3)/3 + (75*a^4*tan(c/2 +
 (d*x)/2)^4)/2 + 48*a^4*tan(c/2 + (d*x)/2)^5 - 73*a^4*tan(c/2 + (d*x)/2)^6
+ 80*a^4*tan(c/2 + (d*x)/2)^7 - (1135*a^4*tan(c/2 + (d*x)/2)^8)/4 + 120*a^4
*tan(c/2 + (d*x)/2)^9 - 119*a^4*tan(c/2 + (d*x)/2)^10 + 104*a^4*tan(c/2 + (
d*x)/2)^11 + a^4/4 + (8*a^4*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2
)^4 + 64*tan(c/2 + (d*x)/2)^6 + 96*tan(c/2 + (d*x)/2)^8 + 64*tan(c/2 + (d*x
)/2)^10 + 16*tan(c/2 + (d*x)/2)^12)) - (9*a^4*tan(c/2 + (d*x)/2)^2)/(16*d)
+ (10*a^4*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.532 $\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=146

$$\frac{a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^5(c+dx)}{5d}-\frac{a^4\csc^4(c+dx)}{d}-\frac{4a^4\csc^3(c+dx)}{3d}+\frac{2a^4\csc}{}$$

```
[Out] 10*a^4*csc(d*x+c)/d+2*a^4*csc(d*x+c)^2/d-4/3*a^4*csc(d*x+c)^3/d-a^4*csc(d*x
+c)^4/d-1/5*a^4*csc(d*x+c)^5/d-4*a^4*ln(sin(d*x+c))/d+4*a^4*sin(d*x+c)/d+2*
a^4*sin(d*x+c)^2/d+1/3*a^4*sin(d*x+c)^3/d
```

Rubi [A] time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^5(c+dx)}{5d}-\frac{a^4\csc^4(c+dx)}{d}-\frac{4a^4\csc^3(c+dx)}{3d}+\frac{2a^4\csc}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (10*a^4*Csc[c + d*x])/d + (2*a^4*Csc[c + d*x]^2)/d - (4*a^4*Csc[c + d*x]^3)
/(3*d) - (a^4*Csc[c + d*x]^4)/d - (a^4*Csc[c + d*x]^5)/(5*d) - (4*a^4*Log[S
in[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (2*a^4*Sin[c + d*x]^2)/d + (a^4*
Sin[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2(a+x)^6}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^6}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(4a^2+\frac{a^8}{x^6}+\frac{4a^7}{x^5}+\frac{4a^6}{x^4}-\frac{4a^5}{x^3}-\frac{10a^4}{x^2}-\frac{4a^3}{x}+4ax+x\right.\right.}{d}$$
$$= \frac{10a^4\csc(c+dx)}{d}+\frac{2a^4\csc^2(c+dx)}{d}-\frac{4a^4\csc^3(c+dx)}{3d}-\frac{a^4}{}$$

Mathematica [A] time = 0.18, size = 96, normalized size = 0.66

$$\frac{a^4\left(5\sin^3(c+dx)+30\sin^2(c+dx)+60\sin(c+dx)-3\csc^5(c+dx)-15\csc^4(c+dx)-20\csc^3(c+dx)+30\csc\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(150*Csc[c + d*x] + 30*Csc[c + d*x]^2 - 20*Csc[c + d*x]^3 - 15*Csc[c +
 d*x]^4 - 3*Csc[c + d*x]^5 - 60*Log[Sin[c + d*x]] + 60*Sin[c + d*x] + 30*Si
n[c + d*x]^2 + 5*Sin[c + d*x]^3))/(15*d)
```

fricas [A] time = 0.59, size = 192, normalized size = 1.32

$$\frac{5\,a^4\cos(dx+c)^8-80\,a^4\cos(dx+c)^6+360\,a^4\cos(dx+c)^4-480\,a^4\cos(dx+c)^2+192\,a^4-60\left(a^4\cos(dx+c\right.}{15\left(d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/15*(5*a^4*cos(d*x + c)^8 - 80*a^4*cos(d*x + c)^6 + 360*a^4*cos(d*x + c)^4
 - 480*a^4*cos(d*x + c)^2 + 192*a^4 - 60*(a^4*cos(d*x + c)^4 - 2*a^4*cos(d*
x + c)^2 + a^4)*log(1/2*sin(d*x + c))*sin(d*x + c) - 15*(2*a^4*cos(d*x + c)
```

```
^6 - 5*a^4*cos(d*x + c)^4 + 6*a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c))/((d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.45, size = 134, normalized size = 0.92

$$\frac{5\,a^4\sin(dx+c)^3+30\,a^4\sin(dx+c)^2-60\,a^4\log(|\sin(dx+c)|)+60\,a^4\sin(dx+c)+\frac{137\,a^4\sin(dx+c)^5+150\,a^4\sin(dx}{}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/15*(5*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^2 - 60*a^4*log(abs(sin(d*x
 + c))) + 60*a^4*sin(d*x + c) + (137*a^4*sin(d*x + c)^5 + 150*a^4*sin(d*x +
 c)^4 + 30*a^4*sin(d*x + c)^3 - 20*a^4*sin(d*x + c)^2 - 15*a^4*sin(d*x + c)
 - 3*a^4)/sin(d*x + c)^5)/d
```

maple [A] time = 0.43, size = 235, normalized size = 1.61

$$\frac{24a^4\left(\cos^6(dx+c)\right)}{5d\sin(dx+c)}+\frac{64a^4\sin(dx+c)}{5d}+\frac{24a^4\sin(dx+c)\left(\cos^4(dx+c)\right)}{5d}+\frac{32a^4\left(\cos^2(dx+c)\right)\sin(dx+c)}{5d}-\frac{2a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 24/5/d*a^4/sin(d*x+c)*cos(d*x+c)^6+64/5*a^4*sin(d*x+c)/d+24/5/d*a^4*sin(d*x
+c)*cos(d*x+c)^4+32/5/d*a^4*cos(d*x+c)^2*sin(d*x+c)-2/d*a^4/sin(d*x+c)^2*co
s(d*x+c)^6-2/d*a^4*cos(d*x+c)^4-4/d*a^4*cos(d*x+c)^2-4*a^4*ln(sin(d*x+c))/d
-29/15/d*a^4/sin(d*x+c)^3*cos(d*x+c)^6-1/d*a^4*cot(d*x+c)^4+2/d*a^4*cot(d*x
+c)^2-1/5/d*a^4/sin(d*x+c)^5*cos(d*x+c)^6
```

maxima [A] time = 0.57, size = 120, normalized size = 0.82

$$\frac{5\,a^4\sin(dx+c)^3+30\,a^4\sin(dx+c)^2-60\,a^4\log(\sin(dx+c))+60\,a^4\sin(dx+c)+\frac{150\,a^4\sin(dx+c)^4+30\,a^4\sin(dx+c}{s}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/15*(5*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^2 - 60*a^4*log(sin(d*x + c
)) + 60*a^4*sin(d*x + c) + (150*a^4*sin(d*x + c)^4 + 30*a^4*sin(d*x + c)^3
- 20*a^4*sin(d*x + c)^2 - 15*a^4*sin(d*x + c) - 3*a^4)/sin(d*x + c)^5)/d
```

mupad [B] time = 9.09, size = 357, normalized size = 2.45

$$\frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4d}-\frac{19\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{96\,d}-\frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{16\,d}-\frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}-\frac{4\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{398\,a^4\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^4)/sin(c + d*x)^6,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^2)/(4*d) - (19*a^4*tan(c/2 + (d*x)/2)^3)/(96*d) - (
a^4*tan(c/2 + (d*x)/2)^4)/(16*d) - (a^4*tan(c/2 + (d*x)/2)^5)/(160*d) - (4*
a^4*log(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x)/2)^3 - (104*a^4*tan
(c/2 + (d*x)/2)^2)/15 + (612*a^4*tan(c/2 + (d*x)/2)^4)/5 + 18*a^4*tan(c/2 +
 (d*x)/2)^5 + (3314*a^4*tan(c/2 + (d*x)/2)^6)/5 + 278*a^4*tan(c/2 + (d*x)/2
)^7 + 1017*a^4*tan(c/2 + (d*x)/2)^8 + 264*a^4*tan(c/2 + (d*x)/2)^9 + 398*a^
4*tan(c/2 + (d*x)/2)^10 - a^4/5 - 2*a^4*tan(c/2 + (d*x)/2))/(d*(32*tan(c/2
+ (d*x)/2)^5 + 96*tan(c/2 + (d*x)/2)^7 + 96*tan(c/2 + (d*x)/2)^9 + 32*tan(c
/2 + (d*x)/2)^11)) + (71*a^4*tan(c/2 + (d*x)/2))/(16*d) + (4*a^4*log(tan(c/
2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.533 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\sin^7(c+dx)}{7ad}-\frac{\sin^6(c+dx)}{6ad}-\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^4(c+dx)}{4ad}$$

```
[Out] 1/4*sin(d*x+c)^4/a/d-1/5*sin(d*x+c)^5/a/d-1/6*sin(d*x+c)^6/a/d+1/7*sin(d*x+
c)^7/a/d
```

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 75}

$$\frac{\sin^7(c+dx)}{7ad}-\frac{\sin^6(c+dx)}{6ad}-\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^4/(4*a*d) - Sin[c + d*x]^5/(5*a*d) - Sin[c + d*x]^6/(6*a*d) +
Sin[c + d*x]^7/(7*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^3(a+x)}{a^3}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^3(a+x)\,dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3x^3-a^2x^4-ax^5+x^6\right)\,dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= \frac{\sin^4(c+dx)}{4ad} - \frac{\sin^5(c+dx)}{5ad} - \frac{\sin^6(c+dx)}{6ad} + \frac{\sin^7(c+dx)}{7ad}$$

Mathematica [A] time = 0.35, size = 48, normalized size = 0.66

$$\frac{\sin^4(c+dx)\left(60\sin^3(c+dx)-70\sin^2(c+dx)-84\sin(c+dx)+105\right)}{420ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^4*(105 - 84*Sin[c + d*x] - 70*Sin[c + d*x]^2 + 60*Sin[c + d*x
]^3))/(420*a*d)
```

fricas [A] time = 0.69, size = 67, normalized size = 0.92

$$\frac{70\cos\left(dx+c\right)^6-105\cos\left(dx+c\right)^4-12\left(5\cos\left(dx+c\right)^6-8\cos\left(dx+c\right)^4+\cos\left(dx+c\right)^2+2\right)\sin\left(dx+c\right)}{420\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(70*cos(d*x + c)^6 - 105*cos(d*x + c)^4 - 12*(5*cos(d*x + c)^6 - 8*co
s(d*x + c)^4 + cos(d*x + c)^2 + 2)*sin(d*x + c))/(a*d)
```

giac [A] time = 0.17, size = 49, normalized size = 0.67

$$\frac{60\sin\left(dx+c\right)^7-70\sin\left(dx+c\right)^6-84\sin\left(dx+c\right)^5+105\sin\left(dx+c\right)^4}{420\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*(60*sin(d*x + c)^7 - 70*sin(d*x + c)^6 - 84*sin(d*x + c)^5 + 105*sin(
d*x + c)^4)/(a*d)
```

maple [A] time = 0.25, size = 49, normalized size = 0.67

$$\frac{\frac{\left(\sin^7(dx+c)\right)}{7} - \frac{\left(\sin^6(dx+c)\right)}{6} - \frac{\left(\sin^5(dx+c)\right)}{5} + \frac{\left(\sin^4(dx+c)\right)}{4}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/7*sin(d*x+c)^7-1/6*sin(d*x+c)^6-1/5*sin(d*x+c)^5+1/4*sin(d*x+c)^4)
```

maxima [A] time = 0.67, size = 49, normalized size = 0.67

$$\frac{60\,\sin\left(dx+c\right)^7 - 70\,\sin\left(dx+c\right)^6 - 84\,\sin\left(dx+c\right)^5 + 105\,\sin\left(dx+c\right)^4}{420\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/420*(60*sin(d*x + c)^7 - 70*sin(d*x + c)^6 - 84*sin(d*x + c)^5 + 105*sin(
d*x + c)^4)/(a*d)
```

mupad [B] time = 0.07, size = 57, normalized size = 0.78

$$\frac{\frac{\sin(c+d\,x)^4}{4\,a} - \frac{\sin(c+d\,x)^5}{5\,a} - \frac{\sin(c+d\,x)^6}{6\,a} + \frac{\sin(c+d\,x)^7}{7\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^4/(4*a) - sin(c + d*x)^5/(5*a) - sin(c + d*x)^6/(6*a) + sin(c
 + d*x)^7/(7*a))/d
```

sympy [A] time = 81.79, size = 981, normalized size = 13.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((420*tan(c/2 + d*x/2)**10/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d
*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 +
d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 7
35*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 672*tan(c/2 + d*x/2)**9/(105*a*d*ta
n(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/
2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205
*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 140*tan
(c/2 + d*x/2)**8/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**
12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*
d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*
x/2)**2 + 105*a*d) + 576*tan(c/2 + d*x/2)**7/(105*a*d*tan(c/2 + d*x/2)**14
+ 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*t
an(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/
2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 140*tan(c/2 + d*x/2)**6/(1
05*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c
/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)*
*6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d)
- 672*tan(c/2 + d*x/2)**5/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 +
 d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8
+ 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan
(c/2 + d*x/2)**2 + 105*a*d) + 420*tan(c/2 + d*x/2)**4/(105*a*d*tan(c/2 + d*
x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3
675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c
/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d), Ne(d, 0)), (x*sin(
c)**3*cos(c)**5/(a*sin(c) + a), True))
```

3.534 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\sin^6(c+dx)}{6ad}-\frac{\sin^5(c+dx)}{5ad}-\frac{\sin^4(c+dx)}{4ad}+\frac{\sin^3(c+dx)}{3ad}$$

```
[Out] 1/3*sin(d*x+c)^3/a/d-1/4*sin(d*x+c)^4/a/d-1/5*sin(d*x+c)^5/a/d+1/6*sin(d*x+
c)^6/a/d
```

Rubi [A] time = 0.16, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2835, 2564, 14}

$$\frac{\sin^6(c+dx)}{6ad}-\frac{\sin^5(c+dx)}{5ad}-\frac{\sin^4(c+dx)}{4ad}+\frac{\sin^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^3/(3*a*d) - Sin[c + d*x]^4/(4*a*d) - Sin[c + d*x]^5/(5*a*d) +
Sin[c + d*x]^6/(6*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^3(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^3(c+dx)\sin^3(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx, x, \sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int x^3\left(1-x^2\right)\,dx, x, \sin(c+d\right.}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(x^2-x^4\right)\,dx, x, \sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \left(x^3-x^5\right)\,dx, x, \sin(c+dx)\right)}{ad}$$
$$= \frac{\sin^3(c+dx)}{3ad} - \frac{\sin^4(c+dx)}{4ad} - \frac{\sin^5(c+dx)}{5ad} + \frac{\sin^6(c+dx)}{6ad}$$

Mathematica [A] time = 0.21, size = 48, normalized size = 0.66

$$\frac{\sin^3(c+dx)\left(10\sin^3(c+dx)-12\sin^2(c+dx)-15\sin(c+dx)+20\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^3*(20 - 15*Sin[c + d*x] - 12*Sin[c + d*x]^2 + 10*Sin[c + d*x]
^3))/(60*a*d)
```

fricas [A] time = 0.64, size = 59, normalized size = 0.81

$$-\frac{10\,\cos\left(dx+c\right)^6-15\,\cos\left(dx+c\right)^4+4\left(3\,\cos\left(dx+c\right)^4-\cos\left(dx+c\right)^2-2\right)\sin\left(dx+c\right)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(10*cos(d*x + c)^6 - 15*cos(d*x + c)^4 + 4*(3*cos(d*x + c)^4 - cos(d*
x + c)^2 - 2)*sin(d*x + c))/(a*d)
```

giac [A] time = 0.18, size = 49, normalized size = 0.67

$$\frac{10\,\sin\left(dx+c\right)^6-12\,\sin\left(dx+c\right)^5-15\,\sin\left(dx+c\right)^4+20\,\sin\left(dx+c\right)^3}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(10*sin(d*x + c)^6 - 12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 20*sin(d*
x + c)^3)/(a*d)
```

maple [A] time = 0.24, size = 49, normalized size = 0.67

$$\frac{\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^4(dx+c)\right)}{4}+\frac{\left(\sin^3(dx+c)\right)}{3}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/6*sin(d*x+c)^6-1/5*sin(d*x+c)^5-1/4*sin(d*x+c)^4+1/3*sin(d*x+c)^3)
```

maxima [A] time = 0.58, size = 49, normalized size = 0.67

$$\frac{10\,\sin\left(dx+c\right)^6-12\,\sin\left(dx+c\right)^5-15\,\sin\left(dx+c\right)^4+20\,\sin\left(dx+c\right)^3}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(10*sin(d*x + c)^6 - 12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 20*sin(d*
x + c)^3)/(a*d)
```

mupad [B] time = 0.06, size = 57, normalized size = 0.78

$$\frac{\frac{\sin(c+d\,x)^3}{3\,a}-\frac{\sin(c+d\,x)^4}{4\,a}-\frac{\sin(c+d\,x)^5}{5\,a}+\frac{\sin(c+d\,x)^6}{6\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^3/(3*a) - sin(c + d*x)^4/(4*a) - sin(c + d*x)^5/(5*a) + sin(c
 + d*x)^6/(6*a))/d
```

sympy [A] time = 51.02, size = 862, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((40*tan(c/2 + d*x/2)**9/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan
(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)*
*6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 6
0*tan(c/2 + d*x/2)**8/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2
)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d
*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 24*tan(c/2 +
d*x/2)**7/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*
a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d
*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 40*tan(c/2 + d*x/2)**6/(1
5*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2
+ d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 9
0*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 24*tan(c/2 + d*x/2)**5/(15*a*d*tan(c/
2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8
+ 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/
2 + d*x/2)**2 + 15*a*d) - 60*tan(c/2 + d*x/2)**4/(15*a*d*tan(c/2 + d*x/2)**
12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*ta
n(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**
2 + 15*a*d) + 40*tan(c/2 + d*x/2)**3/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*
tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/
2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d),
 Ne(d, 0)), (x*sin(c)**2*cos(c)**5/(a*sin(c) + a), True))
```

3.535 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^4(c+dx)}{4ad}$$

```
[Out] -1/4*cos(d*x+c)^4/a/d-1/3*sin(d*x+c)^3/a/d+1/5*sin(d*x+c)^5/a/d
```

Rubi [A] time = 0.11, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2835, 2565, 30, 2564, 14}

$$\frac{\sin^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cos[c + d*x]^4/(4*a*d) - Sin[c + d*x]^3/(3*a*d) + Sin[c + d*x]^5/(5*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^3(c+dx)\sin(c+dx)\,dx}{a}-\frac{\int \cos^3(c+dx)\sin^2(c+dx)\,dx}{a}\\ &= -\frac{\operatorname{Subst}\left(\int x^3\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\operatorname{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\sin(c+dx)\right)}{ad}\\ &= -\frac{\cos^4(c+dx)}{4ad}-\frac{\operatorname{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\sin(c+dx)\right)}{ad}\\ &= -\frac{\cos^4(c+dx)}{4ad}-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A] time = 0.16, size = 48, normalized size = 0.87

$$\frac{\sin^2(c+dx)\left(12\sin^3(c+dx)-15\sin^2(c+dx)-20\sin(c+dx)+30\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^2*(30 - 20*Sin[c + d*x] - 15*Sin[c + d*x]^2 + 12*Sin[c + d*x]
^3))/(60*a*d)
```

fricas [A] time = 0.63, size = 49, normalized size = 0.89

$$-\frac{15\cos\left(dx+c\right)^4-4\left(3\cos\left(dx+c\right)^4-\cos\left(dx+c\right)^2-2\right)\sin\left(dx+c\right)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(15*cos(d*x + c)^4 - 4*(3*cos(d*x + c)^4 - cos(d*x + c)^2 - 2)*sin(d*
x + c))/(a*d)
```

giac [A] time = 0.18, size = 49, normalized size = 0.89

$$\frac{12 \sin(dx+c)^5 - 15 \sin(dx+c)^4 - 20 \sin(dx+c)^3 + 30 \sin(dx+c)^2}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 30*sin(d*
x + c)^2)/(a*d)
```

maple [A] time = 0.19, size = 49, normalized size = 0.89

$$\frac{\frac{\left(\sin^5(dx+c)\right)}{5} - \frac{\left(\sin^4(dx+c)\right)}{4} - \frac{\left(\sin^3(dx+c)\right)}{3} + \frac{\left(\sin^2(dx+c)\right)}{2}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/5*sin(d*x+c)^5-1/4*sin(d*x+c)^4-1/3*sin(d*x+c)^3+1/2*sin(d*x+c)^2)
```

maxima [A] time = 0.55, size = 49, normalized size = 0.89

$$\frac{12 \sin(dx+c)^5 - 15 \sin(dx+c)^4 - 20 \sin(dx+c)^3 + 30 \sin(dx+c)^2}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 30*sin(d*
x + c)^2)/(a*d)
```

mupad [B] time = 0.06, size = 57, normalized size = 1.04

$$\frac{\frac{\sin(c+d\,x)^2}{2\,a} - \frac{\sin(c+d\,x)^3}{3\,a} - \frac{\sin(c+d\,x)^4}{4\,a} + \frac{\sin(c+d\,x)^5}{5\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^2/(2*a) - sin(c + d*x)^3/(3*a) - sin(c + d*x)^4/(4*a) + sin(c
 + d*x)^5/(5*a))/d
```

sympy [A] time = 30.74, size = 741, normalized size = 13.47

$$\begin{cases} \frac{30\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)}{15ad\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+75ad\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+150ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+150ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+75ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+15ad} - \frac{}{15ad\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+75ad\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)} \\ \frac{x\sin(c)\cos^{5}(c)}{a\sin(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((30*tan(c/2 + d*x/2)**8/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan
(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**
4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 40*tan(c/2 + d*x/2)**7/(15*a*d*t
an(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)
**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) +
30*tan(c/2 + d*x/2)**6/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/
2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*
tan(c/2 + d*x/2)**2 + 15*a*d) + 16*tan(c/2 + d*x/2)**5/(15*a*d*tan(c/2 + d*
x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a
*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*tan(c/2
+ d*x/2)**4/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150
*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d
*x/2)**2 + 15*a*d) - 40*tan(c/2 + d*x/2)**3/(15*a*d*tan(c/2 + d*x/2)**10 +
75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2
+ d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*tan(c/2 + d*x/2)**2
/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/
2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 +
15*a*d), Ne(d, 0)), (x*sin(c)*cos(c)**5/(a*sin(c) + a), True))
```

3.536 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=65

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] ln(sin(d*x+c))/a/d-sin(d*x+c)/a/d-1/2*sin(d*x+c)^2/a/d+1/3*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 75}

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d) + Sin
[c + d*x]^3/(3*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2(a+x)}{x}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x}\,dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2+\frac{a^3}{x}-ax+x^2\right)dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{ad} - \frac{\sin^2(c+dx)}{2ad} + \frac{\sin^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.75

$$\frac{2\sin^3(c+dx)-3\sin^2(c+dx)-6\sin(c+dx)+6\log(\sin(c+dx))-2}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2 + 6*Log[Sin[c + d*x]] - 6*Sin[c + d*x] - 3*Sin[c + d*x]^2 + 2*Sin[c + d
*x]^3)/(6*a*d)
```

fricas [A] time = 0.63, size = 48, normalized size = 0.74

$$\frac{3\cos(dx+c)^2-2\left(\cos(dx+c)^2+2\right)\sin(dx+c)+6\log\left(\frac{1}{2}\sin(dx+c)\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 + 2)*sin(d*x + c) + 6*log(1/2*sin
(d*x + c)))/(a*d)
```

giac [A] time = 0.17, size = 61, normalized size = 0.94

$$\frac{\frac{6\log(|\sin(dx+c)|)}{a}+\frac{2a^2\sin(dx+c)^3-3a^2\sin(dx+c)^2-6a^2\sin(dx+c)}{a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c)))/a + (2*a^2*sin(d*x + c)^3 - 3*a^2*sin(d*x + c
)^2 - 6*a^2*sin(d*x + c))/a^3)/d
```

maple [A] time = 0.38, size = 62, normalized size = 0.95

$$\frac{\ln\left(\sin\left(dx+c\right)\right)}{ad}-\frac{\sin\left(dx+c\right)}{ad}-\frac{\sin^{2}\left(dx+c\right)}{2ad}+\frac{\sin^{3}\left(dx+c\right)}{3da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] ln(sin(d*x+c))/a/d-sin(d*x+c)/a/d-1/2*sin(d*x+c)^2/a/d+1/3*sin(d*x+c)^3/d/a
```

maxima [A] time = 0.83, size = 51, normalized size = 0.78

$$\frac{\frac{2\,\sin(dx+c)^{3}-3\,\sin(dx+c)^{2}-6\,\sin(dx+c)}{a}+\frac{6\,\log(\sin(dx+c))}{a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*sin(d*x + c)^3 - 3*sin(d*x + c)^2 - 6*sin(d*x + c))/a + 6*log(sin(d
*x + c))/a)/d
```

mupad [B] time = 9.05, size = 102, normalized size = 1.57

$$\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}-\frac{\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{a\,d}-\frac{2\,\sin\left(c+d\,x\right)}{3\,a\,d}+\frac{{\cos\left(c+d\,x\right)}^2}{2\,a\,d}-\frac{{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{3\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) - log(1/cos(c/2 + (d*x)/2)
^2)/(a*d) - (2*sin(c + d*x))/(3*a*d) + cos(c + d*x)^2/(2*a*d) - (cos(c + d*
x)^2*sin(c + d*x))/(3*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.537 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] -csc(d*x+c)/a/d-ln(sin(d*x+c))/a/d-sin(d*x+c)/a/d+1/2*sin(d*x+c)^2/a/d
```

Rubi [A] time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 75}

$$\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) + Sin[
c + d*x]^2/(2*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2(a+x)}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a+\frac{a^3}{x^2}-\frac{a^2}{x}+x\right)dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= -\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2ad}$$

Mathematica [A] time = 0.06, size = 45, normalized size = 0.73

$$\frac{\sin^2(c+dx)-2\sin(c+dx)-2\csc(c+dx)-2\log(\sin(c+dx))+6}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6 - 2*Csc[c + d*x] - 2*Log[Sin[c + d*x]] - 2*Sin[c + d*x] + Sin[c + d*x]^2
)/(2*a*d)
```

fricas [A] time = 0.76, size = 65, normalized size = 1.05

$$\frac{4\cos(dx+c)^2-\left(2\cos(dx+c)^2-1\right)\sin(dx+c)-4\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-8}{4\,ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*cos(d*x + c)^2 - (2*cos(d*x + c)^2 - 1)*sin(d*x + c) - 4*log(1/2*sin
(d*x + c))*sin(d*x + c) - 8)/(a*d*sin(d*x + c))
```

giac [A] time = 0.19, size = 65, normalized size = 1.05

$$-\frac{\frac{2\log(|\sin(dx+c)|)}{a}-\frac{a\sin(dx+c)^2-2\,a\sin(dx+c)}{a^2}-\frac{2\,(\sin(dx+c)-1)}{a\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(sin(d*x + c)))/a - (a*sin(d*x + c)^2 - 2*a*sin(d*x + c))/a^
2 - 2*(sin(d*x + c) - 1)/(a*sin(d*x + c)))/d
```

maple [A] time = 0.43, size = 63, normalized size = 1.02

$$\frac{\sin^{2}\left(dx+c\right)}{2ad}-\frac{\sin\left(dx+c\right)}{ad}-\frac{1}{da\sin\left(dx+c\right)}-\frac{\ln\left(\sin\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2*sin(d*x+c)^2/a/d-sin(d*x+c)/a/d-1/d/a/sin(d*x+c)-ln(sin(d*x+c))/a/d
```

maxima [A] time = 0.32, size = 52, normalized size = 0.84

$$\frac{\frac{\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)}{a}-\frac{2\,\log\left(\sin\left(dx+c\right)\right)}{a}-\frac{2}{a\sin\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((sin(d*x + c)^2 - 2*sin(d*x + c))/a - 2*log(sin(d*x + c))/a - 2/(a*sin
(d*x + c)))/d
```

mupad [B] time = 8.80, size = 146, normalized size = 2.35

$$\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a\,d}-\frac{5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1}{d\,\left(2\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+4\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - (6*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 +
 (d*x)/2)^3 + 5*tan(c/2 + (d*x)/2)^4 + 1)/(d*(2*a*tan(c/2 + (d*x)/2) + 4*a*
tan(c/2 + (d*x)/2)^3 + 2*a*tan(c/2 + (d*x)/2)^5)) - tan(c/2 + (d*x)/2)/(2*a
*d) - log(tan(c/2 + (d*x)/2))/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.538 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{\sin(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d-ln(sin(d*x+c))/a/d+sin(d*x+c)/a/d
```

Rubi [A] time = 0.11, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 75}

$$\frac{\sin(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - Log[Sin[c + d*x]]/(a*d) + Sin
[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2(a+x)}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^3}{x^3}-\frac{a^2}{x^2}-\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\sin(c+dx)}{ad}$$

Mathematica [A] time = 0.09, size = 45, normalized size = 0.75

$$-\frac{-2\sin(c+dx)+\csc^2(c+dx)-2\csc(c+dx)+2\log(\sin(c+dx))+3}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*(3 - 2*Csc[c + d*x] + Csc[c + d*x]^2 + 2*Log[Sin[c + d*x]] - 2*Sin[c +
 d*x])/(a*d)
```

fricas [A] time = 0.66, size = 61, normalized size = 1.02

$$-\frac{2\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-2\left(\cos(dx+c)^2-2\right)\sin(dx+c)-1}{2\left(ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 2*(cos(d*x + c)^2 - 2)
*sin(d*x + c) - 1)/(a*d*cos(d*x + c)^2 - a*d)
```

giac [A] time = 0.19, size = 63, normalized size = 1.05

$$-\frac{\frac{2\log(|\sin(dx+c)|)}{a}-\frac{2\sin(dx+c)}{a}-\frac{3\sin(dx+c)^2+2\sin(dx+c)-1}{a\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(sin(d*x + c)))/a - 2*sin(d*x + c)/a - (3*sin(d*x + c)^2 + 2
*sin(d*x + c) - 1)/(a*sin(d*x + c)^2))/d
```

maple [A] time = 0.46, size = 61, normalized size = 1.02

$$\frac{\sin \left(dx+c\right)}{ad}+\frac{1}{da\sin \left(dx+c\right)}-\frac{\ln \left(\sin \left(dx+c\right)\right)}{ad}-\frac{1}{2ad\sin \left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] sin(d*x+c)/a/d+1/d/a/sin(d*x+c)-ln(sin(d*x+c))/a/d-1/2/a/d/sin(d*x+c)^2
```

maxima [A] time = 0.91, size = 52, normalized size = 0.87

$$-\frac{\frac{2\,\log\left(\sin\left(dx+c\right)\right)}{a}-\frac{2\,\sin\left(dx+c\right)}{a}-\frac{2\,\sin\left(dx+c\right)-1}{a\sin\left(dx+c\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*log(sin(d*x + c))/a - 2*sin(d*x + c)/a - (2*sin(d*x + c) - 1)/(a*si
n(d*x + c)^2))/d
```

mupad [B] time = 8.93, size = 150, normalized size = 2.50

$$\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{2}}{d\,\left(4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}+\frac{\ln\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (2*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2/2 + 10*tan(c/2 + (d*x)/2)^3 -
1/2)/(d*(4*a*tan(c/2 + (d*x)/2)^2 + 4*a*tan(c/2 + (d*x)/2)^4)) - log(tan(c/
2 + (d*x)/2))/(a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) + tan(c/2 + (d*x)/2)/(2*
a*d) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.539 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=64

$$-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] csc(d*x+c)/a/d+1/2*csc(d*x+c)^2/a/d-1/3*csc(d*x+c)^3/a/d+ln(sin(d*x+c))/a/d
```

Rubi [A] time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 75}

$$-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) + Log[
Sin[c + d*x]]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{a^2}{x^3}-\frac{a}{x^2}+\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{\log(\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 0.75

$$\frac{-2\csc^3(c+dx)+3\csc^2(c+dx)+6\csc(c+dx)+6\log(\sin(c+dx))}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6*Csc[c + d*x] + 3*Csc[c + d*x]^2 - 2*Csc[c + d*x]^3 + 6*Log[Sin[c + d*x]]
)/(6*a*d)
```

fricas [A] time = 0.78, size = 75, normalized size = 1.17

$$\frac{6\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+6\cos(dx+c)^2-3\sin(dx+c)-4}{6\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c))*sin(d*x + c) + 6*cos(d*x
+ c)^2 - 3*sin(d*x + c) - 4)/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

giac [A] time = 0.20, size = 62, normalized size = 0.97

$$\frac{\frac{6\log(|\sin(dx+c)|)}{a}-\frac{11\sin(dx+c)^3-6\sin(dx+c)^2-3\sin(dx+c)+2}{a\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c)))/a - (11*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 3
*sin(d*x + c) + 2)/(a*sin(d*x + c)^3))/d
```

maple [A] time = 0.46, size = 63, normalized size = 0.98

$$\frac{1}{da\sin(dx+c)}+\frac{\ln(\sin(dx+c))}{ad}+\frac{1}{2ad\sin(dx+c)^2}-\frac{1}{3ad\sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a/sin(d*x+c)+ln(sin(d*x+c))/a/d+1/2/a/d/sin(d*x+c)^2-1/3/a/d/sin(d*x+c)
^3
```

maxima [A] time = 0.77, size = 50, normalized size = 0.78

$$\frac{\frac{6\log(\sin(dx+c))}{a}+\frac{6\sin(dx+c)^2+3\sin(dx+c)-2}{a\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*log(sin(d*x + c))/a + (6*sin(d*x + c)^2 + 3*sin(d*x + c) - 2)/(a*sin
(d*x + c)^3))/d
```

mupad [B] time = 8.95, size = 138, normalized size = 2.16

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(3\,t\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - tan(c/2 + (d*x)/2)^3/(24*a*d) + log(tan(c/2
+ (d*x)/2))/(a*d) + (3*tan(c/2 + (d*x)/2))/(8*a*d) - log(tan(c/2 + (d*x)/2)
^2 + 1)/(a*d) + (cot(c/2 + (d*x)/2)^3*(tan(c/2 + (d*x)/2) + 3*tan(c/2 + (d*
x)/2)^2 - 1/3))/(8*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.540 $\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=51

$$-\frac{\cot^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc(c+dx)}{ad}$$

```
[Out] -1/4*cot(d*x+c)^4/a/d-csc(d*x+c)/a/d+1/3*csc(d*x+c)^3/a/d
```

Rubi [A] time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2706, 2607, 30, 2606}

$$-\frac{\cot^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cot[c + d*x]^4/(4*a*d) - Csc[c + d*x]/(a*d) + Csc[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^3(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^3(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int x^3\,dx,x,-\cot(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \left(-1+x^2\right)dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot^4(c+dx)}{4ad}-\frac{\csc(c+dx)}{ad}+\frac{\csc^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.05, size = 30, normalized size = 0.59

$$-\frac{(\csc(c+dx)-1)^3(3\csc(c+dx)+5)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x]),x]
[Out] -1/12*((-1 + Csc[c + d*x])^3*(5 + 3*Csc[c + d*x]))/(a*d)
```

fricas [A] time = 0.61, size = 63, normalized size = 1.24

$$-\frac{6\cos\left(dx+c\right)^2-4\left(3\cos\left(dx+c\right)^2-2\right)\sin\left(dx+c\right)-3}{12\left(ad\cos\left(dx+c\right)^4-2\,ad\cos\left(dx+c\right)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/12*(6*cos(d*x + c)^2 - 4*(3*cos(d*x + c)^2 - 2)*sin(d*x + c) - 3)/(a*d*c
os(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

giac [A] time = 0.20, size = 46, normalized size = 0.90

$$-\frac{12\sin\left(dx+c\right)^3-6\sin\left(dx+c\right)^2-4\sin\left(dx+c\right)+3}{12\,ad\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin(
d*x + c)^4)
```

maple [A] time = 0.46, size = 49, normalized size = 0.96

$$\frac{-\frac{1}{\sin(dx+c)} + \frac{1}{2\sin(dx+c)^{2}} - \frac{1}{4\sin(dx+c)^{4}} + \frac{1}{3\sin(dx+c)^{3}}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/sin(d*x+c)+1/2/sin(d*x+c)^2-1/4/sin(d*x+c)^4+1/3/sin(d*x+c)^3)
```

maxima [A] time = 0.81, size = 46, normalized size = 0.90

$$-\frac{12 \sin(dx+c)^{3} - 6 \sin(dx+c)^{2} - 4 \sin(dx+c) + 3}{12\, ad \sin(dx+c)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin(
d*x + c)^4)
```

mupad [B] time = 8.93, size = 45, normalized size = 0.88

$$\frac{-\sin(c+d\,x)^{3} + \frac{\sin(c+d\,x)^{2}}{2} + \frac{\sin(c+d\,x)}{3} - \frac{1}{4}}{a\,d\sin(c+d\,x)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^5*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x)/3 + sin(c + d*x)^2/2 - sin(c + d*x)^3 - 1/4)/(a*d*sin(c + d*x
)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.541 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\cot^4(c+dx)}{4ad}-\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] 1/4*cot(d*x+c)^4/a/d+1/3*csc(d*x+c)^3/a/d-1/5*csc(d*x+c)^5/a/d
```

Rubi [A] time = 0.14, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2835, 2606, 14, 2607, 30}

$$\frac{\cot^4(c+dx)}{4ad}-\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^4/(4*a*d) + Csc[c + d*x]^3/(3*a*d) - Csc[c + d*x]^5/(5*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^3(c+dx)\csc^2(c+dx)\,dx}{a}+\frac{\int \cot^3(c+dx)\csc^3(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^3\,dx,x,-\cot(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\csc(c+dx)\right)}{ad}\\ &= \frac{\cot^4(c+dx)}{4ad}-\frac{\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\csc(c+dx)\right)}{ad}\\ &= \frac{\cot^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A] time = 0.11, size = 48, normalized size = 0.87

$$\frac{\csc^2(c+dx)\left(-12\csc^3(c+dx)+15\csc^2(c+dx)+20\csc(c+dx)-30\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^2*(-30 + 20*Csc[c + d*x] + 15*Csc[c + d*x]^2 - 12*Csc[c + d*x
]^3))/(60*a*d)
```

fricas [A] time = 0.66, size = 71, normalized size = 1.29

$$-\frac{20\cos(dx+c)^2-15\left(2\cos(dx+c)^2-1\right)\sin(dx+c)-8}{60\left(ad\cos(dx+c)^4-2\,ad\cos(dx+c)^2+ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(20*cos(d*x + c)^2 - 15*(2*cos(d*x + c)^2 - 1)*sin(d*x + c) - 8)/((a*
d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))
```

giac [A] time = 0.23, size = 46, normalized size = 0.84

$$-\frac{30\,\sin\left(dx+c\right)^{3}-20\,\sin\left(dx+c\right)^{2}-15\,\sin\left(dx+c\right)+12}{60\,ad\sin\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(30*sin(d*x + c)^3 - 20*sin(d*x + c)^2 - 15*sin(d*x + c) + 12)/(a*d*s
in(d*x + c)^5)
```

maple [A] time = 0.48, size = 49, normalized size = 0.89

$$\frac{-\frac{1}{5\sin\left(dx+c\right)^{5}}-\frac{1}{2\sin\left(dx+c\right)^{2}}+\frac{1}{4\sin\left(dx+c\right)^{4}}+\frac{1}{3\sin\left(dx+c\right)^{3}}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/5/sin(d*x+c)^5-1/2/sin(d*x+c)^2+1/4/sin(d*x+c)^4+1/3/sin(d*x+c)^3
)
```

maxima [A] time = 0.77, size = 46, normalized size = 0.84

$$-\frac{30\,\sin\left(dx+c\right)^{3}-20\,\sin\left(dx+c\right)^{2}-15\,\sin\left(dx+c\right)+12}{60\,ad\sin\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(30*sin(d*x + c)^3 - 20*sin(d*x + c)^2 - 15*sin(d*x + c) + 12)/(a*d*s
in(d*x + c)^5)
```

mupad [B] time = 8.96, size = 46, normalized size = 0.84

$$\frac{-30\sin\left(c+d\,x\right)^{3}+20\sin\left(c+d\,x\right)^{2}+15\,\sin\left(c+d\,x\right)-12}{60\,a\,d\sin\left(c+d\,x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^6*(a + a*sin(c + d*x))),x)
```

```
[Out] (15*sin(c + d*x) + 20*sin(c + d*x)^2 - 30*sin(c + d*x)^3 - 12)/(60*a*d*sin(
c + d*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.542 $\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^4(c+dx)}{4ad}-\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] -1/3*csc(d*x+c)^3/a/d+1/4*csc(d*x+c)^4/a/d+1/5*csc(d*x+c)^5/a/d-1/6*csc(d*x
+c)^6/a/d
```

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 75}

$$-\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^4(c+dx)}{4ad}-\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Csc[c + d*x]^3/(3*a*d) + Csc[c + d*x]^4/(4*a*d) + Csc[c + d*x]^5/(5*a*d) -
 Csc[c + d*x]^6/(6*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a-x)^2(a+x)}{x^7}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{a^3}{x^7}-\frac{a^2}{x^6}-\frac{a}{x^5}+\frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^4(c+dx)}{4ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{\csc^6(c+dx)}{6ad}$$

Mathematica [A] time = 0.10, size = 48, normalized size = 0.66

$$\frac{\csc^3(c+dx)\left(-10\csc^3(c+dx)+12\csc^2(c+dx)+15\csc(c+dx)-20\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^3*(-20 + 15*Csc[c + d*x] + 12*Csc[c + d*x]^2 - 10*Csc[c + d*x
]^3))/(60*a*d)
```

fricas [A] time = 0.52, size = 76, normalized size = 1.04

$$\frac{15\cos(dx+c)^2-4\left(5\cos(dx+c)^2-2\right)\sin(dx+c)-5}{60\left(ad\cos(dx+c)^6-3\,ad\cos(dx+c)^4+3\,ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*cos(d*x + c)^2 - 4*(5*cos(d*x + c)^2 - 2)*sin(d*x + c) - 5)/(a*d*c
os(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)
```

giac [A] time = 0.22, size = 46, normalized size = 0.63

$$-\frac{20\sin(dx+c)^3-15\sin(dx+c)^2-12\sin(dx+c)+10}{60\,ad\sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(20*sin(d*x + c)^3 - 15*sin(d*x + c)^2 - 12*sin(d*x + c) + 10)/(a*d*s
in(d*x + c)^6)
```

maple [A] time = 0.49, size = 49, normalized size = 0.67

$$\frac{-\frac{1}{6\sin(dx+c)^{6}}+\frac{1}{5\sin(dx+c)^{5}}+\frac{1}{4\sin(dx+c)^{4}}-\frac{1}{3\sin(dx+c)^{3}}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/6/sin(d*x+c)^6+1/5/sin(d*x+c)^5+1/4/sin(d*x+c)^4-1/3/sin(d*x+c)^3
)
```

maxima [A] time = 0.82, size = 46, normalized size = 0.63

$$-\frac{20\,\sin\left(dx+c\right)^{3}-15\,\sin\left(dx+c\right)^{2}-12\,\sin\left(dx+c\right)+10}{60\,ad\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(20*sin(d*x + c)^3 - 15*sin(d*x + c)^2 - 12*sin(d*x + c) + 10)/(a*d*s
in(d*x + c)^6)
```

mupad [B] time = 8.94, size = 46, normalized size = 0.63

$$\frac{-20\sin\left(c+d\,x\right)^{3}+15\sin\left(c+d\,x\right)^{2}+12\,\sin\left(c+d\,x\right)-10}{60\,a\,d\sin\left(c+d\,x\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (12*sin(c + d*x) + 15*sin(c + d*x)^2 - 20*sin(c + d*x)^3 - 10)/(60*a*d*sin(
c + d*x)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.543 $\int \frac{\cot^5(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{\csc^7(c+dx)}{7ad}+\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{\csc^4(c+dx)}{4ad}$$

```
[Out] -1/4*csc(d*x+c)^4/a/d+1/5*csc(d*x+c)^5/a/d+1/6*csc(d*x+c)^6/a/d-1/7*csc(d*x
+c)^7/a/d
```

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 75}

$$-\frac{\csc^7(c+dx)}{7ad}+\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{\csc^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Csc[c + d*x]^4/(4*a*d) + Csc[c + d*x]^5/(5*a*d) + Csc[c + d*x]^6/(6*a*d) -
 Csc[c + d*x]^7/(7*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^8(a-x)^2(a+x)}{x^8}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^8}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{a^3}{x^8}-\frac{a^2}{x^7}-\frac{a}{x^6}+\frac{1}{x^5}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^4(c+dx)}{4ad}+\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^6(c+dx)}{6ad}-\frac{\csc^7(c+dx)}{7ad}$$

Mathematica [A] time = 0.11, size = 48, normalized size = 0.66

$$\frac{\csc^4(c+dx)\left(-60\csc^3(c+dx)+70\csc^2(c+dx)+84\csc(c+dx)-105\right)}{420ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^4*(-105 + 84*Csc[c + d*x] + 70*Csc[c + d*x]^2 - 60*Csc[c + d*
x]^3))/(420*a*d)
```

fricas [A] time = 0.55, size = 84, normalized size = 1.15

$$\frac{84\cos(dx+c)^2-35\left(3\cos(dx+c)^2-1\right)\sin(dx+c)-24}{420\left(ad\cos(dx+c)^6-3\,ad\cos(dx+c)^4+3\,ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(84*cos(d*x + c)^2 - 35*(3*cos(d*x + c)^2 - 1)*sin(d*x + c) - 24)/((a
*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)*sin(
d*x + c))
```

giac [A] time = 0.23, size = 46, normalized size = 0.63

$$-\frac{105\sin(dx+c)^3-84\sin(dx+c)^2-70\sin(dx+c)+60}{420\,ad\sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/420*(105*sin(d*x + c)^3 - 84*sin(d*x + c)^2 - 70*sin(d*x + c) + 60)/(a*d
*sin(d*x + c)^7)
```

maple [A] time = 0.54, size = 49, normalized size = 0.67

$$\frac{\frac{1}{6\sin(dx+c)^6}+\frac{1}{5\sin(dx+c)^5}-\frac{1}{7\sin(dx+c)^7}-\frac{1}{4\sin(dx+c)^4}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^8/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/6/sin(d*x+c)^6+1/5/sin(d*x+c)^5-1/7/sin(d*x+c)^7-1/4/sin(d*x+c)^4)
```

maxima [A] time = 1.00, size = 46, normalized size = 0.63

$$-\frac{105 \sin(dx+c)^3 - 84 \sin(dx+c)^2 - 70 \sin(dx+c) + 60}{420\,ad\sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(105*sin(d*x + c)^3 - 84*sin(d*x + c)^2 - 70*sin(d*x + c) + 60)/(a*d
*sin(d*x + c)^7)
```

mupad [B] time = 8.96, size = 46, normalized size = 0.63

$$\frac{-105\sin(c+d\,x)^3+84\sin(c+d\,x)^2+70\,\sin(c+d\,x)-60}{420\,a\,d\sin(c+d\,x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^8*(a + a*sin(c + d*x))),x)
```

```
[Out] (70*sin(c + d*x) + 84*sin(c + d*x)^2 - 105*sin(c + d*x)^3 - 60)/(420*a*d*si
n(c + d*x)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.544 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^6(c+dx)}{6a^2d} - \frac{2\sin^5(c+dx)}{5a^2d} + \frac{\sin^4(c+dx)}{4a^2d}$$

```
[Out] 1/4*sin(d*x+c)^4/a^2/d-2/5*sin(d*x+c)^5/a^2/d+1/6*sin(d*x+c)^6/a^2/d
```

Rubi [A] time = 0.11, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$\frac{\sin^6(c+dx)}{6a^2d} - \frac{2\sin^5(c+dx)}{5a^2d} + \frac{\sin^4(c+dx)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]^4/(4*a^2*d) - (2*Sin[c + d*x]^5)/(5*a^2*d) + Sin[c + d*x]^6/(6
*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^3}{a^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^3\,dx, x, a\sin(c+dx)\right)}{a^8 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2x^3-2ax^4+x^5\right)\,dx, x, a\sin(c+dx)\right)}{a^8 d}$$
$$= \frac{\sin^4(c+dx)}{4a^2d} - \frac{2\sin^5(c+dx)}{5a^2d} + \frac{\sin^6(c+dx)}{6a^2d}$$

Mathematica [A] time = 0.54, size = 38, normalized size = 0.69

$$\frac{\sin^4(c+dx)\left(10\sin^2(c+dx)-24\sin(c+dx)+15\right)}{60a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^4*(15 - 24*Sin[c + d*x] + 10*Sin[c + d*x]^2))/(60*a^2*d)
```

fricas [A] time = 0.75, size = 67, normalized size = 1.22

$$-\frac{10\cos(dx+c)^6 - 45\cos(dx+c)^4 + 60\cos(dx+c)^2 + 24\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\sin(dx+c)}{60\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/60*(10*cos(d*x + c)^6 - 45*cos(d*x + c)^4 + 60*cos(d*x + c)^2 + 24*(cos(
d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c))/(a^2*d)
```

giac [A] time = 0.23, size = 39, normalized size = 0.71

$$\frac{10\sin(dx+c)^6 - 24\sin(dx+c)^5 + 15\sin(dx+c)^4}{60\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

[Out] 1/60*(10*sin(d*x + c)^6 - 24*sin(d*x + c)^5 + 15*sin(d*x + c)^4)/(a^2*d)

maple [A] time = 0.38, size = 39, normalized size = 0.71

$$\frac{\frac{\left(\sin^{6}(dx+c)\right)}{6}-\frac{2\left(\sin^{5}(dx+c)\right)}{5}+\frac{\left(\sin^{4}(dx+c)\right)}{4}}{d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)

[Out] 1/d/a^2*(1/6*sin(d*x+c)^6-2/5*sin(d*x+c)^5+1/4*sin(d*x+c)^4)

maxima [A] time = 0.72, size = 39, normalized size = 0.71

$$\frac{10\,\sin\left(dx+c\right)^{6}-24\,\sin\left(dx+c\right)^{5}+15\,\sin\left(dx+c\right)^{4}}{60\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")

[Out] 1/60*(10*sin(d*x + c)^6 - 24*sin(d*x + c)^5 + 15*sin(d*x + c)^4)/(a^2*d)

mupad [B] time = 0.06, size = 36, normalized size = 0.65

$$\frac{\sin\left(c+d\,x\right)^{4}\,\left(10\sin\left(c+d\,x\right)^{2}-24\,\sin\left(c+d\,x\right)+15\right)}{60\,a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^5*sin(c + d*x)^3)/(a + a*sin(c + d*x))^2,x)

[Out] (sin(c + d*x)^4*(10*sin(c + d*x)^2 - 24*sin(c + d*x) + 15))/(60*a^2*d)

sympy [A] time = 128.34, size = 682, normalized size = 12.40

$$\begin{cases} \frac{60\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^{2}d\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+90a^{2}d\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+225a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+300a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+225a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+90a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^{2}d} - \frac{}{15a^{2}d\,t} \\ \frac{x\sin^{3}\left(c\right)\cos^{5}\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((60*tan(c/2 + d*x/2)**8/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2
*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c
/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2
)**2 + 15*a**2*d) - 192*tan(c/2 + d*x/2)**7/(15*a**2*d*tan(c/2 + d*x/2)**12
 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a*
*2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c
/2 + d*x/2)**2 + 15*a**2*d) + 280*tan(c/2 + d*x/2)**6/(15*a**2*d*tan(c/2 +
d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**
8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a*
*2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 192*tan(c/2 + d*x/2)**5/(15*a**2*d*
tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2
+ d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)*
*4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 60*tan(c/2 + d*x/2)**4/(1
5*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d
*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2
+ d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d), Ne(d, 0)), (x*sin
(c)**3*cos(c)**5/(a*sin(c) + a)**2, True))
```

3.545 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^5(c+dx)}{5a^2d}-\frac{\sin^4(c+dx)}{2a^2d}+\frac{\sin^3(c+dx)}{3a^2d}$$

```
[Out] 1/3*sin(d*x+c)^3/a^2/d-1/2*sin(d*x+c)^4/a^2/d+1/5*sin(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.10, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$\frac{\sin^5(c+dx)}{5a^2d}-\frac{\sin^4(c+dx)}{2a^2d}+\frac{\sin^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]^3/(3*a^2*d) - Sin[c + d*x]^4/(2*a^2*d) + Sin[c + d*x]^5/(5*a^2
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2}{a^2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^2\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2x^2-2ax^3+x^4\right)\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\sin^3(c+dx)}{3a^2d} - \frac{\sin^4(c+dx)}{2a^2d} + \frac{\sin^5(c+dx)}{5a^2d}$$

Mathematica [A] time = 0.66, size = 38, normalized size = 0.69

$$-\frac{\sin^3(c+dx)(15\sin(c+dx)+3\cos(2(c+dx))-13)}{30a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/30*(Sin[c + d*x]^3*(-13 + 3*Cos[2*(c + d*x)] + 15*Sin[c + d*x]))/(a^2*d)
```

fricas [A] time = 0.61, size = 59, normalized size = 1.07

$$-\frac{15\,\cos\left(dx+c\right)^4-30\,\cos\left(dx+c\right)^2-2\left(3\,\cos\left(dx+c\right)^4-11\,\cos\left(dx+c\right)^2+8\right)\sin\left(dx+c\right)}{30\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/30*(15*cos(d*x + c)^4 - 30*cos(d*x + c)^2 - 2*(3*cos(d*x + c)^4 - 11*cos
(d*x + c)^2 + 8)*sin(d*x + c))/(a^2*d)
```

giac [A] time = 0.18, size = 39, normalized size = 0.71

$$\frac{6\,\sin\left(dx+c\right)^5-15\,\sin\left(dx+c\right)^4+10\,\sin\left(dx+c\right)^3}{30\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

[Out] 1/30*(6*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 10*sin(d*x + c)^3)/(a^2*d)

maple [A] time = 0.36, size = 39, normalized size = 0.71

$$\frac{\frac{\left(\sin^{5}(dx+c)\right)}{5}-\frac{\left(\sin^{4}(dx+c)\right)}{2}+\frac{\left(\sin^{3}(dx+c)\right)}{3}}{d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)

[Out] 1/d/a^2*(1/5*sin(d*x+c)^5-1/2*sin(d*x+c)^4+1/3*sin(d*x+c)^3)

maxima [A] time = 0.57, size = 39, normalized size = 0.71

$$\frac{6\,\sin\left(dx+c\right)^{5}-15\,\sin\left(dx+c\right)^{4}+10\,\sin\left(dx+c\right)^{3}}{30\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")

[Out] 1/30*(6*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 10*sin(d*x + c)^3)/(a^2*d)

mupad [B] time = 0.05, size = 36, normalized size = 0.65

$$\frac{\sin\left(c+d\,x\right)^{3}\,\left(6\sin\left(c+d\,x\right)^{2}-15\,\sin\left(c+d\,x\right)+10\right)}{30\,a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^5*sin(c + d*x)^2)/(a + a*sin(c + d*x))^2,x)

[Out] (sin(c + d*x)^3*(6*sin(c + d*x)^2 - 15*sin(c + d*x) + 10))/(30*a^2*d)

sympy [A] time = 82.06, size = 588, normalized size = 10.69

$$\begin{cases} \frac{40\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^{2}d\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^{2}d} - \frac{}{15a^{2}d\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^{2}d\tan} \\ \frac{x\sin^{2}\left(c\right)\cos^{5}\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((40*tan(c/2 + d*x/2)**7/(15*a**2*d*tan(c/2 + d*x/2)**10 + 75*a**2
*d*tan(c/2 + d*x/2)**8 + 150*a**2*d*tan(c/2 + d*x/2)**6 + 150*a**2*d*tan(c/
2 + d*x/2)**4 + 75*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 120*tan(c/2 +
d*x/2)**6/(15*a**2*d*tan(c/2 + d*x/2)**10 + 75*a**2*d*tan(c/2 + d*x/2)**8 +
 150*a**2*d*tan(c/2 + d*x/2)**6 + 150*a**2*d*tan(c/2 + d*x/2)**4 + 75*a**2*
d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 176*tan(c/2 + d*x/2)**5/(15*a**2*d*tan
(c/2 + d*x/2)**10 + 75*a**2*d*tan(c/2 + d*x/2)**8 + 150*a**2*d*tan(c/2 + d*
x/2)**6 + 150*a**2*d*tan(c/2 + d*x/2)**4 + 75*a**2*d*tan(c/2 + d*x/2)**2 +
15*a**2*d) - 120*tan(c/2 + d*x/2)**4/(15*a**2*d*tan(c/2 + d*x/2)**10 + 75*a
**2*d*tan(c/2 + d*x/2)**8 + 150*a**2*d*tan(c/2 + d*x/2)**6 + 150*a**2*d*tan
(c/2 + d*x/2)**4 + 75*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 40*tan(c/2
+ d*x/2)**3/(15*a**2*d*tan(c/2 + d*x/2)**10 + 75*a**2*d*tan(c/2 + d*x/2)**8
 + 150*a**2*d*tan(c/2 + d*x/2)**6 + 150*a**2*d*tan(c/2 + d*x/2)**4 + 75*a**
2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**5/(a*
sin(c) + a)**2, True))
```

3.546 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^4(c+dx)}{4a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin^2(c+dx)}{2a^2d}$$

```
[Out] 1/2*sin(d*x+c)^2/a^2/d-2/3*sin(d*x+c)^3/a^2/d+1/4*sin(d*x+c)^4/a^2/d
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 43}

$$\frac{\sin^4(c+dx)}{4a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]^2/(2*a^2*d) - (2*Sin[c + d*x]^3)/(3*a^2*d) + Sin[c + d*x]^4/(4
*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x}{a}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x\,dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2x-2ax^2+x^3\right)\,dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{\sin^2(c+dx)}{2a^2d} - \frac{2\sin^3(c+dx)}{3a^2d} + \frac{\sin^4(c+dx)}{4a^2d}$$

Mathematica [A] time = 0.21, size = 38, normalized size = 0.69

$$\frac{\sin^2(c+dx)\left(3\sin^2(c+dx)-8\sin(c+dx)+6\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
[Out] (Sin[c + d*x]^2*(6 - 8*Sin[c + d*x] + 3*Sin[c + d*x]^2))/(12*a^2*d)
```

fricas [A] time = 0.77, size = 47, normalized size = 0.85

$$\frac{3\cos(dx+c)^4 - 12\cos(dx+c)^2 + 8\left(\cos(dx+c)^2-1\right)\sin(dx+c)}{12\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] 1/12*(3*cos(d*x + c)^4 - 12*cos(d*x + c)^2 + 8*(cos(d*x + c)^2 - 1)*sin(d*x
 + c))/(a^2*d)
```

giac [A] time = 0.17, size = 39, normalized size = 0.71

$$\frac{3\sin(dx+c)^4 - 8\sin(dx+c)^3 + 6\sin(dx+c)^2}{12\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] 1/12*(3*sin(d*x + c)^4 - 8*sin(d*x + c)^3 + 6*sin(d*x + c)^2)/(a^2*d)
```

maple [A] time = 0.30, size = 39, normalized size = 0.71

$$\frac{\frac{\left(\sin^{4}(dx+c)\right)}{4}-\frac{2\left(\sin^{3}(dx+c)\right)}{3}+\frac{\left(\sin^{2}(dx+c)\right)}{2}}{d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
[Out] 1/d/a^2*(1/4*sin(d*x+c)^4-2/3*sin(d*x+c)^3+1/2*sin(d*x+c)^2)
```

maxima [A] time = 0.81, size = 39, normalized size = 0.71

$$\frac{3\,\sin\left(dx+c\right)^{4}-8\,\sin\left(dx+c\right)^{3}+6\,\sin\left(dx+c\right)^{2}}{12\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] 1/12*(3*sin(d*x + c)^4 - 8*sin(d*x + c)^3 + 6*sin(d*x + c)^2)/(a^2*d)
```

mupad [B] time = 8.83, size = 36, normalized size = 0.65

$$\frac{\sin\left(c+d\,x\right)^{2}\,\left(3\sin\left(c+d\,x\right)^{2}-8\,\sin\left(c+d\,x\right)+6\right)}{12\,a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x))/(a + a*sin(c + d*x))^2,x)
[Out] (sin(c + d*x)^2*(3*sin(c + d*x)^2 - 8*sin(c + d*x) + 6))/(12*a^2*d)
```

sympy [A] time = 53.00, size = 493, normalized size = 8.96

$$\begin{cases} \frac{6\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{2}d}-\frac{16\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^{2}d\tan^{4}\left(\frac{c}{2}+\right.} \\ \frac{x\sin\left(c\right)\cos^{5}\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((6*tan(c/2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*
tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d
*x/2)**2 + 3*a**2*d) - 16*tan(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**8
 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*
d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 24*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/
2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)*
*4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 16*tan(c/2 + d*x/2)**3/(3*
a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(
c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 6*tan(c/2 + d
*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18
*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d), Ne
(d, 0)), (x*sin(c)*cos(c)**5/(a*sin(c) + a)**2, True))
```

3.547 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$\frac{\sin^2(c+dx)}{2a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}$$

```
[Out] ln(sin(d*x+c))/a^2/d-2*sin(d*x+c)/a^2/d+1/2*sin(d*x+c)^2/a^2/d
```

Rubi [A] time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 43}

$$\frac{\sin^2(c+dx)}{2a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^2*d) - (2*Sin[c + d*x])/(a^2*d) + Sin[c + d*x]^2/(2*a^
2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^2}{x}+x\right)\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\log(\sin(c+dx))}{a^2 d} - \frac{2\sin(c+dx)}{a^2 d} + \frac{\sin^2(c+dx)}{2a^2 d}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.77

$$\frac{\sin^2(c+dx) - 4\sin(c+dx) + 2\log(\sin(c+dx))}{2a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
[Out] (2*Log[Sin[c + d*x]] - 4*Sin[c + d*x] + Sin[c + d*x]^2)/(2*a^2*d)
```

fricas [A] time = 0.51, size = 36, normalized size = 0.77

$$-\frac{\cos\left(dx+c\right)^2 - 2\,\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right) + 4\,\sin\left(dx+c\right)}{2\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] -1/2*(cos(d*x + c)^2 - 2*log(1/2*sin(d*x + c)) + 4*sin(d*x + c))/(a^2*d)
```

giac [A] time = 0.18, size = 47, normalized size = 1.00

$$\frac{\frac{2\,\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^2} + \frac{a^2\sin\left(dx+c\right)^2 - 4\,a^2\sin\left(dx+c\right)}{a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(sin(d*x + c)))/a^2 + (a^2*sin(d*x + c)^2 - 4*a^2*sin(d*x + c
))/a^4)/d
```

maple [A] time = 0.46, size = 46, normalized size = 0.98

$$\frac{\ln\left(\sin\left(dx+c\right)\right)}{a^{2}d}-\frac{2\sin\left(dx+c\right)}{a^{2}d}+\frac{\sin^{2}\left(dx+c\right)}{2a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] ln(sin(d*x+c))/a^2/d-2*sin(d*x+c)/a^2/d+1/2*sin(d*x+c)^2/a^2/d
```

maxima [A] time = 0.51, size = 39, normalized size = 0.83

$$\frac{\frac{\sin\left(dx+c\right)^{2}-4\,\sin\left(dx+c\right)}{a^{2}}+\frac{2\,\log\left(\sin\left(dx+c\right)\right)}{a^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((sin(d*x + c)^2 - 4*sin(d*x + c))/a^2 + 2*log(sin(d*x + c))/a^2)/d
```

mupad [B] time = 9.11, size = 120, normalized size = 2.55

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a^2\,d}-\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+2\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (
4*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2 + 4*tan(c/2 + (d*x)/2)^3)/(d*
(2*a^2*tan(c/2 + (d*x)/2)^2 + a^2*tan(c/2 + (d*x)/2)^4 + a^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.548 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=43

$$\frac{\sin(c+dx)}{a^2d}-\frac{\csc(c+dx)}{a^2d}-\frac{2\log(\sin(c+dx))}{a^2d}$$

```
[Out] -csc(d*x+c)/a^2/d-2*ln(sin(d*x+c))/a^2/d+sin(d*x+c)/a^2/d
```

Rubi [A] time = 0.10, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$\frac{\sin(c+dx)}{a^2d}-\frac{\csc(c+dx)}{a^2d}-\frac{2\log(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]/(a^2*d)) - (2*Log[Sin[c + d*x]])/(a^2*d) + Sin[c + d*x]/(a^2
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^2}{x^2}-\frac{2a}{x}\right)dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= -\frac{\csc(c+dx)}{a^2 d} - \frac{2\log(\sin(c+dx))}{a^2 d} + \frac{\sin(c+dx)}{a^2 d}$$

Mathematica [A] time = 0.04, size = 32, normalized size = 0.74

$$-\frac{-\sin(c+dx)+\csc(c+dx)+2\log(\sin(c+dx))}{a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((Csc[c + d*x] + 2*Log[Sin[c + d*x]] - Sin[c + d*x])/(a^2*d))
```

fricas [A] time = 0.76, size = 42, normalized size = 0.98

$$-\frac{\cos(dx+c)^2 + 2\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)}{a^2 d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(cos(d*x + c)^2 + 2*log(1/2*sin(d*x + c))*sin(d*x + c))/(a^2*d*sin(d*x + c
))
```

giac [A] time = 0.20, size = 53, normalized size = 1.23

$$-\frac{\frac{2\log(|\sin(dx+c)|)}{a^2} - \frac{\sin(dx+c)}{a^2} - \frac{2\sin(dx+c)-1}{a^2\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*log(abs(sin(d*x + c)))/a^2 - sin(d*x + c)/a^2 - (2*sin(d*x + c) - 1)/(a
^2*sin(d*x + c)))/d
```

maple [A] time = 0.48, size = 46, normalized size = 1.07

$$\frac{\sin\left(dx+c\right)}{a^{2}d}-\frac{1}{a^{2}d\sin\left(dx+c\right)}-\frac{2\ln\left(\sin\left(dx+c\right)\right)}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] sin(d*x+c)/a^2/d-1/a^2/d/sin(d*x+c)-2*ln(sin(d*x+c))/a^2/d
```

maxima [A] time = 0.57, size = 41, normalized size = 0.95

$$-\frac{\frac{2\log\left(\sin\left(dx+c\right)\right)}{a^{2}}-\frac{\sin\left(dx+c\right)}{a^{2}}+\frac{1}{a^{2}\sin\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -(2*log(sin(d*x + c))/a^2 - sin(d*x + c)/a^2 + 1/(a^2*sin(d*x + c)))/d
```

mupad [B] time = 8.92, size = 110, normalized size = 2.56

$$\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}{d\,\left(2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}+\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (3*tan(c/2 + (d*x)/2)^2 - 1)/(d*(2*a^2*tan(c/2 + (d*x)/2)^3 + 2*a^2*tan(c/2
 + (d*x)/2))) - (2*log(tan(c/2 + (d*x)/2)))/(a^2*d) - tan(c/2 + (d*x)/2)/(2
*a^2*d) + (2*log(tan(c/2 + (d*x)/2)^2 + 1))/(a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.549 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}$$

```
[Out] 2*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a^2/d+ln(sin(d*x+c))/a^2/d
```

Rubi [A] time = 0.10, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a^2*d) + Log[Sin[c + d*x]]/(a^
2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^3} - \frac{2a}{x^2} + \frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{2\csc(c+dx)}{a^2 d} - \frac{\csc^2(c+dx)}{2a^2 d} + \frac{\log(\sin(c+dx))}{a^2 d}$$

Mathematica [A] time = 0.05, size = 38, normalized size = 0.81

$$\frac{-\csc^2(c+dx) + 4\csc(c+dx) + 2\log(\sin(c+dx))}{2a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*Csc[c + d*x] - Csc[c + d*x]^2 + 2*Log[Sin[c + d*x]])/(2*a^2*d)
```

fricas [A] time = 0.69, size = 55, normalized size = 1.17

$$\frac{2\left(\cos(dx+c)^2 - 1\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 4\sin(dx+c) + 1}{2\left(a^2 d\cos(dx+c)^2 - a^2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 4*sin(d*x + c) + 1)/(a^
2*d*cos(d*x + c)^2 - a^2*d)
```

giac [A] time = 0.22, size = 52, normalized size = 1.11

$$\frac{\frac{2\log(|\sin(dx+c)|)}{a^2} - \frac{3\sin(dx+c)^2 - 4\sin(dx+c) + 1}{a^2\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(sin(d*x + c)))/a^2 - (3*sin(d*x + c)^2 - 4*sin(d*x + c) + 1)
/(a^2*sin(d*x + c)^2))/d
```

maple [A] time = 0.53, size = 48, normalized size = 1.02

$$\frac{2}{a^2 d \sin(dx+c)} + \frac{\ln(\sin(dx+c))}{a^2 d} - \frac{1}{2d\, a^2 \sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/a^2/d/sin(d*x+c)+ln(sin(d*x+c))/a^2/d-1/2/d/a^2/sin(d*x+c)^2
```

maxima [A] time = 0.85, size = 40, normalized size = 0.85

$$\frac{\frac{2\log(\sin(dx+c))}{a^2} + \frac{4\sin(dx+c)-1}{a^2\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/2*(2*log(sin(d*x + c))/a^2 + (4*sin(d*x + c) - 1)/(a^2*sin(d*x + c)^2))/d
```

mupad [B] time = 8.89, size = 104, normalized size = 2.21

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} - \frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,a^2\,d} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2\,d} - \frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a^2\,d} + \frac{{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{8}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - tan(c/2 + (d*x)/2)^2/(8*a^2*d) + tan(c/2
+ (d*x)/2)/(a^2*d) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) + (cot(c/2 + (d*
x)/2)^2*(tan(c/2 + (d*x)/2) - 1/8))/(a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.550 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=31

$$-\frac{\csc^3(c+dx)(a-a\sin(c+dx))^3}{3a^5d}$$

```
[Out] -1/3*csc(d*x+c)^3*(a-a*sin(d*x+c))^3/a^5/d
```

Rubi [A] time = 0.08, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 37}

$$-\frac{\csc^3(c+dx)(a-a\sin(c+dx))^3}{3a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^3*(a - a*Sin[c + d*x])^3)/(3*a^5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{\csc^3(c+dx)(a-a\sin(c+dx))^3}{3a^5d}$$

Mathematica [A] time = 0.04, size = 20, normalized size = 0.65

$$-\frac{(\csc(c+dx)-1)^3}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
[Out] -1/3*(-1 + Csc[c + d*x])^3/(a^2*d)
```

fricas [A] time = 0.63, size = 52, normalized size = 1.68

$$-\frac{3\cos(dx+c)^2 + 3\sin(dx+c) - 4}{3\left(a^2d\cos(dx+c)^2 - a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
[Out] -1/3*(3*cos(d*x + c)^2 + 3*sin(d*x + c) - 4)/((a^2*d*cos(d*x + c)^2 - a^2*d
)*sin(d*x + c))
```

giac [A] time = 0.23, size = 36, normalized size = 1.16

$$-\frac{3\sin(dx+c)^2 - 3\sin(dx+c) + 1}{3a^2d\sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] -1/3*(3*sin(d*x + c)^2 - 3*sin(d*x + c) + 1)/(a^2*d*sin(d*x + c)^3)
```

maple [A] time = 0.54, size = 37, normalized size = 1.19

$$\frac{-\frac{1}{\sin(dx+c)}+\frac{1}{\sin(dx+c)^2}-\frac{1}{3\sin(dx+c)^3}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/sin(d*x+c)+1/sin(d*x+c)^2-1/3/sin(d*x+c)^3)
```

maxima [A] time = 0.31, size = 36, normalized size = 1.16

$$-\frac{3\,\sin\left(dx+c\right)^2-3\,\sin\left(dx+c\right)+1}{3\,a^2 d\sin\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3*(3*sin(d*x + c)^2 - 3*sin(d*x + c) + 1)/(a^2*d*sin(d*x + c)^3)
```

mupad [B] time = 8.93, size = 34, normalized size = 1.10

$$-\frac{\sin\left(c+d\,x\right)^2-\sin\left(c+d\,x\right)+\frac{1}{3}}{a^2\,d\sin\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(sin(c + d*x)^2 - sin(c + d*x) + 1/3)/(a^2*d*sin(c + d*x)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.551 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

```
[Out] -1/2*csc(d*x+c)^2/a^2/d+2/3*csc(d*x+c)^3/a^2/d-1/4*csc(d*x+c)^4/a^2/d
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 43}

$$-\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Csc[c + d*x]^2/(2*a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(
4*a^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^5}-\frac{2a}{x^4}+\frac{1}{x^3}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^4(c+dx)}{4a^2d}$$

Mathematica [A] time = 0.06, size = 38, normalized size = 0.69

$$\frac{\csc^4(c+dx)(8\sin(c+dx)+3\cos(2(c+dx))-6)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^4*(-6 + 3*Cos[2*(c + d*x)] + 8*Sin[c + d*x]))/(12*a^2*d)
```

fricas [A] time = 0.71, size = 57, normalized size = 1.04

$$\frac{6\cos(dx+c)^2+8\sin(dx+c)-9}{12\left(a^2d\cos(dx+c)^4-2a^2d\cos(dx+c)^2+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(6*cos(d*x + c)^2 + 8*sin(d*x + c) - 9)/(a^2*d*cos(d*x + c)^4 - 2*a^2*
d*cos(d*x + c)^2 + a^2*d)
```

giac [A] time = 0.26, size = 36, normalized size = 0.65

$$-\frac{6\sin(dx+c)^2-8\sin(dx+c)+3}{12a^2d\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)
```

maple [A] time = 0.53, size = 39, normalized size = 0.71

$$\frac{-\frac{1}{2\sin(dx+c)^2}-\frac{1}{4\sin(dx+c)^4}+\frac{2}{3\sin(dx+c)^3}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/2/sin(d*x+c)^2-1/4/sin(d*x+c)^4+2/3/sin(d*x+c)^3)
```

maxima [A] time = 0.53, size = 36, normalized size = 0.65

$$-\frac{6\,\sin\left(dx+c\right)^2-8\,\sin\left(dx+c\right)+3}{12\,a^2d\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)
```

mupad [B] time = 8.92, size = 36, normalized size = 0.65

$$-\frac{\frac{\sin(c+d\,x)^2}{2}-\frac{2\,\sin(c+d\,x)}{3}+\frac{1}{4}}{a^2\,d\sin\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^5*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(sin(c + d*x)^2/2 - (2*sin(c + d*x))/3 + 1/4)/(a^2*d*sin(c + d*x)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.552 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\csc^5(c+dx)}{5a^2d}+\frac{\csc^4(c+dx)}{2a^2d}-\frac{\csc^3(c+dx)}{3a^2d}$$

```
[Out] -1/3*csc(d*x+c)^3/a^2/d+1/2*csc(d*x+c)^4/a^2/d-1/5*csc(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 43}

$$-\frac{\csc^5(c+dx)}{5a^2d}+\frac{\csc^4(c+dx)}{2a^2d}-\frac{\csc^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Csc[c + d*x]^3/(3*a^2*d) + Csc[c + d*x]^4/(2*a^2*d) - Csc[c + d*x]^5/(5*a^
2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{a^2}{x^6} - \frac{2a}{x^5} + \frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^3(c+dx)}{3a^2d} + \frac{\csc^4(c+dx)}{2a^2d} - \frac{\csc^5(c+dx)}{5a^2d}$$

Mathematica [A] time = 0.07, size = 38, normalized size = 0.69

$$\frac{\csc^5(c+dx)(15\sin(c+dx) + 5\cos(2(c+dx)) - 11)}{30a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^5*(-11 + 5*Cos[2*(c + d*x)] + 15*Sin[c + d*x]))/(30*a^2*d)
```

fricas [A] time = 0.76, size = 65, normalized size = 1.18

$$\frac{10\cos(dx+c)^2 + 15\sin(dx+c) - 16}{30\left(a^2d\cos(dx+c)^4 - 2a^2d\cos(dx+c)^2 + a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/30*(10*cos(d*x + c)^2 + 15*sin(d*x + c) - 16)/((a^2*d*cos(d*x + c)^4 - 2*
a^2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

giac [A] time = 0.23, size = 36, normalized size = 0.65

$$-\frac{10\sin(dx+c)^2 - 15\sin(dx+c) + 6}{30a^2d\sin(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] -1/30*(10*sin(d*x + c)^2 - 15*sin(d*x + c) + 6)/(a^2*d*sin(d*x + c)^5)
```

maple [A] time = 0.56, size = 39, normalized size = 0.71

$$\frac{-\frac{1}{5\sin(dx+c)^5}+\frac{1}{2\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x)
[Out] 1/d/a^2*(-1/5/sin(d*x+c)^5+1/2/sin(d*x+c)^4-1/3/sin(d*x+c)^3)
```

maxima [A] time = 0.41, size = 36, normalized size = 0.65

$$-\frac{10\,\sin\left(dx+c\right)^2-15\,\sin\left(dx+c\right)+6}{30\,a^2d\sin\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
[Out] -1/30*(10*sin(d*x + c)^2 - 15*sin(d*x + c) + 6)/(a^2*d*sin(d*x + c)^5)
```

mupad [B] time = 8.94, size = 36, normalized size = 0.65

$$-\frac{\frac{\sin(c+d\,x)^2}{3}-\frac{\sin(c+d\,x)}{2}+\frac{1}{5}}{a^2\,d\sin\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^6*(a + a*sin(c + d*x))^2),x)
[Out] -(sin(c + d*x)^2/3 - sin(c + d*x)/2 + 1/5)/(a^2*d*sin(c + d*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
[Out] Timed out
```

3.553 $\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\csc^6(c+dx)}{6a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{\csc^4(c+dx)}{4a^2d}$$

```
[Out] -1/4*csc(d*x+c)^4/a^2/d+2/5*csc(d*x+c)^5/a^2/d-1/6*csc(d*x+c)^6/a^2/d
```

Rubi [A] time = 0.10, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$-\frac{\csc^6(c+dx)}{6a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{\csc^4(c+dx)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Csc[c + d*x]^4/(4*a^2*d) + (2*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6/(
6*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a-x)^2}{x^7}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(a-x)^2}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{a^2}{x^7}-\frac{2a}{x^6}+\frac{1}{x^5}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{\csc^6(c+dx)}{6a^2d}$$

Mathematica [**A**] time = 0.08, size = 38, normalized size = 0.69

$$-\frac{\csc^4(c+dx)\left(10\csc^2(c+dx)-24\csc(c+dx)+15\right)}{60a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/60*(Csc[c + d*x]^4*(15 - 24*Csc[c + d*x] + 10*Csc[c + d*x]^2))/(a^2*d)
```

fricas [**A**] time = 0.70, size = 72, normalized size = 1.31

$$-\frac{15\cos\left(dx+c\right)^2+24\sin\left(dx+c\right)-25}{60\left(a^2d\cos\left(dx+c\right)^6-3\,a^2d\cos\left(dx+c\right)^4+3\,a^2d\cos\left(dx+c\right)^2-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/60*(15*cos(d*x + c)^2 + 24*sin(d*x + c) - 25)/(a^2*d*cos(d*x + c)^6 - 3*
a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)
```

giac [**A**] time = 0.27, size = 36, normalized size = 0.65

$$-\frac{15\sin\left(dx+c\right)^2-24\sin\left(dx+c\right)+10}{60\,a^2d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] -1/60*(15*sin(d*x + c)^2 - 24*sin(d*x + c) + 10)/(a^2*d*sin(d*x + c)^6)
```

maple [A] time = 0.58, size = 39, normalized size = 0.71

$$\frac{-\frac{1}{6\sin(dx+c)^{6}}+\frac{2}{5\sin(dx+c)^{5}}-\frac{1}{4\sin(dx+c)^{4}}}{d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
[Out] 1/d/a^2*(-1/6/sin(d*x+c)^6+2/5/sin(d*x+c)^5-1/4/sin(d*x+c)^4)
```

maxima [A] time = 0.96, size = 36, normalized size = 0.65

$$-\frac{15\,\sin\left(dx+c\right)^{2}-24\,\sin\left(dx+c\right)+10}{60\,a^{2}d\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
[Out] -1/60*(15*sin(d*x + c)^2 - 24*sin(d*x + c) + 10)/(a^2*d*sin(d*x + c)^6)
```

mupad [B] time = 8.95, size = 36, normalized size = 0.65

$$-\frac{\frac{\sin(c+d\,x)^{2}}{4}-\frac{2\,\sin(c+d\,x)}{5}+\frac{1}{6}}{a^{2}\,d\sin\left(c+d\,x\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^7*(a + a*sin(c + d*x))^2),x)
[Out] -(sin(c + d*x)^2/4 - (2*sin(c + d*x))/5 + 1/6)/(a^2*d*sin(c + d*x)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
[Out] Timed out
```

3.554 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{\sin^5(c+dx)}{5a^3d}-\frac{3\sin^4(c+dx)}{4a^3d}+\frac{4\sin^3(c+dx)}{3a^3d}-\frac{2\sin^2(c+dx)}{a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] -4*ln(1+sin(d*x+c))/a^3/d+4*sin(d*x+c)/a^3/d-2*sin(d*x+c)^2/a^3/d+4/3*sin(d
*x+c)^3/a^3/d-3/4*sin(d*x+c)^4/a^3/d+1/5*sin(d*x+c)^5/a^3/d
```

Rubi [A] time = 0.13, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{\sin^5(c+dx)}{5a^3d}-\frac{3\sin^4(c+dx)}{4a^3d}+\frac{4\sin^3(c+dx)}{3a^3d}-\frac{2\sin^2(c+dx)}{a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^3*d) + (4*Sin[c + d*x])/(a^3*d) - (2*Sin[c +
d*x]^2)/(a^3*d) + (4*Sin[c + d*x]^3)/(3*a^3*d) - (3*Sin[c + d*x]^4)/(4*a^3*
d) + Sin[c + d*x]^5/(5*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^3}{a^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2x^3}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^4-4a^3x+4a^2x^2-3ax^3+x^4-\frac{4a^5}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= -\frac{4\log(1+\sin(c+dx))}{a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{2\sin^2(c+dx)}{a^3d}+\frac{4\sin^3(c+dx)}{3a^3d}-$$

Mathematica [A] time = 0.97, size = 71, normalized size = 0.70

$$\frac{192\sin^5(c+dx)-720\sin^4(c+dx)+1280\sin^3(c+dx)-1920\sin^2(c+dx)+3840\sin(c+dx)-3840\log(\sin(}{960a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (45 - 3840*Log[1 + Sin[c + d*x]] + 3840*Sin[c + d*x] - 1920*Sin[c + d*x]^2
+ 1280*Sin[c + d*x]^3 - 720*Sin[c + d*x]^4 + 192*Sin[c + d*x]^5)/(960*a^3*d
)
```

fricas [A] time = 0.67, size = 70, normalized size = 0.69

$$-\frac{45\cos(dx+c)^4-210\cos(dx+c)^2-4\left(3\cos(dx+c)^4-26\cos(dx+c)^2+83\right)\sin(dx+c)+240\log(\sin(}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/60*(45*cos(d*x + c)^4 - 210*cos(d*x + c)^2 - 4*(3*cos(d*x + c)^4 - 26*co
s(d*x + c)^2 + 83)*sin(d*x + c) + 240*log(sin(d*x + c) + 1))/(a^3*d)
```

giac [B] time = 0.28, size = 193, normalized size = 1.89

$$\frac{\frac{60\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^3}-\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{137\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+805\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-640\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/15*(60*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 120*log(abs(tan(1/2*d*x + 1/
2*c) + 1))/a^3 - (137*tan(1/2*d*x + 1/2*c)^10 - 120*tan(1/2*d*x + 1/2*c)^9
+ 805*tan(1/2*d*x + 1/2*c)^8 - 640*tan(1/2*d*x + 1/2*c)^7 + 1910*tan(1/2*d*
x + 1/2*c)^6 - 1136*tan(1/2*d*x + 1/2*c)^5 + 1910*tan(1/2*d*x + 1/2*c)^4 -
640*tan(1/2*d*x + 1/2*c)^3 + 805*tan(1/2*d*x + 1/2*c)^2 - 120*tan(1/2*d*x +
 1/2*c) + 137)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^3))/d
```

maple [A] time = 0.42, size = 97, normalized size = 0.95

$$-\frac{4\ln\left(1+\sin\left(dx+c\right)\right)}{a^{3}d}+\frac{4\sin\left(dx+c\right)}{a^{3}d}-\frac{2\left(\sin^{2}\left(dx+c\right)\right)}{a^{3}d}+\frac{4\left(\sin^{3}\left(dx+c\right)\right)}{3a^{3}d}-\frac{3\left(\sin^{4}\left(dx+c\right)\right)}{4a^{3}d}+\frac{\sin^{5}\left(dx+c\right)}{5a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -4*ln(1+sin(d*x+c))/a^3/d+4*sin(d*x+c)/a^3/d-2*sin(d*x+c)^2/a^3/d+4/3*sin(d
*x+c)^3/a^3/d-3/4*sin(d*x+c)^4/a^3/d+1/5*sin(d*x+c)^5/a^3/d
```

maxima [A] time = 0.42, size = 73, normalized size = 0.72

$$\frac{\frac{12\sin\left(dx+c\right)^{5}-45\sin\left(dx+c\right)^{4}+80\sin\left(dx+c\right)^{3}-120\sin\left(dx+c\right)^{2}+240\sin\left(dx+c\right)}{a^{3}}-\frac{240\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*((12*sin(d*x + c)^5 - 45*sin(d*x + c)^4 + 80*sin(d*x + c)^3 - 120*sin(
d*x + c)^2 + 240*sin(d*x + c))/a^3 - 240*log(sin(d*x + c) + 1)/a^3)/d
```

mupad [B] time = 0.06, size = 83, normalized size = 0.81

$$-\frac{\frac{4\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^{3}}-\frac{4\sin\left(c+d\,x\right)}{a^{3}}+\frac{2\sin\left(c+d\,x\right)^{2}}{a^{3}}-\frac{4\sin\left(c+d\,x\right)^{3}}{3\,a^{3}}+\frac{3\sin\left(c+d\,x\right)^{4}}{4\,a^{3}}-\frac{\sin\left(c+d\,x\right)^{5}}{5\,a^{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^3)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -((4*log(sin(c + d*x) + 1))/a^3 - (4*sin(c + d*x))/a^3 + (2*sin(c + d*x)^2)
/a^3 - (4*sin(c + d*x)^3)/(3*a^3) + (3*sin(c + d*x)^4)/(4*a^3) - sin(c + d*
x)^5/(5*a^3))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.555 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$\frac{\sin^4(c+dx)}{4a^3d}-\frac{\sin^3(c+dx)}{a^3d}+\frac{2\sin^2(c+dx)}{a^3d}-\frac{4\sin(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] 4*ln(1+sin(d*x+c))/a^3/d-4*sin(d*x+c)/a^3/d+2*sin(d*x+c)^2/a^3/d-sin(d*x+c)
^3/a^3/d+1/4*sin(d*x+c)^4/a^3/d
```

Rubi [A] time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{\sin^4(c+dx)}{4a^3d}-\frac{\sin^3(c+dx)}{a^3d}+\frac{2\sin^2(c+dx)}{a^3d}-\frac{4\sin(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*Log[1 + Sin[c + d*x]])/(a^3*d) - (4*Sin[c + d*x])/(a^3*d) + (2*Sin[c + d
*x]^2)/(a^3*d) - Sin[c + d*x]^3/(a^3*d) + Sin[c + d*x]^4/(4*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2}{a^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^3+4a^2x-3ax^2+x^3+\frac{4a^4}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{4\log(1+\sin(c+dx))}{a^3d} - \frac{4\sin(c+dx)}{a^3d} + \frac{2\sin^2(c+dx)}{a^3d} - \frac{\sin^3(c+dx)}{a^3d} + \frac{\sin}{}$$

Mathematica [A] time = 0.94, size = 59, normalized size = 0.72

$$\frac{-152\sin(c+dx)+8\sin(3(c+dx))-36\cos(2(c+dx))+\cos(4(c+dx))+128\log(\sin(c+dx)+1)+35}{32a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (35 - 36*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] + 128*Log[1 + Sin[c + d*x]] -
152*Sin[c + d*x] + 8*Sin[3*(c + d*x)])/(32*a^3*d)
```

fricas [A] time = 0.61, size = 56, normalized size = 0.68

$$\frac{\cos(dx+c)^4 - 10\cos(dx+c)^2 + 4\left(\cos(dx+c)^2-5\right)\sin(dx+c) + 16\log(\sin(dx+c)+1)}{4a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(cos(d*x + c)^4 - 10*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 5)*sin(d*x +
c) + 16*log(sin(d*x + c) + 1))/(a^3*d)
```

giac [B] time = 0.22, size = 167, normalized size = 2.04

$$-\frac{\frac{12\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^3} - \frac{24\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3} - \frac{25\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8 - 24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 124\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 - 96\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 -}{\left(\tan\left(\frac{1}{2}dx\right.\right.}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/3*(12*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 24*log(abs(tan(1/2*d*x + 1/2
*c) + 1))/a^3 - (25*tan(1/2*d*x + 1/2*c)^8 - 24*tan(1/2*d*x + 1/2*c)^7 + 12
4*tan(1/2*d*x + 1/2*c)^6 - 96*tan(1/2*d*x + 1/2*c)^5 + 210*tan(1/2*d*x + 1/
2*c)^4 - 96*tan(1/2*d*x + 1/2*c)^3 + 124*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/
2*d*x + 1/2*c) + 25)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3))/d
```

maple [A] time = 0.44, size = 81, normalized size = 0.99

$$\frac{4\ln\left(1+\sin\left(dx+c\right)\right)}{a^3d}-\frac{4\sin\left(dx+c\right)}{a^3d}+\frac{2\left(\sin^2\left(dx+c\right)\right)}{a^3d}-\frac{\sin^3\left(dx+c\right)}{a^3d}+\frac{\sin^4\left(dx+c\right)}{4a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4*ln(1+sin(d*x+c))/a^3/d-4*sin(d*x+c)/a^3/d+2*sin(d*x+c)^2/a^3/d-sin(d*x+c)
^3/a^3/d+1/4*sin(d*x+c)^4/a^3/d
```

maxima [A] time = 0.41, size = 61, normalized size = 0.74

$$\frac{\frac{\sin(dx+c)^4-4\sin(dx+c)^3+8\sin(dx+c)^2-16\sin(dx+c)}{a^3}+\frac{16\log(\sin(dx+c)+1)}{a^3}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*((sin(d*x + c)^4 - 4*sin(d*x + c)^3 + 8*sin(d*x + c)^2 - 16*sin(d*x + c
))/a^3 + 16*log(sin(d*x + c) + 1)/a^3)/d
```

mupad [B] time = 8.82, size = 69, normalized size = 0.84

$$\frac{\frac{4\ln(\sin(c+d\,x)+1)}{a^3}-\frac{4\sin(c+d\,x)}{a^3}+\frac{2\sin(c+d\,x)^2}{a^3}-\frac{\sin(c+d\,x)^3}{a^3}+\frac{\sin(c+d\,x)^4}{4\,a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((4*log(sin(c + d*x) + 1))/a^3 - (4*sin(c + d*x))/a^3 + (2*sin(c + d*x)^2)/
a^3 - sin(c + d*x)^3/a^3 + sin(c + d*x)^4/(4*a^3))/d
```

sympy [A] time = 135.28, size = 1698, normalized size = 20.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**8/(a**3*d*tan(c/2
+ d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 +
 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 32*log(tan(c/2 + d*x/2) + 1)*tan(
c/2 + d*x/2)**6/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6
+ 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 4
8*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(a**3*d*tan(c/2 + d*x/2)**8
 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*t
an(c/2 + d*x/2)**2 + a**3*d) + 32*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2
)**2/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*
tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 8*log(tan(c/
2 + d*x/2) + 1)/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6
+ 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 4
*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**8/(a**3*d*tan(c/2 + d*x/2)*
*8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d
*tan(c/2 + d*x/2)**2 + a**3*d) - 16*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 +
d*x/2)**6/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a*
*3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 24*log(
tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(a**3*d*tan(c/2 + d*x/2)**8 +
4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(
c/2 + d*x/2)**2 + a**3*d) - 16*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2
)**2/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*
tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 4*log(tan(c/
2 + d*x/2)**2 + 1)/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)*
*6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d)
- 8*tan(c/2 + d*x/2)**7/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*
x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**
3*d) + 8*tan(c/2 + d*x/2)**6/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2
 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2
+ a**3*d) - 32*tan(c/2 + d*x/2)**5/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*t
an(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/
2)**2 + a**3*d) + 20*tan(c/2 + d*x/2)**4/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a*
*3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2
+ d*x/2)**2 + a**3*d) - 32*tan(c/2 + d*x/2)**3/(a**3*d*tan(c/2 + d*x/2)**8
+ 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*ta
n(c/2 + d*x/2)**2 + a**3*d) + 8*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/2 + d*x/2
)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3
*d*tan(c/2 + d*x/2)**2 + a**3*d) - 8*tan(c/2 + d*x/2)/(a**3*d*tan(c/2 + d*x
```

```
/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a*
*3*d*tan(c/2 + d*x/2)**2 + a**3*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**5/(a*si
n(c) + a)**3, True))
```

3.556 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=68

$$\frac{\sin^3(c+dx)}{3a^3d}-\frac{3\sin^2(c+dx)}{2a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] -4*ln(1+sin(d*x+c))/a^3/d+4*sin(d*x+c)/a^3/d-3/2*sin(d*x+c)^2/a^3/d+1/3*sin
(d*x+c)^3/a^3/d
```

Rubi [A] time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 77}

$$\frac{\sin^3(c+dx)}{3a^3d}-\frac{3\sin^2(c+dx)}{2a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^3*d) + (4*Sin[c + d*x])/(a^3*d) - (3*Sin[c +
d*x]^2)/(2*a^3*d) + Sin[c + d*x]^3/(3*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x}{a(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2x}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^2-3ax+x^2-\frac{4a^3}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= -\frac{4\log(1+\sin(c+dx))}{a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{3\sin^2(c+dx)}{2a^3d}+\frac{\sin^3(c+dx)}{3a^3d}$$

Mathematica [A] time = 0.35, size = 51, normalized size = 0.75

$$\frac{32\sin^3(c+dx)-144\sin^2(c+dx)+384\sin(c+dx)-384\log(\sin(c+dx)+1)+15}{96a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (15 - 384*Log[1 + Sin[c + d*x]] + 384*Sin[c + d*x] - 144*Sin[c + d*x]^2 + 3
2*Sin[c + d*x]^3)/(96*a^3*d)
```

fricas [A] time = 0.63, size = 48, normalized size = 0.71

$$\frac{9\cos(dx+c)^2-2\left(\cos(dx+c)^2-13\right)\sin(dx+c)-24\log(\sin(dx+c)+1)}{6a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(9*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 13)*sin(d*x + c) - 24*log(sin(d
*x + c) + 1))/(a^3*d)
```

giac [B] time = 0.21, size = 141, normalized size = 2.07

$$\frac{2\left(\frac{6\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^3}-\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{11\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+42\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-28\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+42}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3a^3}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/3*(6*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 12*log(abs(tan(1/2*d*x + 1/2*c
) + 1))/a^3 - (11*tan(1/2*d*x + 1/2*c)^6 - 12*tan(1/2*d*x + 1/2*c)^5 + 42*t
an(1/2*d*x + 1/2*c)^4 - 28*tan(1/2*d*x + 1/2*c)^3 + 42*tan(1/2*d*x + 1/2*c)
^2 - 12*tan(1/2*d*x + 1/2*c) + 11)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^3))/d
```

maple [A] time = 0.39, size = 65, normalized size = 0.96

$$-\frac{4\ln(1+\sin(dx+c))}{a^3d}+\frac{4\sin(dx+c)}{a^3d}-\frac{3\left(\sin^2(dx+c)\right)}{2a^3d}+\frac{\sin^3(dx+c)}{3a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -4*ln(1+sin(d*x+c))/a^3/d+4*sin(d*x+c)/a^3/d-3/2*sin(d*x+c)^2/a^3/d+1/3*sin
(d*x+c)^3/a^3/d
```

maxima [A] time = 0.60, size = 53, normalized size = 0.78

$$\frac{\frac{2\sin(dx+c)^3-9\sin(dx+c)^2+24\sin(dx+c)}{a^3}-\frac{24\log(\sin(dx+c)+1)}{a^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*((2*sin(d*x + c)^3 - 9*sin(d*x + c)^2 + 24*sin(d*x + c))/a^3 - 24*log(s
in(d*x + c) + 1)/a^3)/d
```

mupad [B] time = 0.05, size = 57, normalized size = 0.84

$$-\frac{\frac{4\ln(\sin(c+dx)+1)}{a^3}-\frac{4\sin(c+dx)}{a^3}+\frac{3\sin(c+dx)^2}{2a^3}-\frac{\sin(c+dx)^3}{3a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -((4*log(sin(c + d*x) + 1))/a^3 - (4*sin(c + d*x))/a^3 + (3*sin(c + d*x)^2)
/(2*a^3) - sin(c + d*x)^3/(3*a^3))/d
```

sympy [A] time = 86.48, size = 1102, normalized size = 16.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-24*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**6/(3*a**3*d*tan(
c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 + d*x/2)*
*2 + 3*a**3*d) - 72*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(3*a**3*d
*tan(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 + d*
x/2)**2 + 3*a**3*d) - 72*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(3*a
**3*d*tan(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2
 + d*x/2)**2 + 3*a**3*d) - 24*log(tan(c/2 + d*x/2) + 1)/(3*a**3*d*tan(c/2 +
 d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 + d*x/2)**2 +
3*a**3*d) + 12*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**6/(3*a**3*d*t
an(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 + d*x/
2)**2 + 3*a**3*d) + 36*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(3*
a**3*d*tan(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/
2 + d*x/2)**2 + 3*a**3*d) + 36*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2
)**2/(3*a**3*d*tan(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*
d*tan(c/2 + d*x/2)**2 + 3*a**3*d) + 12*log(tan(c/2 + d*x/2)**2 + 1)/(3*a**3
*d*tan(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 +
d*x/2)**2 + 3*a**3*d) + 24*tan(c/2 + d*x/2)**5/(3*a**3*d*tan(c/2 + d*x/2)**
6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 3*a**3*d)
 - 18*tan(c/2 + d*x/2)**4/(3*a**3*d*tan(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2
+ d*x/2)**4 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 3*a**3*d) + 56*tan(c/2 + d*x/2
)**3/(3*a**3*d*tan(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*
d*tan(c/2 + d*x/2)**2 + 3*a**3*d) - 18*tan(c/2 + d*x/2)**2/(3*a**3*d*tan(c/
2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 + d*x/2)**2
 + 3*a**3*d) + 24*tan(c/2 + d*x/2)/(3*a**3*d*tan(c/2 + d*x/2)**6 + 9*a**3*d
*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 3*a**3*d), Ne(d, 0)),
 (x*sin(c)*cos(c)**5/(a*sin(c) + a)**3, True))
```

3.557 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=45

$$\frac{\sin(c+dx)}{a^3d}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] ln(sin(d*x+c))/a^3/d-4*ln(1+sin(d*x+c))/a^3/d+sin(d*x+c)/a^3/d
```

Rubi [A] time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 72}

$$\frac{\sin(c+dx)}{a^3d}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - (4*Log[1 + Sin[c + d*x]])/(a^3*d) + Sin[c + d*x
]/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2}{x(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a}{x}-\frac{4a}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\log(\sin(c+dx))}{a^3 d} - \frac{4\log(1+\sin(c+dx))}{a^3 d} + \frac{\sin(c+dx)}{a^3 d}$$

Mathematica [A] time = 0.04, size = 32, normalized size = 0.71

$$\frac{\sin(c+dx)+\log(\sin(c+dx))-4\log(\sin(c+dx)+1)}{a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
[Out] (Log[Sin[c + d*x]] - 4*Log[1 + Sin[c + d*x]] + Sin[c + d*x])/(a^3*d)
```

fricas [A] time = 0.83, size = 34, normalized size = 0.76

$$\frac{\log\left(\frac{1}{2}\sin(dx+c)\right)-4\log(\sin(dx+c)+1)+\sin(dx+c)}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
[Out] (log(1/2*sin(d*x + c)) - 4*log(sin(d*x + c) + 1) + sin(d*x + c))/(a^3*d)
```

giac [B] time = 0.21, size = 103, normalized size = 2.29

$$\frac{\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^3}-\frac{8\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 8*log(abs(tan(1/2*d*x + 1/2*c) + 1
))/a^3 + log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (3*tan(1/2*d*x + 1/2*c)^2 - 2
*tan(1/2*d*x + 1/2*c) + 3)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3))/d
```

maple [A] time = 0.55, size = 46, normalized size = 1.02

$$\frac{\ln\left(\sin\left(dx+c\right)\right)}{a^{3}d}-\frac{4\ln\left(1+\sin\left(dx+c\right)\right)}{a^{3}d}+\frac{\sin\left(dx+c\right)}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] ln(sin(d*x+c))/a^3/d-4*ln(1+sin(d*x+c))/a^3/d+sin(d*x+c)/a^3/d
```

maxima [A] time = 0.49, size = 43, normalized size = 0.96

$$-\frac{\frac{4\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}}-\frac{\log\left(\sin\left(dx+c\right)\right)}{a^{3}}-\frac{\sin\left(dx+c\right)}{a^{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(4*log(sin(d*x + c) + 1)/a^3 - log(sin(d*x + c))/a^3 - sin(d*x + c)/a^3)/d
```

mupad [B] time = 8.92, size = 95, normalized size = 2.11

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{8\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^3\,d}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}+\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) - (8*log(tan(c/2 + (d*x)/2) + 1))/(a^3*d) +
 (2*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 + (d*x)/2)^2 + a^3)) + (3*log(tan(c
/2 + (d*x)/2)^2 + 1))/(a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.558 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=47

$$-\frac{\csc(c+dx)}{a^3d}-\frac{3\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] -csc(d*x+c)/a^3/d-3*ln(sin(d*x+c))/a^3/d+4*ln(1+sin(d*x+c))/a^3/d
```

Rubi [A] time = 0.11, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\csc(c+dx)}{a^3d}-\frac{3\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Csc[c + d*x]/(a^3*d)) - (3*Log[Sin[c + d*x]])/(a^3*d) + (4*Log[1 + Sin[c
+ d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2}{x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{3}{x}+\frac{4}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= -\frac{\csc(c+dx)}{a^3 d} - \frac{3\log(\sin(c+dx))}{a^3 d} + \frac{4\log(1+\sin(c+dx))}{a^3 d}$$

Mathematica [A] time = 0.05, size = 35, normalized size = 0.74

$$-\frac{\csc(c+dx)+3\log(\sin(c+dx))-4\log(\sin(c+dx)+1)}{a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
[Out] -((Csc[c + d*x] + 3*Log[Sin[c + d*x]] - 4*Log[1 + Sin[c + d*x]])/(a^3*d))
```

fricas [A] time = 0.85, size = 52, normalized size = 1.11

$$-\frac{3\,\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-4\,\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+1}{a^3 d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
[Out] -(3*log(1/2*sin(d*x + c))*sin(d*x + c) - 4*log(sin(d*x + c) + 1)*sin(d*x +
c) + 1)/(a^3*d*sin(d*x + c))
```

giac [B] time = 0.25, size = 101, normalized size = 2.15

$$-\frac{\frac{2\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{a^3}-\frac{16\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}+\frac{6\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 16*log(abs(tan(1/2*d*x + 1/2*
c) + 1))/a^3 + 6*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + tan(1/2*d*x + 1/2*c)/
a^3 - (6*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)))/d
```

maple [A] time = 0.58, size = 50, normalized size = 1.06

$$-\frac{1}{a^{3}d\sin\left(dx+c\right)}-\frac{3\ln\left(\sin\left(dx+c\right)\right)}{a^{3}d}+\frac{4\ln\left(1+\sin\left(dx+c\right)\right)}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/a^3/d/sin(d*x+c)-3*ln(sin(d*x+c))/a^3/d+4*ln(1+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.87, size = 44, normalized size = 0.94

$$\frac{\frac{4\log(\sin(dx+c)+1)}{a^{3}}-\frac{3\log(\sin(dx+c))}{a^{3}}-\frac{1}{a^{3}\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] (4*log(sin(d*x + c) + 1)/a^3 - 3*log(sin(d*x + c))/a^3 - 1/(a^3*sin(d*x + c
)))/d
```

mupad [B] time = 8.98, size = 71, normalized size = 1.51

$$-\frac{3\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-8\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(3*log(tan(c/2 + (d*x)/2)) - 8*log(tan(c/2 + (d*x)/2) + 1) + cot(c/2 + (d*
x)/2)/2 + tan(c/2 + (d*x)/2)/2 + log(tan(c/2 + (d*x)/2)^2 + 1))/(a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.559 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=65

$$-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] 3*csc(d*x+c)/a^3/d-1/2*csc(d*x+c)^2/a^3/d+4*ln(sin(d*x+c))/a^3/d-4*ln(1+sin
(d*x+c))/a^3/d
```

Rubi [A] time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^3*d) + (4*Log[Sin[c + d*x]])
/(a^3*d) - (4*Log[1 + Sin[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^3}-\frac{3}{x^2}+\frac{4}{ax}-\frac{4}{a(a+x)}\right)dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{3\csc(c+dx)}{a^3 d} - \frac{\csc^2(c+dx)}{2a^3 d} + \frac{4\log(\sin(c+dx))}{a^3 d} - \frac{4\log(1+\sin(c+dx))}{a^3 d}$$

Mathematica [A] time = 0.08, size = 49, normalized size = 0.75

$$\frac{-\csc^2(c+dx)+6\csc(c+dx)+8\log(\sin(c+dx))-8\log(\sin(c+dx)+1)}{2a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*Csc[c + d*x] - Csc[c + d*x]^2 + 8*Log[Sin[c + d*x]] - 8*Log[1 + Sin[c +
d*x]])/(2*a^3*d)
```

fricas [A] time = 0.77, size = 76, normalized size = 1.17

$$\frac{8\left(\cos\left(dx+c\right)^2-1\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-8\left(\cos\left(dx+c\right)^2-1\right)\log\left(\sin\left(dx+c\right)+1\right)-6\,\sin\left(dx+c\right)+1}{2\left(a^3 d\cos\left(dx+c\right)^2-a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/2*(8*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 8*(cos(d*x + c)^2 - 1)*
log(sin(d*x + c) + 1) - 6*sin(d*x + c) + 1)/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

giac [A] time = 0.25, size = 115, normalized size = 1.77

$$-\frac{\frac{64\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{32\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(64*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 32*log(abs(tan(1/2*d*x +
1/2*c)))/a^3 + (48*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 1)/(a
^3*tan(1/2*d*x + 1/2*c)^2) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d
*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.67, size = 66, normalized size = 1.02

$$-\frac{1}{2a^3d\sin(dx+c)^2}+\frac{3}{a^3d\sin(dx+c)}+\frac{4\ln(\sin(dx+c))}{a^3d}-\frac{4\ln(1+\sin(dx+c))}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2/a^3/d/sin(d*x+c)^2+3/a^3/d/sin(d*x+c)+4*ln(sin(d*x+c))/a^3/d-4*ln(1+si
n(d*x+c))/a^3/d
```

maxima [A] time = 0.80, size = 55, normalized size = 0.85

$$-\frac{\frac{8\log(\sin(dx+c)+1)}{a^3}-\frac{8\log(\sin(dx+c))}{a^3}-\frac{6\sin(dx+c)-1}{a^3\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(8*log(sin(d*x + c) + 1)/a^3 - 8*log(sin(d*x + c))/a^3 - (6*sin(d*x +
c) - 1)/(a^3*sin(d*x + c)^2))/d
```

mupad [B] time = 8.88, size = 107, normalized size = 1.65

$$\frac{4\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}-\frac{8\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^3\,d}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{2}\right)}{4\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (4*log(tan(c/2 + (d*x)/2)))/(a^3*d) - tan(c/2 + (d*x)/2)^2/(8*a^3*d) - (8*l
og(tan(c/2 + (d*x)/2) + 1))/(a^3*d) + (3*tan(c/2 + (d*x)/2))/(2*a^3*d) + (c
ot(c/2 + (d*x)/2)^2*(6*tan(c/2 + (d*x)/2) - 1/2))/(4*a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
[Out] Timed out
```

3.560 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$-\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^2(c+dx)}{2a^3d}-\frac{4\csc(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] -4*csc(d*x+c)/a^3/d+3/2*csc(d*x+c)^2/a^3/d-1/3*csc(d*x+c)^3/a^3/d-4*ln(sin(
d*x+c))/a^3/d+4*ln(1+sin(d*x+c))/a^3/d
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^2(c+dx)}{2a^3d}-\frac{4\csc(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*Csc[c + d*x])/(a^3*d) + (3*Csc[c + d*x]^2)/(2*a^3*d) - Csc[c + d*x]^3/(
3*a^3*d) - (4*Log[Sin[c + d*x]])/(a^3*d) + (4*Log[1 + Sin[c + d*x]])/(a^3*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2}{x^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^4}-\frac{3}{x^3}+\frac{4}{ax^2}-\frac{4}{a^2x}+\frac{4}{a^2(a+x)}\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{4\csc(c+dx)}{a^3d}+\frac{3\csc^2(c+dx)}{2a^3d}-\frac{\csc^3(c+dx)}{3a^3d}-\frac{4\log(\sin(c+dx))}{a^3d}+\frac{4\log(1$$

Mathematica [A] time = 0.12, size = 59, normalized size = 0.71

$$-\frac{2\csc^3(c+dx)-9\csc^2(c+dx)+24\csc(c+dx)+24\log(\sin(c+dx))-24\log(\sin(c+dx)+1)}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/6*(24*Csc[c + d*x] - 9*Csc[c + d*x]^2 + 2*Csc[c + d*x]^3 + 24*Log[Sin[c
+ d*x]] - 24*Log[1 + Sin[c + d*x]])/(a^3*d)
```

fricas [A] time = 0.68, size = 106, normalized size = 1.28

$$-\frac{24\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-24\left(\cos(dx+c)^2-1\right)\log(\sin(dx+c)+1)\sin(dx+c}{6\left(a^3d\cos(dx+c)^2-a^3d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(24*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c))*sin(d*x + c) - 24*(cos(
d*x + c)^2 - 1)*log(sin(d*x + c) + 1)*sin(d*x + c) + 24*cos(d*x + c)^2 + 9*
sin(d*x + c) - 26)/((a^3*d*cos(d*x + c)^2 - a^3*d)*sin(d*x + c))
```

giac [A] time = 0.25, size = 145, normalized size = 1.75

$$\frac{\frac{192\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{96\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{176\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-51\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}-\frac{a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-9a}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(192*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 96*log(abs(tan(1/2*d*x +
 1/2*c)))/a^3 + (176*tan(1/2*d*x + 1/2*c)^3 - 51*tan(1/2*d*x + 1/2*c)^2 + 9
*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)^3) - (a^6*tan(1/2*d*x
+ 1/2*c)^3 - 9*a^6*tan(1/2*d*x + 1/2*c)^2 + 51*a^6*tan(1/2*d*x + 1/2*c))/a^
9)/d
```

maple [A] time = 0.68, size = 82, normalized size = 0.99

$$-\frac{1}{3d\,a^3\sin\left(dx+c\right)^3}+\frac{3}{2a^3d\sin\left(dx+c\right)^2}-\frac{4}{a^3d\sin\left(dx+c\right)}-\frac{4\ln\left(\sin\left(dx+c\right)\right)}{a^3d}+\frac{4\ln\left(1+\sin\left(dx+c\right)\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/3/d/a^3/sin(d*x+c)^3+3/2/a^3/d/sin(d*x+c)^2-4/a^3/d/sin(d*x+c)-4*ln(sin(
d*x+c))/a^3/d+4*ln(1+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.72, size = 65, normalized size = 0.78

$$\frac{\frac{24\,\log\left(\sin\left(dx+c\right)+1\right)}{a^3}-\frac{24\,\log\left(\sin\left(dx+c\right)\right)}{a^3}-\frac{24\,\sin\left(dx+c\right)^2-9\,\sin\left(dx+c\right)+2}{a^3\sin\left(dx+c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6*(24*log(sin(d*x + c) + 1)/a^3 - 24*log(sin(d*x + c))/a^3 - (24*sin(d*x
+ c)^2 - 9*sin(d*x + c) + 2)/(a^3*sin(d*x + c)^3))/d
```

mupad [B] time = 8.99, size = 139, normalized size = 1.67

$$\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^3\,d}-\frac{4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}+\frac{8\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^3\,d}-\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (3*tan(c/2 + (d*x)/2)^2)/(8*a^3*d) - tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (4*l
og(tan(c/2 + (d*x)/2)))/(a^3*d) + (8*log(tan(c/2 + (d*x)/2) + 1))/(a^3*d) -
```

```
(17*tan(c/2 + (d*x)/2))/(8*a^3*d) - (cot(c/2 + (d*x)/2)^3*(17*tan(c/2 + (d
*x)/2)^2 - 3*tan(c/2 + (d*x)/2) + 1/3))/(8*a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.561 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3} dx$

Optimal. Leaf size=96

$$-\frac{\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{4\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] 4*csc(d*x+c)/a^3/d-2*csc(d*x+c)^2/a^3/d+csc(d*x+c)^3/a^3/d-1/4*csc(d*x+c)^4
/a^3/d+4*ln(sin(d*x+c))/a^3/d-4*ln(1+sin(d*x+c))/a^3/d
```

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{4\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*Csc[c + d*x])/(a^3*d) - (2*Csc[c + d*x]^2)/(a^3*d) + Csc[c + d*x]^3/(a^3
*d) - Csc[c + d*x]^4/(4*a^3*d) + (4*Log[Sin[c + d*x]])/(a^3*d) - (4*Log[1 +
 Sin[c + d*x]])/(a^3*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^5} - \frac{3}{x^4} + \frac{4}{ax^3} - \frac{4}{a^2x^2} + \frac{4}{a^3x} - \frac{4}{a^3(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{4\csc(c+dx)}{a^3d} - \frac{2\csc^2(c+dx)}{a^3d} + \frac{\csc^3(c+dx)}{a^3d} - \frac{\csc^4(c+dx)}{4a^3d} + \frac{4\log(\sin(c+dx))}{a^3d} -$$

Mathematica [A] time = 0.31, size = 69, normalized size = 0.72

$$\frac{-\csc^4(c+dx) + 4\csc^3(c+dx) - 8\csc^2(c+dx) + 16\csc(c+dx) + 16\log(\sin(c+dx)) - 16\log(\sin(c+dx)+1)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (16*Csc[c + d*x] - 8*Csc[c + d*x]^2 + 4*Csc[c + d*x]^3 - Csc[c + d*x]^4 + 1
6*Log[Sin[c + d*x]] - 16*Log[1 + Sin[c + d*x]])/(4*a^3*d)
```

fricas [A] time = 0.81, size = 131, normalized size = 1.36

$$\frac{8\cos(dx+c)^2 + 16\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 16\left(\cos(dx+c)^4 - 2\cos(dx+c\right.}{4\left(a^3d\cos(dx+c)^4 - 2a^3d\cos(dx+c)^2 + a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(8*cos(d*x + c)^2 + 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*
sin(d*x + c)) - 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(sin(d*x + c)
 + 1) - 4*(4*cos(d*x + c)^2 - 5)*sin(d*x + c) - 9)/(a^3*d*cos(d*x + c)^4 -
2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

giac [A] time = 0.26, size = 174, normalized size = 1.81

$$-\frac{\frac{1536\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3} - \frac{768\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3} + \frac{1600\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 456\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 108\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/192*(1536*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 768*log(abs(tan(1/2*d
*x + 1/2*c)))/a^3 + (1600*tan(1/2*d*x + 1/2*c)^4 - 456*tan(1/2*d*x + 1/2*c)
^3 + 108*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 3)/(a^3*tan(1/2
*d*x + 1/2*c)^4) + 3*(a^9*tan(1/2*d*x + 1/2*c)^4 - 8*a^9*tan(1/2*d*x + 1/2*
c)^3 + 36*a^9*tan(1/2*d*x + 1/2*c)^2 - 152*a^9*tan(1/2*d*x + 1/2*c))/a^12)/
d
```

maple [A] time = 0.68, size = 97, normalized size = 1.01

$$-\frac{1}{4d\,a^{3}\sin\left(dx+c\right)^{4}}+\frac{1}{d\,a^{3}\sin\left(dx+c\right)^{3}}-\frac{2}{a^{3}d\sin\left(dx+c\right)^{2}}+\frac{4}{a^{3}d\sin\left(dx+c\right)}+\frac{4\ln\left(\sin\left(dx+c\right)\right)}{a^{3}d}-\frac{4\ln\left(1+\sin\left(d\right.\right.}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/4/d/a^3/sin(d*x+c)^4+1/d/a^3/sin(d*x+c)^3-2/a^3/d/sin(d*x+c)^2+4/a^3/d/s
in(d*x+c)+4*ln(sin(d*x+c))/a^3/d-4*ln(1+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.67, size = 75, normalized size = 0.78

$$-\frac{\frac{16\,\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}}-\frac{16\,\log\left(\sin\left(dx+c\right)\right)}{a^{3}}-\frac{16\,\sin\left(dx+c\right)^{3}-8\,\sin\left(dx+c\right)^{2}+4\,\sin\left(dx+c\right)-1}{a^{3}\sin\left(dx+c\right)^{4}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/4*(16*log(sin(d*x + c) + 1)/a^3 - 16*log(sin(d*x + c))/a^3 - (16*sin(d*x
 + c)^3 - 8*sin(d*x + c)^2 + 4*sin(d*x + c) - 1)/(a^3*sin(d*x + c)^4))/d
```

mupad [B] time = 8.88, size = 171, normalized size = 1.78

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8\,a^{3}\,d}-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{16\,a^{3}\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{64\,a^{3}\,d}+\frac{4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^{3}\,d}-\frac{8\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^{3}\,d}+\frac{19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{8\,a^{3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^5*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(8*a^3*d) - (9*tan(c/2 + (d*x)/2)^2)/(16*a^3*d) - tan(
c/2 + (d*x)/2)^4/(64*a^3*d) + (4*log(tan(c/2 + (d*x)/2)))/(a^3*d) - (8*log(
tan(c/2 + (d*x)/2) + 1))/(a^3*d) + (19*tan(c/2 + (d*x)/2))/(8*a^3*d) + (cot
(c/2 + (d*x)/2)^4*(2*tan(c/2 + (d*x)/2) - 9*tan(c/2 + (d*x)/2)^2 + 38*tan(c
/2 + (d*x)/2)^3 - 1/4))/(16*a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.562 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=117

$$-\frac{\csc^5(c+dx)}{5a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}+\frac{2\csc^2(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c}{a^3d}$$

```
[Out] -4*csc(d*x+c)/a^3/d+2*csc(d*x+c)^2/a^3/d-4/3*csc(d*x+c)^3/a^3/d+3/4*csc(d*x
+c)^4/a^3/d-1/5*csc(d*x+c)^5/a^3/d-4*ln(sin(d*x+c))/a^3/d+4*ln(1+sin(d*x+c)
)/a^3/d
```

Rubi [A] time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{\csc^5(c+dx)}{5a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}+\frac{2\csc^2(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*Csc[c + d*x])/(a^3*d) + (2*Csc[c + d*x]^2)/(a^3*d) - (4*Csc[c + d*x]^3)
/(3*a^3*d) + (3*Csc[c + d*x]^4)/(4*a^3*d) - Csc[c + d*x]^5/(5*a^3*d) - (4*L
og[Sin[c + d*x]])/(a^3*d) + (4*Log[1 + Sin[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2}{x^6(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^2}{x^6(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{a}{x^6}-\frac{3}{x^5}+\frac{4}{ax^4}-\frac{4}{a^2x^3}+\frac{4}{a^3x^2}-\frac{4}{a^4x}+\frac{4}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{4\csc(c+dx)}{a^3d}+\frac{2\csc^2(c+dx)}{a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{\csc^5(c+d}{5a^3d}$$

Mathematica [A] time = 0.13, size = 79, normalized size = 0.68

$$-\frac{12\csc^5(c+dx)-45\csc^4(c+dx)+80\csc^3(c+dx)-120\csc^2(c+dx)+240\csc(c+dx)+240\log(\sin(c+dx))}{60a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/60*(240*Csc[c + d*x] - 120*Csc[c + d*x]^2 + 80*Csc[c + d*x]^3 - 45*Csc[c
 + d*x]^4 + 12*Csc[c + d*x]^5 + 240*Log[Sin[c + d*x]] - 240*Log[1 + Sin[c +
 d*x]])/(a^3*d)
```

fricas [A] time = 0.58, size = 161, normalized size = 1.38

$$-\frac{240\cos(dx+c)^4+240\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-240\left(\cos(dx+\right.}{60\left(a^3d\cos(dx+c)^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/60*(240*cos(d*x + c)^4 + 240*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log
(1/2*sin(d*x + c))*sin(d*x + c) - 240*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 +
1)*log(sin(d*x + c) + 1)*sin(d*x + c) - 560*cos(d*x + c)^2 + 15*(8*cos(d*x
+ c)^2 - 11)*sin(d*x + c) + 332)/((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x +
 c)^2 + a^3*d)*sin(d*x + c))
```

giac [A] time = 0.27, size = 204, normalized size = 1.74

$$\frac{\frac{7680 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{3840 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{8768\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2460\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+660\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-190\,\tan\left(\frac{1}{2}\,dx+\right.}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(7680*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 3840*log(abs(tan(1/2*d
*x + 1/2*c)))/a^3 + (8768*tan(1/2*d*x + 1/2*c)^5 - 2460*tan(1/2*d*x + 1/2*c
)^4 + 660*tan(1/2*d*x + 1/2*c)^3 - 190*tan(1/2*d*x + 1/2*c)^2 + 45*tan(1/2*
d*x + 1/2*c) - 6)/(a^3*tan(1/2*d*x + 1/2*c)^5) - (6*a^12*tan(1/2*d*x + 1/2*
c)^5 - 45*a^12*tan(1/2*d*x + 1/2*c)^4 + 190*a^12*tan(1/2*d*x + 1/2*c)^3 - 6
60*a^12*tan(1/2*d*x + 1/2*c)^2 + 2460*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.66, size = 114, normalized size = 0.97

$$-\frac{1}{5d\,a^3\sin\left(dx+c\right)^5}+\frac{3}{4d\,a^3\sin\left(dx+c\right)^4}-\frac{4}{3d\,a^3\sin\left(dx+c\right)^3}+\frac{2}{a^3d\sin\left(dx+c\right)^2}-\frac{4}{a^3d\sin\left(dx+c\right)}-\frac{4\ln\left(\sin\left(dx\right.\right.}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/5/d/a^3/sin(d*x+c)^5+3/4/d/a^3/sin(d*x+c)^4-4/3/d/a^3/sin(d*x+c)^3+2/a^3
/d/sin(d*x+c)^2-4/a^3/d/sin(d*x+c)-4*ln(sin(d*x+c))/a^3/d+4*ln(1+sin(d*x+c)
)/a^3/d
```

maple [A] time = 0.87, size = 85, normalized size = 0.73

$$\frac{\frac{240 \log\left(\sin\left(dx+c\right)+1\right)}{a^3}-\frac{240 \log\left(\sin\left(dx+c\right)\right)}{a^3}-\frac{240\,\sin\left(dx+c\right)^4-120\,\sin\left(dx+c\right)^3+80\,\sin\left(dx+c\right)^2-45\,\sin\left(dx+c\right)+12}{a^3\sin\left(dx+c\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(240*log(sin(d*x + c) + 1)/a^3 - 240*log(sin(d*x + c))/a^3 - (240*sin(
d*x + c)^4 - 120*sin(d*x + c)^3 + 80*sin(d*x + c)^2 - 45*sin(d*x + c) + 12)
/(a^3*sin(d*x + c)^5))/d
```

mupad [B] time = 8.96, size = 203, normalized size = 1.74

$$\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,a^3\,d}-\frac{19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,a^3\,d}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,a^3\,d}-\frac{4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}+\frac{8\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^6*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (11*tan(c/2 + (d*x)/2)^2)/(16*a^3*d) - (19*tan(c/2 + (d*x)/2)^3)/(96*a^3*d)
 + (3*tan(c/2 + (d*x)/2)^4)/(64*a^3*d) - tan(c/2 + (d*x)/2)^5/(160*a^3*d) -
 (4*log(tan(c/2 + (d*x)/2)))/(a^3*d) + (8*log(tan(c/2 + (d*x)/2) + 1))/(a^3
*d) - (41*tan(c/2 + (d*x)/2))/(16*a^3*d) - (cot(c/2 + (d*x)/2)^5*((19*tan(c
/2 + (d*x)/2)^2)/3 - (3*tan(c/2 + (d*x)/2))/2 - 22*tan(c/2 + (d*x)/2)^3 + 8
2*tan(c/2 + (d*x)/2)^4 + 1/5))/(32*a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.563 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{4}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\csc^4(c+dx)}{4a^4d}+\frac{4\csc^3(c+dx)}{3a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{12\csc(c+dx)}{a^4d}+\frac{16\log(\sin(c+dx))}{a^4d}-\frac{16\,\text{l}}{}$$

```
[Out] 12*csc(d*x+c)/a^4/d-4*csc(d*x+c)^2/a^4/d+4/3*csc(d*x+c)^3/a^4/d-1/4*csc(d*x
+c)^4/a^4/d+16*ln(sin(d*x+c))/a^4/d-16*ln(1+sin(d*x+c))/a^4/d+4/d/(a^4+a^4*
sin(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$\frac{4}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\csc^4(c+dx)}{4a^4d}+\frac{4\csc^3(c+dx)}{3a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{12\csc(c+dx)}{a^4d}+\frac{16\log(\sin(c+dx))}{a^4d}-\frac{16\,\text{l}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (12*Csc[c + d*x])/(a^4*d) - (4*Csc[c + d*x]^2)/(a^4*d) + (4*Csc[c + d*x]^3)
/(3*a^4*d) - Csc[c + d*x]^4/(4*a^4*d) + (16*Log[Sin[c + d*x]])/(a^4*d) - (1
6*Log[1 + Sin[c + d*x]])/(a^4*d) + 4/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^5}-\frac{4}{ax^4}+\frac{8}{a^2x^3}-\frac{12}{a^3x^2}+\frac{16}{a^4x}-\frac{4}{a^3(a+x)^2}-\frac{16}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{12\csc(c+dx)}{a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{4\csc^3(c+dx)}{3a^4d}-\frac{\csc^4(c+dx)}{4a^4d}+\frac{16\log(\sin(c+dx}{a^4d}$$

Mathematica [A] time = 0.71, size = 81, normalized size = 0.68

$$\frac{\frac{48}{\sin(c+dx)+1}-3\csc^4(c+dx)+16\csc^3(c+dx)-48\csc^2(c+dx)+144\csc(c+dx)+192\log(\sin(c+dx))-192\log}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (144*Csc[c + d*x] - 48*Csc[c + d*x]^2 + 16*Csc[c + d*x]^3 - 3*Csc[c + d*x]^
4 + 192*Log[Sin[c + d*x]] - 192*Log[1 + Sin[c + d*x]] + 48/(1 + Sin[c + d*x
]))/(12*a^4*d)
```

fricas [B] time = 0.83, size = 235, normalized size = 1.96

$$\frac{192\cos(dx+c)^4-352\cos(dx+c)^2+192\left(\cos(dx+c)^4-2\cos(dx+c)^2+\left(\cos(dx+c)^4-2\cos(dx+c)^2+\right.\right.}{12\left(a^4d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/12*(192*cos(d*x + c)^4 - 352*cos(d*x + c)^2 + 192*(cos(d*x + c)^4 - 2*cos
(d*x + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + 1)*log
(1/2*sin(d*x + c)) - 192*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + (cos(d*x + c)
^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (96*co
s(d*x + c)^2 - 109)*sin(d*x + c) + 157)/(a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos
(d*x + c)^2 + a^4*d + (a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^2 + a^4*
d)*sin(d*x + c))
```

giac [A] time = 0.30, size = 218, normalized size = 1.82

$$-\frac{\frac{6144 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{3072 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}-\frac{1536\left(6 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+11 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\right)}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^2}+\frac{6400 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1248}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/192*(6144*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3072*log(abs(tan(1/2*
d*x + 1/2*c)))/a^4 - 1536*(6*tan(1/2*d*x + 1/2*c)^2 + 11*tan(1/2*d*x + 1/2*
c) + 6)/(a^4*(tan(1/2*d*x + 1/2*c) + 1)^2) + (6400*tan(1/2*d*x + 1/2*c)^4 -
 1248*tan(1/2*d*x + 1/2*c)^3 + 204*tan(1/2*d*x + 1/2*c)^2 - 32*tan(1/2*d*x
+ 1/2*c) + 3)/(a^4*tan(1/2*d*x + 1/2*c)^4) + (3*a^12*tan(1/2*d*x + 1/2*c)^4
 - 32*a^12*tan(1/2*d*x + 1/2*c)^3 + 204*a^12*tan(1/2*d*x + 1/2*c)^2 - 1248*
a^12*tan(1/2*d*x + 1/2*c))/a^16)/d
```

maple [A] time = 0.68, size = 116, normalized size = 0.97

$$-\frac{1}{4d\,a^4 \sin(dx+c)^4}+\frac{4}{3d\,a^4 \sin(dx+c)^3}-\frac{4}{d\,a^4 \sin(dx+c)^2}+\frac{12}{d\,a^4 \sin(dx+c)}+\frac{16 \ln(\sin(dx+c))}{a^4 d}+\frac{4}{d\,a^4 (1+\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/4/d/a^4/sin(d*x+c)^4+4/3/d/a^4/sin(d*x+c)^3-4/d/a^4/sin(d*x+c)^2+12/d/a^
4/sin(d*x+c)+16*ln(sin(d*x+c))/a^4/d+4/d/a^4/(1+sin(d*x+c))-16*ln(1+sin(d*x
+c))/a^4/d
```

maxima [A] time = 0.54, size = 100, normalized size = 0.83

$$\frac{\frac{192 \sin(dx+c)^4+96 \sin(dx+c)^3-32 \sin(dx+c)^2+13 \sin(dx+c)-3}{a^4 \sin(dx+c)^5+a^4 \sin(dx+c)^4}-\frac{192 \log(\sin(dx+c)+1)}{a^4}+\frac{192 \log(\sin(dx+c))}{a^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/12*((192*sin(d*x + c)^4 + 96*sin(d*x + c)^3 - 32*sin(d*x + c)^2 + 13*sin(
d*x + c) - 3)/(a^4*sin(d*x + c)^5 + a^4*sin(d*x + c)^4) - 192*log(sin(d*x +
 c) + 1)/a^4 + 192*log(sin(d*x + c))/a^4)/d
```

mupad [B] time = 8.89, size = 233, normalized size = 1.94

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,a^4\,d}-\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^4\,d}+\frac{16\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{32\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^4\,d}+\frac{-24\tan\left(\frac{c}{2}+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^5*(a + a*sin(c + d*x))^4),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(6*a^4*d) - (17*tan(c/2 + (d*x)/2)^2)/(16*a^4*d) - tan
(c/2 + (d*x)/2)^4/(64*a^4*d) + (16*log(tan(c/2 + (d*x)/2)))/(a^4*d) - (32*l
og(tan(c/2 + (d*x)/2) + 1))/(a^4*d) + ((13*tan(c/2 + (d*x)/2))/6 - (143*tan
(c/2 + (d*x)/2)^2)/12 + (218*tan(c/2 + (d*x)/2)^3)/3 + 191*tan(c/2 + (d*x)/
2)^4 - 24*tan(c/2 + (d*x)/2)^5 - 1/4)/(d*(16*a^4*tan(c/2 + (d*x)/2)^4 + 32*
a^4*tan(c/2 + (d*x)/2)^5 + 16*a^4*tan(c/2 + (d*x)/2)^6)) + (13*tan(c/2 + (d
*x)/2))/(2*a^4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.564 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=135

$$-\frac{4}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\csc^5(c+dx)}{5a^4d}+\frac{\csc^4(c+dx)}{a^4d}-\frac{8\csc^3(c+dx)}{3a^4d}+\frac{6\csc^2(c+dx)}{a^4d}-\frac{16\csc(c+dx)}{a^4d}-\frac{20\log(s\cdots}{a\cdots}$$

```
[Out] -16*csc(d*x+c)/a^4/d+6*csc(d*x+c)^2/a^4/d-8/3*csc(d*x+c)^3/a^4/d+csc(d*x+c)
^4/a^4/d-1/5*csc(d*x+c)^5/a^4/d-20*ln(sin(d*x+c))/a^4/d+20*ln(1+sin(d*x+c))
/a^4/d-4/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{4}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\csc^5(c+dx)}{5a^4d}+\frac{\csc^4(c+dx)}{a^4d}-\frac{8\csc^3(c+dx)}{3a^4d}+\frac{6\csc^2(c+dx)}{a^4d}-\frac{16\csc(c+dx)}{a^4d}-\frac{20\log(s\cdots}{a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-16*Csc[c + d*x])/(a^4*d) + (6*Csc[c + d*x]^2)/(a^4*d) - (8*Csc[c + d*x]^3
)/(3*a^4*d) + Csc[c + d*x]^4/(a^4*d) - Csc[c + d*x]^5/(5*a^4*d) - (20*Log[S
in[c + d*x]])/(a^4*d) + (20*Log[1 + Sin[c + d*x]])/(a^4*d) - 4/(d*(a^4 + a^
4*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
```

```
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2}{x^6(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^2}{x^6(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{x^6} - \frac{4}{ax^5} + \frac{8}{a^2x^4} - \frac{12}{a^3x^3} + \frac{16}{a^4x^2} - \frac{20}{a^5x} + \frac{4}{a^4(a+x)^2} + \frac{20}{a^5(a+x)}\right) dx, x, a\sin(c$$
$$= -\frac{16\csc(c+dx)}{a^4d} + \frac{6\csc^2(c+dx)}{a^4d} - \frac{8\csc^3(c+dx)}{3a^4d} + \frac{\csc^4(c+dx)}{a^4d} - \frac{\csc^5(c+dx}{5a^4d}$$

Mathematica [A] time = 0.28, size = 91, normalized size = 0.67

$$-\frac{\frac{60}{\sin(c+dx)+1} + 3\csc^5(c+dx) - 15\csc^4(c+dx) + 40\csc^3(c+dx) - 90\csc^2(c+dx) + 240\csc(c+dx) + 300\log(\text{si}}{15a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/15*(240*Csc[c + d*x] - 90*Csc[c + d*x]^2 + 40*Csc[c + d*x]^3 - 15*Csc[c
+ d*x]^4 + 3*Csc[c + d*x]^5 + 300*Log[Sin[c + d*x]] - 300*Log[1 + Sin[c + d
*x]] + 60/(1 + Sin[c + d*x]))/(a^4*d)
```

fricas [B] time = 0.76, size = 283, normalized size = 2.10

$$\frac{150\cos(dx+c)^4 - 325\cos(dx+c)^2 - 300\left(\cos(dx+c)^6 - 3\cos(dx+c)^4 + 3\cos(dx+c)^2 - \left(\cos(dx+c)^4 - \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/15*(150*cos(d*x + c)^4 - 325*cos(d*x + c)^2 - 300*(cos(d*x + c)^6 - 3*cos
(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*si
```

```
n(d*x + c) - 1)*log(1/2*sin(d*x + c)) + 300*(cos(d*x + c)^6 - 3*cos(d*x + c
)^4 + 3*cos(d*x + c)^2 - (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x +
c) - 1)*log(sin(d*x + c) + 1) + 2*(150*cos(d*x + c)^4 - 275*cos(d*x + c)^2
+ 119)*sin(d*x + c) + 178)/(a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*x + c)^4 +
 3*a^4*d*cos(d*x + c)^2 - a^4*d - (a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x +
 c)^2 + a^4*d)*sin(d*x + c))
```

giac [A] time = 0.31, size = 248, normalized size = 1.84

$$\frac{\frac{19200 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4} - \frac{9600 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4} - \frac{1920\left(15 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+28 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\right)}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^2} + \frac{21920 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 435}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/480*(19200*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 9600*log(abs(tan(1/2*
d*x + 1/2*c)))/a^4 - 1920*(15*tan(1/2*d*x + 1/2*c)^2 + 28*tan(1/2*d*x + 1/2
*c) + 15)/(a^4*(tan(1/2*d*x + 1/2*c) + 1)^2) + (21920*tan(1/2*d*x + 1/2*c)^
5 - 4350*tan(1/2*d*x + 1/2*c)^4 + 840*tan(1/2*d*x + 1/2*c)^3 - 175*tan(1/2*
d*x + 1/2*c)^2 + 30*tan(1/2*d*x + 1/2*c) - 3)/(a^4*tan(1/2*d*x + 1/2*c)^5)
- (3*a^16*tan(1/2*d*x + 1/2*c)^5 - 30*a^16*tan(1/2*d*x + 1/2*c)^4 + 175*a^1
6*tan(1/2*d*x + 1/2*c)^3 - 840*a^16*tan(1/2*d*x + 1/2*c)^2 + 4350*a^16*tan(
1/2*d*x + 1/2*c))/a^20)/d
```

maple [A] time = 0.68, size = 131, normalized size = 0.97

$$-\frac{1}{5d\,a^4 \sin\left(dx+c\right)^5}+\frac{1}{d\,a^4 \sin\left(dx+c\right)^4}-\frac{8}{3d\,a^4 \sin\left(dx+c\right)^3}+\frac{6}{d\,a^4 \sin\left(dx+c\right)^2}-\frac{16}{d\,a^4 \sin\left(dx+c\right)}-\frac{20 \ln\left(\sin\left(dx\right.\right.}{a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/5/d/a^4/sin(d*x+c)^5+1/d/a^4/sin(d*x+c)^4-8/3/d/a^4/sin(d*x+c)^3+6/d/a^4
/sin(d*x+c)^2-16/d/a^4/sin(d*x+c)-20*ln(sin(d*x+c))/a^4/d-4/d/a^4/(1+sin(d*
x+c))+20*ln(1+sin(d*x+c))/a^4/d
```

maxima [A] time = 0.78, size = 110, normalized size = 0.81

$$-\frac{\frac{300 \sin(dx+c)^5+150 \sin(dx+c)^4-50 \sin(dx+c)^3+25 \sin(dx+c)^2-12 \sin(dx+c)+3}{a^4 \sin(dx+c)^6+a^4 \sin(dx+c)^5} - \frac{300 \log(\sin(dx+c)+1)}{a^4} + \frac{300 \log(\sin(dx+c))}{a^4}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] -1/15*((300*sin(d*x + c)^5 + 150*sin(d*x + c)^4 - 50*sin(d*x + c)^3 + 25*si
n(d*x + c)^2 - 12*sin(d*x + c) + 3)/(a^4*sin(d*x + c)^6 + a^4*sin(d*x + c)^
5) - 300*log(sin(d*x + c) + 1)/a^4 + 300*log(sin(d*x + c))/a^4)/d
```

mupad [B] time = 8.91, size = 266, normalized size = 1.97

$$\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4a^4d}-\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{96a^4d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{16a^4d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160a^4d}-\frac{20\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4d}-\frac{34\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+52}{d\left(3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^6*(a + a*sin(c + d*x))^4),x)
```

```
[Out] (7*tan(c/2 + (d*x)/2)^2)/(4*a^4*d) - (35*tan(c/2 + (d*x)/2)^3)/(96*a^4*d) +
 tan(c/2 + (d*x)/2)^4/(16*a^4*d) - tan(c/2 + (d*x)/2)^5/(160*a^4*d) - (20*l
og(tan(c/2 + (d*x)/2)))/(a^4*d) - ((118*tan(c/2 + (d*x)/2)^2)/15 - (8*tan(c
/2 + (d*x)/2))/5 - (104*tan(c/2 + (d*x)/2)^3)/3 + (569*tan(c/2 + (d*x)/2)^4
)/3 + 524*tan(c/2 + (d*x)/2)^5 + 34*tan(c/2 + (d*x)/2)^6 + 1/5)/(d*(32*a^4*
tan(c/2 + (d*x)/2)^5 + 64*a^4*tan(c/2 + (d*x)/2)^6 + 32*a^4*tan(c/2 + (d*x)
/2)^7)) + (40*log(tan(c/2 + (d*x)/2) + 1))/(a^4*d) - (145*tan(c/2 + (d*x)/2
))/(16*a^4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.565 $\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=181

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}+\frac{a^3\sin^{n+3}(c+dx)}{d(n+3)}-\frac{5a^3\sin^{n+4}(c+dx)}{d(n+4)}-\frac{5a^3\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a^3\sin^{n+6}(c}{d(n+6}$$

```
[Out] a^3*sin(d*x+c)^(1+n)/d/(1+n)+3*a^3*sin(d*x+c)^(2+n)/d/(2+n)+a^3*sin(d*x+c)^
(3+n)/d/(3+n)-5*a^3*sin(d*x+c)^(4+n)/d/(4+n)-5*a^3*sin(d*x+c)^(5+n)/d/(5+n)
+a^3*sin(d*x+c)^(6+n)/d/(6+n)+3*a^3*sin(d*x+c)^(7+n)/d/(7+n)+a^3*sin(d*x+c)
^(8+n)/d/(8+n)
```

Rubi [A] time = 0.18, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 88}

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}+\frac{a^3\sin^{n+3}(c+dx)}{d(n+3)}-\frac{5a^3\sin^{n+4}(c+dx)}{d(n+4)}-\frac{5a^3\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a^3\sin^{n+6}(c}{d(n+6}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (3*a^3*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (a^3*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (5*a^3*Sin[c + d*x]^(4 +
n))/(d*(4 + n)) - (5*a^3*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (a^3*Sin[c + d
*x]^(6 + n))/(d*(6 + n)) + (3*a^3*Sin[c + d*x]^(7 + n))/(d*(7 + n)) + (a^3*
Sin[c + d*x]^(8 + n))/(d*(8 + n))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n(a+x)^5\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(a^7\left(\frac{x}{a}\right)^n + 3a^7\left(\frac{x}{a}\right)^{1+n} + a^7\left(\frac{x}{a}\right)^{2+n} - 5a^7\left(\frac{x}{a}\right)^{3+n} - \right.\right.}{}$$
$$= \frac{a^3\sin^{1+n}(c+dx)}{d(1+n)} + \frac{3a^3\sin^{2+n}(c+dx)}{d(2+n)} + \frac{a^3\sin^{3+n}(c+dx)}{d(3+n)}$$

Mathematica [A] time = 0.59, size = 123, normalized size = 0.68

$$\frac{a^3\sin^{n+1}(c+dx)\left(\frac{\sin^7(c+dx)}{n+8} + \frac{3\sin^6(c+dx)}{n+7} + \frac{\sin^5(c+dx)}{n+6} - \frac{5\sin^4(c+dx)}{n+5} - \frac{5\sin^3(c+dx)}{n+4} + \frac{\sin^2(c+dx)}{n+3} + \frac{3\sin(c+dx)}{n+2} + \frac{1}{n+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (3*Sin[c + d*x])/(2 + n) + Sin[c
+ d*x]^2/(3 + n) - (5*Sin[c + d*x]^3)/(4 + n) - (5*Sin[c + d*x]^4)/(5 + n)
+ Sin[c + d*x]^5/(6 + n) + (3*Sin[c + d*x]^6)/(7 + n) + Sin[c + d*x]^7/(8 +
 n)))/d
```

fricas [B] time = 0.77, size = 616, normalized size = 3.40

$$\frac{\left(\left(a^3n^7 + 28\,a^3n^6 + 322\,a^3n^5 + 1960\,a^3n^4 + 6769\,a^3n^3 + 13132\,a^3n^2 + 13068\,a^3n + 5040\,a^3\right)\cos\left(dx+c\right)^8 + 32\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] ((a^3*n^7 + 28*a^3*n^6 + 322*a^3*n^5 + 1960*a^3*n^4 + 6769*a^3*n^3 + 13132*
a^3*n^2 + 13068*a^3*n + 5040*a^3)*cos(d*x + c)^8 + 32*a^3*n^5 + 720*a^3*n^4
 - (5*a^3*n^7 + 142*a^3*n^6 + 1654*a^3*n^5 + 10180*a^3*n^4 + 35485*a^3*n^3
+ 69358*a^3*n^2 + 69416*a^3*n + 26880*a^3)*cos(d*x + c)^6 + 6080*a^3*n^3 +
23520*a^3*n^2 + 2*(2*a^3*n^7 + 49*a^3*n^6 + 470*a^3*n^5 + 2230*a^3*n^4 + 54
38*a^3*n^3 + 6361*a^3*n^2 + 2730*a^3*n)*cos(d*x + c)^4 + 39968*a^3*n + 2184
0*a^3 + 8*(2*a^3*n^6 + 45*a^3*n^5 + 380*a^3*n^4 + 1470*a^3*n^3 + 2498*a^3*n
^2 + 1365*a^3*n)*cos(d*x + c)^2 + (32*a^3*n^5 + 720*a^3*n^4 - 3*(a^3*n^7 +
29*a^3*n^6 + 343*a^3*n^5 + 2135*a^3*n^4 + 7504*a^3*n^3 + 14756*a^3*n^2 + 14
```

```
832*a^3*n + 5760*a^3)*cos(d*x + c)^6 + 6080*a^3*n^3 + 24000*a^3*n^2 + 2*(2*
a^3*n^7 + 53*a^3*n^6 + 566*a^3*n^5 + 3155*a^3*n^4 + 9908*a^3*n^3 + 17492*a^
3*n^2 + 15984*a^3*n + 5760*a^3)*cos(d*x + c)^4 + 44288*a^3*n + 30720*a^3 +
8*(2*a^3*n^6 + 47*a^3*n^5 + 425*a^3*n^4 + 1880*a^3*n^3 + 4268*a^3*n^2 + 468
8*a^3*n + 1920*a^3)*cos(d*x + c)^2)*sin(d*x + c))*sin(d*x + c)^n/(d*n^8 + 3
6*d*n^7 + 546*d*n^6 + 4536*d*n^5 + 22449*d*n^4 + 67284*d*n^3 + 118124*d*n^2
 + 109584*d*n + 40320*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 21.58, size = 0, normalized size = 0.00

$$\int \left(\cos^{5}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

maxima [A] time = 0.93, size = 161, normalized size = 0.89

$$\frac{\frac{a^{3}\sin(dx+c)^{n+8}}{n+8}+\frac{3\,a^{3}\sin(dx+c)^{n+7}}{n+7}+\frac{a^{3}\sin(dx+c)^{n+6}}{n+6}-\frac{5\,a^{3}\sin(dx+c)^{n+5}}{n+5}-\frac{5\,a^{3}\sin(dx+c)^{n+4}}{n+4}+\frac{a^{3}\sin(dx+c)^{n+3}}{n+3}+\frac{3\,a^{3}\sin(dx+c)^{n+2}}{n+2}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] (a^3*sin(d*x + c)^(n + 8)/(n + 8) + 3*a^3*sin(d*x + c)^(n + 7)/(n + 7) + a^
3*sin(d*x + c)^(n + 6)/(n + 6) - 5*a^3*sin(d*x + c)^(n + 5)/(n + 5) - 5*a^3
*sin(d*x + c)^(n + 4)/(n + 4) + a^3*sin(d*x + c)^(n + 3)/(n + 3) + 3*a^3*si
n(d*x + c)^(n + 2)/(n + 2) + a^3*sin(d*x + c)^(n + 1)/(n + 1))/d
```

mupad [B] time = 15.45, size = 923, normalized size = 5.10

$$\frac{a^{3}\sin\left(c+d\,x\right)^{n}\,\left(27\,n^{7}+1028\,n^{6}+17366\,n^{5}+162200\,n^{4}+870443\,n^{3}+2585492\,n^{2}+3757604\,n+1896720\right)}{128\,d\,\left(n^{8}+36\,n^{7}+546\,n^{6}+4536\,n^{5}+22449\,n^{4}+67284\,n^{3}+118124\,n^{2}+109584\,n+40320\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^n*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (a^3*sin(c + d*x)^n*(3757604*n + 2585492*n^2 + 870443*n^3 + 162200*n^4 + 17
366*n^5 + 1028*n^6 + 27*n^7 + 1896720))/(128*d*(109584*n + 118124*n^2 + 672
84*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a^3*sin
(c + d*x)^n*cos(8*c + 8*d*x)*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 3
22*n^5 + 28*n^6 + n^7 + 5040))/(128*d*(109584*n + 118124*n^2 + 67284*n^3 +
22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a^3*sin(c + d*x)
*sin(c + d*x)^n*(n*3467760i + n^2*2140836i + n^3*675728i + n^4*118935i + n^
5*11975i + n^6*669i + n^7*17i + 2217600i)*1i)/(64*d*(109584*n + 118124*n^2
+ 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a^
3*sin(c + d*x)^n*cos(6*c + 6*d*x)*(43280*n + 43094*n^2 + 21947*n^3 + 6260*n
^4 + 1010*n^5 + 86*n^6 + 3*n^7 + 16800))/(32*d*(109584*n + 118124*n^2 + 672
84*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a^3*sin
(c + d*x)^n*cos(4*c + 4*d*x)*(303180*n + 273336*n^2 + 122023*n^3 + 29520*n^
4 + 3910*n^5 + 264*n^6 + 7*n^7 + 126000))/(32*d*(109584*n + 118124*n^2 + 67
284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a^3*si
n(c + d*x)^n*cos(2*c + 2*d*x)*(596208*n + 333226*n^2 + 75333*n^3 + 5260*n^4
 - 498*n^5 - 86*n^6 - 3*n^7 + 332640))/(32*d*(109584*n + 118124*n^2 + 67284
*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a^3*sin(c
 + d*x)^n*sin(7*c + 7*d*x)*(n*14832i + n^2*14756i + n^3*7504i + n^4*2135i +
 n^5*343i + n^6*29i + n^7*1i + 5760i)*3i)/(64*d*(109584*n + 118124*n^2 + 67
284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a^3*si
n(c + d*x)^n*sin(5*c + 5*d*x)*(n*94608i + n^2*81404i + n^3*33296i + n^4*678
5i + n^5*617i + n^6*11i - n^7*1i + 40320i)*1i)/(64*d*(109584*n + 118124*n^2
 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a
^3*sin(c + d*x)^n*sin(3*c + 3*d*x)*(n*583216i + n^2*567700i + n^3*275824i +
 n^4*72475i + n^5*10339i + n^6*745i + n^7*21i + 228480i)*1i)/(64*d*(109584*
n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8
+ 40320))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.566 $\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=160

$$\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2\sin^{n+2}(c+dx)}{d(n+2)}-\frac{a^2\sin^{n+3}(c+dx)}{d(n+3)}-\frac{4a^2\sin^{n+4}(c+dx)}{d(n+4)}-\frac{a^2\sin^{n+5}(c+dx)}{d(n+5)}+\frac{2a^2\sin^{n+6}(c+dx)}{d(n+6)}$$

```
[Out] a^2*sin(d*x+c)^(1+n)/d/(1+n)+2*a^2*sin(d*x+c)^(2+n)/d/(2+n)-a^2*sin(d*x+c)^
(3+n)/d/(3+n)-4*a^2*sin(d*x+c)^(4+n)/d/(4+n)-a^2*sin(d*x+c)^(5+n)/d/(5+n)+2
*a^2*sin(d*x+c)^(6+n)/d/(6+n)+a^2*sin(d*x+c)^(7+n)/d/(7+n)
```

Rubi [A] time = 0.17, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 88}

$$\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2\sin^{n+2}(c+dx)}{d(n+2)}-\frac{a^2\sin^{n+3}(c+dx)}{d(n+3)}-\frac{4a^2\sin^{n+4}(c+dx)}{d(n+4)}-\frac{a^2\sin^{n+5}(c+dx)}{d(n+5)}+\frac{2a^2\sin^{n+6}(c+dx)}{d(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a^2*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - (a^2*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (4*a^2*Sin[c + d*x]^(4 +
n))/(d*(4 + n)) - (a^2*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (2*a^2*Sin[c + d
*x]^(6 + n))/(d*(6 + n)) + (a^2*Sin[c + d*x]^(7 + n))/(d*(7 + n))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^6\left(\frac{x}{a}\right)^n + 2a^6\left(\frac{x}{a}\right)^{1+n} - a^6\left(\frac{x}{a}\right)^{2+n} - 4a^6\left(\frac{x}{a}\right)^{3+n} - \right.\right.}{a^5 d}$$
$$= \frac{a^2\sin^{1+n}(c+dx)}{d(1+n)} + \frac{2a^2\sin^{2+n}(c+dx)}{d(2+n)} - \frac{a^2\sin^{3+n}(c+dx)}{d(3+n)}$$

Mathematica [A] time = 0.39, size = 110, normalized size = 0.69

$$\frac{a^2\sin^{n+1}(c+dx)\left(\frac{\sin^6(c+dx)}{n+7} + \frac{2\sin^5(c+dx)}{n+6} - \frac{\sin^4(c+dx)}{n+5} - \frac{4\sin^3(c+dx)}{n+4} - \frac{\sin^2(c+dx)}{n+3} + \frac{2\sin(c+dx)}{n+2} + \frac{1}{n+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (2*Sin[c + d*x])/(2 + n) - Sin[c
+ d*x]^2/(3 + n) - (4*Sin[c + d*x]^3)/(4 + n) - Sin[c + d*x]^4/(5 + n) + (2
*Sin[c + d*x]^5)/(6 + n) + Sin[c + d*x]^6/(7 + n)))/d
```

fricas [B] time = 0.83, size = 473, normalized size = 2.96

$$-\frac{\left(2\left(a^2n^6 + 22\,a^2n^5 + 190\,a^2n^4 + 820\,a^2n^3 + 1849\,a^2n^2 + 2038\,a^2n + 840\,a^2\right)\cos(dx+c)^6 - 16\,a^2n^4 - 256\,a^2n^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(2*(a^2*n^6 + 22*a^2*n^5 + 190*a^2*n^4 + 820*a^2*n^3 + 1849*a^2*n^2 + 2038
*a^2*n + 840*a^2)*cos(d*x + c)^6 - 16*a^2*n^4 - 256*a^2*n^3 - 2*(a^2*n^6 +
18*a^2*n^5 + 118*a^2*n^4 + 348*a^2*n^3 + 457*a^2*n^2 + 210*a^2*n)*cos(d*x +
 c)^4 - 1376*a^2*n^2 - 2816*a^2*n - 8*(a^2*n^5 + 16*a^2*n^4 + 86*a^2*n^3 +
176*a^2*n^2 + 105*a^2*n)*cos(d*x + c)^2 - 1680*a^2 + ((a^2*n^6 + 21*a^2*n^5
 + 175*a^2*n^4 + 735*a^2*n^3 + 1624*a^2*n^2 + 1764*a^2*n + 720*a^2)*cos(d*x
 + c)^6 - 16*a^2*n^4 - 256*a^2*n^3 - 2*(a^2*n^6 + 20*a^2*n^5 + 159*a^2*n^4
+ 640*a^2*n^3 + 1364*a^2*n^2 + 1440*a^2*n + 576*a^2)*cos(d*x + c)^4 - 1472*
a^2*n^2 - 3584*a^2*n - 8*(a^2*n^5 + 17*a^2*n^4 + 108*a^2*n^3 + 316*a^2*n^2
+ 416*a^2*n + 192*a^2)*cos(d*x + c)^2 - 3072*a^2)*sin(d*x + c))*sin(d*x + c
```

```
)^n/(d*n^7 + 28*d*n^6 + 322*d*n^5 + 1960*d*n^4 + 6769*d*n^3 + 13132*d*n^2 +
 13068*d*n + 5040*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 16.81, size = 0, normalized size = 0.00

$$\int \left(\cos^{5}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

maxima [A] time = 0.97, size = 143, normalized size = 0.89

$$\frac{\frac{a^{2}\sin(dx+c)^{n+7}}{n+7}+\frac{2\,a^{2}\sin(dx+c)^{n+6}}{n+6}-\frac{a^{2}\sin(dx+c)^{n+5}}{n+5}-\frac{4\,a^{2}\sin(dx+c)^{n+4}}{n+4}-\frac{a^{2}\sin(dx+c)^{n+3}}{n+3}+\frac{2\,a^{2}\sin(dx+c)^{n+2}}{n+2}+\frac{a^{2}\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
[Out] (a^2*sin(d*x + c)^(n + 7)/(n + 7) + 2*a^2*sin(d*x + c)^(n + 6)/(n + 6) - a^
2*sin(d*x + c)^(n + 5)/(n + 5) - 4*a^2*sin(d*x + c)^(n + 4)/(n + 4) - a^2*s
in(d*x + c)^(n + 3)/(n + 3) + 2*a^2*sin(d*x + c)^(n + 2)/(n + 2) + a^2*sin(
d*x + c)^(n + 1)/(n + 1))/d
```

mupad [B] time = 14.66, size = 819, normalized size = 5.12

$$\frac{a^{2}\sin\left(c+d\,x\right)^{n}\,\left(n^{6}\,1\mathrm{i}+n^{5}\,30\mathrm{i}+n^{4}\,398\mathrm{i}+n^{3}\,2788\mathrm{i}+n^{2}\,10137\mathrm{i}+n\,16958\mathrm{i}+9240\mathrm{i}\right)}{8\,d\,\left(n^{7}\,1\mathrm{i}+n^{6}\,28\mathrm{i}+n^{5}\,322\mathrm{i}+n^{4}\,1960\mathrm{i}+n^{3}\,6769\mathrm{i}+n^{2}\,13132\mathrm{i}+n\,13068\mathrm{i}+5040\mathrm{i}\right)}-\frac{a^{2}\sin\left(c+d\,x\right)^{n}\,\sin\left(7\,c\right.}{64\,d\,\left(n^{7}\,1\mathrm{i}+n^{6}\,28\mathrm{i}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^n*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x)^n*(n*16958i + n^2*10137i + n^3*2788i + n^4*398i + n^5*30i
 + n^6*1i + 9240i))/(8*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n
^5*322i + n^6*28i + n^7*1i + 5040i)) - (a^2*sin(c + d*x)^n*sin(7*c + 7*d*x)
*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)*1i)/(64*d*(n*
13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i +
 5040i)) + (a^2*sin(c + d*x)*sin(c + d*x)^n*(296844*n + 148360*n^2 + 36773*
n^3 + 4869*n^4 + 343*n^5 + 11*n^6 + 226800)*1i)/(64*d*(n*13068i + n^2*13132
i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i)) - (a^2*si
n(c + d*x)^n*cos(6*c + 6*d*x)*(n*2038i + n^2*1849i + n^3*820i + n^4*190i +
n^5*22i + n^6*1i + 840i))/(16*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*19
60i + n^5*322i + n^6*28i + n^7*1i + 5040i)) - (a^2*sin(c + d*x)^n*cos(4*c +
 4*d*x)*(n*5694i + n^2*4633i + n^3*1764i + n^4*334i + n^5*30i + n^6*1i + 25
20i))/(8*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*
28i + n^7*1i + 5040i)) - (a^2*sin(c + d*x)^n*cos(2*c + 2*d*x)*(n*20490i + n
^2*9159i + n^3*1228i - n^4*62i - n^5*22i - n^6*1i + 12600i))/(16*d*(n*13068
i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040
i)) + (a^2*sin(c + d*x)^n*sin(5*c + 5*d*x)*(2700*n + 2792*n^2 + 1445*n^3 +
397*n^4 + 55*n^5 + 3*n^6 + 1008)*1i)/(64*d*(n*13068i + n^2*13132i + n^3*676
9i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i)) + (a^2*sin(c + d*x)^
n*sin(3*c + 3*d*x)*(71932*n + 58568*n^2 + 22569*n^3 + 4417*n^4 + 419*n^5 +
15*n^6 + 31920)*1i)/(64*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i +
n^5*322i + n^6*28i + n^7*1i + 5040i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.567 $\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=123

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}-\frac{2a\sin^{n+3}(c+dx)}{d(n+3)}-\frac{2a\sin^{n+4}(c+dx)}{d(n+4)}+\frac{a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a\sin^{n+6}(c+dx)}{d(n+6)}$$

```
[Out] a*sin(d*x+c)^(1+n)/d/(1+n)+a*sin(d*x+c)^(2+n)/d/(2+n)-2*a*sin(d*x+c)^(3+n)/
d/(3+n)-2*a*sin(d*x+c)^(4+n)/d/(4+n)+a*sin(d*x+c)^(5+n)/d/(5+n)+a*sin(d*x+c
)^(6+n)/d/(6+n)
```

Rubi [A] time = 0.12, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2836, 88}

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}-\frac{2a\sin^{n+3}(c+dx)}{d(n+3)}-\frac{2a\sin^{n+4}(c+dx)}{d(n+4)}+\frac{a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a\sin^{n+6}(c+dx)}{d(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (a*Sin[c + d*x]^(2 + n))/(d*(2 + n))
 - (2*a*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (2*a*Sin[c + d*x]^(4 + n))/(d*(
4 + n)) + (a*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (a*Sin[c + d*x]^(6 + n))/(
d*(6 + n))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n (a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5\left(\frac{x}{a}\right)^n + a^5\left(\frac{x}{a}\right)^{1+n} - 2a^5\left(\frac{x}{a}\right)^{2+n} - 2a^5\left(\frac{x}{a}\right)^{3+n} + a\right.\right.}{a^5 d}$$
$$= \frac{a\sin^{1+n}(c+dx)}{d(1+n)} + \frac{a\sin^{2+n}(c+dx)}{d(2+n)} - \frac{2a\sin^{3+n}(c+dx)}{d(3+n)} - \frac{2}{}$$

Mathematica [B] time = 1.38, size = 345, normalized size = 2.80

$$\frac{a\sin^{n+1}(c+dx)\left(2n^5\sin(c+dx) + 3n^5\sin(3(c+dx)) + n^5\sin(5(c+dx)) + 46n^4\sin(c+dx) + 61n^4\sin(3(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n)*(8544 + 10520*n + 4888*n^2 + 1114*n^3 + 128*n^4 + 6
*n^5 + 8*(336 + 692*n + 484*n^2 + 147*n^3 + 20*n^4 + n^5)*Cos[2*(c + d*x)]
+ 2*(144 + 324*n + 260*n^2 + 95*n^3 + 16*n^4 + n^5)*Cos[4*(c + d*x)] + 2640
*Sin[c + d*x] + 4468*n*Sin[c + d*x] + 2258*n^2*Sin[c + d*x] + 474*n^3*Sin[c
 + d*x] + 46*n^4*Sin[c + d*x] + 2*n^5*Sin[c + d*x] + 840*Sin[3*(c + d*x)] +
 1798*n*Sin[3*(c + d*x)] + 1331*n^2*Sin[3*(c + d*x)] + 431*n^3*Sin[3*(c + d
*x)] + 61*n^4*Sin[3*(c + d*x)] + 3*n^5*Sin[3*(c + d*x)] + 120*Sin[5*(c + d*
x)] + 274*n*Sin[5*(c + d*x)] + 225*n^2*Sin[5*(c + d*x)] + 85*n^3*Sin[5*(c +
 d*x)] + 15*n^4*Sin[5*(c + d*x)] + n^5*Sin[5*(c + d*x)]))/(16*d*(1 + n)*(2
+ n)*(3 + n)*(4 + n)*(5 + n)*(6 + n))
```

fricas [B] time = 0.73, size = 282, normalized size = 2.29

$$-\frac{\left(\left(an^5 + 15\,an^4 + 85\,an^3 + 225\,an^2 + 274\,an + 120\,a\right)\cos\left(dx+c\right)^6 - \left(an^5 + 11\,an^4 + 41\,an^3 + 61\,an^2 + 30\,an\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((a*n^5 + 15*a*n^4 + 85*a*n^3 + 225*a*n^2 + 274*a*n + 120*a)*cos(d*x + c)^
6 - (a*n^5 + 11*a*n^4 + 41*a*n^3 + 61*a*n^2 + 30*a*n)*cos(d*x + c)^4 - 8*a*
n^3 - 72*a*n^2 - 4*(a*n^4 + 9*a*n^3 + 23*a*n^2 + 15*a*n)*cos(d*x + c)^2 - 1
84*a*n - ((a*n^5 + 16*a*n^4 + 95*a*n^3 + 260*a*n^2 + 324*a*n + 144*a)*cos(d
*x + c)^4 + 8*a*n^3 + 96*a*n^2 + 4*(a*n^4 + 13*a*n^3 + 56*a*n^2 + 92*a*n +
```

```
48*a)*cos(d*x + c)^2 + 352*a*n + 384*a)*sin(d*x + c) - 120*a)*sin(d*x + c)^
n/(d*n^6 + 21*d*n^5 + 175*d*n^4 + 735*d*n^3 + 1624*d*n^2 + 1764*d*n + 720*d
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 9.86, size = 0, normalized size = 0.00

$$\int \left(\cos^{5}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

maxima [A] time = 0.60, size = 109, normalized size = 0.89

$$\frac{\frac{a\sin(dx+c)^{n+6}}{n+6}+\frac{a\sin(dx+c)^{n+5}}{n+5}-\frac{2\,a\sin(dx+c)^{n+4}}{n+4}-\frac{2\,a\sin(dx+c)^{n+3}}{n+3}+\frac{a\sin(dx+c)^{n+2}}{n+2}+\frac{a\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*sin(d*x + c)^(n + 6)/(n + 6) + a*sin(d*x + c)^(n + 5)/(n + 5) - 2*a*sin(
d*x + c)^(n + 4)/(n + 4) - 2*a*sin(d*x + c)^(n + 3)/(n + 3) + a*sin(d*x + c
)^(n + 2)/(n + 2) + a*sin(d*x + c)^(n + 1)/(n + 1))/d
```

mupad [B] time = 13.56, size = 550, normalized size = 4.47

$$\frac{a\sin\left(c+d\,x\right)^n\,\left(n^5+23\,n^4+237\,n^3+1129\,n^2+2234\,n+1320\right)}{16\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{a\sin\left(c+d\,x\right)^n\,\cos\left(6\,c+6\,d\,x\right)\,\left(n^5+15\,n^4\right.}{32\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^n*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x)^n*(2234*n + 1129*n^2 + 237*n^3 + 23*n^4 + n^5 + 1320))/(16*
d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (a*sin(c
+ d*x)^n*cos(6*c + 6*d*x)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/
(32*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (a*si
n(c + d*x)^n*cos(4*c + 4*d*x)*(762*n + 553*n^2 + 173*n^3 + 23*n^4 + n^5 + 3
60))/(16*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) -
(a*sin(c + d*x)*sin(c + d*x)^n*(n*3876i + n^2*1476i + n^3*263i + n^4*24i +
n^5*1i + 3600i)*1i)/(8*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 +
n^6 + 720)) - (a*sin(c + d*x)^n*cos(2*c + 2*d*x)*(2670*n + 927*n^2 + 43*n^3
 - 15*n^4 - n^5 + 1800))/(32*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*
n^5 + n^6 + 720)) - (a*sin(c + d*x)^n*sin(5*c + 5*d*x)*(n*324i + n^2*260i +
 n^3*95i + n^4*16i + n^5*1i + 144i)*1i)/(16*d*(1764*n + 1624*n^2 + 735*n^3
+ 175*n^4 + 21*n^5 + n^6 + 720)) - (a*sin(c + d*x)^n*sin(3*c + 3*d*x)*(n*24
44i + n^2*1676i + n^3*493i + n^4*64i + n^5*3i + 1200i)*1i)/(16*d*(1764*n +
1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.568 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\sin^{n+1}(c+dx)}{ad(n+1)} - \frac{\sin^{n+2}(c+dx)}{ad(n+2)} - \frac{\sin^{n+3}(c+dx)}{ad(n+3)} + \frac{\sin^{n+4}(c+dx)}{ad(n+4)}$$

```
[Out] sin(d*x+c)^(1+n)/a/d/(1+n)-sin(d*x+c)^(2+n)/a/d/(2+n)-sin(d*x+c)^(3+n)/a/d/
(3+n)+sin(d*x+c)^(4+n)/a/d/(4+n)
```

Rubi [A] time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 75}

$$\frac{\sin^{n+1}(c+dx)}{ad(n+1)} - \frac{\sin^{n+2}(c+dx)}{ad(n+2)} - \frac{\sin^{n+3}(c+dx)}{ad(n+3)} + \frac{\sin^{n+4}(c+dx)}{ad(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a*d*(1 + n)) - Sin[c + d*x]^(2 + n)/(a*d*(2 + n)) - S
in[c + d*x]^(3 + n)/(a*d*(3 + n)) + Sin[c + d*x]^(4 + n)/(a*d*(4 + n))
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n(a+x)\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3\left(\frac{x}{a}\right)^n - a^3\left(\frac{x}{a}\right)^{1+n} - a^3\left(\frac{x}{a}\right)^{2+n} + a^3\left(\frac{x}{a}\right)^{3+n}\right)dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\sin^{1+n}(c+dx)}{ad(1+n)} - \frac{\sin^{2+n}(c+dx)}{ad(2+n)} - \frac{\sin^{3+n}(c+dx)}{ad(3+n)} + \frac{\sin^{4+n}(c+dx)}{ad(4+n)}$$

Mathematica [A] time = 0.70, size = 74, normalized size = 0.81

$$\frac{\sin^{n+1}(c+dx)\left(-\frac{(n+4)\sin^2(c+dx)}{n+3} - \frac{(n+4)\sin(c+dx)}{n+2} + \sin^3(c+dx) + \frac{n+4}{n+1}\right)}{ad(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*((4 + n)/(1 + n) - ((4 + n)*Sin[c + d*x])/(2 + n) - (
(4 + n)*Sin[c + d*x]^2)/(3 + n) + Sin[c + d*x]^3))/(a*d*(4 + n))
```

fricas [A] time = 0.88, size = 134, normalized size = 1.47

$$\frac{\left(\left(n^3+6n^2+11n+6\right)\cos(dx+c)^4 - \left(n^3+4n^2+3n\right)\cos(dx+c)^2 - 2n^2 + \left(\left(n^3+7n^2+14n+8\right)\cos(dx+\right.\right.}{adn^4+10adn^3+35adn^2+50adn+24ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*cos(d*x + c)^4 - (n^3 + 4*n^2 + 3*n)*cos(d*x + c)
^2 - 2*n^2 + ((n^3 + 7*n^2 + 14*n + 8)*cos(d*x + c)^2 + 2*n^2 + 12*n + 16)*
sin(d*x + c) - 8*n - 6)*sin(d*x + c)^n/(a*d*n^4 + 10*a*d*n^3 + 35*a*d*n^2 +
 50*a*d*n + 24*a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n\cos(dx+c)^5}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(a*sin(d*x + c) + a), x)
```

maple [F] time = 5.92, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{5}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)}{a+a\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

maxima [A] time = 0.80, size = 124, normalized size = 1.36

$$\frac{\left(\left(n^{3}+6\,n^{2}+11\,n+6\right)\sin\left(dx+c\right)^{4}-\left(n^{3}+7\,n^{2}+14\,n+8\right)\sin\left(dx+c\right)^{3}-\left(n^{3}+8\,n^{2}+19\,n+12\right)\sin\left(dx+c\right.}{\left(n^{4}+10\,n^{3}+35\,n^{2}+50\,n+24\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*sin(d*x + c)^4 - (n^3 + 7*n^2 + 14*n + 8)*sin(d*x
 + c)^3 - (n^3 + 8*n^2 + 19*n + 12)*sin(d*x + c)^2 + (n^3 + 9*n^2 + 26*n +
24)*sin(d*x + c))*sin(d*x + c)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*a*d)
```

mupad [B] time = 10.21, size = 228, normalized size = 2.51

$$\frac{\sin\left(c+d\,x\right)^{n}\,\left(144\,\sin\left(c+d\,x\right)-43\,n+24\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+16\,\sin\left(3\,c+3\,d\,x\right)+124\,n\,\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^n*(144*sin(c + d*x) - 43*n + 24*cos(2*c + 2*d*x) + 6*cos(4*c
+ 4*d*x) + 16*sin(3*c + 3*d*x) + 124*n*sin(c + d*x) + 32*n*cos(2*c + 2*d*x)
 + 11*n*cos(4*c + 4*d*x) + 28*n*sin(3*c + 3*d*x) + 30*n^2*sin(c + d*x) + 2*
n^3*sin(c + d*x) - 14*n^2 - n^3 + 8*n^2*cos(2*c + 2*d*x) + 6*n^2*cos(4*c +
4*d*x) + n^3*cos(4*c + 4*d*x) + 14*n^2*sin(3*c + 3*d*x) + 2*n^3*sin(3*c + 3
*d*x) - 30))/(8*a*d*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.569 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=68

$$\frac{\sin^{n+1}(c+dx)}{a^2d(n+1)}-\frac{2\sin^{n+2}(c+dx)}{a^2d(n+2)}+\frac{\sin^{n+3}(c+dx)}{a^2d(n+3)}$$

```
[Out] sin(d*x+c)^(1+n)/a^2/d/(1+n)-2*sin(d*x+c)^(2+n)/a^2/d/(2+n)+sin(d*x+c)^(3+n
)/a^2/d/(3+n)
```

Rubi [A] time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 43}

$$\frac{\sin^{n+1}(c+dx)}{a^2d(n+1)}-\frac{2\sin^{n+2}(c+dx)}{a^2d(n+2)}+\frac{\sin^{n+3}(c+dx)}{a^2d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a^2*d*(1 + n)) - (2*Sin[c + d*x]^(2 + n))/(a^2*d*(2 +
 n)) + Sin[c + d*x]^(3 + n)/(a^2*d*(3 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(\frac{x}{a}\right)^n - 2a^2\left(\frac{x}{a}\right)^{1+n} + a^2\left(\frac{x}{a}\right)^{2+n}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\sin^{1+n}(c+dx)}{a^2 d(1+n)} - \frac{2\sin^{2+n}(c+dx)}{a^2 d(2+n)} + \frac{\sin^{3+n}(c+dx)}{a^2 d(3+n)}$$

Mathematica [A] time = 0.12, size = 50, normalized size = 0.74

$$\frac{\sin^{n+1}(c+dx)\left(\frac{\sin^2(c+dx)}{n+3} - \frac{2\sin(c+dx)}{n+2} + \frac{1}{n+1}\right)}{a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*((1 + n)^(-1) - (2*Sin[c + d*x])/(2 + n) + Sin[c + d*
x]^2/(3 + n)))/(a^2*d)
```

fricas [A] time = 0.76, size = 105, normalized size = 1.54

$$\frac{\left(2\left(n^2+4n+3\right)\cos(dx+c)^2 - 2n^2 - \left(\left(n^2+3n+2\right)\cos(dx+c)^2 - 2n^2 - 8n - 8\right)\sin(dx+c) - 8n - 6\right)\sin(}{a^2dn^3 + 6a^2dn^2 + 11a^2dn + 6a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (2*(n^2 + 4*n + 3)*cos(d*x + c)^2 - 2*n^2 - ((n^2 + 3*n + 2)*cos(d*x + c)^2
 - 2*n^2 - 8*n - 8)*sin(d*x + c) - 8*n - 6)*sin(d*x + c)^n/(a^2*d*n^3 + 6*a
^2*d*n^2 + 11*a^2*d*n + 6*a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n\cos(dx+c)^5}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(a*sin(d*x + c) + a)^2, x)
```

maple [F] time = 13.92, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{5}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)}{\left(a+a\sin\left(dx+c\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

maxima [A] time = 0.92, size = 81, normalized size = 1.19

$$\frac{\left(\left(n^{2}+3\,n+2\right)\sin\left(dx+c\right)^{3}-2\,\left(n^{2}+4\,n+3\right)\sin\left(dx+c\right)^{2}+\left(n^{2}+5\,n+6\right)\sin\left(dx+c\right)\right)\sin\left(dx+c\right)^{n}}{\left(n^{3}+6\,n^{2}+11\,n+6\right)a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] ((n^2 + 3*n + 2)*sin(d*x + c)^3 - 2*(n^2 + 4*n + 3)*sin(d*x + c)^2 + (n^2 +
 5*n + 6)*sin(d*x + c))*sin(d*x + c)^n/((n^3 + 6*n^2 + 11*n + 6)*a^2*d)
```

mupad [B] time = 9.54, size = 146, normalized size = 2.15

$$-\frac{\frac{\sin(c+d\,x)^{n}\,\left(24\sin(c+d\,x)^{2}-30\,\sin(c+d\,x)+2\,\sin(3\,c+3\,d\,x)\right)}{4}+\frac{n\sin(c+d\,x)^{n}\,\left(32\sin(c+d\,x)^{2}-29\,\sin(c+d\,x)+3\,\sin(3\,c+3\,d\,x)\right)}{4}+\frac{n^{2}\sin(c+d\,x)^{n}\,(\cdots}{\cdots}}{a^{2}\,d\,\left(n^{3}+6\,n^{2}+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] -((sin(c + d*x)^n*(2*sin(3*c + 3*d*x) - 30*sin(c + d*x) + 24*sin(c + d*x)^2
))/4 + (n*sin(c + d*x)^n*(3*sin(3*c + 3*d*x) - 29*sin(c + d*x) + 32*sin(c +
 d*x)^2))/4 + (n^2*sin(c + d*x)^n*(sin(3*c + 3*d*x) - 7*sin(c + d*x) + 8*si
n(c + d*x)^2))/4)/(a^2*d*(11*n + 6*n^2 + n^3 + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.570 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=85

$$\frac{4\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^3d(n+1)}-\frac{3\sin^{n+1}(c+dx)}{a^3d(n+1)}+\frac{\sin^{n+2}(c+dx)}{a^3d(n+2)}$$

```
[Out] -3*sin(d*x+c)^(1+n)/a^3/d/(1+n)+4*hypergeom([1, 1+n],[2+n],-sin(d*x+c))*sin
(d*x+c)^(1+n)/a^3/d/(1+n)+sin(d*x+c)^(2+n)/a^3/d/(2+n)
```

Rubi [A] time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 88, 64}

$$\frac{4\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^3d(n+1)}-\frac{3\sin^{n+1}(c+dx)}{a^3d(n+1)}+\frac{\sin^{n+2}(c+dx)}{a^3d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*Sin[c + d*x]^(1 + n))/(a^3*d*(1 + n)) + (4*Hypergeometric2F1[1, 1 + n,
2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a^3*d*(1 + n)) + Sin[c + d*x]^
(2 + n)/(a^3*d*(2 + n))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a\left(\frac{x}{a}\right)^n + a\left(\frac{x}{a}\right)^{1+n} + \frac{4a^2\left(\frac{x}{a}\right)^n}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= -\frac{3\sin^{1+n}(c+dx)}{a^3 d(1+n)} + \frac{\sin^{2+n}(c+dx)}{a^3 d(2+n)} + \frac{4\,\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= -\frac{3\sin^{1+n}(c+dx)}{a^3 d(1+n)} + \frac{4\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^3 d(1+n)} + \frac{\sin^{2}}{a^3}$$

Mathematica [A] time = 0.11, size = 64, normalized size = 0.75

$$\frac{\sin^{n+1}(c+dx)(4(n+2)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx)) + (n+1)\sin(c+dx) - 3(n+2))}{a^3 d(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(-3*(2 + n) + 4*(2 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, -Sin[c + d*x]] + (1 + n)*Sin[c + d*x]))/(a^3*d*(1 + n)*(2 + n))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sin(dx+c)^n\cos(dx+c)^5}{3\,a^3\cos(dx+c)^2 - 4\,a^3 + \left(a^3\cos(dx+c)^2 - 4\,a^3\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)^5/(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^
3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin (dx+c)^n \cos (dx+c)^5}{(a\sin (dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(a*sin(d*x + c) + a)^3, x)
```

maple [F] time = 6.21, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^5 (dx+c)\right) (\sin^n (dx+c))}{(a+a\sin (dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin (dx+c)^n \cos (dx+c)^5}{(a\sin (dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(a*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c+d\,x)^5 \sin (c+d\,x)^n}{(a+a\,\sin (c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.571 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=88

$$-\frac{4\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^4d}+\frac{\sin^{n+1}(c+dx)}{a^4d(n+1)}+\frac{4\sin^{n+1}(c+dx)}{d\left(a^4\sin(c+dx)+a^4\right)}$$

```
[Out] sin(d*x+c)^(1+n)/a^4/d/(1+n)-4*hypergeom([1, 1+n],[2+n],-sin(d*x+c))*sin(d*
x+c)^(1+n)/a^4/d+4*sin(d*x+c)^(1+n)/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2836, 89, 80, 64}

$$-\frac{4\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^4d}+\frac{\sin^{n+1}(c+dx)}{a^4d(n+1)}+\frac{4\sin^{n+1}(c+dx)}{d\left(a^4\sin(c+dx)+a^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a^4*d*(1 + n)) - (4*Hypergeometric2F1[1, 1 + n, 2 + n
, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a^4*d) + (4*Sin[c + d*x]^(1 + n))/(
d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2\left(\frac{x}{a}\right)^n}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$

$$= \frac{4\sin^{1+n}(c+dx)}{d\left(a^4+a^4\sin(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{(a(3+4n)-x)\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$

$$= \frac{\sin^{1+n}(c+dx)}{a^4 d(1+n)} + \frac{4\sin^{1+n}(c+dx)}{d\left(a^4+a^4\sin(c+dx)\right)} - \frac{(4(1+n))\,\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin\right.}{a^4 d}$$

$$= \frac{\sin^{1+n}(c+dx)}{a^4 d(1+n)} - \frac{4\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4 d} + \frac{4\sin}{d\left(a^4+\right.}$$

Mathematica [A] time = 0.10, size = 72, normalized size = 0.82

$$\frac{\sin^{n+1}(c+dx)(-4(n+1)(\sin(c+dx)+1)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))+\sin(c+dx)+4n+5)}{a^4 d(n+1)(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(5 + 4*n + Sin[c + d*x] - 4*(1 + n)*Hypergeometric2F1
[1, 1 + n, 2 + n, -Sin[c + d*x]]*(1 + Sin[c + d*x])))/(a^4*d*(1 + n)*(1 + S
in[c + d*x]))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin(dx+c)^n\cos(dx+c)^5}{a^4\cos(dx+c)^4-8\,a^4\cos(dx+c)^2+8\,a^4-4\left(a^4\cos(dx+c)^2-2\,a^4\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^5/(a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x
+ c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(dx+c)^n\cos(dx+c)^5}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(a*sin(d*x + c) + a)^4, x)
```

maple [F] time = 12.28, size = 0, normalized size = 0.00

$$\int\frac{\left(\cos^5(dx+c)\right)\left(\sin^n(dx+c)\right)}{(a+a\sin(dx+c))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(dx+c)^n\cos(dx+c)^5}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(a*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^5 \sin(c+dx)^n}{(a+a\sin(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + a*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.572 $\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=165

$$-\frac{a\cos^{11}(c+dx)}{11d}+\frac{2a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{3a\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{a}{}$$

```
[Out] 3/256*a*x-1/7*a*cos(d*x+c)^7/d+2/9*a*cos(d*x+c)^9/d-1/11*a*cos(d*x+c)^11/d+
3/256*a*cos(d*x+c)*sin(d*x+c)/d+1/128*a*cos(d*x+c)^3*sin(d*x+c)/d+1/160*a*c
os(d*x+c)^5*sin(d*x+c)/d-3/80*a*cos(d*x+c)^7*sin(d*x+c)/d-1/10*a*cos(d*x+c)
^7*sin(d*x+c)^3/d
```

Rubi [A] time = 0.20, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 270}

$$-\frac{a\cos^{11}(c+dx)}{11d}+\frac{2a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{3a\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/256 - (a*Cos[c + d*x]^7)/(7*d) + (2*a*Cos[c + d*x]^9)/(9*d) - (a*Co
s[c + d*x]^11)/(11*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (a*Cos[c
+ d*x]^3*Sin[c + d*x])/(128*d) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(160*d) -
(3*a*Cos[c + d*x]^7*Sin[c + d*x])/(80*d) - (a*Cos[c + d*x]^7*Sin[c + d*x]^3
)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^6(c+dx)\sin^4(c+dx)\,dx + a\int \cos^6(c+dx)\sin^5(c+$$

$$= -\frac{a\cos^7(c+dx)\sin^3(c+dx)}{10d} + \frac{1}{10}(3a)\int \cos^6(c+dx)\sin^2(c$$

$$= -\frac{3a\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{a\cos^7(c+dx)\sin^3(c+dx)}{10d} +$$

$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{2a\cos^9(c+dx)}{9d} - \frac{a\cos^{11}(c+dx)}{11d} + \frac{a\cos$$

$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{2a\cos^9(c+dx)}{9d} - \frac{a\cos^{11}(c+dx)}{11d} + \frac{a\cos$$

$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{2a\cos^9(c+dx)}{9d} - \frac{a\cos^{11}(c+dx)}{11d} + \frac{3a\cos$$

$$= \frac{3ax}{256} - \frac{a\cos^7(c+dx)}{7d} + \frac{2a\cos^9(c+dx)}{9d} - \frac{a\cos^{11}(c+dx)}{11d} +$$

Mathematica [A] time = 0.57, size = 121, normalized size = 0.73

$$\frac{a(13860\sin(2(c+dx))-27720\sin(4(c+dx))-6930\sin(6(c+dx))+3465\sin(8(c+dx))+1386\sin(10(c+dx$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(83160*d*x - 69300*Cos[c + d*x] - 23100*Cos[3*(c + d*x)] + 6930*Cos[5*(c
 + d*x)] + 4950*Cos[7*(c + d*x)] - 770*Cos[9*(c + d*x)] - 630*Cos[11*(c + d
*x)] + 13860*Sin[2*(c + d*x)] - 27720*Sin[4*(c + d*x)] - 6930*Sin[6*(c + d*
x)] + 3465*Sin[8*(c + d*x)] + 1386*Sin[10*(c + d*x)]))/(7096320*d)
```

fricas [A] time = 0.70, size = 106, normalized size = 0.64

$$-\frac{80640\,a\cos(dx+c)^{11}-197120\,a\cos(dx+c)^{9}+126720\,a\cos(dx+c)^{7}-10395\,adx-693\left(128\,a\cos(dx+c\right.}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/887040*(80640*a*cos(d*x + c)^11 - 197120*a*cos(d*x + c)^9 + 126720*a*cos
(d*x + c)^7 - 10395*a*d*x - 693*(128*a*cos(d*x + c)^9 - 176*a*cos(d*x + c)^
7 + 8*a*cos(d*x + c)^5 + 10*a*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x +
 c))/d
```

giac [A] time = 0.30, size = 167, normalized size = 1.01

$$\frac{3}{256}\,ax-\frac{a\cos(11\,dx+11\,c)}{11264\,d}-\frac{a\cos(9\,dx+9\,c)}{9216\,d}+\frac{5\,a\cos(7\,dx+7\,c)}{7168\,d}+\frac{a\cos(5\,dx+5\,c)}{1024\,d}-\frac{5\,a\cos(3\,dx+3\,c)}{1536\,d}-\frac{5\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/256*a*x - 1/11264*a*cos(11*d*x + 11*c)/d - 1/9216*a*cos(9*d*x + 9*c)/d +
5/7168*a*cos(7*d*x + 7*c)/d + 1/1024*a*cos(5*d*x + 5*c)/d - 5/1536*a*cos(3*
d*x + 3*c)/d - 5/512*a*cos(d*x + c)/d + 1/5120*a*sin(10*d*x + 10*c)/d + 1/2
048*a*sin(8*d*x + 8*c)/d - 1/1024*a*sin(6*d*x + 6*c)/d - 1/256*a*sin(4*d*x
+ 4*c)/d + 1/512*a*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.24, size = 134, normalized size = 0.81

$$\frac{a\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^7(dx+c)\right)}{11}-\frac{4\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{99}-\frac{8\left(\cos^7(dx+c)\right)}{693}\right)+a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c)),x)

[Out] 1/d*(a*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/693*cos(d*x+c)^7)+a*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c))

maxima [A] time = 0.38, size = 86, normalized size = 0.52

$$-\frac{10240\left(63\cos\left(dx+c\right)^{11}-154\cos\left(dx+c\right)^{9}+99\cos\left(dx+c\right)^{7}\right)a-693\left(32\sin\left(2\,dx+2\,c\right)^{5}+120\,dx+120\,c\right.}{7096320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/7096320*(10240*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c)^7)*a - 693*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a)/d

mupad [B] time = 11.87, size = 447, normalized size = 2.71

$$\frac{3\,a\,x}{256}+\frac{\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{21}}{128}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}}{4}-\frac{3323\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{640}+\left(\frac{a\,(1715175\,c+1715175\,d\,x-9461760)}{887040}-\frac{495\,a\,(c+d\,x)}{256}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6*sin(c + d*x)^4*(a + a*sin(c + d*x)),x)

[Out] (3*a*x)/256 + ((a*(10395*c + 10395*d*x - 20480))/887040 - (3*a*tan(c/2 + (d*x)/2))/128 - (3*a*(c + d*x))/256 + tan(c/2 + (d*x)/2)^2*((a*(114345*c + 114345*d*x - 225280))/887040 - (33*a*(c + d*x))/256) + tan(c/2 + (d*x)/2)^4*((a*(571725*c + 571725*d*x - 1126400))/887040 - (165*a*(c + d*x))/256) + tan(c/2 + (d*x)/2)^6*((a*(1715175*c + 1715175*d*x + 6082560))/887040 - (495*a*(c + d*x))/256) + tan(c/2 + (d*x)/2)^16*((a*(1715175*c + 1715175*d*x - 9461760))/887040 - (495*a*(c + d*x))/256) + tan(c/2 + (d*x)/2)^14*((a*(3430350*c + 3430350*d*x + 23654400))/887040 - (495*a*(c + d*x))/128) + tan(c/2 + (d*x)/2)^8*((a*(3430350*c + 3430350*d*x - 30412800))/887040 - (495*a*(c + d*x))/128) + tan(c/2 + (d*x)/2)^10*((a*(4802490*c + 4802490*d*x + 42577920))/887040 - (693*a*(c + d*x))/128) + tan(c/2 + (d*x)/2)^12*((a*(4802490*c + 4802490*d*x - 52039680))/887040 - (693*a*(c + d*x))/128) - (a*tan(c/2 + (d*x)/2)^3)/4 + (3323*a*tan(c/2 + (d*x)/2)^5)/640 - (54*a*tan(c/2 + (d*x)/2)^7)/5

```
 + (841*a*tan(c/2 + (d*x)/2)^9)/64 - (841*a*tan(c/2 + (d*x)/2)^13)/64 + (54
*a*tan(c/2 + (d*x)/2)^15)/5 - (3323*a*tan(c/2 + (d*x)/2)^17)/640 + (a*tan(c
/2 + (d*x)/2)^19)/4 + (3*a*tan(c/2 + (d*x)/2)^21)/128)/(d*(tan(c/2 + (d*x)/
2)^2 + 1)^11)
```

sympy [A] time = 44.94, size = 318, normalized size = 1.93

$$\begin{cases} \frac{3ax\sin^{10}(c+dx)}{256} + \frac{15ax\sin^{8}(c+dx)\cos^{2}(c+dx)}{256} + \frac{15ax\sin^{6}(c+dx)\cos^{4}(c+dx)}{128} + \frac{15ax\sin^{4}(c+dx)\cos^{6}(c+dx)}{128} + \frac{15ax\sin^{2}(c+dx)\cos^{8}(c+d}{256} \\ x\left(a\sin(c)+a\right)\sin^{4}(c)\cos^{6}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**10/256 + 15*a*x*sin(c + d*x)**8*cos(c + d*x)
**2/256 + 15*a*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 15*a*x*sin(c + d*x)*
*4*cos(c + d*x)**6/128 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 3*a*x
*cos(c + d*x)**10/256 + 3*a*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a*sin(
c + d*x)**7*cos(c + d*x)**3/(128*d) + a*sin(c + d*x)**5*cos(c + d*x)**5/(10
*d) - a*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 7*a*sin(c + d*x)**3*cos(c +
 d*x)**7/(128*d) - 4*a*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) - 3*a*sin(c +
 d*x)*cos(c + d*x)**9/(256*d) - 8*a*cos(c + d*x)**11/(693*d), Ne(d, 0)), (x
*(a*sin(c) + a)*sin(c)**4*cos(c)**6, True))
```

3.573 $\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=149

$$\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{3a\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{160d}$$

```
[Out] 3/256*a*x-1/7*a*cos(d*x+c)^7/d+1/9*a*cos(d*x+c)^9/d+3/256*a*cos(d*x+c)*sin(
d*x+c)/d+1/128*a*cos(d*x+c)^3*sin(d*x+c)/d+1/160*a*cos(d*x+c)^5*sin(d*x+c)/
d-3/80*a*cos(d*x+c)^7*sin(d*x+c)/d-1/10*a*cos(d*x+c)^7*sin(d*x+c)^3/d
```

Rubi [A] time = 0.21, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2565, 14, 2568, 2635, 8}

$$\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{3a\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{160d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/256 - (a*Cos[c + d*x]^7)/(7*d) + (a*Cos[c + d*x]^9)/(9*d) + (3*a*Co
s[c + d*x]*Sin[c + d*x])/(256*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(128*d)
+ (a*Cos[c + d*x]^5*Sin[c + d*x])/(160*d) - (3*a*Cos[c + d*x]^7*Sin[c + d*x
])/(80*d) - (a*Cos[c + d*x]^7*Sin[c + d*x]^3)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^6(c+dx)\sin^3(c+dx)\,dx + a\int \cos^6(c+dx)\sin^4(c$$

$$= -\frac{a\cos^7(c+dx)\sin^3(c+dx)}{10d} + \frac{1}{10}(3a)\int \cos^6(c+dx)\sin^2$$

$$= -\frac{3a\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{a\cos^7(c+dx)\sin^3(c+dx)}{10d}$$

$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{a\cos^5(c+dx)\sin(c+}{160d}$$

$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{a\cos^3(c+dx)\sin(c+}{128d}$$

$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{3a\cos(c+dx)\sin(c+}{256d}$$

$$= \frac{3ax}{256} - \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{3a\cos(c+dx)\sin}{256d}$$

Mathematica [A] time = 0.35, size = 101, normalized size = 0.68

$$\frac{a(1260\sin(2(c+dx)) - 2520\sin(4(c+dx)) - 630\sin(6(c+dx)) + 315\sin(8(c+dx)) + 126\sin(10(c+dx)) - 1}{645120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(7560*d*x - 15120*Cos[c + d*x] - 6720*Cos[3*(c + d*x)] + 1080*Cos[7*(c +
 d*x)] + 280*Cos[9*(c + d*x)] + 1260*Sin[2*(c + d*x)] - 2520*Sin[4*(c + d*x
)] - 630*Sin[6*(c + d*x)] + 315*Sin[8*(c + d*x)] + 126*Sin[10*(c + d*x)]))/
(645120*d)
```

fricas [A] time = 0.78, size = 95, normalized size = 0.64

$$\frac{8960\,a\cos\left(dx+c\right)^{9}-11520\,a\cos\left(dx+c\right)^{7}+945\,adx+63\left(128\,a\cos\left(dx+c\right)^{9}-176\,a\cos\left(dx+c\right)^{7}+8\,a\cos\left(d\right.\right.}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/80640*(8960*a*cos(d*x + c)^9 - 11520*a*cos(d*x + c)^7 + 945*a*d*x + 63*(1
28*a*cos(d*x + c)^9 - 176*a*cos(d*x + c)^7 + 8*a*cos(d*x + c)^5 + 10*a*cos(
d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.26, size = 137, normalized size = 0.92

$$\frac{3}{256}\,ax+\frac{a\cos\left(9\,dx+9\,c\right)}{2304\,d}+\frac{3\,a\cos\left(7\,dx+7\,c\right)}{1792\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{96\,d}-\frac{3\,a\cos\left(dx+c\right)}{128\,d}+\frac{a\sin\left(10\,dx+10\,c\right)}{5120\,d}+\frac{a\sin\left(8\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/256*a*x + 1/2304*a*cos(9*d*x + 9*c)/d + 3/1792*a*cos(7*d*x + 7*c)/d - 1/9
6*a*cos(3*d*x + 3*c)/d - 3/128*a*cos(d*x + c)/d + 1/5120*a*sin(10*d*x + 10*
c)/d + 1/2048*a*sin(8*d*x + 8*c)/d - 1/1024*a*sin(6*d*x + 6*c)/d - 1/256*a*
sin(4*d*x + 4*c)/d + 1/512*a*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.25, size = 116, normalized size = 0.78

$$\frac{a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{7}(dx+c)\right)}{10}-\frac{3\left(\cos^{7}(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{160}+\frac{3dx}{256}+\frac{3c}{256}\right)+a\left(-\frac{\left(\sin^{2}(d\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/160*
(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*
c)+a*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7))
```

maxima [A] time = 0.42, size = 76, normalized size = 0.51

$$\frac{10240\left(7\cos\left(dx+c\right)^{9}-9\cos\left(dx+c\right)^{7}\right)a+63\left(32\sin\left(2\,dx+2\,c\right)^{5}+120\,dx+120\,c+5\sin\left(8\,dx+8\,c\right)-40\right.}{645120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/645120*(10240*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a + 63*(32*sin(2*d*x
+ 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a)/d
```

mupad [B] time = 12.19, size = 414, normalized size = 2.78

$$\frac{3\,a\,x}{256}+\frac{\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}}{128}+\frac{29\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{128}+\left(\frac{a\,(42525\,c+42525\,d\,x-322560)}{80640}-\frac{135\,a\,(c+d\,x)}{256}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-\frac{867\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{160}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/256 + ((a*(945*c + 945*d*x - 5120))/80640 - (3*a*tan(c/2 + (d*x)/2)
)/128 - (3*a*(c + d*x))/256 + tan(c/2 + (d*x)/2)^2*((a*(9450*c + 9450*d*x -
 51200))/80640 - (15*a*(c + d*x))/128) + tan(c/2 + (d*x)/2)^4*((a*(42525*c
+ 42525*d*x + 92160))/80640 - (135*a*(c + d*x))/256) + tan(c/2 + (d*x)/2)^1
6*((a*(42525*c + 42525*d*x - 322560))/80640 - (135*a*(c + d*x))/256) + tan(
c/2 + (d*x)/2)^14*((a*(113400*c + 113400*d*x + 215040))/80640 - (45*a*(c +
d*x))/32) + tan(c/2 + (d*x)/2)^6*((a*(113400*c + 113400*d*x - 829440))/8064
0 - (45*a*(c + d*x))/32) + tan(c/2 + (d*x)/2)^10*((a*(238140*c + 238140*d*x
 - 645120))/80640 - (189*a*(c + d*x))/64) + tan(c/2 + (d*x)/2)^12*((a*(1984
50*c + 198450*d*x - 1075200))/80640 - (315*a*(c + d*x))/128) - (29*a*tan(c/
2 + (d*x)/2)^3)/128 + (867*a*tan(c/2 + (d*x)/2)^5)/160 - (519*a*tan(c/2 + (
d*x)/2)^7)/32 + (1879*a*tan(c/2 + (d*x)/2)^9)/64 - (1879*a*tan(c/2 + (d*x)/
2)^11)/64 + (519*a*tan(c/2 + (d*x)/2)^13)/32 - (867*a*tan(c/2 + (d*x)/2)^15
)/160 + (29*a*tan(c/2 + (d*x)/2)^17)/128 + (3*a*tan(c/2 + (d*x)/2)^19)/128)
/(d*(tan(c/2 + (d*x)/2)^2 + 1)^10)
```

sympy [A] time = 29.35, size = 294, normalized size = 1.97

$$\begin{cases} \frac{3ax\sin^{10}(c+dx)}{256} + \frac{15ax\sin^{8}(c+dx)\cos^{2}(c+dx)}{256} + \frac{15ax\sin^{6}(c+dx)\cos^{4}(c+dx)}{128} + \frac{15ax\sin^{4}(c+dx)\cos^{6}(c+dx)}{128} + \frac{15ax\sin^{2}(c+dx)\cos^{8}(c+dx)}{256} \\ x(a\sin(c)+a)\sin^{3}(c)\cos^{6}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**10/256 + 15*a*x*sin(c + d*x)**8*cos(c + d*x)
**2/256 + 15*a*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 15*a*x*sin(c + d*x)*
*4*cos(c + d*x)**6/128 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 3*a*x
*cos(c + d*x)**10/256 + 3*a*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a*sin(
c + d*x)**7*cos(c + d*x)**3/(128*d) + a*sin(c + d*x)**5*cos(c + d*x)**5/(10
*d) - 7*a*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) - a*sin(c + d*x)**2*cos(c
 + d*x)**7/(7*d) - 3*a*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 2*a*cos(c + d
*x)**9/(63*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**3*cos(c)**6, True))
```

3.574 $\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=125

$$\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a\sin(c+dx)\cos^3(c+$$

```
[Out] 5/128*a*x-1/7*a*cos(d*x+c)^7/d+1/9*a*cos(d*x+c)^9/d+5/128*a*cos(d*x+c)*sin(
d*x+c)/d+5/192*a*cos(d*x+c)^3*sin(d*x+c)/d+1/48*a*cos(d*x+c)^5*sin(d*x+c)/d
-1/8*a*cos(d*x+c)^7*sin(d*x+c)/d
```

Rubi [A] time = 0.15, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 14}

$$\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a\sin(c+dx)\cos^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/128 - (a*Cos[c + d*x]^7)/(7*d) + (a*Cos[c + d*x]^9)/(9*d) + (5*a*Co
s[c + d*x]*Sin[c + d*x])/(128*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(192*d
) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (a*Cos[c + d*x]^7*Sin[c + d*x]
)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^6(c+dx)\sin^2(c+dx)\,dx + a\int \cos^6(c+dx)\sin^3(c+\\
&= -\frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}a\int \cos^6(c+dx)\,dx - \frac{a\,\mathrm{Sub}}{}\\
&= \frac{a\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{48}\\
&= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{5a\cos^3(c+dx)\sin(c+}{192d}\\
&= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{5a\cos(c+dx)\sin(c+d}{128d}\\
&= \frac{5ax}{128} - \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{5a\cos(c+dx)\sin(}{128d}\end{aligned}$$

Mathematica [A] time = 0.32, size = 91, normalized size = 0.73

$$\frac{a(1008\sin(2(c+dx)) - 504\sin(4(c+dx)) - 336\sin(6(c+dx)) - 63\sin(8(c+dx)) - 1512\cos(c+dx) - 672\cos(}{64512d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(2520*d*x - 1512*Cos[c + d*x] - 672*Cos[3*(c + d*x)] + 108*Cos[7*(c + d*
x)] + 28*Cos[9*(c + d*x)] + 1008*Sin[2*(c + d*x)] - 504*Sin[4*(c + d*x)] -
336*Sin[6*(c + d*x)] - 63*Sin[8*(c + d*x)]))/(64512*d)
```

fricas [A] time = 0.76, size = 84, normalized size = 0.67

$$\frac{896\,a\cos\left(dx+c\right)^{9}-1152\,a\cos\left(dx+c\right)^{7}+315\,adx-21\left(48\,a\cos\left(dx+c\right)^{7}-8\,a\cos\left(dx+c\right)^{5}-10\,a\cos\left(dx+\right.\right.}{8064\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8064*(896*a*cos(d*x + c)^9 - 1152*a*cos(d*x + c)^7 + 315*a*d*x - 21*(48*a
*cos(d*x + c)^7 - 8*a*cos(d*x + c)^5 - 10*a*cos(d*x + c)^3 - 15*a*cos(d*x +
 c))*sin(d*x + c))/d
```

giac [A] time = 0.26, size = 122, normalized size = 0.98

$$\frac{5}{128}\,ax+\frac{a\cos\left(9\,dx+9\,c\right)}{2304\,d}+\frac{3\,a\cos\left(7\,dx+7\,c\right)}{1792\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{96\,d}-\frac{3\,a\cos\left(dx+c\right)}{128\,d}-\frac{a\sin\left(8\,dx+8\,c\right)}{1024\,d}-\frac{a\sin\left(6\,c\right.}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/128*a*x + 1/2304*a*cos(9*d*x + 9*c)/d + 3/1792*a*cos(7*d*x + 7*c)/d - 1/9
6*a*cos(3*d*x + 3*c)/d - 3/128*a*cos(d*x + c)/d - 1/1024*a*sin(8*d*x + 8*c)
/d - 1/192*a*sin(6*d*x + 6*c)/d - 1/128*a*sin(4*d*x + 4*c)/d + 1/64*a*sin(2
*d*x + 2*c)/d
```

maple [A] time = 0.24, size = 98, normalized size = 0.78

$$\frac{a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{7}(dx+c)\right)}{9}-\frac{2\left(\cos^{7}(dx+c)\right)}{63}\right)+a\left(-\frac{\left(\cos^{7}(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+a*(-1/8*cos(d*x+c
)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x
+c)+5/128*d*x+5/128*c))
```

maxima [A] time = 0.50, size = 76, normalized size = 0.61

$$\frac{1024\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a+21\left(64\sin(2dx+2c)^3+120dx+120c-3\sin(8dx+8c)-24\sin\right.}{64512d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/64512*(1024*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a + 21*(64*sin(2*d*x +
2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a)/d
```

mupad [B] time = 12.36, size = 386, normalized size = 3.09

$$\frac{5ax}{128}+\frac{\frac{5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{64}-\frac{191a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{96}+\left(\frac{a(11340c+11340dx-32256)}{8064}-\frac{45a(c+dx)}{32}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+\frac{83a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{32}+\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] (5*a*x)/128 + ((a*(315*c + 315*d*x - 512))/8064 - (5*a*tan(c/2 + (d*x)/2))/
64 - (5*a*(c + d*x))/128 + tan(c/2 + (d*x)/2)^2*((a*(2835*c + 2835*d*x - 46
08))/8064 - (45*a*(c + d*x))/128) + tan(c/2 + (d*x)/2)^4*((a*(11340*c + 113
40*d*x + 13824))/8064 - (45*a*(c + d*x))/32) + tan(c/2 + (d*x)/2)^14*((a*(1
1340*c + 11340*d*x - 32256))/8064 - (45*a*(c + d*x))/32) + tan(c/2 + (d*x)/
2)^12*((a*(26460*c + 26460*d*x + 53760))/8064 - (105*a*(c + d*x))/32) + tan
(c/2 + (d*x)/2)^6*((a*(26460*c + 26460*d*x - 96768))/8064 - (105*a*(c + d*x
))/32) + tan(c/2 + (d*x)/2)^8*((a*(39690*c + 39690*d*x + 96768))/8064 - (31
5*a*(c + d*x))/64) + tan(c/2 + (d*x)/2)^10*((a*(39690*c + 39690*d*x - 16128
0))/8064 - (315*a*(c + d*x))/64) + (191*a*tan(c/2 + (d*x)/2)^3)/96 - (83*a*
tan(c/2 + (d*x)/2)^5)/32 + (145*a*tan(c/2 + (d*x)/2)^7)/32 - (145*a*tan(c/2
 + (d*x)/2)^11)/32 + (83*a*tan(c/2 + (d*x)/2)^13)/32 - (191*a*tan(c/2 + (d*
x)/2)^15)/96 + (5*a*tan(c/2 + (d*x)/2)^17)/64)/(d*(tan(c/2 + (d*x)/2)^2 + 1
)^9)
```

sympy [A] time = 18.97, size = 248, normalized size = 1.98

$$\begin{cases}\frac{5ax\sin^8(c+dx)}{128}+\frac{5ax\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{15ax\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{5ax\sin^2(c+dx)\cos^6(c+dx)}{32}+\frac{5ax\cos^8(c+dx)}{128}+\frac{5a\sin^7(c+d}{12} \\ x(a\sin(c)+a)\sin^2(c)\cos^6(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**8/128 + 5*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 5*a*x*sin(c + d*x)**2*co
s(c + d*x)**6/32 + 5*a*x*cos(c + d*x)**8/128 + 5*a*sin(c + d*x)**7*cos(c +
d*x)/(128*d) + 55*a*sin(c + d*x)**5*cos(c + d*x)**3/(384*d) + 73*a*sin(c +
d*x)**3*cos(c + d*x)**5/(384*d) - a*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) -
 5*a*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 2*a*cos(c + d*x)**9/(63*d), Ne(
d, 0)), (x*(a*sin(c) + a)*sin(c)**2*cos(c)**6, True))
```

3.575 $\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=109

$$-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5a\sin(c+$$

```
[Out] 5/128*a*x-1/7*a*cos(d*x+c)^7/d+5/128*a*cos(d*x+c)*sin(d*x+c)/d+5/192*a*cos(
d*x+c)^3*sin(d*x+c)/d+1/48*a*cos(d*x+c)^5*sin(d*x+c)/d-1/8*a*cos(d*x+c)^7*s
in(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2838, 2565, 30, 2568, 2635, 8}

$$-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/128 - (a*Cos[c + d*x]^7)/(7*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/(1
28*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (a*Cos[c + d*x]^5*Sin[c
 + d*x])/(48*d) - (a*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
```

```
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^6(c+dx)\sin(c+dx)\,dx + a\int \cos^6(c+dx)\sin^2(c+$$

$$= -\frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}a\int \cos^6(c+dx)\,dx - \frac{a\,\mathrm{Sub}}{}$$

$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{a\cos^7(c+d}{8}$$

$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{a\cos^5(c+}{}$$

$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{5a\cos(c+dx)\sin(c+dx)}{128d} + \frac{5a\cos^3(c+}{1}$$

$$= \frac{5ax}{128} - \frac{a\cos^7(c+dx)}{7d} + \frac{5a\cos(c+dx)\sin(c+dx)}{128d} + \frac{5a\cos}{}$$

Mathematica [A] time = 0.26, size = 91, normalized size = 0.83

$$-\frac{a(-336\sin(2(c+dx))+168\sin(4(c+dx))+112\sin(6(c+dx))+21\sin(8(c+dx))+1680\cos(c+dx)+1008}{21504d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

[Out] -1/21504*(a*(-840*d*x + 1680*Cos[c + d*x] + 1008*Cos[3*(c + d*x)] + 336*Cos
[5*(c + d*x)] + 48*Cos[7*(c + d*x)] - 336*Sin[2*(c + d*x)] + 168*Sin[4*(c +
 d*x)] + 112*Sin[6*(c + d*x)] + 21*Sin[8*(c + d*x)]))/d

fricas [A] time = 0.74, size = 73, normalized size = 0.67

$$-\frac{384\,a\cos\left(dx+c\right)^{7}-105\,adx+7\left(48\,a\cos\left(dx+c\right)^{7}-8\,a\cos\left(dx+c\right)^{5}-10\,a\cos\left(dx+c\right)^{3}-15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/2688*(384*a*cos(d*x + c)^7 - 105*a*d*x + 7*(48*a*cos(d*x + c)^7 - 8*a*co
s(d*x + c)^5 - 10*a*cos(d*x + c)^3 - 15*a*cos(d*x + c))*sin(d*x + c))/d

giac [A] time = 0.19, size = 122, normalized size = 1.12

$$\frac{5}{128}\,ax-\frac{a\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{a\cos\left(5\,dx+5\,c\right)}{64\,d}-\frac{3\,a\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{5\,a\cos\left(dx+c\right)}{64\,d}-\frac{a\sin\left(8\,dx+8\,c\right)}{1024\,d}-\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 5/128*a*x - 1/448*a*cos(7*d*x + 7*c)/d - 1/64*a*cos(5*d*x + 5*c)/d - 3/64*a
*cos(3*d*x + 3*c)/d - 5/64*a*cos(d*x + c)/d - 1/1024*a*sin(8*d*x + 8*c)/d -
 1/192*a*sin(6*d*x + 6*c)/d - 1/128*a*sin(4*d*x + 4*c)/d + 1/64*a*sin(2*d*x
 + 2*c)/d

maple [A] time = 0.25, size = 78, normalized size = 0.72

$$\frac{a\left(-\frac{\left(\cos^{7}\left(dx+c\right)\right)\sin\left(dx+c\right)}{8}+\frac{\left(\cos^{5}\left(dx+c\right)+\frac{5\left(\cos^{3}\left(dx+c\right)\right)}{4}+\frac{15\cos\left(dx+c\right)}{8}\right)\sin\left(dx+c\right)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{a\left(\cos^{7}\left(dx+c\right)\right)}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c)),x)

[Out] 1/d*(a*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15
/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-1/7*a*cos(d*x+c)^7)

maxima [A] time = 0.35, size = 63, normalized size = 0.58

$$-\frac{3072\,a\cos\left(dx+c\right)^{7}-7\left(64\,\sin\left(2\,dx+2\,c\right)^{3}+120\,dx+120\,c-3\,\sin\left(8\,dx+8\,c\right)-24\,\sin\left(4\,dx+4\,c\right)\right)a}{21504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/21504*(3072*a*cos(d*x + c)^7 - 7*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*
c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a)/d
```

mupad [B] time = 9.39, size = 96, normalized size = 0.88

$$-\frac{a\left(210\,\cos\left(c+d\,x\right)+126\,\cos\left(3\,c+3\,d\,x\right)+42\,\cos\left(5\,c+5\,d\,x\right)+6\,\cos\left(7\,c+7\,d\,x\right)-42\,\sin\left(2\,c+2\,d\,x\right)+2\right.}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] -(a*(210*cos(c + d*x) + 126*cos(3*c + 3*d*x) + 42*cos(5*c + 5*d*x) + 6*cos(
7*c + 7*d*x) - 42*sin(2*c + 2*d*x) + 21*sin(4*c + 4*d*x) + 14*sin(6*c + 6*d
*x) + (21*sin(8*c + 8*d*x))/8 - 105*d*x))/(2688*d)
```

sympy [A] time = 11.23, size = 223, normalized size = 2.05

$$\begin{cases} \frac{5ax\sin^8(c+dx)}{128} + \frac{5ax\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{15ax\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{5ax\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{5ax\cos^8(c+dx)}{128} + \frac{5a\sin^7(c+ \\ x\left(a\sin(c)+a\right)\sin(c)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**8/128 + 5*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 5*a*x*sin(c + d*x)**2*co
s(c + d*x)**6/32 + 5*a*x*cos(c + d*x)**8/128 + 5*a*sin(c + d*x)**7*cos(c +
d*x)/(128*d) + 55*a*sin(c + d*x)**5*cos(c + d*x)**3/(384*d) + 73*a*sin(c +
d*x)**3*cos(c + d*x)**5/(384*d) - 5*a*sin(c + d*x)*cos(c + d*x)**7/(128*d)
- a*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*cos(c)**6, T
rue))
```

3.576 $\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=127

$$\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c$$

```
[Out] 5/16*a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d+1/5*a*
cos(d*x+c)^5/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d*x+c)^3*sin(d*x+c
)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2838, 2592, 302, 206, 2635, 8}

$$\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/16 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c +
d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/
(16*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c
 + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^6(c+dx)\,dx + a\int \cos^5(c+dx)\cot(c+dx)\,dx$$

$$= \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(5a)\int \cos^4(c+dx)\,dx - \frac{a\,\mathrm{Su}}{}$$

$$= \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{8}$$

$$= \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{5a\cos(c}{}$$

$$= \frac{5ax}{16} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+}{3d}$$

Mathematica [A] time = 0.12, size = 100, normalized size = 0.79

$$\frac{a\Big(225\sin(2(c+dx)) + 45\sin(4(c+dx)) + 5\sin(6(c+dx)) + 1320\cos(c+dx) + 140\cos(3(c+dx)) + 12\cos(5}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(300*c + 300*d*x + 1320*Cos[c + d*x] + 140*Cos[3*(c + d*x)] + 12*Cos[5*(
c + d*x)] - 960*Log[Cos[(c + d*x)/2]] + 960*Log[Sin[(c + d*x)/2]] + 225*Sin
[2*(c + d*x)] + 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(960*d)
```

fricas [A] time = 0.80, size = 110, normalized size = 0.87

$$\frac{48\,a\cos\left(dx+c\right)^{5}+80\,a\cos\left(dx+c\right)^{3}+75\,adx+240\,a\cos\left(dx+c\right)-120\,a\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+120\,a\log\left(\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*a*cos(d*x + c)^5 + 80*a*cos(d*x + c)^3 + 75*a*d*x + 240*a*cos(d*x
 + c) - 120*a*log(1/2*cos(d*x + c) + 1/2) + 120*a*log(-1/2*cos(d*x + c) + 1
/2) + 5*(8*a*cos(d*x + c)^5 + 10*a*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(
d*x + c))/d
```

giac [A] time = 0.19, size = 201, normalized size = 1.58

$$\frac{75\,(dx+c)a+240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(165\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-720\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-2160\,a\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(75*(d*x + c)*a + 240*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(165*a*tan
(1/2*d*x + 1/2*c)^11 - 720*a*tan(1/2*d*x + 1/2*c)^10 - 25*a*tan(1/2*d*x + 1
/2*c)^9 - 2160*a*tan(1/2*d*x + 1/2*c)^8 + 450*a*tan(1/2*d*x + 1/2*c)^7 - 36
80*a*tan(1/2*d*x + 1/2*c)^6 - 450*a*tan(1/2*d*x + 1/2*c)^5 - 3360*a*tan(1/2
*d*x + 1/2*c)^4 + 25*a*tan(1/2*d*x + 1/2*c)^3 - 1488*a*tan(1/2*d*x + 1/2*c)
^2 - 165*a*tan(1/2*d*x + 1/2*c) - 368*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 0.38, size = 131, normalized size = 1.03

$$\frac{a\left(\cos^{5}\left(dx+c\right)\right)\sin\left(dx+c\right)}{6d}+\frac{5a\left(\cos^{3}\left(dx+c\right)\right)\sin\left(dx+c\right)}{24d}+\frac{5a\cos\left(dx+c\right)\sin\left(dx+c\right)}{16d}+\frac{5ax}{16}+\frac{5ca}{16d}+\frac{a\left(\cos^{5}\left(d\right.\right.}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/6*a*cos(d*x+c)^5*sin(d*x+c)/d+5/24*a*cos(d*x+c)^3*sin(d*x+c)/d+5/16*a*cos
(d*x+c)*sin(d*x+c)/d+5/16*a*x+5/16/d*c*a+1/5*a*cos(d*x+c)^5/d+1/3*a*cos(d*x
+c)^3/d+a*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.39, size = 106, normalized size = 0.83

$$\frac{32\left(6\cos\left(dx+c\right)^{5}+10\cos\left(dx+c\right)^{3}+30\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*(32*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(
cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a - 5*(4*sin(2*d*x + 2*c)^3 -
 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

mupad [B] time = 10.71, size = 327, normalized size = 2.57

$$\frac{-\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+18\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{92\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}+\frac{15\,a}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+20\tan\left(\frac{c}{2}+\frac{d}{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] ((46*a)/15 + (11*a*tan(c/2 + (d*x)/2))/8 + (62*a*tan(c/2 + (d*x)/2)^2)/5 -
(5*a*tan(c/2 + (d*x)/2)^3)/24 + 28*a*tan(c/2 + (d*x)/2)^4 + (15*a*tan(c/2 +
 (d*x)/2)^5)/4 + (92*a*tan(c/2 + (d*x)/2)^6)/3 - (15*a*tan(c/2 + (d*x)/2)^7
)/4 + 18*a*tan(c/2 + (d*x)/2)^8 + (5*a*tan(c/2 + (d*x)/2)^9)/24 + 6*a*tan(c
/2 + (d*x)/2)^10 - (11*a*tan(c/2 + (d*x)/2)^11)/8)/(d*(6*tan(c/2 + (d*x)/2)
^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)
/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (a*log(tan(
c/2 + (d*x)/2)))/d + (5*a*atan((25*a^2)/(64*((5*a^2)/4 - (25*a^2*tan(c/2 +
(d*x)/2))/64)) + (5*a^2*tan(c/2 + (d*x)/2))/(4*((5*a^2)/4 - (25*a^2*tan(c/2
 + (d*x)/2))/64))))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.577 $\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=121

$$\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{15a\cot(c+dx)}{8d}+\frac{a\cos^4(c+dx)\cot(c+dx)}{4d}+\frac{5a\cos^2(c+dx)\text{co}}{8d}$$

```
[Out] -15/8*a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d+1/5*a
*cos(d*x+c)^5/d-15/8*a*cot(d*x+c)/d+5/8*a*cos(d*x+c)^2*cot(d*x+c)/d+1/4*a*c
os(d*x+c)^4*cot(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2838, 2591, 288, 321, 203, 2592, 302, 206}

$$\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{15a\cot(c+dx)}{8d}+\frac{a\cos^4(c+dx)\cot(c+dx)}{4d}+\frac{5a\cos^2(c+dx)\text{co}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-15*a*x)/8 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c +
 d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) - (15*a*Cot[c + d*x])/(8*d) + (5*
a*Cos[c + d*x]^2*Cot[c + d*x])/(8*d) + (a*Cos[c + d*x]^4*Cot[c + d*x])/(4*d
)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^5(c+dx)\cot(c+dx)\,dx + a\int \cos^4(c+dx)\cot^2(c+$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^3}\,d\right.}{d}$$

$$= \frac{a\cos^4(c+dx)\cot(c+dx)}{4d} - \frac{a\,\text{Subst}\left(\int\left(-1-x^2-x^4+\frac{1}{1-}\right.\right.}{d}$$

$$= \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{5a\cos^2}{}$$

$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} +$$

$$= -\frac{15ax}{8} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c}{3d}$$

Mathematica [A] time = 0.27, size = 98, normalized size = 0.81

$$-\frac{a\left(240\sin(2(c+dx)) + 15\sin(4(c+dx)) - 660\cos(c+dx) - 70\cos(3(c+dx)) - 6\cos(5(c+dx)) + 480\cot(c\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/480*(a*(900*c + 900*d*x - 660*Cos[c + d*x] - 70*Cos[3*(c + d*x)] - 6*Cos
[5*(c + d*x)] + 480*Cot[c + d*x] + 480*Log[Cos[(c + d*x)/2]] - 480*Log[Sin[
(c + d*x)/2]] + 240*Sin[2*(c + d*x)] + 15*Sin[4*(c + d*x)]))/d
```

fricas [A] time = 0.83, size = 129, normalized size = 1.07

$$\frac{30\,a\cos\left(dx+c\right)^5 + 75\,a\cos\left(dx+c\right)^3 - 60\,a\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right) + 60\,a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\right.}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(30*a*cos(d*x + c)^5 + 75*a*cos(d*x + c)^3 - 60*a*log(1/2*cos(d*x + c
) + 1/2)*sin(d*x + c) + 60*a*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 22
```

```
5*a*cos(d*x + c) + (24*a*cos(d*x + c)^5 + 40*a*cos(d*x + c)^3 - 225*a*d*x +
 120*a*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.20, size = 198, normalized size = 1.64

$$-\frac{225\,(dx+c)a-120\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-60\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{60\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(135\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(225*(d*x + c)*a - 120*a*log(abs(tan(1/2*d*x + 1/2*c))) - 60*a*tan(1
/2*d*x + 1/2*c) + 60*(2*a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c) -
2*(135*a*tan(1/2*d*x + 1/2*c)^9 + 360*a*tan(1/2*d*x + 1/2*c)^8 + 150*a*tan(
1/2*d*x + 1/2*c)^7 + 720*a*tan(1/2*d*x + 1/2*c)^6 + 1120*a*tan(1/2*d*x + 1/
2*c)^4 - 150*a*tan(1/2*d*x + 1/2*c)^3 + 560*a*tan(1/2*d*x + 1/2*c)^2 - 135*
a*tan(1/2*d*x + 1/2*c) + 184*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

maple [A] time = 0.29, size = 153, normalized size = 1.26

$$\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{5d}+\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{3d}+\frac{a\cos\left(dx+c\right)}{d}+\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}-\frac{a\left(\cos^{7}\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}-\frac{a\left(\cos^{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/5*a*cos(d*x+c)^5/d+1/3*a*cos(d*x+c)^3/d+a*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c
)-cot(d*x+c))-1/d*a/sin(d*x+c)*cos(d*x+c)^7-a*cos(d*x+c)^5*sin(d*x+c)/d-5/4
*a*cos(d*x+c)^3*sin(d*x+c)/d-15/8*a*cos(d*x+c)*sin(d*x+c)/d-15/8*a*x-15/8/d
*c*a
```

maxima [A] time = 0.47, size = 121, normalized size = 1.00

$$\frac{4\left(6\,\cos\left(dx+c\right)^{5}+10\,\cos\left(dx+c\right)^{3}+30\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)a-}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/120*(4*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(c
os(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a - 15*(15*d*x + 15*c + (15*ta
```

```
n(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 +
tan(d*x + c)))*a)/d
```

mupad [B] time = 8.92, size = 313, normalized size = 2.59

$$\frac{\frac{7\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{2}+12\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+24\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-10\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{112\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-15\,a\,\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+10\,\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] ((92*a*tan(c/2 + (d*x)/2))/15 - a - (19*a*tan(c/2 + (d*x)/2)^2)/2 + (56*a*t
an(c/2 + (d*x)/2)^3)/3 - 15*a*tan(c/2 + (d*x)/2)^4 + (112*a*tan(c/2 + (d*x)
/2)^5)/3 - 10*a*tan(c/2 + (d*x)/2)^6 + 24*a*tan(c/2 + (d*x)/2)^7 + 12*a*tan
(c/2 + (d*x)/2)^9 + (7*a*tan(c/2 + (d*x)/2)^10)/2)/(d*(2*tan(c/2 + (d*x)/2)
 + 10*tan(c/2 + (d*x)/2)^3 + 20*tan(c/2 + (d*x)/2)^5 + 20*tan(c/2 + (d*x)/2
)^7 + 10*tan(c/2 + (d*x)/2)^9 + 2*tan(c/2 + (d*x)/2)^11)) + (a*tan(c/2 + (d
*x)/2))/(2*d) + (a*log(tan(c/2 + (d*x)/2)))/d + (15*a*atan((225*a^2)/(16*((
15*a^2)/2 + (225*a^2*tan(c/2 + (d*x)/2))/16)) - (15*a^2*tan(c/2 + (d*x)/2))
/(2*((15*a^2)/2 + (225*a^2*tan(c/2 + (d*x)/2))/16))))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.578 $\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$-\frac{5a\cos^3(c+dx)}{6d}-\frac{5a\cos(c+dx)}{2d}-\frac{15a\cot(c+dx)}{8d}+\frac{a\cos^4(c+dx)\cot(c+dx)}{4d}-\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d}+\frac{5a\,c}{}$$

```
[Out] -15/8*a*x+5/2*a*arctanh(cos(d*x+c))/d-5/2*a*cos(d*x+c)/d-5/6*a*cos(d*x+c)^3
/d-15/8*a*cot(d*x+c)/d+5/8*a*cos(d*x+c)^2*cot(d*x+c)/d+1/4*a*cos(d*x+c)^4*c
ot(d*x+c)/d-1/2*a*cos(d*x+c)^3*cot(d*x+c)^2/d
```

Rubi [A] time = 0.14, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2838, 2592, 288, 302, 206, 2591, 321, 203}

$$-\frac{5a\cos^3(c+dx)}{6d}-\frac{5a\cos(c+dx)}{2d}-\frac{15a\cot(c+dx)}{8d}-\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d}+\frac{a\cos^4(c+dx)\cot(c+dx)}{4d}+\frac{5a\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-15*a*x)/8 + (5*a*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a*Cos[c + d*x])/(2*d)
- (5*a*Cos[c + d*x]^3)/(6*d) - (15*a*Cot[c + d*x])/(8*d) + (5*a*Cos[c + d*x
]^2*Cot[c + d*x])/(8*d) + (a*Cos[c + d*x]^4*Cot[c + d*x])/(4*d) - (a*Cos[c
+ d*x]^3*Cot[c + d*x]^2)/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\cot^2(c+dx)\,dx + a\int \cos^3(c+dx)\cot^3(c+$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{x^6}{(1+x^2)^3}\,\right.}{d}$$

$$= \frac{a\cos^4(c+dx)\cot(c+dx)}{4d} - \frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d} - \frac{(5}{}$$

$$= \frac{5a\cos^2(c+dx)\cot(c+dx)}{8d} + \frac{a\cos^4(c+dx)\cot(c+dx)}{4d} - \frac{a}{}$$

$$= -\frac{5a\cos(c+dx)}{2d} - \frac{5a\cos^3(c+dx)}{6d} - \frac{15a\cot(c+dx)}{8d} + \frac{5a\,\mathrm{c}}{}$$

$$= -\frac{15ax}{8} + \frac{5a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{5a\cos(c+dx)}{2d} - \frac{5a\cos^3}{6}$$

Mathematica [A] time = 2.75, size = 117, normalized size = 0.87

$$-\frac{a\left(216\cos(c+dx)+8\cos(3(c+dx))+3\left(16\sin(2(c+dx))+\sin(4(c+dx))+32\cot(c+dx)+4\csc^2\left(\frac{1}{2}(c+d\right.\right.\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/96*(a*(216*Cos[c + d*x] + 8*Cos[3*(c + d*x)] + 3*(60*c + 60*d*x + 32*Cot
[c + d*x] + 4*Csc[(c + d*x)/2]^2 - 80*Log[Cos[(c + d*x)/2]] + 80*Log[Sin[(c
 + d*x)/2]] - 4*Sec[(c + d*x)/2]^2 + 16*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]
)))/d
```

fricas [A] time = 0.71, size = 162, normalized size = 1.21

$$-\frac{8\,a\cos(dx+c)^5 + 45\,adx\cos(dx+c)^2 + 40\,a\cos(dx+c)^3 - 45\,adx - 60\,a\cos(dx+c) - 30\left(a\cos(dx+c)^2 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*a*cos(d*x + c)^5 + 45*a*d*x*cos(d*x + c)^2 + 40*a*cos(d*x + c)^3 -
 45*a*d*x - 60*a*cos(d*x + c) - 30*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x +
 c) + 1/2) + 30*(a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 3*(2*
a*cos(d*x + c)^5 + 5*a*cos(d*x + c)^3 - 15*a*cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^2 - d)
```

giac [A] time = 0.20, size = 214, normalized size = 1.60

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-45\,(dx+c)a-60\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(30\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\cdots}{\tan\left(\frac{1}{2}\,dx+\cdots\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(3*a*tan(1/2*d*x + 1/2*c)^2 - 45*(d*x + c)*a - 60*a*log(abs(tan(1/2*d*
x + 1/2*c))) + 12*a*tan(1/2*d*x + 1/2*c) + 3*(30*a*tan(1/2*d*x + 1/2*c)^2 -
 4*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^2 + 2*(27*a*tan(1/2*d*x
 + 1/2*c)^7 - 72*a*tan(1/2*d*x + 1/2*c)^6 + 3*a*tan(1/2*d*x + 1/2*c)^5 - 16
8*a*tan(1/2*d*x + 1/2*c)^4 - 3*a*tan(1/2*d*x + 1/2*c)^3 - 152*a*tan(1/2*d*x
 + 1/2*c)^2 - 27*a*tan(1/2*d*x + 1/2*c) - 56*a)/(tan(1/2*d*x + 1/2*c)^2 + 1
)^4)/d
```

maple [A] time = 0.37, size = 177, normalized size = 1.32

$$-\frac{a\left(\cos^{7}(dx+c)\right)}{d\sin(dx+c)}-\frac{a\left(\cos^{5}(dx+c)\right)\sin(dx+c)}{d}-\frac{5a\left(\cos^{3}(dx+c)\right)\sin(dx+c)}{4d}-\frac{15a\cos(dx+c)\sin(dx+c)}{8d}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^7-a*cos(d*x+c)^5*sin(d*x+c)/d-5/4*a*cos(d*x+c)
^3*sin(d*x+c)/d-15/8*a*cos(d*x+c)*sin(d*x+c)/d-15/8*a*x-15/8/d*c*a-1/2/d*a/
sin(d*x+c)^2*cos(d*x+c)^7-1/2*a*cos(d*x+c)^5/d-5/6*a*cos(d*x+c)^3/d-5/2*a*c
os(d*x+c)/d-5/2/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.57, size = 131, normalized size = 0.98

$$-\frac{2\left(4\,\cos(dx+c)^{3}-\frac{6\,\cos(dx+c)}{\cos(dx+c)^{2}-1}+24\,\cos(dx+c)-15\,\log(\cos(dx+c)+1)+15\,\log(\cos(dx+c)-1)\right)a+3\cdots}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/24*(2*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a + 3*(15*d*
x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*
tan(d*x + c)^3 + tan(d*x + c)))*a)/d
```

mupad [B] time = 8.86, size = 321, normalized size = 2.40

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{5\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{-7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{49\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}+7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) + (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (5*a*log(ta
n(c/2 + (d*x)/2)))/(2*d) - (a/2 + 2*a*tan(c/2 + (d*x)/2) + (62*a*tan(c/2 +
(d*x)/2)^2)/3 + 17*a*tan(c/2 + (d*x)/2)^3 + (161*a*tan(c/2 + (d*x)/2)^4)/3
+ 13*a*tan(c/2 + (d*x)/2)^5 + 58*a*tan(c/2 + (d*x)/2)^6 + 7*a*tan(c/2 + (d*
x)/2)^7 + (49*a*tan(c/2 + (d*x)/2)^8)/2 - 7*a*tan(c/2 + (d*x)/2)^9)/(d*(4*t
an(c/2 + (d*x)/2)^2 + 16*tan(c/2 + (d*x)/2)^4 + 24*tan(c/2 + (d*x)/2)^6 + 1
6*tan(c/2 + (d*x)/2)^8 + 4*tan(c/2 + (d*x)/2)^10)) - (15*a*atan((225*a^2)/(
16*((75*a^2)/4 - (225*a^2*tan(c/2 + (d*x)/2))/16)) + (75*a^2*tan(c/2 + (d*x
)/2))/(4*((75*a^2)/4 - (225*a^2*tan(c/2 + (d*x)/2))/16))))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.579 $\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=130

$$-\frac{5a\cos^3(c+dx)}{6d}-\frac{5a\cos(c+dx)}{2d}-\frac{5a\cot^3(c+dx)}{6d}+\frac{5a\cot(c+dx)}{2d}-\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d}+\frac{a\cos^2(c+dx)}{2d}$$

```
[Out] 5/2*a*x+5/2*a*arctanh(cos(d*x+c))/d-5/2*a*cos(d*x+c)/d-5/6*a*cos(d*x+c)^3/d
+5/2*a*cot(d*x+c)/d-1/2*a*cos(d*x+c)^3*cot(d*x+c)^2/d-5/6*a*cot(d*x+c)^3/d+
1/2*a*cos(d*x+c)^2*cot(d*x+c)^3/d
```

Rubi [A] time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2591, 288, 302, 203, 2592, 206}

$$-\frac{5a\cos^3(c+dx)}{6d}-\frac{5a\cos(c+dx)}{2d}-\frac{5a\cot^3(c+dx)}{6d}+\frac{5a\cot(c+dx)}{2d}-\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d}+\frac{a\cos^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/2 + (5*a*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a*Cos[c + d*x])/(2*d) -
(5*a*Cos[c + d*x]^3)/(6*d) + (5*a*Cot[c + d*x])/(2*d) - (a*Cos[c + d*x]^3*C
ot[c + d*x]^2)/(2*d) - (5*a*Cot[c + d*x]^3)/(6*d) + (a*Cos[c + d*x]^2*Cot[c
 + d*x]^3)/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^3(c+dx)\cot^3(c+dx)\,dx + a\int \cos^2(c+dx)\cot^4(c$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\right.}{}$$

$$= -\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d} + \frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d}$$

$$= -\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d} + \frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d}$$

$$= -\frac{5a\cos(c+dx)}{2d} - \frac{5a\cos^3(c+dx)}{6d} + \frac{5a\cot(c+dx)}{2d} - \frac{a\,\text{co}}{}$$

$$= \frac{5ax}{2} + \frac{5a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{5a\cos(c+dx)}{2d} - \frac{5a\cos^3(}{6d}$$

Mathematica [A] time = 6.10, size = 174, normalized size = 1.34

$$\frac{5a(c+dx)}{2d} + \frac{a\sin(2(c+dx))}{4d} - \frac{9a\cos(c+dx)}{4d} - \frac{a\cos(3(c+dx))}{12d} + \frac{7a\cot(c+dx)}{3d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*(c + d*x))/(2*d) - (9*a*Cos[c + d*x])/(4*d) - (a*Cos[3*(c + d*x)])/(12
*d) + (7*a*Cot[c + d*x])/(3*d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c +
d*x]*Csc[c + d*x]^2)/(3*d) + (5*a*Log[Cos[(c + d*x)/2]])/(2*d) - (5*a*Log[S
in[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d) + (a*Sin[2*(c + d*x)
])/(4*d)
```

fricas [A] time = 0.84, size = 182, normalized size = 1.40

$$-\frac{6\,a\cos(dx+c)^5 - 40\,a\cos(dx+c)^3 - 15\left(a\cos(dx+c)^2 - a\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) + 15\left(a\,\text{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*cos(d*x + c)^5 - 40*a*cos(d*x + c)^3 - 15*(a*cos(d*x + c)^2 - a)
*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(a*cos(d*x + c)^2 - a)*log(-
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*a*cos(d*x + c) + 2*(2*a*cos(d*x +
 c)^5 - 15*a*d*x*cos(d*x + c)^2 + 10*a*cos(d*x + c)^3 + 15*a*d*x - 15*a*cos
(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.21, size = 220, normalized size = 1.69

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+180\,(dx+c)a-180\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-81\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/72*(3*a*tan(1/2*d*x + 1/2*c)^3 + 9*a*tan(1/2*d*x + 1/2*c)^2 + 180*(d*x +
c)*a - 180*a*log(abs(tan(1/2*d*x + 1/2*c))) - 81*a*tan(1/2*d*x + 1/2*c) + (
110*a*tan(1/2*d*x + 1/2*c)^9 + 9*a*tan(1/2*d*x + 1/2*c)^8 - 111*a*tan(1/2*d
*x + 1/2*c)^7 + 240*a*tan(1/2*d*x + 1/2*c)^6 - 273*a*tan(1/2*d*x + 1/2*c)^5
 + 306*a*tan(1/2*d*x + 1/2*c)^4 - 253*a*tan(1/2*d*x + 1/2*c)^3 + 72*a*tan(1
/2*d*x + 1/2*c)^2 - 9*a*tan(1/2*d*x + 1/2*c) - 3*a)/(tan(1/2*d*x + 1/2*c)^3
 + tan(1/2*d*x + 1/2*c))^3)/d
```

maple [A] time = 0.32, size = 199, normalized size = 1.53

$$-\frac{a\left(\cos^{7}(dx+c)\right)}{2d\sin(dx+c)^{2}}-\frac{a\left(\cos^{5}(dx+c)\right)}{2d}-\frac{5a\left(\cos^{3}(dx+c)\right)}{6d}-\frac{5a\cos(dx+c)}{2d}-\frac{5a\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{a\left(\right.}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^7-1/2*a*cos(d*x+c)^5/d-5/6*a*cos(d*x+c)^3/
d-5/2*a*cos(d*x+c)/d-5/2/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*a/sin(d*x+c)^3
*cos(d*x+c)^7+4/3/d*a/sin(d*x+c)*cos(d*x+c)^7+4/3*a*cos(d*x+c)^5*sin(d*x+c)
/d+5/3*a*cos(d*x+c)^3*sin(d*x+c)/d+5/2*a*cos(d*x+c)*sin(d*x+c)/d+5/2*a*x+5/
2/d*c*a
```

maxima [A] time = 0.60, size = 122, normalized size = 0.94

$$-\frac{\left(4\,\cos(dx+c)^{3}-\frac{6\,\cos(dx+c)}{\cos(dx+c)^{2}-1}+24\,\cos(dx+c)-15\,\log(\cos(dx+c)+1)+15\,\log(\cos(dx+c)-1)\right)a-2\left(15\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x
 + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a - 2*(15*d*x
+ 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(
d*x + c)^3))*a)/d
```

mupad [**B**] time = 8.97, size = 310, normalized size = 2.38

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+49\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\frac{80\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+67\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-34\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a/3 + a*tan(c/2 + (d*x)/2) - 8*a*tan(c/2
+ (d*x)/2)^2 + (121*a*tan(c/2 + (d*x)/2)^3)/3 - 34*a*tan(c/2 + (d*x)/2)^4 +
 67*a*tan(c/2 + (d*x)/2)^5 - (80*a*tan(c/2 + (d*x)/2)^6)/3 + 49*a*tan(c/2 +
 (d*x)/2)^7 - a*tan(c/2 + (d*x)/2)^8)/(d*(8*tan(c/2 + (d*x)/2)^3 + 24*tan(c
/2 + (d*x)/2)^5 + 24*tan(c/2 + (d*x)/2)^7 + 8*tan(c/2 + (d*x)/2)^9)) - (9*a
*tan(c/2 + (d*x)/2))/(8*d) + (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (5*a*log(tan
(c/2 + (d*x)/2)))/(2*d) - (5*a*atan((25*a^2)/(25*a^2 + 25*a^2*tan(c/2 + (d*
x)/2)) - (25*a^2*tan(c/2 + (d*x)/2))/(25*a^2 + 25*a^2*tan(c/2 + (d*x)/2))))
/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.580 $\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$\frac{15a\cos(c+dx)}{8d}-\frac{5a\cot^3(c+dx)}{6d}+\frac{5a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5a\,\mathrm{co}}{}$$

```
[Out] 5/2*a*x-15/8*a*arctanh(cos(d*x+c))/d+15/8*a*cos(d*x+c)/d+5/2*a*cot(d*x+c)/d
+5/8*a*cos(d*x+c)*cot(d*x+c)^2/d-5/6*a*cot(d*x+c)^3/d+1/2*a*cos(d*x+c)^2*co
t(d*x+c)^3/d-1/4*a*cos(d*x+c)*cot(d*x+c)^4/d
```

Rubi [A] time = 0.13, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2838, 2592, 288, 321, 206, 2591, 302, 203}

$$\frac{15a\cos(c+dx)}{8d}-\frac{5a\cot^3(c+dx)}{6d}+\frac{5a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5a\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/2 - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d)
+ (5*a*Cot[c + d*x])/(2*d) + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) - (5*a
*Cot[c + d*x]^3)/(6*d) + (a*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a*Cos[c
 + d*x]*Cot[c + d*x]^4)/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^2(c+dx)\cot^4(c+dx)\,dx + a\int \cos(c+dx)\cot^5(c+
$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,d\right.}{d}$$

$$= \frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d} - \frac{a\cos(c+dx)\cot^4(c+dx)}{4d} + \frac{(5}{}$$

$$= \frac{5a\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d} - \frac{a}{}$$

$$= \frac{15a\cos(c+dx)}{8d} + \frac{5a\cot(c+dx)}{2d} + \frac{5a\cos(c+dx)\cot^2(c+d}{8d}$$

$$= \frac{5ax}{2} - \frac{15a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{15a\cos(c+dx)}{8d} + \frac{5a\cot(c}{2d}$$

Mathematica [A] time = 1.43, size = 138, normalized size = 1.03

$$\frac{a\left(192\cos(c+dx) - 64\cot(c+dx)\left(\csc^2(c+dx)-7\right) + 3\left(16\sin(2(c+dx)) - \csc^4\left(\frac{1}{2}(c+dx)\right) + 18\csc^2\left(\frac{1}{2}(c+\right.\right.\right.}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(192*Cos[c + d*x] - 64*Cot[c + d*x]*(-7 + Csc[c + d*x]^2) + 3*(18*Csc[(c
 + d*x)/2]^2 - Csc[(c + d*x)/2]^4 + 40*(4*c + 4*d*x - 3*Log[Cos[(c + d*x)/2
]] + 3*Log[Sin[(c + d*x)/2]]) - 18*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^4
+ 16*Sin[2*(c + d*x)])))/(192*d)
```

fricas [A] time = 0.70, size = 202, normalized size = 1.51

$$\frac{120\,adx\cos(dx+c)^4 + 48\,a\cos(dx+c)^5 - 240\,adx\cos(dx+c)^2 - 150\,a\cos(dx+c)^3 + 120\,adx + 90\,a\cos(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(120*a*d*x*cos(d*x + c)^4 + 48*a*cos(d*x + c)^5 - 240*a*d*x*cos(d*x +
c)^2 - 150*a*cos(d*x + c)^3 + 120*a*d*x + 90*a*cos(d*x + c) - 45*(a*cos(d*x
 + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 45*(a*cos(d
*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 8*(3*a*c
os(d*x + c)^5 - 20*a*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/(d*c
os(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.24, size = 213, normalized size = 1.59

$$3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+480\,(dx+c)a+360\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*a*tan(1/2*d*x + 1/2*c)^3 - 48*a*tan(1
/2*d*x + 1/2*c)^2 + 480*(d*x + c)*a + 360*a*log(abs(tan(1/2*d*x + 1/2*c)))
- 216*a*tan(1/2*d*x + 1/2*c) - 192*(a*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(1/2*
d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)
^2 - (750*a*tan(1/2*d*x + 1/2*c)^4 - 216*a*tan(1/2*d*x + 1/2*c)^3 - 48*a*ta
n(1/2*d*x + 1/2*c)^2 + 8*a*tan(1/2*d*x + 1/2*c) + 3*a)/tan(1/2*d*x + 1/2*c)
^4)/d
```

maple [A] time = 0.33, size = 221, normalized size = 1.65

$$-\frac{a\left(\cos^{7}(dx+c)\right)}{3d\sin(dx+c)^{3}}+\frac{4a\left(\cos^{7}(dx+c)\right)}{3d\sin(dx+c)}+\frac{4a\left(\cos^{5}(dx+c)\right)\sin(dx+c)}{3d}+\frac{5a\left(\cos^{3}(dx+c)\right)\sin(dx+c)}{3d}+\frac{5a\cos(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^7+4/3/d*a/sin(d*x+c)*cos(d*x+c)^7+4/3*a*co
s(d*x+c)^5*sin(d*x+c)/d+5/3*a*cos(d*x+c)^3*sin(d*x+c)/d+5/2*a*cos(d*x+c)*si
n(d*x+c)/d+5/2*a*x+5/2/d*c*a-1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^7+3/8/d*a/sin(
d*x+c)^2*cos(d*x+c)^7+3/8*a*cos(d*x+c)^5/d+5/8*a*cos(d*x+c)^3/d+15/8*a*cos(
d*x+c)/d+15/8/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.61, size = 136, normalized size = 1.01

$$\frac{8\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^{4}+10\,\tan(dx+c)^{2}-2}{\tan(dx+c)^{5}+\tan(dx+c)^{3}}\right)a-3\,a\left(\frac{2\left(9\,\cos(dx+c)^{3}-7\,\cos(dx+c)\right)}{\cos(dx+c)^{4}-2\,\cos(dx+c)^{2}+1}-16\,\cos(dx+c)+15\,\log(\cos(dx\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(8*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d
*x + c)^5 + tan(d*x + c)^3))*a - 3*a*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))
/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x
 + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.82, size = 300, normalized size = 2.24

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}-\frac{9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{15\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}+\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2)^2)/(4*d) - (9*a*tan
(c/2 + (d*x)/2))/(8*d) + (a*tan(c/2 + (d*x)/2)^4)/(64*d) + (15*a*log(tan(c/
2 + (d*x)/2)))/(8*d) + ((7*a*tan(c/2 + (d*x)/2)^2)/2 - (2*a*tan(c/2 + (d*x)
/2))/3 - a/4 + (50*a*tan(c/2 + (d*x)/2)^3)/3 + (159*a*tan(c/2 + (d*x)/2)^4)
/4 + (154*a*tan(c/2 + (d*x)/2)^5)/3 + 36*a*tan(c/2 + (d*x)/2)^6 + 2*a*tan(c
/2 + (d*x)/2)^7)/(d*(16*tan(c/2 + (d*x)/2)^4 + 32*tan(c/2 + (d*x)/2)^6 + 16
*tan(c/2 + (d*x)/2)^8)) + (5*a*atan((25*a^2)/((75*a^2)/4 - 25*a^2*tan(c/2 +
 (d*x)/2)) + (75*a^2*tan(c/2 + (d*x)/2))/(4*((75*a^2)/4 - 25*a^2*tan(c/2 +
(d*x)/2)))))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.581 $\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=122

$$\frac{15a\cos(c+dx)}{8d}-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5a\cos(c+dx)\cot^2}{8d}$$

```
[Out] -a*x-15/8*a*arctanh(cos(d*x+c))/d+15/8*a*cos(d*x+c)/d-a*cot(d*x+c)/d+5/8*a*
cos(d*x+c)*cot(d*x+c)^2/d+1/3*a*cot(d*x+c)^3/d-1/4*a*cos(d*x+c)*cot(d*x+c)^
4/d-1/5*a*cot(d*x+c)^5/d
```

Rubi [A] time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2710, 2592, 288, 321, 206, 3473, 8}

$$\frac{15a\cos(c+dx)}{8d}-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5a\cos(c+dx)\cot^2}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d) - (
a*Cot[c + d*x])/d + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*
x]^3)/(3*d) - (a*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx = \int \left(a\cos(c+dx)\cot^5(c+dx)+a\cot^6(c+dx)\right)\,dx$$
$$= a\int \cos(c+dx)\cot^5(c+dx)\,dx + a\int \cot^6(c+dx)\,dx$$
$$= -\frac{a\cot^5(c+dx)}{5d} - a\int \cot^4(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3}\,dx, x, \cos(c+\right.}{d}$$
$$= \frac{a\cot^3(c+dx)}{3d} - \frac{a\cos(c+dx)\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx)}{5d} + a\int \cot^2$$
$$= -\frac{a\cot(c+dx)}{d} + \frac{5a\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cot^3(c+dx)}{3d} - \frac{a\cos(c}{}$$
$$= -ax + \frac{15a\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + \frac{5a\cos(c+dx)\cot^2(c+dx)}{8d} +$$
$$= -ax - \frac{15a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{15a\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + \frac{5a}{}$$

Mathematica [C] time = 0.07, size = 164, normalized size = 1.34

$$-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d} + \frac{a\cos(c+dx)}{d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{9a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x])/d + (9*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^
4)/(64*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x
]^2])/(5*d) - (15*a*Log[Cos[(c + d*x)/2]])/(8*d) + (15*a*Log[Sin[(c + d*x)/
2]])/(8*d) - (9*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d
)
```

fricas [B] time = 0.62, size = 222, normalized size = 1.82

$$-\frac{368\,a\cos(dx+c)^5 - 560\,a\cos(dx+c)^3 + 225\left(a\cos(dx+c)^4 - 2\,a\cos(dx+c)^2 + a\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(368*a*cos(d*x + c)^5 - 560*a*cos(d*x + c)^3 + 225*(a*cos(d*x + c)^4
 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(
a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + 240*a*cos(d*x + c) + 30*(8*a*d*x*cos(d*x + c)^4 - 8*a*cos(d*x +
 c)^5 - 16*a*d*x*cos(d*x + c)^2 + 25*a*cos(d*x + c)^3 + 8*a*d*x - 15*a*cos(
d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*
x + c))
```

giac [A] time = 0.22, size = 199, normalized size = 1.63

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-960\,(dx+c)a+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*a*tan(1/2*d*x + 1/2*c)^4 - 70*a*tan(
1/2*d*x + 1/2*c)^3 - 240*a*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a + 1800*
a*log(abs(tan(1/2*d*x + 1/2*c))) + 660*a*tan(1/2*d*x + 1/2*c) + 1920*a/(tan
(1/2*d*x + 1/2*c)^2 + 1) - (4110*a*tan(1/2*d*x + 1/2*c)^5 + 660*a*tan(1/2*d
*x + 1/2*c)^4 - 240*a*tan(1/2*d*x + 1/2*c)^3 - 70*a*tan(1/2*d*x + 1/2*c)^2
+ 15*a*tan(1/2*d*x + 1/2*c) + 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.23, size = 159, normalized size = 1.30

$$-\frac{a\left(\cos^{7}(dx+c)\right)}{4d\sin(dx+c)^{4}}+\frac{3a\left(\cos^{7}(dx+c)\right)}{8d\sin(dx+c)^{2}}+\frac{3a\left(\cos^{5}(dx+c)\right)}{8d}+\frac{5a\left(\cos^{3}(dx+c)\right)}{8d}+\frac{15a\cos(dx+c)}{8d}+\frac{15a\ln(\csc(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^7+3/8/d*a/sin(d*x+c)^2*cos(d*x+c)^7+3/8*a*
cos(d*x+c)^5/d+5/8*a*cos(d*x+c)^3/d+15/8*a*cos(d*x+c)/d+15/8/d*a*ln(csc(d*x
+c)-cot(d*x+c))-1/5*a*cot(d*x+c)^5/d+1/3*a*cot(d*x+c)^3/d-a*cot(d*x+c)/d-a*
x-1/d*c*a
```

maxima [A] time = 0.44, size = 125, normalized size = 1.02

$$-\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^{4}-5\,\tan(dx+c)^{2}+3}{\tan(dx+c)^{5}}\right)a+15\,a\left(\frac{2\left(9\,\cos(dx+c)^{3}-7\,\cos(dx+c)\right)}{\cos(dx+c)^{4}-2\,\cos(dx+c)^{2}+1}-16\,\cos(dx+c)+15\,\log(\cos(d}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a + 15*a*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4
 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*
log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.87, size = 291, normalized size = 2.39

$$\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{22\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-72\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{59\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{32\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{a}{\cdots}}{d\,\left(32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (11*a*tan(c/2 + (d*x)/2))/(16*d) - (a/5 + (a*tan(c/2 + (d*x)/2))/2 - (32*a*
tan(c/2 + (d*x)/2)^2)/15 - (15*a*tan(c/2 + (d*x)/2)^3)/2 + (59*a*tan(c/2 +
(d*x)/2)^4)/3 - 72*a*tan(c/2 + (d*x)/2)^5 + 22*a*tan(c/2 + (d*x)/2)^6)/(d*(
32*tan(c/2 + (d*x)/2)^5 + 32*tan(c/2 + (d*x)/2)^7)) - (a*tan(c/2 + (d*x)/2)
^2)/(4*d) - (7*a*tan(c/2 + (d*x)/2)^3)/(96*d) + (a*tan(c/2 + (d*x)/2)^4)/(6
4*d) + (a*tan(c/2 + (d*x)/2)^5)/(160*d) + (15*a*log(tan(c/2 + (d*x)/2)))/(8
*d) + (2*a*atan((4*a^2)/((15*a^2)/2 + 4*a^2*tan(c/2 + (d*x)/2)) - (15*a^2*t
an(c/2 + (d*x)/2))/(2*((15*a^2)/2 + 4*a^2*tan(c/2 + (d*x)/2)))))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.582 $\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=128

$$-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^5(c+dx)\csc(c+dx)}{6d}+\frac{5a\cot^3(c+$$

```
[Out] -a*x+5/16*a*arctanh(cos(d*x+c))/d-a*cot(d*x+c)/d+1/3*a*cot(d*x+c)^3/d-1/5*a
*cot(d*x+c)^5/d-5/16*a*cot(d*x+c)*csc(d*x+c)/d+5/24*a*cot(d*x+c)^3*csc(d*x+
c)/d-1/6*a*cot(d*x+c)^5*csc(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2838, 2611, 3770, 3473, 8}

$$-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^5(c+dx)\csc(c+dx)}{6d}+\frac{5a\cot^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) + (5*a*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x])/d + (a*Cot[c
 + d*x]^3)/(3*d) - (a*Cot[c + d*x]^5)/(5*d) - (5*a*Cot[c + d*x]*Csc[c + d*x
])/(16*d) + (5*a*Cot[c + d*x]^3*Csc[c + d*x])/(24*d) - (a*Cot[c + d*x]^5*Cs
c[c + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\,dx + a\int \cot^6(c+dx)\csc(c+dx)\,dx$$
$$= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^5(c+dx)\csc(c+dx)}{6d} - \frac{1}{6}(5a)\int \cot^4$$
$$= \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} + \frac{5a\cot^3(c+dx)\csc(c+dx}{24d}$$
$$= -\frac{a\cot(c+dx)}{d} + \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{5a\cot(c}{}$$
$$= -ax + \frac{5a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\cot(c+dx)}{d} + \frac{a\cot^3(c+}{3d}$$

Mathematica [C] time = 0.06, size = 193, normalized size = 1.51

$$-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{11a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + -$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-11*a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c + d*x)/2]^4)/(32*d) - (a*Csc[
(c + d*x)/2]^6)/(384*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2
, -Tan[c + d*x]^2])/(5*d) + (5*a*Log[Cos[(c + d*x)/2]])/(16*d) - (5*a*Log[S
in[(c + d*x)/2]])/(16*d) + (11*a*Sec[(c + d*x)/2]^2)/(64*d) - (a*Sec[(c + d
*x)/2]^4)/(32*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

fricas [B] time = 0.79, size = 254, normalized size = 1.98

$$-\frac{480\,adx\cos(dx+c)^6 - 1440\,adx\cos(dx+c)^4 - 330\,a\cos(dx+c)^5 + 1440\,adx\cos(dx+c)^2 + 400\,a\cos(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/480*(480*a*d*x*cos(d*x + c)^6 - 1440*a*d*x*cos(d*x + c)^4 - 330*a*cos(d*
x + c)^5 + 1440*a*d*x*cos(d*x + c)^2 + 400*a*cos(d*x + c)^3 - 480*a*d*x - 1
50*a*cos(d*x + c) - 75*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x
 + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 75*(a*cos(d*x + c)^6 - 3*a*cos(d
*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) - 32*(23*a
*cos(d*x + c)^5 - 35*a*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.27, size = 208, normalized size = 1.62

$$\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-140\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+225\,a\tan\left(\frac{1}{2}\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*a*tan(1/2*d*x + 1/2*c)^6 + 12*a*tan(1/2*d*x + 1/2*c)^5 - 45*a*tan
(1/2*d*x + 1/2*c)^4 - 140*a*tan(1/2*d*x + 1/2*c)^3 + 225*a*tan(1/2*d*x + 1/
2*c)^2 - 1920*(d*x + c)*a - 600*a*log(abs(tan(1/2*d*x + 1/2*c))) + 1320*a*t
an(1/2*d*x + 1/2*c) + (1470*a*tan(1/2*d*x + 1/2*c)^6 - 1320*a*tan(1/2*d*x +
 1/2*c)^5 - 225*a*tan(1/2*d*x + 1/2*c)^4 + 140*a*tan(1/2*d*x + 1/2*c)^3 + 4
5*a*tan(1/2*d*x + 1/2*c)^2 - 12*a*tan(1/2*d*x + 1/2*c) - 5*a)/tan(1/2*d*x +
 1/2*c)^6)/d
```

maple [A] time = 0.25, size = 181, normalized size = 1.41

$$-\frac{a\left(\cot^{5}\left(dx+c\right)\right)}{5d}+\frac{a\left(\cot^{3}\left(dx+c\right)\right)}{3d}-\frac{a\cot\left(dx+c\right)}{d}-ax-\frac{ca}{d}-\frac{a\left(\cos^{7}\left(dx+c\right)\right)}{6d\sin\left(dx+c\right)^{6}}+\frac{a\left(\cos^{7}\left(dx+c\right)\right)}{24d\sin\left(dx+c\right)^{4}}-\frac{a\left(\cos^{7}\left(dx+\right.\right.}{16d\sin\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/5*a*cot(d*x+c)^5/d+1/3*a*cot(d*x+c)^3/d-a*cot(d*x+c)/d-a*x-1/d*c*a-1/6/d
*a/sin(d*x+c)^6*cos(d*x+c)^7+1/24/d*a/sin(d*x+c)^4*cos(d*x+c)^7-1/16/d*a/si
n(d*x+c)^2*cos(d*x+c)^7-1/16*a*cos(d*x+c)^5/d-5/48*a*cos(d*x+c)^3/d-5/16*a*
cos(d*x+c)/d-5/16/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.43, size = 137, normalized size = 1.07

$$-\frac{32\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a-5\,a\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log\left(\cos\left(dx+\right.\right.\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/480*(32*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a - 5*a*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x
+ c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(c
os(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.46, size = 285, normalized size = 2.23

$$\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{5\,a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{16\,d}-\frac{11\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{2\,a\,\mathrm{atan}\left(\frac{16\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-16\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{15\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^7,x)
```

```
[Out] (11*a*tan(c/2 + (d*x)/2))/(16*d) - (5*a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2)))/(16*d) - (11*a*cot(c/2 + (d*x)/2))/(16*d) - (2*a*atan((16*cos(c/2
+ (d*x)/2) + 5*sin(c/2 + (d*x)/2))/(5*cos(c/2 + (d*x)/2) - 16*sin(c/2 + (d*
x)/2))))/d - (15*a*cot(c/2 + (d*x)/2)^2)/(128*d) + (7*a*cot(c/2 + (d*x)/2)^
3)/(96*d) + (3*a*cot(c/2 + (d*x)/2)^4)/(128*d) - (a*cot(c/2 + (d*x)/2)^5)/(
160*d) - (a*cot(c/2 + (d*x)/2)^6)/(384*d) + (15*a*tan(c/2 + (d*x)/2)^2)/(12
8*d) - (7*a*tan(c/2 + (d*x)/2)^3)/(96*d) - (3*a*tan(c/2 + (d*x)/2)^4)/(128*
d) + (a*tan(c/2 + (d*x)/2)^5)/(160*d) + (a*tan(c/2 + (d*x)/2)^6)/(384*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.583 $\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=96

$$-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^5(c+dx)\csc(c+dx)}{6d}+\frac{5a\cot^3(c+dx)\csc(c+dx)}{24d}-\frac{5a\cot(c+dx)}{16d}$$

```
[Out] 5/16*a*arctanh(cos(d*x+c))/d-1/7*a*cot(d*x+c)^7/d-5/16*a*cot(d*x+c)*csc(d*x
+c)/d+5/24*a*cot(d*x+c)^3*csc(d*x+c)/d-1/6*a*cot(d*x+c)^5*csc(d*x+c)/d
```

Rubi [A] time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2838, 2607, 30, 2611, 3770}

$$-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^5(c+dx)\csc(c+dx)}{6d}+\frac{5a\cot^3(c+dx)\csc(c+dx)}{24d}-\frac{5a\cot(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x]^7)/(7*d) - (5*a*Cot[c
+ d*x]*Csc[c + d*x])/(16*d) + (5*a*Cot[c + d*x]^3*Csc[c + d*x])/(24*d) - (a
*Cot[c + d*x]^5*Csc[c + d*x])/(6*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\csc(c+dx)\,dx + a\int \cot^6(c+dx)\csc^2(c+$$

$$= -\frac{a\cot^5(c+dx)\csc(c+dx)}{6d} - \frac{1}{6}(5a)\int \cot^4(c+dx)\csc(c+$$

$$= -\frac{a\cot^7(c+dx)}{7d} + \frac{5a\cot^3(c+dx)\csc(c+dx)}{24d} - \frac{a\cot^5(c+$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{5a\cot(c+dx)\csc(c+dx)}{16d} + \frac{5a\cot^3(c+$$

$$= \frac{5a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\cot^7(c+dx)}{7d} - \frac{5a\cot(c+dx)\mathrm{c}}{16d}$$

Mathematica [A] time = 0.05, size = 175, normalized size = 1.82

$$-\frac{a\cot^7(c+dx)}{7d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{11a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\sec^4\left(\frac{1}{2}(c\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7*(a*Cot[c + d*x]^7)/d - (11*a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c +
d*x)/2]^4)/(32*d) - (a*Csc[(c + d*x)/2]^6)/(384*d) + (5*a*Log[Cos[(c + d*x)
/2]])/(16*d) - (5*a*Log[Sin[(c + d*x)/2]])/(16*d) + (11*a*Sec[(c + d*x)/2]^
2)/(64*d) - (a*Sec[(c + d*x)/2]^4)/(32*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

fricas [B] time = 0.87, size = 210, normalized size = 2.19

$$\frac{96\,a\cos(dx+c)^7 + 105\left(a\cos(dx+c)^6 - 3\,a\cos(dx+c)^4 + 3\,a\cos(dx+c)^2 - a\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/672*(96*a*cos(d*x + c)^7 + 105*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3
*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 105*(a*co
s(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) + 14*(33*a*cos(d*x + c)^5 - 40*a*cos(d*x + c)^3
+ 15*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4
+ 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [B] time = 0.24, size = 228, normalized size = 2.38

$$3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+7\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-63\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+63\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2688*(3*a*tan(1/2*d*x + 1/2*c)^7 + 7*a*tan(1/2*d*x + 1/2*c)^6 - 21*a*tan(
1/2*d*x + 1/2*c)^5 - 63*a*tan(1/2*d*x + 1/2*c)^4 + 63*a*tan(1/2*d*x + 1/2*c
)^3 + 315*a*tan(1/2*d*x + 1/2*c)^2 - 840*a*log(abs(tan(1/2*d*x + 1/2*c))) -
 105*a*tan(1/2*d*x + 1/2*c) + (2178*a*tan(1/2*d*x + 1/2*c)^7 + 105*a*tan(1/
2*d*x + 1/2*c)^6 - 315*a*tan(1/2*d*x + 1/2*c)^5 - 63*a*tan(1/2*d*x + 1/2*c)
^4 + 63*a*tan(1/2*d*x + 1/2*c)^3 + 21*a*tan(1/2*d*x + 1/2*c)^2 - 7*a*tan(1/
2*d*x + 1/2*c) - 3*a)/tan(1/2*d*x + 1/2*c)^7)/d
```

maple [A] time = 0.27, size = 152, normalized size = 1.58

$$-\frac{a\left(\cos^{7}\left(dx+c\right)\right)}{6d\sin\left(dx+c\right)^{6}}+\frac{a\left(\cos^{7}\left(dx+c\right)\right)}{24d\sin\left(dx+c\right)^{4}}-\frac{a\left(\cos^{7}\left(dx+c\right)\right)}{16d\sin\left(dx+c\right)^{2}}-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{16d}-\frac{5a\left(\cos^{3}\left(dx+c\right)\right)}{48d}-\frac{5a\cos\left(dx+c\right)}{16d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/6/d*a/sin(d*x+c)^6*cos(d*x+c)^7+1/24/d*a/sin(d*x+c)^4*cos(d*x+c)^7-1/16/
d*a/sin(d*x+c)^2*cos(d*x+c)^7-1/16*a*cos(d*x+c)^5/d-5/48*a*cos(d*x+c)^3/d-5
/16*a*cos(d*x+c)/d-5/16/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/7/d*a/sin(d*x+c)^7*
cos(d*x+c)^7
```

maxima [A] time = 0.40, size = 106, normalized size = 1.10

$$\frac{7\,a\left(\frac{2\left(33\,\cos(dx+c)^{5}-40\,\cos(dx+c)^{3}+15\,\cos(dx+c)\right)}{\cos(dx+c)^{6}-3\,\cos(dx+c)^{4}+3\,\cos(dx+c)^{2}-1}+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{96\,a}{\tan(dx+c)^{7}}}{672\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/672*(7*a*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))/(co
s(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(cos(d*x +
c) + 1) - 15*log(cos(d*x + c) - 1)) - 96*a/tan(d*x + c)^7)/d
```

mupad [B] time = 9.94, size = 385, normalized size = 4.01

$$-\frac{a\left(3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-7\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+7\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+21\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^8,x)
```

```
[Out] -(a*(3*cos(c/2 + (d*x)/2)^14 - 3*sin(c/2 + (d*x)/2)^14 - 7*cos(c/2 + (d*x)/
2)*sin(c/2 + (d*x)/2)^13 + 7*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) + 21*
cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 + 63*cos(c/2 + (d*x)/2)^3*sin(c/
2 + (d*x)/2)^11 - 63*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 - 315*cos(c
/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 + 105*cos(c/2 + (d*x)/2)^6*sin(c/2 + (
d*x)/2)^8 - 105*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 315*cos(c/2 + (
d*x)/2)^9*sin(c/2 + (d*x)/2)^5 + 63*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2
)^4 - 63*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 - 21*cos(c/2 + (d*x)/2)
^12*sin(c/2 + (d*x)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*c
os(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7))/(2688*d*cos(c/2 + (d*x)/2)^7*sin
(c/2 + (d*x)/2)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.584 $\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=122

$$-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a\cot^3(c+dx)\csc^3(c+dx)}{48d}-\frac{5a\cot(c+d}{6}$$

[Out] 5/128*a*arctanh(cos(d*x+c))/d-1/7*a*cot(d*x+c)^7/d+5/128*a*cot(d*x+c)*csc(d*x+c)/d-5/64*a*cot(d*x+c)*csc(d*x+c)^3/d+5/48*a*cot(d*x+c)^3*csc(d*x+c)^3/d-1/8*a*cot(d*x+c)^5*csc(d*x+c)^3/d

Rubi [A] time = 0.18, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 30}

$$-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a\cot^3(c+dx)\csc^3(c+dx)}{48d}-\frac{5a\cot(c+d}{6}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]

[Out] (5*a*ArcTanh[Cos[c + d*x]])/(128*d) - (a*Cot[c + d*x]^7)/(7*d) + (5*a*Cot[c + d*x]*Csc[c + d*x])/(128*d) - (5*a*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) + (5*a*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\csc^2(c+dx)\,dx + a\int \cot^6(c+dx)\csc^3(c$$

$$= -\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{1}{8}(5a)\int \cot^4(c+dx)\csc^3(c$$

$$= -\frac{a\cot^7(c+dx)}{7d} + \frac{5a\cot^3(c+dx)\csc^3(c+dx)}{48d} - \frac{a\cot^5(c+$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{5a\cot(c+dx)\csc^3(c+dx)}{64d} + \frac{5a\cot^3(c+$$

$$= -\frac{a\cot^7(c+dx)}{7d} + \frac{5a\cot(c+dx)\csc(c+dx)}{128d} - \frac{5a\cot(c+d$$

$$= \frac{5a\tanh^{-1}(\cos(c+dx))}{128d} - \frac{a\cot^7(c+dx)}{7d} + \frac{5a\cot(c+dx)\,c}{128d}$$

Mathematica [A] time = 0.06, size = 215, normalized size = 1.76

$$-\frac{a\cot^7(c+dx)}{7d} - \frac{a\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d} + \frac{7a\csc^6\left(\frac{1}{2}(c+dx)\right)}{1536d} - \frac{15a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d} + \frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{512d} + \frac{a\sec^8\left(\frac{1}{2}(\right.}{204}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7*(a*Cot[c + d*x]^7)/d + (5*a*Csc[(c + d*x)/2]^2)/(512*d) - (15*a*Csc[(c
 + d*x)/2]^4)/(1024*d) + (7*a*Csc[(c + d*x)/2]^6)/(1536*d) - (a*Csc[(c + d*
x)/2]^8)/(2048*d) + (5*a*Log[Cos[(c + d*x)/2]])/(128*d) - (5*a*Log[Sin[(c +
 d*x)/2]])/(128*d) - (5*a*Sec[(c + d*x)/2]^2)/(512*d) + (15*a*Sec[(c + d*x)
/2]^4)/(1024*d) - (7*a*Sec[(c + d*x)/2]^6)/(1536*d) + (a*Sec[(c + d*x)/2]^8
)/(2048*d)
```

fricas [B] time = 0.78, size = 225, normalized size = 1.84

$$-\frac{768\,a\cos\left(dx+c\right)^{7}\sin\left(dx+c\right)+210\,a\cos\left(dx+c\right)^{7}+1022\,a\cos\left(dx+c\right)^{5}-770\,a\cos\left(dx+c\right)^{3}+210\,a\cos\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/5376*(768*a*cos(d*x + c)^7*sin(d*x + c) + 210*a*cos(d*x + c)^7 + 1022*a*
cos(d*x + c)^5 - 770*a*cos(d*x + c)^3 + 210*a*cos(d*x + c) - 105*(a*cos(d*x
 + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)
*log(1/2*cos(d*x + c) + 1/2) + 105*(a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 +
 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2))
/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x
+ c)^2 + d)
```

giac [B] time = 0.28, size = 256, normalized size = 2.10

$$\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-112\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-336\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+168\,a\tan\left(\frac{1}{2}\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/43008*(21*a*tan(1/2*d*x + 1/2*c)^8 + 48*a*tan(1/2*d*x + 1/2*c)^7 - 112*a*
tan(1/2*d*x + 1/2*c)^6 - 336*a*tan(1/2*d*x + 1/2*c)^5 + 168*a*tan(1/2*d*x +
 1/2*c)^4 + 1008*a*tan(1/2*d*x + 1/2*c)^3 + 336*a*tan(1/2*d*x + 1/2*c)^2 -
1680*a*log(abs(tan(1/2*d*x + 1/2*c))) - 1680*a*tan(1/2*d*x + 1/2*c) + (4566
*a*tan(1/2*d*x + 1/2*c)^8 + 1680*a*tan(1/2*d*x + 1/2*c)^7 - 336*a*tan(1/2*d
*x + 1/2*c)^6 - 1008*a*tan(1/2*d*x + 1/2*c)^5 - 168*a*tan(1/2*d*x + 1/2*c)^
```

```
4 + 336*a*tan(1/2*d*x + 1/2*c)^3 + 112*a*tan(1/2*d*x + 1/2*c)^2 - 48*a*tan(
1/2*d*x + 1/2*c) - 21*a)/tan(1/2*d*x + 1/2*c)^8)/d
```

maple [A] time = 0.27, size = 174, normalized size = 1.43

$$-\frac{a\left(\cos^7(dx+c)\right)}{7d\sin(dx+c)^7}-\frac{a\left(\cos^7(dx+c)\right)}{8d\sin(dx+c)^8}-\frac{a\left(\cos^7(dx+c)\right)}{48d\sin(dx+c)^6}+\frac{a\left(\cos^7(dx+c)\right)}{192d\sin(dx+c)^4}-\frac{a\left(\cos^7(dx+c)\right)}{128d\sin(dx+c)^2}-\frac{a\left(\cos^5(dx+c\right.}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/7/d*a/sin(d*x+c)^7*cos(d*x+c)^7-1/8/d*a/sin(d*x+c)^8*cos(d*x+c)^7-1/48/d
*a/sin(d*x+c)^6*cos(d*x+c)^7+1/192/d*a/sin(d*x+c)^4*cos(d*x+c)^7-1/128/d*a/
sin(d*x+c)^2*cos(d*x+c)^7-1/128*a*cos(d*x+c)^5/d-5/384*a*cos(d*x+c)^3/d-5/1
28*a*cos(d*x+c)/d-5/128/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.37, size = 126, normalized size = 1.03

$$-\frac{7a\left(\frac{2\left(15\cos(dx+c)^7+73\cos(dx+c)^5-55\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1}-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)}{5376d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/5376*(7*a*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3
+ 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 -
4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)
) + 768*a/tan(d*x + c)^7)/d
```

mupad [B] time = 9.15, size = 285, normalized size = 2.34

$$\frac{5a\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{128d}-\frac{5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128d}-\frac{a\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{128d}-\frac{3a\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{128d}-\frac{a\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{256d}+\frac{a\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{128d}+\frac{a\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^9,x)
```

```
[Out] (5*a*cot(c/2 + (d*x)/2))/(128*d) - (5*a*tan(c/2 + (d*x)/2))/(128*d) - (a*co
t(c/2 + (d*x)/2)^2)/(128*d) - (3*a*cot(c/2 + (d*x)/2)^3)/(128*d) - (a*cot(c
/2 + (d*x)/2)^4)/(256*d) + (a*cot(c/2 + (d*x)/2)^5)/(128*d) + (a*cot(c/2 +
(d*x)/2)^6)/(384*d) - (a*cot(c/2 + (d*x)/2)^7)/(896*d) - (a*cot(c/2 + (d*x)
/2)^8)/(2048*d) + (a*tan(c/2 + (d*x)/2)^2)/(128*d) + (3*a*tan(c/2 + (d*x)/2
```

```
)^3)/(128*d) + (a*tan(c/2 + (d*x)/2)^4)/(256*d) - (a*tan(c/2 + (d*x)/2)^5)/
(128*d) - (a*tan(c/2 + (d*x)/2)^6)/(384*d) + (a*tan(c/2 + (d*x)/2)^7)/(896*
d) + (a*tan(c/2 + (d*x)/2)^8)/(2048*d) - (5*a*log(tan(c/2 + (d*x)/2)))/(128
*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.585 $\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=138

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a\cot^3(c+dx)\csc^3(c+$$

```
[Out] 5/128*a*arctanh(cos(d*x+c))/d-1/7*a*cot(d*x+c)^7/d-1/9*a*cot(d*x+c)^9/d+5/1
28*a*cot(d*x+c)*csc(d*x+c)/d-5/64*a*cot(d*x+c)*csc(d*x+c)^3/d+5/48*a*cot(d*
x+c)^3*csc(d*x+c)^3/d-1/8*a*cot(d*x+c)^5*csc(d*x+c)^3/d
```

Rubi [A] time = 0.20, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2607, 14, 2611, 3768, 3770}

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a\cot^3(c+dx)\csc^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*ArcTanh[Cos[c + d*x]])/(128*d) - (a*Cot[c + d*x]^7)/(7*d) - (a*Cot[c +
 d*x]^9)/(9*d) + (5*a*Cot[c + d*x]*Csc[c + d*x])/(128*d) - (5*a*Cot[c + d*x
]*Csc[c + d*x]^3)/(64*d) + (5*a*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a*
Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\csc^3(c+dx)\,dx + a\int \cot^6(c+dx)\csc^4(c+$$

$$= -\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{1}{8}(5a)\int \cot^4(c+dx)\csc^3(c+$$

$$= \frac{5a\cot^3(c+dx)\csc^3(c+dx)}{48d} - \frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d} +$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} - \frac{5a\cot(c+dx)\csc^3(c+d}{64d}$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} + \frac{5a\cot(c+dx)\csc(c+dx}{128d}$$

$$= \frac{5a\tanh^{-1}(\cos(c+dx))}{128d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} +$$

Mathematica [B] time = 0.09, size = 301, normalized size = 2.18

$$\frac{2a\cot(c+dx)}{63d} - \frac{a\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d} + \frac{7a\csc^6\left(\frac{1}{2}(c+dx)\right)}{1536d} - \frac{15a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d} + \frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{512d} + \frac{a\sec^8\left(\frac{1}{2}(c-\right.}{2048d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Cot[c + d*x])/(63*d) + (5*a*Csc[(c + d*x)/2]^2)/(512*d) - (15*a*Csc[(c
 + d*x)/2]^4)/(1024*d) + (7*a*Csc[(c + d*x)/2]^6)/(1536*d) - (a*Csc[(c + d*
x)/2]^8)/(2048*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(63*d) - (5*a*Cot[c + d
*x]*Csc[c + d*x]^4)/(21*d) + (19*a*Cot[c + d*x]*Csc[c + d*x]^6)/(63*d) - (a
*Cot[c + d*x]*Csc[c + d*x]^8)/(9*d) + (5*a*Log[Cos[(c + d*x)/2]])/(128*d) -
 (5*a*Log[Sin[(c + d*x)/2]])/(128*d) - (5*a*Sec[(c + d*x)/2]^2)/(512*d) + (
15*a*Sec[(c + d*x)/2]^4)/(1024*d) - (7*a*Sec[(c + d*x)/2]^6)/(1536*d) + (a*
Sec[(c + d*x)/2]^8)/(2048*d)
```

fricas [B] time = 0.60, size = 259, normalized size = 1.88

$$512\,a\cos\left(dx+c\right)^{9}-2304\,a\cos\left(dx+c\right)^{7}+315\,\Big(a\cos\left(dx+c\right)^{8}-4\,a\cos\left(dx+c\right)^{6}+6\,a\cos\left(dx+c\right)^{4}-4\,a\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/16128*(512*a*cos(d*x + c)^9 - 2304*a*cos(d*x + c)^7 + 315*(a*cos(d*x + c)
^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 315*(a*cos(d*x + c)^8 - 4*a*cos(d*x
+ c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) - 42*(15*a*cos(d*x + c)^7 + 73*a*cos(d*x + c)^5 - 55*a
*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c)
)
```

giac [B] time = 0.27, size = 256, normalized size = 1.86

$$28\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+63\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-108\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-336\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+504\,a\tan\Big(\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/129024*(28*a*tan(1/2*d*x + 1/2*c)^9 + 63*a*tan(1/2*d*x + 1/2*c)^8 - 108*a
*tan(1/2*d*x + 1/2*c)^7 - 336*a*tan(1/2*d*x + 1/2*c)^6 + 504*a*tan(1/2*d*x
+ 1/2*c)^4 + 672*a*tan(1/2*d*x + 1/2*c)^3 + 1008*a*tan(1/2*d*x + 1/2*c)^2 -
```

```
 5040*a*log(abs(tan(1/2*d*x + 1/2*c))) - 1512*a*tan(1/2*d*x + 1/2*c) + (142
58*a*tan(1/2*d*x + 1/2*c)^9 + 1512*a*tan(1/2*d*x + 1/2*c)^8 - 1008*a*tan(1/
2*d*x + 1/2*c)^7 - 672*a*tan(1/2*d*x + 1/2*c)^6 - 504*a*tan(1/2*d*x + 1/2*c
)^5 + 336*a*tan(1/2*d*x + 1/2*c)^3 + 108*a*tan(1/2*d*x + 1/2*c)^2 - 63*a*ta
n(1/2*d*x + 1/2*c) - 28*a)/tan(1/2*d*x + 1/2*c)^9)/d
```

maple [A] time = 0.29, size = 196, normalized size = 1.42

$$-\frac{a\left(\cos^7(dx+c)\right)}{8d\sin(dx+c)^8}-\frac{a\left(\cos^7(dx+c)\right)}{48d\sin(dx+c)^6}+\frac{a\left(\cos^7(dx+c)\right)}{192d\sin(dx+c)^4}-\frac{a\left(\cos^7(dx+c)\right)}{128d\sin(dx+c)^2}-\frac{a\left(\cos^5(dx+c)\right)}{128d}-\frac{5a\left(\cos^3(dx+c\right.}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/8/d*a/sin(d*x+c)^8*cos(d*x+c)^7-1/48/d*a/sin(d*x+c)^6*cos(d*x+c)^7+1/192
/d*a/sin(d*x+c)^4*cos(d*x+c)^7-1/128/d*a/sin(d*x+c)^2*cos(d*x+c)^7-1/128*a*
cos(d*x+c)^5/d-5/384*a*cos(d*x+c)^3/d-5/128*a*cos(d*x+c)/d-5/128/d*a*ln(csc
(d*x+c)-cot(d*x+c))-1/9/d*a/sin(d*x+c)^9*cos(d*x+c)^7-2/63/d*a/sin(d*x+c)^7
*cos(d*x+c)^7
```

maxima [A] time = 0.35, size = 138, normalized size = 1.00

$$-\frac{21\,a\left(\frac{2\left(15\,\cos(dx+c)^7+73\,\cos(dx+c)^5-55\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+}{16128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/16128*(21*a*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^
3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
1)) + 256*(9*tan(d*x + c)^2 + 7)*a/tan(d*x + c)^9)/d
```

mupad [B] time = 9.19, size = 285, normalized size = 2.07

$$\frac{3\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{256\,d}-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{256\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{192\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}+\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}+\frac{3\,a\cot\left(\right.}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^10,x)
```

```
[Out] (3*a*cot(c/2 + (d*x)/2))/(256*d) - (3*a*tan(c/2 + (d*x)/2))/(256*d) - (a*co
t(c/2 + (d*x)/2)^2)/(128*d) - (a*cot(c/2 + (d*x)/2)^3)/(192*d) - (a*cot(c/2
 + (d*x)/2)^4)/(256*d) + (a*cot(c/2 + (d*x)/2)^6)/(384*d) + (3*a*cot(c/2 +
(d*x)/2)^7)/(3584*d) - (a*cot(c/2 + (d*x)/2)^8)/(2048*d) - (a*cot(c/2 + (d*
x)/2)^9)/(4608*d) + (a*tan(c/2 + (d*x)/2)^2)/(128*d) + (a*tan(c/2 + (d*x)/2
)^3)/(192*d) + (a*tan(c/2 + (d*x)/2)^4)/(256*d) - (a*tan(c/2 + (d*x)/2)^6)/
(384*d) - (3*a*tan(c/2 + (d*x)/2)^7)/(3584*d) + (a*tan(c/2 + (d*x)/2)^8)/(2
048*d) + (a*tan(c/2 + (d*x)/2)^9)/(4608*d) - (5*a*log(tan(c/2 + (d*x)/2)))/
(128*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**10*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.586 $\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=160

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{a\cot^7(c+dx)}{7d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d}+\frac{a\cot^3(c+dx)\csc^5(c+dx}{16d}$$

```
[Out] 3/256*a*arctanh(cos(d*x+c))/d-1/7*a*cot(d*x+c)^7/d-1/9*a*cot(d*x+c)^9/d+3/2
56*a*cot(d*x+c)*csc(d*x+c)/d+1/128*a*cot(d*x+c)*csc(d*x+c)^3/d-1/32*a*cot(d
*x+c)*csc(d*x+c)^5/d+1/16*a*cot(d*x+c)^3*csc(d*x+c)^5/d-1/10*a*cot(d*x+c)^5
*csc(d*x+c)^5/d
```

Rubi [A] time = 0.21, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2838, 2611, 3768, 3770, 2607, 14}

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{a\cot^7(c+dx)}{7d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d}+\frac{a\cot^3(c+dx)\csc^5(c+dx}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Cos[c + d*x]])/(256*d) - (a*Cot[c + d*x]^7)/(7*d) - (a*Cot[c +
 d*x]^9)/(9*d) + (3*a*Cot[c + d*x]*Csc[c + d*x])/(256*d) + (a*Cot[c + d*x]*
Csc[c + d*x]^3)/(128*d) - (a*Cot[c + d*x]*Csc[c + d*x]^5)/(32*d) + (a*Cot[c
 + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\csc^4(c+dx)\,dx + a\int \cot^6(c+dx)\csc^5(c+$$

$$= -\frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{1}{2}a\int \cot^4(c+dx)\csc^5(c+$$

$$= \frac{a\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d} +$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} - \frac{a\cot(c+dx)\csc^5(c+d}{32d}$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} + \frac{a\cot(c+dx)\csc^3(c+d}{128d}$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} + \frac{3a\cot(c+dx)\csc(c+d}{256d}$$

$$= \frac{3a\tanh^{-1}(\cos(c+dx))}{256d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} +$$

Mathematica [B] time = 0.09, size = 341, normalized size = 2.13

$$\frac{2a\cot(c+dx)}{63d}-\frac{a\csc^{10}\left(\frac{1}{2}(c+dx)\right)}{10240d}+\frac{3a\csc^{8}\left(\frac{1}{2}(c+dx)\right)}{4096d}-\frac{3a\csc^{6}\left(\frac{1}{2}(c+dx)\right)}{2048d}-\frac{a\csc^{4}\left(\frac{1}{2}(c+dx)\right)}{1024d}+\frac{3a\csc^{2}\left(\frac{1}{2}(c+\right.}{1024d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Cot[c + d*x])/(63*d) + (3*a*Csc[(c + d*x)/2]^2)/(1024*d) - (a*Csc[(c +
 d*x)/2]^4)/(1024*d) - (3*a*Csc[(c + d*x)/2]^6)/(2048*d) + (3*a*Csc[(c + d*
x)/2]^8)/(4096*d) - (a*Csc[(c + d*x)/2]^10)/(10240*d) + (a*Cot[c + d*x]*Csc
[c + d*x]^2)/(63*d) - (5*a*Cot[c + d*x]*Csc[c + d*x]^4)/(21*d) + (19*a*Cot[
c + d*x]*Csc[c + d*x]^6)/(63*d) - (a*Cot[c + d*x]*Csc[c + d*x]^8)/(9*d) + (
3*a*Log[Cos[(c + d*x)/2]])/(256*d) - (3*a*Log[Sin[(c + d*x)/2]])/(256*d) -
(3*a*Sec[(c + d*x)/2]^2)/(1024*d) + (a*Sec[(c + d*x)/2]^4)/(1024*d) + (3*a*
Sec[(c + d*x)/2]^6)/(2048*d) - (3*a*Sec[(c + d*x)/2]^8)/(4096*d) + (a*Sec[(
c + d*x)/2]^10)/(10240*d)
```

fricas [B] time = 0.66, size = 289, normalized size = 1.81

$$-\frac{1890\,a\cos\left(dx+c\right)^{9}-8820\,a\cos\left(dx+c\right)^{7}-16128\,a\cos\left(dx+c\right)^{5}+8820\,a\cos\left(dx+c\right)^{3}-1890\,a\cos\left(dx+c\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/161280*(1890*a*cos(d*x + c)^9 - 8820*a*cos(d*x + c)^7 - 16128*a*cos(d*x
+ c)^5 + 8820*a*cos(d*x + c)^3 - 1890*a*cos(d*x + c) - 945*(a*cos(d*x + c)^
10 - 5*a*cos(d*x + c)^8 + 10*a*cos(d*x + c)^6 - 10*a*cos(d*x + c)^4 + 5*a*c
os(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 945*(a*cos(d*x + c)^10 - 5
*a*cos(d*x + c)^8 + 10*a*cos(d*x + c)^6 - 10*a*cos(d*x + c)^4 + 5*a*cos(d*x
 + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 2560*(2*a*cos(d*x + c)^9 - 9*a*
cos(d*x + c)^7)*sin(d*x + c))/(d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*
d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.29, size = 284, normalized size = 1.78

$$\frac{126\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-315\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-1080\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-630\,a\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1290240*(126*a*tan(1/2*d*x + 1/2*c)^10 + 280*a*tan(1/2*d*x + 1/2*c)^9 - 3
15*a*tan(1/2*d*x + 1/2*c)^8 - 1080*a*tan(1/2*d*x + 1/2*c)^7 - 630*a*tan(1/2
*d*x + 1/2*c)^6 + 2520*a*tan(1/2*d*x + 1/2*c)^4 + 6720*a*tan(1/2*d*x + 1/2*
c)^3 + 1260*a*tan(1/2*d*x + 1/2*c)^2 - 15120*a*log(abs(tan(1/2*d*x + 1/2*c)
)) - 15120*a*tan(1/2*d*x + 1/2*c) + (44286*a*tan(1/2*d*x + 1/2*c)^10 + 1512
0*a*tan(1/2*d*x + 1/2*c)^9 - 1260*a*tan(1/2*d*x + 1/2*c)^8 - 6720*a*tan(1/2
*d*x + 1/2*c)^7 - 2520*a*tan(1/2*d*x + 1/2*c)^6 + 630*a*tan(1/2*d*x + 1/2*c
)^4 + 1080*a*tan(1/2*d*x + 1/2*c)^3 + 315*a*tan(1/2*d*x + 1/2*c)^2 - 280*a*
tan(1/2*d*x + 1/2*c) - 126*a)/tan(1/2*d*x + 1/2*c)^10)/d
```

maple [A] time = 0.29, size = 218, normalized size = 1.36

$$-\frac{a\left(\cos^{7}\left(dx+c\right)\right)}{9d\sin\left(dx+c\right)^{9}}-\frac{2a\left(\cos^{7}\left(dx+c\right)\right)}{63d\sin\left(dx+c\right)^{7}}-\frac{a\left(\cos^{7}\left(dx+c\right)\right)}{10d\sin\left(dx+c\right)^{10}}-\frac{3a\left(\cos^{7}\left(dx+c\right)\right)}{80d\sin\left(dx+c\right)^{8}}-\frac{a\left(\cos^{7}\left(dx+c\right)\right)}{160d\sin\left(dx+c\right)^{6}}+\frac{a\left(\cos^{7}\left(dx\right.\right.}{640d\sin\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/9/d*a/sin(d*x+c)^9*cos(d*x+c)^7-2/63/d*a/sin(d*x+c)^7*cos(d*x+c)^7-1/10/
d*a/sin(d*x+c)^10*cos(d*x+c)^7-3/80/d*a/sin(d*x+c)^8*cos(d*x+c)^7-1/160/d*a
/sin(d*x+c)^6*cos(d*x+c)^7+1/640/d*a/sin(d*x+c)^4*cos(d*x+c)^7-3/1280/d*a/s
in(d*x+c)^2*cos(d*x+c)^7-3/1280*a*cos(d*x+c)^5/d-1/256*a*cos(d*x+c)^3/d-3/2
56*a*cos(d*x+c)/d-3/256/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.57, size = 158, normalized size = 0.99

$$-\frac{63\,a\left(\frac{2\left(15\,\cos(dx+c)^{9}-70\,\cos(dx+c)^{7}-128\,\cos(dx+c)^{5}+70\,\cos(dx+c)^{3}-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^{8}+10\,\cos(dx+c)^{6}-10\,\cos(dx+c)^{4}+5\,\cos(dx+c)^{2}-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(\right.\right.\right.}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/161280*(63*a*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x + c
)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x + c
)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*lo
g(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2560*(9*tan(d*x + c)^2 +
7)*a/tan(d*x + c)^9)/d
```

mupad [B] time = 9.40, size = 319, normalized size = 1.99

$$\frac{3\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{256\,d}-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{256\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{192\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{512\,d}+\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2048\,d}+\frac{3\,a\cot(}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^11,x)
```

```
[Out] (3*a*cot(c/2 + (d*x)/2))/(256*d) - (3*a*tan(c/2 + (d*x)/2))/(256*d) - (a*co
t(c/2 + (d*x)/2)^2)/(1024*d) - (a*cot(c/2 + (d*x)/2)^3)/(192*d) - (a*cot(c/
2 + (d*x)/2)^4)/(512*d) + (a*cot(c/2 + (d*x)/2)^6)/(2048*d) + (3*a*cot(c/2
+ (d*x)/2)^7)/(3584*d) + (a*cot(c/2 + (d*x)/2)^8)/(4096*d) - (a*cot(c/2 + (
d*x)/2)^9)/(4608*d) - (a*cot(c/2 + (d*x)/2)^10)/(10240*d) + (a*tan(c/2 + (d
*x)/2)^2)/(1024*d) + (a*tan(c/2 + (d*x)/2)^3)/(192*d) + (a*tan(c/2 + (d*x)/
2)^4)/(512*d) - (a*tan(c/2 + (d*x)/2)^6)/(2048*d) - (3*a*tan(c/2 + (d*x)/2)
^7)/(3584*d) - (a*tan(c/2 + (d*x)/2)^8)/(4096*d) + (a*tan(c/2 + (d*x)/2)^9)
/(4608*d) + (a*tan(c/2 + (d*x)/2)^10)/(10240*d) - (3*a*log(tan(c/2 + (d*x)/
2)))/(256*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**11*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.587 $\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=176

$$-\frac{a\cot^{11}(c+dx)}{11d}-\frac{2a\cot^{9}(c+dx)}{9d}-\frac{a\cot^{7}(c+dx)}{7d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a\cot^{5}(c+dx)\csc^{5}(c+dx)}{10d}+\frac{a\cot^{3}}{}$$

```
[Out] 3/256*a*arctanh(cos(d*x+c))/d-1/7*a*cot(d*x+c)^7/d-2/9*a*cot(d*x+c)^9/d-1/1
1*a*cot(d*x+c)^11/d+3/256*a*cot(d*x+c)*csc(d*x+c)/d+1/128*a*cot(d*x+c)*csc(
d*x+c)^3/d-1/32*a*cot(d*x+c)*csc(d*x+c)^5/d+1/16*a*cot(d*x+c)^3*csc(d*x+c)^
5/d-1/10*a*cot(d*x+c)^5*csc(d*x+c)^5/d
```

Rubi [A] time = 0.22, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2607, 270, 2611, 3768, 3770}

$$-\frac{a\cot^{11}(c+dx)}{11d}-\frac{2a\cot^{9}(c+dx)}{9d}-\frac{a\cot^{7}(c+dx)}{7d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a\cot^{5}(c+dx)\csc^{5}(c+dx)}{10d}+\frac{a\cot^{3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Cos[c + d*x]])/(256*d) - (a*Cot[c + d*x]^7)/(7*d) - (2*a*Cot[c
 + d*x]^9)/(9*d) - (a*Cot[c + d*x]^11)/(11*d) + (3*a*Cot[c + d*x]*Csc[c + d
*x])/(256*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) - (a*Cot[c + d*x]*Cs
c[c + d*x]^5)/(32*d) + (a*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a*Cot[c
+ d*x]^5*Csc[c + d*x]^5)/(10*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
```

```
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\csc^5(c+dx)\,dx + a\int \cot^6(c+dx)\csc^6(c+$$

$$= -\frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{1}{2}a\int \cot^4(c+dx)\csc^5(c+dx$$

$$= \frac{a\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d} + \frac{1}{1}$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{2a\cot^9(c+dx)}{9d} - \frac{a\cot^{11}(c+dx)}{11d} - \frac{a\cot(}{}$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{2a\cot^9(c+dx)}{9d} - \frac{a\cot^{11}(c+dx)}{11d} + \frac{a\cot(}{}$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{2a\cot^9(c+dx)}{9d} - \frac{a\cot^{11}(c+dx)}{11d} + \frac{3a\cot}{}$$

$$= \frac{3a\tanh^{-1}(\cos(c+dx))}{256d} - \frac{a\cot^7(c+dx)}{7d} - \frac{2a\cot^9(c+dx)}{9d} -$$

Mathematica [B] time = 0.10, size = 363, normalized size = 2.06

$$\frac{8a\cot(c+dx)}{693d}-\frac{a\csc^{10}\left(\frac{1}{2}(c+dx)\right)}{10240d}+\frac{3a\csc^{8}\left(\frac{1}{2}(c+dx)\right)}{4096d}-\frac{3a\csc^{6}\left(\frac{1}{2}(c+dx)\right)}{2048d}-\frac{a\csc^{4}\left(\frac{1}{2}(c+dx)\right)}{1024d}+\frac{3a\csc^{2}\left(\frac{1}{2}(c\right.}{1024d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (8*a*Cot[c + d*x])/(693*d) + (3*a*Csc[(c + d*x)/2]^2)/(1024*d) - (a*Csc[(c
+ d*x)/2]^4)/(1024*d) - (3*a*Csc[(c + d*x)/2]^6)/(2048*d) + (3*a*Csc[(c + d
*x)/2]^8)/(4096*d) - (a*Csc[(c + d*x)/2]^10)/(10240*d) + (4*a*Cot[c + d*x]*
Csc[c + d*x]^2)/(693*d) + (a*Cot[c + d*x]*Csc[c + d*x]^4)/(231*d) - (113*a*
Cot[c + d*x]*Csc[c + d*x]^6)/(693*d) + (23*a*Cot[c + d*x]*Csc[c + d*x]^8)/(
99*d) - (a*Cot[c + d*x]*Csc[c + d*x]^10)/(11*d) + (3*a*Log[Cos[(c + d*x)/2]
])/(256*d) - (3*a*Log[Sin[(c + d*x)/2]])/(256*d) - (3*a*Sec[(c + d*x)/2]^2)
/(1024*d) + (a*Sec[(c + d*x)/2]^4)/(1024*d) + (3*a*Sec[(c + d*x)/2]^6)/(204
8*d) - (3*a*Sec[(c + d*x)/2]^8)/(4096*d) + (a*Sec[(c + d*x)/2]^10)/(10240*d
)
```

fricas [B] time = 0.91, size = 320, normalized size = 1.82

$$\frac{20480\,a\cos\left(dx+c\right)^{11}-112640\,a\cos\left(dx+c\right)^{9}+253440\,a\cos\left(dx+c\right)^{7}+10395\left(a\cos\left(dx+c\right)^{10}-5\,a\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1774080*(20480*a*cos(d*x + c)^11 - 112640*a*cos(d*x + c)^9 + 253440*a*cos
(d*x + c)^7 + 10395*(a*cos(d*x + c)^10 - 5*a*cos(d*x + c)^8 + 10*a*cos(d*x
+ c)^6 - 10*a*cos(d*x + c)^4 + 5*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) - 10395*(a*cos(d*x + c)^10 - 5*a*cos(d*x + c)^8 + 10*a
*cos(d*x + c)^6 - 10*a*cos(d*x + c)^4 + 5*a*cos(d*x + c)^2 - a)*log(-1/2*co
s(d*x + c) + 1/2)*sin(d*x + c) - 1386*(15*a*cos(d*x + c)^9 - 70*a*cos(d*x +
 c)^7 - 128*a*cos(d*x + c)^5 + 70*a*cos(d*x + c)^3 - 15*a*cos(d*x + c))*sin
(d*x + c))/((d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 -
 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [B] time = 0.29, size = 340, normalized size = 1.93

$$\frac{630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1386\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-770\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-3465\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-4950}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/14192640*(630*a*tan(1/2*d*x + 1/2*c)^11 + 1386*a*tan(1/2*d*x + 1/2*c)^10
- 770*a*tan(1/2*d*x + 1/2*c)^9 - 3465*a*tan(1/2*d*x + 1/2*c)^8 - 4950*a*tan
(1/2*d*x + 1/2*c)^7 - 6930*a*tan(1/2*d*x + 1/2*c)^6 + 6930*a*tan(1/2*d*x +
1/2*c)^5 + 27720*a*tan(1/2*d*x + 1/2*c)^4 + 23100*a*tan(1/2*d*x + 1/2*c)^3
+ 13860*a*tan(1/2*d*x + 1/2*c)^2 - 166320*a*log(abs(tan(1/2*d*x + 1/2*c)))
- 69300*a*tan(1/2*d*x + 1/2*c) + (502266*a*tan(1/2*d*x + 1/2*c)^11 + 69300*
a*tan(1/2*d*x + 1/2*c)^10 - 13860*a*tan(1/2*d*x + 1/2*c)^9 - 23100*a*tan(1/
2*d*x + 1/2*c)^8 - 27720*a*tan(1/2*d*x + 1/2*c)^7 - 6930*a*tan(1/2*d*x + 1/
2*c)^6 + 6930*a*tan(1/2*d*x + 1/2*c)^5 + 4950*a*tan(1/2*d*x + 1/2*c)^4 + 34
65*a*tan(1/2*d*x + 1/2*c)^3 + 770*a*tan(1/2*d*x + 1/2*c)^2 - 1386*a*tan(1/2
*d*x + 1/2*c) - 630*a)/tan(1/2*d*x + 1/2*c)^11)/d
```

maple [A] time = 0.29, size = 240, normalized size = 1.36

$$-\frac{a\left(\cos^{7}\left(dx+c\right)\right)}{10d\sin\left(dx+c\right)^{10}}-\frac{3a\left(\cos^{7}\left(dx+c\right)\right)}{80d\sin\left(dx+c\right)^{8}}-\frac{a\left(\cos^{7}\left(dx+c\right)\right)}{160d\sin\left(dx+c\right)^{6}}+\frac{a\left(\cos^{7}\left(dx+c\right)\right)}{640d\sin\left(dx+c\right)^{4}}-\frac{3a\left(\cos^{7}\left(dx+c\right)\right)}{1280d\sin\left(dx+c\right)^{2}}-\frac{3a\left(\cos^{5}\left(d\right.\right.}{1280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/10/d*a/sin(d*x+c)^10*cos(d*x+c)^7-3/80/d*a/sin(d*x+c)^8*cos(d*x+c)^7-1/1
60/d*a/sin(d*x+c)^6*cos(d*x+c)^7+1/640/d*a/sin(d*x+c)^4*cos(d*x+c)^7-3/1280
/d*a/sin(d*x+c)^2*cos(d*x+c)^7-3/1280*a*cos(d*x+c)^5/d-1/256*a*cos(d*x+c)^3
/d-3/256*a*cos(d*x+c)/d-3/256/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/11/d*a/sin(d*
x+c)^11*cos(d*x+c)^7-4/99/d*a/sin(d*x+c)^9*cos(d*x+c)^7-8/693/d*a/sin(d*x+c
)^7*cos(d*x+c)^7
```

maxima [A] time = 0.33, size = 168, normalized size = 0.95

$$-\frac{693\,a\left(\frac{2\left(15\,\cos(dx+c)^{9}-70\,\cos(dx+c)^{7}-128\,\cos(dx+c)^{5}+70\,\cos(dx+c)^{3}-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^{8}+10\,\cos(dx+c)^{6}-10\,\cos(dx+c)^{4}+5\,\cos(dx+c)^{2}-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(\right.\right.\right.}{1774080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/1774080*(693*a*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x +
 c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x +
 c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*
```

```
log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2560*(99*tan(d*x + c)^4
 + 154*tan(d*x + c)^2 + 63)*a/tan(d*x + c)^11)/d
```

mupad [B] time = 9.98, size = 387, normalized size = 2.20

$$\frac{5\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{1024\,d}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{1024\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}-\frac{5\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3072\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{512\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2048\,d}+\frac{a\cot}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^12,x)
```

```
[Out] (5*a*cot(c/2 + (d*x)/2))/(1024*d) - (5*a*tan(c/2 + (d*x)/2))/(1024*d) - (a*
cot(c/2 + (d*x)/2)^2)/(1024*d) - (5*a*cot(c/2 + (d*x)/2)^3)/(3072*d) - (a*c
ot(c/2 + (d*x)/2)^4)/(512*d) - (a*cot(c/2 + (d*x)/2)^5)/(2048*d) + (a*cot(c
/2 + (d*x)/2)^6)/(2048*d) + (5*a*cot(c/2 + (d*x)/2)^7)/(14336*d) + (a*cot(c
/2 + (d*x)/2)^8)/(4096*d) + (a*cot(c/2 + (d*x)/2)^9)/(18432*d) - (a*cot(c/2
 + (d*x)/2)^10)/(10240*d) - (a*cot(c/2 + (d*x)/2)^11)/(22528*d) + (a*tan(c/
2 + (d*x)/2)^2)/(1024*d) + (5*a*tan(c/2 + (d*x)/2)^3)/(3072*d) + (a*tan(c/2
 + (d*x)/2)^4)/(512*d) + (a*tan(c/2 + (d*x)/2)^5)/(2048*d) - (a*tan(c/2 + (
d*x)/2)^6)/(2048*d) - (5*a*tan(c/2 + (d*x)/2)^7)/(14336*d) - (a*tan(c/2 + (
d*x)/2)^8)/(4096*d) - (a*tan(c/2 + (d*x)/2)^9)/(18432*d) + (a*tan(c/2 + (d*
x)/2)^10)/(10240*d) + (a*tan(c/2 + (d*x)/2)^11)/(22528*d) - (3*a*log(tan(c/
2 + (d*x)/2)))/(256*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**12*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.588 $\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=209

$$-\frac{2a^2\cos^{11}(c+dx)}{11d}+\frac{4a^2\cos^9(c+dx)}{9d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^5(c+dx)\cos^7(c+dx)}{12d}-\frac{17a^2\sin^3(c+dx)\cos^7(c+}{120d}$$

```
[Out] 17/1024*a^2*x-2/7*a^2*cos(d*x+c)^7/d+4/9*a^2*cos(d*x+c)^9/d-2/11*a^2*cos(d*
x+c)^11/d+17/1024*a^2*cos(d*x+c)*sin(d*x+c)/d+17/1536*a^2*cos(d*x+c)^3*sin(
d*x+c)/d+17/1920*a^2*cos(d*x+c)^5*sin(d*x+c)/d-17/320*a^2*cos(d*x+c)^7*sin(
d*x+c)/d-17/120*a^2*cos(d*x+c)^7*sin(d*x+c)^3/d-1/12*a^2*cos(d*x+c)^7*sin(d
*x+c)^5/d
```

Rubi [A] time = 0.40, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2568, 2635, 8, 2565, 270}

$$-\frac{2a^2\cos^{11}(c+dx)}{11d}+\frac{4a^2\cos^9(c+dx)}{9d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^5(c+dx)\cos^7(c+dx)}{12d}-\frac{17a^2\sin^3(c+dx)\cos^7(c+}{120d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (17*a^2*x)/1024 - (2*a^2*Cos[c + d*x]^7)/(7*d) + (4*a^2*Cos[c + d*x]^9)/(9*
d) - (2*a^2*Cos[c + d*x]^11)/(11*d) + (17*a^2*Cos[c + d*x]*Sin[c + d*x])/(1
024*d) + (17*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(1536*d) + (17*a^2*Cos[c + d*
x]^5*Sin[c + d*x])/(1920*d) - (17*a^2*Cos[c + d*x]^7*Sin[c + d*x])/(320*d)
- (17*a^2*Cos[c + d*x]^7*Sin[c + d*x]^3)/(120*d) - (a^2*Cos[c + d*x]^7*Sin[
c + d*x]^5)/(12*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^6(c+dx)\sin^4(c+dx) + 2a^2\cos^6(c+dx)\sin^5(c+\right.$$
$$= a^2\int \cos^6(c+dx)\sin^4(c+dx)\,dx + a^2\int \cos^6(c+dx)\sin^6($$
$$= -\frac{a^2\cos^7(c+dx)\sin^3(c+dx)}{10d} - \frac{a^2\cos^7(c+dx)\sin^5(c+dx}{12d}$$
$$= -\frac{3a^2\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{17a^2\cos^7(c+dx)\sin^3(c+}{120d}$$
$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\cos^{11}(c+dx)}{11d}$$
$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\cos^{11}(c+dx)}{11d}$$
$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\cos^{11}(c+dx)}{11d}$$
$$= \frac{3a^2x}{256} - \frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\cos^{11}(c}{11d}$$
$$= \frac{17a^2x}{1024} - \frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\cos^{11}(c}{11d}$$

Mathematica [A] time = 1.39, size = 136, normalized size = 0.65

$$\frac{a^2(55440\sin(2(c+dx)) - 162855\sin(4(c+dx)) - 27720\sin(6(c+dx)) + 24255\sin(8(c+dx)) + 5544\sin(10(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(166320*c + 471240*d*x - 554400*Cos[c + d*x] - 184800*Cos[3*(c + d*x)]
 + 55440*Cos[5*(c + d*x)] + 39600*Cos[7*(c + d*x)] - 6160*Cos[9*(c + d*x)]
- 5040*Cos[11*(c + d*x)] + 55440*Sin[2*(c + d*x)] - 162855*Sin[4*(c + d*x)]
 - 27720*Sin[6*(c + d*x)] + 24255*Sin[8*(c + d*x)] + 5544*Sin[10*(c + d*x)]
 - 1155*Sin[12*(c + d*x)]))/(28385280*d)
```

fricas [A] time = 0.72, size = 137, normalized size = 0.66

$$-\frac{645120\,a^2\cos(dx+c)^{11} - 1576960\,a^2\cos(dx+c)^9 + 1013760\,a^2\cos(dx+c)^7 - 58905\,a^2dx + 231\left(1280\,a^2\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3548160*(645120*a^2*cos(d*x + c)^11 - 1576960*a^2*cos(d*x + c)^9 + 10137
60*a^2*cos(d*x + c)^7 - 58905*a^2*d*x + 231*(1280*a^2*cos(d*x + c)^11 - 473
6*a^2*cos(d*x + c)^9 + 4272*a^2*cos(d*x + c)^7 - 136*a^2*cos(d*x + c)^5 - 1
70*a^2*cos(d*x + c)^3 - 255*a^2*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.43, size = 208, normalized size = 1.00

$$\frac{17}{1024}\,a^2x-\frac{a^2\cos\left(11\,dx+11\,c\right)}{5632\,d}-\frac{a^2\cos\left(9\,dx+9\,c\right)}{4608\,d}+\frac{5\,a^2\cos\left(7\,dx+7\,c\right)}{3584\,d}+\frac{a^2\cos\left(5\,dx+5\,c\right)}{512\,d}-\frac{5\,a^2\cos\left(3\,dx+3\right.}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 17/1024*a^2*x - 1/5632*a^2*cos(11*d*x + 11*c)/d - 1/4608*a^2*cos(9*d*x + 9*
c)/d + 5/3584*a^2*cos(7*d*x + 7*c)/d + 1/512*a^2*cos(5*d*x + 5*c)/d - 5/768
*a^2*cos(3*d*x + 3*c)/d - 5/256*a^2*cos(d*x + c)/d - 1/24576*a^2*sin(12*d*x
 + 12*c)/d + 1/5120*a^2*sin(10*d*x + 10*c)/d + 7/8192*a^2*sin(8*d*x + 8*c)/
d - 1/1024*a^2*sin(6*d*x + 6*c)/d - 47/8192*a^2*sin(4*d*x + 4*c)/d + 1/512*
a^2*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.28, size = 238, normalized size = 1.14

$$\frac{a^2\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^7(dx+c)\right)}{12}-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{24}-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{64}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{384}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*cos(d*x+c)^7-1/
64*cos(d*x+c)^7*sin(d*x+c)+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*
x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c)+2*a^2*(-1/11*sin(d*x+c)^4*cos(d*x+c)^
7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/693*cos(d*x+c)^7)+a^2*(-1/10*sin(d*x+c)^
3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/160*(cos(d*x+c)^5+5/4*cos(d*x
+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c))
```

maxima [A] time = 0.36, size = 138, normalized size = 0.66

$$-\frac{81920\left(63\,\cos\left(dx+c\right)^{11}-154\,\cos\left(dx+c\right)^{9}+99\,\cos\left(dx+c\right)^{7}\right)a^2-2772\left(32\,\sin\left(2\,dx+2\,c\right)^{5}+120\,dx+12\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/28385280*(81920*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x +
c)^7)*a^2 - 2772*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8
*c) - 40*sin(4*d*x + 4*c))*a^2 - 1155*(4*sin(4*d*x + 4*c)^3 + 120*d*x + 120
*c + 9*sin(8*d*x + 8*c) - 48*sin(4*d*x + 4*c))*a^2)/d
```

mupad [B] time = 11.12, size = 518, normalized size = 2.48

$$a^2\left(\frac{17\,c}{1024}-\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{512}-\frac{128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{231}-\frac{595\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{1536}-\frac{64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{21}+\frac{11097\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2560}+\frac{704\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{63}+\frac{27449\tan}{25}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^4*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*((17*c)/1024 - (17*tan(c/2 + (d*x)/2))/512 - (128*tan(c/2 + (d*x)/2)^2
)/231 - (595*tan(c/2 + (d*x)/2)^3)/1536 - (64*tan(c/2 + (d*x)/2)^4)/21 + (1
1097*tan(c/2 + (d*x)/2)^5)/2560 + (704*tan(c/2 + (d*x)/2)^6)/63 + (27449*ta
n(c/2 + (d*x)/2)^7)/2560 - (384*tan(c/2 + (d*x)/2)^8)/7 - (202307*tan(c/2 +
 (d*x)/2)^9)/3840 + (192*tan(c/2 + (d*x)/2)^10)/7 + (28659*tan(c/2 + (d*x)/
2)^11)/256 - (64*tan(c/2 + (d*x)/2)^12)/3 - (28659*tan(c/2 + (d*x)/2)^13)/2
56 - 64*tan(c/2 + (d*x)/2)^14 + (202307*tan(c/2 + (d*x)/2)^15)/3840 + 32*ta
n(c/2 + (d*x)/2)^16 - (27449*tan(c/2 + (d*x)/2)^17)/2560 - (64*tan(c/2 + (d
*x)/2)^18)/3 - (11097*tan(c/2 + (d*x)/2)^19)/2560 + (595*tan(c/2 + (d*x)/2)
^21)/1536 + (17*tan(c/2 + (d*x)/2)^23)/512 + (17*d*x)/1024 + (51*tan(c/2 +
(d*x)/2)^2*(c + d*x))/256 + (561*tan(c/2 + (d*x)/2)^4*(c + d*x))/512 + (935
*tan(c/2 + (d*x)/2)^6*(c + d*x))/256 + (8415*tan(c/2 + (d*x)/2)^8*(c + d*x)
)/1024 + (1683*tan(c/2 + (d*x)/2)^10*(c + d*x))/128 + (3927*tan(c/2 + (d*x)
/2)^12*(c + d*x))/256 + (1683*tan(c/2 + (d*x)/2)^14*(c + d*x))/128 + (8415*
tan(c/2 + (d*x)/2)^16*(c + d*x))/1024 + (935*tan(c/2 + (d*x)/2)^18*(c + d*x
))/256 + (561*tan(c/2 + (d*x)/2)^20*(c + d*x))/512 + (51*tan(c/2 + (d*x)/2)
^22*(c + d*x))/256 + (17*tan(c/2 + (d*x)/2)^24*(c + d*x))/1024 - 32/693))/(
d*(tan(c/2 + (d*x)/2)^2 + 1)^12)
```

sympy [A] time = 63.18, size = 656, normalized size = 3.14

$$\begin{cases}\frac{5a^2x\sin^{12}(c+dx)}{1024}+\frac{15a^2x\sin^{10}(c+dx)\cos^2(c+dx)}{512}+\frac{3a^2x\sin^{10}(c+dx)}{256}+\frac{75a^2x\sin^8(c+dx)\cos^4(c+dx)}{1024}+\frac{15a^2x\sin^8(c+dx)\cos^2(c+dx)}{256}+\frac{25}{} \\ x\left(a\sin(c)+a\right)^2\sin^4(c)\cos^6(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**12/1024 + 15*a**2*x*sin(c + d*x)**10*cos(
c + d*x)**2/512 + 3*a**2*x*sin(c + d*x)**10/256 + 75*a**2*x*sin(c + d*x)**8
*cos(c + d*x)**4/1024 + 15*a**2*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 25*
a**2*x*sin(c + d*x)**6*cos(c + d*x)**6/256 + 15*a**2*x*sin(c + d*x)**6*cos(
c + d*x)**4/128 + 75*a**2*x*sin(c + d*x)**4*cos(c + d*x)**8/1024 + 15*a**2*
x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**2*x*sin(c + d*x)**2*cos(c + d
*x)**10/512 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 5*a**2*x*cos(
c + d*x)**12/1024 + 3*a**2*x*cos(c + d*x)**10/256 + 5*a**2*sin(c + d*x)**11
*cos(c + d*x)/(1024*d) + 85*a**2*sin(c + d*x)**9*cos(c + d*x)**3/(3072*d) +
 3*a**2*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 33*a**2*sin(c + d*x)**7*cos(
c + d*x)**5/(512*d) + 7*a**2*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) - 33*a
**2*sin(c + d*x)**5*cos(c + d*x)**7/(512*d) + a**2*sin(c + d*x)**5*cos(c +
d*x)**5/(10*d) - 2*a**2*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 85*a**2*sin
(c + d*x)**3*cos(c + d*x)**9/(3072*d) - 7*a**2*sin(c + d*x)**3*cos(c + d*x)
**7/(128*d) - 8*a**2*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) - 5*a**2*sin(c
+ d*x)*cos(c + d*x)**11/(1024*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**9/(256
*d) - 16*a**2*cos(c + d*x)**11/(693*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin
(c)**4*cos(c)**6, True))
```

3.589 $\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=183

$$-\frac{a^2\cos^{11}(c+dx)}{11d}+\frac{a^2\cos^9(c+dx)}{3d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^3(c+dx)\cos^7(c+dx)}{5d}-\frac{3a^2\sin(c+dx)\cos^7(c+dx)}{40d}$$

```
[Out] 3/128*a^2*x-2/7*a^2*cos(d*x+c)^7/d+1/3*a^2*cos(d*x+c)^9/d-1/11*a^2*cos(d*x+
c)^11/d+3/128*a^2*cos(d*x+c)*sin(d*x+c)/d+1/64*a^2*cos(d*x+c)^3*sin(d*x+c)/
d+1/80*a^2*cos(d*x+c)^5*sin(d*x+c)/d-3/40*a^2*cos(d*x+c)^7*sin(d*x+c)/d-1/5
*a^2*cos(d*x+c)^7*sin(d*x+c)^3/d
```

Rubi [A] time = 0.27, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{a^2\cos^{11}(c+dx)}{11d}+\frac{a^2\cos^9(c+dx)}{3d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^3(c+dx)\cos^7(c+dx)}{5d}-\frac{3a^2\sin(c+dx)\cos^7(c+dx)}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/128 - (2*a^2*Cos[c + d*x]^7)/(7*d) + (a^2*Cos[c + d*x]^9)/(3*d) -
 (a^2*Cos[c + d*x]^11)/(11*d) + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) +
 (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (a^2*Cos[c + d*x]^5*Sin[c + d*x
])/(80*d) - (3*a^2*Cos[c + d*x]^7*Sin[c + d*x])/(40*d) - (a^2*Cos[c + d*x]^
7*Sin[c + d*x]^3)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^6(c+dx)\sin^3(c+dx)+2a^2\cos^6(c+dx)\sin^4(c+\right.$$
$$= a^2\int \cos^6(c+dx)\sin^3(c+dx)\,dx + a^2\int \cos^6(c+dx)\sin^5($$
$$= -\frac{a^2\cos^7(c+dx)\sin^3(c+dx)}{5d} + \frac{1}{5}\left(3a^2\right)\int \cos^6(c+dx)\sin$$
$$= -\frac{3a^2\cos^7(c+dx)\sin(c+dx)}{40d} - \frac{a^2\cos^7(c+dx)\sin^3(c+dx}{5d}$$
$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{a^2\cos^9(c+dx)}{3d} - \frac{a^2\cos^{11}(c+dx)}{11d} + $$
$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{a^2\cos^9(c+dx)}{3d} - \frac{a^2\cos^{11}(c+dx)}{11d} + $$
$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{a^2\cos^9(c+dx)}{3d} - \frac{a^2\cos^{11}(c+dx)}{11d} + $$
$$= \frac{3a^2x}{128} - \frac{2a^2\cos^7(c+dx)}{7d} + \frac{a^2\cos^9(c+dx)}{3d} - \frac{a^2\cos^{11}(c+d}{11d}$$

Mathematica [A] time = 0.99, size = 126, normalized size = 0.69

$$\frac{a^2(4620\sin(2(c+dx)) - 9240\sin(4(c+dx)) - 2310\sin(6(c+dx)) + 1155\sin(8(c+dx)) + 462\sin(10(c+dx)) -}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(27720*c + 27720*d*x - 39270*Cos[c + d*x] - 16170*Cos[3*(c + d*x)] + 1
155*Cos[5*(c + d*x)] + 2805*Cos[7*(c + d*x)] + 385*Cos[9*(c + d*x)] - 105*C
os[11*(c + d*x)] + 4620*Sin[2*(c + d*x)] - 9240*Sin[4*(c + d*x)] - 2310*Sin
[6*(c + d*x)] + 1155*Sin[8*(c + d*x)] + 462*Sin[10*(c + d*x)]))/(1182720*d)
```

fricas [A] time = 0.80, size = 124, normalized size = 0.68

$$-\frac{13440\,a^2\cos(dx+c)^{11} - 49280\,a^2\cos(dx+c)^9 + 42240\,a^2\cos(dx+c)^7 - 3465\,a^2dx - 231\left(128\,a^2\cos(dx+c\right.}{147840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

[Out] -1/147840*(13440*a^2*cos(d*x + c)^11 - 49280*a^2*cos(d*x + c)^9 + 42240*a^2
*cos(d*x + c)^7 - 3465*a^2*d*x - 231*(128*a^2*cos(d*x + c)^9 - 176*a^2*cos(
d*x + c)^7 + 8*a^2*cos(d*x + c)^5 + 10*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x
+ c))*sin(d*x + c))/d

giac [A] time = 0.37, size = 191, normalized size = 1.04

$$\frac{3}{128}\,a^2x - \frac{a^2\cos\left(11\,dx+11\,c\right)}{11264\,d} + \frac{a^2\cos\left(9\,dx+9\,c\right)}{3072\,d} + \frac{17\,a^2\cos\left(7\,dx+7\,c\right)}{7168\,d} + \frac{a^2\cos\left(5\,dx+5\,c\right)}{1024\,d} - \frac{7\,a^2\cos\left(3\,dx+3\right.}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] 3/128*a^2*x - 1/11264*a^2*cos(11*d*x + 11*c)/d + 1/3072*a^2*cos(9*d*x + 9*c
)/d + 17/7168*a^2*cos(7*d*x + 7*c)/d + 1/1024*a^2*cos(5*d*x + 5*c)/d - 7/51
2*a^2*cos(3*d*x + 3*c)/d - 17/512*a^2*cos(d*x + c)/d + 1/2560*a^2*sin(10*d*
x + 10*c)/d + 1/1024*a^2*sin(8*d*x + 8*c)/d - 1/512*a^2*sin(6*d*x + 6*c)/d
- 1/128*a^2*sin(4*d*x + 4*c)/d + 1/256*a^2*sin(2*d*x + 2*c)/d

maple [A] time = 0.28, size = 172, normalized size = 0.94

$$\frac{a^2\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^7(dx+c)\right)}{11} - \frac{4\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{99} - \frac{8\left(\cos^7(dx+c)\right)}{693}\right) + 2a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10} - \frac{3\left(\cos^7(dx+c)\right)\sin(d\right.}{80}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)

[Out] 1/d*(a^2*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/
693*cos(d*x+c)^7)+2*a^2*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*
sin(d*x+c)+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)
+3/256*d*x+3/256*c)+a^2*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7))

maxima [A] time = 0.41, size = 116, normalized size = 0.63

$$-\frac{5120\left(63\,\cos\left(dx+c\right)^{11} - 154\,\cos\left(dx+c\right)^{9} + 99\,\cos\left(dx+c\right)^{7}\right)a^2 - 56320\left(7\,\cos\left(dx+c\right)^{9} - 9\,\cos\left(dx+c\right)^{7}\right)}{3548160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")

[Out] -1/3548160*(5120*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c)
^7)*a^2 - 56320*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^2 - 693*(32*sin(2*d

*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a
^2)/d

mupad [B] time = 12.05, size = 543, normalized size = 2.97

$$\frac{3\,a^2\,x}{128}-\frac{\frac{3\,a^2\,(c+d\,x)}{128}+\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{3323\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{320}+\frac{108\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{5}-\frac{841\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{32}+\frac{841\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32}-\frac{108\,a^2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/128 - ((3*a^2*(c + d*x))/128 + (a^2*tan(c/2 + (d*x)/2)^3)/2 - (33
23*a^2*tan(c/2 + (d*x)/2)^5)/320 + (108*a^2*tan(c/2 + (d*x)/2)^7)/5 - (841*
a^2*tan(c/2 + (d*x)/2)^9)/32 + (841*a^2*tan(c/2 + (d*x)/2)^13)/32 - (108*a^
2*tan(c/2 + (d*x)/2)^15)/5 + (3323*a^2*tan(c/2 + (d*x)/2)^17)/320 - (a^2*ta
n(c/2 + (d*x)/2)^19)/2 - (3*a^2*tan(c/2 + (d*x)/2)^21)/64 - a^2*((3*c)/128
+ (3*d*x)/128 - 20/231) + tan(c/2 + (d*x)/2)^2*((33*a^2*(c + d*x))/128 - a^
2*((33*c)/128 + (33*d*x)/128 - 20/21)) + tan(c/2 + (d*x)/2)^18*((165*a^2*(c
 + d*x))/128 - a^2*((165*c)/128 + (165*d*x)/128 - 4)) + tan(c/2 + (d*x)/2)^
4*((165*a^2*(c + d*x))/128 - a^2*((165*c)/128 + (165*d*x)/128 - 16/21)) + t
an(c/2 + (d*x)/2)^14*((495*a^2*(c + d*x))/64 - a^2*((495*c)/64 + (495*d*x)/
64 + 16)) + tan(c/2 + (d*x)/2)^16*((495*a^2*(c + d*x))/128 - a^2*((495*c)/1
28 + (495*d*x)/128 - 12)) + tan(c/2 + (d*x)/2)^6*((495*a^2*(c + d*x))/128 -
 a^2*((495*c)/128 + (495*d*x)/128 - 16/7)) + tan(c/2 + (d*x)/2)^8*((495*a^2
*(c + d*x))/64 - a^2*((495*c)/64 + (495*d*x)/64 - 312/7)) + tan(c/2 + (d*x)
/2)^10*((693*a^2*(c + d*x))/64 - a^2*((693*c)/64 + (693*d*x)/64 + 40)) + ta
n(c/2 + (d*x)/2)^12*((693*a^2*(c + d*x))/64 - a^2*((693*c)/64 + (693*d*x)/6
4 - 80)) + (3*a^2*tan(c/2 + (d*x)/2))/64)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^11)
```

sympy [A] time = 40.49, size = 384, normalized size = 2.10

$$\begin{cases}\frac{3a^2x\sin^{10}(c+dx)}{128}+\frac{15a^2x\sin^8(c+dx)\cos^2(c+dx)}{128}+\frac{15a^2x\sin^6(c+dx)\cos^4(c+dx)}{64}+\frac{15a^2x\sin^4(c+dx)\cos^6(c+dx)}{64}+\frac{15a^2x\sin^2(c+dx)\cos^8(c}{128}\\ x\left(a\sin(c)+a\right)^2\sin^3(c)\cos^6(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**10/128 + 15*a**2*x*sin(c + d*x)**8*cos(c
+ d*x)**2/128 + 15*a**2*x*sin(c + d*x)**6*cos(c + d*x)**4/64 + 15*a**2*x*si
```

```
n(c + d*x)**4*cos(c + d*x)**6/64 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**
8/128 + 3*a**2*x*cos(c + d*x)**10/128 + 3*a**2*sin(c + d*x)**9*cos(c + d*x)
/(128*d) + 7*a**2*sin(c + d*x)**7*cos(c + d*x)**3/(64*d) + a**2*sin(c + d*x
)**5*cos(c + d*x)**5/(5*d) - a**2*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 7
*a**2*sin(c + d*x)**3*cos(c + d*x)**7/(64*d) - 4*a**2*sin(c + d*x)**2*cos(c
 + d*x)**9/(63*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 3*a**2*sin
(c + d*x)*cos(c + d*x)**9/(128*d) - 8*a**2*cos(c + d*x)**11/(693*d) - 2*a**
2*cos(c + d*x)**9/(63*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**3*cos(c)*
*6, True))
```

3.590 $\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=165

$$\frac{2a^2\cos^9(c+dx)}{9d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{13a^2\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{13a^2\sin(c+dx}{480}$$

```
[Out] 13/256*a^2*x-2/7*a^2*cos(d*x+c)^7/d+2/9*a^2*cos(d*x+c)^9/d+13/256*a^2*cos(d
*x+c)*sin(d*x+c)/d+13/384*a^2*cos(d*x+c)^3*sin(d*x+c)/d+13/480*a^2*cos(d*x+
c)^5*sin(d*x+c)/d-13/80*a^2*cos(d*x+c)^7*sin(d*x+c)/d-1/10*a^2*cos(d*x+c)^7
*sin(d*x+c)^3/d
```

Rubi [A] time = 0.30, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2568, 2635, 8, 2565, 14}

$$\frac{2a^2\cos^9(c+dx)}{9d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{13a^2\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{13a^2\sin(c+dx}{480}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (13*a^2*x)/256 - (2*a^2*Cos[c + d*x]^7)/(7*d) + (2*a^2*Cos[c + d*x]^9)/(9*d
) + (13*a^2*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (13*a^2*Cos[c + d*x]^3*Sin
[c + d*x])/(384*d) + (13*a^2*Cos[c + d*x]^5*Sin[c + d*x])/(480*d) - (13*a^2
*Cos[c + d*x]^7*Sin[c + d*x])/(80*d) - (a^2*Cos[c + d*x]^7*Sin[c + d*x]^3)/
(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^6(c+dx)\sin^2(c+dx)+2a^2\cos^6(c+dx)\sin^3(c\right.$$

$$= a^2\int \cos^6(c+dx)\sin^2(c+dx)\,dx + a^2\int \cos^6(c+dx)\sin$$

$$= -\frac{a^2\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{a^2\cos^7(c+dx)\sin^3(c+dx}{10d}$$

$$= \frac{a^2\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{13a^2\cos^7(c+dx)\sin(c+dx}{80d}$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{2a^2\cos^9(c+dx)}{9d} + \frac{5a^2\cos^3(c+dx)}{192d}$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{2a^2\cos^9(c+dx)}{9d} + \frac{5a^2\cos(c+dx)\,\mathrm{s}}{128d}$$

$$= \frac{5a^2x}{128} - \frac{2a^2\cos^7(c+dx)}{7d} + \frac{2a^2\cos^9(c+dx)}{9d} + \frac{13a^2\cos(c}{}$$

$$= \frac{13a^2x}{256} - \frac{2a^2\cos^7(c+dx)}{7d} + \frac{2a^2\cos^9(c+dx)}{9d} + \frac{13a^2\cos(}{}$$

Mathematica [A] time = 0.63, size = 106, normalized size = 0.64

$$\frac{a^2(11340\sin(2(c+dx))-7560\sin(4(c+dx))-3990\sin(6(c+dx))-315\sin(8(c+dx))+126\sin(10(c+dx))-}{645120}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(12600*c + 32760*d*x - 30240*Cos[c + d*x] - 13440*Cos[3*(c + d*x)] + 2
160*Cos[7*(c + d*x)] + 560*Cos[9*(c + d*x)] + 11340*Sin[2*(c + d*x)] - 7560
*Sin[4*(c + d*x)] - 3990*Sin[6*(c + d*x)] - 315*Sin[8*(c + d*x)] + 126*Sin[
10*(c + d*x)]))/(645120*d)
```

fricas [A] time = 0.75, size = 111, normalized size = 0.67

$$\frac{17920\,a^2\cos(dx+c)^9-23040\,a^2\cos(dx+c)^7+4095\,a^2dx+21\left(384\,a^2\cos(dx+c)^9-1008\,a^2\cos(dx+c)^7+1\right.}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/80640*(17920*a^2*cos(d*x + c)^9 - 23040*a^2*cos(d*x + c)^7 + 4095*a^2*d*x
 + 21*(384*a^2*cos(d*x + c)^9 - 1008*a^2*cos(d*x + c)^7 + 104*a^2*cos(d*x +
 c)^5 + 130*a^2*cos(d*x + c)^3 + 195*a^2*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.31, size = 157, normalized size = 0.95

$$\frac{13}{256}\,a^2x+\frac{a^2\cos(9\,dx+9\,c)}{1152\,d}+\frac{3\,a^2\cos(7\,dx+7\,c)}{896\,d}-\frac{a^2\cos(3\,dx+3\,c)}{48\,d}-\frac{3\,a^2\cos(dx+c)}{64\,d}+\frac{a^2\sin(10\,dx+10\,c)}{5120\,d}-\frac{a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 13/256*a^2*x + 1/1152*a^2*cos(9*d*x + 9*c)/d + 3/896*a^2*cos(7*d*x + 7*c)/d
 - 1/48*a^2*cos(3*d*x + 3*c)/d - 3/64*a^2*cos(d*x + c)/d + 1/5120*a^2*sin(1
0*d*x + 10*c)/d - 1/2048*a^2*sin(8*d*x + 8*c)/d - 19/3072*a^2*sin(6*d*x + 6
*c)/d - 3/256*a^2*sin(4*d*x + 4*c)/d + 9/512*a^2*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.28, size = 184, normalized size = 1.12

$$\frac{a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{160}+\frac{3dx}{256}+\frac{3c}{256}\right)+2a^2\left(-\frac{(\mathrm{si}}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/16
0*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/25
6*c)+2*a^2*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+a^2*(-1/8*cos
(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*s
in(d*x+c)+5/128*d*x+5/128*c))
```

maxima [A] time = 0.34, size = 128, normalized size = 0.78

$$\frac{20480\left(7\cos\left(dx+c\right)^{9}-9\cos\left(dx+c\right)^{7}\right)a^{2}+63\left(32\sin\left(2\,dx+2\,c\right)^{5}+120\,dx+120\,c+5\sin\left(8\,dx+8\,c\right)-40\right.}{645120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/645120*(20480*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^2 + 63*(32*sin(2*d*
x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a^
2 + 210*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*
sin(4*d*x + 4*c))*a^2)/d
```

mupad [B] time = 12.12, size = 469, normalized size = 2.84

$$\frac{13\,a^2\,x}{256}-\frac{\frac{13\,a^2\,(c+d\,x)}{256}-\frac{647\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{384}-\frac{2311\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{480}+\frac{457\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{32}-\frac{2169\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{64}+\frac{2169\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (13*a^2*x)/256 - ((13*a^2*(c + d*x))/256 - (647*a^2*tan(c/2 + (d*x)/2)^3)/3
84 - (2311*a^2*tan(c/2 + (d*x)/2)^5)/480 + (457*a^2*tan(c/2 + (d*x)/2)^7)/3
2 - (2169*a^2*tan(c/2 + (d*x)/2)^9)/64 + (2169*a^2*tan(c/2 + (d*x)/2)^11)/6
4 - (457*a^2*tan(c/2 + (d*x)/2)^13)/32 + (2311*a^2*tan(c/2 + (d*x)/2)^15)/4
80 + (647*a^2*tan(c/2 + (d*x)/2)^17)/384 - (13*a^2*tan(c/2 + (d*x)/2)^19)/1
28 - (a^2*(4095*c + 4095*d*x - 10240))/80640 + tan(c/2 + (d*x)/2)^2*((65*a^
2*(c + d*x))/128 - (a^2*(40950*c + 40950*d*x - 102400))/80640) + tan(c/2 +
(d*x)/2)^4*((585*a^2*(c + d*x))/256 - (a^2*(184275*c + 184275*d*x + 184320)
)/80640) + tan(c/2 + (d*x)/2)^16*((585*a^2*(c + d*x))/256 - (a^2*(184275*c
+ 184275*d*x - 645120))/80640) + tan(c/2 + (d*x)/2)^14*((195*a^2*(c + d*x))
```

```
/32 - (a^2*(491400*c + 491400*d*x + 430080))/80640) + tan(c/2 + (d*x)/2)^6*
((195*a^2*(c + d*x))/32 - (a^2*(491400*c + 491400*d*x - 1658880))/80640) +
tan(c/2 + (d*x)/2)^10*((819*a^2*(c + d*x))/64 - (a^2*(1031940*c + 1031940*d
*x - 1290240))/80640) + tan(c/2 + (d*x)/2)^12*((1365*a^2*(c + d*x))/128 - (
a^2*(859950*c + 859950*d*x - 2150400))/80640) + (13*a^2*tan(c/2 + (d*x)/2))
/128)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^10)
```

sympy [A] time = 27.32, size = 529, normalized size = 3.21

$$\begin{cases} \frac{3a^2x\sin^{10}(c+dx)}{256} + \frac{15a^2x\sin^8(c+dx)\cos^2(c+dx)}{256} + \frac{5a^2x\sin^8(c+dx)}{128} + \frac{15a^2x\sin^6(c+dx)\cos^4(c+dx)}{128} + \frac{5a^2x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{15a^2 \\ x(a\sin(c)+a)^2\sin^2(c)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**10/256 + 15*a**2*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 5*a**2*x*sin(c + d*x)**8/128 + 15*a**2*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 5*a**2*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 15*a**2*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**2*x*sin(c + d*x)**4*cos(c + d*
x)**4/64 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 5*a**2*x*sin(c +
 d*x)**2*cos(c + d*x)**6/32 + 3*a**2*x*cos(c + d*x)**10/256 + 5*a**2*x*cos(
c + d*x)**8/128 + 3*a**2*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**2*sin(
c + d*x)**7*cos(c + d*x)**3/(128*d) + 5*a**2*sin(c + d*x)**7*cos(c + d*x)/(
128*d) + a**2*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 55*a**2*sin(c + d*x)
**5*cos(c + d*x)**3/(384*d) - 7*a**2*sin(c + d*x)**3*cos(c + d*x)**7/(128*d
) + 73*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) - 2*a**2*sin(c + d*x)**
2*cos(c + d*x)**7/(7*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 5*a
**2*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 4*a**2*cos(c + d*x)**9/(63*d), N
e(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**2*cos(c)**6, True))
```

3.591 $\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=153

$$-\frac{a^2\cos^7(c+dx)}{28d}-\frac{\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{36d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{24d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{96d}$$

```
[Out] 5/64*a^2*x-1/28*a^2*cos(d*x+c)^7/d+5/64*a^2*cos(d*x+c)*sin(d*x+c)/d+5/96*a^
2*cos(d*x+c)^3*sin(d*x+c)/d+1/24*a^2*cos(d*x+c)^5*sin(d*x+c)/d-1/9*cos(d*x+
c)^7*(a+a*sin(d*x+c))^2/d-1/36*cos(d*x+c)^7*(a^2+a^2*sin(d*x+c))/d
```

Rubi [A] time = 0.15, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{a^2\cos^7(c+dx)}{28d}-\frac{\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{36d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{24d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*x)/64 - (a^2*Cos[c + d*x]^7)/(28*d) + (5*a^2*Cos[c + d*x]*Sin[c + d*
x])/(64*d) + (5*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(96*d) + (a^2*Cos[c + d*x]
^5*Sin[c + d*x])/(24*d) - (Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2)/(9*d) - (
Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x]))/(36*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}+\frac{2}{9}\int \cos^6(c+dx)(a+a$$

$$= -\frac{\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}-\frac{\cos^7(c+dx)\left(a^2+a^2\sin\right.}{36d}$$

$$= -\frac{a^2\cos^7(c+dx)}{28d}-\frac{\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}-\frac{\cos^7(}{}$$

$$= -\frac{a^2\cos^7(c+dx)}{28d}+\frac{a^2\cos^5(c+dx)\sin(c+dx)}{24d}-\frac{\cos^7(c+d}{}$$

$$= -\frac{a^2\cos^7(c+dx)}{28d}+\frac{5a^2\cos^3(c+dx)\sin(c+dx)}{96d}+\frac{a^2\cos^5(c}{}$$

$$= -\frac{a^2\cos^7(c+dx)}{28d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{64d}+\frac{5a^2\cos^3(c}{}$$

$$= \frac{5a^2x}{64}-\frac{a^2\cos^7(c+dx)}{28d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{64d}+\frac{5a^2}{}$$

Mathematica [A] time = 0.63, size = 106, normalized size = 0.69

$$\frac{a^2(1008\sin(2(c+dx))-504\sin(4(c+dx))-336\sin(6(c+dx))-63\sin(8(c+dx))-3276\cos(c+dx)-1848\,\mathrm{c}}{32256d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(2520*c + 2520*d*x - 3276*Cos[c + d*x] - 1848*Cos[3*(c + d*x)] - 504*C
os[5*(c + d*x)] - 18*Cos[7*(c + d*x)] + 14*Cos[9*(c + d*x)] + 1008*Sin[2*(c
 + d*x)] - 504*Sin[4*(c + d*x)] - 336*Sin[6*(c + d*x)] - 63*Sin[8*(c + d*x)
]))/(32256*d)
```

fricas [A] time = 0.76, size = 98, normalized size = 0.64

$$\frac{448\,a^2\cos\left(dx+c\right)^9-1152\,a^2\cos\left(dx+c\right)^7+315\,a^2dx-21\,\left(48\,a^2\cos\left(dx+c\right)^7-8\,a^2\cos\left(dx+c\right)^5-10\,a^2\cos\right.}{4032\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4032*(448*a^2*cos(d*x + c)^9 - 1152*a^2*cos(d*x + c)^7 + 315*a^2*d*x - 21
*(48*a^2*cos(d*x + c)^7 - 8*a^2*cos(d*x + c)^5 - 10*a^2*cos(d*x + c)^3 - 15
*a^2*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.26, size = 157, normalized size = 1.03

$$\frac{5}{64}\,a^2x+\frac{a^2\cos\left(9\,dx+9\,c\right)}{2304\,d}-\frac{a^2\cos\left(7\,dx+7\,c\right)}{1792\,d}-\frac{a^2\cos\left(5\,dx+5\,c\right)}{64\,d}-\frac{11\,a^2\cos\left(3\,dx+3\,c\right)}{192\,d}-\frac{13\,a^2\cos\left(dx+c\right)}{128\,d}-\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/64*a^2*x + 1/2304*a^2*cos(9*d*x + 9*c)/d - 1/1792*a^2*cos(7*d*x + 7*c)/d
- 1/64*a^2*cos(5*d*x + 5*c)/d - 11/192*a^2*cos(3*d*x + 3*c)/d - 13/128*a^2*
cos(d*x + c)/d - 1/512*a^2*sin(8*d*x + 8*c)/d - 1/96*a^2*sin(6*d*x + 6*c)/d
 - 1/64*a^2*sin(4*d*x + 4*c)/d + 1/32*a^2*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.25, size = 116, normalized size = 0.76

$$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{9}-\frac{2\left(\cos^7(dx+c)\right)}{63}\right)+2a^2\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

[Out] 1/d*(a^2*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+2*a^2*(-1/8*cos
(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*s
in(d*x+c)+5/128*d*x+5/128*c)-1/7*a^2*cos(d*x+c)^7)

maxima [A] time = 0.98, size = 93, normalized size = 0.61

$$-\frac{4608\,a^2\cos\left(dx+c\right)^7-512\left(7\,\cos\left(dx+c\right)^9-9\,\cos\left(dx+c\right)^7\right)a^2-21\left(64\,\sin\left(2\,dx+2\,c\right)^3+120\,dx+120\,c-3\right.}{32256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/32256*(4608*a^2*cos(d*x + c)^7 - 512*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^
7)*a^2 - 21*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) -
24*sin(4*d*x + 4*c))*a^2)/d

mupad [B] time = 10.85, size = 501, normalized size = 3.27

$$\frac{5\,a^2\,x}{64}-\frac{\frac{83\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{16}-\frac{191\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{48}-\frac{145\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{16}+\frac{145\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{16}-\frac{83\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{16}+\frac{191\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{48}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6*sin(c + d*x)*(a + a*sin(c + d*x))^2,x)

[Out] (5*a^2*x)/64 - ((83*a^2*tan(c/2 + (d*x)/2)^5)/16 - (191*a^2*tan(c/2 + (d*x)
/2)^3)/48 - (145*a^2*tan(c/2 + (d*x)/2)^7)/16 + (145*a^2*tan(c/2 + (d*x)/2)
^11)/16 - (83*a^2*tan(c/2 + (d*x)/2)^13)/16 + (191*a^2*tan(c/2 + (d*x)/2)^1
5)/48 - (5*a^2*tan(c/2 + (d*x)/2)^17)/32 + (a^2*(315*c + 315*d*x))/4032 - (
a^2*(315*c + 315*d*x - 1408))/4032 + tan(c/2 + (d*x)/2)^2*((a^2*(315*c + 31
5*d*x))/448 - (a^2*(2835*c + 2835*d*x - 4608))/4032) + tan(c/2 + (d*x)/2)^1
6*((a^2*(315*c + 315*d*x))/448 - (a^2*(2835*c + 2835*d*x - 8064))/4032) + t
an(c/2 + (d*x)/2)^4*((a^2*(315*c + 315*d*x))/112 - (a^2*(11340*c + 11340*d*
x - 18432))/4032) + tan(c/2 + (d*x)/2)^14*((a^2*(315*c + 315*d*x))/112 - (a
^2*(11340*c + 11340*d*x - 32256))/4032) + tan(c/2 + (d*x)/2)^12*((a^2*(315*
c + 315*d*x))/48 - (a^2*(26460*c + 26460*d*x - 21504))/4032) + tan(c/2 + (d
x)/2)^8((a^2*(315*c + 315*d*x))/32 - (a^2*(39690*c + 39690*d*x - 16128))/
4032) + tan(c/2 + (d*x)/2)^6*((a^2*(315*c + 315*d*x))/48 - (a^2*(26460*c +
26460*d*x - 96768))/4032) + tan(c/2 + (d*x)/2)^10*((a^2*(315*c + 315*d*x))/
32 - (a^2*(39690*c + 39690*d*x - 161280))/4032) + (5*a^2*tan(c/2 + (d*x)/2)
)/32)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^9)

sympy [A] time = 16.87, size = 282, normalized size = 1.84

$$\begin{cases} \frac{5a^2x\sin^8(c+dx)}{64} + \frac{5a^2x\sin^6(c+dx)\cos^2(c+dx)}{16} + \frac{15a^2x\sin^4(c+dx)\cos^4(c+dx)}{32} + \frac{5a^2x\sin^2(c+dx)\cos^6(c+dx)}{16} + \frac{5a^2x\cos^8(c+dx)}{64} + \frac{5a^2 \mathrm{s}}{} \\ x(a\sin(c)+a)^2\sin(c)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**8/64 + 5*a**2*x*sin(c + d*x)**6*cos(c + d
*x)**2/16 + 15*a**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 5*a**2*x*sin(c +
 d*x)**2*cos(c + d*x)**6/16 + 5*a**2*x*cos(c + d*x)**8/64 + 5*a**2*sin(c +
d*x)**7*cos(c + d*x)/(64*d) + 55*a**2*sin(c + d*x)**5*cos(c + d*x)**3/(192*
d) + 73*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(192*d) - a**2*sin(c + d*x)**2
*cos(c + d*x)**7/(7*d) - 5*a**2*sin(c + d*x)*cos(c + d*x)**7/(64*d) - 2*a**
2*cos(c + d*x)**9/(63*d) - a**2*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a*sin
(c) + a)**2*sin(c)*cos(c)**6, True))
```

3.592 $\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=161

$$-\frac{a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{5a^2\sin(c+dx)}{12}$$

```
[Out] 5/8*a^2*x-a^2*arctanh(cos(d*x+c))/d+a^2*cos(d*x+c)/d+1/3*a^2*cos(d*x+c)^3/d
+1/5*a^2*cos(d*x+c)^5/d-1/7*a^2*cos(d*x+c)^7/d+5/8*a^2*cos(d*x+c)*sin(d*x+c
)/d+5/12*a^2*cos(d*x+c)^3*sin(d*x+c)/d+1/3*a^2*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2873, 2635, 8, 2592, 302, 206, 2565, 30}

$$-\frac{a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{5a^2\sin(c+dx)}{12}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*x)/8 - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d + (a^2*C
os[c + d*x]^3)/(3*d) + (a^2*Cos[c + d*x]^5)/(5*d) - (a^2*Cos[c + d*x]^7)/(7
*d) + (5*a^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*a^2*Cos[c + d*x]^3*Sin[c
 + d*x])/(12*d) + (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(2a^2\cos^6(c+dx) + a^2\cos^5(c+dx)\cot(c+dx) + a^2\cos^6(\right.$$
$$= a^2\int \cos^5(c+dx)\cot(c+dx)\,dx + a^2\int \cos^6(c+dx)\sin(c+$$
$$= \frac{a^2\cos^5(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(5a^2\right)\int \cos^4(c+dx)\,dx - \frac{a}{}$$
$$= -\frac{a^2\cos^7(c+dx)}{7d} + \frac{5a^2\cos^3(c+dx)\sin(c+dx)}{12d} + \frac{a^2\cos^5(c}{}$$
$$= \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} + \frac{a^2\cos^5(c+dx)}{5d} - \frac{a^2\cos}{}$$
$$= \frac{5a^2x}{8} - \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c}{3d}$$

Mathematica [A] time = 0.42, size = 112, normalized size = 0.70

$$\frac{a^2\left(3150\sin(2(c+dx)) + 630\sin(4(c+dx)) + 70\sin(6(c+dx)) + 8715\cos(c+dx) + 665\cos(3(c+dx)) - 21\,\mathrm{co}\right.}{6720}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(4200*c + 4200*d*x + 8715*Cos[c + d*x] + 665*Cos[3*(c + d*x)] - 21*Cos
[5*(c + d*x)] - 15*Cos[7*(c + d*x)] - 6720*Log[Cos[(c + d*x)/2]] + 6720*Log
[Sin[(c + d*x)/2]] + 3150*Sin[2*(c + d*x)] + 630*Sin[4*(c + d*x)] + 70*Sin[
6*(c + d*x)]))/(6720*d)
```

fricas [A] time = 0.58, size = 141, normalized size = 0.88

$$-\frac{120\,a^2\cos\left(dx+c\right)^7 - 168\,a^2\cos\left(dx+c\right)^5 - 280\,a^2\cos\left(dx+c\right)^3 - 525\,a^2dx - 840\,a^2\cos\left(dx+c\right) + 420\,a^2\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/840*(120*a^2*cos(d*x + c)^7 - 168*a^2*cos(d*x + c)^5 - 280*a^2*cos(d*x +
 c)^3 - 525*a^2*d*x - 840*a^2*cos(d*x + c) + 420*a^2*log(1/2*cos(d*x + c) +
 1/2) - 420*a^2*log(-1/2*cos(d*x + c) + 1/2) - 35*(8*a^2*cos(d*x + c)^5 + 1
0*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.25, size = 245, normalized size = 1.52

$$525\,(dx+c)a^2 + 840\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - \frac{2\left(1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} - 1680\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12} + 980\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(525*(d*x + c)*a^2 + 840*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(1155
*a^2*tan(1/2*d*x + 1/2*c)^13 - 1680*a^2*tan(1/2*d*x + 1/2*c)^12 + 980*a^2*t
an(1/2*d*x + 1/2*c)^11 - 10080*a^2*tan(1/2*d*x + 1/2*c)^10 + 2975*a^2*tan(1
/2*d*x + 1/2*c)^9 - 16240*a^2*tan(1/2*d*x + 1/2*c)^8 - 24640*a^2*tan(1/2*d*
x + 1/2*c)^6 - 2975*a^2*tan(1/2*d*x + 1/2*c)^5 - 14448*a^2*tan(1/2*d*x + 1/
2*c)^4 - 980*a^2*tan(1/2*d*x + 1/2*c)^3 - 6496*a^2*tan(1/2*d*x + 1/2*c)^2 -
 1155*a^2*tan(1/2*d*x + 1/2*c) - 1168*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/
d
```

maple [A] time = 0.48, size = 165, normalized size = 1.02

$$-\frac{a^2\left(\cos^7(dx+c)\right)}{7d} + \frac{a^2\left(\cos^5(dx+c)\right)\sin(dx+c)}{3d} + \frac{5a^2\left(\cos^3(dx+c)\right)\sin(dx+c)}{12d} + \frac{5a^2\cos(dx+c)\sin(dx+}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/7*a^2*cos(d*x+c)^7/d+1/3*a^2*cos(d*x+c)^5*sin(d*x+c)/d+5/12*a^2*cos(d*x+
c)^3*sin(d*x+c)/d+5/8*a^2*cos(d*x+c)*sin(d*x+c)/d+5/8*a^2*x+5/8/d*a^2*c+1/5
*a^2*cos(d*x+c)^5/d+1/3*a^2*cos(d*x+c)^3/d+a^2*cos(d*x+c)/d+1/d*a^2*ln(csc(
d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.43, size = 123, normalized size = 0.76

$$-\frac{480\,a^2\cos(dx+c)^7 - 112\left(6\,\cos(dx+c)^5 + 10\,\cos(dx+c)^3 + 30\,\cos(dx+c) - 15\,\log(\cos(dx+c)+1) + 1\right.}{336}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*(480*a^2*cos(d*x + c)^7 - 112*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3
 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a
```

```
^2 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin
(2*d*x + 2*c))*a^2)/d
```

mupad [B] time = 10.84, size = 384, normalized size = 2.39

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{5\,a^2\,\mathrm{atan}\left(\frac{25\,a^4}{16\left(\frac{5\,a^4}{2}-\frac{25\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{5\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{5\,a^4}{2}-\frac{25\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}+\frac{-\frac{11\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{4}+4\,a^2\,\tan\left(\frac{c}{2}+\frac{d}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (5*a^2*atan((25*a^4)/(16*((5*a^4)/2 - (25
*a^4*tan(c/2 + (d*x)/2))/16)) + (5*a^4*tan(c/2 + (d*x)/2))/(2*((5*a^4)/2 -
(25*a^4*tan(c/2 + (d*x)/2))/16))))/(4*d) + ((232*a^2*tan(c/2 + (d*x)/2)^2)/
15 + (7*a^2*tan(c/2 + (d*x)/2)^3)/3 + (172*a^2*tan(c/2 + (d*x)/2)^4)/5 + (8
5*a^2*tan(c/2 + (d*x)/2)^5)/12 + (176*a^2*tan(c/2 + (d*x)/2)^6)/3 + (116*a^
2*tan(c/2 + (d*x)/2)^8)/3 - (85*a^2*tan(c/2 + (d*x)/2)^9)/12 + 24*a^2*tan(c
/2 + (d*x)/2)^10 - (7*a^2*tan(c/2 + (d*x)/2)^11)/3 + 4*a^2*tan(c/2 + (d*x)/
2)^12 - (11*a^2*tan(c/2 + (d*x)/2)^13)/4 + (292*a^2)/105 + (11*a^2*tan(c/2
+ (d*x)/2))/4)/(d*(7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 35*ta
n(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 + 7
*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.593 $\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=158

$$\frac{2a^2\cos^5(c+dx)}{5d}+\frac{2a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{7a^2\sin^3(c+$$

```
[Out] -25/16*a^2*x-2*a^2*arctanh(cos(d*x+c))/d+2*a^2*cos(d*x+c)/d+2/3*a^2*cos(d*x
+c)^3/d+2/5*a^2*cos(d*x+c)^5/d-a^2*cot(d*x+c)/d-7/16*a^2*cos(d*x+c)*sin(d*x
+c)/d-7/24*a^2*cos(d*x+c)*sin(d*x+c)^3/d+1/6*a^2*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A] time = 0.23, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2872, 3770, 3767, 8, 2638, 2633, 2635}

$$\frac{2a^2\cos^5(c+dx)}{5d}+\frac{2a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{7a^2\sin^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-25*a^2*x)/16 - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d +
 (2*a^2*Cos[c + d*x]^3)/(3*d) + (2*a^2*Cos[c + d*x]^5)/(5*d) - (a^2*Cot[c +
 d*x])/d - (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (7*a^2*Cos[c + d*x]*S
in[c + d*x]^3)/(24*d) + (a^2*Cos[c + d*x]*Sin[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-2a^8+2a^8\csc(c+dx)+a^8\csc^2(c+dx)-6a^8\sin(c+d\right.}{}$$

$$= -2a^2x + a^2\int \csc^2(c+dx)\,dx - a^2\int \sin^6(c+dx)\,dx + \left(2a\right.$$

$$= -2a^2x - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{6a^2\cos(c+dx)}{d} - \frac{a^2\,\mathrm{co}}{}$$

$$= -2a^2x - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{2a^2}{}$$

$$= -\frac{5a^2x}{4} - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{2a^2}{}$$

$$= -\frac{25a^2x}{16} - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{2a}{}$$

Mathematica [A] time = 0.31, size = 110, normalized size = 0.70

$$\frac{a^2\left(-255\sin(2(c+dx))+15\sin(4(c+dx))+5\sin(6(c+dx))+2640\cos(c+dx)+280\cos(3(c+dx))+24\cos\right.}{960a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-1500*c - 1500*d*x + 2640*Cos[c + d*x] + 280*Cos[3*(c + d*x)] + 24*Co
s[5*(c + d*x)] - 960*Cot[c + d*x] - 1920*Log[Cos[(c + d*x)/2]] + 1920*Log[S
in[(c + d*x)/2]] - 255*Sin[2*(c + d*x)] + 15*Sin[4*(c + d*x)] + 5*Sin[6*(c
+ d*x)]))/(960*d)
```

fricas [A] time = 0.80, size = 161, normalized size = 1.02

$$-\frac{40\,a^2\cos\left(dx+c\right)^7-50\,a^2\cos\left(dx+c\right)^5-125\,a^2\cos\left(dx+c\right)^3+240\,a^2\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(40*a^2*cos(d*x + c)^7 - 50*a^2*cos(d*x + c)^5 - 125*a^2*cos(d*x + c
)^3 + 240*a^2*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 240*a^2*log(-1/2*c
os(d*x + c) + 1/2)*sin(d*x + c) + 375*a^2*cos(d*x + c) - (96*a^2*cos(d*x +
c)^5 + 160*a^2*cos(d*x + c)^3 - 375*a^2*d*x + 480*a^2*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

giac [A] time = 0.27, size = 274, normalized size = 1.73

$$-\frac{375\,(dx+c)a^2-480\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-120\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{120\left(4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(105\,a^2\tan\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/240*(375*(d*x + c)*a^2 - 480*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 120*a^
2*tan(1/2*d*x + 1/2*c) + 120*(4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x
 + 1/2*c) - 2*(105*a^2*tan(1/2*d*x + 1/2*c)^11 + 1440*a^2*tan(1/2*d*x + 1/2
*c)^10 + 595*a^2*tan(1/2*d*x + 1/2*c)^9 + 4320*a^2*tan(1/2*d*x + 1/2*c)^8 -
```

```
150*a^2*tan(1/2*d*x + 1/2*c)^7 + 7360*a^2*tan(1/2*d*x + 1/2*c)^6 + 150*a^2
*tan(1/2*d*x + 1/2*c)^5 + 6720*a^2*tan(1/2*d*x + 1/2*c)^4 - 595*a^2*tan(1/2
*d*x + 1/2*c)^3 + 2976*a^2*tan(1/2*d*x + 1/2*c)^2 - 105*a^2*tan(1/2*d*x + 1
/2*c) + 736*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 0.42, size = 175, normalized size = 1.11

$$-\frac{5a^2\left(\cos^5(dx+c)\right)\sin(dx+c)}{6d}-\frac{25a^2\left(\cos^3(dx+c)\right)\sin(dx+c)}{24d}-\frac{25a^2\cos(dx+c)\sin(dx+c)}{16d}-\frac{25a^2x}{16}-\frac{25a^2c}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -5/6*a^2*cos(d*x+c)^5*sin(d*x+c)/d-25/24*a^2*cos(d*x+c)^3*sin(d*x+c)/d-25/1
6*a^2*cos(d*x+c)*sin(d*x+c)/d-25/16*a^2*x-25/16/d*a^2*c+2/5*a^2*cos(d*x+c)^
5/d+2/3*a^2*cos(d*x+c)^3/d+2*a^2*cos(d*x+c)/d+2/d*a^2*ln(csc(d*x+c)-cot(d*x
+c))-1/d*a^2/sin(d*x+c)*cos(d*x+c)^7
```

maxima [A] time = 0.78, size = 173, normalized size = 1.09

$$\frac{64\left(6\cos(dx+c)^5+10\cos(dx+c)^3+30\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/960*(64*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(
cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 - 5*(4*sin(2*d*x + 2*c)^3
 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 - 120*(15*
d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 +
2*tan(d*x + c)^3 + tan(d*x + c)))*a^2)/d
```

mupad [B] time = 8.97, size = 401, normalized size = 2.54

$$\frac{2a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{25a^2\,\mathrm{atan}\left(\frac{625a^4}{64\left(\frac{25a^4}{2}+\frac{625a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}\right)}-\frac{25a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\left(\frac{25a^4}{2}+\frac{625a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}\right)}\right)}{8d}+\frac{\frac{3a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{4}+24a^2\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (2*a^2*log(tan(c/2 + (d*x)/2)))/d + (25*a^2*atan((625*a^4)/(64*((25*a^4)/2
+ (625*a^4*tan(c/2 + (d*x)/2))/64)) - (25*a^4*tan(c/2 + (d*x)/2))/(2*((25*a
^4)/2 + (625*a^4*tan(c/2 + (d*x)/2))/64))))/(8*d) + ((248*a^2*tan(c/2 + (d*
x)/2)^3)/5 - (31*a^2*tan(c/2 + (d*x)/2)^2)/4 - (299*a^2*tan(c/2 + (d*x)/2)^
4)/12 + 112*a^2*tan(c/2 + (d*x)/2)^5 - (35*a^2*tan(c/2 + (d*x)/2)^6)/2 + (3
68*a^2*tan(c/2 + (d*x)/2)^7)/3 - (35*a^2*tan(c/2 + (d*x)/2)^8)/2 + 72*a^2*t
an(c/2 + (d*x)/2)^9 + (47*a^2*tan(c/2 + (d*x)/2)^10)/12 + 24*a^2*tan(c/2 +
(d*x)/2)^11 + (3*a^2*tan(c/2 + (d*x)/2)^12)/4 - a^2 + (184*a^2*tan(c/2 + (d
*x)/2))/15)/(d*(2*tan(c/2 + (d*x)/2) + 12*tan(c/2 + (d*x)/2)^3 + 30*tan(c/2
 + (d*x)/2)^5 + 40*tan(c/2 + (d*x)/2)^7 + 30*tan(c/2 + (d*x)/2)^9 + 12*tan(
c/2 + (d*x)/2)^11 + 2*tan(c/2 + (d*x)/2)^13)) + (a^2*tan(c/2 + (d*x)/2))/(2
*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.594 $\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=140

$$\frac{a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}+\frac{a^2\sin^3(c+dx)\cos(c+dx)}{2d}-\frac{9a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{3a^2\text{t}}{}$$

```
[Out] -15/4*a^2*x+3/2*a^2*arctanh(cos(d*x+c))/d-a^2*cos(d*x+c)/d+1/5*a^2*cos(d*x+
c)^5/d-2*a^2*cot(d*x+c)/d-1/2*a^2*cot(d*x+c)*csc(d*x+c)/d-9/4*a^2*cos(d*x+c
)*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.21, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2872, 3770, 3767, 8, 3768, 2635, 2633}

$$\frac{a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}+\frac{a^2\sin^3(c+dx)\cos(c+dx)}{2d}-\frac{9a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{3a^2\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-15*a^2*x)/4 + (3*a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (a^2*Cos[c + d*x])/d
+ (a^2*Cos[c + d*x]^5)/(5*d) - (2*a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]*C
sc[c + d*x])/(2*d) - (9*a^2*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^2*Cos[c +
 d*x]*Sin[c + d*x]^3)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-6a^8 - 2a^8\csc(c+dx) + 2a^8\csc^2(c+dx) + a^8\csc^3(c+\right.}{}$$

$$= -6a^2x + a^2\int \csc^3(c+dx)\,dx - a^2\int \sin^5(c+dx)\,dx - \left(2\right.$$

$$= -6a^2x + \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cot(c+dx)\csc(c+}{2d}$$

$$= -3a^2x + \frac{3a^2\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^2\cos(c+dx)}{d} + \frac{a^2 c}{}$$

$$= -\frac{15a^2x}{4} + \frac{3a^2\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^2\cos(c+dx)}{d} + \frac{a^2}{}$$

Mathematica [A] time = 5.53, size = 174, normalized size = 1.24

$$(a\sin(c+dx)+a)^2\Big(-300(c+dx)-80\sin(2(c+dx))-5\sin(4(c+dx))-70\cos(c+dx)+5\cos(3(c+dx))+\mathrm{co}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^2*(-300*(c + d*x) - 70*Cos[c + d*x] + 5*Cos[3*(c + d*
x)] + Cos[5*(c + d*x)] - 80*Cot[(c + d*x)/2] - 10*Csc[(c + d*x)/2]^2 + 120*
Log[Cos[(c + d*x)/2]] - 120*Log[Sin[(c + d*x)/2]] + 10*Sec[(c + d*x)/2]^2 -
 80*Sin[2*(c + d*x)] - 5*Sin[4*(c + d*x)] + 80*Tan[(c + d*x)/2]))/(80*d*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

fricas [A] time = 0.93, size = 199, normalized size = 1.42

$$4\,a^2\cos\left(dx+c\right)^7-4\,a^2\cos\left(dx+c\right)^5-75\,a^2dx\cos\left(dx+c\right)^2-20\,a^2\cos\left(dx+c\right)^3+75\,a^2dx+30\,a^2\cos\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/20*(4*a^2*cos(d*x + c)^7 - 4*a^2*cos(d*x + c)^5 - 75*a^2*d*x*cos(d*x + c)
^2 - 20*a^2*cos(d*x + c)^3 + 75*a^2*d*x + 30*a^2*cos(d*x + c) + 15*(a^2*cos
(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2) - 15*(a^2*cos(d*x + c)^2 - a
^2)*log(-1/2*cos(d*x + c) + 1/2) - 5*(2*a^2*cos(d*x + c)^5 + 5*a^2*cos(d*x
+ c)^3 - 15*a^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.26, size = 244, normalized size = 1.74

$$5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-150\,(dx+c)a^2-60\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+40\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{5\left(18\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\tan\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/40*(5*a^2*tan(1/2*d*x + 1/2*c)^2 - 150*(d*x + c)*a^2 - 60*a^2*log(abs(tan
(1/2*d*x + 1/2*c))) + 40*a^2*tan(1/2*d*x + 1/2*c) + 5*(18*a^2*tan(1/2*d*x +
```

```
 1/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2 + 4*(4
5*a^2*tan(1/2*d*x + 1/2*c)^9 + 50*a^2*tan(1/2*d*x + 1/2*c)^7 - 80*a^2*tan(1
/2*d*x + 1/2*c)^6 - 80*a^2*tan(1/2*d*x + 1/2*c)^4 - 50*a^2*tan(1/2*d*x + 1/
2*c)^3 - 80*a^2*tan(1/2*d*x + 1/2*c)^2 - 45*a^2*tan(1/2*d*x + 1/2*c) - 16*a
^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

maple [A] time = 0.48, size = 199, normalized size = 1.42

$$-\frac{3a^2\left(\cos^5(dx+c)\right)}{10d}-\frac{a^2\left(\cos^3(dx+c)\right)}{2d}-\frac{3a^2\cos(dx+c)}{2d}-\frac{3a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{2a^2\left(\cos^7(dx+\right.}{d\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/10*a^2*cos(d*x+c)^5/d-1/2*a^2*cos(d*x+c)^3/d-3/2*a^2*cos(d*x+c)/d-3/2/d*
a^2*ln(csc(d*x+c)-cot(d*x+c))-2/d*a^2/sin(d*x+c)*cos(d*x+c)^7-2*a^2*cos(d*x
+c)^5*sin(d*x+c)/d-5/2*a^2*cos(d*x+c)^3*sin(d*x+c)/d-15/4*a^2*cos(d*x+c)*si
n(d*x+c)/d-15/4*a^2*x-15/4/d*a^2*c-1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7
```

maxima [A] time = 0.81, size = 191, normalized size = 1.36

$$\frac{2\left(6\cos(dx+c)^5+10\cos(dx+c)^3+30\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/60*(2*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(co
s(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 - 5*(4*cos(d*x + c)^3 - 6*c
os(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c) +
1) + 15*log(cos(d*x + c) - 1))*a^2 - 15*(15*d*x + 15*c + (15*tan(d*x + c)^4
 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x + c)
))*a^2)/d
```

mupad [B] time = 8.94, size = 377, normalized size = 2.69

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d}-\frac{3a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}-\frac{15a^2\,\mathrm{atan}\left(\frac{225a^4}{4\left(\frac{45a^4}{2}-\frac{225a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}\right)}+\frac{45a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\left(\frac{45a^4}{2}-\frac{225a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}\right)}\right)}{2d}-\frac{-14a^2\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*a^2*log(tan(c/2 + (d*x)/2)))/(2*d) -
(15*a^2*atan((225*a^4)/(4*((45*a^4)/2 - (225*a^4*tan(c/2 + (d*x)/2))/4)) +
(45*a^4*tan(c/2 + (d*x)/2))/(2*((45*a^4)/2 - (225*a^4*tan(c/2 + (d*x)/2))/4
))))/(2*d) - ((89*a^2*tan(c/2 + (d*x)/2)^2)/10 + 38*a^2*tan(c/2 + (d*x)/2)^
3 + 37*a^2*tan(c/2 + (d*x)/2)^4 + 60*a^2*tan(c/2 + (d*x)/2)^5 + 37*a^2*tan(
c/2 + (d*x)/2)^6 + 40*a^2*tan(c/2 + (d*x)/2)^7 + (69*a^2*tan(c/2 + (d*x)/2)
^8)/2 + (a^2*tan(c/2 + (d*x)/2)^10)/2 - 14*a^2*tan(c/2 + (d*x)/2)^11 + a^2/
2 + 4*a^2*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 20*tan(c/2 + (d*
x)/2)^4 + 40*tan(c/2 + (d*x)/2)^6 + 40*tan(c/2 + (d*x)/2)^8 + 20*tan(c/2 +
(d*x)/2)^10 + 4*tan(c/2 + (d*x)/2)^12)) + (a^2*tan(c/2 + (d*x)/2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.595 $\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=153

$$-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{4a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{5a^2\sin(c+}{}$$

```
[Out] 5/8*a^2*x+5*a^2*arctanh(cos(d*x+c))/d-4*a^2*cos(d*x+c)/d-2/3*a^2*cos(d*x+c)
^3/d+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d-a^2*cot(d*x+c)*csc(d*x+c)/d-5/
8*a^2*cos(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.22, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2872, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{4a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{5a^2\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*x)/8 + (5*a^2*ArcTanh[Cos[c + d*x]])/d - (4*a^2*Cos[c + d*x])/d - (2
*a^2*Cos[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3
*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d - (5*a^2*Cos[c + d*x]*Sin[c + d*x])
/(8*d) + (a^2*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int\left(-6a^8\csc(c+dx)-2a^8\csc^2(c+dx)+2a^8\csc^3(c+dx)\right.}{}$$

$$= a^2\int\csc^4(c+dx)\,dx - a^2\int\sin^4(c+dx)\,dx - \left(2a^2\right)\int\csc$$

$$= \frac{6a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{6a^2\cos(c+dx)}{d} - \frac{a^2\cot(c+d}{}$$

$$= a^2x + \frac{5a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^2\cos(c+dx)}{d} - \frac{2a^2\mathrm{co}}{}$$

$$= \frac{5a^2x}{8} + \frac{5a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^2\cos(c+dx)}{d} - \frac{2a^2}{}$$

Mathematica [A] time = 3.37, size = 209, normalized size = 1.37

$$\frac{a^2(\sin(c+dx)+1)^2\left(60(c+dx)-24\sin(2(c+dx))-3\sin(4(c+dx))-432\cos(c+dx)-16\cos(3(c+dx))-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(60*(c + d*x) - 432*Cos[c + d*x] - 16*Cos[3*(c +
d*x)] + 64*Cot[(c + d*x)/2] - 24*Csc[(c + d*x)/2]^2 + 480*Log[Cos[(c + d*x)
/2]] - 480*Log[Sin[(c + d*x)/2]] + 24*Sec[(c + d*x)/2]^2 + 32*Csc[c + d*x]^
3*Sin[(c + d*x)/2]^4 - 2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 24*Sin[2*(c + d*
x)] - 3*Sin[4*(c + d*x)] - 64*Tan[(c + d*x)/2]))/(96*d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^4)
```

fricas [A] time = 0.75, size = 219, normalized size = 1.43

$$\frac{6\,a^2\cos\left(dx+c\right)^7-3\,a^2\cos\left(dx+c\right)^5+20\,a^2\cos\left(dx+c\right)^3-15\,a^2\cos\left(dx+c\right)+60\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(6*a^2*cos(d*x + c)^7 - 3*a^2*cos(d*x + c)^5 + 20*a^2*cos(d*x + c)^3 -
 15*a^2*cos(d*x + c) + 60*(a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) +
```

```
 1/2)*sin(d*x + c) - 60*(a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) +
1/2)*sin(d*x + c) - (16*a^2*cos(d*x + c)^5 - 15*a^2*d*x*cos(d*x + c)^2 + 80
*a^2*cos(d*x + c)^3 + 15*a^2*d*x - 120*a^2*cos(d*x + c))*sin(d*x + c))/((d*
cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.28, size = 274, normalized size = 1.79

$$\frac{a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 6\,a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 15\,(dx+c)a^2 - 120\,a^2 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - 15\,a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 + 15*(d*x +
 c)*a^2 - 120*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a^2*tan(1/2*d*x + 1/2
*c) + (220*a^2*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*a
^2*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^3 + 2*(15*a^2*tan(1/2*d
*x + 1/2*c)^7 - 144*a^2*tan(1/2*d*x + 1/2*c)^6 - 9*a^2*tan(1/2*d*x + 1/2*c)
^5 - 336*a^2*tan(1/2*d*x + 1/2*c)^4 + 9*a^2*tan(1/2*d*x + 1/2*c)^3 - 304*a^
2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c) - 112*a^2)/(tan(1/2*
d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.43, size = 223, normalized size = 1.46

$$\frac{a^2\left(\cos^7(dx+c)\right)}{3d\sin(dx+c)} + \frac{a^2\left(\cos^5(dx+c)\right)\sin(dx+c)}{3d} + \frac{5a^2\left(\cos^3(dx+c)\right)\sin(dx+c)}{12d} + \frac{5a^2\cos(dx+c)\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/3/d*a^2/sin(d*x+c)*cos(d*x+c)^7+1/3*a^2*cos(d*x+c)^5*sin(d*x+c)/d+5/12*a^
2*cos(d*x+c)^3*sin(d*x+c)/d+5/8*a^2*cos(d*x+c)*sin(d*x+c)/d+5/8*a^2*x+5/8/d
*a^2*c-1/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-a^2*cos(d*x+c)^5/d-5/3*a^2*cos(d*x
+c)^3/d-5*a^2*cos(d*x+c)/d-5/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*a^2/sin(
d*x+c)^3*cos(d*x+c)^7
```

maxima [A] time = 0.76, size = 190, normalized size = 1.24

$$-\frac{4\left(4\,\cos(dx+c)^3 - \frac{6\,\cos(dx+c)}{\cos(dx+c)^2-1} + 24\,\cos(dx+c) - 15\,\log(\cos(dx+c)+1) + 15\,\log(\cos(dx+c)-1)\right)a^2 + 3}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/24*(4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 + 3*(15*
d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 +
2*tan(d*x + c)^3 + tan(d*x + c)))*a^2 - 4*(15*d*x + 15*c + (15*tan(d*x + c)
^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a^2)/d
```

mupad [B] time = 8.93, size = 384, normalized size = 2.51

$$\frac{a^2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}+\frac{a^2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{5\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{5\,a^2\,\mathrm{atan}\left(\frac{25\,a^4}{16\left(\frac{25\,a^4}{2}+\frac{25\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}-\frac{25\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{25\,a^4}{2}+\frac{25\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(4*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (5*a
^2*log(tan(c/2 + (d*x)/2)))/d - (5*a^2*atan((25*a^4)/(16*((25*a^4)/2 + (25*
a^4*tan(c/2 + (d*x)/2))/16)) - (25*a^4*tan(c/2 + (d*x)/2))/(2*((25*a^4)/2 +
 (25*a^4*tan(c/2 + (d*x)/2))/16))))/(4*d) - (5*a^2*tan(c/2 + (d*x)/2))/(8*d
) - ((248*a^2*tan(c/2 + (d*x)/2)^3)/3 - (11*a^2*tan(c/2 + (d*x)/2)^2)/3 - 8
*a^2*tan(c/2 + (d*x)/2)^4 + (644*a^2*tan(c/2 + (d*x)/2)^5)/3 - (104*a^2*tan
(c/2 + (d*x)/2)^6)/3 + 232*a^2*tan(c/2 + (d*x)/2)^7 - (41*a^2*tan(c/2 + (d*
x)/2)^8)/3 + 98*a^2*tan(c/2 + (d*x)/2)^9 - 15*a^2*tan(c/2 + (d*x)/2)^10 + a
^2/3 + 2*a^2*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 32*tan(c/2 +
(d*x)/2)^5 + 48*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9 + 8*tan(c/2
+ (d*x)/2)^11))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.596 $\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=153

$$-\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{4a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+\frac{5a^2\tanh^{-1}(\cos}{8d}$$

```
[Out] 5*a^2*x+5/8*a^2*arctanh(cos(d*x+c))/d-a^2*cos(d*x+c)/d-1/3*a^2*cos(d*x+c)^3
/d+4*a^2*cot(d*x+c)/d-2/3*a^2*cot(d*x+c)^3/d+5/8*a^2*cot(d*x+c)*csc(d*x+c)/
d-1/4*a^2*cot(d*x+c)*csc(d*x+c)^3/d+a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2872, 3767, 8, 3768, 3770, 2638, 2635, 2633}

$$-\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{4a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+\frac{5a^2\tanh^{-1}(\cos}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] 5*a^2*x + (5*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (a^2*Cos[c + d*x])/d - (a^2
*Cos[c + d*x]^3)/(3*d) + (4*a^2*Cot[c + d*x])/d - (2*a^2*Cot[c + d*x]^3)/(3
*d) + (5*a^2*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d
*x]^3)/(4*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(6a^8-6a^8\csc^2(c+dx)-2a^8\csc^3(c+dx)+2a^8\csc^4(c+d\right.}{}$$
$$= 6a^2x + a^2\int \csc^5(c+dx)\,dx - a^2\int \sin^3(c+dx)\,dx - \left(2a^2\right)$$
$$= 6a^2x - \frac{2a^2\cos(c+dx)}{d} + \frac{a^2\cot(c+dx)\csc(c+dx)}{d} - \frac{a^2\cot}{}$$
$$= 5a^2x + \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cos(c+dx)}{d} - \frac{a^2\cos^3(c}{3d}$$
$$= 5a^2x + \frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^2\cos(c+dx)}{d} - \frac{a^2\cos^3(}{3a}$$

Mathematica [A] time = 1.29, size = 227, normalized size = 1.48

$$\frac{a^2(\sin(c+dx)+1)^2\left(960(c+dx)+96\sin(2(c+dx))-240\cos(c+dx)-16\cos(3(c+dx))-448\tan\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(960*(c + d*x) - 240*Cos[c + d*x] - 16*Cos[3*(c +
 d*x)] + 448*Cot[(c + d*x)/2] + 30*Csc[(c + d*x)/2]^2 - 3*Csc[(c + d*x)/2]^
4 + 120*Log[Cos[(c + d*x)/2]] - 120*Log[Sin[(c + d*x)/2]] - 30*Sec[(c + d*x
)/2]^2 + 3*Sec[(c + d*x)/2]^4 + 128*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 8*C
sc[(c + d*x)/2]^4*Sin[c + d*x] + 96*Sin[2*(c + d*x)] - 448*Tan[(c + d*x)/2]
))/(192*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

fricas [A] time = 0.81, size = 245, normalized size = 1.60

$$-\frac{16\,a^2\cos\left(dx+c\right)^7 - 240\,a^2dx\cos\left(dx+c\right)^4 + 16\,a^2\cos\left(dx+c\right)^5 + 480\,a^2dx\cos\left(dx+c\right)^2 - 50\,a^2\cos\left(dx+c\right)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(16*a^2*cos(d*x + c)^7 - 240*a^2*d*x*cos(d*x + c)^4 + 16*a^2*cos(d*x
+ c)^5 + 480*a^2*d*x*cos(d*x + c)^2 - 50*a^2*cos(d*x + c)^3 - 240*a^2*d*x +
```

```
30*a^2*cos(d*x + c) - 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)
*log(1/2*cos(d*x + c) + 1/2) + 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^
2 + a^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*(3*a^2*cos(d*x + c)^5 - 20*a^2*c
os(d*x + c)^3 + 15*a^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*
cos(d*x + c)^2 + d)
```

giac [A] time = 0.30, size = 259, normalized size = 1.69

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+960\,(dx+c)a^2-120\,a^2\log\left(\left|\tan\left(\frac{1}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*a^
2*tan(1/2*d*x + 1/2*c)^2 + 960*(d*x + c)*a^2 - 120*a^2*log(abs(tan(1/2*d*x
+ 1/2*c))) - 432*a^2*tan(1/2*d*x + 1/2*c) - 128*(3*a^2*tan(1/2*d*x + 1/2*c)
^5 + 6*a^2*tan(1/2*d*x + 1/2*c)^4 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*ta
n(1/2*d*x + 1/2*c) + 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3 + (250*a^2*tan(1
/2*d*x + 1/2*c)^4 + 432*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*a^2*tan(1/2*d*x + 1
/2*c)^2 - 16*a^2*tan(1/2*d*x + 1/2*c) - 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.44, size = 247, normalized size = 1.61

$$-\frac{a^2\left(\cos^7(dx+c)\right)}{8d\sin(dx+c)^2}-\frac{a^2\left(\cos^5(dx+c)\right)}{8d}-\frac{5a^2\left(\cos^3(dx+c)\right)}{24d}-\frac{5a^2\cos(dx+c)}{8d}-\frac{5a^2\ln(\csc(dx+c)-\cot(dx+c}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/8/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-1/8*a^2*cos(d*x+c)^5/d-5/24*a^2*cos(d*
x+c)^3/d-5/8*a^2*cos(d*x+c)/d-5/8/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/3/d*a^2
/sin(d*x+c)^3*cos(d*x+c)^7+8/3/d*a^2/sin(d*x+c)*cos(d*x+c)^7+8/3*a^2*cos(d*
x+c)^5*sin(d*x+c)/d+10/3*a^2*cos(d*x+c)^3*sin(d*x+c)/d+5*a^2*cos(d*x+c)*sin
(d*x+c)/d+5*a^2*x+5/d*a^2*c-1/4/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7
```

maxima [A] time = 0.44, size = 206, normalized size = 1.35

$$-\frac{4\left(4\cos(dx+c)^3-\frac{6\cos(dx+c)}{\cos(dx+c)^2-1}+24\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^2-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/48*(4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 - 16*(15
*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 +
 tan(d*x + c)^3))*a^2 + 3*a^2*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d
*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) +
 1) - 15*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.94, size = 373, normalized size = 2.44

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-62\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{320\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}}{d\,\left(16\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^3)/(12*d) - (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^2
*tan(c/2 + (d*x)/2)^4)/(64*d) + ((5*a^2*tan(c/2 + (d*x)/2)^2)/4 + 32*a^2*ta
n(c/2 + (d*x)/2)^3 - (449*a^2*tan(c/2 + (d*x)/2)^4)/12 + 136*a^2*tan(c/2 +
(d*x)/2)^5 - (233*a^2*tan(c/2 + (d*x)/2)^6)/4 + (320*a^2*tan(c/2 + (d*x)/2)
^7)/3 - 62*a^2*tan(c/2 + (d*x)/2)^8 + 4*a^2*tan(c/2 + (d*x)/2)^9 - a^2/4 -
(4*a^2*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2)^4 + 48*tan(c/2 + (d
*x)/2)^6 + 48*tan(c/2 + (d*x)/2)^8 + 16*tan(c/2 + (d*x)/2)^10)) - (5*a^2*lo
g(tan(c/2 + (d*x)/2)))/(8*d) - (10*a^2*atan((100*a^4)/((25*a^4)/2 + 100*a^4
*tan(c/2 + (d*x)/2)) - (25*a^4*tan(c/2 + (d*x)/2))/(2*((25*a^4)/2 + 100*a^4
*tan(c/2 + (d*x)/2)))))/d - (9*a^2*tan(c/2 + (d*x)/2))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.597 $\int \cot^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=139

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{15a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot}{}$$

```
[Out] 3/2*a^2*x-15/4*a^2*arctanh(cos(d*x+c))/d+2*a^2*cos(d*x+c)/d+a^2*cot(d*x+c)/
d-1/5*a^2*cot(d*x+c)^5/d+9/4*a^2*cot(d*x+c)*csc(d*x+c)/d-1/2*a^2*cot(d*x+c)
*csc(d*x+c)^3/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.25, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 3770, 3768, 3767, 2638, 2635, 8}

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{15a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/2 - (15*a^2*ArcTanh[Cos[c + d*x]])/(4*d) + (2*a^2*Cos[c + d*x])/d
 + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^5)/(5*d) + (9*a^2*Cot[c + d*x]*
Csc[c + d*x])/(4*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(2*d) + (a^2*Cos[c
+ d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(2a^8+6a^8\csc(c+dx)-6a^8\csc^3(c+dx)-2a^8\csc^4(c+dx)+2a^8\csc\right.}{a^6}$$

$$= 2a^2x + a^2\int \csc^6(c+dx)\,dx - a^2\int \sin^2(c+dx)\,dx - \left(2a^2\right)\int \csc^4(c+$$

$$= 2a^2x - \frac{6a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{3a^2\cot(c+dx)\csc}{d}$$

$$= \frac{3a^2x}{2} - \frac{3a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{a^2\cot(c+dx)}{d} -$$

$$= \frac{3a^2x}{2} - \frac{15a^2\tanh^{-1}(\cos(c+dx))}{4d} + \frac{2a^2\cos(c+dx)}{d} + \frac{a^2\cot(c+dx)}{d} -$$

Mathematica [A] time = 1.41, size = 264, normalized size = 1.90

$$\frac{a^2(\sin(c+dx)+1)^2\left(240(c+dx)+40\sin(2(c+dx))+320\cos(c+dx)-64\tan\left(\frac{1}{2}(c+dx)\right)+64\cot\left(\frac{1}{2}(c+dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(240*(c + d*x) + 320*Cos[c + d*x] + 64*Cot[(c + d
*x)/2] + 90*Csc[(c + d*x)/2]^2 - 5*Csc[(c + d*x)/2]^4 - 600*Log[Cos[(c + d*
x)/2]] + 600*Log[Sin[(c + d*x)/2]] - 90*Sec[(c + d*x)/2]^2 + 5*Sec[(c + d*x
)/2]^4 - 56*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + (7*Csc[(c + d*x)/2]^4*Sin[c
 + d*x])/2 - (Csc[(c + d*x)/2]^6*Sin[c + d*x])/2 + 40*Sin[2*(c + d*x)] - 64
*Tan[(c + d*x)/2] + Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(160*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^4)
```

fricas [B] time = 0.81, size = 265, normalized size = 1.91

$$-\frac{20\,a^2\cos\left(dx+c\right)^7-92\,a^2\cos\left(dx+c\right)^5+140\,a^2\cos\left(dx+c\right)^3-60\,a^2\cos\left(dx+c\right)+75\left(a^2\cos\left(dx+c\right)^4-2\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/40*(20*a^2*cos(d*x + c)^7 - 92*a^2*cos(d*x + c)^5 + 140*a^2*cos(d*x + c)
^3 - 60*a^2*cos(d*x + c) + 75*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 +
a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 75*(a^2*cos(d*x + c)^4 - 2*
a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 10*(6
*a^2*d*x*cos(d*x + c)^4 + 8*a^2*cos(d*x + c)^5 - 12*a^2*d*x*cos(d*x + c)^2
- 25*a^2*cos(d*x + c)^3 + 6*a^2*d*x + 15*a^2*cos(d*x + c))*sin(d*x + c))/((
d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [B] time = 0.29, size = 272, normalized size = 1.96

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-80\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+240\,(dx+c)a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/160*(a^2*tan(1/2*d*x + 1/2*c)^5 + 5*a^2*tan(1/2*d*x + 1/2*c)^4 - 5*a^2*ta
n(1/2*d*x + 1/2*c)^3 - 80*a^2*tan(1/2*d*x + 1/2*c)^2 + 240*(d*x + c)*a^2 +
600*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 70*a^2*tan(1/2*d*x + 1/2*c) - 160*
(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*
x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - (1370*a^2*tan(1/2*d*x
+ 1/2*c)^5 - 70*a^2*tan(1/2*d*x + 1/2*c)^4 - 80*a^2*tan(1/2*d*x + 1/2*c)^3
- 5*a^2*tan(1/2*d*x + 1/2*c)^2 + 5*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*
d*x + 1/2*c)^5)/d
```

maple [B] time = 0.45, size = 293, normalized size = 2.11

$$-\frac{a^2\left(\cos^7(dx+c)\right)}{3d\sin(dx+c)^3}+\frac{4a^2\left(\cos^7(dx+c)\right)}{3d\sin(dx+c)}+\frac{4a^2\left(\cos^5(dx+c)\right)\sin(dx+c)}{3d}+\frac{5a^2\left(\cos^3(dx+c)\right)\sin(dx+c)}{3d}+\frac{5a^2 c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d*a^2/sin(d*x+c)^3*cos(d*x+c)^7+4/3/d*a^2/sin(d*x+c)*cos(d*x+c)^7+4/3*
a^2*cos(d*x+c)^5*sin(d*x+c)/d+5/3*a^2*cos(d*x+c)^3*sin(d*x+c)/d+5/2*a^2*cos
(d*x+c)*sin(d*x+c)/d+3/2*a^2*x+3/2/d*a^2*c-1/2/d*a^2/sin(d*x+c)^4*cos(d*x+c
)^7+3/4/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7+3/4*a^2*cos(d*x+c)^5/d+5/4*a^2*cos(
d*x+c)^3/d+15/4*a^2*cos(d*x+c)/d+15/4/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/5*a
^2*cot(d*x+c)^5/d+1/3*a^2*cot(d*x+c)^3/d-a^2*cot(d*x+c)/d
```

maximA [A] time = 0.65, size = 184, normalized size = 1.32

$$\frac{20\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)a^2-8\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2-15\,a^2\left(\frac{2\left(9\,\cos(dx\right.}{\cos(dx+c)^4}\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/120*(20*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan
(d*x + c)^5 + tan(d*x + c)^3))*a^2 - 8*(15*d*x + 15*c + (15*tan(d*x + c)^4
- 5*tan(d*x + c)^2 + 3)/tan(d*x + c)^5)*a^2 - 15*a^2*(2*(9*cos(d*x + c)^3 -
 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c)
+ 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.90, size = 363, normalized size = 2.61

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{32\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{32\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}+\frac{15\,a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4\,d}+\frac{-18\,a^2\tan(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^4)/(32*d) - (a^2*tan(c/2 + (d*x)/2)^3)/(32*d) - (a^
2*tan(c/2 + (d*x)/2)^2)/(2*d) + (a^2*tan(c/2 + (d*x)/2)^5)/(160*d) + (15*a^
2*log(tan(c/2 + (d*x)/2)))/(4*d) + ((3*a^2*tan(c/2 + (d*x)/2)^2)/5 + 14*a^2
*tan(c/2 + (d*x)/2)^3 + (79*a^2*tan(c/2 + (d*x)/2)^4)/5 + 159*a^2*tan(c/2 +
 (d*x)/2)^5 + 61*a^2*tan(c/2 + (d*x)/2)^6 + 144*a^2*tan(c/2 + (d*x)/2)^7 -
18*a^2*tan(c/2 + (d*x)/2)^8 - a^2/5 - a^2*tan(c/2 + (d*x)/2))/(d*(32*tan(c/
2 + (d*x)/2)^5 + 64*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9)) + (3*a
^2*atan((9*a^4)/((45*a^4)/2 - 9*a^4*tan(c/2 + (d*x)/2)) + (45*a^4*tan(c/2 +
 (d*x)/2))/(2*((45*a^4)/2 - 9*a^4*tan(c/2 + (d*x)/2)))))/d - (7*a^2*tan(c/2
 + (d*x)/2))/(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.598 $\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=157

$$\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{2a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot(c+dx)}{d}-\frac{25a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot(c+dx)\,\mathrm{cs}}{6d}$$

```
[Out] -2*a^2*x-25/16*a^2*arctanh(cos(d*x+c))/d+a^2*cos(d*x+c)/d-2*a^2*cot(d*x+c)/
d+2/3*a^2*cot(d*x+c)^3/d-2/5*a^2*cot(d*x+c)^5/d+7/16*a^2*cot(d*x+c)*csc(d*x
+c)/d+7/24*a^2*cot(d*x+c)*csc(d*x+c)^3/d-1/6*a^2*cot(d*x+c)*csc(d*x+c)^5/d
```

Rubi [A] time = 0.23, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2872, 3770, 3767, 8, 3768, 2638}

$$\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{2a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot(c+dx)}{d}-\frac{25a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot(c+dx)\,\mathrm{cs}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -2*a^2*x - (25*a^2*ArcTanh[Cos[c + d*x]])/(16*d) + (a^2*Cos[c + d*x])/d - (
2*a^2*Cot[c + d*x])/d + (2*a^2*Cot[c + d*x]^3)/(3*d) - (2*a^2*Cot[c + d*x]^
5)/(5*d) + (7*a^2*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (7*a^2*Cot[c + d*x]*C
sc[c + d*x]^3)/(24*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-2a^8+2a^8\csc(c+dx)+6a^8\csc^2(c+dx)-6a^8\csc^4(c+\right.}{}$$

$$= -2a^2x + a^2\int \csc^7(c+dx)\,dx - a^2\int \sin(c+dx)\,dx + \left(2a^2\right.$$

$$= -2a^2x - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cot}{}$$

$$= -2a^2x - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^2\cos(c+dx)}{d} - \frac{2a^2\mathrm{c}}{}$$

$$= -2a^2x - \frac{5a^2\tanh^{-1}(\cos(c+dx))}{4d} + \frac{a^2\cos(c+dx)}{d} - \frac{2a^2\mathrm{c}}{}$$

$$= -2a^2x - \frac{25a^2\tanh^{-1}(\cos(c+dx))}{16d} + \frac{a^2\cos(c+dx)}{d} - \frac{2a^2}{}$$

Mathematica [A] time = 1.61, size = 270, normalized size = 1.72

$$-\frac{a^2\sin(c+dx)(\sin(c+dx)+1)^2\left(-1920\cot(c+dx)+\csc^6\left(\frac{1}{2}(c+dx)\right)(5\csc(c+dx)+12)-2\csc^4\left(\frac{1}{2}(c+dx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/1920*(a^2*(-1920*Cot[c + d*x] + Csc[(c + d*x)/2]^2*(1472 - 210*Csc[c + d
*x]) + Csc[(c + d*x)/2]^6*(12 + 5*Csc[c + d*x]) - 2*Csc[(c + d*x)/2]^4*(82
+ 15*Csc[c + d*x]) + 120*Csc[c + d*x]*(32*(c + d*x) + 25*Log[Cos[(c + d*x)/
2]] - 25*Log[Sin[(c + d*x)/2]]) - 2*(241 + 327*Cos[c + d*x] + 92*Cos[2*(c +
 d*x)])*Sec[(c + d*x)/2]^6 + 840*Csc[c + d*x]^3*Sin[(c + d*x)/2]^2 + 480*Cs
c[c + d*x]^5*Sin[(c + d*x)/2]^4 - 320*Csc[c + d*x]^7*Sin[(c + d*x)/2]^6)*Si
n[c + d*x]*(1 + Sin[c + d*x])^2)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4
)
```

fricas [B] time = 0.91, size = 303, normalized size = 1.93

$$-\frac{960\,a^2dx\cos\left(dx+c\right)^6-480\,a^2\cos\left(dx+c\right)^7-2880\,a^2dx\cos\left(dx+c\right)^4+1650\,a^2\cos\left(dx+c\right)^5+2880\,a^2dx\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(960*a^2*d*x*cos(d*x + c)^6 - 480*a^2*cos(d*x + c)^7 - 2880*a^2*d*x*
cos(d*x + c)^4 + 1650*a^2*cos(d*x + c)^5 + 2880*a^2*d*x*cos(d*x + c)^2 - 20
00*a^2*cos(d*x + c)^3 - 960*a^2*d*x + 750*a^2*cos(d*x + c) + 375*(a^2*cos(d
*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(
d*x + c) + 1/2) - 375*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*co
s(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2) - 64*(23*a^2*cos(d*x + c)^
5 - 35*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.31, size = 259, normalized size = 1.65

$$\frac{5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-280\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-255\,a^2\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*tan(1/2*d*x + 1/2*c)^5 - 15*a
^2*tan(1/2*d*x + 1/2*c)^4 - 280*a^2*tan(1/2*d*x + 1/2*c)^3 - 255*a^2*tan(1/
2*d*x + 1/2*c)^2 - 3840*(d*x + c)*a^2 + 3000*a^2*log(abs(tan(1/2*d*x + 1/2*
c))) + 2640*a^2*tan(1/2*d*x + 1/2*c) + 3840*a^2/(tan(1/2*d*x + 1/2*c)^2 + 1
) - (7350*a^2*tan(1/2*d*x + 1/2*c)^6 + 2640*a^2*tan(1/2*d*x + 1/2*c)^5 - 25
5*a^2*tan(1/2*d*x + 1/2*c)^4 - 280*a^2*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(
```

```
1/2*d*x + 1/2*c)^2 + 24*a^2*tan(1/2*d*x + 1/2*c) + 5*a^2)/tan(1/2*d*x + 1/2
*c)^6)/d
```

maple [A] time = 0.37, size = 205, normalized size = 1.31

$$-\frac{5a^2\left(\cos^7(dx+c)\right)}{24d\sin(dx+c)^4}+\frac{5a^2\left(\cos^7(dx+c)\right)}{16d\sin(dx+c)^2}+\frac{5a^2\left(\cos^5(dx+c)\right)}{16d}+\frac{25a^2\left(\cos^3(dx+c)\right)}{48d}+\frac{25a^2\cos(dx+c)}{16d}+\frac{25a^2\
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -5/24/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7+5/16/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7+
5/16*a^2*cos(d*x+c)^5/d+25/48*a^2*cos(d*x+c)^3/d+25/16*a^2*cos(d*x+c)/d+25/
16/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/5*a^2*cot(d*x+c)^5/d+2/3*a^2*cot(d*x+c
)^3/d-2*a^2*cot(d*x+c)/d-2*a^2*x-2/d*a^2*c-1/6/d*a^2/sin(d*x+c)^6*cos(d*x+c
)^7
```

maxima [A] time = 0.46, size = 220, normalized size = 1.40

$$-\frac{64\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2-5\,a^2\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log\left(\cos\left(dx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*(64*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a^2 - 5*a^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(
d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*l
og(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 30*a^2*(2*(9*cos(d*x + c
)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x
+ c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 11.01, size = 657, normalized size = 4.18

$$\frac{5\,a^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-5\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+24\,a^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-24\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\,\sin\left(\frac{c}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] (5*a^2*sin(c/2 + (d*x)/2)^14 - 5*a^2*cos(c/2 + (d*x)/2)^14 + 24*a^2*cos(c/2
 + (d*x)/2)*sin(c/2 + (d*x)/2)^13 - 24*a^2*cos(c/2 + (d*x)/2)^13*sin(c/2 +
(d*x)/2) - 10*a^2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 256*a^2*cos(
c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^11 - 270*a^2*cos(c/2 + (d*x)/2)^4*sin(c
/2 + (d*x)/2)^10 + 2360*a^2*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 - 255
*a^2*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 + 4095*a^2*cos(c/2 + (d*x)/2
)^8*sin(c/2 + (d*x)/2)^6 - 2360*a^2*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)
^5 + 270*a^2*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^4 + 256*a^2*cos(c/2 +
 (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 10*a^2*cos(c/2 + (d*x)/2)^12*sin(c/2 +
(d*x)/2)^2 + 3000*a^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 +
(d*x)/2)^6*sin(c/2 + (d*x)/2)^8 + 3000*a^2*log(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2))*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 7680*a^2*atan((32*co
s(c/2 + (d*x)/2) - 25*sin(c/2 + (d*x)/2))/(25*cos(c/2 + (d*x)/2) + 32*sin(c
/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 + 7680*a^2*atan((
32*cos(c/2 + (d*x)/2) - 25*sin(c/2 + (d*x)/2))/(25*cos(c/2 + (d*x)/2) + 32*
sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6)/(1920*d*cos
(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6*(cos(c/2 + (d*x)/2)^2 + sin(c/2 + (d
*x)/2)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.599 $\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=162

$$-\frac{a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot^5(c+dx)\,\mathrm{cs}}{3d}$$

```
[Out] -a^2*x+5/8*a^2*arctanh(cos(d*x+c))/d-a^2*cot(d*x+c)/d+1/3*a^2*cot(d*x+c)^3/
d-1/5*a^2*cot(d*x+c)^5/d-1/7*a^2*cot(d*x+c)^7/d-5/8*a^2*cot(d*x+c)*csc(d*x+
c)/d+5/12*a^2*cot(d*x+c)^3*csc(d*x+c)/d-1/3*a^2*cot(d*x+c)^5*csc(d*x+c)/d
```

Rubi [A] time = 0.23, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2873, 3473, 8, 2611, 3770, 2607, 30}

$$-\frac{a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot^5(c+dx)\,\mathrm{cs}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (5*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (a^2*Cot[c + d*x])/d + (a^
2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]^5)/(5*d) - (a^2*Cot[c + d*x]^7)
/(7*d) - (5*a^2*Cot[c + d*x]*Csc[c + d*x])/(8*d) + (5*a^2*Cot[c + d*x]^3*Cs
c[c + d*x])/(12*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)+2a^2\cot^6(c+dx)\csc(c+dx)+a^2\cot^6(\right.$$
$$= a^2\int \cot^6(c+dx)\,dx + a^2\int \cot^6(c+dx)\csc^2(c+dx)\,dx +$$
$$= -\frac{a^2\cot^5(c+dx)}{5d} - \frac{a^2\cot^5(c+dx)\csc(c+dx)}{3d} - a^2\int \cot^4($$
$$= \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} - \frac{a^2\cot^7(c+dx)}{7d} + \frac{5a^2\,\mathrm{c}}{}$$
$$= -\frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} - \frac{a^2\,\mathrm{co}}{}$$
$$= -a^2x + \frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(}{3}$$

Mathematica [A] time = 1.09, size = 262, normalized size = 1.62

$$a^2\left(9344\tan\left(\frac{1}{2}(c+dx)\right)-9344\cot\left(\frac{1}{2}(c+dx)\right)-4620\csc^2\left(\frac{1}{2}(c+dx)\right)+70\sec^6\left(\frac{1}{2}(c+dx)\right)-840\sec^4\left(\frac{1}{2}(c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-13440*c - 13440*d*x - 9344*Cot[(c + d*x)/2] - 4620*Csc[(c + d*x)/2]^
2 + 8400*Log[Cos[(c + d*x)/2]] - 8400*Log[Sin[(c + d*x)/2]] + 4620*Sec[(c +
 d*x)/2]^2 - 840*Sec[(c + d*x)/2]^4 + 70*Sec[(c + d*x)/2]^6 - 4624*Csc[c +
d*x]^3*Sin[(c + d*x)/2]^4 - (15*Csc[(c + d*x)/2]^8*Sin[c + d*x])/2 + Csc[(c
 + d*x)/2]^6*(-70 + 33*Sin[c + d*x]) + Csc[(c + d*x)/2]^4*(840 + 289*Sin[c
+ d*x]) + 9344*Tan[(c + d*x)/2] - 66*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2] +
15*Sec[(c + d*x)/2]^6*Tan[(c + d*x)/2]))/(13440*d)
```

fricas [B] time = 0.84, size = 323, normalized size = 1.99

$$-\frac{2336\,a^2\cos(dx+c)^7-6496\,a^2\cos(dx+c)^5+5600\,a^2\cos(dx+c)^3-1680\,a^2\cos(dx+c)-525\left(a^2\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/1680*(2336*a^2*cos(d*x + c)^7 - 6496*a^2*cos(d*x + c)^5 + 5600*a^2*cos(d
*x + c)^3 - 1680*a^2*cos(d*x + c) - 525*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x
 + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 525*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2
 - a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 70*(24*a^2*d*x*cos(d*x
+ c)^6 - 72*a^2*d*x*cos(d*x + c)^4 - 33*a^2*cos(d*x + c)^5 + 72*a^2*d*x*cos
(d*x + c)^2 + 40*a^2*cos(d*x + c)^3 - 24*a^2*d*x - 15*a^2*cos(d*x + c))*sin
(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d
)*sin(d*x + c))
```

giac [A] time = 0.31, size = 270, normalized size = 1.67

$$\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-630\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-665\,a^2\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*a^2*tan(1/2*d*x + 1/2*c)^7 + 70*a^2*tan(1/2*d*x + 1/2*c)^6 - 21
*a^2*tan(1/2*d*x + 1/2*c)^5 - 630*a^2*tan(1/2*d*x + 1/2*c)^4 - 665*a^2*tan(
1/2*d*x + 1/2*c)^3 + 3150*a^2*tan(1/2*d*x + 1/2*c)^2 - 13440*(d*x + c)*a^2
- 8400*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 8715*a^2*tan(1/2*d*x + 1/2*c) +
 (21780*a^2*tan(1/2*d*x + 1/2*c)^7 - 8715*a^2*tan(1/2*d*x + 1/2*c)^6 - 3150
*a^2*tan(1/2*d*x + 1/2*c)^5 + 665*a^2*tan(1/2*d*x + 1/2*c)^4 + 630*a^2*tan(
1/2*d*x + 1/2*c)^3 + 21*a^2*tan(1/2*d*x + 1/2*c)^2 - 70*a^2*tan(1/2*d*x + 1
/2*c) - 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/d
```

maple [A] time = 0.38, size = 229, normalized size = 1.41

$$-\frac{a^2\left(\cot^5(dx+c)\right)}{5d}+\frac{a^2\left(\cot^3(dx+c)\right)}{3d}-\frac{a^2\cot(dx+c)}{d}-a^2x-\frac{a^2c}{d}-\frac{a^2\left(\cos^7(dx+c)\right)}{3d\sin(dx+c)^6}+\frac{a^2\left(\cos^7(dx+c)\right)}{12d\sin(dx+c)^4}-\frac{a^2\left(\cos\right.}{8d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/5*a^2*cot(d*x+c)^5/d+1/3*a^2*cot(d*x+c)^3/d-a^2*cot(d*x+c)/d-a^2*x-1/d*a
^2*c-1/3/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+1/12/d*a^2/sin(d*x+c)^4*cos(d*x+c)
^7-1/8/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-1/8*a^2*cos(d*x+c)^5/d-5/24*a^2*cos(
d*x+c)^3/d-5/8*a^2*cos(d*x+c)/d-5/8/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/7/d*a
^2/sin(d*x+c)^7*cos(d*x+c)^7
```

maxima [A] time = 0.43, size = 154, normalized size = 0.95

$$-\frac{112\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2-35\,a^2\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log\left(\cos\left(dx\right.\right.\right.}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/1680*(112*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/ta
n(d*x + c)^5)*a^2 - 35*a^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*c
os(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 1
5*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 240*a^2/tan(d*x + c)^
7)/d
```

mupad [B] time = 9.84, size = 351, normalized size = 2.17

$$\frac{19\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{384\,d}-\frac{15\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64\,d}+\frac{3\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{192\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{896\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^8,x)
```

```
[Out] (19*a^2*cot(c/2 + (d*x)/2)^3)/(384*d) - (15*a^2*cot(c/2 + (d*x)/2)^2)/(64*d
) + (3*a^2*cot(c/2 + (d*x)/2)^4)/(64*d) + (a^2*cot(c/2 + (d*x)/2)^5)/(640*d
) - (a^2*cot(c/2 + (d*x)/2)^6)/(192*d) - (a^2*cot(c/2 + (d*x)/2)^7)/(896*d)
 + (15*a^2*tan(c/2 + (d*x)/2)^2)/(64*d) - (19*a^2*tan(c/2 + (d*x)/2)^3)/(38
4*d) - (3*a^2*tan(c/2 + (d*x)/2)^4)/(64*d) - (a^2*tan(c/2 + (d*x)/2)^5)/(64
0*d) + (a^2*tan(c/2 + (d*x)/2)^6)/(192*d) + (a^2*tan(c/2 + (d*x)/2)^7)/(896
*d) - (2*a^2*atan((8*cos(c/2 + (d*x)/2) + 5*sin(c/2 + (d*x)/2))/(5*cos(c/2
+ (d*x)/2) - 8*sin(c/2 + (d*x)/2))))/d - (5*a^2*log(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2)))/(8*d) - (83*a^2*cot(c/2 + (d*x)/2))/(128*d) + (83*a^2*tan(
c/2 + (d*x)/2))/(128*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.600 $\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=182

$$-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{45a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{8d}-\frac{a^2\cot^5(c+dx)\csc(c+dx)}{6d}+\frac{5a^2\cot}{}$$

```
[Out] 45/128*a^2*arctanh(cos(d*x+c))/d-2/7*a^2*cot(d*x+c)^7/d-35/128*a^2*cot(d*x+
c)*csc(d*x+c)/d+5/24*a^2*cot(d*x+c)^3*csc(d*x+c)/d-1/6*a^2*cot(d*x+c)^5*csc
(d*x+c)/d-5/64*a^2*cot(d*x+c)*csc(d*x+c)^3/d+5/48*a^2*cot(d*x+c)^3*csc(d*x+
c)^3/d-1/8*a^2*cot(d*x+c)^5*csc(d*x+c)^3/d
```

Rubi [A] time = 0.31, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2873, 2611, 3770, 2607, 30, 3768}

$$-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{45a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a^2\cot^3(c+dx)\csc^3(c+dx)}{48d}-\frac{a^2\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (45*a^2*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (35
*a^2*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d*x
])/(24*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x])/(6*d) - (5*a^2*Cot[c + d*x]*C
sc[c + d*x]^3)/(64*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a^2
*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \Big(a^2\cot^6(c+dx)\csc(c+dx) + 2a^2\cot^6(c+dx)\csc^2(c+$$

$$= a^2\int \cot^6(c+dx)\csc(c+dx)\,dx + a^2\int \cot^6(c+dx)\csc^3($$

$$= -\frac{a^2\cot^5(c+dx)\csc(c+dx)}{6d} - \frac{a^2\cot^5(c+dx)\csc^3(c+dx}{8d}$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} + \frac{5a^2\cot^3(c+dx)\csc(c+dx)}{24d} - \frac{a^2\cot^5}{}$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{5a^2\cot(c+dx)\csc(c+dx)}{16d} + \frac{5a^2\cot}{}$$

$$= \frac{5a^2\tanh^{-1}(\cos(c+dx))}{16d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{35a^2\cot(c-}{}$$

$$= \frac{45a^2\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{35a^2\cot(c}{}$$

Mathematica [B] time = 0.11, size = 401, normalized size = 2.20

$$a^2\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{7d}+\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{7d}-\frac{\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d}+\frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{512d}+\frac{17\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d}-\frac{83\csc^2\left(\frac{1}{2}\right.}{51}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*(Cot[(c + d*x)/2]/(7*d) - (83*Csc[(c + d*x)/2]^2)/(512*d) - (19*Cot[(c
+ d*x)/2]*Csc[(c + d*x)/2]^2)/(224*d) + (17*Csc[(c + d*x)/2]^4)/(1024*d) +
(5*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(224*d) + Csc[(c + d*x)/2]^6/(512*d
) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^6)/(448*d) - Csc[(c + d*x)/2]^8/(204
8*d) + (45*Log[Cos[(c + d*x)/2]])/(128*d) - (45*Log[Sin[(c + d*x)/2]])/(128
*d) + (83*Sec[(c + d*x)/2]^2)/(512*d) - (17*Sec[(c + d*x)/2]^4)/(1024*d) -
Sec[(c + d*x)/2]^6/(512*d) + Sec[(c + d*x)/2]^8/(2048*d) - Tan[(c + d*x)/2]
/(7*d) + (19*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(224*d) - (5*Sec[(c + d*x
)/2]^4*Tan[(c + d*x)/2])/(224*d) + (Sec[(c + d*x)/2]^6*Tan[(c + d*x)/2])/(4
48*d))
```

fricas [A] time = 0.87, size = 255, normalized size = 1.40

$$-\frac{512\,a^2\cos\left(dx+c\right)^7\sin\left(dx+c\right)-1162\,a^2\cos\left(dx+c\right)^7+3066\,a^2\cos\left(dx+c\right)^5-2310\,a^2\cos\left(dx+c\right)^3+630\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/1792*(512*a^2*cos(d*x + c)^7*sin(d*x + c) - 1162*a^2*cos(d*x + c)^7 + 30
66*a^2*cos(d*x + c)^5 - 2310*a^2*cos(d*x + c)^3 + 630*a^2*cos(d*x + c) - 31
5*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2
*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2) + 315*(a^2*cos(d*x + c)^
8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^
2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6
*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.36, size = 260, normalized size = 1.43

$$\frac{7\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+32\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-224\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-280\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+672\,a^2\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/14336*(7*a^2*tan(1/2*d*x + 1/2*c)^8 + 32*a^2*tan(1/2*d*x + 1/2*c)^7 - 224
*a^2*tan(1/2*d*x + 1/2*c)^5 - 280*a^2*tan(1/2*d*x + 1/2*c)^4 + 672*a^2*tan(
1/2*d*x + 1/2*c)^3 + 1792*a^2*tan(1/2*d*x + 1/2*c)^2 - 5040*a^2*log(abs(tan
(1/2*d*x + 1/2*c))) - 1120*a^2*tan(1/2*d*x + 1/2*c) + (13698*a^2*tan(1/2*d*
x + 1/2*c)^8 + 1120*a^2*tan(1/2*d*x + 1/2*c)^7 - 1792*a^2*tan(1/2*d*x + 1/2
*c)^6 - 672*a^2*tan(1/2*d*x + 1/2*c)^5 + 280*a^2*tan(1/2*d*x + 1/2*c)^4 + 2
24*a^2*tan(1/2*d*x + 1/2*c)^3 - 32*a^2*tan(1/2*d*x + 1/2*c) - 7*a^2)/tan(1/
2*d*x + 1/2*c)^8)/d
```

maple [A] time = 0.39, size = 192, normalized size = 1.05

$$-\frac{3a^2\left(\cos^7(dx+c)\right)}{16d\sin(dx+c)^6}+\frac{3a^2\left(\cos^7(dx+c)\right)}{64d\sin(dx+c)^4}-\frac{9a^2\left(\cos^7(dx+c)\right)}{128d\sin(dx+c)^2}-\frac{9a^2\left(\cos^5(dx+c)\right)}{128d}-\frac{15a^2\left(\cos^3(dx+c)\right)}{128d}-\frac{45a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/16/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+3/64/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7-
9/128/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-9/128*a^2*cos(d*x+c)^5/d-15/128*a^2*c
os(d*x+c)^3/d-45/128*a^2*cos(d*x+c)/d-45/128/d*a^2*ln(csc(d*x+c)-cot(d*x+c)
)-2/7/d*a^2/sin(d*x+c)^7*cos(d*x+c)^7-1/8/d*a^2/sin(d*x+c)^8*cos(d*x+c)^7
```

maxima [A] time = 0.54, size = 221, normalized size = 1.21

$$-\frac{7a^2\left(\frac{2\left(15\cos(dx+c)^7+73\cos(dx+c)^5-55\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1}-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)}{53}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/5376*(7*a^2*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^
3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
1)) - 56*a^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))/(
cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(cos(d*x
+ c) + 1) - 15*log(cos(d*x + c) - 1)) + 1536*a^2/tan(d*x + c)^7)/d
```

mupad [B] time = 11.16, size = 387, normalized size = 2.13

$$-\frac{a^2\left(7\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^9,x)
```

```
[Out] -(a^2*(7*cos(c/2 + (d*x)/2)^16 - 7*sin(c/2 + (d*x)/2)^16 - 32*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^15 + 32*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2) +
 224*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^13 + 280*cos(c/2 + (d*x)/2)^4*
sin(c/2 + (d*x)/2)^12 - 672*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^11 - 17
92*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 1120*cos(c/2 + (d*x)/2)^7*s
in(c/2 + (d*x)/2)^9 - 1120*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^7 + 1792
*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^6 + 672*cos(c/2 + (d*x)/2)^11*sin
(c/2 + (d*x)/2)^5 - 280*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 224*co
s(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2)^3 + 5040*log(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8))/(14336*d*cos(c/
2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.601 $\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=152

$$-\frac{a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{64d}-\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{4d}+\frac{5a^2\cot^3(c+dx)\mathrm{cs}}{24d}$$

```
[Out] 5/64*a^2*arctanh(cos(d*x+c))/d-2/7*a^2*cot(d*x+c)^7/d-1/9*a^2*cot(d*x+c)^9/
d+5/64*a^2*cot(d*x+c)*csc(d*x+c)/d-5/32*a^2*cot(d*x+c)*csc(d*x+c)^3/d+5/24*
a^2*cot(d*x+c)^3*csc(d*x+c)^3/d-1/4*a^2*cot(d*x+c)^5*csc(d*x+c)^3/d
```

Rubi [A] time = 0.29, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2607, 30, 2611, 3768, 3770, 14}

$$-\frac{a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{64d}-\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{4d}+\frac{5a^2\cot^3(c+dx)\mathrm{cs}}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*ArcTanh[Cos[c + d*x]])/(64*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (a^2*
Cot[c + d*x]^9)/(9*d) + (5*a^2*Cot[c + d*x]*Csc[c + d*x])/(64*d) - (5*a^2*C
ot[c + d*x]*Csc[c + d*x]^3)/(32*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d*x]^3)/
(24*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^3)/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)\csc^2(c+dx) + 2a^2\cot^6(c+dx)\csc^3(c+\right.$$

$$= a^2\int \cot^6(c+dx)\csc^2(c+dx)\,dx + a^2\int \cot^6(c+dx)\csc^4($$

$$= -\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}\left(5a^2\right)\int \cot^4(c+dx)\csc^3$$

$$= -\frac{a^2\cot^7(c+dx)}{7d} + \frac{5a^2\cot^3(c+dx)\csc^3(c+dx)}{24d} - \frac{a^2\cot^5(}{}$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} - \frac{5a^2\cot(c+dx)\csc^3(}{32d}$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} + \frac{5a^2\cot(c+dx)\csc(c}{64d}$$

$$= \frac{5a^2\tanh^{-1}(\cos(c+dx))}{64d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+d}{9d}$$

Mathematica [B] time = 1.63, size = 313, normalized size = 2.06

$$-\frac{a^2\csc^9(c+dx)\left(36540\sin(2(c+dx))+20916\sin(4(c+dx))+16044\sin(6(c+dx))+630\sin(8(c+dx))+72\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/1032192*(a^2*Csc[c + d*x]^9*(72576*Cos[c + d*x] + 37632*Cos[3*(c + d*x)]
 + 6912*Cos[5*(c + d*x)] - 1728*Cos[7*(c + d*x)] - 704*Cos[9*(c + d*x)] - 3
9690*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 39690*Log[Sin[(c + d*x)/2]]*Sin[c
 + d*x] + 36540*Sin[2*(c + d*x)] + 26460*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d
*x)] - 26460*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 20916*Sin[4*(c + d*x)
] - 11340*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 11340*Log[Sin[(c + d*x)/
2]]*Sin[5*(c + d*x)] + 16044*Sin[6*(c + d*x)] + 2835*Log[Cos[(c + d*x)/2]]*
Sin[7*(c + d*x)] - 2835*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] + 630*Sin[8*
(c + d*x)] - 315*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)] + 315*Log[Sin[(c +
d*x)/2]]*Sin[9*(c + d*x)]))/d
```

fricas [B] time = 0.77, size = 291, normalized size = 1.91

$$\frac{1408\,a^2\cos(dx+c)^9-2304\,a^2\cos(dx+c)^7+315\left(a^2\cos(dx+c)^8-4\,a^2\cos(dx+c)^6+6\,a^2\cos(dx+c)^4-4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/8064*(1408*a^2*cos(d*x + c)^9 - 2304*a^2*cos(d*x + c)^7 + 315*(a^2*cos(d*
x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)
^2 + a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 315*(a^2*cos(d*x + c)^
8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^
2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 42*(15*a^2*cos(d*x + c)^7 +
73*a^2*cos(d*x + c)^5 - 55*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x + c))*sin(d*
x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*
cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [B] time = 0.34, size = 324, normalized size = 2.13

$$\frac{14\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+63\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+18\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-336\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-504\,a^2\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/64512*(14*a^2*tan(1/2*d*x + 1/2*c)^9 + 63*a^2*tan(1/2*d*x + 1/2*c)^8 + 18
*a^2*tan(1/2*d*x + 1/2*c)^7 - 336*a^2*tan(1/2*d*x + 1/2*c)^6 - 504*a^2*tan(
1/2*d*x + 1/2*c)^5 + 504*a^2*tan(1/2*d*x + 1/2*c)^4 + 1848*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 1008*a^2*tan(1/2*d*x + 1/2*c)^2 - 5040*a^2*log(abs(tan(1/2*d*x
 + 1/2*c))) - 3276*a^2*tan(1/2*d*x + 1/2*c) + (14258*a^2*tan(1/2*d*x + 1/2*
c)^9 + 3276*a^2*tan(1/2*d*x + 1/2*c)^8 - 1008*a^2*tan(1/2*d*x + 1/2*c)^7 -
1848*a^2*tan(1/2*d*x + 1/2*c)^6 - 504*a^2*tan(1/2*d*x + 1/2*c)^5 + 504*a^2*
tan(1/2*d*x + 1/2*c)^4 + 336*a^2*tan(1/2*d*x + 1/2*c)^3 - 18*a^2*tan(1/2*d*
x + 1/2*c)^2 - 63*a^2*tan(1/2*d*x + 1/2*c) - 14*a^2)/tan(1/2*d*x + 1/2*c)^9
)/d
```

maple [A] time = 0.42, size = 216, normalized size = 1.42

$$-\frac{11a^2\left(\cos^7(dx+c)\right)}{63d\sin(dx+c)^7}-\frac{a^2\left(\cos^7(dx+c)\right)}{4d\sin(dx+c)^8}-\frac{a^2\left(\cos^7(dx+c)\right)}{24d\sin(dx+c)^6}+\frac{a^2\left(\cos^7(dx+c)\right)}{96d\sin(dx+c)^4}-\frac{a^2\left(\cos^7(dx+c)\right)}{64d\sin(dx+c)^2}-\frac{a^2\left(\cos^5(dx\right.}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -11/63/d*a^2/sin(d*x+c)^7*cos(d*x+c)^7-1/4/d*a^2/sin(d*x+c)^8*cos(d*x+c)^7-
1/24/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+1/96/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7-1
/64/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-1/64*a^2*cos(d*x+c)^5/d-5/192*a^2*cos(d
*x+c)^3/d-5/64*a^2*cos(d*x+c)/d-5/64/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/9/d*
a^2/sin(d*x+c)^9*cos(d*x+c)^7
```

maxima [A] time = 0.37, size = 155, normalized size = 1.02

$$-\frac{21\,a^2\left(\frac{2\left(15\,\cos(dx+c)^7+73\,\cos(dx+c)^5-55\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-15\,\log(\cos(dx+c)+1)+15\,\log(\cos(dx+c)-1)\right)}{8064\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/8064*(21*a^2*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)
^3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
 - 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
 1)) + 1152*a^2/tan(d*x + c)^7 + 128*(9*tan(d*x + c)^2 + 7)*a^2/tan(d*x + c
)^9)/d
```

mupad [B] time = 9.41, size = 357, normalized size = 2.35

$$\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{128\,d}-\frac{11\,a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{384\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{128\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{64\,d}+\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{192\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3584\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^10,x)
```

```
[Out] (a^2*cot(c/2 + (d*x)/2)^5)/(128*d) - (11*a^2*cot(c/2 + (d*x)/2)^3)/(384*d)
- (a^2*cot(c/2 + (d*x)/2)^4)/(128*d) - (a^2*cot(c/2 + (d*x)/2)^2)/(64*d) +
(a^2*cot(c/2 + (d*x)/2)^6)/(192*d) - (a^2*cot(c/2 + (d*x)/2)^7)/(3584*d) -
(a^2*cot(c/2 + (d*x)/2)^8)/(1024*d) - (a^2*cot(c/2 + (d*x)/2)^9)/(4608*d) +
 (a^2*tan(c/2 + (d*x)/2)^2)/(64*d) + (11*a^2*tan(c/2 + (d*x)/2)^3)/(384*d)
+ (a^2*tan(c/2 + (d*x)/2)^4)/(128*d) - (a^2*tan(c/2 + (d*x)/2)^5)/(128*d) -
 (a^2*tan(c/2 + (d*x)/2)^6)/(192*d) + (a^2*tan(c/2 + (d*x)/2)^7)/(3584*d) +
 (a^2*tan(c/2 + (d*x)/2)^8)/(1024*d) + (a^2*tan(c/2 + (d*x)/2)^9)/(4608*d)
- (5*a^2*log(tan(c/2 + (d*x)/2)))/(64*d) + (13*a^2*cot(c/2 + (d*x)/2))/(256
*d) - (13*a^2*tan(c/2 + (d*x)/2))/(256*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**10*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.602 $\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=228

$$-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{13a^2\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{10d}-\frac{a^2\cot^5(c+dx)\csc}{8d}$$

```
[Out] 13/256*a^2*arctanh(cos(d*x+c))/d-2/7*a^2*cot(d*x+c)^7/d-2/9*a^2*cot(d*x+c)^
9/d+13/256*a^2*cot(d*x+c)*csc(d*x+c)/d-9/128*a^2*cot(d*x+c)*csc(d*x+c)^3/d+
5/48*a^2*cot(d*x+c)^3*csc(d*x+c)^3/d-1/8*a^2*cot(d*x+c)^5*csc(d*x+c)^3/d-1/
32*a^2*cot(d*x+c)*csc(d*x+c)^5/d+1/16*a^2*cot(d*x+c)^3*csc(d*x+c)^5/d-1/10*
a^2*cot(d*x+c)^5*csc(d*x+c)^5/d
```

Rubi [A] time = 0.39, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3768, 3770, 2607, 14}

$$-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{13a^2\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{10d}-\frac{a^2\cot^5(c+dx)\csc}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (13*a^2*ArcTanh[Cos[c + d*x]])/(256*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (2*
a^2*Cot[c + d*x]^9)/(9*d) + (13*a^2*Cot[c + d*x]*Csc[c + d*x])/(256*d) - (9
*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d
*x]^3)/(48*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d) - (a^2*Cot[c + d*
x]*Csc[c + d*x]^5)/(32*d) + (a^2*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a
^2*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)\csc^3(c+dx)+2a^2\cot^6(c+dx)\csc^4(c+\right.$$
$$= a^2\int \cot^6(c+dx)\csc^3(c+dx)\,dx + a^2\int \cot^6(c+dx)\csc^5($$
$$= -\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{10d}$$
$$= \frac{5a^2\cot^3(c+dx)\csc^3(c+dx)}{48d} - \frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{8d}$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} - \frac{5a^2\cot(c+dx)\csc^3}{64d}$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} + \frac{5a^2\cot(c+dx)\csc(}{128d}$$
$$= \frac{5a^2\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+}{9d}$$
$$= \frac{13a^2\tanh^{-1}(\cos(c+dx))}{256d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+}{9d}$$

Mathematica [A] time = 1.29, size = 353, normalized size = 1.55

$$-\frac{a^2\csc^{10}(c+dx)\left(1075200\sin(2(c+dx))+1044480\sin(4(c+dx))+414720\sin(6(c+dx))+51200\sin(8(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/41287680*(a^2*Csc[c + d*x]^10*(2732940*Cos[c + d*x] + 1151640*Cos[3*(c +
 d*x)] + 388248*Cos[5*(c + d*x)] - 135870*Cos[7*(c + d*x)] - 8190*Cos[9*(c
+ d*x)] - 515970*Log[Cos[(c + d*x)/2]] + 859950*Cos[2*(c + d*x)]*Log[Cos[(c
 + d*x)/2]] - 491400*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 184275*Cos[6*
(c + d*x)]*Log[Cos[(c + d*x)/2]] - 40950*Cos[8*(c + d*x)]*Log[Cos[(c + d*x)
/2]] + 4095*Cos[10*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 515970*Log[Sin[(c + d
*x)/2]] - 859950*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 491400*Cos[4*(c +
 d*x)]*Log[Sin[(c + d*x)/2]] - 184275*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]
] + 40950*Cos[8*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 4095*Cos[10*(c + d*x)]*L
og[Sin[(c + d*x)/2]] + 1075200*Sin[2*(c + d*x)] + 1044480*Sin[4*(c + d*x)]
+ 414720*Sin[6*(c + d*x)] + 51200*Sin[8*(c + d*x)] - 5120*Sin[10*(c + d*x)]
))/d
```

fricas [A] time = 0.76, size = 327, normalized size = 1.43

$$-\frac{8190\,a^2\cos\left(dx+c\right)^9+15540\,a^2\cos\left(dx+c\right)^7-69888\,a^2\cos\left(dx+c\right)^5+38220\,a^2\cos\left(dx+c\right)^3-8190\,a^2\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/161280*(8190*a^2*cos(d*x + c)^9 + 15540*a^2*cos(d*x + c)^7 - 69888*a^2*c
os(d*x + c)^5 + 38220*a^2*cos(d*x + c)^3 - 8190*a^2*cos(d*x + c) - 4095*(a^
2*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 10*a^2*c
os(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2) + 4
095*(a^2*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 1
0*a^2*cos(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) +
1/2) + 5120*(2*a^2*cos(d*x + c)^9 - 9*a^2*cos(d*x + c)^7)*sin(d*x + c))/(d*
cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x +
 c)^4 + 5*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.36, size = 324, normalized size = 1.42

$$\frac{126\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+560\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+315\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-2160\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-399}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/1290240*(126*a^2*tan(1/2*d*x + 1/2*c)^10 + 560*a^2*tan(1/2*d*x + 1/2*c)^9
 + 315*a^2*tan(1/2*d*x + 1/2*c)^8 - 2160*a^2*tan(1/2*d*x + 1/2*c)^7 - 3990*
a^2*tan(1/2*d*x + 1/2*c)^6 + 7560*a^2*tan(1/2*d*x + 1/2*c)^4 + 13440*a^2*ta
n(1/2*d*x + 1/2*c)^3 + 11340*a^2*tan(1/2*d*x + 1/2*c)^2 - 65520*a^2*log(abs
(tan(1/2*d*x + 1/2*c))) - 30240*a^2*tan(1/2*d*x + 1/2*c) + (191906*a^2*tan(
1/2*d*x + 1/2*c)^10 + 30240*a^2*tan(1/2*d*x + 1/2*c)^9 - 11340*a^2*tan(1/2*
d*x + 1/2*c)^8 - 13440*a^2*tan(1/2*d*x + 1/2*c)^7 - 7560*a^2*tan(1/2*d*x +
1/2*c)^6 + 3990*a^2*tan(1/2*d*x + 1/2*c)^4 + 2160*a^2*tan(1/2*d*x + 1/2*c)^
3 - 315*a^2*tan(1/2*d*x + 1/2*c)^2 - 560*a^2*tan(1/2*d*x + 1/2*c) - 126*a^2
)/tan(1/2*d*x + 1/2*c)^10)/d
```

maple [A] time = 0.40, size = 240, normalized size = 1.05

$$-\frac{13a^2\left(\cos^7\left(dx+c\right)\right)}{80d\sin\left(dx+c\right)^8}-\frac{13a^2\left(\cos^7\left(dx+c\right)\right)}{480d\sin\left(dx+c\right)^6}+\frac{13a^2\left(\cos^7\left(dx+c\right)\right)}{1920d\sin\left(dx+c\right)^4}-\frac{13a^2\left(\cos^7\left(dx+c\right)\right)}{1280d\sin\left(dx+c\right)^2}-\frac{13a^2\left(\cos^5\left(dx+c\right)\right)}{1280d}-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x)

[Out] -13/80/d*a^2/sin(d*x+c)^8*cos(d*x+c)^7-13/480/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+13/1920/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7-13/1280/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-13/1280*a^2*cos(d*x+c)^5/d-13/768*a^2*cos(d*x+c)^3/d-13/256*a^2*cos(d*x+c)/d-13/256/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/9/d*a^2/sin(d*x+c)^9*cos(d*x+c)^7-4/63/d*a^2/sin(d*x+c)^7*cos(d*x+c)^7-1/10/d*a^2/sin(d*x+c)^10*cos(d*x+c)^7

maxima [A] time = 0.32, size = 273, normalized size = 1.20

$$-\frac{63\,a^2\left(\frac{2\left(15\,\cos(dx+c)^9-70\,\cos(dx+c)^7-128\,\cos(dx+c)^5+70\,\cos(dx+c)^3-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^8+10\,\cos(dx+c)^6-10\,\cos(dx+c)^4+5\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/161280*(63*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 210*a^2*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 5120*(9*tan(d*x + c)^2 + 7)*a^2/tan(d*x + c)^9)/d

mupad [B] time = 9.43, size = 357, normalized size = 1.57

$$\frac{19\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{6144\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{3\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{512\,d}-\frac{9\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}+\frac{3\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{1792\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{4096\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^11,x)

[Out] (19*a^2*cot(c/2 + (d*x)/2)^6)/(6144*d) - (a^2*cot(c/2 + (d*x)/2)^3)/(96*d) - (3*a^2*cot(c/2 + (d*x)/2)^4)/(512*d) - (9*a^2*cot(c/2 + (d*x)/2)^2)/(1024*d) + (3*a^2*cot(c/2 + (d*x)/2)^7)/(1792*d) - (a^2*cot(c/2 + (d*x)/2)^8)/(4096*d) - (a^2*cot(c/2 + (d*x)/2)^9)/(2304*d) - (a^2*cot(c/2 + (d*x)/2)^10)/(10240*d) + (9*a^2*tan(c/2 + (d*x)/2)^2)/(1024*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(96*d) + (3*a^2*tan(c/2 + (d*x)/2)^4)/(512*d) - (19*a^2*tan(c/2 + (d*x)

```
/2)^6)/(6144*d) - (3*a^2*tan(c/2 + (d*x)/2)^7)/(1792*d) + (a^2*tan(c/2 + (d
*x)/2)^8)/(4096*d) + (a^2*tan(c/2 + (d*x)/2)^9)/(2304*d) + (a^2*tan(c/2 + (
d*x)/2)^10)/(10240*d) - (13*a^2*log(tan(c/2 + (d*x)/2)))/(256*d) + (3*a^2*c
ot(c/2 + (d*x)/2))/(128*d) - (3*a^2*tan(c/2 + (d*x)/2))/(128*d)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**11*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.603 $\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=194

$$-\frac{a^2\cot^{11}(c+dx)}{11d}-\frac{a^2\cot^9(c+dx)}{3d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{5d}+\frac{a^2}{}$$

```
[Out] 3/128*a^2*arctanh(cos(d*x+c))/d-2/7*a^2*cot(d*x+c)^7/d-1/3*a^2*cot(d*x+c)^9
/d-1/11*a^2*cot(d*x+c)^11/d+3/128*a^2*cot(d*x+c)*csc(d*x+c)/d+1/64*a^2*cot(
d*x+c)*csc(d*x+c)^3/d-1/16*a^2*cot(d*x+c)*csc(d*x+c)^5/d+1/8*a^2*cot(d*x+c)
^3*csc(d*x+c)^5/d-1/5*a^2*cot(d*x+c)^5*csc(d*x+c)^5/d
```

Rubi [A] time = 0.30, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2873, 2607, 14, 2611, 3768, 3770, 270}

$$-\frac{a^2\cot^{11}(c+dx)}{11d}-\frac{a^2\cot^9(c+dx)}{3d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{5d}+\frac{a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (a^2
*Cot[c + d*x]^9)/(3*d) - (a^2*Cot[c + d*x]^11)/(11*d) + (3*a^2*Cot[c + d*x]
*Csc[c + d*x])/(128*d) + (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a^2*Co
t[c + d*x]*Csc[c + d*x]^5)/(16*d) + (a^2*Cot[c + d*x]^3*Csc[c + d*x]^5)/(8*
d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^5)/(5*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)\csc^4(c+dx) + 2a^2\cot^6(c+dx)\csc^5(c+\right.$$
$$= a^2\int \cot^6(c+dx)\csc^4(c+dx)\,dx + a^2\int \cot^6(c+dx)\csc^6($$
$$= -\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{5d} - a^2\int \cot^4(c+dx)\csc^5(c+$$
$$= \frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{5d}$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{3d} - \frac{a^2\cot^{11}(c+dx)}{11d} - \frac{a^2}{}$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{3d} - \frac{a^2\cot^{11}(c+dx)}{11d} + \frac{a^2}{}$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{3d} - \frac{a^2\cot^{11}(c+dx)}{11d} + \frac{3a}{}$$
$$= \frac{3a^2\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+d}{3d}$$

Mathematica [A] time = 3.05, size = 187, normalized size = 0.96

$$\frac{a^2(\sin(c+dx)+1)^2\left(887040\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc^{10}(c+dx)(107322}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(887040*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x
)/2]]) - Cot[c + d*x]*Csc[c + d*x]^10*(1318400 + 1798400*Cos[2*(c + d*x)] +
 440320*Cos[4*(c + d*x)] - 81280*Cos[6*(c + d*x)] - 38400*Cos[8*(c + d*x)]
+ 3200*Cos[10*(c + d*x)] + 1073226*Sin[c + d*x] + 869484*Sin[3*(c + d*x)] +
 727188*Sin[5*(c + d*x)] + 40425*Sin[7*(c + d*x)] - 3465*Sin[9*(c + d*x)]))
)/(37847040*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

fricas [B] time = 0.94, size = 360, normalized size = 1.86

$$\frac{12800\,a^2\cos(dx+c)^{11} - 70400\,a^2\cos(dx+c)^9 + 84480\,a^2\cos(dx+c)^7 + 3465\left(a^2\cos(dx+c)^{10} - 5\,a^2\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/295680*(12800*a^2*cos(d*x + c)^11 - 70400*a^2*cos(d*x + c)^9 + 84480*a^2*
cos(d*x + c)^7 + 3465*(a^2*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*
cos(d*x + c)^6 - 10*a^2*cos(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(1/
2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3465*(a^2*cos(d*x + c)^10 - 5*a^2*cos(
d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 10*a^2*cos(d*x + c)^4 + 5*a^2*cos(d*x
+ c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 462*(15*a^2*cos(d
*x + c)^9 - 70*a^2*cos(d*x + c)^7 - 128*a^2*cos(d*x + c)^5 + 70*a^2*cos(d*x
 + c)^3 - 15*a^2*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^10 - 5*d*cos(
d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2
 - d)*sin(d*x + c))
```

giac [B] time = 0.37, size = 388, normalized size = 2.00

$$\frac{105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+462\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+385\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-28}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/2365440*(105*a^2*tan(1/2*d*x + 1/2*c)^11 + 462*a^2*tan(1/2*d*x + 1/2*c)^1
0 + 385*a^2*tan(1/2*d*x + 1/2*c)^9 - 1155*a^2*tan(1/2*d*x + 1/2*c)^8 - 2805
*a^2*tan(1/2*d*x + 1/2*c)^7 - 2310*a^2*tan(1/2*d*x + 1/2*c)^6 + 1155*a^2*ta
n(1/2*d*x + 1/2*c)^5 + 9240*a^2*tan(1/2*d*x + 1/2*c)^4 + 16170*a^2*tan(1/2*
d*x + 1/2*c)^3 + 4620*a^2*tan(1/2*d*x + 1/2*c)^2 - 55440*a^2*log(abs(tan(1/
2*d*x + 1/2*c))) - 39270*a^2*tan(1/2*d*x + 1/2*c) + (167422*a^2*tan(1/2*d*x
 + 1/2*c)^11 + 39270*a^2*tan(1/2*d*x + 1/2*c)^10 - 4620*a^2*tan(1/2*d*x + 1
/2*c)^9 - 16170*a^2*tan(1/2*d*x + 1/2*c)^8 - 9240*a^2*tan(1/2*d*x + 1/2*c)^
7 - 1155*a^2*tan(1/2*d*x + 1/2*c)^6 + 2310*a^2*tan(1/2*d*x + 1/2*c)^5 + 280
5*a^2*tan(1/2*d*x + 1/2*c)^4 + 1155*a^2*tan(1/2*d*x + 1/2*c)^3 - 385*a^2*ta
n(1/2*d*x + 1/2*c)^2 - 462*a^2*tan(1/2*d*x + 1/2*c) - 105*a^2)/tan(1/2*d*x
+ 1/2*c)^11)/d
```

maple [A] time = 0.39, size = 264, normalized size = 1.36

$$-\frac{5a^2\left(\cos^7(dx+c)\right)}{33d\sin(dx+c)^9}-\frac{10a^2\left(\cos^7(dx+c)\right)}{231d\sin(dx+c)^7}-\frac{a^2\left(\cos^7(dx+c)\right)}{5d\sin(dx+c)^{10}}-\frac{3a^2\left(\cos^7(dx+c)\right)}{40d\sin(dx+c)^8}-\frac{a^2\left(\cos^7(dx+c)\right)}{80d\sin(dx+c)^6}+\frac{a^2\left(\cos\right.}{320d\,\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -5/33/d*a^2/sin(d*x+c)^9*cos(d*x+c)^7-10/231/d*a^2/sin(d*x+c)^7*cos(d*x+c)^
7-1/5/d*a^2/sin(d*x+c)^10*cos(d*x+c)^7-3/40/d*a^2/sin(d*x+c)^8*cos(d*x+c)^7
-1/80/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+1/320/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7
-3/640/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-3/640*a^2*cos(d*x+c)^5/d-1/128*a^2*c
os(d*x+c)^3/d-3/128*a^2*cos(d*x+c)/d-3/128/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-
1/11/d*a^2/sin(d*x+c)^11*cos(d*x+c)^7
```

maxima [A] time = 0.33, size = 197, normalized size = 1.02

$$-\frac{693\,a^2\left(\frac{2\left(15\,\cos(dx+c)^9-70\,\cos(dx+c)^7-128\,\cos(dx+c)^5+70\,\cos(dx+c)^3-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^8+10\,\cos(dx+c)^6-10\,\cos(dx+c)^4+5\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\right.\right.}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/887040*(693*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x
+ c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x
+ c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15
*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 14080*(9*tan(d*x + c)^
2 + 7)*a^2/tan(d*x + c)^9 + 1280*(99*tan(d*x + c)^4 + 154*tan(d*x + c)^2 +
63)*a^2/tan(d*x + c)^11)/d
```

mupad [B] time = 9.97, size = 433, normalized size = 2.23

$$\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{1024\,d}-\frac{7\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{1024\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2048\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{512\,d}+\frac{17\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{14336\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^12,x)
```

```
[Out] (a^2*cot(c/2 + (d*x)/2)^6)/(1024*d) - (7*a^2*cot(c/2 + (d*x)/2)^3)/(1024*d)
 - (a^2*cot(c/2 + (d*x)/2)^4)/(256*d) - (a^2*cot(c/2 + (d*x)/2)^5)/(2048*d)
 - (a^2*cot(c/2 + (d*x)/2)^2)/(512*d) + (17*a^2*cot(c/2 + (d*x)/2)^7)/(1433
6*d) + (a^2*cot(c/2 + (d*x)/2)^8)/(2048*d) - (a^2*cot(c/2 + (d*x)/2)^9)/(61
44*d) - (a^2*cot(c/2 + (d*x)/2)^10)/(5120*d) - (a^2*cot(c/2 + (d*x)/2)^11)/
(22528*d) + (a^2*tan(c/2 + (d*x)/2)^2)/(512*d) + (7*a^2*tan(c/2 + (d*x)/2)^
3)/(1024*d) + (a^2*tan(c/2 + (d*x)/2)^4)/(256*d) + (a^2*tan(c/2 + (d*x)/2)^
5)/(2048*d) - (a^2*tan(c/2 + (d*x)/2)^6)/(1024*d) - (17*a^2*tan(c/2 + (d*x)
/2)^7)/(14336*d) - (a^2*tan(c/2 + (d*x)/2)^8)/(2048*d) + (a^2*tan(c/2 + (d*
x)/2)^9)/(6144*d) + (a^2*tan(c/2 + (d*x)/2)^10)/(5120*d) + (a^2*tan(c/2 + (
```

```
d*x)/2)^11)/(22528*d) - (3*a^2*log(tan(c/2 + (d*x)/2)))/(128*d) + (17*a^2*c
ot(c/2 + (d*x)/2))/(1024*d) - (17*a^2*tan(c/2 + (d*x)/2))/(1024*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**12*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.604 $\int \cot^6(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=270

$$-\frac{2a^2\cot^{11}(c+dx)}{11d}-\frac{4a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{17a^2\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^2\cot^5(c+dx)\csc^7(c+dx)}{12d}-$$

[Out] 17/1024*a^2*arctanh(cos(d*x+c))/d-2/7*a^2*cot(d*x+c)^7/d-4/9*a^2*cot(d*x+c)^9/d-2/11*a^2*cot(d*x+c)^11/d+17/1024*a^2*cot(d*x+c)*csc(d*x+c)/d+17/1536*a^2*cot(d*x+c)*csc(d*x+c)^3/d-11/384*a^2*cot(d*x+c)*csc(d*x+c)^5/d+1/16*a^2*cot(d*x+c)^3*csc(d*x+c)^5/d-1/10*a^2*cot(d*x+c)^5*csc(d*x+c)^5/d-1/64*a^2*cot(d*x+c)*csc(d*x+c)^7/d+1/24*a^2*cot(d*x+c)^3*csc(d*x+c)^7/d-1/12*a^2*cot(d*x+c)^5*csc(d*x+c)^7/d

Rubi [A] time = 0.43, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3768, 3770, 2607, 270}

$$-\frac{2a^2\cot^{11}(c+dx)}{11d}-\frac{4a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{17a^2\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^2\cot^5(c+dx)\csc^7(c+dx)}{12d}-$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]

[Out] (17*a^2*ArcTanh[Cos[c + d*x]])/(1024*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (4*a^2*Cot[c + d*x]^9)/(9*d) - (2*a^2*Cot[c + d*x]^11)/(11*d) + (17*a^2*Cot[c + d*x]*Csc[c + d*x])/(1024*d) + (17*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(1536*d) - (11*a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(384*d) + (a^2*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^7)/(64*d) + (a^2*Cot[c + d*x]^3*Csc[c + d*x]^7)/(24*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^7)/(12*d)

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)\csc^5(c+dx) + 2a^2\cot^6(c+dx)\csc^6(c+\right.$$
$$= a^2\int \cot^6(c+dx)\csc^5(c+dx)\,dx + a^2\int \cot^6(c+dx)\csc^7($$
$$= -\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{a^2\cot^5(c+dx)\csc^7(c+dx)}{12d}$$
$$= \frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{10d} -$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^9(c+dx)}{9d} - \frac{2a^2\cot^{11}(c+dx)}{11d} -$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^9(c+dx)}{9d} - \frac{2a^2\cot^{11}(c+dx)}{11d} +$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^9(c+dx)}{9d} - \frac{2a^2\cot^{11}(c+dx)}{11d} +$$
$$= \frac{3a^2\tanh^{-1}(\cos(c+dx))}{256d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^9(c+}{9d}$$
$$= \frac{17a^2\tanh^{-1}(\cos(c+dx))}{1024d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^9(c+}{9d}$$

Mathematica [A] time = 4.72, size = 197, normalized size = 0.73

$$\frac{a^2(\sin(c+dx)+1)^2\left(30159360\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc^{11}(c+dx)(29655\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(30159360*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d
*x)/2]]) - Cot[c + d*x]*Csc[c + d*x]^11*(65553642 + 67499586*Cos[2*(c + d*x
)] + 25966248*Cos[4*(c + d*x)] - 6944091*Cos[6*(c + d*x)] - 746130*Cos[8*(c
 + d*x)] + 58905*Cos[10*(c + d*x)] + 29655040*Sin[c + d*x] + 51445760*Sin[3
*(c + d*x)] + 25600000*Sin[5*(c + d*x)] + 3235840*Sin[7*(c + d*x)] - 532480
*Sin[9*(c + d*x)] + 40960*Sin[11*(c + d*x)])))/(1816657920*d*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^4)
```

fricas [A] time = 1.02, size = 384, normalized size = 1.42

$$-\frac{117810\,a^2\cos\left(dx+c\right)^{11}-667590\,a^2\cos\left(dx+c\right)^{9}+135828\,a^2\cos\left(dx+c\right)^{7}+1555092\,a^2\cos\left(dx+c\right)^{5}-667}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/7096320*(117810*a^2*cos(d*x + c)^11 - 667590*a^2*cos(d*x + c)^9 + 135828
*a^2*cos(d*x + c)^7 + 1555092*a^2*cos(d*x + c)^5 - 667590*a^2*cos(d*x + c)^
3 + 117810*a^2*cos(d*x + c) - 58905*(a^2*cos(d*x + c)^12 - 6*a^2*cos(d*x +
c)^10 + 15*a^2*cos(d*x + c)^8 - 20*a^2*cos(d*x + c)^6 + 15*a^2*cos(d*x + c)
^4 - 6*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2) + 58905*(a^2*c
os(d*x + c)^12 - 6*a^2*cos(d*x + c)^10 + 15*a^2*cos(d*x + c)^8 - 20*a^2*cos
(d*x + c)^6 + 15*a^2*cos(d*x + c)^4 - 6*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*
cos(d*x + c) + 1/2) + 20480*(8*a^2*cos(d*x + c)^11 - 44*a^2*cos(d*x + c)^9
+ 99*a^2*cos(d*x + c)^7)*sin(d*x + c))/(d*cos(d*x + c)^12 - 6*d*cos(d*x + c
)^10 + 15*d*cos(d*x + c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)^4 - 6*
d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.42, size = 420, normalized size = 1.56

$$\frac{1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+5040\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+5544\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-6160\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/56770560*(1155*a^2*tan(1/2*d*x + 1/2*c)^12 + 5040*a^2*tan(1/2*d*x + 1/2*c
)^11 + 5544*a^2*tan(1/2*d*x + 1/2*c)^10 - 6160*a^2*tan(1/2*d*x + 1/2*c)^9 -
 24255*a^2*tan(1/2*d*x + 1/2*c)^8 - 39600*a^2*tan(1/2*d*x + 1/2*c)^7 - 2772
0*a^2*tan(1/2*d*x + 1/2*c)^6 + 55440*a^2*tan(1/2*d*x + 1/2*c)^5 + 162855*a^
2*tan(1/2*d*x + 1/2*c)^4 + 184800*a^2*tan(1/2*d*x + 1/2*c)^3 + 55440*a^2*ta
n(1/2*d*x + 1/2*c)^2 - 942480*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 554400*a
^2*tan(1/2*d*x + 1/2*c) + (2924714*a^2*tan(1/2*d*x + 1/2*c)^12 + 554400*a^2
*tan(1/2*d*x + 1/2*c)^11 - 55440*a^2*tan(1/2*d*x + 1/2*c)^10 - 184800*a^2*t
an(1/2*d*x + 1/2*c)^9 - 162855*a^2*tan(1/2*d*x + 1/2*c)^8 - 55440*a^2*tan(1
/2*d*x + 1/2*c)^7 + 27720*a^2*tan(1/2*d*x + 1/2*c)^6 + 39600*a^2*tan(1/2*d*
x + 1/2*c)^5 + 24255*a^2*tan(1/2*d*x + 1/2*c)^4 + 6160*a^2*tan(1/2*d*x + 1/
```

```
2*c)^3 - 5544*a^2*tan(1/2*d*x + 1/2*c)^2 - 5040*a^2*tan(1/2*d*x + 1/2*c) -
1155*a^2)/tan(1/2*d*x + 1/2*c)^12)/d
```

maple [A] time = 0.41, size = 288, normalized size = 1.07

$$-\frac{17a^2\left(\cos^7(dx+c)\right)}{120d\sin(dx+c)^{10}}-\frac{17a^2\left(\cos^7(dx+c)\right)}{320d\sin(dx+c)^8}-\frac{17a^2\left(\cos^7(dx+c)\right)}{1920d\sin(dx+c)^6}+\frac{17a^2\left(\cos^7(dx+c)\right)}{7680d\sin(dx+c)^4}-\frac{17a^2\left(\cos^7(dx+c)\right)}{5120d\sin(dx+c)^2}-\frac{17}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -17/120/d*a^2/sin(d*x+c)^10*cos(d*x+c)^7-17/320/d*a^2/sin(d*x+c)^8*cos(d*x+
c)^7-17/1920/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+17/7680/d*a^2/sin(d*x+c)^4*cos
(d*x+c)^7-17/5120/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-17/5120*a^2*cos(d*x+c)^5/
d-17/3072*a^2*cos(d*x+c)^3/d-17/1024*a^2*cos(d*x+c)/d-17/1024/d*a^2*ln(csc(
d*x+c)-cot(d*x+c))-2/11/d*a^2/sin(d*x+c)^11*cos(d*x+c)^7-8/99/d*a^2/sin(d*x
+c)^9*cos(d*x+c)^7-16/693/d*a^2/sin(d*x+c)^7*cos(d*x+c)^7-1/12/d*a^2/sin(d*
x+c)^12*cos(d*x+c)^7
```

maxima [A] time = 0.33, size = 323, normalized size = 1.20

$$-\frac{1155\,a^2\left(\frac{2\left(15\cos(dx+c)^{11}-85\cos(dx+c)^9+198\cos(dx+c)^7+198\cos(dx+c)^5-85\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^{12}-6\cos(dx+c)^{10}+15\cos(dx+c)^8-20\cos(dx+c)^6+15\cos(dx+c)^4-6\cos(dx+c)^2+1}-15\log(\cos(dx+c)+1)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/7096320*(1155*a^2*(2*(15*cos(d*x + c)^11 - 85*cos(d*x + c)^9 + 198*cos(d
*x + c)^7 + 198*cos(d*x + c)^5 - 85*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(
d*x + c)^12 - 6*cos(d*x + c)^10 + 15*cos(d*x + c)^8 - 20*cos(d*x + c)^6 + 1
5*cos(d*x + c)^4 - 6*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*lo
g(cos(d*x + c) - 1)) + 2772*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 -
 128*cos(d*x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10
 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c
)^2 - 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 20480*(99
*tan(d*x + c)^4 + 154*tan(d*x + c)^2 + 63)*a^2/tan(d*x + c)^11)/d
```

mupad [B] time = 10.36, size = 471, normalized size = 1.74

$$\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2048\,d}-\frac{5\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{1536\,d}-\frac{47\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{16384\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{1024\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}+\frac{5\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{7168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^13,x)
```

```
[Out] (a^2*cot(c/2 + (d*x)/2)^6)/(2048*d) - (5*a^2*cot(c/2 + (d*x)/2)^3)/(1536*d)
 - (47*a^2*cot(c/2 + (d*x)/2)^4)/(16384*d) - (a^2*cot(c/2 + (d*x)/2)^5)/(10
24*d) - (a^2*cot(c/2 + (d*x)/2)^2)/(1024*d) + (5*a^2*cot(c/2 + (d*x)/2)^7)/
(7168*d) + (7*a^2*cot(c/2 + (d*x)/2)^8)/(16384*d) + (a^2*cot(c/2 + (d*x)/2)
^9)/(9216*d) - (a^2*cot(c/2 + (d*x)/2)^10)/(10240*d) - (a^2*cot(c/2 + (d*x)
/2)^11)/(11264*d) - (a^2*cot(c/2 + (d*x)/2)^12)/(49152*d) + (a^2*tan(c/2 +
(d*x)/2)^2)/(1024*d) + (5*a^2*tan(c/2 + (d*x)/2)^3)/(1536*d) + (47*a^2*tan(
c/2 + (d*x)/2)^4)/(16384*d) + (a^2*tan(c/2 + (d*x)/2)^5)/(1024*d) - (a^2*ta
n(c/2 + (d*x)/2)^6)/(2048*d) - (5*a^2*tan(c/2 + (d*x)/2)^7)/(7168*d) - (7*a
^2*tan(c/2 + (d*x)/2)^8)/(16384*d) - (a^2*tan(c/2 + (d*x)/2)^9)/(9216*d) +
(a^2*tan(c/2 + (d*x)/2)^10)/(10240*d) + (a^2*tan(c/2 + (d*x)/2)^11)/(11264*
d) + (a^2*tan(c/2 + (d*x)/2)^12)/(49152*d) - (17*a^2*log(tan(c/2 + (d*x)/2)
))/(1024*d) + (5*a^2*cot(c/2 + (d*x)/2))/(512*d) - (5*a^2*tan(c/2 + (d*x)/2
))/(512*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**13*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.605 $\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=224

$$\frac{a^3\cos^{13}(c+dx)}{13d}-\frac{6a^3\cos^{11}(c+dx)}{11d}+\frac{a^3\cos^9(c+dx)}{d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{a^3\sin^5(c+dx)\cos^7(c+dx)}{4d}-\frac{9a^3\sin^3(c$$

```
[Out] 27/1024*a^3*x-4/7*a^3*cos(d*x+c)^7/d+a^3*cos(d*x+c)^9/d-6/11*a^3*cos(d*x+c)
^11/d+1/13*a^3*cos(d*x+c)^13/d+27/1024*a^3*cos(d*x+c)*sin(d*x+c)/d+9/512*a^
3*cos(d*x+c)^3*sin(d*x+c)/d+9/640*a^3*cos(d*x+c)^5*sin(d*x+c)/d-27/320*a^3*
cos(d*x+c)^7*sin(d*x+c)/d-9/40*a^3*cos(d*x+c)^7*sin(d*x+c)^3/d-1/4*a^3*cos(
d*x+c)^7*sin(d*x+c)^5/d
```

Rubi [A] time = 0.42, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2568, 2635, 8, 2565, 270}

$$\frac{a^3\cos^{13}(c+dx)}{13d}-\frac{6a^3\cos^{11}(c+dx)}{11d}+\frac{a^3\cos^9(c+dx)}{d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{a^3\sin^5(c+dx)\cos^7(c+dx)}{4d}-\frac{9a^3\sin^3(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (27*a^3*x)/1024 - (4*a^3*Cos[c + d*x]^7)/(7*d) + (a^3*Cos[c + d*x]^9)/d - (
6*a^3*Cos[c + d*x]^11)/(11*d) + (a^3*Cos[c + d*x]^13)/(13*d) + (27*a^3*Cos[
c + d*x]*Sin[c + d*x])/(1024*d) + (9*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(512*
d) + (9*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(640*d) - (27*a^3*Cos[c + d*x]^7*S
in[c + d*x])/(320*d) - (9*a^3*Cos[c + d*x]^7*Sin[c + d*x]^3)/(40*d) - (a^3*
Cos[c + d*x]^7*Sin[c + d*x]^5)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^6(c+dx)\sin^4(c+dx) + 3a^3\cos^6(c+dx)\sin^5(c+\right.$$

$$= a^3\int \cos^6(c+dx)\sin^4(c+dx)\,dx + a^3\int \cos^6(c+dx)\sin^7($$

$$= -\frac{a^3\cos^7(c+dx)\sin^3(c+dx)}{10d} - \frac{a^3\cos^7(c+dx)\sin^5(c+dx}{4d}$$

$$= -\frac{3a^3\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{9a^3\cos^7(c+dx)\sin^3(c+d}{40d}$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos^{11}(c+dx)}{11d} +$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos^{11}(c+dx)}{11d} +$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos^{11}(c+dx)}{11d} +$$

$$= \frac{3a^3x}{256} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos^{11}(c+}{11d}$$

$$= \frac{27a^3x}{1024} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos^{11}(c-}{11d}$$

Mathematica [A] time = 2.30, size = 146, normalized size = 0.65

$$\frac{a^3(80080\sin(2(c+dx)) - 385385\sin(4(c+dx)) - 40040\sin(6(c+dx)) + 65065\sin(8(c+dx)) + 8008\sin(10(c-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(720720*c + 1081080*d*x - 1401400*Cos[c + d*x] - 450450*Cos[3*(c + d*x
)] + 150150*Cos[5*(c + d*x)] + 94380*Cos[7*(c + d*x)] - 20020*Cos[9*(c + d*
x)] - 11830*Cos[11*(c + d*x)] + 770*Cos[13*(c + d*x)] + 80080*Sin[2*(c + d*
x)] - 385385*Sin[4*(c + d*x)] - 40040*Sin[6*(c + d*x)] + 65065*Sin[8*(c + d
*x)] + 8008*Sin[10*(c + d*x)] - 5005*Sin[12*(c + d*x)]))/(41000960*d)
```

fricas [A] time = 1.07, size = 150, normalized size = 0.67

$$\frac{394240\,a^3\cos(dx+c)^{13} - 2795520\,a^3\cos(dx+c)^{11} + 5125120\,a^3\cos(dx+c)^9 - 2928640\,a^3\cos(dx+c)^7 + 135}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/5125120*(394240*a^3*cos(d*x + c)^13 - 2795520*a^3*cos(d*x + c)^11 + 51251
20*a^3*cos(d*x + c)^9 - 2928640*a^3*cos(d*x + c)^7 + 135135*a^3*d*x - 1001*
(1280*a^3*cos(d*x + c)^11 - 3712*a^3*cos(d*x + c)^9 + 2864*a^3*cos(d*x + c)
^7 - 72*a^3*cos(d*x + c)^5 - 90*a^3*cos(d*x + c)^3 - 135*a^3*cos(d*x + c))*
sin(d*x + c))/d
```

giac [A] time = 0.55, size = 225, normalized size = 1.00

$$\frac{27}{1024}\,a^3x+\frac{a^3\cos\left(13\,dx+13\,c\right)}{53248\,d}-\frac{13\,a^3\cos\left(11\,dx+11\,c\right)}{45056\,d}-\frac{a^3\cos\left(9\,dx+9\,c\right)}{2048\,d}+\frac{33\,a^3\cos\left(7\,dx+7\,c\right)}{14336\,d}+\frac{15\,a^3\cos(5}{409}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 27/1024*a^3*x + 1/53248*a^3*cos(13*d*x + 13*c)/d - 13/45056*a^3*cos(11*d*x
+ 11*c)/d - 1/2048*a^3*cos(9*d*x + 9*c)/d + 33/14336*a^3*cos(7*d*x + 7*c)/d
 + 15/4096*a^3*cos(5*d*x + 5*c)/d - 45/4096*a^3*cos(3*d*x + 3*c)/d - 35/102
4*a^3*cos(d*x + c)/d - 1/8192*a^3*sin(12*d*x + 12*c)/d + 1/5120*a^3*sin(10*
d*x + 10*c)/d + 13/8192*a^3*sin(8*d*x + 8*c)/d - 1/1024*a^3*sin(6*d*x + 6*c
)/d - 77/8192*a^3*sin(4*d*x + 4*c)/d + 1/512*a^3*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.29, size = 308, normalized size = 1.38

$$\frac{a^3\left(-\frac{\left(\sin^6(dx+c)\right)\left(\cos^7(dx+c)\right)}{13}-\frac{6\left(\sin^4(dx+c)\right)\left(\cos^7(dx+c)\right)}{143}-\frac{8\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{429}-\frac{16\left(\cos^7(dx+c)\right)}{3003}\right)+3a^3\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^7\right.}{12}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/13*sin(d*x+c)^6*cos(d*x+c)^7-6/143*sin(d*x+c)^4*cos(d*x+c)^7-8
/429*sin(d*x+c)^2*cos(d*x+c)^7-16/3003*cos(d*x+c)^7)+3*a^3*(-1/12*sin(d*x+c
)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*cos(d*x+c)^7-1/64*cos(d*x+c)^7*sin(d*x+c
)+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1024*d
*x+5/1024*c)+3*a^3*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d
*x+c)^7-8/693*cos(d*x+c)^7)+a^3*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d
*x+c)^7*sin(d*x+c)+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*si
n(d*x+c)+3/256*d*x+3/256*c))
```

maxima [A] time = 0.34, size = 184, normalized size = 0.82

$$\frac{40960\left(231\,\cos\left(dx+c\right)^{13}-819\,\cos\left(dx+c\right)^{11}+1001\,\cos\left(dx+c\right)^{9}-429\,\cos\left(dx+c\right)^{7}\right)a^3-532480\left(63\,\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/123002880*(40960*(231*cos(d*x + c)^13 - 819*cos(d*x + c)^11 + 1001*cos(d*
x + c)^9 - 429*cos(d*x + c)^7)*a^3 - 532480*(63*cos(d*x + c)^11 - 154*cos(d
*x + c)^9 + 99*cos(d*x + c)^7)*a^3 + 12012*(32*sin(2*d*x + 2*c)^5 + 120*d*x
 + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a^3 + 15015*(4*sin(4*d
*x + 4*c)^3 + 120*d*x + 120*c + 9*sin(8*d*x + 8*c) - 48*sin(4*d*x + 4*c))*a
^3)/d
```

mupad [B] time = 12.19, size = 612, normalized size = 2.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^4*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (27*a^3*x)/1024 - ((27*a^3*(c + d*x))/1024 + (171*a^3*tan(c/2 + (d*x)/2)^3)
/256 - (1603*a^3*tan(c/2 + (d*x)/2)^5)/640 - (59523*a^3*tan(c/2 + (d*x)/2)^
7)/1280 + (305539*a^3*tan(c/2 + (d*x)/2)^9)/2560 - (93973*a^3*tan(c/2 + (d*
x)/2)^11)/640 + (93973*a^3*tan(c/2 + (d*x)/2)^15)/640 - (305539*a^3*tan(c/2
 + (d*x)/2)^17)/2560 + (59523*a^3*tan(c/2 + (d*x)/2)^19)/1280 + (1603*a^3*t
an(c/2 + (d*x)/2)^21)/640 - (171*a^3*tan(c/2 + (d*x)/2)^23)/256 - (27*a^3*t
an(c/2 + (d*x)/2)^25)/512 - a^3*((27*c)/1024 + (27*d*x)/1024 - 80/1001) + t
an(c/2 + (d*x)/2)^2*((351*a^3*(c + d*x))/1024 - a^3*((351*c)/1024 + (351*d*
x)/1024 - 80/77)) + tan(c/2 + (d*x)/2)^4*((1053*a^3*(c + d*x))/512 - a^3*((
1053*c)/512 + (1053*d*x)/512 - 480/77)) + tan(c/2 + (d*x)/2)^20*((3861*a^3*
(c + d*x))/512 - a^3*((3861*c)/512 + (3861*d*x)/512 - 32)) + tan(c/2 + (d*x
)/2)^6*((3861*a^3*(c + d*x))/512 - a^3*((3861*c)/512 + (3861*d*x)/512 + 64/
7)) + tan(c/2 + (d*x)/2)^14*((11583*a^3*(c + d*x))/256 - a^3*((11583*c)/256
 + (11583*d*x)/256 - 320)) + tan(c/2 + (d*x)/2)^12*((11583*a^3*(c + d*x))/2
56 - a^3*((11583*c)/256 + (11583*d*x)/256 + 1280/7)) + tan(c/2 + (d*x)/2)^1
8*((19305*a^3*(c + d*x))/1024 - a^3*((19305*c)/1024 + (19305*d*x)/1024 - 16
)) + tan(c/2 + (d*x)/2)^8*((19305*a^3*(c + d*x))/1024 - a^3*((19305*c)/1024
 + (19305*d*x)/1024 - 288/7)) + tan(c/2 + (d*x)/2)^16*((34749*a^3*(c + d*x)
)/1024 - a^3*((34749*c)/1024 + (34749*d*x)/1024 + 48)) + tan(c/2 + (d*x)/2)
^10*((34749*a^3*(c + d*x))/1024 - a^3*((34749*c)/1024 + (34749*d*x)/1024 -
1056/7)) + (27*a^3*tan(c/2 + (d*x)/2))/512)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^1
3)
```

sympy [A] time = 92.94, size = 748, normalized size = 3.34

$$\begin{cases} \frac{15a^3x\sin^{12}(c+dx)}{1024} + \frac{45a^3x\sin^{10}(c+dx)\cos^2(c+dx)}{512} + \frac{3a^3x\sin^{10}(c+dx)}{256} + \frac{225a^3x\sin^8(c+dx)\cos^4(c+dx)}{1024} + \frac{15a^3x\sin^8(c+dx)\cos^2(c+dx)}{256} \\ x(a\sin(c)+a)^3\sin^4(c)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((15*a**3*x*sin(c + d*x)**12/1024 + 45*a**3*x*sin(c + d*x)**10*cos
(c + d*x)**2/512 + 3*a**3*x*sin(c + d*x)**10/256 + 225*a**3*x*sin(c + d*x)*
*8*cos(c + d*x)**4/1024 + 15*a**3*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 7
5*a**3*x*sin(c + d*x)**6*cos(c + d*x)**6/256 + 15*a**3*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 225*a**3*x*sin(c + d*x)**4*cos(c + d*x)**8/1024 + 15*a*
*3*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 45*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**10/512 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 15*a**3*x*
cos(c + d*x)**12/1024 + 3*a**3*x*cos(c + d*x)**10/256 + 15*a**3*sin(c + d*x
)**11*cos(c + d*x)/(1024*d) + 85*a**3*sin(c + d*x)**9*cos(c + d*x)**3/(1024
*d) + 3*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 99*a**3*sin(c + d*x)**7
*cos(c + d*x)**5/(512*d) + 7*a**3*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) -
 a**3*sin(c + d*x)**6*cos(c + d*x)**7/(7*d) - 99*a**3*sin(c + d*x)**5*cos(c
 + d*x)**7/(512*d) + a**3*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) - 2*a**3*s
in(c + d*x)**4*cos(c + d*x)**9/(21*d) - 3*a**3*sin(c + d*x)**4*cos(c + d*x)
**7/(7*d) - 85*a**3*sin(c + d*x)**3*cos(c + d*x)**9/(1024*d) - 7*a**3*sin(c
 + d*x)**3*cos(c + d*x)**7/(128*d) - 8*a**3*sin(c + d*x)**2*cos(c + d*x)**1
1/(231*d) - 4*a**3*sin(c + d*x)**2*cos(c + d*x)**9/(21*d) - 15*a**3*sin(c +
 d*x)*cos(c + d*x)**11/(1024*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**9/(256*
d) - 16*a**3*cos(c + d*x)**13/(3003*d) - 8*a**3*cos(c + d*x)**11/(231*d), N
e(d, 0)), (x*(a*sin(c) + a)**3*sin(c)**4*cos(c)**6, True))
```

3.606 $\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=209

$$-\frac{3a^3\cos^{11}(c+dx)}{11d}+\frac{7a^3\cos^9(c+dx)}{9d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{a^3\sin^5(c+dx)\cos^7(c+dx)}{12d}-\frac{41a^3\sin^3(c+dx)\cos^7(c-}{120d}$$

```
[Out] 41/1024*a^3*x-4/7*a^3*cos(d*x+c)^7/d+7/9*a^3*cos(d*x+c)^9/d-3/11*a^3*cos(d*
x+c)^11/d+41/1024*a^3*cos(d*x+c)*sin(d*x+c)/d+41/1536*a^3*cos(d*x+c)^3*sin(
d*x+c)/d+41/1920*a^3*cos(d*x+c)^5*sin(d*x+c)/d-41/320*a^3*cos(d*x+c)^7*sin(
d*x+c)/d-41/120*a^3*cos(d*x+c)^7*sin(d*x+c)^3/d-1/12*a^3*cos(d*x+c)^7*sin(d
*x+c)^5/d
```

Rubi [A] time = 0.41, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{3a^3\cos^{11}(c+dx)}{11d}+\frac{7a^3\cos^9(c+dx)}{9d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{a^3\sin^5(c+dx)\cos^7(c+dx)}{12d}-\frac{41a^3\sin^3(c+dx)\cos^7(c-}{120d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (41*a^3*x)/1024 - (4*a^3*Cos[c + d*x]^7)/(7*d) + (7*a^3*Cos[c + d*x]^9)/(9*
d) - (3*a^3*Cos[c + d*x]^11)/(11*d) + (41*a^3*Cos[c + d*x]*Sin[c + d*x])/(1
024*d) + (41*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(1536*d) + (41*a^3*Cos[c + d*
x]^5*Sin[c + d*x])/(1920*d) - (41*a^3*Cos[c + d*x]^7*Sin[c + d*x])/(320*d)
- (41*a^3*Cos[c + d*x]^7*Sin[c + d*x]^3)/(120*d) - (a^3*Cos[c + d*x]^7*Sin[
c + d*x]^5)/(12*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^6(c+dx)\sin^3(c+dx) + 3a^3\cos^6(c+dx)\sin^4(c+\right.$$
$$= a^3\int \cos^6(c+dx)\sin^3(c+dx)\,dx + a^3\int \cos^6(c+dx)\sin^6($$
$$= -\frac{3a^3\cos^7(c+dx)\sin^3(c+dx)}{10d} - \frac{a^3\cos^7(c+dx)\sin^5(c+d}{12d}$$
$$= -\frac{9a^3\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{41a^3\cos^7(c+dx)\sin^3(c+}{120d}$$
$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos^{11}(c+dx)}{11d}$$
$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos^{11}(c+dx)}{11d}$$
$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos^{11}(c+dx)}{11d}$$
$$= \frac{9a^3x}{256} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos^{11}(c}{11d}$$
$$= \frac{41a^3x}{1024} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos^{11}(c}{11d}$$

Mathematica [A] time = 1.61, size = 136, normalized size = 0.65

$$\frac{a^3(166320\sin(2(c+dx)) - 384615\sin(4(c+dx)) - 83160\sin(6(c+dx)) + 51975\sin(8(c+dx)) + 16632\sin(10(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1247400*c + 1136520*d*x - 1496880*Cos[c + d*x] - 572880*Cos[3*(c + d*
x)] + 83160*Cos[5*(c + d*x)] + 106920*Cos[7*(c + d*x)] + 3080*Cos[9*(c + d*
x)] - 7560*Cos[11*(c + d*x)] + 166320*Sin[2*(c + d*x)] - 384615*Sin[4*(c +
d*x)] - 83160*Sin[6*(c + d*x)] + 51975*Sin[8*(c + d*x)] + 16632*Sin[10*(c +
 d*x)] - 1155*Sin[12*(c + d*x)]))/(28385280*d)
```

fricas [A] time = 0.83, size = 137, normalized size = 0.66

$$-\frac{967680\,a^3\cos(dx+c)^{11} - 2759680\,a^3\cos(dx+c)^9 + 2027520\,a^3\cos(dx+c)^7 - 142065\,a^3dx + 231\left(1280\,a^3\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/3548160*(967680*a^3*cos(d*x + c)^11 - 2759680*a^3*cos(d*x + c)^9 + 20275
20*a^3*cos(d*x + c)^7 - 142065*a^3*d*x + 231*(1280*a^3*cos(d*x + c)^11 - 78
08*a^3*cos(d*x + c)^9 + 8496*a^3*cos(d*x + c)^7 - 328*a^3*cos(d*x + c)^5 -
410*a^3*cos(d*x + c)^3 - 615*a^3*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.51, size = 208, normalized size = 1.00

$$\frac{41}{1024}\,a^3x - \frac{3\,a^3\cos\left(11\,dx+11\,c\right)}{11264\,d} + \frac{a^3\cos\left(9\,dx+9\,c\right)}{9216\,d} + \frac{27\,a^3\cos\left(7\,dx+7\,c\right)}{7168\,d} + \frac{3\,a^3\cos\left(5\,dx+5\,c\right)}{1024\,d} - \frac{31\,a^3\cos\left(3\,\right.}{1536}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 41/1024*a^3*x - 3/11264*a^3*cos(11*d*x + 11*c)/d + 1/9216*a^3*cos(9*d*x + 9
*c)/d + 27/7168*a^3*cos(7*d*x + 7*c)/d + 3/1024*a^3*cos(5*d*x + 5*c)/d - 31
/1536*a^3*cos(3*d*x + 3*c)/d - 27/512*a^3*cos(d*x + c)/d - 1/24576*a^3*sin(
12*d*x + 12*c)/d + 3/5120*a^3*sin(10*d*x + 10*c)/d + 15/8192*a^3*sin(8*d*x
+ 8*c)/d - 3/1024*a^3*sin(6*d*x + 6*c)/d - 111/8192*a^3*sin(4*d*x + 4*c)/d
+ 3/512*a^3*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.28, size = 272, normalized size = 1.30

$$\frac{a^3\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^7(dx+c)\right)}{12} - \frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{24} - \frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{64} + \frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{384} + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*cos(d*x+c)^7-1/
64*cos(d*x+c)^7*sin(d*x+c)+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*
x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c)+3*a^3*(-1/11*sin(d*x+c)^4*cos(d*x+c)^
7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/693*cos(d*x+c)^7)+3*a^3*(-1/10*sin(d*x+c
)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/160*(cos(d*x+c)^5+5/4*cos(d
*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)+a^3*(-1/9*sin(d*x+c)
^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7))
```

maxima [A] time = 0.35, size = 164, normalized size = 0.78

$$-\frac{122880\left(63\,\cos\left(dx+c\right)^{11} - 154\,\cos\left(dx+c\right)^{9} + 99\,\cos\left(dx+c\right)^{7}\right)a^3 - 450560\left(7\,\cos\left(dx+c\right)^{9} - 9\,\cos\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/28385280*(122880*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x +
 c)^7)*a^3 - 450560*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^3 - 8316*(32*si
n(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*
c))*a^3 - 1155*(4*sin(4*d*x + 4*c)^3 + 120*d*x + 120*c + 9*sin(8*d*x + 8*c)
 - 48*sin(4*d*x + 4*c))*a^3)/d
```

mupad [B] time = 11.02, size = 683, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^3*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (41*a^3*x)/1024 - ((1435*a^3*tan(c/2 + (d*x)/2)^3)/1536 - (36401*a^3*tan(c/
2 + (d*x)/2)^5)/2560 + (1263*a^3*tan(c/2 + (d*x)/2)^7)/2560 + (184331*a^3*t
an(c/2 + (d*x)/2)^9)/3840 - (35387*a^3*tan(c/2 + (d*x)/2)^11)/256 + (35387*
a^3*tan(c/2 + (d*x)/2)^13)/256 - (184331*a^3*tan(c/2 + (d*x)/2)^15)/3840 -
(1263*a^3*tan(c/2 + (d*x)/2)^17)/2560 + (36401*a^3*tan(c/2 + (d*x)/2)^19)/2
560 - (1435*a^3*tan(c/2 + (d*x)/2)^21)/1536 - (41*a^3*tan(c/2 + (d*x)/2)^23
)/512 + a^3*((41*c)/1024 + (41*d*x)/1024) - a^3*((41*c)/1024 + (41*d*x)/102
4 - 92/693) - tan(c/2 + (d*x)/2)^22*(a^3*((123*c)/256 + (123*d*x)/256) - 12
*a^3*((41*c)/1024 + (41*d*x)/1024)) - tan(c/2 + (d*x)/2)^2*(a^3*((123*c)/25
6 + (123*d*x)/256 - 368/231) - 12*a^3*((41*c)/1024 + (41*d*x)/1024)) + tan(
c/2 + (d*x)/2)^20*(66*a^3*((41*c)/1024 + (41*d*x)/1024) - a^3*((1353*c)/512
 + (1353*d*x)/512 - 4)) + tan(c/2 + (d*x)/2)^4*(66*a^3*((41*c)/1024 + (41*d
*x)/1024) - a^3*((1353*c)/512 + (1353*d*x)/512 - 100/21)) + tan(c/2 + (d*x)
/2)^18*(220*a^3*((41*c)/1024 + (41*d*x)/1024) - a^3*((2255*c)/256 + (2255*d
*x)/256 - 112/3)) + tan(c/2 + (d*x)/2)^6*(220*a^3*((41*c)/1024 + (41*d*x)/1
024) - a^3*((2255*c)/256 + (2255*d*x)/256 + 512/63)) + tan(c/2 + (d*x)/2)^1
4*(792*a^3*((41*c)/1024 + (41*d*x)/1024) - a^3*((4059*c)/128 + (4059*d*x)/1
28 - 128)) + tan(c/2 + (d*x)/2)^10*(792*a^3*((41*c)/1024 + (41*d*x)/1024) -
 a^3*((4059*c)/128 + (4059*d*x)/128 + 160/7)) + tan(c/2 + (d*x)/2)^12*(924*
a^3*((41*c)/1024 + (41*d*x)/1024) - a^3*((9471*c)/256 + (9471*d*x)/256 - 18
4/3)) + tan(c/2 + (d*x)/2)^16*(495*a^3*((41*c)/1024 + (41*d*x)/1024) - a^3*
((20295*c)/1024 + (20295*d*x)/1024 + 36)) + tan(c/2 + (d*x)/2)^8*(495*a^3*(
(41*c)/1024 + (41*d*x)/1024) - a^3*((20295*c)/1024 + (20295*d*x)/1024 - 712
/7)) + (41*a^3*tan(c/2 + (d*x)/2))/512)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^12)
```

sympy [A] time = 63.75, size = 699, normalized size = 3.34

$$\begin{cases} \frac{5a^3x\sin^{12}(c+dx)}{1024} + \frac{15a^3x\sin^{10}(c+dx)\cos^2(c+dx)}{512} + \frac{9a^3x\sin^{10}(c+dx)}{256} + \frac{75a^3x\sin^8(c+dx)\cos^4(c+dx)}{1024} + \frac{45a^3x\sin^8(c+dx)\cos^2(c+dx)}{256} + \\ x(a\sin(c)+a)^3\sin^3(c)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((5*a**3*x*sin(c + d*x)**12/1024 + 15*a**3*x*sin(c + d*x)**10*cos(
c + d*x)**2/512 + 9*a**3*x*sin(c + d*x)**10/256 + 75*a**3*x*sin(c + d*x)**8
*cos(c + d*x)**4/1024 + 45*a**3*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 25*
a**3*x*sin(c + d*x)**6*cos(c + d*x)**6/256 + 45*a**3*x*sin(c + d*x)**6*cos(
c + d*x)**4/128 + 75*a**3*x*sin(c + d*x)**4*cos(c + d*x)**8/1024 + 45*a**3*
x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**3*x*sin(c + d*x)**2*cos(c + d
*x)**10/512 + 45*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 5*a**3*x*cos(
c + d*x)**12/1024 + 9*a**3*x*cos(c + d*x)**10/256 + 5*a**3*sin(c + d*x)**11
*cos(c + d*x)/(1024*d) + 85*a**3*sin(c + d*x)**9*cos(c + d*x)**3/(3072*d) +
 9*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 33*a**3*sin(c + d*x)**7*cos(
c + d*x)**5/(512*d) + 21*a**3*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) - 33*
a**3*sin(c + d*x)**5*cos(c + d*x)**7/(512*d) + 3*a**3*sin(c + d*x)**5*cos(c
 + d*x)**5/(10*d) - 3*a**3*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 85*a**3*
sin(c + d*x)**3*cos(c + d*x)**9/(3072*d) - 21*a**3*sin(c + d*x)**3*cos(c +
d*x)**7/(128*d) - 4*a**3*sin(c + d*x)**2*cos(c + d*x)**9/(21*d) - a**3*sin(
c + d*x)**2*cos(c + d*x)**7/(7*d) - 5*a**3*sin(c + d*x)*cos(c + d*x)**11/(1
024*d) - 9*a**3*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 8*a**3*cos(c + d*x)*
*11/(231*d) - 2*a**3*cos(c + d*x)**9/(63*d), Ne(d, 0)), (x*(a*sin(c) + a)**
3*sin(c)**3*cos(c)**6, True))
```

3.607 $\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=183

$$-\frac{a^3\cos^{11}(c+dx)}{11d}+\frac{5a^3\cos^9(c+dx)}{9d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{3a^3\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{19a^3\sin(c+dx)\cos^7(c+dx)}{80d}$$

```
[Out] 19/256*a^3*x-4/7*a^3*cos(d*x+c)^7/d+5/9*a^3*cos(d*x+c)^9/d-1/11*a^3*cos(d*x
+c)^11/d+19/256*a^3*cos(d*x+c)*sin(d*x+c)/d+19/384*a^3*cos(d*x+c)^3*sin(d*x
+c)/d+19/480*a^3*cos(d*x+c)^5*sin(d*x+c)/d-19/80*a^3*cos(d*x+c)^7*sin(d*x+c
)/d-3/10*a^3*cos(d*x+c)^7*sin(d*x+c)^3/d
```

Rubi [A] time = 0.33, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2873, 2568, 2635, 8, 2565, 14, 270}

$$-\frac{a^3\cos^{11}(c+dx)}{11d}+\frac{5a^3\cos^9(c+dx)}{9d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{3a^3\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{19a^3\sin(c+dx)\cos^7(c+dx)}{80d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (19*a^3*x)/256 - (4*a^3*Cos[c + d*x]^7)/(7*d) + (5*a^3*Cos[c + d*x]^9)/(9*d
) - (a^3*Cos[c + d*x]^11)/(11*d) + (19*a^3*Cos[c + d*x]*Sin[c + d*x])/(256*
d) + (19*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + (19*a^3*Cos[c + d*x]^5*
Sin[c + d*x])/(480*d) - (19*a^3*Cos[c + d*x]^7*Sin[c + d*x])/(80*d) - (3*a^
3*Cos[c + d*x]^7*Sin[c + d*x]^3)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^6(c+dx)\sin^2(c+dx) + 3a^3\cos^6(c+dx)\sin^3(c+\right.$$

$$= a^3\int \cos^6(c+dx)\sin^2(c+dx)\,dx + a^3\int \cos^6(c+dx)\sin^5($$

$$= -\frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{3a^3\cos^7(c+dx)\sin^3(c+dx}{10d}$$

$$= \frac{a^3\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{19a^3\cos^7(c+dx)\sin(c+dx)}{80d}$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^9(c+dx)}{9d} - \frac{a^3\cos^{11}(c+dx)}{11d} +$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^9(c+dx)}{9d} - \frac{a^3\cos^{11}(c+dx)}{11d} +$$

$$= \frac{5a^3x}{128} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^9(c+dx)}{9d} - \frac{a^3\cos^{11}(c+}{11d}$$

$$= \frac{19a^3x}{256} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^9(c+dx)}{9d} - \frac{a^3\cos^{11}(c}{11d}$$

Mathematica [A] time = 1.14, size = 126, normalized size = 0.69

$$\frac{a^3(152460\sin(2(c+dx)) - 138600\sin(4(c+dx)) - 57750\sin(6(c+dx)) + 3465\sin(8(c+dx)) + 4158\sin(10(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(415800*c + 526680*d*x - 568260*Cos[c + d*x] - 244860*Cos[3*(c + d*x)]
 + 6930*Cos[5*(c + d*x)] + 40590*Cos[7*(c + d*x)] + 8470*Cos[9*(c + d*x)] -
 630*Cos[11*(c + d*x)] + 152460*Sin[2*(c + d*x)] - 138600*Sin[4*(c + d*x)]
- 57750*Sin[6*(c + d*x)] + 3465*Sin[8*(c + d*x)] + 4158*Sin[10*(c + d*x)]))
/(7096320*d)
```

fricas [A] time = 0.78, size = 124, normalized size = 0.68

$$-\frac{80640\,a^3\cos(dx+c)^{11} - 492800\,a^3\cos(dx+c)^9 + 506880\,a^3\cos(dx+c)^7 - 65835\,a^3dx - 231\left(1152\,a^3\cos(dx\right.}{88704}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/887040*(80640*a^3*cos(d*x + c)^11 - 492800*a^3*cos(d*x + c)^9 + 506880*a
^3*cos(d*x + c)^7 - 65835*a^3*d*x - 231*(1152*a^3*cos(d*x + c)^9 - 2064*a^3
*cos(d*x + c)^7 + 152*a^3*cos(d*x + c)^5 + 190*a^3*cos(d*x + c)^3 + 285*a^3
*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.42, size = 191, normalized size = 1.04

$$\frac{19}{256}\,a^3x - \frac{a^3\cos\left(11\,dx + 11\,c\right)}{11264\,d} + \frac{11\,a^3\cos\left(9\,dx + 9\,c\right)}{9216\,d} + \frac{41\,a^3\cos\left(7\,dx + 7\,c\right)}{7168\,d} + \frac{a^3\cos\left(5\,dx + 5\,c\right)}{1024\,d} - \frac{53\,a^3\cos\left(3\,dx\right.}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 19/256*a^3*x - 1/11264*a^3*cos(11*d*x + 11*c)/d + 11/9216*a^3*cos(9*d*x + 9
*c)/d + 41/7168*a^3*cos(7*d*x + 7*c)/d + 1/1024*a^3*cos(5*d*x + 5*c)/d - 53
/1536*a^3*cos(3*d*x + 3*c)/d - 41/512*a^3*cos(d*x + c)/d + 3/5120*a^3*sin(1
0*d*x + 10*c)/d + 1/2048*a^3*sin(8*d*x + 8*c)/d - 25/3072*a^3*sin(6*d*x + 6
*c)/d - 5/256*a^3*sin(4*d*x + 4*c)/d + 11/512*a^3*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.29, size = 236, normalized size = 1.29

$$\frac{a^3\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^7(dx+c)\right)}{11} - \frac{4\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{99} - \frac{8\left(\cos^7(dx+c)\right)}{693}\right) + 3a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10} - \frac{3\left(\cos^7(dx+c)\right)\sin(dx\right.}{80}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/
693*cos(d*x+c)^7)+3*a^3*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*
sin(d*x+c)+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)
+3/256*d*x+3/256*c)+3*a^3*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7
)+a^3*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/
8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c))
```

maxima [A] time = 0.32, size = 164, normalized size = 0.90

$$-\frac{10240\left(63\,\cos\left(dx+c\right)^{11} - 154\,\cos\left(dx+c\right)^{9} + 99\,\cos\left(dx+c\right)^{7}\right)a^3 - 337920\left(7\,\cos\left(dx+c\right)^{9} - 9\,\cos\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/7096320*(10240*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c
)^7)*a^3 - 337920*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^3 - 2079*(32*sin(
2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c)
)*a^3 - 2310*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c)
- 24*sin(4*d*x + 4*c))*a^3)/d
```

mupad [B] time = 12.12, size = 543, normalized size = 2.97

$$\frac{19\,a^3\,x}{256}-\frac{\frac{19\,a^3\,(c+d\,x)}{256}-\frac{13\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}-\frac{32417\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{1920}+\frac{466\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{15}-\frac{2937\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{64}+\frac{2937\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (19*a^3*x)/256 - ((19*a^3*(c + d*x))/256 - (13*a^3*tan(c/2 + (d*x)/2)^3)/12
 - (32417*a^3*tan(c/2 + (d*x)/2)^5)/1920 + (466*a^3*tan(c/2 + (d*x)/2)^7)/1
5 - (2937*a^3*tan(c/2 + (d*x)/2)^9)/64 + (2937*a^3*tan(c/2 + (d*x)/2)^13)/6
4 - (466*a^3*tan(c/2 + (d*x)/2)^15)/15 + (32417*a^3*tan(c/2 + (d*x)/2)^17)/
1920 + (13*a^3*tan(c/2 + (d*x)/2)^19)/12 - (19*a^3*tan(c/2 + (d*x)/2)^21)/1
28 - a^3*((19*c)/256 + (19*d*x)/256 - 148/693) + tan(c/2 + (d*x)/2)^2*((209
*a^3*(c + d*x))/256 - a^3*((209*c)/256 + (209*d*x)/256 - 148/63)) + tan(c/2
 + (d*x)/2)^18*((1045*a^3*(c + d*x))/256 - a^3*((1045*c)/256 + (1045*d*x)/2
56 - 12)) + tan(c/2 + (d*x)/2)^4*((1045*a^3*(c + d*x))/256 - a^3*((1045*c)/
256 + (1045*d*x)/256 + 16/63)) + tan(c/2 + (d*x)/2)^14*((3135*a^3*(c + d*x)
)/128 - a^3*((3135*c)/128 + (3135*d*x)/128 - 16/3)) + tan(c/2 + (d*x)/2)^16
*((3135*a^3*(c + d*x))/256 - a^3*((3135*c)/256 + (3135*d*x)/256 - 44/3)) +
tan(c/2 + (d*x)/2)^8*((3135*a^3*(c + d*x))/128 - a^3*((3135*c)/128 + (3135*
d*x)/128 - 456/7)) + tan(c/2 + (d*x)/2)^6*((3135*a^3*(c + d*x))/256 - a^3*(
(3135*c)/256 + (3135*d*x)/256 - 144/7)) + tan(c/2 + (d*x)/2)^10*((4389*a^3*
(c + d*x))/128 - a^3*((4389*c)/128 + (4389*d*x)/128 + 24)) + tan(c/2 + (d*x
)/2)^12*((4389*a^3*(c + d*x))/128 - a^3*((4389*c)/128 + (4389*d*x)/128 - 36
8/3)) + (19*a^3*tan(c/2 + (d*x)/2))/128)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^11)
```

sympy [A] time = 41.84, size = 597, normalized size = 3.26

$$\begin{cases} \frac{9a^3x\sin^{10}(c+dx)}{256}+\frac{45a^3x\sin^8(c+dx)\cos^2(c+dx)}{256}+\frac{5a^3x\sin^8(c+dx)}{128}+\frac{45a^3x\sin^6(c+dx)\cos^4(c+dx)}{128}+\frac{5a^3x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{45a^3\cdots}{\cdots} \\ x\,(a\sin(c)+a)^3\sin^2(c)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((9*a**3*x*sin(c + d*x)**10/256 + 45*a**3*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 5*a**3*x*sin(c + d*x)**8/128 + 45*a**3*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 5*a**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 45*a**3*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**3*x*sin(c + d*x)**4*cos(c + d*
x)**4/64 + 45*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 5*a**3*x*sin(c +
 d*x)**2*cos(c + d*x)**6/32 + 9*a**3*x*cos(c + d*x)**10/256 + 5*a**3*x*cos(
c + d*x)**8/128 + 9*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 21*a**3*sin
(c + d*x)**7*cos(c + d*x)**3/(128*d) + 5*a**3*sin(c + d*x)**7*cos(c + d*x)/
(128*d) + 3*a**3*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 55*a**3*sin(c + d
*x)**5*cos(c + d*x)**3/(384*d) - a**3*sin(c + d*x)**4*cos(c + d*x)**7/(7*d)
 - 21*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) + 73*a**3*sin(c + d*x)**
3*cos(c + d*x)**5/(384*d) - 4*a**3*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) -
 3*a**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 9*a**3*sin(c + d*x)*cos(c +
 d*x)**9/(256*d) - 5*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 8*a**3*cos
(c + d*x)**11/(693*d) - 2*a**3*cos(c + d*x)**9/(21*d), Ne(d, 0)), (x*(a*sin
(c) + a)**3*sin(c)**2*cos(c)**6, True))
```

3.608 $\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=181

$$-\frac{33a^3\cos^7(c+dx)}{560d}-\frac{11\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{240d}+\frac{11a^3\sin(c+dx)\cos^5(c+dx)}{160d}+\frac{11a^3\sin(c+dx)\cos^3(}{128d}$$

```
[Out] 33/256*a^3*x-33/560*a^3*cos(d*x+c)^7/d+33/256*a^3*cos(d*x+c)*sin(d*x+c)/d+1
1/128*a^3*cos(d*x+c)^3*sin(d*x+c)/d+11/160*a^3*cos(d*x+c)^5*sin(d*x+c)/d-1/
30*a*cos(d*x+c)^7*(a+a*sin(d*x+c))^2/d-1/10*cos(d*x+c)^7*(a+a*sin(d*x+c))^3
/d-11/240*cos(d*x+c)^7*(a^3+a^3*sin(d*x+c))/d
```

Rubi [A] time = 0.20, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{33a^3\cos^7(c+dx)}{560d}-\frac{11\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{240d}+\frac{11a^3\sin(c+dx)\cos^5(c+dx)}{160d}+\frac{11a^3\sin(c+dx)\cos^3(}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (33*a^3*x)/256 - (33*a^3*Cos[c + d*x]^7)/(560*d) + (33*a^3*Cos[c + d*x]*Sin
[c + d*x])/(256*d) + (11*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) + (11*a^3
*Cos[c + d*x]^5*Sin[c + d*x])/(160*d) - (a*Cos[c + d*x]^7*(a + a*Sin[c + d*
x])^2)/(30*d) - (Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3)/(10*d) - (11*Cos[c
+ d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(240*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{\cos^7(c+dx)(a+a\sin(c+dx))^3}{10d} + \frac{3}{10}\int \cos^6(c+dx)(a+$$

$$= -\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{30d} - \frac{\cos^7(c+dx)(a+a\,\mathrm{si}}{10d}$$

$$= -\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{30d} - \frac{\cos^7(c+dx)(a+a\,\mathrm{si}}{10d}$$

$$= -\frac{33a^3\cos^7(c+dx)}{560d} - \frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{30d} -$$

$$= -\frac{33a^3\cos^7(c+dx)}{560d} + \frac{11a^3\cos^5(c+dx)\sin(c+dx)}{160d} - \frac{a\cos}{}$$

$$= -\frac{33a^3\cos^7(c+dx)}{560d} + \frac{11a^3\cos^3(c+dx)\sin(c+dx)}{128d} + \frac{11a^3}{}$$

$$= -\frac{33a^3\cos^7(c+dx)}{560d} + \frac{33a^3\cos(c+dx)\sin(c+dx)}{256d} + \frac{11a^3}{}$$

$$= \frac{33a^3x}{256} - \frac{33a^3\cos^7(c+dx)}{560d} + \frac{33a^3\cos(c+dx)\sin(c+dx)}{256d}$$

Mathematica [A] time = 0.92, size = 116, normalized size = 0.64

$$\frac{a^3(10500\sin(2(c+dx))-5880\sin(4(c+dx))-3570\sin(6(c+dx))-525\sin(8(c+dx))+42\sin(10(c+dx))-\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(31500*c + 27720*d*x - 31920*Cos[c + d*x] - 16800*Cos[3*(c + d*x)] - 3
360*Cos[5*(c + d*x)] + 600*Cos[7*(c + d*x)] + 280*Cos[9*(c + d*x)] + 10500*
Sin[2*(c + d*x)] - 5880*Sin[4*(c + d*x)] - 3570*Sin[6*(c + d*x)] - 525*Sin[
8*(c + d*x)] + 42*Sin[10*(c + d*x)]))/(215040*d)
```

fricas [A] time = 0.66, size = 111, normalized size = 0.61

$$\frac{8960\,a^3\cos(dx+c)^9-15360\,a^3\cos(dx+c)^7+3465\,a^3dx+21\left(128\,a^3\cos(dx+c)^9-656\,a^3\cos(dx+c)^7+88\ldots\right.}{26880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/26880*(8960*a^3*cos(d*x + c)^9 - 15360*a^3*cos(d*x + c)^7 + 3465*a^3*d*x
+ 21*(128*a^3*cos(d*x + c)^9 - 656*a^3*cos(d*x + c)^7 + 88*a^3*cos(d*x + c)
^5 + 110*a^3*cos(d*x + c)^3 + 165*a^3*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.36, size = 174, normalized size = 0.96

$$\frac{33}{256}\,a^3x+\frac{a^3\cos(9\,dx+9\,c)}{768\,d}+\frac{5\,a^3\cos(7\,dx+7\,c)}{1792\,d}-\frac{a^3\cos(5\,dx+5\,c)}{64\,d}-\frac{5\,a^3\cos(3\,dx+3\,c)}{64\,d}-\frac{19\,a^3\cos(dx+c)}{128\,d}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 33/256*a^3*x + 1/768*a^3*cos(9*d*x + 9*c)/d + 5/1792*a^3*cos(7*d*x + 7*c)/d
 - 1/64*a^3*cos(5*d*x + 5*c)/d - 5/64*a^3*cos(3*d*x + 3*c)/d - 19/128*a^3*c
os(d*x + c)/d + 1/5120*a^3*sin(10*d*x + 10*c)/d - 5/2048*a^3*sin(8*d*x + 8*
c)/d - 17/1024*a^3*sin(6*d*x + 6*c)/d - 7/256*a^3*sin(4*d*x + 4*c)/d + 25/5
12*a^3*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.28, size = 198, normalized size = 1.09

$$\frac{a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{160}+\frac{3dx}{256}+\frac{3c}{256}\right)+3a^3\left(-\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/16
0*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/25
6*c)+3*a^3*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+3*a^3*(-1/8*c
os(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))
*sin(d*x+c)+5/128*d*x+5/128*c)-1/7*a^3*cos(d*x+c)^7)
```

maxima [A] time = 0.32, size = 141, normalized size = 0.78

$$-\frac{30720\,a^3\cos(dx+c)^7-10240\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a^3-21\left(32\sin(2\,dx+2\,c)^5+120\,dx+120\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/215040*(30720*a^3*cos(d*x + c)^7 - 10240*(7*cos(d*x + c)^9 - 9*cos(d*x +
 c)^7)*a^3 - 21*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*
c) - 40*sin(4*d*x + 4*c))*a^3 - 210*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*
c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a^3)/d
```

mupad [B] time = 10.96, size = 572, normalized size = 3.16

$$\frac{33\,a^3\,x}{256}-\frac{\frac{333\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{32}-\frac{577\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160}-\frac{705\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{128}-\frac{2749\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{64}+\frac{2749\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64}-\frac{333\,a^3\tan\left(\frac{c}{2}+\right.}{32}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (33*a^3*x)/256 - ((333*a^3*tan(c/2 + (d*x)/2)^7)/32 - (577*a^3*tan(c/2 + (d
*x)/2)^5)/160 - (705*a^3*tan(c/2 + (d*x)/2)^3)/128 - (2749*a^3*tan(c/2 + (d
*x)/2)^9)/64 + (2749*a^3*tan(c/2 + (d*x)/2)^11)/64 - (333*a^3*tan(c/2 + (d*
x)/2)^13)/32 + (577*a^3*tan(c/2 + (d*x)/2)^15)/160 + (705*a^3*tan(c/2 + (d*
x)/2)^17)/128 - (33*a^3*tan(c/2 + (d*x)/2)^19)/128 + a^3*((33*c)/256 + (33*
d*x)/256) - a^3*((33*c)/256 + (33*d*x)/256 - 10/21) + tan(c/2 + (d*x)/2)^18
*(10*a^3*((33*c)/256 + (33*d*x)/256) - a^3*((165*c)/128 + (165*d*x)/128 - 2
)) + tan(c/2 + (d*x)/2)^2*(10*a^3*((33*c)/256 + (33*d*x)/256) - a^3*((165*c
)/128 + (165*d*x)/128 - 58/21)) + tan(c/2 + (d*x)/2)^14*(120*a^3*((33*c)/25
6 + (33*d*x)/256) - a^3*((495*c)/32 + (495*d*x)/32 - 8)) + tan(c/2 + (d*x)/
2)^6*(120*a^3*((33*c)/256 + (33*d*x)/256) - a^3*((495*c)/32 + (495*d*x)/32
```

```
- 344/7)) + tan(c/2 + (d*x)/2)^16*(45*a^3*((33*c)/256 + (33*d*x)/256) - a^3
*((1485*c)/256 + (1485*d*x)/256 - 18)) + tan(c/2 + (d*x)/2)^4*(45*a^3*((33*
c)/256 + (33*d*x)/256) - a^3*((1485*c)/256 + (1485*d*x)/256 - 24/7)) + tan(
c/2 + (d*x)/2)^10*(252*a^3*((33*c)/256 + (33*d*x)/256) - a^3*((2079*c)/64 +
 (2079*d*x)/64 - 60)) + tan(c/2 + (d*x)/2)^8*(210*a^3*((33*c)/256 + (33*d*x
)/256) - a^3*((3465*c)/128 + (3465*d*x)/128 - 28)) + tan(c/2 + (d*x)/2)^12*
(210*a^3*((33*c)/256 + (33*d*x)/256) - a^3*((3465*c)/128 + (3465*d*x)/128 -
 72)) + (33*a^3*tan(c/2 + (d*x)/2))/128)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^10)
```

sympy [A] time = 26.98, size = 542, normalized size = 2.99

$$\begin{cases} \frac{3a^3x\sin^{10}(c+dx)}{256} + \frac{15a^3x\sin^8(c+dx)\cos^2(c+dx)}{256} + \frac{15a^3x\sin^8(c+dx)}{128} + \frac{15a^3x\sin^6(c+dx)\cos^4(c+dx)}{128} + \frac{15a^3x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{15 \\ x\left(a\sin(c)+a\right)^3\sin(c)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**10/256 + 15*a**3*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 15*a**3*x*sin(c + d*x)**8/128 + 15*a**3*x*sin(c + d*x)**6*c
os(c + d*x)**4/128 + 15*a**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 15*a**3
*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 45*a**3*x*sin(c + d*x)**4*cos(c +
d*x)**4/64 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 15*a**3*x*sin(
c + d*x)**2*cos(c + d*x)**6/32 + 3*a**3*x*cos(c + d*x)**10/256 + 15*a**3*x*
cos(c + d*x)**8/128 + 3*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**3*
sin(c + d*x)**7*cos(c + d*x)**3/(128*d) + 15*a**3*sin(c + d*x)**7*cos(c + d
*x)/(128*d) + a**3*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 55*a**3*sin(c +
 d*x)**5*cos(c + d*x)**3/(128*d) - 7*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(
128*d) + 73*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 3*a**3*sin(c + d
*x)**2*cos(c + d*x)**7/(7*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**9/(256*d)
- 15*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 2*a**3*cos(c + d*x)**9/(21
*d) - a**3*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)*co
s(c)**6, True))
```

3.609 $\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=185

$$-\frac{3a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{25a^3\sin(c-$$

```
[Out] 125/128*a^3*x-a^3*arctanh(cos(d*x+c))/d+a^3*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)
^3/d+1/5*a^3*cos(d*x+c)^5/d-3/7*a^3*cos(d*x+c)^7/d+125/128*a^3*cos(d*x+c)*s
in(d*x+c)/d+125/192*a^3*cos(d*x+c)^3*sin(d*x+c)/d+25/48*a^3*cos(d*x+c)^5*si
n(d*x+c)/d-1/8*a^3*cos(d*x+c)^7*sin(d*x+c)/d
```

Rubi [A] time = 0.24, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2873, 2635, 8, 2592, 302, 206, 2565, 30, 2568}

$$-\frac{3a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{25a^3\sin(c-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (125*a^3*x)/128 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Cos[c + d*x])/d + (a
^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cos[c + d*x]^5)/(5*d) - (3*a^3*Cos[c + d*x]
^7)/(7*d) + (125*a^3*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (125*a^3*Cos[c +
d*x]^3*Sin[c + d*x])/(192*d) + (25*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(48*d)
- (a^3*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx = \int \Big(3a^3\cos^6(c+dx)+a^3\cos^5(c+dx)\cot(c+dx)+3a^3\text{co}$$

$$= a^3\int \cos^5(c+dx)\cot(c+dx)\,dx + a^3\int \cos^6(c+dx)\sin^2($$

$$= \frac{a^3\cos^5(c+dx)\sin(c+dx)}{2d} - \frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d} +$$

$$= -\frac{3a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^3(c+dx)\sin(c+dx)}{8d} + \frac{25a^3\text{c}}{}$$

$$= \frac{a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} + \frac{a^3\cos^5(c+dx)}{5d} - \frac{3a^3\text{c}}{}$$

$$= \frac{15a^3x}{16} - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} + \frac{a^3\text{cos}}{}$$

$$= \frac{125a^3x}{128} - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} + \frac{a^3\text{co}}{}$$

Mathematica [A] time = 0.70, size = 122, normalized size = 0.66

$$\frac{a^3\Big(77280\sin(2(c+dx))+14280\sin(4(c+dx))+1120\sin(6(c+dx))-105\sin(8(c+dx))+122640\cos(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(105000*c + 105000*d*x + 122640*Cos[c + d*x] + 560*Cos[3*(c + d*x)] -
3696*Cos[5*(c + d*x)] - 720*Cos[7*(c + d*x)] - 107520*Log[Cos[(c + d*x)/2]]
 + 107520*Log[Sin[(c + d*x)/2]] + 77280*Sin[2*(c + d*x)] + 14280*Sin[4*(c +
 d*x)] + 1120*Sin[6*(c + d*x)] - 105*Sin[8*(c + d*x)]))/(107520*d)
```

fricas [A] time = 0.84, size = 154, normalized size = 0.83

$$-\frac{5760\,a^3\cos(dx+c)^7-2688\,a^3\cos(dx+c)^5-4480\,a^3\cos(dx+c)^3-13125\,a^3dx-13440\,a^3\cos(dx+c)+6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/13440*(5760*a^3*cos(d*x + c)^7 - 2688*a^3*cos(d*x + c)^5 - 4480*a^3*cos(
d*x + c)^3 - 13125*a^3*d*x - 13440*a^3*cos(d*x + c) + 6720*a^3*log(1/2*cos(
```

```
d*x + c) + 1/2) - 6720*a^3*log(-1/2*cos(d*x + c) + 1/2) + 35*(48*a^3*cos(d*
x + c)^7 - 200*a^3*cos(d*x + c)^5 - 250*a^3*cos(d*x + c)^3 - 375*a^3*cos(d*
x + c))*sin(d*x + c))/d
```

giac [A] time = 0.31, size = 277, normalized size = 1.50

$$13125\,(dx+c)a^3+13440\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(27195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+65135\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-161280\,a^3\tan\left(\frac{1}{2}\,dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/13440*(13125*(d*x + c)*a^3 + 13440*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2
*(27195*a^3*tan(1/2*d*x + 1/2*c)^15 + 65135*a^3*tan(1/2*d*x + 1/2*c)^13 - 1
61280*a^3*tan(1/2*d*x + 1/2*c)^12 + 63595*a^3*tan(1/2*d*x + 1/2*c)^11 - 286
720*a^3*tan(1/2*d*x + 1/2*c)^10 + 133175*a^3*tan(1/2*d*x + 1/2*c)^9 - 51968
0*a^3*tan(1/2*d*x + 1/2*c)^8 - 133175*a^3*tan(1/2*d*x + 1/2*c)^7 - 544768*a
^3*tan(1/2*d*x + 1/2*c)^6 - 63595*a^3*tan(1/2*d*x + 1/2*c)^5 - 254464*a^3*t
an(1/2*d*x + 1/2*c)^4 - 65135*a^3*tan(1/2*d*x + 1/2*c)^3 - 118784*a^3*tan(1
/2*d*x + 1/2*c)^2 - 27195*a^3*tan(1/2*d*x + 1/2*c) - 14848*a^3)/(tan(1/2*d*
x + 1/2*c)^2 + 1)^8)/d
```

maple [A] time = 0.49, size = 187, normalized size = 1.01

$$-\frac{a^3\left(\cos^7(dx+c)\right)\sin(dx+c)}{8d}+\frac{25a^3\left(\cos^5(dx+c)\right)\sin(dx+c)}{48d}+\frac{125a^3\left(\cos^3(dx+c)\right)\sin(dx+c)}{192d}+\frac{125a^3\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/8*a^3*cos(d*x+c)^7*sin(d*x+c)/d+25/48*a^3*cos(d*x+c)^5*sin(d*x+c)/d+125/
192*a^3*cos(d*x+c)^3*sin(d*x+c)/d+125/128*a^3*cos(d*x+c)*sin(d*x+c)/d+125/1
28*a^3*x+125/128/d*a^3*c-3/7*a^3*cos(d*x+c)^7/d+1/5*a^3*cos(d*x+c)^5/d+1/3*
a^3*cos(d*x+c)^3/d+a^3*cos(d*x+c)/d+1/d*a^3*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.45, size = 171, normalized size = 0.92

$$-\frac{46080\,a^3\cos(dx+c)^7-3584\left(6\,\cos(dx+c)^5+10\,\cos(dx+c)^3+30\,\cos(dx+c)-15\,\log(\cos(dx+c)+1)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/107520*(46080*a^3*cos(d*x + c)^7 - 3584*(6*cos(d*x + c)^5 + 10*cos(d*x +
 c)^3 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
1))*a^3 - 35*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c)
- 24*sin(4*d*x + 4*c))*a^3 + 1680*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9
*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3)/d
```

mupad [B] time = 10.85, size = 429, normalized size = 2.32

$$\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{125\,a^3\,\mathrm{atan}\left(\frac{15625\,a^6}{4096\,\left(\frac{125\,a^6}{32}-\frac{15625\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4096}\right)}+\frac{125\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{32\,\left(\frac{125\,a^6}{32}-\frac{15625\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4096}\right)}\right)}{64\,d}+\frac{-\frac{259\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d + (125*a^3*atan((15625*a^6)/(4096*((125*a^6
)/32 - (15625*a^6*tan(c/2 + (d*x)/2))/4096)) + (125*a^6*tan(c/2 + (d*x)/2))
/(32*((125*a^6)/32 - (15625*a^6*tan(c/2 + (d*x)/2))/4096))))/(64*d) + ((185
6*a^3*tan(c/2 + (d*x)/2)^2)/105 + (1861*a^3*tan(c/2 + (d*x)/2)^3)/192 + (56
8*a^3*tan(c/2 + (d*x)/2)^4)/15 + (1817*a^3*tan(c/2 + (d*x)/2)^5)/192 + (121
6*a^3*tan(c/2 + (d*x)/2)^6)/15 + (3805*a^3*tan(c/2 + (d*x)/2)^7)/192 + (232
*a^3*tan(c/2 + (d*x)/2)^8)/3 - (3805*a^3*tan(c/2 + (d*x)/2)^9)/192 + (128*a
^3*tan(c/2 + (d*x)/2)^10)/3 - (1817*a^3*tan(c/2 + (d*x)/2)^11)/192 + 24*a^3
*tan(c/2 + (d*x)/2)^12 - (1861*a^3*tan(c/2 + (d*x)/2)^13)/192 - (259*a^3*ta
n(c/2 + (d*x)/2)^15)/64 + (232*a^3)/105 + (259*a^3*tan(c/2 + (d*x)/2))/64)/
(d*(8*tan(c/2 + (d*x)/2)^2 + 28*tan(c/2 + (d*x)/2)^4 + 56*tan(c/2 + (d*x)/2
)^6 + 70*tan(c/2 + (d*x)/2)^8 + 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*
x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.610 $\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=173

$$-\frac{a^3\cos^7(c+dx)}{7d}+\frac{3a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\sin^5(c+dx)\cos(c+d}{2d}$$

```
[Out] -15/16*a^3*x-3*a^3*arctanh(cos(d*x+c))/d+3*a^3*cos(d*x+c)/d+a^3*cos(d*x+c)^
3/d+3/5*a^3*cos(d*x+c)^5/d-1/7*a^3*cos(d*x+c)^7/d-a^3*cot(d*x+c)/d+15/16*a^
3*cos(d*x+c)*sin(d*x+c)/d-11/8*a^3*cos(d*x+c)*sin(d*x+c)^3/d+1/2*a^3*cos(d*
x+c)*sin(d*x+c)^5/d
```

Rubi [A] time = 0.24, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2872, 3770, 3767, 8, 2638, 2635, 2633}

$$-\frac{a^3\cos^7(c+dx)}{7d}+\frac{3a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\sin^5(c+dx)\cos(c+d}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-15*a^3*x)/16 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d +
 (a^3*Cos[c + d*x]^3)/d + (3*a^3*Cos[c + d*x]^5)/(5*d) - (a^3*Cos[c + d*x]^
7)/(7*d) - (a^3*Cot[c + d*x])/d + (15*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d)
 - (11*a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(8*d) + (a^3*Cos[c + d*x]*Sin[c + d
*x]^5)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(3a^9\csc(c+dx)+a^9\csc^2(c+dx)-8a^9\sin(c+dx)-6a\right.}{}$$

$$= a^3\int \csc^2(c+dx)\,dx - a^3\int \sin^7(c+dx)\,dx + \left(3a^3\right)\int \mathrm{cs}$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{8a^3\cos(c+dx)}{d} + \frac{3a^3\cos(c}{}$$

$$= -3a^3x - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} + \frac{a^3}{}$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c}{d}$$

$$= -\frac{15a^3x}{16} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} + \frac{a}{}$$

Mathematica [A] time = 1.88, size = 168, normalized size = 0.97

$$\frac{(a\sin(c+dx)+a)^3\left(-2100(c+dx)+455\sin(2(c+dx))+245\sin(4(c+dx))+35\sin(6(c+dx))+9065\cos(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-2100*(c + d*x) + 9065*Cos[c + d*x] + 875*Cos[3*(c
 + d*x)] + 49*Cos[5*(c + d*x)] - 5*Cos[7*(c + d*x)] - 1120*Cot[(c + d*x)/2]
 - 6720*Log[Cos[(c + d*x)/2]] + 6720*Log[Sin[(c + d*x)/2]] + 455*Sin[2*(c +
 d*x)] + 245*Sin[4*(c + d*x)] + 35*Sin[6*(c + d*x)] + 1120*Tan[(c + d*x)/2]
))/(2240*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.77, size = 173, normalized size = 1.00

$$-\frac{280\,a^3\cos(dx+c)^7-70\,a^3\cos(dx+c)^5-175\,a^3\cos(dx+c)^3+840\,a^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/560*(280*a^3*cos(d*x + c)^7 - 70*a^3*cos(d*x + c)^5 - 175*a^3*cos(d*x +
c)^3 + 840*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 840*a^3*log(-1/2*
cos(d*x + c) + 1/2)*sin(d*x + c) + 525*a^3*cos(d*x + c) + (80*a^3*cos(d*x +
 c)^7 - 336*a^3*cos(d*x + c)^5 - 560*a^3*cos(d*x + c)^3 + 525*a^3*d*x - 168
0*a^3*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.33, size = 290, normalized size = 1.68

$$-\frac{525\,(dx+c)a^3-1680\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{280\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(525\,a^3\tan\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/560*(525*(d*x + c)*a^3 - 1680*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 280*a
^3*tan(1/2*d*x + 1/2*c) + 280*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*
```

```
x + 1/2*c) + 2*(525*a^3*tan(1/2*d*x + 1/2*c)^13 - 4480*a^3*tan(1/2*d*x + 1/
2*c)^12 - 980*a^3*tan(1/2*d*x + 1/2*c)^11 - 20160*a^3*tan(1/2*d*x + 1/2*c)^
10 + 945*a^3*tan(1/2*d*x + 1/2*c)^9 - 38080*a^3*tan(1/2*d*x + 1/2*c)^8 - 49
280*a^3*tan(1/2*d*x + 1/2*c)^6 - 945*a^3*tan(1/2*d*x + 1/2*c)^5 - 32256*a^3
*tan(1/2*d*x + 1/2*c)^4 + 980*a^3*tan(1/2*d*x + 1/2*c)^3 - 12992*a^3*tan(1/
2*d*x + 1/2*c)^2 - 525*a^3*tan(1/2*d*x + 1/2*c) - 2496*a^3)/(tan(1/2*d*x +
1/2*c)^2 + 1)^7)/d
```

maple [A] time = 0.42, size = 190, normalized size = 1.10

$$-\frac{a^3\left(\cos^7(dx+c)\right)}{7d}-\frac{a^3\left(\cos^5(dx+c)\right)\sin(dx+c)}{2d}-\frac{5a^3\left(\cos^3(dx+c)\right)\sin(dx+c)}{8d}-\frac{15a^3\cos(dx+c)\sin(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/7*a^3*cos(d*x+c)^7/d-1/2*a^3*cos(d*x+c)^5*sin(d*x+c)/d-5/8*a^3*cos(d*x+c
)^3*sin(d*x+c)/d-15/16*a^3*cos(d*x+c)*sin(d*x+c)/d-15/16*a^3*x-15/16/d*a^3*
c+3/5*a^3*cos(d*x+c)^5/d+a^3*cos(d*x+c)^3/d+3*a^3*cos(d*x+c)/d+3/d*a^3*ln(c
sc(d*x+c)-cot(d*x+c))-1/d*a^3/sin(d*x+c)*cos(d*x+c)^7
```

maxima [A] time = 0.41, size = 186, normalized size = 1.08

$$-\frac{320\,a^3\cos(dx+c)^7-224\left(6\cos(dx+c)^5+10\cos(dx+c)^3+30\cos(dx+c)-15\log(\cos(dx+c)+1)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2240*(320*a^3*cos(d*x + c)^7 - 224*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3
 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a
^3 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin
(2*d*x + 2*c))*a^3 + 280*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x +
 c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x + c)))*a^3)/d
```

mupad [B] time = 9.38, size = 429, normalized size = 2.48

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{15\,a^3\,\mathrm{atan}\left(\frac{225\,a^6}{64\left(\frac{45\,a^6}{4}+\frac{225\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}-\frac{45\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\left(\frac{45\,a^6}{4}+\frac{225\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}\right)}{8\,d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{19\,a^3\,\tan\left(\frac{c}{2}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d + (15*a^3*atan((225*a^6)/(64*((45*a^6)/4
+ (225*a^6*tan(c/2 + (d*x)/2))/64)) - (45*a^6*tan(c/2 + (d*x)/2))/(4*((45*a
^6)/4 + (225*a^6*tan(c/2 + (d*x)/2))/64))))/(8*d) + (a^3*tan(c/2 + (d*x)/2)
)/(2*d) - ((13*a^3*tan(c/2 + (d*x)/2)^2)/4 - (464*a^3*tan(c/2 + (d*x)/2)^3)
/5 + 28*a^3*tan(c/2 + (d*x)/2)^4 - (1152*a^3*tan(c/2 + (d*x)/2)^5)/5 + (113
*a^3*tan(c/2 + (d*x)/2)^6)/4 - 352*a^3*tan(c/2 + (d*x)/2)^7 + 35*a^3*tan(c/
2 + (d*x)/2)^8 - 272*a^3*tan(c/2 + (d*x)/2)^9 + (111*a^3*tan(c/2 + (d*x)/2)
^10)/4 - 144*a^3*tan(c/2 + (d*x)/2)^11 - 32*a^3*tan(c/2 + (d*x)/2)^13 + (19
*a^3*tan(c/2 + (d*x)/2)^14)/4 + a^3 - (624*a^3*tan(c/2 + (d*x)/2))/35)/(d*(
2*tan(c/2 + (d*x)/2) + 14*tan(c/2 + (d*x)/2)^3 + 42*tan(c/2 + (d*x)/2)^5 +
70*tan(c/2 + (d*x)/2)^7 + 70*tan(c/2 + (d*x)/2)^9 + 42*tan(c/2 + (d*x)/2)^1
1 + 14*tan(c/2 + (d*x)/2)^13 + 2*tan(c/2 + (d*x)/2)^15))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.611 $\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=181

$$\frac{3a^3\cos^5(c+dx)}{5d}+\frac{2a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{5a^3\sin^3(c+$$

```
[Out] -85/16*a^3*x-1/2*a^3*arctanh(cos(d*x+c))/d+a^3*cos(d*x+c)/d+2/3*a^3*cos(d*x
+c)^3/d+3/5*a^3*cos(d*x+c)^5/d-3*a^3*cot(d*x+c)/d-1/2*a^3*cot(d*x+c)*csc(d*
x+c)/d-43/16*a^3*cos(d*x+c)*sin(d*x+c)/d+5/24*a^3*cos(d*x+c)*sin(d*x+c)^3/d
+1/6*a^3*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A] time = 0.25, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2872, 3767, 8, 3768, 3770, 2638, 2635, 2633}

$$\frac{3a^3\cos^5(c+dx)}{5d}+\frac{2a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{5a^3\sin^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-85*a^3*x)/16 - (a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*Cos[c + d*x])/d +
 (2*a^3*Cos[c + d*x]^3)/(3*d) + (3*a^3*Cos[c + d*x]^5)/(5*d) - (3*a^3*Cot[c
 + d*x])/d - (a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) - (43*a^3*Cos[c + d*x]*S
in[c + d*x])/(16*d) + (5*a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) + (a^3*Cos
[c + d*x]*Sin[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int\left(-8a^9+3a^9\csc^2(c+dx)+a^9\csc^3(c+dx)-6a^9\sin(c+\right.}{}$$

$$= -8a^3x + a^3\int \csc^3(c+dx)\,dx - a^3\int \sin^6(c+dx)\,dx + \left(3\right.$$

$$= -8a^3x + \frac{6a^3\cos(c+dx)}{d} - \frac{a^3\cot(c+dx)\csc(c+dx)}{2d} - \frac{3a}{}$$

$$= -5a^3x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{a^3\cos(c+dx)}{d} + \frac{2a^3 c}{}$$

$$= -5a^3x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{a^3\cos(c+dx)}{d} + \frac{2a^3 c}{}$$

$$= -\frac{85a^3x}{16} - \frac{a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{a^3\cos(c+dx)}{d} + \frac{2a^3}{}$$

Mathematica [B] time = 6.37, size = 664, normalized size = 3.67

$$-\frac{81\sin(2(c+dx))(a\sin(c+dx)+a)^3}{64d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6} - \frac{3\sin(4(c+dx))(a\sin(c+dx)+a)^3}{64d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6} + \frac{\sin(6(c+dx))(a\sin}{192d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-85*(c + d*x)*(a + a*Sin[c + d*x])^3)/(16*d*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^6) + (15*Cos[c + d*x]*(a + a*Sin[c + d*x])^3)/(8*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^6) + (17*Cos[3*(c + d*x)]*(a + a*Sin[c + d*x])^3)/(48
*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + (3*Cos[5*(c + d*x)]*(a + a*Si
n[c + d*x])^3)/(80*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) - (3*Cot[(c +
 d*x)/2]*(a + a*Sin[c + d*x])^3)/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^6) - (Csc[(c + d*x)/2]^2*(a + a*Sin[c + d*x])^3)/(8*d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^6) - (Log[Cos[(c + d*x)/2]]*(a + a*Sin[c + d*x])^3)/(2*d*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + (Log[Sin[(c + d*x)/2]]*(a + a*Si
n[c + d*x])^3)/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + (Sec[(c + d*
x)/2]^2*(a + a*Sin[c + d*x])^3)/(8*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
6) - (81*(a + a*Sin[c + d*x])^3*Sin[2*(c + d*x)])/(64*d*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^6) - (3*(a + a*Sin[c + d*x])^3*Sin[4*(c + d*x)])/(64*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + ((a + a*Sin[c + d*x])^3*Sin[6*(c
+ d*x)])/(192*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + (3*(a + a*Sin[c
+ d*x])^3*Tan[(c + d*x)/2])/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.82, size = 212, normalized size = 1.17

$$\frac{144\,a^3\cos\left(dx+c\right)^7+16\,a^3\cos\left(dx+c\right)^5-1275\,a^3dx\cos\left(dx+c\right)^2+80\,a^3\cos\left(dx+c\right)^3+1275\,a^3dx-120\,a^3\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/240*(144*a^3*cos(d*x + c)^7 + 16*a^3*cos(d*x + c)^5 - 1275*a^3*d*x*cos(d*
x + c)^2 + 80*a^3*cos(d*x + c)^3 + 1275*a^3*d*x - 120*a^3*cos(d*x + c) - 60
*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2) + 60*(a^3*cos(d*x +
 c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) + 5*(8*a^3*cos(d*x + c)^7 - 34*a^
3*cos(d*x + c)^5 - 85*a^3*cos(d*x + c)^3 + 255*a^3*cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.37, size = 306, normalized size = 1.69

$$\frac{30\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1275\,(dx+c)a^3+120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+360\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{30\left(6\,a^3\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(30*a^3*tan(1/2*d*x + 1/2*c)^2 - 1275*(d*x + c)*a^3 + 120*a^3*log(abs
(tan(1/2*d*x + 1/2*c))) + 360*a^3*tan(1/2*d*x + 1/2*c) - 30*(6*a^3*tan(1/2*
d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^2
+ 2*(645*a^3*tan(1/2*d*x + 1/2*c)^11 + 1440*a^3*tan(1/2*d*x + 1/2*c)^10 + 1
735*a^3*tan(1/2*d*x + 1/2*c)^9 + 3360*a^3*tan(1/2*d*x + 1/2*c)^8 + 450*a^3*
tan(1/2*d*x + 1/2*c)^7 + 5440*a^3*tan(1/2*d*x + 1/2*c)^6 - 450*a^3*tan(1/2*
d*x + 1/2*c)^5 + 4800*a^3*tan(1/2*d*x + 1/2*c)^4 - 1735*a^3*tan(1/2*d*x + 1
/2*c)^3 + 1824*a^3*tan(1/2*d*x + 1/2*c)^2 - 645*a^3*tan(1/2*d*x + 1/2*c) +
544*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 0.50, size = 199, normalized size = 1.10

$$-\frac{17a^3\left(\cos^5\left(dx+c\right)\right)\sin\left(dx+c\right)}{6d}-\frac{85a^3\left(\cos^3\left(dx+c\right)\right)\sin\left(dx+c\right)}{24d}-\frac{85a^3\cos\left(dx+c\right)\sin\left(dx+c\right)}{16d}-\frac{85a^3x}{16}-\frac{85a^3}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -17/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d-85/24*a^3*cos(d*x+c)^3*sin(d*x+c)/d-85/
16*a^3*cos(d*x+c)*sin(d*x+c)/d-85/16*a^3*x-85/16/d*a^3*c+1/10*a^3*cos(d*x+c
)^5/d+1/6*a^3*cos(d*x+c)^3/d+1/2*a^3*cos(d*x+c)/d+1/2/d*a^3*ln(csc(d*x+c)-c
ot(d*x+c))-3/d*a^3/sin(d*x+c)*cos(d*x+c)^7-1/2/d*a^3/sin(d*x+c)^2*cos(d*x+c
)^7
```

maxima [A] time = 0.49, size = 239, normalized size = 1.32

$$\frac{96\left(6\cos\left(dx+c\right)^{5}+10\cos\left(dx+c\right)^{3}+30\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/960*(96*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(
cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^3 - 80*(4*cos(d*x + c)^3 -
6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c)
 + 1) + 15*log(cos(d*x + c) - 1))*a^3 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x -
60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3 - 360*(15*d*x + 15*c +
 (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x +
c)^3 + tan(d*x + c)))*a^3)/d
```

mupad [B] time = 8.92, size = 438, normalized size = 2.42

$$\frac{\frac{31\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{2}+\frac{95\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}}{2}+\frac{131\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{6}+109\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-75\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}+\frac{1043\,a^3\,\tan\left(\frac{c}{2}\right.}{6}}{d\,\left(4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+24\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+60\,\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] ((227*a^3*tan(c/2 + (d*x)/2)^2)/15 - (115*a^3*tan(c/2 + (d*x)/2)^3)/2 + (53
3*a^3*tan(c/2 + (d*x)/2)^4)/10 - (887*a^3*tan(c/2 + (d*x)/2)^5)/6 + 150*a^3
*tan(c/2 + (d*x)/2)^6 - 135*a^3*tan(c/2 + (d*x)/2)^7 + (1043*a^3*tan(c/2 +
(d*x)/2)^8)/6 - 75*a^3*tan(c/2 + (d*x)/2)^9 + 109*a^3*tan(c/2 + (d*x)/2)^10
 + (131*a^3*tan(c/2 + (d*x)/2)^11)/6 + (95*a^3*tan(c/2 + (d*x)/2)^12)/2 + (
31*a^3*tan(c/2 + (d*x)/2)^13)/2 - a^3/2 - 6*a^3*tan(c/2 + (d*x)/2))/(d*(4*t
```

```
an(c/2 + (d*x)/2)^2 + 24*tan(c/2 + (d*x)/2)^4 + 60*tan(c/2 + (d*x)/2)^6 + 8
0*tan(c/2 + (d*x)/2)^8 + 60*tan(c/2 + (d*x)/2)^10 + 24*tan(c/2 + (d*x)/2)^1
2 + 4*tan(c/2 + (d*x)/2)^14)) + (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^3*log
(tan(c/2 + (d*x)/2)))/(2*d) + (85*a^3*atan((7225*a^6)/(64*((85*a^6)/8 + (72
25*a^6*tan(c/2 + (d*x)/2))/64)) - (85*a^6*tan(c/2 + (d*x)/2))/(8*((85*a^6)/
8 + (7225*a^6*tan(c/2 + (d*x)/2))/64))))/(8*d) + (3*a^3*tan(c/2 + (d*x)/2))
/(2*d)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.612 $\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=176

$$\frac{a^3\cos^5(c+dx)}{5d}-\frac{2a^3\cos^3(c+dx)}{3d}-\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin^3(c+dx)\cos(c+a}{4d}$$

```
[Out] -25/8*a^3*x+13/2*a^3*arctanh(cos(d*x+c))/d-5*a^3*cos(d*x+c)/d-2/3*a^3*cos(d
*x+c)^3/d+1/5*a^3*cos(d*x+c)^5/d-a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d-3/
2*a^3*cot(d*x+c)*csc(d*x+c)/d-23/8*a^3*cos(d*x+c)*sin(d*x+c)/d+3/4*a^3*cos(
d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.21, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2872, 3770, 3768, 3767, 2638, 2635, 8, 2633}

$$\frac{a^3\cos^5(c+dx)}{5d}-\frac{2a^3\cos^3(c+dx)}{3d}-\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin^3(c+dx)\cos(c+a}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-25*a^3*x)/8 + (13*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a^3*Cos[c + d*x])
/d - (2*a^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cos[c + d*x]^5)/(5*d) - (a^3*Cot[c
 + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])
/(2*d) - (23*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (3*a^3*Cos[c + d*x]*Sin
[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int\left(-6a^9-8a^9\csc(c+dx)+3a^9\csc^3(c+dx)+a^9\csc^4(c+\right.}{}$$

$$= -6a^3x + a^3\int\csc^4(c+dx)\,dx - a^3\int\sin^5(c+dx)\,dx + \left(3\right.$$

$$= -6a^3x + \frac{8a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{6a^3\cos(c+dx)}{d} - \frac{3a^3}{}$$

$$= -2a^3x + \frac{13a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{5a^3\cos(c+dx)}{d} - \frac{2a}{}$$

$$= -\frac{25a^3x}{8} + \frac{13a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{5a^3\cos(c+dx)}{d} - $$

Mathematica [A] time = 1.43, size = 219, normalized size = 1.24

$$\frac{a^3(\sin(c+dx)+1)^3\left(-1500(c+dx)-600\sin(2(c+dx))-45\sin(4(c+dx))-2580\cos(c+dx)-50\cos(3(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-1500*(c + d*x) - 2580*Cos[c + d*x] - 50*Cos[3*(
c + d*x)] + 6*Cos[5*(c + d*x)] - 160*Cot[(c + d*x)/2] - 180*Csc[(c + d*x)/2
]^2 + 3120*Log[Cos[(c + d*x)/2]] - 3120*Log[Sin[(c + d*x)/2]] + 180*Sec[(c
+ d*x)/2]^2 + 160*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 10*Csc[(c + d*x)/2]^4
*Sin[c + d*x] - 600*Sin[2*(c + d*x)] - 45*Sin[4*(c + d*x)] + 160*Tan[(c + d
*x)/2]))/(480*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.70, size = 231, normalized size = 1.31

$$\frac{90\,a^3\cos(dx+c)^7 + 75\,a^3\cos(dx+c)^5 - 500\,a^3\cos(dx+c)^3 + 375\,a^3\cos(dx+c) + 390\left(a^3\cos(dx+c)^2 - a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/120*(90*a^3*cos(d*x + c)^7 + 75*a^3*cos(d*x + c)^5 - 500*a^3*cos(d*x + c)
^3 + 375*a^3*cos(d*x + c) + 390*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x
```

```
+ c) + 1/2)*sin(d*x + c) - 390*(a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) + (24*a^3*cos(d*x + c)^7 - 104*a^3*cos(d*x + c)^5
- 375*a^3*d*x*cos(d*x + c)^2 - 520*a^3*cos(d*x + c)^3 + 375*a^3*d*x + 780*a
^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.38, size = 292, normalized size = 1.66

$$5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-375\,(dx+c)a^3-780\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+45\,a^3\tan\left(\frac{1}{2}\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(5*a^3*tan(1/2*d*x + 1/2*c)^3 + 45*a^3*tan(1/2*d*x + 1/2*c)^2 - 375*(
d*x + c)*a^3 - 780*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 45*a^3*tan(1/2*d*x
+ 1/2*c) + 5*(286*a^3*tan(1/2*d*x + 1/2*c)^3 - 9*a^3*tan(1/2*d*x + 1/2*c)^2
 - 9*a^3*tan(1/2*d*x + 1/2*c) - a^3)/tan(1/2*d*x + 1/2*c)^3 + 2*(345*a^3*ta
n(1/2*d*x + 1/2*c)^9 - 720*a^3*tan(1/2*d*x + 1/2*c)^8 + 330*a^3*tan(1/2*d*x
 + 1/2*c)^7 - 2880*a^3*tan(1/2*d*x + 1/2*c)^6 - 3680*a^3*tan(1/2*d*x + 1/2*
c)^4 - 330*a^3*tan(1/2*d*x + 1/2*c)^3 - 2560*a^3*tan(1/2*d*x + 1/2*c)^2 - 3
45*a^3*tan(1/2*d*x + 1/2*c) - 656*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

maple [A] time = 0.45, size = 223, normalized size = 1.27

$$-\frac{13a^3\left(\cos^5(dx+c)\right)}{10d}-\frac{13a^3\left(\cos^3(dx+c)\right)}{6d}-\frac{13a^3\cos(dx+c)}{2d}-\frac{13a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{5a^3\left(\cos^7(\right.}{3d\sin(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -13/10*a^3*cos(d*x+c)^5/d-13/6*a^3*cos(d*x+c)^3/d-13/2*a^3*cos(d*x+c)/d-13/
2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-5/3/d*a^3/sin(d*x+c)*cos(d*x+c)^7-5/3*a^3
*cos(d*x+c)^5*sin(d*x+c)/d-25/12*a^3*cos(d*x+c)^3*sin(d*x+c)/d-25/8*a^3*cos
(d*x+c)*sin(d*x+c)/d-25/8*a^3*x-25/8/d*a^3*c-3/2/d*a^3/sin(d*x+c)^2*cos(d*x
+c)^7-1/3/d*a^3/sin(d*x+c)^3*cos(d*x+c)^7
```

maxima [A] time = 0.42, size = 246, normalized size = 1.40

$$4\left(6\,\cos(dx+c)^5+10\,\cos(dx+c)^3+30\,\cos(dx+c)-15\,\log(\cos(dx+c)+1)+15\,\log(\cos(dx+c)-1)\right)a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/120*(4*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(c
os(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^3 - 30*(4*cos(d*x + c)^3 - 6
*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c)
+ 1) + 15*log(cos(d*x + c) - 1))*a^3 - 45*(15*d*x + 15*c + (15*tan(d*x + c)
^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x +
c)))*a^3 + 20*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/
(tan(d*x + c)^5 + tan(d*x + c)^3))*a^3)/d
```

mupad [B] time = 8.99, size = 429, normalized size = 2.44

$$\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{13\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{-43\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+99\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] (3*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^3*tan(c/2 + (d*x)/2)^3)/(24*d) - (1
3*a^3*log(tan(c/2 + (d*x)/2)))/(2*d) - ((14*a^3*tan(c/2 + (d*x)/2)^2)/3 + (
1537*a^3*tan(c/2 + (d*x)/2)^3)/15 + (193*a^3*tan(c/2 + (d*x)/2)^4)/3 + (111
4*a^3*tan(c/2 + (d*x)/2)^5)/3 + (232*a^3*tan(c/2 + (d*x)/2)^6)/3 + (1562*a^
3*tan(c/2 + (d*x)/2)^7)/3 + (95*a^3*tan(c/2 + (d*x)/2)^8)/3 + 399*a^3*tan(c
/2 + (d*x)/2)^9 - (86*a^3*tan(c/2 + (d*x)/2)^10)/3 + 99*a^3*tan(c/2 + (d*x)
/2)^11 - 43*a^3*tan(c/2 + (d*x)/2)^12 + a^3/3 + 3*a^3*tan(c/2 + (d*x)/2))/(
d*(8*tan(c/2 + (d*x)/2)^3 + 40*tan(c/2 + (d*x)/2)^5 + 80*tan(c/2 + (d*x)/2)
^7 + 80*tan(c/2 + (d*x)/2)^9 + 40*tan(c/2 + (d*x)/2)^11 + 8*tan(c/2 + (d*x)
/2)^13)) - (25*a^3*atan((625*a^6)/(16*((325*a^6)/4 - (625*a^6*tan(c/2 + (d*
x)/2))/16)) + (325*a^6*tan(c/2 + (d*x)/2))/(4*((325*a^6)/4 - (625*a^6*tan(c
/2 + (d*x)/2))/16))))/(4*d) + (3*a^3*tan(c/2 + (d*x)/2))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.613 $\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=178

$$-\frac{a^3\cos^3(c+dx)}{d}-\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{5a^3\cot(c+dx)}{d}+\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{3a^3\sin(c+dx)}{8}$$

```
[Out] 45/8*a^3*x+45/8*a^3*arctanh(cos(d*x+c))/d-5*a^3*cos(d*x+c)/d-a^3*cos(d*x+c)
^3/d+5*a^3*cot(d*x+c)/d-a^3*cot(d*x+c)^3/d-3/8*a^3*cot(d*x+c)*csc(d*x+c)/d-
1/4*a^3*cot(d*x+c)*csc(d*x+c)^3/d+3/8*a^3*cos(d*x+c)*sin(d*x+c)/d+1/4*a^3*c
os(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.22, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2872, 3770, 3767, 8, 3768, 2638, 2633, 2635}

$$-\frac{a^3\cos^3(c+dx)}{d}-\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{5a^3\cot(c+dx)}{d}+\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{3a^3\sin(c+dx)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (45*a^3*x)/8 + (45*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (5*a^3*Cos[c + d*x])/
d - (a^3*Cos[c + d*x]^3)/d + (5*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/
d - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*Cot[c + d*x]*Csc[c + d*x
]^3)/(4*d) + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*Cos[c + d*x]*Si
n[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(6a^9 - 6a^9\csc(c+dx) - 8a^9\csc^2(c+dx) + 3a^9\csc^4(c+d\right.}{}$$

$$= 6a^3x + a^3\int \csc^5(c+dx)\,dx - a^3\int \sin^4(c+dx)\,dx + \left(3a^3\right)$$

$$= 6a^3x + \frac{6a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{8a^3\cos(c+dx)}{d} - \frac{a^3\cot(}{}$$

$$= 6a^3x + \frac{6a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{5a^3\cos(c+dx)}{d} - \frac{a^3\cos^3}{}$$

$$= \frac{45a^3x}{8} + \frac{45a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{5a^3\cos(c+dx)}{d} - \frac{a^3c}{}$$

Mathematica [A] time = 0.86, size = 235, normalized size = 1.32

$$\frac{a^3(\sin(c+dx)+1)^3\left(360(c+dx) + 16\sin(2(c+dx)) - 2\sin(4(c+dx)) - 368\cos(c+dx) - 16\cos(3(c+dx)) - 1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(360*(c + d*x) - 368*Cos[c + d*x] - 16*Cos[3*(c +
 d*x)] + 192*Cot[(c + d*x)/2] - 6*Csc[(c + d*x)/2]^2 - Csc[(c + d*x)/2]^4 +
 360*Log[Cos[(c + d*x)/2]] - 360*Log[Sin[(c + d*x)/2]] + 6*Sec[(c + d*x)/2]
^2 + Sec[(c + d*x)/2]^4 + 64*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 4*Csc[(c +
 d*x)/2]^4*Sin[c + d*x] + 16*Sin[2*(c + d*x)] - 2*Sin[4*(c + d*x)] - 192*Ta
n[(c + d*x)/2]))/(64*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.81, size = 258, normalized size = 1.45

$$-\frac{16\,a^3\cos(dx+c)^7 - 90\,a^3dx\cos(dx+c)^4 + 48\,a^3\cos(dx+c)^5 + 180\,a^3dx\cos(dx+c)^2 - 150\,a^3\cos(dx+c)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*(16*a^3*cos(d*x + c)^7 - 90*a^3*d*x*cos(d*x + c)^4 + 48*a^3*cos(d*x +
 c)^5 + 180*a^3*d*x*cos(d*x + c)^2 - 150*a^3*cos(d*x + c)^3 - 90*a^3*d*x +
```

```
90*a^3*cos(d*x + c) - 45*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*
log(1/2*cos(d*x + c) + 1/2) + 45*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2
 + a^3)*log(-1/2*cos(d*x + c) + 1/2) + 2*(2*a^3*cos(d*x + c)^7 - 9*a^3*cos(
d*x + c)^5 + 60*a^3*cos(d*x + c)^3 - 45*a^3*cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.40, size = 313, normalized size = 1.76

$$\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+8a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+360(dx+c)a^3-360a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*tan(1/2*d*x + 1/2*c)^4 + 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*tan
(1/2*d*x + 1/2*c)^2 + 360*(d*x + c)*a^3 - 360*a^3*log(abs(tan(1/2*d*x + 1/2
*c))) - 184*a^3*tan(1/2*d*x + 1/2*c) + (250*a^3*tan(1/2*d*x + 1/2*c)^12 + 1
36*a^3*tan(1/2*d*x + 1/2*c)^11 - 32*a^3*tan(1/2*d*x + 1/2*c)^10 + 552*a^3*t
an(1/2*d*x + 1/2*c)^9 - 837*a^3*tan(1/2*d*x + 1/2*c)^8 + 1248*a^3*tan(1/2*d
*x + 1/2*c)^7 - 1100*a^3*tan(1/2*d*x + 1/2*c)^6 + 736*a^3*tan(1/2*d*x + 1/2
*c)^5 - 556*a^3*tan(1/2*d*x + 1/2*c)^4 + 152*a^3*tan(1/2*d*x + 1/2*c)^3 - 1
2*a^3*tan(1/2*d*x + 1/2*c)^2 - 8*a^3*tan(1/2*d*x + 1/2*c) - a^3)/(tan(1/2*d
*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^4)/d
```

maple [A] time = 0.43, size = 247, normalized size = 1.39

$$\frac{3a^3\left(\cos^7(dx+c)\right)}{d\sin(dx+c)}+\frac{3a^3\left(\cos^5(dx+c)\right)\sin(dx+c)}{d}+\frac{15a^3\left(\cos^3(dx+c)\right)\sin(dx+c)}{4d}+\frac{45a^3\cos(dx+c)\sin(d}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 3/d*a^3/sin(d*x+c)*cos(d*x+c)^7+3*a^3*cos(d*x+c)^5*sin(d*x+c)/d+15/4*a^3*co
s(d*x+c)^3*sin(d*x+c)/d+45/8*a^3*cos(d*x+c)*sin(d*x+c)/d+45/8*a^3*x+45/8/d*
a^3*c-9/8/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7-9/8*a^3*cos(d*x+c)^5/d-15/8*a^3*c
os(d*x+c)^3/d-45/8*a^3*cos(d*x+c)/d-45/8/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-1/
d*a^3/sin(d*x+c)^3*cos(d*x+c)^7-1/4/d*a^3/sin(d*x+c)^4*cos(d*x+c)^7
```

maxima [A] time = 0.41, size = 268, normalized size = 1.51

$$-\frac{4\left(4\cos(dx+c)^3-\frac{6\cos(dx+c)}{\cos(dx+c)^2-1}+24\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^3+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^3 + 2*(15*
d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 +
2*tan(d*x + c)^3 + tan(d*x + c)))*a^3 - 8*(15*d*x + 15*c + (15*tan(d*x + c)
^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a^3 + a^3*(2
*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1
) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))
/d
```

mupad [B] time = 8.96, size = 419, normalized size = 2.35

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{45\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{-34\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+258\,a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^3*tan(c/2 + (d*x)/2)^3)/(8*d) + (a^3*
tan(c/2 + (d*x)/2)^4)/(64*d) - (45*a^3*log(tan(c/2 + (d*x)/2)))/(8*d) - (3*
a^3*tan(c/2 + (d*x)/2)^2 - 38*a^3*tan(c/2 + (d*x)/2)^3 + (403*a^3*tan(c/2 +
 (d*x)/2)^4)/2 - 184*a^3*tan(c/2 + (d*x)/2)^5 + 525*a^3*tan(c/2 + (d*x)/2)^
6 - 312*a^3*tan(c/2 + (d*x)/2)^7 + (2337*a^3*tan(c/2 + (d*x)/2)^8)/4 - 138*
a^3*tan(c/2 + (d*x)/2)^9 + 258*a^3*tan(c/2 + (d*x)/2)^10 - 34*a^3*tan(c/2 +
 (d*x)/2)^11 + a^3/4 + 2*a^3*tan(c/2 + (d*x)/2))/(d*(16*tan(c/2 + (d*x)/2)^
4 + 64*tan(c/2 + (d*x)/2)^6 + 96*tan(c/2 + (d*x)/2)^8 + 64*tan(c/2 + (d*x)/
2)^10 + 16*tan(c/2 + (d*x)/2)^12)) - (45*a^3*atan((2025*a^6)/(16*((2025*a^6
)/16 + (2025*a^6*tan(c/2 + (d*x)/2))/16)) - (2025*a^6*tan(c/2 + (d*x)/2))/(
16*((2025*a^6)/16 + (2025*a^6*tan(c/2 + (d*x)/2))/16))))/(4*d) - (23*a^3*ta
n(c/2 + (d*x)/2))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.614 $\int \cot^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=175

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{2a^3\cot^3(c+dx)}{3d}+\frac{5a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] 13/2*a^3*x-25/8*a^3*arctanh(cos(d*x+c))/d+a^3*cos(d*x+c)/d-1/3*a^3*cos(d*x+
c)^3/d+5*a^3*cot(d*x+c)/d-2/3*a^3*cot(d*x+c)^3/d-1/5*a^3*cot(d*x+c)^5/d+23/
8*a^3*cot(d*x+c)*csc(d*x+c)/d-3/4*a^3*cot(d*x+c)*csc(d*x+c)^3/d+3/2*a^3*cos
(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.30, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2709, 3770, 3767, 8, 3768, 2635, 2633}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{2a^3\cot^3(c+dx)}{3d}+\frac{5a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (13*a^3*x)/2 - (25*a^3*ArcTanh[Cos[c + d*x]])/(8*d) + (a^3*Cos[c + d*x])/d
- (a^3*Cos[c + d*x]^3)/(3*d) + (5*a^3*Cot[c + d*x])/d - (2*a^3*Cot[c + d*x]
^3)/(3*d) - (a^3*Cot[c + d*x]^5)/(5*d) + (23*a^3*Cot[c + d*x]*Csc[c + d*x])
/(8*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) + (3*a^3*Cos[c + d*x]*Si
n[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(8a^9+6a^9\csc(c+dx)-6a^9\csc^2(c+dx)-8a^9\csc^3(c+dx)+3a^9\text{cs}\right.}{a^6}$$

$$= 8a^3x+a^3\int \csc^6(c+dx)\,dx-a^3\int \sin^3(c+dx)\,dx+\left(3a^3\right)\int \csc^5(c+$$

$$= 8a^3x-\frac{6a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3\cot(c+dx)\csc(c+dx)}{d}-\frac{3a^3\text{co}}{}$$

$$= \frac{13a^3x}{2}-\frac{2a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}+$$

$$= \frac{13a^3x}{2}-\frac{25a^3\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}$$

Mathematica [A] time = 1.71, size = 271, normalized size = 1.55

$$\frac{a^3(\sin(c+dx)+1)^3\left(6240(c+dx)+720\sin(2(c+dx))+720\cos(c+dx)-80\cos(3(c+dx))-2624\tan\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(6240*(c + d*x) + 720*Cos[c + d*x] - 80*Cos[3*(c
+ d*x)] + 2624*Cot[(c + d*x)/2] + 690*Csc[(c + d*x)/2]^2 - 45*Csc[(c + d*x)
/2]^4 - 3000*Log[Cos[(c + d*x)/2]] + 3000*Log[Sin[(c + d*x)/2]] - 690*Sec[(
c + d*x)/2]^2 + 45*Sec[(c + d*x)/2]^4 + 304*Csc[c + d*x]^3*Sin[(c + d*x)/2]
^4 - 19*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 3*Csc[(c + d*x)/2]^6*Sin[c + d*x]
 + 720*Sin[2*(c + d*x)] - 2624*Tan[(c + d*x)/2] + 6*Sec[(c + d*x)/2]^4*Tan[
(c + d*x)/2]))/(960*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.77, size = 278, normalized size = 1.59

$$-\frac{360\,a^3\cos(dx+c)^7-2392\,a^3\cos(dx+c)^5+3640\,a^3\cos(dx+c)^3-1560\,a^3\cos(dx+c)+375\left(a^3\cos(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/240*(360*a^3*cos(d*x + c)^7 - 2392*a^3*cos(d*x + c)^5 + 3640*a^3*cos(d*x
 + c)^3 - 1560*a^3*cos(d*x + c) + 375*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x +
 c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 375*(a^3*cos(d*x +
c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) + 10*(8*a^3*cos(d*x + c)^7 - 156*a^3*d*x*cos(d*x + c)^4 - 40*a^3*cos(d*x
+ c)^5 + 312*a^3*d*x*cos(d*x + c)^2 + 125*a^3*cos(d*x + c)^3 - 156*a^3*d*x
- 75*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^
2 + d)*sin(d*x + c))
```

giac [A] time = 0.39, size = 276, normalized size = 1.58

$$\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+50\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-600\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6240\,(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*tan(1/2*d*x + 1/2*c)^4 + 50*a^
3*tan(1/2*d*x + 1/2*c)^3 - 600*a^3*tan(1/2*d*x + 1/2*c)^2 + 6240*(d*x + c)*
a^3 + 3000*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2580*a^3*tan(1/2*d*x + 1/2*
c) - 320*(9*a^3*tan(1/2*d*x + 1/2*c)^5 - 12*a^3*tan(1/2*d*x + 1/2*c)^2 - 9*
a^3*tan(1/2*d*x + 1/2*c) - 4*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3 - (6850*a^
3*tan(1/2*d*x + 1/2*c)^5 - 2580*a^3*tan(1/2*d*x + 1/2*c)^4 - 600*a^3*tan(1/
2*d*x + 1/2*c)^3 + 50*a^3*tan(1/2*d*x + 1/2*c)^2 + 45*a^3*tan(1/2*d*x + 1/2
*c) + 6*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.45, size = 293, normalized size = 1.67

$$\frac{5a^3\left(\cos^7(dx+c)\right)}{8d\sin(dx+c)^2}+\frac{5a^3\left(\cos^5(dx+c)\right)}{8d}+\frac{25a^3\left(\cos^3(dx+c)\right)}{24d}+\frac{25a^3\cos(dx+c)}{8d}+\frac{25a^3\ln(\csc(dx+c)-\cot(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 5/8/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7+5/8*a^3*cos(d*x+c)^5/d+25/24*a^3*cos(d*
x+c)^3/d+25/8*a^3*cos(d*x+c)/d+25/8/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-1/d*a^3
/sin(d*x+c)^3*cos(d*x+c)^7+4/d*a^3/sin(d*x+c)*cos(d*x+c)^7+4*a^3*cos(d*x+c)
^5*sin(d*x+c)/d+5*a^3*cos(d*x+c)^3*sin(d*x+c)/d+15/2*a^3*cos(d*x+c)*sin(d*x
+c)/d+13/2*a^3*x+13/2/d*a^3*c-3/4/d*a^3/sin(d*x+c)^4*cos(d*x+c)^7-1/5*a^3*c
ot(d*x+c)^5/d+1/3*a^3*cot(d*x+c)^3/d-a^3*cot(d*x+c)/d
```

maxima [A] time = 0.42, size = 250, normalized size = 1.43

$$-\frac{20\left(4\cos(dx+c)^3-\frac{6\cos(dx+c)}{\cos(dx+c)^2-1}+24\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^3-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/240*(20*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos
(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^3 - 120*
(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^
5 + tan(d*x + c)^3))*a^3 + 16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d
*x + c)^2 + 3)/tan(d*x + c)^5)*a^3 + 45*a^3*(2*(9*cos(d*x + c)^3 - 7*cos(d*
x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(
cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.92, size = 408, normalized size = 2.33

$$\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{25\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}+\frac{13\,a^3\,\mathrm{at}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] (5*a^3*tan(c/2 + (d*x)/2)^3)/(96*d) - (5*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) +
(3*a^3*tan(c/2 + (d*x)/2)^4)/(64*d) + (a^3*tan(c/2 + (d*x)/2)^5)/(160*d) +
(25*a^3*log(tan(c/2 + (d*x)/2)))/(8*d) + (13*a^3*atan((169*a^6)/((325*a^6)/
4 - 169*a^6*tan(c/2 + (d*x)/2)) + (325*a^6*tan(c/2 + (d*x)/2))/(4*((325*a^6
)/4 - 169*a^6*tan(c/2 + (d*x)/2)))))/d + ((31*a^3*tan(c/2 + (d*x)/2)^3)/2 -
 (34*a^3*tan(c/2 + (d*x)/2)^2)/15 + (402*a^3*tan(c/2 + (d*x)/2)^4)/5 + (589
*a^3*tan(c/2 + (d*x)/2)^5)/6 + (1744*a^3*tan(c/2 + (d*x)/2)^6)/5 + (373*a^3
*tan(c/2 + (d*x)/2)^7)/2 + (769*a^3*tan(c/2 + (d*x)/2)^8)/3 + 20*a^3*tan(c/
2 + (d*x)/2)^9 - 10*a^3*tan(c/2 + (d*x)/2)^10 - a^3/5 - (3*a^3*tan(c/2 + (d
*x)/2))/2)/(d*(32*tan(c/2 + (d*x)/2)^5 + 96*tan(c/2 + (d*x)/2)^7 + 96*tan(c
/2 + (d*x)/2)^9 + 32*tan(c/2 + (d*x)/2)^11)) - (43*a^3*tan(c/2 + (d*x)/2))/
(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.615 $\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=182

$$\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3\cot^5(c+dx)}{5d}+\frac{2a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{85a^3\tanh^{-1}(c}{16a}$$

```
[Out] -1/2*a^3*x-85/16*a^3*arctanh(cos(d*x+c))/d+3*a^3*cos(d*x+c)/d-a^3*cot(d*x+c
)/d+2/3*a^3*cot(d*x+c)^3/d-3/5*a^3*cot(d*x+c)^5/d+43/16*a^3*cot(d*x+c)*csc(
d*x+c)/d-5/24*a^3*cot(d*x+c)*csc(d*x+c)^3/d-1/6*a^3*cot(d*x+c)*csc(d*x+c)^5
/d+1/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.25, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2872, 3770, 3767, 8, 3768, 2638, 2635}

$$\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3\cot^5(c+dx)}{5d}+\frac{2a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{85a^3\tanh^{-1}(c}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x)/2 - (85*a^3*ArcTanh[Cos[c + d*x]])/(16*d) + (3*a^3*Cos[c + d*x])/d
 - (a^3*Cot[c + d*x])/d + (2*a^3*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x
]^5)/(5*d) + (43*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (5*a^3*Cot[c + d*x
]*Csc[c + d*x]^3)/(24*d) - (a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(6*d) + (a^3*C
os[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(8a^9\csc(c+dx)+6a^9\csc^2(c+dx)-6a^9\csc^3(c+dx)-8a\right.}{}$$

$$= a^3\int \csc^7(c+dx)\,dx - a^3\int \sin^2(c+dx)\,dx + \left(3a^3\right)\int \csc^6($$

$$= -\frac{8a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} + \frac{3a^3\cot(c+d}{}$$

$$= -\frac{a^3x}{2} - \frac{5a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} - \frac{a^3\cot(}{}$$

$$= -\frac{a^3x}{2} - \frac{5a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} - \frac{a^3\cot(}{}$$

$$= -\frac{a^3x}{2} - \frac{85a^3\tanh^{-1}(\cos(c+dx))}{16d} + \frac{3a^3\cos(c+dx)}{d} - \frac{a^3\mathrm{co}}{}$$

Mathematica [A] time = 1.86, size = 289, normalized size = 1.59

$$\frac{a^3(\sin(c+dx)+1)^3\left(-960(c+dx)+480\sin(2(c+dx))+5760\cos(c+dx)+2176\tan\left(\frac{1}{2}(c+dx)\right)-2176\cot\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-960*(c + d*x) + 5760*Cos[c + d*x] - 2176*Cot[(c
 + d*x)/2] + 1290*Csc[(c + d*x)/2]^2 - 30*Csc[(c + d*x)/2]^4 - 5*Csc[(c + d
*x)/2]^6 - 10200*Log[Cos[(c + d*x)/2]] + 10200*Log[Sin[(c + d*x)/2]] - 1290
*Sec[(c + d*x)/2]^2 + 30*Sec[(c + d*x)/2]^4 + 5*Sec[(c + d*x)/2]^6 - 3296*C
sc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 206*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 18
*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 2176*Tan[(c + d*x
)/2] + 36*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(1920*d*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.66, size = 316, normalized size = 1.74

$$-\frac{240\,a^3dx\cos\left(dx+c\right)^6-1440\,a^3\cos\left(dx+c\right)^7-720\,a^3dx\cos\left(dx+c\right)^4+5610\,a^3\cos\left(dx+c\right)^5+720\,a^3dx\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/480*(240*a^3*d*x*cos(d*x + c)^6 - 1440*a^3*cos(d*x + c)^7 - 720*a^3*d*x*
cos(d*x + c)^4 + 5610*a^3*cos(d*x + c)^5 + 720*a^3*d*x*cos(d*x + c)^2 - 680
0*a^3*cos(d*x + c)^3 - 240*a^3*d*x + 2550*a^3*cos(d*x + c) + 1275*(a^3*cos(
d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos
(d*x + c) + 1/2) - 1275*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*
cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) - 16*(15*a^3*cos(d*x + c
)^7 + 23*a^3*cos(d*x + c)^5 - 35*a^3*cos(d*x + c)^3 + 15*a^3*cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
 d)
```

giac [A] time = 0.43, size = 307, normalized size = 1.69

$$\frac{5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-340\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1215\,a^3\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*tan(1/2*d*x + 1/2*c)^4 - 340*a^3*tan(1/2*d*x + 1/2*c)^3 - 1215*a^3*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a^3 + 10200*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 1800*a^3*tan(1/2*d*x + 1/2*c) - 1920*(a^3*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - (24990*a^3*tan(1/2*d*x + 1/2*c)^6 + 1800*a^3*tan(1/2*d*x + 1/2*c)^5 - 1215*a^3*tan(1/2*d*x + 1/2*c)^4 - 340*a^3*tan(1/2*d*x + 1/2*c)^3 + 45*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a^3*tan(1/2*d*x + 1/2*c) + 5*a^3)/tan(1/2*d*x + 1/2*c)^6)/d

maple [A] time = 0.48, size = 316, normalized size = 1.74

$$\frac{4a^3\left(\cos^5(dx+c)\right)\sin(dx+c)}{3d}+\frac{17a^3\left(\cos^5(dx+c)\right)}{16d}-\frac{3a^3\left(\cot^5(dx+c)\right)}{5d}+\frac{a^3\left(\cot^3(dx+c)\right)}{d}-\frac{3a^3\cot(dx+c)}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x)

[Out] 4/3*a^3*cos(d*x+c)^5*sin(d*x+c)/d+17/16*a^3*cos(d*x+c)^5/d-3/5*a^3*cot(d*x+c)^5/d+a^3*cot(d*x+c)^3/d-3*a^3*cot(d*x+c)/d-17/24/d*a^3/sin(d*x+c)^4*cos(d*x+c)^7+17/16/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7-1/6/d*a^3/sin(d*x+c)^6*cos(d*x+c)^7-1/3/d*a^3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/d*a^3/sin(d*x+c)*cos(d*x+c)^7+5/3*a^3*cos(d*x+c)^3*sin(d*x+c)/d+5/2*a^3*cos(d*x+c)*sin(d*x+c)/d-1/2*a^3*x+85/48*a^3*cos(d*x+c)^3/d+85/16*a^3*cos(d*x+c)/d+85/16/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-1/2/d*a^3*c

maxima [A] time = 0.41, size = 275, normalized size = 1.51

$$\frac{80\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)a^3-96\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3+5\,a^3\left(\frac{2\left(33\,\cos(d\right.}{\cos(dx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/480*(80*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a^3 - 96*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x + c)^5)*a^3 + 5*a^3*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 +

```
 3*cos(d*x + c)^2 - 1) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1
)) - 90*a^3*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d
*x + c) - 1)))/d
```

mupad [B] time = 8.94, size = 396, normalized size = 2.18

$$\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{128\,d}-\frac{17\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{81\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}+\frac{85\,a^3\,\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] (3*a^3*tan(c/2 + (d*x)/2)^4)/(128*d) - (17*a^3*tan(c/2 + (d*x)/2)^3)/(96*d)
 - (81*a^3*tan(c/2 + (d*x)/2)^2)/(128*d) + (3*a^3*tan(c/2 + (d*x)/2)^5)/(16
0*d) + (a^3*tan(c/2 + (d*x)/2)^6)/(384*d) + (85*a^3*log(tan(c/2 + (d*x)/2))
)/(16*d) + (a^3*atan(a^6/((85*a^6)/8 + a^6*tan(c/2 + (d*x)/2)) - (85*a^6*ta
n(c/2 + (d*x)/2))/(8*((85*a^6)/8 + a^6*tan(c/2 + (d*x)/2)))))/d - ((11*a^3*
tan(c/2 + (d*x)/2)^2)/6 - (134*a^3*tan(c/2 + (d*x)/2)^3)/15 - (112*a^3*tan(
c/2 + (d*x)/2)^4)/3 + (578*a^3*tan(c/2 + (d*x)/2)^5)/15 - (927*a^3*tan(c/2
+ (d*x)/2)^6)/2 + (134*a^3*tan(c/2 + (d*x)/2)^7)/3 - (849*a^3*tan(c/2 + (d*
x)/2)^8)/2 + 124*a^3*tan(c/2 + (d*x)/2)^9 + a^3/6 + (6*a^3*tan(c/2 + (d*x)/
2))/5)/(d*(64*tan(c/2 + (d*x)/2)^6 + 128*tan(c/2 + (d*x)/2)^8 + 64*tan(c/2
+ (d*x)/2)^10)) + (15*a^3*tan(c/2 + (d*x)/2))/(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.616 $\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=172

$$\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^7(c+dx)}{7d}-\frac{3a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}-\frac{15a^3\tanh^{-1}(\cos(c+dx))}{16d}-$$

```
[Out] -3*a^3*x-15/16*a^3*arctanh(cos(d*x+c))/d+a^3*cos(d*x+c)/d-3*a^3*cot(d*x+c)/
d+a^3*cot(d*x+c)^3/d-3/5*a^3*cot(d*x+c)^5/d-1/7*a^3*cot(d*x+c)^7/d-15/16*a^
3*cot(d*x+c)*csc(d*x+c)/d+11/8*a^3*cot(d*x+c)*csc(d*x+c)^3/d-1/2*a^3*cot(d*
x+c)*csc(d*x+c)^5/d
```

Rubi [A] time = 0.29, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2872, 3767, 8, 3768, 3770, 2638}

$$\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^7(c+dx)}{7d}-\frac{3a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}-\frac{15a^3\tanh^{-1}(\cos(c+dx))}{16d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -3*a^3*x - (15*a^3*ArcTanh[Cos[c + d*x]])/(16*d) + (a^3*Cos[c + d*x])/d - (
3*a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/d - (3*a^3*Cot[c + d*x]^5)/(5*
d) - (a^3*Cot[c + d*x]^7)/(7*d) - (15*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d)
 + (11*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a^3*Cot[c + d*x]*Csc[c + d
*x]^5)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
```

```
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(-3a^9+8a^9\csc^2(c+dx)+6a^9\csc^3(c+dx)-6a^9\csc^4(\right.}{}$$

$$= -3a^3x+a^3\int \csc^8(c+dx)\,dx-a^3\int \sin(c+dx)\,dx+\left(3a\right.$$

$$= -3a^3x+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{d}+\frac{2a}{}$$

$$= -3a^3x-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3}{}$$

$$= -3a^3x+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\cot^3(c+dx}{d}$$

$$= -3a^3x-\frac{15a^3\tanh^{-1}(\cos(c+dx))}{16d}+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3}{}$$

Mathematica [A] time = 1.40, size = 292, normalized size = 1.70

$$\frac{a^3\left(4480\cos(c+dx)+9984\tan\left(\frac{1}{2}(c+dx)\right)-9984\cot\left(\frac{1}{2}(c+dx)\right)-35\csc^6\left(\frac{1}{2}(c+dx)\right)+350\csc^4\left(\frac{1}{2}(c+dx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-13440*c - 13440*d*x + 4480*Cos[c + d*x] - 9984*Cot[(c + d*x)/2] - 10
50*Csc[(c + d*x)/2]^2 + 350*Csc[(c + d*x)/2]^4 - 35*Csc[(c + d*x)/2]^6 - 42
00*Log[Cos[(c + d*x)/2]] + 4200*Log[Sin[(c + d*x)/2]] + 1050*Sec[(c + d*x)/
2]^2 - 350*Sec[(c + d*x)/2]^4 + 35*Sec[(c + d*x)/2]^6 - 7664*Csc[c + d*x]^3
*Sin[(c + d*x)/2]^4 + 479*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 17*Csc[(c + d*x
)/2]^6*Sin[c + d*x] - (5*Csc[(c + d*x)/2]^8*Sin[c + d*x])/2 + 9984*Tan[(c +
 d*x)/2] + 34*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2] + 5*Sec[(c + d*x)/2]^6*Ta
n[(c + d*x)/2]))/(4480*d)
```

fricas [B] time = 0.76, size = 336, normalized size = 1.95

$$-\frac{4992\,a^3\cos\left(dx+c\right)^7-12992\,a^3\cos\left(dx+c\right)^5+11200\,a^3\cos\left(dx+c\right)^3-3360\,a^3\cos\left(dx+c\right)+525\left(a^3\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/1120*(4992*a^3*cos(d*x + c)^7 - 12992*a^3*cos(d*x + c)^5 + 11200*a^3*cos
(d*x + c)^3 - 3360*a^3*cos(d*x + c) + 525*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d
*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) - 525*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)
^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 70*(48*a^3*d*x*cos(d*
x + c)^6 - 16*a^3*cos(d*x + c)^7 - 144*a^3*d*x*cos(d*x + c)^4 + 33*a^3*cos(
d*x + c)^5 + 144*a^3*d*x*cos(d*x + c)^2 - 40*a^3*cos(d*x + c)^3 - 48*a^3*d*
x + 15*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c
)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.42, size = 291, normalized size = 1.69

$$\frac{5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+49\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-245\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-875\,a^3\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5*a^3*tan(1/2*d*x + 1/2*c)^7 + 35*a^3*tan(1/2*d*x + 1/2*c)^6 + 49*a
^3*tan(1/2*d*x + 1/2*c)^5 - 245*a^3*tan(1/2*d*x + 1/2*c)^4 - 875*a^3*tan(1/
```

```
2*d*x + 1/2*c)^3 + 455*a^3*tan(1/2*d*x + 1/2*c)^2 - 13440*(d*x + c)*a^3 + 4
200*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 9065*a^3*tan(1/2*d*x + 1/2*c) + 89
60*a^3/(tan(1/2*d*x + 1/2*c)^2 + 1) - (10890*a^3*tan(1/2*d*x + 1/2*c)^7 + 9
065*a^3*tan(1/2*d*x + 1/2*c)^6 + 455*a^3*tan(1/2*d*x + 1/2*c)^5 - 875*a^3*t
an(1/2*d*x + 1/2*c)^4 - 245*a^3*tan(1/2*d*x + 1/2*c)^3 + 49*a^3*tan(1/2*d*x
 + 1/2*c)^2 + 35*a^3*tan(1/2*d*x + 1/2*c) + 5*a^3)/tan(1/2*d*x + 1/2*c)^7)/
d
```

maple [A] time = 0.38, size = 228, normalized size = 1.33

$$-\frac{a^3\left(\cos^7(dx+c)\right)}{8d\sin(dx+c)^4}+\frac{3a^3\left(\cos^7(dx+c)\right)}{16d\sin(dx+c)^2}+\frac{3a^3\left(\cos^5(dx+c)\right)}{16d}+\frac{5a^3\left(\cos^3(dx+c)\right)}{16d}+\frac{15a^3\cos(dx+c)}{16d}+\frac{15a^3\ln(\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/8/d*a^3/sin(d*x+c)^4*cos(d*x+c)^7+3/16/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7+3
/16*a^3*cos(d*x+c)^5/d+5/16*a^3*cos(d*x+c)^3/d+15/16*a^3*cos(d*x+c)/d+15/16
/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3/5*a^3*cot(d*x+c)^5/d+a^3*cot(d*x+c)^3/d-
3*a^3*cot(d*x+c)/d-3*a^3*x-3/d*a^3*c-1/2/d*a^3/sin(d*x+c)^6*cos(d*x+c)^7-1/
7/d*a^3/sin(d*x+c)^7*cos(d*x+c)^7
```

maxima [A] time = 0.42, size = 233, normalized size = 1.35

$$-\frac{224\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3-35\,a^3\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log(\cos(\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/1120*(224*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/ta
n(d*x + c)^5)*a^3 - 35*a^3*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*c
os(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 1
5*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 70*a^3*(2*(9*cos(d*x
+ c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d
*x + c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 160*a^3/ta
n(d*x + c)^7)/d
```

mupad [B] time = 9.04, size = 388, normalized size = 2.26

$$\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}-\frac{25\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{128\,d}-\frac{7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{128\,d}+\frac{7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{128\,d}+\frac{a^3\tan\left(\frac{c}{2}+\right.}{896\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^8,x)
```

```
[Out] (13*a^3*tan(c/2 + (d*x)/2)^2)/(128*d) - (25*a^3*tan(c/2 + (d*x)/2)^3)/(128*
d) - (7*a^3*tan(c/2 + (d*x)/2)^4)/(128*d) + (7*a^3*tan(c/2 + (d*x)/2)^5)/(6
40*d) + (a^3*tan(c/2 + (d*x)/2)^6)/(128*d) + (a^3*tan(c/2 + (d*x)/2)^7)/(89
6*d) + (15*a^3*log(tan(c/2 + (d*x)/2)))/(16*d) + (6*a^3*atan((36*a^6)/((45*
a^6)/4 + 36*a^6*tan(c/2 + (d*x)/2)) - (45*a^6*tan(c/2 + (d*x)/2))/(4*((45*a
^6)/4 + 36*a^6*tan(c/2 + (d*x)/2)))))/d - ((54*a^3*tan(c/2 + (d*x)/2)^2)/35
 - 6*a^3*tan(c/2 + (d*x)/2)^3 - (118*a^3*tan(c/2 + (d*x)/2)^4)/5 + 6*a^3*ta
n(c/2 + (d*x)/2)^5 + 234*a^3*tan(c/2 + (d*x)/2)^6 - 243*a^3*tan(c/2 + (d*x)
/2)^7 + 259*a^3*tan(c/2 + (d*x)/2)^8 + a^3/7 + a^3*tan(c/2 + (d*x)/2))/(d*(
128*tan(c/2 + (d*x)/2)^7 + 128*tan(c/2 + (d*x)/2)^9)) + (259*a^3*tan(c/2 +
(d*x)/2))/(128*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.617 $\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=238

$$-\frac{3a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{125a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^3\cot^5(c+dx}{8}$$

```
[Out] -a^3*x+125/128*a^3*arctanh(cos(d*x+c))/d-a^3*cot(d*x+c)/d+1/3*a^3*cot(d*x+c
)^3/d-1/5*a^3*cot(d*x+c)^5/d-3/7*a^3*cot(d*x+c)^7/d-115/128*a^3*cot(d*x+c)*
csc(d*x+c)/d+5/8*a^3*cot(d*x+c)^3*csc(d*x+c)/d-1/2*a^3*cot(d*x+c)^5*csc(d*x
+c)/d-5/64*a^3*cot(d*x+c)*csc(d*x+c)^3/d+5/48*a^3*cot(d*x+c)^3*csc(d*x+c)^3
/d-1/8*a^3*cot(d*x+c)^5*csc(d*x+c)^3/d
```

Rubi [A] time = 0.36, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2873, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$-\frac{3a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{125a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^3\cot^5(c+dx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (125*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (a^3*Cot[c + d*x])/d +
 (a^3*Cot[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x]^5)/(5*d) - (3*a^3*Cot[c + d
*x]^7)/(7*d) - (115*a^3*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (5*a^3*Cot[c +
 d*x]^3*Csc[c + d*x])/(8*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x])/(2*d) - (5*
a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) + (5*a^3*Cot[c + d*x]^3*Csc[c + d*x
]^3)/(48*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx) + 3a^3\cot^6(c+dx)\csc(c+dx) + 3a^3 \text{co}\right.$$
$$= a^3\int \cot^6(c+dx)\,dx + a^3\int \cot^6(c+dx)\csc^3(c+dx)\,dx +$$
$$= -\frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^5(c+dx)\csc(c+dx)}{2d} - \frac{a^3\cot^5(c}{}$$
$$= \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} - \frac{3a^3\cot^7(c+dx)}{7d} + \frac{5a}{}$$
$$= -\frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} - \frac{3a^3}{}$$
$$= -a^3x + \frac{15a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a^3\cot(c+dx)}{d} + \frac{a^3\text{cc}}{}$$
$$= -a^3x + \frac{125a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{a^3\cot(c+dx)}{d} + \frac{a^3\text{c}}{}$$

Mathematica [A] time = 1.21, size = 279, normalized size = 1.17

$$\frac{a^3\left(118784\tan\left(\frac{1}{2}(c+dx)\right) - 118784\cot\left(\frac{1}{2}(c+dx)\right) - 108780\csc^2\left(\frac{1}{2}(c+dx)\right) + 105\sec^8\left(\frac{1}{2}(c+dx)\right) + 700\,\text{se}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-215040*c - 215040*d*x - 118784*Cot[(c + d*x)/2] - 108780*Csc[(c + d*
x)/2]^2 + 210000*Log[Cos[(c + d*x)/2]] - 210000*Log[Sin[(c + d*x)/2]] + 108
780*Sec[(c + d*x)/2]^2 - 17010*Sec[(c + d*x)/2]^4 + 700*Sec[(c + d*x)/2]^6
+ 105*Sec[(c + d*x)/2]^8 + 71936*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + Csc[(c
 + d*x)/2]^4*(17010 - 4496*Sin[c + d*x]) - 15*Csc[(c + d*x)/2]^8*(7 + 24*Si
n[c + d*x]) + 4*Csc[(c + d*x)/2]^6*(-175 + 732*Sin[c + d*x]) + 118784*Tan[(
c + d*x)/2] - 5856*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2] + 720*Sec[(c + d*x)/
2]^6*Tan[(c + d*x)/2]))/(215040*d)
```

fricas [A] time = 0.75, size = 362, normalized size = 1.52

$$-\frac{26880\,a^3dx\cos(dx+c)^8 - 107520\,a^3dx\cos(dx+c)^6 - 54390\,a^3\cos(dx+c)^7 + 161280\,a^3dx\cos(dx+c)^4 + 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/26880*(26880*a^3*d*x*cos(d*x + c)^8 - 107520*a^3*d*x*cos(d*x + c)^6 - 54
390*a^3*cos(d*x + c)^7 + 161280*a^3*d*x*cos(d*x + c)^4 + 127750*a^3*cos(d*x
 + c)^5 - 107520*a^3*d*x*cos(d*x + c)^2 - 96250*a^3*cos(d*x + c)^3 + 26880*
a^3*d*x + 26250*a^3*cos(d*x + c) - 13125*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*
x + c)^6 + 6*a^3*cos(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d
*x + c) + 1/2) + 13125*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*c
os(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2) -
256*(116*a^3*cos(d*x + c)^7 - 406*a^3*cos(d*x + c)^5 + 350*a^3*cos(d*x + c)
^3 - 105*a^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x +
c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.47, size = 302, normalized size = 1.27

$$\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+720\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3696\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1428}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/215040*(105*a^3*tan(1/2*d*x + 1/2*c)^8 + 720*a^3*tan(1/2*d*x + 1/2*c)^7 +
 1120*a^3*tan(1/2*d*x + 1/2*c)^6 - 3696*a^3*tan(1/2*d*x + 1/2*c)^5 - 14280*
a^3*tan(1/2*d*x + 1/2*c)^4 - 560*a^3*tan(1/2*d*x + 1/2*c)^3 + 77280*a^3*tan
(1/2*d*x + 1/2*c)^2 - 215040*(d*x + c)*a^3 - 210000*a^3*log(abs(tan(1/2*d*x
 + 1/2*c))) + 122640*a^3*tan(1/2*d*x + 1/2*c) + (570750*a^3*tan(1/2*d*x + 1
/2*c)^8 - 122640*a^3*tan(1/2*d*x + 1/2*c)^7 - 77280*a^3*tan(1/2*d*x + 1/2*c
)^6 + 560*a^3*tan(1/2*d*x + 1/2*c)^5 + 14280*a^3*tan(1/2*d*x + 1/2*c)^4 + 3
696*a^3*tan(1/2*d*x + 1/2*c)^3 - 1120*a^3*tan(1/2*d*x + 1/2*c)^2 - 720*a^3*
tan(1/2*d*x + 1/2*c) - 105*a^3)/tan(1/2*d*x + 1/2*c)^8)/d
```

maple [A] time = 0.45, size = 253, normalized size = 1.06

$$-\frac{a^3\left(\cot^5(dx+c)\right)}{5d}+\frac{a^3\left(\cot^3(dx+c)\right)}{3d}-\frac{a^3\cot(dx+c)}{d}-a^3x-\frac{a^3c}{d}-\frac{25a^3\left(\cos^7(dx+c)\right)}{48d\sin(dx+c)^6}+\frac{25a^3\left(\cos^7(dx+c)\right)}{192d\sin(dx+c)^4}-\frac{25}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/5*a^3*cot(d*x+c)^5/d+1/3*a^3*cot(d*x+c)^3/d-a^3*cot(d*x+c)/d-a^3*x-1/d*a
^3*c-25/48/d*a^3/sin(d*x+c)^6*cos(d*x+c)^7+25/192/d*a^3/sin(d*x+c)^4*cos(d*
```

x+c)^7-25/128/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7-25/128*a^3*cos(d*x+c)^5/d-125
/384*a^3*cos(d*x+c)^3/d-125/128*a^3*cos(d*x+c)/d-125/128/d*a^3*ln(csc(d*x+c
)-cot(d*x+c))-3/7/d*a^3/sin(d*x+c)^7*cos(d*x+c)^7-1/8/d*a^3/sin(d*x+c)^8*co
s(d*x+c)^7

maxima [A] time = 0.41, size = 265, normalized size = 1.11

$$-\frac{1792\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3+35\,a^3\left(\frac{2\left(15\,\cos(dx+c)^7+73\,\cos(dx+c)^5-55\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/26880*(1792*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/
tan(d*x + c)^5)*a^3 + 35*a^3*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55
*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*c
os(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(c
os(d*x + c) - 1)) - 840*a^3*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*
cos(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) +
15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 11520*a^3/tan(d*x +
c)^7)/d
```

mupad [B] time = 10.30, size = 389, normalized size = 1.63

$$\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{384\,d}-\frac{23\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64\,d}+\frac{17\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}+\frac{11\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{192\,d}-\frac{3\,a^3\cot\left(\frac{c}{2}\right.}{896\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^9,x)
```

```
[Out] (a^3*cot(c/2 + (d*x)/2)^3)/(384*d) - (23*a^3*cot(c/2 + (d*x)/2)^2)/(64*d) +
 (17*a^3*cot(c/2 + (d*x)/2)^4)/(256*d) + (11*a^3*cot(c/2 + (d*x)/2)^5)/(640
*d) - (a^3*cot(c/2 + (d*x)/2)^6)/(192*d) - (3*a^3*cot(c/2 + (d*x)/2)^7)/(89
6*d) - (a^3*cot(c/2 + (d*x)/2)^8)/(2048*d) + (23*a^3*tan(c/2 + (d*x)/2)^2)/
(64*d) - (a^3*tan(c/2 + (d*x)/2)^3)/(384*d) - (17*a^3*tan(c/2 + (d*x)/2)^4)
/(256*d) - (11*a^3*tan(c/2 + (d*x)/2)^5)/(640*d) + (a^3*tan(c/2 + (d*x)/2)^
6)/(192*d) + (3*a^3*tan(c/2 + (d*x)/2)^7)/(896*d) + (a^3*tan(c/2 + (d*x)/2)
^8)/(2048*d) - (2*a^3*atan((128*cos(c/2 + (d*x)/2) + 125*sin(c/2 + (d*x)/2)
)/(125*cos(c/2 + (d*x)/2) - 128*sin(c/2 + (d*x)/2))))/d - (125*a^3*log(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(128*d) - (73*a^3*cot(c/2 + (d*x)/2))/(
128*d) + (73*a^3*tan(c/2 + (d*x)/2))/(128*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.618 $\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=200

$$-\frac{a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{55a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{3a^3\cot^5(c+dx)\csc^3(c+dx)}{8d}-\frac{a^3\cot^5(c+dx)\,\mathrm{c}}{6d}$$

```
[Out] 55/128*a^3*arctanh(cos(d*x+c))/d-4/7*a^3*cot(d*x+c)^7/d-1/9*a^3*cot(d*x+c)^
9/d-25/128*a^3*cot(d*x+c)*csc(d*x+c)/d+5/24*a^3*cot(d*x+c)^3*csc(d*x+c)/d-1
/6*a^3*cot(d*x+c)^5*csc(d*x+c)/d-15/64*a^3*cot(d*x+c)*csc(d*x+c)^3/d+5/16*a
^3*cot(d*x+c)^3*csc(d*x+c)^3/d-3/8*a^3*cot(d*x+c)^5*csc(d*x+c)^3/d
```

Rubi [A] time = 0.36, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2611, 3770, 2607, 30, 3768, 14}

$$-\frac{a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{55a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{3a^3\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a^3\cot^3(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (55*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (a^
3*Cot[c + d*x]^9)/(9*d) - (25*a^3*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (5*a
^3*Cot[c + d*x]^3*Csc[c + d*x])/(24*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x])/
(6*d) - (15*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) + (5*a^3*Cot[c + d*x]^3
*Csc[c + d*x]^3)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)\csc(c+dx)+3a^3\cot^6(c+dx)\csc^2(c+\right.$$
$$= a^3\int \cot^6(c+dx)\csc(c+dx)\,dx + a^3\int \cot^6(c+dx)\csc^4($$
$$= -\frac{a^3\cot^5(c+dx)\csc(c+dx)}{6d} - \frac{3a^3\cot^5(c+dx)\csc^3(c+d}{8d}$$
$$= -\frac{3a^3\cot^7(c+dx)}{7d} + \frac{5a^3\cot^3(c+dx)\csc(c+dx)}{24d} - \frac{a^3\cot^5}{}$$
$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{9d} - \frac{5a^3\cot(c+dx)\csc(}{16d}$$
$$= \frac{5a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+}{9d}$$
$$= \frac{55a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+}{9d}$$

Mathematica [B] time = 0.14, size = 459, normalized size = 2.30

$$a^3\left(-\frac{29\tan\left(\frac{1}{2}(c+dx)\right)}{126d} + \frac{29\cot\left(\frac{1}{2}(c+dx)\right)}{126d} - \frac{3\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d} + \frac{17\csc^6\left(\frac{1}{2}(c+dx)\right)}{1536d} - \frac{13\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((29*Cot[(c + d*x)/2])/(126*d) - (73*Csc[(c + d*x)/2]^2)/(512*d) - (416
3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(32256*d) - (13*Csc[(c + d*x)/2]^4)/
(1024*d) + (319*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(10752*d) + (17*Csc[(c
 + d*x)/2]^6)/(1536*d) - (53*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^6)/(32256*d)
 - (3*Csc[(c + d*x)/2]^8)/(2048*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^8)/
(4608*d) + (55*Log[Cos[(c + d*x)/2]])/(128*d) - (55*Log[Sin[(c + d*x)/2]])/
(128*d) + (73*Sec[(c + d*x)/2]^2)/(512*d) + (13*Sec[(c + d*x)/2]^4)/(1024*d
) - (17*Sec[(c + d*x)/2]^6)/(1536*d) + (3*Sec[(c + d*x)/2]^8)/(2048*d) - (2
9*Tan[(c + d*x)/2])/(126*d) + (4163*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3
2256*d) - (319*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(10752*d) + (53*Sec[(c
+ d*x)/2]^6*Tan[(c + d*x)/2])/(32256*d) + (Sec[(c + d*x)/2]^8*Tan[(c + d*x)
/2])/(4608*d))
```

fricas [A] time = 1.02, size = 291, normalized size = 1.46

$$7424\,a^3\cos(dx+c)^9 - 9216\,a^3\cos(dx+c)^7 + 3465\left(a^3\cos(dx+c)^8 - 4\,a^3\cos(dx+c)^6 + 6\,a^3\cos(dx+c)^4 -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] 1/16128*(7424*a^3*cos(d*x + c)^9 - 9216*a^3*cos(d*x + c)^7 + 3465*(a^3*cos(
d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x + c)^4 - 4*a^3*cos(d*x +
c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3465*(a^3*cos(d*x +
c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 +
 a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 42*(219*a^3*cos(d*x + c)^
7 - 803*a^3*cos(d*x + c)^5 + 605*a^3*cos(d*x + c)^3 - 165*a^3*cos(d*x + c))
*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4
 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.42, size = 324, normalized size = 1.62

$$\frac{28\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+189\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+324\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-672\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3024\,a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/129024*(28*a^3*tan(1/2*d*x + 1/2*c)^9 + 189*a^3*tan(1/2*d*x + 1/2*c)^8 +
324*a^3*tan(1/2*d*x + 1/2*c)^7 - 672*a^3*tan(1/2*d*x + 1/2*c)^6 - 3024*a^3*
tan(1/2*d*x + 1/2*c)^5 - 1512*a^3*tan(1/2*d*x + 1/2*c)^4 + 9744*a^3*tan(1/2
*d*x + 1/2*c)^3 + 18144*a^3*tan(1/2*d*x + 1/2*c)^2 - 55440*a^3*log(abs(tan(
1/2*d*x + 1/2*c))) - 16632*a^3*tan(1/2*d*x + 1/2*c) + (156838*a^3*tan(1/2*d
*x + 1/2*c)^9 + 16632*a^3*tan(1/2*d*x + 1/2*c)^8 - 18144*a^3*tan(1/2*d*x +
1/2*c)^7 - 9744*a^3*tan(1/2*d*x + 1/2*c)^6 + 1512*a^3*tan(1/2*d*x + 1/2*c)^
5 + 3024*a^3*tan(1/2*d*x + 1/2*c)^4 + 672*a^3*tan(1/2*d*x + 1/2*c)^3 - 324*
a^3*tan(1/2*d*x + 1/2*c)^2 - 189*a^3*tan(1/2*d*x + 1/2*c) - 28*a^3)/tan(1/2
*d*x + 1/2*c)^9)/d
```

maple [A] time = 0.40, size = 216, normalized size = 1.08

$$-\frac{11a^3\left(\cos^7(dx+c)\right)}{48d\sin(dx+c)^6}+\frac{11a^3\left(\cos^7(dx+c)\right)}{192d\sin(dx+c)^4}-\frac{11a^3\left(\cos^7(dx+c)\right)}{128d\sin(dx+c)^2}-\frac{11a^3\left(\cos^5(dx+c)\right)}{128d}-\frac{55a^3\left(\cos^3(dx+c)\right)}{384d}-\frac{55}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x)
```

[Out] -11/48/d*a^3/sin(d*x+c)^6*cos(d*x+c)^7+11/192/d*a^3/sin(d*x+c)^4*cos(d*x+c)^7-11/128/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7-11/128*a^3*cos(d*x+c)^5/d-55/384*a^3*cos(d*x+c)^3/d-55/128*a^3*cos(d*x+c)/d-55/128/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-29/63/d*a^3/sin(d*x+c)^7*cos(d*x+c)^7-3/8/d*a^3/sin(d*x+c)^8*cos(d*x+c)^7-1/9/d*a^3/sin(d*x+c)^9*cos(d*x+c)^7

maxima [A] time = 0.43, size = 246, normalized size = 1.23

$$-\frac{63\,a^3\left(\frac{2\left(15\cos(dx+c)^7+73\cos(dx+c)^5-55\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1}-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -1/16128*(63*a^3*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) - 168*a^3*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 6912*a^3/tan(d*x + c)^7 + 256*(9*tan(d*x + c)^2 + 7)*a^3/tan(d*x + c)^9)/d

mupad [B] time = 9.40, size = 357, normalized size = 1.78

$$\frac{3\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}-\frac{29\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{384\,d}-\frac{9\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64\,d}+\frac{3\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{128\,d}+\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{192\,d}-\frac{9\,a^3\cot\left(\frac{c}{2}+\right.}{3584\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^10,x)

[Out] (3*a^3*cot(c/2 + (d*x)/2)^4)/(256*d) - (29*a^3*cot(c/2 + (d*x)/2)^3)/(384*d) - (9*a^3*cot(c/2 + (d*x)/2)^2)/(64*d) + (3*a^3*cot(c/2 + (d*x)/2)^5)/(128*d) + (a^3*cot(c/2 + (d*x)/2)^6)/(192*d) - (9*a^3*cot(c/2 + (d*x)/2)^7)/(3584*d) - (3*a^3*cot(c/2 + (d*x)/2)^8)/(2048*d) - (a^3*cot(c/2 + (d*x)/2)^9)/(4608*d) + (9*a^3*tan(c/2 + (d*x)/2)^2)/(64*d) + (29*a^3*tan(c/2 + (d*x)/2)^3)/(384*d) - (3*a^3*tan(c/2 + (d*x)/2)^4)/(256*d) - (3*a^3*tan(c/2 + (d*x)/2)^5)/(128*d) - (a^3*tan(c/2 + (d*x)/2)^6)/(192*d) + (9*a^3*tan(c/2 + (d*x)/2)^7)/(3584*d) + (3*a^3*tan(c/2 + (d*x)/2)^8)/(2048*d) + (a^3*tan(c/2 + (d*x)/2)^9)/(4608*d) - (55*a^3*log(tan(c/2 + (d*x)/2)))/(128*d) + (33*a^3*cot(c/2 + (d*x)/2))/(256*d) - (33*a^3*tan(c/2 + (d*x)/2))/(256*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**10*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.619 $\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=228

$$-\frac{a^3\cot^9(c+dx)}{3d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{33a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^3\cot^5(c+dx)\csc^5(c+dx)}{10d}-\frac{3a^3\cot^5(c+dx)\,c}{8d}$$

```
[Out] 33/256*a^3*arctanh(cos(d*x+c))/d-4/7*a^3*cot(d*x+c)^7/d-1/3*a^3*cot(d*x+c)^
9/d+33/256*a^3*cot(d*x+c)*csc(d*x+c)/d-29/128*a^3*cot(d*x+c)*csc(d*x+c)^3/d
+5/16*a^3*cot(d*x+c)^3*csc(d*x+c)^3/d-3/8*a^3*cot(d*x+c)^5*csc(d*x+c)^3/d-1
/32*a^3*cot(d*x+c)*csc(d*x+c)^5/d+1/16*a^3*cot(d*x+c)^3*csc(d*x+c)^5/d-1/10
*a^3*cot(d*x+c)^5*csc(d*x+c)^5/d
```

Rubi [A] time = 0.43, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2607, 30, 2611, 3768, 3770, 14}

$$-\frac{a^3\cot^9(c+dx)}{3d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{33a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^3\cot^5(c+dx)\csc^5(c+dx)}{10d}-\frac{3a^3\cot^5(c+dx)\,c}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (33*a^3*ArcTanh[Cos[c + d*x]])/(256*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (a^
3*Cot[c + d*x]^9)/(3*d) + (33*a^3*Cot[c + d*x]*Csc[c + d*x])/(256*d) - (29*
a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (5*a^3*Cot[c + d*x]^3*Csc[c + d*
x]^3)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d) - (a^3*Cot[c + d
*x]*Csc[c + d*x]^5)/(32*d) + (a^3*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (
a^3*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)\csc^2(c+dx)+3a^3\cot^6(c+dx)\csc^3(c+\right.$$

$$= a^3\int \cot^6(c+dx)\csc^2(c+dx)\,dx + a^3\int \cot^6(c+dx)\csc^5$$

$$= -\frac{3a^3\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{a^3\cot^5(c+dx)\csc^5(c+}{10d}$$

$$= -\frac{a^3\cot^7(c+dx)}{7d} + \frac{5a^3\cot^3(c+dx)\csc^3(c+dx)}{16d} - \frac{3a^3\mathrm{co}}{}$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{3d} - \frac{15a^3\cot(c+dx)\mathrm{csc}}{64d}$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{3d} + \frac{15a^3\cot(c+dx)\mathrm{csc}}{128d}$$

$$= \frac{15a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+}{3d}$$

$$= \frac{33a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+}{3d}$$

Mathematica [A] time = 2.06, size = 365, normalized size = 1.60

$$\frac{a^3(\sin(c+dx)+1)^3\left(-51200\tan\left(\frac{1}{2}(c+dx)\right)+51200\cot\left(\frac{1}{2}(c+dx)\right)+13860\csc^2\left(\frac{1}{2}(c+dx)\right)+42\sec^{10}\left(\frac{1}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(51200*Cot[(c + d*x)/2] + 13860*Csc[(c + d*x)/2]^
2 + 55440*Log[Cos[(c + d*x)/2]] - 55440*Log[Sin[(c + d*x)/2]] - 13860*Sec[(
c + d*x)/2]^2 + 19320*Sec[(c + d*x)/2]^4 - 5250*Sec[(c + d*x)/2]^6 + 315*Se
c[(c + d*x)/2]^8 + 42*Sec[(c + d*x)/2]^10 + 164800*Csc[c + d*x]^3*Sin[(c +
d*x)/2]^4 + 3840*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + Csc[(c + d*x)/2]^6*(52
50 - 60*Sin[c + d*x]) - 14*Csc[(c + d*x)/2]^10*(3 + 10*Sin[c + d*x]) + 5*Cs
c[(c + d*x)/2]^8*(-63 + 172*Sin[c + d*x]) - 20*Csc[(c + d*x)/2]^4*(966 + 51
5*Sin[c + d*x]) - 51200*Tan[(c + d*x)/2] - 1720*Sec[(c + d*x)/2]^6*Tan[(c +
 d*x)/2] + 280*Sec[(c + d*x)/2]^8*Tan[(c + d*x)/2]))/(430080*d*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.74, size = 327, normalized size = 1.43

$$-\frac{6930\,a^{3}\cos\left(dx+c\right)^{9}+21420\,a^{3}\cos\left(dx+c\right)^{7}-59136\,a^{3}\cos\left(dx+c\right)^{5}+32340\,a^{3}\cos\left(dx+c\right)^{3}-6930\,a^{3}\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] -1/53760*(6930*a^3*cos(d*x + c)^9 + 21420*a^3*cos(d*x + c)^7 - 59136*a^3*co
s(d*x + c)^5 + 32340*a^3*cos(d*x + c)^3 - 6930*a^3*cos(d*x + c) - 3465*(a^3
*cos(d*x + c)^10 - 5*a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10*a^3*co
s(d*x + c)^4 + 5*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2) + 34
65*(a^3*cos(d*x + c)^10 - 5*a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10
*a^3*cos(d*x + c)^4 + 5*a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1
/2) + 2560*(5*a^3*cos(d*x + c)^9 - 12*a^3*cos(d*x + c)^7)*sin(d*x + c))/(d*
cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x +
 c)^4 + 5*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.47, size = 356, normalized size = 1.56

$$\frac{42\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+280\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+525\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-600\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3570\,a^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/430080*(42*a^3*tan(1/2*d*x + 1/2*c)^10 + 280*a^3*tan(1/2*d*x + 1/2*c)^9 +
 525*a^3*tan(1/2*d*x + 1/2*c)^8 - 600*a^3*tan(1/2*d*x + 1/2*c)^7 - 3570*a^3
*tan(1/2*d*x + 1/2*c)^6 - 3360*a^3*tan(1/2*d*x + 1/2*c)^5 + 5880*a^3*tan(1/
2*d*x + 1/2*c)^4 + 16800*a^3*tan(1/2*d*x + 1/2*c)^3 + 10500*a^3*tan(1/2*d*x
 + 1/2*c)^2 - 55440*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 31920*a^3*tan(1/2*
d*x + 1/2*c) + (162382*a^3*tan(1/2*d*x + 1/2*c)^10 + 31920*a^3*tan(1/2*d*x
+ 1/2*c)^9 - 10500*a^3*tan(1/2*d*x + 1/2*c)^8 - 16800*a^3*tan(1/2*d*x + 1/2
*c)^7 - 5880*a^3*tan(1/2*d*x + 1/2*c)^6 + 3360*a^3*tan(1/2*d*x + 1/2*c)^5 +
 3570*a^3*tan(1/2*d*x + 1/2*c)^4 + 600*a^3*tan(1/2*d*x + 1/2*c)^3 - 525*a^3
*tan(1/2*d*x + 1/2*c)^2 - 280*a^3*tan(1/2*d*x + 1/2*c) - 42*a^3)/tan(1/2*d*
x + 1/2*c)^10)/d
```

maple [A] time = 0.40, size = 240, normalized size = 1.05

$$-\frac{5a^3\left(\cos^7(dx+c)\right)}{21d\sin(dx+c)^7}-\frac{33a^3\left(\cos^7(dx+c)\right)}{80d\sin(dx+c)^8}-\frac{11a^3\left(\cos^7(dx+c)\right)}{160d\sin(dx+c)^6}+\frac{11a^3\left(\cos^7(dx+c)\right)}{640d\sin(dx+c)^4}-\frac{33a^3\left(\cos^7(dx+c)\right)}{1280d\sin(dx+c)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -5/21/d*a^3/sin(d*x+c)^7*cos(d*x+c)^7-33/80/d*a^3/sin(d*x+c)^8*cos(d*x+c)^7
-11/160/d*a^3/sin(d*x+c)^6*cos(d*x+c)^7+11/640/d*a^3/sin(d*x+c)^4*cos(d*x+c
)^7-33/1280/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7-33/1280*a^3*cos(d*x+c)^5/d-11/2
56*a^3*cos(d*x+c)^3/d-33/256*a^3*cos(d*x+c)/d-33/256/d*a^3*ln(csc(d*x+c)-co
t(d*x+c))-1/3/d*a^3/sin(d*x+c)^9*cos(d*x+c)^7-1/10/d*a^3/sin(d*x+c)^10*cos(
d*x+c)^7
```

maxima [A] time = 0.34, size = 286, normalized size = 1.25

$$-\frac{21\,a^3\left(\frac{2\left(15\,\cos(dx+c)^9-70\,\cos(dx+c)^7-128\,\cos(dx+c)^5+70\,\cos(dx+c)^3-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^8+10\,\cos(dx+c)^6-10\,\cos(dx+c)^4+5\,\cos(dx+c)^2-1}-15\,\log\left(\cos(dx+c)+1\right)+15\,\log(\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] -1/53760*(21*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x +
c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x +
c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*l
og(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 210*a^3*(2*(15*cos(d*x +
 c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x +
 c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*lo
g(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 7680*a^3/tan(d*x + c)^7 +
 2560*(9*tan(d*x + c)^2 + 7)*a^3/tan(d*x + c)^9)/d
```

mupad [B] time = 9.65, size = 395, normalized size = 1.73

$$\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{128\,d}-\frac{5\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{128\,d}-\frac{7\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{512\,d}-\frac{25\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}+\frac{17\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2048\,d}+\frac{5\,a^3\cot\left(\frac{c}{2}\right.}{3584}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^11,x)
```

```
[Out] (a^3*cot(c/2 + (d*x)/2)^5)/(128*d) - (5*a^3*cot(c/2 + (d*x)/2)^3)/(128*d) -
 (7*a^3*cot(c/2 + (d*x)/2)^4)/(512*d) - (25*a^3*cot(c/2 + (d*x)/2)^2)/(1024
*d) + (17*a^3*cot(c/2 + (d*x)/2)^6)/(2048*d) + (5*a^3*cot(c/2 + (d*x)/2)^7)
/(3584*d) - (5*a^3*cot(c/2 + (d*x)/2)^8)/(4096*d) - (a^3*cot(c/2 + (d*x)/2)
^9)/(1536*d) - (a^3*cot(c/2 + (d*x)/2)^10)/(10240*d) + (25*a^3*tan(c/2 + (d
*x)/2)^2)/(1024*d) + (5*a^3*tan(c/2 + (d*x)/2)^3)/(128*d) + (7*a^3*tan(c/2
+ (d*x)/2)^4)/(512*d) - (a^3*tan(c/2 + (d*x)/2)^5)/(128*d) - (17*a^3*tan(c/
2 + (d*x)/2)^6)/(2048*d) - (5*a^3*tan(c/2 + (d*x)/2)^7)/(3584*d) + (5*a^3*t
an(c/2 + (d*x)/2)^8)/(4096*d) + (a^3*tan(c/2 + (d*x)/2)^9)/(1536*d) + (a^3*
tan(c/2 + (d*x)/2)^10)/(10240*d) - (33*a^3*log(tan(c/2 + (d*x)/2)))/(256*d)
 + (19*a^3*cot(c/2 + (d*x)/2))/(256*d) - (19*a^3*tan(c/2 + (d*x)/2))/(256*d
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**11*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.620 $\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=246

$$-\frac{a^3\cot^{11}(c+dx)}{11d}-\frac{5a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{19a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{3a^3\cot^5(c+dx)\csc^5(c+dx)}{10d}$$

```
[Out] 19/256*a^3*arctanh(cos(d*x+c))/d-4/7*a^3*cot(d*x+c)^7/d-5/9*a^3*cot(d*x+c)^
9/d-1/11*a^3*cot(d*x+c)^11/d+19/256*a^3*cot(d*x+c)*csc(d*x+c)/d-7/128*a^3*c
ot(d*x+c)*csc(d*x+c)^3/d+5/48*a^3*cot(d*x+c)^3*csc(d*x+c)^3/d-1/8*a^3*cot(d
*x+c)^5*csc(d*x+c)^3/d-3/32*a^3*cot(d*x+c)*csc(d*x+c)^5/d+3/16*a^3*cot(d*x+
c)^3*csc(d*x+c)^5/d-3/10*a^3*cot(d*x+c)^5*csc(d*x+c)^5/d
```

Rubi [A] time = 0.44, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2611, 3768, 3770, 2607, 14, 270}

$$-\frac{a^3\cot^{11}(c+dx)}{11d}-\frac{5a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{19a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{3a^3\cot^5(c+dx)\csc^5(c+dx)}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (19*a^3*ArcTanh[Cos[c + d*x]])/(256*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (5*
a^3*Cot[c + d*x]^9)/(9*d) - (a^3*Cot[c + d*x]^11)/(11*d) + (19*a^3*Cot[c +
d*x]*Csc[c + d*x])/(256*d) - (7*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) +
(5*a^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a^3*Cot[c + d*x]^5*Csc[c +
d*x]^3)/(8*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(32*d) + (3*a^3*Cot[c +
 d*x]^3*Csc[c + d*x]^5)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*
d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)\csc^3(c+dx)+3a^3\cot^6(c+dx)\csc^4(c+\right.$$
$$= a^3\int \cot^6(c+dx)\csc^3(c+dx)\,dx + a^3\int \cot^6(c+dx)\csc^6$$
$$= -\frac{a^3\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{3a^3\cot^5(c+dx)\csc^5(c+}{10d}$$
$$= \frac{5a^3\cot^3(c+dx)\csc^3(c+dx)}{48d} - \frac{a^3\cot^5(c+dx)\csc^3(c+dx}{8d}$$
$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{5a^3\cot^9(c+dx)}{9d} - \frac{a^3\cot^{11}(c+dx)}{11d} -$$
$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{5a^3\cot^9(c+dx)}{9d} - \frac{a^3\cot^{11}(c+dx)}{11d} +$$
$$= \frac{5a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{5a^3\cot^9(c+}{9d}$$
$$= \frac{19a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{5a^3\cot^9(c}{9d}$$

Mathematica [A] time = 3.58, size = 187, normalized size = 0.76

$$\frac{a^3(\sin(c+dx)+1)^3\left(16853760\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc^{10}(c+dx)(144}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(16853760*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d
*x)/2]]) - Cot[c + d*x]*Csc[c + d*x]^10*(10050560 + 12423680*Cos[2*(c + d*x
)] + 839680*Cos[4*(c + d*x)] - 2149120*Cos[6*(c + d*x)] - 568320*Cos[8*(c +
 d*x)] + 47360*Cos[10*(c + d*x)] + 14477694*Sin[c + d*x] + 5875716*Sin[3*(c
 + d*x)] + 7902972*Sin[5*(c + d*x)] - 414645*Sin[7*(c + d*x)] - 65835*Sin[9
*(c + d*x)])))/(227082240*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.69, size = 360, normalized size = 1.46

$$\frac{189440\,a^3\cos(dx+c)^{11} - 1041920\,a^3\cos(dx+c)^9 + 1013760\,a^3\cos(dx+c)^7 + 65835\left(a^3\cos(dx+c)^{10} - 5\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] 1/1774080*(189440*a^3*cos(d*x + c)^11 - 1041920*a^3*cos(d*x + c)^9 + 101376
0*a^3*cos(d*x + c)^7 + 65835*(a^3*cos(d*x + c)^10 - 5*a^3*cos(d*x + c)^8 +
10*a^3*cos(d*x + c)^6 - 10*a^3*cos(d*x + c)^4 + 5*a^3*cos(d*x + c)^2 - a^3)
*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 65835*(a^3*cos(d*x + c)^10 - 5*
a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10*a^3*cos(d*x + c)^4 + 5*a^3*
cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 462*(285*
a^3*cos(d*x + c)^9 - 50*a^3*cos(d*x + c)^7 - 2432*a^3*cos(d*x + c)^5 + 1330
*a^3*cos(d*x + c)^3 - 285*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^
10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*c
os(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.46, size = 388, normalized size = 1.58

$$\frac{630\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+4158\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+8470\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-3465\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/14192640*(630*a^3*tan(1/2*d*x + 1/2*c)^11 + 4158*a^3*tan(1/2*d*x + 1/2*c)
^10 + 8470*a^3*tan(1/2*d*x + 1/2*c)^9 - 3465*a^3*tan(1/2*d*x + 1/2*c)^8 - 4
0590*a^3*tan(1/2*d*x + 1/2*c)^7 - 57750*a^3*tan(1/2*d*x + 1/2*c)^6 + 6930*a
^3*tan(1/2*d*x + 1/2*c)^5 + 138600*a^3*tan(1/2*d*x + 1/2*c)^4 + 244860*a^3*
tan(1/2*d*x + 1/2*c)^3 + 152460*a^3*tan(1/2*d*x + 1/2*c)^2 - 1053360*a^3*lo
g(abs(tan(1/2*d*x + 1/2*c))) - 568260*a^3*tan(1/2*d*x + 1/2*c) + (3181018*a
^3*tan(1/2*d*x + 1/2*c)^11 + 568260*a^3*tan(1/2*d*x + 1/2*c)^10 - 152460*a^
3*tan(1/2*d*x + 1/2*c)^9 - 244860*a^3*tan(1/2*d*x + 1/2*c)^8 - 138600*a^3*t
an(1/2*d*x + 1/2*c)^7 - 6930*a^3*tan(1/2*d*x + 1/2*c)^6 + 57750*a^3*tan(1/2
*d*x + 1/2*c)^5 + 40590*a^3*tan(1/2*d*x + 1/2*c)^4 + 3465*a^3*tan(1/2*d*x +
 1/2*c)^3 - 8470*a^3*tan(1/2*d*x + 1/2*c)^2 - 4158*a^3*tan(1/2*d*x + 1/2*c)
 - 630*a^3)/tan(1/2*d*x + 1/2*c)^11)/d
```

maple [A] time = 0.41, size = 264, normalized size = 1.07

$$-\frac{19a^3\left(\cos^7(dx+c)\right)}{80d\sin(dx+c)^8}-\frac{19a^3\left(\cos^7(dx+c)\right)}{480d\sin(dx+c)^6}+\frac{19a^3\left(\cos^7(dx+c)\right)}{1920d\sin(dx+c)^4}-\frac{19a^3\left(\cos^7(dx+c)\right)}{1280d\sin(dx+c)^2}-\frac{19a^3\left(\cos^5(dx+c)\right)}{1280d}-\frac{19}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -19/80/d*a^3/sin(d*x+c)^8*cos(d*x+c)^7-19/480/d*a^3/sin(d*x+c)^6*cos(d*x+c)
^7+19/1920/d*a^3/sin(d*x+c)^4*cos(d*x+c)^7-19/1280/d*a^3/sin(d*x+c)^2*cos(d
*x+c)^7-19/1280*a^3*cos(d*x+c)^5/d-19/768*a^3*cos(d*x+c)^3/d-19/256*a^3*cos
(d*x+c)/d-19/256/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-37/99/d*a^3/sin(d*x+c)^9*c
os(d*x+c)^7-74/693/d*a^3/sin(d*x+c)^7*cos(d*x+c)^7-3/10/d*a^3/sin(d*x+c)^10
*cos(d*x+c)^7-1/11/d*a^3/sin(d*x+c)^11*cos(d*x+c)^7
```

maxima [A] time = 0.49, size = 308, normalized size = 1.25

$$-\frac{2079\,a^3\left(\frac{2\left(15\,\cos(dx+c)^9-70\,\cos(dx+c)^7-128\,\cos(dx+c)^5+70\,\cos(dx+c)^3-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^8+10\,\cos(dx+c)^6-10\,\cos(dx+c)^4+5\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] -1/1774080*(2079*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*
x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*
x + c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) -
15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2310*a^3*(2*(15*cos(
d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(
d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) -
15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 84480*(9*tan(d*x + c
)^2 + 7)*a^3/tan(d*x + c)^9 + 2560*(99*tan(d*x + c)^4 + 154*tan(d*x + c)^2
+ 63)*a^3/tan(d*x + c)^11)/d
```

mupad [B] time = 9.98, size = 433, normalized size = 1.76

$$\frac{25\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{6144\,d}-\frac{53\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3072\,d}-\frac{5\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{512\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2048\,d}-\frac{11\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}+\frac{41\,a^3\cot\left(\right.}{143}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^12,x)
```

```
[Out] (25*a^3*cot(c/2 + (d*x)/2)^6)/(6144*d) - (53*a^3*cot(c/2 + (d*x)/2)^3)/(307
2*d) - (5*a^3*cot(c/2 + (d*x)/2)^4)/(512*d) - (a^3*cot(c/2 + (d*x)/2)^5)/(2
048*d) - (11*a^3*cot(c/2 + (d*x)/2)^2)/(1024*d) + (41*a^3*cot(c/2 + (d*x)/2
)^7)/(14336*d) + (a^3*cot(c/2 + (d*x)/2)^8)/(4096*d) - (11*a^3*cot(c/2 + (d
*x)/2)^9)/(18432*d) - (3*a^3*cot(c/2 + (d*x)/2)^10)/(10240*d) - (a^3*cot(c/
2 + (d*x)/2)^11)/(22528*d) + (11*a^3*tan(c/2 + (d*x)/2)^2)/(1024*d) + (53*a
^3*tan(c/2 + (d*x)/2)^3)/(3072*d) + (5*a^3*tan(c/2 + (d*x)/2)^4)/(512*d) +
```

```
(a^3*tan(c/2 + (d*x)/2)^5)/(2048*d) - (25*a^3*tan(c/2 + (d*x)/2)^6)/(6144*d
) - (41*a^3*tan(c/2 + (d*x)/2)^7)/(14336*d) - (a^3*tan(c/2 + (d*x)/2)^8)/(4
096*d) + (11*a^3*tan(c/2 + (d*x)/2)^9)/(18432*d) + (3*a^3*tan(c/2 + (d*x)/2
)^10)/(10240*d) + (a^3*tan(c/2 + (d*x)/2)^11)/(22528*d) - (19*a^3*log(tan(c
/2 + (d*x)/2)))/(256*d) + (41*a^3*cot(c/2 + (d*x)/2))/(1024*d) - (41*a^3*ta
n(c/2 + (d*x)/2))/(1024*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**12*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.621 $\int \cot^6(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=270

$$-\frac{3a^3\cot^{11}(c+dx)}{11d}-\frac{7a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{41a^3\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^3\cot^5(c+dx)\csc^7(c+dx)}{12d}$$

```
[Out] 41/1024*a^3*arctanh(cos(d*x+c))/d-4/7*a^3*cot(d*x+c)^7/d-7/9*a^3*cot(d*x+c)
^9/d-3/11*a^3*cot(d*x+c)^11/d+41/1024*a^3*cot(d*x+c)*csc(d*x+c)/d+41/1536*a
^3*cot(d*x+c)*csc(d*x+c)^3/d-35/384*a^3*cot(d*x+c)*csc(d*x+c)^5/d+3/16*a^3*
cot(d*x+c)^3*csc(d*x+c)^5/d-3/10*a^3*cot(d*x+c)^5*csc(d*x+c)^5/d-1/64*a^3*c
ot(d*x+c)*csc(d*x+c)^7/d+1/24*a^3*cot(d*x+c)^3*csc(d*x+c)^7/d-1/12*a^3*cot(
d*x+c)^5*csc(d*x+c)^7/d
```

Rubi [A] time = 0.46, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2873, 2607, 14, 2611, 3768, 3770, 270}

$$-\frac{3a^3\cot^{11}(c+dx)}{11d}-\frac{7a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{41a^3\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^3\cot^5(c+dx)\csc^7(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (41*a^3*ArcTanh[Cos[c + d*x]])/(1024*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (7
*a^3*Cot[c + d*x]^9)/(9*d) - (3*a^3*Cot[c + d*x]^11)/(11*d) + (41*a^3*Cot[c
 + d*x]*Csc[c + d*x])/(1024*d) + (41*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(1536
*d) - (35*a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(384*d) + (3*a^3*Cot[c + d*x]^3*
Csc[c + d*x]^5)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d) - (a^
3*Cot[c + d*x]*Csc[c + d*x]^7)/(64*d) + (a^3*Cot[c + d*x]^3*Csc[c + d*x]^7)
/(24*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x]^7)/(12*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)\csc^4(c+dx)+3a^3\cot^6(c+dx)\csc^5(c+\right.$$

$$= a^3\int \cot^6(c+dx)\csc^4(c+dx)\,dx + a^3\int \cot^6(c+dx)\csc^7$$

$$= -\frac{3a^3\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{a^3\cot^5(c+dx)\csc^7(c+}{12d}$$

$$= \frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{3a^3\cot^5(c+dx)\csc^5(c+}{10d}$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot^9(c+dx)}{9d} - \frac{3a^3\cot^{11}(c+dx)}{11d}$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot^9(c+dx)}{9d} - \frac{3a^3\cot^{11}(c+dx)}{11d}$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot^9(c+dx)}{9d} - \frac{3a^3\cot^{11}(c+dx)}{11d}$$

$$= \frac{9a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot^9(c+}{9d}$$

$$= \frac{41a^3\tanh^{-1}(\cos(c+dx))}{1024d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot^9(c}{9d}$$

Mathematica [A] time = 4.72, size = 197, normalized size = 0.73

$$\frac{a^3(\sin(c+dx)+1)^3\left(72737280\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc^{11}(c+dx)(497}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(72737280*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d
*x)/2]]) - Cot[c + d*x]*Csc[c + d*x]^11*(91311066 + 62609778*Cos[2*(c + d*x
)] + 22551144*Cos[4*(c + d*x)] - 23426403*Cos[6*(c + d*x)] - 1799490*Cos[8*
(c + d*x)] + 142065*Cos[10*(c + d*x)] + 49776640*Sin[c + d*x] + 84039680*Si
n[3*(c + d*x)] + 38118400*Sin[5*(c + d*x)] + 2206720*Sin[7*(c + d*x)] - 153
0880*Sin[9*(c + d*x)] + 117760*Sin[11*(c + d*x)])))/(1816657920*d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 1.01, size = 384, normalized size = 1.42

$$-\frac{284130\, a^3 \cos\left(dx+c\right)^{11} - 1610070\, a^3 \cos\left(dx+c\right)^{9} - 507276\, a^3 \cos\left(dx+c\right)^{7} + 3750516\, a^3 \cos\left(dx+c\right)^{5} - 161\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] -1/7096320*(284130*a^3*cos(d*x + c)^11 - 1610070*a^3*cos(d*x + c)^9 - 50727
6*a^3*cos(d*x + c)^7 + 3750516*a^3*cos(d*x + c)^5 - 1610070*a^3*cos(d*x + c
)^3 + 284130*a^3*cos(d*x + c) - 142065*(a^3*cos(d*x + c)^12 - 6*a^3*cos(d*x
 + c)^10 + 15*a^3*cos(d*x + c)^8 - 20*a^3*cos(d*x + c)^6 + 15*a^3*cos(d*x +
 c)^4 - 6*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2) + 142065*(a
^3*cos(d*x + c)^12 - 6*a^3*cos(d*x + c)^10 + 15*a^3*cos(d*x + c)^8 - 20*a^3
*cos(d*x + c)^6 + 15*a^3*cos(d*x + c)^4 - 6*a^3*cos(d*x + c)^2 + a^3)*log(-
1/2*cos(d*x + c) + 1/2) + 10240*(46*a^3*cos(d*x + c)^11 - 253*a^3*cos(d*x +
 c)^9 + 396*a^3*cos(d*x + c)^7)*sin(d*x + c))/(d*cos(d*x + c)^12 - 6*d*cos(
d*x + c)^10 + 15*d*cos(d*x + c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)
^4 - 6*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.54, size = 420, normalized size = 1.56

$$\frac{1155\, a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12} + 7560\, a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 16632\, a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10} + 3080\, a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/56770560*(1155*a^3*tan(1/2*d*x + 1/2*c)^12 + 7560*a^3*tan(1/2*d*x + 1/2*c
)^11 + 16632*a^3*tan(1/2*d*x + 1/2*c)^10 + 3080*a^3*tan(1/2*d*x + 1/2*c)^9
- 51975*a^3*tan(1/2*d*x + 1/2*c)^8 - 106920*a^3*tan(1/2*d*x + 1/2*c)^7 - 83
160*a^3*tan(1/2*d*x + 1/2*c)^6 + 83160*a^3*tan(1/2*d*x + 1/2*c)^5 + 384615*
a^3*tan(1/2*d*x + 1/2*c)^4 + 572880*a^3*tan(1/2*d*x + 1/2*c)^3 + 166320*a^3
*tan(1/2*d*x + 1/2*c)^2 - 2273040*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 1496
880*a^3*tan(1/2*d*x + 1/2*c) + (7053722*a^3*tan(1/2*d*x + 1/2*c)^12 + 14968
80*a^3*tan(1/2*d*x + 1/2*c)^11 - 166320*a^3*tan(1/2*d*x + 1/2*c)^10 - 57288
0*a^3*tan(1/2*d*x + 1/2*c)^9 - 384615*a^3*tan(1/2*d*x + 1/2*c)^8 - 83160*a^
3*tan(1/2*d*x + 1/2*c)^7 + 83160*a^3*tan(1/2*d*x + 1/2*c)^6 + 106920*a^3*ta
n(1/2*d*x + 1/2*c)^5 + 51975*a^3*tan(1/2*d*x + 1/2*c)^4 - 3080*a^3*tan(1/2*
```

```
d*x + 1/2*c)^3 - 16632*a^3*tan(1/2*d*x + 1/2*c)^2 - 7560*a^3*tan(1/2*d*x +
1/2*c) - 1155*a^3)/tan(1/2*d*x + 1/2*c)^12)/d
```

maple [A] time = 0.41, size = 288, normalized size = 1.07

$$-\frac{23a^3\left(\cos^7(dx+c)\right)}{99d\sin(dx+c)^9}-\frac{46a^3\left(\cos^7(dx+c)\right)}{693d\sin(dx+c)^7}-\frac{41a^3\left(\cos^7(dx+c)\right)}{120d\sin(dx+c)^{10}}-\frac{41a^3\left(\cos^7(dx+c)\right)}{320d\sin(dx+c)^8}-\frac{41a^3\left(\cos^7(dx+c)\right)}{1920d\sin(dx+c)^6}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -23/99/d*a^3/sin(d*x+c)^9*cos(d*x+c)^7-46/693/d*a^3/sin(d*x+c)^7*cos(d*x+c)
^7-41/120/d*a^3/sin(d*x+c)^10*cos(d*x+c)^7-41/320/d*a^3/sin(d*x+c)^8*cos(d*
x+c)^7-41/1920/d*a^3/sin(d*x+c)^6*cos(d*x+c)^7+41/7680/d*a^3/sin(d*x+c)^4*c
os(d*x+c)^7-41/5120/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7-41/5120*a^3*cos(d*x+c)^
5/d-41/3072*a^3*cos(d*x+c)^3/d-41/1024*a^3*cos(d*x+c)/d-41/1024/d*a^3*ln(cs
c(d*x+c)-cot(d*x+c))-3/11/d*a^3/sin(d*x+c)^11*cos(d*x+c)^7-1/12/d*a^3/sin(d
*x+c)^12*cos(d*x+c)^7
```

maxima [A] time = 0.40, size = 348, normalized size = 1.29

$$-\frac{1155\,a^3\left(\frac{2\left(15\cos(dx+c)^{11}-85\cos(dx+c)^9+198\cos(dx+c)^7+198\cos(dx+c)^5-85\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^{12}-6\cos(dx+c)^{10}+15\cos(dx+c)^8-20\cos(dx+c)^6+15\cos(dx+c)^4-6\cos(dx+c)^2+1}-15\log(\cos(dx+c)+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] -1/7096320*(1155*a^3*(2*(15*cos(d*x + c)^11 - 85*cos(d*x + c)^9 + 198*cos(d
*x + c)^7 + 198*cos(d*x + c)^5 - 85*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(
d*x + c)^12 - 6*cos(d*x + c)^10 + 15*cos(d*x + c)^8 - 20*cos(d*x + c)^6 + 1
5*cos(d*x + c)^4 - 6*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*lo
g(cos(d*x + c) - 1)) + 8316*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 -
 128*cos(d*x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10
 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c
)^2 - 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 112640*(9
*tan(d*x + c)^2 + 7)*a^3/tan(d*x + c)^9 + 30720*(99*tan(d*x + c)^4 + 154*ta
n(d*x + c)^2 + 63)*a^3/tan(d*x + c)^11)/d
```

mupad [B] time = 10.40, size = 471, normalized size = 1.74

$$\frac{3\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2048\,d}-\frac{31\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3072\,d}-\frac{111\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{16384\,d}-\frac{3\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2048\,d}-\frac{3\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}+\frac{27\,a^3\,\mathrm{co}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^13,x)
```

```
[Out] (3*a^3*cot(c/2 + (d*x)/2)^6)/(2048*d) - (31*a^3*cot(c/2 + (d*x)/2)^3)/(3072
*d) - (111*a^3*cot(c/2 + (d*x)/2)^4)/(16384*d) - (3*a^3*cot(c/2 + (d*x)/2)^
5)/(2048*d) - (3*a^3*cot(c/2 + (d*x)/2)^2)/(1024*d) + (27*a^3*cot(c/2 + (d*
x)/2)^7)/(14336*d) + (15*a^3*cot(c/2 + (d*x)/2)^8)/(16384*d) - (a^3*cot(c/2
 + (d*x)/2)^9)/(18432*d) - (3*a^3*cot(c/2 + (d*x)/2)^10)/(10240*d) - (3*a^3
*cot(c/2 + (d*x)/2)^11)/(22528*d) - (a^3*cot(c/2 + (d*x)/2)^12)/(49152*d) +
 (3*a^3*tan(c/2 + (d*x)/2)^2)/(1024*d) + (31*a^3*tan(c/2 + (d*x)/2)^3)/(307
2*d) + (111*a^3*tan(c/2 + (d*x)/2)^4)/(16384*d) + (3*a^3*tan(c/2 + (d*x)/2)
^5)/(2048*d) - (3*a^3*tan(c/2 + (d*x)/2)^6)/(2048*d) - (27*a^3*tan(c/2 + (d
*x)/2)^7)/(14336*d) - (15*a^3*tan(c/2 + (d*x)/2)^8)/(16384*d) + (a^3*tan(c/
2 + (d*x)/2)^9)/(18432*d) + (3*a^3*tan(c/2 + (d*x)/2)^10)/(10240*d) + (3*a^
3*tan(c/2 + (d*x)/2)^11)/(22528*d) + (a^3*tan(c/2 + (d*x)/2)^12)/(49152*d)
- (41*a^3*log(tan(c/2 + (d*x)/2)))/(1024*d) + (27*a^3*cot(c/2 + (d*x)/2))/(
1024*d) - (27*a^3*tan(c/2 + (d*x)/2))/(1024*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**13*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.622 $\int \cot^6(c+dx)\csc^8(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=286

$$-\frac{a^3\cot^{13}(c+dx)}{13d}-\frac{6a^3\cot^{11}(c+dx)}{11d}-\frac{a^3\cot^9(c+dx)}{d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{27a^3\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^3\cot^5(c+$$

```
[Out] 27/1024*a^3*arctanh(cos(d*x+c))/d-4/7*a^3*cot(d*x+c)^7/d-a^3*cot(d*x+c)^9/d
-6/11*a^3*cot(d*x+c)^11/d-1/13*a^3*cot(d*x+c)^13/d+27/1024*a^3*cot(d*x+c)*c
sc(d*x+c)/d+9/512*a^3*cot(d*x+c)*csc(d*x+c)^3/d-3/128*a^3*cot(d*x+c)*csc(d*
x+c)^5/d+1/16*a^3*cot(d*x+c)^3*csc(d*x+c)^5/d-1/10*a^3*cot(d*x+c)^5*csc(d*x
+c)^5/d-3/64*a^3*cot(d*x+c)*csc(d*x+c)^7/d+1/8*a^3*cot(d*x+c)^3*csc(d*x+c)^
7/d-1/4*a^3*cot(d*x+c)^5*csc(d*x+c)^7/d
```

Rubi [A] time = 0.46, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3768, 3770, 2607, 270}

$$-\frac{a^3\cot^{13}(c+dx)}{13d}-\frac{6a^3\cot^{11}(c+dx)}{11d}-\frac{a^3\cot^9(c+dx)}{d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{27a^3\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^3\cot^5(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^8*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (27*a^3*ArcTanh[Cos[c + d*x]])/(1024*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (a
^3*Cot[c + d*x]^9)/d - (6*a^3*Cot[c + d*x]^11)/(11*d) - (a^3*Cot[c + d*x]^1
3)/(13*d) + (27*a^3*Cot[c + d*x]*Csc[c + d*x])/(1024*d) + (9*a^3*Cot[c + d*
x]*Csc[c + d*x]^3)/(512*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(128*d) +
(a^3*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*
x]^5)/(10*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^7)/(64*d) + (a^3*Cot[c + d*
x]^3*Csc[c + d*x]^7)/(8*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x]^7)/(4*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^8(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)\csc^5(c+dx) + 3a^3\cot^6(c+dx)\csc^6(c+\right.$$
$$= a^3\int \cot^6(c+dx)\csc^5(c+dx)\,dx + a^3\int \cot^6(c+dx)\csc^8$$
$$= -\frac{a^3\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{a^3\cot^5(c+dx)\csc^7(c+d}{4d}$$
$$= \frac{a^3\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{a^3\cot^5(c+dx)\csc^5(c+dx)}{10d}$$
$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{d} - \frac{6a^3\cot^{11}(c+dx)}{11d} -$$
$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{d} - \frac{6a^3\cot^{11}(c+dx)}{11d} -$$
$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{d} - \frac{6a^3\cot^{11}(c+dx)}{11d} -$$
$$= \frac{3a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+}{d}$$
$$= \frac{27a^3\tanh^{-1}(\cos(c+dx))}{1024d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+}{d}$$

Mathematica [A] time = 6.50, size = 283, normalized size = 0.99

$$\frac{27(a\sin(c+dx)+a)^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{1024d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6} - \frac{27(a\sin(c+dx)+a)^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1024d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6} + \frac{\cot(c+dx)\csc^{12}(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^8*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (27*Log[Cos[(c + d*x)/2]]*(a + a*Sin[c + d*x])^3)/(1024*d*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])^6) - (27*Log[Sin[(c + d*x)/2]]*(a + a*Sin[c + d*x])^3)
/(1024*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + (Cot[c + d*x]*Csc[c + d
*x]^12*(a + a*Sin[c + d*x])^3*(-200294400 - 243712000*Cos[2*(c + d*x)] - 11
079680*Cos[4*(c + d*x)] + 43294720*Cos[6*(c + d*x)] + 9420800*Cos[8*(c + d*
x)] - 1433600*Cos[10*(c + d*x)] + 102400*Cos[12*(c + d*x)] - 194159966*Sin[
c + d*x] - 182107926*Sin[3*(c + d*x)] - 123736613*Sin[5*(c + d*x)] + 457156
7*Sin[7*(c + d*x)] + 1846845*Sin[9*(c + d*x)] - 135135*Sin[11*(c + d*x)]))/
(5248122880*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.99, size = 417, normalized size = 1.46

$$409600\,a^3\cos(dx+c)^{13} - 2662400\,a^3\cos(dx+c)^{11} + 7321600\,a^3\cos(dx+c)^9 - 5857280\,a^3\cos(dx+c)^7 + 135$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^14*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] 1/10250240*(409600*a^3*cos(d*x + c)^13 - 2662400*a^3*cos(d*x + c)^11 + 7321
600*a^3*cos(d*x + c)^9 - 5857280*a^3*cos(d*x + c)^7 + 135135*(a^3*cos(d*x +
 c)^12 - 6*a^3*cos(d*x + c)^10 + 15*a^3*cos(d*x + c)^8 - 20*a^3*cos(d*x + c
)^6 + 15*a^3*cos(d*x + c)^4 - 6*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 135135*(a^3*cos(d*x + c)^12 - 6*a^3*cos(d*x + c)^
10 + 15*a^3*cos(d*x + c)^8 - 20*a^3*cos(d*x + c)^6 + 15*a^3*cos(d*x + c)^4
- 6*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2
002*(135*a^3*cos(d*x + c)^11 - 765*a^3*cos(d*x + c)^9 + 758*a^3*cos(d*x + c
)^7 + 1782*a^3*cos(d*x + c)^5 - 765*a^3*cos(d*x + c)^3 + 135*a^3*cos(d*x +
c))*sin(d*x + c))/((d*cos(d*x + c)^12 - 6*d*cos(d*x + c)^10 + 15*d*cos(d*x
+ c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)^4 - 6*d*cos(d*x + c)^2 + d
)*sin(d*x + c))
```

giac [A] time = 0.50, size = 452, normalized size = 1.58

$$770\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} + 5005\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12} + 11830\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 8008\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^14*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/82001920*(770*a^3*tan(1/2*d*x + 1/2*c)^13 + 5005*a^3*tan(1/2*d*x + 1/2*c)
^12 + 11830*a^3*tan(1/2*d*x + 1/2*c)^11 + 8008*a^3*tan(1/2*d*x + 1/2*c)^10
- 20020*a^3*tan(1/2*d*x + 1/2*c)^9 - 65065*a^3*tan(1/2*d*x + 1/2*c)^8 - 943
80*a^3*tan(1/2*d*x + 1/2*c)^7 - 40040*a^3*tan(1/2*d*x + 1/2*c)^6 + 150150*a
^3*tan(1/2*d*x + 1/2*c)^5 + 385385*a^3*tan(1/2*d*x + 1/2*c)^4 + 450450*a^3*
tan(1/2*d*x + 1/2*c)^3 + 80080*a^3*tan(1/2*d*x + 1/2*c)^2 - 2162160*a^3*log
(abs(tan(1/2*d*x + 1/2*c))) - 1401400*a^3*tan(1/2*d*x + 1/2*c) + (6875958*a
^3*tan(1/2*d*x + 1/2*c)^13 + 1401400*a^3*tan(1/2*d*x + 1/2*c)^12 - 80080*a^
3*tan(1/2*d*x + 1/2*c)^11 - 450450*a^3*tan(1/2*d*x + 1/2*c)^10 - 385385*a^3
*tan(1/2*d*x + 1/2*c)^9 - 150150*a^3*tan(1/2*d*x + 1/2*c)^8 + 40040*a^3*tan
```

```
(1/2*d*x + 1/2*c)^7 + 94380*a^3*tan(1/2*d*x + 1/2*c)^6 + 65065*a^3*tan(1/2*
d*x + 1/2*c)^5 + 20020*a^3*tan(1/2*d*x + 1/2*c)^4 - 8008*a^3*tan(1/2*d*x +
1/2*c)^3 - 11830*a^3*tan(1/2*d*x + 1/2*c)^2 - 5005*a^3*tan(1/2*d*x + 1/2*c)
 - 770*a^3)/tan(1/2*d*x + 1/2*c)^13)/d
```

maple [A] time = 0.41, size = 312, normalized size = 1.09

$$-\frac{9a^3\left(\cos^7(dx+c)\right)}{640d\sin(dx+c)^6}+\frac{9a^3\left(\cos^7(dx+c)\right)}{2560d\sin(dx+c)^4}-\frac{27a^3\left(\cos^7(dx+c)\right)}{5120d\sin(dx+c)^2}-\frac{45a^3\left(\cos^7(dx+c)\right)}{143d\sin(dx+c)^{11}}-\frac{20a^3\left(\cos^7(dx+c)\right)}{143d\sin(dx+c)^9}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^14*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -9/640/d*a^3/sin(d*x+c)^6*cos(d*x+c)^7+9/2560/d*a^3/sin(d*x+c)^4*cos(d*x+c)
^7-27/5120/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7-45/143/d*a^3/sin(d*x+c)^11*cos(d
*x+c)^7-20/143/d*a^3/sin(d*x+c)^9*cos(d*x+c)^7-9/40/d*a^3/sin(d*x+c)^10*cos
(d*x+c)^7-27/320/d*a^3/sin(d*x+c)^8*cos(d*x+c)^7-1/4/d*a^3/sin(d*x+c)^12*co
s(d*x+c)^7-1/13/d*a^3/sin(d*x+c)^13*cos(d*x+c)^7-40/1001/d*a^3/sin(d*x+c)^7
*cos(d*x+c)^7-27/5120*a^3*cos(d*x+c)^5/d-9/1024*a^3*cos(d*x+c)^3/d-27/1024*
a^3*cos(d*x+c)/d-27/1024/d*a^3*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.36, size = 368, normalized size = 1.29

$$-\frac{15015\,a^3\left(\frac{2\left(15\,\cos(dx+c)^{11}-85\,\cos(dx+c)^9+198\,\cos(dx+c)^7+198\,\cos(dx+c)^5-85\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^{12}-6\,\cos(dx+c)^{10}+15\,\cos(dx+c)^8-20\,\cos(dx+c)^6+15\,\cos(dx+c)^4-6\,\cos(dx+c)^2+1}-15\,\log\left(\cos(dx+c)+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^14*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] -1/30750720*(15015*a^3*(2*(15*cos(d*x + c)^11 - 85*cos(d*x + c)^9 + 198*cos
(d*x + c)^7 + 198*cos(d*x + c)^5 - 85*cos(d*x + c)^3 + 15*cos(d*x + c))/(co
s(d*x + c)^12 - 6*cos(d*x + c)^10 + 15*cos(d*x + c)^8 - 20*cos(d*x + c)^6 +
 15*cos(d*x + c)^4 - 6*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*
log(cos(d*x + c) - 1)) + 12012*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^
7 - 128*cos(d*x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)
^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x
+ c)^2 - 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 133120
*(99*tan(d*x + c)^4 + 154*tan(d*x + c)^2 + 63)*a^3/tan(d*x + c)^11 + 10240*
(429*tan(d*x + c)^6 + 1001*tan(d*x + c)^4 + 819*tan(d*x + c)^2 + 231)*a^3/t
an(d*x + c)^13)/d
```

mupad [B] time = 10.90, size = 509, normalized size = 1.78

$$\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{2048\,d}-\frac{45\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{8192\,d}-\frac{77\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{16384\,d}-\frac{15\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{8192\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{1024\,d}+\frac{33\,a^3\cot\left(\frac{c}{2}+\right.}{28672\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^14,x)
```

```
[Out] (a^3*cot(c/2 + (d*x)/2)^6)/(2048*d) - (45*a^3*cot(c/2 + (d*x)/2)^3)/(8192*d
) - (77*a^3*cot(c/2 + (d*x)/2)^4)/(16384*d) - (15*a^3*cot(c/2 + (d*x)/2)^5)
/(8192*d) - (a^3*cot(c/2 + (d*x)/2)^2)/(1024*d) + (33*a^3*cot(c/2 + (d*x)/2
)^7)/(28672*d) + (13*a^3*cot(c/2 + (d*x)/2)^8)/(16384*d) + (a^3*cot(c/2 + (
d*x)/2)^9)/(4096*d) - (a^3*cot(c/2 + (d*x)/2)^10)/(10240*d) - (13*a^3*cot(c
/2 + (d*x)/2)^11)/(90112*d) - (a^3*cot(c/2 + (d*x)/2)^12)/(16384*d) - (a^3*
cot(c/2 + (d*x)/2)^13)/(106496*d) + (a^3*tan(c/2 + (d*x)/2)^2)/(1024*d) + (
45*a^3*tan(c/2 + (d*x)/2)^3)/(8192*d) + (77*a^3*tan(c/2 + (d*x)/2)^4)/(1638
4*d) + (15*a^3*tan(c/2 + (d*x)/2)^5)/(8192*d) - (a^3*tan(c/2 + (d*x)/2)^6)/
(2048*d) - (33*a^3*tan(c/2 + (d*x)/2)^7)/(28672*d) - (13*a^3*tan(c/2 + (d*x
)/2)^8)/(16384*d) - (a^3*tan(c/2 + (d*x)/2)^9)/(4096*d) + (a^3*tan(c/2 + (d
*x)/2)^10)/(10240*d) + (13*a^3*tan(c/2 + (d*x)/2)^11)/(90112*d) + (a^3*tan(
c/2 + (d*x)/2)^12)/(16384*d) + (a^3*tan(c/2 + (d*x)/2)^13)/(106496*d) - (27
*a^3*log(tan(c/2 + (d*x)/2)))/(1024*d) + (35*a^3*cot(c/2 + (d*x)/2))/(2048*
d) - (35*a^3*tan(c/2 + (d*x)/2))/(2048*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**14*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.623 $\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=178

$$\frac{4a^4\cos^5(c+dx)}{5d}-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{4a^4\cot(c+dx)}{d}+\frac{a^4\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{23a^4\sin^3(c}{}$$

```
[Out] -135/16*a^4*x+6*a^4*arctanh(cos(d*x+c))/d-4*a^4*cos(d*x+c)/d+4/5*a^4*cos(d*
x+c)^5/d-4*a^4*cot(d*x+c)/d-1/3*a^4*cot(d*x+c)^3/d-2*a^4*cot(d*x+c)*csc(d*x
+c)/d-89/16*a^4*cos(d*x+c)*sin(d*x+c)/d+23/24*a^4*cos(d*x+c)*sin(d*x+c)^3/d
+1/6*a^4*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A] time = 0.28, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2872, 3770, 3767, 8, 3768, 2635, 2633}

$$\frac{4a^4\cos^5(c+dx)}{5d}-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{4a^4\cot(c+dx)}{d}+\frac{a^4\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{23a^4\sin^3(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-135*a^4*x)/16 + (6*a^4*ArcTanh[Cos[c + d*x]])/d - (4*a^4*Cos[c + d*x])/d
+ (4*a^4*Cos[c + d*x]^5)/(5*d) - (4*a^4*Cot[c + d*x])/d - (a^4*Cot[c + d*x]
^3)/(3*d) - (2*a^4*Cot[c + d*x]*Csc[c + d*x])/d - (89*a^4*Cos[c + d*x]*Sin[
c + d*x])/(16*d) + (23*a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) + (a^4*Cos[c
 + d*x]*Sin[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(-14a^{10}-8a^{10}\csc(c+dx)+3a^{10}\csc^2(c+dx)+4a^{10}\csc\right.}{}$$

$$= -14a^4x + a^4\int \csc^4(c+dx)\,dx - a^4\int \sin^6(c+dx)\,dx + \left(3a\right.$$

$$= -14a^4x + \frac{8a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{2a^4\cot(c+dx)\csc(c}{d}$$

$$= -7a^4x + \frac{6a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^4\cos(c+dx)}{d} + \frac{4a^4}{}$$

$$= -\frac{65a^4x}{8} + \frac{6a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^4\cos(c+dx)}{d} + \frac{4a}{}$$

$$= -\frac{135a^4x}{16} + \frac{6a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^4\cos(c+dx)}{d} + \frac{4}{}$$

Mathematica [A] time = 1.66, size = 229, normalized size = 1.29

$$a^4(\sin(c+dx)+1)^4\left(-8100(c+dx)-2415\sin(2(c+dx))-135\sin(4(c+dx))+5\sin(6(c+dx))-3360\cos(c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Sin[c + d*x])^4*(-8100*(c + d*x) - 3360*Cos[c + d*x] + 240*Cos[3*
(c + d*x)] + 48*Cos[5*(c + d*x)] - 1760*Cot[(c + d*x)/2] - 480*Csc[(c + d*x
)/2]^2 + 5760*Log[Cos[(c + d*x)/2]] - 5760*Log[Sin[(c + d*x)/2]] + 480*Sec[
(c + d*x)/2]^2 + 320*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 20*Csc[(c + d*x)/2
]^4*Sin[c + d*x] - 2415*Sin[2*(c + d*x)] - 135*Sin[4*(c + d*x)] + 5*Sin[6*(
c + d*x)] + 1760*Tan[(c + d*x)/2]))/(960*d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^8)
```

fricas [A] time = 0.89, size = 245, normalized size = 1.38

$$-\frac{40\,a^4\cos\left(dx+c\right)^9-390\,a^4\cos\left(dx+c\right)^7-405\,a^4\cos\left(dx+c\right)^5+2700\,a^4\cos\left(dx+c\right)^3-2025\,a^4\cos\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/240*(40*a^4*cos(d*x + c)^9 - 390*a^4*cos(d*x + c)^7 - 405*a^4*cos(d*x +
c)^5 + 2700*a^4*cos(d*x + c)^3 - 2025*a^4*cos(d*x + c) - 720*(a^4*cos(d*x +
 c)^2 - a^4)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 720*(a^4*cos(d*x +
c)^2 - a^4)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(64*a^4*cos(d*x +
 c)^7 - 64*a^4*cos(d*x + c)^5 - 675*a^4*d*x*cos(d*x + c)^2 - 320*a^4*cos(d*
x + c)^3 + 675*a^4*d*x + 480*a^4*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x +
c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.49, size = 324, normalized size = 1.82

$$\frac{10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2025\,(dx+c)a^4-1440\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+450\,a^4\,\mathrm{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(10*a^4*tan(1/2*d*x + 1/2*c)^3 + 120*a^4*tan(1/2*d*x + 1/2*c)^2 - 202
5*(d*x + c)*a^4 - 1440*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 450*a^4*tan(1/2
*d*x + 1/2*c) + 10*(264*a^4*tan(1/2*d*x + 1/2*c)^3 - 45*a^4*tan(1/2*d*x + 1
/2*c)^2 - 12*a^4*tan(1/2*d*x + 1/2*c) - a^4)/tan(1/2*d*x + 1/2*c)^3 + 2*(13
35*a^4*tan(1/2*d*x + 1/2*c)^11 + 3085*a^4*tan(1/2*d*x + 1/2*c)^9 - 3840*a^4
*tan(1/2*d*x + 1/2*c)^8 + 1110*a^4*tan(1/2*d*x + 1/2*c)^7 - 7680*a^4*tan(1/
2*d*x + 1/2*c)^6 - 1110*a^4*tan(1/2*d*x + 1/2*c)^5 - 7680*a^4*tan(1/2*d*x +
 1/2*c)^4 - 3085*a^4*tan(1/2*d*x + 1/2*c)^3 - 4608*a^4*tan(1/2*d*x + 1/2*c)
^2 - 1335*a^4*tan(1/2*d*x + 1/2*c) - 768*a^4)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
6)/d
```

maple [A] time = 0.44, size = 223, normalized size = 1.25

$$-\frac{9a^4\left(\cos^5(dx+c)\right)\sin(dx+c)}{2d}-\frac{6a^4\left(\cos^5(dx+c)\right)}{5d}-\frac{a^4\left(\cos^7(dx+c)\right)}{3d\sin(dx+c)^3}-\frac{14a^4\left(\cos^7(dx+c)\right)}{3d\sin(dx+c)}-\frac{2a^4\left(\cos^7(dx+c)\right.}{d\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -9/2*a^4*cos(d*x+c)^5*sin(d*x+c)/d-6/5*a^4*cos(d*x+c)^5/d-1/3/d*a^4/sin(d*x
+c)^3*cos(d*x+c)^7-14/3/d*a^4/sin(d*x+c)*cos(d*x+c)^7-2/d*a^4/sin(d*x+c)^2*
cos(d*x+c)^7-135/16*a^4*x-45/8*a^4*cos(d*x+c)^3*sin(d*x+c)/d-135/16*a^4*cos
(d*x+c)*sin(d*x+c)/d-135/16/d*a^4*c-2*a^4*cos(d*x+c)^3/d-6*a^4*cos(d*x+c)/d
-6/d*a^4*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.41, size = 294, normalized size = 1.65

$$\frac{128\left(6\cos(dx+c)^5+10\cos(dx+c)^3+30\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/960*(128*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log
(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^4 - 320*(4*cos(d*x + c)^3
- 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x +
c) + 1) + 15*log(cos(d*x + c) - 1))*a^4 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x
- 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^4 - 720*(15*d*x + 15*c
 + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x
+ c)^3 + tan(d*x + c)))*a^4 + 160*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*
tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a^4)/d
```

mupad [B] time = 8.98, size = 474, normalized size = 2.66

$$\frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,d}+\frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{6\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{135\,a^4\,\mathrm{atan}\left(\frac{18225\,a^8}{64\left(\frac{405\,a^8}{2}-\frac{18225\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}+\frac{405\,a^8\,\mathrm{t}}{2\left(\frac{405\,a^8}{2}-\frac{182}{}\right.}\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^4)/sin(c + d*x)^4,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^2)/(2*d) + (a^4*tan(c/2 + (d*x)/2)^3)/(24*d) - (6*a
^4*log(tan(c/2 + (d*x)/2)))/d - (135*a^4*atan((18225*a^8)/(64*((405*a^8)/2
- (18225*a^8*tan(c/2 + (d*x)/2))/64)) + (405*a^8*tan(c/2 + (d*x)/2))/(2*((4
05*a^8)/2 - (18225*a^8*tan(c/2 + (d*x)/2))/64))))/(8*d) - (17*a^4*tan(c/2 +
 (d*x)/2)^2 + (376*a^4*tan(c/2 + (d*x)/2)^3)/5 + 184*a^4*tan(c/2 + (d*x)/2)
^4 + (1836*a^4*tan(c/2 + (d*x)/2)^5)/5 + (1312*a^4*tan(c/2 + (d*x)/2)^6)/3
+ 592*a^4*tan(c/2 + (d*x)/2)^7 + 379*a^4*tan(c/2 + (d*x)/2)^8 + 572*a^4*tan
(c/2 + (d*x)/2)^9 + 153*a^4*tan(c/2 + (d*x)/2)^10 + 280*a^4*tan(c/2 + (d*x)
/2)^11 - (346*a^4*tan(c/2 + (d*x)/2)^12)/3 + 4*a^4*tan(c/2 + (d*x)/2)^13 -
74*a^4*tan(c/2 + (d*x)/2)^14 + a^4/3 + 4*a^4*tan(c/2 + (d*x)/2))/(d*(8*tan(
c/2 + (d*x)/2)^3 + 48*tan(c/2 + (d*x)/2)^5 + 120*tan(c/2 + (d*x)/2)^7 + 160
*tan(c/2 + (d*x)/2)^9 + 120*tan(c/2 + (d*x)/2)^11 + 48*tan(c/2 + (d*x)/2)^1
3 + 8*tan(c/2 + (d*x)/2)^15)) + (15*a^4*tan(c/2 + (d*x)/2))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.624 $\int \frac{\cos^6(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=159

$$\frac{\cos^9(c+dx)}{9ad}-\frac{2\cos^7(c+dx)}{7ad}+\frac{\cos^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{16ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{64ad}$$

```
[Out] 3/128*x/a+1/5*cos(d*x+c)^5/a/d-2/7*cos(d*x+c)^7/a/d+1/9*cos(d*x+c)^9/a/d+3/
128*cos(d*x+c)*sin(d*x+c)/a/d+1/64*cos(d*x+c)^3*sin(d*x+c)/a/d-1/16*cos(d*x
+c)^5*sin(d*x+c)/a/d-1/8*cos(d*x+c)^5*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.22, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2568, 2635, 8, 2565, 270}

$$\frac{\cos^9(c+dx)}{9ad}-\frac{2\cos^7(c+dx)}{7ad}+\frac{\cos^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{16ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{64ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(128*a) + Cos[c + d*x]^5/(5*a*d) - (2*Cos[c + d*x]^7)/(7*a*d) + Cos[c
 + d*x]^9/(9*a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(128*a*d) + (Cos[c + d*x]
^3*Sin[c + d*x])/(64*a*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(16*a*d) - (Cos[c
 + d*x]^5*Sin[c + d*x]^3)/(8*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a} - \frac{\int \cos^4(c+dx)\sin^5(c+dx)\,dx}{a}$$

$$= -\frac{\cos^5(c+dx)\sin^3(c+dx)}{8ad} + \frac{3\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{8a} + \frac{\mathrm{Subst}\left(\int x^4$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{16ad} - \frac{\cos^5(c+dx)\sin^3(c+dx)}{8ad} + \frac{\int \cos^4(c+dx)\,dx}{16a} +$$

$$= \frac{\cos^5(c+dx)}{5ad} - \frac{2\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{64ad} -$$

$$= \frac{\cos^5(c+dx)}{5ad} - \frac{2\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} + \frac{3\cos(c+dx)\sin(c+dx)}{128ad} +$$

$$= \frac{3x}{128a} + \frac{\cos^5(c+dx)}{5ad} - \frac{2\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} + \frac{3\cos(c+dx)\sin(c-}{128ad}$$

Mathematica [B] time = 8.41, size = 429, normalized size = 2.70

$$\frac{15120dx\sin\left(\frac{c}{2}\right)-7560\sin\left(\frac{c}{2}+dx\right)+7560\sin\left(\frac{3c}{2}+dx\right)-1680\sin\left(\frac{5c}{2}+3dx\right)+1680\sin\left(\frac{7c}{2}+3dx\right)-2520\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2520*(5*c + 6*d*x)*Cos[c/2] + 7560*Cos[c/2 + d*x] + 7560*Cos[(3*c)/2 + d*x
] + 1680*Cos[(5*c)/2 + 3*d*x] + 1680*Cos[(7*c)/2 + 3*d*x] - 2520*Cos[(7*c)/
2 + 4*d*x] + 2520*Cos[(9*c)/2 + 4*d*x] - 1008*Cos[(9*c)/2 + 5*d*x] - 1008*C
os[(11*c)/2 + 5*d*x] - 180*Cos[(13*c)/2 + 7*d*x] - 180*Cos[(15*c)/2 + 7*d*x
] + 315*Cos[(15*c)/2 + 8*d*x] - 315*Cos[(17*c)/2 + 8*d*x] + 140*Cos[(17*c)/
2 + 9*d*x] + 140*Cos[(19*c)/2 + 9*d*x] + 12600*Sin[c/2] + 12600*c*Sin[c/2]
+ 15120*d*x*Sin[c/2] - 7560*Sin[c/2 + d*x] + 7560*Sin[(3*c)/2 + d*x] - 1680
*Sin[(5*c)/2 + 3*d*x] + 1680*Sin[(7*c)/2 + 3*d*x] - 2520*Sin[(7*c)/2 + 4*d*
x] - 2520*Sin[(9*c)/2 + 4*d*x] + 1008*Sin[(9*c)/2 + 5*d*x] - 1008*Sin[(11*c
)/2 + 5*d*x] + 180*Sin[(13*c)/2 + 7*d*x] - 180*Sin[(15*c)/2 + 7*d*x] + 315*
Sin[(15*c)/2 + 8*d*x] + 315*Sin[(17*c)/2 + 8*d*x] - 140*Sin[(17*c)/2 + 9*d*
x] + 140*Sin[(19*c)/2 + 9*d*x])/(645120*a*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.91, size = 90, normalized size = 0.57

$$\frac{4480\cos\left(dx+c\right)^{9}-11520\cos\left(dx+c\right)^{7}+8064\cos\left(dx+c\right)^{5}+945\,dx+315\left(16\cos\left(dx+c\right)^{7}-24\cos\left(dx+c\right.\right.}{40320\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40320*(4480*cos(d*x + c)^9 - 11520*cos(d*x + c)^7 + 8064*cos(d*x + c)^5 +
 945*d*x + 315*(16*cos(d*x + c)^7 - 24*cos(d*x + c)^5 + 2*cos(d*x + c)^3 +
3*cos(d*x + c))*sin(d*x + c))/(a*d)
```

giac [A] time = 0.21, size = 218, normalized size = 1.37

$$\frac{\frac{945\,(dx+c)}{a}+\frac{2\left(945\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}+8190\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-97650\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+215040\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+106470\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-322560\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40320*(945*(d*x + c)/a + 2*(945*tan(1/2*d*x + 1/2*c)^17 + 8190*tan(1/2*d*
x + 1/2*c)^15 - 97650*tan(1/2*d*x + 1/2*c)^13 + 215040*tan(1/2*d*x + 1/2*c)
^12 + 106470*tan(1/2*d*x + 1/2*c)^11 - 322560*tan(1/2*d*x + 1/2*c)^10 + 451
584*tan(1/2*d*x + 1/2*c)^8 - 106470*tan(1/2*d*x + 1/2*c)^7 - 129024*tan(1/2
*d*x + 1/2*c)^6 + 97650*tan(1/2*d*x + 1/2*c)^5 + 36864*tan(1/2*d*x + 1/2*c)
^4 - 8190*tan(1/2*d*x + 1/2*c)^3 + 9216*tan(1/2*d*x + 1/2*c)^2 - 945*tan(1/
2*d*x + 1/2*c) + 1024)/((tan(1/2*d*x + 1/2*c)^2 + 1)^9*a))/d
```

maple [B] time = 0.30, size = 517, normalized size = 3.25

$$\frac{16}{315ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}+\frac{16\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 16/315/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9-3/64/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*t
an(1/2*d*x+1/2*c)+16/35/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^2
-13/32/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^3+64/35/a/d/(1+tan
(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^4+155/32/a/d/(1+tan(1/2*d*x+1/2*c)^
2)^9*tan(1/2*d*x+1/2*c)^5-32/5/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1
/2*c)^6-169/32/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^7+112/5/a/
d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^8-16/a/d/(1+tan(1/2*d*x+1/2
*c)^2)^9*tan(1/2*d*x+1/2*c)^10+169/32/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/
2*d*x+1/2*c)^11+32/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^12-1
55/32/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^13+13/32/a/d/(1+tan
(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^15+3/64/a/d/(1+tan(1/2*d*x+1/2*c)^2
)^9*tan(1/2*d*x+1/2*c)^17+3/64/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.48, size = 502, normalized size = 3.16

$$-\frac{\frac{\frac{945\sin(dx+c)}{\cos(dx+c)+1}-\frac{9216\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8190\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{36864\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{97650\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{129024\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{106470\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{451584\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{322560\sin(dx}{(\cos(dx+c)+}}{a+\frac{9a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{36a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{84a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{126a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{126a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{84a\sin(dx+c)^{1}}{(\cos(dx+c)+1)^{1}}}}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/20160*((945*sin(d*x + c)/(cos(d*x + c) + 1) - 9216*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 8190*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 36864*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 97650*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 12
```

```
9024*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 106470*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 - 451584*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 322560*sin(d*x +
c)^10/(cos(d*x + c) + 1)^10 - 106470*sin(d*x + c)^11/(cos(d*x + c) + 1)^11
- 215040*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 97650*sin(d*x + c)^13/(cos
(d*x + c) + 1)^13 - 8190*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 945*sin(d*
x + c)^17/(cos(d*x + c) + 1)^17 - 1024)/(a + 9*a*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 36*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 84*a*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 + 126*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 126*a*si
n(d*x + c)^10/(cos(d*x + c) + 1)^10 + 84*a*sin(d*x + c)^12/(cos(d*x + c) +
1)^12 + 36*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 9*a*sin(d*x + c)^16/(c
os(d*x + c) + 1)^16 + a*sin(d*x + c)^18/(cos(d*x + c) + 1)^18) - 945*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 11.41, size = 211, normalized size = 1.33

$$\frac{3\,x}{128\,a}+\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{64}+\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{32}-\frac{155\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32}+\frac{32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}+\frac{169\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{32}-16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{112\tan\cdots}{\cdots}}{a\,d\left(\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^4)/(a + a*sin(c + d*x)),x)
```

```
[Out] (3*x)/(128*a) + ((16*tan(c/2 + (d*x)/2)^2)/35 - (3*tan(c/2 + (d*x)/2))/64 -
 (13*tan(c/2 + (d*x)/2)^3)/32 + (64*tan(c/2 + (d*x)/2)^4)/35 + (155*tan(c/2
 + (d*x)/2)^5)/32 - (32*tan(c/2 + (d*x)/2)^6)/5 - (169*tan(c/2 + (d*x)/2)^7
)/32 + (112*tan(c/2 + (d*x)/2)^8)/5 - 16*tan(c/2 + (d*x)/2)^10 + (169*tan(c
/2 + (d*x)/2)^11)/32 + (32*tan(c/2 + (d*x)/2)^12)/3 - (155*tan(c/2 + (d*x)/
2)^13)/32 + (13*tan(c/2 + (d*x)/2)^15)/32 + (3*tan(c/2 + (d*x)/2)^17)/64 +
16/315)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.625 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=141

$$\frac{\cos^7(c+dx)}{7ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)\cos^5(c+dx)}{8ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{16ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{64ad}-\frac{3\,\mathrm{s}}{}$$

```
[Out] -3/128*x/a-1/5*cos(d*x+c)^5/a/d+1/7*cos(d*x+c)^7/a/d-3/128*cos(d*x+c)*sin(d
*x+c)/a/d-1/64*cos(d*x+c)^3*sin(d*x+c)/a/d+1/16*cos(d*x+c)^5*sin(d*x+c)/a/d
+1/8*cos(d*x+c)^5*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.21, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2565, 14, 2568, 2635, 8}

$$\frac{\cos^7(c+dx)}{7ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)\cos^5(c+dx)}{8ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{16ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{64ad}-\frac{3\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(128*a) - Cos[c + d*x]^5/(5*a*d) + Cos[c + d*x]^7/(7*a*d) - (3*Cos[c
 + d*x]*Sin[c + d*x])/(128*a*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(64*a*d) +
(Cos[c + d*x]^5*Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^5*Sin[c + d*x]^3)/(8
*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^4(c+dx)\sin^3(c+dx)\,dx}{a} - \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a}$$
$$= \frac{\cos^5(c+dx)\sin^3(c+dx)}{8ad} - \frac{3\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{8a} - \frac{\text{Subst}\left(\int x^4\left(1\right.\right.}{}$$
$$= \frac{\cos^5(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^5(c+dx)\sin^3(c+dx)}{8ad} - \frac{\int \cos^4(c+dx)\,dx}{16a} - \frac{\text{Su}}{}$$
$$= -\frac{\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{64ad} + \frac{\cos^5(c+dx)\sin(c}{16ad}$$
$$= -\frac{\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} - \frac{3\cos(c+dx)\sin(c+dx)}{128ad} - \frac{\cos^3(c+dx)\sin(c}{64ad}$$
$$= -\frac{3x}{128a} - \frac{\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} - \frac{3\cos(c+dx)\sin(c+dx)}{128ad} - \frac{\cos^3(c+}{}$$

Mathematica [B] time = 8.73, size = 375, normalized size = 2.66

$$\frac{-1680dx\sin\left(\frac{c}{2}\right) + 1680\sin\left(\frac{c}{2}+dx\right) - 1680\sin\left(\frac{3c}{2}+dx\right) + 560\sin\left(\frac{5c}{2}+3dx\right) - 560\sin\left(\frac{7c}{2}+3dx\right) + 280\sin\left(\frac{7}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (1680*(c - d*x)*Cos[c/2] - 1680*Cos[c/2 + d*x] - 1680*Cos[(3*c)/2 + d*x] -
560*Cos[(5*c)/2 + 3*d*x] - 560*Cos[(7*c)/2 + 3*d*x] + 280*Cos[(7*c)/2 + 4*d
*x] - 280*Cos[(9*c)/2 + 4*d*x] + 112*Cos[(9*c)/2 + 5*d*x] + 112*Cos[(11*c)/
2 + 5*d*x] + 80*Cos[(13*c)/2 + 7*d*x] + 80*Cos[(15*c)/2 + 7*d*x] - 35*Cos[(
15*c)/2 + 8*d*x] + 35*Cos[(17*c)/2 + 8*d*x] - 3360*Sin[c/2] + 1680*c*Sin[c/
2] - 1680*d*x*Sin[c/2] + 1680*Sin[c/2 + d*x] - 1680*Sin[(3*c)/2 + d*x] + 56
0*Sin[(5*c)/2 + 3*d*x] - 560*Sin[(7*c)/2 + 3*d*x] + 280*Sin[(7*c)/2 + 4*d*x
] + 280*Sin[(9*c)/2 + 4*d*x] - 112*Sin[(9*c)/2 + 5*d*x] + 112*Sin[(11*c)/2
+ 5*d*x] - 80*Sin[(13*c)/2 + 7*d*x] + 80*Sin[(15*c)/2 + 7*d*x] - 35*Sin[(15
*c)/2 + 8*d*x] - 35*Sin[(17*c)/2 + 8*d*x])/(71680*a*d*(Cos[c/2] + Sin[c/2])
)
```

fricas [A] time = 0.63, size = 80, normalized size = 0.57

$$\frac{640\,\cos\left(dx+c\right)^{7}-896\,\cos\left(dx+c\right)^{5}-105\,dx-35\left(16\,\cos\left(dx+c\right)^{7}-24\,\cos\left(dx+c\right)^{5}+2\,\cos\left(dx+c\right)^{3}+3\,\cdots}{4480\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4480*(640*cos(d*x + c)^7 - 896*cos(d*x + c)^5 - 105*d*x - 35*(16*cos(d*x
+ c)^7 - 24*cos(d*x + c)^5 + 2*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x + c
))/(a*d)
```

giac [A] time = 0.17, size = 205, normalized size = 1.45

$$-\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+805\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+8960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-11655\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+23485\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+8960\,\tan\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4480*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^15 + 805*tan(1/2*d*x
 + 1/2*c)^13 + 8960*tan(1/2*d*x + 1/2*c)^12 - 11655*tan(1/2*d*x + 1/2*c)^11
 + 23485*tan(1/2*d*x + 1/2*c)^9 + 8960*tan(1/2*d*x + 1/2*c)^8 - 23485*tan(1
/2*d*x + 1/2*c)^7 + 14336*tan(1/2*d*x + 1/2*c)^6 + 11655*tan(1/2*d*x + 1/2*
c)^5 - 1792*tan(1/2*d*x + 1/2*c)^4 - 805*tan(1/2*d*x + 1/2*c)^3 + 2048*tan(
1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 256)/((tan(1/2*d*x + 1/2*c)
^2 + 1)^8*a))/d
```

maple [B] time = 0.28, size = 483, normalized size = 3.43

$$-\frac{4}{35ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{32\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{23\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -4/35/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8+3/64/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*ta
n(1/2*d*x+1/2*c)-32/35/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^2+
23/64/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3+4/5/a/d/(1+tan(1/
2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^4-333/64/a/d/(1+tan(1/2*d*x+1/2*c)^2)^
8*tan(1/2*d*x+1/2*c)^5-32/5/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*
c)^6+671/64/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^7-4/a/d/(1+ta
n(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^8-671/64/a/d/(1+tan(1/2*d*x+1/2*c)
^2)^8*tan(1/2*d*x+1/2*c)^9+333/64/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*
x+1/2*c)^11-4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^12-23/64/a/
d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13-3/64/a/d/(1+tan(1/2*d*x+
1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15-3/64/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.52, size = 461, normalized size = 3.27

$$\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{2048\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1792\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{11655\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14336\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{23485\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{8960\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{23485\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+}{a+\frac{8a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+}}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2240*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 2048*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1792*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 11655*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14336*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 23485*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 - 8960*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 23485*sin(d*x + c)^9/(cos
(d*x + c) + 1)^9 + 11655*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 8960*sin(d
*x + c)^12/(cos(d*x + c) + 1)^12 - 805*sin(d*x + c)^13/(cos(d*x + c) + 1)^1
3 - 105*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 256)/(a + 8*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 28*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 56*a*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8 + 56*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a*sin(d*x + c)^12/(cos(
d*x + c) + 1)^12 + 8*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + a*sin(d*x +
```

```
c)^16/(cos(d*x + c) + 1)^16) - 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/
a)/d
```

mupad [B] time = 11.62, size = 199, normalized size = 1.41

$$-\frac{3\,x}{128\,a}-\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+\frac{23\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-\frac{333\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64}+\frac{671\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{64}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-\frac{67}{}}{a\,d\left(\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] - (3*x)/(128*a) - ((32*tan(c/2 + (d*x)/2)^2)/35 - (3*tan(c/2 + (d*x)/2))/64
 - (23*tan(c/2 + (d*x)/2)^3)/64 - (4*tan(c/2 + (d*x)/2)^4)/5 + (333*tan(c/2
 + (d*x)/2)^5)/64 + (32*tan(c/2 + (d*x)/2)^6)/5 - (671*tan(c/2 + (d*x)/2)^7
)/64 + 4*tan(c/2 + (d*x)/2)^8 + (671*tan(c/2 + (d*x)/2)^9)/64 - (333*tan(c/
2 + (d*x)/2)^11)/64 + 4*tan(c/2 + (d*x)/2)^12 + (23*tan(c/2 + (d*x)/2)^13)/
64 + (3*tan(c/2 + (d*x)/2)^15)/64 + 4/35)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^8
)
```

sympy [A] time = 122.27, size = 3580, normalized size = 25.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-105*d*x*tan(c/2 + d*x/2)**16/(4480*a*d*tan(c/2 + d*x/2)**16 + 3
5840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*
d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/
2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)
**2 + 4480*a*d) - 840*d*x*tan(c/2 + d*x/2)**14/(4480*a*d*tan(c/2 + d*x/2)**
16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*tan(c/2 + d*x/2)**12 + 250
880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*
tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 +
d*x/2)**2 + 4480*a*d) - 2940*d*x*tan(c/2 + d*x/2)**12/(4480*a*d*tan(c/2 + d
*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*tan(c/2 + d*x/2)**1
2 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2 + d*x/2)**8 + 2508
80*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)**4 + 35840*a*d*tan
(c/2 + d*x/2)**2 + 4480*a*d) - 5880*d*x*tan(c/2 + d*x/2)**10/(4480*a*d*tan(
c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*tan(c/2 + d*
x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2 + d*x/2)**8
```

```
 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)**4 + 35840*
a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 7350*d*x*tan(c/2 + d*x/2)**8/(4480*a*
d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*tan(c/
2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2 + d*x
/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)**4 +
35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 5880*d*x*tan(c/2 + d*x/2)**6/(4
480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*
tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2
 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)
**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 2940*d*x*tan(c/2 + d*x/2)
**4/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 12544
0*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*t
an(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 +
d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 840*d*x*tan(c/2 + d
*x/2)**2/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 +
125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*
a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c
/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 105*d*x/(4480*
a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*tan(
c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2 + d
*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)**4
+ 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 210*tan(c/2 + d*x/2)**15/(448
0*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*ta
n(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2 +
 d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)**
4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 1610*tan(c/2 + d*x/2)**13/(
4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d
*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/
2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2
)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 17920*tan(c/2 + d*x/2)**
12/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440
*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*ta
n(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d
*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) + 23310*tan(c/2 + d*x/
2)**11/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 12
5440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*
d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2
 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 46970*tan(c/2 +
d*x/2)**9/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 +
 125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600
*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(
c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 17920*tan(c/2
 + d*x/2)**8/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**1
4 + 125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313
600*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*t
```

an(c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) + 46970*tan(
c/2 + d*x/2)**7/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)
14 + 125440*a*d*tan(c/2 + d*x/2)12 + 250880*a*d*tan(c/2 + d*x/2)**10 +
313600*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*
d*tan(c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 28672*t
an(c/2 + d*x/2)**6/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x
/2)**14 + 125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10
 + 313600*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440
*a*d*tan(c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 2331
0*tan(c/2 + d*x/2)**5/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 +
d*x/2)**14 + 125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)*
*10 + 313600*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125
440*a*d*tan(c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) + 3
584*tan(c/2 + d*x/2)**4/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2
+ d*x/2)**14 + 125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2
)**10 + 313600*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 1
25440*a*d*tan(c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) +
 1610*tan(c/2 + d*x/2)**3/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/
2 + d*x/2)**14 + 125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x
/2)**10 + 313600*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 +
 125440*a*d*tan(c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d)
 - 4096*tan(c/2 + d*x/2)**2/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(
c/2 + d*x/2)**14 + 125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d
*x/2)**10 + 313600*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6
 + 125440*a*d*tan(c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*
d) + 210*tan(c/2 + d*x/2)/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/
2 + d*x/2)**14 + 125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x
/2)**10 + 313600*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 +
 125440*a*d*tan(c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d)
 - 512/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 12
5440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*
d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2
 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d), Ne(d, 0)), (x*sin
(c)**3*cos(c)**6/(a*sin(c) + a), True))

3.626 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=115

$$-\frac{\cos^7(c+dx)}{7ad}+\frac{\cos^5(c+dx)}{5ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{24ad}+\frac{\sin(c+dx)\cos(c+dx)}{16ad}+\frac{x}{16a}$$

```
[Out] 1/16*x/a+1/5*cos(d*x+c)^5/a/d-1/7*cos(d*x+c)^7/a/d+1/16*cos(d*x+c)*sin(d*x+
c)/a/d+1/24*cos(d*x+c)^3*sin(d*x+c)/a/d-1/6*cos(d*x+c)^5*sin(d*x+c)/a/d
```

Rubi [A] time = 0.18, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2568, 2635, 8, 2565, 14}

$$-\frac{\cos^7(c+dx)}{7ad}+\frac{\cos^5(c+dx)}{5ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{24ad}+\frac{\sin(c+dx)\cos(c+dx)}{16ad}+\frac{x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(16*a) + Cos[c + d*x]^5/(5*a*d) - Cos[c + d*x]^7/(7*a*d) + (Cos[c + d*x]*
Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) - (Cos[c +
d*x]^5*Sin[c + d*x])/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
```

```
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^4(c+dx)\sin^3(c+dx)\,dx}{a}$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{6ad} + \frac{\int \cos^4(c+dx)\,dx}{6a} + \frac{\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx, x,\right.}{ad}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{24ad} - \frac{\cos^5(c+dx)\sin(c+dx)}{6ad} + \frac{\int \cos^2(c+dx)\,dx}{8a} + \frac{\text{S}}{}$$

$$= \frac{\cos^5(c+dx)}{5ad} - \frac{\cos^7(c+dx)}{7ad} + \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(c}{24ad}$$

$$= \frac{x}{16a} + \frac{\cos^5(c+dx)}{5ad} - \frac{\cos^7(c+dx)}{7ad} + \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)}{24}$$

Mathematica [B] time = 11.39, size = 715, normalized size = 6.22

$$\frac{5\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{64d(a\sin(c+dx)+a)} - \frac{-\frac{50\sin(c)\sin(dx)}{d} + \frac{10\sin(3c)\sin(3dx)}{d} - \frac{2\sin(5c)\sin(5dx)}{d} + \frac{50\cos(c}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/160*(30*x + (50*Cos[c]*Cos[d*x])/d - (10*Cos[3*c]*Cos[3*d*x])/d + (2*Cos
[5*c]*Cos[5*d*x])/d - (20*Cos[2*d*x]*Sin[2*c])/d + (5*Cos[4*d*x]*Sin[4*c])/
d - (50*Sin[c]*Sin[d*x])/d - (20*Cos[2*c]*Sin[2*d*x])/d + (10*Sin[3*c]*Sin[
3*d*x])/d + (5*Cos[4*c]*Sin[4*d*x])/d - (2*Sin[5*c]*Sin[5*d*x])/d - (10*Sin
[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))
/a + (x + (Cos[c]*Cos[d*x])/d - (Sin[c]*Sin[d*x])/d - Sin[(d*x)/2]/(d*(Cos[
c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(16*a) - (-6*x - (
9*Cos[c]*Cos[d*x])/d + (Cos[3*c]*Cos[3*d*x])/d + (3*Cos[2*d*x]*Sin[2*c])/d
+ (9*Sin[c]*Sin[d*x])/d + (3*Cos[2*c]*Sin[2*d*x])/d - (Sin[3*c]*Sin[3*d*x])
/d + (3*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])))/(48*a) - (-420*x - (735*Cos[c]*Cos[d*x])/d + (175*Cos[3*c]*Cos[
3*d*x])/d - (63*Cos[5*c]*Cos[5*d*x])/d + (15*Cos[7*c]*Cos[7*d*x])/d + (315*
Cos[2*d*x]*Sin[2*c])/d - (105*Cos[4*d*x]*Sin[4*c])/d + (35*Cos[6*d*x]*Sin[6
*c])/d + (735*Sin[c]*Sin[d*x])/d + (315*Cos[2*c]*Sin[2*d*x])/d - (175*Sin[3
*c]*Sin[3*d*x])/d - (105*Cos[4*c]*Sin[4*d*x])/d + (63*Sin[5*c]*Sin[5*d*x])/
d + (35*Cos[6*c]*Sin[6*d*x])/d - (15*Sin[7*c]*Sin[7*d*x])/d + (105*Sin[(d*x
)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(672
0*a) + (5*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(64*d*(a
+ a*Sin[c + d*x]))
```

fricas [A] time = 0.72, size = 70, normalized size = 0.61

$$-\frac{240\cos\left(dx+c\right)^{7}-336\cos\left(dx+c\right)^{5}-105\,dx+35\left(8\cos\left(dx+c\right)^{5}-2\cos\left(dx+c\right)^{3}-3\cos\left(dx+c\right)\right)\sin\left(dx\right.}{1680\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/1680*(240*cos(d*x + c)^7 - 336*cos(d*x + c)^5 - 105*d*x + 35*(8*cos(d*x
+ c)^5 - 2*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c))/(a*d)
```

giac [A] time = 0.16, size = 179, normalized size = 1.56

$$\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-1540\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+3360\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+1085\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-3360\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+6720\tan\left(\frac{1}{2}\,dx\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right.}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1680*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^13 - 1540*tan(1/2*d*x
 + 1/2*c)^11 + 3360*tan(1/2*d*x + 1/2*c)^10 + 1085*tan(1/2*d*x + 1/2*c)^9 -
```

```
 3360*tan(1/2*d*x + 1/2*c)^8 + 6720*tan(1/2*d*x + 1/2*c)^6 - 1085*tan(1/2*d
*x + 1/2*c)^5 - 1344*tan(1/2*d*x + 1/2*c)^4 + 1540*tan(1/2*d*x + 1/2*c)^3 +
 672*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 96)/((tan(1/2*d*x
+ 1/2*c)^2 + 1)^7*a))/d
```

maple [B] time = 0.27, size = 415, normalized size = 3.61

$$\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{11\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{4\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{31\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{4}{ad(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/8/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13-11/6/a/d/(1+tan(1/
2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11+4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7*ta
n(1/2*d*x+1/2*c)^10+31/24/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)
^9-4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8+8/a/d/(1+tan(1/2*d
*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6-31/24/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7*ta
n(1/2*d*x+1/2*c)^5-8/5/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4+
11/6/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^3+4/5/a/d/(1+tan(1/2
*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2-1/8/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7*ta
n(1/2*d*x+1/2*c)+4/35/a/d/(1+tan(1/2*d*x+1/2*c)^2)^7+1/8/a/d*arctan(tan(1/2
*d*x+1/2*c))
```

maxima [B] time = 0.56, size = 400, normalized size = 3.48

$$-\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{672\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1540\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1344\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1085\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{6720\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{3360\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{1085\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{3360\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{1540}{(\cos}}{a+\frac{7a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{21a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{21a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{7a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 672*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 1540*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1344*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 1085*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 6720*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 3360*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8 - 1085*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 3360*sin(d*x + c)^10/(cos(d*
x + c) + 1)^10 + 1540*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 105*sin(d*x +
 c)^13/(cos(d*x + c) + 1)^13 - 96)/(a + 7*a*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 21*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 35*a*sin(d*x + c)^6/(cos(
```

```
d*x + c) + 1)^6 + 35*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 21*a*sin(d*x +
 c)^10/(cos(d*x + c) + 1)^10 + 7*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 +
a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14) - 105*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a)/d
```

mupad [B] time = 12.75, size = 172, normalized size = 1.50

$$\frac{x}{16\,a}+\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}-\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{6}+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{31\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-\frac{31\,\tan\left(\cdots\right.}{24\cdots}}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] x/(16*a) + ((4*tan(c/2 + (d*x)/2)^2)/5 - tan(c/2 + (d*x)/2)/8 + (11*tan(c/2
 + (d*x)/2)^3)/6 - (8*tan(c/2 + (d*x)/2)^4)/5 - (31*tan(c/2 + (d*x)/2)^5)/2
4 + 8*tan(c/2 + (d*x)/2)^6 - 4*tan(c/2 + (d*x)/2)^8 + (31*tan(c/2 + (d*x)/2
)^9)/24 + 4*tan(c/2 + (d*x)/2)^10 - (11*tan(c/2 + (d*x)/2)^11)/6 + tan(c/2
+ (d*x)/2)^13/8 + 4/35)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [A] time = 78.79, size = 2773, normalized size = 24.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((105*d*x*tan(c/2 + d*x/2)**14/(1680*a*d*tan(c/2 + d*x/2)**14 + 11
760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*t
an(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*
x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 735*d*x*tan(c/2 + d*x
/2)**12/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 3
5280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*t
an(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*
x/2)**2 + 1680*a*d) + 2205*d*x*tan(c/2 + d*x/2)**10/(1680*a*d*tan(c/2 + d*x
/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 +
 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*
tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 3675*d*x*
tan(c/2 + d*x/2)**8/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*
x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 +
 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*
tan(c/2 + d*x/2)**2 + 1680*a*d) + 3675*d*x*tan(c/2 + d*x/2)**6/(1680*a*d*ta
```

n(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 2205*d*x*tan(c/2 + d*x/2)**4/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 735*d*x*tan(c/2 + d*x/2)**2/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 105*d*x/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 210*tan(c/2 + d*x/2)**13/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 3080*tan(c/2 + d*x/2)**11/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 6720*tan(c/2 + d*x/2)**10/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 2170*tan(c/2 + d*x/2)**9/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 6720*tan(c/2 + d*x/2)**8/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 13440*tan(c/2 + d*x/2)**6/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 2170*tan(c/2 + d*x/2)**5/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 2688*tan(c/2 + d*x/2)**4/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 3080*tan(c/2 + d*x/2)**3/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 +

1680*a*d) + 1344*tan(c/2 + d*x/2)**2/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 210*tan(c/2 + d*x/2)/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 192/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**6/(a*sin(c) + a), True))

3.627 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{24ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

```
[Out] -1/16*x/a-1/5*cos(d*x+c)^5/a/d-1/16*cos(d*x+c)*sin(d*x+c)/a/d-1/24*cos(d*x+
c)^3*sin(d*x+c)/a/d+1/6*cos(d*x+c)^5*sin(d*x+c)/a/d
```

Rubi [A] time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2565, 30, 2568, 2635, 8}

$$-\frac{\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{24ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -x/(16*a) - Cos[c + d*x]^5/(5*a*d) - (Cos[c + d*x]*Sin[c + d*x])/(16*a*d) -
 (Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(6*
a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
```

```
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^4(c+dx)\sin(c+dx)\,dx}{a} - \frac{\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{a}$$
$$= \frac{\cos^5(c+dx)\sin(c+dx)}{6ad} - \frac{\int \cos^4(c+dx)\,dx}{6a} - \frac{\text{Subst}\left(\int x^4\,dx, x, \cos(c+dx)\right)}{ad}$$
$$= -\frac{\cos^5(c+dx)}{5ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{24ad} + \frac{\cos^5(c+dx)\sin(c+dx)}{6ad} - \frac{\int \cos^2(}{}$$
$$= -\frac{\cos^5(c+dx)}{5ad} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{24ad} + \frac{\cos^5(c+}{}$$
$$= -\frac{x}{16a} - \frac{\cos^5(c+dx)}{5ad} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{24ad} + \frac{\mathrm{co}}{}$$

Mathematica [B] time = 5.08, size = 377, normalized size = 3.89

$$-\frac{120dx\sin\left(\frac{c}{2}\right) - 120\sin\left(\frac{c}{2}+dx\right) + 120\sin\left(\frac{3c}{2}+dx\right) + 15\sin\left(\frac{3c}{2}+2dx\right) + 15\sin\left(\frac{5c}{2}+2dx\right) - 60\sin\left(\frac{5c}{2}+3dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/1920*(-30*(5*c - 4*d*x)*Cos[c/2] + 120*Cos[c/2 + d*x] + 120*Cos[(3*c)/2
+ d*x] + 15*Cos[(3*c)/2 + 2*d*x] - 15*Cos[(5*c)/2 + 2*d*x] + 60*Cos[(5*c)/2
 + 3*d*x] + 60*Cos[(7*c)/2 + 3*d*x] - 15*Cos[(7*c)/2 + 4*d*x] + 15*Cos[(9*c
)/2 + 4*d*x] + 12*Cos[(9*c)/2 + 5*d*x] + 12*Cos[(11*c)/2 + 5*d*x] - 5*Cos[(
11*c)/2 + 6*d*x] + 5*Cos[(13*c)/2 + 6*d*x] + 300*Sin[c/2] - 150*c*Sin[c/2]
+ 120*d*x*Sin[c/2] - 120*Sin[c/2 + d*x] + 120*Sin[(3*c)/2 + d*x] + 15*Sin[(
3*c)/2 + 2*d*x] + 15*Sin[(5*c)/2 + 2*d*x] - 60*Sin[(5*c)/2 + 3*d*x] + 60*Si
n[(7*c)/2 + 3*d*x] - 15*Sin[(7*c)/2 + 4*d*x] - 15*Sin[(9*c)/2 + 4*d*x] - 12
*Sin[(9*c)/2 + 5*d*x] + 12*Sin[(11*c)/2 + 5*d*x] - 5*Sin[(11*c)/2 + 6*d*x]
- 5*Sin[(13*c)/2 + 6*d*x])/(a*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.85, size = 60, normalized size = 0.62

$$-\frac{48\,\cos\left(dx+c\right)^{5}+15\,dx-5\,\left(8\,\cos\left(dx+c\right)^{5}-2\,\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(48*cos(d*x + c)^5 + 15*d*x - 5*(8*cos(d*x + c)^5 - 2*cos(d*x + c)^3
 - 3*cos(d*x + c))*sin(d*x + c))/(a*d)
```

giac [B] time = 0.15, size = 179, normalized size = 1.85

$$-\frac{\frac{15\,(dx+c)}{a}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-235\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+390\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}\cdots}{\cdots\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}\cdots}}{240\,d}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(15*(d*x + c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^11 + 240*tan(1/2*d*x +
1/2*c)^10 - 235*tan(1/2*d*x + 1/2*c)^9 + 240*tan(1/2*d*x + 1/2*c)^8 + 390*t
an(1/2*d*x + 1/2*c)^7 + 480*tan(1/2*d*x + 1/2*c)^6 - 390*tan(1/2*d*x + 1/2*
c)^5 + 480*tan(1/2*d*x + 1/2*c)^4 + 235*tan(1/2*d*x + 1/2*c)^3 + 48*tan(1/2
*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 48)/((tan(1/2*d*x + 1/2*c)^2 +
1)^6*a))/d
```

maple [B] time = 0.21, size = 415, normalized size = 4.28

$$-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}-\frac{2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}+\frac{47\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}-\frac{2\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}-\frac{1\cdots}{4ad\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/8/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11-2/a/d/(1+tan(1/2*
d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10+47/24/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*
tan(1/2*d*x+1/2*c)^9-2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8-
13/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7-4/a/d/(1+tan(1/2*d
*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6+13/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan
(1/2*d*x+1/2*c)^5-4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4-47/
24/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3-2/5/a/d/(1+tan(1/2*d
*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2+1/8/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(
1/2*d*x+1/2*c)-2/5/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6-1/8/a/d*arctan(tan(1/2*d*
x+1/2*c))
```

maxima [B] time = 0.52, size = 379, normalized size = 3.91

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{235\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{480\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{390\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{480\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{390\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{240\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{235\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{240\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-}{a+\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 235*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 480*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 390*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 480*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6 - 390*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 240*
sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 235*sin(d*x + c)^9/(cos(d*x + c) + 1)
^9 - 240*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 15*sin(d*x + c)^11/(cos(d*
x + c) + 1)^11 - 48)/(a + 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a*sin(d*x + c)^10/(cos(d*x
 + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 15*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 12.49, size = 173, normalized size = 1.78

$$-\frac{x}{16a}-\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{8}+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-\frac{47\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{24}+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+\frac{13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{4}+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-\frac{13\tan(}{}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] - x/(16*a) - ((2*tan(c/2 + (d*x)/2)^2)/5 - tan(c/2 + (d*x)/2)/8 + (47*tan(c
/2 + (d*x)/2)^3)/24 + 4*tan(c/2 + (d*x)/2)^4 - (13*tan(c/2 + (d*x)/2)^5)/4
+ 4*tan(c/2 + (d*x)/2)^6 + (13*tan(c/2 + (d*x)/2)^7)/4 + 2*tan(c/2 + (d*x)/
2)^8 - (47*tan(c/2 + (d*x)/2)^9)/24 + 2*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (
d*x)/2)^11/8 + 2/5)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [A] time = 49.69, size = 2307, normalized size = 23.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-15*d*x*tan(c/2 + d*x/2)**12/(240*a*d*tan(c/2 + d*x/2)**12 + 144
0*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/
2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2
 + 240*a*d) - 90*d*x*tan(c/2 + d*x/2)**10/(240*a*d*tan(c/2 + d*x/2)**12 + 1
440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(
c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)*
*2 + 240*a*d) - 225*d*x*tan(c/2 + d*x/2)**8/(240*a*d*tan(c/2 + d*x/2)**12 +
 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*ta
n(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2
)**2 + 240*a*d) - 300*d*x*tan(c/2 + d*x/2)**6/(240*a*d*tan(c/2 + d*x/2)**12
 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*
tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x
/2)**2 + 240*a*d) - 225*d*x*tan(c/2 + d*x/2)**4/(240*a*d*tan(c/2 + d*x/2)**
12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*
d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d
*x/2)**2 + 240*a*d) - 90*d*x*tan(c/2 + d*x/2)**2/(240*a*d*tan(c/2 + d*x/2)*
*12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a
*d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 +
d*x/2)**2 + 240*a*d) - 15*d*x/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(
c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)
**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d
) - 30*tan(c/2 + d*x/2)**11/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/
2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**
6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d)
- 480*tan(c/2 + d*x/2)**10/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2
 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6
 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) +
 470*tan(c/2 + d*x/2)**9/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 +
 d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 +
 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 4
```

```
80*tan(c/2 + d*x/2)**8/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d
*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3
600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 780
*tan(c/2 + d*x/2)**7/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x
/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 360
0*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 960*t
an(c/2 + d*x/2)**6/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2
)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*
a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) + 780*tan
(c/2 + d*x/2)**5/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)*
*10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*
d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 960*tan(c
/2 + d*x/2)**4/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**1
0 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d*
tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 470*tan(c/2
 + d*x/2)**3/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**10
+ 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d*ta
n(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 96*tan(c/2 +
d*x/2)**2/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3
600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c
/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) + 30*tan(c/2 + d*x
/2)/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*
d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d
*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 96/(240*a*d*tan(c/2 +
d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 +
 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan
(c/2 + d*x/2)**2 + 240*a*d), Ne(d, 0)), (x*sin(c)*cos(c)**6/(a*sin(c) + a),
 True))
```

3.628 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{3\sin(c+dx)\cos(c+dx)}{8ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{3x}{8a}$$

```
[Out] -3/8*x/a-arctanh(cos(d*x+c))/a/d+cos(d*x+c)/a/d+1/3*cos(d*x+c)^3/a/d-3/8*co
s(d*x+c)*sin(d*x+c)/a/d-1/4*cos(d*x+c)^3*sin(d*x+c)/a/d
```

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2592, 302, 206, 2635, 8}

$$\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{3\sin(c+dx)\cos(c+dx)}{8ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(8*a) - ArcTanh[Cos[c + d*x]]/(a*d) + Cos[c + d*x]/(a*d) + Cos[c + d
*x]^3/(3*a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) - (Cos[c + d*x]^3*Sin
[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^4(c+dx)\,dx}{a}+\frac{\int \cos^3(c+dx)\cot(c+dx)\,dx}{a}$$
$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}-\frac{3\int \cos^2(c+dx)\,dx}{4a}-\frac{\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,x,\cos(c+$$
$$= -\frac{3\cos(c+dx)\sin(c+dx)}{8ad}-\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}-\frac{3\int 1\,dx}{8a}-\frac{\text{Subst}\left(\int\left(-\right.\right.$$
$$= -\frac{3x}{8a}+\frac{\cos(c+dx)}{ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{3\cos(c+dx)\sin(c+dx)}{8ad}-\frac{\cos^3(c+dx)\,\text{s}}{4ad}$$
$$= -\frac{3x}{8a}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\cos(c+dx)}{ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{3\cos(c+dx)\sin(c}{8ad}$$

Mathematica [A] time = 0.37, size = 86, normalized size = 0.85

$$\frac{120\cos(c+dx)+8\cos(3(c+dx))-3\left(8\sin(2(c+dx))+\sin(4(c+dx))+4\left(-8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+8\log\left(\text{c}\right.\right.\right.}{96ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (120*Cos[c + d*x] + 8*Cos[3*(c + d*x)] - 3*(4*(3*c + 3*d*x + 8*Log[Cos[(c +
 d*x)/2]] - 8*Log[Sin[(c + d*x)/2]]) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)
]))/(96*a*d)
```

fricas [A] time = 0.69, size = 84, normalized size = 0.83

$$\frac{8\,\cos\left(dx+c\right)^{3}-9\,dx-3\left(2\,\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+24\,\cos\left(dx+c\right)-12\,\log\left(\frac{1}{2}\,\cos\left(dx+\right.\right.}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*cos(d*x + c)^3 - 9*d*x - 3*(2*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(
d*x + c) + 24*cos(d*x + c) - 12*log(1/2*cos(d*x + c) + 1/2) + 12*log(-1/2*c
os(d*x + c) + 1/2))/(a*d)
```

giac [A] time = 0.16, size = 143, normalized size = 1.42

$$-\frac{\frac{9\,(dx+c)}{a}-\frac{24\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+96\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+8}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(9*(d*x + c)/a - 24*log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(15*tan(1/2*
d*x + 1/2*c)^7 + 48*tan(1/2*d*x + 1/2*c)^6 - 9*tan(1/2*d*x + 1/2*c)^5 + 96*
tan(1/2*d*x + 1/2*c)^4 + 9*tan(1/2*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)
^2 - 15*tan(1/2*d*x + 1/2*c) + 32)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

maple [B] time = 0.41, size = 296, normalized size = 2.93

$$\frac{5\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{4\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{8\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{3\left(\right.}{4ad\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 5/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+4/a/d/(1+tan(1/2*d*
x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6-3/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1
/2*d*x+1/2*c)^5+8/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4+3/4/a
/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3+20/3/a/d/(1+tan(1/2*d*x+
1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2-5/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2
*d*x+1/2*c)+8/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4-3/4/a/d*arctan(tan(1/2*d*x+1
/2*c))+1/a/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.54, size = 280, normalized size = 2.77

$$-\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{80\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{96\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{48\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-32}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{12\log\left(\frac{\text{si}}{\text{cos}}\right.}{a}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 80*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 9*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 96*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 9*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 48*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 32)/(a + 4*
a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin(d*x + c)^8/(cos(d*x
+ c) + 1)^8) + 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 12*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 10.43, size = 225, normalized size = 2.23

$$\frac{3\,\mathrm{atan}\left(\frac{9}{16\left(\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}+\frac{3}{2}\right)}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\left(\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}+\frac{3}{2}\right)}\right)}{4\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d}+\frac{\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4}+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4}+8}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+4\,a\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*atan(9/(16*((9*tan(c/2 + (d*x)/2))/16 + 3/2)) - (3*tan(c/2 + (d*x)/2))/(
2*((9*tan(c/2 + (d*x)/2))/16 + 3/2))))/(4*a*d) + log(tan(c/2 + (d*x)/2))/(a
*d) + ((20*tan(c/2 + (d*x)/2)^2)/3 - (5*tan(c/2 + (d*x)/2))/4 + (3*tan(c/2
+ (d*x)/2)^3)/4 + 8*tan(c/2 + (d*x)/2)^4 - (3*tan(c/2 + (d*x)/2)^5)/4 + 4*t
```

```
an(c/2 + (d*x)/2)^6 + (5*tan(c/2 + (d*x)/2)^7)/4 + 8/3)/(d*(a + 4*a*tan(c/2
 + (d*x)/2)^2 + 6*a*tan(c/2 + (d*x)/2)^4 + 4*a*tan(c/2 + (d*x)/2)^6 + a*tan
(c/2 + (d*x)/2)^8))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^6(c+dx)\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**6*csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

3.629 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}-\frac{3\cot(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot(c+dx)}{2ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{3x}{2a}$$

```
[Out] -3/2*x/a+arctanh(cos(d*x+c))/a/d-cos(d*x+c)/a/d-1/3*cos(d*x+c)^3/a/d-3/2*co
t(d*x+c)/a/d+1/2*cos(d*x+c)^2*cot(d*x+c)/a/d
```

Rubi [A] time = 0.16, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2839, 2591, 288, 321, 203, 2592, 302, 206}

$$-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}-\frac{3\cot(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot(c+dx)}{2ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(2*a) + ArcTanh[Cos[c + d*x]]/(a*d) - Cos[c + d*x]/(a*d) - Cos[c + d
*x]^3/(3*a*d) - (3*Cot[c + d*x])/(2*a*d) + (Cos[c + d*x]^2*Cot[c + d*x])/(2
*a*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^3(c+dx)\cot(c+dx)\,dx}{a} + \frac{\int \cos^2(c+dx)\cot^2(c+dx)\,dx}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{ad}$$

$$= \frac{\cos^2(c+dx)\cot(c+dx)}{2ad} + \frac{\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \cos(c+dx)\right)}{ad} - \frac{3\,\text{S}}{}$$

$$= -\frac{\cos(c+dx)}{ad} - \frac{\cos^3(c+dx)}{3ad} - \frac{3\cot(c+dx)}{2ad} + \frac{\cos^2(c+dx)\cot(c+dx)}{2ad} + \frac{\text{Su}}{}$$

$$= -\frac{3x}{2a} + \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\cos(c+dx)}{ad} - \frac{\cos^3(c+dx)}{3ad} - \frac{3\cot(c+dx)}{2ad} + \frac{\text{c}}{}$$

Mathematica [A] time = 0.79, size = 122, normalized size = 1.28

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)^2\left(27\cos(c+dx)+(2\sin(c+dx)-3)\cos(3(c+dx))+6\sin(c+dx)\left(5\,\text{co}\right.\right.}{48ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/48*((1 + Cot[(c + d*x)/2])^2*(27*Cos[c + d*x] + 6*(6*c + 6*d*x + 5*Cos[c
 + d*x] - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]])*Sin[c + d*x] +
 Cos[3*(c + d*x)]*(-3 + 2*Sin[c + d*x]))*Tan[(c + d*x)/2])/(a*d*(1 + Sin[c
+ d*x]))
```

fricas [A] time = 0.63, size = 104, normalized size = 1.09

$$\frac{3\cos(dx+c)^3 - \left(2\cos(dx+c)^3 + 9\,dx + 6\cos(dx+c)\right)\sin(dx+c) + 3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) -}{6\,ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cos(d*x + c)^3 - (2*cos(d*x + c)^3 + 9*d*x + 6*cos(d*x + c))*sin(d*x
 + c) + 3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*log(-1/2*cos(d*x + c
) + 1/2)*sin(d*x + c) - 9*cos(d*x + c))/(a*d*sin(d*x + c))
```

giac [A] time = 0.17, size = 147, normalized size = 1.55

$$-\frac{\frac{9\,(dx+c)}{a}+\frac{6\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{3\left(2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)/a + 6*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3*tan(1/2*d*x +
1/2*c)/a - 3*(2*tan(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)) - 2*(3*t
an(1/2*d*x + 1/2*c)^5 - 12*tan(1/2*d*x + 1/2*c)^4 - 12*tan(1/2*d*x + 1/2*c)
^2 - 3*tan(1/2*d*x + 1/2*c) - 8)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

maple [B] time = 0.44, size = 230, normalized size = 2.42

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{4\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{4\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)+1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2
*c)^5-4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4-4/a/d/(1+tan(1/
2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan
(1/2*d*x+1/2*c)-8/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3-3/a/d*arctan(tan(1/2*d*x
+1/2*c))-1/2/a/d/tan(1/2*d*x+1/2*c)-1/a/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.49, size = 277, normalized size = 2.92

$$-\frac{\frac{\frac{16\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{15\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{24\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{9\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{24\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{3\,\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+3}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3\,a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}+\frac{18\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*((16*sin(d*x + c)/(cos(d*x + c) + 1) + 15*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 24*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9*sin(d*x + c)^4/(cos(d*
```

```
x + c) + 1)^4 + 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 3)/(a*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 3*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7) + 18*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a + 6*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - 3*sin(d*x + c)/(a*(cos(
d*x + c) + 1)))/d
```

mupad [B] time = 9.03, size = 229, normalized size = 2.41

$$\frac{3\,\mathrm{atan}\left(\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-6}+\frac{9}{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-6}\right)}{a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+}{d\left(2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*atan((6*tan(c/2 + (d*x)/2))/(9*tan(c/2 + (d*x)/2) - 6) + 9/(9*tan(c/2 +
(d*x)/2) - 6)))/(a*d) - log(tan(c/2 + (d*x)/2))/(a*d) - ((16*tan(c/2 + (d*x
)/2))/3 + 5*tan(c/2 + (d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^3 + 3*tan(c/2 + (d*
x)/2)^4 + 8*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6 + 1)/(d*(2*a*tan(c/
2 + (d*x)/2) + 6*a*tan(c/2 + (d*x)/2)^3 + 6*a*tan(c/2 + (d*x)/2)^5 + 2*a*ta
n(c/2 + (d*x)/2)^7)) + tan(c/2 + (d*x)/2)/(2*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.630 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{3\cos(c+dx)}{2ad}+\frac{3\cot(c+dx)}{2ad}-\frac{\cos^2(c+dx)\cot(c+dx)}{2ad}-\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{3x}{2a}$$

```
[Out] 3/2*x/a+3/2*arctanh(cos(d*x+c))/a/d-3/2*cos(d*x+c)/a/d+3/2*cot(d*x+c)/a/d-1
/2*cos(d*x+c)^2*cot(d*x+c)/a/d-1/2*cos(d*x+c)*cot(d*x+c)^2/a/d
```

Rubi [A] time = 0.16, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2839, 2592, 288, 321, 206, 2591, 203}

$$-\frac{3\cos(c+dx)}{2ad}+\frac{3\cot(c+dx)}{2ad}-\frac{\cos^2(c+dx)\cot(c+dx)}{2ad}-\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(2*a) + (3*ArcTanh[Cos[c + d*x]])/(2*a*d) - (3*Cos[c + d*x])/(2*a*d)
+ (3*Cot[c + d*x])/(2*a*d) - (Cos[c + d*x]^2*Cot[c + d*x])/(2*a*d) - (Cos[c
 + d*x]*Cot[c + d*x]^2)/(2*a*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^2(c+dx)\cot^2(c+dx)\,dx}{a} + \frac{\int \cos(c+dx)\cot^3(c+dx)\,dx}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{ad}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)}{2ad} - \frac{\cos(c+dx)\cot^2(c+dx)}{2ad} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x$$

$$= -\frac{3\cos(c+dx)}{2ad} + \frac{3\cot(c+dx)}{2ad} - \frac{\cos^2(c+dx)\cot(c+dx)}{2ad} - \frac{\cos(c+dx)\cot^2}{2ad}$$

$$= \frac{3x}{2a} + \frac{3\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{3\cos(c+dx)}{2ad} + \frac{3\cot(c+dx)}{2ad} - \frac{\cos^2(c+dx)\,c}{2ad}$$

Mathematica [A] time = 0.48, size = 152, normalized size = 1.43

$$-\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-10\sin(2(c+dx))+\sin(4(c+dx))+12\cos(c+dx)-4\cos(3(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/64*((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(-12*c - 12*d*x + 12*Cos[c +
 d*x] - 4*Cos[3*(c + d*x)] - 12*Log[Cos[(c + d*x)/2]] + 12*Cos[2*(c + d*x)]
*(c + d*x + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + 12*Log[Sin[(c
+ d*x)/2]] - 10*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(a*d*(1 + Sin[c + d*x
]))
```

fricas [A] time = 0.82, size = 126, normalized size = 1.19

$$\frac{6\,dx\cos(dx+c)^2 - 4\cos(dx+c)^3 - 6\,dx + 3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - 3\left(\cos(dx+c)^2-1\right.}{4\left(ad\cos(dx+c)^2-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(6*d*x*cos(d*x + c)^2 - 4*cos(d*x + c)^3 - 6*d*x + 3*(cos(d*x + c)^2 -
1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x +
```

```
c) + 1/2) + 2*(cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c) + 6*cos(d*x +
c))/(a*d*cos(d*x + c)^2 - a*d)
```

giac [A] time = 0.18, size = 167, normalized size = 1.58

$$\frac{\frac{12\,(dx+c)}{a} - \frac{12\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} + \frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2} + \frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 16\,\tan\left(\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + \tan\left(\frac{1}{2}\right.\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(12*(d*x + c)/a - 12*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a*tan(1/2*d*x
+ 1/2*c)^2 - 4*a*tan(1/2*d*x + 1/2*c))/a^2 + (6*tan(1/2*d*x + 1/2*c)^6 - 4*
tan(1/2*d*x + 1/2*c)^5 - 5*tan(1/2*d*x + 1/2*c)^4 + 16*tan(1/2*d*x + 1/2*c)
^3 - 12*tan(1/2*d*x + 1/2*c)^2 + 4*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x
+ 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2*a))/d
```

maple [B] time = 0.47, size = 234, normalized size = 2.21

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad} - \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad} - \frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} - \frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} + \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} - \frac{}{ad\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/8/a/d*tan(1/2*d*x+1/2*c)^2-1/2/a/d*tan(1/2*d*x+1/2*c)-1/a/d/(1+tan(1/2*d*
x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2
*d*x+1/2*c)^2+1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-2/a/d/(1+
tan(1/2*d*x+1/2*c)^2)^2+3/a/d*arctan(tan(1/2*d*x+1/2*c))-1/8/a/d/tan(1/2*d*
x+1/2*c)^2+1/2/a/d/tan(1/2*d*x+1/2*c)-3/2/a/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.58, size = 261, normalized size = 2.46

$$-\frac{\frac{\frac{4\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a} - \frac{\frac{4\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{18\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{16\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{17\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{4\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - 1}{\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{2\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}} - \frac{24\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{12\,\log\left(\frac{\mathrm{si}}{\mathrm{cos}}\right.}{a}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)/a - (4*sin(d*x + c)/(cos(d*x + c) + 1) - 18*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 17*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 2*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6) - 24*arctan(sin(d*x + c)/(cos(d*x + c) +
1))/a + 12*log(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 8.98, size = 223, normalized size = 2.10

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{3\,\mathrm{atan}\left(\frac{9}{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+9}-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+9}\right)}{a\,d}-\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}-8}{d\,\left(4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - (3*atan(9/(9*tan(c/2 + (d*x)/2) + 9) - (9*ta
n(c/2 + (d*x)/2))/(9*tan(c/2 + (d*x)/2) + 9)))/(a*d) - (3*log(tan(c/2 + (d*
x)/2)))/(2*a*d) - (9*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2) - 8*tan(c/
2 + (d*x)/2)^3 + (17*tan(c/2 + (d*x)/2)^4)/2 + 2*tan(c/2 + (d*x)/2)^5 + 1/2
)/(d*(4*a*tan(c/2 + (d*x)/2)^2 + 8*a*tan(c/2 + (d*x)/2)^4 + 4*a*tan(c/2 + (
d*x)/2)^6)) - tan(c/2 + (d*x)/2)/(2*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.631 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=94

$$\frac{3\cos(c+dx)}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{x}{a}$$

```
[Out] x/a-3/2*arctanh(cos(d*x+c))/a/d+3/2*cos(d*x+c)/a/d+cot(d*x+c)/a/d+1/2*cos(d
*x+c)*cot(d*x+c)^2/a/d-1/3*cot(d*x+c)^3/a/d
```

Rubi [A] time = 0.14, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2839, 3473, 8, 2592, 288, 321, 206}

$$\frac{3\cos(c+dx)}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a - (3*ArcTanh[Cos[c + d*x]])/(2*a*d) + (3*Cos[c + d*x])/(2*a*d) + Cot[c
+ d*x]/(a*d) + (Cos[c + d*x]*Cot[c + d*x]^2)/(2*a*d) - Cot[c + d*x]^3/(3*a*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos(c+dx)\cot^3(c+dx)\,dx}{a} + \frac{\int \cot^4(c+dx)\,dx}{a}$$

$$= -\frac{\cot^3(c+dx)}{3ad} - \frac{\int \cot^2(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{ad}$$

$$= \frac{\cot(c+dx)}{ad} + \frac{\cos(c+dx)\cot^2(c+dx)}{2ad} - \frac{\cot^3(c+dx)}{3ad} + \frac{\int 1\,dx}{a} - \frac{3\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{x}{a} + \frac{3\cos(c+dx)}{2ad} + \frac{\cot(c+dx)}{ad} + \frac{\cos(c+dx)\cot^2(c+dx)}{2ad} - \frac{\cot^3(c+dx)}{3ad} -$$

$$= \frac{x}{a} - \frac{3\tanh^{-1}(\cos(c+dx))}{2ad} + \frac{3\cos(c+dx)}{2ad} + \frac{\cot(c+dx)}{ad} + \frac{\cos(c+dx)\cot^2(c}{2ad}$$

Mathematica [A] time = 0.91, size = 138, normalized size = 1.47

$$\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(9\sin(2(c+dx))-2(3\sin(c+dx)+4)\cos(3\right.}{192ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*
(12*(2*c + 2*d*x - 3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2]])*Sin[c
 + d*x]^3 - 2*Cos[3*(c + d*x)]*(4 + 3*Sin[c + d*x]) + 9*Sin[2*(c + d*x)]))/
(192*a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.81, size = 148, normalized size = 1.57

$$\frac{16\cos(dx+c)^3 - 9\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) + 9\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(\right.}{12\left(ad\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(16*cos(d*x + c)^3 - 9*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 9*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*
x + c) + 6*(2*d*x*cos(d*x + c)^2 + 2*cos(d*x + c)^3 - 2*d*x - 3*cos(d*x + c
```

```
))*sin(d*x + c) - 12*cos(d*x + c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c)
)
```

giac [A] time = 0.19, size = 157, normalized size = 1.67

$$\frac{\frac{24\,(dx+c)}{a}+\frac{36\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{3}}+\frac{48}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a}-\frac{66\,\tan\left(\frac{1}{2}\,dx\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)/a + 36*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a^2*tan(1/2*d
*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c))
/a^3 + 48/((tan(1/2*d*x + 1/2*c)^2 + 1)*a) - (66*tan(1/2*d*x + 1/2*c)^3 - 1
5*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 1)/(a*tan(1/2*d*x + 1/2
*c)^3))/d
```

maple [A] time = 0.50, size = 173, normalized size = 1.84

$$\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{2}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{24ad\tan\left(\frac{dx}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/24/a/d*tan(1/2*d*x+1/2*c)^3-1/8/a/d*tan(1/2*d*x+1/2*c)^2-5/8/a/d*tan(1/2*
d*x+1/2*c)+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)+2/a/d*arctan(tan(1/2*d*x+1/2*c))-
1/24/a/d/tan(1/2*d*x+1/2*c)^3+1/8/a/d/tan(1/2*d*x+1/2*c)^2+5/8/a/d/tan(1/2*
d*x+1/2*c)+3/2/a/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.53, size = 240, normalized size = 2.55

$$-\frac{\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a}-\frac{\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{51\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{15\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-1}{\frac{a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}-\frac{48\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{36\,\log\left(\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/24*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - (3*sin(d*x + c)/(cos(d*x
 + c) + 1) + 14*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 51*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1)/(a*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) - 48*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 36*log(sin(d*x + c)/(cos(d*x + c
) + 1))/a)/d
```

mupad [B] time = 8.97, size = 212, normalized size = 2.26

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{2\,\mathrm{atan}\left(\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-6}+\frac{4}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-6}\right)}{a\,d}+\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+17\cdots}{d\,\left(8\,a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - (2*atan((6*t
an(c/2 + (d*x)/2))/(4*tan(c/2 + (d*x)/2) - 6) + 4/(4*tan(c/2 + (d*x)/2) - 6
)))/(a*d) + (3*log(tan(c/2 + (d*x)/2)))/(2*a*d) + (tan(c/2 + (d*x)/2) + (14
*tan(c/2 + (d*x)/2)^2)/3 + 17*tan(c/2 + (d*x)/2)^3 + 5*tan(c/2 + (d*x)/2)^4
 - 1/3)/(d*(8*a*tan(c/2 + (d*x)/2)^3 + 8*a*tan(c/2 + (d*x)/2)^5)) - (5*tan(
c/2 + (d*x)/2))/(8*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.632 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=102

$$\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}+\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{x}{a}$$

```
[Out] -x/a-3/8*arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d+1/3*cot(d*x+c)^3/a/d+3/8*co
t(d*x+c)*csc(d*x+c)/a/d-1/4*cot(d*x+c)^3*csc(d*x+c)/a/d
```

Rubi [A] time = 0.13, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2611, 3770, 3473, 8}

$$\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}+\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) - (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]/(a*d) + Cot[c + d
*x]^3/(3*a*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) - (Cot[c + d*x]^3*Csc
[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^4(c+dx)\,dx}{a} + \frac{\int \cot^4(c+dx)\csc(c+dx)\,dx}{a}$$
$$= \frac{\cot^3(c+dx)}{3ad} - \frac{\cot^3(c+dx)\csc(c+dx)}{4ad} - \frac{3\int \cot^2(c+dx)\csc(c+dx)\,dx}{4a} + \frac{\int c$$
$$= -\frac{\cot(c+dx)}{ad} + \frac{\cot^3(c+dx)}{3ad} + \frac{3\cot(c+dx)\csc(c+dx)}{8ad} - \frac{\cot^3(c+dx)\csc(c+$$
$$= -\frac{x}{a} - \frac{3\tanh^{-1}(\cos(c+dx))}{8ad} - \frac{\cot(c+dx)}{ad} + \frac{\cot^3(c+dx)}{3ad} + \frac{3\cot(c+dx)\csc(c$$

Mathematica [B] time = 0.67, size = 232, normalized size = 2.27

$$-\frac{\csc^4(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(32\sin(2(c+dx))-32\sin(4(c+dx))+24c\cos(4(c+dx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/192*(Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(72*c + 72*d
*x + 18*Cos[c + d*x] + 30*Cos[3*(c + d*x)] + 24*c*Cos[4*(c + d*x)] + 24*d*x
*Cos[4*(c + d*x)] + 27*Log[Cos[(c + d*x)/2]] + 9*Cos[4*(c + d*x)]*Log[Cos[(
c + d*x)/2]] - 12*Cos[2*(c + d*x)]*(8*c + 8*d*x + 3*Log[Cos[(c + d*x)/2]] -
 3*Log[Sin[(c + d*x)/2]]) - 27*Log[Sin[(c + d*x)/2]] - 9*Cos[4*(c + d*x)]*L
og[Sin[(c + d*x)/2]] + 32*Sin[2*(c + d*x)] - 32*Sin[4*(c + d*x)]))/(a*d*(1
+ Sin[c + d*x]))
```

fricas [A] time = 0.91, size = 171, normalized size = 1.68

$$-\frac{48\,dx\cos\left(dx+c\right)^{4}-96\,dx\cos\left(dx+c\right)^{2}+30\,\cos\left(dx+c\right)^{3}+48\,dx+9\,\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(48*d*x*cos(d*x + c)^4 - 96*d*x*cos(d*x + c)^2 + 30*cos(d*x + c)^3 +
48*d*x + 9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1
/2) - 9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2
) - 16*(4*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c) - 18*cos(d*x + c))/
(a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

giac [A] time = 0.20, size = 167, normalized size = 1.64

$$-\frac{\frac{192\,(dx+c)}{a}-\frac{72\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{3\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-8\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-24\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+120\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{4}}+\frac{150\,\tan\left(\frac{1}{2}\right.}{}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(192*(d*x + c)/a - 72*log(abs(tan(1/2*d*x + 1/2*c)))/a - (3*a^3*tan(
1/2*d*x + 1/2*c)^4 - 8*a^3*tan(1/2*d*x + 1/2*c)^3 - 24*a^3*tan(1/2*d*x + 1/
2*c)^2 + 120*a^3*tan(1/2*d*x + 1/2*c))/a^4 + (150*tan(1/2*d*x + 1/2*c)^4 +
120*tan(1/2*d*x + 1/2*c)^3 - 24*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/
2*c) + 3)/(a*tan(1/2*d*x + 1/2*c)^4))/d
```

maple [A] time = 0.48, size = 188, normalized size = 1.84

$$\frac{\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{64ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}+-\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/64/a/d*tan(1/2*d*x+1/2*c)^4-1/24/a/d*tan(1/2*d*x+1/2*c)^3-1/8/a/d*tan(1/2
*d*x+1/2*c)^2+5/8/a/d*tan(1/2*d*x+1/2*c)-2/a/d*arctan(tan(1/2*d*x+1/2*c))-1
```

/64/a/d/tan(1/2*d*x+1/2*c)^4+1/24/a/d/tan(1/2*d*x+1/2*c)^3+1/8/a/d/tan(1/2*d*x+1/2*c)^2-5/8/a/d/tan(1/2*d*x+1/2*c)+3/8/a/d*ln(tan(1/2*d*x+1/2*c))

maxima [B] time = 0.48, size = 217, normalized size = 2.13

$$\frac{\frac{\frac{120 \sin(dx+c)}{\cos(dx+c)+1} - \frac{24 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{8 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a} - \frac{384 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{72 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{\left(\frac{8 \sin(dx+c)}{\cos(dx+c)+1} + \frac{24 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{120 \sin}{(\cos(}\right.}{a \sin(dx+}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/192*((120*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4)/a - 384*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 72*log(
sin(d*x + c)/(cos(d*x + c) + 1))/a + (8*sin(d*x + c)/(cos(d*x + c) + 1) + 2
4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 120*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 - 3)*(cos(d*x + c) + 1)^4/(a*sin(d*x + c)^4))/d
```

mupad [B] time = 9.42, size = 317, normalized size = 3.11

$$\frac{3 \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - 3 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - 8\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 + 8 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 24 \cos\left(\frac{c}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^5*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*sin(c/2 + (d*x)/2)^8 - 3*cos(c/2 + (d*x)/2)^8 - 8*cos(c/2 + (d*x)/2)*sin
(c/2 + (d*x)/2)^7 + 8*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2) - 24*cos(c/2
+ (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 120*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x
)/2)^5 - 120*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3 + 24*cos(c/2 + (d*x)
/2)^6*sin(c/2 + (d*x)/2)^2 + 384*atan((8*cos(c/2 + (d*x)/2) - 3*sin(c/2 + (
d*x)/2))/(3*cos(c/2 + (d*x)/2) + 8*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^
4*sin(c/2 + (d*x)/2)^4 + 72*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(
c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4)/(192*a*d*cos(c/2 + (d*x)/2)^4*sin(c/
2 + (d*x)/2)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.633 $\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\cot^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

```
[Out] 3/8*arctanh(cos(d*x+c))/a/d-1/5*cot(d*x+c)^5/a/d-3/8*cot(d*x+c)*csc(d*x+c)/
a/d+1/4*cot(d*x+c)^3*csc(d*x+c)/a/d
```

Rubi [A] time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]^5/(5*a*d) - (3*Cot[c + d*x
]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]^3*Csc[c + d*x])/(4*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^4(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= \frac{\cot^3(c+dx)\csc(c+dx)}{4ad}+\frac{3\int \cot^2(c+dx)\csc(c+dx)\,dx}{4a}+\frac{\operatorname{Subst}\left(\int x^4\,dx,x,-\cot(\right.}{ad}\\ &= -\frac{\cot^5(c+dx)}{5ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\int \csc(c+d}{8a}\\ &= \frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot^5(c+dx)}{5ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot^3(c+dx)\csc(}{4ad}\end{aligned}$$

Mathematica [B] time = 0.78, size = 189, normalized size = 2.30

$$-\frac{\csc^5(c+dx)\left(20\sin(2(c+dx))-50\sin(4(c+dx))+80\cos(c+dx)+40\cos(3(c+dx))+8\cos(5(c+dx))+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/640*(Csc[c + d*x]^5*(80*Cos[c + d*x] + 40*Cos[3*(c + d*x)] + 8*Cos[5*(c
+ d*x)] - 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 150*Log[Sin[(c + d*x)/2]
]*Sin[c + d*x] + 20*Sin[2*(c + d*x)] + 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c +
d*x)] - 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 50*Sin[4*(c + d*x)] - 1
5*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*Log[Sin[(c + d*x)/2]]*Sin[5*(
c + d*x)]))/(a*d)
```

fricas [B] time = 0.76, size = 155, normalized size = 1.89

$$-\frac{16\cos(dx+c)^5-15\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+15\left(\cos(dx-\right.}{80\left(ad\cos(dx+c)^4-2a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/80*(16*cos(d*x + c)^5 - 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2
 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 10*(5*cos(d*x + c)^3 - 3*
cos(d*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a
*d)*sin(d*x + c))
```

giac [B] time = 0.20, size = 187, normalized size = 2.28

$$-\frac{\frac{120\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{2\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-5\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^5}-\frac{274\ \cdots}{\cdots}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/320*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - (2*a^4*tan(1/2*d*x + 1/2*c)^
5 - 5*a^4*tan(1/2*d*x + 1/2*c)^4 - 10*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^4*t
an(1/2*d*x + 1/2*c)^2 + 20*a^4*tan(1/2*d*x + 1/2*c))/a^5 - (274*tan(1/2*d*x
 + 1/2*c)^5 - 20*tan(1/2*d*x + 1/2*c)^4 - 40*tan(1/2*d*x + 1/2*c)^3 + 10*ta
n(1/2*d*x + 1/2*c)^2 + 5*tan(1/2*d*x + 1/2*c) - 2)/(a*tan(1/2*d*x + 1/2*c)^
5))/d
```

maple [B] time = 0.50, size = 208, normalized size = 2.54

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{32ad}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{1}{16ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\cdots\right.\right.}{8ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/160/a/d*tan(1/2*d*x+1/2*c)^5-1/64/a/d*tan(1/2*d*x+1/2*c)^4-1/32/a/d*tan(1
/2*d*x+1/2*c)^3+1/8/a/d*tan(1/2*d*x+1/2*c)^2+1/16/a/d*tan(1/2*d*x+1/2*c)-1/
16/a/d/tan(1/2*d*x+1/2*c)-3/8/a/d*ln(tan(1/2*d*x+1/2*c))-1/160/a/d/tan(1/2*
d*x+1/2*c)^5-1/8/a/d/tan(1/2*d*x+1/2*c)^2+1/64/a/d/tan(1/2*d*x+1/2*c)^4+1/3
2/a/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.66, size = 234, normalized size = 2.85

$$\frac{\frac{\frac{20 \sin(dx+c)}{\cos(dx+c)+1}+\frac{40 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}-\frac{120 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{5 \sin(dx+c)}{\cos(dx+c)+1}+\frac{10 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{40 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\dots\right.}{a \sin(dx+c)^5}}{320\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/320*((20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 120*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a + (5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2
0*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2)*(cos(d*x + c) + 1)^5/(a*sin(d*x
+ c)^5))/d
```

mupad [B] time = 9.04, size = 183, normalized size = 2.23

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,a\,d}-\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a\,d}-\frac{\cot\left(\frac{c}{2}+\dots\right.}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^6*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - tan(c/2 + (d*x)/2)^3/(32*a*d) - tan(c/2 + (d
*x)/2)^4/(64*a*d) + tan(c/2 + (d*x)/2)^5/(160*a*d) - (3*log(tan(c/2 + (d*x)
/2)))/(8*a*d) + tan(c/2 + (d*x)/2)/(16*a*d) - (cot(c/2 + (d*x)/2)^5*(4*tan(
c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)/2 + 2*tan(c/2
+ (d*x)/2)^4 + 1/5))/(32*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.634 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=135

$$-\frac{\cos^7(c+dx)}{7a^2d}+\frac{3\cos^5(c+dx)}{5a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}-\frac{\sin(c+dx)}{8}$$

```
[Out] -1/8*x/a^2-2/3*cos(d*x+c)^3/a^2/d+3/5*cos(d*x+c)^5/a^2/d-1/7*cos(d*x+c)^7/a
^2/d-1/8*cos(d*x+c)*sin(d*x+c)/a^2/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a^2/d+1/3*
cos(d*x+c)^3*sin(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.35, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{\cos^7(c+dx)}{7a^2d}+\frac{3\cos^5(c+dx)}{5a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}-\frac{\sin(c+dx)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -x/(8*a^2) - (2*Cos[c + d*x]^3)/(3*a^2*d) + (3*Cos[c + d*x]^5)/(5*a^2*d) -
Cos[c + d*x]^7/(7*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) + (Cos[c +
 d*x]^3*Sin[c + d*x])/(4*a^2*d) + (Cos[c + d*x]^3*Sin[c + d*x]^3)/(3*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^2(c+dx)\sin^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cos^2(c+dx)\sin^3(c+dx) - 2a^2\cos^2(c+dx)\sin^4(c+dx) + a^2\cos^2(c+d\right.}{a^4}$$
$$= \frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a^2} + \frac{\int \cos^2(c+dx)\sin^5(c+dx)\,dx}{a^2} - \frac{2\int \cos^2(c-}{}$$
$$= \frac{\cos^3(c+dx)\sin^3(c+dx)}{3a^2d} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a^2} - \frac{\text{Subst}\left(\int x^2\left(1-\right.\right.}{}$$
$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{3a^2d} - \frac{\int \cos^2(c+dx)\,dx}{4a^2} - \frac{\text{Su}}{}$$
$$= -\frac{2\cos^3(c+dx)}{3a^2d} + \frac{3\cos^5(c+dx)}{5a^2d} - \frac{\cos^7(c+dx)}{7a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{8a^2d} +$$
$$= -\frac{x}{8a^2} - \frac{2\cos^3(c+dx)}{3a^2d} + \frac{3\cos^5(c+dx)}{5a^2d} - \frac{\cos^7(c+dx)}{7a^2d} - \frac{\cos(c+dx)\sin(c+}{8a^2d}$$

Mathematica [B] time = 3.15, size = 418, normalized size = 3.10

$$-\frac{1680dx\sin\left(\frac{c}{2}\right) - 1365\sin\left(\frac{c}{2}+dx\right) + 1365\sin\left(\frac{3c}{2}+dx\right) - 210\sin\left(\frac{3c}{2}+2dx\right) - 210\sin\left(\frac{5c}{2}+2dx\right) - 175\sin\left(\frac{5c}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/13440*(210*(1 + 8*d*x)*Cos[c/2] + 1365*Cos[c/2 + d*x] + 1365*Cos[(3*c)/2
 + d*x] - 210*Cos[(3*c)/2 + 2*d*x] + 210*Cos[(5*c)/2 + 2*d*x] + 175*Cos[(5*
c)/2 + 3*d*x] + 175*Cos[(7*c)/2 + 3*d*x] - 210*Cos[(7*c)/2 + 4*d*x] + 210*C
os[(9*c)/2 + 4*d*x] - 147*Cos[(9*c)/2 + 5*d*x] - 147*Cos[(11*c)/2 + 5*d*x]
+ 70*Cos[(11*c)/2 + 6*d*x] - 70*Cos[(13*c)/2 + 6*d*x] + 15*Cos[(13*c)/2 + 7
*d*x] + 15*Cos[(15*c)/2 + 7*d*x] - 210*Sin[c/2] + 1680*d*x*Sin[c/2] - 1365*
Sin[c/2 + d*x] + 1365*Sin[(3*c)/2 + d*x] - 210*Sin[(3*c)/2 + 2*d*x] - 210*S
in[(5*c)/2 + 2*d*x] - 175*Sin[(5*c)/2 + 3*d*x] + 175*Sin[(7*c)/2 + 3*d*x] -
 210*Sin[(7*c)/2 + 4*d*x] - 210*Sin[(9*c)/2 + 4*d*x] + 147*Sin[(9*c)/2 + 5*
d*x] - 147*Sin[(11*c)/2 + 5*d*x] + 70*Sin[(11*c)/2 + 6*d*x] + 70*Sin[(13*c)
/2 + 6*d*x] - 15*Sin[(13*c)/2 + 7*d*x] + 15*Sin[(15*c)/2 + 7*d*x])/(a^2*d*(
Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.71, size = 80, normalized size = 0.59

$$-\frac{120 \cos\left(dx+c\right)^{7}-504 \cos\left(dx+c\right)^{5}+560 \cos\left(dx+c\right)^{3}+105\,dx+35\left(8 \cos\left(dx+c\right)^{5}-14 \cos\left(dx+c\right)^{3}+\right.}{840\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/840*(120*cos(d*x + c)^7 - 504*cos(d*x + c)^5 + 560*cos(d*x + c)^3 + 105*
d*x + 35*(8*cos(d*x + c)^5 - 14*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x +
c))/(a^2*d)
```

giac [A] time = 0.21, size = 179, normalized size = 1.33

$$-\frac{\frac{105\,(dx+c)}{a^{2}}+\frac{2\left(105 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+700 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1680 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-3395 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+7280 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-1120 \tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(105*(d*x + c)/a^2 + 2*(105*tan(1/2*d*x + 1/2*c)^13 + 700*tan(1/2*d*
x + 1/2*c)^11 + 1680*tan(1/2*d*x + 1/2*c)^10 - 3395*tan(1/2*d*x + 1/2*c)^9
+ 7280*tan(1/2*d*x + 1/2*c)^8 - 1120*tan(1/2*d*x + 1/2*c)^6 + 3395*tan(1/2*
d*x + 1/2*c)^5 + 2016*tan(1/2*d*x + 1/2*c)^4 - 700*tan(1/2*d*x + 1/2*c)^3 +
 1232*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 176)/((tan(1/2*d*
x + 1/2*c)^2 + 1)^7*a^2))/d
```

maple [B] time = 0.34, size = 415, normalized size = 3.07

$$-\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}-\frac{5\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}-\frac{4\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}+\frac{97\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12a^{2}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13-5/3/d/a^2/(1+ta
n(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11-4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^7*tan(1/2*d*x+1/2*c)^10+97/12/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*
```

```
x+1/2*c)^9-52/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8+8/3/d
/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6-97/12/d/a^2/(1+tan(1/2
*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^5-24/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7
*tan(1/2*d*x+1/2*c)^4+5/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*
c)^3-44/15/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2+1/4/d/a^2/
(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)-44/105/d/a^2/(1+tan(1/2*d*x+1
/2*c)^2)^7-1/4/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 416, normalized size = 3.08

$$\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{1232\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{700\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2016\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3395\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1120\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{7280\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{3395\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{1680\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{700\sin(}{(\cos(dx+}}{a^2+\frac{7a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{21a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{21a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{7a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/420*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 1232*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 700*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2016*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 3395*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1120*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 7280*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 + 3395*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 1680*sin(d*x + c)^10/(cos(d*x
 + c) + 1)^10 - 700*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 105*sin(d*x + c
)^13/(cos(d*x + c) + 1)^13 - 176)/(a^2 + 7*a^2*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 21*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 35*a^2*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 + 35*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 21*a^
2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 7*a^2*sin(d*x + c)^12/(cos(d*x +
c) + 1)^12 + a^2*sin(d*x + c)^14/(cos(d*x + c) + 1)^14) - 105*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 12.66, size = 173, normalized size = 1.28

$$-\frac{x}{8\,a^2}-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{4}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{97\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{12}+\frac{52\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3}-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}+\frac{97\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{12}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^3)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - x/(8*a^2) - ((44*tan(c/2 + (d*x)/2)^2)/15 - tan(c/2 + (d*x)/2)/4 - (5*tan
(c/2 + (d*x)/2)^3)/3 + (24*tan(c/2 + (d*x)/2)^4)/5 + (97*tan(c/2 + (d*x)/2)
```

```
^5)/12 - (8*tan(c/2 + (d*x)/2)^6)/3 + (52*tan(c/2 + (d*x)/2)^8)/3 - (97*tan
(c/2 + (d*x)/2)^9)/12 + 4*tan(c/2 + (d*x)/2)^10 + (5*tan(c/2 + (d*x)/2)^11)
/3 + tan(c/2 + (d*x)/2)^13/4 + 44/105)/(a^2*d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.635 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$\frac{\cos^3(c+dx)(a-a\sin(c+dx))^3}{6a^5d}+\frac{\cos^5(c+dx)}{10a^2d}+\frac{\sin(c+dx)\cos^3(c+dx)}{8a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{3x}{16a^2}$$

```
[Out] 3/16*x/a^2+1/10*cos(d*x+c)^5/a^2/d+3/16*cos(d*x+c)*sin(d*x+c)/a^2/d+1/8*cos
(d*x+c)^3*sin(d*x+c)/a^2/d+1/6*cos(d*x+c)^3*(a-a*sin(d*x+c))^3/a^5/d
```

Rubi [A] time = 0.27, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2875, 2870, 2669, 2635, 8}

$$\frac{\cos^5(c+dx)}{10a^2d}+\frac{\cos^3(c+dx)(a-a\sin(c+dx))^3}{6a^5d}+\frac{\sin(c+dx)\cos^3(c+dx)}{8a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{3x}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*x)/(16*a^2) + Cos[c + d*x]^5/(10*a^2*d) + (3*Cos[c + d*x]*Sin[c + d*x])/
(16*a^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(8*a^2*d) + (Cos[c + d*x]^3*(a -
 a*Sin[c + d*x])^3)/(6*a^5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2870

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(2*b*f*g*(m + 1)), x] + Dist[a/(2*g^2)
, Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cos^2(c+dx)\sin^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\cos^3(c+dx)(a-a\sin(c+dx))^3}{6a^5d} + \frac{\int \cos^4(c+dx)(a-a\sin(c+dx))\,dx}{2a^3} \\
&= \frac{\cos^5(c+dx)}{10a^2d} + \frac{\cos^3(c+dx)(a-a\sin(c+dx))^3}{6a^5d} + \frac{\int \cos^4(c+dx)\,dx}{2a^2} \\
&= \frac{\cos^5(c+dx)}{10a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^3(c+dx)(a-a\sin(c+dx))^3}{6a^5d} + \\
&= \frac{\cos^5(c+dx)}{10a^2d} + \frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{8a^2d} + \cos^3(\\
&= \frac{3x}{16a^2} + \frac{\cos^5(c+dx)}{10a^2d} + \frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{8a^2d}
\end{aligned}
$$

Mathematica [B] time = 2.06, size = 362, normalized size = 3.48

$$360dx\sin\left(\frac{c}{2}\right) - 240\sin\left(\frac{c}{2}+dx\right) + 240\sin\left(\frac{3c}{2}+dx\right) - 15\sin\left(\frac{3c}{2}+2dx\right) - 15\sin\left(\frac{5c}{2}+2dx\right) - 40\sin\left(\frac{5c}{2}+3dx\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (360*d*x*Cos[c/2] + 240*Cos[c/2 + d*x] + 240*Cos[(3*c)/2 + d*x] - 15*Cos[(3
*c)/2 + 2*d*x] + 15*Cos[(5*c)/2 + 2*d*x] + 40*Cos[(5*c)/2 + 3*d*x] + 40*Cos
```

```
[(7*c)/2 + 3*d*x] - 45*Cos[(7*c)/2 + 4*d*x] + 45*Cos[(9*c)/2 + 4*d*x] - 24*
Cos[(9*c)/2 + 5*d*x] - 24*Cos[(11*c)/2 + 5*d*x] + 5*Cos[(11*c)/2 + 6*d*x] -
 5*Cos[(13*c)/2 + 6*d*x] + 50*Sin[c/2] + 360*d*x*Sin[c/2] - 240*Sin[c/2 + d
*x] + 240*Sin[(3*c)/2 + d*x] - 15*Sin[(3*c)/2 + 2*d*x] - 15*Sin[(5*c)/2 + 2
*d*x] - 40*Sin[(5*c)/2 + 3*d*x] + 40*Sin[(7*c)/2 + 3*d*x] - 45*Sin[(7*c)/2
+ 4*d*x] - 45*Sin[(9*c)/2 + 4*d*x] + 24*Sin[(9*c)/2 + 5*d*x] - 24*Sin[(11*c
)/2 + 5*d*x] + 5*Sin[(11*c)/2 + 6*d*x] + 5*Sin[(13*c)/2 + 6*d*x])/(1920*a^2
*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.82, size = 70, normalized size = 0.67

$$-\frac{96\cos(dx+c)^5 - 160\cos(dx+c)^3 - 45\,dx - 5\left(8\cos(dx+c)^5 - 26\cos(dx+c)^3 + 9\cos(dx+c)\right)\sin(dx+}{240\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(96*cos(d*x + c)^5 - 160*cos(d*x + c)^3 - 45*d*x - 5*(8*cos(d*x + c)
^5 - 26*cos(d*x + c)^3 + 9*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

giac [A] time = 0.18, size = 153, normalized size = 1.47

$$\frac{\frac{45\,(dx+c)}{a^2} + \frac{2\left(45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11} - 65\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9 + 960\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8 - 750\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 640\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 + 750\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 65\,\mathrm{t}}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^6 a^2}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(45*(d*x + c)/a^2 + 2*(45*tan(1/2*d*x + 1/2*c)^11 - 65*tan(1/2*d*x +
1/2*c)^9 + 960*tan(1/2*d*x + 1/2*c)^8 - 750*tan(1/2*d*x + 1/2*c)^7 + 640*ta
n(1/2*d*x + 1/2*c)^6 + 750*tan(1/2*d*x + 1/2*c)^5 + 65*tan(1/2*d*x + 1/2*c)
^3 + 384*tan(1/2*d*x + 1/2*c)^2 - 45*tan(1/2*d*x + 1/2*c) + 64)/((tan(1/2*d
*x + 1/2*c)^2 + 1)^6*a^2))/d
```

maple [B] time = 0.33, size = 347, normalized size = 3.34

$$\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6} - \frac{13\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6} + \frac{8\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6} - \frac{25\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 3/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11-13/24/d/a^2/(1+t
an(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9+8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)^8-25/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+
1/2*c)^7+16/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6+25/4/d/
a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5+13/24/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+16/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*
tan(1/2*d*x+1/2*c)^2-3/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)+8/15/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6+3/8/d/a^2*arctan(tan(1/2*d*x+1/2*c)
)
```

maxima [B] time = 0.48, size = 353, normalized size = 3.39

$$-\frac{\frac{\frac{45\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{384\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{65\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{750\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{640\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{750\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{960\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{65\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{45\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-64}{a^2+\frac{6\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20\,a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15\,a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6\,a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{45\,a}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*((45*sin(d*x + c)/(cos(d*x + c) + 1) - 384*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 65*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 750*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 - 640*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 750*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 - 960*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 65*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 45*sin(d*x + c)^11/(cos(d*x + c) + 1)
^11 - 64)/(a^2 + 6*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^2*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 + 20*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^2*sin(d*x + c)^10/(cos(d*
x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 45*arctan(sin
(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 11.70, size = 146, normalized size = 1.40

$$\frac{3\,x}{16\,a^2}+\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}+\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{24}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (3*x)/(16*a^2) + ((16*tan(c/2 + (d*x)/2)^2)/5 - (3*tan(c/2 + (d*x)/2))/8 +
(13*tan(c/2 + (d*x)/2)^3)/24 + (25*tan(c/2 + (d*x)/2)^5)/4 + (16*tan(c/2 +
(d*x)/2)^6)/3 - (25*tan(c/2 + (d*x)/2)^7)/4 + 8*tan(c/2 + (d*x)/2)^8 - (13*
tan(c/2 + (d*x)/2)^9)/24 + (3*tan(c/2 + (d*x)/2)^11)/8 + 8/15)/(a^2*d*(tan(
c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [A] time = 139.91, size = 2271, normalized size = 21.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((45*d*x*tan(c/2 + d*x/2)**12/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1
440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a*
*2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*ta
n(c/2 + d*x/2)**2 + 240*a**2*d) + 270*d*x*tan(c/2 + d*x/2)**10/(240*a**2*d*
tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c
/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*
x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 675*d*x*tan(c/2 +
 d*x/2)**8/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)*
*10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3
600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2
*d) + 900*d*x*tan(c/2 + d*x/2)**6/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a
**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*
tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2
 + d*x/2)**2 + 240*a**2*d) + 675*d*x*tan(c/2 + d*x/2)**4/(240*a**2*d*tan(c/
2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d
*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**
4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 270*d*x*tan(c/2 + d*x/2
)**2/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 +
3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a*
*2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) +
45*d*x/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10
+ 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*
a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d)
+ 90*tan(c/2 + d*x/2)**11/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*ta
n(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2
+ d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2
)**2 + 240*a**2*d) - 130*tan(c/2 + d*x/2)**9/(240*a**2*d*tan(c/2 + d*x/2)**
12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4
800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**
2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 1920*tan(c/2 + d*x/2)**8/(240*a**2*
d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan
(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 +
```

```
d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) - 1500*tan(c/2 +
d*x/2)**7/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**
10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 36
00*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*
d) + 1280*tan(c/2 + d*x/2)**6/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*
d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(
c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d
*x/2)**2 + 240*a**2*d) + 1500*tan(c/2 + d*x/2)**5/(240*a**2*d*tan(c/2 + d*x
/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**
8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 144
0*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 130*tan(c/2 + d*x/2)**3/(240*a
**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d
*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/
2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 768*tan(c/2
 + d*x/2)**2/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2
)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 +
 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a*
*2*d) - 90*tan(c/2 + d*x/2)/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*
tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/
2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x
/2)**2 + 240*a**2*d) + 128/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*t
an(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2
 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/
2)**2 + 240*a**2*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**6/(a*sin(c) + a)**2, T
rue))
```

3.636 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$-\frac{2\cos^5(c+dx)}{15a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{6a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{x}{4a^2}-\frac{\cos^7(c+dx)}{3d(a\sin(c+dx)+a)^2}$$

```
[Out] -1/4*x/a^2-2/15*cos(d*x+c)^5/a^2/d-1/4*cos(d*x+c)*sin(d*x+c)/a^2/d-1/6*cos(
d*x+c)^3*sin(d*x+c)/a^2/d-1/3*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^2
```

Rubi [A] time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2859, 2682, 2635, 8}

$$-\frac{2\cos^5(c+dx)}{15a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{6a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{x}{4a^2}-\frac{\cos^7(c+dx)}{3d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -x/(4*a^2) - (2*Cos[c + d*x]^5)/(15*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(4
*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(6*a^2*d) - Cos[c + d*x]^7/(3*d*(a
+ a*Sin[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\cos^7(c+dx)}{3d(a+a\sin(c+dx))^2} - \frac{2\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx}{3a}$$
$$= -\frac{2\cos^5(c+dx)}{15a^2d} - \frac{\cos^7(c+dx)}{3d(a+a\sin(c+dx))^2} - \frac{2\int \cos^4(c+dx)\,dx}{3a^2}$$
$$= -\frac{2\cos^5(c+dx)}{15a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{6a^2d} - \frac{\cos^7(c+dx)}{3d(a+a\sin(c+dx))^2} - \frac{\int \cos^2$$
$$= -\frac{2\cos^5(c+dx)}{15a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{4a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{6a^2d} - \frac{c}{3d(a+}$$
$$= -\frac{x}{4a^2} - \frac{2\cos^5(c+dx)}{15a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{4a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{6a^2d} -$$

Mathematica [B] time = 1.13, size = 262, normalized size = 2.62

$$\frac{-120dx\sin\left(\frac{c}{2}\right)+90\sin\left(\frac{c}{2}+dx\right)-90\sin\left(\frac{3c}{2}+dx\right)+25\sin\left(\frac{5c}{2}+3dx\right)-25\sin\left(\frac{7c}{2}+3dx\right)+15\sin\left(\frac{7c}{2}+4dx\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*(5 + 24*d*x)*Cos[c/2] - 90*Cos[c/2 + d*x] - 90*Cos[(3*c)/2 + d*x] - 25*
Cos[(5*c)/2 + 3*d*x] - 25*Cos[(7*c)/2 + 3*d*x] + 15*Cos[(7*c)/2 + 4*d*x] -
15*Cos[(9*c)/2 + 4*d*x] + 3*Cos[(9*c)/2 + 5*d*x] + 3*Cos[(11*c)/2 + 5*d*x]
+ 25*Sin[c/2] - 120*d*x*Sin[c/2] + 90*Sin[c/2 + d*x] - 90*Sin[(3*c)/2 + d*x
] + 25*Sin[(5*c)/2 + 3*d*x] - 25*Sin[(7*c)/2 + 3*d*x] + 15*Sin[(7*c)/2 + 4*
d*x] + 15*Sin[(9*c)/2 + 4*d*x] - 3*Sin[(9*c)/2 + 5*d*x] + 3*Sin[(11*c)/2 +
5*d*x])/(480*a^2*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.53, size = 60, normalized size = 0.60

$$\frac{12 \cos(dx+c)^5 - 40 \cos(dx+c)^3 - 15\,dx + 15\left(2 \cos(dx+c)^3 - \cos(dx+c)\right)\sin(dx+c)}{60\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*cos(d*x + c)^5 - 40*cos(d*x + c)^3 - 15*d*x + 15*(2*cos(d*x + c)^3
 - cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

giac [A] time = 0.19, size = 140, normalized size = 1.40

$$-\frac{\frac{15\,(dx+c)}{a^2} + \frac{2\left(15 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 60 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8 - 90 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 240 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 + 40 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 90 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 80 \tan}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 1\right)^5 a^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(15*(d*x + c)/a^2 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 60*tan(1/2*d*x + 1
/2*c)^8 - 90*tan(1/2*d*x + 1/2*c)^7 + 240*tan(1/2*d*x + 1/2*c)^6 + 40*tan(1
/2*d*x + 1/2*c)^4 + 90*tan(1/2*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)^2 -
 15*tan(1/2*d*x + 1/2*c) + 28)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^2))/d
```

maple [B] time = 0.30, size = 313, normalized size = 3.13

$$-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5} - \frac{2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5} + \frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5} - \frac{8\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5} - \frac{}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9-2/d/a^2/(1+tan(1
/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8+3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*
tan(1/2*d*x+1/2*c)^7-8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^
6-4/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4-3/d/a^2/(1+tan(
1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3-8/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)
^5*tan(1/2*d*x+1/2*c)^2+1/2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/
2*c)-14/15/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5-1/2/d/a^2*arctan(tan(1/2*d*x+1/
2*c))
```

maxima [B] time = 0.42, size = 310, normalized size = 3.10

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{80\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{90\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{40\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{240\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{90\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{60\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{15\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-28}{a^2+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 80*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 90*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 40*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 240*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 90*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 - 60*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 15*sin(d*
x + c)^9/(cos(d*x + c) + 1)^9 - 28)/(a^2 + 5*a^2*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^2*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^2*
sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 15*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a^2)/d
```

mupad [B] time = 9.00, size = 81, normalized size = 0.81

$$\frac{\cos(c+dx)^5}{5a^2d}-\frac{2\cos(c+dx)^3}{3a^2d}-\frac{x}{4a^2}+\frac{\cos(c+dx)^3\sin(c+dx)}{2a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{4a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] cos(c + d*x)^5/(5*a^2*d) - (2*cos(c + d*x)^3)/(3*a^2*d) - x/(4*a^2) + (cos(
c + d*x)^3*sin(c + d*x))/(2*a^2*d) - (cos(c + d*x)*sin(c + d*x))/(4*a^2*d)
```

sympy [A] time = 87.47, size = 1720, normalized size = 17.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-15*d*x*tan(c/2 + d*x/2)**10/(60*a**2*d*tan(c/2 + d*x/2)**10 + 3
00*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d
*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) - 75*d*x
*tan(c/2 + d*x/2)**8/(60*a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 +
 d*x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**
```

```
4 + 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) - 150*d*x*tan(c/2 + d*x/2)*
*6/(60*a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a
**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan
(c/2 + d*x/2)**2 + 60*a**2*d) - 150*d*x*tan(c/2 + d*x/2)**4/(60*a**2*d*tan(
c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 + d*
x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**2 +
 60*a**2*d) - 75*d*x*tan(c/2 + d*x/2)**2/(60*a**2*d*tan(c/2 + d*x/2)**10 +
300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*
d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) - 15*d*
x/(60*a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a*
*2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(
c/2 + d*x/2)**2 + 60*a**2*d) - 30*tan(c/2 + d*x/2)**9/(60*a**2*d*tan(c/2 +
d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**
6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a*
*2*d) - 120*tan(c/2 + d*x/2)**8/(60*a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*
d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2
 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) + 180*tan(c/2 +
d*x/2)**7/(60*a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8
+ 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**
2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) - 480*tan(c/2 + d*x/2)**6/(60*a**2*d*t
an(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 +
 d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**
2 + 60*a**2*d) - 80*tan(c/2 + d*x/2)**4/(60*a**2*d*tan(c/2 + d*x/2)**10 + 3
00*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d
*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) - 180*ta
n(c/2 + d*x/2)**3/(60*a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*
x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 +
 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) - 160*tan(c/2 + d*x/2)**2/(60*
a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*t
an(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 +
d*x/2)**2 + 60*a**2*d) + 30*tan(c/2 + d*x/2)/(60*a**2*d*tan(c/2 + d*x/2)**1
0 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a
**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) - 5
6/(60*a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a*
*2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(
c/2 + d*x/2)**2 + 60*a**2*d), Ne(d, 0)), (x*sin(c)*cos(c)**6/(a*sin(c) + a)
**2, True))
```

3.637 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{x}{a^2}$$

```
[Out] -x/a^2-arctanh(cos(d*x+c))/a^2/d+cos(d*x+c)/a^2/d-1/3*cos(d*x+c)^3/a^2/d-co
s(d*x+c)*sin(d*x+c)/a^2/d
```

Rubi [A] time = 0.20, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2875, 2873, 2635, 8, 2592, 321, 206, 2565, 30}

$$-\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(a^2*d) - Cos[c + d
*x]^3/(3*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos(c+dx)\cot(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(-2a^2\cos^2(c+dx)+a^2\cos(c+dx)\cot(c+dx)+a^2\cos^2(c+dx)\sin(c+dx\right.}{a^4}$$
$$= \frac{\int \cos(c+dx)\cot(c+dx)\,dx}{a^2} + \frac{\int \cos^2(c+dx)\sin(c+dx)\,dx}{a^2} - \frac{2\int \cos^2(c+a}{a^2}$$
$$= -\frac{\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{\int 1\,dx}{a^2} - \frac{\text{Subst}\left(\int x^2\,dx, x, \cos(c+dx)\right)}{a^2d} - \frac{\text{Subst}\left(\right.}{}$$
$$= -\frac{x}{a^2} + \frac{\cos(c+dx)}{a^2d} - \frac{\cos^3(c+dx)}{3a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\right.}{}$$
$$= -\frac{x}{a^2} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{a^2d} - \frac{\cos^3(c+dx)}{3a^2d} - \frac{\cos(c+dx)\sin(c}{a^2d}$$

Mathematica [A] time = 0.32, size = 69, normalized size = 0.95

$$-\frac{-9\cos(c+dx)+\cos(3(c+dx))+6\left(\sin(2(c+dx))+2\left(-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+c+\right.\right.}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/12*(-9*Cos[c + d*x] + Cos[3*(c + d*x)] + 6*(2*(c + d*x + Log[Cos[(c + d*
x)/2]] - Log[Sin[(c + d*x)/2]]) + Sin[2*(c + d*x)]))/(a^2*d)
```

fricas [A] time = 0.85, size = 71, normalized size = 0.97

$$-\frac{2\cos(dx+c)^3+6\,dx+6\cos(dx+c)\sin(dx+c)-6\cos(dx+c)+3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\log\left(-\frac{1}{2}\right.}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*cos(d*x + c)^3 + 6*d*x + 6*cos(d*x + c)*sin(d*x + c) - 6*cos(d*x +
c) + 3*log(1/2*cos(d*x + c) + 1/2) - 3*log(-1/2*cos(d*x + c) + 1/2))/(a^2*d
)
```

giac [A] time = 0.17, size = 91, normalized size = 1.25

$$-\frac{\frac{3\,(dx+c)}{a^2} - \frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} - \frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3 a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^2 - 3*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 2*(3*tan(1/2
*d*x + 1/2*c)^5 + 6*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 2)/((
tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2))/d
```

maple [B] time = 0.51, size = 160, normalized size = 2.19

$$\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4}{3d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{2\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+4/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan
(1/2*d*x+1/2*c)+4/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3-2/d/a^2*arctan(tan(1/2
*d*x+1/2*c))+1/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.41, size = 188, normalized size = 2.58

$$-\frac{\frac{2\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-2\right)}{a^2+\frac{3\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{6\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(2*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2)/(a^2 + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
```

```
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 6*arctan(sin(d*x + c)/(cos(d*x + c
) + 1))/a^2 - 3*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 9.47, size = 167, normalized size = 2.29

$$\frac{2\,\mathrm{atan}\left(\frac{4}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4}\right)}{a^2\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\tan\left(\frac{c}{2}+\right.}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^2\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (2*atan(4/(4*tan(c/2 + (d*x)/2) + 4) - (4*tan(c/2 + (d*x)/2))/(4*tan(c/2 +
(d*x)/2) + 4)))/(a^2*d) + log(tan(c/2 + (d*x)/2))/(a^2*d) + (4*tan(c/2 + (d
*x)/2)^2 - 2*tan(c/2 + (d*x)/2) + 2*tan(c/2 + (d*x)/2)^5 + 4/3)/(d*(3*a^2*t
an(c/2 + (d*x)/2)^2 + 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6
 + a^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.638 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=74

$$-\frac{2\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{x}{2a^2}$$

```
[Out] -1/2*x/a^2+2*arctanh(cos(d*x+c))/a^2/d-2*cos(d*x+c)/a^2/d-cot(d*x+c)/a^2/d+
1/2*cos(d*x+c)*sin(d*x+c)/a^2/d
```

Rubi [A] time = 0.20, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2709, 3770, 3767, 8, 2638, 2635}

$$-\frac{2\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -x/(2*a^2) + (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - (2*Cos[c + d*x])/(a^2*d) -
 Cot[c + d*x]/(a^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
```

```
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cot^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\ &= \frac{\int \left(-2a^4\csc(c+dx)+a^4\csc^2(c+dx)+2a^4\sin(c+dx)-a^4\sin^2(c+dx)\right)\,dx}{a^6}\\ &= \frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{\int \sin^2(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\,dx}{a^2}+\frac{2\int \sin(c+dx)\,dx}{a^2}\\ &= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{\int 1\,dx}{2a^2}-\\ &= -\frac{x}{2a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}\end{aligned}$$

Mathematica [A] time = 0.53, size = 116, normalized size = 1.57

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(-2(c+dx)+\sin(2(c+dx))-8\cos(c+dx)+2\tan\left(\frac{1}{2}(c+dx)\right)-2\cot\left(\frac{1}{2}\right.\right.}{4d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(-2*(c + d*x) - 8*Cos[c + d*x] - 2
*Cot[(c + d*x)/2] + 8*Log[Cos[(c + d*x)/2]] - 8*Log[Sin[(c + d*x)/2]] + Sin
[2*(c + d*x)] + 2*Tan[(c + d*x)/2]))/(4*d*(a + a*Sin[c + d*x])^2)
```

fricas [A] time = 0.82, size = 88, normalized size = 1.19

$$-\frac{\cos\left(dx+c\right)^{3}+\left(dx+4\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)-2\,\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\,\log\left(-\frac{1}{2}\,\cos\left(dx\right.\right.}{2\,a^{2}d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(cos(d*x + c)^3 + (d*x + 4*cos(d*x + c))*sin(d*x + c) - 2*log(1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) + 2*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
+ cos(d*x + c))/(a^2*d*sin(d*x + c))
```

giac [A] time = 0.21, size = 131, normalized size = 1.77

$$-\frac{\frac{dx+c}{a^{2}}+\frac{4\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}}-\frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/a^2 + 4*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - tan(1/2*d*x +
1/2*c)/a^2 - (4*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)) + 2*(t
an(1/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 4
)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

maple [B] time = 0.53, size = 196, normalized size = 2.65

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^{2}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{4\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{2}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^{2}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{4}{a^{2}d\left(1+\tan^{2}\left(\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x
+1/2*c)^3-4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+1/a^2/d/(
1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-4/a^2/d/(1+tan(1/2*d*x+1/2*c)^
2)^2-1/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/2/d/a^2/tan(1/2*d*x+1/2*c)-2/d/a^
2*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.41, size = 202, normalized size = 2.73

$$-\frac{\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1}{\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{2a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*((8*sin(d*x + c)/(cos(d*x + c) + 1) + 8*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)/(a^2*sin(d*x + c)/(cos(d
*x + c) + 1) + 2*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^2*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5) + 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 +
4*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - sin(d*x + c)/(a^2*(cos(d*x + c
) + 1)))/d
```

mupad [B] time = 9.06, size = 175, normalized size = 2.36

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1}{d\left(2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{2\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{\mathrm{atan}\left(\frac{}{\tan}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^2*d) - (8*tan(c/2 + (d*x)/2) + 8*tan(c/2 + (d*x)/2)
^3 + 3*tan(c/2 + (d*x)/2)^4 + 1)/(d*(4*a^2*tan(c/2 + (d*x)/2)^3 + 2*a^2*tan
(c/2 + (d*x)/2)^5 + 2*a^2*tan(c/2 + (d*x)/2))) - (2*log(tan(c/2 + (d*x)/2))
)/(a^2*d) + atan(1/(tan(c/2 + (d*x)/2) - 4) + (4*tan(c/2 + (d*x)/2))/(tan(c
/2 + (d*x)/2) - 4))/(a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.639 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$\frac{\cos(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2x}{a^2}$$

```
[Out] 2*x/a^2-1/2*arctanh(cos(d*x+c))/a^2/d+cos(d*x+c)/a^2/d+2*cot(d*x+c)/a^2/d-1
/2*cot(d*x+c)*csc(d*x+c)/a^2/d
```

Rubi [A] time = 0.22, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2872, 3767, 8, 3768, 3770, 2638}

$$\frac{\cos(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*x)/a^2 - ArcTanh[Cos[c + d*x]]/(2*a^2*d) + Cos[c + d*x]/(a^2*d) + (2*Cot
[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
```

```
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cot^2(c+dx)\csc(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\
&= \frac{\int \left(2a^4-2a^4\csc^2(c+dx)+a^4\csc^3(c+dx)-a^4\sin(c+dx)\right)dx}{a^6}\\
&= \frac{2x}{a^2}+\frac{\int \csc^3(c+dx)\,dx}{a^2}-\frac{\int \sin(c+dx)\,dx}{a^2}-\frac{2\int \csc^2(c+dx)\,dx}{a^2}\\
&= \frac{2x}{a^2}+\frac{\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{\int \csc(c+dx)\,dx}{2a^2}+\frac{2\,\mathrm{Subst}(\int 1
\end{aligned}$$

$$= \frac{2x}{a^2}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{\cos(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+}{2a^2d}$$

Mathematica [A] time = 0.58, size = 134, normalized size = 1.84

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(16(c+dx)+8\cos(c+dx)-8\tan\left(\frac{1}{2}(c+dx)\right)+8\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}\right.\right.}{8d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(16*(c + d*x) + 8*Cos[c + d*x] + 8
*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Si
n[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 - 8*Tan[(c + d*x)/2]))/(8*d*(a + a*Sin
[c + d*x])^2)
```

fricas [A] time = 0.76, size = 118, normalized size = 1.62

$$\frac{8\,dx\cos\left(dx+c\right)^{2}+4\,\cos\left(dx+c\right)^{3}-8\,dx-\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\cos\left(dx+c\right)^{2}-1\right)\log}{4\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4*(8*d*x*cos(d*x + c)^2 + 4*cos(d*x + c)^3 - 8*d*x - (cos(d*x + c)^2 - 1)
*log(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) +
 1/2) - 8*cos(d*x + c)*sin(d*x + c) - 2*cos(d*x + c))/(a^2*d*cos(d*x + c)^2
 - a^2*d)
```

giac [A] time = 0.21, size = 128, normalized size = 1.75

$$\frac{\frac{16\,(dx+c)}{a^{2}}+\frac{4\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}}+\frac{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-8\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{4}}+\frac{16}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{2}}-\frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(16*(d*x + c)/a^2 + 4*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (a^2*tan(1/2
*d*x + 1/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^4 + 16/((tan(1/2*d*x + 1/2*
c)^2 + 1)*a^2) - (6*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a
^2*tan(1/2*d*x + 1/2*c)^2))/d
```

maple [A] time = 0.56, size = 134, normalized size = 1.84

$$\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^{2}d}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^{2}}+\frac{2}{d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{2}}-\frac{1}{8a^{2}d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{}{d\,a^{2}\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^2*tan(1/2*d*x+1/2*c)+2/d/a^2/(1+tan(1/
2*d*x+1/2*c)^2)+4/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/8/a^2/d/tan(1/2*d*x+1/
2*c)^2+1/d/a^2/tan(1/2*d*x+1/2*c)+1/2/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.41, size = 204, normalized size = 2.79

$$\frac{\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}+\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1}{\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^2}+\frac{32\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*((8*sin(d*x + c)/(cos(d*x + c) + 1) + 15*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a^2*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - (8*sin(d*x +
c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^2 + 32*arcta
n(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 4*log(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2)/d
```

mupad [B] time = 9.10, size = 186, normalized size = 2.55

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}-\frac{4\,\mathrm{atan}\left(\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4}+\frac{16}{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4}\right)}{a^2\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^2\,d}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+4\,t}{d\,\left(4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a^2\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (4*atan((4*tan(c/2 + (d*x)/2))/(16*tan(c/2
 + (d*x)/2) - 4) + 16/(16*tan(c/2 + (d*x)/2) - 4)))/(a^2*d) + log(tan(c/2 +
 (d*x)/2))/(2*a^2*d) + (4*tan(c/2 + (d*x)/2) + (15*tan(c/2 + (d*x)/2)^2)/2
+ 4*tan(c/2 + (d*x)/2)^3 - 1/2)/(d*(4*a^2*tan(c/2 + (d*x)/2)^2 + 4*a^2*tan(
c/2 + (d*x)/2)^4)) - tan(c/2 + (d*x)/2)/(a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.640 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

```
[Out] -x/a^2-arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d+co
t(d*x+c)*csc(d*x+c)/a^2/d
```

Rubi [A] time = 0.32, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 3473, 8, 2611, 3770, 2607, 30}

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - ArcTanh[Cos[c + d*x]]/(a^2*d) - Cot[c + d*x]/(a^2*d) - Cot[c + d
*x]^3/(3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^2(c+dx)\csc^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^2(c+dx) - 2a^2\cot^2(c+dx)\csc(c+dx) + a^2\cot^2(c+dx)\csc^2(c+dx\right.}{a^4}$$
$$= \frac{\int \cot^2(c+dx)\,dx}{a^2} + \frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a^2} - \frac{2\int \cot^2(c+dx)\csc(c+}{a^2}$$
$$= -\frac{\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{a^2d} - \frac{\int 1\,dx}{a^2} + \frac{\int \csc(c+dx)\,dx}{a^2} + \frac{\text{Subst}\left(\right.}{}$$
$$= -\frac{x}{a^2} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d} + \frac{\cot(c+dx)\csc(c+}{a^2d}$$

Mathematica [A] time = 1.28, size = 124, normalized size = 1.70

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)^4\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-6\sin(2(c+dx))+6\cos(c+dx)-2\cos(3(c+dx))+\right.}{96a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/96*((1 + Cot[(c + d*x)/2])^4*Sec[(c + d*x)/2]^2*(6*Cos[c + d*x] - 2*Cos[
3*(c + d*x)] + 12*(c + d*x + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])
*Sin[c + d*x]^3 - 6*Sin[2*(c + d*x)])*Tan[(c + d*x)/2])/(a^2*d*(1 + Sin[c +
 d*x])^2)
```

fricas [A] time = 0.49, size = 139, normalized size = 1.90

$$-\frac{4\cos(dx+c)^3 + 3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 3\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos\right.}{6\left(a^2d\cos(dx+c)^2 - a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/6*(4*cos(d*x + c)^3 + 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)
*sin(d*x + c) - 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) + 6*(d*x*cos(d*x + c)^2 - d*x + cos(d*x + c))*sin(d*x + c) - 6*cos(d*
x + c))/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

giac [A] time = 0.24, size = 137, normalized size = 1.88

$$-\frac{\frac{24\,(dx+c)}{a^2}-\frac{24\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{44\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}-\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^6}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(24*(d*x + c)/a^2 - 24*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (44*tan(1
/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c)^2 - 6*tan(1/2*d*x + 1/2*c) + 1)/
(a^2*tan(1/2*d*x + 1/2*c)^3) - (a^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^4*tan(1/2*
d*x + 1/2*c)^2 + 9*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [B] time = 0.57, size = 149, normalized size = 2.04

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^2}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2d}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^2}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}-\frac{1}{24a^2d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{1}{4a^2d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^2*tan(1/2*d*x+1/2*c)^2+3/8/d/a^2*ta
n(1/2*d*x+1/2*c)-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/24/a^2/d/tan(1/2*d*x+
1/2*c)^3+1/4/a^2/d/tan(1/2*d*x+1/2*c)^2-3/8/d/a^2/tan(1/2*d*x+1/2*c)+1/d/a^
2*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.45, size = 176, normalized size = 2.41

$$\frac{\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{48\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{24\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{6\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a^2\sin(dx+c)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 48*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/a^2 + 24*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + (6*s
```

```
in(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)
*(cos(d*x + c) + 1)^3/(a^2*sin(d*x + c)^3))/d
```

mupad [B] time = 9.30, size = 261, normalized size = 3.58

$$\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-6\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+9\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (sin(c/2 + (d*x)/2)^6 - cos(c/2 + (d*x)/2)^6 - 6*cos(c/2 + (d*x)/2)*sin(c/2
 + (d*x)/2)^5 + 6*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2) + 9*cos(c/2 + (d*
x)/2)^2*sin(c/2 + (d*x)/2)^4 - 9*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2
+ 48*atan((cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2) + s
in(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^3 + 24*log(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^
3)/(24*a^2*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.641 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=82

$$\frac{2\cot^3(c+dx)}{3a^2d}+\frac{5\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8a^2d}$$

```
[Out] 5/8*arctanh(cos(d*x+c))/a^2/d+2/3*cot(d*x+c)^3/a^2/d-3/8*cot(d*x+c)*csc(d*x
+c)/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.30, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2873, 2611, 3770, 2607, 30, 3768}

$$\frac{2\cot^3(c+dx)}{3a^2d}+\frac{5\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*ArcTanh[Cos[c + d*x]])/(8*a^2*d) + (2*Cot[c + d*x]^3)/(3*a^2*d) - (3*Cot
[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^2(c+dx)\csc^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^2(c+dx)\csc(c+dx)-2a^2\cot^2(c+dx)\csc^2(c+dx)+a^2\cot^2(c+dx)\right.}{a^4}$$
$$= \frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a^2}+\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a^2}-\frac{2\int \cot^2(c+d}{}$$
$$= -\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{\int \csc^3(c+dx)\,dx}{4a^2}-\frac{\int cs}{}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{\cot(c+dx)}{4}$$
$$= \frac{5\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{\cot(c+d}{}$$

Mathematica [A] time = 1.42, size = 116, normalized size = 1.41

$$\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^4\left(24\sin(2(c+dx))-33\cos(c+dx)+(16\sin(c+dx)+9)\cos(3(c+dx))+\right.}{1536a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^4*(-33*Cos[c + d*x] + 60*(Log[Cos[(c
 + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 + Cos[3*(c + d*x)]*(9 +
 16*Sin[c + d*x]) + 24*Sin[2*(c + d*x)]))/(1536*a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.72, size = 138, normalized size = 1.68

$$\frac{32\cos(dx+c)^3\sin(dx+c)+18\cos(dx+c)^3+15\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\right.}{48\left(a^2d\cos(dx+c)^4-2a^2d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/48*(32*cos(d*x + c)^3*sin(d*x + c) + 18*cos(d*x + c)^3 + 15*(cos(d*x + c)
^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2) - 15*(cos(d*x + c)^4
 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2) - 30*cos(d*x + c))/(a
^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)
```

giac [B] time = 0.25, size = 158, normalized size = 1.93

$$-\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{250\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-48\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+16\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}-\frac{3a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-16a^6\tan\ldots}{}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/192*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (250*tan(1/2*d*x + 1/2*c)^
4 - 48*tan(1/2*d*x + 1/2*c)^3 - 24*tan(1/2*d*x + 1/2*c)^2 + 16*tan(1/2*d*x
+ 1/2*c) - 3)/(a^2*tan(1/2*d*x + 1/2*c)^4) - (3*a^6*tan(1/2*d*x + 1/2*c)^4
- 16*a^6*tan(1/2*d*x + 1/2*c)^3 + 24*a^6*tan(1/2*d*x + 1/2*c)^2 + 48*a^6*ta
n(1/2*d*x + 1/2*c))/a^8)/d
```

maple [B] time = 0.63, size = 170, normalized size = 2.07

$$\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a^2d}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{12d\,a^2}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2d}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^2}-\frac{1}{4d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^2}-\frac{1}{8a^2d\tan\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/64/d/a^2*tan(1/2*d*x+1/2*c)^4-1/12/d/a^2*tan(1/2*d*x+1/2*c)^3+1/8/d/a^2*t
an(1/2*d*x+1/2*c)^2+1/4/d/a^2*tan(1/2*d*x+1/2*c)-1/4/d/a^2/tan(1/2*d*x+1/2*
c)-5/8/d/a^2*ln(tan(1/2*d*x+1/2*c))-1/8/a^2/d/tan(1/2*d*x+1/2*c)^2-1/64/a^2
/d/tan(1/2*d*x+1/2*c)^4+1/12/a^2/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.33, size = 194, normalized size = 2.37

$$\frac{\frac{\frac{48\sin(dx+c)}{\cos(dx+c)+1}+\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^2}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{16\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{48\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-3\right)(\cos(dx+c)+1)^4}{a^2\sin(dx+c)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/192*((48*sin(d*x + c)/(cos(d*x + c) + 1) + 24*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4)/a^2 - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + (16*si
n(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 48
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3)*(cos(d*x + c) + 1)^4/(a^2*sin(d*x
 + c)^4))/d
```

mupad [B] time = 9.03, size = 151, normalized size = 1.84

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^2\,d}-\frac{5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^2\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(4\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^5*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) - tan(c/2 + (d*x)/2)^3/(12*a^2*d) + tan(c/2
+ (d*x)/2)^4/(64*a^2*d) - (5*log(tan(c/2 + (d*x)/2)))/(8*a^2*d) + tan(c/2 +
```

```
 (d*x)/2)/(4*a^2*d) - (cot(c/2 + (d*x)/2)^4*(2*tan(c/2 + (d*x)/2)^2 - (4*ta
n(c/2 + (d*x)/2))/3 + 4*tan(c/2 + (d*x)/2)^3 + 1/4))/(16*a^2*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.642 $\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^3(c+dx)}{3a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{4a^2d}$$

```
[Out] -1/4*arctanh(cos(d*x+c))/a^2/d-2/3*cot(d*x+c)^3/a^2/d-1/5*cot(d*x+c)^5/a^2/
d-1/4*cot(d*x+c)*csc(d*x+c)/a^2/d+1/2*cot(d*x+c)*csc(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.15, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2709, 3767, 8, 3768, 3770}

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^3(c+dx)}{3a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(4*a^2*d) - (2*Cot[c + d*x]^3)/(3*a^2*d) - Cot[c + d
*x]^5/(5*a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(4*a^2*d) + (Cot[c + d*x]*Csc
[c + d*x]^3)/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int\left(-a^4\csc^2(c+dx)+2a^4\csc^3(c+dx)-2a^4\csc^5(c+dx)+a^4\csc^6(c+dx)\right)dx}{a^6}$$
$$= -\frac{\int\csc^2(c+dx)\,dx}{a^2}+\frac{\int\csc^6(c+dx)\,dx}{a^2}+\frac{2\int\csc^3(c+dx)\,dx}{a^2}-\frac{2\int\csc^5(c+dx)\,d}{a^2}$$
$$= -\frac{\cot(c+dx)\csc(c+dx)}{a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}+\frac{\int\csc(c+dx)\,dx}{a^2}-\frac{3\int\csc^3}{2}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{4a^2d}+$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}-\frac{2\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{4a^2d}+$$

Mathematica [A] time = 0.56, size = 189, normalized size = 1.89

$$-\frac{\csc^5(c+dx)\left(-180\sin(2(c+dx))-30\sin(4(c+dx))+200\cos(c+dx)+20\cos(3(c+dx))-28\cos(5(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/960*(Csc[c + d*x]^5*(200*Cos[c + d*x] + 20*Cos[3*(c + d*x)] - 28*Cos[5*(
c + d*x)] + 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 150*Log[Sin[(c + d*x)/
2]]*Sin[c + d*x] - 180*Sin[2*(c + d*x)] - 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c
 + d*x)] + 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 30*Sin[4*(c + d*x)]
+ 15*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin[
5*(c + d*x)]))/(a^2*d)
```

fricas [A] time = 0.75, size = 167, normalized size = 1.67

$$\frac{56 \cos\left(dx+c\right)^{5} - 80 \cos\left(dx+c\right)^{3} - 15 \left(\cos\left(dx+c\right)^{4} - 2 \cos\left(dx+c\right)^{2} + 1\right) \log\left(\frac{1}{2} \cos\left(dx+c\right) + \frac{1}{2}\right) \sin\left(dx+c\right.}{120 \left(a^{2} d \cos\left(dx+c\right)^{4} -\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(56*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 15*(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)^
4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*(c
os(d*x + c)^3 + cos(d*x + c))*sin(d*x + c))/((a^2*d*cos(d*x + c)^4 - 2*a^2*
d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

giac [A] time = 0.25, size = 157, normalized size = 1.57

$$\frac{\frac{120 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^{2}} - \frac{274 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5} - 90 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4} + 25 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2} - 15 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right) + 3}{a^{2} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}} + \frac{3 a^{8} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5} - 15 a^{8} \tan\left(\right.}{}}{480 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (274*tan(1/2*d*x + 1/2*c)^5
 - 90*tan(1/2*d*x + 1/2*c)^4 + 25*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x +
 1/2*c) + 3)/(a^2*tan(1/2*d*x + 1/2*c)^5) + (3*a^8*tan(1/2*d*x + 1/2*c)^5 -
 15*a^8*tan(1/2*d*x + 1/2*c)^4 + 25*a^8*tan(1/2*d*x + 1/2*c)^3 - 90*a^8*tan
(1/2*d*x + 1/2*c))/a^10)/d
```

maple [A] time = 0.66, size = 170, normalized size = 1.70

$$\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{160 a^{2} d} - \frac{\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)}{32 a^{2} d} + \frac{5 \left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96 d\, a^{2}} - \frac{3 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16 d\, a^{2}} + \frac{3}{16 d\, a^{2} \tan\left(\frac{dx}{2}+\frac{c}{2}\right)} + \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4 d\, a^{2}} - \frac{}{160 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/160/d/a^2*tan(1/2*d*x+1/2*c)^5-1/32/d/a^2*tan(1/2*d*x+1/2*c)^4+5/96/d/a^2
*tan(1/2*d*x+1/2*c)^3-3/16/d/a^2*tan(1/2*d*x+1/2*c)+3/16/d/a^2/tan(1/2*d*x+
```

```
1/2*c)+1/4/d/a^2*ln(tan(1/2*d*x+1/2*c))-1/160/a^2/d/tan(1/2*d*x+1/2*c)^5+1/
32/a^2/d/tan(1/2*d*x+1/2*c)^4-5/96/a^2/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.32, size = 195, normalized size = 1.95

$$-\frac{\frac{\frac{90 \sin(dx+c)}{\cos(dx+c)+1}-\frac{25 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}-\frac{120 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{15 \sin(dx+c)}{\cos(dx+c)+1}-\frac{25 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{90 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-3\right)(\cos(dx+c)+}{a^2 \sin(dx+c)^5}}{480\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*((90*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5)/a^2 - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - (15*s
in(d*x + c)/(cos(d*x + c) + 1) - 25*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 9
0*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a^2*sin(d*
x + c)^5))/d
```

mupad [B] time = 9.02, size = 149, normalized size = 1.49

$$\frac{5 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32\,a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,a^2\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^2\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a^2\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(6\tan\left(\frac{c}{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^6*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (5*tan(c/2 + (d*x)/2)^3)/(96*a^2*d) - tan(c/2 + (d*x)/2)^4/(32*a^2*d) + tan
(c/2 + (d*x)/2)^5/(160*a^2*d) + log(tan(c/2 + (d*x)/2))/(4*a^2*d) - (3*tan(
c/2 + (d*x)/2))/(16*a^2*d) + (cot(c/2 + (d*x)/2)^5*(tan(c/2 + (d*x)/2) - (5
*tan(c/2 + (d*x)/2)^2)/3 + 6*tan(c/2 + (d*x)/2)^4 - 1/5))/(32*a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.643 $\int \frac{\cot^6(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$\frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}-\frac{5\cot(c+dx)\csc^3(c+dx)}{24a^2d}+\frac{3\,\mathrm{c}}{}$$

```
[Out] 3/16*arctanh(cos(d*x+c))/a^2/d+2/3*cot(d*x+c)^3/a^2/d+2/5*cot(d*x+c)^5/a^2/
d+3/16*cot(d*x+c)*csc(d*x+c)/a^2/d-5/24*cot(d*x+c)*csc(d*x+c)^3/a^2/d-1/6*c
ot(d*x+c)*csc(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.33, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2875, 2873, 2611, 3768, 3770, 2607, 14}

$$\frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}-\frac{5\cot(c+dx)\csc^3(c+dx)}{24a^2d}+\frac{3\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(16*a^2*d) + (2*Cot[c + d*x]^3)/(3*a^2*d) + (2*Co
t[c + d*x]^5)/(5*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(16*a^2*d) - (5*Cot
[c + d*x]*Csc[c + d*x]^3)/(24*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a^2
*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^2(c+dx)\csc^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cot^2(c+dx)\csc^3(c+dx) - 2a^2\cot^2(c+dx)\csc^4(c+dx) + a^2\cot^2(c+dx)\right.}{a^4}$$

$$= \frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a^2} + \frac{\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{a^2} - \frac{2\int \cot^2(c+}{}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d} - \frac{\int \csc^5(c+dx)\,dx}{6a^2} - \frac{\int \mathrm{c}}{}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{8a^2d} - \frac{5\cot(c+dx)\csc^3(c+dx)}{24a^2d} - \frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}$$

$$= \frac{\tanh^{-1}(\cos(c+dx))}{8a^2d} + \frac{2\cot^3(c+dx)}{3a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{3\cot(c+dx)\csc(c+}{16a^2d}$$

$$= \frac{3\tanh^{-1}(\cos(c+dx))}{16a^2d} + \frac{2\cot^3(c+dx)}{3a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{3\cot(c+dx)\csc(c+}{16a^2d}$$

Mathematica [A] time = 0.68, size = 229, normalized size = 1.85

$$-\frac{\csc^6(c+dx)\left(-960\sin(2(c+dx)) - 384\sin(4(c+dx)) + 64\sin(6(c+dx)) + 1500\cos(c+dx) - 130\cos(3(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/7680*(Csc[c + d*x]^6*(1500*Cos[c + d*x] - 130*Cos[3*(c + d*x)] - 90*Cos[
5*(c + d*x)] - 450*Log[Cos[(c + d*x)/2]] + 675*Cos[2*(c + d*x)]*Log[Cos[(c
+ d*x)/2]] - 270*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 45*Cos[6*(c + d*x
)]*Log[Cos[(c + d*x)/2]] + 450*Log[Sin[(c + d*x)/2]] - 675*Cos[2*(c + d*x)]
*Log[Sin[(c + d*x)/2]] + 270*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 45*Co
s[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 960*Sin[2*(c + d*x)] - 384*Sin[4*(c
+ d*x)] + 64*Sin[6*(c + d*x)]))/(a^2*d)
```

fricas [A] time = 0.77, size = 196, normalized size = 1.58

$$-\frac{90\cos(dx+c)^5 - 80\cos(dx+c)^3 - 45\left(\cos(dx+c)^6 - 3\cos(dx+c)^4 + 3\cos(dx+c)^2 - 1\right)\log\left(\frac{1}{2}\cos(dx+\right.}{480\left(a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(90*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 45*(cos(d*x + c)^6 - 3*cos(
d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 45*(cos(d*
x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) +
 1/2) - 64*(2*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*sin(d*x + c) - 90*cos(d*x
+ c))/(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x + c)
^2 - a^2*d)
```

giac [A] time = 0.29, size = 216, normalized size = 1.74

$$-\frac{\frac{360 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} - \frac{882 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 240 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 15 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 40 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 45 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 24 \tan\left(\frac{1}{2}\right.}{a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/1920*(360*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (882*tan(1/2*d*x + 1/2*c)
^6 - 240*tan(1/2*d*x + 1/2*c)^5 + 15*tan(1/2*d*x + 1/2*c)^4 + 40*tan(1/2*d*
x + 1/2*c)^3 - 45*tan(1/2*d*x + 1/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) - 5)/(a^
2*tan(1/2*d*x + 1/2*c)^6) - (5*a^10*tan(1/2*d*x + 1/2*c)^6 - 24*a^10*tan(1/
2*d*x + 1/2*c)^5 + 45*a^10*tan(1/2*d*x + 1/2*c)^4 - 40*a^10*tan(1/2*d*x + 1
/2*c)^3 - 15*a^10*tan(1/2*d*x + 1/2*c)^2 + 240*a^10*tan(1/2*d*x + 1/2*c))/a
^12)/d
```

maple [B] time = 0.68, size = 246, normalized size = 1.98

$$\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{384d\,a^2} - \frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{80a^2d} + \frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128a^2d} - \frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{48d\,a^2} - \frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{128a^2d} + \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^2} - \frac{1}{384d\,a^2\tan\left(\frac{dx}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/384/d/a^2*tan(1/2*d*x+1/2*c)^6-1/80/d/a^2*tan(1/2*d*x+1/2*c)^5+3/128/d/a^
2*tan(1/2*d*x+1/2*c)^4-1/48/d/a^2*tan(1/2*d*x+1/2*c)^3-1/128/d/a^2*tan(1/2*
d*x+1/2*c)^2+1/8/d/a^2*tan(1/2*d*x+1/2*c)-1/384/d/a^2/tan(1/2*d*x+1/2*c)^6-
1/8/d/a^2/tan(1/2*d*x+1/2*c)-3/16/d/a^2*ln(tan(1/2*d*x+1/2*c))+1/80/a^2/d/t
an(1/2*d*x+1/2*c)^5+1/128/a^2/d/tan(1/2*d*x+1/2*c)^2-3/128/a^2/d/tan(1/2*d*
x+1/2*c)^4+1/48/a^2/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.32, size = 274, normalized size = 2.21

$$\frac{\frac{\frac{240\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{24\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a^2}-\frac{360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{24\sin(dx+c)}{\cos(dx+c)+1}-\frac{45\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{40}{(c}\right.}{}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/1920*((240*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6)/a^2 - 360*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2
 + (24*sin(d*x + c)/(cos(d*x + c) + 1) - 45*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 - 240*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x + c) +
1)^6/(a^2*sin(d*x + c)^6))/d
```

mupad [B] time = 10.11, size = 339, normalized size = 2.73

$$-\frac{5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+24\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-45\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^7*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 + 24*cos(c/2 + (d*x)/2)
*sin(c/2 + (d*x)/2)^11 - 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) - 45*c
os(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 40*cos(c/2 + (d*x)/2)^3*sin(c/2
 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 - 240*cos(c/2
+ (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 240*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x
)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 40*cos(c/2 + (d*x)/
2)^9*sin(c/2 + (d*x)/2)^3 + 45*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2 +
 360*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*sin(c/
2 + (d*x)/2)^6)/(1920*a^2*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.644 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=129

$$-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{4\cos(c+dx)}{a^3d}+\frac{\sin^5(c+dx)\cos(c+dx)}{6a^3d}+\frac{23\sin^3(c+dx)\cos(c+dx)}{24a^3d}+\frac{23\sin(c$$

```
[Out] -23/16*x/a^3-4*cos(d*x+c)/a^3/d+7/3*cos(d*x+c)^3/a^3/d-3/5*cos(d*x+c)^5/a^3
/d+23/16*cos(d*x+c)*sin(d*x+c)/a^3/d+23/24*cos(d*x+c)*sin(d*x+c)^3/a^3/d+1/
6*cos(d*x+c)*sin(d*x+c)^5/a^3/d
```

Rubi [A] time = 0.24, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 2633, 2635, 8}

$$-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{4\cos(c+dx)}{a^3d}+\frac{\sin^5(c+dx)\cos(c+dx)}{6a^3d}+\frac{23\sin^3(c+dx)\cos(c+dx)}{24a^3d}+\frac{23\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-23*x)/(16*a^3) - (4*Cos[c + d*x])/(a^3*d) + (7*Cos[c + d*x]^3)/(3*a^3*d)
- (3*Cos[c + d*x]^5)/(5*a^3*d) + (23*Cos[c + d*x]*Sin[c + d*x])/(16*a^3*d)
+ (23*Cos[c + d*x]*Sin[c + d*x]^3)/(24*a^3*d) + (Cos[c + d*x]*Sin[c + d*x]^
5)/(6*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(a^3\sin^3(c+dx)-3a^3\sin^4(c+dx)+3a^3\sin^5(c+dx)-a^3\sin^6(c+dx)\right)\,dx}{a^6}$$
$$= \frac{\int \sin^3(c+dx)\,dx}{a^3}-\frac{\int \sin^6(c+dx)\,dx}{a^3}-\frac{3\int \sin^4(c+dx)\,dx}{a^3}+\frac{3\int \sin^5(c+d}{a^3}$$
$$= \frac{3\cos(c+dx)\sin^3(c+dx)}{4a^3d}+\frac{\cos(c+dx)\sin^5(c+dx)}{6a^3d}-\frac{5\int \sin^4(c+dx)\,dx}{6a^3}$$
$$= -\frac{4\cos(c+dx)}{a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{9\cos(c+dx)\sin(c+dx}{8a^3d}$$
$$= -\frac{9x}{8a^3}-\frac{4\cos(c+dx)}{a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{23\cos(c+dx)\sin}{16a^3d}$$
$$= -\frac{23x}{16a^3}-\frac{4\cos(c+dx)}{a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{23\cos(c+dx)\,\mathrm{si}}{16a^3d}$$

Mathematica [B] time = 2.19, size = 366, normalized size = 2.84

$$\frac{-2760dx\sin\left(\frac{c}{2}\right)+2520\sin\left(\frac{c}{2}+dx\right)-2520\sin\left(\frac{3c}{2}+dx\right)+945\sin\left(\frac{3c}{2}+2dx\right)+945\sin\left(\frac{5c}{2}+2dx\right)-380\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*(3 + 920*d*x)*Cos[c/2] - 2520*Cos[c/2 + d*x] - 2520*Cos[(3*c)/2 + d*x]
+ 945*Cos[(3*c)/2 + 2*d*x] - 945*Cos[(5*c)/2 + 2*d*x] + 380*Cos[(5*c)/2 + 3
*d*x] + 380*Cos[(7*c)/2 + 3*d*x] - 135*Cos[(7*c)/2 + 4*d*x] + 135*Cos[(9*c)
/2 + 4*d*x] - 36*Cos[(9*c)/2 + 5*d*x] - 36*Cos[(11*c)/2 + 5*d*x] + 5*Cos[(1
1*c)/2 + 6*d*x] - 5*Cos[(13*c)/2 + 6*d*x] + 9*Sin[c/2] - 2760*d*x*Sin[c/2]
+ 2520*Sin[c/2 + d*x] - 2520*Sin[(3*c)/2 + d*x] + 945*Sin[(3*c)/2 + 2*d*x]
+ 945*Sin[(5*c)/2 + 2*d*x] - 380*Sin[(5*c)/2 + 3*d*x] + 380*Sin[(7*c)/2 + 3
*d*x] - 135*Sin[(7*c)/2 + 4*d*x] - 135*Sin[(9*c)/2 + 4*d*x] + 36*Sin[(9*c)/
2 + 5*d*x] - 36*Sin[(11*c)/2 + 5*d*x] + 5*Sin[(11*c)/2 + 6*d*x] + 5*Sin[(13
*c)/2 + 6*d*x])/(1920*a^3*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.94, size = 78, normalized size = 0.60

$$-\frac{144\cos(dx+c)^5 - 560\cos(dx+c)^3 + 345\,dx - 5\left(8\cos(dx+c)^5 - 62\cos(dx+c)^3 + 123\cos(dx+c)\right)\sin(\ldots}{240\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/240*(144*cos(d*x + c)^5 - 560*cos(d*x + c)^3 + 345*d*x - 5*(8*cos(d*x +
c)^5 - 62*cos(d*x + c)^3 + 123*cos(d*x + c))*sin(d*x + c) + 960*cos(d*x + c
))/(a^3*d)
```

giac [A] time = 0.26, size = 166, normalized size = 1.29

$$-\frac{\frac{345\,(dx+c)}{a^3} + \frac{2\left(345\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11} + 1955\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9 + 480\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8 + 2250\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 5440\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 - 2250\tan\left(\frac{1}{2}dx+\ldots\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^6 a^3}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(345*(d*x + c)/a^3 + 2*(345*tan(1/2*d*x + 1/2*c)^11 + 1955*tan(1/2*d
*x + 1/2*c)^9 + 480*tan(1/2*d*x + 1/2*c)^8 + 2250*tan(1/2*d*x + 1/2*c)^7 +
5440*tan(1/2*d*x + 1/2*c)^6 - 2250*tan(1/2*d*x + 1/2*c)^5 + 7680*tan(1/2*d*
x + 1/2*c)^4 - 1955*tan(1/2*d*x + 1/2*c)^3 + 3264*tan(1/2*d*x + 1/2*c)^2 -
345*tan(1/2*d*x + 1/2*c) + 544)/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a^3))/d
```

maple [B] time = 0.47, size = 381, normalized size = 2.95

$$-\frac{23\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{391\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{4\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{75\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -23/8/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11-391/24/a^3/d/(
1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9-4/a^3/d/(1+tan(1/2*d*x+1/2*c
)^2)^6*tan(1/2*d*x+1/2*c)^8-75/4/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d
*x+1/2*c)^7-136/3/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6+75/
4/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5-64/a^3/d/(1+tan(1/2
*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4+391/24/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)
^6*tan(1/2*d*x+1/2*c)^3-136/5/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+
1/2*c)^2+23/8/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-68/15/a^3
/d/(1+tan(1/2*d*x+1/2*c)^2)^6-23/8/a^3/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 373, normalized size = 2.89

$$\frac{\frac{\frac{345\sin(dx+c)}{\cos(dx+c)+1}-\frac{3264\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1955\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{7680\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2250\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5440\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2250\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{480\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{1955\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{345\sin(\ldots}{(\cos(dx\ldots}}{a^3+\frac{6a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}\ldots}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/120*((345*sin(d*x + c)/(cos(d*x + c) + 1) - 3264*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 1955*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 7680*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 2250*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5440*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2250*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7 - 480*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 1955*sin(d*x + c)^9/(cos(d*x
+ c) + 1)^9 - 345*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 544)/(a^3 + 6*a^3
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 20*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 + 6*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*s
in(d*x + c)^12/(cos(d*x + c) + 1)^12) - 345*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a^3)/d
```

mupad [B] time = 11.56, size = 160, normalized size = 1.24

$$-\frac{23\,x}{16\,a^3}-\frac{\frac{23\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{391\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{75\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{136\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}-\frac{75\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+64\tan\ldots}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^3)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (23*x)/(16*a^3) - ((136*tan(c/2 + (d*x)/2)^2)/5 - (23*tan(c/2 + (d*x)/2))
/8 - (391*tan(c/2 + (d*x)/2)^3)/24 + 64*tan(c/2 + (d*x)/2)^4 - (75*tan(c/2
+ (d*x)/2)^5)/4 + (136*tan(c/2 + (d*x)/2)^6)/3 + (75*tan(c/2 + (d*x)/2)^7)/
4 + 4*tan(c/2 + (d*x)/2)^8 + (391*tan(c/2 + (d*x)/2)^9)/24 + (23*tan(c/2 +
(d*x)/2)^11)/8 + 68/15)/(a^3*d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.645 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=105

$$\frac{\cos^5(c+dx)}{5a^3d}-\frac{5\cos^3(c+dx)}{3a^3d}+\frac{4\cos(c+dx)}{a^3d}-\frac{3\sin^3(c+dx)\cos(c+dx)}{4a^3d}-\frac{13\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{13x}{8a^3}$$

```
[Out] 13/8*x/a^3+4*cos(d*x+c)/a^3/d-5/3*cos(d*x+c)^3/a^3/d+1/5*cos(d*x+c)^5/a^3/d
-13/8*cos(d*x+c)*sin(d*x+c)/a^3/d-3/4*cos(d*x+c)*sin(d*x+c)^3/a^3/d
```

Rubi [A] time = 0.22, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 2635, 8, 2633}

$$\frac{\cos^5(c+dx)}{5a^3d}-\frac{5\cos^3(c+dx)}{3a^3d}+\frac{4\cos(c+dx)}{a^3d}-\frac{3\sin^3(c+dx)\cos(c+dx)}{4a^3d}-\frac{13\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{13x}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (13*x)/(8*a^3) + (4*Cos[c + d*x])/(a^3*d) - (5*Cos[c + d*x]^3)/(3*a^3*d) +
Cos[c + d*x]^5/(5*a^3*d) - (13*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d) - (3*Co
s[c + d*x]*Sin[c + d*x]^3)/(4*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
```

```
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sin^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(a^3\sin^2(c+dx) - 3a^3\sin^3(c+dx) + 3a^3\sin^4(c+dx) - a^3\sin^5(c+dx)\right)dx}{a^6}$$

$$= \frac{\int \sin^2(c+dx)\,dx}{a^3} - \frac{\int \sin^5(c+dx)\,dx}{a^3} - \frac{3\int \sin^3(c+dx)\,dx}{a^3} + \frac{3\int \sin^4(c+dx}{a^3}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{3\cos(c+dx)\sin^3(c+dx)}{4a^3d} + \frac{\int 1\,dx}{2a^3} + \frac{9\int \sin^2(c+}{4a^3}$$

$$= \frac{x}{2a^3} + \frac{4\cos(c+dx)}{a^3d} - \frac{5\cos^3(c+dx)}{3a^3d} + \frac{\cos^5(c+dx)}{5a^3d} - \frac{13\cos(c+dx)\sin(c+}{8a^3d}$$

$$= \frac{13x}{8a^3} + \frac{4\cos(c+dx)}{a^3d} - \frac{5\cos^3(c+dx)}{3a^3d} + \frac{\cos^5(c+dx)}{5a^3d} - \frac{13\cos(c+dx)\sin(c+}{8a^3d}$$

Mathematica [B] time = 1.77, size = 310, normalized size = 2.95

$$\frac{1560dx\sin\left(\frac{c}{2}\right) - 1380\sin\left(\frac{c}{2}+dx\right) + 1380\sin\left(\frac{3c}{2}+dx\right) - 480\sin\left(\frac{3c}{2}+2dx\right) - 480\sin\left(\frac{5c}{2}+2dx\right) + 170\sin\left(\frac{5c}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (1560*d*x*Cos[c/2] + 1380*Cos[c/2 + d*x] + 1380*Cos[(3*c)/2 + d*x] - 480*Co
s[(3*c)/2 + 2*d*x] + 480*Cos[(5*c)/2 + 2*d*x] - 170*Cos[(5*c)/2 + 3*d*x] -
170*Cos[(7*c)/2 + 3*d*x] + 45*Cos[(7*c)/2 + 4*d*x] - 45*Cos[(9*c)/2 + 4*d*x
] + 6*Cos[(9*c)/2 + 5*d*x] + 6*Cos[(11*c)/2 + 5*d*x] + 10*Sin[c/2] + 1560*d
*x*Sin[c/2] - 1380*Sin[c/2 + d*x] + 1380*Sin[(3*c)/2 + d*x] - 480*Sin[(3*c)
```

```
/2 + 2*d*x] - 480*Sin[(5*c)/2 + 2*d*x] + 170*Sin[(5*c)/2 + 3*d*x] - 170*Sin
[(7*c)/2 + 3*d*x] + 45*Sin[(7*c)/2 + 4*d*x] + 45*Sin[(9*c)/2 + 4*d*x] - 6*S
in[(9*c)/2 + 5*d*x] + 6*Sin[(11*c)/2 + 5*d*x])/(960*a^3*d*(Cos[c/2] + Sin[c
/2]))
```

fricas [A] time = 0.69, size = 68, normalized size = 0.65

$$\frac{24 \cos (dx+c)^5 - 200 \cos (dx+c)^3 + 195\,dx + 15\left(6 \cos (dx+c)^3 - 19 \cos (dx+c)\right)\sin (dx+c) + 480 \cos (d}{120\,a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/120*(24*cos(d*x + c)^5 - 200*cos(d*x + c)^3 + 195*d*x + 15*(6*cos(d*x + c
)^3 - 19*cos(d*x + c))*sin(d*x + c) + 480*cos(d*x + c))/(a^3*d)
```

giac [A] time = 0.21, size = 127, normalized size = 1.21

$$\frac{\frac{195\,(dx+c)}{a^3} + \frac{2\left(195 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 750 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 720 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 + 2320 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 750 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 1520 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5 a^3}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(195*(d*x + c)/a^3 + 2*(195*tan(1/2*d*x + 1/2*c)^9 + 750*tan(1/2*d*x
+ 1/2*c)^7 + 720*tan(1/2*d*x + 1/2*c)^6 + 2320*tan(1/2*d*x + 1/2*c)^4 - 750
*tan(1/2*d*x + 1/2*c)^3 + 1520*tan(1/2*d*x + 1/2*c)^2 - 195*tan(1/2*d*x + 1
/2*c) + 304)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^3))/d
```

maple [B] time = 0.43, size = 279, normalized size = 2.66

$$\frac{13\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5} + \frac{25\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5} + \frac{12\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5} + \frac{116\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5} - \frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 13/4/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9+25/2/a^3/d/(1+ta
n(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7+12/a^3/d/(1+tan(1/2*d*x+1/2*c)^2
```

```
)^5*tan(1/2*d*x+1/2*c)^6+116/3/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x
+1/2*c)^4-25/2/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3+76/3/a
^3/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2-13/4/a^3/d/(1+tan(1/2*
d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)+76/15/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^5+1
3/4/a^3/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 1.47, size = 290, normalized size = 2.76

$$-\frac{\frac{\frac{195\sin(dx+c)}{\cos(dx+c)+1}-\frac{1520\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{750\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2320\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{720\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{750\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{195\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-304}{a^3+\frac{5a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{195\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/60*((195*sin(d*x + c)/(cos(d*x + c) + 1) - 1520*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 750*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2320*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 720*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 750*sin(d
*x + c)^7/(cos(d*x + c) + 1)^7 - 195*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
304)/(a^3 + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^
3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^3*sin(d*x + c)^10/(cos(d*x + c) +
 1)^10) - 195*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 9.03, size = 95, normalized size = 0.90

$$\frac{13\,x}{8\,a^3}+\frac{4\,\cos\left(c+d\,x\right)}{a^3\,d}-\frac{5\cos\left(c+d\,x\right)^3}{3\,a^3\,d}+\frac{\cos\left(c+d\,x\right)^5}{5\,a^3\,d}+\frac{3\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,a^3\,d}-\frac{19\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (13*x)/(8*a^3) + (4*cos(c + d*x))/(a^3*d) - (5*cos(c + d*x)^3)/(3*a^3*d) +
cos(c + d*x)^5/(5*a^3*d) + (3*cos(c + d*x)^3*sin(c + d*x))/(4*a^3*d) - (19*
cos(c + d*x)*sin(c + d*x))/(8*a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.646 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=84

$$\frac{\cos^3(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d}+\frac{\sin^3(c+dx)\cos(c+dx)}{4a^3d}+\frac{15\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{15x}{8a^3}$$

```
[Out] -15/8*x/a^3-4*cos(d*x+c)/a^3/d+cos(d*x+c)^3/a^3/d+15/8*cos(d*x+c)*sin(d*x+c
)/a^3/d+1/4*cos(d*x+c)*sin(d*x+c)^3/a^3/d
```

Rubi [A] time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2859, 2679, 2682, 2635, 8}

$$-\frac{5\cos^3(c+dx)}{4a^3d}-\frac{3\cos^5(c+dx)}{4d\left(a^3\sin(c+dx)+a^3\right)}-\frac{15\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{15x}{8a^3}-\frac{\cos^7(c+dx)}{d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-15*x)/(8*a^3) - (5*Cos[c + d*x]^3)/(4*a^3*d) - (15*Cos[c + d*x]*Sin[c + d
*x])/(8*a^3*d) - Cos[c + d*x]^7/(d*(a + a*Sin[c + d*x])^3) - (3*Cos[c + d*x
]^5)/(4*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
```

```
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\cos^7(c+dx)}{d(a+a\sin(c+dx))^3} - \frac{3\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{a}$$
$$= -\frac{\cos^7(c+dx)}{d(a+a\sin(c+dx))^3} - \frac{3\cos^5(c+dx)}{4d\left(a^3+a^3\sin(c+dx)\right)} - \frac{15\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx}{4a^2}$$
$$= -\frac{5\cos^3(c+dx)}{4a^3d} - \frac{\cos^7(c+dx)}{d(a+a\sin(c+dx))^3} - \frac{3\cos^5(c+dx)}{4d\left(a^3+a^3\sin(c+dx)\right)} - \frac{15\int \cos^2(}{4}$$
$$= -\frac{5\cos^3(c+dx)}{4a^3d} - \frac{15\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{\cos^7(c+dx)}{d(a+a\sin(c+dx))^3} - \frac{3\,c}{4d\left(a^3+\right.}$$
$$= -\frac{15x}{8a^3} - \frac{5\cos^3(c+dx)}{4a^3d} - \frac{15\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{\cos^7(c+dx)}{d(a+a\sin(c+dx))^3} - \frac{}{4}$$

Mathematica [B] time = 1.37, size = 255, normalized size = 3.04

$$-\frac{120dx\sin\left(\frac{c}{2}\right) - 104\sin\left(\frac{c}{2}+dx\right) + 104\sin\left(\frac{3c}{2}+dx\right) - 32\sin\left(\frac{3c}{2}+2dx\right) - 32\sin\left(\frac{5c}{2}+2dx\right) + 8\sin\left(\frac{5c}{2}+3dx\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/64*((1 + 120*d*x)*Cos[c/2] + 104*Cos[c/2 + d*x] + 104*Cos[(3*c)/2 + d*x]
 - 32*Cos[(3*c)/2 + 2*d*x] + 32*Cos[(5*c)/2 + 2*d*x] - 8*Cos[(5*c)/2 + 3*d*
x] - 8*Cos[(7*c)/2 + 3*d*x] + Cos[(7*c)/2 + 4*d*x] - Cos[(9*c)/2 + 4*d*x] -
 Sin[c/2] + 120*d*x*Sin[c/2] - 104*Sin[c/2 + d*x] + 104*Sin[(3*c)/2 + d*x]
- 32*Sin[(3*c)/2 + 2*d*x] - 32*Sin[(5*c)/2 + 2*d*x] + 8*Sin[(5*c)/2 + 3*d*x
] - 8*Sin[(7*c)/2 + 3*d*x] + Sin[(7*c)/2 + 4*d*x] + Sin[(9*c)/2 + 4*d*x])/(
a^3*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.72, size = 58, normalized size = 0.69

$$\frac{8\cos\left(dx+c\right)^{3}-15\,dx-\left(2\cos\left(dx+c\right)^{3}-17\cos\left(dx+c\right)\right)\sin\left(dx+c\right)-32\cos\left(dx+c\right)}{8\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*cos(d*x + c)^3 - 15*d*x - (2*cos(d*x + c)^3 - 17*cos(d*x + c))*sin(d
*x + c) - 32*cos(d*x + c))/(a^3*d)
```

giac [A] time = 0.19, size = 127, normalized size = 1.51

$$-\frac{\frac{15\,(dx+c)}{a^{3}}+\frac{2\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+23\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+72\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-23\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+88\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\tan}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}a^{3}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*(d*x + c)/a^3 + 2*(15*tan(1/2*d*x + 1/2*c)^7 + 8*tan(1/2*d*x + 1/2
*c)^6 + 23*tan(1/2*d*x + 1/2*c)^5 + 72*tan(1/2*d*x + 1/2*c)^4 - 23*tan(1/2*
d*x + 1/2*c)^3 + 88*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 24)/
((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3))/d
```

maple [B] time = 0.43, size = 279, normalized size = 3.32

$$-\frac{15\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{3}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{3}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{23\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{3}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{18\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{3}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -15/4/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-2/a^3/d/(1+tan(
1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6-23/4/a^3/d/(1+tan(1/2*d*x+1/2*c)^2
)^4*tan(1/2*d*x+1/2*c)^5-18/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/
2*c)^4+23/4/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-22/a^3/d/
(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2+15/4/a^3/d/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)-6/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^4-15/4/a^3/d
*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.44, size = 267, normalized size = 3.18

$$\frac{\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{88\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{23\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{72\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{23\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{8\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-24}{a^3+\frac{4\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{15\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 88*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 23*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 72*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 23*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 24)/(a^3 + 4
*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^8/
(cos(d*x + c) + 1)^8) - 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 9.00, size = 78, normalized size = 0.93

$$\frac{\cos\left(c+d\,x\right)^3}{a^3\,d}-\frac{4\,\cos\left(c+d\,x\right)}{a^3\,d}-\frac{15\,x}{8\,a^3}-\frac{\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,a^3\,d}+\frac{17\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] cos(c + d*x)^3/(a^3*d) - (4*cos(c + d*x))/(a^3*d) - (15*x)/(8*a^3) - (cos(c
 + d*x)^3*sin(c + d*x))/(4*a^3*d) + (17*cos(c + d*x)*sin(c + d*x))/(8*a^3*d
)
```

sympy [A] time = 166.36, size = 1246, normalized size = 14.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-15*d*x*tan(c/2 + d*x/2)**8/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a
**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c
/2 + d*x/2)**2 + 8*a**3*d) - 60*d*x*tan(c/2 + d*x/2)**6/(8*a**3*d*tan(c/2 +
 d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**4
+ 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d) - 90*d*x*tan(c/2 + d*x/2)**4/(8
*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan
(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d) - 60*d*x*tan(c
/2 + d*x/2)**2/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**
6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*
d) - 15*d*x/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**6 +
 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d)
- 30*tan(c/2 + d*x/2)**7/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2
+ d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2
 + 8*a**3*d) - 16*tan(c/2 + d*x/2)**6/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a*
*3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/
2 + d*x/2)**2 + 8*a**3*d) - 46*tan(c/2 + d*x/2)**5/(8*a**3*d*tan(c/2 + d*x/
2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*
a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d) - 144*tan(c/2 + d*x/2)**4/(8*a**3*d*
tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d
*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d) + 46*tan(c/2 + d*x/2)*
*3/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*
d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d) - 176*tan
(c/2 + d*x/2)**2/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)
**6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**
3*d) + 30*tan(c/2 + d*x/2)/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/
2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)*
*2 + 8*a**3*d) - 48/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x
/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*
a**3*d), Ne(d, 0)), (x*sin(c)*cos(c)**6/(a*sin(c) + a)**3, True))
```

3.647 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=60

$$-\frac{3\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{7x}{2a^3}$$

```
[Out] -7/2*x/a^3-arctanh(cos(d*x+c))/a^3/d-3*cos(d*x+c)/a^3/d+1/2*cos(d*x+c)*sin(
d*x+c)/a^3/d
```

Rubi [A] time = 0.15, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2869, 2757, 3770, 2638, 2635, 8}

$$-\frac{3\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{7x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-7*x)/(2*a^3) - ArcTanh[Cos[c + d*x]]/(a^3*d) - (3*Cos[c + d*x])/(a^3*d) +
 (Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
```

```
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-3a^3+a^3\csc(c+dx)+3a^3\sin(c+dx)-a^3\sin^2(c+dx)\right)dx}{a^6}$$
$$= -\frac{3x}{a^3}+\frac{\int \csc(c+dx)\,dx}{a^3}-\frac{\int \sin^2(c+dx)\,dx}{a^3}+\frac{3\int \sin(c+dx)\,dx}{a^3}$$
$$= -\frac{3x}{a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{\int 1\,d}{2a^3}$$
$$= -\frac{7x}{2a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}$$

Mathematica [A] time = 0.21, size = 63, normalized size = 1.05

$$\frac{\sin(2(c+dx))-12\cos(c+dx)-2\left(-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+7c+7dx\right)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-12*Cos[c + d*x] - 2*(7*c + 7*d*x + 2*Log[Cos[(c + d*x)/2]] - 2*Log[Sin[(c
 + d*x)/2]]) + Sin[2*(c + d*x)])/(4*a^3*d)
```

fricas [A] time = 0.66, size = 59, normalized size = 0.98

$$-\frac{7\,dx-\cos\left(dx+c\right)\sin\left(dx+c\right)+6\,\cos\left(dx+c\right)+\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(7*d*x - cos(d*x + c)*sin(d*x + c) + 6*cos(d*x + c) + log(1/2*cos(d*x
+ c) + 1/2) - log(-1/2*cos(d*x + c) + 1/2))/(a^3*d)
```

giac [A] time = 0.20, size = 89, normalized size = 1.48

$$-\frac{\frac{7\,(dx+c)}{a^{3}}-\frac{2\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{3}}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(7*(d*x + c)/a^3 - 2*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 2*(tan(1/2*d
*x + 1/2*c)^3 + 6*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 6)/((tan(
1/2*d*x + 1/2*c)^2 + 1)^2*a^3))/d
```

maple [B] time = 0.62, size = 159, normalized size = 2.65

$$-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^{3}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{6\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{3}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^{3}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{6}{a^{3}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{7\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-6/a^3/d/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+1/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^2*ta
n(1/2*d*x+1/2*c)-6/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^2-7/a^3/d*arctan(tan(1/2*
d*x+1/2*c))+1/a^3/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 161, normalized size = 2.68

$$\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-6}{a^{3}+\frac{2\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}-\frac{7\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] ((sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
 sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 6)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 7*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a^3 + log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)
/d
```

mupad [B] time = 9.44, size = 150, normalized size = 2.50

$$\frac{7\,\mathrm{atan}\left(\frac{49}{49\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+14}-\frac{14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{49\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+14}\right)}{a^3\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (7*atan(49/(49*tan(c/2 + (d*x)/2) + 14) - (14*tan(c/2 + (d*x)/2))/(49*tan(c
/2 + (d*x)/2) + 14)))/(a^3*d) + log(tan(c/2 + (d*x)/2))/(a^3*d) - (6*tan(c/
2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^3 + 6)/(d*(2*a^3*t
an(c/2 + (d*x)/2)^2 + a^3*tan(c/2 + (d*x)/2)^4 + a^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.648 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=49

$$\frac{\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{3x}{a^3}$$

```
[Out] 3*x/a^3+3*arctanh(cos(d*x+c))/a^3/d+cos(d*x+c)/a^3/d-cot(d*x+c)/a^3/d
```

Rubi [A] time = 0.16, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2869, 2757, 3770, 3767, 8, 2638}

$$\frac{\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{3x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*x)/a^3 + (3*ArcTanh[Cos[c + d*x]])/(a^3*d) + Cos[c + d*x]/(a^3*d) - Cot[
c + d*x]/(a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
```

```
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\ &= \frac{\int \left(3a^3-3a^3\csc(c+dx)+a^3\csc^2(c+dx)-a^3\sin(c+dx)\right)dx}{a^6}\\ &= \frac{3x}{a^3}+\frac{\int \csc^2(c+dx)\,dx}{a^3}-\frac{\int \sin(c+dx)\,dx}{a^3}-\frac{3\int \csc(c+dx)\,dx}{a^3}\\ &= \frac{3x}{a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx)}{a^3d}-\frac{\mathrm{Subst}(\int 1\,dx,x,\cot(c+dx))}{a^3d}\\ &= \frac{3x}{a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}\end{aligned}$$

Mathematica [B] time = 0.50, size = 106, normalized size = 2.16

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(6(c+dx)+2\cos(c+dx)+\tan\left(\frac{1}{2}(c+dx)\right)-\cot\left(\frac{1}{2}(c+dx)\right)-6\log\left(\sin\right.\right.}{2d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(6*(c + d*x) + 2*Cos[c + d*x] - Co
t[(c + d*x)/2] + 6*Log[Cos[(c + d*x)/2]] - 6*Log[Sin[(c + d*x)/2]] + Tan[(c
 + d*x)/2]))/(2*d*(a + a*Sin[c + d*x])^3)
```

fricas [A] time = 0.69, size = 82, normalized size = 1.67

$$\frac{2\,(3\,dx+\cos(dx+c))\sin(dx+c)+3\,\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-3\,\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx}{2\,a^3d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(3*d*x + cos(d*x + c))*sin(d*x + c) + 3*log(1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*cos(d*x + c)
)/(a^3*d*sin(d*x + c))
```

giac [B] time = 0.21, size = 111, normalized size = 2.27

$$\frac{\frac{6\,(dx+c)}{a^3}-\frac{6\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}+\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*(d*x + c)/a^3 - 6*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + tan(1/2*d*x +
 1/2*c)/a^3 + (2*tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + 6*tan(1/
2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a^3))/
d
```

maple [A] time = 0.66, size = 97, normalized size = 1.98

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}+\frac{2}{a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d}-\frac{1}{2d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)+2/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)+6/a^3/d*arcta
n(tan(1/2*d*x+1/2*c))-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/a^3/d*ln(tan(1/2*d*x+1
/2*c))
```

maxima [B] time = 0.43, size = 158, normalized size = 3.22

$$\frac{\frac{\frac{4 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1}{\frac{a^3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}} + \frac{12 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} - \frac{6 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} + \frac{\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/2*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 1)/(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + a^3*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3) + 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 6*log(sin(d
*x + c)/(cos(d*x + c) + 1))/a^3 + sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 9.48, size = 151, normalized size = 3.08

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d} - \frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d} - \frac{6\,\mathrm{atan}\left(\frac{36}{36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+36} - \frac{36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+36}\right)}{a^3\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 4\tan\left(\frac{c}{2}+\ldots\right.}{d\left(2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 2\,a^3\tan\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - (3*log(tan(c/2 + (d*x)/2)))/(a^3*d) - (6*ata
n(36/(36*tan(c/2 + (d*x)/2) + 36) - (36*tan(c/2 + (d*x)/2))/(36*tan(c/2 + (
d*x)/2) + 36)))/(a^3*d) - (tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) + 1)
/(d*(2*a^3*tan(c/2 + (d*x)/2)^3 + 2*a^3*tan(c/2 + (d*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.649 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=60

$$\frac{3\cot(c+dx)}{a^3d}-\frac{7\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{x}{a^3}$$

```
[Out] -x/a^3-7/2*arctanh(cos(d*x+c))/a^3/d+3*cot(d*x+c)/a^3/d-1/2*cot(d*x+c)*csc(
d*x+c)/a^3/d
```

Rubi [A] time = 0.18, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2869, 2757, 3770, 3767, 8, 3768}

$$\frac{3\cot(c+dx)}{a^3d}-\frac{7\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - (7*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cot[c + d*x])/(a^3*d) -
 (Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\ &= \frac{\int \left(-a^3+3a^3\csc(c+dx)-3a^3\csc^2(c+dx)+a^3\csc^3(c+dx)\right)dx}{a^6}\\ &= -\frac{x}{a^3}+\frac{\int \csc^3(c+dx)\,dx}{a^3}+\frac{3\int \csc(c+dx)\,dx}{a^3}-\frac{3\int \csc^2(c+dx)\,dx}{a^3}\\ &= -\frac{x}{a^3}-\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{\int \csc(c+dx)\,dx}{2a^3}+\\ &= -\frac{x}{a^3}-\frac{7\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}\end{aligned}$$

Mathematica [B] time = 0.47, size = 126, normalized size = 2.10

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(-8(c+dx)-12\tan\left(\frac{1}{2}(c+dx)\right)+12\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)+\right.}{8d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(-8*(c + d*x) + 12*Cot[(c + d*x)/2
] - Csc[(c + d*x)/2]^2 - 28*Log[Cos[(c + d*x)/2]] + 28*Log[Sin[(c + d*x)/2]
] + Sec[(c + d*x)/2]^2 - 12*Tan[(c + d*x)/2]))/(8*d*(a + a*Sin[c + d*x])^3)
```

fricas [A] time = 0.80, size = 109, normalized size = 1.82

$$-\frac{4\,dx\cos\left(dx+c\right)^{2}-4\,dx+7\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-7\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(-\frac{1}{2}\,\cos\left(dx\right.\right.}{4\left(a^{3}d\cos\left(dx+c\right)^{2}-a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/4*(4*d*x*cos(d*x + c)^2 - 4*d*x + 7*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x
 + c) + 1/2) - 7*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) + 12*cos
(d*x + c)*sin(d*x + c) - 2*cos(d*x + c))/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

giac [A] time = 0.25, size = 108, normalized size = 1.80

$$-\frac{\frac{8\,(dx+c)}{a^{3}}-\frac{28\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{3}}+\frac{42\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}-\frac{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(8*(d*x + c)/a^3 - 28*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + (42*tan(1/2
*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c)^2)
 - (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.68, size = 112, normalized size = 1.87

$$\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^{3}d}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{3}}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{3}d}-\frac{1}{8a^{3}d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{3}{2d\,a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{7\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{2a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/8/a^3/d*tan(1/2*d*x+1/2*c)^2-3/2/d/a^3*tan(1/2*d*x+1/2*c)-2/a^3/d*arctan(
tan(1/2*d*x+1/2*c))-1/8/a^3/d/tan(1/2*d*x+1/2*c)^2+3/2/d/a^3/tan(1/2*d*x+1/
2*c)+7/2/a^3/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.41, size = 138, normalized size = 2.30

$$-\frac{\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^3}+\frac{16\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{28\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(\frac{12\sin(dx+c)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)^2}{a^3\sin(dx+c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/8*((12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) +
1)^2)/a^3 + 16*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 28*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^3 - (12*sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(c
os(d*x + c) + 1)^2/(a^3*sin(d*x + c)^2))/d
```

mupad [B] time = 9.35, size = 161, normalized size = 2.68

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8a^3d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8a^3d}+\frac{2\,\mathrm{atan}\left(\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{7\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a^3d}+\frac{7\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2a^3d}+\frac{3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^3d}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) - cot(c/2 + (d*x)/2)^2/(8*a^3*d) + (2*atan((
2*cos(c/2 + (d*x)/2) - 7*sin(c/2 + (d*x)/2))/(7*cos(c/2 + (d*x)/2) + 2*sin(
c/2 + (d*x)/2))))/(a^3*d) + (7*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
(2*a^3*d) + (3*cot(c/2 + (d*x)/2))/(2*a^3*d) - (3*tan(c/2 + (d*x)/2))/(2*a^
3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.650 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=72

$$-\frac{\cot^3(c+dx)}{3a^3d}-\frac{4\cot(c+dx)}{a^3d}+\frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}$$

```
[Out] 5/2*arctanh(cos(d*x+c))/a^3/d-4*cot(d*x+c)/a^3/d-1/3*cot(d*x+c)^3/a^3/d+3/2
*cot(d*x+c)*csc(d*x+c)/a^3/d
```

Rubi [A] time = 0.19, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2869, 2757, 3770, 3767, 8, 3768}

$$-\frac{\cot^3(c+dx)}{3a^3d}-\frac{4\cot(c+dx)}{a^3d}+\frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*ArcTanh[Cos[c + d*x]])/(2*a^3*d) - (4*Cot[c + d*x])/(a^3*d) - Cot[c + d*
x]^3/(3*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\csc(c+dx)+3a^3\csc^2(c+dx)-3a^3\csc^3(c+dx)+a^3\csc^4(c+dx)\right) d}{a^6}$$
$$= -\frac{\int \csc(c+dx)\,dx}{a^3}+\frac{\int \csc^4(c+dx)\,dx}{a^3}+\frac{3\int \csc^2(c+dx)\,dx}{a^3}-\frac{3\int \csc^3(c+}{a^3}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{3\int \csc(c+dx)\,dx}{2a^3}-\frac{\text{Subs}}{}$$
$$= \frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{4\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{3\cot(c+dx)\csc(c+}{2a^3d}$$

Mathematica [A] time = 1.30, size = 115, normalized size = 1.60

$$-\frac{\csc^3(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(-18\sin(2(c+dx))+30\cos(c+dx)-22\cos(3(c+dx))-6\right.}{24a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/24*(Csc[c + d*x]^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(30*Cos[c + d
*x] - 22*Cos[3*(c + d*x)] - 60*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2
]])*Sin[c + d*x]^3 - 18*Sin[2*(c + d*x)]))/(a^3*d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.77, size = 123, normalized size = 1.71

$$-\frac{44\cos(dx+c)^3-15\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+15\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}c\right.}{12\left(a^3d\cos(dx+c)^2-a^3d\right)\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(44*cos(d*x + c)^3 - 15*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) + 15*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + 18*cos(d*x + c)*sin(d*x + c) - 48*cos(d*x + c))/((a^3*d*cos(d*x
 + c)^2 - a^3*d)*sin(d*x + c))
```

giac [A] time = 0.24, size = 128, normalized size = 1.78

$$-\frac{\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{110\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}-\frac{a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-9a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+45a^6\tan\left(\frac{1}{2}\right.}{a^9}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/24*(60*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (110*tan(1/2*d*x + 1/2*c)^3
- 45*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x
+ 1/2*c)^3) - (a^6*tan(1/2*d*x + 1/2*c)^3 - 9*a^6*tan(1/2*d*x + 1/2*c)^2 +
45*a^6*tan(1/2*d*x + 1/2*c))/a^9)/d
```

maple [A] time = 0.68, size = 132, normalized size = 1.83

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^3}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^3d}+\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^3}-\frac{15}{8d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^3d}+\frac{3}{8a^3d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d/a^3*tan(1/2*d*x+1/2*c)^3-3/8/a^3/d*tan(1/2*d*x+1/2*c)^2+15/8/d/a^3*t
an(1/2*d*x+1/2*c)-15/8/d/a^3/tan(1/2*d*x+1/2*c)-5/2/a^3/d*ln(tan(1/2*d*x+1/
2*c))+3/8/a^3/d/tan(1/2*d*x+1/2*c)^2-1/24/d/a^3/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.32, size = 153, normalized size = 2.12

$$\frac{\frac{\frac{45\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\left(\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{45\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a^3\sin(dx+c)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/24*((45*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^3 - 60*log(sin(d*x + c)/(co
s(d*x + c) + 1))/a^3 + (9*sin(d*x + c)/(cos(d*x + c) + 1) - 45*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a^3*sin(d*x + c)^3))/d
```

mupad [B] time = 9.25, size = 119, normalized size = 1.65

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^3\,d}-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}-\frac{5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^3\,d}+\frac{15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-3\,\mathrm{ta}\right.}{8\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (3*tan(c/2 + (d*x)/2)^2)/(8*a^3*d) - (5*l
og(tan(c/2 + (d*x)/2)))/(2*a^3*d) + (15*tan(c/2 + (d*x)/2))/(8*a^3*d) - (co
t(c/2 + (d*x)/2)^3*(15*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2) + 1/3))/
(8*a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.651 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^3} dx$

Optimal. Leaf size=93

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{4\cot(c+dx)}{a^3d}-\frac{15\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{15\cot(c+dx)\csc(c+dx)}{8a^3d}$$

```
[Out] -15/8*arctanh(cos(d*x+c))/a^3/d+4*cot(d*x+c)/a^3/d+cot(d*x+c)^3/a^3/d-15/8*
cot(d*x+c)*csc(d*x+c)/a^3/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a^3/d
```

Rubi [A] time = 0.20, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2869, 2757, 3767, 8, 3768, 3770}

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{4\cot(c+dx)}{a^3d}-\frac{15\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{15\cot(c+dx)\csc(c+dx)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-15*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + (4*Cot[c + d*x])/(a^3*d) + Cot[c +
d*x]^3/(a^3*d) - (15*Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) - (Cot[c + d*x]*C
sc[c + d*x]^3)/(4*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^5(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\csc^2(c+dx)+3a^3\csc^3(c+dx)-3a^3\csc^4(c+dx)+a^3\csc^5(c+dx)\right)\,d}{a^6}$$
$$= -\frac{\int \csc^2(c+dx)\,dx}{a^3}+\frac{\int \csc^5(c+dx)\,dx}{a^3}+\frac{3\int \csc^3(c+dx)\,dx}{a^3}-\frac{3\int \csc^4(c+}{a^3}$$
$$= -\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{3\int \csc^3(c+dx)\,dx}{4a^3}+$$
$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{4\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{15\cot(c+dx)\csc(c}{8a^3d}$$
$$= -\frac{15\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{4\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{15\cot(c+dx)\csc(}{8a^3d}$$

Mathematica [A] time = 2.10, size = 125, normalized size = 1.34

$$-\frac{\csc^4(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(-56\sin(2(c+dx))+46\cos(c+dx)+6(8\sin(c+dx)-5)\right.}{64a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/64*(Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(46*Cos[c + d
*x] + 120*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 +
6*Cos[3*(c + d*x)]*(-5 + 8*Sin[c + d*x]) - 56*Sin[2*(c + d*x)]))/(a^3*d*(1
+ Sin[c + d*x])^3)
```

fricas [A] time = 0.72, size = 149, normalized size = 1.60

$$\frac{30\cos\left(dx+c\right)^{3}-15\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+15\left(\cos\left(dx+c\right)^{4}-2\cos\left(\right.\right.}{16\left(a^{3}d\cos\left(dx+c\right)^{4}-2\,a^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(30*cos(d*x + c)^3 - 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/
2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2
*cos(d*x + c) + 1/2) - 16*(3*cos(d*x + c)^3 - 4*cos(d*x + c))*sin(d*x + c)
- 34*cos(d*x + c))/(a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

giac [A] time = 0.26, size = 156, normalized size = 1.68

$$\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{3}}-\frac{250\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-104\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+32\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}}+\frac{a^{9}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-8a^{9}\tan\left(\frac{1}{2}d\right.}{}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (250*tan(1/2*d*x + 1/2*c)^4
- 104*tan(1/2*d*x + 1/2*c)^3 + 32*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x +
1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c)^4) + (a^9*tan(1/2*d*x + 1/2*c)^4 - 8*
a^9*tan(1/2*d*x + 1/2*c)^3 + 32*a^9*tan(1/2*d*x + 1/2*c)^2 - 104*a^9*tan(1/
2*d*x + 1/2*c))/a^12)/d
```

maple [A] time = 0.68, size = 170, normalized size = 1.83

$$\frac{\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a^{3}d}-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{3}}+\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^{3}d}-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{3}}+\frac{13}{8d\,a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{3}d}-\frac{}{2a^{3}d\,t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/64/a^3/d*tan(1/2*d*x+1/2*c)^4-1/8/d/a^3*tan(1/2*d*x+1/2*c)^3+1/2/a^3/d*ta
n(1/2*d*x+1/2*c)^2-13/8/d/a^3*tan(1/2*d*x+1/2*c)+13/8/d/a^3/tan(1/2*d*x+1/2
*c)+15/8/a^3/d*ln(tan(1/2*d*x+1/2*c))-1/2/a^3/d/tan(1/2*d*x+1/2*c)^2-1/64/a
^3/d/tan(1/2*d*x+1/2*c)^4+1/8/d/a^3/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.32, size = 195, normalized size = 2.10

$$-\frac{\frac{\frac{104 \sin(dx+c)}{\cos(dx+c)+1}-\frac{32 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^3}-\frac{120 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(\frac{8 \sin(dx+c)}{\cos(dx+c)+1}-\frac{32 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{104 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1\right)(\cos(dx+c)-}{a^3 \sin(dx+c)^4}}{64 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/64*((104*sin(d*x + c)/(cos(d*x + c) + 1) - 32*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4)/a^3 - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (8*sin(d
*x + c)/(cos(d*x + c) + 1) - 32*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 104*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)*(cos(d*x + c) + 1)^4/(a^3*sin(d*x +
 c)^4))/d
```

mupad [B] time = 9.29, size = 151, normalized size = 1.62

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,a^3\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^3\,d}+\frac{15\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^3\,d}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(26\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^5*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(2*a^3*d) - tan(c/2 + (d*x)/2)^3/(8*a^3*d) + tan(c/2 +
 (d*x)/2)^4/(64*a^3*d) + (15*log(tan(c/2 + (d*x)/2)))/(8*a^3*d) - (13*tan(c
/2 + (d*x)/2))/(8*a^3*d) + (cot(c/2 + (d*x)/2)^4*(2*tan(c/2 + (d*x)/2) - 8*
tan(c/2 + (d*x)/2)^2 + 26*tan(c/2 + (d*x)/2)^3 - 1/4))/(16*a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.652 $\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^3} dx$

Optimal. Leaf size=114

$$-\frac{\cot^5(c+dx)}{5a^3d}-\frac{5\cot^3(c+dx)}{3a^3d}-\frac{4\cot(c+dx)}{a^3d}+\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{13\cot(c+dx}{8a}$$

```
[Out] 13/8*arctanh(cos(d*x+c))/a^3/d-4*cot(d*x+c)/a^3/d-5/3*cot(d*x+c)^3/a^3/d-1/
5*cot(d*x+c)^5/a^3/d+13/8*cot(d*x+c)*csc(d*x+c)/a^3/d+3/4*cot(d*x+c)*csc(d*
x+c)^3/a^3/d
```

Rubi [A] time = 0.18, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2708, 2757, 3768, 3770, 3767}

$$-\frac{\cot^5(c+dx)}{5a^3d}-\frac{5\cot^3(c+dx)}{3a^3d}-\frac{4\cot(c+dx)}{a^3d}+\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{13\cot(c+dx}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (13*ArcTanh[Cos[c + d*x]])/(8*a^3*d) - (4*Cot[c + d*x])/(a^3*d) - (5*Cot[c
+ d*x]^3)/(3*a^3*d) - Cot[c + d*x]^5/(5*a^3*d) + (13*Cot[c + d*x]*Csc[c + d
*x])/(8*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^3*d)
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^6(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\csc^3(c+dx)+3a^3\csc^4(c+dx)-3a^3\csc^5(c+dx)+a^3\csc^6(c+dx)\right)dx}{a^6}$$
$$= -\frac{\int \csc^3(c+dx)\,dx}{a^3}+\frac{\int \csc^6(c+dx)\,dx}{a^3}+\frac{3\int \csc^4(c+dx)\,dx}{a^3}-\frac{3\int \csc^5(c+dx)\,d}{a^3}$$
$$= \frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{\int \csc(c+dx)\,dx}{2a^3}-\frac{9\int \csc^3}{4}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{4\cot(c+dx)}{a^3d}-\frac{5\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{13\cot(c+}{}$$
$$= \frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{4\cot(c+dx)}{a^3d}-\frac{5\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{13\cot(c}{}$$

Mathematica [A] time = 1.80, size = 189, normalized size = 1.66

$$\frac{\csc^5(c+dx)\left(1500\sin(2(c+dx))-390\sin(4(c+dx))-1600\cos(c+dx)+1520\cos(3(c+dx))-304\cos(5(c-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^5*(-1600*Cos[c + d*x] + 1520*Cos[3*(c + d*x)] - 304*Cos[5*(c
+ d*x)] + 1950*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 1950*Log[Sin[(c + d*x)/
```

```
2]]*Sin[c + d*x] + 1500*Sin[2*(c + d*x)] - 975*Log[Cos[(c + d*x)/2]]*Sin[3*
(c + d*x)] + 975*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 390*Sin[4*(c + d*
x)] + 195*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 195*Log[Sin[(c + d*x)/2]
]*Sin[5*(c + d*x)]))/(1920*a^3*d)
```

fricas [A] time = 0.80, size = 179, normalized size = 1.57

$$-\frac{608\cos(dx+c)^5-1520\cos(dx+c)^3-195\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(}{240\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/240*(608*cos(d*x + c)^5 - 1520*cos(d*x + c)^3 - 195*(cos(d*x + c)^4 - 2*
cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 195*(cos(d*x
 + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
 30*(13*cos(d*x + c)^3 - 19*cos(d*x + c))*sin(d*x + c) + 960*cos(d*x + c))/
((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))
```

giac [A] time = 0.28, size = 187, normalized size = 1.64

$$-\frac{\frac{1560\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{3562\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-1380\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+480\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-170\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-6}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}-\frac{6\,a}{}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/960*(1560*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (3562*tan(1/2*d*x + 1/2*c
)^5 - 1380*tan(1/2*d*x + 1/2*c)^4 + 480*tan(1/2*d*x + 1/2*c)^3 - 170*tan(1/
2*d*x + 1/2*c)^2 + 45*tan(1/2*d*x + 1/2*c) - 6)/(a^3*tan(1/2*d*x + 1/2*c)^5
) - (6*a^12*tan(1/2*d*x + 1/2*c)^5 - 45*a^12*tan(1/2*d*x + 1/2*c)^4 + 170*a
^12*tan(1/2*d*x + 1/2*c)^3 - 480*a^12*tan(1/2*d*x + 1/2*c)^2 + 1380*a^12*ta
n(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.68, size = 208, normalized size = 1.82

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{160d\,a^3}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a^3d}+\frac{17\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d\,a^3}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3d}+\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^3}-\frac{23}{16d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{13}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/160/d/a^3*tan(1/2*d*x+1/2*c)^5-3/64/a^3/d*tan(1/2*d*x+1/2*c)^4+17/96/d/a^
3*tan(1/2*d*x+1/2*c)^3-1/2/a^3/d*tan(1/2*d*x+1/2*c)^2+23/16/d/a^3*tan(1/2*d
*x+1/2*c)-23/16/d/a^3/tan(1/2*d*x+1/2*c)-13/8/a^3/d*ln(tan(1/2*d*x+1/2*c))-
1/160/d/a^3/tan(1/2*d*x+1/2*c)^5+1/2/a^3/d/tan(1/2*d*x+1/2*c)^2+3/64/a^3/d/
tan(1/2*d*x+1/2*c)^4-17/96/d/a^3/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.32, size = 234, normalized size = 2.05

$$\frac{\frac{\frac{1380 \sin(dx+c)}{\cos(dx+c)+1} - \frac{480 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{170 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{45 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{6 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{1560 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} + \frac{\left(\frac{45 \sin(dx+c)}{\cos(dx+c)+1} - \frac{170 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{480 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \cdots}{a^3 \sin(dx+ \cdots}}{960\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/960*((1380*sin(d*x + c)/(cos(d*x + c) + 1) - 480*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 170*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 45*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 1560*log
(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + (45*sin(d*x + c)/(cos(d*x + c) + 1)
 - 170*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 480*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 1380*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6)*(cos(d*x + c) + 1
)^5/(a^3*sin(d*x + c)^5))/d
```

mupad [B] time = 9.85, size = 291, normalized size = 2.55

$$-\frac{6 \cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{10} - 6 \sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{10} + 45\,\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right) \sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{9} - 45 \cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{9}\,\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right) - 170\,\mathrm{c} \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^6*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(6*cos(c/2 + (d*x)/2)^10 - 6*sin(c/2 + (d*x)/2)^10 + 45*cos(c/2 + (d*x)/2)
*sin(c/2 + (d*x)/2)^9 - 45*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) - 170*co
s(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 480*cos(c/2 + (d*x)/2)^3*sin(c/2
+ (d*x)/2)^7 - 1380*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 + 1380*cos(c/
2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 - 480*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d
*x)/2)^3 + 170*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 1560*log(sin(c/2
```

```
 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)/
(960*a^3*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.653 $\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=267

$$\frac{a^3\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{3a^3\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] a^3*cos(d*x+c)*hypergeom([-5/2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*
x+c)^(1+n)/d/(1+n)/(cos(d*x+c)^2)^(1/2)+3*a^3*cos(d*x+c)*hypergeom([-5/2, 1
+1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)/(cos(d*x+c)^2)^(1/
2)+3*a^3*cos(d*x+c)*hypergeom([-5/2, 3/2+1/2*n],[5/2+1/2*n],sin(d*x+c)^2)*s
in(d*x+c)^(3+n)/d/(3+n)/(cos(d*x+c)^2)^(1/2)+a^3*cos(d*x+c)*hypergeom([-5/2
, 1/2*n+2],[1/2*n+3],sin(d*x+c)^2)*sin(d*x+c)^(4+n)/d/(4+n)/(cos(d*x+c)^2)^
(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2873, 2577}

$$\frac{a^3\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{3a^3\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (3*a^3*Cos[c
+ d*x]*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2]) + (3*a^3*Cos[c + d*x]*Hype
rgeometric2F1[-5/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3 +
 n))/(d*(3 + n)*Sqrt[Cos[c + d*x]^2]) + (a^3*Cos[c + d*x]*Hypergeometric2F1
[-5/2, (4 + n)/2, (6 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(4 + n))/(d*(4 +
n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^6(c+dx)\sin^n(c+dx)+3a^3\cos^6(c+dx)\sin^{1+n}(c\right.$$

$$= a^3\int \cos^6(c+dx)\sin^n(c+dx)\,dx + a^3\int \cos^6(c+dx)\sin^{3}$$

$$= \frac{a^3\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+d}{d(1+n)\sqrt{\cos^2(c+dx)}}$$

Mathematica [A] time = 0.62, size = 188, normalized size = 0.70

$$\frac{a^3\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\sin^{n+1}(c+dx)\left(\frac{{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{n+1}+\sin(c+dx)\left(\frac{3\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{n+2}+\sin\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*(Hypergeometric
2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]/(1 + n) + Sin[c + d*x]*((3*
Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2])/(2 + n) + Si
n[c + d*x]*((3*Hypergeometric2F1[-5/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2
])/(3 + n) + (Hypergeometric2F1[-5/2, (4 + n)/2, (6 + n)/2, Sin[c + d*x]^2]
*Sin[c + d*x])/(4 + n)))))/d
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,a^3\cos(dx+c)^8-4\,a^3\cos(dx+c)^6+\left(a^3\cos(dx+c)^8-4\,a^3\cos(dx+c)^6\right)\sin(dx+c)\right)\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + (a^3*cos(d*x + c)^
8 - 4*a^3*cos(d*x + c)^6)*sin(d*x + c))*sin(d*x + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^3\sin(dx+c)^n\cos(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^3*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

maple [F] time = 27.62, size = 0, normalized size = 0.00

$$\int \left(\cos^6(dx+c)\right)\left(\sin^n(dx+c)\right)(a+a\sin(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
[Out] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^3\sin(dx+c)^n\cos(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
[Out] integrate((a*sin(d*x + c) + a)^3*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^6\sin(c+d\,x)^n\,(a+a\,\sin(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^n*(a + a*sin(c + d*x))^3,x)
[Out] int(cos(c + d*x)^6*sin(c + d*x)^n*(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**n*(a+a*sin(d*x+c))**3,x)
[Out] Timed out
```

3.654 $\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=200

$$\frac{a^2\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{2a^2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};s\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] a^2*cos(d*x+c)*hypergeom([-5/2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*
x+c)^(1+n)/d/(1+n)/(cos(d*x+c)^2)^(1/2)+2*a^2*cos(d*x+c)*hypergeom([-5/2, 1
+1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)/(cos(d*x+c)^2)^(1/
2)+a^2*cos(d*x+c)*hypergeom([-5/2, 3/2+1/2*n],[5/2+1/2*n],sin(d*x+c)^2)*sin
(d*x+c)^(3+n)/d/(3+n)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2873, 2577}

$$\frac{a^2\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{2a^2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};s\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (2*a^2*Cos[c
+ d*x]*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2]) + (a^2*Cos[c + d*x]*Hyperg
eometric2F1[-5/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3 + n
))/(d*(3 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
```

```
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^6(c+dx)\sin^n(c+dx)+2a^2\cos^6(c+dx)\sin^{1+n}\right.$$
$$= a^2\int \cos^6(c+dx)\sin^n(c+dx)\,dx + a^2\int \cos^6(c+dx)\sin$$
$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+}{d(1+n)\sqrt{\cos^2(c+dx)}}$$

Mathematica [A] time = 0.29, size = 164, normalized size = 0.82

$$\frac{a^2\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\sin^{n+1}(c+dx)\left(\left(n^2+5n+6\right){}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+(n+1)\sin(c+dx\right.}{d(n+1)(n+2)(n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*((6 + 5*n + n^2
)*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2] + (1 + n)*S
in[c + d*x]*(2*(3 + n)*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c
+ d*x]^2] + (2 + n)*Hypergeometric2F1[-5/2, (3 + n)/2, (5 + n)/2, Sin[c + d
*x]^2]*Sin[c + d*x])))/(d*(1 + n)*(2 + n)*(3 + n))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos(dx+c)^8-2\,a^2\cos(dx+c)^6\sin(dx+c)-2\,a^2\cos(dx+c)^6\right)\sin(dx+c)^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral(-(a^2*cos(d*x + c)^8 - 2*a^2*cos(d*x + c)^6*sin(d*x + c) - 2*a^2*c
os(d*x + c)^6)*sin(d*x + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^2\sin(dx+c)^n\cos(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

maple [F] time = 20.09, size = 0, normalized size = 0.00

$$\int \left(\cos^{6}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(dx+c\right)+a\right)^{2}\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{6}\,\sin\left(c+d\,x\right)^{n}\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^n*(a + a*sin(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^6*sin(c + d*x)^n*(a + a*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.655 $\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{a\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\sin\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] a*cos(d*x+c)*hypergeom([-5/2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+
c)^(1+n)/d/(1+n)/(cos(d*x+c)^2)^(1/2)+a*cos(d*x+c)*hypergeom([-5/2, 1+1/2*n
],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2838, 2577}

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{a\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\sin\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^
2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (a*Cos[c + d*x]
*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^6(c+dx)\sin^n(c+dx)\,dx + a\int \cos^6(c+dx)\sin^{1+n}($$

$$= \frac{a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)\sqrt{\cos^2(c+dx)}}$$

Mathematica [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
[Out] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x]), x]
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a\cos\left(dx+c\right)^6\sin\left(dx+c\right)+a\cos\left(dx+c\right)^6\right)\sin\left(dx+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((a*cos(d*x + c)^6*sin(d*x + c) + a*cos(d*x + c)^6)*sin(d*x + c)^n,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)^n\cos\left(dx+c\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

maple [F] time = 11.87, size = 0, normalized size = 0.00

$$\int \left(\cos^6\left(dx+c\right)\right)\left(\sin^n\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sin(dx + c) + a) \sin(dx + c)^n \cos(dx + c)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + d\,x)^6 \sin(c + d\,x)^n \, (a + a\,\sin(c + d\,x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^n*(a + a*sin(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^6*sin(c + d*x)^n*(a + a*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.656 $\int \cos^7(c+dx)\sin^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$-\frac{a\sin^{14}(c+dx)}{14d}-\frac{a\sin^{13}(c+dx)}{13d}+\frac{a\sin^{12}(c+dx)}{4d}+\frac{3a\sin^{11}(c+dx)}{11d}-\frac{3a\sin^{10}(c+dx)}{10d}-\frac{a\sin^{9}(c+dx)}{3d}+\frac{a\sin^{8}(c+}{8d}$$

```
[Out] 1/7*a*sin(d*x+c)^7/d+1/8*a*sin(d*x+c)^8/d-1/3*a*sin(d*x+c)^9/d-3/10*a*sin(d
*x+c)^10/d+3/11*a*sin(d*x+c)^11/d+1/4*a*sin(d*x+c)^12/d-1/13*a*sin(d*x+c)^1
3/d-1/14*a*sin(d*x+c)^14/d
```

Rubi [A] time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\sin^{14}(c+dx)}{14d}-\frac{a\sin^{13}(c+dx)}{13d}+\frac{a\sin^{12}(c+dx)}{4d}+\frac{3a\sin^{11}(c+dx)}{11d}-\frac{3a\sin^{10}(c+dx)}{10d}-\frac{a\sin^{9}(c+dx)}{3d}+\frac{a\sin^{8}(c+}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c + d*x]^8)/(8*d) - (a*Sin[c + d*x]^9)/(3
*d) - (3*a*Sin[c + d*x]^10)/(10*d) + (3*a*Sin[c + d*x]^11)/(11*d) + (a*Sin[
c + d*x]^12)/(4*d) - (a*Sin[c + d*x]^13)/(13*d) - (a*Sin[c + d*x]^14)/(14*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)\sin^6(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3x^6(a+x)^4}{a^6}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^3x^6(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^{13}d}$$
$$= \frac{\text{Subst}\left(\int \left(a^7x^6+a^6x^7-3a^5x^8-3a^4x^9+3a^3x^{10}+3a^2x^{11}\right.\right.}{a^{13}d}$$
$$= \frac{a\sin^7(c+dx)}{7d}+\frac{a\sin^8(c+dx)}{8d}-\frac{a\sin^9(c+dx)}{3d}-\frac{3a\sin^{10}}{10}$$

Mathematica [A] time = 1.03, size = 117, normalized size = 0.91

$$-\frac{a(-1201200\sin(c+dx)+300300\sin(3(c+dx))+180180\sin(5(c+dx))-51480\sin(7(c+dx))-40040\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/246005760*(a*(525525*Cos[2*(c + d*x)] - 105105*Cos[6*(c + d*x)] + 21021*
Cos[10*(c + d*x)] - 2145*Cos[14*(c + d*x)] - 1201200*Sin[c + d*x] + 300300*
Sin[3*(c + d*x)] + 180180*Sin[5*(c + d*x)] - 51480*Sin[7*(c + d*x)] - 40040
*Sin[9*(c + d*x)] + 5460*Sin[11*(c + d*x)] + 4620*Sin[13*(c + d*x)]))/d
```

fricas [A] time = 0.73, size = 128, normalized size = 0.99

$$\frac{8580\,a\cos(dx+c)^{14}-30030\,a\cos(dx+c)^{12}+36036\,a\cos(dx+c)^{10}-15015\,a\cos(dx+c)^{8}-40\left(231\,a\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120120*(8580*a*cos(d*x + c)^14 - 30030*a*cos(d*x + c)^12 + 36036*a*cos(d*
x + c)^10 - 15015*a*cos(d*x + c)^8 - 40*(231*a*cos(d*x + c)^12 - 567*a*cos(
d*x + c)^10 + 371*a*cos(d*x + c)^8 - 5*a*cos(d*x + c)^6 - 6*a*cos(d*x + c)^
4 - 8*a*cos(d*x + c)^2 - 16*a)*sin(d*x + c))/d
```

giac [A] time = 0.41, size = 163, normalized size = 1.26

$$\frac{a\cos(14\,dx+14\,c)}{114688\,d}-\frac{7\,a\cos(10\,dx+10\,c)}{81920\,d}+\frac{7\,a\cos(6\,dx+6\,c)}{16384\,d}-\frac{35\,a\cos(2\,dx+2\,c)}{16384\,d}-\frac{a\sin(13\,dx+13\,c)}{53248\,d}-\frac{a\sin(}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/114688*a*cos(14*d*x + 14*c)/d - 7/81920*a*cos(10*d*x + 10*c)/d + 7/16384*
a*cos(6*d*x + 6*c)/d - 35/16384*a*cos(2*d*x + 2*c)/d - 1/53248*a*sin(13*d*x
 + 13*c)/d - 1/45056*a*sin(11*d*x + 11*c)/d + 1/6144*a*sin(9*d*x + 9*c)/d +
 3/14336*a*sin(7*d*x + 7*c)/d - 3/4096*a*sin(5*d*x + 5*c)/d - 5/4096*a*sin(
3*d*x + 3*c)/d + 5/1024*a*sin(d*x + c)/d
```

maple [A] time = 0.24, size = 166, normalized size = 1.29

$$\frac{a\left(-\frac{\left(\sin^{6}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{14}-\frac{\left(\sin^{4}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{28}-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{70}-\frac{\left(\cos^{8}(dx+c)\right)}{280}\right)+a\left(-\frac{\left(\sin^{5}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{13}-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/14*sin(d*x+c)^6*cos(d*x+c)^8-1/28*sin(d*x+c)^4*cos(d*x+c)^8-1/70
*sin(d*x+c)^2*cos(d*x+c)^8-1/280*cos(d*x+c)^8)+a*(-1/13*sin(d*x+c)^5*cos(d*
x+c)^8-5/143*sin(d*x+c)^3*cos(d*x+c)^8-5/429*sin(d*x+c)*cos(d*x+c)^8+5/3003
*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.32, size = 94, normalized size = 0.73

$$-\frac{8580\,a\sin(dx+c)^{14}+9240\,a\sin(dx+c)^{13}-30030\,a\sin(dx+c)^{12}-32760\,a\sin(dx+c)^{11}+36036\,a\sin(dx}{120120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/120120*(8580*a*sin(d*x + c)^14 + 9240*a*sin(d*x + c)^13 - 30030*a*sin(d*
x + c)^12 - 32760*a*sin(d*x + c)^11 + 36036*a*sin(d*x + c)^10 + 40040*a*sin
(d*x + c)^9 - 15015*a*sin(d*x + c)^8 - 17160*a*sin(d*x + c)^7)/d
```

mupad [B] time = 0.10, size = 93, normalized size = 0.72

$$\frac{-\frac{a\sin(c+d\,x)^{14}}{14}-\frac{a\sin(c+d\,x)^{13}}{13}+\frac{a\sin(c+d\,x)^{12}}{4}+\frac{3\,a\sin(c+d\,x)^{11}}{11}-\frac{3\,a\sin(c+d\,x)^{10}}{10}-\frac{a\sin(c+d\,x)^{9}}{3}+\frac{a\sin(c+d\,x)^{8}}{8}+\frac{a\sin(c+d\,x)^{7}}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^6*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^7)/7 + (a*sin(c + d*x)^8)/8 - (a*sin(c + d*x)^9)/3 - (3*a*
sin(c + d*x)^10)/10 + (3*a*sin(c + d*x)^11)/11 + (a*sin(c + d*x)^12)/4 - (a
*sin(c + d*x)^13)/13 - (a*sin(c + d*x)^14)/14)/d
```

sympy [A] time = 117.15, size = 184, normalized size = 1.43

$$\begin{cases} \frac{16a\sin^{13}(c+dx)}{3003d} + \frac{8a\sin^{11}(c+dx)\cos^{2}(c+dx)}{231d} + \frac{2a\sin^{9}(c+dx)\cos^{4}(c+dx)}{21d} + \frac{a\sin^{7}(c+dx)\cos^{6}(c+dx)}{7d} - \frac{a\sin^{6}(c+dx)\cos^{8}(c+dx)}{8d} - \frac{3a\sin \\ x\left(a\sin(c)+a\right)\sin^{6}(c)\cos^{7}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**13/(3003*d) + 8*a*sin(c + d*x)**11*cos(c + d*
x)**2/(231*d) + 2*a*sin(c + d*x)**9*cos(c + d*x)**4/(21*d) + a*sin(c + d*x)
**7*cos(c + d*x)**6/(7*d) - a*sin(c + d*x)**6*cos(c + d*x)**8/(8*d) - 3*a*s
in(c + d*x)**4*cos(c + d*x)**10/(40*d) - a*sin(c + d*x)**2*cos(c + d*x)**12
/(40*d) - a*cos(c + d*x)**14/(280*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**
6*cos(c)**7, True))
```

3.657 $\int \cos^7(c+dx)\sin^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=113

$$-\frac{a\sin^{13}(c+dx)}{13d}+\frac{3a\sin^{11}(c+dx)}{11d}-\frac{a\sin^9(c+dx)}{3d}+\frac{a\sin^7(c+dx)}{7d}-\frac{a\cos^{12}(c+dx)}{12d}+\frac{a\cos^{10}(c+dx)}{5d}-\frac{a\cos^8(c+}{8d}$$

```
[Out] -1/8*a*cos(d*x+c)^8/d+1/5*a*cos(d*x+c)^10/d-1/12*a*cos(d*x+c)^12/d+1/7*a*si
n(d*x+c)^7/d-1/3*a*sin(d*x+c)^9/d+3/11*a*sin(d*x+c)^11/d-1/13*a*sin(d*x+c)^
13/d
```

Rubi [A] time = 0.14, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2834, 2565, 266, 43, 2564, 270}

$$-\frac{a\sin^{13}(c+dx)}{13d}+\frac{3a\sin^{11}(c+dx)}{11d}-\frac{a\sin^9(c+dx)}{3d}+\frac{a\sin^7(c+dx)}{7d}-\frac{a\cos^{12}(c+dx)}{12d}+\frac{a\cos^{10}(c+dx)}{5d}-\frac{a\cos^8(c+}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Cos[c + d*x]^10)/(5*d) - (a*Cos[c + d*x]^12)
/(12*d) + (a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^9)/(3*d) + (3*a*Sin[c
+ d*x]^11)/(11*d) - (a*Sin[c + d*x]^13)/(13*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)\sin^5(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^7(c+dx)\sin^5(c+dx)\,dx + a\int \cos^7(c+dx)\sin^6(c\\ &= -\frac{a\,\mathrm{Subst}\left(\int x^7\left(1-x^2\right)^2\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\mathrm{Subst}\left(\int x\right.}{}\\ &= -\frac{a\,\mathrm{Subst}\left(\int (1-x)^2x^3\,dx, x, \cos^2(c+dx)\right)}{2d} + \frac{a\,\mathrm{Subst}\left(\int \left(x\right.\right.}{}\\ &= \frac{a\sin^7(c+dx)}{7d} - \frac{a\sin^9(c+dx)}{3d} + \frac{3a\sin^{11}(c+dx)}{11d} - \frac{a\sin^1}{}\\ &= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\cos^{10}(c+dx)}{5d} - \frac{a\cos^{12}(c+dx)}{12d} + \frac{a\,\mathrm{si}}{}\end{aligned}$$

Mathematica [A] time = 0.73, size = 137, normalized size = 1.21

$$-\frac{a(-600600\sin(c+dx)+150150\sin(3(c+dx))+90090\sin(5(c+dx))-25740\sin(7(c+dx))-20020\sin(9(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/123002880*(a*(600600*Cos[2*(c + d*x)] + 75075*Cos[4*(c + d*x)] - 100100*
Cos[6*(c + d*x)] - 30030*Cos[8*(c + d*x)] + 12012*Cos[10*(c + d*x)] + 5005*
Cos[12*(c + d*x)] - 600600*Sin[c + d*x] + 150150*Sin[3*(c + d*x)] + 90090*S
in[5*(c + d*x)] - 25740*Sin[7*(c + d*x)] - 20020*Sin[9*(c + d*x)] + 2730*Si
n[11*(c + d*x)] + 2310*Sin[13*(c + d*x)]))/d
```

fricas [A] time = 0.60, size = 117, normalized size = 1.04

$$-\frac{10010\,a\cos\left(dx+c\right)^{12}-24024\,a\cos\left(dx+c\right)^{10}+15015\,a\cos\left(dx+c\right)^{8}+40\left(231\,a\cos\left(dx+c\right)^{12}-567\,a\cos\left(dx\right.\right.}{120120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/120120*(10010*a*cos(d*x + c)^12 - 24024*a*cos(d*x + c)^10 + 15015*a*cos(
d*x + c)^8 + 40*(231*a*cos(d*x + c)^12 - 567*a*cos(d*x + c)^10 + 371*a*cos(
d*x + c)^8 - 5*a*cos(d*x + c)^6 - 6*a*cos(d*x + c)^4 - 8*a*cos(d*x + c)^2 -
 16*a)*sin(d*x + c))/d
```

giac [A] time = 0.37, size = 193, normalized size = 1.71

$$-\frac{a\cos\left(12\,dx+12\,c\right)}{24576\,d}-\frac{a\cos\left(10\,dx+10\,c\right)}{10240\,d}+\frac{a\cos\left(8\,dx+8\,c\right)}{4096\,d}+\frac{5\,a\cos\left(6\,dx+6\,c\right)}{6144\,d}-\frac{5\,a\cos\left(4\,dx+4\,c\right)}{8192\,d}-\frac{5\,a\cos\left(2\right.}{102}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24576*a*cos(12*d*x + 12*c)/d - 1/10240*a*cos(10*d*x + 10*c)/d + 1/4096*a
*cos(8*d*x + 8*c)/d + 5/6144*a*cos(6*d*x + 6*c)/d - 5/8192*a*cos(4*d*x + 4*
c)/d - 5/1024*a*cos(2*d*x + 2*c)/d - 1/53248*a*sin(13*d*x + 13*c)/d - 1/450
56*a*sin(11*d*x + 11*c)/d + 1/6144*a*sin(9*d*x + 9*c)/d + 3/14336*a*sin(7*d
*x + 7*c)/d - 3/4096*a*sin(5*d*x + 5*c)/d - 5/4096*a*sin(3*d*x + 3*c)/d + 5
/1024*a*sin(d*x + c)/d
```

maple [A] time = 0.23, size = 148, normalized size = 1.31

$$\frac{a\left(-\frac{\left(\sin^{5}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{13}-\frac{5\left(\sin^{3}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{143}-\frac{5\sin(dx+c)\left(\cos^{8}(dx+c)\right)}{429}+\frac{5\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}(dx+c)\right)}{5}+\frac{8\left(\cos^{2}(dx+c)\right)}{5}\right)\sin(dx\right.}{3003}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/13*sin(d*x+c)^5*cos(d*x+c)^8-5/143*sin(d*x+c)^3*cos(d*x+c)^8-5/4
29*sin(d*x+c)*cos(d*x+c)^8+5/3003*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*c
os(d*x+c)^2)*sin(d*x+c))+a*(-1/12*sin(d*x+c)^4*cos(d*x+c)^8-1/30*sin(d*x+c)
^2*cos(d*x+c)^8-1/120*cos(d*x+c)^8))
```

maxima [A] time = 0.32, size = 94, normalized size = 0.83

$$-\frac{9240\,a\sin\left(dx+c\right)^{13}+10010\,a\sin\left(dx+c\right)^{12}-32760\,a\sin\left(dx+c\right)^{11}-36036\,a\sin\left(dx+c\right)^{10}+40040\,a\sin(\ldots}{120120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/120120*(9240*a*sin(d*x + c)^13 + 10010*a*sin(d*x + c)^12 - 32760*a*sin(d
*x + c)^11 - 36036*a*sin(d*x + c)^10 + 40040*a*sin(d*x + c)^9 + 45045*a*sin
(d*x + c)^8 - 17160*a*sin(d*x + c)^7 - 20020*a*sin(d*x + c)^6)/d
```

mupad [B] time = 8.82, size = 93, normalized size = 0.82

$$\frac{-\frac{a\sin(c+d\,x)^{13}}{13}-\frac{a\sin(c+d\,x)^{12}}{12}+\frac{3\,a\sin(c+d\,x)^{11}}{11}+\frac{3\,a\sin(c+d\,x)^{10}}{10}-\frac{a\sin(c+d\,x)^{9}}{3}-\frac{3\,a\sin(c+d\,x)^{8}}{8}+\frac{a\sin(c+d\,x)^{7}}{7}+\frac{a\sin(c+d\,x)^{6}}{6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^5*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^6)/6 + (a*sin(c + d*x)^7)/7 - (3*a*sin(c + d*x)^8)/8 - (a*
sin(c + d*x)^9)/3 + (3*a*sin(c + d*x)^10)/10 + (3*a*sin(c + d*x)^11)/11 - (
a*sin(c + d*x)^12)/12 - (a*sin(c + d*x)^13)/13)/d
```

sympy [A] time = 83.46, size = 160, normalized size = 1.42

$$\begin{cases}\frac{16a\sin^{13}(c+dx)}{3003d}+\frac{8a\sin^{11}(c+dx)\cos^{2}(c+dx)}{231d}+\frac{2a\sin^{9}(c+dx)\cos^{4}(c+dx)}{21d}+\frac{a\sin^{7}(c+dx)\cos^{6}(c+dx)}{7d}-\frac{a\sin^{4}(c+dx)\cos^{8}(c+dx)}{8d}-\frac{a\sin^{2}\ldots}{\ldots} \\ x\left(a\sin(c)+a\right)\sin^{5}(c)\cos^{7}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**13/(3003*d) + 8*a*sin(c + d*x)**11*cos(c + d*
x)**2/(231*d) + 2*a*sin(c + d*x)**9*cos(c + d*x)**4/(21*d) + a*sin(c + d*x)
**7*cos(c + d*x)**6/(7*d) - a*sin(c + d*x)**4*cos(c + d*x)**8/(8*d) - a*sin
(c + d*x)**2*cos(c + d*x)**10/(20*d) - a*cos(c + d*x)**12/(120*d), Ne(d, 0)
), (x*(a*sin(c) + a)*sin(c)**5*cos(c)**7, True))
```

3.658 $\int \cos^7(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=113

$$-\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^{9}(c+dx)}{3d}-\frac{3a\sin^{7}(c+dx)}{7d}+\frac{a\sin^{5}(c+dx)}{5d}-\frac{a\cos^{12}(c+dx)}{12d}+\frac{a\cos^{10}(c+dx)}{5d}-\frac{a\cos^{8}(c+dx)}{8d}$$

```
[Out] -1/8*a*cos(d*x+c)^8/d+1/5*a*cos(d*x+c)^10/d-1/12*a*cos(d*x+c)^12/d+1/5*a*si
n(d*x+c)^5/d-3/7*a*sin(d*x+c)^7/d+1/3*a*sin(d*x+c)^9/d-1/11*a*sin(d*x+c)^11
/d
```

Rubi [A] time = 0.14, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2834, 2564, 270, 2565, 266, 43}

$$-\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^{9}(c+dx)}{3d}-\frac{3a\sin^{7}(c+dx)}{7d}+\frac{a\sin^{5}(c+dx)}{5d}-\frac{a\cos^{12}(c+dx)}{12d}+\frac{a\cos^{10}(c+dx)}{5d}-\frac{a\cos^{8}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Cos[c + d*x]^10)/(5*d) - (a*Cos[c + d*x]^12)
/(12*d) + (a*Sin[c + d*x]^5)/(5*d) - (3*a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c
+ d*x]^9)/(3*d) - (a*Sin[c + d*x]^11)/(11*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^7(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^7(c+dx)\sin^4(c+dx)\,dx + a\int \cos^7(c+dx)\sin^5(c$$

$$= -\frac{a\,\text{Subst}\left(\int x^7\left(1-x^2\right)^2\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x\right.}{}$$

$$= -\frac{a\,\text{Subst}\left(\int (1-x)^2x^3\,dx, x, \cos^2(c+dx)\right)}{2d} + \frac{a\,\text{Subst}\left(\int \left(x\right.\right.}{}$$

$$= \frac{a\sin^5(c+dx)}{5d} - \frac{3a\sin^7(c+dx)}{7d} + \frac{a\sin^9(c+dx)}{3d} - \frac{a\sin^{11}}{1}$$

$$= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\cos^{10}(c+dx)}{5d} - \frac{a\cos^{12}(c+dx)}{12d} + \frac{a\,\text{si}}{}$$

Mathematica [A] time = 0.56, size = 127, normalized size = 1.12

$$-\frac{a(-129360\sin(c+dx)+18480\sin(3(c+dx))+20328\sin(5(c+dx))+1320\sin(7(c+dx))-3080\sin(9(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/9461760*(a*(46200*Cos[2*(c + d*x)] + 5775*Cos[4*(c + d*x)] - 7700*Cos[6*
(c + d*x)] - 2310*Cos[8*(c + d*x)] + 924*Cos[10*(c + d*x)] + 385*Cos[12*(c
+ d*x)] - 129360*Sin[c + d*x] + 18480*Sin[3*(c + d*x)] + 20328*Sin[5*(c + d
*x)] + 1320*Sin[7*(c + d*x)] - 3080*Sin[9*(c + d*x)] - 840*Sin[11*(c + d*x)
]))/d
```

fricas [A] time = 0.86, size = 106, normalized size = 0.94

$$-\frac{770\,a\cos\left(dx+c\right)^{12}-1848\,a\cos\left(dx+c\right)^{10}+1155\,a\cos\left(dx+c\right)^{8}-8\left(105\,a\cos\left(dx+c\right)^{10}-140\,a\cos\left(dx+c\right)^{8}\right.}{9240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/9240*(770*a*cos(d*x + c)^12 - 1848*a*cos(d*x + c)^10 + 1155*a*cos(d*x +
c)^8 - 8*(105*a*cos(d*x + c)^10 - 140*a*cos(d*x + c)^8 + 5*a*cos(d*x + c)^6
 + 6*a*cos(d*x + c)^4 + 8*a*cos(d*x + c)^2 + 16*a)*sin(d*x + c))/d
```

giac [A] time = 0.32, size = 178, normalized size = 1.58

$$-\frac{a\cos\left(12\,dx+12\,c\right)}{24576\,d}-\frac{a\cos\left(10\,dx+10\,c\right)}{10240\,d}+\frac{a\cos\left(8\,dx+8\,c\right)}{4096\,d}+\frac{5\,a\cos\left(6\,dx+6\,c\right)}{6144\,d}-\frac{5\,a\cos\left(4\,dx+4\,c\right)}{8192\,d}-\frac{5\,a\cos\left(2\right.}{102}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24576*a*cos(12*d*x + 12*c)/d - 1/10240*a*cos(10*d*x + 10*c)/d + 1/4096*a
*cos(8*d*x + 8*c)/d + 5/6144*a*cos(6*d*x + 6*c)/d - 5/8192*a*cos(4*d*x + 4*
c)/d - 5/1024*a*cos(2*d*x + 2*c)/d + 1/11264*a*sin(11*d*x + 11*c)/d + 1/307
2*a*sin(9*d*x + 9*c)/d - 1/7168*a*sin(7*d*x + 7*c)/d - 11/5120*a*sin(5*d*x
+ 5*c)/d - 1/512*a*sin(3*d*x + 3*c)/d + 7/512*a*sin(d*x + c)/d
```

maple [A] time = 0.24, size = 130, normalized size = 1.15

$$\frac{a\left(-\frac{\left(\sin^{4}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{12}-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{30}-\frac{\left(\cos^{8}(dx+c)\right)}{120}\right)+a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{11}-\frac{\sin(dx+c)\left(\cos^{8}(dx+c)\right)}{33}+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/12*sin(d*x+c)^4*cos(d*x+c)^8-1/30*sin(d*x+c)^2*cos(d*x+c)^8-1/12
0*cos(d*x+c)^8)+a*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/33*sin(d*x+c)*cos(d*x+
c)^8+1/231*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
))
```

maxima [A] time = 0.31, size = 94, normalized size = 0.83

$$-\frac{770\,a\sin\left(dx+c\right)^{12}+840\,a\sin\left(dx+c\right)^{11}-2772\,a\sin\left(dx+c\right)^{10}-3080\,a\sin\left(dx+c\right)^{9}+3465\,a\sin\left(dx+c\right)^{8}}{9240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/9240*(770*a*sin(d*x + c)^12 + 840*a*sin(d*x + c)^11 - 2772*a*sin(d*x + c
)^10 - 3080*a*sin(d*x + c)^9 + 3465*a*sin(d*x + c)^8 + 3960*a*sin(d*x + c)^
7 - 1540*a*sin(d*x + c)^6 - 1848*a*sin(d*x + c)^5)/d
```

mupad [B] time = 0.08, size = 93, normalized size = 0.82

$$\frac{-\frac{a\sin(c+d\,x)^{12}}{12}-\frac{a\sin(c+d\,x)^{11}}{11}+\frac{3\,a\sin(c+d\,x)^{10}}{10}+\frac{a\sin(c+d\,x)^{9}}{3}-\frac{3\,a\sin(c+d\,x)^{8}}{8}-\frac{3\,a\sin(c+d\,x)^{7}}{7}+\frac{a\sin(c+d\,x)^{6}}{6}+\frac{a\sin(c+d\,x)^{5}}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^4*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^5)/5 + (a*sin(c + d*x)^6)/6 - (3*a*sin(c + d*x)^7)/7 - (3*
a*sin(c + d*x)^8)/8 + (a*sin(c + d*x)^9)/3 + (3*a*sin(c + d*x)^10)/10 - (a*
sin(c + d*x)^11)/11 - (a*sin(c + d*x)^12)/12)/d
```

sympy [A] time = 56.20, size = 160, normalized size = 1.42

$$\begin{cases}\frac{16a\sin^{11}(c+dx)}{1155d}+\frac{8a\sin^{9}(c+dx)\cos^{2}(c+dx)}{105d}+\frac{6a\sin^{7}(c+dx)\cos^{4}(c+dx)}{35d}+\frac{a\sin^{5}(c+dx)\cos^{6}(c+dx)}{5d}-\frac{a\sin^{4}(c+dx)\cos^{8}(c+dx)}{8d}-\frac{a\sin^{2}(}{} \\ x\left(a\sin\left(c\right)+a\right)\sin^{4}\left(c\right)\cos^{7}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**11/(1155*d) + 8*a*sin(c + d*x)**9*cos(c + d*x
)**2/(105*d) + 6*a*sin(c + d*x)**7*cos(c + d*x)**4/(35*d) + a*sin(c + d*x)*
*5*cos(c + d*x)**6/(5*d) - a*sin(c + d*x)**4*cos(c + d*x)**8/(8*d) - a*sin(
c + d*x)**2*cos(c + d*x)**10/(20*d) - a*cos(c + d*x)**12/(120*d), Ne(d, 0))
, (x*(a*sin(c) + a)*sin(c)**4*cos(c)**7, True))
```

3.659 $\int \cos^7(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^9(c+dx)}{3d}-\frac{3a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\cos^{10}(c+dx)}{10d}-\frac{a\cos^8(c+dx)}{8d}$$

```
[Out] -1/8*a*cos(d*x+c)^8/d+1/10*a*cos(d*x+c)^10/d+1/5*a*sin(d*x+c)^5/d-3/7*a*sin
(d*x+c)^7/d+1/3*a*sin(d*x+c)^9/d-1/11*a*sin(d*x+c)^11/d
```

Rubi [A] time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2565, 14, 2564, 270}

$$-\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^9(c+dx)}{3d}-\frac{3a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\cos^{10}(c+dx)}{10d}-\frac{a\cos^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Cos[c + d*x]^10)/(10*d) + (a*Sin[c + d*x]^5)
/(5*d) - (3*a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c + d*x]^9)/(3*d) - (a*Sin[c +
 d*x]^11)/(11*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^7(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^7(c+dx)\sin^3(c+dx)\,dx + a\int \cos^7(c+dx)\sin^4(c$$

$$= -\frac{a\,\text{Subst}\left(\int x^7\left(1-x^2\right)\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^4\right.}{}$$

$$= -\frac{a\,\text{Subst}\left(\int \left(x^7-x^9\right)\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \left(x^4\right.\right.}{}$$

$$= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\cos^{10}(c+dx)}{10d} + \frac{a\sin^5(c+dx)}{5d} - \frac{3a\,\text{si}}{}$$

Mathematica [A] time = 0.61, size = 117, normalized size = 1.21

$$\frac{a(16170\sin(c+dx) - 2310\sin(3(c+dx)) - 2541\sin(5(c+dx)) - 165\sin(7(c+dx)) + 385\sin(9(c+dx)) + 105}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-16170*Cos[2*(c + d*x)] - 4620*Cos[4*(c + d*x)] + 1155*Cos[6*(c + d*x)]
 + 1155*Cos[8*(c + d*x)] + 231*Cos[10*(c + d*x)] + 16170*Sin[c + d*x] - 231
0*Sin[3*(c + d*x)] - 2541*Sin[5*(c + d*x)] - 165*Sin[7*(c + d*x)] + 385*Sin
[9*(c + d*x)] + 105*Sin[11*(c + d*x)]))/(1182720*d)
```

fricas [A] time = 0.89, size = 95, normalized size = 0.98

$$\frac{924\,a\cos(dx+c)^{10} - 1155\,a\cos(dx+c)^8 + 8\left(105\,a\cos(dx+c)^{10} - 140\,a\cos(dx+c)^8 + 5\,a\cos(dx+c)^6 + 6\right.}{9240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/9240*(924*a*cos(d*x + c)^10 - 1155*a*cos(d*x + c)^8 + 8*(105*a*cos(d*x +
c)^10 - 140*a*cos(d*x + c)^8 + 5*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 + 8*
a*cos(d*x + c)^2 + 16*a)*sin(d*x + c))/d
```

giac [A] time = 0.29, size = 163, normalized size = 1.68

$$\frac{a\cos(10\,dx+10\,c)}{5120\,d}+\frac{a\cos(8\,dx+8\,c)}{1024\,d}+\frac{a\cos(6\,dx+6\,c)}{1024\,d}-\frac{a\cos(4\,dx+4\,c)}{256\,d}-\frac{7\,a\cos(2\,dx+2\,c)}{512\,d}+\frac{a\sin(11\,dx+}{11264\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/5120*a*cos(10*d*x + 10*c)/d + 1/1024*a*cos(8*d*x + 8*c)/d + 1/1024*a*cos(
6*d*x + 6*c)/d - 1/256*a*cos(4*d*x + 4*c)/d - 7/512*a*cos(2*d*x + 2*c)/d +
1/11264*a*sin(11*d*x + 11*c)/d + 1/3072*a*sin(9*d*x + 9*c)/d - 1/7168*a*sin
(7*d*x + 7*c)/d - 11/5120*a*sin(5*d*x + 5*c)/d - 1/512*a*sin(3*d*x + 3*c)/d
 + 7/512*a*sin(d*x + c)/d
```

maple [A] time = 0.24, size = 112, normalized size = 1.15

$$\frac{a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^8(dx+c)\right)}{11}-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{33}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{231}\right)+a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^8(\right.}{10}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/33*sin(d*x+c)*cos(d*x+c)^8+1/231*
(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/10*
sin(d*x+c)^2*cos(d*x+c)^8-1/40*cos(d*x+c)^8))
```

maxima [A] time = 0.31, size = 94, normalized size = 0.97

$$-\frac{840\,a\sin(dx+c)^{11}+924\,a\sin(dx+c)^{10}-3080\,a\sin(dx+c)^{9}-3465\,a\sin(dx+c)^{8}+3960\,a\sin(dx+c)^{7}+}{9240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/9240*(840*a*sin(d*x + c)^11 + 924*a*sin(d*x + c)^10 - 3080*a*sin(d*x + c
)^9 - 3465*a*sin(d*x + c)^8 + 3960*a*sin(d*x + c)^7 + 4620*a*sin(d*x + c)^6
 - 1848*a*sin(d*x + c)^5 - 2310*a*sin(d*x + c)^4)/d
```

mupad [B] time = 0.08, size = 93, normalized size = 0.96

$$\frac{-\frac{a\sin(c+d\,x)^{11}}{11}-\frac{a\sin(c+d\,x)^{10}}{10}+\frac{a\sin(c+d\,x)^9}{3}+\frac{3\,a\sin(c+d\,x)^8}{8}-\frac{3\,a\sin(c+d\,x)^7}{7}-\frac{a\sin(c+d\,x)^6}{2}+\frac{a\sin(c+d\,x)^5}{5}+\frac{a\sin(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^4)/4 + (a*sin(c + d*x)^5)/5 - (a*sin(c + d*x)^6)/2 - (3*a*
sin(c + d*x)^7)/7 + (3*a*sin(c + d*x)^8)/8 + (a*sin(c + d*x)^9)/3 - (a*sin(
c + d*x)^10)/10 - (a*sin(c + d*x)^11)/11)/d
```

sympy [A] time = 37.47, size = 138, normalized size = 1.42

$$\begin{cases} \frac{16a\sin^{11}(c+dx)}{1155d}+\frac{8a\sin^{9}(c+dx)\cos^{2}(c+dx)}{105d}+\frac{6a\sin^{7}(c+dx)\cos^{4}(c+dx)}{35d}+\frac{a\sin^{5}(c+dx)\cos^{6}(c+dx)}{5d}-\frac{a\sin^{2}(c+dx)\cos^{8}(c+dx)}{8d}-\frac{a\cos^{10}\ldots}{4\ldots} \\ x\left(a\sin(c)+a\right)\sin^{3}(c)\cos^{7}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**11/(1155*d) + 8*a*sin(c + d*x)**9*cos(c + d*x
)**2/(105*d) + 6*a*sin(c + d*x)**7*cos(c + d*x)**4/(35*d) + a*sin(c + d*x)*
*5*cos(c + d*x)**6/(5*d) - a*sin(c + d*x)**2*cos(c + d*x)**8/(8*d) - a*cos(
c + d*x)**10/(40*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**3*cos(c)**7, True
))
```

3.660 $\int \cos^7(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\sin^9(c+dx)}{9d}+\frac{3a\sin^7(c+dx)}{7d}-\frac{3a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{a\cos^{10}(c+dx)}{10d}-\frac{a\cos^8(c+dx)}{8d}$$

```
[Out] -1/8*a*cos(d*x+c)^8/d+1/10*a*cos(d*x+c)^10/d+1/3*a*sin(d*x+c)^3/d-3/5*a*sin
(d*x+c)^5/d+3/7*a*sin(d*x+c)^7/d-1/9*a*sin(d*x+c)^9/d
```

Rubi [A] time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2564, 270, 2565, 14}

$$-\frac{a\sin^9(c+dx)}{9d}+\frac{3a\sin^7(c+dx)}{7d}-\frac{3a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{a\cos^{10}(c+dx)}{10d}-\frac{a\cos^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Cos[c + d*x]^10)/(10*d) + (a*Sin[c + d*x]^3)
/(3*d) - (3*a*Sin[c + d*x]^5)/(5*d) + (3*a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c
 + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^7(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^7(c+dx)\sin^2(c+dx)\,dx + a\int \cos^7(c+dx)\sin^3(c$$

$$= -\frac{a\,\text{Subst}\left(\int x^7\left(1-x^2\right)\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^2\right.}{}$$

$$= \frac{a\,\text{Subst}\left(\int \left(x^2-3x^4+3x^6-x^8\right)\,dx, x, \sin(c+dx)\right)}{d} - \frac{a\,\text{Su}}{}$$

$$= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\cos^{10}(c+dx)}{10d} + \frac{a\sin^3(c+dx)}{3d} - \frac{3a\,\text{si}}{}$$

Mathematica [A] time = 0.45, size = 97, normalized size = 1.00

$$-\frac{a(-17640\sin(c+dx)+2016\sin(5(c+dx))+900\sin(7(c+dx))+140\sin(9(c+dx))+4410\cos(2(c+dx))+}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/322560*(a*(4410*Cos[2*(c + d*x)] + 1260*Cos[4*(c + d*x)] - 315*Cos[6*(c
+ d*x)] - 315*Cos[8*(c + d*x)] - 63*Cos[10*(c + d*x)] - 17640*Sin[c + d*x]
+ 2016*Sin[5*(c + d*x)] + 900*Sin[7*(c + d*x)] + 140*Sin[9*(c + d*x)]))/d
```

fricas [A] time = 0.83, size = 84, normalized size = 0.87

$$\frac{252\,a\cos\left(dx+c\right)^{10}-315\,a\cos\left(dx+c\right)^{8}-8\left(35\,a\cos\left(dx+c\right)^{8}-5\,a\cos\left(dx+c\right)^{6}-6\,a\cos\left(dx+c\right)^{4}-8\,a\cos\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2520*(252*a*cos(d*x + c)^10 - 315*a*cos(d*x + c)^8 - 8*(35*a*cos(d*x + c)
^8 - 5*a*cos(d*x + c)^6 - 6*a*cos(d*x + c)^4 - 8*a*cos(d*x + c)^2 - 16*a)*s
in(d*x + c))/d
```

giac [A] time = 0.26, size = 133, normalized size = 1.37

$$\frac{a\cos(10\,dx+10\,c)}{5120\,d}+\frac{a\cos(8\,dx+8\,c)}{1024\,d}+\frac{a\cos(6\,dx+6\,c)}{1024\,d}-\frac{a\cos(4\,dx+4\,c)}{256\,d}-\frac{7\,a\cos(2\,dx+2\,c)}{512\,d}-\frac{a\sin(9\,dx+9}{2304\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/5120*a*cos(10*d*x + 10*c)/d + 1/1024*a*cos(8*d*x + 8*c)/d + 1/1024*a*cos(
6*d*x + 6*c)/d - 1/256*a*cos(4*d*x + 4*c)/d - 7/512*a*cos(2*d*x + 2*c)/d -
1/2304*a*sin(9*d*x + 9*c)/d - 5/1792*a*sin(7*d*x + 7*c)/d - 1/160*a*sin(5*d
*x + 5*c)/d + 7/128*a*sin(d*x + c)/d
```

maple [A] time = 0.24, size = 94, normalized size = 0.97

$$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^8(dx+c)\right)}{10}-\frac{\left(\cos^8(dx+c)\right)}{40}\right)+a\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8-1/40*cos(d*x+c)^8)+a*(-1/9*sin(d*x+
c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*
sin(d*x+c)))
```

maxima [A] time = 0.31, size = 94, normalized size = 0.97

$$-\frac{252\,a\sin(dx+c)^{10}+280\,a\sin(dx+c)^{9}-945\,a\sin(dx+c)^{8}-1080\,a\sin(dx+c)^{7}+1260\,a\sin(dx+c)^{6}+151}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2520*(252*a*sin(d*x + c)^10 + 280*a*sin(d*x + c)^9 - 945*a*sin(d*x + c)^
8 - 1080*a*sin(d*x + c)^7 + 1260*a*sin(d*x + c)^6 + 1512*a*sin(d*x + c)^5 -
 630*a*sin(d*x + c)^4 - 840*a*sin(d*x + c)^3)/d
```

mupad [B] time = 8.95, size = 93, normalized size = 0.96

$$\frac{-\frac{a\sin(c+d\,x)^{10}}{10}-\frac{a\sin(c+d\,x)^9}{9}+\frac{3\,a\sin(c+d\,x)^8}{8}+\frac{3\,a\sin(c+d\,x)^7}{7}-\frac{a\sin(c+d\,x)^6}{2}-\frac{3\,a\sin(c+d\,x)^5}{5}+\frac{a\sin(c+d\,x)^4}{4}+\frac{a\sin(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^3)/3 + (a*sin(c + d*x)^4)/4 - (3*a*sin(c + d*x)^5)/5 - (a*
sin(c + d*x)^6)/2 + (3*a*sin(c + d*x)^7)/7 + (3*a*sin(c + d*x)^8)/8 - (a*si
n(c + d*x)^9)/9 - (a*sin(c + d*x)^10)/10)/d
```

sympy [A] time = 24.12, size = 138, normalized size = 1.42

$$\begin{cases} \frac{16a\sin^9(c+dx)}{315d}+\frac{8a\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{2a\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{a\sin^3(c+dx)\cos^6(c+dx)}{3d}-\frac{a\sin^2(c+dx)\cos^8(c+dx)}{8d}-\frac{a\cos^{10}}{40} \\ x\left(a\sin(c)+a\right)\sin^2(c)\cos^7(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**9/(315*d) + 8*a*sin(c + d*x)**7*cos(c + d*x)*
*2/(35*d) + 2*a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + a*sin(c + d*x)**3*c
os(c + d*x)**6/(3*d) - a*sin(c + d*x)**2*cos(c + d*x)**8/(8*d) - a*cos(c +
d*x)**10/(40*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**2*cos(c)**7, True))
```

3.661 $\int \cos^7(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\sin^9(c+dx)}{9d}+\frac{3a\sin^7(c+dx)}{7d}-\frac{3a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^8(c+dx)}{8d}$$

```
[Out] -1/8*a*cos(d*x+c)^8/d+1/3*a*sin(d*x+c)^3/d-3/5*a*sin(d*x+c)^5/d+3/7*a*sin(d
*x+c)^7/d-1/9*a*sin(d*x+c)^9/d
```

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2834, 2565, 30, 2564, 270}

$$-\frac{a\sin^9(c+dx)}{9d}+\frac{3a\sin^7(c+dx)}{7d}-\frac{3a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^3)/(3*d) - (3*a*Sin[c + d*x]^5)
/(5*d) + (3*a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^9)/(9*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^7(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^7(c+dx)\sin(c+dx)\,dx + a\int \cos^7(c+dx)\sin^2(c+$$

$$= -\frac{a\,\text{Subst}\left(\int x^7\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^2\left(1-x^2\right)^3\right.}{d}$$

$$= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\,\text{Subst}\left(\int \left(x^2-3x^4+3x^6-x^8\right)dx, x, \text{si}\right.}{d}$$

$$= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\sin^3(c+dx)}{3d} - \frac{3a\sin^5(c+dx)}{5d} + \frac{3a\sin}{}$$

Mathematica [A] time = 0.38, size = 60, normalized size = 0.74

$$\frac{a\left(\sin^3(c+dx)(1389\cos(2(c+dx))+330\cos(4(c+dx))+35\cos(6(c+dx))+1606)-1260\cos^8(c+dx)\right)}{10080d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-1260*Cos[c + d*x]^8 + (1606 + 1389*Cos[2*(c + d*x)] + 330*Cos[4*(c + d
*x)] + 35*Cos[6*(c + d*x)])*Sin[c + d*x]^3))/(10080*d)
```

fricas [A] time = 0.65, size = 73, normalized size = 0.90

$$-\frac{315\,a\cos(dx+c)^8+8\left(35\,a\cos(dx+c)^8-5\,a\cos(dx+c)^6-6\,a\cos(dx+c)^4-8\,a\cos(dx+c)^2-16\,a\right)\sin}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2520*(315*a*cos(d*x + c)^8 + 8*(35*a*cos(d*x + c)^8 - 5*a*cos(d*x + c)^6
 - 6*a*cos(d*x + c)^4 - 8*a*cos(d*x + c)^2 - 16*a)*sin(d*x + c))/d
```

giac [A] time = 0.23, size = 118, normalized size = 1.46

$$-\frac{a\cos(8\,dx+8\,c)}{1024\,d}-\frac{a\cos(6\,dx+6\,c)}{128\,d}-\frac{7\,a\cos(4\,dx+4\,c)}{256\,d}-\frac{7\,a\cos(2\,dx+2\,c)}{128\,d}-\frac{a\sin(9\,dx+9\,c)}{2304\,d}-\frac{5\,a\sin(7\,dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1024*a*cos(8*d*x + 8*c)/d - 1/128*a*cos(6*d*x + 6*c)/d - 7/256*a*cos(4*d
*x + 4*c)/d - 7/128*a*cos(2*d*x + 2*c)/d - 1/2304*a*sin(9*d*x + 9*c)/d - 5/
1792*a*sin(7*d*x + 7*c)/d - 1/160*a*sin(5*d*x + 5*c)/d + 7/128*a*sin(d*x +
c)/d
```

maple [A] time = 0.24, size = 74, normalized size = 0.91

$$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)-\frac{a\left(\cos^8(dx+c)\right)}{8}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)
^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/8*a*cos(d*x+c)^8)
```

maxima [A] time = 0.38, size = 94, normalized size = 1.16

$$-\frac{280\,a\sin(dx+c)^9+315\,a\sin(dx+c)^8-1080\,a\sin(dx+c)^7-1260\,a\sin(dx+c)^6+1512\,a\sin(dx+c)^5+18}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2520*(280*a*sin(d*x + c)^9 + 315*a*sin(d*x + c)^8 - 1080*a*sin(d*x + c)^
7 - 1260*a*sin(d*x + c)^6 + 1512*a*sin(d*x + c)^5 + 1890*a*sin(d*x + c)^4 -
 840*a*sin(d*x + c)^3 - 1260*a*sin(d*x + c)^2)/d
```

mupad [B] time = 9.02, size = 93, normalized size = 1.15

$$\frac{-\frac{a\sin(c+d\,x)^9}{9}-\frac{a\sin(c+d\,x)^8}{8}+\frac{3\,a\sin(c+d\,x)^7}{7}+\frac{a\sin(c+d\,x)^6}{2}-\frac{3\,a\sin(c+d\,x)^5}{5}-\frac{3\,a\sin(c+d\,x)^4}{4}+\frac{a\sin(c+d\,x)^3}{3}+\frac{a\sin(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^2)/2 + (a*sin(c + d*x)^3)/3 - (3*a*sin(c + d*x)^4)/4 - (3*
a*sin(c + d*x)^5)/5 + (a*sin(c + d*x)^6)/2 + (3*a*sin(c + d*x)^7)/7 - (a*si
n(c + d*x)^8)/8 - (a*sin(c + d*x)^9)/9)/d
```

sympy [A] time = 15.73, size = 114, normalized size = 1.41

$$\begin{cases} \frac{16a\sin^9(c+dx)}{315d} + \frac{8a\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{2a\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{a\sin^3(c+dx)\cos^6(c+dx)}{3d} - \frac{a\cos^8(c+dx)}{8d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)\sin(c)\cos^7(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**9/(315*d) + 8*a*sin(c + d*x)**7*cos(c + d*x)*
*2/(35*d) + 2*a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + a*sin(c + d*x)**3*c
os(c + d*x)**6/(3*d) - a*cos(c + d*x)**8/(8*d), Ne(d, 0)), (x*(a*sin(c) + a
)*sin(c)*cos(c)**7, True))
```

3.662 $\int \cos^6(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=118

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^6(c+dx)}{6d}+\frac{3a\sin^5(c+dx)}{5d}+\frac{3a\sin^4(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d-3/2*a*sin(d*x+c)^2/d-a*sin(d*x+c)^3/d+3/4
*a*sin(d*x+c)^4/d+3/5*a*sin(d*x+c)^5/d-1/6*a*sin(d*x+c)^6/d-1/7*a*sin(d*x+c
)^7/d
```

Rubi [A] time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2836, 12, 88}

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^6(c+dx)}{6d}+\frac{3a\sin^5(c+dx)}{5d}+\frac{3a\sin^4(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (3*a*Sin[c + d*x]^2)/(2*d) -
 (a*Sin[c + d*x]^3)/d + (3*a*Sin[c + d*x]^4)/(4*d) + (3*a*Sin[c + d*x]^5)/(
5*d) - (a*Sin[c + d*x]^6)/(6*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^3(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^6+\frac{a^7}{x}-3a^5x-3a^4x^2+3a^3x^3+3a^2x^4-ax^5-\right.\right.}{a^6 d}$$
$$= \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}-\frac{a\sin}{}$$

Mathematica [A] time = 0.13, size = 106, normalized size = 0.90

$$-\frac{a\sin^7(c+dx)}{7d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{d}+\frac{a\sin(c+dx)}{d}+\frac{a\left(-2\sin^6(c+dx)+9\sin^4(c+dx)-18\sin^2(c\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d + (3*a*Sin[c + d*x]^5)/(5*d) - (a
*Sin[c + d*x]^7)/(7*d) + (a*(12*Log[Sin[c + d*x]] - 18*Sin[c + d*x]^2 + 9*S
in[c + d*x]^4 - 2*Sin[c + d*x]^6))/(12*d)
```

fricas [A] time = 0.82, size = 96, normalized size = 0.81

$$\frac{70\,a\cos(dx+c)^6+105\,a\cos(dx+c)^4+210\,a\cos(dx+c)^2+420\,a\log\left(\frac{1}{2}\sin(dx+c)\right)+12\left(5\,a\cos(dx+c)^6\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(70*a*cos(d*x + c)^6 + 105*a*cos(d*x + c)^4 + 210*a*cos(d*x + c)^2 +
420*a*log(1/2*sin(d*x + c)) + 12*(5*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 +
 8*a*cos(d*x + c)^2 + 16*a)*sin(d*x + c))/d
```

giac [A] time = 0.27, size = 92, normalized size = 0.78

$$-\frac{60\,a\sin(dx+c)^7+70\,a\sin(dx+c)^6-252\,a\sin(dx+c)^5-315\,a\sin(dx+c)^4+420\,a\sin(dx+c)^3+630\,a\,\mathrm{s}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/420*(60*a*sin(d*x + c)^7 + 70*a*sin(d*x + c)^6 - 252*a*sin(d*x + c)^5 -
315*a*sin(d*x + c)^4 + 420*a*sin(d*x + c)^3 + 630*a*sin(d*x + c)^2 - 420*a*
log(abs(sin(d*x + c))) - 420*a*sin(d*x + c))/d
```

maple [A] time = 0.34, size = 128, normalized size = 1.08

$$\frac{16a\sin(dx+c)}{35d}+\frac{\left(\cos^6(dx+c)\right)\sin(dx+c)\,a}{7d}+\frac{6a\sin(dx+c)\left(\cos^4(dx+c)\right)}{35d}+\frac{8\left(\cos^2(dx+c)\right)\sin(dx+c)\,a}{35d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 16/35*a*sin(d*x+c)/d+1/7/d*cos(d*x+c)^6*sin(d*x+c)*a+6/35/d*a*sin(d*x+c)*co
s(d*x+c)^4+8/35/d*a*sin(d*x+c)*cos(d*x+c)^2+1/6*a*cos(d*x+c)^6/d+1/4*a*cos(
d*x+c)^4/d+1/2*a*cos(d*x+c)^2/d+a*ln(sin(d*x+c))/d
```

maxima [A] time = 0.34, size = 91, normalized size = 0.77

$$-\frac{60\,a\sin(dx+c)^7+70\,a\sin(dx+c)^6-252\,a\sin(dx+c)^5-315\,a\sin(dx+c)^4+420\,a\sin(dx+c)^3+630\,a\sin}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(60*a*sin(d*x + c)^7 + 70*a*sin(d*x + c)^6 - 252*a*sin(d*x + c)^5 -
315*a*sin(d*x + c)^4 + 420*a*sin(d*x + c)^3 + 630*a*sin(d*x + c)^2 - 420*a*
log(sin(d*x + c)) - 420*a*sin(d*x + c))/d
```

mupad [B] time = 9.16, size = 160, normalized size = 1.36

$$\frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}+\frac{a\cos\left(c+d\,x\right)^2}{2\,d}+\frac{a\cos\left(c+d\,x\right)^4}{4\,d}+\frac{a\cos\left(c+d\,x\right)^6}{6\,d}+\frac{16\,a\,\sin\left(c+d\,x\right)}{35\,d}+\frac{8\,a\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*log(1/cos(c/2 + (d*x)
/2)^2))/d + (a*cos(c + d*x)^2)/(2*d) + (a*cos(c + d*x)^4)/(4*d) + (a*cos(c
```

```
+ d*x)^6)/(6*d) + (16*a*sin(c + d*x))/(35*d) + (8*a*cos(c + d*x)^2*sin(c +
d*x))/(35*d) + (6*a*cos(c + d*x)^4*sin(c + d*x))/(35*d) + (a*cos(c + d*x)^6
*sin(c + d*x))/(7*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)*(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.663 $\int \cos^5(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=114

$$-\frac{a\sin^6(c+dx)}{6d}-\frac{a\sin^5(c+dx)}{5d}+\frac{3a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}-\frac{3a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}$$

```
[Out] -a*csc(d*x+c)/d+a*ln(sin(d*x+c))/d-3*a*sin(d*x+c)/d-3/2*a*sin(d*x+c)^2/d+a*
sin(d*x+c)^3/d+3/4*a*sin(d*x+c)^4/d-1/5*a*sin(d*x+c)^5/d-1/6*a*sin(d*x+c)^6
/d
```

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\sin^6(c+dx)}{6d}-\frac{a\sin^5(c+dx)}{5d}+\frac{3a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}-\frac{3a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (3*a*Sin[c + d*x])/d - (3
*a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/d + (3*a*Sin[c + d*x]^4)/(4*d
) - (a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^6)/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^3(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^5+\frac{a^7}{x^2}+\frac{a^6}{x}-3a^4x+3a^3x^2+3a^2x^3-ax^4-\right.\right.}{a^5 d}$$
$$= -\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}-\frac{3a\sin(c+dx)}{d}-\frac{3a}{}$$

Mathematica [A] time = 0.13, size = 102, normalized size = 0.89

$$-\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{d}-\frac{3a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\left(-2\sin^6(c+dx)+9\sin^4(c+dx)-18\sin^2(c\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (3*a*Sin[c + d*x])/d + (a*Sin[c + d*x]^3)/d - (a*Si
n[c + d*x]^5)/(5*d) + (a*(12*Log[Sin[c + d*x]] - 18*Sin[c + d*x]^2 + 9*Sin[
c + d*x]^4 - 2*Sin[c + d*x]^6))/(12*d)
```

fricas [A] time = 0.70, size = 113, normalized size = 0.99

$$\frac{48\,a\cos\left(dx+c\right)^{6}+96\,a\cos\left(dx+c\right)^{4}+384\,a\cos\left(dx+c\right)^{2}+240\,a\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+5\,\left(8\,a\cos\right.}{240\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*a*cos(d*x + c)^6 + 96*a*cos(d*x + c)^4 + 384*a*cos(d*x + c)^2 + 2
40*a*log(1/2*sin(d*x + c))*sin(d*x + c) + 5*(8*a*cos(d*x + c)^6 + 12*a*cos(
d*x + c)^4 + 24*a*cos(d*x + c)^2 - 19*a)*sin(d*x + c) - 768*a)/(d*sin(d*x +
 c))
```

giac [A] time = 0.21, size = 101, normalized size = 0.89

$$-\frac{10\,a\sin\left(dx+c\right)^{6}+12\,a\sin\left(dx+c\right)^{5}-45\,a\sin\left(dx+c\right)^{4}-60\,a\sin\left(dx+c\right)^{3}+90\,a\sin\left(dx+c\right)^{2}-60\,a\log\left(|\mathrm{s}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(10*a*sin(d*x + c)^6 + 12*a*sin(d*x + c)^5 - 45*a*sin(d*x + c)^4 - 60
*a*sin(d*x + c)^3 + 90*a*sin(d*x + c)^2 - 60*a*log(abs(sin(d*x + c))) + 180
*a*sin(d*x + c) + 60*(a*sin(d*x + c) + a)/sin(d*x + c))/d
```

maple [A] time = 0.29, size = 150, normalized size = 1.32

$$\frac{a\left(\cos^6(dx+c)\right)}{6d}+\frac{a\left(\cos^4(dx+c)\right)}{4d}+\frac{a\left(\cos^2(dx+c)\right)}{2d}+\frac{a\ln(\sin(dx+c))}{d}-\frac{a\left(\cos^8(dx+c)\right)}{d\sin(dx+c)}-\frac{16a\sin(dx+c)}{5d}-\frac{(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/6*a*cos(d*x+c)^6/d+1/4*a*cos(d*x+c)^4/d+1/2*a*cos(d*x+c)^2/d+a*ln(sin(d*x
+c))/d-1/d*a/sin(d*x+c)*cos(d*x+c)^8-16/5*a*sin(d*x+c)/d-1/d*cos(d*x+c)^6*s
in(d*x+c)*a-6/5/d*a*sin(d*x+c)*cos(d*x+c)^4-8/5/d*a*sin(d*x+c)*cos(d*x+c)^2
```

maxima [A] time = 0.36, size = 91, normalized size = 0.80

$$-\frac{10\,a\sin(dx+c)^6+12\,a\sin(dx+c)^5-45\,a\sin(dx+c)^4-60\,a\sin(dx+c)^3+90\,a\sin(dx+c)^2-60\,a\log(\sin}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(10*a*sin(d*x + c)^6 + 12*a*sin(d*x + c)^5 - 45*a*sin(d*x + c)^4 - 60
*a*sin(d*x + c)^3 + 90*a*sin(d*x + c)^2 - 60*a*log(sin(d*x + c)) + 180*a*si
n(d*x + c) + 60*a/sin(d*x + c))/d
```

mupad [B] time = 9.33, size = 340, normalized size = 2.98

$$\frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}-\frac{6\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d}+\frac{18\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}-\frac{104\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3\,d}+\frac{44\,a\cos\left(\frac{c}{2}+\frac{}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*log(1/cos(c/2 + (d*x)
/2)^2))/d - (6*a*cos(c/2 + (d*x)/2)^2)/d + (18*a*cos(c/2 + (d*x)/2)^4)/d -
```

```
(104*a*cos(c/2 + (d*x)/2)^6)/(3*d) + (44*a*cos(c/2 + (d*x)/2)^8)/d - (32*a*
cos(c/2 + (d*x)/2)^10)/d + (32*a*cos(c/2 + (d*x)/2)^12)/(3*d) - (13*a*cos(c
/2 + (d*x)/2))/(2*d*sin(c/2 + (d*x)/2)) - (a*sin(c/2 + (d*x)/2))/(2*d*cos(c
/2 + (d*x)/2)) + (14*a*cos(c/2 + (d*x)/2)^3)/(d*sin(c/2 + (d*x)/2)) - (112*
a*cos(c/2 + (d*x)/2)^5)/(5*d*sin(c/2 + (d*x)/2)) + (136*a*cos(c/2 + (d*x)/2
)^7)/(5*d*sin(c/2 + (d*x)/2)) - (96*a*cos(c/2 + (d*x)/2)^9)/(5*d*sin(c/2 +
(d*x)/2)) + (32*a*cos(c/2 + (d*x)/2)^11)/(5*d*sin(c/2 + (d*x)/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.664 $\int \cos^4(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^2(c+dx)}{2d}-\frac{3a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-$$

```
[Out] -a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-3*a*ln(sin(d*x+c))/d-3*a*sin(d*x+c)/d+
3/2*a*sin(d*x+c)^2/d+a*sin(d*x+c)^3/d-1/4*a*sin(d*x+c)^4/d-1/5*a*sin(d*x+c)
^5/d
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^2(c+dx)}{2d}-\frac{3a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (3*a*Log[Sin[c + d*x]])/
d - (3*a*Sin[c + d*x])/d + (3*a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/
d - (a*Sin[c + d*x]^4)/(4*d) - (a*Sin[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^3(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^4+\frac{a^7}{x^3}+\frac{a^6}{x^2}-\frac{3a^5}{x}+3a^3x+3a^2x^2-ax^3-x^4\right.\right.}{a^4 d}$$
$$= -\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{3a\log(\sin(c+dx))}{d}-\frac{3a}{}$$

Mathematica [A] time = 0.12, size = 100, normalized size = 0.87

$$-\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{d}-\frac{3a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}-\frac{a\left(\sin^4(c+dx)-6\sin^2(c+dx)+2\csc^2(c+dx\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (3*a*Sin[c + d*x])/d + (a*Sin[c + d*x]^3)/d - (a*Si
n[c + d*x]^5)/(5*d) - (a*(2*Csc[c + d*x]^2 + 12*Log[Sin[c + d*x]] - 6*Sin[c
 + d*x]^2 + Sin[c + d*x]^4))/(4*d)
```

fricas [A] time = 0.88, size = 124, normalized size = 1.08

$$-\frac{40\,a\cos\left(dx+c\right)^6+120\,a\cos\left(dx+c\right)^4-255\,a\cos\left(dx+c\right)^2+480\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+}{160\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/160*(40*a*cos(d*x + c)^6 + 120*a*cos(d*x + c)^4 - 255*a*cos(d*x + c)^2 +
 480*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 32*(a*cos(d*x + c)^6 +
2*a*cos(d*x + c)^4 + 8*a*cos(d*x + c)^2 - 16*a)*sin(d*x + c) + 15*a)/(d*cos
(d*x + c)^2 - d)
```

giac [A] time = 0.24, size = 104, normalized size = 0.90

$$-\frac{4\,a\sin\left(dx+c\right)^5+5\,a\sin\left(dx+c\right)^4-20\,a\sin\left(dx+c\right)^3-30\,a\sin\left(dx+c\right)^2+60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,a\sin}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/20*(4*a*sin(d*x + c)^5 + 5*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 30*a
*sin(d*x + c)^2 + 60*a*log(abs(sin(d*x + c))) + 60*a*sin(d*x + c) - 10*(9*a
*sin(d*x + c)^2 - 2*a*sin(d*x + c) - a)/sin(d*x + c)^2)/d
```

maple [A] time = 0.35, size = 173, normalized size = 1.50

$$-\frac{a\left(\cos^{8}(dx+c)\right)}{d\sin(dx+c)}-\frac{16a\sin(dx+c)}{5d}-\frac{\left(\cos^{6}(dx+c)\right)\sin(dx+c)\,a}{d}-\frac{6a\sin(dx+c)\left(\cos^{4}(dx+c)\right)}{5d}-\frac{8\left(\cos^{2}(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^8-16/5*a*sin(d*x+c)/d-1/d*cos(d*x+c)^6*sin(d*x
+c)*a-6/5/d*a*sin(d*x+c)*cos(d*x+c)^4-8/5/d*a*sin(d*x+c)*cos(d*x+c)^2-1/2/d
*a/sin(d*x+c)^2*cos(d*x+c)^8-1/2*a*cos(d*x+c)^6/d-3/4*a*cos(d*x+c)^4/d-3/2*
a*cos(d*x+c)^2/d-3*a*ln(sin(d*x+c))/d
```

maxima [A] time = 0.36, size = 90, normalized size = 0.78

$$-\frac{4\,a\sin(dx+c)^{5}+5\,a\sin(dx+c)^{4}-20\,a\sin(dx+c)^{3}-30\,a\sin(dx+c)^{2}+60\,a\log(\sin(dx+c))+60\,a\sin(d}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/20*(4*a*sin(d*x + c)^5 + 5*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 30*a
*sin(d*x + c)^2 + 60*a*log(sin(d*x + c)) + 60*a*sin(d*x + c) + 10*(2*a*sin(
d*x + c) + a)/sin(d*x + c)^2)/d
```

mupad [B] time = 9.17, size = 311, normalized size = 2.70

$$\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}{d}-\frac{26\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-\frac{47\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{2}+74\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}-\frac{107\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{2}+\frac{628\,a\tan}{5}}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (3*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a/2 + 2*a*tan(c/2 + (d*x)/2) + (5*
a*tan(c/2 + (d*x)/2)^2)/2 + 34*a*tan(c/2 + (d*x)/2)^3 - 19*a*tan(c/2 + (d*x
)/2)^4 + 84*a*tan(c/2 + (d*x)/2)^5 - 51*a*tan(c/2 + (d*x)/2)^6 + (628*a*tan
(c/2 + (d*x)/2)^7)/5 - (107*a*tan(c/2 + (d*x)/2)^8)/2 + 74*a*tan(c/2 + (d*x
)/2)^9 - (47*a*tan(c/2 + (d*x)/2)^10)/2 + 26*a*tan(c/2 + (d*x)/2)^11)/(d*(4
*tan(c/2 + (d*x)/2)^2 + 20*tan(c/2 + (d*x)/2)^4 + 40*tan(c/2 + (d*x)/2)^6 +
 40*tan(c/2 + (d*x)/2)^8 + 20*tan(c/2 + (d*x)/2)^10 + 4*tan(c/2 + (d*x)/2)^
12)) - (a*tan(c/2 + (d*x)/2))/(2*d) - (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*a
*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.665 $\int \cos^3(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=118

$$-\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{3d}+\frac{3a\sin^2(c+dx)}{2d}+\frac{3a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}+\frac{3a\csc(c+dx)}{d}$$

```
[Out] 3*a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-3*a*ln(sin(d*x+c
))/d+3*a*sin(d*x+c)/d+3/2*a*sin(d*x+c)^2/d-1/3*a*sin(d*x+c)^3/d-1/4*a*sin(d
*x+c)^4/d
```

Rubi [A] time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{3d}+\frac{3a\sin^2(c+dx)}{2d}+\frac{3a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}+\frac{3a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
- (3*a*Log[Sin[c + d*x]])/d + (3*a*Sin[c + d*x])/d + (3*a*Sin[c + d*x]^2)/(
2*d) - (a*Sin[c + d*x]^3)/(3*d) - (a*Sin[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^3(a+x)^4}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(3a^3+\frac{a^7}{x^4}+\frac{a^6}{x^3}-\frac{3a^5}{x^2}-\frac{3a^4}{x}+3a^2x-ax^2-x^3\right)dx,$$
$$= \frac{3a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{3a\log(s$$

Mathematica [A] time = 0.21, size = 103, normalized size = 0.87

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{3a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{3a\csc(c+dx)}{d}-\frac{a\left(\sin^4(c+dx)-6\sin^2(c+dx)+2\csc^2(c+d\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (3*a*Sin[c + d*x])/d - (a
*Sin[c + d*x]^3)/(3*d) - (a*(2*Csc[c + d*x]^2 + 12*Log[Sin[c + d*x]] - 6*Si
n[c + d*x]^2 + Sin[c + d*x]^4))/(4*d)
```

fricas [A] time = 0.68, size = 139, normalized size = 1.18

$$-\frac{32\,a\cos(dx+c)^6+192\,a\cos(dx+c)^4-768\,a\cos(dx+c)^2+288\left(a\cos(dx+c)^2-a\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\text{si}}{96\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/96*(32*a*cos(d*x + c)^6 + 192*a*cos(d*x + c)^4 - 768*a*cos(d*x + c)^2 +
288*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c))*sin(d*x + c) + 3*(8*a*cos(
d*x + c)^6 + 24*a*cos(d*x + c)^4 - 51*a*cos(d*x + c)^2 + 3*a)*sin(d*x + c)
+ 512*a)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.22, size = 104, normalized size = 0.88

$$-\frac{3\,a\sin(dx+c)^4+4\,a\sin(dx+c)^3-18\,a\sin(dx+c)^2+36\,a\log(|\sin(dx+c)|)-36\,a\sin(dx+c)-\frac{2\left(33\,a\sin(\right.}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 18*a*sin(d*x + c)^2 + 36*a
*log(abs(sin(d*x + c))) - 36*a*sin(d*x + c) - 2*(33*a*sin(d*x + c)^3 + 18*a
*sin(d*x + c)^2 - 3*a*sin(d*x + c) - 2*a)/sin(d*x + c)^3)/d
```

maple [A] time = 0.31, size = 195, normalized size = 1.65

$$-\frac{a\left(\cos^{8}(dx+c)\right)}{2d\sin(dx+c)^{2}}-\frac{a\left(\cos^{6}(dx+c)\right)}{2d}-\frac{3a\left(\cos^{4}(dx+c)\right)}{4d}-\frac{3a\left(\cos^{2}(dx+c)\right)}{2d}-\frac{3a\ln(\sin(dx+c))}{d}-\frac{a\left(\cos^{8}(dx+\right.}{3d\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^8-1/2*a*cos(d*x+c)^6/d-3/4*a*cos(d*x+c)^4/
d-3/2*a*cos(d*x+c)^2/d-3*a*ln(sin(d*x+c))/d-1/3/d*a/sin(d*x+c)^3*cos(d*x+c)
^8+5/3/d*a/sin(d*x+c)*cos(d*x+c)^8+16/3*a*sin(d*x+c)/d+5/3/d*cos(d*x+c)^6*s
in(d*x+c)*a+2/d*a*sin(d*x+c)*cos(d*x+c)^4+8/3/d*a*sin(d*x+c)*cos(d*x+c)^2
```

maxima [A] time = 0.33, size = 92, normalized size = 0.78

$$-\frac{3\,a\sin(dx+c)^{4}+4\,a\sin(dx+c)^{3}-18\,a\sin(dx+c)^{2}+36\,a\log(\sin(dx+c))-36\,a\sin(dx+c)-\frac{2\left(18\,a\sin(dx+\right.}{\mathrm{s}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 18*a*sin(d*x + c)^2 + 36*a
*log(sin(d*x + c)) - 36*a*sin(d*x + c) - 2*(18*a*sin(d*x + c)^2 - 3*a*sin(d
*x + c) - 2*a)/sin(d*x + c)^3)/d
```

mupad [B] time = 9.07, size = 300, normalized size = 2.54

$$\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{8\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24\,d}-\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{59\,a\,\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] (11*a*tan(c/2 + (d*x)/2))/(8*d) + (3*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (
a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (3*a*log(
tan(c/2 + (d*x)/2)))/d + ((29*a*tan(c/2 + (d*x)/2)^2)/3 - a*tan(c/2 + (d*x)
/2) - a/3 - 4*a*tan(c/2 + (d*x)/2)^3 + 90*a*tan(c/2 + (d*x)/2)^4 + 42*a*tan
(c/2 + (d*x)/2)^5 + (562*a*tan(c/2 + (d*x)/2)^6)/3 + 60*a*tan(c/2 + (d*x)/2
)^7 + (499*a*tan(c/2 + (d*x)/2)^8)/3 + 47*a*tan(c/2 + (d*x)/2)^9 + 59*a*tan
(c/2 + (d*x)/2)^10)/(d*(8*tan(c/2 + (d*x)/2)^3 + 32*tan(c/2 + (d*x)/2)^5 +
48*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9 + 8*tan(c/2 + (d*x)/2)^11
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.666 $\int \cos^2(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=118

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{2d}+\frac{3a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{3a\csc^2(c+dx)}{2d}+\frac{3a\csc(c+dx)}{d}$$

```
[Out] 3*a*csc(d*x+c)/d+3/2*a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)
^4/d+3*a*ln(sin(d*x+c))/d+3*a*sin(d*x+c)/d-1/2*a*sin(d*x+c)^2/d-1/3*a*sin(d
*x+c)^3/d
```

Rubi [A] time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{2d}+\frac{3a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{3a\csc^2(c+dx)}{2d}+\frac{3a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*Csc[c + d*x])/d + (3*a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d
) - (a*Csc[c + d*x]^4)/(4*d) + (3*a*Log[Sin[c + d*x]])/d + (3*a*Sin[c + d*x
])/d - (a*Sin[c + d*x]^2)/(2*d) - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^5(a-x)^3(a+x)^4}{x^5}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^5}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$

$$= \frac{\text{Subst}\left(\int \left(3a^2+\frac{a^7}{x^5}+\frac{a^6}{x^4}-\frac{3a^5}{x^3}-\frac{3a^4}{x^2}+\frac{3a^3}{x}-ax-x^2\right)dx, x,\right.}{a^2 d}$$

$$= \frac{3a\csc(c+dx)}{d}+\frac{3a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^4(}{4}$$

Mathematica [A] time = 0.54, size = 105, normalized size = 0.89

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{3a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{3a\csc(c+dx)}{d}+\frac{a\left(-2\sin^2(c+dx)-\csc^4(c+dx)+6\csc^2(c+\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (3*a*Sin[c + d*x])/d - (a
*Sin[c + d*x]^3)/(3*d) + (a*(6*Csc[c + d*x]^2 - Csc[c + d*x]^4 + 12*Log[Sin
[c + d*x]] - 2*Sin[c + d*x]^2))/(4*d)
```

fricas [A] time = 0.73, size = 142, normalized size = 1.20

$$\frac{6\,a\cos(dx+c)^6-15\,a\cos(dx+c)^4-6\,a\cos(dx+c)^2+36\left(a\cos(dx+c)^4-2\,a\cos(dx+c)^2+a\right)\log\left(\frac{1}{2}\sin\right.}{12\left(d\cos(dx+c)^4-2\,d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*cos(d*x + c)^6 - 15*a*cos(d*x + c)^4 - 6*a*cos(d*x + c)^2 + 36*(a
*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*sin(d*x + c)) + 4*(a*cos(
d*x + c)^6 + 6*a*cos(d*x + c)^4 - 24*a*cos(d*x + c)^2 + 16*a)*sin(d*x + c)
+ 12*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.25, size = 103, normalized size = 0.87

$$-\frac{4\,a\sin(dx+c)^3+6\,a\sin(dx+c)^2-36\,a\log(|\sin(dx+c)|)-36\,a\sin(dx+c)+\frac{75\,a\sin(dx+c)^4-36\,a\sin(dx+c)^3-18\,a}{\sin(dx+c}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(4*a*sin(d*x + c)^3 + 6*a*sin(d*x + c)^2 - 36*a*log(abs(sin(d*x + c))
) - 36*a*sin(d*x + c) + (75*a*sin(d*x + c)^4 - 36*a*sin(d*x + c)^3 - 18*a*s
in(d*x + c)^2 + 4*a*sin(d*x + c) + 3*a)/sin(d*x + c)^4)/d
```

maple [A] time = 0.30, size = 217, normalized size = 1.84

$$-\frac{a\left(\cos^{8}\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)^{3}}+\frac{5a\left(\cos^{8}\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)}+\frac{16a\sin\left(dx+c\right)}{3d}+\frac{5\left(\cos^{6}\left(dx+c\right)\right)\sin\left(dx+c\right)a}{3d}+\frac{2a\sin\left(dx+c\right)\left(\cos^{4}\left(a\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^8+5/3/d*a/sin(d*x+c)*cos(d*x+c)^8+16/3*a*s
in(d*x+c)/d+5/3/d*cos(d*x+c)^6*sin(d*x+c)*a+2/d*a*sin(d*x+c)*cos(d*x+c)^4+8
/3/d*a*sin(d*x+c)*cos(d*x+c)^2-1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^8+1/2/d*a/si
n(d*x+c)^2*cos(d*x+c)^8+1/2*a*cos(d*x+c)^6/d+3/4*a*cos(d*x+c)^4/d+3/2*a*cos
(d*x+c)^2/d+3*a*ln(sin(d*x+c))/d
```

maxima [A] time = 0.35, size = 92, normalized size = 0.78

$$-\frac{4\,a\sin\left(dx+c\right)^{3}+6\,a\sin\left(dx+c\right)^{2}-36\,a\log\left(\sin\left(dx+c\right)\right)-36\,a\sin\left(dx+c\right)-\frac{36\,a\sin\left(dx+c\right)^{3}+18\,a\sin\left(dx+c\right)^{2}-4\,a\sin\left(\right.}{\sin\left(dx+c\right)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*a*sin(d*x + c)^3 + 6*a*sin(d*x + c)^2 - 36*a*log(sin(d*x + c)) - 3
6*a*sin(d*x + c) - (36*a*sin(d*x + c)^3 + 18*a*sin(d*x + c)^2 - 4*a*sin(d*x
 + c) - 3*a)/sin(d*x + c)^4)/d
```

mupad [B] time = 9.01, size = 290, normalized size = 2.46

$$\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{118\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}-27\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{644\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{3}-\frac{69\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{4}+160\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{d\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+48\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (11*a*tan(c/2 + (d*x)/2))/(8*d) + ((17*a*tan(c/2 + (d*x)/2)^2)/4 - (2*a*tan
(c/2 + (d*x)/2))/3 - a/4 + 20*a*tan(c/2 + (d*x)/2)^3 + (57*a*tan(c/2 + (d*x
)/2)^4)/4 + 160*a*tan(c/2 + (d*x)/2)^5 - (69*a*tan(c/2 + (d*x)/2)^6)/4 + (6
44*a*tan(c/2 + (d*x)/2)^7)/3 - 27*a*tan(c/2 + (d*x)/2)^8 + 118*a*tan(c/2 +
(d*x)/2)^9)/(d*(16*tan(c/2 + (d*x)/2)^4 + 48*tan(c/2 + (d*x)/2)^6 + 48*tan(
c/2 + (d*x)/2)^8 + 16*tan(c/2 + (d*x)/2)^10)) - (3*a*log(tan(c/2 + (d*x)/2)
^2 + 1))/d + (5*a*tan(c/2 + (d*x)/2)^2)/(16*d) - (a*tan(c/2 + (d*x)/2)^3)/(
24*d) - (a*tan(c/2 + (d*x)/2)^4)/(64*d) + (3*a*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.667 $\int \cos(c+dx)\cot^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}+$$

```
[Out] -3*a*csc(d*x+c)/d+3/2*a*csc(d*x+c)^2/d+a*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/
d-1/5*a*csc(d*x+c)^5/d+3*a*ln(sin(d*x+c))/d-a*sin(d*x+c)/d-1/2*a*sin(d*x+c)
^2/d
```

Rubi [A] time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2836, 12, 88}

$$-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d + (3*a*Csc[c + d*x]^2)/(2*d) + (a*Csc[c + d*x]^3)/d -
 (a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) + (3*a*Log[Sin[c + d*x
]])/d - (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^6(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^3(a+x)^4}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^6}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-a+\frac{a^7}{x^6}+\frac{a^6}{x^5}-\frac{3a^5}{x^4}-\frac{3a^4}{x^3}+\frac{3a^3}{x^2}+\frac{3a^2}{x}-x\right)dx, x, a$$
$$= -\frac{3a\csc(c+dx)}{d}+\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc^3(c+dx)}{d}-\frac{a\csc^4}{}$$

Mathematica [A] time = 0.19, size = 102, normalized size = 0.89

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc(c+dx)}{d}+\frac{a\left(-2\sin^2(c+dx)-\csc^4(c+dx)+6\csc^2(c+\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^3)/d - (a*Csc[c + d*x]^5)/(5*d) - (
a*Sin[c + d*x])/d + (a*(6*Csc[c + d*x]^2 - Csc[c + d*x]^4 + 12*Log[Sin[c +
d*x]] - 2*Sin[c + d*x]^2))/(4*d)
```

fricas [A] time = 0.72, size = 157, normalized size = 1.37

$$\frac{20\,a\cos(dx+c)^6-120\,a\cos(dx+c)^4+160\,a\cos(dx+c)^2+60\left(a\cos(dx+c)^4-2\,a\cos(dx+c)^2+a\right)\log\left(\frac{1}{2}\right.}{20\left(d\cos(dx+c)^4-2\,d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/20*(20*a*cos(d*x + c)^6 - 120*a*cos(d*x + c)^4 + 160*a*cos(d*x + c)^2 + 6
0*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*sin(d*x + c))*sin(d*x
 + c) + 5*(2*a*cos(d*x + c)^6 - 5*a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + 4
*a)*sin(d*x + c) - 64*a)/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d
*x + c))
```

giac [A] time = 0.24, size = 103, normalized size = 0.90

$$-\frac{10\,a\sin\left(dx+c\right)^{2}-60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+20\,a\sin\left(dx+c\right)+\frac{137\,a\sin(dx+c)^{5}+60\,a\sin(dx+c)^{4}-30\,a\sin(dx+c)^{3}-20\,a\sin(dx}{\sin(dx+c)^{5}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/20*(10*a*sin(d*x + c)^2 - 60*a*log(abs(sin(d*x + c))) + 20*a*sin(d*x + c
) + (137*a*sin(d*x + c)^5 + 60*a*sin(d*x + c)^4 - 30*a*sin(d*x + c)^3 - 20*
a*sin(d*x + c)^2 + 5*a*sin(d*x + c) + 4*a)/sin(d*x + c)^5)/d
```

maple [B] time = 0.31, size = 239, normalized size = 2.08

$$-\frac{a\left(\cos^{8}\left(dx+c\right)\right)}{4d\sin\left(dx+c\right)^{4}}+\frac{a\left(\cos^{8}\left(dx+c\right)\right)}{2d\sin\left(dx+c\right)^{2}}+\frac{a\left(\cos^{6}\left(dx+c\right)\right)}{2d}+\frac{3a\left(\cos^{4}\left(dx+c\right)\right)}{4d}+\frac{3a\left(\cos^{2}\left(dx+c\right)\right)}{2d}+\frac{3a\ln\left(\sin\left(dx+\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^8+1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^8+1/2*a*
cos(d*x+c)^6/d+3/4*a*cos(d*x+c)^4/d+3/2*a*cos(d*x+c)^2/d+3*a*ln(sin(d*x+c))
/d-1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^8+1/5/d*a/sin(d*x+c)^3*cos(d*x+c)^8-1/d*
a/sin(d*x+c)*cos(d*x+c)^8-16/5*a*sin(d*x+c)/d-1/d*cos(d*x+c)^6*sin(d*x+c)*a
-6/5/d*a*sin(d*x+c)*cos(d*x+c)^4-8/5/d*a*sin(d*x+c)*cos(d*x+c)^2
```

maxima [A] time = 0.34, size = 91, normalized size = 0.79

$$-\frac{10\,a\sin\left(dx+c\right)^{2}-60\,a\log\left(\sin\left(dx+c\right)\right)+20\,a\sin\left(dx+c\right)+\frac{60\,a\sin(dx+c)^{4}-30\,a\sin(dx+c)^{3}-20\,a\sin(dx+c)^{2}+5\,a\sin(dx+c)}{\sin(dx+c)^{5}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/20*(10*a*sin(d*x + c)^2 - 60*a*log(sin(d*x + c)) + 20*a*sin(d*x + c) + (
60*a*sin(d*x + c)^4 - 30*a*sin(d*x + c)^3 - 20*a*sin(d*x + c)^2 + 5*a*sin(d
*x + c) + 4*a)/sin(d*x + c)^5)/d
```

mupad [B] time = 9.10, size = 281, normalized size = 2.44

$$\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{16\,d}-\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}{d}-\frac{19\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{32\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{64\,d}-\frac{a\tan\left(\frac{c}{2}+\right.}{160}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (5*a*tan(c/2 + (d*x)/2)^2)/(16*d) - (3*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d -
 (19*a*tan(c/2 + (d*x)/2))/(16*d) + (3*a*tan(c/2 + (d*x)/2)^3)/(32*d) - (a*
tan(c/2 + (d*x)/2)^4)/(64*d) - (a*tan(c/2 + (d*x)/2)^5)/(160*d) + (3*a*log(
tan(c/2 + (d*x)/2)))/d - (a/5 + (a*tan(c/2 + (d*x)/2))/2 - (13*a*tan(c/2 +
(d*x)/2)^2)/5 - 9*a*tan(c/2 + (d*x)/2)^3 + (161*a*tan(c/2 + (d*x)/2)^4)/5 -
 (39*a*tan(c/2 + (d*x)/2)^5)/2 + 137*a*tan(c/2 + (d*x)/2)^6 + 54*a*tan(c/2
+ (d*x)/2)^7 + 102*a*tan(c/2 + (d*x)/2)^8)/(d*(32*tan(c/2 + (d*x)/2)^5 + 64
*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.668 $\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}$$

```
[Out] -3*a*csc(d*x+c)/d-3/2*a*csc(d*x+c)^2/d+a*csc(d*x+c)^3/d+3/4*a*csc(d*x+c)^4/
d-1/5*a*csc(d*x+c)^5/d-1/6*a*csc(d*x+c)^6/d-a*ln(sin(d*x+c))/d-a*sin(d*x+c)
/d
```

Rubi [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 88}

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d - (3*a*Csc[c + d*x]^2)/(2*d) + (a*Csc[c + d*x]^3)/d +
 (3*a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)
/(6*d) - (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^7}{x^7}+\frac{a^6}{x^6}-\frac{3a^5}{x^5}-\frac{3a^4}{x^4}+\frac{3a^3}{x^3}+\frac{3a^2}{x^2}-\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{3a\csc(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^4(c+dx)}{4d}-$$

Mathematica [A] time = 0.39, size = 111, normalized size = 0.97

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc(c+dx)}{d}-\frac{a\left(2\cot^6(c+dx)-3\cot^4(c+dx)+6\cot^2(c+\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^3)/d - (a*Csc[c + d*x]^5)/(5*d) - (
a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c + d*x]^6 + 12*Log[Cos[c +
d*x]] + 12*Log[Tan[c + d*x]]))/(12*d) - (a*Sin[c + d*x])/d
```

fricas [A] time = 0.92, size = 158, normalized size = 1.37

$$\frac{90\,a\cos(dx+c)^4-135\,a\cos(dx+c)^2-60\left(a\cos(dx+c)^6-3\,a\cos(dx+c)^4+3\,a\cos(dx+c)^2-a\right)\log\left(\frac{1}{2}\,\mathrm{s}\right.}{60\left(d\cos(dx+c)^6-3\,d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(90*a*cos(d*x + c)^4 - 135*a*cos(d*x + c)^2 - 60*(a*cos(d*x + c)^6 - 3
*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) - 12*(5*a
*cos(d*x + c)^6 - 30*a*cos(d*x + c)^4 + 40*a*cos(d*x + c)^2 - 16*a)*sin(d*x
 + c) + 55*a)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
 d)
```

giac [A] time = 0.28, size = 104, normalized size = 0.90

$$-\frac{60\,a\log(|\sin(dx+c)|)+60\,a\sin(dx+c)-\frac{147\,a\sin(dx+c)^6-180\,a\sin(dx+c)^5-90\,a\sin(dx+c)^4+60\,a\sin(dx+c)^3+45\,a\sin(dx+c)^2-}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*a*log(abs(sin(d*x + c))) + 60*a*sin(d*x + c) - (147*a*sin(d*x + c
)^6 - 180*a*sin(d*x + c)^5 - 90*a*sin(d*x + c)^4 + 60*a*sin(d*x + c)^3 + 45
*a*sin(d*x + c)^2 - 12*a*sin(d*x + c) - 10*a)/sin(d*x + c)^6)/d
```

maple [A] time = 0.34, size = 195, normalized size = 1.70

$$-\frac{a\left(\cos^{8}\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)^{5}}+\frac{a\left(\cos^{8}\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)^{3}}-\frac{a\left(\cos^{8}\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}-\frac{16a\sin\left(dx+c\right)}{5d}-\frac{\left(\cos^{6}\left(dx+c\right)\right)\sin\left(dx+c\right)a}{d}-\frac{6a\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^7*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^8+1/5/d*a/sin(d*x+c)^3*cos(d*x+c)^8-1/d*a/
sin(d*x+c)*cos(d*x+c)^8-16/5*a*sin(d*x+c)/d-1/d*cos(d*x+c)^6*sin(d*x+c)*a-6
/5/d*a*sin(d*x+c)*cos(d*x+c)^4-8/5/d*a*sin(d*x+c)*cos(d*x+c)^2-1/6*a*cot(d*
x+c)^6/d+1/4/d*a*cot(d*x+c)^4-1/2*a*cot(d*x+c)^2/d-a*ln(sin(d*x+c))/d
```

maxima [A] time = 0.32, size = 91, normalized size = 0.79

$$-\frac{60\,a\log\left(\sin\left(dx+c\right)\right)+60\,a\sin\left(dx+c\right)+\frac{180\,a\sin\left(dx+c\right)^{5}+90\,a\sin\left(dx+c\right)^{4}-60\,a\sin\left(dx+c\right)^{3}-45\,a\sin\left(dx+c\right)^{2}+12\,a\sin\left(dx+c\right)+10\,a}{\sin\left(dx+c\right)^{6}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(60*a*log(sin(d*x + c)) + 60*a*sin(d*x + c) + (180*a*sin(d*x + c)^5 +
 90*a*sin(d*x + c)^4 - 60*a*sin(d*x + c)^3 - 45*a*sin(d*x + c)^2 + 12*a*sin
(d*x + c) + 10*a)/sin(d*x + c)^6)/d
```

mupad [B] time = 10.12, size = 267, normalized size = 2.32

$$\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{32\,d}-\frac{29\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{128\,d}-\frac{19\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{32\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{160\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{384\,d}-\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^7,x)
```

```
[Out] (3*a*tan(c/2 + (d*x)/2)^3)/(32*d) - (29*a*tan(c/2 + (d*x)/2)^2)/(128*d) - (
19*a*tan(c/2 + (d*x)/2))/(16*d) + (a*tan(c/2 + (d*x)/2)^4)/(32*d) - (a*tan(
c/2 + (d*x)/2)^5)/(160*d) - (a*tan(c/2 + (d*x)/2)^6)/(384*d) - (a*(1920*log
(tan(c/2 + (d*x)/2)) - 1920*log(tan(c/2 + (d*x)/2)^2 + 1)))/(1920*d) - (cot
(c/2 + (d*x)/2)^6*(a/384 + (a*tan(c/2 + (d*x)/2))/160 - (11*a*tan(c/2 + (d*
x)/2)^2)/384 - (7*a*tan(c/2 + (d*x)/2)^3)/80 + (25*a*tan(c/2 + (d*x)/2)^4)/
128 + (35*a*tan(c/2 + (d*x)/2)^5)/32 + (29*a*tan(c/2 + (d*x)/2)^6)/128 + (5
1*a*tan(c/2 + (d*x)/2)^7)/16))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.669 $\int \cot^7(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}+\frac{3a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc(c+dx)}{d}$$

```
[Out] a*csc(d*x+c)/d-3/2*a*csc(d*x+c)^2/d-a*csc(d*x+c)^3/d+3/4*a*csc(d*x+c)^4/d+3
/5*a*csc(d*x+c)^5/d-1/6*a*csc(d*x+c)^6/d-1/7*a*csc(d*x+c)^7/d-a*ln(sin(d*x+
c))/d
```

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2836, 12, 88}

$$-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}+\frac{3a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x])/d - (3*a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/d + (3
*a*Csc[c + d*x]^4)/(4*d) + (3*a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)/
(6*d) - (a*Csc[c + d*x]^7)/(7*d) - (a*Log[Sin[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^7(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^8(a-x)^3(a+x)^4}{x^8}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^8}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{a^7}{x^8}+\frac{a^6}{x^7}-\frac{3a^5}{x^6}-\frac{3a^4}{x^5}+\frac{3a^3}{x^4}+\frac{3a^2}{x^3}-\frac{a}{x^2}-\frac{1}{x}\right)dx, x, a\right.}{d}$$
$$= \frac{a\csc(c+dx)}{d} - \frac{3a\csc^2(c+dx)}{2d} - \frac{a\csc^3(c+dx)}{d} + \frac{3a\csc^4(c}{4d}$$

Mathematica [A] time = 0.39, size = 115, normalized size = 0.97

$$-\frac{a\csc^7(c+dx)}{7d}+\frac{3a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{d}+\frac{a\csc(c+dx)}{d}-\frac{a\left(2\cot^6(c+dx)-3\cot^4(c+dx)+6\cot^2(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*a*Csc[c + d*x]^5)/(5*d) - (a
*Csc[c + d*x]^7)/(7*d) - (a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c
+ d*x]^6 + 12*Log[Cos[c + d*x]] + 12*Log[Tan[c + d*x]]))/(12*d)
```

fricas [A] time = 0.57, size = 172, normalized size = 1.45

$$\frac{420\,a\cos(dx+c)^6-840\,a\cos(dx+c)^4+672\,a\cos(dx+c)^2-420\left(a\cos(dx+c)^6-3\,a\cos(dx+c)^4+3\,a\,\text{co}\right.}{420\left(d\cos(dx+c)^6-3\,d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(420*a*cos(d*x + c)^6 - 840*a*cos(d*x + c)^4 + 672*a*cos(d*x + c)^2 -
 420*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(1
/2*sin(d*x + c))*sin(d*x + c) + 35*(18*a*cos(d*x + c)^4 - 27*a*cos(d*x + c)
^2 + 11*a)*sin(d*x + c) - 192*a)/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 +
3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.26, size = 106, normalized size = 0.89

$$-\frac{420\,a\log\left(|\sin\left(dx+c\right)|\right)-\frac{1089\,a\sin(dx+c)^7+420\,a\sin(dx+c)^6-630\,a\sin(dx+c)^5-420\,a\sin(dx+c)^4+315\,a\sin(dx+c)^3+252\,a\sin(dx+c)^2-70}{\sin(dx+c)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/420*(420*a*log(abs(sin(d*x + c))) - (1089*a*sin(d*x + c)^7 + 420*a*sin(d
*x + c)^6 - 630*a*sin(d*x + c)^5 - 420*a*sin(d*x + c)^4 + 315*a*sin(d*x + c
)^3 + 252*a*sin(d*x + c)^2 - 70*a*sin(d*x + c) - 60*a)/sin(d*x + c)^7)/d
```

maple [A] time = 0.36, size = 217, normalized size = 1.82

$$-\frac{a\left(\cot^6\left(dx+c\right)\right)}{6d}+\frac{a\left(\cot^4\left(dx+c\right)\right)}{4d}-\frac{a\left(\cot^2\left(dx+c\right)\right)}{2d}-\frac{a\ln\left(\sin\left(dx+c\right)\right)}{d}-\frac{a\left(\cos^8\left(dx+c\right)\right)}{7d\sin\left(dx+c\right)^7}+\frac{a\left(\cos^8\left(dx+c\right)\right)}{35d\sin\left(dx+c\right)^5}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^8*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/6*a*cot(d*x+c)^6/d+1/4/d*a*cot(d*x+c)^4-1/2*a*cot(d*x+c)^2/d-a*ln(sin(d*
x+c))/d-1/7/d*a/sin(d*x+c)^7*cos(d*x+c)^8+1/35/d*a/sin(d*x+c)^5*cos(d*x+c)^
8-1/35/d*a/sin(d*x+c)^3*cos(d*x+c)^8+1/7/d*a/sin(d*x+c)*cos(d*x+c)^8+16/35*
a*sin(d*x+c)/d+1/7/d*cos(d*x+c)^6*sin(d*x+c)*a+6/35/d*a*sin(d*x+c)*cos(d*x+
c)^4+8/35/d*a*sin(d*x+c)*cos(d*x+c)^2
```

maxima [A] time = 0.32, size = 94, normalized size = 0.79

$$-\frac{420\,a\log\left(\sin\left(dx+c\right)\right)-\frac{420\,a\sin(dx+c)^6-630\,a\sin(dx+c)^5-420\,a\sin(dx+c)^4+315\,a\sin(dx+c)^3+252\,a\sin(dx+c)^2-70\,a\sin(dx+c)-60\,a}{\sin(dx+c)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(420*a*log(sin(d*x + c)) - (420*a*sin(d*x + c)^6 - 630*a*sin(d*x + c
)^5 - 420*a*sin(d*x + c)^4 + 315*a*sin(d*x + c)^3 + 252*a*sin(d*x + c)^2 -
70*a*sin(d*x + c) - 60*a)/sin(d*x + c)^7)/d
```

mupad [B] time = 9.32, size = 270, normalized size = 2.27

$$\frac{35\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}+\frac{35\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{29\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}-\frac{7\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{128\,d}+\frac{a\cot\left(\frac{c}{2}\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^8,x)
```

```
[Out] (35*a*cot(c/2 + (d*x)/2))/(128*d) + (35*a*tan(c/2 + (d*x)/2))/(128*d) + (a*
log(tan(c/2 + (d*x)/2)^2 + 1))/d - (29*a*cot(c/2 + (d*x)/2)^2)/(128*d) - (7
*a*cot(c/2 + (d*x)/2)^3)/(128*d) + (a*cot(c/2 + (d*x)/2)^4)/(32*d) + (7*a*c
ot(c/2 + (d*x)/2)^5)/(640*d) - (a*cot(c/2 + (d*x)/2)^6)/(384*d) - (a*cot(c/
2 + (d*x)/2)^7)/(896*d) - (29*a*tan(c/2 + (d*x)/2)^2)/(128*d) - (7*a*tan(c/
2 + (d*x)/2)^3)/(128*d) + (a*tan(c/2 + (d*x)/2)^4)/(32*d) + (7*a*tan(c/2 +
(d*x)/2)^5)/(640*d) - (a*tan(c/2 + (d*x)/2)^6)/(384*d) - (a*tan(c/2 + (d*x)
/2)^7)/(896*d) - (a*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**8*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.670 $\int \cot^7(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^7(c+dx)}{7d}+\frac{3a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{d}+\frac{a\csc(c+dx)}{d}$$

```
[Out] -1/8*a*cot(d*x+c)^8/d+a*csc(d*x+c)/d-a*csc(d*x+c)^3/d+3/5*a*csc(d*x+c)^5/d-
1/7*a*csc(d*x+c)^7/d
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2607, 30, 2606, 194}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^7(c+dx)}{7d}+\frac{3a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{d}+\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) + (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*
a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^7(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc(c+dx)\,dx + a\int \cot^7(c+dx)\csc^2(c+$$

$$= -\frac{a\,\text{Subst}\left(\int x^7\,dx, x, -\cot(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(-1+x^2\right)}{c}$$

$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\,\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx, x, c\right.}{d}$$

$$= -\frac{a\cot^8(c+dx)}{8d} + \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{d} + \frac{3a\csc^5}{5}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 1.00

$$-\frac{a\cot^8(c+dx)}{8d} - \frac{a\csc^7(c+dx)}{7d} + \frac{3a\csc^5(c+dx)}{5d} - \frac{a\csc^3(c+dx)}{d} + \frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cot[c + d*x]^8)/d + (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*
a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

fricas [A] time = 0.64, size = 131, normalized size = 1.77

$$-\frac{140\,a\cos(dx+c)^6 - 210\,a\cos(dx+c)^4 + 140\,a\cos(dx+c)^2 + 8\left(35\,a\cos(dx+c)^6 - 70\,a\cos(dx+c)^4 + 56\right.}{280\left(d\cos(dx+c)^8 - 4\,d\cos(dx+c)^6 + 6\,d\cos(dx+c)^4 - 4\,d\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/280*(140*a*cos(d*x + c)^6 - 210*a*cos(d*x + c)^4 + 140*a*cos(d*x + c)^2
+ 8*(35*a*cos(d*x + c)^6 - 70*a*cos(d*x + c)^4 + 56*a*cos(d*x + c)^2 - 16*a
)*sin(d*x + c) - 35*a)/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x
 + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.28, size = 92, normalized size = 1.24

$$\frac{280\,a\sin\left(dx+c\right)^{7}+140\,a\sin\left(dx+c\right)^{6}-280\,a\sin\left(dx+c\right)^{5}-210\,a\sin\left(dx+c\right)^{4}+168\,a\sin\left(dx+c\right)^{3}+140\,a\sin}{280\,d\sin\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/280*(280*a*sin(d*x + c)^7 + 140*a*sin(d*x + c)^6 - 280*a*sin(d*x + c)^5 -
 210*a*sin(d*x + c)^4 + 168*a*sin(d*x + c)^3 + 140*a*sin(d*x + c)^2 - 40*a*
sin(d*x + c) - 35*a)/(d*sin(d*x + c)^8)
```

maple [B] time = 0.36, size = 138, normalized size = 1.86

$$\frac{a\left(-\frac{\cos^{8}(dx+c)}{7\sin(dx+c)^{7}}+\frac{\cos^{8}(dx+c)}{35\sin(dx+c)^{5}}-\frac{\cos^{8}(dx+c)}{35\sin(dx+c)^{3}}+\frac{\cos^{8}(dx+c)}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}(dx+c)\right)}{5}+\frac{8\left(\cos^{2}(dx+c)\right)}{5}\right)\sin(dx+c)}{7}\right)-\frac{a\left(\cos^{8}(dx+c)\right.}{8\sin(dx+c)^{8}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^9*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/
sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)^
6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/8*a/sin(d*x+c)^8*cos(d*x
+c)^8)
```

maxima [A] time = 0.33, size = 92, normalized size = 1.24

$$\frac{280\,a\sin\left(dx+c\right)^{7}+140\,a\sin\left(dx+c\right)^{6}-280\,a\sin\left(dx+c\right)^{5}-210\,a\sin\left(dx+c\right)^{4}+168\,a\sin\left(dx+c\right)^{3}+140\,a\sin}{280\,d\sin\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/280*(280*a*sin(d*x + c)^7 + 140*a*sin(d*x + c)^6 - 280*a*sin(d*x + c)^5 -
 210*a*sin(d*x + c)^4 + 168*a*sin(d*x + c)^3 + 140*a*sin(d*x + c)^2 - 40*a*
sin(d*x + c) - 35*a)/(d*sin(d*x + c)^8)
```

mupad [B] time = 9.39, size = 91, normalized size = 1.23

$$-\frac{-a\sin\left(c+d\,x\right)^7-\frac{a\sin(c+d\,x)^6}{2}+a\sin\left(c+d\,x\right)^5+\frac{3\,a\sin(c+d\,x)^4}{4}-\frac{3\,a\sin(c+d\,x)^3}{5}-\frac{a\sin(c+d\,x)^2}{2}+\frac{a\,\sin(c+d\,x)}{7}+\frac{a}{8}}{d\sin\left(c+d\,x\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^9,x)
```

```
[Out] -(a/8 + (a*sin(c + d*x))/7 - (a*sin(c + d*x)^2)/2 - (3*a*sin(c + d*x)^3)/5
+ (3*a*sin(c + d*x)^4)/4 + a*sin(c + d*x)^5 - (a*sin(c + d*x)^6)/2 - a*sin(
c + d*x)^7)/(d*sin(c + d*x)^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**9*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.671 $\int \cot^7(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^9(c+dx)}{9d}+\frac{3a\csc^7(c+dx)}{7d}-\frac{3a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{3d}$$

```
[Out] -1/8*a*cot(d*x+c)^8/d+1/3*a*csc(d*x+c)^3/d-3/5*a*csc(d*x+c)^5/d+3/7*a*csc(d
*x+c)^7/d-1/9*a*csc(d*x+c)^9/d
```

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2606, 270, 2607, 30}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^9(c+dx)}{9d}+\frac{3a\csc^7(c+dx)}{7d}-\frac{3a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) + (a*Csc[c + d*x]^3)/(3*d) - (3*a*Csc[c + d*x]^5)
/(5*d) + (3*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^7(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc^2(c+dx)\,dx + a\int \cot^7(c+dx)\csc^3(c$$

$$= -\frac{a\,\mathrm{Subst}\left(\int x^7\,dx, x, -\cot(c+dx)\right)}{d} - \frac{a\,\mathrm{Subst}\left(\int x^2\left(-1+\right.\right.}{}$$

$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\,\mathrm{Subst}\left(\int\left(-x^2+3x^4-3x^6+x^8\right)dx, x,\right.}{d}$$

$$= -\frac{a\cot^8(c+dx)}{8d} + \frac{a\csc^3(c+dx)}{3d} - \frac{3a\csc^5(c+dx)}{5d} + \frac{3a\,\mathrm{cs}}{}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 1.00

$$-\frac{a\cot^8(c+dx)}{8d} - \frac{a\csc^9(c+dx)}{9d} + \frac{3a\csc^7(c+dx)}{7d} - \frac{3a\csc^5(c+dx)}{5d} + \frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cot[c + d*x]^8)/d + (a*Csc[c + d*x]^3)/(3*d) - (3*a*Csc[c + d*x]^5)
/(5*d) + (3*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

fricas [A] time = 0.83, size = 139, normalized size = 1.72

$$-\frac{840\,a\cos(dx+c)^6 - 1008\,a\cos(dx+c)^4 + 576\,a\cos(dx+c)^2 + 315\left(4\,a\cos(dx+c)^6 - 6\,a\cos(dx+c)^4 + 4\right.}{2520\left(d\cos(dx+c)^8 - 4\,d\cos(dx+c)^6 + 6\,d\cos(dx+c)^4 - 4\,d\cos(dx+c)^2 + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2520*(840*a*cos(d*x + c)^6 - 1008*a*cos(d*x + c)^4 + 576*a*cos(d*x + c)^
2 + 315*(4*a*cos(d*x + c)^6 - 6*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 - a)*
sin(d*x + c) - 128*a)/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x
 + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.28, size = 92, normalized size = 1.14

$$\frac{1260\,a\sin\left(dx+c\right)^{7}+840\,a\sin\left(dx+c\right)^{6}-1890\,a\sin\left(dx+c\right)^{5}-1512\,a\sin\left(dx+c\right)^{4}+1260\,a\sin\left(dx+c\right)^{3}+108}{2520\,d\sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2520*(1260*a*sin(d*x + c)^7 + 840*a*sin(d*x + c)^6 - 1890*a*sin(d*x + c)^
5 - 1512*a*sin(d*x + c)^4 + 1260*a*sin(d*x + c)^3 + 1080*a*sin(d*x + c)^2 -
 315*a*sin(d*x + c) - 280*a)/(d*sin(d*x + c)^9)
```

maple [B] time = 0.38, size = 156, normalized size = 1.93

$$\frac{-\frac{a\left(\cos^{8}\left(dx+c\right)\right)}{8\sin\left(dx+c\right)^{8}}+a\left(-\frac{\cos^{8}\left(dx+c\right)}{9\sin\left(dx+c\right)^{9}}-\frac{\cos^{8}\left(dx+c\right)}{63\sin\left(dx+c\right)^{7}}+\frac{\cos^{8}\left(dx+c\right)}{315\sin\left(dx+c\right)^{5}}-\frac{\cos^{8}\left(dx+c\right)}{315\sin\left(dx+c\right)^{3}}+\frac{\cos^{8}\left(dx+c\right)}{63\sin\left(dx+c\right)}+\frac{\left(\frac{16}{5}+\cos^{6}\left(dx+c\right)+\frac{6\left(\cos^{4}\left(dx+c\right)\right)}{5}+\frac{8}{}\right.}{63}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^10*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/8*a/sin(d*x+c)^8*cos(d*x+c)^8+a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^8-1/6
3/sin(d*x+c)^7*cos(d*x+c)^8+1/315/sin(d*x+c)^5*cos(d*x+c)^8-1/315/sin(d*x+c
)^3*cos(d*x+c)^8+1/63/sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*c
os(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.35, size = 92, normalized size = 1.14

$$\frac{1260\,a\sin\left(dx+c\right)^{7}+840\,a\sin\left(dx+c\right)^{6}-1890\,a\sin\left(dx+c\right)^{5}-1512\,a\sin\left(dx+c\right)^{4}+1260\,a\sin\left(dx+c\right)^{3}+108}{2520\,d\sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/2520*(1260*a*sin(d*x + c)^7 + 840*a*sin(d*x + c)^6 - 1890*a*sin(d*x + c)^
5 - 1512*a*sin(d*x + c)^4 + 1260*a*sin(d*x + c)^3 + 1080*a*sin(d*x + c)^2 -
 315*a*sin(d*x + c) - 280*a)/(d*sin(d*x + c)^9)
```

mupad [B] time = 9.23, size = 92, normalized size = 1.14

$$-\frac{-\frac{a\sin(c+d\,x)^7}{2}-\frac{a\sin(c+d\,x)^6}{3}+\frac{3\,a\sin(c+d\,x)^5}{4}+\frac{3\,a\sin(c+d\,x)^4}{5}-\frac{a\sin(c+d\,x)^3}{2}-\frac{3\,a\sin(c+d\,x)^2}{7}+\frac{a\,\sin(c+d\,x)}{8}+\frac{a}{9}}{d\sin\left(c+d\,x\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^10,x)
```

```
[Out] -(a/9 + (a*sin(c + d*x))/8 - (3*a*sin(c + d*x)^2)/7 - (a*sin(c + d*x)^3)/2
+ (3*a*sin(c + d*x)^4)/5 + (3*a*sin(c + d*x)^5)/4 - (a*sin(c + d*x)^6)/3 -
(a*sin(c + d*x)^7)/2)/(d*sin(c + d*x)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**10*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.672 $\int \cot^7(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\cot^{10}(c+dx)}{10d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^9(c+dx)}{9d}+\frac{3a\csc^7(c+dx)}{7d}-\frac{3a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{3d}$$

```
[Out] -1/8*a*cot(d*x+c)^8/d-1/10*a*cot(d*x+c)^10/d+1/3*a*csc(d*x+c)^3/d-3/5*a*csc
(d*x+c)^5/d+3/7*a*csc(d*x+c)^7/d-1/9*a*csc(d*x+c)^9/d
```

Rubi [A] time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2607, 14, 2606, 270}

$$-\frac{a\cot^{10}(c+dx)}{10d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^9(c+dx)}{9d}+\frac{3a\csc^7(c+dx)}{7d}-\frac{3a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) - (a*Cot[c + d*x]^10)/(10*d) + (a*Csc[c + d*x]^3)
/(3*d) - (3*a*Csc[c + d*x]^5)/(5*d) + (3*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c
 + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^7(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc^3(c+dx)\,dx + a\int \cot^7(c+dx)\csc^4(c$$
$$= -\frac{a\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{a\,\text{Subst}\left(\int \left(-x^2+3x^4-3x^6+x^8\right)dx, x, \csc(c+dx)\right)}{d} - \frac{a}{}$$
$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\cot^{10}(c+dx)}{10d} + \frac{a\csc^3(c+dx)}{3d} - \frac{3a\csc}{}$$

Mathematica [A] time = 0.18, size = 86, normalized size = 0.89

$$-\frac{a\csc^3(c+dx)\left(252\csc^7(c+dx)+280\csc^6(c+dx)-945\csc^5(c+dx)-1080\csc^4(c+dx)+1260\csc^3(c+dx)\right.}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2520*(a*Csc[c + d*x]^3*(-840 - 630*Csc[c + d*x] + 1512*Csc[c + d*x]^2 +
1260*Csc[c + d*x]^3 - 1080*Csc[c + d*x]^4 - 945*Csc[c + d*x]^5 + 280*Csc[c
+ d*x]^6 + 252*Csc[c + d*x]^7))/d
```

fricas [A] time = 1.07, size = 144, normalized size = 1.48

$$\frac{630\,a\cos(dx+c)^6 - 630\,a\cos(dx+c)^4 + 315\,a\cos(dx+c)^2 + 8\left(105\,a\cos(dx+c)^6 - 126\,a\cos(dx+c)^4 + 72\right.}{2520\left(d\cos(dx+c)^{10} - 5\,d\cos(dx+c)^8 + 10\,d\cos(dx+c)^6 - 10\,d\cos(dx+c)^4 + 5\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/2520*(630*a*cos(d*x + c)^6 - 630*a*cos(d*x + c)^4 + 315*a*cos(d*x + c)^2
+ 8*(105*a*cos(d*x + c)^6 - 126*a*cos(d*x + c)^4 + 72*a*cos(d*x + c)^2 - 16
*a)*sin(d*x + c) - 63*a)/(d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos
(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.29, size = 92, normalized size = 0.95

$$\frac{840\,a\sin\left(dx+c\right)^{7}+630\,a\sin\left(dx+c\right)^{6}-1512\,a\sin\left(dx+c\right)^{5}-1260\,a\sin\left(dx+c\right)^{4}+1080\,a\sin\left(dx+c\right)^{3}+945}{2520\,d\sin\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2520*(840*a*sin(d*x + c)^7 + 630*a*sin(d*x + c)^6 - 1512*a*sin(d*x + c)^5
 - 1260*a*sin(d*x + c)^4 + 1080*a*sin(d*x + c)^3 + 945*a*sin(d*x + c)^2 - 2
80*a*sin(d*x + c) - 252*a)/(d*sin(d*x + c)^10)
```

maple [B] time = 0.38, size = 176, normalized size = 1.81

$$\frac{a\left(-\frac{\cos^{8}(dx+c)}{9\sin(dx+c)^{9}}-\frac{\cos^{8}(dx+c)}{63\sin(dx+c)^{7}}+\frac{\cos^{8}(dx+c)}{315\sin(dx+c)^{5}}-\frac{\cos^{8}(dx+c)}{315\sin(dx+c)^{3}}+\frac{\cos^{8}(dx+c)}{63\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}(dx+c)\right)}{5}+\frac{8\left(\cos^{2}(dx+c)\right)}{5}\right)\sin(dx+}{63}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^11*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^8-1/63/sin(d*x+c)^7*cos(d*x+c)^8+1/315
/sin(d*x+c)^5*cos(d*x+c)^8-1/315/sin(d*x+c)^3*cos(d*x+c)^8+1/63/sin(d*x+c)*
cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin
(d*x+c))+a*(-1/10/sin(d*x+c)^10*cos(d*x+c)^8-1/40/sin(d*x+c)^8*cos(d*x+c)^8
))
```

maxima [A] time = 0.37, size = 92, normalized size = 0.95

$$\frac{840\,a\sin\left(dx+c\right)^{7}+630\,a\sin\left(dx+c\right)^{6}-1512\,a\sin\left(dx+c\right)^{5}-1260\,a\sin\left(dx+c\right)^{4}+1080\,a\sin\left(dx+c\right)^{3}+945}{2520\,d\sin\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/2520*(840*a*sin(d*x + c)^7 + 630*a*sin(d*x + c)^6 - 1512*a*sin(d*x + c)^5
 - 1260*a*sin(d*x + c)^4 + 1080*a*sin(d*x + c)^3 + 945*a*sin(d*x + c)^2 - 2
80*a*sin(d*x + c) - 252*a)/(d*sin(d*x + c)^10)
```

mupad [B] time = 9.21, size = 92, normalized size = 0.95

$$-\frac{-\frac{a\sin(c+d\,x)^7}{3}-\frac{a\sin(c+d\,x)^6}{4}+\frac{3\,a\sin(c+d\,x)^5}{5}+\frac{a\sin(c+d\,x)^4}{2}-\frac{3\,a\sin(c+d\,x)^3}{7}-\frac{3\,a\sin(c+d\,x)^2}{8}+\frac{a\,\sin(c+d\,x)}{9}+\frac{a}{10}}{d\sin\left(c+d\,x\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^11,x)
```

```
[Out] -(a/10 + (a*sin(c + d*x))/9 - (3*a*sin(c + d*x)^2)/8 - (3*a*sin(c + d*x)^3)
/7 + (a*sin(c + d*x)^4)/2 + (3*a*sin(c + d*x)^5)/5 - (a*sin(c + d*x)^6)/4 -
 (a*sin(c + d*x)^7)/3)/(d*sin(c + d*x)^10)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**11*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.673 $\int \cot^7(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\cot^{10}(c+dx)}{10d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^{11}(c+dx)}{11d}+\frac{a\csc^9(c+dx)}{3d}-\frac{3a\csc^7(c+dx)}{7d}+\frac{a\csc^5(c+dx)}{5d}$$

```
[Out] -1/8*a*cot(d*x+c)^8/d-1/10*a*cot(d*x+c)^10/d+1/5*a*csc(d*x+c)^5/d-3/7*a*csc
(d*x+c)^7/d+1/3*a*csc(d*x+c)^9/d-1/11*a*csc(d*x+c)^11/d
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2606, 270, 2607, 14}

$$-\frac{a\cot^{10}(c+dx)}{10d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^{11}(c+dx)}{11d}+\frac{a\csc^9(c+dx)}{3d}-\frac{3a\csc^7(c+dx)}{7d}+\frac{a\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) - (a*Cot[c + d*x]^10)/(10*d) + (a*Csc[c + d*x]^5)
/(5*d) - (3*a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c + d*x]^9)/(3*d) - (a*Csc[c +
 d*x]^11)/(11*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^7(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc^4(c+dx)\,dx + a\int \cot^7(c+dx)\csc^5(c\cdots$$

$$= -\frac{a\,\text{Subst}\left(\int x^4\left(-1+x^2\right)^3\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int\cdots\right.}{\cdots}$$

$$= -\frac{a\,\text{Subst}\left(\int \left(x^7+x^9\right)dx, x, -\cot(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int\left(\cdots\right.\right.}{\cdots}$$

$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\cot^{10}(c+dx)}{10d} + \frac{a\csc^5(c+dx)}{5d} - \frac{3a\csc\cdots}{\cdots}$$

Mathematica [A] time = 0.14, size = 86, normalized size = 0.89

$$-\frac{a\csc^4(c+dx)\left(840\csc^7(c+dx)+924\csc^6(c+dx)-3080\csc^5(c+dx)-3465\csc^4(c+dx)+3960\csc^3(c+d\cdots\right.}{9240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/9240*(a*Csc[c + d*x]^4*(-2310 - 1848*Csc[c + d*x] + 4620*Csc[c + d*x]^2
+ 3960*Csc[c + d*x]^3 - 3465*Csc[c + d*x]^4 - 3080*Csc[c + d*x]^5 + 924*Csc
[c + d*x]^6 + 840*Csc[c + d*x]^7))/d
```

fricas [A] time = 0.93, size = 152, normalized size = 1.57

$$\frac{1848\,a\cos\left(dx+c\right)^6-1584\,a\cos\left(dx+c\right)^4+704\,a\cos\left(dx+c\right)^2+231\left(10\,a\cos\left(dx+c\right)^6-10\,a\cos\left(dx+c\right)^4+\cdots\right.}{9240\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^8+10\,d\cos\left(dx+c\right)^6-10\,d\cos\left(dx+c\right)^4+5\,d\cos\left(d\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/9240*(1848*a*cos(d*x + c)^6 - 1584*a*cos(d*x + c)^4 + 704*a*cos(d*x + c)^
2 + 231*(10*a*cos(d*x + c)^6 - 10*a*cos(d*x + c)^4 + 5*a*cos(d*x + c)^2 - a
)*sin(d*x + c) - 128*a)/((d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos
(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.28, size = 92, normalized size = 0.95

$$\frac{2310\,a\sin\left(dx+c\right)^{7}+1848\,a\sin\left(dx+c\right)^{6}-4620\,a\sin\left(dx+c\right)^{5}-3960\,a\sin\left(dx+c\right)^{4}+3465\,a\sin\left(dx+c\right)^{3}+3}{9240\,d\sin\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/9240*(2310*a*sin(d*x + c)^7 + 1848*a*sin(d*x + c)^6 - 4620*a*sin(d*x + c)
^5 - 3960*a*sin(d*x + c)^4 + 3465*a*sin(d*x + c)^3 + 3080*a*sin(d*x + c)^2
- 924*a*sin(d*x + c) - 840*a)/(d*sin(d*x + c)^11)
```

maple [B] time = 0.39, size = 194, normalized size = 2.00

$$\frac{a\left(-\frac{\cos^{8}(dx+c)}{10\sin(dx+c)^{10}}-\frac{\cos^{8}(dx+c)}{40\sin(dx+c)^{8}}\right)+a\left(-\frac{\cos^{8}(dx+c)}{11\sin(dx+c)^{11}}-\frac{\cos^{8}(dx+c)}{33\sin(dx+c)^{9}}-\frac{\cos^{8}(dx+c)}{231\sin(dx+c)^{7}}+\frac{\cos^{8}(dx+c)}{1155\sin(dx+c)^{5}}-\frac{\cos^{8}(dx+c)}{1155\sin(dx+c)^{3}}+\frac{\text{cc}}{231}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^12*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/10/sin(d*x+c)^10*cos(d*x+c)^8-1/40/sin(d*x+c)^8*cos(d*x+c)^8)+a*
(-1/11/sin(d*x+c)^11*cos(d*x+c)^8-1/33/sin(d*x+c)^9*cos(d*x+c)^8-1/231/sin(
d*x+c)^7*cos(d*x+c)^8+1/1155/sin(d*x+c)^5*cos(d*x+c)^8-1/1155/sin(d*x+c)^3*
cos(d*x+c)^8+1/231/sin(d*x+c)*cos(d*x+c)^8+1/231*(16/5+cos(d*x+c)^6+6/5*cos
(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.33, size = 92, normalized size = 0.95

$$\frac{2310\,a\sin\left(dx+c\right)^{7}+1848\,a\sin\left(dx+c\right)^{6}-4620\,a\sin\left(dx+c\right)^{5}-3960\,a\sin\left(dx+c\right)^{4}+3465\,a\sin\left(dx+c\right)^{3}+3}{9240\,d\sin\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/9240*(2310*a*sin(d*x + c)^7 + 1848*a*sin(d*x + c)^6 - 4620*a*sin(d*x + c)
^5 - 3960*a*sin(d*x + c)^4 + 3465*a*sin(d*x + c)^3 + 3080*a*sin(d*x + c)^2
- 924*a*sin(d*x + c) - 840*a)/(d*sin(d*x + c)^11)
```

mupad [B] time = 9.28, size = 92, normalized size = 0.95

$$-\frac{-\frac{a\sin(c+d\,x)^7}{4}-\frac{a\sin(c+d\,x)^6}{5}+\frac{a\sin(c+d\,x)^5}{2}+\frac{3\,a\sin(c+d\,x)^4}{7}-\frac{3\,a\sin(c+d\,x)^3}{8}-\frac{a\sin(c+d\,x)^2}{3}+\frac{a\,\sin(c+d\,x)}{10}+\frac{a}{11}}{d\sin\left(c+d\,x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^12,x)
```

```
[Out] -(a/11 + (a*sin(c + d*x))/10 - (a*sin(c + d*x)^2)/3 - (3*a*sin(c + d*x)^3)/
8 + (3*a*sin(c + d*x)^4)/7 + (a*sin(c + d*x)^5)/2 - (a*sin(c + d*x)^6)/5 -
(a*sin(c + d*x)^7)/4)/(d*sin(c + d*x)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**12*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.674 $\int \cot^7(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=113

$$-\frac{a\cot^{12}(c+dx)}{12d}-\frac{a\cot^{10}(c+dx)}{5d}-\frac{a\cot^{8}(c+dx)}{8d}-\frac{a\csc^{11}(c+dx)}{11d}+\frac{a\csc^{9}(c+dx)}{3d}-\frac{3a\csc^{7}(c+dx)}{7d}+\frac{a\csc^{5}(c+d}{5d}$$

```
[Out] -1/8*a*cot(d*x+c)^8/d-1/5*a*cot(d*x+c)^10/d-1/12*a*cot(d*x+c)^12/d+1/5*a*cs
c(d*x+c)^5/d-3/7*a*csc(d*x+c)^7/d+1/3*a*csc(d*x+c)^9/d-1/11*a*csc(d*x+c)^11
/d
```

Rubi [A] time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2834, 2607, 266, 43, 2606, 270}

$$-\frac{a\cot^{12}(c+dx)}{12d}-\frac{a\cot^{10}(c+dx)}{5d}-\frac{a\cot^{8}(c+dx)}{8d}-\frac{a\csc^{11}(c+dx)}{11d}+\frac{a\csc^{9}(c+dx)}{3d}-\frac{3a\csc^{7}(c+dx)}{7d}+\frac{a\csc^{5}(c+d}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) - (a*Cot[c + d*x]^10)/(5*d) - (a*Cot[c + d*x]^12)
/(12*d) + (a*Csc[c + d*x]^5)/(5*d) - (3*a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c
+ d*x]^9)/(3*d) - (a*Csc[c + d*x]^11)/(11*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^7(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc^5(c+dx)\,dx + a\int \cot^7(c+dx)\csc^6(c$$
$$= -\frac{a\,\text{Subst}\left(\int x^4\left(-1+x^2\right)^3\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{a\,\text{Subst}\left(\int x^3(1+x)^2\,dx, x, \cot^2(c+dx)\right)}{2d} - \frac{a\,\text{Subst}\left(\int\left(-\right.\right.}{}$$
$$= \frac{a\csc^5(c+dx)}{5d} - \frac{3a\csc^7(c+dx)}{7d} + \frac{a\csc^9(c+dx)}{3d} - \frac{a\csc^{11}}{1}$$
$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\cot^{10}(c+dx)}{5d} - \frac{a\cot^{12}(c+dx)}{12d} + \frac{a\csc}{}$$

Mathematica [A] time = 0.23, size = 86, normalized size = 0.76

$$-\frac{a\csc^{12}(c+dx)(-45\sin(c+dx)+1111\sin(3(c+dx))+363\sin(5(c+dx))+231\sin(7(c+dx))+3003\cos(2(}{73920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/73920*(a*Csc[c + d*x]^12*(1617 + 3003*Cos[2*(c + d*x)] + 1155*Cos[4*(c +
 d*x)] + 385*Cos[6*(c + d*x)] - 45*Sin[c + d*x] + 1111*Sin[3*(c + d*x)] + 3
63*Sin[5*(c + d*x)] + 231*Sin[7*(c + d*x)]))/d
```

fricas [A] time = 1.03, size = 153, normalized size = 1.35

$$-\frac{1540\,a\cos\left(dx+c\right)^{6}-1155\,a\cos\left(dx+c\right)^{4}+462\,a\cos\left(dx+c\right)^{2}+8\left(231\,a\cos\left(dx+c\right)^{6}-198\,a\cos\left(dx+c\right)^{4}+\right.}{9240\left(d\cos\left(dx+c\right)^{12}-6\,d\cos\left(dx+c\right)^{10}+15\,d\cos\left(dx+c\right)^{8}-20\,d\cos\left(dx+c\right)^{6}+15\,d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/9240*(1540*a*cos(d*x + c)^6 - 1155*a*cos(d*x + c)^4 + 462*a*cos(d*x + c)
^2 + 8*(231*a*cos(d*x + c)^6 - 198*a*cos(d*x + c)^4 + 88*a*cos(d*x + c)^2 -
 16*a)*sin(d*x + c) - 77*a)/(d*cos(d*x + c)^12 - 6*d*cos(d*x + c)^10 + 15*d
*cos(d*x + c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)^4 - 6*d*cos(d*x +
 c)^2 + d)
```

giac [A] time = 0.30, size = 92, normalized size = 0.81

$$\frac{1848\,a\sin\left(dx+c\right)^{7}+1540\,a\sin\left(dx+c\right)^{6}-3960\,a\sin\left(dx+c\right)^{5}-3465\,a\sin\left(dx+c\right)^{4}+3080\,a\sin\left(dx+c\right)^{3}+2}{9240\,d\sin\left(dx+c\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/9240*(1848*a*sin(d*x + c)^7 + 1540*a*sin(d*x + c)^6 - 3960*a*sin(d*x + c)
^5 - 3465*a*sin(d*x + c)^4 + 3080*a*sin(d*x + c)^3 + 2772*a*sin(d*x + c)^2
- 840*a*sin(d*x + c) - 770*a)/(d*sin(d*x + c)^12)
```

maple [B] time = 0.38, size = 212, normalized size = 1.88

$$\frac{a\left(-\frac{\cos^{8}(dx+c)}{11\sin(dx+c)^{11}}-\frac{\cos^{8}(dx+c)}{33\sin(dx+c)^{9}}-\frac{\cos^{8}(dx+c)}{231\sin(dx+c)^{7}}+\frac{\cos^{8}(dx+c)}{1155\sin(dx+c)^{5}}-\frac{\cos^{8}(dx+c)}{1155\sin(dx+c)^{3}}+\frac{\cos^{8}(dx+c)}{231\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}(dx+c\right.}{5}\right.}{23\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^13*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/11/sin(d*x+c)^11*cos(d*x+c)^8-1/33/sin(d*x+c)^9*cos(d*x+c)^8-1/2
31/sin(d*x+c)^7*cos(d*x+c)^8+1/1155/sin(d*x+c)^5*cos(d*x+c)^8-1/1155/sin(d*
x+c)^3*cos(d*x+c)^8+1/231/sin(d*x+c)*cos(d*x+c)^8+1/231*(16/5+cos(d*x+c)^6+
6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/12/sin(d*x+c)^12*cos(d
*x+c)^8-1/30/sin(d*x+c)^10*cos(d*x+c)^8-1/120/sin(d*x+c)^8*cos(d*x+c)^8))
```

maxima [A] time = 0.35, size = 92, normalized size = 0.81

$$\frac{1848\,a\sin\left(dx+c\right)^{7}+1540\,a\sin\left(dx+c\right)^{6}-3960\,a\sin\left(dx+c\right)^{5}-3465\,a\sin\left(dx+c\right)^{4}+3080\,a\sin\left(dx+c\right)^{3}+}{9240\,d\sin\left(dx+c\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/9240*(1848*a*sin(d*x + c)^7 + 1540*a*sin(d*x + c)^6 - 3960*a*sin(d*x + c)
^5 - 3465*a*sin(d*x + c)^4 + 3080*a*sin(d*x + c)^3 + 2772*a*sin(d*x + c)^2
- 840*a*sin(d*x + c) - 770*a)/(d*sin(d*x + c)^12)
```

mupad [B] time = 9.19, size = 92, normalized size = 0.81

$$-\frac{-\frac{a\sin(c+d\,x)^7}{5}-\frac{a\sin(c+d\,x)^6}{6}+\frac{3\,a\sin(c+d\,x)^5}{7}+\frac{3\,a\sin(c+d\,x)^4}{8}-\frac{a\sin(c+d\,x)^3}{3}-\frac{3\,a\sin(c+d\,x)^2}{10}+\frac{a\,\sin(c+d\,x)}{11}+\frac{a}{12}}{d\sin\left(c+d\,x\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^13,x)
```

```
[Out] -(a/12 + (a*sin(c + d*x))/11 - (3*a*sin(c + d*x)^2)/10 - (a*sin(c + d*x)^3)
/3 + (3*a*sin(c + d*x)^4)/8 + (3*a*sin(c + d*x)^5)/7 - (a*sin(c + d*x)^6)/6
 - (a*sin(c + d*x)^7)/5)/(d*sin(c + d*x)^12)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**13*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.675 $\int \cot^7(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=113

$$-\frac{a\cot^{12}(c+dx)}{12d}-\frac{a\cot^{10}(c+dx)}{5d}-\frac{a\cot^{8}(c+dx)}{8d}-\frac{a\csc^{13}(c+dx)}{13d}+\frac{3a\csc^{11}(c+dx)}{11d}-\frac{a\csc^{9}(c+dx)}{3d}+\frac{a\csc^{7}(c+}{7d}$$

```
[Out] -1/8*a*cot(d*x+c)^8/d-1/5*a*cot(d*x+c)^10/d-1/12*a*cot(d*x+c)^12/d+1/7*a*cs
c(d*x+c)^7/d-1/3*a*csc(d*x+c)^9/d+3/11*a*csc(d*x+c)^11/d-1/13*a*csc(d*x+c)^
13/d
```

Rubi [A] time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2834, 2606, 270, 2607, 266, 43}

$$-\frac{a\cot^{12}(c+dx)}{12d}-\frac{a\cot^{10}(c+dx)}{5d}-\frac{a\cot^{8}(c+dx)}{8d}-\frac{a\csc^{13}(c+dx)}{13d}+\frac{3a\csc^{11}(c+dx)}{11d}-\frac{a\csc^{9}(c+dx)}{3d}+\frac{a\csc^{7}(c+}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) - (a*Cot[c + d*x]^10)/(5*d) - (a*Cot[c + d*x]^12)
/(12*d) + (a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(3*d) + (3*a*Csc[c
+ d*x]^11)/(11*d) - (a*Csc[c + d*x]^13)/(13*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^7(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc^6(c+dx)\,dx + a\int \cot^7(c+dx)\csc^7(c$$

$$= -\frac{a\,\text{Subst}\left(\int x^6\left(-1+x^2\right)^3\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a\,\text{Subst}\left(\int x^3(1+x)^2\,dx, x, \cot^2(c+dx)\right)}{2d} - \frac{a\,\text{Subst}\left(\int (-\right.}{}$$

$$= \frac{a\csc^7(c+dx)}{7d} - \frac{a\csc^9(c+dx)}{3d} + \frac{3a\csc^{11}(c+dx)}{11d} - \frac{a\csc^1}{}$$

$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\cot^{10}(c+dx)}{5d} - \frac{a\cot^{12}(c+dx)}{12d} + \frac{a\csc}{}$$

Mathematica [A] time = 0.22, size = 86, normalized size = 0.76

$$-\frac{a\csc^{13}(c+dx)(3003\sin(c+dx)+24024\sin(3(c+dx))+10010\sin(5(c+dx))+5005\sin(7(c+dx))+7046}{1921920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/1921920*(a*Csc[c + d*x]^13*(40200 + 70460*Cos[2*(c + d*x)] + 28600*Cos[4
*(c + d*x)] + 8580*Cos[6*(c + d*x)] + 3003*Sin[c + d*x] + 24024*Sin[3*(c +
d*x)] + 10010*Sin[5*(c + d*x)] + 5005*Sin[7*(c + d*x)]))/d
```

fricas [A] time = 0.67, size = 161, normalized size = 1.42

$$-\frac{17160\,a\cos\left(dx+c\right)^{6}-11440\,a\cos\left(dx+c\right)^{4}+4160\,a\cos\left(dx+c\right)^{2}+1001\left(20\,a\cos\left(dx+c\right)^{6}-15\,a\cos\left(dx+c\right.\right.}{120120\left(d\cos\left(dx+c\right)^{12}-6\,d\cos\left(dx+c\right)^{10}+15\,d\cos\left(dx+c\right)^{8}-20\,d\cos\left(dx+c\right)^{6}+15\,d\cos\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^14*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/120120*(17160*a*cos(d*x + c)^6 - 11440*a*cos(d*x + c)^4 + 4160*a*cos(d*x
 + c)^2 + 1001*(20*a*cos(d*x + c)^6 - 15*a*cos(d*x + c)^4 + 6*a*cos(d*x + c
)^2 - a)*sin(d*x + c) - 640*a)/((d*cos(d*x + c)^12 - 6*d*cos(d*x + c)^10 +
15*d*cos(d*x + c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)^4 - 6*d*cos(d
*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.32, size = 92, normalized size = 0.81

$$\frac{20020\,a\sin\left(dx+c\right)^{7}+17160\,a\sin\left(dx+c\right)^{6}-45045\,a\sin\left(dx+c\right)^{5}-40040\,a\sin\left(dx+c\right)^{4}+36036\,a\sin\left(dx+c\right.}{120120\,d\sin\left(dx+c\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^14*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120120*(20020*a*sin(d*x + c)^7 + 17160*a*sin(d*x + c)^6 - 45045*a*sin(d*x
 + c)^5 - 40040*a*sin(d*x + c)^4 + 36036*a*sin(d*x + c)^3 + 32760*a*sin(d*x
 + c)^2 - 10010*a*sin(d*x + c) - 9240*a)/(d*sin(d*x + c)^13)
```

maple [B] time = 0.39, size = 230, normalized size = 2.04

$$\frac{a\left(-\frac{\cos^{8}(dx+c)}{12\sin(dx+c)^{12}}-\frac{\cos^{8}(dx+c)}{30\sin(dx+c)^{10}}-\frac{\cos^{8}(dx+c)}{120\sin(dx+c)^{8}}\right)+a\left(-\frac{\cos^{8}(dx+c)}{13\sin(dx+c)^{13}}-\frac{5\left(\cos^{8}(dx+c)\right)}{143\sin(dx+c)^{11}}-\frac{5\left(\cos^{8}(dx+c)\right)}{429\sin(dx+c)^{9}}-\frac{5\left(\cos^{8}(dx+c)\right)}{3003\sin(dx+c)^{7}}+\ldots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^14*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/12/sin(d*x+c)^12*cos(d*x+c)^8-1/30/sin(d*x+c)^10*cos(d*x+c)^8-1/
120/sin(d*x+c)^8*cos(d*x+c)^8)+a*(-1/13/sin(d*x+c)^13*cos(d*x+c)^8-5/143/si
n(d*x+c)^11*cos(d*x+c)^8-5/429/sin(d*x+c)^9*cos(d*x+c)^8-5/3003/sin(d*x+c)^
7*cos(d*x+c)^8+1/3003/sin(d*x+c)^5*cos(d*x+c)^8-1/3003/sin(d*x+c)^3*cos(d*x
+c)^8+5/3003/sin(d*x+c)*cos(d*x+c)^8+5/3003*(16/5+cos(d*x+c)^6+6/5*cos(d*x+
c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.36, size = 92, normalized size = 0.81

$$\frac{20020\,a\sin\left(dx+c\right)^{7}+17160\,a\sin\left(dx+c\right)^{6}-45045\,a\sin\left(dx+c\right)^{5}-40040\,a\sin\left(dx+c\right)^{4}+36036\,a\sin\left(dx+\right.}{120120\,d\sin\left(dx+c\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^14*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/120120*(20020*a*sin(d*x + c)^7 + 17160*a*sin(d*x + c)^6 - 45045*a*sin(d*x
 + c)^5 - 40040*a*sin(d*x + c)^4 + 36036*a*sin(d*x + c)^3 + 32760*a*sin(d*x
 + c)^2 - 10010*a*sin(d*x + c) - 9240*a)/(d*sin(d*x + c)^13)
```

mupad [B] time = 9.35, size = 92, normalized size = 0.81

$$-\frac{-\frac{a\sin(c+d\,x)^7}{6}-\frac{a\sin(c+d\,x)^6}{7}+\frac{3\,a\sin(c+d\,x)^5}{8}+\frac{a\sin(c+d\,x)^4}{3}-\frac{3\,a\sin(c+d\,x)^3}{10}-\frac{3\,a\sin(c+d\,x)^2}{11}+\frac{a\,\sin(c+d\,x)}{12}+\frac{a}{13}}{d\sin\left(c+d\,x\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^14,x)
```

```
[Out] -(a/13 + (a*sin(c + d*x))/12 - (3*a*sin(c + d*x)^2)/11 - (3*a*sin(c + d*x)^
3)/10 + (a*sin(c + d*x)^4)/3 + (3*a*sin(c + d*x)^5)/8 - (a*sin(c + d*x)^6)/
7 - (a*sin(c + d*x)^7)/6)/(d*sin(c + d*x)^13)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**14*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.676 $\int \cot^7(c+dx)\csc^8(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$-\frac{a\csc^{14}(c+dx)}{14d}-\frac{a\csc^{13}(c+dx)}{13d}+\frac{a\csc^{12}(c+dx)}{4d}+\frac{3a\csc^{11}(c+dx)}{11d}-\frac{3a\csc^{10}(c+dx)}{10d}-\frac{a\csc^{9}(c+dx)}{3d}+\frac{a\csc^{8}(c}{8d}$$

```
[Out] 1/7*a*csc(d*x+c)^7/d+1/8*a*csc(d*x+c)^8/d-1/3*a*csc(d*x+c)^9/d-3/10*a*csc(d
*x+c)^10/d+3/11*a*csc(d*x+c)^11/d+1/4*a*csc(d*x+c)^12/d-1/13*a*csc(d*x+c)^1
3/d-1/14*a*csc(d*x+c)^14/d
```

Rubi [A] time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\csc^{14}(c+dx)}{14d}-\frac{a\csc^{13}(c+dx)}{13d}+\frac{a\csc^{12}(c+dx)}{4d}+\frac{3a\csc^{11}(c+dx)}{11d}-\frac{3a\csc^{10}(c+dx)}{10d}-\frac{a\csc^{9}(c+dx)}{3d}+\frac{a\csc^{8}(c}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^8*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^9)/(3
*d) - (3*a*Csc[c + d*x]^10)/(10*d) + (3*a*Csc[c + d*x]^11)/(11*d) + (a*Csc[
c + d*x]^12)/(4*d) - (a*Csc[c + d*x]^13)/(13*d) - (a*Csc[c + d*x]^14)/(14*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^7(c+dx)\csc^8(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^{15}(a-x)^3(a+x)^4}{x^{15}}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$

$$= \frac{a^8\,\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^{15}}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^8\,\text{Subst}\left(\int \left(\frac{a^7}{x^{15}} + \frac{a^6}{x^{14}} - \frac{3a^5}{x^{13}} - \frac{3a^4}{x^{12}} + \frac{3a^3}{x^{11}} + \frac{3a^2}{x^{10}} - \frac{a}{x^9} - \frac{1}{x^8}\right) dx}{d}$$

$$= \frac{a\csc^7(c+dx)}{7d} + \frac{a\csc^8(c+dx)}{8d} - \frac{a\csc^9(c+dx)}{3d} - \frac{3a\csc^{10}}{10}$$

Mathematica [A] time = 0.24, size = 86, normalized size = 0.67

$$-\frac{a\csc^{14}(c+dx)(9940\sin(c+dx) + 41860\sin(3(c+dx)) + 20020\sin(5(c+dx)) + 8580\sin(7(c+dx)) + 12912}{3843840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^8*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3843840*(a*Csc[c + d*x]^14*(76362 + 129129*Cos[2*(c + d*x)] + 54054*Cos[
4*(c + d*x)] + 15015*Cos[6*(c + d*x)] + 9940*Sin[c + d*x] + 41860*Sin[3*(c
+ d*x)] + 20020*Sin[5*(c + d*x)] + 8580*Sin[7*(c + d*x)]))/d
```

fricas [A] time = 0.57, size = 166, normalized size = 1.29

$$\frac{15015\,a\cos(dx+c)^6 - 9009\,a\cos(dx+c)^4 + 3003\,a\cos(dx+c)^2 + 40\left(429\,a\cos(dx+c)^6 - 286\,a\cos(dx+c\right.}{120120\left(d\cos(dx+c)^{14} - 7\,d\cos(dx+c)^{12} + 21\,d\cos(dx+c)^{10} - 35\,d\cos(dx+c)^8 + 35\,d\cos(dx+c)^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^15*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/120120*(15015*a*cos(d*x + c)^6 - 9009*a*cos(d*x + c)^4 + 3003*a*cos(d*x +
 c)^2 + 40*(429*a*cos(d*x + c)^6 - 286*a*cos(d*x + c)^4 + 104*a*cos(d*x + c
)^2 - 16*a)*sin(d*x + c) - 429*a)/(d*cos(d*x + c)^14 - 7*d*cos(d*x + c)^12
+ 21*d*cos(d*x + c)^10 - 35*d*cos(d*x + c)^8 + 35*d*cos(d*x + c)^6 - 21*d*c
os(d*x + c)^4 + 7*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.34, size = 92, normalized size = 0.71

$$\frac{17160\,a\sin\left(dx+c\right)^{7}+15015\,a\sin\left(dx+c\right)^{6}-40040\,a\sin\left(dx+c\right)^{5}-36036\,a\sin\left(dx+c\right)^{4}+32760\,a\sin\left(dx+c\right.}{120120\,d\sin\left(dx+c\right)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^15*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120120*(17160*a*sin(d*x + c)^7 + 15015*a*sin(d*x + c)^6 - 40040*a*sin(d*x
 + c)^5 - 36036*a*sin(d*x + c)^4 + 32760*a*sin(d*x + c)^3 + 30030*a*sin(d*x
 + c)^2 - 9240*a*sin(d*x + c) - 8580*a)/(d*sin(d*x + c)^14)
```

maple [B] time = 0.38, size = 248, normalized size = 1.92

$$\frac{a\left(-\frac{\cos^{8}(dx+c)}{13\sin(dx+c)^{13}}-\frac{5\left(\cos^{8}(dx+c)\right)}{143\sin(dx+c)^{11}}-\frac{5\left(\cos^{8}(dx+c)\right)}{429\sin(dx+c)^{9}}-\frac{5\left(\cos^{8}(dx+c)\right)}{3003\sin(dx+c)^{7}}+\frac{\cos^{8}(dx+c)}{3003\sin(dx+c)^{5}}-\frac{\cos^{8}(dx+c)}{3003\sin(dx+c)^{3}}+\frac{5\left(\cos^{8}(dx+c)\right)}{3003\sin(dx+c)}+\frac{5\left(\frac{16}{5}\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^15*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/13/sin(d*x+c)^13*cos(d*x+c)^8-5/143/sin(d*x+c)^11*cos(d*x+c)^8-5
/429/sin(d*x+c)^9*cos(d*x+c)^8-5/3003/sin(d*x+c)^7*cos(d*x+c)^8+1/3003/sin(
d*x+c)^5*cos(d*x+c)^8-1/3003/sin(d*x+c)^3*cos(d*x+c)^8+5/3003/sin(d*x+c)*co
s(d*x+c)^8+5/3003*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin
(d*x+c))+a*(-1/14/sin(d*x+c)^14*cos(d*x+c)^8-1/28/sin(d*x+c)^12*cos(d*x+c)^
8-1/70/sin(d*x+c)^10*cos(d*x+c)^8-1/280/sin(d*x+c)^8*cos(d*x+c)^8))
```

maxima [A] time = 0.35, size = 92, normalized size = 0.71

$$\frac{17160\,a\sin\left(dx+c\right)^{7}+15015\,a\sin\left(dx+c\right)^{6}-40040\,a\sin\left(dx+c\right)^{5}-36036\,a\sin\left(dx+c\right)^{4}+32760\,a\sin\left(dx+c\right.}{120120\,d\sin\left(dx+c\right)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^15*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/120120*(17160*a*sin(d*x + c)^7 + 15015*a*sin(d*x + c)^6 - 40040*a*sin(d*x
 + c)^5 - 36036*a*sin(d*x + c)^4 + 32760*a*sin(d*x + c)^3 + 30030*a*sin(d*x
 + c)^2 - 9240*a*sin(d*x + c) - 8580*a)/(d*sin(d*x + c)^14)
```

mupad [B] time = 9.24, size = 92, normalized size = 0.71

$$-\frac{-\frac{a\sin(c+d\,x)^7}{7}-\frac{a\sin(c+d\,x)^6}{8}+\frac{a\sin(c+d\,x)^5}{3}+\frac{3\,a\sin(c+d\,x)^4}{10}-\frac{3\,a\sin(c+d\,x)^3}{11}-\frac{a\sin(c+d\,x)^2}{4}+\frac{a\,\sin(c+d\,x)}{13}+\frac{a}{14}}{d\sin\left(c+d\,x\right)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^15,x)
```

```
[Out] -(a/14 + (a*sin(c + d*x))/13 - (a*sin(c + d*x)^2)/4 - (3*a*sin(c + d*x)^3)/
11 + (3*a*sin(c + d*x)^4)/10 + (a*sin(c + d*x)^5)/3 - (a*sin(c + d*x)^6)/8
- (a*sin(c + d*x)^7)/7)/(d*sin(c + d*x)^14)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**15*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.677 $\int \frac{\cos^7(c+dx)\sin^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\sin^{12}(c+dx)}{12ad}+\frac{\sin^{11}(c+dx)}{11ad}+\frac{\sin^{10}(c+dx)}{5ad}-\frac{2\sin^9(c+dx)}{9ad}-\frac{\sin^8(c+dx)}{8ad}+\frac{\sin^7(c+dx)}{7ad}$$

```
[Out] 1/7*sin(d*x+c)^7/a/d-1/8*sin(d*x+c)^8/a/d-2/9*sin(d*x+c)^9/a/d+1/5*sin(d*x+
c)^10/a/d+1/11*sin(d*x+c)^11/a/d-1/12*sin(d*x+c)^12/a/d
```

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\sin^{12}(c+dx)}{12ad}+\frac{\sin^{11}(c+dx)}{11ad}+\frac{\sin^{10}(c+dx)}{5ad}-\frac{2\sin^9(c+dx)}{9ad}-\frac{\sin^8(c+dx)}{8ad}+\frac{\sin^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^7/(7*a*d) - Sin[c + d*x]^8/(8*a*d) - (2*Sin[c + d*x]^9)/(9*a*d
) + Sin[c + d*x]^10/(5*a*d) + Sin[c + d*x]^11/(11*a*d) - Sin[c + d*x]^12/(1
2*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3x^6(a+x)^2}{a^6}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^3x^6(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^{13}d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5x^6-a^4x^7-2a^3x^8+2a^2x^9+ax^{10}-x^{11}\right)dx, x, a\sin(c+dx)\right)}{a^{13}d}$$
$$= \frac{\sin^7(c+dx)}{7ad}-\frac{\sin^8(c+dx)}{8ad}-\frac{2\sin^9(c+dx)}{9ad}+\frac{\sin^{10}(c+dx)}{5ad}+\frac{\sin^{11}(c+dx)}{11ad}$$

Mathematica [A] time = 0.60, size = 68, normalized size = 0.62

$$\frac{\sin^7(c+dx)\left(-2310\sin^5(c+dx)+2520\sin^4(c+dx)+5544\sin^3(c+dx)-6160\sin^2(c+dx)-3465\sin(c+dx)\right.}{27720ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^7*(3960 - 3465*Sin[c + d*x] - 6160*Sin[c + d*x]^2 + 5544*Sin[
c + d*x]^3 + 2520*Sin[c + d*x]^4 - 2310*Sin[c + d*x]^5))/(27720*a*d)
```

fricas [A] time = 0.52, size = 109, normalized size = 1.00

$$-\frac{2310\cos(dx+c)^{12}-8316\cos(dx+c)^{10}+10395\cos(dx+c)^8-4620\cos(dx+c)^6+40\left(63\cos(dx+c)^{10}\right.}{27720\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/27720*(2310*cos(d*x + c)^12 - 8316*cos(d*x + c)^10 + 10395*cos(d*x + c)^
8 - 4620*cos(d*x + c)^6 + 40*(63*cos(d*x + c)^10 - 161*cos(d*x + c)^8 + 113
*cos(d*x + c)^6 - 3*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 8)*sin(d*x + c))/(a
*d)
```

giac [A] time = 0.21, size = 69, normalized size = 0.63

$$-\frac{2310\sin(dx+c)^{12}-2520\sin(dx+c)^{11}-5544\sin(dx+c)^{10}+6160\sin(dx+c)^9+3465\sin(dx+c)^8-39}{27720\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/27720*(2310*sin(d*x + c)^12 - 2520*sin(d*x + c)^11 - 5544*sin(d*x + c)^1
0 + 6160*sin(d*x + c)^9 + 3465*sin(d*x + c)^8 - 3960*sin(d*x + c)^7)/(a*d)
```

maple [A] time = 0.32, size = 69, normalized size = 0.63

$$\frac{-\frac{\left(\sin^{12}(dx+c)\right)}{12}+\frac{\left(\sin^{11}(dx+c)\right)}{11}+\frac{\left(\sin^{10}(dx+c)\right)}{5}-\frac{2\left(\sin^{9}(dx+c)\right)}{9}-\frac{\left(\sin^{8}(dx+c)\right)}{8}+\frac{\left(\sin^{7}(dx+c)\right)}{7}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/12*sin(d*x+c)^12+1/11*sin(d*x+c)^11+1/5*sin(d*x+c)^10-2/9*sin(d*x
+c)^9-1/8*sin(d*x+c)^8+1/7*sin(d*x+c)^7)
```

maxima [A] time = 0.38, size = 69, normalized size = 0.63

$$-\frac{2310 \sin(dx+c)^{12} - 2520 \sin(dx+c)^{11} - 5544 \sin(dx+c)^{10} + 6160 \sin(dx+c)^{9} + 3465 \sin(dx+c)^{8} - 396}{27720\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/27720*(2310*sin(d*x + c)^12 - 2520*sin(d*x + c)^11 - 5544*sin(d*x + c)^1
0 + 6160*sin(d*x + c)^9 + 3465*sin(d*x + c)^8 - 3960*sin(d*x + c)^7)/(a*d)
```

mupad [B] time = 0.08, size = 83, normalized size = 0.76

$$\frac{\frac{\sin(c+d\,x)^7}{7\,a}-\frac{\sin(c+d\,x)^8}{8\,a}-\frac{2\sin(c+d\,x)^9}{9\,a}+\frac{\sin(c+d\,x)^{10}}{5\,a}+\frac{\sin(c+d\,x)^{11}}{11\,a}-\frac{\sin(c+d\,x)^{12}}{12\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^6)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^7/(7*a) - sin(c + d*x)^8/(8*a) - (2*sin(c + d*x)^9)/(9*a) + s
in(c + d*x)^10/(5*a) + sin(c + d*x)^11/(11*a) - sin(c + d*x)^12/(12*a))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.678 $\int \frac{\cos^7(c+dx)\sin^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\sin^{11}(c+dx)}{11ad}+\frac{\sin^{10}(c+dx)}{10ad}+\frac{2\sin^9(c+dx)}{9ad}-\frac{\sin^8(c+dx)}{4ad}-\frac{\sin^7(c+dx)}{7ad}+\frac{\sin^6(c+dx)}{6ad}$$

```
[Out] 1/6*sin(d*x+c)^6/a/d-1/7*sin(d*x+c)^7/a/d-1/4*sin(d*x+c)^8/a/d+2/9*sin(d*x+
c)^9/a/d+1/10*sin(d*x+c)^10/a/d-1/11*sin(d*x+c)^11/a/d
```

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\sin^{11}(c+dx)}{11ad}+\frac{\sin^{10}(c+dx)}{10ad}+\frac{2\sin^9(c+dx)}{9ad}-\frac{\sin^8(c+dx)}{4ad}-\frac{\sin^7(c+dx)}{7ad}+\frac{\sin^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^6/(6*a*d) - Sin[c + d*x]^7/(7*a*d) - Sin[c + d*x]^8/(4*a*d) +
(2*Sin[c + d*x]^9)/(9*a*d) + Sin[c + d*x]^10/(10*a*d) - Sin[c + d*x]^11/(11
*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3x^5(a+x)^2}{a^5}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^3x^5(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^{12}d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5x^5-a^4x^6-2a^3x^7+2a^2x^8+ax^9-x^{10}\right)dx, x, a\sin(c+dx)\right)}{a^{12}d}$$
$$= \frac{\sin^6(c+dx)}{6ad}-\frac{\sin^7(c+dx)}{7ad}-\frac{\sin^8(c+dx)}{4ad}+\frac{2\sin^9(c+dx)}{9ad}+\frac{\sin^{10}(c+dx)}{10ad}$$

Mathematica [A] time = 0.92, size = 68, normalized size = 0.62

$$\frac{\sin^6(c+dx)\left(-1260\sin^5(c+dx)+1386\sin^4(c+dx)+3080\sin^3(c+dx)-3465\sin^2(c+dx)-1980\sin(c+dx\right.}{13860ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^6*(2310 - 1980*Sin[c + d*x] - 3465*Sin[c + d*x]^2 + 3080*Sin[
c + d*x]^3 + 1386*Sin[c + d*x]^4 - 1260*Sin[c + d*x]^5))/(13860*a*d)
```

fricas [A] time = 0.47, size = 99, normalized size = 0.91

$$-\frac{1386\cos(dx+c)^{10}-3465\cos(dx+c)^8+2310\cos(dx+c)^6-20\left(63\cos(dx+c)^{10}-161\cos(dx+c)^8+1\right.}{13860\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/13860*(1386*cos(d*x + c)^10 - 3465*cos(d*x + c)^8 + 2310*cos(d*x + c)^6
- 20*(63*cos(d*x + c)^10 - 161*cos(d*x + c)^8 + 113*cos(d*x + c)^6 - 3*cos(
d*x + c)^4 - 4*cos(d*x + c)^2 - 8)*sin(d*x + c))/(a*d)
```

giac [A] time = 0.23, size = 69, normalized size = 0.63

$$-\frac{1260\sin(dx+c)^{11}-1386\sin(dx+c)^{10}-3080\sin(dx+c)^9+3465\sin(dx+c)^8+1980\sin(dx+c)^7-23}{13860\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/13860*(1260*sin(d*x + c)^11 - 1386*sin(d*x + c)^10 - 3080*sin(d*x + c)^9
 + 3465*sin(d*x + c)^8 + 1980*sin(d*x + c)^7 - 2310*sin(d*x + c)^6)/(a*d)
```

maple [A] time = 0.31, size = 69, normalized size = 0.63

$$\frac{-\frac{\left(\sin^{11}(dx+c)\right)}{11}+\frac{\left(\sin^{10}(dx+c)\right)}{10}+\frac{2\left(\sin^{9}(dx+c)\right)}{9}-\frac{\left(\sin^{8}(dx+c)\right)}{4}-\frac{\left(\sin^{7}(dx+c)\right)}{7}+\frac{\left(\sin^{6}(dx+c)\right)}{6}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/11*sin(d*x+c)^11+1/10*sin(d*x+c)^10+2/9*sin(d*x+c)^9-1/4*sin(d*x+
c)^8-1/7*sin(d*x+c)^7+1/6*sin(d*x+c)^6)
```

maxima [A] time = 0.42, size = 69, normalized size = 0.63

$$-\frac{1260\,\sin\left(dx+c\right)^{11}-1386\,\sin\left(dx+c\right)^{10}-3080\,\sin\left(dx+c\right)^{9}+3465\,\sin\left(dx+c\right)^{8}+1980\,\sin\left(dx+c\right)^{7}-2310}{13860\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/13860*(1260*sin(d*x + c)^11 - 1386*sin(d*x + c)^10 - 3080*sin(d*x + c)^9
 + 3465*sin(d*x + c)^8 + 1980*sin(d*x + c)^7 - 2310*sin(d*x + c)^6)/(a*d)
```

mupad [B] time = 8.99, size = 83, normalized size = 0.76

$$\frac{\frac{\sin(c+d\,x)^6}{6\,a}-\frac{\sin(c+d\,x)^7}{7\,a}-\frac{\sin(c+d\,x)^8}{4\,a}+\frac{2\,\sin(c+d\,x)^9}{9\,a}+\frac{\sin(c+d\,x)^{10}}{10\,a}-\frac{\sin(c+d\,x)^{11}}{11\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^5)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^6/(6*a) - sin(c + d*x)^7/(7*a) - sin(c + d*x)^8/(4*a) + (2*si
n(c + d*x)^9)/(9*a) + sin(c + d*x)^10/(10*a) - sin(c + d*x)^11/(11*a))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.679 $\int \frac{\cos^7(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\sin^{10}(c+dx)}{10ad}+\frac{\sin^9(c+dx)}{9ad}+\frac{\sin^8(c+dx)}{4ad}-\frac{2\sin^7(c+dx)}{7ad}-\frac{\sin^6(c+dx)}{6ad}+\frac{\sin^5(c+dx)}{5ad}$$

```
[Out] 1/5*sin(d*x+c)^5/a/d-1/6*sin(d*x+c)^6/a/d-2/7*sin(d*x+c)^7/a/d+1/4*sin(d*x+
c)^8/a/d+1/9*sin(d*x+c)^9/a/d-1/10*sin(d*x+c)^10/a/d
```

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\sin^{10}(c+dx)}{10ad}+\frac{\sin^9(c+dx)}{9ad}+\frac{\sin^8(c+dx)}{4ad}-\frac{2\sin^7(c+dx)}{7ad}-\frac{\sin^6(c+dx)}{6ad}+\frac{\sin^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^5/(5*a*d) - Sin[c + d*x]^6/(6*a*d) - (2*Sin[c + d*x]^7)/(7*a*d
) + Sin[c + d*x]^8/(4*a*d) + Sin[c + d*x]^9/(9*a*d) - Sin[c + d*x]^10/(10*a
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3x^4(a+x)^2}{a^4}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^3x^4(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^{11}d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5x^4-a^4x^5-2a^3x^6+2a^2x^7+ax^8-x^9\right)dx, x, a\sin(c+dx)\right)}{a^{11}d}$$
$$= \frac{\sin^5(c+dx)}{5ad}-\frac{\sin^6(c+dx)}{6ad}-\frac{2\sin^7(c+dx)}{7ad}+\frac{\sin^8(c+dx)}{4ad}+\frac{\sin^9(c+dx)}{9ad}-$$

Mathematica [A] time = 0.41, size = 68, normalized size = 0.62

$$\frac{\sin^5(c+dx)\left(-126\sin^5(c+dx)+140\sin^4(c+dx)+315\sin^3(c+dx)-360\sin^2(c+dx)-210\sin(c+dx)+252\right.}{1260ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^5*(252 - 210*Sin[c + d*x] - 360*Sin[c + d*x]^2 + 315*Sin[c +
d*x]^3 + 140*Sin[c + d*x]^4 - 126*Sin[c + d*x]^5))/(1260*a*d)
```

fricas [A] time = 0.48, size = 89, normalized size = 0.82

$$\frac{126\cos(dx+c)^{10}-315\cos(dx+c)^8+210\cos(dx+c)^6+4\left(35\cos(dx+c)^8-50\cos(dx+c)^6+3\cos(dx+\right.}{1260\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1260*(126*cos(d*x + c)^10 - 315*cos(d*x + c)^8 + 210*cos(d*x + c)^6 + 4*(
35*cos(d*x + c)^8 - 50*cos(d*x + c)^6 + 3*cos(d*x + c)^4 + 4*cos(d*x + c)^2
 + 8)*sin(d*x + c))/(a*d)
```

giac [A] time = 0.20, size = 69, normalized size = 0.63

$$-\frac{126\sin(dx+c)^{10}-140\sin(dx+c)^9-315\sin(dx+c)^8+360\sin(dx+c)^7+210\sin(dx+c)^6-252\sin(dx}{1260\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1260*(126*sin(d*x + c)^10 - 140*sin(d*x + c)^9 - 315*sin(d*x + c)^8 + 36
0*sin(d*x + c)^7 + 210*sin(d*x + c)^6 - 252*sin(d*x + c)^5)/(a*d)
```

maple [A] time = 0.30, size = 69, normalized size = 0.63

$$\frac{-\frac{\left(\sin^{10}(dx+c)\right)}{10}+\frac{\left(\sin^{9}(dx+c)\right)}{9}+\frac{\left(\sin^{8}(dx+c)\right)}{4}-\frac{2\left(\sin^{7}(dx+c)\right)}{7}-\frac{\left(\sin^{6}(dx+c)\right)}{6}+\frac{\left(\sin^{5}(dx+c)\right)}{5}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/10*sin(d*x+c)^10+1/9*sin(d*x+c)^9+1/4*sin(d*x+c)^8-2/7*sin(d*x+c)
^7-1/6*sin(d*x+c)^6+1/5*sin(d*x+c)^5)
```

maxima [A] time = 0.37, size = 69, normalized size = 0.63

$$-\frac{126 \sin (dx+c)^{10}-140 \sin (dx+c)^{9}-315 \sin (dx+c)^{8}+360 \sin (dx+c)^{7}+210 \sin (dx+c)^{6}-252 \sin (d}{1260\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/1260*(126*sin(d*x + c)^10 - 140*sin(d*x + c)^9 - 315*sin(d*x + c)^8 + 36
0*sin(d*x + c)^7 + 210*sin(d*x + c)^6 - 252*sin(d*x + c)^5)/(a*d)
```

mupad [B] time = 8.94, size = 83, normalized size = 0.76

$$\frac{\frac{\sin(c+d\,x)^5}{5\,a}-\frac{\sin(c+d\,x)^6}{6\,a}-\frac{2\sin(c+d\,x)^7}{7\,a}+\frac{\sin(c+d\,x)^8}{4\,a}+\frac{\sin(c+d\,x)^9}{9\,a}-\frac{\sin(c+d\,x)^{10}}{10\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^4)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^5/(5*a) - sin(c + d*x)^6/(6*a) - (2*sin(c + d*x)^7)/(7*a) + s
in(c + d*x)^8/(4*a) + sin(c + d*x)^9/(9*a) - sin(c + d*x)^10/(10*a))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.680 $\int \frac{\cos^7(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$-\frac{\sin^9(c+dx)}{9ad}+\frac{2\sin^7(c+dx)}{7ad}-\frac{\sin^5(c+dx)}{5ad}+\frac{\cos^8(c+dx)}{8ad}-\frac{\cos^6(c+dx)}{6ad}$$

```
[Out] -1/6*cos(d*x+c)^6/a/d+1/8*cos(d*x+c)^8/a/d-1/5*sin(d*x+c)^5/a/d+2/7*sin(d*x
+c)^7/a/d-1/9*sin(d*x+c)^9/a/d
```

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2835, 2565, 14, 2564, 270}

$$-\frac{\sin^9(c+dx)}{9ad}+\frac{2\sin^7(c+dx)}{7ad}-\frac{\sin^5(c+dx)}{5ad}+\frac{\cos^8(c+dx)}{8ad}-\frac{\cos^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cos[c + d*x]^6/(6*a*d) + Cos[c + d*x]^8/(8*a*d) - Sin[c + d*x]^5/(5*a*d) +
 (2*Sin[c + d*x]^7)/(7*a*d) - Sin[c + d*x]^9/(9*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^5(c+dx)\sin^3(c+dx)\,dx}{a} - \frac{\int \cos^5(c+dx)\sin^4(c+dx)\,dx}{a}$$
$$= -\frac{\text{Subst}\left(\int x^5\left(1-x^2\right)\,dx, x, \cos(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int x^4\left(1-x^2\right)^2\,dx, x, \sin(c\right.}{ad}$$
$$= -\frac{\text{Subst}\left(\int \left(x^5-x^7\right)\,dx, x, \cos(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx, x, \sin\right.}{ad}$$
$$= -\frac{\cos^6(c+dx)}{6ad} + \frac{\cos^8(c+dx)}{8ad} - \frac{\sin^5(c+dx)}{5ad} + \frac{2\sin^7(c+dx)}{7ad} - \frac{\sin^9(c+dx}{9ad}$$

Mathematica [A] time = 0.56, size = 68, normalized size = 0.75

$$\frac{\sin^4(c+dx)\left(-280\sin^5(c+dx)+315\sin^4(c+dx)+720\sin^3(c+dx)-840\sin^2(c+dx)-504\sin(c+dx)+63\right.}{2520ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^4*(630 - 504*Sin[c + d*x] - 840*Sin[c + d*x]^2 + 720*Sin[c +
d*x]^3 + 315*Sin[c + d*x]^4 - 280*Sin[c + d*x]^5))/(2520*a*d)
```

fricas [A] time = 0.51, size = 79, normalized size = 0.87

$$\frac{315\cos(dx+c)^8 - 420\cos(dx+c)^6 - 8\left(35\cos(dx+c)^8 - 50\cos(dx+c)^6 + 3\cos(dx+c)^4 + 4\cos(dx+c\right.}{2520\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*cos(d*x + c)^8 - 420*cos(d*x + c)^6 - 8*(35*cos(d*x + c)^8 - 50
*cos(d*x + c)^6 + 3*cos(d*x + c)^4 + 4*cos(d*x + c)^2 + 8)*sin(d*x + c))/(a
*d)
```

giac [A] time = 0.20, size = 69, normalized size = 0.76

$$-\frac{280 \sin(dx+c)^9 - 315 \sin(dx+c)^8 - 720 \sin(dx+c)^7 + 840 \sin(dx+c)^6 + 504 \sin(dx+c)^5 - 630 \sin(dx}{2520\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2520*(280*sin(d*x + c)^9 - 315*sin(d*x + c)^8 - 720*sin(d*x + c)^7 + 840
*sin(d*x + c)^6 + 504*sin(d*x + c)^5 - 630*sin(d*x + c)^4)/(a*d)
```

maple [A] time = 0.26, size = 69, normalized size = 0.76

$$\frac{-\frac{\left(\sin^9(dx+c)\right)}{9}+\frac{\left(\sin^8(dx+c)\right)}{8}+\frac{2\left(\sin^7(dx+c)\right)}{7}-\frac{\left(\sin^6(dx+c)\right)}{3}-\frac{\left(\sin^5(dx+c)\right)}{5}+\frac{\left(\sin^4(dx+c)\right)}{4}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/9*sin(d*x+c)^9+1/8*sin(d*x+c)^8+2/7*sin(d*x+c)^7-1/3*sin(d*x+c)^6
-1/5*sin(d*x+c)^5+1/4*sin(d*x+c)^4)
```

maxima [A] time = 0.33, size = 69, normalized size = 0.76

$$-\frac{280 \sin(dx+c)^9 - 315 \sin(dx+c)^8 - 720 \sin(dx+c)^7 + 840 \sin(dx+c)^6 + 504 \sin(dx+c)^5 - 630 \sin(dx}{2520\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2520*(280*sin(d*x + c)^9 - 315*sin(d*x + c)^8 - 720*sin(d*x + c)^7 + 840
*sin(d*x + c)^6 + 504*sin(d*x + c)^5 - 630*sin(d*x + c)^4)/(a*d)
```

mupad [B] time = 0.06, size = 83, normalized size = 0.91

$$\frac{\frac{\sin(c+d\,x)^4}{4\,a}-\frac{\sin(c+d\,x)^5}{5\,a}-\frac{\sin(c+d\,x)^6}{3\,a}+\frac{2\sin(c+d\,x)^7}{7\,a}+\frac{\sin(c+d\,x)^8}{8\,a}-\frac{\sin(c+d\,x)^9}{9\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^4/(4*a) - sin(c + d*x)^5/(5*a) - sin(c + d*x)^6/(3*a) + (2*si
n(c + d*x)^7)/(7*a) + sin(c + d*x)^8/(8*a) - sin(c + d*x)^9/(9*a))/d
```

sympy [A] time = 169.58, size = 1906, normalized size = 20.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((1260*tan(c/2 + d*x/2)**14/(315*a*d*tan(c/2 + d*x/2)**18 + 2835*a
*d*tan(c/2 + d*x/2)**16 + 11340*a*d*tan(c/2 + d*x/2)**14 + 26460*a*d*tan(c/
2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d*x/2)**10 + 39690*a*d*tan(c/2 + d*x/2
)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 + 11340*a*d*tan(c/2 + d*x/2)**4 + 2835
*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) - 2016*tan(c/2 + d*x/2)**13/(315*a*d*ta
n(c/2 + d*x/2)**18 + 2835*a*d*tan(c/2 + d*x/2)**16 + 11340*a*d*tan(c/2 + d*
x/2)**14 + 26460*a*d*tan(c/2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d*x/2)**10
+ 39690*a*d*tan(c/2 + d*x/2)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 + 11340*a*d
*tan(c/2 + d*x/2)**4 + 2835*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) - 420*tan(c/
2 + d*x/2)**12/(315*a*d*tan(c/2 + d*x/2)**18 + 2835*a*d*tan(c/2 + d*x/2)**1
6 + 11340*a*d*tan(c/2 + d*x/2)**14 + 26460*a*d*tan(c/2 + d*x/2)**12 + 39690
*a*d*tan(c/2 + d*x/2)**10 + 39690*a*d*tan(c/2 + d*x/2)**8 + 26460*a*d*tan(c
/2 + d*x/2)**6 + 11340*a*d*tan(c/2 + d*x/2)**4 + 2835*a*d*tan(c/2 + d*x/2)*
*2 + 315*a*d) + 3456*tan(c/2 + d*x/2)**11/(315*a*d*tan(c/2 + d*x/2)**18 + 2
835*a*d*tan(c/2 + d*x/2)**16 + 11340*a*d*tan(c/2 + d*x/2)**14 + 26460*a*d*t
an(c/2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d*x/2)**10 + 39690*a*d*tan(c/2 +
d*x/2)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 + 11340*a*d*tan(c/2 + d*x/2)**4 +
 2835*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) + 2520*tan(c/2 + d*x/2)**10/(315*a
*d*tan(c/2 + d*x/2)**18 + 2835*a*d*tan(c/2 + d*x/2)**16 + 11340*a*d*tan(c/2
 + d*x/2)**14 + 26460*a*d*tan(c/2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d*x/2)
**10 + 39690*a*d*tan(c/2 + d*x/2)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 + 1134
0*a*d*tan(c/2 + d*x/2)**4 + 2835*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) - 6976*
tan(c/2 + d*x/2)**9/(315*a*d*tan(c/2 + d*x/2)**18 + 2835*a*d*tan(c/2 + d*x/
2)**16 + 11340*a*d*tan(c/2 + d*x/2)**14 + 26460*a*d*tan(c/2 + d*x/2)**12 +
39690*a*d*tan(c/2 + d*x/2)**10 + 39690*a*d*tan(c/2 + d*x/2)**8 + 26460*a*d*
tan(c/2 + d*x/2)**6 + 11340*a*d*tan(c/2 + d*x/2)**4 + 2835*a*d*tan(c/2 + d*
x/2)**2 + 315*a*d) + 2520*tan(c/2 + d*x/2)**8/(315*a*d*tan(c/2 + d*x/2)**18
 + 2835*a*d*tan(c/2 + d*x/2)**16 + 11340*a*d*tan(c/2 + d*x/2)**14 + 26460*a
*d*tan(c/2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d*x/2)**10 + 39690*a*d*tan(c/
2 + d*x/2)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 + 11340*a*d*tan(c/2 + d*x/2)*
*4 + 2835*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) + 3456*tan(c/2 + d*x/2)**7/(31
```

```
5*a*d*tan(c/2 + d*x/2)**18 + 2835*a*d*tan(c/2 + d*x/2)**16 + 11340*a*d*tan(
c/2 + d*x/2)**14 + 26460*a*d*tan(c/2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d*x
/2)**10 + 39690*a*d*tan(c/2 + d*x/2)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 + 1
1340*a*d*tan(c/2 + d*x/2)**4 + 2835*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) - 42
0*tan(c/2 + d*x/2)**6/(315*a*d*tan(c/2 + d*x/2)**18 + 2835*a*d*tan(c/2 + d*
x/2)**16 + 11340*a*d*tan(c/2 + d*x/2)**14 + 26460*a*d*tan(c/2 + d*x/2)**12
+ 39690*a*d*tan(c/2 + d*x/2)**10 + 39690*a*d*tan(c/2 + d*x/2)**8 + 26460*a*
d*tan(c/2 + d*x/2)**6 + 11340*a*d*tan(c/2 + d*x/2)**4 + 2835*a*d*tan(c/2 +
d*x/2)**2 + 315*a*d) - 2016*tan(c/2 + d*x/2)**5/(315*a*d*tan(c/2 + d*x/2)**
18 + 2835*a*d*tan(c/2 + d*x/2)**16 + 11340*a*d*tan(c/2 + d*x/2)**14 + 26460
*a*d*tan(c/2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d*x/2)**10 + 39690*a*d*tan(
c/2 + d*x/2)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 + 11340*a*d*tan(c/2 + d*x/2
)**4 + 2835*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) + 1260*tan(c/2 + d*x/2)**4/(
315*a*d*tan(c/2 + d*x/2)**18 + 2835*a*d*tan(c/2 + d*x/2)**16 + 11340*a*d*ta
n(c/2 + d*x/2)**14 + 26460*a*d*tan(c/2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d
*x/2)**10 + 39690*a*d*tan(c/2 + d*x/2)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 +
 11340*a*d*tan(c/2 + d*x/2)**4 + 2835*a*d*tan(c/2 + d*x/2)**2 + 315*a*d), N
e(d, 0)), (x*sin(c)**3*cos(c)**7/(a*sin(c) + a), True))
```

3.681 $\int \frac{\cos^7(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\sin^7(c+dx)}{7ad}-\frac{2\sin^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^8(c+dx)}{8ad}+\frac{\cos^6(c+dx)}{6ad}$$

```
[Out] 1/6*cos(d*x+c)^6/a/d-1/8*cos(d*x+c)^8/a/d+1/3*sin(d*x+c)^3/a/d-2/5*sin(d*x+
c)^5/a/d+1/7*sin(d*x+c)^7/a/d
```

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2835, 2564, 270, 2565, 14}

$$\frac{\sin^7(c+dx)}{7ad}-\frac{2\sin^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^8(c+dx)}{8ad}+\frac{\cos^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^6/(6*a*d) - Cos[c + d*x]^8/(8*a*d) + Sin[c + d*x]^3/(3*a*d) -
(2*Sin[c + d*x]^5)/(5*a*d) + Sin[c + d*x]^7/(7*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^5(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^5(c+dx)\sin^3(c+dx)\,dx}{a}$$

$$= \frac{\text{Subst}\left(\int x^5\left(1-x^2\right)\,dx, x, \cos(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx, x, \sin(c+$$

$$= \frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \sin(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int \left(x^5-x^7\right)\,dx, x, \cos(c$$

$$= \frac{\cos^6(c+dx)}{6ad} - \frac{\cos^8(c+dx)}{8ad} + \frac{\sin^3(c+dx)}{3ad} - \frac{2\sin^5(c+dx)}{5ad} + \frac{\sin^7(c+dx)}{7ad}$$

Mathematica [A] time = 0.29, size = 68, normalized size = 0.75

$$\frac{\sin^3(c+dx)\left(-105\sin^5(c+dx)+120\sin^4(c+dx)+280\sin^3(c+dx)-336\sin^2(c+dx)-210\sin(c+dx)+280\right.}{840ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^3*(280 - 210*Sin[c + d*x] - 336*Sin[c + d*x]^2 + 280*Sin[c +
d*x]^3 + 120*Sin[c + d*x]^4 - 105*Sin[c + d*x]^5))/(840*a*d)
```

fricas [A] time = 0.47, size = 69, normalized size = 0.76

$$-\frac{105\cos(dx+c)^8 - 140\cos(dx+c)^6 + 8\left(15\cos(dx+c)^6 - 3\cos(dx+c)^4 - 4\cos(dx+c)^2 - 8\right)\sin(dx+c)}{840\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/840*(105*cos(d*x + c)^8 - 140*cos(d*x + c)^6 + 8*(15*cos(d*x + c)^6 - 3*
cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 8)*sin(d*x + c))/(a*d)
```

giac [A] time = 0.18, size = 69, normalized size = 0.76

$$-\frac{105 \sin\left(dx+c\right)^{8}-120 \sin\left(dx+c\right)^{7}-280 \sin\left(dx+c\right)^{6}+336 \sin\left(dx+c\right)^{5}+210 \sin\left(dx+c\right)^{4}-280 \sin\left(dx\right.}{840\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/840*(105*sin(d*x + c)^8 - 120*sin(d*x + c)^7 - 280*sin(d*x + c)^6 + 336*
sin(d*x + c)^5 + 210*sin(d*x + c)^4 - 280*sin(d*x + c)^3)/(a*d)
```

maple [A] time = 0.22, size = 69, normalized size = 0.76

$$\frac{-\frac{\left(\sin^{8}(dx+c)\right)}{8}+\frac{\left(\sin^{7}(dx+c)\right)}{7}+\frac{\left(\sin^{6}(dx+c)\right)}{3}-\frac{2\left(\sin^{5}(dx+c)\right)}{5}-\frac{\left(\sin^{4}(dx+c)\right)}{4}+\frac{\left(\sin^{3}(dx+c)\right)}{3}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/8*sin(d*x+c)^8+1/7*sin(d*x+c)^7+1/3*sin(d*x+c)^6-2/5*sin(d*x+c)^5
-1/4*sin(d*x+c)^4+1/3*sin(d*x+c)^3)
```

maxima [A] time = 0.33, size = 69, normalized size = 0.76

$$-\frac{105 \sin\left(dx+c\right)^{8}-120 \sin\left(dx+c\right)^{7}-280 \sin\left(dx+c\right)^{6}+336 \sin\left(dx+c\right)^{5}+210 \sin\left(dx+c\right)^{4}-280 \sin\left(dx\right.}{840\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*(105*sin(d*x + c)^8 - 120*sin(d*x + c)^7 - 280*sin(d*x + c)^6 + 336*
sin(d*x + c)^5 + 210*sin(d*x + c)^4 - 280*sin(d*x + c)^3)/(a*d)
```

mupad [B] time = 8.96, size = 83, normalized size = 0.91

$$\frac{\frac{\sin(c+d\,x)^3}{3\,a}-\frac{\sin(c+d\,x)^4}{4\,a}-\frac{2\,\sin(c+d\,x)^5}{5\,a}+\frac{\sin(c+d\,x)^6}{3\,a}+\frac{\sin(c+d\,x)^7}{7\,a}-\frac{\sin(c+d\,x)^8}{8\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^7*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)

[Out] (sin(c + d*x)^3/(3*a) - sin(c + d*x)^4/(4*a) - (2*sin(c + d*x)^5)/(5*a) + sin(c + d*x)^6/(3*a) + sin(c + d*x)^7/(7*a) - sin(c + d*x)^8/(8*a))/d

sympy [A] time = 114.13, size = 1719, normalized size = 18.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**7*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)

[Out] Piecewise((280*tan(c/2 + d*x/2)**13/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 420*tan(c/2 + d*x/2)**12/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 56*tan(c/2 + d*x/2)**11/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 560*tan(c/2 + d*x/2)**10/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 688*tan(c/2 + d*x/2)**9/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 1400*tan(c/2 + d*x/2)**8/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 688*tan(c/2 + d*x/2)**7/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 560*tan(c/2 + d*x/2)**6/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 56*tan(c/2 + d*x/2)**5/(105*a*

```
d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 +
d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 +
 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(
c/2 + d*x/2)**2 + 105*a*d) - 420*tan(c/2 + d*x/2)**4/(105*a*d*tan(c/2 + d*x
/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 58
80*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c
/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2
 + 105*a*d) + 280*tan(c/2 + d*x/2)**3/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a
*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2
+ d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6
+ 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d), Ne
(d, 0)), (x*sin(c)**2*cos(c)**7/(a*sin(c) + a), True))
```

3.682 $\int \frac{\cos^7(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{\sin^7(c+dx)}{7ad}+\frac{2\sin^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^6(c+dx)}{6ad}$$

```
[Out] -1/6*cos(d*x+c)^6/a/d-1/3*sin(d*x+c)^3/a/d+2/5*sin(d*x+c)^5/a/d-1/7*sin(d*x
+c)^7/a/d
```

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2835, 2565, 30, 2564, 270}

$$-\frac{\sin^7(c+dx)}{7ad}+\frac{2\sin^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cos[c + d*x]^6/(6*a*d) - Sin[c + d*x]^3/(3*a*d) + (2*Sin[c + d*x]^5)/(5*a*
d) - Sin[c + d*x]^7/(7*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^7(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^5(c+dx)\sin(c+dx)\,dx}{a} - \frac{\int \cos^5(c+dx)\sin^2(c+dx)\,dx}{a} \\
&= -\frac{\text{Subst}\left(\int x^5\,dx, x, \cos(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx, x, \sin(c+dx)\right)}{ad} \\
&= -\frac{\cos^6(c+dx)}{6ad} - \frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \sin(c+dx)\right)}{ad} \\
&= -\frac{\cos^6(c+dx)}{6ad} - \frac{\sin^3(c+dx)}{3ad} + \frac{2\sin^5(c+dx)}{5ad} - \frac{\sin^7(c+dx)}{7ad}
\end{aligned}
$$

Mathematica [A] time = 0.26, size = 68, normalized size = 0.93

$$
\frac{\sin^2(c+dx)\left(-30\sin^5(c+dx)+35\sin^4(c+dx)+84\sin^3(c+dx)-105\sin^2(c+dx)-70\sin(c+dx)+105\right)}{210ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^2*(105 - 70*Sin[c + d*x] - 105*Sin[c + d*x]^2 + 84*Sin[c + d*
x]^3 + 35*Sin[c + d*x]^4 - 30*Sin[c + d*x]^5))/(210*a*d)
```

fricas [A] time = 0.45, size = 59, normalized size = 0.81

$$
-\frac{35\,\cos\left(dx+c\right)^6 - 2\left(15\,\cos\left(dx+c\right)^6 - 3\,\cos\left(dx+c\right)^4 - 4\,\cos\left(dx+c\right)^2 - 8\right)\sin\left(dx+c\right)}{210\,ad}
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/210*(35*cos(d*x + c)^6 - 2*(15*cos(d*x + c)^6 - 3*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 8)*sin(d*x + c))/(a*d)

giac [A] time = 0.17, size = 69, normalized size = 0.95

$$-\frac{30 \sin\left(dx+c\right)^{7} - 35 \sin\left(dx+c\right)^{6} - 84 \sin\left(dx+c\right)^{5} + 105 \sin\left(dx+c\right)^{4} + 70 \sin\left(dx+c\right)^{3} - 105 \sin\left(dx+c\right)^{2}}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/210*(30*sin(d*x + c)^7 - 35*sin(d*x + c)^6 - 84*sin(d*x + c)^5 + 105*sin(d*x + c)^4 + 70*sin(d*x + c)^3 - 105*sin(d*x + c)^2)/(a*d)

maple [A] time = 0.19, size = 69, normalized size = 0.95

$$\frac{-\frac{\left(\sin^{7}\left(dx+c\right)\right)}{7}+\frac{\left(\sin^{6}\left(dx+c\right)\right)}{6}+\frac{2\left(\sin^{5}\left(dx+c\right)\right)}{5}-\frac{\left(\sin^{4}\left(dx+c\right)\right)}{2}-\frac{\left(\sin^{3}\left(dx+c\right)\right)}{3}+\frac{\left(\sin^{2}\left(dx+c\right)\right)}{2}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x)

[Out] 1/d/a*(-1/7*sin(d*x+c)^7+1/6*sin(d*x+c)^6+2/5*sin(d*x+c)^5-1/2*sin(d*x+c)^4-1/3*sin(d*x+c)^3+1/2*sin(d*x+c)^2)

maxima [A] time = 0.33, size = 69, normalized size = 0.95

$$-\frac{30 \sin\left(dx+c\right)^{7} - 35 \sin\left(dx+c\right)^{6} - 84 \sin\left(dx+c\right)^{5} + 105 \sin\left(dx+c\right)^{4} + 70 \sin\left(dx+c\right)^{3} - 105 \sin\left(dx+c\right)^{2}}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/210*(30*sin(d*x + c)^7 - 35*sin(d*x + c)^6 - 84*sin(d*x + c)^5 + 105*sin(d*x + c)^4 + 70*sin(d*x + c)^3 - 105*sin(d*x + c)^2)/(a*d)

mupad [B] time = 9.13, size = 83, normalized size = 1.14

$$\frac{\frac{\sin(c+d\,x)^2}{2\,a}-\frac{\sin(c+d\,x)^3}{3\,a}-\frac{\sin(c+d\,x)^4}{2\,a}+\frac{2\sin(c+d\,x)^5}{5\,a}+\frac{\sin(c+d\,x)^6}{6\,a}-\frac{\sin(c+d\,x)^7}{7\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^2/(2*a) - sin(c + d*x)^3/(3*a) - sin(c + d*x)^4/(2*a) + (2*si
n(c + d*x)^5)/(5*a) + sin(c + d*x)^6/(6*a) - sin(c + d*x)^7/(7*a))/d
```

sympy [A] time = 76.74, size = 1530, normalized size = 20.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((210*tan(c/2 + d*x/2)**12/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d
*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 +
d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 7
35*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 280*tan(c/2 + d*x/2)**11/(105*a*d*t
an(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x
/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 220
5*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 210*ta
n(c/2 + d*x/2)**10/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)
**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*
a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 +
d*x/2)**2 + 105*a*d) + 224*tan(c/2 + d*x/2)**9/(105*a*d*tan(c/2 + d*x/2)**1
4 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d
*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*
x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 700*tan(c/2 + d*x/2)**8/
(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan
(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2
)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d
) - 912*tan(c/2 + d*x/2)**7/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2
 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**
8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*t
an(c/2 + d*x/2)**2 + 105*a*d) + 700*tan(c/2 + d*x/2)**6/(105*a*d*tan(c/2 +
d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 +
 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan
(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 224*tan(c/2 + d
*x/2)**5/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 220
5*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/
2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2
+ 105*a*d) + 210*tan(c/2 + d*x/2)**4/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*
d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 +
 d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 +
735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 280*tan(c/2 + d*x/2)**3/(105*a*d*t
an(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x
/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 220
```

```
5*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 210*ta
n(c/2 + d*x/2)**2/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)*
*12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a
*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d
*x/2)**2 + 105*a*d), Ne(d, 0)), (x*sin(c)*cos(c)**7/(a*sin(c) + a), True))
```

3.683 $\int \frac{\cos^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=68

$$-\frac{(a-a\sin(c+dx))^6}{6a^7d}+\frac{4(a-a\sin(c+dx))^5}{5a^6d}-\frac{(a-a\sin(c+dx))^4}{a^5d}$$

```
[Out] -(a-a*sin(d*x+c))^4/a^5/d+4/5*(a-a*sin(d*x+c))^5/a^6/d-1/6*(a-a*sin(d*x+c))
^6/a^7/d
```

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$-\frac{(a-a\sin(c+dx))^6}{6a^7d}+\frac{4(a-a\sin(c+dx))^5}{5a^6d}-\frac{(a-a\sin(c+dx))^4}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a - a*Sin[c + d*x])^4/(a^5*d)) + (4*(a - a*Sin[c + d*x])^5)/(5*a^6*d) -
(a - a*Sin[c + d*x])^6/(6*a^7*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^2(a-x)^3-4a(a-x)^4+(a-x)^5\right)\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= -\frac{(a-a\sin(c+dx))^4}{a^5d}+\frac{4(a-a\sin(c+dx))^5}{5a^6d}-\frac{(a-a\sin(c+dx))^6}{6a^7d}$$

Mathematica [A] time = 0.19, size = 66, normalized size = 0.97

$$-\frac{\sin(c+dx)\left(5\sin^5(c+dx)-6\sin^4(c+dx)-15\sin^3(c+dx)+20\sin^2(c+dx)+15\sin(c+dx)-30\right)}{30ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/30*(Sin[c + d*x]*(-30 + 15*Sin[c + d*x] + 20*Sin[c + d*x]^2 - 15*Sin[c +
 d*x]^3 - 6*Sin[c + d*x]^4 + 5*Sin[c + d*x]^5))/(a*d)
```

fricas [A] time = 0.45, size = 49, normalized size = 0.72

$$\frac{5\cos(dx+c)^6+2\left(3\cos(dx+c)^4+4\cos(dx+c)^2+8\right)\sin(dx+c)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(5*cos(d*x + c)^6 + 2*(3*cos(d*x + c)^4 + 4*cos(d*x + c)^2 + 8)*sin(d*
x + c))/(a*d)
```

giac [A] time = 0.15, size = 67, normalized size = 0.99

$$-\frac{5\sin(dx+c)^6-6\sin(dx+c)^5-15\sin(dx+c)^4+20\sin(dx+c)^3+15\sin(dx+c)^2-30\sin(dx+c)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(5*sin(d*x + c)^6 - 6*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 20*sin(d*x
 + c)^3 + 15*sin(d*x + c)^2 - 30*sin(d*x + c))/(a*d)
```

maple [A] time = 0.31, size = 65, normalized size = 0.96

$$\frac{-\frac{\left(\sin^6(dx+c)\right)}{6}+\frac{\left(\sin^5(dx+c)\right)}{5}+\frac{\left(\sin^4(dx+c)\right)}{2}-\frac{2\left(\sin^3(dx+c)\right)}{3}-\frac{\left(\sin^2(dx+c)\right)}{2}+\sin\left(dx+c\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/6*sin(d*x+c)^6+1/5*sin(d*x+c)^5+1/2*sin(d*x+c)^4-2/3*sin(d*x+c)^3
-1/2*sin(d*x+c)^2+sin(d*x+c))
```

maxima [A] time = 0.33, size = 67, normalized size = 0.99

$$-\frac{5\sin\left(dx+c\right)^6-6\sin\left(dx+c\right)^5-15\sin\left(dx+c\right)^4+20\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)^2-30\sin\left(dx+c\right)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(5*sin(d*x + c)^6 - 6*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 20*sin(d*x
 + c)^3 + 15*sin(d*x + c)^2 - 30*sin(d*x + c))/(a*d)
```

mupad [B] time = 9.14, size = 80, normalized size = 1.18

$$\frac{\frac{\sin(c+d\,x)}{a}-\frac{\sin(c+d\,x)^2}{2\,a}-\frac{2\sin(c+d\,x)^3}{3\,a}+\frac{\sin(c+d\,x)^4}{2\,a}+\frac{\sin(c+d\,x)^5}{5\,a}-\frac{\sin(c+d\,x)^6}{6\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)/a - sin(c + d*x)^2/(2*a) - (2*sin(c + d*x)^3)/(3*a) + sin(c +
 d*x)^4/(2*a) + sin(c + d*x)^5/(5*a) - sin(c + d*x)^6/(6*a))/d
```

sympy [A] time = 49.95, size = 1096, normalized size = 16.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((30*tan(c/2 + d*x/2)**11/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*ta
n(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)
```

```
**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) -
30*tan(c/2 + d*x/2)**10/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x
/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a
*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 70*tan(c/2
+ d*x/2)**9/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 22
5*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 +
 d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 156*tan(c/2 + d*x/2)**7
/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c
/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4
+ 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 100*tan(c/2 + d*x/2)**6/(15*a*d*ta
n(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)
**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*ta
n(c/2 + d*x/2)**2 + 15*a*d) + 156*tan(c/2 + d*x/2)**5/(15*a*d*tan(c/2 + d*x
/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a
*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x
/2)**2 + 15*a*d) + 70*tan(c/2 + d*x/2)**3/(15*a*d*tan(c/2 + d*x/2)**12 + 90
*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 +
 d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*
a*d) - 30*tan(c/2 + d*x/2)**2/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2
 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 +
 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*ta
n(c/2 + d*x/2)/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 +
 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/
2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d), Ne(d, 0)), (x*cos(c)*
*7/(a*sin(c) + a), True))
```

3.684 $\int \frac{\cos^6(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=99

$$-\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^4(c+dx)}{4ad}+\frac{2\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] ln(sin(d*x+c))/a/d-sin(d*x+c)/a/d-sin(d*x+c)^2/a/d+2/3*sin(d*x+c)^3/a/d+1/4
*sin(d*x+c)^4/a/d-1/5*sin(d*x+c)^5/a/d
```

Rubi [A] time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^4(c+dx)}{4ad}+\frac{2\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(a*d) + (2*Si
n[c + d*x]^3)/(3*a*d) + Sin[c + d*x]^4/(4*a*d) - Sin[c + d*x]^5/(5*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^3(a+x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^4+\frac{a^5}{x}-2a^3x+2a^2x^2+ax^3-x^4\right)dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= \frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{ad}+\frac{2\sin^3(c+dx)}{3ad}+\frac{\sin^4(c+dx)}{4ad}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 0.69

$$\frac{-12\sin^5(c+dx)+15\sin^4(c+dx)+40\sin^3(c+dx)-60\sin^2(c+dx)-60\sin(c+dx)+60\log(\sin(c+dx))}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (60*Log[Sin[c + d*x]] - 60*Sin[c + d*x] - 60*Sin[c + d*x]^2 + 40*Sin[c + d*
x]^3 + 15*Sin[c + d*x]^4 - 12*Sin[c + d*x]^5)/(60*a*d)
```

fricas [A] time = 0.46, size = 70, normalized size = 0.71

$$\frac{15\cos(dx+c)^4+30\cos(dx+c)^2-4\left(3\cos(dx+c)^4+4\cos(dx+c)^2+8\right)\sin(dx+c)+60\log\left(\frac{1}{2}\sin(dx+\right.}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*cos(d*x + c)^4 + 30*cos(d*x + c)^2 - 4*(3*cos(d*x + c)^4 + 4*cos(d
*x + c)^2 + 8)*sin(d*x + c) + 60*log(1/2*sin(d*x + c)))/(a*d)
```

giac [A] time = 0.19, size = 88, normalized size = 0.89

$$\frac{\frac{60\log(|\sin(dx+c)|)}{a}-\frac{12a^4\sin(dx+c)^5-15a^4\sin(dx+c)^4-40a^4\sin(dx+c)^3+60a^4\sin(dx+c)^2+60a^4\sin(dx+c)}{a^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(sin(d*x + c)))/a - (12*a^4*sin(d*x + c)^5 - 15*a^4*sin(d*x
 + c)^4 - 40*a^4*sin(d*x + c)^3 + 60*a^4*sin(d*x + c)^2 + 60*a^4*sin(d*x +
c))/a^5)/d
```

maple [A] time = 0.37, size = 94, normalized size = 0.95

$$\frac{\ln\left(\sin\left(dx+c\right)\right)}{ad}-\frac{\sin\left(dx+c\right)}{ad}-\frac{\sin^{2}\left(dx+c\right)}{ad}+\frac{2\left(\sin^{3}\left(dx+c\right)\right)}{3da}+\frac{\sin^{4}\left(dx+c\right)}{4da}-\frac{\sin^{5}\left(dx+c\right)}{5da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] ln(sin(d*x+c))/a/d-sin(d*x+c)/a/d-sin(d*x+c)^2/a/d+2/3*sin(d*x+c)^3/d/a+1/4
*sin(d*x+c)^4/d/a-1/5*sin(d*x+c)^5/d/a
```

maxima [A] time = 0.37, size = 71, normalized size = 0.72

$$-\frac{\frac{12\sin\left(dx+c\right)^{5}-15\sin\left(dx+c\right)^{4}-40\sin\left(dx+c\right)^{3}+60\sin\left(dx+c\right)^{2}+60\sin\left(dx+c\right)}{a}-\frac{60\log\left(\sin\left(dx+c\right)\right)}{a}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 40*sin(d*x + c)^3 + 60*sin(
d*x + c)^2 + 60*sin(d*x + c))/a - 60*log(sin(d*x + c))/a)/d
```

mupad [B] time = 9.29, size = 140, normalized size = 1.41

$$\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}-\frac{\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{a\,d}-\frac{8\,\sin\left(c+d\,x\right)}{15\,a\,d}+\frac{{\cos\left(c+d\,x\right)}^2}{2\,a\,d}+\frac{{\cos\left(c+d\,x\right)}^4}{4\,a\,d}-\frac{4\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{15\,a\,d}-\frac{c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) - log(1/cos(c/2 + (d*x)/2)
^2)/(a*d) - (8*sin(c + d*x))/(15*a*d) + cos(c + d*x)^2/(2*a*d) + cos(c + d*
x)^4/(4*a*d) - (4*cos(c + d*x)^2*sin(c + d*x))/(15*a*d) - (cos(c + d*x)^4*s
in(c + d*x))/(5*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.685 $\int \frac{\cos^5(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{\sin^4(c+dx)}{4ad}+\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{ad}-\frac{2\sin(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] -csc(d*x+c)/a/d-ln(sin(d*x+c))/a/d-2*sin(d*x+c)/a/d+sin(d*x+c)^2/a/d+1/3*si
n(d*x+c)^3/a/d-1/4*sin(d*x+c)^4/a/d
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\sin^4(c+dx)}{4ad}+\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{ad}-\frac{2\sin(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) - (2*Sin[c + d*x])/(a*d) +
Sin[c + d*x]^2/(a*d) + Sin[c + d*x]^3/(3*a*d) - Sin[c + d*x]^4/(4*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^3(a+x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^3+\frac{a^5}{x^2}-\frac{a^4}{x}+2a^2x+ax^2-x^3\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= -\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}-\frac{2\sin(c+dx)}{ad}+\frac{\sin^2(c+dx)}{ad}+\frac{\sin^3(c+dx}{3ad}$$

Mathematica [A] time = 0.13, size = 66, normalized size = 0.69

$$-\frac{3\sin^4(c+dx)-4\sin^3(c+dx)-12\sin^2(c+dx)+24\sin(c+dx)+12\csc(c+dx)+12\log(\sin(c+dx))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/12*(12*Csc[c + d*x] + 12*Log[Sin[c + d*x]] + 24*Sin[c + d*x] - 12*Sin[c
+ d*x]^2 - 4*Sin[c + d*x]^3 + 3*Sin[c + d*x]^4)/(a*d)
```

fricas [A] time = 0.48, size = 85, normalized size = 0.89

$$\frac{32\cos(dx+c)^4+128\cos(dx+c)^2-3\left(8\cos(dx+c)^4+16\cos(dx+c)^2-11\right)\sin(dx+c)-96\log\left(\frac{1}{2}\sin(dx\right.}{96\,ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(32*cos(d*x + c)^4 + 128*cos(d*x + c)^2 - 3*(8*cos(d*x + c)^4 + 16*cos
(d*x + c)^2 - 11)*sin(d*x + c) - 96*log(1/2*sin(d*x + c))*sin(d*x + c) - 25
6)/(a*d*sin(d*x + c))
```

giac [A] time = 0.21, size = 95, normalized size = 1.00

$$-\frac{\frac{12\log(|\sin(dx+c)|)}{a}-\frac{12(\sin(dx+c)-1)}{a\sin(dx+c)}+\frac{3a^3\sin(dx+c)^4-4a^3\sin(dx+c)^3-12a^3\sin(dx+c)^2+24a^3\sin(dx+c)}{a^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(12*log(abs(sin(d*x + c)))/a - 12*(sin(d*x + c) - 1)/(a*sin(d*x + c))
 + (3*a^3*sin(d*x + c)^4 - 4*a^3*sin(d*x + c)^3 - 12*a^3*sin(d*x + c)^2 + 2
4*a^3*sin(d*x + c))/a^4)/d
```

maple [A] time = 0.42, size = 94, normalized size = 0.99

$$-\frac{\sin^4(dx+c)}{4da}+\frac{\sin^3(dx+c)}{3da}+\frac{\sin^2(dx+c)}{ad}-\frac{2\sin(dx+c)}{ad}-\frac{1}{da\sin(dx+c)}-\frac{\ln(\sin(dx+c))}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4*sin(d*x+c)^4/d/a+1/3*sin(d*x+c)^3/d/a+sin(d*x+c)^2/a/d-2*sin(d*x+c)/a/
d-1/d/a/sin(d*x+c)-ln(sin(d*x+c))/a/d
```

maxima [A] time = 0.35, size = 74, normalized size = 0.78

$$-\frac{\frac{3\sin(dx+c)^4-4\sin(dx+c)^3-12\sin(dx+c)^2+24\sin(dx+c)}{a}+\frac{12\log(\sin(dx+c))}{a}+\frac{12}{a\sin(dx+c)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((3*sin(d*x + c)^4 - 4*sin(d*x + c)^3 - 12*sin(d*x + c)^2 + 24*sin(d*
x + c))/a + 12*log(sin(d*x + c))/a + 12/(a*sin(d*x + c)))/d
```

mupad [B] time = 9.36, size = 272, normalized size = 2.86

$$\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{ad}-\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{ad}+\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{ad}-\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{ad}+\frac{\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}\right)}{ad}-\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{ad}+\frac{20\cos}{3ad\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] (4*cos(c/2 + (d*x)/2)^2)/(a*d) - (8*cos(c/2 + (d*x)/2)^4)/(a*d) + (8*cos(c/
2 + (d*x)/2)^6)/(a*d) - (4*cos(c/2 + (d*x)/2)^8)/(a*d) + log(1/cos(c/2 + (d
*x)/2)^2)/(a*d) - log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) + (20*co
s(c/2 + (d*x)/2)^3)/(3*a*d*sin(c/2 + (d*x)/2)) - (16*cos(c/2 + (d*x)/2)^5)/
(3*a*d*sin(c/2 + (d*x)/2)) + (8*cos(c/2 + (d*x)/2)^7)/(3*a*d*sin(c/2 + (d*x
```

```
)/2)) - (9*cos(c/2 + (d*x)/2))/(2*a*d*sin(c/2 + (d*x)/2)) - sin(c/2 + (d*x)
/2)/(2*a*d*cos(c/2 + (d*x)/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.686 $\int \frac{\cos^4(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{2ad}+\frac{2\sin(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}-\frac{2\log(\sin(c+dx))}{ad}$$

```
[Out] csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d-2*ln(sin(d*x+c))/a/d+2*sin(d*x+c)/a/d+1
/2*sin(d*x+c)^2/a/d-1/3*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{2ad}+\frac{2\sin(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}-\frac{2\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - (2*Log[Sin[c + d*x]])/(a*d) +
 (2*Sin[c + d*x])/(a*d) + Sin[c + d*x]^2/(2*a*d) - Sin[c + d*x]^3/(3*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^3(a+x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(2a^2+\frac{a^5}{x^3}-\frac{a^4}{x^2}-\frac{2a^3}{x}+ax-x^2\right)dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}-\frac{2\log(\sin(c+dx))}{ad}+\frac{2\sin(c+dx)}{ad}+\frac{\sin^2(c+dx}{2ad}$$

Mathematica [A] time = 0.11, size = 66, normalized size = 0.68

$$\frac{-2\sin^3(c+dx)+3\sin^2(c+dx)+12\sin(c+dx)-3\csc^2(c+dx)+6\csc(c+dx)-12\log(\sin(c+dx))}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6*Csc[c + d*x] - 3*Csc[c + d*x]^2 - 12*Log[Sin[c + d*x]] + 12*Sin[c + d*x]
 + 3*Sin[c + d*x]^2 - 2*Sin[c + d*x]^3)/(6*a*d)
```

fricas [A] time = 0.51, size = 91, normalized size = 0.94

$$-\frac{6\cos(dx+c)^4-9\cos(dx+c)^2+24\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-4\left(\cos(dx+c)^4+4\cos(dx+c\right.}{12\left(ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*cos(d*x + c)^4 - 9*cos(d*x + c)^2 + 24*(cos(d*x + c)^2 - 1)*log(1/
2*sin(d*x + c)) - 4*(cos(d*x + c)^4 + 4*cos(d*x + c)^2 - 8)*sin(d*x + c) -
3)/(a*d*cos(d*x + c)^2 - a*d)
```

giac [A] time = 0.21, size = 94, normalized size = 0.97

$$-\frac{\frac{12\log(|\sin(dx+c)|)}{a}+\frac{2a^2\sin(dx+c)^3-3a^2\sin(dx+c)^2-12a^2\sin(dx+c)}{a^3}-\frac{3\left(6\sin(dx+c)^2+2\sin(dx+c)-1\right)}{a\sin(dx+c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(12*log(abs(sin(d*x + c)))/a + (2*a^2*sin(d*x + c)^3 - 3*a^2*sin(d*x +
 c)^2 - 12*a^2*sin(d*x + c))/a^3 - 3*(6*sin(d*x + c)^2 + 2*sin(d*x + c) - 1
)/(a*sin(d*x + c)^2))/d
```

maple [A] time = 0.47, size = 94, normalized size = 0.97

$$-\frac{\sin^3(dx+c)}{3da}+\frac{\sin^2(dx+c)}{2ad}+\frac{2\sin(dx+c)}{ad}+\frac{1}{da\sin(dx+c)}-\frac{2\ln(\sin(dx+c))}{ad}-\frac{1}{2ad\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3*sin(d*x+c)^3/d/a+1/2*sin(d*x+c)^2/a/d+2*sin(d*x+c)/a/d+1/d/a/sin(d*x+c
)-2*ln(sin(d*x+c))/a/d-1/2/a/d/sin(d*x+c)^2
```

maxima [A] time = 0.36, size = 74, normalized size = 0.76

$$-\frac{\frac{2\sin(dx+c)^3-3\sin(dx+c)^2-12\sin(dx+c)}{a}+\frac{12\log(\sin(dx+c))}{a}-\frac{3(2\sin(dx+c)-1)}{a\sin(dx+c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*((2*sin(d*x + c)^3 - 3*sin(d*x + c)^2 - 12*sin(d*x + c))/a + 12*log(si
n(d*x + c))/a - 3*(2*sin(d*x + c) - 1)/(a*sin(d*x + c)^2))/d
```

mupad [B] time = 9.23, size = 231, normalized size = 2.38

$$\frac{18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\frac{15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{2}+\frac{82\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{3}+\frac{13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2}+22\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\cdots}{d\left(4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (2*tan(c/2 + (d*x)/2) - (3*tan(c/2 + (d*x)/2)^2)/2 + 22*tan(c/2 + (d*x)/2)^
3 + (13*tan(c/2 + (d*x)/2)^4)/2 + (82*tan(c/2 + (d*x)/2)^5)/3 + (15*tan(c/2
 + (d*x)/2)^6)/2 + 18*tan(c/2 + (d*x)/2)^7 - 1/2)/(d*(4*a*tan(c/2 + (d*x)/2
```

```
)^2 + 12*a*tan(c/2 + (d*x)/2)^4 + 12*a*tan(c/2 + (d*x)/2)^6 + 4*a*tan(c/2 +
 (d*x)/2)^8)) - (2*log(tan(c/2 + (d*x)/2)))/(a*d) - tan(c/2 + (d*x)/2)^2/(8
*a*d) + tan(c/2 + (d*x)/2)/(2*a*d) + (2*log(tan(c/2 + (d*x)/2)^2 + 1))/(a*d
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.687 $\int \frac{\cos^3(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}+\frac{2\csc(c+dx)}{ad}+\frac{2\log(\sin(c+dx))}{ad}$$

```
[Out] 2*csc(d*x+c)/a/d+1/2*csc(d*x+c)^2/a/d-1/3*csc(d*x+c)^3/a/d+2*ln(sin(d*x+c))
/a/d+sin(d*x+c)/a/d-1/2*sin(d*x+c)^2/a/d
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}+\frac{2\csc(c+dx)}{ad}+\frac{2\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*Csc[c + d*x])/(a*d) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) +
(2*Log[Sin[c + d*x]])/(a*d) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^3(a+x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(a+\frac{a^5}{x^4}-\frac{a^4}{x^3}-\frac{2a^3}{x^2}+\frac{2a^2}{x}-x\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{2\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{2\log(\sin(c+dx))}{ad}+\frac{\sin(c+dx)}{ad}$$

Mathematica [A] time = 0.17, size = 66, normalized size = 0.68

$$\frac{-3\sin^2(c+dx)+6\sin(c+dx)-2\csc^3(c+dx)+3\csc^2(c+dx)+12\csc(c+dx)+12\log(\sin(c+dx))}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (12*Csc[c + d*x] + 3*Csc[c + d*x]^2 - 2*Csc[c + d*x]^3 + 12*Log[Sin[c + d*x
]] + 6*Sin[c + d*x] - 3*Sin[c + d*x]^2)/(6*a*d)
```

fricas [A] time = 0.49, size = 107, normalized size = 1.10

$$-\frac{12\cos(dx+c)^4-24\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-48\cos(dx+c)^2-3\left(2\cos(dx+c)^4\right.}{12\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*cos(d*x + c)^4 - 24*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c))*si
n(d*x + c) - 48*cos(d*x + c)^2 - 3*(2*cos(d*x + c)^4 - 3*cos(d*x + c)^2 - 1
)*sin(d*x + c) + 32)/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

giac [A] time = 0.29, size = 87, normalized size = 0.90

$$\frac{\frac{12\log(|\sin(dx+c)|)}{a}-\frac{3\left(a\sin(dx+c)^2-2a\sin(dx+c)\right)}{a^2}-\frac{22\sin(dx+c)^3-12\sin(dx+c)^2-3\sin(dx+c)+2}{a\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(12*log(abs(sin(d*x + c)))/a - 3*(a*sin(d*x + c)^2 - 2*a*sin(d*x + c))/
a^2 - (22*sin(d*x + c)^3 - 12*sin(d*x + c)^2 - 3*sin(d*x + c) + 2)/(a*sin(d
*x + c)^3))/d
```

maple [A] time = 0.47, size = 94, normalized size = 0.97

$$-\frac{\sin^2(dx+c)}{2ad}+\frac{\sin(dx+c)}{ad}+\frac{2}{da\sin(dx+c)}+\frac{2\ln(\sin(dx+c))}{ad}+\frac{1}{2ad\sin(dx+c)^2}-\frac{1}{3ad\sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2*sin(d*x+c)^2/a/d+sin(d*x+c)/a/d+2/d/a/sin(d*x+c)+2*ln(sin(d*x+c))/a/d+
1/2/a/d/sin(d*x+c)^2-1/3/a/d/sin(d*x+c)^3
```

maxima [A] time = 0.32, size = 73, normalized size = 0.75

$$-\frac{\frac{3\left(\sin(dx+c)^2-2\sin(dx+c)\right)}{a}-\frac{12\log(\sin(dx+c))}{a}-\frac{12\sin(dx+c)^2+3\sin(dx+c)-2}{a\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*(sin(d*x + c)^2 - 2*sin(d*x + c))/a - 12*log(sin(d*x + c))/a - (12*
sin(d*x + c)^2 + 3*sin(d*x + c) - 2)/(a*sin(d*x + c)^3))/d
```

mupad [B] time = 9.17, size = 221, normalized size = 2.28

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8ad}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24ad}+\frac{2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8ad}+\frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{89}{}}{d\left(8a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - tan(c/2 + (d*x)/2)^3/(24*a*d) + (2*log(tan(c
/2 + (d*x)/2)))/(a*d) + (7*tan(c/2 + (d*x)/2))/(8*a*d) + (tan(c/2 + (d*x)/2
) + (19*tan(c/2 + (d*x)/2)^2)/3 + 2*tan(c/2 + (d*x)/2)^3 + (89*tan(c/2 + (d
```

```
*x)/2)^4)/3 - 15*tan(c/2 + (d*x)/2)^5 + 23*tan(c/2 + (d*x)/2)^6 - 1/3)/(d*(
8*a*tan(c/2 + (d*x)/2)^3 + 16*a*tan(c/2 + (d*x)/2)^5 + 8*a*tan(c/2 + (d*x)/
2)^7)) - (2*log(tan(c/2 + (d*x)/2)^2 + 1))/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.688 $\int \frac{\cos^2(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{\sin(c+dx)}{ad}-\frac{\csc^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{ad}-\frac{2\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] -2*csc(d*x+c)/a/d+csc(d*x+c)^2/a/d+1/3*csc(d*x+c)^3/a/d-1/4*csc(d*x+c)^4/a/
d+ln(sin(d*x+c))/a/d-sin(d*x+c)/a/d
```

Rubi [A] time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\sin(c+dx)}{ad}-\frac{\csc^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{ad}-\frac{2\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*Csc[c + d*x])/(a*d) + Csc[c + d*x]^2/(a*d) + Csc[c + d*x]^3/(3*a*d) - C
sc[c + d*x]^4/(4*a*d) + Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^5(a-x)^3(a+x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^5}{x^5}-\frac{a^4}{x^4}-\frac{2a^3}{x^3}+\frac{2a^2}{x^2}+\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= -\frac{2\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^4(c+dx)}{4ad}+\frac{\log(\sin(c+dx}{ad}$$

Mathematica [A] time = 0.30, size = 66, normalized size = 0.70

$$-\frac{12\sin(c+dx)+3\csc^4(c+dx)-4\csc^3(c+dx)-12\csc^2(c+dx)+24\csc(c+dx)-12\log(\sin(c+dx))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/12*(24*Csc[c + d*x] - 12*Csc[c + d*x]^2 - 4*Csc[c + d*x]^3 + 3*Csc[c + d
*x]^4 - 12*Log[Sin[c + d*x]] + 12*Sin[c + d*x])/(a*d)
```

fricas [A] time = 0.48, size = 104, normalized size = 1.11

$$-\frac{12\cos(dx+c)^2-12\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)+4\left(3\cos(dx+c)^4-12\cos(dx\right.}{12\left(ad\cos(dx+c)^4-2\,ad\cos(dx+c)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*cos(d*x + c)^2 - 12*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1
/2*sin(d*x + c)) + 4*(3*cos(d*x + c)^4 - 12*cos(d*x + c)^2 + 8)*sin(d*x + c
) - 9)/(a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

giac [A] time = 0.24, size = 83, normalized size = 0.88

$$\frac{\frac{12\log(|\sin(dx+c)|)}{a}-\frac{12\sin(dx+c)}{a}-\frac{25\sin(dx+c)^4+24\sin(dx+c)^3-12\sin(dx+c)^2-4\sin(dx+c)+3}{a\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*log(abs(sin(d*x + c)))/a - 12*sin(d*x + c)/a - (25*sin(d*x + c)^4
+ 24*sin(d*x + c)^3 - 12*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*sin(d*x +
c)^4))/d
```

maple [A] time = 0.46, size = 93, normalized size = 0.99

$$-\frac{\sin(dx+c)}{ad}-\frac{2}{da\sin(dx+c)}+\frac{\ln(\sin(dx+c))}{ad}+\frac{1}{ad\sin(dx+c)^2}-\frac{1}{4ad\sin(dx+c)^4}+\frac{1}{3ad\sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] -sin(d*x+c)/a/d-2/d/a/sin(d*x+c)+ln(sin(d*x+c))/a/d+1/a/d/sin(d*x+c)^2-1/4/
a/d/sin(d*x+c)^4+1/3/a/d/sin(d*x+c)^3
```

maxima [A] time = 0.33, size = 72, normalized size = 0.77

$$\frac{\frac{12\log(\sin(dx+c))}{a}-\frac{12\sin(dx+c)}{a}-\frac{24\sin(dx+c)^3-12\sin(dx+c)^2-4\sin(dx+c)+3}{a\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*log(sin(d*x + c))/a - 12*sin(d*x + c)/a - (24*sin(d*x + c)^3 - 12*
sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*sin(d*x + c)^4))/d
```

mupad [B] time = 9.34, size = 214, normalized size = 2.28

$$\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{-46\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{40}{}}{d\left(16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^5*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*tan(c/2 + (d*x)/2)^2)/(16*a*d) + tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2
 + (d*x)/2)^4/(64*a*d) + log(tan(c/2 + (d*x)/2))/(a*d) + ((2*tan(c/2 + (d*x
)/2))/3 + (11*tan(c/2 + (d*x)/2)^2)/4 - (40*tan(c/2 + (d*x)/2)^3)/3 + 3*tan
```

```
(c/2 + (d*x)/2)^4 - 46*tan(c/2 + (d*x)/2)^5 - 1/4)/(d*(16*a*tan(c/2 + (d*x)
/2)^4 + 16*a*tan(c/2 + (d*x)/2)^6)) - (7*tan(c/2 + (d*x)/2))/(8*a*d) - log(
tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.689 $\int \frac{\cos(c+dx)\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=100

$$-\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^4(c+dx)}{4ad}+\frac{2\csc^3(c+dx)}{3ad}-\frac{\csc^2(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] -csc(d*x+c)/a/d-csc(d*x+c)^2/a/d+2/3*csc(d*x+c)^3/a/d+1/4*csc(d*x+c)^4/a/d-
1/5*csc(d*x+c)^5/a/d-ln(sin(d*x+c))/a/d
```

Rubi [A] time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^4(c+dx)}{4ad}+\frac{2\csc^3(c+dx)}{3ad}-\frac{\csc^2(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Csc[c + d*x]^2/(a*d) + (2*Csc[c + d*x]^3)/(3*a*d) +
 Csc[c + d*x]^4/(4*a*d) - Csc[c + d*x]^5/(5*a*d) - Log[Sin[c + d*x]]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^3(a+x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6}-\frac{a^4}{x^5}-\frac{2a^3}{x^4}+\frac{2a^2}{x^3}+\frac{a}{x^2}-\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{ad}+\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc^4(c+dx)}{4ad}-\frac{\csc^5(c+dx)}{5ad}-$$

Mathematica [A] time = 0.10, size = 68, normalized size = 0.68

$$-\frac{12\csc^5(c+dx)-15\csc^4(c+dx)-40\csc^3(c+dx)+60\csc^2(c+dx)+60\csc(c+dx)+60\log(\sin(c+dx))}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/60*(60*Csc[c + d*x] + 60*Csc[c + d*x]^2 - 40*Csc[c + d*x]^3 - 15*Csc[c +
 d*x]^4 + 12*Csc[c + d*x]^5 + 60*Log[Sin[c + d*x]])/(a*d)
```

fricas [A] time = 0.47, size = 118, normalized size = 1.18

$$-\frac{60\cos(dx+c)^4+60\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-80\cos(dx+c)^2-}{60\left(ad\cos(dx+c)^4-2ad\cos(dx+c)^2+ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(60*cos(d*x + c)^4 + 60*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1
/2*sin(d*x + c))*sin(d*x + c) - 80*cos(d*x + c)^2 - 15*(4*cos(d*x + c)^2 -
3)*sin(d*x + c) + 32)/((a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)*si
n(d*x + c))
```

giac [A] time = 0.22, size = 82, normalized size = 0.82

$$-\frac{\frac{60\log(|\sin(dx+c)|)}{a}-\frac{137\sin(dx+c)^5-60\sin(dx+c)^4-60\sin(dx+c)^3+40\sin(dx+c)^2+15\sin(dx+c)-12}{a\sin(dx+c)^5}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*log(abs(sin(d*x + c)))/a - (137*sin(d*x + c)^5 - 60*sin(d*x + c)^
4 - 60*sin(d*x + c)^3 + 40*sin(d*x + c)^2 + 15*sin(d*x + c) - 12)/(a*sin(d*
x + c)^5))/d
```

maple [A] time = 0.48, size = 97, normalized size = 0.97

$$-\frac{1}{da\sin\left(dx+c\right)}-\frac{\ln\left(\sin\left(dx+c\right)\right)}{ad}-\frac{1}{5ad\sin\left(dx+c\right)^{5}}-\frac{1}{ad\sin\left(dx+c\right)^{2}}+\frac{1}{4ad\sin\left(dx+c\right)^{4}}+\frac{2}{3ad\sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d/a/sin(d*x+c)-ln(sin(d*x+c))/a/d-1/5/a/d/sin(d*x+c)^5-1/a/d/sin(d*x+c)^
2+1/4/a/d/sin(d*x+c)^4+2/3/a/d/sin(d*x+c)^3
```

maxima [A] time = 0.36, size = 70, normalized size = 0.70

$$-\frac{\frac{60\log\left(\sin\left(dx+c\right)\right)}{a}+\frac{60\sin\left(dx+c\right)^{4}+60\sin\left(dx+c\right)^{3}-40\sin\left(dx+c\right)^{2}-15\sin\left(dx+c\right)+12}{a\sin\left(dx+c\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(60*log(sin(d*x + c))/a + (60*sin(d*x + c)^4 + 60*sin(d*x + c)^3 - 40
*sin(d*x + c)^2 - 15*sin(d*x + c) + 12)/(a*sin(d*x + c)^5))/d
```

mupad [B] time = 9.24, size = 204, normalized size = 2.04

$$\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,a\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a\,d}+\frac{\ln\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^6*(a + a*sin(c + d*x))),x)
```

```
[Out] (5*tan(c/2 + (d*x)/2)^3)/(96*a*d) - (3*tan(c/2 + (d*x)/2)^2)/(16*a*d) + tan
(c/2 + (d*x)/2)^4/(64*a*d) - tan(c/2 + (d*x)/2)^5/(160*a*d) - log(tan(c/2 +
 (d*x)/2))/(a*d) - (5*tan(c/2 + (d*x)/2))/(16*a*d) + log(tan(c/2 + (d*x)/2)
^2 + 1)/(a*d) - (cot(c/2 + (d*x)/2)^5*(6*tan(c/2 + (d*x)/2)^3 - (5*tan(c/2
```

```
+ (d*x)/2)^2)/3 - tan(c/2 + (d*x)/2)/2 + 10*tan(c/2 + (d*x)/2)^4 + 1/5))/(3
2*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.690 $\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=68

$$-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc(c+dx)}{ad}$$

```
[Out] -1/6*cot(d*x+c)^6/a/d+csc(d*x+c)/a/d-2/3*csc(d*x+c)^3/a/d+1/5*csc(d*x+c)^5/
a/d
```

Rubi [A] time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2606, 194}

$$-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cot[c + d*x]^6/(6*a*d) + Csc[c + d*x]/(a*d) - (2*Csc[c + d*x]^3)/(3*a*d) +
 Csc[c + d*x]^5/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^5(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int x^5\,dx,x,-\cot(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot^6(c+dx)}{6ad}+\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot^6(c+dx)}{6ad}+\frac{\csc(c+dx)}{ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A] time = 0.13, size = 61, normalized size = 0.90

$$\frac{\csc^6(c+dx)(78\sin(c+dx)-5(7\sin(3(c+dx))-3\sin(5(c+dx))+5)-15\cos(4(c+dx)))}{240ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^6*(-15*Cos[4*(c + d*x)] + 78*Sin[c + d*x] - 5*(5 + 7*Sin[3*(c
 + d*x)] - 3*Sin[5*(c + d*x)])))/(240*a*d)
```

fricas [A] time = 0.47, size = 96, normalized size = 1.41

$$\frac{15\,\cos\left(dx+c\right)^4-15\,\cos\left(dx+c\right)^2-2\left(15\,\cos\left(dx+c\right)^4-20\,\cos\left(dx+c\right)^2+8\right)\sin\left(dx+c\right)+5}{30\left(ad\cos\left(dx+c\right)^6-3\,ad\cos\left(dx+c\right)^4+3\,ad\cos\left(dx+c\right)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

[Out] 1/30*(15*cos(d*x + c)^4 - 15*cos(d*x + c)^2 - 2*(15*cos(d*x + c)^4 - 20*cos(d*x + c)^2 + 8)*sin(d*x + c) + 5)/(a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)

giac [A] time = 0.22, size = 66, normalized size = 0.97

$$\frac{30 \sin\left(dx+c\right)^{5} - 15 \sin\left(dx+c\right)^{4} - 20 \sin\left(dx+c\right)^{3} + 15 \sin\left(dx+c\right)^{2} + 6 \sin\left(dx+c\right) - 5}{30 \, ad \sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d*x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)

maple [A] time = 0.47, size = 67, normalized size = 0.99

$$\frac{-\frac{1}{6\sin(dx+c)^{6}} + \frac{1}{\sin(dx+c)} + \frac{1}{5\sin(dx+c)^{5}} - \frac{1}{2\sin(dx+c)^{2}} + \frac{1}{2\sin(dx+c)^{4}} - \frac{2}{3\sin(dx+c)^{3}}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^7*csc(d*x+c)^7/(a+a*sin(d*x+c)),x)

[Out] 1/d/a*(-1/6/sin(d*x+c)^6+1/sin(d*x+c)+1/5/sin(d*x+c)^5-1/2/sin(d*x+c)^2+1/2/sin(d*x+c)^4-2/3/sin(d*x+c)^3)

maxima [A] time = 0.34, size = 66, normalized size = 0.97

$$\frac{30 \sin\left(dx+c\right)^{5} - 15 \sin\left(dx+c\right)^{4} - 20 \sin\left(dx+c\right)^{3} + 15 \sin\left(dx+c\right)^{2} + 6 \sin\left(dx+c\right) - 5}{30 \, ad \sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d*x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)

mupad [B] time = 9.04, size = 63, normalized size = 0.93

$$\frac{\sin\left(c+d\,x\right)^5-\frac{\sin\left(c+d\,x\right)^4}{2}-\frac{2\,\sin\left(c+d\,x\right)^3}{3}+\frac{\sin\left(c+d\,x\right)^2}{2}+\frac{\sin\left(c+d\,x\right)}{5}-\frac{1}{6}}{a\,d\,\sin\left(c+d\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x)/5 + sin(c + d*x)^2/2 - (2*sin(c + d*x)^3)/3 - sin(c + d*x)^4/
2 + sin(c + d*x)^5 - 1/6)/(a*d*sin(c + d*x)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.691 $\int \frac{\cot^7(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\cot^6(c+dx)}{6ad}-\frac{\csc^7(c+dx)}{7ad}+\frac{2\csc^5(c+dx)}{5ad}-\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] 1/6*cot(d*x+c)^6/a/d-1/3*csc(d*x+c)^3/a/d+2/5*csc(d*x+c)^5/a/d-1/7*csc(d*x+
c)^7/a/d
```

Rubi [A] time = 0.14, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2835, 2606, 270, 2607, 30}

$$\frac{\cot^6(c+dx)}{6ad}-\frac{\csc^7(c+dx)}{7ad}+\frac{2\csc^5(c+dx)}{5ad}-\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^6/(6*a*d) - Csc[c + d*x]^3/(3*a*d) + (2*Csc[c + d*x]^5)/(5*a*d
) - Csc[c + d*x]^7/(7*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^7(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^5(c+dx)\csc^2(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^3(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \csc(c+dx)\right)}{ad}\\ &= \frac{\cot^6(c+dx)}{6ad}-\frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \csc(c+dx)\right)}{ad}\\ &= \frac{\cot^6(c+dx)}{6ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{2\csc^5(c+dx)}{5ad}-\frac{\csc^7(c+dx)}{7ad}\end{aligned}$$

Mathematica [A] time = 0.16, size = 68, normalized size = 0.93

$$\frac{\csc^2(c+dx)\left(-30\csc^5(c+dx)+35\csc^4(c+dx)+84\csc^3(c+dx)-105\csc^2(c+dx)-70\csc(c+dx)+105\right)}{210ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^2*(105 - 70*Csc[c + d*x] - 105*Csc[c + d*x]^2 + 84*Csc[c + d*
x]^3 + 35*Csc[c + d*x]^4 - 30*Csc[c + d*x]^5))/(210*a*d)
```

fricas [A] time = 0.47, size = 104, normalized size = 1.42

$$\frac{70\cos\left(dx+c\right)^4-56\cos\left(dx+c\right)^2-35\left(3\cos\left(dx+c\right)^4-3\cos\left(dx+c\right)^2+1\right)\sin\left(dx+c\right)+16}{210\left(ad\cos\left(dx+c\right)^6-3\,ad\cos\left(dx+c\right)^4+3\,ad\cos\left(dx+c\right)^2-ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/210*(70*cos(d*x + c)^4 - 56*cos(d*x + c)^2 - 35*(3*cos(d*x + c)^4 - 3*cos
(d*x + c)^2 + 1)*sin(d*x + c) + 16)/((a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x +
c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

giac [A] time = 0.24, size = 66, normalized size = 0.90

$$\frac{105 \sin\left(dx+c\right)^{5}-70 \sin\left(dx+c\right)^{4}-105 \sin\left(dx+c\right)^{3}+84 \sin\left(dx+c\right)^{2}+35 \sin\left(dx+c\right)-30}{210 \, ad \sin\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/210*(105*sin(d*x + c)^5 - 70*sin(d*x + c)^4 - 105*sin(d*x + c)^3 + 84*sin
(d*x + c)^2 + 35*sin(d*x + c) - 30)/(a*d*sin(d*x + c)^7)
```

maple [A] time = 0.52, size = 69, normalized size = 0.95

$$\frac{\frac{1}{6 \sin(dx+c)^{6}}+\frac{2}{5 \sin(dx+c)^{5}}-\frac{1}{7 \sin(dx+c)^{7}}+\frac{1}{2 \sin(dx+c)^{2}}-\frac{1}{2 \sin(dx+c)^{4}}-\frac{1}{3 \sin(dx+c)^{3}}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^8/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/6/sin(d*x+c)^6+2/5/sin(d*x+c)^5-1/7/sin(d*x+c)^7+1/2/sin(d*x+c)^2-
1/2/sin(d*x+c)^4-1/3/sin(d*x+c)^3)
```

maxima [A] time = 0.38, size = 66, normalized size = 0.90

$$\frac{105 \sin\left(dx+c\right)^{5}-70 \sin\left(dx+c\right)^{4}-105 \sin\left(dx+c\right)^{3}+84 \sin\left(dx+c\right)^{2}+35 \sin\left(dx+c\right)-30}{210 \, ad \sin\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/210*(105*sin(d*x + c)^5 - 70*sin(d*x + c)^4 - 105*sin(d*x + c)^3 + 84*sin
(d*x + c)^2 + 35*sin(d*x + c) - 30)/(a*d*sin(d*x + c)^7)
```

mupad [B] time = 8.97, size = 66, normalized size = 0.90

$$\frac{105 \sin\left(c+d\,x\right)^{5}-70 \sin\left(c+d\,x\right)^{4}-105 \sin\left(c+d\,x\right)^{3}+84 \sin\left(c+d\,x\right)^{2}+35 \sin\left(c+d\,x\right)-30}{210\,a\,d\,\sin\left(c+d\,x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^8*(a + a*sin(c + d*x))),x)
```

```
[Out] (35*sin(c + d*x) + 84*sin(c + d*x)^2 - 105*sin(c + d*x)^3 - 70*sin(c + d*x)
^4 + 105*sin(c + d*x)^5 - 30)/(210*a*d*sin(c + d*x)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.692 $\int \frac{\cot^7(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$-\frac{\cot^8(c+dx)}{8ad}-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^7(c+dx)}{7ad}-\frac{2\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] -1/6*cot(d*x+c)^6/a/d-1/8*cot(d*x+c)^8/a/d+1/3*csc(d*x+c)^3/a/d-2/5*csc(d*x
+c)^5/a/d+1/7*csc(d*x+c)^7/a/d
```

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2835, 2607, 14, 2606, 270}

$$-\frac{\cot^8(c+dx)}{8ad}-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^7(c+dx)}{7ad}-\frac{2\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cot[c + d*x]^6/(6*a*d) - Cot[c + d*x]^8/(8*a*d) + Csc[c + d*x]^3/(3*a*d) -
 (2*Csc[c + d*x]^5)/(5*a*d) + Csc[c + d*x]^7/(7*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\int \frac{\cot^7(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^5(c+dx)\csc^3(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^4(c+dx)\,dx}{a}$$
$$=\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^5\left(1+x^2\right)\,dx,x,-\cot(c+dx)\right)}{ad}$$
$$=\frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(x^5+x^7\right)\,dx,x,-\cot(c+dx)\right)}{ad}$$
$$=-\frac{\cot^6(c+dx)}{6ad}-\frac{\cot^8(c+dx)}{8ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{2\csc^5(c+dx)}{5ad}+\frac{\csc^7(c+dx)}{7ad}$$

Mathematica [A] time = 0.14, size = 68, normalized size = 0.75

$$\frac{\csc^3(c+dx)\left(-105\csc^5(c+dx)+120\csc^4(c+dx)+280\csc^3(c+dx)-336\csc^2(c+dx)-210\csc(c+dx)+280\right)}{840ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^3*(280 - 210*Csc[c + d*x] - 336*Csc[c + d*x]^2 + 280*Csc[c +
d*x]^3 + 120*Csc[c + d*x]^4 - 105*Csc[c + d*x]^5))/(840*a*d)
```

fricas [A] time = 0.48, size = 107, normalized size = 1.18

$$-\frac{210\cos(dx+c)^4-140\cos(dx+c)^2-8\left(35\cos(dx+c)^4-28\cos(dx+c)^2+8\right)\sin(dx+c)+35}{840\left(ad\cos(dx+c)^8-4\,ad\cos(dx+c)^6+6\,ad\cos(dx+c)^4-4\,ad\cos(dx+c)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/840*(210*cos(d*x + c)^4 - 140*cos(d*x + c)^2 - 8*(35*cos(d*x + c)^4 - 28
*cos(d*x + c)^2 + 8)*sin(d*x + c) + 35)/(a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x
 + c)^6 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos(d*x + c)^2 + a*d)
```

giac [A] time = 0.25, size = 66, normalized size = 0.73

$$\frac{280 \sin(dx+c)^5 - 210 \sin(dx+c)^4 - 336 \sin(dx+c)^3 + 280 \sin(dx+c)^2 + 120 \sin(dx+c) - 105}{840 \, ad \sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/840*(280*sin(d*x + c)^5 - 210*sin(d*x + c)^4 - 336*sin(d*x + c)^3 + 280*s
in(d*x + c)^2 + 120*sin(d*x + c) - 105)/(a*d*sin(d*x + c)^8)
```

maple [A] time = 0.53, size = 69, normalized size = 0.76

$$\frac{\frac{1}{3\sin(dx+c)^6} - \frac{2}{5\sin(dx+c)^5} + \frac{1}{7\sin(dx+c)^7} - \frac{1}{8\sin(dx+c)^8} - \frac{1}{4\sin(dx+c)^4} + \frac{1}{3\sin(dx+c)^3}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/3/sin(d*x+c)^6-2/5/sin(d*x+c)^5+1/7/sin(d*x+c)^7-1/8/sin(d*x+c)^8-
1/4/sin(d*x+c)^4+1/3/sin(d*x+c)^3)
```

maxima [A] time = 0.33, size = 66, normalized size = 0.73

$$\frac{280 \sin(dx+c)^5 - 210 \sin(dx+c)^4 - 336 \sin(dx+c)^3 + 280 \sin(dx+c)^2 + 120 \sin(dx+c) - 105}{840 \, ad \sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/840*(280*sin(d*x + c)^5 - 210*sin(d*x + c)^4 - 336*sin(d*x + c)^3 + 280*s
in(d*x + c)^2 + 120*sin(d*x + c) - 105)/(a*d*sin(d*x + c)^8)
```

mupad [B] time = 9.04, size = 66, normalized size = 0.73

$$\frac{280 \sin(c+d\,x)^5 - 210 \sin(c+d\,x)^4 - 336 \sin(c+d\,x)^3 + 280 \sin(c+d\,x)^2 + 120 \sin(c+d\,x) - 105}{840\,a\,d \sin(c+d\,x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (120*sin(c + d*x) + 280*sin(c + d*x)^2 - 336*sin(c + d*x)^3 - 210*sin(c + d
*x)^4 + 280*sin(c + d*x)^5 - 105)/(840*a*d*sin(c + d*x)^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.693 $\int \frac{\cot^7(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\cot^8(c+dx)}{8ad}+\frac{\cot^6(c+dx)}{6ad}-\frac{\csc^9(c+dx)}{9ad}+\frac{2\csc^7(c+dx)}{7ad}-\frac{\csc^5(c+dx)}{5ad}$$

```
[Out] 1/6*cot(d*x+c)^6/a/d+1/8*cot(d*x+c)^8/a/d-1/5*csc(d*x+c)^5/a/d+2/7*csc(d*x+
c)^7/a/d-1/9*csc(d*x+c)^9/a/d
```

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2835, 2606, 270, 2607, 14}

$$\frac{\cot^8(c+dx)}{8ad}+\frac{\cot^6(c+dx)}{6ad}-\frac{\csc^9(c+dx)}{9ad}+\frac{2\csc^7(c+dx)}{7ad}-\frac{\csc^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^6/(6*a*d) + Cot[c + d*x]^8/(8*a*d) - Csc[c + d*x]^5/(5*a*d) +
(2*Csc[c + d*x]^7)/(7*a*d) - Csc[c + d*x]^9/(9*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\int \frac{\cot^7(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^5(c+dx)\csc^4(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^5(c+dx)\,dx}{a}$$
$$= -\frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx, x, \csc(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int x^5\left(1+x^2\right)\,dx, x, -\text{co}$$
$$= \frac{\text{Subst}\left(\int \left(x^5+x^7\right)\,dx, x, -\cot(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx, x, \text{csc}(}{ad}$$
$$= \frac{\cot^6(c+dx)}{6ad}+\frac{\cot^8(c+dx)}{8ad}-\frac{\csc^5(c+dx)}{5ad}+\frac{2\csc^7(c+dx)}{7ad}-\frac{\csc^9(c+dx)}{9ad}$$

Mathematica [A] time = 0.18, size = 68, normalized size = 0.75

$$\frac{\csc^4(c+dx)\left(-280\csc^5(c+dx)+315\csc^4(c+dx)+720\csc^3(c+dx)-840\csc^2(c+dx)-504\csc(c+dx)+630\right.}{2520ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^4*(630 - 504*Csc[c + d*x] - 840*Csc[c + d*x]^2 + 720*Csc[c +
d*x]^3 + 315*Csc[c + d*x]^4 - 280*Csc[c + d*x]^5))/(2520*a*d)
```

fricas [A] time = 0.45, size = 115, normalized size = 1.26

$$-\frac{504\cos(dx+c)^4-288\cos(dx+c)^2-105\left(6\cos(dx+c)^4-4\cos(dx+c)^2+1\right)\sin(dx+c)+64}{2520\left(ad\cos(dx+c)^8-4ad\cos(dx+c)^6+6ad\cos(dx+c)^4-4ad\cos(dx+c)^2+ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2520*(504*cos(d*x + c)^4 - 288*cos(d*x + c)^2 - 105*(6*cos(d*x + c)^4 -
4*cos(d*x + c)^2 + 1)*sin(d*x + c) + 64)/((a*d*cos(d*x + c)^8 - 4*a*d*cos(d
*x + c)^6 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c)
)
```

giac [A] time = 0.23, size = 66, normalized size = 0.73

$$\frac{630 \sin\left(dx+c\right)^{5}-504 \sin\left(dx+c\right)^{4}-840 \sin\left(dx+c\right)^{3}+720 \sin\left(dx+c\right)^{2}+315 \sin\left(dx+c\right)-280}{2520 \, ad \sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2520*(630*sin(d*x + c)^5 - 504*sin(d*x + c)^4 - 840*sin(d*x + c)^3 + 720*
sin(d*x + c)^2 + 315*sin(d*x + c) - 280)/(a*d*sin(d*x + c)^9)
```

maple [A] time = 0.56, size = 69, normalized size = 0.76

$$\frac{-\frac{1}{3 \sin\left(dx+c\right)^{6}}-\frac{1}{5 \sin\left(dx+c\right)^{5}}+\frac{2}{7 \sin\left(dx+c\right)^{7}}-\frac{1}{9 \sin\left(dx+c\right)^{9}}+\frac{1}{8 \sin\left(dx+c\right)^{8}}+\frac{1}{4 \sin\left(dx+c\right)^{4}}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^10/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/3/sin(d*x+c)^6-1/5/sin(d*x+c)^5+2/7/sin(d*x+c)^7-1/9/sin(d*x+c)^9
+1/8/sin(d*x+c)^8+1/4/sin(d*x+c)^4)
```

maxima [A] time = 0.31, size = 66, normalized size = 0.73

$$\frac{630 \sin\left(dx+c\right)^{5}-504 \sin\left(dx+c\right)^{4}-840 \sin\left(dx+c\right)^{3}+720 \sin\left(dx+c\right)^{2}+315 \sin\left(dx+c\right)-280}{2520 \, ad \sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/2520*(630*sin(d*x + c)^5 - 504*sin(d*x + c)^4 - 840*sin(d*x + c)^3 + 720*
sin(d*x + c)^2 + 315*sin(d*x + c) - 280)/(a*d*sin(d*x + c)^9)
```

mupad [B] time = 9.04, size = 65, normalized size = 0.71

$$\frac{\frac{\sin(c+d\,x)^5}{4}-\frac{\sin(c+d\,x)^4}{5}-\frac{\sin(c+d\,x)^3}{3}+\frac{2\sin(c+d\,x)^2}{7}+\frac{\sin(c+d\,x)}{8}-\frac{1}{9}}{a\,d\sin\left(c+d\,x\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^10*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x)/8 + (2*sin(c + d*x)^2)/7 - sin(c + d*x)^3/3 - sin(c + d*x)^4/
5 + sin(c + d*x)^5/4 - 1/9)/(a*d*sin(c + d*x)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**10/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.694 $\int \frac{\cot^7(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\csc^{10}(c+dx)}{10ad}+\frac{\csc^9(c+dx)}{9ad}+\frac{\csc^8(c+dx)}{4ad}-\frac{2\csc^7(c+dx)}{7ad}-\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}$$

```
[Out] 1/5*csc(d*x+c)^5/a/d-1/6*csc(d*x+c)^6/a/d-2/7*csc(d*x+c)^7/a/d+1/4*csc(d*x+
c)^8/a/d+1/9*csc(d*x+c)^9/a/d-1/10*csc(d*x+c)^10/a/d
```

Rubi [A] time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\csc^{10}(c+dx)}{10ad}+\frac{\csc^9(c+dx)}{9ad}+\frac{\csc^8(c+dx)}{4ad}-\frac{2\csc^7(c+dx)}{7ad}-\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^5/(5*a*d) - Csc[c + d*x]^6/(6*a*d) - (2*Csc[c + d*x]^7)/(7*a*d
) + Csc[c + d*x]^8/(4*a*d) + Csc[c + d*x]^9/(9*a*d) - Csc[c + d*x]^10/(10*a
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^7(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^{11}(a-x)^3(a+x)^2}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \left(\frac{a^5}{x^{11}}-\frac{a^4}{x^{10}}-\frac{2a^3}{x^9}+\frac{2a^2}{x^8}+\frac{a}{x^7}-\frac{1}{x^6}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^5(c+dx)}{5ad}-\frac{\csc^6(c+dx)}{6ad}-\frac{2\csc^7(c+dx)}{7ad}+\frac{\csc^8(c+dx)}{4ad}+\frac{\csc^9(c+dx)}{9ad}-$$

Mathematica [A] time = 0.12, size = 68, normalized size = 0.62

$$\frac{\csc^5(c+dx)\left(-126\csc^5(c+dx)+140\csc^4(c+dx)+315\csc^3(c+dx)-360\csc^2(c+dx)-210\csc(c+dx)+252\right.}{1260ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^5*(252 - 210*Csc[c + d*x] - 360*Csc[c + d*x]^2 + 315*Csc[c +
d*x]^3 + 140*Csc[c + d*x]^4 - 126*Csc[c + d*x]^5))/(1260*a*d)
```

fricas [A] time = 0.46, size = 120, normalized size = 1.10

$$\frac{210\cos(dx+c)^4-105\cos(dx+c)^2-4\left(63\cos(dx+c)^4-36\cos(dx+c)^2+8\right)\sin(dx+c)+21}{1260\left(ad\cos(dx+c)^{10}-5ad\cos(dx+c)^8+10ad\cos(dx+c)^6-10ad\cos(dx+c)^4+5ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1260*(210*cos(d*x + c)^4 - 105*cos(d*x + c)^2 - 4*(63*cos(d*x + c)^4 - 36
*cos(d*x + c)^2 + 8)*sin(d*x + c) + 21)/(a*d*cos(d*x + c)^10 - 5*a*d*cos(d*
x + c)^8 + 10*a*d*cos(d*x + c)^6 - 10*a*d*cos(d*x + c)^4 + 5*a*d*cos(d*x +
c)^2 - a*d)
```

giac [A] time = 0.26, size = 66, normalized size = 0.61

$$\frac{252\sin(dx+c)^5-210\sin(dx+c)^4-360\sin(dx+c)^3+315\sin(dx+c)^2+140\sin(dx+c)-126}{1260\,ad\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1260*(252*sin(d*x + c)^5 - 210*sin(d*x + c)^4 - 360*sin(d*x + c)^3 + 315*
sin(d*x + c)^2 + 140*sin(d*x + c) - 126)/(a*d*sin(d*x + c)^10)
```

maple [A] time = 0.58, size = 69, normalized size = 0.63

$$\frac{-\frac{1}{6\sin(dx+c)^6}+\frac{1}{5\sin(dx+c)^5}-\frac{2}{7\sin(dx+c)^7}+\frac{1}{9\sin(dx+c)^9}+\frac{1}{4\sin(dx+c)^8}-\frac{1}{10\sin(dx+c)^{10}}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^11/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/6/sin(d*x+c)^6+1/5/sin(d*x+c)^5-2/7/sin(d*x+c)^7+1/9/sin(d*x+c)^9
+1/4/sin(d*x+c)^8-1/10/sin(d*x+c)^10)
```

maxima [A] time = 0.32, size = 66, normalized size = 0.61

$$\frac{252\sin(dx+c)^5-210\sin(dx+c)^4-360\sin(dx+c)^3+315\sin(dx+c)^2+140\sin(dx+c)-126}{1260\,ad\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/1260*(252*sin(d*x + c)^5 - 210*sin(d*x + c)^4 - 360*sin(d*x + c)^3 + 315*
sin(d*x + c)^2 + 140*sin(d*x + c) - 126)/(a*d*sin(d*x + c)^10)
```

mupad [B] time = 9.08, size = 66, normalized size = 0.61

$$\frac{252\sin(c+d\,x)^5-210\sin(c+d\,x)^4-360\sin(c+d\,x)^3+315\sin(c+d\,x)^2+140\sin(c+d\,x)-126}{1260\,a\,d\sin(c+d\,x)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^11*(a + a*sin(c + d*x))),x)
```

```
[Out] (140*sin(c + d*x) + 315*sin(c + d*x)^2 - 360*sin(c + d*x)^3 - 210*sin(c + d
*x)^4 + 252*sin(c + d*x)^5 - 126)/(1260*a*d*sin(c + d*x)^10)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**11/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.695 $\int \frac{\cot^7(c+dx)\csc^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\csc^{11}(c+dx)}{11ad}+\frac{\csc^{10}(c+dx)}{10ad}+\frac{2\csc^9(c+dx)}{9ad}-\frac{\csc^8(c+dx)}{4ad}-\frac{\csc^7(c+dx)}{7ad}+\frac{\csc^6(c+dx)}{6ad}$$

```
[Out] 1/6*csc(d*x+c)^6/a/d-1/7*csc(d*x+c)^7/a/d-1/4*csc(d*x+c)^8/a/d+2/9*csc(d*x+
c)^9/a/d+1/10*csc(d*x+c)^10/a/d-1/11*csc(d*x+c)^11/a/d
```

Rubi [A] time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\csc^{11}(c+dx)}{11ad}+\frac{\csc^{10}(c+dx)}{10ad}+\frac{2\csc^9(c+dx)}{9ad}-\frac{\csc^8(c+dx)}{4ad}-\frac{\csc^7(c+dx)}{7ad}+\frac{\csc^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^6/(6*a*d) - Csc[c + d*x]^7/(7*a*d) - Csc[c + d*x]^8/(4*a*d) +
(2*Csc[c + d*x]^9)/(9*a*d) + Csc[c + d*x]^10/(10*a*d) - Csc[c + d*x]^11/(11
*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^7(c+dx)\csc^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^{12}(a-x)^3(a+x)^2}{x^{12}}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{a^5\,\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^{12}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{a^5}{x^{12}} - \frac{a^4}{x^{11}} - \frac{2a^3}{x^{10}} + \frac{2a^2}{x^9} + \frac{a}{x^8} - \frac{1}{x^7}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^6(c+dx)}{6ad} - \frac{\csc^7(c+dx)}{7ad} - \frac{\csc^8(c+dx)}{4ad} + \frac{2\csc^9(c+dx)}{9ad} + \frac{\csc^{10}(c+dx)}{10ad} -$$

Mathematica [A] time = 0.13, size = 68, normalized size = 0.62

$$\frac{\csc^6(c+dx)\left(-1260\csc^5(c+dx) + 1386\csc^4(c+dx) + 3080\csc^3(c+dx) - 3465\csc^2(c+dx) - 1980\csc(c+dx)\right.}{13860ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^6*(2310 - 1980*Csc[c + d*x] - 3465*Csc[c + d*x]^2 + 3080*Csc[
c + d*x]^3 + 1386*Csc[c + d*x]^4 - 1260*Csc[c + d*x]^5))/(13860*a*d)
```

fricas [A] time = 0.47, size = 128, normalized size = 1.17

$$\frac{1980\cos(dx+c)^4 - 880\cos(dx+c)^2 - 231\left(10\cos(dx+c)^4 - 5\cos(dx+c)^2 + 1\right)\sin(dx+c) + 16}{13860\left(ad\cos(dx+c)^{10} - 5ad\cos(dx+c)^8 + 10ad\cos(dx+c)^6 - 10ad\cos(dx+c)^4 + 5ad\cos(dx+c)^2 - ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/13860*(1980*cos(d*x + c)^4 - 880*cos(d*x + c)^2 - 231*(10*cos(d*x + c)^4
- 5*cos(d*x + c)^2 + 1)*sin(d*x + c) + 160)/((a*d*cos(d*x + c)^10 - 5*a*d*c
os(d*x + c)^8 + 10*a*d*cos(d*x + c)^6 - 10*a*d*cos(d*x + c)^4 + 5*a*d*cos(d
*x + c)^2 - a*d)*sin(d*x + c))
```

giac [A] time = 0.25, size = 66, normalized size = 0.61

$$\frac{2310\sin(dx+c)^5 - 1980\sin(dx+c)^4 - 3465\sin(dx+c)^3 + 3080\sin(dx+c)^2 + 1386\sin(dx+c) - 1260}{13860\,ad\sin(dx+c)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13860*(2310*sin(d*x + c)^5 - 1980*sin(d*x + c)^4 - 3465*sin(d*x + c)^3 +
3080*sin(d*x + c)^2 + 1386*sin(d*x + c) - 1260)/(a*d*sin(d*x + c)^11)
```

maple [A] time = 0.68, size = 69, normalized size = 0.63

$$\frac{\frac{1}{6\sin(dx+c)^{6}}-\frac{1}{7\sin(dx+c)^{7}}+\frac{2}{9\sin(dx+c)^{9}}-\frac{1}{4\sin(dx+c)^{8}}-\frac{1}{11\sin(dx+c)^{11}}+\frac{1}{10\sin(dx+c)^{10}}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^12/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/6/sin(d*x+c)^6-1/7/sin(d*x+c)^7+2/9/sin(d*x+c)^9-1/4/sin(d*x+c)^8-
1/11/sin(d*x+c)^11+1/10/sin(d*x+c)^10)
```

maxima [A] time = 0.32, size = 66, normalized size = 0.61

$$\frac{2310\,\sin\left(dx+c\right)^{5}-1980\,\sin\left(dx+c\right)^{4}-3465\,\sin\left(dx+c\right)^{3}+3080\,\sin\left(dx+c\right)^{2}+1386\,\sin\left(dx+c\right)-1260}{13860\,ad\sin\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/13860*(2310*sin(d*x + c)^5 - 1980*sin(d*x + c)^4 - 3465*sin(d*x + c)^3 +
3080*sin(d*x + c)^2 + 1386*sin(d*x + c) - 1260)/(a*d*sin(d*x + c)^11)
```

mupad [B] time = 9.04, size = 65, normalized size = 0.60

$$\frac{\frac{\sin(c+d\,x)^5}{6}-\frac{\sin(c+d\,x)^4}{7}-\frac{\sin(c+d\,x)^3}{4}+\frac{2\sin(c+d\,x)^2}{9}+\frac{\sin(c+d\,x)}{10}-\frac{1}{11}}{a\,d\sin\left(c+d\,x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^12*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x)/10 + (2*sin(c + d*x)^2)/9 - sin(c + d*x)^3/4 - sin(c + d*x)^4
/7 + sin(c + d*x)^5/6 - 1/11)/(a*d*sin(c + d*x)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**12/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.696 $\int \frac{\cot^7(c+dx)\csc^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\csc^{12}(c+dx)}{12ad}+\frac{\csc^{11}(c+dx)}{11ad}+\frac{\csc^{10}(c+dx)}{5ad}-\frac{2\csc^9(c+dx)}{9ad}-\frac{\csc^8(c+dx)}{8ad}+\frac{\csc^7(c+dx)}{7ad}$$

```
[Out] 1/7*csc(d*x+c)^7/a/d-1/8*csc(d*x+c)^8/a/d-2/9*csc(d*x+c)^9/a/d+1/5*csc(d*x+
c)^10/a/d+1/11*csc(d*x+c)^11/a/d-1/12*csc(d*x+c)^12/a/d
```

Rubi [A] time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\csc^{12}(c+dx)}{12ad}+\frac{\csc^{11}(c+dx)}{11ad}+\frac{\csc^{10}(c+dx)}{5ad}-\frac{2\csc^9(c+dx)}{9ad}-\frac{\csc^8(c+dx)}{8ad}+\frac{\csc^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^7/(7*a*d) - Csc[c + d*x]^8/(8*a*d) - (2*Csc[c + d*x]^9)/(9*a*d
) + Csc[c + d*x]^10/(5*a*d) + Csc[c + d*x]^11/(11*a*d) - Csc[c + d*x]^12/(1
2*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^7(c+dx)\csc^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^{13}(a-x)^3(a+x)^2}{x^{13}}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^{13}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(\frac{a^5}{x^{13}}-\frac{a^4}{x^{12}}-\frac{2a^3}{x^{11}}+\frac{2a^2}{x^{10}}+\frac{a}{x^9}-\frac{1}{x^8}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^7(c+dx)}{7ad}-\frac{\csc^8(c+dx)}{8ad}-\frac{2\csc^9(c+dx)}{9ad}+\frac{\csc^{10}(c+dx)}{5ad}+\frac{\csc^{11}(c+dx)}{11ad}$$

Mathematica [A] time = 0.13, size = 68, normalized size = 0.62

$$\frac{\csc^7(c+dx)\left(-2310\csc^5(c+dx)+2520\csc^4(c+dx)+5544\csc^3(c+dx)-6160\csc^2(c+dx)-3465\csc(c+dx)\right.}{27720ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^7*(3960 - 3465*Csc[c + d*x] - 6160*Csc[c + d*x]^2 + 5544*Csc[
c + d*x]^3 + 2520*Csc[c + d*x]^4 - 2310*Csc[c + d*x]^5))/(27720*a*d)
```

fricas [A] time = 0.50, size = 131, normalized size = 1.20

$$-\frac{3465\cos(dx+c)^4-1386\cos(dx+c)^2-40\left(99\cos(dx+c)^4-44\cos(dx+c)^2+8\right)\sin(dx+}{27720\left(ad\cos(dx+c)^{12}-6\,ad\cos(dx+c)^{10}+15\,ad\cos(dx+c)^8-20\,ad\cos(dx+c)^6+15\,ad\cos(dx+c)^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/27720*(3465*cos(d*x + c)^4 - 1386*cos(d*x + c)^2 - 40*(99*cos(d*x + c)^4
 - 44*cos(d*x + c)^2 + 8)*sin(d*x + c) + 231)/(a*d*cos(d*x + c)^12 - 6*a*d*
cos(d*x + c)^10 + 15*a*d*cos(d*x + c)^8 - 20*a*d*cos(d*x + c)^6 + 15*a*d*co
s(d*x + c)^4 - 6*a*d*cos(d*x + c)^2 + a*d)
```

giac [A] time = 0.27, size = 66, normalized size = 0.61

$$\frac{3960\sin(dx+c)^5-3465\sin(dx+c)^4-6160\sin(dx+c)^3+5544\sin(dx+c)^2+2520\sin(dx+c)-2310}{27720\,ad\sin(dx+c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/27720*(3960*sin(d*x + c)^5 - 3465*sin(d*x + c)^4 - 6160*sin(d*x + c)^3 +
5544*sin(d*x + c)^2 + 2520*sin(d*x + c) - 2310)/(a*d*sin(d*x + c)^12)
```

maple [A] time = 0.66, size = 69, normalized size = 0.63

$$\frac{\frac{1}{7\sin(dx+c)^7}-\frac{2}{9\sin(dx+c)^9}-\frac{1}{8\sin(dx+c)^8}+\frac{1}{11\sin(dx+c)^{11}}-\frac{1}{12\sin(dx+c)^{12}}+\frac{1}{5\sin(dx+c)^{10}}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^13/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/7/sin(d*x+c)^7-2/9/sin(d*x+c)^9-1/8/sin(d*x+c)^8+1/11/sin(d*x+c)^1
1-1/12/sin(d*x+c)^12+1/5/sin(d*x+c)^10)
```

maxima [A] time = 0.32, size = 66, normalized size = 0.61

$$\frac{3960\sin(dx+c)^5-3465\sin(dx+c)^4-6160\sin(dx+c)^3+5544\sin(dx+c)^2+2520\sin(dx+c)-2310}{27720\,ad\sin(dx+c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/27720*(3960*sin(d*x + c)^5 - 3465*sin(d*x + c)^4 - 6160*sin(d*x + c)^3 +
5544*sin(d*x + c)^2 + 2520*sin(d*x + c) - 2310)/(a*d*sin(d*x + c)^12)
```

mupad [B] time = 9.14, size = 65, normalized size = 0.60

$$\frac{\frac{\sin(c+d\,x)^5}{7}-\frac{\sin(c+d\,x)^4}{8}-\frac{2\sin(c+d\,x)^3}{9}+\frac{\sin(c+d\,x)^2}{5}+\frac{\sin(c+d\,x)}{11}-\frac{1}{12}}{a\,d\sin\left(c+d\,x\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^13*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x)/11 + sin(c + d*x)^2/5 - (2*sin(c + d*x)^3)/9 - sin(c + d*x)^4
/8 + sin(c + d*x)^5/7 - 1/12)/(a*d*sin(c + d*x)^12)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**13/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.697 $\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=184

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}-\frac{8a^3\sin^{n+4}(c+dx)}{d(n+4)}-\frac{6a^3\sin^{n+5}(c+dx)}{d(n+5)}+\frac{6a^3\sin^{n+6}(c+dx)}{d(n+6)}+\frac{8a^3\sin^{n+7}}{d(n+}$$

```
[Out] a^3*sin(d*x+c)^(1+n)/d/(1+n)+3*a^3*sin(d*x+c)^(2+n)/d/(2+n)-8*a^3*sin(d*x+c
)^(4+n)/d/(4+n)-6*a^3*sin(d*x+c)^(5+n)/d/(5+n)+6*a^3*sin(d*x+c)^(6+n)/d/(6+
n)+8*a^3*sin(d*x+c)^(7+n)/d/(7+n)-3*a^3*sin(d*x+c)^(9+n)/d/(9+n)-a^3*sin(d*
x+c)^(10+n)/d/(10+n)
```

Rubi [A] time = 0.18, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 88}

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}-\frac{8a^3\sin^{n+4}(c+dx)}{d(n+4)}-\frac{6a^3\sin^{n+5}(c+dx)}{d(n+5)}+\frac{6a^3\sin^{n+6}(c+dx)}{d(n+6)}+\frac{8a^3\sin^{n+7}}{d(n+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (3*a^3*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - (8*a^3*Sin[c + d*x]^(4 + n))/(d*(4 + n)) - (6*a^3*Sin[c + d*x]^(5
+ n))/(d*(5 + n)) + (6*a^3*Sin[c + d*x]^(6 + n))/(d*(6 + n)) + (8*a^3*Sin[c
 + d*x]^(7 + n))/(d*(7 + n)) - (3*a^3*Sin[c + d*x]^(9 + n))/(d*(9 + n)) - (
a^3*Sin[c + d*x]^(10 + n))/(d*(10 + n))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n(a+x)^6\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^9\left(\frac{x}{a}\right)^n + 3a^9\left(\frac{x}{a}\right)^{1+n} - 8a^9\left(\frac{x}{a}\right)^{3+n} - 6a^9\left(\frac{x}{a}\right)^{4+n} +\right.\right.}{}$$
$$= \frac{a^3\sin^{1+n}(c+dx)}{d(1+n)} + \frac{3a^3\sin^{2+n}(c+dx)}{d(2+n)} - \frac{8a^3\sin^{4+n}(c+dx}{d(4+n)}$$

Mathematica [A] time = 0.96, size = 126, normalized size = 0.68

$$\frac{a^3\sin^{n+1}(c+dx)\left(-\frac{\sin^9(c+dx)}{n+10} - \frac{3\sin^8(c+dx)}{n+9} + \frac{8\sin^6(c+dx)}{n+7} + \frac{6\sin^5(c+dx)}{n+6} - \frac{6\sin^4(c+dx)}{n+5} - \frac{8\sin^3(c+dx)}{n+4} + \frac{3\sin(c+dx)}{n+2} + \frac{1}{n+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (3*Sin[c + d*x])/(2 + n) - (8*Sin
[c + d*x]^3)/(4 + n) - (6*Sin[c + d*x]^4)/(5 + n) + (6*Sin[c + d*x]^5)/(6 +
 n) + (8*Sin[c + d*x]^6)/(7 + n) - (3*Sin[c + d*x]^8)/(9 + n) - Sin[c + d*x
]^9/(10 + n)))/d
```

fricas [B] time = 0.58, size = 697, normalized size = 3.79

$$\frac{\left(\left(a^3n^7 + 34\,a^3n^6 + 472\,a^3n^5 + 3442\,a^3n^4 + 14083\,a^3n^3 + 31804\,a^3n^2 + 35844\,a^3n + 15120\,a^3\right)\cos\left(dx+c\right)^{10} - 5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] ((a^3*n^7 + 34*a^3*n^6 + 472*a^3*n^5 + 3442*a^3*n^4 + 14083*a^3*n^3 + 31804
*a^3*n^2 + 35844*a^3*n + 15120*a^3)*cos(d*x + c)^10 - 5*(a^3*n^7 + 34*a^3*n
^6 + 472*a^3*n^5 + 3442*a^3*n^4 + 14083*a^3*n^3 + 31804*a^3*n^2 + 35844*a^3
*n + 15120*a^3)*cos(d*x + c)^8 + 192*a^3*n^4 + 4*(a^3*n^7 + 28*a^3*n^6 + 30
4*a^3*n^5 + 1618*a^3*n^4 + 4375*a^3*n^3 + 5554*a^3*n^2 + 2520*a^3*n)*cos(d*
x + c)^6 + 4224*a^3*n^3 + 31488*a^3*n^2 + 24*(a^3*n^6 + 24*a^3*n^5 + 208*a^
3*n^4 + 786*a^3*n^3 + 1231*a^3*n^2 + 630*a^3*n)*cos(d*x + c)^4 + 87936*a^3*
n + 60480*a^3 + 96*(a^3*n^5 + 22*a^3*n^4 + 164*a^3*n^3 + 458*a^3*n^2 + 315*
a^3*n)*cos(d*x + c)^2 - (3*(a^3*n^7 + 35*a^3*n^6 + 497*a^3*n^5 + 3689*a^3*n
```

```
^4 + 15302*a^3*n^3 + 34916*a^3*n^2 + 39640*a^3*n + 16800*a^3)*cos(d*x + c)^
8 - 192*a^3*n^4 - 4*(a^3*n^7 + 31*a^3*n^6 + 385*a^3*n^5 + 2485*a^3*n^4 + 89
74*a^3*n^3 + 18004*a^3*n^2 + 18360*a^3*n + 7200*a^3)*cos(d*x + c)^6 - 4224*
a^3*n^3 - 31488*a^3*n^2 - 24*(a^3*n^6 + 26*a^3*n^5 + 255*a^3*n^4 + 1210*a^3
*n^3 + 2924*a^3*n^2 + 3384*a^3*n + 1440*a^3)*cos(d*x + c)^4 - 93696*a^3*n -
 92160*a^3 - 96*(a^3*n^5 + 23*a^3*n^4 + 186*a^3*n^3 + 652*a^3*n^2 + 968*a^3
*n + 480*a^3)*cos(d*x + c)^2)*sin(d*x + c))*sin(d*x + c)^n/(d*n^8 + 44*d*n^
7 + 812*d*n^6 + 8162*d*n^5 + 48503*d*n^4 + 172634*d*n^3 + 353884*d*n^2 + 37
3560*d*n + 151200*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 39.12Not invertible Error:
Bad Argument Value
```

maple [F] time = 41.96, size = 0, normalized size = 0.00

$$\int \left(\cos^{7}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

maxima [A] time = 0.35, size = 165, normalized size = 0.90

$$-\frac{\frac{a^{3}\sin(dx+c)^{n+10}}{n+10}+\frac{3\,a^{3}\sin(dx+c)^{n+9}}{n+9}-\frac{8\,a^{3}\sin(dx+c)^{n+7}}{n+7}-\frac{6\,a^{3}\sin(dx+c)^{n+6}}{n+6}+\frac{6\,a^{3}\sin(dx+c)^{n+5}}{n+5}+\frac{8\,a^{3}\sin(dx+c)^{n+4}}{n+4}-\frac{3\,a^{3}\sin(dx+c)^{n}}{n+2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -(a^3*sin(d*x + c)^(n + 10)/(n + 10) + 3*a^3*sin(d*x + c)^(n + 9)/(n + 9) -
 8*a^3*sin(d*x + c)^(n + 7)/(n + 7) - 6*a^3*sin(d*x + c)^(n + 6)/(n + 6) +
6*a^3*sin(d*x + c)^(n + 5)/(n + 5) + 8*a^3*sin(d*x + c)^(n + 4)/(n + 4) - 3
*a^3*sin(d*x + c)^(n + 2)/(n + 2) - a^3*sin(d*x + c)^(n + 1)/(n + 1))/d
```

mupad [B] time = 17.09, size = 1130, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^n*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (3*a^3*sin(c + d*x)^n*(6117676*n + 3058196*n^2 + 755233*n^3 + 109542*n^4 +
9800*n^5 + 502*n^6 + 11*n^7 + 3714480))/(256*d*(373560*n + 353884*n^2 + 172
634*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151200)) - (5*a^3
*sin(c + d*x)^n*cos(8*c + 8*d*x)*(35844*n + 31804*n^2 + 14083*n^3 + 3442*n^
4 + 472*n^5 + 34*n^6 + n^7 + 15120))/(256*d*(373560*n + 353884*n^2 + 172634
*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151200)) + (a^3*sin(
c + d*x)^n*cos(10*c + 10*d*x)*(35844*n + 31804*n^2 + 14083*n^3 + 3442*n^4 +
 472*n^5 + 34*n^6 + n^7 + 15120))/(512*d*(373560*n + 353884*n^2 + 172634*n^
3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151200)) - (a^3*sin(c +
 d*x)*sin(c + d*x)^n*(n*16168200i + n^2*7143148i + n^3*1614322i + n^4*21508
3i + n^5*18019i + n^6*889i + n^7*19i + 13759200i)*1i)/(128*d*(373560*n + 35
3884*n^2 + 172634*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151
200)) - (3*a^3*sin(c + d*x)^n*cos(6*c + 6*d*x)*(1320260*n + 1100668*n^2 + 4
46515*n^3 + 97426*n^4 + 11608*n^5 + 706*n^6 + 17*n^7 + 579600))/(512*d*(373
560*n + 353884*n^2 + 172634*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 +
 n^8 + 151200)) - (a^3*sin(c + d*x)^n*cos(4*c + 4*d*x)*(1729500*n + 1246276
*n^2 + 413653*n^3 + 71710*n^4 + 6760*n^5 + 334*n^6 + 7*n^7 + 831600))/(64*d
*(373560*n + 353884*n^2 + 172634*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*
n^7 + n^8 + 151200)) + (a^3*sin(c + d*x)^n*cos(2*c + 2*d*x)*(122059*n^3 - 2
395364*n^2 - 9293340*n + 119842*n^4 + 17176*n^5 + 1042*n^6 + 25*n^7 - 68796
00))/(256*d*(373560*n + 353884*n^2 + 172634*n^3 + 48503*n^4 + 8162*n^5 + 81
2*n^6 + 44*n^7 + n^8 + 151200)) + (a^3*sin(c + d*x)^n*sin(9*c + 9*d*x)*(n*3
9640i + n^2*34916i + n^3*15302i + n^4*3689i + n^5*497i + n^6*35i + n^7*1i +
 16800i)*3i)/(256*d*(373560*n + 353884*n^2 + 172634*n^3 + 48503*n^4 + 8162*
n^5 + 812*n^6 + 44*n^7 + n^8 + 151200)) - (a^3*sin(c + d*x)^n*sin(5*c + 5*d
*x)*(n*97464i + n^2*117044i + n^3*66110i + n^4*18845i + n^5*2741i + n^6*191
i + n^7*5i + 30240i)*1i)/(64*d*(373560*n + 353884*n^2 + 172634*n^3 + 48503*
n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151200)) + (a^3*sin(c + d*x)^n*si
n(7*c + 7*d*x)*(n*538680i + n^2*445172i + n^3*177758i + n^4*37709i + n^5*42
77i + n^6*239i + n^7*5i + 237600i)*1i)/(256*d*(373560*n + 353884*n^2 + 1726
34*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151200)) - (a^3*si
n(c + d*x)^n*sin(3*c + 3*d*x)*(n*763320i + n^2*586164i + n^3*211966i + n^4*
40253i + n^5*4149i + n^6*223i + n^7*5i + 352800i)*3i)/(64*d*(373560*n + 353
884*n^2 + 172634*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 1512
00))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.698 $\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=184

$$\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2\sin^{n+2}(c+dx)}{d(n+2)}-\frac{2a^2\sin^{n+3}(c+dx)}{d(n+3)}-\frac{6a^2\sin^{n+4}(c+dx)}{d(n+4)}+\frac{6a^2\sin^{n+6}(c+dx)}{d(n+6)}+\frac{2a^2\sin^{n+7}(c}{d(n+7}$$

```
[Out] a^2*sin(d*x+c)^(1+n)/d/(1+n)+2*a^2*sin(d*x+c)^(2+n)/d/(2+n)-2*a^2*sin(d*x+c
)^(3+n)/d/(3+n)-6*a^2*sin(d*x+c)^(4+n)/d/(4+n)+6*a^2*sin(d*x+c)^(6+n)/d/(6+
n)+2*a^2*sin(d*x+c)^(7+n)/d/(7+n)-2*a^2*sin(d*x+c)^(8+n)/d/(8+n)-a^2*sin(d*
x+c)^(9+n)/d/(9+n)
```

Rubi [A] time = 0.18, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 88}

$$\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2\sin^{n+2}(c+dx)}{d(n+2)}-\frac{2a^2\sin^{n+3}(c+dx)}{d(n+3)}-\frac{6a^2\sin^{n+4}(c+dx)}{d(n+4)}+\frac{6a^2\sin^{n+6}(c+dx)}{d(n+6)}+\frac{2a^2\sin^{n+7}(c}{d(n+7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a^2*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - (2*a^2*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (6*a^2*Sin[c + d*x]^(4
+ n))/(d*(4 + n)) + (6*a^2*Sin[c + d*x]^(6 + n))/(d*(6 + n)) + (2*a^2*Sin[c
 + d*x]^(7 + n))/(d*(7 + n)) - (2*a^2*Sin[c + d*x]^(8 + n))/(d*(8 + n)) - (
a^2*Sin[c + d*x]^(9 + n))/(d*(9 + n))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n (a+x)^5\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^8\left(\frac{x}{a}\right)^n + 2a^8\left(\frac{x}{a}\right)^{1+n} - 2a^8\left(\frac{x}{a}\right)^{2+n} - 6a^8\left(\frac{x}{a}\right)^{3+n}\right.\right.}{}$$
$$= \frac{a^2\sin^{1+n}(c+dx)}{d(1+n)} + \frac{2a^2\sin^{2+n}(c+dx)}{d(2+n)} - \frac{2a^2\sin^{3+n}(c+a}{d(3+n)}$$

Mathematica [A] time = 0.78, size = 126, normalized size = 0.68

$$\frac{a^2\sin^{n+1}(c+dx)\left(-\frac{\sin^8(c+dx)}{n+9} - \frac{2\sin^7(c+dx)}{n+8} + \frac{2\sin^6(c+dx)}{n+7} + \frac{6\sin^5(c+dx)}{n+6} - \frac{6\sin^3(c+dx)}{n+4} - \frac{2\sin^2(c+dx)}{n+3} + \frac{2\sin(c+dx)}{n+2} + \frac{1}{n+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (2*Sin[c + d*x])/(2 + n) - (2*Sin
[c + d*x]^2)/(3 + n) - (6*Sin[c + d*x]^3)/(4 + n) + (6*Sin[c + d*x]^5)/(6 +
 n) + (2*Sin[c + d*x]^6)/(7 + n) - (2*Sin[c + d*x]^7)/(8 + n) - Sin[c + d*x
]^8/(9 + n)))/d
```

fricas [B] time = 0.57, size = 628, normalized size = 3.41

$$-\frac{\left(2\left(a^2n^7 + 32\,a^2n^6 + 414\,a^2n^5 + 2788\,a^2n^4 + 10469\,a^2n^3 + 21708\,a^2n^2 + 22716\,a^2n + 9072\,a^2\right)\cos\left(dx+c\right)^8 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(2*(a^2*n^7 + 32*a^2*n^6 + 414*a^2*n^5 + 2788*a^2*n^4 + 10469*a^2*n^3 + 21
708*a^2*n^2 + 22716*a^2*n + 9072*a^2)*cos(d*x + c)^8 - 2*(a^2*n^7 + 26*a^2*
n^6 + 258*a^2*n^5 + 1240*a^2*n^4 + 3029*a^2*n^3 + 3534*a^2*n^2 + 1512*a^2*n
)*cos(d*x + c)^6 - 96*a^2*n^4 - 1920*a^2*n^3 - 12*(a^2*n^6 + 22*a^2*n^5 + 1
70*a^2*n^4 + 560*a^2*n^3 + 789*a^2*n^2 + 378*a^2*n)*cos(d*x + c)^4 - 12480*
a^2*n^2 - 28800*a^2*n - 48*(a^2*n^5 + 20*a^2*n^4 + 130*a^2*n^3 + 300*a^2*n^
2 + 189*a^2*n)*cos(d*x + c)^2 - 18144*a^2 + ((a^2*n^7 + 31*a^2*n^6 + 391*a^
2*n^5 + 2581*a^2*n^4 + 9544*a^2*n^3 + 19564*a^2*n^2 + 20304*a^2*n + 8064*a^
2)*cos(d*x + c)^8 - 2*(a^2*n^7 + 29*a^2*n^6 + 343*a^2*n^5 + 2135*a^2*n^4 +
```

```
7504*a^2*n^3 + 14756*a^2*n^2 + 14832*a^2*n + 5760*a^2)*cos(d*x + c)^6 - 96*
a^2*n^4 - 1920*a^2*n^3 - 12*(a^2*n^6 + 24*a^2*n^5 + 223*a^2*n^4 + 1020*a^2*
n^3 + 2404*a^2*n^2 + 2736*a^2*n + 1152*a^2)*cos(d*x + c)^4 - 13440*a^2*n^2
- 38400*a^2*n - 48*(a^2*n^5 + 21*a^2*n^4 + 160*a^2*n^3 + 540*a^2*n^2 + 784*
a^2*n + 384*a^2)*cos(d*x + c)^2 - 36864*a^2)*sin(d*x + c))*sin(d*x + c)^n/(
d*n^8 + 40*d*n^7 + 670*d*n^6 + 6100*d*n^5 + 32773*d*n^4 + 105460*d*n^3 + 19
6380*d*n^2 + 190800*d*n + 72576*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 28.83, size = 0, normalized size = 0.00

$$\int \left(\cos^{7}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

maxima [A] time = 0.32, size = 165, normalized size = 0.90

$$-\frac{\frac{a^{2}\sin(dx+c)^{n+9}}{n+9}+\frac{2\,a^{2}\sin(dx+c)^{n+8}}{n+8}-\frac{2\,a^{2}\sin(dx+c)^{n+7}}{n+7}-\frac{6\,a^{2}\sin(dx+c)^{n+6}}{n+6}+\frac{6\,a^{2}\sin(dx+c)^{n+4}}{n+4}+\frac{2\,a^{2}\sin(dx+c)^{n+3}}{n+3}-\frac{2\,a^{2}\sin(dx+c)^{n+2}}{n+2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -(a^2*sin(d*x + c)^(n + 9)/(n + 9) + 2*a^2*sin(d*x + c)^(n + 8)/(n + 8) - 2
*a^2*sin(d*x + c)^(n + 7)/(n + 7) - 6*a^2*sin(d*x + c)^(n + 6)/(n + 6) + 6*
a^2*sin(d*x + c)^(n + 4)/(n + 4) + 2*a^2*sin(d*x + c)^(n + 3)/(n + 3) - 2*a
^2*sin(d*x + c)^(n + 2)/(n + 2) - a^2*sin(d*x + c)^(n + 1)/(n + 1))/d
```

mupad [B] time = 16.45, size = 1142, normalized size = 6.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^n*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x)^n*(n*1507788i + n^2*868332i + n^3*238585i + n^4*37844i +
n^5*3702i + n^6*208i + n^7*5i + 843696i))/(64*d*(n*190800i + n^2*196380i +
n^3*105460i + n^4*32773i + n^5*6100i + n^6*670i + n^7*40i + n^8*1i + 72576i
)) - (a^2*sin(c + d*x)^n*sin(9*c + 9*d*x)*(20304*n + 19564*n^2 + 9544*n^3 +
 2581*n^4 + 391*n^5 + 31*n^6 + n^7 + 8064)*1i)/(256*d*(n*190800i + n^2*1963
80i + n^3*105460i + n^4*32773i + n^5*6100i + n^6*670i + n^7*40i + n^8*1i +
72576i)) + (a^2*sin(c + d*x)*sin(c + d*x)^n*(6799248*n + 3169500*n^2 + 7706
32*n^3 + 111993*n^4 + 10267*n^5 + 555*n^6 + 13*n^7 + 5588352)*1i)/(128*d*(n
*190800i + n^2*196380i + n^3*105460i + n^4*32773i + n^5*6100i + n^6*670i +
n^7*40i + n^8*1i + 72576i)) - (a^2*sin(c + d*x)^n*cos(8*c + 8*d*x)*(n*22716
i + n^2*21708i + n^3*10469i + n^4*2788i + n^5*414i + n^6*32i + n^7*1i + 907
2i))/(64*d*(n*190800i + n^2*196380i + n^3*105460i + n^4*32773i + n^5*6100i
+ n^6*670i + n^7*40i + n^8*1i + 72576i)) - (a^2*sin(c + d*x)^n*cos(6*c + 6*
d*x)*(n*43920i + n^2*39882i + n^3*17909i + n^4*4336i + n^5*570i + n^6*38i +
 n^7*1i + 18144i))/(16*d*(n*190800i + n^2*196380i + n^3*105460i + n^4*32773
i + n^5*6100i + n^6*670i + n^7*40i + n^8*1i + 72576i)) - (a^2*sin(c + d*x)^
n*cos(4*c + 4*d*x)*(n*140868i + n^2*111816i + n^3*41669i + n^4*7996i + n^5*
822i + n^6*44i + n^7*1i + 63504i))/(16*d*(n*190800i + n^2*196380i + n^3*105
460i + n^4*32773i + n^5*6100i + n^6*670i + n^7*40i + n^8*1i + 72576i)) + (a
^2*sin(c + d*x)^n*cos(2*c + 2*d*x)*(n^3*2549i - n^2*59958i - n*186480i + n^
4*3568i + n^5*570i + n^6*38i + n^7*1i - 127008i))/(16*d*(n*190800i + n^2*19
6380i + n^3*105460i + n^4*32773i + n^5*6100i + n^6*670i + n^7*40i + n^8*1i
+ 72576i)) - (a^2*sin(c + d*x)^n*sin(7*c + 7*d*x)*(23472*n + 18900*n^2 + 67
76*n^3 + 987*n^4 - 7*n^5 - 15*n^6 - n^7 + 10368)*1i)/(256*d*(n*190800i + n^
2*196380i + n^3*105460i + n^4*32773i + n^5*6100i + n^6*670i + n^7*40i + n^8
*1i + 72576i)) + (a^2*sin(c + d*x)^n*sin(5*c + 5*d*x)*(178128*n + 165132*n^
2 + 76280*n^3 + 19149*n^4 + 2627*n^5 + 183*n^6 + 5*n^7 + 72576)*1i)/(64*d*(
n*190800i + n^2*196380i + n^3*105460i + n^4*32773i + n^5*6100i + n^6*670i +
 n^7*40i + n^8*1i + 72576i)) + (a^2*sin(c + d*x)^n*sin(3*c + 3*d*x)*(112094
4*n + 889556*n^2 + 338024*n^3 + 68603*n^4 + 7661*n^5 + 449*n^6 + 11*n^7 + 5
08032)*1i)/(64*d*(n*190800i + n^2*196380i + n^3*105460i + n^4*32773i + n^5*
6100i + n^6*670i + n^7*40i + n^8*1i + 72576i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.699 $\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=167

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}-\frac{3a\sin^{n+3}(c+dx)}{d(n+3)}-\frac{3a\sin^{n+4}(c+dx)}{d(n+4)}+\frac{3a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{3a\sin^{n+6}(c+dx)}{d(n+6)}-$$

```
[Out] a*sin(d*x+c)^(1+n)/d/(1+n)+a*sin(d*x+c)^(2+n)/d/(2+n)-3*a*sin(d*x+c)^(3+n)/
d/(3+n)-3*a*sin(d*x+c)^(4+n)/d/(4+n)+3*a*sin(d*x+c)^(5+n)/d/(5+n)+3*a*sin(d
*x+c)^(6+n)/d/(6+n)-a*sin(d*x+c)^(7+n)/d/(7+n)-a*sin(d*x+c)^(8+n)/d/(8+n)
```

Rubi [A] time = 0.14, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2836, 88}

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}-\frac{3a\sin^{n+3}(c+dx)}{d(n+3)}-\frac{3a\sin^{n+4}(c+dx)}{d(n+4)}+\frac{3a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{3a\sin^{n+6}(c+dx)}{d(n+6)}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (a*Sin[c + d*x]^(2 + n))/(d*(2 + n))
 - (3*a*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (3*a*Sin[c + d*x]^(4 + n))/(d*(
4 + n)) + (3*a*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (3*a*Sin[c + d*x]^(6 + n
))/(d*(6 + n)) - (a*Sin[c + d*x]^(7 + n))/(d*(7 + n)) - (a*Sin[c + d*x]^(8
+ n))/(d*(8 + n))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n (a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(a^7\left(\frac{x}{a}\right)^n + a^7\left(\frac{x}{a}\right)^{1+n} - 3a^7\left(\frac{x}{a}\right)^{2+n} - 3a^7\left(\frac{x}{a}\right)^{3+n} + \right.\right.}{}$$

$$= \frac{a\sin^{1+n}(c+dx)}{d(1+n)} + \frac{a\sin^{2+n}(c+dx)}{d(2+n)} - \frac{3a\sin^{3+n}(c+dx)}{d(3+n)} -$$

Mathematica [B] time = 3.24, size = 659, normalized size = 3.95

$$\frac{a\sin^{n+1}(c+dx)\left(5n^7\sin(c+dx) + 9n^7\sin(3(c+dx)) + 5n^7\sin(5(c+dx)) + n^7\sin(7(c+dx)) + 2n^7\cos(6(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n)*(1755648 + 2521536*n + 1486096*n^2 + 481280*n^3 + 9
4012*n^4 + 11084*n^5 + 724*n^6 + 20*n^7 + 6*(114816 + 262064*n + 219828*n^2
 + 90640*n^3 + 20499*n^4 + 2611*n^5 + 177*n^6 + 5*n^7)*Cos[2*(c + d*x)] + 1
2*(10368 + 25776*n + 24372*n^2 + 11584*n^3 + 3027*n^4 + 439*n^5 + 33*n^6 +
n^7)*Cos[4*(c + d*x)] + 11520*Cos[6*(c + d*x)] + 29664*n*Cos[6*(c + d*x)] +
 29512*n^2*Cos[6*(c + d*x)] + 15008*n^3*Cos[6*(c + d*x)] + 4270*n^4*Cos[6*(
c + d*x)] + 686*n^5*Cos[6*(c + d*x)] + 58*n^6*Cos[6*(c + d*x)] + 2*n^7*Cos[
6*(c + d*x)] + 468720*Sin[c + d*x] + 879324*n*Sin[c + d*x] + 552236*n^2*Sin
[c + d*x] + 167669*n^3*Sin[c + d*x] + 28904*n^4*Sin[c + d*x] + 3050*n^5*Sin
[c + d*x] + 188*n^6*Sin[c + d*x] + 5*n^7*Sin[c + d*x] + 186480*Sin[3*(c + d
*x)] + 439836*n*Sin[3*(c + d*x)] + 384948*n^2*Sin[3*(c + d*x)] + 165273*n^3
*Sin[3*(c + d*x)] + 38232*n^4*Sin[3*(c + d*x)] + 4866*n^5*Sin[3*(c + d*x)]
+ 324*n^6*Sin[3*(c + d*x)] + 9*n^7*Sin[3*(c + d*x)] + 45360*Sin[5*(c + d*x)
] + 114252*n*Sin[5*(c + d*x)] + 110036*n^2*Sin[5*(c + d*x)] + 53525*n^3*Sin
[5*(c + d*x)] + 14360*n^4*Sin[5*(c + d*x)] + 2138*n^5*Sin[5*(c + d*x)] + 16
4*n^6*Sin[5*(c + d*x)] + 5*n^7*Sin[5*(c + d*x)] + 5040*Sin[7*(c + d*x)] + 1
3068*n*Sin[7*(c + d*x)] + 13132*n^2*Sin[7*(c + d*x)] + 6769*n^3*Sin[7*(c +
d*x)] + 1960*n^4*Sin[7*(c + d*x)] + 322*n^5*Sin[7*(c + d*x)] + 28*n^6*Sin[7
*(c + d*x)] + n^7*Sin[7*(c + d*x)]))/(64*d*(1 + n)*(2 + n)*(3 + n)*(4 + n)*
(5 + n)*(6 + n)*(7 + n)*(8 + n))
```

fricas [B] time = 0.55, size = 445, normalized size = 2.66

$$-\frac{\left(\left(an^7 + 28\,an^6 + 322\,an^5 + 1960\,an^4 + 6769\,an^3 + 13132\,an^2 + 13068\,an + 5040\,a\right)\cos(dx+c)^8 - \left(an^7 + 22\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((a*n^7 + 28*a*n^6 + 322*a*n^5 + 1960*a*n^4 + 6769*a*n^3 + 13132*a*n^2 + 1
3068*a*n + 5040*a)*cos(d*x + c)^8 - (a*n^7 + 22*a*n^6 + 190*a*n^5 + 820*a*n
^4 + 1849*a*n^3 + 2038*a*n^2 + 840*a*n)*cos(d*x + c)^6 - 48*a*n^4 - 6*(a*n^
6 + 18*a*n^5 + 118*a*n^4 + 348*a*n^3 + 457*a*n^2 + 210*a*n)*cos(d*x + c)^4
- 768*a*n^3 - 4128*a*n^2 - 24*(a*n^5 + 16*a*n^4 + 86*a*n^3 + 176*a*n^2 + 10
5*a*n)*cos(d*x + c)^2 - 8448*a*n - ((a*n^7 + 29*a*n^6 + 343*a*n^5 + 2135*a*
n^4 + 7504*a*n^3 + 14756*a*n^2 + 14832*a*n + 5760*a)*cos(d*x + c)^6 + 48*a*
n^4 + 6*(a*n^6 + 24*a*n^5 + 223*a*n^4 + 1020*a*n^3 + 2404*a*n^2 + 2736*a*n
+ 1152*a)*cos(d*x + c)^4 + 960*a*n^3 + 6720*a*n^2 + 24*(a*n^5 + 21*a*n^4 +
160*a*n^3 + 540*a*n^2 + 784*a*n + 384*a)*cos(d*x + c)^2 + 19200*a*n + 18432
*a)*sin(d*x + c) - 5040*a)*sin(d*x + c)^n/(d*n^8 + 36*d*n^7 + 546*d*n^6 + 4
536*d*n^5 + 22449*d*n^4 + 67284*d*n^3 + 118124*d*n^2 + 109584*d*n + 40320*d
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 17.35, size = 0, normalized size = 0.00

$$\int \left(\cos^{7}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

maxima [A] time = 0.32, size = 148, normalized size = 0.89

$$-\frac{\frac{a\sin(dx+c)^{n+8}}{n+8}+\frac{a\sin(dx+c)^{n+7}}{n+7}-\frac{3\,a\sin(dx+c)^{n+6}}{n+6}-\frac{3\,a\sin(dx+c)^{n+5}}{n+5}+\frac{3\,a\sin(dx+c)^{n+4}}{n+4}+\frac{3\,a\sin(dx+c)^{n+3}}{n+3}-\frac{a\sin(dx+c)^{n+2}}{n+2}-\frac{a\sin(dx}{n}\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*sin(d*x + c)^(n + 8)/(n + 8) + a*sin(d*x + c)^(n + 7)/(n + 7) - 3*a*sin
(d*x + c)^(n + 6)/(n + 6) - 3*a*sin(d*x + c)^(n + 5)/(n + 5) + 3*a*sin(d*x
+ c)^(n + 4)/(n + 4) + 3*a*sin(d*x + c)^(n + 3)/(n + 3) - a*sin(d*x + c)^(n
 + 2)/(n + 2) - a*sin(d*x + c)^(n + 1)/(n + 1))/d
```

mupad [B] time = 15.90, size = 901, normalized size = 5.40

$$\frac{a\sin(c+d\,x)^n\,\left(5\,n^7+188\,n^6+3050\,n^5+28904\,n^4+167669\,n^3+552236\,n^2+879324\,n+468720\right)}{128\,d\,\left(n^8+36\,n^7+546\,n^6+4536\,n^5+22449\,n^4+67284\,n^3+118124\,n^2+109584\,n+40320\right)}-\frac{a\sin(c}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^n*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x)^n*(879324*n + 552236*n^2 + 167669*n^3 + 28904*n^4 + 3050*n^
5 + 188*n^6 + 5*n^7 + 468720))/(128*d*(109584*n + 118124*n^2 + 67284*n^3 +
22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c + d*x)^n
*cos(2*c + 2*d*x)*(109872*n + 41822*n^2 + 599*n^3 - 2332*n^4 - 454*n^5 - 34
*n^6 - n^7 + 70560))/(32*d*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 +
 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c + d*x)^n*sin(7*c +
7*d*x)*(n*14832i + n^2*14756i + n^3*7504i + n^4*2135i + n^5*343i + n^6*29i
+ n^7*1i + 5760i)*1i)/(64*d*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4
+ 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c + d*x)^n*sin(5*c +
 5*d*x)*(n*139824i + n^2*131476i + n^3*62000i + n^4*16027i + n^5*2291i + n^
6*169i + n^7*5i + 56448i)*1i)/(64*d*(109584*n + 118124*n^2 + 67284*n^3 + 22
449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c + d*x)^n*s
in(3*c + 3*d*x)*(n*210512i + n^2*171084i + n^3*67472i + n^4*14445i + n^5*17
33i + n^6*111i + n^7*3i + 94080i)*3i)/(64*d*(109584*n + 118124*n^2 + 67284*
n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c +
d*x)^n*cos(8*c + 8*d*x)*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^
5 + 28*n^6 + n^7 + 5040))/(128*d*(109584*n + 118124*n^2 + 67284*n^3 + 22449
*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c + d*x)^n*cos(
6*c + 6*d*x)*(25296*n + 24226*n^2 + 11689*n^3 + 3100*n^4 + 454*n^5 + 34*n^6
 + n^7 + 10080))/(32*d*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 453
6*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c + d*x)^n*cos(4*c + 4*d*
x)*(81396*n + 68728*n^2 + 27937*n^3 + 5968*n^4 + 682*n^5 + 40*n^6 + n^7 + 3
5280))/(32*d*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 54
6*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c + d*x)*sin(c + d*x)^n*(n*1735344i
 + n^2*826612i + n^3*209360i + n^4*32515i + n^5*3251i + n^6*193i + n^7*5i +
 1411200i)*1i)/(64*d*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*
n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.700 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$\frac{\sin^{n+1}(c+dx)}{ad(n+1)}-\frac{\sin^{n+2}(c+dx)}{ad(n+2)}-\frac{2\sin^{n+3}(c+dx)}{ad(n+3)}+\frac{2\sin^{n+4}(c+dx)}{ad(n+4)}+\frac{\sin^{n+5}(c+dx)}{ad(n+5)}-\frac{\sin^{n+6}(c+dx)}{ad(n+6)}$$

```
[Out] sin(d*x+c)^(1+n)/a/d/(1+n)-sin(d*x+c)^(2+n)/a/d/(2+n)-2*sin(d*x+c)^(3+n)/a/
d/(3+n)+2*sin(d*x+c)^(4+n)/a/d/(4+n)+sin(d*x+c)^(5+n)/a/d/(5+n)-sin(d*x+c)^
(6+n)/a/d/(6+n)
```

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 88}

$$\frac{\sin^{n+1}(c+dx)}{ad(n+1)}-\frac{\sin^{n+2}(c+dx)}{ad(n+2)}-\frac{2\sin^{n+3}(c+dx)}{ad(n+3)}+\frac{2\sin^{n+4}(c+dx)}{ad(n+4)}+\frac{\sin^{n+5}(c+dx)}{ad(n+5)}-\frac{\sin^{n+6}(c+dx)}{ad(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a*d*(1 + n)) - Sin[c + d*x]^(2 + n)/(a*d*(2 + n)) - (
2*Sin[c + d*x]^(3 + n))/(a*d*(3 + n)) + (2*Sin[c + d*x]^(4 + n))/(a*d*(4 +
n)) + Sin[c + d*x]^(5 + n)/(a*d*(5 + n)) - Sin[c + d*x]^(6 + n)/(a*d*(6 + n
))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n (a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5\left(\frac{x}{a}\right)^n - a^5\left(\frac{x}{a}\right)^{1+n} - 2a^5\left(\frac{x}{a}\right)^{2+n} + 2a^5\left(\frac{x}{a}\right)^{3+n} + a^5\left(\frac{x}{a}\right)^{4+n} - a^5\left(\frac{x}{a}\right)^{5\cdots}\right.\right.}{a^7 d}$$
$$= \frac{\sin^{1+n}(c+dx)}{ad(1+n)} - \frac{\sin^{2+n}(c+dx)}{ad(2+n)} - \frac{2\sin^{3+n}(c+dx)}{ad(3+n)} + \frac{2\sin^{4+n}(c+dx)}{ad(4+n)} + \frac{\sin^{5}\cdots}{ad\cdots}$$

Mathematica [A] time = 0.33, size = 95, normalized size = 0.69

$$\frac{\sin^{n+1}(c+dx)\left(-\frac{\sin^5(c+dx)}{n+6} + \frac{\sin^4(c+dx)}{n+5} + \frac{2\sin^3(c+dx)}{n+4} - \frac{2\sin^2(c+dx)}{n+3} - \frac{\sin(c+dx)}{n+2} + \frac{1}{n+1}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*((1 + n)^(-1) - Sin[c + d*x]/(2 + n) - (2*Sin[c + d*x
]^2)/(3 + n) + (2*Sin[c + d*x]^3)/(4 + n) + Sin[c + d*x]^4/(5 + n) - Sin[c
+ d*x]^5/(6 + n)))/(a*d)
```

fricas [A] time = 0.49, size = 243, normalized size = 1.77

$$\frac{\left(\left(n^5 + 15\,n^4 + 85\,n^3 + 225\,n^2 + 274\,n + 120\right)\cos\left(dx+c\right)^6 - \left(n^5 + 11\,n^4 + 41\,n^3 + 61\,n^2 + 30\,n\right)\cos\left(dx+c\right)^4 - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*cos(d*x + c)^6 - (n^5 + 11
*n^4 + 41*n^3 + 61*n^2 + 30*n)*cos(d*x + c)^4 - 8*n^3 - 4*(n^4 + 9*n^3 + 23
*n^2 + 15*n)*cos(d*x + c)^2 - 72*n^2 + ((n^5 + 16*n^4 + 95*n^3 + 260*n^2 +
324*n + 144)*cos(d*x + c)^4 + 8*n^3 + 4*(n^4 + 13*n^3 + 56*n^2 + 92*n + 48)
*cos(d*x + c)^2 + 96*n^2 + 352*n + 384)*sin(d*x + c) - 184*n - 120)*sin(d*x
 + c)^n/(a*d*n^6 + 21*a*d*n^5 + 175*a*d*n^4 + 735*a*d*n^3 + 1624*a*d*n^2 +
1764*a*d*n + 720*a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^n \cos\left(dx+c\right)^7}{a\sin\left(dx+c\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a), x)
```

maple [F] time = 9.34, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{7}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)}{a+a\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

maxima [A] time = 0.37, size = 241, normalized size = 1.76

$$-\frac{\left(\left(n^{5}+15\,n^{4}+85\,n^{3}+225\,n^{2}+274\,n+120\right)\sin\left(dx+c\right)^{6}-\left(n^{5}+16\,n^{4}+95\,n^{3}+260\,n^{2}+324\,n+144\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*sin(d*x + c)^6 - (n^5 + 1
6*n^4 + 95*n^3 + 260*n^2 + 324*n + 144)*sin(d*x + c)^5 - 2*(n^5 + 17*n^4 +
107*n^3 + 307*n^2 + 396*n + 180)*sin(d*x + c)^4 + 2*(n^5 + 18*n^4 + 121*n^3
 + 372*n^2 + 508*n + 240)*sin(d*x + c)^3 + (n^5 + 19*n^4 + 137*n^3 + 461*n^
2 + 702*n + 360)*sin(d*x + c)^2 - (n^5 + 20*n^4 + 155*n^3 + 580*n^2 + 1044*
n + 720)*sin(d*x + c))*sin(d*x + c)^n/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 +
1624*n^2 + 1764*n + 720)*a*d)
```

mupad [B] time = 14.29, size = 568, normalized size = 4.15

$$\frac{\sin\left(c+d\,x\right)^{n}\,\cos\left(6\,c+6\,d\,x\right)\,\left(n^{5}+15\,n^{4}+85\,n^{3}+225\,n^{2}+274\,n+120\right)}{32\,a\,d\,\left(n^{6}+21\,n^{5}+175\,n^{4}+735\,n^{3}+1624\,n^{2}+1764\,n+720\right)}-\frac{\sin\left(c+d\,x\right)^{n}\,\left(4\,n^{5}+92\,n^{4}+948\right.}{64\,a\,d\,\left(n^{6}+21\,n^{5}+175\,n^{4}+735\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^n)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^n*cos(6*c + 6*d*x)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 +
 120))/(32*a*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)
) - (sin(c + d*x)^n*(8936*n + 4516*n^2 + 948*n^3 + 92*n^4 + 4*n^5 + 5280))/
```

```
(64*a*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (si
n(c + d*x)*sin(c + d*x)^n*(n*15504i + n^2*5904i + n^3*1052i + n^4*96i + n^5
*4i + 14400i)*1i)/(32*a*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 +
 n^6 + 720)) + (sin(c + d*x)^n*cos(4*c + 4*d*x)*(1524*n + 1106*n^2 + 346*n^
3 + 46*n^4 + 2*n^5 + 720))/(32*a*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 +
 21*n^5 + n^6 + 720)) + (sin(c + d*x)^n*cos(2*c + 2*d*x)*(2670*n + 927*n^2
+ 43*n^3 - 15*n^4 - n^5 + 1800))/(32*a*d*(1764*n + 1624*n^2 + 735*n^3 + 175
*n^4 + 21*n^5 + n^6 + 720)) - (sin(c + d*x)^n*sin(5*c + 5*d*x)*(n*648i + n^
2*520i + n^3*190i + n^4*32i + n^5*2i + 288i)*1i)/(32*a*d*(1764*n + 1624*n^2
 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (sin(c + d*x)^n*sin(3*c + 3*d
*x)*(n*4888i + n^2*3352i + n^3*986i + n^4*128i + n^5*6i + 2400i)*1i)/(32*a*
d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.701 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=92

$$\frac{\sin^{n+1}(c+dx)}{a^2d(n+1)}-\frac{2\sin^{n+2}(c+dx)}{a^2d(n+2)}+\frac{2\sin^{n+4}(c+dx)}{a^2d(n+4)}-\frac{\sin^{n+5}(c+dx)}{a^2d(n+5)}$$

```
[Out] sin(d*x+c)^(1+n)/a^2/d/(1+n)-2*sin(d*x+c)^(2+n)/a^2/d/(2+n)+2*sin(d*x+c)^(4
+n)/a^2/d/(4+n)-sin(d*x+c)^(5+n)/a^2/d/(5+n)
```

Rubi [A] time = 0.14, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 75}

$$\frac{\sin^{n+1}(c+dx)}{a^2d(n+1)}-\frac{2\sin^{n+2}(c+dx)}{a^2d(n+2)}+\frac{2\sin^{n+4}(c+dx)}{a^2d(n+4)}-\frac{\sin^{n+5}(c+dx)}{a^2d(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a^2*d*(1 + n)) - (2*Sin[c + d*x]^(2 + n))/(a^2*d*(2 +
 n)) + (2*Sin[c + d*x]^(4 + n))/(a^2*d*(4 + n)) - Sin[c + d*x]^(5 + n)/(a^2
*d*(5 + n))
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n(a+x)\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4\left(\frac{x}{a}\right)^n - 2a^4\left(\frac{x}{a}\right)^{1+n} + 2a^4\left(\frac{x}{a}\right)^{3+n} - a^4\left(\frac{x}{a}\right)^{4+n}\right)dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\sin^{1+n}(c+dx)}{a^2 d(1+n)} - \frac{2\sin^{2+n}(c+dx)}{a^2 d(2+n)} + \frac{2\sin^{4+n}(c+dx)}{a^2 d(4+n)} - \frac{\sin^{5+n}(c+dx)}{a^2 d(5+n)}$$

Mathematica [A] time = 0.36, size = 117, normalized size = 1.27

$$\frac{\sin^{n+1}(c+dx)\left(-\left(\left(n^3+7n^2+14n+8\right)\sin^4(c+dx)\right)+2\left(n^3+8n^2+17n+10\right)\sin^3(c+dx)-2\left(n^3+10n^2+2\right.\right.}{a^2 d(n+1)(n+2)(n+4)(n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(40 + 38*n + 11*n^2 + n^3 - 2*(20 + 29*n + 10*n^2 + n
^3)*Sin[c + d*x] + 2*(10 + 17*n + 8*n^2 + n^3)*Sin[c + d*x]^3 - (8 + 14*n +
 7*n^2 + n^3)*Sin[c + d*x]^4))/(a^2*d*(1 + n)*(2 + n)*(4 + n)*(5 + n))
```

fricas [A] time = 0.47, size = 169, normalized size = 1.84

$$\frac{\left(2\left(n^3+8n^2+17n+10\right)\cos(dx+c)^4-2\left(n^3+6n^2+5n\right)\cos(dx+c)^2-4n^2-\left(\left(n^3+7n^2+14n+8\right)\cos(d\right.\right.}{a^2dn^4+12a^2dn^3+49a^2dn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (2*(n^3 + 8*n^2 + 17*n + 10)*cos(d*x + c)^4 - 2*(n^3 + 6*n^2 + 5*n)*cos(d*x
 + c)^2 - 4*n^2 - ((n^3 + 7*n^2 + 14*n + 8)*cos(d*x + c)^4 - 2*(n^3 + 7*n^2
 + 14*n + 8)*cos(d*x + c)^2 - 4*n^2 - 24*n - 32)*sin(d*x + c) - 24*n - 20)*
sin(d*x + c)^n/(a^2*d*n^4 + 12*a^2*d*n^3 + 49*a^2*d*n^2 + 78*a^2*d*n + 40*a
^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n\cos(dx+c)^7}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a)^2, x)
```

maple [F] time = 22.12, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{7}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)}{\left(a+a\sin\left(dx+c\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

maxima [A] time = 0.37, size = 126, normalized size = 1.37

$$-\frac{\left(\left(n^{3}+7\,n^{2}+14\,n+8\right)\sin\left(dx+c\right)^{5}-2\left(n^{3}+8\,n^{2}+17\,n+10\right)\sin\left(dx+c\right)^{4}+2\left(n^{3}+10\,n^{2}+29\,n+20\right)\sin}{\left(n^{4}+12\,n^{3}+49\,n^{2}+78\,n+40\right)a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -((n^3 + 7*n^2 + 14*n + 8)*sin(d*x + c)^5 - 2*(n^3 + 8*n^2 + 17*n + 10)*sin
(d*x + c)^4 + 2*(n^3 + 10*n^2 + 29*n + 20)*sin(d*x + c)^2 - (n^3 + 11*n^2 +
 38*n + 40)*sin(d*x + c))*sin(d*x + c)^n/((n^4 + 12*n^3 + 49*n^2 + 78*n + 4
0)*a^2*d)
```

mupad [B] time = 11.26, size = 280, normalized size = 3.04

$$\frac{\sin\left(c+d\,x\right)^{n}\,\left(560\,\sin\left(c+d\,x\right)-260\,n+160\,\cos\left(2\,c+2\,d\,x\right)+40\,\cos\left(4\,c+4\,d\,x\right)+40\,\sin\left(3\,c+3\,d\,x\right)-8\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^n)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^n*(560*sin(c + d*x) - 260*n + 160*cos(2*c + 2*d*x) + 40*cos(4
*c + 4*d*x) + 40*sin(3*c + 3*d*x) - 8*sin(5*c + 5*d*x) + 468*n*sin(c + d*x)
 + 192*n*cos(2*c + 2*d*x) + 68*n*cos(4*c + 4*d*x) + 70*n*sin(3*c + 3*d*x) -
 14*n*sin(5*c + 5*d*x) + 106*n^2*sin(c + d*x) + 6*n^3*sin(c + d*x) - 64*n^2
 - 4*n^3 + 32*n^2*cos(2*c + 2*d*x) + 32*n^2*cos(4*c + 4*d*x) + 4*n^3*cos(4*
```

```
c + 4*d*x) + 35*n^2*sin(3*c + 3*d*x) + 5*n^3*sin(3*c + 3*d*x) - 7*n^2*sin(5
*c + 5*d*x) - n^3*sin(5*c + 5*d*x) - 200))/(16*a^2*d*(78*n + 49*n^2 + 12*n^
3 + n^4 + 40))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.702 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=92

$$\frac{\sin^{n+1}(c+dx)}{a^3d(n+1)}-\frac{3\sin^{n+2}(c+dx)}{a^3d(n+2)}+\frac{3\sin^{n+3}(c+dx)}{a^3d(n+3)}-\frac{\sin^{n+4}(c+dx)}{a^3d(n+4)}$$

```
[Out] sin(d*x+c)^(1+n)/a^3/d/(1+n)-3*sin(d*x+c)^(2+n)/a^3/d/(2+n)+3*sin(d*x+c)^(3
+n)/a^3/d/(3+n)-sin(d*x+c)^(4+n)/a^3/d/(4+n)
```

Rubi [A] time = 0.14, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 43}

$$\frac{\sin^{n+1}(c+dx)}{a^3d(n+1)}-\frac{3\sin^{n+2}(c+dx)}{a^3d(n+2)}+\frac{3\sin^{n+3}(c+dx)}{a^3d(n+3)}-\frac{\sin^{n+4}(c+dx)}{a^3d(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a^3*d*(1 + n)) - (3*Sin[c + d*x]^(2 + n))/(a^3*d*(2 +
 n)) + (3*Sin[c + d*x]^(3 + n))/(a^3*d*(3 + n)) - Sin[c + d*x]^(4 + n)/(a^3
*d*(4 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int (a-x)^3 \left(\frac{x}{a}\right)^n\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3\left(\frac{x}{a}\right)^n - 3a^3\left(\frac{x}{a}\right)^{1+n} + 3a^3\left(\frac{x}{a}\right)^{2+n} - a^3\left(\frac{x}{a}\right)^{3+n}\right)dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\sin^{1+n}(c+dx)}{a^3 d(1+n)} - \frac{3\sin^{2+n}(c+dx)}{a^3 d(2+n)} + \frac{3\sin^{3+n}(c+dx)}{a^3 d(3+n)} - \frac{\sin^{4+n}(c+dx)}{a^3 d(4+n)}$$

Mathematica [A] time = 0.20, size = 66, normalized size = 0.72

$$\frac{\sin^{n+1}(c+dx)\left(-\frac{\sin^3(c+dx)}{n+4} + \frac{3\sin^2(c+dx)}{n+3} - \frac{3\sin(c+dx)}{n+2} + \frac{1}{n+1}\right)}{a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*((1 + n)^(-1) - (3*Sin[c + d*x])/(2 + n) + (3*Sin[c +
 d*x]^2)/(3 + n) - Sin[c + d*x]^3/(4 + n)))/(a^3*d)
```

fricas [A] time = 0.50, size = 160, normalized size = 1.74

$$-\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)\cos\left(dx+c\right)^4+4\,n^3-\left(5\,n^3+36\,n^2+79\,n+48\right)\cos\left(dx+c\right)^2+30\,n^2-\left(4\,n^3-3\,\left(n^3+\right.\right.}{a^3dn^4+10\,a^3dn^3+35\,a^3dn^2+50\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -((n^3 + 6*n^2 + 11*n + 6)*cos(d*x + c)^4 + 4*n^3 - (5*n^3 + 36*n^2 + 79*n
+ 48)*cos(d*x + c)^2 + 30*n^2 - (4*n^3 - 3*(n^3 + 7*n^2 + 14*n + 8)*cos(d*x
 + c)^2 + 30*n^2 + 68*n + 48)*sin(d*x + c) + 68*n + 42)*sin(d*x + c)^n/(a^3
*d*n^4 + 10*a^3*d*n^3 + 35*a^3*d*n^2 + 50*a^3*d*n + 24*a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^n\cos\left(dx+c\right)^7}{\left(a\sin\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a)^3, x)
```

maple [F] time = 5.84, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{7}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)}{\left(a+a\sin\left(dx+c\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

maxima [A] time = 0.37, size = 126, normalized size = 1.37

$$-\frac{\left(\left(n^{3}+6\,n^{2}+11\,n+6\right)\sin\left(dx+c\right)^{4}-3\left(n^{3}+7\,n^{2}+14\,n+8\right)\sin\left(dx+c\right)^{3}+3\left(n^{3}+8\,n^{2}+19\,n+12\right)\sin\left(d\right.\right.}{\left(n^{4}+10\,n^{3}+35\,n^{2}+50\,n+24\right)a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -((n^3 + 6*n^2 + 11*n + 6)*sin(d*x + c)^4 - 3*(n^3 + 7*n^2 + 14*n + 8)*sin(
d*x + c)^3 + 3*(n^3 + 8*n^2 + 19*n + 12)*sin(d*x + c)^2 - (n^3 + 9*n^2 + 26
*n + 24)*sin(d*x + c))*sin(d*x + c)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*
a^3*d)
```

mupad [B] time = 10.61, size = 242, normalized size = 2.63

$$-\frac{\sin\left(c+d\,x\right)^{n}\,\left(261\,n-336\,\sin\left(c+d\,x\right)-168\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+48\,\sin\left(3\,c+3\,d\,x\right)-46\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^n)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -(sin(c + d*x)^n*(261*n - 336*sin(c + d*x) - 168*cos(2*c + 2*d*x) + 6*cos(4
*c + 4*d*x) + 48*sin(3*c + 3*d*x) - 460*n*sin(c + d*x) - 272*n*cos(2*c + 2*
d*x) + 11*n*cos(4*c + 4*d*x) + 84*n*sin(3*c + 3*d*x) - 198*n^2*sin(c + d*x)
 - 26*n^3*sin(c + d*x) + 114*n^2 + 15*n^3 - 120*n^2*cos(2*c + 2*d*x) - 16*n
^3*cos(2*c + 2*d*x) + 6*n^2*cos(4*c + 4*d*x) + n^3*cos(4*c + 4*d*x) + 42*n^
2*sin(3*c + 3*d*x) + 6*n^3*sin(3*c + 3*d*x) + 162))/(8*a^3*d*(50*n + 35*n^2
 + 10*n^3 + n^4 + 24))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.703 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=109

$$\frac{8\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^4d(n+1)}-\frac{7\sin^{n+1}(c+dx)}{a^4d(n+1)}+\frac{4\sin^{n+2}(c+dx)}{a^4d(n+2)}-\frac{\sin^{n+3}(c+dx)}{a^4d(n+3)}$$

```
[Out] -7*sin(d*x+c)^(1+n)/a^4/d/(1+n)+8*hypergeom([1, 1+n],[2+n],-sin(d*x+c))*sin
(d*x+c)^(1+n)/a^4/d/(1+n)+4*sin(d*x+c)^(2+n)/a^4/d/(2+n)-sin(d*x+c)^(3+n)/a
^4/d/(3+n)
```

Rubi [A] time = 0.17, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2836, 88, 43, 64}

$$\frac{8\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^4d(n+1)}-\frac{7\sin^{n+1}(c+dx)}{a^4d(n+1)}+\frac{4\sin^{n+2}(c+dx)}{a^4d(n+2)}-\frac{\sin^{n+3}(c+dx)}{a^4d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-7*Sin[c + d*x]^(1 + n))/(a^4*d*(1 + n)) + (8*Hypergeometric2F1[1, 1 + n,
2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a^4*d*(1 + n)) + (4*Sin[c + d*
x]^(2 + n))/(a^4*d*(2 + n)) - Sin[c + d*x]^(3 + n)/(a^4*d*(3 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(-4a^2\left(\frac{x}{a}\right)^n - 2a(a-x)\left(\frac{x}{a}\right)^n - (a-x)^2\left(\frac{x}{a}\right)^n + \frac{8a^3\left(\frac{x}{a}\right)^n}{a+x}\right) dx, x, a\sin(c +$$

$$= -\frac{4\sin^{1+n}(c+dx)}{a^4 d(1+n)} - \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n\,dx, x, a\sin(c+dx)\right)}{a^7 d} - \frac{2\,\text{Subst}\left(\int (\right.}{}$$

$$= -\frac{4\sin^{1+n}(c+dx)}{a^4 d(1+n)} + \frac{8\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4 d(1+n)} - \frac{\text{Subs}}{}$$

$$= -\frac{7\sin^{1+n}(c+dx)}{a^4 d(1+n)} + \frac{8\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4 d(1+n)} + \frac{4\sin}{a^4}$$

Mathematica [A] time = 0.27, size = 104, normalized size = 0.95

$$\frac{\frac{8a^3\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{n+1} - \frac{7a^3\sin^{n+1}(c+dx)}{n+1} + \frac{4a^3\sin^{n+2}(c+dx)}{n+2} - \frac{a^3\sin^{n+3}(c+dx)}{n+3}}{a^7 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] ((-7*a^3*Sin[c + d*x]^(1 + n))/(1 + n) + (8*a^3*Hypergeometric2F1[1, 1 + n,
 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(1 + n) + (4*a^3*Sin[c + d*x]^
(2 + n))/(2 + n) - (a^3*Sin[c + d*x]^(3 + n))/(3 + n))/(a^7*d)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)^{7}}{a^{4}\cos\left(dx+c\right)^{4}-8\,a^{4}\cos\left(dx+c\right)^{2}+8\,a^{4}-4\,\left(a^{4}\cos\left(dx+c\right)^{2}-2\,a^{4}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^7/(a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x
+ c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)^{7}}{\left(a\sin\left(dx+c\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a)^4, x)
```

maple [F] time = 9.07, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{7}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)}{\left(a+a\sin\left(dx+c\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)^{7}}{\left(a\sin\left(dx+c\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^7 \sin(c+dx)^n}{(a+a\sin(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^n)/(a + a*sin(c + d*x))^4,x)
[Out] int((cos(c + d*x)^7*sin(c + d*x)^n)/(a + a*sin(c + d*x))^4, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c))**4,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.704 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^5}\,dx$

Optimal. Leaf size=160

$$-\frac{(a-a\sin(c+dx))^2\sin^{n+1}(c+dx)}{d(n+2)\left(a^7\sin(c+dx)+a^7\right)}+\frac{\sin^{n+1}(c+dx)\left(a(2n+7)\sin(c+dx)+a\left(8n^2+30n+27\right)\right)}{d\left(n^2+3n+2\right)\left(a^6\sin(c+dx)+a^6\right)}-\frac{4(2n+3)\sin$$

```
[Out] -4*(3+2*n)*hypergeom([1, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(1+n)/a^5/d/(1+
n)-sin(d*x+c)^(1+n)*(a-a*sin(d*x+c))^2/d/(2+n)/(a^7+a^7*sin(d*x+c))+sin(d*x
+c)^(1+n)*(a*(8*n^2+30*n+27)+a*(7+2*n)*sin(d*x+c))/d/(n^2+3*n+2)/(a^6+a^6*s
in(d*x+c))
```

Rubi [A] time = 0.20, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2836, 100, 146, 64}

$$-\frac{4(2n+3)\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^5d(n+1)}+\frac{\sin^{n+1}(c+dx)\left(a(2n+7)\sin(c+dx)+a\left(8n^2+3\right.\right.}{d\left(n^2+3n+2\right)\left(a^6\sin(c+dx)+a^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^5,x]
```

```
[Out] (-4*(3 + 2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x
]^(1 + n))/(a^5*d*(1 + n)) - (Sin[c + d*x]^(1 + n)*(a - a*Sin[c + d*x])^2)/
(d*(2 + n)*(a^7 + a^7*Sin[c + d*x])) + (Sin[c + d*x]^(1 + n)*(a*(27 + 30*n
+ 8*n^2) + a*(7 + 2*n)*Sin[c + d*x]))/(d*(2 + 3*n + n^2)*(a^6 + a^6*Sin[c +
 d*x]))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^5}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3\left(\frac{x}{a}\right)^n}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$

$$= -\frac{\sin^{1+n}(c+dx)(a-a\sin(c+dx))^2}{d(2+n)\left(a^7+a^7\sin(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{(a-x)\left(\frac{x}{a}\right)^n(a(3+2n)+(-7-2n)x)}{(a+x)^2}\,dx, x,\right.}{a^6 d(2+n)}$$

$$= -\frac{\sin^{1+n}(c+dx)(a-a\sin(c+dx))^2}{d(2+n)\left(a^7+a^7\sin(c+dx)\right)} + \frac{\sin^{1+n}(c+dx)\left(a\left(27+30n+8n^2\right)+a(\right.}{d(1+n)(2+n)\left(a^6+a^6\sin(\right.}$$

$$= -\frac{4(3+2n)\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^5 d(1+n)} - \frac{\sin^{1+n}(c+dx)(}{d(2+n)\left(a^7+\right.}$$

Mathematica [A] time = 0.22, size = 108, normalized size = 0.68

$$\frac{\sin^{n+1}(c+dx)\left(-4\left(2n^2+7n+6\right)(\sin(c+dx)+1)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))-(n+1)\sin^2(c+dx)+(4\ldots\right.}{a^5d(n+1)(n+2)(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^5,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(26 + 29*n + 8*n^2 + (9 + 4*n)*Sin[c + d*x] - (1 + n)
*Sin[c + d*x]^2 - 4*(6 + 7*n + 2*n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, -S
in[c + d*x]]*(1 + Sin[c + d*x])))/(a^5*d*(1 + n)*(2 + n)*(1 + Sin[c + d*x])
)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sin(dx+c)^n\cos(dx+c)^7}{5\,a^5\cos(dx+c)^4-20\,a^5\cos(dx+c)^2+16\,a^5+\left(a^5\cos(dx+c)^4-12\,a^5\cos(dx+c)^2+16\,a^5\right)\sin(d\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x, algorithm="fricas
")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^7/(5*a^5*cos(d*x + c)^4 - 20*a^5*cos(d
*x + c)^2 + 16*a^5 + (a^5*cos(d*x + c)^4 - 12*a^5*cos(d*x + c)^2 + 16*a^5)*
sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n\cos(dx+c)^7}{(a\sin(dx+c)+a)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a)^5, x)
```

maple [F] time = 2.80, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^7(dx+c)\right)(\sin^n(dx+c))}{(a+a\sin(dx+c))^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)^{7}}{\left(a\sin\left(dx+c\right)+a\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^7\,{\sin\left(c+d\,x\right)}^n}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^n)/(a + a*sin(c + d*x))^5,x)
```

```
[Out] int((cos(c + d*x)^7*sin(c + d*x)^n)/(a + a*sin(c + d*x))^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

3.705 $\int \frac{\cos^8(c+dx)\sin^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=209

$$-\frac{\cos^{11}(c+dx)}{11ad}+\frac{2\cos^9(c+dx)}{9ad}-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin^5(c+dx)\cos^7(c+dx)}{12ad}+\frac{\sin^3(c+dx)\cos^7(c+dx)}{24ad}+\frac{\sin(c+d}{}$$

```
[Out] -5/1024*x/a-1/7*cos(d*x+c)^7/a/d+2/9*cos(d*x+c)^9/a/d-1/11*cos(d*x+c)^11/a/
d-5/1024*cos(d*x+c)*sin(d*x+c)/a/d-5/1536*cos(d*x+c)^3*sin(d*x+c)/a/d-1/384
*cos(d*x+c)^5*sin(d*x+c)/a/d+1/64*cos(d*x+c)^7*sin(d*x+c)/a/d+1/24*cos(d*x+
c)^7*sin(d*x+c)^3/a/d+1/12*cos(d*x+c)^7*sin(d*x+c)^5/a/d
```

Rubi [A] time = 0.28, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2565, 270, 2568, 2635, 8}

$$-\frac{\cos^{11}(c+dx)}{11ad}+\frac{2\cos^9(c+dx)}{9ad}-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin^5(c+dx)\cos^7(c+dx)}{12ad}+\frac{\sin^3(c+dx)\cos^7(c+dx)}{24ad}+\frac{\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*x)/(1024*a) - Cos[c + d*x]^7/(7*a*d) + (2*Cos[c + d*x]^9)/(9*a*d) - Cos
[c + d*x]^11/(11*a*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(1024*a*d) - (5*Cos[c
 + d*x]^3*Sin[c + d*x])/(1536*a*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(384*a*d
) + (Cos[c + d*x]^7*Sin[c + d*x])/(64*a*d) + (Cos[c + d*x]^7*Sin[c + d*x]^3
)/(24*a*d) + (Cos[c + d*x]^7*Sin[c + d*x]^5)/(12*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^6(c+dx)\sin^5(c+dx)\,dx}{a} - \frac{\int \cos^6(c+dx)\sin^6(c+dx)\,dx}{a}$$
$$= \frac{\cos^7(c+dx)\sin^5(c+dx)}{12ad} - \frac{5\int \cos^6(c+dx)\sin^4(c+dx)\,dx}{12a} - \frac{\text{Subst}\left(\int x^6\left(\right.\right.}{}$$
$$= \frac{\cos^7(c+dx)\sin^3(c+dx)}{24ad} + \frac{\cos^7(c+dx)\sin^5(c+dx)}{12ad} - \frac{\int \cos^6(c+dx)\sin^2(}{8a}$$
$$= -\frac{\cos^7(c+dx)}{7ad} + \frac{2\cos^9(c+dx)}{9ad} - \frac{\cos^{11}(c+dx)}{11ad} + \frac{\cos^7(c+dx)\sin(c+dx)}{64ad}$$
$$= -\frac{\cos^7(c+dx)}{7ad} + \frac{2\cos^9(c+dx)}{9ad} - \frac{\cos^{11}(c+dx)}{11ad} - \frac{\cos^5(c+dx)\sin(c+dx)}{384ad}$$
$$= -\frac{\cos^7(c+dx)}{7ad} + \frac{2\cos^9(c+dx)}{9ad} - \frac{\cos^{11}(c+dx)}{11ad} - \frac{5\cos^3(c+dx)\sin(c+dx)}{1536ad}$$
$$= -\frac{\cos^7(c+dx)}{7ad} + \frac{2\cos^9(c+dx)}{9ad} - \frac{\cos^{11}(c+dx)}{11ad} - \frac{5\cos(c+dx)\sin(c+dx)}{1024ad}$$
$$= -\frac{5x}{1024a} - \frac{\cos^7(c+dx)}{7ad} + \frac{2\cos^9(c+dx)}{9ad} - \frac{\cos^{11}(c+dx)}{11ad} - \frac{5\cos(c+dx)\sin}{1024ad}$$

Mathematica [B] time = 14.28, size = 518, normalized size = 2.48

$$-\frac{55440dx\sin\left(\frac{c}{2}\right) - 55440\sin\left(\frac{c}{2}+dx\right) + 55440\sin\left(\frac{3c}{2}+dx\right) - 18480\sin\left(\frac{5c}{2}+3dx\right) + 18480\sin\left(\frac{7c}{2}+3dx\right) -}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/11354112*(55440*d*x*Cos[c/2] + 55440*Cos[c/2 + d*x] + 55440*Cos[(3*c)/2
+ d*x] + 18480*Cos[(5*c)/2 + 3*d*x] + 18480*Cos[(7*c)/2 + 3*d*x] - 10395*Co
s[(7*c)/2 + 4*d*x] + 10395*Cos[(9*c)/2 + 4*d*x] - 5544*Cos[(9*c)/2 + 5*d*x]
 - 5544*Cos[(11*c)/2 + 5*d*x] - 3960*Cos[(13*c)/2 + 7*d*x] - 3960*Cos[(15*c
)/2 + 7*d*x] + 2079*Cos[(15*c)/2 + 8*d*x] - 2079*Cos[(17*c)/2 + 8*d*x] + 61
6*Cos[(17*c)/2 + 9*d*x] + 616*Cos[(19*c)/2 + 9*d*x] + 504*Cos[(21*c)/2 + 11
*d*x] + 504*Cos[(23*c)/2 + 11*d*x] - 231*Cos[(23*c)/2 + 12*d*x] + 231*Cos[(
25*c)/2 + 12*d*x] + 99792*Sin[c/2] + 55440*d*x*Sin[c/2] - 55440*Sin[c/2 + d
*x] + 55440*Sin[(3*c)/2 + d*x] - 18480*Sin[(5*c)/2 + 3*d*x] + 18480*Sin[(7*
c)/2 + 3*d*x] - 10395*Sin[(7*c)/2 + 4*d*x] - 10395*Sin[(9*c)/2 + 4*d*x] + 5
544*Sin[(9*c)/2 + 5*d*x] - 5544*Sin[(11*c)/2 + 5*d*x] + 3960*Sin[(13*c)/2 +
 7*d*x] - 3960*Sin[(15*c)/2 + 7*d*x] + 2079*Sin[(15*c)/2 + 8*d*x] + 2079*Si
```

```
n[(17*c)/2 + 8*d*x] - 616*Sin[(17*c)/2 + 9*d*x] + 616*Sin[(19*c)/2 + 9*d*x]
 - 504*Sin[(21*c)/2 + 11*d*x] + 504*Sin[(23*c)/2 + 11*d*x] - 231*Sin[(23*c)
/2 + 12*d*x] - 231*Sin[(25*c)/2 + 12*d*x])/(a*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.48, size = 110, normalized size = 0.53

$$-\frac{64512 \cos(dx+c)^{11} - 157696 \cos(dx+c)^{9} + 101376 \cos(dx+c)^{7} + 3465\,dx - 231\left(256 \cos(dx+c)^{11} - 640\right.}{709632\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/709632*(64512*cos(d*x + c)^11 - 157696*cos(d*x + c)^9 + 101376*cos(d*x +
 c)^7 + 3465*d*x - 231*(256*cos(d*x + c)^11 - 640*cos(d*x + c)^9 + 432*cos(
d*x + c)^7 - 8*cos(d*x + c)^5 - 10*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*
x + c))/(a*d)
```

giac [A] time = 0.23, size = 309, normalized size = 1.48

$$-\frac{\frac{3465\,(dx+c)}{a} + \frac{2\left(3465 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{23} + 40425 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{21} + 215523 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{19} + 3784704 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{18} - 5794173 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/709632*(3465*(d*x + c)/a + 2*(3465*tan(1/2*d*x + 1/2*c)^23 + 40425*tan(1
/2*d*x + 1/2*c)^21 + 215523*tan(1/2*d*x + 1/2*c)^19 + 3784704*tan(1/2*d*x +
 1/2*c)^18 - 5794173*tan(1/2*d*x + 1/2*c)^17 - 5677056*tan(1/2*d*x + 1/2*c)
^16 + 19523658*tan(1/2*d*x + 1/2*c)^15 + 11354112*tan(1/2*d*x + 1/2*c)^14 -
 35058870*tan(1/2*d*x + 1/2*c)^13 + 3784704*tan(1/2*d*x + 1/2*c)^12 + 35058
870*tan(1/2*d*x + 1/2*c)^11 - 4866048*tan(1/2*d*x + 1/2*c)^10 - 19523658*ta
n(1/2*d*x + 1/2*c)^9 + 9732096*tan(1/2*d*x + 1/2*c)^8 + 5794173*tan(1/2*d*x
 + 1/2*c)^7 - 1982464*tan(1/2*d*x + 1/2*c)^6 - 215523*tan(1/2*d*x + 1/2*c)^
5 + 540672*tan(1/2*d*x + 1/2*c)^4 - 40425*tan(1/2*d*x + 1/2*c)^3 + 98304*ta
n(1/2*d*x + 1/2*c)^2 - 3465*tan(1/2*d*x + 1/2*c) + 8192)/((tan(1/2*d*x + 1/
2*c)^2 + 1)^12*a))/d
```

maple [B] time = 0.34, size = 755, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c)),x)

[Out] -16/693/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12+5/512/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)-64/231/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^2+175/1536/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^3-32/21/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^4+311/512/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^5+352/63/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^6-8361/512/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^7-192/7/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^8+42259/768/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^9+96/7/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^10-25295/256/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^11-32/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^12+25295/256/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^13-32/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^14-42259/768/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^15+16/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^16+8361/512/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^17-32/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^18-311/512/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^19-175/1536/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^21-5/512/a/d/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^23-5/512/a/d*arctan(tan(1/2*d*x+1/2*c))

maxima [B] time = 0.44, size = 705, normalized size = 3.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/354816*((3465*sin(d*x + c)/(cos(d*x + c) + 1) - 98304*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 40425*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 540672*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 215523*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1982464*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 5794173*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 9732096*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 19523658*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 4866048*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 35058870*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 3784704*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 35058870*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 11354112*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 19523658*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + 5677056*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 + 5794173*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 3784704*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 - 215523*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 - 40425*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 - 3465*sin(d*x + c)^23/(cos(d*x + c) + 1)^23 - 8192)/(a + 12*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 66*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 220*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 495*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 792*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 924*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 792*a*sin(d

```
*x + c)^14/(cos(d*x + c) + 1)^14 + 495*a*sin(d*x + c)^16/(cos(d*x + c) + 1)
^16 + 220*a*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + 66*a*sin(d*x + c)^20/(c
os(d*x + c) + 1)^20 + 12*a*sin(d*x + c)^22/(cos(d*x + c) + 1)^22 + a*sin(d*
x + c)^24/(cos(d*x + c) + 1)^24) - 3465*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a)/d
```

mupad [B] time = 11.85, size = 303, normalized size = 1.45

$$-\frac{5\,x}{1024\,a}-\frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{23}}{512}+\frac{175\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{21}}{1536}+\frac{311\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}}{512}+\frac{32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}}{3}-\frac{8361\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{512}-16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^5)/(a + a*sin(c + d*x)),x)
```

```
[Out] - (5*x)/(1024*a) - ((64*tan(c/2 + (d*x)/2)^2)/231 - (5*tan(c/2 + (d*x)/2))/
512 - (175*tan(c/2 + (d*x)/2)^3)/1536 + (32*tan(c/2 + (d*x)/2)^4)/21 - (311
*tan(c/2 + (d*x)/2)^5)/512 - (352*tan(c/2 + (d*x)/2)^6)/63 + (8361*tan(c/2
+ (d*x)/2)^7)/512 + (192*tan(c/2 + (d*x)/2)^8)/7 - (42259*tan(c/2 + (d*x)/2
)^9)/768 - (96*tan(c/2 + (d*x)/2)^10)/7 + (25295*tan(c/2 + (d*x)/2)^11)/256
 + (32*tan(c/2 + (d*x)/2)^12)/3 - (25295*tan(c/2 + (d*x)/2)^13)/256 + 32*ta
n(c/2 + (d*x)/2)^14 + (42259*tan(c/2 + (d*x)/2)^15)/768 - 16*tan(c/2 + (d*x
)/2)^16 - (8361*tan(c/2 + (d*x)/2)^17)/512 + (32*tan(c/2 + (d*x)/2)^18)/3 +
 (311*tan(c/2 + (d*x)/2)^19)/512 + (175*tan(c/2 + (d*x)/2)^21)/1536 + (5*ta
n(c/2 + (d*x)/2)^23)/512 + 16/693)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^12)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.706 $\int \frac{\cos^8(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=183

$$\frac{\cos^{11}(c+dx)}{11ad}-\frac{2\cos^9(c+dx)}{9ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{\sin^3(c+dx)\cos^7(c+dx)}{10ad}-\frac{3\sin(c+dx)\cos^7(c+dx)}{80ad}+\frac{\sin(c+dx)}{1}$$

```
[Out] 3/256*x/a+1/7*cos(d*x+c)^7/a/d-2/9*cos(d*x+c)^9/a/d+1/11*cos(d*x+c)^11/a/d+
3/256*cos(d*x+c)*sin(d*x+c)/a/d+1/128*cos(d*x+c)^3*sin(d*x+c)/a/d+1/160*cos
(d*x+c)^5*sin(d*x+c)/a/d-3/80*cos(d*x+c)^7*sin(d*x+c)/a/d-1/10*cos(d*x+c)^7
*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.24, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2568, 2635, 8, 2565, 270}

$$\frac{\cos^{11}(c+dx)}{11ad}-\frac{2\cos^9(c+dx)}{9ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{\sin^3(c+dx)\cos^7(c+dx)}{10ad}-\frac{3\sin(c+dx)\cos^7(c+dx)}{80ad}+\frac{\sin(c+dx)}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(256*a) + Cos[c + d*x]^7/(7*a*d) - (2*Cos[c + d*x]^9)/(9*a*d) + Cos[c
 + d*x]^11/(11*a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(256*a*d) + (Cos[c + d*
x]^3*Sin[c + d*x])/(128*a*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(160*a*d) - (3
*Cos[c + d*x]^7*Sin[c + d*x])/(80*a*d) - (Cos[c + d*x]^7*Sin[c + d*x]^3)/(1
0*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^6(c+dx)\sin^4(c+dx)\,dx}{a} - \frac{\int \cos^6(c+dx)\sin^5(c+dx)\,dx}{a}$$

$$= -\frac{\cos^7(c+dx)\sin^3(c+dx)}{10ad} + \frac{3\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{10a} + \frac{\text{Subst}\left(\int x^6\right.}{}$$

$$= -\frac{3\cos^7(c+dx)\sin(c+dx)}{80ad} - \frac{\cos^7(c+dx)\sin^3(c+dx)}{10ad} + \frac{3\int \cos^6(c+dx)\,d}{80a}$$

$$= \frac{\cos^7(c+dx)}{7ad} - \frac{2\cos^9(c+dx)}{9ad} + \frac{\cos^{11}(c+dx)}{11ad} + \frac{\cos^5(c+dx)\sin(c+dx)}{160ad} -$$

$$= \frac{\cos^7(c+dx)}{7ad} - \frac{2\cos^9(c+dx)}{9ad} + \frac{\cos^{11}(c+dx)}{11ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{128ad} +$$

$$= \frac{\cos^7(c+dx)}{7ad} - \frac{2\cos^9(c+dx)}{9ad} + \frac{\cos^{11}(c+dx)}{11ad} + \frac{3\cos(c+dx)\sin(c+dx)}{256ad} +$$

$$= \frac{3x}{256a} + \frac{\cos^7(c+dx)}{7ad} - \frac{2\cos^9(c+dx)}{9ad} + \frac{\cos^{11}(c+dx)}{11ad} + \frac{3\cos(c+dx)\sin(c}{256ad}$$

Mathematica [B] time = 12.16, size = 573, normalized size = 3.13

$$\frac{97020\sin^2\left(\frac{1}{2}(c+dx)\right)}{d(a\sin(c+dx)+a)} + \frac{103950\sin(c)\sin(dx)}{ad} - \frac{66990\sin(3c)\sin(3dx)}{ad} + \frac{24948\sin(5c)\sin(5dx)}{ad} - \frac{1980\sin(7c)\sin(7dx)}{ad} - \frac{76230\sin(2(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((97020*c)/(a*d) + (83160*x)/a - (103950*Cos[c]*Cos[d*x])/(a*d) + (66990*Co
s[3*c]*Cos[3*d*x])/(a*d) - (24948*Cos[5*c]*Cos[5*d*x])/(a*d) + (1980*Cos[7*
c]*Cos[7*d*x])/(a*d) + (173250*Cos[c + d*x])/(a*d) - (43890*Cos[3*(c + d*x)
])/(a*d) + (18018*Cos[5*(c + d*x)])/(a*d) - (6930*Cos[7*(c + d*x)])/(a*d) +
 (770*Cos[9*(c + d*x)])/(a*d) + (630*Cos[11*(c + d*x)])/(a*d) + (90090*Cos[
2*d*x]*Sin[2*c])/(a*d) - (55440*Cos[4*d*x]*Sin[4*c])/(a*d) + (4620*Cos[6*d*
x]*Sin[6*c])/(a*d) + (103950*Sin[c]*Sin[d*x])/(a*d) + (90090*Cos[2*c]*Sin[2
*d*x])/(a*d) - (66990*Sin[3*c]*Sin[3*d*x])/(a*d) - (55440*Cos[4*c]*Sin[4*d*
x])/(a*d) + (24948*Sin[5*c]*Sin[5*d*x])/(a*d) + (4620*Cos[6*c]*Sin[6*d*x])/
(a*d) - (1980*Sin[7*c]*Sin[7*d*x])/(a*d) - (76230*Sin[(d*x)/2])/(a*d*(Cos[c
/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) - (20790*Sin[(c + d*
x)/2])/(a*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (48510*Sin[c + d*x])/(
a*d*(1 + Sin[c + d*x])) + (97020*Sin[(c + d*x)/2]^2)/(d*(a + a*Sin[c + d*x]
)) - (76230*Sin[2*(c + d*x)])/(a*d) + (27720*Sin[4*(c + d*x)])/(a*d) - (115
```

```
50*Sin[6*(c + d*x)])/(a*d) + (3465*Sin[8*(c + d*x)])/(a*d) + (1386*Sin[10*(
c + d*x)])/(a*d))/7096320
```

fricas [A] time = 0.47, size = 100, normalized size = 0.55

$$\frac{80640\cos(dx+c)^{11} - 197120\cos(dx+c)^{9} + 126720\cos(dx+c)^{7} + 10395\,dx + 693\left(128\cos(dx+c)^{9} - 176\right.}{887040\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/887040*(80640*cos(d*x + c)^11 - 197120*cos(d*x + c)^9 + 126720*cos(d*x +
c)^7 + 10395*d*x + 693*(128*cos(d*x + c)^9 - 176*cos(d*x + c)^7 + 8*cos(d*x
 + c)^5 + 10*cos(d*x + c)^3 + 15*cos(d*x + c))*sin(d*x + c))/(a*d)
```

giac [A] time = 0.22, size = 270, normalized size = 1.48

$$\frac{10395\,(dx+c)}{a} + \frac{2\left(10395\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{21} + 110880\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{19} - 2302839\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17} + 4730880\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16} + 4790016\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/887040*(10395*(d*x + c)/a + 2*(10395*tan(1/2*d*x + 1/2*c)^21 + 110880*tan
(1/2*d*x + 1/2*c)^19 - 2302839*tan(1/2*d*x + 1/2*c)^17 + 4730880*tan(1/2*d*
x + 1/2*c)^16 + 4790016*tan(1/2*d*x + 1/2*c)^15 - 11827200*tan(1/2*d*x + 1/
2*c)^14 - 5828130*tan(1/2*d*x + 1/2*c)^13 + 26019840*tan(1/2*d*x + 1/2*c)^1
2 - 21288960*tan(1/2*d*x + 1/2*c)^10 + 5828130*tan(1/2*d*x + 1/2*c)^9 + 152
06400*tan(1/2*d*x + 1/2*c)^8 - 4790016*tan(1/2*d*x + 1/2*c)^7 - 3041280*tan
(1/2*d*x + 1/2*c)^6 + 2302839*tan(1/2*d*x + 1/2*c)^5 + 563200*tan(1/2*d*x +
 1/2*c)^4 - 110880*tan(1/2*d*x + 1/2*c)^3 + 112640*tan(1/2*d*x + 1/2*c)^2 -
 10395*tan(1/2*d*x + 1/2*c) + 10240)/((tan(1/2*d*x + 1/2*c)^2 + 1)^11*a))/d
```

maple [B] time = 0.32, size = 653, normalized size = 3.57

$$\frac{16}{693ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}} - \frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{128ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}} + \frac{16\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{63ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}} - \frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

[Out] 16/693/a/d/(1+tan(1/2*d*x+1/2*c)^2)^11-3/128/a/d/(1+tan(1/2*d*x+1/2*c)^2)^1
1*tan(1/2*d*x+1/2*c)+16/63/a/d/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*
c)^2-1/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^3+80/63/a/d/(1+
tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^4+3323/640/a/d/(1+tan(1/2*d*x+1
/2*c)^2)^11*tan(1/2*d*x+1/2*c)^5-48/7/a/d/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1
/2*d*x+1/2*c)^6-54/5/a/d/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^7+2
40/7/a/d/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^8+841/64/a/d/(1+tan
(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^9-48/a/d/(1+tan(1/2*d*x+1/2*c)^2)^
11*tan(1/2*d*x+1/2*c)^10+176/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+
1/2*c)^12-841/64/a/d/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^13-80/3
/a/d/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^14+54/5/a/d/(1+tan(1/2*
d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^15+32/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^11
*tan(1/2*d*x+1/2*c)^16-3323/640/a/d/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x
+1/2*c)^17+1/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^19+3/128/
a/d/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^21+3/128/a/d*arctan(tan(
1/2*d*x+1/2*c))

maxima [B] time = 0.44, size = 624, normalized size = 3.41

$$-\frac{\frac{10395\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{112640\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{110880\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{563200\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2302839\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3041280\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{4790016\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{15206400\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}{a+\frac{11\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{55\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{165\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{330\,a\sin(dx+c)}{(\cos(dx+c)+}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/443520*((10395*sin(d*x + c)/(cos(d*x + c) + 1) - 112640*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 110880*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 563200*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2302839*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5 + 3041280*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 4790016*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 - 15206400*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 582
8130*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 21288960*sin(d*x + c)^10/(cos(d*
x + c) + 1)^10 - 26019840*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 5828130*s
in(d*x + c)^13/(cos(d*x + c) + 1)^13 + 11827200*sin(d*x + c)^14/(cos(d*x +
c) + 1)^14 - 4790016*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 4730880*sin(d*
x + c)^16/(cos(d*x + c) + 1)^16 + 2302839*sin(d*x + c)^17/(cos(d*x + c) + 1
)^17 - 110880*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 - 10395*sin(d*x + c)^21
/(cos(d*x + c) + 1)^21 - 10240)/(a + 11*a*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 55*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 165*a*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + 330*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 462*a*sin(d*x
+ c)^10/(cos(d*x + c) + 1)^10 + 462*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12
 + 330*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 165*a*sin(d*x + c)^16/(cos
(d*x + c) + 1)^16 + 55*a*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + 11*a*sin(d

```
*x + c)^20/(cos(d*x + c) + 1)^20 + a*sin(d*x + c)^22/(cos(d*x + c) + 1)^22)
 - 10395*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 11.60, size = 263, normalized size = 1.44

$$\frac{3x}{256a}+\frac{\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{21}}{128}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}}{4}-\frac{3323\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{640}+\frac{32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{3}+\frac{54\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{5}-\frac{80\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{3}-\frac{841\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^4)/(a + a*sin(c + d*x)),x)
```

```
[Out] (3*x)/(256*a) + ((16*tan(c/2 + (d*x)/2)^2)/63 - (3*tan(c/2 + (d*x)/2))/128
- tan(c/2 + (d*x)/2)^3/4 + (80*tan(c/2 + (d*x)/2)^4)/63 + (3323*tan(c/2 + (
d*x)/2)^5)/640 - (48*tan(c/2 + (d*x)/2)^6)/7 - (54*tan(c/2 + (d*x)/2)^7)/5
+ (240*tan(c/2 + (d*x)/2)^8)/7 + (841*tan(c/2 + (d*x)/2)^9)/64 - 48*tan(c/2
 + (d*x)/2)^10 + (176*tan(c/2 + (d*x)/2)^12)/3 - (841*tan(c/2 + (d*x)/2)^13
)/64 - (80*tan(c/2 + (d*x)/2)^14)/3 + (54*tan(c/2 + (d*x)/2)^15)/5 + (32*ta
n(c/2 + (d*x)/2)^16)/3 - (3323*tan(c/2 + (d*x)/2)^17)/640 + tan(c/2 + (d*x)
/2)^19/4 + (3*tan(c/2 + (d*x)/2)^21)/128 + 16/693)/(a*d*(tan(c/2 + (d*x)/2)
^2 + 1)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.707 $\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=165

$$\frac{\cos^9(c+dx)}{9ad}-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin^3(c+dx)\cos^7(c+dx)}{10ad}+\frac{3\sin(c+dx)\cos^7(c+dx)}{80ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{160ad}-\text{s}$$

```
[Out] -3/256*x/a-1/7*cos(d*x+c)^7/a/d+1/9*cos(d*x+c)^9/a/d-3/256*cos(d*x+c)*sin(d
*x+c)/a/d-1/128*cos(d*x+c)^3*sin(d*x+c)/a/d-1/160*cos(d*x+c)^5*sin(d*x+c)/a
/d+3/80*cos(d*x+c)^7*sin(d*x+c)/a/d+1/10*cos(d*x+c)^7*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.24, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2565, 14, 2568, 2635, 8}

$$\frac{\cos^9(c+dx)}{9ad}-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin^3(c+dx)\cos^7(c+dx)}{10ad}+\frac{3\sin(c+dx)\cos^7(c+dx)}{80ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{160ad}-\text{s}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(256*a) - Cos[c + d*x]^7/(7*a*d) + Cos[c + d*x]^9/(9*a*d) - (3*Cos[c
 + d*x]*Sin[c + d*x])/(256*a*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(128*a*d) -
 (Cos[c + d*x]^5*Sin[c + d*x])/(160*a*d) + (3*Cos[c + d*x]^7*Sin[c + d*x])/
(80*a*d) + (Cos[c + d*x]^7*Sin[c + d*x]^3)/(10*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^6(c+dx)\sin^3(c+dx)\,dx}{a} - \frac{\int \cos^6(c+dx)\sin^4(c+dx)\,dx}{a}$$

$$= \frac{\cos^7(c+dx)\sin^3(c+dx)}{10ad} - \frac{3\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{10a} - \frac{\text{Subst}\left(\int x^6\left(1\right.\right.}{}$$

$$= \frac{3\cos^7(c+dx)\sin(c+dx)}{80ad} + \frac{\cos^7(c+dx)\sin^3(c+dx)}{10ad} - \frac{3\int \cos^6(c+dx)\,dx}{80a} -$$

$$= -\frac{\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} - \frac{\cos^5(c+dx)\sin(c+dx)}{160ad} + \frac{3\cos^7(c+dx)\sin}{80ad}$$

$$= -\frac{\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{128ad} - \frac{\cos^5(c+dx)\sin(c}{160ad}$$

$$= -\frac{\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} - \frac{3\cos(c+dx)\sin(c+dx)}{256ad} - \frac{\cos^3(c+dx)\sin(}{128ad}$$

$$= -\frac{3x}{256a} - \frac{\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} - \frac{3\cos(c+dx)\sin(c+dx)}{256ad} - \frac{\cos^3(c+}{}$$

Mathematica [B] time = 14.16, size = 533, normalized size = 3.23

$$-\frac{15120dx\sin\left(\frac{c}{2}\right)-15120\sin\left(\frac{c}{2}+dx\right)+15120\sin\left(\frac{3c}{2}+dx\right)+1260\sin\left(\frac{3c}{2}+2dx\right)+1260\sin\left(\frac{5c}{2}+2dx\right)-67}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/1290240*(-1260*(25*c - 12*d*x)*Cos[c/2] + 15120*Cos[c/2 + d*x] + 15120*C
os[(3*c)/2 + d*x] + 1260*Cos[(3*c)/2 + 2*d*x] - 1260*Cos[(5*c)/2 + 2*d*x] +
 6720*Cos[(5*c)/2 + 3*d*x] + 6720*Cos[(7*c)/2 + 3*d*x] - 2520*Cos[(7*c)/2 +
 4*d*x] + 2520*Cos[(9*c)/2 + 4*d*x] - 630*Cos[(11*c)/2 + 6*d*x] + 630*Cos[(
13*c)/2 + 6*d*x] - 1080*Cos[(13*c)/2 + 7*d*x] - 1080*Cos[(15*c)/2 + 7*d*x]
+ 315*Cos[(15*c)/2 + 8*d*x] - 315*Cos[(17*c)/2 + 8*d*x] - 280*Cos[(17*c)/2
+ 9*d*x] - 280*Cos[(19*c)/2 + 9*d*x] + 126*Cos[(19*c)/2 + 10*d*x] - 126*Cos
[(21*c)/2 + 10*d*x] + 37800*Sin[c/2] - 31500*c*Sin[c/2] + 15120*d*x*Sin[c/2
] - 15120*Sin[c/2 + d*x] + 15120*Sin[(3*c)/2 + d*x] + 1260*Sin[(3*c)/2 + 2*
d*x] + 1260*Sin[(5*c)/2 + 2*d*x] - 6720*Sin[(5*c)/2 + 3*d*x] + 6720*Sin[(7*
c)/2 + 3*d*x] - 2520*Sin[(7*c)/2 + 4*d*x] - 2520*Sin[(9*c)/2 + 4*d*x] - 630
*Sin[(11*c)/2 + 6*d*x] - 630*Sin[(13*c)/2 + 6*d*x] + 1080*Sin[(13*c)/2 + 7*
d*x] - 1080*Sin[(15*c)/2 + 7*d*x] + 315*Sin[(15*c)/2 + 8*d*x] + 315*Sin[(17
*c)/2 + 8*d*x] + 280*Sin[(17*c)/2 + 9*d*x] - 280*Sin[(19*c)/2 + 9*d*x] + 12
6*Sin[(19*c)/2 + 10*d*x] + 126*Sin[(21*c)/2 + 10*d*x])/(a*d*(Cos[c/2] + Sin
[c/2]))
```

fricas [A] time = 0.47, size = 90, normalized size = 0.55

$$\frac{8960\cos\left(dx+c\right)^{9}-11520\cos\left(dx+c\right)^{7}-945\,dx-63\left(128\cos\left(dx+c\right)^{9}-176\cos\left(dx+c\right)^{7}+8\cos\left(dx+c\right)^{5}\right.}{80640\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/80640*(8960*cos(d*x + c)^9 - 11520*cos(d*x + c)^7 - 945*d*x - 63*(128*cos
(d*x + c)^9 - 176*cos(d*x + c)^7 + 8*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 1
5*cos(d*x + c))*sin(d*x + c))/(a*d)
```

giac [A] time = 0.21, size = 257, normalized size = 1.56

$$-\frac{\frac{945\,(dx+c)}{a}+\frac{2\left(945\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{19}+9135\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}+161280\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16}-218484\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-107520\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}+65\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80640*(945*(d*x + c)/a + 2*(945*tan(1/2*d*x + 1/2*c)^19 + 9135*tan(1/2*d
*x + 1/2*c)^17 + 161280*tan(1/2*d*x + 1/2*c)^16 - 218484*tan(1/2*d*x + 1/2*
c)^15 - 107520*tan(1/2*d*x + 1/2*c)^14 + 653940*tan(1/2*d*x + 1/2*c)^13 + 5
37600*tan(1/2*d*x + 1/2*c)^12 - 1183770*tan(1/2*d*x + 1/2*c)^11 + 322560*ta
n(1/2*d*x + 1/2*c)^10 + 1183770*tan(1/2*d*x + 1/2*c)^9 - 653940*tan(1/2*d*x
 + 1/2*c)^7 + 414720*tan(1/2*d*x + 1/2*c)^6 + 218484*tan(1/2*d*x + 1/2*c)^5
 - 46080*tan(1/2*d*x + 1/2*c)^4 - 9135*tan(1/2*d*x + 1/2*c)^3 + 25600*tan(1
/2*d*x + 1/2*c)^2 - 945*tan(1/2*d*x + 1/2*c) + 2560)/((tan(1/2*d*x + 1/2*c)
^2 + 1)^10*a))/d
```

maple [B] time = 0.31, size = 619, normalized size = 3.75

$$-\frac{4}{63ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{128ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{40\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{63ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}+\frac{29\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad\left(1+\tan^{2}\left(\frac{dx}{2}+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -4/63/a/d/(1+tan(1/2*d*x+1/2*c)^2)^10+3/128/a/d/(1+tan(1/2*d*x+1/2*c)^2)^10
*tan(1/2*d*x+1/2*c)-40/63/a/d/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c
)^2+29/128/a/d/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^3+8/7/a/d/(1+
tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^4-867/160/a/d/(1+tan(1/2*d*x+1/
2*c)^2)^10*tan(1/2*d*x+1/2*c)^5-72/7/a/d/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/
2*d*x+1/2*c)^6+519/32/a/d/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^7-
1879/64/a/d/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^9-8/a/d/(1+tan(1
/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^10+1879/64/a/d/(1+tan(1/2*d*x+1/2*c)
^2)^10*tan(1/2*d*x+1/2*c)^11-40/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d
*x+1/2*c)^12-519/32/a/d/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^13+8
/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^14+867/160/a/d/(1+tan
(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^15-4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^
10*tan(1/2*d*x+1/2*c)^16-29/128/a/d/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x
+1/2*c)^17-3/128/a/d/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^19-3/12
8/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.44, size = 583, normalized size = 3.53

$$\frac{\frac{945\sin(dx+c)}{\cos(dx+c)+1}-\frac{25600\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{9135\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{46080\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{218484\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{414720\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{653940\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{1183770\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{322560\sin(dx+\cdots}{(\cos(dx+c)+1\cdots}}{a+\frac{10a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{45a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{120a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{210a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{252a\sin(dx+\cdots}{(\cos(dx+c)+1\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/40320*((945*sin(d*x + c)/(cos(d*x + c) + 1) - 25600*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 9135*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 46080*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 218484*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4
14720*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 653940*sin(d*x + c)^7/(cos(d*x
+ c) + 1)^7 - 1183770*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 322560*sin(d*x
+ c)^10/(cos(d*x + c) + 1)^10 + 1183770*sin(d*x + c)^11/(cos(d*x + c) + 1)^
11 - 537600*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 653940*sin(d*x + c)^13/
(cos(d*x + c) + 1)^13 + 107520*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 2184
84*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 161280*sin(d*x + c)^16/(cos(d*x
+ c) + 1)^16 - 9135*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 945*sin(d*x + c
)^19/(cos(d*x + c) + 1)^19 - 2560)/(a + 10*a*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 45*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 120*a*sin(d*x + c)^6/(co
s(d*x + c) + 1)^6 + 210*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 252*a*sin(d
*x + c)^10/(cos(d*x + c) + 1)^10 + 210*a*sin(d*x + c)^12/(cos(d*x + c) + 1)
^12 + 120*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 45*a*sin(d*x + c)^16/(c
os(d*x + c) + 1)^16 + 10*a*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + a*sin(d*
x + c)^20/(cos(d*x + c) + 1)^20) - 945*arctan(sin(d*x + c)/(cos(d*x + c) +
1))/a)/d
```

mupad [B] time = 11.52, size = 251, normalized size = 1.52

$$-\frac{3\,x}{256\,a}-\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}}{128}+\frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{128}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-\frac{867\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{160}-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}}{3}+\frac{519\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32}+\frac{40\tan \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] - (3*x)/(256*a) - ((40*tan(c/2 + (d*x)/2)^2)/63 - (3*tan(c/2 + (d*x)/2))/12
8 - (29*tan(c/2 + (d*x)/2)^3)/128 - (8*tan(c/2 + (d*x)/2)^4)/7 + (867*tan(c
/2 + (d*x)/2)^5)/160 + (72*tan(c/2 + (d*x)/2)^6)/7 - (519*tan(c/2 + (d*x)/2
)^7)/32 + (1879*tan(c/2 + (d*x)/2)^9)/64 + 8*tan(c/2 + (d*x)/2)^10 - (1879*
tan(c/2 + (d*x)/2)^11)/64 + (40*tan(c/2 + (d*x)/2)^12)/3 + (519*tan(c/2 + (
d*x)/2)^13)/32 - (8*tan(c/2 + (d*x)/2)^14)/3 - (867*tan(c/2 + (d*x)/2)^15)/
160 + 4*tan(c/2 + (d*x)/2)^16 + (29*tan(c/2 + (d*x)/2)^17)/128 + (3*tan(c/2
 + (d*x)/2)^19)/128 + 4/63)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^10)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.708 $\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=139

$$-\frac{\cos^9(c+dx)}{9ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{\sin(c+dx)\cos^7(c+dx)}{8ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{48ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{192ad}+$$

```
[Out] 5/128*x/a+1/7*cos(d*x+c)^7/a/d-1/9*cos(d*x+c)^9/a/d+5/128*cos(d*x+c)*sin(d*
x+c)/a/d+5/192*cos(d*x+c)^3*sin(d*x+c)/a/d+1/48*cos(d*x+c)^5*sin(d*x+c)/a/d
-1/8*cos(d*x+c)^7*sin(d*x+c)/a/d
```

Rubi [A] time = 0.20, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2568, 2635, 8, 2565, 14}

$$-\frac{\cos^9(c+dx)}{9ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{\sin(c+dx)\cos^7(c+dx)}{8ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{48ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{192ad}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*x)/(128*a) + Cos[c + d*x]^7/(7*a*d) - Cos[c + d*x]^9/(9*a*d) + (5*Cos[c
+ d*x]*Sin[c + d*x])/(128*a*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(192*a*d)
+ (Cos[c + d*x]^5*Sin[c + d*x])/(48*a*d) - (Cos[c + d*x]^7*Sin[c + d*x])/(8
*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^6(c+dx)\sin^3(c+dx)\,dx}{a}$$

$$= -\frac{\cos^7(c+dx)\sin(c+dx)}{8ad} + \frac{\int \cos^6(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\int x^6\left(1-x^2\right)\,dx, x, \text{co}$$

$$= \frac{\cos^5(c+dx)\sin(c+dx)}{48ad} - \frac{\cos^7(c+dx)\sin(c+dx)}{8ad} + \frac{5\int \cos^4(c+dx)\,dx}{48a} + \text{S}$$

$$= \frac{\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{192ad} + \frac{\cos^5(c+dx)\sin(c}{48ad}$$

$$= \frac{\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{5\cos(c+dx)\sin(c+dx)}{128ad} + \frac{5\cos^3(c+dx)\sin(}{192ad}$$

$$= \frac{5x}{128a} + \frac{\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{5\cos(c+dx)\sin(c+dx)}{128ad} + \frac{5\cos^3(c+}{1}$$

Mathematica [B] time = 9.64, size = 479, normalized size = 3.45

$$-\frac{-5040dx\sin\left(\frac{c}{2}\right) + 1512\sin\left(\frac{c}{2}+dx\right) - 1512\sin\left(\frac{3c}{2}+dx\right) - 1008\sin\left(\frac{3c}{2}+2dx\right) - 1008\sin\left(\frac{5c}{2}+2dx\right) + 672\,\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/129024*(2520*(c - 2*d*x)*Cos[c/2] - 1512*Cos[c/2 + d*x] - 1512*Cos[(3*c)
/2 + d*x] - 1008*Cos[(3*c)/2 + 2*d*x] + 1008*Cos[(5*c)/2 + 2*d*x] - 672*Cos
[(5*c)/2 + 3*d*x] - 672*Cos[(7*c)/2 + 3*d*x] + 504*Cos[(7*c)/2 + 4*d*x] - 5
04*Cos[(9*c)/2 + 4*d*x] + 336*Cos[(11*c)/2 + 6*d*x] - 336*Cos[(13*c)/2 + 6*
d*x] + 108*Cos[(13*c)/2 + 7*d*x] + 108*Cos[(15*c)/2 + 7*d*x] + 63*Cos[(15*c
)/2 + 8*d*x] - 63*Cos[(17*c)/2 + 8*d*x] + 28*Cos[(17*c)/2 + 9*d*x] + 28*Cos
[(19*c)/2 + 9*d*x] - 7560*Sin[c/2] + 2520*c*Sin[c/2] - 5040*d*x*Sin[c/2] +
1512*Sin[c/2 + d*x] - 1512*Sin[(3*c)/2 + d*x] - 1008*Sin[(3*c)/2 + 2*d*x] -
 1008*Sin[(5*c)/2 + 2*d*x] + 672*Sin[(5*c)/2 + 3*d*x] - 672*Sin[(7*c)/2 + 3
*d*x] + 504*Sin[(7*c)/2 + 4*d*x] + 504*Sin[(9*c)/2 + 4*d*x] + 336*Sin[(11*c
)/2 + 6*d*x] + 336*Sin[(13*c)/2 + 6*d*x] - 108*Sin[(13*c)/2 + 7*d*x] + 108*
Sin[(15*c)/2 + 7*d*x] + 63*Sin[(15*c)/2 + 8*d*x] + 63*Sin[(17*c)/2 + 8*d*x]
 - 28*Sin[(17*c)/2 + 9*d*x] + 28*Sin[(19*c)/2 + 9*d*x])/(a*d*(Cos[c/2] + Si
n[c/2]))
```

fricas [A] time = 0.47, size = 80, normalized size = 0.58

$$-\frac{896\cos\left(dx+c\right)^{9}-1152\cos\left(dx+c\right)^{7}-315\,dx+21\left(48\cos\left(dx+c\right)^{7}-8\cos\left(dx+c\right)^{5}-10\cos\left(dx+c\right)^{3}-\right.}{8064\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8064*(896*cos(d*x + c)^9 - 1152*cos(d*x + c)^7 - 315*d*x + 21*(48*cos(d*
x + c)^7 - 8*cos(d*x + c)^5 - 10*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*x
+ c))/(a*d)
```

giac [A] time = 0.17, size = 231, normalized size = 1.66

$$\frac{315\,(dx+c)}{a}+\frac{2\left(315\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}-8022\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+16128\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}+10458\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-26880\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-18270\,\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8064*(315*(d*x + c)/a + 2*(315*tan(1/2*d*x + 1/2*c)^17 - 8022*tan(1/2*d*x
 + 1/2*c)^15 + 16128*tan(1/2*d*x + 1/2*c)^14 + 10458*tan(1/2*d*x + 1/2*c)^1
3 - 26880*tan(1/2*d*x + 1/2*c)^12 - 18270*tan(1/2*d*x + 1/2*c)^11 + 80640*t
```

```
an(1/2*d*x + 1/2*c)^10 - 48384*tan(1/2*d*x + 1/2*c)^8 + 18270*tan(1/2*d*x +
 1/2*c)^7 + 48384*tan(1/2*d*x + 1/2*c)^6 - 10458*tan(1/2*d*x + 1/2*c)^5 - 6
912*tan(1/2*d*x + 1/2*c)^4 + 8022*tan(1/2*d*x + 1/2*c)^3 + 2304*tan(1/2*d*x
 + 1/2*c)^2 - 315*tan(1/2*d*x + 1/2*c) + 256)/((tan(1/2*d*x + 1/2*c)^2 + 1)
^9*a))/d
```

maple [B] time = 0.31, size = 551, normalized size = 3.96

$$\frac{4}{63ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}}+\frac{4\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}}+\frac{191\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 4/63/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9-5/64/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan
(1/2*d*x+1/2*c)+4/7/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^2+191
/96/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^3-12/7/a/d/(1+tan(1/2
*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^4-83/32/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*
tan(1/2*d*x+1/2*c)^5+12/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^6
+145/32/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^7-12/a/d/(1+tan(1
/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^8+20/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*t
an(1/2*d*x+1/2*c)^10-145/32/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*
c)^11-20/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^12+83/32/a/d/(
1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^13+4/a/d/(1+tan(1/2*d*x+1/2*c)
^2)^9*tan(1/2*d*x+1/2*c)^14-191/96/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d
*x+1/2*c)^15+5/64/a/d/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^17+5/64
/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 522, normalized size = 3.76

$$-\frac{\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}-\frac{2304\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8022\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{6912\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{10458\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{48384\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{18270\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{48384\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{80640\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+}{a+\frac{9a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{36a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{84a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{126a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{126a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{84a\sin(dx+}{(\cos(dx+c)+}}}{4032d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4032*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 2304*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 8022*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6912*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 10458*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4838
4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 18270*sin(d*x + c)^7/(cos(d*x + c)
```

```
+ 1)^7 + 48384*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 80640*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 + 18270*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 26880
*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 10458*sin(d*x + c)^13/(cos(d*x + c
) + 1)^13 - 16128*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 8022*sin(d*x + c)
^15/(cos(d*x + c) + 1)^15 - 315*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 256
)/(a + 9*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 36*a*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 84*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 126*a*sin(d*x +
 c)^8/(cos(d*x + c) + 1)^8 + 126*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
84*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 36*a*sin(d*x + c)^14/(cos(d*x
+ c) + 1)^14 + 9*a*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 + a*sin(d*x + c)^1
8/(cos(d*x + c) + 1)^18) - 315*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 11.60, size = 224, normalized size = 1.61

$$\frac{5\,x}{128\,a}+\frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{64}-\frac{191\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{96}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+\frac{83\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32}-\frac{20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}-\frac{145\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{32}+20\,\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] (5*x)/(128*a) + ((4*tan(c/2 + (d*x)/2)^2)/7 - (5*tan(c/2 + (d*x)/2))/64 + (
191*tan(c/2 + (d*x)/2)^3)/96 - (12*tan(c/2 + (d*x)/2)^4)/7 - (83*tan(c/2 +
(d*x)/2)^5)/32 + 12*tan(c/2 + (d*x)/2)^6 + (145*tan(c/2 + (d*x)/2)^7)/32 -
12*tan(c/2 + (d*x)/2)^8 + 20*tan(c/2 + (d*x)/2)^10 - (145*tan(c/2 + (d*x)/2
)^11)/32 - (20*tan(c/2 + (d*x)/2)^12)/3 + (83*tan(c/2 + (d*x)/2)^13)/32 + 4
*tan(c/2 + (d*x)/2)^14 - (191*tan(c/2 + (d*x)/2)^15)/96 + (5*tan(c/2 + (d*x
)/2)^17)/64 + 4/63)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

sympy [A] time = 170.69, size = 4490, normalized size = 32.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((315*d*x*tan(c/2 + d*x/2)**18/(8064*a*d*tan(c/2 + d*x/2)**18 + 72
576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d
*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(
c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x
/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 2835*d*x*tan(c/2 + d*x
/2)**16/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 2
```

90304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*
a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan
(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x
/2)**2 + 8064*a*d) + 11340*d*x*tan(c/2 + d*x/2)**14/(8064*a*d*tan(c/2 + d*x
/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14
+ 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016
064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*t
an(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 26460*d*x*
tan(c/2 + d*x/2)**12/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d
*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**
12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 6
77376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*
tan(c/2 + d*x/2)**2 + 8064*a*d) + 39690*d*x*tan(c/2 + d*x/2)**10/(8064*a*d*
tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2
+ d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/
2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 +
290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d)
+ 39690*d*x*tan(c/2 + d*x/2)**8/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d
*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/
2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d
*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4
+ 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 26460*d*x*tan(c/2 + d*x/2)**6
/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a
*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan
(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 +
d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2
+ 8064*a*d) + 11340*d*x*tan(c/2 + d*x/2)**4/(8064*a*d*tan(c/2 + d*x/2)**18
+ 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376
*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*
tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 +
d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 2835*d*x*tan(c/2 +
d*x/2)**2/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16
+ 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 10160
64*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*
tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 +
d*x/2)**2 + 8064*a*d) + 315*d*x/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*
tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2
+ d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*
x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 +
72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 630*tan(c/2 + d*x/2)**17/(8064
*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan
(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 +
d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)
6 + 290304*a*d*tan(c/2 + d*x/2)4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064
*a*d) - 16044*tan(c/2 + d*x/2)**15/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a

*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(
c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 +
 d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**
4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 32256*tan(c/2 + d*x/2)**14/
(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*
d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(
c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d
*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 +
 8064*a*d) + 20916*tan(c/2 + d*x/2)**13/(8064*a*d*tan(c/2 + d*x/2)**18 + 72
576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d
*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(
c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x
/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) - 53760*tan(c/2 + d*x/2)
12/(8064*a*d*tan(c/2 + d*x/2)18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 2903
04*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d
*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/
2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)
2 + 8064*a*d) - 36540*tan(c/2 + d*x/2)11/(8064*a*d*tan(c/2 + d*x/2)**18
 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 67737
6*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d
*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2
+ d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 161280*tan(c/2 +
d*x/2)**10/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16
+ 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 10160
64*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*
tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 +
d*x/2)**2 + 8064*a*d) - 96768*tan(c/2 + d*x/2)**8/(8064*a*d*tan(c/2 + d*x/2
)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 +
677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 101606
4*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan
(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 36540*tan(c/
2 + d*x/2)**7/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**
16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 10
16064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a
*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2
 + d*x/2)**2 + 8064*a*d) + 96768*tan(c/2 + d*x/2)**6/(8064*a*d*tan(c/2 + d*
x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14
 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 101
6064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*
tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) - 20916*tan
(c/2 + d*x/2)**5/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2
)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 +
 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 67737
6*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(
c/2 + d*x/2)**2 + 8064*a*d) - 13824*tan(c/2 + d*x/2)**4/(8064*a*d*tan(c/2 +

```
d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)*
*14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 +
1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a
*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 16044*
tan(c/2 + d*x/2)**3/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*
x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**1
2 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 67
7376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*t
an(c/2 + d*x/2)**2 + 8064*a*d) + 4608*tan(c/2 + d*x/2)**2/(8064*a*d*tan(c/2
 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2
)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10
+ 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304
*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) - 630*
tan(c/2 + d*x/2)/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2
)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 +
 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 67737
6*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(
c/2 + d*x/2)**2 + 8064*a*d) + 512/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*
d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c
/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 +
d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4
 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d), Ne(d, 0)), (x*sin(c)**2*cos(c
)**8/(a*sin(c) + a), True))
```

3.709 $\int \frac{\cos^8(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=121

$$-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin(c+dx)\cos^7(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{48ad}-\frac{5\sin(c+dx)\cos^3(c+dx)}{192ad}-\frac{5\sin(c+dx)\,\mathrm{co}}{128a}$$

```
[Out] -5/128*x/a-1/7*cos(d*x+c)^7/a/d-5/128*cos(d*x+c)*sin(d*x+c)/a/d-5/192*cos(d
*x+c)^3*sin(d*x+c)/a/d-1/48*cos(d*x+c)^5*sin(d*x+c)/a/d+1/8*cos(d*x+c)^7*si
n(d*x+c)/a/d
```

Rubi [A] time = 0.15, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2565, 30, 2568, 2635, 8}

$$-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin(c+dx)\cos^7(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{48ad}-\frac{5\sin(c+dx)\cos^3(c+dx)}{192ad}-\frac{5\sin(c+dx)\,\mathrm{co}}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*x)/(128*a) - Cos[c + d*x]^7/(7*a*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(12
8*a*d) - (5*Cos[c + d*x]^3*Sin[c + d*x])/(192*a*d) - (Cos[c + d*x]^5*Sin[c
+ d*x])/(48*a*d) + (Cos[c + d*x]^7*Sin[c + d*x])/(8*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
```

```
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^8(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^6(c+dx)\sin(c+dx)\,dx}{a} - \frac{\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{a} \\
&= \frac{\cos^7(c+dx)\sin(c+dx)}{8ad} - \frac{\int \cos^6(c+dx)\,dx}{8a} - \frac{\operatorname{Subst}\left(\int x^6\,dx, x, \cos(c+dx)\right)}{ad} \\
&= -\frac{\cos^7(c+dx)}{7ad} - \frac{\cos^5(c+dx)\sin(c+dx)}{48ad} + \frac{\cos^7(c+dx)\sin(c+dx)}{8ad} - \frac{5\int \cos
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\cos^7(c+dx)}{7ad} - \frac{5\cos^3(c+dx)\sin(c+dx)}{192ad} - \frac{\cos^5(c+dx)\sin(c+dx)}{48ad} + \frac{\cos^7(
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\cos^7(c+dx)}{7ad} - \frac{5\cos(c+dx)\sin(c+dx)}{128ad} - \frac{5\cos^3(c+dx)\sin(c+dx)}{192ad} - \frac{\cos^5(
\end{aligned}
$$

$$
= -\frac{5x}{128a} - \frac{\cos^7(c+dx)}{7ad} - \frac{5\cos(c+dx)\sin(c+dx)}{128ad} - \frac{5\cos^3(c+dx)\sin(c+dx)}{192ad}
$$

Mathematica [**B**] time = 10.35, size = 481, normalized size = 3.98

$$
-\frac{1680dx\sin\left(\frac{c}{2}\right) - 1680\sin\left(\frac{c}{2}+dx\right) + 1680\sin\left(\frac{3c}{2}+dx\right) + 336\sin\left(\frac{3c}{2}+2dx\right) + 336\sin\left(\frac{5c}{2}+2dx\right) - 1008\sin\left(
}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/43008*(-336*(7*c - 5*d*x)*Cos[c/2] + 1680*Cos[c/2 + d*x] + 1680*Cos[(3*c
)/2 + d*x] + 336*Cos[(3*c)/2 + 2*d*x] - 336*Cos[(5*c)/2 + 2*d*x] + 1008*Cos
[(5*c)/2 + 3*d*x] + 1008*Cos[(7*c)/2 + 3*d*x] - 168*Cos[(7*c)/2 + 4*d*x] +
168*Cos[(9*c)/2 + 4*d*x] + 336*Cos[(9*c)/2 + 5*d*x] + 336*Cos[(11*c)/2 + 5*
d*x] - 112*Cos[(11*c)/2 + 6*d*x] + 112*Cos[(13*c)/2 + 6*d*x] + 48*Cos[(13*c
)/2 + 7*d*x] + 48*Cos[(15*c)/2 + 7*d*x] - 21*Cos[(15*c)/2 + 8*d*x] + 21*Cos
[(17*c)/2 + 8*d*x] + 4704*Sin[c/2] - 2352*c*Sin[c/2] + 1680*d*x*Sin[c/2] -
1680*Sin[c/2 + d*x] + 1680*Sin[(3*c)/2 + d*x] + 336*Sin[(3*c)/2 + 2*d*x] +
336*Sin[(5*c)/2 + 2*d*x] - 1008*Sin[(5*c)/2 + 3*d*x] + 1008*Sin[(7*c)/2 + 3
*d*x] - 168*Sin[(7*c)/2 + 4*d*x] - 168*Sin[(9*c)/2 + 4*d*x] - 336*Sin[(9*c)
/2 + 5*d*x] + 336*Sin[(11*c)/2 + 5*d*x] - 112*Sin[(11*c)/2 + 6*d*x] - 112*S
in[(13*c)/2 + 6*d*x] - 48*Sin[(13*c)/2 + 7*d*x] + 48*Sin[(15*c)/2 + 7*d*x]
- 21*Sin[(15*c)/2 + 8*d*x] - 21*Sin[(17*c)/2 + 8*d*x])/(a*d*(Cos[c/2] + Sin
[c/2]))
```

fricas [A] time = 0.47, size = 70, normalized size = 0.58

$$-\frac{384\,\cos\left(dx+c\right)^{7}+105\,dx-7\left(48\,\cos\left(dx+c\right)^{7}-8\,\cos\left(dx+c\right)^{5}-10\,\cos\left(dx+c\right)^{3}-15\,\cos\left(dx+c\right)\right)\sin\left(d\right.}{2688\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2688*(384*cos(d*x + c)^7 + 105*d*x - 7*(48*cos(d*x + c)^7 - 8*cos(d*x +
c)^5 - 10*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*x + c))/(a*d)
```

giac [B] time = 0.16, size = 231, normalized size = 1.91

$$-\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+2688\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}-2779\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+2688\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+6265\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+13440\,\tan\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2688*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^15 + 2688*tan(1/2*d*
x + 1/2*c)^14 - 2779*tan(1/2*d*x + 1/2*c)^13 + 2688*tan(1/2*d*x + 1/2*c)^12
 + 6265*tan(1/2*d*x + 1/2*c)^11 + 13440*tan(1/2*d*x + 1/2*c)^10 - 12355*tan
(1/2*d*x + 1/2*c)^9 + 13440*tan(1/2*d*x + 1/2*c)^8 + 12355*tan(1/2*d*x + 1/
```

```
2*c)^7 + 8064*tan(1/2*d*x + 1/2*c)^6 - 6265*tan(1/2*d*x + 1/2*c)^5 + 8064*t
an(1/2*d*x + 1/2*c)^4 + 2779*tan(1/2*d*x + 1/2*c)^3 + 384*tan(1/2*d*x + 1/2
*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 384)/((tan(1/2*d*x + 1/2*c)^2 + 1)^8*a))
/d
```

maple [B] time = 0.21, size = 551, normalized size = 4.55

$$-\frac{2}{7ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{397\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/7/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8+5/64/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan
(1/2*d*x+1/2*c)-2/7/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^2-397
/192/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3-6/a/d/(1+tan(1/2*d
*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^4+895/192/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*
tan(1/2*d*x+1/2*c)^5-6/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^6-
1765/192/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^7-10/a/d/(1+tan(
1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^8+1765/192/a/d/(1+tan(1/2*d*x+1/2*c)
^2)^8*tan(1/2*d*x+1/2*c)^9-10/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/
2*c)^10-895/192/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11-2/a/d/
(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^12+397/192/a/d/(1+tan(1/2*d*x
+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13-2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2
*d*x+1/2*c)^14-5/64/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15-5/
64/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 501, normalized size = 4.14

$$\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{384\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2779\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{8064\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6265\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{8064\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{12355\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{13440\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{12355\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{13440}{(\cos(}}{a+\frac{8a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a\sin(d}{(\cos(dx+c}}}{1344\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/1344*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 384*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 2779*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 8064*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 6265*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8064*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 12355*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 - 13440*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 12355*sin(d*x + c)^9/(cos(
```

```
d*x + c) + 1)^9 - 13440*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6265*sin(d*
x + c)^11/(cos(d*x + c) + 1)^11 - 2688*sin(d*x + c)^12/(cos(d*x + c) + 1)^1
2 + 2779*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 2688*sin(d*x + c)^14/(cos(
d*x + c) + 1)^14 - 105*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 384)/(a + 8*
a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 + 56*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a*sin(d*x + c)^8/(cos
(d*x + c) + 1)^8 + 56*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a*sin(d*
x + c)^12/(cos(d*x + c) + 1)^12 + 8*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14
 + a*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 105*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a)/d
```

mupad [B] time = 11.68, size = 225, normalized size = 1.86

$$-\frac{5\,x}{128\,a}-\frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-\frac{397\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\frac{895\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{192}+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] - (5*x)/(128*a) - ((2*tan(c/2 + (d*x)/2)^2)/7 - (5*tan(c/2 + (d*x)/2))/64 +
 (397*tan(c/2 + (d*x)/2)^3)/192 + 6*tan(c/2 + (d*x)/2)^4 - (895*tan(c/2 + (
d*x)/2)^5)/192 + 6*tan(c/2 + (d*x)/2)^6 + (1765*tan(c/2 + (d*x)/2)^7)/192 +
 10*tan(c/2 + (d*x)/2)^8 - (1765*tan(c/2 + (d*x)/2)^9)/192 + 10*tan(c/2 + (
d*x)/2)^10 + (895*tan(c/2 + (d*x)/2)^11)/192 + 2*tan(c/2 + (d*x)/2)^12 - (3
97*tan(c/2 + (d*x)/2)^13)/192 + 2*tan(c/2 + (d*x)/2)^14 + (5*tan(c/2 + (d*x
)/2)^15)/64 + 2/7)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^8)
```

sympy [A] time = 111.64, size = 3888, normalized size = 32.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-105*d*x*tan(c/2 + d*x/2)**16/(2688*a*d*tan(c/2 + d*x/2)**16 + 2
1504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**12 + 150528*a*d
*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 150528*a*d*tan(c/2
 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(c/2 + d*x/2)**
2 + 2688*a*d) - 840*d*x*tan(c/2 + d*x/2)**14/(2688*a*d*tan(c/2 + d*x/2)**16
 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**12 + 150528
*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 150528*a*d*tan
```

(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(c/2 + d*x/
2)**2 + 2688*a*d) - 2940*d*x*tan(c/2 + d*x/2)**12/(2688*a*d*tan(c/2 + d*x/2
)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**12 + 1
50528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 150528*a*
d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(c/2 +
 d*x/2)**2 + 2688*a*d) - 5880*d*x*tan(c/2 + d*x/2)**10/(2688*a*d*tan(c/2 +
d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**1
2 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 1505
28*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(
c/2 + d*x/2)**2 + 2688*a*d) - 7350*d*x*tan(c/2 + d*x/2)**8/(2688*a*d*tan(c/
2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2
)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 +
150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*
tan(c/2 + d*x/2)**2 + 2688*a*d) - 5880*d*x*tan(c/2 + d*x/2)**6/(2688*a*d*ta
n(c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d
*x/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**
8 + 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*
a*d*tan(c/2 + d*x/2)**2 + 2688*a*d) - 2940*d*x*tan(c/2 + d*x/2)**4/(2688*a*
d*tan(c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2
 + d*x/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/
2)**8 + 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21
504*a*d*tan(c/2 + d*x/2)**2 + 2688*a*d) - 840*d*x*tan(c/2 + d*x/2)**2/(2688
*a*d*tan(c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(
c/2 + d*x/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d
*x/2)**8 + 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 +
 21504*a*d*tan(c/2 + d*x/2)**2 + 2688*a*d) - 105*d*x/(2688*a*d*tan(c/2 + d*
x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**12
+ 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 150528
*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(c/
2 + d*x/2)**2 + 2688*a*d) - 210*tan(c/2 + d*x/2)**15/(2688*a*d*tan(c/2 + d*
x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**12
+ 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 150528
*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(c/
2 + d*x/2)**2 + 2688*a*d) - 5376*tan(c/2 + d*x/2)**14/(2688*a*d*tan(c/2 + d
*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**12
 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 15052
8*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(c
/2 + d*x/2)**2 + 2688*a*d) + 5558*tan(c/2 + d*x/2)**13/(2688*a*d*tan(c/2 +
d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**1
2 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 1505
28*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(
c/2 + d*x/2)**2 + 2688*a*d) - 5376*tan(c/2 + d*x/2)**12/(2688*a*d*tan(c/2 +
 d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**
12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 150
528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan

(c/2 + d*x/2)**2 + 2688*a*d) - 12530*tan(c/2 + d*x/2)**11/(2688*a*d*tan(c/2
 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)
12 + 150528*a*d*tan(c/2 + d*x/2)10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 1
50528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*t
an(c/2 + d*x/2)**2 + 2688*a*d) - 26880*tan(c/2 + d*x/2)**10/(2688*a*d*tan(c
/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/
2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 +
 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d
*tan(c/2 + d*x/2)**2 + 2688*a*d) + 24710*tan(c/2 + d*x/2)**9/(2688*a*d*tan(
c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x
/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8
+ 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*
d*tan(c/2 + d*x/2)**2 + 2688*a*d) - 26880*tan(c/2 + d*x/2)**8/(2688*a*d*tan
(c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*
x/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8
 + 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a
*d*tan(c/2 + d*x/2)**2 + 2688*a*d) - 24710*tan(c/2 + d*x/2)**7/(2688*a*d*ta
n(c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d
*x/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**
8 + 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*
a*d*tan(c/2 + d*x/2)**2 + 2688*a*d) - 16128*tan(c/2 + d*x/2)**6/(2688*a*d*t
an(c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 +
d*x/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)*
*8 + 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504
*a*d*tan(c/2 + d*x/2)**2 + 2688*a*d) + 12530*tan(c/2 + d*x/2)**5/(2688*a*d*
tan(c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 +
 d*x/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)
8 + 150528*a*d*tan(c/2 + d*x/2)6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 2150
4*a*d*tan(c/2 + d*x/2)**2 + 2688*a*d) - 16128*tan(c/2 + d*x/2)**4/(2688*a*d
*tan(c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2
+ d*x/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2
)**8 + 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 215
04*a*d*tan(c/2 + d*x/2)**2 + 2688*a*d) - 5558*tan(c/2 + d*x/2)**3/(2688*a*d
*tan(c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2
+ d*x/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2
)**8 + 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 215
04*a*d*tan(c/2 + d*x/2)**2 + 2688*a*d) - 768*tan(c/2 + d*x/2)**2/(2688*a*d*
tan(c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 +
 d*x/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)
8 + 150528*a*d*tan(c/2 + d*x/2)6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 2150
4*a*d*tan(c/2 + d*x/2)**2 + 2688*a*d) + 210*tan(c/2 + d*x/2)/(2688*a*d*tan(
c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x
/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8
+ 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*
d*tan(c/2 + d*x/2)**2 + 2688*a*d) - 768/(2688*a*d*tan(c/2 + d*x/2)**16 + 21

```
504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**12 + 150528*a*d*
tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 150528*a*d*tan(c/2
+ d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(c/2 + d*x/2)**2
 + 2688*a*d), Ne(d, 0)), (x*sin(c)*cos(c)**8/(a*sin(c) + a), True))
```

3.710 $\int \frac{\cos^7(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=143

$$\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}-\frac{5\sin(c+dx)\cos^3(c+dx)}{24ad}-\frac{5\sin(c+dx)\,c}{16aa}$$

```
[Out] -5/16*x/a-arctanh(cos(d*x+c))/a/d+cos(d*x+c)/a/d+1/3*cos(d*x+c)^3/a/d+1/5*c
os(d*x+c)^5/a/d-5/16*cos(d*x+c)*sin(d*x+c)/a/d-5/24*cos(d*x+c)^3*sin(d*x+c)
/a/d-1/6*cos(d*x+c)^5*sin(d*x+c)/a/d
```

Rubi [A] time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2592, 302, 206, 2635, 8}

$$\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}-\frac{5\sin(c+dx)\cos^3(c+dx)}{24ad}-\frac{5\sin(c+dx)\,c}{16aa}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*x)/(16*a) - ArcTanh[Cos[c + d*x]]/(a*d) + Cos[c + d*x]/(a*d) + Cos[c +
d*x]^3/(3*a*d) + Cos[c + d*x]^5/(5*a*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(16
*a*d) - (5*Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) - (Cos[c + d*x]^5*Sin[c +
d*x])/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^6(c+dx)\,dx}{a} + \frac{\int \cos^5(c+dx)\cot(c+dx)\,dx}{a}$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{6ad} - \frac{5\int \cos^4(c+dx)\,dx}{6a} - \frac{\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \cos(c+\right.}{ad}$$

$$= -\frac{5\cos^3(c+dx)\sin(c+dx)}{24ad} - \frac{\cos^5(c+dx)\sin(c+dx)}{6ad} - \frac{5\int \cos^2(c+dx)\,dx}{8a} -$$

$$= \frac{\cos(c+dx)}{ad} + \frac{\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{5\cos(c+dx)\sin(c+dx)}{16ad} - \frac{5\cos}{}$$

$$= -\frac{5x}{16a} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\cos(c+dx)}{ad} + \frac{\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{5}{}$$

Mathematica [A] time = 0.27, size = 102, normalized size = 0.71

$$-\frac{225\sin(2(c+dx)) + 45\sin(4(c+dx)) + 5\sin(6(c+dx)) - 1320\cos(c+dx) - 140\cos(3(c+dx)) - 12\cos(5(c-}{960ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/960*(300*c + 300*d*x - 1320*Cos[c + d*x] - 140*Cos[3*(c + d*x)] - 12*Cos
[5*(c + d*x)] + 960*Log[Cos[(c + d*x)/2]] - 960*Log[Sin[(c + d*x)/2]] + 225
*Sin[2*(c + d*x)] + 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)])/(a*d)
```

fricas [A] time = 0.50, size = 104, normalized size = 0.73

$$\frac{48\cos(dx+c)^5+80\cos(dx+c)^3-75\,dx-5\left(8\cos(dx+c)^5+10\cos(dx+c)^3+15\cos(dx+c)\right)\sin(dx+}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*cos(d*x + c)^5 + 80*cos(d*x + c)^3 - 75*d*x - 5*(8*cos(d*x + c)^5
 + 10*cos(d*x + c)^3 + 15*cos(d*x + c))*sin(d*x + c) + 240*cos(d*x + c) - 1
20*log(1/2*cos(d*x + c) + 1/2) + 120*log(-1/2*cos(d*x + c) + 1/2))/(a*d)
```

giac [A] time = 0.17, size = 195, normalized size = 1.36

$$-\frac{\frac{75(dx+c)}{a}-\frac{240\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{2\left(165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+720\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-25\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+2160\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+450\tan\left(\frac{1}{2}\right.\right.}{}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(75*(d*x + c)/a - 240*log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(165*tan(
1/2*d*x + 1/2*c)^11 + 720*tan(1/2*d*x + 1/2*c)^10 - 25*tan(1/2*d*x + 1/2*c)
^9 + 2160*tan(1/2*d*x + 1/2*c)^8 + 450*tan(1/2*d*x + 1/2*c)^7 + 3680*tan(1/
2*d*x + 1/2*c)^6 - 450*tan(1/2*d*x + 1/2*c)^5 + 3360*tan(1/2*d*x + 1/2*c)^4
 + 25*tan(1/2*d*x + 1/2*c)^3 + 1488*tan(1/2*d*x + 1/2*c)^2 - 165*tan(1/2*d*
x + 1/2*c) + 368)/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a))/d
```

maple [B] time = 0.44, size = 432, normalized size = 3.02

$$\frac{11\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{6\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{5\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{18\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{15}{4ad\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 11/8/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11+6/a/d/(1+tan(1/2*
d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10-5/24/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*t
an(1/2*d*x+1/2*c)^9+18/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8+
15/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7+92/3/a/d/(1+tan(1/
2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6-15/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*
tan(1/2*d*x+1/2*c)^5+28/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4
+5/24/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+62/5/a/d/(1+tan(1
/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2-11/8/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6
*tan(1/2*d*x+1/2*c)+46/15/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6-5/8/a/d*arctan(tan
(1/2*d*x+1/2*c))+1/a/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 402, normalized size = 2.81

$$-\frac{\frac{\frac{165 \sin(dx+c)}{\cos(dx+c)+1} - \frac{1488 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{25 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{3360 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{450 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{3680 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{450 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{2160 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{25 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{720 \sin(dx+c)}{(\cos(dx+c)}}{a + \frac{6 a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{15 a \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{20 a \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{15 a \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{6 a \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{a \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}}{120 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/120*((165*sin(d*x + c)/(cos(d*x + c) + 1) - 1488*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 25*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3360*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 450*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3680*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 - 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
 2160*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 25*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 - 720*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 165*sin(d*x + c)^11/(
cos(d*x + c) + 1)^11 - 368)/(a + 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
15*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a*sin(d*x + c)^10/(
cos(d*x + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) + 75*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a - 120*log(sin(d*x + c)/(cos(d*x + c) +
1))/a)/d
```

mupad [B] time = 11.83, size = 305, normalized size = 2.13

$$\frac{5 \operatorname{atan}\left(\frac{25}{64 \left(\frac{25 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)}{64}+\frac{5}{4}\right)} - \frac{5 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)}{4 \left(\frac{25 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)}{64}+\frac{5}{4}\right)}\right)}{8 a d} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d x}{2}\right)\right)}{a d} + \frac{\frac{11 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{11}}{8} + 6 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{10} - \frac{5 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{9}}{24}}{d \left(a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{12} +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] (5*atan(25/(64*((25*tan(c/2 + (d*x)/2))/64 + 5/4)) - (5*tan(c/2 + (d*x)/2))
/(4*((25*tan(c/2 + (d*x)/2))/64 + 5/4))))/(8*a*d) + log(tan(c/2 + (d*x)/2))
/(a*d) + ((62*tan(c/2 + (d*x)/2)^2)/5 - (11*tan(c/2 + (d*x)/2))/8 + (5*tan(
c/2 + (d*x)/2)^3)/24 + 28*tan(c/2 + (d*x)/2)^4 - (15*tan(c/2 + (d*x)/2)^5)/
4 + (92*tan(c/2 + (d*x)/2)^6)/3 + (15*tan(c/2 + (d*x)/2)^7)/4 + 18*tan(c/2
+ (d*x)/2)^8 - (5*tan(c/2 + (d*x)/2)^9)/24 + 6*tan(c/2 + (d*x)/2)^10 + (11*
tan(c/2 + (d*x)/2)^11)/8 + 46/15)/(d*(a + 6*a*tan(c/2 + (d*x)/2)^2 + 15*a*t
an(c/2 + (d*x)/2)^4 + 20*a*tan(c/2 + (d*x)/2)^6 + 15*a*tan(c/2 + (d*x)/2)^8
 + 6*a*tan(c/2 + (d*x)/2)^10 + a*tan(c/2 + (d*x)/2)^12))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.711 $\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$-\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}-\frac{15\cot(c+dx)}{8ad}+\frac{\cos^4(c+dx)\cot(c+dx)}{4ad}+\frac{5\cos^2(c+dx)\cot(c+dx)}{8ad}$$

```
[Out] -15/8*x/a+arctanh(cos(d*x+c))/a/d-cos(d*x+c)/a/d-1/3*cos(d*x+c)^3/a/d-1/5*c
os(d*x+c)^5/a/d-15/8*cot(d*x+c)/a/d+5/8*cos(d*x+c)^2*cot(d*x+c)/a/d+1/4*cos
(d*x+c)^4*cot(d*x+c)/a/d
```

Rubi [A] time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2839, 2591, 288, 321, 203, 2592, 302, 206}

$$-\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}-\frac{15\cot(c+dx)}{8ad}+\frac{\cos^4(c+dx)\cot(c+dx)}{4ad}+\frac{5\cos^2(c+dx)\cot(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-15*x)/(8*a) + ArcTanh[Cos[c + d*x]]/(a*d) - Cos[c + d*x]/(a*d) - Cos[c +
d*x]^3/(3*a*d) - Cos[c + d*x]^5/(5*a*d) - (15*Cot[c + d*x])/(8*a*d) + (5*Co
s[c + d*x]^2*Cot[c + d*x])/(8*a*d) + (Cos[c + d*x]^4*Cot[c + d*x])/(4*a*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^5(c+dx)\cot(c+dx)\,dx}{a} + \frac{\int \cos^4(c+dx)\cot^2(c+dx)\,dx}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \cos(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^3}\,dx, x, \cot(c+dx)\right)}{ad}$$

$$= \frac{\cos^4(c+dx)\cot(c+dx)}{4ad} + \frac{\text{Subst}\left(\int \left(-1-x^2-x^4+\frac{1}{1-x^2}\right)dx, x, \cos(c+dx)\right)}{ad}$$

$$= -\frac{\cos(c+dx)}{ad} - \frac{\cos^3(c+dx)}{3ad} - \frac{\cos^5(c+dx)}{5ad} + \frac{5\cos^2(c+dx)\cot(c+dx)}{8ad} + \frac{\text{co}}{}$$

$$= \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\cos(c+dx)}{ad} - \frac{\cos^3(c+dx)}{3ad} - \frac{\cos^5(c+dx)}{5ad} - \frac{15\cot(c+}{8ad}$$

$$= -\frac{15x}{8a} + \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\cos(c+dx)}{ad} - \frac{\cos^3(c+dx)}{3ad} - \frac{\cos^5(c+dx)}{5ad} - \frac{1}{}$$

Mathematica [A] time = 0.70, size = 146, normalized size = 1.07

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(1800c\sin(c+dx)+1800dx\sin(c+dx)+590\sin(2(c+dx))+64\sin(4(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/1920*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(1200*Cos[c + d*x] - 225*Cos[3*(
c + d*x)] - 15*Cos[5*(c + d*x)] + 1800*c*Sin[c + d*x] + 1800*d*x*Sin[c + d*
x] - 960*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 960*Log[Sin[(c + d*x)/2]]*Sin
[c + d*x] + 590*Sin[2*(c + d*x)] + 64*Sin[4*(c + d*x)] + 6*Sin[6*(c + d*x)]
))/(a*d)
```

fricas [A] time = 0.48, size = 124, normalized size = 0.91

$$\frac{30\cos(dx+c)^5 + 75\cos(dx+c)^3 - \left(24\cos(dx+c)^5 + 40\cos(dx+c)^3 + 225dx + 120\cos(dx+c)\right)\sin(dx+}{120\,ad\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(30*cos(d*x + c)^5 + 75*cos(d*x + c)^3 - (24*cos(d*x + c)^5 + 40*cos(
d*x + c)^3 + 225*d*x + 120*cos(d*x + c))*sin(d*x + c) + 60*log(1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) - 60*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2
25*cos(d*x + c))/(a*d*sin(d*x + c))
```

giac [A] time = 0.19, size = 199, normalized size = 1.45

$$-\frac{\frac{225\,(dx+c)}{a}+\frac{120\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{60\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{60\left(2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-360\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+150}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(225*(d*x + c)/a + 120*log(abs(tan(1/2*d*x + 1/2*c)))/a - 60*tan(1/2
*d*x + 1/2*c)/a - 60*(2*tan(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c))
- 2*(135*tan(1/2*d*x + 1/2*c)^9 - 360*tan(1/2*d*x + 1/2*c)^8 + 150*tan(1/2*
d*x + 1/2*c)^7 - 720*tan(1/2*d*x + 1/2*c)^6 - 1120*tan(1/2*d*x + 1/2*c)^4 -
 150*tan(1/2*d*x + 1/2*c)^3 - 560*tan(1/2*d*x + 1/2*c)^2 - 135*tan(1/2*d*x
+ 1/2*c) - 184)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a))/d
```

maple [B] time = 0.45, size = 367, normalized size = 2.68

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{9\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{6\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{5\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{12\left(\tan^{6}\left(\frac{dx}{2}+\right.\right.}{ad\left(1+\tan^{2}\left(\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)+9/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1
/2*c)^9-6/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8+5/2/a/d/(1+ta
n(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7-12/a/d/(1+tan(1/2*d*x+1/2*c)^2)^
5*tan(1/2*d*x+1/2*c)^6-56/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*
c)^4-5/2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3-28/3/a/d/(1+ta
n(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2-9/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)
^5*tan(1/2*d*x+1/2*c)-46/15/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5-15/4/a/d*arctan(
tan(1/2*d*x+1/2*c))-1/2/a/d/tan(1/2*d*x+1/2*c)-1/a/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 379, normalized size = 2.77

$$-\frac{\frac{\frac{184\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{285\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{560\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{450\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1120\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{300\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{720\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{360\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{105\,\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+30}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10\,a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{10\,a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{5\,a\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{a\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}+\frac{225}{\cdots}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((184*sin(d*x + c)/(cos(d*x + c) + 1) + 285*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 560*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 450*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 1120*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 300*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 720*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3
60*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 105*sin(d*x + c)^10/(cos(d*x + c)
+ 1)^10 + 30)/(a*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 10*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*a*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 5*a*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + a*si
n(d*x + c)^11/(cos(d*x + c) + 1)^11) + 225*arctan(sin(d*x + c)/(cos(d*x + c
) + 1))/a + 60*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - 30*sin(d*x + c)/(a*
(cos(d*x + c) + 1)))/d
```

mupad [B] time = 9.04, size = 296, normalized size = 2.16

$$\frac{15\,\mathrm{atan}\left(\frac{15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{225\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}-\frac{15}{2}\right)}+\frac{225}{16\left(\frac{225\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}-\frac{15}{2}\right)}\right)}{4\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{2}+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+24\tan\cdots}{d\left(2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+10\,a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] (15*atan((15*tan(c/2 + (d*x)/2))/(2*((225*tan(c/2 + (d*x)/2))/16 - 15/2)) +
 225/(16*((225*tan(c/2 + (d*x)/2))/16 - 15/2))))/(4*a*d) - log(tan(c/2 + (d
*x)/2))/(a*d) - ((92*tan(c/2 + (d*x)/2))/15 + (19*tan(c/2 + (d*x)/2)^2)/2 +
 (56*tan(c/2 + (d*x)/2)^3)/3 + 15*tan(c/2 + (d*x)/2)^4 + (112*tan(c/2 + (d*
x)/2)^5)/3 + 10*tan(c/2 + (d*x)/2)^6 + 24*tan(c/2 + (d*x)/2)^7 + 12*tan(c/2
 + (d*x)/2)^9 - (7*tan(c/2 + (d*x)/2)^10)/2 + 1)/(d*(2*a*tan(c/2 + (d*x)/2)
 + 10*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^5 + 20*a*tan(c/2 + (
d*x)/2)^7 + 10*a*tan(c/2 + (d*x)/2)^9 + 2*a*tan(c/2 + (d*x)/2)^11)) + tan(c
/2 + (d*x)/2)/(2*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.712 $\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=150

$$-\frac{5\cos^3(c+dx)}{6ad}-\frac{5\cos(c+dx)}{2ad}+\frac{15\cot(c+dx)}{8ad}-\frac{\cos^4(c+dx)\cot(c+dx)}{4ad}-\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}-\frac{5\cos^2(c+$$

```
[Out] 15/8*x/a+5/2*arctanh(cos(d*x+c))/a/d-5/2*cos(d*x+c)/a/d-5/6*cos(d*x+c)^3/a/
d+15/8*cot(d*x+c)/a/d-5/8*cos(d*x+c)^2*cot(d*x+c)/a/d-1/4*cos(d*x+c)^4*cot(
d*x+c)/a/d-1/2*cos(d*x+c)^3*cot(d*x+c)^2/a/d
```

Rubi [A] time = 0.19, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2839, 2592, 288, 302, 206, 2591, 321, 203}

$$-\frac{5\cos^3(c+dx)}{6ad}-\frac{5\cos(c+dx)}{2ad}+\frac{15\cot(c+dx)}{8ad}-\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}-\frac{\cos^4(c+dx)\cot(c+dx)}{4ad}-\frac{5\cos^2(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (15*x)/(8*a) + (5*ArcTanh[Cos[c + d*x]])/(2*a*d) - (5*Cos[c + d*x])/(2*a*d)
 - (5*Cos[c + d*x]^3)/(6*a*d) + (15*Cot[c + d*x])/(8*a*d) - (5*Cos[c + d*x]
^2*Cot[c + d*x])/(8*a*d) - (Cos[c + d*x]^4*Cot[c + d*x])/(4*a*d) - (Cos[c +
 d*x]^3*Cot[c + d*x]^2)/(2*a*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^4(c+dx)\cot^2(c+dx)\,dx}{a} + \frac{\int \cos^3(c+dx)\cot^3(c+dx)\,dx}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^3}\,dx, x, \cot(c+dx)\right)}{ad}$$

$$= -\frac{\cos^4(c+dx)\cot(c+dx)}{4ad} - \frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx\right.}{4ad}$$

$$= -\frac{5\cos^2(c+dx)\cot(c+dx)}{8ad} - \frac{\cos^4(c+dx)\cot(c+dx)}{4ad} - \frac{\cos^3(c+dx)\cot^2(c+}{2ad}$$

$$= -\frac{5\cos(c+dx)}{2ad} - \frac{5\cos^3(c+dx)}{6ad} + \frac{15\cot(c+dx)}{8ad} - \frac{5\cos^2(c+dx)\cot(c+dx)}{8ad}$$

$$= \frac{15x}{8a} + \frac{5\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{5\cos(c+dx)}{2ad} - \frac{5\cos^3(c+dx)}{6ad} + \frac{15\cot(c+d}{8ad}$$

Mathematica [A] time = 0.53, size = 179, normalized size = 1.19

$$-\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-285\sin(2(c+dx))+42\sin(4(c+dx))+3\sin(6(c+dx))+400\cos(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/1536*((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(-360*c - 360*d*x + 400*Co
s[c + d*x] - 200*Cos[3*(c + d*x)] - 8*Cos[5*(c + d*x)] - 480*Log[Cos[(c + d
*x)/2]] + 120*Cos[2*(c + d*x)]*(3*c + 3*d*x + 4*Log[Cos[(c + d*x)/2]] - 4*L
og[Sin[(c + d*x)/2]]) + 480*Log[Sin[(c + d*x)/2]] - 285*Sin[2*(c + d*x)] +
42*Sin[4*(c + d*x)] + 3*Sin[6*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.49, size = 148, normalized size = 0.99

$$-\frac{8\cos(dx+c)^5 - 45\,dx\cos(dx+c)^2 + 40\cos(dx+c)^3 + 45\,dx - 30\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*cos(d*x + c)^5 - 45*d*x*cos(d*x + c)^2 + 40*cos(d*x + c)^3 + 45*d*
x - 30*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 30*(cos(d*x + c)^
2 - 1)*log(-1/2*cos(d*x + c) + 1/2) - 3*(2*cos(d*x + c)^5 + 5*cos(d*x + c)^
3 - 15*cos(d*x + c))*sin(d*x + c) - 60*cos(d*x + c))/(a*d*cos(d*x + c)^2 -
a*d)
```

giac [A] time = 0.21, size = 216, normalized size = 1.44

$$\frac{\frac{45\,(dx+c)}{a} - \frac{60\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} + \frac{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^2} + \frac{3\left(30\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2} - \frac{2\left(27\,\tan\left(\frac{1}{2}\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(45*(d*x + c)/a - 60*log(abs(tan(1/2*d*x + 1/2*c)))/a + 3*(a*tan(1/2*d
*x + 1/2*c)^2 - 4*a*tan(1/2*d*x + 1/2*c))/a^2 + 3*(30*tan(1/2*d*x + 1/2*c)^
2 + 4*tan(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)^2) - 2*(27*tan(1/2*
d*x + 1/2*c)^7 + 72*tan(1/2*d*x + 1/2*c)^6 + 3*tan(1/2*d*x + 1/2*c)^5 + 168
*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*x + 1/2*c)^3 + 152*tan(1/2*d*x + 1/2*
c)^2 - 27*tan(1/2*d*x + 1/2*c) + 56)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

maple [B] time = 0.56, size = 371, normalized size = 2.47

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad} - \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad} - \frac{9\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4} - \frac{6\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4} - \frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/8/a/d*tan(1/2*d*x+1/2*c)^2-1/2/a/d*tan(1/2*d*x+1/2*c)-9/4/a/d/(1+tan(1/2*
d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-6/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1
/2*d*x+1/2*c)^6-1/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-14/
a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4+1/4/a/d/(1+tan(1/2*d*x+
1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-38/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/
2*d*x+1/2*c)^2+9/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-14/3/a
/d/(1+tan(1/2*d*x+1/2*c)^2)^4+15/4/a/d*arctan(tan(1/2*d*x+1/2*c))-1/8/a/d/t
an(1/2*d*x+1/2*c)^2+1/2/a/d/tan(1/2*d*x+1/2*c)-5/2/a/d*ln(tan(1/2*d*x+1/2*c
))
```

maxima [B] time = 0.42, size = 383, normalized size = 2.55

$$\frac{\frac{\frac{12 \sin(dx+c)}{\cos(dx+c)+1} - \frac{124 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{102 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{322 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{78 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{348 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{42 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{147 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{42 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} - 3}{\frac{a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{4 a \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{6 a \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{4 a \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{a \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}} - \frac{3\left(\frac{4 \sin(d\cdots}{\cos(dx+\cdots}\right.}{\cdots}}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/24*((12*sin(d*x + c)/(cos(d*x + c) + 1) - 124*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 102*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 322*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 78*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 348*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 42*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 147*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 - 42*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
- 3)/(a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 + 6*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 4*a*sin(d*x + c)^8/(
cos(d*x + c) + 1)^8 + a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 3*(4*sin(d
*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a + 90*ar
ctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 60*log(sin(d*x + c)/(cos(d*x + c)
 + 1))/a)/d
```

mupad [B] time = 9.02, size = 303, normalized size = 2.02

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d} - \frac{15\,\mathrm{atan}\left(\frac{225}{16\left(\frac{225\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}+\frac{75}{4}\right)} - \frac{75\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\left(\frac{225\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}+\frac{75}{4}\right)}\right)}{4\,a\,d} - \frac{5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d} - \frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9 + \frac{49\tan\left(\cdots\right.}{2} \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - (15*atan(225/(16*((225*tan(c/2 + (d*x)/2))/1
6 + 75/4)) - (75*tan(c/2 + (d*x)/2))/(4*((225*tan(c/2 + (d*x)/2))/16 + 75/4
))))/(4*a*d) - (5*log(tan(c/2 + (d*x)/2)))/(2*a*d) - ((62*tan(c/2 + (d*x)/2
)^2)/3 - 2*tan(c/2 + (d*x)/2) - 17*tan(c/2 + (d*x)/2)^3 + (161*tan(c/2 + (d
*x)/2)^4)/3 - 13*tan(c/2 + (d*x)/2)^5 + 58*tan(c/2 + (d*x)/2)^6 - 7*tan(c/2
 + (d*x)/2)^7 + (49*tan(c/2 + (d*x)/2)^8)/2 + 7*tan(c/2 + (d*x)/2)^9 + 1/2)
/(d*(4*a*tan(c/2 + (d*x)/2)^2 + 16*a*tan(c/2 + (d*x)/2)^4 + 24*a*tan(c/2 +
(d*x)/2)^6 + 16*a*tan(c/2 + (d*x)/2)^8 + 4*a*tan(c/2 + (d*x)/2)^10)) - tan(
c/2 + (d*x)/2)/(2*a*d)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.713 $\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=146

$$\frac{5\cos^3(c+dx)}{6ad}+\frac{5\cos(c+dx)}{2ad}-\frac{5\cot^3(c+dx)}{6ad}+\frac{5\cot(c+dx)}{2ad}+\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot^3(c+d}{2ad}$$

```
[Out] 5/2*x/a-5/2*arctanh(cos(d*x+c))/a/d+5/2*cos(d*x+c)/a/d+5/6*cos(d*x+c)^3/a/d
+5/2*cot(d*x+c)/a/d+1/2*cos(d*x+c)^3*cot(d*x+c)^2/a/d-5/6*cot(d*x+c)^3/a/d+
1/2*cos(d*x+c)^2*cot(d*x+c)^3/a/d
```

Rubi [A] time = 0.18, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2839, 2591, 288, 302, 203, 2592, 206}

$$\frac{5\cos^3(c+dx)}{6ad}+\frac{5\cos(c+dx)}{2ad}-\frac{5\cot^3(c+dx)}{6ad}+\frac{5\cot(c+dx)}{2ad}+\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot^3(c+d}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*x)/(2*a) - (5*ArcTanh[Cos[c + d*x]])/(2*a*d) + (5*Cos[c + d*x])/(2*a*d)
+ (5*Cos[c + d*x]^3)/(6*a*d) + (5*Cot[c + d*x])/(2*a*d) + (Cos[c + d*x]^3*C
ot[c + d*x]^2)/(2*a*d) - (5*Cot[c + d*x]^3)/(6*a*d) + (Cos[c + d*x]^2*Cot[c
 + d*x]^3)/(2*a*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^3(c+dx)\cot^3(c+dx)\,dx}{a} + \frac{\int \cos^2(c+dx)\cot^4(c+dx)\,dx}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{ad}$$

$$= \frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad} + \frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x,\right.}{2ad}$$

$$= \frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad} + \frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad} - \frac{5\,\text{Subst}\left(\int\left(-1-x^2+\right.\right.}{}$$

$$= \frac{5\cos(c+dx)}{2ad} + \frac{5\cos^3(c+dx)}{6ad} + \frac{5\cot(c+dx)}{2ad} + \frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad} -$$

$$= \frac{5x}{2a} - \frac{5\tanh^{-1}(\cos(c+dx))}{2ad} + \frac{5\cos(c+dx)}{2ad} + \frac{5\cos^3(c+dx)}{6ad} + \frac{5\cot(c+dx)}{2ad}$$

Mathematica [A] time = 0.81, size = 197, normalized size = 1.35

$$-\frac{\csc^3(c+dx)\left(-180c\sin(c+dx) - 180dx\sin(c+dx) - 75\sin(2(c+dx)) + 60c\sin(3(c+dx)) + 60dx\sin(3(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/96*(Csc[c + d*x]^3*(-30*Cos[c + d*x] + 65*Cos[3*(c + d*x)] - 3*Cos[5*(c
+ d*x)] - 180*c*Sin[c + d*x] - 180*d*x*Sin[c + d*x] + 180*Log[Cos[(c + d*x)
/2]]*Sin[c + d*x] - 180*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] - 75*Sin[2*(c +
d*x)] + 60*c*Sin[3*(c + d*x)] + 60*d*x*Sin[3*(c + d*x)] - 60*Log[Cos[(c + d
*x)/2]]*Sin[3*(c + d*x)] + 60*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 24*S
in[4*(c + d*x)] + Sin[6*(c + d*x)]))/(a*d)
```

fricas [A] time = 0.49, size = 168, normalized size = 1.15

$$-\frac{6\cos(dx+c)^5 - 40\cos(dx+c)^3 + 15\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 15\left(\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*(cos(d*x + c)^2 - 1)*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(cos(d*x + c)^2 - 1)*log(-1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) - 2*(2*cos(d*x + c)^5 + 15*d*x*cos(d*x + c)^2
+ 10*cos(d*x + c)^3 - 15*d*x - 15*cos(d*x + c))*sin(d*x + c) + 30*cos(d*x +
 c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

giac [A] time = 0.38, size = 228, normalized size = 1.56

$$\frac{180\,(dx+c)}{a}+\frac{180\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{3\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-27\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^3}-\frac{110\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-9\,\tan\left(\frac{1}{2}\,dx\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/72*(180*(d*x + c)/a + 180*log(abs(tan(1/2*d*x + 1/2*c)))/a + 3*(a^2*tan(1
/2*d*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 27*a^2*tan(1/2*d*x + 1/2
*c))/a^3 - (110*tan(1/2*d*x + 1/2*c)^9 - 9*tan(1/2*d*x + 1/2*c)^8 - 111*tan
(1/2*d*x + 1/2*c)^7 - 240*tan(1/2*d*x + 1/2*c)^6 - 273*tan(1/2*d*x + 1/2*c)
^5 - 306*tan(1/2*d*x + 1/2*c)^4 - 253*tan(1/2*d*x + 1/2*c)^3 - 72*tan(1/2*d
*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 3)/((tan(1/2*d*x + 1/2*c)^3 + tan(
1/2*d*x + 1/2*c))^3*a))/d
```

maple [B] time = 0.64, size = 306, normalized size = 2.10

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{6\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{8\left(\tan^2\left(\cdots\right.\right.}{ad\left(1+\tan^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/24/a/d*tan(1/2*d*x+1/2*c)^3-1/8/a/d*tan(1/2*d*x+1/2*c)^2-9/8/a/d*tan(1/2*
d*x+1/2*c)-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+6/a/d/(1+t
an(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4+8/a/d/(1+tan(1/2*d*x+1/2*c)^2)^
3*tan(1/2*d*x+1/2*c)^2+1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)+
14/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3+5/a/d*arctan(tan(1/2*d*x+1/2*c))-1/24/a
/d/tan(1/2*d*x+1/2*c)^3+1/8/a/d/tan(1/2*d*x+1/2*c)^2+9/8/a/d/tan(1/2*d*x+1/
2*c)+5/2/a/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 362, normalized size = 2.48

$$-\frac{\frac{\frac{27\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a}-\frac{\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{24\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{121\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{102\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{201\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{80\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{147\,\sin(dx+c)}{(\cos(dx+c)+1)}}{\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\,a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/24*((27*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - (3*sin(d*x + c)/(cos(d*x
 + c) + 1) + 24*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 121*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 + 102*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 201*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 + 80*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 147*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 - 1)/(a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 3*a*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a*sin(d*x + c)^9/(c
os(d*x + c) + 1)^9) - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 60*lo
g(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 9.06, size = 290, normalized size = 1.99

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{5\,\mathrm{atan}\left(\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-25}+\frac{25}{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-25}\right)}{a\,d}+\frac{5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - (5*atan((25*
tan(c/2 + (d*x)/2))/(25*tan(c/2 + (d*x)/2) - 25) + 25/(25*tan(c/2 + (d*x)/2
) - 25)))/(a*d) + (5*log(tan(c/2 + (d*x)/2)))/(2*a*d) + (tan(c/2 + (d*x)/2)
 + 8*tan(c/2 + (d*x)/2)^2 + (121*tan(c/2 + (d*x)/2)^3)/3 + 34*tan(c/2 + (d*
x)/2)^4 + 67*tan(c/2 + (d*x)/2)^5 + (80*tan(c/2 + (d*x)/2)^6)/3 + 49*tan(c/
2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8 - 1/3)/(d*(8*a*tan(c/2 + (d*x)/2)^3 +
 24*a*tan(c/2 + (d*x)/2)^5 + 24*a*tan(c/2 + (d*x)/2)^7 + 8*a*tan(c/2 + (d*x
)/2)^9)) - (9*tan(c/2 + (d*x)/2))/(8*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.714 $\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=150

$$\frac{15\cos(c+dx)}{8ad}+\frac{5\cot^3(c+dx)}{6ad}-\frac{5\cot(c+dx)}{2ad}-\frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad}-\frac{\cos(c+dx)\cot^4(c+dx)}{4ad}+\frac{5\cos(c+dx)}{8}$$

```
[Out] -5/2*x/a-15/8*arctanh(cos(d*x+c))/a/d+15/8*cos(d*x+c)/a/d-5/2*cot(d*x+c)/a/
d+5/8*cos(d*x+c)*cot(d*x+c)^2/a/d+5/6*cot(d*x+c)^3/a/d-1/2*cos(d*x+c)^2*cot
(d*x+c)^3/a/d-1/4*cos(d*x+c)*cot(d*x+c)^4/a/d
```

Rubi [A] time = 0.18, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2839, 2592, 288, 321, 206, 2591, 302, 203}

$$\frac{15\cos(c+dx)}{8ad}+\frac{5\cot^3(c+dx)}{6ad}-\frac{5\cot(c+dx)}{2ad}-\frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad}-\frac{\cos(c+dx)\cot^4(c+dx)}{4ad}+\frac{5\cos(c+dx)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*x)/(2*a) - (15*ArcTanh[Cos[c + d*x]])/(8*a*d) + (15*Cos[c + d*x])/(8*a*
d) - (5*Cot[c + d*x])/(2*a*d) + (5*Cos[c + d*x]*Cot[c + d*x]^2)/(8*a*d) + (
5*Cot[c + d*x]^3)/(6*a*d) - (Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*a*d) - (Cos[
c + d*x]*Cot[c + d*x]^4)/(4*a*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^2(c+dx)\cot^4(c+dx)\,dx}{a} + \frac{\int \cos(c+dx)\cot^5(c+dx)\,dx}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{ad}$$

$$= -\frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad} - \frac{\cos(c+dx)\cot^4(c+dx)}{4ad} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx\right.}{4ad}$$

$$= \frac{5\cos(c+dx)\cot^2(c+dx)}{8ad} - \frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad} - \frac{\cos(c+dx)\cot^4(c+}{4ad}$$

$$= \frac{15\cos(c+dx)}{8ad} - \frac{5\cot(c+dx)}{2ad} + \frac{5\cos(c+dx)\cot^2(c+dx)}{8ad} + \frac{5\cot^3(c+dx)}{6ad} -$$

$$= -\frac{5x}{2a} - \frac{15\tanh^{-1}(\cos(c+dx))}{8ad} + \frac{15\cos(c+dx)}{8ad} - \frac{5\cot(c+dx)}{2ad} + \frac{5\cos(c+d}{}$$

Mathematica [A] time = 0.75, size = 252, normalized size = 1.68

$$-\frac{\csc^4(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(95\sin(2(c+dx))-68\sin(4(c+dx))+3\sin(6(c+dx))+60\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/192*(Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(180*c + 180
*d*x - 30*Cos[c + d*x] + 90*Cos[3*(c + d*x)] + 60*c*Cos[4*(c + d*x)] + 60*d
*x*Cos[4*(c + d*x)] - 12*Cos[5*(c + d*x)] + 135*Log[Cos[(c + d*x)/2]] + 45*
Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 60*Cos[2*(c + d*x)]*(4*c + 4*d*x +
 3*Log[Cos[(c + d*x)/2]] - 3*Log[Sin[(c + d*x)/2]]) - 135*Log[Sin[(c + d*x)
/2]] - 45*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 95*Sin[2*(c + d*x)] - 68
*Sin[4*(c + d*x)] + 3*Sin[6*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.46, size = 191, normalized size = 1.27

$$-\frac{120\,dx\cos(dx+c)^4 - 48\cos(dx+c)^5 - 240\,dx\cos(dx+c)^2 + 150\cos(dx+c)^3 + 120\,dx + 45\left(\cos(dx+c)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(120*d*x*cos(d*x + c)^4 - 48*cos(d*x + c)^5 - 240*d*x*cos(d*x + c)^2
+ 150*cos(d*x + c)^3 + 120*d*x + 45*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)
*log(1/2*cos(d*x + c) + 1/2) - 45*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*l
og(-1/2*cos(d*x + c) + 1/2) + 8*(3*cos(d*x + c)^5 - 20*cos(d*x + c)^3 + 15*
cos(d*x + c))*sin(d*x + c) - 90*cos(d*x + c))/(a*d*cos(d*x + c)^4 - 2*a*d*c
os(d*x + c)^2 + a*d)
```

giac [A] time = 0.22, size = 224, normalized size = 1.49

$$-\frac{\frac{480\,(dx+c)}{a} - \frac{360\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} - \frac{192\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a} - \frac{3\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-8\,a^{3}\tan\left(\frac{1}{2}\,dx+\right.}{}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(480*(d*x + c)/a - 360*log(abs(tan(1/2*d*x + 1/2*c)))/a - 192*(tan(1
/2*d*x + 1/2*c)^3 + 2*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 2)/((
tan(1/2*d*x + 1/2*c)^2 + 1)^2*a) - (3*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 48*a^3*tan(1/2*d*x + 1/2*c)^2 + 216*a^3*tan(1/2*d*x
+ 1/2*c))/a^4 + (750*tan(1/2*d*x + 1/2*c)^4 + 216*tan(1/2*d*x + 1/2*c)^3 -
48*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*d*x + 1/
2*c)^4))/d
```

maple [B] time = 0.55, size = 310, normalized size = 2.07

$$\frac{\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{2\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/64/a/d*tan(1/2*d*x+1/2*c)^4-1/24/a/d*tan(1/2*d*x+1/2*c)^3-1/4/a/d*tan(1/2
*d*x+1/2*c)^2+9/8/a/d*tan(1/2*d*x+1/2*c)+1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*t
an(1/2*d*x+1/2*c)^3+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2-1
/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+2/a/d/(1+tan(1/2*d*x+1/2
*c)^2)^2-5/a/d*arctan(tan(1/2*d*x+1/2*c))-1/64/a/d/tan(1/2*d*x+1/2*c)^4+1/2
4/a/d/tan(1/2*d*x+1/2*c)^3+1/4/a/d/tan(1/2*d*x+1/2*c)^2-9/8/a/d/tan(1/2*d*x
+1/2*c)+15/8/a/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.42, size = 340, normalized size = 2.27

$$\frac{\frac{\frac{216 \sin(dx+c)}{\cos(dx+c)+1}-\frac{48 \sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8 \sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3 \sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}{a}+\frac{\frac{8 \sin(dx+c)}{\cos(dx+c)+1}+\frac{42 \sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{200 \sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{477 \sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{616 \sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{432 \sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}{\frac{a \sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{2 a \sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a \sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}}{192\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/192*((216*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4)/a + (8*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 477*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 - 616*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4
32*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 24*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 - 3)/(a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*a*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 960*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a + 360*log(sin(d*x + c)/(cos(d*x + c) + 1))/a)
/d
```

mupad [B] time = 9.03, size = 286, normalized size = 1.91

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a\,d}+\frac{5\,\mathrm{atan}\left(\frac{25}{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{75}{4}}-\frac{75\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\left(25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{75}{4}\right)}\right)}{a\,d}+\frac{15\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^5*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^4/(64*a*d) - tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2 + (
d*x)/2)^2/(4*a*d) + (5*atan(25/(25*tan(c/2 + (d*x)/2) + 75/4) - (75*tan(c/2
 + (d*x)/2))/(4*(25*tan(c/2 + (d*x)/2) + 75/4))))/(a*d) + (15*log(tan(c/2 +
 (d*x)/2)))/(8*a*d) + ((2*tan(c/2 + (d*x)/2))/3 + (7*tan(c/2 + (d*x)/2)^2)/
2 - (50*tan(c/2 + (d*x)/2)^3)/3 + (159*tan(c/2 + (d*x)/2)^4)/4 - (154*tan(c
/2 + (d*x)/2)^5)/3 + 36*tan(c/2 + (d*x)/2)^6 - 2*tan(c/2 + (d*x)/2)^7 - 1/4
)/(d*(16*a*tan(c/2 + (d*x)/2)^4 + 32*a*tan(c/2 + (d*x)/2)^6 + 16*a*tan(c/2
+ (d*x)/2)^8)) + (9*tan(c/2 + (d*x)/2))/(8*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.715 $\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=138

$$-\frac{15\cos(c+dx)}{8ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^4(c+dx)}{4ad}-\frac{5\cos(c+dx)\cot^2(c+dx)}{8ad}-$$

```
[Out] -x/a+15/8*arctanh(cos(d*x+c))/a/d-15/8*cos(d*x+c)/a/d-cot(d*x+c)/a/d-5/8*co
s(d*x+c)*cot(d*x+c)^2/a/d+1/3*cot(d*x+c)^3/a/d+1/4*cos(d*x+c)*cot(d*x+c)^4/
a/d-1/5*cot(d*x+c)^5/a/d
```

Rubi [A] time = 0.15, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2839, 3473, 8, 2592, 288, 321, 206}

$$-\frac{15\cos(c+dx)}{8ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^4(c+dx)}{4ad}-\frac{5\cos(c+dx)\cot^2(c+dx)}{8ad}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) + (15*ArcTanh[Cos[c + d*x]])/(8*a*d) - (15*Cos[c + d*x])/(8*a*d) - C
ot[c + d*x]/(a*d) - (5*Cos[c + d*x]*Cot[c + d*x]^2)/(8*a*d) + Cot[c + d*x]^
3/(3*a*d) + (Cos[c + d*x]*Cot[c + d*x]^4)/(4*a*d) - Cot[c + d*x]^5/(5*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos(c+dx)\cot^5(c+dx)\,dx}{a} + \frac{\int \cot^6(c+dx)\,dx}{a}$$

$$= -\frac{\cot^5(c+dx)}{5ad} - \frac{\int \cot^4(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos(c+dx)\right)}{ad}$$

$$= \frac{\cot^3(c+dx)}{3ad} + \frac{\cos(c+dx)\cot^4(c+dx)}{4ad} - \frac{\cot^5(c+dx)}{5ad} + \frac{\int \cot^2(c+dx)\,dx}{a} -$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{5\cos(c+dx)\cot^2(c+dx)}{8ad} + \frac{\cot^3(c+dx)}{3ad} + \frac{\cos(c+dx)\cot^4(c$$

$$= -\frac{x}{a} - \frac{15\cos(c+dx)}{8ad} - \frac{\cot(c+dx)}{ad} - \frac{5\cos(c+dx)\cot^2(c+dx)}{8ad} + \frac{\cot^3(c+dx}{3ad}$$

$$= -\frac{x}{a} + \frac{15\tanh^{-1}(\cos(c+dx))}{8ad} - \frac{15\cos(c+dx)}{8ad} - \frac{\cot(c+dx)}{ad} - \frac{5\cos(c+dx)\,c}{8aa}$$

Mathematica [A] time = 1.00, size = 264, normalized size = 1.91

$$-\frac{\csc^5(c+dx)\left(1200c\sin(c+dx) + 1200dx\sin(c+dx) + 600\sin(2(c+dx)) - 600c\sin(3(c+dx)) - 600dx\sin(3(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/1920*(Csc[c + d*x]^5*(400*Cos[c + d*x] - 200*Cos[3*(c + d*x)] + 184*Cos[
5*(c + d*x)] + 1200*c*Sin[c + d*x] + 1200*d*x*Sin[c + d*x] - 2250*Log[Cos[(
c + d*x)/2]]*Sin[c + d*x] + 2250*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] + 600*S
in[2*(c + d*x)] - 600*c*Sin[3*(c + d*x)] - 600*d*x*Sin[3*(c + d*x)] + 1125*
Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] - 1125*Log[Sin[(c + d*x)/2]]*Sin[3*(
c + d*x)] - 510*Sin[4*(c + d*x)] + 120*c*Sin[5*(c + d*x)] + 120*d*x*Sin[5*(
c + d*x)] - 225*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 225*Log[Sin[(c + d
*x)/2]]*Sin[5*(c + d*x)] + 60*Sin[6*(c + d*x)]))/(a*d)
```

fricas [A] time = 0.48, size = 211, normalized size = 1.53

$$-\frac{368\cos(dx+c)^5 - 560\cos(dx+c)^3 - 225\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(368*cos(d*x + c)^5 - 560*cos(d*x + c)^3 - 225*(cos(d*x + c)^4 - 2*c
os(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 225*(cos(d*x
+ c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
30*(8*d*x*cos(d*x + c)^4 + 8*cos(d*x + c)^5 - 16*d*x*cos(d*x + c)^2 - 25*co
s(d*x + c)^3 + 8*d*x + 15*cos(d*x + c))*sin(d*x + c) + 240*cos(d*x + c))/((
a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))
```

giac [A] time = 0.22, size = 217, normalized size = 1.57

$$-\frac{\frac{960\,(dx+c)}{a}+\frac{1800\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{1920}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a}-\frac{6\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-70\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+240\,a^{4}}{a^{5}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/960*(960*(d*x + c)/a + 1800*log(abs(tan(1/2*d*x + 1/2*c)))/a + 1920/((ta
n(1/2*d*x + 1/2*c)^2 + 1)*a) - (6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^4*tan(1
/2*d*x + 1/2*c)^4 - 70*a^4*tan(1/2*d*x + 1/2*c)^3 + 240*a^4*tan(1/2*d*x + 1
/2*c)^2 + 660*a^4*tan(1/2*d*x + 1/2*c))/a^5 - (4110*tan(1/2*d*x + 1/2*c)^5
- 660*tan(1/2*d*x + 1/2*c)^4 - 240*tan(1/2*d*x + 1/2*c)^3 + 70*tan(1/2*d*x
+ 1/2*c)^2 + 15*tan(1/2*d*x + 1/2*c) - 6)/(a*tan(1/2*d*x + 1/2*c)^5))/d
```

maple [A] time = 0.52, size = 249, normalized size = 1.80

$$\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad}-\frac{\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{7\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96ad}+\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{2}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/160/a/d*tan(1/2*d*x+1/2*c)^5-1/64/a/d*tan(1/2*d*x+1/2*c)^4-7/96/a/d*tan(1
/2*d*x+1/2*c)^3+1/4/a/d*tan(1/2*d*x+1/2*c)^2+11/16/a/d*tan(1/2*d*x+1/2*c)-2
/a/d/(1+tan(1/2*d*x+1/2*c)^2)-2/a/d*arctan(tan(1/2*d*x+1/2*c))-1/160/a/d/ta
n(1/2*d*x+1/2*c)^5+1/64/a/d/tan(1/2*d*x+1/2*c)^4+7/96/a/d/tan(1/2*d*x+1/2*c
)^3-1/4/a/d/tan(1/2*d*x+1/2*c)^2-11/16/a/d/tan(1/2*d*x+1/2*c)-15/8/a/d*ln(t
an(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 319, normalized size = 2.31

$$\frac{\frac{\frac{660\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{240\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{70\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{15\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}+\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{64\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{225\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{590\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2160\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{\frac{a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*((660*sin(d*x + c)/(cos(d*x + c) + 1) + 240*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 70*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a + (15*sin(d*x
+ c)/(cos(d*x + c) + 1) + 64*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 225*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 - 590*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 -
 2160*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 660*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 - 6)/(a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7) - 1920*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 1800*
log(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 9.03, size = 279, normalized size = 2.02

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a\,d}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,a\,d}+\frac{2\,\mathrm{atan}\left(\frac{15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{15}{2}\right)}+\frac{4}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{15}{2}}\right)}{a\,d}-\frac{15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^6*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(4*a*d) - (7*tan(c/2 + (d*x)/2)^3)/(96*a*d) - tan(c/2
+ (d*x)/2)^4/(64*a*d) + tan(c/2 + (d*x)/2)^5/(160*a*d) + (2*atan((15*tan(c/
2 + (d*x)/2))/(2*(4*tan(c/2 + (d*x)/2) - 15/2)) + 4/(4*tan(c/2 + (d*x)/2) -
 15/2)))/(a*d) - (15*log(tan(c/2 + (d*x)/2)))/(8*a*d) - ((15*tan(c/2 + (d*x
)/2)^3)/2 - (32*tan(c/2 + (d*x)/2)^2)/15 - tan(c/2 + (d*x)/2)/2 + (59*tan(c
/2 + (d*x)/2)^4)/3 + 72*tan(c/2 + (d*x)/2)^5 + 22*tan(c/2 + (d*x)/2)^6 + 1/
5)/(d*(32*a*tan(c/2 + (d*x)/2)^5 + 32*a*tan(c/2 + (d*x)/2)^7)) + (11*tan(c/
2 + (d*x)/2))/(16*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.716 $\int \frac{\cos(c+dx)\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=142

$$\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}+\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}+\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}$$

```
[Out] x/a+5/16*arctanh(cos(d*x+c))/a/d+cot(d*x+c)/a/d-1/3*cot(d*x+c)^3/a/d+1/5*co
t(d*x+c)^5/a/d-5/16*cot(d*x+c)*csc(d*x+c)/a/d+5/24*cot(d*x+c)^3*csc(d*x+c)/
a/d-1/6*cot(d*x+c)^5*csc(d*x+c)/a/d
```

Rubi [A] time = 0.18, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2611, 3770, 3473, 8}

$$\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}+\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}+\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + (5*ArcTanh[Cos[c + d*x]])/(16*a*d) + Cot[c + d*x]/(a*d) - Cot[c + d*x
]^3/(3*a*d) + Cot[c + d*x]^5/(5*a*d) - (5*Cot[c + d*x]*Csc[c + d*x])/(16*a*
d) + (5*Cot[c + d*x]^3*Csc[c + d*x])/(24*a*d) - (Cot[c + d*x]^5*Csc[c + d*x
])/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^6(c+dx)\,dx}{a}+\frac{\int \cot^6(c+dx)\csc(c+dx)\,dx}{a}\\
&= \frac{\cot^5(c+dx)}{5ad}-\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\int \cot^4(c+dx)\csc(c+dx)\,dx}{6a}+\frac{\int
\\
&= -\frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}+\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}-\frac{\cot^5(c+dx)\csc(}{6ad}\\
&= \frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}-\frac{5\cot(c+dx)\csc(c+dx)}{16ad}+\frac{5\cot
\\
&= \frac{x}{a}+\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}-\frac{5\,c
\end{aligned}
$$

Mathematica [B] time = 1.02, size = 317, normalized size = 2.23

$$
-\frac{\csc^6(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-1200\sin(2(c+dx))+768\sin(4(c+dx))-368\sin(6(c+
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7680*(Csc[c + d*x]^6*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(-2400*c -
2400*d*x + 900*Cos[c + d*x] + 50*Cos[3*(c + d*x)] - 1440*c*Cos[4*(c + d*x)]
 - 1440*d*x*Cos[4*(c + d*x)] + 330*Cos[5*(c + d*x)] + 240*c*Cos[6*(c + d*x)
] + 240*d*x*Cos[6*(c + d*x)] - 750*Log[Cos[(c + d*x)/2]] - 450*Cos[4*(c + d
*x)]*Log[Cos[(c + d*x)/2]] + 75*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 22
```

```
5*Cos[2*(c + d*x)]*(16*(c + d*x) + 5*Log[Cos[(c + d*x)/2]] - 5*Log[Sin[(c +
 d*x)/2]]) + 750*Log[Sin[(c + d*x)/2]] + 450*Cos[4*(c + d*x)]*Log[Sin[(c +
d*x)/2]] - 75*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 1200*Sin[2*(c + d*x)
] + 768*Sin[4*(c + d*x)] - 368*Sin[6*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.49, size = 236, normalized size = 1.66

$$\frac{480\,dx\cos(dx+c)^6 - 1440\,dx\cos(dx+c)^4 + 330\,\cos(dx+c)^5 + 1440\,dx\cos(dx+c)^2 - 400\,\cos(dx+c)^3 - 48}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/480*(480*d*x*cos(d*x + c)^6 - 1440*d*x*cos(d*x + c)^4 + 330*cos(d*x + c)^
5 + 1440*d*x*cos(d*x + c)^2 - 400*cos(d*x + c)^3 - 480*d*x + 75*(cos(d*x +
c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)
 - 75*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*c
os(d*x + c) + 1/2) - 32*(23*cos(d*x + c)^5 - 35*cos(d*x + c)^3 + 15*cos(d*x
 + c))*sin(d*x + c) + 150*cos(d*x + c))/(a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x
 + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)
```

giac [A] time = 0.24, size = 224, normalized size = 1.58

$$\frac{\frac{1920\,(dx+c)}{a} - \frac{600\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} + \frac{5\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 12\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 45\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 140\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 225\,a^5\tan\left(\frac{1}{2}\right.}{a^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1920*(1920*(d*x + c)/a - 600*log(abs(tan(1/2*d*x + 1/2*c)))/a + (5*a^5*ta
n(1/2*d*x + 1/2*c)^6 - 12*a^5*tan(1/2*d*x + 1/2*c)^5 - 45*a^5*tan(1/2*d*x +
 1/2*c)^4 + 140*a^5*tan(1/2*d*x + 1/2*c)^3 + 225*a^5*tan(1/2*d*x + 1/2*c)^2
 - 1320*a^5*tan(1/2*d*x + 1/2*c))/a^6 + (1470*tan(1/2*d*x + 1/2*c)^6 + 1320
*tan(1/2*d*x + 1/2*c)^5 - 225*tan(1/2*d*x + 1/2*c)^4 - 140*tan(1/2*d*x + 1/
2*c)^3 + 45*tan(1/2*d*x + 1/2*c)^2 + 12*tan(1/2*d*x + 1/2*c) - 5)/(a*tan(1/
2*d*x + 1/2*c)^6))/d
```

maple [B] time = 0.55, size = 264, normalized size = 1.86

$$\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{384ad} - \frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad} - \frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad} + \frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96ad} + \frac{15\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad} - \frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad} + \frac{2\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/384/a/d*tan(1/2*d*x+1/2*c)^6-1/160/a/d*tan(1/2*d*x+1/2*c)^5-3/128/a/d*tan
(1/2*d*x+1/2*c)^4+7/96/a/d*tan(1/2*d*x+1/2*c)^3+15/128/a/d*tan(1/2*d*x+1/2*
c)^2-11/16/a/d*tan(1/2*d*x+1/2*c)+2/a/d*arctan(tan(1/2*d*x+1/2*c))-1/384/a/
d/tan(1/2*d*x+1/2*c)^6+1/160/a/d/tan(1/2*d*x+1/2*c)^5+3/128/a/d/tan(1/2*d*x
+1/2*c)^4-7/96/a/d/tan(1/2*d*x+1/2*c)^3-15/128/a/d/tan(1/2*d*x+1/2*c)^2+11/
16/a/d/tan(1/2*d*x+1/2*c)-5/16/a/d*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.45, size = 298, normalized size = 2.10

$$-\frac{\frac{\frac{1320\sin(dx+c)}{\cos(dx+c)+1}-\frac{225\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{140\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{12\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a}-\frac{3840\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{600\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/1920*((1320*sin(d*x + c)/(cos(d*x + c) + 1) - 225*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 140*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 12*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6)/a - 3840*arctan(sin(d*x + c)/(cos(d*x + c) + 1
))/a + 600*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (12*sin(d*x + c)/(cos(d
*x + c) + 1) + 45*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 140*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 - 225*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1320*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x + c) + 1)^6/(a*sin(d*x + c)^6)
)/d
```

mupad [B] time = 10.39, size = 413, normalized size = 2.91

$$-\frac{5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+12\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+45\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] -(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 + 12*cos(c/2 + (d*x)/2)
*sin(c/2 + (d*x)/2)^11 - 12*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) + 45*c
os(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 140*cos(c/2 + (d*x)/2)^3*sin(c/
2 + (d*x)/2)^9 - 225*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 1320*cos(c
```

```
/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 1320*cos(c/2 + (d*x)/2)^7*sin(c/2 +
(d*x)/2)^5 + 225*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 140*cos(c/2 +
(d*x)/2)^9*sin(c/2 + (d*x)/2)^3 - 45*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/
2)^2 + 3840*atan((16*cos(c/2 + (d*x)/2) - 5*sin(c/2 + (d*x)/2))/(5*cos(c/2
+ (d*x)/2) + 16*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2
)^6 + 600*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*s
in(c/2 + (d*x)/2)^6)/(1920*a*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.717 $\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}$$

```
[Out] -5/16*arctanh(cos(d*x+c))/a/d-1/7*cot(d*x+c)^7/a/d+5/16*cot(d*x+c)*csc(d*x+
c)/a/d-5/24*cot(d*x+c)^3*csc(d*x+c)/a/d+1/6*cot(d*x+c)^5*csc(d*x+c)/a/d
```

Rubi [A] time = 0.15, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(16*a*d) - Cot[c + d*x]^7/(7*a*d) + (5*Cot[c + d
*x]*Csc[c + d*x])/(16*a*d) - (5*Cot[c + d*x]^3*Csc[c + d*x])/(24*a*d) + (Co
t[c + d*x]^5*Csc[c + d*x])/(6*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^6(c+dx)\csc(c+dx)\,dx}{a} + \frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a}$$
$$= \frac{\cot^5(c+dx)\csc(c+dx)}{6ad} + \frac{5\int \cot^4(c+dx)\csc(c+dx)\,dx}{6a} + \frac{\operatorname{Subst}\left(\int x^6\,dx, x, -\cot(c\right.}{ad}$$
$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{5\cot^3(c+dx)\csc(c+dx)}{24ad} + \frac{\cot^5(c+dx)\csc(c+dx)}{6ad} - \frac{5\int \cot^2(c+d}{}$$
$$= -\frac{\cot^7(c+dx)}{7ad} + \frac{5\cot(c+dx)\csc(c+dx)}{16ad} - \frac{5\cot^3(c+dx)\csc(c+dx)}{24ad} + \frac{\cot^5(c+dx)\,c}{6ad}$$
$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{16ad} - \frac{\cot^7(c+dx)}{7ad} + \frac{5\cot(c+dx)\csc(c+dx)}{16ad} - \frac{5\cot^3(c+dx)\csc}{24ad}$$

Mathematica [B] time = 0.97, size = 284, normalized size = 2.68

$$-\frac{\csc^5(c+dx)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-1190\sin(2(c+dx))+392\sin(4(c+dx))-462\sin(6(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/86016*(Csc[c + d*x]^5*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(1680*Cos[
c + d*x] + 1008*Cos[3*(c + d*x)] + 336*Cos[5*(c + d*x)] + 48*Cos[7*(c + d*x
)] + 3675*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 3675*Log[Sin[(c + d*x)/2]]*S
in[c + d*x] - 1190*Sin[2*(c + d*x)] - 2205*Log[Cos[(c + d*x)/2]]*Sin[3*(c +
 d*x)] + 2205*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 392*Sin[4*(c + d*x)]
 + 735*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 735*Log[Sin[(c + d*x)/2]]*S
```

```
in[5*(c + d*x)] - 462*Sin[6*(c + d*x)] - 105*Log[Cos[(c + d*x)/2]]*Sin[7*(c
 + d*x)] + 105*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)]))/(a*d*(1 + Sin[c + d
*x]))
```

fricas [B] time = 0.48, size = 198, normalized size = 1.87

$$\frac{96 \cos\left(dx+c\right)^{7} - 105 \left(\cos\left(dx+c\right)^{6} - 3 \cos\left(dx+c\right)^{4} + 3 \cos\left(dx+c\right)^{2} - 1\right) \log\left(\frac{1}{2} \cos\left(dx+c\right) + \frac{1}{2}\right) \sin\left(dx+\right.}{672 \left(ad\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/672*(96*cos(d*x + c)^7 - 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d
*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 105*(cos(d*x + c)
^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 14*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))
*sin(d*x + c))/((a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x
+ c)^2 - a*d)*sin(d*x + c))
```

giac [B] time = 0.23, size = 244, normalized size = 2.30

$$\frac{\frac{840 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a} + \frac{3 a^{6} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{7} - 7 a^{6} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{6} - 21 a^{6} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{5} + 63 a^{6} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{4} + 63 a^{6} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{3} - 315 a^{6}}{a^{7}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2688*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a + (3*a^6*tan(1/2*d*x + 1/2*c)^
7 - 7*a^6*tan(1/2*d*x + 1/2*c)^6 - 21*a^6*tan(1/2*d*x + 1/2*c)^5 + 63*a^6*t
an(1/2*d*x + 1/2*c)^4 + 63*a^6*tan(1/2*d*x + 1/2*c)^3 - 315*a^6*tan(1/2*d*x
 + 1/2*c)^2 - 105*a^6*tan(1/2*d*x + 1/2*c))/a^7 - (2178*tan(1/2*d*x + 1/2*c
)^7 - 105*tan(1/2*d*x + 1/2*c)^6 - 315*tan(1/2*d*x + 1/2*c)^5 + 63*tan(1/2*
d*x + 1/2*c)^4 + 63*tan(1/2*d*x + 1/2*c)^3 - 21*tan(1/2*d*x + 1/2*c)^2 - 7*
tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*d*x + 1/2*c)^7))/d
```

maple [B] time = 0.55, size = 284, normalized size = 2.68

$$\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{896ad} - \frac{\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)}{384ad} - \frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{128ad} + \frac{3\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad} + \frac{3\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad} - \frac{15\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad} - \frac{5 \tan}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/896/a/d*tan(1/2*d*x+1/2*c)^7-1/384/a/d*tan(1/2*d*x+1/2*c)^6-1/128/a/d*tan
(1/2*d*x+1/2*c)^5+3/128/a/d*tan(1/2*d*x+1/2*c)^4+3/128/a/d*tan(1/2*d*x+1/2*
c)^3-15/128/a/d*tan(1/2*d*x+1/2*c)^2-5/128/a/d*tan(1/2*d*x+1/2*c)+1/384/a/d
/tan(1/2*d*x+1/2*c)^6+5/128/a/d/tan(1/2*d*x+1/2*c)+5/16/a/d*ln(tan(1/2*d*x+
1/2*c))+1/128/a/d/tan(1/2*d*x+1/2*c)^5-1/896/a/d/tan(1/2*d*x+1/2*c)^7+15/12
8/a/d/tan(1/2*d*x+1/2*c)^2-3/128/a/d/tan(1/2*d*x+1/2*c)^4-3/128/a/d/tan(1/2
*d*x+1/2*c)^3
```

maxima [B] time = 0.36, size = 315, normalized size = 2.97

$$-\frac{\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1}+\frac{315 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{63 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{63 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{7 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a}-\frac{840 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{7 \sin(dx+c)}{\cos(dx+c)+1}+\cdots}{\cdots}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2688*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 315*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 63*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 7*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a - 840*
log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (7*sin(d*x + c)/(cos(d*x + c) + 1)
 + 21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 - 63*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 315*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 + 105*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3)*(cos(d*x + c
) + 1)^7/(a*sin(d*x + c)^7))/d
```

mupad [B] time = 10.62, size = 387, normalized size = 3.65

$$\frac{3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+7\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-21\cos\left(\frac{c}{2}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^8*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*sin(c/2 + (d*x)/2)^14 - 3*cos(c/2 + (d*x)/2)^14 - 7*cos(c/2 + (d*x)/2)*s
in(c/2 + (d*x)/2)^13 + 7*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) - 21*cos(
c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 + 63*cos(c/2 + (d*x)/2)^3*sin(c/2 +
```

```
(d*x)/2)^11 + 63*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 - 315*cos(c/2 +
 (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 - 105*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)
/2)^8 + 105*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 315*cos(c/2 + (d*x)
/2)^9*sin(c/2 + (d*x)/2)^5 - 63*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^4
- 63*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 21*cos(c/2 + (d*x)/2)^12*
sin(c/2 + (d*x)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c
/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)/(2688*a*d*cos(c/2 + (d*x)/2)^7*sin(c/
2 + (d*x)/2)^7)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.718 $\int \frac{\cot^8(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{\cot^7(c+dx)}{7ad}+\frac{5\tanh^{-1}(\cos(c+dx))}{128ad}-\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad}+\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad}-\frac{5\cot(c+dx)\csc^3(c}{64ad}$$

```
[Out] 5/128*arctanh(cos(d*x+c))/a/d+1/7*cot(d*x+c)^7/a/d+5/128*cot(d*x+c)*csc(d*x
+c)/a/d-5/64*cot(d*x+c)*csc(d*x+c)^3/a/d+5/48*cot(d*x+c)^3*csc(d*x+c)^3/a/d
-1/8*cot(d*x+c)^5*csc(d*x+c)^3/a/d
```

Rubi [A] time = 0.22, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2611, 3768, 3770, 2607, 30}

$$\frac{\cot^7(c+dx)}{7ad}+\frac{5\tanh^{-1}(\cos(c+dx))}{128ad}-\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad}+\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad}-\frac{5\cot(c+dx)\csc^3(c}{64ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Cos[c + d*x]])/(128*a*d) + Cot[c + d*x]^7/(7*a*d) + (5*Cot[c + d
*x]*Csc[c + d*x])/(128*a*d) - (5*Cot[c + d*x]*Csc[c + d*x]^3)/(64*a*d) + (5
*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*a*d) - (Cot[c + d*x]^5*Csc[c + d*x]^3)/
(8*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a} + \frac{\int \cot^6(c+dx)\csc^3(c+dx)\,dx}{a}$$
$$= -\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad} - \frac{5\int \cot^4(c+dx)\csc^3(c+dx)\,dx}{8a} - \frac{\text{Subst}\left(\int x^6\,d\right.}{}$$
$$= \frac{\cot^7(c+dx)}{7ad} + \frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad} - \frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad} + \frac{5\int c}{}$$
$$= \frac{\cot^7(c+dx)}{7ad} - \frac{5\cot(c+dx)\csc^3(c+dx)}{64ad} + \frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad} - \frac{\cot^5}{}$$
$$= \frac{\cot^7(c+dx)}{7ad} + \frac{5\cot(c+dx)\csc(c+dx)}{128ad} - \frac{5\cot(c+dx)\csc^3(c+dx)}{64ad} + \frac{5\cot^3}{}$$
$$= \frac{5\tanh^{-1}(\cos(c+dx))}{128ad} + \frac{\cot^7(c+dx)}{7ad} + \frac{5\cot(c+dx)\csc(c+dx)}{128ad} - \frac{5\cot(c+}{}$$

Mathematica [B] time = 1.00, size = 291, normalized size = 2.17

$$\csc^8(c+dx)\Big(5376\sin(2(c+dx))+5376\sin(4(c+dx))+2304\sin(6(c+dx))+384\sin(8(c+dx))-24710\cos($$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^8*(-24710*Cos[c + d*x] - 12530*Cos[3*(c + d*x)] - 5558*Cos[5*
(c + d*x)] - 210*Cos[7*(c + d*x)] + 3675*Log[Cos[(c + d*x)/2]] - 5880*Cos[2
*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 2940*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)
/2]] - 840*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 105*Cos[8*(c + d*x)]*Lo
g[Cos[(c + d*x)/2]] - 3675*Log[Sin[(c + d*x)/2]] + 5880*Cos[2*(c + d*x)]*Lo
g[Sin[(c + d*x)/2]] - 2940*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 840*Cos
[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 105*Cos[8*(c + d*x)]*Log[Sin[(c + d*x
)/2]] + 5376*Sin[2*(c + d*x)] + 5376*Sin[4*(c + d*x)] + 2304*Sin[6*(c + d*x
)] + 384*Sin[8*(c + d*x)]))/(344064*a*d)
```

fricas [A] time = 0.47, size = 216, normalized size = 1.61

$$768\cos(dx+c)^7\sin(dx+c)-210\cos(dx+c)^7-1022\cos(dx+c)^5+770\cos(dx+c)^3+105\Big(\cos(dx+c)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/5376*(768*cos(d*x + c)^7*sin(d*x + c) - 210*cos(d*x + c)^7 - 1022*cos(d*x
 + c)^5 + 770*cos(d*x + c)^3 + 105*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*c
os(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2) - 105*(co
s(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*
log(-1/2*cos(d*x + c) + 1/2) - 210*cos(d*x + c))/(a*d*cos(d*x + c)^8 - 4*a*
d*cos(d*x + c)^6 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos(d*x + c)^2 + a*d)
```

giac [B] time = 0.26, size = 274, normalized size = 2.04

$$-\frac{\frac{1680\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{21a^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-48a^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-112a^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+336a^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+168a^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/43008*(1680*log(abs(tan(1/2*d*x + 1/2*c)))/a - (21*a^7*tan(1/2*d*x + 1/2
*c)^8 - 48*a^7*tan(1/2*d*x + 1/2*c)^7 - 112*a^7*tan(1/2*d*x + 1/2*c)^6 + 33
6*a^7*tan(1/2*d*x + 1/2*c)^5 + 168*a^7*tan(1/2*d*x + 1/2*c)^4 - 1008*a^7*ta
n(1/2*d*x + 1/2*c)^3 + 336*a^7*tan(1/2*d*x + 1/2*c)^2 + 1680*a^7*tan(1/2*d*
x + 1/2*c))/a^8 - (4566*tan(1/2*d*x + 1/2*c)^8 - 1680*tan(1/2*d*x + 1/2*c)^
7 - 336*tan(1/2*d*x + 1/2*c)^6 + 1008*tan(1/2*d*x + 1/2*c)^5 - 168*tan(1/2*
d*x + 1/2*c)^4 - 336*tan(1/2*d*x + 1/2*c)^3 + 112*tan(1/2*d*x + 1/2*c)^2 +
48*tan(1/2*d*x + 1/2*c) - 21)/(a*tan(1/2*d*x + 1/2*c)^8))/d
```

maple [B] time = 0.56, size = 322, normalized size = 2.40

$$\frac{\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2048ad}-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{896ad}-\frac{\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)}{384ad}+\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{128ad}+\frac{\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)}{256ad}-\frac{3\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}+\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{128ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2048/a/d*tan(1/2*d*x+1/2*c)^8-1/896/a/d*tan(1/2*d*x+1/2*c)^7-1/384/a/d*ta
n(1/2*d*x+1/2*c)^6+1/128/a/d*tan(1/2*d*x+1/2*c)^5+1/256/a/d*tan(1/2*d*x+1/2
*c)^4-3/128/a/d*tan(1/2*d*x+1/2*c)^3+1/128/a/d*tan(1/2*d*x+1/2*c)^2+5/128/a
/d*tan(1/2*d*x+1/2*c)+1/384/a/d/tan(1/2*d*x+1/2*c)^6-5/128/a/d/tan(1/2*d*x+
1/2*c)-5/128/a/d*ln(tan(1/2*d*x+1/2*c))-1/128/a/d/tan(1/2*d*x+1/2*c)^5+1/89
6/a/d/tan(1/2*d*x+1/2*c)^7-1/128/a/d/tan(1/2*d*x+1/2*c)^2-1/2048/a/d/tan(1/
2*d*x+1/2*c)^8-1/256/a/d/tan(1/2*d*x+1/2*c)^4+3/128/a/d/tan(1/2*d*x+1/2*c)^
3
```

maxima [B] time = 0.33, size = 354, normalized size = 2.64

$$\frac{\frac{\frac{1680\sin(dx+c)}{\cos(dx+c)+1}+\frac{336\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{1008\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{168\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{336\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{112\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{48\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{21\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}{a}-\frac{1680\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{43008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/43008*((1680*sin(d*x + c)/(cos(d*x + c) + 1) + 336*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 1008*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 168*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 336*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 112*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 48*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 +
 21*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)/a - 1680*log(sin(d*x + c)/(cos(d*x
 + c) + 1))/a + (48*sin(d*x + c)/(cos(d*x + c) + 1) + 112*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 336*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 168*sin(d*x
```

```
+ c)^4/(cos(d*x + c) + 1)^4 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 33
6*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1680*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 - 21)*(cos(d*x + c) + 1)^8/(a*sin(d*x + c)^8))/d
```

mupad [B] time = 11.39, size = 435, normalized size = 3.25

$$-\frac{21\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-21\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+48\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}-48\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+112}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] -(21*cos(c/2 + (d*x)/2)^16 - 21*sin(c/2 + (d*x)/2)^16 + 48*cos(c/2 + (d*x)/
2)*sin(c/2 + (d*x)/2)^15 - 48*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2) + 11
2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^14 - 336*cos(c/2 + (d*x)/2)^3*sin
(c/2 + (d*x)/2)^13 - 168*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^12 + 1008*
cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^11 - 336*cos(c/2 + (d*x)/2)^6*sin(c
/2 + (d*x)/2)^10 - 1680*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^9 + 1680*co
s(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^7 + 336*cos(c/2 + (d*x)/2)^10*sin(c/2
 + (d*x)/2)^6 - 1008*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^5 + 168*cos(c
/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 + 336*cos(c/2 + (d*x)/2)^13*sin(c/2 +
 (d*x)/2)^3 - 112*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2 + 1680*log(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)
^8)/(43008*a*d*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.719 $\int \frac{\cot^8(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=152

$$-\frac{\cot^9(c+dx)}{9ad}-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{128ad}+\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad}-\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad}+\frac{5}{}$$

```
[Out] -5/128*arctanh(cos(d*x+c))/a/d-1/7*cot(d*x+c)^7/a/d-1/9*cot(d*x+c)^9/a/d-5/
128*cot(d*x+c)*csc(d*x+c)/a/d+5/64*cot(d*x+c)*csc(d*x+c)^3/a/d-5/48*cot(d*x
+c)^3*csc(d*x+c)^3/a/d+1/8*cot(d*x+c)^5*csc(d*x+c)^3/a/d
```

Rubi [A] time = 0.24, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2607, 14, 2611, 3768, 3770}

$$-\frac{\cot^9(c+dx)}{9ad}-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{128ad}+\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad}-\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad}+\frac{5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(128*a*d) - Cot[c + d*x]^7/(7*a*d) - Cot[c + d*x
]^9/(9*a*d) - (5*Cot[c + d*x]*Csc[c + d*x])/(128*a*d) + (5*Cot[c + d*x]*Csc
[c + d*x]^3)/(64*a*d) - (5*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*a*d) + (Cot[c
 + d*x]^5*Csc[c + d*x]^3)/(8*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^8(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^6(c+dx)\csc^3(c+dx)\,dx}{a}+\frac{\int \cot^6(c+dx)\csc^4(c+dx)\,dx}{a}\\
&= \frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad}+\frac{5\int \cot^4(c+dx)\csc^3(c+dx)\,dx}{8a}+\frac{\operatorname{Subst}\left(\int x^6\left(1\right.\right.}{}\\
&= -\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad}+\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad}-\frac{5\int \cot^2(c+dx)\operatorname{csc}}{16a}\\
&= -\frac{\cot^7(c+dx)}{7ad}-\frac{\cot^9(c+dx)}{9ad}+\frac{5\cot(c+dx)\csc^3(c+dx)}{64ad}-\frac{5\cot^3(c+dx)\operatorname{csc}}{48ad}\\
&= -\frac{\cot^7(c+dx)}{7ad}-\frac{\cot^9(c+dx)}{9ad}-\frac{5\cot(c+dx)\csc(c+dx)}{128ad}+\frac{5\cot(c+dx)\csc^3(}{64ad}\\
&= -\frac{5\tanh^{-1}(\cos(c+dx))}{128ad}-\frac{\cot^7(c+dx)}{7ad}-\frac{\cot^9(c+dx)}{9ad}-\frac{5\cot(c+dx)\csc(c+}{128ad}
\end{aligned}$$

Mathematica [B] time = 1.37, size = 313, normalized size = 2.06

$$-\frac{\csc^9(c+dx)\left(-36540\sin(2(c+dx))-20916\sin(4(c+dx))-16044\sin(6(c+dx))-630\sin(8(c+dx))+129\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2064384*(Csc[c + d*x]^9*(129024*Cos[c + d*x] + 75264*Cos[3*(c + d*x)] +
23040*Cos[5*(c + d*x)] + 2304*Cos[7*(c + d*x)] - 256*Cos[9*(c + d*x)] + 396
90*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 39690*Log[Sin[(c + d*x)/2]]*Sin[c +
 d*x] - 36540*Sin[2*(c + d*x)] - 26460*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x
)] + 26460*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 20916*Sin[4*(c + d*x)]
+ 11340*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 11340*Log[Sin[(c + d*x)/2]
]*Sin[5*(c + d*x)] - 16044*Sin[6*(c + d*x)] - 2835*Log[Cos[(c + d*x)/2]]*Si
n[7*(c + d*x)] + 2835*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] - 630*Sin[8*(c
 + d*x)] + 315*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)] - 315*Log[Sin[(c + d*
x)/2]]*Sin[9*(c + d*x)]))/(a*d)
```

fricas [A] time = 0.48, size = 249, normalized size = 1.64

$$\frac{512\cos(dx+c)^9-2304\cos(dx+c)^7-315\left(\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/16128*(512*cos(d*x + c)^9 - 2304*cos(d*x + c)^7 - 315*(cos(d*x + c)^8 - 4
*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) + 315*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d
*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
 + 42*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d
*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x + c)^6 + 6*a*d*
cos(d*x + c)^4 - 4*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))
```

giac [A] time = 0.24, size = 273, normalized size = 1.80

$$\frac{\frac{5040\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{28a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-63a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-108a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+336a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-504a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}{a^9}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/129024*(5040*log(abs(tan(1/2*d*x + 1/2*c)))/a + (28*a^8*tan(1/2*d*x + 1/2
*c)^9 - 63*a^8*tan(1/2*d*x + 1/2*c)^8 - 108*a^8*tan(1/2*d*x + 1/2*c)^7 + 33
6*a^8*tan(1/2*d*x + 1/2*c)^6 - 504*a^8*tan(1/2*d*x + 1/2*c)^4 + 672*a^8*tan
(1/2*d*x + 1/2*c)^3 - 1008*a^8*tan(1/2*d*x + 1/2*c)^2 - 1512*a^8*tan(1/2*d*
x + 1/2*c))/a^9 - (14258*tan(1/2*d*x + 1/2*c)^9 - 1512*tan(1/2*d*x + 1/2*c)
^8 - 1008*tan(1/2*d*x + 1/2*c)^7 + 672*tan(1/2*d*x + 1/2*c)^6 - 504*tan(1/2
*d*x + 1/2*c)^5 + 336*tan(1/2*d*x + 1/2*c)^3 - 108*tan(1/2*d*x + 1/2*c)^2 -
 63*tan(1/2*d*x + 1/2*c) + 28)/(a*tan(1/2*d*x + 1/2*c)^9))/d
```

maple [B] time = 0.60, size = 322, normalized size = 2.12

$$\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{4608ad}-\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{2048ad}-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3584ad}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{384ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{256ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{128ad}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4608/a/d*tan(1/2*d*x+1/2*c)^9-1/2048/a/d*tan(1/2*d*x+1/2*c)^8-3/3584/a/d*
tan(1/2*d*x+1/2*c)^7+1/384/a/d*tan(1/2*d*x+1/2*c)^6-1/256/a/d*tan(1/2*d*x+1
/2*c)^4+1/192/a/d*tan(1/2*d*x+1/2*c)^3-1/128/a/d*tan(1/2*d*x+1/2*c)^2-3/256
/a/d*tan(1/2*d*x+1/2*c)-1/384/a/d/tan(1/2*d*x+1/2*c)^6+3/256/a/d/tan(1/2*d*
x+1/2*c)+5/128/a/d*ln(tan(1/2*d*x+1/2*c))+3/3584/a/d/tan(1/2*d*x+1/2*c)^7+1
/128/a/d/tan(1/2*d*x+1/2*c)^2-1/4608/a/d/tan(1/2*d*x+1/2*c)^9+1/2048/a/d/ta
n(1/2*d*x+1/2*c)^8+1/256/a/d/tan(1/2*d*x+1/2*c)^4-1/192/a/d/tan(1/2*d*x+1/2
*c)^3
```

maxima [B] time = 0.33, size = 355, normalized size = 2.34

$$-\frac{\frac{\frac{1512\sin(dx+c)}{\cos(dx+c)+1}+\frac{1008\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{672\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{504\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{336\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{108\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{63\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{28\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a}-\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{129024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/129024*((1512*sin(d*x + c)/(cos(d*x + c) + 1) + 1008*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 672*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 504*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 - 336*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 108*s
```

```
in(d*x + c)^7/(cos(d*x + c) + 1)^7 + 63*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 - 28*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a - 5040*log(sin(d*x + c)/(cos(d
*x + c) + 1))/a - (63*sin(d*x + c)/(cos(d*x + c) + 1) + 108*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 336*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 504*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5 - 672*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1
008*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 1512*sin(d*x + c)^8/(cos(d*x + c)
 + 1)^8 - 28)*(cos(d*x + c) + 1)^9/(a*sin(d*x + c)^9))/d
```

mupad [B] time = 12.55, size = 435, normalized size = 2.86

$$\frac{28\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-28\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-63\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}+63\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^10*(a + a*sin(c + d*x))),x)
```

```
[Out] (28*sin(c/2 + (d*x)/2)^18 - 28*cos(c/2 + (d*x)/2)^18 - 63*cos(c/2 + (d*x)/2
)*sin(c/2 + (d*x)/2)^17 + 63*cos(c/2 + (d*x)/2)^17*sin(c/2 + (d*x)/2) - 108
*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^16 + 336*cos(c/2 + (d*x)/2)^3*sin(
c/2 + (d*x)/2)^15 - 504*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^13 + 672*co
s(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^12 - 1008*cos(c/2 + (d*x)/2)^7*sin(c/
2 + (d*x)/2)^11 - 1512*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^10 + 1512*co
s(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^8 + 1008*cos(c/2 + (d*x)/2)^11*sin(c
/2 + (d*x)/2)^7 - 672*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^6 + 504*cos(
c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2)^5 - 336*cos(c/2 + (d*x)/2)^15*sin(c/2
+ (d*x)/2)^3 + 108*cos(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^2 + 5040*log(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2
)^9)/(129024*a*d*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**10/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.720 $\int \frac{\cot^8(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=176

$$\frac{\cot^9(c+dx)}{9ad}+\frac{\cot^7(c+dx)}{7ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{256ad}-\frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad}+\frac{\cot^3(c+dx)\csc^5(c+dx)}{16ad}-\frac{\cot(c+$$

```
[Out] 3/256*arctanh(cos(d*x+c))/a/d+1/7*cot(d*x+c)^7/a/d+1/9*cot(d*x+c)^9/a/d+3/2
56*cot(d*x+c)*csc(d*x+c)/a/d+1/128*cot(d*x+c)*csc(d*x+c)^3/a/d-1/32*cot(d*x
+c)*csc(d*x+c)^5/a/d+1/16*cot(d*x+c)^3*csc(d*x+c)^5/a/d-1/10*cot(d*x+c)^5*c
sc(d*x+c)^5/a/d
```

Rubi [A] time = 0.25, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2611, 3768, 3770, 2607, 14}

$$\frac{\cot^9(c+dx)}{9ad}+\frac{\cot^7(c+dx)}{7ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{256ad}-\frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad}+\frac{\cot^3(c+dx)\csc^5(c+dx)}{16ad}-\frac{\cot(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(256*a*d) + Cot[c + d*x]^7/(7*a*d) + Cot[c + d*x]
^9/(9*a*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(256*a*d) + (Cot[c + d*x]*Csc[c
+ d*x]^3)/(128*a*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(32*a*d) + (Cot[c + d*x
]^3*Csc[c + d*x]^5)/(16*a*d) - (Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
```

```
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^8(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^6(c+dx)\csc^4(c+dx)\,dx}{a}+\frac{\int \cot^6(c+dx)\csc^5(c+dx)\,dx}{a}\\ &= -\frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad}-\frac{\int \cot^4(c+dx)\csc^5(c+dx)\,dx}{2a}-\frac{\text{Subst}\left(\int x^6\left(1\right.\right.}{}\\ &= \frac{\cot^3(c+dx)\csc^5(c+dx)}{16ad}-\frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad}+\frac{3\int \cot^2(c+dx)\csc^5}{16a}\\ &= \frac{\cot^7(c+dx)}{7ad}+\frac{\cot^9(c+dx)}{9ad}-\frac{\cot(c+dx)\csc^5(c+dx)}{32ad}+\frac{\cot^3(c+dx)\csc^5(c}{16ad}\\ &= \frac{\cot^7(c+dx)}{7ad}+\frac{\cot^9(c+dx)}{9ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{128ad}-\frac{\cot(c+dx)\csc^5(c}{32ad}\\ &= \frac{\cot^7(c+dx)}{7ad}+\frac{\cot^9(c+dx)}{9ad}+\frac{3\cot(c+dx)\csc(c+dx)}{256ad}+\frac{\cot(c+dx)\csc^3(c}{128ad}\\ &= \frac{3\tanh^{-1}(\cos(c+dx))}{256ad}+\frac{\cot^7(c+dx)}{7ad}+\frac{\cot^9(c+dx)}{9ad}+\frac{3\cot(c+dx)\csc(c+}{256ad}\end{aligned}$$

Mathematica [B] time = 1.55, size = 386, normalized size = 2.19

$$-\frac{\csc^9(c+dx)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-537600\sin(2(c+dx))-522240\sin(4(c+dx))-207360\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/165150720*(Csc[c + d*x]^9*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(23675
40*Cos[c + d*x] + 1307880*Cos[3*(c + d*x)] + 436968*Cos[5*(c + d*x)] + 1827
0*Cos[7*(c + d*x)] - 1890*Cos[9*(c + d*x)] - 119070*Log[Cos[(c + d*x)/2]] +
 198450*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 113400*Cos[4*(c + d*x)]*Lo
g[Cos[(c + d*x)/2]] + 42525*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 9450*C
os[8*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 945*Cos[10*(c + d*x)]*Log[Cos[(c +
d*x)/2]] + 119070*Log[Sin[(c + d*x)/2]] - 198450*Cos[2*(c + d*x)]*Log[Sin[(
c + d*x)/2]] + 113400*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 42525*Cos[6*
(c + d*x)]*Log[Sin[(c + d*x)/2]] + 9450*Cos[8*(c + d*x)]*Log[Sin[(c + d*x)/
2]] - 945*Cos[10*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 537600*Sin[2*(c + d*x)]
 - 522240*Sin[4*(c + d*x)] - 207360*Sin[6*(c + d*x)] - 25600*Sin[8*(c + d*x
)] + 2560*Sin[10*(c + d*x)]))/(a*d*(1 + Csc[c + d*x]))
```

fricas [A] time = 0.49, size = 272, normalized size = 1.55

$$-\frac{1890\cos(dx+c)^9-8820\cos(dx+c)^7-16128\cos(dx+c)^5+8820\cos(dx+c)^3-945\left(\cos(dx+c)^{10}-5\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/161280*(1890*cos(d*x + c)^9 - 8820*cos(d*x + c)^7 - 16128*cos(d*x + c)^5
 + 8820*cos(d*x + c)^3 - 945*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d
*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c)
+ 1/2) + 945*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*c
os(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) - 2560*(
2*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*sin(d*x + c) - 1890*cos(d*x + c))/(a*d
*cos(d*x + c)^10 - 5*a*d*cos(d*x + c)^8 + 10*a*d*cos(d*x + c)^6 - 10*a*d*co
s(d*x + c)^4 + 5*a*d*cos(d*x + c)^2 - a*d)
```

giac [A] time = 0.29, size = 303, normalized size = 1.72

$$-\frac{\frac{15120 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{126\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-280\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-315\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+1080\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-630\,a^9\tan\left(\frac{1}{2}\,dx \right.}{a^{10}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1290240*(15120*log(abs(tan(1/2*d*x + 1/2*c)))/a - (126*a^9*tan(1/2*d*x +
 1/2*c)^10 - 280*a^9*tan(1/2*d*x + 1/2*c)^9 - 315*a^9*tan(1/2*d*x + 1/2*c)^
8 + 1080*a^9*tan(1/2*d*x + 1/2*c)^7 - 630*a^9*tan(1/2*d*x + 1/2*c)^6 + 2520
*a^9*tan(1/2*d*x + 1/2*c)^4 - 6720*a^9*tan(1/2*d*x + 1/2*c)^3 + 1260*a^9*ta
n(1/2*d*x + 1/2*c)^2 + 15120*a^9*tan(1/2*d*x + 1/2*c))/a^10 - (44286*tan(1/
2*d*x + 1/2*c)^10 - 15120*tan(1/2*d*x + 1/2*c)^9 - 1260*tan(1/2*d*x + 1/2*c
)^8 + 6720*tan(1/2*d*x + 1/2*c)^7 - 2520*tan(1/2*d*x + 1/2*c)^6 + 630*tan(1
/2*d*x + 1/2*c)^4 - 1080*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c)^
2 + 280*tan(1/2*d*x + 1/2*c) - 126)/(a*tan(1/2*d*x + 1/2*c)^10))/d
```

maple [B] time = 0.62, size = 360, normalized size = 2.05

$$\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{10240ad}-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4608ad}-\frac{\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4096ad}+\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3584ad}-\frac{\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2048ad}+\frac{\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)}{512ad}-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/10240/a/d*tan(1/2*d*x+1/2*c)^10-1/4608/a/d*tan(1/2*d*x+1/2*c)^9-1/4096/a/
d*tan(1/2*d*x+1/2*c)^8+3/3584/a/d*tan(1/2*d*x+1/2*c)^7-1/2048/a/d*tan(1/2*d
*x+1/2*c)^6+1/512/a/d*tan(1/2*d*x+1/2*c)^4-1/192/a/d*tan(1/2*d*x+1/2*c)^3+1
/1024/a/d*tan(1/2*d*x+1/2*c)^2+3/256/a/d*tan(1/2*d*x+1/2*c)+1/2048/a/d/tan(
1/2*d*x+1/2*c)^6-3/256/a/d/tan(1/2*d*x+1/2*c)-3/256/a/d*ln(tan(1/2*d*x+1/2*
c))-3/3584/a/d/tan(1/2*d*x+1/2*c)^7-1/1024/a/d/tan(1/2*d*x+1/2*c)^2+1/4608/
a/d/tan(1/2*d*x+1/2*c)^9+1/4096/a/d/tan(1/2*d*x+1/2*c)^8-1/512/a/d/tan(1/2*
d*x+1/2*c)^4+1/192/a/d/tan(1/2*d*x+1/2*c)^3-1/10240/a/d/tan(1/2*d*x+1/2*c)^
10
```

maxima [B] time = 0.36, size = 394, normalized size = 2.24

$$\frac{\frac{\frac{15120\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{1260\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6720\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2520\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{630\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{1080\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{315\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{280\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{126\,\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}{a}-151\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/1290240*((15120*sin(d*x + c)/(cos(d*x + c) + 1) + 1260*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 6720*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2520*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - 630*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 10
80*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 315*sin(d*x + c)^8/(cos(d*x + c) +
 1)^8 - 280*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 126*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10)/a - 15120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a + (280*s
in(d*x + c)/(cos(d*x + c) + 1) + 315*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
1080*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 630*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 2520*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 6720*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7 - 1260*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 15120*sin(d
*x + c)^9/(cos(d*x + c) + 1)^9 - 126)*(cos(d*x + c) + 1)^10/(a*sin(d*x + c)
^10))/d
```

mupad [B] time = 14.00, size = 483, normalized size = 2.74

$$-\frac{126\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}-126\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}+280\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}-280\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^11*(a + a*sin(c + d*x))),x)
```

```
[Out] -(126*cos(c/2 + (d*x)/2)^20 - 126*sin(c/2 + (d*x)/2)^20 + 280*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^19 - 280*cos(c/2 + (d*x)/2)^19*sin(c/2 + (d*x)/2)
+ 315*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^18 - 1080*cos(c/2 + (d*x)/2)^
3*sin(c/2 + (d*x)/2)^17 + 630*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^16 -
2520*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^14 + 6720*cos(c/2 + (d*x)/2)^7
*sin(c/2 + (d*x)/2)^13 - 1260*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^12 -
15120*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^11 + 15120*cos(c/2 + (d*x)/2)
^11*sin(c/2 + (d*x)/2)^9 + 1260*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^8
- 6720*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2)^7 + 2520*cos(c/2 + (d*x)/2)
^14*sin(c/2 + (d*x)/2)^6 - 630*cos(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^4 +
 1080*cos(c/2 + (d*x)/2)^17*sin(c/2 + (d*x)/2)^3 - 315*cos(c/2 + (d*x)/2)^1
8*sin(c/2 + (d*x)/2)^2 + 15120*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*c
os(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^10)/(1290240*a*d*cos(c/2 + (d*x)/2)
^10*sin(c/2 + (d*x)/2)^10)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**11/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.721 $\int \frac{\cot^8(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$-\frac{\cot^{11}(c+dx)}{11ad}-\frac{2\cot^9(c+dx)}{9ad}-\frac{\cot^7(c+dx)}{7ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{256ad}+\frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad}-\frac{\cot^3(c+dx)\csc^5(c+dx)}{16ad}$$

```
[Out] -3/256*arctanh(cos(d*x+c))/a/d-1/7*cot(d*x+c)^7/a/d-2/9*cot(d*x+c)^9/a/d-1/
11*cot(d*x+c)^11/a/d-3/256*cot(d*x+c)*csc(d*x+c)/a/d-1/128*cot(d*x+c)*csc(d
*x+c)^3/a/d+1/32*cot(d*x+c)*csc(d*x+c)^5/a/d-1/16*cot(d*x+c)^3*csc(d*x+c)^5
/a/d+1/10*cot(d*x+c)^5*csc(d*x+c)^5/a/d
```

Rubi [A] time = 0.25, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2607, 270, 2611, 3768, 3770}

$$-\frac{\cot^{11}(c+dx)}{11ad}-\frac{2\cot^9(c+dx)}{9ad}-\frac{\cot^7(c+dx)}{7ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{256ad}+\frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad}-\frac{\cot^3(c+dx)\csc^5(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(256*a*d) - Cot[c + d*x]^7/(7*a*d) - (2*Cot[c +
d*x]^9)/(9*a*d) - Cot[c + d*x]^11/(11*a*d) - (3*Cot[c + d*x]*Csc[c + d*x])/
(256*a*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(128*a*d) + (Cot[c + d*x]*Csc[c +
 d*x]^5)/(32*a*d) - (Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*a*d) + (Cot[c + d*x
]^5*Csc[c + d*x]^5)/(10*a*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
```

```
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^6(c+dx)\csc^5(c+dx)\,dx}{a} + \frac{\int \cot^6(c+dx)\csc^6(c+dx)\,dx}{a}$$

$$= \frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad} + \frac{\int \cot^4(c+dx)\csc^5(c+dx)\,dx}{2a} + \frac{\text{Subst}\left(\int x^6\left(1+\right.\right.}{}$$

$$= -\frac{\cot^3(c+dx)\csc^5(c+dx)}{16ad} + \frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad} - \frac{3\int \cot^2(c+dx)\csc^5}{16a}$$

$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{2\cot^9(c+dx)}{9ad} - \frac{\cot^{11}(c+dx)}{11ad} + \frac{\cot(c+dx)\csc^5(c+dx)}{32ad} - \frac{c}{}$$

$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{2\cot^9(c+dx)}{9ad} - \frac{\cot^{11}(c+dx)}{11ad} - \frac{\cot(c+dx)\csc^3(c+dx)}{128ad} + \frac{c}{}$$

$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{2\cot^9(c+dx)}{9ad} - \frac{\cot^{11}(c+dx)}{11ad} - \frac{3\cot(c+dx)\csc(c+dx)}{256ad} - \frac{c}{}$$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{256ad} - \frac{\cot^7(c+dx)}{7ad} - \frac{2\cot^9(c+dx)}{9ad} - \frac{\cot^{11}(c+dx)}{11ad} - \frac{3\,\text{co}}{}$$

Mathematica [A] time = 2.94, size = 187, normalized size = 0.96

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-2661120\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(-2661120*(Log[Cos[(c + d*x)/2]] -
 Log[Sin[(c + d*x)/2]]) - Cot[c + d*x]*Csc[c + d*x]^10*(6840320 + 9973760*C
os[2*(c + d*x)] + 3543040*Cos[4*(c + d*x)] + 343040*Cos[6*(c + d*x)] - 6144
0*Cos[8*(c + d*x)] + 5120*Cos[10*(c + d*x)] - 3219678*Sin[c + d*x] - 260845
2*Sin[3*(c + d*x)] - 2181564*Sin[5*(c + d*x)] - 121275*Sin[7*(c + d*x)] + 1
0395*Sin[9*(c + d*x)])))/(227082240*a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.53, size = 302, normalized size = 1.56

$$\frac{20480\cos(dx+c)^{11} - 112640\cos(dx+c)^9 + 253440\cos(dx+c)^7 - 10395\left(\cos(dx+c)^{10} - 5\cos(dx+c)^8 +\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1774080*(20480*cos(d*x + c)^11 - 112640*cos(d*x + c)^9 + 253440*cos(d*x +
 c)^7 - 10395*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*
cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) + 10395*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*c
os(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) + 1386*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x + c)^5 + 7
0*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^10 - 5
*a*d*cos(d*x + c)^8 + 10*a*d*cos(d*x + c)^6 - 10*a*d*cos(d*x + c)^4 + 5*a*d
*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

giac [B] time = 0.27, size = 360, normalized size = 1.86

$$\frac{\frac{166320 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{630a^{10}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-1386a^{10}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-770a^{10}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+3465a^{10}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-4950a^{10}\tan\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/14192640*(166320*log(abs(tan(1/2*d*x + 1/2*c)))/a + (630*a^10*tan(1/2*d*x
 + 1/2*c)^11 - 1386*a^10*tan(1/2*d*x + 1/2*c)^10 - 770*a^10*tan(1/2*d*x + 1
/2*c)^9 + 3465*a^10*tan(1/2*d*x + 1/2*c)^8 - 4950*a^10*tan(1/2*d*x + 1/2*c)
^7 + 6930*a^10*tan(1/2*d*x + 1/2*c)^6 + 6930*a^10*tan(1/2*d*x + 1/2*c)^5 -
27720*a^10*tan(1/2*d*x + 1/2*c)^4 + 23100*a^10*tan(1/2*d*x + 1/2*c)^3 - 138
60*a^10*tan(1/2*d*x + 1/2*c)^2 - 69300*a^10*tan(1/2*d*x + 1/2*c))/a^11 - (5
02266*tan(1/2*d*x + 1/2*c)^11 - 69300*tan(1/2*d*x + 1/2*c)^10 - 13860*tan(1
/2*d*x + 1/2*c)^9 + 23100*tan(1/2*d*x + 1/2*c)^8 - 27720*tan(1/2*d*x + 1/2*
c)^7 + 6930*tan(1/2*d*x + 1/2*c)^6 + 6930*tan(1/2*d*x + 1/2*c)^5 - 4950*tan
(1/2*d*x + 1/2*c)^4 + 3465*tan(1/2*d*x + 1/2*c)^3 - 770*tan(1/2*d*x + 1/2*c
)^2 - 1386*tan(1/2*d*x + 1/2*c) + 630)/(a*tan(1/2*d*x + 1/2*c)^11))/d
```

maple [B] time = 0.63, size = 436, normalized size = 2.25

$$\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{22528ad}-\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{10240ad}-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{18432ad}+\frac{\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4096ad}-\frac{5\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14336ad}+\frac{\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2048ad}+\frac{\tan^{5}\left(\frac{dx}{2}+\cdots\right)}{2048ad}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/22528/a/d*tan(1/2*d*x+1/2*c)^11-1/10240/a/d*tan(1/2*d*x+1/2*c)^10-1/18432
/a/d*tan(1/2*d*x+1/2*c)^9+1/4096/a/d*tan(1/2*d*x+1/2*c)^8-5/14336/a/d*tan(1
/2*d*x+1/2*c)^7+1/2048/a/d*tan(1/2*d*x+1/2*c)^6+1/2048/a/d*tan(1/2*d*x+1/2*
c)^5-1/512/a/d*tan(1/2*d*x+1/2*c)^4+5/3072/a/d*tan(1/2*d*x+1/2*c)^3-1/1024/
a/d*tan(1/2*d*x+1/2*c)^2-5/1024/a/d*tan(1/2*d*x+1/2*c)-1/2048/a/d/tan(1/2*d
*x+1/2*c)^6+5/1024/a/d/tan(1/2*d*x+1/2*c)+3/256/a/d*ln(tan(1/2*d*x+1/2*c))-
1/2048/a/d/tan(1/2*d*x+1/2*c)^5+5/14336/a/d/tan(1/2*d*x+1/2*c)^7+1/1024/a/d
/tan(1/2*d*x+1/2*c)^2+1/18432/a/d/tan(1/2*d*x+1/2*c)^9-1/4096/a/d/tan(1/2*d
*x+1/2*c)^8+1/512/a/d/tan(1/2*d*x+1/2*c)^4-1/22528/a/d/tan(1/2*d*x+1/2*c)^1
1-5/3072/a/d/tan(1/2*d*x+1/2*c)^3+1/10240/a/d/tan(1/2*d*x+1/2*c)^10
```

maxima [B] time = 0.33, size = 475, normalized size = 2.45

$$-\frac{\frac{\frac{69300 \sin(dx+c)}{\cos(dx+c)+1}+\frac{13860 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{23100 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{27720 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6930 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{6930 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{4950 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3465 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{770 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\cdots}{a}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/14192640*((69300*sin(d*x + c)/(cos(d*x + c) + 1) + 13860*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 23100*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 27720*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6930*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5 - 6930*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 4950*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 3465*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 770*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9 + 1386*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 630*s
in(d*x + c)^11/(cos(d*x + c) + 1)^11)/a - 166320*log(sin(d*x + c)/(cos(d*x
+ c) + 1))/a - (1386*sin(d*x + c)/(cos(d*x + c) + 1) + 770*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 - 3465*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4950*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 6930*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
 6930*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 27720*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 - 23100*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 13860*sin(d*x + c)
^9/(cos(d*x + c) + 1)^9 + 69300*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 630
)*(cos(d*x + c) + 1)^11/(a*sin(d*x + c)^11))/d
```

mupad [B] time = 16.27, size = 579, normalized size = 2.98

$$\frac{630 \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{22}-630 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{22}-1386\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{21}+1386 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{21}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^12*(a + a*sin(c + d*x))),x)
```

```
[Out] (630*sin(c/2 + (d*x)/2)^22 - 630*cos(c/2 + (d*x)/2)^22 - 1386*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^21 + 1386*cos(c/2 + (d*x)/2)^21*sin(c/2 + (d*x)/2)
 - 770*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^20 + 3465*cos(c/2 + (d*x)/2)
^3*sin(c/2 + (d*x)/2)^19 - 4950*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^18
+ 6930*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^17 + 6930*cos(c/2 + (d*x)/2)
^6*sin(c/2 + (d*x)/2)^16 - 27720*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^15
 + 23100*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^14 - 13860*cos(c/2 + (d*x)
/2)^9*sin(c/2 + (d*x)/2)^13 - 69300*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2
)^12 + 69300*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^10 + 13860*cos(c/2 +
(d*x)/2)^13*sin(c/2 + (d*x)/2)^9 - 23100*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d
*x)/2)^8 + 27720*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2)^7 - 6930*cos(c/2
+ (d*x)/2)^16*sin(c/2 + (d*x)/2)^6 - 6930*cos(c/2 + (d*x)/2)^17*sin(c/2 + (
d*x)/2)^5 + 4950*cos(c/2 + (d*x)/2)^18*sin(c/2 + (d*x)/2)^4 - 3465*cos(c/2
+ (d*x)/2)^19*sin(c/2 + (d*x)/2)^3 + 770*cos(c/2 + (d*x)/2)^20*sin(c/2 + (d
*x)/2)^2 + 166320*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x
)/2)^11*sin(c/2 + (d*x)/2)^11)/(14192640*a*d*cos(c/2 + (d*x)/2)^11*sin(c/2
+ (d*x)/2)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**12/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.722 $\int \frac{\cos^8(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=203

$$\frac{\cos^{11}(c+dx)}{11a^2d}-\frac{4\cos^9(c+dx)}{9a^2d}+\frac{5\cos^7(c+dx)}{7a^2d}-\frac{2\cos^5(c+dx)}{5a^2d}+\frac{\sin^5(c+dx)\cos^5(c+dx)}{5a^2d}+\frac{\sin^3(c+dx)\cos^5(c+}{8a^2d}$$

```
[Out] -3/128*x/a^2-2/5*cos(d*x+c)^5/a^2/d+5/7*cos(d*x+c)^7/a^2/d-4/9*cos(d*x+c)^9
/a^2/d+1/11*cos(d*x+c)^11/a^2/d-3/128*cos(d*x+c)*sin(d*x+c)/a^2/d-1/64*cos(
d*x+c)^3*sin(d*x+c)/a^2/d+1/16*cos(d*x+c)^5*sin(d*x+c)/a^2/d+1/8*cos(d*x+c)
^5*sin(d*x+c)^3/a^2/d+1/5*cos(d*x+c)^5*sin(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.41, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2873, 2565, 270, 2568, 2635, 8}

$$\frac{\cos^{11}(c+dx)}{11a^2d}-\frac{4\cos^9(c+dx)}{9a^2d}+\frac{5\cos^7(c+dx)}{7a^2d}-\frac{2\cos^5(c+dx)}{5a^2d}+\frac{\sin^5(c+dx)\cos^5(c+dx)}{5a^2d}+\frac{\sin^3(c+dx)\cos^5(c+}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*x)/(128*a^2) - (2*Cos[c + d*x]^5)/(5*a^2*d) + (5*Cos[c + d*x]^7)/(7*a^2
*d) - (4*Cos[c + d*x]^9)/(9*a^2*d) + Cos[c + d*x]^11/(11*a^2*d) - (3*Cos[c
+ d*x]*Sin[c + d*x])/(128*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(64*a^2*d)
 + (Cos[c + d*x]^5*Sin[c + d*x])/(16*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x]^
3)/(8*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x]^5)/(5*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^4(c+dx)\sin^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cos^4(c+dx)\sin^5(c+dx) - 2a^2\cos^4(c+dx)\sin^6(c+dx) + a^2\cos^4(c+d\right.}{a^4}$$
$$= \frac{\int \cos^4(c+dx)\sin^5(c+dx)\,dx}{a^2} + \frac{\int \cos^4(c+dx)\sin^7(c+dx)\,dx}{a^2} - \frac{2\int \cos^4(c-}{}$$
$$= \frac{\cos^5(c+dx)\sin^5(c+dx)}{5a^2d} - \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a^2} - \frac{\text{Subst}\left(\int x^4\left(1-\right.\right.}{}$$
$$= \frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} + \frac{\cos^5(c+dx)\sin^5(c+dx)}{5a^2d} - \frac{3\int \cos^4(c+dx)\sin^2(}{8a^2}$$
$$= -\frac{2\cos^5(c+dx)}{5a^2d} + \frac{5\cos^7(c+dx)}{7a^2d} - \frac{4\cos^9(c+dx)}{9a^2d} + \frac{\cos^{11}(c+dx)}{11a^2d} + \frac{\cos^5(c+}{}$$
$$= -\frac{2\cos^5(c+dx)}{5a^2d} + \frac{5\cos^7(c+dx)}{7a^2d} - \frac{4\cos^9(c+dx)}{9a^2d} + \frac{\cos^{11}(c+dx)}{11a^2d} - \frac{\cos^3(c+}{}$$
$$= -\frac{2\cos^5(c+dx)}{5a^2d} + \frac{5\cos^7(c+dx)}{7a^2d} - \frac{4\cos^9(c+dx)}{9a^2d} + \frac{\cos^{11}(c+dx)}{11a^2d} - \frac{3\cos(c-}{}$$
$$= -\frac{3x}{128a^2} - \frac{2\cos^5(c+dx)}{5a^2d} + \frac{5\cos^7(c+dx)}{7a^2d} - \frac{4\cos^9(c+dx)}{9a^2d} + \frac{\cos^{11}(c+dx)}{11a^2d} -$$

Mathematica [B] time = 10.59, size = 1453, normalized size = 7.16

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*Cos[c + d*x]*(1 + 2*Sin[c + d*x]))/(3072*a^2*d*(1 + Sin[c + d*x])^2) +
(27720*(c + d*x) + 41580*Cos[c + d*x] - 7056*Cos[3*(c + d*x)] + 1764*Cos[5*
(c + d*x)] - 360*Cos[7*(c + d*x)] + 28*Cos[9*(c + d*x)] + (42*Sin[(c + d*x)
/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 21/(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2 - (15204*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]) - 15120*Sin[2*(c + d*x)] + 3528*Sin[4*(c + d*x)] - 840*Sin[6*(c + d*x)
] + 126*Sin[8*(c + d*x)])/(86016*a^2*d) + (-360360*(c + d*x) - 566280*Cos[c
 + d*x] + 108900*Cos[3*(c + d*x)] - 33264*Cos[5*(c + d*x)] + 9900*Cos[7*(c
+ d*x)] - 2200*Cos[9*(c + d*x)] + 180*Cos[11*(c + d*x)] - (330*Sin[(c + d*x
)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 165/(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^2 + (166980*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]) + 217800*Sin[2*(c + d*x)] - 59400*Sin[4*(c + d*x)] + 18480*Sin[6*(c
```

```
 + d*x)] - 4950*Sin[8*(c + d*x)] + 792*Sin[10*(c + d*x)])/(2027520*a^2*d) +
 (25*(36*d*x*Cos[(d*x)/2] - 21*Cos[c + (d*x)/2] + 35*Cos[c + (3*d*x)/2] - 1
2*d*x*Cos[2*c + (3*d*x)/2] - 3*Cos[3*c + (5*d*x)/2] - 57*Sin[(d*x)/2] + 36*
d*x*Sin[c + (d*x)/2] + 12*d*x*Sin[c + (3*d*x)/2] + 9*Sin[2*c + (3*d*x)/2] +
 3*Sin[2*c + (5*d*x)/2]))/(12288*a^2*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^3) + (5*(180*d*x*Cos[(d*x)/2] - 21*Cos[c + (d*x)/2]
 + 147*Cos[c + (3*d*x)/2] - 60*d*x*Cos[2*c + (3*d*x)/2] - 15*Cos[3*c + (5*d
*x)/2] + 3*Cos[3*c + (7*d*x)/2] + Cos[5*c + (9*d*x)/2] - 201*Sin[(d*x)/2] +
 180*d*x*Sin[c + (d*x)/2] + 60*d*x*Sin[c + (3*d*x)/2] + 73*Sin[2*c + (3*d*x
)/2] + 15*Sin[2*c + (5*d*x)/2] + 3*Sin[4*c + (7*d*x)/2] - Sin[4*c + (9*d*x)
/2]))/(12288*a^2*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^3) - (7*(2520*d*x*Cos[(d*x)/2] + 165*Cos[c + (d*x)/2] + 1905*Cos[c + (3
*d*x)/2] - 840*d*x*Cos[2*c + (3*d*x)/2] - 210*Cos[3*c + (5*d*x)/2] + 42*Cos
[3*c + (7*d*x)/2] + 14*Cos[5*c + (9*d*x)/2] - 6*Cos[5*c + (11*d*x)/2] - 3*C
os[7*c + (13*d*x)/2] - 2355*Sin[(d*x)/2] + 2520*d*x*Sin[c + (d*x)/2] + 840*
d*x*Sin[c + (3*d*x)/2] + 1175*Sin[2*c + (3*d*x)/2] + 210*Sin[2*c + (5*d*x)/
2] + 42*Sin[4*c + (7*d*x)/2] - 14*Sin[4*c + (9*d*x)/2] - 6*Sin[6*c + (11*d*
x)/2] + 3*Sin[6*c + (13*d*x)/2]))/(30720*a^2*d*(Cos[c/2] + Sin[c/2])*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^3) + (7560*d*x*Cos[(d*x)/2] + 1239*Cos[c +
(d*x)/2] + 5467*Cos[c + (3*d*x)/2] - 2520*d*x*Cos[2*c + (3*d*x)/2] - 630*Co
s[3*c + (5*d*x)/2] + 126*Cos[3*c + (7*d*x)/2] + 42*Cos[5*c + (9*d*x)/2] - 1
8*Cos[5*c + (11*d*x)/2] - 9*Cos[7*c + (13*d*x)/2] + 5*Cos[7*c + (15*d*x)/2]
 + 3*Cos[9*c + (17*d*x)/2] - 6321*Sin[(d*x)/2] + 7560*d*x*Sin[c + (d*x)/2]
+ 2520*d*x*Sin[c + (3*d*x)/2] + 3773*Sin[2*c + (3*d*x)/2] + 630*Sin[2*c + (
5*d*x)/2] + 126*Sin[4*c + (7*d*x)/2] - 42*Sin[4*c + (9*d*x)/2] - 18*Sin[6*c
 + (11*d*x)/2] + 9*Sin[6*c + (13*d*x)/2] + 5*Sin[8*c + (15*d*x)/2] - 3*Sin[
8*c + (17*d*x)/2])/(43008*a^2*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^3)
```

fricas [A] time = 0.51, size = 110, normalized size = 0.54

$$\frac{40320\,\cos\left(dx+c\right)^{11} - 197120\,\cos\left(dx+c\right)^{9} + 316800\,\cos\left(dx+c\right)^{7} - 177408\,\cos\left(dx+c\right)^{5} - 10395\,dx + 693\,(}{443520}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/443520*(40320*cos(d*x + c)^11 - 197120*cos(d*x + c)^9 + 316800*cos(d*x +
c)^7 - 177408*cos(d*x + c)^5 - 10395*d*x + 693*(128*cos(d*x + c)^9 - 336*co
s(d*x + c)^7 + 248*cos(d*x + c)^5 - 10*cos(d*x + c)^3 - 15*cos(d*x + c))*si
n(d*x + c))/(a^2*d)
```

giac [A] time = 0.31, size = 270, normalized size = 1.33

$$-\frac{\frac{10395\,(dx+c)}{a^2}+\frac{2\left(10395\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{21}+110880\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{19}+535689\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}+2365440\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16}-6564096\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/443520*(10395*(d*x + c)/a^2 + 2*(10395*tan(1/2*d*x + 1/2*c)^21 + 110880*
tan(1/2*d*x + 1/2*c)^19 + 535689*tan(1/2*d*x + 1/2*c)^17 + 2365440*tan(1/2*
d*x + 1/2*c)^16 - 6564096*tan(1/2*d*x + 1/2*c)^15 + 8279040*tan(1/2*d*x + 1
/2*c)^14 + 8364510*tan(1/2*d*x + 1/2*c)^13 - 12536832*tan(1/2*d*x + 1/2*c)^
12 + 20579328*tan(1/2*d*x + 1/2*c)^10 - 8364510*tan(1/2*d*x + 1/2*c)^9 - 25
34400*tan(1/2*d*x + 1/2*c)^8 + 6564096*tan(1/2*d*x + 1/2*c)^7 + 506880*tan(
1/2*d*x + 1/2*c)^6 - 535689*tan(1/2*d*x + 1/2*c)^5 + 957440*tan(1/2*d*x + 1
/2*c)^4 - 110880*tan(1/2*d*x + 1/2*c)^3 + 191488*tan(1/2*d*x + 1/2*c)^2 - 1
0395*tan(1/2*d*x + 1/2*c) + 17408)/((tan(1/2*d*x + 1/2*c)^2 + 1)^11*a^2))/d
```

maple [B] time = 0.51, size = 653, normalized size = 3.22

$$-\frac{272}{3465d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}}-\frac{272\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{315d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}}+\frac{\tan^3\left(\frac{dx}{2}\right.}{2d\,a^2\left(1+\tan^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -272/3465/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11+3/64/d/a^2/(1+tan(1/2*d*x+1/2*c
)^2)^11*tan(1/2*d*x+1/2*c)-272/315/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/
2*d*x+1/2*c)^2+1/2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^3-2
72/63/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^4+773/320/d/a^2/
(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^5-16/7/d/a^2/(1+tan(1/2*d*x+
1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^6-148/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*t
an(1/2*d*x+1/2*c)^7+80/7/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*
c)^8+1207/32/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^9-464/5/d
/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^10+848/15/d/a^2/(1+tan(
1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^12-1207/32/d/a^2/(1+tan(1/2*d*x+1/2
*c)^2)^11*tan(1/2*d*x+1/2*c)^13-112/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan
(1/2*d*x+1/2*c)^14+148/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*
c)^15-32/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^16-773/320/
```

```
d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^17-1/2/d/a^2/(1+tan(1/
2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^19-3/64/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^11*tan(1/2*d*x+1/2*c)^21-3/64/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.49, size = 648, normalized size = 3.19

$$\frac{\frac{10395\sin(dx+c)}{\cos(dx+c)+1}-\frac{191488\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{110880\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{957440\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{535689\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{506880\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6564096\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{2534400\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{8364\ldots}{(\mathrm{c}\ldots}}{a^2+\frac{11a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{55a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{165a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{330a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/221760*((10395*sin(d*x + c)/(cos(d*x + c) + 1) - 191488*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 110880*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 957440*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 535689*sin(d*x + c)^5/(cos(d*x + c) + 1
)^5 - 506880*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6564096*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 2534400*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 8364510
*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 20579328*sin(d*x + c)^10/(cos(d*x +
c) + 1)^10 + 12536832*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8364510*sin(d
*x + c)^13/(cos(d*x + c) + 1)^13 - 8279040*sin(d*x + c)^14/(cos(d*x + c) +
1)^14 + 6564096*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 2365440*sin(d*x + c
)^16/(cos(d*x + c) + 1)^16 - 535689*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 -
 110880*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 - 10395*sin(d*x + c)^21/(cos(
d*x + c) + 1)^21 - 17408)/(a^2 + 11*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 55*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 165*a^2*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + 330*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 462*a^2*si
n(d*x + c)^10/(cos(d*x + c) + 1)^10 + 462*a^2*sin(d*x + c)^12/(cos(d*x + c)
 + 1)^12 + 330*a^2*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 165*a^2*sin(d*x
+ c)^16/(cos(d*x + c) + 1)^16 + 55*a^2*sin(d*x + c)^18/(cos(d*x + c) + 1)^1
8 + 11*a^2*sin(d*x + c)^20/(cos(d*x + c) + 1)^20 + a^2*sin(d*x + c)^22/(cos
(d*x + c) + 1)^22) - 10395*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 11.51, size = 264, normalized size = 1.30

$$-\frac{3x}{128a^2}-\frac{\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{21}}{64}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}}{2}+\frac{773\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{320}+\frac{32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{3}-\frac{148\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{5}+\frac{112\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{3}+\frac{1207\tan\ldots}{3\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^5)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - (3*x)/(128*a^2) - ((272*tan(c/2 + (d*x)/2)^2)/315 - (3*tan(c/2 + (d*x)/2)
)/64 - tan(c/2 + (d*x)/2)^3/2 + (272*tan(c/2 + (d*x)/2)^4)/63 - (773*tan(c/
2 + (d*x)/2)^5)/320 + (16*tan(c/2 + (d*x)/2)^6)/7 + (148*tan(c/2 + (d*x)/2)
^7)/5 - (80*tan(c/2 + (d*x)/2)^8)/7 - (1207*tan(c/2 + (d*x)/2)^9)/32 + (464
*tan(c/2 + (d*x)/2)^10)/5 - (848*tan(c/2 + (d*x)/2)^12)/15 + (1207*tan(c/2
+ (d*x)/2)^13)/32 + (112*tan(c/2 + (d*x)/2)^14)/3 - (148*tan(c/2 + (d*x)/2)
^15)/5 + (32*tan(c/2 + (d*x)/2)^16)/3 + (773*tan(c/2 + (d*x)/2)^17)/320 + t
an(c/2 + (d*x)/2)^19/2 + (3*tan(c/2 + (d*x)/2)^21)/64 + 272/3465)/(a^2*d*(t
an(c/2 + (d*x)/2)^2 + 1)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.723 $\int \frac{\cos^8(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=185

$$\frac{2\cos^9(c+dx)}{9a^2d}-\frac{4\cos^7(c+dx)}{7a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{\sin^5(c+dx)\cos^5(c+dx)}{10a^2d}-\frac{3\sin^3(c+dx)\cos^5(c+dx)}{16a^2d}-\frac{3\sin(c$$

```
[Out] 9/256*x/a^2+2/5*cos(d*x+c)^5/a^2/d-4/7*cos(d*x+c)^7/a^2/d+2/9*cos(d*x+c)^9/
a^2/d+9/256*cos(d*x+c)*sin(d*x+c)/a^2/d+3/128*cos(d*x+c)^3*sin(d*x+c)/a^2/d
-3/32*cos(d*x+c)^5*sin(d*x+c)/a^2/d-3/16*cos(d*x+c)^5*sin(d*x+c)^3/a^2/d-1/
10*cos(d*x+c)^5*sin(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.46, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2873, 2568, 2635, 8, 2565, 270}

$$\frac{2\cos^9(c+dx)}{9a^2d}-\frac{4\cos^7(c+dx)}{7a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{\sin^5(c+dx)\cos^5(c+dx)}{10a^2d}-\frac{3\sin^3(c+dx)\cos^5(c+dx)}{16a^2d}-\frac{3\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (9*x)/(256*a^2) + (2*Cos[c + d*x]^5)/(5*a^2*d) - (4*Cos[c + d*x]^7)/(7*a^2*
d) + (2*Cos[c + d*x]^9)/(9*a^2*d) + (9*Cos[c + d*x]*Sin[c + d*x])/(256*a^2*
d) + (3*Cos[c + d*x]^3*Sin[c + d*x])/(128*a^2*d) - (3*Cos[c + d*x]^5*Sin[c
+ d*x])/(32*a^2*d) - (3*Cos[c + d*x]^5*Sin[c + d*x]^3)/(16*a^2*d) - (Cos[c
+ d*x]^5*Sin[c + d*x]^5)/(10*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^4(c+dx)\sin^4(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cos^4(c+dx)\sin^4(c+dx) - 2a^2\cos^4(c+dx)\sin^5(c+dx) + a^2\cos^4(c+\right.}{a^4}$$

$$= \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a^2} + \frac{\int \cos^4(c+dx)\sin^6(c+dx)\,dx}{a^2} - \frac{2\int \cos^4(c}{}$$

$$= -\frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{\cos^5(c+dx)\sin^5(c+dx)}{10a^2d} + \frac{3\int \cos^4(c+dx)\,\mathrm{si}}{8a^2}$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} - \frac{3\cos^5(c+dx)\sin^3(c+dx)}{16a^2d} - \frac{\cos^5(c+dx)\sin^5(}{10a^2d}$$

$$= \frac{2\cos^5(c+dx)}{5a^2d} - \frac{4\cos^7(c+dx)}{7a^2d} + \frac{2\cos^9(c+dx)}{9a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{64a^2d}$$

$$= \frac{2\cos^5(c+dx)}{5a^2d} - \frac{4\cos^7(c+dx)}{7a^2d} + \frac{2\cos^9(c+dx)}{9a^2d} + \frac{3\cos(c+dx)\sin(c+dx)}{128a^2d}$$

$$= \frac{3x}{128a^2} + \frac{2\cos^5(c+dx)}{5a^2d} - \frac{4\cos^7(c+dx)}{7a^2d} + \frac{2\cos^9(c+dx)}{9a^2d} + \frac{9\cos(c+dx)\,\mathrm{si}}{256a^2}$$

$$= \frac{9x}{256a^2} + \frac{2\cos^5(c+dx)}{5a^2d} - \frac{4\cos^7(c+dx)}{7a^2d} + \frac{2\cos^9(c+dx)}{9a^2d} + \frac{9\cos(c+dx)\,\mathrm{si}}{256a^2}$$

Mathematica [B] time = 8.38, size = 585, normalized size = 3.16

$$\frac{45360dx\sin\left(\frac{c}{2}\right) - 30240\sin\left(\frac{c}{2}+dx\right) + 30240\sin\left(\frac{3c}{2}+dx\right) - 1260\sin\left(\frac{3c}{2}+2dx\right) - 1260\sin\left(\frac{5c}{2}+2dx\right) - 672}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2520*(187*c - 18*d*x)*Cos[c/2] + 30240*Cos[c/2 + d*x] + 30240*Cos[(3*c)/2
 + d*x] - 1260*Cos[(3*c)/2 + 2*d*x] + 1260*Cos[(5*c)/2 + 2*d*x] + 6720*Cos[
(5*c)/2 + 3*d*x] + 6720*Cos[(7*c)/2 + 3*d*x] - 7560*Cos[(7*c)/2 + 4*d*x] +
7560*Cos[(9*c)/2 + 4*d*x] - 4032*Cos[(9*c)/2 + 5*d*x] - 4032*Cos[(11*c)/2 +
 5*d*x] + 630*Cos[(11*c)/2 + 6*d*x] - 630*Cos[(13*c)/2 + 6*d*x] - 720*Cos[(
13*c)/2 + 7*d*x] - 720*Cos[(15*c)/2 + 7*d*x] + 945*Cos[(15*c)/2 + 8*d*x] -
945*Cos[(17*c)/2 + 8*d*x] + 560*Cos[(17*c)/2 + 9*d*x] + 560*Cos[(19*c)/2 +
9*d*x] - 126*Cos[(19*c)/2 + 10*d*x] + 126*Cos[(21*c)/2 + 10*d*x] + 327180*S
in[c/2] - 471240*c*Sin[c/2] + 45360*d*x*Sin[c/2] - 30240*Sin[c/2 + d*x] + 3
```

```
0240*Sin[(3*c)/2 + d*x] - 1260*Sin[(3*c)/2 + 2*d*x] - 1260*Sin[(5*c)/2 + 2*
d*x] - 6720*Sin[(5*c)/2 + 3*d*x] + 6720*Sin[(7*c)/2 + 3*d*x] - 7560*Sin[(7*
c)/2 + 4*d*x] - 7560*Sin[(9*c)/2 + 4*d*x] + 4032*Sin[(9*c)/2 + 5*d*x] - 403
2*Sin[(11*c)/2 + 5*d*x] + 630*Sin[(11*c)/2 + 6*d*x] + 630*Sin[(13*c)/2 + 6*
d*x] + 720*Sin[(13*c)/2 + 7*d*x] - 720*Sin[(15*c)/2 + 7*d*x] + 945*Sin[(15*
c)/2 + 8*d*x] + 945*Sin[(17*c)/2 + 8*d*x] - 560*Sin[(17*c)/2 + 9*d*x] + 560
*Sin[(19*c)/2 + 9*d*x] - 126*Sin[(19*c)/2 + 10*d*x] - 126*Sin[(21*c)/2 + 10
*d*x])/(1290240*a^2*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.46, size = 100, normalized size = 0.54

$$\frac{17920 \cos(dx+c)^9 - 46080 \cos(dx+c)^7 + 32256 \cos(dx+c)^5 + 2835\,dx - 63\left(128 \cos(dx+c)^9 - 496 \cos(d\right.}{80640\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/80640*(17920*cos(d*x + c)^9 - 46080*cos(d*x + c)^7 + 32256*cos(d*x + c)^5
 + 2835*d*x - 63*(128*cos(d*x + c)^9 - 496*cos(d*x + c)^7 + 488*cos(d*x + c
)^5 - 30*cos(d*x + c)^3 - 45*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

giac [A] time = 0.26, size = 257, normalized size = 1.39

$$\frac{\frac{2835(dx+c)}{a^2} + \frac{2\left(2835 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{19} + 27405 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{17} - 139356 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15} + 860160 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{14} - 618660 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13} - 43\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/80640*(2835*(d*x + c)/a^2 + 2*(2835*tan(1/2*d*x + 1/2*c)^19 + 27405*tan(1
/2*d*x + 1/2*c)^17 - 139356*tan(1/2*d*x + 1/2*c)^15 + 860160*tan(1/2*d*x +
1/2*c)^14 - 618660*tan(1/2*d*x + 1/2*c)^13 - 430080*tan(1/2*d*x + 1/2*c)^12
 + 1609650*tan(1/2*d*x + 1/2*c)^11 + 516096*tan(1/2*d*x + 1/2*c)^10 - 16096
50*tan(1/2*d*x + 1/2*c)^9 + 1290240*tan(1/2*d*x + 1/2*c)^8 + 618660*tan(1/2
*d*x + 1/2*c)^7 - 368640*tan(1/2*d*x + 1/2*c)^6 + 139356*tan(1/2*d*x + 1/2*
c)^5 + 184320*tan(1/2*d*x + 1/2*c)^4 - 27405*tan(1/2*d*x + 1/2*c)^3 + 40960
*tan(1/2*d*x + 1/2*c)^2 - 2835*tan(1/2*d*x + 1/2*c) + 4096)/((tan(1/2*d*x +
 1/2*c)^2 + 1)^10*a^2))/d
```

maple [B] time = 0.46, size = 619, normalized size = 3.35

$$\frac{32}{315 d\,a^{2} \left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{128 d\,a^{2} \left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}+\frac{64\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{63 d\,a^{2} \left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{87\left(\tan^{3}\left(\frac{dx}{2}\right.\right.}{128 d\,a^{2} \left(1+\tan^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 32/315/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10-9/128/d/a^2/(1+tan(1/2*d*x+1/2*c)^
2)^10*tan(1/2*d*x+1/2*c)+64/63/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*
x+1/2*c)^2-87/128/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^3+32
/7/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^4+553/160/d/a^2/(1+
tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^5-64/7/d/a^2/(1+tan(1/2*d*x+1/2
*c)^2)^10*tan(1/2*d*x+1/2*c)^6+491/32/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan
(1/2*d*x+1/2*c)^7+32/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^8
-2555/64/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^9+64/5/d/a^2/
(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^10+2555/64/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^11-32/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^
10*tan(1/2*d*x+1/2*c)^12-491/32/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d
*x+1/2*c)^13+64/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^14-5
53/160/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^15+87/128/d/a^2
/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^17+9/128/d/a^2/(1+tan(1/2*d
*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^19+9/128/d/a^2*arctan(tan(1/2*d*x+1/2*c)
)
```

maxima [B] time = 0.44, size = 605, normalized size = 3.27

$$-\frac{\frac{2835\sin(dx+c)}{\cos(dx+c)+1}-\frac{40960\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{27405\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{184320\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{139356\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{368640\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{618660\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{1290240\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{160965}{(\cos}}{a^{2}+\frac{10a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{45a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{120a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{210a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{252a^{2}}{(\cos(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/40320*((2835*sin(d*x + c)/(cos(d*x + c) + 1) - 40960*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 27405*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 184320*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 139356*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
 + 368640*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 618660*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7 - 1290240*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1609650*sin
(d*x + c)^9/(cos(d*x + c) + 1)^9 - 516096*sin(d*x + c)^10/(cos(d*x + c) + 1
```

```
)^10 - 1609650*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 430080*sin(d*x + c)^
12/(cos(d*x + c) + 1)^12 + 618660*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 8
60160*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 139356*sin(d*x + c)^15/(cos(d
*x + c) + 1)^15 - 27405*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 2835*sin(d*
x + c)^19/(cos(d*x + c) + 1)^19 - 4096)/(a^2 + 10*a^2*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 45*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 120*a^2*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 210*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8 + 252*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 210*a^2*sin(d*x + c)^12
/(cos(d*x + c) + 1)^12 + 120*a^2*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 45
*a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 + 10*a^2*sin(d*x + c)^18/(cos(d*
x + c) + 1)^18 + a^2*sin(d*x + c)^20/(cos(d*x + c) + 1)^20) - 2835*arctan(s
in(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 11.63, size = 250, normalized size = 1.35

$$\frac{9\,x}{256\,a^2}+\frac{\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}}{128}+\frac{87\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{128}-\frac{553\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{160}+\frac{64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}}{3}-\frac{491\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32}-\frac{32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}+\frac{2555\tan\left(\frac{c}{2}\right.}{64}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^4)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (9*x)/(256*a^2) + ((64*tan(c/2 + (d*x)/2)^2)/63 - (9*tan(c/2 + (d*x)/2))/12
8 - (87*tan(c/2 + (d*x)/2)^3)/128 + (32*tan(c/2 + (d*x)/2)^4)/7 + (553*tan(
c/2 + (d*x)/2)^5)/160 - (64*tan(c/2 + (d*x)/2)^6)/7 + (491*tan(c/2 + (d*x)/
2)^7)/32 + 32*tan(c/2 + (d*x)/2)^8 - (2555*tan(c/2 + (d*x)/2)^9)/64 + (64*t
an(c/2 + (d*x)/2)^10)/5 + (2555*tan(c/2 + (d*x)/2)^11)/64 - (32*tan(c/2 + (
d*x)/2)^12)/3 - (491*tan(c/2 + (d*x)/2)^13)/32 + (64*tan(c/2 + (d*x)/2)^14)
/3 - (553*tan(c/2 + (d*x)/2)^15)/160 + (87*tan(c/2 + (d*x)/2)^17)/128 + (9*
tan(c/2 + (d*x)/2)^19)/128 + 32/315)/(a^2*d*(tan(c/2 + (d*x)/2)^2 + 1)^10)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.724 $\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=159

$$-\frac{\cos^9(c+dx)}{9a^2d}+\frac{3\cos^7(c+dx)}{7a^2d}-\frac{2\cos^5(c+dx)}{5a^2d}+\frac{\sin^3(c+dx)\cos^5(c+dx)}{4a^2d}+\frac{\sin(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{\sin(c+d}{}$$

```
[Out] -3/64*x/a^2-2/5*cos(d*x+c)^5/a^2/d+3/7*cos(d*x+c)^7/a^2/d-1/9*cos(d*x+c)^9/
a^2/d-3/64*cos(d*x+c)*sin(d*x+c)/a^2/d-1/32*cos(d*x+c)^3*sin(d*x+c)/a^2/d+1
/8*cos(d*x+c)^5*sin(d*x+c)/a^2/d+1/4*cos(d*x+c)^5*sin(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.36, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{\cos^9(c+dx)}{9a^2d}+\frac{3\cos^7(c+dx)}{7a^2d}-\frac{2\cos^5(c+dx)}{5a^2d}+\frac{\sin^3(c+dx)\cos^5(c+dx)}{4a^2d}+\frac{\sin(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*x)/(64*a^2) - (2*Cos[c + d*x]^5)/(5*a^2*d) + (3*Cos[c + d*x]^7)/(7*a^2*
d) - Cos[c + d*x]^9/(9*a^2*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(64*a^2*d) -
(Cos[c + d*x]^3*Sin[c + d*x])/(32*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(8
*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x]^3)/(4*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^4(c+dx)\sin^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cos^4(c+dx)\sin^3(c+dx) - 2a^2\cos^4(c+dx)\sin^4(c+dx) + a^2\cos^4(c+\right.}{a^4}$$
$$= \frac{\int \cos^4(c+dx)\sin^3(c+dx)\,dx}{a^2} + \frac{\int \cos^4(c+dx)\sin^5(c+dx)\,dx}{a^2} - \frac{2\int \cos^4(c}{}$$
$$= \frac{\cos^5(c+dx)\sin^3(c+dx)}{4a^2d} - \frac{3\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{4a^2} - \frac{\text{Subst}\left(\int x^4\left(\right.\right.}{}$$
$$= \frac{\cos^5(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^5(c+dx)\sin^3(c+dx)}{4a^2d} - \frac{\int \cos^4(c+dx)\,dx}{8a^2} - $$
$$= -\frac{2\cos^5(c+dx)}{5a^2d} + \frac{3\cos^7(c+dx)}{7a^2d} - \frac{\cos^9(c+dx)}{9a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{32a^2d}$$
$$= -\frac{2\cos^5(c+dx)}{5a^2d} + \frac{3\cos^7(c+dx)}{7a^2d} - \frac{\cos^9(c+dx)}{9a^2d} - \frac{3\cos(c+dx)\sin(c+dx)}{64a^2d}$$
$$= -\frac{3x}{64a^2} - \frac{2\cos^5(c+dx)}{5a^2d} + \frac{3\cos^7(c+dx)}{7a^2d} - \frac{\cos^9(c+dx)}{9a^2d} - \frac{3\cos(c+dx)\sin(}{64a^2d}$$

Mathematica [B] time = 6.91, size = 430, normalized size = 2.70

$$-\frac{15120dx\sin\left(\frac{c}{2}\right) - 11340\sin\left(\frac{c}{2}+dx\right) + 11340\sin\left(\frac{3c}{2}+dx\right) - 3360\sin\left(\frac{5c}{2}+3dx\right) + 3360\sin\left(\frac{7c}{2}+3dx\right) - 25}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/322560*(420*(7 + 330*c + 36*d*x)*Cos[c/2] + 11340*Cos[c/2 + d*x] + 11340
*Cos[(3*c)/2 + d*x] + 3360*Cos[(5*c)/2 + 3*d*x] + 3360*Cos[(7*c)/2 + 3*d*x]
 - 2520*Cos[(7*c)/2 + 4*d*x] + 2520*Cos[(9*c)/2 + 4*d*x] - 1008*Cos[(9*c)/2
 + 5*d*x] - 1008*Cos[(11*c)/2 + 5*d*x] - 450*Cos[(13*c)/2 + 7*d*x] - 450*Co
s[(15*c)/2 + 7*d*x] + 315*Cos[(15*c)/2 + 8*d*x] - 315*Cos[(17*c)/2 + 8*d*x]
 + 70*Cos[(17*c)/2 + 9*d*x] + 70*Cos[(19*c)/2 + 9*d*x] - 78960*Sin[c/2] + 1
38600*c*Sin[c/2] + 15120*d*x*Sin[c/2] - 11340*Sin[c/2 + d*x] + 11340*Sin[(3
*c)/2 + d*x] - 3360*Sin[(5*c)/2 + 3*d*x] + 3360*Sin[(7*c)/2 + 3*d*x] - 2520
*Sin[(7*c)/2 + 4*d*x] - 2520*Sin[(9*c)/2 + 4*d*x] + 1008*Sin[(9*c)/2 + 5*d*
x] - 1008*Sin[(11*c)/2 + 5*d*x] + 450*Sin[(13*c)/2 + 7*d*x] - 450*Sin[(15*c
)/2 + 7*d*x] + 315*Sin[(15*c)/2 + 8*d*x] + 315*Sin[(17*c)/2 + 8*d*x] - 70*S
```

```
in[(17*c)/2 + 9*d*x] + 70*Sin[(19*c)/2 + 9*d*x])/(a^2*d*(Cos[c/2] + Sin[c/2
]))
```

fricas [A] time = 0.47, size = 90, normalized size = 0.57

$$-\frac{2240\cos(dx+c)^9 - 8640\cos(dx+c)^7 + 8064\cos(dx+c)^5 + 945\,dx + 315\left(16\cos(dx+c)^7 - 24\cos(dx+c\right.}{20160\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20160*(2240*cos(d*x + c)^9 - 8640*cos(d*x + c)^7 + 8064*cos(d*x + c)^5 +
 945*d*x + 315*(16*cos(d*x + c)^7 - 24*cos(d*x + c)^5 + 2*cos(d*x + c)^3 +
3*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

giac [A] time = 0.29, size = 231, normalized size = 1.45

$$-\frac{\frac{945\,(dx+c)}{a^2} + \frac{2\left(945\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{17} + 8190\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15} + 40320\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{14} - 97650\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13} + 147840\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12} + 10647\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20160*(945*(d*x + c)/a^2 + 2*(945*tan(1/2*d*x + 1/2*c)^17 + 8190*tan(1/2
*d*x + 1/2*c)^15 + 40320*tan(1/2*d*x + 1/2*c)^14 - 97650*tan(1/2*d*x + 1/2*
c)^13 + 147840*tan(1/2*d*x + 1/2*c)^12 + 106470*tan(1/2*d*x + 1/2*c)^11 - 1
20960*tan(1/2*d*x + 1/2*c)^10 + 330624*tan(1/2*d*x + 1/2*c)^8 - 106470*tan(
1/2*d*x + 1/2*c)^7 - 8064*tan(1/2*d*x + 1/2*c)^6 + 97650*tan(1/2*d*x + 1/2*
c)^5 + 19584*tan(1/2*d*x + 1/2*c)^4 - 8190*tan(1/2*d*x + 1/2*c)^3 + 14976*t
an(1/2*d*x + 1/2*c)^2 - 945*tan(1/2*d*x + 1/2*c) + 1664)/((tan(1/2*d*x + 1/
2*c)^2 + 1)^9*a^2))/d
```

maple [B] time = 0.44, size = 551, normalized size = 3.47

$$-\frac{52}{315d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9} + \frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9} - \frac{52\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9} + \frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{16d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -52/315/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9+3/32/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^9*tan(1/2*d*x+1/2*c)-52/35/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1
/2*c)^2+13/16/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^3-68/35/d
/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^4-155/16/d/a^2/(1+tan(1/
2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^5+4/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9
*tan(1/2*d*x+1/2*c)^6+169/16/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1
/2*c)^7-164/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^8+12/d/a^
2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^10-169/16/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^11-44/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9
*tan(1/2*d*x+1/2*c)^12+155/16/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+
1/2*c)^13-4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^14-13/16/d/
a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^15-3/32/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^17-3/32/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 542, normalized size = 3.41

$$\frac{\frac{\frac{945\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{14976\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8190\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{19584\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{97650\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{8064\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{106470\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{330624\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120960\,\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\cdots}{a^2+\frac{9\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{36\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{84\,a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{126\,a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{126\,a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{84\,a^2\cdots}{\cdots}}}{1008\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/10080*((945*sin(d*x + c)/(cos(d*x + c) + 1) - 14976*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 8190*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 19584*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 97650*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 80
64*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 106470*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 - 330624*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 120960*sin(d*x + c)
^10/(cos(d*x + c) + 1)^10 - 106470*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 -
147840*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 97650*sin(d*x + c)^13/(cos(d
*x + c) + 1)^13 - 40320*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 8190*sin(d*
x + c)^15/(cos(d*x + c) + 1)^15 - 945*sin(d*x + c)^17/(cos(d*x + c) + 1)^17
 - 1664)/(a^2 + 9*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 36*a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 84*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
126*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 126*a^2*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10 + 84*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 36*a^2*si
n(d*x + c)^14/(cos(d*x + c) + 1)^14 + 9*a^2*sin(d*x + c)^16/(cos(d*x + c) +
 1)^16 + a^2*sin(d*x + c)^18/(cos(d*x + c) + 1)^18) - 945*arctan(sin(d*x +
c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 11.68, size = 225, normalized size = 1.42

$$-\frac{3\,x}{64\,a^2}-\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{32}+\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{16}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-\frac{155\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{16}+\frac{44\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}+\frac{169\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{16}-12\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^3)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - (3*x)/(64*a^2) - ((52*tan(c/2 + (d*x)/2)^2)/35 - (3*tan(c/2 + (d*x)/2))/3
2 - (13*tan(c/2 + (d*x)/2)^3)/16 + (68*tan(c/2 + (d*x)/2)^4)/35 + (155*tan(
c/2 + (d*x)/2)^5)/16 - (4*tan(c/2 + (d*x)/2)^6)/5 - (169*tan(c/2 + (d*x)/2)
^7)/16 + (164*tan(c/2 + (d*x)/2)^8)/5 - 12*tan(c/2 + (d*x)/2)^10 + (169*tan
(c/2 + (d*x)/2)^11)/16 + (44*tan(c/2 + (d*x)/2)^12)/3 - (155*tan(c/2 + (d*x
)/2)^13)/16 + 4*tan(c/2 + (d*x)/2)^14 + (13*tan(c/2 + (d*x)/2)^15)/16 + (3*
tan(c/2 + (d*x)/2)^17)/32 + 52/315)/(a^2*d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.725 $\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=141

$$-\frac{2\cos^7(c+dx)}{7a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{11\sin(c+dx)\cos^5(c+dx)}{48a^2d}+\frac{11\sin(c+dx)\cos^3(c}{192a^2d}$$

```
[Out] 11/128*x/a^2+2/5*cos(d*x+c)^5/a^2/d-2/7*cos(d*x+c)^7/a^2/d+11/128*cos(d*x+c
)*sin(d*x+c)/a^2/d+11/192*cos(d*x+c)^3*sin(d*x+c)/a^2/d-11/48*cos(d*x+c)^5*
sin(d*x+c)/a^2/d-1/8*cos(d*x+c)^5*sin(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.37, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2873, 2568, 2635, 8, 2565, 14}

$$-\frac{2\cos^7(c+dx)}{7a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{11\sin(c+dx)\cos^5(c+dx)}{48a^2d}+\frac{11\sin(c+dx)\cos^3(c}{192a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (11*x)/(128*a^2) + (2*Cos[c + d*x]^5)/(5*a^2*d) - (2*Cos[c + d*x]^7)/(7*a^2
*d) + (11*Cos[c + d*x]*Sin[c + d*x])/(128*a^2*d) + (11*Cos[c + d*x]^3*Sin[c
 + d*x])/(192*a^2*d) - (11*Cos[c + d*x]^5*Sin[c + d*x])/(48*a^2*d) - (Cos[c
 + d*x]^5*Sin[c + d*x]^3)/(8*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^4(c+dx)\sin^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cos^4(c+dx)\sin^2(c+dx) - 2a^2\cos^4(c+dx)\sin^3(c+dx) + a^2\cos^4(c+\right.}{a^4}$$
$$= \frac{\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{a^2} + \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a^2} - 2\int\cos^4(c$$
$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{6a^2d} - \frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} + \frac{\int \cos^4(c+dx)\,dx}{6a^2} +$$
$$= \frac{\cos^3(c+dx)\sin(c+dx)}{24a^2d} - \frac{11\cos^5(c+dx)\sin(c+dx)}{48a^2d} - \frac{\cos^5(c+dx)\sin^3(c}{8a^2d}$$
$$= \frac{2\cos^5(c+dx)}{5a^2d} - \frac{2\cos^7(c+dx)}{7a^2d} + \frac{\cos(c+dx)\sin(c+dx)}{16a^2d} + \frac{11\cos^3(c+dx)}{192a^2}$$
$$= \frac{x}{16a^2} + \frac{2\cos^5(c+dx)}{5a^2d} - \frac{2\cos^7(c+dx)}{7a^2d} + \frac{11\cos(c+dx)\sin(c+dx)}{128a^2d} + 11\,\mathrm{co}$$
$$= \frac{11x}{128a^2} + \frac{2\cos^5(c+dx)}{5a^2d} - \frac{2\cos^7(c+dx)}{7a^2d} + \frac{11\cos(c+dx)\sin(c+dx)}{128a^2d} + 11\,\mathrm{c}$$

Mathematica [B] time = 3.88, size = 481, normalized size = 3.41

$$\frac{18480dx\sin\left(\frac{c}{2}\right) - 10080\sin\left(\frac{c}{2}+dx\right) + 10080\sin\left(\frac{3c}{2}+dx\right) + 1680\sin\left(\frac{3c}{2}+2dx\right) + 1680\sin\left(\frac{5c}{2}+2dx\right) - 336}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (9240*(15*c + 2*d*x)*Cos[c/2] + 10080*Cos[c/2 + d*x] + 10080*Cos[(3*c)/2 +
d*x] + 1680*Cos[(3*c)/2 + 2*d*x] - 1680*Cos[(5*c)/2 + 2*d*x] + 3360*Cos[(5*
c)/2 + 3*d*x] + 3360*Cos[(7*c)/2 + 3*d*x] - 2520*Cos[(7*c)/2 + 4*d*x] + 252
0*Cos[(9*c)/2 + 4*d*x] - 672*Cos[(9*c)/2 + 5*d*x] - 672*Cos[(11*c)/2 + 5*d*
x] - 560*Cos[(11*c)/2 + 6*d*x] + 560*Cos[(13*c)/2 + 6*d*x] - 480*Cos[(13*c)
/2 + 7*d*x] - 480*Cos[(15*c)/2 + 7*d*x] + 105*Cos[(15*c)/2 + 8*d*x] - 105*C
os[(17*c)/2 + 8*d*x] - 79800*Sin[c/2] + 138600*c*Sin[c/2] + 18480*d*x*Sin[c
/2] - 10080*Sin[c/2 + d*x] + 10080*Sin[(3*c)/2 + d*x] + 1680*Sin[(3*c)/2 +
2*d*x] + 1680*Sin[(5*c)/2 + 2*d*x] - 3360*Sin[(5*c)/2 + 3*d*x] + 3360*Sin[(
7*c)/2 + 3*d*x] - 2520*Sin[(7*c)/2 + 4*d*x] - 2520*Sin[(9*c)/2 + 4*d*x] + 6
72*Sin[(9*c)/2 + 5*d*x] - 672*Sin[(11*c)/2 + 5*d*x] - 560*Sin[(11*c)/2 + 6*
d*x] - 560*Sin[(13*c)/2 + 6*d*x] + 480*Sin[(13*c)/2 + 7*d*x] - 480*Sin[(15*
```

```
c)/2 + 7*d*x] + 105*Sin[(15*c)/2 + 8*d*x] + 105*Sin[(17*c)/2 + 8*d*x])/(215
040*a^2*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.49, size = 80, normalized size = 0.57

$$-\frac{3840\cos(dx+c)^{7}-5376\cos(dx+c)^{5}-1155\,dx-35\left(48\cos(dx+c)^{7}-136\cos(dx+c)^{5}+22\cos(dx+c)\right.}{13440\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/13440*(3840*cos(d*x + c)^7 - 5376*cos(d*x + c)^5 - 1155*d*x - 35*(48*cos
(d*x + c)^7 - 136*cos(d*x + c)^5 + 22*cos(d*x + c)^3 + 33*cos(d*x + c))*sin
(d*x + c))/(a^2*d)
```

giac [A] time = 0.24, size = 205, normalized size = 1.45

$$\frac{\frac{1155\,(dx+c)}{a^{2}}+\frac{2\left(1155\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15}-9065\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+53760\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}-38605\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+79135\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+53760\,\mathrm{ta}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(1155*(d*x + c)/a^2 + 2*(1155*tan(1/2*d*x + 1/2*c)^15 - 9065*tan(1/
2*d*x + 1/2*c)^13 + 53760*tan(1/2*d*x + 1/2*c)^12 - 38605*tan(1/2*d*x + 1/2
*c)^11 + 79135*tan(1/2*d*x + 1/2*c)^9 + 53760*tan(1/2*d*x + 1/2*c)^8 - 7913
5*tan(1/2*d*x + 1/2*c)^7 + 86016*tan(1/2*d*x + 1/2*c)^6 + 38605*tan(1/2*d*x
 + 1/2*c)^5 - 10752*tan(1/2*d*x + 1/2*c)^4 + 9065*tan(1/2*d*x + 1/2*c)^3 +
12288*tan(1/2*d*x + 1/2*c)^2 - 1155*tan(1/2*d*x + 1/2*c) + 1536)/((tan(1/2*
d*x + 1/2*c)^2 + 1)^8*a^2))/d
```

maple [B] time = 0.38, size = 483, normalized size = 3.43

$$\frac{8}{35d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{8}}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{8}}+\frac{64\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{8}}+\frac{259\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 8/35/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8-11/64/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^
8*tan(1/2*d*x+1/2*c)+64/35/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2
*c)^2+259/192/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3-8/5/d/a
^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^4+1103/192/d/a^2/(1+tan(1/
2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5+64/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^
8*tan(1/2*d*x+1/2*c)^6-2261/192/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*
x+1/2*c)^7+8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^8+2261/192
/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^9-1103/192/d/a^2/(1+ta
n(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11+8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^8*tan(1/2*d*x+1/2*c)^12-259/192/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*
d*x+1/2*c)^13+11/64/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15+
11/64/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.44, size = 479, normalized size = 3.40

$$-\frac{\frac{\frac{1155 \sin(dx+c)}{\cos(dx+c)+1} - \frac{12288 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{9065 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{10752 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{38605 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{86016 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{79135 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{53760 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{79135 \sin(dx+c}{(\cos(dx+c)+1)}}{a^2 + \frac{8a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{28a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{56a^2 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{70a^2 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{56a^2 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{28a^2 \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}}{6720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/6720*((1155*sin(d*x + c)/(cos(d*x + c) + 1) - 12288*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 9065*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10752*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 38605*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8
6016*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 79135*sin(d*x + c)^7/(cos(d*x +
c) + 1)^7 - 53760*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 79135*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9 + 38605*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 5376
0*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 9065*sin(d*x + c)^13/(cos(d*x + c
) + 1)^13 - 1155*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 1536)/(a^2 + 8*a^2
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^2*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 56*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^2*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 + 56*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a
^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a^2*sin(d*x + c)^14/(cos(d*x +
 c) + 1)^14 + a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 1155*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 11.70, size = 198, normalized size = 1.40

$$\frac{11\,x}{128\,a^2} + \frac{\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64} - \frac{259\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192} + 8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - \frac{1103\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{192} + \frac{2261\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{192} + 8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{a^2\, d\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (11*x)/(128*a^2) + ((64*tan(c/2 + (d*x)/2)^2)/35 - (11*tan(c/2 + (d*x)/2))/
64 + (259*tan(c/2 + (d*x)/2)^3)/192 - (8*tan(c/2 + (d*x)/2)^4)/5 + (1103*ta
n(c/2 + (d*x)/2)^5)/192 + (64*tan(c/2 + (d*x)/2)^6)/5 - (2261*tan(c/2 + (d*
x)/2)^7)/192 + 8*tan(c/2 + (d*x)/2)^8 + (2261*tan(c/2 + (d*x)/2)^9)/192 - (
1103*tan(c/2 + (d*x)/2)^11)/192 + 8*tan(c/2 + (d*x)/2)^12 - (259*tan(c/2 +
(d*x)/2)^13)/192 + (11*tan(c/2 + (d*x)/2)^15)/64 + 8/35)/(a^2*d*(tan(c/2 +
(d*x)/2)^2 + 1)^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.726 $\int \frac{\cos^8(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{2\cos^7(c+dx)}{35a^2d}-\frac{\sin(c+dx)\cos^5(c+dx)}{15a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{12a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{8a^2d}-\frac{x}{8a^2}-\frac{\cos^9(c}{5d(a\sin(c}$$

```
[Out] -1/8*x/a^2-2/35*cos(d*x+c)^7/a^2/d-1/8*cos(d*x+c)*sin(d*x+c)/a^2/d-1/12*cos
(d*x+c)^3*sin(d*x+c)/a^2/d-1/15*cos(d*x+c)^5*sin(d*x+c)/a^2/d-1/5*cos(d*x+c
)^9/d/(a+a*sin(d*x+c))^2
```

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2859, 2682, 2635, 8}

$$-\frac{2\cos^7(c+dx)}{35a^2d}-\frac{\sin(c+dx)\cos^5(c+dx)}{15a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{12a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{8a^2d}-\frac{x}{8a^2}-\frac{\cos^9(c}{5d(a\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -x/(8*a^2) - (2*Cos[c + d*x]^7)/(35*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(8
*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(12*a^2*d) - (Cos[c + d*x]^5*Sin[c
+ d*x])/(15*a^2*d) - Cos[c + d*x]^9/(5*d*(a + a*Sin[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\cos^9(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\int \frac{\cos^8(c+dx)}{a+a\sin(c+dx)}\,dx}{5a}$$
$$= -\frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos^9(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\int \cos^6(c+dx)\,dx}{5a^2}$$
$$= -\frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{15a^2d} - \frac{\cos^9(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{\int \cos^4(c$$
$$= -\frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{12a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{15a^2d} - \frac{c}{5d(a+}$$
$$= -\frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{8a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{12a^2d} - \frac{\cos^5(c}{}$$
$$= -\frac{x}{8a^2} - \frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{8a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{12a^2d} - $$

Mathematica [B] time = 4.86, size = 418, normalized size = 3.37

$$-\frac{1680dx\sin\left(\frac{c}{2}\right) - 1155\sin\left(\frac{c}{2}+dx\right) + 1155\sin\left(\frac{3c}{2}+dx\right) + 210\sin\left(\frac{3c}{2}+2dx\right) + 210\sin\left(\frac{5c}{2}+2dx\right) - 525\sin\left(\frac{5}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/13440*(70*(7 + 24*d*x)*Cos[c/2] + 1155*Cos[c/2 + d*x] + 1155*Cos[(3*c)/2
 + d*x] + 210*Cos[(3*c)/2 + 2*d*x] - 210*Cos[(5*c)/2 + 2*d*x] + 525*Cos[(5*
c)/2 + 3*d*x] + 525*Cos[(7*c)/2 + 3*d*x] - 210*Cos[(7*c)/2 + 4*d*x] + 210*C
os[(9*c)/2 + 4*d*x] + 63*Cos[(9*c)/2 + 5*d*x] + 63*Cos[(11*c)/2 + 5*d*x] -
70*Cos[(11*c)/2 + 6*d*x] + 70*Cos[(13*c)/2 + 6*d*x] - 15*Cos[(13*c)/2 + 7*d
*x] - 15*Cos[(15*c)/2 + 7*d*x] - 490*Sin[c/2] + 1680*d*x*Sin[c/2] - 1155*Si
```

```
n[c/2 + d*x] + 1155*Sin[(3*c)/2 + d*x] + 210*Sin[(3*c)/2 + 2*d*x] + 210*Sin
[(5*c)/2 + 2*d*x] - 525*Sin[(5*c)/2 + 3*d*x] + 525*Sin[(7*c)/2 + 3*d*x] - 2
10*Sin[(7*c)/2 + 4*d*x] - 210*Sin[(9*c)/2 + 4*d*x] - 63*Sin[(9*c)/2 + 5*d*x
] + 63*Sin[(11*c)/2 + 5*d*x] - 70*Sin[(11*c)/2 + 6*d*x] - 70*Sin[(13*c)/2 +
 6*d*x] + 15*Sin[(13*c)/2 + 7*d*x] - 15*Sin[(15*c)/2 + 7*d*x])/(a^2*d*(Cos[
c/2] + Sin[c/2]))
```

fricas [A] time = 0.46, size = 70, normalized size = 0.56

$$\frac{120 \cos(dx+c)^7 - 336 \cos(dx+c)^5 - 105\,dx + 35\left(8 \cos(dx+c)^5 - 2 \cos(dx+c)^3 - 3 \cos(dx+c)\right)\sin(dx}{840\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*cos(d*x + c)^7 - 336*cos(d*x + c)^5 - 105*d*x + 35*(8*cos(d*x +
c)^5 - 2*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

giac [A] time = 0.19, size = 192, normalized size = 1.55

$$-\frac{\frac{105\,(dx+c)}{a^2} + \frac{2\left(105 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} + 840 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12} - 1540 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 3360 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10} + 1085 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} + 840 \tan\left(\frac{1}{2}\right.}{\left(\mathrm{ta}\right.}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(105*(d*x + c)/a^2 + 2*(105*tan(1/2*d*x + 1/2*c)^13 + 840*tan(1/2*d*
x + 1/2*c)^12 - 1540*tan(1/2*d*x + 1/2*c)^11 + 3360*tan(1/2*d*x + 1/2*c)^10
 + 1085*tan(1/2*d*x + 1/2*c)^9 + 840*tan(1/2*d*x + 1/2*c)^8 + 6720*tan(1/2*
d*x + 1/2*c)^6 - 1085*tan(1/2*d*x + 1/2*c)^5 + 1176*tan(1/2*d*x + 1/2*c)^4
+ 1540*tan(1/2*d*x + 1/2*c)^3 + 672*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*
x + 1/2*c) + 216)/((tan(1/2*d*x + 1/2*c)^2 + 1)^7*a^2))/d
```

maple [B] time = 0.31, size = 449, normalized size = 3.62

$$-\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7} - \frac{2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7} + \frac{11\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7} - \frac{8\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13-2/d/a^2/(1+tan(
1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12+11/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^
2)^7*tan(1/2*d*x+1/2*c)^11-8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1
/2*c)^10-31/12/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^9-2/d/a^
2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8-16/d/a^2/(1+tan(1/2*d*x+1
/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6+31/12/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(
1/2*d*x+1/2*c)^5-14/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4
-11/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^3-8/5/d/a^2/(1+ta
n(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2+1/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^
2)^7*tan(1/2*d*x+1/2*c)-18/35/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7-1/4/d/a^2*ar
ctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.46, size = 436, normalized size = 3.52

$$\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{672\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1540\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1176\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1085\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{6720\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{840\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{1085\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{3360\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{1540\sin(\cdots}{(\cos(dx+c\cdots}}{a^2+\frac{7a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{21a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{21a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{7a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a^2\sin(dx+\cdots}{(\cos(dx+c)\cdots}}\cdots}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/420*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 672*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 1540*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1176*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 1085*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 6720*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 840*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
- 1085*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 3360*sin(d*x + c)^10/(cos(d*x
+ c) + 1)^10 + 1540*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 840*sin(d*x + c
)^12/(cos(d*x + c) + 1)^12 - 105*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 21
6)/(a^2 + 7*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 21*a^2*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 35*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 35*a^2
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 21*a^2*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10 + 7*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + a^2*sin(d*x + c)^1
4/(cos(d*x + c) + 1)^14) - 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)
/d
```

mupad [B] time = 12.65, size = 186, normalized size = 1.50

$$-\frac{x}{8a^2}-\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{4}+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-\frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{3}+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\frac{31\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{12}+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+16\,\mathrm{ta}\cdots}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - x/(8*a^2) - ((8*tan(c/2 + (d*x)/2)^2)/5 - tan(c/2 + (d*x)/2)/4 + (11*tan(
c/2 + (d*x)/2)^3)/3 + (14*tan(c/2 + (d*x)/2)^4)/5 - (31*tan(c/2 + (d*x)/2)^
5)/12 + 16*tan(c/2 + (d*x)/2)^6 + 2*tan(c/2 + (d*x)/2)^8 + (31*tan(c/2 + (d
*x)/2)^9)/12 + 8*tan(c/2 + (d*x)/2)^10 - (11*tan(c/2 + (d*x)/2)^11)/3 + 2*t
an(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^13/4 + 18/35)/(a^2*d*(tan(c/2 + (
d*x)/2)^2 + 1)^7)
```

sympy [A] time = 172.04, size = 3196, normalized size = 25.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-105*d*x*tan(c/2 + d*x/2)**14/(840*a**2*d*tan(c/2 + d*x/2)**14 +
 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 294
00*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a*
*2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) -
735*d*x*tan(c/2 + d*x/2)**12/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d
*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*ta
n(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2
 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 2205*d*x*tan
(c/2 + d*x/2)**10/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 +
d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x
/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**
4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 3675*d*x*tan(c/2 + d*x/
2)**8/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 +
 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 294
00*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**
2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 3675*d*x*tan(c/2 + d*x/2)**6/(840*a
**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*
d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*ta
n(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2
+ d*x/2)**2 + 840*a**2*d) - 2205*d*x*tan(c/2 + d*x/2)**4/(840*a**2*d*tan(c/
2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 +
d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/
2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2
+ 840*a**2*d) - 735*d*x*tan(c/2 + d*x/2)**2/(840*a**2*d*tan(c/2 + d*x/2)**1
4 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 +
29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640
*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d)
 - 105*d*x/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)*
*12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8
```

+ 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 588
0*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 210*tan(c/2 + d*x/2)**13/(840*
a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2
*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*t
an(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2
 + d*x/2)**2 + 840*a**2*d) - 1680*tan(c/2 + d*x/2)**12/(840*a**2*d*tan(c/2
+ d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*
x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)
6 + 17640*a2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 +
840*a**2*d) + 3080*tan(c/2 + d*x/2)**11/(840*a**2*d*tan(c/2 + d*x/2)**14 +
5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 2940
0*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**
2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 6
720*tan(c/2 + d*x/2)**10/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan
(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/
2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d
*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 2170*tan(c/2 + d
*x/2)**9/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**1
2 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 +
29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*
a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 1680*tan(c/2 + d*x/2)**8/(840*a*
*2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d
*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan
(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 +
 d*x/2)**2 + 840*a**2*d) - 13440*tan(c/2 + d*x/2)**6/(840*a**2*d*tan(c/2 +
d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/
2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**
6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 84
0*a**2*d) + 2170*tan(c/2 + d*x/2)**5/(840*a**2*d*tan(c/2 + d*x/2)**14 + 588
0*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a
2*d*tan(c/2 + d*x/2)8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d
*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 2352
*tan(c/2 + d*x/2)**4/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2
 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 +
d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2
)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 3080*tan(c/2 + d*x/2
)**3/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 +
17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 2940
0*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2
*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 1344*tan(c/2 + d*x/2)**2/(840*a**2*d
*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan
(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2
 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x
/2)**2 + 840*a**2*d) + 210*tan(c/2 + d*x/2)/(840*a**2*d*tan(c/2 + d*x/2)**1
4 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 +

```
29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640
*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d)
 - 432/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12
+ 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29
400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a*
*2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d), Ne(d, 0)), (x*sin(c)*cos(c)**8/(a*s
in(c) + a)**2, True))
```

3.727 $\int \frac{\cos^7(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=119

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{2a^2d}-\frac{3\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{\tanh^{-1}(\cos(c+}{a^2d}$$

```
[Out] -3/4*x/a^2-arctanh(cos(d*x+c))/a^2/d+cos(d*x+c)/a^2/d+1/3*cos(d*x+c)^3/a^2/
d-1/5*cos(d*x+c)^5/a^2/d-3/4*cos(d*x+c)*sin(d*x+c)/a^2/d-1/2*cos(d*x+c)^3*s
in(d*x+c)/a^2/d
```

Rubi [A] time = 0.24, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2875, 2873, 2635, 8, 2592, 302, 206, 2565, 30}

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{2a^2d}-\frac{3\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{\tanh^{-1}(\cos(c+}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*x)/(4*a^2) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(a^2*d) + Cos
[c + d*x]^3/(3*a^2*d) - Cos[c + d*x]^5/(5*a^2*d) - (3*Cos[c + d*x]*Sin[c +
d*x])/(4*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^3(c+dx)\cot(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(-2a^2\cos^4(c+dx)+a^2\cos^3(c+dx)\cot(c+dx)+a^2\cos^4(c+dx)\sin(c+dx\right.}{a^4}$$
$$= \frac{\int \cos^3(c+dx)\cot(c+dx)\,dx}{a^2} + \frac{\int \cos^4(c+dx)\sin(c+dx)\,dx}{a^2} - \frac{2\int \cos^4(c+d}{a^2}$$
$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{2a^2d} - \frac{3\int \cos^2(c+dx)\,dx}{2a^2} - \frac{\text{Subst}\left(\int x^4\,dx, x, \cos(c+dx\right.}{a^2d}$$
$$= -\frac{\cos^5(c+dx)}{5a^2d} - \frac{3\cos(c+dx)\sin(c+dx)}{4a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{2a^2d} - \frac{3\int 1\,d}{4a^2}$$
$$= -\frac{3x}{4a^2} + \frac{\cos(c+dx)}{a^2d} + \frac{\cos^3(c+dx)}{3a^2d} - \frac{\cos^5(c+dx)}{5a^2d} - \frac{3\cos(c+dx)\sin(c+dx)}{4a^2d}$$
$$= -\frac{3x}{4a^2} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{a^2d} + \frac{\cos^3(c+dx)}{3a^2d} - \frac{\cos^5(c+dx)}{5a^2d} - 3$$

Mathematica [A] time = 0.71, size = 93, normalized size = 0.78

$$\frac{270\cos(c+dx)+5\cos(3(c+dx))-3\left(40\sin(2(c+dx))+5\sin(4(c+dx))+\cos(5(c+dx))-80\log\left(\sin\left(\frac{1}{2}(c+\right.\right.\right.}{240a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (270*Cos[c + d*x] + 5*Cos[3*(c + d*x)] - 3*(60*c + 60*d*x + Cos[5*(c + d*x)
] + 80*Log[Cos[(c + d*x)/2]] - 80*Log[Sin[(c + d*x)/2]] + 40*Sin[2*(c + d*x
)] + 5*Sin[4*(c + d*x)]))/(240*a^2*d)
```

fricas [A] time = 0.50, size = 94, normalized size = 0.79

$$-\frac{12\cos(dx+c)^5-20\cos(dx+c)^3+45\,dx+15\left(2\cos(dx+c)^3+3\cos(dx+c)\right)\sin(dx+c)-60\cos(dx+c}{60\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(12*cos(d*x + c)^5 - 20*cos(d*x + c)^3 + 45*d*x + 15*(2*cos(d*x + c)^
3 + 3*cos(d*x + c))*sin(d*x + c) - 60*cos(d*x + c) + 30*log(1/2*cos(d*x + c
) + 1/2) - 30*log(-1/2*cos(d*x + c) + 1/2))/(a^2*d)
```

giac [A] time = 0.22, size = 156, normalized size = 1.31

$$-\frac{\frac{45\,(dx+c)}{a^2} - \frac{60\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} - \frac{2\left(75\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+60\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+30\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+360\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+320\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5 a^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(45*(d*x + c)/a^2 - 60*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 2*(75*tan
(1/2*d*x + 1/2*c)^9 + 60*tan(1/2*d*x + 1/2*c)^8 + 30*tan(1/2*d*x + 1/2*c)^7
 + 360*tan(1/2*d*x + 1/2*c)^6 + 320*tan(1/2*d*x + 1/2*c)^4 - 30*tan(1/2*d*x
 + 1/2*c)^3 + 280*tan(1/2*d*x + 1/2*c)^2 - 75*tan(1/2*d*x + 1/2*c) + 68)/((
tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^2))/d
```

maple [B] time = 0.54, size = 329, normalized size = 2.76

$$\frac{5\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{12\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 5/2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9+2/d/a^2/(1+tan(1/
2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*t
an(1/2*d*x+1/2*c)^7+12/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^
6+32/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4-1/d/a^2/(1+tan
(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3+28/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^
2)^5*tan(1/2*d*x+1/2*c)^2-5/2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+
1/2*c)+34/15/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5-3/2/d/a^2*arctan(tan(1/2*d*x+
1/2*c))+1/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 333, normalized size = 2.80

$$-\frac{\frac{\frac{75\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{280\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{30\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{320\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{360\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{30\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{60\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{75\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-68}{a^2+\frac{5\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10\,a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5\,a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{45\,\arctan\left(\frac{\sin(dx+c}{\cos(dx+c)}\right.}{a^2}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*((75*sin(d*x + c)/(cos(d*x + c) + 1) - 280*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 320*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 360*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 30*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 - 60*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 75*sin
(d*x + c)^9/(cos(d*x + c) + 1)^9 - 68)/(a^2 + 5*a^2*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^2*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a
^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 45*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a^2 - 30*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 10.76, size = 262, normalized size = 2.20

$$\frac{3\,\mathrm{atan}\left(\frac{9}{4\left(\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+3\right)}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+3}\right)}{2\,a^2\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+12\,t}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (3*atan(9/(4*((9*tan(c/2 + (d*x)/2))/4 + 3)) - (3*tan(c/2 + (d*x)/2))/((9*t
an(c/2 + (d*x)/2))/4 + 3)))/(2*a^2*d) + log(tan(c/2 + (d*x)/2))/(a^2*d) + (
(28*tan(c/2 + (d*x)/2)^2)/3 - (5*tan(c/2 + (d*x)/2))/2 - tan(c/2 + (d*x)/2)
^3 + (32*tan(c/2 + (d*x)/2)^4)/3 + 12*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x
)/2)^7 + 2*tan(c/2 + (d*x)/2)^8 + (5*tan(c/2 + (d*x)/2)^9)/2 + 34/15)/(d*(5
*a^2*tan(c/2 + (d*x)/2)^2 + 10*a^2*tan(c/2 + (d*x)/2)^4 + 10*a^2*tan(c/2 +
(d*x)/2)^6 + 5*a^2*tan(c/2 + (d*x)/2)^8 + a^2*tan(c/2 + (d*x)/2)^10 + a^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.728 $\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{2\tanh^{-1}(\cos(}{a^2d}$$

```
[Out] -9/8*x/a^2+2*arctanh(cos(d*x+c))/a^2/d-2*cos(d*x+c)/a^2/d-2/3*cos(d*x+c)^3/
a^2/d-cot(d*x+c)/a^2/d+1/8*cos(d*x+c)*sin(d*x+c)/a^2/d-1/4*cos(d*x+c)*sin(d
*x+c)^3/a^2/d
```

Rubi [A] time = 0.30, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2872, 3770, 3767, 8, 2638, 2635, 2633}

$$-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{2\tanh^{-1}(\cos(}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-9*x)/(8*a^2) + (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - (2*Cos[c + d*x])/(a^2*
d) - (2*Cos[c + d*x]^3)/(3*a^2*d) - Cot[c + d*x]/(a^2*d) + (Cos[c + d*x]*Si
n[c + d*x])/(8*a^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^2(c+dx)\cot^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(-a^6-2a^6\csc(c+dx)+a^6\csc^2(c+dx)+4a^6\sin(c+dx)-a^6\sin^2(c+dx\right.}{a^8}$$
$$= -\frac{x}{a^2}+\frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{\int \sin^2(c+dx)\,dx}{a^2}+\frac{\int \sin^4(c+dx)\,dx}{a^2}-\frac{2\int \csc(}{}$$
$$= -\frac{x}{a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{4\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-$$
$$= -\frac{3x}{2a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\cot(c+dx}{a^2d}$$
$$= -\frac{9x}{8a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\cot(c+dx}{a^2d}$$

Mathematica [A] time = 1.52, size = 128, normalized size = 1.10

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(-108(c+dx)+3\sin(4(c+dx))-240\cos(c+dx)-16\cos(3(c+dx))+4\right.}{96d(a\sin(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(-108*(c + d*x) - 240*Cos[c + d*x]
 - 16*Cos[3*(c + d*x)] - 48*Cot[(c + d*x)/2] + 192*Log[Cos[(c + d*x)/2]] -
192*Log[Sin[(c + d*x)/2]] + 3*Sin[4*(c + d*x)] + 48*Tan[(c + d*x)/2]))/(96*
d*(a + a*Sin[c + d*x])^2)
```

fricas [A] time = 0.50, size = 113, normalized size = 0.97

$$-\frac{6\cos(dx+c)^5-9\cos(dx+c)^3+\left(16\cos(dx+c)^3+27\,dx+48\cos(dx+c)\right)\sin(dx+c)-24\log\left(\frac{1}{2}\cos(\right.}{24\,a^2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*cos(d*x + c)^5 - 9*cos(d*x + c)^3 + (16*cos(d*x + c)^3 + 27*d*x +
48*cos(d*x + c))*sin(d*x + c) - 24*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
```

```
 + 24*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 27*cos(d*x + c))/(a^2*d*s
in(d*x + c))
```

giac [A] time = 0.22, size = 186, normalized size = 1.60

$$-\frac{\frac{27\,(dx+c)}{a^2}+\frac{48\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}-\frac{12\left(4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+96\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-21\,\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(27*(d*x + c)/a^2 + 48*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 12*tan(1/
2*d*x + 1/2*c)/a^2 - 12*(4*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2
*c)) + 2*(3*tan(1/2*d*x + 1/2*c)^7 + 96*tan(1/2*d*x + 1/2*c)^6 - 21*tan(1/2
*d*x + 1/2*c)^5 + 192*tan(1/2*d*x + 1/2*c)^4 + 21*tan(1/2*d*x + 1/2*c)^3 +
160*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 64)/((tan(1/2*d*x + 1
/2*c)^2 + 1)^4*a^2))/d
```

maple [B] time = 0.54, size = 333, normalized size = 2.87

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{8\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{16\left(\tan^4\left(\frac{dx}{2}\right.\right.}{a^2d\left(1+\tan^2\left(\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d
*x+1/2*c)^7-8/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6+7/4/a^2
/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-16/a^2/d/(1+tan(1/2*d*x+
1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4-7/4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1
/2*d*x+1/2*c)^3-40/3/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2+
1/4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-16/3/a^2/d/(1+tan(1
/2*d*x+1/2*c)^2)^4-9/4/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/2/d/a^2/tan(1/2*d
*x+1/2*c)-2/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.44, size = 348, normalized size = 3.00

$$-\frac{\frac{\frac{64\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{21\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{160\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{57\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{192\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{96\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{9\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+6}{\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{4\,a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6\,a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{4\,a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}+\frac{27\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right.}{a^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*((64*sin(d*x + c)/(cos(d*x + c) + 1) + 21*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 160*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 57*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 192*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 96*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 9*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 + 6)/(a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 4
*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a^2*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 + 4*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a^2*sin(d*x + c)^9/
(cos(d*x + c) + 1)^9) + 27*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 24
*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 6*sin(d*x + c)/(a^2*(cos(d*x +
c) + 1)))/d
```

mupad [B] time = 9.21, size = 279, normalized size = 2.41

$$\frac{9\,\mathrm{atan}\left(\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\frac{81\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}-9}+\frac{81}{16\left(\frac{81\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}-9\right)}\right)}{4\,a^2\,d}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+ \cdots}{d\left(2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+8\,a^2\tan\left(\frac{c}{2}+ \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (9*atan((9*tan(c/2 + (d*x)/2))/((81*tan(c/2 + (d*x)/2))/16 - 9) + 81/(16*((
81*tan(c/2 + (d*x)/2))/16 - 9))))/(4*a^2*d) - (2*log(tan(c/2 + (d*x)/2)))/(
a^2*d) - ((32*tan(c/2 + (d*x)/2))/3 + (7*tan(c/2 + (d*x)/2)^2)/2 + (80*tan(
c/2 + (d*x)/2)^3)/3 + (19*tan(c/2 + (d*x)/2)^4)/2 + 32*tan(c/2 + (d*x)/2)^5
 + tan(c/2 + (d*x)/2)^6/2 + 16*tan(c/2 + (d*x)/2)^7 + (3*tan(c/2 + (d*x)/2)
^8)/2 + 1)/(d*(8*a^2*tan(c/2 + (d*x)/2)^3 + 12*a^2*tan(c/2 + (d*x)/2)^5 + 8
*a^2*tan(c/2 + (d*x)/2)^7 + 2*a^2*tan(c/2 + (d*x)/2)^9 + 2*a^2*tan(c/2 + (d
*x)/2))) + tan(c/2 + (d*x)/2)/(2*a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.729 $\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=97

$$\frac{\cos^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{3x}{a^2}$$

```
[Out] 3*x/a^2+1/2*arctanh(cos(d*x+c))/a^2/d+1/3*cos(d*x+c)^3/a^2/d+2*cot(d*x+c)/a
^2/d-1/2*cot(d*x+c)*csc(d*x+c)/a^2/d+cos(d*x+c)*sin(d*x+c)/a^2/d
```

Rubi [A] time = 0.25, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2875, 2872, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$\frac{\cos^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{3x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*x)/a^2 + ArcTanh[Cos[c + d*x]]/(2*a^2*d) + Cos[c + d*x]^3/(3*a^2*d) + (2
*Cot[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) + (Cos[c + d
*x]*Sin[c + d*x])/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos(c+dx)\cot^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(4a^6 - a^6\csc(c+dx) - 2a^6\csc^2(c+dx) + a^6\csc^3(c+dx) - a^6\sin(c+dx) -}{a^8}$$
$$= \frac{4x}{a^2} - \frac{\int \csc(c+dx)\,dx}{a^2} + \frac{\int \csc^3(c+dx)\,dx}{a^2} - \frac{\int \sin(c+dx)\,dx}{a^2} + \frac{\int \sin^3(c+dx}{a^2}$$
$$= \frac{4x}{a^2} + \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cos(c+}{}$$
$$= \frac{3x}{a^2} + \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} + \frac{\cos^3(c+dx)}{3a^2d} + \frac{2\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c}{2a^2d}$$

Mathematica [A] time = 1.99, size = 158, normalized size = 1.63

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)^4 \left(6\cos(c+dx) + 2\cos(3(c+dx)) + 3\left(4\sin(2(c+dx)) - 8\tan\left(\frac{1}{2}(c+dx)\right) +\right.\right.}{24a^2d(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(6*Cos[c + d*x] + 2*Cos[3*(c + d*x
)] + 3*(24*c + 24*d*x + 8*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 + 4*Log[Cos
[(c + d*x)/2]] - 4*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 + 4*Sin[2*(c
+ d*x)] - 8*Tan[(c + d*x)/2])))/(24*a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.53, size = 140, normalized size = 1.44

$$\frac{4\cos(dx+c)^5 + 36\,dx\cos(dx+c)^2 - 4\cos(dx+c)^3 - 36\,dx + 3\left(\cos(dx+c)^2 - 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) -}{12\left(a^2d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(4*cos(d*x + c)^5 + 36*d*x*cos(d*x + c)^2 - 4*cos(d*x + c)^3 - 36*d*x
+ 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^2 -
1)*log(-1/2*cos(d*x + c) + 1/2) + 12*(cos(d*x + c)^3 - 3*cos(d*x + c))*sin(
d*x + c) + 6*cos(d*x + c))/(a^2*d*cos(d*x + c)^2 - a^2*d)
```

giac [A] time = 0.23, size = 168, normalized size = 1.73

$$\frac{\frac{72\,(dx+c)}{a^2}-\frac{12\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{3\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^4}+\frac{3\left(6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-\frac{16\left(3\,\tan\left(\frac{1}{2}\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(72*(d*x + c)/a^2 - 12*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 3*(a^2*tan
(1/2*d*x + 1/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^4 + 3*(6*tan(1/2*d*x +
1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)^2) - 16*(3
*tan(1/2*d*x + 1/2*c)^5 - 3*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*x + 1/2*c)
 - 1)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2))/d
```

maple [B] time = 0.62, size = 234, normalized size = 2.41

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2d}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^2*tan(1/2*d*x+1/2*c)-2/d/a^2/(1+tan(1/
2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*t
an(1/2*d*x+1/2*c)^4+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)+2
/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3+6/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/8/
a^2/d/tan(1/2*d*x+1/2*c)^2+1/d/a^2/tan(1/2*d*x+1/2*c)-1/2/d/a^2*ln(tan(1/2*
d*x+1/2*c))
```

maxima [B] time = 0.44, size = 330, normalized size = 3.40

$$\frac{\frac{\frac{24\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{120\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{9\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{72\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{45\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{24\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-3}{\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3\,a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{3\left(\frac{8\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2}+\frac{144}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((24*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 9*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 72*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 45*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 - 24*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3)/(a^2*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 3*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*x + c)^8/(cos(d*x
 + c) + 1)^8) - 3*(8*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(
d*x + c) + 1)^2)/a^2 + 144*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 12
*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 9.07, size = 270, normalized size = 2.78

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}-\frac{6\,\mathrm{atan}\left(\frac{36}{36\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6}-\frac{6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{36\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6}\right)}{a^2\,d}+\frac{-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\cdots}{d\,\left(4\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+12\,a^2\,\tan\left(\frac{c}{2}+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (6*atan(36/(36*tan(c/2 + (d*x)/2) + 6) - (
6*tan(c/2 + (d*x)/2))/(36*tan(c/2 + (d*x)/2) + 6)))/(a^2*d) + (4*tan(c/2 +
(d*x)/2) + (7*tan(c/2 + (d*x)/2)^2)/6 + 20*tan(c/2 + (d*x)/2)^3 - (3*tan(c/
2 + (d*x)/2)^4)/2 + 12*tan(c/2 + (d*x)/2)^5 + (15*tan(c/2 + (d*x)/2)^6)/2 -
 4*tan(c/2 + (d*x)/2)^7 - 1/2)/(d*(4*a^2*tan(c/2 + (d*x)/2)^2 + 12*a^2*tan(
c/2 + (d*x)/2)^4 + 12*a^2*tan(c/2 + (d*x)/2)^6 + 4*a^2*tan(c/2 + (d*x)/2)^8
)) - log(tan(c/2 + (d*x)/2))/(2*a^2*d) - tan(c/2 + (d*x)/2)/(a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.730 $\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=97

$$\frac{2\cos(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{x}{2a^2}$$

```
[Out] -1/2*x/a^2-3*arctanh(cos(d*x+c))/a^2/d+2*cos(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/
a^2/d+cot(d*x+c)*csc(d*x+c)/a^2/d-1/2*cos(d*x+c)*sin(d*x+c)/a^2/d
```

Rubi [A] time = 0.24, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2709, 3770, 3767, 8, 3768, 2638, 2635}

$$\frac{2\cos(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -x/(2*a^2) - (3*ArcTanh[Cos[c + d*x]])/(a^2*d) + (2*Cos[c + d*x])/(a^2*d) -
 Cot[c + d*x]^3/(3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d) - (Cos[c +
d*x]*Sin[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
```

```
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(-a^6+4a^6\csc(c+dx)-a^6\csc^2(c+dx)-2a^6\csc^3(c+dx)+a^6\csc^4(c+d\right.}{a^8}$$
$$= -\frac{x}{a^2}-\frac{\int \csc^2(c+dx)\,dx}{a^2}+\frac{\int \csc^4(c+dx)\,dx}{a^2}+\frac{\int \sin^2(c+dx)\,dx}{a^2}-\frac{2\int \csc^3}{}$$
$$= -\frac{x}{a^2}-\frac{4\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{2\cos(c+dx)}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{\text{co}}{}$$
$$= -\frac{x}{2a^2}-\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2}$$

Mathematica [A] time = 2.45, size = 184, normalized size = 1.90

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)^4\sec^2\left(\frac{1}{2}(c+dx)\right)\left(30\cos(c+dx)-\cos(3(c+dx))+3\left(\cos(5(c+dx))+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/768*((1 + Cot[(c + d*x)/2])^4*Sec[(c + d*x)/2]^2*(30*Cos[c + d*x] - Cos[
3*(c + d*x)] + 3*(Cos[5*(c + d*x)] + 8*(c + d*x - 6*Cos[c + d*x] + 2*Cos[3*
(c + d*x)] + 6*Log[Cos[(c + d*x)/2]] - Cos[2*(c + d*x)]*(c + d*x + 6*Log[Co
s[(c + d*x)/2]] - 6*Log[Sin[(c + d*x)/2]]) - 6*Log[Sin[(c + d*x)/2]])*Sin[c
 + d*x]))*Tan[(c + d*x)/2])/(a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.48, size = 161, normalized size = 1.66

$$\frac{3\cos(dx+c)^5-4\cos(dx+c)^3-9\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+9\left(\cos(dx+c)^2\right.}{6\left(a^2d\,\text{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*cos(d*x + c)^5 - 4*cos(d*x + c)^3 - 9*(cos(d*x + c)^2 - 1)*log(1/2*c
os(d*x + c) + 1/2)*sin(d*x + c) + 9*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 3*(d*x*cos(d*x + c)^2 - 4*cos(d*x + c)^3 - d*x +
```

```
6*cos(d*x + c))*sin(d*x + c) + 3*cos(d*x + c))/((a^2*d*cos(d*x + c)^2 - a^2
*d)*sin(d*x + c))
```

giac [B] time = 0.23, size = 194, normalized size = 2.00

$$-\frac{\frac{12\,(dx+c)}{a^2}-\frac{72\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{24\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}+\frac{132\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(d*x + c)/a^2 - 72*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 24*(tan(1
/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 4)/((
tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (132*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/
2*d*x + 1/2*c)^2 - 6*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d*x + 1/2*c)^3)
 - (a^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^4*tan(1/2*d*x + 1/2*c)^2 - 3*a^4*tan(1
/2*d*x + 1/2*c))/a^6)/d
```

maple [B] time = 0.65, size = 272, normalized size = 2.80

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^2}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2d}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^2}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{\tan\left(\cdots\right.}{d\,a^2\left(1+\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^2*tan(1/2*d*x+1/2*c)^2-1/8/d/a^2*ta
n(1/2*d*x+1/2*c)+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+4/
d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2-1/d/a^2/(1+tan(1/2*d*
x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2-1/d/a^2
*arctan(tan(1/2*d*x+1/2*c))-1/24/a^2/d/tan(1/2*d*x+1/2*c)^3+1/4/a^2/d/tan(1
/2*d*x+1/2*c)^2+1/8/d/a^2/tan(1/2*d*x+1/2*c)+3/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.45, size = 306, normalized size = 3.15

$$\frac{\frac{\frac{6\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{108\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{19\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{102\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{27\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-1}{\frac{a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2\,a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}-\frac{\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\,\mathrm{arctan}\cdots}{\cdots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((6*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 108*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 19*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 102*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 27*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 - 1)/(a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*a^2*s
in(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7) - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(c
os(d*x + c) + 1))/a^2 + 72*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 9.16, size = 253, normalized size = 2.61

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a^2\,d}+\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+34\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+36\,\mathrm{ta}\cdots}{d\left(8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+16\,a^2\tan\left(\frac{c}{2}+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^2*d) - tan(c/2 + (d*x)/2)^2/(4*a^2*d) + (3*log(t
an(c/2 + (d*x)/2)))/(a^2*d) + (2*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2/
3 + 36*tan(c/2 + (d*x)/2)^3 - (19*tan(c/2 + (d*x)/2)^4)/3 + 34*tan(c/2 + (d
*x)/2)^5 + 9*tan(c/2 + (d*x)/2)^6 - 1/3)/(d*(8*a^2*tan(c/2 + (d*x)/2)^3 + 1
6*a^2*tan(c/2 + (d*x)/2)^5 + 8*a^2*tan(c/2 + (d*x)/2)^7)) - tan(c/2 + (d*x)
/2)/(8*a^2*d) + atan(1/(tan(c/2 + (d*x)/2) + 6) - (6*tan(c/2 + (d*x)/2))/(t
an(c/2 + (d*x)/2) + 6))/(a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.731 $\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$-\frac{\cos(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{9\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{\cot(c+dx)\csc(c}{8a^2d}$$

```
[Out] -2*x/a^2+9/8*arctanh(cos(d*x+c))/a^2/d-cos(d*x+c)/a^2/d-2*cot(d*x+c)/a^2/d+
2/3*cot(d*x+c)^3/a^2/d+1/8*cot(d*x+c)*csc(d*x+c)/a^2/d-1/4*cot(d*x+c)*csc(d
*x+c)^3/a^2/d
```

Rubi [A] time = 0.28, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2872, 3770, 3767, 8, 3768, 2638}

$$-\frac{\cos(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{9\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{\cot(c+dx)\csc(c}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*x)/a^2 + (9*ArcTanh[Cos[c + d*x]])/(8*a^2*d) - Cos[c + d*x]/(a^2*d) - (
2*Cot[c + d*x])/(a^2*d) + (2*Cot[c + d*x]^3)/(3*a^2*d) + (Cot[c + d*x]*Csc[
c + d*x])/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cot^4(c+dx)\csc(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(-2a^6-a^6\csc(c+dx)+4a^6\csc^2(c+dx)-a^6\csc^3(c+dx)-2a^6\csc^4(c+\right.}{a^8} \\
&= -\frac{2x}{a^2}-\frac{\int \csc(c+dx)\,dx}{a^2}-\frac{\int \csc^3(c+dx)\,dx}{a^2}+\frac{\int \csc^5(c+dx)\,dx}{a^2}+\frac{\int \sin(c+}{a^2} \\
&= -\frac{2x}{a^2}+\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cos(c+dx)}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c}{} \\
&= -\frac{2x}{a^2}+\frac{3\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cos(c+dx)}{a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d} \\
&= -\frac{2x}{a^2}+\frac{9\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cos(c+dx)}{a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 1.88, size = 219, normalized size = 1.89

$$-\frac{\sin^5(c+dx)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^4\left(192\cot(c+dx)+(3\csc(c+dx)-8)\csc^4\left(\frac{1}{2}(c+dx)\right)+(128\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/3072*((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^4*(192*Cot[c + d*x] + Csc[(c
 + d*x)/2]^2*(128 - 6*Csc[c + d*x]) + Csc[(c + d*x)/2]^4*(-8 + 3*Csc[c + d*
x]) + 8*(3*Csc[c + d*x]*(16*(c + d*x) - 9*Log[Cos[(c + d*x)/2]] + 9*Log[Sin
[(c + d*x)/2]]) - (7 + 8*Cos[c + d*x])*Sec[(c + d*x)/2]^4 + 3*Csc[c + d*x]^
3*Sin[(c + d*x)/2]^2 - 6*Csc[c + d*x]^5*Sin[(c + d*x)/2]^4))*Sin[c + d*x]^5
)/(a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.47, size = 187, normalized size = 1.61

$$-\frac{96\,dx\cos\left(dx+c\right)^4+48\,\cos\left(dx+c\right)^5-192\,dx\cos\left(dx+c\right)^2-90\,\cos\left(dx+c\right)^3+96\,dx-27\left(\cos\left(dx+c\right)^4-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(96*d*x*cos(d*x + c)^4 + 48*cos(d*x + c)^5 - 192*d*x*cos(d*x + c)^2 -
 90*cos(d*x + c)^3 + 96*d*x - 27*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*lo
g(1/2*cos(d*x + c) + 1/2) + 27*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(
-1/2*cos(d*x + c) + 1/2) - 32*(4*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x +
 c) + 54*cos(d*x + c))/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2
*d)
```

giac [A] time = 0.25, size = 159, normalized size = 1.37

$$-\frac{\frac{384\,(dx+c)}{a^2}+\frac{216\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{384}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}-\frac{450\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}-\frac{3\,a^6\tan}{}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/192*(384*(d*x + c)/a^2 + 216*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 384/((
tan(1/2*d*x + 1/2*c)^2 + 1)*a^2) - (450*tan(1/2*d*x + 1/2*c)^4 - 240*tan(1/
2*d*x + 1/2*c)^3 + 16*tan(1/2*d*x + 1/2*c) - 3)/(a^2*tan(1/2*d*x + 1/2*c)^4
) - (3*a^6*tan(1/2*d*x + 1/2*c)^4 - 16*a^6*tan(1/2*d*x + 1/2*c)^3 + 240*a^6
*tan(1/2*d*x + 1/2*c))/a^8)/d
```

maple [A] time = 0.71, size = 173, normalized size = 1.49

$$\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a^2d}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{12d\,a^2}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^2}-\frac{2}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}-\frac{1}{64a^2d\tan\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/64/d/a^2*tan(1/2*d*x+1/2*c)^4-1/12/d/a^2*tan(1/2*d*x+1/2*c)^3+5/4/d/a^2*t
an(1/2*d*x+1/2*c)-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^2*arctan(tan(1/2*d
*x+1/2*c))-1/64/a^2/d/tan(1/2*d*x+1/2*c)^4+1/12/a^2/d/tan(1/2*d*x+1/2*c)^3-
5/4/d/a^2/tan(1/2*d*x+1/2*c)-9/8/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 263, normalized size = 2.27

$$\frac{\frac{\frac{16\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{224\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{384\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{240\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-3}{\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{240\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^2}-\frac{768\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+\cdots}\right.}{a^2}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/192*((16*sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 224*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 384*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 240*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3)/(a^2*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) +
(240*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/a^2 - 768*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/a^2 - 216*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 9.08, size = 232, normalized size = 2.00

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,a^2\,d}+\frac{4\,\mathrm{atan}\left(\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9}+\frac{16}{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9}\right)}{a^2\,d}-\frac{9\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^2\,d}-\frac{20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^5*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^4/(64*a^2*d) - tan(c/2 + (d*x)/2)^3/(12*a^2*d) + (4*atan
((9*tan(c/2 + (d*x)/2))/(16*tan(c/2 + (d*x)/2) - 9) + 16/(16*tan(c/2 + (d*x
)/2) - 9)))/(a^2*d) - (9*log(tan(c/2 + (d*x)/2)))/(8*a^2*d) - (tan(c/2 + (d
*x)/2)^2/4 - (4*tan(c/2 + (d*x)/2))/3 + (56*tan(c/2 + (d*x)/2)^3)/3 + 32*ta
n(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^5 + 1/4)/(d*(16*a^2*tan(c/2 + (d
*x)/2)^4 + 16*a^2*tan(c/2 + (d*x)/2)^6)) + (5*tan(c/2 + (d*x)/2))/(4*a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.732 $\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=118

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot^3(c+dx)\csc(c+dx)}{2a^2d}-\frac{3\cot(c+dx)\csc(c}{4a^2d}$$

```
[Out] x/a^2+3/4*arctanh(cos(d*x+c))/a^2/d+cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d
-1/5*cot(d*x+c)^5/a^2/d-3/4*cot(d*x+c)*csc(d*x+c)/a^2/d+1/2*cot(d*x+c)^3*cs
c(d*x+c)/a^2/d
```

Rubi [A] time = 0.32, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2875, 2873, 3473, 8, 2611, 3770, 2607, 30}

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot^3(c+dx)\csc(c+dx)}{2a^2d}-\frac{3\cot(c+dx)\csc(c}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] x/a^2 + (3*ArcTanh[Cos[c + d*x]])/(4*a^2*d) + Cot[c + d*x]/(a^2*d) - Cot[c
+ d*x]^3/(3*a^2*d) - Cot[c + d*x]^5/(5*a^2*d) - (3*Cot[c + d*x]*Csc[c + d*x
])/(4*a^2*d) + (Cot[c + d*x]^3*Csc[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
```

```
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cot^4(c+dx) - 2a^2\cot^4(c+dx)\csc(c+dx) + a^2\cot^4(c+dx)\csc^2(c+d\right.}{a^4}$$

$$= \frac{\int \cot^4(c+dx)\,dx}{a^2} + \frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a^2} - \frac{2\int \cot^4(c+dx)\csc(c+}{a^2}$$

$$= -\frac{\cot^3(c+dx)}{3a^2d} + \frac{\cot^3(c+dx)\csc(c+dx)}{2a^2d} - \frac{\int \cot^2(c+dx)\,dx}{a^2} + \frac{3\int \cot^2(c+}{}$$

$$= \frac{\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5a^2d} - \frac{3\cot(c+dx)\csc(c+dx)}{4a^2d} + \cot^3$$

$$= \frac{x}{a^2} + \frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d} + \frac{\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5a^2d} - 3$$

Mathematica [B] time = 1.12, size = 254, normalized size = 2.15

$$\frac{\csc^5(c+dx)\left(600c\sin(c+dx) + 600dx\sin(c+dx) - 60\sin(2(c+dx)) - 300c\sin(3(c+dx)) - 300dx\sin(3(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^5*(-40*Cos[c + d*x] - 220*Cos[3*(c + d*x)] + 68*Cos[5*(c + d*
x)] + 600*c*Sin[c + d*x] + 600*d*x*Sin[c + d*x] + 450*Log[Cos[(c + d*x)/2]]
*Sin[c + d*x] - 450*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] - 60*Sin[2*(c + d*x)
] - 300*c*Sin[3*(c + d*x)] - 300*d*x*Sin[3*(c + d*x)] - 225*Log[Cos[(c + d*
x)/2]]*Sin[3*(c + d*x)] + 225*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 150*
Sin[4*(c + d*x)] + 60*c*Sin[5*(c + d*x)] + 60*d*x*Sin[5*(c + d*x)] + 45*Log
[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 45*Log[Sin[(c + d*x)/2]]*Sin[5*(c + d
*x)]))/(960*a^2*d)
```

fricas [A] time = 0.48, size = 207, normalized size = 1.75

$$\frac{136\cos(dx+c)^5 - 280\cos(dx+c)^3 + 45\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(136*cos(d*x + c)^5 - 280*cos(d*x + c)^3 + 45*(cos(d*x + c)^4 - 2*cos
(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 45*(cos(d*x + c
)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*
(4*d*x*cos(d*x + c)^4 - 8*d*x*cos(d*x + c)^2 + 5*cos(d*x + c)^3 + 4*d*x - 3
*cos(d*x + c))*sin(d*x + c) + 120*cos(d*x + c))/((a^2*d*cos(d*x + c)^4 - 2*
a^2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

giac [A] time = 0.26, size = 195, normalized size = 1.65

$$\frac{\frac{480\,(dx+c)}{a^2} - \frac{360\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} + \frac{822\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+270\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(480*(d*x + c)/a^2 - 360*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (822*ta
n(1/2*d*x + 1/2*c)^5 + 270*tan(1/2*d*x + 1/2*c)^4 - 120*tan(1/2*d*x + 1/2*c
)^3 - 5*tan(1/2*d*x + 1/2*c)^2 + 15*tan(1/2*d*x + 1/2*c) - 3)/(a^2*tan(1/2*
d*x + 1/2*c)^5) + (3*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*a^8*tan(1/2*d*x + 1/2*
c)^4 + 5*a^8*tan(1/2*d*x + 1/2*c)^3 + 120*a^8*tan(1/2*d*x + 1/2*c)^2 - 270*
a^8*tan(1/2*d*x + 1/2*c))/a^10)/d
```

maple [B] time = 0.74, size = 226, normalized size = 1.92

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{160a^2d}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{32a^2d}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{96d\,a^2}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2d}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^2}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}-\frac{}{160a^2d\,\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/160/d/a^2*tan(1/2*d*x+1/2*c)^5-1/32/d/a^2*tan(1/2*d*x+1/2*c)^4+1/96/d/a^2
*tan(1/2*d*x+1/2*c)^3+1/4/d/a^2*tan(1/2*d*x+1/2*c)^2-9/16/d/a^2*tan(1/2*d*x
+1/2*c)+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/160/a^2/d/tan(1/2*d*x+1/2*c)^5
-1/96/a^2/d/tan(1/2*d*x+1/2*c)^3+1/32/a^2/d/tan(1/2*d*x+1/2*c)^4-1/4/a^2/d/
tan(1/2*d*x+1/2*c)^2+9/16/d/a^2/tan(1/2*d*x+1/2*c)-3/4/d/a^2*ln(tan(1/2*d*x
+1/2*c))
```

maxima [B] time = 0.43, size = 258, normalized size = 2.19

$$-\frac{\frac{\frac{270\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{120\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}-\frac{960\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{360\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{5}{(\mathrm{co}}\right.}{}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*((270*sin(d*x + c)/(cos(d*x + c) + 1) - 120*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 - 960*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 360*log(sin(d*x + c)/(cos(d*x + c)
+ 1))/a^2 - (15*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 270*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a^2*sin(d*x + c)^5))/d
```

mupad [B] time = 9.85, size = 365, normalized size = 3.09

$$-\frac{3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}-15\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-5\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^6*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(3*cos(c/2 + (d*x)/2)^10 - 3*sin(c/2 + (d*x)/2)^10 + 15*cos(c/2 + (d*x)/2)
*sin(c/2 + (d*x)/2)^9 - 15*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) - 5*cos(
c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 - 120*cos(c/2 + (d*x)/2)^3*sin(c/2 +
(d*x)/2)^7 + 270*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 - 270*cos(c/2 +
(d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 120*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/
2)^3 + 5*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 960*atan((4*cos(c/2 +
(d*x)/2) - 3*sin(c/2 + (d*x)/2))/(3*cos(c/2 + (d*x)/2) + 4*sin(c/2 + (d*x)/
2)))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5 + 360*log(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)/(480*a^2*d*
cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.733 $\int \frac{\cos(c+dx)\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=132

$$\frac{2\cot^5(c+dx)}{5a^2d}-\frac{7\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d}-\frac{\cot^3(c+dx)\csc(c+dx)}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+}{8a^2d}$$

```
[Out] -7/16*arctanh(cos(d*x+c))/a^2/d+2/5*cot(d*x+c)^5/a^2/d+5/16*cot(d*x+c)*csc(
d*x+c)/a^2/d-1/4*cot(d*x+c)^3*csc(d*x+c)/a^2/d+1/8*cot(d*x+c)*csc(d*x+c)^3/
a^2/d-1/6*cot(d*x+c)^3*csc(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.34, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2875, 2873, 2611, 3770, 2607, 30, 3768}

$$\frac{2\cot^5(c+dx)}{5a^2d}-\frac{7\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d}-\frac{\cot^3(c+dx)\csc(c+dx)}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*ArcTanh[Cos[c + d*x]])/(16*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) + (5*C
ot[c + d*x]*Csc[c + d*x])/(16*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x])/(4*a^2
*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(8*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x
]^3)/(6*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^4(c+dx)\csc(c+dx) - 2a^2\cot^4(c+dx)\csc^2(c+dx) + a^2\cot^4(c+dx)\right.}{a^4}$$
$$= \frac{\int \cot^4(c+dx)\csc(c+dx)\,dx}{a^2} + \frac{\int \cot^4(c+dx)\csc^3(c+dx)\,dx}{a^2} - \frac{2\int \cot^4(c+d}{}$$
$$= -\frac{\cot^3(c+dx)\csc(c+dx)}{4a^2d} - \frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d} - \frac{\int \cot^2(c+dx)\csc^3(c+}{2a^2}$$
$$= \frac{2\cot^5(c+dx)}{5a^2d} + \frac{3\cot(c+dx)\csc(c+dx)}{8a^2d} - \frac{\cot^3(c+dx)\csc(c+dx)}{4a^2d} + \frac{\cot(c+}{}$$
$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{8a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{5\cot(c+dx)\csc(c+dx)}{16a^2d} - \frac{\cot^3(c+}{}$$
$$= -\frac{7\tanh^{-1}(\cos(c+dx))}{16a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{5\cot(c+dx)\csc(c+dx)}{16a^2d} - \frac{\cot^3(c+}{}$$

Mathematica [A] time = 1.89, size = 145, normalized size = 1.10

$$\frac{\csc^6(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(60(32\sin(c+dx)-11)\cos(c+dx)+6(32\sin(c+dx)+45)\mathrm{c}\right.}{7680a^2\mathrm{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^6*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(3360*(-Log[Cos[(c
+ d*x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^6 + 60*Cos[c + d*x]*(-11 +
 32*Sin[c + d*x]) + 6*Cos[5*(c + d*x)]*(45 + 32*Sin[c + d*x]) + 10*Cos[3*(c
 + d*x)]*(-89 + 96*Sin[c + d*x])))/(7680*a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.46, size = 183, normalized size = 1.39

$$-\frac{192\cos(dx+c)^5\sin(dx+c)+270\cos(dx+c)^5-560\cos(dx+c)^3+105\left(\cos(dx+c)^6-3\cos(dx+c)^4+\right.}{480\left(a^2d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(192*cos(d*x + c)^5*sin(d*x + c) + 270*cos(d*x + c)^5 - 560*cos(d*x
+ c)^3 + 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log
(1/2*cos(d*x + c) + 1/2) - 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d
*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) + 210*cos(d*x + c))/(a^2*d*cos(
d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)
```

giac [A] time = 0.30, size = 215, normalized size = 1.63

$$\frac{\frac{840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{2058\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-240\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-255\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-24\tan\left(\frac{1}{2}\right.}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (2058*tan(1/2*d*x + 1/2*c)
^6 - 240*tan(1/2*d*x + 1/2*c)^5 - 255*tan(1/2*d*x + 1/2*c)^4 + 120*tan(1/2*
d*x + 1/2*c)^3 + 15*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 5)/(
a^2*tan(1/2*d*x + 1/2*c)^6) + (5*a^10*tan(1/2*d*x + 1/2*c)^6 - 24*a^10*tan(
1/2*d*x + 1/2*c)^5 + 15*a^10*tan(1/2*d*x + 1/2*c)^4 + 120*a^10*tan(1/2*d*x
+ 1/2*c)^3 - 255*a^10*tan(1/2*d*x + 1/2*c)^2 - 240*a^10*tan(1/2*d*x + 1/2*c
))/a^12)/d
```

maple [B] time = 0.66, size = 246, normalized size = 1.86

$$\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{384d\,a^2}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{80a^2d}+\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{128a^2d}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^2}-\frac{17\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128a^2d}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^2}-\frac{1}{384d\,a^2\tan\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/384/d/a^2*tan(1/2*d*x+1/2*c)^6-1/80/d/a^2*tan(1/2*d*x+1/2*c)^5+1/128/d/a^
2*tan(1/2*d*x+1/2*c)^4+1/16/d/a^2*tan(1/2*d*x+1/2*c)^3-17/128/d/a^2*tan(1/2
*d*x+1/2*c)^2-1/8/d/a^2*tan(1/2*d*x+1/2*c)-1/384/d/a^2/tan(1/2*d*x+1/2*c)^6
+1/8/d/a^2/tan(1/2*d*x+1/2*c)+7/16/d/a^2*ln(tan(1/2*d*x+1/2*c))+1/80/a^2/d/
tan(1/2*d*x+1/2*c)^5+17/128/a^2/d/tan(1/2*d*x+1/2*c)^2-1/128/a^2/d/tan(1/2*
d*x+1/2*c)^4-1/16/a^2/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.33, size = 275, normalized size = 2.08

$$-\frac{\frac{\frac{240\sin(dx+c)}{\cos(dx+c)+1}+\frac{255\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{120\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{24\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a^2}-\frac{840\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{24\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right.}{}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/1920*((240*sin(d*x + c)/(cos(d*x + c) + 1) + 255*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6)/a^2 - 840*log(sin(d*x + c)/(cos(d*x + c) + 1))/
a^2 - (24*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 255*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 240*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x +
c) + 1)^6/(a^2*sin(d*x + c)^6))/d
```

mupad [B] time = 10.34, size = 339, normalized size = 2.57

$$\frac{5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-24\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+15\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^7*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (5*sin(c/2 + (d*x)/2)^12 - 5*cos(c/2 + (d*x)/2)^12 - 24*cos(c/2 + (d*x)/2)*
sin(c/2 + (d*x)/2)^11 + 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) + 15*co
s(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 120*cos(c/2 + (d*x)/2)^3*sin(c/2
 + (d*x)/2)^9 - 255*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 - 240*cos(c/2
 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 240*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*
x)/2)^5 + 255*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 120*cos(c/2 + (d*
x)/2)^9*sin(c/2 + (d*x)/2)^3 - 15*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^
2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*sin
(c/2 + (d*x)/2)^6)/(1920*a^2*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.734 $\int \frac{\cot^8(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{\cot(c+dx)\csc^5(c+dx)}{3a^2d}-\frac{7\cot(c+dx)\csc^3(c+dx)}{12a^2d}+\text{cot}$$

```
[Out] 1/8*arctanh(cos(d*x+c))/a^2/d-2/5*cot(d*x+c)^5/a^2/d-1/7*cot(d*x+c)^7/a^2/d
+1/8*cot(d*x+c)*csc(d*x+c)/a^2/d-7/12*cot(d*x+c)*csc(d*x+c)^3/a^2/d+1/3*cot
(d*x+c)*csc(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.26, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2709, 3767, 8, 3768, 3770}

$$-\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{\cot(c+dx)\csc^5(c+dx)}{3a^2d}-\frac{7\cot(c+dx)\csc^3(c+dx)}{12a^2d}+\text{cot}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(8*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - Cot[c + d*
x]^7/(7*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (7*Cot[c + d*x]*Cs
c[c + d*x]^3)/(12*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(3*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^8(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \left(a^6\csc^2(c+dx)-2a^6\csc^3(c+dx)-a^6\csc^4(c+dx)+4a^6\csc^5(c+dx)-a^6\csc^6(\right.}{a^8}\\ &= \frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{\int \csc^4(c+dx)\,dx}{a^2}-\frac{\int \csc^6(c+dx)\,dx}{a^2}+\frac{\int \csc^8(c+dx)\,dx}{a^2}-\frac{2}{}\\ &= \frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{a^2d}+\frac{\cot(c+dx)\csc^5(c+dx)}{3a^2d}-\frac{\int cs}{}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}-\frac{\cot^7(c+dx)}{7a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{7}{}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}-\frac{\cot^7(c+dx)}{7a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{7}{}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}-\frac{\cot^7(c+dx)}{7a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{7}{}\end{aligned}$$

Mathematica [B] time = 1.11, size = 251, normalized size = 2.02

$$-\frac{\csc^7(c+dx)\left(-2170\sin(2(c+dx))-3080\sin(4(c+dx))-210\sin(6(c+dx))+5880\cos(c+dx)+2184\cos(3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/53760*(Csc[c + d*x]^7*(5880*Cos[c + d*x] + 2184*Cos[3*(c + d*x)] - 168*C
os[5*(c + d*x)] - 216*Cos[7*(c + d*x)] - 3675*Log[Cos[(c + d*x)/2]]*Sin[c +
 d*x] + 3675*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] - 2170*Sin[2*(c + d*x)] + 2
205*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] - 2205*Log[Sin[(c + d*x)/2]]*Sin
[3*(c + d*x)] - 3080*Sin[4*(c + d*x)] - 735*Log[Cos[(c + d*x)/2]]*Sin[5*(c
```

```
+ d*x)] + 735*Log[Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 210*Sin[6*(c + d*x)]
 + 105*Log[Cos[(c + d*x)/2]]*Sin[7*(c + d*x)] - 105*Log[Sin[(c + d*x)/2]]*S
in[7*(c + d*x)]))/(a^2*d)
```

fricas [A] time = 0.48, size = 216, normalized size = 1.74

$$-\frac{432 \cos(dx+c)^7 - 672 \cos(dx+c)^5 - 105\left(\cos(dx+c)^6 - 3\cos(dx+c)^4 + 3\cos(dx+c)^2 - 1\right)\log\left(\frac{1}{2}\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/1680*(432*cos(d*x + c)^7 - 672*cos(d*x + c)^5 - 105*(cos(d*x + c)^6 - 3*
cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) + 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 70*(3*cos(d*x + c)^5 + 8*cos(d*x + c
)^3 - 3*cos(d*x + c))*sin(d*x + c))/((a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*
x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

giac [B] time = 0.28, size = 245, normalized size = 1.98

$$-\frac{\frac{1680\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2} - \frac{4356\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 1155\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 - 210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 525\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 63}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/13440*(1680*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (4356*tan(1/2*d*x + 1/2
*c)^7 - 1155*tan(1/2*d*x + 1/2*c)^6 - 210*tan(1/2*d*x + 1/2*c)^5 + 525*tan(
1/2*d*x + 1/2*c)^4 - 210*tan(1/2*d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c)^2
 + 70*tan(1/2*d*x + 1/2*c) - 15)/(a^2*tan(1/2*d*x + 1/2*c)^7) - (15*a^12*ta
n(1/2*d*x + 1/2*c)^7 - 70*a^12*tan(1/2*d*x + 1/2*c)^6 + 63*a^12*tan(1/2*d*x
 + 1/2*c)^5 + 210*a^12*tan(1/2*d*x + 1/2*c)^4 - 525*a^12*tan(1/2*d*x + 1/2*
c)^3 + 210*a^12*tan(1/2*d*x + 1/2*c)^2 + 1155*a^12*tan(1/2*d*x + 1/2*c))/a^
14)/d
```

maple [B] time = 0.74, size = 284, normalized size = 2.29

$$\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{896d\,a^2} - \frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{192d\,a^2} + \frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{640a^2d} + \frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a^2d} - \frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d\,a^2} + \frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a^2d} + \frac{11\tan\left(\frac{dx}{2}\right.}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/896/d/a^2*tan(1/2*d*x+1/2*c)^7-1/192/d/a^2*tan(1/2*d*x+1/2*c)^6+3/640/d/a
^2*tan(1/2*d*x+1/2*c)^5+1/64/d/a^2*tan(1/2*d*x+1/2*c)^4-5/128/d/a^2*tan(1/2
*d*x+1/2*c)^3+1/64/d/a^2*tan(1/2*d*x+1/2*c)^2+11/128/d/a^2*tan(1/2*d*x+1/2*
c)+1/192/d/a^2/tan(1/2*d*x+1/2*c)^6-11/128/d/a^2/tan(1/2*d*x+1/2*c)-1/8/d/a
^2*ln(tan(1/2*d*x+1/2*c))-3/640/a^2/d/tan(1/2*d*x+1/2*c)^5-1/896/d/a^2/tan(
1/2*d*x+1/2*c)^7-1/64/a^2/d/tan(1/2*d*x+1/2*c)^2-1/64/a^2/d/tan(1/2*d*x+1/2
*c)^4+5/128/a^2/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.33, size = 314, normalized size = 2.53

$$\frac{\frac{\frac{1155 \sin(dx+c)}{\cos(dx+c)+1}+\frac{210 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{525 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{210 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{63 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{70 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2}-\frac{1680 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{70 \sin(dx+c)}{\cos(dx+c)+1}-\right.}{}}{13440\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/13440*((1155*sin(d*x + c)/(cos(d*x + c) + 1) + 210*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 525*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 210*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 70*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^2
 - 1680*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + (70*sin(d*x + c)/(cos(d*
x + c) + 1) - 63*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 210*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + 525*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 210*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 1155*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1
5)*(cos(d*x + c) + 1)^7/(a^2*sin(d*x + c)^7))/d
```

mupad [B] time = 10.84, size = 387, normalized size = 3.12

$$-\frac{15 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-15 \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+70 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right) \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-70 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-63}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^8*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(15*cos(c/2 + (d*x)/2)^14 - 15*sin(c/2 + (d*x)/2)^14 + 70*cos(c/2 + (d*x)/
2)*sin(c/2 + (d*x)/2)^13 - 70*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) - 63
```

```
*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 210*cos(c/2 + (d*x)/2)^3*sin(
c/2 + (d*x)/2)^11 + 525*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 - 210*co
s(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 - 1155*cos(c/2 + (d*x)/2)^6*sin(c/2
 + (d*x)/2)^8 + 1155*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 210*cos(c/
2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5 - 525*cos(c/2 + (d*x)/2)^10*sin(c/2 + (
d*x)/2)^4 + 210*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 63*cos(c/2 + (
d*x)/2)^12*sin(c/2 + (d*x)/2)^2 + 1680*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)/(13440*a^2*d*cos(c/2 + (d
*x)/2)^7*sin(c/2 + (d*x)/2)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**8/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.735 $\int \frac{\cot^8(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=176

$$\frac{2\cot^7(c+dx)}{7a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{11\tanh^{-1}(\cos(c+dx))}{128a^2d}-\frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d}-\frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d}+\frac{\text{co}}{}$$

```
[Out] -11/128*arctanh(cos(d*x+c))/a^2/d+2/5*cot(d*x+c)^5/a^2/d+2/7*cot(d*x+c)^7/a
^2/d-11/128*cot(d*x+c)*csc(d*x+c)/a^2/d+7/64*cot(d*x+c)*csc(d*x+c)^3/a^2/d-
1/6*cot(d*x+c)^3*csc(d*x+c)^3/a^2/d+1/16*cot(d*x+c)*csc(d*x+c)^5/a^2/d-1/8*
cot(d*x+c)^3*csc(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.41, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2873, 2611, 3768, 3770, 2607, 14}

$$\frac{2\cot^7(c+dx)}{7a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{11\tanh^{-1}(\cos(c+dx))}{128a^2d}-\frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d}-\frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d}+\frac{\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(128*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) + (2
*Cot[c + d*x]^7)/(7*a^2*d) - (11*Cot[c + d*x]*Csc[c + d*x])/(128*a^2*d) + (
7*Cot[c + d*x]*Csc[c + d*x]^3)/(64*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x]^3)
/(6*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(16*a^2*d) - (Cot[c + d*x]^3*Csc
[c + d*x]^5)/(8*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
```

```
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^4(c+dx)\csc^3(c+dx) - 2a^2\cot^4(c+dx)\csc^4(c+dx) + a^2\cot^4(c+dx)\right.}{a^4}$$
$$= \frac{\int \cot^4(c+dx)\csc^3(c+dx)\,dx}{a^2} + \frac{\int \cot^4(c+dx)\csc^5(c+dx)\,dx}{a^2} - \frac{2\int \cot^4(c+}{}$$
$$= -\frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d} - \frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d} - \frac{3\int \cot^2(c+dx)\csc^5(}{8a^2}$$
$$= \frac{\cot(c+dx)\csc^3(c+dx)}{8a^2d} - \frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d} + \frac{\cot(c+dx)\csc^5(c+dx)}{16a^2d}$$
$$= \frac{2\cot^5(c+dx)}{5a^2d} + \frac{2\cot^7(c+dx)}{7a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{16a^2d} + \frac{7\cot(c+dx)\csc^3(}{64a^2d}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{16a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{2\cot^7(c+dx)}{7a^2d} - \frac{11\cot(c+dx)\csc(c}{128a^2d}$$
$$= -\frac{11\tanh^{-1}(\cos(c+dx))}{128a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{2\cot^7(c+dx)}{7a^2d} - \frac{11\cot(c+dx)\csc}{128a^2d}$$

Mathematica [A] time = 0.95, size = 291, normalized size = 1.65

$$-\frac{\csc^8(c+dx)\left(-86016\sin(2(c+dx)) - 64512\sin(4(c+dx)) - 12288\sin(6(c+dx)) + 1536\sin(8(c+dx)) + 158\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/1720320*(Csc[c + d*x]^8*(158270*Cos[c + d*x] + 77210*Cos[3*(c + d*x)] -
18130*Cos[5*(c + d*x)] - 2310*Cos[7*(c + d*x)] + 40425*Log[Cos[(c + d*x)/2]
] - 64680*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 32340*Cos[4*(c + d*x)]*L
og[Cos[(c + d*x)/2]] - 9240*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 1155*C
os[8*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 40425*Log[Sin[(c + d*x)/2]] + 64680
*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 32340*Cos[4*(c + d*x)]*Log[Sin[(c
 + d*x)/2]] + 9240*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 1155*Cos[8*(c +
 d*x)]*Log[Sin[(c + d*x)/2]] - 86016*Sin[2*(c + d*x)] - 64512*Sin[4*(c + d*
x)] - 12288*Sin[6*(c + d*x)] + 1536*Sin[8*(c + d*x)]))/(a^2*d)
```

fricas [A] time = 0.48, size = 239, normalized size = 1.36

$$\frac{2310\cos(dx+c)^7 + 490\cos(dx+c)^5 - 8470\cos(dx+c)^3 - 1155\left(\cos(dx+c)^8 - 4\cos(dx+c)^6 + 6\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/26880*(2310*cos(d*x + c)^7 + 490*cos(d*x + c)^5 - 8470*cos(d*x + c)^3 - 1
155*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^
2 + 1)*log(1/2*cos(d*x + c) + 1/2) + 1155*(cos(d*x + c)^8 - 4*cos(d*x + c)^
6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2) -
 1536*(2*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*sin(d*x + c) + 2310*cos(d*x + c
))/(a^2*d*cos(d*x + c)^8 - 4*a^2*d*cos(d*x + c)^6 + 6*a^2*d*cos(d*x + c)^4
- 4*a^2*d*cos(d*x + c)^2 + a^2*d)
```

giac [A] time = 0.31, size = 273, normalized size = 1.55

$$\frac{\frac{18480 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{50226\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-10080\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-1680\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+3360\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2520\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}{a^2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/215040*(18480*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (50226*tan(1/2*d*x + 1
/2*c)^8 - 10080*tan(1/2*d*x + 1/2*c)^7 - 1680*tan(1/2*d*x + 1/2*c)^6 + 3360
*tan(1/2*d*x + 1/2*c)^5 - 2520*tan(1/2*d*x + 1/2*c)^4 + 672*tan(1/2*d*x + 1
/2*c)^3 + 560*tan(1/2*d*x + 1/2*c)^2 - 480*tan(1/2*d*x + 1/2*c) + 105)/(a^2
*tan(1/2*d*x + 1/2*c)^8) + (105*a^14*tan(1/2*d*x + 1/2*c)^8 - 480*a^14*tan(
1/2*d*x + 1/2*c)^7 + 560*a^14*tan(1/2*d*x + 1/2*c)^6 + 672*a^14*tan(1/2*d*x
 + 1/2*c)^5 - 2520*a^14*tan(1/2*d*x + 1/2*c)^4 + 3360*a^14*tan(1/2*d*x + 1/
2*c)^3 - 1680*a^14*tan(1/2*d*x + 1/2*c)^2 - 10080*a^14*tan(1/2*d*x + 1/2*c)
)/a^16)/d
```

maple [B] time = 0.70, size = 322, normalized size = 1.83

$$\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{2048d\,a^2}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{448d\,a^2}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{384d\,a^2}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{320a^2d}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256a^2d}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d\,a^2}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{128a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2048/d/a^2*tan(1/2*d*x+1/2*c)^8-1/448/d/a^2*tan(1/2*d*x+1/2*c)^7+1/384/d/
a^2*tan(1/2*d*x+1/2*c)^6+1/320/d/a^2*tan(1/2*d*x+1/2*c)^5-3/256/d/a^2*tan(1
```

/2*d*x+1/2*c)^4+1/64/d/a^2*tan(1/2*d*x+1/2*c)^3-1/128/d/a^2*tan(1/2*d*x+1/2*c)^2-3/64/d/a^2*tan(1/2*d*x+1/2*c)-1/384/d/a^2/tan(1/2*d*x+1/2*c)^6+3/64/d/a^2/tan(1/2*d*x+1/2*c)+11/128/d/a^2*ln(tan(1/2*d*x+1/2*c))-1/320/a^2/d/tan(1/2*d*x+1/2*c)^5+1/448/d/a^2/tan(1/2*d*x+1/2*c)^7+1/128/a^2/d/tan(1/2*d*x+1/2*c)^2-1/2048/d/a^2/tan(1/2*d*x+1/2*c)^8+3/256/a^2/d/tan(1/2*d*x+1/2*c)^4-1/64/a^2/d/tan(1/2*d*x+1/2*c)^3

maxima [B] time = 0.33, size = 355, normalized size = 2.02

$$-\frac{\frac{\frac{10080\sin(dx+c)}{\cos(dx+c)+1}+\frac{1680\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3360\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2520\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{672\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{560\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{480\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{105\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}{a^2}-\frac{18480\log\left(\frac{\sin(dx+c}{\cos(dx+c)}\right.}{a^2}}{215040\,}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/215040*((10080*sin(d*x + c)/(cos(d*x + c) + 1) + 1680*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 3360*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2520*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 672*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 560*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 480*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 105*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)/a^2 - 18480*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - (480*sin(d*x + c)/(cos(d*x + c) + 1) - 560*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 672*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2520*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3360*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1680*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 10080*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 105)*(cos(d*x + c) + 1)^8/(a^2*sin(d*x + c)^8))/d

mupad [B] time = 11.96, size = 435, normalized size = 2.47

$$105\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-105\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-480\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}+480\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^8/(sin(c + d*x)^9*(a + a*sin(c + d*x))^2),x)

[Out] (105*sin(c/2 + (d*x)/2)^16 - 105*cos(c/2 + (d*x)/2)^16 - 480*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^15 + 480*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2) + 560*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 672*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^13 - 2520*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^12 + 3360*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^11 - 1680*cos(c/2 + (d*x)/2)^6*

```
sin(c/2 + (d*x)/2)^10 - 10080*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^9 + 1
0080*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^7 + 1680*cos(c/2 + (d*x)/2)^10
*sin(c/2 + (d*x)/2)^6 - 3360*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^5 + 2
520*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 672*cos(c/2 + (d*x)/2)^13*
sin(c/2 + (d*x)/2)^3 - 560*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2 + 184
80*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^8*sin(c/2
+ (d*x)/2)^8)/(215040*a^2*d*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**9/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.736 $\int \frac{\cot^8(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=168

$$-\frac{\cot^9(c+dx)}{9a^2d}-\frac{3\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{64a^2d}+\frac{\cot^3(c+dx)\csc^5(c+dx)}{4a^2d}-\frac{\cot(c+dx)\csc}{8a^2d}$$

```
[Out] 3/64*arctanh(cos(d*x+c))/a^2/d-2/5*cot(d*x+c)^5/a^2/d-3/7*cot(d*x+c)^7/a^2/
d-1/9*cot(d*x+c)^9/a^2/d+3/64*cot(d*x+c)*csc(d*x+c)/a^2/d+1/32*cot(d*x+c)*c
sc(d*x+c)^3/a^2/d-1/8*cot(d*x+c)*csc(d*x+c)^5/a^2/d+1/4*cot(d*x+c)^3*csc(d*
x+c)^5/a^2/d
```

Rubi [A] time = 0.39, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2607, 14, 2611, 3768, 3770, 270}

$$-\frac{\cot^9(c+dx)}{9a^2d}-\frac{3\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{64a^2d}+\frac{\cot^3(c+dx)\csc^5(c+dx)}{4a^2d}-\frac{\cot(c+dx)\csc}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(64*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - (3*Co
t[c + d*x]^7)/(7*a^2*d) - Cot[c + d*x]^9/(9*a^2*d) + (3*Cot[c + d*x]*Csc[c
+ d*x])/(64*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(32*a^2*d) - (Cot[c + d*
x]*Csc[c + d*x]^5)/(8*a^2*d) + (Cot[c + d*x]^3*Csc[c + d*x]^5)/(4*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cot^4(c+dx)\csc^4(c+dx) - 2a^2\cot^4(c+dx)\csc^5(c+dx) + a^2\cot^4(c+dx\right.}{a^4}$$

$$= \frac{\int \cot^4(c+dx)\csc^4(c+dx)\,dx}{a^2} + \frac{\int \cot^4(c+dx)\csc^6(c+dx)\,dx}{a^2} - \frac{2\int \cot^4(c+}{}$$

$$= \frac{\cot^3(c+dx)\csc^5(c+dx)}{4a^2d} + \frac{3\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{4a^2} + \frac{\text{Subst}\left(\int x^4\left(1\right.\right.}{}$$

$$= -\frac{\cot(c+dx)\csc^5(c+dx)}{8a^2d} + \frac{\cot^3(c+dx)\csc^5(c+dx)}{4a^2d} - \frac{\int \csc^5(c+dx)\,dx}{8a^2} + \frac{\text{S}}{}$$

$$= -\frac{2\cot^5(c+dx)}{5a^2d} - \frac{3\cot^7(c+dx)}{7a^2d} - \frac{\cot^9(c+dx)}{9a^2d} + \frac{\cot(c+dx)\csc^3(c+dx)}{32a^2d} - $$

$$= -\frac{2\cot^5(c+dx)}{5a^2d} - \frac{3\cot^7(c+dx)}{7a^2d} - \frac{\cot^9(c+dx)}{9a^2d} + \frac{3\cot(c+dx)\csc(c+dx)}{64a^2d} + $$

$$= \frac{3\tanh^{-1}(\cos(c+dx))}{64a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{3\cot^7(c+dx)}{7a^2d} - \frac{\cot^9(c+dx)}{9a^2d} + \frac{3\,\text{co}}{}$$

Mathematica [A] time = 1.87, size = 313, normalized size = 1.86

$$\frac{\csc^9(c+dx)\left(212940\sin(2(c+dx)) + 195300\sin(4(c+dx)) + 16380\sin(6(c+dx)) - 1890\sin(8(c+dx)) - 4515\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^9*(-451584*Cos[c + d*x] - 155904*Cos[3*(c + d*x)] + 20736*Cos
[5*(c + d*x)] + 14976*Cos[7*(c + d*x)] - 1664*Cos[9*(c + d*x)] + 119070*Log
[Cos[(c + d*x)/2]]*Sin[c + d*x] - 119070*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]
 + 212940*Sin[2*(c + d*x)] - 79380*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] +
 79380*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 195300*Sin[4*(c + d*x)] + 3
4020*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 34020*Log[Sin[(c + d*x)/2]]*S
in[5*(c + d*x)] + 16380*Sin[6*(c + d*x)] - 8505*Log[Cos[(c + d*x)/2]]*Sin[7
*(c + d*x)] + 8505*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] - 1890*Sin[8*(c +
 d*x)] + 945*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)] - 945*Log[Sin[(c + d*x)
/2]]*Sin[9*(c + d*x)]))/(5160960*a^2*d)
```

fricas [A] time = 0.49, size = 269, normalized size = 1.60

$$-\frac{3328\,\cos\left(dx+c\right)^{9}-14976\,\cos\left(dx+c\right)^{7}+16128\,\cos\left(dx+c\right)^{5}-945\left(\cos\left(dx+c\right)^{8}-4\,\cos\left(dx+c\right)^{6}+6\,\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/40320*(3328*cos(d*x + c)^9 - 14976*cos(d*x + c)^7 + 16128*cos(d*x + c)^5
 - 945*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x +
c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 945*(cos(d*x + c)^8 -
4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) + 630*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11
*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x + c))/((a^2*d*cos(d*x + c)^8 - 4*
a^2*d*cos(d*x + c)^6 + 6*a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^2 + a^
2*d)*sin(d*x + c))
```

giac [A] time = 0.30, size = 245, normalized size = 1.46

$$-\frac{\frac{15120\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}}-\frac{42774\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-11340\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+3360\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-2520\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1008\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] -1/322560*(15120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (42774*tan(1/2*d*x +
1/2*c)^9 - 11340*tan(1/2*d*x + 1/2*c)^8 + 3360*tan(1/2*d*x + 1/2*c)^6 - 252
0*tan(1/2*d*x + 1/2*c)^5 + 1008*tan(1/2*d*x + 1/2*c)^4 - 450*tan(1/2*d*x +
1/2*c)^2 + 315*tan(1/2*d*x + 1/2*c) - 70)/(a^2*tan(1/2*d*x + 1/2*c)^9) - (7
0*a^16*tan(1/2*d*x + 1/2*c)^9 - 315*a^16*tan(1/2*d*x + 1/2*c)^8 + 450*a^16*
tan(1/2*d*x + 1/2*c)^7 - 1008*a^16*tan(1/2*d*x + 1/2*c)^5 + 2520*a^16*tan(1
/2*d*x + 1/2*c)^4 - 3360*a^16*tan(1/2*d*x + 1/2*c)^3 + 11340*a^16*tan(1/2*d
*x + 1/2*c))/a^18)/d
```

maple [A] time = 0.71, size = 284, normalized size = 1.69

$$\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4608d\,a^{2}}-\frac{\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)}{1024d\,a^{2}}+\frac{5\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3584d\,a^{2}}-\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{320a^{2}d}+\frac{\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)}{128a^{2}d}-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{96d\,a^{2}}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{256d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c))^2,x)

[Out] 1/4608/d/a^2*tan(1/2*d*x+1/2*c)^9-1/1024/d/a^2*tan(1/2*d*x+1/2*c)^8+5/3584/d/a^2*tan(1/2*d*x+1/2*c)^7-1/320/d/a^2*tan(1/2*d*x+1/2*c)^5+1/128/d/a^2*tan(1/2*d*x+1/2*c)^4-1/96/d/a^2*tan(1/2*d*x+1/2*c)^3+9/256/d/a^2*tan(1/2*d*x+1/2*c)-9/256/d/a^2/tan(1/2*d*x+1/2*c)-3/64/d/a^2*ln(tan(1/2*d*x+1/2*c))+1/320/a^2/d/tan(1/2*d*x+1/2*c)^5-5/3584/d/a^2/tan(1/2*d*x+1/2*c)^7-1/4608/d/a^2/tan(1/2*d*x+1/2*c)^9+1/1024/d/a^2/tan(1/2*d*x+1/2*c)^8-1/128/a^2/d/tan(1/2*d*x+1/2*c)^4+1/96/a^2/d/tan(1/2*d*x+1/2*c)^3

maxima [B] time = 0.34, size = 314, normalized size = 1.87

$$\frac{\frac{\frac{11340 \sin(dx+c)}{\cos(dx+c)+1}-\frac{3360 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2520 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{1008 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{450 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{315 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{70 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^2}-\frac{15120 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{315 \sin(d\ldots}{\cos(dx+c\ldots}\right.}{\ldots}}{322560\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/322560*((11340*sin(d*x + c)/(cos(d*x + c) + 1) - 3360*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2520*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 315*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 70*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^2 - 15120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + (315*sin(d*x + c)/(cos(d*x + c) + 1) - 450*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1008*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2520*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3360*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 11340*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 70)*(cos(d*x + c) + 1)^9/(a^2*sin(d*x + c)^9))/d

mupad [B] time = 12.56, size = 387, normalized size = 2.30

$$-\frac{70 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-70 \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+315 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right) \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}-315 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^8/(sin(c + d*x)^10*(a + a*sin(c + d*x))^2),x)

[Out] -(70*cos(c/2 + (d*x)/2)^18 - 70*sin(c/2 + (d*x)/2)^18 + 315*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^17 - 315*cos(c/2 + (d*x)/2)^17*sin(c/2 + (d*x)/2) -

```
450*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^16 + 1008*cos(c/2 + (d*x)/2)^4*
sin(c/2 + (d*x)/2)^14 - 2520*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^13 + 3
360*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^12 - 11340*cos(c/2 + (d*x)/2)^8
*sin(c/2 + (d*x)/2)^10 + 11340*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^8 -
 3360*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^6 + 2520*cos(c/2 + (d*x)/2)^
13*sin(c/2 + (d*x)/2)^5 - 1008*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^4 +
 450*cos(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^2 + 15120*log(sin(c/2 + (d*x)
/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^9)/(322560*
a^2*d*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**10/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.737 $\int \frac{\cot^8(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=218

$$\frac{2\cot^9(c+dx)}{9a^2d}+\frac{4\cot^7(c+dx)}{7a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{9\tanh^{-1}(\cos(c+dx))}{256a^2d}-\frac{\cot^3(c+dx)\csc^7(c+dx)}{10a^2d}-\frac{\cot^3(c+dx)\,\mathrm{c}}{8a^2}$$

```
[Out] -9/256*arctanh(cos(d*x+c))/a^2/d+2/5*cot(d*x+c)^5/a^2/d+4/7*cot(d*x+c)^7/a^
2/d+2/9*cot(d*x+c)^9/a^2/d-9/256*cot(d*x+c)*csc(d*x+c)/a^2/d-3/128*cot(d*x+
c)*csc(d*x+c)^3/a^2/d+9/160*cot(d*x+c)*csc(d*x+c)^5/a^2/d-1/8*cot(d*x+c)^3*
csc(d*x+c)^5/a^2/d+3/80*cot(d*x+c)*csc(d*x+c)^7/a^2/d-1/10*cot(d*x+c)^3*csc
(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.49, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2873, 2611, 3768, 3770, 2607, 270}

$$\frac{2\cot^9(c+dx)}{9a^2d}+\frac{4\cot^7(c+dx)}{7a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{9\tanh^{-1}(\cos(c+dx))}{256a^2d}-\frac{\cot^3(c+dx)\csc^7(c+dx)}{10a^2d}-\frac{\cot^3(c+dx)\,\mathrm{c}}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-9*ArcTanh[Cos[c + d*x]])/(256*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) + (4*
Cot[c + d*x]^7)/(7*a^2*d) + (2*Cot[c + d*x]^9)/(9*a^2*d) - (9*Cot[c + d*x]*
Csc[c + d*x])/(256*a^2*d) - (3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*a^2*d) + (
9*Cot[c + d*x]*Csc[c + d*x]^5)/(160*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x]^5
)/(8*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x]^7)/(80*a^2*d) - (Cot[c + d*x]^3*
Csc[c + d*x]^7)/(10*a^2*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc^7(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^4(c+dx)\csc^5(c+dx)-2a^2\cot^4(c+dx)\csc^6(c+dx)+a^2\cot^4(c+dx\right.}{a^4}$$
$$= \frac{\int \cot^4(c+dx)\csc^5(c+dx)\,dx}{a^2}+\frac{\int \cot^4(c+dx)\csc^7(c+dx)\,dx}{a^2}-\frac{2\int \cot^4(c+}{}$$
$$= -\frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d}-\frac{\cot^3(c+dx)\csc^7(c+dx)}{10a^2d}-\frac{3\int \cot^2(c+dx)\csc^7}{10a^2}$$
$$= \frac{\cot(c+dx)\csc^5(c+dx)}{16a^2d}-\frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d}+\frac{3\cot(c+dx)\csc^7(c+d}{80a^2d}$$
$$= \frac{2\cot^5(c+dx)}{5a^2d}+\frac{4\cot^7(c+dx)}{7a^2d}+\frac{2\cot^9(c+dx)}{9a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{64a^2d}+$$
$$= \frac{2\cot^5(c+dx)}{5a^2d}+\frac{4\cot^7(c+dx)}{7a^2d}+\frac{2\cot^9(c+dx)}{9a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{128a^2d}-$$
$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{128a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}+\frac{4\cot^7(c+dx)}{7a^2d}+\frac{2\cot^9(c+dx)}{9a^2d}-$$
$$= -\frac{9\tanh^{-1}(\cos(c+dx))}{256a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}+\frac{4\cot^7(c+dx)}{7a^2d}+\frac{2\cot^9(c+dx)}{9a^2d}-$$

Mathematica [A] time = 1.81, size = 353, normalized size = 1.62

$$\frac{\csc^{10}(c+dx)\left(1720320\sin(2(c+dx))+1228800\sin(4(c+dx))+184320\sin(6(c+dx))-40960\sin(8(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^10*(-3219300*Cos[c + d*x] - 1237320*Cos[3*(c + d*x)] + 278712
*Cos[5*(c + d*x)] + 54810*Cos[7*(c + d*x)] - 5670*Cos[9*(c + d*x)] - 357210
*Log[Cos[(c + d*x)/2]] + 595350*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 34
0200*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 127575*Cos[6*(c + d*x)]*Log[C
os[(c + d*x)/2]] - 28350*Cos[8*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 2835*Cos[
10*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 357210*Log[Sin[(c + d*x)/2]] - 595350
*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 340200*Cos[4*(c + d*x)]*Log[Sin[(
c + d*x)/2]] - 127575*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 28350*Cos[8*
(c + d*x)]*Log[Sin[(c + d*x)/2]] - 2835*Cos[10*(c + d*x)]*Log[Sin[(c + d*x)
/2]] + 1720320*Sin[2*(c + d*x)] + 1228800*Sin[4*(c + d*x)] + 184320*Sin[6*(
```

```
c + d*x)] - 40960*Sin[8*(c + d*x)] + 4096*Sin[10*(c + d*x)]))/(41287680*a^2
*d)
```

fricas [A] time = 0.50, size = 294, normalized size = 1.35

$$\frac{5670 \cos(dx+c)^9 - 26460 \cos(dx+c)^7 + 16128 \cos(dx+c)^5 + 26460 \cos(dx+c)^3 - 2835 \left(\cos(dx+c)^{10} - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/161280*(5670*cos(d*x + c)^9 - 26460*cos(d*x + c)^7 + 16128*cos(d*x + c)^5
 + 26460*cos(d*x + c)^3 - 2835*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos
(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c
) + 1/2) + 2835*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 1
0*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) - 102
4*(8*cos(d*x + c)^9 - 36*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*sin(d*x + c) -
 5670*cos(d*x + c))/(a^2*d*cos(d*x + c)^10 - 5*a^2*d*cos(d*x + c)^8 + 10*a^
2*d*cos(d*x + c)^6 - 10*a^2*d*cos(d*x + c)^4 + 5*a^2*d*cos(d*x + c)^2 - a^2
*d)
```

giac [A] time = 0.34, size = 331, normalized size = 1.52

$$\frac{\frac{45360 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2} - \frac{132858 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10} - 30240 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9 + 1260 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8 + 6720 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 7560 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{a^2 \tan\left(\frac{1}{2}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/1290240*(45360*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (132858*tan(1/2*d*x +
 1/2*c)^10 - 30240*tan(1/2*d*x + 1/2*c)^9 + 1260*tan(1/2*d*x + 1/2*c)^8 + 6
720*tan(1/2*d*x + 1/2*c)^7 - 7560*tan(1/2*d*x + 1/2*c)^6 + 4032*tan(1/2*d*x
 + 1/2*c)^5 - 630*tan(1/2*d*x + 1/2*c)^4 - 720*tan(1/2*d*x + 1/2*c)^3 + 945
*tan(1/2*d*x + 1/2*c)^2 - 560*tan(1/2*d*x + 1/2*c) + 126)/(a^2*tan(1/2*d*x
+ 1/2*c)^10) + (126*a^18*tan(1/2*d*x + 1/2*c)^10 - 560*a^18*tan(1/2*d*x + 1
/2*c)^9 + 945*a^18*tan(1/2*d*x + 1/2*c)^8 - 720*a^18*tan(1/2*d*x + 1/2*c)^7
 - 630*a^18*tan(1/2*d*x + 1/2*c)^6 + 4032*a^18*tan(1/2*d*x + 1/2*c)^5 - 756
0*a^18*tan(1/2*d*x + 1/2*c)^4 + 6720*a^18*tan(1/2*d*x + 1/2*c)^3 + 1260*a^1
8*tan(1/2*d*x + 1/2*c)^2 - 30240*a^18*tan(1/2*d*x + 1/2*c))/a^20)/d
```

maple [B] time = 0.75, size = 398, normalized size = 1.83

$$\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{10240d\,a^2}-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2304d\,a^2}+\frac{3\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4096d\,a^2}-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{1792d\,a^2}-\frac{\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2048d\,a^2}+\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{320a^2d}-\frac{3\left(\tan^{4}\left(\frac{dx}{2}+\right.\right.}{512a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/10240/d/a^2*tan(1/2*d*x+1/2*c)^10-1/2304/d/a^2*tan(1/2*d*x+1/2*c)^9+3/409
6/d/a^2*tan(1/2*d*x+1/2*c)^8-1/1792/d/a^2*tan(1/2*d*x+1/2*c)^7-1/2048/d/a^2
*tan(1/2*d*x+1/2*c)^6+1/320/d/a^2*tan(1/2*d*x+1/2*c)^5-3/512/d/a^2*tan(1/2*
d*x+1/2*c)^4+1/192/d/a^2*tan(1/2*d*x+1/2*c)^3+1/1024/d/a^2*tan(1/2*d*x+1/2*
c)^2-3/128/d/a^2*tan(1/2*d*x+1/2*c)+1/2048/d/a^2/tan(1/2*d*x+1/2*c)^6+3/128
/d/a^2/tan(1/2*d*x+1/2*c)+9/256/d/a^2*ln(tan(1/2*d*x+1/2*c))-1/320/a^2/d/ta
n(1/2*d*x+1/2*c)^5+1/1792/d/a^2/tan(1/2*d*x+1/2*c)^7-1/1024/a^2/d/tan(1/2*d
*x+1/2*c)^2+1/2304/d/a^2/tan(1/2*d*x+1/2*c)^9-3/4096/d/a^2/tan(1/2*d*x+1/2*
c)^8+3/512/a^2/d/tan(1/2*d*x+1/2*c)^4-1/192/a^2/d/tan(1/2*d*x+1/2*c)^3-1/10
240/d/a^2/tan(1/2*d*x+1/2*c)^10
```

maxima [B] time = 0.43, size = 435, normalized size = 2.00

$$-\frac{\frac{\frac{30240\sin(dx+c)}{\cos(dx+c)+1}-\frac{1260\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6720\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{7560\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4032\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{630\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{720\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{945\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{560\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{126\sin(\cdots}{(\cos(dx\cdots}}{a^2}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/1290240*((30240*sin(d*x + c)/(cos(d*x + c) + 1) - 1260*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 6720*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7560*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 - 4032*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
630*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 720*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7 - 945*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 560*sin(d*x + c)^9/(cos(
d*x + c) + 1)^9 - 126*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)/a^2 - 45360*lo
g(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - (560*sin(d*x + c)/(cos(d*x + c) +
1) - 945*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 720*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 630*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4032*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 + 7560*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6720*sin
(d*x + c)^7/(cos(d*x + c) + 1)^7 - 1260*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 + 30240*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 126)*(cos(d*x + c) + 1)^10/(
a^2*sin(d*x + c)^10))/d
```

mupad [B] time = 14.34, size = 531, normalized size = 2.44

$$126\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}-126\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}-560\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}+560\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^11*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (126*sin(c/2 + (d*x)/2)^20 - 126*cos(c/2 + (d*x)/2)^20 - 560*cos(c/2 + (d*x
)/2)*sin(c/2 + (d*x)/2)^19 + 560*cos(c/2 + (d*x)/2)^19*sin(c/2 + (d*x)/2) +
 945*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^18 - 720*cos(c/2 + (d*x)/2)^3*
sin(c/2 + (d*x)/2)^17 - 630*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^16 + 40
32*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^15 - 7560*cos(c/2 + (d*x)/2)^6*s
in(c/2 + (d*x)/2)^14 + 6720*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^13 + 12
60*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^12 - 30240*cos(c/2 + (d*x)/2)^9*
sin(c/2 + (d*x)/2)^11 + 30240*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^9 -
1260*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^8 - 6720*cos(c/2 + (d*x)/2)^1
3*sin(c/2 + (d*x)/2)^7 + 7560*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^6 -
4032*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2)^5 + 630*cos(c/2 + (d*x)/2)^16
*sin(c/2 + (d*x)/2)^4 + 720*cos(c/2 + (d*x)/2)^17*sin(c/2 + (d*x)/2)^3 - 94
5*cos(c/2 + (d*x)/2)^18*sin(c/2 + (d*x)/2)^2 + 45360*log(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^10)/(1290240*
a^2*d*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^10)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**11/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.738 $\int \frac{\cot^8(c+dx)\csc^4(c+dx)}{(a+a\sin(c+dx))^2} dx$

Optimal. Leaf size=210

$$-\frac{\cot^{11}(c+dx)}{11a^2d}-\frac{4\cot^9(c+dx)}{9a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{128a^2d}+\frac{\cot^3(c+dx)\csc^7(c+dx}{5a^2d}$$

```
[Out] 3/128*arctanh(cos(d*x+c))/a^2/d-2/5*cot(d*x+c)^5/a^2/d-5/7*cot(d*x+c)^7/a^2
/d-4/9*cot(d*x+c)^9/a^2/d-1/11*cot(d*x+c)^11/a^2/d+3/128*cot(d*x+c)*csc(d*x
+c)/a^2/d+1/64*cot(d*x+c)*csc(d*x+c)^3/a^2/d+1/80*cot(d*x+c)*csc(d*x+c)^5/a
^2/d-3/40*cot(d*x+c)*csc(d*x+c)^7/a^2/d+1/5*cot(d*x+c)^3*csc(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.43, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2873, 2607, 270, 2611, 3768, 3770}

$$-\frac{\cot^{11}(c+dx)}{11a^2d}-\frac{4\cot^9(c+dx)}{9a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{128a^2d}+\frac{\cot^3(c+dx)\csc^7(c+dx}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(128*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - (5*C
ot[c + d*x]^7)/(7*a^2*d) - (4*Cot[c + d*x]^9)/(9*a^2*d) - Cot[c + d*x]^11/(
11*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(128*a^2*d) + (Cot[c + d*x]*Csc[c
 + d*x]^3)/(64*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(80*a^2*d) - (3*Cot[c
 + d*x]*Csc[c + d*x]^7)/(40*a^2*d) + (Cot[c + d*x]^3*Csc[c + d*x]^7)/(5*a^2
*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc^8(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cot^4(c+dx)\csc^6(c+dx) - 2a^2\cot^4(c+dx)\csc^7(c+dx) + a^2\cot^4(c+dx\right.}{a^4}$$

$$= \frac{\int \cot^4(c+dx)\csc^6(c+dx)\,dx}{a^2} + \frac{\int \cot^4(c+dx)\csc^8(c+dx)\,dx}{a^2} - \frac{2\int \cot^4(c+}{}$$

$$= \frac{\cot^3(c+dx)\csc^7(c+dx)}{5a^2d} + \frac{3\int \cot^2(c+dx)\csc^7(c+dx)\,dx}{5a^2} + \frac{\text{Subst}\left(\int x^4\left(1\right.\right.}{}$$

$$= -\frac{3\cot(c+dx)\csc^7(c+dx)}{40a^2d} + \frac{\cot^3(c+dx)\csc^7(c+dx)}{5a^2d} - \frac{3\int \csc^7(c+dx)\,dx}{40a^2}$$

$$= -\frac{2\cot^5(c+dx)}{5a^2d} - \frac{5\cot^7(c+dx)}{7a^2d} - \frac{4\cot^9(c+dx)}{9a^2d} - \frac{\cot^{11}(c+dx)}{11a^2d} + \frac{\cot(c+d}{8}$$

$$= -\frac{2\cot^5(c+dx)}{5a^2d} - \frac{5\cot^7(c+dx)}{7a^2d} - \frac{4\cot^9(c+dx)}{9a^2d} - \frac{\cot^{11}(c+dx)}{11a^2d} + \frac{\cot(c+d}{6}$$

$$= -\frac{2\cot^5(c+dx)}{5a^2d} - \frac{5\cot^7(c+dx)}{7a^2d} - \frac{4\cot^9(c+dx)}{9a^2d} - \frac{\cot^{11}(c+dx)}{11a^2d} + \frac{3\cot(c+}{1}$$

$$= \frac{3\tanh^{-1}(\cos(c+dx))}{128a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{5\cot^7(c+dx)}{7a^2d} - \frac{4\cot^9(c+dx)}{9a^2d} - \text{co}$$

Mathematica [A] time = 4.23, size = 186, normalized size = 0.89

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(2661120\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cot(c+dx)\csc^1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(2661120*(Log[Cos[(c + d*x)/2]] -
Log[Sin[(c + d*x)/2]]) + Cot[c + d*x]*Csc[c + d*x]^10*(-5402624 - 5752832*C
os[2*(c + d*x)] + 346112*Cos[4*(c + d*x)] + 583168*Cos[6*(c + d*x)] - 10444
8*Cos[8*(c + d*x)] + 8704*Cos[10*(c + d*x)] + 2457378*Sin[c + d*x] + 590713
2*Sin[3*(c + d*x)] + 656964*Sin[5*(c + d*x)] - 121275*Sin[7*(c + d*x)] + 10
395*Sin[9*(c + d*x)])))/(113541120*a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.52, size = 324, normalized size = 1.54

$$-\frac{34816\cos(dx+c)^{11} - 191488\cos(dx+c)^9 + 430848\cos(dx+c)^7 - 354816\cos(dx+c)^5 - 10395\left(\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/887040*(34816*cos(d*x + c)^11 - 191488*cos(d*x + c)^9 + 430848*cos(d*x +
 c)^7 - 354816*cos(d*x + c)^5 - 10395*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 +
 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) + 10395*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 +
10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(-1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) + 1386*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7
+ 128*cos(d*x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*x + c))/(
(a^2*d*cos(d*x + c)^10 - 5*a^2*d*cos(d*x + c)^8 + 10*a^2*d*cos(d*x + c)^6 -
 10*a^2*d*cos(d*x + c)^4 + 5*a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

giac [A] time = 0.34, size = 361, normalized size = 1.72

$$-\frac{\frac{166320 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^2} - \frac{502266 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{11} - 131670 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{10} + 13860 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{9} + 25410 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{8} - 27720 \tan(}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] -1/7096320*(166320*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (502266*tan(1/2*d*x
 + 1/2*c)^11 - 131670*tan(1/2*d*x + 1/2*c)^10 + 13860*tan(1/2*d*x + 1/2*c)^
9 + 25410*tan(1/2*d*x + 1/2*c)^8 - 27720*tan(1/2*d*x + 1/2*c)^7 + 18711*tan
(1/2*d*x + 1/2*c)^6 - 6930*tan(1/2*d*x + 1/2*c)^5 - 1485*tan(1/2*d*x + 1/2*
c)^4 + 3465*tan(1/2*d*x + 1/2*c)^3 - 2695*tan(1/2*d*x + 1/2*c)^2 + 1386*tan
(1/2*d*x + 1/2*c) - 315)/(a^2*tan(1/2*d*x + 1/2*c)^11) - (315*a^20*tan(1/2*
d*x + 1/2*c)^11 - 1386*a^20*tan(1/2*d*x + 1/2*c)^10 + 2695*a^20*tan(1/2*d*x
 + 1/2*c)^9 - 3465*a^20*tan(1/2*d*x + 1/2*c)^8 + 1485*a^20*tan(1/2*d*x + 1/
2*c)^7 + 6930*a^20*tan(1/2*d*x + 1/2*c)^6 - 18711*a^20*tan(1/2*d*x + 1/2*c)
^5 + 27720*a^20*tan(1/2*d*x + 1/2*c)^4 - 25410*a^20*tan(1/2*d*x + 1/2*c)^3
- 13860*a^20*tan(1/2*d*x + 1/2*c)^2 + 131670*a^20*tan(1/2*d*x + 1/2*c))/a^2
2)/d
```

maple [B] time = 0.76, size = 436, normalized size = 2.08

$$\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{22528 d\, a^2} - \frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{5120 d\, a^2} + \frac{7\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{18432 d\, a^2} - \frac{\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2048 d\, a^2} + \frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14336 d\, a^2} + \frac{\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)}{1024 d\, a^2} - \frac{27\left(\tan\right.}{102}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/22528/d/a^2*tan(1/2*d*x+1/2*c)^11-1/5120/d/a^2*tan(1/2*d*x+1/2*c)^10+7/18
432/d/a^2*tan(1/2*d*x+1/2*c)^9-1/2048/d/a^2*tan(1/2*d*x+1/2*c)^8+3/14336/d/
a^2*tan(1/2*d*x+1/2*c)^7+1/1024/d/a^2*tan(1/2*d*x+1/2*c)^6-27/10240/d/a^2*t
an(1/2*d*x+1/2*c)^5+1/256/d/a^2*tan(1/2*d*x+1/2*c)^4-11/3072/d/a^2*tan(1/2*
d*x+1/2*c)^3-1/512/d/a^2*tan(1/2*d*x+1/2*c)^2+19/1024/d/a^2*tan(1/2*d*x+1/2
*c)-1/1024/d/a^2/tan(1/2*d*x+1/2*c)^6-19/1024/d/a^2/tan(1/2*d*x+1/2*c)-3/12
8/d/a^2*ln(tan(1/2*d*x+1/2*c))+27/10240/a^2/d/tan(1/2*d*x+1/2*c)^5-3/14336/
d/a^2/tan(1/2*d*x+1/2*c)^7+1/512/a^2/d/tan(1/2*d*x+1/2*c)^2-7/18432/d/a^2/t
an(1/2*d*x+1/2*c)^9+1/2048/d/a^2/tan(1/2*d*x+1/2*c)^8-1/256/a^2/d/tan(1/2*d
*x+1/2*c)^4-1/22528/d/a^2/tan(1/2*d*x+1/2*c)^11+11/3072/a^2/d/tan(1/2*d*x+1
/2*c)^3+1/5120/d/a^2/tan(1/2*d*x+1/2*c)^10
```

maxima [B] time = 0.33, size = 474, normalized size = 2.26

$$\frac{\frac{131670 \sin(dx+c)}{\cos(dx+c)+1} - \frac{13860 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{25410 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{27720 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{18711 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{6930 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{1485 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{3465 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{2695 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} - 1}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] 1/7096320*((131670*sin(d*x + c)/(cos(d*x + c) + 1) - 13860*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 - 25410*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 27720*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 - 18711*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5 + 6930*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1485*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 3465*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 2695*sin(d*x + c)
^9/(cos(d*x + c) + 1)^9 - 1386*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 315*
sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^2 - 166320*log(sin(d*x + c)/(cos(d
*x + c) + 1))/a^2 + (1386*sin(d*x + c)/(cos(d*x + c) + 1) - 2695*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 3465*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1485
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6930*sin(d*x + c)^5/(cos(d*x + c) +
1)^5 + 18711*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 27720*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7 + 25410*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 13860*sin(d
*x + c)^9/(cos(d*x + c) + 1)^9 - 131670*sin(d*x + c)^10/(cos(d*x + c) + 1)^
10 - 315)*(cos(d*x + c) + 1)^11/(a^2*sin(d*x + c)^11))/d
```

mupad [B] time = 16.55, size = 579, normalized size = 2.76

$$-\frac{315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{22}-315\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{22}+1386\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{21}-1386\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{21}\,\sin\left(\frac{c}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^12*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(315*cos(c/2 + (d*x)/2)^22 - 315*sin(c/2 + (d*x)/2)^22 + 1386*cos(c/2 + (d
*x)/2)*sin(c/2 + (d*x)/2)^21 - 1386*cos(c/2 + (d*x)/2)^21*sin(c/2 + (d*x)/2
) - 2695*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^20 + 3465*cos(c/2 + (d*x)/
2)^3*sin(c/2 + (d*x)/2)^19 - 1485*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^1
8 - 6930*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^17 + 18711*cos(c/2 + (d*x)
/2)^6*sin(c/2 + (d*x)/2)^16 - 27720*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)
^15 + 25410*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^14 + 13860*cos(c/2 + (d
*x)/2)^9*sin(c/2 + (d*x)/2)^13 - 131670*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*
x)/2)^12 + 131670*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^10 - 13860*cos(c
/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2)^9 - 25410*cos(c/2 + (d*x)/2)^14*sin(c/2
 + (d*x)/2)^8 + 27720*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2)^7 - 18711*co
s(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^6 + 6930*cos(c/2 + (d*x)/2)^17*sin(c
/2 + (d*x)/2)^5 + 1485*cos(c/2 + (d*x)/2)^18*sin(c/2 + (d*x)/2)^4 - 3465*co
s(c/2 + (d*x)/2)^19*sin(c/2 + (d*x)/2)^3 + 2695*cos(c/2 + (d*x)/2)^20*sin(c
/2 + (d*x)/2)^2 + 166320*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2
 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^11)/(7096320*a^2*d*cos(c/2 + (d*x)/2)^11*
sin(c/2 + (d*x)/2)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**12/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.739 $\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=161

$$-\frac{3\cos^7(c+dx)}{7a^3d}+\frac{7\cos^5(c+dx)}{5a^3d}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{\sin^5(c+dx)\cos^3(c+dx)}{8a^3d}+\frac{29\sin^3(c+dx)\cos^3(c+dx)}{48a^3d}+\frac{29\sin}{}$$

```
[Out] -29/128*x/a^3-4/3*cos(d*x+c)^3/a^3/d+7/5*cos(d*x+c)^5/a^3/d-3/7*cos(d*x+c)^
7/a^3/d-29/128*cos(d*x+c)*sin(d*x+c)/a^3/d+29/64*cos(d*x+c)^3*sin(d*x+c)/a^
3/d+29/48*cos(d*x+c)^3*sin(d*x+c)^3/a^3/d+1/8*cos(d*x+c)^3*sin(d*x+c)^5/a^3
/d
```

Rubi [A] time = 0.48, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{3\cos^7(c+dx)}{7a^3d}+\frac{7\cos^5(c+dx)}{5a^3d}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{\sin^5(c+dx)\cos^3(c+dx)}{8a^3d}+\frac{29\sin^3(c+dx)\cos^3(c+dx)}{48a^3d}+\frac{29\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-29*x)/(128*a^3) - (4*Cos[c + d*x]^3)/(3*a^3*d) + (7*Cos[c + d*x]^5)/(5*a^
3*d) - (3*Cos[c + d*x]^7)/(7*a^3*d) - (29*Cos[c + d*x]*Sin[c + d*x])/(128*a
^3*d) + (29*Cos[c + d*x]^3*Sin[c + d*x])/(64*a^3*d) + (29*Cos[c + d*x]^3*Si
n[c + d*x]^3)/(48*a^3*d) + (Cos[c + d*x]^3*Sin[c + d*x]^5)/(8*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cos^2(c+dx)\sin^3(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(a^3\cos^2(c+dx)\sin^3(c+dx) - 3a^3\cos^2(c+dx)\sin^4(c+dx) + 3a^3\cos^2(c+\right.}{a^6}$$
$$= \frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a^3} - \frac{\int \cos^2(c+dx)\sin^6(c+dx)\,dx}{a^3} - \frac{3\int \cos^2(c+}{}$$
$$= \frac{\cos^3(c+dx)\sin^3(c+dx)}{2a^3d} + \frac{\cos^3(c+dx)\sin^5(c+dx)}{8a^3d} - \frac{5\int \cos^2(c+dx)\sin^4(}{8a^3}$$
$$= \frac{3\cos^3(c+dx)\sin(c+dx)}{8a^3d} + \frac{29\cos^3(c+dx)\sin^3(c+dx)}{48a^3d} + \frac{\cos^3(c+dx)\sin^5(}{8a^3d}$$
$$= -\frac{4\cos^3(c+dx)}{3a^3d} + \frac{7\cos^5(c+dx)}{5a^3d} - \frac{3\cos^7(c+dx)}{7a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{16a^3d}$$
$$= -\frac{3x}{16a^3} - \frac{4\cos^3(c+dx)}{3a^3d} + \frac{7\cos^5(c+dx)}{5a^3d} - \frac{3\cos^7(c+dx)}{7a^3d} - \frac{29\cos(c+dx)\sin}{128a^3d}$$
$$= -\frac{29x}{128a^3} - \frac{4\cos^3(c+dx)}{3a^3d} + \frac{7\cos^5(c+dx)}{5a^3d} - \frac{3\cos^7(c+dx)}{7a^3d} - \frac{29\cos(c+dx)\,s}{128a^3}$$

Mathematica [B] time = 3.99, size = 482, normalized size = 2.99

$$\frac{-48720dx\sin\left(\frac{c}{2}\right) + 38640\sin\left(\frac{c}{2}+dx\right) - 38640\sin\left(\frac{3c}{2}+dx\right) + 6720\sin\left(\frac{3c}{2}+2dx\right) + 6720\sin\left(\frac{5c}{2}+2dx\right) + 39}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (84*(-7 + 12870*c - 580*d*x)*Cos[c/2] - 38640*Cos[c/2 + d*x] - 38640*Cos[(3
*c)/2 + d*x] + 6720*Cos[(3*c)/2 + 2*d*x] - 6720*Cos[(5*c)/2 + 2*d*x] - 3920
*Cos[(5*c)/2 + 3*d*x] - 3920*Cos[(7*c)/2 + 3*d*x] + 5880*Cos[(7*c)/2 + 4*d*
x] - 5880*Cos[(9*c)/2 + 4*d*x] + 4368*Cos[(9*c)/2 + 5*d*x] + 4368*Cos[(11*c
)/2 + 5*d*x] - 2240*Cos[(11*c)/2 + 6*d*x] + 2240*Cos[(13*c)/2 + 6*d*x] - 72
0*Cos[(13*c)/2 + 7*d*x] - 720*Cos[(15*c)/2 + 7*d*x] + 105*Cos[(15*c)/2 + 8*
d*x] - 105*Cos[(17*c)/2 + 8*d*x] - 998928*Sin[c/2] + 1081080*c*Sin[c/2] - 4
8720*d*x*Sin[c/2] + 38640*Sin[c/2 + d*x] - 38640*Sin[(3*c)/2 + d*x] + 6720*
Sin[(3*c)/2 + 2*d*x] + 6720*Sin[(5*c)/2 + 2*d*x] + 3920*Sin[(5*c)/2 + 3*d*x
] - 3920*Sin[(7*c)/2 + 3*d*x] + 5880*Sin[(7*c)/2 + 4*d*x] + 5880*Sin[(9*c)/
2 + 4*d*x] - 4368*Sin[(9*c)/2 + 5*d*x] + 4368*Sin[(11*c)/2 + 5*d*x] - 2240*
Sin[(11*c)/2 + 6*d*x] - 2240*Sin[(13*c)/2 + 6*d*x] + 720*Sin[(13*c)/2 + 7*d
```

```
*x] - 720*Sin[(15*c)/2 + 7*d*x] + 105*Sin[(15*c)/2 + 8*d*x] + 105*Sin[(17*c
)/2 + 8*d*x])/(215040*a^3*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.46, size = 90, normalized size = 0.56

$$-\frac{5760\cos(dx+c)^7-18816\cos(dx+c)^5+17920\cos(dx+c)^3+3045\,dx-35\left(48\cos(dx+c)^7-328\cos(\ldots\right.}{13440\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/13440*(5760*cos(d*x + c)^7 - 18816*cos(d*x + c)^5 + 17920*cos(d*x + c)^3
 + 3045*d*x - 35*(48*cos(d*x + c)^7 - 328*cos(d*x + c)^5 + 454*cos(d*x + c)
^3 - 87*cos(d*x + c))*sin(d*x + c))/(a^3*d)
```

giac [A] time = 0.29, size = 218, normalized size = 1.35

$$-\frac{\frac{3045\,(dx+c)}{a^3}+\frac{2\left(3045\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15}+23345\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+26880\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}-51275\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+286720\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-\ldots\right.}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/13440*(3045*(d*x + c)/a^3 + 2*(3045*tan(1/2*d*x + 1/2*c)^15 + 23345*tan(
1/2*d*x + 1/2*c)^13 + 26880*tan(1/2*d*x + 1/2*c)^12 - 51275*tan(1/2*d*x + 1
/2*c)^11 + 286720*tan(1/2*d*x + 1/2*c)^10 - 179095*tan(1/2*d*x + 1/2*c)^9 +
 170240*tan(1/2*d*x + 1/2*c)^8 + 179095*tan(1/2*d*x + 1/2*c)^7 - 14336*tan(
1/2*d*x + 1/2*c)^6 + 51275*tan(1/2*d*x + 1/2*c)^5 + 109312*tan(1/2*d*x + 1/
2*c)^4 - 23345*tan(1/2*d*x + 1/2*c)^3 + 38912*tan(1/2*d*x + 1/2*c)^2 - 3045
*tan(1/2*d*x + 1/2*c) + 4864)/((tan(1/2*d*x + 1/2*c)^2 + 1)^8*a^3))/d
```

maple [B] time = 0.41, size = 517, normalized size = 3.21

$$-\frac{76}{105d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{608\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{667\left(\tan^3\left(\frac{dx}{2}\ldots\right.\right.}{192d\,a^3\left(1+\tan^2\left(\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

[Out] -76/105/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8+29/64/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)-608/105/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^2+667/192/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3-244/15/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^4-1465/192/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5+32/15/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^6-5117/192/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^7-76/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^8+5117/192/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^9-128/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^10+1465/192/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11-4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^12-667/192/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13-29/64/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15-29/64/a^3/d*arctan(tan(1/2*d*x+1/2*c))

maxima [B] time = 0.43, size = 499, normalized size = 3.10

$$\frac{\frac{\frac{3045 \sin(dx+c)}{\cos(dx+c)+1} - \frac{38912 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{23345 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{109312 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{51275 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{14336 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{179095 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{170240 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{179095 \sin(dx+\cdots}{(\cos(dx+c)+1\cdots}}{a^3 + \frac{8 a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{28 a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{56 a^3 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{70 a^3 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{56 a^3 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{28 a^3 \sin\cdots}{(\cos(dx\cdots}}}{6720 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/6720*((3045*sin(d*x + c)/(cos(d*x + c) + 1) - 38912*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 23345*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 109312*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 51275*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 14336*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 179095*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 170240*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 179095*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 286720*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 51275*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 26880*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 23345*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 3045*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 4864)/(a^3 + 8*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 56*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a^3*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 3045*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d

mupad [B] time = 11.94, size = 212, normalized size = 1.32

$$-\frac{29\,x}{128\,a^3}-\frac{\frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+\frac{667\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-\frac{1465\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{192}+\frac{128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{3}-\frac{5117\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{192}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^3)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (29*x)/(128*a^3) - ((608*tan(c/2 + (d*x)/2)^2)/105 - (29*tan(c/2 + (d*x)/
2))/64 - (667*tan(c/2 + (d*x)/2)^3)/192 + (244*tan(c/2 + (d*x)/2)^4)/15 + (
1465*tan(c/2 + (d*x)/2)^5)/192 - (32*tan(c/2 + (d*x)/2)^6)/15 + (5117*tan(c
/2 + (d*x)/2)^7)/192 + (76*tan(c/2 + (d*x)/2)^8)/3 - (5117*tan(c/2 + (d*x)/
2)^9)/192 + (128*tan(c/2 + (d*x)/2)^10)/3 - (1465*tan(c/2 + (d*x)/2)^11)/19
2 + 4*tan(c/2 + (d*x)/2)^12 + (667*tan(c/2 + (d*x)/2)^13)/192 + (29*tan(c/2
 + (d*x)/2)^15)/64 + 76/105)/(a^3*d*(tan(c/2 + (d*x)/2)^2 + 1)^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.740 $\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=133

$$\frac{\cos^7(c+dx)}{7a^3d}-\frac{\cos^5(c+dx)}{a^3d}+\frac{4\cos^3(c+dx)}{3a^3d}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{2a^3d}-\frac{5\sin(c+dx)\cos^3(c+dx)}{8a^3d}+\frac{5\sin(c+dx}{16}$$

```
[Out] 5/16*x/a^3+4/3*cos(d*x+c)^3/a^3/d-cos(d*x+c)^5/a^3/d+1/7*cos(d*x+c)^7/a^3/d
+5/16*cos(d*x+c)*sin(d*x+c)/a^3/d-5/8*cos(d*x+c)^3*sin(d*x+c)/a^3/d-1/2*cos
(d*x+c)^3*sin(d*x+c)^3/a^3/d
```

Rubi [A] time = 0.40, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2568, 2635, 8, 2565, 14, 270}

$$\frac{\cos^7(c+dx)}{7a^3d}-\frac{\cos^5(c+dx)}{a^3d}+\frac{4\cos^3(c+dx)}{3a^3d}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{2a^3d}-\frac{5\sin(c+dx)\cos^3(c+dx)}{8a^3d}+\frac{5\sin(c+dx}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*x)/(16*a^3) + (4*Cos[c + d*x]^3)/(3*a^3*d) - Cos[c + d*x]^5/(a^3*d) + Co
s[c + d*x]^7/(7*a^3*d) + (5*Cos[c + d*x]*Sin[c + d*x])/(16*a^3*d) - (5*Cos[
c + d*x]^3*Sin[c + d*x])/(8*a^3*d) - (Cos[c + d*x]^3*Sin[c + d*x]^3)/(2*a^3
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cos^2(c+dx)\sin^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(a^3\cos^2(c+dx)\sin^2(c+dx) - 3a^3\cos^2(c+dx)\sin^3(c+dx) + 3a^3\cos^2(c+\right.}{a^6}$$

$$= \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a^3} - \frac{\int \cos^2(c+dx)\sin^5(c+dx)\,dx}{a^3} - \frac{3\int \cos^2(c+}{}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d} - \frac{\cos^3(c+dx)\sin^3(c+dx)}{2a^3d} + \frac{\int \cos^2(c+dx)\,dx}{4a^3} +$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{5\cos^3(c+dx)\sin(c+dx)}{8a^3d} - \frac{\cos^3(c+dx)\sin^3(c+d}{2a^3d}$$

$$= \frac{x}{8a^3} + \frac{4\cos^3(c+dx)}{3a^3d} - \frac{\cos^5(c+dx)}{a^3d} + \frac{\cos^7(c+dx)}{7a^3d} + \frac{5\cos(c+dx)\sin(c+d}{16a^3d}$$

$$= \frac{5x}{16a^3} + \frac{4\cos^3(c+dx)}{3a^3d} - \frac{\cos^5(c+dx)}{a^3d} + \frac{\cos^7(c+dx)}{7a^3d} + \frac{5\cos(c+dx)\sin(c+}{16a^3d}$$

Mathematica [B] time = 8.92, size = 429, normalized size = 3.23

$$\frac{840dx\sin\left(\frac{c}{2}\right) - 609\sin\left(\frac{c}{2}+dx\right) + 609\sin\left(\frac{3c}{2}+dx\right) - 63\sin\left(\frac{3c}{2}+2dx\right) - 63\sin\left(\frac{5c}{2}+2dx\right) - 91\sin\left(\frac{5c}{2}+3dx\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-168*(99*c - 5*d*x)*Cos[c/2] + 609*Cos[c/2 + d*x] + 609*Cos[(3*c)/2 + d*x]
 - 63*Cos[(3*c)/2 + 2*d*x] + 63*Cos[(5*c)/2 + 2*d*x] + 91*Cos[(5*c)/2 + 3*d
*x] + 91*Cos[(7*c)/2 + 3*d*x] - 105*Cos[(7*c)/2 + 4*d*x] + 105*Cos[(9*c)/2
+ 4*d*x] - 63*Cos[(9*c)/2 + 5*d*x] - 63*Cos[(11*c)/2 + 5*d*x] + 21*Cos[(11*
c)/2 + 6*d*x] - 21*Cos[(13*c)/2 + 6*d*x] + 3*Cos[(13*c)/2 + 7*d*x] + 3*Cos[
(15*c)/2 + 7*d*x] + 16996*Sin[c/2] - 16632*c*Sin[c/2] + 840*d*x*Sin[c/2] -
609*Sin[c/2 + d*x] + 609*Sin[(3*c)/2 + d*x] - 63*Sin[(3*c)/2 + 2*d*x] - 63*
Sin[(5*c)/2 + 2*d*x] - 91*Sin[(5*c)/2 + 3*d*x] + 91*Sin[(7*c)/2 + 3*d*x] -
105*Sin[(7*c)/2 + 4*d*x] - 105*Sin[(9*c)/2 + 4*d*x] + 63*Sin[(9*c)/2 + 5*d*
x] - 63*Sin[(11*c)/2 + 5*d*x] + 21*Sin[(11*c)/2 + 6*d*x] + 21*Sin[(13*c)/2
+ 6*d*x] - 3*Sin[(13*c)/2 + 7*d*x] + 3*Sin[(15*c)/2 + 7*d*x])/(2688*a^3*d*(
Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.45, size = 80, normalized size = 0.60

$$\frac{48\cos(dx+c)^7 - 336\cos(dx+c)^5 + 448\cos(dx+c)^3 + 105\,dx + 21\left(8\cos(dx+c)^5 - 18\cos(dx+c)^3 + 5\,\mathrm{c}\right.}{336\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/336*(48*cos(d*x + c)^7 - 336*cos(d*x + c)^5 + 448*cos(d*x + c)^3 + 105*d*
x + 21*(8*cos(d*x + c)^5 - 18*cos(d*x + c)^3 + 5*cos(d*x + c))*sin(d*x + c)
)/(a^3*d)
```

giac [A] time = 0.24, size = 179, normalized size = 1.35

$$\frac{\frac{105\,(dx+c)}{a^3} + \frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+252\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+2016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-2499\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+5152\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+448\,\tan\left(\frac{1}{2}\,dx+\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{336\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/336*(105*(d*x + c)/a^3 + 2*(105*tan(1/2*d*x + 1/2*c)^13 + 252*tan(1/2*d*x
 + 1/2*c)^11 + 2016*tan(1/2*d*x + 1/2*c)^10 - 2499*tan(1/2*d*x + 1/2*c)^9 +
 5152*tan(1/2*d*x + 1/2*c)^8 + 448*tan(1/2*d*x + 1/2*c)^6 + 2499*tan(1/2*d*
x + 1/2*c)^5 + 1344*tan(1/2*d*x + 1/2*c)^4 - 252*tan(1/2*d*x + 1/2*c)^3 + 1
120*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 160)/((tan(1/2*d*x
+ 1/2*c)^2 + 1)^7*a^3))/d
```

maple [B] time = 0.40, size = 415, normalized size = 3.12

$$\frac{5\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{12\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{119\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 5/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13+3/2/d/a^3/(1+tan
(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11+12/d/a^3/(1+tan(1/2*d*x+1/2*c)^2
)^7*tan(1/2*d*x+1/2*c)^10-119/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*
x+1/2*c)^9+92/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8+8/3/d
/a^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6+119/8/d/a^3/(1+tan(1/2
*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^5+8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^7*ta
n(1/2*d*x+1/2*c)^4-3/2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^
3+20/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2-5/8/d/a^3/(1+t
```

```
an(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)+20/21/d/a^3/(1+tan(1/2*d*x+1/2*c)
^2)^7+5/8/a^3/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.47, size = 416, normalized size = 3.13

$$-\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{1120\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{252\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1344\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2499\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{448\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{5152\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{2499\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{2016\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{252\sin\cdots}{(\cos(dx\cdots}}{a^3+\frac{7a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{21a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{21a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{7a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a^3\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/168*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 1120*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 252*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1344*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 2499*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 448*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 5152*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 + 2499*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 2016*sin(d*x + c)^10/(cos(d*x
 + c) + 1)^10 - 252*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 105*sin(d*x + c
)^13/(cos(d*x + c) + 1)^13 - 160)/(a^3 + 7*a^3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 21*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 35*a^3*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 + 35*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 21*a^
3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 7*a^3*sin(d*x + c)^12/(cos(d*x +
c) + 1)^12 + a^3*sin(d*x + c)^14/(cos(d*x + c) + 1)^14) - 105*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 12.64, size = 172, normalized size = 1.29

$$\frac{5x}{16a^3}+\frac{\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{8}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{2}+12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-\frac{119\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{8}+\frac{92\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{3}+\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3}+\frac{119\tan\left(\frac{c}{2}+\frac{d\cdots}{2}\right.}{8}\cdots}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (5*x)/(16*a^3) + ((20*tan(c/2 + (d*x)/2)^2)/3 - (5*tan(c/2 + (d*x)/2))/8 -
(3*tan(c/2 + (d*x)/2)^3)/2 + 8*tan(c/2 + (d*x)/2)^4 + (119*tan(c/2 + (d*x)/
2)^5)/8 + (8*tan(c/2 + (d*x)/2)^6)/3 + (92*tan(c/2 + (d*x)/2)^8)/3 - (119*t
an(c/2 + (d*x)/2)^9)/8 + 12*tan(c/2 + (d*x)/2)^10 + (3*tan(c/2 + (d*x)/2)^1
1)/2 + (5*tan(c/2 + (d*x)/2)^13)/8 + 20/21)/(a^3*d*(tan(c/2 + (d*x)/2)^2 +
1)^7)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.741 $\int \frac{\cos^8(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=131

$$-\frac{7\cos^5(c+dx)}{30a^3d}-\frac{\cos^7(c+dx)}{6d\left(a^3\sin(c+dx)+a^3\right)}-\frac{7\sin(c+dx)\cos^3(c+dx)}{24a^3d}-\frac{7\sin(c+dx)\cos(c+dx)}{16a^3d}-\frac{7x}{16a^3}-\frac{\cos}{3d(a\sin}$$

```
[Out] -7/16*x/a^3-7/30*cos(d*x+c)^5/a^3/d-7/16*cos(d*x+c)*sin(d*x+c)/a^3/d-7/24*c
os(d*x+c)^3*sin(d*x+c)/a^3/d-1/3*cos(d*x+c)^9/d/(a+a*sin(d*x+c))^3-1/6*cos(
d*x+c)^7/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2859, 2679, 2682, 2635, 8}

$$-\frac{7\cos^5(c+dx)}{30a^3d}-\frac{\cos^7(c+dx)}{6d\left(a^3\sin(c+dx)+a^3\right)}-\frac{7\sin(c+dx)\cos^3(c+dx)}{24a^3d}-\frac{7\sin(c+dx)\cos(c+dx)}{16a^3d}-\frac{7x}{16a^3}-\frac{\cos}{3d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-7*x)/(16*a^3) - (7*Cos[c + d*x]^5)/(30*a^3*d) - (7*Cos[c + d*x]*Sin[c + d
*x])/(16*a^3*d) - (7*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^3*d) - Cos[c + d*x]
^9/(3*d*(a + a*Sin[c + d*x])^3) - Cos[c + d*x]^7/(6*d*(a^3 + a^3*Sin[c + d*
x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
```

```
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\cos^9(c+dx)}{3d(a+a\sin(c+dx))^3}-\frac{\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{a}$$

$$= -\frac{\cos^9(c+dx)}{3d(a+a\sin(c+dx))^3}-\frac{\cos^7(c+dx)}{6d\left(a^3+a^3\sin(c+dx)\right)}-\frac{7\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx}{6a^2}$$

$$= -\frac{7\cos^5(c+dx)}{30a^3d}-\frac{\cos^9(c+dx)}{3d(a+a\sin(c+dx))^3}-\frac{\cos^7(c+dx)}{6d\left(a^3+a^3\sin(c+dx)\right)}-\frac{7\int \cos$$

$$= -\frac{7\cos^5(c+dx)}{30a^3d}-\frac{7\cos^3(c+dx)\sin(c+dx)}{24a^3d}-\frac{\cos^9(c+dx)}{3d(a+a\sin(c+dx))^3}-\frac{}{6d\left(a\right.}$$

$$= -\frac{7\cos^5(c+dx)}{30a^3d}-\frac{7\cos(c+dx)\sin(c+dx)}{16a^3d}-\frac{7\cos^3(c+dx)\sin(c+dx)}{24a^3d}-\frac{}{3a}$$

$$= -\frac{7x}{16a^3}-\frac{7\cos^5(c+dx)}{30a^3d}-\frac{7\cos(c+dx)\sin(c+dx)}{16a^3d}-\frac{7\cos^3(c+dx)\sin(c+a}{24a^3d}$$

Mathematica [B] time = 1.99, size = 366, normalized size = 2.79

$$\frac{-840dx\sin\left(\frac{c}{2}\right)+600\sin\left(\frac{c}{2}+dx\right)-600\sin\left(\frac{3c}{2}+dx\right)+15\sin\left(\frac{3c}{2}+2dx\right)+15\sin\left(\frac{5c}{2}+2dx\right)+140\sin\left(\frac{5c}{2}+3d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-21*(1 + 40*d*x)*Cos[c/2] - 600*Cos[c/2 + d*x] - 600*Cos[(3*c)/2 + d*x] +
15*Cos[(3*c)/2 + 2*d*x] - 15*Cos[(5*c)/2 + 2*d*x] - 140*Cos[(5*c)/2 + 3*d*x
] - 140*Cos[(7*c)/2 + 3*d*x] + 105*Cos[(7*c)/2 + 4*d*x] - 105*Cos[(9*c)/2 +
 4*d*x] + 36*Cos[(9*c)/2 + 5*d*x] + 36*Cos[(11*c)/2 + 5*d*x] - 5*Cos[(11*c)
/2 + 6*d*x] + 5*Cos[(13*c)/2 + 6*d*x] + 21*Sin[c/2] - 840*d*x*Sin[c/2] + 60
0*Sin[c/2 + d*x] - 600*Sin[(3*c)/2 + d*x] + 15*Sin[(3*c)/2 + 2*d*x] + 15*Si
n[(5*c)/2 + 2*d*x] + 140*Sin[(5*c)/2 + 3*d*x] - 140*Sin[(7*c)/2 + 3*d*x] +
105*Sin[(7*c)/2 + 4*d*x] + 105*Sin[(9*c)/2 + 4*d*x] - 36*Sin[(9*c)/2 + 5*d*
x] + 36*Sin[(11*c)/2 + 5*d*x] - 5*Sin[(11*c)/2 + 6*d*x] - 5*Sin[(13*c)/2 +
6*d*x])/(1920*a^3*d*(Cos[c/2] + Sin[c/2]))
```

fricas [A] time = 0.47, size = 70, normalized size = 0.53

$$\frac{144\cos\left(dx+c\right)^{5}-320\cos\left(dx+c\right)^{3}-105\,dx-5\left(8\cos\left(dx+c\right)^{5}-50\cos\left(dx+c\right)^{3}+21\cos\left(dx+c\right)\right)\sin\left(dx\right.}{240\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(144*cos(d*x + c)^5 - 320*cos(d*x + c)^3 - 105*d*x - 5*(8*cos(d*x + c
)^5 - 50*cos(d*x + c)^3 + 21*cos(d*x + c))*sin(d*x + c))/(a^3*d)
```

giac [A] time = 0.24, size = 179, normalized size = 1.37

$$-\frac{\frac{105\,(dx+c)}{a^{3}}+\frac{2\left(105\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+240\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-365\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+2160\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-1110\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+1760\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\right.}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(105*(d*x + c)/a^3 + 2*(105*tan(1/2*d*x + 1/2*c)^11 + 240*tan(1/2*d*
x + 1/2*c)^10 - 365*tan(1/2*d*x + 1/2*c)^9 + 2160*tan(1/2*d*x + 1/2*c)^8 -
```

```
1110*tan(1/2*d*x + 1/2*c)^7 + 1760*tan(1/2*d*x + 1/2*c)^6 + 1110*tan(1/2*d*
x + 1/2*c)^5 + 480*tan(1/2*d*x + 1/2*c)^4 + 365*tan(1/2*d*x + 1/2*c)^3 + 81
6*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 176)/((tan(1/2*d*x +
1/2*c)^2 + 1)^6*a^3))/d
```

maple [B] time = 0.39, size = 415, normalized size = 3.17

$$-\frac{7\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{73\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{18\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -7/8/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11-2/d/a^3/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10+73/24/a^3/d/(1+tan(1/2*d*x+1/2*c)
^2)^6*tan(1/2*d*x+1/2*c)^9-18/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+
1/2*c)^8+37/4/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7-44/3/a^
3/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6-37/4/a^3/d/(1+tan(1/2*d
*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5-4/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(
1/2*d*x+1/2*c)^4-73/24/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^
3-34/5/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2+7/8/a^3/d/(1+t
an(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-22/15/a^3/d/(1+tan(1/2*d*x+1/2*c)
^2)^6-7/8/a^3/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.47, size = 393, normalized size = 3.00

$$\frac{\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{816\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{365\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{480\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{1110\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1760\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{1110\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{2160\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{365\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{240\,\sin(dx}{(\cos(dx+c)}}{a^3+\frac{6\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15\,a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6\,a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/120*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 816*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 365*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 480*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 1110*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1760*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 1110*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
 2160*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 365*sin(d*x + c)^9/(cos(d*x + c
) + 1)^9 - 240*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 105*sin(d*x + c)^11/
(cos(d*x + c) + 1)^11 - 176)/(a^3 + 6*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^3*sin(d*x + c)^6/(co
```

```
s(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^3*sin(
d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^
12) - 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 12.49, size = 173, normalized size = 1.32

$$-\frac{7x}{16a^3}-\frac{\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{8}+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-\frac{73\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{24}+18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-\frac{37\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{4}+\frac{44\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{3}+\frac{37\tan\left(\right.}{}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (7*x)/(16*a^3) - ((34*tan(c/2 + (d*x)/2)^2)/5 - (7*tan(c/2 + (d*x)/2))/8
+ (73*tan(c/2 + (d*x)/2)^3)/24 + 4*tan(c/2 + (d*x)/2)^4 + (37*tan(c/2 + (d*
x)/2)^5)/4 + (44*tan(c/2 + (d*x)/2)^6)/3 - (37*tan(c/2 + (d*x)/2)^7)/4 + 18
*tan(c/2 + (d*x)/2)^8 - (73*tan(c/2 + (d*x)/2)^9)/24 + 2*tan(c/2 + (d*x)/2)
^10 + (7*tan(c/2 + (d*x)/2)^11)/8 + 22/15)/(a^3*d*(tan(c/2 + (d*x)/2)^2 + 1
)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.742 $\int \frac{\cos^7(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=99

$$-\frac{\cos^3(c+dx)}{a^3d}+\frac{\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^3d}-\frac{13\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{13x}{8a^3}$$

```
[Out] -13/8*x/a^3-arctanh(cos(d*x+c))/a^3/d+cos(d*x+c)/a^3/d-cos(d*x+c)^3/a^3/d-1
3/8*cos(d*x+c)*sin(d*x+c)/a^3/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a^3/d
```

Rubi [A] time = 0.24, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2875, 2873, 2635, 8, 2592, 321, 206, 2565, 30, 2568}

$$-\frac{\cos^3(c+dx)}{a^3d}+\frac{\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^3d}-\frac{13\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{13x}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-13*x)/(8*a^3) - ArcTanh[Cos[c + d*x]]/(a^3*d) + Cos[c + d*x]/(a^3*d) - Co
s[c + d*x]^3/(a^3*d) - (13*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d) + (Cos[c +
d*x]^3*Sin[c + d*x])/(4*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
```

```
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cos(c+dx)\cot(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-3a^3\cos^2(c+dx)+a^3\cos(c+dx)\cot(c+dx)+3a^3\cos^2(c+dx)\sin(c+a\right.}{a^6}$$
$$= \frac{\int \cos(c+dx)\cot(c+dx)\,dx}{a^3} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a^3} - \frac{3\int \cos^2(c+}{a^3}$$
$$= -\frac{3\cos(c+dx)\sin(c+dx)}{2a^3d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d} - \frac{\int \cos^2(c+dx)\,dx}{4a^3} -$$
$$= -\frac{3x}{2a^3} + \frac{\cos(c+dx)}{a^3d} - \frac{\cos^3(c+dx)}{a^3d} - \frac{13\cos(c+dx)\sin(c+dx)}{8a^3d} + \frac{\cos^3(c+a}{4}$$
$$= -\frac{13x}{8a^3} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{a^3d} - \frac{\cos^3(c+dx)}{a^3d} - \frac{13\cos(c+dx)\,\mathrm{s}}{8a^3d}$$

Mathematica [A] time = 0.41, size = 80, normalized size = 0.81

$$\frac{-24\sin(2(c+dx))+\sin(4(c+dx))+8\cos(c+dx)-8\cos(3(c+dx))+32\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-32\log\left(\cos\right.}{32a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-52*c - 52*d*x + 8*Cos[c + d*x] - 8*Cos[3*(c + d*x)] - 32*Log[Cos[(c + d*x
)/2]] + 32*Log[Sin[(c + d*x)/2]] - 24*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])/
(32*a^3*d)
```

fricas [A] time = 0.48, size = 84, normalized size = 0.85

$$-\frac{8\cos(dx+c)^3+13\,dx-\left(2\cos(dx+c)^3-13\cos(dx+c)\right)\sin(dx+c)-8\cos(dx+c)+4\log\left(\frac{1}{2}\cos(dx\right.}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(8*cos(d*x + c)^3 + 13*d*x - (2*cos(d*x + c)^3 - 13*cos(d*x + c))*sin(
d*x + c) - 8*cos(d*x + c) + 4*log(1/2*cos(d*x + c) + 1/2) - 4*log(-1/2*cos(
d*x + c) + 1/2))/(a^3*d)
```

giac [A] time = 0.25, size = 129, normalized size = 1.30

$$-\frac{\frac{13\,(dx+c)}{a^3}-\frac{8\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{2\left(11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+19\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-19\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(13*(d*x + c)/a^3 - 8*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 2*(11*tan(1
/2*d*x + 1/2*c)^7 - 16*tan(1/2*d*x + 1/2*c)^6 + 19*tan(1/2*d*x + 1/2*c)^5 -
 19*tan(1/2*d*x + 1/2*c)^3 + 16*tan(1/2*d*x + 1/2*c)^2 - 11*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3))/d
```

maple [B] time = 0.57, size = 239, normalized size = 2.41

$$\frac{11\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{19\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{19\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 11/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-4/d/a^3/(1+tan(1
/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6+19/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)
^4*tan(1/2*d*x+1/2*c)^5-19/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)^3+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2-11/4/d/a^3/
(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-13/4/a^3/d*arctan(tan(1/2*d*x
+1/2*c))+1/d/a^3*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.60, size = 269, normalized size = 2.72

$$-\frac{\frac{\frac{11\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{19\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{19\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{16\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{11\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3+\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{13\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((11*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 19*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 19*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 + 16*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 11*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/(a^3 + 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6
*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^3*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 13*arctan(sin(d*x +
c)/(cos(d*x + c) + 1))/a^3 - 4*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 11.00, size = 222, normalized size = 2.24

$$\frac{13\,\mathrm{atan}\left(\frac{169}{16\left(\frac{169\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}+\frac{13}{2}\right)}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{169\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}+\frac{13}{2}\right)}\right)}{4\,a^3\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{19\tan\left(\frac{c}{2}\right.}{4}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (13*atan(169/(16*((169*tan(c/2 + (d*x)/2))/16 + 13/2)) - (13*tan(c/2 + (d*x
)/2))/(2*((169*tan(c/2 + (d*x)/2))/16 + 13/2))))/(4*a^3*d) + log(tan(c/2 +
(d*x)/2))/(a^3*d) - ((11*tan(c/2 + (d*x)/2))/4 - 4*tan(c/2 + (d*x)/2)^2 + (
19*tan(c/2 + (d*x)/2)^3)/4 - (19*tan(c/2 + (d*x)/2)^5)/4 + 4*tan(c/2 + (d*x
)/2)^6 - (11*tan(c/2 + (d*x)/2)^7)/4)/(d*(4*a^3*tan(c/2 + (d*x)/2)^2 + 6*a^
3*tan(c/2 + (d*x)/2)^4 + 4*a^3*tan(c/2 + (d*x)/2)^6 + a^3*tan(c/2 + (d*x)/2
)^8 + a^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.743 $\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=92

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{3\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{3\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{x}{2a^3}$$

```
[Out] 1/2*x/a^3+3*arctanh(cos(d*x+c))/a^3/d-3*cos(d*x+c)/a^3/d+1/3*cos(d*x+c)^3/a
^3/d-cot(d*x+c)/a^3/d+3/2*cos(d*x+c)*sin(d*x+c)/a^3/d
```

Rubi [A] time = 0.22, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2709, 3770, 3767, 8, 2638, 2635, 2633}

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{3\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{3\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] x/(2*a^3) + (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - (3*Cos[c + d*x])/(a^3*d) +
Cos[c + d*x]^3/(3*a^3*d) - Cot[c + d*x]/(a^3*d) + (3*Cos[c + d*x]*Sin[c + d
*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(2a^5 - 3a^5\csc(c+dx) + a^5\csc^2(c+dx) + 2a^5\sin(c+dx) - 3a^5\sin^2(c+dx)\right.}{a^8}$$

$$= \frac{2x}{a^3} + \frac{\int \csc^2(c+dx)\,dx}{a^3} + \frac{\int \sin^3(c+dx)\,dx}{a^3} + \frac{2\int \sin(c+dx)\,dx}{a^3} - \frac{3\int \csc(c}{a^3}$$

$$= \frac{2x}{a^3} + \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cos(c+dx)}{a^3d} + \frac{3\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{3\int}{2}$$

$$= \frac{x}{2a^3} + \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{3\cos(c+dx)}{a^3d} + \frac{\cos^3(c+dx)}{3a^3d} - \frac{\cot(c+dx)}{a^3d} +$$

Mathematica [A] time = 0.99, size = 126, normalized size = 1.37

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(6(c+dx)+9\sin(2(c+dx))-33\cos(c+dx)+\cos(3(c+dx))+6\tan\left(\frac{1}{2}(c\right.\right.}{12d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(6*(c + d*x) - 33*Cos[c + d*x] + C
os[3*(c + d*x)] - 6*Cot[(c + d*x)/2] + 36*Log[Cos[(c + d*x)/2]] - 36*Log[Si
n[(c + d*x)/2]] + 9*Sin[2*(c + d*x)] + 6*Tan[(c + d*x)/2]))/(12*d*(a + a*Si
n[c + d*x])^3)
```

fricas [A] time = 0.47, size = 104, normalized size = 1.13

$$-\frac{9\cos(dx+c)^3 - \left(2\cos(dx+c)^3 + 3\,dx - 18\cos(dx+c)\right)\sin(dx+c) - 9\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c}{6\,a^3d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(9*cos(d*x + c)^3 - (2*cos(d*x + c)^3 + 3*d*x - 18*cos(d*x + c))*sin(d
*x + c) - 9*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 9*log(-1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 3*cos(d*x + c))/(a^3*d*sin(d*x + c))
```

giac [A] time = 0.25, size = 147, normalized size = 1.60

$$\frac{\frac{3(dx+c)}{a^3} - \frac{18\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3} + \frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3} + \frac{3\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)} - \frac{2\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+36\tan\left(\frac{1}{2}dx+\frac{1}{2}\cdots\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3a^3}\cdots}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)/a^3 - 18*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 3*tan(1/2*d*
x + 1/2*c)/a^3 + 3*(6*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c))
- 2*(9*tan(1/2*d*x + 1/2*c)^5 + 12*tan(1/2*d*x + 1/2*c)^4 + 36*tan(1/2*d*x
+ 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 16)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a
^3))/d
```

maple [B] time = 0.61, size = 230, normalized size = 2.50

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3} - \frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3} - \frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3} - \frac{12\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3} + \frac{3\tan\left(\frac{dx}{2}+\cdots\right.}{a^3d\left(1+\tan^2\left(\frac{dx}{2}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-3/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x
+1/2*c)^5-4/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4-12/a^3/d/
(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2+3/a^3/d/(1+tan(1/2*d*x+1/2*
c)^2)^3*tan(1/2*d*x+1/2*c)-16/3/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^3+1/a^3/d*ar
ctan(tan(1/2*d*x+1/2*c))-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^3*ln(tan(1/2*d*
x+1/2*c))
```

maxima [B] time = 0.43, size = 285, normalized size = 3.10

$$-\frac{\frac{\frac{32\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{72\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{24\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{21\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+3}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}} - \frac{6\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} + \frac{18\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} - \cdots}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*((32*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 72*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 21*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 3)/(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^3*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
+ a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) - 6*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a^3 + 18*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 3*sin(d*x +
 c)/(a^3*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 9.12, size = 231, normalized size = 2.51

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{32\tan}{}}{d\left(2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - ((32*tan(c/2 + (d*x)/2))/3 - 3*tan(c/2 + (d*
x)/2)^2 + 24*tan(c/2 + (d*x)/2)^3 + 3*tan(c/2 + (d*x)/2)^4 + 8*tan(c/2 + (d
*x)/2)^5 + 7*tan(c/2 + (d*x)/2)^6 + 1)/(d*(6*a^3*tan(c/2 + (d*x)/2)^3 + 6*a
^3*tan(c/2 + (d*x)/2)^5 + 2*a^3*tan(c/2 + (d*x)/2)^7 + 2*a^3*tan(c/2 + (d*x
)/2))) - (3*log(tan(c/2 + (d*x)/2)))/(a^3*d) - atan(1/(tan(c/2 + (d*x)/2) +
 6) - (6*tan(c/2 + (d*x)/2))/(tan(c/2 + (d*x)/2) + 6))/(a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.744 $\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=98

$$\frac{3\cos(c+dx)}{a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{5x}{2a^3}$$

```
[Out] 5/2*x/a^3-5/2*arctanh(cos(d*x+c))/a^3/d+3*cos(d*x+c)/a^3/d+3*cot(d*x+c)/a^3
/d-1/2*cot(d*x+c)*csc(d*x+c)/a^3/d-1/2*cos(d*x+c)*sin(d*x+c)/a^3/d
```

Rubi [A] time = 0.25, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2872, 3770, 3767, 8, 3768, 2638, 2635}

$$\frac{3\cos(c+dx)}{a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{5x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*x)/(2*a^3) - (5*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cos[c + d*x])/(a^3
*d) + (3*Cot[c + d*x])/(a^3*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d) - (C
os[c + d*x]*Sin[c + d*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
```

```
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)\csc(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(2a^5 + 2a^5\csc(c+dx) - 3a^5\csc^2(c+dx) + a^5\csc^3(c+dx) - 3a^5\sin(c+d\right.}{a^8}$$

$$= \frac{2x}{a^3} + \frac{\int \csc^3(c+dx)\,dx}{a^3} + \frac{\int \sin^2(c+dx)\,dx}{a^3} + \frac{2\int \csc(c+dx)\,dx}{a^3} - \frac{3\int \csc^2(}{}$$

$$= \frac{2x}{a^3} - \frac{2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{3\cos(c+dx)}{a^3d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^3d} - \cos$$

$$= \frac{5x}{2a^3} - \frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{3\cos(c+dx)}{a^3d} + \frac{3\cot(c+dx)}{a^3d} - \frac{\cot(c+dx)\,c}{2a^3}$$

Mathematica [A] time = 0.89, size = 144, normalized size = 1.47

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(20(c+dx)-2\sin(2(c+dx))+24\cos(c+dx)-12\tan\left(\frac{1}{2}(c+dx)\right)+12\,c\right.}{8d(a\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(20*(c + d*x) + 24*Cos[c + d*x] +
12*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 - 20*Log[Cos[(c + d*x)/2]] + 20*Lo
g[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 - 2*Sin[2*(c + d*x)] - 12*Tan[(c +
 d*x)/2]))/(8*d*(a + a*Sin[c + d*x])^3)
```

fricas [A] time = 0.48, size = 130, normalized size = 1.33

$$\frac{10\,dx\cos(dx+c)^2 + 12\cos(dx+c)^3 - 10\,dx - 5\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) + 5\left(\cos(dx+c)^2\right.}{4\left(a^3d\cos(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(10*d*x*cos(d*x + c)^2 + 12*cos(d*x + c)^3 - 10*d*x - 5*(cos(d*x + c)^2
 - 1)*log(1/2*cos(d*x + c) + 1/2) + 5*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x
 + c) + 1/2) - 2*(cos(d*x + c)^3 + 5*cos(d*x + c))*sin(d*x + c) - 10*cos(d*
x + c))/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

giac [A] time = 0.26, size = 172, normalized size = 1.76

$$\frac{\frac{20\,(dx+c)}{a^3}+\frac{20\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-20\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-16\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-36\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,\mathrm{ta}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^2a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(20*(d*x + c)/a^3 + 20*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (10*tan(1/2
*d*x + 1/2*c)^6 - 20*tan(1/2*d*x + 1/2*c)^5 - 27*tan(1/2*d*x + 1/2*c)^4 - 1
6*tan(1/2*d*x + 1/2*c)^3 - 36*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2
*c) + 1)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2*a^3) + (a^3*tan
(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [B] time = 0.68, size = 234, normalized size = 2.39

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3d}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^3*tan(1/2*d*x+1/2*c)+1/d/a^3/(1+tan(
1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+6/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2
*tan(1/2*d*x+1/2*c)^2-1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)
+6/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2+5/a^3/d*arctan(tan(1/2*d*x+1/2*c))-1/8/
d/a^3/tan(1/2*d*x+1/2*c)^2+3/2/d/a^3/tan(1/2*d*x+1/2*c)+5/2/d/a^3*ln(tan(1/
2*d*x+1/2*c))
```

maxima [B] time = 0.57, size = 267, normalized size = 2.72

$$\frac{\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}+\frac{46\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{47\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-1}{\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^3}+\frac{40\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{20\log\left(\frac{\mathrm{sin}}{\mathrm{cos}(}\right.}{a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8*((12*sin(d*x + c)/(cos(d*x + c) + 1) + 46*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 47*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 20*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a^3*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - (12*sin(d*x + c)/(cos(d*x + c) + 1)
 - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^3 + 40*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a^3 + 20*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 9.24, size = 228, normalized size = 2.33

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}-\frac{5\,\mathrm{atan}\left(\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-25}+\frac{25}{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-25}\right)}{a^3\,d}+\frac{5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^3\,d}+\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{47\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\left(4\,a^3\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) - (5*atan((25*tan(c/2 + (d*x)/2))/(25*tan(c/
2 + (d*x)/2) - 25) + 25/(25*tan(c/2 + (d*x)/2) - 25)))/(a^3*d) + (5*log(tan
(c/2 + (d*x)/2)))/(2*a^3*d) + (6*tan(c/2 + (d*x)/2) + 23*tan(c/2 + (d*x)/2)
^2 + 8*tan(c/2 + (d*x)/2)^3 + (47*tan(c/2 + (d*x)/2)^4)/2 + 10*tan(c/2 + (d
*x)/2)^5 - 1/2)/(d*(4*a^3*tan(c/2 + (d*x)/2)^2 + 8*a^3*tan(c/2 + (d*x)/2)^4
 + 4*a^3*tan(c/2 + (d*x)/2)^6)) - (3*tan(c/2 + (d*x)/2))/(2*a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.745 $\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=92

$$-\frac{\cos(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}-\frac{3\cot(c+dx)}{a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{3x}{a^3}$$

```
[Out] -3*x/a^3-1/2*arctanh(cos(d*x+c))/a^3/d-cos(d*x+c)/a^3/d-3*cot(d*x+c)/a^3/d-
1/3*cot(d*x+c)^3/a^3/d+3/2*cot(d*x+c)*csc(d*x+c)/a^3/d
```

Rubi [A] time = 0.27, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2872, 3770, 3767, 8, 3768, 2638}

$$-\frac{\cos(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}-\frac{3\cot(c+dx)}{a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{3x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*x)/a^3 - ArcTanh[Cos[c + d*x]]/(2*a^3*d) - Cos[c + d*x]/(a^3*d) - (3*Co
t[c + d*x])/(a^3*d) - Cot[c + d*x]^3/(3*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*
x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)\csc^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(-3a^5+2a^5\csc(c+dx)+2a^5\csc^2(c+dx)-3a^5\csc^3(c+dx)+a^5\csc^4(c+\right.}{a^8}$$

$$= -\frac{3x}{a^3}+\frac{\int \csc^4(c+dx)\,dx}{a^3}+\frac{\int \sin(c+dx)\,dx}{a^3}+\frac{2\int \csc(c+dx)\,dx}{a^3}+\frac{2\int \csc^2}{}$$

$$= -\frac{3x}{a^3}-\frac{2\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cos(c+dx)}{a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{3}{}$$

$$= -\frac{3x}{a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cos(c+dx)}{a^3d}-\frac{3\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+$$

Mathematica [A] time = 2.65, size = 132, normalized size = 1.43

$$\frac{\csc^3(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(2(3\sin(c+dx)+8)\cos(3(c+dx))+6(5\sin(c+dx)-4)\cos(\right.}{24a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(-12*(6*(c + d*x) +
 Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3 + 2*Cos[3*(c
 + d*x)]*(8 + 3*Sin[c + d*x]) + 6*Cos[c + d*x]*(-4 + 5*Sin[c + d*x])))/(24*
a^3*d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.48, size = 150, normalized size = 1.63

$$-\frac{32\cos\left(dx+c\right)^3+3\left(\cos\left(dx+c\right)^2-1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\left(\cos\left(dx+c\right)^2-1\right)\log\left(-\frac{1}{2}\cos\right.}{12\left(a^3d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(32*cos(d*x + c)^3 + 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/
2)*sin(d*x + c) - 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d
*x + c) + 6*(6*d*x*cos(d*x + c)^2 + 2*cos(d*x + c)^3 - 6*d*x + cos(d*x + c)
)*sin(d*x + c) - 36*cos(d*x + c))/((a^3*d*cos(d*x + c)^2 - a^3*d)*sin(d*x +
 c))
```

giac [A] time = 0.27, size = 157, normalized size = 1.71

$$-\frac{\frac{72\,(dx+c)}{a^3}-\frac{12\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{48}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}+\frac{22\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}-\frac{a^6\tan\left(\frac{1}{2}\,dx+\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/24*(72*(d*x + c)/a^3 - 12*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 48/((tan(
1/2*d*x + 1/2*c)^2 + 1)*a^3) + (22*tan(1/2*d*x + 1/2*c)^3 + 33*tan(1/2*d*x
+ 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c)^3) - (a^
```

```
6*tan(1/2*d*x + 1/2*c)^3 - 9*a^6*tan(1/2*d*x + 1/2*c)^2 + 33*a^6*tan(1/2*d*
x + 1/2*c))/a^9)/d
```

maple [A] time = 0.70, size = 173, normalized size = 1.88

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^3}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^3d}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^3}-\frac{2}{a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d}-\frac{}{24d\,a^3\,\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d/a^3*tan(1/2*d*x+1/2*c)^3-3/8/d/a^3*tan(1/2*d*x+1/2*c)^2+11/8/d/a^3*t
an(1/2*d*x+1/2*c)-2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)-6/a^3/d*arctan(tan(1/2*d
*x+1/2*c))-1/24/d/a^3/tan(1/2*d*x+1/2*c)^3+3/8/d/a^3/tan(1/2*d*x+1/2*c)^2-1
1/8/d/a^3/tan(1/2*d*x+1/2*c)+1/2/d/a^3*ln(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 242, normalized size = 2.63

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{34\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{39\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{33\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-1}{\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{\frac{33\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{144\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{12\log\left(\frac{}{\mathrm{c}}\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/24*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 34*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 39*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 33*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 1)/(a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^3*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5) + (33*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^3 -
 144*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + 12*log(sin(d*x + c)/(cos
(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 9.26, size = 219, normalized size = 2.38

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^3\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}+\frac{6\,\mathrm{atan}\left(\frac{36}{36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6}-\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6}\right)}{a^3\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^3\,d}+\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (3*tan(c/2 + (d*x)/2)^2)/(8*a^3*d) + (6*a
tan(36/(36*tan(c/2 + (d*x)/2) + 6) - (6*tan(c/2 + (d*x)/2))/(36*tan(c/2 + (
d*x)/2) + 6)))/(a^3*d) + log(tan(c/2 + (d*x)/2))/(2*a^3*d) + (11*tan(c/2 +
(d*x)/2))/(8*a^3*d) - ((34*tan(c/2 + (d*x)/2)^2)/3 - 3*tan(c/2 + (d*x)/2) +
 13*tan(c/2 + (d*x)/2)^3 + 11*tan(c/2 + (d*x)/2)^4 + 1/3)/(d*(8*a^3*tan(c/2
 + (d*x)/2)^3 + 8*a^3*tan(c/2 + (d*x)/2)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.746 $\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=97

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{11\cot(c+dx)\csc(c+dx)}{8a^3d}+\frac{x}{a^3}$$

```
[Out] x/a^3+13/8*arctanh(cos(d*x+c))/a^3/d+cot(d*x+c)/a^3/d+cot(d*x+c)^3/a^3/d-11
/8*cot(d*x+c)*csc(d*x+c)/a^3/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a^3/d
```

Rubi [A] time = 0.32, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2875, 2873, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{11\cot(c+dx)\csc(c+dx)}{8a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] x/a^3 + (13*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + Cot[c + d*x]/(a^3*d) + Cot[c
 + d*x]^3/(a^3*d) - (11*Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) - (Cot[c + d*x
]*Csc[c + d*x]^3)/(4*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
```

```
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)\csc^3(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\cot^2(c+dx)+3a^3\cot^2(c+dx)\csc(c+dx)-3a^3\cot^2(c+dx)\csc^2(c+\right.}{a^6}$$
$$= -\frac{\int \cot^2(c+dx)\,dx}{a^3}+\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^2(c+dx)\csc(c}{a^3}$$
$$= \frac{\cot(c+dx)}{a^3d}-\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{\int \csc^3(c}{4}$$
$$= \frac{x}{a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{11\cot(c+dx)\csc}{8a^3d}$$
$$= \frac{x}{a^3}+\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{11\cot(c+dx)c}{8a^3d}$$

Mathematica [A] time = 2.45, size = 165, normalized size = 1.70

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(-22\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)+22\sec^2\left(\frac{1}{2}(c+dx)\right)+(4\sin(c\right.}{64}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(-22*Csc[(c + d*x)/2]^2 + 22*Sec[(
c + d*x)/2]^2 + Sec[(c + d*x)/2]^4 + 8*(8*c + 8*d*x + 13*Log[Cos[(c + d*x)/
2]] - 13*Log[Sin[(c + d*x)/2]] - 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4) + Csc
[(c + d*x)/2]^4*(-1 + 4*Sin[c + d*x])))/(64*a^3*d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.48, size = 164, normalized size = 1.69

$$\frac{16\,dx\cos(dx+c)^4-32\,dx\cos(dx+c)^2+22\,\cos(dx+c)^3+16\,dx+13\left(\cos(dx+c)^4-2\,\cos(dx+c)^2+1\right)\mathrm{lo}}{16\left(a^3d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(16*d*x*cos(d*x + c)^4 - 32*d*x*cos(d*x + c)^2 + 22*cos(d*x + c)^3 + 1
6*d*x + 13*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1
```

```
/2) - 13*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/
2) + 16*cos(d*x + c)*sin(d*x + c) - 26*cos(d*x + c))/(a^3*d*cos(d*x + c)^4
- 2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

giac [A] time = 0.31, size = 166, normalized size = 1.71

$$\frac{\frac{192\,(dx+c)}{a^3} - \frac{312\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} + \frac{650\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-72\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4} + \frac{3\left(a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/192*(192*(d*x + c)/a^3 - 312*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + (650*ta
n(1/2*d*x + 1/2*c)^4 + 24*tan(1/2*d*x + 1/2*c)^3 - 72*tan(1/2*d*x + 1/2*c)^
2 + 24*tan(1/2*d*x + 1/2*c) - 3)/(a^3*tan(1/2*d*x + 1/2*c)^4) + 3*(a^9*tan(
1/2*d*x + 1/2*c)^4 - 8*a^9*tan(1/2*d*x + 1/2*c)^3 + 24*a^9*tan(1/2*d*x + 1/
2*c)^2 - 8*a^9*tan(1/2*d*x + 1/2*c))/a^12)/d
```

maple [B] time = 0.71, size = 188, normalized size = 1.94

$$\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d\,a^3} - \frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^3} + \frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^3d} - \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^3} + \frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d} - \frac{1}{64d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/64/d/a^3*tan(1/2*d*x+1/2*c)^4-1/8/d/a^3*tan(1/2*d*x+1/2*c)^3+3/8/d/a^3*ta
n(1/2*d*x+1/2*c)^2-1/8/d/a^3*tan(1/2*d*x+1/2*c)+2/a^3/d*arctan(tan(1/2*d*x+
1/2*c))-1/64/d/a^3/tan(1/2*d*x+1/2*c)^4+1/8/d/a^3/tan(1/2*d*x+1/2*c)^3-3/8/
d/a^3/tan(1/2*d*x+1/2*c)^2+1/8/d/a^3/tan(1/2*d*x+1/2*c)-13/8/d/a^3*ln(tan(1
/2*d*x+1/2*c))
```

maxima [B] time = 0.50, size = 218, normalized size = 2.25

$$-\frac{\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^3} - \frac{128\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} + \frac{104\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} - \frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\,\mathrm{s}}{(\mathrm{cos}}\right.}{a^3\sin(d}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/64*((8*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4)/a^3 - 128*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + 104*log
(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (8*sin(d*x + c)/(cos(d*x + c) + 1)
- 24*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 - 1)*(cos(d*x + c) + 1)^4/(a^3*sin(d*x + c)^4))/d
```

mupad [B] time = 9.53, size = 315, normalized size = 3.25

$$-\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+8\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-24\cos\left(\frac{c}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^5*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(cos(c/2 + (d*x)/2)^8 - sin(c/2 + (d*x)/2)^8 + 8*cos(c/2 + (d*x)/2)*sin(c/
2 + (d*x)/2)^7 - 8*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2) - 24*cos(c/2 + (
d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 8*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^
5 - 8*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3 + 24*cos(c/2 + (d*x)/2)^6*s
in(c/2 + (d*x)/2)^2 + 128*atan((8*cos(c/2 + (d*x)/2) - 13*sin(c/2 + (d*x)/2
))/(13*cos(c/2 + (d*x)/2) + 8*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^4 + 104*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2
+ (d*x)/2)^4*sin(c/2 + (d*x)/2)^4)/(64*a^3*d*cos(c/2 + (d*x)/2)^4*sin(c/2 +
 (d*x)/2)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.747 $\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=100

$$-\frac{\cot^5(c+dx)}{5a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{7\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^3d}$$

```
[Out] -7/8*arctanh(cos(d*x+c))/a^3/d-4/3*cot(d*x+c)^3/a^3/d-1/5*cot(d*x+c)^5/a^3/
d+1/8*cot(d*x+c)*csc(d*x+c)/a^3/d+3/4*cot(d*x+c)*csc(d*x+c)^3/a^3/d
```

Rubi [A] time = 0.34, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2611, 3770, 2607, 30, 3768, 14}

$$-\frac{\cot^5(c+dx)}{5a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{7\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-7*ArcTanh[Cos[c + d*x]])/(8*a^3*d) - (4*Cot[c + d*x]^3)/(3*a^3*d) - Cot[c
 + d*x]^5/(5*a^3*d) + (Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) + (3*Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)\csc^4(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(-a^3\cot^2(c+dx)\csc(c+dx)+3a^3\cot^2(c+dx)\csc^2(c+dx)-3a^3\cot^2(c+\right.}{a^6}$$

$$= -\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a^3}+\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a^3}+\frac{3\int \cot^2(c-}{}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{\int \csc(c+dx)\,dx}{2a^3}+\frac{3\int}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot^3(c+dx)}{a^3d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^3d}+\frac{3\cot(c+dx}{4}$$

$$= -\frac{7\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot(c+dx)\csc(c+}{8a^3d}$$

Mathematica [A] time = 1.76, size = 189, normalized size = 1.89

$$-\frac{\csc^5(c+dx)\left(-780\sin(2(c+dx))+30\sin(4(c+dx))+560\cos(c+dx)-40\cos(3(c+dx))-136\cos(5(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/1920*(Csc[c + d*x]^5*(560*Cos[c + d*x] - 40*Cos[3*(c + d*x)] - 136*Cos[5
*(c + d*x)] + 1050*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 1050*Log[Sin[(c + d
*x)/2]]*Sin[c + d*x] - 780*Sin[2*(c + d*x)] - 525*Log[Cos[(c + d*x)/2]]*Sin
[3*(c + d*x)] + 525*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 30*Sin[4*(c +
d*x)] + 105*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 105*Log[Sin[(c + d*x)/
2]]*Sin[5*(c + d*x)]))/(a^3*d)
```

fricas [A] time = 0.46, size = 169, normalized size = 1.69

$$\frac{272\cos(dx+c)^5-320\cos(dx+c)^3-105\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx}{240\left(a^3d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/240*(272*cos(d*x + c)^5 - 320*cos(d*x + c)^3 - 105*(cos(d*x + c)^4 - 2*co
s(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 105*(cos(d*x +
 c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3
0*(cos(d*x + c)^3 - 7*cos(d*x + c))*sin(d*x + c))/((a^3*d*cos(d*x + c)^4 -
2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))
```

giac [B] time = 0.31, size = 186, normalized size = 1.86

$$\frac{\frac{840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{1918\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-420\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+130\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}+\frac{6a^{12}\cdots}{\cdots}}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (1918*tan(1/2*d*x + 1/2*c)^
5 - 420*tan(1/2*d*x + 1/2*c)^4 - 120*tan(1/2*d*x + 1/2*c)^3 + 130*tan(1/2*d
*x + 1/2*c)^2 - 45*tan(1/2*d*x + 1/2*c) + 6)/(a^3*tan(1/2*d*x + 1/2*c)^5) +
 (6*a^12*tan(1/2*d*x + 1/2*c)^5 - 45*a^12*tan(1/2*d*x + 1/2*c)^4 + 130*a^12
*tan(1/2*d*x + 1/2*c)^3 - 120*a^12*tan(1/2*d*x + 1/2*c)^2 - 420*a^12*tan(1/
2*d*x + 1/2*c))/a^15)/d
```

maple [B] time = 0.71, size = 208, normalized size = 2.08

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{160d\,a^3}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d\,a^3}+\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d\,a^3}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3d}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^3}+\frac{7}{16d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{7\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/160/d/a^3*tan(1/2*d*x+1/2*c)^5-3/64/d/a^3*tan(1/2*d*x+1/2*c)^4+13/96/d/a^
3*tan(1/2*d*x+1/2*c)^3-1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-7/16/d/a^3*tan(1/2*d*
x+1/2*c)+7/16/d/a^3/tan(1/2*d*x+1/2*c)+7/8/d/a^3*ln(tan(1/2*d*x+1/2*c))-1/1
60/d/a^3/tan(1/2*d*x+1/2*c)^5+1/8/d/a^3/tan(1/2*d*x+1/2*c)^2+3/64/d/a^3/tan
(1/2*d*x+1/2*c)^4-13/96/d/a^3/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.40, size = 235, normalized size = 2.35

$$-\frac{\frac{\frac{420\sin(dx+c)}{\cos(dx+c)+1}+\frac{120\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{130\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{840\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{130\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{120\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\cdots\right.}{a^3\sin(dx+\cdots}}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/960*((420*sin(d*x + c)/(cos(d*x + c) + 1) + 120*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 130*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 840*log(
sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (45*sin(d*x + c)/(cos(d*x + c) + 1)
- 130*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 120*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 420*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6)*(cos(d*x + c) + 1)^
5/(a^3*sin(d*x + c)^5))/d
```

mupad [B] time = 9.60, size = 291, normalized size = 2.91

$$6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-6\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-45\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+45\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+130\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^6*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (6*sin(c/2 + (d*x)/2)^10 - 6*cos(c/2 + (d*x)/2)^10 - 45*cos(c/2 + (d*x)/2)*
sin(c/2 + (d*x)/2)^9 + 45*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) + 130*cos
(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 - 120*cos(c/2 + (d*x)/2)^3*sin(c/2 +
 (d*x)/2)^7 - 420*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 + 420*cos(c/2 +
 (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 120*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)
/2)^3 - 130*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 840*log(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)/(960
*a^3*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.748 $\int \frac{\cos(c+dx)\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=124

$$\frac{3\cot^5(c+dx)}{5a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\tanh^{-1}(\cos(c+dx))}{16a^3d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^3d}-\frac{17\cot(c+dx)\csc^3(c+dx)}{24a^3d}+$$

```
[Out] 7/16*arctanh(cos(d*x+c))/a^3/d+4/3*cot(d*x+c)^3/a^3/d+3/5*cot(d*x+c)^5/a^3/
d+7/16*cot(d*x+c)*csc(d*x+c)/a^3/d-17/24*cot(d*x+c)*csc(d*x+c)^3/a^3/d-1/6*
cot(d*x+c)*csc(d*x+c)^5/a^3/d
```

Rubi [A] time = 0.36, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2875, 2873, 2607, 30, 2611, 3768, 3770, 14}

$$\frac{3\cot^5(c+dx)}{5a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\tanh^{-1}(\cos(c+dx))}{16a^3d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^3d}-\frac{17\cot(c+dx)\csc^3(c+dx)}{24a^3d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*ArcTanh[Cos[c + d*x]])/(16*a^3*d) + (4*Cot[c + d*x]^3)/(3*a^3*d) + (3*Co
t[c + d*x]^5)/(5*a^3*d) + (7*Cot[c + d*x]*Csc[c + d*x])/(16*a^3*d) - (17*Co
t[c + d*x]*Csc[c + d*x]^3)/(24*a^3*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a^
3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)\csc^5(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(-a^3\cot^2(c+dx)\csc^2(c+dx)+3a^3\cot^2(c+dx)\csc^3(c+dx)-3a^3\cot^2(c+\right.}{a^6}$$

$$= -\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a^3}+\frac{\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{a^3}+\frac{3\int \cot^2(c}{}$$

$$= -\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^3d}-\frac{\int \csc^5(c+dx)\,dx}{6a^3}-$$

$$= \frac{\cot^3(c+dx)}{3a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{8a^3d}-\frac{17\cot(c+dx)\csc^3(c+dx)}{24a^3d}-\frac{\cot(c}{}$$

$$= \frac{3\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}+\frac{7\cot(c+dx)\csc(c}{16a^3d}$$

$$= \frac{7\tanh^{-1}(\cos(c+dx))}{16a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}+\frac{7\cot(c+dx)\csc(c}{16a^3d}$$

Mathematica [A] time = 1.01, size = 242, normalized size = 1.95

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(704\tan\left(\frac{1}{2}(c+dx)\right)-704\cot\left(\frac{1}{2}(c+dx)\right)+210\csc^2\left(\frac{1}{2}(c+dx)\right)+5\sec^6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(-704*Cot[(c + d*x)/2] + 210*Csc[(
c + d*x)/2]^2 + 840*Log[Cos[(c + d*x)/2]] - 840*Log[Sin[(c + d*x)/2]] - 210
*Sec[(c + d*x)/2]^2 + 90*Sec[(c + d*x)/2]^4 + 5*Sec[(c + d*x)/2]^6 - 544*Cs
c[c + d*x]^3*Sin[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^6*(-5 + 18*Sin[c + d*x])
 + Csc[(c + d*x)/2]^4*(-90 + 34*Sin[c + d*x]) + 704*Tan[(c + d*x)/2] - 36*S
ec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(1920*a^3*d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.45, size = 196, normalized size = 1.58

$$-\frac{210\cos(dx+c)^5-80\cos(dx+c)^3-105\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(\right.}{480}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")

[Out] -1/480*(210*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 105*(cos(d*x + c)^6 - 3*co
s(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 105*(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c
) + 1/2) - 32*(11*cos(d*x + c)^5 - 20*cos(d*x + c)^3)*sin(d*x + c) - 210*co
s(d*x + c))/(a^3*d*cos(d*x + c)^6 - 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*
x + c)^2 - a^3*d)

giac [A] time = 0.33, size = 216, normalized size = 1.74

$$-\frac{\frac{840 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3} - \frac{2058 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 - 600 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 15 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 + 140 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 105 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 36 \tan\left(\frac{1}{2}\right.}{a^3 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/1920*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (2058*tan(1/2*d*x + 1/2*c
)^6 - 600*tan(1/2*d*x + 1/2*c)^5 + 15*tan(1/2*d*x + 1/2*c)^4 + 140*tan(1/2*
d*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c)^2 + 36*tan(1/2*d*x + 1/2*c) - 5)/
(a^3*tan(1/2*d*x + 1/2*c)^6) - (5*a^15*tan(1/2*d*x + 1/2*c)^6 - 36*a^15*tan
(1/2*d*x + 1/2*c)^5 + 105*a^15*tan(1/2*d*x + 1/2*c)^4 - 140*a^15*tan(1/2*d*
x + 1/2*c)^3 - 15*a^15*tan(1/2*d*x + 1/2*c)^2 + 600*a^15*tan(1/2*d*x + 1/2*
c))/a^18)/d

maple [B] time = 0.72, size = 246, normalized size = 1.98

$$\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{384d\,a^3} - \frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d\,a^3} + \frac{7\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d\,a^3} - \frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d\,a^3} - \frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{128a^3d} + \frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^3} - \frac{}{384d\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^3,x)

[Out] 1/384/d/a^3*tan(1/2*d*x+1/2*c)^6-3/160/d/a^3*tan(1/2*d*x+1/2*c)^5+7/128/d/a
^3*tan(1/2*d*x+1/2*c)^4-7/96/d/a^3*tan(1/2*d*x+1/2*c)^3-1/128/d/a^3*tan(1/2
*d*x+1/2*c)^2+5/16/d/a^3*tan(1/2*d*x+1/2*c)-1/384/d/a^3/tan(1/2*d*x+1/2*c)^
6-5/16/d/a^3/tan(1/2*d*x+1/2*c)-7/16/d/a^3*ln(tan(1/2*d*x+1/2*c))+3/160/d/a
^3/tan(1/2*d*x+1/2*c)^5+1/128/d/a^3/tan(1/2*d*x+1/2*c)^2-7/128/d/a^3/tan(1/
2*d*x+1/2*c)^4+7/96/d/a^3/tan(1/2*d*x+1/2*c)^3

maxima [B] time = 0.33, size = 274, normalized size = 2.21

$$\frac{\frac{\frac{600 \sin(dx+c)}{\cos(dx+c)+1} - \frac{15 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{140 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{105 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{36 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a^3} - \frac{840 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} + \frac{\left(\frac{36 \sin(dx+c)}{\cos(dx+c)+1} - \frac{105 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \cdots\right.}{\cdots}}{1920\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/1920*((600*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 140*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 105*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 36*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6)/a^3 - 840*log(sin(d*x + c)/(cos(d*x + c) + 1))/a
^3 + (36*sin(d*x + c)/(cos(d*x + c) + 1) - 105*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 140*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 600*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x + c
) + 1)^6/(a^3*sin(d*x + c)^6))/d
```

mupad [B] time = 10.18, size = 339, normalized size = 2.73

$$-\frac{5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - 5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} + 36\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11} - 36\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 105 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^7*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 + 36*cos(c/2 + (d*x)/2)
*sin(c/2 + (d*x)/2)^11 - 36*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) - 105*
cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 140*cos(c/2 + (d*x)/2)^3*sin(c
/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 - 600*cos(c/
2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 600*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d
*x)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 140*cos(c/2 + (d*
x)/2)^9*sin(c/2 + (d*x)/2)^3 + 105*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)
^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*si
n(c/2 + (d*x)/2)^6)/(1920*a^3*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**7/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.749 $\int \frac{\cot^8(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=140

$$-\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{5\tanh^{-1}(\cos(c+dx))}{16a^3d}+\frac{\cot(c+dx)\csc^5(c+dx)}{2a^3d}+\frac{\cot(c+dx)\csc^3}{8a^3d}$$

```
[Out] -5/16*arctanh(cos(d*x+c))/a^3/d-4/3*cot(d*x+c)^3/a^3/d-cot(d*x+c)^5/a^3/d-1
/7*cot(d*x+c)^7/a^3/d-5/16*cot(d*x+c)*csc(d*x+c)/a^3/d+1/8*cot(d*x+c)*csc(d
*x+c)^3/a^3/d+1/2*cot(d*x+c)*csc(d*x+c)^5/a^3/d
```

Rubi [A] time = 0.23, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2709, 3768, 3770, 3767}

$$-\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{5\tanh^{-1}(\cos(c+dx))}{16a^3d}+\frac{\cot(c+dx)\csc^5(c+dx)}{2a^3d}+\frac{\cot(c+dx)\csc^3}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(16*a^3*d) - (4*Cot[c + d*x]^3)/(3*a^3*d) - Cot[
c + d*x]^5/(a^3*d) - Cot[c + d*x]^7/(7*a^3*d) - (5*Cot[c + d*x]*Csc[c + d*x
])/(16*a^3*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(8*a^3*d) + (Cot[c + d*x]*Csc
[c + d*x]^5)/(2*a^3*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \left(a^5\csc^3(c+dx) - 3a^5\csc^4(c+dx) + 2a^5\csc^5(c+dx) + 2a^5\csc^6(c+dx) - 3a^5\csc\right.}{a^8}$$

$$= \frac{\int \csc^3(c+dx)\,dx}{a^3} + \frac{\int \csc^8(c+dx)\,dx}{a^3} + \frac{2\int \csc^5(c+dx)\,dx}{a^3} + \frac{2\int \csc^6(c+dx)\,dx}{a^3}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2a^3d} - \frac{\cot(c+dx)\csc^3(c+dx)}{2a^3d} + \frac{\cot(c+dx)\csc^5(c+dx)}{2a^3d} + \int$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{4\cot^3(c+dx)}{3a^3d} - \frac{\cot^5(c+dx)}{a^3d} - \frac{\cot^7(c+dx)}{7a^3d} - \frac{5\cot(c+dx}{4}$$

$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{4a^3d} - \frac{4\cot^3(c+dx)}{3a^3d} - \frac{\cot^5(c+dx)}{a^3d} - \frac{\cot^7(c+dx)}{7a^3d} - \frac{5\cot(c+}{}$$

$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{16a^3d} - \frac{4\cot^3(c+dx)}{3a^3d} - \frac{\cot^5(c+dx)}{a^3d} - \frac{\cot^7(c+dx)}{7a^3d} - \frac{5\cot(c+}{}$$

Mathematica [A] time = 0.99, size = 251, normalized size = 1.79

$$\frac{\csc^7(c+dx)\left(4998\sin(2(c+dx)) + 504\sin(4(c+dx)) - 210\sin(6(c+dx)) - 4704\cos(c+dx) + 672\cos(3(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^7*(-4704*Cos[c + d*x] + 672*Cos[3*(c + d*x)] + 1120*Cos[5*(c
+ d*x)] - 160*Cos[7*(c + d*x)] - 3675*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] +
3675*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] + 4998*Sin[2*(c + d*x)] + 2205*Log[
Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] - 2205*Log[Sin[(c + d*x)/2]]*Sin[3*(c +
d*x)] + 504*Sin[4*(c + d*x)] - 735*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] +
 735*Log[Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 210*Sin[6*(c + d*x)] + 105*Lo
g[Cos[(c + d*x)/2]]*Sin[7*(c + d*x)] - 105*Log[Sin[(c + d*x)/2]]*Sin[7*(c +
 d*x)]))/(21504*a^3*d)
```

fricas [A] time = 0.46, size = 226, normalized size = 1.61

$$\frac{320 \cos(dx+c)^7 - 1120 \cos(dx+c)^5 + 896 \cos(dx+c)^3 - 105\left(\cos(dx+c)^6 - 3\cos(dx+c)^4 + 3\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/672*(320*cos(d*x + c)^7 - 1120*cos(d*x + c)^5 + 896*cos(d*x + c)^3 - 105*
(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) + 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d
*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 42*(5*cos(d*x +
c)^5 - 8*cos(d*x + c)^3 - 5*cos(d*x + c))*sin(d*x + c))/((a^3*d*cos(d*x + c
)^6 - 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^2 - a^3*d)*sin(d*x + c)
)
```

giac [A] time = 0.35, size = 244, normalized size = 1.74

$$\frac{\frac{840 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3} - \frac{2178 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 609 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 + 63 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 91 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 105 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 63 \tan\left(\frac{1}{2}\right.}{a^3 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2688*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (2178*tan(1/2*d*x + 1/2*c)
^7 - 609*tan(1/2*d*x + 1/2*c)^6 + 63*tan(1/2*d*x + 1/2*c)^5 + 91*tan(1/2*d*
x + 1/2*c)^4 - 105*tan(1/2*d*x + 1/2*c)^3 + 63*tan(1/2*d*x + 1/2*c)^2 - 21*
tan(1/2*d*x + 1/2*c) + 3)/(a^3*tan(1/2*d*x + 1/2*c)^7) + (3*a^18*tan(1/2*d*
x + 1/2*c)^7 - 21*a^18*tan(1/2*d*x + 1/2*c)^6 + 63*a^18*tan(1/2*d*x + 1/2*c
)^5 - 105*a^18*tan(1/2*d*x + 1/2*c)^4 + 91*a^18*tan(1/2*d*x + 1/2*c)^3 + 63
*a^18*tan(1/2*d*x + 1/2*c)^2 - 609*a^18*tan(1/2*d*x + 1/2*c))/a^21)/d
```

maple [B] time = 0.73, size = 284, normalized size = 2.03

$$\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{896d\,a^3} - \frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{128d\,a^3} + \frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d\,a^3} - \frac{5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d\,a^3} + \frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d\,a^3} + \frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128a^3d} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/896/d/a^3*tan(1/2*d*x+1/2*c)^7-1/128/d/a^3*tan(1/2*d*x+1/2*c)^6+3/128/d/a
^3*tan(1/2*d*x+1/2*c)^5-5/128/d/a^3*tan(1/2*d*x+1/2*c)^4+13/384/d/a^3*tan(1
/2*d*x+1/2*c)^3+3/128/d/a^3*tan(1/2*d*x+1/2*c)^2-29/128/d/a^3*tan(1/2*d*x+1
/2*c)+1/128/d/a^3/tan(1/2*d*x+1/2*c)^6+29/128/d/a^3/tan(1/2*d*x+1/2*c)+5/16
/d/a^3*ln(tan(1/2*d*x+1/2*c))-3/128/d/a^3/tan(1/2*d*x+1/2*c)^5-1/896/d/a^3/
tan(1/2*d*x+1/2*c)^7-3/128/d/a^3/tan(1/2*d*x+1/2*c)^2+5/128/d/a^3/tan(1/2*d
*x+1/2*c)^4-13/384/d/a^3/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.33, size = 315, normalized size = 2.25

$$-\frac{\frac{\frac{609\sin(dx+c)}{\cos(dx+c)+1}-\frac{63\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{91\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{105\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{21\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3}-\frac{840\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\cdots\right.}{}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2688*((609*sin(d*x + c)/(cos(d*x + c) + 1) - 63*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 91*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 105*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 21*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^3 - 8
40*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (21*sin(d*x + c)/(cos(d*x + c
) + 1) - 63*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 105*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - 91*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 63*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 + 609*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3)*(cos(d
*x + c) + 1)^7/(a^3*sin(d*x + c)^7))/d
```

mupad [B] time = 10.86, size = 387, normalized size = 2.76

$$\frac{3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+63\cos\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^8*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (3*sin(c/2 + (d*x)/2)^14 - 3*cos(c/2 + (d*x)/2)^14 - 21*cos(c/2 + (d*x)/2)*
sin(c/2 + (d*x)/2)^13 + 21*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) + 63*co
s(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 105*cos(c/2 + (d*x)/2)^3*sin(c/2
 + (d*x)/2)^11 + 91*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 + 63*cos(c/2
```

```
 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 - 609*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*
x)/2)^8 + 609*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 - 63*cos(c/2 + (d*x
)/2)^9*sin(c/2 + (d*x)/2)^5 - 91*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^4
 + 105*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 - 63*cos(c/2 + (d*x)/2)^1
2*sin(c/2 + (d*x)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos
(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)/(2688*a^3*d*cos(c/2 + (d*x)/2)^7*si
n(c/2 + (d*x)/2)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**8/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.750 $\int \frac{\cot^8(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=166

$$\frac{3\cot^7(c+dx)}{7a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{29\tanh^{-1}(\cos(c+dx))}{128a^3d}-\frac{\cot(c+dx)\csc^7(c+dx)}{8a^3d}-\frac{23\cot(c+dx)}{48a}$$

```
[Out] 29/128*arctanh(cos(d*x+c))/a^3/d+4/3*cot(d*x+c)^3/a^3/d+7/5*cot(d*x+c)^5/a^
3/d+3/7*cot(d*x+c)^7/a^3/d+29/128*cot(d*x+c)*csc(d*x+c)/a^3/d+29/192*cot(d*
x+c)*csc(d*x+c)^3/a^3/d-23/48*cot(d*x+c)*csc(d*x+c)^5/a^3/d-1/8*cot(d*x+c)*
csc(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.41, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2875, 2873, 2607, 14, 2611, 3768, 3770, 270}

$$\frac{3\cot^7(c+dx)}{7a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{29\tanh^{-1}(\cos(c+dx))}{128a^3d}-\frac{\cot(c+dx)\csc^7(c+dx)}{8a^3d}-\frac{23\cot(c+dx)}{48a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (29*ArcTanh[Cos[c + d*x]])/(128*a^3*d) + (4*Cot[c + d*x]^3)/(3*a^3*d) + (7*
Cot[c + d*x]^5)/(5*a^3*d) + (3*Cot[c + d*x]^7)/(7*a^3*d) + (29*Cot[c + d*x]
*Csc[c + d*x])/(128*a^3*d) + (29*Cot[c + d*x]*Csc[c + d*x]^3)/(192*a^3*d) -
 (23*Cot[c + d*x]*Csc[c + d*x]^5)/(48*a^3*d) - (Cot[c + d*x]*Csc[c + d*x]^7
)/(8*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)\csc^7(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\cot^2(c+dx)\csc^4(c+dx)+3a^3\cot^2(c+dx)\csc^5(c+dx)-3a^3\cot^2(c+\right.}{a^6}$$
$$= -\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a^3}+\frac{\int \cot^2(c+dx)\csc^7(c+dx)\,dx}{a^3}+\frac{3\int \cot^2(c+}{}$$
$$= -\frac{\cot(c+dx)\csc^5(c+dx)}{2a^3d}-\frac{\cot(c+dx)\csc^7(c+dx)}{8a^3d}-\frac{\int \csc^7(c+dx)\,dx}{8a^3}-\frac{\int \mathrm{c}}{}$$
$$= \frac{\cot(c+dx)\csc^3(c+dx)}{8a^3d}-\frac{23\cot(c+dx)\csc^5(c+dx)}{48a^3d}-\frac{\cot(c+dx)\csc^7(c+dx}{8a^3d}$$
$$= \frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{3\cot^7(c+dx)}{7a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{16a^3d}+$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{16a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{3\cot^7(c+dx)}{7a^3d}+\frac{29}{}$$
$$= \frac{29\tanh^{-1}(\cos(c+dx))}{128a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{3\cot^7(c+dx)}{7a^3d}+\frac{2}{}$$

Mathematica [A] time = 5.51, size = 317, normalized size = 1.91

$$-\frac{\sin^7(c+dx)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^6\left(15(7\csc(c+dx)-24)\csc^8\left(\frac{1}{2}(c+dx)\right)+4(455\csc(c+dx)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/13762560*((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^6*(Csc[(c + d*x)/2]^4*(1
328 - 210*Csc[c + d*x]) + 15*Csc[(c + d*x)/2]^8*(-24 + 7*Csc[c + d*x]) + 4*
Csc[(c + d*x)/2]^6*(-276 + 455*Csc[c + d*x]) - 4*Csc[(c + d*x)/2]^2*(-4864
+ 3045*Csc[c + d*x]) - 8*(6090*Csc[c + d*x]*(Log[Cos[(c + d*x)/2]] - Log[Si
n[(c + d*x)/2]]) + ((2833 + 4616*Cos[c + d*x] + 1907*Cos[2*(c + d*x)] + 304
*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^8)/4 - 6090*Csc[c + d*x]^3*Sin[(c + d*x
)/2]^2 - 420*Csc[c + d*x]^5*Sin[(c + d*x)/2]^4 + 14560*Csc[c + d*x]^7*Sin[(
c + d*x)/2]^6 + 3360*Csc[c + d*x]^9*Sin[(c + d*x)/2]^8))*Sin[c + d*x]^7)/(a
^3*d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.50, size = 249, normalized size = 1.50

$$-\frac{6090\cos(dx+c)^7-22330\cos(dx+c)^5+13510\cos(dx+c)^3-3045\left(\cos(dx+c)^8-4\cos(dx+c)^6+6\,\mathrm{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/26880*(6090*cos(d*x + c)^7 - 22330*cos(d*x + c)^5 + 13510*cos(d*x + c)^3
 - 3045*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x +
 c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2) + 3045*(cos(d*x + c)^8 - 4*cos(d*x +
 c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/
2) - 256*(38*cos(d*x + c)^7 - 133*cos(d*x + c)^5 + 140*cos(d*x + c)^3)*sin(
d*x + c) + 6090*cos(d*x + c))/(a^3*d*cos(d*x + c)^8 - 4*a^3*d*cos(d*x + c)^
6 + 6*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2 + a^3*d)
```

giac [A] time = 0.36, size = 274, normalized size = 1.65

$$-\frac{\frac{48720\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{132414\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-38640\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+6720\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+3920\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-5880\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/215040*(48720*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (132414*tan(1/2*d*x +
 1/2*c)^8 - 38640*tan(1/2*d*x + 1/2*c)^7 + 6720*tan(1/2*d*x + 1/2*c)^6 + 39
20*tan(1/2*d*x + 1/2*c)^5 - 5880*tan(1/2*d*x + 1/2*c)^4 + 4368*tan(1/2*d*x
+ 1/2*c)^3 - 2240*tan(1/2*d*x + 1/2*c)^2 + 720*tan(1/2*d*x + 1/2*c) - 105)/
(a^3*tan(1/2*d*x + 1/2*c)^8) - (105*a^21*tan(1/2*d*x + 1/2*c)^8 - 720*a^21*
tan(1/2*d*x + 1/2*c)^7 + 2240*a^21*tan(1/2*d*x + 1/2*c)^6 - 4368*a^21*tan(1
/2*d*x + 1/2*c)^5 + 5880*a^21*tan(1/2*d*x + 1/2*c)^4 - 3920*a^21*tan(1/2*d*
x + 1/2*c)^3 - 6720*a^21*tan(1/2*d*x + 1/2*c)^2 + 38640*a^21*tan(1/2*d*x +
1/2*c))/a^24)/d
```

maple [B] time = 0.76, size = 322, normalized size = 1.94

$$\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{2048d\,a^3}-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{896d\,a^3}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{96d\,a^3}-\frac{13\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{640d\,a^3}+\frac{7\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256d\,a^3}-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d\,a^3}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2048/d/a^3*tan(1/2*d*x+1/2*c)^8-3/896/d/a^3*tan(1/2*d*x+1/2*c)^7+1/96/d/a
^3*tan(1/2*d*x+1/2*c)^6-13/640/d/a^3*tan(1/2*d*x+1/2*c)^5+7/256/d/a^3*tan(1
/2*d*x+1/2*c)^4-7/384/d/a^3*tan(1/2*d*x+1/2*c)^3-1/32/d/a^3*tan(1/2*d*x+1/2
*c)^2+23/128/d/a^3*tan(1/2*d*x+1/2*c)-1/96/d/a^3/tan(1/2*d*x+1/2*c)^6-23/12
8/d/a^3/tan(1/2*d*x+1/2*c)-29/128/d/a^3*ln(tan(1/2*d*x+1/2*c))+13/640/d/a^3
/tan(1/2*d*x+1/2*c)^5+3/896/d/a^3/tan(1/2*d*x+1/2*c)^7+1/32/d/a^3/tan(1/2*d
*x+1/2*c)^2-1/2048/d/a^3/tan(1/2*d*x+1/2*c)^8-7/256/d/a^3/tan(1/2*d*x+1/2*c
)^4+7/384/d/a^3/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.33, size = 354, normalized size = 2.13

$$\frac{\frac{\frac{38640 \sin(dx+c)}{\cos(dx+c)+1} - \frac{6720 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{3920 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5880 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{4368 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{2240 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{720 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{105 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}}{a^3} - \frac{48720 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+\cdots}\right.}{a^3}}{215040\, a \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/215040*((38640*sin(d*x + c)/(cos(d*x + c) + 1) - 6720*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 3920*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5880*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 4368*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 22
40*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 720*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 + 105*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)/a^3 - 48720*log(sin(d*x +
c)/(cos(d*x + c) + 1))/a^3 + (720*sin(d*x + c)/(cos(d*x + c) + 1) - 2240*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4368*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 - 5880*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3920*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 + 6720*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 38640*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 - 105)*(cos(d*x + c) + 1)^8/(a^3*sin(d*x + c)^8))/
d
```

mupad [B] time = 11.50, size = 435, normalized size = 2.62

$$-\frac{105 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16} - 105 \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16} + 720 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right) \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15} - 720 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^9*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(105*cos(c/2 + (d*x)/2)^16 - 105*sin(c/2 + (d*x)/2)^16 + 720*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^15 - 720*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2)
- 2240*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 4368*cos(c/2 + (d*x)/2)
^3*sin(c/2 + (d*x)/2)^13 - 5880*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^12
+ 3920*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^11 + 6720*cos(c/2 + (d*x)/2)
^6*sin(c/2 + (d*x)/2)^10 - 38640*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^9
+ 38640*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^7 - 6720*cos(c/2 + (d*x)/2)
^10*sin(c/2 + (d*x)/2)^6 - 3920*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^5
+ 5880*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 4368*cos(c/2 + (d*x)/2)
^13*sin(c/2 + (d*x)/2)^3 + 2240*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2
+ 48720*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^8*sin
(c/2 + (d*x)/2)^8)/(215040*a^3*d*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**9/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.751 $\int \sin^2(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=82

$$-\frac{a\cos^3(c+dx)}{3d}+\frac{2a\cos(c+dx)}{d}+\frac{3a\tan(c+dx)}{2d}+\frac{a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}$$

```
[Out] -3/2*a*x+2*a*cos(d*x+c)/d-1/3*a*cos(d*x+c)^3/d+a*sec(d*x+c)/d+3/2*a*tan(d*x
+c)/d-1/2*a*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2591, 288, 321, 203, 2590, 270}

$$-\frac{a\cos^3(c+dx)}{3d}+\frac{2a\cos(c+dx)}{d}+\frac{3a\tan(c+dx)}{2d}+\frac{a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*x)/2 + (2*a*Cos[c + d*x])/d - (a*Cos[c + d*x]^3)/(3*d) + (a*Sec[c + d
*x])/d + (3*a*Tan[c + d*x])/(2*d) - (a*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sin^2(c+dx)\tan^2(c+dx)\,dx + a\int \sin^3(c+dx)\tan^2(c$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\right.}{}$$

$$= -\frac{a\sin^2(c+dx)\tan(c+dx)}{2d} - \frac{a\,\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)d\right.}{d}$$

$$= \frac{2a\cos(c+dx)}{d} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\sec(c+dx)}{d} + \frac{3a\tan(c}{2d}$$

$$= -\frac{3ax}{2} + \frac{2a\cos(c+dx)}{d} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\sec(c+dx)}{d} +$$

Mathematica [A] time = 0.31, size = 82, normalized size = 1.00

$$-\frac{3a(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}+\frac{7a\cos(c+dx)}{4d}-\frac{a\cos(3(c+dx))}{12d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*(c + d*x))/(2*d) + (7*a*Cos[c + d*x])/(4*d) - (a*Cos[3*(c + d*x)])/(1
2*d) + (a*Sec[c + d*x])/d + (a*Sin[2*(c + d*x)])/(4*d) + (a*Tan[c + d*x])/d
```

fricas [A] time = 0.45, size = 130, normalized size = 1.59

$$-\frac{2\,a\cos\left(dx+c\right)^{4}-a\cos\left(dx+c\right)^{3}+9\,adx-12\,a\cos\left(dx+c\right)^{2}+3\left(3\,adx-5\,a\right)\cos\left(dx+c\right)-\left(2\,a\cos\left(dx+c\right)^{3}\right.}{6\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*cos(d*x + c)^4 - a*cos(d*x + c)^3 + 9*a*d*x - 12*a*cos(d*x + c)^2
 + 3*(3*a*d*x - 5*a)*cos(d*x + c) - (2*a*cos(d*x + c)^3 + 9*a*d*x + 3*a*cos
(d*x + c)^2 - 9*a*cos(d*x + c) + 6*a)*sin(d*x + c) - 6*a)/(d*cos(d*x + c) -
 d*sin(d*x + c) + d)
```

giac [A] time = 0.18, size = 105, normalized size = 1.28

$$-\frac{9\left(dx+c\right)a+\frac{12\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-10\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)*a + 12*a/(tan(1/2*d*x + 1/2*c) - 1) + 2*(3*a*tan(1/2*d*x
+ 1/2*c)^5 - 6*a*tan(1/2*d*x + 1/2*c)^4 - 24*a*tan(1/2*d*x + 1/2*c)^2 - 3*a
*tan(1/2*d*x + 1/2*c) - 10*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.40, size = 104, normalized size = 1.27

$$\frac{a\left(\frac{\sin^{6}(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^{4}\left(dx+c\right)+\frac{4\left(\sin^{2}(dx+c)\right)}{3}\right)\cos\left(dx+c\right)\right)+a\left(\frac{\sin^{5}(dx+c)}{\cos(dx+c)}+\left(\sin^{3}\left(dx+c\right)+\frac{3\sin(dx+c)}{2}\right)\cos\left(dx\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x
+c))+a*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/
2*d*x-3/2*c))
```

maxima [A] time = 0.44, size = 75, normalized size = 0.91

$$-\frac{2\left(\cos\left(dx+c\right)^{3}-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)a+3\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^{2}+1}-2\,\tan\left(dx+c\right)\right)a}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a + 3*(3*d*x + 3
*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a)/d
```

mupad [B] time = 14.82, size = 257, normalized size = 3.13

$$\frac{\left(\frac{a\,(9\,c+9\,d\,x-18)}{6}-\frac{3\,a\,(c+d\,x)}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\left(\frac{9\,a\,(c+d\,x)}{2}-\frac{a\,(27\,c+27\,d\,x-18)}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+\left(\frac{a\,(27\,c+27\,d\,x-48)}{6}-\frac{9\,a\,(c+d}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] ((a*(9*c + 9*d*x - 32))/6 - tan(c/2 + (d*x)/2)*((a*(9*c + 9*d*x - 14))/6 -
(3*a*(c + d*x))/2) - (3*a*(c + d*x))/2 + tan(c/2 + (d*x)/2)^6*((a*(9*c + 9*
d*x - 18))/6 - (3*a*(c + d*x))/2) - tan(c/2 + (d*x)/2)^5*((a*(27*c + 27*d*x
 - 18))/6 - (9*a*(c + d*x))/2) - tan(c/2 + (d*x)/2)^3*((a*(27*c + 27*d*x -
48))/6 - (9*a*(c + d*x))/2) + tan(c/2 + (d*x)/2)^4*((a*(27*c + 27*d*x - 48)
)/6 - (9*a*(c + d*x))/2) + tan(c/2 + (d*x)/2)^2*((a*(27*c + 27*d*x - 78))/6
 - (9*a*(c + d*x))/2))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 +
1)^3) - (3*a*x)/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.752 $\int \sin(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=65

$$\frac{a\cos(c+dx)}{d}+\frac{3a\tan(c+dx)}{2d}+\frac{a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}$$

```
[Out] -3/2*a*x+a*cos(d*x+c)/d+a*sec(d*x+c)/d+3/2*a*tan(d*x+c)/d-1/2*a*sin(d*x+c)^
2*tan(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2838, 2590, 14, 2591, 288, 321, 203}

$$\frac{a\cos(c+dx)}{d}+\frac{3a\tan(c+dx)}{2d}+\frac{a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*x)/2 + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (3*a*Tan[c + d*x])/(
2*d) - (a*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sin(c+dx)\tan^2(c+dx)\,dx + a\int \sin^2(c+dx)\tan^2(c+$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx\right.}{d}$$

$$= -\frac{a\sin^2(c+dx)\tan(c+dx)}{2d} - \frac{a\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, \text{co}\right.}{d}$$

$$= \frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{3a\tan(c+dx)}{2d} - \frac{a\sin^2(c+}{}$$

$$= -\frac{3ax}{2} + \frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{3a\tan(c+dx)}{2d} - \frac{a}{}$$

Mathematica [A] time = 0.11, size = 63, normalized size = 0.97

$$-\frac{3a(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\cos(c+dx)}{d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*(c + d*x))/(2*d) + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (a*Sin[2
*(c + d*x)])/(4*d) + (a*Tan[c + d*x])/d
```

fricas [A] time = 0.45, size = 104, normalized size = 1.60

$$\frac{a\cos(dx+c)^3-3\,adx+2\,a\cos(dx+c)^2-3\,(adx-a)\cos(dx+c)+\left(3\,adx+a\cos(dx+c)^2-a\cos(dx+c)+\right.}{2\,(d\cos(dx+c)-d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)^3 - 3*a*d*x + 2*a*cos(d*x + c)^2 - 3*(a*d*x - a)*cos(d*
x + c) + (3*a*d*x + a*cos(d*x + c)^2 - a*cos(d*x + c) + 2*a)*sin(d*x + c) +
 2*a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [A] time = 0.17, size = 90, normalized size = 1.38

$$-\frac{3\,(dx+c)a+\frac{4\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(3*(d*x + c)*a + 4*a/(tan(1/2*d*x + 1/2*c) - 1) + 2*(a*tan(1/2*d*x + 1
/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(1
/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.39, size = 94, normalized size = 1.45

$$\frac{a\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/
2*d*x-3/2*c)+a*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 0.42, size = 62, normalized size = 0.95

$$-\frac{\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)a-2\,a\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a
- 2*a*(1/cos(d*x + c) + cos(d*x + c)))/d
```

mupad [B] time = 11.42, size = 160, normalized size = 2.46

$$\frac{\left(\frac{a\,(3\,d\,x-6)}{2}-\frac{3\,a\,d\,x}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(3\,a\,d\,x-\frac{a\,(6\,d\,x-6)}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{a\,(6\,d\,x-10)}{2}-3\,a\,d\,x\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\left(\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] ((a*(3*d*x - 8))/2 - tan(c/2 + (d*x)/2)*((a*(3*d*x - 2))/2 - (3*a*d*x)/2) -
 tan(c/2 + (d*x)/2)^3*((a*(6*d*x - 6))/2 - 3*a*d*x) + tan(c/2 + (d*x)/2)^4*
((a*(3*d*x - 6))/2 - (3*a*d*x)/2) + tan(c/2 + (d*x)/2)^2*((a*(6*d*x - 10))/
2 - 3*a*d*x) - (3*a*d*x)/2)/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)
^2 + 1)^2) - (3*a*x)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin^3\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\sin^4\left(c+dx\right)\sec^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)**3*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**4*
sec(c + d*x)**2, x))
```

3.753 $\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=39

$$\frac{a\cos(c+dx)}{d}+\frac{a\cos(c+dx)}{d(1-\sin(c+dx))}-ax$$

```
[Out] -a*x+a*cos(d*x+c)/d+a*cos(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2708, 2746, 12, 2735, 2648}

$$\frac{a\cos(c+dx)}{d}+\frac{a\cos(c+dx)}{d(1-\sin(c+dx))}-ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) + (a*Cos[c + d*x])/d + (a*Cos[c + d*x])/(d*(1 - Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx &= a^2\int \frac{\sin^2(c+dx)}{a-a\sin(c+dx)}\,dx\\ &= \frac{a\cos(c+dx)}{d}+a\int \frac{a\sin(c+dx)}{a-a\sin(c+dx)}\,dx\\ &= \frac{a\cos(c+dx)}{d}+a^2\int \frac{\sin(c+dx)}{a-a\sin(c+dx)}\,dx\\ &= -ax+\frac{a\cos(c+dx)}{d}+a^2\int \frac{1}{a-a\sin(c+dx)}\,dx\\ &= -ax+\frac{a\cos(c+dx)}{d}+\frac{a^2\cos(c+dx)}{d(a-a\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 1.21

$$\frac{a\cos(c+dx)}{d}-\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (
a*Tan[c + d*x])/d
```

fricas [B] time = 0.44, size = 80, normalized size = 2.05

$$-\frac{adx-a\cos\left(dx+c\right)^2+(adx-2\,a)\cos\left(dx+c\right)-(adx-a\cos\left(dx+c\right)+a)\sin\left(dx+c\right)-a}{d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*d*x - a*cos(d*x + c)^2 + (a*d*x - 2*a)*cos(d*x + c) - (a*d*x - a*cos(d*
x + c) + a)*sin(d*x + c) - a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [B] time = 0.17, size = 81, normalized size = 2.08

$$-\frac{(dx+c)a+\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*a + 2*(a*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) + 2*a)
/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) -
1))/d
```

maple [A] time = 0.30, size = 59, normalized size = 1.51

$$\frac{a\left(\frac{\sin^{4}(dx+c)}{\cos(dx+c)}+\left(2+\sin^{2}(dx+c)\right)\cos(dx+c)\right)+a\left(\tan(dx+c)-dx-c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+a*(tan(d*x+c)-
d*x-c))
```

maxima [A] time = 0.42, size = 39, normalized size = 1.00

$$-\frac{(dx+c-\tan(dx+c))a-a\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a - a*(1/cos(d*x + c) + cos(d*x + c)))/d
```

mupad [B] time = 9.21, size = 99, normalized size = 2.54

$$\frac{\left(a\,\left(d\,x-2\right)-a\,d\,x\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\left(a\,d\,x-a\,\left(d\,x-2\right)\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\,\left(d\,x-4\right)-a\,d\,x}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] (a*(d*x - 4) - tan(c/2 + (d*x)/2)*(a*(d*x - 2) - a*d*x) + tan(c/2 + (d*x)/2
)^2*(a*(d*x - 2) - a*d*x) - a*d*x)/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (
d*x)/2)^2 + 1)) - a*x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin^2(c+dx)\sec^2(c+dx)\,dx + \int \sin^3(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)**2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**3*
sec(c + d*x)**2, x))
```

3.754 $\int \sec(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=27

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-ax$$

```
[Out] -a*x+a*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2838, 2606, 8, 3473}

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -(a*x) + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx = a\int \sec(c+dx)\tan(c+dx)\,dx + a\int \tan^2(c+dx)\,dx$$
$$= \frac{a\tan(c+dx)}{d} - a\int 1\,dx + \frac{a\,\mathrm{Subst}(\int 1\,dx, x, \sec(c+dx))}{d}$$
$$= -ax + \frac{a\sec(c+dx)}{d} + \frac{a\tan(c+dx)}{d}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.33

$$-\frac{a\tan^{-1}(\tan(c+dx))}{d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

fricas [B] time = 0.45, size = 60, normalized size = 2.22

$$-\frac{adx + (adx - a)\cos(dx+c) - (adx + a)\sin(dx+c) - a}{d\cos(dx+c) - d\sin(dx+c) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -(a*d*x + (a*d*x - a)*cos(d*x + c) - (a*d*x + a)*sin(d*x + c) - a)/(d*cos(d
*x + c) - d*sin(d*x + c) + d)
```

giac [A] time = 0.16, size = 29, normalized size = 1.07

$$-\frac{(dx+c)a + \frac{2a}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -((d*x + c)*a + 2*a/(tan(1/2*d*x + 1/2*c) - 1))/d
```

maple [A] time = 0.14, size = 32, normalized size = 1.19

$$\frac{a(\tan(dx+c) - dx - c) + \frac{a}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(tan(d*x+c)-d*x-c)+a/cos(d*x+c))
```

maxima [A] time = 0.41, size = 32, normalized size = 1.19

$$-\frac{(dx+c-\tan(dx+c))a-\frac{a}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a - a/cos(d*x + c))/d
```

mupad [B] time = 8.98, size = 24, normalized size = 0.89

$$-a\,x-\frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] - a*x - (2*a)/(d*(tan(c/2 + (d*x)/2) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin(c+dx)\sec^2(c+dx)\,dx+\int\sin^2(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**2*sec
(c + d*x)**2, x))
```

3.755 $\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=36

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d+a*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2838, 2622, 321, 207, 3767, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \sec^2(c+dx)\,dx + a\int \csc(c+dx)\sec^2(c+dx)\,dx$$

$$= -\frac{a\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d} + \frac{a\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{se}\right.}{d}$$

$$= \frac{a\sec(c+dx)}{d} + \frac{a\tan(c+dx)}{d} + \frac{a\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c\right.}{d}$$

$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\tan(c+dx)}{d}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.56

$$\frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[(c + d*x)/2]])/d) + (a*Log[Sin[(c + d*x)/2]])/d + (a*Sec[c + d
*x])/d + (a*Tan[c + d*x])/d
```

fricas [B] time = 0.46, size = 108, normalized size = 3.00

$$\frac{2\,a\cos(dx+c) - (a\cos(dx+c) - a\sin(dx+c) + a)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) + (a\cos(dx+c) - a\sin(dx+c) +}{2\,(d\cos(dx+c) - d\sin(dx+c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*cos(d*x + c) - (a*cos(d*x + c) - a*sin(d*x + c) + a)*log(1/2*cos(d
*x + c) + 1/2) + (a*cos(d*x + c) - a*sin(d*x + c) + a)*log(-1/2*cos(d*x + c
) + 1/2) + 2*a*sin(d*x + c) + 2*a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [A] time = 0.17, size = 34, normalized size = 0.94

$$\frac{a \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - \frac{2\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*a/(tan(1/2*d*x + 1/2*c) - 1))/d
```

maple [A] time = 0.41, size = 47, normalized size = 1.31

$$\frac{a\tan\left(dx+c\right)}{d} + \frac{a}{d\cos\left(dx+c\right)} + \frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*tan(d*x+c)/d+1/d*a/cos(d*x+c)+1/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.34, size = 48, normalized size = 1.33

$$\frac{a\left(\frac{2}{\cos(dx+c)} - \log\left(\cos\left(dx+c\right)+1\right) + \log\left(\cos\left(dx+c\right)-1\right)\right) + 2\,a\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(a*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 2
*a*tan(d*x + c))/d
```

mupad [B] time = 8.95, size = 35, normalized size = 0.97

$$\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^2*sin(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)))/d - (2*a)/(d*(tan(c/2 + (d*x)/2) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \csc(c+dx)\sec^2(c+dx)\,dx+\int \sin(c+dx)\csc(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)*sec(c + d*x)**2, x) + Integral(sin(c + d*x)*csc(c
+ d*x)*sec(c + d*x)**2, x))
```

3.756 $\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=48

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d-a*cot(d*x+c)/d+a*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2620, 14, 2622, 321, 207}

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) - (a*Cot[c + d*x])/d + (a*Sec[c + d*x])/d +
(a*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx &= a\int \csc(c+dx)\sec^2(c+dx)\,dx + a\int \csc^2(c+dx)\sec^2(c+\\
&= \frac{a\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x,\right.}{d}\\
&= \frac{a\sec(c+dx)}{d} + \frac{a\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, \tan(c+dx)\right)}{d} + \frac{a\,\text{S}}{}\\
&= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} - \frac{a\cot(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a}{}\end{aligned}$$

Mathematica [A] time = 0.08, size = 68, normalized size = 1.42

$$\frac{a\tan(c+dx)}{d} - \frac{a\cot(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/
2]])/d + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

fricas [B] time = 0.46, size = 165, normalized size = 3.44

$$-\frac{4\,a\cos\left(dx+c\right)^{2}+2\,a\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)^{2}+\left(a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right.\right.}{2\,\left(d\cos\left(dx+c\right)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) + (a*cos(d*x + c)^2 + (a*cos(d*
x + c) + a)*sin(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) - (a*cos(d*x + c)
^2 + (a*cos(d*x + c) + a)*sin(d*x + c) - a)*log(-1/2*cos(d*x + c) + 1/2) -
2*(2*a*cos(d*x + c) + a)*sin(d*x + c) - 2*a)/(d*cos(d*x + c)^2 + (d*cos(d*x
 + c) + d)*sin(d*x + c) - d)
```

giac [A] time = 0.21, size = 87, normalized size = 1.81

$$\frac{2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*log(abs(tan(1/2*d*x + 1/2*c))) + a*tan(1/2*d*x + 1/2*c) - (a*tan(1
/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c) - a)/(tan(1/2*d*x + 1/2*c)^2 -
 tan(1/2*d*x + 1/2*c)))/d
```

maple [A] time = 0.38, size = 69, normalized size = 1.44

$$\frac{a}{d\cos\left(dx+c\right)}+\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}+\frac{a}{d\sin\left(dx+c\right)\cos\left(dx+c\right)}-\frac{2a\cot\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*a/cos(d*x+c)+1/d*a*ln(csc(d*x+c)-cot(d*x+c))+1/d*a/sin(d*x+c)/cos(d*x+c
)-2*a*cot(d*x+c)/d
```

maxima [A] time = 0.45, size = 59, normalized size = 1.23

$$\frac{a\left(\frac{2}{\cos\left(dx+c\right)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-2\,a\left(\frac{1}{\tan\left(dx+c\right)}-\tan\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(a*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 2
*a*(1/tan(d*x + c) - tan(d*x + c)))/d
```

mupad [B] time = 8.97, size = 77, normalized size = 1.60

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^2*sin(c + d*x)^2),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) + (a*log(tan(c/2 + (d*x)/2)))/d - (a - 5*a*tan
(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.757 $\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=75

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}$$

```
[Out] -3/2*a*arctanh(cos(d*x+c))/d-a*cot(d*x+c)/d+3/2*a*sec(d*x+c)/d-1/2*a*csc(d*
x+c)^2*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2622, 288, 321, 207, 2620, 14}

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x])/d + (3*a*Sec[c + d*x]
)/(2*d) - (a*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (a*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \csc^2(c+dx)\sec^2(c+dx)\,dx + a\int \csc^3(c+dx)\sec^2(c+$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx,\right.}{d}$$

$$= -\frac{a\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{a\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, \tan\right.}{d}$$

$$= -\frac{a\cot(c+dx)}{d} + \frac{3a\sec(c+dx)}{2d} - \frac{a\csc^2(c+dx)\sec(c+dx)}{2d}$$

$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot(c+dx)}{d} + \frac{3a\sec(c+dx)}{2d} -$$

Mathematica [B] time = 1.50, size = 172, normalized size = 2.29

$$-\frac{2a\cot(2(c+dx))}{d}-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*Cot[2*(c + d*x)])/d - (a*Csc[(c + d*x)/2]^2)/(8*d) - (3*a*Log[Cos[(c
+ d*x)/2]])/(2*d) + (3*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]
^2)/(8*d) + (a*Sin[(c + d*x)/2])/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
- (a*Sin[(c + d*x)/2])/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [B] time = 0.47, size = 261, normalized size = 3.48

$$8\,a\cos\left(dx+c\right)^3+6\,a\cos\left(dx+c\right)^2-6\,a\cos\left(dx+c\right)-3\left(a\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)-\left(a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(8*a*cos(d*x + c)^3 + 6*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 3*(a*cos(
d*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*sin
(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) + 3*(a*cos(d*x + c)^3 + a*cos(d*
x + c)^2 - a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*sin(d*x + c) - a)*log(-1
/2*cos(d*x + c) + 1/2) + 2*(4*a*cos(d*x + c)^2 + a*cos(d*x + c) - 2*a)*sin(
d*x + c) - 4*a)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - (d*
cos(d*x + c)^2 - d)*sin(d*x + c) - d)
```

giac [A] time = 0.21, size = 102, normalized size = 1.36

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{16\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}-\frac{18\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,a\tan}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 + 12*a*log(abs(tan(1/2*d*x + 1/2*c))) + 4*a*t
an(1/2*d*x + 1/2*c) - 16*a/(tan(1/2*d*x + 1/2*c) - 1) - (18*a*tan(1/2*d*x +
 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c)^2)/d
```

maple [A] time = 0.46, size = 93, normalized size = 1.24

$$\frac{a}{d\sin(dx+c)\cos(dx+c)}-\frac{2a\cot(dx+c)}{d}-\frac{a}{2d\sin(dx+c)^2\cos(dx+c)}+\frac{3a}{2d\cos(dx+c)}+\frac{3a\ln(\csc(dx+c)-c}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*a/sin(d*x+c)/cos(d*x+c)-2*a*cot(d*x+c)/d-1/2/d*a/sin(d*x+c)^2/cos(d*x+c
)+3/2/d*a/cos(d*x+c)+3/2/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.32, size = 84, normalized size = 1.12

$$\frac{a\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)-4a\left(\frac{1}{\tan(dx+c)}-\tan(dx+c)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(co
s(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 4*a*(1/tan(d*x + c) - tan(d*x
+ c)))/d
```

mupad [B] time = 8.90, size = 113, normalized size = 1.51

$$\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}-\frac{-10a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}+\frac{a}{2}}{d\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d}+\frac{3a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^2*sin(c + d*x)^3),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) - (a/2 + (3*a*tan(c/2 + (d*x)/2))/2 - 10*a*tan
(c/2 + (d*x)/2)^2)/(d*(4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^3)) +
(a*tan(c/2 + (d*x)/2)^2)/(8*d) + (3*a*log(tan(c/2 + (d*x)/2)))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.758 $\int \csc^4(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=91

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot^3(c+dx)}{3d}-\frac{2a\cot(c+dx)}{d}+\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+a}{2d}$$

```
[Out] -3/2*a*arctanh(cos(d*x+c))/d-2*a*cot(d*x+c)/d-1/3*a*cot(d*x+c)^3/d+3/2*a*se
c(d*x+c)/d-1/2*a*csc(d*x+c)^2*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2620, 270, 2622, 288, 321, 207}

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot^3(c+dx)}{3d}-\frac{2a\cot(c+dx)}{d}+\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+a}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (2*a*Cot[c + d*x])/d - (a*Cot[c + d*x]
^3)/(3*d) + (3*a*Sec[c + d*x])/(2*d) - (a*Csc[c + d*x]^2*Sec[c + d*x])/(2*d
) + (a*Tan[c + d*x])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \csc^4(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \csc^3(c+dx)\sec^2(c+dx)\,dx + a\int \csc^4(c+dx)\sec^2(c+$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4}\,d\right.}{d}$$

$$= -\frac{a\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{a\,\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx, x\right.}{d}$$

$$= -\frac{2a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} + \frac{3a\sec(c+dx)}{2d} - \frac{a\csc^2(c}{}$$

$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} +$$

Mathematica [B] time = 4.72, size = 205, normalized size = 2.25

$$\frac{a\tan(c+dx)}{d}-\frac{5a\cot(c+dx)}{3d}-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{3a\log\left(\cos\left(\frac{1}{2}\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*a*Cot[c + d*x])/(3*d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]*
Csc[c + d*x]^2)/(3*d) - (3*a*Log[Cos[(c + d*x)/2]])/(2*d) + (3*a*Log[Sin[(c
 + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d) + (a*Sin[(c + d*x)/2])/(d
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (a*Sin[(c + d*x)/2])/(d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])) + (a*Tan[c + d*x])/d
```

fricas [B] time = 0.45, size = 308, normalized size = 3.38

$$-\frac{32\,a\cos\left(dx+c\right)^4+14\,a\cos\left(dx+c\right)^3-48\,a\cos\left(dx+c\right)^2-18\,a\cos\left(dx+c\right)+9\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(32*a*cos(d*x + c)^4 + 14*a*cos(d*x + c)^3 - 48*a*cos(d*x + c)^2 - 18
*a*cos(d*x + c) + 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + (a*cos(d*x + c
)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*log(1/2*cos(
d*x + c) + 1/2) - 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + (a*cos(d*x + c
)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*log(-1/2*cos
(d*x + c) + 1/2) - 2*(16*a*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 - 15*a*cos(d
*x + c) - 6*a)*sin(d*x + c) + 12*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2
+ (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c) +
 d)
```

giac [A] time = 0.21, size = 130, normalized size = 1.43

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{48\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 + 36*a*log(abs(
tan(1/2*d*x + 1/2*c))) + 21*a*tan(1/2*d*x + 1/2*c) - 48*a/(tan(1/2*d*x + 1/
2*c) - 1) - (66*a*tan(1/2*d*x + 1/2*c)^3 + 21*a*tan(1/2*d*x + 1/2*c)^2 + 3*
a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.44, size = 116, normalized size = 1.27

$$-\frac{a}{2d\sin(dx+c)^2\cos(dx+c)}+\frac{3a}{2d\cos(dx+c)}+\frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{a}{3d\sin(dx+c)^3\cos(dx+c)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2/cos(d*x+c)+3/2/d*a/cos(d*x+c)+3/2/d*a*ln(csc(d*x+c)-c
ot(d*x+c))-1/3/d*a/sin(d*x+c)^3/cos(d*x+c)+4/3/d*a/sin(d*x+c)/cos(d*x+c)-8/
3*a*cot(d*x+c)/d
```

maxima [A] time = 0.31, size = 98, normalized size = 1.08

$$\frac{3\,a\left(\frac{2\left(3\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\,\log(\cos(dx+c)+1)+3\,\log(\cos(dx+c)-1)\right)-4\,a\left(\frac{6\,\tan(dx+c)^2+1}{\tan(dx+c)^3}-3\,\tan(dx+c)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log
(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 4*a*((6*tan(d*x + c)^2 + 1)
/tan(d*x + c)^3 - 3*tan(d*x + c)))/d
```

mupad [B] time = 8.99, size = 144, normalized size = 1.58

$$\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{-23\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{a}{3}}{d\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^2*sin(c + d*x)^4),x)
```

```
[Out] (7*a*tan(c/2 + (d*x)/2))/(8*d) - (a/3 + (2*a*tan(c/2 + (d*x)/2))/3 + 6*a*ta
n(c/2 + (d*x)/2)^2 - 23*a*tan(c/2 + (d*x)/2)^3)/(d*(8*tan(c/2 + (d*x)/2)^3
- 8*tan(c/2 + (d*x)/2)^4)) + (a*tan(c/2 + (d*x)/2)^2)/(8*d) + (a*tan(c/2 +
(d*x)/2)^3)/(24*d) + (3*a*log(tan(c/2 + (d*x)/2)))/(2*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.759 $\int \sin(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=89

$$-\frac{a^2\cos^3(c+dx)}{3d}+\frac{3a^2\cos(c+dx)}{d}+\frac{3a^2\tan(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\sin^2(c+dx)\tan(c+dx)}{d}-3a^2x$$

```
[Out] -3*a^2*x+3*a^2*cos(d*x+c)/d-1/3*a^2*cos(d*x+c)^3/d+2*a^2*sec(d*x+c)/d+3*a^2
*tan(d*x+c)/d-a^2*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.21, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2873, 2590, 14, 2591, 288, 321, 203, 270}

$$-\frac{a^2\cos^3(c+dx)}{3d}+\frac{3a^2\cos(c+dx)}{d}+\frac{3a^2\tan(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\sin^2(c+dx)\tan(c+dx)}{d}-3a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -3*a^2*x + (3*a^2*Cos[c + d*x])/d - (a^2*Cos[c + d*x]^3)/(3*d) + (2*a^2*Sec
[c + d*x])/d + (3*a^2*Tan[c + d*x])/d - (a^2*Sin[c + d*x]^2*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx = \int \left(a^2\sin(c+dx)\tan^2(c+dx)+2a^2\sin^2(c+dx)\tan^2(c+d\right.$$
$$= a^2\int \sin(c+dx)\tan^2(c+dx)\,dx + a^2\int \sin^3(c+dx)\tan^2(c$$
$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a^2\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2}\,d\right.}{d}$$
$$= -\frac{a^2\sin^2(c+dx)\tan(c+dx)}{d} - \frac{a^2\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x,\right.}{d}$$
$$= \frac{3a^2\cos(c+dx)}{d} - \frac{a^2\cos^3(c+dx)}{3d} + \frac{2a^2\sec(c+dx)}{d} + \frac{3a^2\,\text{t}}{}$$
$$= -3a^2x + \frac{3a^2\cos(c+dx)}{d} - \frac{a^2\cos^3(c+dx)}{3d} + \frac{2a^2\sec(c+dx}{d}$$

Mathematica [A] time = 0.52, size = 161, normalized size = 1.81

$$-\frac{a^2(\sin(c+dx)+1)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)(-6\sin(2(c+dx))-33\cos(c+dx)+\cos(3(c+dx))+36c+36dx)-\sin\right.}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -1/12*(a^2*(1 + Sin[c + d*x])^2*(Cos[(c + d*x)/2]*(36*c + 36*d*x - 33*Cos[c
 + d*x] + Cos[3*(c + d*x)] - 6*Sin[2*(c + d*x)]) - Sin[(c + d*x)/2]*(48 + 3
6*c + 36*d*x - 33*Cos[c + d*x] + Cos[3*(c + d*x)] - 6*Sin[2*(c + d*x)])))/(
d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^4)
```

fricas [A] time = 0.43, size = 152, normalized size = 1.71

$$-\frac{a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^3+9\,a^2dx-9\,a^2\cos\left(dx+c\right)^2-6\,a^2+3\left(3\,a^2dx-4\,a^2\right)\cos\left(dx+c\right)-\left(a^2\,\text{co}\right.}{3\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^3 + 9*a^2*d*x - 9*a^2*cos(d*x
 + c)^2 - 6*a^2 + 3*(3*a^2*d*x - 4*a^2)*cos(d*x + c) - (a^2*cos(d*x + c)^3
+ 9*a^2*d*x + 3*a^2*cos(d*x + c)^2 - 6*a^2*cos(d*x + c) + 6*a^2)*sin(d*x +
c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [A] time = 0.20, size = 119, normalized size = 1.34

$$-\frac{9\,(dx+c)a^2+\frac{12\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-18\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(9*(d*x + c)*a^2 + 12*a^2/(tan(1/2*d*x + 1/2*c) - 1) + 2*(3*a^2*tan(1/
2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^4 - 18*a^2*tan(1/2*d*x + 1/2*
c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c) - 8*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/
d
```

maple [A] time = 0.50, size = 148, normalized size = 1.66

$$\frac{a^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+2a^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d
*x+c))+2*a^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x
+c)-3/2*d*x-3/2*c)+a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)
))
```

maxima [A] time = 0.62, size = 98, normalized size = 1.10

$$-\frac{\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\,\cos(dx+c)\right)a^2+3\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)a^2-3\,a^2\left(\frac{1}{\cos(dx+c)}\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3*((cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^2 + 3*(3*d*x + 3
*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^2 - 3*a^2*(1/cos
(d*x + c) + cos(d*x + c)))/d
```

mupad [B] time = 14.71, size = 288, normalized size = 3.24

$$-3\,a^2\,x-\frac{3\,a^2\,\left(c+d\,x\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^2\,\left(c+d\,x\right)-\frac{a^2\,\left(9\,c+9\,d\,x-10\right)}{3}\right)-\frac{a^2\,\left(9\,c+9\,d\,x-28\right)}{3}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(3\,a^2\,\left(c+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] - 3*a^2*x - (3*a^2*(c + d*x) - tan(c/2 + (d*x)/2)*(3*a^2*(c + d*x) - (a^2*(
9*c + 9*d*x - 10))/3) - (a^2*(9*c + 9*d*x - 28))/3 + tan(c/2 + (d*x)/2)^6*(
3*a^2*(c + d*x) - (a^2*(9*c + 9*d*x - 18))/3) - tan(c/2 + (d*x)/2)^5*(9*a^2
*(c + d*x) - (a^2*(27*c + 27*d*x - 18))/3) - tan(c/2 + (d*x)/2)^3*(9*a^2*(c
 + d*x) - (a^2*(27*c + 27*d*x - 36))/3) + tan(c/2 + (d*x)/2)^4*(9*a^2*(c +
d*x) - (a^2*(27*c + 27*d*x - 48))/3) + tan(c/2 + (d*x)/2)^2*(9*a^2*(c + d*x
) - (a^2*(27*c + 27*d*x - 66))/3))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (
d*x)/2)^2 + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.760 $\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=71

$$\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{5a^2x}{2}$$

```
[Out] -5/2*a^2*x+2*a^2*cos(d*x+c)/d+2*a^2*cos(d*x+c)/d/(1-sin(d*x+c))+1/2*a^2*cos
(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2709, 2648, 2638, 2635, 8}

$$\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{5a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (-5*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (2*a^2*Cos[c + d*x])/(d*(1 - Sin[c
+ d*x])) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx = a^2\int\left(-2-\frac{2}{-1+\sin(c+dx)}-2\sin(c+dx)-\sin^2(c+dx)\right)dx$$
$$= -2a^2x-a^2\int\sin^2(c+dx)\,dx-\left(2a^2\right)\int\frac{1}{-1+\sin(c+dx)}\,dx-\left(2a^2\right)\int$$
$$= -2a^2x+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin(c+a}{2d}$$
$$= -\frac{5a^2x}{2}+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin(c+}{2d}$$

Mathematica [B] time = 0.39, size = 145, normalized size = 2.04

$$-\frac{a^2(\sin(c+dx)+1)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)(10(c+dx)-\sin(2(c+dx))-8\cos(c+dx))+\sin\left(\frac{1}{2}(c+dx)\right)(-2(5c+5a\right.}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -1/4*(a^2*(1 + Sin[c + d*x])^2*(Cos[(c + d*x)/2]*(10*(c + d*x) - 8*Cos[c +
d*x] - Sin[2*(c + d*x)]) + Sin[(c + d*x)/2]*(-2*(8 + 5*c + 5*d*x) + 8*Cos[c
 + d*x] + Sin[2*(c + d*x)])))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

fricas [A] time = 0.47, size = 125, normalized size = 1.76

$$\frac{a^2\cos(dx+c)^3-5a^2dx+4a^2\cos(dx+c)^2+4a^2-\left(5a^2dx-7a^2\right)\cos(dx+c)+\left(5a^2dx+a^2\cos(dx+c)^2-3\right.}{2(d\cos(dx+c)-d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/2*(a^2*cos(d*x + c)^3 - 5*a^2*d*x + 4*a^2*cos(d*x + c)^2 + 4*a^2 - (5*a^2
*d*x - 7*a^2)*cos(d*x + c) + (5*a^2*d*x + a^2*cos(d*x + c)^2 - 3*a^2*cos(d*
x + c) + 4*a^2)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [A] time = 0.20, size = 102, normalized size = 1.44

$$-\frac{5\,(dx+c)a^2+\frac{8\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*(d*x + c)*a^2 + 8*a^2/(tan(1/2*d*x + 1/2*c) - 1) + 2*(a^2*tan(1/2*d
*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*x + 1/2*c) - 4
*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.42, size = 117, normalized size = 1.65

$$\frac{a^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-
3/2*d*x-3/2*c)+2*a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+
a^2*(tan(d*x+c)-d*x-c))
```

maxima [A] time = 0.52, size = 84, normalized size = 1.18

$$-\frac{\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)a^2+2\,(dx+c-\tan(dx+c))a^2-4\,a^2\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^
2 + 2*(d*x + c - tan(d*x + c))*a^2 - 4*a^2*(1/cos(d*x + c) + cos(d*x + c)))
/d
```

mupad [B] time = 11.39, size = 183, normalized size = 2.58

$$-\frac{5\,a^2\,x}{2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^2\,\left(5\,d\,x-6\right)}{2}-\frac{5\,a^2\,d\,x}{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a^2\,\left(5\,d\,x-10\right)}{2}-\frac{5\,a^2\,d\,x}{2}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{a^2\,\left(10\,d\,x-10\right)}{2}-\dots\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\dots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] - (5*a^2*x)/2 - (tan(c/2 + (d*x)/2)*((a^2*(5*d*x - 6))/2 - (5*a^2*d*x)/2) -
 tan(c/2 + (d*x)/2)^4*((a^2*(5*d*x - 10))/2 - (5*a^2*d*x)/2) + tan(c/2 + (d
*x)/2)^3*((a^2*(10*d*x - 10))/2 - 5*a^2*d*x) - tan(c/2 + (d*x)/2)^2*((a^2*(
10*d*x - 22))/2 - 5*a^2*d*x) - (a^2*(5*d*x - 16))/2 + (5*a^2*d*x)/2)/(d*(ta
n(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sin^2(c+dx)\sec^2(c+dx)\,dx+\int2\sin^3(c+dx)\sec^2(c+dx)\,dx+\int\sin^4(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sin(c + d*x)**2*sec(c + d*x)**2, x) + Integral(2*sin(c + d*x
)**3*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**4*sec(c + d*x)**2, x))
```

3.761 $\int \sec(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=43

$$\frac{2a^2\cos(c+dx)}{d}-2a^2x+\frac{\sec(c+dx)(a\sin(c+dx)+a)^2}{d}$$

```
[Out] -2*a^2*x+2*a^2*cos(d*x+c)/d+sec(d*x+c)*(a+a*sin(d*x+c))^2/d
```

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2855, 2638}

$$\frac{2a^2\cos(c+dx)}{d}-2a^2x+\frac{\sec(c+dx)(a\sin(c+dx)+a)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -2*a^2*x + (2*a^2*Cos[c + d*x])/d + (Sec[c + d*x]*(a + a*Sin[c + d*x])^2)/d
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx &= \frac{\sec(c+dx)(a+a\sin(c+dx))^2}{d}-(2a)\int(a+a\sin(c+dx))\\ &= -2a^2x+\frac{\sec(c+dx)(a+a\sin(c+dx))^2}{d}-\left(2a^2\right)\int\sin(c+\\ &= -2a^2x+\frac{2a^2\cos(c+dx)}{d}+\frac{\sec(c+dx)(a+a\sin(c+dx))^2}{d}\end{aligned}$$

Mathematica [B] time = 0.37, size = 90, normalized size = 2.09

$$\frac{(a\sin(c+dx)+a)^2\left(-2(c+dx)+\cos(c+dx)+\frac{4\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] ((-2*(c + d*x) + Cos[c + d*x] + (4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]))*(a + a*Sin[c + d*x])^2)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^4)
```

fricas [B] time = 0.44, size = 101, normalized size = 2.35

$$-\frac{2\,a^2dx-a^2\cos\left(dx+c\right)^2-2\,a^2+\left(2\,a^2dx-3\,a^2\right)\cos\left(dx+c\right)-\left(2\,a^2dx-a^2\cos\left(dx+c\right)+2\,a^2\right)\sin\left(dx+c\right)}{d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*d*x - a^2*cos(d*x + c)^2 - 2*a^2 + (2*a^2*d*x - 3*a^2)*cos(d*x + c)
 - (2*a^2*d*x - a^2*cos(d*x + c) + 2*a^2)*sin(d*x + c))/(d*cos(d*x + c) - d
*sin(d*x + c) + d)
```

giac [B] time = 0.19, size = 89, normalized size = 2.07

$$-\frac{2\left((dx+c)a^2+\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((d*x + c)*a^2 + (2*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*x + 1/2*c
) + 3*a^2)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x +
 1/2*c) - 1))/d
```

maple [A] time = 0.32, size = 76, normalized size = 1.77

$$\frac{a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2\left(dx+c\right)\right)\cos\left(dx+c\right)\right)+2a^2\left(\tan\left(dx+c\right)-dx-c\right)+\frac{a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(tan(d
*x+c)-d*x-c)+a^2/cos(d*x+c))
```

maxima [A] time = 1.07, size = 57, normalized size = 1.33

$$-\frac{2\,(dx+c-\tan(dx+c))a^2 - a^2\left(\frac{1}{\cos(dx+c)} + \cos(dx+c)\right) - \frac{a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*(d*x + c - tan(d*x + c))*a^2 - a^2*(1/cos(d*x + c) + cos(d*x + c)) - a^
2/cos(d*x + c))/d
```

mupad [B] time = 9.18, size = 117, normalized size = 2.72

$$-2\,a^2\,x-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2\,\left(d\,x-1\right)-2\,a^2\,d\,x\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2\,\left(d\,x-2\right)-2\,a^2\,d\,x\right)-2\,a^2\,\left(d\,x-3\right)+2\,a^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] - 2*a^2*x - (tan(c/2 + (d*x)/2)*(2*a^2*(d*x - 1) - 2*a^2*d*x) - tan(c/2 + (
d*x)/2)^2*(2*a^2*(d*x - 2) - 2*a^2*d*x) - 2*a^2*(d*x - 3) + 2*a^2*d*x)/(d*(
tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \sin(c+dx)\sec^2(c+dx)\,dx + \int 2\sin^2(c+dx)\sec^2(c+dx)\,dx + \int \sin^3(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sin(c + d*x)*sec(c + d*x)**2, x) + Integral(2*sin(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**3*sec(c + d*x)**2, x))
```

3.762 $\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=44

$$\frac{2a^2\tan(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -a^2*arctanh(cos(d*x+c))/d+2*a^2*sec(d*x+c)/d+2*a^2*tan(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2873, 3767, 8, 2622, 321, 207, 2606}

$$\frac{2a^2\tan(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*ArcTanh[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d + (2*a^2*Tan[c +
d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(2a^2\sec^2(c+dx)+a^2\csc(c+dx)\sec^2(c+dx)+a^2\sec(\right.\\ &= a^2\int \csc(c+dx)\sec^2(c+dx)\,dx+a^2\int \sec(c+dx)\tan(c-\\ &= \frac{a^2\,\text{Subst}(\int 1\,dx,x,\sec(c+dx))}{d}+\frac{a^2\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,s\right.}{d}\\ &= \frac{2a^2\sec(c+dx)}{d}+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,\right.}{d}\\ &= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}+\frac{2a^2\tan(c+a}{d}\end{aligned}$$

Mathematica [A] time = 0.11, size = 69, normalized size = 1.57

$$\frac{a^2\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{4\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]] + (4*Sin[(c + d*x)/2])
/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])))/d
```

fricas [B] time = 0.46, size = 126, normalized size = 2.86

$$\frac{4\,a^2\cos\left(dx+c\right)+4\,a^2\sin\left(dx+c\right)+4\,a^2-\left(a^2\cos\left(dx+c\right)-a^2\sin\left(dx+c\right)+a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a^2\right.}{2\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*cos(d*x + c) + 4*a^2*sin(d*x + c) + 4*a^2 - (a^2*cos(d*x + c) -
a^2*sin(d*x + c) + a^2)*log(1/2*cos(d*x + c) + 1/2) + (a^2*cos(d*x + c) - a
^2*sin(d*x + c) + a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c) - d*si
n(d*x + c) + d)
```

giac [A] time = 0.20, size = 38, normalized size = 0.86

$$\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{4\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 4*a^2/(tan(1/2*d*x + 1/2*c) - 1))/d
```

maple [A] time = 0.55, size = 55, normalized size = 1.25

$$\frac{2a^2}{d\cos\left(dx+c\right)}+\frac{2a^2\tan\left(dx+c\right)}{d}+\frac{a^2\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/d*a^2/cos(d*x+c)+2*a^2*tan(d*x+c)/d+1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.31, size = 65, normalized size = 1.48

$$\frac{a^2\left(\frac{2}{\cos\left(dx+c\right)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+4\,a^2\tan\left(dx+c\right)+\frac{2\,a^2}{\cos\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) +
 4*a^2*tan(d*x + c) + 2*a^2/cos(d*x + c))/d
```

mupad [B] time = 8.89, size = 39, normalized size = 0.89

$$\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{4\,a^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^2*sin(c + d*x)),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d - (4*a^2)/(d*(tan(c/2 + (d*x)/2) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.763 $\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=58

$$\frac{2a^2\tan(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -2*a^2*arctanh(cos(d*x+c))/d-a^2*cot(d*x+c)/d+2*a^2*sec(d*x+c)/d+2*a^2*tan(
d*x+c)/d
```

Rubi [A] time = 0.22, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2873, 3767, 8, 2622, 321, 207, 2620, 14}

$$\frac{2a^2\tan(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*ArcTanh[Cos[c + d*x]])/d - (a^2*Cot[c + d*x])/d + (2*a^2*Sec[c + d*
x])/d + (2*a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\sec^2(c+dx)+2a^2\csc(c+dx)\sec^2(c+dx)+a^2\csc\right.$$

$$= a^2\int \sec^2(c+dx)\,dx + a^2\int \csc^2(c+dx)\sec^2(c+dx)\,dx$$

$$= -\frac{a^2\,\mathrm{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1+x^2}{x^2}\,dx\right.}{d}$$

$$= \frac{2a^2\sec(c+dx)}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{a^2\,\mathrm{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\right.}{d}$$

$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cot(c+dx)}{d} + \frac{2a^2\sec(c+}{d}$$

Mathematica [A] time = 0.39, size = 96, normalized size = 1.66

$$\frac{a^2\left(\tan\left(\frac{1}{2}(c+dx)\right)-\cot\left(\frac{1}{2}(c+dx)\right)+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{8\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-Cot[(c + d*x)/2] - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]]
 + (8*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + Tan[(c + d*
x)/2]))/(2*d)
```

fricas [B] time = 0.45, size = 192, normalized size = 3.31

$$-\frac{3a^2\cos(dx+c)^2+a^2\cos(dx+c)-2a^2+\left(a^2\cos(dx+c)^2-a^2+\left(a^2\cos(dx+c)+a^2\right)\sin(dx+c)\right)\log\left(\frac{1}{2}\,c\right.}{d\cos(dx\,-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(3*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) - 2*a^2 + (a^2*cos(d*x + c)^2 - a
^2 + (a^2*cos(d*x + c) + a^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (
a^2*cos(d*x + c)^2 - a^2 + (a^2*cos(d*x + c) + a^2)*sin(d*x + c))*log(-1/2*
cos(d*x + c) + 1/2) - (3*a^2*cos(d*x + c) + 2*a^2)*sin(d*x + c))/(d*cos(d*x
 + c)^2 + (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

giac [A] time = 0.20, size = 98, normalized size = 1.69

$$\frac{4a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)+a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{2a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+7a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^2}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + a^2*tan(1/2*d*x + 1/2*c) - (2*a
^2*tan(1/2*d*x + 1/2*c)^2 + 7*a^2*tan(1/2*d*x + 1/2*c) - a^2)/(tan(1/2*d*x
+ 1/2*c)^2 - tan(1/2*d*x + 1/2*c)))/d
```

maple [A] time = 0.60, size = 92, normalized size = 1.59

$$\frac{a^2\tan(dx+c)}{d}+\frac{2a^2}{d\cos(dx+c)}+\frac{2a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{a^2}{d\sin(dx+c)\cos(dx+c)}-\frac{2a^2\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] a^2*tan(d*x+c)/d+2/d*a^2/cos(d*x+c)+2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+1/d*a
^2/sin(d*x+c)/cos(d*x+c)-2*a^2*cot(d*x+c)/d
```

maxima [A] time = 0.48, size = 72, normalized size = 1.24

$$\frac{a^2\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-a^2\left(\frac{1}{\tan(dx+c)}-\tan(dx+c)\right)+a^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] (a^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - a^2
*(1/tan(d*x + c) - tan(d*x + c)) + a^2*tan(d*x + c))/d
```

mupad [B] time = 8.93, size = 86, normalized size = 1.48

$$\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2-9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^2*sin(c + d*x)^2),x)
```

```
[Out] (2*a^2*log(tan(c/2 + (d*x)/2)))/d - (a^2 - 9*a^2*tan(c/2 + (d*x)/2))/(d*(2*
tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2)) + (a^2*tan(c/2 + (d*x)/2))/(2
*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.764 $\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=86

$$\frac{2a^2\tan(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}+\frac{5a^2\sec(c+dx)}{2d}-\frac{5a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d}$$

```
[Out] -5/2*a^2*arctanh(cos(d*x+c))/d-2*a^2*cot(d*x+c)/d+5/2*a^2*sec(d*x+c)/d-1/2*
a^2*csc(d*x+c)^2*sec(d*x+c)/d+2*a^2*tan(d*x+c)/d
```

Rubi [A] time = 0.21, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2622, 321, 207, 2620, 14, 288}

$$\frac{2a^2\tan(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}+\frac{5a^2\sec(c+dx)}{2d}-\frac{5a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (2*a^2*Cot[c + d*x])/d + (5*a^2*Sec[
c + d*x])/(2*d) - (a^2*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (2*a^2*Tan[c +
d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\csc(c+dx)\sec^2(c+dx)+2a^2\csc^2(c+dx)\sec^2(c+a\right.$$

$$= a^2\int \csc(c+dx)\sec^2(c+dx)\,dx + a^2\int \csc^3(c+dx)\sec^2(c$$

$$= \frac{a^2\operatorname{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\right.}{}$$

$$= \frac{a^2\sec(c+dx)}{d} - \frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{a^2\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{2a^2\cot(c+dx)}{d} + \frac{5a^2\sec(c+dx}{2d}$$

$$= -\frac{5a^2\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a^2\cot(c+dx)}{d} + \frac{5a^2\sec(c+}{2d}$$

Mathematica [A] time = 1.10, size = 124, normalized size = 1.44

$$\frac{a^2\left(8\tan\left(\frac{1}{2}(c+dx)\right)-8\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)+20\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-20\log\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-8*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 - 20*Log[Cos[(c + d*x)/2]] +
 20*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 + (32*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]) + 8*Tan[(c + d*x)/2]))/(8*d)
```

fricas [B] time = 0.45, size = 300, normalized size = 3.49

$$\frac{16\,a^2\cos(dx+c)^3+10\,a^2\cos(dx+c)^2-14\,a^2\cos(dx+c)-8\,a^2-5\left(a^2\cos(dx+c)^3+a^2\cos(dx+c)^2-a^2\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4*(16*a^2*cos(d*x + c)^3 + 10*a^2*cos(d*x + c)^2 - 14*a^2*cos(d*x + c) -
8*a^2 - 5*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - a^2
```

```
 - (a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 5
*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - a^2 - (a^2*c
os(d*x + c)^2 - a^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(8*a^2*
cos(d*x + c)^2 + 3*a^2*cos(d*x + c) - 4*a^2)*sin(d*x + c))/(d*cos(d*x + c)^
3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - (d*cos(d*x + c)^2 - d)*sin(d*x + c)
 - d)
```

giac [A] time = 0.23, size = 116, normalized size = 1.35

$$\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+20a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)+8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{32a^2}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}-\frac{30a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+8}{\tan\left(\frac{1}{2}dx\right.}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 + 20*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 8
*a^2*tan(1/2*d*x + 1/2*c) - 32*a^2/(tan(1/2*d*x + 1/2*c) - 1) - (30*a^2*tan
(1/2*d*x + 1/2*c)^2 + 8*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c
)^2)/d
```

maple [A] time = 0.60, size = 104, normalized size = 1.21

$$\frac{5a^2}{2d\cos(dx+c)}+\frac{5a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d}+\frac{2a^2}{d\sin(dx+c)\cos(dx+c)}-\frac{4a^2\cot(dx+c)}{d}-\frac{}{2d\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 5/2/d*a^2/cos(d*x+c)+5/2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+2/d*a^2/sin(d*x+c)
/cos(d*x+c)-4*a^2*cot(d*x+c)/d-1/2/d*a^2/sin(d*x+c)^2/cos(d*x+c)
```

maxima [A] time = 0.33, size = 124, normalized size = 1.44

$$\frac{a^2\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)+2a^2\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*(a^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(
cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 2*a^2*(2/cos(d*x + c) - log(
```

```
cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 8*a^2*(1/tan(d*x + c) - tan(d*
x + c)))/d
```

mupad [B] time = 8.96, size = 124, normalized size = 1.44

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d}+\frac{5a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{-20a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{7a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}+\frac{a^2}{2}}{d\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^2*sin(c + d*x)^3),x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (5*a^2*log(tan(c/2 + (d*x)/2)))/(2*d) +
(a^2*tan(c/2 + (d*x)/2))/d - (a^2/2 - 20*a^2*tan(c/2 + (d*x)/2)^2 + (7*a^2*
tan(c/2 + (d*x)/2))/2)/(d*(4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^3)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.765 $\int \sin(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=111

$$-\frac{a^3\cos^3(c+dx)}{d}+\frac{7a^3\cos(c+dx)}{d}+\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{19a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}$$

```
[Out] -51/8*a^3*x+7*a^3*cos(d*x+c)/d-a^3*cos(d*x+c)^3/d+4*a^3*cos(d*x+c)/d/(1-sin
(d*x+c))+19/8*a^3*cos(d*x+c)*sin(d*x+c)/d+1/4*a^3*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.17, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2872, 2648, 2638, 2635, 8, 2633}

$$-\frac{a^3\cos^3(c+dx)}{d}+\frac{7a^3\cos(c+dx)}{d}+\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{19a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (-51*a^3*x)/8 + (7*a^3*Cos[c + d*x])/d - (a^3*Cos[c + d*x]^3)/d + (4*a^3*Co
s[c + d*x])/(d*(1 - Sin[c + d*x])) + (19*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*
d) + (a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = a^2\int\left(-4a-\frac{4a}{-1+\sin(c+dx)}-4a\sin(c+dx)-4a\sin^2(c+\right.$$

$$= -4a^3x - a^3\int\sin^4(c+dx)\,dx - \left(3a^3\right)\int\sin^3(c+dx)\,dx - \left(4\right.$$

$$= -4a^3x + \frac{4a^3\cos(c+dx)}{d} + \frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))} + \frac{2a^3\cos(c+}{}$$

$$= -6a^3x + \frac{7a^3\cos(c+dx)}{d} - \frac{a^3\cos^3(c+dx)}{d} + \frac{4a^3\cos(c+d}{d(1-\sin(c+d}$$

$$= -\frac{51a^3x}{8} + \frac{7a^3\cos(c+dx)}{d} - \frac{a^3\cos^3(c+dx)}{d} + \frac{4a^3\cos(c+}{d(1-\sin(c+}$$

Mathematica [A] time = 0.79, size = 125, normalized size = 1.13

$$\frac{(a\sin(c+dx)+a)^3\left(-204(c+dx)+40\sin(2(c+dx))-\sin(4(c+dx))+200\cos(c+dx)-8\cos(3(c+dx))+\frac{}{\cos}\right.}{32d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-204*(c + d*x) + 200*Cos[c + d*x] - 8*Cos[3*(c + d
*x)] + (256*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 40*Si
```

```
n[2*(c + d*x)] - Sin[4*(c + d*x)]))/(32*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^6)
```

fricas [A] time = 0.44, size = 178, normalized size = 1.60

$$-\frac{2\,a^{3}\cos\left(dx+c\right)^{5}+8\,a^{3}\cos\left(dx+c\right)^{4}-15\,a^{3}\cos\left(dx+c\right)^{3}+51\,a^{3}dx-56\,a^{3}\cos\left(dx+c\right)^{2}-32\,a^{3}+\left(51\,a^{3}dx-\right.}{8\,(d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/8*(2*a^3*cos(d*x + c)^5 + 8*a^3*cos(d*x + c)^4 - 15*a^3*cos(d*x + c)^3 +
 51*a^3*d*x - 56*a^3*cos(d*x + c)^2 - 32*a^3 + (51*a^3*d*x - 67*a^3)*cos(d*
x + c) + (2*a^3*cos(d*x + c)^4 - 6*a^3*cos(d*x + c)^3 - 51*a^3*d*x - 21*a^3
*cos(d*x + c)^2 + 35*a^3*cos(d*x + c) - 32*a^3)*sin(d*x + c))/(d*cos(d*x +
c) - d*sin(d*x + c) + d)
```

giac [A] time = 0.21, size = 167, normalized size = 1.50

$$-\frac{51\,(dx+c)a^{3}+\frac{64\,a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\left(19\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-32\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+27\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-144\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-27\,a^{3}\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(51*(d*x + c)*a^3 + 64*a^3/(tan(1/2*d*x + 1/2*c) - 1) + 2*(19*a^3*tan(
1/2*d*x + 1/2*c)^7 - 32*a^3*tan(1/2*d*x + 1/2*c)^6 + 27*a^3*tan(1/2*d*x + 1
/2*c)^5 - 144*a^3*tan(1/2*d*x + 1/2*c)^4 - 27*a^3*tan(1/2*d*x + 1/2*c)^3 -
160*a^3*tan(1/2*d*x + 1/2*c)^2 - 19*a^3*tan(1/2*d*x + 1/2*c) - 48*a^3)/(tan
(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [B] time = 0.60, size = 212, normalized size = 1.91

$$\frac{a^{3}\left(\frac{\sin^{7}(dx+c)}{\cos(dx+c)}+\left(\sin^{5}\left(dx+c\right)+\frac{5\left(\sin^{3}(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos\left(dx+c\right)-\frac{15dx}{8}-\frac{15c}{8}\right)+3a^{3}\left(\frac{\sin^{6}(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^{4}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d
*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+3*a^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(
d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(
d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+a^3*(sin(d*x+c)^4/cos(d*
x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 0.41, size = 162, normalized size = 1.46

$$-\frac{8\left(\cos\left(dx+c\right)^{3}-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)a^{3}+\left(15\,dx+15\,c-\frac{9\,\tan(dx+c)^{3}+7\,\tan(dx+c)}{\tan(dx+c)^{4}+2\,\tan(dx+c)^{2}+1}-8\,\tan\left(dx+c\right)\right)a^{3}+12}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/8*(8*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^3 + (15*d*x +
15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)
^2 + 1) - 8*tan(d*x + c))*a^3 + 12*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c
)^2 + 1) - 2*tan(d*x + c))*a^3 - 8*a^3*(1/cos(d*x + c) + cos(d*x + c)))/d
```

mupad [B] time = 14.70, size = 363, normalized size = 3.27

$$-\frac{51\,a^3\,x}{8}-\frac{\frac{51\,a^3\,(c+d\,x)}{8}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{51\,a^3\,(c+d\,x)}{8}-\frac{a^3\,(51\,c+51\,d\,x-58)}{8}\right)-\frac{a^3\,(51\,c+51\,d\,x-160)}{8}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(\frac{51\,a^3\,(c+d\,x)}{8}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] - (51*a^3*x)/8 - ((51*a^3*(c + d*x))/8 - tan(c/2 + (d*x)/2)*((51*a^3*(c + d
*x))/8 - (a^3*(51*c + 51*d*x - 58))/8) - (a^3*(51*c + 51*d*x - 160))/8 + ta
n(c/2 + (d*x)/2)^8*((51*a^3*(c + d*x))/8 - (a^3*(51*c + 51*d*x - 102))/8) -
 tan(c/2 + (d*x)/2)^7*((51*a^3*(c + d*x))/2 - (a^3*(204*c + 204*d*x - 102))
/8) - tan(c/2 + (d*x)/2)^3*((51*a^3*(c + d*x))/2 - (a^3*(204*c + 204*d*x -
266))/8) + tan(c/2 + (d*x)/2)^6*((51*a^3*(c + d*x))/2 - (a^3*(204*c + 204*d
*x - 374))/8) + tan(c/2 + (d*x)/2)^2*((51*a^3*(c + d*x))/2 - (a^3*(204*c +
204*d*x - 538))/8) - tan(c/2 + (d*x)/2)^5*((153*a^3*(c + d*x))/4 - (a^3*(30
6*c + 306*d*x - 342))/8) + tan(c/2 + (d*x)/2)^4*((153*a^3*(c + d*x))/4 - (a
^3*(306*c + 306*d*x - 618))/8))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x
)/2)^2 + 1)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.766 $\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=89

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3x}{2}$$

```
[Out] -11/2*a^3*x+5*a^3*cos(d*x+c)/d-1/3*a^3*cos(d*x+c)^3/d+4*a^3*cos(d*x+c)/d/(1
-sin(d*x+c))+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 2648, 2638, 2635, 8, 2633}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (-11*a^3*x)/2 + (5*a^3*Cos[c + d*x])/d - (a^3*Cos[c + d*x]^3)/(3*d) + (4*a^
3*Cos[c + d*x])/(d*(1 - Sin[c + d*x])) + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/
(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = a^2\int\left(-4a-\frac{4a}{-1+\sin(c+dx)}-4a\sin(c+dx)-3a\sin^2(c+dx)-a\,\text{si}\right.$$

$$= -4a^3x-a^3\int\sin^3(c+dx)\,dx-\left(3a^3\right)\int\sin^2(c+dx)\,dx-\left(4a^3\right)\int\frac{}{-1}$$

$$= -4a^3x+\frac{4a^3\cos(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{3a^3\cos(c+dx)\sin(c}{2d}$$

$$= -\frac{11a^3x}{2}+\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{3a}{}$$

Mathematica [A] time = 0.48, size = 115, normalized size = 1.29

$$\frac{(a\sin(c+dx)+a)^3\left(-66(c+dx)+9\sin(2(c+dx))+57\cos(c+dx)-\cos(3(c+dx))+\frac{96\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{12d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-66*(c + d*x) + 57*Cos[c + d*x] - Cos[3*(c + d*x)]
 + (96*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 9*Sin[2*(c
 + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.45, size = 154, normalized size = 1.73

$$-\frac{2\,a^3\cos\left(dx+c\right)^4-7\,a^3\cos\left(dx+c\right)^3+33\,a^3dx-30\,a^3\cos\left(dx+c\right)^2-24\,a^3+3\left(11\,a^3dx-15\,a^3\right)\cos\left(dx+c\right)-}{6\left(d\cos\left(dx+c\right)-d\sin\left(dx-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(2*a^3*cos(d*x + c)^4 - 7*a^3*cos(d*x + c)^3 + 33*a^3*d*x - 30*a^3*cos
(d*x + c)^2 - 24*a^3 + 3*(11*a^3*d*x - 15*a^3)*cos(d*x + c) - (2*a^3*cos(d*
x + c)^3 + 33*a^3*d*x + 9*a^3*cos(d*x + c)^2 - 21*a^3*cos(d*x + c) + 24*a^3
)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [A] time = 0.22, size = 119, normalized size = 1.34

$$-\frac{33\,(dx+c)a^3+\frac{48\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-60\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-28\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(33*(d*x + c)*a^3 + 48*a^3/(tan(1/2*d*x + 1/2*c) - 1) + 2*(9*a^3*tan(1
/2*d*x + 1/2*c)^5 - 24*a^3*tan(1/2*d*x + 1/2*c)^4 - 60*a^3*tan(1/2*d*x + 1/
2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) - 28*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
3)/d
```

maple [A] time = 0.51, size = 167, normalized size = 1.88

$$\frac{a^3\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4\left(dx+c\right)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos\left(dx+c\right)\right)+3a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3\left(dx+c\right)+\frac{3\sin(dx+c)}{2}\right)\cos\left(\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d
*x+c))+3*a^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x
+c)-3/2*d*x-3/2*c)+3*a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+
c))+a^3*(tan(d*x+c)-d*x-c))
```

maxima [A] time = 0.42, size = 117, normalized size = 1.31

$$-\frac{2\left(\cos\left(dx+c\right)^{3}-\frac{3}{\cos(dx+c)}-6\cos\left(dx+c\right)\right)a^{3}+9\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^{2}+1}-2\tan\left(dx+c\right)\right)a^{3}+6\left(dx+c-t\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^3 + 9*(3*d*x +
 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^3 + 6*(d*x + c
 - tan(d*x + c))*a^3 - 18*a^3*(1/cos(d*x + c) + cos(d*x + c)))/d
```

mupad [B] time = 14.76, size = 288, normalized size = 3.24

$$-\frac{11\,a^3\,x}{2}-\frac{\frac{11\,a^3\,(c+d\,x)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{11\,a^3\,(c+d\,x)}{2}-\frac{a^3\,(33\,c+33\,d\,x-38)}{6}\right)-\frac{a^3\,(33\,c+33\,d\,x-104)}{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{11\,a^3\,(c+d\right.}{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] - (11*a^3*x)/2 - ((11*a^3*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((11*a^3*(c + d
*x))/2 - (a^3*(33*c + 33*d*x - 38))/6) - (a^3*(33*c + 33*d*x - 104))/6 + ta
n(c/2 + (d*x)/2)^6*((11*a^3*(c + d*x))/2 - (a^3*(33*c + 33*d*x - 66))/6) -
tan(c/2 + (d*x)/2)^5*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 66))/6)
- tan(c/2 + (d*x)/2)^3*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 120))/
6) + tan(c/2 + (d*x)/2)^4*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 192
))/6) + tan(c/2 + (d*x)/2)^2*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x -
246))/6))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.767 $\int \sec(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=67

$$\frac{6a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{9a^3x}{2}+\frac{\sec(c+dx)(a\sin(c+dx)+a)^3}{d}$$

```
[Out] -9/2*a^3*x+6*a^3*cos(d*x+c)/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d+sec(d*x+c)*(a
+a*sin(d*x+c))^3/d
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2855, 2644}

$$\frac{6a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{9a^3x}{2}+\frac{\sec(c+dx)(a\sin(c+dx)+a)^3}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (-9*a^3*x)/2 + (6*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(
2*d) + (Sec[c + d*x]*(a + a*Sin[c + d*x])^3)/d
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx &= \frac{\sec(c+dx)(a+a\sin(c+dx))^3}{d}-(3a)\int (a+a\sin(c+dx))^2\,d\\ &= -\frac{9a^3x}{2}+\frac{6a^3\cos(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{\sec}{}\end{aligned}$$

Mathematica [B] time = 0.50, size = 145, normalized size = 2.16

$$-\frac{a^3(\sin(c+dx)+1)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)(18(c+dx)-\sin(2(c+dx))-12\cos(c+dx))+\sin\left(\frac{1}{2}(c+dx)\right)(-2(9c+\right.}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -1/4*(a^3*(1 + Sin[c + d*x])^3*(Cos[(c + d*x)/2]*(18*(c + d*x) - 12*Cos[c +
 d*x] - Sin[2*(c + d*x)]) + Sin[(c + d*x)/2]*(-2*(16 + 9*c + 9*d*x) + 12*Co
s[c + d*x] + Sin[2*(c + d*x)])))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.44, size = 125, normalized size = 1.87

$$\frac{a^3\cos(dx+c)^3-9a^3dx+6a^3\cos(dx+c)^2+8a^3-\left(9a^3dx-13a^3\right)\cos(dx+c)+\left(9a^3dx+a^3\cos(dx+c)^2\right.}{2\left(d\cos(dx+c)-d\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*cos(d*x + c)^3 - 9*a^3*d*x + 6*a^3*cos(d*x + c)^2 + 8*a^3 - (9*a^3
*d*x - 13*a^3)*cos(d*x + c) + (9*a^3*d*x + a^3*cos(d*x + c)^2 - 5*a^3*cos(d
*x + c) + 8*a^3)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [A] time = 0.18, size = 102, normalized size = 1.52

$$-\frac{9(dx+c)a^3+\frac{16a^3}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}+\frac{2\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-6a^3\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(9*(d*x + c)*a^3 + 16*a^3/(tan(1/2*d*x + 1/2*c) - 1) + 2*(a^3*tan(1/2*
d*x + 1/2*c)^3 - 6*a^3*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) -
6*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [B] time = 0.42, size = 130, normalized size = 1.94

$$\frac{a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+3a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-
3/2*d*x-3/2*c)+3*a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+
3*a^3*(tan(d*x+c)-d*x-c)+a^3/cos(d*x+c))
```

maxima [A] time = 0.42, size = 97, normalized size = 1.45

$$-\frac{\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)a^3+6\left(dx+c-\tan\left(dx+c\right)\right)a^3-6\,a^3\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)-\frac{2}{\cos(\ldots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^
3 + 6*(d*x + c - tan(d*x + c))*a^3 - 6*a^3*(1/cos(d*x + c) + cos(d*x + c))
- 2*a^3/cos(d*x + c))/d
```

mupad [B] time = 11.39, size = 183, normalized size = 2.73

$$-\frac{9\,a^3\,x}{2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{a^3\,(9\,d\,x-10)}{2}-\frac{9\,a^3\,d\,x}{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{a^3\,(9\,d\,x-18)}{2}-\frac{9\,a^3\,d\,x}{2}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{a^3\,(18\,d\,x-14)}{2}-\ldots\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] - (9*a^3*x)/2 - (tan(c/2 + (d*x)/2)*((a^3*(9*d*x - 10))/2 - (9*a^3*d*x)/2)
- tan(c/2 + (d*x)/2)^4*((a^3*(9*d*x - 18))/2 - (9*a^3*d*x)/2) + tan(c/2 + (
d*x)/2)^3*((a^3*(18*d*x - 14))/2 - 9*a^3*d*x) - tan(c/2 + (d*x)/2)^2*((a^3*
(18*d*x - 42))/2 - 9*a^3*d*x) - (a^3*(9*d*x - 28))/2 + (9*a^3*d*x)/2)/(d*(t
an(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\sin\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int 3\sin^3\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int \mathrm{s}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sin(c + d*x)*sec(c + d*x)**2, x) + Integral(3*sin(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(3*sin(c + d*x)**3*sec(c + d*x)**2, x) + In
tegral(sin(c + d*x)**4*sec(c + d*x)**2, x))
```

3.768 $\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=48

$$\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+a^3(-x)$$

```
[Out] -a^3*x-a^3*arctanh(cos(d*x+c))/d+4*a^3*cos(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2872, 3770, 2648}

$$\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+a^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) - (a^3*ArcTanh[Cos[c + d*x]])/d + (4*a^3*Cos[c + d*x])/(d*(1 - Sin
[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx = a^2\int\left(-a+a\csc(c+dx)-\frac{4a}{-1+\sin(c+dx)}\right)dx$$
$$= -a^3x + a^3\int\csc(c+dx)\,dx - \left(4a^3\right)\int\frac{1}{-1+\sin(c+dx)}\,dx$$
$$= -a^3x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}$$

Mathematica [A] time = 0.14, size = 74, normalized size = 1.54

$$-\frac{a^3\left(-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\frac{8\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+c+dx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*(c + d*x + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]] - (8*Sin[(c
 + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])))/d)
```

fricas [B] time = 0.46, size = 151, normalized size = 3.15

$$-\frac{2\,a^3dx-8\,a^3+2\left(a^3dx-4\,a^3\right)\cos\left(dx+c\right)+\left(a^3\cos\left(dx+c\right)-a^3\sin\left(dx+c\right)+a^3\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(d\cos\left(dx+c\right)-d\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*d*x - 8*a^3 + 2*(a^3*d*x - 4*a^3)*cos(d*x + c) + (a^3*cos(d*x +
 c) - a^3*sin(d*x + c) + a^3)*log(1/2*cos(d*x + c) + 1/2) - (a^3*cos(d*x +
c) - a^3*sin(d*x + c) + a^3)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*d*x + 4*
a^3)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [A] time = 0.19, size = 49, normalized size = 1.02

$$-\frac{(dx+c)a^3-a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{8\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((d*x + c)*a^3 - a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 8*a^3/(tan(1/2*d*x +
 1/2*c) - 1))/d
```

maple [A] time = 0.56, size = 70, normalized size = 1.46

$$-a^3x + \frac{4a^3 \tan(dx+c)}{d} - \frac{a^3c}{d} + \frac{4a^3}{d\cos(dx+c)} + \frac{a^3 \ln(\csc(dx+c) - \cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -a^3*x+4*a^3*tan(d*x+c)/d-1/d*a^3*c+4/d*a^3/cos(d*x+c)+1/d*a^3*ln(csc(d*x+c
)-cot(d*x+c))
```

maxima [A] time = 0.43, size = 84, normalized size = 1.75

$$-\frac{2(dx+c-\tan(dx+c))a^3 - a^3\left(\frac{2}{\cos(dx+c)} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) - 6a^3\tan(dx+c) -}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c - tan(d*x + c))*a^3 - a^3*(2/cos(d*x + c) - log(cos(d*x +
c) + 1) + log(cos(d*x + c) - 1)) - 6*a^3*tan(d*x + c) - 6*a^3/cos(d*x + c))
/d
```

mupad [B] time = 8.93, size = 112, normalized size = 2.33

$$\frac{a^3 \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{2a^3 \operatorname{atan}\left(\frac{4a^6}{4a^6+4a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{4a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^6+4a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} - \frac{8a^3}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d + (2*a^3*atan((4*a^6)/(4*a^6 + 4*a^6*tan(c/
2 + (d*x)/2)) - (4*a^6*tan(c/2 + (d*x)/2))/(4*a^6 + 4*a^6*tan(c/2 + (d*x)/2
))))/d - (8*a^3)/(d*(tan(c/2 + (d*x)/2) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.769 $\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=56

$$-\frac{a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -3*a^3*arctanh(cos(d*x+c))/d-a^3*cot(d*x+c)/d+4*a^3*cos(d*x+c)/d/(1-sin(d*x
+c))
```

Rubi [A] time = 0.14, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2872, 3770, 3767, 8, 2648}

$$-\frac{a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/d - (a^3*Cot[c + d*x])/d + (4*a^3*Cos[c + d*
x])/(d*(1 - Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx = a^2\int\left(3a\csc(c+dx)+a\csc^2(c+dx)-\frac{4a}{-1+\sin(c+dx)}\right.$$

$$= a^3\int\csc^2(c+dx)\,dx+\left(3a^3\right)\int\csc(c+dx)\,dx-\left(4a^3\right)\int$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{a^3\,\mathrm{Subs}}{}$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+}{d(1-\sin(c}$$

Mathematica [A] time = 0.37, size = 96, normalized size = 1.71

$$\frac{a^3\left(\tan\left(\frac{1}{2}(c+dx)\right)-\cot\left(\frac{1}{2}(c+dx)\right)+6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{16\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-Cot[(c + d*x)/2] - 6*Log[Cos[(c + d*x)/2]] + 6*Log[Sin[(c + d*x)/2]]
 + (16*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + Tan[(c + d
*x)/2]))/(2*d)
```

fricas [B] time = 0.46, size = 194, normalized size = 3.46

$$-\frac{10\,a^3\cos(dx+c)^2+2\,a^3\cos(dx+c)-8\,a^3+3\left(a^3\cos(dx+c)^2-a^3+\left(a^3\cos(dx+c)+a^3\right)\sin(dx+c)\right)\log}{2\left(d\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/2*(10*a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) - 8*a^3 + 3*(a^3*cos(d*x +
 c)^2 - a^3 + (a^3*cos(d*x + c) + a^3)*sin(d*x + c))*log(1/2*cos(d*x + c) +
 1/2) - 3*(a^3*cos(d*x + c)^2 - a^3 + (a^3*cos(d*x + c) + a^3)*sin(d*x + c)
)*log(-1/2*cos(d*x + c) + 1/2) - 2*(5*a^3*cos(d*x + c) + 4*a^3)*sin(d*x + c
))/(d*cos(d*x + c)^2 + (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

giac [A] time = 0.21, size = 98, normalized size = 1.75

$$\frac{6\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+14\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + a^3*tan(1/2*d*x + 1/2*c) - (3*a
^3*tan(1/2*d*x + 1/2*c)^2 + 14*a^3*tan(1/2*d*x + 1/2*c) - a^3)/(tan(1/2*d*x
 + 1/2*c)^2 - tan(1/2*d*x + 1/2*c)))/d
```

maple [A] time = 0.61, size = 93, normalized size = 1.66

$$\frac{4a^{3}}{d\cos\left(dx+c\right)}+\frac{3a^{3}\tan\left(dx+c\right)}{d}+\frac{3a^{3}\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}+\frac{a^{3}}{d\sin\left(dx+c\right)\cos\left(dx+c\right)}-\frac{2a^{3}\cot\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4/d*a^3/cos(d*x+c)+3*a^3*tan(d*x+c)/d+3/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+1/d
*a^3/sin(d*x+c)/cos(d*x+c)-2*a^3*cot(d*x+c)/d
```

maxima [A] time = 0.33, size = 88, normalized size = 1.57

$$\frac{3\,a^{3}\left(\frac{2}{\cos\left(dx+c\right)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-2\,a^{3}\left(\frac{1}{\tan\left(dx+c\right)}-\tan\left(dx+c\right)\right)+6\,a^{3}\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/2*(3*a^3*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1))
 - 2*a^3*(1/tan(d*x + c) - tan(d*x + c)) + 6*a^3*tan(d*x + c) + 2*a^3/cos(d
*x + c))/d
```

mupad [B] time = 8.97, size = 86, normalized size = 1.54

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^3-17\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)^2),x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d - (a^3 - 17*a^3*tan(c/2 + (d*x)/2))/(d*(2
*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2)) + (a^3*tan(c/2 + (d*x)/2))/(
2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.770 $\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=80

$$-\frac{3a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{9a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -9/2*a^3*arctanh(cos(d*x+c))/d-3*a^3*cot(d*x+c)/d-1/2*a^3*cot(d*x+c)*csc(d*
x+c)/d+4*a^3*cos(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A] time = 0.16, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2872, 3770, 3767, 8, 3768, 2648}

$$-\frac{3a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{9a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-9*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c
+ d*x]*Csc[c + d*x])/(2*d) + (4*a^3*Cos[c + d*x])/(d*(1 - Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx = a^2\int \Big(4a\csc(c+dx)+3a\csc^2(c+dx)+a\csc^3(c+dx)-$$

$$= a^3\int \csc^3(c+dx)\,dx+\left(3a^3\right)\int \csc^2(c+dx)\,dx+\left(4a^3\right)\int$$

$$= -\frac{4a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+$$

$$= -\frac{9a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\cot(c+$$

Mathematica [A] time = 1.09, size = 124, normalized size = 1.55

$$\frac{a^3\left(12\tan\left(\frac{1}{2}(c+dx)\right)-12\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)+36\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-12*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 - 36*Log[Cos[(c + d*x)/2]]
+ 36*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 + (64*Sin[(c + d*x)/2])/(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]) + 12*Tan[(c + d*x)/2]))/(8*d)
```

fricas [B] time = 0.47, size = 300, normalized size = 3.75

$$\frac{28\,a^3\cos(dx+c)^3+18\,a^3\cos(dx+c)^2-26\,a^3\cos(dx+c)-16\,a^3-9\left(a^3\cos(dx+c)^3+a^3\cos(dx+c)^2-a^3\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(28*a^3*cos(d*x + c)^3 + 18*a^3*cos(d*x + c)^2 - 26*a^3*cos(d*x + c) -
16*a^3 - 9*(a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - a^
3 - (a^3*cos(d*x + c)^2 - a^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) +
9*(a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - a^3 - (a^3*
cos(d*x + c)^2 - a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(14*a^
3*cos(d*x + c)^2 + 5*a^3*cos(d*x + c) - 8*a^3)*sin(d*x + c))/(d*cos(d*x + c
)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - (d*cos(d*x + c)^2 - d)*sin(d*x +
c) - d)
```

giac [A] time = 0.23, size = 116, normalized size = 1.45

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{64\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}-\frac{54\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}{\tan\left(\frac{1}{2}\,dx\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 1
2*a^3*tan(1/2*d*x + 1/2*c) - 64*a^3/(tan(1/2*d*x + 1/2*c) - 1) - (54*a^3*ta
n(1/2*d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2
*c)^2)/d
```

maple [A] time = 0.70, size = 117, normalized size = 1.46

$$\frac{a^3\tan(dx+c)}{d}+\frac{9a^3}{2d\cos(dx+c)}+\frac{9a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d}+\frac{3a^3}{d\sin(dx+c)\cos(dx+c)}-\frac{6a^3\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] a^3*tan(d*x+c)/d+9/2/d*a^3/cos(d*x+c)+9/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+3
/d*a^3/sin(d*x+c)/cos(d*x+c)-6*a^3*cot(d*x+c)/d-1/2/d*a^3/sin(d*x+c)^2/cos(
d*x+c)
```

maxima [A] time = 0.34, size = 135, normalized size = 1.69

$$\frac{a^3\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)+6a^3\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+\right.\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*(a^3*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(
cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 6*a^3*(2/cos(d*x + c) - log(
cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 12*a^3*(1/tan(d*x + c) - tan(d
*x + c)) + 4*a^3*tan(d*x + c))/d
```

mupad [B] time = 8.95, size = 125, normalized size = 1.56

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{9\,a^3\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}+\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{-38\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{11\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a^3}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)^3),x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (9*a^3*log(tan(c/2 + (d*x)/2)))/(2*d) +
(3*a^3*tan(c/2 + (d*x)/2))/(2*d) - (a^3/2 - 38*a^3*tan(c/2 + (d*x)/2)^2 + (
11*a^3*tan(c/2 + (d*x)/2))/2)/(d*(4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x
)/2)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.771 $\int \csc^4(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=98

$$-\frac{a^3\cot^3(c+dx)}{3d}-\frac{5a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -11/2*a^3*arctanh(cos(d*x+c))/d-5*a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d-3
/2*a^3*cot(d*x+c)*csc(d*x+c)/d+4*a^3*cos(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A] time = 0.18, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2872, 3770, 3767, 8, 3768, 2648}

$$-\frac{a^3\cot^3(c+dx)}{3d}-\frac{5a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a^3*Cot[c + d*x])/d - (a^3*Cot[c
 + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (4*a^3*Cos[c +
 d*x])/(d*(1 - Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^4(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx = a^2\int \Big(4a\csc(c+dx)+4a\csc^2(c+dx)+3a\csc^3(c+dx)+$$

$$= a^3\int \csc^4(c+dx)\,dx+\left(3a^3\right)\int \csc^3(c+dx)\,dx+\left(4a^3\right)\int$$

$$= -\frac{4a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}+$$

$$= -\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{5a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c}{3d}$$

Mathematica [B] time = 6.14, size = 211, normalized size = 2.15

$$a^3\left(\frac{7\tan\left(\frac{1}{2}(c+dx)\right)}{3d}-\frac{7\cot\left(\frac{1}{2}(c+dx)\right)}{3d}-\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{11\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((-7*Cot[(c + d*x)/2])/(3*d) - (3*Csc[(c + d*x)/2]^2)/(8*d) - (Cot[(c +
 d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) - (11*Log[Cos[(c + d*x)/2]])/(2*d) + (1
1*Log[Sin[(c + d*x)/2]])/(2*d) + (3*Sec[(c + d*x)/2]^2)/(8*d) + (8*Sin[(c +
```

```
 d*x)/2])/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (7*Tan[(c + d*x)/2])/
(3*d) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d))
```

fricas [B] time = 0.47, size = 354, normalized size = 3.61

$$-\frac{104\,a^3\cos\left(dx+c\right)^4+38\,a^3\cos\left(dx+c\right)^3-156\,a^3\cos\left(dx+c\right)^2-42\,a^3\cos\left(dx+c\right)+48\,a^3+33\,\left(a^3\cos\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(104*a^3*cos(d*x + c)^4 + 38*a^3*cos(d*x + c)^3 - 156*a^3*cos(d*x + c
)^2 - 42*a^3*cos(d*x + c) + 48*a^3 + 33*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x
 + c)^2 + a^3 + (a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 - a^3*cos(d*x + c)
 - a^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 33*(a^3*cos(d*x + c)^4
- 2*a^3*cos(d*x + c)^2 + a^3 + (a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 - a
^3*cos(d*x + c) - a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(52*a
^3*cos(d*x + c)^3 + 33*a^3*cos(d*x + c)^2 - 45*a^3*cos(d*x + c) - 24*a^3)*s
in(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + (d*cos(d*x + c)^3 + d
*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c) + d)
```

giac [A] time = 0.23, size = 148, normalized size = 1.51

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+132\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+57\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{192\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^3*tan(1/2*d*x + 1/2*c)^2 + 132*a^3*l
og(abs(tan(1/2*d*x + 1/2*c))) + 57*a^3*tan(1/2*d*x + 1/2*c) - 192*a^3/(tan(
1/2*d*x + 1/2*c) - 1) - (242*a^3*tan(1/2*d*x + 1/2*c)^3 + 57*a^3*tan(1/2*d*
x + 1/2*c)^2 + 9*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.59, size = 128, normalized size = 1.31

$$\frac{11a^3}{2d\cos\left(dx+c\right)}+\frac{11a^3\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{2d}+\frac{13a^3}{3d\sin\left(dx+c\right)\cos\left(dx+c\right)}-\frac{26a^3\cot\left(dx+c\right)}{3d}-\frac{}{2d\sin\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 11/2/d*a^3/cos(d*x+c)+11/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+13/3/d*a^3/sin(d
*x+c)/cos(d*x+c)-26/3*a^3*cot(d*x+c)/d-3/2/d*a^3/sin(d*x+c)^2/cos(d*x+c)-1/
3/d*a^3/sin(d*x+c)^3/cos(d*x+c)
```

maxima [A] time = 0.33, size = 160, normalized size = 1.63

$$\frac{9\,a^3\left(\frac{2\left(3\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+6\,a^3\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)\right.\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(9*a^3*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*l
og(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 6*a^3*(2/cos(d*x + c) - l
og(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 36*a^3*(1/tan(d*x + c) - ta
n(d*x + c)) - 4*a^3*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 - 3*tan(d*x + c)
))/d
```

mupad [B] time = 8.97, size = 160, normalized size = 1.63

$$\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{-83\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+16\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{8\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{a^3}{3}}{d\,\left(8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}+\frac{11\,a^3\,\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)^4),x)
```

```
[Out] (3*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^3*tan(c/2 + (d*x)/2)^3)/(24*d) - (1
6*a^3*tan(c/2 + (d*x)/2)^2 - 83*a^3*tan(c/2 + (d*x)/2)^3 + a^3/3 + (8*a^3*t
an(c/2 + (d*x)/2))/3)/(d*(8*tan(c/2 + (d*x)/2)^3 - 8*tan(c/2 + (d*x)/2)^4))
 + (11*a^3*log(tan(c/2 + (d*x)/2)))/(2*d) + (19*a^3*tan(c/2 + (d*x)/2))/(8*
d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.772 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{\cos(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{2\sec(c+dx)}{ad}+\frac{x}{a}$$

```
[Out] x/a+cos(d*x+c)/a/d+2*sec(d*x+c)/a/d-1/3*sec(d*x+c)^3/a/d-tan(d*x+c)/a/d+1/3
*tan(d*x+c)^3/a/d
```

Rubi [A] time = 0.14, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2839, 3473, 8, 2590, 270}

$$\frac{\cos(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{2\sec(c+dx)}{ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + Cos[c + d*x]/(a*d) + (2*Sec[c + d*x])/(a*d) - Sec[c + d*x]^3/(3*a*d)
- Tan[c + d*x]/(a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
```

```
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \tan^4(c+dx)\,dx}{a} - \frac{\int \sin(c+dx)\tan^4(c+dx)\,dx}{a}$$

$$= \frac{\tan^3(c+dx)}{3ad} - \frac{\int \tan^2(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^4}\,dx, x, \cos(c+dx)\right)}{ad}$$

$$= -\frac{\tan(c+dx)}{ad} + \frac{\tan^3(c+dx)}{3ad} + \frac{\int 1\,dx}{a} + \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right) dx, x, \cos(c$$

$$= \frac{x}{a} + \frac{\cos(c+dx)}{ad} + \frac{2\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{3ad} - \frac{\tan(c+dx)}{ad} + \frac{\tan^3(c+dx}{3ad}$$

Mathematica [A] time = 0.39, size = 148, normalized size = 1.78

$$\frac{11\sin(c+dx)+6c\sin(2(c+dx))+6dx\sin(2(c+dx))-11\sin(2(c+dx))+3\sin(3(c+dx))+2(6c+6dx-11)}{12ad(\sin(c+dx)+1)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (18 + 2*(-11 + 6*c + 6*d*x)*Cos[c + d*x] + 14*Cos[2*(c + d*x)] + 11*Sin[c +
 d*x] - 11*Sin[2*(c + d*x)] + 6*c*Sin[2*(c + d*x)] + 6*d*x*Sin[2*(c + d*x)]
 + 3*Sin[3*(c + d*x)])/(12*a*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.46, size = 80, normalized size = 0.96

$$\frac{3\,dx\cos\left(dx+c\right)+7\,\cos\left(dx+c\right)^2+\left(3\,dx\cos\left(dx+c\right)+3\,\cos\left(dx+c\right)^2+2\right)\sin\left(dx+c\right)+1}{3\left(ad\cos\left(dx+c\right)\sin\left(dx+c\right)+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*x*cos(d*x + c) + 7*cos(d*x + c)^2 + (3*d*x*cos(d*x + c) + 3*cos(d*
x + c)^2 + 2)*sin(d*x + c) + 1)/(a*d*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*
x + c))
```

giac [A] time = 0.19, size = 125, normalized size = 1.51

$$\frac{\frac{6\,(dx+c)}{a}-\frac{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)a}+\frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+36\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)/a - 3*(tan(1/2*d*x + 1/2*c)^2 - 4*tan(1/2*d*x + 1/2*c) + 5
)/((tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c)
- 1)*a) + (15*tan(1/2*d*x + 1/2*c)^2 + 36*tan(1/2*d*x + 1/2*c) + 17)/(a*(ta
n(1/2*d*x + 1/2*c) + 1)^3))/d
```

maple [A] time = 0.36, size = 126, normalized size = 1.52

$$-\frac{1}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2}{3ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/a/d/(tan(1/2*d*x+1/2*c)-1)+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)+2/a/d*arctan
(tan(1/2*d*x+1/2*c))-2/3/a/d/(tan(1/2*d*x+1/2*c)+1)^3+1/a/d/(tan(1/2*d*x+1/
2*c)+1)^2+5/2/a/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.43, size = 236, normalized size = 2.84

$$\frac{2\left(\frac{\frac{13\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{2\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{6\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{3\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+8}{a+\frac{2\,a\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{2\,a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}+\frac{3\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*((13*sin(d*x + c)/(cos(d*x + c) + 1) + 2*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 6*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 8)/(a + 2*a*sin(d*x +
c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - a*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 2*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - a*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6) + 3*arctan(sin(d*x + c)/(cos(d*x + c) +
1))/a)/d
```

mupad [B] time = 13.52, size = 129, normalized size = 1.55

$$\frac{x}{a}-\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{26\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{16}{3}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] x/a - ((26*tan(c/2 + (d*x)/2))/3 + (4*tan(c/2 + (d*x)/2)^2)/3 - (4*tan(c/2
+ (d*x)/2)^3)/3 - 4*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^5 + 16/3)/(
a*d*(tan(c/2 + (d*x)/2) + 1)^3*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2 +
 tan(c/2 + (d*x)/2)^3 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin^4(c+dx)\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**4*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

3.773 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=70

$$-\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}-\frac{x}{a}$$

```
[Out] -x/a-sec(d*x+c)/a/d+1/3*sec(d*x+c)^3/a/d+tan(d*x+c)/a/d-1/3*tan(d*x+c)^3/a/
d
```

Rubi [A] time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2839, 2606, 3473, 8}

$$-\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) - Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/(3*a*d) + Tan[c + d*x]/(a*d) -
 Tan[c + d*x]^3/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec(c+dx)\tan^3(c+dx)\,dx}{a} - \frac{\int \tan^4(c+dx)\,dx}{a} \\
&= -\frac{\tan^3(c+dx)}{3ad} + \frac{\int \tan^2(c+dx)\,dx}{a} + \frac{\mathrm{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \sec(c+dx)\right)}{ad} \\
&= -\frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} + \frac{\tan(c+dx)}{ad} - \frac{\tan^3(c+dx)}{3ad} - \frac{\int 1\,dx}{a} \\
&= -\frac{x}{a} - \frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} + \frac{\tan(c+dx)}{ad} - \frac{\tan^3(c+dx)}{3ad}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 111, normalized size = 1.59

$$\frac{-2\sin(c+dx)+4\cos(2(c+dx))+(6c+6dx-5)(\sin(c+dx)+1)\cos(c+dx)}{6ad(\sin(c+dx)+1)\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (4*Cos[2*(c + d*x)] - 2*Sin[c + d*x] + (-5 + 6*c + 6*d*x)*Cos[c + d*x]*(1 +
 Sin[c + d*x]))/(6*a*d*(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.46, size = 70, normalized size = 1.00

$$-\frac{3\,dx\cos\left(dx+c\right)+4\,\cos\left(dx+c\right)^2+\left(3\,dx\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)-2}{3\,\left(ad\cos\left(dx+c\right)\sin\left(dx+c\right)+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*d*x*cos(d*x + c) + 4*cos(d*x + c)^2 + (3*d*x*cos(d*x + c) - 1)*sin(
d*x + c) - 2)/(a*d*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*x + c))
```

giac [A] time = 0.23, size = 77, normalized size = 1.10

$$-\frac{\frac{6\,(dx+c)}{a}+\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)/a + 3/(a*(tan(1/2*d*x + 1/2*c) - 1)) + (9*tan(1/2*d*x + 1
/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) + 11)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

maple [A] time = 0.36, size = 104, normalized size = 1.49

$$-\frac{1}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2}{3ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{1}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{\ }{2ad\left(\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/a/d/(tan(1/2*d*x+1/2*c)-1)-2/a/d*arctan(tan(1/2*d*x+1/2*c))+2/3/a/d/(t
an(1/2*d*x+1/2*c)+1)^3-1/a/d/(tan(1/2*d*x+1/2*c)+1)^2-3/2/a/d/(tan(1/2*d*x+
1/2*c)+1)
```

maxima [B] time = 0.42, size = 154, normalized size = 2.20

$$-\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{3\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+2}{a+\frac{2\,a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{3\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2/3*((sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 - 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2)/(a + 2*a*sin(d*x + c)/(cos
(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4) + 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 11.18, size = 79, normalized size = 1.13

$$\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{4}{3}}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3}-\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2))/3 - 4*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^3
 + 4/3)/(a*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1)^3) - x/a
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.774 $\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] sec(d*x+c)/a/d-1/3*sec(d*x+c)^3/a/d+1/3*tan(d*x+c)^3/a/d
```

Rubi [A] time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2706, 2607, 30, 2606}

$$\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(3*a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^3(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^2\,dx, x, \tan(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \sec(c+dx)\right)}{ad}\\ &= \frac{\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{3ad} + \frac{\tan^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [B] time = 0.14, size = 106, normalized size = 2.12

$$\frac{8\sin(c+dx)-5\sin(2(c+dx))-10\cos(c+dx)+2\cos(2(c+dx))+6}{12ad(\sin(c+dx)+1)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6 - 10*Cos[c + d*x] + 2*Cos[2*(c + d*x)] + 8*Sin[c + d*x] - 5*Sin[2*(c + d
*x)])/(12*a*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.47, size = 47, normalized size = 0.94

$$\frac{\cos\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1}{3\left(ad\cos\left(dx+c\right)\sin\left(dx+c\right)+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^2 + 2*sin(d*x + c) + 1)/(a*d*cos(d*x + c)*sin(d*x + c) +
a*d*cos(d*x + c))
```

giac [A] time = 0.20, size = 68, normalized size = 1.36

$$-\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) - (3*tan(1/2*d*x + 1/2*c)^2 + 12*tan
(1/2*d*x + 1/2*c) + 5)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

maple [A] time = 0.32, size = 70, normalized size = 1.40

$$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{8}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 8/d/a*(-1/16/(tan(1/2*d*x+1/2*c)-1)-1/12/(tan(1/2*d*x+1/2*c)+1)^3+1/8/(tan(
1/2*d*x+1/2*c)+1)^2+1/16/(tan(1/2*d*x+1/2*c)+1))
```

maxima [A] time = 0.32, size = 90, normalized size = 1.80

$$\frac{4\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{3\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4/3*(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x
 + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4)*d)
```

mupad [B] time = 8.90, size = 47, normalized size = 0.94

$$-\frac{4\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{3ad\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] -(4*(2*tan(c/2 + (d*x)/2) + 1))/(3*a*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 +
(d*x)/2) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^2(c+dx)\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

3.775 $\int \frac{\sec(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\sec^3(c+dx)}{3ad}-\frac{\tan^3(c+dx)}{3ad}$$

[Out] 1/3*sec(d*x+c)^3/a/d-1/3*tan(d*x+c)^3/a/d

Rubi [A] time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2839, 2606, 30, 2607}

$$\frac{\sec^3(c+dx)}{3ad}-\frac{\tan^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]

[Out] Sec[c + d*x]^3/(3*a*d) - Tan[c + d*x]^3/(3*a*d)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
```

```
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^3(c+dx)\tan(c+dx)\,dx}{a}-\frac{\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^2\,dx,x,\sec(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\,dx,x,\tan(c+dx)\right)}{ad}\\ &= \frac{\sec^3(c+dx)}{3ad}-\frac{\tan^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [B] time = 0.14, size = 104, normalized size = 2.81

$$\frac{-2\sin(c+dx)+\frac{1}{2}\sin(2(c+dx))+\cos(c+dx)+\cos(2(c+dx))-3}{6ad(\sin(c+dx)+1)\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3 + Cos[c + d*x] + Cos[2*(c + d*x)] - 2*Sin[c + d*x] + Sin[2*(c + d*x)]/2
)/(6*a*d*(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.43, size = 47, normalized size = 1.27

$$-\frac{\cos(dx+c)^2-\sin(dx+c)-2}{3\left(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(d*x + c)^2 - sin(d*x + c) - 2)/(a*d*cos(d*x + c)*sin(d*x + c) + a
*d*cos(d*x + c))
```

giac [A] time = 0.19, size = 57, normalized size = 1.54

$$-\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) - (3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a*
(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

maple [B] time = 0.28, size = 70, normalized size = 1.89

$$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{4}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 4/d/a*(-1/8/(tan(1/2*d*x+1/2*c)-1)+1/6/(tan(1/2*d*x+1/2*c)+1)^3-1/4/(tan(1/
2*d*x+1/2*c)+1)^2+1/8/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.32, size = 110, normalized size = 2.97

$$\frac{2\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)}{3\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*(2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d)
```

mupad [B] time = 8.91, size = 60, normalized size = 1.62

$$-\frac{2\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] -(2*(2*tan(c/2 + (d*x)/2) + 3*tan(c/2 + (d*x)/2)^2 + 1))/(3*a*d*(tan(c/2 +
(d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin(c+dx)\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

3.776 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=79

$$-\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -arctanh(cos(d*x+c))/a/d+sec(d*x+c)/a/d+1/3*sec(d*x+c)^3/a/d-tan(d*x+c)/a/d
-1/3*tan(d*x+c)^3/a/d
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2622, 302, 207, 3767}

$$-\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/(3*a*d
) - Tan[c + d*x]/(a*d) - Tan[c + d*x]^3/(3*a*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \sec^4(c+dx)\,dx}{a}+\frac{\int \csc(c+dx)\sec^4(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{ad}\\ &= -\frac{\tan(c+dx)}{ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx,x,\sec(c+dx)\right)}{ad}\\ &= \frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx \right.}{ad}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}-\frac{\tan^3(c+}{3ad}\end{aligned}$$

Mathematica [A] time = 0.61, size = 149, normalized size = 1.89

$$\frac{6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\left(-\frac{11}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}-\frac{2}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}\right.\right.}\right.}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-6*Log[Cos[(c + d*x)/2]] + 6*Log[Sin[(c + d*x)/2]] + (Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^(-2) + Sin[(c + d*x)/2]*(3/(Cos[(c + d*x)/2] - Sin[(c + d*
```

```
x)/2]) - 2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 11/(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])))/(6*a*d)
```

fricas [A] time = 0.48, size = 115, normalized size = 1.46

$$\frac{4\cos(dx+c)^2 - 3(\cos(dx+c)\sin(dx+c)+\cos(dx+c))\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) + 3(\cos(dx+c)\sin(dx+c)}{6(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(4*cos(d*x + c)^2 - 3*(cos(d*x + c)*sin(d*x + c) + cos(d*x + c))*log(1/
2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)*sin(d*x + c) + cos(d*x + c))*log(-1
/2*cos(d*x + c) + 1/2) + 2*sin(d*x + c) + 4)/(a*d*cos(d*x + c)*sin(d*x + c)
 + a*d*cos(d*x + c))
```

giac [A] time = 0.18, size = 83, normalized size = 1.05

$$\frac{\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a} - \frac{3}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)} + \frac{15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+13}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3/(a*(tan(1/2*d*x + 1/2*c) - 1))
+ (15*tan(1/2*d*x + 1/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) + 13)/(a*(tan(1/2*d*
x + 1/2*c) + 1)^3))/d
```

maple [A] time = 0.42, size = 103, normalized size = 1.30

$$-\frac{1}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2}{3ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{5}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/a/d/(tan(1/2*d*x+1/2*c)-1)+1/a/d*ln(tan(1/2*d*x+1/2*c))+2/3/a/d/(tan(1
/2*d*x+1/2*c)+1)^3-1/a/d/(tan(1/2*d*x+1/2*c)+1)^2+5/2/a/d/(tan(1/2*d*x+1/2*
c)+1)
```

maxima [A] time = 0.32, size = 136, normalized size = 1.72

$$\frac{\frac{2\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+4\right)}{a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(2*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 4)/(a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + 3*log(sin(d*
x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 9.90, size = 92, normalized size = 1.16

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{8}{3}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) + ((10*tan(c/2 + (d*x)/2))/3 - 2*tan(c/2 + (d
*x)/2)^3 + 8/3)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 2*a*tan(c/2 + (d*x)/2)^3 -
 a*tan(c/2 + (d*x)/2)^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc(c+dx)\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

3.777 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{\tan^3(c+dx)}{3ad}+\frac{2\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d-sec(d*x+c)/a/d-1/3*sec(d*x+c)^3/a/d+
2*tan(d*x+c)/a/d+1/3*tan(d*x+c)^3/a/d
```

Rubi [A] time = 0.19, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2620, 270, 2622, 302, 207}

$$\frac{\tan^3(c+dx)}{3ad}+\frac{2\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d) - Sec[c + d*x]/(a*d) - Sec
[c + d*x]^3/(3*a*d) + (2*Tan[c + d*x])/(a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \csc(c+dx)\sec^4(c+dx)\,dx}{a} + \frac{\int \csc^2(c+dx)\sec^4(c+dx)\,dx}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx, x, \tan(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x\right.}{ad}$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{3ad} + \frac{2\tan(c+dx)}{ad} + \frac{\tan^3(c+dx)}{3ad} -$$

$$= \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\cot(c+dx)}{ad} - \frac{\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{3ad} + \frac{2\tan(c+}{ad}$$

Mathematica [B] time = 0.63, size = 245, normalized size = 2.63

$$-\frac{\csc^3(c+dx)\left(4\sin(c+dx)-16\sin(2(c+dx))+8\sin(3(c+dx))+10\cos(2(c+dx))+8\cos(3(c+dx))+6\,\text{si}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(Csc[c + d*x]^3*(2 + 10*Cos[2*(c + d*x)] + 8*Cos[3*(c + d*x)] + 3*Cos[
3*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 3*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2
]] + Cos[c + d*x]*(-8 - 3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2]])
+ 4*Sin[c + d*x] - 16*Sin[2*(c + d*x)] - 6*Log[Cos[(c + d*x)/2]]*Sin[2*(c +
 d*x)] + 6*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)]))/(a
*d*(Csc[(c + d*x)/2] - Sec[(c + d*x)/2])*(Csc[(c + d*x)/2] + Sec[(c + d*x)/
2])*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.47, size = 162, normalized size = 1.74

$$\frac{10\cos(dx+c)^2 + 3\left(\cos(dx+c)^3 - \cos(dx+c)\sin(dx+c) - \cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) - 3\left(\cos(d\right.}{6\left(ad\cos(dx+c)^3 - ad\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(10*cos(d*x + c)^2 + 3*(cos(d*x + c)^3 - cos(d*x + c)*sin(d*x + c) - co
s(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^3 - cos(d*x + c)*
sin(d*x + c) - cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(8*cos(d*x +
c)^2 - 1)*sin(d*x + c) - 4)/(a*d*cos(d*x + c)^3 - a*d*cos(d*x + c)*sin(d*x
+ c) - a*d*cos(d*x + c))
```

giac [A] time = 0.18, size = 133, normalized size = 1.43

$$-\frac{\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a} - \frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} - \frac{3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a} + \frac{21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+36\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+19}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3*tan(1/2*d*x + 1/2*c)/a - 3*(ta
n(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 1)/((tan(1/2*d*x + 1/2*c)^2
 - tan(1/2*d*x + 1/2*c))*a) + (21*tan(1/2*d*x + 1/2*c)^2 + 36*tan(1/2*d*x +
 1/2*c) + 19)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

maple [A] time = 0.43, size = 139, normalized size = 1.49

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad} - \frac{1}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} - \frac{1}{2ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad} - \frac{2}{3ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3} + \frac{}{ad\left(\tan\left(\frac{}{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)-1/2/a/d/(tan(1/2*d*x+1/2*c)-1)-1/2/a/d/tan(1/2*d
*x+1/2*c)-1/a/d*ln(tan(1/2*d*x+1/2*c))-2/3/a/d/(tan(1/2*d*x+1/2*c)+1)^3+1/a
/d/(tan(1/2*d*x+1/2*c)+1)^2-7/2/a/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.33, size = 215, normalized size = 2.31

$$-\frac{\frac{\frac{22\sin(dx+c)}{\cos(dx+c)+1}+\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{30\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*((22*sin(d*x + c)/(cos(d*x + c) + 1) + 8*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 3)/(a*sin(d*x + c)/(cos(d*x + c) + 1) + 2*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 2*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5) + 6*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - 3
*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 9.11, size = 150, normalized size = 1.61

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}-\frac{-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{22\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+1}{d\left(-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\right.\right.}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a*d) - ((22*tan(c/2 + (d*x)/2))/3 + (8*tan(c/2 + (d*x
)/2)^2)/3 - 10*tan(c/2 + (d*x)/2)^3 - 9*tan(c/2 + (d*x)/2)^4 + 1)/(d*(2*a*t
an(c/2 + (d*x)/2) + 4*a*tan(c/2 + (d*x)/2)^2 - 4*a*tan(c/2 + (d*x)/2)^4 - 2
*a*tan(c/2 + (d*x)/2)^5)) - log(tan(c/2 + (d*x)/2))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^2(c+dx)\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

3.778 $\int \frac{\sin^4(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=149

$$-\frac{2\cos(c+dx)}{a^2d}+\frac{9\tan^5(c+dx)}{10a^2d}-\frac{3\tan^3(c+dx)}{2a^2d}+\frac{9\tan(c+dx)}{2a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}-$$

```
[Out] -9/2*x/a^2-2*cos(d*x+c)/a^2/d-6*sec(d*x+c)/a^2/d+2*sec(d*x+c)^3/a^2/d-2/5*s
ec(d*x+c)^5/a^2/d+9/2*tan(d*x+c)/a^2/d-3/2*tan(d*x+c)^3/a^2/d+9/10*tan(d*x+
c)^5/a^2/d-1/2*sin(d*x+c)^2*tan(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.31, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2875, 2710, 3473, 8, 2590, 270, 2591, 288, 302, 203}

$$-\frac{2\cos(c+dx)}{a^2d}+\frac{9\tan^5(c+dx)}{10a^2d}-\frac{3\tan^3(c+dx)}{2a^2d}+\frac{9\tan(c+dx)}{2a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^4*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-9*x)/(2*a^2) - (2*Cos[c + d*x])/(a^2*d) - (6*Sec[c + d*x])/(a^2*d) + (2*S
ec[c + d*x]^3)/(a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) + (9*Tan[c + d*x])/(2
*a^2*d) - (3*Tan[c + d*x]^3)/(2*a^2*d) + (9*Tan[c + d*x]^5)/(10*a^2*d) - (S
in[c + d*x]^2*Tan[c + d*x]^5)/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int (a-a\sin(c+dx))^2\tan^6(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\tan^6(c+dx)-2a^2\sin(c+dx)\tan^6(c+dx)+a^2\sin^2(c+dx)\tan^6(c+a\right.}{a^4}$$
$$= \frac{\int \tan^6(c+dx)\,dx}{a^2}+\frac{\int \sin^2(c+dx)\tan^6(c+dx)\,dx}{a^2}-\frac{2\int \sin(c+dx)\tan^6(c}{a^2}$$
$$= \frac{\tan^5(c+dx)}{5a^2d}-\frac{\int \tan^4(c+dx)\,dx}{a^2}+\frac{\mathrm{Subst}\left(\int \frac{x^8}{(1+x^2)^2}\,dx,x,\tan(c+dx)\right)}{a^2d}+\frac{2}{}$$
$$= -\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan^5(c+dx)}{5a^2d}-\frac{\sin^2(c+dx)\tan^5(c+dx)}{2a^2d}+\frac{\int \tan^2(c+dx)\,}{a^2}$$
$$= -\frac{2\cos(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\tan(c+}{a^2d}$$
$$= -\frac{x}{a^2}-\frac{2\cos(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{9\,\mathrm{ta}}{}$$
$$= -\frac{9x}{2a^2}-\frac{2\cos(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{9}{}$$

Mathematica [A] time = 0.59, size = 191, normalized size = 1.28

$$-\frac{250\sin(c+dx)+720c\sin(2(c+dx))+720dx\sin(2(c+dx))-824\sin(2(c+dx))+351\sin(3(c+dx))+5\sin}{160a^2d\left(\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^4*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/160*(500 + 10*(-103 + 90*c + 90*d*x)*Cos[c + d*x] + 544*Cos[2*(c + d*x)]
 + 206*Cos[3*(c + d*x)] - 180*c*Cos[3*(c + d*x)] - 180*d*x*Cos[3*(c + d*x)]
 - 20*Cos[4*(c + d*x)] + 250*Sin[c + d*x] - 824*Sin[2*(c + d*x)] + 720*c*Si
n[2*(c + d*x)] + 720*d*x*Sin[2*(c + d*x)] + 351*Sin[3*(c + d*x)] + 5*Sin[5*
(c + d*x)])/(a^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^5)
```

fricas [A] time = 0.46, size = 132, normalized size = 0.89

$$-\frac{45\,dx\cos\left(dx+c\right)^{3}+10\,\cos\left(dx+c\right)^{4}-90\,dx\cos\left(dx+c\right)-78\,\cos\left(dx+c\right)^{2}-\left(5\,\cos\left(dx+c\right)^{4}+90\,dx\cos\left(dx\right.\right.}{10\left(a^{2}d\cos\left(dx+c\right)^{3}-2\,a^{2}d\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a^{2}d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/10*(45*d*x*cos(d*x + c)^3 + 10*cos(d*x + c)^4 - 90*d*x*cos(d*x + c) - 78
*cos(d*x + c)^2 - (5*cos(d*x + c)^4 + 90*d*x*cos(d*x + c) + 84*cos(d*x + c)
^2 - 6)*sin(d*x + c) + 4)/(a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(
d*x + c) - 2*a^2*d*cos(d*x + c))
```

giac [A] time = 0.23, size = 160, normalized size = 1.07

$$-\frac{\frac{90\,(dx+c)}{a^{2}}+\frac{20\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{2}}+\frac{5}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{155\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+690\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1}{a^{2}\left(\tan\left(\frac{1}{2}\right.\right.}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(90*(d*x + c)/a^2 + 20*(tan(1/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*
c)^2 - tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + 5/(
a^2*(tan(1/2*d*x + 1/2*c) - 1)) + (155*tan(1/2*d*x + 1/2*c)^4 + 690*tan(1/2
*d*x + 1/2*c)^3 + 1120*tan(1/2*d*x + 1/2*c)^2 + 750*tan(1/2*d*x + 1/2*c) +
181)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.47, size = 267, normalized size = 1.79

$$-\frac{1}{4a^{2}d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^{2}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{4\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{2}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^{2}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{}{a^{2}d\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/a^2/d/(tan(1/2*d*x+1/2*c)-1)-1/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/
2*d*x+1/2*c)^3-4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+1/a^
2/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-4/a^2/d/(1+tan(1/2*d*x+1/
```

```
2*c)^2)^2-9/d/a^2*arctan(tan(1/2*d*x+1/2*c))-4/5/a^2/d/(tan(1/2*d*x+1/2*c)+
1)^5+2/a^2/d/(tan(1/2*d*x+1/2*c)+1)^4+1/a^2/d/(tan(1/2*d*x+1/2*c)+1)^3-7/2/
a^2/d/(tan(1/2*d*x+1/2*c)+1)^2-31/4/a^2/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.43, size = 421, normalized size = 2.83

$$-\frac{\frac{\frac{211\sin(dx+c)}{\cos(dx+c)+1}+\frac{268\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{212\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{84\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{174\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{300\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{300\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{180\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{45\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+64}{a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{7a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{7a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{45\ldots}{\ldots}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/5*((211*sin(d*x + c)/(cos(d*x + c) + 1) + 268*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 212*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 174*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 300*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6 - 300*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 180*
sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 45*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 + 64)/(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 7*a^2*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a^2*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6 - 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*a^2*sin(d*x + c)^8/(cos
(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*sin(d*x
+ c)^10/(cos(d*x + c) + 1)^10) + 45*arctan(sin(d*x + c)/(cos(d*x + c) + 1))
/a^2)/d
```

mupad [B] time = 17.67, size = 172, normalized size = 1.15

$$\frac{-9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-36\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-60\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-60\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-\frac{174\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{5}+\frac{84\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{5}+\frac{21\ldots}{\ldots}}{a^2 d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^5\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((211*tan(c/2 + (d*x)/2))/5 + (268*tan(c/2 + (d*x)/2)^2)/5 + (212*tan(c/2 +
 (d*x)/2)^3)/5 + (84*tan(c/2 + (d*x)/2)^4)/5 - (174*tan(c/2 + (d*x)/2)^5)/5
 - 60*tan(c/2 + (d*x)/2)^6 - 60*tan(c/2 + (d*x)/2)^7 - 36*tan(c/2 + (d*x)/2
)^8 - 9*tan(c/2 + (d*x)/2)^9 + 64/5)/(a^2*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c
/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (9*x)/(2*a^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.779 $\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{\cos(c+dx)}{a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\tan^3(c+dx)}{3a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{5\sec^3(c+dx)}{3a^2d}+\frac{4\sec(c+dx)}{a^2d}+\frac{2x}{a^2}$$

```
[Out] 2*x/a^2+cos(d*x+c)/a^2/d+4*sec(d*x+c)/a^2/d-5/3*sec(d*x+c)^3/a^2/d+2/5*sec(
d*x+c)^5/a^2/d-2*tan(d*x+c)/a^2/d+2/3*tan(d*x+c)^3/a^2/d-2/5*tan(d*x+c)^5/a
^2/d
```

Rubi [A] time = 0.28, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2875, 2873, 2606, 194, 3473, 8, 2590, 270}

$$\frac{\cos(c+dx)}{a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\tan^3(c+dx)}{3a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{5\sec^3(c+dx)}{3a^2d}+\frac{4\sec(c+dx)}{a^2d}+\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*x)/a^2 + Cos[c + d*x]/(a^2*d) + (4*Sec[c + d*x])/(a^2*d) - (5*Sec[c + d*
x]^3)/(3*a^2*d) + (2*Sec[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x])/(a^2*d) +
 (2*Tan[c + d*x]^3)/(3*a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec(c+dx)(a-a\sin(c+dx))^2\tan^5(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec(c+dx)\tan^5(c+dx) - 2a^2\tan^6(c+dx) + a^2\sin(c+dx)\tan^6(c+d\right.}{a^4}$$
$$= \frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a^2} + \frac{\int \sin(c+dx)\tan^6(c+dx)\,dx}{a^2} - \frac{2\int \tan^6(c+}{a^2}$$
$$= -\frac{2\tan^5(c+dx)}{5a^2d} + \frac{2\int \tan^4(c+dx)\,dx}{a^2} - \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= \frac{2\tan^3(c+dx)}{3a^2d} - \frac{2\tan^5(c+dx)}{5a^2d} - \frac{2\int \tan^2(c+dx)\,dx}{a^2} - \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^6}-\right.\right.}{}$$
$$= \frac{\cos(c+dx)}{a^2d} + \frac{4\sec(c+dx)}{a^2d} - \frac{5\sec^3(c+dx)}{3a^2d} + \frac{2\sec^5(c+dx)}{5a^2d} - \frac{2\tan(c+d}{a^2d}$$
$$= \frac{2x}{a^2} + \frac{\cos(c+dx)}{a^2d} + \frac{4\sec(c+dx)}{a^2d} - \frac{5\sec^3(c+dx)}{3a^2d} + \frac{2\sec^5(c+dx)}{5a^2d} - \frac{2\tan(}{a}$$

Mathematica [A] time = 0.57, size = 148, normalized size = 1.23

$$\frac{\sec(c+dx)(400\sin(c+dx) + 480c\sin(2(c+dx)) + 480dx\sin(2(c+dx)) - 796\sin(2(c+dx)) + 304\sin(3(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(550 + (-995 + 600*c + 600*d*x)*Cos[c + d*x] + 376*Cos[2*(c +
 d*x)] + 199*Cos[3*(c + d*x)] - 120*c*Cos[3*(c + d*x)] - 120*d*x*Cos[3*(c +
 d*x)] - 30*Cos[4*(c + d*x)] + 400*Sin[c + d*x] - 796*Sin[2*(c + d*x)] + 48
0*c*Sin[2*(c + d*x)] + 480*d*x*Sin[2*(c + d*x)] + 304*Sin[3*(c + d*x)]))/(2
40*a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.47, size = 122, normalized size = 1.02

$$\frac{30\,dx\cos(dx+c)^3 + 15\cos(dx+c)^4 - 60\,dx\cos(dx+c) - 62\cos(dx+c)^2 - 2\left(30\,dx\cos(dx+c) + 38\cos(\right.}{15\left(a^2d\cos(dx+c)^3 - 2\,a^2d\cos(dx+c)\sin(dx+c) - 2\,a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/15*(30*d*x*cos(d*x + c)^3 + 15*cos(d*x + c)^4 - 60*d*x*cos(d*x + c) - 62*
cos(d*x + c)^2 - 2*(30*d*x*cos(d*x + c) + 38*cos(d*x + c)^2 + 3)*sin(d*x +
c) - 9)/(a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d
*cos(d*x + c))
```

giac [A] time = 0.23, size = 151, normalized size = 1.26

$$\frac{\frac{120\,(dx+c)}{a^2} - \frac{15\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)a^2} + \frac{255\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 1170\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 1960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 1310\,\cdots}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(120*(d*x + c)/a^2 - 15*(tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*
c) + 9)/((tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1
/2*c) - 1)*a^2) + (255*tan(1/2*d*x + 1/2*c)^4 + 1170*tan(1/2*d*x + 1/2*c)^3
 + 1960*tan(1/2*d*x + 1/2*c)^2 + 1310*tan(1/2*d*x + 1/2*c) + 313)/(a^2*(tan
(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.44, size = 169, normalized size = 1.41

$$-\frac{1}{4a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{2}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} + \frac{4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2} + \frac{4}{5a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5} - \frac{\cdots}{a^2d\left(\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/a^2/d/(tan(1/2*d*x+1/2*c)-1)+2/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)+4/d/a^2*
arctan(tan(1/2*d*x+1/2*c))+4/5/a^2/d/(tan(1/2*d*x+1/2*c)+1)^5-2/a^2/d/(tan(
1/2*d*x+1/2*c)+1)^4-1/3/a^2/d/(tan(1/2*d*x+1/2*c)+1)^3+5/2/a^2/d/(tan(1/2*d
*x+1/2*c)+1)^2+17/4/a^2/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.43, size = 335, normalized size = 2.79

$$\frac{4\left(\frac{\frac{97\,\sin(dx+c)}{\cos(dx+c)+1} + \frac{108\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{27\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{40\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{85\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{60\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + 28}{a^2 + \frac{4\,a^2\sin(dx+c)}{\cos(dx+c)+1} + \frac{6\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{4\,a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{4\,a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{6\,a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{4\,a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}} + \frac{15\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 4/15*((97*sin(d*x + c)/(cos(d*x + c) + 1) + 108*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 27*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 40*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 85*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 60*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 28)/(a^2
+ 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 6*a^2*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 4*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 4*a^2*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 6*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 4*a^2*s
in(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 15.38, size = 156, normalized size = 1.30

$$\frac{2\,x}{a^2}-\frac{-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{68\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-\frac{32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{5}+\frac{144\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\frac{388\tan\left(\cdots\right.}{1\cdots}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (2*x)/a^2 - ((388*tan(c/2 + (d*x)/2))/15 + (144*tan(c/2 + (d*x)/2)^2)/5 + (
36*tan(c/2 + (d*x)/2)^3)/5 - (32*tan(c/2 + (d*x)/2)^4)/3 - (68*tan(c/2 + (d
*x)/2)^5)/3 - 16*tan(c/2 + (d*x)/2)^6 - 4*tan(c/2 + (d*x)/2)^7 + 112/15)/(a
^2*d*(tan(c/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2
+ tan(c/2 + (d*x)/2)^3 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.780 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=106

$$\frac{2\tan^5(c+dx)}{5a^2d}-\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{x}{a^2}$$

```
[Out] -x/a^2-2*sec(d*x+c)/a^2/d+4/3*sec(d*x+c)^3/a^2/d-2/5*sec(d*x+c)^5/a^2/d+tan
(d*x+c)/a^2/d-1/3*tan(d*x+c)^3/a^2/d+2/5*tan(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.28, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2607, 30, 2606, 194, 3473, 8}

$$\frac{2\tan^5(c+dx)}{5a^2d}-\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*Sec[c + d*x])/(a^2*d) + (4*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec
[c + d*x]^5)/(5*a^2*d) + Tan[c + d*x]/(a^2*d) - Tan[c + d*x]^3/(3*a^2*d) +
(2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec^2(c+dx)(a-a\sin(c+dx))^2\tan^4(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec^2(c+dx)\tan^4(c+dx)-2a^2\sec(c+dx)\tan^5(c+dx)+a^2\tan^6(c+dx\right.}{a^4}$$
$$= \frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a^2}+\frac{\int \tan^6(c+dx)\,dx}{a^2}-\frac{2\int \sec(c+dx)\tan^5(c+}{a^2}$$
$$= \frac{\tan^5(c+dx)}{5a^2d}-\frac{\int \tan^4(c+dx)\,dx}{a^2}+\frac{\text{Subst}\left(\int x^4\,dx,x,\tan(c+dx)\right)}{a^2d}-\frac{2\,\text{Subst}}{}$$
$$= -\frac{\tan^3(c+dx)}{3a^2d}+\frac{2\tan^5(c+dx)}{5a^2d}+\frac{\int \tan^2(c+dx)\,dx}{a^2}-\frac{2\,\text{Subst}\left(\int \left(1-2x^2+\right.\right.}{a^2}$$
$$= -\frac{2\sec(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{\tan^3(c+dx}{3a^2d}$$
$$= -\frac{x}{a^2}-\frac{2\sec(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{\tan^3(c}{3a}$$

Mathematica [A] time = 0.56, size = 143, normalized size = 1.35

$$-\frac{\sec(c+dx)\left(-10\sin(c+dx)+60c\sin(2(c+dx))+60dx\sin(2(c+dx))-89\sin(2(c+dx))+26\sin(3(c+dx))\right.}{60a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/60*(Sec[c + d*x]*(20 + (5*(-89 + 60*c + 60*d*x)*Cos[c + d*x])/4 + 44*Cos
[2*(c + d*x)] + (89*Cos[3*(c + d*x)])/4 - 15*c*Cos[3*(c + d*x)] - 15*d*x*Co
s[3*(c + d*x)] - 10*Sin[c + d*x] - 89*Sin[2*(c + d*x)] + 60*c*Sin[2*(c + d*
x)] + 60*d*x*Sin[2*(c + d*x)] + 26*Sin[3*(c + d*x)]))/(a^2*d*(1 + Sin[c + d
*x])^2)
```

fricas [A] time = 0.44, size = 112, normalized size = 1.06

$$-\frac{15\,dx\cos(dx+c)^3-30\,dx\cos(dx+c)-22\,\cos(dx+c)^2-\left(30\,dx\cos(dx+c)+26\,\cos(dx+c)^2-9\right)\sin(dx}{15\left(a^2d\cos(dx+c)^3-2\,a^2d\cos(dx+c)\sin(dx+c)-2\,a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/15*(15*d*x*cos(d*x + c)^3 - 30*d*x*cos(d*x + c) - 22*cos(d*x + c)^2 - (3
0*d*x*cos(d*x + c) + 26*cos(d*x + c)^2 - 9)*sin(d*x + c) + 6)/(a^2*d*cos(d*
x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

giac [A] time = 0.22, size = 103, normalized size = 0.97

$$-\frac{\frac{60\,(dx+c)}{a^2}+\frac{15}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+510\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+920\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+610\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+143}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(60*(d*x + c)/a^2 + 15/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) + (105*tan(1/
2*d*x + 1/2*c)^4 + 510*tan(1/2*d*x + 1/2*c)^3 + 920*tan(1/2*d*x + 1/2*c)^2
+ 610*tan(1/2*d*x + 1/2*c) + 143)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.43, size = 146, normalized size = 1.38

$$-\frac{1}{4a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}-\frac{4}{5a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{2}{a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{}{3a^2d\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/a^2/d/(tan(1/2*d*x+1/2*c)-1)-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-4/5/a^
2/d/(tan(1/2*d*x+1/2*c)+1)^5+2/a^2/d/(tan(1/2*d*x+1/2*c)+1)^4-1/3/a^2/d/(ta
n(1/2*d*x+1/2*c)+1)^3-3/2/a^2/d/(tan(1/2*d*x+1/2*c)+1)^2-7/4/a^2/d/(tan(1/2
*d*x+1/2*c)+1)
```

maxima [B] time = 0.43, size = 249, normalized size = 2.35

$$-\frac{2\left(\frac{\frac{49\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{70\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{60\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{15\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+16}{a^2+\frac{4\,a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4\,a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{15\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -2/15*((49*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 70*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 60*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 16)/(a^2 + 4*a^2
*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 15*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 14.09, size = 105, normalized size = 0.99

$$\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{98\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+\frac{32}{15}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}-\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((98*tan(c/2 + (d*x)/2))/15 + (8*tan(c/2 + (d*x)/2)^2)/3 - (28*tan(c/2 + (d
*x)/2)^3)/3 - 8*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^5 + 32/15)/(a^2
*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1)^5) - x/a^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^4(c+dx)\sec^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**4*sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

3.781 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{\sec^3(c+dx)}{a^2d}+\frac{\sec(c+dx)}{a^2d}$$

```
[Out] sec(d*x+c)/a^2/d-sec(d*x+c)^3/a^2/d+2/5*sec(d*x+c)^5/a^2/d-2/5*tan(d*x+c)^5
/a^2/d
```

Rubi [A] time = 0.26, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2873, 2606, 14, 2607, 30, 194}

$$-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{\sec^3(c+dx)}{a^2d}+\frac{\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sec[c + d*x]/(a^2*d) - Sec[c + d*x]^3/(a^2*d) + (2*Sec[c + d*x]^5)/(5*a^2*d
) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec^3(c+dx)(a-a\sin(c+dx))^2\tan^3(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec^3(c+dx)\tan^3(c+dx)-2a^2\sec^2(c+dx)\tan^4(c+dx)+a^2\sec(c+dx)\right.}{a^4}$$
$$= \frac{\int \sec^3(c+dx)\tan^3(c+dx)\,dx}{a^2}+\frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a^2}-\frac{2\int \sec^2(c+}{}$$
$$= \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int \left(-1+x^2\right)^2 dx,x,\sec(c+\right.}{a^2d}$$
$$= -\frac{2\tan^5(c+dx)}{5a^2d}+\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int \left(-x^2\right.\right.}{}$$
$$= \frac{\sec(c+dx)}{a^2d}-\frac{\sec^3(c+dx)}{a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}$$

Mathematica [A] time = 0.28, size = 84, normalized size = 1.27

$$\frac{\sec(c+dx)(40\sin(c+dx)-52\sin(2(c+dx))+8\sin(3(c+dx))-65\cos(c+dx)-8\cos(2(c+dx))+13\cos(3}{80a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(40 - 65*Cos[c + d*x] - 8*Cos[2*(c + d*x)] + 13*Cos[3*(c + d*
x)] + 40*Sin[c + d*x] - 52*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)]))/(80*a^2*
d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.48, size = 76, normalized size = 1.15

$$\frac{\cos(dx+c)^2-2\left(\cos(dx+c)^2+1\right)\sin(dx+c)-3}{5\left(a^2d\cos(dx+c)^3-2\,a^2d\cos(dx+c)\sin(dx+c)-2\,a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/5*(cos(d*x + c)^2 - 2*(cos(d*x + c)^2 + 1)*sin(d*x + c) - 3)/(a^2*d*cos(d
*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

giac [A] time = 0.22, size = 94, normalized size = 1.42

$$-\frac{\frac{5}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+30\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+50\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) - (5*tan(1/2*d*x + 1/2*c)^4 + 30*
tan(1/2*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)^2 + 50*tan(1/2*d*x + 1/2*c
) + 11)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.43, size = 100, normalized size = 1.52

$$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{16}{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+64}}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 16/d/a^2*(-1/64/(tan(1/2*d*x+1/2*c)-1)+1/20/(tan(1/2*d*x+1/2*c)+1)^5-1/8/(t
an(1/2*d*x+1/2*c)+1)^4+1/16/(tan(1/2*d*x+1/2*c)+1)^3+1/32/(tan(1/2*d*x+1/2*
c)+1)^2+1/64/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.33, size = 164, normalized size = 2.48

$$\frac{4\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{5\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 4/5*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 1)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^2
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6)*d)
```

mupad [B] time = 9.27, size = 111, normalized size = 1.68

$$\frac{\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{5}+\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{5}+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^2\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((4*cos(c/2 + (d*x)/2)^6)/5 + (16*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2))/
5 + 4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2)/(a^2*d*(cos(c/2 + (d*x)/2)
 - sin(c/2 + (d*x)/2))*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.782 $\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$\frac{2\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{2\sec^3(c+dx)}{3a^2d}$$

```
[Out] 2/3*sec(d*x+c)^3/a^2/d-2/5*sec(d*x+c)^5/a^2/d+1/3*tan(d*x+c)^3/a^2/d+2/5*ta
n(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.19, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2711, 2607, 14, 2606, 30}

$$\frac{2\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{2\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) + Tan[c + d*x]^
3/(3*a^2*d) + (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \left(a^2\sec^4(c+dx)\tan^2(c+dx) - 2a^2\sec^3(c+dx)\tan^3(c+dx) + a^2\sec^2(c+dx)\tan\right.}{a^4}$$
$$= \frac{\int \sec^4(c+dx)\tan^2(c+dx)\,dx}{a^2} + \frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a^2} - \frac{2\int \sec^3(c+dx)}{a^2}$$
$$= \frac{\text{Subst}\left(\int x^4\,dx, x, \tan(c+dx)\right)}{a^2d} + \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)dx, x, \tan(c+dx)\right)}{a^2d} - \frac{2\,\text{Subst}}{}$$
$$= \frac{\tan^5(c+dx)}{5a^2d} + \frac{\text{Subst}\left(\int \left(x^2+x^4\right)dx, x, \tan(c+dx)\right)}{a^2d} - \frac{2\,\text{Subst}\left(\int \left(-x^2+x^4\right)dx,\right.}{a^2d}$$
$$= \frac{2\sec^3(c+dx)}{3a^2d} - \frac{2\sec^5(c+dx)}{5a^2d} + \frac{\tan^3(c+dx)}{3a^2d} + \frac{2\tan^5(c+dx)}{5a^2d}$$

Mathematica [A] time = 0.26, size = 86, normalized size = 1.18

$$-\frac{\sec(c+dx)\left(-35\sin(c+dx) + 11\sin(2(c+dx)) + \sin(3(c+dx)) + \frac{55}{4}\cos(c+dx) + 4\cos(2(c+dx)) - \frac{11}{4}\cos(3\right.}{60a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/60*(Sec[c + d*x]*(-20 + (55*Cos[c + d*x])/4 + 4*Cos[2*(c + d*x)] - (11*C
os[3*(c + d*x)])/4 - 35*Sin[c + d*x] + 11*Sin[2*(c + d*x)] + Sin[3*(c + d*x
)]))/(a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.43, size = 77, normalized size = 1.05

$$\frac{2\cos(dx+c)^2 + \left(\cos(dx+c)^2 - 9\right)\sin(dx+c) - 6}{15\left(a^2d\cos(dx+c)^3 - 2a^2d\cos(dx+c)\sin(dx+c) - 2a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/15*(2*cos(d*x + c)^2 + (cos(d*x + c)^2 - 9)*sin(d*x + c) - 6)/(a^2*d*cos(
d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

giac [A] time = 0.21, size = 94, normalized size = 1.29

$$-\frac{\frac{15}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+90\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+70\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(15/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) - (15*tan(1/2*d*x + 1/2*c)^4 + 9
0*tan(1/2*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)^2 + 70*tan(1/2*d*x + 1/2
*c) + 17)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.41, size = 100, normalized size = 1.37

$$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{5}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{8}{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32}}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 8/d/a^2*(-1/32/(tan(1/2*d*x+1/2*c)-1)-1/10/(tan(1/2*d*x+1/2*c)+1)^5+1/4/(ta
n(1/2*d*x+1/2*c)+1)^4-5/24/(tan(1/2*d*x+1/2*c)+1)^3+1/16/(tan(1/2*d*x+1/2*c
)+1)^2+1/32/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.33, size = 184, normalized size = 2.52

$$\frac{8\left(\frac{4\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+1\right)}{15\left(a^2+\frac{4\,a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4\,a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 8/15*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/((a^2 + 4*a^2*sin(d*x + c
)/(cos(d*x + c) + 1) + 5*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 5*a^2*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d)
```

mupad [B] time = 9.31, size = 132, normalized size = 1.81

$$\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+5\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}{15\,a^2\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (8*cos(c/2 + (d*x)/2)^3*(cos(c/2 + (d*x)/2)^3 + 5*sin(c/2 + (d*x)/2)^3 + 5*
cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^2 + 4*cos(c/2 + (d*x)/2)^2*sin(c/2 +
(d*x)/2)))/(15*a^2*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))*(cos(c/2 + (
d*x)/2) + sin(c/2 + (d*x)/2))^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^2(c+dx)\sec^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

3.783 $\int \frac{\sec(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=71

$$\frac{4\tan(c+dx)}{15a^2d}-\frac{2\sec(c+dx)}{15d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\sec(c+dx)}{5d(a\sin(c+dx)+a)^2}$$

```
[Out] 1/5*sec(d*x+c)/d/(a+a*sin(d*x+c))^2-2/15*sec(d*x+c)/d/(a^2+a^2*sin(d*x+c))+
4/15*tan(d*x+c)/a^2/d
```

Rubi [A] time = 0.13, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2859, 2672, 3767, 8}

$$\frac{4\tan(c+dx)}{15a^2d}-\frac{2\sec(c+dx)}{15d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\sec(c+dx)}{5d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sec[c + d*x]/(5*d*(a + a*Sin[c + d*x])^2) - (2*Sec[c + d*x])/(15*d*(a^2 + a
^2*Sin[c + d*x])) + (4*Tan[c + d*x])/(15*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
```

```
) && NeQ[2*m + p + 1, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2} + \frac{2\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx}{5a}\\ &= \frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\sec(c+dx)}{15d\left(a^2+a^2\sin(c+dx)\right)} + \frac{4\int \sec^2(c+dx)\,dx}{15a^2}\\ &= \frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\sec(c+dx)}{15d\left(a^2+a^2\sin(c+dx)\right)} - \frac{4\,\text{Subst}(\int 1\,dx, x, -\tan(c+\dots}{15a^2d}\\ &= \frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\sec(c+dx)}{15d\left(a^2+a^2\sin(c+dx)\right)} + \frac{4\tan(c+dx)}{15a^2d}\end{aligned}$$

Mathematica [A] time = 0.25, size = 82, normalized size = 1.15

$$-\frac{\sec(c+dx)(-80\sin(c+dx)-4\sin(2(c+dx))+16\sin(3(c+dx))-5\cos(c+dx)+64\cos(2(c+dx))+\cos(3(\dots}{240a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/240*(Sec[c + d*x]*(-80 - 5*Cos[c + d*x] + 64*Cos[2*(c + d*x)] + Cos[3*(c
 + d*x)] - 80*Sin[c + d*x] - 4*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)]))/(a^
2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.48, size = 80, normalized size = 1.13

$$\frac{8\cos(dx+c)^2+2\left(2\cos(dx+c)^2-3\right)\sin(dx+c)-9}{15\left(a^2d\cos(dx+c)^3-2\,a^2d\cos(dx+c)\sin(dx+c)-2\,a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(d*x + c)^2 + 2*(2*cos(d*x + c)^2 - 3)*sin(d*x + c) - 9)/(a^2*d*
cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

giac [A] time = 0.19, size = 94, normalized size = 1.32

$$-\frac{\frac{15}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-50\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(15/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) - (15*tan(1/2*d*x + 1/2*c)^4 - 3
0*tan(1/2*d*x + 1/2*c)^3 - 40*tan(1/2*d*x + 1/2*c)^2 - 50*tan(1/2*d*x + 1/2
*c) - 7)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.36, size = 100, normalized size = 1.41

$$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{7}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{4}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16}}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 4/d/a^2*(-1/16/(tan(1/2*d*x+1/2*c)-1)+1/5/(tan(1/2*d*x+1/2*c)+1)^5-1/2/(tan
(1/2*d*x+1/2*c)+1)^4+7/12/(tan(1/2*d*x+1/2*c)+1)^3-3/8/(tan(1/2*d*x+1/2*c)+
1)^2+1/16/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.33, size = 204, normalized size = 2.87

$$\frac{2\left(\frac{4\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}{15\left(a^2+\frac{4\,a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4\,a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/15*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x
```

```
 + c) + 1)^4 + 1)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 - 4*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6)*d)
```

mupad [B] time = 9.39, size = 159, normalized size = 2.24

$$\frac{\frac{2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{15}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{a^2\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((2*cos(c/2 + (d*x)/2)^6)/15 + (8*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2))/
15 + 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + (8*cos(c/2 + (d*x)/2)^3*
sin(c/2 + (d*x)/2)^3)/3 + (8*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2)/3)/
(a^2*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))*(cos(c/2 + (d*x)/2) + sin(
c/2 + (d*x)/2))^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin{\left(c+dx \right)}\sec^{2}{\left(c+dx \right)}}{\sin^{2}{\left(c+dx \right)}+2\sin{\left(c+dx \right)}+1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

3.784 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=115

$$-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{4\tan^3(c+dx)}{3a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

```
[Out] -arctanh(cos(d*x+c))/a^2/d+sec(d*x+c)/a^2/d+1/3*sec(d*x+c)^3/a^2/d+2/5*sec(
d*x+c)^5/a^2/d-2*tan(d*x+c)/a^2/d-4/3*tan(d*x+c)^3/a^2/d-2/5*tan(d*x+c)^5/a
^2/d
```

Rubi [A] time = 0.27, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2606, 30}

$$-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{4\tan^3(c+dx)}{3a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + Sec[c + d*x]/(a^2*d) + Sec[c + d*x]^3/(3
*a^2*d) + (2*Sec[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x])/(a^2*d) - (4*Tan[
c + d*x]^3)/(3*a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc(c+dx)\sec^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(-2a^2\sec^6(c+dx)+a^2\csc(c+dx)\sec^6(c+dx)+a^2\sec^5(c+dx)\tan(c+d\right.}{a^4}$$
$$= \frac{\int \csc(c+dx)\sec^6(c+dx)\,dx}{a^2} + \frac{\int \sec^5(c+dx)\tan(c+dx)\,dx}{a^2} - \frac{2\int \sec^6(c+}{a^2}$$
$$= \frac{\text{Subst}\left(\int x^4\,dx, x, \sec(c+dx)\right)}{a^2 d} + \frac{\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^2 d} + \frac{2\,\text{Subs}}{}$$
$$= \frac{\sec^5(c+dx)}{5a^2 d} - \frac{2\tan(c+dx)}{a^2 d} - \frac{4\tan^3(c+dx)}{3a^2 d} - \frac{2\tan^5(c+dx)}{5a^2 d} + \frac{\text{Subst}\left(\int \left(1\right.\right.}{}$$
$$= \frac{\sec(c+dx)}{a^2 d} + \frac{\sec^3(c+dx)}{3a^2 d} + \frac{2\sec^5(c+dx)}{5a^2 d} - \frac{2\tan(c+dx)}{a^2 d} - \frac{4\tan^3(c+dx)}{3a^2 d}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2 d} + \frac{\sec(c+dx)}{a^2 d} + \frac{\sec^3(c+dx)}{3a^2 d} + \frac{2\sec^5(c+dx)}{5a^2 d} - \frac{2\tan(}{a}$$

Mathematica [A] time = 0.52, size = 196, normalized size = 1.70

$$\frac{\sec(c+dx)\left(160\sin(c+dx)-316\sin(2(c+dx))+64\sin(3(c+dx))+136\cos(2(c+dx))+79\cos(3(c+dx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(280 + 136*Cos[2*(c + d*x)] + 79*Cos[3*(c + d*x)] + 60*Cos[3*
(c + d*x)]*Log[Cos[(c + d*x)/2]] - 5*Cos[c + d*x]*(79 + 60*Log[Cos[(c + d*x
)/2]] - 60*Log[Sin[(c + d*x)/2]]) - 60*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2
]] + 160*Sin[c + d*x] - 316*Sin[2*(c + d*x)] - 240*Log[Cos[(c + d*x)/2]]*Si
n[2*(c + d*x)] + 240*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] + 64*Sin[3*(c +
 d*x)]))/(240*a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.45, size = 168, normalized size = 1.46

$$-\frac{34\cos(dx+c)^2+15\left(\cos(dx+c)^3-2\cos(dx+c)\sin(dx+c)-2\cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-}{30\left(a^2 d\cos(dx+c)^3-2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(34*cos(d*x + c)^2 + 15*(cos(d*x + c)^3 - 2*cos(d*x + c)*sin(d*x + c)
 - 2*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(cos(d*x + c)^3 - 2*cos
(d*x + c)*sin(d*x + c) - 2*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(
8*cos(d*x + c)^2 + 3)*sin(d*x + c) + 18)/(a^2*d*cos(d*x + c)^3 - 2*a^2*d*co
s(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

giac [A] time = 0.19, size = 109, normalized size = 0.95

$$\frac{\frac{60 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} - \frac{15}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)} + \frac{255\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+810\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+710\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+193}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 15/(a^2*(tan(1/2*d*x + 1/2*c)
 - 1)) + (255*tan(1/2*d*x + 1/2*c)^4 + 810*tan(1/2*d*x + 1/2*c)^3 + 1120*ta
n(1/2*d*x + 1/2*c)^2 + 710*tan(1/2*d*x + 1/2*c) + 193)/(a^2*(tan(1/2*d*x +
1/2*c) + 1)^5))/d
```

maple [A] time = 0.56, size = 145, normalized size = 1.26

$$-\frac{1}{4a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}+\frac{4}{5a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{2}{a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{11}{3a^2d\left(\tan\left(\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/a^2/d/(tan(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c))+4/5/a^2/d/
(tan(1/2*d*x+1/2*c)+1)^5-2/a^2/d/(tan(1/2*d*x+1/2*c)+1)^4+11/3/a^2/d/(tan(1
/2*d*x+1/2*c)+1)^3-7/2/a^2/d/(tan(1/2*d*x+1/2*c)+1)^2+17/4/a^2/d/(tan(1/2*d
*x+1/2*c)+1)
```

maxima [B] time = 0.33, size = 250, normalized size = 2.17

$$\frac{\frac{4\left(\frac{37\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{30\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{15\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+13\right)}{a^2+\frac{4\,a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4\,a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{15\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/15*(4*(37*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 30*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 13)/(a^2 + 4*a^
2*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 15*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 10.78, size = 117, normalized size = 1.02

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{148\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+\frac{52}{15}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - ((148*tan(c/2 + (d*x)/2))/15 + (28*tan(c/
2 + (d*x)/2)^2)/3 - (8*tan(c/2 + (d*x)/2)^3)/3 - 8*tan(c/2 + (d*x)/2)^4 - 4
*tan(c/2 + (d*x)/2)^5 + 52/15)/(a^2*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (
d*x)/2) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc\left(c+dx\right)\sec^2\left(c+dx\right)}{\sin^2\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

3.785 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=130

$$\frac{2\tan^5(c+dx)}{5a^2d}+\frac{5\tan^3(c+dx)}{3a^2d}+\frac{4\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}+\frac{2\tan}{}$$

```
[Out] 2*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a^2/d-2*sec(d*x+c)/a^2/d-2/3*sec(d*x
+c)^3/a^2/d-2/5*sec(d*x+c)^5/a^2/d+4*tan(d*x+c)/a^2/d+5/3*tan(d*x+c)^3/a^2/
d+2/5*tan(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.31, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2620, 270}

$$\frac{2\tan^5(c+dx)}{5a^2d}+\frac{5\tan^3(c+dx)}{3a^2d}+\frac{4\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}+\frac{2\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Sec[c + d*x])
/(a^2*d) - (2*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) + (4
*Tan[c + d*x])/(a^2*d) + (5*Tan[c + d*x]^3)/(3*a^2*d) + (2*Tan[c + d*x]^5)/
(5*a^2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^2(c+dx)\sec^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec^6(c+dx)-2a^2\csc(c+dx)\sec^6(c+dx)+a^2\csc^2(c+dx)\sec^6(c+dx\right.}{a^4}$$
$$= \frac{\int \sec^6(c+dx)\,dx}{a^2}+\frac{\int \csc^2(c+dx)\sec^6(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\sec^6(c+}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^2}\,dx,x,\tan(c+dx)\right)}{a^2d}-\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(c+\right.}{a^2d}$$
$$= \frac{\tan(c+dx)}{a^2d}+\frac{2\tan^3(c+dx)}{3a^2d}+\frac{\tan^5(c+dx)}{5a^2d}+\frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+3x^2+x^4\right)\right.}{a^2d}$$
$$= -\frac{\cot(c+dx)}{a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{4\tan(c+dx}{a^2d}$$
$$= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5}{5}$$

Mathematica [B] time = 0.83, size = 289, normalized size = 2.22

$$-\frac{\csc^3(c+dx)\left(58\sin(c+dx)-168\sin(2(c+dx))+82\sin(3(c+dx))+28\sin(4(c+dx))+48\cos(2(c+dx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/15*(Csc[c + d*x]^3*(40 + 48*Cos[2*(c + d*x)] + 112*Cos[3*(c + d*x)] - 28
*Cos[4*(c + d*x)] + 60*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 4*Cos[c + d
*x]*(28 + 15*Log[Cos[(c + d*x)/2]] - 15*Log[Sin[(c + d*x)/2]]) - 60*Cos[3*(
c + d*x)]*Log[Sin[(c + d*x)/2]] + 58*Sin[c + d*x] - 168*Sin[2*(c + d*x)] -
90*Log[Cos[(c + d*x)/2]]*Sin[2*(c + d*x)] + 90*Log[Sin[(c + d*x)/2]]*Sin[2*
(c + d*x)] + 82*Sin[3*(c + d*x)] + 28*Sin[4*(c + d*x)] + 15*Log[Cos[(c + d*
x)/2]]*Sin[4*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)]))/(a^2*
d*(Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2)*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.46, size = 218, normalized size = 1.68

$$-\frac{56\cos(dx+c)^4-80\cos(dx+c)^2-15\left(2\cos(dx+c)^3+\left(\cos(dx+c)^3-2\cos(dx+c)\right)\sin(dx+c)-2\cos\right.}{15\left(2a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/15*(56*cos(d*x + c)^4 - 80*cos(d*x + c)^2 - 15*(2*cos(d*x + c)^3 + (cos(
d*x + c)^3 - 2*cos(d*x + c))*sin(d*x + c) - 2*cos(d*x + c))*log(1/2*cos(d*x
 + c) + 1/2) + 15*(2*cos(d*x + c)^3 + (cos(d*x + c)^3 - 2*cos(d*x + c))*sin
(d*x + c) - 2*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(41*cos(d*x +
c)^2 - 3)*sin(d*x + c) + 9)/(2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)
+ (a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.23, size = 161, normalized size = 1.24

$$-\frac{\frac{120 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2} - \frac{30 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2} - \frac{15\left(4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a^2} + \frac{465\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1590\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2}{a^2\left(\tan\left(\frac{1}{2}\right.\right.}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 30*tan(1/2*d*x + 1/2*c)/a^2
 - 15*(4*tan(1/2*d*x + 1/2*c)^2 - 7*tan(1/2*d*x + 1/2*c) + 2)/((tan(1/2*d*x
 + 1/2*c)^2 - tan(1/2*d*x + 1/2*c))*a^2) + (465*tan(1/2*d*x + 1/2*c)^4 + 15
90*tan(1/2*d*x + 1/2*c)^3 + 2240*tan(1/2*d*x + 1/2*c)^2 + 1450*tan(1/2*d*x
+ 1/2*c) + 383)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.56, size = 182, normalized size = 1.40

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2} - \frac{1}{4a^2 d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} - \frac{1}{2d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2} - \frac{4}{5a^2 d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5} + \frac{}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/4/a^2/d/(tan(1/2*d*x+1/2*c)-1)-1/2/d/a^2/tan
(1/2*d*x+1/2*c)-2/d/a^2*ln(tan(1/2*d*x+1/2*c))-4/5/a^2/d/(tan(1/2*d*x+1/2*c
)+1)^5+2/a^2/d/(tan(1/2*d*x+1/2*c)+1)^4-13/3/a^2/d/(tan(1/2*d*x+1/2*c)+1)^3
+9/2/a^2/d/(tan(1/2*d*x+1/2*c)+1)^2-31/4/a^2/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.34, size = 309, normalized size = 2.38

$$-\frac{\frac{\frac{244\sin(dx+c)}{\cos(dx+c)+1}+\frac{571\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{320\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{475\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{660\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{255\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+15}{\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{15\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/30*((244*sin(d*x + c)/(cos(d*x + c) + 1) + 571*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 320*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 475*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 660*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 255*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 + 15)/(a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 4
*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*a^2*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 5*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4*a^2*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 - a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 60*log(
sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 15*sin(d*x + c)/(a^2*(cos(d*x + c) +
 1)))/d
```

mupad [B] time = 10.78, size = 216, normalized size = 1.66

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{-17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-44\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{95\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{64\tan\left(\frac{c}{2}\right.}{3}}{d\left(-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+10\,a^2\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^2*d) - (2*log(tan(c/2 + (d*x)/2)))/(a^2*d) - ((244*
tan(c/2 + (d*x)/2))/15 + (571*tan(c/2 + (d*x)/2)^2)/15 + (64*tan(c/2 + (d*x
)/2)^3)/3 - (95*tan(c/2 + (d*x)/2)^4)/3 - 44*tan(c/2 + (d*x)/2)^5 - 17*tan(
c/2 + (d*x)/2)^6 + 1)/(d*(8*a^2*tan(c/2 + (d*x)/2)^2 + 10*a^2*tan(c/2 + (d*
x)/2)^3 - 10*a^2*tan(c/2 + (d*x)/2)^5 - 8*a^2*tan(c/2 + (d*x)/2)^6 - 2*a^2*
tan(c/2 + (d*x)/2)^7 + 2*a^2*tan(c/2 + (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2*sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

3.786 $\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=158

$$-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{a^2d}-\frac{6\tan(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{9\sec^5(c+dx)}{10a^2d}+\frac{3\sec^3(c+dx)}{2a^2d}+\frac{9\sec(c+dx)}{2a^2d}-\frac{9}{}$$

```
[Out] -9/2*arctanh(cos(d*x+c))/a^2/d+2*cot(d*x+c)/a^2/d+9/2*sec(d*x+c)/a^2/d+3/2*
sec(d*x+c)^3/a^2/d+9/10*sec(d*x+c)^5/a^2/d-1/2*csc(d*x+c)^2*sec(d*x+c)^5/a^
2/d-6*tan(d*x+c)/a^2/d-2*tan(d*x+c)^3/a^2/d-2/5*tan(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.35, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2622, 302, 207, 2620, 270, 288}

$$-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{a^2d}-\frac{6\tan(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{9\sec^5(c+dx)}{10a^2d}+\frac{3\sec^3(c+dx)}{2a^2d}+\frac{9\sec(c+dx)}{2a^2d}-\frac{9}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-9*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (9*Sec[c
+ d*x])/(2*a^2*d) + (3*Sec[c + d*x]^3)/(2*a^2*d) + (9*Sec[c + d*x]^5)/(10*a
^2*d) - (Csc[c + d*x]^2*Sec[c + d*x]^5)/(2*a^2*d) - (6*Tan[c + d*x])/(a^2*d
) - (2*Tan[c + d*x]^3)/(a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^3(c+dx)\sec^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\csc(c+dx)\sec^6(c+dx) - 2a^2\csc^2(c+dx)\sec^6(c+dx) + a^2\csc^3(c+dx\right.}{a^4}$$
$$= \frac{\int \csc(c+dx)\sec^6(c+dx)\,dx}{a^2} + \frac{\int \csc^3(c+dx)\sec^6(c+dx)\,dx}{a^2} - \frac{2\int \csc^2(c+}{}$$
$$= \frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{a^2d} + \frac{\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^2d} - \frac{2\,\text{S}}{}$$
$$= -\frac{\csc^2(c+dx)\sec^5(c+dx)}{2a^2d} + \frac{\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right)dx, x, \sec(c+dx\right.}{a^2d}$$
$$= \frac{2\cot(c+dx)}{a^2d} + \frac{\sec(c+dx)}{a^2d} + \frac{\sec^3(c+dx)}{3a^2d} + \frac{\sec^5(c+dx)}{5a^2d} - \frac{\csc^2(c+dx)\sec^5}{2a^2d}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{2\cot(c+dx)}{a^2d} + \frac{9\sec(c+dx)}{2a^2d} + \frac{3\sec^3(c+dx)}{2a^2d} + \frac{9\,\text{se}}{}$$
$$= -\frac{9\tanh^{-1}(\cos(c+dx))}{2a^2d} + \frac{2\cot(c+dx)}{a^2d} + \frac{9\sec(c+dx)}{2a^2d} + \frac{3\sec^3(c+dx)}{2a^2d} + \frac{9}{}$$

Mathematica [B] time = 0.73, size = 328, normalized size = 2.08

$$-\frac{\csc^2(c+dx)\sec(c+dx)\left(-432\sin(c+dx) + 744\sin(2(c+dx)) - 176\sin(3(c+dx)) - 372\sin(4(c+dx)) + 128\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/320*(Csc[c + d*x]^2*Sec[c + d*x]*(-348 + 176*Cos[2*(c + d*x)] - 651*Cos[
3*(c + d*x)] + 332*Cos[4*(c + d*x)] + 93*Cos[5*(c + d*x)] - 630*Cos[3*(c +
d*x)]*Log[Cos[(c + d*x)/2]] + 90*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 1
8*Cos[c + d*x]*(31 + 30*Log[Cos[(c + d*x)/2]] - 30*Log[Sin[(c + d*x)/2]]) +
 630*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 90*Cos[5*(c + d*x)]*Log[Sin[(
c + d*x)/2]] - 432*Sin[c + d*x] + 744*Sin[2*(c + d*x)] + 720*Log[Cos[(c + d
*x)/2]]*Sin[2*(c + d*x)] - 720*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] - 176
*Sin[3*(c + d*x)] - 372*Sin[4*(c + d*x)] - 360*Log[Cos[(c + d*x)/2]]*Sin[4*
(c + d*x)] + 360*Log[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)] + 128*Sin[5*(c + d*
x)]))/(a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.47, size = 260, normalized size = 1.65

$$-\frac{166 \cos(dx+c)^4 - 144 \cos(dx+c)^2 + 45\left(\cos(dx+c)^5 - 3 \cos(dx+c)^3 - 2\left(\cos(dx+c)^3 - \cos(dx+c)\right)\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20*(166*cos(d*x + c)^4 - 144*cos(d*x + c)^2 + 45*(cos(d*x + c)^5 - 3*cos
(d*x + c)^3 - 2*(cos(d*x + c)^3 - cos(d*x + c))*sin(d*x + c) + 2*cos(d*x +
c))*log(1/2*cos(d*x + c) + 1/2) - 45*(cos(d*x + c)^5 - 3*cos(d*x + c)^3 - 2
*(cos(d*x + c)^3 - cos(d*x + c))*sin(d*x + c) + 2*cos(d*x + c))*log(-1/2*co
s(d*x + c) + 1/2) + 4*(32*cos(d*x + c)^4 - 35*cos(d*x + c)^2 - 2)*sin(d*x +
 c) - 12)/(a^2*d*cos(d*x + c)^5 - 3*a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x
+ c) - 2*(a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.23, size = 187, normalized size = 1.18

$$\frac{\frac{180 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2} + \frac{5\left(a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^4} - \frac{10}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)} - \frac{5\left(54\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/40*(180*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 5*(a^2*tan(1/2*d*x + 1/2*c)^
2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^4 - 10/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) -
 5*(54*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d*
x + 1/2*c)^2) + 2*(245*tan(1/2*d*x + 1/2*c)^4 + 870*tan(1/2*d*x + 1/2*c)^3
+ 1240*tan(1/2*d*x + 1/2*c)^2 + 810*tan(1/2*d*x + 1/2*c) + 211)/(a^2*(tan(1
/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.62, size = 219, normalized size = 1.39

$$-\frac{1}{4a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2d}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2}-\frac{1}{8a^2d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{1}{d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{9\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{2d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/a^2/d/(tan(1/2*d*x+1/2*c)-1)+1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^2*ta
n(1/2*d*x+1/2*c)-1/8/a^2/d/tan(1/2*d*x+1/2*c)^2+1/d/a^2/tan(1/2*d*x+1/2*c)+
9/2/d/a^2*ln(tan(1/2*d*x+1/2*c))+4/5/a^2/d/(tan(1/2*d*x+1/2*c)+1)^5-2/a^2/d
/(tan(1/2*d*x+1/2*c)+1)^4+5/a^2/d/(tan(1/2*d*x+1/2*c)+1)^3-11/2/a^2/d/(tan(
1/2*d*x+1/2*c)+1)^2+49/4/a^2/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.34, size = 354, normalized size = 2.24

$$\frac{\frac{\frac{20\sin(dx+c)}{\cos(dx+c)+1}+\frac{567\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1448\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{985\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{820\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1355\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{520\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-5}{\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{5a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{4a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{5\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2}+\frac{180}{}}{40d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/40*((20*sin(d*x + c)/(cos(d*x + c) + 1) + 567*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 1448*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 985*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 820*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1355*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 - 520*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5)
/(a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^2*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 5*a^2*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 - 4*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^2*s
in(d*x + c)^8/(cos(d*x + c) + 1)^8) - 5*(8*sin(d*x + c)/(cos(d*x + c) + 1)
- sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^2 + 180*log(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^2)/d
```

mupad [B] time = 10.09, size = 191, normalized size = 1.21

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8a^2d}+\frac{9\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2a^2d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-\frac{271\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{8}-\frac{41\tan\left(\frac{c}{2}+\right.}{2}\right.}{a^2d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) + (9*log(tan(c/2 + (d*x)/2)))/(2*a^2*d) - ta
n(c/2 + (d*x)/2)/(a^2*d) - (cot(c/2 + (d*x)/2)^2*(tan(c/2 + (d*x)/2)/2 + (5
67*tan(c/2 + (d*x)/2)^2)/40 + (181*tan(c/2 + (d*x)/2)^3)/5 + (197*tan(c/2 +
```

```
(d*x)/2)^4)/8 - (41*tan(c/2 + (d*x)/2)^5)/2 - (271*tan(c/2 + (d*x)/2)^6)/8
- 13*tan(c/2 + (d*x)/2)^7 - 1/8))/(a^2*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2
+ (d*x)/2) + 1)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.787 $\int \frac{\sin^4(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=151

$$\frac{\cos(c+dx)}{a^3d}+\frac{4\tan^7(c+dx)}{7a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{a^3d}-\frac{3\tan(c+dx)}{a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{13\sec^5(c+dx)}{5a^3d}-\frac{5\,\text{se}}{}$$

```
[Out] 3*x/a^3+cos(d*x+c)/a^3/d+7*sec(d*x+c)/a^3/d-5*sec(d*x+c)^3/a^3/d+13/5*sec(d
*x+c)^5/a^3/d-4/7*sec(d*x+c)^7/a^3/d-3*tan(d*x+c)/a^3/d+tan(d*x+c)^3/a^3/d-
3/5*tan(d*x+c)^5/a^3/d+4/7*tan(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.34, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2875, 2873, 2607, 30, 2606, 194, 3473, 8, 2590, 270}

$$\frac{\cos(c+dx)}{a^3d}+\frac{4\tan^7(c+dx)}{7a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{a^3d}-\frac{3\tan(c+dx)}{a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{13\sec^5(c+dx)}{5a^3d}-\frac{5\,\text{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^4*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*x)/a^3 + Cos[c + d*x]/(a^3*d) + (7*Sec[c + d*x])/(a^3*d) - (5*Sec[c + d*
x]^3)/(a^3*d) + (13*Sec[c + d*x]^5)/(5*a^3*d) - (4*Sec[c + d*x]^7)/(7*a^3*d
) - (3*Tan[c + d*x])/(a^3*d) + Tan[c + d*x]^3/(a^3*d) - (3*Tan[c + d*x]^5)/
(5*a^3*d) + (4*Tan[c + d*x]^7)/(7*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^2(c+dx)(a-a\sin(c+dx))^3\tan^6(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(a^3\sec^2(c+dx)\tan^6(c+dx) - 3a^3\sec(c+dx)\tan^7(c+dx) + 3a^3\tan^8(c+ \ldots\right.}{a^6}$$

$$= \frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a^3} - \frac{\int \sin(c+dx)\tan^8(c+dx)\,dx}{a^3} - \frac{3\int \sec(c+ \ldots}{}$$

$$= \frac{3\tan^7(c+dx)}{7a^3d} - \frac{3\int \tan^6(c+dx)\,dx}{a^3} + \frac{\text{Subst}\left(\int x^6\,dx, x, \tan(c+dx)\right)}{a^3d} + \frac{\text{Subs}\ldots}{}$$

$$= -\frac{3\tan^5(c+dx)}{5a^3d} + \frac{4\tan^7(c+dx)}{7a^3d} + \frac{3\int \tan^4(c+dx)\,dx}{a^3} + \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^8} - \ldots\right.\right.}{}$$

$$= \frac{\cos(c+dx)}{a^3d} + \frac{7\sec(c+dx)}{a^3d} - \frac{5\sec^3(c+dx)}{a^3d} + \frac{13\sec^5(c+dx)}{5a^3d} - \frac{4\sec^7(c+d\ldots}{7a^3d}$$

$$= \frac{\cos(c+dx)}{a^3d} + \frac{7\sec(c+dx)}{a^3d} - \frac{5\sec^3(c+dx)}{a^3d} + \frac{13\sec^5(c+dx)}{5a^3d} - \frac{4\sec^7(c+d\ldots}{7a^3d}$$

$$= \frac{3x}{a^3} + \frac{\cos(c+dx)}{a^3d} + \frac{7\sec(c+dx)}{a^3d} - \frac{5\sec^3(c+dx)}{a^3d} + \frac{13\sec^5(c+dx)}{5a^3d} - \frac{4\sec^7\ldots}{7\ldots}$$

Mathematica [A] time = 0.68, size = 224, normalized size = 1.48

$$\frac{8008\sin(c+dx) + 11760c\sin(2(c+dx)) + 11760dx\sin(2(c+dx)) - 20762\sin(2(c+dx)) + 6588\sin(3(c+dx)) \ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^4*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (8400 + 14*(-1483 + 840*c + 840*d*x)*Cos[c + d*x] + 5152*Cos[2*(c + d*x)] +
 8898*Cos[3*(c + d*x)] - 5040*c*Cos[3*(c + d*x)] - 5040*d*x*Cos[3*(c + d*x)
] - 2288*Cos[4*(c + d*x)] + 8008*Sin[c + d*x] - 20762*Sin[2*(c + d*x)] + 11
760*c*Sin[2*(c + d*x)] + 11760*d*x*Sin[2*(c + d*x)] + 6588*Sin[3*(c + d*x)]
 + 1483*Sin[4*(c + d*x)] - 840*c*Sin[4*(c + d*x)] - 840*d*x*Sin[4*(c + d*x)
] - 140*Sin[5*(c + d*x)])/(2240*a^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

fricas [A] time = 0.46, size = 159, normalized size = 1.05

$$\frac{315\,dx\cos(dx+c)^3 + 286\cos(dx+c)^4 - 420\,dx\cos(dx+c) - 447\cos(dx+c)^2 + \left(105\,dx\cos(dx+c)^3 + 35\,c\ldots\right.}{35\left(3\,a^3d\cos(dx+c)^3 - 4\,a^3d\cos(dx+c) + \left(a^3d\cos(dx+c)^3 - 4\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/35*(315*d*x*cos(d*x + c)^3 + 286*cos(d*x + c)^4 - 420*d*x*cos(d*x + c) -
447*cos(d*x + c)^2 + (105*d*x*cos(d*x + c)^3 + 35*cos(d*x + c)^4 - 420*d*x*
cos(d*x + c) - 438*cos(d*x + c)^2 - 20)*sin(d*x + c) - 15)/(3*a^3*d*cos(d*x
 + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x +
c))*sin(d*x + c))
```

giac [A] time = 0.30, size = 177, normalized size = 1.17

$$\frac{\frac{840\,(dx+c)}{a^3} - \frac{35\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)a^3} + \frac{1715\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+11480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+31815\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+4}{a^3\left(\tan\left(\frac{1}{2}\right.\right.}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/280*(840*(d*x + c)/a^3 - 35*(tan(1/2*d*x + 1/2*c)^2 - 16*tan(1/2*d*x + 1/
2*c) + 17)/((tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x
+ 1/2*c) - 1)*a^3) + (1715*tan(1/2*d*x + 1/2*c)^6 + 11480*tan(1/2*d*x + 1/2
*c)^5 + 31815*tan(1/2*d*x + 1/2*c)^4 + 45920*tan(1/2*d*x + 1/2*c)^3 + 35161
*tan(1/2*d*x + 1/2*c)^2 + 13832*tan(1/2*d*x + 1/2*c) + 2221)/(a^3*(tan(1/2*
d*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.53, size = 211, normalized size = 1.40

$$-\frac{1}{8a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2}{a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d}-\frac{8}{7a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{}{a^3d\left(\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/8/a^3/d/(tan(1/2*d*x+1/2*c)-1)+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)+6/a^3/d*
arctan(tan(1/2*d*x+1/2*c))-8/7/a^3/d/(tan(1/2*d*x+1/2*c)+1)^7+4/a^3/d/(tan(
1/2*d*x+1/2*c)+1)^6-14/5/a^3/d/(tan(1/2*d*x+1/2*c)+1)^5-3/a^3/d/(tan(1/2*d*
x+1/2*c)+1)^4+1/2/a^3/d/(tan(1/2*d*x+1/2*c)+1)^3+17/4/a^3/d/(tan(1/2*d*x+1/
2*c)+1)^2+49/8/a^3/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.44, size = 421, normalized size = 2.79

$$\frac{2\left(\frac{\frac{951\sin(dx+c)}{\cos(dx+c)+1}+\frac{2010\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1980\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{574\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{966\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1890\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{1540\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{630\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{105\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+176}{a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{15a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{20a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{14a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{14a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{20a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{15a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{6a^3\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\cdots\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 2/35*((951*sin(d*x + c)/(cos(d*x + c) + 1) + 2010*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1980*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 574*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 - 966*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1890*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 - 1540*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
 630*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 105*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 + 176)/(a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 15*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 20*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
+ 14*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 14*a^3*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 - 20*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 15*a^3*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - 6*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
 a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 105*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^3)/d
```

mupad [B] time = 15.50, size = 182, normalized size = 1.21

$$\frac{3x}{a^3}-\frac{-6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-36\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-88\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-108\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-\frac{276\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{5}+\frac{164\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{5}\cdots}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^7\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (3*x)/a^3 - ((1902*tan(c/2 + (d*x)/2))/35 + (804*tan(c/2 + (d*x)/2)^2)/7 +
(792*tan(c/2 + (d*x)/2)^3)/7 + (164*tan(c/2 + (d*x)/2)^4)/5 - (276*tan(c/2
+ (d*x)/2)^5)/5 - 108*tan(c/2 + (d*x)/2)^6 - 88*tan(c/2 + (d*x)/2)^7 - 36*t
an(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^9 + 352/35)/(a^3*d*(tan(c/2 + (d
*x)/2) + 1)^7*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/
2)^3 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.788 $\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=142

$$-\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{\tan^3(c+dx)}{3a^3d}+\frac{\tan(c+dx)}{a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{11\sec^5(c+dx)}{5a^3d}+\frac{10\sec^3(c+dx)}{3a^3d}-\frac{3}{}$$

```
[Out] -x/a^3-3*sec(d*x+c)/a^3/d+10/3*sec(d*x+c)^3/a^3/d-11/5*sec(d*x+c)^5/a^3/d+4
/7*sec(d*x+c)^7/a^3/d+tan(d*x+c)/a^3/d-1/3*tan(d*x+c)^3/a^3/d+1/5*tan(d*x+c
)^5/a^3/d-4/7*tan(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.34, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2875, 2873, 2606, 270, 2607, 30, 194, 3473, 8}

$$-\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{\tan^3(c+dx)}{3a^3d}+\frac{\tan(c+dx)}{a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{11\sec^5(c+dx)}{5a^3d}+\frac{10\sec^3(c+dx)}{3a^3d}-\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - (3*Sec[c + d*x])/(a^3*d) + (10*Sec[c + d*x]^3)/(3*a^3*d) - (11*S
ec[c + d*x]^5)/(5*a^3*d) + (4*Sec[c + d*x]^7)/(7*a^3*d) + Tan[c + d*x]/(a^3
*d) - Tan[c + d*x]^3/(3*a^3*d) + Tan[c + d*x]^5/(5*a^3*d) - (4*Tan[c + d*x]
^7)/(7*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^3(c+dx)(a-a\sin(c+dx))^3\tan^5(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(a^3\sec^3(c+dx)\tan^5(c+dx) - 3a^3\sec^2(c+dx)\tan^6(c+dx) + 3a^3\sec(c+ a\right.}{a^6}$$
$$= \frac{\int \sec^3(c+dx)\tan^5(c+dx)\,dx}{a^3} - \frac{\int \tan^8(c+dx)\,dx}{a^3} - \frac{3\int \sec^2(c+dx)\tan^6(c-}{a^3}$$
$$= -\frac{\tan^7(c+dx)}{7a^3d} + \frac{\int \tan^6(c+dx)\,dx}{a^3} + \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \sec(c+dx\right.}{a^3d}$$
$$= \frac{\tan^5(c+dx)}{5a^3d} - \frac{4\tan^7(c+dx)}{7a^3d} - \frac{\int \tan^4(c+dx)\,dx}{a^3} + \frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\right.}{a^3d}$$
$$= -\frac{3\sec(c+dx)}{a^3d} + \frac{10\sec^3(c+dx)}{3a^3d} - \frac{11\sec^5(c+dx)}{5a^3d} + \frac{4\sec^7(c+dx)}{7a^3d} - \frac{\tan^3(c}{3a^3}$$
$$= -\frac{3\sec(c+dx)}{a^3d} + \frac{10\sec^3(c+dx)}{3a^3d} - \frac{11\sec^5(c+dx)}{5a^3d} + \frac{4\sec^7(c+dx)}{7a^3d} + \frac{\tan(c-}{a^3d}$$
$$= -\frac{x}{a^3} - \frac{3\sec(c+dx)}{a^3d} + \frac{10\sec^3(c+dx)}{3a^3d} - \frac{11\sec^5(c+dx)}{5a^3d} + \frac{4\sec^7(c+dx)}{7a^3d} + \frac{ta}{}$$

Mathematica [A] time = 0.80, size = 214, normalized size = 1.51

$$-\frac{2688\sin(c+dx) + 11760c\sin(2(c+dx)) + 11760dx\sin(2(c+dx)) - 23282\sin(2(c+dx)) + 5568\sin(3(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/6720*(4200 + 14*(-1663 + 840*c + 840*d*x)*Cos[c + d*x] + 6272*Cos[2*(c +
 d*x)] + 9978*Cos[3*(c + d*x)] - 5040*c*Cos[3*(c + d*x)] - 5040*d*x*Cos[3*(
c + d*x)] - 1768*Cos[4*(c + d*x)] + 2688*Sin[c + d*x] - 23282*Sin[2*(c + d*
x)] + 11760*c*Sin[2*(c + d*x)] + 11760*d*x*Sin[2*(c + d*x)] + 5568*Sin[3*(c
 + d*x)] + 1663*Sin[4*(c + d*x)] - 840*c*Sin[4*(c + d*x)] - 840*d*x*Sin[4*(
c + d*x)])/(a^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^7)
```

fricas [A] time = 0.45, size = 150, normalized size = 1.06

$$-\frac{315\,dx\cos(dx+c)^3 + 221\,\cos(dx+c)^4 - 420\,dx\cos(dx+c) - 417\,\cos(dx+c)^2 + 3\left(35\,dx\cos(dx+c)^3 - 14\right.}{105\left(3\,a^3d\cos(dx+c)^3 - 4\,a^3d\cos(dx+c) + \left(a^3d\cos(dx+c)^3 - 4\,a^3d\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/105*(315*d*x*cos(d*x + c)^3 + 221*cos(d*x + c)^4 - 420*d*x*cos(d*x + c)
- 417*cos(d*x + c)^2 + 3*(35*d*x*cos(d*x + c)^3 - 140*d*x*cos(d*x + c) - 11
6*cos(d*x + c)^2 + 15)*sin(d*x + c) + 60)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d
*cos(d*x + c) + (a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.30, size = 129, normalized size = 0.91

$$-\frac{\frac{840\,(dx+c)}{a^3}+\frac{105}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{1575\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+10920\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+31675\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+48160\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36981\,\tan}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/840*(840*(d*x + c)/a^3 + 105/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) + (1575*ta
n(1/2*d*x + 1/2*c)^6 + 10920*tan(1/2*d*x + 1/2*c)^5 + 31675*tan(1/2*d*x + 1
/2*c)^4 + 48160*tan(1/2*d*x + 1/2*c)^3 + 36981*tan(1/2*d*x + 1/2*c)^2 + 143
92*tan(1/2*d*x + 1/2*c) + 2281)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.50, size = 187, normalized size = 1.32

$$-\frac{1}{8a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d}+\frac{8}{7a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}-\frac{4}{a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{}{5a^3d\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/8/a^3/d/(tan(1/2*d*x+1/2*c)-1)-2/a^3/d*arctan(tan(1/2*d*x+1/2*c))+8/7/a^
3/d/(tan(1/2*d*x+1/2*c)+1)^7-4/a^3/d/(tan(1/2*d*x+1/2*c)+1)^6+18/5/a^3/d/(t
an(1/2*d*x+1/2*c)+1)^5+1/a^3/d/(tan(1/2*d*x+1/2*c)+1)^4-5/6/a^3/d/(tan(1/2*
d*x+1/2*c)+1)^3-7/4/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2-15/8/a^3/d/(tan(1/2*d*x+
1/2*c)+1)
```

maxima [B] time = 0.43, size = 335, normalized size = 2.36

$$-\frac{2\left(\frac{\frac{711\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{1274\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{469\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1260\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{1435\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{630\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{105\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+136}{a^3+\frac{6\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14\,a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6\,a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{105\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -2/105*((711*sin(d*x + c)/(cos(d*x + c) + 1) + 1274*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 469*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1260*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 1435*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 630*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 105*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
+ 136)/(a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^3
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x + c)
+ 1)^6 - 6*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/(co
s(d*x + c) + 1)^8) + 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 15.65, size = 131, normalized size = 0.92

$$\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{82\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{134\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{15}+\frac{364\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{474\tan\left(\cdots\right.}{35}\cdots}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] ((474*tan(c/2 + (d*x)/2))/35 + (364*tan(c/2 + (d*x)/2)^2)/15 + (134*tan(c/2
 + (d*x)/2)^3)/15 - 24*tan(c/2 + (d*x)/2)^4 - (82*tan(c/2 + (d*x)/2)^5)/3 -
 12*tan(c/2 + (d*x)/2)^6 - 2*tan(c/2 + (d*x)/2)^7 + 272/105)/(a^3*d*(tan(c/
2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1)^7) - x/a^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.789 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3} dx$

Optimal. Leaf size=102

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{9\sec^5(c+dx)}{5a^3d}-\frac{2\sec^3(c+dx)}{a^3d}+\frac{\sec(c+dx)}{a^3d}$$

```
[Out] sec(d*x+c)/a^3/d-2*sec(d*x+c)^3/a^3/d+9/5*sec(d*x+c)^5/a^3/d-4/7*sec(d*x+c)
^7/a^3/d+1/5*tan(d*x+c)^5/a^3/d+4/7*tan(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.33, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2607, 14, 2606, 270, 30, 194}

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{9\sec^5(c+dx)}{5a^3d}-\frac{2\sec^3(c+dx)}{a^3d}+\frac{\sec(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]/(a^3*d) - (2*Sec[c + d*x]^3)/(a^3*d) + (9*Sec[c + d*x]^5)/(5*a
^3*d) - (4*Sec[c + d*x]^7)/(7*a^3*d) + Tan[c + d*x]^5/(5*a^3*d) + (4*Tan[c
+ d*x]^7)/(7*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^4(c+dx)(a-a\sin(c+dx))^3\tan^4(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(a^3\sec^4(c+dx)\tan^4(c+dx) - 3a^3\sec^3(c+dx)\tan^5(c+dx) + 3a^3\sec^2(c\right.}{a^6}$$
$$= \frac{\int \sec^4(c+dx)\tan^4(c+dx)\,dx}{a^3} - \frac{\int \sec(c+dx)\tan^7(c+dx)\,dx}{a^3} - \frac{3\int \sec^3(c}{}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\text{Subst}\left(\int x^4\left(1+x^2\right)\,dx, x, \tan(c\right.}{a^3d}$$
$$= \frac{3\tan^7(c+dx)}{7a^3d} - \frac{\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\text{Subs}}{}$$
$$= \frac{\sec(c+dx)}{a^3d} - \frac{2\sec^3(c+dx)}{a^3d} + \frac{9\sec^5(c+dx)}{5a^3d} - \frac{4\sec^7(c+dx)}{7a^3d} + \frac{\tan^5(c+d}{5a^3d}$$

Mathematica [A] time = 0.50, size = 104, normalized size = 1.02

$$\frac{\sec(c+dx)(1344\sin(c+dx) - 1946\sin(2(c+dx)) + 64\sin(3(c+dx)) + 139\sin(4(c+dx)) - 1946\cos(c+dx)}{2240a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(840 - 1946*Cos[c + d*x] - 224*Cos[2*(c + d*x)] + 834*Cos[3*(
c + d*x)] - 104*Cos[4*(c + d*x)] + 1344*Sin[c + d*x] - 1946*Sin[2*(c + d*x)
] + 64*Sin[3*(c + d*x)] + 139*Sin[4*(c + d*x)]))/(2240*a^3*d*(1 + Sin[c + d
*x])^3)
```

fricas [A] time = 0.42, size = 104, normalized size = 1.02

$$\frac{13\cos(dx+c)^4 - 6\cos(dx+c)^2 - 4\left(\cos(dx+c)^2+5\right)\sin(dx+c) - 15}{35\left(3a^3d\cos(dx+c)^3 - 4a^3d\cos(dx+c) + \left(a^3d\cos(dx+c)^3 - 4a^3d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/35*(13*cos(d*x + c)^4 - 6*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + 5)*sin(d*x
 + c) - 15)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x
 + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.26, size = 120, normalized size = 1.18

$$-\frac{\frac{35}{a^{3}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+280\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1015\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+2240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1673\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+616\,\tan\left(\frac{1}{2}\right.}{a^{3}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{7}}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) - (35*tan(1/2*d*x + 1/2*c)^6 +
280*tan(1/2*d*x + 1/2*c)^5 + 1015*tan(1/2*d*x + 1/2*c)^4 + 2240*tan(1/2*d*x
 + 1/2*c)^3 + 1673*tan(1/2*d*x + 1/2*c)^2 + 616*tan(1/2*d*x + 1/2*c) + 93)/
(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.50, size = 130, normalized size = 1.27

$$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{22}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 32/d/a^3*(-1/256/(tan(1/2*d*x+1/2*c)-1)-1/28/(tan(1/2*d*x+1/2*c)+1)^7+1/8/(
tan(1/2*d*x+1/2*c)+1)^6-11/80/(tan(1/2*d*x+1/2*c)+1)^5+1/32/(tan(1/2*d*x+1/
2*c)+1)^4+1/64/(tan(1/2*d*x+1/2*c)+1)^3+1/128/(tan(1/2*d*x+1/2*c)+1)^2+1/25
6/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.34, size = 230, normalized size = 2.25

$$\frac{16\left(\frac{6\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{14\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+1\right)}{35\left(a^{3}+\frac{6\,a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{14\,a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14\,a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{14\,a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{6\,a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 16/35*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 14*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 14*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/((a^3 + 6*a^3*sin(d*x
+ c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 14*a
^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^3*sin(d*x + c)^5/(cos(d*x + c
```

```
) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*a^3*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

mupad [B] time = 9.69, size = 135, normalized size = 1.32

$$\frac{\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{35}+\frac{96\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35}+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{5}}{a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] ((16*cos(c/2 + (d*x)/2)^8)/35 + (96*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)
)/35 + (32*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3)/5 + (32*cos(c/2 + (d*
x)/2)^6*sin(c/2 + (d*x)/2)^2)/5)/(a^3*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*
x)/2))*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.790 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$-\frac{4\tan^7(c+dx)}{7a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{7\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx)}{a^3d}$$

```
[Out] sec(d*x+c)^3/a^3/d-7/5*sec(d*x+c)^5/a^3/d+4/7*sec(d*x+c)^7/a^3/d-3/5*tan(d*
x+c)^5/a^3/d-4/7*tan(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.31, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2873, 2606, 14, 2607, 270, 30}

$$-\frac{4\tan^7(c+dx)}{7a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{7\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]^3/(a^3*d) - (7*Sec[c + d*x]^5)/(5*a^3*d) + (4*Sec[c + d*x]^7)/
(7*a^3*d) - (3*Tan[c + d*x]^5)/(5*a^3*d) - (4*Tan[c + d*x]^7)/(7*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^5(c+dx)(a-a\sin(c+dx))^3\tan^3(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(a^3\sec^5(c+dx)\tan^3(c+dx)-3a^3\sec^4(c+dx)\tan^4(c+dx)+3a^3\sec^3(c+\right.}{a^6}$$

$$= \frac{\int \sec^5(c+dx)\tan^3(c+dx)\,dx}{a^3}-\frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a^3}-\frac{3\int \sec^4(c}{}$$

$$= -\frac{\text{Subst}\left(\int x^6\,dx,x,\tan(c+dx)\right)}{a^3d}+\frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)dx,x,\sec(c+dx)\right)}{a^3d}$$

$$= -\frac{\tan^7(c+dx)}{7a^3d}+\frac{\text{Subst}\left(\int\left(-x^4+x^6\right)dx,x,\sec(c+dx)\right)}{a^3d}+\frac{3\,\text{Subst}\left(\int\left(x^2-2\right.\right.}{}$$

$$= \frac{\sec^3(c+dx)}{a^3d}-\frac{7\sec^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}-\frac{4\tan^7(c+}{7a^3d}$$

Mathematica [A] time = 0.38, size = 104, normalized size = 1.18

$$\frac{\sec(c+dx)(1008\sin(c+dx)-602\sin(2(c+dx))+48\sin(3(c+dx))+43\sin(4(c+dx))-602\cos(c+dx)-448}{2240a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(840 - 602*Cos[c + d*x] - 448*Cos[2*(c + d*x)] + 258*Cos[3*(c
 + d*x)] - 8*Cos[4*(c + d*x)] + 1008*Sin[c + d*x] - 602*Sin[2*(c + d*x)] +
48*Sin[3*(c + d*x)] + 43*Sin[4*(c + d*x)]))/(2240*a^3*d*(1 + Sin[c + d*x])^
3)
```

fricas [A] time = 0.43, size = 102, normalized size = 1.16

$$\frac{\cos(dx+c)^4+13\cos(dx+c)^2-3\left(\cos(dx+c)^2+5\right)\sin(dx+c)-20}{35\left(3a^3d\cos(dx+c)^3-4a^3d\cos(dx+c)+\left(a^3d\cos(dx+c)^3-4a^3d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/35*(cos(d*x + c)^4 + 13*cos(d*x + c)^2 - 3*(cos(d*x + c)^2 + 5)*sin(d*x +
 c) - 20)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x +
 c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.25, size = 120, normalized size = 1.36

$$-\frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}-\frac{35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+280\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+1015\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+1001\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+392\tan\left(\frac{1}{2}\right.}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) - (35*tan(1/2*d*x + 1/2*c)^6 +
280*tan(1/2*d*x + 1/2*c)^5 + 1015*tan(1/2*d*x + 1/2*c)^4 + 1120*tan(1/2*d*x
 + 1/2*c)^3 + 1001*tan(1/2*d*x + 1/2*c)^2 + 392*tan(1/2*d*x + 1/2*c) + 61)/
(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.49, size = 130, normalized size = 1.48

$$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{26}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 16/d/a^3*(-1/128/(tan(1/2*d*x+1/2*c)-1)+1/14/(tan(1/2*d*x+1/2*c)+1)^7-1/4/(
tan(1/2*d*x+1/2*c)+1)^6+13/40/(tan(1/2*d*x+1/2*c)+1)^5-3/16/(tan(1/2*d*x+1/
2*c)+1)^4+1/32/(tan(1/2*d*x+1/2*c)+1)^3+1/64/(tan(1/2*d*x+1/2*c)+1)^2+1/128
/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.34, size = 250, normalized size = 2.84

$$\frac{4\left(\frac{18\sin(dx+c)}{\cos(dx+c)+1}+\frac{42\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{42\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{35\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3\right)}{35\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 4/35*(18*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 42*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 35*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 3)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 - 14*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 - 6*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

mupad [B] time = 9.73, size = 158, normalized size = 1.80

$$\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4+18\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+42\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+42\cos\left(\right.\right.}{35a^3d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (4*cos(c/2 + (d*x)/2)^4*(3*cos(c/2 + (d*x)/2)^4 + 35*sin(c/2 + (d*x)/2)^4 +
 42*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^3 + 18*cos(c/2 + (d*x)/2)^3*sin(c
/2 + (d*x)/2) + 42*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2))/(35*a^3*d*(c
os(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*
x)/2))^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.791 $\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\sec^5(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{3a^3d}$$

```
[Out] -1/3*sec(d*x+c)^3/a^3/d+sec(d*x+c)^5/a^3/d-4/7*sec(d*x+c)^7/a^3/d+1/3*tan(d
*x+c)^3/a^3/d+tan(d*x+c)^5/a^3/d+4/7*tan(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.24, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2711, 2607, 270, 2606, 14}

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\sec^5(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -Sec[c + d*x]^3/(3*a^3*d) + Sec[c + d*x]^5/(a^3*d) - (4*Sec[c + d*x]^7)/(7*
a^3*d) + Tan[c + d*x]^3/(3*a^3*d) + Tan[c + d*x]^5/(a^3*d) + (4*Tan[c + d*x
]^7)/(7*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \left(a^3\sec^6(c+dx)\tan^2(c+dx) - 3a^3\sec^5(c+dx)\tan^3(c+dx) + 3a^3\sec^4(c+dx)\,\mathrm{ta}\right.}{a^6}$$

$$= \frac{\int \sec^6(c+dx)\tan^2(c+dx)\,dx}{a^3} - \frac{\int \sec^3(c+dx)\tan^5(c+dx)\,dx}{a^3} - \frac{3\int \sec^5(c+dx)\,\mathrm{t}}{a^3}$$

$$= -\frac{\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\mathrm{Subst}\left(\int x^2\left(1+x^2\right)^2\,dx, x, \tan(c+\right.}{a^3d}$$

$$= -\frac{\mathrm{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\mathrm{Subst}\left(\int \left(x^2+2x^4+x^6\right)\,dx, x, \tan\right.}{a^3d}$$

$$= -\frac{\sec^3(c+dx)}{3a^3d} + \frac{\sec^5(c+dx)}{a^3d} - \frac{4\sec^7(c+dx)}{7a^3d} + \frac{\tan^3(c+dx)}{3a^3d} + \frac{\tan^5(c+dx)}{a^3d} + \frac{4\,\mathrm{t}}{}$$

Mathematica [A] time = 0.35, size = 104, normalized size = 1.01

$$\frac{\sec(c+dx)(672\sin(c+dx) - 70\sin(2(c+dx)) - 96\sin(3(c+dx)) + 5\sin(4(c+dx)) - 70\cos(c+dx) - 224\cos}{1344a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(336 - 70*Cos[c + d*x] - 224*Cos[2*(c + d*x)] + 30*Cos[3*(c +
 d*x)] + 16*Cos[4*(c + d*x)] + 672*Sin[c + d*x] - 70*Sin[2*(c + d*x)] - 96*
Sin[3*(c + d*x)] + 5*Sin[4*(c + d*x)]))/(1344*a^3*d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.43, size = 104, normalized size = 1.01

$$-\frac{2\cos(dx+c)^4 - 9\cos(dx+c)^2 - 6\left(\cos(dx+c)^2-2\right)\sin(dx+c) + 9}{21\left(3a^3d\cos(dx+c)^3 - 4a^3d\cos(dx+c) + \left(a^3d\cos(dx+c)^3 - 4a^3d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/21*(2*cos(d*x + c)^4 - 9*cos(d*x + c)^2 - 6*(cos(d*x + c)^2 - 2)*sin(d*x
 + c) + 9)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x
+ c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.27, size = 120, normalized size = 1.17

$$-\frac{\frac{21}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+168\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+161\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+224\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+63\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+56\,\tan\left(\frac{1}{2}\,dx+\right.}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/168*(21/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) - (21*tan(1/2*d*x + 1/2*c)^6 +
168*tan(1/2*d*x + 1/2*c)^5 + 161*tan(1/2*d*x + 1/2*c)^4 + 224*tan(1/2*d*x +
 1/2*c)^3 + 63*tan(1/2*d*x + 1/2*c)^2 + 56*tan(1/2*d*x + 1/2*c) + 11)/(a^3*
(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.47, size = 130, normalized size = 1.26

$$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}-\frac{6}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{5}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{13}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 8/d/a^3*(-1/64/(tan(1/2*d*x+1/2*c)-1)-1/7/(tan(1/2*d*x+1/2*c)+1)^7+1/2/(tan
(1/2*d*x+1/2*c)+1)^6-3/4/(tan(1/2*d*x+1/2*c)+1)^5+5/8/(tan(1/2*d*x+1/2*c)+1
)^4-13/48/(tan(1/2*d*x+1/2*c)+1)^3+1/32/(tan(1/2*d*x+1/2*c)+1)^2+1/64/(tan(
1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.34, size = 270, normalized size = 2.62

$$\frac{4\left(\frac{6\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+1\right)}{21\left(a^3+\frac{6\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14\,a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6\,a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")

[Out] 4/21*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 14*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 28*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 14*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a^3 + 6*a^3*si
n(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 14*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^3*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*a^3*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)

mupad [B] time = 10.15, size = 183, normalized size = 1.78

$$\frac{\frac{4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{21}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{7}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d}{2}\right.}{a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c + d*x)^2/(cos(c + d*x)^2*(a + a*sin(c + d*x))^3),x)

[Out] ((4*cos(c/2 + (d*x)/2)^8)/21 + (8*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2))/
7 + (8*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^5)/3 + 4*cos(c/2 + (d*x)/2)^
4*sin(c/2 + (d*x)/2)^4 + (16*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3)/3 +
 (8*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2)/3)/(a^3*d*(cos(c/2 + (d*x)/2
) - sin(c/2 + (d*x)/2))*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^7)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^2(c+dx)\sec^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)

[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)*
*2 + 3*sin(c + d*x) + 1), x)/a**3

3.792 $\int \frac{\sec(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=99

$$\frac{6\tan(c+dx)}{35a^3d}-\frac{3\sec(c+dx)}{35d\left(a^3\sin(c+dx)+a^3\right)}-\frac{3\sec(c+dx)}{35ad(a\sin(c+dx)+a)^2}+\frac{\sec(c+dx)}{7d(a\sin(c+dx)+a)^3}$$

```
[Out] 1/7*sec(d*x+c)/d/(a+a*sin(d*x+c))^3-3/35*sec(d*x+c)/a/d/(a+a*sin(d*x+c))^2-
3/35*sec(d*x+c)/d/(a^3+a^3*sin(d*x+c))+6/35*tan(d*x+c)/a^3/d
```

Rubi [A] time = 0.14, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2859, 2672, 3767, 8}

$$\frac{6\tan(c+dx)}{35a^3d}-\frac{3\sec(c+dx)}{35d\left(a^3\sin(c+dx)+a^3\right)}-\frac{3\sec(c+dx)}{35ad(a\sin(c+dx)+a)^2}+\frac{\sec(c+dx)}{7d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]/(7*d*(a + a*Sin[c + d*x])^3) - (3*Sec[c + d*x])/(35*a*d*(a + a
*Sin[c + d*x])^2) - (3*Sec[c + d*x])/(35*d*(a^3 + a^3*Sin[c + d*x])) + (6*T
an[c + d*x])/(35*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} + \frac{3\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{7a}$$

$$= \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{3\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} + \frac{9\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx}{35a^2}$$

$$= \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{3\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} - \frac{3\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)} +$$

$$= \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{3\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} - \frac{3\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)} -$$

$$= \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{3\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} - \frac{3\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)} +$$

Mathematica [A] time = 0.38, size = 104, normalized size = 1.05

$$\frac{\sec(c+dx)(672\sin(c+dx)+182\sin(2(c+dx))-288\sin(3(c+dx))-13\sin(4(c+dx))+182\cos(c+dx)-672}{2240a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(560 + 182*Cos[c + d*x] - 672*Cos[2*(c + d*x)] - 78*Cos[3*(c
+ d*x)] + 48*Cos[4*(c + d*x)] + 672*Sin[c + d*x] + 182*Sin[2*(c + d*x)] - 2
88*Sin[3*(c + d*x)] - 13*Sin[4*(c + d*x)]))/(2240*a^3*d*(1 + Sin[c + d*x])^
3)
```

fricas [A] time = 0.45, size = 106, normalized size = 1.07

$$-\frac{6\cos(dx+c)^4 - 27\cos(dx+c)^2 - 3\left(6\cos(dx+c)^2-5\right)\sin(dx+c) + 20}{35\left(3a^3d\cos(dx+c)^3 - 4a^3d\cos(dx+c) + \left(a^3d\cos(dx+c)^3 - 4a^3d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/35*(6*cos(d*x + c)^4 - 27*cos(d*x + c)^2 - 3*(6*cos(d*x + c)^2 - 5)*sin(
d*x + c) + 20)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(
d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.27, size = 120, normalized size = 1.21

$$-\frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-280\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-665\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-791\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-392\,\tan\left(\frac{1}{2}\,\cdots\right.}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) - (35*tan(1/2*d*x + 1/2*c)^6 -
280*tan(1/2*d*x + 1/2*c)^5 - 665*tan(1/2*d*x + 1/2*c)^4 - 1120*tan(1/2*d*x
+ 1/2*c)^3 - 791*tan(1/2*d*x + 1/2*c)^2 - 392*tan(1/2*d*x + 1/2*c) - 51)/(a
^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.45, size = 130, normalized size = 1.31

$$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{34}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{7}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{9}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{7}{4\left(\tan\left(\frac{dx}{2}+\cdots\right.\right.}}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4/d/a^3*(-1/32/(tan(1/2*d*x+1/2*c)-1)+2/7/(tan(1/2*d*x+1/2*c)+1)^7-1/(tan(1
/2*d*x+1/2*c)+1)^6+17/10/(tan(1/2*d*x+1/2*c)+1)^5-7/4/(tan(1/2*d*x+1/2*c)+1
)^4+9/8/(tan(1/2*d*x+1/2*c)+1)^3-7/16/(tan(1/2*d*x+1/2*c)+1)^2+1/32/(tan(1/
2*d*x+1/2*c)+1))
```

maxima [B] time = 0.34, size = 290, normalized size = 2.93

$$-\frac{2\left(\frac{6\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{21\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{56\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{105\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{70\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{35\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}{35\left(a^3+\frac{6\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14\,a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6\,a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^3\sin(dx+c)\cdots}{(\cos(dx+c)+1\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/35*(6*sin(d*x + c)/(cos(d*x + c) + 1) - 21*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 56*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 105*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 70*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 35*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + 1)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) +
14*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 - 14*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 - 6*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a
^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

mupad [B] time = 10.01, size = 206, normalized size = 2.08

$$\frac{2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-6\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+21\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+56\cos\left(\frac{c}{2}+\right.\right.}{35\,a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (2*cos(c/2 + (d*x)/2)^2*(35*sin(c/2 + (d*x)/2)^6 - cos(c/2 + (d*x)/2)^6 + 7
0*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^5 - 6*cos(c/2 + (d*x)/2)^5*sin(c/2
+ (d*x)/2) + 105*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 56*cos(c/2 + (
d*x)/2)^3*sin(c/2 + (d*x)/2)^3 + 21*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)
^2))/(35*a^3*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))*(cos(c/2 + (d*x)/2
) + sin(c/2 + (d*x)/2))^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin\left(c+dx\right)\sec^2\left(c+dx\right)}{\sin^3\left(c+dx\right)+3\sin^2\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

3.793 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=151

$$-\frac{4\tan^7(c+dx)}{7a^3d}-\frac{11\tan^5(c+dx)}{5a^3d}-\frac{10\tan^3(c+dx)}{3a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx)}{3a^3d}$$

```
[Out] -arctanh(cos(d*x+c))/a^3/d+sec(d*x+c)/a^3/d+1/3*sec(d*x+c)^3/a^3/d+1/5*sec(
d*x+c)^5/a^3/d+4/7*sec(d*x+c)^7/a^3/d-3*tan(d*x+c)/a^3/d-10/3*tan(d*x+c)^3/
a^3/d-11/5*tan(d*x+c)^5/a^3/d-4/7*tan(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.29, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2606, 30, 2607, 270}

$$-\frac{4\tan^7(c+dx)}{7a^3d}-\frac{11\tan^5(c+dx)}{5a^3d}-\frac{10\tan^3(c+dx)}{3a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^3*d)) + Sec[c + d*x]/(a^3*d) + Sec[c + d*x]^3/(3
*a^3*d) + Sec[c + d*x]^5/(5*a^3*d) + (4*Sec[c + d*x]^7)/(7*a^3*d) - (3*Tan[
c + d*x])/(a^3*d) - (10*Tan[c + d*x]^3)/(3*a^3*d) - (11*Tan[c + d*x]^5)/(5*
a^3*d) - (4*Tan[c + d*x]^7)/(7*a^3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-3a^3\sec^8(c+dx)+a^3\csc(c+dx)\sec^8(c+dx)+3a^3\sec^7(c+dx)\tan(c+\right.}{a^6}$$
$$= \frac{\int \csc(c+dx)\sec^8(c+dx)\,dx}{a^3} - \frac{\int \sec^6(c+dx)\tan^2(c+dx)\,dx}{a^3} - \frac{3\int \sec^8(c+}{a^3}$$
$$= \frac{\text{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^3 d} - \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^2\,dx, x, \tan(c+dx)\right)}{a^3 d}$$
$$= \frac{3\sec^7(c+dx)}{7a^3d} - \frac{3\tan(c+dx)}{a^3d} - \frac{3\tan^3(c+dx)}{a^3d} - \frac{9\tan^5(c+dx)}{5a^3d} - \frac{3\tan^7(c+}{7a^3d}$$
$$= \frac{\sec(c+dx)}{a^3d} + \frac{\sec^3(c+dx)}{3a^3d} + \frac{\sec^5(c+dx)}{5a^3d} + \frac{4\sec^7(c+dx)}{7a^3d} - \frac{3\tan(c+dx)}{a^3d} -$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\sec(c+dx)}{a^3d} + \frac{\sec^3(c+dx)}{3a^3d} + \frac{\sec^5(c+dx)}{5a^3d} + \frac{4\sec^7(}{7a}$$

Mathematica [B] time = 0.43, size = 341, normalized size = 2.26

$$\frac{\frac{105\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} - 2281\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5 + 353\left(\sin\left(\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (60 - (120*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 324*Si
n[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 162*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2 - 706*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^3 + 353*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - 2281*Sin[(c + d
*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 - 840*Log[Cos[(c + d*x)/2]]*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6 + 840*Log[Sin[(c + d*x)/2]]*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^6 + (105*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^6)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(840*d*(a + a
*Sin[c + d*x])^3)
```

fricas [A] time = 0.46, size = 218, normalized size = 1.44

$$\frac{272\,\cos(dx+c)^4 - 594\,\cos(dx+c)^2 - 105\left(3\,\cos(dx+c)^3 + \left(\cos(dx+c)^3 - 4\,\cos(dx+c)\right)\sin(dx+c) - 4\,\cos \cdots}{210\left(3\,\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/210*(272*cos(d*x + c)^4 - 594*cos(d*x + c)^2 - 105*(3*cos(d*x + c)^3 + (c
os(d*x + c)^3 - 4*cos(d*x + c))*sin(d*x + c) - 4*cos(d*x + c))*log(1/2*cos(
d*x + c) + 1/2) + 105*(3*cos(d*x + c)^3 + (cos(d*x + c)^3 - 4*cos(d*x + c))
*sin(d*x + c) - 4*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 6*(101*cos(d
*x + c)^2 + 15)*sin(d*x + c) - 120)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d
*x + c) + (a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.27, size = 135, normalized size = 0.89

$$\frac{\frac{840\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} - \frac{105}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)} + \frac{5145\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24360\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+54005\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+66080\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 \cdots}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 105/(a^3*(tan(1/2*d*x + 1/2
*c) - 1)) + (5145*tan(1/2*d*x + 1/2*c)^6 + 24360*tan(1/2*d*x + 1/2*c)^5 + 5
4005*tan(1/2*d*x + 1/2*c)^4 + 66080*tan(1/2*d*x + 1/2*c)^3 + 47691*tan(1/2*
d*x + 1/2*c)^2 + 18872*tan(1/2*d*x + 1/2*c) + 3431)/(a^3*(tan(1/2*d*x + 1/2
*c) + 1)^7))/d
```

maple [A] time = 0.65, size = 187, normalized size = 1.24

$$-\frac{1}{8a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}+\frac{8}{7a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}-\frac{4}{a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{42}{5a^3d\left(\tan\left(\frac{dx}{2} \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/8/a^3/d/(tan(1/2*d*x+1/2*c)-1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c))+8/7/a^3/d/
(tan(1/2*d*x+1/2*c)+1)^7-4/a^3/d/(tan(1/2*d*x+1/2*c)+1)^6+42/5/a^3/d/(tan(1
```

```
/2*d*x+1/2*c)+1)^5-11/a^3/d/(tan(1/2*d*x+1/2*c)+1)^4+67/6/a^3/d/(tan(1/2*d*
x+1/2*c)+1)^3-31/4/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2+49/8/a^3/d/(tan(1/2*d*x+1
/2*c)+1)
```

maxima [B] time = 0.35, size = 336, normalized size = 2.23

$$\frac{\frac{2\left(\frac{1011\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{1939\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1379\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{525\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{1715\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1155\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{315\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+221\right)}{a^3+\frac{6\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14\,a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6\,a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{105\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/105*(2*(1011*sin(d*x + c)/(cos(d*x + c) + 1) + 1939*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 1379*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 525*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 - 1715*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1155*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 315*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 + 221)/(a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*
a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 - 6*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/
(cos(d*x + c) + 1)^8) + 105*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 11.33, size = 143, normalized size = 0.95

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-22\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{98\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{394\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{15}+\frac{55}{\cdots}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) - ((674*tan(c/2 + (d*x)/2))/35 + (554*tan(c
/2 + (d*x)/2)^2)/15 + (394*tan(c/2 + (d*x)/2)^3)/15 - 10*tan(c/2 + (d*x)/2)
^4 - (98*tan(c/2 + (d*x)/2)^5)/3 - 22*tan(c/2 + (d*x)/2)^6 - 6*tan(c/2 + (d
*x)/2)^7 + 442/105)/(a^3*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1
)^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc(c+dx)\sec^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

3.794 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=162

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{13\tan^5(c+dx)}{5a^3d}+\frac{5\tan^3(c+dx)}{a^3d}+\frac{7\tan(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}-\frac{3\sec^5(c+dx)}{5a^3d}-$$

```
[Out] 3*arctanh(cos(d*x+c))/a^3/d-cot(d*x+c)/a^3/d-3*sec(d*x+c)/a^3/d-sec(d*x+c)^
3/a^3/d-3/5*sec(d*x+c)^5/a^3/d-4/7*sec(d*x+c)^7/a^3/d+7*tan(d*x+c)/a^3/d+5*
tan(d*x+c)^3/a^3/d+13/5*tan(d*x+c)^5/a^3/d+4/7*tan(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.35, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2620, 270, 2606, 30}

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{13\tan^5(c+dx)}{5a^3d}+\frac{5\tan^3(c+dx)}{a^3d}+\frac{7\tan(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}-\frac{3\sec^5(c+dx)}{5a^3d}-$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) - (3*Sec[c + d*x])
/(a^3*d) - Sec[c + d*x]^3/(a^3*d) - (3*Sec[c + d*x]^5)/(5*a^3*d) - (4*Sec[c
 + d*x]^7)/(7*a^3*d) + (7*Tan[c + d*x])/(a^3*d) + (5*Tan[c + d*x]^3)/(a^3*d
) + (13*Tan[c + d*x]^5)/(5*a^3*d) + (4*Tan[c + d*x]^7)/(7*a^3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^2(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(3a^3\sec^8(c+dx) - 3a^3\csc(c+dx)\sec^8(c+dx) + a^3\csc^2(c+dx)\sec^8(c+\right.}{a^6}$$
$$= \frac{\int \csc^2(c+dx)\sec^8(c+dx)\,dx}{a^3} - \frac{\int \sec^7(c+dx)\tan(c+dx)\,dx}{a^3} + \frac{3\int \sec^8(c-}{a^3}$$
$$= -\frac{\text{Subst}\left(\int x^6\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^2}\,dx, x, \tan(c+dx)\right)}{a^3d} - \frac{3\,\text{Su}}{}$$
$$= -\frac{\sec^7(c+dx)}{7a^3d} + \frac{3\tan(c+dx)}{a^3d} + \frac{3\tan^3(c+dx)}{a^3d} + \frac{9\tan^5(c+dx)}{5a^3d} + \frac{3\tan^7(c}{7a^3}$$
$$= -\frac{\cot(c+dx)}{a^3d} - \frac{3\sec(c+dx)}{a^3d} - \frac{\sec^3(c+dx)}{a^3d} - \frac{3\sec^5(c+dx)}{5a^3d} - \frac{4\sec^7(c+d}{7a^3d}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{\cot(c+dx)}{a^3d} - \frac{3\sec(c+dx)}{a^3d} - \frac{\sec^3(c+dx)}{a^3d} - \frac{3\sec^5}{5}$$

Mathematica [B] time = 0.88, size = 351, normalized size = 2.17

$$\frac{\csc^3(c+dx)\left(-1316\sin(c+dx) + 3520\sin(2(c+dx)) - 1380\sin(3(c+dx)) - 1056\sin(4(c+dx)) + 176\sin(5(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(-966 - 440*Cos[2*(c + d*x)] - 2640*Cos[3*(c + d*x)] + 846*
Cos[4*(c + d*x)] + 176*Cos[5*(c + d*x)] - 1575*Cos[3*(c + d*x)]*Log[Cos[(c
+ d*x)/2]] + 105*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 14*Cos[c + d*x]*(
176 + 105*Log[Cos[(c + d*x)/2]] - 105*Log[Sin[(c + d*x)/2]]) + 1575*Cos[3*(
c + d*x)]*Log[Sin[(c + d*x)/2]] - 105*Cos[5*(c + d*x)]*Log[Sin[(c + d*x)/2]
] - 1316*Sin[c + d*x] + 3520*Sin[2*(c + d*x)] + 2100*Log[Cos[(c + d*x)/2]]*
Sin[2*(c + d*x)] - 2100*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] - 1380*Sin[3
*(c + d*x)] - 1056*Sin[4*(c + d*x)] - 630*Log[Cos[(c + d*x)/2]]*Sin[4*(c +
d*x)] + 630*Log[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)] + 176*Sin[5*(c + d*x)]))
/(140*a^3*d*(Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2)*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.49, size = 265, normalized size = 1.64

$$\frac{846 \cos(dx+c)^4 - 956 \cos(dx+c)^2 + 105\left(\cos(dx+c)^5 - 5 \cos(dx+c)^3 - \left(3 \cos(dx+c)^3 - 4 \cos(dx+c)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/70*(846*cos(d*x + c)^4 - 956*cos(d*x + c)^2 + 105*(cos(d*x + c)^5 - 5*cos
(d*x + c)^3 - (3*cos(d*x + c)^3 - 4*cos(d*x + c))*sin(d*x + c) + 4*cos(d*x
+ c))*log(1/2*cos(d*x + c) + 1/2) - 105*(cos(d*x + c)^5 - 5*cos(d*x + c)^3
- (3*cos(d*x + c)^3 - 4*cos(d*x + c))*sin(d*x + c) + 4*cos(d*x + c))*log(-1
/2*cos(d*x + c) + 1/2) + 2*(176*cos(d*x + c)^4 - 477*cos(d*x + c)^2 + 15)*s
in(d*x + c) + 40)/(a^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^3 + 4*a^3*d*
cos(d*x + c) - (3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c)
)
```

giac [A] time = 0.22, size = 187, normalized size = 1.15

$$-\frac{\frac{840 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} - \frac{140 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3} - \frac{35\left(12 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 17 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a^3} + \frac{3885 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 + 19880 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 140*tan(1/2*d*x + 1/2*c)/a
^3 - 35*(12*tan(1/2*d*x + 1/2*c)^2 - 17*tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2
*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c))*a^3) + (3885*tan(1/2*d*x + 1/2*c)^6
 + 19880*tan(1/2*d*x + 1/2*c)^5 + 45465*tan(1/2*d*x + 1/2*c)^4 + 57120*tan(
1/2*d*x + 1/2*c)^3 + 41671*tan(1/2*d*x + 1/2*c)^2 + 16632*tan(1/2*d*x + 1/2
*c) + 2931)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.65, size = 224, normalized size = 1.38

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3} - \frac{1}{8a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} - \frac{1}{2d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3} - \frac{8}{7a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7} + \frac{}{a^3d\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-1/8/a^3/d/(tan(1/2*d*x+1/2*c)-1)-1/2/d/a^3/tan
(1/2*d*x+1/2*c)-3/d/a^3*ln(tan(1/2*d*x+1/2*c))-8/7/a^3/d/(tan(1/2*d*x+1/2*c
)+1)^7+4/a^3/d/(tan(1/2*d*x+1/2*c)+1)^6-46/5/a^3/d/(tan(1/2*d*x+1/2*c)+1)^5
+13/a^3/d/(tan(1/2*d*x+1/2*c)+1)^4-31/2/a^3/d/(tan(1/2*d*x+1/2*c)+1)^3+49/4
/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2-111/8/a^3/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.34, size = 395, normalized size = 2.44

$$-\frac{\frac{\frac{934\sin(dx+c)}{\cos(dx+c)+1}+\frac{3854\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6566\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3556\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3710\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{7070\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{4270\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{1015\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+35}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{6a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{14a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{14a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{14a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{6a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{a^3\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}+\frac{210\log\left(\frac{\sin\cdots}{\cos\cdots}\right.}{a^3}\cdots}{70\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/70*((934*sin(d*x + c)/(cos(d*x + c) + 1) + 3854*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 6566*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3556*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 3710*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 7070*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 4270*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7 - 1015*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 35)/(a^3*sin(d*x + c)/(cos(d
*x + c) + 1) + 6*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 14*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1
4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 14*a^3*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 6*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - a^3*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9) + 210*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 35
*sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 10.84, size = 274, normalized size = 1.69

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}-\frac{-29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-122\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-202\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-106\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\cdots}{d\left(-2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-12\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-28\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-28\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+28\,a^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - ((934*tan(c/2 + (d*x)/2))/35 + (3854*tan(c/2
 + (d*x)/2)^2)/35 + (938*tan(c/2 + (d*x)/2)^3)/5 + (508*tan(c/2 + (d*x)/2)^
4)/5 - 106*tan(c/2 + (d*x)/2)^5 - 202*tan(c/2 + (d*x)/2)^6 - 122*tan(c/2 +
```

```
(d*x)/2)^7 - 29*tan(c/2 + (d*x)/2)^8 + 1)/(d*(12*a^3*tan(c/2 + (d*x)/2)^2 +
 28*a^3*tan(c/2 + (d*x)/2)^3 + 28*a^3*tan(c/2 + (d*x)/2)^4 - 28*a^3*tan(c/2
 + (d*x)/2)^6 - 28*a^3*tan(c/2 + (d*x)/2)^7 - 12*a^3*tan(c/2 + (d*x)/2)^8 -
 2*a^3*tan(c/2 + (d*x)/2)^9 + 2*a^3*tan(c/2 + (d*x)/2))) - (3*log(tan(c/2 +
 (d*x)/2)))/(a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.795 $\int \sin^2(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=117

$$\frac{a\cos^3(c+dx)}{3d}-\frac{3a\cos(c+dx)}{d}+\frac{5a\tan^3(c+dx)}{6d}-\frac{5a\tan(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}-\frac{3a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx}{2}$$

```
[Out] 5/2*a*x-3*a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d-3*a*sec(d*x+c)/d+1/3*a*sec(d*
x+c)^3/d-5/2*a*tan(d*x+c)/d+5/6*a*tan(d*x+c)^3/d-1/2*a*sin(d*x+c)^2*tan(d*x
+c)^3/d
```

Rubi [**A**] time = 0.13, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2591, 288, 302, 203, 2590, 270}

$$\frac{a\cos^3(c+dx)}{3d}-\frac{3a\cos(c+dx)}{d}+\frac{5a\tan^3(c+dx)}{6d}-\frac{5a\tan(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}-\frac{3a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (5*a*x)/2 - (3*a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d) - (3*a*Sec[c +
d*x])/d + (a*Sec[c + d*x]^3)/(3*d) - (5*a*Tan[c + d*x])/(2*d) + (5*a*Tan[c
+ d*x]^3)/(6*d) - (a*Sin[c + d*x]^2*Tan[c + d*x]^3)/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx = a\int \sin^2(c+dx)\tan^4(c+dx)\,dx + a\int \sin^3(c+dx)\tan^4(c$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{(1-x^2)^3}{x^4}\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\right.}{}$$

$$= -\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d} - \frac{a\,\text{Subst}\left(\int \left(3+\frac{1}{x^4}-\frac{3}{x^2}-x\right.\right.}{d}$$

$$= -\frac{3a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{3a\sec(c+dx)}{d} + \frac{a\sec}{}$$

$$= -\frac{3a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{3a\sec(c+dx)}{d} + \frac{a\sec}{}$$

$$= \frac{5ax}{2} - \frac{3a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{3a\sec(c+dx)}{d} +$$

Mathematica [A] time = 0.41, size = 84, normalized size = 0.72

$$\frac{a\left(-3\sin(2(c+dx)) - 33\cos(c+dx) + \cos(3(c+dx)) - 28\tan(c+dx) + 4\sec^3(c+dx) - 36\sec(c+dx) + 4\,\text{ta}\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*(30*c + 30*d*x - 33*Cos[c + d*x] + Cos[3*(c + d*x)] - 36*Sec[c + d*x] +
4*Sec[c + d*x]^3 - 3*Sin[2*(c + d*x)] - 28*Tan[c + d*x] + 4*Sec[c + d*x]^2*
Tan[c + d*x]))/(12*d)
```

fricas [A] time = 0.47, size = 108, normalized size = 0.92

$$\frac{a\cos(dx+c)^4 - 15\,adx\cos(dx+c) + 29\,a\cos(dx+c)^2 + \left(2\,a\cos(dx+c)^4 + 15\,adx\cos(dx+c) - 15\,a\cos(dx\right.}{6\,(d\cos(dx+c)\sin(dx+c) - d\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(a*cos(d*x + c)^4 - 15*a*d*x*cos(d*x + c) + 29*a*cos(d*x + c)^2 + (2*a*
cos(d*x + c)^4 + 15*a*d*x*cos(d*x + c) - 15*a*cos(d*x + c)^2 - 4*a)*sin(d*x
 + c) + 2*a)/(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

giac [A] time = 0.20, size = 182, normalized size = 1.56

$$\frac{15\,(dx+c)a-\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{33\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-102\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+200\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-330\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+402\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,d\right.\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)*a - 3*a/(tan(1/2*d*x + 1/2*c) + 1) + (33*a*tan(1/2*d*x +
1/2*c)^8 - 102*a*tan(1/2*d*x + 1/2*c)^7 + 200*a*tan(1/2*d*x + 1/2*c)^6 - 33
0*a*tan(1/2*d*x + 1/2*c)^5 + 402*a*tan(1/2*d*x + 1/2*c)^4 - 410*a*tan(1/2*d
*x + 1/2*c)^3 + 264*a*tan(1/2*d*x + 1/2*c)^2 - 150*a*tan(1/2*d*x + 1/2*c) +
 61*a)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2
*c) - 1)^3)/d
```

maple [A] time = 0.46, size = 164, normalized size = 1.40

$$\frac{a\left(\frac{\sin^{8}(dx+c)}{3\cos(dx+c)^{3}}-\frac{5\left(\sin^{8}(dx+c)\right)}{3\cos(dx+c)}-\frac{5\left(\frac{16}{5}+\sin^{6}(dx+c)+\frac{6\left(\sin^{4}(dx+c)\right)}{5}+\frac{8\left(\sin^{2}(dx+c)\right)}{5}\right)\cos(dx+c)}{3}\right)+a\left(\frac{\sin^{7}(dx+c)}{3\cos(dx+c)^{3}}-\frac{4\left(\sin^{7}(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^{5}(dx\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16/5
+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+a*(1/3*sin(d*x
+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*sin(d*
x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c))
```

maxima [A] time = 0.42, size = 96, normalized size = 0.82

$$\frac{2\left(\cos\left(dx+c\right)^{3}-\frac{9\,\cos(dx+c)^{2}-1}{\cos(dx+c)^{3}}-9\,\cos\left(dx+c\right)\right)a+\left(2\,\tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^{2}+1}-12\,\tan\left(dx+\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x
+ c))*a + (2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^
2 + 1) - 12*tan(d*x + c))*a)/d
```

mupad [B] time = 14.96, size = 306, normalized size = 2.62

$$\frac{5\,a\,x}{2}+\frac{\left(5\,a\,d\,x-\frac{a\,(30\,d\,x-30)}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{a\,(45\,d\,x-60)}{6}-\frac{15\,a\,d\,x}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\left(10\,a\,d\,x-\frac{a\,(60\,d\,x-80)}{6}\right)\,\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^6*(a + a*sin(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] (5*a*x)/2 + (tan(c/2 + (d*x)/2)*((a*(30*d*x - 98))/6 - 5*a*d*x) - (a*(15*d*
x - 64))/6 - tan(c/2 + (d*x)/2)^4*((a*(30*d*x - 28))/6 - 5*a*d*x) - tan(c/2
 + (d*x)/2)^9*((a*(30*d*x - 30))/6 - 5*a*d*x) + tan(c/2 + (d*x)/2)^8*((a*(4
5*d*x - 60))/6 - (15*a*d*x)/2) + tan(c/2 + (d*x)/2)^6*((a*(30*d*x - 100))/6
 - 5*a*d*x) - tan(c/2 + (d*x)/2)^7*((a*(60*d*x - 80))/6 - 10*a*d*x) - tan(c
/2 + (d*x)/2)^2*((a*(45*d*x - 132))/6 - (15*a*d*x)/2) + tan(c/2 + (d*x)/2)^
3*((a*(60*d*x - 176))/6 - 10*a*d*x) + 6*a*tan(c/2 + (d*x)/2)^5 + (5*a*d*x)/
2)/(d*(tan(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2 +
 tan(c/2 + (d*x)/2)^3 - 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.796 $\int \sin(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=101

$$-\frac{a\cos(c+dx)}{d}+\frac{5a\tan^3(c+dx)}{6d}-\frac{5a\tan(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d}+5$$

```
[Out] 5/2*a*x-a*cos(d*x+c)/d-2*a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d-5/2*a*tan(d*x+
c)/d+5/6*a*tan(d*x+c)^3/d-1/2*a*sin(d*x+c)^2*tan(d*x+c)^3/d
```

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2838, 2590, 270, 2591, 288, 302, 203}

$$-\frac{a\cos(c+dx)}{d}+\frac{5a\tan^3(c+dx)}{6d}-\frac{5a\tan(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d}+5$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (5*a*x)/2 - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/
(3*d) - (5*a*Tan[c + d*x])/(2*d) + (5*a*Tan[c + d*x]^3)/(6*d) - (a*Sin[c +
d*x]^2*Tan[c + d*x]^3)/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx = a\int \sin(c+dx)\tan^4(c+dx)\,dx + a\int \sin^2(c+dx)\tan^4(c+$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^4}\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^2}\right.}{d}$$

$$= -\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d} - \frac{a\,\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx,\right.}{d}$$

$$= -\frac{a\cos(c+dx)}{d} - \frac{2a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\sin^2(c}{}$$

$$= -\frac{a\cos(c+dx)}{d} - \frac{2a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{5a\tan(}{2}$$

$$= \frac{5ax}{2} - \frac{a\cos(c+dx)}{d} - \frac{2a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{5a}{}$$

Mathematica [A] time = 0.25, size = 76, normalized size = 0.75

$$\frac{a\left(-3\sin(2(c+dx))-12\cos(c+dx)-28\tan(c+dx)+4\sec^3(c+dx)-24\sec(c+dx)+4\tan(c+dx)\sec^2(c+\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*(30*c + 30*d*x - 12*Cos[c + d*x] - 24*Sec[c + d*x] + 4*Sec[c + d*x]^3 -
3*Sin[2*(c + d*x)] - 28*Tan[c + d*x] + 4*Sec[c + d*x]^2*Tan[c + d*x]))/(12*
d)
```

fricas [A] time = 0.46, size = 98, normalized size = 0.97

$$\frac{3\,a\cos\left(dx+c\right)^4-15\,adx\cos\left(dx+c\right)+17\,a\cos\left(dx+c\right)^2+\left(15\,adx\cos\left(dx+c\right)-3\,a\cos\left(dx+c\right)^2+2\,a\right)\sin\left(dx\right.}{6\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*cos(d*x + c)^4 - 15*a*d*x*cos(d*x + c) + 17*a*cos(d*x + c)^2 + (15
*a*d*x*cos(d*x + c) - 3*a*cos(d*x + c)^2 + 2*a)*sin(d*x + c) - 4*a)/(d*cos(
d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

giac [A] time = 0.20, size = 134, normalized size = 1.33

$$\frac{15\left(dx+c\right)a+\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{6\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}+\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)*a + 3*a/(tan(1/2*d*x + 1/2*c) + 1) + 6*(a*tan(1/2*d*x + 1
/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(1
/2*d*x + 1/2*c)^2 + 1)^2 + (21*a*tan(1/2*d*x + 1/2*c)^2 - 48*a*tan(1/2*d*x
+ 1/2*c) + 23*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

maple [A] time = 0.46, size = 154, normalized size = 1.52

$$\frac{a\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+a\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(
d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+a*(1/3
*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*si
n(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 0.42, size = 87, normalized size = 0.86

$$\frac{\left(2\,\tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^{2}+1}-12\,\tan\left(dx+c\right)\right)a-2\,a\left(\frac{6\,\cos(dx+c)^{2}-1}{\cos(dx+c)^{3}}+3\,\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1
) - 12*tan(d*x + c))*a - 2*a*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos
(d*x + c)))/d
```

mupad [B] time = 14.62, size = 243, normalized size = 2.41

$$\frac{5\,a\,x}{2}-\frac{\left(\frac{a\,(30\,d\,x-30)}{6}-5\,a\,d\,x\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(5\,a\,d\,x-\frac{a\,(30\,d\,x-60)}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{a\,(30\,d\,x-50)}{6}-5\,a\,d\,x\right)\,\tan\left(\frac{c}{2}\right.}{d\,\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^5*(a + a*sin(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] (5*a*x)/2 - ((a*(15*d*x - 32))/6 - tan(c/2 + (d*x)/2)*((a*(30*d*x - 34))/6
- 5*a*d*x) + tan(c/2 + (d*x)/2)^2*((a*(30*d*x - 4))/6 - 5*a*d*x) - tan(c/2
+ (d*x)/2)^3*((a*(30*d*x - 14))/6 - 5*a*d*x) + tan(c/2 + (d*x)/2)^7*((a*(30
*d*x - 30))/6 - 5*a*d*x) + tan(c/2 + (d*x)/2)^5*((a*(30*d*x - 50))/6 - 5*a*
d*x) - tan(c/2 + (d*x)/2)^6*((a*(30*d*x - 60))/6 - 5*a*d*x) + (20*a*tan(c/2
 + (d*x)/2)^4)/3 - (5*a*d*x)/2)/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d
*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**5*(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.797 $\int (a+a\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=72

$$-\frac{a\cos(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}+ax$$

```
[Out] a*x-a*cos(d*x+c)/d-2*a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d-a*tan(d*x+c)/d+1/3
*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2710, 3473, 8, 2590, 270}

$$-\frac{a\cos(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d)
- (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
```

```
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+a\sin(c+dx))\tan^4(c+dx)\,dx = \int \left(a\tan^4(c+dx)+a\sin(c+dx)\tan^4(c+dx)\right)\,dx$$
$$= a\int \tan^4(c+dx)\,dx + a\int \sin(c+dx)\tan^4(c+dx)\,dx$$
$$= \frac{a\tan^3(c+dx)}{3d} - a\int \tan^2(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^4}\,dx, x, \cos(c+d$$
$$= -\frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d} + a\int 1\,dx - \frac{a\,\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)\,d$$
$$= ax - \frac{a\cos(c+dx)}{d} - \frac{2a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\tan(c+dx)}{d} + \frac{a$$

Mathematica [A] time = 0.04, size = 81, normalized size = 1.12

$$-\frac{a\cos(c+dx)}{d}+\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*ArcTan[Tan[c + d*x]])/d - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a
*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.48, size = 88, normalized size = 1.22

$$-\frac{3\,adx\cos(dx+c)-7\,a\cos(dx+c)^2-\left(3\,adx\cos(dx+c)-3\,a\cos(dx+c)^2-2\,a\right)\sin(dx+c)-a}{3\,(d\cos(dx+c)\sin(dx+c)-d\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*d*x*cos(d*x + c) - 7*a*cos(d*x + c)^2 - (3*a*d*x*cos(d*x + c) - 3
*a*cos(d*x + c)^2 - 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)*sin(d*x + c) - d
*cos(d*x + c))
```

giac [A] time = 0.19, size = 124, normalized size = 1.72

$$\frac{6\,(dx+c)a-\frac{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-36\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*a - 3*(a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)
 + 5*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/
2*c) + 1) + (15*a*tan(1/2*d*x + 1/2*c)^2 - 36*a*tan(1/2*d*x + 1/2*c) + 17*a
)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

maple [A] time = 0.36, size = 98, normalized size = 1.36

$$\frac{a\left(\frac{\sin^{6}(dx+c)}{3\cos(dx+c)^{3}}-\frac{\sin^{6}(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^{4}(dx+c)+\frac{4\left(\sin^{2}(dx+c)\right)}{3}\right)\cos(dx+c)\right)+a\left(\frac{\left(\tan^{3}(dx+c)\right)}{3}-\tan(dx+c)+dx+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+
c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

maxima [A] time = 0.42, size = 65, normalized size = 0.90

$$\frac{\left(\tan(dx+c)^{3}+3\,dx+3\,c-3\,\tan(dx+c)\right)a-a\left(\frac{6\,\cos(dx+c)^{2}-1}{\cos(dx+c)^{3}}+3\,\cos(dx+c)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - a*((6*cos(d*x + c)
^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

mupad [B] time = 12.72, size = 185, normalized size = 2.57

$$ax+\frac{\left(2\,a\,d\,x-\frac{a\,\left(6\,d\,x-6\right)}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{a\,\left(3\,d\,x-12\right)}{3}-a\,d\,x\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\left(a\,d\,x-\frac{a\,\left(3\,d\,x-4\right)}{3}\right)\,\mathrm{ta}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] a*x + (tan(c/2 + (d*x)/2)*((a*(6*d*x - 26))/3 - 2*a*d*x) - (a*(3*d*x - 16))
/3 - tan(c/2 + (d*x)/2)^2*((a*(3*d*x - 4))/3 - a*d*x) - tan(c/2 + (d*x)/2)^
5*((a*(6*d*x - 6))/3 - 2*a*d*x) + tan(c/2 + (d*x)/2)^4*((a*(3*d*x - 12))/3
- a*d*x) + (4*a*tan(c/2 + (d*x)/2)^3)/3 + a*d*x)/(d*(tan(c/2 + (d*x)/2) - 1
)^3*(tan(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.798 $\int \sec(c+dx)(a+a\sin(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=60

$$\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\sec(c+dx)}{d}+ax$$

```
[Out] a*x-a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d-a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2838, 2606, 3473, 8}

$$\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\sec(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] a*x - (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d +
(a*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))\tan^3(c+dx)\,dx &= a\int \sec(c+dx)\tan^3(c+dx)\,dx + a\int \tan^4(c+dx)\,dx\\ &= \frac{a\tan^3(c+dx)}{3d} - a\int \tan^2(c+dx)\,dx + \frac{a\,\text{Subst}\left(\int \left(-1+x^2\right)\right.}{d}\\ &= -\frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+}{3d}\\ &= ax - \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(}{3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 1.15

$$\frac{a\tan^{-1}(\tan(c+dx))}{d} + \frac{a\tan^3(c+dx)}{3d} - \frac{a\tan(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
[Out] (a*ArcTan[Tan[c + d*x]])/d - (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d)
- (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.46, size = 75, normalized size = 1.25

$$-\frac{3\,adx\cos(dx+c) - 4\,a\cos(dx+c)^2 - (3\,adx\cos(dx+c)+a)\sin(dx+c) + 2\,a}{3\,(d\cos(dx+c)\sin(dx+c) - d\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/3*(3*a*d*x*cos(d*x + c) - 4*a*cos(d*x + c)^2 - (3*a*d*x*cos(d*x + c) + a
)*sin(d*x + c) + 2*a)/(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

giac [A] time = 0.18, size = 74, normalized size = 1.23

$$\frac{6\,(dx+c)a + \frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1} + \frac{9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*a + 3*a/(tan(1/2*d*x + 1/2*c) + 1) + (9*a*tan(1/2*d*x + 1/
2*c)^2 - 24*a*tan(1/2*d*x + 1/2*c) + 11*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

maple [A] time = 0.37, size = 88, normalized size = 1.47

$$\frac{a\left(\frac{\left(\tan^{3}(dx+c)\right)}{3}-\tan\left(dx+c\right)+dx+c\right)+a\left(\frac{\sin^{4}(dx+c)}{3\cos(dx+c)^{3}}-\frac{\sin^{4}(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^{2}(dx+c)\right)\cos(dx+c)}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+a*(1/3*sin(d*x+c)^4/cos(d*x+c)^3
-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 0.42, size = 55, normalized size = 0.92

$$\frac{\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a-\frac{\left(3\,\cos(dx+c)^{2}-1\right)a}{\cos(dx+c)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - (3*cos(d*x + c)^2
- 1)*a/cos(d*x + c)^3)/d
```

mupad [B] time = 10.14, size = 117, normalized size = 1.95

$$a\,x-\frac{\left(\frac{a\,(6\,d\,x-6)}{3}-2\,a\,d\,x\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+\left(2\,a\,d\,x-\frac{a\,(6\,d\,x-2)}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a\,(3\,d\,x-4)}{3}-a\,d\,x}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^{3}\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] a*x - ((a*(3*d*x - 4))/3 - tan(c/2 + (d*x)/2)*((a*(6*d*x - 2))/3 - 2*a*d*x)
 + tan(c/2 + (d*x)/2)^3*((a*(6*d*x - 6))/3 - 2*a*d*x) + 4*a*tan(c/2 + (d*x)
/2)^2 - a*d*x)/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.799 $\int \sec^2(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=45

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\sec(c+dx)}{d}$$

```
[Out] -a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2838, 2607, 30, 2606}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
```

```
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sec^2(c+dx)\tan^2(c+dx)\,dx + a\int \sec(c+dx)\tan^3(c+$$
$$= \frac{a\,\text{Subst}\left(\int x^2\,dx, x, \tan(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \left(-1+x^2\right)\,dx,\right.}{d}$$
$$= -\frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{a\tan^3(c+dx)}{3d}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 1.00

$$\frac{a\tan^3(c+dx)}{3d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.45, size = 49, normalized size = 1.09

$$\frac{a\cos\left(dx+c\right)^2 - 2\,a\sin\left(dx+c\right) + a}{3\left(d\cos\left(dx+c\right)\sin\left(dx+c\right) - d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(d*x + c)^2 - 2*a*sin(d*x + c) + a)/(d*cos(d*x + c)*sin(d*x + c)
- d*cos(d*x + c))
```

giac [A] time = 0.18, size = 67, normalized size = 1.49

$$-\frac{\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1} - \frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 5\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*a/(tan(1/2*d*x + 1/2*c) + 1) - (3*a*tan(1/2*d*x + 1/2*c)^2 - 12*a*t
an(1/2*d*x + 1/2*c) + 5*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

maple [A] time = 0.36, size = 82, normalized size = 1.82

$$\frac{a\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+\frac{a\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+si
n(d*x+c)^2)*cos(d*x+c))+1/3*a*sin(d*x+c)^3/cos(d*x+c)^3)
```

maxima [A] time = 0.32, size = 39, normalized size = 0.87

$$\frac{a\tan\left(dx+c\right)^3-\frac{\left(3\cos(dx+c)^2-1\right)a}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(a*tan(d*x + c)^3 - (3*cos(d*x + c)^2 - 1)*a/cos(d*x + c)^3)/d
```

mupad [B] time = 9.03, size = 74, normalized size = 1.64

$$-\frac{4\,a\,\left(\sin\left(c+d\,x\right)^2+2\,\sin\left(c+d\,x\right)+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\sin\left(2\,c+2\,d\,x\right)-4\right)}{3\,d\,\left(8\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,\sin\left(2\,c+2\,d\,x\right)-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] -(4*a*(2*sin(c + d*x) + sin(2*c + 2*d*x) + 4*sin(c/2 + (d*x)/2)^2 + sin(c +
 d*x)^2 - 4))/(3*d*(2*sin(2*c + 2*d*x) + 8*sin(c/2 + (d*x)/2)^2 - 4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.800 $\int \sec^3(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=33

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\sec^3(c+dx)}{3d}$$

```
[Out] 1/3*a*sec(d*x+c)^3/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2838, 2606, 30, 2607}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
```

```
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx = a\int \sec^3(c+dx)\tan(c+dx)\,dx + a\int \sec^2(c+dx)\tan^2(c+d$$
$$= \frac{a\,\text{Subst}\left(\int x^2\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^2\,dx, x, \tan(c\right.}{d}$$
$$= \frac{a\sec^3(c+dx)}{3d} + \frac{a\tan^3(c+dx)}{3d}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{a\tan^3(c+dx)}{3d} + \frac{a\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.44, size = 50, normalized size = 1.52

$$\frac{a\cos(dx+c)^2 + a\sin(dx+c) - 2a}{3(d\cos(dx+c)\sin(dx+c) - d\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(d*x + c)^2 + a*sin(d*x + c) - 2*a)/(d*cos(d*x + c)*sin(d*x + c)
- d*cos(d*x + c))
```

giac [A] time = 0.17, size = 53, normalized size = 1.61

$$\frac{\frac{3a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1} - \frac{3a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a/(tan(1/2*d*x + 1/2*c) + 1) - (3*a*tan(1/2*d*x + 1/2*c)^2 + a)/(tan
(1/2*d*x + 1/2*c) - 1)^3)/d
```

maple [A] time = 0.20, size = 36, normalized size = 1.09

$$\frac{\frac{a\left(\sin^{3}(dx+c)\right)}{3\cos(dx+c)^{3}}+\frac{a}{3\cos(dx+c)^{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*sin(d*x+c)^3/cos(d*x+c)^3+1/3*a/cos(d*x+c)^3)
```

maxima [A] time = 0.32, size = 26, normalized size = 0.79

$$\frac{a\tan\left(dx+c\right)^{3}+\frac{a}{\cos(dx+c)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(a*tan(d*x + c)^3 + a/cos(d*x + c)^3)/d
```

mupad [B] time = 9.03, size = 50, normalized size = 1.52

$$-\frac{2\,a\,\left(\cos\left(c+d\,x\right)+1\right)\,\left(\cos\left(c+d\,x\right)+\sin\left(c+d\,x\right)-2\right)}{3\,d\,\left(2\,\cos\left(c+d\,x\right)-\sin\left(2\,c+2\,d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] -(2*a*(cos(c + d*x) + 1)*(cos(c + d*x) + sin(c + d*x) - 2))/(3*d*(2*cos(c +
 d*x) - sin(2*c + 2*d*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\sec^{4}\left(c+dx\right)dx+\int\sin^{2}\left(c+dx\right)\sec^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**4, x) + Integral(sin(c + d*x)**2*sec
(c + d*x)**4, x))
```

3.801 $\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=68

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d+a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d+a*tan(d*x+c)/d
+1/3*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2838, 2622, 302, 207, 3767}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3
*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \sec^4(c+dx)\,dx + a\int \csc(c+dx)\sec^4(c+dx)\,dx$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(1+x^2\right)\,d\right.}{d}$$

$$= \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d} + \frac{a\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right.\right.}{d}$$

$$= \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+}{3d}$$

$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} +$$

Mathematica [A] time = 0.12, size = 85, normalized size = 1.25

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[(c + d*x)/2]])/d) + (a*Log[Sin[(c + d*x)/2]])/d + (a*Sec[c + d
*x])/d + (a*Sec[c + d*x]^3)/(3*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 0.48, size = 126, normalized size = 1.85

$$-\frac{4\,a\cos\left(dx+c\right)^2+3\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)-a\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(a\cos\left(dx+c\right)\text{si}\right.}{6\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(4*a*cos(d*x + c)^2 + 3*(a*cos(d*x + c)*sin(d*x + c) - a*cos(d*x + c))
*log(1/2*cos(d*x + c) + 1/2) - 3*(a*cos(d*x + c)*sin(d*x + c) - a*cos(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2) - 2*a*sin(d*x + c) + 4*a)/(d*cos(d*x + c)
*sin(d*x + c) - d*cos(d*x + c))
```

giac [A] time = 0.20, size = 81, normalized size = 1.19

$$\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}-\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*a*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a/(tan(1/2*d*x + 1/2*c) + 1) -
(15*a*tan(1/2*d*x + 1/2*c)^2 - 24*a*tan(1/2*d*x + 1/2*c) + 13*a)/(tan(1/2*d
*x + 1/2*c) - 1)^3)/d
```

maple [A] time = 0.43, size = 82, normalized size = 1.21

$$\frac{2a\tan\left(dx+c\right)}{3d}+\frac{a\tan\left(dx+c\right)\left(\sec^{2}\left(dx+c\right)\right)}{3d}+\frac{a}{3d\cos\left(dx+c\right)^{3}}+\frac{a}{d\cos\left(dx+c\right)}+\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 2/3*a*tan(d*x+c)/d+1/3/d*a*tan(d*x+c)*sec(d*x+c)^2+1/3/d*a/cos(d*x+c)^3+1/d
*a/cos(d*x+c)+1/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.32, size = 73, normalized size = 1.07

$$\frac{2\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a+a\left(\frac{2\left(3\,\cos\left(dx+c\right)^{2}+1\right)}{\cos\left(dx+c\right)^{3}}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a + a*(2*(3*cos(d*x + c)^2 + 1)/co
s(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.94, size = 90, normalized size = 1.32

$$\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\frac{10\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}+\frac{8\,a}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^4*sin(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)))/d - ((8*a)/3 - (10*a*tan(c/2 + (d*x)/2))/3 + 2*
a*tan(c/2 + (d*x)/2)^3)/(d*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^3 +
 tan(c/2 + (d*x)/2)^4 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.802 $\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\tan^3(c+dx)}{3d}+\frac{2a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d-a*cot(d*x+c)/d+a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d
+2*a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2620, 270, 2622, 302, 207}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{2a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) - (a*Cot[c + d*x])/d + (a*Sec[c + d*x])/d +
(a*Sec[c + d*x]^3)/(3*d) + (2*a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \csc(c+dx)\sec^4(c+dx)\,dx + a\int \csc^2(c+dx)\sec^4(c+$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2}\,dx\right.}{d}$$

$$= \frac{a\,\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx, x, \tan(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{a\cot(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{2a\tan(}{d}$$

$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} - \frac{a\cot(c+dx)}{d} + \frac{a\sec(c+dx)}{d} +$$

Mathematica [A] time = 0.06, size = 109, normalized size = 1.35

$$\frac{5a\tan(c+dx)}{3d} - \frac{a\cot(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/
2]])/d + (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d) + (5*a*Tan[c + d*x])
/(3*d) + (a*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

fricas [B] time = 0.48, size = 170, normalized size = 2.10

$$-\frac{10\,a\cos\left(dx+c\right)^{2}+3\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)\sin\left(dx+c\right)-a\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-3}{6\left(d\cos\left(dx+c\right)^{3}+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(10*a*cos(d*x + c)^2 + 3*(a*cos(d*x + c)^3 + a*cos(d*x + c)*sin(d*x +
c) - a*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(a*cos(d*x + c)^3 + a*
cos(d*x + c)*sin(d*x + c) - a*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) -
2*(8*a*cos(d*x + c)^2 - a)*sin(d*x + c) - 4*a)/(d*cos(d*x + c)^3 + d*cos(d*
x + c)*sin(d*x + c) - d*cos(d*x + c))
```

giac [A] time = 0.21, size = 129, normalized size = 1.59

$$\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-36\,a\tan\left(\frac{1}{2}\,d\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*a*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a*tan(1/2*d*x + 1/2*c) - 3*(a*t
an(1/2*d*x + 1/2*c)^2 + 3*a*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2*d*x + 1/2*c)
^2 + tan(1/2*d*x + 1/2*c)) - (21*a*tan(1/2*d*x + 1/2*c)^2 - 36*a*tan(1/2*d*
x + 1/2*c) + 19*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

maple [A] time = 0.46, size = 106, normalized size = 1.31

$$\frac{a}{3d\cos\left(dx+c\right)^{3}}+\frac{a}{d\cos\left(dx+c\right)}+\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}+\frac{a}{3d\sin\left(dx+c\right)\cos\left(dx+c\right)^{3}}+\frac{4a}{3d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/3/d*a/cos(d*x+c)^3+1/d*a/cos(d*x+c)+1/d*a*ln(csc(d*x+c)-cot(d*x+c))+1/3/d
*a/sin(d*x+c)/cos(d*x+c)^3+4/3/d*a/sin(d*x+c)/cos(d*x+c)-8/3*a*cot(d*x+c)/d
```

maxima [A] time = 0.32, size = 83, normalized size = 1.02

$$\frac{2\left(\tan\left(dx+c\right)^{3}-\frac{3}{\tan(dx+c)}+6\,\tan\left(dx+c\right)\right)a+a\left(\frac{2\left(3\,\cos(dx+c)^{2}+1\right)}{\cos(dx+c)^{3}}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+\right.\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a + a*(2*(3*cos(d
*x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c
) - 1)))/d
```

mupad [B] time = 9.10, size = 145, normalized size = 1.79

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+\frac{8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{3}-\frac{22\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+}{d\left(-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^4*sin(c + d*x)^2),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) + (a*log(tan(c/2 + (d*x)/2)))/d - (a - (22*a*t
an(c/2 + (d*x)/2))/3 + (8*a*tan(c/2 + (d*x)/2)^2)/3 + 10*a*tan(c/2 + (d*x)/
2)^3 - 9*a*tan(c/2 + (d*x)/2)^4)/(d*(2*tan(c/2 + (d*x)/2) - 4*tan(c/2 + (d*
x)/2)^2 + 4*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.803 $\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=110

$$\frac{a\tan^3(c+dx)}{3d}+\frac{2a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{5a\sec^3(c+dx)}{6d}+\frac{5a\sec(c+dx)}{2d}-\frac{5a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(}{}$$

```
[Out] -5/2*a*arctanh(cos(d*x+c))/d-a*cot(d*x+c)/d+5/2*a*sec(d*x+c)/d+5/6*a*sec(d*
x+c)^3/d-1/2*a*csc(d*x+c)^2*sec(d*x+c)^3/d+2*a*tan(d*x+c)/d+1/3*a*tan(d*x+c
)^3/d
```

Rubi [A] time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2622, 288, 302, 207, 2620, 270}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{2a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{5a\sec^3(c+dx)}{6d}+\frac{5a\sec(c+dx)}{2d}-\frac{5a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x])/d + (5*a*Sec[c + d*x]
)/(2*d) + (5*a*Sec[c + d*x]^3)/(6*d) - (a*Csc[c + d*x]^2*Sec[c + d*x]^3)/(2
*d) + (2*a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \csc^2(c+dx)\sec^4(c+dx)\,dx + a\int \csc^3(c+dx)\sec^4(c+$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2}\,d\right.}{d}$$

$$= -\frac{a\csc^2(c+dx)\sec^3(c+dx)}{2d} + \frac{a\,\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)\,dx,\right.}{d}$$

$$= -\frac{a\cot(c+dx)}{d} - \frac{a\csc^2(c+dx)\sec^3(c+dx)}{2d} + \frac{2a\tan(c+dx}{d}$$

$$= -\frac{a\cot(c+dx)}{d} + \frac{5a\sec(c+dx)}{2d} + \frac{5a\sec^3(c+dx)}{6d} - \frac{a\csc^2(}{}$$

$$= -\frac{5a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot(c+dx)}{d} + \frac{5a\sec(c+dx)}{2d} +$$

Mathematica [B] time = 6.10, size = 359, normalized size = 3.26

$$\frac{5a\tan(c+dx)}{3d} - \frac{a\cot(c+dx)}{d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{5a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{5a\log\left(\cos\left(\frac{1}{2}(\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (5*a*Log[Cos[(c + d*
x)/2]])/(2*d) + (5*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/
(8*d) + a/(12*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (a*Sin[(c + d*x)
/2])/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (13*a*Sin[(c + d*x)/2]
)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (a*Sin[(c + d*x)/2])/(6*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + a/(12*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2) - (13*a*Sin[(c + d*x)/2])/(6*d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])) + (5*a*Tan[c + d*x])/(3*d) + (a*Sec[c + d*x]^2*Tan[c + d*x])/(3*
d)
```

fricas [B] time = 0.48, size = 222, normalized size = 2.02

$$\frac{32\,a\cos\left(dx+c\right)^4 - 18\,a\cos\left(dx+c\right)^2 - 15\left(a\cos\left(dx+c\right)^3 - a\cos\left(dx+c\right) - \left(a\cos\left(dx+c\right)^3 - a\cos\left(dx+c\right)\right)\sin\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(32*a*cos(d*x + c)^4 - 18*a*cos(d*x + c)^2 - 15*(a*cos(d*x + c)^3 - a*
cos(d*x + c) - (a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*x + c))*log(1/2*co
s(d*x + c) + 1/2) + 15*(a*cos(d*x + c)^3 - a*cos(d*x + c) - (a*cos(d*x + c)
^3 - a*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(a*cos(
d*x + c)^2 + 2*a)*sin(d*x + c) - 8*a)/(d*cos(d*x + c)^3 - d*cos(d*x + c) -
(d*cos(d*x + c)^3 - d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.23, size = 148, normalized size = 1.35

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+60\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{12\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}-\frac{3\left(30\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 \cdots\right.}{\tan\left(\frac{1}{2}\,dx \cdots\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(3*a*tan(1/2*d*x + 1/2*c)^2 + 60*a*log(abs(tan(1/2*d*x + 1/2*c))) + 12
*a*tan(1/2*d*x + 1/2*c) + 12*a/(tan(1/2*d*x + 1/2*c) + 1) - 3*(30*a*tan(1/2
*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c)^2 - 4*
(27*a*tan(1/2*d*x + 1/2*c)^2 - 48*a*tan(1/2*d*x + 1/2*c) + 25*a)/(tan(1/2*d
*x + 1/2*c) - 1)^3)/d
```

maple [A] time = 0.51, size = 138, normalized size = 1.25

$$\frac{a}{3d\sin\left(dx+c\right)\cos\left(dx+c\right)^3}+\frac{4a}{3d\sin\left(dx+c\right)\cos\left(dx+c\right)}-\frac{8a\cot\left(dx+c\right)}{3d}+\frac{a}{3d\sin\left(dx+c\right)^2\cos\left(dx+c\right)^3}-\frac{\cdots}{6d\sin \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/3/d*a/sin(d*x+c)/cos(d*x+c)^3+4/3/d*a/sin(d*x+c)/cos(d*x+c)-8/3*a*cot(d*x
+c)/d+1/3/d*a/sin(d*x+c)^2/cos(d*x+c)^3-5/6/d*a/sin(d*x+c)^2/cos(d*x+c)+5/2
/d*a/cos(d*x+c)+5/2/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.32, size = 106, normalized size = 0.96

$$\frac{4\left(\tan\left(dx+c\right)^3-\frac{3}{\tan\left(dx+c\right)}+6\,\tan\left(dx+c\right)\right)a+a\left(\frac{2\left(15\,\cos\left(dx+c\right)^4-10\,\cos\left(dx+c\right)^2-2\right)}{\cos\left(dx+c\right)^5-\cos\left(dx+c\right)^3}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15 \cdots\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a + a*(2*(15*cos
(d*x + c)^4 - 10*cos(d*x + c)^2 - 2)/(cos(d*x + c)^5 - cos(d*x + c)^3) - 15
*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 8.98, size = 180, normalized size = 1.64

$$\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}-\frac{-18\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{23\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2}+\frac{67\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}-\frac{68\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{a}{2}}{d\left(-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}+\frac{a\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^4*sin(c + d*x)^3),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) - (a/2 + a*tan(c/2 + (d*x)/2) - (68*a*tan(c/2
+ (d*x)/2)^2)/3 + (67*a*tan(c/2 + (d*x)/2)^3)/3 + (23*a*tan(c/2 + (d*x)/2)^
4)/2 - 18*a*tan(c/2 + (d*x)/2)^5)/(d*(4*tan(c/2 + (d*x)/2)^2 - 8*tan(c/2 +
(d*x)/2)^3 + 8*tan(c/2 + (d*x)/2)^5 - 4*tan(c/2 + (d*x)/2)^6)) + (a*tan(c/2
 + (d*x)/2)^2)/(8*d) + (5*a*log(tan(c/2 + (d*x)/2)))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.804 $\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=101

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{11a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{7a^2x}{2}$$

```
[Out] 7/2*a^2*x-2*a^2*cos(d*x+c)/d+1/3*a^2*cos(d*x+c)/d/(1-sin(d*x+c))^2-11/3*a^2
*cos(d*x+c)/d/(1-sin(d*x+c))-1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.21, antiderivative size = 120, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2708, 2765, 2977, 2734}

$$-\frac{16a^2\cos(c+dx)}{3d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{8a^2\sin^2(c+dx)\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{7a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (7*a^2*x)/2 - (16*a^2*Cos[c + d*x])/(3*d) - (7*a^2*Cos[c + d*x]*Sin[c + d*x
])/(2*d) - (8*a^2*Cos[c + d*x]*Sin[c + d*x]^2)/(3*d*(1 - Sin[c + d*x])) + (
a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*(a - a*Sin[c + d*x])^2)
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
```

```
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx = a^4\int \frac{\sin^4(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cos(c+dx)\sin^3(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{\sin^2(c+dx)(-3a-5a\sin(c+dx))}{a-a\sin(c+dx)}$$
$$= -\frac{8a^2\cos(c+dx)\sin^2(c+dx)}{3d(1-\sin(c+dx))} + \frac{a^4\cos(c+dx)\sin^3(c+dx)}{3d(a-a\sin(c+dx))^2} - \frac{1}{3}\int \sin($$
$$= \frac{7a^2x}{2} - \frac{16a^2\cos(c+dx)}{3d} - \frac{7a^2\cos(c+dx)\sin(c+dx)}{2d} - \frac{8a^2\cos(c+dx}{3d(1-\sin}$$

Mathematica [A] time = 1.26, size = 159, normalized size = 1.57

$$-\frac{a^2\left(-21(12c+12dx+7)\cos\left(\frac{1}{2}(c+dx)\right)+(84c+84dx+239)\cos\left(\frac{3}{2}(c+dx)\right)+3\left(-5\cos\left(\frac{5}{2}(c+dx)\right)+\cos\left(\frac{7}{2}\right.\right.\right.}{48d\left(\cos\left(\frac{1}{2}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] -1/48*(a^2*(-21*(7 + 12*c + 12*d*x)*Cos[(c + d*x)/2] + (239 + 84*c + 84*d*x
)*Cos[(3*(c + d*x))/2] + 3*(-5*Cos[(5*(c + d*x))/2] + Cos[(7*(c + d*x))/2]
+ 2*(50 + 56*c + 56*d*x + (-27 + 28*c + 28*d*x)*Cos[c + d*x] - 6*Cos[2*(c +
```

```
d*x)] - Cos[3*(c + d*x)])*Sin[(c + d*x)/2])))/(d*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^3)
```

fricas [B] time = 0.50, size = 196, normalized size = 1.94

$$\frac{3\,a^2\cos\left(dx+c\right)^4-6\,a^2\cos\left(dx+c\right)^3-42\,a^2dx+\left(21\,a^2dx+31\,a^2\right)\cos\left(dx+c\right)^2-2\,a^2-\left(21\,a^2dx-38\,a^2\right)\cos}{6\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^4 - 6*a^2*cos(d*x + c)^3 - 42*a^2*d*x + (21*a^2*d*x
 + 31*a^2)*cos(d*x + c)^2 - 2*a^2 - (21*a^2*d*x - 38*a^2)*cos(d*x + c) - (3
*a^2*cos(d*x + c)^3 - 42*a^2*d*x + 9*a^2*cos(d*x + c)^2 + 2*a^2 - (21*a^2*d
*x - 40*a^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c)
 + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.21, size = 135, normalized size = 1.34

$$\frac{21\,(dx+c)a^2+\frac{6\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}+\frac{4\left(9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+10\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(21*(d*x + c)*a^2 + 6*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x +
 1/2*c)^2 - a^2*tan(1/2*d*x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
2 + 4*(9*a^2*tan(1/2*d*x + 1/2*c)^2 - 21*a^2*tan(1/2*d*x + 1/2*c) + 10*a^2)
/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

maple [A] time = 0.49, size = 186, normalized size = 1.84

$$\frac{a^2\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+2a^2\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(si
n(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+2*a^
2*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+
4/3*sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

maxima [A] time = 0.43, size = 120, normalized size = 1.19

$$\frac{\left(2\,\tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^{2}+1}-12\,\tan\left(dx+c\right)\right)a^{2}+2\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/6*((2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1
) - 12*tan(d*x + c))*a^2 + 2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c)
)*a^2 - 4*a^2*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

mupad [B] time = 14.69, size = 287, normalized size = 2.84

$$\frac{7\,a^2\,x}{2}+\frac{\frac{7\,a^2\,(c+d\,x)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{21\,a^2\,(c+d\,x)}{2}-\frac{a^2\,(63\,c+63\,d\,x-150)}{6}\right)-\frac{a^2\,(21\,c+21\,d\,x-64)}{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{21\,a^2\,(c+d\,x)}{2}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x))^2)/cos(c + d*x)^4,x)
```

```
[Out] (7*a^2*x)/2 + ((7*a^2*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((21*a^2*(c + d*x))
/2 - (a^2*(63*c + 63*d*x - 150))/6) - (a^2*(21*c + 21*d*x - 64))/6 + tan(c/
2 + (d*x)/2)^6*((21*a^2*(c + d*x))/2 - (a^2*(63*c + 63*d*x - 42))/6) - tan(
c/2 + (d*x)/2)^5*((35*a^2*(c + d*x))/2 - (a^2*(105*c + 105*d*x - 126))/6) +
 tan(c/2 + (d*x)/2)^2*((35*a^2*(c + d*x))/2 - (a^2*(105*c + 105*d*x - 194))
/6) + tan(c/2 + (d*x)/2)^4*((49*a^2*(c + d*x))/2 - (a^2*(147*c + 147*d*x -
196))/6) - tan(c/2 + (d*x)/2)^3*((49*a^2*(c + d*x))/2 - (a^2*(147*c + 147*d
*x - 252))/6))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.805 $\int \sec(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=86

$$\frac{a^4\sin^2(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{4a^2\cos(c+dx)}{3d}-\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+2a^2x$$

```
[Out] 2*a^2*x-4/3*a^2*cos(d*x+c)/d-2*a^2*cos(d*x+c)/d/(1-sin(d*x+c))+1/3*a^4*cos(
d*x+c)*sin(d*x+c)^2/d/(a-a*sin(d*x+c))^2
```

Rubi [A] time = 0.25, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2869, 2765, 2968, 3023, 12, 2735, 2648}

$$-\frac{4a^2\cos(c+dx)}{3d}+\frac{a^4\sin^2(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] 2*a^2*x - (4*a^2*Cos[c + d*x])/(3*d) - (2*a^2*Cos[c + d*x])/(d*(1 - Sin[c +
 d*x])) + (a^4*Cos[c + d*x]*Sin[c + d*x]^2)/(3*d*(a - a*Sin[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
```

```
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx = a^4\int \frac{\sin^3(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{\sin(c+dx)(-2a-4a}{a-a\sin(c+}$$
$$= \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{-2a\sin(c+dx)-4a}{a-a\sin(c+}$$
$$= -\frac{4a^2\cos(c+dx)}{3d} + \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} - \frac{1}{3}a\int \frac{6}{a-}$$
$$= -\frac{4a^2\cos(c+dx)}{3d} + \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} - \left(2a^3\right)\int$$
$$= 2a^2x - \frac{4a^2\cos(c+dx)}{3d} + \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} - \left(2a\right.$$
$$= 2a^2x - \frac{4a^2\cos(c+dx)}{3d} + \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} - \frac{2}{d(a}$$

Mathematica [A] time = 1.02, size = 131, normalized size = 1.52

$$\frac{a^2(\sin(c+dx)+1)^2\left(-3\cos(c+dx)+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(8\sin(c+dx)-7)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{1}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+6c+6dx\right)}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(6*c + 6*d*x - 3*Cos[c + d*x] + (Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^(-2) + (2*Sin[(c + d*x)/2]*(-7 + 8*Sin[c + d*x]))/(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2])^3))/(3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^4)
```

fricas [B] time = 0.46, size = 168, normalized size = 1.95

$$-\frac{3\,a^2\cos(dx+c)^3+12\,a^2dx-\left(6\,a^2dx+11\,a^2\right)\cos(dx+c)^2+a^2+\left(6\,a^2dx-13\,a^2\right)\cos(dx+c)-\left(12\,a^2dx-3\right.}{3\left(d\cos(dx+c)^2-d\cos(dx+c)+(d\cos(dx+c)+2\,d)\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(3*a^2*cos(d*x + c)^3 + 12*a^2*d*x - (6*a^2*d*x + 11*a^2)*cos(d*x + c)
^2 + a^2 + (6*a^2*d*x - 13*a^2)*cos(d*x + c) - (12*a^2*d*x - 3*a^2*cos(d*x
+ c)^2 - a^2 + 2*(3*a^2*d*x - 7*a^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.21, size = 86, normalized size = 1.00

$$\frac{2\left(3\,(dx+c)a^2-\frac{3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\frac{6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*(d*x + c)*a^2 - 3*a^2/(tan(1/2*d*x + 1/2*c)^2 + 1) + (6*a^2*tan(1/2*
d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + 7*a^2)/(tan(1/2*d*x + 1/2*c)
 - 1)^3)/d
```

maple [A] time = 0.48, size = 162, normalized size = 1.88

$$\frac{a^2\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+2a^2\Big(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*
x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x
+c)+a^2*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+s
in(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 0.42, size = 95, normalized size = 1.10

$$\frac{2\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^2-a^2\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos(dx+c)\right)-\frac{\left(3\,\cos(dx+c)^2-1\right)a^2}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2 - a^2*((6*cos(d*
x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)) - (3*cos(d*x + c)^2 - 1)*a^2
/cos(d*x + c)^3)/d
```

mupad [B] time = 12.16, size = 182, normalized size = 2.12

$$2\,a^2\,x+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{2\,a^2\,(9\,d\,x-24)}{3}-6\,a^2\,d\,x\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{2\,a^2\,(9\,d\,x-6)}{3}-6\,a^2\,d\,x\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{2\,a^2\,(12\,d\,x-}{3}\right.}{d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3\,\left(\tan\left(\frac{c}{2}+\frac{d}{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x))^2)/cos(c + d*x)^4,x)
```

```
[Out] 2*a^2*x + (tan(c/2 + (d*x)/2)*((2*a^2*(9*d*x - 24))/3 - 6*a^2*d*x) - tan(c/
2 + (d*x)/2)^4*((2*a^2*(9*d*x - 6))/3 - 6*a^2*d*x) + tan(c/2 + (d*x)/2)^3*(
(2*a^2*(12*d*x - 18))/3 - 8*a^2*d*x) - tan(c/2 + (d*x)/2)^2*((2*a^2*(12*d*x
 - 22))/3 - 8*a^2*d*x) - (2*a^2*(3*d*x - 10))/3 + 2*a^2*d*x)/(d*(tan(c/2 +
(d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.806 $\int \sec^2(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=63

$$\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{5a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+a^2x$$

```
[Out] a^2*x-5/3*a^2*cos(d*x+c)/d/(1-sin(d*x+c))+1/3*a^4*cos(d*x+c)/d/(a-a*sin(d*x
+c))^2
```

Rubi [A] time = 0.21, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2869, 2758, 2735, 2648}

$$\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{5a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] a^2*x - (5*a^2*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) + (a^4*Cos[c + d*x])/
(3*d*(a - a*Sin[c + d*x])^2)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx &= a^4\int \frac{\sin^2(c+dx)}{(a-a\sin(c+dx))^2}\,dx\\ &= \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{1}{3}a^2\int \frac{-2a-3a\sin(c+dx)}{a-a\sin(c+dx)}\,dx\\ &= a^2x+\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{1}{3}\left(5a^3\right)\int \frac{1}{a-a\sin(c+dx)}\\ &= a^2x+\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{5a^3\cos(c+dx)}{3d(a-a\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.05, size = 79, normalized size = 1.25

$$\frac{a^2\tan^{-1}(\tan(c+dx))}{d}+\frac{2a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\sec^3(c+dx)}{3d}-\frac{2a^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (a^2*ArcTan[Tan[c + d*x]])/d - (2*a^2*Sec[c + d*x])/d + (2*a^2*Sec[c + d*x]
^3)/(3*d) - (a^2*Tan[c + d*x])/d + (2*a^2*Tan[c + d*x]^3)/(3*d)
```

fricas [B] time = 0.45, size = 141, normalized size = 2.24

$$-\frac{6\,a^2dx-\left(3\,a^2dx+5\,a^2\right)\cos\left(dx+c\right)^2+a^2+\left(3\,a^2dx-4\,a^2\right)\cos\left(dx+c\right)-\left(6\,a^2dx-a^2+\left(3\,a^2dx-5\,a^2\right)\cos\right.}{3\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(6*a^2*d*x - (3*a^2*d*x + 5*a^2)*cos(d*x + c)^2 + a^2 + (3*a^2*d*x - 4
*a^2)*cos(d*x + c) - (6*a^2*d*x - a^2 + (3*a^2*d*x - 5*a^2)*cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*si
n(d*x + c) - 2*d)
```

giac [A] time = 0.21, size = 67, normalized size = 1.06

$$\frac{3\,(dx+c)a^2+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^2 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*tan(1/2*d*x
+ 1/2*c) + 4*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

maple [A] time = 0.40, size = 114, normalized size = 1.81

$$\frac{a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+2a^2\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+\frac{a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+2*a^2*(1/3*sin(d*x+c)^4/cos(d*
x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+1/3*a^2
*sin(d*x+c)^3/cos(d*x+c)^3)
```

maxima [A] time = 0.42, size = 71, normalized size = 1.13

$$\frac{a^2\tan(dx+c)^3+\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^2-\frac{2\left(3\,\cos(dx+c)^2-1\right)a^2}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(a^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a
^2 - 2*(3*cos(d*x + c)^2 - 1)*a^2/cos(d*x + c)^3)/d
```

mupad [B] time = 9.29, size = 102, normalized size = 1.62

$$a^2\,x+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^2\,(9\,d\,x-18)}{3}-3\,a^2\,d\,x\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2\,(9\,d\,x-6)}{3}-3\,a^2\,d\,x\right)-\frac{a^2\,(3\,d\,x-8)}{3}+a^2\,d\,x}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x))^2)/cos(c + d*x)^4,x)
```

```
[Out] a^2*x + (tan(c/2 + (d*x)/2)*((a^2*(9*d*x - 18))/3 - 3*a^2*d*x) - tan(c/2 +
(d*x)/2)^2*((a^2*(9*d*x - 6))/3 - 3*a^2*d*x) - (a^2*(3*d*x - 8))/3 + a^2*d*
x)/(d*(tan(c/2 + (d*x)/2) - 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.807 $\int \sec^3(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=60

$$-\frac{2a^2\tan(c+dx)}{3d}-\frac{2a^2\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^2}{3d}$$

```
[Out] -2/3*a^2*sec(d*x+c)/d+1/3*sec(d*x+c)^3*(a+a*sin(d*x+c))^2/d-2/3*a^2*tan(d*x
+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2855, 2669, 3767, 8}

$$-\frac{2a^2\tan(c+dx)}{3d}-\frac{2a^2\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] (-2*a^2*Sec[c + d*x])/(3*d) + (Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2)/(3*d)
 - (2*a^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx = \frac{\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} - \frac{1}{3}(2a)\int \sec^2(c+dx)(a$$
$$= -\frac{2a^2\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} - \frac{1}{3}\big(2$$
$$= -\frac{2a^2\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} + \frac{(2a}{}$$
$$= -\frac{2a^2\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} - \frac{2a^2}{}$$

Mathematica [A] time = 0.29, size = 72, normalized size = 1.20

$$\frac{a^2\left(-3\sin\left(\frac{1}{2}(c+dx)\right)+3\cos\left(\frac{1}{2}(c+dx)\right)-2\cos\left(\frac{3}{2}(c+dx)\right)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] (a^2*(3*Cos[(c + d*x)/2] - 2*Cos[(3*(c + d*x))/2] - 3*Sin[(c + d*x)/2]))/(3
*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)
```

fricas [A] time = 0.44, size = 98, normalized size = 1.63

$$\frac{2\,a^2\cos(dx+c)^2+a^2\cos(dx+c)-a^2-\left(2\,a^2\cos(dx+c)+a^2\right)\sin(dx+c)}{3\left(d\cos(dx+c)^2-d\cos(dx+c)+(d\cos(dx+c)+2\,d)\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) - a^2 - (2*a^2*cos(d*x + c) +
a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2
*d)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.20, size = 38, normalized size = 0.63

$$-\frac{2\left(3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^{2}\right)}{3\,d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^2*tan(1/2*d*x + 1/2*c) - a^2)/(d*(tan(1/2*d*x + 1/2*c) - 1)^3)
```

maple [A] time = 0.37, size = 99, normalized size = 1.65

$$\frac{a^{2}\left(\frac{\sin^{4}(dx+c)}{3\cos(dx+c)^{3}}-\frac{\sin^{4}(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^{2}(dx+c)\right)\cos(dx+c)}{3}\right)+\frac{2a^{2}\left(\sin^{3}(dx+c)\right)}{3\cos(dx+c)^{3}}+\frac{a^{2}}{3\cos(dx+c)^{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+
sin(d*x+c)^2)*cos(d*x+c))+2/3*a^2*sin(d*x+c)^3/cos(d*x+c)^3+1/3*a^2/cos(d*x
+c)^3)
```

maxima [A] time = 0.32, size = 56, normalized size = 0.93

$$\frac{2\,a^{2}\tan\left(dx+c\right)^{3}-\frac{\left(3\,\cos(dx+c)^{2}-1\right)a^{2}}{\cos(dx+c)^{3}}+\frac{a^{2}}{\cos(dx+c)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(2*a^2*tan(d*x + c)^3 - (3*cos(d*x + c)^2 - 1)*a^2/cos(d*x + c)^3 + a^2
/cos(d*x + c)^3)/d
```

mupad [B] time = 9.14, size = 34, normalized size = 0.57

$$-\frac{2\,a^{2}\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x))^2)/cos(c + d*x)^4,x)
```

```
[Out] -(2*a^2*(3*tan(c/2 + (d*x)/2) - 1))/(3*d*(tan(c/2 + (d*x)/2) - 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.808 $\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=73

$$\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -a^2*arctanh(cos(d*x+c))/d+4/3*a^2*cos(d*x+c)/d/(1-sin(d*x+c))+1/3*a^4*cos(
d*x+c)/d/(a-a*sin(d*x+c))^2
```

Rubi [A] time = 0.19, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2869, 2766, 2978, 12, 3770}

$$\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*ArcTanh[Cos[c + d*x]])/d) + (4*a^2*Cos[c + d*x])/(3*d*(1 - Sin[c + d
*x])) + (a^4*Cos[c + d*x])/(3*d*(a - a*Sin[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
```

```
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx &= a^4\int \frac{\csc(c+dx)}{(a-a\sin(c+dx))^2}\,dx\\ &= \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{1}{3}a^2\int \frac{\csc(c+dx)(3a+a\sin(c+\\ &\quad a-a\sin(c+dx)\\ &= \frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{1}{3}\int 3a^2\csc\\ &= \frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+a^2\int \csc(c\\ &= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^4\cos}{3d(a-a}\end{aligned}$$

Mathematica [A] time = 0.51, size = 142, normalized size = 1.95

$$\frac{a^2(\sin(c+dx)+1)^2\left(3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(4\sin(c+dx)-5)}{\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{}{\left(\cos\left(\frac{1}{2}(c+dx)\right.\right.}\right.}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(-3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2
]] + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2) + (2*Sin[(c + d*x)/2]*(-5 +
 4*Sin[c + d*x]))/(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3))/(3*d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^4)
```

fricas [B] time = 0.46, size = 231, normalized size = 3.16

$$-\frac{8\,a^2\cos\left(dx+c\right)^2+10\,a^2\cos\left(dx+c\right)+2\,a^2+3\left(a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)-2\,a^2+\left(a^2\cos\left(dx+c\right)+2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(8*a^2*cos(d*x + c)^2 + 10*a^2*cos(d*x + c) + 2*a^2 + 3*(a^2*cos(d*x +
 c)^2 - a^2*cos(d*x + c) - 2*a^2 + (a^2*cos(d*x + c) + 2*a^2)*sin(d*x + c))
*log(1/2*cos(d*x + c) + 1/2) - 3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2
*a^2 + (a^2*cos(d*x + c) + 2*a^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2
) - 2*(4*a^2*cos(d*x + c) - a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*
x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.20, size = 73, normalized size = 1.00

$$\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(6*a^2*tan(1/2*d*x + 1/2*c)^2
 - 9*a^2*tan(1/2*d*x + 1/2*c) + 5*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

maple [A] time = 0.60, size = 92, normalized size = 1.26

$$\frac{2a^2}{3d\cos\left(dx+c\right)^3}+\frac{4a^2\tan\left(dx+c\right)}{3d}+\frac{2a^2\tan\left(dx+c\right)\left(\sec^2\left(dx+c\right)\right)}{3d}+\frac{a^2}{d\cos\left(dx+c\right)}+\frac{a^2\ln\left(\csc\left(dx+c\right)-\cot\left(dx\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/3/d*a^2/cos(d*x+c)^3+4/3*a^2*tan(d*x+c)/d+2/3/d*a^2*tan(d*x+c)*sec(d*x+c)
^2+1/d*a^2/cos(d*x+c)+1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.32, size = 90, normalized size = 1.23

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{2}+a^{2}\left(\frac{2\left(3\,\cos(dx+c)^{2}+1\right)}{\cos(dx+c)^{3}}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + a^2*(2*(3*cos(d*x + c)^2 + 1
)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 2*a
^2/cos(d*x + c)^3)/d
```

mupad [B] time = 9.50, size = 98, normalized size = 1.34

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{4\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-6\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{10\,a^2}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^4*sin(c + d*x)),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d - (4*a^2*tan(c/2 + (d*x)/2)^2 + (10*a^2)/3
- 6*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/2) - 3*tan(c/2 + (d*x)/2)
^2 + tan(c/2 + (d*x)/2)^3 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.809 $\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=87

$$\frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{10a^2\cot(c+dx)}{3d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))}$$

```
[Out] -2*a^2*arctanh(cos(d*x+c))/d-10/3*a^2*cot(d*x+c)/d+2*a^2*cot(d*x+c)/d/(1-si
n(d*x+c))+1/3*a^4*cot(d*x+c)/d/(a-a*sin(d*x+c))^2
```

Rubi [A] time = 0.27, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2869, 2766, 2978, 2748, 3767, 8, 3770}

$$-\frac{10a^2\cot(c+dx)}{3d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*ArcTanh[Cos[c + d*x]])/d - (10*a^2*Cot[c + d*x])/(3*d) + (2*a^2*Cot
[c + d*x])/(d*(1 - Sin[c + d*x])) + (a^4*Cot[c + d*x])/(3*d*(a - a*Sin[c +
d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx = a^4\int \frac{\csc^2(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{\csc^2(c+dx)(4a+2a\sin(c\ldots}{a-a\sin(c+dx)}$$
$$= \frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))} + \frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}\int \csc^2(c+\ldots$$
$$= \frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))} + \frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2} + \left(2a^2\right)\int \csc(\ldots$$
$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))} + \frac{a^4\cot\ldots}{3d(a-a\,s\ldots}$$
$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{10a^2\cot(c+dx)}{3d} + \frac{2a^2\cot(c\ldots}{d(1-\sin(c\ldots}$$

Mathematica [A] time = 0.92, size = 135, normalized size = 1.55

$$\frac{a^2\left(3\tan\left(\frac{1}{2}(c+dx)\right) - 3\cot\left(\frac{1}{2}(c+dx)\right) + 12\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \frac{4\sin\left(\frac{1}{2}(c+dx)\right)(7\sin\ldots}{\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\ldots\right.\right.}\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-3*Cot[(c + d*x)/2] - 12*Log[Cos[(c + d*x)/2]] + 12*Log[Sin[(c + d*x)
/2]] + 2/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*Sin[(c + d*x)/2]*(-8
+ 7*Sin[c + d*x]))/(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 3*Tan[(c + d*
x)/2]))/(6*d)
```

fricas [B] time = 0.48, size = 329, normalized size = 3.78

$$\frac{10\,a^2\cos(dx+c)^3 - 4\,a^2\cos(dx+c)^2 - 13\,a^2\cos(dx+c) + a^2 - 3\left(a^2\cos(dx+c)^3 + 2\,a^2\cos(dx+c)^2 - a^2\cos\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/3*(10*a^2*cos(d*x + c)^3 - 4*a^2*cos(d*x + c)^2 - 13*a^2*cos(d*x + c) + a
^2 - 3*(a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2*a^
```

```
2 - (a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2*a^2)*sin(d*x + c))*log(1/2*c
os(d*x + c) + 1/2) + 3*(a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x + c)^2 - a^2*cos
(d*x + c) - 2*a^2 - (a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2*a^2)*sin(d*x
 + c))*log(-1/2*cos(d*x + c) + 1/2) + (10*a^2*cos(d*x + c)^2 + 14*a^2*cos(d
*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - d*cos
(d*x + c) - (d*cos(d*x + c)^2 - d*cos(d*x + c) - 2*d)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.22, size = 118, normalized size = 1.36

$$\frac{12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{4\left(9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(12*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^2*tan(1/2*d*x + 1/2*c) - 3
*(4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c) - 4*(9*a^2*tan(1/2
*d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + 8*a^2)/(tan(1/2*d*x + 1/2*c
) - 1)^3)/d
```

maple [A] time = 0.73, size = 156, normalized size = 1.79

$$\frac{2a^2\tan(dx+c)}{3d}+\frac{a^2\tan(dx+c)\left(\sec^2(dx+c)\right)}{3d}+\frac{2a^2}{3d\cos(dx+c)^3}+\frac{2a^2}{d\cos(dx+c)}+\frac{2a^2\ln(\csc(dx+c)-\cot(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/3*a^2*tan(d*x+c)/d+1/3/d*a^2*tan(d*x+c)*sec(d*x+c)^2+2/3/d*a^2/cos(d*x+c)
^3+2/d*a^2/cos(d*x+c)+2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+1/3/d*a^2/sin(d*x+c
)/cos(d*x+c)^3+4/3/d*a^2/sin(d*x+c)/cos(d*x+c)-8/3*a^2*cot(d*x+c)/d
```

maxima [A] time = 0.33, size = 107, normalized size = 1.23

$$\frac{\left(\tan(dx+c)^3-\frac{3}{\tan(dx+c)}+6\,\tan(dx+c)\right)a^2+\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^2+a^2\left(\frac{2\left(3\,\cos(dx+c)^2+1\right)}{\cos(dx+c)^3}-3\,\log\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*((tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^2 + (tan(d*x + c)
^3 + 3*tan(d*x + c))*a^2 + a^2*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3
*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.45, size = 144, normalized size = 1.66

$$\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-13\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+23\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{41\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+a^2}{d\,\left(-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{a^2\,\tan\left(\frac{c}{2}+\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^4*sin(c + d*x)^2),x)
```

```
[Out] (2*a^2*log(tan(c/2 + (d*x)/2)))/d - (23*a^2*tan(c/2 + (d*x)/2)^2 - 13*a^2*t
an(c/2 + (d*x)/2)^3 + a^2 - (41*a^2*tan(c/2 + (d*x)/2))/3)/(d*(2*tan(c/2 +
(d*x)/2) - 6*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^3 - 2*tan(c/2 + (d
*x)/2)^4)) + (a^2*tan(c/2 + (d*x)/2))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.810 $\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=125

$$\frac{a^4\cot(c+dx)\csc(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{16a^2\cot(c+dx)}{3d}-\frac{7a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{7a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{8a^2\cot(c+}{3d(1-}$$

```
[Out] -7/2*a^2*arctanh(cos(d*x+c))/d-16/3*a^2*cot(d*x+c)/d-7/2*a^2*cot(d*x+c)*csc
(d*x+c)/d+8/3*a^2*cot(d*x+c)*csc(d*x+c)/d/(1-sin(d*x+c))+1/3*a^4*cot(d*x+c)
*csc(d*x+c)/d/(a-a*sin(d*x+c))^2
```

Rubi [A] time = 0.31, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2869, 2766, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{16a^2\cot(c+dx)}{3d}-\frac{7a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{7a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{a^4\cot(c+dx)\csc(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{8a^2\cot(c}{3d(1}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (16*a^2*Cot[c + d*x])/(3*d) - (7*a^2
*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (8*a^2*Cot[c + d*x]*Csc[c + d*x])/(3*d*
(1 - Sin[c + d*x])) + (a^4*Cot[c + d*x]*Csc[c + d*x])/(3*d*(a - a*Sin[c + d
*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
```

```
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx = a^4\int \frac{\csc^3(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cot(c+dx)\csc(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{\csc^3(c+dx)(5a+3a\,\mathrm{s}}{a-a\sin(c+}$$
$$= \frac{8a^2\cot(c+dx)\csc(c+dx)}{3d(1-\sin(c+dx))} + \frac{a^4\cot(c+dx)\csc(c+dx)}{3d(a-a\sin(c+dx))^2} +$$
$$= \frac{8a^2\cot(c+dx)\csc(c+dx)}{3d(1-\sin(c+dx))} + \frac{a^4\cot(c+dx)\csc(c+dx)}{3d(a-a\sin(c+dx))^2} +$$
$$= -\frac{7a^2\cot(c+dx)\csc(c+dx)}{2d} + \frac{8a^2\cot(c+dx)\csc(c+dx)}{3d(1-\sin(c+dx))}$$
$$= -\frac{7a^2\tanh^{-1}(\cos(c+dx))}{2d} - \frac{16a^2\cot(c+dx)}{3d} - \frac{7a^2\cot(c}{}$$

Mathematica [A] time = 1.99, size = 190, normalized size = 1.52

$$\frac{a^2\left(24\tan\left(\frac{1}{2}(c+dx)\right) - 24\cot\left(\frac{1}{2}(c+dx)\right) - 3\csc^2\left(\frac{1}{2}(c+dx)\right) + 3\sec^2\left(\frac{1}{2}(c+dx)\right) + 84\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-24*Cot[(c + d*x)/2] - 3*Csc[(c + d*x)/2]^2 - 84*Log[Cos[(c + d*x)/2]
] + 84*Log[Sin[(c + d*x)/2]] + 3*Sec[(c + d*x)/2]^2 + 8/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2 + (16*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^3 + (160*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) +
24*Tan[(c + d*x)/2]))/(24*d)
```

fricas [B] time = 0.51, size = 428, normalized size = 3.42

$$-\frac{64\,a^2\cos(dx+c)^4 + 86\,a^2\cos(dx+c)^3 - 54\,a^2\cos(dx+c)^2 - 80\,a^2\cos(dx+c) - 4\,a^2 + 21\left(a^2\cos(dx+c)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/12*(64*a^2*cos(d*x + c)^4 + 86*a^2*cos(d*x + c)^3 - 54*a^2*cos(d*x + c)^
2 - 80*a^2*cos(d*x + c) - 4*a^2 + 21*(a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)
^3 - 3*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) + 2*a^2 + (a^2*cos(d*x + c)^3
+ 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2*a^2)*sin(d*x + c))*log(1/2*co
s(d*x + c) + 1/2) - 21*(a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 3*a^2*cos
(d*x + c)^2 + a^2*cos(d*x + c) + 2*a^2 + (a^2*cos(d*x + c)^3 + 2*a^2*cos(d*
x + c)^2 - a^2*cos(d*x + c) - 2*a^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) +
1/2) - 2*(32*a^2*cos(d*x + c)^3 - 11*a^2*cos(d*x + c)^2 - 38*a^2*cos(d*x +
c) + 2*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4 - d*cos(d*x + c)^3 - 3*d*cos(d*
x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - d*cos(
d*x + c) - 2*d)*sin(d*x + c) + 2*d)
```

giac [A] time = 0.25, size = 150, normalized size = 1.20

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+84\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(42\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*tan(1/2*d*x + 1/2*c)^2 + 84*a^2*log(abs(tan(1/2*d*x + 1/2*c)))
+ 24*a^2*tan(1/2*d*x + 1/2*c) - 3*(42*a^2*tan(1/2*d*x + 1/2*c)^2 + 8*a^2*ta
n(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c)^2 - 16*(12*a^2*tan(1/2*d*x +
 1/2*c)^2 - 21*a^2*tan(1/2*d*x + 1/2*c) + 11*a^2)/(tan(1/2*d*x + 1/2*c) - 1
)^3)/d
```

maple [A] time = 0.71, size = 168, normalized size = 1.34

$$\frac{a^2}{3d\cos(dx+c)^3}+\frac{7a^2}{2d\cos(dx+c)}+\frac{7a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d}+\frac{2a^2}{3d\sin(dx+c)\cos(dx+c)^3}+\frac{}{3d\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/3/d*a^2/cos(d*x+c)^3+7/2/d*a^2/cos(d*x+c)+7/2/d*a^2*ln(csc(d*x+c)-cot(d*x
+c))+2/3/d*a^2/sin(d*x+c)/cos(d*x+c)^3+8/3/d*a^2/sin(d*x+c)/cos(d*x+c)-16/3
*a^2*cot(d*x+c)/d+1/3/d*a^2/sin(d*x+c)^2/cos(d*x+c)^3-5/6/d*a^2/sin(d*x+c)^
2/cos(d*x+c)
```

maxima [A] time = 0.33, size = 160, normalized size = 1.28

$$\frac{8\left(\tan(dx+c)^3-\frac{3}{\tan(dx+c)}+6\,\tan(dx+c)\right)a^2+a^2\left(\frac{2\left(15\,\cos(dx+c)^4-10\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^5-\cos(dx+c)^3}-15\,\log(\cos(dx+c)+1)+15\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(8*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^2 + a^2*(2*(15
*cos(d*x + c)^4 - 10*cos(d*x + c)^2 - 2)/(cos(d*x + c)^5 - cos(d*x + c)^3)
- 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2*a^2*(2*(3*cos(d*
x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c)
 - 1)))/d
```

mupad [B] time = 9.07, size = 182, normalized size = 1.46

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d}+\frac{7a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}-\frac{-36a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{135a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2}-\frac{239a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{6}+\frac{5a^2\tan\left(\frac{c}{2}\right.}{2}}{d\left(-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^4*sin(c + d*x)^3),x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (7*a^2*log(tan(c/2 + (d*x)/2)))/(2*d) -
((135*a^2*tan(c/2 + (d*x)/2)^3)/2 - (239*a^2*tan(c/2 + (d*x)/2)^2)/6 - 36*a
^2*tan(c/2 + (d*x)/2)^4 + a^2/2 + (5*a^2*tan(c/2 + (d*x)/2))/2)/(d*(4*tan(c
/2 + (d*x)/2)^2 - 12*tan(c/2 + (d*x)/2)^3 + 12*tan(c/2 + (d*x)/2)^4 - 4*tan
(c/2 + (d*x)/2)^5)) + (a^2*tan(c/2 + (d*x)/2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.811 $\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=119

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{6a^3\cos(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{25a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{17a^3x}{2}$$

```
[Out] 17/2*a^3*x-6*a^3*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3/d+2/3*a^3*cos(d*x+c)/d/(
1-sin(d*x+c))^2-25/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))-3/2*a^3*cos(d*x+c)*sin
(d*x+c)/d
```

Rubi [A] time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2709, 2650, 2648, 2638, 2635, 8, 2633}

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{6a^3\cos(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{25a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{17a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (17*a^3*x)/2 - (6*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) + (2*a^3
*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (25*a^3*Cos[c + d*x])/(3*d*(1 -
 Sin[c + d*x])) - (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx = a^4\int\left(\frac{7}{a}+\frac{2}{a(-1+\sin(c+dx))^2}+\frac{9}{a(-1+\sin(c+dx))}+\frac{5\sin(c+dx)}{a}\right.$$

$$= 7a^3x + a^3\int\sin^3(c+dx)\,dx + \left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx + \left(3a^3\right)$$

$$= 7a^3x - \frac{5a^3\cos(c+dx)}{d} + \frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{9a^3\cos(c+dx)}{d(1-\sin(c+dx))} -$$

$$= \frac{17a^3x}{2} - \frac{6a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} + \frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2} -$$

Mathematica [A] time = 2.13, size = 177, normalized size = 1.49

$$\frac{(a\sin(c+dx)+a)^3\left(102(c+dx)-9\sin(2(c+dx))-69\cos(c+dx)+\cos(3(c+dx))-\frac{200\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\right.}{12d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(102*(c + d*x) - 69*Cos[c + d*x] + Cos[3*(c + d*x)]
 + 8/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (16*Sin[(c + d*x)/2])/(Cos[(
c + d*x)/2] - Sin[(c + d*x)/2])^3 - (200*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]) - 9*Sin[2*(c + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^6)
```

fricas [B] time = 0.49, size = 220, normalized size = 1.85

$$\frac{2a^3\cos(dx+c)^5+7a^3\cos(dx+c)^4-22a^3\cos(dx+c)^3-102a^3dx-4a^3+\left(51a^3dx+77a^3\right)\cos(dx+c)^2-(}{6\left(d\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^5 + 7*a^3*cos(d*x + c)^4 - 22*a^3*cos(d*x + c)^3 -
102*a^3*d*x - 4*a^3 + (51*a^3*d*x + 77*a^3)*cos(d*x + c)^2 - (51*a^3*d*x -
100*a^3)*cos(d*x + c) + (2*a^3*cos(d*x + c)^4 - 5*a^3*cos(d*x + c)^3 + 102*
a^3*d*x - 27*a^3*cos(d*x + c)^2 - 4*a^3 + (51*a^3*d*x - 104*a^3)*cos(d*x +
c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*
d)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.26, size = 187, normalized size = 1.57

$$\frac{51(dx+c)a^3+\frac{2\left(51a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-153a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+289a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-459a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+501a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-511a^3\mathrm{t}\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(51*(d*x + c)*a^3 + 2*(51*a^3*tan(1/2*d*x + 1/2*c)^8 - 153*a^3*tan(1/2*
d*x + 1/2*c)^7 + 289*a^3*tan(1/2*d*x + 1/2*c)^6 - 459*a^3*tan(1/2*d*x + 1/2
*c)^5 + 501*a^3*tan(1/2*d*x + 1/2*c)^4 - 511*a^3*tan(1/2*d*x + 1/2*c)^3 + 3
27*a^3*tan(1/2*d*x + 1/2*c)^2 - 189*a^3*tan(1/2*d*x + 1/2*c) + 80*a^3)/(tan
(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) - 1)^3)
/d
```

maple [B] time = 0.61, size = 266, normalized size = 2.24

$$\frac{a^{3}\left(\frac{\sin^{8}(dx+c)}{3\cos(dx+c)^{3}}-\frac{5\left(\sin^{8}(dx+c)\right)}{3\cos(dx+c)}-\frac{5\left(\frac{16}{5}+\sin^{6}(dx+c)+\frac{6\left(\sin^{4}(dx+c)\right)}{5}+\frac{8\left(\sin^{2}(dx+c)\right)}{5}\right)\cos(dx+c)}{3}\right)+3a^{3}\left(\frac{\sin^{7}(dx+c)}{3\cos(dx+c)^{3}}-\frac{4\left(\sin^{7}(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(s\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16
/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/3*s
in(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*
sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+3*a^3*(1/3*sin(d*x+
c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^
2)*cos(d*x+c))+a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

maxima [A] time = 0.42, size = 165, normalized size = 1.39

$$\frac{2\left(\cos\left(dx+c\right)^{3}-\frac{9\cos(dx+c)^{2}-1}{\cos(dx+c)^{3}}-9\cos\left(dx+c\right)\right)a^{3}+3\left(2\tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3\tan(dx+c)}{\tan(dx+c)^{2}+1}-12\tan\left(d\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x
 + c))*a^3 + 3*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x +
 c)^2 + 1) - 12*tan(d*x + c))*a^3 + 2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan
(d*x + c))*a^3 - 6*a^3*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x +
 c)))/d
```

mupad [B] time = 14.92, size = 317, normalized size = 2.66

$$\frac{17\,a^3\,x}{2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^3\,(153\,d\,x-378)}{6}-\frac{51\,a^3\,d\,x}{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(\frac{a^3\,(153\,d\,x-102)}{6}-\frac{51\,a^3\,d\,x}{2}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{a^3\,(306}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x))^3)/cos(c + d*x)^4,x)
```

```
[Out] (17*a^3*x)/2 + (tan(c/2 + (d*x)/2)*((a^3*(153*d*x - 378))/6 - (51*a^3*d*x)/
2) - tan(c/2 + (d*x)/2)^8*((a^3*(153*d*x - 102))/6 - (51*a^3*d*x)/2) + tan(
c/2 + (d*x)/2)^7*((a^3*(306*d*x - 306))/6 - 51*a^3*d*x) - tan(c/2 + (d*x)/2
)^2*((a^3*(306*d*x - 654))/6 - 51*a^3*d*x) - tan(c/2 + (d*x)/2)^6*((a^3*(51
0*d*x - 578))/6 - 85*a^3*d*x) + tan(c/2 + (d*x)/2)^3*((a^3*(510*d*x - 1022)
)/6 - 85*a^3*d*x) + tan(c/2 + (d*x)/2)^5*((a^3*(612*d*x - 918))/6 - 102*a^3
*d*x) - tan(c/2 + (d*x)/2)^4*((a^3*(612*d*x - 1002))/6 - 102*a^3*d*x) - (a^
3*(51*d*x - 160))/6 + (17*a^3*d*x)/2)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d
*x)/2)^2 + tan(c/2 + (d*x)/2)^3 - 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.812 $\int \sec(c+dx)(a+a\sin(c+dx))^3 \tan^3(c+dx)\,dx$

Optimal. Leaf size=101

$$-\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{19a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{11a^3x}{2}$$

```
[Out] 11/2*a^3*x-3*a^3*cos(d*x+c)/d+2/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))^2-19/3*a^
3*cos(d*x+c)/d/(1-sin(d*x+c))-1/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.16, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2872, 2650, 2648, 2638, 2635, 8}

$$-\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{19a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{11a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (11*a^3*x)/2 - (3*a^3*Cos[c + d*x])/d + (2*a^3*Cos[c + d*x])/(3*d*(1 - Sin[
c + d*x])^2) - (19*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) - (a^3*Cos[c
+ d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^3\tan^3(c+dx)\,dx = a^4\int\left(\frac{5}{a}+\frac{2}{a(-1+\sin(c+dx))^2}+\frac{7}{a(-1+\sin(c+dx))}+\frac{3\,\text{s}}{}\right.$$

$$= 5a^3x + a^3\int\sin^2(c+dx)\,dx + \left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,d$$

$$= 5a^3x - \frac{3a^3\cos(c+dx)}{d} + \frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{7a^3\cos(c}{d(1-\sin(c}$$

$$= \frac{11a^3x}{2} - \frac{3a^3\cos(c+dx)}{d} + \frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{19a^3\cos}{3d(1-\sin}$$

Mathematica [A] time = 1.48, size = 159, normalized size = 1.57

$$-\frac{a^3\left(-3(132c+132dx+89)\cos\left(\frac{1}{2}(c+dx)\right)+(132c+132dx+403)\cos\left(\frac{3}{2}(c+dx)\right)+3\left(-9\cos\left(\frac{5}{2}(c+dx)\right)+\text{c}\right.\right.}{48d\left(\cos\left(\frac{1}{2}(c\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] -1/48*(a^3*(-3*(89 + 132*c + 132*d*x)*Cos[(c + d*x)/2] + (403 + 132*c + 132
*d*x)*Cos[(3*(c + d*x))/2] + 3*(-9*Cos[(5*(c + d*x))/2] + Cos[(7*(c + d*x))
```

```
/2] + 2*(86 + 88*c + 88*d*x + (-43 + 44*c + 44*d*x)*Cos[c + d*x] - 10*Cos[2
*(c + d*x)] - Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(d*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^3)
```

fricas [B] time = 0.46, size = 196, normalized size = 1.94

$$\frac{3\,a^3\cos\left(dx+c\right)^4-12\,a^3\cos\left(dx+c\right)^3-66\,a^3dx-4\,a^3+\left(33\,a^3dx+53\,a^3\right)\cos\left(dx+c\right)^2-\left(33\,a^3dx-64\,a^3\right)\cos}{6\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*a^3*cos(d*x + c)^4 - 12*a^3*cos(d*x + c)^3 - 66*a^3*d*x - 4*a^3 + (3
3*a^3*d*x + 53*a^3)*cos(d*x + c)^2 - (33*a^3*d*x - 64*a^3)*cos(d*x + c) - (
3*a^3*cos(d*x + c)^3 - 66*a^3*d*x + 15*a^3*cos(d*x + c)^2 + 4*a^3 - (33*a^3
*d*x - 68*a^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x +
c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.26, size = 135, normalized size = 1.34

$$\frac{33\left(dx+c\right)a^3+\frac{6\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}+\frac{4\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(33*(d*x + c)*a^3 + 6*(a^3*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*tan(1/2*d*x +
 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
2 + 4*(15*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*a^3*tan(1/2*d*x + 1/2*c) + 17*a^3
)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

maple [B] time = 0.58, size = 246, normalized size = 2.44

$$\frac{a^3\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+3a^3\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(si
n(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+3*a^
3*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+
4/3*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+a^3
*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+
c)^2)*cos(d*x+c)))
```

maxima [A] time = 0.43, size = 145, normalized size = 1.44

$$\frac{\left(2\,\tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^{2}+1}-12\,\tan\left(dx+c\right)\right)a^{3}+6\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6*((2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1
) - 12*tan(d*x + c))*a^3 + 6*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c)
)*a^3 - 6*a^3*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)) - 2*
(3*cos(d*x + c)^2 - 1)*a^3/cos(d*x + c)^3)/d
```

mupad [B] time = 14.96, size = 287, normalized size = 2.84

$$\frac{11\,a^{3}\,x}{2}+\frac{\frac{11\,a^{3}\,(c+d\,x)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{33\,a^{3}\,(c+d\,x)}{2}-\frac{a^{3}\,(99\,c+99\,d\,x-246)}{6}\right)-\frac{a^{3}\,(33\,c+33\,d\,x-104)}{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\left(\frac{33\,a^{3}\,(c+d\,x)}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x))^3)/cos(c + d*x)^4,x)
```

```
[Out] (11*a^3*x)/2 + ((11*a^3*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((33*a^3*(c + d*x
))/2 - (a^3*(99*c + 99*d*x - 246))/6) - (a^3*(33*c + 33*d*x - 104))/6 + tan
(c/2 + (d*x)/2)^6*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 66))/6) - t
an(c/2 + (d*x)/2)^5*((55*a^3*(c + d*x))/2 - (a^3*(165*c + 165*d*x - 198))/6
) + tan(c/2 + (d*x)/2)^2*((55*a^3*(c + d*x))/2 - (a^3*(165*c + 165*d*x - 32
2))/6) + tan(c/2 + (d*x)/2)^4*((77*a^3*(c + d*x))/2 - (a^3*(231*c + 231*d*x
 - 308))/6) - tan(c/2 + (d*x)/2)^3*((77*a^3*(c + d*x))/2 - (a^3*(231*c + 23
1*d*x - 420))/6))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1)^
2)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
[Out] Timed out
```

3.813 $\int \sec^2(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=77

$$-\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}-\frac{3a^3\cos(c+dx)}{d}+3a^3x+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

```
[Out] 3*a^3*x-3*a^3*cos(d*x+c)/d-2*a^5*cos(d*x+c)^3/d/(a-a*sin(d*x+c))^2+1/3*sec(
d*x+c)^3*(a+a*sin(d*x+c))^3/d
```

Rubi [A] time = 0.22, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2871, 2670, 2680, 2682, 8}

$$-\frac{3a^3\cos(c+dx)}{d}-\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}+3a^3x+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] 3*a^3*x - (3*a^3*Cos[c + d*x])/d - (2*a^5*Cos[c + d*x]^3)/(d*(a - a*Sin[c +
 d*x])^2) + (Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2871

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*m), x] - Dist[1/g^2, Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && EqQ[a^2 - b^2, 0] && EqQ[m + p + 1, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \int \sec^2(c+dx)(a+a\,\mathrm{si}$$

$$= \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - a^6\int \frac{\cos^4(c+dx)}{(a-a\sin(c+dx)}$$

$$= -\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2} + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d}$$

$$= -\frac{3a^3\cos(c+dx)}{d} - \frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2} + \frac{\sec^3(c+dx)(a}{}$$

$$= 3a^3x - \frac{3a^3\cos(c+dx)}{d} - \frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2} + \frac{\sec^3(c+}{}$$

Mathematica [A] time = 1.24, size = 133, normalized size = 1.73

$$\frac{a^3(\sin(c+dx)+1)^3\left(-3\cos(c+dx)+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(13\sin(c+dx)-11)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+9c+9dx\right)}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(9*c + 9*d*x - 3*Cos[c + d*x] + 2/(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-11 + 13*Sin[c + d*x]))/(Co
```

```
s[(c + d*x)/2] - Sin[(c + d*x)/2])^3))/(3*d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^6)
```

fricas [B] time = 0.47, size = 169, normalized size = 2.19

$$-\frac{3\,a^3\cos\left(dx+c\right)^3+18\,a^3dx+2\,a^3-\left(9\,a^3dx+16\,a^3\right)\cos\left(dx+c\right)^2+\left(9\,a^3dx-17\,a^3\right)\cos\left(dx+c\right)-\left(18\,a^3dx-\right.}{3\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/3*(3*a^3*cos(d*x + c)^3 + 18*a^3*d*x + 2*a^3 - (9*a^3*d*x + 16*a^3)*cos(
d*x + c)^2 + (9*a^3*d*x - 17*a^3)*cos(d*x + c) - (18*a^3*d*x - 3*a^3*cos(d*
x + c)^2 - 2*a^3 + (9*a^3*d*x - 19*a^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.23, size = 87, normalized size = 1.13

$$\frac{9\,(dx+c)a^3-\frac{6\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(9*(d*x + c)*a^3 - 6*a^3/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(9*a^3*tan(1/
2*d*x + 1/2*c)^2 - 24*a^3*tan(1/2*d*x + 1/2*c) + 11*a^3)/(tan(1/2*d*x + 1/2
*c) - 1)^3)/d
```

maple [B] time = 0.50, size = 184, normalized size = 2.39

$$\frac{a^3\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4\left(dx+c\right)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos\left(dx+c\right)\right)+3a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan\left(dx+c\right)+dx\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*
x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x
+c)+3*a^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2
+sin(d*x+c)^2)*cos(d*x+c))+1/3*a^3*sin(d*x+c)^3/cos(d*x+c)^3)
```

maxima [A] time = 0.42, size = 107, normalized size = 1.39

$$\frac{a^3\tan(dx+c)^3+3\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^3-a^3\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos(dx+c)\right)-\frac{3\left(3\,\cos\right.}{\cos}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/3*(a^3*tan(d*x + c)^3 + 3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))
*a^3 - a^3*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)) - 3*(3*
cos(d*x + c)^2 - 1)*a^3/cos(d*x + c)^3)/d
```

mupad [B] time = 12.30, size = 182, normalized size = 2.36

$$3\,a^3\,x+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^3\,\left(27\,d\,x-66\right)}{3}-9\,a^3\,d\,x\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a^3\,\left(27\,d\,x-18\right)}{3}-9\,a^3\,d\,x\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{a^3\,\left(36\,d\,x-5\right.}{3}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x))^3)/cos(c + d*x)^4,x)
```

```
[Out] 3*a^3*x + (tan(c/2 + (d*x)/2)*((a^3*(27*d*x - 66))/3 - 9*a^3*d*x) - tan(c/2
 + (d*x)/2)^4*((a^3*(27*d*x - 18))/3 - 9*a^3*d*x) + tan(c/2 + (d*x)/2)^3*((
a^3*(36*d*x - 54))/3 - 12*a^3*d*x) - tan(c/2 + (d*x)/2)^2*((a^3*(36*d*x - 5
8))/3 - 12*a^3*d*x) - (a^3*(9*d*x - 28))/3 + 3*a^3*d*x)/(d*(tan(c/2 + (d*x)
/2) - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.814 $\int \sec^3(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=64

$$a^3x-\frac{2a^5\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

```
[Out] a^3*x+1/3*sec(d*x+c)^3*(a+a*sin(d*x+c))^3/d-2*a^5*cos(d*x+c)/d/(a^2-a^2*sin
(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2855, 2670, 2680, 8}

$$-\frac{2a^5\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}+a^3x+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] a^3*x + (Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*d) - (2*a^5*Cos[c + d*x]
)/(d*(a^2 - a^2*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx = \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - a\int \sec^2(c+dx)(a+a\,\mathrm{s}$$

$$= \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - a^5\int \frac{\cos^2(c+dx)}{(a-a\sin(c+dx))}$$

$$= \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \frac{2a^5\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}$$

$$= a^3x + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \frac{2a^5\cos(c+dx}{d\left(a^2-a^2\sin(c+\right.}$$

Mathematica [A] time = 0.72, size = 107, normalized size = 1.67

$$-\frac{a^3\left(-9(c+dx+2)\cos\left(\frac{1}{2}(c+dx)\right)+(3c+3dx+14)\cos\left(\frac{3}{2}(c+dx)\right)+6\sin\left(\frac{1}{2}(c+dx)\right)(2(c+dx+2)+(c+\right.}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -1/6*(a^3*(-9*(2 + c + d*x)*Cos[(c + d*x)/2] + (14 + 3*c + 3*d*x)*Cos[(3*(c
 + d*x))/2] + 6*(2*(2 + c + d*x) + (c + d*x)*Cos[c + d*x])*Sin[(c + d*x)/2]
))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)
```

fricas [B] time = 0.45, size = 143, normalized size = 2.23

$$-\frac{6a^3dx+2a^3-\left(3a^3dx+7a^3\right)\cos(dx+c)^2+\left(3a^3dx-5a^3\right)\cos(dx+c)-\left(6a^3dx-2a^3+\left(3a^3dx-7a^3\right)\mathrm{c}\right.}{3\left(d\cos(dx+c)^2-d\cos(dx+c)+(d\cos(dx+c)+2d)\sin(dx+c)-2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^3*d*x + 2*a^3 - (3*a^3*d*x + 7*a^3)*cos(d*x + c)^2 + (3*a^3*d*x -
 5*a^3)*cos(d*x + c) - (6*a^3*d*x - 2*a^3 + (3*a^3*d*x - 7*a^3)*cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d
)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.20, size = 67, normalized size = 1.05

$$\frac{3\,(dx+c)a^3 + \frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 5\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^3 + 2*(3*a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x
 + 1/2*c) + 5*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

maple [B] time = 0.40, size = 126, normalized size = 1.97

$$\frac{a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3} - \tan(dx+c) + dx + c\right) + 3a^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3} - \frac{\sin^4(dx+c)}{3\cos(dx+c)} - \frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right) + \frac{a^3\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3} + \frac{}{3\,\mathrm{co}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+3*a^3*(1/3*sin(d*x+c)^4/cos(d*
x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+a^3*sin
(d*x+c)^3/cos(d*x+c)^3+1/3*a^3/cos(d*x+c)^3)
```

maxima [A] time = 0.42, size = 84, normalized size = 1.31

$$\frac{3\,a^3\tan(dx+c)^3 + \left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a^3 - \frac{3\left(3\,\cos(dx+c)^2-1\right)a^3}{\cos(dx+c)^3} + \frac{a^3}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^3*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))
*a^3 - 3*(3*cos(d*x + c)^2 - 1)*a^3/cos(d*x + c)^3 + a^3/cos(d*x + c)^3)/d
```

mupad [B] time = 9.26, size = 102, normalized size = 1.59

$$a^3\,x+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^3\,\left(9\,d\,x-24\right)}{3}-3\,a^3\,d\,x\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^3\,\left(9\,d\,x-6\right)}{3}-3\,a^3\,d\,x\right)-\frac{a^3\,\left(3\,d\,x-10\right)}{3}+a^3\,d\,x}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x))^3)/cos(c + d*x)^4,x)
```

```
[Out] a^3*x + (tan(c/2 + (d*x)/2)*((a^3*(9*d*x - 24))/3 - 3*a^3*d*x) - tan(c/2 +
(d*x)/2)^2*((a^3*(9*d*x - 6))/3 - 3*a^3*d*x) - (a^3*(3*d*x - 10))/3 + a^3*d
*x)/(d*(tan(c/2 + (d*x)/2) - 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.815 $\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=72

$$\frac{5a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -a^3*arctanh(cos(d*x+c))/d+2/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))^2+5/3*a^3*co
s(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2872, 3770, 2650, 2648}

$$\frac{5a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*ArcTanh[Cos[c + d*x]])/d) + (2*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d
*x])^2) + (5*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx = a^4\int\left(\frac{\csc(c+dx)}{a}+\frac{2}{a(-1+\sin(c+dx))^2}-\frac{1}{a(-1+\sin(c+\right.$$
$$= a^3\int\csc(c+dx)\,dx - a^3\int\frac{1}{-1+\sin(c+dx)}\,dx+\left(2a^3\right)\int$$
$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{a^3\cos}{d(1-\sin}$$
$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{5a^3\cos}{3d(1-\sin}$$

Mathematica [A] time = 0.63, size = 144, normalized size = 2.00

$$\frac{a^3(\sin(c+dx)+1)^3\left(3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(5\sin(c+dx)-7)}{\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{}{\left(\cos\left(\frac{1}{2}(c+dx)\right.\right.}\right.}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2
]] + 2/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-7 +
5*Sin[c + d*x]))/(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3))/(3*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [B] time = 0.47, size = 231, normalized size = 3.21

$$-\frac{10\,a^3\cos(dx+c)^2+14\,a^3\cos(dx+c)+4\,a^3+3\left(a^3\cos(dx+c)^2-a^3\cos(dx+c)-2\,a^3+\left(a^3\cos(dx+c)+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

[Out] -1/6*(10*a^3*cos(d*x + c)^2 + 14*a^3*cos(d*x + c) + 4*a^3 + 3*(a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3 + (a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3 + (a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(5*a^3*cos(d*x + c) - 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)

giac [A] time = 0.25, size = 73, normalized size = 1.01

$$\frac{3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/3*(3*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(9*a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x + 1/2*c) + 7*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d

maple [A] time = 0.64, size = 115, normalized size = 1.60

$$\frac{a^3\left(\sin^3(dx+c)\right)}{3d\cos(dx+c)^3}+\frac{4a^3}{3d\cos(dx+c)^3}+\frac{2a^3\tan(dx+c)}{d}+\frac{a^3\tan(dx+c)\left(\sec^2(dx+c)\right)}{d}+\frac{a^3}{d\cos(dx+c)}+\frac{a^3\ln(\csc$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x)

[Out] 1/3/d*a^3*sin(d*x+c)^3/cos(d*x+c)^3+4/3/d*a^3/cos(d*x+c)^3+2*a^3*tan(d*x+c)/d+1/d*a^3*tan(d*x+c)*sec(d*x+c)^2+1/d*a^3/cos(d*x+c)+1/d*a^3*ln(csc(d*x+c)-cot(d*x+c))

maxima [A] time = 0.33, size = 103, normalized size = 1.43

$$\frac{2\,a^3\tan(dx+c)^3+6\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^3+a^3\left(\frac{2\left(3\,\cos(dx+c)^2+1\right)}{\cos(dx+c)^3}-3\,\log(\cos(dx+c)+1)+3\,\log(\cos}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/6*(2*a^3*tan(d*x + c)^3 + 6*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 + a^3*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 6*a^3/cos(d*x + c)^3)/d

mupad [B] time = 9.23, size = 98, normalized size = 1.36

$$\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{6\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-8\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{14\,a^3}{3}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^4*sin(c + d*x)),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d - (6*a^3*tan(c/2 + (d*x)/2)^2 + (14*a^3)/3
- 8*a^3*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/2) - 3*tan(c/2 + (d*x)/2)
^2 + tan(c/2 + (d*x)/2)^3 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.816 $\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=86

$$-\frac{a^3\cot(c+dx)}{d}+\frac{11a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -3*a^3*arctanh(cos(d*x+c))/d-a^3*cot(d*x+c)/d+2/3*a^3*cos(d*x+c)/d/(1-sin(d
*x+c))^2+11/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2872, 3770, 3767, 8, 2650, 2648}

$$-\frac{a^3\cot(c+dx)}{d}+\frac{11a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/d - (a^3*Cot[c + d*x])/d + (2*a^3*Cos[c + d*
x])/(3*d*(1 - Sin[c + d*x])^2) + (11*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*
x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
```

```
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx = a^4\int\left(\frac{3\csc(c+dx)}{a}+\frac{\csc^2(c+dx)}{a}+\frac{2}{a(-1+\sin(c+dx))}\right.$$

$$= a^3\int\csc^2(c+dx)\,dx+\left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx+($$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{3a^3\,c}{d(1-}$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{2a^3\cos(c}{3d(1-\sin(c}$$

Mathematica [A] time = 0.96, size = 135, normalized size = 1.57

$$\frac{a^3\left(3\tan\left(\frac{1}{2}(c+dx)\right)-3\cot\left(\frac{1}{2}(c+dx)\right)+18\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{4\sin\left(\frac{1}{2}(c+dx)\right)(11}{\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\right.}\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-3*Cot[(c + d*x)/2] - 18*Log[Cos[(c + d*x)/2]] + 18*Log[Sin[(c + d*x)
/2]] + 4/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*Sin[(c + d*x)/2]*(-13
```

```
+ 11*Sin[c + d*x]))/(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 3*Tan[(c +
d*x)/2]))/(6*d)
```

fricas [B] time = 0.49, size = 334, normalized size = 3.88

$$\frac{28\,a^3\cos\left(dx+c\right)^3-10\,a^3\cos\left(dx+c\right)^2-34\,a^3\cos\left(dx+c\right)+4\,a^3-9\left(a^3\cos\left(dx+c\right)^3+2\,a^3\cos\left(dx+c\right)^2-a^3\cos\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(28*a^3*cos(d*x + c)^3 - 10*a^3*cos(d*x + c)^2 - 34*a^3*cos(d*x + c) +
4*a^3 - 9*(a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2
*a^3 - (a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3)*sin(d*x + c))*log(1/
2*cos(d*x + c) + 1/2) + 9*(a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - a^3*
cos(d*x + c) - 2*a^3 - (a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3)*sin(
d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(14*a^3*cos(d*x + c)^2 + 19*a^3*
cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2
- d*cos(d*x + c) - (d*cos(d*x + c)^2 - d*cos(d*x + c) - 2*d)*sin(d*x + c) -
 2*d)
```

giac [A] time = 0.25, size = 118, normalized size = 1.37

$$\frac{18\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{4\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\ldots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(18*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/2*d*x + 1/2*c) - 3
*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c) - 4*(15*a^3*tan(1/
2*d*x + 1/2*c)^2 - 24*a^3*tan(1/2*d*x + 1/2*c) + 13*a^3)/(tan(1/2*d*x + 1/2
*c) - 1)^3)/d
```

maple [A] time = 0.72, size = 155, normalized size = 1.80

$$\frac{4a^3}{3d\cos\left(dx+c\right)^3}+\frac{2a^3\tan\left(dx+c\right)}{d}+\frac{a^3\tan\left(dx+c\right)\left(\sec^2\left(dx+c\right)\right)}{d}+\frac{3a^3}{d\cos\left(dx+c\right)}+\frac{3a^3\ln\left(\csc\left(dx+c\right)-\cot\left(dx\ldots\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4/3/d*a^3/cos(d*x+c)^3+2*a^3*tan(d*x+c)/d+1/d*a^3*tan(d*x+c)*sec(d*x+c)^2+3
/d*a^3/cos(d*x+c)+3/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+1/3/d*a^3/sin(d*x+c)/co
s(d*x+c)^3+4/3/d*a^3/sin(d*x+c)/cos(d*x+c)-8/3*a^3*cot(d*x+c)/d
```

maxima [A] time = 0.32, size = 123, normalized size = 1.43

$$\frac{2\left(\tan\left(dx+c\right)^{3}-\frac{3}{\tan(dx+c)}+6\tan\left(dx+c\right)\right)a^{3}+6\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)a^{3}+3\,a^{3}\left(\frac{2\left(3\cos(dx+c)^{2}+1\right)}{\cos(dx+c)^{3}}-3\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^3 + 6*(tan(d*x
+ c)^3 + 3*tan(d*x + c))*a^3 + 3*a^3*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)
^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 2*a^3/cos(d*x + c
)^3)/d
```

mupad [B] time = 9.21, size = 144, normalized size = 1.67

$$\frac{3\,a^{3}\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-21\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+35\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-\frac{61\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+a^{3}}{d\left(-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{a^{3}\tan\left(\frac{c}{2}\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^4*sin(c + d*x)^2),x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d - (35*a^3*tan(c/2 + (d*x)/2)^2 - 21*a^3*t
an(c/2 + (d*x)/2)^3 + a^3 - (61*a^3*tan(c/2 + (d*x)/2))/3)/(d*(2*tan(c/2 +
(d*x)/2) - 6*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^3 - 2*tan(c/2 + (d
*x)/2)^4)) + (a^3*tan(c/2 + (d*x)/2))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.817 $\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=110

$$-\frac{3a^3\cot(c+dx)}{d}+\frac{17a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -11/2*a^3*arctanh(cos(d*x+c))/d-3*a^3*cot(d*x+c)/d-1/2*a^3*cot(d*x+c)*csc(d
*x+c)/d+2/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))^2+17/3*a^3*cos(d*x+c)/d/(1-sin(
d*x+c))
```

Rubi [A] time = 0.19, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2872, 3770, 3767, 8, 3768, 2650, 2648}

$$-\frac{3a^3\cot(c+dx)}{d}+\frac{17a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c
 + d*x]*Csc[c + d*x])/(2*d) + (2*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^
2) + (17*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx &= a^4\int\left(\frac{5\csc(c+dx)}{a}+\frac{3\csc^2(c+dx)}{a}+\frac{\csc^3(c+dx)}{a}+\frac{}{a(}\right.\\ &= a^3\int\csc^3(c+dx)\,dx+\left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx+\\ &= -\frac{5a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+\\ &= -\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\cot(c+}{}\end{aligned}$$

Mathematica [A] time = 2.02, size = 190, normalized size = 1.73

$$\frac{a^3\left(36\tan\left(\frac{1}{2}(c+dx)\right)-36\cot\left(\frac{1}{2}(c+dx)\right)-3\csc^2\left(\frac{1}{2}(c+dx)\right)+3\sec^2\left(\frac{1}{2}(c+dx)\right)+132\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-36*Cot[(c + d*x)/2] - 3*Csc[(c + d*x)/2]^2 - 132*Log[Cos[(c + d*x)/2
]] + 132*Log[Sin[(c + d*x)/2]] + 3*Sec[(c + d*x)/2]^2 + 16/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2 + (32*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2])^3 + (272*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
 + 36*Tan[(c + d*x)/2]))/(24*d)
```

fricas [B] time = 0.48, size = 428, normalized size = 3.89

$$-\frac{104\,a^3\cos\left(dx+c\right)^4+142\,a^3\cos\left(dx+c\right)^3-90\,a^3\cos\left(dx+c\right)^2-136\,a^3\cos\left(dx+c\right)-8\,a^3+33\left(a^3\cos\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(104*a^3*cos(d*x + c)^4 + 142*a^3*cos(d*x + c)^3 - 90*a^3*cos(d*x + c
)^2 - 136*a^3*cos(d*x + c) - 8*a^3 + 33*(a^3*cos(d*x + c)^4 - a^3*cos(d*x +
 c)^3 - 3*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*a^3 + (a^3*cos(d*x + c)
^3 + 2*a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3)*sin(d*x + c))*log(1/2
*cos(d*x + c) + 1/2) - 33*(a^3*cos(d*x + c)^4 - a^3*cos(d*x + c)^3 - 3*a^3*
cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*a^3 + (a^3*cos(d*x + c)^3 + 2*a^3*cos
(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c)
 + 1/2) - 2*(52*a^3*cos(d*x + c)^3 - 19*a^3*cos(d*x + c)^2 - 64*a^3*cos(d*x
 + c) + 4*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4 - d*cos(d*x + c)^3 - 3*d*cos
(d*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - d*c
os(d*x + c) - 2*d)*sin(d*x + c) + 2*d)
```

giac [A] time = 0.27, size = 150, normalized size = 1.36

$$\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+132\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(66\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^3*tan(1/2*d*x + 1/2*c)^2 + 132*a^3*log(abs(tan(1/2*d*x + 1/2*c)))
 + 36*a^3*tan(1/2*d*x + 1/2*c) - 3*(66*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*
tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^2 - 16*(21*a^3*tan(1/2*d*x
```

```
+ 1/2*c)^2 - 36*a^3*tan(1/2*d*x + 1/2*c) + 19*a^3)/(tan(1/2*d*x + 1/2*c) -
1)^3)/d
```

maple [A] time = 0.84, size = 202, normalized size = 1.84

$$\frac{2a^3\tan(dx+c)}{3d}+\frac{a^3\tan(dx+c)\left(\sec^2(dx+c)\right)}{3d}+\frac{a^3}{d\cos(dx+c)^3}+\frac{11a^3}{2d\cos(dx+c)}+\frac{11a^3\ln(\csc(dx+c)-\cot(d}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/3*a^3*tan(d*x+c)/d+1/3/d*a^3*tan(d*x+c)*sec(d*x+c)^2+1/d*a^3/cos(d*x+c)^3
+11/2/d*a^3/cos(d*x+c)+11/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+1/d*a^3/sin(d*x
+c)/cos(d*x+c)^3+4/d*a^3/sin(d*x+c)/cos(d*x+c)-8*a^3*cot(d*x+c)/d+1/3/d*a^3
/sin(d*x+c)^2/cos(d*x+c)^3-5/6/d*a^3/sin(d*x+c)^2/cos(d*x+c)
```

maxima [A] time = 0.33, size = 182, normalized size = 1.65

$$\frac{12\left(\tan(dx+c)^3-\frac{3}{\tan(dx+c)}+6\tan(dx+c)\right)a^3+4\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3+a^3\left(\frac{2\left(15\cos(dx+c)^4-10\cos}{\cos(dx+c)^5-\cos(dx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(12*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^3 + 4*(tan(d*
x + c)^3 + 3*tan(d*x + c))*a^3 + a^3*(2*(15*cos(d*x + c)^4 - 10*cos(d*x + c
)^2 - 2)/(cos(d*x + c)^5 - cos(d*x + c)^3) - 15*log(cos(d*x + c) + 1) + 15*
log(cos(d*x + c) - 1)) + 6*a^3*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3
*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.14, size = 183, normalized size = 1.66

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d}+\frac{11a^3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}-\frac{-62a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{227a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2}-\frac{403a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{6}+\frac{9a^3\tan(}{2}}{d\left(-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^4*sin(c + d*x)^3),x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (11*a^3*log(tan(c/2 + (d*x)/2)))/(2*d) -
 ((227*a^3*tan(c/2 + (d*x)/2)^3)/2 - (403*a^3*tan(c/2 + (d*x)/2)^2)/6 - 62*
```

```
a^3*tan(c/2 + (d*x)/2)^4 + a^3/2 + (9*a^3*tan(c/2 + (d*x)/2))/2)/(d*(4*tan(
c/2 + (d*x)/2)^2 - 12*tan(c/2 + (d*x)/2)^3 + 12*tan(c/2 + (d*x)/2)^4 - 4*ta
n(c/2 + (d*x)/2)^5)) + (3*a^3*tan(c/2 + (d*x)/2))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.818 $\int \csc^4(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=128

$$-\frac{a^3\cot^3(c+dx)}{3d}-\frac{6a^3\cot(c+dx)}{d}+\frac{23a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{17a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\text{c}}{}$$

```
[Out] -17/2*a^3*arctanh(cos(d*x+c))/d-6*a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d-3
/2*a^3*cot(d*x+c)*csc(d*x+c)/d+2/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))^2+23/3*a
^3*cos(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A] time = 0.21, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2872, 3770, 3767, 8, 3768, 2650, 2648}

$$-\frac{a^3\cot^3(c+dx)}{3d}-\frac{6a^3\cot(c+dx)}{d}+\frac{23a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{17a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\text{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-17*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (6*a^3*Cot[c + d*x])/d - (a^3*Cot[c
 + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (2*a^3*Cos[c +
d*x])/(3*d*(1 - Sin[c + d*x])^2) + (23*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c +
d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx &= a^4\int\left(\frac{7\csc(c+dx)}{a}+\frac{5\csc^2(c+dx)}{a}+\frac{3\csc^3(c+dx)}{a}+\frac{\mathrm{cs}}{}\right.\\ &= a^3\int\csc^4(c+dx)\,dx+\left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx+\left(3\right.\\ &= -\frac{7a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}+\\ &= -\frac{17a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{6a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+}{3d}\end{aligned}$$

Mathematica [B] time = 6.18, size = 287, normalized size = 2.24

$$a^3\left(\frac{17\tan\left(\frac{1}{2}(c+dx)\right)}{6d}-\frac{17\cot\left(\frac{1}{2}(c+dx)\right)}{6d}-\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{17\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((-17*Cot[(c + d*x)/2])/(6*d) - (3*Csc[(c + d*x)/2]^2)/(8*d) - (Cot[(c
+ d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) - (17*Log[Cos[(c + d*x)/2]])/(2*d) + (
17*Log[Sin[(c + d*x)/2]])/(2*d) + (3*Sec[(c + d*x)/2]^2)/(8*d) + 2/(3*d*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*Sin[(c + d*x)/2])/(3*d*(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])^3) + (46*Sin[(c + d*x)/2])/(3*d*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])) + (17*Tan[(c + d*x)/2])/(6*d) + (Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2])/(24*d))
```

fricas [B] time = 0.50, size = 528, normalized size = 4.12

$$\frac{160\,a^3\cos(dx+c)^5 - 58\,a^3\cos(dx+c)^4 - 356\,a^3\cos(dx+c)^3 + 70\,a^3\cos(dx+c)^2 + 200\,a^3\cos(dx+c) - 8\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(160*a^3*cos(d*x + c)^5 - 58*a^3*cos(d*x + c)^4 - 356*a^3*cos(d*x + c)
^3 + 70*a^3*cos(d*x + c)^2 + 200*a^3*cos(d*x + c) - 8*a^3 - 51*(a^3*cos(d*x
 + c)^5 + 2*a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 - 4*a^3*cos(d*x + c)^
2 + a^3*cos(d*x + c) + 2*a^3 - (a^3*cos(d*x + c)^4 - a^3*cos(d*x + c)^3 - 3
*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))*log(1/2*cos(d
*x + c) + 1/2) + 51*(a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 2*a^3*cos(
d*x + c)^3 - 4*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*a^3 - (a^3*cos(d*x
 + c)^4 - a^3*cos(d*x + c)^3 - 3*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*
a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(80*a^3*cos(d*x + c)^4
+ 109*a^3*cos(d*x + c)^3 - 69*a^3*cos(d*x + c)^2 - 104*a^3*cos(d*x + c) - 4
*a^3)*sin(d*x + c))/(d*cos(d*x + c)^5 + 2*d*cos(d*x + c)^4 - 2*d*cos(d*x +
c)^3 - 4*d*cos(d*x + c)^2 + d*cos(d*x + c) - (d*cos(d*x + c)^4 - d*cos(d*x
+ c)^3 - 3*d*cos(d*x + c)^2 + d*cos(d*x + c) + 2*d)*sin(d*x + c) + 2*d)
```

giac [A] time = 0.31, size = 194, normalized size = 1.52

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 204\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) + 69\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - \frac{187\,a^3\tan}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^3*tan(1/2*d*x + 1/2*c)^2 + 204*a^3*l
og(abs(tan(1/2*d*x + 1/2*c))) + 69*a^3*tan(1/2*d*x + 1/2*c) - (187*a^3*tan(
1/2*d*x + 1/2*c)^6 - 60*a^3*tan(1/2*d*x + 1/2*c)^5 - 405*a^3*tan(1/2*d*x +
1/2*c)^4 + 394*a^3*tan(1/2*d*x + 1/2*c)^3 - 45*a^3*tan(1/2*d*x + 1/2*c)^2 -
 6*a^3*tan(1/2*d*x + 1/2*c) - a^3)/(tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x +
1/2*c))^3)/d
```

maple [A] time = 0.70, size = 214, normalized size = 1.67

$$\frac{a^3}{3d\cos(dx+c)^3}+\frac{17a^3}{2d\cos(dx+c)}+\frac{17a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d}+\frac{a^3}{d\sin(dx+c)\cos(dx+c)^3}+\frac{2}{3d\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/3/d*a^3/cos(d*x+c)^3+17/2/d*a^3/cos(d*x+c)+17/2/d*a^3*ln(csc(d*x+c)-cot(d
*x+c))+1/d*a^3/sin(d*x+c)/cos(d*x+c)^3+20/3/d*a^3/sin(d*x+c)/cos(d*x+c)-40/
3*a^3*cot(d*x+c)/d+1/d*a^3/sin(d*x+c)^2/cos(d*x+c)^3-5/2/d*a^3/sin(d*x+c)^2
/cos(d*x+c)+1/3/d*a^3/sin(d*x+c)^3/cos(d*x+c)^3-2/3/d*a^3/sin(d*x+c)^3/cos(
d*x+c)
```

maxima [A] time = 0.34, size = 205, normalized size = 1.60

$$12\left(\tan(dx+c)^3-\frac{3}{\tan(dx+c)}+6\tan(dx+c)\right)a^3+4\left(\tan(dx+c)^3-\frac{9\tan(dx+c)^2+1}{\tan(dx+c)^3}+9\tan(dx+c)\right)a^3+3a^3\left(\frac{2(15}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(12*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^3 + 4*(tan(d*
x + c)^3 - (9*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 + 9*tan(d*x + c))*a^3 + 3*
a^3*(2*(15*cos(d*x + c)^4 - 10*cos(d*x + c)^2 - 2)/(cos(d*x + c)^5 - cos(d*
x + c)^3) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2*a^3*(2
*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(co
s(d*x + c) - 1)))/d
```

mupad [B] time = 11.41, size = 239, normalized size = 1.87

$$a^3\left(6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+45\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-581\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+897\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-303\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-181\tan\left(\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^4*sin(c + d*x)^4),x)
```

```
[Out] (a^3*(6*tan(c/2 + (d*x)/2) + 45*tan(c/2 + (d*x)/2)^2 - 581*tan(c/2 + (d*x)/
2)^3 + 897*tan(c/2 + (d*x)/2)^4 - 303*tan(c/2 + (d*x)/2)^5 - 181*tan(c/2 +
(d*x)/2)^6 + 45*tan(c/2 + (d*x)/2)^7 + 6*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (
d*x)/2)^9 - 204*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 612*log(tan(
c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^4 - 612*log(tan(c/2 + (d*x)/2))*tan(c/2
+ (d*x)/2)^5 + 204*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 + 1))/(24*d
*tan(c/2 + (d*x)/2)^3*(tan(c/2 + (d*x)/2) - 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.819 $\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx$

Optimal. Leaf size=143

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{16a^4\cos(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{35a^4\sin(c+dx)\cos(c+dx)}{8d}-\frac{56a^4\cos(c+dx}{3d(1-\sin(c+dx}$$

```
[Out] 163/8*a^4*x-16*a^4*cos(d*x+c)/d+4/3*a^4*cos(d*x+c)^3/d+4/3*a^4*cos(d*x+c)/d
/(1-sin(d*x+c))^2-56/3*a^4*cos(d*x+c)/d/(1-sin(d*x+c))-35/8*a^4*cos(d*x+c)*
sin(d*x+c)/d-1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 2650, 2648, 2638, 2635, 8, 2633}

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{16a^4\cos(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{35a^4\sin(c+dx)\cos(c+dx)}{8d}-\frac{56a^4\cos(c+dx}{3d(1-\sin(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

```
[Out] (163*a^4*x)/8 - (16*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) + (4
*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (56*a^4*Cos[c + d*x])/(3*d*
(1 - Sin[c + d*x])) - (35*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c
 + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx = a^4\int\Bigg(16+\frac{4}{(-1+\sin(c+dx))^2}+\frac{20}{-1+\sin(c+dx)}+12\sin(c+dx)+$$

$$= 16a^4x+a^4\int\sin^4(c+dx)\,dx+\left(4a^4\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx+\Big(4a^4$$

$$= 16a^4x-\frac{12a^4\cos(c+dx)}{d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{20a^4\cos(c+dx)}{d(1-\sin(c+dx))}$$

$$= 20a^4x-\frac{16a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-$$

$$= \frac{163a^4x}{8}-\frac{16a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}$$

Mathematica [A] time = 1.57, size = 252, normalized size = 1.76

$$\frac{a^4\left(-11736c\sin\left(\frac{1}{2}(c+dx)\right)-11736dx\sin\left(\frac{1}{2}(c+dx)\right)-16488\sin\left(\frac{1}{2}(c+dx)\right)-3912c\sin\left(\frac{3}{2}(c+dx)\right)-3912dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

```
[Out] (a^4*(24*(209 + 489*c + 489*d*x)*Cos[(c + d*x)/2] - 24*(453 + 163*c + 163*d
*x)*Cos[(3*(c + d*x))/2] + 885*Cos[(5*(c + d*x))/2] - 129*Cos[(7*(c + d*x))
/2] - 23*Cos[(9*(c + d*x))/2] + 3*Cos[(11*(c + d*x))/2] - 16488*Sin[(c + d*
x)/2] - 11736*c*Sin[(c + d*x)/2] - 11736*d*x*Sin[(c + d*x)/2] + 3704*Sin[(3
*(c + d*x))/2] - 3912*c*Sin[(3*(c + d*x))/2] - 3912*d*x*Sin[(3*(c + d*x))/2
] + 885*Sin[(5*(c + d*x))/2] + 129*Sin[(7*(c + d*x))/2] - 23*Sin[(9*(c + d*
x))/2] - 3*Sin[(11*(c + d*x))/2]))/(384*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^3)
```

fricas [A] time = 0.48, size = 247, normalized size = 1.73

$$-\frac{6\,a^4\cos\left(dx+c\right)^6-20\,a^4\cos\left(dx+c\right)^5-85\,a^4\cos\left(dx+c\right)^4+214\,a^4\cos\left(dx+c\right)^3+978\,a^4dx+32\,a^4-\left(489\,a^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^6 - 20*a^4*cos(d*x + c)^5 - 85*a^4*cos(d*x + c)^4
 + 214*a^4*cos(d*x + c)^3 + 978*a^4*d*x + 32*a^4 - (489*a^4*d*x + 721*a^4)*
cos(d*x + c)^2 + (489*a^4*d*x - 962*a^4)*cos(d*x + c) - (6*a^4*cos(d*x + c)
^5 + 26*a^4*cos(d*x + c)^4 - 59*a^4*cos(d*x + c)^3 + 978*a^4*d*x - 273*a^4*
cos(d*x + c)^2 - 32*a^4 + (489*a^4*d*x - 994*a^4)*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c)
 - 2*d)
```

giac [A] time = 0.27, size = 200, normalized size = 1.40

$$\frac{489\,(dx+c)a^4+\frac{64\left(12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-27\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-288\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+129\,a^4\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(489*(d*x + c)*a^4 + 64*(12*a^4*tan(1/2*d*x + 1/2*c)^2 - 27*a^4*tan(1/
2*d*x + 1/2*c) + 13*a^4)/(tan(1/2*d*x + 1/2*c) - 1)^3 + 2*(105*a^4*tan(1/2*
d*x + 1/2*c)^7 - 288*a^4*tan(1/2*d*x + 1/2*c)^6 + 129*a^4*tan(1/2*d*x + 1/2
*c)^5 - 1056*a^4*tan(1/2*d*x + 1/2*c)^4 - 129*a^4*tan(1/2*d*x + 1/2*c)^3 -
1120*a^4*tan(1/2*d*x + 1/2*c)^2 - 105*a^4*tan(1/2*d*x + 1/2*c) - 352*a^4)/(
tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [B] time = 0.68, size = 360, normalized size = 2.52

$$a^4\left(\frac{\sin^9(dx+c)}{3\cos(dx+c)^3} - \frac{2(\sin^9(dx+c))}{\cos(dx+c)} - 2\left(\sin^7(dx+c) + \frac{7(\sin^5(dx+c))}{6} + \frac{35(\sin^3(dx+c))}{24} + \frac{35\sin(dx+c)}{16}\right)\cos(dx+c) + \frac{35dx}{8} + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(1/3*sin(d*x+c)^9/cos(d*x+c)^3-2*sin(d*x+c)^9/cos(d*x+c)-2*(sin(d*
x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/
8*d*x+35/8*c)+4*a^4*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x
+c)-5/3*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+6
*a^4*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*
x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+4*a^4*(1
/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*
sin(d*x+c)^2)*cos(d*x+c))+a^4*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

maxima [A] time = 0.43, size = 238, normalized size = 1.66

$$32\left(\cos(dx+c)^3 - \frac{9\cos(dx+c)^2-1}{\cos(dx+c)^3} - 9\cos(dx+c)\right)a^4 + \left(8\tan(dx+c)^3 + 105\,dx + 105\,c - \frac{3\left(13\tan(dx+c)^3 + 11\tan(d\cdots\right.}{\tan(dx+c)^4 + 2\tan(dx+c)^2\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/24*(32*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*
x + c))*a^4 + (8*tan(d*x + c)^3 + 105*d*x + 105*c - 3*(13*tan(d*x + c)^3 +
11*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 72*tan(d*x + c))
*a^4 + 24*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^
2 + 1) - 12*tan(d*x + c))*a^4 + 8*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x
```

```
 + c))*a^4 - 32*a^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)
))/d
```

mupad [B] time = 16.83, size = 437, normalized size = 3.06

$$\frac{163\,a^4\,x}{8}+\frac{\frac{163\,a^4\,(c+d\,x)}{8}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{489\,a^4\,(c+d\,x)}{8}-\frac{a^4\,(1467\,c+1467\,d\,x-3630)}{24}\right)-\frac{a^4\,(489\,c+489\,d\,x-1536)}{24}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x))^4)/cos(c + d*x)^4,x)
```

```
[Out] (163*a^4*x)/8 + ((163*a^4*(c + d*x))/8 - tan(c/2 + (d*x)/2)*((489*a^4*(c +
d*x))/8 - (a^4*(1467*c + 1467*d*x - 3630))/24) - (a^4*(489*c + 489*d*x - 15
36))/24 + tan(c/2 + (d*x)/2)^10*((489*a^4*(c + d*x))/8 - (a^4*(1467*c + 146
7*d*x - 978))/24) - tan(c/2 + (d*x)/2)^9*((1141*a^4*(c + d*x))/8 - (a^4*(34
23*c + 3423*d*x - 2934))/24) + tan(c/2 + (d*x)/2)^2*((1141*a^4*(c + d*x))/8
 - (a^4*(3423*c + 3423*d*x - 7818))/24) + tan(c/2 + (d*x)/2)^8*((2119*a^4*(
c + d*x))/8 - (a^4*(6357*c + 6357*d*x - 6520))/24) - tan(c/2 + (d*x)/2)^3*(
(2119*a^4*(c + d*x))/8 - (a^4*(6357*c + 6357*d*x - 13448))/24) - tan(c/2 +
(d*x)/2)^7*((1467*a^4*(c + d*x))/4 - (a^4*(8802*c + 8802*d*x - 11736))/24)
+ tan(c/2 + (d*x)/2)^4*((1467*a^4*(c + d*x))/4 - (a^4*(8802*c + 8802*d*x -
15912))/24) + tan(c/2 + (d*x)/2)^6*((1793*a^4*(c + d*x))/4 - (a^4*(10758*c
+ 10758*d*x - 15364))/24) - tan(c/2 + (d*x)/2)^5*((1793*a^4*(c + d*x))/4 -
(a^4*(10758*c + 10758*d*x - 18428))/24))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan
(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.820 $\int \sec^2(c+dx)(a+a\sin(c+dx))^4\tan^2(c+dx)\,dx$

Optimal. Leaf size=101

$$-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{32a^4\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{17a^4x}{2}$$

```
[Out] 17/2*a^4*x-4*a^4*cos(d*x+c)/d+4/3*a^4*cos(d*x+c)/d/(1-sin(d*x+c))^2-32/3*a^
4*cos(d*x+c)/d/(1-sin(d*x+c))-1/2*a^4*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.16, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2872, 2650, 2648, 2638, 2635, 8}

$$-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{32a^4\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{17a^4x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```

```
[Out] (17*a^4*x)/2 - (4*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x])/(3*d*(1 - Sin[
c + d*x])^2) - (32*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) - (a^4*Cos[c
+ d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^4\tan^2(c+dx)\,dx = a^4\int\left(8+\frac{4}{(-1+\sin(c+dx))^2}+\frac{12}{-1+\sin(c+dx)}+4\sin(c\right.$$

$$= 8a^4x + a^4\int\sin^2(c+dx)\,dx + \left(4a^4\right)\int\frac{1}{(-1+\sin(c+dx))^2}$$

$$= 8a^4x - \frac{4a^4\cos(c+dx)}{d} + \frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{12a^4\cos(}{d(1-\sin(}$$

$$= \frac{17a^4x}{2} - \frac{4a^4\cos(c+dx)}{d} + \frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{32a^4\mathrm{c}}{3d(1-\mathrm{s}}$$

Mathematica [A] time = 1.80, size = 158, normalized size = 1.56

$$-\frac{a^4\left(-3(204c+204dx+161)\cos\left(\frac{1}{2}(c+dx)\right)+(204c+204dx+647)\cos\left(\frac{3}{2}(c+dx)\right)-39\cos\left(\frac{5}{2}(c+dx)\right)+3\,\mathrm{c}\right.}{48d\left(\cos\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```

```
[Out] -1/48*(a^4*(-3*(161 + 204*c + 204*d*x)*Cos[(c + d*x)/2] + (647 + 204*c + 20
4*d*x)*Cos[(3*(c + d*x))/2] - 39*Cos[(5*(c + d*x))/2] + 3*Cos[(7*(c + d*x))
```

```
/2] + 6*(146 + 136*c + 136*d*x + (-59 + 68*c + 68*d*x)*Cos[c + d*x] - 14*Co
s[2*(c + d*x)] - Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(d*(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])^3)
```

fricas [B] time = 0.46, size = 197, normalized size = 1.95

$$\frac{3\,a^4\cos(dx+c)^4 - 18\,a^4\cos(dx+c)^3 - 102\,a^4dx - 8\,a^4 + 17\left(3\,a^4dx + 5\,a^4\right)\cos(dx+c)^2 - \left(51\,a^4dx - 98\,a^4\right)\cos}{6\left(d\cos(dx+c)^2 - d\cos(dx+c) +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*a^4*cos(d*x + c)^4 - 18*a^4*cos(d*x + c)^3 - 102*a^4*d*x - 8*a^4 + 1
7*(3*a^4*d*x + 5*a^4)*cos(d*x + c)^2 - (51*a^4*d*x - 98*a^4)*cos(d*x + c) -
 (3*a^4*cos(d*x + c)^3 - 102*a^4*d*x + 21*a^4*cos(d*x + c)^2 + 8*a^4 - (51*
a^4*d*x - 106*a^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*
x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.24, size = 135, normalized size = 1.34

$$\frac{51\,(dx+c)a^4 + \frac{6\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 8\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 8\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2} + \frac{16\left(6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 7\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(51*(d*x + c)*a^4 + 6*(a^4*tan(1/2*d*x + 1/2*c)^3 - 8*a^4*tan(1/2*d*x +
 1/2*c)^2 - a^4*tan(1/2*d*x + 1/2*c) - 8*a^4)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
2 + 16*(6*a^4*tan(1/2*d*x + 1/2*c)^2 - 15*a^4*tan(1/2*d*x + 1/2*c) + 7*a^4)
/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

maple [B] time = 0.57, size = 268, normalized size = 2.65

$$\frac{a^4\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3} - \frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)} - \frac{4\left(\sin^5(dx+c) + \frac{5\left(\sin^3(dx+c)\right)}{4} + \frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{3} + \frac{5dx}{2} + \frac{5c}{2}\right) + 4a^4\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3} - \frac{\sin^6(dx+c)}{\cos(dx+c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(si
n(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+4*a^
4*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+
4/3*sin(d*x+c)^2)*cos(d*x+c))+6*a^4*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+4*a
^4*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*
x+c)^2)*cos(d*x+c))+1/3*a^4*sin(d*x+c)^3/cos(d*x+c)^3)
```

maxima [A] time = 0.42, size = 158, normalized size = 1.56

$$\frac{2\,a^4\tan\left(dx+c\right)^3+\left(2\,\tan\left(dx+c\right)^3+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^2+1}-12\,\tan\left(dx+c\right)\right)a^4+12\left(\tan\left(dx+c\right)^3+3\,dx\cdots\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*a^4*tan(d*x + c)^3 + (2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x +
 c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x + c))*a^4 + 12*(tan(d*x + c)^3 + 3*d*
x + 3*c - 3*tan(d*x + c))*a^4 - 8*a^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^
3 + 3*cos(d*x + c)) - 8*(3*cos(d*x + c)^2 - 1)*a^4/cos(d*x + c)^3)/d
```

mupad [B] time = 14.85, size = 287, normalized size = 2.84

$$\frac{17\,a^4\,x}{2}+\frac{\frac{17\,a^4\,(c+d\,x)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{51\,a^4\,(c+d\,x)}{2}-\frac{a^4\,(153\,c+153\,d\,x-378)}{6}\right)-\frac{a^4\,(51\,c+51\,d\,x-160)}{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{51\,a^4\,(c+d\cdots}{2}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x))^4)/cos(c + d*x)^4,x)
```

```
[Out] (17*a^4*x)/2 + ((17*a^4*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((51*a^4*(c + d*x
))/2 - (a^4*(153*c + 153*d*x - 378))/6) - (a^4*(51*c + 51*d*x - 160))/6 + t
an(c/2 + (d*x)/2)^6*((51*a^4*(c + d*x))/2 - (a^4*(153*c + 153*d*x - 102))/6
) - tan(c/2 + (d*x)/2)^5*((85*a^4*(c + d*x))/2 - (a^4*(255*c + 255*d*x - 30
6))/6) + tan(c/2 + (d*x)/2)^2*((85*a^4*(c + d*x))/2 - (a^4*(255*c + 255*d*x
 - 494))/6) + tan(c/2 + (d*x)/2)^4*((119*a^4*(c + d*x))/2 - (a^4*(357*c + 3
57*d*x - 460))/6) - tan(c/2 + (d*x)/2)^3*((119*a^4*(c + d*x))/2 - (a^4*(357
*c + 357*d*x - 660))/6))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2)^
2 + 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.821 $\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{\cos(c+dx)}{ad}+\frac{\tan^5(c+dx)}{5ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{ad}-\frac{3\sec(c+dx)}{ad}-\frac{x}{a}$$

```
[Out] -x/a-cos(d*x+c)/a/d-3*sec(d*x+c)/a/d+sec(d*x+c)^3/a/d-1/5*sec(d*x+c)^5/a/d+
tan(d*x+c)/a/d-1/3*tan(d*x+c)^3/a/d+1/5*tan(d*x+c)^5/a/d
```

Rubi [A] time = 0.16, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2839, 3473, 8, 2590, 270}

$$-\frac{\cos(c+dx)}{ad}+\frac{\tan^5(c+dx)}{5ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{ad}-\frac{3\sec(c+dx)}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) - Cos[c + d*x]/(a*d) - (3*Sec[c + d*x])/(a*d) + Sec[c + d*x]^3/(a*d)
 - Sec[c + d*x]^5/(5*a*d) + Tan[c + d*x]/(a*d) - Tan[c + d*x]^3/(3*a*d) + T
an[c + d*x]^5/(5*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
```

```
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \tan^6(c+dx)\,dx}{a} - \frac{\int \sin(c+dx)\tan^6(c+dx)\,dx}{a}$$

$$= \frac{\tan^5(c+dx)}{5ad} - \frac{\int \tan^4(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx, x, \cos(c+dx)\right)}{ad}$$

$$= -\frac{\tan^3(c+dx)}{3ad} + \frac{\tan^5(c+dx)}{5ad} + \frac{\int \tan^2(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^6}-\frac{3}{x^4}\right.\right.}{}$$

$$= -\frac{\cos(c+dx)}{ad} - \frac{3\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{\tan(c+dx)}{ad}$$

$$= -\frac{x}{a} - \frac{\cos(c+dx)}{ad} - \frac{3\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{\tan(c+a}{ad}$$

Mathematica [A] time = 0.68, size = 224, normalized size = 1.91

$$-\frac{216\sin(c+dx)+240c\sin(2(c+dx))+240dx\sin(2(c+dx))-618\sin(2(c+dx))+532\sin(3(c+dx))+120c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/960*(1200 + 18*(-103 + 40*c + 40*d*x)*Cos[c + d*x] + 1568*Cos[2*(c + d*x
)] - 618*Cos[3*(c + d*x)] + 240*c*Cos[3*(c + d*x)] + 240*d*x*Cos[3*(c + d*x
)] + 304*Cos[4*(c + d*x)] + 216*Sin[c + d*x] - 618*Sin[2*(c + d*x)] + 240*c
*Sin[2*(c + d*x)] + 240*d*x*Sin[2*(c + d*x)] + 532*Sin[3*(c + d*x)] - 309*S
in[4*(c + d*x)] + 120*c*Sin[4*(c + d*x)] + 120*d*x*Sin[4*(c + d*x)] + 60*Si
```

```
n[5*(c + d*x)])/(a*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^5)
```

fricas [A] time = 0.48, size = 108, normalized size = 0.92

$$-\frac{15\,dx\cos\left(dx+c\right)^{3}+38\,\cos\left(dx+c\right)^{4}+11\,\cos\left(dx+c\right)^{2}+\left(15\,dx\cos\left(dx+c\right)^{3}+15\,\cos\left(dx+c\right)^{4}+22\,\cos\left(dx\right.\right.}{15\left(ad\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(15*d*x*cos(d*x + c)^3 + 38*cos(d*x + c)^4 + 11*cos(d*x + c)^2 + (15*
d*x*cos(d*x + c)^3 + 15*cos(d*x + c)^4 + 22*cos(d*x + c)^2 - 4)*sin(d*x + c
) - 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

giac [A] time = 0.25, size = 149, normalized size = 1.27

$$-\frac{\frac{120\,(dx+c)}{a}+\frac{240}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a}-\frac{5\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+23\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}+\frac{345\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+1560\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2570\,\tan\left(\frac{1}{2}\right.}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(120*(d*x + c)/a + 240/((tan(1/2*d*x + 1/2*c)^2 + 1)*a) - 5*(21*tan(
1/2*d*x + 1/2*c)^2 - 48*tan(1/2*d*x + 1/2*c) + 23)/(a*(tan(1/2*d*x + 1/2*c)
 - 1)^3) + (345*tan(1/2*d*x + 1/2*c)^4 + 1560*tan(1/2*d*x + 1/2*c)^3 + 2570
*tan(1/2*d*x + 1/2*c)^2 + 1720*tan(1/2*d*x + 1/2*c) + 413)/(a*(tan(1/2*d*x
+ 1/2*c) + 1)^5))/d
```

maple [A] time = 0.44, size = 210, normalized size = 1.79

$$-\frac{1}{6ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{7}{8ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2\arctan\left(\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/6/a/d/(tan(1/2*d*x+1/2*c)-1)^3-1/4/a/d/(tan(1/2*d*x+1/2*c)-1)^2+7/8/a/d/
(tan(1/2*d*x+1/2*c)-1)-2/a/d/(1+tan(1/2*d*x+1/2*c)^2)-2/a/d*arctan(tan(1/2*
```

```
d*x+1/2*c))-2/5/a/d/(tan(1/2*d*x+1/2*c)+1)^5+1/a/d/(tan(1/2*d*x+1/2*c)+1)^4
+1/3/a/d/(tan(1/2*d*x+1/2*c)+1)^3-3/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2-23/8/a/d
/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.42, size = 400, normalized size = 3.42

$$-\frac{2\left(\frac{\frac{81\sin(dx+c)}{\cos(dx+c)+1}-\frac{78\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{172\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{26\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{22\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{70\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{20\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{30\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{15\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+48}{a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{4a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2a\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\cdots\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2/15*((81*sin(d*x + c)/(cos(d*x + c) + 1) - 78*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 172*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 26*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 22*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 70*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 20*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 30*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8 - 15*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 4
8)/(a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - a*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 4*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2*a*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 4*a*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a*si
n(d*x + c)^9/(cos(d*x + c) + 1)^9 - a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10
) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 18.67, size = 172, normalized size = 1.47

$$\frac{-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3}+\frac{28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3}+\frac{44\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{15}-\frac{52\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{15}-\frac{344\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{15}\cdots}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^5\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] ((54*tan(c/2 + (d*x)/2))/5 - (52*tan(c/2 + (d*x)/2)^2)/5 - (344*tan(c/2 + (
d*x)/2)^3)/15 - (52*tan(c/2 + (d*x)/2)^4)/15 + (44*tan(c/2 + (d*x)/2)^5)/15
 + (28*tan(c/2 + (d*x)/2)^6)/3 + (8*tan(c/2 + (d*x)/2)^7)/3 - 4*tan(c/2 + (
d*x)/2)^8 - 2*tan(c/2 + (d*x)/2)^9 + 32/5)/(a*d*(tan(c/2 + (d*x)/2) - 1)^3*
(tan(c/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1)) - x/a
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.822 $\int \frac{\sin(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{2\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}+\frac{x}{a}$$

```
[Out] x/a+sec(d*x+c)/a/d-2/3*sec(d*x+c)^3/a/d+1/5*sec(d*x+c)^5/a/d-tan(d*x+c)/a/d
+1/3*tan(d*x+c)^3/a/d-1/5*tan(d*x+c)^5/a/d
```

Rubi [A] time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2606, 194, 3473, 8}

$$-\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{2\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + Sec[c + d*x]/(a*d) - (2*Sec[c + d*x]^3)/(3*a*d) + Sec[c + d*x]^5/(5*a
*d) - Tan[c + d*x]/(a*d) + Tan[c + d*x]^3/(3*a*d) - Tan[c + d*x]^5/(5*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
```

```
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a} - \frac{\int \tan^6(c+dx)\,dx}{a}$$

$$= -\frac{\tan^5(c+dx)}{5ad} + \frac{\int \tan^4(c+dx)\,dx}{a} + \frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= \frac{\tan^3(c+dx)}{3ad} - \frac{\tan^5(c+dx)}{5ad} - \frac{\int \tan^2(c+dx)\,dx}{a} + \frac{\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx\right.}{ad}$$

$$= \frac{\sec(c+dx)}{ad} - \frac{2\sec^3(c+dx)}{3ad} + \frac{\sec^5(c+dx)}{5ad} - \frac{\tan(c+dx)}{ad} + \frac{\tan^3(c+dx)}{3ad} - \text{ta}$$

$$= \frac{x}{a} + \frac{\sec(c+dx)}{ad} - \frac{2\sec^3(c+dx)}{3ad} + \frac{\sec^5(c+dx)}{5ad} - \frac{\tan(c+dx)}{ad} + \frac{\tan^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.61, size = 191, normalized size = 1.82

$$-\frac{\sec^3(c+dx)\left(8\sin(c+dx) - 30c\sin(2(c+dx)) - 30dx\sin(2(c+dx)) + \frac{89}{4}\sin(2(c+dx)) + 16\sin(3(c+dx)) - \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/120*(Sec[c + d*x]^3*(-25 + (267/4 - 90*c - 90*d*x)*Cos[c + d*x] - 16*Cos
[2*(c + d*x)] + (89*Cos[3*(c + d*x)])/4 - 30*c*Cos[3*(c + d*x)] - 30*d*x*Co
s[3*(c + d*x)] - 23*Cos[4*(c + d*x)] + 8*Sin[c + d*x] + (89*Sin[2*(c + d*x)
])/4 - 30*c*Sin[2*(c + d*x)] - 30*d*x*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)
] + (89*Sin[4*(c + d*x)])/8 - 15*c*Sin[4*(c + d*x)] - 15*d*x*Sin[4*(c + d*x
)]))/(a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.45, size = 98, normalized size = 0.93

$$\frac{15\,dx\cos\left(dx+c\right)^{3}+23\,\cos\left(dx+c\right)^{4}-19\,\cos\left(dx+c\right)^{2}+\left(15\,dx\cos\left(dx+c\right)^{3}-8\,\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+\right.}{15\left(ad\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(15*d*x*cos(d*x + c)^3 + 23*cos(d*x + c)^4 - 19*cos(d*x + c)^2 + (15*d
*x*cos(d*x + c)^3 - 8*cos(d*x + c)^2 + 1)*sin(d*x + c) + 4)/(a*d*cos(d*x +
c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

giac [A] time = 0.23, size = 130, normalized size = 1.24

$$\frac{\frac{120\,(dx+c)}{a}+\frac{5\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-36\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}+\frac{3\left(55\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+260\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+450\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+300\,\tan\left(\frac{1}{2}\,dx+\right.\right.}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(120*(d*x + c)/a + 5*(15*tan(1/2*d*x + 1/2*c)^2 - 36*tan(1/2*d*x + 1/
2*c) + 17)/(a*(tan(1/2*d*x + 1/2*c) - 1)^3) + 3*(55*tan(1/2*d*x + 1/2*c)^4
+ 260*tan(1/2*d*x + 1/2*c)^3 + 450*tan(1/2*d*x + 1/2*c)^2 + 300*tan(1/2*d*x
 + 1/2*c) + 71)/(a*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.43, size = 166, normalized size = 1.58

$$-\frac{1}{6ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{5}{8ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{}{5ad\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/6/a/d/(tan(1/2*d*x+1/2*c)-1)^3-1/4/a/d/(tan(1/2*d*x+1/2*c)-1)^2+5/8/a/d/
(tan(1/2*d*x+1/2*c)-1)+2/a/d*arctan(tan(1/2*d*x+1/2*c))+2/5/a/d/(tan(1/2*d*
x+1/2*c)+1)^5-1/a/d/(tan(1/2*d*x+1/2*c)+1)^4+1/a/d/(tan(1/2*d*x+1/2*c)+1)^2
+11/8/a/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.43, size = 318, normalized size = 3.03

$$\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{46\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{13\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{100\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{30\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+8}{a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/15*((sin(d*x + c)/(cos(d*x + c) + 1) - 46*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 - 13*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 100*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 35*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 30*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 8)/(a + 2*a*
sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 15*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 16.14, size = 131, normalized size = 1.25

$$\frac{x}{a}-\frac{-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\frac{14\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{3}+\frac{40\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}-\frac{26\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{15}-\frac{92\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{15}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{15}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] x/a - ((2*tan(c/2 + (d*x)/2))/15 - (92*tan(c/2 + (d*x)/2)^2)/15 - (26*tan(c
/2 + (d*x)/2)^3)/15 + (40*tan(c/2 + (d*x)/2)^4)/3 + (14*tan(c/2 + (d*x)/2)^
5)/3 - 4*tan(c/2 + (d*x)/2)^6 - 2*tan(c/2 + (d*x)/2)^7 + 16/15)/(a*d*(tan(c
/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.823 $\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{\tan^5(c+dx)}{5ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}$$

```
[Out] -sec(d*x+c)/a/d+2/3*sec(d*x+c)^3/a/d-1/5*sec(d*x+c)^5/a/d+1/5*tan(d*x+c)^5/
a/d
```

Rubi [A] time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2606, 194}

$$\frac{\tan^5(c+dx)}{5ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]/(a*d)) + (2*Sec[c + d*x]^3)/(3*a*d) - Sec[c + d*x]^5/(5*a*d)
 + Tan[c + d*x]^5/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a}-\frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a}\\ &= \frac{\operatorname{Subst}\left(\int x^4\,dx,x,\tan(c+dx)\right)}{ad}-\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\sec(c+dx)\right)}{ad}\\ &= \frac{\tan^5(c+dx)}{5ad}-\frac{\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\sec(c+dx)\right)}{ad}\\ &= -\frac{\sec(c+dx)}{ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\tan^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A] time = 0.27, size = 106, normalized size = 1.54

$$-\frac{\sec^3(c+dx)(-64\sin(c+dx)-178\sin(2(c+dx))+192\sin(3(c+dx))-89\sin(4(c+dx))-534\cos(c+dx)+}{960ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/960*(Sec[c + d*x]^3*(200 - 534*Cos[c + d*x] + 288*Cos[2*(c + d*x)] - 178
*Cos[3*(c + d*x)] + 24*Cos[4*(c + d*x)] - 64*Sin[c + d*x] - 178*Sin[2*(c +
d*x)] + 192*Sin[3*(c + d*x)] - 89*Sin[4*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]
))
```

fricas [A] time = 0.43, size = 75, normalized size = 1.09

$$-\frac{3\cos\left(dx+c\right)^4+6\cos\left(dx+c\right)^2+4\left(3\cos\left(dx+c\right)^2-1\right)\sin\left(dx+c\right)-1}{15\left(ad\cos\left(dx+c\right)^3\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(d*x + c)^4 + 6*cos(d*x + c)^2 + 4*(3*cos(d*x + c)^2 - 1)*sin(d
*x + c) - 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

giac [A] time = 0.24, size = 120, normalized size = 1.74

$$\frac{\frac{5\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}-\frac{45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+490\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+320\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+73}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(9*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 11)/(a*(tan(
1/2*d*x + 1/2*c) - 1)^3) - (45*tan(1/2*d*x + 1/2*c)^4 + 240*tan(1/2*d*x + 1
/2*c)^3 + 490*tan(1/2*d*x + 1/2*c)^2 + 320*tan(1/2*d*x + 1/2*c) + 73)/(a*(t
an(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [B] time = 0.41, size = 130, normalized size = 1.88

$$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\right.\right.}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 32/d/a*(-1/192/(tan(1/2*d*x+1/2*c)-1)^3-1/128/(tan(1/2*d*x+1/2*c)-1)^2+3/25
6/(tan(1/2*d*x+1/2*c)-1)-1/80/(tan(1/2*d*x+1/2*c)+1)^5+1/32/(tan(1/2*d*x+1/
2*c)+1)^4-1/96/(tan(1/2*d*x+1/2*c)+1)^3-1/64/(tan(1/2*d*x+1/2*c)+1)^2-3/256
/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.33, size = 214, normalized size = 3.10

$$-\frac{16\left(\frac{2\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+1\right)}{15\left(a+\frac{2\,a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6\,a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{6\,a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{2\,a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{2\,a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -16/15*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 6*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/((a + 2*a*sin(d*x + c)/
(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*a
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1
)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

mupad **[B]** time = 9.55, size = 73, normalized size = 1.06

$$\frac{16\left(-6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] (16*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2)^3
 + 1))/(15*a*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^5)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.824 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$-\frac{\tan^5(c+dx)}{5ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{3ad}$$

```
[Out] -1/3*sec(d*x+c)^3/a/d+1/5*sec(d*x+c)^5/a/d-1/5*tan(d*x+c)^5/a/d
```

Rubi [A] time = 0.14, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2606, 14, 2607, 30}

$$-\frac{\tan^5(c+dx)}{5ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Sec[c + d*x]^3/(3*a*d) + Sec[c + d*x]^5/(5*a*d) - Tan[c + d*x]^5/(5*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^3(c+dx)\tan^3(c+dx)\,dx}{a}-\frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int x^4\,dx, x, \tan(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx, x, \sec(c+dx)\right)}{ad}\\
&= -\frac{\tan^5(c+dx)}{5ad}+\frac{\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx, x, \sec(c+dx)\right)}{ad}\\
&= -\frac{\sec^3(c+dx)}{3ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{\tan^5(c+dx)}{5ad}
\end{aligned}
$$

Mathematica [A] time = 0.29, size = 106, normalized size = 1.93

$$
\frac{\sec^3(c+dx)(16\sin(c+dx)+22\sin(2(c+dx))-48\sin(3(c+dx))+11\sin(4(c+dx))+66\cos(c+dx)-192\cos}{960ad(\sin(c+dx)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^3*(40 + 66*Cos[c + d*x] - 192*Cos[2*(c + d*x)] + 22*Cos[3*(c
+ d*x)] + 24*Cos[4*(c + d*x)] + 16*Sin[c + d*x] + 22*Sin[2*(c + d*x)] - 48*
Sin[3*(c + d*x)] + 11*Sin[4*(c + d*x)]))/(960*a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.44, size = 75, normalized size = 1.36

$$
\frac{3\cos\left(dx+c\right)^4-9\cos\left(dx+c\right)^2-\left(3\cos\left(dx+c\right)^2-1\right)\sin\left(dx+c\right)+4}{15\left(ad\cos\left(dx+c\right)^3\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - 1)*sin(d*x
+ c) + 4)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

giac [B] time = 0.21, size = 120, normalized size = 2.18

$$\frac{\frac{5\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}-\frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+60\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+20\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(3*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 5)/(a*(tan(1
/2*d*x + 1/2*c) - 1)^3) - (15*tan(1/2*d*x + 1/2*c)^4 + 60*tan(1/2*d*x + 1/2
*c)^3 + 10*tan(1/2*d*x + 1/2*c)^2 + 20*tan(1/2*d*x + 1/2*c) + 7)/(a*(tan(1/
2*d*x + 1/2*c) + 1)^5))/d
```

maple [B] time = 0.41, size = 115, normalized size = 2.09

$$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{16}{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-128}+\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{\cdots}{8\left(\tan\left(\cdots\right.\right.}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 16/d/a*(-1/96/(tan(1/2*d*x+1/2*c)-1)^3-1/64/(tan(1/2*d*x+1/2*c)-1)^2+1/128/
(tan(1/2*d*x+1/2*c)-1)+1/40/(tan(1/2*d*x+1/2*c)+1)^5-1/16/(tan(1/2*d*x+1/2*
c)+1)^4+1/24/(tan(1/2*d*x+1/2*c)+1)^3-1/128/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.34, size = 234, normalized size = 4.25

$$-\frac{4\left(\frac{2\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{15\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}{15\left(a+\frac{2\,a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6\,a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{6\,a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{2\,a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{2\,a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -4/15*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 6*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15*sin(d*x + c)^4/(cos(d*x
```

```
+ c) + 1)^4 + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*s
in(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 - 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c
) + 1)^8)*d)
```

mupad [B] time = 9.86, size = 86, normalized size = 1.56

$$-\frac{4\left(15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] -(4*(2*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3
 + 15*tan(c/2 + (d*x)/2)^4 - 1))/(15*a*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/
2 + (d*x)/2) + 1)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.825 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{3ad}$$

```
[Out] 1/3*sec(d*x+c)^3/a/d-1/5*sec(d*x+c)^5/a/d+1/3*tan(d*x+c)^3/a/d+1/5*tan(d*x+
c)^5/a/d
```

Rubi [A] time = 0.16, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2839, 2607, 14, 2606}

$$\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]^3/(3*a*d) - Sec[c + d*x]^5/(5*a*d) + Tan[c + d*x]^3/(3*a*d) +
Tan[c + d*x]^5/(5*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^4(c+dx)\tan^2(c+dx)\,dx}{a}-\frac{\int \sec^3(c+dx)\tan^3(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\sec(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,\tan(c\right.}{ad}\\ &= -\frac{\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\sec(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,\tan(c+\right.}{ad}\\ &= \frac{\sec^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{3ad}+\frac{\tan^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A] time = 0.34, size = 106, normalized size = 1.45

$$-\frac{\sec^3(c+dx)(-224\sin(c+dx)+22\sin(2(c+dx))+32\sin(3(c+dx))+11\sin(4(c+dx))+66\cos(c+dx)-32}{960ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/960*(Sec[c + d*x]^3*(-80 + 66*Cos[c + d*x] - 32*Cos[2*(c + d*x)] + 22*Co
s[3*(c + d*x)] - 16*Cos[4*(c + d*x)] - 224*Sin[c + d*x] + 22*Sin[2*(c + d*x
)] + 32*Sin[3*(c + d*x)] + 11*Sin[4*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.45, size = 73, normalized size = 1.00

$$\frac{2\cos(dx+c)^4-\cos(dx+c)^2-2\left(\cos(dx+c)^2-2\right)\sin(dx+c)+1}{15\left(ad\cos(dx+c)^3\sin(dx+c)+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(2*cos(d*x + c)^4 - cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 2)*sin(d*x +
c) + 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

giac [A] time = 0.23, size = 109, normalized size = 1.49

$$-\frac{\frac{5\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}-\frac{3\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+50\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+9\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(5*(3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a*(tan(1/2*d*x + 1/2*c) - 1)^3) -
 3*(5*tan(1/2*d*x + 1/2*c)^4 + 40*tan(1/2*d*x + 1/2*c)^3 + 50*tan(1/2*d*x +
 1/2*c)^2 + 40*tan(1/2*d*x + 1/2*c) + 9)/(a*(tan(1/2*d*x + 1/2*c) + 1)^5))/
d
```

maple [A] time = 0.38, size = 130, normalized size = 1.78

$$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 8/d/a*(-1/48/(tan(1/2*d*x+1/2*c)-1)^3-1/32/(tan(1/2*d*x+1/2*c)-1)^2-1/64/(t
an(1/2*d*x+1/2*c)-1)-1/20/(tan(1/2*d*x+1/2*c)+1)^5+1/8/(tan(1/2*d*x+1/2*c)+
1)^4-1/8/(tan(1/2*d*x+1/2*c)+1)^3+1/16/(tan(1/2*d*x+1/2*c)+1)^2+1/64/(tan(1
/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.34, size = 254, normalized size = 3.48

$$\frac{4\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{4\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{5\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{10\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+1\right)}{15\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{6a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{2a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{2a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4/15*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 10*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a + 2*a*sin(d*x +
 c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(
```

```
d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

mupad [B] time = 10.16, size = 99, normalized size = 1.36

$$-\frac{4\left(10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] -(4*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2 + 4*tan(c/2 + (d*x)/2)^3
 + 5*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^5 + 1))/(15*a*d*(tan(c/2
+ (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^2(c+dx)\sec^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

3.826 $\int \frac{\sec^3(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$-\frac{\tan^5(c+dx)}{5ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\sec^5(c+dx)}{5ad}$$

```
[Out] 1/5*sec(d*x+c)^5/a/d-1/3*tan(d*x+c)^3/a/d-1/5*tan(d*x+c)^5/a/d
```

Rubi [A] time = 0.11, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2606, 30, 2607, 14}

$$-\frac{\tan^5(c+dx)}{5ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\sec^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]^5/(5*a*d) - Tan[c + d*x]^3/(3*a*d) - Tan[c + d*x]^5/(5*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^5(c+dx)\tan(c+dx)\,dx}{a}-\frac{\int \sec^4(c+dx)\tan^2(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^4\,dx,x,\sec(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,\tan(c+dx)\right)}{ad}\\ &= \frac{\sec^5(c+dx)}{5ad}-\frac{\text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,\tan(c+dx)\right)}{ad}\\ &= \frac{\sec^5(c+dx)}{5ad}-\frac{\tan^3(c+dx)}{3ad}-\frac{\tan^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A] time = 0.26, size = 106, normalized size = 1.93

$$-\frac{\sec^3(c+dx)(-96\sin(c+dx)+18\sin(2(c+dx))-32\sin(3(c+dx))+9\sin(4(c+dx))+54\cos(c+dx)+32\,\text{co}}{960ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/960*(Sec[c + d*x]^3*(-240 + 54*Cos[c + d*x] + 32*Cos[2*(c + d*x)] + 18*C
os[3*(c + d*x)] + 16*Cos[4*(c + d*x)] - 96*Sin[c + d*x] + 18*Sin[2*(c + d*x
)] - 32*Sin[3*(c + d*x)] + 9*Sin[4*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.42, size = 75, normalized size = 1.36

$$-\frac{2\cos(dx+c)^4-\cos(dx+c)^2-\left(2\cos(dx+c)^2+1\right)\sin(dx+c)-4}{15\left(ad\cos(dx+c)^3\sin(dx+c)+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/15*(2*cos(d*x + c)^4 - cos(d*x + c)^2 - (2*cos(d*x + c)^2 + 1)*sin(d*x + c) - 4)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)

giac [B] time = 0.24, size = 120, normalized size = 2.18

$$-\frac{\frac{5\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}-\frac{45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+60\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+70\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+20\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/120*(5*(9*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 7)/(a*(tan(1/2*d*x + 1/2*c) - 1)^3) - (45*tan(1/2*d*x + 1/2*c)^4 + 60*tan(1/2*d*x + 1/2*c)^3 + 70*tan(1/2*d*x + 1/2*c)^2 + 20*tan(1/2*d*x + 1/2*c) + 13)/(a*(tan(1/2*d*x + 1/2*c) + 1)^5))/d

maple [B] time = 0.34, size = 130, normalized size = 2.36

$$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{4}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x)

[Out] 4/d/a*(-1/24/(tan(1/2*d*x+1/2*c)-1)^3-1/16/(tan(1/2*d*x+1/2*c)-1)^2-3/32/(tan(1/2*d*x+1/2*c)-1)-1/4/(tan(1/2*d*x+1/2*c)+1)^4+1/10/(tan(1/2*d*x+1/2*c)+1)^5+1/3/(tan(1/2*d*x+1/2*c)+1)^3-1/4/(tan(1/2*d*x+1/2*c)+1)^2+3/32/(tan(1/2*d*x+1/2*c)+1))

maxima [B] time = 0.33, size = 274, normalized size = 4.98

$$\frac{2\left(\frac{6\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{5\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{10\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{15\,\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+3\right)}{15\left(a+\frac{2\,a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6\,a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{6\,a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{2\,a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{2\,a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")

```
[Out] 2/15*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 10*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + 3)/((a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
6*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c)
+ 1)^6 - 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*
x + c) + 1)^8)*d)
```

mupad [B] time = 10.85, size = 112, normalized size = 2.04

$$-\frac{2\left(15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right.}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] -(2*(6*tan(c/2 + (d*x)/2) + 9*tan(c/2 + (d*x)/2)^2 - 8*tan(c/2 + (d*x)/2)^3
 + 5*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^5 + 15*tan(c/2 + (d*x)/2)
^6 + 3))/(15*a*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin(c+dx)\sec^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

3.827 $\int \frac{\csc(c+dx)\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=115

$$-\frac{\tan^5(c+dx)}{5ad}-\frac{2\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -arctanh(cos(d*x+c))/a/d+sec(d*x+c)/a/d+1/3*sec(d*x+c)^3/a/d+1/5*sec(d*x+c)
^5/a/d-tan(d*x+c)/a/d-2/3*tan(d*x+c)^3/a/d-1/5*tan(d*x+c)^5/a/d
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2622, 302, 207, 3767}

$$-\frac{\tan^5(c+dx)}{5ad}-\frac{2\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/(3*a*d
) + Sec[c + d*x]^5/(5*a*d) - Tan[c + d*x]/(a*d) - (2*Tan[c + d*x]^3)/(3*a*d
) - Tan[c + d*x]^5/(5*a*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \sec^6(c+dx)\,dx}{a}+\frac{\int \csc(c+dx)\sec^6(c+dx)\,dx}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(c+d$$

$$= -\frac{\tan(c+dx)}{ad}-\frac{2\tan^3(c+dx)}{3ad}-\frac{\tan^5(c+dx)}{5ad}+\frac{\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right.\right.}{ad}$$

$$= \frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{\tan(c+dx)}{ad}-\frac{2\tan^3(c+dx)}{3ad}-\frac{\text{ta}}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{\tan(c+d}{ad}$$

Mathematica [B] time = 0.66, size = 267, normalized size = 2.32

$$-\frac{\sec^3(c+dx)\left(-22\sin(c+dx)+\frac{149}{4}\sin(2(c+dx))-14\sin(3(c+dx))+\frac{149}{8}\sin(4(c+dx))-76\cos(2(c+dx))-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/120*(Sec[c + d*x]^3*(-100 - 76*Cos[2*(c + d*x)] + (149*Cos[3*(c + d*x)])
/4 - 8*Cos[4*(c + d*x)] + 30*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + Cos[c
 + d*x]*(447/4 + 90*Log[Cos[(c + d*x)/2]] - 90*Log[Sin[(c + d*x)/2]]) - 30*
Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 22*Sin[c + d*x] + (149*Sin[2*(c +
```

```
d*x)])/4 + 30*Log[Cos[(c + d*x)/2]]*Sin[2*(c + d*x)] - 30*Log[Sin[(c + d*x)
/2]]*Sin[2*(c + d*x)] - 14*Sin[3*(c + d*x)] + (149*Sin[4*(c + d*x)])/8 + 15
*Log[Cos[(c + d*x)/2]]*Sin[4*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin[4*(c
 + d*x)]))/(a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.46, size = 149, normalized size = 1.30

$$\frac{16\cos(dx+c)^4+22\cos(dx+c)^2-15\left(\cos(dx+c)^3\sin(dx+c)+\cos(dx+c)^3\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+1}{30\left(ad\cos(dx+c)^3\sin(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(16*cos(d*x + c)^4 + 22*cos(d*x + c)^2 - 15*(cos(d*x + c)^3*sin(d*x +
c) + cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^3*sin(d
*x + c) + cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) + 2*(7*cos(d*x + c)^
2 + 1)*sin(d*x + c) + 8)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c
)^3)
```

giac [A] time = 0.21, size = 136, normalized size = 1.18

$$\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{5\left(21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-36\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+19\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}+\frac{3\left(115\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+380\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+530\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right.}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - 5*(21*tan(1/2*d*x + 1/2*c)^2
- 36*tan(1/2*d*x + 1/2*c) + 19)/(a*(tan(1/2*d*x + 1/2*c) - 1)^3) + 3*(115*t
an(1/2*d*x + 1/2*c)^4 + 380*tan(1/2*d*x + 1/2*c)^3 + 530*tan(1/2*d*x + 1/2*
c)^2 + 340*tan(1/2*d*x + 1/2*c) + 91)/(a*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.50, size = 187, normalized size = 1.63

$$-\frac{1}{6ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{4ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{7}{8ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2}{5ad\left(\tan\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/6/a/d/(tan(1/2*d*x+1/2*c)-1)^3-1/4/a/d/(tan(1/2*d*x+1/2*c)-1)^2-7/8/a/d/
(tan(1/2*d*x+1/2*c)-1)+1/a/d*ln(tan(1/2*d*x+1/2*c))+2/5/a/d/(tan(1/2*d*x+1/
2*c)+1)^5-1/a/d/(tan(1/2*d*x+1/2*c)+1)^4+2/a/d/(tan(1/2*d*x+1/2*c)+1)^3-2/a
/d/(tan(1/2*d*x+1/2*c)+1)^2+23/8/a/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.34, size = 320, normalized size = 2.78

$$\frac{\frac{2\left(\frac{31\sin(dx+c)}{\cos(dx+c)+1}-\frac{31\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{73\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{25\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{65\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+23\right)}{a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/15*(2*(31*sin(d*x + c)/(cos(d*x + c) + 1) - 31*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 73*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 25*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 65*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 23)/(a +
 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 - 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 15*log(sin
(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 11.53, size = 143, normalized size = 1.24

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{26\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}+\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{146\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{15}-\frac{62\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{15}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) - ((62*tan(c/2 + (d*x)/2))/15 - (62*tan(c/2 +
 (d*x)/2)^2)/15 - (146*tan(c/2 + (d*x)/2)^3)/15 + (10*tan(c/2 + (d*x)/2)^4)
/3 + (26*tan(c/2 + (d*x)/2)^5)/3 + 2*tan(c/2 + (d*x)/2)^6 - 2*tan(c/2 + (d*
x)/2)^7 + 46/15)/(a*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^5
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.828 $\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{ad}+\frac{3\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d-sec(d*x+c)/a/d-1/3*sec(d*x+c)^3/a/d-
1/5*sec(d*x+c)^5/a/d+3*tan(d*x+c)/a/d+tan(d*x+c)^3/a/d+1/5*tan(d*x+c)^5/a/d
```

Rubi [A] time = 0.17, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2620, 270, 2622, 302, 207}

$$\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{ad}+\frac{3\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d) - Sec[c + d*x]/(a*d) - Sec
[c + d*x]^3/(3*a*d) - Sec[c + d*x]^5/(5*a*d) + (3*Tan[c + d*x])/(a*d) + Tan
[c + d*x]^3/(a*d) + Tan[c + d*x]^5/(5*a*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \csc(c+dx)\sec^6(c+dx)\,dx}{a} + \frac{\int \csc^2(c+dx)\sec^6(c+dx)\,dx}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^2}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+3x^2+x^4\right)dx, x, \tan(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \left(1+x^2+x^4+\right.\right.}{a}$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{3ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{3\tan(c+dx)}{ad} +$$

$$= \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\cot(c+dx)}{ad} - \frac{\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{3ad} - \frac{\sec^5(c+}{5ad}$$

Mathematica [B] time = 0.60, size = 341, normalized size = 2.71

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(352\sin(c+dx)-596\sin(2(c+dx))+864\sin(3(c+dx))-298\,\mathrm{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3840*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(176 + 1216*Cos[2
*(c + d*x)] + 149*Cos[3*(c + d*x)] + 528*Cos[4*(c + d*x)] + 149*Cos[5*(c +
d*x)] + 120*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 120*Cos[5*(c + d*x)]*L
og[Cos[(c + d*x)/2]] - 120*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 120*Cos
[5*(c + d*x)]*Log[Sin[(c + d*x)/2]] + Cos[c + d*x]*(-298 - 240*Log[Cos[(c +
 d*x)/2]] + 240*Log[Sin[(c + d*x)/2]]) + 352*Sin[c + d*x] - 596*Sin[2*(c +
d*x)] - 480*Log[Cos[(c + d*x)/2]]*Sin[2*(c + d*x)] + 480*Log[Sin[(c + d*x)/
2]]*Sin[2*(c + d*x)] + 864*Sin[3*(c + d*x)] - 298*Sin[4*(c + d*x)] - 240*Lo
g[Cos[(c + d*x)/2]]*Sin[4*(c + d*x)] + 240*Log[Sin[(c + d*x)/2]]*Sin[4*(c +
 d*x)] + 384*Sin[5*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.47, size = 194, normalized size = 1.54

$$\frac{66\,\cos\left(dx+c\right)^{4}-28\,\cos\left(dx+c\right)^{2}+15\left(\cos\left(dx+c\right)^{5}-\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)-\cos\left(dx+c\right)^{3}\right)\log\left(\frac{1}{2}\,\cos\left(d\right.\right.}{30\left(ad\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(66*cos(d*x + c)^4 - 28*cos(d*x + c)^2 + 15*(cos(d*x + c)^5 - cos(d*x
+ c)^3*sin(d*x + c) - cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) - 15*(cos
(d*x + c)^5 - cos(d*x + c)^3*sin(d*x + c) - cos(d*x + c)^3)*log(-1/2*cos(d*
x + c) + 1/2) + 2*(48*cos(d*x + c)^4 - 9*cos(d*x + c)^2 - 1)*sin(d*x + c) -
 8)/(a*d*cos(d*x + c)^5 - a*d*cos(d*x + c)^3*sin(d*x + c) - a*d*cos(d*x + c
)^3)
```

giac [A] time = 0.22, size = 178, normalized size = 1.41

$$-\frac{\frac{120\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{60\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{60\left(2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{5\left(27\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+25\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}+\frac{585\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - 60*tan(1/2*d*x + 1/2*c)/a -
60*(2*tan(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)) + 5*(27*tan(1/2*d*
x + 1/2*c)^2 - 48*tan(1/2*d*x + 1/2*c) + 25)/(a*(tan(1/2*d*x + 1/2*c) - 1)^
3) + (585*tan(1/2*d*x + 1/2*c)^4 + 2040*tan(1/2*d*x + 1/2*c)^3 + 2890*tan(1
/2*d*x + 1/2*c)^2 + 1880*tan(1/2*d*x + 1/2*c) + 493)/(a*(tan(1/2*d*x + 1/2*
c) + 1)^5))/d
```

maple [A] time = 0.50, size = 223, normalized size = 1.77

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{1}{6ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{9}{8ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{2ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)-1/6/a/d/(tan(1/2*d*x+1/2*c)-1)^3-1/4/a/d/(tan(1/
2*d*x+1/2*c)-1)^2-9/8/a/d/(tan(1/2*d*x+1/2*c)-1)-1/2/a/d/tan(1/2*d*x+1/2*c)
-1/a/d*ln(tan(1/2*d*x+1/2*c))-2/5/a/d/(tan(1/2*d*x+1/2*c)+1)^5+1/a/d/(tan(1
/2*d*x+1/2*c)+1)^4-7/3/a/d/(tan(1/2*d*x+1/2*c)+1)^3+5/2/a/d/(tan(1/2*d*x+1/
2*c)+1)^2-39/8/a/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.34, size = 379, normalized size = 3.01

$$-\frac{\frac{\frac{122\sin(dx+c)}{\cos(dx+c)+1}-\frac{26\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{454\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{252\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{510\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{330\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{210\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{195\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+15}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{2a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{6a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{6a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{2a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{2a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{a\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right.}{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*((122*sin(d*x + c)/(cos(d*x + c) + 1) - 26*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 454*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 252*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 510*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 330*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 - 210*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 195
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 15)/(a*sin(d*x + c)/(cos(d*x + c) +
1) + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*a*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 6*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 6*a*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 + 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 2*a*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - a*sin(d*x + c)^9/(cos(d*x + c) + 1)^9) + 30
*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - 15*sin(d*x + c)/(a*(cos(d*x + c)
+ 1)))/d
```

mupad [B] time = 10.82, size = 257, normalized size = 2.04

$$\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}-22\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-34\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+\frac{84\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{5}+\frac{454\tan\left(\right.}{1}}{d\left(-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}-4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-4\,a\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] ((26*tan(c/2 + (d*x)/2)^2)/15 - (122*tan(c/2 + (d*x)/2))/15 + (454*tan(c/2
+ (d*x)/2)^3)/15 + (84*tan(c/2 + (d*x)/2)^4)/5 - 34*tan(c/2 + (d*x)/2)^5 -
22*tan(c/2 + (d*x)/2)^6 + 14*tan(c/2 + (d*x)/2)^7 + 13*tan(c/2 + (d*x)/2)^8
 - 1)/(d*(2*a*tan(c/2 + (d*x)/2) + 4*a*tan(c/2 + (d*x)/2)^2 - 4*a*tan(c/2 +
 (d*x)/2)^3 - 12*a*tan(c/2 + (d*x)/2)^4 + 12*a*tan(c/2 + (d*x)/2)^6 + 4*a*t
an(c/2 + (d*x)/2)^7 - 4*a*tan(c/2 + (d*x)/2)^8 - 2*a*tan(c/2 + (d*x)/2)^9))
 - log(tan(c/2 + (d*x)/2))/(a*d) + tan(c/2 + (d*x)/2)/(2*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.829 $\int \frac{\sin^3(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=155

$$-\frac{\cos(c+dx)}{a^2d}-\frac{2\tan^7(c+dx)}{7a^2d}+\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{3a^2d}+\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{7\sec^5(c+dx)}{5a^2d}+$$

```
[Out] -2*x/a^2-cos(d*x+c)/a^2/d-5*sec(d*x+c)/a^2/d+3*sec(d*x+c)^3/a^2/d-7/5*sec(d
*x+c)^5/a^2/d+2/7*sec(d*x+c)^7/a^2/d+2*tan(d*x+c)/a^2/d-2/3*tan(d*x+c)^3/a^
2/d+2/5*tan(d*x+c)^5/a^2/d-2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.29, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2606, 194, 3473, 8, 2590, 270}

$$-\frac{\cos(c+dx)}{a^2d}-\frac{2\tan^7(c+dx)}{7a^2d}+\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{3a^2d}+\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{7\sec^5(c+dx)}{5a^2d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*x)/a^2 - Cos[c + d*x]/(a^2*d) - (5*Sec[c + d*x])/(a^2*d) + (3*Sec[c + d
*x]^3)/(a^2*d) - (7*Sec[c + d*x]^5)/(5*a^2*d) + (2*Sec[c + d*x]^7)/(7*a^2*d
) + (2*Tan[c + d*x])/(a^2*d) - (2*Tan[c + d*x]^3)/(3*a^2*d) + (2*Tan[c + d*
x]^5)/(5*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec(c+dx)(a-a\sin(c+dx))^2\tan^7(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec(c+dx)\tan^7(c+dx) - 2a^2\tan^8(c+dx) + a^2\sin(c+dx)\tan^8(c+d\right.}{a^4}$$
$$= \frac{\int \sec(c+dx)\tan^7(c+dx)\,dx}{a^2} + \frac{\int \sin(c+dx)\tan^8(c+dx)\,dx}{a^2} - \frac{2\int \tan^8(c+}{a^2}$$
$$= -\frac{2\tan^7(c+dx)}{7a^2d} + \frac{2\int \tan^6(c+dx)\,dx}{a^2} - \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{x^8}\,dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= \frac{2\tan^5(c+dx)}{5a^2d} - \frac{2\tan^7(c+dx)}{7a^2d} - \frac{2\int \tan^4(c+dx)\,dx}{a^2} - \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^8} - \right.\right.}{}$$
$$= -\frac{\cos(c+dx)}{a^2d} - \frac{5\sec(c+dx)}{a^2d} + \frac{3\sec^3(c+dx)}{a^2d} - \frac{7\sec^5(c+dx)}{5a^2d} + \frac{2\sec^7(c+}{7a^2d}$$
$$= -\frac{\cos(c+dx)}{a^2d} - \frac{5\sec(c+dx)}{a^2d} + \frac{3\sec^3(c+dx)}{a^2d} - \frac{7\sec^5(c+dx)}{5a^2d} + \frac{2\sec^7(c+}{7a^2d}$$
$$= -\frac{2x}{a^2} - \frac{\cos(c+dx)}{a^2d} - \frac{5\sec(c+dx)}{a^2d} + \frac{3\sec^3(c+dx)}{a^2d} - \frac{7\sec^5(c+dx)}{5a^2d} + \frac{2\sec}{}$$

Mathematica [A] time = 0.79, size = 267, normalized size = 1.72

$$-\frac{5488\sin(c+dx) + 6720c\sin(2(c+dx)) + 6720dx\sin(2(c+dx)) - 13224\sin(2(c+dx)) + 8376\sin(3(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/6720*(11172 + 42*(-551 + 280*c + 280*d*x)*Cos[c + d*x] + 14834*Cos[2*(c
+ d*x)] - 4959*Cos[3*(c + d*x)] + 2520*c*Cos[3*(c + d*x)] + 2520*d*x*Cos[3*
(c + d*x)] + 1852*Cos[4*(c + d*x)] + 1653*Cos[5*(c + d*x)] - 840*c*Cos[5*(c
 + d*x)] - 840*d*x*Cos[5*(c + d*x)] - 210*Cos[6*(c + d*x)] + 5488*Sin[c + d
*x] - 13224*Sin[2*(c + d*x)] + 6720*c*Sin[2*(c + d*x)] + 6720*d*x*Sin[2*(c
+ d*x)] + 8376*Sin[3*(c + d*x)] - 6612*Sin[4*(c + d*x)] + 3360*c*Sin[4*(c +
 d*x)] + 3360*d*x*Sin[4*(c + d*x)] + 2248*Sin[5*(c + d*x)])/(a^2*d*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

fricas [A] time = 0.48, size = 150, normalized size = 0.97

$$-\frac{210\,dx\cos(dx+c)^5 + 105\,\cos(dx+c)^6 - 420\,dx\cos(dx+c)^3 - 389\,\cos(dx+c)^4 - 173\,\cos(dx+c)^2 - 2\left(21\right.}{105\left(a^2d\cos(dx+c)^5 - 2\,a^2d\cos(dx+c)^3\sin(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(210*d*x*cos(d*x + c)^5 + 105*cos(d*x + c)^6 - 420*d*x*cos(d*x + c)^
3 - 389*cos(d*x + c)^4 - 173*cos(d*x + c)^2 - 2*(210*d*x*cos(d*x + c)^3 + 2
81*cos(d*x + c)^4 + 51*cos(d*x + c)^2 - 5)*sin(d*x + c) + 25)/(a^2*d*cos(d*
x + c)^5 - 2*a^2*d*cos(d*x + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.30, size = 175, normalized size = 1.13

$$-\frac{\frac{1680\,(dx+c)}{a^2}+\frac{1680}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}-\frac{35\left(12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-27\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\right)}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{3780\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+25095\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6884}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(1680*(d*x + c)/a^2 + 1680/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^2) - 35*(
12*tan(1/2*d*x + 1/2*c)^2 - 27*tan(1/2*d*x + 1/2*c) + 13)/(a^2*(tan(1/2*d*x
 + 1/2*c) - 1)^3) + (3780*tan(1/2*d*x + 1/2*c)^6 + 25095*tan(1/2*d*x + 1/2*
c)^5 + 68845*tan(1/2*d*x + 1/2*c)^4 + 98350*tan(1/2*d*x + 1/2*c)^3 + 75222*
tan(1/2*d*x + 1/2*c)^2 + 29659*tan(1/2*d*x + 1/2*c) + 4777)/(a^2*(tan(1/2*d
*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.58, size = 253, normalized size = 1.63

$$-\frac{1}{12d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/12/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3-1/8/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2+1/2
/a^2/d/(tan(1/2*d*x+1/2*c)-1)-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^2*arct
an(tan(1/2*d*x+1/2*c))+4/7/d/a^2/(tan(1/2*d*x+1/2*c)+1)^7-2/d/a^2/(tan(1/2*
d*x+1/2*c)+1)^6+6/5/a^2/d/(tan(1/2*d*x+1/2*c)+1)^5+2/a^2/d/(tan(1/2*d*x+1/2
*c)+1)^4-1/12/a^2/d/(tan(1/2*d*x+1/2*c)+1)^3-23/8/a^2/d/(tan(1/2*d*x+1/2*c)
+1)^2-9/2/a^2/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.43, size = 507, normalized size = 3.27

$$-\frac{4\left(\frac{\frac{759\sin(dx+c)}{\cos(dx+c)+1}+\frac{444\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1249\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1816\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{454\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{616\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{1274\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{560\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{385\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{420}{(\cos}}{a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{11a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{8a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{11a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{4a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{4a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-}\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -4/105*((759*sin(d*x + c)/(cos(d*x + c) + 1) + 444*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 1249*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1816*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 454*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 616*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 1274*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
+ 560*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 385*sin(d*x + c)^9/(cos(d*x + c
) + 1)^9 - 420*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 105*sin(d*x + c)^11/
(cos(d*x + c) + 1)^11 + 216)/(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) +
 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*a^2*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 11*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 8*a^2*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 11*
a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 4*a^2*sin(d*x + c)^9/(cos(d*x + c
) + 1)^9 - 4*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 4*a^2*sin(d*x + c)
^11/(cos(d*x + c) + 1)^11 - a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) + 10
5*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 17.73, size = 198, normalized size = 1.28

$$\frac{-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{44\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{3}+\frac{64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}+\frac{728\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{15}+\frac{352\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{15}-\frac{1816\tan\left(\frac{c}{2}\right.}{105}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^7\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((1012*tan(c/2 + (d*x)/2))/35 + (592*tan(c/2 + (d*x)/2)^2)/35 - (4996*tan(c
/2 + (d*x)/2)^3)/105 - (7264*tan(c/2 + (d*x)/2)^4)/105 - (1816*tan(c/2 + (d
*x)/2)^5)/105 + (352*tan(c/2 + (d*x)/2)^6)/15 + (728*tan(c/2 + (d*x)/2)^7)/
15 + (64*tan(c/2 + (d*x)/2)^8)/3 - (44*tan(c/2 + (d*x)/2)^9)/3 - 16*tan(c/2
 + (d*x)/2)^10 - 4*tan(c/2 + (d*x)/2)^11 + 288/35)/(a^2*d*(tan(c/2 + (d*x)/
2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^7*(tan(c/2 + (d*x)/2)^2 + 1)) - (2*x)/a^
2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.830 $\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=140

$$\frac{2\tan^7(c+dx)}{7a^2d}-\frac{\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{6\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{a^2d}+\frac{2\,\mathrm{se}}{}$$

```
[Out] x/a^2+2*sec(d*x+c)/a^2/d-2*sec(d*x+c)^3/a^2/d+6/5*sec(d*x+c)^5/a^2/d-2/7*se
c(d*x+c)^7/a^2/d-tan(d*x+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d-1/5*tan(d*x+c)^5/a
^2/d+2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.30, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2607, 30, 2606, 194, 3473, 8}

$$\frac{2\tan^7(c+dx)}{7a^2d}-\frac{\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{6\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{a^2d}+\frac{2\,\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] x/a^2 + (2*Sec[c + d*x])/(a^2*d) - (2*Sec[c + d*x]^3)/(a^2*d) + (6*Sec[c +
d*x]^5)/(5*a^2*d) - (2*Sec[c + d*x]^7)/(7*a^2*d) - Tan[c + d*x]/(a^2*d) + T
an[c + d*x]^3/(3*a^2*d) - Tan[c + d*x]^5/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*
a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec^2(c+dx)(a-a\sin(c+dx))^2\tan^6(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec^2(c+dx)\tan^6(c+dx) - 2a^2\sec(c+dx)\tan^7(c+dx) + a^2\tan^8(c+$$
$$= \frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a^2} + \frac{\int \tan^8(c+dx)\,dx}{a^2} - \frac{2\int \sec(c+dx)\tan^7(c}{a^2}$$
$$= \frac{\tan^7(c+dx)}{7a^2d} - \frac{\int \tan^6(c+dx)\,dx}{a^2} + \frac{\mathrm{Subst}\left(\int x^6\,dx, x, \tan(c+dx)\right)}{a^2d} - \frac{2\,\mathrm{Subs}}{}$$
$$= -\frac{\tan^5(c+dx)}{5a^2d} + \frac{2\tan^7(c+dx)}{7a^2d} + \frac{\int \tan^4(c+dx)\,dx}{a^2} - \frac{2\,\mathrm{Subst}\left(\int \left(-1+3x^2\right.\right.}{}$$
$$= \frac{2\sec(c+dx)}{a^2d} - \frac{2\sec^3(c+dx)}{a^2d} + \frac{6\sec^5(c+dx)}{5a^2d} - \frac{2\sec^7(c+dx)}{7a^2d} + \frac{\tan^3(c+}{3a^2d}$$
$$= \frac{2\sec(c+dx)}{a^2d} - \frac{2\sec^3(c+dx)}{a^2d} + \frac{6\sec^5(c+dx)}{5a^2d} - \frac{2\sec^7(c+dx)}{7a^2d} - \frac{\tan(c+d}{a^2d}$$
$$= \frac{x}{a^2} + \frac{2\sec(c+dx)}{a^2d} - \frac{2\sec^3(c+dx)}{a^2d} + \frac{6\sec^5(c+dx)}{5a^2d} - \frac{2\sec^7(c+dx)}{7a^2d} - \tan$$

Mathematica [A] time = 0.57, size = 257, normalized size = 1.84

$$\frac{2128\sin(c+dx) + 6720c\sin(2(c+dx)) + 6720dx\sin(2(c+dx)) - 9144\sin(2(c+dx)) + 456\sin(3(c+dx)) + 3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4032 + 42*(-381 + 280*c + 280*d*x)*Cos[c + d*x] + 5504*Cos[2*(c + d*x)] -
3429*Cos[3*(c + d*x)] + 2520*c*Cos[3*(c + d*x)] + 2520*d*x*Cos[3*(c + d*x)]
 + 2752*Cos[4*(c + d*x)] + 1143*Cos[5*(c + d*x)] - 840*c*Cos[5*(c + d*x)] -
 840*d*x*Cos[5*(c + d*x)] + 2128*Sin[c + d*x] - 9144*Sin[2*(c + d*x)] + 672
0*c*Sin[2*(c + d*x)] + 6720*d*x*Sin[2*(c + d*x)] + 456*Sin[3*(c + d*x)] - 4
572*Sin[4*(c + d*x)] + 3360*c*Sin[4*(c + d*x)] + 3360*d*x*Sin[4*(c + d*x)]
+ 1528*Sin[5*(c + d*x)])/(13440*a^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

fricas [A] time = 0.48, size = 140, normalized size = 1.00

$$\frac{105\,dx\cos(dx+c)^5 - 210\,dx\cos(dx+c)^3 - 172\,\cos(dx+c)^4 + 86\,\cos(dx+c)^2 - \left(210\,dx\cos(dx+c)^3 + 191\right.}{105\left(a^2d\cos(dx+c)^5 - 2\,a^2d\cos(dx+c)^3\sin(dx+c) - 2\,a^2d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/105*(105*d*x*cos(d*x + c)^5 - 210*d*x*cos(d*x + c)^3 - 172*cos(d*x + c)^4
 + 86*cos(d*x + c)^2 - (210*d*x*cos(d*x + c)^3 + 191*cos(d*x + c)^4 - 129*c
os(d*x + c)^2 + 25)*sin(d*x + c) - 10)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(
d*x + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.29, size = 155, normalized size = 1.11

$$\frac{\frac{840\,(dx+c)}{a^2} + \frac{35\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+10\right)}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3} + \frac{1365\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 + 9345\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 26600\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 39410\,\tan\left(\frac{1}{2}\right.}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(840*(d*x + c)/a^2 + 35*(9*tan(1/2*d*x + 1/2*c)^2 - 21*tan(1/2*d*x +
1/2*c) + 10)/(a^2*(tan(1/2*d*x + 1/2*c) - 1)^3) + (1365*tan(1/2*d*x + 1/2*c
)^6 + 9345*tan(1/2*d*x + 1/2*c)^5 + 26600*tan(1/2*d*x + 1/2*c)^4 + 39410*ta
n(1/2*d*x + 1/2*c)^3 + 30261*tan(1/2*d*x + 1/2*c)^2 + 11837*tan(1/2*d*x + 1
/2*c) + 1886)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.56, size = 230, normalized size = 1.64

$$-\frac{1}{12d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{3}{8a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}-\frac{}{7d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/12/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3-1/8/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2+3/8
/a^2/d/(tan(1/2*d*x+1/2*c)-1)+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-4/7/d/a^2/
(tan(1/2*d*x+1/2*c)+1)^7+2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^6-8/5/a^2/d/(tan(1/
2*d*x+1/2*c)+1)^5-1/a^2/d/(tan(1/2*d*x+1/2*c)+1)^4+5/12/a^2/d/(tan(1/2*d*x+
1/2*c)+1)^3+11/8/a^2/d/(tan(1/2*d*x+1/2*c)+1)^2+13/8/a^2/d/(tan(1/2*d*x+1/2
*c)+1)
```

maxima [B] time = 0.43, size = 421, normalized size = 3.01

$$\frac{2\left(\frac{\frac{279\sin(dx+c)}{\cos(dx+c)+1}-\frac{132\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1048\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{364\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1554\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{980\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{280\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{420\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{105\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+96}{a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\cdots\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 2/105*((279*sin(d*x + c)/(cos(d*x + c) + 1) - 132*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 1048*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 364*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 1554*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 980*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 - 280*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
420*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 105*sin(d*x + c)^9/(cos(d*x + c)
+ 1)^9 + 96)/(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14
*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*s
in(d*x + c)^10/(cos(d*x + c) + 1)^10) + 105*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a^2)/d
```

mupad [B] time = 16.89, size = 156, normalized size = 1.11

$$\frac{x}{a^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{56\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}-\frac{148\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}+\frac{104\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15}+\frac{2096\tan\left(\frac{c}{2}+\cdots\right)}{105}\cdots}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] x/a^2 + ((88*tan(c/2 + (d*x)/2)^2)/35 - (186*tan(c/2 + (d*x)/2))/35 + (2096
*tan(c/2 + (d*x)/2)^3)/105 + (104*tan(c/2 + (d*x)/2)^4)/15 - (148*tan(c/2 +
 (d*x)/2)^5)/5 - (56*tan(c/2 + (d*x)/2)^6)/3 + (16*tan(c/2 + (d*x)/2)^7)/3
+ 8*tan(c/2 + (d*x)/2)^8 + 2*tan(c/2 + (d*x)/2)^9 - 64/35)/(a^2*d*(tan(c/2
+ (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.831 $\int \frac{\sin(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{2\tan^7(c+dx)}{7a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{\sec^5(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)}{a^2d}$$

```
[Out] -sec(d*x+c)/a^2/d+4/3*sec(d*x+c)^3/a^2/d-sec(d*x+c)^5/a^2/d+2/7*sec(d*x+c)^
7/a^2/d-2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.27, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2873, 2606, 270, 2607, 30, 194}

$$-\frac{2\tan^7(c+dx)}{7a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{\sec^5(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]/(a^2*d)) + (4*Sec[c + d*x]^3)/(3*a^2*d) - Sec[c + d*x]^5/(a^
2*d) + (2*Sec[c + d*x]^7)/(7*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sec^3(c+dx)(a-a\sin(c+dx))^2\tan^5(c+dx)\,dx}{a^4}\\
&= \frac{\int \left(a^2\sec^3(c+dx)\tan^5(c+dx)-2a^2\sec^2(c+dx)\tan^6(c+dx)+a^2\sec(c+dx)\right.}{a^4}\\
&= \frac{\int \sec^3(c+dx)\tan^5(c+dx)\,dx}{a^2}+\frac{\int \sec(c+dx)\tan^7(c+dx)\,dx}{a^2}-\frac{2\int \sec^2(c+}{}\\
&= \frac{\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\mathrm{Subst}\left(\int \left(-1+x^2\right)^3\,dx,x,\sec(c+\right.}{a^2d}\\
&= -\frac{2\tan^7(c+dx)}{7a^2d}+\frac{\mathrm{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\mathrm{Subst}}{}\\
&= -\frac{\sec(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec^5(c+dx)}{a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\tan^7(c+dx}{7a^2d}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 126, normalized size = 1.48

$$-\frac{\sec^3(c+dx)(28\sin(c+dx)-104\sin(2(c+dx))+66\sin(3(c+dx))-52\sin(4(c+dx))+6\sin(5(c+dx))-1}{336a^2d(\sin(c+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/336*(Sec[c + d*x]^3*(42 - 182*Cos[c + d*x] + 104*Cos[2*(c + d*x)] - 39*C
os[3*(c + d*x)] - 18*Cos[4*(c + d*x)] + 13*Cos[5*(c + d*x)] + 28*Sin[c + d*
x] - 104*Sin[2*(c + d*x)] + 66*Sin[3*(c + d*x)] - 52*Sin[4*(c + d*x)] + 6*S
in[5*(c + d*x)]))/(a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.48, size = 104, normalized size = 1.22

$$-\frac{9\cos(dx+c)^4-22\cos(dx+c)^2-2\left(3\cos(dx+c)^4+6\cos(dx+c)^2-1\right)\sin(dx+c)+5}{21\left(a^2d\cos(dx+c)^5-2a^2d\cos(dx+c)^3\sin(dx+c)-2a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/21*(9*cos(d*x + c)^4 - 22*cos(d*x + c)^2 - 2*(3*cos(d*x + c)^4 + 6*cos(d
*x + c)^2 - 1)*sin(d*x + c) + 5)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x +
c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.30, size = 146, normalized size = 1.72

$$\frac{\frac{7\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{42\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+1015\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1750\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+1344\tan\left(\frac{1}{2}\right.}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{168d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/168*(7*(6*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 7)/(a^2*(tan
(1/2*d*x + 1/2*c) - 1)^3) - (42*tan(1/2*d*x + 1/2*c)^6 + 315*tan(1/2*d*x +
1/2*c)^5 + 1015*tan(1/2*d*x + 1/2*c)^4 + 1750*tan(1/2*d*x + 1/2*c)^3 + 1344
*tan(1/2*d*x + 1/2*c)^2 + 511*tan(1/2*d*x + 1/2*c) + 79)/(a^2*(tan(1/2*d*x
+ 1/2*c) + 1)^7))/d
```

maple [A] time = 0.56, size = 145, normalized size = 1.71

$$-\frac{\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{64}{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-256}+\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{5}{12\left(\tan\left(\frac{dx}{2}\right.\right.}}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 64/d/a^2*(-1/768/(tan(1/2*d*x+1/2*c)-1)^3-1/512/(tan(1/2*d*x+1/2*c)-1)^2+1/
256/(tan(1/2*d*x+1/2*c)-1)+1/112/(tan(1/2*d*x+1/2*c)+1)^7-1/32/(tan(1/2*d*x
+1/2*c)+1)^6+1/32/(tan(1/2*d*x+1/2*c)+1)^5-5/768/(tan(1/2*d*x+1/2*c)+1)^3-3
/512/(tan(1/2*d*x+1/2*c)+1)^2-1/256/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.34, size = 296, normalized size = 3.48

$$-\frac{16\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}{21\left(a^{2}+\frac{4a^{2}\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8a^{2}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{14a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{8a^{2}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{3a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -16/21*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 1)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 - 14*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
 a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*d)
```

mupad [B] time = 12.37, size = 160, normalized size = 1.88

$$-\frac{\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{21}+\frac{64\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{21}+\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{7}-\frac{128\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{21}-\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}}{a^{2}d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{3}\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -((16*cos(c/2 + (d*x)/2)^10)/21 + (64*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/
2))/21 - (32*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4)/3 - (128*cos(c/2 +
(d*x)/2)^7*sin(c/2 + (d*x)/2)^3)/21 + (16*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d
*x)/2)^2)/7)/(a^2*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 +
(d*x)/2) + sin(c/2 + (d*x)/2))^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.832 $\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{\tan^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{4\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}$$

```
[Out] -2/3*sec(d*x+c)^3/a^2/d+4/5*sec(d*x+c)^5/a^2/d-2/7*sec(d*x+c)^7/a^2/d+1/5*t
an(d*x+c)^5/a^2/d+2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2711, 2607, 14, 2606, 270, 30}

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{\tan^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{4\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*Sec[c + d*x]^3)/(3*a^2*d) + (4*Sec[c + d*x]^5)/(5*a^2*d) - (2*Sec[c + d
*x]^7)/(7*a^2*d) + Tan[c + d*x]^5/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \left(a^2\sec^4(c+dx)\tan^4(c+dx)-2a^2\sec^3(c+dx)\tan^5(c+dx)+a^2\sec^2(c+dx)\,\mathrm{ta}\right.}{a^4}\\ &= \frac{\int \sec^4(c+dx)\tan^4(c+dx)\,dx}{a^2}+\frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a^2}-\frac{2\int \sec^3(c+dx}{}\\ &= \frac{\mathrm{Subst}\left(\int x^6\,dx,x,\tan(c+dx)\right)}{a^2d}+\frac{\mathrm{Subst}\left(\int x^4\left(1+x^2\right)dx,x,\tan(c+dx)\right)}{a^2d}-\frac{2\,\mathrm{Sub}}{}\\ &= \frac{\tan^7(c+dx)}{7a^2d}+\frac{\mathrm{Subst}\left(\int \left(x^4+x^6\right)dx,x,\tan(c+dx)\right)}{a^2d}-\frac{2\,\mathrm{Subst}\left(\int \left(x^2-2x^4+x^6\right.\right.}{a^2d}\\ &= -\frac{2\sec^3(c+dx)}{3a^2d}+\frac{4\sec^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{\tan^5(c+dx)}{5a^2d}+\frac{2\tan^7(c+dx}{7a^2d}\end{aligned}$$

Mathematica [A] time = 0.26, size = 126, normalized size = 1.38

$$-\frac{\sec^3(c+dx)(-1232\sin(c+dx)-824\sin(2(c+dx))+1896\sin(3(c+dx))-412\sin(4(c+dx))-72\sin(5(c+}{13440a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/13440*(Sec[c + d*x]^3*(672 - 1442*Cos[c + d*x] + 1664*Cos[2*(c + d*x)] -
 309*Cos[3*(c + d*x)] - 288*Cos[4*(c + d*x)] + 103*Cos[5*(c + d*x)] - 1232*
```

```
Sin[c + d*x] - 824*Sin[2*(c + d*x)] + 1896*Sin[3*(c + d*x)] - 412*Sin[4*(c
+ d*x)] - 72*Sin[5*(c + d*x)]))/(a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.47, size = 103, normalized size = 1.13

$$-\frac{18 \cos\left(dx+c\right)^{4}-44 \cos\left(dx+c\right)^{2}+\left(9 \cos\left(dx+c\right)^{4}-66 \cos\left(dx+c\right)^{2}+25\right) \sin\left(dx+c\right)+10}{105 \left(a^{2} d \cos\left(dx+c\right)^{5}-2 \, a^{2} d \cos\left(dx+c\right)^{3} \sin\left(dx+c\right)-2 \, a^{2} d \cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(18*cos(d*x + c)^4 - 44*cos(d*x + c)^2 + (9*cos(d*x + c)^4 - 66*cos(
d*x + c)^2 + 25)*sin(d*x + c) + 10)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x
 + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.27, size = 146, normalized size = 1.60

$$\frac{\frac{35 \left(3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-9 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+4\right)}{a^{2} \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)^{3}}-\frac{105 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}+735 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}+2030 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}+2030 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+1701 \tan\left(\frac{1}{2}\right.}{a^{2} \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)^{7}}}{840 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(35*(3*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 4)/(a^2*(tan
(1/2*d*x + 1/2*c) - 1)^3) - (105*tan(1/2*d*x + 1/2*c)^6 + 735*tan(1/2*d*x +
 1/2*c)^5 + 2030*tan(1/2*d*x + 1/2*c)^4 + 2030*tan(1/2*d*x + 1/2*c)^3 + 170
1*tan(1/2*d*x + 1/2*c)^2 + 707*tan(1/2*d*x + 1/2*c) + 116)/(a^2*(tan(1/2*d*
x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.57, size = 160, normalized size = 1.76

$$\frac{-\frac{1}{12 \left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8 \left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{32}{256 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)-256}-\frac{4}{7 \left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{12}{5 \left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tan\left(\frac{dx}{2}\right.\right.}}{a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 32/d/a^2*(-1/384/(tan(1/2*d*x+1/2*c)-1)^3-1/256/(tan(1/2*d*x+1/2*c)-1)^2+1/
256/(tan(1/2*d*x+1/2*c)-1)-1/56/(tan(1/2*d*x+1/2*c)+1)^7+1/16/(tan(1/2*d*x+
```

```
1/2*c)+1)^6-3/40/(tan(1/2*d*x+1/2*c)+1)^5+1/32/(tan(1/2*d*x+1/2*c)+1)^4+1/3
84/(tan(1/2*d*x+1/2*c)+1)^3-1/256/(tan(1/2*d*x+1/2*c)+1)^2-1/256/(tan(1/2*d
*x+1/2*c)+1))
```

maxima [B] time = 0.34, size = 316, normalized size = 3.47

$$-\frac{32\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\cdots\right.}{105\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c\cdots}{(\cos(dx+c)+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -32/105*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a^2 + 4*a^2*s
in(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^2*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 8*a^2*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4
*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*sin(d*x + c)^10/(cos(d*x + c
) + 1)^10)*d)
```

mupad [B] time = 12.38, size = 184, normalized size = 2.02

$$-\frac{\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{105}+\frac{128\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{105}+\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{35}-\frac{256\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{105}-\frac{64\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\cdots\right.}{15}\cdots}{a^2\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -((32*cos(c/2 + (d*x)/2)^10)/105 + (128*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x
)/2))/105 - (32*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)/5 - (64*cos(c/2
+ (d*x)/2)^6*sin(c/2 + (d*x)/2)^4)/15 - (256*cos(c/2 + (d*x)/2)^7*sin(c/2 +
 (d*x)/2)^3)/105 + (32*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2)/35)/(a^2*
d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2
 + (d*x)/2))^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.833 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{3\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

```
[Out] 1/3*sec(d*x+c)^3/a^2/d-3/5*sec(d*x+c)^5/a^2/d+2/7*sec(d*x+c)^7/a^2/d-2/5*ta
n(d*x+c)^5/a^2/d-2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.29, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2875, 2873, 2606, 14, 2607, 270}

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{3\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sec[c + d*x]^3/(3*a^2*d) - (3*Sec[c + d*x]^5)/(5*a^2*d) + (2*Sec[c + d*x]^7
)/(7*a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sec^5(c+dx)(a-a\sin(c+dx))^2\tan^3(c+dx)\,dx}{a^4}\\
&= \frac{\int \left(a^2\sec^5(c+dx)\tan^3(c+dx)-2a^2\sec^4(c+dx)\tan^4(c+dx)+a^2\sec^3(c+dx\right.}{a^4}\\
&= \frac{\int \sec^5(c+dx)\tan^3(c+dx)\,dx}{a^2}+\frac{\int \sec^3(c+dx)\tan^5(c+dx)\,dx}{a^2}-\frac{2\int \sec^4(c+}{}\\
&= \frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\sec(c\right.}{a^2d}\\
&= \frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int \left(-x^4+x^6\right)\,dx,x,\sec(\right.}{a^2d}\\
&= \frac{\sec^3(c+dx)}{3a^2d}-\frac{3\sec^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan^7(c+d}{7a^2d}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 126, normalized size = 1.38

$$\frac{\sec^3(c+dx)(448\sin(c+dx)-104\sin(2(c+dx))-144\sin(3(c+dx))-52\sin(4(c+dx))+48\sin(5(c+dx))-1}{6720a^2d(\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^3*(672 - 182*Cos[c + d*x] - 736*Cos[2*(c + d*x)] - 39*Cos[3*(
c + d*x)] + 192*Cos[4*(c + d*x)] + 13*Cos[5*(c + d*x)] + 448*Sin[c + d*x] -
 104*Sin[2*(c + d*x)] - 144*Sin[3*(c + d*x)] - 52*Sin[4*(c + d*x)] + 48*Sin
[5*(c + d*x)]))/(6720*a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.45, size = 104, normalized size = 1.14

$$-\frac{24 \cos(dx+c)^4 - 47 \cos(dx+c)^2 + 2\left(6 \cos(dx+c)^4 - 9 \cos(dx+c)^2 + 5\right)\sin(dx+c) + 25}{105\left(a^2 d\cos(dx+c)^5 - 2\,a^2 d\cos(dx+c)^3\sin(dx+c) - 2\,a^2 d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(24*cos(d*x + c)^4 - 47*cos(d*x + c)^2 + 2*(6*cos(d*x + c)^4 - 9*cos
(d*x + c)^2 + 5)*sin(d*x + c) + 25)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x
 + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.28, size = 120, normalized size = 1.32

$$-\frac{\frac{35\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3} - \frac{105\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1015\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1330\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1302\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+469\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+67}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(35*(3*tan(1/2*d*x + 1/2*c) - 1)/(a^2*(tan(1/2*d*x + 1/2*c) - 1)^3)
- (105*tan(1/2*d*x + 1/2*c)^5 + 1015*tan(1/2*d*x + 1/2*c)^4 + 1330*tan(1/2*
d*x + 1/2*c)^3 + 1302*tan(1/2*d*x + 1/2*c)^2 + 469*tan(1/2*d*x + 1/2*c) + 6
7)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.51, size = 130, normalized size = 1.43

$$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3} - \frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} + \frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7} - \frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6} + \frac{14}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5} - \frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4} + \frac{\cdots}{12\left(\tan\left(\cdots\right.\right.}}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 16/d/a^2*(-1/192/(tan(1/2*d*x+1/2*c)-1)^3-1/128/(tan(1/2*d*x+1/2*c)-1)^2+1/
28/(tan(1/2*d*x+1/2*c)+1)^7-1/8/(tan(1/2*d*x+1/2*c)+1)^6+7/40/(tan(1/2*d*x+
1/2*c)+1)^5-1/8/(tan(1/2*d*x+1/2*c)+1)^4+7/192/(tan(1/2*d*x+1/2*c)+1)^3+1/1
28/(tan(1/2*d*x+1/2*c)+1)^2)
```

maxima [B] time = 0.34, size = 336, normalized size = 3.69

$$\frac{4\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{91\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{84\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{105\sin(dx+c)}{(\cos(dx+c)+1)}\right.}{105\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 4/105*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 91*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 84*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 105*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 1)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*
a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 14*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) +
 1)^9 - a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*d)
```

mupad [B] time = 14.07, size = 207, normalized size = 2.27

$$\frac{\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{105}+\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{105}+\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{35}-\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{105}+\frac{52\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{15}+}{a^2\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{d}{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((4*cos(c/2 + (d*x)/2)^10)/105 + (16*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2
))/105 + 4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 + (16*cos(c/2 + (d*x)/
2)^5*sin(c/2 + (d*x)/2)^5)/5 + (52*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^
4)/15 - (32*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3)/105 + (4*cos(c/2 + (
d*x)/2)^8*sin(c/2 + (d*x)/2)^2)/35)/(a^2*d*(cos(c/2 + (d*x)/2) - sin(c/2 +
(d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^7)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.834 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{3\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}$$

```
[Out] 2/5*sec(d*x+c)^5/a^2/d-2/7*sec(d*x+c)^7/a^2/d+1/3*tan(d*x+c)^3/a^2/d+3/5*ta
n(d*x+c)^5/a^2/d+2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.31, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2875, 2873, 2607, 270, 2606, 14}

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{3\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^5)/(5*a^2*d) - (2*Sec[c + d*x]^7)/(7*a^2*d) + Tan[c + d*x]^
3/(3*a^2*d) + (3*Tan[c + d*x]^5)/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec^6(c+dx)(a-a\sin(c+dx))^2\tan^2(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec^6(c+dx)\tan^2(c+dx)-2a^2\sec^5(c+dx)\tan^3(c+dx)+a^2\sec^4(c+\right.}{a^4}$$
$$= \frac{\int \sec^6(c+dx)\tan^2(c+dx)\,dx}{a^2}+\frac{\int \sec^4(c+dx)\tan^4(c+dx)\,dx}{a^2}-\frac{2\int \sec^5(c}{}$$
$$= \frac{\operatorname{Subst}\left(\int x^4\left(1+x^2\right)\,dx,x,\tan(c+dx)\right)}{a^2d}+\frac{\operatorname{Subst}\left(\int x^2\left(1+x^2\right)^2\,dx,x,\tan(c\right.}{a^2d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(x^4+x^6\right)\,dx,x,\tan(c+dx)\right)}{a^2d}+\frac{\operatorname{Subst}\left(\int \left(x^2+2x^4+x^6\right)\,dx,x,\tan\right.}{a^2d}$$
$$= \frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{\tan^3(c+dx)}{3a^2d}+\frac{3\tan^5(c+dx)}{5a^2d}+\frac{2\tan^7(c}{7a^2d}$$

Mathematica [A] time = 0.37, size = 126, normalized size = 1.38

$$\frac{\sec^3(c+dx)(3136\sin(c+dx)-408\sin(2(c+dx))-48\sin(3(c+dx))-204\sin(4(c+dx))+16\sin(5(c+dx))}{13440a^2d(\sin(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^3*(1344 - 714*Cos[c + d*x] + 128*Cos[2*(c + d*x)] - 153*Cos[3
*(c + d*x)] + 64*Cos[4*(c + d*x)] + 51*Cos[5*(c + d*x)] + 3136*Sin[c + d*x]
 - 408*Sin[2*(c + d*x)] - 48*Sin[3*(c + d*x)] - 204*Sin[4*(c + d*x)] + 16*S
in[5*(c + d*x)]))/(13440*a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.44, size = 103, normalized size = 1.13

$$-\frac{4\cos(dx+c)^4-2\cos(dx+c)^2+\left(2\cos(dx+c)^4-3\cos(dx+c)^2+25\right)\sin(dx+c)+10}{105\left(a^2d\cos(dx+c)^5-2a^2d\cos(dx+c)^3\sin(dx+c)-2a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(4*cos(d*x + c)^4 - 2*cos(d*x + c)^2 + (2*cos(d*x + c)^4 - 3*cos(d*x
 + c)^2 + 25)*sin(d*x + c) + 10)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x +
c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.26, size = 146, normalized size = 1.60

$$-\frac{\frac{35\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+945\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+1820\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+1617\tan\left(\right.}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(35*(3*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 2)/(a^2*(ta
n(1/2*d*x + 1/2*c) - 1)^3) - (105*tan(1/2*d*x + 1/2*c)^6 + 945*tan(1/2*d*x
+ 1/2*c)^5 + 1820*tan(1/2*d*x + 1/2*c)^4 + 2450*tan(1/2*d*x + 1/2*c)^3 + 16
17*tan(1/2*d*x + 1/2*c)^2 + 749*tan(1/2*d*x + 1/2*c) + 122)/(a^2*(tan(1/2*d
*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.50, size = 160, normalized size = 1.76

$$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}-\frac{16}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 8/d/a^2*(-1/96/(tan(1/2*d*x+1/2*c)-1)^3-1/64/(tan(1/2*d*x+1/2*c)-1)^2-1/64/
(tan(1/2*d*x+1/2*c)-1)-1/14/(tan(1/2*d*x+1/2*c)+1)^7+1/4/(tan(1/2*d*x+1/2*c
)+1)^6-2/5/(tan(1/2*d*x+1/2*c)+1)^5+3/8/(tan(1/2*d*x+1/2*c)+1)^4-19/96/(tan
(1/2*d*x+1/2*c)+1)^3+3/64/(tan(1/2*d*x+1/2*c)+1)^2+1/64/(tan(1/2*d*x+1/2*c)
+1))
```

maxima [B] time = 0.34, size = 356, normalized size = 3.91

$$\frac{8\left(\frac{12\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{11\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{7\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{42\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{35\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35}{(\cos}\right.}{105\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 8/105*(12*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 11*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 7*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 42*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 35*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 35*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3)/((a^2 + 4
*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 8*a^2*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)
^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*sin(d*x + c)^10/(cos(d
*x + c) + 1)^10)*d)
```

mupad [B] time = 14.34, size = 231, normalized size = 2.54

$$\frac{\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{35}+\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{35}+\frac{24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{35}+\frac{88\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{105}-\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{15}-}{a^2\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((8*cos(c/2 + (d*x)/2)^10)/35 + (32*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)
)/35 + (8*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^7)/3 + (8*cos(c/2 + (d*x)
/2)^4*sin(c/2 + (d*x)/2)^6)/3 + (16*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)
^5)/5 - (8*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4)/15 + (88*cos(c/2 + (d
*x)/2)^7*sin(c/2 + (d*x)/2)^3)/105 + (24*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*
```

```
x)/2)^2)/35)/(a^2*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 +
(d*x)/2) + sin(c/2 + (d*x)/2))^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^2(c+dx)\sec^4(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**4/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

3.835 $\int \frac{\sec^3(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=93

$$\frac{8\tan^3(c+dx)}{105a^2d}+\frac{8\tan(c+dx)}{35a^2d}-\frac{2\sec^3(c+dx)}{35d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\sec^3(c+dx)}{7d(a\sin(c+dx)+a)^2}$$

```
[Out] 1/7*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^2-2/35*sec(d*x+c)^3/d/(a^2+a^2*sin(d*x+
c))+8/35*tan(d*x+c)/a^2/d+8/105*tan(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2859, 2672, 3767}

$$\frac{8\tan^3(c+dx)}{105a^2d}+\frac{8\tan(c+dx)}{35a^2d}-\frac{2\sec^3(c+dx)}{35d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\sec^3(c+dx)}{7d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sec[c + d*x]^3/(7*d*(a + a*Sin[c + d*x])^2) - (2*Sec[c + d*x]^3)/(35*d*(a^2
 + a^2*Sin[c + d*x])) + (8*Tan[c + d*x])/(35*a^2*d) + (8*Tan[c + d*x]^3)/(1
05*a^2*d)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2} + \frac{2\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx}{7a}$$
$$= \frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2} - \frac{2\sec^3(c+dx)}{35d\left(a^2+a^2\sin(c+dx)\right)} + \frac{8\int \sec^4(c+dx)\,dx}{35a^2}$$
$$= \frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2} - \frac{2\sec^3(c+dx)}{35d\left(a^2+a^2\sin(c+dx)\right)} - \frac{8\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x,$$
$$= \frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2} - \frac{2\sec^3(c+dx)}{35d\left(a^2+a^2\sin(c+dx)\right)} + \frac{8\tan(c+dx)}{35a^2d} + \frac{8\tan^3(c$$

Mathematica [A] time = 0.30, size = 134, normalized size = 1.44

$$-\frac{\sec^3(c+dx)\left(-56\sin(c+dx)+3\sin(2(c+dx))-12\sin(3(c+dx))+\frac{3}{2}\sin(4(c+dx))+4\sin(5(c+dx))+\frac{21}{4}c}{420a^2d(\sin(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/420*(Sec[c + d*x]^3*(-84 + (21*Cos[c + d*x])/4 + 32*Cos[2*(c + d*x)] + (
9*Cos[3*(c + d*x)])/8 + 16*Cos[4*(c + d*x)] - (3*Cos[5*(c + d*x)])/8 - 56*S
in[c + d*x] + 3*Sin[2*(c + d*x)] - 12*Sin[3*(c + d*x)] + (3*Sin[4*(c + d*x)
])/2 + 4*Sin[5*(c + d*x)]))/(a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.45, size = 104, normalized size = 1.12

$$\frac{32\cos(dx+c)^4-16\cos(dx+c)^2+2\left(8\cos(dx+c)^4-12\cos(dx+c)^2-5\right)\sin(dx+c)-25}{105\left(a^2d\cos(dx+c)^5-2\,a^2d\cos(dx+c)^3\sin(dx+c)-2\,a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(32*cos(d*x + c)^4 - 16*cos(d*x + c)^2 + 2*(8*cos(d*x + c)^4 - 12*cos
(d*x + c)^2 - 5)*sin(d*x + c) - 25)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x
 + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.26, size = 146, normalized size = 1.57

$$-\frac{\frac{35\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5\right)}{a^{2}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}-\frac{210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-175\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-910\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-756\tan\left(\frac{1}{2}\right.}{a^{2}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{7}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(35*(6*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 5)/(a^2*(ta
n(1/2*d*x + 1/2*c) - 1)^3) - (210*tan(1/2*d*x + 1/2*c)^6 + 105*tan(1/2*d*x
+ 1/2*c)^5 - 175*tan(1/2*d*x + 1/2*c)^4 - 910*tan(1/2*d*x + 1/2*c)^3 - 756*
tan(1/2*d*x + 1/2*c)^2 - 427*tan(1/2*d*x + 1/2*c) - 31)/(a^2*(tan(1/2*d*x +
 1/2*c) + 1)^7))/d
```

maple [A] time = 0.46, size = 160, normalized size = 1.72

$$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}+\frac{18}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\right.\right.}}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 4/d/a^2*(-1/48/(tan(1/2*d*x+1/2*c)-1)^3-1/32/(tan(1/2*d*x+1/2*c)-1)^2-1/16/
(tan(1/2*d*x+1/2*c)-1)+1/7/(tan(1/2*d*x+1/2*c)+1)^7-1/2/(tan(1/2*d*x+1/2*c)
+1)^6+9/10/(tan(1/2*d*x+1/2*c)+1)^5-1/(tan(1/2*d*x+1/2*c)+1)^4+35/48/(tan(1
/2*d*x+1/2*c)+1)^3-11/32/(tan(1/2*d*x+1/2*c)+1)^2+1/16/(tan(1/2*d*x+1/2*c)+
1))
```

maxima [B] time = 0.33, size = 376, normalized size = 4.04

$$\frac{2\left(\frac{36\sin(dx+c)}{\cos(dx+c)+1}+\frac{132\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{68\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{14\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{84\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{140\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{140\sin(dx+}{(\cos(dx+c)+}\right.}{105\left(a^{2}+\frac{4a^{2}\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8a^{2}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{14a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{8a^{2}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{3a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/105*(36*sin(d*x + c)/(cos(d*x + c) + 1) + 132*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 68*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 14*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 84*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 140*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 140*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 105*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 + 9)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x +
 c) + 1) + 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 - 14*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 - 3*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(c
os(d*x + c) + 1)^9 - a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*d)
```

mupad [B] time = 14.35, size = 254, normalized size = 2.73

$$\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(9\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8+36\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+132\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+68\cos\left(\frac{c}{2}\right.\right.}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (2*cos(c/2 + (d*x)/2)^2*(9*cos(c/2 + (d*x)/2)^8 + 105*sin(c/2 + (d*x)/2)^8
+ 140*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^7 + 36*cos(c/2 + (d*x)/2)^7*sin
(c/2 + (d*x)/2) + 140*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 - 84*cos(c/
2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^5 + 14*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*
x)/2)^4 + 68*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3 + 132*cos(c/2 + (d*x
)/2)^6*sin(c/2 + (d*x)/2)^2))/(105*a^2*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d
*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin(c+dx)\sec^4(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**4/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

3.836 $\int \frac{\csc(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=149

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{6\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}+\frac{\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+$$

```
[Out] -arctanh(cos(d*x+c))/a^2/d+sec(d*x+c)/a^2/d+1/3*sec(d*x+c)^3/a^2/d+1/5*sec(
d*x+c)^5/a^2/d+2/7*sec(d*x+c)^7/a^2/d-2*tan(d*x+c)/a^2/d-2*tan(d*x+c)^3/a^2
/d-6/5*tan(d*x+c)^5/a^2/d-2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.25, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2606, 30}

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{6\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}+\frac{\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + Sec[c + d*x]/(a^2*d) + Sec[c + d*x]^3/(3
*a^2*d) + Sec[c + d*x]^5/(5*a^2*d) + (2*Sec[c + d*x]^7)/(7*a^2*d) - (2*Tan[
c + d*x])/(a^2*d) - (2*Tan[c + d*x]^3)/(a^2*d) - (6*Tan[c + d*x]^5)/(5*a^2*
d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(-2a^2\sec^8(c+dx)+a^2\csc(c+dx)\sec^8(c+dx)+a^2\sec^7(c+dx)\tan(c+d\right.}{a^4}$$

$$= \frac{\int \csc(c+dx)\sec^8(c+dx)\,dx}{a^2}+\frac{\int \sec^7(c+dx)\tan(c+dx)\,dx}{a^2}-\frac{2\int \sec^8(c+}{a^2}$$

$$= \frac{\text{Subst}\left(\int x^6\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{2\,\text{Subs}}{}$$

$$= \frac{\sec^7(c+dx)}{7a^2d}-\frac{2\tan(c+dx)}{a^2d}-\frac{2\tan^3(c+dx)}{a^2d}-\frac{6\tan^5(c+dx)}{5a^2d}-\frac{2\tan^7(c+}{7a^2d}$$

$$= \frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\tan(c+dx)}{a^2d}-$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec^5(c+dx)}{5a^2d}+\frac{2\sec^7(}{7a}$$

Mathematica [B] time = 0.61, size = 352, normalized size = 2.36

$$\frac{2464\sin(c+dx)-4472\sin(2(c+dx))+2208\sin(3(c+dx))-2236\sin(4(c+dx))+384\sin(5(c+dx))+5312}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (6216 + 5312*Cos[2*(c + d*x)] - 1677*Cos[3*(c + d*x)] + 696*Cos[4*(c + d*x)
] + 559*Cos[5*(c + d*x)] - 1260*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 42
0*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 14*Cos[c + d*x]*(559 + 420*Log[C
os[(c + d*x)/2]] - 420*Log[Sin[(c + d*x)/2]]) + 1260*Cos[3*(c + d*x)]*Log[S
in[(c + d*x)/2]] - 420*Cos[5*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 2464*Sin[c
+ d*x] - 4472*Sin[2*(c + d*x)] - 3360*Log[Cos[(c + d*x)/2]]*Sin[2*(c + d*x)
] + 3360*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] + 2208*Sin[3*(c + d*x)] - 2
236*Sin[4*(c + d*x)] - 1680*Log[Cos[(c + d*x)/2]]*Sin[4*(c + d*x)] + 1680*L
og[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)] + 384*Sin[5*(c + d*x)])/(6720*a^2*d*(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^7)
```

fricas [A] time = 0.46, size = 200, normalized size = 1.34

$$-\frac{174\,\cos\left(dx+c\right)^{4}+158\,\cos\left(dx+c\right)^{2}+105\,\left(\cos\left(dx+c\right)^{5}-2\,\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)^{3}\right)\log\left(\cdots\right.}{210\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/210*(174*cos(d*x + c)^4 + 158*cos(d*x + c)^2 + 105*(cos(d*x + c)^5 - 2*c
os(d*x + c)^3*sin(d*x + c) - 2*cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2)
- 105*(cos(d*x + c)^5 - 2*cos(d*x + c)^3*sin(d*x + c) - 2*cos(d*x + c)^3)*l
og(-1/2*cos(d*x + c) + 1/2) + 4*(48*cos(d*x + c)^4 + 33*cos(d*x + c)^2 + 5)
*sin(d*x + c) + 50)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x + c)^3*sin(d*x
+ c) - 2*a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.22, size = 161, normalized size = 1.08

$$\frac{\frac{840\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}}-\frac{35\left(12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\right)}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}+\frac{3780\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+18585\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+41755\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\cdots\right.}{a^{2}\left(\tan\cdots\right.}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 35*(12*tan(1/2*d*x + 1/2*c)
^2 - 21*tan(1/2*d*x + 1/2*c) + 11)/(a^2*(tan(1/2*d*x + 1/2*c) - 1)^3) + (37
80*tan(1/2*d*x + 1/2*c)^6 + 18585*tan(1/2*d*x + 1/2*c)^5 + 41755*tan(1/2*d*
x + 1/2*c)^4 + 51730*tan(1/2*d*x + 1/2*c)^3 + 37506*tan(1/2*d*x + 1/2*c)^2
+ 14917*tan(1/2*d*x + 1/2*c) + 2671)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.76, size = 229, normalized size = 1.54

$$-\frac{1}{12d\,a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8d\,a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{2a^{2}d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{2}}+\frac{\cdots}{7d\,a^{2}\left(\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/12/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3-1/8/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2
/a^2/d/(tan(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c))+4/7/d/a^2/(tan
```

```
(1/2*d*x+1/2*c)+1)^7-2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^6+22/5/a^2/d/(tan(1/2*d
*x+1/2*c)+1)^5-6/a^2/d/(tan(1/2*d*x+1/2*c)+1)^4+79/12/a^2/d/(tan(1/2*d*x+1/
2*c)+1)^3-39/8/a^2/d/(tan(1/2*d*x+1/2*c)+1)^2+9/2/a^2/d/(tan(1/2*d*x+1/2*c)
+1)
```

maxima [B] time = 0.34, size = 422, normalized size = 2.83

$$\frac{\frac{2\left(\frac{554\sin(dx+c)}{\cos(dx+c)+1}+\frac{258\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1108\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1204\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{504\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1470\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{420\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{315\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{210\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+191\right)}{a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(2*(554*sin(d*x + c)/(cos(d*x + c) + 1) + 258*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 1108*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1204*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 504*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1470*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 420*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 - 315*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 210*sin(d*x + c)^9/(cos(d*x +
c) + 1)^9 + 191)/(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
- 14*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x
+ c)^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a
^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 105*log(sin(d*x + c)/(cos(d*x +
 c) + 1))/a^2)/d
```

mupad [B] time = 12.34, size = 169, normalized size = 1.13

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}-\cdots}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - ((1108*tan(c/2 + (d*x)/2))/105 + (172*tan
(c/2 + (d*x)/2)^2)/35 - (2216*tan(c/2 + (d*x)/2)^3)/105 - (344*tan(c/2 + (d
*x)/2)^4)/15 + (48*tan(c/2 + (d*x)/2)^5)/5 + 28*tan(c/2 + (d*x)/2)^6 + 8*ta
n(c/2 + (d*x)/2)^7 - 6*tan(c/2 + (d*x)/2)^8 - 4*tan(c/2 + (d*x)/2)^9 + 382/
105)/(a^2*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.837 $\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=164

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{7\tan^5(c+dx)}{5a^2d}+\frac{3\tan^3(c+dx)}{a^2d}+\frac{5\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-2$$

```
[Out] 2*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a^2/d-2*sec(d*x+c)/a^2/d-2/3*sec(d*x
+c)^3/a^2/d-2/5*sec(d*x+c)^5/a^2/d-2/7*sec(d*x+c)^7/a^2/d+5*tan(d*x+c)/a^2/
d+3*tan(d*x+c)^3/a^2/d+7/5*tan(d*x+c)^5/a^2/d+2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.33, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2620, 270}

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{7\tan^5(c+dx)}{5a^2d}+\frac{3\tan^3(c+dx)}{a^2d}+\frac{5\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-2$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Sec[c + d*x])
/(a^2*d) - (2*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) - (2
*Sec[c + d*x]^7)/(7*a^2*d) + (5*Tan[c + d*x])/(a^2*d) + (3*Tan[c + d*x]^3)/
(a^2*d) + (7*Tan[c + d*x]^5)/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^2(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\sec^8(c+dx) - 2a^2\csc(c+dx)\sec^8(c+dx) + a^2\csc^2(c+dx)\sec^8(c+d\right.}{a^4}$$

$$= \frac{\int \sec^8(c+dx)\,dx}{a^2} + \frac{\int \csc^2(c+dx)\sec^8(c+dx)\,dx}{a^2} - \frac{2\int \csc(c+dx)\sec^8(c-}{a^2}$$

$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^2}\,dx, x, \tan(c+dx)\right)}{a^2d} - \frac{\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\,dx, x, -\right.}{a^2d}$$

$$= \frac{\tan(c+dx)}{a^2d} + \frac{\tan^3(c+dx)}{a^2d} + \frac{3\tan^5(c+dx)}{5a^2d} + \frac{\tan^7(c+dx)}{7a^2d} + \frac{\text{Subst}\left(\int \left(4\right.\right.}{}$$

$$= -\frac{\cot(c+dx)}{a^2d} - \frac{2\sec(c+dx)}{a^2d} - \frac{2\sec^3(c+dx)}{3a^2d} - \frac{2\sec^5(c+dx)}{5a^2d} - \frac{2\sec^7(c+}{7a^2d}$$

$$= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{a^2d} - \frac{2\sec(c+dx)}{a^2d} - \frac{2\sec^3(c+dx)}{3a^2d} - \frac{2\sec}{}$$

Mathematica [B] time = 6.10, size = 442, normalized size = 2.70

$$16\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{13\sin\left(\frac{1}{2}(c+dx)\right)}{384d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{\sin\left(\frac{1}{2}(c\right.}{384d\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (16*(-1/32*Cot[(c + d*x)/2]/d + Log[Cos[(c + d*x)/2]]/(8*d) - Log[Sin[(c +
d*x)/2]]/(8*d) + 1/(768*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + Sin[(c
 + d*x)/2]/(384*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (13*Sin[(c + d
*x)/2])/(384*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/(2
24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7) - 1/(448*d*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^6) + (3*Sin[(c + d*x)/2])/(140*d*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^5) - 3/(280*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + (9
97*Sin[(c + d*x)/2])/(13440*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - 99
7/(26880*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4777*Sin[(c + d*x)/2
])/(13440*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + Tan[(c + d*x)/2]/(32*d
)))/a^2
```

fricas [A] time = 0.48, size = 250, normalized size = 1.52

$$-\frac{432\,\cos\left(dx+c\right)^{6}-660\,\cos\left(dx+c\right)^{4}+98\,\cos\left(dx+c\right)^{2}-105\left(2\,\cos\left(dx+c\right)^{5}-2\,\cos\left(dx+c\right)^{3}+\left(\cos\left(dx+c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(432*cos(d*x + c)^6 - 660*cos(d*x + c)^4 + 98*cos(d*x + c)^2 - 105*(
2*cos(d*x + c)^5 - 2*cos(d*x + c)^3 + (cos(d*x + c)^5 - 2*cos(d*x + c)^3)*s
in(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 105*(2*cos(d*x + c)^5 - 2*cos(d*
x + c)^3 + (cos(d*x + c)^5 - 2*cos(d*x + c)^3)*sin(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2) - 2*(327*cos(d*x + c)^4 - 41*cos(d*x + c)^2 - 5)*sin(d*x + c
) + 25)/(2*a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x + c)^3 + (a^2*d*cos(d*x +
 c)^5 - 2*a^2*d*cos(d*x + c)^3)*sin(d*x + c))
```

giac [A] time = 0.25, size = 204, normalized size = 1.24

$$-\frac{\frac{1680\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}}-\frac{420\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}}-\frac{420\left(4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{35\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-27\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+14\right)}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}+\frac{7875\,\tan\left(\frac{1}{2}\right.}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(1680*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 420*tan(1/2*d*x + 1/2*c)/
a^2 - 420*(4*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)) + 35*(15*
tan(1/2*d*x + 1/2*c)^2 - 27*tan(1/2*d*x + 1/2*c) + 14)/(a^2*(tan(1/2*d*x +
1/2*c) - 1)^3) + (7875*tan(1/2*d*x + 1/2*c)^6 + 41055*tan(1/2*d*x + 1/2*c)^
5 + 94640*tan(1/2*d*x + 1/2*c)^4 + 119630*tan(1/2*d*x + 1/2*c)^3 + 87507*ta
n(1/2*d*x + 1/2*c)^2 + 34979*tan(1/2*d*x + 1/2*c) + 6122)/(a^2*(tan(1/2*d*x
 + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.64, size = 266, normalized size = 1.62

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}-\frac{1}{12d\,a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8d\,a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{5}{8a^{2}d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{2d\,a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/12/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3-1/8/d/a^2/
(tan(1/2*d*x+1/2*c)-1)^2-5/8/a^2/d/(tan(1/2*d*x+1/2*c)-1)-1/2/d/a^2/tan(1/2
*d*x+1/2*c)-2/d/a^2*ln(tan(1/2*d*x+1/2*c))-4/7/d/a^2/(tan(1/2*d*x+1/2*c)+1)
^7+2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^6-24/5/a^2/d/(tan(1/2*d*x+1/2*c)+1)^5+7/a
^2/d/(tan(1/2*d*x+1/2*c)+1)^4-107/12/a^2/d/(tan(1/2*d*x+1/2*c)+1)^3+59/8/a^
2/d/(tan(1/2*d*x+1/2*c)+1)^2-75/8/a^2/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.34, size = 481, normalized size = 2.93

$$-\frac{\frac{\frac{1828 \sin(dx+c)}{\cos(dx+c)+1}+\frac{3847 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1656 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{12734 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{7952 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{9702 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{12600 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{315 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{5460 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}-22}{\frac{a^2 \sin(dx+c)}{\cos(dx+c)+1}+\frac{4 a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3 a^2 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{8 a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{14 a^2 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{14 a^2 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{8 a^2 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{3 a^2 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{4 a^2 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-a}}{210\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/210*((1828*sin(d*x + c)/(cos(d*x + c) + 1) + 3847*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 1656*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 12734*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 - 7952*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 9702
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 12600*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 - 315*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 5460*sin(d*x + c)^9/(cos(
d*x + c) + 1)^9 - 2205*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 105)/(a^2*si
n(d*x + c)/(cos(d*x + c) + 1) + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 3*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 8*a^2*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 - 14*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 14*a^2*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 8*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 3*
a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 4*a^2*sin(d*x + c)^10/(cos(d*x +
c) + 1)^10 - a^2*sin(d*x + c)^11/(cos(d*x + c) + 1)^11) + 420*log(sin(d*x +
 c)/(cos(d*x + c) + 1))/a^2 - 105*sin(d*x + c)/(a^2*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 11.24, size = 331, normalized size = 2.02

$$\frac{21 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+52 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+3 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-120 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}-\frac{462 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{5}}{d\left(-2\,a^2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-8\,a^2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-6\,a^2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+16\,a^2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+28\,a^2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((552*tan(c/2 + (d*x)/2)^3)/35 - (3847*tan(c/2 + (d*x)/2)^2)/105 - (1828*ta
n(c/2 + (d*x)/2))/105 + (12734*tan(c/2 + (d*x)/2)^4)/105 + (1136*tan(c/2 +
(d*x)/2)^5)/15 - (462*tan(c/2 + (d*x)/2)^6)/5 - 120*tan(c/2 + (d*x)/2)^7 +
3*tan(c/2 + (d*x)/2)^8 + 52*tan(c/2 + (d*x)/2)^9 + 21*tan(c/2 + (d*x)/2)^10
 - 1)/(d*(8*a^2*tan(c/2 + (d*x)/2)^2 + 6*a^2*tan(c/2 + (d*x)/2)^3 - 16*a^2*
tan(c/2 + (d*x)/2)^4 - 28*a^2*tan(c/2 + (d*x)/2)^5 + 28*a^2*tan(c/2 + (d*x)
/2)^7 + 16*a^2*tan(c/2 + (d*x)/2)^8 - 6*a^2*tan(c/2 + (d*x)/2)^9 - 8*a^2*ta
n(c/2 + (d*x)/2)^10 - 2*a^2*tan(c/2 + (d*x)/2)^11 + 2*a^2*tan(c/2 + (d*x)/2
))) - (2*log(tan(c/2 + (d*x)/2)))/(a^2*d) + tan(c/2 + (d*x)/2)/(2*a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.838 $\int \frac{\csc^3(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=194

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{8\tan^5(c+dx)}{5a^2d}-\frac{4\tan^3(c+dx)}{a^2d}-\frac{8\tan(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{11\sec^7(c+dx)}{14a^2d}+\frac{11\sec^5(c+dx)}{10a^2d}$$

```
[Out] -11/2*arctanh(cos(d*x+c))/a^2/d+2*cot(d*x+c)/a^2/d+11/2*sec(d*x+c)/a^2/d+11
/6*sec(d*x+c)^3/a^2/d+11/10*sec(d*x+c)^5/a^2/d+11/14*sec(d*x+c)^7/a^2/d-1/2
*csc(d*x+c)^2*sec(d*x+c)^7/a^2/d-8*tan(d*x+c)/a^2/d-4*tan(d*x+c)^3/a^2/d-8/
5*tan(d*x+c)^5/a^2/d-2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.37, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2622, 302, 207, 2620, 270, 288}

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{8\tan^5(c+dx)}{5a^2d}-\frac{4\tan^3(c+dx)}{a^2d}-\frac{8\tan(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{11\sec^7(c+dx)}{14a^2d}+\frac{11\sec^5(c+dx)}{10a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (11*Sec[
c + d*x])/(2*a^2*d) + (11*Sec[c + d*x]^3)/(6*a^2*d) + (11*Sec[c + d*x]^5)/(
10*a^2*d) + (11*Sec[c + d*x]^7)/(14*a^2*d) - (Csc[c + d*x]^2*Sec[c + d*x]^7
)/(2*a^2*d) - (8*Tan[c + d*x])/(a^2*d) - (4*Tan[c + d*x]^3)/(a^2*d) - (8*Ta
n[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^3(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\csc(c+dx)\sec^8(c+dx) - 2a^2\csc^2(c+dx)\sec^8(c+dx) + a^2\csc^3(c+d\right.}{a^4}$$

$$= \frac{\int \csc(c+dx)\sec^8(c+dx)\,dx}{a^2} + \frac{\int \csc^3(c+dx)\sec^8(c+dx)\,dx}{a^2} - \frac{2\int \csc^2(c-}{}$$

$$= \frac{\text{Subst}\left(\int \frac{x^{10}}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{a^2 d} + \frac{\text{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^2 d} - $$

$$= -\frac{\csc^2(c+dx)\sec^7(c+dx)}{2a^2 d} + \frac{\text{Subst}\left(\int \left(1+x^2+x^4+x^6+\frac{1}{-1+x^2}\right)dx, x, \sec(\right.}{a^2 d}$$

$$= \frac{2\cot(c+dx)}{a^2 d} + \frac{\sec(c+dx)}{a^2 d} + \frac{\sec^3(c+dx)}{3a^2 d} + \frac{\sec^5(c+dx)}{5a^2 d} + \frac{\sec^7(c+dx)}{7a^2 d} - $$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2 d} + \frac{2\cot(c+dx)}{a^2 d} + \frac{11\sec(c+dx)}{2a^2 d} + \frac{11\sec^3(c+dx)}{6a^2 d} + $$

$$= -\frac{11\tanh^{-1}(\cos(c+dx))}{2a^2 d} + \frac{2\cot(c+dx)}{a^2 d} + \frac{11\sec(c+dx)}{2a^2 d} + \frac{11\sec^3(c+dx)}{6a^2 d}$$

Mathematica [A] time = 0.57, size = 277, normalized size = 1.43

$$\frac{-36960\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^7 + 36960\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-36960*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7 + 369
60*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7 + (Csc[c +
 d*x]^2*(4510 - 6908*Cos[c + d*x] - 563*Cos[2*(c + d*x)] + 4396*Cos[3*(c +
d*x)] - 5390*Cos[4*(c + d*x)] + 3140*Cos[5*(c + d*x)] - 1917*Cos[6*(c + d*x
)] - 628*Cos[7*(c + d*x)] + 4488*Sin[c + d*x] - 7536*Sin[2*(c + d*x)] + 383
6*Sin[3*(c + d*x)] - 780*Sin[5*(c + d*x)] + 2512*Sin[6*(c + d*x)] - 768*Sin
[7*(c + d*x)]))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)/(6720*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^3*(a + a*Sin[c + d*x])^2)
```

fricas [A] time = 0.48, size = 292, normalized size = 1.51

$$-\frac{3834\cos(dx+c)^6-3056\cos(dx+c)^4-468\cos(dx+c)^2+1155\left(\cos(dx+c)^7-3\cos(dx+c)^5+2\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/420*(3834*cos(d*x + c)^6 - 3056*cos(d*x + c)^4 - 468*cos(d*x + c)^2 + 11
55*(cos(d*x + c)^7 - 3*cos(d*x + c)^5 + 2*cos(d*x + c)^3 - 2*(cos(d*x + c)^
5 - cos(d*x + c)^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 1155*(cos(d
*x + c)^7 - 3*cos(d*x + c)^5 + 2*cos(d*x + c)^3 - 2*(cos(d*x + c)^5 - cos(d
*x + c)^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(768*cos(d*x + c)
^6 - 765*cos(d*x + c)^4 - 98*cos(d*x + c)^2 - 10)*sin(d*x + c) - 100)/(a^2*
d*cos(d*x + c)^7 - 3*a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^3 - 2*(a^2
*d*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^3)*sin(d*x + c))
```

giac [A] time = 0.27, size = 238, normalized size = 1.23

$$\frac{\frac{4620\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{105\left(a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^4}-\frac{105\left(66\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}-\frac{35\left(18\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}{a^2\left(\tan\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(4620*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 105*(a^2*tan(1/2*d*x + 1/2
*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^4 - 105*(66*tan(1/2*d*x + 1/2*c)^2 -
8*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d*x + 1/2*c)^2) - 35*(18*tan(1/2*d
*x + 1/2*c)^2 - 33*tan(1/2*d*x + 1/2*c) + 17)/(a^2*(tan(1/2*d*x + 1/2*c) -
1)^3) + (14070*tan(1/2*d*x + 1/2*c)^6 + 75705*tan(1/2*d*x + 1/2*c)^5 + 1772
05*tan(1/2*d*x + 1/2*c)^4 + 226450*tan(1/2*d*x + 1/2*c)^3 + 166488*tan(1/2*
d*x + 1/2*c)^2 + 66661*tan(1/2*d*x + 1/2*c) + 11533)/(a^2*(tan(1/2*d*x + 1/
2*c) + 1)^7))/d
```

maple [A] time = 0.70, size = 303, normalized size = 1.56

$$-\frac{1}{12d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3}{4a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2d}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/12/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3-1/8/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2-3/4
/a^2/d/(tan(1/2*d*x+1/2*c)-1)+1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^2*tan(1/
2*d*x+1/2*c)-1/8/a^2/d/tan(1/2*d*x+1/2*c)^2+1/d/a^2/tan(1/2*d*x+1/2*c)+11/2
/d/a^2*ln(tan(1/2*d*x+1/2*c))+4/7/d/a^2/(tan(1/2*d*x+1/2*c)+1)^7-2/d/a^2/(t
an(1/2*d*x+1/2*c)+1)^6+26/5/a^2/d/(tan(1/2*d*x+1/2*c)+1)^5-8/a^2/d/(tan(1/2
*d*x+1/2*c)+1)^4+139/12/a^2/d/(tan(1/2*d*x+1/2*c)+1)^3-83/8/a^2/d/(tan(1/2*
d*x+1/2*c)+1)^2+67/4/a^2/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.34, size = 526, normalized size = 2.71

$$\frac{\frac{\frac{420 \sin(dx+c)}{\cos(dx+c)+1}+\frac{15173 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{38432 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{894 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{95344 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{77182 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{61992 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{101115 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{11340 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{\frac{a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4 a^2 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3 a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{8 a^2 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14 a^2 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{14 a^2 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{8 a^2 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{3 a^2 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{4 a^2 \sin}{(\cos(dx}}}{840 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/840*((420*sin(d*x + c)/(cos(d*x + c) + 1) + 15173*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 38432*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 894*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 - 95344*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 77182
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 61992*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 + 101115*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 11340*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9 - 33495*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 14280*s
in(d*x + c)^11/(cos(d*x + c) + 1)^11 - 105)/(a^2*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 4*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^2*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 8*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^2
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 14*a^2*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 + 8*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 3*a^2*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 - 4*a^2*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a^2*s
in(d*x + c)^12/(cos(d*x + c) + 1)^12) - 105*(8*sin(d*x + c)/(cos(d*x + c) +
 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^2 + 4620*log(sin(d*x + c)/(cos
(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 10.77, size = 243, normalized size = 1.25

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}+\frac{11\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(-17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-\frac{319\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{8}+\frac{27\,\mathrm{ta}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) + (11*log(tan(c/2 + (d*x)/2)))/(2*a^2*d) - t
an(c/2 + (d*x)/2)/(a^2*d) - (cot(c/2 + (d*x)/2)^2*(tan(c/2 + (d*x)/2)/2 + (
15173*tan(c/2 + (d*x)/2)^2)/840 + (4804*tan(c/2 + (d*x)/2)^3)/105 + (149*ta
n(c/2 + (d*x)/2)^4)/140 - (11918*tan(c/2 + (d*x)/2)^5)/105 - (5513*tan(c/2
+ (d*x)/2)^6)/60 + (369*tan(c/2 + (d*x)/2)^7)/5 + (963*tan(c/2 + (d*x)/2)^8
)/8 + (27*tan(c/2 + (d*x)/2)^9)/2 - (319*tan(c/2 + (d*x)/2)^10)/8 - 17*tan(
c/2 + (d*x)/2)^11 - 1/8))/(a^2*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x
)/2) + 1)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.839 $\int \frac{\sin^3(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=178

$$-\frac{4\tan^9(c+dx)}{9a^3d}+\frac{\tan^7(c+dx)}{7a^3d}-\frac{\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{15\sec^7(c+dx)}{7a^3d}+\frac{21}{}$$

```
[Out] x/a^3+3*sec(d*x+c)/a^3/d-13/3*sec(d*x+c)^3/a^3/d+21/5*sec(d*x+c)^5/a^3/d-15
/7*sec(d*x+c)^7/a^3/d+4/9*sec(d*x+c)^9/a^3/d-tan(d*x+c)/a^3/d+1/3*tan(d*x+c
)^3/a^3/d-1/5*tan(d*x+c)^5/a^3/d+1/7*tan(d*x+c)^7/a^3/d-4/9*tan(d*x+c)^9/a^
3/d
```

Rubi [A] time = 0.37, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2875, 2873, 2606, 270, 2607, 30, 194, 3473, 8}

$$-\frac{4\tan^9(c+dx)}{9a^3d}+\frac{\tan^7(c+dx)}{7a^3d}-\frac{\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{15\sec^7(c+dx)}{7a^3d}+\frac{21}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] x/a^3 + (3*Sec[c + d*x])/(a^3*d) - (13*Sec[c + d*x]^3)/(3*a^3*d) + (21*Sec[
c + d*x]^5)/(5*a^3*d) - (15*Sec[c + d*x]^7)/(7*a^3*d) + (4*Sec[c + d*x]^9)/
(9*a^3*d) - Tan[c + d*x]/(a^3*d) + Tan[c + d*x]^3/(3*a^3*d) - Tan[c + d*x]^
5/(5*a^3*d) + Tan[c + d*x]^7/(7*a^3*d) - (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^3(c+dx)(a-a\sin(c+dx))^3\tan^7(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(a^3\sec^3(c+dx)\tan^7(c+dx) - 3a^3\sec^2(c+dx)\tan^8(c+dx) + 3a^3\sec(c+\right.}{a^6}$$

$$= \frac{\int \sec^3(c+dx)\tan^7(c+dx)\,dx}{a^3} - \frac{\int \tan^{10}(c+dx)\,dx}{a^3} - \frac{3\int \sec^2(c+dx)\tan^8(}{a^3}$$

$$= -\frac{\tan^9(c+dx)}{9a^3d} + \frac{\int \tan^8(c+dx)\,dx}{a^3} + \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^3 dx, x, \sec(c+\right.}{a^3d}$$

$$= \frac{\tan^7(c+dx)}{7a^3d} - \frac{4\tan^9(c+dx)}{9a^3d} - \frac{\int \tan^6(c+dx)\,dx}{a^3} + \frac{\text{Subst}\left(\int\left(-x^2+3x^4-\right.\right.}{}$$

$$= \frac{3\sec(c+dx)}{a^3d} - \frac{13\sec^3(c+dx)}{3a^3d} + \frac{21\sec^5(c+dx)}{5a^3d} - \frac{15\sec^7(c+dx)}{7a^3d} + \frac{4\sec^9}{9}$$

$$= \frac{3\sec(c+dx)}{a^3d} - \frac{13\sec^3(c+dx)}{3a^3d} + \frac{21\sec^5(c+dx)}{5a^3d} - \frac{15\sec^7(c+dx)}{7a^3d} + \frac{4\sec^9}{9}$$

$$= \frac{3\sec(c+dx)}{a^3d} - \frac{13\sec^3(c+dx)}{3a^3d} + \frac{21\sec^5(c+dx)}{5a^3d} - \frac{15\sec^7(c+dx)}{7a^3d} + \frac{4\sec^9}{9}$$

$$= \frac{x}{a^3} + \frac{3\sec(c+dx)}{a^3d} - \frac{13\sec^3(c+dx)}{3a^3d} + \frac{21\sec^5(c+dx)}{5a^3d} - \frac{15\sec^7(c+dx)}{7a^3d} +$$

Mathematica [A] time = 0.51, size = 273, normalized size = 1.53

$$\frac{93312\sin(c+dx) + 272160(c+dx)\sin(2(c+dx)) - 506277\sin(2(c+dx)) + 125248\sin(3(c+dx)) + 120960(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (169344 - 675036*Cos[c + d*x] + 362880*(c + d*x)*Cos[c + d*x] + 173952*Cos[
2*(c + d*x)] - 37502*Cos[3*(c + d*x)] + 20160*(c + d*x)*Cos[3*(c + d*x)] +
54912*Cos[4*(c + d*x)] + 112506*Cos[5*(c + d*x)] - 60480*(c + d*x)*Cos[5*(c
 + d*x)] - 21376*Cos[6*(c + d*x)] + 93312*Sin[c + d*x] - 506277*Sin[2*(c +
d*x)] + 272160*(c + d*x)*Sin[2*(c + d*x)] + 125248*Sin[3*(c + d*x)] - 22501
2*Sin[4*(c + d*x)] + 120960*(c + d*x)*Sin[4*(c + d*x)] + 67776*Sin[5*(c + d
*x)] + 18751*Sin[6*(c + d*x)] - 10080*(c + d*x)*Sin[6*(c + d*x)])/(322560*d
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^3*(a + a*Sin[c + d*x])^3)
```

fricas [A] time = 0.48, size = 177, normalized size = 0.99

$$\frac{945\,dx\cos\left(dx+c\right)^{5}+668\,\cos\left(dx+c\right)^{6}-1260\,dx\cos\left(dx+c\right)^{3}-1431\,\cos\left(dx+c\right)^{4}+465\,\cos\left(dx+c\right)^{2}+\left(315\right.}{315\left(3\,a^{3}d\cos\left(dx+c\right)^{5}-4\,a^{3}d\cos\left(dx+c\right)^{3}+\left(a^{3}d\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/315*(945*d*x*cos(d*x + c)^5 + 668*cos(d*x + c)^6 - 1260*d*x*cos(d*x + c)^
3 - 1431*cos(d*x + c)^4 + 465*cos(d*x + c)^2 + (315*d*x*cos(d*x + c)^5 - 12
60*d*x*cos(d*x + c)^3 - 1059*cos(d*x + c)^4 + 305*cos(d*x + c)^2 - 35)*sin(
d*x + c) - 70)/(3*a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*co
s(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*sin(d*x + c))
```

giac [A] time = 0.41, size = 181, normalized size = 1.02

$$\frac{\frac{10080\,(dx+c)}{a^{3}}+\frac{105\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+23\right)}{a^{3}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}+\frac{17955\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+160020\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+624960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+138\cdots}{\cdots}}{10080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/10080*(10080*(d*x + c)/a^3 + 105*(21*tan(1/2*d*x + 1/2*c)^2 - 48*tan(1/2*
d*x + 1/2*c) + 23)/(a^3*(tan(1/2*d*x + 1/2*c) - 1)^3) + (17955*tan(1/2*d*x
+ 1/2*c)^8 + 160020*tan(1/2*d*x + 1/2*c)^7 + 624960*tan(1/2*d*x + 1/2*c)^6
+ 1387260*tan(1/2*d*x + 1/2*c)^5 + 1884582*tan(1/2*d*x + 1/2*c)^4 + 1556268
*tan(1/2*d*x + 1/2*c)^3 + 774792*tan(1/2*d*x + 1/2*c)^2 + 215748*tan(1/2*d*
x + 1/2*c) + 25967)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^9))/d
```

maple [A] time = 0.57, size = 272, normalized size = 1.53

$$-\frac{1}{24d\,a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{16d\,a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{7}{32a^{3}d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{3}d}+\frac{\cdots}{9a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/24/d/a^3/(tan(1/2*d*x+1/2*c)-1)^3-1/16/d/a^3/(tan(1/2*d*x+1/2*c)-1)^2+7/
32/a^3/d/(tan(1/2*d*x+1/2*c)-1)+2/a^3/d*arctan(tan(1/2*d*x+1/2*c))+8/9/d/a^
```

```
3/(tan(1/2*d*x+1/2*c)+1)^9-4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^8+40/7/a^3/d/(tan
(1/2*d*x+1/2*c)+1)^7-4/3/a^3/d/(tan(1/2*d*x+1/2*c)+1)^6-21/10/a^3/d/(tan(1/
2*d*x+1/2*c)+1)^5-3/4/a^3/d/(tan(1/2*d*x+1/2*c)+1)^4+3/4/a^3/d/(tan(1/2*d*x
+1/2*c)+1)^3+13/8/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2+57/32/a^3/d/(tan(1/2*d*x+1
/2*c)+1)
```

maxima [B] time = 0.44, size = 487, normalized size = 2.74

$$\frac{2\left(\frac{\frac{1893\sin(dx+c)}{\cos(dx+c)+1}+\frac{2526\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2939\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{9936\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3546\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{11172\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{9702\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3675\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{1890\sin(dx}{(\cos(dx+c)+}}{a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{36a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{27a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2a^3\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{12a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 2/315*((1893*sin(d*x + c)/(cos(d*x + c) + 1) + 2526*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 2939*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 9936*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 - 3546*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 11172*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 9702*sin(d*x + c)^7/(cos(d*x + c) + 1
)^7 - 3675*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 1890*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10 - 315*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 368)/(a^3 +
6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 36*a^3
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*sin(d*x + c)^10
/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a^3*
sin(d*x + c)^12/(cos(d*x + c) + 1)^12) + 315*arctan(sin(d*x + c)/(cos(d*x +
 c) + 1))/a^3)/d
```

mupad [B] time = 17.81, size = 169, normalized size = 0.95

$$\frac{x}{a^3}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{70\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{3}-\frac{308\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{5}-\frac{1064\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{15}+\frac{788\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{35}+\frac{2208}{}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] x/a^3 + ((5878*tan(c/2 + (d*x)/2)^3)/315 - (1684*tan(c/2 + (d*x)/2)^2)/105
- (1262*tan(c/2 + (d*x)/2))/105 + (2208*tan(c/2 + (d*x)/2)^4)/35 + (788*tan
```

```
(c/2 + (d*x)/2)^5)/35 - (1064*tan(c/2 + (d*x)/2)^6)/15 - (308*tan(c/2 + (d*
x)/2)^7)/5 + (70*tan(c/2 + (d*x)/2)^9)/3 + 12*tan(c/2 + (d*x)/2)^10 + 2*tan
(c/2 + (d*x)/2)^11 - 736/315)/(a^3*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 +
(d*x)/2) + 1)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**7/(a+a*sin(d*x+c))**3,x)
[Out] Timed out
```

3.840 $\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=121

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{\tan^7(c+dx)}{7a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{13\sec^7(c+dx)}{7a^3d}-\frac{3\sec^5(c+dx)}{a^3d}+\frac{7\sec^3(c+dx)}{3a^3d}-\frac{\sec(c+dx)}{a^3d}$$

```
[Out] -sec(d*x+c)/a^3/d+7/3*sec(d*x+c)^3/a^3/d-3*sec(d*x+c)^5/a^3/d+13/7*sec(d*x+
c)^7/a^3/d-4/9*sec(d*x+c)^9/a^3/d+1/7*tan(d*x+c)^7/a^3/d+4/9*tan(d*x+c)^9/a
^3/d
```

Rubi [A] time = 0.35, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2607, 14, 2606, 270, 30, 194}

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{\tan^7(c+dx)}{7a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{13\sec^7(c+dx)}{7a^3d}-\frac{3\sec^5(c+dx)}{a^3d}+\frac{7\sec^3(c+dx)}{3a^3d}-\frac{\sec(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Sec[c + d*x]/(a^3*d)) + (7*Sec[c + d*x]^3)/(3*a^3*d) - (3*Sec[c + d*x]^5)
/(a^3*d) + (13*Sec[c + d*x]^7)/(7*a^3*d) - (4*Sec[c + d*x]^9)/(9*a^3*d) + T
an[c + d*x]^7/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^4(c+dx)(a-a\sin(c+dx))^3\tan^6(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(a^3\sec^4(c+dx)\tan^6(c+dx) - 3a^3\sec^3(c+dx)\tan^7(c+dx) + 3a^3\sec^2(c\right.}{a^6}$$
$$= \frac{\int \sec^4(c+dx)\tan^6(c+dx)\,dx}{a^3} - \frac{\int \sec(c+dx)\tan^9(c+dx)\,dx}{a^3} - \frac{3\int \sec^3(c}{}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+x^2\right)^4\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\text{Subst}\left(\int x^6\left(1+x^2\right)\,dx, x, \tan(c\right.}{a^3d}$$
$$= \frac{\tan^9(c+dx)}{3a^3d} - \frac{\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\text{Su}}{}$$
$$= -\frac{\sec(c+dx)}{a^3d} + \frac{7\sec^3(c+dx)}{3a^3d} - \frac{3\sec^5(c+dx)}{a^3d} + \frac{13\sec^7(c+dx)}{7a^3d} - \frac{4\sec^9(c}{9a^3}$$

Mathematica [A] time = 0.38, size = 185, normalized size = 1.53

$$\frac{-2304\sin(c+dx) + 27189\sin(2(c+dx)) - 16256\sin(3(c+dx)) + 12084\sin(4(c+dx)) + 384\sin(5(c+dx)) -}{64512d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-9408 + 36252*Cos[c + d*x] - 12384*Cos[2*(c + d*x)] + 2014*Cos[3*(c + d*x)
] + 4800*Cos[4*(c + d*x)] - 6042*Cos[5*(c + d*x)] + 608*Cos[6*(c + d*x)] -
2304*Sin[c + d*x] + 27189*Sin[2*(c + d*x)] - 16256*Sin[3*(c + d*x)] + 12084
*Sin[4*(c + d*x)] + 384*Sin[5*(c + d*x)] - 1007*Sin[6*(c + d*x)])/(64512*d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3*(a + a*Sin[c + d*x])^3)
```

fricas [A] time = 0.47, size = 130, normalized size = 1.07

$$-\frac{19\cos(dx+c)^6 + 9\cos(dx+c)^4 - 51\cos(dx+c)^2 + 2\left(3\cos(dx+c)^4 - 34\cos(dx+c)^2 + 7\right)\sin(dx+c)}{63\left(3a^3d\cos(dx+c)^5 - 4a^3d\cos(dx+c)^3 + \left(a^3d\cos(dx+c)^5 - 4a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/63*(19*cos(d*x + c)^6 + 9*cos(d*x + c)^4 - 51*cos(d*x + c)^2 + 2*(3*cos(
d*x + c)^4 - 34*cos(d*x + c)^2 + 7)*sin(d*x + c) + 7)/(3*a^3*d*cos(d*x + c)
^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^
3)*sin(d*x + c))
```

giac [A] time = 0.39, size = 172, normalized size = 1.42

$$\frac{\frac{21\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-36\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+17\right)}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}-\frac{315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+3024\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+13020\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+32760\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+5128}{a^{3}\left(\tan\left(\frac{1}{2}dx+\right.\right.}}{2016\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2016*(21*(15*tan(1/2*d*x + 1/2*c)^2 - 36*tan(1/2*d*x + 1/2*c) + 17)/(a^3*
(tan(1/2*d*x + 1/2*c) - 1)^3) - (315*tan(1/2*d*x + 1/2*c)^8 + 3024*tan(1/2*
d*x + 1/2*c)^7 + 13020*tan(1/2*d*x + 1/2*c)^6 + 32760*tan(1/2*d*x + 1/2*c)^
5 + 51282*tan(1/2*d*x + 1/2*c)^4 + 43008*tan(1/2*d*x + 1/2*c)^3 + 20988*tan
(1/2*d*x + 1/2*c)^2 + 5688*tan(1/2*d*x + 1/2*c) + 667)/(a^3*(tan(1/2*d*x +
1/2*c) + 1)^9))/d
```

maple [A] time = 0.57, size = 190, normalized size = 1.57

$$\frac{-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{5}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}-\frac{44}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{1}{3\left(\tan\left(\frac{dx}{2}\right.\right.}}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 128/d/a^3*(-1/3072/(tan(1/2*d*x+1/2*c)-1)^3-1/2048/(tan(1/2*d*x+1/2*c)-1)^2
+5/4096/(tan(1/2*d*x+1/2*c)-1)-1/144/(tan(1/2*d*x+1/2*c)+1)^9+1/32/(tan(1/2
*d*x+1/2*c)+1)^8-11/224/(tan(1/2*d*x+1/2*c)+1)^7+5/192/(tan(1/2*d*x+1/2*c)+
1)^6+1/256/(tan(1/2*d*x+1/2*c)+1)^5-1/512/(tan(1/2*d*x+1/2*c)+1)^4-1/384/(t
an(1/2*d*x+1/2*c)+1)^3-1/512/(tan(1/2*d*x+1/2*c)+1)^2-5/4096/(tan(1/2*d*x+1
/2*c)+1))
```

maxima [B] time = 0.34, size = 362, normalized size = 2.99

$$-\frac{32\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{12\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\right.}{63\left(a^{3}+\frac{6a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{36a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{27a^{3}\sin(dx+c)}{(\cos(dx+c)+1)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -32/63*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 12*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 36*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a^3 + 6*a^3*s
in(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 36*a^3*sin(d*
x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
- 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*sin(d*x + c)^10/(cos(d
*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a^3*sin(d*x
 + c)^12/(cos(d*x + c) + 1)^12)*d)
```

mupad [B] time = 13.17, size = 184, normalized size = 1.52

$$-\frac{\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{63}+\frac{64\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}+\frac{128\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{21}+\frac{64\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{63}-\frac{96\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\sin\left(\frac{c}{2}+\cdots\right.}{7}\cdots}{a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^3\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -((32*cos(c/2 + (d*x)/2)^12)/63 + (64*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)
/2))/21 - (128*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)/7 - (96*cos(c/2 +
 (d*x)/2)^8*sin(c/2 + (d*x)/2)^4)/7 + (64*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d
*x)/2)^3)/63 + (128*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2)/21)/(a^3*d*
(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 +
 (d*x)/2))^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.841 $\int \frac{\sin(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=105

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{3\tan^7(c+dx)}{7a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{11\sec^7(c+dx)}{7a^3d}+\frac{2\sec^5(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{a^3d}$$

```
[Out] -sec(d*x+c)^3/a^3/d+2*sec(d*x+c)^5/a^3/d-11/7*sec(d*x+c)^7/a^3/d+4/9*sec(d*
x+c)^9/a^3/d-3/7*tan(d*x+c)^7/a^3/d-4/9*tan(d*x+c)^9/a^3/d
```

Rubi [A] time = 0.34, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2873, 2606, 270, 2607, 14, 30}

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{3\tan^7(c+dx)}{7a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{11\sec^7(c+dx)}{7a^3d}+\frac{2\sec^5(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Sec[c + d*x]^3/(a^3*d)) + (2*Sec[c + d*x]^5)/(a^3*d) - (11*Sec[c + d*x]^7
)/(7*a^3*d) + (4*Sec[c + d*x]^9)/(9*a^3*d) - (3*Tan[c + d*x]^7)/(7*a^3*d) -
 (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^5(c+dx)(a-a\sin(c+dx))^3\tan^5(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(a^3\sec^5(c+dx)\tan^5(c+dx) - 3a^3\sec^4(c+dx)\tan^6(c+dx) + 3a^3\sec^3(c+\right.}{a^6}$$

$$= \frac{\int \sec^5(c+dx)\tan^5(c+dx)\,dx}{a^3} - \frac{\int \sec^2(c+dx)\tan^8(c+dx)\,dx}{a^3} - \frac{3\int \sec^4(c}{}$$

$$= -\frac{\text{Subst}\left(\int x^8\,dx, x, \tan(c+dx)\right)}{a^3d} + \frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx, x, \sec(c+dx)\right)}{a^3d}$$

$$= -\frac{\tan^9(c+dx)}{9a^3d} + \frac{\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{3\,\text{Subst}\left(\int \left(-\right.\right.}{}$$

$$= -\frac{\sec^3(c+dx)}{a^3d} + \frac{2\sec^5(c+dx)}{a^3d} - \frac{11\sec^7(c+dx)}{7a^3d} + \frac{4\sec^9(c+dx)}{9a^3d} - \frac{3\tan^7(c}{7a^3}$$

Mathematica [A] time = 0.21, size = 185, normalized size = 1.76

$$\frac{-1152\sin(c+dx)+6507\sin(2(c+dx))-8128\sin(3(c+dx))+2892\sin(4(c+dx))+192\sin(5(c+dx))-241\,\mathrm{s}}{64512d(a\sin(c+dx)+a)^3\left(\cos\right.}$$

Antiderivative was successfully verified.

[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]

[Out] (-1344 + 8676*Cos[c + d*x] - 11232*Cos[2*(c + d*x)] + 482*Cos[3*(c + d*x)] + 4416*Cos[4*(c + d*x)] - 1446*Cos[5*(c + d*x)] - 32*Cos[6*(c + d*x)] - 1152*Sin[c + d*x] + 6507*Sin[2*(c + d*x)] - 8128*Sin[3*(c + d*x)] + 2892*Sin[4*(c + d*x)] + 192*Sin[5*(c + d*x)] - 241*Sin[6*(c + d*x)])/(64512*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a + a*Sin[c + d*x])^3)

fricas [A] time = 0.47, size = 128, normalized size = 1.22

$$\frac{\cos(dx+c)^6-36\cos(dx+c)^4+57\cos(dx+c)^2-\left(3\cos(dx+c)^4-34\cos(dx+c)^2+7\right)\sin(dx+c)-14}{63\left(3a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3+\left(a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/63*(cos(d*x + c)^6 - 36*cos(d*x + c)^4 + 57*cos(d*x + c)^2 - (3*cos(d*x + c)^4 - 34*cos(d*x + c)^2 + 7)*sin(d*x + c) - 14)/(3*a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*sin(d*x + c))

giac [A] time = 0.39, size = 172, normalized size = 1.64

$$\frac{\frac{21\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+11\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{189\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+1764\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+7224\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+16380\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+19026\,\mathrm{t}}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}}{2016\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/2016*(21*(9*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 11)/(a^3*(tan(1/2*d*x + 1/2*c) - 1)^3) - (189*tan(1/2*d*x + 1/2*c)^8 + 1764*tan(1/2*d*x + 1/2*c)^7 + 7224*tan(1/2*d*x + 1/2*c)^6 + 16380*tan(1/2*d*x + 1/2*c)^5

```
+ 19026*tan(1/2*d*x + 1/2*c)^4 + 16380*tan(1/2*d*x + 1/2*c)^3 + 8352*tan(1/
2*d*x + 1/2*c)^2 + 2340*tan(1/2*d*x + 1/2*c) + 281)/(a^3*(tan(1/2*d*x + 1/2
*c) + 1)^9))/d
```

maple [A] time = 0.56, size = 190, normalized size = 1.81

$$\frac{-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}+\frac{48}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}-\frac{}{3\left(\tan\left(\right.\right.}}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 64/d/a^3*(-1/1536/(tan(1/2*d*x+1/2*c)-1)^3-1/1024/(tan(1/2*d*x+1/2*c)-1)^2+
3/2048/(tan(1/2*d*x+1/2*c)-1)+1/72/(tan(1/2*d*x+1/2*c)+1)^9-1/16/(tan(1/2*d
*x+1/2*c)+1)^8+3/28/(tan(1/2*d*x+1/2*c)+1)^7-1/12/(tan(1/2*d*x+1/2*c)+1)^6+
3/128/(tan(1/2*d*x+1/2*c)+1)^5+1/256/(tan(1/2*d*x+1/2*c)+1)^4-1/768/(tan(1/
2*d*x+1/2*c)+1)^3-1/512/(tan(1/2*d*x+1/2*c)+1)^2-3/2048/(tan(1/2*d*x+1/2*c)
+1))
```

maxima [B] time = 0.34, size = 382, normalized size = 3.64

$$-\frac{16\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{12\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36\sin(}{(\cos(dx-}\right.}{63\left(a^{3}+\frac{6a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{36a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{27a^{3}\sin(dx+}{(\cos(dx+c)+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -16/63*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 12*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 36*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 42*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 + 1)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 1
2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 - 27*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27
*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x +
c) + 1)^9 - 12*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x +
c)^11/(cos(d*x + c) + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

mupad [B] time = 13.19, size = 208, normalized size = 1.98

$$-\frac{\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{63}+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}+\frac{64\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{21}+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{63}-\frac{48\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{7}}{a^3\,d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^3\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -((16*cos(c/2 + (d*x)/2)^12)/63 + (32*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)
/2))/21 - (32*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)/3 - (64*cos(c/2 +
(d*x)/2)^7*sin(c/2 + (d*x)/2)^5)/7 - (48*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*
x)/2)^4)/7 + (32*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3)/63 + (64*cos(c/
2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2)/21)/(a^3*d*(cos(c/2 + (d*x)/2) - sin(
c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.842 $\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=127

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{5\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{9\sec^7(c+dx)}{7a^3d}-\frac{6\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx)}{3a^3d}$$

```
[Out] 1/3*sec(d*x+c)^3/a^3/d-6/5*sec(d*x+c)^5/a^3/d+9/7*sec(d*x+c)^7/a^3/d-4/9*se
c(d*x+c)^9/a^3/d+1/5*tan(d*x+c)^5/a^3/d+5/7*tan(d*x+c)^7/a^3/d+4/9*tan(d*x+
c)^9/a^3/d
```

Rubi [A] time = 0.22, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2711, 2607, 270, 2606, 14}

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{5\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{9\sec^7(c+dx)}{7a^3d}-\frac{6\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]^3/(3*a^3*d) - (6*Sec[c + d*x]^5)/(5*a^3*d) + (9*Sec[c + d*x]^7
)/(7*a^3*d) - (4*Sec[c + d*x]^9)/(9*a^3*d) + Tan[c + d*x]^5/(5*a^3*d) + (5*
Tan[c + d*x]^7)/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \left(a^3\sec^6(c+dx)\tan^4(c+dx)-3a^3\sec^5(c+dx)\tan^5(c+dx)+3a^3\sec^4(c+dx)\tan\right.}{a^6}$$

$$= \frac{\int \sec^6(c+dx)\tan^4(c+dx)\,dx}{a^3} - \frac{\int \sec^3(c+dx)\tan^7(c+dx)\,dx}{a^3} - \frac{3\int \sec^5(c+dx)\,}{a^3}$$

$$= -\frac{\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\mathrm{Subst}\left(\int x^4\left(1+x^2\right)^2\,dx, x, \tan(c+\right.}{a^3d}$$

$$= -\frac{\mathrm{Subst}\left(\int \left(-x^2+3x^4-3x^6+x^8\right)dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\mathrm{Subst}\left(\int \left(x^4+2x^6+x^8\right)d\right.}{a^3d}$$

$$= \frac{\sec^3(c+dx)}{3a^3d} - \frac{6\sec^5(c+dx)}{5a^3d} + \frac{9\sec^7(c+dx)}{7a^3d} - \frac{4\sec^9(c+dx)}{9a^3d} + \frac{\tan^5(c+dx)}{5a^3d} + \frac{5}{}$$

Mathematica [A] time = 0.29, size = 185, normalized size = 1.46

$$\frac{39168\sin(c+dx)+837\sin(2(c+dx))-28288\sin(3(c+dx))+372\sin(4(c+dx))+4224\sin(5(c+dx))-31\sin}{322560d(a\sin(c+dx)+a)^3\left(\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5376 + 1116*Cos[c + d*x] - 21312*Cos[2*(c + d*x)] + 62*Cos[3*(c + d*x)] +
8448*Cos[4*(c + d*x)] - 186*Cos[5*(c + d*x)] - 704*Cos[6*(c + d*x)] + 39168
*Sin[c + d*x] + 837*Sin[2*(c + d*x)] - 28288*Sin[3*(c + d*x)] + 372*Sin[4*(
```

```
c + d*x)] + 4224*Sin[5*(c + d*x)] - 31*Sin[6*(c + d*x)])/(322560*d*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a
+ a*Sin[c + d*x])^3)
```

fricas [A] time = 0.45, size = 130, normalized size = 1.02

$$\frac{22\cos(dx+c)^6-99\cos(dx+c)^4+120\cos(dx+c)^2-2\left(33\cos(dx+c)^4-80\cos(dx+c)^2+35\right)\sin(dx+\cdots}{315\left(3a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3+\left(a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/315*(22*cos(d*x + c)^6 - 99*cos(d*x + c)^4 + 120*cos(d*x + c)^2 - 2*(33*c
os(d*x + c)^4 - 80*cos(d*x + c)^2 + 35)*sin(d*x + c) - 35)/(3*a^3*d*cos(d*x
 + c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x
+ c)^3)*sin(d*x + c))
```

giac [A] time = 0.35, size = 159, normalized size = 1.25

$$\frac{\frac{105\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+2520\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+7140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-1638\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-8232\,\mathrm{t}\cdots}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^9}}{10080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/10080*(105*(3*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 5)/(a^3*
(tan(1/2*d*x + 1/2*c) - 1)^3) - (315*tan(1/2*d*x + 1/2*c)^8 + 2520*tan(1/2*
d*x + 1/2*c)^7 + 7140*tan(1/2*d*x + 1/2*c)^6 - 1638*tan(1/2*d*x + 1/2*c)^4
- 8232*tan(1/2*d*x + 1/2*c)^3 - 2988*tan(1/2*d*x + 1/2*c)^2 - 432*tan(1/2*d
*x + 1/2*c) - 13)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^9))/d
```

maple [A] time = 0.55, size = 175, normalized size = 1.38

$$\frac{-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{32}{1024\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1024}-\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}-\frac{52}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{\cdots}{3(\mathrm{t}\cdots}}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 32/d/a^3*(-1/768/(tan(1/2*d*x+1/2*c)-1)^3-1/512/(tan(1/2*d*x+1/2*c)-1)^2+1/
1024/(tan(1/2*d*x+1/2*c)-1)-1/36/(tan(1/2*d*x+1/2*c)+1)^9+1/8/(tan(1/2*d*x+
1/2*c)+1)^8-13/56/(tan(1/2*d*x+1/2*c)+1)^7+11/48/(tan(1/2*d*x+1/2*c)+1)^6-3
9/320/(tan(1/2*d*x+1/2*c)+1)^5+3/128/(tan(1/2*d*x+1/2*c)+1)^4+1/192/(tan(1/
2*d*x+1/2*c)+1)^3-1/1024/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.34, size = 402, normalized size = 3.17

$$-\frac{16\left(\frac{6\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{12\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{162\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\cdots\right.}{315\left(a^3+\frac{6\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{12\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36\,a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{36\,a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{27\,a^3\sin(dx+\cdots}{(\cos(dx+c)+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -16/315*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 12*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 162*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 126*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 - 126*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 1)/((a^
3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 36
*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8/(cos(d*x +
 c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*sin(d*x + c
)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 -
a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

mupad [B] time = 14.33, size = 232, normalized size = 1.83

$$-\frac{\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{315}+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{105}+\frac{64\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{105}+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{315}-\frac{48\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\cdots\right.}{35}\cdots}{a^3\,d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^3\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -((16*cos(c/2 + (d*x)/2)^12)/315 + (32*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x
)/2))/105 - (32*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7)/5 - (32*cos(c/2
+ (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)/5 - (288*cos(c/2 + (d*x)/2)^7*sin(c/2 +
(d*x)/2)^5)/35 - (48*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4)/35 + (32*co
s(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3)/315 + (64*cos(c/2 + (d*x)/2)^10*si
```

```
n(c/2 + (d*x)/2)^2)/105)/(a^3*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3
*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.843 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=105

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{\tan^7(c+dx)}{a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{\sec^7(c+dx)}{a^3d}+\frac{3\sec^5(c+dx)}{5a^3d}$$

```
[Out] 3/5*sec(d*x+c)^5/a^3/d-sec(d*x+c)^7/a^3/d+4/9*sec(d*x+c)^9/a^3/d-3/5*tan(d*
x+c)^5/a^3/d-tan(d*x+c)^7/a^3/d-4/9*tan(d*x+c)^9/a^3/d
```

Rubi [A] time = 0.34, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2875, 2873, 2606, 14, 2607, 270}

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{\tan^7(c+dx)}{a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{\sec^7(c+dx)}{a^3d}+\frac{3\sec^5(c+dx)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*Sec[c + d*x]^5)/(5*a^3*d) - Sec[c + d*x]^7/(a^3*d) + (4*Sec[c + d*x]^9)/
(9*a^3*d) - (3*Tan[c + d*x]^5)/(5*a^3*d) - Tan[c + d*x]^7/(a^3*d) - (4*Tan[
c + d*x]^9)/(9*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^7(c+dx)(a-a\sin(c+dx))^3\tan^3(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(a^3\sec^7(c+dx)\tan^3(c+dx) - 3a^3\sec^6(c+dx)\tan^4(c+dx) + 3a^3\sec^5(c+\right.}{a^6}$$

$$= \frac{\int \sec^7(c+dx)\tan^3(c+dx)\,dx}{a^3} - \frac{\int \sec^4(c+dx)\tan^6(c+dx)\,dx}{a^3} - \frac{3\int \sec^6(c}{}$$

$$= \frac{\mathrm{Subst}\left(\int x^6\left(-1+x^2\right)dx, x, \sec(c+dx)\right)}{a^3d} - \frac{\mathrm{Subst}\left(\int x^6\left(1+x^2\right)dx, x, \tan(c\right.}{a^3d}$$

$$= \frac{\mathrm{Subst}\left(\int \left(-x^6+x^8\right)dx, x, \sec(c+dx)\right)}{a^3d} - \frac{\mathrm{Subst}\left(\int \left(x^6+x^8\right)dx, x, \tan(c+d\right.}{a^3d}$$

$$= \frac{3\sec^5(c+dx)}{5a^3d} - \frac{\sec^7(c+dx)}{a^3d} + \frac{4\sec^9(c+dx)}{9a^3d} - \frac{3\tan^5(c+dx)}{5a^3d} - \frac{\tan^7(c+d}{a^3d}$$

Mathematica [A] time = 0.32, size = 185, normalized size = 1.76

$$\frac{4608\sin(c+dx) - 1323\sin(2(c+dx)) - 128\sin(3(c+dx)) - 588\sin(4(c+dx)) + 384\sin(5(c+dx)) + 49\sin(}{46080d(a\sin(c+dx)+a)^3\left(\cos\left(\frac{1}{2}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5376 - 1764*Cos[c + d*x] - 4032*Cos[2*(c + d*x)] - 98*Cos[3*(c + d*x)] + 7
68*Cos[4*(c + d*x)] + 294*Cos[5*(c + d*x)] - 64*Cos[6*(c + d*x)] + 4608*Sin
[c + d*x] - 1323*Sin[2*(c + d*x)] - 128*Sin[3*(c + d*x)] - 588*Sin[4*(c + d
*x)] + 384*Sin[5*(c + d*x)] + 49*Sin[6*(c + d*x)])/(46080*d*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a + a*Sin
[c + d*x])^3)
```

fricas [A] time = 0.46, size = 130, normalized size = 1.24

$$\frac{2\cos(dx+c)^6-9\cos(dx+c)^4+15\cos(dx+c)^2-\left(6\cos(dx+c)^4-5\cos(dx+c)^2+5\right)\sin(dx+c)-10}{45\left(3a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3+\left(a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/45*(2*cos(d*x + c)^6 - 9*cos(d*x + c)^4 + 15*cos(d*x + c)^2 - (6*cos(d*x
+ c)^4 - 5*cos(d*x + c)^2 + 5)*sin(d*x + c) - 10)/(3*a^3*d*cos(d*x + c)^5 -
 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*s
in(d*x + c))
```

giac [A] time = 0.35, size = 161, normalized size = 1.53

$$-\frac{\frac{15\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+540\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+3120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+5940\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+8298\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+6372\,\mathrm{t}}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^9}}{1440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/1440*(15*(3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^3*(tan(1/2*d*x + 1/2*c) - 1)^
3) - (45*tan(1/2*d*x + 1/2*c)^8 + 540*tan(1/2*d*x + 1/2*c)^7 + 3120*tan(1/2
*d*x + 1/2*c)^6 + 5940*tan(1/2*d*x + 1/2*c)^5 + 8298*tan(1/2*d*x + 1/2*c)^4
 + 6372*tan(1/2*d*x + 1/2*c)^3 + 3528*tan(1/2*d*x + 1/2*c)^2 + 972*tan(1/2*
d*x + 1/2*c) + 113)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^9))/d
```

maple [A] time = 0.56, size = 190, normalized size = 1.81

$$\frac{-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}-\frac{28}{3\left(\tan\left(\frac{dx}{2}+\right.\right.}}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 16/d/a^3*(-1/384/(tan(1/2*d*x+1/2*c)-1)^3-1/256/(tan(1/2*d*x+1/2*c)-1)^2-1/
512/(tan(1/2*d*x+1/2*c)-1)+1/18/(tan(1/2*d*x+1/2*c)+1)^9-1/4/(tan(1/2*d*x+1
/2*c)+1)^8+1/2/(tan(1/2*d*x+1/2*c)+1)^7-7/12/(tan(1/2*d*x+1/2*c)+1)^6+67/16
0/(tan(1/2*d*x+1/2*c)+1)^5-11/64/(tan(1/2*d*x+1/2*c)+1)^4+5/192/(tan(1/2*d*
x+1/2*c)+1)^3+1/128/(tan(1/2*d*x+1/2*c)+1)^2+1/512/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.34, size = 422, normalized size = 4.02

$$\frac{4\left(\frac{6\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{12\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{18\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{18\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{84\,\sin(dx+\cdots}{(\cos(dx+c)\cdots}\right.}{45\left(a^3+\frac{6\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{12\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36\,a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{36\,a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{27\,a^3\sin(dx+c\cdots}{(\cos(dx+c)+1)\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 4/45*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 12*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 18*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 18*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 84*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + 54*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 45*sin(d*x +
c)^8/(cos(d*x + c) + 1)^8 + 1)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1
) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 - 27*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 + 27*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9 - 12*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^1
2)*d)
```

mupad [B] time = 16.12, size = 255, normalized size = 2.43

$$\frac{\frac{4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{45}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{45}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{5}+\cdots}{a^3\,d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^3\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] ((4*cos(c/2 + (d*x)/2)^12)/45 + (8*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)
)/15 + 4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + (24*cos(c/2 + (d*x)/2)
^5*sin(c/2 + (d*x)/2)^7)/5 + (112*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6
)/15 + (8*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)/5 + (8*cos(c/2 + (d*x)
/2)^8*sin(c/2 + (d*x)/2)^4)/5 + (8*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^
3)/45 + (16*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2)/15)/(a^3*d*(cos(c/2
 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2
))^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.844 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=127

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{9\tan^7(c+dx)}{7a^3d}+\frac{6\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{5\sec^7(c+dx)}{7a^3d}-\frac{\sec^5(c+dx)}{5a^3d}$$

```
[Out] -1/5*sec(d*x+c)^5/a^3/d+5/7*sec(d*x+c)^7/a^3/d-4/9*sec(d*x+c)^9/a^3/d+1/3*t
an(d*x+c)^3/a^3/d+6/5*tan(d*x+c)^5/a^3/d+9/7*tan(d*x+c)^7/a^3/d+4/9*tan(d*x
+c)^9/a^3/d
```

Rubi [A] time = 0.36, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2875, 2873, 2607, 270, 2606, 14}

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{9\tan^7(c+dx)}{7a^3d}+\frac{6\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{5\sec^7(c+dx)}{7a^3d}-\frac{\sec^5(c+dx)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -Sec[c + d*x]^5/(5*a^3*d) + (5*Sec[c + d*x]^7)/(7*a^3*d) - (4*Sec[c + d*x]^
9)/(9*a^3*d) + Tan[c + d*x]^3/(3*a^3*d) + (6*Tan[c + d*x]^5)/(5*a^3*d) + (9
*Tan[c + d*x]^7)/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^8(c+dx)(a-a\sin(c+dx))^3\tan^2(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(a^3\sec^8(c+dx)\tan^2(c+dx)-3a^3\sec^7(c+dx)\tan^3(c+dx)+3a^3\sec^6(c+\right.}{a^6}$$

$$= \frac{\int \sec^8(c+dx)\tan^2(c+dx)\,dx}{a^3}-\frac{\int \sec^5(c+dx)\tan^5(c+dx)\,dx}{a^3}-\frac{3\int \sec^7(c}{}$$

$$= -\frac{\operatorname{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx,x,\sec(c+dx)\right)}{a^3d}+\frac{\operatorname{Subst}\left(\int x^2\left(1+x^2\right)^3\,dx,x,\tan\right.}{a^3d}$$

$$= -\frac{\operatorname{Subst}\left(\int \left(x^4-2x^6+x^8\right)dx,x,\sec(c+dx)\right)}{a^3d}+\frac{\operatorname{Subst}\left(\int \left(x^2+3x^4+3x^6+x\right.\right.}{a^3d}$$

$$= -\frac{\sec^5(c+dx)}{5a^3d}+\frac{5\sec^7(c+dx)}{7a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{\tan^3(c+dx)}{3a^3d}+\frac{6\tan^5(c+}{5a^3d}$$

Mathematica [A] time = 0.33, size = 185, normalized size = 1.46

$$\frac{73728\sin(c+dx)-7263\sin(2(c+dx))+512\sin(3(c+dx))-3228\sin(4(c+dx))-1536\sin(5(c+dx))+269}{322560d(a\sin(c+dx)+a)^3\left(c\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (32256 - 9684*Cos[c + d*x] - 6912*Cos[2*(c + d*x)] - 538*Cos[3*(c + d*x)] -
 3072*Cos[4*(c + d*x)] + 1614*Cos[5*(c + d*x)] + 256*Cos[6*(c + d*x)] + 737
28*Sin[c + d*x] - 7263*Sin[2*(c + d*x)] + 512*Sin[3*(c + d*x)] - 3228*Sin[4
*(c + d*x)] - 1536*Sin[5*(c + d*x)] + 269*Sin[6*(c + d*x)])/(322560*d*(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*
(a + a*Sin[c + d*x])^3)
```

fricas [A] time = 0.45, size = 130, normalized size = 1.02

$$-\frac{8\cos(dx+c)^6-36\cos(dx+c)^4+15\cos(dx+c)^2-2\left(12\cos(dx+c)^4-10\cos(dx+c)^2-35\right)\sin(dx+c}{315\left(3a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3+\left(a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/315*(8*cos(d*x + c)^6 - 36*cos(d*x + c)^4 + 15*cos(d*x + c)^2 - 2*(12*co
s(d*x + c)^4 - 10*cos(d*x + c)^2 - 35)*sin(d*x + c) + 35)/(3*a^3*d*cos(d*x
+ c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x +
 c)^3)*sin(d*x + c))
```

giac [A] time = 0.34, size = 172, normalized size = 1.35

$$-\frac{\frac{105\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{945\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+10080\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+23940\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+42840\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+4}{a^3\left(\tan\left(\frac{1}{2}c\right.\right.}}{10080d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/10080*(105*(9*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 7)/(a^3
*(tan(1/2*d*x + 1/2*c) - 1)^3) - (945*tan(1/2*d*x + 1/2*c)^8 + 10080*tan(1/
2*d*x + 1/2*c)^7 + 23940*tan(1/2*d*x + 1/2*c)^6 + 42840*tan(1/2*d*x + 1/2*c
```

```
)^5 + 41958*tan(1/2*d*x + 1/2*c)^4 + 32592*tan(1/2*d*x + 1/2*c)^3 + 14148*t
an(1/2*d*x + 1/2*c)^2 + 5112*tan(1/2*d*x + 1/2*c) + 673)/(a^3*(tan(1/2*d*x
+ 1/2*c) + 1)^9))/d
```

maple [A] time = 0.54, size = 190, normalized size = 1.50

$$\frac{-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}-\frac{60}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{3}{3\left(\tan\left(\frac{dx}{2}\right.\right.}}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 8/d/a^3*(-1/192/(tan(1/2*d*x+1/2*c)-1)^3-1/128/(tan(1/2*d*x+1/2*c)-1)^2-3/2
56/(tan(1/2*d*x+1/2*c)-1)-1/9/(tan(1/2*d*x+1/2*c)+1)^9+1/2/(tan(1/2*d*x+1/2
*c)+1)^8-15/14/(tan(1/2*d*x+1/2*c)+1)^7+17/12/(tan(1/2*d*x+1/2*c)+1)^6-99/8
0/(tan(1/2*d*x+1/2*c)+1)^5+23/32/(tan(1/2*d*x+1/2*c)+1)^4-1/4/(tan(1/2*d*x+
1/2*c)+1)^3+1/32/(tan(1/2*d*x+1/2*c)+1)^2+3/256/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.34, size = 442, normalized size = 3.48

$$\frac{4\left(\frac{66\sin(dx+c)}{\cos(dx+c)+1}+\frac{132\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{232\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{18\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{108\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{84\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{50}{(\mathrm{cc}}\right.}{315\left(a^{3}+\frac{6a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{36a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{27a^{3}\sin(dx+c)}{(\cos(dx+c)+1)^{8}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 4/315*(66*sin(d*x + c)/(cos(d*x + c) + 1) + 132*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 232*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 18*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 108*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 84*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 + 504*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 315*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 210*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
 + 11)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*
sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c)
 + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

mupad [B] time = 14.87, size = 279, normalized size = 2.20

$$\frac{\frac{44\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{315}+\frac{88\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{105}+\frac{176\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{105}+\frac{928\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{315}+\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{\cdots}{}\right.}{35}}{a^3\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{si}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] ((44*cos(c/2 + (d*x)/2)^12)/315 + (88*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)
/2))/105 + (8*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^9)/3 + 4*cos(c/2 + (d
*x)/2)^4*sin(c/2 + (d*x)/2)^8 + (32*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)
^7)/5 + (16*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)/15 + (48*cos(c/2 + (
d*x)/2)^7*sin(c/2 + (d*x)/2)^5)/35 + (8*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x
)/2)^4)/35 + (928*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3)/315 + (176*cos
(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2)/105)/(a^3*d*(cos(c/2 + (d*x)/2) -
sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.845 $\int \frac{\sec^3(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=123

$$\frac{4\tan^3(c+dx)}{63a^3d}+\frac{4\tan(c+dx)}{21a^3d}-\frac{\sec^3(c+dx)}{21d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\sec^3(c+dx)}{21ad(a\sin(c+dx)+a)^2}+\frac{\sec^3(c+dx)}{9d(a\sin(c+dx)+a)^3}$$

```
[Out] 1/9*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^3-1/21*sec(d*x+c)^3/a/d/(a+a*sin(d*x+c)
)^2-1/21*sec(d*x+c)^3/d/(a^3+a^3*sin(d*x+c))+4/21*tan(d*x+c)/a^3/d+4/63*tan
(d*x+c)^3/a^3/d
```

Rubi [A] time = 0.16, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2859, 2672, 3767}

$$\frac{4\tan^3(c+dx)}{63a^3d}+\frac{4\tan(c+dx)}{21a^3d}-\frac{\sec^3(c+dx)}{21d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\sec^3(c+dx)}{21ad(a\sin(c+dx)+a)^2}+\frac{\sec^3(c+dx)}{9d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]^3/(9*d*(a + a*Sin[c + d*x])^3) - Sec[c + d*x]^3/(21*a*d*(a + a
*Sin[c + d*x])^2) - Sec[c + d*x]^3/(21*d*(a^3 + a^3*Sin[c + d*x])) + (4*Tan
[c + d*x])/(21*a^3*d) + (4*Tan[c + d*x]^3)/(63*a^3*d)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} + \frac{\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{3a}$$
$$= \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} - \frac{\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2} + \frac{5\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx}{21a^2}$$
$$= \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} - \frac{\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2} - \frac{\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+dx)\right)}$$
$$= \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} - \frac{\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2} - \frac{\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+dx)\right)}$$
$$= \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} - \frac{\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2} - \frac{\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+dx)\right)}$$

Mathematica [A] time = 0.28, size = 185, normalized size = 1.50

$$\frac{9216\sin(c+dx)+675\sin(2(c+dx))+512\sin(3(c+dx))+300\sin(4(c+dx))-1536\sin(5(c+dx))-25\sin(}{64512d(a\sin(c+dx)+a)^3\left(\cos\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (10752 + 900*Cos[c + d*x] - 6912*Cos[2*(c + d*x)] + 50*Cos[3*(c + d*x)] - 3
072*Cos[4*(c + d*x)] - 150*Cos[5*(c + d*x)] + 256*Cos[6*(c + d*x)] + 9216*S
in[c + d*x] + 675*Sin[2*(c + d*x)] + 512*Sin[3*(c + d*x)] + 300*Sin[4*(c +
d*x)] - 1536*Sin[5*(c + d*x)] - 25*Sin[6*(c + d*x)])/(64512*d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a + a*S
in[c + d*x])^3)
```

fricas [A] time = 0.43, size = 130, normalized size = 1.06

$$-\frac{8\cos(dx+c)^6-36\cos(dx+c)^4+15\cos(dx+c)^2-\left(24\cos(dx+c)^4-20\cos(dx+c)^2-7\right)\sin(dx+c)-}{63\left(3a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3+\left(a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/63*(8*cos(d*x + c)^6 - 36*cos(d*x + c)^4 + 15*cos(d*x + c)^2 - (24*cos(d
*x + c)^4 - 20*cos(d*x + c)^2 - 7)*sin(d*x + c) + 14)/(3*a^3*d*cos(d*x + c)
^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^
3)*sin(d*x + c))
```

giac [A] time = 0.33, size = 172, normalized size = 1.40

$$-\frac{\frac{21\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+13\right)}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}-\frac{315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-756\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-4200\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-11340\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-14994\cdots}{a^{3}\left(\tan\left(\frac{1}{2}dx+\cdots\right.\right.}}{2016\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2016*(21*(15*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 13)/(a^3
*(tan(1/2*d*x + 1/2*c) - 1)^3) - (315*tan(1/2*d*x + 1/2*c)^8 - 756*tan(1/2*
d*x + 1/2*c)^7 - 4200*tan(1/2*d*x + 1/2*c)^6 - 11340*tan(1/2*d*x + 1/2*c)^5
 - 14994*tan(1/2*d*x + 1/2*c)^4 - 13356*tan(1/2*d*x + 1/2*c)^3 - 6768*tan(1
/2*d*x + 1/2*c)^2 - 2196*tan(1/2*d*x + 1/2*c) - 209)/(a^3*(tan(1/2*d*x + 1/
2*c) + 1)^9))/d
```

maple [A] time = 0.60, size = 190, normalized size = 1.54

$$\frac{-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{5}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}+\frac{64}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}-\frac{4\cdots}{3\left(\tan\left(\frac{dx}{2}\cdots\right.\right.}}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4/d/a^3*(-1/96/(tan(1/2*d*x+1/2*c)-1)^3-1/64/(tan(1/2*d*x+1/2*c)-1)^2-5/128
/(tan(1/2*d*x+1/2*c)-1)+2/9/(tan(1/2*d*x+1/2*c)+1)^9-1/(tan(1/2*d*x+1/2*c)+
1)^8+16/7/(tan(1/2*d*x+1/2*c)+1)^7-10/3/(tan(1/2*d*x+1/2*c)+1)^6+27/8/(tan(
1/2*d*x+1/2*c)+1)^5-39/16/(tan(1/2*d*x+1/2*c)+1)^4+59/48/(tan(1/2*d*x+1/2*c
)+1)^3-13/32/(tan(1/2*d*x+1/2*c)+1)^2+5/128/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.34, size = 442, normalized size = 3.59

$$\frac{2\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{75\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{128\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{162\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{42\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{189\cdots}{(\cos\cdots}\right.}{63\left(a^{3}+\frac{6a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{36a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{27a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2/63*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 75*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 128*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 162*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 36*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 42*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + 189*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 126*sin(d
*x + c)^9/(cos(d*x + c) + 1)^9 + 63*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 1)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1
)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*sin
(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c) +
1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

mupad [B] time = 15.00, size = 279, normalized size = 2.27

$$\frac{\frac{2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{63}+\frac{4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}+\frac{50\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{21}+\frac{256\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{63}+\frac{36\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{7}}{a^3\,d\left(\cos\left(\frac{c}{2}+\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] ((2*cos(c/2 + (d*x)/2)^12)/63 + (4*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)
)/21 + 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 4*cos(c/2 + (d*x)/2)^
3*sin(c/2 + (d*x)/2)^9 + 6*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 - (4*c
os(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)/3 - (8*cos(c/2 + (d*x)/2)^7*sin(c
/2 + (d*x)/2)^5)/7 + (36*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4)/7 + (25
6*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3)/63 + (50*cos(c/2 + (d*x)/2)^10
*sin(c/2 + (d*x)/2)^2)/21)/(a^3*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))
^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin(c+dx)\sec^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

3.846 $\int \frac{\csc(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=187

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{15\tan^7(c+dx)}{7a^3d}-\frac{21\tan^5(c+dx)}{5a^3d}-\frac{13\tan^3(c+dx)}{3a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}+\frac{\sec^7(c+d}{7a^3d}$$

```
[Out] -arctanh(cos(d*x+c))/a^3/d+sec(d*x+c)/a^3/d+1/3*sec(d*x+c)^3/a^3/d+1/5*sec(
d*x+c)^5/a^3/d+1/7*sec(d*x+c)^7/a^3/d+4/9*sec(d*x+c)^9/a^3/d-3*tan(d*x+c)/a
^3/d-13/3*tan(d*x+c)^3/a^3/d-21/5*tan(d*x+c)^5/a^3/d-15/7*tan(d*x+c)^7/a^3/
d-4/9*tan(d*x+c)^9/a^3/d
```

Rubi [A] time = 0.36, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2606, 30, 2607, 270}

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{15\tan^7(c+dx)}{7a^3d}-\frac{21\tan^5(c+dx)}{5a^3d}-\frac{13\tan^3(c+dx)}{3a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}+\frac{\sec^7(c+d}{7a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^3*d)) + Sec[c + d*x]/(a^3*d) + Sec[c + d*x]^3/(3
*a^3*d) + Sec[c + d*x]^5/(5*a^3*d) + Sec[c + d*x]^7/(7*a^3*d) + (4*Sec[c +
d*x]^9)/(9*a^3*d) - (3*Tan[c + d*x])/(a^3*d) - (13*Tan[c + d*x]^3)/(3*a^3*d
) - (21*Tan[c + d*x]^5)/(5*a^3*d) - (15*Tan[c + d*x]^7)/(7*a^3*d) - (4*Tan[
c + d*x]^9)/(9*a^3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc(c+dx)\sec^{10}(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-3a^3\sec^{10}(c+dx)+a^3\csc(c+dx)\sec^{10}(c+dx)+3a^3\sec^9(c+dx)\tan(c+\right.}{a^6}$$
$$= \frac{\int \csc(c+dx)\sec^{10}(c+dx)\,dx}{a^3} - \frac{\int \sec^8(c+dx)\tan^2(c+dx)\,dx}{a^3} - \frac{3\int \sec^{10}(c}{a^3}$$
$$= \frac{\text{Subst}\left(\int \frac{x^{10}}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^3d} - \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^3\,dx, x, \tan(c+dx)\right)}{a^3d}$$
$$= \frac{\sec^9(c+dx)}{3a^3d} - \frac{3\tan(c+dx)}{a^3d} - \frac{4\tan^3(c+dx)}{a^3d} - \frac{18\tan^5(c+dx)}{5a^3d} - \frac{12\tan^7(c-}{7a^3d}$$
$$= \frac{\sec(c+dx)}{a^3d} + \frac{\sec^3(c+dx)}{3a^3d} + \frac{\sec^5(c+dx)}{5a^3d} + \frac{\sec^7(c+dx)}{7a^3d} + \frac{4\sec^9(c+dx)}{9a^3d} -$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\sec(c+dx)}{a^3d} + \frac{\sec^3(c+dx)}{3a^3d} + \frac{\sec^5(c+dx)}{5a^3d} + \frac{\sec^7(c}{7a^3}$$

Mathematica [A] time = 1.38, size = 204, normalized size = 1.09

$$322560\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 322560\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \frac{196992\sin(c+dx)-383157\sin(2(c+dx))+211648\sin(3(c+dx))-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-322560*Log[Cos[(c + d*x)/2]] + 322560*Log[Sin[(c + d*x)/2]] + (357504 - 5
10876*Cos[c + d*x] + 317952*Cos[2*(c + d*x)] - 28382*Cos[3*(c + d*x)] + 203
52*Cos[4*(c + d*x)] + 85146*Cos[5*(c + d*x)] - 11776*Cos[6*(c + d*x)] + 196
992*Sin[c + d*x] - 383157*Sin[2*(c + d*x)] + 211648*Sin[3*(c + d*x)] - 1702
92*Sin[4*(c + d*x)] + 50496*Sin[5*(c + d*x)] + 14191*Sin[6*(c + d*x)])/((Co
```

```
s[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
9))/(322560*a^3*d)
```

fricas [A] time = 0.48, size = 250, normalized size = 1.34

$$736\cos(dx+c)^6 - 1422\cos(dx+c)^4 - 510\cos(dx+c)^2 - 315\Big(3\cos(dx+c)^5 - 4\cos(dx+c)^3 + \big(\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/630*(736*cos(d*x + c)^6 - 1422*cos(d*x + c)^4 - 510*cos(d*x + c)^2 - 315*
(3*cos(d*x + c)^5 - 4*cos(d*x + c)^3 + (cos(d*x + c)^5 - 4*cos(d*x + c)^3)*
sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 315*(3*cos(d*x + c)^5 - 4*cos(d
*x + c)^3 + (cos(d*x + c)^5 - 4*cos(d*x + c)^3)*sin(d*x + c))*log(-1/2*cos(
d*x + c) + 1/2) - 2*(789*cos(d*x + c)^4 + 235*cos(d*x + c)^2 + 35)*sin(d*x
+ c) - 140)/(3*a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d
*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*sin(d*x + c))
```

giac [A] time = 0.24, size = 187, normalized size = 1.00

$$\frac{\frac{10080\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3} - \frac{105\left(27\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-48\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+25\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3} + \frac{63315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+412020\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+1273440\tan\left(\cdots\right.}{\cdots}}{1008\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/10080*(10080*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 105*(27*tan(1/2*d*x + 1
/2*c)^2 - 48*tan(1/2*d*x + 1/2*c) + 25)/(a^3*(tan(1/2*d*x + 1/2*c) - 1)^3)
+ (63315*tan(1/2*d*x + 1/2*c)^8 + 412020*tan(1/2*d*x + 1/2*c)^7 + 1273440*t
an(1/2*d*x + 1/2*c)^6 + 2324700*tan(1/2*d*x + 1/2*c)^5 + 2731302*tan(1/2*d*
x + 1/2*c)^4 + 2097228*tan(1/2*d*x + 1/2*c)^3 + 1032552*tan(1/2*d*x + 1/2*c
)^2 + 297828*tan(1/2*d*x + 1/2*c) + 40127)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^
9))/d
```

maple [A] time = 0.79, size = 271, normalized size = 1.45

$$-\frac{1}{24d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{16d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{9}{32a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}+\frac{\cdots}{9d\,a^3\left(\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/24/d/a^3/(tan(1/2*d*x+1/2*c)-1)^3-1/16/d/a^3/(tan(1/2*d*x+1/2*c)-1)^2-9/
32/a^3/d/(tan(1/2*d*x+1/2*c)-1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c))+8/9/d/a^3/(t
an(1/2*d*x+1/2*c)+1)^9-4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^8+72/7/a^3/d/(tan(1/2
*d*x+1/2*c)+1)^7-52/3/a^3/d/(tan(1/2*d*x+1/2*c)+1)^6+219/10/a^3/d/(tan(1/2*
d*x+1/2*c)+1)^5-83/4/a^3/d/(tan(1/2*d*x+1/2*c)+1)^4+193/12/a^3/d/(tan(1/2*d
*x+1/2*c)+1)^3-75/8/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2+201/32/a^3/d/(tan(1/2*d*
x+1/2*c)+1)
```

maxima [B] time = 0.34, size = 508, normalized size = 2.72

$$\frac{\frac{2\left(\frac{3063\sin(dx+c)}{\cos(dx+c)+1}+\frac{4866\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1289\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{11736\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{10566\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5292\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{13482\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{6300\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2625\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\cdots\right.}{a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{36a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{27a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2a^3\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{12a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\cdots}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/315*(2*(3063*sin(d*x + c)/(cos(d*x + c) + 1) + 4866*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 - 1289*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 11736*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 10566*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 52
92*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 13482*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 6300*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2625*sin(d*x + c)^9/(c
os(d*x + c) + 1)^9 - 3150*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 945*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11 + 668)/(a^3 + 6*a^3*sin(d*x + c)/(cos(d*x
+ c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 - 27*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a
^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 27*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9 - 12*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a
^3*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c
) + 1)^12) + 315*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 12.56, size = 195, normalized size = 1.04

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{50\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{3}+40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{428\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{5}+\cdots}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) - ((2042*tan(c/2 + (d*x)/2))/105 + (3244*ta
n(c/2 + (d*x)/2)^2)/105 - (2578*tan(c/2 + (d*x)/2)^3)/315 - (2608*tan(c/2 +
 (d*x)/2)^4)/35 - (2348*tan(c/2 + (d*x)/2)^5)/35 + (168*tan(c/2 + (d*x)/2)^
6)/5 + (428*tan(c/2 + (d*x)/2)^7)/5 + 40*tan(c/2 + (d*x)/2)^8 - (50*tan(c/2
 + (d*x)/2)^9)/3 - 20*tan(c/2 + (d*x)/2)^10 - 6*tan(c/2 + (d*x)/2)^11 + 133
6/315)/(a^3*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.847 $\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=200

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{17\tan^7(c+dx)}{7a^3d}+\frac{28\tan^5(c+dx)}{5a^3d}+\frac{22\tan^3(c+dx)}{3a^3d}+\frac{8\tan(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}$$

```
[Out] 3*arctanh(cos(d*x+c))/a^3/d-cot(d*x+c)/a^3/d-3*sec(d*x+c)/a^3/d-sec(d*x+c)^
3/a^3/d-3/5*sec(d*x+c)^5/a^3/d-3/7*sec(d*x+c)^7/a^3/d-4/9*sec(d*x+c)^9/a^3/
d+8*tan(d*x+c)/a^3/d+22/3*tan(d*x+c)^3/a^3/d+28/5*tan(d*x+c)^5/a^3/d+17/7*t
an(d*x+c)^7/a^3/d+4/9*tan(d*x+c)^9/a^3/d
```

Rubi [A] time = 0.39, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2620, 270, 2606, 30}

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{17\tan^7(c+dx)}{7a^3d}+\frac{28\tan^5(c+dx)}{5a^3d}+\frac{22\tan^3(c+dx)}{3a^3d}+\frac{8\tan(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) - (3*Sec[c + d*x])
/(a^3*d) - Sec[c + d*x]^3/(a^3*d) - (3*Sec[c + d*x]^5)/(5*a^3*d) - (3*Sec[c
 + d*x]^7)/(7*a^3*d) - (4*Sec[c + d*x]^9)/(9*a^3*d) + (8*Tan[c + d*x])/(a^3
*d) + (22*Tan[c + d*x]^3)/(3*a^3*d) + (28*Tan[c + d*x]^5)/(5*a^3*d) + (17*T
an[c + d*x]^7)/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^2(c+dx)\sec^{10}(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(3a^3\sec^{10}(c+dx) - 3a^3\csc(c+dx)\sec^{10}(c+dx) + a^3\csc^2(c+dx)\sec^{10}(c\right.}{a^6}$$
$$= \frac{\int \csc^2(c+dx)\sec^{10}(c+dx)\,dx}{a^3} - \frac{\int \sec^9(c+dx)\tan(c+dx)\,dx}{a^3} + \frac{3\int \sec^{10}(}{a}$$
$$= -\frac{\text{Subst}\left(\int x^8\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\text{Subst}\left(\int \frac{(1+x^2)^5}{x^2}\,dx, x, \tan(c+dx)\right)}{a^3d} - \frac{3\,\text{Su}}{}$$
$$= -\frac{\sec^9(c+dx)}{9a^3d} + \frac{3\tan(c+dx)}{a^3d} + \frac{4\tan^3(c+dx)}{a^3d} + \frac{18\tan^5(c+dx)}{5a^3d} + \frac{12\tan^7}{7a}$$
$$= -\frac{\cot(c+dx)}{a^3d} - \frac{3\sec(c+dx)}{a^3d} - \frac{\sec^3(c+dx)}{a^3d} - \frac{3\sec^5(c+dx)}{5a^3d} - \frac{3\sec^7(c+d}{7a^3d}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{\cot(c+dx)}{a^3d} - \frac{3\sec(c+dx)}{a^3d} - \frac{\sec^3(c+dx)}{a^3d} - \frac{3\sec^5}{5}$$

Mathematica [A] time = 0.64, size = 230, normalized size = 1.15

$$-1935360\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 1935360\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \frac{\csc(c+dx)(-707328\sin(c+dx)+1364182\sin(2(c+dx))-116}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (1935360*Log[Cos[(c + d*x)/2]] - 1935360*Log[Sin[(c + d*x)/2]] + (Csc[c + d
*x]*(-590976 + 1083321*Cos[c + d*x] - 653248*Cos[2*(c + d*x)] - 601845*Cos[
3*(c + d*x)] + 340096*Cos[4*(c + d*x)] - 521599*Cos[5*(c + d*x)] + 259008*C
os[6*(c + d*x)] + 40123*Cos[7*(c + d*x)] - 707328*Sin[c + d*x] + 1364182*Si
n[2*(c + d*x)] - 1161600*Sin[3*(c + d*x)] + 320984*Sin[4*(c + d*x)] - 32934
4*Sin[5*(c + d*x)] - 240738*Sin[6*(c + d*x)] + 53248*Sin[7*(c + d*x)]))/((C
```

```
os[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^9))/(645120*a^3*d)
```

fricas [A] time = 0.49, size = 297, normalized size = 1.48

$$\frac{8094\cos(dx+c)^6 - 9484\cos(dx+c)^4 + 620\cos(dx+c)^2 + 945\left(\cos(dx+c)^7 - 5\cos(dx+c)^5 + 4\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/630*(8094*cos(d*x + c)^6 - 9484*cos(d*x + c)^4 + 620*cos(d*x + c)^2 + 945
*(cos(d*x + c)^7 - 5*cos(d*x + c)^5 + 4*cos(d*x + c)^3 - (3*cos(d*x + c)^5
- 4*cos(d*x + c)^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 945*(cos(d*
x + c)^7 - 5*cos(d*x + c)^5 + 4*cos(d*x + c)^3 - (3*cos(d*x + c)^5 - 4*cos(
d*x + c)^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(1664*cos(d*x +
c)^6 - 4653*cos(d*x + c)^4 + 285*cos(d*x + c)^2 + 35)*sin(d*x + c) + 140)/(
a^3*d*cos(d*x + c)^7 - 5*a^3*d*cos(d*x + c)^5 + 4*a^3*d*cos(d*x + c)^3 - (3
*a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*sin(d*x + c))
```

giac [A] time = 0.28, size = 230, normalized size = 1.15

$$-\frac{\frac{30240\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3} - \frac{5040\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3} - \frac{5040\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)} + \frac{105\left(33\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-60\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+31\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3} + \frac{157815}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/10080*(30240*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 5040*tan(1/2*d*x + 1/2
*c)/a^3 - 5040*(6*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)) + 10
5*(33*tan(1/2*d*x + 1/2*c)^2 - 60*tan(1/2*d*x + 1/2*c) + 31)/(a^3*(tan(1/2*
d*x + 1/2*c) - 1)^3) + (157815*tan(1/2*d*x + 1/2*c)^8 + 1093680*tan(1/2*d*x
 + 1/2*c)^7 + 3488940*tan(1/2*d*x + 1/2*c)^6 + 6524280*tan(1/2*d*x + 1/2*c)
^5 + 7788186*tan(1/2*d*x + 1/2*c)^4 + 6052704*tan(1/2*d*x + 1/2*c)^3 + 2995
596*tan(1/2*d*x + 1/2*c)^2 + 864504*tan(1/2*d*x + 1/2*c) + 113591)/(a^3*(ta
n(1/2*d*x + 1/2*c) + 1)^9))/d
```

maple [A] time = 0.80, size = 308, normalized size = 1.54

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3} - \frac{1}{24d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3} - \frac{1}{16d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} - \frac{11}{32a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} - \frac{1}{2d\,a^3\tan\left(\frac{dx}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-1/24/d/a^3/(tan(1/2*d*x+1/2*c)-1)^3-1/16/d/a^3
/(tan(1/2*d*x+1/2*c)-1)^2-11/32/a^3/d/(tan(1/2*d*x+1/2*c)-1)-1/2/d/a^3/tan(
1/2*d*x+1/2*c)-3/d/a^3*ln(tan(1/2*d*x+1/2*c))-8/9/d/a^3/(tan(1/2*d*x+1/2*c)
+1)^9+4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^8-76/7/a^3/d/(tan(1/2*d*x+1/2*c)+1)^7+
58/3/a^3/d/(tan(1/2*d*x+1/2*c)+1)^6-267/10/a^3/d/(tan(1/2*d*x+1/2*c)+1)^5+1
11/4/a^3/d/(tan(1/2*d*x+1/2*c)+1)^4-25/a^3/d/(tan(1/2*d*x+1/2*c)+1)^3+67/4/
a^3/d/(tan(1/2*d*x+1/2*c)+1)^2-501/32/a^3/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.35, size = 567, normalized size = 2.84

$$-\frac{\frac{\frac{8786 \sin(dx+c)}{\cos(dx+c)+1}+\frac{35076 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{43062 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{41753 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{152172 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{99072 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{93324 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{157689 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{44730 \sin(}{(\cos(dx+c)}}{\frac{a^3 \sin(dx+c)}{\cos(dx+c)+1}+\frac{6 a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{12 a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2 a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{27 a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{36 a^3 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{36 a^3 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{27 a^3 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{2 a^3 \sin(dx+}{(\cos(dx+c)+}}}{630 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/630*((8786*sin(d*x + c)/(cos(d*x + c) + 1) + 35076*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 43062*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 41753*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 152172*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
99072*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 93324*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 157689*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 44730*sin(d*x + c
)^9/(cos(d*x + c) + 1)^9 - 50820*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 42
210*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 10395*sin(d*x + c)^12/(cos(d*x
+ c) + 1)^12 + 315)/(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 6*a^3*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 12*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*
a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 27*a^3*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 - 36*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 36*a^3*sin(d*x + c
)^8/(cos(d*x + c) + 1)^8 + 27*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 2*a
^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 12*a^3*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 - 6*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - a^3*sin(d*x +
c)^13/(cos(d*x + c) + 1)^13) + 1890*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^
3 - 315*sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 11.32, size = 390, normalized size = 1.95

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}-\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{-33\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-134\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-\frac{484\,\tan\left(\frac{c}{2}\right.}{3}}{d\,\left(-2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-12\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-24\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-4\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - (3*log(tan(c/2 + (d*x)/2)))/(a^3*d) - ((8786
*tan(c/2 + (d*x)/2))/315 + (11692*tan(c/2 + (d*x)/2)^2)/105 + (14354*tan(c/
2 + (d*x)/2)^3)/105 - (41753*tan(c/2 + (d*x)/2)^4)/315 - (16908*tan(c/2 + (
d*x)/2)^5)/35 - (11008*tan(c/2 + (d*x)/2)^6)/35 + (4444*tan(c/2 + (d*x)/2)^
7)/15 + (2503*tan(c/2 + (d*x)/2)^8)/5 + 142*tan(c/2 + (d*x)/2)^9 - (484*tan
(c/2 + (d*x)/2)^10)/3 - 134*tan(c/2 + (d*x)/2)^11 - 33*tan(c/2 + (d*x)/2)^1
2 + 1)/(d*(12*a^3*tan(c/2 + (d*x)/2)^2 + 24*a^3*tan(c/2 + (d*x)/2)^3 + 4*a^
3*tan(c/2 + (d*x)/2)^4 - 54*a^3*tan(c/2 + (d*x)/2)^5 - 72*a^3*tan(c/2 + (d*
x)/2)^6 + 72*a^3*tan(c/2 + (d*x)/2)^8 + 54*a^3*tan(c/2 + (d*x)/2)^9 - 4*a^3
*tan(c/2 + (d*x)/2)^10 - 24*a^3*tan(c/2 + (d*x)/2)^11 - 12*a^3*tan(c/2 + (d
*x)/2)^12 - 2*a^3*tan(c/2 + (d*x)/2)^13 + 2*a^3*tan(c/2 + (d*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.848 $\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^4} dx$

Optimal. Leaf size=145

$$\frac{8\tan^{11}(c+dx)}{11a^4d}+\frac{16\tan^9(c+dx)}{9a^4d}+\frac{9\tan^7(c+dx)}{7a^4d}+\frac{\tan^5(c+dx)}{5a^4d}-\frac{8\sec^{11}(c+dx)}{11a^4d}+\frac{20\sec^9(c+dx)}{9a^4d}-\frac{16\sec^7(c+$$

```
[Out] 4/5*sec(d*x+c)^5/a^4/d-16/7*sec(d*x+c)^7/a^4/d+20/9*sec(d*x+c)^9/a^4/d-8/11
*sec(d*x+c)^11/a^4/d+1/5*tan(d*x+c)^5/a^4/d+9/7*tan(d*x+c)^7/a^4/d+16/9*tan
(d*x+c)^9/a^4/d+8/11*tan(d*x+c)^11/a^4/d
```

Rubi [A] time = 0.31, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2711, 2607, 270, 2606, 14}

$$\frac{8\tan^{11}(c+dx)}{11a^4d}+\frac{16\tan^9(c+dx)}{9a^4d}+\frac{9\tan^7(c+dx)}{7a^4d}+\frac{\tan^5(c+dx)}{5a^4d}-\frac{8\sec^{11}(c+dx)}{11a^4d}+\frac{20\sec^9(c+dx)}{9a^4d}-\frac{16\sec^7(c+$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (4*Sec[c + d*x]^5)/(5*a^4*d) - (16*Sec[c + d*x]^7)/(7*a^4*d) + (20*Sec[c +
d*x]^9)/(9*a^4*d) - (8*Sec[c + d*x]^11)/(11*a^4*d) + Tan[c + d*x]^5/(5*a^4*
d) + (9*Tan[c + d*x]^7)/(7*a^4*d) + (16*Tan[c + d*x]^9)/(9*a^4*d) + (8*Tan[
c + d*x]^11)/(11*a^4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \left(a^4\sec^8(c+dx)\tan^4(c+dx)-4a^4\sec^7(c+dx)\tan^5(c+dx)+6a^4\sec^6(c+dx)\,\mathrm{ta}\right.}{a^8}$$

$$= \frac{\int \sec^8(c+dx)\tan^4(c+dx)\,dx}{a^4}+\frac{\int \sec^4(c+dx)\tan^8(c+dx)\,dx}{a^4}-\frac{4\int \sec^7(c+dx)}{a^4}$$

$$= \frac{\mathrm{Subst}\left(\int x^8\left(1+x^2\right)dx, x, \tan(c+dx)\right)}{a^4d}+\frac{\mathrm{Subst}\left(\int x^4\left(1+x^2\right)^3dx, x, \tan(c+dx)\right)}{a^4d}$$

$$= \frac{\mathrm{Subst}\left(\int \left(x^8+x^{10}\right)dx, x, \tan(c+dx)\right)}{a^4d}+\frac{\mathrm{Subst}\left(\int \left(x^4+3x^6+3x^8+x^{10}\right)dx, x, \mathrm{tan}\right.}{a^4d}$$

$$= \frac{4\sec^5(c+dx)}{5a^4d}-\frac{16\sec^7(c+dx)}{7a^4d}+\frac{20\sec^9(c+dx)}{9a^4d}-\frac{8\sec^{11}(c+dx)}{11a^4d}+\frac{\tan^5(c+dx}{5a^4d}$$

Mathematica [A] time = 0.45, size = 166, normalized size = 1.14

$$\frac{\sec^3(c+dx)(501600\sin(c+dx)-70136\sin(2(c+dx))-200288\sin(3(c+dx))-25504\sin(4(c+dx))+48800}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*(168960 - 78903*Cos[c + d*x] - 183040*Cos[2*(c + d*x)] + 87
67*Cos[3*(c + d*x)] + 62464*Cos[4*(c + d*x)] + 19925*Cos[5*(c + d*x)] - 156
16*Cos[6*(c + d*x)] - 797*Cos[7*(c + d*x)] + 501600*Sin[c + d*x] - 70136*Si
n[2*(c + d*x)] - 200288*Sin[3*(c + d*x)] - 25504*Sin[4*(c + d*x)] + 48800*S
```

```
in[5*(c + d*x)] + 6376*Sin[6*(c + d*x)] - 1952*Sin[7*(c + d*x)]))/(3548160*
a^4*d*(1 + Sin[c + d*x])^4)
```

fricas [A] time = 0.44, size = 153, normalized size = 1.06

$$-\frac{488\,\cos\left(dx+c\right)^{6}-1220\,\cos\left(dx+c\right)^{4}+1120\,\cos\left(dx+c\right)^{2}+\left(122\,\cos\left(dx+c\right)^{6}-915\,\cos\left(dx+c\right)^{4}+1400\,\cdots\right.}{3465\left(a^{4}d\cos\left(dx+c\right)^{7}-8\,a^{4}d\cos\left(dx+c\right)^{5}+8\,a^{4}d\cos\left(dx+c\right)^{3}-4\left(a^{4}d\cos\left(dx+c\right)^{5}-2\,a^{4}d\,\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/3465*(488*cos(d*x + c)^6 - 1220*cos(d*x + c)^4 + 1120*cos(d*x + c)^2 + (
122*cos(d*x + c)^6 - 915*cos(d*x + c)^4 + 1400*cos(d*x + c)^2 - 735)*sin(d*
x + c) - 420)/(a^4*d*cos(d*x + c)^7 - 8*a^4*d*cos(d*x + c)^5 + 8*a^4*d*cos(
d*x + c)^3 - 4*(a^4*d*cos(d*x + c)^5 - 2*a^4*d*cos(d*x + c)^3)*sin(d*x + c)
)
```

giac [A] time = 0.46, size = 172, normalized size = 1.19

$$-\frac{\frac{1155\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^{4}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}-\frac{3465\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+47355\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+309540\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+588588\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+891198\,\tan\left(\frac{1}{2}\,dx\cdots\right.}{a^{4}\left(\tan\left(\frac{1}{2}\,dx\cdots\right.\right.}}{110880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/110880*(1155*(3*tan(1/2*d*x + 1/2*c) - 1)/(a^4*(tan(1/2*d*x + 1/2*c) - 1
)^3) - (3465*tan(1/2*d*x + 1/2*c)^9 + 47355*tan(1/2*d*x + 1/2*c)^8 + 309540
*tan(1/2*d*x + 1/2*c)^7 + 588588*tan(1/2*d*x + 1/2*c)^6 + 891198*tan(1/2*d*
x + 1/2*c)^5 + 747450*tan(1/2*d*x + 1/2*c)^4 + 481140*tan(1/2*d*x + 1/2*c)^
3 + 172700*tan(1/2*d*x + 1/2*c)^2 + 35233*tan(1/2*d*x + 1/2*c) + 3203)/(a^4
*(tan(1/2*d*x + 1/2*c) + 1)^11))/d
```

maple [A] time = 0.72, size = 190, normalized size = 1.31

$$\frac{-\frac{1}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{16}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{10}}-\frac{176}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}+\frac{28}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}-\frac{\cdots}{7\left(\tan\cdots\right.}}{a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 32/d/a^4*(-1/1536/(tan(1/2*d*x+1/2*c)-1)^3-1/1024/(tan(1/2*d*x+1/2*c)-1)^2-
1/22/(tan(1/2*d*x+1/2*c)+1)^11+1/4/(tan(1/2*d*x+1/2*c)+1)^10-11/18/(tan(1/2
*d*x+1/2*c)+1)^9+7/8/(tan(1/2*d*x+1/2*c)+1)^8-179/224/(tan(1/2*d*x+1/2*c)+1
)^7+89/192/(tan(1/2*d*x+1/2*c)+1)^6-49/320/(tan(1/2*d*x+1/2*c)+1)^5+1/64/(t
an(1/2*d*x+1/2*c)+1)^4+7/1536/(tan(1/2*d*x+1/2*c)+1)^3+1/1024/(tan(1/2*d*x+
1/2*c)+1)^2)
```

maxima [B] time = 0.35, size = 488, normalized size = 3.37

$$\frac{32\left(\frac{16\sin(dx+c)}{\cos(dx+c)+1}+\frac{50\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{64\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{22\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{517\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\cdots\right.}{3465\left(a^4+\frac{8a^4\sin(dx+c)}{\cos(dx+c)+1}+\frac{25a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{32a^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{11a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{88a^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{99a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{99a^4\sin(dx+\cdots}{(\cos(dx+c)+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 32/3465*(16*sin(d*x + c)/(cos(d*x + c) + 1) + 50*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 64*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 22*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 517*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 726*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 1650*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 924*
sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 693*sin(d*x + c)^9/(cos(d*x + c) + 1)
^9 + 2)/((a^4 + 8*a^4*sin(d*x + c)/(cos(d*x + c) + 1) + 25*a^4*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 32*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 11*a
^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 88*a^4*sin(d*x + c)^5/(cos(d*x + c
) + 1)^5 - 99*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 99*a^4*sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 + 88*a^4*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 11*a
^4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 32*a^4*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 - 25*a^4*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^4*sin(d*x
 + c)^13/(cos(d*x + c) + 1)^13 - a^4*sin(d*x + c)^14/(cos(d*x + c) + 1)^14)
*d)
```

mupad [B] time = 16.85, size = 279, normalized size = 1.92

$$\frac{\frac{64\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{3465}+\frac{512\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3465}+\frac{320\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{693}+\frac{2048\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3465}-\frac{64\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\cdots\right.}{315}\cdots}{a^4\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^4*(a + a*sin(c + d*x))^4),x)
```

```
[Out] ((64*cos(c/2 + (d*x)/2)^14)/3465 + (512*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*
x)/2))/3465 + (32*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9)/5 + (128*cos(c
```

```
/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8)/15 + (320*cos(c/2 + (d*x)/2)^7*sin(c/
2 + (d*x)/2)^7)/21 + (704*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6)/105 +
(1504*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5)/315 - (64*cos(c/2 + (d*x)/
2)^10*sin(c/2 + (d*x)/2)^4)/315 + (2048*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*
x)/2)^3)/3465 + (320*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2)/693)/(a^4*
d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2
 + (d*x)/2))^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.849 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=145

$$-\frac{8\tan^{11}(c+dx)}{11a^4d}-\frac{20\tan^9(c+dx)}{9a^4d}-\frac{16\tan^7(c+dx)}{7a^4d}-\frac{4\tan^5(c+dx)}{5a^4d}+\frac{8\sec^{11}(c+dx)}{11a^4d}-\frac{16\sec^9(c+dx)}{9a^4d}+\frac{9\sec^7(c}{7a^4d}$$

```
[Out] -1/5*sec(d*x+c)^5/a^4/d+9/7*sec(d*x+c)^7/a^4/d-16/9*sec(d*x+c)^9/a^4/d+8/11
*sec(d*x+c)^11/a^4/d-4/5*tan(d*x+c)^5/a^4/d-16/7*tan(d*x+c)^7/a^4/d-20/9*ta
n(d*x+c)^9/a^4/d-8/11*tan(d*x+c)^11/a^4/d
```

Rubi [A] time = 0.41, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2875, 2873, 2606, 14, 2607, 270}

$$-\frac{8\tan^{11}(c+dx)}{11a^4d}-\frac{20\tan^9(c+dx)}{9a^4d}-\frac{16\tan^7(c+dx)}{7a^4d}-\frac{4\tan^5(c+dx)}{5a^4d}+\frac{8\sec^{11}(c+dx)}{11a^4d}-\frac{16\sec^9(c+dx)}{9a^4d}+\frac{9\sec^7(c}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -Sec[c + d*x]^5/(5*a^4*d) + (9*Sec[c + d*x]^7)/(7*a^4*d) - (16*Sec[c + d*x]
^9)/(9*a^4*d) + (8*Sec[c + d*x]^11)/(11*a^4*d) - (4*Tan[c + d*x]^5)/(5*a^4*
d) - (16*Tan[c + d*x]^7)/(7*a^4*d) - (20*Tan[c + d*x]^9)/(9*a^4*d) - (8*Tan
[c + d*x]^11)/(11*a^4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \sec^9(c+dx)(a-a\sin(c+dx))^4\tan^3(c+dx)\,dx}{a^8}$$

$$= \frac{\int \left(a^4\sec^9(c+dx)\tan^3(c+dx) - 4a^4\sec^8(c+dx)\tan^4(c+dx) + 6a^4\sec^7(c+\right.}{}$$

$$= \frac{\int \sec^9(c+dx)\tan^3(c+dx)\,dx}{a^4} + \frac{\int \sec^5(c+dx)\tan^7(c+dx)\,dx}{a^4} - \frac{4\int \sec^8(c}{}$$

$$= \frac{\text{Subst}\left(\int x^8\left(-1+x^2\right)dx, x, \sec(c+dx)\right)}{a^4 d} + \frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)^3 dx, x, \sec(\right.}{a^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(-x^4+3x^6-3x^8+x^{10}\right)dx, x, \sec(c+dx)\right)}{a^4 d} + \frac{\text{Subst}\left(\int \left(-x^8+x^{10}\right)\right.}{a^4 d}$$

$$= -\frac{\sec^5(c+dx)}{5a^4 d} + \frac{9\sec^7(c+dx)}{7a^4 d} - \frac{16\sec^9(c+dx)}{9a^4 d} + \frac{8\sec^{11}(c+dx)}{11a^4 d} - \frac{4\tan^5(}{5a}$$

Mathematica [A] time = 0.46, size = 166, normalized size = 1.14

$$\frac{\sec^3(c+dx)(844800\sin(c+dx) - 191752\sin(2(c+dx)) + 11264\sin(3(c+dx)) - 69728\sin(4(c+dx)) + 2560}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*(844800 - 215721*Cos[c + d*x] - 619520*Cos[2*(c + d*x)] + 2
3969*Cos[3*(c + d*x)] + 32768*Cos[4*(c + d*x)] + 54475*Cos[5*(c + d*x)] - 8
192*Cos[6*(c + d*x)] - 2179*Cos[7*(c + d*x)] + 844800*Sin[c + d*x] - 191752
*Sin[2*(c + d*x)] + 11264*Sin[3*(c + d*x)] - 69728*Sin[4*(c + d*x)] + 25600
*Sin[5*(c + d*x)] + 17432*Sin[6*(c + d*x)] - 1024*Sin[7*(c + d*x)]))/(70963
20*a^4*d*(1 + Sin[c + d*x])^4)
```

fricas [A] time = 0.44, size = 154, normalized size = 1.06

$$-\frac{128\cos\left(dx+c\right)^{6}-320\cos\left(dx+c\right)^{4}+805\cos\left(dx+c\right)^{2}+4\left(8\cos\left(dx+c\right)^{6}-60\cos\left(dx+c\right)^{4}+35\cos\left(dx\right.\right.}{3465\left(a^{4}d\cos\left(dx+c\right)^{7}-8a^{4}d\cos\left(dx+c\right)^{5}+8a^{4}d\cos\left(dx+c\right)^{3}-4\left(a^{4}d\cos\left(dx+c\right)^{5}-2a^{4}d\cos\left(d\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/3465*(128*cos(d*x + c)^6 - 320*cos(d*x + c)^4 + 805*cos(d*x + c)^2 + 4*(
8*cos(d*x + c)^6 - 60*cos(d*x + c)^4 + 35*cos(d*x + c)^2 - 105)*sin(d*x + c
) - 735)/(a^4*d*cos(d*x + c)^7 - 8*a^4*d*cos(d*x + c)^5 + 8*a^4*d*cos(d*x +
 c)^3 - 4*(a^4*d*cos(d*x + c)^5 - 2*a^4*d*cos(d*x + c)^3)*sin(d*x + c))
```

giac [A] time = 0.44, size = 198, normalized size = 1.37

$$-\frac{\frac{1155\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)}{a^{4}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}-\frac{3465\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+45045\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+279510\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+669900\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/110880*(1155*(3*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 2)/(a^
4*(tan(1/2*d*x + 1/2*c) - 1)^3) - (3465*tan(1/2*d*x + 1/2*c)^10 + 45045*tan
(1/2*d*x + 1/2*c)^9 + 279510*tan(1/2*d*x + 1/2*c)^8 + 669900*tan(1/2*d*x +
1/2*c)^7 + 1205358*tan(1/2*d*x + 1/2*c)^6 + 1334718*tan(1/2*d*x + 1/2*c)^5
+ 1144440*tan(1/2*d*x + 1/2*c)^4 + 627660*tan(1/2*d*x + 1/2*c)^3 + 257345*t
an(1/2*d*x + 1/2*c)^2 + 57013*tan(1/2*d*x + 1/2*c) + 5498)/(a^4*(tan(1/2*d*
x + 1/2*c) + 1)^11))/d
```

maple [A] time = 0.73, size = 220, normalized size = 1.52

$$-\frac{1}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{16}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{10}}+\frac{184}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}-\frac{}{\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 16/d/a^4*(-1/768/(tan(1/2*d*x+1/2*c)-1)^3-1/512/(tan(1/2*d*x+1/2*c)-1)^2-1/
512/(tan(1/2*d*x+1/2*c)-1)+1/11/(tan(1/2*d*x+1/2*c)+1)^11-1/2/(tan(1/2*d*x+
1/2*c)+1)^10+23/18/(tan(1/2*d*x+1/2*c)+1)^9-2/(tan(1/2*d*x+1/2*c)+1)^8+235/
112/(tan(1/2*d*x+1/2*c)+1)^7-145/96/(tan(1/2*d*x+1/2*c)+1)^6+29/40/(tan(1/2
*d*x+1/2*c)+1)^5-13/64/(tan(1/2*d*x+1/2*c)+1)^4+13/768/(tan(1/2*d*x+1/2*c)+
1)^3+3/512/(tan(1/2*d*x+1/2*c)+1)^2+1/512/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.35, size = 508, normalized size = 3.50

$$\frac{4\left(\frac{488\sin(dx+c)}{\cos(dx+c)+1}+\frac{1525\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{1952\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{2794\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{176\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{4818\sin}{(\cos(dx+}\right.}{3465\left(a^{4}+\frac{8a^{4}\sin(dx+c)}{\cos(dx+c)+1}+\frac{25a^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{32a^{4}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{11a^{4}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{88a^{4}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{99a^{4}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{99a^{4}\sin(d}{(\cos(dx+c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 4/3465*(488*sin(d*x + c)/(cos(d*x + c) + 1) + 1525*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 1952*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2794*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 176*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4818*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5280*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 + 10857*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 5544*sin(d*x + c)^9/(cos(d*x
 + c) + 1)^9 + 3465*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 61)/((a^4 + 8*a
^4*sin(d*x + c)/(cos(d*x + c) + 1) + 25*a^4*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 32*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 11*a^4*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 88*a^4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 99*a^4*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 99*a^4*sin(d*x + c)^8/(cos(d*x + c) +
1)^8 + 88*a^4*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 11*a^4*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 - 32*a^4*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 25*a
^4*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^4*sin(d*x + c)^13/(cos(d*x +
 c) + 1)^13 - a^4*sin(d*x + c)^14/(cos(d*x + c) + 1)^14)*d)
```

mupad [B] time = 15.89, size = 303, normalized size = 2.09

$$\frac{\frac{244\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{3465}+\frac{1952\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3465}+\frac{1220\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{693}+\frac{7808\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{3465}+\frac{1016\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{315}}{a^4\,d\,\Big(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^4*(a + a*sin(c + d*x))^4),x)
```

```
[Out] ((244*cos(c/2 + (d*x)/2)^14)/3465 + (1952*cos(c/2 + (d*x)/2)^13*sin(c/2 + (
d*x)/2))/3465 + 4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 + (32*cos(c/2
+ (d*x)/2)^5*sin(c/2 + (d*x)/2)^9)/5 + (188*cos(c/2 + (d*x)/2)^6*sin(c/2 +
(d*x)/2)^8)/15 + (128*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)/21 + (584*
cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6)/105 + (64*cos(c/2 + (d*x)/2)^9*s
in(c/2 + (d*x)/2)^5)/315 + (1016*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^4
)/315 + (7808*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3)/3465 + (1220*cos(
c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2)/693)/(a^4*d*(cos(c/2 + (d*x)/2) - s
in(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.850 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=143

$$\frac{8\tan^{11}(c+dx)}{11a^4d}+\frac{8\tan^9(c+dx)}{3a^4d}+\frac{25\tan^7(c+dx)}{7a^4d}+\frac{2\tan^5(c+dx)}{a^4d}+\frac{\tan^3(c+dx)}{3a^4d}-\frac{8\sec^{11}(c+dx)}{11a^4d}+\frac{4\sec^9(c+d}{3a^4d}$$

```
[Out] -4/7*sec(d*x+c)^7/a^4/d+4/3*sec(d*x+c)^9/a^4/d-8/11*sec(d*x+c)^11/a^4/d+1/3
*tan(d*x+c)^3/a^4/d+2*tan(d*x+c)^5/a^4/d+25/7*tan(d*x+c)^7/a^4/d+8/3*tan(d*
x+c)^9/a^4/d+8/11*tan(d*x+c)^11/a^4/d
```

Rubi [A] time = 0.36, antiderivative size = 184, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2870, 2672, 3767, 8}

$$\frac{8\tan(c+dx)}{231a^4d}-\frac{4\sec(c+dx)}{231d\left(a^4\sin(c+dx)+a^4\right)}-\frac{4\sec(c+dx)}{231d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{\sec^3(c+dx)}{6ad(a\sin(c+dx)+a)^3}-\frac{5\sec(c}{231ad(a\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(a*Sec[c + d*x])/(22*d*(a + a*Sin[c + d*x])^5) - Sec[c + d*x]/(33*d*(a + a
*Sin[c + d*x])^4) - (5*Sec[c + d*x])/(231*a*d*(a + a*Sin[c + d*x])^3) + Sec
[c + d*x]^3/(6*a*d*(a + a*Sin[c + d*x])^3) - (4*Sec[c + d*x])/(231*d*(a^2 +
 a^2*Sin[c + d*x])^2) - (4*Sec[c + d*x])/(231*d*(a^4 + a^4*Sin[c + d*x])) +
 (8*Tan[c + d*x])/(231*a^4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2870

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
```

```
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(2*b*f*g*(m + 1)), x] + Dist[a/(2*g^2)
, Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\sec^3(c+dx)}{6ad(a+a\sin(c+dx))^3} + \frac{1}{2}a\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^5}\,dx$$
$$= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} + \frac{\sec^3(c+dx)}{6ad(a+a\sin(c+dx))^3} + \frac{3}{11}\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx)}$$
$$= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} + \frac{\sec^3(c+dx)}{6ad(a+a\sin(c+dx))^3}$$
$$= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+dx)}{231ad(a+a\sin(c+dx)}$$
$$= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+dx)}{231ad(a+a\sin(c+dx)}$$
$$= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+dx)}{231ad(a+a\sin(c+dx)}$$
$$= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+dx)}{231ad(a+a\sin(c+dx)}$$
$$= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+dx)}{231ad(a+a\sin(c+dx)}$$

Mathematica [A] time = 0.49, size = 166, normalized size = 1.16

$$\frac{\sec^3(c+dx)(26048\sin(c+dx) - 1144\sin(2(c+dx)) - 704\sin(3(c+dx)) - 416\sin(4(c+dx)) - 1600\sin(5(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*(11264 - 1287*Cos[c + d*x] - 5632*Cos[2*(c + d*x)] + 143*Co
s[3*(c + d*x)] - 2048*Cos[4*(c + d*x)] + 325*Cos[5*(c + d*x)] + 512*Cos[6*(
c + d*x)] - 13*Cos[7*(c + d*x)] + 26048*Sin[c + d*x] - 1144*Sin[2*(c + d*x)
] - 704*Sin[3*(c + d*x)] - 416*Sin[4*(c + d*x)] - 1600*Sin[5*(c + d*x)] + 1
04*Sin[6*(c + d*x)] + 64*Sin[7*(c + d*x)]))/(118272*a^4*d*(1 + Sin[c + d*x]
)^4)
```

fricas [A] time = 0.45, size = 153, normalized size = 1.07

$$\frac{32\cos(dx+c)^6-80\cos(dx+c)^4+28\cos(dx+c)^2+\left(8\cos(dx+c)^6-60\cos(dx+c)^4+35\cos(dx+c)^2\right.}{231\left(a^4d\cos(dx+c)^7-8a^4d\cos(dx+c)^5+8a^4d\cos(dx+c)^3-4\left(a^4d\cos(dx+c)^5-2a^4d\cos(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/231*(32*cos(d*x + c)^6 - 80*cos(d*x + c)^4 + 28*cos(d*x + c)^2 + (8*cos(d
*x + c)^6 - 60*cos(d*x + c)^4 + 35*cos(d*x + c)^2 + 49)*sin(d*x + c) + 28)/
(a^4*d*cos(d*x + c)^7 - 8*a^4*d*cos(d*x + c)^5 + 8*a^4*d*cos(d*x + c)^3 - 4
*(a^4*d*cos(d*x + c)^5 - 2*a^4*d*cos(d*x + c)^3)*sin(d*x + c))
```

giac [A] time = 0.40, size = 198, normalized size = 1.38

$$-\frac{\frac{77\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5\right)}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{462\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+5775\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+14399\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+29260\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+308}{}}{7392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/7392*(77*(6*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 5)/(a^4*(t
an(1/2*d*x + 1/2*c) - 1)^3) - (462*tan(1/2*d*x + 1/2*c)^10 + 5775*tan(1/2*d
*x + 1/2*c)^9 + 14399*tan(1/2*d*x + 1/2*c)^8 + 29260*tan(1/2*d*x + 1/2*c)^7
 + 30800*tan(1/2*d*x + 1/2*c)^6 + 27874*tan(1/2*d*x + 1/2*c)^5 + 12650*tan(
1/2*d*x + 1/2*c)^4 + 6556*tan(1/2*d*x + 1/2*c)^3 + 1210*tan(1/2*d*x + 1/2*c
)^2 + 935*tan(1/2*d*x + 1/2*c) + 127)/(a^4*(tan(1/2*d*x + 1/2*c) + 1)^11))/
d
```

maple [A] time = 0.73, size = 218, normalized size = 1.52

$$-\frac{1}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{16}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{10}}-\frac{64}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}+\frac{}{(\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 8/d/a^4*(-1/384/(tan(1/2*d*x+1/2*c)-1)^3-1/256/(tan(1/2*d*x+1/2*c)-1)^2-1/1
28/(tan(1/2*d*x+1/2*c)-1)-2/11/(tan(1/2*d*x+1/2*c)+1)^11+1/(tan(1/2*d*x+1/2
*c)+1)^10-8/3/(tan(1/2*d*x+1/2*c)+1)^9+9/2/(tan(1/2*d*x+1/2*c)+1)^8-295/56/
(tan(1/2*d*x+1/2*c)+1)^7+71/16/(tan(1/2*d*x+1/2*c)+1)^6-43/16/(tan(1/2*d*x+
1/2*c)+1)^5+9/8/(tan(1/2*d*x+1/2*c)+1)^4-109/384/(tan(1/2*d*x+1/2*c)+1)^3+5
/256/(tan(1/2*d*x+1/2*c)+1)^2+1/128/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.35, size = 528, normalized size = 3.69

$$\frac{8\left(\frac{16\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{50\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{141\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{132\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{132\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{44\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\cdots\right.}{231\left(a^4+\frac{8\,a^4\sin(dx+c)}{\cos(dx+c)+1}+\frac{25\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{32\,a^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{11\,a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{88\,a^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{99\,a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{99\,a^4\sin(dx+c}{(\cos(dx+c)+1)}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 8/231*(16*sin(d*x + c)/(cos(d*x + c) + 1) + 50*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 141*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 132*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 132*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 44*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 + 110*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 154*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 308*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
 + 154*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 77*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 + 2)/((a^4 + 8*a^4*sin(d*x + c)/(cos(d*x + c) + 1) + 25*a^4*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 32*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 - 11*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 88*a^4*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 - 99*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 99*a^4*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 88*a^4*sin(d*x + c)^9/(cos(d*x + c) + 1
)^9 + 11*a^4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 32*a^4*sin(d*x + c)^11
/(cos(d*x + c) + 1)^11 - 25*a^4*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a
^4*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - a^4*sin(d*x + c)^14/(cos(d*x + c
) + 1)^14)*d)
```

mupad [B] time = 16.00, size = 327, normalized size = 2.29

$$\frac{\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}}{231}+\frac{128\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{231}+\frac{400\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{231}+\frac{376\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{77}+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}\cdots\right.}{7}\cdots}{a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^4*(a + a*sin(c + d*x))^4),x)
```

```
[Out] ((16*cos(c/2 + (d*x)/2)^14)/231 + (128*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x
)/2))/231 + (8*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^11)/3 + (16*cos(c/2
+ (d*x)/2)^4*sin(c/2 + (d*x)/2)^10)/3 + (32*cos(c/2 + (d*x)/2)^5*sin(c/2 +
(d*x)/2)^9)/3 + (16*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8)/3 + (80*cos(
c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)/21 - (32*cos(c/2 + (d*x)/2)^8*sin(c/
2 + (d*x)/2)^6)/21 + (32*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5)/7 + (32
*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^4)/7 + (376*cos(c/2 + (d*x)/2)^11
*sin(c/2 + (d*x)/2)^3)/77 + (400*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2
)/231)/(a^4*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/
2) + sin(c/2 + (d*x)/2))^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.851 $\int \sin(c+dx)(a+a\sin(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=133

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{5a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{39a\log(1-s}{16}$$

```
[Out] -39/16*a*ln(1-sin(d*x+c))/d-9/16*a*ln(1+sin(d*x+c))/d-a*sin(d*x+c)/d-1/2*a*
sin(d*x+c)^2/d+1/8*a^3/d/(a-a*sin(d*x+c))^2-5/4*a^2/d/(a-a*sin(d*x+c))-1/8*
a^2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2836, 12, 88}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{5a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{39a\log(1-s}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (-39*a*Log[1 - Sin[c + d*x]])/(16*d) - (9*a*Log[1 + Sin[c + d*x]])/(16*d) -
 (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d) + a^3/(8*d*(a - a*Sin[c + d*
x])^2) - (5*a^2)/(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x])
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))\tan^5(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x^6}{a^6(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^6}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-a+\frac{a^4}{4(a-x)^3}-\frac{5a^3}{4(a-x)^2}+\frac{39a^2}{16(a-x)}-x+\frac{a^3}{8(a+x)^2}-\frac{9a}{16(a}\right.\right.}{ad}$$
$$= -\frac{39a\log(1-\sin(c+dx))}{16d}-\frac{9a\log(1+\sin(c+dx))}{16d}-\frac{a\sin}{}$$

Mathematica [A] time = 0.45, size = 133, normalized size = 1.00

$$-\frac{a\sin(c+dx)\tan^4(c+dx)}{d}-\frac{a\left(2\sin^2(c+dx)-\sec^4(c+dx)+6\sec^2(c+dx)+12\log(\cos(c+dx))\right)}{4d}-\frac{5a\left(6\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -1/4*(a*(12*Log[Cos[c + d*x]] + 6*Sec[c + d*x]^2 - Sec[c + d*x]^4 + 2*Sin[c
 + d*x]^2))/d - (a*Sin[c + d*x]*Tan[c + d*x]^4)/d - (5*a*(6*Sec[c + d*x]^3*
Tan[c + d*x] - 8*Sec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + S
ec[c + d*x]*Tan[c + d*x])))/(8*d)
```

fricas [A] time = 0.49, size = 172, normalized size = 1.29

$$\frac{8\,a\cos(dx+c)^4+6\,a\cos(dx+c)^2-9\left(a\cos(dx+c)^2\sin(dx+c)-a\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-39}{16\left(d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*a*cos(d*x + c)^4 + 6*a*cos(d*x + c)^2 - 9*(a*cos(d*x + c)^2*sin(d*x
 + c) - a*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 39*(a*cos(d*x + c)^2*sin(
d*x + c) - a*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^4
```

```
 + 6*a*cos(d*x + c)^2 - 3*a)*sin(d*x + c) + 2*a)/(d*cos(d*x + c)^2*sin(d*x
+ c) - d*cos(d*x + c)^2)
```

giac [A] time = 0.26, size = 113, normalized size = 0.85

$$-\frac{16\,a\sin\left(dx+c\right)^{2}+18\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+78\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+32\,a\sin\left(dx+c\right)-\frac{2\,(9\,a\sin(dx+c)+7}{\sin(dx+c)+1}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(16*a*sin(d*x + c)^2 + 18*a*log(abs(sin(d*x + c) + 1)) + 78*a*log(abs
(sin(d*x + c) - 1)) + 32*a*sin(d*x + c) - 2*(9*a*sin(d*x + c) + 7*a)/(sin(d
*x + c) + 1) - (117*a*sin(d*x + c)^2 - 194*a*sin(d*x + c) + 81*a)/(sin(d*x
+ c) - 1)^2)/d
```

maple [A] time = 0.28, size = 205, normalized size = 1.54

$$\frac{a\left(\sin^{8}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}-\frac{a\left(\sin^{8}\left(dx+c\right)\right)}{2d\cos\left(dx+c\right)^{2}}-\frac{a\left(\sin^{6}\left(dx+c\right)\right)}{2d}-\frac{3a\left(\sin^{4}\left(dx+c\right)\right)}{4d}-\frac{3a\left(\sin^{2}\left(dx+c\right)\right)}{2d}-\frac{3a\ln\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*a*sin(d*x+c)^8/cos(d*x+c)^2-1/2*a*s
in(d*x+c)^6/d-3/4*a*sin(d*x+c)^4/d-3/2*a*sin(d*x+c)^2/d-3*a*ln(cos(d*x+c))/
d+1/4/d*a*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a*sin(d*x+c)^7/cos(d*x+c)^2-3/8*a
*sin(d*x+c)^5/d-5/8*a*sin(d*x+c)^3/d-15/8*a*sin(d*x+c)/d+15/8/d*a*ln(sec(d*
x+c)+tan(d*x+c))
```

maxima [A] time = 0.32, size = 106, normalized size = 0.80

$$-\frac{8\,a\sin\left(dx+c\right)^{2}+9\,a\log\left(\sin\left(dx+c\right)+1\right)+39\,a\log\left(\sin\left(dx+c\right)-1\right)+16\,a\sin\left(dx+c\right)-\frac{2\left(9\,a\sin(dx+c)^{2}+3\,a\sin}{\sin(dx+c)^{3}-\sin(dx+c)^{2}-}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(8*a*sin(d*x + c)^2 + 9*a*log(sin(d*x + c) + 1) + 39*a*log(sin(d*x +
c) - 1) + 16*a*sin(d*x + c) - 2*(9*a*sin(d*x + c)^2 + 3*a*sin(d*x + c) - 10
*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x + c) + 1))/d
```

mupad [B] time = 9.82, size = 286, normalized size = 2.15

$$\frac{-\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}+7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2}-\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+7\,a\tan\left(\frac{c}{2}+\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^6*(a + a*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] ((3*a*tan(c/2 + (d*x)/2)^2)/2 - (15*a*tan(c/2 + (d*x)/2))/4 + 7*a*tan(c/2 +
 (d*x)/2)^3 - (7*a*tan(c/2 + (d*x)/2)^4)/2 + (11*a*tan(c/2 + (d*x)/2)^5)/2
- (7*a*tan(c/2 + (d*x)/2)^6)/2 + 7*a*tan(c/2 + (d*x)/2)^7 + (3*a*tan(c/2 +
(d*x)/2)^8)/2 - (15*a*tan(c/2 + (d*x)/2)^9)/4)/(d*(tan(c/2 + (d*x)/2)^2 - 2
*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^5 - 2*t
an(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 - 2*tan(c/2 + (d*x)/2)^9 + tan(c
/2 + (d*x)/2)^10 + 1)) - (9*a*log(tan(c/2 + (d*x)/2) + 1))/(8*d) - (39*a*lo
g(tan(c/2 + (d*x)/2) - 1))/(8*d) + (3*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.852 $\int (a+a\sin(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=115

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\sin(c+dx)}{d}-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{7a\,]}{}$$

```
[Out] -23/16*a*ln(1-sin(d*x+c))/d+7/16*a*ln(1+sin(d*x+c))/d-a*sin(d*x+c)/d+1/8*a^
3/d/(a-a*sin(d*x+c))^2-a^2/d/(a-a*sin(d*x+c))+1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 88}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\sin(c+dx)}{d}-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{7a\,]}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (-23*a*Log[1 - Sin[c + d*x]])/(16*d) + (7*a*Log[1 + Sin[c + d*x]])/(16*d) -
 (a*Sin[c + d*x])/d + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/(d*(a - a*Sin[
c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^3}{4(a-x)^3}-\frac{a^2}{(a-x)^2}+\frac{23a}{16(a-x)}-\frac{a^2}{8(a+x)^2}+\frac{7a}{16(a+x)}\right)dx, x, a\sin(\right.}{d}$$

$$= -\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{7a\log(1+\sin(c+dx))}{16d}-\frac{a\sin(c+dx)}{d}+$$

Mathematica [A] time = 0.27, size = 123, normalized size = 1.07

$$-\frac{a\sin(c+dx)\tan^4(c+dx)}{d}-\frac{a\left(-\tan^4(c+dx)+2\tan^2(c+dx)+4\log(\cos(c+dx))\right)}{4d}-\frac{5a\left(6\tan(c+dx)\sec^3(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((a*Sin[c + d*x]*Tan[c + d*x]^4)/d) - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c +
d*x]^2 - Tan[c + d*x]^4))/(4*d) - (5*a*(6*Sec[c + d*x]^3*Tan[c + d*x] - 8*S
ec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c
+ d*x])))/(8*d)
```

fricas [A] time = 0.47, size = 159, normalized size = 1.38

$$\frac{16\,a\cos(dx+c)^4+2\,a\cos(dx+c)^2+7\left(a\cos(dx+c)^2\sin(dx+c)-a\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-2}{16\left(d\cos(dx+c)^2\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*cos(d*x + c)^4 + 2*a*cos(d*x + c)^2 + 7*(a*cos(d*x + c)^2*sin(d*
x + c) - a*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 23*(a*cos(d*x + c)^2*sin
(d*x + c) - a*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(8*a*cos(d*x + c)^
2 + a)*sin(d*x + c) - 6*a)/(d*cos(d*x + c)^2*sin(d*x + c) - d*cos(d*x + c)^
2)
```

giac [A] time = 0.25, size = 101, normalized size = 0.88

$$\frac{14\,a\log(|\sin(dx+c)+1|)-46\,a\log(|\sin(dx+c)-1|)-32\,a\sin(dx+c)-\frac{2(7\,a\sin(dx+c)+5\,a)}{\sin(dx+c)+1}+\frac{69\,a\sin(dx+c)^2-106}{(\sin(dx+}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(14*a*log(abs(sin(d*x + c) + 1)) - 46*a*log(abs(sin(d*x + c) - 1)) - 3
2*a*sin(d*x + c) - 2*(7*a*sin(d*x + c) + 5*a)/(sin(d*x + c) + 1) + (69*a*si
n(d*x + c)^2 - 106*a*sin(d*x + c) + 41*a)/(sin(d*x + c) - 1)^2)/d
```

maple [A] time = 0.27, size = 147, normalized size = 1.28

$$\frac{a\left(\sin^{7}(dx+c)\right)}{4d\cos(dx+c)^{4}}-\frac{3a\left(\sin^{7}(dx+c)\right)}{8d\cos(dx+c)^{2}}-\frac{3a\left(\sin^{5}(dx+c)\right)}{8d}-\frac{5a\left(\sin^{3}(dx+c)\right)}{8d}-\frac{15a\sin(dx+c)}{8d}+\frac{15a\ln(\sec(dx+c}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a*sin(d*x+c)^7/cos(d*x+c)^2-3/8*a*s
in(d*x+c)^5/d-5/8*a*sin(d*x+c)^3/d-15/8*a*sin(d*x+c)/d+15/8/d*a*ln(sec(d*x+
c)+tan(d*x+c))+1/4*a*tan(d*x+c)^4/d-1/2*a*tan(d*x+c)^2/d-a*ln(cos(d*x+c))/d
```

maxima [A] time = 0.31, size = 95, normalized size = 0.83

$$\frac{7\,a\log(\sin(dx+c)+1)-23\,a\log(\sin(dx+c)-1)-16\,a\sin(dx+c)+\frac{2\left(9\,a\sin(dx+c)^{2}-a\sin(dx+c)-6\,a\right)}{\sin(dx+c)^{3}-\sin(dx+c)^{2}-\sin(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(7*a*log(sin(d*x + c) + 1) - 23*a*log(sin(d*x + c) - 1) - 16*a*sin(d*x
 + c) + 2*(9*a*sin(d*x + c)^2 - a*sin(d*x + c) - 6*a)/(sin(d*x + c)^3 - sin
(d*x + c)^2 - sin(d*x + c) + 1))/d
```

mupad [B] time = 9.29, size = 235, normalized size = 2.04

$$-\frac{\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{2}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{4}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{2}-\frac{15\,a\tan\left(\frac{c}{2}+\frac{d}{}\right.}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^5*(a + a*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] ((11*a*tan(c/2 + (d*x)/2)^2)/2 - (15*a*tan(c/2 + (d*x)/2))/4 + (11*a*tan(c/
2 + (d*x)/2)^3)/4 - 5*a*tan(c/2 + (d*x)/2)^4 + (11*a*tan(c/2 + (d*x)/2)^5)/
4 + (11*a*tan(c/2 + (d*x)/2)^6)/2 - (15*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(2*ta
n(c/2 + (d*x)/2)^3 - 2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^4 + 2*tan(
c/2 + (d*x)/2)^5 - 2*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8 + 1)) - (2
3*a*log(tan(c/2 + (d*x)/2) - 1))/(8*d) + (7*a*log(tan(c/2 + (d*x)/2) + 1))/
(8*d) + (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.853 $\int \sec(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=105

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{3a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{11a\log(1-\sin(c+dx))}{16d}-\frac{5a\log(\sin(c+dx$$

```
[Out] -11/16*a*ln(1-sin(d*x+c))/d-5/16*a*ln(1+sin(d*x+c))/d+1/8*a^3/d/(a-a*sin(d*
x+c))^2-3/4*a^2/d/(a-a*sin(d*x+c))-1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2836, 12, 88}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{3a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{11a\log(1-\sin(c+dx))}{16d}-\frac{5a\log(\sin(c+dx$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (-11*a*Log[1 - Sin[c + d*x]])/(16*d) - (5*a*Log[1 + Sin[c + d*x]])/(16*d) +
 a^3/(8*d*(a - a*Sin[c + d*x])^2) - (3*a^2)/(4*d*(a - a*Sin[c + d*x])) - a^
2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x^4}{a^4(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{a^2}{4(a-x)^3} - \frac{3a}{4(a-x)^2} + \frac{11}{16(a-x)} + \frac{a}{8(a+x)^2} - \frac{5}{16(a+x)}\right) dx \right.}{d}$$
$$= -\frac{11a\log(1-\sin(c+dx))}{16d} - \frac{5a\log(1+\sin(c+dx))}{16d} + \frac{}{8d(a}$$

Mathematica [A] time = 0.24, size = 106, normalized size = 1.01

$$\frac{a\tan^3(c+dx)\sec(c+dx)}{d} - \frac{a\left(-\tan^4(c+dx)+2\tan^2(c+dx)+4\log(\cos(c+dx))\right)}{4d} - \frac{a\left(6\tan(c+dx)\sec^3(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*Sec[c + d*x]*Tan[c + d*x]^3)/d - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x
]^2 - Tan[c + d*x]^4))/(4*d) - (a*(6*Sec[c + d*x]^3*Tan[c + d*x] - 3*(ArcTa
nh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x])))/(8*d)
```

fricas [A] time = 0.48, size = 136, normalized size = 1.30

$$\frac{10\,a\cos(dx+c)^2 - 5\left(a\cos(dx+c)^2\sin(dx+c) - a\cos(dx+c)^2\right)\log(\sin(dx+c)+1) - 11\left(a\cos(dx+c)^2 \text{s}\right.}{16\left(d\cos(dx+c)^2\sin(dx+c) - d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(10*a*cos(d*x + c)^2 - 5*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x +
c)^2)*log(sin(d*x + c) + 1) - 11*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x
 + c)^2)*log(-sin(d*x + c) + 1) - 6*a*sin(d*x + c) + 2*a)/(d*cos(d*x + c)^2
*sin(d*x + c) - d*cos(d*x + c)^2)
```

giac [A] time = 0.23, size = 93, normalized size = 0.89

$$-\frac{10\,a\log(|\sin(dx+c)+1|) + 22\,a\log(|\sin(dx+c)-1|) - \frac{2(5\,a\sin(dx+c)+3\,a)}{\sin(dx+c)+1} - \frac{33\,a\sin(dx+c)^2 - 42\,a\sin(dx+c)+13\,a}{(\sin(dx+c)-1)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(10*a*log(abs(sin(d*x + c) + 1)) + 22*a*log(abs(sin(d*x + c) - 1)) -
2*(5*a*sin(d*x + c) + 3*a)/(sin(d*x + c) + 1) - (33*a*sin(d*x + c)^2 - 42*a
*sin(d*x + c) + 13*a)/(sin(d*x + c) - 1)^2)/d
```

maple [A] time = 0.25, size = 133, normalized size = 1.27

$$\frac{a\left(\tan^4(dx+c)\right)}{4d}-\frac{a\left(\tan^2(dx+c)\right)}{2d}-\frac{a\ln(\cos(dx+c))}{d}+\frac{a\left(\sin^5(dx+c)\right)}{4d\cos(dx+c)^4}-\frac{a\left(\sin^5(dx+c)\right)}{8d\cos(dx+c)^2}-\frac{a\left(\sin^3(dx+c)\right)}{8d}-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4*a*tan(d*x+c)^4/d-1/2*a*tan(d*x+c)^2/d-a*ln(cos(d*x+c))/d+1/4/d*a*sin(d*
x+c)^5/cos(d*x+c)^4-1/8/d*a*sin(d*x+c)^5/cos(d*x+c)^2-1/8*a*sin(d*x+c)^3/d-
3/8*a*sin(d*x+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.31, size = 86, normalized size = 0.82

$$-\frac{5\,a\log(\sin(dx+c)+1)+11\,a\log(\sin(dx+c)-1)-\frac{2\left(5\,a\sin(dx+c)^2+3\,a\sin(dx+c)-6\,a\right)}{\sin(dx+c)^3-\sin(dx+c)^2-\sin(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(5*a*log(sin(d*x + c) + 1) + 11*a*log(sin(d*x + c) - 1) - 2*(5*a*sin(
d*x + c)^2 + 3*a*sin(d*x + c) - 6*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin
(d*x + c) + 1))/d
```

mupad [B] time = 9.26, size = 205, normalized size = 1.95

$$\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{5\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{8\,d}-\frac{11\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{8\,d}+\frac{\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{4}}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (5*a*log(tan(c/2 + (d*x)/2) + 1))/(8*
d) - (11*a*log(tan(c/2 + (d*x)/2) - 1))/(8*d) + ((3*a*tan(c/2 + (d*x)/2))/4
 + (a*tan(c/2 + (d*x)/2)^2)/2 - (9*a*tan(c/2 + (d*x)/2)^3)/2 + (a*tan(c/2 +
 (d*x)/2)^4)/2 + (3*a*tan(c/2 + (d*x)/2)^5)/4)/(d*(2*tan(c/2 + (d*x)/2) + t
an(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4 + 2*tan
(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.854 $\int \sec^2(c+dx)(a+a\sin(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=84

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+1/8*a^3/d/(a-a*sin(d*x+c))^2-1/2*a^2/d/(a-a*sin
(d*x+c))+1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2836, 12, 88, 206}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/
(2*d*(a - a*Sin[c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))\tan^3(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x^3}{a^3(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{x^3}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{a}{4(a-x)^3} - \frac{1}{2(a-x)^2} - \frac{1}{8(a+x)^2} + \frac{3}{8\left(a^2-x^2\right)}\right) dx, x, a\,\text{si}}{d}$$
$$= \frac{a^3}{8d(a-a\sin(c+dx))^2} - \frac{a^2}{2d(a-a\sin(c+dx))} + \frac{}{8d(a+a\,\text{s}}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3}{8d(a-a\sin(c+dx))^2} - \frac{}{2d(a-}$$

Mathematica [A] time = 0.21, size = 84, normalized size = 1.00

$$\frac{a\tan^4(c+dx)}{4d}+\frac{a\tan^3(c+dx)\sec(c+dx)}{d}-\frac{a\left(6\tan(c+dx)\sec^3(c+dx)-3\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (a*Sec[c + d*x]*Tan[c + d*x]^3)/d + (a*Tan[c + d*x]^4)/(4*d) - (a*(6*Sec[c
+ d*x]^3*Tan[c + d*x] - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x
])))/(8*d)
```

fricas [A] time = 0.46, size = 136, normalized size = 1.62

$$\frac{10\,a\cos(dx+c)^2 + 3\left(a\cos(dx+c)^2\sin(dx+c) - a\cos(dx+c)^2\right)\log(\sin(dx+c)+1) - 3\left(a\cos(dx+c)^2\,\text{si}\right.}{16\left(d\cos(dx+c)^2\sin(dx+c) - d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(10*a*cos(d*x + c)^2 + 3*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x +
c)^2)*log(sin(d*x + c) + 1) - 3*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x
+ c)^2)*log(-sin(d*x + c) + 1) + 2*a*sin(d*x + c) - 6*a)/(d*cos(d*x + c)^2*
sin(d*x + c) - d*cos(d*x + c)^2)
```

giac [A] time = 0.22, size = 90, normalized size = 1.07

$$\frac{6\,a\log\left(|\sin\left(dx+c\right)+1|\right)-6\,a\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\,(3\,a\sin(dx+c)+a)}{\sin(dx+c)+1}+\frac{9\,a\sin(dx+c)^{2}-2\,a\sin(dx+c)-3\,a}{(\sin(dx+c)-1)^{2}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(6*a*log(abs(sin(d*x + c) + 1)) - 6*a*log(abs(sin(d*x + c) - 1)) - 2*(
3*a*sin(d*x + c) + a)/(sin(d*x + c) + 1) + (9*a*sin(d*x + c)^2 - 2*a*sin(d*
x + c) - 3*a)/(sin(d*x + c) - 1)^2)/d
```

maple [A] time = 0.24, size = 114, normalized size = 1.36

$$\frac{a\left(\sin^{5}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}-\frac{a\left(\sin^{5}\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^{2}}-\frac{a\left(\sin^{3}\left(dx+c\right)\right)}{8d}-\frac{3a\sin\left(dx+c\right)}{8d}+\frac{3a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{a\left(\sin\right.}{4d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a*sin(d*x+c)^5/cos(d*x+c)^2-1/8*a*s
in(d*x+c)^3/d-3/8*a*sin(d*x+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a*
sin(d*x+c)^4/cos(d*x+c)^4
```

maxima [A] time = 0.31, size = 86, normalized size = 1.02

$$\frac{3\,a\log\left(\sin\left(dx+c\right)+1\right)-3\,a\log\left(\sin\left(dx+c\right)-1\right)+\frac{2\left(5\,a\sin(dx+c)^{2}-a\sin(dx+c)-2\,a\right)}{\sin(dx+c)^{3}-\sin(dx+c)^{2}-\sin(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(3*a*log(sin(d*x + c) + 1) - 3*a*log(sin(d*x + c) - 1) + 2*(5*a*sin(d*
x + c)^2 - a*sin(d*x + c) - 2*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x
 + c) + 1))/d
```

mupad [B] time = 14.46, size = 167, normalized size = 1.99

$$\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}-\frac{3\,a\,\mathrm{ta}\cdots}{\cdots}}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((3*a*tan(c/2 + (d*x)/2)^2)/2 - (3*
a*tan(c/2 + (d*x)/2))/4 + (a*tan(c/2 + (d*x)/2)^3)/2 + (3*a*tan(c/2 + (d*x)
/2)^4)/2 - (3*a*tan(c/2 + (d*x)/2)^5)/4)/(d*(2*tan(c/2 + (d*x)/2) + tan(c/2
 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4 + 2*tan(c/2 +
 (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.855 $\int \sec^3(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=84

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] -1/8*a*arctanh(sin(d*x+c))/d+1/8*a^3/d/(a-a*sin(d*x+c))^2-1/4*a^2/d/(a-a*si
n(d*x+c))-1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2836, 12, 88, 206}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(8*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/(
4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x^2}{a^2(a-x)^3(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{x^2}{(a-x)^3(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{4(a-x)^3}-\frac{1}{4a(a-x)^2}+\frac{1}{8a(a+x)^2}-\frac{1}{8a\left(a^2-x^2\right)}\right)dx,x,\right.}{d}$$
$$= \frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{}{8d(a+a\,\text{s}}$$
$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{}{4d(a-}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 0.88

$$\frac{a\tan^4(c+dx)}{4d}-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{a\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -1/8*(a*ArcTanh[Sin[c + d*x]])/d - (a*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*x]^4)/(4*d)
```

fricas [A] time = 0.47, size = 135, normalized size = 1.61

$$\frac{2\,a\cos(dx+c)^2-\left(a\cos(dx+c)^2\sin(dx+c)-a\cos(dx+c)^2\right)\log(\sin(dx+c)+1)+\left(a\cos(dx+c)^2\sin(dx\right.}{16\left(d\cos(dx+c)^2\sin(dx+c)-d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^
2)*log(sin(d*x + c) + 1) + (a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^
2)*log(-sin(d*x + c) + 1) - 6*a*sin(d*x + c) + 2*a)/(d*cos(d*x + c)^2*sin(d
*x + c) - d*cos(d*x + c)^2)
```

giac [A] time = 0.22, size = 91, normalized size = 1.08

$$-\frac{2\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-2\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,\left(a\sin\left(dx+c\right)-a\right)}{\sin\left(dx+c\right)+1}+\frac{3\,a\sin\left(dx+c\right)^{2}-14\,a\sin\left(dx+c\right)+7\,a}{\left(\sin\left(dx+c\right)-1\right)^{2}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(2*a*log(abs(sin(d*x + c) + 1)) - 2*a*log(abs(sin(d*x + c) - 1)) - 2*
(a*sin(d*x + c) - a)/(sin(d*x + c) + 1) + (3*a*sin(d*x + c)^2 - 14*a*sin(d*
x + c) + 7*a)/(sin(d*x + c) - 1)^2)/d
```

maple [A] time = 0.24, size = 100, normalized size = 1.19

$$\frac{a\left(\sin^{4}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}+\frac{a\left(\sin^{3}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}+\frac{a\left(\sin^{3}\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^{2}}+\frac{a\sin\left(dx+c\right)}{8d}-\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^4/cos(d*x+c)^4+1/4/d*a*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a
*sin(d*x+c)^3/cos(d*x+c)^2+1/8*a*sin(d*x+c)/d-1/8/d*a*ln(sec(d*x+c)+tan(d*x
+c))
```

maxima [A] time = 0.31, size = 84, normalized size = 1.00

$$-\frac{a\log\left(\sin\left(dx+c\right)+1\right)-a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\,\left(a\sin\left(dx+c\right)^{2}+3\,a\sin\left(dx+c\right)-2\,a\right)}{\sin\left(dx+c\right)^{3}-\sin\left(dx+c\right)^{2}-\sin\left(dx+c\right)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*(a*sin(d*x + c
)^2 + 3*a*sin(d*x + c) - 2*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x +
c) + 1))/d
```

mupad [B] time = 14.46, size = 167, normalized size = 1.99

$$-\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{a\tan\left(\frac{c}{2}\right.}{4}}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] - (a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a*tan(c/2 + (d*x)/2))/4 - (a*tan(
c/2 + (d*x)/2)^2)/2 + (5*a*tan(c/2 + (d*x)/2)^3)/2 - (a*tan(c/2 + (d*x)/2)^
4)/2 + (a*tan(c/2 + (d*x)/2)^5)/4)/(d*(2*tan(c/2 + (d*x)/2) + tan(c/2 + (d*
x)/2)^2 - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4 + 2*tan(c/2 + (d*x)
/2)^5 - tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.856 $\int \sec^4(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=61

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] -1/8*a*arctanh(sin(d*x+c))/d+1/8*a^3/d/(a-a*sin(d*x+c))^2+1/8*a^2/d/(a+a*si
n(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2836, 12, 77, 206}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(8*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/(
8*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x}{a(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^4\,\text{Subst}\left(\int \frac{x}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^4\,\text{Subst}\left(\int \left(\frac{1}{4a(a-x)^3} - \frac{1}{8a^2(a+x)^2} - \frac{1}{8a^2\left(a^2-x^2\right)}\right) dx, x, a\sin(c+\right.}{d}$$

$$= \frac{a^3}{8d(a-a\sin(c+dx))^2} + \frac{a^2}{8d(a+a\sin(c+dx))} - \frac{a^2\,\text{Subst}\left(\right.}{}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3}{8d(a-a\sin(c+dx))^2} + \frac{}{8d(a+a}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 1.21

$$\frac{a\sec^4(c+dx)}{4d} - \frac{a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)\sec^3(c+dx)}{4d} - \frac{a\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -1/8*(a*ArcTanh[Sin[c + d*x]])/d + (a*Sec[c + d*x]^4)/(4*d) - (a*Sec[c + d*
x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

fricas [B] time = 0.47, size = 135, normalized size = 2.21

$$\frac{2\,a\cos(dx+c)^2 - \left(a\cos(dx+c)^2\sin(dx+c) - a\cos(dx+c)^2\right)\log(\sin(dx+c)+1) + \left(a\cos(dx+c)^2\sin(dx\right.}{16\left(d\cos(dx+c)^2\sin(dx+c) - d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^
2)*log(sin(d*x + c) + 1) + (a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^
2)*log(-sin(d*x + c) + 1) + 2*a*sin(d*x + c) - 6*a)/(d*cos(d*x + c)^2*sin(d
*x + c) - d*cos(d*x + c)^2)
```

giac [A] time = 0.20, size = 91, normalized size = 1.49

$$-\frac{2\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-2\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,(a\sin(dx+c)+3\,a)}{\sin(dx+c)+1}+\frac{3\,a\sin(dx+c)^2-6\,a\sin(dx+c)-a}{(\sin(dx+c)-1)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(2*a*log(abs(sin(d*x + c) + 1)) - 2*a*log(abs(sin(d*x + c) - 1)) - 2*
(a*sin(d*x + c) + 3*a)/(sin(d*x + c) + 1) + (3*a*sin(d*x + c)^2 - 6*a*sin(d
*x + c) - a)/(sin(d*x + c) - 1)^2)/d
```

maple [A] time = 0.18, size = 92, normalized size = 1.51

$$\frac{a\left(\sin^3\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4}+\frac{a\left(\sin^3\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^2}+\frac{a\sin\left(dx+c\right)}{8d}-\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{a}{4d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a*sin(d*x+c)^3/cos(d*x+c)^2+1/8*a*s
in(d*x+c)/d-1/8/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a/cos(d*x+c)^4
```

maxima [A] time = 0.31, size = 84, normalized size = 1.38

$$-\frac{a\log\left(\sin\left(dx+c\right)+1\right)-a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(a\sin(dx+c)^2-a\sin(dx+c)+2\,a\right)}{\sin(dx+c)^3-\sin(dx+c)^2-\sin(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*(a*sin(d*x + c
)^2 - a*sin(d*x + c) + 2*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x + c)
 + 1))/d
```

mupad [B] time = 14.59, size = 167, normalized size = 2.74

$$-\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{a\tan(}{}}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] - (a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a*tan(c/2 + (d*x)/2))/4 + (3*a*ta
n(c/2 + (d*x)/2)^2)/2 - (3*a*tan(c/2 + (d*x)/2)^3)/2 + (3*a*tan(c/2 + (d*x)
/2)^4)/2 + (a*tan(c/2 + (d*x)/2)^5)/4)/(d*(2*tan(c/2 + (d*x)/2) + tan(c/2 +
 (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4 + 2*tan(c/2 + (
d*x)/2)^5 - tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.857 $\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=117

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{11a\log(1-\sin(c+dx))}{16d}+\frac{a\log(\sin(c+dx)}{d}$$

```
[Out] -11/16*a*ln(1-sin(d*x+c))/d+a*ln(sin(d*x+c))/d-5/16*a*ln(1+sin(d*x+c))/d+1/
8*a^3/d/(a-a*sin(d*x+c))^2+1/2*a^2/d/(a-a*sin(d*x+c))+1/8*a^2/d/(a+a*sin(d*
x+c))
```

Rubi [A] time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2836, 12, 88}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{11a\log(1-\sin(c+dx))}{16d}+\frac{a\log(\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-11*a*Log[1 - Sin[c + d*x]])/(16*d) + (a*Log[Sin[c + d*x]])/d - (5*a*Log[1
 + Sin[c + d*x]])/(16*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/(2*d*(a -
 a*Sin[c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a}{(a-x)^3x(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{1}{(a-x)^3x(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(\frac{1}{4a^3(a-x)^3} + \frac{1}{2a^4(a-x)^2} + \frac{11}{16a^5(a-x)} + \frac{1}{a^5x} - \frac{1}{8a^4(a+x)^2}\right.\right.}{d}$$
$$= -\frac{11a\log(1-\sin(c+dx))}{16d} + \frac{a\log(\sin(c+dx))}{d} - \frac{5a\log(1+}{1}$$

Mathematica [A] time = 0.22, size = 99, normalized size = 0.85

$$\frac{a\tan(c+dx)\sec^3(c+dx)}{4d} - \frac{a\left(-\sec^4(c+dx) - 2\sec^2(c+dx) - 4\log(\sin(c+dx)) + 4\log(\cos(c+dx))\right)}{4d} + \frac{3a\left(\text{ta}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/4*(a*(4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]] - 2*Sec[c + d*x]^2 - Sec
[c + d*x]^4))/d + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(ArcTanh[Sin
[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

fricas [A] time = 0.49, size = 175, normalized size = 1.50

$$-\frac{6\,a\cos(dx+c)^2 - 16\left(a\cos(dx+c)^2\sin(dx+c) - a\cos(dx+c)^2\right)\log\left(\frac{1}{2}\sin(dx+c)\right) + 5\left(a\cos(dx+c)^2\,\text{si}\right.}{16\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(6*a*cos(d*x + c)^2 - 16*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x +
 c)^2)*log(1/2*sin(d*x + c)) + 5*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x
 + c)^2)*log(sin(d*x + c) + 1) + 11*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(
d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*a*sin(d*x + c) + 6*a)/(d*cos(d*x + c
)^2*sin(d*x + c) - d*cos(d*x + c)^2)
```

giac [A] time = 0.24, size = 104, normalized size = 0.89

$$-\frac{10\,a\log\left(|\sin\left(dx+c\right)+1|\right)+22\,a\log\left(|\sin\left(dx+c\right)-1|\right)-32\,a\log\left(|\sin\left(dx+c\right)|\right)-\frac{2\,(5\,a\sin(dx+c)+7\,a)}{\sin(dx+c)+1}-\frac{33\,a\sin(dx\cdots}{(\cdots}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(10*a*log(abs(sin(d*x + c) + 1)) + 22*a*log(abs(sin(d*x + c) - 1)) -
32*a*log(abs(sin(d*x + c))) - 2*(5*a*sin(d*x + c) + 7*a)/(sin(d*x + c) + 1)
 - (33*a*sin(d*x + c)^2 - 82*a*sin(d*x + c) + 53*a)/(sin(d*x + c) - 1)^2)/d
```

maple [A] time = 0.40, size = 100, normalized size = 0.85

$$\frac{a\left(\sec^{3}\left(dx+c\right)\right)\tan\left(dx+c\right)}{4d}+\frac{3a\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{3a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{a}{4d\cos\left(dx+c\right)^{4}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4*a*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*ln(se
c(d*x+c)+tan(d*x+c))+1/4/d*a/cos(d*x+c)^4+1/2/d*a/cos(d*x+c)^2+a*ln(tan(d*x
+c))/d
```

maxima [A] time = 0.32, size = 95, normalized size = 0.81

$$-\frac{5\,a\log\left(\sin\left(dx+c\right)+1\right)+11\,a\log\left(\sin\left(dx+c\right)-1\right)-16\,a\log\left(\sin\left(dx+c\right)\right)+\frac{2\left(3\,a\sin(dx+c)^{2}+a\sin(dx+c)-6\,a\right)}{\sin(dx+c)^{3}-\sin(dx+c)^{2}-\sin(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(5*a*log(sin(d*x + c) + 1) + 11*a*log(sin(d*x + c) - 1) - 16*a*log(si
n(d*x + c)) + 2*(3*a*sin(d*x + c)^2 + a*sin(d*x + c) - 6*a)/(sin(d*x + c)^3
 - sin(d*x + c)^2 - sin(d*x + c) + 1))/d
```

mupad [B] time = 0.10, size = 99, normalized size = 0.85

$$\frac{a\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{\frac{3\,a\,\sin(c+d\,x)^{2}}{8}+\frac{a\,\sin(c+d\,x)}{8}-\frac{3\,a}{4}}{d\,\left(\cos\left(c+d\,x\right)^{2}+\sin\left(c+d\,x\right)^{3}-\sin\left(c+d\,x\right)\right)}-\frac{11\,a\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{16\,d}-\frac{5\,a\,\ln\left(\sin\left(c+\cdots\right.\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)),x)
```

```
[Out] (a*log(sin(c + d*x)))/d - ((a*sin(c + d*x))/8 - (3*a)/4 + (3*a*sin(c + d*x)
^2)/8)/(d*(cos(c + d*x)^2 - sin(c + d*x) + sin(c + d*x)^3)) - (11*a*log(sin
(c + d*x) - 1))/(16*d) - (5*a*log(sin(c + d*x) + 1))/(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.858 $\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{3a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc(c+dx)}{d}-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{a}{}$$

```
[Out] -a*csc(d*x+c)/d-23/16*a*ln(1-sin(d*x+c))/d+a*ln(sin(d*x+c))/d+7/16*a*ln(1+s
in(d*x+c))/d+1/8*a^3/d/(a-a*sin(d*x+c))^2+3/4*a^2/d/(a-a*sin(d*x+c))-1/8*a^
2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{3a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc(c+dx)}{d}-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (23*a*Log[1 - Sin[c + d*x]])/(16*d) + (a*Log[Sin[c
+ d*x]])/d + (7*a*Log[1 + Sin[c + d*x]])/(16*d) + a^3/(8*d*(a - a*Sin[c + d
*x])^2) + (3*a^2)/(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a^2}{(a-x)^3x^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \frac{1}{(a-x)^3x^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{1}{4a^4(a-x)^3}+\frac{3}{4a^5(a-x)^2}+\frac{23}{16a^6(a-x)}+\frac{1}{a^5x^2}+\frac{1}{a^6x}+\right.\right.}{d}$$
$$= -\frac{a\csc(c+dx)}{d}-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{a\log(\sin(c+}{d}$$

Mathematica [C] time = 0.19, size = 76, normalized size = 0.59

$$-\frac{a\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\sin^2(c+dx)\right)}{d}-\frac{a\left(-\sec^4(c+dx)-2\sec^2(c+dx)-4\log(\sin(c+dx))+4\log(\cos(\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[c + d*x]^2])/d) - (a*
(4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]] - 2*Sec[c + d*x]^2 - Sec[c + d*x
]^4))/(4*d)
```

fricas [A] time = 0.47, size = 229, normalized size = 1.78

$$-\frac{22\,a\cos(dx+c)^2-16\left(a\cos(dx+c)^4+a\cos(dx+c)^2\sin(dx+c)-a\cos(dx+c)^2\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(22*a*cos(d*x + c)^2 - 16*(a*cos(d*x + c)^4 + a*cos(d*x + c)^2*sin(d*
x + c) - a*cos(d*x + c)^2)*log(1/2*sin(d*x + c)) - 7*(a*cos(d*x + c)^4 + a*
cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + 23*
```

```
(a*cos(d*x + c)^4 + a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^2)*log(-
sin(d*x + c) + 1) - 2*(15*a*cos(d*x + c)^2 - a)*sin(d*x + c) - 6*a)/(d*cos(
d*x + c)^4 + d*cos(d*x + c)^2*sin(d*x + c) - d*cos(d*x + c)^2)
```

giac [A] time = 0.28, size = 121, normalized size = 0.94

$$\frac{14\,a\log\left(|\sin\left(dx+c\right)+1|\right)-46\,a\log\left(|\sin\left(dx+c\right)-1|\right)+32\,a\log\left(|\sin\left(dx+c\right)|\right)-\frac{23\,a\sin(dx+c)^2+59\,a\sin(dx+c)+32\,a}{\sin(dx+c)^2+\sin(dx+c)}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(14*a*log(abs(sin(d*x + c) + 1)) - 46*a*log(abs(sin(d*x + c) - 1)) + 3
2*a*log(abs(sin(d*x + c))) - (23*a*sin(d*x + c)^2 + 59*a*sin(d*x + c) + 32*
a)/(sin(d*x + c)^2 + sin(d*x + c)) + (69*a*sin(d*x + c)^2 - 162*a*sin(d*x +
 c) + 97*a)/(sin(d*x + c) - 1)^2)/d
```

maple [A] time = 0.35, size = 120, normalized size = 0.93

$$\frac{a}{4d\cos\left(dx+c\right)^{4}}+\frac{a}{2d\cos\left(dx+c\right)^{2}}+\frac{a\ln\left(\tan\left(dx+c\right)\right)}{d}+\frac{a}{4d\sin\left(dx+c\right)\cos\left(dx+c\right)^{4}}+\frac{5a}{8d\sin\left(dx+c\right)\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/cos(d*x+c)^4+1/2/d*a/cos(d*x+c)^2+a*ln(tan(d*x+c))/d+1/4/d*a/sin(d*
x+c)/cos(d*x+c)^4+5/8/d*a/sin(d*x+c)/cos(d*x+c)^2-15/8*a/d/sin(d*x+c)+15/8/
d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.35, size = 114, normalized size = 0.88

$$\frac{7\,a\log\left(\sin\left(dx+c\right)+1\right)-23\,a\log\left(\sin\left(dx+c\right)-1\right)+16\,a\log\left(\sin\left(dx+c\right)\right)-\frac{2\left(15\,a\sin(dx+c)^3-11\,a\sin(dx+c)^2-14\,a\sin(}{\sin(dx+c)^4-\sin(dx+c)^3-\sin(dx+c)^2+\sin}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(7*a*log(sin(d*x + c) + 1) - 23*a*log(sin(d*x + c) - 1) + 16*a*log(sin
(d*x + c)) - 2*(15*a*sin(d*x + c)^3 - 11*a*sin(d*x + c)^2 - 14*a*sin(d*x +
c) + 8*a)/(sin(d*x + c)^4 - sin(d*x + c)^3 - sin(d*x + c)^2 + sin(d*x + c))
)/d
```

mupad [B] time = 0.10, size = 118, normalized size = 0.91

$$\frac{a\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{\frac{15\,a\,\sin(c+d\,x)^3}{8}-\frac{11\,a\,\sin(c+d\,x)^2}{8}-\frac{7\,a\,\sin(c+d\,x)}{4}+a}{d\,\left(\sin\left(c+d\,x\right)^4-\sin\left(c+d\,x\right)^3-\sin\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)\right)}-\frac{23\,a\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)^2),x)
```

```
[Out] (a*log(sin(c + d*x)))/d - (a - (7*a*sin(c + d*x))/4 - (11*a*sin(c + d*x)^2)
/8 + (15*a*sin(c + d*x)^3)/8)/(d*(sin(c + d*x) - sin(c + d*x)^2 - sin(c + d
*x)^3 + sin(c + d*x)^4)) - (23*a*log(sin(c + d*x) - 1))/(16*d) + (7*a*log(s
in(c + d*x) + 1))/(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.859 $\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=143

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{39a\log(1-\mathrm{s}}{16a}$$

```
[Out] -a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-39/16*a*ln(1-sin(d*x+c))/d+3*a*ln(sin(
d*x+c))/d-9/16*a*ln(1+sin(d*x+c))/d+1/8*a^3/d/(a-a*sin(d*x+c))^2+a^2/d/(a-a
*sin(d*x+c))+1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{39a\log(1-\mathrm{s}}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (39*a*Log[1 - Sin[c + d*
x]])/(16*d) + (3*a*Log[Sin[c + d*x]])/d - (9*a*Log[1 + Sin[c + d*x]])/(16*d
) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/(d*(a - a*Sin[c + d*x])) + a^2/(
8*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^5\,\mathrm{Subst}\left(\int \frac{a^3}{(a-x)^3x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^3x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\,\mathrm{Subst}\left(\int \left(\frac{1}{4a^5(a-x)^3}+\frac{1}{a^6(a-x)^2}+\frac{39}{16a^7(a-x)}+\frac{1}{a^5x^3}+\frac{1}{a^6x^2}+\right.\right.}{d}$$
$$= -\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{39a\log(1-\sin(c+dx))}{16d}$$

Mathematica [C] time = 0.74, size = 86, normalized size = 0.60

$$-\frac{a\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\sin^2(c+dx)\right)}{d}-\frac{a\left(2\csc^2(c+dx)-\sec^4(c+dx)-4\sec^2(c+dx)-12\log(\sin(c+dx\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[c + d*x]^2])/d) - (a*
(2*Csc[c + d*x]^2 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]] - 4*Sec[c +
 d*x]^2 - Sec[c + d*x]^4))/(4*d)
```

fricas [B] time = 0.48, size = 294, normalized size = 2.06

$$30\,a\cos(dx+c)^4-16\,a\cos(dx+c)^2+48\left(a\cos(dx+c)^4-a\cos(dx+c)^2-\left(a\cos(dx+c)^4-a\cos(dx+c)^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(30*a*cos(d*x + c)^4 - 16*a*cos(d*x + c)^2 + 48*(a*cos(d*x + c)^4 - a*
cos(d*x + c)^2 - (a*cos(d*x + c)^4 - a*cos(d*x + c)^2)*sin(d*x + c))*log(1/
2*sin(d*x + c)) - 9*(a*cos(d*x + c)^4 - a*cos(d*x + c)^2 - (a*cos(d*x + c)^
```

```
4 - a*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 39*(a*cos(d*x +
 c)^4 - a*cos(d*x + c)^2 - (a*cos(d*x + c)^4 - a*cos(d*x + c)^2)*sin(d*x +
c))*log(-sin(d*x + c) + 1) + 2*(3*a*cos(d*x + c)^2 + a)*sin(d*x + c) - 6*a)
/(d*cos(d*x + c)^4 - d*cos(d*x + c)^2 - (d*cos(d*x + c)^4 - d*cos(d*x + c)^
2)*sin(d*x + c))
```

giac [A] time = 0.28, size = 125, normalized size = 0.87

$$-\frac{36\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+156\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-192\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{4\,(9\,a\sin(dx+c)+11\,a)}{\sin(dx+c)+1}+\frac{27\,a\sin}{}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/64*(36*a*log(abs(sin(d*x + c) + 1)) + 156*a*log(abs(sin(d*x + c) - 1)) -
 192*a*log(abs(sin(d*x + c))) - 4*(9*a*sin(d*x + c) + 11*a)/(sin(d*x + c) +
 1) + (27*a*sin(d*x + c)^4 + 74*a*sin(d*x + c)^3 - 141*a*sin(d*x + c)^2 + 3
2*a)/(sin(d*x + c)^2 - sin(d*x + c))^2)/d
```

maple [A] time = 0.37, size = 151, normalized size = 1.06

$$\frac{a}{4d\sin\left(dx+c\right)\cos\left(dx+c\right)^{4}}+\frac{5a}{8d\sin\left(dx+c\right)\cos\left(dx+c\right)^{2}}-\frac{15a}{8d\sin\left(dx+c\right)}+\frac{15a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/sin(d*x+c)/cos(d*x+c)^4+5/8/d*a/sin(d*x+c)/cos(d*x+c)^2-15/8*a/d/si
n(d*x+c)+15/8/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a/sin(d*x+c)^2/cos(d*x+c)
^4+3/4/d*a/sin(d*x+c)^2/cos(d*x+c)^2-3/2/d*a/sin(d*x+c)^2+3*a*ln(tan(d*x+c)
)/d
```

maxima [A] time = 0.31, size = 127, normalized size = 0.89

$$-\frac{9\,a\log\left(\sin\left(dx+c\right)+1\right)+39\,a\log\left(\sin\left(dx+c\right)-1\right)-48\,a\log\left(\sin\left(dx+c\right)\right)+\frac{2\left(15\,a\sin(dx+c)^{4}-3\,a\sin(dx+c)^{3}-22\,a\sin\right.}{\sin(dx+c)^{5}-\sin(dx+c)^{4}-\sin(dx}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(9*a*log(sin(d*x + c) + 1) + 39*a*log(sin(d*x + c) - 1) - 48*a*log(si
n(d*x + c)) + 2*(15*a*sin(d*x + c)^4 - 3*a*sin(d*x + c)^3 - 22*a*sin(d*x +
```

```
c)^2 + 4*a*sin(d*x + c) + 4*a)/(sin(d*x + c)^5 - sin(d*x + c)^4 - sin(d*x +
 c)^3 + sin(d*x + c)^2))/d
```

mupad [B] time = 9.31, size = 134, normalized size = 0.94

$$\frac{3\,a\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{39\,a\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{16\,d}-\frac{9\,a\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{16\,d}-\frac{\frac{15\,a\,\sin(c+d\,x)^4}{8}-\frac{3\,a\,\sin(c+d\,x)^3}{8}-\frac{11\,a\,\mathrm{s}}{}}{d\,\left(\sin\left(c+d\,x\right)^5-\sin\left(c+d\,x\right)^4-\mathrm{si}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)^3),x)
```

```
[Out] (3*a*log(sin(c + d*x)))/d - (39*a*log(sin(c + d*x) - 1))/(16*d) - (9*a*log(
sin(c + d*x) + 1))/(16*d) - (a/2 + (a*sin(c + d*x))/2 - (11*a*sin(c + d*x)^
2)/4 - (3*a*sin(c + d*x)^3)/8 + (15*a*sin(c + d*x)^4)/8)/(d*(sin(c + d*x)^2
 - sin(c + d*x)^3 - sin(c + d*x)^4 + sin(c + d*x)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.860 $\int \csc^4(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=162

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{5a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+}{d}$$

```
[Out] -3*a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-59/16*a*ln(1-si
n(d*x+c))/d+3*a*ln(sin(d*x+c))/d+11/16*a*ln(1+sin(d*x+c))/d+1/8*a^3/d/(a-a*
sin(d*x+c))^2+5/4*a^2/d/(a-a*sin(d*x+c))-1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{5a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
 - (59*a*Log[1 - Sin[c + d*x]])/(16*d) + (3*a*Log[Sin[c + d*x]])/d + (11*a*
Log[1 + Sin[c + d*x]])/(16*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + (5*a^2)/
(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^4(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a^4}{(a-x)^3x^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^9\,\text{Subst}\left(\int \frac{1}{(a-x)^3x^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^9\,\text{Subst}\left(\int \left(\frac{1}{4a^6(a-x)^3}+\frac{5}{4a^7(a-x)^2}+\frac{59}{16a^8(a-x)}+\frac{1}{a^5x^4}+\frac{1}{a^6x^3}+\right.\right.}{d}$$
$$= -\frac{3a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{59a\,\text{lo}}{}$$

Mathematica [C] time = 1.16, size = 90, normalized size = 0.56

$$-\frac{a\csc^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\sin^2(c+dx)\right)}{3d}-\frac{a\left(2\csc^2(c+dx)-\sec^4(c+dx)-4\sec^2(c+dx)-12\log(\sin(c+\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(a*Csc[c + d*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[c + d*x]^2])/d
- (a*(2*Csc[c + d*x]^2 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]] - 4*Se
c[c + d*x]^2 - Sec[c + d*x]^4))/(4*d)
```

fricas [B] time = 0.47, size = 343, normalized size = 2.12

$$-\frac{138\,a\cos(dx+c)^4-172\,a\cos(dx+c)^2-144\left(a\cos(dx+c)^6-2\,a\cos(dx+c)^4+a\cos(dx+c)^2+\left(a\cos(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(138*a*cos(d*x + c)^4 - 172*a*cos(d*x + c)^2 - 144*(a*cos(d*x + c)^6
- 2*a*cos(d*x + c)^4 + a*cos(d*x + c)^2 + (a*cos(d*x + c)^4 - a*cos(d*x + c
)^2)*sin(d*x + c))*log(1/2*sin(d*x + c)) - 33*(a*cos(d*x + c)^6 - 2*a*cos(d
```

```
*x + c)^4 + a*cos(d*x + c)^2 + (a*cos(d*x + c)^4 - a*cos(d*x + c)^2)*sin(d*
x + c))*log(sin(d*x + c) + 1) + 177*(a*cos(d*x + c)^6 - 2*a*cos(d*x + c)^4
+ a*cos(d*x + c)^2 + (a*cos(d*x + c)^4 - a*cos(d*x + c)^2)*sin(d*x + c))*lo
g(-sin(d*x + c) + 1) - 2*(105*a*cos(d*x + c)^4 - 104*a*cos(d*x + c)^2 + 3*a
)*sin(d*x + c) + 18*a)/(d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^4 + d*cos(d*x +
 c)^2 + (d*cos(d*x + c)^4 - d*cos(d*x + c)^2)*sin(d*x + c))
```

giac [A] time = 0.29, size = 149, normalized size = 0.92

$$\frac{66\,a\log\left(|\sin\left(dx+c\right)+1|\right)-354\,a\log\left(|\sin\left(dx+c\right)-1|\right)+288\,a\log\left(|\sin\left(dx+c\right)|\right)-\frac{6\,(11\,a\sin(dx+c)+13\,a)}{\sin(dx+c)+1}+\frac{3\,(177\,a}{}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/96*(66*a*log(abs(sin(d*x + c) + 1)) - 354*a*log(abs(sin(d*x + c) - 1)) +
288*a*log(abs(sin(d*x + c))) - 6*(11*a*sin(d*x + c) + 13*a)/(sin(d*x + c) +
 1) + 3*(177*a*sin(d*x + c)^2 - 394*a*sin(d*x + c) + 221*a)/(sin(d*x + c) -
 1)^2 - 16*(33*a*sin(d*x + c)^3 + 18*a*sin(d*x + c)^2 + 3*a*sin(d*x + c) +
2*a)/sin(d*x + c)^3)/d
```

maple [A] time = 0.40, size = 173, normalized size = 1.07

$$\frac{a}{4d\sin\left(dx+c\right)^{2}\cos\left(dx+c\right)^{4}}+\frac{3a}{4d\sin\left(dx+c\right)^{2}\cos\left(dx+c\right)^{2}}-\frac{3a}{2d\sin\left(dx+c\right)^{2}}+\frac{3a\ln\left(\tan\left(dx+c\right)\right)}{d}+\frac{}{4d\sin\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/sin(d*x+c)^2/cos(d*x+c)^4+3/4/d*a/sin(d*x+c)^2/cos(d*x+c)^2-3/2/d*a
/sin(d*x+c)^2+3*a*ln(tan(d*x+c))/d+1/4/d*a/sin(d*x+c)^3/cos(d*x+c)^4-7/12/d
*a/sin(d*x+c)^3/cos(d*x+c)^2+35/24/d*a/sin(d*x+c)/cos(d*x+c)^2-35/8*a/d/sin
(d*x+c)+35/8/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.37, size = 138, normalized size = 0.85

$$\frac{33\,a\log\left(\sin\left(dx+c\right)+1\right)-177\,a\log\left(\sin\left(dx+c\right)-1\right)+144\,a\log\left(\sin\left(dx+c\right)\right)-\frac{2\left(105\,a\sin(dx+c)^{5}-69\,a\sin(dx+c)^{4}-10\right.}{\sin(dx+c)^{6}-\sin(dx}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(33*a*log(sin(d*x + c) + 1) - 177*a*log(sin(d*x + c) - 1) + 144*a*log(
sin(d*x + c)) - 2*(105*a*sin(d*x + c)^5 - 69*a*sin(d*x + c)^4 - 106*a*sin(d
*x + c)^3 + 52*a*sin(d*x + c)^2 + 4*a*sin(d*x + c) + 8*a)/(sin(d*x + c)^6 -
 sin(d*x + c)^5 - sin(d*x + c)^4 + sin(d*x + c)^3))/d
```

mupad [**B**] time = 0.11, size = 145, normalized size = 0.90

$$\frac{3\,a\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{\frac{35\,a\,\sin(c+d\,x)^5}{8}-\frac{23\,a\,\sin(c+d\,x)^4}{8}-\frac{53\,a\,\sin(c+d\,x)^3}{12}+\frac{13\,a\,\sin(c+d\,x)^2}{6}+\frac{a\,\sin(c+d\,x)}{6}+\frac{a}{3}}{d\,\left(\sin\left(c+d\,x\right)^6-\sin\left(c+d\,x\right)^5-\sin\left(c+d\,x\right)^4+\sin\left(c+d\,x\right)^3\right)}-\frac{59\,a\,\ln\left(\sin\left(c\right.\right.}{16\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)^4),x)
```

```
[Out] (3*a*log(sin(c + d*x)))/d - (a/3 + (a*sin(c + d*x))/6 + (13*a*sin(c + d*x)^
2)/6 - (53*a*sin(c + d*x)^3)/12 - (23*a*sin(c + d*x)^4)/8 + (35*a*sin(c + d
*x)^5)/8)/(d*(sin(c + d*x)^3 - sin(c + d*x)^4 - sin(c + d*x)^5 + sin(c + d*
x)^6)) - (59*a*log(sin(c + d*x) - 1))/(16*d) + (11*a*log(sin(c + d*x) + 1))
/(16*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.861 $\int (a+a\sin(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=119

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\sin^2(c+dx)}{2d}-\frac{2a^2\sin(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}-a^2\log$$

```
[Out] -31/8*a^2*ln(1-sin(d*x+c))/d-1/8*a^2*ln(1+sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-
1/2*a^2*sin(d*x+c)^2/d+1/4*a^4/d/(a-a*sin(d*x+c))^2-9/4*a^3/d/(a-a*sin(d*x+
c))
```

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{a^2\sin^2(c+dx)}{2d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{2a^2\sin(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}-a^2\,\mathrm{l}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (-31*a^2*Log[1 - Sin[c + d*x]])/(8*d) - (a^2*Log[1 + Sin[c + d*x]])/(8*d) -
 (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d) + a^4/(4*d*(a - a*Sin[
c + d*x])^2) - (9*a^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^4}{2(a-x)^3}-\frac{9a^3}{4(a-x)^2}+\frac{31a^2}{8(a-x)}-x-\frac{a^2}{8(a+x)}\right)dx, x, a\sin(c+\right.}{d}$$
$$= -\frac{31a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(1+\sin(c+dx))}{8d}-\frac{2a^2\sin(c+dx)}{d}$$

Mathematica [A] time = 0.24, size = 75, normalized size = 0.63

$$-\frac{a^2\left(4\sin^2(c+dx)+16\sin(c+dx)-\frac{18}{\sin(c+dx)-1}-\frac{2}{(\sin(c+dx)-1)^2}+31\log(1-\sin(c+dx))+\log(\sin(c+dx)+1)\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -1/8*(a^2*(31*Log[1 - Sin[c + d*x]] + Log[1 + Sin[c + d*x]] - 2/(-1 + Sin[c
 + d*x])^2 - 18/(-1 + Sin[c + d*x]) + 16*Sin[c + d*x] + 4*Sin[c + d*x]^2))/
d
```

fricas [A] time = 0.46, size = 168, normalized size = 1.41

$$\frac{4a^2\cos(dx+c)^4+22a^2\cos(dx+c)^2-12a^2-\left(a^2\cos(dx+c)^2+2a^2\sin(dx+c)-2a^2\right)\log(\sin(dx+c)+1}{8\left(d\cos(dx+c)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(4*a^2*cos(d*x + c)^4 + 22*a^2*cos(d*x + c)^2 - 12*a^2 - (a^2*cos(d*x +
 c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(sin(d*x + c) + 1) - 31*(a^2*cos(d*x
 + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(-sin(d*x + c) + 1) - 2*(4*a^2*cos
(d*x + c)^2 - 5*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2
*d)
```

giac [A] time = 0.29, size = 102, normalized size = 0.86

$$-\frac{8a^2\sin(dx+c)^2+2a^2\log(|\sin(dx+c)+1|)+62a^2\log(|\sin(dx+c)-1|)+32a^2\sin(dx+c)-\frac{93a^2\sin(dx+c}{(s}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(8*a^2*sin(d*x + c)^2 + 2*a^2*log(abs(sin(d*x + c) + 1)) + 62*a^2*log
(abs(sin(d*x + c) - 1)) + 32*a^2*sin(d*x + c) - (93*a^2*sin(d*x + c)^2 - 15
0*a^2*sin(d*x + c) + 61*a^2)/(sin(d*x + c) - 1)^2)/d
```

maple [B] time = 0.30, size = 261, normalized size = 2.19

$$\frac{a^2\left(\sin^8(dx+c)\right)}{4d\cos(dx+c)^4}-\frac{a^2\left(\sin^8(dx+c)\right)}{2d\cos(dx+c)^2}-\frac{a^2\left(\sin^6(dx+c)\right)}{2d}-\frac{3a^2\left(\sin^4(dx+c)\right)}{4d}-\frac{3a^2\left(\sin^2(dx+c)\right)}{2d}-\frac{4a^2\ln(\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*a^2*sin(d*x+c)^8/cos(d*x+c)^2-1/2
*a^2*sin(d*x+c)^6/d-3/4*a^2*sin(d*x+c)^4/d-3/2*a^2*sin(d*x+c)^2/d-4/d*a^2*l
n(cos(d*x+c))+1/2/d*a^2*sin(d*x+c)^7/cos(d*x+c)^4-3/4/d*a^2*sin(d*x+c)^7/co
s(d*x+c)^2-3/4*a^2*sin(d*x+c)^5/d-5/4*a^2*sin(d*x+c)^3/d-15/4*a^2*sin(d*x+c
)/d+15/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a^2*tan(d*x+c)^4-1/2/d*a^2*t
an(d*x+c)^2
```

maxima [A] time = 0.42, size = 96, normalized size = 0.81

$$-\frac{4\,a^2\sin(dx+c)^2+a^2\log(\sin(dx+c)+1)+31\,a^2\log(\sin(dx+c)-1)+16\,a^2\sin(dx+c)-\frac{2\left(9\,a^2\sin(dx+c)-8\right.}{\sin(dx+c)^2-2\sin(dx-}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*(4*a^2*sin(d*x + c)^2 + a^2*log(sin(d*x + c) + 1) + 31*a^2*log(sin(d*x
 + c) - 1) + 16*a^2*sin(d*x + c) - 2*(9*a^2*sin(d*x + c) - 8*a^2)/(sin(d*x
+ c)^2 - 2*sin(d*x + c) + 1))/d
```

mupad [B] time = 9.23, size = 283, normalized size = 2.38

$$\frac{4\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{4\,d}-\frac{\frac{15\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}-22\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{61\,a^2\tan}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^5*(a + a*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (4*a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a^2*log(tan(c/2 + (d*x)/2) + 1))
/(4*d) - ((61*a^2*tan(c/2 + (d*x)/2)^3)/2 - 22*a^2*tan(c/2 + (d*x)/2)^2 - 3
6*a^2*tan(c/2 + (d*x)/2)^4 + (61*a^2*tan(c/2 + (d*x)/2)^5)/2 - 22*a^2*tan(c
/2 + (d*x)/2)^6 + (15*a^2*tan(c/2 + (d*x)/2)^7)/2 + (15*a^2*tan(c/2 + (d*x)
/2))/2)/(d*(8*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 12*tan(c/2 + (d
*x)/2)^3 + 14*tan(c/2 + (d*x)/2)^4 - 12*tan(c/2 + (d*x)/2)^5 + 8*tan(c/2 +
(d*x)/2)^6 - 4*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8 + 1)) - (31*a^2*
log(tan(c/2 + (d*x)/2) - 1))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.862 $\int \sec(c+dx)(a+a\sin(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=101

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{7a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\sin(c+dx)}{d}-\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

```
[Out] -17/8*a^2*ln(1-sin(d*x+c))/d+1/8*a^2*ln(1+sin(d*x+c))/d-a^2*sin(d*x+c)/d+1/
4*a^4/d/(a-a*sin(d*x+c))^2-7/4*a^3/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{7a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\sin(c+dx)}{d}-\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (-17*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (a^2*Log[1 + Sin[c + d*x]])/(8*d) -
 (a^2*Sin[c + d*x])/d + a^4/(4*d*(a - a*Sin[c + d*x])^2) - (7*a^3)/(4*d*(a
- a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^2\tan^4(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x^4}{a^4(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a\,\text{Subst}\left(\int \left(-1+\frac{a^3}{2(a-x)^3}-\frac{7a^2}{4(a-x)^2}+\frac{17a}{8(a-x)}+\frac{a}{8(a+x)}\right)dx, x, a$$

$$= -\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(1+\sin(c+dx))}{8d}-\frac{a^2}{}$$

Mathematica [A] time = 0.12, size = 67, normalized size = 0.66

$$-\frac{a^2\left(8\sin(c+dx)-\frac{14}{\sin(c+dx)-1}-\frac{2}{(\sin(c+dx)-1)^2}+17\log(1-\sin(c+dx))-\log(\sin(c+dx)+1)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] -1/8*(a^2*(17*Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]] - 2/(-1 + Sin[c
 + d*x])^2 - 14/(-1 + Sin[c + d*x]) + 8*Sin[c + d*x]))/d
```

fricas [A] time = 0.46, size = 154, normalized size = 1.52

$$\frac{16\,a^2\cos(dx+c)^2-4\,a^2+\left(a^2\cos(dx+c)^2+2\,a^2\sin(dx+c)-2\,a^2\right)\log(\sin(dx+c)+1)-17\left(a^2\cos(dx+\right.}{8\left(d\cos(dx+c)^2+2\,d\sin(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(16*a^2*cos(d*x + c)^2 - 4*a^2 + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x +
c) - 2*a^2)*log(sin(d*x + c) + 1) - 17*(a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x
+ c) - 2*a^2)*log(-sin(d*x + c) + 1) - 2*(4*a^2*cos(d*x + c)^2 - a^2)*sin(d
*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

giac [A] time = 0.29, size = 88, normalized size = 0.87

$$\frac{2\,a^2\log(|\sin(dx+c)+1|)-34\,a^2\log(|\sin(dx+c)-1|)-16\,a^2\sin(dx+c)+\frac{51\,a^2\sin(dx+c)^2-74\,a^2\sin(dx+c)+27\,a^2}{(\sin(dx+c)-1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) - 34*a^2*log(abs(sin(d*x + c) - 1))
- 16*a^2*sin(d*x + c) + (51*a^2*sin(d*x + c)^2 - 74*a^2*sin(d*x + c) + 27*a
^2)/(sin(d*x + c) - 1)^2)/d
```

maple [B] time = 0.29, size = 213, normalized size = 2.11

$$\frac{a^2\left(\sin^7(dx+c)\right)}{4d\cos(dx+c)^4}-\frac{3a^2\left(\sin^7(dx+c)\right)}{8d\cos(dx+c)^2}-\frac{3a^2\left(\sin^5(dx+c)\right)}{8d}-\frac{3a^2\left(\sin^3(dx+c)\right)}{4d}-\frac{9a^2\sin(dx+c)}{4d}+\frac{9a^2\ln(\sec(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a^2*sin(d*x+c)^7/cos(d*x+c)^2-3/8
*a^2*sin(d*x+c)^5/d-3/4*a^2*sin(d*x+c)^3/d-9/4*a^2*sin(d*x+c)/d+9/4/d*a^2*l
n(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2*tan(d*x+c)^4-1/d*a^2*tan(d*x+c)^2-2/d*a^
2*ln(cos(d*x+c))+1/4/d*a^2*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a^2*sin(d*x+c)^5
/cos(d*x+c)^2
```

maxima [A] time = 0.31, size = 83, normalized size = 0.82

$$\frac{a^2\log(\sin(dx+c)+1)-17\,a^2\log(\sin(dx+c)-1)-8\,a^2\sin(dx+c)+\frac{2\left(7\,a^2\sin(dx+c)-6\,a^2\right)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*(a^2*log(sin(d*x + c) + 1) - 17*a^2*log(sin(d*x + c) - 1) - 8*a^2*sin(d
*x + c) + 2*(7*a^2*sin(d*x + c) - 6*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) +
 1))/d
```

mupad [B] time = 9.27, size = 225, normalized size = 2.23

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{4\,d}-\frac{17\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{4\,d}-\frac{\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2}-14\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+17\,a^2\tan\left(\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2) + 1))/(4*d) - (17*a^2*log(tan(c/2 + (d*x)/2) -
1))/(4*d) - (17*a^2*tan(c/2 + (d*x)/2)^3 - 14*a^2*tan(c/2 + (d*x)/2)^2 - 14
*a^2*tan(c/2 + (d*x)/2)^4 + (9*a^2*tan(c/2 + (d*x)/2)^5)/2 + (9*a^2*tan(c/2
 + (d*x)/2))/2)/(d*(7*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 8*tan(c
/2 + (d*x)/2)^3 + 7*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^5 + tan(c/2
 + (d*x)/2)^6 + 1)) + (2*a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.863 $\int \sec^2(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=87

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{5a^3}{4d(a-a\sin(c+dx))}-\frac{7a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

```
[Out] -7/8*a^2*ln(1-sin(d*x+c))/d-1/8*a^2*ln(1+sin(d*x+c))/d+1/4*a^4/d/(a-a*sin(d
*x+c))^2-5/4*a^3/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{5a^3}{4d(a-a\sin(c+dx))}-\frac{7a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (-7*a^2*Log[1 - Sin[c + d*x]])/(8*d) - (a^2*Log[1 + Sin[c + d*x]])/(8*d) +
a^4/(4*d*(a - a*Sin[c + d*x])^2) - (5*a^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x^3}{a^3(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{x^3}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{a^2}{2(a-x)^3}-\frac{5a}{4(a-x)^2}+\frac{7}{8(a-x)}-\frac{1}{8(a+x)}\right)dx, x, a\sin\right.}{d}$$
$$= -\frac{7a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(1+\sin(c+dx))}{8d}+\frac{}{4d(}$$

Mathematica [A] time = 0.39, size = 91, normalized size = 1.05

$$\frac{a^2\left(3\tanh^{-1}(\sin(c+dx))-6\tan(c+dx)\sec^3(c+dx)+\tan(c+dx)\left(8\tan^2(c+dx)+3\right)\sec(c+dx)-2\left(-\tan\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*(3*ArcTanh[Sin[c + d*x]] - 6*Sec[c + d*x]^3*Tan[c + d*x] + Sec[c + d*x
]*Tan[c + d*x]*(3 + 8*Tan[c + d*x]^2) - 2*(2*Log[Cos[c + d*x]] + Tan[c + d*
x]^2 - Tan[c + d*x]^4)))/(4*d)
```

fricas [A] time = 0.49, size = 125, normalized size = 1.44

$$-\frac{10\,a^2\sin(dx+c)-8\,a^2+\left(a^2\cos(dx+c)^2+2\,a^2\sin(dx+c)-2\,a^2\right)\log(\sin(dx+c)+1)+7\left(a^2\cos(dx+c\right.}{8\left(d\cos(dx+c)^2+2\,d\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/8*(10*a^2*sin(d*x + c) - 8*a^2 + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c
) - 2*a^2)*log(sin(d*x + c) + 1) + 7*(a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x +
c) - 2*a^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) -
2*d)
```

giac [A] time = 0.26, size = 78, normalized size = 0.90

$$-\frac{2\,a^2\log(|\sin(dx+c)+1|)+14\,a^2\log(|\sin(dx+c)-1|)-\frac{21\,a^2\sin(dx+c)^2-22\,a^2\sin(dx+c)+5\,a^2}{(\sin(dx+c)-1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) + 14*a^2*log(abs(sin(d*x + c) - 1))
 - (21*a^2*sin(d*x + c)^2 - 22*a^2*sin(d*x + c) + 5*a^2)/(sin(d*x + c) - 1)
^2)/d
```

maple [B] time = 0.29, size = 173, normalized size = 1.99

$$\frac{a^2 \left(\tan^4 (dx + c)\right)}{4d} - \frac{a^2 \left(\tan^2 (dx + c)\right)}{2d} - \frac{a^2 \ln (\cos (dx + c))}{d} + \frac{a^2 \left(\sin^5 (dx + c)\right)}{2d \cos (dx + c)^4} - \frac{a^2 \left(\sin^5 (dx + c)\right)}{4d \cos (dx + c)^2} - \frac{a^2 \left(\sin^3 (dx +\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*tan(d*x+c)^4-1/2/d*a^2*tan(d*x+c)^2-1/d*a^2*ln(cos(d*x+c))+1/2/d*
a^2*sin(d*x+c)^5/cos(d*x+c)^4-1/4/d*a^2*sin(d*x+c)^5/cos(d*x+c)^2-1/4*a^2*s
in(d*x+c)^3/d-3/4*a^2*sin(d*x+c)/d+3/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/
d*a^2*sin(d*x+c)^4/cos(d*x+c)^4
```

maxima [A] time = 0.32, size = 72, normalized size = 0.83

$$-\frac{a^2 \log (\sin (dx + c) + 1) + 7\,a^2 \log (\sin (dx + c) - 1) - \frac{2\left(5\,a^2 \sin(dx+c) - 4\,a^2\right)}{\sin(dx+c)^2 - 2\,\sin(dx+c) + 1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*(a^2*log(sin(d*x + c) + 1) + 7*a^2*log(sin(d*x + c) - 1) - 2*(5*a^2*si
n(d*x + c) - 4*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

mupad [B] time = 9.30, size = 166, normalized size = 1.91

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d} - \frac{7\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{4\,d} - \frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{4\,d} - \frac{\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2} - 4}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 4\,\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (7*a^2*log(tan(c/2 + (d*x)/2) - 1))
/(4*d) - (a^2*log(tan(c/2 + (d*x)/2) + 1))/(4*d) - ((3*a^2*tan(c/2 + (d*x)/
2)^3)/2 - 4*a^2*tan(c/2 + (d*x)/2)^2 + (3*a^2*tan(c/2 + (d*x)/2))/2)/(d*(6*
tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 4*tan(c/2 + (d*x)/2)^3 + tan(
c/2 + (d*x)/2)^4 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.864 $\int \sec^3(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=64

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{3a^3}{4d(a-a\sin(c+dx))}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

```
[Out] 1/4*a^2*arctanh(sin(d*x+c))/d+1/4*a^4/d/(a-a*sin(d*x+c))^2-3/4*a^3/d/(a-a*s
in(d*x+c))
```

Rubi [A] time = 0.12, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2836, 12, 88, 206}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{3a^3}{4d(a-a\sin(c+dx))}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) - (3*a
^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x^2}{a^2(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{x^2}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{a}{2(a-x)^3}-\frac{3}{4(a-x)^2}+\frac{1}{4(a^2-x^2)}\right)dx, x, a\sin(c+d$$
$$= \frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{3a^3}{4d(a-a\sin(c+dx))}+\frac{a^3\,\text{Subst}}{}$$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{}{4d(a-}$$

Mathematica [A] time = 0.11, size = 39, normalized size = 0.61

$$\frac{a^2\left(\frac{3\sin(c+dx)-2}{(\sin(c+dx)-1)^2}+\tanh^{-1}(\sin(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (a^2*(ArcTanh[Sin[c + d*x]] + (-2 + 3*Sin[c + d*x])/(-1 + Sin[c + d*x])^2))
/(4*d)
```

fricas [B] time = 0.46, size = 125, normalized size = 1.95

$$-\frac{6\,a^2\sin(dx+c)-4\,a^2-\left(a^2\cos(dx+c)^2+2\,a^2\sin(dx+c)-2\,a^2\right)\log(\sin(dx+c)+1)+\left(a^2\cos(dx+c)^2\right.}{8\left(d\cos(dx+c)^2+2\,d\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/8*(6*a^2*sin(d*x + c) - 4*a^2 - (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c)
 - 2*a^2)*log(sin(d*x + c) + 1) + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c)
- 2*a^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d
)
```

giac [A] time = 0.23, size = 77, normalized size = 1.20

$$\frac{2\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-2\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+\frac{3\,a^2\sin\left(dx+c\right)^2+6\,a^2\sin\left(dx+c\right)-5\,a^2}{\left(\sin\left(dx+c\right)-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) - 2*a^2*log(abs(sin(d*x + c) - 1)) +
 (3*a^2*sin(d*x + c)^2 + 6*a^2*sin(d*x + c) - 5*a^2)/(sin(d*x + c) - 1)^2)/
d
```

maple [B] time = 0.27, size = 174, normalized size = 2.72

$$\frac{a^2\left(\sin^5\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4}-\frac{a^2\left(\sin^5\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^2}-\frac{a^2\left(\sin^3\left(dx+c\right)\right)}{8d}-\frac{a^2\sin\left(dx+c\right)}{4d}+\frac{a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{4d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a^2*sin(d*x+c)^5/cos(d*x+c)^2-1/8
*a^2*sin(d*x+c)^3/d-1/4*a^2*sin(d*x+c)/d+1/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c)
)+1/2/d*a^2*sin(d*x+c)^4/cos(d*x+c)^4+1/4/d*a^2*sin(d*x+c)^3/cos(d*x+c)^4+1
/8/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2
```

maxima [A] time = 0.31, size = 72, normalized size = 1.12

$$\frac{a^2\log\left(\sin\left(dx+c\right)+1\right)-a^2\log\left(\sin\left(dx+c\right)-1\right)+\frac{2\left(3\,a^2\sin\left(dx+c\right)-2\,a^2\right)}{\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*(a^2*log(sin(d*x + c) + 1) - a^2*log(sin(d*x + c) - 1) + 2*(3*a^2*sin(d
*x + c) - 2*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

mupad [B] time = 11.55, size = 123, normalized size = 1.92

$$\frac{a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (a^2*atanh(tan(c/2 + (d*x)/2)))/(2*d) - ((a^2*tan(c/2 + (d*x)/2)^3)/2 - 2*a
^2*tan(c/2 + (d*x)/2)^2 + (a^2*tan(c/2 + (d*x)/2))/2)/(d*(6*tan(c/2 + (d*x)
/2)^2 - 4*tan(c/2 + (d*x)/2) - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^
4 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.865 $\int \sec^4(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=64

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

```
[Out] -1/4*a^2*arctanh(sin(d*x+c))/d+1/4*a^4/d/(a-a*sin(d*x+c))^2-1/4*a^3/d/(a-a*
sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2836, 12, 77, 206}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -(a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) - a^3
/(4*d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x}{a(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^4\,\text{Subst}\left(\int \frac{x}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^4\,\text{Subst}\left(\int \left(\frac{1}{2(a-x)^3} - \frac{1}{4a(a-x)^2} - \frac{1}{4a\left(a^2-x^2\right)}\right)dx, x, a\sin(c+d$$

$$= \frac{a^4}{4d(a-a\sin(c+dx))^2} - \frac{a^3}{4d(a-a\sin(c+dx))} - \frac{a^3\,\text{Subst}\left(\right.}{}$$

$$= -\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d} + \frac{a^4}{4d(a-a\sin(c+dx))^2} - \frac{}{4d(a-}$$

Mathematica [A] time = 0.08, size = 36, normalized size = 0.56

$$-\frac{a^2\left(\tanh^{-1}(\sin(c+dx)) - \frac{\sin(c+dx)}{(\sin(c+dx)-1)^2}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
[Out] -1/4*(a^2*(ArcTanh[Sin[c + d*x]] - Sin[c + d*x]/(-1 + Sin[c + d*x])^2))/d
```

fricas [A] time = 0.48, size = 120, normalized size = 1.88

$$-\frac{2\,a^2\sin(dx+c) + \left(a^2\cos(dx+c)^2 + 2\,a^2\sin(dx+c) - 2\,a^2\right)\log(\sin(dx+c)+1) - \left(a^2\cos(dx+c)^2 + 2\,a^2\,\text{s}\right.}{8\left(d\cos(dx+c)^2 + 2\,d\sin(dx+c) - 2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a^2*sin(d*x + c) + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2
)*log(sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)
*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

giac [A] time = 0.22, size = 95, normalized size = 1.48

$$-\frac{a^2\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)+2\right|\right)-a^2\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)-2\right|\right)+\frac{a^2\left(\frac{1}{\sin(dx+c)}+\sin(dx+c)\right)-6a^2}{\frac{1}{\sin(dx+c)}+\sin(dx+c)-2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(a^2*log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - a^2*log(abs(1/sin(
d*x + c) + sin(d*x + c) - 2)) + (a^2*(1/sin(d*x + c) + sin(d*x + c)) - 6*a^
2)/(1/sin(d*x + c) + sin(d*x + c) - 2))/d
```

maple [B] time = 0.26, size = 126, normalized size = 1.97

$$\frac{a^2\left(\sin^4(dx+c)\right)}{4d\cos(dx+c)^4}+\frac{a^2\left(\sin^3(dx+c)\right)}{2d\cos(dx+c)^4}+\frac{a^2\left(\sin^3(dx+c)\right)}{4d\cos(dx+c)^2}+\frac{a^2\sin(dx+c)}{4d}-\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{4d}+\frac{}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^4/cos(d*x+c)^4+1/2/d*a^2*sin(d*x+c)^3/cos(d*x+c)^4+1/4
/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/4*a^2*sin(d*x+c)/d-1/4/d*a^2*ln(sec(d*x+
c)+tan(d*x+c))+1/4/d*a^2/cos(d*x+c)^4
```

maxima [A] time = 0.31, size = 64, normalized size = 1.00

$$-\frac{a^2\log(\sin(dx+c)+1)-a^2\log(\sin(dx+c)-1)-\frac{2a^2\sin(dx+c)}{\sin(dx+c)^2-2\sin(dx+c)+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a^2*log(sin(d*x + c) + 1) - a^2*log(sin(d*x + c) - 1) - 2*a^2*sin(d*x
 + c)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

mupad [B] time = 11.13, size = 106, normalized size = 1.66

$$\frac{\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}-\frac{a^2\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] ((a^2*tan(c/2 + (d*x)/2)^3)/2 + (a^2*tan(c/2 + (d*x)/2))/2)/(d*(6*tan(c/2 +
 (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*
x)/2)^4 + 1)) - (a^2*atanh(tan(c/2 + (d*x)/2)))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.866 $\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=101

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{3a^3}{4d(a-a\sin(c+dx))}-\frac{7a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(\sin(c+dx))}{d}-\frac{a^2\log(\sin(c+dx)}{8d}$$

```
[Out] -7/8*a^2*ln(1-sin(d*x+c))/d+a^2*ln(sin(d*x+c))/d-1/8*a^2*ln(1+sin(d*x+c))/d
+1/4*a^4/d/(a-a*sin(d*x+c))^2+3/4*a^3/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 72}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{3a^3}{4d(a-a\sin(c+dx))}-\frac{7a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(\sin(c+dx))}{d}-\frac{a^2\log(\sin(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (a^2*Log[Sin[c + d*x]])/d - (a^2*Log
[1 + Sin[c + d*x]])/(8*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + (3*a^3)/(4*d
*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a}{(a-x)^3x(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{1}{(a-x)^3x(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(\frac{1}{2a^2(a-x)^3}+\frac{3}{4a^3(a-x)^2}+\frac{7}{8a^4(a-x)}+\frac{1}{a^4x}-\frac{1}{8a^4(a+x)}\right)\right.}{d}$$
$$= -\frac{7a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(\sin(c+dx))}{d}-\frac{a^2\log(1}{}$$

Mathematica [A] time = 0.29, size = 66, normalized size = 0.65

$$-\frac{a^2\left(\frac{6}{\sin(c+dx)-1}-\frac{2}{(\sin(c+dx)-1)^2}+7\log(1-\sin(c+dx))-8\log(\sin(c+dx))+\log(\sin(c+dx)+1)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/8*(a^2*(7*Log[1 - Sin[c + d*x]] - 8*Log[Sin[c + d*x]] + Log[1 + Sin[c +
d*x]] - 2/(-1 + Sin[c + d*x])^2 + 6/(-1 + Sin[c + d*x])))/d
```

fricas [A] time = 0.48, size = 166, normalized size = 1.64

$$\frac{6\,a^2\sin(dx+c)-8\,a^2+8\left(a^2\cos(dx+c)^2+2\,a^2\sin(dx+c)-2\,a^2\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-\left(a^2\cos(dx+c)^2+\right.}{8\left(d\cos(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*a^2*sin(d*x + c) - 8*a^2 + 8*(a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c
) - 2*a^2)*log(1/2*sin(d*x + c)) - (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c)
 - 2*a^2)*log(sin(d*x + c) + 1) - 7*(a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c
) - 2*a^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2
*d)
```

giac [A] time = 0.28, size = 91, normalized size = 0.90

$$-\frac{2\,a^2\log(|\sin(dx+c)+1|)+14\,a^2\log(|\sin(dx+c)-1|)-16\,a^2\log(|\sin(dx+c)|)-\frac{21\,a^2\sin(dx+c)^2-54\,a^2\sin(dx+c}{(\sin(dx+c)-1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) + 14*a^2*log(abs(sin(d*x + c) - 1))
 - 16*a^2*log(abs(sin(d*x + c))) - (21*a^2*sin(d*x + c)^2 - 54*a^2*sin(d*x
+ c) + 37*a^2)/(sin(d*x + c) - 1)^2)/d
```

maple [A] time = 0.66, size = 112, normalized size = 1.11

$$\frac{a^2}{2d\cos(dx+c)^4}+\frac{a^2\tan(dx+c)\left(\sec^3(dx+c)\right)}{2d}+\frac{3a^2\tan(dx+c)\sec(dx+c)}{4d}+\frac{3a^2\ln(\sec(dx+c)+\tan(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d*a^2/cos(d*x+c)^4+1/2/d*a^2*tan(d*x+c)*sec(d*x+c)^3+3/4/d*a^2*tan(d*x+
c)*sec(d*x+c)+3/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2/cos(d*x+c)^2+1/
d*a^2*ln(tan(d*x+c))
```

maxima [A] time = 0.48, size = 84, normalized size = 0.83

$$-\frac{a^2\log(\sin(dx+c)+1)+7\,a^2\log(\sin(dx+c)-1)-8\,a^2\log(\sin(dx+c))+\frac{2\left(3\,a^2\sin(dx+c)-4\,a^2\right)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a^2*log(sin(d*x + c) + 1) + 7*a^2*log(sin(d*x + c) - 1) - 8*a^2*log(s
in(d*x + c)) + 2*(3*a^2*sin(d*x + c) - 4*a^2)/(sin(d*x + c)^2 - 2*sin(d*x +
 c) + 1))/d
```

mupad [B] time = 0.08, size = 91, normalized size = 0.90

$$\frac{a^2\,\ln(\sin(c+d\,x))}{d}-\frac{a^2\,\ln(\sin(c+d\,x)+1)}{8\,d}-\frac{\frac{3\,a^2\,\sin(c+d\,x)}{4}-a^2}{d\,\left(\sin(c+d\,x)^2-2\,\sin(c+d\,x)+1\right)}-\frac{7\,a^2\,\ln(\sin(c+d\,x)-1)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^5*sin(c + d*x)),x)
```

```
[Out] (a^2*log(sin(c + d*x)))/d - (a^2*log(sin(c + d*x) + 1))/(8*d) - ((3*a^2*sin
(c + d*x))/4 - a^2)/(d*(sin(c + d*x)^2 - 2*sin(c + d*x) + 1)) - (7*a^2*log(
sin(c + d*x) - 1))/(8*d)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
[Out] Timed out
```

3.867 $\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{5a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc(c+dx)}{d}-\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{2a^2\log(\sin(c+dx))}{d}+$$

```
[Out] -a^2*csc(d*x+c)/d-17/8*a^2*ln(1-sin(d*x+c))/d+2*a^2*ln(sin(d*x+c))/d+1/8*a^
2*ln(1+sin(d*x+c))/d+1/4*a^4/d/(a-a*sin(d*x+c))^2+5/4*a^3/d/(a-a*sin(d*x+c)
)
```

Rubi [A] time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{5a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc(c+dx)}{d}-\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{2a^2\log(\sin(c+dx))}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) - (17*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (2*a^2*Log
[Sin[c + d*x]])/d + (a^2*Log[1 + Sin[c + d*x]])/(8*d) + a^4/(4*d*(a - a*Sin
[c + d*x])^2) + (5*a^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a^2}{(a-x)^3x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \frac{1}{(a-x)^3x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{1}{2a^3(a-x)^3}+\frac{5}{4a^4(a-x)^2}+\frac{17}{8a^5(a-x)}+\frac{1}{a^4x^2}+\frac{2}{a^5x}+\right.\right.}{d}$$
$$= -\frac{a^2\csc(c+dx)}{d}-\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{2a^2\log(\sin}{d}$$

Mathematica [A] time = 0.27, size = 74, normalized size = 0.64

$$\frac{a^2\left(-\frac{10}{\sin(c+dx)-1}+\frac{2}{(\sin(c+dx)-1)^2}-8\csc(c+dx)-17\log(1-\sin(c+dx))+16\log(\sin(c+dx))+\log(\sin(c+dx)\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-8*Csc[c + d*x] - 17*Log[1 - Sin[c + d*x]] + 16*Log[Sin[c + d*x]] + L
og[1 + Sin[c + d*x]] + 2/(-1 + Sin[c + d*x])^2 - 10/(-1 + Sin[c + d*x])))/(
8*d)
```

fricas [B] time = 0.51, size = 240, normalized size = 2.07

$$\frac{18\,a^2\cos(dx+c)^2+28\,a^2\sin(dx+c)-26\,a^2+16\left(2\,a^2\cos(dx+c)^2-2\,a^2-\left(a^2\cos(dx+c)^2-2\,a^2\right)\sin(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(18*a^2*cos(d*x + c)^2 + 28*a^2*sin(d*x + c) - 26*a^2 + 16*(2*a^2*cos(d
*x + c)^2 - 2*a^2 - (a^2*cos(d*x + c)^2 - 2*a^2)*sin(d*x + c))*log(1/2*sin(
d*x + c)) + (2*a^2*cos(d*x + c)^2 - 2*a^2 - (a^2*cos(d*x + c)^2 - 2*a^2)*si
n(d*x + c))*log(sin(d*x + c) + 1) - 17*(2*a^2*cos(d*x + c)^2 - 2*a^2 - (a^2
```

```
*cos(d*x + c)^2 - 2*a^2)*sin(d*x + c))*log(-sin(d*x + c) + 1))/(2*d*cos(d*x
 + c)^2 - (d*cos(d*x + c)^2 - 2*d)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.30, size = 115, normalized size = 0.99

$$\frac{2\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-34\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+32\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{16\left(2\,a^2\sin(dx+c)+a^2\right)}{\sin(dx+c)}+\frac{51\,a^2\sin}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) - 34*a^2*log(abs(sin(d*x + c) - 1))
+ 32*a^2*log(abs(sin(d*x + c))) - 16*(2*a^2*sin(d*x + c) + a^2)/sin(d*x + c
) + (51*a^2*sin(d*x + c)^2 - 122*a^2*sin(d*x + c) + 75*a^2)/(sin(d*x + c) -
 1)^2)/d
```

maple [A] time = 0.62, size = 176, normalized size = 1.52

$$\frac{a^2\tan\left(dx+c\right)\left(\sec^3\left(dx+c\right)\right)}{4d}+\frac{3a^2\tan\left(dx+c\right)\sec\left(dx+c\right)}{8d}+\frac{9a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{4d}+\frac{a^2}{2d\cos\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^2*tan(d*x+c)*sec(d*x+c)+9/4/d*a^2
*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2/cos(d*x+c)^4+1/d*a^2/cos(d*x+c)^2+2/d*
a^2*ln(tan(d*x+c))+1/4/d*a^2/sin(d*x+c)/cos(d*x+c)^4+5/8/d*a^2/sin(d*x+c)/c
os(d*x+c)^2-15/8/d*a^2/sin(d*x+c)
```

maxima [A] time = 0.31, size = 104, normalized size = 0.90

$$\frac{a^2\log\left(\sin\left(dx+c\right)+1\right)-17\,a^2\log\left(\sin\left(dx+c\right)-1\right)+16\,a^2\log\left(\sin\left(dx+c\right)\right)-\frac{2\left(9\,a^2\sin(dx+c)^2-14\,a^2\sin(dx+c)+4\,a^2\right)}{\sin(dx+c)^3-2\,\sin(dx+c)^2+\sin(dx+c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*(a^2*log(sin(d*x + c) + 1) - 17*a^2*log(sin(d*x + c) - 1) + 16*a^2*log(
sin(d*x + c)) - 2*(9*a^2*sin(d*x + c)^2 - 14*a^2*sin(d*x + c) + 4*a^2)/(sin
(d*x + c)^3 - 2*sin(d*x + c)^2 + sin(d*x + c)))/d
```

mupad [**B**] time = 9.05, size = 110, normalized size = 0.95

$$\frac{a^2\ \ln\left(\sin\left(c+d\,x\right)+1\right)}{8\,d}-\frac{17\,a^2\ \ln\left(\sin\left(c+d\,x\right)-1\right)}{8\,d}+\frac{2\,a^2\ \ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{\frac{9\,a^2\,\sin(c+d\,x)^2}{4}-\frac{7\,a^2\,\sin(c+d\,x)}{2}}{d\,\left(\sin\left(c+d\,x\right)^3-2\,\sin\left(c+d\,x\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^5*sin(c + d*x)^2),x)
```

```
[Out] (a^2*log(sin(c + d*x) + 1))/(8*d) - (17*a^2*log(sin(c + d*x) - 1))/(8*d) +
(2*a^2*log(sin(c + d*x)))/d - (a^2 - (7*a^2*sin(c + d*x))/2 + (9*a^2*sin(c
+ d*x)^2)/4)/(d*(sin(c + d*x) - 2*sin(c + d*x)^2 + sin(c + d*x)^3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.868 $\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=134

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{7a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}+\frac{4a^2\,\mathrm{l}}{}$$

```
[Out] -2*a^2*csc(d*x+c)/d-1/2*a^2*csc(d*x+c)^2/d-31/8*a^2*ln(1-sin(d*x+c))/d+4*a^
2*ln(sin(d*x+c))/d-1/8*a^2*ln(1+sin(d*x+c))/d+1/4*a^4/d/(a-a*sin(d*x+c))^2+
7/4*a^3/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{7a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}+\frac{4a^2\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - (31*a^2*Log[1 - Sin[
c + d*x]])/(8*d) + (4*a^2*Log[Sin[c + d*x]])/d - (a^2*Log[1 + Sin[c + d*x]]
)/(8*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + (7*a^3)/(4*d*(a - a*Sin[c + d*
x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a^3}{(a-x)^3x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\,\text{Subst}\left(\int \frac{1}{(a-x)^3x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\,\text{Subst}\left(\int \left(\frac{1}{2a^4(a-x)^3} + \frac{7}{4a^5(a-x)^2} + \frac{31}{8a^6(a-x)} + \frac{1}{a^4x^3} + \frac{2}{a^5x^2} + \right.\right.}{d}$$
$$= -\frac{2a^2\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{2d} - \frac{31a^2\log(1-\sin(c+}{8d}$$

Mathematica [A] time = 1.13, size = 84, normalized size = 0.63

$$-\frac{a^2\left(\frac{14}{\sin(c+dx)-1} - \frac{2}{(\sin(c+dx)-1)^2} + 4\csc^2(c+dx) + 16\csc(c+dx) + 31\log(1-\sin(c+dx)) - 32\log(\sin(c+dx))\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/8*(a^2*(16*Csc[c + d*x] + 4*Csc[c + d*x]^2 + 31*Log[1 - Sin[c + d*x]] -
32*Log[Sin[c + d*x]] + Log[1 + Sin[c + d*x]] - 2/(-1 + Sin[c + d*x])^2 + 14
/(-1 + Sin[c + d*x])))/d
```

fricas [B] time = 0.47, size = 302, normalized size = 2.25

$$-\frac{44\,a^2\cos\left(dx+c\right)^2 - 40\,a^2 - 32\left(a^2\cos\left(dx+c\right)^4 - 3\,a^2\cos\left(dx+c\right)^2 + 2\,a^2 + 2\left(a^2\cos\left(dx+c\right)^2 - a^2\right)\sin\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/8*(44*a^2*cos(d*x + c)^2 - 40*a^2 - 32*(a^2*cos(d*x + c)^4 - 3*a^2*cos(d
*x + c)^2 + 2*a^2 + 2*(a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))*log(1/2*sin(
```

```
d*x + c)) + (a^2*cos(d*x + c)^4 - 3*a^2*cos(d*x + c)^2 + 2*a^2 + 2*(a^2*cos
(d*x + c)^2 - a^2)*sin(d*x + c))*log(sin(d*x + c) + 1) + 31*(a^2*cos(d*x +
c)^4 - 3*a^2*cos(d*x + c)^2 + 2*a^2 + 2*(a^2*cos(d*x + c)^2 - a^2)*sin(d*x
+ c))*log(-sin(d*x + c) + 1) - 2*(15*a^2*cos(d*x + c)^2 - 19*a^2)*sin(d*x +
 c))/(d*cos(d*x + c)^4 - 3*d*cos(d*x + c)^2 + 2*(d*cos(d*x + c)^2 - d)*sin(
d*x + c) + 2*d)
```

giac [A] time = 0.30, size = 125, normalized size = 0.93

$$-\frac{4\,a^2\log\left(|\sin\left(dx+c\right)+1|\right)+124\,a^2\log\left(|\sin\left(dx+c\right)-1|\right)-128\,a^2\log\left(|\sin\left(dx+c\right)|\right)+\frac{3\,a^2\sin(dx+c)^4+114\,a^2\sin(dx\cdots}{(\sin(\cdots}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/32*(4*a^2*log(abs(sin(d*x + c) + 1)) + 124*a^2*log(abs(sin(d*x + c) - 1)
) - 128*a^2*log(abs(sin(d*x + c))) + (3*a^2*sin(d*x + c)^4 + 114*a^2*sin(d*
x + c)^3 - 173*a^2*sin(d*x + c)^2 + 32*a^2*sin(d*x + c) + 16*a^2)/(sin(d*x
+ c)^2 - sin(d*x + c))^2)/d
```

maple [A] time = 0.57, size = 199, normalized size = 1.49

$$\frac{a^2}{4d\cos\left(dx+c\right)^4}+\frac{a^2}{2d\cos\left(dx+c\right)^2}+\frac{4a^2\ln\left(\tan\left(dx+c\right)\right)}{d}+\frac{a^2}{2d\sin\left(dx+c\right)\cos\left(dx+c\right)^4}+\frac{5a^2}{4d\sin\left(dx+c\right)\cos\left(dx+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2/cos(d*x+c)^4+1/2/d*a^2/cos(d*x+c)^2+4/d*a^2*ln(tan(d*x+c))+1/2/d*
a^2/sin(d*x+c)/cos(d*x+c)^4+5/4/d*a^2/sin(d*x+c)/cos(d*x+c)^2-15/4/d*a^2/si
n(d*x+c)+15/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a^2/sin(d*x+c)^2/cos(d*
x+c)^4+3/4/d*a^2/sin(d*x+c)^2/cos(d*x+c)^2-3/2/d*a^2/sin(d*x+c)^2
```

maxima [A] time = 0.31, size = 119, normalized size = 0.89

$$-\frac{a^2\log\left(\sin\left(dx+c\right)+1\right)+31\,a^2\log\left(\sin\left(dx+c\right)-1\right)-32\,a^2\log\left(\sin\left(dx+c\right)\right)+\frac{2\left(15\,a^2\sin(dx+c)^3-22\,a^2\sin(dx+c)^2+4\cdots\right.}{\sin(dx+c)^4-2\,\sin(dx+c)^3+\sin\cdots}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*(a^2*log(sin(d*x + c) + 1) + 31*a^2*log(sin(d*x + c) - 1) - 32*a^2*log
(sin(d*x + c)) + 2*(15*a^2*sin(d*x + c)^3 - 22*a^2*sin(d*x + c)^2 + 4*a^2*s
in(d*x + c) + 2*a^2)/(sin(d*x + c)^4 - 2*sin(d*x + c)^3 + sin(d*x + c)^2))/
d
```

mupad [B] time = 0.10, size = 126, normalized size = 0.94

$$\frac{4\,a^2\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{a^2\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{8\,d}-\frac{\frac{15\,a^2\,{\sin\left(c+d\,x\right)}^3}{4}-\frac{11\,a^2\,{\sin\left(c+d\,x\right)}^2}{2}+a^2\,\sin\left(c+d\,x\right)+\frac{a^2}{2}}{d\,\left({\sin\left(c+d\,x\right)}^4-2\,{\sin\left(c+d\,x\right)}^3+{\sin\left(c+d\,x\right)}^2\right)}-\frac{31\,a^2\,\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^5*sin(c + d*x)^3),x)
```

```
[Out] (4*a^2*log(sin(c + d*x)))/d - (a^2*log(sin(c + d*x) + 1))/(8*d) - (a^2*sin(
c + d*x) + a^2/2 - (11*a^2*sin(c + d*x)^2)/2 + (15*a^2*sin(c + d*x)^3)/4)/(
d*(sin(c + d*x)^2 - 2*sin(c + d*x)^3 + sin(c + d*x)^4)) - (31*a^2*log(sin(c
 + d*x) - 1))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.869 $\int \csc^4(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=150

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{d}-\frac{4a^2\csc(c+dx)}{d}-\frac{49a^2\log(1-\sin(c+dx))}{8d}$$

```
[Out] -4*a^2*csc(d*x+c)/d-a^2*csc(d*x+c)^2/d-1/3*a^2*csc(d*x+c)^3/d-49/8*a^2*ln(1
-sin(d*x+c))/d+6*a^2*ln(sin(d*x+c))/d+1/8*a^2*ln(1+sin(d*x+c))/d+1/4*a^4/d/
(a-a*sin(d*x+c))^2+9/4*a^3/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{d}-\frac{4a^2\csc(c+dx)}{d}-\frac{49a^2\log(1-\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-4*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/d - (a^2*Csc[c + d*x]^3)/(3*
d) - (49*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (6*a^2*Log[Sin[c + d*x]])/d + (
a^2*Log[1 + Sin[c + d*x]])/(8*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + (9*a^
3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^4(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a^4}{(a-x)^3x^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^9\,\text{Subst}\left(\int \frac{1}{(a-x)^3x^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^9\,\text{Subst}\left(\int \left(\frac{1}{2a^5(a-x)^3}+\frac{9}{4a^6(a-x)^2}+\frac{49}{8a^7(a-x)}+\frac{1}{a^4x^4}+\frac{2}{a^5x^3}+\right.\right.}{d}$$
$$= -\frac{4a^2\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{49}{}$$

Mathematica [A] time = 6.06, size = 133, normalized size = 0.89

$$\frac{a^9\left(-\frac{\csc^3(c+dx)}{3a^7}-\frac{\csc^2(c+dx)}{a^7}-\frac{4\csc(c+dx)}{a^7}-\frac{49\log(1-\sin(c+dx))}{8a^7}+\frac{6\log(\sin(c+dx))}{a^7}+\frac{\log(\sin(c+dx)+1)}{8a^7}+\frac{9}{4a^6(a-a\sin(c+dx))}+\frac{}{4a^5(}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^9*((-4*Csc[c + d*x])/a^7 - Csc[c + d*x]^2/a^7 - Csc[c + d*x]^3/(3*a^7) -
 (49*Log[1 - Sin[c + d*x]])/(8*a^7) + (6*Log[Sin[c + d*x]])/a^7 + Log[1 + S
in[c + d*x]]/(8*a^7) + 1/(4*a^5*(a - a*Sin[c + d*x])^2) + 9/(4*a^6*(a - a*S
in[c + d*x]))))/d
```

fricas [B] time = 0.50, size = 370, normalized size = 2.47

$$\frac{150\,a^2\cos(dx+c)^4-356\,a^2\cos(dx+c)^2+214\,a^2+144\left(2\,a^2\cos(dx+c)^4-4\,a^2\cos(dx+c)^2+2\,a^2-\left(a^2\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(150*a^2*cos(d*x + c)^4 - 356*a^2*cos(d*x + c)^2 + 214*a^2 + 144*(2*a^
2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + 2*a^2 - (a^2*cos(d*x + c)^4 - 3*a
```

```
^2*cos(d*x + c)^2 + 2*a^2)*sin(d*x + c))*log(1/2*sin(d*x + c)) + 3*(2*a^2*c
os(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + 2*a^2 - (a^2*cos(d*x + c)^4 - 3*a^2*
cos(d*x + c)^2 + 2*a^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 147*(2*a^2*co
s(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + 2*a^2 - (a^2*cos(d*x + c)^4 - 3*a^2*c
os(d*x + c)^2 + 2*a^2)*sin(d*x + c))*log(-sin(d*x + c) + 1) + 4*(57*a^2*cos
(d*x + c)^2 - 55*a^2)*sin(d*x + c))/(2*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^
2 - (d*cos(d*x + c)^4 - 3*d*cos(d*x + c)^2 + 2*d)*sin(d*x + c) + 2*d)
```

giac [A] time = 0.35, size = 142, normalized size = 0.95

$$\frac{6\,a^2\log\left(|\sin\left(dx+c\right)+1|\right)-294\,a^2\log\left(|\sin\left(dx+c\right)-1|\right)+288\,a^2\log\left(|\sin\left(dx+c\right)|\right)+\frac{3\left(147\,a^2\sin(dx+c)^2-330\,a^2\sin(\cdots}{(\sin(dx+c)-1)^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/48*(6*a^2*log(abs(sin(d*x + c) + 1)) - 294*a^2*log(abs(sin(d*x + c) - 1))
 + 288*a^2*log(abs(sin(d*x + c))) + 3*(147*a^2*sin(d*x + c)^2 - 330*a^2*sin
(d*x + c) + 187*a^2)/(sin(d*x + c) - 1)^2 - 16*(33*a^2*sin(d*x + c)^3 + 12*
a^2*sin(d*x + c)^2 + 3*a^2*sin(d*x + c) + a^2)/sin(d*x + c)^3)/d
```

maple [A] time = 0.56, size = 215, normalized size = 1.43

$$\frac{a^2}{4d\sin\left(dx+c\right)\cos\left(dx+c\right)^4}+\frac{25a^2}{12d\sin\left(dx+c\right)\cos\left(dx+c\right)^2}-\frac{25a^2}{4d\sin\left(dx+c\right)}+\frac{25a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\cdots}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2/sin(d*x+c)/cos(d*x+c)^4+25/12/d*a^2/sin(d*x+c)/cos(d*x+c)^2-25/4/
d*a^2/sin(d*x+c)+25/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2/sin(d*x+c)^
2/cos(d*x+c)^4+3/2/d*a^2/sin(d*x+c)^2/cos(d*x+c)^2-3/d*a^2/sin(d*x+c)^2+6/d
*a^2*ln(tan(d*x+c))+1/4/d*a^2/sin(d*x+c)^3/cos(d*x+c)^4-7/12/d*a^2/sin(d*x+
c)^3/cos(d*x+c)^2
```

maxima [A] time = 0.45, size = 133, normalized size = 0.89

$$\frac{3\,a^2\log\left(\sin\left(dx+c\right)+1\right)-147\,a^2\log\left(\sin\left(dx+c\right)-1\right)+144\,a^2\log\left(\sin\left(dx+c\right)\right)-\frac{2\left(75\,a^2\sin(dx+c)^4-114\,a^2\sin(dx+c)\cdots}{\sin(dx+c)^5-2\sin\cdots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*(3*a^2*log(sin(d*x + c) + 1) - 147*a^2*log(sin(d*x + c) - 1) + 144*a^2
*log(sin(d*x + c)) - 2*(75*a^2*sin(d*x + c)^4 - 114*a^2*sin(d*x + c)^3 + 28
*a^2*sin(d*x + c)^2 + 4*a^2*sin(d*x + c) + 4*a^2)/(sin(d*x + c)^5 - 2*sin(d
*x + c)^4 + sin(d*x + c)^3))/d
```

mupad [B] time = 9.05, size = 140, normalized size = 0.93

$$\frac{a^2\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{8\,d}-\frac{49\,a^2\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{8\,d}-\frac{\frac{25\,a^2\,\sin(c+d\,x)^4}{4}-\frac{19\,a^2\,\sin(c+d\,x)^3}{2}+\frac{7\,a^2\,\sin(c+d\,x)^2}{3}+\frac{a^2\,\sin(c+d\,x)}{3}+\cdots}{d\,\left(\sin\left(c+d\,x\right)^5-2\,\sin\left(c+d\,x\right)^4+\sin\left(c+d\,x\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^5*sin(c + d*x)^4),x)
```

```
[Out] (a^2*log(sin(c + d*x) + 1))/(8*d) - (49*a^2*log(sin(c + d*x) - 1))/(8*d) -
((a^2*sin(c + d*x))/3 + a^2/3 + (7*a^2*sin(c + d*x)^2)/3 - (19*a^2*sin(c +
d*x)^3)/2 + (25*a^2*sin(c + d*x)^4)/4)/(d*(sin(c + d*x)^3 - 2*sin(c + d*x)^
4 + sin(c + d*x)^5)) + (6*a^2*log(sin(c + d*x)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.870 $\int (a+a\sin(c+dx))^3\tan^5(c+dx)\,dx$

Optimal. Leaf size=114

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{5a^4}{d(a-a\sin(c+dx))}-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{6a^3\sin(c+dx)}{d}-\frac{10a^3\log(1-\sin}{d}$$

```
[Out] -10*a^3*ln(1-sin(d*x+c))/d-6*a^3*sin(d*x+c)/d-3/2*a^3*sin(d*x+c)^2/d-1/3*a^
3*sin(d*x+c)^3/d+1/2*a^5/d/(a-a*sin(d*x+c))^2-5*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 43}

$$-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{5a^4}{d(a-a\sin(c+dx))}-\frac{6a^3\sin(c+dx)}{d}-\frac{10a^3\log(1-s}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] (-10*a^3*Log[1 - Sin[c + d*x]])/d - (6*a^3*Sin[c + d*x])/d - (3*a^3*Sin[c +
 d*x]^2)/(2*d) - (a^3*Sin[c + d*x]^3)/(3*d) + a^5/(2*d*(a - a*Sin[c + d*x])
^2) - (5*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-6a^2+\frac{a^5}{(a-x)^3}-\frac{5a^4}{(a-x)^2}+\frac{10a^3}{a-x}-3ax-x^2\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{10a^3\log(1-\sin(c+dx))}{d}-\frac{6a^3\sin(c+dx)}{d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{a^3 s}{}$$

Mathematica [A] time = 0.31, size = 73, normalized size = 0.64

$$-\frac{a^3\left(2\sin^3(c+dx)+9\sin^2(c+dx)+36\sin(c+dx)+\frac{27-30\sin(c+dx)}{(\sin(c+dx)-1)^2}+60\log(1-\sin(c+dx))\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -1/6*(a^3*(60*Log[1 - Sin[c + d*x]] + (27 - 30*Sin[c + d*x])/(-1 + Sin[c +
d*x])^2 + 36*Sin[c + d*x] + 9*Sin[c + d*x]^2 + 2*Sin[c + d*x]^3))/d
```

fricas [A] time = 0.50, size = 141, normalized size = 1.24

$$\frac{10\,a^3\cos(dx+c)^4+115\,a^3\cos(dx+c)^2-80\,a^3-120\left(a^3\cos(dx+c)^2+2\,a^3\sin(dx+c)-2\,a^3\right)\log(-\sin(dx}{12\left(d\cos(dx+c)^2+2\,d\sin(dx+c)-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(10*a^3*cos(d*x + c)^4 + 115*a^3*cos(d*x + c)^2 - 80*a^3 - 120*(a^3*co
s(d*x + c)^2 + 2*a^3*sin(d*x + c) - 2*a^3)*log(-sin(d*x + c) + 1) + 2*(2*a^
3*cos(d*x + c)^4 - 24*a^3*cos(d*x + c)^2 + 37*a^3)*sin(d*x + c))/(d*cos(d*x
 + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

giac [B] time = 0.29, size = 242, normalized size = 2.12

$$\frac{30\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-60\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+183\,a^3\tan}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(30*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 60*a^3*log(abs(tan(1/2*d*x +
1/2*c) - 1)) - (55*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^3*tan(1/2*d*x + 1/2*c)
^5 + 183*a^3*tan(1/2*d*x + 1/2*c)^4 + 80*a^3*tan(1/2*d*x + 1/2*c)^3 + 183*a
^3*tan(1/2*d*x + 1/2*c)^2 + 36*a^3*tan(1/2*d*x + 1/2*c) + 55*a^3)/(tan(1/2*
d*x + 1/2*c)^2 + 1)^3 + (125*a^3*tan(1/2*d*x + 1/2*c)^4 - 524*a^3*tan(1/2*d
*x + 1/2*c)^3 + 804*a^3*tan(1/2*d*x + 1/2*c)^2 - 524*a^3*tan(1/2*d*x + 1/2*
c) + 125*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

maple [B] time = 0.31, size = 325, normalized size = 2.85

$$\frac{a^3\left(\sin^9(dx+c)\right)}{4d\cos(dx+c)^4}-\frac{5a^3\left(\sin^9(dx+c)\right)}{8d\cos(dx+c)^2}-\frac{5a^3\left(\sin^7(dx+c)\right)}{8d}-\frac{2a^3\left(\sin^5(dx+c)\right)}{d}-\frac{10a^3\left(\sin^3(dx+c)\right)}{3d}-\frac{10a^3\sin(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^9/cos(d*x+c)^4-5/8/d*a^3*sin(d*x+c)^9/cos(d*x+c)^2-5/8
*a^3*sin(d*x+c)^7/d-2*a^3*sin(d*x+c)^5/d-10/3*a^3*sin(d*x+c)^3/d-10*a^3*sin
(d*x+c)/d+10/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a^3*sin(d*x+c)^8/cos(d*x
+c)^4-3/2/d*a^3*sin(d*x+c)^8/cos(d*x+c)^2-3/2*a^3*sin(d*x+c)^6/d-9/4*a^3*si
n(d*x+c)^4/d-9/2*a^3*sin(d*x+c)^2/d-10/d*a^3*ln(cos(d*x+c))+3/4/d*a^3*sin(d
*x+c)^7/cos(d*x+c)^4-9/8/d*a^3*sin(d*x+c)^7/cos(d*x+c)^2+1/4/d*a^3*tan(d*x+
c)^4-1/2/d*a^3*tan(d*x+c)^2
```

maxima [A] time = 0.43, size = 96, normalized size = 0.84

$$-\frac{2\,a^3\sin(dx+c)^3+9\,a^3\sin(dx+c)^2+60\,a^3\log(\sin(dx+c)-1)+36\,a^3\sin(dx+c)-\frac{3\left(10\,a^3\sin(dx+c)-9\,a^3\right)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 60*a^3*log(sin(d*x + c)
 - 1) + 36*a^3*sin(d*x + c) - 3*(10*a^3*sin(d*x + c) - 9*a^3)/(sin(d*x + c)
^2 - 2*sin(d*x + c) + 1))/d
```

mupad [B] time = 11.56, size = 321, normalized size = 2.82

$$\frac{10\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{20\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-60\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{320\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{500\,a^3\,\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+22\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^5*(a + a*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (10*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d - ((320*a^3*tan(c/2 + (d*x)/2)^3)/
3 - 60*a^3*tan(c/2 + (d*x)/2)^2 - (500*a^3*tan(c/2 + (d*x)/2)^4)/3 + 184*a^
3*tan(c/2 + (d*x)/2)^5 - (500*a^3*tan(c/2 + (d*x)/2)^6)/3 + (320*a^3*tan(c/
2 + (d*x)/2)^7)/3 - 60*a^3*tan(c/2 + (d*x)/2)^8 + 20*a^3*tan(c/2 + (d*x)/2)
^9 + 20*a^3*tan(c/2 + (d*x)/2))/(d*(9*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d
*x)/2) - 16*tan(c/2 + (d*x)/2)^3 + 22*tan(c/2 + (d*x)/2)^4 - 24*tan(c/2 + (
d*x)/2)^5 + 22*tan(c/2 + (d*x)/2)^6 - 16*tan(c/2 + (d*x)/2)^7 + 9*tan(c/2 +
 (d*x)/2)^8 - 4*tan(c/2 + (d*x)/2)^9 + tan(c/2 + (d*x)/2)^10 + 1)) - (20*a^
3*log(tan(c/2 + (d*x)/2) - 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.871 $\int \sec(c+dx)(a+a\sin(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=96

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{4a^4}{d(a-a\sin(c+dx))}-\frac{a^3\sin^2(c+dx)}{2d}-\frac{3a^3\sin(c+dx)}{d}-\frac{6a^3\log(1-\sin(c+dx))}{d}$$

```
[Out] -6*a^3*ln(1-sin(d*x+c))/d-3*a^3*sin(d*x+c)/d-1/2*a^3*sin(d*x+c)^2/d+1/2*a^5
/d/(a-a*sin(d*x+c))^2-4*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 43}

$$-\frac{a^3\sin^2(c+dx)}{2d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{4a^4}{d(a-a\sin(c+dx))}-\frac{3a^3\sin(c+dx)}{d}-\frac{6a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (-6*a^3*Log[1 - Sin[c + d*x]])/d - (3*a^3*Sin[c + d*x])/d - (a^3*Sin[c + d*
x]^2)/(2*d) + a^5/(2*d*(a - a*Sin[c + d*x])^2) - (4*a^4)/(d*(a - a*Sin[c +
d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^3\tan^4(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x^4}{a^4(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(-3a+\frac{a^4}{(a-x)^3}-\frac{4a^3}{(a-x)^2}+\frac{6a^2}{a-x}-x\right)dx, x, a\sin(c+\right.}{d}$$
$$= -\frac{6a^3\log(1-\sin(c+dx))}{d}-\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\sin^2(c+}{2d}$$

Mathematica [A] time = 0.24, size = 61, normalized size = 0.64

$$-\frac{a^3\left(\sin^2(c+dx)+6\sin(c+dx)+\frac{7-8\sin(c+dx)}{(\sin(c+dx)-1)^2}+12\log(1-\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] -1/2*(a^3*(12*Log[1 - Sin[c + d*x]] + (7 - 8*Sin[c + d*x])/(-1 + Sin[c + d*
x])^2 + 6*Sin[c + d*x] + Sin[c + d*x]^2))/d
```

fricas [A] time = 0.46, size = 128, normalized size = 1.33

$$\frac{2\,a^3\cos(dx+c)^4+19\,a^3\cos(dx+c)^2-8\,a^3-24\left(a^3\cos(dx+c)^2+2\,a^3\sin(dx+c)-2\,a^3\right)\log(-\sin(dx+c}{4\left(d\cos(dx+c)^2+2\,d\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(2*a^3*cos(d*x + c)^4 + 19*a^3*cos(d*x + c)^2 - 8*a^3 - 24*(a^3*cos(d*x
 + c)^2 + 2*a^3*sin(d*x + c) - 2*a^3)*log(-sin(d*x + c) + 1) - 2*(4*a^3*cos
(d*x + c)^2 - 3*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2
*d)
```

giac [B] time = 0.30, size = 209, normalized size = 2.18

$$\frac{6\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,a^3\tan\left(\frac{1}{2}\,dx\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (6*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*a^3*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) - (9*a^3*tan(1/2*d*x + 1/2*c)^4 + 6*a^3*tan(1/2*d*x + 1/2*c)^3 + 20
*a^3*tan(1/2*d*x + 1/2*c)^2 + 6*a^3*tan(1/2*d*x + 1/2*c) + 9*a^3)/(tan(1/2*
d*x + 1/2*c)^2 + 1)^2 + (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 106*a^3*tan(1/2*d*
x + 1/2*c)^3 + 164*a^3*tan(1/2*d*x + 1/2*c)^2 - 106*a^3*tan(1/2*d*x + 1/2*c
) + 25*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

maple [B] time = 0.30, size = 309, normalized size = 3.22

$$\frac{a^3\left(\sin^8(dx+c)\right)}{4d\cos(dx+c)^4}-\frac{a^3\left(\sin^8(dx+c)\right)}{2d\cos(dx+c)^2}-\frac{a^3\left(\sin^6(dx+c)\right)}{2d}-\frac{3a^3\left(\sin^4(dx+c)\right)}{4d}-\frac{3a^3\left(\sin^2(dx+c)\right)}{2d}-\frac{6a^3\ln(\cos(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*a^3*sin(d*x+c)^8/cos(d*x+c)^2-1/2
*a^3*sin(d*x+c)^6/d-3/4*a^3*sin(d*x+c)^4/d-3/2*a^3*sin(d*x+c)^2/d-6/d*a^3*l
n(cos(d*x+c))+3/4/d*a^3*sin(d*x+c)^7/cos(d*x+c)^4-9/8/d*a^3*sin(d*x+c)^7/co
s(d*x+c)^2-9/8*a^3*sin(d*x+c)^5/d-2*a^3*sin(d*x+c)^3/d-6*a^3*sin(d*x+c)/d+6
/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a^3*tan(d*x+c)^4-3/2/d*a^3*tan(d*x+c
)^2+1/4/d*a^3*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a^3*sin(d*x+c)^5/cos(d*x+c)^2
```

maxima [A] time = 0.31, size = 82, normalized size = 0.85

$$-\frac{a^3\sin(dx+c)^2+12\,a^3\log(\sin(dx+c)-1)+6\,a^3\sin(dx+c)-\frac{8\,a^3\sin(dx+c)-7\,a^3}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(a^3*sin(d*x + c)^2 + 12*a^3*log(sin(d*x + c) - 1) + 6*a^3*sin(d*x + c
) - (8*a^3*sin(d*x + c) - 7*a^3)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

mupad [B] time = 10.93, size = 263, normalized size = 2.74

$$\frac{6\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{12\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-36\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+52\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-64\,a^3\,\tan\left(\frac{c}{2}+\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+14\,\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (6*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (52*a^3*tan(c/2 + (d*x)/2)^3 - 36
*a^3*tan(c/2 + (d*x)/2)^2 - 64*a^3*tan(c/2 + (d*x)/2)^4 + 52*a^3*tan(c/2 +
(d*x)/2)^5 - 36*a^3*tan(c/2 + (d*x)/2)^6 + 12*a^3*tan(c/2 + (d*x)/2)^7 + 12
*a^3*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)
- 12*tan(c/2 + (d*x)/2)^3 + 14*tan(c/2 + (d*x)/2)^4 - 12*tan(c/2 + (d*x)/2)
^5 + 8*tan(c/2 + (d*x)/2)^6 - 4*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8
 + 1)) - (12*a^3*log(tan(c/2 + (d*x)/2) - 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.872 $\int \sec^2(c+dx)(a+a\sin(c+dx))^3\tan^3(c+dx)\,dx$

Optimal. Leaf size=78

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{3a^4}{d(a-a\sin(c+dx))}-\frac{a^3\sin(c+dx)}{d}-\frac{3a^3\log(1-\sin(c+dx))}{d}$$

```
[Out] -3*a^3*ln(1-sin(d*x+c))/d-a^3*sin(d*x+c)/d+1/2*a^5/d/(a-a*sin(d*x+c))^2-3*a
^4/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.12, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{3a^4}{d(a-a\sin(c+dx))}-\frac{a^3\sin(c+dx)}{d}-\frac{3a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (-3*a^3*Log[1 - Sin[c + d*x]])/d - (a^3*Sin[c + d*x])/d + a^5/(2*d*(a - a*S
in[c + d*x])^2) - (3*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^3\tan^3(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x^3}{a^3(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{x^3}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(-1+\frac{a^3}{(a-x)^3}-\frac{3a^2}{(a-x)^2}+\frac{3a}{a-x}\right)dx, x, a\sin(c+dx\right.}{d}$$
$$= -\frac{3a^3\log(1-\sin(c+dx))}{d} - \frac{a^3\sin(c+dx)}{d} + \frac{a^5}{2d(a-a\sin}$$

Mathematica [A] time = 0.23, size = 53, normalized size = 0.68

$$-\frac{a^3\left(2\sin(c+dx)+\frac{5-6\sin(c+dx)}{(\sin(c+dx)-1)^2}+6\log(1-\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] -1/2*(a^3*(6*Log[1 - Sin[c + d*x]] + (5 - 6*Sin[c + d*x])/(-1 + Sin[c + d*x
])^2 + 2*Sin[c + d*x]))/d
```

fricas [A] time = 0.48, size = 110, normalized size = 1.41

$$\frac{4\,a^3\cos(dx+c)^2+a^3-6\left(a^3\cos(dx+c)^2+2\,a^3\sin(dx+c)-2\,a^3\right)\log(-\sin(dx+c)+1)-2\left(a^3\cos(dx+c\right.}{2\left(d\cos(dx+c)^2+2\,d\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/2*(4*a^3*cos(d*x + c)^2 + a^3 - 6*(a^3*cos(d*x + c)^2 + 2*a^3*sin(d*x + c
) - 2*a^3)*log(-sin(d*x + c) + 1) - 2*(a^3*cos(d*x + c)^2 + a^3)*sin(d*x +
c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

giac [B] time = 0.27, size = 178, normalized size = 2.28

$$\frac{6\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\frac{25}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*a^3*log(abs(tan(1/2*d*x + 1
/2*c) - 1)) - 2*(3*a^3*tan(1/2*d*x + 1/2*c)^2 + 2*a^3*tan(1/2*d*x + 1/2*c)
+ 3*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1) + (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 10
8*a^3*tan(1/2*d*x + 1/2*c)^3 + 170*a^3*tan(1/2*d*x + 1/2*c)^2 - 108*a^3*tan
(1/2*d*x + 1/2*c) + 25*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

maple [B] time = 0.32, size = 237, normalized size = 3.04

$$\frac{a^3\left(\sin^7(dx+c)\right)}{4d\cos(dx+c)^4}-\frac{3a^3\left(\sin^7(dx+c)\right)}{8d\cos(dx+c)^2}-\frac{3a^3\left(\sin^5(dx+c)\right)}{8d}-\frac{a^3\left(\sin^3(dx+c)\right)}{d}-\frac{3a^3\sin(dx+c)}{d}+\frac{3a^3\ln(\sec(dx-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a^3*sin(d*x+c)^7/cos(d*x+c)^2-3/8
*a^3*sin(d*x+c)^5/d-a^3*sin(d*x+c)^3/d-3*a^3*sin(d*x+c)/d+3/d*a^3*ln(sec(d*
x+c)+tan(d*x+c))+3/4/d*a^3*tan(d*x+c)^4-3/2/d*a^3*tan(d*x+c)^2-3/d*a^3*ln(c
os(d*x+c))+3/4/d*a^3*sin(d*x+c)^5/cos(d*x+c)^4-3/8/d*a^3*sin(d*x+c)^5/cos(d
*x+c)^2+1/4/d*a^3*sin(d*x+c)^4/cos(d*x+c)^4
```

maxima [A] time = 0.35, size = 70, normalized size = 0.90

$$-\frac{6\,a^3\log\left(\sin\left(dx+c\right)-1\right)+2\,a^3\sin\left(dx+c\right)-\frac{6\,a^3\sin(dx+c)-5\,a^3}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(6*a^3*log(sin(d*x + c) - 1) + 2*a^3*sin(d*x + c) - (6*a^3*sin(d*x + c
) - 5*a^3)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

mupad [B] time = 10.13, size = 205, normalized size = 2.63

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{6\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d}-\frac{6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-18\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+20\,a^3\tan}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (6*a^3*log(tan(c/2 + (d*x)/2) - 1
))/d - (20*a^3*tan(c/2 + (d*x)/2)^3 - 18*a^3*tan(c/2 + (d*x)/2)^2 - 18*a^3*
tan(c/2 + (d*x)/2)^4 + 6*a^3*tan(c/2 + (d*x)/2)^5 + 6*a^3*tan(c/2 + (d*x)/2
))/(d*(7*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 8*tan(c/2 + (d*x)/2)
^3 + 7*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^6
 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.873 $\int \sec^3(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=64

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{2a^4}{d(a-a\sin(c+dx))}-\frac{a^3\log(1-\sin(c+dx))}{d}$$

```
[Out] -a^3*ln(1-sin(d*x+c))/d+1/2*a^5/d/(a-a*sin(d*x+c))^2-2*a^4/d/(a-a*sin(d*x+c
))
```

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{2a^4}{d(a-a\sin(c+dx))}-\frac{a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -((a^3*Log[1 - Sin[c + d*x]])/d) + a^5/(2*d*(a - a*Sin[c + d*x])^2) - (2*a^
4)/(d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x^2}{a^2(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{x^2}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{a^2}{(a-x)^3}-\frac{2a}{(a-x)^2}+\frac{1}{a-x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^3\log(1-\sin(c+dx))}{d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{}{d(a-}$$

Mathematica [A] time = 0.14, size = 45, normalized size = 0.70

$$-\frac{a^3\left(\frac{3-4\sin(c+dx)}{(\sin(c+dx)-1)^2}+2\log(1-\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -1/2*(a^3*(2*Log[1 - Sin[c + d*x]] + (3 - 4*Sin[c + d*x])/(-1 + Sin[c + d*x
])^2))/d
```

fricas [A] time = 0.47, size = 86, normalized size = 1.34

$$-\frac{4\,a^3\sin(dx+c)-3\,a^3+2\left(a^3\cos(dx+c)^2+2\,a^3\sin(dx+c)-2\,a^3\right)\log(-\sin(dx+c)+1)}{2\left(d\cos(dx+c)^2+2\,d\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/2*(4*a^3*sin(d*x + c) - 3*a^3 + 2*(a^3*cos(d*x + c)^2 + 2*a^3*sin(d*x +
c) - 2*a^3)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) -
2*d)
```

giac [A] time = 0.27, size = 125, normalized size = 1.95

$$\frac{6\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{25\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-112\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+186\,a^3\,\text{ta}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(6*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*a^3*log(abs(tan(1/2*d*x + 1
/2*c) - 1)) + (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 112*a^3*tan(1/2*d*x + 1/2*c)
^3 + 186*a^3*tan(1/2*d*x + 1/2*c)^2 - 112*a^3*tan(1/2*d*x + 1/2*c) + 25*a^3
)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

maple [B] time = 0.29, size = 220, normalized size = 3.44

$$\frac{a^3\left(\tan^4(dx+c)\right)}{4d}-\frac{a^3\left(\tan^2(dx+c)\right)}{2d}-\frac{a^3\ln(\cos(dx+c))}{d}+\frac{3a^3\left(\sin^5(dx+c)\right)}{4d\cos(dx+c)^4}-\frac{3a^3\left(\sin^5(dx+c)\right)}{8d\cos(dx+c)^2}-\frac{3a^3\left(\sin^3(d\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*tan(d*x+c)^4-1/2/d*a^3*tan(d*x+c)^2-1/d*a^3*ln(cos(d*x+c))+3/4/d*
a^3*sin(d*x+c)^5/cos(d*x+c)^4-3/8/d*a^3*sin(d*x+c)^5/cos(d*x+c)^2-3/8*a^3*s
in(d*x+c)^3/d-a^3*sin(d*x+c)/d+1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a^3*
sin(d*x+c)^4/cos(d*x+c)^4+1/4/d*a^3*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a^3*sin
(d*x+c)^3/cos(d*x+c)^2
```

maxima [A] time = 0.31, size = 59, normalized size = 0.92

$$-\frac{2\,a^3\log\left(\sin\left(dx+c\right)-1\right)-\frac{4\,a^3\sin(dx+c)-3\,a^3}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(2*a^3*log(sin(d*x + c) - 1) - (4*a^3*sin(d*x + c) - 3*a^3)/(sin(d*x +
 c)^2 - 2*sin(d*x + c) + 1))/d
```

mupad [B] time = 9.71, size = 313, normalized size = 4.89

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\,a^3\,\left(2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)+a^3\,\left(4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)-8\,\ln\left(t\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] - (tan(c/2 + (d*x)/2)*(4*a^3*(2*log(tan(c/2 + (d*x)/2) - 1) - log(tan(c/2 +
 (d*x)/2)^2 + 1)) + a^3*(4*log(tan(c/2 + (d*x)/2)^2 + 1) - 8*log(tan(c/2 +
(d*x)/2) - 1) + 2)) + tan(c/2 + (d*x)/2)^3*(4*a^3*(2*log(tan(c/2 + (d*x)/2)
 - 1) - log(tan(c/2 + (d*x)/2)^2 + 1)) + a^3*(4*log(tan(c/2 + (d*x)/2)^2 +
1) - 8*log(tan(c/2 + (d*x)/2) - 1) + 2)) - tan(c/2 + (d*x)/2)^2*(6*a^3*(2*l
og(tan(c/2 + (d*x)/2) - 1) - log(tan(c/2 + (d*x)/2)^2 + 1)) + a^3*(6*log(ta
n(c/2 + (d*x)/2)^2 + 1) - 12*log(tan(c/2 + (d*x)/2) - 1) + 6)))/(d*(tan(c/2
 + (d*x)/2) - 1)^4) - (a^3*(2*log(tan(c/2 + (d*x)/2) - 1) - log(tan(c/2 + (
d*x)/2)^2 + 1)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.874 $\int \sec^4(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=31

$$\frac{a^5\sin^2(c+dx)}{2d(a-a\sin(c+dx))^2}$$

```
[Out] 1/2*a^5*sin(d*x+c)^2/d/(a-a*sin(d*x+c))^2
```

Rubi [A] time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 37}

$$\frac{a^5\sin^2(c+dx)}{2d(a-a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (a^5*Sin[c + d*x]^2)/(2*d*(a - a*Sin[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x}{a(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \frac{x}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\sin^2(c+dx)}{2d(a-a\sin(c+dx))^2}$$

Mathematica [A] time = 0.03, size = 30, normalized size = 0.97

$$\frac{a^3\sin^2(c+dx)}{2d(1-\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
[Out] (a^3*Sin[c + d*x]^2)/(2*d*(1 - Sin[c + d*x])^2)
```

fricas [A] time = 0.44, size = 44, normalized size = 1.42

$$-\frac{2\,a^3\sin(dx+c)-a^3}{2\left(d\cos(dx+c)^2+2\,d\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
[Out] -1/2*(2*a^3*sin(d*x + c) - a^3)/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

giac [A] time = 0.23, size = 32, normalized size = 1.03

$$\frac{2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] 2*a^3*tan(1/2*d*x + 1/2*c)^2/(d*(tan(1/2*d*x + 1/2*c) - 1)^4)
```

maple [B] time = 0.27, size = 154, normalized size = 4.97

$$\frac{a^3\left(\sin^5(dx+c)\right)}{4d\cos(dx+c)^4}-\frac{a^3\left(\sin^5(dx+c)\right)}{8d\cos(dx+c)^2}-\frac{a^3\left(\sin^3(dx+c)\right)}{8d}+\frac{3a^3\left(\sin^4(dx+c)\right)}{4d\cos(dx+c)^4}+\frac{3a^3\left(\sin^3(dx+c)\right)}{4d\cos(dx+c)^4}+\frac{3a^3\left(\sin^3(dx\right.}{8d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a^3*sin(d*x+c)^5/cos(d*x+c)^2-1/8
*a^3*sin(d*x+c)^3/d+3/4/d*a^3*sin(d*x+c)^4/cos(d*x+c)^4+3/4/d*a^3*sin(d*x+c
)^3/cos(d*x+c)^4+3/8/d*a^3*sin(d*x+c)^3/cos(d*x+c)^2+1/4/d*a^3/cos(d*x+c)^4
```

maxima [A] time = 0.33, size = 42, normalized size = 1.35

$$\frac{2\,a^3\sin(dx+c)-a^3}{2\left(\sin(dx+c)^2-2\,\sin(dx+c)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^3*sin(d*x + c) - a^3)/((sin(d*x + c)^2 - 2*sin(d*x + c) + 1)*d)
```

mupad [B] time = 9.29, size = 30, normalized size = 0.97

$$\frac{a^3\sin(c+d\,x)^2}{8\,d\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (a^3*sin(c + d*x)^2)/(8*d*cos(c/2 + pi/4 + (d*x)/2)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.875 $\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=77

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{a^4}{d(a-a\sin(c+dx))}-\frac{a^3\log(1-\sin(c+dx))}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

```
[Out] -a^3*ln(1-sin(d*x+c))/d+a^3*ln(sin(d*x+c))/d+1/2*a^5/d/(a-a*sin(d*x+c))^2+a
^4/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 44}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{a^4}{d(a-a\sin(c+dx))}-\frac{a^3\log(1-\sin(c+dx))}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Log[1 - Sin[c + d*x]])/d) + (a^3*Log[Sin[c + d*x]])/d + a^5/(2*d*(a
- a*Sin[c + d*x])^2) + a^4/(d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a}{(a-x)^3x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{1}{(a-x)^3x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(\frac{1}{a(a-x)^3}+\frac{1}{a^2(a-x)^2}+\frac{1}{a^3(a-x)}+\frac{1}{a^3x}\right)dx, x, a\sin(c+\right.}{d}$$
$$= -\frac{a^3\log(1-\sin(c+dx))}{d}+\frac{a^3\log(\sin(c+dx))}{d}+\frac{a}{2d(a-a\,\text{si}}$$

Mathematica [A] time = 0.23, size = 54, normalized size = 0.70

$$\frac{a^3\left(\frac{3-2\sin(c+dx)}{(\sin(c+dx)-1)^2}-2\log(1-\sin(c+dx))+2\log(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-2*Log[1 - Sin[c + d*x]] + 2*Log[Sin[c + d*x]] + (3 - 2*Sin[c + d*x])
/(-1 + Sin[c + d*x])^2))/(2*d)
```

fricas [A] time = 0.49, size = 126, normalized size = 1.64

$$\frac{2\,a^3\sin(dx+c)-3\,a^3+2\left(a^3\cos(dx+c)^2+2\,a^3\sin(dx+c)-2\,a^3\right)\log\left(\frac{1}{2}\,\sin(dx+c)\right)-2\left(a^3\cos(dx+c)^2+\right.}{2\left(d\cos(dx+c)^2+2\,d\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*sin(d*x + c) - 3*a^3 + 2*(a^3*cos(d*x + c)^2 + 2*a^3*sin(d*x + c
) - 2*a^3)*log(1/2*sin(d*x + c)) - 2*(a^3*cos(d*x + c)^2 + 2*a^3*sin(d*x +
c) - 2*a^3)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) -
2*d)
```

giac [A] time = 0.27, size = 123, normalized size = 1.60

$$-\frac{12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-6\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{25\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-76\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+114\,a^3\tan\left(\frac{1}{2}\,dx\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(12*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 6*a^3*log(abs(tan(1/2*d*x
 + 1/2*c))) - (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 76*a^3*tan(1/2*d*x + 1/2*c)^
3 + 114*a^3*tan(1/2*d*x + 1/2*c)^2 - 76*a^3*tan(1/2*d*x + 1/2*c) + 25*a^3)/
(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

maple [B] time = 0.61, size = 172, normalized size = 2.23

$$\frac{a^3\left(\sin^3(dx+c)\right)}{4d\cos(dx+c)^4}+\frac{a^3\left(\sin^3(dx+c)\right)}{8d\cos(dx+c)^2}+\frac{a^3\sin(dx+c)}{8d}+\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^3}{d\cos(dx+c)^4}+\frac{3a^3}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a^3*sin(d*x+c)^3/cos(d*x+c)^2+1/8
*a^3*sin(d*x+c)/d+1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^3/cos(d*x+c)^4+3/
4/d*a^3*tan(d*x+c)*sec(d*x+c)^3+9/8/d*a^3*tan(d*x+c)*sec(d*x+c)+1/2/d*a^3/c
os(d*x+c)^2+1/d*a^3*ln(tan(d*x+c))
```

maxima [A] time = 0.48, size = 70, normalized size = 0.91

$$-\frac{2\,a^3\log\left(\sin\left(dx+c\right)-1\right)-2\,a^3\log\left(\sin\left(dx+c\right)\right)+\frac{2\,a^3\sin(dx+c)-3\,a^3}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^3*log(sin(d*x + c) - 1) - 2*a^3*log(sin(d*x + c)) + (2*a^3*sin(d*
x + c) - 3*a^3)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

mupad [B] time = 9.12, size = 61, normalized size = 0.79

$$\frac{2\,a^3\,\mathrm{atanh}\left(2\,\sin\left(c+d\,x\right)-1\right)}{d}-\frac{a^3\,\sin\left(c+d\,x\right)-\frac{3\,a^3}{2}}{d\,\left({\sin\left(c+d\,x\right)}^2-2\,\sin\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^5*sin(c + d*x)),x)
```

```
[Out] (2*a^3*atanh(2*sin(c + d*x) - 1))/d - (a^3*sin(c + d*x) - (3*a^3)/2)/(d*(si
n(c + d*x)^2 - 2*sin(c + d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
[Out] Timed out
```

3.876 $\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=93

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{2a^4}{d(a-a\sin(c+dx))}-\frac{a^3\csc(c+dx)}{d}-\frac{3a^3\log(1-\sin(c+dx))}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

```
[Out] -a^3*csc(d*x+c)/d-3*a^3*ln(1-sin(d*x+c))/d+3*a^3*ln(sin(d*x+c))/d+1/2*a^5/d
/(a-a*sin(d*x+c))^2+2*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.16, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 44}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{2a^4}{d(a-a\sin(c+dx))}-\frac{a^3\csc(c+dx)}{d}-\frac{3a^3\log(1-\sin(c+dx))}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) - (3*a^3*Log[1 - Sin[c + d*x]])/d + (3*a^3*Log[Sin[
c + d*x]])/d + a^5/(2*d*(a - a*Sin[c + d*x])^2) + (2*a^4)/(d*(a - a*Sin[c +
 d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a^2}{(a-x)^3x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \frac{1}{(a-x)^3x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{1}{a^2(a-x)^3}+\frac{2}{a^3(a-x)^2}+\frac{3}{a^4(a-x)}+\frac{1}{a^3x^2}+\frac{3}{a^4x}\right)dx, x,$$
$$= -\frac{a^3\csc(c+dx)}{d}-\frac{3a^3\log(1-\sin(c+dx))}{d}+\frac{3a^3\log(\sin(c$$

Mathematica [A] time = 0.23, size = 63, normalized size = 0.68

$$\frac{a^3\left(-\frac{4}{\sin(c+dx)-1}+\frac{1}{(\sin(c+dx)-1)^2}-2\csc(c+dx)-6\log(1-\sin(c+dx))+6\log(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-2*Csc[c + d*x] - 6*Log[1 - Sin[c + d*x]] + 6*Log[Sin[c + d*x]] + (-1
 + Sin[c + d*x])^(-2) - 4/(-1 + Sin[c + d*x])))/(2*d)
```

fricas [A] time = 0.46, size = 185, normalized size = 1.99

$$\frac{6\,a^3\cos(dx+c)^2+9\,a^3\sin(dx+c)-8\,a^3+6\left(2\,a^3\cos(dx+c)^2-2\,a^3-\left(a^3\cos(dx+c)^2-2\,a^3\right)\sin(dx+c)\right)\text{l}}{2\left(2\,d\cos(dx+c)^2-\left(d\cos(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/2*(6*a^3*cos(d*x + c)^2 + 9*a^3*sin(d*x + c) - 8*a^3 + 6*(2*a^3*cos(d*x +
 c)^2 - 2*a^3 - (a^3*cos(d*x + c)^2 - 2*a^3)*sin(d*x + c))*log(1/2*sin(d*x
+ c)) - 6*(2*a^3*cos(d*x + c)^2 - 2*a^3 - (a^3*cos(d*x + c)^2 - 2*a^3)*sin(
d*x + c))*log(-sin(d*x + c) + 1))/(2*d*cos(d*x + c)^2 - (d*cos(d*x + c)^2 -
 2*d)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.31, size = 166, normalized size = 1.78

$$-\frac{12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-6\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{25}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(12*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 6*a^3*log(abs(tan(1/2*d*x
 + 1/2*c))) + a^3*tan(1/2*d*x + 1/2*c) + (6*a^3*tan(1/2*d*x + 1/2*c) + a^3)
/tan(1/2*d*x + 1/2*c) - (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 88*a^3*tan(1/2*d*x
 + 1/2*c)^3 + 130*a^3*tan(1/2*d*x + 1/2*c)^2 - 88*a^3*tan(1/2*d*x + 1/2*c)
+ 25*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

maple [A] time = 0.62, size = 176, normalized size = 1.89

$$\frac{a^3}{d\cos(dx+c)^4}+\frac{3a^3\tan(dx+c)\left(\sec^3(dx+c)\right)}{4d}+\frac{9a^3\tan(dx+c)\sec(dx+c)}{8d}+\frac{3a^3\ln(\sec(dx+c)+\tan(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*a^3/cos(d*x+c)^4+3/4/d*a^3*tan(d*x+c)*sec(d*x+c)^3+9/8/d*a^3*tan(d*x+c)
*sec(d*x+c)+3/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*a^3/cos(d*x+c)^2+3/d*a^
3*ln(tan(d*x+c))+1/4/d*a^3/sin(d*x+c)/cos(d*x+c)^4+5/8/d*a^3/sin(d*x+c)/cos
(d*x+c)^2-15/8/d*a^3/sin(d*x+c)
```

maxima [A] time = 0.32, size = 90, normalized size = 0.97

$$-\frac{6\,a^3\log(\sin(dx+c)-1)-6\,a^3\log(\sin(dx+c))+\frac{6\,a^3\sin(dx+c)^2-9\,a^3\sin(dx+c)+2\,a^3}{\sin(dx+c)^3-2\,\sin(dx+c)^2+\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(6*a^3*log(sin(d*x + c) - 1) - 6*a^3*log(sin(d*x + c)) + (6*a^3*sin(d*
x + c)^2 - 9*a^3*sin(d*x + c) + 2*a^3)/(sin(d*x + c)^3 - 2*sin(d*x + c)^2 +
 sin(d*x + c)))/d
```

mupad [B] time = 0.08, size = 80, normalized size = 0.86

$$\frac{6\,a^3\,\mathrm{atanh}\left(2\,\sin\left(c+d\,x\right)-1\right)}{d}-\frac{3\,a^3\,{\sin\left(c+d\,x\right)}^2-\frac{9\,a^3\,\sin\left(c+d\,x\right)}{2}+a^3}{d\,\left({\sin\left(c+d\,x\right)}^3-2\,{\sin\left(c+d\,x\right)}^2+\sin\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^5*sin(c + d*x)^2),x)
```

```
[Out] (6*a^3*atanh(2*sin(c + d*x) - 1))/d - (a^3 - (9*a^3*sin(c + d*x))/2 + 3*a^3
*sin(c + d*x)^2)/(d*(sin(c + d*x) - 2*sin(c + d*x)^2 + sin(c + d*x)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.877 $\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=111

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{3a^4}{d(a-a\sin(c+dx))}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}-\frac{6a^3\log(1-\sin(c+dx))}{d}+6a^3\,\mathrm{lc}$$

```
[Out] -3*a^3*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d-6*a^3*ln(1-sin(d*x+c))/d+6*a^3*l
n(sin(d*x+c))/d+1/2*a^5/d/(a-a*sin(d*x+c))^2+3*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 44}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{3a^4}{d(a-a\sin(c+dx))}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}-\frac{6a^3\log(1-\sin(c+dx))}{d}+6a^3\,\mathrm{lc}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (6*a^3*Log[1 - Sin[c
 + d*x]])/d + (6*a^3*Log[Sin[c + d*x]])/d + a^5/(2*d*(a - a*Sin[c + d*x])^2
) + (3*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a^3}{(a-x)^3x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\,\text{Subst}\left(\int \frac{1}{(a-x)^3x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\,\text{Subst}\left(\int \left(\frac{1}{a^3(a-x)^3} + \frac{3}{a^4(a-x)^2} + \frac{6}{a^5(a-x)} + \frac{1}{a^3x^3} + \frac{3}{a^4x^2} + \frac{6}{a^5x}\right)\right.}{d}$$
$$= -\frac{3a^3\csc(c+dx)}{d} - \frac{a^3\csc^2(c+dx)}{2d} - \frac{6a^3\log(1-\sin(c+dx}{d}$$

Mathematica [A] time = 0.79, size = 73, normalized size = 0.66

$$-\frac{a^3\left(\frac{6}{\sin(c+dx)-1} - \frac{1}{(\sin(c+dx)-1)^2} + \csc^2(c+dx) + 6\csc(c+dx) + 12\log(1-\sin(c+dx)) - 12\log(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*(a^3*(6*Csc[c + d*x] + Csc[c + d*x]^2 + 12*Log[1 - Sin[c + d*x]] - 12*
Log[Sin[c + d*x]] - (-1 + Sin[c + d*x])^(-2) + 6/(-1 + Sin[c + d*x])))/d
```

fricas [B] time = 0.47, size = 235, normalized size = 2.12

$$-\frac{18\,a^3\cos(dx+c)^2 - 17\,a^3 - 12\left(a^3\cos(dx+c)^4 - 3\,a^3\cos(dx+c)^2 + 2\,a^3 + 2\left(a^3\cos(dx+c)^2 - a^3\right)\sin(dx+\right.}{2\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/2*(18*a^3*cos(d*x + c)^2 - 17*a^3 - 12*(a^3*cos(d*x + c)^4 - 3*a^3*cos(d
*x + c)^2 + 2*a^3 + 2*(a^3*cos(d*x + c)^2 - a^3)*sin(d*x + c))*log(1/2*sin(
d*x + c)) + 12*(a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^2 + 2*a^3 + 2*(a^3*
cos(d*x + c)^2 - a^3)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 4*(3*a^3*cos(d
*x + c)^2 - 4*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4 - 3*d*cos(d*x + c)^2 + 2
*(d*cos(d*x + c)^2 - d)*sin(d*x + c) + 2*d)
```

giac [A] time = 0.33, size = 198, normalized size = 1.78

$$-\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+96\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-48\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 96*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 48*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 12*a^3*tan(1/2*d*x + 1/2*c) + (
72*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*
d*x + 1/2*c)^2 - 8*(25*a^3*tan(1/2*d*x + 1/2*c)^4 - 92*a^3*tan(1/2*d*x + 1/
2*c)^3 + 136*a^3*tan(1/2*d*x + 1/2*c)^2 - 92*a^3*tan(1/2*d*x + 1/2*c) + 25*
a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

maple [B] time = 0.69, size = 241, normalized size = 2.17

$$\frac{a^3\tan(dx+c)\left(\sec^3(dx+c)\right)}{4d}+\frac{3a^3\tan(dx+c)\sec(dx+c)}{8d}+\frac{6a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3a^3}{4d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^3*tan(d*x+c)*sec(d*x+c)+6/d*a^3*l
n(sec(d*x+c)+tan(d*x+c))+3/4/d*a^3/cos(d*x+c)^4+3/2/d*a^3/cos(d*x+c)^2+6/d*
a^3*ln(tan(d*x+c))+3/4/d*a^3/sin(d*x+c)/cos(d*x+c)^4+15/8/d*a^3/sin(d*x+c)/
cos(d*x+c)^2-45/8/d*a^3/sin(d*x+c)+1/4/d*a^3/sin(d*x+c)^2/cos(d*x+c)^4+3/4/
d*a^3/sin(d*x+c)^2/cos(d*x+c)^2-3/2/d*a^3/sin(d*x+c)^2
```

maxima [A] time = 0.32, size = 103, normalized size = 0.93

$$-\frac{12\,a^3\log(\sin(dx+c)-1)-12\,a^3\log(\sin(dx+c))+\frac{12\,a^3\sin(dx+c)^3-18\,a^3\sin(dx+c)^2+4\,a^3\sin(dx+c)+a^3}{\sin(dx+c)^4-2\,\sin(dx+c)^3+\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(12*a^3*log(sin(d*x + c) - 1) - 12*a^3*log(sin(d*x + c)) + (12*a^3*sin
(d*x + c)^3 - 18*a^3*sin(d*x + c)^2 + 4*a^3*sin(d*x + c) + a^3)/(sin(d*x +
c)^4 - 2*sin(d*x + c)^3 + sin(d*x + c)^2))/d
```

mupad [B] time = 0.09, size = 97, normalized size = 0.87

$$\frac{12\,a^3\,\mathrm{atanh}\left(2\,\sin\left(c+d\,x\right)-1\right)}{d}-\frac{6\,a^3\,\sin\left(c+d\,x\right)^3-9\,a^3\,\sin\left(c+d\,x\right)^2+2\,a^3\,\sin\left(c+d\,x\right)+\frac{a^3}{2}}{d\,\left(\sin\left(c+d\,x\right)^4-2\,\sin\left(c+d\,x\right)^3+\sin\left(c+d\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^5*sin(c + d*x)^3),x)
```

```
[Out] (12*a^3*atanh(2*sin(c + d*x) - 1))/d - (2*a^3*sin(c + d*x) + a^3/2 - 9*a^3*
sin(c + d*x)^2 + 6*a^3*sin(c + d*x)^3)/(d*(sin(c + d*x)^2 - 2*sin(c + d*x)^
3 + sin(c + d*x)^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.878 $\int \frac{\sin^4(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=236

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{3a^2}{16d(a\sin(c+dx)+a)^3}+\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{2ad}-\frac{}{128d(a}$$

```
[Out] 515/256*ln(1-sin(d*x+c))/a/d-1795/256*ln(1+sin(d*x+c))/a/d+5*sin(d*x+c)/a/d
-1/2*sin(d*x+c)^2/a/d+1/3*sin(d*x+c)^3/a/d+1/96*a^2/d/(a-a*sin(d*x+c))^3-17
/128*a/d/(a-a*sin(d*x+c))^2+125/128/d/(a-a*sin(d*x+c))+1/64*a^3/d/(a+a*sin(
d*x+c))^4-3/16*a^2/d/(a+a*sin(d*x+c))^3+71/64*a/d/(a+a*sin(d*x+c))^2-5/d/(a
+a*sin(d*x+c))
```

Rubi [A] time = 0.25, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{3a^2}{16d(a\sin(c+dx)+a)^3}+\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{2ad}-\frac{}{128d(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^4*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (515*Log[1 - Sin[c + d*x]])/(256*a*d) - (1795*Log[1 + Sin[c + d*x]])/(256*a
*d) + (5*Sin[c + d*x])/(a*d) - Sin[c + d*x]^2/(2*a*d) + Sin[c + d*x]^3/(3*a
*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) - (17*a)/(128*d*(a - a*Sin[c + d*x]
)^2) + 125/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*(a + a*Sin[c + d*x])^4)
 - (3*a^2)/(16*d*(a + a*Sin[c + d*x])^3) + (71*a)/(64*d*(a + a*Sin[c + d*x]
)^2) - 5/(d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{x^{11}}{a^{11}(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^{11}}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(5a^2+\frac{a^6}{32(a-x)^4}-\frac{17a^5}{64(a-x)^3}+\frac{125a^4}{128(a-x)^2}-\frac{515a^3}{256(a-x)}-ax+x^2-\frac{a^7}{16(a+x)^5}+\right.\right.}{a^4d}$$

$$= \frac{515\log(1-\sin(c+dx))}{256ad}-\frac{1795\log(1+\sin(c+dx))}{256ad}+\frac{5\sin(c+dx)}{ad}-\frac{\sin^2(c}{2a}$$

Mathematica [A] time = 6.13, size = 153, normalized size = 0.65

$$\frac{256\sin^3(c+dx)-384\sin^2(c+dx)+3840\sin(c+dx)+\frac{750}{1-\sin(c+dx)}-\frac{3840}{\sin(c+dx)+1}-\frac{102}{(1-\sin(c+dx))^2}+\frac{852}{(\sin(c+dx)+1)^2}+}{768ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^4*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (1545*Log[1 - Sin[c + d*x]] - 5385*Log[1 + Sin[c + d*x]] + 8/(1 - Sin[c + d
*x])^3 - 102/(1 - Sin[c + d*x])^2 + 750/(1 - Sin[c + d*x]) + 3840*Sin[c + d
*x] - 384*Sin[c + d*x]^2 + 256*Sin[c + d*x]^3 + 12/(1 + Sin[c + d*x])^4 - 1
44/(1 + Sin[c + d*x])^3 + 852/(1 + Sin[c + d*x])^2 - 3840/(1 + Sin[c + d*x]
))/(768*a*d)
```

fricas [A] time = 0.54, size = 207, normalized size = 0.88

$$\frac{256\cos(dx+c)^{10}-3968\cos(dx+c)^{8}-686\cos(dx+c)^{6}+2810\cos(dx+c)^{4}-796\cos(dx+c)^{2}-5385\left(\text{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/768*(256*cos(d*x + c)^10 - 3968*cos(d*x + c)^8 - 686*cos(d*x + c)^6 + 281
0*cos(d*x + c)^4 - 796*cos(d*x + c)^2 - 5385*(cos(d*x + c)^6*sin(d*x + c) +
 cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 1545*(cos(d*x + c)^6*sin(d*x + c)
+ cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(64*cos(d*x + c)^8 + 1952*cos(
d*x + c)^6 + 375*cos(d*x + c)^4 - 70*cos(d*x + c)^2 + 8)*sin(d*x + c) + 112
)/(a*d*cos(d*x + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

giac [A] time = 0.36, size = 179, normalized size = 0.76

$$-\frac{\frac{21540\log(|\sin(dx+c)+1|)}{a}-\frac{6180\log(|\sin(dx+c)-1|)}{a}-\frac{512\left(2a^2\sin(dx+c)^3-3a^2\sin(dx+c)^2+30a^2\sin(dx+c)\right)}{a^3}+\frac{2\left(5665\sin(dx+c)^3-15495\,\mathrm{s}\right.}{a(\sin}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(21540*log(abs(sin(d*x + c) + 1))/a - 6180*log(abs(sin(d*x + c) - 1
))/a - 512*(2*a^2*sin(d*x + c)^3 - 3*a^2*sin(d*x + c)^2 + 30*a^2*sin(d*x +
c))/a^3 + 2*(5665*sin(d*x + c)^3 - 15495*sin(d*x + c)^2 + 14199*sin(d*x + c
) - 4353)/(a*(sin(d*x + c) - 1)^3) - (44875*sin(d*x + c)^4 + 164140*sin(d*x
 + c)^3 + 226578*sin(d*x + c)^2 + 139660*sin(d*x + c) + 32395)/(a*(sin(d*x
+ c) + 1)^4))/d
```

maple [A] time = 0.48, size = 208, normalized size = 0.88

$$-\frac{1}{96ad\left(\sin(dx+c)-1\right)^3}-\frac{17}{128ad\left(\sin(dx+c)-1\right)^2}-\frac{125}{128ad\left(\sin(dx+c)-1\right)}+\frac{515\ln\left(\sin(dx+c)-1\right)}{256ad}+\frac{\sin^3(d}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^11/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3-17/128/a/d/(sin(d*x+c)-1)^2-125/128/a/d/(sin(d*x
+c)-1)+515/256/a/d*ln(sin(d*x+c)-1)+1/3*sin(d*x+c)^3/d/a-1/2*sin(d*x+c)^2/a
/d+5*sin(d*x+c)/a/d+1/64/a/d/(1+sin(d*x+c))^4-3/16/a/d/(1+sin(d*x+c))^3+71/
64/a/d/(1+sin(d*x+c))^2-5/a/d/(1+sin(d*x+c))-1795/256*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.33, size = 209, normalized size = 0.89

$$-\frac{\frac{2\left(2295\sin(dx+c)^6+375\sin(dx+c)^5-5480\sin(dx+c)^4-680\sin(dx+c)^3+4473\sin(dx+c)^2+313\sin(dx+c)-1232\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{128\left(2\sin(dx+c)^3-3\sin(dx+\right.}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/768*(2*(2295*sin(d*x + c)^6 + 375*sin(d*x + c)^5 - 5480*sin(d*x + c)^4 -
 680*sin(d*x + c)^3 + 4473*sin(d*x + c)^2 + 313*sin(d*x + c) - 1232)/(a*sin
(d*x + c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 +
3*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 128*(2*sin(
d*x + c)^3 - 3*sin(d*x + c)^2 + 30*sin(d*x + c))/a + 5385*log(sin(d*x + c)
+ 1)/a - 1545*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 10.64, size = 567, normalized size = 2.40

$$\frac{515\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{128\,a\,d}-\frac{1795\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{128\,a\,d}-\frac{-\frac{1155\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}}{64}-\frac{835\tan(}{3}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}-2\,a\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^11/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (515*log(tan(c/2 + (d*x)/2) - 1))/(128*a*d) - (1795*log(tan(c/2 + (d*x)/2)
+ 1))/(128*a*d) - ((3205*tan(c/2 + (d*x)/2)^3)/64 - (835*tan(c/2 + (d*x)/2)
^2)/32 - (1155*tan(c/2 + (d*x)/2))/64 + (305*tan(c/2 + (d*x)/2)^4)/4 + (41*
tan(c/2 + (d*x)/2)^5)/16 + (53*tan(c/2 + (d*x)/2)^6)/24 - (5521*tan(c/2 + (
d*x)/2)^7)/48 - (2387*tan(c/2 + (d*x)/2)^8)/12 + (6697*tan(c/2 + (d*x)/2)^9
)/96 + (6901*tan(c/2 + (d*x)/2)^10)/48 + (6697*tan(c/2 + (d*x)/2)^11)/96 -
(2387*tan(c/2 + (d*x)/2)^12)/12 - (5521*tan(c/2 + (d*x)/2)^13)/48 + (53*tan
(c/2 + (d*x)/2)^14)/24 + (41*tan(c/2 + (d*x)/2)^15)/16 + (305*tan(c/2 + (d*
x)/2)^16)/4 + (3205*tan(c/2 + (d*x)/2)^17)/64 - (835*tan(c/2 + (d*x)/2)^18)
/32 - (1155*tan(c/2 + (d*x)/2)^19)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 2*a
*tan(c/2 + (d*x)/2)^2 - 6*a*tan(c/2 + (d*x)/2)^3 - 3*a*tan(c/2 + (d*x)/2)^4
 + 8*a*tan(c/2 + (d*x)/2)^6 + 16*a*tan(c/2 + (d*x)/2)^7 + 2*a*tan(c/2 + (d*
x)/2)^8 - 12*a*tan(c/2 + (d*x)/2)^9 - 12*a*tan(c/2 + (d*x)/2)^10 - 12*a*tan
(c/2 + (d*x)/2)^11 + 2*a*tan(c/2 + (d*x)/2)^12 + 16*a*tan(c/2 + (d*x)/2)^13
 + 8*a*tan(c/2 + (d*x)/2)^14 - 3*a*tan(c/2 + (d*x)/2)^16 - 6*a*tan(c/2 + (d
*x)/2)^17 - 2*a*tan(c/2 + (d*x)/2)^18 + 2*a*tan(c/2 + (d*x)/2)^19 + a*tan(c
/2 + (d*x)/2)^20)) + (5*log(tan(c/2 + (d*x)/2)^2 + 1))/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**11/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.879 $\int \frac{\sin^3(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=220

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{6d(a\sin(c+dx)+a)^3}+\frac{\sin^2(c+dx)}{2ad}-\frac{15a}{128d(a-a\sin(c+dx$$

[Out] 325/256*ln(1-sin(d*x+c))/a/d+955/256*ln(1+sin(d*x+c))/a/d-sin(d*x+c)/a/d+1/2*sin(d*x+c)^2/a/d+1/96*a^2/d/(a-a*sin(d*x+c))^3-15/128*a/d/(a-a*sin(d*x+c))^2+95/128/d/(a-a*sin(d*x+c))-1/64*a^3/d/(a+a*sin(d*x+c))^4+1/6*a^2/d/(a+a*sin(d*x+c))^3-55/64*a/d/(a+a*sin(d*x+c))^2+105/32/d/(a+a*sin(d*x+c))

Rubi [A] time = 0.23, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{6d(a\sin(c+dx)+a)^3}+\frac{\sin^2(c+dx)}{2ad}-\frac{15a}{128d(a-a\sin(c+dx$$

Antiderivative was successfully verified.

[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]

[Out] (325*Log[1 - Sin[c + d*x]])/(256*a*d) + (955*Log[1 + Sin[c + d*x]])/(256*a*d) - Sin[c + d*x]/(a*d) + Sin[c + d*x]^2/(2*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) - (15*a)/(128*d*(a - a*Sin[c + d*x])^2) + 95/(128*d*(a - a*Sin[c + d*x])) - a^3/(64*d*(a + a*Sin[c + d*x])^4) + a^2/(6*d*(a + a*Sin[c + d*x])^3) - (55*a)/(64*d*(a + a*Sin[c + d*x])^2) + 105/(32*d*(a + a*Sin[c + d*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{x^{10}}{a^{10}(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^{10}}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$

$$= \frac{\text{Subst}\left(\int \left(-a+\frac{a^5}{32(a-x)^4}-\frac{15a^4}{64(a-x)^3}+\frac{95a^3}{128(a-x)^2}-\frac{325a^2}{256(a-x)}+x+\frac{a^6}{16(a+x)^5}-\frac{a^5}{2(a+x)^4}\right.\right.}{a^3 d}$$

$$= \frac{325\log(1-\sin(c+dx))}{256ad}+\frac{955\log(1+\sin(c+dx))}{256ad}-\frac{\sin(c+dx)}{ad}+\frac{\sin^2(c+}{2ad}$$

Mathematica [A] time = 6.15, size = 143, normalized size = 0.65

$$\frac{384\sin^2(c+dx)-768\sin(c+dx)+\frac{570}{1-\sin(c+dx)}+\frac{2520}{\sin(c+dx)+1}-\frac{90}{(1-\sin(c+dx))^2}-\frac{660}{(\sin(c+dx)+1)^2}+\frac{8}{(1-\sin(c+dx))^3}+\frac{12}{(\sin(c+d}}{768ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (975*Log[1 - Sin[c + d*x]] + 2865*Log[1 + Sin[c + d*x]] + 8/(1 - Sin[c + d*
x])^3 - 90/(1 - Sin[c + d*x])^2 + 570/(1 - Sin[c + d*x]) - 768*Sin[c + d*x]
 + 384*Sin[c + d*x]^2 - 12/(1 + Sin[c + d*x])^4 + 128/(1 + Sin[c + d*x])^3
- 660/(1 + Sin[c + d*x])^2 + 2520/(1 + Sin[c + d*x]))/(768*a*d)
```

fricas [A] time = 0.55, size = 197, normalized size = 0.90

$$\frac{384\cos(dx+c)^8+1374\cos(dx+c)^6+630\cos(dx+c)^4-132\cos(dx+c)^2+2865\left(\cos(dx+c)^6\sin(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/768*(384*cos(d*x + c)^8 + 1374*cos(d*x + c)^6 + 630*cos(d*x + c)^4 - 132*
cos(d*x + c)^2 + 2865*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(si
n(d*x + c) + 1) + 975*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-s
in(d*x + c) + 1) - 2*(192*cos(d*x + c)^8 + 288*cos(d*x + c)^6 - 945*cos(d*x
 + c)^4 + 330*cos(d*x + c)^2 - 56)*sin(d*x + c) + 16)/(a*d*cos(d*x + c)^6*s
in(d*x + c) + a*d*cos(d*x + c)^6)
```

giac [A] time = 0.36, size = 161, normalized size = 0.73

$$\frac{\frac{11460 \log(|\sin(dx+c)+1|)}{a} + \frac{3900 \log(|\sin(dx+c)-1|)}{a} + \frac{1536\left(a\sin(dx+c)^2 - 2a\sin(dx+c)\right)}{a^2} - \frac{2\left(3575\sin(dx+c)^3 - 9585\sin(dx+c)^2 + 8625\sin(dx\right.}{a(\sin(dx+c)-1)^3}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(11460*log(abs(sin(d*x + c) + 1))/a + 3900*log(abs(sin(d*x + c) - 1)
)/a + 1536*(a*sin(d*x + c)^2 - 2*a*sin(d*x + c))/a^2 - 2*(3575*sin(d*x + c)
^3 - 9585*sin(d*x + c)^2 + 8625*sin(d*x + c) - 2599)/(a*(sin(d*x + c) - 1)^
3) - (23875*sin(d*x + c)^4 + 85420*sin(d*x + c)^3 + 115650*sin(d*x + c)^2 +
 70028*sin(d*x + c) + 15971)/(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.47, size = 192, normalized size = 0.87

$$-\frac{1}{96ad(\sin(dx+c)-1)^3} - \frac{15}{128ad(\sin(dx+c)-1)^2} - \frac{95}{128ad(\sin(dx+c)-1)} + \frac{325\ln(\sin(dx+c)-1)}{256ad} + \frac{\sin^2(d}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^10/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3-15/128/a/d/(sin(d*x+c)-1)^2-95/128/a/d/(sin(d*x+
c)-1)+325/256/a/d*ln(sin(d*x+c)-1)+1/2*sin(d*x+c)^2/a/d-sin(d*x+c)/a/d-1/64
/a/d/(1+sin(d*x+c))^4+1/6/a/d/(1+sin(d*x+c))^3-55/64/a/d/(1+sin(d*x+c))^2+1
05/32/a/d/(1+sin(d*x+c))+955/256*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.40, size = 197, normalized size = 0.90

$$\frac{\frac{2\left(975\sin(dx+c)^6 - 945\sin(dx+c)^5 - 3240\sin(dx+c)^4 + 1560\sin(dx+c)^3 + 3489\sin(dx+c)^2 - 671\sin(dx+c) - 1232\right)}{a\sin(dx+c)^7 + a\sin(dx+c)^6 - 3a\sin(dx+c)^5 - 3a\sin(dx+c)^4 + 3a\sin(dx+c)^3 + 3a\sin(dx+c)^2 - a\sin(dx+c) - a} + \frac{384\left(\sin(dx+c)^2 - 2\sin(dx+c)\right)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/768*(2*(975*sin(d*x + c)^6 - 945*sin(d*x + c)^5 - 3240*sin(d*x + c)^4 + 1
560*sin(d*x + c)^3 + 3489*sin(d*x + c)^2 - 671*sin(d*x + c) - 1232)/(a*sin(
d*x + c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3
*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 384*(sin(d*x
 + c)^2 - 2*sin(d*x + c))/a + 2865*log(sin(d*x + c) + 1)/a + 975*log(sin(d*
x + c) - 1)/a)/d
```

mupad [B] time = 10.34, size = 512, normalized size = 2.33

$$\frac{325\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{128\,a\,d}+\frac{955\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{128\,a\,d}+\frac{-\frac{315\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{64}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}}{32}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}-3\,a\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^10/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (325*log(tan(c/2 + (d*x)/2) - 1))/(128*a*d) + (955*log(tan(c/2 + (d*x)/2) +
 1))/(128*a*d) + ((5*tan(c/2 + (d*x)/2)^2)/32 - (315*tan(c/2 + (d*x)/2))/64
 + (265*tan(c/2 + (d*x)/2)^3)/8 + (195*tan(c/2 + (d*x)/2)^4)/32 - (1217*tan
(c/2 + (d*x)/2)^5)/16 - (2389*tan(c/2 + (d*x)/2)^6)/96 + (1189*tan(c/2 + (d
*x)/2)^7)/24 + (767*tan(c/2 + (d*x)/2)^8)/32 + (6845*tan(c/2 + (d*x)/2)^9)/
96 + (767*tan(c/2 + (d*x)/2)^10)/32 + (1189*tan(c/2 + (d*x)/2)^11)/24 - (23
89*tan(c/2 + (d*x)/2)^12)/96 - (1217*tan(c/2 + (d*x)/2)^13)/16 + (195*tan(c
/2 + (d*x)/2)^14)/32 + (265*tan(c/2 + (d*x)/2)^15)/8 + (5*tan(c/2 + (d*x)/2
)^16)/32 - (315*tan(c/2 + (d*x)/2)^17)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2) -
 3*a*tan(c/2 + (d*x)/2)^2 - 8*a*tan(c/2 + (d*x)/2)^3 + 8*a*tan(c/2 + (d*x)/
2)^5 + 8*a*tan(c/2 + (d*x)/2)^6 + 8*a*tan(c/2 + (d*x)/2)^7 - 6*a*tan(c/2 +
(d*x)/2)^8 - 20*a*tan(c/2 + (d*x)/2)^9 - 6*a*tan(c/2 + (d*x)/2)^10 + 8*a*ta
n(c/2 + (d*x)/2)^11 + 8*a*tan(c/2 + (d*x)/2)^12 + 8*a*tan(c/2 + (d*x)/2)^13
 - 8*a*tan(c/2 + (d*x)/2)^15 - 3*a*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d
*x)/2)^17 + a*tan(c/2 + (d*x)/2)^18)) - (5*log(tan(c/2 + (d*x)/2)^2 + 1))/(
a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**10/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.880 $\int \frac{\sin^2(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=199

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{7a^2}{48d(a\sin(c+dx)+a)^3}-\frac{13a}{128d(a-a\sin(c+dx))^2}+\frac{}{64d(a\sin}$$

```
[Out] 187/256*ln(1-sin(d*x+c))/a/d-443/256*ln(1+sin(d*x+c))/a/d+sin(d*x+c)/a/d+1/
96*a^2/d/(a-a*sin(d*x+c))^3-13/128*a/d/(a-a*sin(d*x+c))^2+69/128/d/(a-a*sin
(d*x+c))+1/64*a^3/d/(a+a*sin(d*x+c))^4-7/48*a^2/d/(a+a*sin(d*x+c))^3+41/64*
a/d/(a+a*sin(d*x+c))^2-2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.21, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{7a^2}{48d(a\sin(c+dx)+a)^3}-\frac{13a}{128d(a-a\sin(c+dx))^2}+\frac{}{64d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (187*Log[1 - Sin[c + d*x]])/(256*a*d) - (443*Log[1 + Sin[c + d*x]])/(256*a*
d) + Sin[c + d*x]/(a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) - (13*a)/(128*d
*(a - a*Sin[c + d*x])^2) + 69/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*(a +
 a*Sin[c + d*x])^4) - (7*a^2)/(48*d*(a + a*Sin[c + d*x])^3) + (41*a)/(64*d*
(a + a*Sin[c + d*x])^2) - 2/(d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{x^9}{a^9(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^9}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^4}{32(a-x)^4}-\frac{13a^3}{64(a-x)^3}+\frac{69a^2}{128(a-x)^2}-\frac{187a}{256(a-x)}-\frac{a^5}{16(a+x)^5}+\frac{7a^4}{16(a+x)^4}-\frac{4}{32(}\right.\right.}{a^2d}$$

$$= \frac{187\log(1-\sin(c+dx))}{256ad}-\frac{443\log(1+\sin(c+dx))}{256ad}+\frac{\sin(c+dx)}{ad}+\frac{}{96d(a-}$$

Mathematica [A] time = 6.14, size = 133, normalized size = 0.67

$$\frac{768\sin(c+dx)+\frac{414}{1-\sin(c+dx)}-\frac{1536}{\sin(c+dx)+1}-\frac{78}{(1-\sin(c+dx))^2}+\frac{492}{(\sin(c+dx)+1)^2}+\frac{8}{(1-\sin(c+dx))^3}-\frac{112}{(\sin(c+dx)+1)^3}+\frac{12}{(\sin(c+dx)+1)}}{768ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (561*Log[1 - Sin[c + d*x]] - 1329*Log[1 + Sin[c + d*x]] + 8/(1 - Sin[c + d*
x])^3 - 78/(1 - Sin[c + d*x])^2 + 414/(1 - Sin[c + d*x]) + 768*Sin[c + d*x]
 + 12/(1 + Sin[c + d*x])^4 - 112/(1 + Sin[c + d*x])^3 + 492/(1 + Sin[c + d*
x])^2 - 1536/(1 + Sin[c + d*x]))/(768*a*d)
```

fricas [A] time = 0.52, size = 187, normalized size = 0.94

$$-\frac{768\cos(dx+c)^8+1182\cos(dx+c)^6-1674\cos(dx+c)^4+636\cos(dx+c)^2+1329\left(\cos(dx+c)^6\sin(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(768*cos(d*x + c)^8 + 1182*cos(d*x + c)^6 - 1674*cos(d*x + c)^4 + 63
6*cos(d*x + c)^2 + 1329*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(
```

```
sin(d*x + c) + 1) - 561*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(
-sin(d*x + c) + 1) - 2*(384*cos(d*x + c)^6 + 207*cos(d*x + c)^4 - 54*cos(d*
x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x + c)^6*sin(d*x + c) + a*d*co
s(d*x + c)^6)
```

giac [A] time = 0.41, size = 147, normalized size = 0.74

$$-\frac{\frac{5316\log\left(|\sin(dx+c)+1|\right)}{a}-\frac{2244\log\left(|\sin(dx+c)-1|\right)}{a}-\frac{3072\sin(dx+c)}{a}+\frac{2\left(2057\sin(dx+c)^3-5343\sin(dx+c)^2+4671\sin(dx+c)-1369\right)}{a(\sin(dx+c)-1)^3}-110\cdots}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(5316*log(abs(sin(d*x + c) + 1))/a - 2244*log(abs(sin(d*x + c) - 1)
)/a - 3072*sin(d*x + c)/a + 2*(2057*sin(d*x + c)^3 - 5343*sin(d*x + c)^2 +
4671*sin(d*x + c) - 1369)/(a*(sin(d*x + c) - 1)^3) - (11075*sin(d*x + c)^4
+ 38156*sin(d*x + c)^3 + 49986*sin(d*x + c)^2 + 29356*sin(d*x + c) + 6499)/
(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.46, size = 175, normalized size = 0.88

$$\frac{\sin(dx+c)}{ad}-\frac{1}{96ad\left(\sin(dx+c)-1\right)^3}-\frac{13}{128ad\left(\sin(dx+c)-1\right)^2}-\frac{69}{128ad\left(\sin(dx+c)-1\right)}+\frac{187\ln\left(\sin(dx+c)-\cdots\right)}{256ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] sin(d*x+c)/a/d-1/96/a/d/(sin(d*x+c)-1)^3-13/128/a/d/(sin(d*x+c)-1)^2-69/128
/a/d/(sin(d*x+c)-1)+187/256/a/d*ln(sin(d*x+c)-1)+1/64/a/d/(1+sin(d*x+c))^4-
7/48/a/d/(1+sin(d*x+c))^3+41/64/a/d/(1+sin(d*x+c))^2-2/a/d/(1+sin(d*x+c))-4
43/256*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.33, size = 186, normalized size = 0.93

$$-\frac{\frac{2\left(975\sin(dx+c)^6+207\sin(dx+c)^5-2088\sin(dx+c)^4-360\sin(dx+c)^3+1569\sin(dx+c)^2+161\sin(dx+c)-400\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}+\frac{1329\log\left(\sin(dx+c)+1\right)}{a}-561\cdots}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(975*sin(d*x + c)^6 + 207*sin(d*x + c)^5 - 2088*sin(d*x + c)^4 -
360*sin(d*x + c)^3 + 1569*sin(d*x + c)^2 + 161*sin(d*x + c) - 400)/(a*sin(d
```

```
*x + c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*
a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 1329*log(sin(
d*x + c) + 1)/a - 561*log(sin(d*x + c) - 1)/a - 768*sin(d*x + c)/a)/d
```

mupad [B] time = 10.14, size = 485, normalized size = 2.44

$$\frac{\frac{315\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64}+\frac{251\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{32}-\frac{1411\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}-\frac{607\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{16}+\cdots}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}-4\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-10\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+4\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+18\,a\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] ((315*tan(c/2 + (d*x)/2))/64 + (251*tan(c/2 + (d*x)/2)^2)/32 - (1411*tan(c/
2 + (d*x)/2)^3)/64 - (607*tan(c/2 + (d*x)/2)^4)/16 + (2183*tan(c/2 + (d*x)/
2)^5)/64 + (6287*tan(c/2 + (d*x)/2)^6)/96 - (2749*tan(c/2 + (d*x)/2)^7)/192
 - (803*tan(c/2 + (d*x)/2)^8)/24 - (2749*tan(c/2 + (d*x)/2)^9)/192 + (6287*
tan(c/2 + (d*x)/2)^10)/96 + (2183*tan(c/2 + (d*x)/2)^11)/64 - (607*tan(c/2
+ (d*x)/2)^12)/16 - (1411*tan(c/2 + (d*x)/2)^13)/64 + (251*tan(c/2 + (d*x)/
2)^14)/32 + (315*tan(c/2 + (d*x)/2)^15)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2)
- 4*a*tan(c/2 + (d*x)/2)^2 - 10*a*tan(c/2 + (d*x)/2)^3 + 4*a*tan(c/2 + (d*x
)/2)^4 + 18*a*tan(c/2 + (d*x)/2)^5 + 4*a*tan(c/2 + (d*x)/2)^6 - 10*a*tan(c/
2 + (d*x)/2)^7 - 10*a*tan(c/2 + (d*x)/2)^8 - 10*a*tan(c/2 + (d*x)/2)^9 + 4*
a*tan(c/2 + (d*x)/2)^10 + 18*a*tan(c/2 + (d*x)/2)^11 + 4*a*tan(c/2 + (d*x)/
2)^12 - 10*a*tan(c/2 + (d*x)/2)^13 - 4*a*tan(c/2 + (d*x)/2)^14 + 2*a*tan(c/
2 + (d*x)/2)^15 + a*tan(c/2 + (d*x)/2)^16)) + (187*log(tan(c/2 + (d*x)/2) -
 1))/(128*a*d) - (443*log(tan(c/2 + (d*x)/2) + 1))/(128*a*d) + log(tan(c/2
+ (d*x)/2)^2 + 1)/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.881 $\int \frac{\sin(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=188

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{8d(a\sin(c+dx)+a)^3}-\frac{11a}{128d(a-a\sin(c+dx))^2}-\frac{}{64d(a\sin}$$

[Out] 93/256*ln(1-sin(d*x+c))/a/d+163/256*ln(1+sin(d*x+c))/a/d+1/96*a^2/d/(a-a*sin(d*x+c))^3-11/128*a/d/(a-a*sin(d*x+c))^2+47/128/d/(a-a*sin(d*x+c))-1/64*a^3/d/(a+a*sin(d*x+c))^4+1/8*a^2/d/(a+a*sin(d*x+c))^3-29/64*a/d/(a+a*sin(d*x+c))^2+35/32/d/(a+a*sin(d*x+c))

Rubi [A] time = 0.18, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{8d(a\sin(c+dx)+a)^3}-\frac{11a}{128d(a-a\sin(c+dx))^2}-\frac{}{64d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (93*Log[1 - Sin[c + d*x]])/(256*a*d) + (163*Log[1 + Sin[c + d*x]])/(256*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) - (11*a)/(128*d*(a - a*Sin[c + d*x])^2) + 47/(128*d*(a - a*Sin[c + d*x])) - a^3/(64*d*(a + a*Sin[c + d*x])^4) + a^2/(8*d*(a + a*Sin[c + d*x])^3) - (29*a)/(64*d*(a + a*Sin[c + d*x])^2) + 35/(32*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{x^8}{a^8(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^8}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^3}{32(a-x)^4} - \frac{11a^2}{64(a-x)^3} + \frac{47a}{128(a-x)^2} - \frac{93}{256(a-x)} + \frac{a^4}{16(a+x)^5} - \frac{3a^3}{8(a+x)^4} + \frac{29a^2}{32(a+x)^3} \right.\right.}{ad}$$

$$= \frac{93\log(1-\sin(c+dx))}{256ad} + \frac{163\log(1+\sin(c+dx))}{256ad} + \frac{a^2}{96d(a-a\sin(c+dx))^3} -$$

Mathematica [A] time = 3.78, size = 117, normalized size = 0.62

$$\frac{\frac{2\left(279\sin^6(c+dx)-489\sin^5(c+dx)-1000\sin^4(c+dx)+728\sin^3(c+dx)+1113\sin^2(c+dx)-295\sin(c+dx)-400\right)}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^4} + 279\log(1-\sin(c+dx)) + 48}{768ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (279*Log[1 - Sin[c + d*x]] + 489*Log[1 + Sin[c + d*x]] + (2*(-400 - 295*Sin
[c + d*x] + 1113*Sin[c + d*x]^2 + 728*Sin[c + d*x]^3 - 1000*Sin[c + d*x]^4
- 489*Sin[c + d*x]^5 + 279*Sin[c + d*x]^6))/((-1 + Sin[c + d*x])^3*(1 + Sin
[c + d*x])^4))/(768*a*d)
```

fricas [A] time = 0.53, size = 167, normalized size = 0.89

$$\frac{558\cos(dx+c)^6 + 326\cos(dx+c)^4 - 100\cos(dx+c)^2 + 489\left(\cos(dx+c)^6\sin(dx+c) + \cos(dx+c)^6\right)\log(}{768(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

[Out] 1/768*(558*cos(d*x + c)^6 + 326*cos(d*x + c)^4 - 100*cos(d*x + c)^2 + 489*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 279*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(4
89*cos(d*x + c)^4 - 250*cos(d*x + c)^2 + 56)*sin(d*x + c) + 16)/(a*d*cos(d*
x + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)

giac [A] time = 0.36, size = 136, normalized size = 0.72

$$\frac{\frac{1956 \log(|\sin(dx+c)+1|)}{a} + \frac{1116 \log(|\sin(dx+c)-1|)}{a} - \frac{2\left(1023 \sin(dx+c)^3 - 2505 \sin(dx+c)^2 + 2073 \sin(dx+c) - 575\right)}{a(\sin(dx+c)-1)^3} - \frac{4075 \sin(dx+c)^4 + 12940 \text{ s}}{}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^7*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/3072*(1956*log(abs(sin(d*x + c) + 1))/a + 1116*log(abs(sin(d*x + c) - 1))
/a - 2*(1023*sin(d*x + c)^3 - 2505*sin(d*x + c)^2 + 2073*sin(d*x + c) - 575
)/(a*(sin(d*x + c) - 1)^3) - (4075*sin(d*x + c)^4 + 12940*sin(d*x + c)^3 +
15762*sin(d*x + c)^2 + 8620*sin(d*x + c) + 1771)/(a*(sin(d*x + c) + 1)^4))/
d

maple [A] time = 0.45, size = 162, normalized size = 0.86

$$-\frac{1}{96ad\left(\sin\left(dx+c\right)-1\right)^{3}}-\frac{11}{128ad\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{47}{128ad\left(\sin\left(dx+c\right)-1\right)}+\frac{93\ln\left(\sin\left(dx+c\right)-1\right)}{256ad}-\frac{}{64ad\,(1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^7*sin(d*x+c)^8/(a+a*sin(d*x+c)),x)

[Out] -1/96/a/d/(sin(d*x+c)-1)^3-11/128/a/d/(sin(d*x+c)-1)^2-47/128/a/d/(sin(d*x+
c)-1)+93/256/a/d*ln(sin(d*x+c)-1)-1/64/a/d/(1+sin(d*x+c))^4+1/8/a/d/(1+sin(
d*x+c))^3-29/64/a/d/(1+sin(d*x+c))^2+35/32/a/d/(1+sin(d*x+c))+163/256*ln(1+
sin(d*x+c))/a/d

maxima [A] time = 0.32, size = 175, normalized size = 0.93

$$\frac{\frac{2\left(279 \sin(dx+c)^6 - 489 \sin(dx+c)^5 - 1000 \sin(dx+c)^4 + 728 \sin(dx+c)^3 + 1113 \sin(dx+c)^2 - 295 \sin(dx+c) - 400\right)}{a \sin(dx+c)^7 + a \sin(dx+c)^6 - 3 a \sin(dx+c)^5 - 3 a \sin(dx+c)^4 + 3 a \sin(dx+c)^3 + 3 a \sin(dx+c)^2 - a \sin(dx+c) - a} + \frac{489 \log(\sin(dx+c)+1)}{a} + \frac{279 \log}{}}{768\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^7*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/768*(2*(279*sin(d*x + c)^6 - 489*sin(d*x + c)^5 - 1000*sin(d*x + c)^4 + 7
28*sin(d*x + c)^3 + 1113*sin(d*x + c)^2 - 295*sin(d*x + c) - 400)/(a*sin(d*

```
x + c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a
*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 489*log(sin(d*
x + c) + 1)/a + 279*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 9.36, size = 432, normalized size = 2.30

$$\frac{-\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}+\frac{29\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{32}+\frac{629\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{96}-\frac{365\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{96}-\frac{5399}{\cdots}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-12\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+9\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+30\,a\,\mathrm{ta}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] ((29*tan(c/2 + (d*x)/2)^2)/32 - (35*tan(c/2 + (d*x)/2))/64 + (629*tan(c/2 +
 (d*x)/2)^3)/96 - (365*tan(c/2 + (d*x)/2)^4)/96 - (5399*tan(c/2 + (d*x)/2)^
5)/192 + (203*tan(c/2 + (d*x)/2)^6)/48 + (3019*tan(c/2 + (d*x)/2)^7)/48 + (
203*tan(c/2 + (d*x)/2)^8)/48 - (5399*tan(c/2 + (d*x)/2)^9)/192 - (365*tan(c
/2 + (d*x)/2)^10)/96 + (629*tan(c/2 + (d*x)/2)^11)/96 + (29*tan(c/2 + (d*x)
/2)^12)/32 - (35*tan(c/2 + (d*x)/2)^13)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2)
- 5*a*tan(c/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 9*a*tan(c/2 + (d*x
)/2)^4 + 30*a*tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 + (d*x)/2)^6 - 40*a*tan(c/
2 + (d*x)/2)^7 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*tan(c/2 + (d*x)/2)^9 + 9*a
*tan(c/2 + (d*x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11 - 5*a*tan(c/2 + (d*x)/2
)^12 + 2*a*tan(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x)/2)^14)) + (93*log(tan(
c/2 + (d*x)/2) - 1))/(128*a*d) + (163*log(tan(c/2 + (d*x)/2) + 1))/(128*a*d
) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.882 $\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{\tan^8(c+dx)}{8ad}-\frac{35\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^7(c+dx)\sec(c+dx)}{8ad}+\frac{7\tan^5(c+dx)\sec(c+dx)}{48ad}-\frac{35\tan^3(c+dx)\sec}{192ad}$$

```
[Out] -35/128*arctanh(sin(d*x+c))/a/d+35/128*sec(d*x+c)*tan(d*x+c)/a/d-35/192*sec
(d*x+c)*tan(d*x+c)^3/a/d+7/48*sec(d*x+c)*tan(d*x+c)^5/a/d-1/8*sec(d*x+c)*ta
n(d*x+c)^7/a/d+1/8*tan(d*x+c)^8/a/d
```

Rubi [A] time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\tan^8(c+dx)}{8ad}-\frac{35\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^7(c+dx)\sec(c+dx)}{8ad}+\frac{7\tan^5(c+dx)\sec(c+dx)}{48ad}-\frac{35\tan^3(c+dx)\sec}{192ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-35*ArcTanh[Sin[c + d*x]])/(128*a*d) + (35*Sec[c + d*x]*Tan[c + d*x])/(128
*a*d) - (35*Sec[c + d*x]*Tan[c + d*x]^3)/(192*a*d) + (7*Sec[c + d*x]*Tan[c
+ d*x]^5)/(48*a*d) - (Sec[c + d*x]*Tan[c + d*x]^7)/(8*a*d) + Tan[c + d*x]^8
/(8*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^7(c+dx)\,dx}{a}-\frac{\int \sec(c+dx)\tan^8(c+dx)\,dx}{a}\\ &= -\frac{\sec(c+dx)\tan^7(c+dx)}{8ad}+\frac{7\int \sec(c+dx)\tan^6(c+dx)\,dx}{8a}+\frac{\mathrm{Subst}\left(\int x^7\,dx,x,\tan(c\right.}{ad}\\ &= \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad}-\frac{\sec(c+dx)\tan^7(c+dx)}{8ad}+\frac{\tan^8(c+dx)}{8ad}-\frac{35\int \sec(c+d}{}\\ &= -\frac{35\sec(c+dx)\tan^3(c+dx)}{192ad}+\frac{7\sec(c+dx)\tan^5(c+dx)}{48ad}-\frac{\sec(c+dx)\tan^7(c+dx)}{8ad}\\ &= \frac{35\sec(c+dx)\tan(c+dx)}{128ad}-\frac{35\sec(c+dx)\tan^3(c+dx)}{192ad}+\frac{7\sec(c+dx)\tan^5(c+dx)}{48ad}\\ &= -\frac{35\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{35\sec(c+dx)\tan(c+dx)}{128ad}-\frac{35\sec(c+dx)\tan^3(c+dx)}{192ad}+\end{aligned}$$

Mathematica [A] time = 0.92, size = 101, normalized size = 0.78

$$-\frac{\frac{279\sin^6(c+dx)+87\sin^5(c+dx)-424\sin^4(c+dx)-136\sin^3(c+dx)+249\sin^2(c+dx)+57\sin(c+dx)-48}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^4}+105\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/384*(105*ArcTanh[Sin[c + d*x]] + (-48 + 57*Sin[c + d*x] + 249*Sin[c + d*
x]^2 - 136*Sin[c + d*x]^3 - 424*Sin[c + d*x]^4 + 87*Sin[c + d*x]^5 + 279*Si
n[c + d*x]^6)/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(a*d)
```

fricas [A] time = 0.50, size = 167, normalized size = 1.28

$$-\frac{558\,\cos\left(dx+c\right)^{6}-826\,\cos\left(dx+c\right)^{4}+476\,\cos\left(dx+c\right)^{2}+105\left(\cos\left(dx+c\right)^{6}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{6}\right)\log\ldots}{768\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(558*cos(d*x + c)^6 - 826*cos(d*x + c)^4 + 476*cos(d*x + c)^2 + 105*
(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) - 105*
(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(
87*cos(d*x + c)^4 - 38*cos(d*x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x
 + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

giac [A] time = 0.34, size = 136, normalized size = 1.05

$$-\frac{\frac{420\,\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a}-\frac{420\,\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a}+\frac{2\left(385\,\sin\left(dx+c\right)^{3}-807\,\sin\left(dx+c\right)^{2}+567\,\sin\left(dx+c\right)-129\right)}{a\left(\sin\left(dx+c\right)-1\right)^{3}}-\frac{875\,\sin\left(dx+c\right)^{4}+1964\,\sin\left(dx+\ldots\right.}{a\left(\mathrm{s}\ldots\right.}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/
a + 2*(385*sin(d*x + c)^3 - 807*sin(d*x + c)^2 + 567*sin(d*x + c) - 129)/(a
*(sin(d*x + c) - 1)^3) - (875*sin(d*x + c)^4 + 1964*sin(d*x + c)^3 + 1554*s
in(d*x + c)^2 + 396*sin(d*x + c) - 21)/(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.40, size = 162, normalized size = 1.25

$$-\frac{1}{96ad\left(\sin\left(dx+c\right)-1\right)^{3}}-\frac{9}{128ad\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{29}{128ad\left(\sin\left(dx+c\right)-1\right)}+\frac{35\ln\left(\sin\left(dx+c\right)-1\right)}{256ad}+\frac{\ldots}{64ad\left(1\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3-9/128/a/d/(sin(d*x+c)-1)^2-29/128/a/d/(sin(d*x+c
)-1)+35/256/a/d*ln(sin(d*x+c)-1)+1/64/a/d/(1+sin(d*x+c))^4-5/48/a/d/(1+sin(
d*x+c))^3+19/64/a/d/(1+sin(d*x+c))^2-1/2/a/d/(1+sin(d*x+c))-35/256*ln(1+sin
(d*x+c))/a/d
```

maxima [A] time = 0.32, size = 175, normalized size = 1.35

$$-\frac{\frac{2\left(279\,\sin\left(dx+c\right)^{6}+87\,\sin\left(dx+c\right)^{5}-424\,\sin\left(dx+c\right)^{4}-136\,\sin\left(dx+c\right)^{3}+249\,\sin\left(dx+c\right)^{2}+57\,\sin\left(dx+c\right)-48\right)}{a\sin\left(dx+c\right)^{7}+a\sin\left(dx+c\right)^{6}-3\,a\sin\left(dx+c\right)^{5}-3\,a\sin\left(dx+c\right)^{4}+3\,a\sin\left(dx+c\right)^{3}+3\,a\sin\left(dx+c\right)^{2}-a\sin\left(dx+c\right)-a}+\frac{105\,\log\left(\sin\left(dx+c\right)+1\right)}{a}-\frac{105\,\log\left(\mathrm{si}\ldots\right.}{\ldots}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(279*sin(d*x + c)^6 + 87*sin(d*x + c)^5 - 424*sin(d*x + c)^4 - 13
6*sin(d*x + c)^3 + 249*sin(d*x + c)^2 + 57*sin(d*x + c) - 48)/(a*sin(d*x +
c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin
(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 105*log(sin(d*x +
c) + 1)/a - 105*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 17.05, size = 388, normalized size = 2.98

$$\frac{\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{32}-\frac{245\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{96}-\frac{595\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{96}+\frac{791}{}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+30\,a\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] ((35*tan(c/2 + (d*x)/2))/64 + (35*tan(c/2 + (d*x)/2)^2)/32 - (245*tan(c/2 +
 (d*x)/2)^3)/96 - (595*tan(c/2 + (d*x)/2)^4)/96 + (791*tan(c/2 + (d*x)/2)^5
)/192 + (231*tan(c/2 + (d*x)/2)^6)/16 - (25*tan(c/2 + (d*x)/2)^7)/16 + (231
*tan(c/2 + (d*x)/2)^8)/16 + (791*tan(c/2 + (d*x)/2)^9)/192 - (595*tan(c/2 +
 (d*x)/2)^10)/96 - (245*tan(c/2 + (d*x)/2)^11)/96 + (35*tan(c/2 + (d*x)/2)^
12)/32 + (35*tan(c/2 + (d*x)/2)^13)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*
a*tan(c/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 9*a*tan(c/2 + (d*x)/2)
^4 + 30*a*tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 + (d*x)/2)^6 - 40*a*tan(c/2 +
(d*x)/2)^7 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*tan(c/2 + (d*x)/2)^9 + 9*a*tan
(c/2 + (d*x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11 - 5*a*tan(c/2 + (d*x)/2)^12
 + 2*a*tan(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x)/2)^14)) - (35*atanh(tan(c/
2 + (d*x)/2)))/(64*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.883 $\int \frac{\sec(c+dx)\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=134

$$-\frac{\tan^8(c+dx)}{8ad}-\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan^5(c+dx)\sec^3(c+dx)}{8ad}-\frac{5\tan^3(c+dx)\sec^3(c+dx)}{48ad}+\frac{5\tan(c+dx)\,\mathrm{se}}{64a}$$

```
[Out] -5/128*arctanh(sin(d*x+c))/a/d-5/128*sec(d*x+c)*tan(d*x+c)/a/d+5/64*sec(d*x
+c)^3*tan(d*x+c)/a/d-5/48*sec(d*x+c)^3*tan(d*x+c)^3/a/d+1/8*sec(d*x+c)^3*ta
n(d*x+c)^5/a/d-1/8*tan(d*x+c)^8/a/d
```

Rubi [A] time = 0.24, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2835, 2611, 3768, 3770, 2607, 30}

$$-\frac{\tan^8(c+dx)}{8ad}-\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan^5(c+dx)\sec^3(c+dx)}{8ad}-\frac{5\tan^3(c+dx)\sec^3(c+dx)}{48ad}+\frac{5\tan(c+dx)\,\mathrm{se}}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(128*a*d) - (5*Sec[c + d*x]*Tan[c + d*x])/(128*a
*d) + (5*Sec[c + d*x]^3*Tan[c + d*x])/(64*a*d) - (5*Sec[c + d*x]^3*Tan[c +
d*x]^3)/(48*a*d) + (Sec[c + d*x]^3*Tan[c + d*x]^5)/(8*a*d) - Tan[c + d*x]^8
/(8*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{a} - \frac{\int \sec^2(c+dx)\tan^7(c+dx)\,dx}{a}$$

$$= \frac{\sec^3(c+dx)\tan^5(c+dx)}{8ad} - \frac{5\int \sec^3(c+dx)\tan^4(c+dx)\,dx}{8a} - \frac{\text{Subst}\left(\int x^7\,dx, \cdots\right.}{a\cdots}$$

$$= -\frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad} + \frac{\sec^3(c+dx)\tan^5(c+dx)}{8ad} - \frac{\tan^8(c+dx)}{8ad} + \frac{5\int \cdots}{\cdots}$$

$$= \frac{5\sec^3(c+dx)\tan(c+dx)}{64ad} - \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad} + \frac{\sec^3(c+dx)\tan^5(c\cdots}{8ad}$$

$$= -\frac{5\sec(c+dx)\tan(c+dx)}{128ad} + \frac{5\sec^3(c+dx)\tan(c+dx)}{64ad} - \frac{5\sec^3(c+dx)\tan^3(\cdots}{48ad}$$

$$= -\frac{5\tanh^{-1}(\sin(c+dx))}{128ad} - \frac{5\sec(c+dx)\tan(c+dx)}{128ad} + \frac{5\sec^3(c+dx)\tan(c+dx\cdots}{64ad}$$

Mathematica [A] time = 0.90, size = 101, normalized size = 0.75

$$-\frac{\frac{-15\sin^6(c+dx)+177\sin^5(c+dx)+104\sin^4(c+dx)-184\sin^3(c+dx)-129\sin^2(c+dx)+63\sin(c+dx)+48}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^4}+15\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/384*(15*ArcTanh[Sin[c + d*x]] + (48 + 63*Sin[c + d*x] - 129*Sin[c + d*x]
^2 - 184*Sin[c + d*x]^3 + 104*Sin[c + d*x]^4 + 177*Sin[c + d*x]^5 - 15*Sin[
c + d*x]^6)/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(a*d)
```

fricas [A] time = 0.47, size = 167, normalized size = 1.25

$$\frac{30\cos(dx+c)^6+118\cos(dx+c)^4-68\cos(dx+c)^2-15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin}{768\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(30*cos(d*x + c)^6 + 118*cos(d*x + c)^4 - 68*cos(d*x + c)^2 - 15*(cos
(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*(cos(
d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(177*c
os(d*x + c)^4 - 170*cos(d*x + c)^2 + 56)*sin(d*x + c) + 16)/(a*d*cos(d*x +
c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

giac [A] time = 0.31, size = 136, normalized size = 1.01

$$-\frac{\frac{60\log(|\sin(dx+c)+1|)}{a}-\frac{60\log(|\sin(dx+c)-1|)}{a}+\frac{2\left(55\sin(dx+c)^3+15\sin(dx+c)^2-111\sin(dx+c)+57\right)}{a(\sin(dx+c)-1)^3}-\frac{125\sin(dx+c)^4+980\sin(dx+c)^3+16}{a(\sin(dx}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a
 + 2*(55*sin(d*x + c)^3 + 15*sin(d*x + c)^2 - 111*sin(d*x + c) + 57)/(a*(sin
(d*x + c) - 1)^3) - (125*sin(d*x + c)^4 + 980*sin(d*x + c)^3 + 1662*sin(d*x
 + c)^2 + 1140*sin(d*x + c) + 285)/(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.40, size = 162, normalized size = 1.21

$$-\frac{1}{96ad\left(\sin(dx+c)-1\right)^3}-\frac{7}{128ad\left(\sin(dx+c)-1\right)^2}-\frac{15}{128ad\left(\sin(dx+c)-1\right)}+\frac{5\ln\left(\sin(dx+c)-1\right)}{256ad}-\frac{}{64ad\,(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3-7/128/a/d/(sin(d*x+c)-1)^2-15/128/a/d/(sin(d*x+c
)-1)+5/256/a/d*ln(sin(d*x+c)-1)-1/64/a/d/(1+sin(d*x+c))^4+1/12/a/d/(1+sin(d
*x+c))^3-11/64/a/d/(1+sin(d*x+c))^2+5/32/a/d/(1+sin(d*x+c))-5/256*ln(1+sin(
d*x+c))/a/d
```

maxima [A] time = 0.32, size = 175, normalized size = 1.31

$$\frac{\frac{2\left(15 \sin(dx+c)^6-177 \sin(dx+c)^5-104 \sin(dx+c)^4+184 \sin(dx+c)^3+129 \sin(dx+c)^2-63 \sin(dx+c)-48\right)}{a \sin(dx+c)^7+a \sin(dx+c)^6-3 a \sin(dx+c)^5-3 a \sin(dx+c)^4+3 a \sin(dx+c)^3+3 a \sin(dx+c)^2-a \sin(dx+c)-a}-\frac{15 \log(\sin(dx+c)+1)}{a}+\frac{15 \log(\sin(dx+c}{a}}{768\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(15*sin(d*x + c)^6 - 177*sin(d*x + c)^5 - 104*sin(d*x + c)^4 + 184
*sin(d*x + c)^3 + 129*sin(d*x + c)^2 - 63*sin(d*x + c) - 48)/(a*sin(d*x + c
)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(
d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 15*log(sin(d*x + c)
 + 1)/a + 15*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 17.12, size = 388, normalized size = 2.90

$$\frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{32}-\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{96}-\frac{85\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{96}+\frac{113}{}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+30\,a\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] ((5*tan(c/2 + (d*x)/2))/64 + (5*tan(c/2 + (d*x)/2)^2)/32 - (35*tan(c/2 + (d
*x)/2)^3)/96 - (85*tan(c/2 + (d*x)/2)^4)/96 + (113*tan(c/2 + (d*x)/2)^5)/19
2 + (33*tan(c/2 + (d*x)/2)^6)/16 + (289*tan(c/2 + (d*x)/2)^7)/16 + (33*tan(
c/2 + (d*x)/2)^8)/16 + (113*tan(c/2 + (d*x)/2)^9)/192 - (85*tan(c/2 + (d*x)
/2)^10)/96 - (35*tan(c/2 + (d*x)/2)^11)/96 + (5*tan(c/2 + (d*x)/2)^12)/32 +
 (5*tan(c/2 + (d*x)/2)^13)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*a*tan(c/2
 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 9*a*tan(c/2 + (d*x)/2)^4 + 30*a
*tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 + (d*x)/2)^6 - 40*a*tan(c/2 + (d*x)/2)^
7 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*tan(c/2 + (d*x)/2)^9 + 9*a*tan(c/2 + (d
*x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11 - 5*a*tan(c/2 + (d*x)/2)^12 + 2*a*ta
```

```
n(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x)/2)^14)) - (5*atanh(tan(c/2 + (d*x)/
2)))/(64*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.884 $\int \frac{\sec^2(c+dx)\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=152

$$\frac{\tan^8(c+dx)}{8ad}+\frac{\tan^6(c+dx)}{6ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^5(c+dx)\sec^3(c+dx)}{8ad}+\frac{5\tan^3(c+dx)\sec^3(c+dx)}{48ad}-\frac{5\tan$$

```
[Out] 5/128*arctanh(sin(d*x+c))/a/d+5/128*sec(d*x+c)*tan(d*x+c)/a/d-5/64*sec(d*x+
c)^3*tan(d*x+c)/a/d+5/48*sec(d*x+c)^3*tan(d*x+c)^3/a/d-1/8*sec(d*x+c)^3*tan
(d*x+c)^5/a/d+1/6*tan(d*x+c)^6/a/d+1/8*tan(d*x+c)^8/a/d
```

Rubi [A] time = 0.24, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2835, 2607, 14, 2611, 3768, 3770}

$$\frac{\tan^8(c+dx)}{8ad}+\frac{\tan^6(c+dx)}{6ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^5(c+dx)\sec^3(c+dx)}{8ad}+\frac{5\tan^3(c+dx)\sec^3(c+dx)}{48ad}-\frac{5\tan$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(128*a*d) + (5*Sec[c + d*x]*Tan[c + d*x])/(128*a*
d) - (5*Sec[c + d*x]^3*Tan[c + d*x])/(64*a*d) + (5*Sec[c + d*x]^3*Tan[c + d
*x]^3)/(48*a*d) - (Sec[c + d*x]^3*Tan[c + d*x]^5)/(8*a*d) + Tan[c + d*x]^6/
(6*a*d) + Tan[c + d*x]^8/(8*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^4(c+dx)\tan^5(c+dx)\,dx}{a} - \frac{\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{a}$$

$$= -\frac{\sec^3(c+dx)\tan^5(c+dx)}{8ad} + \frac{5\int \sec^3(c+dx)\tan^4(c+dx)\,dx}{8a} + \frac{\mathrm{Subst}\left(\int x^5\right.}{}$$

$$= \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad} - \frac{\sec^3(c+dx)\tan^5(c+dx)}{8ad} - \frac{5\int \sec^3(c+dx)\,\mathrm{ta}}{16a}$$

$$= -\frac{5\sec^3(c+dx)\tan(c+dx)}{64ad} + \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad} - \frac{\sec^3(c+dx)\tan}{8ad}$$

$$= \frac{5\sec(c+dx)\tan(c+dx)}{128ad} - \frac{5\sec^3(c+dx)\tan(c+dx)}{64ad} + \frac{5\sec^3(c+dx)\tan^3(}{48ad}$$

$$= \frac{5\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{5\sec(c+dx)\tan(c+dx)}{128ad} - \frac{5\sec^3(c+dx)\tan(c+d}{64ad}$$

Mathematica [A] time = 1.12, size = 92, normalized size = 0.61

$$\frac{-\frac{15}{\sin(c+dx)-1}-\frac{15}{(\sin(c+dx)-1)^2}+\frac{30}{(\sin(c+dx)+1)^2}-\frac{4}{(\sin(c+dx)-1)^3}-\frac{24}{(\sin(c+dx)+1)^3}+\frac{6}{(\sin(c+dx)+1)^4}+15\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]] - 4/(-1 + Sin[c + d*x])^3 - 15/(-1 + Sin[c + d*x]
)^2 - 15/(-1 + Sin[c + d*x]) + 6/(1 + Sin[c + d*x])^4 - 24/(1 + Sin[c + d*x
])^3 + 30/(1 + Sin[c + d*x])^2)/(384*a*d)
```

fricas [A] time = 0.48, size = 167, normalized size = 1.10

$$-\frac{30\cos(dx+c)^6-266\cos(dx+c)^4+316\cos(dx+c)^2-15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\mathrm{si}}{768\left(ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(30*cos(d*x + c)^6 - 266*cos(d*x + c)^4 + 316*cos(d*x + c)^2 - 15*(c
os(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*(co
s(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(15*
cos(d*x + c)^4 - 22*cos(d*x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x +
c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

giac [A] time = 0.33, size = 136, normalized size = 0.89

$$\frac{\frac{60\log(|\sin(dx+c)+1|)}{a}-\frac{60\log(|\sin(dx+c)-1|)}{a}+\frac{2\left(55\sin(dx+c)^3-225\sin(dx+c)^2+225\sin(dx+c)-71\right)}{a(\sin(dx+c)-1)^3}-\frac{125\sin(dx+c)^4+500\sin(dx+c)^3+510\,\mathrm{s}}{a(\sin(dx+c)+}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a +
 2*(55*sin(d*x + c)^3 - 225*sin(d*x + c)^2 + 225*sin(d*x + c) - 71)/(a*(sin
(d*x + c) - 1)^3) - (125*sin(d*x + c)^4 + 500*sin(d*x + c)^3 + 510*sin(d*x
+ c)^2 + 212*sin(d*x + c) + 29)/(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.40, size = 144, normalized size = 0.95

$$-\frac{1}{96ad(\sin(dx+c)-1)^3}-\frac{5}{128ad(\sin(dx+c)-1)^2}-\frac{5}{128ad(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{256ad}+\frac{}{64ad(1+\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3-5/128/a/d/(sin(d*x+c)-1)^2-5/128/a/d/(sin(d*x+c)
-1)-5/256/a/d*ln(sin(d*x+c)-1)+1/64/a/d/(1+sin(d*x+c))^4-1/16/a/d/(1+sin(d*
x+c))^3+5/64/a/d/(1+sin(d*x+c))^2+5/256*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.32, size = 173, normalized size = 1.14

$$-\frac{\frac{2\left(15\sin(dx+c)^6+15\sin(dx+c)^5+88\sin(dx+c)^4-8\sin(dx+c)^3-63\sin(dx+c)^2+\sin(dx+c)+16\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{15\log(\sin(dx+c)+1)}{a}+\frac{15\log(\sin(\ldots}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(15*sin(d*x + c)^6 + 15*sin(d*x + c)^5 + 88*sin(d*x + c)^4 - 8*si
n(d*x + c)^3 - 63*sin(d*x + c)^2 + sin(d*x + c) + 16)/(a*sin(d*x + c)^7 + a
*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x + c
)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 15*log(sin(d*x + c) + 1)/a
 + 15*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 17.21, size = 388, normalized size = 2.55

$$\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,a\,d}+\frac{-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{32}+\frac{35\tan\left(\frac{c}{2}\ldots\right.}{96}\ldots}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(64*a*d) + ((35*tan(c/2 + (d*x)/2)^3)/96 - (5
*tan(c/2 + (d*x)/2)^2)/32 - (5*tan(c/2 + (d*x)/2))/64 + (85*tan(c/2 + (d*x)
/2)^4)/96 - (113*tan(c/2 + (d*x)/2)^5)/192 + (413*tan(c/2 + (d*x)/2)^6)/48
+ (157*tan(c/2 + (d*x)/2)^7)/48 + (413*tan(c/2 + (d*x)/2)^8)/48 - (113*tan(
c/2 + (d*x)/2)^9)/192 + (85*tan(c/2 + (d*x)/2)^10)/96 + (35*tan(c/2 + (d*x)
/2)^11)/96 - (5*tan(c/2 + (d*x)/2)^12)/32 - (5*tan(c/2 + (d*x)/2)^13)/64)/(
d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*a*tan(c/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (
d*x)/2)^3 + 9*a*tan(c/2 + (d*x)/2)^4 + 30*a*tan(c/2 + (d*x)/2)^5 - 5*a*tan(
c/2 + (d*x)/2)^6 - 40*a*tan(c/2 + (d*x)/2)^7 - 5*a*tan(c/2 + (d*x)/2)^8 + 3
0*a*tan(c/2 + (d*x)/2)^9 + 9*a*tan(c/2 + (d*x)/2)^10 - 12*a*tan(c/2 + (d*x)
/2)^11 - 5*a*tan(c/2 + (d*x)/2)^12 + 2*a*tan(c/2 + (d*x)/2)^13 + a*tan(c/2
+ (d*x)/2)^14))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.885 $\int \frac{\sec^3(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=150

$$-\frac{\tan^8(c+dx)}{8ad}-\frac{\tan^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan^3(c+dx)\sec^5(c+dx)}{8ad}-\frac{\tan(c+dx)\sec^5(c+dx)}{16ad}+\frac{\tan}{}$$

```
[Out] 3/128*arctanh(sin(d*x+c))/a/d+3/128*sec(d*x+c)*tan(d*x+c)/a/d+1/64*sec(d*x+
c)^3*tan(d*x+c)/a/d-1/16*sec(d*x+c)^5*tan(d*x+c)/a/d+1/8*sec(d*x+c)^5*tan(d
*x+c)^3/a/d-1/6*tan(d*x+c)^6/a/d-1/8*tan(d*x+c)^8/a/d
```

Rubi [A] time = 0.23, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2835, 2611, 3768, 3770, 2607, 14}

$$-\frac{\tan^8(c+dx)}{8ad}-\frac{\tan^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan^3(c+dx)\sec^5(c+dx)}{8ad}-\frac{\tan(c+dx)\sec^5(c+dx)}{16ad}+\frac{\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(128*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(128*a*
d) + (Sec[c + d*x]^3*Tan[c + d*x])/(64*a*d) - (Sec[c + d*x]^5*Tan[c + d*x])
/(16*a*d) + (Sec[c + d*x]^5*Tan[c + d*x]^3)/(8*a*d) - Tan[c + d*x]^6/(6*a*d
) - Tan[c + d*x]^8/(8*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^5(c+dx)\tan^4(c+dx)\,dx}{a} - \frac{\int \sec^4(c+dx)\tan^5(c+dx)\,dx}{a}\\
&= \frac{\sec^5(c+dx)\tan^3(c+dx)}{8ad} - \frac{3\int \sec^5(c+dx)\tan^2(c+dx)\,dx}{8a} - \frac{\text{Subst}\left(\int x^5 \left(1\right.\right.}{}\\
&= -\frac{\sec^5(c+dx)\tan(c+dx)}{16ad} + \frac{\sec^5(c+dx)\tan^3(c+dx)}{8ad} + \frac{\int \sec^5(c+dx)\,dx}{16a} -\\
&= \frac{\sec^3(c+dx)\tan(c+dx)}{64ad} - \frac{\sec^5(c+dx)\tan(c+dx)}{16ad} + \frac{\sec^5(c+dx)\tan^3(c+d}{8ad}\\
&= \frac{3\sec(c+dx)\tan(c+dx)}{128ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{64ad} - \frac{\sec^5(c+dx)\tan(c+d}{16ad}\\
&= \frac{3\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{3\sec(c+dx)\tan(c+dx)}{128ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{64ad}
\end{aligned}$$

Mathematica [A] time = 0.67, size = 101, normalized size = 0.67

$$\frac{\frac{-9\sin^6(c+dx)-9\sin^5(c+dx)+24\sin^4(c+dx)-72\sin^3(c+dx)-39\sin^2(c+dx)+25\sin(c+dx)+16}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^4}+9\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (9*ArcTanh[Sin[c + d*x]] + (16 + 25*Sin[c + d*x] - 39*Sin[c + d*x]^2 - 72*S
in[c + d*x]^3 + 24*Sin[c + d*x]^4 - 9*Sin[c + d*x]^5 - 9*Sin[c + d*x]^6)/((
-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(384*a*d)
```

fricas [A] time = 0.50, size = 167, normalized size = 1.11

$$-\frac{18\cos(dx+c)^6-6\cos(dx+c)^4+36\cos(dx+c)^2-9\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(\ldots}{768\left(ad\cos\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(18*cos(d*x + c)^6 - 6*cos(d*x + c)^4 + 36*cos(d*x + c)^2 - 9*(cos(d
*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 9*(cos(d*x
 + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(9*cos(d*
x + c)^4 - 90*cos(d*x + c)^2 + 56)*sin(d*x + c) - 16)/(a*d*cos(d*x + c)^6*s
in(d*x + c) + a*d*cos(d*x + c)^6)
```

giac [A] time = 0.31, size = 136, normalized size = 0.91

$$\frac{\frac{36\log(|\sin(dx+c)+1|)}{a}-\frac{36\log(|\sin(dx+c)-1|)}{a}+\frac{2\left(33\sin(dx+c)^3-87\sin(dx+c)^2+39\sin(dx+c)-1\right)}{a(\sin(dx+c)-1)^3}-\frac{75\sin(dx+c)^4+396\sin(dx+c)^3+786\sin(\ldots}{a(\sin(dx+c)+1\ldots}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(36*log(abs(sin(d*x + c) + 1))/a - 36*log(abs(sin(d*x + c) - 1))/a +
 2*(33*sin(d*x + c)^3 - 87*sin(d*x + c)^2 + 39*sin(d*x + c) - 1)/(a*(sin(d*
x + c) - 1)^3) - (75*sin(d*x + c)^4 + 396*sin(d*x + c)^3 + 786*sin(d*x + c)
^2 + 556*sin(d*x + c) + 139)/(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.39, size = 162, normalized size = 1.08

$$-\frac{1}{96ad(\sin(dx+c)-1)^3}-\frac{3}{128ad(\sin(dx+c)-1)^2}+\frac{1}{128ad(\sin(dx+c)-1)}-\frac{3\ln(\sin(dx+c)-1)}{256ad}-\frac{\ldots}{64ad(1+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3-3/128/a/d/(sin(d*x+c)-1)^2+1/128/a/d/(sin(d*x+c)
-1)-3/256/a/d*ln(sin(d*x+c)-1)-1/64/a/d/(1+sin(d*x+c))^4+1/24/a/d/(1+sin(d*
x+c))^3-1/64/a/d/(1+sin(d*x+c))^2-1/32/a/d/(1+sin(d*x+c))+3/256*ln(1+sin(d*
x+c))/a/d
```

maxima [A] time = 0.36, size = 175, normalized size = 1.17

$$-\frac{\frac{2\left(9\,\sin(dx+c)^{6}+9\,\sin(dx+c)^{5}-24\,\sin(dx+c)^{4}+72\,\sin(dx+c)^{3}+39\,\sin(dx+c)^{2}-25\,\sin(dx+c)-16\right)}{a\sin(dx+c)^{7}+a\sin(dx+c)^{6}-3\,a\sin(dx+c)^{5}-3\,a\sin(dx+c)^{4}+3\,a\sin(dx+c)^{3}+3\,a\sin(dx+c)^{2}-a\sin(dx+c)-a}-\frac{9\,\log(\sin(dx+c)+1)}{a}+\frac{9\,\log(\sin(dx+c}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(9*sin(d*x + c)^6 + 9*sin(d*x + c)^5 - 24*sin(d*x + c)^4 + 72*sin
(d*x + c)^3 + 39*sin(d*x + c)^2 - 25*sin(d*x + c) - 16)/(a*sin(d*x + c)^7 +
 a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x +
 c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 9*log(sin(d*x + c) + 1)/
a + 9*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 17.15, size = 388, normalized size = 2.59

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,a\,d}+\frac{-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{32}+\frac{7\,\tan\left(\frac{c}{2}+\right.}{32}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(64*a*d) + ((7*tan(c/2 + (d*x)/2)^3)/32 - (3*
tan(c/2 + (d*x)/2)^2)/32 - (3*tan(c/2 + (d*x)/2))/64 + (17*tan(c/2 + (d*x)/
2)^4)/32 + (387*tan(c/2 + (d*x)/2)^5)/64 + (43*tan(c/2 + (d*x)/2)^6)/48 + (
299*tan(c/2 + (d*x)/2)^7)/48 + (43*tan(c/2 + (d*x)/2)^8)/48 + (387*tan(c/2
+ (d*x)/2)^9)/64 + (17*tan(c/2 + (d*x)/2)^10)/32 + (7*tan(c/2 + (d*x)/2)^11
)/32 - (3*tan(c/2 + (d*x)/2)^12)/32 - (3*tan(c/2 + (d*x)/2)^13)/64)/(d*(a +
 2*a*tan(c/2 + (d*x)/2) - 5*a*tan(c/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2
)^3 + 9*a*tan(c/2 + (d*x)/2)^4 + 30*a*tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 +
(d*x)/2)^6 - 40*a*tan(c/2 + (d*x)/2)^7 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*ta
n(c/2 + (d*x)/2)^9 + 9*a*tan(c/2 + (d*x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11
```

```
 - 5*a*tan(c/2 + (d*x)/2)^12 + 2*a*tan(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x
)/2)^14))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.886 $\int \frac{\sec^4(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=150

$$\frac{\sec^8(c+dx)}{8ad}-\frac{\sec^6(c+dx)}{6ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^3(c+dx)\sec^5(c+dx)}{8ad}+\frac{\tan(c+dx)\sec^5(c+dx)}{16ad}-\frac{\tan(c+$$

```
[Out] -3/128*arctanh(sin(d*x+c))/a/d-1/6*sec(d*x+c)^6/a/d+1/8*sec(d*x+c)^8/a/d-3/
128*sec(d*x+c)*tan(d*x+c)/a/d-1/64*sec(d*x+c)^3*tan(d*x+c)/a/d+1/16*sec(d*x
+c)^5*tan(d*x+c)/a/d-1/8*sec(d*x+c)^5*tan(d*x+c)^3/a/d
```

Rubi [A] time = 0.22, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2835, 2606, 14, 2611, 3768, 3770}

$$\frac{\sec^8(c+dx)}{8ad}-\frac{\sec^6(c+dx)}{6ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^3(c+dx)\sec^5(c+dx)}{8ad}+\frac{\tan(c+dx)\sec^5(c+dx)}{16ad}-\frac{\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(128*a*d) - Sec[c + d*x]^6/(6*a*d) + Sec[c + d*x
]^8/(8*a*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(128*a*d) - (Sec[c + d*x]^3*Tan
[c + d*x])/(64*a*d) + (Sec[c + d*x]^5*Tan[c + d*x])/(16*a*d) - (Sec[c + d*x
]^5*Tan[c + d*x]^3)/(8*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^6(c+dx)\tan^3(c+dx)\,dx}{a} - \frac{\int \sec^5(c+dx)\tan^4(c+dx)\,dx}{a}$$

$$= -\frac{\sec^5(c+dx)\tan^3(c+dx)}{8ad} + \frac{3\int \sec^5(c+dx)\tan^2(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\int x^5 \right.}{}$$

$$= \frac{\sec^5(c+dx)\tan(c+dx)}{16ad} - \frac{\sec^5(c+dx)\tan^3(c+dx)}{8ad} - \frac{\int \sec^5(c+dx)\,dx}{16a} +$$

$$= -\frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{8ad} - \frac{\sec^3(c+dx)\tan(c+dx)}{64ad} + \frac{\sec^5(c+dx)\tan}{16ad}$$

$$= -\frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{8ad} - \frac{3\sec(c+dx)\tan(c+dx)}{128ad} - \frac{\sec^3(c+dx)\tan}{64ad}$$

$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{128ad} - \frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{8ad} - \frac{3\sec(c+dx)\tan(c}{128ad}$$

Mathematica [A] time = 0.85, size = 92, normalized size = 0.61

$$-\frac{-\frac{9}{\sin(c+dx)-1}+\frac{3}{(\sin(c+dx)-1)^2}+\frac{6}{(\sin(c+dx)+1)^2}+\frac{4}{(\sin(c+dx)-1)^3}+\frac{8}{(\sin(c+dx)+1)^3}-\frac{6}{(\sin(c+dx)+1)^4}+9\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/384*(9*ArcTanh[Sin[c + d*x]] + 4/(-1 + Sin[c + d*x])^3 + 3/(-1 + Sin[c +
 d*x])^2 - 9/(-1 + Sin[c + d*x]) - 6/(1 + Sin[c + d*x])^4 + 8/(1 + Sin[c +
d*x])^3 + 6/(1 + Sin[c + d*x])^2)/(a*d)
```

fricas [A] time = 0.48, size = 167, normalized size = 1.11

$$\frac{18\cos(dx+c)^6-6\cos(dx+c)^4-156\cos(dx+c)^2-9\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx}{768\left(ad\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(18*cos(d*x + c)^6 - 6*cos(d*x + c)^4 - 156*cos(d*x + c)^2 - 9*(cos(d
*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 9*(cos(d*x
 + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(9*cos(d*
x + c)^4 + 6*cos(d*x + c)^2 - 8)*sin(d*x + c) + 112)/(a*d*cos(d*x + c)^6*si
n(d*x + c) + a*d*cos(d*x + c)^6)
```

giac [A] time = 0.31, size = 136, normalized size = 0.91

$$-\frac{\frac{36\log(|\sin(dx+c)+1|)}{a}-\frac{36\log(|\sin(dx+c)-1|)}{a}+\frac{2\left(33\sin(dx+c)^3-135\sin(dx+c)^2+183\sin(dx+c)-65\right)}{a(\sin(dx+c)-1)^3}-\frac{75\sin(dx+c)^4+300\sin(dx+c)^3+402}{a(\sin(dx+c)}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(36*log(abs(sin(d*x + c) + 1))/a - 36*log(abs(sin(d*x + c) - 1))/a
+ 2*(33*sin(d*x + c)^3 - 135*sin(d*x + c)^2 + 183*sin(d*x + c) - 65)/(a*(si
n(d*x + c) - 1)^3) - (75*sin(d*x + c)^4 + 300*sin(d*x + c)^3 + 402*sin(d*x
+ c)^2 + 140*sin(d*x + c) + 11)/(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.39, size = 144, normalized size = 0.96

$$-\frac{1}{96ad(\sin(dx+c)-1)^3}-\frac{1}{128ad(\sin(dx+c)-1)^2}+\frac{3}{128ad(\sin(dx+c)-1)}+\frac{3\ln(\sin(dx+c)-1)}{256ad}+\frac{}{64ad(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3-1/128/a/d/(sin(d*x+c)-1)^2+3/128/a/d/(sin(d*x+c)
-1)+3/256/a/d*ln(sin(d*x+c)-1)+1/64/a/d/(1+sin(d*x+c))^4-1/48/a/d/(1+sin(d*
x+c))^3-1/64/a/d/(1+sin(d*x+c))^2-3/256*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.32, size = 175, normalized size = 1.17

$$\frac{\frac{2\left(9\sin(dx+c)^6+9\sin(dx+c)^5-24\sin(dx+c)^4-24\sin(dx+c)^3-57\sin(dx+c)^2+7\sin(dx+c)+16\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{9\log(\sin(dx+c)+1)}{a}+\frac{9\log(\sin(dx+c}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(9*sin(d*x + c)^6 + 9*sin(d*x + c)^5 - 24*sin(d*x + c)^4 - 24*sin(
d*x + c)^3 - 57*sin(d*x + c)^2 + 7*sin(d*x + c) + 16)/(a*sin(d*x + c)^7 + a
*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x + c
)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 9*log(sin(d*x + c) + 1)/a
+ 9*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 17.02, size = 388, normalized size = 2.59

$$\frac{\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{32}-\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{32}+\frac{111\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{32}+\frac{12}{}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+9a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+30a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] ((3*tan(c/2 + (d*x)/2))/64 + (3*tan(c/2 + (d*x)/2)^2)/32 - (7*tan(c/2 + (d*
x)/2)^3)/32 + (111*tan(c/2 + (d*x)/2)^4)/32 + (125*tan(c/2 + (d*x)/2)^5)/64
 + (277*tan(c/2 + (d*x)/2)^6)/48 - (43*tan(c/2 + (d*x)/2)^7)/48 + (277*tan(
c/2 + (d*x)/2)^8)/48 + (125*tan(c/2 + (d*x)/2)^9)/64 + (111*tan(c/2 + (d*x)
/2)^10)/32 - (7*tan(c/2 + (d*x)/2)^11)/32 + (3*tan(c/2 + (d*x)/2)^12)/32 +
(3*tan(c/2 + (d*x)/2)^13)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*a*tan(c/2
+ (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 9*a*tan(c/2 + (d*x)/2)^4 + 30*a*
tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 + (d*x)/2)^6 - 40*a*tan(c/2 + (d*x)/2)^7
 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*tan(c/2 + (d*x)/2)^9 + 9*a*tan(c/2 + (d*
x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11 - 5*a*tan(c/2 + (d*x)/2)^12 + 2*a*tan
(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x)/2)^14)) - (3*atanh(tan(c/2 + (d*x)/2
)))/(64*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.887 $\int \frac{\sec^5(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{\sec^8(c+dx)}{8ad}+\frac{\sec^6(c+dx)}{6ad}-\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan(c+dx)\sec^7(c+dx)}{8ad}-\frac{\tan(c+dx)\sec^5(c+dx)}{48ad}-\frac{5\tan$$

```
[Out] -5/128*arctanh(sin(d*x+c))/a/d+1/6*sec(d*x+c)^6/a/d-1/8*sec(d*x+c)^8/a/d-5/
128*sec(d*x+c)*tan(d*x+c)/a/d-5/192*sec(d*x+c)^3*tan(d*x+c)/a/d-1/48*sec(d*
x+c)^5*tan(d*x+c)/a/d+1/8*sec(d*x+c)^7*tan(d*x+c)/a/d
```

Rubi [A] time = 0.20, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2835, 2611, 3768, 3770, 2606, 14}

$$-\frac{\sec^8(c+dx)}{8ad}+\frac{\sec^6(c+dx)}{6ad}-\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan(c+dx)\sec^7(c+dx)}{8ad}-\frac{\tan(c+dx)\sec^5(c+dx)}{48ad}-\frac{5\tan$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(128*a*d) + Sec[c + d*x]^6/(6*a*d) - Sec[c + d*x
]^8/(8*a*d) - (5*Sec[c + d*x]*Tan[c + d*x])/(128*a*d) - (5*Sec[c + d*x]^3*T
an[c + d*x])/(192*a*d) - (Sec[c + d*x]^5*Tan[c + d*x])/(48*a*d) + (Sec[c +
d*x]^7*Tan[c + d*x])/(8*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^7(c+dx)\tan^2(c+dx)\,dx}{a} - \frac{\int \sec^6(c+dx)\tan^3(c+dx)\,dx}{a}$$

$$= \frac{\sec^7(c+dx)\tan(c+dx)}{8ad} - \frac{\int \sec^7(c+dx)\,dx}{8a} - \frac{\operatorname{Subst}\left(\int x^5\left(-1+x^2\right)\,dx, x, \text{se}\right.}{ad}$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{48ad} + \frac{\sec^7(c+dx)\tan(c+dx)}{8ad} - \frac{5\int \sec^5(c+dx)\,dx}{48a} -$$

$$= \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{8ad} - \frac{5\sec^3(c+dx)\tan(c+dx)}{192ad} - \frac{\sec^5(c+dx)\tan(c}{48ad}$$

$$= \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{8ad} - \frac{5\sec(c+dx)\tan(c+dx)}{128ad} - \frac{5\sec^3(c+dx)\tan(c}{192ad}$$

$$= -\frac{5\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{8ad} - \frac{5\sec(c+dx)\tan(c+}{128ad}$$

Mathematica [A] time = 0.52, size = 92, normalized size = 0.62

$$-\frac{-\frac{3}{\sin(c+dx)-1}-\frac{12}{\sin(c+dx)+1}-\frac{3}{(\sin(c+dx)-1)^2}-\frac{6}{(\sin(c+dx)+1)^2}+\frac{4}{(\sin(c+dx)-1)^3}+\frac{6}{(\sin(c+dx)+1)^4}+15\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/384*(15*ArcTanh[Sin[c + d*x]] + 4/(-1 + Sin[c + d*x])^3 - 3/(-1 + Sin[c
+ d*x])^2 - 3/(-1 + Sin[c + d*x]) + 6/(1 + Sin[c + d*x])^4 - 6/(1 + Sin[c +
 d*x])^2 - 12/(1 + Sin[c + d*x]))/(a*d)
```

fricas [A] time = 0.48, size = 167, normalized size = 1.13

$$\frac{30\cos(dx+c)^6-10\cos(dx+c)^4-4\cos(dx+c)^2-15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(d}{768\left(ad\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(30*cos(d*x + c)^6 - 10*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 15*(cos(d
*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*(cos(d*
x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(15*cos(
d*x + c)^4 + 10*cos(d*x + c)^2 - 56)*sin(d*x + c) + 16)/(a*d*cos(d*x + c)^6
*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

giac [A] time = 0.28, size = 136, normalized size = 0.92

$$-\frac{\frac{60\log(|\sin(dx+c)+1|)}{a}-\frac{60\log(|\sin(dx+c)-1|)}{a}+\frac{2\left(55\sin(dx+c)^3-177\sin(dx+c)^2+177\sin(dx+c)-39\right)}{a(\sin(dx+c)-1)^3}-\frac{125\sin(dx+c)^4+596\sin(dx+c)^3+1}{a(\sin(dx}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a
+ 2*(55*sin(d*x + c)^3 - 177*sin(d*x + c)^2 + 177*sin(d*x + c) - 39)/(a*(si
n(d*x + c) - 1)^3) - (125*sin(d*x + c)^4 + 596*sin(d*x + c)^3 + 1086*sin(d*
x + c)^2 + 884*sin(d*x + c) + 221)/(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.36, size = 144, normalized size = 0.97

$$-\frac{1}{96ad(\sin(dx+c)-1)^3}+\frac{1}{128ad(\sin(dx+c)-1)^2}+\frac{1}{128ad(\sin(dx+c)-1)}+\frac{5\ln(\sin(dx+c)-1)}{256ad}-\frac{}{64ad(1-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3+1/128/a/d/(sin(d*x+c)-1)^2+1/128/a/d/(sin(d*x+c)
-1)+5/256/a/d*ln(sin(d*x+c)-1)-1/64/a/d/(1+sin(d*x+c))^4+1/64/a/d/(1+sin(d*
x+c))^2+1/32/a/d/(1+sin(d*x+c))-5/256*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.31, size = 175, normalized size = 1.18

$$\frac{\frac{2\left(15\,\sin(dx+c)^6+15\,\sin(dx+c)^5-40\,\sin(dx+c)^4-40\,\sin(dx+c)^3+33\,\sin(dx+c)^2-31\,\sin(dx+c)-16\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3\,a\sin(dx+c)^5-3\,a\sin(dx+c)^4+3\,a\sin(dx+c)^3+3\,a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{15\,\log(\sin(dx+c)+1)}{a}+\frac{15\,\log(\sin(dx+c)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(15*sin(d*x + c)^6 + 15*sin(d*x + c)^5 - 40*sin(d*x + c)^4 - 40*si
n(d*x + c)^3 + 33*sin(d*x + c)^2 - 31*sin(d*x + c) - 16)/(a*sin(d*x + c)^7
+ a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x
+ c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 15*log(sin(d*x + c) + 1
)/a + 15*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 17.12, size = 388, normalized size = 2.62

$$\frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{32}+\frac{221\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{96}+\frac{43\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{96}+\frac{625}{}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+30\,a\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] ((5*tan(c/2 + (d*x)/2))/64 + (5*tan(c/2 + (d*x)/2)^2)/32 + (221*tan(c/2 + (
d*x)/2)^3)/96 + (43*tan(c/2 + (d*x)/2)^4)/96 + (625*tan(c/2 + (d*x)/2)^5)/1
92 + (35*tan(c/2 + (d*x)/2)^6)/48 + (355*tan(c/2 + (d*x)/2)^7)/48 + (35*tan
(c/2 + (d*x)/2)^8)/48 + (625*tan(c/2 + (d*x)/2)^9)/192 + (43*tan(c/2 + (d*x
)/2)^10)/96 + (221*tan(c/2 + (d*x)/2)^11)/96 + (5*tan(c/2 + (d*x)/2)^12)/32
 + (5*tan(c/2 + (d*x)/2)^13)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*a*tan(c
/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 9*a*tan(c/2 + (d*x)/2)^4 + 30
*a*tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 + (d*x)/2)^6 - 40*a*tan(c/2 + (d*x)/2
)^7 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*tan(c/2 + (d*x)/2)^9 + 9*a*tan(c/2 +
(d*x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11 - 5*a*tan(c/2 + (d*x)/2)^12 + 2*a*
tan(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x)/2)^14)) - (5*atanh(tan(c/2 + (d*x
)/2)))/(64*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.888 $\int \frac{\sec^6(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{\sec^8(c+dx)}{8ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan(c+dx)\sec^7(c+dx)}{8ad}+\frac{\tan(c+dx)\sec^5(c+dx)}{48ad}+\frac{5\tan(c+dx)\sec^3(c+$$

```
[Out] 5/128*arctanh(sin(d*x+c))/a/d+1/8*sec(d*x+c)^8/a/d+5/128*sec(d*x+c)*tan(d*x
+c)/a/d+5/192*sec(d*x+c)^3*tan(d*x+c)/a/d+1/48*sec(d*x+c)^5*tan(d*x+c)/a/d-
1/8*sec(d*x+c)^7*tan(d*x+c)/a/d
```

Rubi [A] time = 0.15, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2835, 2606, 30, 2611, 3768, 3770}

$$\frac{\sec^8(c+dx)}{8ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan(c+dx)\sec^7(c+dx)}{8ad}+\frac{\tan(c+dx)\sec^5(c+dx)}{48ad}+\frac{5\tan(c+dx)\sec^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^6*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(128*a*d) + Sec[c + d*x]^8/(8*a*d) + (5*Sec[c + d
*x]*Tan[c + d*x])/(128*a*d) + (5*Sec[c + d*x]^3*Tan[c + d*x])/(192*a*d) + (
Sec[c + d*x]^5*Tan[c + d*x])/(48*a*d) - (Sec[c + d*x]^7*Tan[c + d*x])/(8*a*
d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^6(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^8(c+dx)\tan(c+dx)\,dx}{a} - \frac{\int \sec^7(c+dx)\tan^2(c+dx)\,dx}{a} \\
&= -\frac{\sec^7(c+dx)\tan(c+dx)}{8ad} + \frac{\int \sec^7(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\int x^7\,dx, x, \sec(c+dx\right.}{ad} \\
&= \frac{\sec^8(c+dx)}{8ad} + \frac{\sec^5(c+dx)\tan(c+dx)}{48ad} - \frac{\sec^7(c+dx)\tan(c+dx)}{8ad} + \frac{5\int \sec}{} \\
&= \frac{\sec^8(c+dx)}{8ad} + \frac{5\sec^3(c+dx)\tan(c+dx)}{192ad} + \frac{\sec^5(c+dx)\tan(c+dx)}{48ad} - \frac{\sec^7(}{} \\
&= \frac{\sec^8(c+dx)}{8ad} + \frac{5\sec(c+dx)\tan(c+dx)}{128ad} + \frac{5\sec^3(c+dx)\tan(c+dx)}{192ad} + \frac{\sec^5(}{} \\
&= \frac{5\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{\sec^8(c+dx)}{8ad} + \frac{5\sec(c+dx)\tan(c+dx)}{128ad} + \frac{5\sec^3(c}{}
\end{aligned}$$

Mathematica [A] time = 0.95, size = 92, normalized size = 0.71

$$\frac{-\frac{15}{\sin(c+dx)-1}+\frac{9}{(\sin(c+dx)-1)^2}+\frac{6}{(\sin(c+dx)+1)^2}-\frac{4}{(\sin(c+dx)-1)^3}+\frac{8}{(\sin(c+dx)+1)^3}+\frac{6}{(\sin(c+dx)+1)^4}+15\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^6*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]] - 4/(-1 + Sin[c + d*x])^3 + 9/(-1 + Sin[c + d*x])
^2 - 15/(-1 + Sin[c + d*x]) + 6/(1 + Sin[c + d*x])^4 + 8/(1 + Sin[c + d*x])
^3 + 6/(1 + Sin[c + d*x])^2)/(384*a*d)
```

fricas [A] time = 0.49, size = 167, normalized size = 1.28

$$-\frac{30\cos(dx+c)^6-10\cos(dx+c)^4-4\cos(dx+c)^2-15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(d}{768\left(ad\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(30*cos(d*x + c)^6 - 10*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 15*(cos(
d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*(cos(d
*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(15*cos
(d*x + c)^4 + 10*cos(d*x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x + c)^
6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

giac [A] time = 0.29, size = 136, normalized size = 1.05

$$\frac{\frac{60\log(|\sin(dx+c)+1|)}{a}-\frac{60\log(|\sin(dx+c)-1|)}{a}+\frac{2\left(55\sin(dx+c)^3-225\sin(dx+c)^2+321\sin(dx+c)-167\right)}{a(\sin(dx+c)-1)^3}-\frac{125\sin(dx+c)^4+500\sin(dx+c)^3+702}{a(\sin(dx+c)}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a +
 2*(55*sin(d*x + c)^3 - 225*sin(d*x + c)^2 + 321*sin(d*x + c) - 167)/(a*(si
n(d*x + c) - 1)^3) - (125*sin(d*x + c)^4 + 500*sin(d*x + c)^3 + 702*sin(d*x
 + c)^2 + 340*sin(d*x + c) - 35)/(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.33, size = 144, normalized size = 1.11

$$-\frac{1}{96ad(\sin(dx+c)-1)^3}+\frac{3}{128ad(\sin(dx+c)-1)^2}-\frac{5}{128ad(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{256ad}+\frac{}{64ad(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3+3/128/a/d/(sin(d*x+c)-1)^2-5/128/a/d/(sin(d*x+c)
-1)-5/256/a/d*ln(sin(d*x+c)-1)+1/64/a/d/(1+sin(d*x+c))^4+1/48/a/d/(1+sin(d*
x+c))^3+1/64/a/d/(1+sin(d*x+c))^2+5/256*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.31, size = 175, normalized size = 1.35

$$-\frac{\frac{2\left(15\sin(dx+c)^6+15\sin(dx+c)^5-40\sin(dx+c)^4-40\sin(dx+c)^3+33\sin(dx+c)^2+33\sin(dx+c)+48\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{15\log(\sin(dx+c)+1)}{a}+\frac{15\log(\sin(\cdots}{a}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(15*sin(d*x + c)^6 + 15*sin(d*x + c)^5 - 40*sin(d*x + c)^4 - 40*s
in(d*x + c)^3 + 33*sin(d*x + c)^2 + 33*sin(d*x + c) + 48)/(a*sin(d*x + c)^7
 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x
 + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 15*log(sin(d*x + c) +
1)/a + 15*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 17.04, size = 388, normalized size = 2.98

$$\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,a\,d}+\frac{-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+\frac{59\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{32}+\frac{163\tan\left(\frac{c}{2}\cdots\right.}{96\cdots}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(64*a*d) + ((59*tan(c/2 + (d*x)/2)^2)/32 - (5
*tan(c/2 + (d*x)/2))/64 + (163*tan(c/2 + (d*x)/2)^3)/96 + (149*tan(c/2 + (d
*x)/2)^4)/96 - (625*tan(c/2 + (d*x)/2)^5)/192 + (95*tan(c/2 + (d*x)/2)^6)/1
6 + (95*tan(c/2 + (d*x)/2)^7)/16 + (95*tan(c/2 + (d*x)/2)^8)/16 - (625*tan(
c/2 + (d*x)/2)^9)/192 + (149*tan(c/2 + (d*x)/2)^10)/96 + (163*tan(c/2 + (d*
x)/2)^11)/96 + (59*tan(c/2 + (d*x)/2)^12)/32 - (5*tan(c/2 + (d*x)/2)^13)/64
)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*a*tan(c/2 + (d*x)/2)^2 - 12*a*tan(c/2
+ (d*x)/2)^3 + 9*a*tan(c/2 + (d*x)/2)^4 + 30*a*tan(c/2 + (d*x)/2)^5 - 5*a*t
an(c/2 + (d*x)/2)^6 - 40*a*tan(c/2 + (d*x)/2)^7 - 5*a*tan(c/2 + (d*x)/2)^8
+ 30*a*tan(c/2 + (d*x)/2)^9 + 9*a*tan(c/2 + (d*x)/2)^10 - 12*a*tan(c/2 + (d
*x)/2)^11 - 5*a*tan(c/2 + (d*x)/2)^12 + 2*a*tan(c/2 + (d*x)/2)^13 + a*tan(c
/2 + (d*x)/2)^14))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.889 $\int \frac{\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=165

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{24d(a\sin(c+dx)+a)^3}+\frac{5a}{128d(a-a\sin(c+dx))^2}-\frac{}{64d(a\sin}$$

```
[Out] 35/128*arctanh(sin(d*x+c))/a/d+1/96*a^2/d/(a-a*sin(d*x+c))^3+5/128*a/d/(a-a
*sin(d*x+c))^2+15/128/d/(a-a*sin(d*x+c))-1/64*a^3/d/(a+a*sin(d*x+c))^4-1/24
*a^2/d/(a+a*sin(d*x+c))^3-5/64*a/d/(a+a*sin(d*x+c))^2-5/32/d/(a+a*sin(d*x+c
))
```

Rubi [A] time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{24d(a\sin(c+dx)+a)^3}+\frac{5a}{128d(a-a\sin(c+dx))^2}-\frac{}{64d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] (35*ArcTanh[Sin[c + d*x]])/(128*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) +
(5*a)/(128*d*(a - a*Sin[c + d*x])^2) + 15/(128*d*(a - a*Sin[c + d*x])) - a^
3/(64*d*(a + a*Sin[c + d*x])^4) - a^2/(24*d*(a + a*Sin[c + d*x])^3) - (5*a)
/(64*d*(a + a*Sin[c + d*x])^2) - 5/(32*d*(a + a*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^4} + \frac{5}{64a^6(a-x)^3} + \frac{15}{128a^7(a-x)^2} + \frac{1}{16a^4(a+x)^5} + \frac{1}{8a^5(a+x)^4} + \frac{5}{32a^6(a+x)^3} + \frac{}{32a}\right.\right.}{d}$$

$$= \frac{a^2}{96d(a-a\sin(c+dx))^3} + \frac{5a}{128d(a-a\sin(c+dx))^2} + \frac{15}{128d(a-a\sin(c+dx))} - \frac{}{64d(a}$$

$$= \frac{35\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{a^2}{96d(a-a\sin(c+dx))^3} + \frac{5a}{128d(a-a\sin(c+dx))^2} + \frac{}{128d(a}$$

Mathematica [A] time = 0.51, size = 145, normalized size = 0.88

$$-\frac{\sec^6(c+dx)\left(-105\sin^6(c+dx) - 105\sin^5(c+dx) + 280\sin^4(c+dx) + 280\sin^3(c+dx) - 231\sin^2(c+dx) - 2\right.}{384ad(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/384*(Sec[c + d*x]^6*(48 - 105*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^6*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8 - 231*Sin[c + d
*x] - 231*Sin[c + d*x]^2 + 280*Sin[c + d*x]^3 + 280*Sin[c + d*x]^4 - 105*Si
n[c + d*x]^5 - 105*Sin[c + d*x]^6))/(a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.48, size = 167, normalized size = 1.01

$$-\frac{210\cos(dx+c)^6 - 70\cos(dx+c)^4 - 28\cos(dx+c)^2 - 105\left(\cos(dx+c)^6\sin(dx+c) + \cos(dx+c)^6\right)\log(\text{si}}{768\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(210*cos(d*x + c)^6 - 70*cos(d*x + c)^4 - 28*cos(d*x + c)^2 - 105*(c
os(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 105*(c
os(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 14*(1
5*cos(d*x + c)^4 + 10*cos(d*x + c)^2 + 8)*sin(d*x + c) - 16)/(a*d*cos(d*x +
 c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

giac [A] time = 0.26, size = 136, normalized size = 0.82

$$\frac{\frac{420 \log(|\sin(dx+c)+1|)}{a} - \frac{420 \log(|\sin(dx+c)-1|)}{a} + \frac{2\left(385 \sin(dx+c)^3 - 1335 \sin(dx+c)^2 + 1575 \sin(dx+c) - 641\right)}{a(\sin(dx+c)-1)^3} - \frac{875 \sin(dx+c)^4 + 3980 \sin(dx+}{}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/a
 + 2*(385*sin(d*x + c)^3 - 1335*sin(d*x + c)^2 + 1575*sin(d*x + c) - 641)/(
a*(sin(d*x + c) - 1)^3) - (875*sin(d*x + c)^4 + 3980*sin(d*x + c)^3 + 6930*
sin(d*x + c)^2 + 5548*sin(d*x + c) + 1771)/(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.42, size = 162, normalized size = 0.98

$$-\frac{1}{96ad\left(\sin\left(dx+c\right)-1\right)^{3}}+\frac{5}{128ad\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{15}{128ad\left(\sin\left(dx+c\right)-1\right)}-\frac{35\ln\left(\sin\left(dx+c\right)-1\right)}{256ad}-\frac{}{64ad\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3+5/128/a/d/(sin(d*x+c)-1)^2-15/128/a/d/(sin(d*x+c
)-1)-35/256/a/d*ln(sin(d*x+c)-1)-1/64/a/d/(1+sin(d*x+c))^4-1/24/a/d/(1+sin(
d*x+c))^3-5/64/a/d/(1+sin(d*x+c))^2-5/32/a/d/(1+sin(d*x+c))+35/256*ln(1+sin
(d*x+c))/a/d
```

maxima [A] time = 0.31, size = 175, normalized size = 1.06

$$-\frac{\frac{2\left(105 \sin(dx+c)^6 + 105 \sin(dx+c)^5 - 280 \sin(dx+c)^4 - 280 \sin(dx+c)^3 + 231 \sin(dx+c)^2 + 231 \sin(dx+c) - 48\right)}{a \sin(dx+c)^7 + a \sin(dx+c)^6 - 3\, a \sin(dx+c)^5 - 3\, a \sin(dx+c)^4 + 3\, a \sin(dx+c)^3 + 3\, a \sin(dx+c)^2 - a \sin(dx+c) - a} - \frac{105 \log(\sin(dx+c)+1)}{a} + \frac{105 \log(}{}}{768\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(105*sin(d*x + c)^6 + 105*sin(d*x + c)^5 - 280*sin(d*x + c)^4 - 2
80*sin(d*x + c)^3 + 231*sin(d*x + c)^2 + 231*sin(d*x + c) - 48)/(a*sin(d*x
```

```
+ c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*s
in(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 105*log(sin(d*x
+ c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 0.24, size = 158, normalized size = 0.96

$$\frac{35\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)}{128\,a\,d}+\frac{\frac{35\,\sin(c+d\,x)^6}{128}+\frac{35\,\sin(c+d\,x)^5}{128}-\frac{35\,\sin(c+d\,x)^4}{48}-\frac{35\,\sin(c+d\,x)^3}{48}+\frac{77\,\sin(c+}{128}}{d\,\left(-a\,\sin\left(c+d\,x\right)^7-a\,\sin\left(c+d\,x\right)^6+3\,a\,\sin\left(c+d\,x\right)^5+3\,a\,\sin\left(c+d\,x\right)^4-3\,a\,\sin\left(c-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (35*atanh(sin(c + d*x)))/(128*a*d) + ((77*sin(c + d*x))/128 + (77*sin(c + d
*x)^2)/128 - (35*sin(c + d*x)^3)/48 - (35*sin(c + d*x)^4)/48 + (35*sin(c +
d*x)^5)/128 + (35*sin(c + d*x)^6)/128 - 1/8)/(d*(a + a*sin(c + d*x) - 3*a*s
in(c + d*x)^2 - 3*a*sin(c + d*x)^3 + 3*a*sin(c + d*x)^4 + 3*a*sin(c + d*x)^
5 - a*sin(c + d*x)^6 - a*sin(c + d*x)^7))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^7(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**7/(sin(c + d*x) + 1), x)/a
```

3.890 $\int \frac{\csc(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=202

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{16d(a\sin(c+dx)+a)^3}+\frac{7a}{128d(a-a\sin(c+dx))^2}+\frac{}{64d(a\sin}$$

```
[Out] -93/256*ln(1-sin(d*x+c))/a/d+ln(sin(d*x+c))/a/d-163/256*ln(1+sin(d*x+c))/a/
d+1/96*a^2/d/(a-a*sin(d*x+c))^3+7/128*a/d/(a-a*sin(d*x+c))^2+29/128/d/(a-a*
sin(d*x+c))+1/64*a^3/d/(a+a*sin(d*x+c))^4+1/16*a^2/d/(a+a*sin(d*x+c))^3+11/
64*a/d/(a+a*sin(d*x+c))^2+1/2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.20, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{16d(a\sin(c+dx)+a)^3}+\frac{7a}{128d(a-a\sin(c+dx))^2}+\frac{}{64d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-93*Log[1 - Sin[c + d*x]])/(256*a*d) + Log[Sin[c + d*x]]/(a*d) - (163*Log[
1 + Sin[c + d*x]])/(256*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) + (7*a)/(1
28*d*(a - a*Sin[c + d*x])^2) + 29/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*
(a + a*Sin[c + d*x])^4) + a^2/(16*d*(a + a*Sin[c + d*x])^3) + (11*a)/(64*d*
(a + a*Sin[c + d*x])^2) + 1/(2*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\operatorname{Subst}\left(\int \frac{a}{(a-x)^4x(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^8\operatorname{Subst}\left(\int \frac{1}{(a-x)^4x(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^8\operatorname{Subst}\left(\int \left(\frac{1}{32a^6(a-x)^4}+\frac{7}{64a^7(a-x)^3}+\frac{29}{128a^8(a-x)^2}+\frac{93}{256a^9(a-x)}+\frac{1}{a^9x}-\frac{1}{16a^5(a+x)^5}-\right.\right.}{d}$$

$$= -\frac{93\log(1-\sin(c+dx))}{256ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{163\log(1+\sin(c+dx))}{256ad}+\frac{}{96d(}$$

Mathematica [A] time = 6.14, size = 189, normalized size = 0.94

$$\frac{a^8\left(-\frac{93\log(1-\sin(c+dx))}{256a^9}+\frac{\log(\sin(c+dx))}{a^9}-\frac{163\log(\sin(c+dx)+1)}{256a^9}+\frac{29}{128a^8(a-a\sin(c+dx))}+\frac{1}{2a^8(a\sin(c+dx)+a)}+\frac{7}{128a^7(a-a\sin(c+dx))^2}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^8*((-93*Log[1 - Sin[c + d*x]])/(256*a^9) + Log[Sin[c + d*x]]/a^9 - (163*
Log[1 + Sin[c + d*x]])/(256*a^9) + 1/(96*a^6*(a - a*Sin[c + d*x])^3) + 7/(1
28*a^7*(a - a*Sin[c + d*x])^2) + 29/(128*a^8*(a - a*Sin[c + d*x])) + 1/(64*
a^5*(a + a*Sin[c + d*x])^4) + 1/(16*a^6*(a + a*Sin[c + d*x])^3) + 11/(64*a^
7*(a + a*Sin[c + d*x])^2) + 1/(2*a^8*(a + a*Sin[c + d*x]))))/d
```

fricas [A] time = 0.52, size = 202, normalized size = 1.00

$$\frac{210\cos(dx+c)^6+314\cos(dx+c)^4+164\cos(dx+c)^2+768\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(210*cos(d*x + c)^6 + 314*cos(d*x + c)^4 + 164*cos(d*x + c)^2 + 768*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(1/2*sin(d*x + c)) - 489*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) - 279*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(8
7*cos(d*x + c)^4 + 26*cos(d*x + c)^2 + 8)*sin(d*x + c) + 112)/(a*d*cos(d*x
+ c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

giac [A] time = 0.27, size = 149, normalized size = 0.74

$$-\frac{\frac{1956\log(|\sin(dx+c)+1|)}{a}+\frac{1116\log(|\sin(dx+c)-1|)}{a}-\frac{3072\log(|\sin(dx+c)|)}{a}-\frac{2\left(1023\sin(dx+c)^3-3417\sin(dx+c)^2+3849\sin(dx+c)-1471\right)}{a(\sin(dx+c)-1)^3}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(1956*log(abs(sin(d*x + c) + 1))/a + 1116*log(abs(sin(d*x + c) - 1)
)/a - 3072*log(abs(sin(d*x + c)))/a - 2*(1023*sin(d*x + c)^3 - 3417*sin(d*x
 + c)^2 + 3849*sin(d*x + c) - 1471)/(a*(sin(d*x + c) - 1)^3) - (4075*sin(d*
x + c)^4 + 17836*sin(d*x + c)^3 + 29586*sin(d*x + c)^2 + 22156*sin(d*x + c)
 + 6379)/(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.45, size = 176, normalized size = 0.87

$$-\frac{1}{96ad\left(\sin\left(dx+c\right)-1\right)^{3}}+\frac{7}{128ad\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{29}{128ad\left(\sin\left(dx+c\right)-1\right)}-\frac{93\ln\left(\sin\left(dx+c\right)-1\right)}{256ad}+\frac{\ln\left(\sin\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3+7/128/a/d/(sin(d*x+c)-1)^2-29/128/a/d/(sin(d*x+c
)-1)-93/256/a/d*ln(sin(d*x+c)-1)+ln(sin(d*x+c))/a/d+1/64/a/d/(1+sin(d*x+c))
^4+1/16/a/d/(1+sin(d*x+c))^3+11/64/a/d/(1+sin(d*x+c))^2+1/2/a/d/(1+sin(d*x+
c))-163/256*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.33, size = 187, normalized size = 0.93

$$\frac{\frac{2\left(105\sin(dx+c)^6-87\sin(dx+c)^5-472\sin(dx+c)^4+200\sin(dx+c)^3+711\sin(dx+c)^2-121\sin(dx+c)-400\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{489\log(\sin(dx+c)+1)}{a}-\frac{279\log(\sin}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(105*sin(d*x + c)^6 - 87*sin(d*x + c)^5 - 472*sin(d*x + c)^4 + 200
*sin(d*x + c)^3 + 711*sin(d*x + c)^2 - 121*sin(d*x + c) - 400)/(a*sin(d*x +
 c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*si
n(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 489*log(sin(d*x +
 c) + 1)/a - 279*log(sin(d*x + c) - 1)/a + 768*log(sin(d*x + c))/a)/d
```

mupad [B] time = 0.16, size = 191, normalized size = 0.95

$$\frac{\ln(\sin(c+d\,x))}{a\,d}-\frac{163\,\ln(\sin(c+d\,x)+1)}{256\,a\,d}-\frac{93\,\ln(\sin(c+d\,x)-1)}{256\,a\,d}+\frac{-\frac{35\sin(c+d\,x)^6}{128}+\frac{29\,\mathrm{si}}{}}{d\,\left(-a\sin(c+d\,x)^7-a\sin(c+d\,x)^6+3\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] log(sin(c + d*x))/(a*d) - (163*log(sin(c + d*x) + 1))/(256*a*d) - (93*log(s
in(c + d*x) - 1))/(256*a*d) + ((121*sin(c + d*x))/384 - (237*sin(c + d*x)^2
)/128 - (25*sin(c + d*x)^3)/48 + (59*sin(c + d*x)^4)/48 + (29*sin(c + d*x)^
5)/128 - (35*sin(c + d*x)^6)/128 + 25/24)/(d*(a + a*sin(c + d*x) - 3*a*sin(
c + d*x)^2 - 3*a*sin(c + d*x)^3 + 3*a*sin(c + d*x)^4 + 3*a*sin(c + d*x)^5 -
 a*sin(c + d*x)^6 - a*sin(c + d*x)^7))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.891 $\int \frac{\csc^2(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=217

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{12d(a\sin(c+dx)+a)^3}+\frac{9a}{128d(a-a\sin(c+dx))^2}-\frac{}{64d(a\sin}$$

[Out] -csc(d*x+c)/a/d-187/256*ln(1-sin(d*x+c))/a/d-ln(sin(d*x+c))/a/d+443/256*ln(1+sin(d*x+c))/a/d+1/96*a^2/d/(a-a*sin(d*x+c))^3+9/128*a/d/(a-a*sin(d*x+c))^2+47/128/d/(a-a*sin(d*x+c))-1/64*a^3/d/(a+a*sin(d*x+c))^4-1/12*a^2/d/(a+a*sin(d*x+c))^3-19/64*a/d/(a+a*sin(d*x+c))^2-35/32/d/(a+a*sin(d*x+c))

Rubi [A] time = 0.24, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{12d(a\sin(c+dx)+a)^3}+\frac{9a}{128d(a-a\sin(c+dx))^2}-\frac{}{64d(a\sin}$$

Antiderivative was successfully verified.

[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]

[Out] -(Csc[c + d*x]/(a*d)) - (187*Log[1 - Sin[c + d*x]])/(256*a*d) - Log[Sin[c + d*x]]/(a*d) + (443*Log[1 + Sin[c + d*x]])/(256*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) + (9*a)/(128*d*(a - a*Sin[c + d*x])^2) + 47/(128*d*(a - a*Sin[c + d*x])) - a^3/(64*d*(a + a*Sin[c + d*x])^4) - a^2/(12*d*(a + a*Sin[c + d*x])^3) - (19*a)/(64*d*(a + a*Sin[c + d*x])^2) - 35/(32*d*(a + a*Sin[c + d*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{a^2}{(a-x)^4x^2(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^9\,\text{Subst}\left(\int \frac{1}{(a-x)^4x^2(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^9\,\text{Subst}\left(\int \left(\frac{1}{32a^7(a-x)^4} + \frac{9}{64a^8(a-x)^3} + \frac{47}{128a^9(a-x)^2} + \frac{187}{256a^{10}(a-x)} + \frac{1}{a^9x^2} - \frac{1}{a^{10}x} + \frac{}{16a}\right.\right.}{d}$$

$$= -\frac{\csc(c+dx)}{ad} - \frac{187\log(1-\sin(c+dx))}{256ad} - \frac{\log(\sin(c+dx))}{ad} + \frac{443\log(1+\sin}{256ad}$$

Mathematica [A] time = 6.14, size = 201, normalized size = 0.93

$$\frac{a^9\left(-\frac{\csc(c+dx)}{a^{10}} - \frac{187\log(1-\sin(c+dx))}{256a^{10}} - \frac{\log(\sin(c+dx))}{a^{10}} + \frac{443\log(\sin(c+dx)+1)}{256a^{10}} + \frac{47}{128a^9(a-a\sin(c+dx))} - \frac{35}{32a^9(a\sin(c+dx)+a)} + \frac{}{128a^8(a}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^9*(-(Csc[c + d*x]/a^10) - (187*Log[1 - Sin[c + d*x]])/(256*a^10) - Log[S
in[c + d*x]]/a^10 + (443*Log[1 + Sin[c + d*x]])/(256*a^10) + 1/(96*a^7*(a -
 a*Sin[c + d*x])^3) + 9/(128*a^8*(a - a*Sin[c + d*x])^2) + 47/(128*a^9*(a -
 a*Sin[c + d*x])) - 1/(64*a^6*(a + a*Sin[c + d*x])^4) - 1/(12*a^7*(a + a*Si
n[c + d*x])^3) - 19/(64*a^8*(a + a*Sin[c + d*x])^2) - 35/(32*a^9*(a + a*Sin
[c + d*x]))))/d
```

fricas [A] time = 0.50, size = 258, normalized size = 1.19

$$\frac{1506\cos(dx+c)^6 - 438\cos(dx+c)^4 - 188\cos(dx+c)^2 - 768\left(\cos(dx+c)^8 - \cos(dx+c)^6\sin(dx+c) - \cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(1506*cos(d*x + c)^6 - 438*cos(d*x + c)^4 - 188*cos(d*x + c)^2 - 768*
(cos(d*x + c)^8 - cos(d*x + c)^6*sin(d*x + c) - cos(d*x + c)^6)*log(1/2*sin
(d*x + c)) + 1329*(cos(d*x + c)^8 - cos(d*x + c)^6*sin(d*x + c) - cos(d*x +
 c)^6)*log(sin(d*x + c) + 1) - 561*(cos(d*x + c)^8 - cos(d*x + c)^6*sin(d*x
 + c) - cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(945*cos(d*x + c)^6 - 12
3*cos(d*x + c)^4 - 30*cos(d*x + c)^2 - 8)*sin(d*x + c) - 112)/(a*d*cos(d*x
+ c)^8 - a*d*cos(d*x + c)^6*sin(d*x + c) - a*d*cos(d*x + c)^6)
```

giac [A] time = 0.26, size = 170, normalized size = 0.78

$$\frac{\frac{5316\log(|\sin(dx+c)+1|)}{a}-\frac{2244\log(|\sin(dx+c)-1|)}{a}-\frac{3072\log(|\sin(dx+c)|)}{a}+\frac{3072(\sin(dx+c)-1)}{a\sin(dx+c)}+\frac{2\left(2057\sin(dx+c)^3-6735\sin(dx+c)^2+74\right.}{a(\sin(dx+c)-1)^3}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(5316*log(abs(sin(d*x + c) + 1))/a - 2244*log(abs(sin(d*x + c) - 1))
/a - 3072*log(abs(sin(d*x + c)))/a + 3072*(sin(d*x + c) - 1)/(a*sin(d*x + c
)) + 2*(2057*sin(d*x + c)^3 - 6735*sin(d*x + c)^2 + 7407*sin(d*x + c) - 274
5)/(a*(sin(d*x + c) - 1)^3) - (11075*sin(d*x + c)^4 + 47660*sin(d*x + c)^3
+ 77442*sin(d*x + c)^2 + 56460*sin(d*x + c) + 15651)/(a*(sin(d*x + c) + 1)^
4))/d
```

maple [A] time = 0.48, size = 193, normalized size = 0.89

$$-\frac{1}{96ad\left(\sin\left(dx+c\right)-1\right)^{3}}+\frac{9}{128ad\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{47}{128ad\left(\sin\left(dx+c\right)-1\right)}-\frac{187\ln\left(\sin\left(dx+c\right)-1\right)}{256ad}-\frac{}{da\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3+9/128/a/d/(sin(d*x+c)-1)^2-47/128/a/d/(sin(d*x+c
)-1)-187/256/a/d*ln(sin(d*x+c)-1)-1/d/a/sin(d*x+c)-ln(sin(d*x+c))/a/d-1/64/
a/d/(1+sin(d*x+c))^4-1/12/a/d/(1+sin(d*x+c))^3-19/64/a/d/(1+sin(d*x+c))^2-3
5/32/a/d/(1+sin(d*x+c))+443/256*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.32, size = 205, normalized size = 0.94

$$-\frac{\frac{2\left(945\sin(dx+c)^7+753\sin(dx+c)^6-2712\sin(dx+c)^5-2040\sin(dx+c)^4+2559\sin(dx+c)^3+1727\sin(dx+c)^2-784\sin(dx+c)-384\right)}{a\sin(dx+c)^8+a\sin(dx+c)^7-3a\sin(dx+c)^6-3a\sin(dx+c)^5+3a\sin(dx+c)^4+3a\sin(dx+c)^3-a\sin(dx+c)^2-a\sin(dx+c)}-\frac{1329\log(\sin}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(945*sin(d*x + c)^7 + 753*sin(d*x + c)^6 - 2712*sin(d*x + c)^5 -
2040*sin(d*x + c)^4 + 2559*sin(d*x + c)^3 + 1727*sin(d*x + c)^2 - 784*sin(d
*x + c) - 384)/(a*sin(d*x + c)^8 + a*sin(d*x + c)^7 - 3*a*sin(d*x + c)^6 -
3*a*sin(d*x + c)^5 + 3*a*sin(d*x + c)^4 + 3*a*sin(d*x + c)^3 - a*sin(d*x +
c)^2 - a*sin(d*x + c)) - 1329*log(sin(d*x + c) + 1)/a + 561*log(sin(d*x + c
) - 1)/a + 768*log(sin(d*x + c))/a)/d
```

mupad [B] time = 9.33, size = 212, normalized size = 0.98

$$\frac{443\ln(\sin(c+dx)+1)}{256\,a\,d}-\frac{187\ln(\sin(c+dx)-1)}{256\,a\,d}-\frac{-\frac{315\sin(c+dx)^7}{128}-\frac{251\sin(c+dx)^6}{128}+\frac{113\sin(c}{16}}{d\left(-a\sin(c+dx)^8-a\sin(c+dx)^7+3\,a\sin(c+dx)^6+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] (443*log(sin(c + d*x) + 1))/(256*a*d) - (187*log(sin(c + d*x) - 1))/(256*a*
d) - ((49*sin(c + d*x))/24 - (1727*sin(c + d*x)^2)/384 - (853*sin(c + d*x)^
3)/128 + (85*sin(c + d*x)^4)/16 + (113*sin(c + d*x)^5)/16 - (251*sin(c + d*
x)^6)/128 - (315*sin(c + d*x)^7)/128 + 1)/(d*(a*sin(c + d*x) + a*sin(c + d*
x)^2 - 3*a*sin(c + d*x)^3 - 3*a*sin(c + d*x)^4 + 3*a*sin(c + d*x)^5 + 3*a*s
in(c + d*x)^6 - a*sin(c + d*x)^7 - a*sin(c + d*x)^8)) - log(sin(c + d*x))/(
a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.892 $\int \frac{\csc^3(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=232

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{5a^2}{48d(a\sin(c+dx)+a)^3}+\frac{11a}{128d(a-a\sin(c+dx))^2}+\frac{\ }{64d(a\sin \cdots}$$

```
[Out] csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d-325/256*ln(1-sin(d*x+c))/a/d+5*ln(sin(d
*x+c))/a/d-955/256*ln(1+sin(d*x+c))/a/d+1/96*a^2/d/(a-a*sin(d*x+c))^3+11/12
8*a/d/(a-a*sin(d*x+c))^2+69/128/d/(a-a*sin(d*x+c))+1/64*a^3/d/(a+a*sin(d*x+
c))^4+5/48*a^2/d/(a+a*sin(d*x+c))^3+29/64*a/d/(a+a*sin(d*x+c))^2+2/d/(a+a*s
in(d*x+c))
```

Rubi [A] time = 0.25, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{5a^2}{48d(a\sin(c+dx)+a)^3}+\frac{11a}{128d(a-a\sin(c+dx))^2}+\frac{\ }{64d(a\sin \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - (325*Log[1 - Sin[c + d*x]])/(
256*a*d) + (5*Log[Sin[c + d*x]])/(a*d) - (955*Log[1 + Sin[c + d*x]])/(256*a
*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) + (11*a)/(128*d*(a - a*Sin[c + d*x]
)^2) + 69/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*(a + a*Sin[c + d*x])^4)
+ (5*a^2)/(48*d*(a + a*Sin[c + d*x])^3) + (29*a)/(64*d*(a + a*Sin[c + d*x])
^2) + 2/(d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{a^3}{(a-x)^4x^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{10}\,\text{Subst}\left(\int \frac{1}{(a-x)^4x^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{10}\,\text{Subst}\left(\int \left(\frac{1}{32a^8(a-x)^4} + \frac{11}{64a^9(a-x)^3} + \frac{69}{128a^{10}(a-x)^2} + \frac{325}{256a^{11}(a-x)} + \frac{1}{a^9x^3} - \frac{1}{a^{10}x^2} + \right.\right.}{}$$

$$= \frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad} - \frac{325\log(1-\sin(c+dx))}{256ad} + \frac{5\log(\sin(c+dx))}{ad} -$$

Mathematica [A] time = 6.19, size = 213, normalized size = 0.92

$$\frac{a^{10}\left(-\frac{\csc^2(c+dx)}{2a^{11}} + \frac{\csc(c+dx)}{a^{11}} - \frac{325\log(1-\sin(c+dx))}{256a^{11}} + \frac{5\log(\sin(c+dx))}{a^{11}} - \frac{955\log(\sin(c+dx)+1)}{256a^{11}} + \frac{69}{128a^{10}(a-a\sin(c+dx))} + \frac{2}{a^{10}(a\sin(c+a}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^10*(Csc[c + d*x]/a^11 - Csc[c + d*x]^2/(2*a^11) - (325*Log[1 - Sin[c + d
*x]])/(256*a^11) + (5*Log[Sin[c + d*x]])/a^11 - (955*Log[1 + Sin[c + d*x]])
/(256*a^11) + 1/(96*a^8*(a - a*Sin[c + d*x])^3) + 11/(128*a^9*(a - a*Sin[c
+ d*x])^2) + 69/(128*a^10*(a - a*Sin[c + d*x])) + 1/(64*a^7*(a + a*Sin[c +
d*x])^4) + 5/(48*a^8*(a + a*Sin[c + d*x])^3) + 29/(64*a^9*(a + a*Sin[c + d*
x])^2) + 2/(a^10*(a + a*Sin[c + d*x]))))/d
```

fricas [A] time = 0.53, size = 311, normalized size = 1.34

$$\frac{1890\cos(dx+c)^8 - 600\cos(dx+c)^6 - 582\cos(dx+c)^4 - 212\cos(dx+c)^2 + 3840\left(\cos(dx+c)^8 - \cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(1890*cos(d*x + c)^8 - 600*cos(d*x + c)^6 - 582*cos(d*x + c)^4 - 212*
cos(d*x + c)^2 + 3840*(cos(d*x + c)^8 - cos(d*x + c)^6 + (cos(d*x + c)^8 -
cos(d*x + c)^6)*sin(d*x + c))*log(1/2*sin(d*x + c)) - 2865*(cos(d*x + c)^8
- cos(d*x + c)^6 + (cos(d*x + c)^8 - cos(d*x + c)^6)*sin(d*x + c))*log(sin(
d*x + c) + 1) - 975*(cos(d*x + c)^8 - cos(d*x + c)^6 + (cos(d*x + c)^8 - co
s(d*x + c)^6)*sin(d*x + c))*log(-sin(d*x + c) + 1) + 2*(15*cos(d*x + c)^6 -
 165*cos(d*x + c)^4 - 34*cos(d*x + c)^2 - 8)*sin(d*x + c) - 112)/(a*d*cos(d
*x + c)^8 - a*d*cos(d*x + c)^6 + (a*d*cos(d*x + c)^8 - a*d*cos(d*x + c)^6)*
sin(d*x + c))
```

giac [A] time = 0.30, size = 182, normalized size = 0.78

$$-\frac{\frac{11460 \log(|\sin(dx+c)+1|)}{a} + \frac{3900 \log(|\sin(dx+c)-1|)}{a} - \frac{15360 \log(|\sin(dx+c)|)}{a} + \frac{1536\left(15 \sin(dx+c)^2-2 \sin(dx+c)+1\right)}{a \sin(dx+c)^2} - \frac{2\left(3575 \sin(dx+c)^3 \cdots\right.}{\cdots}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(11460*log(abs(sin(d*x + c) + 1))/a + 3900*log(abs(sin(d*x + c) - 1
))/a - 15360*log(abs(sin(d*x + c)))/a + 1536*(15*sin(d*x + c)^2 - 2*sin(d*x
 + c) + 1)/(a*sin(d*x + c)^2) - 2*(3575*sin(d*x + c)^3 - 11553*sin(d*x + c)
^2 + 12513*sin(d*x + c) - 4551)/(a*(sin(d*x + c) - 1)^3) - (23875*sin(d*x +
 c)^4 + 101644*sin(d*x + c)^3 + 163074*sin(d*x + c)^2 + 117036*sin(d*x + c)
 + 31779)/(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.53, size = 208, normalized size = 0.90

$$-\frac{1}{96ad\left(\sin\left(dx+c\right)-1\right)^{3}}+\frac{11}{128ad\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{69}{128ad\left(\sin\left(dx+c\right)-1\right)}-\frac{325\ln\left(\sin\left(dx+c\right)-1\right)}{256ad}-\frac{\cdots}{2ad\sin\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3+11/128/a/d/(sin(d*x+c)-1)^2-69/128/a/d/(sin(d*x+
c)-1)-325/256/a/d*ln(sin(d*x+c)-1)-1/2/a/d/sin(d*x+c)^2+1/d/a/sin(d*x+c)+5*
ln(sin(d*x+c))/a/d+1/64/a/d/(1+sin(d*x+c))^4+5/48/a/d/(1+sin(d*x+c))^3+29/6
4/a/d/(1+sin(d*x+c))^2+2/a/d/(1+sin(d*x+c))-955/256*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.34, size = 217, normalized size = 0.94

$$\frac{\frac{2\left(945 \sin(dx+c)^8-15 \sin(dx+c)^7-3480 \sin(dx+c)^6-120 \sin(dx+c)^5+4479 \sin(dx+c)^4+319 \sin(dx+c)^3-2192 \sin(dx+c)^2-192 \sin(dx+c)+192\right)}{a \sin(dx+c)^9+a \sin(dx+c)^8-3 a \sin(dx+c)^7-3 a \sin(dx+c)^6+3 a \sin(dx+c)^5+3 a \sin(dx+c)^4-a \sin(dx+c)^3-a \sin(dx+c)^2} - \cdots}{768\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(945*sin(d*x + c)^8 - 15*sin(d*x + c)^7 - 3480*sin(d*x + c)^6 - 12
0*sin(d*x + c)^5 + 4479*sin(d*x + c)^4 + 319*sin(d*x + c)^3 - 2192*sin(d*x
+ c)^2 - 192*sin(d*x + c) + 192)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 3*a
*sin(d*x + c)^7 - 3*a*sin(d*x + c)^6 + 3*a*sin(d*x + c)^5 + 3*a*sin(d*x + c
)^4 - a*sin(d*x + c)^3 - a*sin(d*x + c)^2) - 2865*log(sin(d*x + c) + 1)/a -
 975*log(sin(d*x + c) - 1)/a + 3840*log(sin(d*x + c))/a)/d
```

mupad [B] time = 9.24, size = 223, normalized size = 0.96

$$\frac{5\ln\left(\sin\left(c+d\,x\right)\right)}{a\,d}-\frac{955\ln\left(\sin\left(c+d\,x\right)+1\right)}{256\,a\,d}-\frac{325\ln\left(\sin\left(c+d\,x\right)-1\right)}{256\,a\,d}+\frac{-\frac{315\sin(c+d\,x)^8}{128}+\frac{5\sin(c\cdots}{12\cdots}}{d\left(-a\sin\left(c+d\,x\right)^9-a\sin\left(c+d\,x\right)^8+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (5*log(sin(c + d*x)))/(a*d) - (955*log(sin(c + d*x) + 1))/(256*a*d) - (325*
log(sin(c + d*x) - 1))/(256*a*d) + (sin(c + d*x)/2 + (137*sin(c + d*x)^2)/2
4 - (319*sin(c + d*x)^3)/384 - (1493*sin(c + d*x)^4)/128 + (5*sin(c + d*x)^
5)/16 + (145*sin(c + d*x)^6)/16 + (5*sin(c + d*x)^7)/128 - (315*sin(c + d*x
)^8)/128 - 1/2)/(d*(a*sin(c + d*x)^2 + a*sin(c + d*x)^3 - 3*a*sin(c + d*x)^
4 - 3*a*sin(c + d*x)^5 + 3*a*sin(c + d*x)^6 + 3*a*sin(c + d*x)^7 - a*sin(c
+ d*x)^8 - a*sin(c + d*x)^9))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.893 $\int \frac{\csc^4(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=253

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{8d(a\sin(c+dx)+a)^3}+\frac{13a}{128d(a-a\sin(c+dx))^2}-\frac{}{64d(a\sin}$$

```
[Out] -5*csc(d*x+c)/a/d+1/2*csc(d*x+c)^2/a/d-1/3*csc(d*x+c)^3/a/d-515/256*ln(1-si
n(d*x+c))/a/d-5*ln(sin(d*x+c))/a/d+1795/256*ln(1+sin(d*x+c))/a/d+1/96*a^2/d
/(a-a*sin(d*x+c))^3+13/128*a/d/(a-a*sin(d*x+c))^2+95/128/d/(a-a*sin(d*x+c))
-1/64*a^3/d/(a+a*sin(d*x+c))^4-1/8*a^2/d/(a+a*sin(d*x+c))^3-41/64*a/d/(a+a*
sin(d*x+c))^2-105/32/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.26, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{8d(a\sin(c+dx)+a)^3}+\frac{13a}{128d(a-a\sin(c+dx))^2}-\frac{}{64d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^4*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*Csc[c + d*x])/(a*d) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) -
 (515*Log[1 - Sin[c + d*x]])/(256*a*d) - (5*Log[Sin[c + d*x]])/(a*d) + (179
5*Log[1 + Sin[c + d*x]])/(256*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) + (1
3*a)/(128*d*(a - a*Sin[c + d*x])^2) + 95/(128*d*(a - a*Sin[c + d*x])) - a^3
/(64*d*(a + a*Sin[c + d*x])^4) - a^2/(8*d*(a + a*Sin[c + d*x])^3) - (41*a)/
(64*d*(a + a*Sin[c + d*x])^2) - 105/(32*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\mathrm{Subst}\left(\int \frac{a^4}{(a-x)^4x^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{11}\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^4x^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{11}\,\mathrm{Subst}\left(\int \left(\frac{1}{32a^9(a-x)^4} + \frac{13}{64a^{10}(a-x)^3} + \frac{95}{128a^{11}(a-x)^2} + \frac{515}{256a^{12}(a-x)} + \frac{1}{a^9x^4} - \frac{1}{a^{10}x^3} + \right.\right.}{}$$

$$= -\frac{5\csc(c+dx)}{ad} + \frac{\csc^2(c+dx)}{2ad} - \frac{\csc^3(c+dx)}{3ad} - \frac{515\log(1-\sin(c+dx))}{256ad} - \frac{5\,\mathrm{l}}{}$$

Mathematica [A] time = 6.14, size = 231, normalized size = 0.91

$$\frac{a^{11}\left(-\frac{\csc^3(c+dx)}{3a^{12}} + \frac{\csc^2(c+dx)}{2a^{12}} - \frac{5\csc(c+dx)}{a^{12}} - \frac{515\log(1-\sin(c+dx))}{256a^{12}} - \frac{5\log(\sin(c+dx))}{a^{12}} + \frac{1795\log(\sin(c+dx)+1)}{256a^{12}} + \frac{95}{128a^{11}(a-a\sin(c+dx))}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^4*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^11*((-5*Csc[c + d*x])/a^12 + Csc[c + d*x]^2/(2*a^12) - Csc[c + d*x]^3/(3
*a^12) - (515*Log[1 - Sin[c + d*x]])/(256*a^12) - (5*Log[Sin[c + d*x]])/a^1
2 + (1795*Log[1 + Sin[c + d*x]])/(256*a^12) + 1/(96*a^9*(a - a*Sin[c + d*x]
)^3) + 13/(128*a^10*(a - a*Sin[c + d*x])^2) + 95/(128*a^11*(a - a*Sin[c + d
*x])) - 1/(64*a^8*(a + a*Sin[c + d*x])^4) - 1/(8*a^9*(a + a*Sin[c + d*x])^3
) - 41/(64*a^10*(a + a*Sin[c + d*x])^2) - 105/(32*a^11*(a + a*Sin[c + d*x])
)))/d
```

fricas [A] time = 0.53, size = 360, normalized size = 1.42

$$\frac{5010\cos(dx+c)^8 - 6360\cos(dx+c)^6 + 746\cos(dx+c)^4 + 236\cos(dx+c)^2 - 3840\left(\cos(dx+c)^{10} - 2\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(5010*cos(d*x + c)^8 - 6360*cos(d*x + c)^6 + 746*cos(d*x + c)^4 + 236
*cos(d*x + c)^2 - 3840*(cos(d*x + c)^10 - 2*cos(d*x + c)^8 + cos(d*x + c)^6
 - (cos(d*x + c)^8 - cos(d*x + c)^6)*sin(d*x + c))*log(1/2*sin(d*x + c)) +
5385*(cos(d*x + c)^10 - 2*cos(d*x + c)^8 + cos(d*x + c)^6 - (cos(d*x + c)^8
 - cos(d*x + c)^6)*sin(d*x + c))*log(sin(d*x + c) + 1) - 1545*(cos(d*x + c)
^10 - 2*cos(d*x + c)^8 + cos(d*x + c)^6 - (cos(d*x + c)^8 - cos(d*x + c)^6)
*sin(d*x + c))*log(-sin(d*x + c) + 1) + 2*(3465*cos(d*x + c)^8 - 3660*cos(d
*x + c)^6 + 213*cos(d*x + c)^4 + 38*cos(d*x + c)^2 + 8)*sin(d*x + c) + 112)
/(a*d*cos(d*x + c)^10 - 2*a*d*cos(d*x + c)^8 + a*d*cos(d*x + c)^6 - (a*d*co
s(d*x + c)^8 - a*d*cos(d*x + c)^6)*sin(d*x + c))
```

giac [A] time = 0.30, size = 187, normalized size = 0.74

$$\frac{\frac{21540 \log(|\sin(dx+c)+1|)}{a} - \frac{6180 \log(|\sin(dx+c)-1|)}{a} - \frac{15360 \log(|\sin(dx+c)|)}{a} + \frac{19745 \sin(dx+c)^6 - 76875 \sin(dx+c)^5 + 111723 \sin(dx+c)^4 - 740}{(\sin(dx+c)^2 - \sin}}{307}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(21540*log(abs(sin(d*x + c) + 1))/a - 6180*log(abs(sin(d*x + c) - 1)
)/a - 15360*log(abs(sin(d*x + c)))/a + (19745*sin(d*x + c)^6 - 76875*sin(d*
x + c)^5 + 111723*sin(d*x + c)^4 - 74081*sin(d*x + c)^3 + 23040*sin(d*x + c
)^2 - 4608*sin(d*x + c) + 1024)/((sin(d*x + c)^2 - sin(d*x + c))^3*a) - (44
875*sin(d*x + c)^4 + 189580*sin(d*x + c)^3 + 301458*sin(d*x + c)^2 + 214060
*sin(d*x + c) + 57355)/(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.51, size = 225, normalized size = 0.89

$$-\frac{1}{96ad\left(\sin\left(dx+c\right)-1\right)^{3}}+\frac{13}{128ad\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{95}{128ad\left(\sin\left(dx+c\right)-1\right)}-\frac{515\ln\left(\sin\left(dx+c\right)-1\right)}{256ad}-\frac{}{3ad\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3+13/128/a/d/(sin(d*x+c)-1)^2-95/128/a/d/(sin(d*x+
c)-1)-515/256/a/d*ln(sin(d*x+c)-1)-1/3/a/d/sin(d*x+c)^3+1/2/a/d/sin(d*x+c)^
2-5/d/a/sin(d*x+c)-5*ln(sin(d*x+c))/a/d-1/64/a/d/(1+sin(d*x+c))^4-1/8/a/d/(
1+sin(d*x+c))^3-41/64/a/d/(1+sin(d*x+c))^2-105/32/a/d/(1+sin(d*x+c))+1795/2
56*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.33, size = 227, normalized size = 0.90

$$-\frac{\frac{2\left(3465 \sin(dx+c)^9+2505 \sin(dx+c)^8-10200 \sin(dx+c)^7-6840 \sin(dx+c)^6+10023 \sin(dx+c)^5+5863 \sin(dx+c)^4-3344 \sin(dx+c)^3-1344 \sin(dx+c)^2}{a\sin(dx+c)^{10}+a\sin(dx+c)^9-3\,a\sin(dx+c)^8-3\,a\sin(dx+c)^7+3\,a\sin(dx+c)^6+3\,a\sin(dx+c)^5-a\sin(dx+c)^4-a\sin(dx+c)^3}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(3465*sin(d*x + c)^9 + 2505*sin(d*x + c)^8 - 10200*sin(d*x + c)^7
 - 6840*sin(d*x + c)^6 + 10023*sin(d*x + c)^5 + 5863*sin(d*x + c)^4 - 3344*
sin(d*x + c)^3 - 1344*sin(d*x + c)^2 + 64*sin(d*x + c) - 128)/(a*sin(d*x +
c)^10 + a*sin(d*x + c)^9 - 3*a*sin(d*x + c)^8 - 3*a*sin(d*x + c)^7 + 3*a*si
n(d*x + c)^6 + 3*a*sin(d*x + c)^5 - a*sin(d*x + c)^4 - a*sin(d*x + c)^3) -
5385*log(sin(d*x + c) + 1)/a + 1545*log(sin(d*x + c) - 1)/a + 3840*log(sin(
d*x + c))/a)/d
```

mupad [B] time = 9.26, size = 233, normalized size = 0.92

$$\frac{\frac{1155\sin(c+d\,x)^9}{128}+\frac{835\sin(c+d\,x)^8}{128}-\frac{425\sin(c+d\,x)^7}{16}-\frac{285\sin(c+d\,x)^6}{16}+\frac{3341\sin(c+d\,x)^5}{128}+\frac{5863\sin(c+d\,x)^4}{384}-\frac{209\sin(c+d\,x)^3}{24}}{d\,\left(-a\sin\left(c+d\,x\right)^{10}-a\sin\left(c+d\,x\right)^9+3\,a\sin\left(c+d\,x\right)^8+3\,a\sin\left(c+d\,x\right)^7-3\,a\sin\left(c+d\,x\right)^6-3\,a\sin\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x)/6 - (7*sin(c + d*x)^2)/2 - (209*sin(c + d*x)^3)/24 + (5863*si
n(c + d*x)^4)/384 + (3341*sin(c + d*x)^5)/128 - (285*sin(c + d*x)^6)/16 - (
425*sin(c + d*x)^7)/16 + (835*sin(c + d*x)^8)/128 + (1155*sin(c + d*x)^9)/1
28 - 1/3)/(d*(a*sin(c + d*x)^3 + a*sin(c + d*x)^4 - 3*a*sin(c + d*x)^5 - 3*
a*sin(c + d*x)^6 + 3*a*sin(c + d*x)^7 + 3*a*sin(c + d*x)^8 - a*sin(c + d*x)
^9 - a*sin(c + d*x)^10)) - (515*log(sin(c + d*x) - 1))/(256*a*d) + (1795*lo
g(sin(c + d*x) + 1))/(256*a*d) - (5*log(sin(c + d*x)))/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.894 $\int \sec^5(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=91

$$\frac{2a^2\tan^7(c+dx)}{7d}+\frac{2a^2\tan^5(c+dx)}{5d}+\frac{2a^2\sec^7(c+dx)}{7d}-\frac{3a^2\sec^5(c+dx)}{5d}+\frac{a^2\sec^3(c+dx)}{3d}$$

```
[Out] 1/3*a^2*sec(d*x+c)^3/d-3/5*a^2*sec(d*x+c)^5/d+2/7*a^2*sec(d*x+c)^7/d+2/5*a^
2*tan(d*x+c)^5/d+2/7*a^2*tan(d*x+c)^7/d
```

Rubi [A] time = 0.21, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2873, 2606, 14, 2607, 270}

$$\frac{2a^2\tan^7(c+dx)}{7d}+\frac{2a^2\tan^5(c+dx)}{5d}+\frac{2a^2\sec^7(c+dx)}{7d}-\frac{3a^2\sec^5(c+dx)}{5d}+\frac{a^2\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*Sec[c + d*x]^3)/(3*d) - (3*a^2*Sec[c + d*x]^5)/(5*d) + (2*a^2*Sec[c +
d*x]^7)/(7*d) + (2*a^2*Tan[c + d*x]^5)/(5*d) + (2*a^2*Tan[c + d*x]^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx = \int \left(a^2\sec^5(c+dx)\tan^3(c+dx)+2a^2\sec^4(c+dx)\tan^4(c+$$

$$= a^2\int \sec^5(c+dx)\tan^3(c+dx)\,dx + a^2\int \sec^3(c+dx)\tan^5$$

$$= \frac{a^2\,\mathrm{Subst}\left(\int x^4\left(-1+x^2\right)dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{a^2\,\mathrm{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\,\mathrm{Subst}}{}$$

$$= \frac{a^2\sec^3(c+dx)}{3d} - \frac{3a^2\sec^5(c+dx)}{5d} + \frac{2a^2\sec^7(c+dx)}{7d} + \frac{2a}{}$$

Mathematica [A] time = 1.02, size = 139, normalized size = 1.53

$$-\frac{a^2\sec^7(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4(448\sin(c+dx)-104\sin(2(c+dx))-144\sin(3(c+dx))-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] -1/6720*(a^2*Sec[c + d*x]^7*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(-672 +
 182*Cos[c + d*x] + 736*Cos[2*(c + d*x)] + 39*Cos[3*(c + d*x)] - 192*Cos[4*
(c + d*x)] - 13*Cos[5*(c + d*x)] + 448*Sin[c + d*x] - 104*Sin[2*(c + d*x)]
- 144*Sin[3*(c + d*x)] - 52*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)]))/d
```

fricas [A] time = 0.44, size = 115, normalized size = 1.26

$$-\frac{24\,a^2\cos(dx+c)^4-47\,a^2\cos(dx+c)^2+25\,a^2-2\left(6\,a^2\cos(dx+c)^4-9\,a^2\cos(dx+c)^2+5\,a^2\right)\sin(dx+c)}{105\left(d\cos(dx+c)^5+2\,d\cos(dx+c)^3\sin(dx+c)-2\,d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(24*a^2*cos(d*x + c)^4 - 47*a^2*cos(d*x + c)^2 + 25*a^2 - 2*(6*a^2*c
os(d*x + c)^4 - 9*a^2*cos(d*x + c)^2 + 5*a^2)*sin(d*x + c))/(d*cos(d*x + c)
^5 + 2*d*cos(d*x + c)^3*sin(d*x + c) - 2*d*cos(d*x + c)^3)
```

giac [A] time = 0.24, size = 138, normalized size = 1.52

$$-\frac{\frac{35\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}-\frac{105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1015\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1330\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1302\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+469\,a^2\tan\left(\frac{1}{2}\,d\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(35*(3*a^2*tan(1/2*d*x + 1/2*c) + a^2)/(tan(1/2*d*x + 1/2*c) + 1)^3
- (105*a^2*tan(1/2*d*x + 1/2*c)^5 - 1015*a^2*tan(1/2*d*x + 1/2*c)^4 + 1330*
a^2*tan(1/2*d*x + 1/2*c)^3 - 1302*a^2*tan(1/2*d*x + 1/2*c)^2 + 469*a^2*tan(
1/2*d*x + 1/2*c) - 67*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^7)/d
```

maple [B] time = 0.48, size = 248, normalized size = 2.73

$$\frac{a^2\left(\frac{\sin^6(dx+c)}{7\cos(dx+c)^7}+\frac{\sin^6(dx+c)}{35\cos(dx+c)^5}-\frac{\sin^6(dx+c)}{105\cos(dx+c)^3}+\frac{\sin^6(dx+c)}{35\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{35}\right)+2a^2\left(\frac{\sin^5(dx+c)}{7\cos(dx+c)^7}+\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/7*sin(d*x+c)^6/cos(d*x+c)^7+1/35*sin(d*x+c)^6/cos(d*x+c)^5-1/10
5*sin(d*x+c)^6/cos(d*x+c)^3+1/35*sin(d*x+c)^6/cos(d*x+c)+1/35*(8/3+sin(d*x+
c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/3
5*sin(d*x+c)^5/cos(d*x+c)^5)+a^2*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*sin(d*
x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/cos(d*
x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 0.32, size = 91, normalized size = 1.00

$$\frac{6\left(5\,\tan\left(dx+c\right)^7+7\,\tan\left(dx+c\right)^5\right)a^2+\frac{\left(35\,\cos(dx+c)^4-42\,\cos(dx+c)^2+15\right)a^2}{\cos(dx+c)^7}-\frac{3\left(7\,\cos(dx+c)^2-5\right)a^2}{\cos(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/105*(6*(5*tan(d*x + c)^7 + 7*tan(d*x + c)^5)*a^2 + (35*cos(d*x + c)^4 - 4
2*cos(d*x + c)^2 + 15)*a^2/cos(d*x + c)^7 - 3*(7*cos(d*x + c)^2 - 5)*a^2/co
s(d*x + c)^7)/d
```

mupad [B] time = 13.52, size = 215, normalized size = 2.36

$$\frac{4\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+8\cos\left(\frac{c}{2}+\right.\right.}{105\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x))^2)/cos(c + d*x)^8,x)
```

```
[Out] (4*a^2*cos(c/2 + (d*x)/2)^3*(cos(c/2 + (d*x)/2)^7 + 105*cos(c/2 + (d*x)/2)*
sin(c/2 + (d*x)/2)^6 - 4*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2) - 84*cos(c
/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^5 + 91*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d
*x)/2)^4 + 8*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^3 + 3*cos(c/2 + (d*x)/
2)^5*sin(c/2 + (d*x)/2)^2))/(105*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)
)^7*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.895 $\int \frac{\sin^3(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=264

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{19a^3}{256d(a\sin(c+dx)+a)^4}-\frac{3a^2}{64d(a-a\sin(c+dx))^3}-\frac{}{128d(}$$

```
[Out] -843/512*ln(1-sin(d*x+c))/a/d-2229/512*ln(1+sin(d*x+c))/a/d+sin(d*x+c)/a/d-
1/2*sin(d*x+c)^2/a/d+1/256*a^3/d/(a-a*sin(d*x+c))^4-3/64*a^2/d/(a-a*sin(d*x
+c))^3+141/512*a/d/(a-a*sin(d*x+c))^2-39/32/d/(a-a*sin(d*x+c))-1/160*a^4/d/
(a+a*sin(d*x+c))^5+19/256*a^3/d/(a+a*sin(d*x+c))^4-53/128*a^2/d/(a+a*sin(d*
x+c))^3+765/512*a/d/(a+a*sin(d*x+c))^2-1155/256/d/(a+a*sin(d*x+c))
```

Rubi [**A**] time = 0.28, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{19a^3}{256d(a\sin(c+dx)+a)^4}-\frac{3a^2}{64d(a-a\sin(c+dx))^3}-\frac{}{128d(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-843*Log[1 - Sin[c + d*x]])/(512*a*d) - (2229*Log[1 + Sin[c + d*x]])/(512*
a*d) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d) + a^3/(256*d*(a - a*Sin[
c + d*x])^4) - (3*a^2)/(64*d*(a - a*Sin[c + d*x])^3) + (141*a)/(512*d*(a -
a*Sin[c + d*x])^2) - 39/(32*d*(a - a*Sin[c + d*x])) - a^4/(160*d*(a + a*Sin
[c + d*x])^5) + (19*a^3)/(256*d*(a + a*Sin[c + d*x])^4) - (53*a^2)/(128*d*(
a + a*Sin[c + d*x])^3) + (765*a)/(512*d*(a + a*Sin[c + d*x])^2) - 1155/(256
*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\,\text{Subst}\left(\int \frac{x^{12}}{a^{12}(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^{12}}{(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$

$$= \frac{\text{Subst}\left(\int \left(a + \frac{a^6}{64(a-x)^5} - \frac{9a^5}{64(a-x)^4} + \frac{141a^4}{256(a-x)^3} - \frac{39a^3}{32(a-x)^2} + \frac{843a^2}{512(a-x)} - x + \frac{a^7}{32(a+x)^6} - \right.\right.}{a^3 d}$$

$$= -\frac{843\log(1-\sin(c+dx))}{512ad} - \frac{2229\log(1+\sin(c+dx))}{512ad} + \frac{\sin(c+dx)}{ad} - \frac{\sin^2(c}{2a}$$

Mathematica [A] time = 6.17, size = 169, normalized size = 0.64

$$-\frac{1280\sin^2(c+dx) - 2560\sin(c+dx) + \frac{3120}{1-\sin(c+dx)} + \frac{11550}{\sin(c+dx)+1} - \frac{705}{(1-\sin(c+dx))^2} - \frac{3825}{(\sin(c+dx)+1)^2} + \frac{120}{(1-\sin(c+dx))^3} + \frac{}{(\sin}}{256}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2560*(4215*Log[1 - Sin[c + d*x]] + 11145*Log[1 + Sin[c + d*x]] - 10/(1 -
 Sin[c + d*x])^4 + 120/(1 - Sin[c + d*x])^3 - 705/(1 - Sin[c + d*x])^2 + 31
20/(1 - Sin[c + d*x]) - 2560*Sin[c + d*x] + 1280*Sin[c + d*x]^2 + 16/(1 + S
in[c + d*x])^5 - 190/(1 + Sin[c + d*x])^4 + 1060/(1 + Sin[c + d*x])^3 - 382
5/(1 + Sin[c + d*x])^2 + 11550/(1 + Sin[c + d*x]))/(a*d)
```

fricas [A] time = 0.56, size = 217, normalized size = 0.82

$$-\frac{1280\cos(dx+c)^{10} + 6510\cos(dx+c)^8 + 3590\cos(dx+c)^6 - 1124\cos(dx+c)^4 + 272\cos(dx+c)^2 + 1114}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2560*(1280*cos(d*x + c)^10 + 6510*cos(d*x + c)^8 + 3590*cos(d*x + c)^6 -
 1124*cos(d*x + c)^4 + 272*cos(d*x + c)^2 + 11145*(cos(d*x + c)^8*sin(d*x +
 c) + cos(d*x + c)^8)*log(sin(d*x + c) + 1) + 4215*(cos(d*x + c)^8*sin(d*x
+ c) + cos(d*x + c)^8)*log(-sin(d*x + c) + 1) - 2*(640*cos(d*x + c)^10 + 96
0*cos(d*x + c)^8 - 5385*cos(d*x + c)^6 + 2810*cos(d*x + c)^4 - 952*cos(d*x
+ c)^2 + 144)*sin(d*x + c) - 32)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos
(d*x + c)^8)
```

giac [A] time = 0.41, size = 181, normalized size = 0.69

$$-\frac{\frac{44580 \log(|\sin(dx+c)+1|)}{a}+\frac{16860 \log(|\sin(dx+c)-1|)}{a}+\frac{5120\left(a\sin(dx+c)^2-2a\sin(dx+c)\right)}{a^2}-\frac{5\left(7025\sin(dx+c)^4-25604\sin(dx+c)^3+35226\,\mathrm{s}\cdots\right.}{a(\sin(dx+c)-1\cdots}}{102\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/10240*(44580*log(abs(sin(d*x + c) + 1))/a + 16860*log(abs(sin(d*x + c) -
 1))/a + 5120*(a*sin(d*x + c)^2 - 2*a*sin(d*x + c))/a^2 - 5*(7025*sin(d*x +
 c)^4 - 25604*sin(d*x + c)^3 + 35226*sin(d*x + c)^2 - 21644*sin(d*x + c) +
5005)/(a*(sin(d*x + c) - 1)^4) - (101791*sin(d*x + c)^5 + 462755*sin(d*x +
c)^4 + 848410*sin(d*x + c)^3 + 782370*sin(d*x + c)^2 + 362335*sin(d*x + c)
+ 67347)/(a*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.50, size = 227, normalized size = 0.86

$$\frac{1}{256ad\left(\sin(dx+c)-1\right)^4}+\frac{3}{64ad\left(\sin(dx+c)-1\right)^3}+\frac{141}{512ad\left(\sin(dx+c)-1\right)^2}+\frac{39}{32ad\left(\sin(dx+c)-1\right)}-\frac{843\ln(\mathrm{s}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^12/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4+3/64/a/d/(sin(d*x+c)-1)^3+141/512/a/d/(sin(d*x+c
)-1)^2+39/32/a/d/(sin(d*x+c)-1)-843/512/a/d*ln(sin(d*x+c)-1)-1/2*sin(d*x+c)
^2/a/d+sin(d*x+c)/a/d-1/160/a/d/(1+sin(d*x+c))^5+19/256/a/d/(1+sin(d*x+c))^
4-53/128/a/d/(1+sin(d*x+c))^3+765/512/a/d/(1+sin(d*x+c))^2-1155/256/a/d/(1+
sin(d*x+c))-2229/512*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.34, size = 236, normalized size = 0.89

$$-\frac{\frac{2\left(4215\sin(dx+c)^8-5385\sin(dx+c)^7-18655\sin(dx+c)^6+13345\sin(dx+c)^5+30113\sin(dx+c)^4-11487\sin(dx+c)^3-21257\sin(dx+c)^2+3383\sin(\cdots\right.}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+\cdots}\cdots}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/2560*(2*(4215*sin(d*x + c)^8 - 5385*sin(d*x + c)^7 - 18655*sin(d*x + c)^
6 + 13345*sin(d*x + c)^5 + 30113*sin(d*x + c)^4 - 11487*sin(d*x + c)^3 - 21
257*sin(d*x + c)^2 + 3383*sin(d*x + c) + 5568)/(a*sin(d*x + c)^9 + a*sin(d*
x + c)^8 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6
*a*sin(d*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c
) + a) + 1280*(sin(d*x + c)^2 - 2*sin(d*x + c))/a + 11145*log(sin(d*x + c)
+ 1)/a + 4215*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 11.20, size = 648, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^12/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] ((693*tan(c/2 + (d*x)/2))/128 - (75*tan(c/2 + (d*x)/2)^2)/64 - (3153*tan(c/
2 + (d*x)/2)^3)/64 - (87*tan(c/2 + (d*x)/2)^4)/64 + (111333*tan(c/2 + (d*x)
/2)^5)/640 + (1331*tan(c/2 + (d*x)/2)^6)/40 - (4559*tan(c/2 + (d*x)/2)^7)/1
6 - (1823*tan(c/2 + (d*x)/2)^8)/20 + (42953*tan(c/2 + (d*x)/2)^9)/320 + (11
713*tan(c/2 + (d*x)/2)^10)/160 + (43457*tan(c/2 + (d*x)/2)^11)/160 + (11713
*tan(c/2 + (d*x)/2)^12)/160 + (42953*tan(c/2 + (d*x)/2)^13)/320 - (1823*tan
(c/2 + (d*x)/2)^14)/20 - (4559*tan(c/2 + (d*x)/2)^15)/16 + (1331*tan(c/2 +
(d*x)/2)^16)/40 + (111333*tan(c/2 + (d*x)/2)^17)/640 - (87*tan(c/2 + (d*x)/
2)^18)/64 - (3153*tan(c/2 + (d*x)/2)^19)/64 - (75*tan(c/2 + (d*x)/2)^20)/64
 + (693*tan(c/2 + (d*x)/2)^21)/128)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*a*ta
n(c/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 7*a*tan(c/2 + (d*x)/2)^4 +
 26*a*tan(c/2 + (d*x)/2)^5 + 5*a*tan(c/2 + (d*x)/2)^6 - 16*a*tan(c/2 + (d*x
)/2)^7 - 22*a*tan(c/2 + (d*x)/2)^8 - 28*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c
/2 + (d*x)/2)^10 + 56*a*tan(c/2 + (d*x)/2)^11 + 14*a*tan(c/2 + (d*x)/2)^12
- 28*a*tan(c/2 + (d*x)/2)^13 - 22*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 +
(d*x)/2)^15 + 5*a*tan(c/2 + (d*x)/2)^16 + 26*a*tan(c/2 + (d*x)/2)^17 + 7*a*
tan(c/2 + (d*x)/2)^18 - 12*a*tan(c/2 + (d*x)/2)^19 - 5*a*tan(c/2 + (d*x)/2)
^20 + 2*a*tan(c/2 + (d*x)/2)^21 + a*tan(c/2 + (d*x)/2)^22)) - (843*log(tan(
c/2 + (d*x)/2) - 1))/(256*a*d) - (2229*log(tan(c/2 + (d*x)/2) + 1))/(256*a*
d) + (6*log(tan(c/2 + (d*x)/2)^2 + 1))/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**12/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.896 $\int \frac{\sin^2(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=247

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{17a^3}{256d(a\sin(c+dx)+a)^4}-\frac{a^2}{24d(a-a\sin(c+dx))^3}+\frac{}{384d(a\,\mathrm{s}}$$

```
[Out] -437/512*ln(1-sin(d*x+c))/a/d+949/512*ln(1+sin(d*x+c))/a/d-sin(d*x+c)/a/d+1
/256*a^3/d/(a-a*sin(d*x+c))^4-1/24*a^2/d/(a-a*sin(d*x+c))^3+109/512*a/d/(a-
a*sin(d*x+c))^2-203/256/d/(a-a*sin(d*x+c))+1/160*a^4/d/(a+a*sin(d*x+c))^5-1
7/256*a^3/d/(a+a*sin(d*x+c))^4+125/384*a^2/d/(a+a*sin(d*x+c))^3-515/512*a/d
/(a+a*sin(d*x+c))^2+5/2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.26, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{17a^3}{256d(a\sin(c+dx)+a)^4}-\frac{a^2}{24d(a-a\sin(c+dx))^3}+\frac{}{384d(a\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-437*Log[1 - Sin[c + d*x]])/(512*a*d) + (949*Log[1 + Sin[c + d*x]])/(512*a
*d) - Sin[c + d*x]/(a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) - a^2/(24*d*(
a - a*Sin[c + d*x])^3) + (109*a)/(512*d*(a - a*Sin[c + d*x])^2) - 203/(256*
d*(a - a*Sin[c + d*x])) + a^4/(160*d*(a + a*Sin[c + d*x])^5) - (17*a^3)/(25
6*d*(a + a*Sin[c + d*x])^4) + (125*a^2)/(384*d*(a + a*Sin[c + d*x])^3) - (5
15*a)/(512*d*(a + a*Sin[c + d*x])^2) + 5/(2*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\,\text{Subst}\left(\int \frac{x^{11}}{a^{11}(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^{11}}{(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$

$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^5}{64(a-x)^5}-\frac{a^4}{8(a-x)^4}+\frac{109a^3}{256(a-x)^3}-\frac{203a^2}{256(a-x)^2}+\frac{437a}{512(a-x)}-\frac{a^6}{32(a+x)^6}+\right.\right.}{a^2d}$$

$$= -\frac{437\log(1-\sin(c+dx))}{512ad}+\frac{949\log(1+\sin(c+dx))}{512ad}-\frac{\sin(c+dx)}{ad}+\frac{}{256d(}$$

Mathematica [A] time = 6.19, size = 159, normalized size = 0.64

$$-\frac{7680\sin(c+dx)+\frac{6090}{1-\sin(c+dx)}-\frac{19200}{\sin(c+dx)+1}-\frac{1635}{(1-\sin(c+dx))^2}+\frac{7725}{(\sin(c+dx)+1)^2}+\frac{320}{(1-\sin(c+dx))^3}-\frac{2500}{(\sin(c+dx)+1)^3}-\frac{30}{(1-\sin(c}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7680*(6555*Log[1 - Sin[c + d*x]] - 14235*Log[1 + Sin[c + d*x]] - 30/(1 -
 Sin[c + d*x])^4 + 320/(1 - Sin[c + d*x])^3 - 1635/(1 - Sin[c + d*x])^2 + 6
090/(1 - Sin[c + d*x]) + 7680*Sin[c + d*x] - 48/(1 + Sin[c + d*x])^5 + 510/
(1 + Sin[c + d*x])^4 - 2500/(1 + Sin[c + d*x])^3 + 7725/(1 + Sin[c + d*x])^
2 - 19200/(1 + Sin[c + d*x]))/(a*d)
```

fricas [A] time = 0.55, size = 207, normalized size = 0.84

$$\frac{7680\cos(dx+c)^{10}+17610\cos(dx+c)^{8}-27630\cos(dx+c)^{6}+15828\cos(dx+c)^{4}-5584\cos(dx+c)^{2}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^9*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="fricas"
)

[Out] 1/7680*(7680*cos(d*x + c)^10 + 17610*cos(d*x + c)^8 - 27630*cos(d*x + c)^6
+ 15828*cos(d*x + c)^4 - 5584*cos(d*x + c)^2 + 14235*(cos(d*x + c)^8*sin(d*
x + c) + cos(d*x + c)^8)*log(sin(d*x + c) + 1) - 6555*(cos(d*x + c)^8*sin(d
*x + c) + cos(d*x + c)^8)*log(-sin(d*x + c) + 1) - 2*(3840*cos(d*x + c)^8 +
3045*cos(d*x + c)^6 - 1170*cos(d*x + c)^4 + 344*cos(d*x + c)^2 - 48)*sin(d
*x + c) + 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x + c)^8)

giac [A] time = 0.40, size = 167, normalized size = 0.68

$$\frac{\frac{56940 \log(|\sin(dx+c)+1|)}{a} - \frac{26220 \log(|\sin(dx+c)-1|)}{a} - \frac{30720 \sin(dx+c)}{a} + \frac{5\left(10925 \sin(dx+c)^4 - 38828 \sin(dx+c)^3 + 52242 \sin(dx+c)^2 - 31444 \sin\right.}{a(\sin(dx+c)-1)^4}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^9*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/30720*(56940*log(abs(sin(d*x + c) + 1))/a - 26220*log(abs(sin(d*x + c) -
1))/a - 30720*sin(d*x + c)/a + 5*(10925*sin(d*x + c)^4 - 38828*sin(d*x + c)
^3 + 52242*sin(d*x + c)^2 - 31444*sin(d*x + c) + 7129)/(a*(sin(d*x + c) - 1
)^4) - (130013*sin(d*x + c)^5 + 573265*sin(d*x + c)^4 + 1023830*sin(d*x + c
)^3 + 922030*sin(d*x + c)^2 + 417605*sin(d*x + c) + 75961)/(a*(sin(d*x + c)
+ 1)^5))/d

maple [A] time = 0.48, size = 212, normalized size = 0.86

$$-\frac{\sin(dx+c)}{ad} + \frac{1}{256ad(\sin(dx+c)-1)^4} + \frac{1}{24ad(\sin(dx+c)-1)^3} + \frac{109}{512ad(\sin(dx+c)-1)^2} + \frac{203}{256ad(\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^9*sin(d*x+c)^11/(a+a*sin(d*x+c)),x)

[Out] -sin(d*x+c)/a/d+1/256/a/d/(sin(d*x+c)-1)^4+1/24/a/d/(sin(d*x+c)-1)^3+109/51
2/a/d/(sin(d*x+c)-1)^2+203/256/a/d/(sin(d*x+c)-1)-437/512/a/d*ln(sin(d*x+c)
-1)+1/160/a/d/(1+sin(d*x+c))^5-17/256/a/d/(1+sin(d*x+c))^4+125/384/a/d/(1+s
in(d*x+c))^3-515/512/a/d/(1+sin(d*x+c))^2+5/2/a/d/(1+sin(d*x+c))+949/512*ln
(1+sin(d*x+c))/a/d

maxima [A] time = 0.38, size = 225, normalized size = 0.91

$$\frac{\frac{2\left(12645 \sin(dx+c)^8 + 3045 \sin(dx+c)^7 - 36765 \sin(dx+c)^6 - 7965 \sin(dx+c)^5 + 42339 \sin(dx+c)^4 + 7139 \sin(dx+c)^3 - 22171 \sin(dx+c)^2 - 2171 \sin(dx+c)\right.}{a \sin(dx+c)^9 + a \sin(dx+c)^8 - 4 a \sin(dx+c)^7 - 4 a \sin(dx+c)^6 + 6 a \sin(dx+c)^5 + 6 a \sin(dx+c)^4 - 4 a \sin(dx+c)^3 - 4 a \sin(dx+c)^2 + a \sin(dx+c) + a}}{7680\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/7680*(2*(12645*sin(d*x + c)^8 + 3045*sin(d*x + c)^7 - 36765*sin(d*x + c)^
6 - 7965*sin(d*x + c)^5 + 42339*sin(d*x + c)^4 + 7139*sin(d*x + c)^3 - 2217
1*sin(d*x + c)^2 - 2171*sin(d*x + c) + 4384)/(a*sin(d*x + c)^9 + a*sin(d*x
+ c)^8 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a
*sin(d*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c)
+ a) + 14235*log(sin(d*x + c) + 1)/a - 6555*log(sin(d*x + c) - 1)/a - 7680*
sin(d*x + c)/a)/d
```

mupad [B] time = 10.85, size = 595, normalized size = 2.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^11/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (949*log(tan(c/2 + (d*x)/2) + 1))/(256*a*d) - (437*log(tan(c/2 + (d*x)/2) -
 1))/(256*a*d) - ((693*tan(c/2 + (d*x)/2))/128 + (565*tan(c/2 + (d*x)/2)^2)
/64 - (4439*tan(c/2 + (d*x)/2)^3)/128 - (963*tan(c/2 + (d*x)/2)^4)/16 + (70
91*tan(c/2 + (d*x)/2)^5)/80 + (40031*tan(c/2 + (d*x)/2)^6)/240 - (12829*tan
(c/2 + (d*x)/2)^7)/120 - (17969*tan(c/2 + (d*x)/2)^8)/80 + (39491*tan(c/2 +
 (d*x)/2)^9)/960 + (49513*tan(c/2 + (d*x)/2)^10)/480 + (39491*tan(c/2 + (d*
x)/2)^11)/960 - (17969*tan(c/2 + (d*x)/2)^12)/80 - (12829*tan(c/2 + (d*x)/2
)^13)/120 + (40031*tan(c/2 + (d*x)/2)^14)/240 + (7091*tan(c/2 + (d*x)/2)^15
)/80 - (963*tan(c/2 + (d*x)/2)^16)/16 - (4439*tan(c/2 + (d*x)/2)^17)/128 +
(565*tan(c/2 + (d*x)/2)^18)/64 + (693*tan(c/2 + (d*x)/2)^19)/128)/(d*(a + 2
*a*tan(c/2 + (d*x)/2) - 6*a*tan(c/2 + (d*x)/2)^2 - 14*a*tan(c/2 + (d*x)/2)^
3 + 13*a*tan(c/2 + (d*x)/2)^4 + 40*a*tan(c/2 + (d*x)/2)^5 - 8*a*tan(c/2 + (
d*x)/2)^6 - 56*a*tan(c/2 + (d*x)/2)^7 - 14*a*tan(c/2 + (d*x)/2)^8 + 28*a*ta
n(c/2 + (d*x)/2)^9 + 28*a*tan(c/2 + (d*x)/2)^10 + 28*a*tan(c/2 + (d*x)/2)^1
1 - 14*a*tan(c/2 + (d*x)/2)^12 - 56*a*tan(c/2 + (d*x)/2)^13 - 8*a*tan(c/2 +
 (d*x)/2)^14 + 40*a*tan(c/2 + (d*x)/2)^15 + 13*a*tan(c/2 + (d*x)/2)^16 - 14
*a*tan(c/2 + (d*x)/2)^17 - 6*a*tan(c/2 + (d*x)/2)^18 + 2*a*tan(c/2 + (d*x)/
2)^19 + a*tan(c/2 + (d*x)/2)^20)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**11/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.897 $\int \frac{\sin(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)} dx$

Optimal. Leaf size=233

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{15a^3}{256d(a\sin(c+dx)+a)^4}-\frac{7a^2}{192d(a-a\sin(c+dx))^3}-384a$$

```
[Out] -193/512*ln(1-sin(d*x+c))/a/d-319/512*ln(1+sin(d*x+c))/a/d+1/256*a^3/d/(a-a
*sin(d*x+c))^4-7/192*a^2/d/(a-a*sin(d*x+c))^3+81/512*a/d/(a-a*sin(d*x+c))^2
-61/128/d/(a-a*sin(d*x+c))-1/160*a^4/d/(a+a*sin(d*x+c))^5+15/256*a^3/d/(a+a
*sin(d*x+c))^4-95/384*a^2/d/(a+a*sin(d*x+c))^3+325/512*a/d/(a+a*sin(d*x+c))
^2-315/256/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.23, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{15a^3}{256d(a\sin(c+dx)+a)^4}-\frac{7a^2}{192d(a-a\sin(c+dx))^3}-384a$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-193*Log[1 - Sin[c + d*x]])/(512*a*d) - (319*Log[1 + Sin[c + d*x]])/(512*a
*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) - (7*a^2)/(192*d*(a - a*Sin[c + d*
x])^3) + (81*a)/(512*d*(a - a*Sin[c + d*x])^2) - 61/(128*d*(a - a*Sin[c + d
*x])) - a^4/(160*d*(a + a*Sin[c + d*x])^5) + (15*a^3)/(256*d*(a + a*Sin[c +
 d*x])^4) - (95*a^2)/(384*d*(a + a*Sin[c + d*x])^3) + (325*a)/(512*d*(a + a
*Sin[c + d*x])^2) - 315/(256*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\,\text{Subst}\left(\int \frac{x^{10}}{a^{10}(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^{10}}{(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^4}{64(a-x)^5} - \frac{7a^3}{64(a-x)^4} + \frac{81a^2}{256(a-x)^3} - \frac{61a}{128(a-x)^2} + \frac{193}{512(a-x)} + \frac{a^5}{32(a+x)^6} - \frac{15a^4}{64(a+x)^5}\right.\right.}{ad}$$

$$= -\frac{193\log(1-\sin(c+dx))}{512ad} - \frac{319\log(1+\sin(c+dx))}{512ad} + \frac{a^3}{256d(a-a\sin(c+dx))}$$

Mathematica [A] time = 4.97, size = 137, normalized size = 0.59

$$-\frac{\frac{2\left(2895\sin^8(c+dx)-6705\sin^7(c+dx)-13815\sin^6(c+dx)+14985\sin^5(c+dx)+23049\sin^4(c+dx)-12151\sin^3(c+dx)-16561\sin^2(c+dx)+3439\sin(c+dx)\right.}{(\sin(c+dx)-1)^4(\sin(c+dx)+1)^5}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7680*(2895*Log[1 - Sin[c + d*x]] + 4785*Log[1 + Sin[c + d*x]] + (2*(4384
 + 3439*Sin[c + d*x] - 16561*Sin[c + d*x]^2 - 12151*Sin[c + d*x]^3 + 23049*
Sin[c + d*x]^4 + 14985*Sin[c + d*x]^5 - 13815*Sin[c + d*x]^6 - 6705*Sin[c +
 d*x]^7 + 2895*Sin[c + d*x]^8))/((-1 + Sin[c + d*x])^4*(1 + Sin[c + d*x])^5
))/(a*d)
```

fricas [A] time = 0.54, size = 187, normalized size = 0.80

$$-\frac{5790\cos(dx+c)^8 + 4470\cos(dx+c)^6 - 2052\cos(dx+c)^4 + 656\cos(dx+c)^2 + 4785\left(\cos(dx+c)^8\sin(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/7680*(5790*cos(d*x + c)^8 + 4470*cos(d*x + c)^6 - 2052*cos(d*x + c)^4 +
656*cos(d*x + c)^2 + 4785*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*lo
g(sin(d*x + c) + 1) + 2895*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*l
og(-sin(d*x + c) + 1) + 2*(6705*cos(d*x + c)^6 - 5130*cos(d*x + c)^4 + 2296
*cos(d*x + c)^2 - 432)*sin(d*x + c) - 96)/(a*d*cos(d*x + c)^8*sin(d*x + c)
+ a*d*cos(d*x + c)^8)
```

giac [A] time = 0.37, size = 156, normalized size = 0.67

$$-\frac{\frac{19140 \log(|\sin(dx+c)+1|)}{a}+\frac{11580 \log(|\sin(dx+c)-1|)}{a}-\frac{5\left(4825 \sin(dx+c)^4-16372 \sin(dx+c)^3+21138 \sin(dx+c)^2-12236 \sin(dx+c)+2669\right)}{a(\sin(dx+c)-1)^4}-}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30720*(19140*log(abs(sin(d*x + c) + 1))/a + 11580*log(abs(sin(d*x + c) -
 1))/a - 5*(4825*sin(d*x + c)^4 - 16372*sin(d*x + c)^3 + 21138*sin(d*x + c)
^2 - 12236*sin(d*x + c) + 2669)/(a*(sin(d*x + c) - 1)^4) - (43703*sin(d*x +
 c)^5 + 180715*sin(d*x + c)^4 + 305330*sin(d*x + c)^3 + 261130*sin(d*x + c)
^2 + 112415*sin(d*x + c) + 19411)/(a*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.48, size = 198, normalized size = 0.85

$$\frac{1}{256ad\left(\sin\left(dx+c\right)-1\right)^{4}}+\frac{7}{192ad\left(\sin\left(dx+c\right)-1\right)^{3}}+\frac{81}{512ad\left(\sin\left(dx+c\right)-1\right)^{2}}+\frac{61}{128ad\left(\sin\left(dx+c\right)-1\right)}-\frac{193\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^10/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4+7/192/a/d/(sin(d*x+c)-1)^3+81/512/a/d/(sin(d*x+c
)-1)^2+61/128/a/d/(sin(d*x+c)-1)-193/512/a/d*ln(sin(d*x+c)-1)-1/160/a/d/(1+
sin(d*x+c))^5+15/256/a/d/(1+sin(d*x+c))^4-95/384/a/d/(1+sin(d*x+c))^3+325/5
12/a/d/(1+sin(d*x+c))^2-315/256/a/d/(1+sin(d*x+c))-319/512*ln(1+sin(d*x+c))
/a/d
```

maxima [A] time = 0.53, size = 214, normalized size = 0.92

$$-\frac{\frac{2\left(2895 \sin(dx+c)^8-6705 \sin(dx+c)^7-13815 \sin(dx+c)^6+14985 \sin(dx+c)^5+23049 \sin(dx+c)^4-12151 \sin(dx+c)^3-16561 \sin(dx+c)^2+3439 \sin(d\right.}{a \sin(dx+c)^9+a \sin(dx+c)^8-4\,a \sin(dx+c)^7-4\,a \sin(dx+c)^6+6\,a \sin(dx+c)^5+6\,a \sin(dx+c)^4-4\,a \sin(dx+c)^3-4\,a \sin(dx+c)^2+a \sin(dx+c)+}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/7680*(2*(2895*sin(d*x + c)^8 - 6705*sin(d*x + c)^7 - 13815*sin(d*x + c)^
6 + 14985*sin(d*x + c)^5 + 23049*sin(d*x + c)^4 - 12151*sin(d*x + c)^3 - 16
561*sin(d*x + c)^2 + 3439*sin(d*x + c) + 4384)/(a*sin(d*x + c)^9 + a*sin(d*
x + c)^8 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6
*a*sin(d*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c
) + a) + 4785*log(sin(d*x + c) + 1)/a + 2895*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 9.40, size = 539, normalized size = 2.31

$$\frac{\frac{63\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{128}-\frac{65\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{64}-\frac{233\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{32}+\frac{413\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{1}}{64}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}-7\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-16\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+20\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+56\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^10/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] ((63*tan(c/2 + (d*x)/2))/128 - (65*tan(c/2 + (d*x)/2)^2)/64 - (233*tan(c/2
+ (d*x)/2)^3)/32 + (413*tan(c/2 + (d*x)/2)^4)/64 + (6527*tan(c/2 + (d*x)/2)
^5)/160 - (14911*tan(c/2 + (d*x)/2)^6)/960 - (59737*tan(c/2 + (d*x)/2)^7)/4
80 + (12763*tan(c/2 + (d*x)/2)^8)/960 + (45791*tan(c/2 + (d*x)/2)^9)/192 +
(12763*tan(c/2 + (d*x)/2)^10)/960 - (59737*tan(c/2 + (d*x)/2)^11)/480 - (14
911*tan(c/2 + (d*x)/2)^12)/960 + (6527*tan(c/2 + (d*x)/2)^13)/160 + (413*ta
n(c/2 + (d*x)/2)^14)/64 - (233*tan(c/2 + (d*x)/2)^15)/32 - (65*tan(c/2 + (d
*x)/2)^16)/64 + (63*tan(c/2 + (d*x)/2)^17)/128)/(d*(a + 2*a*tan(c/2 + (d*x)
/2) - 7*a*tan(c/2 + (d*x)/2)^2 - 16*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 +
 (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)/2)^5 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a
*tan(c/2 + (d*x)/2)^7 + 14*a*tan(c/2 + (d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2
)^9 + 14*a*tan(c/2 + (d*x)/2)^10 - 112*a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c
/2 + (d*x)/2)^12 + 56*a*tan(c/2 + (d*x)/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14
- 16*a*tan(c/2 + (d*x)/2)^15 - 7*a*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d
*x)/2)^17 + a*tan(c/2 + (d*x)/2)^18)) - (193*log(tan(c/2 + (d*x)/2) - 1))/(
256*a*d) - (319*log(tan(c/2 + (d*x)/2) + 1))/(256*a*d) + log(tan(c/2 + (d*x
)/2)^2 + 1)/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**10/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.898 $\int \frac{\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{\tan^{10}(c+dx)}{10ad}+\frac{63\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^9(c+dx)\sec(c+dx)}{10ad}+\frac{9\tan^7(c+dx)\sec(c+dx)}{80ad}-\frac{21\tan^5(c+dx)\,\mathrm{s}}{160ad}$$

```
[Out] 63/256*arctanh(sin(d*x+c))/a/d-63/256*sec(d*x+c)*tan(d*x+c)/a/d+21/128*sec(
d*x+c)*tan(d*x+c)^3/a/d-21/160*sec(d*x+c)*tan(d*x+c)^5/a/d+9/80*sec(d*x+c)*
tan(d*x+c)^7/a/d-1/10*sec(d*x+c)*tan(d*x+c)^9/a/d+1/10*tan(d*x+c)^10/a/d
```

Rubi [A] time = 0.19, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\tan^{10}(c+dx)}{10ad}+\frac{63\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^9(c+dx)\sec(c+dx)}{10ad}+\frac{9\tan^7(c+dx)\sec(c+dx)}{80ad}-\frac{21\tan^5(c+dx)\,\mathrm{s}}{160ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] (63*ArcTanh[Sin[c + d*x]])/(256*a*d) - (63*Sec[c + d*x]*Tan[c + d*x])/(256*
a*d) + (21*Sec[c + d*x]*Tan[c + d*x]^3)/(128*a*d) - (21*Sec[c + d*x]*Tan[c
+ d*x]^5)/(160*a*d) + (9*Sec[c + d*x]*Tan[c + d*x]^7)/(80*a*d) - (Sec[c + d
*x]*Tan[c + d*x]^9)/(10*a*d) + Tan[c + d*x]^10/(10*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^2(c+dx)\tan^9(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^{10}(c+dx)\,dx}{a}$$

$$= -\frac{\sec(c+dx)\tan^9(c+dx)}{10ad} + \frac{9\int \sec(c+dx)\tan^8(c+dx)\,dx}{10a} + \frac{\text{Subst}\left(\int x^9\,dx, x, \tan(c$$

$$= \frac{9\sec(c+dx)\tan^7(c+dx)}{80ad} - \frac{\sec(c+dx)\tan^9(c+dx)}{10ad} + \frac{\tan^{10}(c+dx)}{10ad} - \frac{63\int \sec(c+$$

$$= -\frac{21\sec(c+dx)\tan^5(c+dx)}{160ad} + \frac{9\sec(c+dx)\tan^7(c+dx)}{80ad} - \frac{\sec(c+dx)\tan^9(c+dx)}{10ad}$$

$$= \frac{21\sec(c+dx)\tan^3(c+dx)}{128ad} - \frac{21\sec(c+dx)\tan^5(c+dx)}{160ad} + \frac{9\sec(c+dx)\tan^7(c+dx)}{80ad}$$

$$= -\frac{63\sec(c+dx)\tan(c+dx)}{256ad} + \frac{21\sec(c+dx)\tan^3(c+dx)}{128ad} - \frac{21\sec(c+dx)\tan^5(c+dx}{160ad}$$

$$= \frac{63\tanh^{-1}(\sin(c+dx))}{256ad} - \frac{63\sec(c+dx)\tan(c+dx)}{256ad} + \frac{21\sec(c+dx)\tan^3(c+dx)}{128ad} - 2$$

Mathematica [A] time = 2.42, size = 122, normalized size = 0.79

$$\frac{\frac{2\left(965\sin^8(c+dx)+325\sin^7(c+dx)-2045\sin^6(c+dx)-765\sin^5(c+dx)+1923\sin^4(c+dx)+643\sin^3(c+dx)-827\sin^2(c+dx)-187\sin(c+dx)+128\right)}{(\sin(c+dx)-1)^4(\sin(c+dx)+1)^5}+630}{2560ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] (630*ArcTanh[Sin[c + d*x]] + (2*(128 - 187*Sin[c + d*x] - 827*Sin[c + d*x]^
2 + 643*Sin[c + d*x]^3 + 1923*Sin[c + d*x]^4 - 765*Sin[c + d*x]^5 - 2045*Si
n[c + d*x]^6 + 325*Sin[c + d*x]^7 + 965*Sin[c + d*x]^8))/((-1 + Sin[c + d*x
])^4*(1 + Sin[c + d*x])^5))/(2560*a*d)
```

fricas [A] time = 0.53, size = 187, normalized size = 1.21

$$\frac{1930 \cos(dx+c)^8 - 3630 \cos(dx+c)^6 + 3156 \cos(dx+c)^4 - 1488 \cos(dx+c)^2 + 315 \left(\cos(dx+c)^8 \sin(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2560*(1930*cos(d*x + c)^8 - 3630*cos(d*x + c)^6 + 3156*cos(d*x + c)^4 - 1
488*cos(d*x + c)^2 + 315*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log
(sin(d*x + c) + 1) - 315*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log
(-sin(d*x + c) + 1) - 2*(325*cos(d*x + c)^6 - 210*cos(d*x + c)^4 + 88*cos(d
*x + c)^2 - 16)*sin(d*x + c) + 288)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*
cos(d*x + c)^8)
```

giac [A] time = 0.37, size = 156, normalized size = 1.01

$$\frac{\frac{1260 \log(|\sin(dx+c)+1|)}{a} - \frac{1260 \log(|\sin(dx+c)-1|)}{a} + \frac{5\left(525 \sin(dx+c)^4 - 1580 \sin(dx+c)^3 + 1818 \sin(dx+c)^2 - 932 \sin(dx+c) + 177\right)}{a(\sin(dx+c)-1)^4} - \frac{2877 \sin(}{}}{10240\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/10240*(1260*log(abs(sin(d*x + c) + 1))/a - 1260*log(abs(sin(d*x + c) - 1)
)/a + 5*(525*sin(d*x + c)^4 - 1580*sin(d*x + c)^3 + 1818*sin(d*x + c)^2 - 9
32*sin(d*x + c) + 177)/(a*(sin(d*x + c) - 1)^4) - (2877*sin(d*x + c)^5 + 92
65*sin(d*x + c)^4 + 12030*sin(d*x + c)^3 + 7430*sin(d*x + c)^2 + 1965*sin(d
*x + c) + 113)/(a*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.46, size = 198, normalized size = 1.29

$$\frac{1}{256ad(\sin(dx+c)-1)^4} + \frac{1}{32ad(\sin(dx+c)-1)^3} + \frac{57}{512ad(\sin(dx+c)-1)^2} + \frac{65}{256ad(\sin(dx+c)-1)} - \frac{63 \ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4+1/32/a/d/(sin(d*x+c)-1)^3+57/512/a/d/(sin(d*x+c)
-1)^2+65/256/a/d/(sin(d*x+c)-1)-63/512/a/d*ln(sin(d*x+c)-1)+1/160/a/d/(1+si
n(d*x+c))^5-13/256/a/d/(1+sin(d*x+c))^4+23/128/a/d/(1+sin(d*x+c))^3-187/512
/a/d/(1+sin(d*x+c))^2+1/2/a/d/(1+sin(d*x+c))+63/512*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.52, size = 214, normalized size = 1.39

$$\frac{\frac{2\left(965 \sin(dx+c)^{8}+325 \sin(dx+c)^{7}-2045 \sin(dx+c)^{6}-765 \sin(dx+c)^{5}+1923 \sin(dx+c)^{4}+643 \sin(dx+c)^{3}-827 \sin(dx+c)^{2}-187 \sin(dx+c)+128\right)}{a \sin(dx+c)^{9}+a \sin(dx+c)^{8}-4 a \sin(dx+c)^{7}-4 a \sin(dx+c)^{6}+6 a \sin(dx+c)^{5}+6 a \sin(dx+c)^{4}-4 a \sin(dx+c)^{3}-4 a \sin(dx+c)^{2}+a \sin(dx+c)+a} + \frac{31}{}}{2560\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2560*(2*(965*sin(d*x + c)^8 + 325*sin(d*x + c)^7 - 2045*sin(d*x + c)^6 -
765*sin(d*x + c)^5 + 1923*sin(d*x + c)^4 + 643*sin(d*x + c)^3 - 827*sin(d*x
 + c)^2 - 187*sin(d*x + c) + 128)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*
a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x +
c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) + 315*
log(sin(d*x + c) + 1)/a - 315*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 17.05, size = 497, normalized size = 3.23

$$\frac{63\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{128\,a\,d} - \frac{\frac{63\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{128}+\frac{63\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}}{64} - \cdots}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}-7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (63*atanh(tan(c/2 + (d*x)/2)))/(128*a*d) - ((63*tan(c/2 + (d*x)/2))/128 + (
63*tan(c/2 + (d*x)/2)^2)/64 - (105*tan(c/2 + (d*x)/2)^3)/32 - (483*tan(c/2
+ (d*x)/2)^4)/64 + (1407*tan(c/2 + (d*x)/2)^5)/160 + (8043*tan(c/2 + (d*x)/
2)^6)/320 - (1779*tan(c/2 + (d*x)/2)^7)/160 - (15159*tan(c/2 + (d*x)/2)^8)/
320 + (245*tan(c/2 + (d*x)/2)^9)/64 - (15159*tan(c/2 + (d*x)/2)^10)/320 - (
1779*tan(c/2 + (d*x)/2)^11)/160 + (8043*tan(c/2 + (d*x)/2)^12)/320 + (1407*
tan(c/2 + (d*x)/2)^13)/160 - (483*tan(c/2 + (d*x)/2)^14)/64 - (105*tan(c/2
+ (d*x)/2)^15)/32 + (63*tan(c/2 + (d*x)/2)^16)/64 + (63*tan(c/2 + (d*x)/2)^
17)/128)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 7*a*tan(c/2 + (d*x)/2)^2 - 16*a*t
an(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)/2)^5
 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a*tan(c/2 + (d*x)/2)^7 + 14*a*tan(c/2 +
(d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c/2 + (d*x)/2)^10 - 112*
a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)^12 + 56*a*tan(c/2 + (d*x)
```

```
/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 + (d*x)/2)^15 - 7*a*tan(
c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d*x)/2)^17 + a*tan(c/2 + (d*x)/2)^18))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.899 $\int \frac{\sec(c+dx)\tan^8(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=160

$$-\frac{\tan^{10}(c+dx)}{10ad}+\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^7(c+dx)\sec^3(c+dx)}{10ad}-\frac{7\tan^5(c+dx)\sec^3(c+dx)}{80ad}+\frac{7\tan^3(c+dx)\,\mathrm{s}}{96a\mathit{d}}$$

```
[Out] 7/256*arctanh(sin(d*x+c))/a/d+7/256*sec(d*x+c)*tan(d*x+c)/a/d-7/128*sec(d*x
+c)^3*tan(d*x+c)/a/d+7/96*sec(d*x+c)^3*tan(d*x+c)^3/a/d-7/80*sec(d*x+c)^3*t
an(d*x+c)^5/a/d+1/10*sec(d*x+c)^3*tan(d*x+c)^7/a/d-1/10*tan(d*x+c)^10/a/d
```

Rubi [A] time = 0.27, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2835, 2611, 3768, 3770, 2607, 30}

$$-\frac{\tan^{10}(c+dx)}{10ad}+\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^7(c+dx)\sec^3(c+dx)}{10ad}-\frac{7\tan^5(c+dx)\sec^3(c+dx)}{80ad}+\frac{7\tan^3(c+dx)\,\mathrm{s}}{96a\mathit{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^8)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(256*a*d) + (7*Sec[c + d*x]*Tan[c + d*x])/(256*a*
d) - (7*Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) + (7*Sec[c + d*x]^3*Tan[c +
d*x]^3)/(96*a*d) - (7*Sec[c + d*x]^3*Tan[c + d*x]^5)/(80*a*d) + (Sec[c + d*
x]^3*Tan[c + d*x]^7)/(10*a*d) - Tan[c + d*x]^10/(10*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^8(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^3(c+dx)\tan^8(c+dx)\,dx}{a} - \frac{\int \sec^2(c+dx)\tan^9(c+dx)\,dx}{a}$$

$$= \frac{\sec^3(c+dx)\tan^7(c+dx)}{10ad} - \frac{7\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{10a} - \frac{\text{Subst}\left(\int x^9\,dx$$

$$= -\frac{7\sec^3(c+dx)\tan^5(c+dx)}{80ad} + \frac{\sec^3(c+dx)\tan^7(c+dx)}{10ad} - \frac{\tan^{10}(c+dx)}{10ad} + \frac{7}{}$$

$$= \frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad} - \frac{7\sec^3(c+dx)\tan^5(c+dx)}{80ad} + \frac{\sec^3(c+dx)\tan^7}{10ad}$$

$$= -\frac{7\sec^3(c+dx)\tan(c+dx)}{128ad} + \frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad} - \frac{7\sec^3(c+dx)\tan}{80ad}$$

$$= \frac{7\sec(c+dx)\tan(c+dx)}{256ad} - \frac{7\sec^3(c+dx)\tan(c+dx)}{128ad} + \frac{7\sec^3(c+dx)\tan^3(c}{96ad}$$

$$= \frac{7\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{7\sec(c+dx)\tan(c+dx)}{256ad} - \frac{7\sec^3(c+dx)\tan(c+dx}{128ad}$$

Mathematica [A] time = 2.42, size = 121, normalized size = 0.76

$$\frac{\frac{-210\sin^8(c+dx)+3630\sin^7(c+dx)+2050\sin^6(c+dx)-5630\sin^5(c+dx)-3838\sin^4(c+dx)+3842\sin^3(c+dx)+2862\sin^2(c+dx)-978\sin(c+dx)-768}{(\sin(c+dx)-1)^4(\sin(c+dx)+1)^5}+21}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^8)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (210*ArcTanh[Sin[c + d*x]] + (-768 - 978*Sin[c + d*x] + 2862*Sin[c + d*x]^2
 + 3842*Sin[c + d*x]^3 - 3838*Sin[c + d*x]^4 - 5630*Sin[c + d*x]^5 + 2050*S
in[c + d*x]^6 + 3630*Sin[c + d*x]^7 - 210*Sin[c + d*x]^8)/((-1 + Sin[c + d*
x])^4*(1 + Sin[c + d*x])^5))/(7680*a*d)
```

fricas [A] time = 0.52, size = 187, normalized size = 1.17

$$-\frac{210\cos(dx+c)^8+1210\cos(dx+c)^6-1052\cos(dx+c)^4+496\cos(dx+c)^2-105\left(\cos(dx+c)^8\sin(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/7680*(210*cos(d*x + c)^8 + 1210*cos(d*x + c)^6 - 1052*cos(d*x + c)^4 + 4
96*cos(d*x + c)^2 - 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(
sin(d*x + c) + 1) + 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(
-sin(d*x + c) + 1) + 2*(1815*cos(d*x + c)^6 - 2630*cos(d*x + c)^4 + 1736*co
s(d*x + c)^2 - 432)*sin(d*x + c) - 96)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a
*d*cos(d*x + c)^8)
```

giac [A] time = 0.35, size = 156, normalized size = 0.98

$$\frac{\frac{420\log(|\sin(dx+c)+1|)}{a}-\frac{420\log(|\sin(dx+c)-1|)}{a}+\frac{5\left(175\sin(dx+c)^4-28\sin(dx+c)^3-522\sin(dx+c)^2+588\sin(dx+c)-189\right)}{a(\sin(dx+c)-1)^4}-\frac{959\sin(dx+c)^5+89}{}}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/
a + 5*(175*sin(d*x + c)^4 - 28*sin(d*x + c)^3 - 522*sin(d*x + c)^2 + 588*si
n(d*x + c) - 189)/(a*(sin(d*x + c) - 1)^4) - (959*sin(d*x + c)^5 + 8995*sin
(d*x + c)^4 + 20810*sin(d*x + c)^3 + 21810*sin(d*x + c)^2 + 11055*sin(d*x +
 c) + 2211)/(a*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.44, size = 198, normalized size = 1.24

$$\frac{1}{256ad\left(\sin\left(dx+c\right)-1\right)^{4}}+\frac{5}{192ad\left(\sin\left(dx+c\right)-1\right)^{3}}+\frac{37}{512ad\left(\sin\left(dx+c\right)-1\right)^{2}}+\frac{7}{64ad\left(\sin\left(dx+c\right)-1\right)}-\frac{7\ln(\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^8/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4+5/192/a/d/(sin(d*x+c)-1)^3+37/512/a/d/(sin(d*x+c
)-1)^2+7/64/a/d/(sin(d*x+c)-1)-7/512/a/d*ln(sin(d*x+c)-1)-1/160/a/d/(1+sin(
d*x+c))^5+11/256/a/d/(1+sin(d*x+c))^4-47/384/a/d/(1+sin(d*x+c))^3+93/512/a/
d/(1+sin(d*x+c))^2-35/256/a/d/(1+sin(d*x+c))+7/512*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.34, size = 214, normalized size = 1.34

$$-\frac{\frac{2\left(105\sin(dx+c)^{8}-1815\sin(dx+c)^{7}-1025\sin(dx+c)^{6}+2815\sin(dx+c)^{5}+1919\sin(dx+c)^{4}-1921\sin(dx+c)^{3}-1431\sin(dx+c)^{2}+489\sin(dx+c)+38}{a\sin(dx+c)^{9}+a\sin(dx+c)^{8}-4a\sin(dx+c)^{7}-4a\sin(dx+c)^{6}+6a\sin(dx+c)^{5}+6a\sin(dx+c)^{4}-4a\sin(dx+c)^{3}-4a\sin(dx+c)^{2}+a\sin(dx+c)+a}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(105*sin(d*x + c)^8 - 1815*sin(d*x + c)^7 - 1025*sin(d*x + c)^6
+ 2815*sin(d*x + c)^5 + 1919*sin(d*x + c)^4 - 1921*sin(d*x + c)^3 - 1431*si
n(d*x + c)^2 + 489*sin(d*x + c) + 384)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8
 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d
*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) -
 105*log(sin(d*x + c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 16.77, size = 496, normalized size = 3.10

$$\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{128\,a\,d}+\frac{-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{128}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{1}}{64}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}-7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(128*a*d) + ((35*tan(c/2 + (d*x)/2)^3)/96 - (
7*tan(c/2 + (d*x)/2)^2)/64 - (7*tan(c/2 + (d*x)/2))/128 + (161*tan(c/2 + (d
*x)/2)^4)/192 - (469*tan(c/2 + (d*x)/2)^5)/480 - (2681*tan(c/2 + (d*x)/2)^6
)/960 + (593*tan(c/2 + (d*x)/2)^7)/480 + (5053*tan(c/2 + (d*x)/2)^8)/960 +
```

```
(10841*tan(c/2 + (d*x)/2)^9)/192 + (5053*tan(c/2 + (d*x)/2)^10)/960 + (593*
tan(c/2 + (d*x)/2)^11)/480 - (2681*tan(c/2 + (d*x)/2)^12)/960 - (469*tan(c/
2 + (d*x)/2)^13)/480 + (161*tan(c/2 + (d*x)/2)^14)/192 + (35*tan(c/2 + (d*x
)/2)^15)/96 - (7*tan(c/2 + (d*x)/2)^16)/64 - (7*tan(c/2 + (d*x)/2)^17)/128)
/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 7*a*tan(c/2 + (d*x)/2)^2 - 16*a*tan(c/2 +
 (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)/2)^5 - 28*a*
tan(c/2 + (d*x)/2)^6 - 112*a*tan(c/2 + (d*x)/2)^7 + 14*a*tan(c/2 + (d*x)/2)
^8 + 140*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c/2 + (d*x)/2)^10 - 112*a*tan(c/
2 + (d*x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)^12 + 56*a*tan(c/2 + (d*x)/2)^13 +
 20*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 + (d*x)/2)^15 - 7*a*tan(c/2 + (d
*x)/2)^16 + 2*a*tan(c/2 + (d*x)/2)^17 + a*tan(c/2 + (d*x)/2)^18))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**8/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.900 $\int \frac{\sec^2(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{\tan^{10}(c+dx)}{10ad}+\frac{\tan^8(c+dx)}{8ad}-\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^7(c+dx)\sec^3(c+dx)}{10ad}+\frac{7\tan^5(c+dx)\sec^3(c+dx)}{80ad}-7$$

```
[Out] -7/256*arctanh(sin(d*x+c))/a/d-7/256*sec(d*x+c)*tan(d*x+c)/a/d+7/128*sec(d*
x+c)^3*tan(d*x+c)/a/d-7/96*sec(d*x+c)^3*tan(d*x+c)^3/a/d+7/80*sec(d*x+c)^3*
tan(d*x+c)^5/a/d-1/10*sec(d*x+c)^3*tan(d*x+c)^7/a/d+1/8*tan(d*x+c)^8/a/d+1/
10*tan(d*x+c)^10/a/d
```

Rubi [A] time = 0.28, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2835, 2607, 14, 2611, 3768, 3770}

$$\frac{\tan^{10}(c+dx)}{10ad}+\frac{\tan^8(c+dx)}{8ad}-\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^7(c+dx)\sec^3(c+dx)}{10ad}+\frac{7\tan^5(c+dx)\sec^3(c+dx)}{80ad}-7$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-7*ArcTanh[Sin[c + d*x]])/(256*a*d) - (7*Sec[c + d*x]*Tan[c + d*x])/(256*a
*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) - (7*Sec[c + d*x]^3*Tan[c +
 d*x]^3)/(96*a*d) + (7*Sec[c + d*x]^3*Tan[c + d*x]^5)/(80*a*d) - (Sec[c + d
*x]^3*Tan[c + d*x]^7)/(10*a*d) + Tan[c + d*x]^8/(8*a*d) + Tan[c + d*x]^10/(
10*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
```

```
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^4(c+dx)\tan^7(c+dx)\,dx}{a} - \frac{\int \sec^3(c+dx)\tan^8(c+dx)\,dx}{a}$$
$$= -\frac{\sec^3(c+dx)\tan^7(c+dx)}{10ad} + \frac{7\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{10a} + \frac{\text{Subst}\left(\int x^7\right.}{}$$
$$= \frac{7\sec^3(c+dx)\tan^5(c+dx)}{80ad} - \frac{\sec^3(c+dx)\tan^7(c+dx)}{10ad} - \frac{7\int \sec^3(c+dx)\text{ta}}{16a}$$
$$= -\frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad} + \frac{7\sec^3(c+dx)\tan^5(c+dx)}{80ad} - \frac{\sec^3(c+dx)\text{ta}}{10ad}$$
$$= \frac{7\sec^3(c+dx)\tan(c+dx)}{128ad} - \frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad} + \frac{7\sec^3(c+dx)\tan}{80ad}$$
$$= -\frac{7\sec(c+dx)\tan(c+dx)}{256ad} + \frac{7\sec^3(c+dx)\tan(c+dx)}{128ad} - \frac{7\sec^3(c+dx)\tan}{96ad}$$
$$= -\frac{7\tanh^{-1}(\sin(c+dx))}{256ad} - \frac{7\sec(c+dx)\tan(c+dx)}{256ad} + \frac{7\sec^3(c+dx)\tan(c+}{128ad}$$

Mathematica [A] time = 1.68, size = 124, normalized size = 0.70

$$-\frac{\frac{210}{1-\sin(c+dx)} - \frac{315}{(1-\sin(c+dx))^2} + \frac{525}{(\sin(c+dx)+1)^2} + \frac{160}{(1-\sin(c+dx))^3} - \frac{580}{(\sin(c+dx)+1)^3} - \frac{30}{(1-\sin(c+dx))^4} + \frac{270}{(\sin(c+dx)+1)^4} - \frac{48}{(\sin(c+dx)+}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7680*(210*ArcTanh[Sin[c + d*x]] - 30/(1 - Sin[c + d*x])^4 + 160/(1 - Sin
[c + d*x])^3 - 315/(1 - Sin[c + d*x])^2 + 210/(1 - Sin[c + d*x]) - 48/(1 +
Sin[c + d*x])^5 + 270/(1 + Sin[c + d*x])^4 - 580/(1 + Sin[c + d*x])^3 + 525
/(1 + Sin[c + d*x])^2)/(a*d)
```

fricas [A] time = 0.52, size = 187, normalized size = 1.05

$$\frac{210\cos(dx+c)^8 - 2630\cos(dx+c)^6 + 4708\cos(dx+c)^4 - 3344\cos(dx+c)^2 - 105\left(\cos(dx+c)^8\sin(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/7680*(210*cos(d*x + c)^8 - 2630*cos(d*x + c)^6 + 4708*cos(d*x + c)^4 - 33
44*cos(d*x + c)^2 - 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(
```

```
sin(d*x + c) + 1) + 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(
-sin(d*x + c) + 1) - 2*(105*cos(d*x + c)^6 - 250*cos(d*x + c)^4 + 184*cos(d
*x + c)^2 - 48)*sin(d*x + c) + 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*
cos(d*x + c)^8)
```

giac [A] time = 0.35, size = 156, normalized size = 0.88

$$-\frac{\frac{420 \log(|\sin(dx+c)+1|)}{a} - \frac{420 \log(|\sin(dx+c)-1|)}{a} + \frac{5\left(175 \sin(dx+c)^4 - 868 \sin(dx+c)^3 + 1302 \sin(dx+c)^2 - 828 \sin(dx+c) + 195\right)}{a(\sin(dx+c)-1)^4} - \frac{959 \sin(dx+c)}{}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30720*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))
/a + 5*(175*sin(d*x + c)^4 - 868*sin(d*x + c)^3 + 1302*sin(d*x + c)^2 - 828
*sin(d*x + c) + 195)/(a*(sin(d*x + c) - 1)^4) - (959*sin(d*x + c)^5 + 4795*
sin(d*x + c)^4 + 7490*sin(d*x + c)^3 + 5610*sin(d*x + c)^2 + 2055*sin(d*x +
 c) + 291)/(a*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.40, size = 180, normalized size = 1.01

$$\frac{1}{256ad\left(\sin\left(dx+c\right)-1\right)^{4}}+\frac{1}{48ad\left(\sin\left(dx+c\right)-1\right)^{3}}+\frac{21}{512ad\left(\sin\left(dx+c\right)-1\right)^{2}}+\frac{7}{256ad\left(\sin\left(dx+c\right)-1\right)}+\frac{7\ln\left(\sin\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4+1/48/a/d/(sin(d*x+c)-1)^3+21/512/a/d/(sin(d*x+c)
-1)^2+7/256/a/d/(sin(d*x+c)-1)+7/512/a/d*ln(sin(d*x+c)-1)+1/160/a/d/(1+sin(
d*x+c))^5-9/256/a/d/(1+sin(d*x+c))^4+29/384/a/d/(1+sin(d*x+c))^3-35/512/a/d
/(1+sin(d*x+c))^2-7/512*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.41, size = 214, normalized size = 1.20

$$\frac{\frac{2\left(105 \sin(dx+c)^8 + 105 \sin(dx+c)^7 + 895 \sin(dx+c)^6 - 65 \sin(dx+c)^5 - 961 \sin(dx+c)^4 - \sin(dx+c)^3 + 489 \sin(dx+c)^2 + 9 \sin(dx+c) - 96\right)}{a \sin(dx+c)^9 + a \sin(dx+c)^8 - 4 a \sin(dx+c)^7 - 4 a \sin(dx+c)^6 + 6 a \sin(dx+c)^5 + 6 a \sin(dx+c)^4 - 4 a \sin(dx+c)^3 - 4 a \sin(dx+c)^2 + a \sin(dx+c) + a} - \frac{105 \log(s}{}}{7680\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/7680*(2*(105*sin(d*x + c)^8 + 105*sin(d*x + c)^7 + 895*sin(d*x + c)^6 - 6
5*sin(d*x + c)^5 - 961*sin(d*x + c)^4 - sin(d*x + c)^3 + 489*sin(d*x + c)^2
```

```
 + 9*sin(d*x + c) - 96)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*sin(d*x
+ c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4 - 4*a
*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 105*log(sin(d*
x + c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 16.70, size = 496, normalized size = 2.79

$$\frac{\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{128}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{64}-\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{96}-\frac{161\tan\left(\frac{c}{2}\right.}{19}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}-7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+20a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+56a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] ((7*tan(c/2 + (d*x)/2))/128 + (7*tan(c/2 + (d*x)/2)^2)/64 - (35*tan(c/2 + (
d*x)/2)^3)/96 - (161*tan(c/2 + (d*x)/2)^4)/192 + (469*tan(c/2 + (d*x)/2)^5)
/480 + (2681*tan(c/2 + (d*x)/2)^6)/960 - (593*tan(c/2 + (d*x)/2)^7)/480 + (
25667*tan(c/2 + (d*x)/2)^8)/960 + (1447*tan(c/2 + (d*x)/2)^9)/192 + (25667*
tan(c/2 + (d*x)/2)^10)/960 - (593*tan(c/2 + (d*x)/2)^11)/480 + (2681*tan(c/
2 + (d*x)/2)^12)/960 + (469*tan(c/2 + (d*x)/2)^13)/480 - (161*tan(c/2 + (d*
x)/2)^14)/192 - (35*tan(c/2 + (d*x)/2)^15)/96 + (7*tan(c/2 + (d*x)/2)^16)/6
4 + (7*tan(c/2 + (d*x)/2)^17)/128)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 7*a*tan
(c/2 + (d*x)/2)^2 - 16*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 +
 56*a*tan(c/2 + (d*x)/2)^5 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a*tan(c/2 + (d
*x)/2)^7 + 14*a*tan(c/2 + (d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2)^9 + 14*a*ta
n(c/2 + (d*x)/2)^10 - 112*a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)
^12 + 56*a*tan(c/2 + (d*x)/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/
2 + (d*x)/2)^15 - 7*a*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d*x)/2)^17 + a
*tan(c/2 + (d*x)/2)^18)) - (7*atanh(tan(c/2 + (d*x)/2)))/(128*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.901 $\int \frac{\sec^3(c+dx)\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=176

$$-\frac{\tan^{10}(c+dx)}{10ad}-\frac{\tan^8(c+dx)}{8ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^5(c+dx)\sec^5(c+dx)}{10ad}-\frac{\tan^3(c+dx)\sec^5(c+dx)}{16ad}+\frac{\tan}{}$$

```
[Out] -3/256*arctanh(sin(d*x+c))/a/d-3/256*sec(d*x+c)*tan(d*x+c)/a/d-1/128*sec(d*
x+c)^3*tan(d*x+c)/a/d+1/32*sec(d*x+c)^5*tan(d*x+c)/a/d-1/16*sec(d*x+c)^5*ta
n(d*x+c)^3/a/d+1/10*sec(d*x+c)^5*tan(d*x+c)^5/a/d-1/8*tan(d*x+c)^8/a/d-1/10
*tan(d*x+c)^10/a/d
```

Rubi [A] time = 0.27, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2835, 2611, 3768, 3770, 2607, 14}

$$-\frac{\tan^{10}(c+dx)}{10ad}-\frac{\tan^8(c+dx)}{8ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^5(c+dx)\sec^5(c+dx)}{10ad}-\frac{\tan^3(c+dx)\sec^5(c+dx)}{16ad}+\frac{\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(256*a*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(256*a
*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) + (Sec[c + d*x]^5*Tan[c + d*x
])/(32*a*d) - (Sec[c + d*x]^5*Tan[c + d*x]^3)/(16*a*d) + (Sec[c + d*x]^5*Ta
n[c + d*x]^5)/(10*a*d) - Tan[c + d*x]^8/(8*a*d) - Tan[c + d*x]^10/(10*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
```

```
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^5(c+dx)\tan^6(c+dx)\,dx}{a} - \frac{\int \sec^4(c+dx)\tan^7(c+dx)\,dx}{a}$$

$$= \frac{\sec^5(c+dx)\tan^5(c+dx)}{10ad} - \frac{\int \sec^5(c+dx)\tan^4(c+dx)\,dx}{2a} - \text{Subst}\left(\int x^7\left(1+\right.\right.$$

$$= -\frac{\sec^5(c+dx)\tan^3(c+dx)}{16ad} + \frac{\sec^5(c+dx)\tan^5(c+dx)}{10ad} + \frac{3\int \sec^5(c+dx)\tan}{16a}$$

$$= \frac{\sec^5(c+dx)\tan(c+dx)}{32ad} - \frac{\sec^5(c+dx)\tan^3(c+dx)}{16ad} + \frac{\sec^5(c+dx)\tan^5(c+}{10ad}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{128ad} + \frac{\sec^5(c+dx)\tan(c+dx)}{32ad} - \frac{\sec^5(c+dx)\tan^3(c+}{16ad}$$

$$= -\frac{3\sec(c+dx)\tan(c+dx)}{256ad} - \frac{\sec^3(c+dx)\tan(c+dx)}{128ad} + \frac{\sec^5(c+dx)\tan(c+}{32ad}$$

$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{256ad} - \frac{3\sec(c+dx)\tan(c+dx)}{256ad} - \frac{\sec^3(c+dx)\tan(c+dx)}{128ad}$$

Mathematica [A] time = 2.76, size = 122, normalized size = 0.69

$$-\frac{30\tanh^{-1}(\sin(c+dx)) - \frac{2\left(15\sin^8(c+dx)+15\sin^7(c+dx)-55\sin^6(c+dx)+265\sin^5(c+dx)+137\sin^4(c+dx)-183\sin^3(c+dx)-113\sin^2(c+dx)+\right.}{(\sin(c+dx)-1)^4(\sin(c+dx)+1)^5}}{2560ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2560*(30*ArcTanh[Sin[c + d*x]] - (2*(32 + 47*Sin[c + d*x] - 113*Sin[c +
d*x]^2 - 183*Sin[c + d*x]^3 + 137*Sin[c + d*x]^4 + 265*Sin[c + d*x]^5 - 55*
Sin[c + d*x]^6 + 15*Sin[c + d*x]^7 + 15*Sin[c + d*x]^8))/((-1 + Sin[c + d*x
])^4*(1 + Sin[c + d*x])^5))/(a*d)
```

fricas [A] time = 0.51, size = 187, normalized size = 1.06

$$\frac{30\cos(dx+c)^8 - 10\cos(dx+c)^6 + 124\cos(dx+c)^4 - 112\cos(dx+c)^2 - 15\left(\cos(dx+c)^8\sin(dx+c) + \cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2560*(30*cos(d*x + c)^8 - 10*cos(d*x + c)^6 + 124*cos(d*x + c)^4 - 112*co
s(d*x + c)^2 - 15*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d*
```

```
x + c) + 1) + 15*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(d*
x + c) + 1) - 2*(15*cos(d*x + c)^6 - 310*cos(d*x + c)^4 + 392*cos(d*x + c)^
2 - 144)*sin(d*x + c) + 32)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x
+ c)^8)
```

giac [A] time = 0.36, size = 156, normalized size = 0.89

$$-\frac{\frac{60\log(|\sin(dx+c)+1|)}{a}-\frac{60\log(|\sin(dx+c)-1|)}{a}+\frac{5\left(25\sin(dx+c)^4-84\sin(dx+c)^3+66\sin(dx+c)^2-12\sin(dx+c)-3\right)}{a(\sin(dx+c)-1)^4}-\frac{137\sin(dx+c)^5+885\sin}{}}{10240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/10240*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a
 + 5*(25*sin(d*x + c)^4 - 84*sin(d*x + c)^3 + 66*sin(d*x + c)^2 - 12*sin(d*
x + c) - 3)/(a*(sin(d*x + c) - 1)^4) - (137*sin(d*x + c)^5 + 885*sin(d*x +
c)^4 + 2270*sin(d*x + c)^3 + 2470*sin(d*x + c)^2 + 1265*sin(d*x + c) + 253)
/(a*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.41, size = 198, normalized size = 1.12

$$\frac{1}{256ad\left(\sin\left(dx+c\right)-1\right)^{4}}+\frac{1}{64ad\left(\sin\left(dx+c\right)-1\right)^{3}}+\frac{9}{512ad\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{1}{128ad\left(\sin\left(dx+c\right)-1\right)}+\frac{3\ln\left(\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4+1/64/a/d/(sin(d*x+c)-1)^3+9/512/a/d/(sin(d*x+c)-
1)^2-1/128/a/d/(sin(d*x+c)-1)+3/512/a/d*ln(sin(d*x+c)-1)-1/160/a/d/(1+sin(d
*x+c))^5+7/256/a/d/(1+sin(d*x+c))^4-5/128/a/d/(1+sin(d*x+c))^3+5/512/a/d/(1
+sin(d*x+c))^2+5/256/a/d/(1+sin(d*x+c))-3/512*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.33, size = 214, normalized size = 1.22

$$\frac{\frac{2\left(15\sin(dx+c)^8+15\sin(dx+c)^7-55\sin(dx+c)^6+265\sin(dx+c)^5+137\sin(dx+c)^4-183\sin(dx+c)^3-113\sin(dx+c)^2+47\sin(dx+c)+32\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{15\log(}{}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2560*(2*(15*sin(d*x + c)^8 + 15*sin(d*x + c)^7 - 55*sin(d*x + c)^6 + 265*
sin(d*x + c)^5 + 137*sin(d*x + c)^4 - 183*sin(d*x + c)^3 - 113*sin(d*x + c)
```

```
^2 + 47*sin(d*x + c) + 32)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*sin(d
*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4 -
4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 15*log(sin(
d*x + c) + 1)/a + 15*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 16.64, size = 496, normalized size = 2.82

$$\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{128}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}}{64}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{32}-\frac{23\tan\left(\right.}{}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}-7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}+20\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+56\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] ((3*tan(c/2 + (d*x)/2))/128 + (3*tan(c/2 + (d*x)/2)^2)/64 - (5*tan(c/2 + (d
*x)/2)^3)/32 - (23*tan(c/2 + (d*x)/2)^4)/64 + (67*tan(c/2 + (d*x)/2)^5)/160
 + (383*tan(c/2 + (d*x)/2)^6)/320 + (2841*tan(c/2 + (d*x)/2)^7)/160 + (741*
tan(c/2 + (d*x)/2)^8)/320 + (1377*tan(c/2 + (d*x)/2)^9)/64 + (741*tan(c/2 +
 (d*x)/2)^10)/320 + (2841*tan(c/2 + (d*x)/2)^11)/160 + (383*tan(c/2 + (d*x)
/2)^12)/320 + (67*tan(c/2 + (d*x)/2)^13)/160 - (23*tan(c/2 + (d*x)/2)^14)/6
4 - (5*tan(c/2 + (d*x)/2)^15)/32 + (3*tan(c/2 + (d*x)/2)^16)/64 + (3*tan(c/
2 + (d*x)/2)^17)/128)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 7*a*tan(c/2 + (d*x)/
2)^2 - 16*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2
 + (d*x)/2)^5 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a*tan(c/2 + (d*x)/2)^7 + 14
*a*tan(c/2 + (d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c/2 + (d*x)
/2)^10 - 112*a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)^12 + 56*a*ta
n(c/2 + (d*x)/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 + (d*x)/2)^
15 - 7*a*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d*x)/2)^17 + a*tan(c/2 + (d
*x)/2)^18)) - (3*atanh(tan(c/2 + (d*x)/2)))/(128*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.902 $\int \frac{\sec^4(c+dx)\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{\sec^{10}(c+dx)}{10ad}-\frac{\sec^8(c+dx)}{4ad}+\frac{\sec^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^5(c+dx)\sec^5(c+dx)}{10ad}+\frac{\tan^3(c+dx)\sec}{16ad}$$

```
[Out] 3/256*arctanh(sin(d*x+c))/a/d+1/6*sec(d*x+c)^6/a/d-1/4*sec(d*x+c)^8/a/d+1/1
0*sec(d*x+c)^10/a/d+3/256*sec(d*x+c)*tan(d*x+c)/a/d+1/128*sec(d*x+c)^3*tan(
d*x+c)/a/d-1/32*sec(d*x+c)^5*tan(d*x+c)/a/d+1/16*sec(d*x+c)^5*tan(d*x+c)^3/
a/d-1/10*sec(d*x+c)^5*tan(d*x+c)^5/a/d
```

Rubi [A] time = 0.27, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2835, 2606, 266, 43, 2611, 3768, 3770}

$$\frac{\sec^{10}(c+dx)}{10ad}-\frac{\sec^8(c+dx)}{4ad}+\frac{\sec^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^5(c+dx)\sec^5(c+dx)}{10ad}+\frac{\tan^3(c+dx)\sec}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*Tan[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(256*a*d) + Sec[c + d*x]^6/(6*a*d) - Sec[c + d*x]
^8/(4*a*d) + Sec[c + d*x]^10/(10*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(256*
a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) - (Sec[c + d*x]^5*Tan[c + d*
x])/(32*a*d) + (Sec[c + d*x]^5*Tan[c + d*x]^3)/(16*a*d) - (Sec[c + d*x]^5*T
an[c + d*x]^5)/(10*a*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^6(c+dx)\tan^5(c+dx)\,dx}{a} - \frac{\int \sec^5(c+dx)\tan^6(c+dx)\,dx}{a}$$

$$= -\frac{\sec^5(c+dx)\tan^5(c+dx)}{10ad} + \frac{\int \sec^5(c+dx)\tan^4(c+dx)\,dx}{2a} + \frac{\text{Subst}\left(\int x^5 (\right.}{}$$

$$= \frac{\sec^5(c+dx)\tan^3(c+dx)}{16ad} - \frac{\sec^5(c+dx)\tan^5(c+dx)}{10ad} - \frac{3\int \sec^5(c+dx)\tan}{16a}$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{32ad} + \frac{\sec^5(c+dx)\tan^3(c+dx)}{16ad} - \frac{\sec^5(c+dx)\tan^5(c}{10ad}$$

$$= \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{4ad} + \frac{\sec^{10}(c+dx)}{10ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{128ad} - \frac{\text{se}}{}$$

$$= \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{4ad} + \frac{\sec^{10}(c+dx)}{10ad} + \frac{3\sec(c+dx)\tan(c+dx)}{256ad} + \frac{\text{s}}{}$$

$$= \frac{3\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{4ad} + \frac{\sec^{10}(c+dx)}{10ad} + \frac{3\,\text{se}}{}$$

Mathematica [A] time = 5.49, size = 116, normalized size = 0.60

$$\frac{-\frac{90}{\sin(c+dx)-1} + \frac{15}{(\sin(c+dx)-1)^2} + \frac{75}{(\sin(c+dx)+1)^2} + \frac{80}{(\sin(c+dx)-1)^3} + \frac{100}{(\sin(c+dx)+1)^3} + \frac{30}{(\sin(c+dx)-1)^4} - \frac{150}{(\sin(c+dx)+1)^4} + \frac{48}{(\sin(c+dx)-}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*Tan[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (90*ArcTanh[Sin[c + d*x]] + 30/(-1 + Sin[c + d*x])^4 + 80/(-1 + Sin[c + d*x
])^3 + 15/(-1 + Sin[c + d*x])^2 - 90/(-1 + Sin[c + d*x]) + 48/(1 + Sin[c +
d*x])^5 - 150/(1 + Sin[c + d*x])^4 + 100/(1 + Sin[c + d*x])^3 + 75/(1 + Sin
[c + d*x])^2)/(7680*a*d)
```

fricas [A] time = 0.51, size = 187, normalized size = 0.96

$$-\frac{90\cos(dx+c)^8 - 30\cos(dx+c)^6 - 1548\cos(dx+c)^4 + 2224\cos(dx+c)^2 - 45\left(\cos(dx+c)^8\sin(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/7680*(90*cos(d*x + c)^8 - 30*cos(d*x + c)^6 - 1548*cos(d*x + c)^4 + 2224
*cos(d*x + c)^2 - 45*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin
```

```
(d*x + c) + 1) + 45*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin
(d*x + c) + 1) - 2*(45*cos(d*x + c)^6 + 30*cos(d*x + c)^4 - 104*cos(d*x + c
)^2 + 48)*sin(d*x + c) - 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*
x + c)^8)
```

giac [A] time = 0.32, size = 156, normalized size = 0.80

$$\frac{\frac{180 \log(|\sin(dx+c)+1|)}{a} - \frac{180 \log(|\sin(dx+c)-1|)}{a} + \frac{5\left(75 \sin(dx+c)^4 - 372 \sin(dx+c)^3 + 678 \sin(dx+c)^2 - 476 \sin(dx+c) + 119\right)}{a(\sin(dx+c)-1)^4} - \frac{411 \sin(dx+c)^5 + 20}{}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(180*log(abs(sin(d*x + c) + 1))/a - 180*log(abs(sin(d*x + c) - 1))/
a + 5*(75*sin(d*x + c)^4 - 372*sin(d*x + c)^3 + 678*sin(d*x + c)^2 - 476*si
n(d*x + c) + 119)/(a*(sin(d*x + c) - 1)^4) - (411*sin(d*x + c)^5 + 2055*sin
(d*x + c)^4 + 3810*sin(d*x + c)^3 + 2810*sin(d*x + c)^2 + 955*sin(d*x + c)
+ 119)/(a*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.40, size = 180, normalized size = 0.93

$$\frac{1}{256ad\left(\sin\left(dx+c\right)-1\right)^4}+\frac{1}{96ad\left(\sin\left(dx+c\right)-1\right)^3}+\frac{1}{512ad\left(\sin\left(dx+c\right)-1\right)^2}-\frac{3}{256ad\left(\sin\left(dx+c\right)-1\right)}-\frac{3\ln\left(\sin\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4+1/96/a/d/(sin(d*x+c)-1)^3+1/512/a/d/(sin(d*x+c)-
1)^2-3/256/a/d/(sin(d*x+c)-1)-3/512/a/d*ln(sin(d*x+c)-1)+1/160/a/d/(1+sin(d
*x+c))^5-5/256/a/d/(1+sin(d*x+c))^4+5/384/a/d/(1+sin(d*x+c))^3+5/512/a/d/(1
+sin(d*x+c))^2+3/512*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.48, size = 214, normalized size = 1.10

$$-\frac{\frac{2\left(45 \sin(dx+c)^8 + 45 \sin(dx+c)^7 - 165 \sin(dx+c)^6 - 165 \sin(dx+c)^5 - 549 \sin(dx+c)^4 + 91 \sin(dx+c)^3 + 301 \sin(dx+c)^2 - 19 \sin(dx+c) - 64\right)}{a\sin(dx+c)^9 + a\sin(dx+c)^8 - 4a\sin(dx+c)^7 - 4a\sin(dx+c)^6 + 6a\sin(dx+c)^5 + 6a\sin(dx+c)^4 - 4a\sin(dx+c)^3 - 4a\sin(dx+c)^2 + a\sin(dx+c) + a} - \frac{45 \log(}{}}{7680\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(45*sin(d*x + c)^8 + 45*sin(d*x + c)^7 - 165*sin(d*x + c)^6 - 16
5*sin(d*x + c)^5 - 549*sin(d*x + c)^4 + 91*sin(d*x + c)^3 + 301*sin(d*x + c
```

```
)^2 - 19*sin(d*x + c) - 64)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*sin(
d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4 -
 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 45*log(sin
(d*x + c) + 1)/a + 45*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 16.62, size = 496, normalized size = 2.56

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{128\,a\,d}+\frac{-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{128}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{64}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}-7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(128*a*d) + ((5*tan(c/2 + (d*x)/2)^3)/32 - (3
*tan(c/2 + (d*x)/2)^2)/64 - (3*tan(c/2 + (d*x)/2))/128 + (23*tan(c/2 + (d*x
)/2)^4)/64 - (67*tan(c/2 + (d*x)/2)^5)/160 + (9091*tan(c/2 + (d*x)/2)^6)/96
0 + (1717*tan(c/2 + (d*x)/2)^7)/480 + (18257*tan(c/2 + (d*x)/2)^8)/960 - (3
5*tan(c/2 + (d*x)/2)^9)/192 + (18257*tan(c/2 + (d*x)/2)^10)/960 + (1717*tan
(c/2 + (d*x)/2)^11)/480 + (9091*tan(c/2 + (d*x)/2)^12)/960 - (67*tan(c/2 +
(d*x)/2)^13)/160 + (23*tan(c/2 + (d*x)/2)^14)/64 + (5*tan(c/2 + (d*x)/2)^15
)/32 - (3*tan(c/2 + (d*x)/2)^16)/64 - (3*tan(c/2 + (d*x)/2)^17)/128)/(d*(a
+ 2*a*tan(c/2 + (d*x)/2) - 7*a*tan(c/2 + (d*x)/2)^2 - 16*a*tan(c/2 + (d*x)/
2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)/2)^5 - 28*a*tan(c/2
 + (d*x)/2)^6 - 112*a*tan(c/2 + (d*x)/2)^7 + 14*a*tan(c/2 + (d*x)/2)^8 + 14
0*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c/2 + (d*x)/2)^10 - 112*a*tan(c/2 + (d*
x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)^12 + 56*a*tan(c/2 + (d*x)/2)^13 + 20*a*t
an(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 + (d*x)/2)^15 - 7*a*tan(c/2 + (d*x)/2)^
16 + 2*a*tan(c/2 + (d*x)/2)^17 + a*tan(c/2 + (d*x)/2)^18))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.903 $\int \frac{\sec^5(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=192

$$-\frac{\sec^{10}(c+dx)}{10ad}+\frac{\sec^8(c+dx)}{4ad}-\frac{\sec^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^3(c+dx)\sec^7(c+dx)}{10ad}-\frac{3\tan(c+dx)\,\mathrm{se}}{80ad}$$

```
[Out] 3/256*arctanh(sin(d*x+c))/a/d-1/6*sec(d*x+c)^6/a/d+1/4*sec(d*x+c)^8/a/d-1/1
0*sec(d*x+c)^10/a/d+3/256*sec(d*x+c)*tan(d*x+c)/a/d+1/128*sec(d*x+c)^3*tan(
d*x+c)/a/d+1/160*sec(d*x+c)^5*tan(d*x+c)/a/d-3/80*sec(d*x+c)^7*tan(d*x+c)/a
/d+1/10*sec(d*x+c)^7*tan(d*x+c)^3/a/d
```

Rubi [A] time = 0.25, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2835, 2611, 3768, 3770, 2606, 266, 43}

$$-\frac{\sec^{10}(c+dx)}{10ad}+\frac{\sec^8(c+dx)}{4ad}-\frac{\sec^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^3(c+dx)\sec^7(c+dx)}{10ad}-\frac{3\tan(c+dx)\,\mathrm{se}}{80ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(256*a*d) - Sec[c + d*x]^6/(6*a*d) + Sec[c + d*x]
^8/(4*a*d) - Sec[c + d*x]^10/(10*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(256*
a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) + (Sec[c + d*x]^5*Tan[c + d*
x])/(160*a*d) - (3*Sec[c + d*x]^7*Tan[c + d*x])/(80*a*d) + (Sec[c + d*x]^7*
Tan[c + d*x]^3)/(10*a*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^7(c+dx)\tan^4(c+dx)\,dx}{a} - \frac{\int \sec^6(c+dx)\tan^5(c+dx)\,dx}{a}$$

$$= \frac{\sec^7(c+dx)\tan^3(c+dx)}{10ad} - \frac{3\int \sec^7(c+dx)\tan^2(c+dx)\,dx}{10a} - \frac{\text{Subst}\left(\int x^5\left(-\right.\right.}{}$$

$$= -\frac{3\sec^7(c+dx)\tan(c+dx)}{80ad} + \frac{\sec^7(c+dx)\tan^3(c+dx)}{10ad} + \frac{3\int \sec^7(c+dx)\,dx}{80a}$$

$$= \frac{\sec^5(c+dx)\tan(c+dx)}{160ad} - \frac{3\sec^7(c+dx)\tan(c+dx)}{80ad} + \frac{\sec^7(c+dx)\tan^3(c+}{10ad}$$

$$= -\frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{4ad} - \frac{\sec^{10}(c+dx)}{10ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{128ad} + \text{se}$$

$$= -\frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{4ad} - \frac{\sec^{10}(c+dx)}{10ad} + \frac{3\sec(c+dx)\tan(c+dx)}{256ad} + \text{s}$$

$$= \frac{3\tanh^{-1}(\sin(c+dx))}{256ad} - \frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{4ad} - \frac{\sec^{10}(c+dx)}{10ad} + \frac{3\sec(}{}$$

Mathematica [A] time = 5.40, size = 116, normalized size = 0.60

$$\frac{-\frac{90}{\sin(c+dx)+1} - \frac{45}{(\sin(c+dx)-1)^2} - \frac{45}{(\sin(c+dx)+1)^2} + \frac{40}{(\sin(c+dx)-1)^3} + \frac{20}{(\sin(c+dx)+1)^3} + \frac{30}{(\sin(c+dx)-1)^4} + \frac{90}{(\sin(c+dx)+1)^4} - \frac{48}{(\sin(c+dx)+1}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (90*ArcTanh[Sin[c + d*x]] + 30/(-1 + Sin[c + d*x])^4 + 40/(-1 + Sin[c + d*x
])^3 - 45/(-1 + Sin[c + d*x])^2 - 48/(1 + Sin[c + d*x])^5 + 90/(1 + Sin[c +
 d*x])^4 + 20/(1 + Sin[c + d*x])^3 - 45/(1 + Sin[c + d*x])^2 - 90/(1 + Sin[
c + d*x]))/(7680*a*d)
```

fricas [A] time = 0.50, size = 187, normalized size = 0.97

$$-\frac{90\cos(dx+c)^8 - 30\cos(dx+c)^6 - 12\cos(dx+c)^4 + 176\cos(dx+c)^2 - 45\left(\cos(dx+c)^8\sin(dx+c) + \text{cos}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/7680*(90*cos(d*x + c)^8 - 30*cos(d*x + c)^6 - 12*cos(d*x + c)^4 + 176*co
s(d*x + c)^2 - 45*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d*
```

```
x + c) + 1) + 45*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(d*
x + c) + 1) - 2*(45*cos(d*x + c)^6 + 30*cos(d*x + c)^4 - 616*cos(d*x + c)^2
 + 432)*sin(d*x + c) - 96)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x +
 c)^8)
```

giac [A] time = 0.32, size = 156, normalized size = 0.81

$$\frac{\frac{180 \log\left(|\sin(dx+c)+1|\right)}{a} - \frac{180 \log\left(|\sin(dx+c)-1|\right)}{a} + \frac{5\left(75 \sin(dx+c)^4-300 \sin(dx+c)^3+414 \sin(dx+c)^2-196 \sin(dx+c)+31\right)}{a(\sin(dx+c)-1)^4} - \frac{411 \sin(dx+c)^5+2}{}}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(180*log(abs(sin(d*x + c) + 1))/a - 180*log(abs(sin(d*x + c) - 1))/
a + 5*(75*sin(d*x + c)^4 - 300*sin(d*x + c)^3 + 414*sin(d*x + c)^2 - 196*si
n(d*x + c) + 31)/(a*(sin(d*x + c) - 1)^4) - (411*sin(d*x + c)^5 + 2415*sin(
d*x + c)^4 + 5730*sin(d*x + c)^3 + 6730*sin(d*x + c)^2 + 3515*sin(d*x + c)
+ 703)/(a*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.40, size = 180, normalized size = 0.94

$$\frac{1}{256ad\left(\sin\left(dx+c\right)-1\right)^{4}}+\frac{1}{192ad\left(\sin\left(dx+c\right)-1\right)^{3}}-\frac{3}{512ad\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{3\ln\left(\sin\left(dx+c\right)-1\right)}{512ad}-\frac{}{160ad\,(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4+1/192/a/d/(sin(d*x+c)-1)^3-3/512/a/d/(sin(d*x+c)
-1)^2-3/512/a/d*ln(sin(d*x+c)-1)-1/160/a/d/(1+sin(d*x+c))^5+3/256/a/d/(1+si
n(d*x+c))^4+1/384/a/d/(1+sin(d*x+c))^3-3/512/a/d/(1+sin(d*x+c))^2-3/256/a/d
/(1+sin(d*x+c))+3/512*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.43, size = 214, normalized size = 1.11

$$-\frac{\frac{2\left(45 \sin(dx+c)^8+45 \sin(dx+c)^7-165 \sin(dx+c)^6-165 \sin(dx+c)^5+219 \sin(dx+c)^4-421 \sin(dx+c)^3-211 \sin(dx+c)^2+109 \sin(dx+c)+64\right)}{a \sin(dx+c)^9+a \sin(dx+c)^8-4 a \sin(dx+c)^7-4 a \sin(dx+c)^6+6 a \sin(dx+c)^5+6 a \sin(dx+c)^4-4 a \sin(dx+c)^3-4 a \sin(dx+c)^2+a \sin(dx+c)+a} - \frac{45 \mathrm{l}}{}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(45*sin(d*x + c)^8 + 45*sin(d*x + c)^7 - 165*sin(d*x + c)^6 - 16
5*sin(d*x + c)^5 + 219*sin(d*x + c)^4 - 421*sin(d*x + c)^3 - 211*sin(d*x +
```

```
c)^2 + 109*sin(d*x + c) + 64)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*si
n(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4
 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 45*log(s
in(d*x + c) + 1)/a + 45*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 16.71, size = 496, normalized size = 2.58

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{128\,a\,d}+\frac{-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{128}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}-7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(128*a*d) + ((5*tan(c/2 + (d*x)/2)^3)/32 - (3
*tan(c/2 + (d*x)/2)^2)/64 - (3*tan(c/2 + (d*x)/2))/128 + (23*tan(c/2 + (d*x
)/2)^4)/64 + (957*tan(c/2 + (d*x)/2)^5)/160 + (899*tan(c/2 + (d*x)/2)^6)/96
0 + (5813*tan(c/2 + (d*x)/2)^7)/480 + (1873*tan(c/2 + (d*x)/2)^8)/960 + (40
61*tan(c/2 + (d*x)/2)^9)/192 + (1873*tan(c/2 + (d*x)/2)^10)/960 + (5813*tan
(c/2 + (d*x)/2)^11)/480 + (899*tan(c/2 + (d*x)/2)^12)/960 + (957*tan(c/2 +
(d*x)/2)^13)/160 + (23*tan(c/2 + (d*x)/2)^14)/64 + (5*tan(c/2 + (d*x)/2)^15
)/32 - (3*tan(c/2 + (d*x)/2)^16)/64 - (3*tan(c/2 + (d*x)/2)^17)/128)/(d*(a
+ 2*a*tan(c/2 + (d*x)/2) - 7*a*tan(c/2 + (d*x)/2)^2 - 16*a*tan(c/2 + (d*x)/
2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)/2)^5 - 28*a*tan(c/2
 + (d*x)/2)^6 - 112*a*tan(c/2 + (d*x)/2)^7 + 14*a*tan(c/2 + (d*x)/2)^8 + 14
0*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c/2 + (d*x)/2)^10 - 112*a*tan(c/2 + (d*
x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)^12 + 56*a*tan(c/2 + (d*x)/2)^13 + 20*a*t
an(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 + (d*x)/2)^15 - 7*a*tan(c/2 + (d*x)/2)^
16 + 2*a*tan(c/2 + (d*x)/2)^17 + a*tan(c/2 + (d*x)/2)^18))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.904 $\int \frac{\sec^6(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{\sec^{10}(c+dx)}{10ad}-\frac{\sec^8(c+dx)}{8ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^3(c+dx)\sec^7(c+dx)}{10ad}+\frac{3\tan(c+dx)\sec^7(c+dx)}{80ad}-\tan$$

```
[Out] -3/256*arctanh(sin(d*x+c))/a/d-1/8*sec(d*x+c)^8/a/d+1/10*sec(d*x+c)^10/a/d-
3/256*sec(d*x+c)*tan(d*x+c)/a/d-1/128*sec(d*x+c)^3*tan(d*x+c)/a/d-1/160*sec
(d*x+c)^5*tan(d*x+c)/a/d+3/80*sec(d*x+c)^7*tan(d*x+c)/a/d-1/10*sec(d*x+c)^7
*tan(d*x+c)^3/a/d
```

Rubi [A] time = 0.24, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2835, 2606, 14, 2611, 3768, 3770}

$$\frac{\sec^{10}(c+dx)}{10ad}-\frac{\sec^8(c+dx)}{8ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^3(c+dx)\sec^7(c+dx)}{10ad}+\frac{3\tan(c+dx)\sec^7(c+dx)}{80ad}-\tan$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^6*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(256*a*d) - Sec[c + d*x]^8/(8*a*d) + Sec[c + d*x
]^10/(10*a*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(256*a*d) - (Sec[c + d*x]^3*T
an[c + d*x])/(128*a*d) - (Sec[c + d*x]^5*Tan[c + d*x])/(160*a*d) + (3*Sec[c
 + d*x]^7*Tan[c + d*x])/(80*a*d) - (Sec[c + d*x]^7*Tan[c + d*x]^3)/(10*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
```

```
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}dx = \frac{\int \sec^8(c+dx)\tan^3(c+dx)\,dx}{a} - \frac{\int \sec^7(c+dx)\tan^4(c+dx)\,dx}{a}$$
$$= -\frac{\sec^7(c+dx)\tan^3(c+dx)}{10ad} + \frac{3\int \sec^7(c+dx)\tan^2(c+dx)\,dx}{10a} + \frac{\text{Subst}\left(\int x^7 \right.}{}$$
$$= \frac{3\sec^7(c+dx)\tan(c+dx)}{80ad} - \frac{\sec^7(c+dx)\tan^3(c+dx)}{10ad} - \frac{3\int \sec^7(c+dx)\,dx}{80a}$$
$$= -\frac{\sec^8(c+dx)}{8ad} + \frac{\sec^{10}(c+dx)}{10ad} - \frac{\sec^5(c+dx)\tan(c+dx)}{160ad} + \frac{3\sec^7(c+dx)\,\mathrm{t}}{80ad}$$
$$= -\frac{\sec^8(c+dx)}{8ad} + \frac{\sec^{10}(c+dx)}{10ad} - \frac{\sec^3(c+dx)\tan(c+dx)}{128ad} - \frac{\sec^5(c+dx)\tan}{160ad}$$
$$= -\frac{\sec^8(c+dx)}{8ad} + \frac{\sec^{10}(c+dx)}{10ad} - \frac{3\sec(c+dx)\tan(c+dx)}{256ad} - \frac{\sec^3(c+dx)\tan}{128ad}$$
$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{256ad} - \frac{\sec^8(c+dx)}{8ad} + \frac{\sec^{10}(c+dx)}{10ad} - \frac{3\sec(c+dx)\tan(c}{256ad}$$

Mathematica [A] time = 2.81, size = 104, normalized size = 0.60

$$-\frac{-\frac{30}{\sin(c+dx)-1} + \frac{15}{(\sin(c+dx)-1)^2} + \frac{15}{(\sin(c+dx)+1)^2} + \frac{20}{(\sin(c+dx)+1)^3} - \frac{10}{(\sin(c+dx)-1)^4} + \frac{10}{(\sin(c+dx)+1)^4} - \frac{16}{(\sin(c+dx)+1)^5} + 30\tanh}{2560ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^6*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2560*(30*ArcTanh[Sin[c + d*x]] - 10/(-1 + Sin[c + d*x])^4 + 15/(-1 + Sin
[c + d*x])^2 - 30/(-1 + Sin[c + d*x]) - 16/(1 + Sin[c + d*x])^5 + 10/(1 + S
in[c + d*x])^4 + 20/(1 + Sin[c + d*x])^3 + 15/(1 + Sin[c + d*x])^2)/(a*d)
```

fricas [A] time = 0.51, size = 187, normalized size = 1.07

$$\frac{30\cos(dx+c)^8 - 10\cos(dx+c)^6 - 4\cos(dx+c)^4 - 368\cos(dx+c)^2 - 15\left(\cos(dx+c)^8\sin(dx+c) + \cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2560*(30*cos(d*x + c)^8 - 10*cos(d*x + c)^6 - 4*cos(d*x + c)^4 - 368*cos(
d*x + c)^2 - 15*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d*x
+ c) + 1) + 15*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(d*x
```

```
+ c) + 1) - 2*(15*cos(d*x + c)^6 + 10*cos(d*x + c)^4 + 8*cos(d*x + c)^2 - 1
6)*sin(d*x + c) + 288)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x + c)^
8)
```

giac [A] time = 0.31, size = 156, normalized size = 0.90

$$-\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a} - \frac{60 \log(|\sin(dx+c)-1|)}{a} + \frac{5\left(25 \sin(dx+c)^4-124 \sin(dx+c)^3+234 \sin(dx+c)^2-196 \sin(dx+c)+53\right)}{a(\sin(dx+c)-1)^4} - \frac{137 \sin(dx+c)^5+685}{}}{10240\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/10240*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a
 + 5*(25*sin(d*x + c)^4 - 124*sin(d*x + c)^3 + 234*sin(d*x + c)^2 - 196*sin
(d*x + c) + 53)/(a*(sin(d*x + c) - 1)^4) - (137*sin(d*x + c)^5 + 685*sin(d*
x + c)^4 + 1310*sin(d*x + c)^3 + 1110*sin(d*x + c)^2 + 305*sin(d*x + c) + 2
1)/(a*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.38, size = 162, normalized size = 0.93

$$\frac{1}{256ad\left(\sin\left(dx+c\right)-1\right)^{4}}-\frac{3}{512ad\left(\sin\left(dx+c\right)-1\right)^{2}}+\frac{3}{256ad\left(\sin\left(dx+c\right)-1\right)}+\frac{3\ln\left(\sin\left(dx+c\right)-1\right)}{512ad}+\frac{}{160ad\,(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4-3/512/a/d/(sin(d*x+c)-1)^2+3/256/a/d/(sin(d*x+c)
-1)+3/512/a/d*ln(sin(d*x+c)-1)+1/160/a/d/(1+sin(d*x+c))^5-1/256/a/d/(1+sin(
d*x+c))^4-1/128/a/d/(1+sin(d*x+c))^3-3/512/a/d/(1+sin(d*x+c))^2-3/512*ln(1+
sin(d*x+c))/a/d
```

maxima [A] time = 0.34, size = 214, normalized size = 1.23

$$\frac{\frac{2\left(15 \sin(dx+c)^8+15 \sin(dx+c)^7-55 \sin(dx+c)^6-55 \sin(dx+c)^5+73 \sin(dx+c)^4+73 \sin(dx+c)^3+143 \sin(dx+c)^2-17 \sin(dx+c)-32\right)}{a \sin(dx+c)^9+a \sin(dx+c)^8-4\,a \sin(dx+c)^7-4\,a \sin(dx+c)^6+6\,a \sin(dx+c)^5+6\,a \sin(dx+c)^4-4\,a \sin(dx+c)^3-4\,a \sin(dx+c)^2+a \sin(dx+c)+a} - \frac{15 \log(\sin}{}}{2560\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2560*(2*(15*sin(d*x + c)^8 + 15*sin(d*x + c)^7 - 55*sin(d*x + c)^6 - 55*s
in(d*x + c)^5 + 73*sin(d*x + c)^4 + 73*sin(d*x + c)^3 + 143*sin(d*x + c)^2
- 17*sin(d*x + c) - 32)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*sin(d*x
```

```
+ c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4 - 4*a
*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 15*log(sin(d*x
 + c) + 1)/a + 15*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 17.34, size = 496, normalized size = 2.85

$$\frac{\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{128}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{64}-\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{32}+\frac{233\tan\cdots}{\cdots}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}-7\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-16\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+20\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+56\,\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] ((3*tan(c/2 + (d*x)/2))/128 + (3*tan(c/2 + (d*x)/2)^2)/64 - (5*tan(c/2 + (d
*x)/2)^3)/32 + (233*tan(c/2 + (d*x)/2)^4)/64 + (323*tan(c/2 + (d*x)/2)^5)/1
60 + (2687*tan(c/2 + (d*x)/2)^6)/320 - (231*tan(c/2 + (d*x)/2)^7)/160 + (53
49*tan(c/2 + (d*x)/2)^8)/320 + (353*tan(c/2 + (d*x)/2)^9)/64 + (5349*tan(c/
2 + (d*x)/2)^10)/320 - (231*tan(c/2 + (d*x)/2)^11)/160 + (2687*tan(c/2 + (d
*x)/2)^12)/320 + (323*tan(c/2 + (d*x)/2)^13)/160 + (233*tan(c/2 + (d*x)/2)^
14)/64 - (5*tan(c/2 + (d*x)/2)^15)/32 + (3*tan(c/2 + (d*x)/2)^16)/64 + (3*t
an(c/2 + (d*x)/2)^17)/128)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 7*a*tan(c/2 + (
d*x)/2)^2 - 16*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 + 56*a*ta
n(c/2 + (d*x)/2)^5 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a*tan(c/2 + (d*x)/2)^7
 + 14*a*tan(c/2 + (d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c/2 +
(d*x)/2)^10 - 112*a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)^12 + 56
*a*tan(c/2 + (d*x)/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 + (d*x
)/2)^15 - 7*a*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d*x)/2)^17 + a*tan(c/2
 + (d*x)/2)^18)) - (3*atanh(tan(c/2 + (d*x)/2)))/(128*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.905 $\int \frac{\sec^7(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=172

$$-\frac{\sec^{10}(c+dx)}{10ad}+\frac{\sec^8(c+dx)}{8ad}-\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan(c+dx)\sec^9(c+dx)}{10ad}-\frac{\tan(c+dx)\sec^7(c+dx)}{80ad}-\frac{7\tan$$

```
[Out] -7/256*arctanh(sin(d*x+c))/a/d+1/8*sec(d*x+c)^8/a/d-1/10*sec(d*x+c)^10/a/d-
7/256*sec(d*x+c)*tan(d*x+c)/a/d-7/384*sec(d*x+c)^3*tan(d*x+c)/a/d-7/480*sec
(d*x+c)^5*tan(d*x+c)/a/d-1/80*sec(d*x+c)^7*tan(d*x+c)/a/d+1/10*sec(d*x+c)^9
*tan(d*x+c)/a/d
```

Rubi [A] time = 0.22, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2835, 2611, 3768, 3770, 2606, 14}

$$-\frac{\sec^{10}(c+dx)}{10ad}+\frac{\sec^8(c+dx)}{8ad}-\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan(c+dx)\sec^9(c+dx)}{10ad}-\frac{\tan(c+dx)\sec^7(c+dx)}{80ad}-\frac{7\tan$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-7*ArcTanh[Sin[c + d*x]])/(256*a*d) + Sec[c + d*x]^8/(8*a*d) - Sec[c + d*x
]^10/(10*a*d) - (7*Sec[c + d*x]*Tan[c + d*x])/(256*a*d) - (7*Sec[c + d*x]^3
*Tan[c + d*x])/(384*a*d) - (7*Sec[c + d*x]^5*Tan[c + d*x])/(480*a*d) - (Sec
[c + d*x]^7*Tan[c + d*x])/(80*a*d) + (Sec[c + d*x]^9*Tan[c + d*x])/(10*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
```

```
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^9(c+dx)\tan^2(c+dx)\,dx}{a} - \frac{\int \sec^8(c+dx)\tan^3(c+dx)\,dx}{a}$$
$$= \frac{\sec^9(c+dx)\tan(c+dx)}{10ad} - \frac{\int \sec^9(c+dx)\,dx}{10a} - \frac{\text{Subst}\left(\int x^7\left(-1+x^2\right)dx, x, \text{se}\right.}{ad}$$
$$= -\frac{\sec^7(c+dx)\tan(c+dx)}{80ad} + \frac{\sec^9(c+dx)\tan(c+dx)}{10ad} - \frac{7\int \sec^7(c+dx)\,dx}{80a} -$$
$$= \frac{\sec^8(c+dx)}{8ad} - \frac{\sec^{10}(c+dx)}{10ad} - \frac{7\sec^5(c+dx)\tan(c+dx)}{480ad} - \frac{\sec^7(c+dx)\tan(}{80ad}$$
$$= \frac{\sec^8(c+dx)}{8ad} - \frac{\sec^{10}(c+dx)}{10ad} - \frac{7\sec^3(c+dx)\tan(c+dx)}{384ad} - \frac{7\sec^5(c+dx)\tan}{480ad}$$
$$= \frac{\sec^8(c+dx)}{8ad} - \frac{\sec^{10}(c+dx)}{10ad} - \frac{7\sec(c+dx)\tan(c+dx)}{256ad} - \frac{7\sec^3(c+dx)\tan(}{384ad}$$
$$= -\frac{7\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{\sec^8(c+dx)}{8ad} - \frac{\sec^{10}(c+dx)}{10ad} - \frac{7\sec(c+dx)\tan(c+}{256ad}$$

Mathematica [A] time = 2.62, size = 122, normalized size = 0.71

$$-\frac{210\tanh^{-1}(\sin(c+dx)) - \frac{2\left(105\sin^8(c+dx)+105\sin^7(c+dx)-385\sin^6(c+dx)-385\sin^5(c+dx)+511\sin^4(c+dx)+511\sin^3(c+dx)-279\sin^2(c+\right.}{(\sin(c+dx)-1)^4(\sin(c+dx)+1)^5}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^7*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7680*(210*ArcTanh[Sin[c + d*x]] - (2*(96 + 201*Sin[c + d*x] - 279*Sin[c
+ d*x]^2 + 511*Sin[c + d*x]^3 + 511*Sin[c + d*x]^4 - 385*Sin[c + d*x]^5 - 3
85*Sin[c + d*x]^6 + 105*Sin[c + d*x]^7 + 105*Sin[c + d*x]^8))/((-1 + Sin[c
+ d*x])^4*(1 + Sin[c + d*x])^5))/(a*d)
```

fricas [A] time = 0.53, size = 187, normalized size = 1.09

$$\frac{210\cos(dx+c)^8 - 70\cos(dx+c)^6 - 28\cos(dx+c)^4 - 16\cos(dx+c)^2 - 105\left(\cos(dx+c)^8\sin(dx+c) + \cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/7680*(210*cos(d*x + c)^8 - 70*cos(d*x + c)^6 - 28*cos(d*x + c)^4 - 16*cos
(d*x + c)^2 - 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d*
```

```
x + c) + 1) + 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(d
*x + c) + 1) - 2*(105*cos(d*x + c)^6 + 70*cos(d*x + c)^4 + 56*cos(d*x + c)^
2 - 432)*sin(d*x + c) + 96)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x
+ c)^8)
```

giac [A] time = 0.29, size = 156, normalized size = 0.91

$$-\frac{\frac{420 \log(|\sin(dx+c)+1|)}{a} - \frac{420 \log(|\sin(dx+c)-1|)}{a} + \frac{5\left(175 \sin(dx+c)^4 - 748 \sin(dx+c)^3 + 1182 \sin(dx+c)^2 - 788 \sin(dx+c) + 155\right)}{a(\sin(dx+c)-1)^4} - \frac{959 \sin(dx+}{}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30720*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))
/a + 5*(175*sin(d*x + c)^4 - 748*sin(d*x + c)^3 + 1182*sin(d*x + c)^2 - 788
*sin(d*x + c) + 155)/(a*(sin(d*x + c) - 1)^4) - (959*sin(d*x + c)^5 + 5395*
sin(d*x + c)^4 + 12290*sin(d*x + c)^3 + 14170*sin(d*x + c)^2 + 8135*sin(d*x
 + c) + 1627)/(a*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.35, size = 198, normalized size = 1.15

$$\frac{1}{256ad\left(\sin\left(dx+c\right)-1\right)^4} - \frac{1}{192ad\left(\sin\left(dx+c\right)-1\right)^3} + \frac{1}{512ad\left(\sin\left(dx+c\right)-1\right)^2} + \frac{1}{128ad\left(\sin\left(dx+c\right)-1\right)} + \frac{7\ln(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4-1/192/a/d/(sin(d*x+c)-1)^3+1/512/a/d/(sin(d*x+c)
-1)^2+1/128/a/d/(sin(d*x+c)-1)+7/512/a/d*ln(sin(d*x+c)-1)-1/160/a/d/(1+sin(
d*x+c))^5-1/256/a/d/(1+sin(d*x+c))^4+1/384/a/d/(1+sin(d*x+c))^3+5/512/a/d/(
1+sin(d*x+c))^2+5/256/a/d/(1+sin(d*x+c))-7/512*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.49, size = 214, normalized size = 1.24

$$\frac{\frac{2\left(105 \sin(dx+c)^8 + 105 \sin(dx+c)^7 - 385 \sin(dx+c)^6 - 385 \sin(dx+c)^5 + 511 \sin(dx+c)^4 + 511 \sin(dx+c)^3 - 279 \sin(dx+c)^2 + 201 \sin(dx+c) + 96\right)}{a \sin(dx+c)^9 + a \sin(dx+c)^8 - 4\,a \sin(dx+c)^7 - 4\,a \sin(dx+c)^6 + 6\,a \sin(dx+c)^5 + 6\,a \sin(dx+c)^4 - 4\,a \sin(dx+c)^3 - 4\,a \sin(dx+c)^2 + a \sin(dx+c) + a} - \frac{105}{}}{7680\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/7680*(2*(105*sin(d*x + c)^8 + 105*sin(d*x + c)^7 - 385*sin(d*x + c)^6 - 3
85*sin(d*x + c)^5 + 511*sin(d*x + c)^4 + 511*sin(d*x + c)^3 - 279*sin(d*x +
```

```
 c)^2 + 201*sin(d*x + c) + 96)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*s
in(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^
4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 105*log
(sin(d*x + c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 16.88, size = 496, normalized size = 2.88

$$\frac{\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{128}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}}{64}+\frac{221\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{96}+\frac{95\tan\left(\frac{c}{2}+\right.}{192}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}-7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}+20\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+56\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] ((7*tan(c/2 + (d*x)/2))/128 + (7*tan(c/2 + (d*x)/2)^2)/64 + (221*tan(c/2 +
(d*x)/2)^3)/96 + (95*tan(c/2 + (d*x)/2)^4)/192 + (2261*tan(c/2 + (d*x)/2)^5
)/480 + (889*tan(c/2 + (d*x)/2)^6)/960 + (7343*tan(c/2 + (d*x)/2)^7)/480 +
(1603*tan(c/2 + (d*x)/2)^8)/960 + (2471*tan(c/2 + (d*x)/2)^9)/192 + (1603*t
an(c/2 + (d*x)/2)^10)/960 + (7343*tan(c/2 + (d*x)/2)^11)/480 + (889*tan(c/2
 + (d*x)/2)^12)/960 + (2261*tan(c/2 + (d*x)/2)^13)/480 + (95*tan(c/2 + (d*x
)/2)^14)/192 + (221*tan(c/2 + (d*x)/2)^15)/96 + (7*tan(c/2 + (d*x)/2)^16)/6
4 + (7*tan(c/2 + (d*x)/2)^17)/128)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 7*a*tan
(c/2 + (d*x)/2)^2 - 16*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 +
 56*a*tan(c/2 + (d*x)/2)^5 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a*tan(c/2 + (d
*x)/2)^7 + 14*a*tan(c/2 + (d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2)^9 + 14*a*ta
n(c/2 + (d*x)/2)^10 - 112*a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)
^12 + 56*a*tan(c/2 + (d*x)/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/
2 + (d*x)/2)^15 - 7*a*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d*x)/2)^17 + a
*tan(c/2 + (d*x)/2)^18)) - (7*atanh(tan(c/2 + (d*x)/2)))/(128*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.906 $\int \frac{\sec^8(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{\sec^{10}(c+dx)}{10ad}+\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan(c+dx)\sec^9(c+dx)}{10ad}+\frac{\tan(c+dx)\sec^7(c+dx)}{80ad}+\frac{7\tan(c+dx)\sec^5(c}{480ad}$$

```
[Out] 7/256*arctanh(sin(d*x+c))/a/d+1/10*sec(d*x+c)^10/a/d+7/256*sec(d*x+c)*tan(d
*x+c)/a/d+7/384*sec(d*x+c)^3*tan(d*x+c)/a/d+7/480*sec(d*x+c)^5*tan(d*x+c)/a
/d+1/80*sec(d*x+c)^7*tan(d*x+c)/a/d-1/10*sec(d*x+c)^9*tan(d*x+c)/a/d
```

Rubi [A] time = 0.17, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2835, 2606, 30, 2611, 3768, 3770}

$$\frac{\sec^{10}(c+dx)}{10ad}+\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan(c+dx)\sec^9(c+dx)}{10ad}+\frac{\tan(c+dx)\sec^7(c+dx)}{80ad}+\frac{7\tan(c+dx)\sec^5(c}{480ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^8*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(256*a*d) + Sec[c + d*x]^10/(10*a*d) + (7*Sec[c +
 d*x]*Tan[c + d*x])/(256*a*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/(384*a*d) +
 (7*Sec[c + d*x]^5*Tan[c + d*x])/(480*a*d) + (Sec[c + d*x]^7*Tan[c + d*x])/
(80*a*d) - (Sec[c + d*x]^9*Tan[c + d*x])/(10*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^8(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^{10}(c+dx)\tan(c+dx)\,dx}{a} - \frac{\int \sec^9(c+dx)\tan^2(c+dx)\,dx}{a}$$
$$= -\frac{\sec^9(c+dx)\tan(c+dx)}{10ad} + \frac{\int \sec^9(c+dx)\,dx}{10a} + \frac{\text{Subst}\left(\int x^9\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= \frac{\sec^{10}(c+dx)}{10ad} + \frac{\sec^7(c+dx)\tan(c+dx)}{80ad} - \frac{\sec^9(c+dx)\tan(c+dx)}{10ad} + \frac{7\int \sec^7}{}$$
$$= \frac{\sec^{10}(c+dx)}{10ad} + \frac{7\sec^5(c+dx)\tan(c+dx)}{480ad} + \frac{\sec^7(c+dx)\tan(c+dx)}{80ad} - \frac{\sec^9(c}{}$$
$$= \frac{\sec^{10}(c+dx)}{10ad} + \frac{7\sec^3(c+dx)\tan(c+dx)}{384ad} + \frac{7\sec^5(c+dx)\tan(c+dx)}{480ad} + \frac{\sec^7}{}$$
$$= \frac{\sec^{10}(c+dx)}{10ad} + \frac{7\sec(c+dx)\tan(c+dx)}{256ad} + \frac{7\sec^3(c+dx)\tan(c+dx)}{384ad} + \frac{7\sec}{}$$
$$= \frac{7\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{\sec^{10}(c+dx)}{10ad} + \frac{7\sec(c+dx)\tan(c+dx)}{256ad} + \frac{7\sec^3(c+}{}$$

Mathematica [A] time = 5.53, size = 116, normalized size = 0.75

$$\frac{-\frac{210}{\sin(c+dx)-1}+\frac{135}{(\sin(c+dx)-1)^2}+\frac{75}{(\sin(c+dx)+1)^2}-\frac{80}{(\sin(c+dx)-1)^3}+\frac{100}{(\sin(c+dx)+1)^3}+\frac{30}{(\sin(c+dx)-1)^4}+\frac{90}{(\sin(c+dx)+1)^4}+\frac{48}{(\sin(c+dx)-}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^8*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (210*ArcTanh[Sin[c + d*x]] + 30/(-1 + Sin[c + d*x])^4 - 80/(-1 + Sin[c + d*
x])^3 + 135/(-1 + Sin[c + d*x])^2 - 210/(-1 + Sin[c + d*x]) + 48/(1 + Sin[c
 + d*x])^5 + 90/(1 + Sin[c + d*x])^4 + 100/(1 + Sin[c + d*x])^3 + 75/(1 + S
in[c + d*x])^2)/(7680*a*d)
```

fricas [A] time = 0.51, size = 187, normalized size = 1.21

$$-\frac{210\cos(dx+c)^8-70\cos(dx+c)^6-28\cos(dx+c)^4-16\cos(dx+c)^2-105\left(\cos(dx+c)^8\sin(dx+c)+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/7680*(210*cos(d*x + c)^8 - 70*cos(d*x + c)^6 - 28*cos(d*x + c)^4 - 16*co
s(d*x + c)^2 - 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d
*x + c) + 1) + 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(
d*x + c) + 1) - 2*(105*cos(d*x + c)^6 + 70*cos(d*x + c)^4 + 56*cos(d*x + c)
^2 + 48)*sin(d*x + c) - 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x
 + c)^8)
```

giac [A] time = 0.31, size = 156, normalized size = 1.01

$$\frac{\frac{420\log(|\sin(dx+c)+1|)}{a}-\frac{420\log(|\sin(dx+c)-1|)}{a}+\frac{5\left(175\sin(dx+c)^4-868\sin(dx+c)^3+1662\sin(dx+c)^2-1484\sin(dx+c)+539\right)}{a(\sin(dx+c)-1)^4}-\frac{959\sin(dx+c}{}}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/
a + 5*(175*sin(d*x + c)^4 - 868*sin(d*x + c)^3 + 1662*sin(d*x + c)^2 - 1484
*sin(d*x + c) + 539)/(a*(sin(d*x + c) - 1)^4) - (959*sin(d*x + c)^5 + 4795*
sin(d*x + c)^4 + 9290*sin(d*x + c)^3 + 8290*sin(d*x + c)^2 + 2735*sin(d*x +
 c) - 293)/(a*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.32, size = 180, normalized size = 1.17

$$\frac{1}{256ad\left(\sin\left(dx+c\right)-1\right)^{4}}-\frac{1}{96ad\left(\sin\left(dx+c\right)-1\right)^{3}}+\frac{9}{512ad\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{7}{256ad\left(\sin\left(dx+c\right)-1\right)}-\frac{7\ln\left(\sin\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4-1/96/a/d/(sin(d*x+c)-1)^3+9/512/a/d/(sin(d*x+c)-
1)^2-7/256/a/d/(sin(d*x+c)-1)-7/512/a/d*ln(sin(d*x+c)-1)+1/160/a/d/(1+sin(d
*x+c))^5+3/256/a/d/(1+sin(d*x+c))^4+5/384/a/d/(1+sin(d*x+c))^3+5/512/a/d/(1
+sin(d*x+c))^2+7/512*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.34, size = 214, normalized size = 1.39

$$-\frac{\frac{2\left(105\sin(dx+c)^{8}+105\sin(dx+c)^{7}-385\sin(dx+c)^{6}-385\sin(dx+c)^{5}+511\sin(dx+c)^{4}+511\sin(dx+c)^{3}-279\sin(dx+c)^{2}-279\sin(dx+c)-384\right)}{a\sin(dx+c)^{9}+a\sin(dx+c)^{8}-4a\sin(dx+c)^{7}-4a\sin(dx+c)^{6}+6a\sin(dx+c)^{5}+6a\sin(dx+c)^{4}-4a\sin(dx+c)^{3}-4a\sin(dx+c)^{2}+a\sin(dx+c)+a}-\frac{10}{}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(105*sin(d*x + c)^8 + 105*sin(d*x + c)^7 - 385*sin(d*x + c)^6 -
385*sin(d*x + c)^5 + 511*sin(d*x + c)^4 + 511*sin(d*x + c)^3 - 279*sin(d*x
+ c)^2 - 279*sin(d*x + c) - 384)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a
*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c
)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 105*l
og(sin(d*x + c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 16.75, size = 496, normalized size = 3.22

$$\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{128\,a\,d}+\frac{-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{128}+\frac{121\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}}{64}+\frac{16}{}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}-7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(128*a*d) + ((121*tan(c/2 + (d*x)/2)^2)/64 -
(7*tan(c/2 + (d*x)/2))/128 + (163*tan(c/2 + (d*x)/2)^3)/96 + (289*tan(c/2 +
 (d*x)/2)^4)/192 - (2261*tan(c/2 + (d*x)/2)^5)/480 + (12551*tan(c/2 + (d*x)
/2)^6)/960 + (6097*tan(c/2 + (d*x)/2)^7)/480 + (11837*tan(c/2 + (d*x)/2)^8)
```

```
/960 - (2471*tan(c/2 + (d*x)/2)^9)/192 + (11837*tan(c/2 + (d*x)/2)^10)/960
+ (6097*tan(c/2 + (d*x)/2)^11)/480 + (12551*tan(c/2 + (d*x)/2)^12)/960 - (2
261*tan(c/2 + (d*x)/2)^13)/480 + (289*tan(c/2 + (d*x)/2)^14)/192 + (163*tan
(c/2 + (d*x)/2)^15)/96 + (121*tan(c/2 + (d*x)/2)^16)/64 - (7*tan(c/2 + (d*x
)/2)^17)/128)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 7*a*tan(c/2 + (d*x)/2)^2 - 1
6*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)
/2)^5 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a*tan(c/2 + (d*x)/2)^7 + 14*a*tan(c
/2 + (d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c/2 + (d*x)/2)^10 -
 112*a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)^12 + 56*a*tan(c/2 +
(d*x)/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 + (d*x)/2)^15 - 7*a
*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d*x)/2)^17 + a*tan(c/2 + (d*x)/2)^1
8))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.907 $\int \frac{\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=210

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{5a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{64d(a-a\sin(c+dx))^3}-\frac{}{128d(a}$$

```
[Out] 63/256*arctanh(sin(d*x+c))/a/d+1/256*a^3/d/(a-a*sin(d*x+c))^4+1/64*a^2/d/(a
-a*sin(d*x+c))^3+21/512*a/d/(a-a*sin(d*x+c))^2+7/64/d/(a-a*sin(d*x+c))-1/16
0*a^4/d/(a+a*sin(d*x+c))^5-5/256*a^3/d/(a+a*sin(d*x+c))^4-5/128*a^2/d/(a+a*
sin(d*x+c))^3-35/512*a/d/(a+a*sin(d*x+c))^2-35/256/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{5a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{64d(a-a\sin(c+dx))^3}-\frac{}{128d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] (63*ArcTanh[Sin[c + d*x]])/(256*a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) +
 a^2/(64*d*(a - a*Sin[c + d*x])^3) + (21*a)/(512*d*(a - a*Sin[c + d*x])^2)
+ 7/(64*d*(a - a*Sin[c + d*x])) - a^4/(160*d*(a + a*Sin[c + d*x])^5) - (5*a
^3)/(256*d*(a + a*Sin[c + d*x])^4) - (5*a^2)/(128*d*(a + a*Sin[c + d*x])^3)
 - (35*a)/(512*d*(a + a*Sin[c + d*x])^2) - 35/(256*d*(a + a*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\,\text{Subst}\left(\int \frac{1}{(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^9\,\text{Subst}\left(\int \left(\frac{1}{64a^6(a-x)^5} + \frac{3}{64a^7(a-x)^4} + \frac{21}{256a^8(a-x)^3} + \frac{7}{64a^9(a-x)^2} + \frac{1}{32a^5(a+x)^6} + \frac{5}{64a^6(a+x)^5} + \right.\right.}{d}$$

$$= \frac{a^3}{256d(a-a\sin(c+dx))^4} + \frac{a^2}{64d(a-a\sin(c+dx))^3} + \frac{21a}{512d(a-a\sin(c+dx))^2} + \frac{}{64d}$$

$$= \frac{63\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{a^3}{256d(a-a\sin(c+dx))^4} + \frac{a^2}{64d(a-a\sin(c+dx))^3} + \frac{}{512d}$$

Mathematica [A] time = 1.40, size = 165, normalized size = 0.79

$$\frac{\sec^8(c+dx)\left(-315\sin^8(c+dx) - 315\sin^7(c+dx) + 1155\sin^6(c+dx) + 1155\sin^5(c+dx) - 1533\sin^4(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^8*(-128 + 315*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^8*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^10 + 837*Sin[c + d*x]
+ 837*Sin[c + d*x]^2 - 1533*Sin[c + d*x]^3 - 1533*Sin[c + d*x]^4 + 1155*Sin
[c + d*x]^5 + 1155*Sin[c + d*x]^6 - 315*Sin[c + d*x]^7 - 315*Sin[c + d*x]^8
))/(1280*a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.52, size = 187, normalized size = 0.89

$$-\frac{630\cos(dx+c)^8 - 210\cos(dx+c)^6 - 84\cos(dx+c)^4 - 48\cos(dx+c)^2 - 315\left(\cos(dx+c)^8\sin(dx+c) + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2560*(630*cos(d*x + c)^8 - 210*cos(d*x + c)^6 - 84*cos(d*x + c)^4 - 48*c
os(d*x + c)^2 - 315*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(
d*x + c) + 1) + 315*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin
(d*x + c) + 1) - 6*(105*cos(d*x + c)^6 + 70*cos(d*x + c)^4 + 56*cos(d*x + c
)^2 + 48)*sin(d*x + c) - 32)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x
 + c)^8)
```

giac [A] time = 0.24, size = 156, normalized size = 0.74

$$\frac{\frac{1260 \log(|\sin(dx+c)+1|)}{a} - \frac{1260 \log(|\sin(dx+c)-1|)}{a} + \frac{5\left(525 \sin(dx+c)^4 - 2324 \sin(dx+c)^3 + 3906 \sin(dx+c)^2 - 2972 \sin(dx+c) + 873\right)}{a(\sin(dx+c)-1)^4} - \frac{2877 \sin(dx}{}}{10240\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/10240*(1260*log(abs(sin(d*x + c) + 1))/a - 1260*log(abs(sin(d*x + c) - 1)
)/a + 5*(525*sin(d*x + c)^4 - 2324*sin(d*x + c)^3 + 3906*sin(d*x + c)^2 - 2
972*sin(d*x + c) + 873)/(a*(sin(d*x + c) - 1)^4) - (2877*sin(d*x + c)^5 + 1
5785*sin(d*x + c)^4 + 35070*sin(d*x + c)^3 + 39670*sin(d*x + c)^2 + 23085*s
in(d*x + c) + 5641)/(a*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.41, size = 198, normalized size = 0.94

$$\frac{1}{256ad\left(\sin\left(dx+c\right)-1\right)^{4}}-\frac{1}{64ad\left(\sin\left(dx+c\right)-1\right)^{3}}+\frac{21}{512ad\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{7}{64ad\left(\sin\left(dx+c\right)-1\right)}-\frac{63\ln\left(\sin\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4-1/64/a/d/(sin(d*x+c)-1)^3+21/512/a/d/(sin(d*x+c)
-1)^2-7/64/a/d/(sin(d*x+c)-1)-63/512/a/d*ln(sin(d*x+c)-1)-1/160/a/d/(1+sin(
d*x+c))^5-5/256/a/d/(1+sin(d*x+c))^4-5/128/a/d/(1+sin(d*x+c))^3-35/512/a/d/
(1+sin(d*x+c))^2-35/256/a/d/(1+sin(d*x+c))+63/512*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.40, size = 214, normalized size = 1.02

$$-\frac{\frac{2\left(315 \sin(dx+c)^8+315 \sin(dx+c)^7-1155 \sin(dx+c)^6-1155 \sin(dx+c)^5+1533 \sin(dx+c)^4+1533 \sin(dx+c)^3-837 \sin(dx+c)^2-837 \sin(dx+c)+128\right)}{a \sin(dx+c)^9+a \sin(dx+c)^8-4\, a \sin(dx+c)^7-4\, a \sin(dx+c)^6+6\, a \sin(dx+c)^5+6\, a \sin(dx+c)^4-4\, a \sin(dx+c)^3-4\, a \sin(dx+c)^2+a \sin(dx+c)+a}}{2560\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2560*(2*(315*sin(d*x + c)^8 + 315*sin(d*x + c)^7 - 1155*sin(d*x + c)^6 -
 1155*sin(d*x + c)^5 + 1533*sin(d*x + c)^4 + 1533*sin(d*x + c)^3 - 837*sin(
d*x + c)^2 - 837*sin(d*x + c) + 128)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 -
 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x
 + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 3
15*log(sin(d*x + c) + 1)/a + 315*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 9.41, size = 199, normalized size = 0.95

$$\frac{63\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)}{256\,a\,d}-\frac{\frac{63\sin(c+d\,x)^8}{256}+\frac{63\sin(c+d\,x)^7}{256}-\frac{231\sin(c+d\,x)^6}{256}-\frac{231\sin(c+d\,x)^5}{256}+\frac{1533}{}}{d\,\left(a\sin\left(c+d\,x\right)^9+a\sin\left(c+d\,x\right)^8-4\,a\sin\left(c+d\,x\right)^7-4\,a\sin\left(c+d\,x\right)^6+6\,a\sin\left(c+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (63*atanh(sin(c + d*x)))/(256*a*d) - ((1533*sin(c + d*x)^3)/1280 - (837*sin
(c + d*x)^2)/1280 - (837*sin(c + d*x))/1280 + (1533*sin(c + d*x)^4)/1280 -
(231*sin(c + d*x)^5)/256 - (231*sin(c + d*x)^6)/256 + (63*sin(c + d*x)^7)/2
56 + (63*sin(c + d*x)^8)/256 + 1/10)/(d*(a + a*sin(c + d*x) - 4*a*sin(c + d
*x)^2 - 4*a*sin(c + d*x)^3 + 6*a*sin(c + d*x)^4 + 6*a*sin(c + d*x)^5 - 4*a*
sin(c + d*x)^6 - 4*a*sin(c + d*x)^7 + a*sin(c + d*x)^8 + a*sin(c + d*x)^9))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.908 $\int \frac{\csc(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=247

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{7a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{48d(a-a\sin(c+dx))^3}+\frac{}{384d(a\,\mathrm{s}}$$

```
[Out] -193/512*ln(1-sin(d*x+c))/a/d+ln(sin(d*x+c))/a/d-319/512*ln(1+sin(d*x+c))/a
/d+1/256*a^3/d/(a-a*sin(d*x+c))^4+1/48*a^2/d/(a-a*sin(d*x+c))^3+37/512*a/d/
(a-a*sin(d*x+c))^2+65/256/d/(a-a*sin(d*x+c))+1/160*a^4/d/(a+a*sin(d*x+c))^5
+7/256*a^3/d/(a+a*sin(d*x+c))^4+29/384*a^2/d/(a+a*sin(d*x+c))^3+93/512*a/d/
(a+a*sin(d*x+c))^2+1/2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.24, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{7a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{48d(a-a\sin(c+dx))^3}+\frac{}{384d(a\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-193*Log[1 - Sin[c + d*x]])/(512*a*d) + Log[Sin[c + d*x]]/(a*d) - (319*Log
[1 + Sin[c + d*x]])/(512*a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) + a^2/(4
8*d*(a - a*Sin[c + d*x])^3) + (37*a)/(512*d*(a - a*Sin[c + d*x])^2) + 65/(2
56*d*(a - a*Sin[c + d*x])) + a^4/(160*d*(a + a*Sin[c + d*x])^5) + (7*a^3)/(
256*d*(a + a*Sin[c + d*x])^4) + (29*a^2)/(384*d*(a + a*Sin[c + d*x])^3) + (
93*a)/(512*d*(a + a*Sin[c + d*x])^2) + 1/(2*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\,\text{Subst}\left(\int \frac{a}{(a-x)^5x(a+x)^6}\,dx,x,a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{10}\,\text{Subst}\left(\int \frac{1}{(a-x)^5x(a+x)^6}\,dx,x,a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{10}\,\text{Subst}\left(\int \left(\frac{1}{64a^7(a-x)^5}+\frac{1}{16a^8(a-x)^4}+\frac{37}{256a^9(a-x)^3}+\frac{65}{256a^{10}(a-x)^2}+\frac{193}{512a^{11}(a-x)}+\frac{}{a^1}\right.\right.}{}$$

$$= -\frac{193\log(1-\sin(c+dx))}{512ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{319\log(1+\sin(c+dx))}{512ad}+\frac{}{25}$$

Mathematica [A] time = 6.21, size = 228, normalized size = 0.92

$$\frac{a^{10}\left(-\frac{193\log(1-\sin(c+dx))}{512a^{11}}+\frac{\log(\sin(c+dx))}{a^{11}}-\frac{319\log(\sin(c+dx)+1)}{512a^{11}}+\frac{65}{256a^{10}(a-a\sin(c+dx))}+\frac{1}{2a^{10}(a\sin(c+dx)+a)}+\frac{37}{512a^9(a-a\sin(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^10*((-193*Log[1 - Sin[c + d*x]])/(512*a^11) + Log[Sin[c + d*x]]/a^11 - (
319*Log[1 + Sin[c + d*x]])/(512*a^11) + 1/(256*a^7*(a - a*Sin[c + d*x])^4)
+ 1/(48*a^8*(a - a*Sin[c + d*x])^3) + 37/(512*a^9*(a - a*Sin[c + d*x])^2) +
 65/(256*a^10*(a - a*Sin[c + d*x])) + 1/(160*a^6*(a + a*Sin[c + d*x])^5) +
7/(256*a^7*(a + a*Sin[c + d*x])^4) + 29/(384*a^8*(a + a*Sin[c + d*x])^3) +
93/(512*a^9*(a + a*Sin[c + d*x])^2) + 1/(2*a^10*(a + a*Sin[c + d*x]))))/d
```

fricas [A] time = 0.57, size = 222, normalized size = 0.90

$$\frac{1890\cos(dx+c)^8+3210\cos(dx+c)^6+1668\cos(dx+c)^4+1136\cos(dx+c)^2+7680\left(\cos(dx+c)^8\sin(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/7680*(1890*cos(d*x + c)^8 + 3210*cos(d*x + c)^6 + 1668*cos(d*x + c)^4 + 1
136*cos(d*x + c)^2 + 7680*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*lo
g(1/2*sin(d*x + c)) - 4785*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*l
og(sin(d*x + c) + 1) - 2895*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*
log(-sin(d*x + c) + 1) + 2*(975*cos(d*x + c)^6 + 330*cos(d*x + c)^4 + 136*c
os(d*x + c)^2 + 48)*sin(d*x + c) + 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) +
a*d*cos(d*x + c)^8)
```

giac [A] time = 0.24, size = 169, normalized size = 0.68

$$-\frac{\frac{19140 \log(|\sin(dx+c)+1|)}{a} + \frac{11580 \log(|\sin(dx+c)-1|)}{a} - \frac{30720 \log(|\sin(dx+c)|)}{a} - \frac{5\left(4825 \sin(dx+c)^4 - 20860 \sin(dx+c)^3 + 34074 \sin(dx+c)^2 - 24\right.}{a(\sin(dx+c)-1)^4}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30720*(19140*log(abs(sin(d*x + c) + 1))/a + 11580*log(abs(sin(d*x + c) -
 1))/a - 30720*log(abs(sin(d*x + c)))/a - 5*(4825*sin(d*x + c)^4 - 20860*si
n(d*x + c)^3 + 34074*sin(d*x + c)^2 - 24996*sin(d*x + c) + 6981)/(a*(sin(d*
x + c) - 1)^4) - (43703*sin(d*x + c)^5 + 233875*sin(d*x + c)^4 + 504050*sin
(d*x + c)^3 + 548250*sin(d*x + c)^2 + 302175*sin(d*x + c) + 67995)/(a*(sin(
d*x + c) + 1)^5))/d
```

maple [A] time = 0.46, size = 212, normalized size = 0.86

$$\frac{1}{256ad\left(\sin\left(dx+c\right)-1\right)^{4}} - \frac{1}{48ad\left(\sin\left(dx+c\right)-1\right)^{3}} + \frac{37}{512ad\left(\sin\left(dx+c\right)-1\right)^{2}} - \frac{65}{256ad\left(\sin\left(dx+c\right)-1\right)} - \frac{193\ln\left(s\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4-1/48/a/d/(sin(d*x+c)-1)^3+37/512/a/d/(sin(d*x+c)
-1)^2-65/256/a/d/(sin(d*x+c)-1)-193/512/a/d*ln(sin(d*x+c)-1)+ln(sin(d*x+c))
/a/d+1/160/a/d/(1+sin(d*x+c))^5+7/256/a/d/(1+sin(d*x+c))^4+29/384/a/d/(1+si
n(d*x+c))^3+93/512/a/d/(1+sin(d*x+c))^2+1/2/a/d/(1+sin(d*x+c))-319/512*ln(1
+sin(d*x+c))/a/d
```

maxima [A] time = 0.49, size = 226, normalized size = 0.91

$$\frac{\frac{2\left(945 \sin(dx+c)^8 - 975 \sin(dx+c)^7 - 5385 \sin(dx+c)^6 + 3255 \sin(dx+c)^5 + 11319 \sin(dx+c)^4 - 3721 \sin(dx+c)^3 - 10831 \sin(dx+c)^2 + 1489 \sin(dx+c) + 438\right.}{a \sin(dx+c)^9 + a \sin(dx+c)^8 - 4a \sin(dx+c)^7 - 4a \sin(dx+c)^6 + 6a \sin(dx+c)^5 + 6a \sin(dx+c)^4 - 4a \sin(dx+c)^3 - 4a \sin(dx+c)^2 + a \sin(dx+c) + a}}{7680\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/7680*(2*(945*sin(d*x + c)^8 - 975*sin(d*x + c)^7 - 5385*sin(d*x + c)^6 +
3255*sin(d*x + c)^5 + 11319*sin(d*x + c)^4 - 3721*sin(d*x + c)^3 - 10831*si
n(d*x + c)^2 + 1489*sin(d*x + c) + 4384)/(a*sin(d*x + c)^9 + a*sin(d*x + c)
^8 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin
(d*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a)
 - 4785*log(sin(d*x + c) + 1)/a - 2895*log(sin(d*x + c) - 1)/a + 7680*log(s
in(d*x + c))/a)/d
```

mupad [B] time = 0.23, size = 231, normalized size = 0.94

$$\frac{\frac{63\sin(c+d\,x)^8}{256}-\frac{65\sin(c+d\,x)^7}{256}-\frac{359\sin(c+d\,x)^6}{256}+\frac{217\sin(c+d\,x)^5}{256}+\frac{3773\sin(c+d\,x)^4}{1280}-\frac{3721\sin(c+d\,x)^3}{3840}-}{d\,\left(a\sin\left(c+d\,x\right)^9+a\sin\left(c+d\,x\right)^8-4\,a\sin\left(c+d\,x\right)^7-4\,a\sin\left(c+d\,x\right)^6+6\,a\sin\left(c+d\,x\right)^5+6\,a\sin\left(c+d\,x\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^9*sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] ((1489*sin(c + d*x))/3840 - (10831*sin(c + d*x)^2)/3840 - (3721*sin(c + d*x
)^3)/3840 + (3773*sin(c + d*x)^4)/1280 + (217*sin(c + d*x)^5)/256 - (359*si
n(c + d*x)^6)/256 - (65*sin(c + d*x)^7)/256 + (63*sin(c + d*x)^8)/256 + 137
/120)/(d*(a + a*sin(c + d*x) - 4*a*sin(c + d*x)^2 - 4*a*sin(c + d*x)^3 + 6*
a*sin(c + d*x)^4 + 6*a*sin(c + d*x)^5 - 4*a*sin(c + d*x)^6 - 4*a*sin(c + d*
x)^7 + a*sin(c + d*x)^8 + a*sin(c + d*x)^9)) - (319*log(sin(c + d*x) + 1))/
(512*a*d) - (193*log(sin(c + d*x) - 1))/(512*a*d) + log(sin(c + d*x))/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.909 $\int \frac{\csc^2(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=262

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{9a^3}{256d(a\sin(c+dx)+a)^4}+\frac{5a^2}{192d(a-a\sin(c+dx))^3}-\frac{}{384d(}$$

```
[Out] -csc(d*x+c)/a/d-437/512*ln(1-sin(d*x+c))/a/d-ln(sin(d*x+c))/a/d+949/512*ln(
1+sin(d*x+c))/a/d+1/256*a^3/d/(a-a*sin(d*x+c))^4+5/192*a^2/d/(a-a*sin(d*x+c
))^3+57/512*a/d/(a-a*sin(d*x+c))^2+61/128/d/(a-a*sin(d*x+c))-1/160*a^4/d/(a
+a*sin(d*x+c))^5-9/256*a^3/d/(a+a*sin(d*x+c))^4-47/384*a^2/d/(a+a*sin(d*x+c
))^3-187/512*a/d/(a+a*sin(d*x+c))^2-315/256/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.29, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{9a^3}{256d(a\sin(c+dx)+a)^4}+\frac{5a^2}{192d(a-a\sin(c+dx))^3}-\frac{}{384d(}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (437*Log[1 - Sin[c + d*x]])/(512*a*d) - Log[Sin[c +
 d*x]]/(a*d) + (949*Log[1 + Sin[c + d*x]])/(512*a*d) + a^3/(256*d*(a - a*Si
n[c + d*x])^4) + (5*a^2)/(192*d*(a - a*Sin[c + d*x])^3) + (57*a)/(512*d*(a
- a*Sin[c + d*x])^2) + 61/(128*d*(a - a*Sin[c + d*x])) - a^4/(160*d*(a + a*
Sin[c + d*x])^5) - (9*a^3)/(256*d*(a + a*Sin[c + d*x])^4) - (47*a^2)/(384*d
*(a + a*Sin[c + d*x])^3) - (187*a)/(512*d*(a + a*Sin[c + d*x])^2) - 315/(25
6*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\,\text{Subst}\left(\int \frac{a^2}{(a-x)^5x^2(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{11}\,\text{Subst}\left(\int \frac{1}{(a-x)^5x^2(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{11}\,\text{Subst}\left(\int \left(\frac{1}{64a^8(a-x)^5} + \frac{5}{64a^9(a-x)^4} + \frac{57}{256a^{10}(a-x)^3} + \frac{61}{128a^{11}(a-x)^2} + \frac{437}{512a^{12}(a-x)} + \right.\right.}{}$$

$$= -\frac{\csc(c+dx)}{ad} - \frac{437\log(1-\sin(c+dx))}{512ad} - \frac{\log(\sin(c+dx))}{ad} + \frac{949\log(1+\text{si}}{512a}$$

Mathematica [A] time = 6.21, size = 240, normalized size = 0.92

$$\frac{a^{11}\left(-\frac{\csc(c+dx)}{a^{12}} - \frac{437\log(1-\sin(c+dx))}{512a^{12}} - \frac{\log(\sin(c+dx))}{a^{12}} + \frac{949\log(\sin(c+dx)+1)}{512a^{12}} + \frac{61}{128a^{11}(a-a\sin(c+dx))} - \frac{315}{256a^{11}(a\sin(c+dx)+a)} + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^11*(-(Csc[c + d*x]/a^12) - (437*Log[1 - Sin[c + d*x]])/(512*a^12) - Log[
Sin[c + d*x]]/a^12 + (949*Log[1 + Sin[c + d*x]])/(512*a^12) + 1/(256*a^8*(a
 - a*Sin[c + d*x])^4) + 5/(192*a^9*(a - a*Sin[c + d*x])^3) + 57/(512*a^10*(
a - a*Sin[c + d*x])^2) + 61/(128*a^11*(a - a*Sin[c + d*x])) - 1/(160*a^7*(a
 + a*Sin[c + d*x])^5) - 9/(256*a^8*(a + a*Sin[c + d*x])^4) - 47/(384*a^9*(a
 + a*Sin[c + d*x])^3) - 187/(512*a^10*(a + a*Sin[c + d*x])^2) - 315/(256*a^
11*(a + a*Sin[c + d*x]))))/d
```

fricas [A] time = 0.56, size = 278, normalized size = 1.06

$$\frac{16950\cos(dx+c)^8 - 5010\cos(dx+c)^6 - 2132\cos(dx+c)^4 - 1264\cos(dx+c)^2 - 7680\left(\cos(dx+c)^{10} - \text{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/7680*(16950*cos(d*x + c)^8 - 5010*cos(d*x + c)^6 - 2132*cos(d*x + c)^4 -
1264*cos(d*x + c)^2 - 7680*(cos(d*x + c)^10 - cos(d*x + c)^8*sin(d*x + c) -
 cos(d*x + c)^8)*log(1/2*sin(d*x + c)) + 14235*(cos(d*x + c)^10 - cos(d*x +
 c)^8*sin(d*x + c) - cos(d*x + c)^8)*log(sin(d*x + c) + 1) - 6555*(cos(d*x
+ c)^10 - cos(d*x + c)^8*sin(d*x + c) - cos(d*x + c)^8)*log(-sin(d*x + c) +
 1) + 2*(10395*cos(d*x + c)^8 - 1545*cos(d*x + c)^6 - 426*cos(d*x + c)^4 -
152*cos(d*x + c)^2 - 48)*sin(d*x + c) - 864)/(a*d*cos(d*x + c)^10 - a*d*cos
(d*x + c)^8*sin(d*x + c) - a*d*cos(d*x + c)^8)
```

giac [**A**] time = 0.27, size = 190, normalized size = 0.73

$$\frac{\frac{56940\,\log(|\sin(dx+c)+1|)}{a}-\frac{26220\,\log(|\sin(dx+c)-1|)}{a}-\frac{30720\,\log(|\sin(dx+c)|)}{a}+\frac{30720\,(\sin(dx+c)-1)}{a\sin(dx+c)}+\frac{5\left(10925\,\sin(dx+c)^4-46628\,\sin(dx+c)\right.}{a(\sin}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(56940*log(abs(sin(d*x + c) + 1))/a - 26220*log(abs(sin(d*x + c) -
1))/a - 30720*log(abs(sin(d*x + c)))/a + 30720*(sin(d*x + c) - 1)/(a*sin(d*
x + c)) + 5*(10925*sin(d*x + c)^4 - 46628*sin(d*x + c)^3 + 75018*sin(d*x +
c)^2 - 54012*sin(d*x + c) + 14721)/(a*(sin(d*x + c) - 1)^4) - (130013*sin(d
*x + c)^5 + 687865*sin(d*x + c)^4 + 1462550*sin(d*x + c)^3 + 1564350*sin(d*
x + c)^2 + 843525*sin(d*x + c) + 184065)/(a*(sin(d*x + c) + 1)^5))/d
```

maple [**A**] time = 0.47, size = 229, normalized size = 0.87

$$\frac{1}{256ad\left(\sin\left(dx+c\right)-1\right)^{4}}-\frac{5}{192ad\left(\sin\left(dx+c\right)-1\right)^{3}}+\frac{57}{512ad\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{61}{128ad\left(\sin\left(dx+c\right)-1\right)}-\frac{437\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4-5/192/a/d/(sin(d*x+c)-1)^3+57/512/a/d/(sin(d*x+c
)-1)^2-61/128/a/d/(sin(d*x+c)-1)-437/512/a/d*ln(sin(d*x+c)-1)-1/d/a/sin(d*x
+c)-ln(sin(d*x+c))/a/d-1/160/a/d/(1+sin(d*x+c))^5-9/256/a/d/(1+sin(d*x+c))^
4-47/384/a/d/(1+sin(d*x+c))^3-187/512/a/d/(1+sin(d*x+c))^2-315/256/a/d/(1+s
in(d*x+c))+949/512*ln(1+sin(d*x+c))/a/d
```

maxima [**A**] time = 0.35, size = 245, normalized size = 0.94

$$-\frac{\frac{2\left(10395\,\sin(dx+c)^9+8475\,\sin(dx+c)^8-40035\,\sin(dx+c)^7-31395\,\sin(dx+c)^6+57309\,\sin(dx+c)^5+42269\,\sin(dx+c)^4-35941\,\sin(dx+c)^3-23621\,\sin(\right.}{a\sin(dx+c)^{10}+a\sin(dx+c)^9-4\,a\sin(dx+c)^8-4\,a\sin(dx+c)^7+6\,a\sin(dx+c)^6+6\,a\sin(dx+c)^5-4\,a\sin(dx+c)^4-4\,a\sin(dx+c)^3+a\sin(dx+}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(10395*sin(d*x + c)^9 + 8475*sin(d*x + c)^8 - 40035*sin(d*x + c)
^7 - 31395*sin(d*x + c)^6 + 57309*sin(d*x + c)^5 + 42269*sin(d*x + c)^4 - 3
5941*sin(d*x + c)^3 - 23621*sin(d*x + c)^2 + 8224*sin(d*x + c) + 3840)/(a*s
in(d*x + c)^10 + a*sin(d*x + c)^9 - 4*a*sin(d*x + c)^8 - 4*a*sin(d*x + c)^7
 + 6*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 - 4*a*sin(d*x + c)^4 - 4*a*sin(d
*x + c)^3 + a*sin(d*x + c)^2 + a*sin(d*x + c)) - 14235*log(sin(d*x + c) + 1
)/a + 6555*log(sin(d*x + c) - 1)/a + 7680*log(sin(d*x + c))/a)/d
```

mupad [B] time = 9.46, size = 252, normalized size = 0.96

$$\frac{949\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{512\,a\,d}-\frac{437\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{512\,a\,d}-\frac{\ln\left(\sin\left(c+d\,x\right)\right)}{a\,d}-\frac{\frac{693\,\sin\left(c+d\,x\right)^9}{256}+\frac{565}{}}{d\,\left(a\,\sin\left(c+d\,x\right)^{10}+a\,\sin\left(c+d\,x\right)^9-4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^9*sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] (949*log(sin(c + d*x) + 1))/(512*a*d) - (437*log(sin(c + d*x) - 1))/(512*a*
d) - log(sin(c + d*x))/(a*d) - ((257*sin(c + d*x))/120 - (23621*sin(c + d*x
)^2)/3840 - (35941*sin(c + d*x)^3)/3840 + (42269*sin(c + d*x)^4)/3840 + (19
103*sin(c + d*x)^5)/1280 - (2093*sin(c + d*x)^6)/256 - (2669*sin(c + d*x)^7
)/256 + (565*sin(c + d*x)^8)/256 + (693*sin(c + d*x)^9)/256 + 1)/(d*(a*sin(
c + d*x) + a*sin(c + d*x)^2 - 4*a*sin(c + d*x)^3 - 4*a*sin(c + d*x)^4 + 6*a
*sin(c + d*x)^5 + 6*a*sin(c + d*x)^6 - 4*a*sin(c + d*x)^7 - 4*a*sin(c + d*x
)^8 + a*sin(c + d*x)^9 + a*sin(c + d*x)^10))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.910 $\int \frac{\csc^3(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=279

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{11a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{32d(a-a\sin(c+dx))^3}+\frac{}{128d(a\,\mathrm{s}}$$

```
[Out] csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d-843/512*ln(1-sin(d*x+c))/a/d+6*ln(sin(d
*x+c))/a/d-2229/512*ln(1+sin(d*x+c))/a/d+1/256*a^3/d/(a-a*sin(d*x+c))^4+1/3
2*a^2/d/(a-a*sin(d*x+c))^3+81/512*a/d/(a-a*sin(d*x+c))^2+203/256/d/(a-a*sin
(d*x+c))+1/160*a^4/d/(a+a*sin(d*x+c))^5+11/256*a^3/d/(a+a*sin(d*x+c))^4+23/
128*a^2/d/(a+a*sin(d*x+c))^3+325/512*a/d/(a+a*sin(d*x+c))^2+5/2/d/(a+a*sin(
d*x+c))
```

Rubi [A] time = 0.30, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{11a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{32d(a-a\sin(c+dx))^3}+\frac{}{128d(a\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - (843*Log[1 - Sin[c + d*x]])/(
512*a*d) + (6*Log[Sin[c + d*x]])/(a*d) - (2229*Log[1 + Sin[c + d*x]])/(512*
a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) + a^2/(32*d*(a - a*Sin[c + d*x])^
3) + (81*a)/(512*d*(a - a*Sin[c + d*x])^2) + 203/(256*d*(a - a*Sin[c + d*x]
)) + a^4/(160*d*(a + a*Sin[c + d*x])^5) + (11*a^3)/(256*d*(a + a*Sin[c + d*
x])^4) + (23*a^2)/(128*d*(a + a*Sin[c + d*x])^3) + (325*a)/(512*d*(a + a*Si
n[c + d*x])^2) + 5/(2*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\,\text{Subst}\left(\int \frac{a^3}{(a-x)^5x^3(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{12}\,\text{Subst}\left(\int \frac{1}{(a-x)^5x^3(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{12}\,\text{Subst}\left(\int \left(\frac{1}{64a^9(a-x)^5} + \frac{3}{32a^{10}(a-x)^4} + \frac{81}{256a^{11}(a-x)^3} + \frac{203}{256a^{12}(a-x)^2} + \frac{843}{512a^{13}(a-x)} + \right.\right.}{}$$

$$= \frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad} - \frac{843\log(1-\sin(c+dx))}{512ad} + \frac{6\log(\sin(c+dx))}{ad} -$$

Mathematica [A] time = 6.23, size = 254, normalized size = 0.91

$$a^{12}\left(-\frac{\csc^2(c+dx)}{2a^{13}} + \frac{\csc(c+dx)}{a^{13}} - \frac{843\log(1-\sin(c+dx))}{512a^{13}} + \frac{6\log(\sin(c+dx))}{a^{13}} - \frac{2229\log(\sin(c+dx)+1)}{512a^{13}} + \frac{203}{256a^{12}(a-a\sin(c+dx))} + \frac{5}{2a^{12}(a\sin\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^12*(Csc[c + d*x]/a^13 - Csc[c + d*x]^2/(2*a^13) - (843*Log[1 - Sin[c + d
*x]])/(512*a^13) + (6*Log[Sin[c + d*x]])/a^13 - (2229*Log[1 + Sin[c + d*x]]
)/(512*a^13) + 1/(256*a^9*(a - a*Sin[c + d*x])^4) + 1/(32*a^10*(a - a*Sin[c
 + d*x])^3) + 81/(512*a^11*(a - a*Sin[c + d*x])^2) + 203/(256*a^12*(a - a*S
in[c + d*x])) + 1/(160*a^8*(a + a*Sin[c + d*x])^5) + 11/(256*a^9*(a + a*Sin
[c + d*x])^4) + 23/(128*a^10*(a + a*Sin[c + d*x])^3) + 325/(512*a^11*(a + a
*Sin[c + d*x])^2) + 5/(2*a^12*(a + a*Sin[c + d*x]))))/d
```

fricas [A] time = 0.54, size = 331, normalized size = 1.19

$$\frac{6930\cos(dx+c)^{10} - 1560\cos(dx+c)^8 - 2454\cos(dx+c)^6 - 884\cos(dx+c)^4 - 464\cos(dx+c)^2 + 15360}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2560*(6930*cos(d*x + c)^10 - 1560*cos(d*x + c)^8 - 2454*cos(d*x + c)^6 -
884*cos(d*x + c)^4 - 464*cos(d*x + c)^2 + 15360*(cos(d*x + c)^10 - cos(d*x
+ c)^8 + (cos(d*x + c)^10 - cos(d*x + c)^8)*sin(d*x + c))*log(1/2*sin(d*x +
 c)) - 11145*(cos(d*x + c)^10 - cos(d*x + c)^8 + (cos(d*x + c)^10 - cos(d*x
 + c)^8)*sin(d*x + c))*log(sin(d*x + c) + 1) - 4215*(cos(d*x + c)^10 - cos(
d*x + c)^8 + (cos(d*x + c)^10 - cos(d*x + c)^8)*sin(d*x + c))*log(-sin(d*x
+ c) + 1) + 2*(375*cos(d*x + c)^8 - 765*cos(d*x + c)^6 - 178*cos(d*x + c)^4
 - 56*cos(d*x + c)^2 - 16)*sin(d*x + c) - 288)/(a*d*cos(d*x + c)^10 - a*d*c
os(d*x + c)^8 + (a*d*cos(d*x + c)^10 - a*d*cos(d*x + c)^8)*sin(d*x + c))
```

giac [A] time = 0.29, size = 202, normalized size = 0.72

$$-\frac{\frac{44580\log(|\sin(dx+c)+1|)}{a}+\frac{16860\log(|\sin(dx+c)-1|)}{a}-\frac{61440\log(|\sin(dx+c)|)}{a}+\frac{5120\left(18\sin(dx+c)^2-2\sin(dx+c)+1\right)}{a\sin(dx+c)^2}-\frac{5\left(7025\sin(dx+c)^4\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/10240*(44580*log(abs(sin(d*x + c) + 1))/a + 16860*log(abs(sin(d*x + c) -
 1))/a - 61440*log(abs(sin(d*x + c)))/a + 5120*(18*sin(d*x + c)^2 - 2*sin(d
*x + c) + 1)/(a*sin(d*x + c)^2) - 5*(7025*sin(d*x + c)^4 - 29724*sin(d*x +
c)^3 + 47346*sin(d*x + c)^2 - 33684*sin(d*x + c) + 9045)/(a*(sin(d*x + c) -
 1)^4) - (101791*sin(d*x + c)^5 + 534555*sin(d*x + c)^4 + 1126810*sin(d*x +
 c)^3 + 1192850*sin(d*x + c)^2 + 634975*sin(d*x + c) + 136235)/(a*(sin(d*x
+ c) + 1)^5))/d
```

maple [A] time = 0.52, size = 244, normalized size = 0.87

$$\frac{1}{256ad\left(\sin\left(dx+c\right)-1\right)^{4}}-\frac{1}{32ad\left(\sin\left(dx+c\right)-1\right)^{3}}+\frac{81}{512ad\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{203}{256ad\left(\sin\left(dx+c\right)-1\right)}-\frac{843\ln(\mathrm{s}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/a/d/(sin(d*x+c)-1)^4-1/32/a/d/(sin(d*x+c)-1)^3+81/512/a/d/(sin(d*x+c)
-1)^2-203/256/a/d/(sin(d*x+c)-1)-843/512/a/d*ln(sin(d*x+c)-1)-1/2/a/d/sin(d
*x+c)^2+1/d/a/sin(d*x+c)+6*ln(sin(d*x+c))/a/d+1/160/a/d/(1+sin(d*x+c))^5+11
/256/a/d/(1+sin(d*x+c))^4+23/128/a/d/(1+sin(d*x+c))^3+325/512/a/d/(1+sin(d*
x+c))^2+5/2/a/d/(1+sin(d*x+c))-2229/512*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.33, size = 257, normalized size = 0.92

$$\frac{\frac{2\left(3465\sin(dx+c)^{10}-375\sin(dx+c)^{9}-16545\sin(dx+c)^{8}+735\sin(dx+c)^{7}+30303\sin(dx+c)^{6}+223\sin(dx+c)^{5}-25847\sin(dx+c)^{4}-1207\sin(dx+c)^{3}+\cdots\right.}{a\sin(dx+c)^{11}+a\sin(dx+c)^{10}-4a\sin(dx+c)^{9}-4a\sin(dx+c)^{8}+6a\sin(dx+c)^{7}+6a\sin(dx+c)^{6}-4a\sin(dx+c)^{5}-4a\sin(dx+c)^{4}+a\sin(\cdots}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2560*(2*(3465*sin(d*x + c)^10 - 375*sin(d*x + c)^9 - 16545*sin(d*x + c)^8
 + 735*sin(d*x + c)^7 + 30303*sin(d*x + c)^6 + 223*sin(d*x + c)^5 - 25847*s
in(d*x + c)^4 - 1207*sin(d*x + c)^3 + 9408*sin(d*x + c)^2 + 640*sin(d*x + c
) - 640)/(a*sin(d*x + c)^11 + a*sin(d*x + c)^10 - 4*a*sin(d*x + c)^9 - 4*a*
sin(d*x + c)^8 + 6*a*sin(d*x + c)^7 + 6*a*sin(d*x + c)^6 - 4*a*sin(d*x + c)
^5 - 4*a*sin(d*x + c)^4 + a*sin(d*x + c)^3 + a*sin(d*x + c)^2) - 11145*log(
sin(d*x + c) + 1)/a - 4215*log(sin(d*x + c) - 1)/a + 15360*log(sin(d*x + c)
)/a)/d
```

mupad [B] time = 9.64, size = 263, normalized size = 0.94

$$\frac{\frac{693\sin(c+d\,x)^{10}}{256}-\frac{75\sin(c+d\,x)^{9}}{256}-\frac{3309\sin(c+d\,x)^{8}}{256}+\frac{147\sin(c+d\,x)^{7}}{256}+\frac{30303\sin(c+d\,x)^{6}}{1280}+\frac{223\sin(c+d\,x)^{5}}{1280}-\cdots}{d\,\left(a\sin(c+d\,x)^{11}+a\sin(c+d\,x)^{10}-4\,a\sin(c+d\,x)^{9}-4\,a\sin(c+d\,x)^{8}+6\,a\sin(c+d\,x)^{7}+6\,a\sin(c+d\,x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^9*sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x)/2 + (147*sin(c + d*x)^2)/20 - (1207*sin(c + d*x)^3)/1280 - (2
5847*sin(c + d*x)^4)/1280 + (223*sin(c + d*x)^5)/1280 + (30303*sin(c + d*x)
^6)/1280 + (147*sin(c + d*x)^7)/256 - (3309*sin(c + d*x)^8)/256 - (75*sin(c
 + d*x)^9)/256 + (693*sin(c + d*x)^10)/256 - 1/2)/(d*(a*sin(c + d*x)^2 + a*
sin(c + d*x)^3 - 4*a*sin(c + d*x)^4 - 4*a*sin(c + d*x)^5 + 6*a*sin(c + d*x)
^6 + 6*a*sin(c + d*x)^7 - 4*a*sin(c + d*x)^8 - 4*a*sin(c + d*x)^9 + a*sin(c
 + d*x)^10 + a*sin(c + d*x)^11)) - (2229*log(sin(c + d*x) + 1))/(512*a*d) -
 (843*log(sin(c + d*x) - 1))/(512*a*d) + (6*log(sin(c + d*x)))/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.911 $\int (g\sec(e+fx))^p(d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=127

$$\frac{\sec(e+fx)(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{n+1}(g\sec(e+fx))^p(\sin(e+fx)+1)^{\frac{1}{2}(-2m+p+1)}F}{df(n+1)}$$

```
[Out] AppellF1(1+n,1/2-m+1/2*p,1/2+1/2*p,2+n,-sin(f*x+e),sin(f*x+e))*sec(f*x+e)*(
g*sec(f*x+e))^p*(1-sin(f*x+e))^(1/2+1/2*p)*(d*sin(f*x+e))^(1+n)*(1+sin(f*x+
e))^(1/2-m+1/2*p)*(a+a*sin(f*x+e))^m/d/f/(1+n)
```

Rubi [A] time = 0.35, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2926, 2886, 135, 133}

$$\frac{\sec(e+fx)(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{n+1}(g\sec(e+fx))^p(\sin(e+fx)+1)^{\frac{1}{2}(-2m+p+1)}F}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p*(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 + n, (1 + p)/2, (1 - 2*m + p)/2, 2 + n, Sin[e + f*x], -Sin[e +
f*x]]*Sec[e + f*x]*(g*Sec[e + f*x])^p*(1 - Sin[e + f*x])^((1 + p)/2)*(d*Sin
[e + f*x])^(1 + n)*(1 + Sin[e + f*x])^((1 - 2*m + p)/2)*(a + a*Sin[e + f*x]
)^m)/(d*f*(1 + n))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2926

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]*(g*Sec[e + f*x])^FracPart[p
], Int[((a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(g*Cos[e + f*x])^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int (g\sec(e+fx))^p(d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx = \left((g\cos(e+fx))^p(g\sec(e+fx))^p\right)\int (g\cos(e+fx))$$

$$= \frac{\left(\sec(e+fx)(g\sec(e+fx))^p(a-a\sin(e+fx))^{\frac{1+p}{2}}(\right.}{}$$

$$= \frac{\left(\sec(e+fx)(g\sec(e+fx))^p(1-\sin(e+fx))^{\frac{1}{2}+\frac{p}{2}}(a\right.}{}$$

$$= \frac{\left(\sec(e+fx)(g\sec(e+fx))^p(1-\sin(e+fx))^{\frac{1}{2}+\frac{p}{2}}(1\right.}{}$$

$$= \frac{F_1\left(1+n;\frac{1+p}{2},\frac{1}{2}(1-2m+p);2+n;\sin(e+fx),-\right.}{}$$

Mathematica [B] time = 3.32, size = 347, normalized size = 2.73

$$\frac{g(p-3)(a(\sin(e+fx)+}{f(p-1)\left(2\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(nF_1\left(\frac{3-p}{2};1-n,m+n-p+1;\frac{5-p}{2};\cot^2\left(\frac{1}{4}(2e+2fx+\pi)\right),-\tan^2\left(\frac{1}{4}(2e+\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^p*(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (g*(-3 + p)*AppellF1[(1 - p)/2, -n, 1 + m + n - p, (3 - p)/2, Cot[(2*e + Pi
 + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*(g*Sec[e + f*x])^(-1 + p)*(d*
Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^m)/(f*(-1 + p)*((-3 + p)*AppellF1[(1
 - p)/2, -n, 1 + m + n - p, (3 - p)/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2
*e - Pi + 2*f*x)/4]^2] + 2*(n*AppellF1[(3 - p)/2, 1 - n, 1 + m + n - p, (5
- p)/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m
 + n - p)*AppellF1[(3 - p)/2, -n, 2 + m + n - p, (5 - p)/2, Cot[(2*e + Pi +
 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan[(2*e - Pi + 2*f*x)/4]^2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g\sec\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\sec\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

maple [F] time = 19.46, size = 0, normalized size = 0.00

$$\int\left(g\sec\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\sec\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^n\,{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^p\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(g/cos(e + f*x))^p*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int((d*sin(e + f*x))^n*(g/cos(e + f*x))^p*(a + a*sin(e + f*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(d*sin(f*x+e))**n*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.912 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=88

$$\frac{(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}{}_2F_1\left(m+1,-n;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)}$$

```
[Out] hypergeom([-n, 1+m],[2+m],-d*(1+sin(f*x+e))/(c-d))*(a+a*sin(f*x+e))^(1+m)*(
c+d*sin(f*x+e))^n/a/f/(1+m)/(((c+d*sin(f*x+e))/(c-d))^n)
```

Rubi [A] time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2833, 70, 69}

$$\frac{(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}{}_2F_1\left(m+1,-n;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a
+ a*Sin[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(1 + m)*((c + d*Sin[
e + f*x])/(c - d))^n)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\text{Subst}\left(\int (a+x)^m\left(c+\frac{dx}{a}\right)^n dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{\left((c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\right)\text{Subst}\left(\int (a+\right.}{af}$$

$$= \frac{{}_2F_1\left(1+m,-n;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e-}{af(1+r}$$

Mathematica [A] time = 0.14, size = 88, normalized size = 1.00

$$\frac{(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\,{}_2F_1\left(m+1,-n;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a
+ a*Sin[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(1 + m)*((c + d*Sin[
e + f*x])/(c - d))^n)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.913 $\int \cos(e+fx)(a+a\sin(e+fx))^4(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=175

$$\frac{a^4(c-d)^4(c+d\sin(e+fx))^{n+1}}{d^5f(n+1)}-\frac{4a^4(c-d)^3(c+d\sin(e+fx))^{n+2}}{d^5f(n+2)}+\frac{6a^4(c-d)^2(c+d\sin(e+fx))^{n+3}}{d^5f(n+3)}-\frac{4a^4(c-d)}{}$$

```
[Out] a^4*(c-d)^4*(c+d*sin(f*x+e))^(1+n)/d^5/f/(1+n)-4*a^4*(c-d)^3*(c+d*sin(f*x+e
))^(2+n)/d^5/f/(2+n)+6*a^4*(c-d)^2*(c+d*sin(f*x+e))^(3+n)/d^5/f/(3+n)-4*a^4
*(c-d)*(c+d*sin(f*x+e))^(4+n)/d^5/f/(4+n)+a^4*(c+d*sin(f*x+e))^(5+n)/d^5/f/
(5+n)
```

Rubi [A] time = 0.21, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 43}

$$\frac{a^4(c-d)^4(c+d\sin(e+fx))^{n+1}}{d^5f(n+1)}-\frac{4a^4(c-d)^3(c+d\sin(e+fx))^{n+2}}{d^5f(n+2)}+\frac{6a^4(c-d)^2(c+d\sin(e+fx))^{n+3}}{d^5f(n+3)}-\frac{4a^4(c-d)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^4*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^4*(c - d)^4*(c + d*Sin[e + f*x])^(1 + n))/(d^5*f*(1 + n)) - (4*a^4*(c -
d)^3*(c + d*Sin[e + f*x])^(2 + n))/(d^5*f*(2 + n)) + (6*a^4*(c - d)^2*(c +
d*Sin[e + f*x])^(3 + n))/(d^5*f*(3 + n)) - (4*a^4*(c - d)*(c + d*Sin[e + f*
x])^(4 + n))/(d^5*f*(4 + n)) + (a^4*(c + d*Sin[e + f*x])^(5 + n))/(d^5*f*(5
 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^4(c+d\sin(e+fx))^n\,dx = \frac{\text{Subst}\left(\int (a+x)^4\left(c+\frac{dx}{a}\right)^n dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{\text{Subst}\left(\int\left(\frac{a^4(c-d)^4\left(c+\frac{dx}{a}\right)^n}{d^4} - \frac{4a^4(c-d)^3\left(c+\frac{dx}{a}\right)^{1+n}}{d^4} + \frac{6a^4(c-d}{}\right.\right.}{}$$

$$= \frac{a^4(c-d)^4(c+d\sin(e+fx))^{1+n}}{d^5f(1+n)} - \frac{4a^4(c-d)^3(c+d}{d^5f(2}$$

Mathematica [A] time = 0.60, size = 130, normalized size = 0.74

$$\frac{a^4(c+d\sin(e+fx))^{n+1}\left(-\frac{4(c-d)^3(c+d\sin(e+fx))}{n+2} + \frac{6(c-d)^2(c+d\sin(e+fx))^2}{n+3} - \frac{4(c-d)(c+d\sin(e+fx))^3}{n+4} + \frac{(c+d\sin(e+fx))^4}{n+5} + \frac{(c-d)^4}{n+1}\right.}{d^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^4*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^4*(c + d*Sin[e + f*x])^(1 + n)*((c - d)^4/(1 + n) - (4*(c - d)^3*(c + d*
Sin[e + f*x]))/(2 + n) + (6*(c - d)^2*(c + d*Sin[e + f*x])^2)/(3 + n) - (4*
(c - d)*(c + d*Sin[e + f*x])^3)/(4 + n) + (c + d*Sin[e + f*x])^4/(5 + n)))/
(d^5*f)
```

fricas [B] time = 0.59, size = 902, normalized size = 5.15

$$\frac{\left(24\,a^4c^5 - 120\,a^4c^4d + 240\,a^4c^3d^2 - 240\,a^4c^2d^3 + 120\,a^4cd^4 + 360\,a^4d^5 + 8\left(a^4cd^4 + a^4d^5\right)n^4 + \left(120\,a^4d^5 + \left(a^4c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] (24*a^4*c^5 - 120*a^4*c^4*d + 240*a^4*c^3*d^2 - 240*a^4*c^2*d^3 + 120*a^4*c
*d^4 + 360*a^4*d^5 + 8*(a^4*c*d^4 + a^4*d^5)*n^4 + (120*a^4*d^5 + (a^4*c*d^
4 + 4*a^4*d^5)*n^4 + 2*(3*a^4*c*d^4 + 22*a^4*d^5)*n^3 + (11*a^4*c*d^4 + 164
*a^4*d^5)*n^2 + 2*(3*a^4*c*d^4 + 122*a^4*d^5)*n)*cos(f*x + e)^4 - 16*(a^4*c
^2*d^3 - 5*a^4*c*d^4 - 6*a^4*d^5)*n^3 + 8*(3*a^4*c^3*d^2 - 15*a^4*c^2*d^3 +
 32*a^4*c*d^4 + 50*a^4*d^5)*n^2 - 4*(120*a^4*d^5 + (2*a^4*c*d^4 + 3*a^4*d^5
```

```
)*n^4 - (3*a^4*c^2*d^3 - 18*a^4*c*d^4 - 35*a^4*d^5)*n^3 + (3*a^4*c^3*d^2 -
18*a^4*c^2*d^3 + 49*a^4*c*d^4 + 141*a^4*d^5)*n^2 + (3*a^4*c^3*d^2 - 15*a^4*
c^2*d^3 + 33*a^4*c*d^4 + 229*a^4*d^5)*n)*cos(f*x + e)^2 - 8*(3*a^4*c^4*d -
15*a^4*c^3*d^2 + 31*a^4*c^2*d^3 - 35*a^4*c*d^4 - 84*a^4*d^5)*n + (384*a^4*d
^5 + 8*(a^4*c*d^4 + a^4*d^5)*n^4 + (a^4*d^5*n^4 + 10*a^4*d^5*n^3 + 35*a^4*d
^5*n^2 + 50*a^4*d^5*n + 24*a^4*d^5)*cos(f*x + e)^4 - 16*(a^4*c^2*d^3 - 5*a^
4*c*d^4 - 6*a^4*d^5)*n^3 + 8*(3*a^4*c^3*d^2 - 15*a^4*c^2*d^3 + 32*a^4*c*d^4
 + 50*a^4*d^5)*n^2 - 4*(72*a^4*d^5 + (a^4*c*d^4 + 2*a^4*d^5)*n^4 - (a^4*c^2
*d^3 - 8*a^4*c*d^4 - 23*a^4*d^5)*n^3 - (3*a^4*c^2*d^3 - 17*a^4*c*d^4 - 91*a
^4*d^5)*n^2 - 2*(a^4*c^2*d^3 - 5*a^4*c*d^4 - 71*a^4*d^5)*n)*cos(f*x + e)^2
- 8*(3*a^4*c^4*d - 15*a^4*c^3*d^2 + 31*a^4*c^2*d^3 - 35*a^4*c*d^4 - 84*a^4*
d^5)*n)*sin(f*x + e))*(d*sin(f*x + e) + c)^n/(d^5*f*n^5 + 15*d^5*f*n^4 + 85
*d^5*f*n^3 + 225*d^5*f*n^2 + 274*d^5*f*n + 120*d^5*f)
```

giac [B] time = 0.24, size = 1840, normalized size = 10.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] (((d*sin(f*x + e) + c)^5*(d*sin(f*x + e) + c)^n*n^4 - 4*(d*sin(f*x + e) + c
)^4*(d*sin(f*x + e) + c)^n*c*n^4 + 6*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e)
 + c)^n*c^2*n^4 - 4*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^3*n^4 +
 (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^4*n^4 + 10*(d*sin(f*x + e) +
 c)^5*(d*sin(f*x + e) + c)^n*n^3 - 44*(d*sin(f*x + e) + c)^4*(d*sin(f*x + e
) + c)^n*c*n^3 + 72*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*c^2*n^3 -
 52*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^3*n^3 + 14*(d*sin(f*x +
 e) + c)*(d*sin(f*x + e) + c)^n*c^4*n^3 + 35*(d*sin(f*x + e) + c)^5*(d*sin(
f*x + e) + c)^n*n^2 - 164*(d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*c*n
^2 + 294*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*c^2*n^2 - 236*(d*sin
(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^3*n^2 + 71*(d*sin(f*x + e) + c)*(
d*sin(f*x + e) + c)^n*c^4*n^2 + 50*(d*sin(f*x + e) + c)^5*(d*sin(f*x + e) +
 c)^n*n - 244*(d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*c*n + 468*(d*si
n(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*c^2*n - 428*(d*sin(f*x + e) + c)^2
*(d*sin(f*x + e) + c)^n*c^3*n + 154*(d*sin(f*x + e) + c)*(d*sin(f*x + e) +
c)^n*c^4*n + 24*(d*sin(f*x + e) + c)^5*(d*sin(f*x + e) + c)^n - 120*(d*sin(
f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*c + 240*(d*sin(f*x + e) + c)^3*(d*si
n(f*x + e) + c)^n*c^2 - 240*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c
^3 + 120*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^4)*a^4/(d^4*n^5 + 15
*d^4*n^4 + 85*d^4*n^3 + 225*d^4*n^2 + 274*d^4*n + 120*d^4) + 4*((d*sin(f*x
+ e) + c)^4*(d*sin(f*x + e) + c)^n*n^3 - 3*(d*sin(f*x + e) + c)^3*(d*sin(f*
x + e) + c)^n*c*n^3 + 3*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2*n
^3 - (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n^3 + 6*(d*sin(f*x + e
```

```
) + c)^4*(d*sin(f*x + e) + c)^n*n^2 - 21*(d*sin(f*x + e) + c)^3*(d*sin(f*x
+ e) + c)^n*c*n^2 + 24*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2*n^
2 - 9*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n^2 + 11*(d*sin(f*x +
 e) + c)^4*(d*sin(f*x + e) + c)^n*n - 42*(d*sin(f*x + e) + c)^3*(d*sin(f*x
+ e) + c)^n*c*n + 57*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2*n -
26*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n + 6*(d*sin(f*x + e) +
c)^4*(d*sin(f*x + e) + c)^n - 24*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c
)^n*c + 36*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2 - 24*(d*sin(f*
x + e) + c)*(d*sin(f*x + e) + c)^n*c^3)*a^4/(d^3*n^4 + 10*d^3*n^3 + 35*d^3*
n^2 + 50*d^3*n + 24*d^3) + 6*((d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n
*n^2 - 2*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c*n^2 + (d*sin(f*x +
 e) + c)*(d*sin(f*x + e) + c)^n*c^2*n^2 + 3*(d*sin(f*x + e) + c)^3*(d*sin(f
*x + e) + c)^n*n - 8*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c*n + 5*
(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2*n + 2*(d*sin(f*x + e) + c)^
3*(d*sin(f*x + e) + c)^n - 6*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*
c + 6*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2)*a^4/(d^2*n^3 + 6*d^2
*n^2 + 11*d^2*n + 6*d^2) + (d*sin(f*x + e) + c)^(n + 1)*a^4/(n + 1) + 4*((d
*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*n - (d*sin(f*x + e) + c)*(d*sin
(f*x + e) + c)^n*c*n + (d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n - 2*(d
*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c)*a^4/((n^2 + 3*n + 2)*d))/(d*f)
```

maple [F] time = 4.38, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^4\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x)
```

maxima [B] time = 1.30, size = 486, normalized size = 2.78

$$\frac{\frac{4\left(d^2(n+1)\sin\left(fx+e\right)^2+cdn\sin\left(fx+e\right)-c^2\right)\left(d\sin\left(fx+e\right)+c\right)^n a^4}{\left(n^2+3n+2\right)d^2}+\frac{\left(d\sin\left(fx+e\right)+c\right)^{n+1}a^4}{d(n+1)}+\frac{6\left(\left(n^2+3n+2\right)d^3\sin\left(fx+e\right)^3+\left(n^2+n\right)cd^2\sin\left(fx+e\right)^2-\right.}{\left(n^3+6n^2+11n+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] (4*(d^2*(n + 1)*sin(f*x + e)^2 + c*d*n*sin(f*x + e) - c^2)*(d*sin(f*x + e)
+ c)^n*a^4/((n^2 + 3*n + 2)*d^2) + (d*sin(f*x + e) + c)^(n + 1)*a^4/(d*(n +
 1)) + 6*((n^2 + 3*n + 2)*d^3*sin(f*x + e)^3 + (n^2 + n)*c*d^2*sin(f*x + e)
```

```
^2 - 2*c^2*d*n*sin(f*x + e) + 2*c^3)*(d*sin(f*x + e) + c)^n*a^4/((n^3 + 6*n
^2 + 11*n + 6)*d^3) + 4*((n^3 + 6*n^2 + 11*n + 6)*d^4*sin(f*x + e)^4 + (n^3
 + 3*n^2 + 2*n)*c*d^3*sin(f*x + e)^3 - 3*(n^2 + n)*c^2*d^2*sin(f*x + e)^2 +
 6*c^3*d*n*sin(f*x + e) - 6*c^4)*(d*sin(f*x + e) + c)^n*a^4/((n^4 + 10*n^3
+ 35*n^2 + 50*n + 24)*d^4) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*d^5*sin(f
*x + e)^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*c*d^4*sin(f*x + e)^4 - 4*(n^3 + 3*
n^2 + 2*n)*c^2*d^3*sin(f*x + e)^3 + 12*(n^2 + n)*c^3*d^2*sin(f*x + e)^2 - 2
4*c^4*d*n*sin(f*x + e) + 24*c^5)*(d*sin(f*x + e) + c)^n*a^4/((n^5 + 15*n^4
+ 85*n^3 + 225*n^2 + 274*n + 120)*d^5))/f
```

mupad [B] time = 18.14, size = 863, normalized size = 4.93

$$\frac{a^4\,\sin\left(5\,e+5\,f\,x\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)\,1{}\mathrm{i}}{16\,f\,\left(n^5\,1{}\mathrm{i}+n^4\,15{}\mathrm{i}+n^3\,85{}\mathrm{i}+n^2\,225{}\mathrm{i}+n\,274{}\mathrm{i}+120{}\mathrm{i}\right)}+\frac{a^4\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n\,\left(c^5\,192{}\mathrm{i}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^4*(c + d*sin(e + f*x))^n,x)
```

```
[Out] (a^4*sin(5*e + 5*f*x)*(c + d*sin(e + f*x))^n*(50*n + 35*n^2 + 10*n^3 + n^4
+ 24)*1i)/(16*f*(n*274i + n^2*225i + n^3*85i + n^4*15i + n^5*1i + 120i)) +
(a^4*(c + d*sin(e + f*x))^n*(c*d^4*960i - c^4*d*960i + d^5*n*2444i + c^5*19
2i + d^5*1320i - c^2*d^3*1920i + c^3*d^2*1920i + d^5*n^2*1436i + d^5*n^3*34
0i + d^5*n^4*28i - c^2*d^3*n*1744i + c^3*d^2*n*912i + c*d^4*n^2*1297i + c*d
^4*n^3*370i + c*d^4*n^4*35i - c^2*d^3*n^2*672i + c^3*d^2*n^2*144i - c^2*d^3
*n^3*80i + c*d^4*n*1730i - c^4*d*n*192i))/(8*d^5*f*(n*274i + n^2*225i + n^3
*85i + n^4*15i + n^5*1i + 120i)) + (a^4*sin(e + f*x)*(c + d*sin(e + f*x))^n
*(4290*d^4*n - 192*c^4*n + 2520*d^4 + 2507*d^4*n^2 + 594*d^4*n^3 + 49*d^4*n
^4 - 1968*c^2*d^2*n + 1912*c*d^3*n^2 + 192*c^3*d*n^2 + 576*c*d^3*n^3 + 56*c
*d^3*n^4 - 936*c^2*d^2*n^2 - 120*c^2*d^2*n^3 + 2160*c*d^3*n + 960*c^3*d*n)*
1i)/(8*d^4*f*(n*274i + n^2*225i + n^3*85i + n^4*15i + n^5*1i + 120i)) + (a^
4*cos(4*e + 4*f*x)*(c + d*sin(e + f*x))^n*(d*20i + c*n*1i + d*n*4i)*(11*n +
 6*n^2 + n^3 + 6))/(8*d*f*(n*274i + n^2*225i + n^3*85i + n^4*15i + n^5*1i +
 120i)) - (a^4*sin(3*e + 3*f*x)*(c + d*sin(e + f*x))^n*(3*n + n^2 + 2)*(251
*d^2*n - 16*c^2*n + 540*d^2 + 29*d^2*n^2 + 80*c*d*n + 16*c*d*n^2)*1i)/(16*d
^2*f*(n*274i + n^2*225i + n^3*85i + n^4*15i + n^5*1i + 120i)) - (a^4*cos(2*
e + 2*f*x)*(n + 1)*(c + d*sin(e + f*x))^n*(c^3*n*12i + d^3*n*312i + d^3*360
i + d^3*n^2*88i + d^3*n^3*8i + c*d^2*n^2*59i - c^2*d*n^2*12i + c*d^2*n^3*7i
 + c*d^2*n*126i - c^2*d*n*60i))/(2*d^3*f*(n*274i + n^2*225i + n^3*85i + n^4
*15i + n^5*1i + 120i))
```

sympy [A] time = 138.04, size = 11900, normalized size = 68.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**4*(c+d*sin(f*x+e))**n,x)

[Out] Piecewise((c**n*(a**4*sin(e + f*x)**5/(5*f) + a**4*sin(e + f*x)**4/f + 2*a*
*4*sin(e + f*x)**3/f + 2*a**4*sin(e + f*x)**2/f + a**4*sin(e + f*x)/f), Eq(
d, 0)), (x*(c + d*sin(e))**n*(a*sin(e) + a)**4*cos(e), Eq(f, 0)), (12*a**4*
c**4*log(c/d + sin(e + f*x))/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x)
+ 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*
sin(e + f*x)**4) + 25*a**4*c**4/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*
x) + 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9
*f*sin(e + f*x)**4) + 48*a**4*c**3*d*log(c/d + sin(e + f*x))*sin(e + f*x)/(
12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)*
*2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)**4) + 88*a**4*c**
3*d*sin(e + f*x)/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d*
*7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)
4) - 12*a4*c**3*d/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c*
*2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e +
f*x)**4) + 72*a**4*c**2*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)**2/(12*c
4*d5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)**2 +
48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)**4) + 108*a**4*c**2*d
2*sin(e + f*x)2/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2
*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f
*x)**4) - 48*a**4*c**2*d**2*sin(e + f*x)/(12*c**4*d**5*f + 48*c**3*d**6*f*s
in(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3
+ 12*d**9*f*sin(e + f*x)**4) - 6*a**4*c**2*d**2/(12*c**4*d**5*f + 48*c**3*d
6*f*sin(e + f*x) + 72*c2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f
*x)**3 + 12*d**9*f*sin(e + f*x)**4) + 48*a**4*c*d**3*log(c/d + sin(e + f*x)
)*sin(e + f*x)**3/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d
7*f*sin(e + f*x)2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x
)**4) + 48*a**4*c*d**3*sin(e + f*x)**3/(12*c**4*d**5*f + 48*c**3*d**6*f*sin
(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 +
12*d**9*f*sin(e + f*x)**4) - 72*a**4*c*d**3*sin(e + f*x)**2/(12*c**4*d**5*f
+ 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8
*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)**4) - 24*a**4*c*d**3*sin(e + f*
x)/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e + f
*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)**4) - 4*a**4*
c*d**3/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e
+ f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)**4) + 12*
a**4*d**4*log(c/d + sin(e + f*x))*sin(e + f*x)**4/(12*c**4*d**5*f + 48*c**3
*d**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e +
f*x)**3 + 12*d**9*f*sin(e + f*x)**4) - 48*a**4*d**4*sin(e + f*x)**3/(12*c*
*4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)**2 +
48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)**4) - 36*a**4*d**4*sin
(e + f*x)**2/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d**7*f
*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)**4)
- 16*a**4*d**4*sin(e + f*x)/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x)

+ 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*
sin(e + f*x)**4) - 3*a**4*d**4/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x
) + 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*
f*sin(e + f*x)**4), Eq(n, -5)), (-12*a**4*c**4*log(c/d + sin(e + f*x))/(3*c
3*d5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d*
*8*f*sin(e + f*x)**3) - 22*a**4*c**4/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e +
f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) - 36*a**4*c*
*3*d*log(c/d + sin(e + f*x))*sin(e + f*x)/(3*c**3*d**5*f + 9*c**2*d**6*f*si
n(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) + 12*a*
*4*c**3*d*log(c/d + sin(e + f*x))/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*
x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) - 54*a**4*c**3*
d*sin(e + f*x)/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin
(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) + 22*a**4*c**3*d/(3*c**3*d**5*f +
9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e +
f*x)**3) - 36*a**4*c**2*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)**2/(3*c**
3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8
*f*sin(e + f*x)**3) + 36*a**4*c**2*d**2*log(c/d + sin(e + f*x))*sin(e + f*x
)/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2
+ 3*d**8*f*sin(e + f*x)**3) - 36*a**4*c**2*d**2*sin(e + f*x)**2/(3*c**3*d**
5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*si
n(e + f*x)**3) + 54*a**4*c**2*d**2*sin(e + f*x)/(3*c**3*d**5*f + 9*c**2*d**
6*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) -
6*a**4*c**2*d**2/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*
sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) - 12*a**4*c*d**3*log(c/d + sin(
e + f*x))*sin(e + f*x)**3/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c
*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) + 36*a**4*c*d**3*log(c/
d + sin(e + f*x))*sin(e + f*x)**2/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*
x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) + 36*a**4*c*d**
3*sin(e + f*x)**2/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*
sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) - 18*a**4*c*d**3*sin(e + f*x)/(
3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3
*d**8*f*sin(e + f*x)**3) - 2*a**4*c*d**3/(3*c**3*d**5*f + 9*c**2*d**6*f*sin
(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) + 12*a**
4*d**4*log(c/d + sin(e + f*x))*sin(e + f*x)**3/(3*c**3*d**5*f + 9*c**2*d**6
*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) +
3*a**4*d**4*sin(e + f*x)**4/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9
*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) - 18*a**4*d**4*sin(e
+ f*x)**2/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin(e +
f*x)**2 + 3*d**8*f*sin(e + f*x)**3) - 6*a**4*d**4*sin(e + f*x)/(3*c**3*d**5
*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin
(e + f*x)**3) - a**4*d**4/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c
*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3), Eq(n, -4)), (12*a**4*c
4*log(c/d + sin(e + f*x))/(2*c2*d**5*f + 4*c*d**6*f*sin(e + f*x) + 2*d*
*7*f*sin(e + f*x)**2) + 18*a**4*c**4/(2*c**2*d**5*f + 4*c*d**6*f*sin(e + f*
x) + 2*d**7*f*sin(e + f*x)**2) + 24*a**4*c**3*d*log(c/d + sin(e + f*x))*sin

(e + f*x)/(2*c**2*d**5*f + 4*c*d**6*f*sin(e + f*x) + 2*d**7*f*sin(e + f*x)*
*2) - 24*a**4*c**3*d*log(c/d + sin(e + f*x))/(2*c**2*d**5*f + 4*c*d**6*f*si
n(e + f*x) + 2*d**7*f*sin(e + f*x)**2) + 24*a**4*c**3*d*sin(e + f*x)/(2*c**
2*d**5*f + 4*c*d**6*f*sin(e + f*x) + 2*d**7*f*sin(e + f*x)**2) - 36*a**4*c*
*3*d/(2*c**2*d**5*f + 4*c*d**6*f*sin(e + f*x) + 2*d**7*f*sin(e + f*x)**2) +
 12*a**4*c**2*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)**2/(2*c**2*d**5*f +
 4*c*d**6*f*sin(e + f*x) + 2*d**7*f*sin(e + f*x)**2) - 48*a**4*c**2*d**2*lo
g(c/d + sin(e + f*x))*sin(e + f*x)/(2*c**2*d**5*f + 4*c*d**6*f*sin(e + f*x)
 + 2*d**7*f*sin(e + f*x)**2) + 12*a**4*c**2*d**2*log(c/d + sin(e + f*x))/(2
*c**2*d**5*f + 4*c*d**6*f*sin(e + f*x) + 2*d**7*f*sin(e + f*x)**2) - 48*a**
4*c**2*d**2*sin(e + f*x)/(2*c**2*d**5*f + 4*c*d**6*f*sin(e + f*x) + 2*d**7*
f*sin(e + f*x)**2) + 18*a**4*c**2*d**2/(2*c**2*d**5*f + 4*c*d**6*f*sin(e +
f*x) + 2*d**7*f*sin(e + f*x)**2) - 24*a**4*c*d**3*log(c/d + sin(e + f*x))*s
in(e + f*x)**2/(2*c**2*d**5*f + 4*c*d**6*f*sin(e + f*x) + 2*d**7*f*sin(e +
f*x)**2) + 24*a**4*c*d**3*log(c/d + sin(e + f*x))*sin(e + f*x)/(2*c**2*d**5
*f + 4*c*d**6*f*sin(e + f*x) + 2*d**7*f*sin(e + f*x)**2) - 4*a**4*c*d**3*si
n(e + f*x)**3/(2*c**2*d**5*f + 4*c*d**6*f*sin(e + f*x) + 2*d**7*f*sin(e + f
*x)**2) + 24*a**4*c*d**3*sin(e + f*x)/(2*c**2*d**5*f + 4*c*d**6*f*sin(e + f
*x) + 2*d**7*f*sin(e + f*x)**2) - 4*a**4*c*d**3/(2*c**2*d**5*f + 4*c*d**6*f
*sin(e + f*x) + 2*d**7*f*sin(e + f*x)**2) + 12*a**4*d**4*log(c/d + sin(e +
f*x))*sin(e + f*x)**2/(2*c**2*d**5*f + 4*c*d**6*f*sin(e + f*x) + 2*d**7*f*s
in(e + f*x)**2) + a**4*d**4*sin(e + f*x)**4/(2*c**2*d**5*f + 4*c*d**6*f*sin
(e + f*x) + 2*d**7*f*sin(e + f*x)**2) + 8*a**4*d**4*sin(e + f*x)**3/(2*c**2
*d**5*f + 4*c*d**6*f*sin(e + f*x) + 2*d**7*f*sin(e + f*x)**2) - 8*a**4*d**4
*sin(e + f*x)/(2*c**2*d**5*f + 4*c*d**6*f*sin(e + f*x) + 2*d**7*f*sin(e + f
*x)**2) - a**4*d**4/(2*c**2*d**5*f + 4*c*d**6*f*sin(e + f*x) + 2*d**7*f*sin
(e + f*x)**2), Eq(n, -3)), (-12*a**4*c**4*log(c/d + sin(e + f*x))/(3*c*d**5
*f + 3*d**6*f*sin(e + f*x)) - 12*a**4*c**4/(3*c*d**5*f + 3*d**6*f*sin(e + f
*x)) - 12*a**4*c**3*d*log(c/d + sin(e + f*x))*sin(e + f*x)/(3*c*d**5*f + 3*
d**6*f*sin(e + f*x)) + 36*a**4*c**3*d*log(c/d + sin(e + f*x))/(3*c*d**5*f +
 3*d**6*f*sin(e + f*x)) + 36*a**4*c**3*d/(3*c*d**5*f + 3*d**6*f*sin(e + f*x
)) + 36*a**4*c**2*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)/(3*c*d**5*f + 3
*d**6*f*sin(e + f*x)) - 36*a**4*c**2*d**2*log(c/d + sin(e + f*x))/(3*c*d**5
*f + 3*d**6*f*sin(e + f*x)) + 6*a**4*c**2*d**2*sin(e + f*x)**2/(3*c*d**5*f
+ 3*d**6*f*sin(e + f*x)) - 36*a**4*c**2*d**2/(3*c*d**5*f + 3*d**6*f*sin(e +
 f*x)) - 36*a**4*c*d**3*log(c/d + sin(e + f*x))*sin(e + f*x)/(3*c*d**5*f +
3*d**6*f*sin(e + f*x)) + 12*a**4*c*d**3*log(c/d + sin(e + f*x))/(3*c*d**5*f
 + 3*d**6*f*sin(e + f*x)) - 2*a**4*c*d**3*sin(e + f*x)**3/(3*c*d**5*f + 3*d
6*f*sin(e + f*x)) - 18*a4*c*d**3*sin(e + f*x)**2/(3*c*d**5*f + 3*d**6*f
*sin(e + f*x)) + 12*a**4*c*d**3/(3*c*d**5*f + 3*d**6*f*sin(e + f*x)) + 12*a
4*d4*log(c/d + sin(e + f*x))*sin(e + f*x)/(3*c*d**5*f + 3*d**6*f*sin(e
+ f*x)) + a**4*d**4*sin(e + f*x)**4/(3*c*d**5*f + 3*d**6*f*sin(e + f*x)) +
6*a**4*d**4*sin(e + f*x)**3/(3*c*d**5*f + 3*d**6*f*sin(e + f*x)) + 18*a**4*
d**4*sin(e + f*x)**2/(3*c*d**5*f + 3*d**6*f*sin(e + f*x)) - 3*a**4*d**4/(3*
c*d**5*f + 3*d**6*f*sin(e + f*x)), Eq(n, -2)), (a**4*c**4*log(c/d + sin(e +

f*x))/(d**5*f) - 4*a**4*c**3*log(c/d + sin(e + f*x))/(d**4*f) - a**4*c**3*
sin(e + f*x)/(d**4*f) + 6*a**4*c**2*log(c/d + sin(e + f*x))/(d**3*f) + a**4
*c**2*sin(e + f*x)**2/(2*d**3*f) + 4*a**4*c**2*sin(e + f*x)/(d**3*f) - 4*a*
*4*c*log(c/d + sin(e + f*x))/(d**2*f) - a**4*c*sin(e + f*x)**3/(3*d**2*f) -
2*a**4*c*sin(e + f*x)**2/(d**2*f) - 6*a**4*c*sin(e + f*x)/(d**2*f) + a**4*
log(c/d + sin(e + f*x))/(d*f) + a**4*sin(e + f*x)**4/(4*d*f) + 4*a**4*sin(e
 + f*x)**3/(3*d*f) + 3*a**4*sin(e + f*x)**2/(d*f) + 4*a**4*sin(e + f*x)/(d*
f), Eq(n, -1)), (24*a**4*c**5*(c + d*sin(e + f*x))**n/(d**5*f*n**5 + 15*d**
5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) -
24*a**4*c**4*d*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**5*f*n**5 + 15*d**
5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) -
24*a**4*c**4*d*n*(c + d*sin(e + f*x))**n/(d**5*f*n**5 + 15*d**5*f*n**4 + 85
*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) - 120*a**4*c**4
d(c + d*sin(e + f*x))**n/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 +
 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 12*a**4*c**3*d**2*n**2*(c +
 d*sin(e + f*x))**n*sin(e + f*x)**2/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5
*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 24*a**4*c**3*d**2*
n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**5*f*n**5 + 15*d**5*f*n**4 + 8
5*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 12*a**4*c**3
*d**2*n**2*(c + d*sin(e + f*x))**n/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*
f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 12*a**4*c**3*d**2*n
*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**5*f*n**5 + 15*d**5*f*n**4 + 85
*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 120*a**4*c**3
*d**2*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**5*f*n**5 + 15*d**5*f*n**4
+ 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 108*a**4*
c**3*d**2*n*(c + d*sin(e + f*x))**n/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5
*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 240*a**4*c**3*d**2
*(c + d*sin(e + f*x))**n/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 2
25*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) - 4*a**4*c**2*d**3*n**3*(c + d*
sin(e + f*x))**n*sin(e + f*x)**3/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*
n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) - 12*a**4*c**2*d**3*n**
3*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**5*f*n**5 + 15*d**5*f*n**4 + 8
5*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) - 12*a**4*c**2
*d**3*n**3*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**5*f*n**5 + 15*d**5*f*n*
*4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) - 4*a**4
*c**2*d**3*n**3*(c + d*sin(e + f*x))**n/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*
d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) - 12*a**4*c**2*d
3*n2*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**5*f*n**5 + 15*d**5*f*n
4 + 85*d5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) - 72*a*
*4*c**2*d**3*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**5*f*n**5 + 15
*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f
) - 108*a**4*c**2*d**3*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**5*f*n*
*5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120
*d**5*f) - 48*a**4*c**2*d**3*n**2*(c + d*sin(e + f*x))**n/(d**5*f*n**5 + 15
*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f

) - 8*a**4*c**2*d**3*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) - 60*a**4*c**2*d**3*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) - 240*a**4*c**2*d**3*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) - 188*a**4*c**2*d**3*n*(c + d*sin(e + f*x))**n/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) - 240*a**4*c**2*d**3*(c + d*sin(e + f*x))**n/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + a**4*c*d**4*n**4*(c + d*sin(e + f*x))**n*sin(e + f*x)**4/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 4*a**4*c*d**4*n**4*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 6*a**4*c*d**4*n**4*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 4*a**4*c*d**4*n**4*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + a**4*c*d**4*n**4*(c + d*sin(e + f*x))**n/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 6*a**4*c*d**4*n**3*(c + d*sin(e + f*x))**n*sin(e + f*x)**4/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 32*a**4*c*d**4*n**3*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 60*a**4*c*d**4*n**3*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 48*a**4*c*d**4*n**3*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 14*a**4*c*d**4*n**3*(c + d*sin(e + f*x))**n/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 11*a**4*c*d**4*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**4/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 68*a**4*c*d**4*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 174*a**4*c*d**4*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 188*a**4*c*d**4*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 71*a**4*c*d**4*n**2*(c + d*sin(e + f*x))**n/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 6*a**4*c*d**4*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**4/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 40*a**4*c*d**4*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 120*a**4*c*d**4*n*(c + d*sin(e + f*x))**n*sin(

e + f*x)**2/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**
2 + 274*d**5*f*n + 120*d**5*f) + 240*a**4*c*d**4*n*(c + d*sin(e + f*x))**n*
sin(e + f*x)/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n*
*2 + 274*d**5*f*n + 120*d**5*f) + 154*a**4*c*d**4*n*(c + d*sin(e + f*x))**n
/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**
5*f*n + 120*d**5*f) + 120*a**4*c*d**4*(c + d*sin(e + f*x))**n/(d**5*f*n**5
+ 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d*
*5*f) + a**4*d**5*n**4*(c + d*sin(e + f*x))**n*sin(e + f*x)**5/(d**5*f*n**5
 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d
5*f) + 4*a4*d**5*n**4*(c + d*sin(e + f*x))**n*sin(e + f*x)**4/(d**5*f*n
5 + 15*d5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 12
0*d**5*f) + 6*a**4*d**5*n**4*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**5*
f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n +
 120*d**5*f) + 4*a**4*d**5*n**4*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d*
*5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*
n + 120*d**5*f) + a**4*d**5*n**4*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**5
*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n
+ 120*d**5*f) + 10*a**4*d**5*n**3*(c + d*sin(e + f*x))**n*sin(e + f*x)**5/(
d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*
f*n + 120*d**5*f) + 44*a**4*d**5*n**3*(c + d*sin(e + f*x))**n*sin(e + f*x)*
*4/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 274*d
5*f*n + 120*d5*f) + 72*a**4*d**5*n**3*(c + d*sin(e + f*x))**n*sin(e + f
*x)**3/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2 + 2
74*d**5*f*n + 120*d**5*f) + 52*a**4*d**5*n**3*(c + d*sin(e + f*x))**n*sin(e
 + f*x)**2/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**2
 + 274*d**5*f*n + 120*d**5*f) + 14*a**4*d**5*n**3*(c + d*sin(e + f*x))**n*s
in(e + f*x)/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f*n**
2 + 274*d**5*f*n + 120*d**5*f) + 35*a**4*d**5*n**2*(c + d*sin(e + f*x))**n*
sin(e + f*x)**5/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d**5*f
*n**2 + 274*d**5*f*n + 120*d**5*f) + 164*a**4*d**5*n**2*(c + d*sin(e + f*x)
)**n*sin(e + f*x)**4/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 + 225*d
5*f*n2 + 274*d**5*f*n + 120*d**5*f) + 294*a**4*d**5*n**2*(c + d*sin(e +
 f*x))**n*sin(e + f*x)**3/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n**3 +
225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 236*a**4*d**5*n**2*(c + d*si
n(e + f*x))**n*sin(e + f*x)**2/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n*
*3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 71*a**4*d**5*n**2*(c +
d*sin(e + f*x))**n*sin(e + f*x)/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*n
3 + 225*d5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 50*a**4*d**5*n*(c + d*
sin(e + f*x))**n*sin(e + f*x)**5/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f*
n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 244*a**4*d**5*n*(c +
d*sin(e + f*x))**n*sin(e + f*x)**4/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*
f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 468*a**4*d**5*n*(c
+ d*sin(e + f*x))**n*sin(e + f*x)**3/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**
5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 428*a**4*d**5*n*(
c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d

```
**5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 154*a**4*d**5*n
*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d*
*5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 24*a**4*d**5*(c
+ d*sin(e + f*x))**n*sin(e + f*x)**5/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**
5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 120*a**4*d**5*(c
+ d*sin(e + f*x))**n*sin(e + f*x)**4/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**
5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 240*a**4*d**5*(c
+ d*sin(e + f*x))**n*sin(e + f*x)**3/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**
5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 240*a**4*d**5*(c
+ d*sin(e + f*x))**n*sin(e + f*x)**2/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**
5*f*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f) + 120*a**4*d**5*(c
+ d*sin(e + f*x))**n*sin(e + f*x)/(d**5*f*n**5 + 15*d**5*f*n**4 + 85*d**5*f
*n**3 + 225*d**5*f*n**2 + 274*d**5*f*n + 120*d**5*f), True))
```

3.914 $\int \cos(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=139

$$-\frac{a^3(c-d)^3(c+d\sin(e+fx))^{n+1}}{d^4f(n+1)}+\frac{3a^3(c-d)^2(c+d\sin(e+fx))^{n+2}}{d^4f(n+2)}-\frac{3a^3(c-d)(c+d\sin(e+fx))^{n+3}}{d^4f(n+3)}+\frac{a^3(c+d\sin(e+fx))^{n+4}}{d^4f(n+4)}$$

```
[Out] -a^3*(c-d)^3*(c+d*sin(f*x+e))^(1+n)/d^4/f/(1+n)+3*a^3*(c-d)^2*(c+d*sin(f*x+
e))^(2+n)/d^4/f/(2+n)-3*a^3*(c-d)*(c+d*sin(f*x+e))^(3+n)/d^4/f/(3+n)+a^3*(c
+d*sin(f*x+e))^(4+n)/d^4/f/(4+n)
```

Rubi [A] time = 0.17, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 43}

$$-\frac{a^3(c-d)^3(c+d\sin(e+fx))^{n+1}}{d^4f(n+1)}+\frac{3a^3(c-d)^2(c+d\sin(e+fx))^{n+2}}{d^4f(n+2)}-\frac{3a^3(c-d)(c+d\sin(e+fx))^{n+3}}{d^4f(n+3)}+\frac{a^3(c+d\sin(e+fx))^{n+4}}{d^4f(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -((a^3*(c - d)^3*(c + d*Sin[e + f*x])^(1 + n))/(d^4*f*(1 + n))) + (3*a^3*(c
 - d)^2*(c + d*Sin[e + f*x])^(2 + n))/(d^4*f*(2 + n)) - (3*a^3*(c - d)*(c +
 d*Sin[e + f*x])^(3 + n))/(d^4*f*(3 + n)) + (a^3*(c + d*Sin[e + f*x])^(4 +
n))/(d^4*f*(4 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx = \frac{\text{Subst}\left(\int (a+x)^3\left(c+\frac{dx}{a}\right)^n\,dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{\text{Subst}\left(\int\left(-\frac{a^3(c-d)^3\left(c+\frac{dx}{a}\right)^n}{d^3}+\frac{3a^3(c-d)^2\left(c+\frac{dx}{a}\right)^{1+n}}{d^3}-\frac{3a^3(c\cdot}{}\right.\right.}{af}$$

$$= -\frac{a^3(c-d)^3(c+d\sin(e+fx))^{1+n}}{d^4f(1+n)}+\frac{3a^3(c-d)^2(c+}{d^4f(}$$

Mathematica [A] time = 0.34, size = 105, normalized size = 0.76

$$\frac{a^3(c+d\sin(e+fx))^{n+1}\left(\frac{3(c-d)^2(c+d\sin(e+fx))}{n+2}-\frac{3(c-d)(c+d\sin(e+fx))^2}{n+3}+\frac{(c+d\sin(e+fx))^3}{n+4}-\frac{(c-d)^3}{n+1}\right)}{d^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^3*(c + d*Sin[e + f*x])^(1 + n)*(-((c - d)^3/(1 + n)) + (3*(c - d)^2*(c +
 d*Sin[e + f*x]))/(2 + n) - (3*(c - d)*(c + d*Sin[e + f*x])^2)/(3 + n) + (c
 + d*Sin[e + f*x])^3/(4 + n)))/(d^4*f)
```

fricas [B] time = 0.54, size = 545, normalized size = 3.92

$$-\frac{\left(6\,a^3c^4-24\,a^3c^3d+36\,a^3c^2d^2-24\,a^3cd^3-42\,a^3d^4-\left(a^3d^4n^3+6\,a^3d^4n^2+11\,a^3d^4n+6\,a^3d^4\right)\cos\left(fx+e\right)^4-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] -(6*a^3*c^4 - 24*a^3*c^3*d + 36*a^3*c^2*d^2 - 24*a^3*c*d^3 - 42*a^3*d^4 - (
a^3*d^4*n^3 + 6*a^3*d^4*n^2 + 11*a^3*d^4*n + 6*a^3*d^4)*cos(f*x + e)^4 - 4*
(a^3*c*d^3 + a^3*d^4)*n^3 + 6*(a^3*c^2*d^2 - 4*a^3*c*d^3 - 5*a^3*d^4)*n^2 +
 (48*a^3*d^4 + (3*a^3*c*d^3 + 5*a^3*d^4)*n^3 - 3*(a^3*c^2*d^2 - 5*a^3*c*d^3
 - 12*a^3*d^4)*n^2 - (3*a^3*c^2*d^2 - 12*a^3*c*d^3 - 79*a^3*d^4)*n)*cos(f*x
 + e)^2 - 2*(3*a^3*c^3*d - 12*a^3*c^2*d^2 + 19*a^3*c*d^3 + 34*a^3*d^4)*n -
(48*a^3*d^4 + 4*(a^3*c*d^3 + a^3*d^4)*n^3 - 6*(a^3*c^2*d^2 - 4*a^3*c*d^3 -
```

```
5*a^3*d^4)*n^2 - (24*a^3*d^4 + (a^3*c*d^3 + 3*a^3*d^4)*n^3 + 3*(a^3*c*d^3 +
 7*a^3*d^4)*n^2 + 2*(a^3*c*d^3 + 21*a^3*d^4)*n)*cos(f*x + e)^2 + 2*(3*a^3*c
^3*d - 12*a^3*c^2*d^2 + 19*a^3*c*d^3 + 34*a^3*d^4)*n)*sin(f*x + e))*(d*sin(
f*x + e) + c)^n/(d^4*f*n^4 + 10*d^4*f*n^3 + 35*d^4*f*n^2 + 50*d^4*f*n + 24*
d^4*f)
```

giac [B] time = 0.20, size = 1001, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] (((d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*n^3 - 3*(d*sin(f*x + e) + c
)^3*(d*sin(f*x + e) + c)^n*c*n^3 + 3*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e)
 + c)^n*c^2*n^3 - (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n^3 + 6*(
d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*n^2 - 21*(d*sin(f*x + e) + c)^
3*(d*sin(f*x + e) + c)^n*c*n^2 + 24*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e)
+ c)^n*c^2*n^2 - 9*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n^2 + 11
*(d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*n - 42*(d*sin(f*x + e) + c)^
3*(d*sin(f*x + e) + c)^n*c*n + 57*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) +
c)^n*c^2*n - 26*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n + 6*(d*si
n(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n - 24*(d*sin(f*x + e) + c)^3*(d*sin
(f*x + e) + c)^n*c + 36*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2 -
 24*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3)*a^3/(d^3*n^4 + 10*d^3*
n^3 + 35*d^3*n^2 + 50*d^3*n + 24*d^3) + 3*((d*sin(f*x + e) + c)^3*(d*sin(f*
x + e) + c)^n*n^2 - 2*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c*n^2 +
 (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2*n^2 + 3*(d*sin(f*x + e) +
c)^3*(d*sin(f*x + e) + c)^n*n - 8*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) +
c)^n*c*n + 5*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2*n + 2*(d*sin(f
*x + e) + c)^3*(d*sin(f*x + e) + c)^n - 6*(d*sin(f*x + e) + c)^2*(d*sin(f*x
 + e) + c)^n*c + 6*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2)*a^3/(d^
2*n^3 + 6*d^2*n^2 + 11*d^2*n + 6*d^2) + (d*sin(f*x + e) + c)^(n + 1)*a^3/(n
 + 1) + 3*((d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*n - (d*sin(f*x + e
) + c)*(d*sin(f*x + e) + c)^n*c*n + (d*sin(f*x + e) + c)^2*(d*sin(f*x + e)
+ c)^n - 2*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c)*a^3/((n^2 + 3*n +
 2)*d))/(d*f)
```

maple [F] time = 4.44, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{3}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

maxima [B] time = 0.66, size = 313, normalized size = 2.25

$$\frac{\frac{3\left(d^2(n+1)\sin\left(fx+e\right)^2+cdn\sin\left(fx+e\right)-c^2\right)\left(d\sin\left(fx+e\right)+c\right)^n a^3}{\left(n^2+3n+2\right)d^2}+\frac{\left(d\sin\left(fx+e\right)+c\right)^{n+1}a^3}{d(n+1)}+\frac{3\left(\left(n^2+3n+2\right)d^3\sin\left(fx+e\right)^3+\left(n^2+n\right)cd^2\sin\left(fx+e\right)^2-\right.}{\left(n^3+6n^2+11n+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] (3*(d^2*(n + 1)*sin(f*x + e)^2 + c*d*n*sin(f*x + e) - c^2)*(d*sin(f*x + e)
+ c)^n*a^3/((n^2 + 3*n + 2)*d^2) + (d*sin(f*x + e) + c)^(n + 1)*a^3/(d*(n +
 1)) + 3*((n^2 + 3*n + 2)*d^3*sin(f*x + e)^3 + (n^2 + n)*c*d^2*sin(f*x + e)
^2 - 2*c^2*d*n*sin(f*x + e) + 2*c^3)*(d*sin(f*x + e) + c)^n*a^3/((n^3 + 6*n
^2 + 11*n + 6)*d^3) + ((n^3 + 6*n^2 + 11*n + 6)*d^4*sin(f*x + e)^4 + (n^3 +
 3*n^2 + 2*n)*c*d^3*sin(f*x + e)^3 - 3*(n^2 + n)*c^2*d^2*sin(f*x + e)^2 + 6
*c^3*d*n*sin(f*x + e) - 6*c^4)*(d*sin(f*x + e) + c)^n*a^3/((n^4 + 10*n^3 +
35*n^2 + 50*n + 24)*d^4))/f
```

mupad [B] time = 13.58, size = 617, normalized size = 4.44

$$\frac{a^3\left(c+d\,\sin\left(e+f\,x\right)\right)^n\,\left(192\,c\,d^3+192\,c^3\,d+261\,d^4\,n+336\,d^4\,\sin\left(e+f\,x\right)-48\,c^4+162\,d^4-288\,c^2\,d^2-168\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^n,x)
```

```
[Out] (a^3*(c + d*sin(e + f*x))^n*(192*c*d^3 + 192*c^3*d + 261*d^4*n + 336*d^4*si
n(e + f*x) - 48*c^4 + 162*d^4 - 288*c^2*d^2 - 168*d^4*cos(2*e + 2*f*x) + 6*
d^4*cos(4*e + 4*f*x) + 114*d^4*n^2 + 15*d^4*n^3 - 48*d^4*sin(3*e + 3*f*x) +
 460*d^4*n*sin(e + f*x) - 180*c^2*d^2*n + 132*c*d^3*n^2 + 20*c*d^3*n^3 - 27
2*d^4*n*cos(2*e + 2*f*x) + 11*d^4*n*cos(4*e + 4*f*x) - 84*d^4*n*sin(3*e + 3
*f*x) + 198*d^4*n^2*sin(e + f*x) + 26*d^4*n^3*sin(e + f*x) - 36*c^2*d^2*n^2
 - 120*d^4*n^2*cos(2*e + 2*f*x) - 16*d^4*n^3*cos(2*e + 2*f*x) + 6*d^4*n^2*c
os(4*e + 4*f*x) + d^4*n^3*cos(4*e + 4*f*x) - 42*d^4*n^2*sin(3*e + 3*f*x) -
6*d^4*n^3*sin(3*e + 3*f*x) + 256*c*d^3*n + 48*c^3*d*n + 12*c^2*d^2*n*cos(2*
e + 2*f*x) - 60*c*d^3*n^2*cos(2*e + 2*f*x) - 12*c*d^3*n^3*cos(2*e + 2*f*x)
- 6*c*d^3*n^2*sin(3*e + 3*f*x) - 2*c*d^3*n^3*sin(3*e + 3*f*x) - 48*c^2*d^2*
n^2*sin(e + f*x) + 300*c*d^3*n*sin(e + f*x) + 48*c^3*d*n*sin(e + f*x) + 12*
c^2*d^2*n^2*cos(2*e + 2*f*x) - 48*c*d^3*n*cos(2*e + 2*f*x) - 4*c*d^3*n*sin(
```

```
3*e + 3*f*x) - 192*c^2*d^2*n*sin(e + f*x) + 186*c*d^3*n^2*sin(e + f*x) + 30
*c*d^3*n^3*sin(e + f*x)))/(8*d^4*f*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))
```

sympy [A] time = 54.77, size = 5596, normalized size = 40.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Piecewise((c**n*(a**3*sin(e + f*x)**4/(4*f) + a**3*sin(e + f*x)**3/f + 3*a*
*3*sin(e + f*x)**2/(2*f) + a**3*sin(e + f*x)/f), Eq(d, 0)), (x*(c + d*sin(e
))**n*(a*sin(e) + a)**3*cos(e), Eq(f, 0)), (6*a**3*c**3*log(c/d + sin(e + f
*x))/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e + f*x) + 18*c*d**6*f*sin(e + f*x
)**2 + 6*d**7*f*sin(e + f*x)**3) + 11*a**3*c**3/(6*c**3*d**4*f + 18*c**2*d*
*5*f*sin(e + f*x) + 18*c*d**6*f*sin(e + f*x)**2 + 6*d**7*f*sin(e + f*x)**3)
 + 18*a**3*c**2*d*log(c/d + sin(e + f*x))*sin(e + f*x)/(6*c**3*d**4*f + 18*
c**2*d**5*f*sin(e + f*x) + 18*c*d**6*f*sin(e + f*x)**2 + 6*d**7*f*sin(e + f
*x)**3) + 27*a**3*c**2*d*sin(e + f*x)/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e
 + f*x) + 18*c*d**6*f*sin(e + f*x)**2 + 6*d**7*f*sin(e + f*x)**3) - 6*a**3*
c**2*d/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e + f*x) + 18*c*d**6*f*sin(e + f
*x)**2 + 6*d**7*f*sin(e + f*x)**3) + 18*a**3*c*d**2*log(c/d + sin(e + f*x))
*sin(e + f*x)**2/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e + f*x) + 18*c*d**6*f
*sin(e + f*x)**2 + 6*d**7*f*sin(e + f*x)**3) + 18*a**3*c*d**2*sin(e + f*x)*
*2/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e + f*x) + 18*c*d**6*f*sin(e + f*x)*
*2 + 6*d**7*f*sin(e + f*x)**3) - 18*a**3*c*d**2*sin(e + f*x)/(6*c**3*d**4*f
 + 18*c**2*d**5*f*sin(e + f*x) + 18*c*d**6*f*sin(e + f*x)**2 + 6*d**7*f*sin
(e + f*x)**3) - 3*a**3*c*d**2/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e + f*x)
+ 18*c*d**6*f*sin(e + f*x)**2 + 6*d**7*f*sin(e + f*x)**3) + 6*a**3*d**3*log
(c/d + sin(e + f*x))*sin(e + f*x)**3/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e
+ f*x) + 18*c*d**6*f*sin(e + f*x)**2 + 6*d**7*f*sin(e + f*x)**3) - 18*a**3*
d**3*sin(e + f*x)**2/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e + f*x) + 18*c*d*
*6*f*sin(e + f*x)**2 + 6*d**7*f*sin(e + f*x)**3) - 9*a**3*d**3*sin(e + f*x)
/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e + f*x) + 18*c*d**6*f*sin(e + f*x)**2
 + 6*d**7*f*sin(e + f*x)**3) - 2*a**3*d**3/(6*c**3*d**4*f + 18*c**2*d**5*f*
sin(e + f*x) + 18*c*d**6*f*sin(e + f*x)**2 + 6*d**7*f*sin(e + f*x)**3), Eq(
n, -4)), (-6*a**3*c**3*log(c/d + sin(e + f*x))/(2*c**2*d**4*f + 4*c*d**5*f*
sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) - 9*a**3*c**3/(2*c**2*d**4*f + 4*c
*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) - 12*a**3*c**2*d*log(c/d +
 sin(e + f*x))*sin(e + f*x)/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d*
*6*f*sin(e + f*x)**2) + 6*a**3*c**2*d*log(c/d + sin(e + f*x))/(2*c**2*d**4*
f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) - 12*a**3*c**2*d*si
n(e + f*x)/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)
**2) + 9*a**3*c**2*d/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*si
n(e + f*x)**2) - 6*a**3*c*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)**2/(2*c
```

```
**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) + 12*a**3*
c*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)/(2*c**2*d**4*f + 4*c*d**5*f*sin
(e + f*x) + 2*d**6*f*sin(e + f*x)**2) + 12*a**3*c*d**2*sin(e + f*x)/(2*c**2
*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) - 3*a**3*c*d*
*2/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) + 6
*a**3*d**3*log(c/d + sin(e + f*x))*sin(e + f*x)**2/(2*c**2*d**4*f + 4*c*d**
5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) + 2*a**3*d**3*sin(e + f*x)**3/
(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) - 6*a*
*3*d**3*sin(e + f*x)/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*si
n(e + f*x)**2) - a**3*d**3/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**
6*f*sin(e + f*x)**2), Eq(n, -3)), (6*a**3*c**3*log(c/d + sin(e + f*x))/(2*c
*d**4*f + 2*d**5*f*sin(e + f*x)) + 6*a**3*c**3/(2*c*d**4*f + 2*d**5*f*sin(e
 + f*x)) + 6*a**3*c**2*d*log(c/d + sin(e + f*x))*sin(e + f*x)/(2*c*d**4*f +
 2*d**5*f*sin(e + f*x)) - 12*a**3*c**2*d*log(c/d + sin(e + f*x))/(2*c*d**4*
f + 2*d**5*f*sin(e + f*x)) - 12*a**3*c**2*d/(2*c*d**4*f + 2*d**5*f*sin(e +
f*x)) - 12*a**3*c*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)/(2*c*d**4*f + 2
*d**5*f*sin(e + f*x)) + 6*a**3*c*d**2*log(c/d + sin(e + f*x))/(2*c*d**4*f +
 2*d**5*f*sin(e + f*x)) - 3*a**3*c*d**2*sin(e + f*x)**2/(2*c*d**4*f + 2*d**
5*f*sin(e + f*x)) + 6*a**3*c*d**2/(2*c*d**4*f + 2*d**5*f*sin(e + f*x)) + 6*
a**3*d**3*log(c/d + sin(e + f*x))*sin(e + f*x)/(2*c*d**4*f + 2*d**5*f*sin(e
 + f*x)) + a**3*d**3*sin(e + f*x)**3/(2*c*d**4*f + 2*d**5*f*sin(e + f*x)) +
 6*a**3*d**3*sin(e + f*x)**2/(2*c*d**4*f + 2*d**5*f*sin(e + f*x)) - 2*a**3*
d**3/(2*c*d**4*f + 2*d**5*f*sin(e + f*x)), Eq(n, -2)), (-a**3*c**3*log(c/d
+ sin(e + f*x))/(d**4*f) + 3*a**3*c**2*log(c/d + sin(e + f*x))/(d**3*f) + a
**3*c**2*sin(e + f*x)/(d**3*f) - 3*a**3*c*log(c/d + sin(e + f*x))/(d**2*f)
- a**3*c*sin(e + f*x)**2/(2*d**2*f) - 3*a**3*c*sin(e + f*x)/(d**2*f) + a**3
*log(c/d + sin(e + f*x))/(d*f) + a**3*sin(e + f*x)**3/(3*d*f) + 3*a**3*sin(
e + f*x)**2/(2*d*f) + 3*a**3*sin(e + f*x)/(d*f), Eq(n, -1)), (-6*a**3*c**4*
(c + d*sin(e + f*x))**n/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50
*d**4*f*n + 24*d**4*f) + 6*a**3*c**3*d*n*(c + d*sin(e + f*x))**n*sin(e + f*
x)/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f
) + 6*a**3*c**3*d*n*(c + d*sin(e + f*x))**n/(d**4*f*n**4 + 10*d**4*f*n**3 +
 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 24*a**3*c**3*d*(c + d*sin(e +
f*x))**n/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*
d**4*f) - 3*a**3*c**2*d**2*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d*
*4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) - 6*
a**3*c**2*d**2*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**4*f*n**4 + 10*
d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) - 3*a**3*c**2*d**2*
n**2*(c + d*sin(e + f*x))**n/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2
 + 50*d**4*f*n + 24*d**4*f) - 3*a**3*c**2*d**2*n*(c + d*sin(e + f*x))**n*si
n(e + f*x)**2/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n
+ 24*d**4*f) - 24*a**3*c**2*d**2*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d*
*4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) - 21
*a**3*c**2*d**2*n*(c + d*sin(e + f*x))**n/(d**4*f*n**4 + 10*d**4*f*n**3 + 3
5*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) - 36*a**3*c**2*d**2*(c + d*sin(e +
```

```
f*x))**n/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24
*d**4*f) + a**3*c*d**3*n**3*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**4*f
*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 3*a**3
*c*d**3*n**3*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**4*f*n**4 + 10*d**4
*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 3*a**3*c*d**3*n**3*(c
 + d*sin(e + f*x))**n*sin(e + f*x)/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*
f*n**2 + 50*d**4*f*n + 24*d**4*f) + a**3*c*d**3*n**3*(c + d*sin(e + f*x))**
n/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f)
 + 3*a**3*c*d**3*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**4*f*n**4
+ 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 15*a**3*c*d*
*3*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**4*f*n**4 + 10*d**4*f*n*
*3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 21*a**3*c*d**3*n**2*(c + d
*sin(e + f*x))**n*sin(e + f*x)/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n*
*2 + 50*d**4*f*n + 24*d**4*f) + 9*a**3*c*d**3*n**2*(c + d*sin(e + f*x))**n/
(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) +
 2*a**3*c*d**3*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**4*f*n**4 + 10*
d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 12*a**3*c*d**3*n*
(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*
d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 36*a**3*c*d**3*n*(c + d*sin(e + f*
x))**n*sin(e + f*x)/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**
4*f*n + 24*d**4*f) + 26*a**3*c*d**3*n*(c + d*sin(e + f*x))**n/(d**4*f*n**4
+ 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 24*a**3*c*d*
*3*(c + d*sin(e + f*x))**n/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 +
 50*d**4*f*n + 24*d**4*f) + a**3*d**4*n**3*(c + d*sin(e + f*x))**n*sin(e +
f*x)**4/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d
**4*f) + 3*a**3*d**4*n**3*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**4*f*n
**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 3*a**3*d
**4*n**3*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**4*f*n**4 + 10*d**4*f*n
**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + a**3*d**4*n**3*(c + d*sin
(e + f*x))**n*sin(e + f*x)/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 +
 50*d**4*f*n + 24*d**4*f) + 6*a**3*d**4*n**2*(c + d*sin(e + f*x))**n*sin(e
+ f*x)**4/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24
*d**4*f) + 21*a**3*d**4*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**4*
f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 24*a*
*3*d**4*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**4*f*n**4 + 10*d**4
*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 9*a**3*d**4*n**2*(c +
 d*sin(e + f*x))**n*sin(e + f*x)/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*
n**2 + 50*d**4*f*n + 24*d**4*f) + 11*a**3*d**4*n*(c + d*sin(e + f*x))**n*si
n(e + f*x)**4/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n
+ 24*d**4*f) + 42*a**3*d**4*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**4
*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 57*a
**3*d**4*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**4*f*n**4 + 10*d**4*f
*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 26*a**3*d**4*n*(c + d*s
in(e + f*x))**n*sin(e + f*x)/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2
 + 50*d**4*f*n + 24*d**4*f) + 6*a**3*d**4*(c + d*sin(e + f*x))**n*sin(e + f
```

```
*x)**4/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d*
*4*f) + 24*a**3*d**4*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**4*f*n**4 +
 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 36*a**3*d**4*
(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*
d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 24*a**3*d**4*(c + d*sin(e + f*x))*
*n*sin(e + f*x)/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*
n + 24*d**4*f), True))
```

3.915 $\int \cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=101

$$\frac{a^2(c-d)^2(c+d\sin(e+fx))^{n+1}}{d^3f(n+1)}-\frac{2a^2(c-d)(c+d\sin(e+fx))^{n+2}}{d^3f(n+2)}+\frac{a^2(c+d\sin(e+fx))^{n+3}}{d^3f(n+3)}$$

```
[Out] a^2*(c-d)^2*(c+d*sin(f*x+e))^(1+n)/d^3/f/(1+n)-2*a^2*(c-d)*(c+d*sin(f*x+e))
^(2+n)/d^3/f/(2+n)+a^2*(c+d*sin(f*x+e))^(3+n)/d^3/f/(3+n)
```

Rubi [A] time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 43}

$$\frac{a^2(c-d)^2(c+d\sin(e+fx))^{n+1}}{d^3f(n+1)}-\frac{2a^2(c-d)(c+d\sin(e+fx))^{n+2}}{d^3f(n+2)}+\frac{a^2(c+d\sin(e+fx))^{n+3}}{d^3f(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^2*(c - d)^2*(c + d*Sin[e + f*x])^(1 + n))/(d^3*f*(1 + n)) - (2*a^2*(c -
d)*(c + d*Sin[e + f*x])^(2 + n))/(d^3*f*(2 + n)) + (a^2*(c + d*Sin[e + f*x]
)^(3 + n))/(d^3*f*(3 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(c+\frac{dx}{a}\right)^n\,dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2(c-d)^2\left(c+\frac{dx}{a}\right)^n}{d^2} - \frac{2a^2(c-d)\left(c+\frac{dx}{a}\right)^{1+n}}{d^2} + \frac{a^2\left(c+\frac{dx}{a}\right.}{d^2}\right.\right.}{af}$$

$$= \frac{a^2(c-d)^2(c+d\sin(e+fx))^{1+n}}{d^3f(1+n)} - \frac{2a^2(c-d)(c+d\,\mathrm{s}}{d^3f(2}$$

Mathematica [A] time = 0.36, size = 78, normalized size = 0.77

$$\frac{a^2(c+d\sin(e+fx))^{n+1}\left(-\frac{2(c-d)(c+d\sin(e+fx))}{n+2}+\frac{(c+d\sin(e+fx))^2}{n+3}+\frac{(c-d)^2}{n+1}\right)}{d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^2*(c + d*Sin[e + f*x])^(1 + n)*((c - d)^2/(1 + n) - (2*(c - d)*(c + d*Si
n[e + f*x]))/(2 + n) + (c + d*Sin[e + f*x])^2/(3 + n)))/(d^3*f)
```

fricas [B] time = 0.51, size = 294, normalized size = 2.91

$$\frac{\left(2\,a^2c^3 - 6\,a^2c^2d + 6\,a^2cd^2 + 6\,a^2d^3 + 2\left(a^2cd^2 + a^2d^3\right)n^2 - \left(6\,a^2d^3 + \left(a^2cd^2 + 2\,a^2d^3\right)n^2 + \left(a^2cd^2 + 8\,a^2d^3\right)n\right)\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] (2*a^2*c^3 - 6*a^2*c^2*d + 6*a^2*c*d^2 + 6*a^2*d^3 + 2*(a^2*c*d^2 + a^2*d^3
)*n^2 - (6*a^2*d^3 + (a^2*c*d^2 + 2*a^2*d^3)*n^2 + (a^2*c*d^2 + 8*a^2*d^3)*
n)*cos(f*x + e)^2 - 2*(a^2*c^2*d - 3*a^2*c*d^2 - 4*a^2*d^3)*n + (8*a^2*d^3
+ 2*(a^2*c*d^2 + a^2*d^3)*n^2 - (a^2*d^3*n^2 + 3*a^2*d^3*n + 2*a^2*d^3)*cos
(f*x + e)^2 - 2*(a^2*c^2*d - 3*a^2*c*d^2 - 4*a^2*d^3)*n)*sin(f*x + e))*(d*s
in(f*x + e) + c)^n/(d^3*f*n^3 + 6*d^3*f*n^2 + 11*d^3*f*n + 6*d^3*f)
```

giac [B] time = 0.19, size = 463, normalized size = 4.58

$$\frac{\left(\left(d\sin\left(fx+e\right)+c\right)^{3}\left(d\sin\left(fx+e\right)+c\right)^{n}n^{2}-2\left(d\sin\left(fx+e\right)+c\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{n}cn^{2}+\left(d\sin\left(fx+e\right)+c\right)\left(d\sin\left(fx+e\right)+c\right)^{n}c^{2}n^{2}+3\left(d\sin\left(fx+e\right)+c\right)^{3}\left(d\sin\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] (((d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*n^2 - 2*(d*sin(f*x + e) + c
)^2*(d*sin(f*x + e) + c)^n*c*n^2 + (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c
)^n*c^2*n^2 + 3*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*n - 8*(d*sin(
f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c*n + 5*(d*sin(f*x + e) + c)*(d*sin(
f*x + e) + c)^n*c^2*n + 2*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n - 6
*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c + 6*(d*sin(f*x + e) + c)*(
d*sin(f*x + e) + c)^n*c^2)*a^2/(d^2*n^3 + 6*d^2*n^2 + 11*d^2*n + 6*d^2) + (
d*sin(f*x + e) + c)^(n + 1)*a^2/(n + 1) + 2*((d*sin(f*x + e) + c)^2*(d*sin(
f*x + e) + c)^n*n - (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c*n + (d*si
n(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n - 2*(d*sin(f*x + e) + c)*(d*sin(f*
x + e) + c)^n*c)*a^2/((n^2 + 3*n + 2)*d))/(d*f)
```

maple [F] time = 3.25, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{2}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

maxima [A] time = 0.71, size = 183, normalized size = 1.81

$$\frac{\frac{2\left(d^{2}(n+1)\sin\left(fx+e\right)^{2}+cdn\sin\left(fx+e\right)-c^{2}\right)\left(d\sin\left(fx+e\right)+c\right)^{n}a^{2}}{\left(n^{2}+3n+2\right)d^{2}}+\frac{\left(d\sin\left(fx+e\right)+c\right)^{n+1}a^{2}}{d(n+1)}+\frac{\left(\left(n^{2}+3n+2\right)d^{3}\sin\left(fx+e\right)^{3}+\left(n^{2}+n\right)cd^{2}\sin\left(fx+e\right)^{2}-2c^{2}\right.}{\left(n^{3}+6n^{2}+11n+6\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] (2*(d^2*(n + 1)*sin(f*x + e)^2 + c*d*n*sin(f*x + e) - c^2)*(d*sin(f*x + e)
+ c)^n*a^2/((n^2 + 3*n + 2)*d^2) + (d*sin(f*x + e) + c)^(n + 1)*a^2/(d*(n +
```

1)) + ((n^2 + 3*n + 2)*d^3*sin(f*x + e)^3 + (n^2 + n)*c*d^2*sin(f*x + e)^2 - 2*c^2*d*n*sin(f*x + e) + 2*c^3)*(d*sin(f*x + e) + c)^n*a^2/((n^3 + 6*n^2 + 11*n + 6)*d^3))/f

mupad [B] time = 11.26, size = 302, normalized size = 2.99

$$\frac{a^2\left(c+d\,\sin\left(e+f\,x\right)\right)^n\,\left(24\,c\,d^2-24\,c^2\,d+16\,d^3\,n+30\,d^3\,\sin\left(e+f\,x\right)+8\,c^3+12\,d^3-12\,d^3\,\cos\left(2\,e+2\,f\,x\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)

[Out] (a^2*(c + d*sin(e + f*x))^n*(24*c*d^2 - 24*c^2*d + 16*d^3*n + 30*d^3*sin(e + f*x) + 8*c^3 + 12*d^3 - 12*d^3*cos(2*e + 2*f*x) + 4*d^3*n^2 - 2*d^3*sin(3*e + 3*f*x) + 29*d^3*n*sin(e + f*x) + 6*c*d^2*n^2 - 16*d^3*n*cos(2*e + 2*f*x) - 3*d^3*n*sin(3*e + 3*f*x) + 7*d^3*n^2*sin(e + f*x) - 4*d^3*n^2*cos(2*e + 2*f*x) - d^3*n^2*sin(3*e + 3*f*x) + 22*c*d^2*n - 8*c^2*d*n - 2*c*d^2*n^2*cos(2*e + 2*f*x) + 24*c*d^2*n*sin(e + f*x) - 8*c^2*d*n*sin(e + f*x) - 2*c*d^2*n*cos(2*e + 2*f*x) + 8*c*d^2*n^2*sin(e + f*x)))/(4*d^3*f*(11*n + 6*n^2 + n^3 + 6))

sympy [A] time = 20.59, size = 2159, normalized size = 21.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)

[Out] Piecewise((c**n*(a**2*sin(e + f*x)**3/(3*f) + a**2*sin(e + f*x)**2/f + a**2*sin(e + f*x)/f), Eq(d, 0)), (x*(c + d*sin(e))**n*(a*sin(e) + a)**2*cos(e), Eq(f, 0)), (2*a**2*c**2*log(c/d + sin(e + f*x))/(2*c**2*d**3*f + 4*c*d**4*f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) + 3*a**2*c**2/(2*c**2*d**3*f + 4*c*d**4*f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) + 4*a**2*c*d*log(c/d + sin(e + f*x))*sin(e + f*x)/(2*c**2*d**3*f + 4*c*d**4*f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) + 4*a**2*c*d*sin(e + f*x)/(2*c**2*d**3*f + 4*c*d**4*f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) - 2*a**2*c*d/(2*c**2*d**3*f + 4*c*d**4*f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) + 2*a**2*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)**2/(2*c**2*d**3*f + 4*c*d**4*f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) - 4*a**2*d**2*sin(e + f*x)/(2*c**2*d**3*f + 4*c*d**4*f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) - a**2*d**2/(2*c**2*d**3*f + 4*c*d**4*f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2), Eq(n, -3)), (-2*a**2*c**2*log(c/d + sin(e + f*x))/(c*d**3*f + d**4*f*sin(e + f*x)) - 2*a**2*c**2/(c*d**3*f + d**4*f*sin(e + f*x)) - 2*a**2*c*d*log(c/d + sin(e + f*x))*sin(e + f*x)/(c*d**3*f + d**4*f*sin(e + f*x)) + 2*a**2*c*d*log(c/d + sin(e + f*x))/(c

```
*d**3*f + d**4*f*sin(e + f*x)) + 2*a**2*c*d/(c*d**3*f + d**4*f*sin(e + f*x)
) + 2*a**2*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)/(c*d**3*f + d**4*f*sin
(e + f*x)) + a**2*d**2*sin(e + f*x)**2/(c*d**3*f + d**4*f*sin(e + f*x)) - a
**2*d**2/(c*d**3*f + d**4*f*sin(e + f*x)), Eq(n, -2)), (a**2*c**2*log(c/d +
 sin(e + f*x))/(d**3*f) - 2*a**2*c*log(c/d + sin(e + f*x))/(d**2*f) - a**2*
c*sin(e + f*x)/(d**2*f) + a**2*log(c/d + sin(e + f*x))/(d*f) + a**2*sin(e +
 f*x)**2/(2*d*f) + 2*a**2*sin(e + f*x)/(d*f), Eq(n, -1)), (2*a**2*c**3*(c +
 d*sin(e + f*x))**n/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f)
- 2*a**2*c**2*d*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**3*f*n**3 + 6*d**
3*f*n**2 + 11*d**3*f*n + 6*d**3*f) - 2*a**2*c**2*d*n*(c + d*sin(e + f*x))**
n/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) - 6*a**2*c**2*d*(c
 + d*sin(e + f*x))**n/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f
) + a**2*c*d**2*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**3*f*n**3 +
 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 2*a**2*c*d**2*n**2*(c + d*sin(e
+ f*x))**n*sin(e + f*x)/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3
*f) + a**2*c*d**2*n**2*(c + d*sin(e + f*x))**n/(d**3*f*n**3 + 6*d**3*f*n**2
 + 11*d**3*f*n + 6*d**3*f) + a**2*c*d**2*n*(c + d*sin(e + f*x))**n*sin(e +
f*x)**2/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 6*a**2*c*d
**2*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**3*f*n**3 + 6*d**3*f*n**2 + 1
1*d**3*f*n + 6*d**3*f) + 5*a**2*c*d**2*n*(c + d*sin(e + f*x))**n/(d**3*f*n*
*3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 6*a**2*c*d**2*(c + d*sin(e +
 f*x))**n/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + a**2*d**
3*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**3*f*n**3 + 6*d**3*f*n**2
 + 11*d**3*f*n + 6*d**3*f) + 2*a**2*d**3*n**2*(c + d*sin(e + f*x))**n*sin(e
 + f*x)**2/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + a**2*d*
*3*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**3*f*n**3 + 6*d**3*f*n**2 +
 11*d**3*f*n + 6*d**3*f) + 3*a**2*d**3*n*(c + d*sin(e + f*x))**n*sin(e + f*
x)**3/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 8*a**2*d**3*
n*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**3*f*n**3 + 6*d**3*f*n**2 + 11
*d**3*f*n + 6*d**3*f) + 5*a**2*d**3*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/
(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 2*a**2*d**3*(c + d
*sin(e + f*x))**n*sin(e + f*x)**3/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*
n + 6*d**3*f) + 6*a**2*d**3*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**3*f
*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 6*a**2*d**3*(c + d*sin(e
+ f*x))**n*sin(e + f*x)/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3
*f), True))
```

3.916 $\int \cos(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=61

$$\frac{a(c+d\sin(e+fx))^{n+2}}{d^2f(n+2)}-\frac{a(c-d)(c+d\sin(e+fx))^{n+1}}{d^2f(n+1)}$$

```
[Out] -a*(c-d)*(c+d*sin(f*x+e))^(1+n)/d^2/f/(1+n)+a*(c+d*sin(f*x+e))^(2+n)/d^2/f/
(2+n)
```

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{a(c+d\sin(e+fx))^{n+2}}{d^2f(n+2)}-\frac{a(c-d)(c+d\sin(e+fx))^{n+1}}{d^2f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -((a*(c - d)*(c + d*Sin[e + f*x])^(1 + n))/(d^2*f*(1 + n))) + (a*(c + d*Sin
[e + f*x])^(2 + n))/(d^2*f*(2 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{\text{Subst}\left(\int (a+x)\left(c+\frac{dx}{a}\right)^n\,dx, x, a\sin(e+fx)\right)}{af}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a(c-d)\left(c+\frac{dx}{a}\right)^n}{d}+\frac{a\left(c+\frac{dx}{a}\right)^{1+n}}{d}\right)dx, x, a\sin(e+
$$
$$= -\frac{a(c-d)(c+d\sin(e+fx))^{1+n}}{d^2f(1+n)}+\frac{a(c+d\sin(e+fx))^{
}}{d^2f(2+n)}$$

Mathematica [A] time = 0.50, size = 52, normalized size = 0.85

$$\frac{a(c+d\sin(e+fx))^{n+1}(-c+d(n+1)\sin(e+fx)+d(n+2))}{d^2f(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a*(c + d*Sin[e + f*x])^(1 + n)*(-c + d*(2 + n) + d*(1 + n)*Sin[e + f*x]))/
(d^2*f*(1 + n)*(2 + n))
```

fricas [A] time = 0.49, size = 116, normalized size = 1.90

$$-\frac{\left(ac^2-2\,acd-ad^2+\left(ad^2n+ad^2\right)\cos\left(fx+e\right)^2-\left(acd+ad^2\right)n-\left(2\,ad^2+\left(acd+ad^2\right)n\right)\sin\left(fx+e\right)\right)\left(d\sin\left(f
\right.\right.}{d^2fn^2+3\,d^2fn+2\,d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fric
as")
```

```
[Out] -(a*c^2 - 2*a*c*d - a*d^2 + (a*d^2*n + a*d^2)*cos(f*x + e)^2 - (a*c*d + a*d
^2)*n - (2*a*d^2 + (a*c*d + a*d^2)*n)*sin(f*x + e))*(d*sin(f*x + e) + c)^n/
(d^2*f*n^2 + 3*d^2*f*n + 2*d^2*f)
```

giac [B] time = 0.16, size = 156, normalized size = 2.56

$$\frac{\frac{\left(d\sin\left(fx+e\right)+c\right)^{n+1}a}{n+1}+\frac{\left(\left(d\sin\left(fx+e\right)+c\right)^2\left(d\sin\left(fx+e\right)+c\right)^n n-\left(d\sin\left(fx+e\right)+c\right)\left(d\sin\left(fx+e\right)+c\right)^n cn+\left(d\sin\left(fx+e\right)+c\right)^2\left(d\sin\left(fx+e\right)+c\right)^n-2\left(d\sin\left(fx
\right.\right.}{\left(n^2+3\,n+2\right)d}}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="giac
")
```

```
[Out] ((d*sin(f*x + e) + c)^(n + 1)*a/(n + 1) + ((d*sin(f*x + e) + c)^2*(d*sin(f*
x + e) + c)^n*n - (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c*n + (d*sin(
f*x + e) + c)^2*(d*sin(f*x + e) + c)^n - 2*(d*sin(f*x + e) + c)*(d*sin(f*x
+ e) + c)^n*c)*a/((n^2 + 3*n + 2)*d))/(d*f)
```

maple [F] time = 1.16, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

maxima [A] time = 0.69, size = 87, normalized size = 1.43

$$\frac{\frac{\left(d^{2}(n+1)\sin\left(fx+e\right)^{2}+cdn\sin\left(fx+e\right)-c^{2}\right)\left(d\sin\left(fx+e\right)+c\right)^{n}a}{\left(n^{2}+3\,n+2\right)d^{2}}+\frac{\left(d\sin\left(fx+e\right)+c\right)^{n+1}a}{d(n+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="maxi
ma")
```

```
[Out] ((d^2*(n + 1)*sin(f*x + e)^2 + c*d*n*sin(f*x + e) - c^2)*(d*sin(f*x + e) +
c)^n*a/((n^2 + 3*n + 2)*d^2) + (d*sin(f*x + e) + c)^(n + 1)*a/(d*(n + 1)))/
f
```

mupad [B] time = 10.08, size = 121, normalized size = 1.98

$$\frac{a\left(c+d\,\sin\left(e+f\,x\right)\right)^{n}\,\left(4\,c\,d+d^{2}\,n+4\,d^{2}\,\sin\left(e+f\,x\right)+d^{2}\,\left(2\sin\left(e+f\,x\right)^{2}-1\right)-2\,c^{2}+d^{2}+2\,d^{2}\,n\,\sin\left(e+\right.\right.}{2\,d^{2}\,f\,\left(n^{2}+3\,n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
```

```
[Out] (a*(c + d*sin(e + f*x))^n*(4*c*d + d^2*n + 4*d^2*sin(e + f*x) + d^2*(2*sin(
e + f*x)^2 - 1) - 2*c^2 + d^2 + 2*d^2*n*sin(e + f*x) + d^2*n*(2*sin(e + f*x
)^2 - 1) + 2*c*d*n + 2*c*d*n*sin(e + f*x)))/(2*d^2*f*(3*n + n^2 + 2))
```

sympy [A] time = 7.13, size = 586, normalized size = 9.61

$$\begin{cases} c^n \left(\frac{a\sin^2(e+fx)}{2f} + \frac{a\sin(e+fx)}{f}\right) \\ x(c+d\sin(e))^n (a\sin(e)+a)\cos(e) \\ \frac{ac\log\left(\frac{c}{d}+\sin(e+fx)\right)}{cd^2f+d^3f\sin(e+fx)} + \frac{ac}{cd^2f+d^3f\sin(e+fx)} + \frac{ad\log\left(\frac{c}{d}+\sin(e+fx)\right)\sin(e+fx)}{cd^2f+d^3f\sin(e+fx)} - \frac{ad}{cd^2f+d^3f\sin(e+fx)} \\ -\frac{ac\log\left(\frac{c}{d}+\sin(e+fx)\right)}{d^2f} + \frac{a\log\left(\frac{c}{d}+\sin(e+fx)\right)}{df} + \frac{a\sin(e+fx)}{df} \\ -\frac{ac^2(c+d\sin(e+fx))^n}{d^2fn^2+3d^2fn+2d^2f} + \frac{acdn(c+d\sin(e+fx))^n\sin(e+fx)}{d^2fn^2+3d^2fn+2d^2f} + \frac{acdn(c+d\sin(e+fx))^n}{d^2fn^2+3d^2fn+2d^2f} + \frac{2acd(c+d\sin(e+fx))^n}{d^2fn^2+3d^2fn+2d^2f} + \frac{ad^2n(c+d\sin(e+fx))^n\sin^2(}{d^2fn^2+3d^2fn+2d^2f} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Piecewise((c**n*(a*sin(e + f*x)**2/(2*f) + a*sin(e + f*x)/f), Eq(d, 0)), (x
*(c + d*sin(e))**n*(a*sin(e) + a)*cos(e), Eq(f, 0)), (a*c*log(c/d + sin(e +
 f*x))/(c*d**2*f + d**3*f*sin(e + f*x)) + a*c/(c*d**2*f + d**3*f*sin(e + f*
x)) + a*d*log(c/d + sin(e + f*x))*sin(e + f*x)/(c*d**2*f + d**3*f*sin(e + f
*x)) - a*d/(c*d**2*f + d**3*f*sin(e + f*x)), Eq(n, -2)), (-a*c*log(c/d + si
n(e + f*x))/(d**2*f) + a*log(c/d + sin(e + f*x))/(d*f) + a*sin(e + f*x)/(d*
f), Eq(n, -1)), (-a*c**2*(c + d*sin(e + f*x))**n/(d**2*f*n**2 + 3*d**2*f*n
+ 2*d**2*f) + a*c*d*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**2*f*n**2 + 3
*d**2*f*n + 2*d**2*f) + a*c*d*n*(c + d*sin(e + f*x))**n/(d**2*f*n**2 + 3*d*
*2*f*n + 2*d**2*f) + 2*a*c*d*(c + d*sin(e + f*x))**n/(d**2*f*n**2 + 3*d**2*
f*n + 2*d**2*f) + a*d**2*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**2*f*
n**2 + 3*d**2*f*n + 2*d**2*f) + a*d**2*n*(c + d*sin(e + f*x))**n*sin(e + f*
x)/(d**2*f*n**2 + 3*d**2*f*n + 2*d**2*f) + a*d**2*(c + d*sin(e + f*x))**n*s
in(e + f*x)**2/(d**2*f*n**2 + 3*d**2*f*n + 2*d**2*f) + 2*a*d**2*(c + d*sin(
e + f*x))**n*sin(e + f*x)/(d**2*f*n**2 + 3*d**2*f*n + 2*d**2*f), True))
```

3.917 $\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{af(n+1)(c-d)}$$

```
[Out] -hypergeom([1, 1+n],[2+n],(c+d*sin(f*x+e))/(c-d))*(c+d*sin(f*x+e))^(1+n)/a/
(c-d)/f/(1+n)
```

Rubi [A] time = 0.12, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 68}

$$-\frac{(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{af(n+1)(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c + d*
Sin[e + f*x])^(1 + n))/(a*(c - d)*f*(1 + n)))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(c+\frac{dx}{a}\right)^n}{a+x}\,dx, x, a\sin(e+fx)\right)}{af}$$
$$= -\frac{{}_2F_1\left(1,1+n;2+n;\frac{c+d\sin(e+fx)}{c-d}\right)(c+d\sin(e+fx))^{1+n}}{a(c-d)f(1+n)}$$

Mathematica [**A**] time = 0.07, size = 60, normalized size = 1.00

$$-\frac{(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{af(n+1)(c-d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c + d*
Sin[e + f*x])^(1 + n))/(a*(c - d)*f*(1 + n)))
```

fricas [**F**] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)}{a\sin\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a), x)
```

maple [F] time = 2.42, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
[Out] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="maxi
ma")
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{n}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)),x)
[Out] int((cos(e + f*x)*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
[Out] Timed out
```

3.918 $\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=60

$$\frac{d(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{a^2f(n+1)(c-d)^2}$$

```
[Out] d*hypergeom([2, 1+n],[2+n],(c+d*sin(f*x+e))/(c-d))*(c+d*sin(f*x+e))^(1+n)/a
^2/(c-d)^2/f/(1+n)
```

Rubi [A] time = 0.11, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 68}

$$\frac{d(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{a^2f(n+1)(c-d)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d*Hypergeometric2F1[2, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c + d*
Sin[e + f*x])^(1 + n))/(a^2*(c - d)^2*f*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(c+\frac{dx}{a}\right)^n}{(a+x)^2}\,dx, x, a\sin(e+fx)\right)}{af}$$
$$= \frac{d\,{}_2F_1\left(2,1+n;2+n;\frac{c+d\sin(e+fx)}{c-d}\right)(c+d\sin(e+fx))^{1+n}}{a^2(c-d)^2f(1+n)}$$

Mathematica [A] time = 0.07, size = 61, normalized size = 1.02

$$\frac{d(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;-\frac{c+d\sin(e+fx)}{d-c}\right)}{a^2f(n+1)(d-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d*Hypergeometric2F1[2, 1 + n, 2 + n, -((c + d*Sin[e + f*x])/(-c + d))]*(c
+ d*Sin[e + f*x])^(1 + n))/(a^2*(-c + d)^2*f*(1 + n))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)/(a^2*cos(f*x + e)^2 - 2*a^2*s
in(f*x + e) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)}{\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a)^2, x)
```

maple [F] time = 3.90, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{n}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((cos(e + f*x)*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.919 $\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=63

$$-\frac{d^2(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{a^3f(n+1)(c-d)^3}$$

```
[Out] -d^2*hypergeom([3, 1+n],[2+n],(c+d*sin(f*x+e))/(c-d))*(c+d*sin(f*x+e))^(1+n
)/a^3/(c-d)^3/f/(1+n)
```

Rubi [A] time = 0.12, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 68}

$$-\frac{d^2(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{a^3f(n+1)(c-d)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((d^2*Hypergeometric2F1[3, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c
+ d*Sin[e + f*x])^(1 + n))/(a^3*(c - d)^3*f*(1 + n)))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(c+\frac{dx}{a}\right)^n}{(a+x)^3}\,dx, x, a\sin(e+fx)\right)}{af}$$
$$= -\frac{d^2\,{}_2F_1\left(3,1+n;2+n;\frac{c+d\sin(e+fx)}{c-d}\right)(c+d\sin(e+fx))^{1+n}}{a^3(c-d)^3f(1+n)}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 1.00

$$\frac{d^2(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(3,n+1;n+2;-\frac{c+d\sin(e+fx)}{d-c}\right)}{a^3f(n+1)(d-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (d^2*Hypergeometric2F1[3, 1 + n, 2 + n, -((c + d*Sin[e + f*x])/(-c + d))]*(
c + d*Sin[e + f*x])^(1 + n))/(a^3*(-c + d)^3*f*(1 + n))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)/(3*a^3*cos(f*x + e)^2 - 4*a^3
 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)}{\left(a\sin\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a)^3, x)
```

maple [F] time = 3.97, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] int((cos(e + f*x)*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.920 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^4\,dx$

Optimal. Leaf size=170

$$\frac{d^4(a\sin(e+fx)+a)^{m+5}}{a^5f(m+5)}+\frac{4d^3(c-d)(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{6d^2(c-d)^2(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{4d(c-d)^3(a\sin\ldots}{a^2f(m\ldots}$$

```
[Out] (c-d)^4*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)+4*(c-d)^3*d*(a+a*sin(f*x+e))^(2+m)
/a^2/f/(2+m)+6*(c-d)^2*d^2*(a+a*sin(f*x+e))^(3+m)/a^3/f/(3+m)+4*(c-d)*d^3*(
a+a*sin(f*x+e))^(4+m)/a^4/f/(4+m)+d^4*(a+a*sin(f*x+e))^(5+m)/a^5/f/(5+m)
```

Rubi [A] time = 0.18, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 43}

$$\frac{6d^2(c-d)^2(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{4d^3(c-d)(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{4d(c-d)^3(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{d^4(a\sin\ldots}{a^5\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] ((c - d)^4*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (4*(c - d)^3*d*(a
+ a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) + (6*(c - d)^2*d^2*(a + a*Sin[e
+ f*x])^(3 + m))/(a^3*f*(3 + m)) + (4*(c - d)*d^3*(a + a*Sin[e + f*x])^(4 +
 m))/(a^4*f*(4 + m)) + (d^4*(a + a*Sin[e + f*x])^(5 + m))/(a^5*f*(5 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^4\,dx = \frac{\text{Subst}\left(\int (a+x)^m\left(c+\frac{dx}{a}\right)^4 dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{\text{Subst}\left(\int \left((c-d)^4(a+x)^m + \frac{4(c-d)^3d(a+x)^{1+m}}{a} + \frac{6(c-d}{}\right.\right.}{}$$

$$= \frac{(c-d)^4(a+a\sin(e+fx))^{1+m}}{af(1+m)} + \frac{4(c-d)^3d(a+a\sin}{a^2f(2+}$$

Mathematica [A] time = 0.69, size = 143, normalized size = 0.84

$$\frac{(a(\sin(e+fx)+1))^{m+1}\left(\frac{4a^4d^3(c-d)(\sin(e+fx)+1)^3}{m+4} + \frac{6a^4d^2(c-d)^2(\sin(e+fx)+1)^2}{m+3} + \frac{4a^4d(c-d)^3(\sin(e+fx)+1)}{m+2} + \frac{a^4(c-d)^4}{m+1} + \frac{d^4(a\sin(}{m}\right.}{a^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*((a^4*(c - d)^4)/(1 + m) + (4*a^4*(c - d)^3
*d*(1 + Sin[e + f*x]))/(2 + m) + (6*a^4*(c - d)^2*d^2*(1 + Sin[e + f*x])^2)
/(3 + m) + (4*a^4*(c - d)*d^3*(1 + Sin[e + f*x])^3)/(4 + m) + (d^4*(a + a*S
in[e + f*x])^4)/(5 + m)))/(a^5*f)
```

fricas [B] time = 0.55, size = 744, normalized size = 4.38

$$\frac{\left(\left(c^4+4c^3d+6c^2d^2+4cd^3+d^4\right)m^4+\left(\left(4cd^3+d^4\right)m^4+120cd^3+2\left(22cd^3+3d^4\right)m^3+\left(164cd^3+11d^4\right)m^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] ((c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*m^4 + ((4*c*d^3 + d^4)*m^4 + 1
20*c*d^3 + 2*(22*c*d^3 + 3*d^4)*m^3 + (164*c*d^3 + 11*d^4)*m^2 + 2*(122*c*d
^3 + 3*d^4)*m)*cos(f*x + e)^4 + 120*c^4 + 240*c^2*d^2 + 24*d^4 + 2*(7*c^4 +
 24*c^3*d + 30*c^2*d^2 + 16*c*d^3 + 3*d^4)*m^3 + (71*c^4 + 188*c^3*d + 186*
c^2*d^2 + 92*c*d^3 + 23*d^4)*m^2 - 2*((2*c^3*d + 3*c^2*d^2 + 4*c*d^3 + d^4)
*m^4 + 120*c^3*d + 120*c*d^3 + 2*(13*c^3*d + 15*c^2*d^2 + 19*c*d^3 + 3*d^4)
*m^3 + (118*c^3*d + 87*c^2*d^2 + 128*c*d^3 + 17*d^4)*m^2 + 2*(107*c^3*d + 3
0*c^2*d^2 + 107*c*d^3 + 6*d^4)*m)*cos(f*x + e)^2 + 2*(77*c^4 + 120*c^3*d +
```

```
114*c^2*d^2 + 80*c*d^3 + 9*d^4)*m + ((c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 +
 d^4)*m^4 + (d^4*m^4 + 10*d^4*m^3 + 35*d^4*m^2 + 50*d^4*m + 24*d^4)*cos(f*x
 + e)^4 + 120*c^4 + 240*c^2*d^2 + 24*d^4 + 2*(7*c^4 + 24*c^3*d + 30*c^2*d^2
 + 16*c*d^3 + 3*d^4)*m^3 + (71*c^4 + 188*c^3*d + 186*c^2*d^2 + 92*c*d^3 + 2
3*d^4)*m^2 - 2*((3*c^2*d^2 + 2*c*d^3 + d^4)*m^4 + 120*c^2*d^2 + 24*d^4 + 4*
(9*c^2*d^2 + 4*c*d^3 + 2*d^4)*m^3 + (147*c^2*d^2 + 34*c*d^3 + 29*d^4)*m^2 +
 2*(117*c^2*d^2 + 10*c*d^3 + 23*d^4)*m)*cos(f*x + e)^2 + 2*(77*c^4 + 120*c^
3*d + 114*c^2*d^2 + 80*c*d^3 + 9*d^4)*m)*sin(f*x + e))*(a*sin(f*x + e) + a)
^m/(f*m^5 + 15*f*m^4 + 85*f*m^3 + 225*f*m^2 + 274*f*m + 120*f)
```

giac [B] time = 0.28, size = 1845, normalized size = 10.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] (6*((a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m^2 - 2*(a*sin(f*x + e) +
 a)^2*(a*sin(f*x + e) + a)^m*a*m^2 + (a*sin(f*x + e) + a)*(a*sin(f*x + e) +
 a)^m*a^2*m^2 + 3*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m - 8*(a*si
n(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a*m + 5*(a*sin(f*x + e) + a)*(a*si
n(f*x + e) + a)^m*a^2*m + 2*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m -
 6*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a + 6*(a*sin(f*x + e) + a)
*(a*sin(f*x + e) + a)^m*a^2)*c^2*d^2/(a^2*m^3 + 6*a^2*m^2 + 11*a^2*m + 6*a^
2) + 4*((a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^3 - 3*(a*sin(f*x +
e) + a)^3*(a*sin(f*x + e) + a)^m*a*m^3 + 3*(a*sin(f*x + e) + a)^2*(a*sin(f*
x + e) + a)^m*a^2*m^3 - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m^3
 + 6*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^2 - 21*(a*sin(f*x + e)
 + a)^3*(a*sin(f*x + e) + a)^m*a*m^2 + 24*(a*sin(f*x + e) + a)^2*(a*sin(f*x
 + e) + a)^m*a^2*m^2 - 9*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m^
2 + 11*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m - 42*(a*sin(f*x + e)
 + a)^3*(a*sin(f*x + e) + a)^m*a*m + 57*(a*sin(f*x + e) + a)^2*(a*sin(f*x +
 e) + a)^m*a^2*m - 26*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m + 6
*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m - 24*(a*sin(f*x + e) + a)^3*
(a*sin(f*x + e) + a)^m*a + 36*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m
*a^2 - 24*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3)*c*d^3/(a^3*m^4 +
 10*a^3*m^3 + 35*a^3*m^2 + 50*a^3*m + 24*a^3) + ((a*sin(f*x + e) + a)^5*(a*
sin(f*x + e) + a)^m*m^4 - 4*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a
*m^4 + 6*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a^2*m^4 - 4*(a*sin(f
*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^3*m^4 + (a*sin(f*x + e) + a)*(a*sin
(f*x + e) + a)^m*a^4*m^4 + 10*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m
*m^3 - 44*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a*m^3 + 72*(a*sin(f
*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a^2*m^3 - 52*(a*sin(f*x + e) + a)^2*(
a*sin(f*x + e) + a)^m*a^3*m^3 + 14*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a
```

```
)^m*a^4*m^3 + 35*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*m^2 - 164*(a
*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a*m^2 + 294*(a*sin(f*x + e) + a
)^3*(a*sin(f*x + e) + a)^m*a^2*m^2 - 236*(a*sin(f*x + e) + a)^2*(a*sin(f*x
+ e) + a)^m*a^3*m^2 + 71*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^4*m^
2 + 50*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*m - 244*(a*sin(f*x + e
) + a)^4*(a*sin(f*x + e) + a)^m*a*m + 468*(a*sin(f*x + e) + a)^3*(a*sin(f*x
 + e) + a)^m*a^2*m - 428*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^3*
m + 154*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^4*m + 24*(a*sin(f*x +
 e) + a)^5*(a*sin(f*x + e) + a)^m - 120*(a*sin(f*x + e) + a)^4*(a*sin(f*x +
 e) + a)^m*a + 240*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a^2 - 240*
(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^3 + 120*(a*sin(f*x + e) + a
)*(a*sin(f*x + e) + a)^m*a^4)*d^4/(a^4*m^5 + 15*a^4*m^4 + 85*a^4*m^3 + 225*
a^4*m^2 + 274*a^4*m + 120*a^4) + (a*sin(f*x + e) + a)^(m + 1)*c^4/(m + 1) +
 4*((a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*m - (a*sin(f*x + e) + a)*
(a*sin(f*x + e) + a)^m*a*m + (a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m
- 2*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a)*c^3*d/((m^2 + 3*m + 2)*a
))/(a*f)
```

maple [F] time = 19.37, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{4}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x)
```

maxima [B] time = 0.70, size = 457, normalized size = 2.69

$$\frac{4\left(a^{m}(m+1)\sin\left(fx+e\right)^{2}+a^{m}m\sin\left(fx+e\right)-a^{m}\right)c^{3}d\left(\sin\left(fx+e\right)+1\right)^{m}}{m^{2}+3m+2}+\frac{6\left(\left(m^{2}+3m+2\right)a^{m}\sin\left(fx+e\right)^{3}+\left(m^{2}+m\right)a^{m}\sin\left(fx+e\right)^{2}-2a^{m}m\sin\left(fx+e\right)+2a}{m^{3}+6m^{2}+11m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] (4*(a^m*(m + 1)*sin(f*x + e)^2 + a^m*m*sin(f*x + e) - a^m)*c^3*d*(sin(f*x +
 e) + 1)^m/(m^2 + 3*m + 2) + 6*((m^2 + 3*m + 2)*a^m*sin(f*x + e)^3 + (m^2 +
 m)*a^m*sin(f*x + e)^2 - 2*a^m*m*sin(f*x + e) + 2*a^m)*c^2*d^2*(sin(f*x + e
) + 1)^m/(m^3 + 6*m^2 + 11*m + 6) + 4*((m^3 + 6*m^2 + 11*m + 6)*a^m*sin(f*x
 + e)^4 + (m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^3 - 3*(m^2 + m)*a^m*sin(f*x
+ e)^2 + 6*a^m*m*sin(f*x + e) - 6*a^m)*c*d^3*(sin(f*x + e) + 1)^m/(m^4 + 10
*m^3 + 35*m^2 + 50*m + 24) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*a^m*sin(f
```

```
*x + e)^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*a^m*sin(f*x + e)^4 - 4*(m^3 + 3*m^
2 + 2*m)*a^m*sin(f*x + e)^3 + 12*(m^2 + m)*a^m*sin(f*x + e)^2 - 24*a^m*m*si
n(f*x + e) + 24*a^m)*d^4*(sin(f*x + e) + 1)^m/(m^5 + 15*m^4 + 85*m^3 + 225*
m^2 + 274*m + 120) + (a*sin(f*x + e) + a)^(m + 1)*c^4/(a*(m + 1)))/f
```

mupad [B] time = 16.87, size = 1656, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^4,x)
```

```
[Out] exp(- e*5i - f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*
1i)*1i)/2))^m*((exp(e*6i + f*x*6i)*(2464*c^4*m + 20*d^4*m + 1920*c^4 + 240*
d^4 + 2880*c^2*d^2 + 1136*c^4*m^2 + 224*c^4*m^3 + 16*c^4*m^4 + 206*d^4*m^2
+ 52*d^4*m^3 + 10*d^4*m^4 + 1776*c^2*d^2*m + 1200*c*d^3*m^2 + 3008*c^3*d*m^
2 + 384*c*d^3*m^3 + 768*c^3*d*m^3 + 48*c*d^3*m^4 + 64*c^3*d*m^4 + 1800*c^2*
d^2*m^2 + 672*c^2*d^2*m^3 + 72*c^2*d^2*m^4 + 2400*c*d^3*m + 3840*c^3*d*m))/
(32*f*(m*274i + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 120i)) - (exp(e*4i
+ f*x*4i)*(2464*c^4*m + 20*d^4*m + 1920*c^4 + 240*d^4 + 2880*c^2*d^2 + 1136
*c^4*m^2 + 224*c^4*m^3 + 16*c^4*m^4 + 206*d^4*m^2 + 52*d^4*m^3 + 10*d^4*m^4
 + 1776*c^2*d^2*m + 1200*c*d^3*m^2 + 3008*c^3*d*m^2 + 384*c*d^3*m^3 + 768*c
^3*d*m^3 + 48*c*d^3*m^4 + 64*c^3*d*m^4 + 1800*c^2*d^2*m^2 + 672*c^2*d^2*m^3
 + 72*c^2*d^2*m^4 + 2400*c*d^3*m + 3840*c^3*d*m))/(32*f*(m*274i + m^2*225i
+ m^3*85i + m^4*15i + m^5*1i + 120i)) - (d^4*(50*m + 35*m^2 + 10*m^3 + m^4
+ 24))/(32*f*(m*274i + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 120i)) + (ex
p(e*5i + f*x*5i)*(c^4*m*4928i - c^3*d*3840i - c*d^3*2400i + d^4*m*264i + c^
4*3840i + d^4*768i + c^2*d^2*7680i + c^4*m^2*2272i + c^4*m^3*448i + c^4*m^4
*32i + d^4*m^2*324i + d^4*m^3*72i + d^4*m^4*12i + c^2*d^2*m*5376i + c*d^3*m
^2*816i + c^3*d*m^2*2240i + c*d^3*m^3*336i + c^3*d*m^3*704i + c*d^3*m^4*48i
 + c^3*d*m^4*64i + c^2*d^2*m^2*3168i + c^2*d^2*m^3*960i + c^2*d^2*m^4*96i +
 c*d^3*m*1200i + c^3*d*m*832i))/(32*f*(m*274i + m^2*225i + m^3*85i + m^4*15
i + m^5*1i + 120i)) + (d^4*exp(e*10i + f*x*10i)*(50*m + 35*m^2 + 10*m^3 + m
^4 + 24))/(32*f*(m*274i + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 120i)) +
(d^2*exp(e*2i + f*x*2i)*(3*m + m^2 + 2)*(216*c^2*m + 19*d^2*m + 480*c^2 + 6
0*d^2 + 24*c^2*m^2 + 5*d^2*m^2 + 80*c*d*m + 16*c*d*m^2))/(32*f*(m*274i + m^
2*225i + m^3*85i + m^4*15i + m^5*1i + 120i)) - (d^2*exp(e*8i + f*x*8i)*(3*m
 + m^2 + 2)*(216*c^2*m + 19*d^2*m + 480*c^2 + 60*d^2 + 24*c^2*m^2 + 5*d^2*m
^2 + 80*c*d*m + 16*c*d*m^2))/(32*f*(m*274i + m^2*225i + m^3*85i + m^4*15i +
 m^5*1i + 120i)) - (d*exp(e*3i + f*x*3i)*(m + 1)*(c*d^2*120i + c^3*m*188i +
 d^3*m*18i + c^3*240i + c^3*m^2*48i + c^3*m^3*4i + d^3*m^2*5i + d^3*m^3*1i
+ c*d^2*m^2*28i + c^2*d*m^2*54i + c*d^2*m^3*4i + c^2*d*m^3*6i + c*d^2*m*64i
 + c^2*d*m*120i))/(4*f*(m*274i + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 12
0i)) - (d*exp(e*7i + f*x*7i)*(m + 1)*(c*d^2*120i + c^3*m*188i + d^3*m*18i +
 c^3*240i + c^3*m^2*48i + c^3*m^3*4i + d^3*m^2*5i + d^3*m^3*1i + c*d^2*m^2*
```

```
28i + c^2*d*m^2*54i + c*d^2*m^3*4i + c^2*d*m^3*6i + c*d^2*m*64i + c^2*d*m*1
20i))/(4*f*(m*274i + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 120i)) + (d^3*
exp(e*1i + f*x*1i)*(c*20i + c*m*4i + d*m*1i)*(11*m + 6*m^2 + m^3 + 6))/(16*
f*(m*274i + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 120i)) + (d^3*exp(e*9i
+ f*x*9i)*(c*20i + c*m*4i + d*m*1i)*(11*m + 6*m^2 + m^3 + 6))/(16*f*(m*274i
 + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 120i)))
```

sympy [A] time = 102.31, size = 9238, normalized size = 54.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((x*(c + d*sin(e))**4*(a*sin(e) + a)**m*cos(e), Eq(f, 0)), (-3*c**
4/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e
+ f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) - 16*c**3*d*sin(e + f*x)/(1
2*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*
x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) - 4*c**3*d/(12*a**5*f*sin(e + f
*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*
sin(e + f*x) + 12*a**5*f) - 36*c**2*d**2*sin(e + f*x)**2/(12*a**5*f*sin(e +
 f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*
f*sin(e + f*x) + 12*a**5*f) - 24*c**2*d**2*sin(e + f*x)/(12*a**5*f*sin(e +
f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f
*sin(e + f*x) + 12*a**5*f) - 6*c**2*d**2/(12*a**5*f*sin(e + f*x)**4 + 48*a*
*5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) +
 12*a**5*f) - 48*c*d**3*sin(e + f*x)**3/(12*a**5*f*sin(e + f*x)**4 + 48*a**
5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) +
12*a**5*f) - 72*c*d**3*sin(e + f*x)**2/(12*a**5*f*sin(e + f*x)**4 + 48*a**5
*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 1
2*a**5*f) - 48*c*d**3*sin(e + f*x)/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*s
in(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a*
*5*f) - 12*c*d**3/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 +
72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 12*d**4*l
og(sin(e + f*x) + 1)*sin(e + f*x)**4/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f
*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*
a**5*f) + 48*d**4*log(sin(e + f*x) + 1)*sin(e + f*x)**3/(12*a**5*f*sin(e +
f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f
*sin(e + f*x) + 12*a**5*f) + 72*d**4*log(sin(e + f*x) + 1)*sin(e + f*x)**2/
(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e +
f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 48*d**4*log(sin(e + f*x) +
1)*sin(e + f*x)/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72
*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 12*d**4*log
(sin(e + f*x) + 1)/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 +
 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 48*d**4*
```

sin(e + f*x)**3/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72
*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 108*d**4*si
n(e + f*x)**2/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a
5*f*sin(e + f*x)2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 88*d**4*sin(e
 + f*x)/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*
sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 25*d**4/(12*a**5*f*
sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 4
8*a**5*f*sin(e + f*x) + 12*a**5*f), Eq(m, -5)), (-c**4/(3*a**4*f*sin(e + f*
x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) - 6*c*
*3*d*sin(e + f*x)/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*
a**4*f*sin(e + f*x) + 3*a**4*f) - 2*c**3*d/(3*a**4*f*sin(e + f*x)**3 + 9*a*
*4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) - 18*c**2*d**2*sin
(e + f*x)**2/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*
f*sin(e + f*x) + 3*a**4*f) - 18*c**2*d**2*sin(e + f*x)/(3*a**4*f*sin(e + f*
x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) - 6*c*
*2*d**2/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin
(e + f*x) + 3*a**4*f) + 12*c*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)**3/(3*
a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) +
 3*a**4*f) + 36*c*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)**2/(3*a**4*f*sin(
e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f)
 + 36*c*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)/(3*a**4*f*sin(e + f*x)**3 +
9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) + 12*c*d**3*lo
g(sin(e + f*x) + 1)/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 +
9*a**4*f*sin(e + f*x) + 3*a**4*f) + 36*c*d**3*sin(e + f*x)**2/(3*a**4*f*sin
(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f)
 + 54*c*d**3*sin(e + f*x)/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)
2 + 9*a4*f*sin(e + f*x) + 3*a**4*f) + 22*c*d**3/(3*a**4*f*sin(e + f*x)*
*3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) - 12*d**4
*log(sin(e + f*x) + 1)*sin(e + f*x)**3/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f
*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) - 36*d**4*log(sin(e +
f*x) + 1)*sin(e + f*x)**2/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)
2 + 9*a4*f*sin(e + f*x) + 3*a**4*f) - 36*d**4*log(sin(e + f*x) + 1)*sin
(e + f*x)/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*s
in(e + f*x) + 3*a**4*f) - 12*d**4*log(sin(e + f*x) + 1)/(3*a**4*f*sin(e + f
*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) + 3*d
4*sin(e + f*x)4/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 +
9*a**4*f*sin(e + f*x) + 3*a**4*f) - 36*d**4*sin(e + f*x)**2/(3*a**4*f*sin(e
 + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) -
 54*d**4*sin(e + f*x)/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2
+ 9*a**4*f*sin(e + f*x) + 3*a**4*f) - 22*d**4/(3*a**4*f*sin(e + f*x)**3 + 9
*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f), Eq(m, -4)), (-
c**4/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 8*c**3
*d*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*
f) - 4*c**3*d/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f)
 + 12*c**2*d**2*log(sin(e + f*x) + 1)*sin(e + f*x)**2/(2*a**3*f*sin(e + f*x

)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 24*c**2*d**2*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 12*c**2*d**2*log(sin(e + f*x) + 1)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 24*c**2*d**2*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 18*c**2*d**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 24*c*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 48*c*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 24*c*d**3*log(sin(e + f*x) + 1)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 8*c*d**3*sin(e + f*x)**3/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 48*c*d**3*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 36*c*d**3/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 12*d**4*log(sin(e + f*x) + 1)*sin(e + f*x)**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 24*d**4*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 12*d**4*log(sin(e + f*x) + 1)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + d**4*sin(e + f*x)**4/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 4*d**4*sin(e + f*x)**3/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 24*d**4*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 18*d**4/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f), Eq(m, -3)), (-3*c**4/(3*a**2*f*sin(e + f*x) + 3*a**2*f) + 12*c**3*d*log(sin(e + f*x) + 1)*sin(e + f*x)/(3*a**2*f*sin(e + f*x) + 3*a**2*f) + 12*c**3*d*log(sin(e + f*x) + 1)/(3*a**2*f*sin(e + f*x) + 3*a**2*f) + 12*c**3*d/(3*a**2*f*sin(e + f*x) + 3*a**2*f) - 36*c**2*d**2*log(sin(e + f*x) + 1)*sin(e + f*x)/(3*a**2*f*sin(e + f*x) + 3*a**2*f) - 36*c**2*d**2*log(sin(e + f*x) + 1)/(3*a**2*f*sin(e + f*x) + 3*a**2*f) + 18*c**2*d**2*sin(e + f*x)**2/(3*a**2*f*sin(e + f*x) + 3*a**2*f) - 36*c**2*d**2/(3*a**2*f*sin(e + f*x) + 3*a**2*f) + 36*c*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)/(3*a**2*f*sin(e + f*x) + 3*a**2*f) + 36*c*d**3*log(sin(e + f*x) + 1)/(3*a**2*f*sin(e + f*x) + 3*a**2*f) + 6*c*d**3*sin(e + f*x)**3/(3*a**2*f*sin(e + f*x) + 3*a**2*f) - 18*c*d**3*sin(e + f*x)**2/(3*a**2*f*sin(e + f*x) + 3*a**2*f) + 36*c*d**3/(3*a**2*f*sin(e + f*x) + 3*a**2*f) - 12*d**4*log(sin(e + f*x) + 1)*sin(e + f*x)/(3*a**2*f*sin(e + f*x) + 3*a**2*f) - 12*d**4*log(sin(e + f*x) + 1)/(3*a**2*f*sin(e + f*x) + 3*a**2*f) + d**4*sin(e + f*x)**4/(3*a**2*f*sin(e + f*x) + 3*a**2*f) - 2*d**4*sin(e + f*x)**3/(3*a**2*f*sin(e + f*x) + 3*a**2*f) + 6*d**4*sin(e + f*x)**2/(3*a**2*f*sin(e + f*x) + 3*a**2*f) - 12*d**4/(3*a**2*f*sin(e + f*x) + 3*a**2*f), Eq(m, -2)), (c**4*log(sin(e + f*x) + 1)/(a*f) - 4*c**3*d*log(sin(e + f*x) + 1)/(a*f) + 4*c**3*d*sin(e + f*x)/(a*f) + 6*c**2*d**2*log(sin(e + f*x) + 1)/(a*f) + 3*c**2*d**2*sin(e + f*x)**2/(a*f) - 6*c**2*d**2*sin(e + f*x)/(a*f) - 4*c*d**3*log(sin(e + f*x) + 1)/(a*f) + 4*c*d**3*sin(e + f*x)**3/(3*a*f) - 2*c*d**3*sin(e + f*x)**2/(a*f) + 4*c*d**3*sin(e + f*x)/(a*f) + d**4*log(sin(e + f*x) + 1)/(a*f) + d**4*sin(e + f*x)**4/(4*a*f) - d**4*sin(e + f*x)**3/(3*a*f) + d**4*sin(e + f*x)**2/(2*a*f) - d**4*sin(e + f*x)/(a*f), Eq(m, -1)

), (c**4*m**4*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**5 + 15*f*m**4 + 85
*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + c**4*m**4*(a*sin(e + f*x) + a)**m
/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 14*c**4*
m**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**5 + 15*f*m**4 + 85*f*m**3 +
 225*f*m**2 + 274*f*m + 120*f) + 14*c**4*m**3*(a*sin(e + f*x) + a)**m/(f*m*
*5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 71*c**4*m**2*(
a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f
*m**2 + 274*f*m + 120*f) + 71*c**4*m**2*(a*sin(e + f*x) + a)**m/(f*m**5 + 1
5*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 154*c**4*m*(a*sin(e
+ f*x) + a)**m*sin(e + f*x)/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 +
274*f*m + 120*f) + 154*c**4*m*(a*sin(e + f*x) + a)**m/(f*m**5 + 15*f*m**4 +
 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 120*c**4*(a*sin(e + f*x) + a)*
*m*sin(e + f*x)/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 12
0*f) + 120*c**4*(a*sin(e + f*x) + a)**m/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 2
25*f*m**2 + 274*f*m + 120*f) + 4*c**3*d*m**4*(a*sin(e + f*x) + a)**m*sin(e
+ f*x)**2/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) +
 4*c**3*d*m**4*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**5 + 15*f*m**4 + 8
5*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 52*c**3*d*m**3*(a*sin(e + f*x) +
 a)**m*sin(e + f*x)**2/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f
*m + 120*f) + 48*c**3*d*m**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**5 +
 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) - 4*c**3*d*m**3*(a*s
in(e + f*x) + a)**m/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m
+ 120*f) + 236*c**3*d*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**5
+ 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 188*c**3*d*m**2*(
a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f
*m**2 + 274*f*m + 120*f) - 48*c**3*d*m**2*(a*sin(e + f*x) + a)**m/(f*m**5 +
 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 428*c**3*d*m*(a*si
n(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*
m**2 + 274*f*m + 120*f) + 240*c**3*d*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)
/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) - 188*c**3
*d*m*(a*sin(e + f*x) + a)**m/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 +
 274*f*m + 120*f) + 240*c**3*d*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m
5 + 15*f*m4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) - 240*c**3*d*(a
*sin(e + f*x) + a)**m/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*
m + 120*f) + 6*c**2*d**2*m**4*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m*
*5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 6*c**2*d**2*m*
4(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**5 + 15*f*m**4 + 85*f*m**3
+ 225*f*m**2 + 274*f*m + 120*f) + 72*c**2*d**2*m**3*(a*sin(e + f*x) + a)**m
*sin(e + f*x)**3/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 1
20*f) + 60*c**2*d**2*m**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**5 +
 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) - 12*c**2*d**2*m**3*
(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*
f*m**2 + 274*f*m + 120*f) + 294*c**2*d**2*m**2*(a*sin(e + f*x) + a)**m*sin(
e + f*x)**3/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f)
 + 174*c**2*d**2*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**5 + 15*

```
f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) - 108*c**2*d**2*m**2*(a*
sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m
**2 + 274*f*m + 120*f) + 12*c**2*d**2*m**2*(a*sin(e + f*x) + a)**m/(f*m**5
+ 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 468*c**2*d**2*m*(
a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 22
5*f*m**2 + 274*f*m + 120*f) + 120*c**2*d**2*m*(a*sin(e + f*x) + a)**m*sin(e
 + f*x)**2/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f)
- 240*c**2*d**2*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**5 + 15*f*m**4
+ 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 108*c**2*d**2*m*(a*sin(e + f*
x) + a)**m/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f)
+ 240*c**2*d**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**5 + 15*f*m**4
 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 240*c**2*d**2*(a*sin(e + f*x
) + a)**m/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) +
 4*c*d**3*m**4*(a*sin(e + f*x) + a)**m*sin(e + f*x)**4/(f*m**5 + 15*f*m**4
+ 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 4*c*d**3*m**4*(a*sin(e + f*x)
 + a)**m*sin(e + f*x)**3/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274
*f*m + 120*f) + 44*c*d**3*m**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)**4/(f*m
**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 32*c*d**3*m**
3*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**5 + 15*f*m**4 + 85*f*m**3 +
 225*f*m**2 + 274*f*m + 120*f) - 12*c*d**3*m**3*(a*sin(e + f*x) + a)**m*sin
(e + f*x)**2/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f
) + 164*c*d**3*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**4/(f*m**5 + 15*f*
m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 68*c*d**3*m**2*(a*sin(e
+ f*x) + a)**m*sin(e + f*x)**3/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2
 + 274*f*m + 120*f) - 72*c*d**3*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**
2/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 24*c*d*
*3*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**5 + 15*f*m**4 + 85*f*m**
3 + 225*f*m**2 + 274*f*m + 120*f) + 244*c*d**3*m*(a*sin(e + f*x) + a)**m*si
n(e + f*x)**4/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*
f) + 40*c*d**3*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**5 + 15*f*m**
4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) - 60*c*d**3*m*(a*sin(e + f*x)
 + a)**m*sin(e + f*x)**2/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274
*f*m + 120*f) + 120*c*d**3*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**5 +
 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) - 24*c*d**3*m*(a*sin
(e + f*x) + a)**m/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m +
120*f) + 120*c*d**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)**4/(f*m**5 + 15*f*
m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) - 120*c*d**3*(a*sin(e + f*
x) + a)**m/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f)
+ d**4*m**4*(a*sin(e + f*x) + a)**m*sin(e + f*x)**5/(f*m**5 + 15*f*m**4 + 8
5*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + d**4*m**4*(a*sin(e + f*x) + a)**
m*sin(e + f*x)**4/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m +
120*f) + 10*d**4*m**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)**5/(f*m**5 + 15*
f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 6*d**4*m**3*(a*sin(e +
 f*x) + a)**m*sin(e + f*x)**4/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2
+ 274*f*m + 120*f) - 4*d**4*m**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f
```

```
*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 35*d**4*m**
2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**5/(f*m**5 + 15*f*m**4 + 85*f*m**3 +
 225*f*m**2 + 274*f*m + 120*f) + 11*d**4*m**2*(a*sin(e + f*x) + a)**m*sin(e
 + f*x)**4/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f)
- 12*d**4*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**5 + 15*f*m**4
+ 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 12*d**4*m**2*(a*sin(e + f*x)
+ a)**m*sin(e + f*x)**2/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*
f*m + 120*f) + 50*d**4*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**5/(f*m**5 +
15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 6*d**4*m*(a*sin(e +
 f*x) + a)**m*sin(e + f*x)**4/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2
+ 274*f*m + 120*f) - 8*d**4*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m*
*5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 12*d**4*m*(a*s
in(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f
*m**2 + 274*f*m + 120*f) - 24*d**4*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(
f*m**5 + 15*f*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f) + 24*d**4*(a
*sin(e + f*x) + a)**m*sin(e + f*x)**5/(f*m**5 + 15*f*m**4 + 85*f*m**3 + 225
*f*m**2 + 274*f*m + 120*f) + 24*d**4*(a*sin(e + f*x) + a)**m/(f*m**5 + 15*f
*m**4 + 85*f*m**3 + 225*f*m**2 + 274*f*m + 120*f), True))
```

3.921 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=133

$$\frac{d^3(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{3d^2(c-d)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{3d(c-d)^2(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(c-d)^3(a\sin(e}{af(m}$$

```
[Out] (c-d)^3*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)+3*(c-d)^2*d*(a+a*sin(f*x+e))^(2+m)
/a^2/f/(2+m)+3*(c-d)*d^2*(a+a*sin(f*x+e))^(3+m)/a^3/f/(3+m)+d^3*(a+a*sin(f*
x+e))^(4+m)/a^4/f/(4+m)
```

Rubi [A] time = 0.14, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 43}

$$\frac{3d^2(c-d)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{3d(c-d)^2(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{d^3(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{(c-d)^3(a\sin(e}{af(m}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((c - d)^3*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (3*(c - d)^2*d*(a
+ a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) + (3*(c - d)*d^2*(a + a*Sin[e +
f*x])^(3 + m))/(a^3*f*(3 + m)) + (d^3*(a + a*Sin[e + f*x])^(4 + m))/(a^4*f*
(4 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^3\,dx = \frac{\text{Subst}\left(\int (a+x)^m\left(c+\frac{dx}{a}\right)^3 dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{\text{Subst}\left(\int \left((c-d)^3(a+x)^m + \frac{3(c-d)^2d(a+x)^{1+m}}{a} + \frac{3(c-d)d}{\cdots}\right.\right.}{af}$$

$$= \frac{(c-d)^3(a+a\sin(e+fx))^{1+m}}{af(1+m)} + \frac{3(c-d)^2d(a+a\sin\cdots}{a^2f(2+\cdots}$$

Mathematica [A] time = 0.39, size = 113, normalized size = 0.85

$$\frac{(a(\sin(e+fx)+1))^{m+1}\left(\frac{3a^3d^2(c-d)(\sin(e+fx)+1)^2}{m+3}+\frac{3a^3d(c-d)^2(\sin(e+fx)+1)}{m+2}+\frac{a^3(c-d)^3}{m+1}+\frac{d^3(a\sin(e+fx)+a)^3}{m+4}\right)}{a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*((a^3*(c - d)^3)/(1 + m) + (3*a^3*(c - d)^2
*d*(1 + Sin[e + f*x]))/(2 + m) + (3*a^3*(c - d)*d^2*(1 + Sin[e + f*x])^2)/(
3 + m) + (d^3*(a + a*Sin[e + f*x])^3)/(4 + m)))/(a^4*f)
```

fricas [B] time = 0.51, size = 403, normalized size = 3.03

$$\frac{\left(\left(d^3m^3+6\,d^3m^2+11\,d^3m+6\,d^3\right)\cos\left(fx+e\right)^4+\left(c^3+3\,c^2d+3\,cd^2+d^3\right)m^3+24\,c^3+24\,cd^2+3\left(3\,c^3+7\,c^2d\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] ((d^3*m^3 + 6*d^3*m^2 + 11*d^3*m + 6*d^3)*cos(f*x + e)^4 + (c^3 + 3*c^2*d +
 3*c*d^2 + d^3)*m^3 + 24*c^3 + 24*c*d^2 + 3*(3*c^3 + 7*c^2*d + 5*c*d^2 + d^
3)*m^2 - ((3*c^2*d + 3*c*d^2 + 2*d^3)*m^3 + 36*c^2*d + 12*d^3 + 3*(8*c^2*d
+ 5*c*d^2 + 3*d^3)*m^2 + (57*c^2*d + 12*c*d^2 + 19*d^3)*m)*cos(f*x + e)^2 +
 2*(13*c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*m + ((c^3 + 3*c^2*d + 3*c*d^2 + d^
3)*m^3 + 24*c^3 + 24*c*d^2 + 3*(3*c^3 + 7*c^2*d + 5*c*d^2 + d^3)*m^2 - ((3*
c*d^2 + d^3)*m^3 + 24*c*d^2 + 3*(7*c*d^2 + d^3)*m^2 + 2*(21*c*d^2 + d^3)*m)
*cos(f*x + e)^2 + 2*(13*c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*m)*sin(f*x + e))*
(a*sin(f*x + e) + a)^m/(f*m^4 + 10*f*m^3 + 35*f*m^2 + 50*f*m + 24*f)
```

giac [B] time = 0.24, size = 1003, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] (3*((a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m^2 - 2*(a*sin(f*x + e) +
 a)^2*(a*sin(f*x + e) + a)^m*a*m^2 + (a*sin(f*x + e) + a)*(a*sin(f*x + e) +
 a)^m*a^2*m^2 + 3*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m - 8*(a*si
n(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a*m + 5*(a*sin(f*x + e) + a)*(a*si
n(f*x + e) + a)^m*a^2*m + 2*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m -
 6*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a + 6*(a*sin(f*x + e) + a)
*(a*sin(f*x + e) + a)^m*a^2)*c*d^2/(a^2*m^3 + 6*a^2*m^2 + 11*a^2*m + 6*a^2)
 + ((a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^3 - 3*(a*sin(f*x + e) +
 a)^3*(a*sin(f*x + e) + a)^m*a*m^3 + 3*(a*sin(f*x + e) + a)^2*(a*sin(f*x +
e) + a)^m*a^2*m^3 - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m^3 + 6
*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^2 - 21*(a*sin(f*x + e) + a
)^3*(a*sin(f*x + e) + a)^m*a*m^2 + 24*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e
) + a)^m*a^2*m^2 - 9*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m^2 +
11*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m - 42*(a*sin(f*x + e) + a
)^3*(a*sin(f*x + e) + a)^m*a*m + 57*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e)
+ a)^m*a^2*m - 26*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m + 6*(a*
sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m - 24*(a*sin(f*x + e) + a)^3*(a*s
in(f*x + e) + a)^m*a + 36*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^2
 - 24*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3)*d^3/(a^3*m^4 + 10*a^
3*m^3 + 35*a^3*m^2 + 50*a^3*m + 24*a^3) + (a*sin(f*x + e) + a)^(m + 1)*c^3/
(m + 1) + 3*((a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*m - (a*sin(f*x +
 e) + a)*(a*sin(f*x + e) + a)^m*a*m + (a*sin(f*x + e) + a)^2*(a*sin(f*x + e
) + a)^m - 2*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a)*c^2*d/((m^2 + 3
*m + 2)*a))/(a*f)
```

maple [F] time = 8.89, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x)
```

maxima [B] time = 0.75, size = 294, normalized size = 2.21

$$\frac{3\left(a^m(m+1)\sin\left(fx+e\right)^2+a^m m\sin\left(fx+e\right)-a^m\right)c^2d\left(\sin\left(fx+e\right)+1\right)^m}{m^2+3m+2}+\frac{3\left(\left(m^2+3m+2\right)a^m\sin\left(fx+e\right)^3+\left(m^2+m\right)a^m\sin\left(fx+e\right)^2-2a^m m\sin\left(fx+e\right)+2a^m\right.}{m^3+6m^2+11m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] (3*(a^m*(m + 1)*sin(f*x + e)^2 + a^m*m*sin(f*x + e) - a^m)*c^2*d*(sin(f*x +
 e) + 1)^m/(m^2 + 3*m + 2) + 3*((m^2 + 3*m + 2)*a^m*sin(f*x + e)^3 + (m^2 +
 m)*a^m*sin(f*x + e)^2 - 2*a^m*m*sin(f*x + e) + 2*a^m)*c*d^2*(sin(f*x + e)
+ 1)^m/(m^3 + 6*m^2 + 11*m + 6) + ((m^3 + 6*m^2 + 11*m + 6)*a^m*sin(f*x + e
)^4 + (m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^3 - 3*(m^2 + m)*a^m*sin(f*x + e)
^2 + 6*a^m*m*sin(f*x + e) - 6*a^m)*d^3*(sin(f*x + e) + 1)^m/(m^4 + 10*m^3 +
 35*m^2 + 50*m + 24) + (a*sin(f*x + e) + a)^(m + 1)*c^3/(a*(m + 1)))/f
```

mupad [B] time = 13.44, size = 703, normalized size = 5.29

$$\frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^m\left(192\,c\,d^2-144\,c^2\,d+208\,c^3\,m+21\,d^3\,m+192\,c^3\sin\left(e+fx\right)+192\,c^3-30\,d^3-24\,d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^3,x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(192*c*d^2 - 144*c^2*d + 208*c^3*m + 21*d^3*m + 1
92*c^3*sin(e + f*x) + 192*c^3 - 30*d^3 - 24*d^3*cos(2*e + 2*f*x) + 6*d^3*co
s(4*e + 4*f*x) + 72*c^3*m^2 + 8*c^3*m^3 + 6*d^3*m^2 + 3*d^3*m^3 + 208*c^3*m
*sin(e + f*x) + 60*d^3*m*sin(e + f*x) - 144*c^2*d*cos(2*e + 2*f*x) + 60*c*d
^2*m^2 + 72*c^2*d*m^2 + 12*c*d^2*m^3 + 12*c^2*d*m^3 - 32*d^3*m*cos(2*e + 2*
f*x) + 11*d^3*m*cos(4*e + 4*f*x) - 48*c*d^2*sin(3*e + 3*f*x) + 72*c^3*m^2*s
in(e + f*x) + 8*c^3*m^3*sin(e + f*x) - 4*d^3*m*sin(3*e + 3*f*x) + 18*d^3*m^
2*sin(e + f*x) + 6*d^3*m^3*sin(e + f*x) - 12*d^3*m^2*cos(2*e + 2*f*x) - 4*d
^3*m^3*cos(2*e + 2*f*x) + 6*d^3*m^2*cos(4*e + 4*f*x) + d^3*m^3*cos(4*e + 4*
f*x) - 6*d^3*m^2*sin(3*e + 3*f*x) - 2*d^3*m^3*sin(3*e + 3*f*x) + 96*c*d^2*m
 + 60*c^2*d*m + 144*c*d^2*sin(e + f*x) - 60*c*d^2*m^2*cos(2*e + 2*f*x) - 96
*c^2*d*m^2*cos(2*e + 2*f*x) - 12*c*d^2*m^3*cos(2*e + 2*f*x) - 12*c^2*d*m^3*
cos(2*e + 2*f*x) - 42*c*d^2*m^2*sin(3*e + 3*f*x) - 6*c*d^2*m^3*sin(3*e + 3*
f*x) + 60*c*d^2*m*sin(e + f*x) + 288*c^2*d*m*sin(e + f*x) - 48*c*d^2*m*cos(
2*e + 2*f*x) - 228*c^2*d*m*cos(2*e + 2*f*x) - 84*c*d^2*m*sin(3*e + 3*f*x) +
 78*c*d^2*m^2*sin(e + f*x) + 168*c^2*d*m^2*sin(e + f*x) + 18*c*d^2*m^3*sin(
```

```
e + f*x) + 24*c^2*d*m^3*sin(e + f*x)))/(8*f*(50*m + 35*m^2 + 10*m^3 + m^4 +
 24))
```

sympy [A] time = 41.43, size = 4310, normalized size = 32.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((x*(c + d*sin(e))**3*(a*sin(e) + a)**m*cos(e), Eq(f, 0)), (-2*c**
3/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e +
 f*x) + 6*a**4*f) - 9*c**2*d*sin(e + f*x)/(6*a**4*f*sin(e + f*x)**3 + 18*a*
*4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 3*c**2*d/(6*a**
4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) +
6*a**4*f) - 18*c*d**2*sin(e + f*x)**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f
*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 18*c*d**2*sin(e + f
*x)/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e
 + f*x) + 6*a**4*f) - 6*c*d**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e
+ f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) + 6*d**3*log(sin(e + f*x) +
1)*sin(e + f*x)**3/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 +
18*a**4*f*sin(e + f*x) + 6*a**4*f) + 18*d**3*log(sin(e + f*x) + 1)*sin(e +
f*x)**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*s
in(e + f*x) + 6*a**4*f) + 18*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)/(6*a**
4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) +
6*a**4*f) + 6*d**3*log(sin(e + f*x) + 1)/(6*a**4*f*sin(e + f*x)**3 + 18*a**
4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) + 18*d**3*sin(e +
f*x)**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*s
in(e + f*x) + 6*a**4*f) + 27*d**3*sin(e + f*x)/(6*a**4*f*sin(e + f*x)**3 +
18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) + 11*d**3/(6
*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x
) + 6*a**4*f), Eq(m, -4)), (-c**3/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(
e + f*x) + 2*a**3*f) - 6*c**2*d*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*
a**3*f*sin(e + f*x) + 2*a**3*f) - 3*c**2*d/(2*a**3*f*sin(e + f*x)**2 + 4*a*
*3*f*sin(e + f*x) + 2*a**3*f) + 6*c*d**2*log(sin(e + f*x) + 1)*sin(e + f*x)
**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 12*c*d*
*2*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*
sin(e + f*x) + 2*a**3*f) + 6*c*d**2*log(sin(e + f*x) + 1)/(2*a**3*f*sin(e +
 f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 12*c*d**2*sin(e + f*x)/(2*a*
*3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 9*c*d**2/(2*a**3
*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 6*d**3*log(sin(e +
 f*x) + 1)*sin(e + f*x)**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x
) + 2*a**3*f) - 12*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**3*f*sin(e
+ f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 6*d**3*log(sin(e + f*x) + 1
)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 2*d**3*si
```

n(e + f*x)**3/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f)
 - 12*d**3*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) +
 2*a**3*f) - 9*d**3/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a
3*f), Eq(m, -3)), (-2*c3/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*c**2*d*
log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*c
2*d*log(sin(e + f*x) + 1)/(2*a2*f*sin(e + f*x) + 2*a**2*f) + 6*c**2*d/(
2*a**2*f*sin(e + f*x) + 2*a**2*f) - 12*c*d**2*log(sin(e + f*x) + 1)*sin(e +
 f*x)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) - 12*c*d**2*log(sin(e + f*x) + 1)/
(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*c*d**2*sin(e + f*x)**2/(2*a**2*f*sin
(e + f*x) + 2*a**2*f) - 12*c*d**2/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*d*
*3*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) +
6*d**3*log(sin(e + f*x) + 1)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + d**3*sin(
e + f*x)**3/(2*a**2*f*sin(e + f*x) + 2*a**2*f) - 3*d**3*sin(e + f*x)**2/(2*
a**2*f*sin(e + f*x) + 2*a**2*f) + 6*d**3/(2*a**2*f*sin(e + f*x) + 2*a**2*f)
, Eq(m, -2)), (c**3*log(sin(e + f*x) + 1)/(a*f) - 3*c**2*d*log(sin(e + f*x)
 + 1)/(a*f) + 3*c**2*d*sin(e + f*x)/(a*f) + 3*c*d**2*log(sin(e + f*x) + 1)/
(a*f) + 3*c*d**2*sin(e + f*x)**2/(2*a*f) - 3*c*d**2*sin(e + f*x)/(a*f) - d*
*3*log(sin(e + f*x) + 1)/(a*f) + d**3*sin(e + f*x)**3/(3*a*f) - d**3*sin(e
+ f*x)**2/(2*a*f) + d**3*sin(e + f*x)/(a*f), Eq(m, -1)), (c**3*m**3*(a*sin(
e + f*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24
*f) + c**3*m**3*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 5
0*f*m + 24*f) + 9*c**3*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**4 +
10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 9*c**3*m**2*(a*sin(e + f*x) + a)**
m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 26*c**3*m*(a*sin(e + f
*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) +
 26*c**3*m*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m
 + 24*f) + 24*c**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3
 + 35*f*m**2 + 50*f*m + 24*f) + 24*c**3*(a*sin(e + f*x) + a)**m/(f*m**4 + 1
0*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 3*c**2*d*m**3*(a*sin(e + f*x) + a)*
*m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 3*c**
2*d*m**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m*
*2 + 50*f*m + 24*f) + 24*c**2*d*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**
2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 21*c**2*d*m**2*(a*sin(
e + f*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24
*f) - 3*c**2*d*m**2*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m**2
 + 50*f*m + 24*f) + 57*c**2*d*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*
m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 36*c**2*d*m*(a*sin(e + f*x)
 + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) - 21
*c**2*d*m*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m
+ 24*f) + 36*c**2*d*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*
m**3 + 35*f*m**2 + 50*f*m + 24*f) - 36*c**2*d*(a*sin(e + f*x) + a)**m/(f*m*
*4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 3*c*d**2*m**3*(a*sin(e + f*x)
 + a)**m*sin(e + f*x)**3/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) +
 3*c*d**2*m**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3
+ 35*f*m**2 + 50*f*m + 24*f) + 21*c*d**2*m**2*(a*sin(e + f*x) + a)**m*sin(e

```
 + f*x)**3/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 15*c*d**2*m**
2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 +
 50*f*m + 24*f) - 6*c*d**2*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**
4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 42*c*d**2*m*(a*sin(e + f*x) +
a)**m*sin(e + f*x)**3/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 12
*c*d**2*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*
f*m**2 + 50*f*m + 24*f) - 24*c*d**2*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/
(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 6*c*d**2*m*(a*sin(e + f*
x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 24*c*d**2*(a*
sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f
*m + 24*f) + 24*c*d**2*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m
**2 + 50*f*m + 24*f) + d**3*m**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)**4/(f
*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + d**3*m**3*(a*sin(e + f*x)
+ a)**m*sin(e + f*x)**3/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) +
6*d**3*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**4/(f*m**4 + 10*f*m**3 + 3
5*f*m**2 + 50*f*m + 24*f) + 3*d**3*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x
)**3/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) - 3*d**3*m**2*(a*sin(
e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m +
 24*f) + 11*d**3*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**4/(f*m**4 + 10*f*m
**3 + 35*f*m**2 + 50*f*m + 24*f) + 2*d**3*m*(a*sin(e + f*x) + a)**m*sin(e +
 f*x)**3/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) - 3*d**3*m*(a*sin
(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m
+ 24*f) + 6*d**3*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3
 + 35*f*m**2 + 50*f*m + 24*f) + 6*d**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)
**4/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) - 6*d**3*(a*sin(e + f*
x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f), True))
```

3.922 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=96

$$\frac{d^2(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{2d(c-d)(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(c-d)^2(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

```
[Out] (c-d)^2*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)+2*(c-d)*d*(a+a*sin(f*x+e))^(2+m)/a
^2/f/(2+m)+d^2*(a+a*sin(f*x+e))^(3+m)/a^3/f/(3+m)
```

Rubi [A] time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 43}

$$\frac{2d(c-d)(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{d^2(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{(c-d)^2(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((c - d)^2*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (2*(c - d)*d*(a +
a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) + (d^2*(a + a*Sin[e + f*x])^(3 + m
))/(a^3*f*(3 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx = \frac{\text{Subst}\left(\int (a+x)^m\left(c+\frac{dx}{a}\right)^2\,dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{\text{Subst}\left(\int\left((c-d)^2(a+x)^m + \frac{2(c-d)d(a+x)^{1+m}}{a} + \frac{d^2(a+x}{a^2}\right.\right.}{af}$$

$$= \frac{(c-d)^2(a+a\sin(e+fx))^{1+m}}{af(1+m)} + \frac{2(c-d)d(a+a\sin}{a^2f(2+}$$

Mathematica [A] time = 0.41, size = 83, normalized size = 0.86

$$\frac{(a(\sin(e+fx)+1))^{m+1}\left(\frac{2a^2d(c-d)(\sin(e+fx)+1)}{m+2}+\frac{a^2(c-d)^2}{m+1}+\frac{d^2(a\sin(e+fx)+a)^2}{m+3}\right)}{a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*((a^2*(c - d)^2)/(1 + m) + (2*a^2*(c - d)*d
*(1 + Sin[e + f*x]))/(2 + m) + (d^2*(a + a*Sin[e + f*x])^2)/(3 + m)))/(a^3*
f)
```

fricas [A] time = 0.48, size = 189, normalized size = 1.97

$$\frac{\left(\left(c^2+2\,cd+d^2\right)m^2-\left(\left(2\,cd+d^2\right)m^2+6\,cd+\left(8\,cd+d^2\right)m\right)\cos\left(fx+e\right)^2+6\,c^2+2\,d^2+\left(5\,c^2+6\,cd+d^2\right)m\right.}{fm^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] ((c^2 + 2*c*d + d^2)*m^2 - ((2*c*d + d^2)*m^2 + 6*c*d + (8*c*d + d^2)*m)*co
s(f*x + e)^2 + 6*c^2 + 2*d^2 + (5*c^2 + 6*c*d + d^2)*m + ((c^2 + 2*c*d + d^
2)*m^2 - (d^2*m^2 + 3*d^2*m + 2*d^2)*cos(f*x + e)^2 + 6*c^2 + 2*d^2 + (5*c^
2 + 6*c*d + d^2)*m)*sin(f*x + e))*(a*sin(f*x + e) + a)^m/(f*m^3 + 6*f*m^2 +
 11*f*m + 6*f)
```

giac [B] time = 0.16, size = 462, normalized size = 4.81

$$\frac{\left(\left(a\sin\left(fx+e\right)+a\right)^3\left(a\sin\left(fx+e\right)+a\right)^m m^2-2\left(a\sin\left(fx+e\right)+a\right)^2\left(a\sin\left(fx+e\right)+a\right)^m am^2+\left(a\sin\left(fx+e\right)+a\right)\left(a\sin\left(fx+e\right)+a\right)^m a^2m^2+3\left(a\sin\left(fx+e\right)+a\right)^3\left(a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] (((a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m^2 - 2*(a*sin(f*x + e) + a
)^2*(a*sin(f*x + e) + a)^m*a*m^2 + (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a
)^m*a^2*m^2 + 3*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m - 8*(a*sin(
f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a*m + 5*(a*sin(f*x + e) + a)*(a*sin(
f*x + e) + a)^m*a^2*m + 2*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m - 6
*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a + 6*(a*sin(f*x + e) + a)*(
a*sin(f*x + e) + a)^m*a^2)*d^2/(a^2*m^3 + 6*a^2*m^2 + 11*a^2*m + 6*a^2) + (
a*sin(f*x + e) + a)^(m + 1)*c^2/(m + 1) + 2*((a*sin(f*x + e) + a)^2*(a*sin(
f*x + e) + a)^m*m - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a*m + (a*si
n(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m - 2*(a*sin(f*x + e) + a)*(a*sin(f*
x + e) + a)^m*a)*c*d/((m^2 + 3*m + 2)*a))/(a*f)
```

maple [F] time = 6.35, size = 0, normalized size = 0.00

$$\int \cos \left(f x+e\right) \left(a+a \sin \left(f x+e\right)\right)^{m} \left(c+d \sin \left(f x+e\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

maxima [A] time = 0.67, size = 171, normalized size = 1.78

$$\frac{\frac{2\left(a^{m}(m+1)\sin\left(fx+e\right)^{2}+a^{m}m\sin\left(fx+e\right)-a^{m}\right)cd\left(\sin\left(fx+e\right)+1\right)^{m}}{m^{2}+3m+2}+\frac{\left(\left(m^{2}+3m+2\right)a^{m}\sin\left(fx+e\right)^{3}+\left(m^{2}+m\right)a^{m}\sin\left(fx+e\right)^{2}-2a^{m}m\sin\left(fx+e\right)+2a^{m}\right)d^{2}}{m^{3}+6m^{2}+11m+6}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] (2*(a^m*(m + 1)*sin(f*x + e)^2 + a^m*m*sin(f*x + e) - a^m)*c*d*(sin(f*x + e
) + 1)^m/(m^2 + 3*m + 2) + ((m^2 + 3*m + 2)*a^m*sin(f*x + e)^3 + (m^2 + m)*
a^m*sin(f*x + e)^2 - 2*a^m*m*sin(f*x + e) + 2*a^m)*d^2*(sin(f*x + e) + 1)^m
/(m^3 + 6*m^2 + 11*m + 6) + (a*sin(f*x + e) + a)^(m + 1)*c^2/(a*(m + 1)))/f
```

mupad [B] time = 11.48, size = 305, normalized size = 3.18

$$\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(20\,c^2\,m-12\,c\,d+2\,d^2\,m+24\,c^2\,\sin\left(e+f\,x\right)+6\,d^2\,\sin\left(e+f\,x\right)+24\,c^2+8\,d^2+4\,c^2\,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2,x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(20*c^2*m - 12*c*d + 2*d^2*m + 24*c^2*sin(e + f*x
) + 6*d^2*sin(e + f*x) + 24*c^2 + 8*d^2 + 4*c^2*m^2 + 2*d^2*m^2 - 2*d^2*sin
(3*e + 3*f*x) + 20*c^2*m*sin(e + f*x) + d^2*m*sin(e + f*x) - 2*d^2*m*cos(2*
e + 2*f*x) + 4*c^2*m^2*sin(e + f*x) - 3*d^2*m*sin(3*e + 3*f*x) + 3*d^2*m^2*
sin(e + f*x) + 8*c*d*m - 2*d^2*m^2*cos(2*e + 2*f*x) - d^2*m^2*sin(3*e + 3*f
*x) - 12*c*d*cos(2*e + 2*f*x) + 4*c*d*m^2 + 24*c*d*m*sin(e + f*x) - 16*c*d*
m*cos(2*e + 2*f*x) + 8*c*d*m^2*sin(e + f*x) - 4*c*d*m^2*cos(2*e + 2*f*x)))/
(4*f*(11*m + 6*m^2 + m^3 + 6))
```

sympy [A] time = 15.95, size = 1622, normalized size = 16.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((x*(c + d*sin(e))**2*(a*sin(e) + a)**m*cos(e), Eq(f, 0)), (-c**2/
(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 4*c*d*sin(e
 + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 2*c
*d/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 2*d**2*l
og(sin(e + f*x) + 1)*sin(e + f*x)**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*s
in(e + f*x) + 2*a**3*f) + 4*d**2*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**3
*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 2*d**2*log(sin(e +
 f*x) + 1)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) +
4*d**2*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a
**3*f) + 3*d**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*
f), Eq(m, -3)), (-c**2/(a**2*f*sin(e + f*x) + a**2*f) + 2*c*d*log(sin(e + f
*x) + 1)*sin(e + f*x)/(a**2*f*sin(e + f*x) + a**2*f) + 2*c*d*log(sin(e + f*
x) + 1)/(a**2*f*sin(e + f*x) + a**2*f) + 2*c*d/(a**2*f*sin(e + f*x) + a**2*
f) - 2*d**2*log(sin(e + f*x) + 1)*sin(e + f*x)/(a**2*f*sin(e + f*x) + a**2*
f) - 2*d**2*log(sin(e + f*x) + 1)/(a**2*f*sin(e + f*x) + a**2*f) + d**2*sin
(e + f*x)**2/(a**2*f*sin(e + f*x) + a**2*f) - 2*d**2/(a**2*f*sin(e + f*x) +
 a**2*f), Eq(m, -2)), (c**2*log(sin(e + f*x) + 1)/(a*f) - 2*c*d*log(sin(e +
 f*x) + 1)/(a*f) + 2*c*d*sin(e + f*x)/(a*f) + d**2*log(sin(e + f*x) + 1)/(a
*f) + d**2*sin(e + f*x)**2/(2*a*f) - d**2*sin(e + f*x)/(a*f), Eq(m, -1)), (
c**2*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**3 + 6*f*m**2 + 11*f*m
+ 6*f) + c**2*m**2*(a*sin(e + f*x) + a)**m/(f*m**3 + 6*f*m**2 + 11*f*m + 6*
f) + 5*c**2*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**3 + 6*f*m**2 + 11*
f*m + 6*f) + 5*c**2*m*(a*sin(e + f*x) + a)**m/(f*m**3 + 6*f*m**2 + 11*f*m +
 6*f) + 6*c**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**3 + 6*f*m**2 + 11
*f*m + 6*f) + 6*c**2*(a*sin(e + f*x) + a)**m/(f*m**3 + 6*f*m**2 + 11*f*m +
```

```
6*f) + 2*c*d*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**3 + 6*f*m**
2 + 11*f*m + 6*f) + 2*c*d*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**3
 + 6*f*m**2 + 11*f*m + 6*f) + 8*c*d*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)*
*2/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 6*c*d*m*(a*sin(e + f*x) + a)**m*sin
(e + f*x)/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) - 2*c*d*m*(a*sin(e + f*x) + a)
**m/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 6*c*d*(a*sin(e + f*x) + a)**m*sin(
e + f*x)**2/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) - 6*c*d*(a*sin(e + f*x) + a)
**m/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + d**2*m**2*(a*sin(e + f*x) + a)**m*
sin(e + f*x)**3/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + d**2*m**2*(a*sin(e + f
*x) + a)**m*sin(e + f*x)**2/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 3*d**2*m*(
a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) +
 d**2*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**3 + 6*f*m**2 + 11*f*m
 + 6*f) - 2*d**2*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**3 + 6*f*m**2
+ 11*f*m + 6*f) + 2*d**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**3 +
6*f*m**2 + 11*f*m + 6*f) + 2*d**2*(a*sin(e + f*x) + a)**m/(f*m**3 + 6*f*m**
2 + 11*f*m + 6*f), True))
```

3.923 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=59

$$\frac{d(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(c-d)(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

```
[Out] (c-d)*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)+d*(a+a*sin(f*x+e))^(2+m)/a^2/f/(2+m)
```

Rubi [A] time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{d(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(c-d)(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((c - d)*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (d*(a + a*Sin[e + f*
x])^(2 + m))/(a^2*f*(2 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))\,dx = \frac{\text{Subst}\left(\int (a+x)^m\left(c+\frac{dx}{a}\right)dx, x, a\sin(e+fx)\right)}{af}$$
$$= \frac{\text{Subst}\left(\int \left((c-d)(a+x)^m+\frac{d(a+x)^{1+m}}{a}\right)dx, x, a\sin(e+ \right.}{af}$$
$$= \frac{(c-d)(a+a\sin(e+fx))^{1+m}}{af(1+m)} + \frac{d(a+a\sin(e+fx))^{2+}}{a^2f(2+m)}$$

Mathematica [A] time = 0.12, size = 51, normalized size = 0.86

$$\frac{(a(\sin(e+fx)+1))^{m+1}(c(m+2)+d(m+1)\sin(e+fx)-d)}{af(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*(-d + c*(2 + m) + d*(1 + m)*Sin[e + f*x]))/
(a*f*(1 + m)*(2 + m))
```

fricas [A] time = 0.48, size = 70, normalized size = 1.19

$$-\frac{\left((dm+d)\cos\left(fx+e\right)^2-(c+d)m-((c+d)m+2c)\sin\left(fx+e\right)-2c\right)\left(a\sin\left(fx+e\right)+a\right)^m}{fm^2+3fm+2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -((d*m + d)*cos(f*x + e)^2 - (c + d)*m - ((c + d)*m + 2*c)*sin(f*x + e) - 2
*c)*(a*sin(f*x + e) + a)^m/(f*m^2 + 3*f*m + 2*f)
```

giac [B] time = 0.15, size = 156, normalized size = 2.64

$$\frac{\frac{\left(a\sin\left(fx+e\right)+a\right)^{m+1}c}{m+1} + \frac{\left(\left(a\sin\left(fx+e\right)+a\right)^2\left(a\sin\left(fx+e\right)+a\right)^m m-\left(a\sin\left(fx+e\right)+a\right)\left(a\sin\left(fx+e\right)+a\right)^m am+\left(a\sin\left(fx+e\right)+a\right)^2\left(a\sin\left(fx+e\right)+a\right)^m-2\left(a\sin\right.\right.}{\left(m^2+3m+2\right)a}}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] ((a*sin(f*x + e) + a)^(m + 1)*c/(m + 1) + ((a*sin(f*x + e) + a)^2*(a*sin(f*
x + e) + a)^m*m - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a*m + (a*sin(
f*x + e) + a)^2*(a*sin(f*x + e) + a)^m - 2*(a*sin(f*x + e) + a)*(a*sin(f*x
+ e) + a)^m*a)*d/((m^2 + 3*m + 2)*a))/(a*f)
```

maple [F] time = 4.62, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

maxima [A] time = 0.83, size = 83, normalized size = 1.41

$$\frac{\frac{\left(a^{m}(m+1)\sin\left(fx+e\right)^{2}+a^{m}m\sin\left(fx+e\right)-a^{m}\right)d\left(\sin\left(fx+e\right)+1\right)^{m}}{m^{2}+3m+2}+\frac{\left(a\sin\left(fx+e\right)+a\right)^{m+1}c}{a(m+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] ((a^m*(m + 1)*sin(f*x + e)^2 + a^m*m*sin(f*x + e) - a^m)*d*(sin(f*x + e) +
1)^m/(m^2 + 3*m + 2) + (a*sin(f*x + e) + a)^(m + 1)*c/(a*(m + 1)))/f
```

mupad [B] time = 9.76, size = 99, normalized size = 1.68

$$\frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(4c-d+2cm+dm+4c\sin\left(e+fx\right)+d\left(2\sin\left(e+fx\right)^{2}-1\right)+2cm\sin\left(e+fx\right.\right.}{2f\left(m^{2}+3m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)),x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(4*c - d + 2*c*m + d*m + 4*c*sin(e + f*x) + d*(2*
sin(e + f*x)^2 - 1) + 2*c*m*sin(e + f*x) + 2*d*m*sin(e + f*x) + d*m*(2*sin(
e + f*x)^2 - 1)))/(2*f*(3*m + m^2 + 2))
```

sympy [A] time = 5.59, size = 428, normalized size = 7.25

$$\begin{cases} x\left(c+d\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^{m}\cos\left(e\right) \\ -\frac{c}{a^{2}f\sin\left(e+fx\right)+a^{2}f}+\frac{d\log\left(\sin\left(e+fx\right)+1\right)\sin\left(e+fx\right)}{a^{2}f\sin\left(e+fx\right)+a^{2}f}+\frac{d\log\left(\sin\left(e+fx\right)+1\right)}{a^{2}f\sin\left(e+fx\right)+a^{2}f}+\frac{d}{a^{2}f\sin\left(e+fx\right)+a^{2}f} \\ \frac{c\log\left(\sin\left(e+fx\right)+1\right)}{af}-\frac{d\log\left(\sin\left(e+fx\right)+1\right)}{af}+\frac{d\sin\left(e+fx\right)}{af} \\ \frac{cm\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)}{fm^{2}+3fm+2f}+\frac{cm\left(a\sin\left(e+fx\right)+a\right)^{m}}{fm^{2}+3fm+2f}+\frac{2c\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)}{fm^{2}+3fm+2f}+\frac{2c\left(a\sin\left(e+fx\right)+a\right)^{m}}{fm^{2}+3fm+2f}+\frac{dm\left(a\sin\left(e+fx\right)+a\right)^{m}\sin}{fm^{2}+3fm+2f} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((x*(c + d*sin(e))*(a*sin(e) + a)**m*cos(e), Eq(f, 0)), (-c/(a**2*
f*sin(e + f*x) + a**2*f) + d*log(sin(e + f*x) + 1)*sin(e + f*x)/(a**2*f*sin
(e + f*x) + a**2*f) + d*log(sin(e + f*x) + 1)/(a**2*f*sin(e + f*x) + a**2*f
) + d/(a**2*f*sin(e + f*x) + a**2*f), Eq(m, -2)), (c*log(sin(e + f*x) + 1)/
(a*f) - d*log(sin(e + f*x) + 1)/(a*f) + d*sin(e + f*x)/(a*f), Eq(m, -1)), (
c*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**2 + 3*f*m + 2*f) + c*m*(a*si
n(e + f*x) + a)**m/(f*m**2 + 3*f*m + 2*f) + 2*c*(a*sin(e + f*x) + a)**m*sin
(e + f*x)/(f*m**2 + 3*f*m + 2*f) + 2*c*(a*sin(e + f*x) + a)**m/(f*m**2 + 3*
f*m + 2*f) + d*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**2 + 3*f*m +
2*f) + d*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**2 + 3*f*m + 2*f) + d*
(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**2 + 3*f*m + 2*f) - d*(a*sin(e
 + f*x) + a)**m/(f*m**2 + 3*f*m + 2*f), True))
```

3.924 $\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=59

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)}$$

```
[Out] hypergeom([1, 1+m],[2+m],-d*(1+sin(f*x+e))/(c-d))*(a+a*sin(f*x+e))^(1+m)/a/
(c-d)/f/(1+m)
```

Rubi [A] time = 0.10, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 68}

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x]),x]
```

```
[Out] (Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)*f*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^m}{c+\frac{dx}{a}}\,dx, x, a\sin(e+fx)\right)}{af}$$
$$= \frac{{}_2F_1\left(1, 1+m; 2+m; -\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)f(1+m)}$$

Mathematica [A] time = 0.10, size = 59, normalized size = 1.00

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1, m+1; m+2; -\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x]),x]
```

```
[Out] (Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)*f*(1 + m))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)}{d\sin\left(fx+e\right)+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c), x)
```

maple [F] time = 2.11, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}}{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="maxi
ma")
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)),x)
[Out] int((cos(e + f*x)*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m/(c+d*sin(f*x+e)),x)
[Out] Timed out
```

3.925 $\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=59

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(2,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)^2}$$

```
[Out] hypergeom([2, 1+m],[2+m],-d*(1+sin(f*x+e))/(c-d))*(a+a*sin(f*x+e))^(1+m)/a/
(c-d)^2/f/(1+m)
```

Rubi [A] time = 0.10, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 68}

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(2,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)^2*f*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^m}{\left(c+\frac{dx}{a}\right)^2}\,dx, x, a\sin(e+fx)\right)}{af}$$
$$= \frac{{}_2F_1\left(2,1+m;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)^2f(1+m)}$$

Mathematica [A] time = 0.09, size = 59, normalized size = 1.00

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(2,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)^2*f*(1 + m))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)/(d^2*cos(f*x + e)^2 - 2*c*d*s
in(f*x + e) - c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)}{\left(d\sin\left(fx+e\right)+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c)^2, x)
```

maple [F] time = 4.58, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c+d\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] int((cos(e + f*x)*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.926 $\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=59

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(3,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)^3}$$

```
[Out] hypergeom([3, 1+m],[2+m],-d*(1+sin(f*x+e))/(c-d))*(a+a*sin(f*x+e))^(1+m)/a/
(c-d)^3/f/(1+m)
```

Rubi [A] time = 0.10, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 68}

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(3,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (Hypergeometric2F1[3, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)^3*f*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^m}{\left(c+\frac{dx}{a}\right)^3}\,dx, x, a\sin(e+fx)\right)}{af}$$
$$= \frac{{}_2F_1\left(3,1+m;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)^3 f(1+m)}$$

Mathematica [A] time = 0.08, size = 59, normalized size = 1.00

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(3,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (Hypergeometric2F1[3, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)^3*f*(1 + m))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)/(3*c*d^2*cos(f*x + e)^2 - c^3
 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)}{\left(d\sin\left(fx+e\right)+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c)^3, x)
```

maple [F] time = 4.35, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c+d\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)}{\left(d\sin\left(fx+e\right)+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3,x)
```

```
[Out] int((cos(e + f*x)*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.927 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=54

$$-\frac{\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(1,m+n+2;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

```
[Out] -hypergeom([1, 2+m+n],[2+m],1+sin(d*x+c))*sin(d*x+c)^(1+n)*(a+a*sin(d*x+c))
^(1+m)/a/d/(1+m)
```

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 66, 64}

$$\frac{(\sin(c+dx)+1)^{-m}\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^m\,{}_2F_1(-m,n+1;n+2;-\sin(c+dx))}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[-m, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a
 + a*Sin[c + d*x])^m)/(d*(1 + n)*(1 + Sin[c + d*x])^m)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (a+x)^m\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{((1+\sin(c+dx))^{-m}(a+a\sin(c+dx))^m)\,\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (1$$
$$= \frac{{}_2F_1(-m,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)(1+\sin}{d(1+n)}$$

Mathematica [A] time = 0.07, size = 61, normalized size = 1.13

$$\frac{(\sin(c+dx)+1)^{-m}\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^m\,{}_2F_1(-m,n+1;n+2;-\sin(c+dx))}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[-m, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a
 + a*Sin[c + d*x])^m)/(d*(1 + n)*(1 + Sin[c + d*x])^m)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^m\sin(dx+c)^n\cos(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*sin(d*x + c)^n*cos(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m\sin(dx+c)^n\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sin(d*x + c)^n*cos(d*x + c), x)
```

maple [F] time = 5.19, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\left(\sin^{n}(dx+c)\right)(a+a\sin(dx+c))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x)
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{m}\sin(dx+c)^{n}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^m*sin(d*x + c)^n*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos(c+d\,x)\,\sin(c+d\,x)^{n}\,(a+a\,\sin(c+d\,x))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^n*(a + a*sin(c + d*x))^m,x)
[Out] int(cos(c + d*x)*sin(c + d*x)^n*(a + a*sin(c + d*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{m}\sin^{n}(c+dx)\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c))**m,x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*sin(c + d*x)**n*cos(c + d*x), x)
```

3.928 $\int \cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=134

$$\frac{(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}-\frac{4(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}+\frac{6(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}-\frac{4(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}+\frac{(a\sin(c+}{ad(m}$$

```
[Out] (a+a*sin(d*x+c))^(1+m)/a/d/(1+m)-4*(a+a*sin(d*x+c))^(2+m)/a^2/d/(2+m)+6*(a+
a*sin(d*x+c))^(3+m)/a^3/d/(3+m)-4*(a+a*sin(d*x+c))^(4+m)/a^4/d/(4+m)+(a+a*s
in(d*x+c))^(5+m)/a^5/d/(5+m)
```

Rubi [A] time = 0.12, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{4(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}+\frac{6(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}-\frac{4(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}+\frac{(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}+\frac{(a\sin(c+}{ad(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m)) - (4*(a + a*Sin[c + d*x])^(2 + m
))/(a^2*d*(2 + m)) + (6*(a + a*Sin[c + d*x])^(3 + m))/(a^3*d*(3 + m)) - (4*
(a + a*Sin[c + d*x])^(4 + m))/(a^4*d*(4 + m)) + (a + a*Sin[c + d*x])^(5 + m
)/(a^5*d*(5 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+x)^m}{a^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^4(a+x)^m\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4(a+x)^m - 4a^3(a+x)^{1+m} + 6a^2(a+x)^{2+m} - 4a\right.\right.}{a^5 d}$$
$$= \frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)} - \frac{4(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)} + \frac{6(a+a\,\mathrm{s}}{a^3}$$

Mathematica [A] time = 1.40, size = 150, normalized size = 1.12

$$\frac{(a(\sin(c+dx)+1))^{m+1}\left(\frac{3\left(-2\left(m^2+3m+2\right)\cos(2(c+dx))-8(m+1)\sin(c+dx)+m^2+m+6\right)}{(m+1)(m+2)(m+3)} + \frac{16(\sin(c+dx)+1)^4}{m+5} - \frac{64(\sin(c+dx)+1)^3}{m+4} + \frac{84(\sin}{}\right.}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(1 + m)*(7/(1 + m) - (40*(1 + Sin[c + d*x]))/(2 + m
) + (84*(1 + Sin[c + d*x])^2)/(3 + m) - (64*(1 + Sin[c + d*x])^3)/(4 + m) +
 (16*(1 + Sin[c + d*x])^4)/(5 + m) + (3*(6 + m + m^2 - 2*(2 + 3*m + m^2)*Co
s[2*(c + d*x)] - 8*(1 + m)*Sin[c + d*x]))/((1 + m)*(2 + m)*(3 + m))))/(16*a
*d)
```

fricas [A] time = 0.48, size = 197, normalized size = 1.47

$$\frac{\left(\left(m^4+6\,m^3+11\,m^2+6\,m\right)\cos\left(dx+c\right)^4+m^4+6\,m^3-2\left(m^4+6\,m^3+17\,m^2+12\,m\right)\cos\left(dx+c\right)^2+23\,m^2+\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] ((m^4 + 6*m^3 + 11*m^2 + 6*m)*cos(d*x + c)^4 + m^4 + 6*m^3 - 2*(m^4 + 6*m^3
 + 17*m^2 + 12*m)*cos(d*x + c)^2 + 23*m^2 + ((m^4 + 10*m^3 + 35*m^2 + 50*m
+ 24)*cos(d*x + c)^4 + m^4 + 6*m^3 - 2*(m^4 + 8*m^3 + 29*m^2 + 46*m + 24)*c
os(d*x + c)^2 + 23*m^2 + 18*m + 24)*sin(d*x + c) + 18*m + 24)*(a*sin(d*x +
c) + a)^m/(d*m^5 + 15*d*m^4 + 85*d*m^3 + 225*d*m^2 + 274*d*m + 120*d)
```

giac [B] time = 0.27, size = 402, normalized size = 3.00

$$\frac{(a\sin(dx+c)+a)^m m^4\sin(dx+c)^5+(a\sin(dx+c)+a)^m m^4\sin(dx+c)^4+10\,(a\sin(dx+c)+a)^m m^3\sin(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] ((a*sin(d*x + c) + a)^m*m^4*sin(d*x + c)^5 + (a*sin(d*x + c) + a)^m*m^4*sin
(d*x + c)^4 + 10*(a*sin(d*x + c) + a)^m*m^3*sin(d*x + c)^5 + 6*(a*sin(d*x +
 c) + a)^m*m^3*sin(d*x + c)^4 + 35*(a*sin(d*x + c) + a)^m*m^2*sin(d*x + c)^
5 - 4*(a*sin(d*x + c) + a)^m*m^3*sin(d*x + c)^3 + 11*(a*sin(d*x + c) + a)^m
*m^2*sin(d*x + c)^4 + 50*(a*sin(d*x + c) + a)^m*m*sin(d*x + c)^5 - 12*(a*si
n(d*x + c) + a)^m*m^2*sin(d*x + c)^3 + 6*(a*sin(d*x + c) + a)^m*m*sin(d*x +
 c)^4 + 24*(a*sin(d*x + c) + a)^m*sin(d*x + c)^5 + 12*(a*sin(d*x + c) + a)^
m*m^2*sin(d*x + c)^2 - 8*(a*sin(d*x + c) + a)^m*m*sin(d*x + c)^3 + 12*(a*si
n(d*x + c) + a)^m*m*sin(d*x + c)^2 - 24*(a*sin(d*x + c) + a)^m*m*sin(d*x +
c) + 24*(a*sin(d*x + c) + a)^m)/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m +
 120)*d)
```

maple [F] time = 6.76, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\left(\sin^4(dx+c)\right)(a+a\sin(dx+c))^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
```

maxima [A] time = 0.91, size = 159, normalized size = 1.19

$$\frac{\left(\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)a^m\sin(dx+c)^5+\left(m^4+6\,m^3+11\,m^2+6\,m\right)a^m\sin(dx+c)^4-4\left(m^3+3\,m\right.}{\left(m^5+15\,m^4+85\,m^3+225\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*a^m*sin(d*x + c)^5 + (m^4 + 6*m^3 + 11
*m^2 + 6*m)*a^m*sin(d*x + c)^4 - 4*(m^3 + 3*m^2 + 2*m)*a^m*sin(d*x + c)^3 +
 12*(m^2 + m)*a^m*sin(d*x + c)^2 - 24*a^m*m*sin(d*x + c) + 24*a^m)*(sin(d*x
 + c) + 1)^m/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*d)
```

mupad [B] time = 12.19, size = 349, normalized size = 2.60

$$\frac{(a\ (\sin(c+d\,x)+1))^m\ \left(132\,m+240\ \sin(c+d\,x)-120\ \sin(3\,c+3\,d\,x)+24\ \sin(5\,c+5\,d\,x)+20\,m\ \sin(c+d\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)*sin(c + d*x)^4*(a + a*sin(c + d*x))^m,x)

[Out] ((a*(sin(c + d*x) + 1))^m*(132*m + 240*sin(c + d*x) - 120*sin(3*c + 3*d*x) + 24*sin(5*c + 5*d*x) + 20*m*sin(c + d*x) - 144*m*cos(2*c + 2*d*x) + 12*m*cos(4*c + 4*d*x) - 218*m*sin(3*c + 3*d*x) + 50*m*sin(5*c + 5*d*x) + 206*m^2*sin(c + d*x) + 52*m^3*sin(c + d*x) + 10*m^4*sin(c + d*x) + 162*m^2 + 36*m^3 + 6*m^4 - 184*m^2*cos(2*c + 2*d*x) - 48*m^3*cos(2*c + 2*d*x) - 8*m^4*cos(2*c + 2*d*x) + 22*m^2*cos(4*c + 4*d*x) + 12*m^3*cos(4*c + 4*d*x) + 2*m^4*cos(4*c + 4*d*x) - 127*m^2*sin(3*c + 3*d*x) - 34*m^3*sin(3*c + 3*d*x) - 5*m^4*sin(3*c + 3*d*x) + 35*m^2*sin(5*c + 5*d*x) + 10*m^3*sin(5*c + 5*d*x) + m^4*sin(5*c + 5*d*x) + 384))/(16*d*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))

sympy [A] time = 67.34, size = 2747, normalized size = 20.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*sin(d*x+c)**4*(a+a*sin(d*x+c))**m,x)

[Out] Piecewise((x*(a*sin(c) + a)**m*sin(c)**4*cos(c), Eq(d, 0)), (12*log(sin(c + d*x) + 1)*sin(c + d*x)**4/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 48*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 72*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 48*log(sin(c + d*x) + 1)*sin(c + d*x)/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 12*log(sin(c + d*x) + 1)/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 48*sin(c + d*x)**3/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 108*sin(c + d*x)**2/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 88*sin(c + d*x)/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12

```
*a**5*d) + 25/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a
**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d), Eq(m, -5)), (-
12*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(3*a**4*d*sin(c + d*x)**3 + 9*a**4
*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 36*log(sin(c + d*x
) + 1)*sin(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2
 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 36*log(sin(c + d*x) + 1)*sin(c + d*x
)/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d
*x) + 3*a**4*d) - 12*log(sin(c + d*x) + 1)/(3*a**4*d*sin(c + d*x)**3 + 9*a*
*4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) + 3*sin(c + d*x)**
4/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d
*x) + 3*a**4*d) - 36*sin(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*s
in(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 54*sin(c + d*x)/(3*a**
4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*
a**4*d) - 22/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*
d*sin(c + d*x) + 3*a**4*d), Eq(m, -4)), (12*log(sin(c + d*x) + 1)*sin(c + d
*x)**2/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 24*l
og(sin(c + d*x) + 1)*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(
c + d*x) + 2*a**3*d) + 12*log(sin(c + d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 +
 4*a**3*d*sin(c + d*x) + 2*a**3*d) + sin(c + d*x)**4/(2*a**3*d*sin(c + d*x)
**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 4*sin(c + d*x)**3/(2*a**3*d*sin(c
 + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 24*sin(c + d*x)/(2*a**3*d*
sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 18/(2*a**3*d*sin(c +
d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d), Eq(m, -3)), (-12*log(sin(c + d
*x) + 1)*sin(c + d*x)/(3*a**2*d*sin(c + d*x) + 3*a**2*d) - 12*log(sin(c + d
*x) + 1)/(3*a**2*d*sin(c + d*x) + 3*a**2*d) + sin(c + d*x)**4/(3*a**2*d*sin
(c + d*x) + 3*a**2*d) - 2*sin(c + d*x)**3/(3*a**2*d*sin(c + d*x) + 3*a**2*d
) + 6*sin(c + d*x)**2/(3*a**2*d*sin(c + d*x) + 3*a**2*d) - 12/(3*a**2*d*sin
(c + d*x) + 3*a**2*d), Eq(m, -2)), (log(sin(c + d*x) + 1)/(a*d) + sin(c + d
*x)**4/(4*a*d) - sin(c + d*x)**3/(3*a*d) - sin(c + d*x)/(a*d) - cos(c + d*x
)**2/(2*a*d), Eq(m, -1)), (m**4*(a*sin(c + d*x) + a)**m*sin(c + d*x)**5/(d*
m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + m**4*(a*sin(
c + d*x) + a)**m*sin(c + d*x)**4/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m*
*2 + 274*d*m + 120*d) + 10*m**3*(a*sin(c + d*x) + a)**m*sin(c + d*x)**5/(d*
m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 6*m**3*(a*si
n(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*
m**2 + 274*d*m + 120*d) - 4*m**3*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d
*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 35*m**2*(a*
sin(c + d*x) + a)**m*sin(c + d*x)**5/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*
d*m**2 + 274*d*m + 120*d) + 11*m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4
/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) - 12*m**2*
(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 2
25*d*m**2 + 274*d*m + 120*d) + 12*m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)
**2/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 50*m*
(a*sin(c + d*x) + a)**m*sin(c + d*x)**5/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 2
25*d*m**2 + 274*d*m + 120*d) + 6*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/
```

```
(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) - 8*m*(a*si
n(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*
m**2 + 274*d*m + 120*d) + 12*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**2/(d*m
**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) - 24*m*(a*sin(c
 + d*x) + a)**m*sin(c + d*x)/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 +
 274*d*m + 120*d) + 24*(a*sin(c + d*x) + a)**m*sin(c + d*x)**5/(d*m**5 + 15
*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 24*(a*sin(c + d*x) +
a)**m/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d), True
))
```

3.929 $\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=108

$$\frac{(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}-\frac{3(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}+\frac{3(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}-\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

```
[Out] -(a+a*sin(d*x+c))^(1+m)/a/d/(1+m)+3*(a+a*sin(d*x+c))^(2+m)/a^2/d/(2+m)-3*(a
+a*sin(d*x+c))^(3+m)/a^3/d/(3+m)+(a+a*sin(d*x+c))^(4+m)/a^4/d/(4+m)
```

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{3(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}-\frac{3(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}+\frac{(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}-\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m))) + (3*(a + a*Sin[c + d*x])^(2
+ m))/(a^2*d*(2 + m)) - (3*(a + a*Sin[c + d*x])^(3 + m))/(a^3*d*(3 + m)) +
(a + a*Sin[c + d*x])^(4 + m)/(a^4*d*(4 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^m}{a^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^3(a+x)^m\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^3(a+x)^m + 3a^2(a+x)^{1+m} - 3a(a+x)^{2+m} + (a\right.\right.}{a^4 d}$$
$$= -\frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)} + \frac{3(a+a\sin(c+dx))^{2+m}}{a^2 d(2+m)} - \frac{3(a+a}{a}$$

Mathematica [A] time = 0.57, size = 94, normalized size = 0.87

$$\frac{\left(-3\left(m^2+3m+2\right)\sin^2(c+dx)+\left(m^3+6m^2+11m+6\right)\sin^3(c+dx)+6(m+1)\sin(c+dx)-6\right)(a(\sin(c+dx)}{ad(m+1)(m+2)(m+3)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(1 + m)*(-6 + 6*(1 + m)*Sin[c + d*x] - 3*(2 + 3*m +
 m^2)*Sin[c + d*x]^2 + (6 + 11*m + 6*m^2 + m^3)*Sin[c + d*x]^3))/(a*d*(1 +
m)*(2 + m)*(3 + m)*(4 + m))
```

fricas [A] time = 0.48, size = 140, normalized size = 1.30

$$\frac{\left(\left(m^3+6\,m^2+11\,m+6\right)\cos\left(dx+c\right)^4+m^3-\left(2\,m^3+9\,m^2+19\,m+12\right)\cos\left(dx+c\right)^2+3\,m^2+\left(m^3-\left(m^3+3\,m\right.\right.\right.}{dm^4+10\,dm^3+35\,dm^2+50\,dm+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] ((m^3 + 6*m^2 + 11*m + 6)*cos(d*x + c)^4 + m^3 - (2*m^3 + 9*m^2 + 19*m + 12
)*cos(d*x + c)^2 + 3*m^2 + (m^3 - (m^3 + 3*m^2 + 2*m)*cos(d*x + c)^2 + 3*m^
2 + 8*m)*sin(d*x + c) + 8*m)*(a*sin(d*x + c) + a)^m/(d*m^4 + 10*d*m^3 + 35*
d*m^2 + 50*d*m + 24*d)
```

giac [B] time = 0.28, size = 274, normalized size = 2.54

$$\frac{(a\sin\left(dx+c\right)+a)^m m^3\sin\left(dx+c\right)^4+(a\sin\left(dx+c\right)+a)^m m^3\sin\left(dx+c\right)^3+6\,(a\sin\left(dx+c\right)+a)^m m^2\sin\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="giac")

[Out] ((a*sin(d*x + c) + a)^m*m^3*sin(d*x + c)^4 + (a*sin(d*x + c) + a)^m*m^3*sin(d*x + c)^3 + 6*(a*sin(d*x + c) + a)^m*m^2*sin(d*x + c)^4 + 3*(a*sin(d*x + c) + a)^m*m^2*sin(d*x + c)^3 + 11*(a*sin(d*x + c) + a)^m*m*sin(d*x + c)^4 - 3*(a*sin(d*x + c) + a)^m*m^2*sin(d*x + c)^2 + 2*(a*sin(d*x + c) + a)^m*m*sin(d*x + c)^3 + 6*(a*sin(d*x + c) + a)^m*sin(d*x + c)^4 - 3*(a*sin(d*x + c) + a)^m*m*sin(d*x + c)^2 + 6*(a*sin(d*x + c) + a)^m*m*sin(d*x + c) - 6*(a*sin(d*x + c) + a)^m)/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*d)

maple [F] time = 3.23, size = 0, normalized size = 0.00

$$\int \cos \left(dx+c\right) \left(\sin^{3}\left(dx+c\right)\right) \left(a+a\sin \left(dx+c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x)

[Out] int(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x)

maxima [A] time = 0.69, size = 119, normalized size = 1.10

$$\frac{\left(\left(m^{3}+6\,m^{2}+11\,m+6\right)a^{m}\sin \left(dx+c\right)^{4}+\left(m^{3}+3\,m^{2}+2\,m\right)a^{m}\sin \left(dx+c\right)^{3}-3\,\left(m^{2}+m\right)a^{m}\sin \left(dx+c\right)^{2}+6}{\left(m^{4}+10\,m^{3}+35\,m^{2}+50\,m+24\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="maxima")

[Out] ((m^3 + 6*m^2 + 11*m + 6)*a^m*sin(d*x + c)^4 + (m^3 + 3*m^2 + 2*m)*a^m*sin(d*x + c)^3 - 3*(m^2 + m)*a^m*sin(d*x + c)^2 + 6*a^m*m*sin(d*x + c) - 6*a^m)*(sin(d*x + c) + 1)^m/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*d)

mupad [B] time = 10.74, size = 224, normalized size = 2.07

$$\frac{\left(a\,\left(\sin \left(c+d\,x\right)+1\right)\right)^{m}\,\left(21\,m-24\,\cos \left(2\,c+2\,d\,x\right)+6\,\cos \left(4\,c+4\,d\,x\right)+60\,m\,\sin \left(c+d\,x\right)-32\,m\,\cos \left(2\,c+2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)*sin(c + d*x)^3*(a + a*sin(c + d*x))^m,x)

[Out] ((a*(sin(c + d*x) + 1))^m*(21*m - 24*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x) + 60*m*sin(c + d*x) - 32*m*cos(2*c + 2*d*x) + 11*m*cos(4*c + 4*d*x) - 4*m*s

```
in(3*c + 3*d*x) + 18*m^2*sin(c + d*x) + 6*m^3*sin(c + d*x) + 6*m^2 + 3*m^3
- 12*m^2*cos(2*c + 2*d*x) - 4*m^3*cos(2*c + 2*d*x) + 6*m^2*cos(4*c + 4*d*x)
 + m^3*cos(4*c + 4*d*x) - 6*m^2*sin(3*c + 3*d*x) - 2*m^3*sin(3*c + 3*d*x) -
 30))/(8*d*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
```

sympy [A] time = 28.58, size = 1508, normalized size = 13.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**m*sin(c)**3*cos(c), Eq(d, 0)), (6*log(sin(c +
d*x) + 1)*sin(c + d*x)**3/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x
)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 18*log(sin(c + d*x) + 1)*sin(c
+ d*x)**2/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d
*sin(c + d*x) + 6*a**4*d) + 18*log(sin(c + d*x) + 1)*sin(c + d*x)/(6*a**4*d
*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a
**4*d) + 6*log(sin(c + d*x) + 1)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(
c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 18*sin(c + d*x)**2/(6*a*
*4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) +
 6*a**4*d) + 27*sin(c + d*x)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c +
d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 11/(6*a**4*d*sin(c + d*x)**3
 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d), Eq(m, -4
)), (-6*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)**2 + 4
*a**3*d*sin(c + d*x) + 2*a**3*d) - 12*log(sin(c + d*x) + 1)*sin(c + d*x)/(2
*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 6*log(sin(c +
 d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) +
2*sin(c + d*x)**3/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**
3*d) - 12*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) +
2*a**3*d) - 9/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d)
, Eq(m, -3)), (6*log(sin(c + d*x) + 1)*sin(c + d*x)/(2*a**2*d*sin(c + d*x)
+ 2*a**2*d) + 6*log(sin(c + d*x) + 1)/(2*a**2*d*sin(c + d*x) + 2*a**2*d) +
sin(c + d*x)**3/(2*a**2*d*sin(c + d*x) + 2*a**2*d) - 3*sin(c + d*x)**2/(2*a
**2*d*sin(c + d*x) + 2*a**2*d) + 6/(2*a**2*d*sin(c + d*x) + 2*a**2*d), Eq(m
, -2)), (-log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)**3/(3*a*d) + sin(c + d
*x)/(a*d) + cos(c + d*x)**2/(2*a*d), Eq(m, -1)), (m**3*(a*sin(c + d*x) + a)
**m*sin(c + d*x)**4/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m + 24*d) + m**3
*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**4 + 10*d*m**3 + 35*d*m**2 +
50*d*m + 24*d) + 6*m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**4 + 1
0*d*m**3 + 35*d*m**2 + 50*d*m + 24*d) + 3*m**2*(a*sin(c + d*x) + a)**m*sin(
c + d*x)**3/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m + 24*d) - 3*m**2*(a*si
n(c + d*x) + a)**m*sin(c + d*x)**2/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m
 + 24*d) + 11*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**4 + 10*d*m**3
 + 35*d*m**2 + 50*d*m + 24*d) + 2*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3
```

```
/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m + 24*d) - 3*m*(a*sin(c + d*x) + a
)**m*sin(c + d*x)**2/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m + 24*d) + 6*m
*(a*sin(c + d*x) + a)**m*sin(c + d*x)/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*
d*m + 24*d) + 6*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**4 + 10*d*m**3
 + 35*d*m**2 + 50*d*m + 24*d) - 6*(a*sin(c + d*x) + a)**m/(d*m**4 + 10*d*m*
*3 + 35*d*m**2 + 50*d*m + 24*d), True))
```

3.930 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=80

$$\frac{(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}-\frac{2(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}+\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

```
[Out] (a+a*sin(d*x+c))^(1+m)/a/d/(1+m)-2*(a+a*sin(d*x+c))^(2+m)/a^2/d/(2+m)+(a+a*
sin(d*x+c))^(3+m)/a^3/d/(3+m)
```

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{2(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}+\frac{(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}+\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m)) - (2*(a + a*Sin[c + d*x])^(2 + m
))/(a^2*d*(2 + m)) + (a + a*Sin[c + d*x])^(3 + m)/(a^3*d*(3 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+x)^m}{a^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^2(a+x)^m\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2(a+x)^m - 2a(a+x)^{1+m} + (a+x)^{2+m}\right)dx, x, a\right.}{a^3d}$$
$$= \frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)} - \frac{2(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)} + \frac{(a+a}{a}$$

Mathematica [A] time = 0.19, size = 77, normalized size = 0.96

$$-\frac{(a(\sin(c+dx)+1))^{m+1}\left((m^2+3m+2)\cos(2(c+dx))+4(m+1)\sin(c+dx)-m^2-3m-6\right)}{2ad(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -1/2*((a*(1 + Sin[c + d*x]))^(1 + m)*(-6 - 3*m - m^2 + (2 + 3*m + m^2)*Cos[
2*(c + d*x)] + 4*(1 + m)*Sin[c + d*x]))/(a*d*(1 + m)*(2 + m)*(3 + m))
```

fricas [A] time = 0.45, size = 93, normalized size = 1.16

$$-\frac{\left((m^2+m)\cos(dx+c)^2-m^2+\left((m^2+3m+2)\cos(dx+c)^2-m^2-m-2\right)\sin(dx+c)-m-2\right)(a\sin(dx+}{dm^3+6dm^2+11dm+6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -((m^2 + m)*cos(d*x + c)^2 - m^2 + ((m^2 + 3*m + 2)*cos(d*x + c)^2 - m^2 -
m - 2)*sin(d*x + c) - m - 2)*(a*sin(d*x + c) + a)^m/(d*m^3 + 6*d*m^2 + 11*d
*m + 6*d)
```

giac [B] time = 0.26, size = 170, normalized size = 2.12

$$\frac{(a\sin(dx+c)+a)^m m^2\sin(dx+c)^3 + (a\sin(dx+c)+a)^m m^2\sin(dx+c)^2 + 3(a\sin(dx+c)+a)^m m\sin(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] ((a*sin(d*x + c) + a)^m*m^2*sin(d*x + c)^3 + (a*sin(d*x + c) + a)^m*m^2*sin
(d*x + c)^2 + 3*(a*sin(d*x + c) + a)^m*m*sin(d*x + c)^3 + (a*sin(d*x + c) +
 a)^m*m*sin(d*x + c)^2 + 2*(a*sin(d*x + c) + a)^m*sin(d*x + c)^3 - 2*(a*sin
(d*x + c) + a)^m*m*sin(d*x + c) + 2*(a*sin(d*x + c) + a)^m)/((m^3 + 6*m^2 +
 11*m + 6)*d)
```

maple [F] time = 4.78, size = 0, normalized size = 0.00

$$\int \cos\left(dx+c\right)\left(\sin^{2}\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

maxima [A] time = 0.64, size = 84, normalized size = 1.05

$$\frac{\left(\left(m^{2}+3\,m+2\right)a^{m}\sin\left(dx+c\right)^{3}+\left(m^{2}+m\right)a^{m}\sin\left(dx+c\right)^{2}-2\,a^{m}m\sin\left(dx+c\right)+2\,a^{m}\right)\left(\sin\left(dx+c\right)+1\right)^{m}}{\left(m^{3}+6\,m^{2}+11\,m+6\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((m^2 + 3*m + 2)*a^m*sin(d*x + c)^3 + (m^2 + m)*a^m*sin(d*x + c)^2 - 2*a^m*
m*sin(d*x + c) + 2*a^m)*(sin(d*x + c) + 1)^m/((m^3 + 6*m^2 + 11*m + 6)*d)
```

mupad [B] time = 9.92, size = 138, normalized size = 1.72

$$\frac{\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)^{m}\,\left(2\,m+6\,\sin\left(c+d\,x\right)-2\,\sin\left(3\,c+3\,d\,x\right)+m\,\sin\left(c+d\,x\right)+2\,m\,\left(2\sin\left(c+d\,x\right)^{2}-1\right)-}{4\,d\,\left(m^{3}+6\,m^{2}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^2*(a + a*sin(c + d*x))^m,x)
```

```
[Out] ((a*(sin(c + d*x) + 1))^m*(2*m + 6*sin(c + d*x) - 2*sin(3*c + 3*d*x) + m*si
n(c + d*x) + 2*m*(2*sin(c + d*x)^2 - 1) - 3*m*sin(3*c + 3*d*x) + 3*m^2*sin(
c + d*x) + 2*m^2*(2*sin(c + d*x)^2 - 1) + 2*m^2 - m^2*sin(3*c + 3*d*x) + 8)
)/(4*d*(11*m + 6*m^2 + m^3 + 6))
```

sympy [A] time = 11.85, size = 697, normalized size = 8.71

$$\begin{cases} x(a\sin(c)+a)^m\sin^2(c)\cos(c) \\ \frac{2\log(\sin(c+dx)+1)\sin^2(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d}+\frac{4\log(\sin(c+dx)+1)\sin(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d}+\frac{2\log(\sin(c+dx)+1)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d}+\frac{4}{2a^3d\sin^2(c+d} \\ -\frac{2\log(\sin(c+dx)+1)\sin(c+dx)}{a^2d\sin(c+dx)+a^2d}-\frac{2\log(\sin(c+dx)+1)}{a^2d\sin(c+dx)+a^2d}+\frac{\sin^2(c+dx)}{a^2d\sin(c+dx)+a^2d}-\frac{2}{a^2d\sin(c+dx)+a^2d} \\ \frac{\log(\sin(c+dx)+1)}{ad}-\frac{\sin(c+dx)}{ad}-\frac{\cos^2(c+dx)}{2ad} \\ \frac{m^2(a\sin(c+dx)+a)^m\sin^3(c+dx)}{dm^3+6dm^2+11dm+6d}+\frac{m^2(a\sin(c+dx)+a)^m\sin^2(c+dx)}{dm^3+6dm^2+11dm+6d}+\frac{3m(a\sin(c+dx)+a)^m\sin^3(c+dx)}{dm^3+6dm^2+11dm+6d}+\frac{m(a\sin(c+dx)+a)^m\sin^2(c+dx)}{dm^3+6dm^2+11dm+6d}- \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**m*sin(c)**2*cos(c), Eq(d, 0)), (2*log(sin(c +
d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x)
 + 2*a**3*d) + 4*log(sin(c + d*x) + 1)*sin(c + d*x)/(2*a**3*d*sin(c + d*x)*
*2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 2*log(sin(c + d*x) + 1)/(2*a**3*d*
sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 4*sin(c + d*x)/(2*a**
3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 3/(2*a**3*d*sin(c
 + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d), Eq(m, -3)), (-2*log(sin(c +
 d*x) + 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**2*d) - 2*log(sin(c + d*x)
 + 1)/(a**2*d*sin(c + d*x) + a**2*d) + sin(c + d*x)**2/(a**2*d*sin(c + d*x)
 + a**2*d) - 2/(a**2*d*sin(c + d*x) + a**2*d), Eq(m, -2)), (log(sin(c + d*x
) + 1)/(a*d) - sin(c + d*x)/(a*d) - cos(c + d*x)**2/(2*a*d), Eq(m, -1)), (m
**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**3 + 6*d*m**2 + 11*d*m + 6
*d) + m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**2/(d*m**3 + 6*d*m**2 + 11*
d*m + 6*d) + 3*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**3 + 6*d*m**2
 + 11*d*m + 6*d) + m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**2/(d*m**3 + 6*d*
m**2 + 11*d*m + 6*d) - 2*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)/(d*m**3 + 6
*d*m**2 + 11*d*m + 6*d) + 2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**3
 + 6*d*m**2 + 11*d*m + 6*d) + 2*(a*sin(c + d*x) + a)**m/(d*m**3 + 6*d*m**2
+ 11*d*m + 6*d), True))
```

3.931 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=54

$$\frac{(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}-\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

```
[Out] -(a+a*sin(d*x+c))^(1+m)/a/d/(1+m)+(a+a*sin(d*x+c))^(2+m)/a^2/d/(2+m)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 43}

$$\frac{(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}-\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m))) + (a + a*Sin[c + d*x])^(2 + m
)/(a^2*d*(2 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^m}{a}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x(a+x)^m\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-a(a+x)^m+(a+x)^{1+m}\right)dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= -\frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)} + \frac{(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.80

$$\frac{((m+1)\sin(c+dx)-1)(a(\sin(c+dx)+1))^{m+1}}{ad(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(1 + m)*(-1 + (1 + m)*Sin[c + d*x]))/(a*d*(1 + m)*(
2 + m))
```

fricas [A] time = 0.45, size = 54, normalized size = 1.00

$$-\frac{\left((m+1)\cos\left(dx+c\right)^2 - m\sin\left(dx+c\right) - m\right)\left(a\sin\left(dx+c\right)+a\right)^m}{dm^2+3\,dm+2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -((m + 1)*cos(d*x + c)^2 - m*sin(d*x + c) - m)*(a*sin(d*x + c) + a)^m/(d*m^
2 + 3*d*m + 2*d)
```

giac [A] time = 0.26, size = 92, normalized size = 1.70

$$\frac{\left(a\sin\left(dx+c\right)+a\right)^m m\sin\left(dx+c\right)^2 + \left(a\sin\left(dx+c\right)+a\right)^m m\sin\left(dx+c\right) + \left(a\sin\left(dx+c\right)+a\right)^m \sin\left(dx+c\right)^2 -}{\left(m^2+3\,m+2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] ((a*sin(d*x + c) + a)^m*m*sin(d*x + c)^2 + (a*sin(d*x + c) + a)^m*m*sin(d*x
 + c) + (a*sin(d*x + c) + a)^m*sin(d*x + c)^2 - (a*sin(d*x + c) + a)^m)/((m
^2 + 3*m + 2)*d)
```

maple [F] time = 1.78, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\sin(dx+c)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

maxima [A] time = 0.91, size = 56, normalized size = 1.04

$$\frac{\left(a^m(m+1)\sin(dx+c)^2+a^m m\sin(dx+c)-a^m\right)(\sin(dx+c)+1)^m}{\left(m^2+3m+2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] (a^m*(m + 1)*sin(d*x + c)^2 + a^m*m*sin(d*x + c) - a^m)*(sin(d*x + c) + 1)^
m/((m^2 + 3*m + 2)*d)
```

mupad [B] time = 9.42, size = 62, normalized size = 1.15

$$\frac{(a\ (\sin(c+d\,x)+1))^m\ \left(\frac{m}{2}+m\ \sin(c+d\,x)+\frac{m\left(2\sin(c+d\,x)^2-1\right)}{2}+\sin(c+d\,x)^2-1\right)}{d\ \left(m^2+3\,m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] ((a*(sin(c + d*x) + 1))^m*(m/2 + m*sin(c + d*x) + (m*(2*sin(c + d*x)^2 - 1)
)/2 + sin(c + d*x)^2 - 1))/(d*(3*m + m^2 + 2))
```

sympy [A] time = 4.80, size = 248, normalized size = 4.59

$$\begin{cases} x(a\sin(c)+a)^m\sin(c)\cos(c) & \text{for } d=0 \\ \frac{\log(\sin(c+dx)+1)\sin(c+dx)}{a^2d\sin(c+dx)+a^2d}+\frac{\log(\sin(c+dx)+1)}{a^2d\sin(c+dx)+a^2d}+\frac{1}{a^2d\sin(c+dx)+a^2d} & \text{for } m=-2 \\ -\frac{\log(\sin(c+dx)+1)}{ad}+\frac{\sin(c+dx)}{ad} & \text{for } m=-1 \\ \frac{m(a\sin(c+dx)+a)^m\sin^2(c+dx)}{dm^2+3dm+2d}+\frac{m(a\sin(c+dx)+a)^m\sin(c+dx)}{dm^2+3dm+2d}+\frac{(a\sin(c+dx)+a)^m\sin^2(c+dx)}{dm^2+3dm+2d}-\frac{(a\sin(c+dx)+a)^m}{dm^2+3dm+2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**m*sin(c)*cos(c), Eq(d, 0)), (log(sin(c + d*x)
+ 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**2*d) + log(sin(c + d*x) + 1)/(a
**2*d*sin(c + d*x) + a**2*d) + 1/(a**2*d*sin(c + d*x) + a**2*d), Eq(m, -2))
, (-log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)/(a*d), Eq(m, -1)), (m*(a*sin
(c + d*x) + a)**m*sin(c + d*x)**2/(d*m**2 + 3*d*m + 2*d) + m*(a*sin(c + d*x
) + a)**m*sin(c + d*x)/(d*m**2 + 3*d*m + 2*d) + (a*sin(c + d*x) + a)**m*sin
(c + d*x)**2/(d*m**2 + 3*d*m + 2*d) - (a*sin(c + d*x) + a)**m/(d*m**2 + 3*d
*m + 2*d), True))
```

3.932 $\int \cot(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=43

$$-\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(1,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

[Out] -hypergeom([1, 1+m],[2+m],1+sin(d*x+c))*(a+a*sin(d*x+c))^(1+m)/a/d/(1+m)

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 65}

$$-\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(1,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x])^m,x]

[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x])^(1 + m))/(a*d*(1 + m)))

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^m}{x}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{{}_2F_1(1,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+dx))^{1+m}}{ad(1+m)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 43, normalized size = 1.00

$$-\frac{(a\sin(c+dx)+a)^{m+1}{}_2F_1(1,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x]
)^(1 + m))/(a*d*(1 + m)))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c), x)
```

maple [F] time = 1.88, size = 0, normalized size = 0.00

$$\int\cos(dx+c)\csc(dx+c)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos (c+d\,x)\ (a+a\,\sin (c+d\,x))^m}{\sin (c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^m)/sin(c + d*x),x)
[Out] int((cos(c + d*x)*(a + a*sin(c + d*x))^m)/sin(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\,(\sin (c+dx)+1))^m \cos (c+dx) \csc (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**m,x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*cos(c + d*x)*csc(c + d*x), x)
```

3.933 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=42

$$\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(2,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

```
[Out] hypergeom([2, 1+m],[2+m],1+sin(d*x+c))*(a+a*sin(d*x+c))^(1+m)/a/d/(1+m)
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 65}

$$\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(2,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x])^
(1 + m))/(a*d*(1 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a+x)^m}{x^2}\,dx,x,a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a+x)^m}{x^2}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{{}_2F_1(2,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+dx))^{1+m}}{ad(1+m)}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 1.00

$$\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(2,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^m,x]
[Out] (Hypergeometric2F1[2, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x])^
(1 + m))/(a*d*(1 + m))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^2, x)
```

maple [F] time = 1.14, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\left(\csc^2(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m \cos(dx+c)\csc(dx+c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^m}{\sin(c+d\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^m)/sin(c + d*x)^2,x)
```

```
[Out] int((cos(c + d*x)*(a + a*sin(c + d*x))^m)/sin(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^m \cos(c+dx)\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*cos(c + d*x)*csc(c + d*x)**2, x)
```

3.934 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=43

$$-\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(3,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

```
[Out] -hypergeom([3, 1+m],[2+m],1+sin(d*x+c))*(a+a*sin(d*x+c))^(1+m)/a/d/(1+m)
```

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 65}

$$-\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(3,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[3, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x]
)^(1 + m))/(a*d*(1 + m)))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a+x)^m}{x^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(a+x)^m}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{{}_2F_1(3,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+dx))^{1+}}{ad(1+m)}$$

Mathematica [A] time = 0.07, size = 43, normalized size = 1.00

$$-\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(3,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
[Out] -((Hypergeometric2F1[3, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x]
)^(1 + m))/(a*d*(1 + m)))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^3, x)
```

maple [F] time = 1.20, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\left(\csc^3(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^m}{\sin(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^m)/sin(c + d*x)^3,x)
```

```
[Out] int((cos(c + d*x)*(a + a*sin(c + d*x))^m)/sin(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.935 $\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=79

$$-\frac{a(c+d)\cos^3(e+fx)}{3f}+\frac{a(4c+d)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}ax(4c+d)-\frac{ad\sin(e+fx)\cos^3(e+fx)}{4f}$$

```
[Out] 1/8*a*(4*c+d)*x-1/3*a*(c+d)*cos(f*x+e)^3/f+1/8*a*(4*c+d)*cos(f*x+e)*sin(f*x
+e)/f-1/4*a*d*cos(f*x+e)^3*sin(f*x+e)/f
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2860, 2669, 2635, 8}

$$-\frac{a(4c+d)\cos^3(e+fx)}{12f}+\frac{a(4c+d)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}ax(4c+d)-\frac{d\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(4*c + d)*x)/8 - (a*(4*c + d)*Cos[e + f*x]^3)/(12*f) + (a*(4*c + d)*Cos[
e + f*x]*Sin[e + f*x])/(8*f) - (d*Cos[e + f*x]^3*(a + a*Sin[e + f*x]))/(4*f
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{d\cos^3(e+fx)(a+a\sin(e+fx))}{4f}+\frac{1}{4}(4c+d)\int \cos$$
$$= -\frac{a(4c+d)\cos^3(e+fx)}{12f}-\frac{d\cos^3(e+fx)(a+a\sin(e+$$
$$= -\frac{a(4c+d)\cos^3(e+fx)}{12f}+\frac{a(4c+d)\cos(e+fx)\sin(e}{8f}$$
$$= \frac{1}{8}a(4c+d)x-\frac{a(4c+d)\cos^3(e+fx)}{12f}+\frac{a(4c+d)\cos(}{}$$

Mathematica [A] time = 0.60, size = 64, normalized size = 0.81

$$-\frac{a(24(c+d)\cos(e+fx)+8(c+d)\cos(3(e+fx))-12fx(4c+d)-24c\sin(2(e+fx))+3d\sin(4(e+fx)))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] -1/96*(a*(-12*(4*c + d)*f*x + 24*(c + d)*Cos[e + f*x] + 8*(c + d)*Cos[3*(e
+ f*x)] - 24*c*Sin[2*(e + f*x)] + 3*d*Sin[4*(e + f*x)]))/f
```

fricas [A] time = 0.44, size = 72, normalized size = 0.91

$$-\frac{8\,(ac+ad)\cos\left(fx+e\right)^{3}-3\,(4\,ac+ad)fx+3\left(2\,ad\cos\left(fx+e\right)^{3}-(4\,ac+ad)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/24*(8*(a*c + a*d)*cos(f*x + e)^3 - 3*(4*a*c + a*d)*f*x + 3*(2*a*d*cos(f*
x + e)^3 - (4*a*c + a*d)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.17, size = 87, normalized size = 1.10

$$\frac{1}{8}(4ac+ad)x-\frac{ad\sin\left(4fx+4e\right)}{32f}+\frac{ac\sin\left(2fx+2e\right)}{4f}-\frac{(ac+ad)\cos\left(3fx+3e\right)}{12f}-\frac{(ac+ad)\cos\left(fx+e\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/8*(4*a*c + a*d)*x - 1/32*a*d*sin(4*f*x + 4*e)/f + 1/4*a*c*sin(2*f*x + 2*e
)/f - 1/12*(a*c + a*d)*cos(3*f*x + 3*e)/f - 1/4*(a*c + a*d)*cos(f*x + e)/f
```

maple [A] time = 0.36, size = 96, normalized size = 1.22

$$\frac{da\left(-\frac{\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right)\right)}{4}+\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{8}+\frac{fx}{8}+\frac{e}{8}\right)-\frac{\left(\cos^{3}\left(fx+e\right)\right)ac}{3}-\frac{da\left(\cos^{3}\left(fx+e\right)\right)}{3}+ca\left(\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(d*a*(-1/4*sin(f*x+e)*cos(f*x+e)^3+1/8*sin(f*x+e)*cos(f*x+e)+1/8*f*x+1/
8*e)-1/3*cos(f*x+e)^3*a*c-1/3*d*a*cos(f*x+e)^3+c*a*(1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e))
```

maxima [A] time = 0.34, size = 74, normalized size = 0.94

$$-\frac{32\,ac\cos\left(fx+e\right)^{3}+32\,ad\cos\left(fx+e\right)^{3}-24\left(2\,fx+2\,e+\sin\left(2\,fx+2\,e\right)\right)ac-3\left(4\,fx+4\,e-\sin\left(4\,fx+4\right.\right.}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*a*c*cos(f*x + e)^3 + 32*a*d*cos(f*x + e)^3 - 24*(2*f*x + 2*e + si
n(2*f*x + 2*e))*a*c - 3*(4*f*x + 4*e - sin(4*f*x + 4*e))*a*d)/f
```

mupad [B] time = 10.24, size = 276, normalized size = 3.49

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4c+d)}{4\left(ac+\frac{ad}{4}\right)}\right)(4\,c+d)}{4\,f}-\frac{\left(a\,c-\frac{a\,d}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}+(2\,a\,c+2\,a\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+\left(a\,c+\frac{7\,a\,d}{4}\right)\,\tan\left(\right.}{f\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))*(c + d*sin(e + f*x)),x)
```

```
[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(4*c + d))/(4*(a*c + (a*d)/4)))*(4*c + d))/(4
*f) - ((2*a*c)/3 + (2*a*d)/3 + tan(e/2 + (f*x)/2)^4*(2*a*c + 2*a*d) + tan(e
/2 + (f*x)/2)^6*(2*a*c + 2*a*d) + tan(e/2 + (f*x)/2)^2*((2*a*c)/3 + (2*a*d)
/3) + tan(e/2 + (f*x)/2)^7*(a*c - (a*d)/4) - tan(e/2 + (f*x)/2)^3*(a*c + (7
*a*d)/4) + tan(e/2 + (f*x)/2)^5*(a*c + (7*a*d)/4) - tan(e/2 + (f*x)/2)*(a*c
 - (a*d)/4))/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/
2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) - (a*(4*c + d)*(atan(tan(e/2 +
(f*x)/2)) - (f*x)/2))/(4*f)
```

sympy [A] time = 0.99, size = 199, normalized size = 2.52

$$\begin{cases} \frac{acx\sin^2\left(e+fx\right)}{2} + \frac{acx\cos^2\left(e+fx\right)}{2} + \frac{ac\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} - \frac{ac\cos^3\left(e+fx\right)}{3f} + \frac{adx\sin^4\left(e+fx\right)}{8} + \frac{adx\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4} + \frac{adx\cos^{4}}{8} \\ x\left(c+d\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)\cos^2\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x*sin(e + f*x)**2/2 + a*c*x*cos(e + f*x)**2/2 + a*c*sin(e +
f*x)*cos(e + f*x)/(2*f) - a*c*cos(e + f*x)**3/(3*f) + a*d*x*sin(e + f*x)**4
/8 + a*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + a*d*x*cos(e + f*x)**4/8 + a*
d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - a*d*sin(e + f*x)*cos(e + f*x)**3/(8*
f) - a*d*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))*(a*sin(e) + a)
*cos(e)**2, True))
```

3.936 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=123

$$\frac{2\sqrt{c+d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}\sqrt{d}\,f(c-d)}-\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)}$$

```
[Out] -2*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/a
^(3/2)/(c-d)/f+2*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*
x+e))^(1/2))*(c+d)^(1/2)/a^(3/2)/(c-d)/f/d^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2916, 2985, 2649, 206, 2773, 208}

$$\frac{2\sqrt{c+d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}\sqrt{d}\,f(c-d)}-\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(a^(3/2)*(c - d)*f) + (2*Sqrt[c + d]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e +
f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(c - d)*Sqrt[d]*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/b^2, Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ
[2*m, 2*n]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = \frac{\int \frac{a-a\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx}{a^2}$$

$$= \frac{2\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{a(c-d)} - \frac{(c+d)\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{a^2(c-d)}$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a(c-d)f} + \frac{(2(c+d))\,\text{Subst}\left(\int \frac{\ldots}{ac+\ldots}\right.}{a(\ldots}$$

$$= -\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)f} + \frac{2\sqrt{c+d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(\ldots}{\sqrt{c+d}\,\sqrt{a+a\sin\ldots}}\right.}{a^{3/2}(c-d)\sqrt{d}\,f}$$

Mathematica [C] time = 2.80, size = 220, normalized size = 1.79

$$\frac{(-1)^{3/4}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sqrt[4]{-1}\sqrt{c+d}\left(\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),
x]
```

```
[Out] ((-1)^(3/4)*((-4 - 4*I)*Sqrt[d]*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e
 + f*x)/4])] + (-1)^(1/4)*Sqrt[c + d]*(Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d]
+ Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])] - Log[Sec[(e + f*x)
/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(Sqrt[d]*(-c + d)*f*(a*(1 + Sin[
e + f*x]))^(3/2))
```

fricas [B] time = 0.59, size = 667, normalized size = 5.42

$$\left[-\frac{\sqrt{\frac{c+d}{ad}}\log\left(\frac{d^2\cos(fx+e)^3-(6cd+7d^2)\cos(fx+e)^2-c^2-2cd-d^2-4\left(d^2\cos(fx+e)^2-cd-3d^2-(cd+2d^2)\cos(fx+e)+(d^2\cos(fx+e)+cd+3d^2)\sin\ldots\right.}{d^2\cos(fx+e)^3+(2cd+d^2)\cos(fx+e)^2-c^2-2cd-d^2-(c^2+d^2)\cos(\ldots}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt((c + d)/(a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*
x + e)^2 - c^2 - 2*c*d - d^2 - 4*(d^2*cos(f*x + e)^2 - c*d - 3*d^2 - (c*d +
 2*d^2)*cos(f*x + e) + (d^2*cos(f*x + e) + c*d + 3*d^2)*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt((c + d)/(a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e
) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e
))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 -
 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f
*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 2*sqrt(2)*log(-(cos(f*x + e)^
2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(c
os(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)
```

```
^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c -
a*d)*f), (sqrt(-(c + d)/(a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f
*x + e) - c - 2*d)*sqrt(-(c + d)/(a*d))/((c + d)*cos(f*x + e))) - sqrt(2)*l
og(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*si
n(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e)
+ 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))
/sqrt(a))/((a*c - a*d)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning
, integration of abs or sign assumes constant sign by intervals (correct if
 the argument is real):Check [abs(cos((f*t_nostep+exp(1))/2-pi/4))]Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos((f*t_nost
ep+exp(1))/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
```

ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep+1)]Warning, assuming -c^3*d^5-c^2*d^6+c*d^7+d^8-c*d^7-d^8 is positive. Hint: run assume to make assumptions on a variableWarning, assuming -a*c^3*d^5-a*c^2*d^6+a*c*d^7+a*d^8-a*c*d^7-a*d^8 is positive. Hint: run assume to make assumptions on a variableWarning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.Non regular value [0] was discarded and replaced randomly by 0=[-90]Evaluation time: 65.87 sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 1.86, size = 160, normalized size = 1.30

$$-\frac{2\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{a\left(c+d\right)d}\,-\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\right.}}{\sqrt{a}}\right.\right.}{a^{\frac{3}{2}}\left(c-d\right)\sqrt{a\left(c+d\right)d}\,\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/a^(3/2)*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(2^(1/2)*arctanh(1/2*(-
a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)-arctanh((-a*(sin
(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1/2)*c-arctanh((-a*(sin(f*x+e)-1)
)^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1/2)*d)/(c-d)/(a*(c+d)*d)^(1/2)/cos(f*x+e)/
(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(e+f\,x\right)}^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))),x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.937 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}\sqrt{d}f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f\sqrt{c-d}}$$

```
[Out] -2*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2
)/(c+d*sin(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/f/(c-d)^(1/2)+2*arctan(cos(f*x+e)
*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(3/2)/f/d
^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2916, 2982, 2782, 208, 2775, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}\sqrt{d}f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c +
 d*Sin[e + f*x]])])/(a^(3/2)*Sqrt[d]*f) - (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[
c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]])])/(a^(3/2)*Sqrt[c - d]*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
```

```
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/b^2, Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ
[2*m, 2*n]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{\int \frac{a-a\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a^2} + \frac{2\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f} + \frac{2\,\text{S}}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}\sqrt{d}\,f} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{}}{\sqrt{2}\sqrt{a+a\sin(}}\right.}{a^{3/2}\sqrt{c-}}$$

Mathematica [C] time = 33.31, size = 208404, normalized size = 1478.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 1.11, size = 2071, normalized size = 14.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*a*d*log(-((c - 3*d)*cos(f*x + e)^2 - 2*sqrt(2)*((c - d)*cos
(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*s
in(f*x + e) + c)/sqrt(a*c - a*d) + (3*c - d)*cos(f*x + e) - ((c - 3*d)*cos(
f*x + e) - 2*c - 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f*x
+ e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a*c - a*d) - sqrt(-a*d)*lo
g((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 +
 a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*
d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*
a*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x
```

+ e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d
^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (16
*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d
^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x
+ e))*sqrt(-a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (a*c^4
 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (12
8*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^
4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*
a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*c
os(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)))/(a^2*d*f), 1
/2*(2*sqrt(2)*a*d*log(-((c - 3*d)*cos(f*x + e)^2 - 2*sqrt(2)*((c - d)*cos(f
*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin
(f*x + e) + c)/sqrt(a*c - a*d) + (3*c - d)*cos(f*x + e) - ((c - 3*d)*cos(f*
x + e) - 2*c - 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a*c - a*d) - sqrt(a*d)*arcta
n(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x +
 e))*sqrt(a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(2*a*d^3*c
os(f*x + e)^3 - (3*a*c*d^2 - a*d^3)*cos(f*x + e)*sin(f*x + e) - (a*c^2*d -
a*c*d^2 + 2*a*d^3)*cos(f*x + e))))/(a^2*d*f), 1/4*(8*sqrt(2)*a*d*sqrt(-1/(a
*c - a*d))*arctan(sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)
*sqrt(-1/(a*c - a*d))/cos(f*x + e)) - sqrt(-a*d)*log((128*a*d^4*cos(f*x + e
)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3
 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x
+ e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 -
8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d
 - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^
3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^
3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(
5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a*d)*sqrt(a*
sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (a*c^4 - 28*a*c^3*d + 230*a*c^
2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 +
 a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4
)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 +
32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e
))/(cos(f*x + e) + sin(f*x + e) + 1)))/(a^2*d*f), 1/2*(4*sqrt(2)*a*d*sqrt(-
1/(a*c - a*d))*arctan(sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)*sqrt(-1/(a*c - a*d))/cos(f*x + e)) - sqrt(a*d)*arctan(1/4*(8*d^2*cos(f
*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*d)*sqr
t(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(2*a*d^3*cos(f*x + e)^3 - (3
*a*c*d^2 - a*d^3)*cos(f*x + e)*sin(f*x + e) - (a*c^2*d - a*c*d^2 + 2*a*d^3)
*cos(f*x + e))))/(a^2*d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) +
c)), x)
```

maple [B] time = 0.60, size = 4463, normalized size = 31.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(2*(d^2/c^2)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)
*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2
)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*c^2*d^2+(d^2/c^2)^(1/2)*(((d^
2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*
d^4)*c)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*a
rctan(((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2
/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*
x+e)+d)*((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c
^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))
*(2*c-2*d)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*c*cos(f*x+e)-(d^2/c^2)^(1/2)*((
(d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2
-4*d^4)*c)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2
)*arctan(((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((
d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos
(f*x+e)+d)*((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2
)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/
2))*(2*c-2*d)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*c*sin(f*x+e)-(d^2/c^2)^(1/2)
*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*s
in(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(
1/2))*(2*c-2*d)^(1/2)*c*d^3+((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)
+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(
1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*d^4-4*2^(1/2)*((c+d*sin(f*
x+e))/(cos(f*x+e)+1))^(1/2)*ln(-2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e)
```

)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d
*cos(f*x+e)-c+d)/(-1+cos(f*x+e)-sin(f*x+e)))*(-(d^2/c^2)^(1/2)*c)^(1/2)*c^2
*d^2*sin(f*x+e)+8*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(-2*((2
*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*
sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(-1+cos(f*x+e)-sin(f
x+e)))(-(d^2/c^2)^(1/2)*c)^(1/2)*c*d^3*sin(f*x+e)-(d^2/c^2)^(1/2)*(((d^2/
c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^
4)*c)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arc
tan(((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c
^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+
e)+d)*((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4
+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*(
2*c-2*d)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*c+(d^2/c^2)^(1/2)*((c+d*sin(f*x+e
))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2
/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^
(1/2)*c^3*d*cos(f*x+e)-2*(d^2/c^2)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)
*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f
*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*c^2*d^2*cos(f
x+e)+(d^2/c^2)^(1/2)((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)
^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(
-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*c*d^3*cos(f*x+e)+(d^2/c^2)^(1/2)
*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*s
in(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(
1/2))*(2*c-2*d)^(1/2)*c^3*d*sin(f*x+e)-2*(d^2/c^2)^(1/2)*((c+d*sin(f*x+e))/
((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^
2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/
2)*c^2*d^2*sin(f*x+e)+(d^2/c^2)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*
sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+
e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*c*d^3*sin(f*x+e)
+(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*
d^2-4*d^4)*c)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(
1/2)*arctan(((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))
/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*
cos(f*x+e)+d)*((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(
1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^
(1/2))*(2*c-2*d)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*d*cos(f*x+e)-(((d^2/c^2)^
(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)
^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan((
(d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c^2)^(
1/2)*c*sin(f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d)
*((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d
^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*(2*c-2
d)^(1/2)(-(d^2/c^2)^(1/2)*c)^(1/2)*d*sin(f*x+e)-(((d^2/c^2)^(1/2)*c^4+6*(
d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*((c+d*
sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((d^2/c^2)^(1/

```
2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*
x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)^(
1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)
*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*(2*c-2*d)^(1/2)*(-(
d^2/c^2)^(1/2)*c)^(1/2)*d-4*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)
*ln(-2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin
(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(-1+cos(f*
x+e)-sin(f*x+e)))*(-(d^2/c^2)^(1/2)*c)^(1/2)*d^4*sin(f*x+e)-(d^2/c^2)^(1/2)
*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*s
in(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(
1/2))*(2*c-2*d)^(1/2)*c^3*d-((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)
+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(
1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*c^2*d^2*cos(f*x+e)-2*((c+d
*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin(f*x
+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*
(2*c-2*d)^(1/2)*c*d^3+((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)
^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(
-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*c^2*d^2-((c+d*sin(f*x+e))/((d^2/
c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/
2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*d^4
*cos(f*x+e)-((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arc
tan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)
^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*d^4*sin(f*x+e)+2*((c+d*sin(f*x+e))/((d^2/c
^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2
)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*c*d^
3*cos(f*x+e)-((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*ar
ctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2
)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*c^2*d^2*sin(f*x+e)+2*((c+d*sin(f*x+e))/((
d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)
^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)
*c*d^3*sin(f*x+e))*(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x
+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)/(c+d*sin(f*x+e))^(1/2)/d^2/
(-(d^2/c^2)^(1/2)*c)^(1/2)/(c^2-2*c*d+d^2)/(2*c-2*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) +
c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}{\left(e+fx \right)}}{\left(a \left(\sin{\left(e+fx \right)} + 1\right)\right)^{\frac{3}{2}} \sqrt{c+d\sin{\left(e+fx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(c + d*sin(e +
f*x))), x)
```

3.938 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=135

$$\frac{2\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{3}{2};-\frac{1}{2},-n;m+\frac{5}{2};\frac{1}{2}(\sin(e+f\ldots\right.}{af(2m+3)\sqrt{1-\sin(e+fx)}}$$

```
[Out] 2*AppellF1(3/2+m,-n,-1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*
cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(3+2*m)/((
(c+d*sin(f*x+e))/(c-d))^n)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2918, 140, 139, 138}

$$\frac{2\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{3}{2};-\frac{1}{2},-n;m+\frac{5}{2};\frac{1}{2}(\sin(e+f\ldots\right.}{af(2m+3)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (2*Sqrt[2]*AppellF1[3/2 + m, -1/2, -n, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*
(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c
+ d*Sin[e + f*x])^n)/(a*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e +
f*x])/(c - d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2918

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[e + f*
x]/(a^(p - 2)*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[I
nt[(a + b*x)^(m + p/2 - 1/2)*(a - b*x)^(p/2 - 1/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&
 IntegerQ[p/2] && !IntegerQ[m]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \sqrt{a-ax}\,(a+ax)^{\frac{1}{2}+m}(c+dx)\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+}}$$

$$= \frac{\left(\sqrt{2}\cos(e+fx)\right)\text{Subst}\left(\int \sqrt{\frac{1}{2}-\frac{x}{2}}\,(a+ax)^{\frac{1}{2}+m}(c\right.}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{a+a\sin(e+}}$$

$$= \frac{\left(\sqrt{2}\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{a(c+d\sin(e+fx)}{ac-ad}\right.\right.}{f\sqrt{\frac{a-a}{}}}$$

$$= \frac{2\sqrt{2}\,F_1\left(\frac{3}{2}+m;-\frac{1}{2},-n;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx)),\right.}{}$$

Mathematica [A] time = 0.74, size = 158, normalized size = 1.17

$$-\frac{4\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\cos(e+fx)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{-m-\frac{1}{2}}(a(\sin(e+fx)+1))^m(c+d\sin(e+fx))^n\left(\frac{c+d}{\cdots}\right.}{3f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*AppellF1[3/2, -1/2 - m, -n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[
(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c
+ d*Sin[e + f*x])^n*Sin[(2*e - Pi + 2*f*x)/4]^2*(Sin[(2*e + Pi + 2*f*x)/4]^
2)^(-1/2 - m))/(3*f*((c + d*Sin[e + f*x])/(c + d))^n)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

maple [F] time = 1.11, size = 0, normalized size = 0.00

$$\int\left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.939 $\int \cos^2(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=119

$$-\frac{16\sqrt{2}\,a^3(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{7}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1}{}\right.}{3f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -16/3*a^3*AppellF1(3/2,-n,-7/2,5/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e
))*cos(f*x+e)*(1-sin(f*x+e))*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e)
)/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{16\sqrt{2}\,a^3(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{7}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1}{}\right.}{3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-16*Sqrt[2]*a^3*AppellF1[3/2, -7/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 -
 Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x
])^n)/(3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx = \frac{\left(a^3\cos(e+fx)\right)\text{Subst}\left(\int\sqrt{1-x}\,(1+x)^{7/2}(c+dx)\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+f}$$

$$= \frac{\left(a^3\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)\right.}{f\sqrt{1-\sin(}$$

$$= -\frac{16\sqrt{2}\,a^3F_1\left(\frac{3}{2};-\frac{7}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\text{si}}{c}\right.}{}$$

Mathematica [F] time = 116.95, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n, x]
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2+\left(a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)\right)\left(d\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^
4 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*(d*sin(f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

maple [F] time = 3.48, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{3}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^{2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.940 $\int \cos^2(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=119

$$-\frac{8\sqrt{2}\,a^2(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-}{}\right.}{3f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -8/3*a^2*AppellF1(3/2,-n,-5/2,5/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e)
)*cos(f*x+e)*(1-sin(f*x+e))*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))
/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{8\sqrt{2}\,a^2(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-}{}\right.}{3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-8*Sqrt[2]*a^2*AppellF1[3/2, -5/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n)/(3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int\sqrt{1-x}\,(1+x)^{5/2}(c+dx)\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+f}$$

$$= \frac{\left(a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)\right.}{f\sqrt{1-\sin(}$$

$$= -\frac{8\sqrt{2}\,a^2F_1\left(\frac{3}{2};-\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin}{c+}\right.}{}$$

Mathematica [F] time = 77.43, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n, x]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2\sin\left(fx+e\right)-2\,a^2\cos\left(fx+e\right)^2\right)\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*c
os(f*x + e)^2)*(d*sin(f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{2} \left(d \sin\left(fx+e\right)+c\right)^{n} \cos\left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

maple [F] time = 3.28, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right) \left(a+a \sin\left(fx+e\right)\right)^{2} \left(c+d \sin\left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{2} \left(d \sin\left(fx+e\right)+c\right)^{n} \cos\left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
[Out] Timed out
```

3.941 $\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=117

$$-\frac{4\sqrt{2}\,a(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-s}{}\right.}{3f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -4/3*a*AppellF1(3/2,-n,-3/2,5/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*
cos(f*x+e)*(1-sin(f*x+e))*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(
c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2868, 139, 138}

$$-\frac{4\sqrt{2}\,a(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-s}{}\right.}{3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*Sqrt[2]*a*AppellF1[3/2, -3/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Si
n[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^
n)/(3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2868

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(c*g*(g
*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(1 + (d*x)/c)^((p + 1)/2)*(1 - (d*x)/c)^((p - 1)/
2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{(a\cos(e+fx))\,\text{Subst}\left(\int \sqrt{1-x}\,(1+x)^{3/2}(c+dx)^n\, \right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx}}$$

$$= \frac{\left(a\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right.}{f\sqrt{1-\sin(e}}$$

$$= -\frac{4\sqrt{2}\,aF_1\left(\frac{3}{2};-\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+}{c+d}\right.}{3}$$

Mathematica [F] time = 16.12, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\cos\left(fx+e\right)^2\sin\left(fx+e\right)+a\cos\left(fx+e\right)^2\right)\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((a*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*(d*sin(f*x + e)
 + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin \left(f x + e\right) + a\right) \left(d \sin \left(f x + e\right) + c\right)^{n} \cos \left(f x + e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

maple [F] time = 1.35, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(f x + e\right)\right) \left(a + a \sin \left(f x + e\right)\right) \left(c + d \sin \left(f x + e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin \left(f x + e\right) + a\right) \left(d \sin \left(f x + e\right) + c\right)^{n} \cos \left(f x + e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos \left(e+f\,x\right)}^2\,\left(a+a\,\sin \left(e+f\,x\right)\right)\,{\left(c+d\,\sin \left(e+f\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.942 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=119

$$-\frac{\sqrt{2}\,(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{1}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+}{c+d}\right.}{3af\sqrt{\sin(e+fx)+1}}$$

```
[Out] -1/3*AppellF1(3/2,-n,1/2,5/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos
(f*x+e)*(1-sin(f*x+e))*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(((c+d*sin(f*x+e))/(c
+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2914, 2755, 139, 138}

$$-\frac{\sqrt{2}\,(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{1}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+}{c+d}\right.}{3af\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(Sqrt[2]*AppellF1[3/2, 1/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e +
 f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(3
*a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2755

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*S
qrt[1 - Sin[e + f*x]]), Subst[Int[((a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 -
(d*x)/c], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rule 2914

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^(2*m), I
nt[(c + d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\int (a-a\sin(e+fx))(c+d\sin(e+fx))^n\,dx}{a^2}$$

$$= \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{\sqrt{1-x}\,(c+dx)^n}{\sqrt{1+x}}\,dx, x, \sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\sqrt{1-x}\,\left(-\frac{c}{-c}\right.}{\sqrt{1}}\right.}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\sqrt{2}\,F_1\left(\frac{3}{2};\frac{1}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\text{s}}{3af\sqrt{1+\sin(e+fx)}}$$

Mathematica [A] time = 1.01, size = 229, normalized size = 1.92

$$-\frac{\sec(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{-\frac{d(\sin(e+fx)-1)}{c+d}}\,(c+d\sin(e+fx))^{n+1}\left((n+1)(c+d\sin(e+\right.}{adf(n+1)(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Sec[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[-((d*(-1 + Sin
[e + f*x]))/(c + d))]*(c + d*Sin[e + f*x])^(1 + n)*(-((c + d)*(2 + n)*Appel
lF1[1 + n, 1/2, 1/2, 2 + n, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin[e + f*
x])/(c + d)]) + (1 + n)*AppellF1[2 + n, 1/2, 1/2, 3 + n, (c + d*Sin[e + f*x
])/(c - d), (c + d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])))/(a*d*(-c +
 d)*f*(1 + n)*(2 + n)*Sqrt[(d*(1 + Sin[e + f*x]))/(-c + d)]))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a), x)
```

maple [F] time = 1.33, size = 0, normalized size = 0.00

$$\int\frac{\left(\cos^{2}\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^{2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{n}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)),x)
```

```
[Out] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.943 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=119

$$-\frac{(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{3\sqrt{2}\,a^2f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -1/6*AppellF1(3/2,-n,3/2,5/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos
(f*x+e)*(1-sin(f*x+e))*(c+d*sin(f*x+e))^n/a^2/f/(((c+d*sin(f*x+e))/(c+d))^n
)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{3\sqrt{2}\,a^2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(AppellF1[3/2, 3/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(3*Sqrt[2]
*a^2*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\cos(e+fx)\,\mathrm{Subst}\left(\int \frac{\sqrt{1-x}\,(c+dx)^n}{(1+x)^{3/2}}\,dx, x, \sin(e+fx)\right)}{a^2 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1-x}\left(-\frac{c}{-c}\right.}{(1+x}\right.}{a^2 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{3}{2};\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(}{3\sqrt{2}\,a^2 f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 8.83, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2, x
]
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a^2*cos(f*x + e)^2 - 2*a^2
*sin(f*x + e) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a)^2, x)
```

maple [F] time = 5.73, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,x)
[Out] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
[Out] Timed out
```

3.944 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=119

$$-\frac{(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{6\sqrt{2}\,a^3f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -1/12*AppellF1(3/2,-n,5/2,5/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(1-sin(f*x+e))*(c+d*sin(f*x+e))^n/a^3/f/(((c+d*sin(f*x+e))/(c+d))^
n)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{6\sqrt{2}\,a^3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(AppellF1[3/2, 5/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(6*Sqrt[2]
*a^3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{\sqrt{1-x}\,(c+dx)^n}{(1+x)^{5/2}}\,dx, x, \sin(e+fx)\right)}{a^3 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\sqrt{1-x}\left(-\frac{c}{-c}\right.}{(1+x}\right.}{a^3 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{3}{2};\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(}{6\sqrt{2}\,a^3 f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 20.69, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3, x
]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(3*a^3*cos(f*x + e)^2 - 4*a
^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a)^3, x)
```

maple [F] time = 4.02, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3,x)
[Out] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
[Out] Timed out
```

3.945 $\int \cos^4(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=135

$$\frac{4\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^{m+2}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{5}{2};-\frac{3}{2},-n;m+\frac{7}{2};\frac{1}{2}(\sin(e+f\right.}{a^2f(2m+5)\sqrt{1-\sin(e+fx)}}$$

```
[Out] 4*AppellF1(5/2+m,-n,-3/2,7/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*
cos(f*x+e)*(a+a*sin(f*x+e))^(2+m)*(c+d*sin(f*x+e))^n*2^(1/2)/a^2/f/(5+2*m)/
(((c+d*sin(f*x+e))/(c-d))^n)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2918, 140, 139, 138}

$$\frac{4\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^{m+2}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{5}{2};-\frac{3}{2},-n;m+\frac{7}{2};\frac{1}{2}(\sin(e+f\right.}{a^2f(2m+5)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (4*Sqrt[2]*AppellF1[5/2 + m, -3/2, -n, 7/2 + m, (1 + Sin[e + f*x])/2, -((d*
(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^(2 + m)*(c
+ d*Sin[e + f*x])^n)/(a^2*f*(5 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e
+ f*x])/(c - d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2918

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[e + f*
x]/(a^(p - 2)*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[I
nt[(a + b*x)^(m + p/2 - 1/2)*(a - b*x)^(p/2 - 1/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&
 IntegerQ[p/2] && !IntegerQ[m]
```

Rubi steps

$$\int \cos^4(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int (a-ax)^{3/2}(a+ax)^{\frac{3}{2}+m}(c+dx\right.}{a^2 f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e}$$

$$= \frac{\left(2\sqrt{2}\,\cos(e+fx)\right)\text{Subst}\left(\int \left(\frac{1}{2}-\frac{x}{2}\right)^{3/2}(a+ax)^{\frac{3}{2}+}\right.}{af\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{a+a\sin(}}$$

$$= \frac{\left(2\sqrt{2}\,\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{a(c+d\sin(e+f}{ac-ad}\right.\right.}{af\sqrt{}}$$

$$= \frac{4\sqrt{2}\,F_1\left(\frac{5}{2}+m;-\frac{3}{2},-n;\frac{7}{2}+m;\frac{1}{2}(1+\sin(e+fx)),\right.}{}$$

Mathematica [A] time = 1.38, size = 160, normalized size = 1.19

$$-\frac{4\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\cos^3(e+fx)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{-m-\frac{3}{2}}(a(\sin(e+fx)+1))^m(c+d\sin(e+fx))^n\left(\frac{c+}{}\right.}{5f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*AppellF1[5/2, -3/2 - m, -n, 7/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[
(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[e + f*x]^3*(a*(1 + Sin[e + f*x]))^m*(
c + d*Sin[e + f*x])^n*Sin[(2*e - Pi + 2*f*x)/4]^2*(Sin[(2*e + Pi + 2*f*x)/4
]^2)^(-3/2 - m))/(5*f*((c + d*Sin[e + f*x])/(c + d))^n)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^4, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [F] time = 1.24, size = 0, normalized size = 0.00

$$\int\left(\cos^4\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^4\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)^4*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.946 $\int \cos^4(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n \, dx$

Optimal. Leaf size=121

$$-\frac{16\sqrt{2}\, a^2(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n} F_1\left(\frac{5}{2};-\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(\ldots}{}\right.}{5f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -16/5*a^2*AppellF1(5/2,-n,-7/2,7/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e
))*cos(f*x+e)*(1-sin(f*x+e))^2*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+
e))/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{16\sqrt{2}\, a^2(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n} F_1\left(\frac{5}{2};-\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(\ldots}{}\right.}{5f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-16*Sqrt[2]*a^2*AppellF1[5/2, -7/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 -
 Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f
*x])^n)/(5*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \cos^4(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int(1-x)^{3/2}(1+x)^{7/2}(c+dx\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+}}$$

$$= \frac{\left(a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)\right.}{f\sqrt{1-\sin}}$$

$$= -\frac{16\sqrt{2}\,a^2F_1\left(\frac{5}{2};-\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin}{c}\right.}{}$$

Mathematica [F] time = 1.63, size = 0, normalized size = 0.00

$$\int \cos^4(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n, x]
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos\left(fx+e\right)^6-2\,a^2\cos\left(fx+e\right)^4\sin\left(fx+e\right)-2\,a^2\cos\left(fx+e\right)^4\right)\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^6 - 2*a^2*cos(f*x + e)^4*sin(f*x + e) - 2*a^2*c
os(f*x + e)^4)*(d*sin(f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin \left(fx+e\right)+a\right)^{2} \left(d \sin \left(fx+e\right)+c\right)^{n} \cos \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^4, x)
```

maple [F] time = 3.58, size = 0, normalized size = 0.00

$$\int \left(\cos^{4}\left(fx+e\right)\right) \left(a+a \sin \left(fx+e\right)\right)^{2} \left(c+d \sin \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos \left(e+f\,x\right)}^4\,{\left(a+a\,\sin \left(e+f\,x\right)\right)}^2\,{\left(c+d\,\sin \left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)^4*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
[Out] Timed out
```

3.947 $\int \cos^4(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=119

$$-\frac{8\sqrt{2}\,a(1-\sin(e+fx))\cos^3(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};-\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-}{}\right.}{5f(\sin(e+fx)+1)^{3/2}}$$

```
[Out] -8/5*a*AppellF1(5/2,-n,-5/2,7/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*
cos(f*x+e)^3*(1-sin(f*x+e))*(c+d*sin(f*x+e))^n*2^(1/2)/f/(1+sin(f*x+e))^(3/
2)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Rubi [A] time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2868, 139, 138}

$$-\frac{8\sqrt{2}\,a(1-\sin(e+fx))\cos^3(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};-\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-}{}\right.}{5f(\sin(e+fx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-8*Sqrt[2]*a*AppellF1[5/2, -5/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Si
n[e + f*x]))/(c + d)]*Cos[e + f*x]^3*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n)/(5*f*(1 + Sin[e + f*x])^(3/2)*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2868

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(c*g*(g
*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(1 + (d*x)/c)^((p + 1)/2)*(1 - (d*x)/c)^((p - 1)/
2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^4(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{\left(a\cos^3(e+fx)\right)\text{Subst}\left(\int(1-x)^{3/2}(1+x)^{5/2}(c+dx\right.}{f(1-\sin(e+fx))^{3/2}(1+\sin(e+fx}$$

$$= \frac{\left(a\cos^3(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right.}{f(1-\sin(e+}$$

$$= -\frac{8\sqrt{2}\,aF_1\left(\frac{5}{2};-\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+}{c+d}\right.}{5}$$

Mathematica [F] time = 0.75, size = 0, normalized size = 0.00

$$\int \cos^4(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\cos\left(fx+e\right)^4\sin\left(fx+e\right)+a\cos\left(fx+e\right)^4\right)\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((a*cos(f*x + e)^4*sin(f*x + e) + a*cos(f*x + e)^4)*(d*sin(f*x + e)
 + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)\left(d \sin\left(fx+e\right)+c\right)^{n} \cos\left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^4, x)
```

maple [F] time = 1.56, size = 0, normalized size = 0.00

$$\int \left(\cos^{4}\left(fx+e\right)\right)\left(a+a \sin\left(fx+e\right)\right)\left(c+d \sin\left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(e+f\,x\right)}^4\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)^4*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.948 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=121

$$-\frac{2\sqrt{2}\,(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};-\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5af\sqrt{\sin(e+fx)+1}}$$

```
[Out] -2/5*AppellF1(5/2,-n,-1/2,7/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(1-sin(f*x+e))^2*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(((c+d*sin(f*x+e))
/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{2\sqrt{2}\,(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};-\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5af\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-2*Sqrt[2]*AppellF1[5/2, -1/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[
e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^
n)/(5*a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int (1-x)^{3/2}\sqrt{1+x}\,(c+dx)^n\,dx, x, \sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int (1-x)^{3/2}\right.}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e}}$$

$$= -\frac{2\sqrt{2}F_1\left(\frac{5}{2};-\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1}{5af\sqrt{1+\sin(e+f}}$$

Mathematica [F] time = 30.71, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]), x]
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^4}{a\sin\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin \left(f x+e\right)+c\right)^{n} \cos \left(f x+e\right)^{4}}{a \sin \left(f x+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a), x)
```

maple [F] time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{4}\left(f x+e\right)\right) \left(c+d \sin \left(f x+e\right)\right)^{n}}{a+a \sin \left(f x+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin \left(f x+e\right)+c\right)^{n} \cos \left(f x+e\right)^{4}}{a \sin \left(f x+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos \left(e+f\,x\right)}^4\,{\left(c+d\,\sin \left(e+f\,x\right)\right)}^n}{a+a\,\sin \left(e+f\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)),x)
```

```
[Out] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.949 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=121

$$-\frac{\sqrt{2}\,(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e}{c+d}\right.}{5a^2f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -1/5*AppellF1(5/2,-n,1/2,7/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos
(f*x+e)*(1-sin(f*x+e))^2*(c+d*sin(f*x+e))^n*2^(1/2)/a^2/f/(((c+d*sin(f*x+e)
)/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2914, 2784, 139, 138}

$$-\frac{\sqrt{2}\,(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e}{c+d}\right.}{5a^2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(Sqrt[2]*AppellF1[5/2, 1/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e +
 f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/
(5*a^2*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 2914

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^(2*m), I
nt[(c + d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\int (a-a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx}{a^4}$$
$$= \frac{\cos(e+fx)\,\mathrm{Subst}\left(\int \frac{(1-x)^{3/2}(c+dx)^n}{\sqrt{1+x}}\,dx, x, \sin(e+fx)\right)}{a^2 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\mathrm{Subst}\left(\int \frac{(1-x)^{3/2}\left(-\right.}{\sqrt{}}\right.}{a^2 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{\sqrt{2}F_1\left(\frac{5}{2};\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\mathrm{s}}{5a^2 f\sqrt{1+\sin(e+fx}}$$

Mathematica [F] time = 17.61, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2, x
]
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a^2*cos(f*x + e)^2 - 2*a^2
*sin(f*x + e) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^2, x)
```

maple [F] time = 5.53, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{4}\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^2, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^4\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.950 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=121

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{3}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5\sqrt{2}a^3f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -1/10*AppellF1(5/2,-n,3/2,7/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(1-sin(f*x+e))^2*(c+d*sin(f*x+e))^n/a^3/f/(((c+d*sin(f*x+e))/(c+d)
)^n)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{3}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5\sqrt{2}a^3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(AppellF1[5/2, 3/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/(5*Sqrt[
2]*a^3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{3/2}(c+dx)^n}{(1+x)^{3/2}}\,dx, x, \sin(e+fx)\right)}{a^3 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{(1-x)^{3/2}\left(-\right.}{(1+}\right.}{a^3 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{5}{2};\frac{3}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(}{5\sqrt{2}\,a^3 f\sqrt{1+\sin(e+fx}}$$

Mathematica [F] time = 19.80, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3, x
]
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^4}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(3*a^3*cos(f*x + e)^2 - 4*a
^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{n} \cos\left(fx+e\right)^{4}}{\left(a \sin\left(fx+e\right)+a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^3, x)
```

maple [F] time = 3.89, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{4}\left(fx+e\right)\right)\left(c+d \sin\left(fx+e\right)\right)^{n}}{\left(a+a \sin\left(fx+e\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{n} \cos\left(fx+e\right)^{4}}{\left(a \sin\left(fx+e\right)+a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^4\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3,x)
[Out] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
[Out] Timed out
```

3.951 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx$

Optimal. Leaf size=121

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{10\sqrt{2}\,a^4f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -1/20*AppellF1(5/2,-n,5/2,7/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(1-sin(f*x+e))^2*(c+d*sin(f*x+e))^n/a^4/f/(((c+d*sin(f*x+e))/(c+d)
)^n)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{10\sqrt{2}\,a^4f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^4,x]
```

```
[Out] -(AppellF1[5/2, 5/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/(10*Sqrt
[2]*a^4*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx = \frac{\cos(e+fx)\,\mathrm{Subst}\left(\int \frac{(1-x)^{3/2}(c+dx)^n}{(1+x)^{5/2}}\,dx, x, \sin(e+fx)\right)}{a^4 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\mathrm{Subst}\left(\int \frac{(1-x)^{3/2}\left(-\right.}{(1+}\right.}{a^4 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{5}{2};\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(}{10\sqrt{2}\,a^4 f\sqrt{1+\sin(e+fx}}$$

Mathematica [F] time = 25.13, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^4,x]
```

```
[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^4, x
]
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^4}{a^4\cos\left(fx+e\right)^4-8\,a^4\cos\left(fx+e\right)^2+8\,a^4-4\left(a^4\cos\left(fx+e\right)^2-2\,a^4\right)\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorithm="
fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a^4*cos(f*x + e)^4 - 8*a^4*
cos(f*x + e)^2 + 8*a^4 - 4*(a^4*cos(f*x + e)^2 - 2*a^4)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^4, x)
```

maple [F] time = 6.31, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{4}\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x)
```

```
[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^4\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^4,x)
[Out] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**4,x)
[Out] Timed out
```

3.952 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^5}\,dx$

Optimal. Leaf size=121

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{20\sqrt{2}\,a^5f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -1/40*AppellF1(5/2,-n,7/2,7/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(1-sin(f*x+e))^2*(c+d*sin(f*x+e))^n/a^5/f/(((c+d*sin(f*x+e))/(c+d)
)^n)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{20\sqrt{2}\,a^5f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^5,x]
```

```
[Out] -(AppellF1[5/2, 7/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/(20*Sqrt
[2]*a^5*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^5}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{3/2}(c+dx)^n}{(1+x)^{7/2}}\,dx, x, \sin(e+fx)\right)}{a^5 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{(1-x)^{3/2}\left(-\right.}{(1+}\right.}{a^5 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{5}{2};\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(}{20\sqrt{2}\,a^5 f\sqrt{1+\sin(e+f}}$$

Mathematica [F] time = 2.43, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^5,x]
```

```
[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^5, x
]
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^4}{5\,a^5\cos\left(fx+e\right)^4-20\,a^5\cos\left(fx+e\right)^2+16\,a^5+\left(a^5\cos\left(fx+e\right)^4-12\,a^5\cos\left(fx+e\right)^2+16\,a^5\right)\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x, algorithm="
fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(5*a^5*cos(f*x + e)^4 - 20*a
^5*cos(f*x + e)^2 + 16*a^5 + (a^5*cos(f*x + e)^4 - 12*a^5*cos(f*x + e)^2 +
16*a^5)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^5, x)
```

maple [F] time = 4.48, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{4}\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x)
```

```
[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^4\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^5,x)
[Out] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**5,x)
[Out] Timed out
```

3.953 $\int \cos^7(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$-\frac{B(a\sin(c+dx)+a)^9}{9a^8d}-\frac{(A-7B)(a\sin(c+dx)+a)^8}{8a^7d}+\frac{6(A-3B)(a\sin(c+dx)+a)^7}{7a^6d}-\frac{2(3A-5B)(a\sin(c+dx)\cdots}{3a^5d}$$

```
[Out] 8/5*(A-B)*(a+a*sin(d*x+c))^5/a^4/d-2/3*(3*A-5*B)*(a+a*sin(d*x+c))^6/a^5/d+6
/7*(A-3*B)*(a+a*sin(d*x+c))^7/a^6/d-1/8*(A-7*B)*(a+a*sin(d*x+c))^8/a^7/d-1/
9*B*(a+a*sin(d*x+c))^9/a^8/d
```

Rubi [A] time = 0.14, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 77}

$$-\frac{(A-7B)(a\sin(c+dx)+a)^8}{8a^7d}+\frac{6(A-3B)(a\sin(c+dx)+a)^7}{7a^6d}-\frac{2(3A-5B)(a\sin(c+dx)+a)^6}{3a^5d}+\frac{8(A-B)(a\sin(\cdots}{5a^4\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (8*(A - B)*(a + a*Sin[c + d*x])^5)/(5*a^4*d) - (2*(3*A - 5*B)*(a + a*Sin[c
 + d*x])^6)/(3*a^5*d) + (6*(A - 3*B)*(a + a*Sin[c + d*x])^7)/(7*a^6*d) - ((A
 - 7*B)*(a + a*Sin[c + d*x])^8)/(8*a^7*d) - (B*(a + a*Sin[c + d*x])^9)/(9*a
^8*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^4\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+d\right.}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(8a^3(A-B)(a+x)^4 - 4a^2(3A-5B)(a+x\right.\right.}{}$$

$$= \frac{8(A-B)(a+a\sin(c+dx))^5}{5a^4 d} - \frac{2(3A-5B)(a+a\sin}{3a^5 d}$$

Mathematica [A] time = 0.81, size = 194, normalized size = 1.45

$$\frac{a(\sin(c+dx)+1)(-17640(A+B)\cos(2(c+dx)) - 8820(A+B)\cos(4(c+dx)) + 176400A\sin(c+dx) + 3528}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(1 + Sin[c + d*x])*(-17640*(A + B)*Cos[2*(c + d*x)] - 8820*(A + B)*Cos[4
*(c + d*x)] - 2520*A*Cos[6*(c + d*x)] - 2520*B*Cos[6*(c + d*x)] - 315*A*Cos
[8*(c + d*x)] - 315*B*Cos[8*(c + d*x)] + 176400*A*Sin[c + d*x] + 17640*B*Si
n[c + d*x] + 35280*A*Sin[3*(c + d*x)] + 7056*A*Sin[5*(c + d*x)] - 2016*B*Si
n[5*(c + d*x)] + 720*A*Sin[7*(c + d*x)] - 900*B*Sin[7*(c + d*x)] - 140*B*Si
n[9*(c + d*x)]))/(322560*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)
```

fricas [A] time = 0.61, size = 97, normalized size = 0.72

$$-\frac{315\,(A+B)a\cos\left(dx+c\right)^8 + 8\left(35\,Ba\cos\left(dx+c\right)^8 - 5\,(9\,A+B)a\cos\left(dx+c\right)^6 - 6\,(9\,A+B)a\cos\left(dx+c\right)^4 -\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2520*(315*(A + B)*a*cos(d*x + c)^8 + 8*(35*B*a*cos(d*x + c)^8 - 5*(9*A +
 B)*a*cos(d*x + c)^6 - 6*(9*A + B)*a*cos(d*x + c)^4 - 8*(9*A + B)*a*cos(d*x
 + c)^2 - 16*(9*A + B)*a)*sin(d*x + c))/d
```

giac [A] time = 0.30, size = 182, normalized size = 1.36

$$-\frac{Ba\sin\left(9\,dx+9\,c\right)}{2304\,d} + \frac{7\,Aa\sin\left(3\,dx+3\,c\right)}{64\,d} - \frac{(Aa+Ba)\cos\left(8\,dx+8\,c\right)}{1024\,d} - \frac{(Aa+Ba)\cos\left(6\,dx+6\,c\right)}{128\,d} - \frac{7\,(Aa+Ba)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2304*B*a*sin(9*d*x + 9*c)/d + 7/64*A*a*sin(3*d*x + 3*c)/d - 1/1024*(A*a
+ B*a)*cos(8*d*x + 8*c)/d - 1/128*(A*a + B*a)*cos(6*d*x + 6*c)/d - 7/256*(A
*a + B*a)*cos(4*d*x + 4*c)/d - 7/128*(A*a + B*a)*cos(2*d*x + 2*c)/d + 1/179
2*(4*A*a - 5*B*a)*sin(7*d*x + 7*c)/d + 1/320*(7*A*a - 2*B*a)*sin(5*d*x + 5*
c)/d + 7/128*(10*A*a + B*a)*sin(d*x + c)/d
```

maple [A] time = 0.46, size = 128, normalized size = 0.96

$$\frac{aB\left(-\frac{\left(\cos^{8}(dx+c)\right)\sin(dx+c)}{9}+\frac{\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}(dx+c)\right)}{5}+\frac{8\left(\cos^{2}(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)-\frac{aA\left(\cos^{8}(dx+c)\right)}{8}-\frac{aB\left(\cos^{8}(dx+c)\right)}{8}+\frac{aA\left(\frac{16}{5}+\cos^{6}\cdots\right.}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(-1/9*cos(d*x+c)^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+
c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/8*a*A*cos(d*x+c)^8-1/8*a*B*cos(d*x+c)^
8+1/7*a*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.36, size = 134, normalized size = 1.00

$$-\frac{280\,Ba\sin(dx+c)^{9}+315\,(A+B)a\sin(dx+c)^{8}+360\,(A-3\,B)a\sin(dx+c)^{7}-1260\,(A+B)a\sin(dx+c)^{6}-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2520*(280*B*a*sin(d*x + c)^9 + 315*(A + B)*a*sin(d*x + c)^8 + 360*(A - 3
*B)*a*sin(d*x + c)^7 - 1260*(A + B)*a*sin(d*x + c)^6 - 1512*(A - B)*a*sin(d
*x + c)^5 + 1890*(A + B)*a*sin(d*x + c)^4 + 840*(3*A - B)*a*sin(d*x + c)^3
- 1260*(A + B)*a*sin(d*x + c)^2 - 2520*A*a*sin(d*x + c))/d
```

mupad [B] time = 0.12, size = 134, normalized size = 1.00

$$-\frac{\frac{B\,a\sin(c+d\,x)^{9}}{9}+\frac{a\,(A+B)\sin(c+d\,x)^{8}}{8}+\frac{a\,(A-3\,B)\sin(c+d\,x)^{7}}{7}-\frac{a\,(A+B)\sin(c+d\,x)^{6}}{2}-\frac{3\,a\,(A-B)\sin(c+d\,x)^{5}}{5}+\frac{3\,a\,(A+B)\sin(c+d\,x)^{4}}{4}+\frac{a\,(3\cdots}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(A + B*sin(c + d*x))*(a + a*sin(c + d*x)),x)
```

```
[Out] -((a*sin(c + d*x)^3*(3*A - B))/3 - A*a*sin(c + d*x) - (a*sin(c + d*x)^2*(A
+ B))/2 + (3*a*sin(c + d*x)^4*(A + B))/4 - (a*sin(c + d*x)^6*(A + B))/2 + (
a*sin(c + d*x)^8*(A + B))/8 - (3*a*sin(c + d*x)^5*(A - B))/5 + (a*sin(c + d
*x)^7*(A - 3*B))/7 + (B*a*sin(c + d*x)^9)/9)/d
```

sympy [A] time = 13.88, size = 228, normalized size = 1.70

$$\begin{cases} \frac{16Aa\sin^7(c+dx)}{35d} + \frac{8Aa\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2Aa\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{Aa\sin(c+dx)\cos^6(c+dx)}{d} - \frac{Aa\cos^8(c+dx)}{8d} + \frac{16Ba\sin^9(c+dx)}{315d} \\ x(A+B\sin(c))(a\sin(c)+a)\cos^7(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*A*a*sin(c + d*x)**7/(35*d) + 8*A*a*sin(c + d*x)**5*cos(c + d*
x)**2/(5*d) + 2*A*a*sin(c + d*x)**3*cos(c + d*x)**4/d + A*a*sin(c + d*x)*co
s(c + d*x)**6/d - A*a*cos(c + d*x)**8/(8*d) + 16*B*a*sin(c + d*x)**9/(315*d
) + 8*B*a*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 2*B*a*sin(c + d*x)**5*co
s(c + d*x)**4/(5*d) + B*a*sin(c + d*x)**3*cos(c + d*x)**6/(3*d) - B*a*cos(c
 + d*x)**8/(8*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)*cos(c)**7, Tr
ue))
```

3.954 $\int \cos^5(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=102

$$\frac{B(a\sin(c+dx)+a)^7}{7a^6d}+\frac{(A-5B)(a\sin(c+dx)+a)^6}{6a^5d}-\frac{4(A-2B)(a\sin(c+dx)+a)^5}{5a^4d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{a^3d}$$

```
[Out] (A-B)*(a+a*sin(d*x+c))^4/a^3/d-4/5*(A-2*B)*(a+a*sin(d*x+c))^5/a^4/d+1/6*(A-
5*B)*(a+a*sin(d*x+c))^6/a^5/d+1/7*B*(a+a*sin(d*x+c))^7/a^6/d
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 77}

$$\frac{(A-5B)(a\sin(c+dx)+a)^6}{6a^5d}-\frac{4(A-2B)(a\sin(c+dx)+a)^5}{5a^4d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{a^3d}+\frac{B(a\sin(c+dx)+a)^7}{7a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^4)/(a^3*d) - (4*(A - 2*B)*(a + a*Sin[c + d*x]
)^5)/(5*a^4*d) + ((A - 5*B)*(a + a*Sin[c + d*x])^6)/(6*a^5*d) + (B*(a + a*S
in[c + d*x])^7)/(7*a^6*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^3\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+d\right.}{a^5d}$$

$$= \frac{\text{Subst}\left(\int \left(4a^2(A-B)(a+x)^3 - 4a(A-2B)(a+x)^4\right.\right.}{a^5}$$

$$= \frac{(A-B)(a+a\sin(c+dx))^4}{a^3d} - \frac{4(A-2B)(a+a\sin(c}{5a^4d}$$

Mathematica [A] time = 0.66, size = 130, normalized size = 1.27

$$-\frac{a(525(A+B)\cos(2(c+dx))+210(A+B)\cos(4(c+dx))-4200A\sin(c+dx)-700A\sin(3(c+dx))-84A\,\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/6720*(a*(525*(A + B)*Cos[2*(c + d*x)] + 210*(A + B)*Cos[4*(c + d*x)] + 3
5*A*Cos[6*(c + d*x)] + 35*B*Cos[6*(c + d*x)] - 4200*A*Sin[c + d*x] - 525*B*
Sin[c + d*x] - 700*A*Sin[3*(c + d*x)] + 35*B*Sin[3*(c + d*x)] - 84*A*Sin[5*
(c + d*x)] + 63*B*Sin[5*(c + d*x)] + 15*B*Sin[7*(c + d*x)]))/d
```

fricas [A] time = 0.72, size = 81, normalized size = 0.79

$$-\frac{35\,(A+B)a\cos\left(dx+c\right)^6+2\left(15\,Ba\cos\left(dx+c\right)^6-3\,(7\,A+B)a\cos\left(dx+c\right)^4-4\,(7\,A+B)a\cos\left(dx+c\right)^2-8\right.}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/210*(35*(A + B)*a*cos(d*x + c)^6 + 2*(15*B*a*cos(d*x + c)^6 - 3*(7*A + B
)*a*cos(d*x + c)^4 - 4*(7*A + B)*a*cos(d*x + c)^2 - 8*(7*A + B)*a)*sin(d*x
+ c))/d
```

giac [A] time = 0.24, size = 145, normalized size = 1.42

$$-\frac{Ba\sin\left(7\,dx+7\,c\right)}{448\,d}-\frac{(Aa+Ba)\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{(Aa+Ba)\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\,(Aa+Ba)\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{(4\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/448*B*a*sin(7*d*x + 7*c)/d - 1/192*(A*a + B*a)*cos(6*d*x + 6*c)/d - 1/32
*(A*a + B*a)*cos(4*d*x + 4*c)/d - 5/64*(A*a + B*a)*cos(2*d*x + 2*c)/d + 1/3
20*(4*A*a - 3*B*a)*sin(5*d*x + 5*c)/d + 1/192*(20*A*a - B*a)*sin(3*d*x + 3*
c)/d + 5/64*(8*A*a + B*a)*sin(d*x + c)/d
```

maple [A] time = 0.45, size = 108, normalized size = 1.06

$$\frac{aB\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{aA\left(\cos^6(dx+c)\right)}{6}-\frac{aB\left(\cos^6(dx+c)\right)}{6}+\frac{aA\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(d\right.}{3}\right.}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-1/6*a*A*cos(d*x+c)^6-1/6*a*B*cos(d*x+c)^6+1/5*a*A*(8/3+cos
(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.35, size = 104, normalized size = 1.02

$$\frac{30\,Ba\sin\left(dx+c\right)^{7}+35\,(A+B)a\sin\left(dx+c\right)^{6}+42\,(A-2\,B)a\sin\left(dx+c\right)^{5}-105\,(A+B)a\sin\left(dx+c\right)^{4}-70\,(2}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/210*(30*B*a*sin(d*x + c)^7 + 35*(A + B)*a*sin(d*x + c)^6 + 42*(A - 2*B)*a
*sin(d*x + c)^5 - 105*(A + B)*a*sin(d*x + c)^4 - 70*(2*A - B)*a*sin(d*x + c
)^3 + 105*(A + B)*a*sin(d*x + c)^2 + 210*A*a*sin(d*x + c))/d
```

mupad [B] time = 0.08, size = 102, normalized size = 1.00

$$\frac{\frac{B\,a\,\sin(c+d\,x)^7}{7}+\frac{a\,(A+B)\,\sin(c+d\,x)^6}{6}+\frac{a\,(A-2\,B)\,\sin(c+d\,x)^5}{5}-\frac{a\,(A+B)\,\sin(c+d\,x)^4}{2}-\frac{a\,(2\,A-B)\,\sin(c+d\,x)^3}{3}+\frac{a\,(A+B)\,\sin(c+d\,x)^2}{2}+A\,a\,\mathrm{si}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B*sin(c + d*x))*(a + a*sin(c + d*x)),x)
```

```
[Out] (A*a*sin(c + d*x) - (a*sin(c + d*x)^3*(2*A - B))/3 + (a*sin(c + d*x)^2*(A +
 B))/2 - (a*sin(c + d*x)^4*(A + B))/2 + (a*sin(c + d*x)^6*(A + B))/6 + (a*s
in(c + d*x)^5*(A - 2*B))/5 + (B*a*sin(c + d*x)^7)/7)/d
```

sympy [A] time = 5.33, size = 178, normalized size = 1.75

$$\begin{cases} \frac{8Aa\sin^5(c+dx)}{15d} + \frac{4Aa\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{Aa\sin(c+dx)\cos^4(c+dx)}{d} - \frac{Aa\cos^6(c+dx)}{6d} + \frac{8Ba\sin^7(c+dx)}{105d} + \frac{4Ba\sin^5(c+dx)\cos^2(c+ \\ x(A+B\sin(c))(a\sin(c)+a)\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*A*a*sin(c + d*x)**5/(15*d) + 4*A*a*sin(c + d*x)**3*cos(c + d*x
)**2/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**4/d - A*a*cos(c + d*x)**6/(6*d)
 + 8*B*a*sin(c + d*x)**7/(105*d) + 4*B*a*sin(c + d*x)**5*cos(c + d*x)**2/(1
5*d) + B*a*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - B*a*cos(c + d*x)**6/(6*d
), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)*cos(c)**5, True))
```

3.955 $\int \cos^3(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$-\frac{B(a\sin(c+dx)+a)^5}{5a^4d}-\frac{(A-3B)(a\sin(c+dx)+a)^4}{4a^3d}+\frac{2(A-B)(a\sin(c+dx)+a)^3}{3a^2d}$$

```
[Out] 2/3*(A-B)*(a+a*sin(d*x+c))^3/a^2/d-1/4*(A-3*B)*(a+a*sin(d*x+c))^4/a^3/d-1/5
*B*(a+a*sin(d*x+c))^5/a^4/d
```

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 77}

$$-\frac{(A-3B)(a\sin(c+dx)+a)^4}{4a^3d}+\frac{2(A-B)(a\sin(c+dx)+a)^3}{3a^2d}-\frac{B(a\sin(c+dx)+a)^5}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[c + d*x])^3)/(3*a^2*d) - ((A - 3*B)*(a + a*Sin[c + d*
x])^4)/(4*a^3*d) - (B*(a + a*Sin[c + d*x])^5)/(5*a^4*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^2\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+dx\right.}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(2a(A-B)(a+x)^2+(-A+3B)(a+x)^3-\right.\right.}{a^3d}$$
$$= \frac{2(A-B)(a+a\sin(c+dx))^3}{3a^2d} - \frac{(A-3B)(a+a\sin(c}{4a^3d}$$

Mathematica [A] time = 0.81, size = 78, normalized size = 1.00

$$-\frac{a(-4(100A+11B)\sin(c+dx)+3\cos(4(c+dx))(5(A+B)+4B\sin(c+dx))+\cos(2(c+dx))((32B-80A)\text{si}}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/480*(a*(-4*(100*A + 11*B)*Sin[c + d*x] + 3*Cos[4*(c + d*x)]*(5*(A + B) +
 4*B*Sin[c + d*x]) + Cos[2*(c + d*x)]*(60*(A + B) + (-80*A + 32*B)*Sin[c +
d*x])))/d
```

fricas [A] time = 0.79, size = 65, normalized size = 0.83

$$-\frac{15\,(A+B)a\cos\left(dx+c\right)^4+4\left(3\,Ba\cos\left(dx+c\right)^4-(5\,A+B)a\cos\left(dx+c\right)^2-2\,(5\,A+B)a\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/60*(15*(A + B)*a*cos(d*x + c)^4 + 4*(3*B*a*cos(d*x + c)^4 - (5*A + B)*a*
cos(d*x + c)^2 - 2*(5*A + B)*a)*sin(d*x + c))/d
```

giac [A] time = 0.20, size = 100, normalized size = 1.28

$$-\frac{12\,Ba\sin\left(dx+c\right)^5+15\,Aa\sin\left(dx+c\right)^4+15\,Ba\sin\left(dx+c\right)^4+20\,Aa\sin\left(dx+c\right)^3-20\,Ba\sin\left(dx+c\right)^3-30}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/60*(12*B*a*sin(d*x + c)^5 + 15*A*a*sin(d*x + c)^4 + 15*B*a*sin(d*x + c)^
4 + 20*A*a*sin(d*x + c)^3 - 20*B*a*sin(d*x + c)^3 - 30*A*a*sin(d*x + c)^2 -
 30*B*a*sin(d*x + c)^2 - 60*A*a*sin(d*x + c))/d
```

maple [A] time = 0.46, size = 88, normalized size = 1.13

$$\frac{aB\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{aA\left(\cos^4(dx+c)\right)}{4}-\frac{aB\left(\cos^4(dx+c)\right)}{4}+\frac{aA\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-1/
4*a*A*cos(d*x+c)^4-1/4*a*B*cos(d*x+c)^4+1/3*a*A*(2+cos(d*x+c)^2)*sin(d*x+c)
)
```

maxima [A] time = 0.35, size = 72, normalized size = 0.92

$$-\frac{12\,Ba\sin\left(dx+c\right)^5+15\,(A+B)a\sin\left(dx+c\right)^4+20\,(A-B)a\sin\left(dx+c\right)^3-30\,(A+B)a\sin\left(dx+c\right)^2-60\,Aa\,s}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/60*(12*B*a*sin(d*x + c)^5 + 15*(A + B)*a*sin(d*x + c)^4 + 20*(A - B)*a*s
in(d*x + c)^3 - 30*(A + B)*a*sin(d*x + c)^2 - 60*A*a*sin(d*x + c))/d
```

mupad [B] time = 9.01, size = 72, normalized size = 0.92

$$-\frac{\frac{B\,a\,\sin(c+d\,x)^5}{5}+\frac{a\,(A+B)\,\sin(c+d\,x)^4}{4}+\frac{a\,(A-B)\,\sin(c+d\,x)^3}{3}-\frac{a\,(A+B)\,\sin(c+d\,x)^2}{2}-A\,a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*sin(c + d*x))*(a + a*sin(c + d*x)),x)
```

```
[Out] -((a*sin(c + d*x)^4*(A + B))/4 - (a*sin(c + d*x)^2*(A + B))/2 - A*a*sin(c +
 d*x) + (a*sin(c + d*x)^3*(A - B))/3 + (B*a*sin(c + d*x)^5)/5)/d
```

sympy [A] time = 1.77, size = 128, normalized size = 1.64

$$\begin{cases} \frac{2Aa\sin^3(c+dx)}{3d}+\frac{Aa\sin(c+dx)\cos^2(c+dx)}{d}-\frac{Aa\cos^4(c+dx)}{4d}+\frac{2Ba\sin^5(c+dx)}{15d}+\frac{Ba\sin^3(c+dx)\cos^2(c+dx)}{3d}-\frac{Ba\cos^4(c+dx)}{4d} & \text{for } d \neq \\ x\left(A+B\sin(c)\right)\left(a\sin(c)+a\right)\cos^3(c) & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*A*a*sin(c + d*x)**3/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**2/d
 - A*a*cos(c + d*x)**4/(4*d) + 2*B*a*sin(c + d*x)**5/(15*d) + B*a*sin(c + d
*x)**3*cos(c + d*x)**2/(3*d) - B*a*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(A
+ B*sin(c))*(a*sin(c) + a)*cos(c)**3, True))
```

3.956 $\int \cos(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=49

$$\frac{a(A+B)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}+\frac{aB\sin^3(c+dx)}{3d}$$

```
[Out] a*A*sin(d*x+c)/d+1/2*a*(A+B)*sin(d*x+c)^2/d+1/3*a*B*sin(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 43}

$$\frac{a(A+B)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}+\frac{aB\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + (a*(A + B)*Sin[c + d*x]^2)/(2*d) + (a*B*Sin[c + d*x]
^3)/(3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(aA+(A+B)x+\frac{Bx^2}{a}\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{aA\sin(c+dx)}{d}+\frac{a(A+B)\sin^2(c+dx)}{2d}+\frac{aB\sin^3(c+dx)}{3d}$$

Mathematica [A] time = 0.39, size = 46, normalized size = 0.94

$$-\frac{a(\cos(2(c+dx))(3(A+B)+2B\sin(c+dx))-2(6A+B)\sin(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
[Out] -1/12*(a*(-2*(6*A + B)*Sin[c + d*x] + Cos[2*(c + d*x)]*(3*(A + B) + 2*B*Sin
[c + d*x])))/d
```

fricas [A] time = 0.70, size = 48, normalized size = 0.98

$$-\frac{3\,(A+B)a\cos\left(dx+c\right)^{2}+2\left(Ba\cos\left(dx+c\right)^{2}-(3\,A+B)a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fricas
")
[Out] -1/6*(3*(A + B)*a*cos(d*x + c)^2 + 2*(B*a*cos(d*x + c)^2 - (3*A + B)*a)*sin
(d*x + c))/d
```

giac [A] time = 0.14, size = 52, normalized size = 1.06

$$\frac{2\,Ba\sin\left(dx+c\right)^{3}+3\,Aa\sin\left(dx+c\right)^{2}+3\,Ba\sin\left(dx+c\right)^{2}+6\,Aa\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac")
[Out] 1/6*(2*B*a*sin(d*x + c)^3 + 3*A*a*sin(d*x + c)^2 + 3*B*a*sin(d*x + c)^2 + 6
*A*a*sin(d*x + c))/d
```

maple [A] time = 0.23, size = 44, normalized size = 0.90

$$\frac{\frac{aB\left(\sin^3(dx+c)\right)}{3}+\frac{(aA+aB)\left(\sin^2(dx+c)\right)}{2}+A\sin(dx+c)\,a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*B*sin(d*x+c)^3+1/2*(A*a+B*a)*sin(d*x+c)^2+A*sin(d*x+c)*a)
```

maxima [A] time = 0.35, size = 42, normalized size = 0.86

$$\frac{2\,Ba\sin(dx+c)^3+3\,(A+B)a\sin(dx+c)^2+6\,Aa\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/6*(2*B*a*sin(d*x + c)^3 + 3*(A + B)*a*sin(d*x + c)^2 + 6*A*a*sin(d*x + c)
)/d
```

mupad [B] time = 0.07, size = 40, normalized size = 0.82

$$\frac{\frac{B\,a\sin(c+d\,x)^3}{3}+\frac{a\,(A+B)\sin(c+d\,x)^2}{2}+A\,a\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*sin(c + d*x))*(a + a*sin(c + d*x)),x)
```

```
[Out] (A*a*sin(c + d*x) + (a*sin(c + d*x)^2*(A + B))/2 + (B*a*sin(c + d*x)^3)/3)/
d
```

sympy [A] time = 0.44, size = 75, normalized size = 1.53

$$\begin{cases}\frac{Aa\sin(c+dx)}{d}-\frac{Aa\cos^2(c+dx)}{2d}+\frac{Ba\sin^3(c+dx)}{3d}-\frac{Ba\cos^2(c+dx)}{2d} & \text{for } d\neq 0\\ x\,(A+B\sin(c))\,(a\sin(c)+a)\cos(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a*sin(c + d*x)/d - A*a*cos(c + d*x)**2/(2*d) + B*a*sin(c + d*x
)**3/(3*d) - B*a*cos(c + d*x)**2/(2*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin
(c) + a)*cos(c), True))
```

3.957 $\int \sec(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=34

$$-\frac{a(A+B)\log(1-\sin(c+dx))}{d}-\frac{aB\sin(c+dx)}{d}$$

```
[Out] -a*(A+B)*ln(1-sin(d*x+c))/d-a*B*sin(d*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2836, 43}

$$-\frac{a(A+B)\log(1-\sin(c+dx))}{d}-\frac{aB\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -((a*(A + B)*Log[1 - Sin[c + d*x]])/d) - (a*B*Sin[c + d*x])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{a\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(-\frac{B}{a}+\frac{A+B}{a-x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a(A+B)\log(1-\sin(c+dx))}{d} - \frac{aB\sin(c+dx)}{d}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 2.00

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d} - \frac{aA\log(\cos(c+dx))}{d} - \frac{aB\sin(c+dx)}{d} + \frac{aB\tanh^{-1}(\sin(c+dx))}{d} - \frac{aB\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d - (a*A*Log[Co
s[c + d*x]])/d - (a*B*Log[Cos[c + d*x]])/d - (a*B*Sin[c + d*x])/d
```

fricas [A] time = 0.84, size = 31, normalized size = 0.91

$$-\frac{(A+B)a\log(-\sin(dx+c)+1)+Ba\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] -((A + B)*a*log(-sin(d*x + c) + 1) + B*a*sin(d*x + c))/d
```

giac [B] time = 0.17, size = 114, normalized size = 3.35

$$\frac{(Aa+Ba)\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right) - 2(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 \cdots}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((A*a + B*a)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*(A*a + B*a)*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) - (A*a*tan(1/2*d*x + 1/2*c)^2 + B*a*tan(1/2*d*x + 1/2*
c)^2 + 2*B*a*tan(1/2*d*x + 1/2*c) + A*a + B*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)
)/d
```

maple [A] time = 0.36, size = 47, normalized size = 1.38

$$-\frac{a\ln\left(\sin\left(dx+c\right)-1\right)A}{d}-\frac{aB\sin\left(dx+c\right)}{d}-\frac{a\ln\left(\sin\left(dx+c\right)-1\right)B}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] -a/d*ln(sin(d*x+c)-1)*A-a*B*sin(d*x+c)/d-a/d*ln(sin(d*x+c)-1)*B
```

maxima [A] time = 0.36, size = 29, normalized size = 0.85

$$-\frac{(A+B)a\log\left(\sin\left(dx+c\right)-1\right)+Ba\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -((A + B)*a*log(sin(d*x + c) - 1) + B*a*sin(d*x + c))/d
```

mupad [B] time = 0.07, size = 35, normalized size = 1.03

$$-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(A\,a+B\,a\right)}{d}-\frac{B\,a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x),x)
```

```
[Out] - (log(sin(c + d*x) - 1)*(A*a + B*a))/d - (B*a*sin(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec\left(c+dx\right)dx+\int A\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\sin^{2}\left(c+dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x), x) + Integral(A*sin(c + d*x)*sec(c + d*x), x) +
 Integral(B*sin(c + d*x)*sec(c + d*x), x) + Integral(B*sin(c + d*x)**2*sec(
c + d*x), x))
```

3.958 $\int \sec^3(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=47

$$\frac{a^2(A+B)}{2d(a-a\sin(c+dx))}+\frac{a(A-B)\tanh^{-1}(\sin(c+dx))}{2d}$$

```
[Out] 1/2*a*(A-B)*arctanh(sin(d*x+c))/d+1/2*a^2*(A+B)/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 77, 206}

$$\frac{a^2(A+B)}{2d(a-a\sin(c+dx))}+\frac{a(A-B)\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(A - B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(A + B))/(2*d*(a - a*Sin[c +
 d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{a^3\operatorname{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^2(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\operatorname{Subst}\left(\int \left(\frac{A+B}{2a(a-x)^2}+\frac{A-B}{2a\left(a^2-x^2\right)}\right)dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2(A+B)}{2d(a-a\sin(c+dx))}+\frac{\left(a^2(A-B)\right)\operatorname{Subst}\left(\int \frac{1}{a^2-x^2}\,d$$
$$= \frac{a(A-B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(A+B)}{2d(a-a\sin(c+dx)}$$

Mathematica [C] time = 0.63, size = 260, normalized size = 5.53

$$\frac{a\left(2i(A-B)(\sin(c+dx)-1)\tan^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)+(A-B)\sin(c+dx)\left(2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(2*A + 2*B + I*A*d*x - I*B*d*x - 2*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 2*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*Log[(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^2] - B*Log[(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2]
 + (2*I)*(A - B)*ArcTan[Tan[(c + d*x)/2]]*(-1 + Sin[c + d*x]) + (A - B)*((-
I)*d*x + 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])^2])*Sin[c + d*x]))/(4*d*(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^2)
```

fricas [B] time = 0.71, size = 90, normalized size = 1.91

$$-\frac{2(A+B)a-((A-B)a\sin(dx+c)-(A-B)a)\log(\sin(dx+c)+1)+((A-B)a\sin(dx+c)-(A-B)a)\log}{4(d\sin(dx+c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/4*(2*(A + B)*a - ((A - B)*a*sin(d*x + c) - (A - B)*a)*log(sin(d*x + c) +
 1) + ((A - B)*a*sin(d*x + c) - (A - B)*a)*log(-sin(d*x + c) + 1))/(d*sin(d
*x + c) - d)
```

giac [A] time = 0.23, size = 84, normalized size = 1.79

$$\frac{(Aa - Ba)\log\left(\left|\sin\left(dx + c\right) + 1\right|\right) - (Aa - Ba)\log\left(\left|\sin\left(dx + c\right) - 1\right|\right) + \frac{Aa\sin(dx+c) - Ba\sin(dx+c) - 3\,Aa - Ba}{\sin(dx+c) - 1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/4*((A*a - B*a)*log(abs(sin(d*x + c) + 1)) - (A*a - B*a)*log(abs(sin(d*x +
 c) - 1)) + (A*a*sin(d*x + c) - B*a*sin(d*x + c) - 3*A*a - B*a)/(sin(d*x +
c) - 1))/d
```

maple [B] time = 0.57, size = 129, normalized size = 2.74

$$\frac{aA}{2d\cos\left(dx + c\right)^{2}} + \frac{aB\left(\sin^{3}\left(dx + c\right)\right)}{2d\cos\left(dx + c\right)^{2}} + \frac{aB\sin\left(dx + c\right)}{2d} - \frac{aB\ln\left(\sec\left(dx + c\right) + \tan\left(dx + c\right)\right)}{2d} + \frac{aA\sec\left(dx + c\right)\tan\left(d\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/2/d*a*A/cos(d*x+c)^2+1/2/d*a*B*sin(d*x+c)^3/cos(d*x+c)^2+1/2*a*B*sin(d*x+
c)/d-1/2/d*a*B*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*A*sec(d*x+c)*tan(d*x+c)+1/
2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*B/cos(d*x+c)^2
```

maxima [A] time = 0.34, size = 55, normalized size = 1.17

$$\frac{(A - B)a\log\left(\sin\left(dx + c\right) + 1\right) - (A - B)a\log\left(\sin\left(dx + c\right) - 1\right) - \frac{2\,(A+B)a}{\sin(dx+c) - 1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*((A - B)*a*log(sin(d*x + c) + 1) - (A - B)*a*log(sin(d*x + c) - 1) - 2*
(A + B)*a/(sin(d*x + c) - 1))/d
```

mupad [B] time = 9.12, size = 43, normalized size = 0.91

$$\frac{a\operatorname{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(A-B\right)}{2\,d}-\frac{\frac{A\,a}{2}+\frac{B\,a}{2}}{d\,\left(\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] (a*atanh(sin(c + d*x))*(A - B))/(2*d) - ((A*a)/2 + (B*a)/2)/(d*(sin(c + d*x
) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^{3}\left(c+dx\right)dx+\int A\sin\left(c+dx\right)\sec^{3}\left(c+dx\right)dx+\int B\sin\left(c+dx\right)\sec^{3}\left(c+dx\right)dx+\int B\sin^{2}\left(c+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**3, x) + Integral(A*sin(c + d*x)*sec(c + d*x)**3
, x) + Integral(B*sin(c + d*x)*sec(c + d*x)**3, x) + Integral(B*sin(c + d*x
)**2*sec(c + d*x)**3, x))
```

3.959 $\int \sec^5(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=100

$$\frac{a^3(A+B)}{8d(a-a\sin(c+dx))^2}-\frac{a^2(A-B)}{8d(a\sin(c+dx)+a)}+\frac{a^2A}{4d(a-a\sin(c+dx))}+\frac{a(3A-B)\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] 1/8*a*(3*A-B)*arctanh(sin(d*x+c))/d+1/8*a^3*(A+B)/d/(a-a*sin(d*x+c))^2+1/4*
a^2*A/d/(a-a*sin(d*x+c))-1/8*a^2*(A-B)/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 77, 206}

$$\frac{a^3(A+B)}{8d(a-a\sin(c+dx))^2}-\frac{a^2(A-B)}{8d(a\sin(c+dx)+a)}+\frac{a^2A}{4d(a-a\sin(c+dx))}+\frac{a(3A-B)\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(3*A - B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(A + B))/(8*d*(a - a*Sin[c
 + d*x])^2) + (a^2*A)/(4*d*(a - a*Sin[c + d*x])) - (a^2*(A - B))/(8*d*(a +
a*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{A+B}{4a^2(a-x)^3} + \frac{A}{4a^3(a-x)^2} + \frac{A-B}{8a^3(a+x)^2} + \frac{3A-}{8a^3(a^2-}\right.\right.}{d}$$

$$= \frac{a^3(A+B)}{8d(a-a\sin(c+dx))^2} + \frac{a^2A}{4d(a-a\sin(c+dx))} - \frac{}{8d(}$$

$$= \frac{a(3A-B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(A+B)}{8d(a-a\sin(c+d}$$

Mathematica [C] time = 1.56, size = 357, normalized size = 3.57

$$\frac{a(\sin(c+dx)+1)\left(ix(3A-B)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2 + \frac{2(A+B)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4} - \frac{2(3A-B)(s}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*((2*(-A + B))/d + I*(3*A - B)*x*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2
- ((2*I)*(3*A - B)*ArcTan[Tan[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^2)/d - (2*(3*A - B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^2)/d + ((3*A - B)*Log[(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/d + (2*(A + B)*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])^4) + (4*A*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(d*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])^2))*(1 + Sin[c + d*x]))/(16*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^4)
```

fricas [A] time = 0.85, size = 182, normalized size = 1.82

$$-\frac{2\,(3\,A-B)a\cos\left(dx+c\right)^{2}+2\,(3\,A-B)a\sin\left(dx+c\right)-2\,(A-3\,B)a-\Big((3\,A-B)a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-(3}{16\,\Big(d\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/16*(2*(3*A - B)*a*cos(d*x + c)^2 + 2*(3*A - B)*a*sin(d*x + c) - 2*(A - 3
*B)*a - ((3*A - B)*a*cos(d*x + c)^2*sin(d*x + c) - (3*A - B)*a*cos(d*x + c)
^2)*log(sin(d*x + c) + 1) + ((3*A - B)*a*cos(d*x + c)^2*sin(d*x + c) - (3*A
 - B)*a*cos(d*x + c)^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2*sin(d*x +
 c) - d*cos(d*x + c)^2)
```

giac [A] time = 0.22, size = 152, normalized size = 1.52

$$\frac{2\,(3\,Aa-Ba)\log\left(|\sin\left(dx+c\right)+1|\right)-2\,(3\,Aa-Ba)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\,(3\,Aa\sin(dx+c)-Ba\sin(dx+c)+5\,Aa-3\,Ba)}{\sin(dx+c)+1}+}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/32*(2*(3*A*a - B*a)*log(abs(sin(d*x + c) + 1)) - 2*(3*A*a - B*a)*log(abs(
sin(d*x + c) - 1)) - 2*(3*A*a*sin(d*x + c) - B*a*sin(d*x + c) + 5*A*a - 3*B
*a)/(sin(d*x + c) + 1) + (9*A*a*sin(d*x + c)^2 - 3*B*a*sin(d*x + c)^2 - 26*
A*a*sin(d*x + c) + 6*B*a*sin(d*x + c) + 21*A*a + B*a)/(sin(d*x + c) - 1)^2)
/d
```

maple [A] time = 0.60, size = 173, normalized size = 1.73

$$\frac{aA}{4d\cos\left(dx+c\right)^{4}}+\frac{aB\left(\sin^{3}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}+\frac{aB\left(\sin^{3}\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^{2}}+\frac{aB\sin\left(dx+c\right)}{8d}-\frac{aB\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{aA}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*A/cos(d*x+c)^4+1/4/d*a*B*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a*B*sin(d*
x+c)^3/cos(d*x+c)^2+1/8*a*B*sin(d*x+c)/d-1/8/d*a*B*ln(sec(d*x+c)+tan(d*x+c)
)+1/4/d*a*A*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a*A*sec(d*x+c)*tan(d*x+c)+3/8/d*a
*A*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a*B/cos(d*x+c)^4
```

maxima [A] time = 0.35, size = 115, normalized size = 1.15

$$\frac{(3A-B)a\log(\sin(dx+c)+1)-(3A-B)a\log(\sin(dx+c)-1)-\frac{2\left((3A-B)a\sin(dx+c)^2-(3A-B)a\sin(dx+c)-2(A+B)a\right)}{\sin(dx+c)^3-\sin(dx+c)^2-\sin(dx+c)+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/16*((3*A - B)*a*log(sin(d*x + c) + 1) - (3*A - B)*a*log(sin(d*x + c) - 1)
 - 2*((3*A - B)*a*sin(d*x + c)^2 - (3*A - B)*a*sin(d*x + c) - 2*(A + B)*a)/
(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x + c) + 1))/d
```

mupad [B] time = 0.14, size = 98, normalized size = 0.98

$$\frac{a\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(3\,A-B\right)}{8\,d}-\frac{\left(\frac{B\,a}{8}-\frac{3\,A\,a}{8}\right)\,\sin\left(c+d\,x\right)^2+\left(\frac{3\,A\,a}{8}-\frac{B\,a}{8}\right)\,\sin\left(c+d\,x\right)+\frac{A\,a}{4}+\frac{B\,a}{4}}{d\,\left(-\sin\left(c+d\,x\right)^3+\sin\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (a*atanh(sin(c + d*x))*(3*A - B))/(8*d) - ((A*a)/4 + (B*a)/4 + sin(c + d*x)
*((3*A*a)/8 - (B*a)/8) - sin(c + d*x)^2*((3*A*a)/8 - (B*a)/8))/(d*(sin(c +
d*x) + sin(c + d*x)^2 - sin(c + d*x)^3 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.960 $\int \sec^7(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=157

$$\frac{a^4(A+B)}{24d(a-a\sin(c+dx))^3}+\frac{a^3(3A+B)}{32d(a-a\sin(c+dx))^2}-\frac{a^3(A-B)}{32d(a\sin(c+dx)+a)^2}-\frac{a^2(2A-B)}{16d(a\sin(c+dx)+a)}+\frac{3a^2A}{16d(a-a\sin \dots}$$

```
[Out] 1/16*a*(5*A-B)*arctanh(sin(d*x+c))/d+1/24*a^4*(A+B)/d/(a-a*sin(d*x+c))^3+1/
32*a^3*(3*A+B)/d/(a-a*sin(d*x+c))^2+3/16*a^2*A/d/(a-a*sin(d*x+c))-1/32*a^3*
(A-B)/d/(a+a*sin(d*x+c))^2-1/16*a^2*(2*A-B)/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 77, 206}

$$\frac{a^4(A+B)}{24d(a-a\sin(c+dx))^3}+\frac{a^3(3A+B)}{32d(a-a\sin(c+dx))^2}-\frac{a^3(A-B)}{32d(a\sin(c+dx)+a)^2}-\frac{a^2(2A-B)}{16d(a\sin(c+dx)+a)}+\frac{3a^2A}{16d(a-a\sin \dots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(5*A - B)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^4*(A + B))/(24*d*(a - a*Sin
[c + d*x])^3) + (a^3*(3*A + B))/(32*d*(a - a*Sin[c + d*x])^2) + (3*a^2*A)/(
16*d*(a - a*Sin[c + d*x])) - (a^3*(A - B))/(32*d*(a + a*Sin[c + d*x])^2) -
(a^2*(2*A - B))/(16*d*(a + a*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\,\text{Subst}\left(\int\left(\frac{A+B}{8a^3(a-x)^4}+\frac{3A+B}{16a^4(a-x)^3}+\frac{3A}{16a^5(a-x)^2}+\frac{A\cdot}{16a^4(}\right.\right.}{d}$$

$$= \frac{a^4(A+B)}{24d(a-a\sin(c+dx))^3}+\frac{a^3(3A+B)}{32d(a-a\sin(c+dx))^2}+$$

$$= \frac{a(5A-B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(A+B)}{24d(a-a\sin(c+a}$$

Mathematica [C] time = 1.75, size = 451, normalized size = 2.87

$$\frac{a(\sin(c+dx)+1)\left(3ix(5A-B)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4+\frac{3(3A+B)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}+\frac{4(A+B)}{d(\text{co}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*((3*(-A + B))/d - (6*(2*A - B)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/
d + (3*I)*(5*A - B)*x*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - ((6*I)*(5*A
 - B)*ArcTan[Tan[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/d -
 (6*(5*A - B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^4)/d + (3*(5*A - B)*Log[(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/d + (4*(A + B)*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^4)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^6) +
 (3*(3*A + B)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(d*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^4) + (18*A*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(d
```

```
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2))*(1 + Sin[c + d*x]))/(96*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.73, size = 222, normalized size = 1.41

$$-\frac{6\,(5\,A-B)a\cos\left(dx+c\right)^{4}-2\,(5\,A-B)a\cos\left(dx+c\right)^{2}-4\,(A-5\,B)a-3\,\left((5\,A-B)a\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/96*(6*(5*A - B)*a*cos(d*x + c)^4 - 2*(5*A - B)*a*cos(d*x + c)^2 - 4*(A -
 5*B)*a - 3*((5*A - B)*a*cos(d*x + c)^4*sin(d*x + c) - (5*A - B)*a*cos(d*x
+ c)^4)*log(sin(d*x + c) + 1) + 3*((5*A - B)*a*cos(d*x + c)^4*sin(d*x + c)
- (5*A - B)*a*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) + 2*(3*(5*A - B)*a*cos
(d*x + c)^2 + 2*(5*A - B)*a)*sin(d*x + c))/(d*cos(d*x + c)^4*sin(d*x + c) -
 d*cos(d*x + c)^4)
```

giac [A] time = 0.23, size = 201, normalized size = 1.28

$$\frac{6\,(5\,Aa-Ba)\log\left(|\sin\left(dx+c\right)+1|\right)-6\,(5\,Aa-Ba)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{3\left(15\,Aa\sin(dx+c)^{2}-3\,Ba\sin(dx+c)^{2}+38\,Aa\sin\right.}{(\sin(dx+c)+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/192*(6*(5*A*a - B*a)*log(abs(sin(d*x + c) + 1)) - 6*(5*A*a - B*a)*log(abs
(sin(d*x + c) - 1)) - 3*(15*A*a*sin(d*x + c)^2 - 3*B*a*sin(d*x + c)^2 + 38*
A*a*sin(d*x + c) - 10*B*a*sin(d*x + c) + 25*A*a - 9*B*a)/(sin(d*x + c) + 1)
^2 + (55*A*a*sin(d*x + c)^3 - 11*B*a*sin(d*x + c)^3 - 201*A*a*sin(d*x + c)^
2 + 33*B*a*sin(d*x + c)^2 + 255*A*a*sin(d*x + c) - 27*B*a*sin(d*x + c) - 11
7*A*a - 3*B*a)/(sin(d*x + c) - 1)^3)/d
```

maple [A] time = 0.60, size = 217, normalized size = 1.38

$$\frac{aA}{6d\cos\left(dx+c\right)^{6}}+\frac{aB\left(\sin^{3}\left(dx+c\right)\right)}{6d\cos\left(dx+c\right)^{6}}+\frac{aB\left(\sin^{3}\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^{4}}+\frac{aB\left(\sin^{3}\left(dx+c\right)\right)}{16d\cos\left(dx+c\right)^{2}}+\frac{aB\sin\left(dx+c\right)}{16d}-\frac{aB\ln\left(\sec\left(dx+c\right)\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/6/d*a*A/cos(d*x+c)^6+1/6/d*a*B*sin(d*x+c)^3/cos(d*x+c)^6+1/8/d*a*B*sin(d*
x+c)^3/cos(d*x+c)^4+1/16/d*a*B*sin(d*x+c)^3/cos(d*x+c)^2+1/16*a*B*sin(d*x+c
)/d-1/16/d*a*B*ln(sec(d*x+c)+tan(d*x+c))+1/6/d*a*A*tan(d*x+c)*sec(d*x+c)^5+
5/24/d*a*A*tan(d*x+c)*sec(d*x+c)^3+5/16/d*a*A*sec(d*x+c)*tan(d*x+c)+5/16/d*
a*A*ln(sec(d*x+c)+tan(d*x+c))+1/6/d*a*B/cos(d*x+c)^6
```

maxima [A] time = 0.35, size = 171, normalized size = 1.09

$$\frac{3\,(5\,A-B)a\log\left(\sin\left(dx+c\right)+1\right)-3\,(5\,A-B)a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,(5\,A-B)a\sin(dx+c)^{4}-3\,(5\,A-B)a\sin(dx+c)^{3}-5\cdots\right.}{\sin(dx+c)^{5}-\sin(dx+c)^{4}-2\,\sin(\cdots}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(3*(5*A - B)*a*log(sin(d*x + c) + 1) - 3*(5*A - B)*a*log(sin(d*x + c)
- 1) - 2*(3*(5*A - B)*a*sin(d*x + c)^4 - 3*(5*A - B)*a*sin(d*x + c)^3 - 5*(
5*A - B)*a*sin(d*x + c)^2 + 5*(5*A - B)*a*sin(d*x + c) + 8*(A + B)*a)/(sin(
d*x + c)^5 - sin(d*x + c)^4 - 2*sin(d*x + c)^3 + 2*sin(d*x + c)^2 + sin(d*x
 + c) - 1))/d
```

mupad [B] time = 9.26, size = 155, normalized size = 0.99

$$\frac{a\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(5\,A-B\right)}{16\,d}-\frac{\left(\frac{5\,A\,a}{16}-\frac{B\,a}{16}\right)\,\sin\left(c+d\,x\right)^{4}+\left(\frac{B\,a}{16}-\frac{5\,A\,a}{16}\right)\,\sin\left(c+d\,x\right)^{3}+\left(\frac{5\,B\,a}{48}-\frac{25\,A\,a}{48}\right)\,\sin\left(c\cdots\right.}{d\,\left(\sin\left(c+d\,x\right)^{5}-\sin\left(c+d\,x\right)^{4}-2\,\sin\left(c+d\,x\right)^{3}+2\,\sin\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^7,x)
```

```
[Out] (a*atanh(sin(c + d*x))*(5*A - B))/(16*d) - ((A*a)/6 + (B*a)/6 + sin(c + d*x
)*((25*A*a)/48 - (5*B*a)/48) - sin(c + d*x)^3*((5*A*a)/16 - (B*a)/16) + sin
(c + d*x)^4*((5*A*a)/16 - (B*a)/16) - sin(c + d*x)^2*((25*A*a)/48 - (5*B*a)
/48))/(d*(sin(c + d*x) + 2*sin(c + d*x)^2 - 2*sin(c + d*x)^3 - sin(c + d*x)
^4 + sin(c + d*x)^5 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.961 $\int \cos^6(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=138

$$-\frac{a(8A+B)\cos^7(c+dx)}{56d}+\frac{a(8A+B)\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a(8A+B)\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5a(8A+B}{}$$

```
[Out] 5/128*a*(8*A+B)*x-1/56*a*(8*A+B)*cos(d*x+c)^7/d+5/128*a*(8*A+B)*cos(d*x+c)*
sin(d*x+c)/d+5/192*a*(8*A+B)*cos(d*x+c)^3*sin(d*x+c)/d+1/48*a*(8*A+B)*cos(d
*x+c)^5*sin(d*x+c)/d-1/8*B*cos(d*x+c)^7*(a+a*sin(d*x+c))/d
```

Rubi [A] time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2860, 2669, 2635, 8}

$$-\frac{a(8A+B)\cos^7(c+dx)}{56d}+\frac{a(8A+B)\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a(8A+B)\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5a(8A+B}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (5*a*(8*A + B)*x)/128 - (a*(8*A + B)*Cos[c + d*x]^7)/(56*d) + (5*a*(8*A + B
)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (5*a*(8*A + B)*Cos[c + d*x]^3*Sin[c
+ d*x])/(192*d) + (a*(8*A + B)*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (B*Cos
[c + d*x]^7*(a + a*Sin[c + d*x]))/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = -\frac{B\cos^7(c+dx)(a+a\sin(c+dx))}{8d}+\frac{1}{8}(8A+B)\int$$
$$= -\frac{a(8A+B)\cos^7(c+dx)}{56d}-\frac{B\cos^7(c+dx)(a+a\sin}{8d}$$
$$= -\frac{a(8A+B)\cos^7(c+dx)}{56d}+\frac{a(8A+B)\cos^5(c+dx)}{48d}$$
$$= -\frac{a(8A+B)\cos^7(c+dx)}{56d}+\frac{5a(8A+B)\cos^3(c+dx)}{192d}$$
$$= -\frac{a(8A+B)\cos^7(c+dx)}{56d}+\frac{5a(8A+B)\cos(c+dx)}{128d}$$
$$= \frac{5}{128}a(8A+B)x-\frac{a(8A+B)\cos^7(c+dx)}{56d}+\frac{5a(8A}{}$$

Mathematica [A] time = 0.84, size = 164, normalized size = 1.19

$$-\frac{a(1680(A+B)\cos(c+dx)+1008(A+B)\cos(3(c+dx))-5040A\sin(2(c+dx))-1008A\sin(4(c+dx))-11}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/21504*(a*(-6720*A*d*x - 840*B*d*x + 1680*(A + B)*Cos[c + d*x] + 1008*(A
+ B)*Cos[3*(c + d*x)] + 336*A*Cos[5*(c + d*x)] + 336*B*Cos[5*(c + d*x)] + 4
8*A*Cos[7*(c + d*x)] + 48*B*Cos[7*(c + d*x)] - 5040*A*Sin[2*(c + d*x)] - 33
6*B*Sin[2*(c + d*x)] - 1008*A*Sin[4*(c + d*x)] + 168*B*Sin[4*(c + d*x)] - 1
12*A*Sin[6*(c + d*x)] + 112*B*Sin[6*(c + d*x)] + 21*B*Sin[8*(c + d*x)]))/d
```

fricas [A] time = 0.91, size = 97, normalized size = 0.70

$$-\frac{384\,(A+B)a\cos(dx+c)^7-105\,(8\,A+B)adx+7\left(48\,Ba\cos(dx+c)^7-8\,(8\,A+B)a\cos(dx+c)^5-10\,(8\,A\right.}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2688*(384*(A + B)*a*cos(d*x + c)^7 - 105*(8*A + B)*a*d*x + 7*(48*B*a*cos
(d*x + c)^7 - 8*(8*A + B)*a*cos(d*x + c)^5 - 10*(8*A + B)*a*cos(d*x + c)^3
- 15*(8*A + B)*a*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.25, size = 176, normalized size = 1.28

$$\frac{5}{128}(8\,Aa+Ba)x-\frac{Ba\sin(8\,dx+8\,c)}{1024\,d}-\frac{(Aa+Ba)\cos(7\,dx+7\,c)}{448\,d}-\frac{(Aa+Ba)\cos(5\,dx+5\,c)}{64\,d}-\frac{3\,(Aa+Ba)\cos(3}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 5/128*(8*A*a + B*a)*x - 1/1024*B*a*sin(8*d*x + 8*c)/d - 1/448*(A*a + B*a)*c
os(7*d*x + 7*c)/d - 1/64*(A*a + B*a)*cos(5*d*x + 5*c)/d - 3/64*(A*a + B*a)*
cos(3*d*x + 3*c)/d - 5/64*(A*a + B*a)*cos(d*x + c)/d + 1/192*(A*a - B*a)*si
n(6*d*x + 6*c)/d + 1/128*(6*A*a - B*a)*sin(4*d*x + 4*c)/d + 1/64*(15*A*a +
B*a)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.50, size = 138, normalized size = 1.00

$$\frac{aB\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{aA\left(\cos^7(dx+c)\right)}{7}-\frac{aB\left(\cos^7(dx+c)\right)}{7}+a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-1/7*a*A*cos(d*x+c)^7-1/7*a*B
*cos(d*x+c)^7+a*A*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(
d*x+c)+5/16*d*x+5/16*c))
```

maxima [A] time = 0.34, size = 124, normalized size = 0.90

$$-\frac{3072\,Aa\cos(dx+c)^7+3072\,Ba\cos(dx+c)^7+112\left(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48}{21504}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/21504*(3072*A*a*cos(d*x + c)^7 + 3072*B*a*cos(d*x + c)^7 + 112*(4*sin(2*
d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*A*
a - 7*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*si
n(4*d*x + 4*c))*B*a)/d
```

mupad [B] time = 10.72, size = 504, normalized size = 3.65

$$\frac{5\,a\,\mathrm{atan}\left(\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(8\,A+B)}{64\left(\frac{5\,A\,a}{8}+\frac{5\,B\,a}{64}\right)}\right)(8\,A+B)}{64\,d}-\frac{5\,a\,\left(8\,A+B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{64\,d}-\frac{\left(\frac{11\,A\,a}{8}-\frac{5\,B\,a}{64}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(A + B*sin(c + d*x))*(a + a*sin(c + d*x)),x)
```

```
[Out] (5*a*atan((5*a*tan(c/2 + (d*x)/2)*(8*A + B))/(64*((5*A*a)/8 + (5*B*a)/64)))
*(8*A + B))/(64*d) - (5*a*(8*A + B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(
64*d) - ((2*A*a)/7 + (2*B*a)/7 - tan(c/2 + (d*x)/2)*((11*A*a)/8 - (5*B*a)/6
4) + tan(c/2 + (d*x)/2)^4*(6*A*a + 6*B*a) + tan(c/2 + (d*x)/2)^12*(2*A*a +
2*B*a) + tan(c/2 + (d*x)/2)^6*(6*A*a + 6*B*a) + tan(c/2 + (d*x)/2)^14*(2*A*
a + 2*B*a) + tan(c/2 + (d*x)/2)^2*((2*A*a)/7 + (2*B*a)/7) + tan(c/2 + (d*x)
/2)^8*(10*A*a + 10*B*a) + tan(c/2 + (d*x)/2)^10*(10*A*a + 10*B*a) + tan(c/2
 + (d*x)/2)^15*((11*A*a)/8 - (5*B*a)/64) - tan(c/2 + (d*x)/2)^3*((61*A*a)/2
4 + (397*B*a)/192) + tan(c/2 + (d*x)/2)^13*((61*A*a)/24 + (397*B*a)/192) -
tan(c/2 + (d*x)/2)^5*((113*A*a)/24 - (895*B*a)/192) + tan(c/2 + (d*x)/2)^11
*((113*A*a)/24 - (895*B*a)/192) - tan(c/2 + (d*x)/2)^7*((85*A*a)/24 + (1765
*B*a)/192) + tan(c/2 + (d*x)/2)^9*((85*A*a)/24 + (1765*B*a)/192))/(d*(8*tan
(c/2 + (d*x)/2)^2 + 28*tan(c/2 + (d*x)/2)^4 + 56*tan(c/2 + (d*x)/2)^6 + 70*
tan(c/2 + (d*x)/2)^8 + 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12
+ 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1))
```

sympy [A] time = 9.70, size = 416, normalized size = 3.01

$$\begin{cases} \frac{5Aax\sin^6(c+dx)}{16}+\frac{15Aax\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{15Aax\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{5Aax\cos^6(c+dx)}{16}+\frac{5Aa\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{5A}{} \\ x\left(A+B\sin(c)\right)\left(a\sin(c)+a\right)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*A*a*x*sin(c + d*x)**6/16 + 15*A*a*x*sin(c + d*x)**4*cos(c + d*
x)**2/16 + 15*A*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*A*a*x*cos(c + d*
x)**6/16 + 5*A*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*A*a*sin(c + d*x)**
3*cos(c + d*x)**3/(6*d) + 11*A*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - A*a*
cos(c + d*x)**7/(7*d) + 5*B*a*x*sin(c + d*x)**8/128 + 5*B*a*x*sin(c + d*x)*
*6*cos(c + d*x)**2/32 + 15*B*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 5*B*a
*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 5*B*a*x*cos(c + d*x)**8/128 + 5*B*a
*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*B*a*sin(c + d*x)**5*cos(c + d*x)
**3/(384*d) + 73*B*a*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) - 5*B*a*sin(c
+ d*x)*cos(c + d*x)**7/(128*d) - B*a*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(
A + B*sin(c))*(a*sin(c) + a)*cos(c)**6, True))
```

3.962 $\int \cos^4(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=111

$$-\frac{a(6A+B)\cos^5(c+dx)}{30d}+\frac{a(6A+B)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a(6A+B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}ax(6A+B$$

```
[Out] 1/16*a*(6*A+B)*x-1/30*a*(6*A+B)*cos(d*x+c)^5/d+1/16*a*(6*A+B)*cos(d*x+c)*si
n(d*x+c)/d+1/24*a*(6*A+B)*cos(d*x+c)^3*sin(d*x+c)/d-1/6*B*cos(d*x+c)^5*(a+a
*sin(d*x+c))/d
```

Rubi [A] time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2860, 2669, 2635, 8}

$$-\frac{a(6A+B)\cos^5(c+dx)}{30d}+\frac{a(6A+B)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a(6A+B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}ax(6A+B$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(6*A + B)*x)/16 - (a*(6*A + B)*Cos[c + d*x]^5)/(30*d) + (a*(6*A + B)*Cos
[c + d*x]*Sin[c + d*x])/(16*d) + (a*(6*A + B)*Cos[c + d*x]^3*Sin[c + d*x])/
(24*d) - (B*Cos[c + d*x]^5*(a + a*Sin[c + d*x]))/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = -\frac{B\cos^5(c+dx)(a+a\sin(c+dx))}{6d}+\frac{1}{6}(6A+B)\int \mathrm{co}$$
$$= -\frac{a(6A+B)\cos^5(c+dx)}{30d}-\frac{B\cos^5(c+dx)(a+a\sin(c}{6d}$$
$$= -\frac{a(6A+B)\cos^5(c+dx)}{30d}+\frac{a(6A+B)\cos^3(c+dx)\,\mathrm{si}}{24d}$$
$$= -\frac{a(6A+B)\cos^5(c+dx)}{30d}+\frac{a(6A+B)\cos(c+dx)\,\mathrm{sin}}{16d}$$
$$= \frac{1}{16}a(6A+B)x-\frac{a(6A+B)\cos^5(c+dx)}{30d}+\frac{a(6A+B)}{}$$

Mathematica [A] time = 0.59, size = 120, normalized size = 1.08

$$-\frac{a(120(A+B)\cos(c+dx)+60(A+B)\cos(3(c+dx))-240A\sin(2(c+dx))-30A\sin(4(c+dx))+12A\cos(5(}{96}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/960*(a*(-360*A*d*x - 60*B*d*x + 120*(A + B)*Cos[c + d*x] + 60*(A + B)*Co
s[3*(c + d*x)] + 12*A*Cos[5*(c + d*x)] + 12*B*Cos[5*(c + d*x)] - 240*A*Sin[
2*(c + d*x)] - 15*B*Sin[2*(c + d*x)] - 30*A*Sin[4*(c + d*x)] + 15*B*Sin[4*(
c + d*x)] + 5*B*Sin[6*(c + d*x)]))/d
```

fricas [A] time = 0.81, size = 81, normalized size = 0.73

$$-\frac{48\,(A+B)a\cos\left(dx+c\right)^5-15\,(6\,A+B)adx+5\left(8\,Ba\cos\left(dx+c\right)^5-2\,(6\,A+B)a\cos\left(dx+c\right)^3-3\,(6\,A+B)a\,\mathrm{c}\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/240*(48*(A + B)*a*cos(d*x + c)^5 - 15*(6*A + B)*a*d*x + 5*(8*B*a*cos(d*x
 + c)^5 - 2*(6*A + B)*a*cos(d*x + c)^3 - 3*(6*A + B)*a*cos(d*x + c))*sin(d*
x + c))/d
```

giac [A] time = 0.21, size = 133, normalized size = 1.20

$$\frac{1}{16}(6\,Aa+Ba)x-\frac{Ba\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{(Aa+Ba)\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{(Aa+Ba)\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{(Aa+Ba)\cos\left(dx\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/16*(6*A*a + B*a)*x - 1/192*B*a*sin(6*d*x + 6*c)/d - 1/80*(A*a + B*a)*cos(
5*d*x + 5*c)/d - 1/16*(A*a + B*a)*cos(3*d*x + 3*c)/d - 1/8*(A*a + B*a)*cos(
d*x + c)/d + 1/64*(2*A*a - B*a)*sin(4*d*x + 4*c)/d + 1/64*(16*A*a + B*a)*si
n(2*d*x + 2*c)/d
```

maple [A] time = 0.49, size = 118, normalized size = 1.06

$$\frac{aB\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{aA\left(\cos^5(dx+c)\right)}{5}-\frac{aB\left(\cos^5(dx+c)\right)}{5}+aA\left(\frac{\left(\cos^3(dx+c)\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)-1/5*a*A*cos(d*x+c)^5-1/5*a*B*cos(d*x+c)^5+a*A*(1
/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.40, size = 98, normalized size = 0.88

$$-\frac{192\,Aa\cos\left(dx+c\right)^{5}+192\,Ba\cos\left(dx+c\right)^{5}-30\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa-5\left(4\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/960*(192*A*a*cos(d*x + c)^5 + 192*B*a*cos(d*x + c)^5 - 30*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 5*(4*sin(2*d*x + 2*c)^3 + 1
2*d*x + 12*c - 3*sin(4*d*x + 4*c))*B*a)/d
```

mupad [B] time = 10.51, size = 391, normalized size = 3.52

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(6\,A+B)}{8\left(\frac{3\,A\,a}{4}+\frac{B\,a}{8}\right)}\right)(6\,A+B)}{8\,d}-\frac{a\,(6\,A+B)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{8\,d}-\frac{\left(\frac{5\,A\,a}{4}-\frac{B\,a}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B*sin(c + d*x))*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(6*A + B))/(8*((3*A*a)/4 + (B*a)/8)))*(6*A +
B))/(8*d) - (a*(6*A + B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(8*d) - ((2*
A*a)/5 + (2*B*a)/5 - tan(c/2 + (d*x)/2)*((5*A*a)/4 - (B*a)/8) + tan(c/2 + (
d*x)/2)^4*(4*A*a + 4*B*a) + tan(c/2 + (d*x)/2)^8*(2*A*a + 2*B*a) + tan(c/2
+ (d*x)/2)^6*(4*A*a + 4*B*a) + tan(c/2 + (d*x)/2)^10*(2*A*a + 2*B*a) + tan(
c/2 + (d*x)/2)^2*((2*A*a)/5 + (2*B*a)/5) - tan(c/2 + (d*x)/2)^5*((A*a)/2 -
(13*B*a)/4) + tan(c/2 + (d*x)/2)^7*((A*a)/2 - (13*B*a)/4) + tan(c/2 + (d*x)
/2)^11*((5*A*a)/4 - (B*a)/8) - tan(c/2 + (d*x)/2)^3*((7*A*a)/4 + (47*B*a)/2
4) + tan(c/2 + (d*x)/2)^9*((7*A*a)/4 + (47*B*a)/24))/(d*(6*tan(c/2 + (d*x)/
2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*
x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

sympy [A] time = 3.50, size = 306, normalized size = 2.76

$$\begin{cases} \frac{3Aax\sin^4(c+dx)}{8}+\frac{3Aax\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3Aax\cos^4(c+dx)}{8}+\frac{3Aa\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5Aa\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{Aa\cos^5(\ldots}{5d} \\ x\,(A+B\sin(c))\,(a\sin(c)+a)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise(((3*A*a*x*sin(c + d*x)**4/8 + 3*A*a*x*sin(c + d*x)**2*cos(c + d*x)
**2/4 + 3*A*a*x*cos(c + d*x)**4/8 + 3*A*a*sin(c + d*x)**3*cos(c + d*x)/(8*d
) + 5*A*a*sin(c + d*x)*cos(c + d*x)**3/(8*d) - A*a*cos(c + d*x)**5/(5*d) +
B*a*x*sin(c + d*x)**6/16 + 3*B*a*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*B
*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + B*a*x*cos(c + d*x)**6/16 + B*a*si
n(c + d*x)**5*cos(c + d*x)/(16*d) + B*a*sin(c + d*x)**3*cos(c + d*x)**3/(6*
d) - B*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - B*a*cos(c + d*x)**5/(5*d), N
e(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)*cos(c)**4, True))
```

3.963 $\int \cos^2(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=84

$$-\frac{a(4A+B)\cos^3(c+dx)}{12d}+\frac{a(4A+B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+B)-\frac{B\cos^3(c+dx)(a\sin(c+dx)+a)}{4d}$$

```
[Out] 1/8*a*(4*A+B)*x-1/12*a*(4*A+B)*cos(d*x+c)^3/d+1/8*a*(4*A+B)*cos(d*x+c)*sin(
d*x+c)/d-1/4*B*cos(d*x+c)^3*(a+a*sin(d*x+c))/d
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2860, 2669, 2635, 8}

$$-\frac{a(4A+B)\cos^3(c+dx)}{12d}+\frac{a(4A+B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+B)-\frac{B\cos^3(c+dx)(a\sin(c+dx)+a)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(4*A + B)*x)/8 - (a*(4*A + B)*Cos[c + d*x]^3)/(12*d) + (a*(4*A + B)*Cos[
c + d*x]*Sin[c + d*x])/(8*d) - (B*Cos[c + d*x]^3*(a + a*Sin[c + d*x]))/(4*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = -\frac{B\cos^3(c+dx)(a+a\sin(c+dx))}{4d}+\frac{1}{4}(4A+B)\int \text{co}$$
$$= -\frac{a(4A+B)\cos^3(c+dx)}{12d}-\frac{B\cos^3(c+dx)(a+a\sin(c}{4d}$$
$$= -\frac{a(4A+B)\cos^3(c+dx)}{12d}+\frac{a(4A+B)\cos(c+dx)\sin}{8d}$$
$$= \frac{1}{8}a(4A+B)x-\frac{a(4A+B)\cos^3(c+dx)}{12d}+\frac{a(4A+B)}{}$$

Mathematica [A] time = 0.67, size = 64, normalized size = 0.76

$$-\frac{a(24(A+B)\cos(c+dx)+8(A+B)\cos(3(c+dx))-12dx(4A+B)-24A\sin(2(c+dx))+3B\sin(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/96*(a*(-12*(4*A + B)*d*x + 24*(A + B)*Cos[c + d*x] + 8*(A + B)*Cos[3*(c
+ d*x)] - 24*A*Sin[2*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/d
```

fricas [A] time = 0.79, size = 65, normalized size = 0.77

$$-\frac{8\,(A+B)a\cos\left(dx+c\right)^3-3\,(4\,A+B)adx+3\,\left(2\,Ba\cos\left(dx+c\right)^3-(4\,A+B)a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/24*(8*(A + B)*a*cos(d*x + c)^3 - 3*(4*A + B)*a*d*x + 3*(2*B*a*cos(d*x +
c)^3 - (4*A + B)*a*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.18, size = 83, normalized size = 0.99

$$\frac{1}{8}(4Aa+Ba)x-\frac{Ba\sin(4dx+4c)}{32d}+\frac{Aa\sin(2dx+2c)}{4d}-\frac{(Aa+Ba)\cos(3dx+3c)}{12d}-\frac{(Aa+Ba)\cos(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/8*(4*A*a + B*a)*x - 1/32*B*a*sin(4*d*x + 4*c)/d + 1/4*A*a*sin(2*d*x + 2*c
)/d - 1/12*(A*a + B*a)*cos(3*d*x + 3*c)/d - 1/4*(A*a + B*a)*cos(d*x + c)/d
```

maple [A] time = 0.36, size = 96, normalized size = 1.14

$$\frac{aB\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{aA\left(\cos^3(dx+c)\right)}{3}-\frac{aB\left(\cos^3(dx+c)\right)}{3}+aA\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)-1/3*a*A*cos(d*x+c)^3-1/3*a*B*cos(d*x+c)^3+a*A*(1/2*cos(d*x+c)*sin(d*x+
c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.38, size = 74, normalized size = 0.88

$$-\frac{32\,Aa\cos(dx+c)^3+32\,Ba\cos(dx+c)^3-24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa-3\,(4\,dx+4\,c-\sin(4\,dx+4\,c}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*A*a*cos(d*x + c)^3 + 32*B*a*cos(d*x + c)^3 - 24*(2*d*x + 2*c + si
n(2*d*x + 2*c))*A*a - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*B*a)/d
```

mupad [B] time = 10.55, size = 276, normalized size = 3.29

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(4\,A+B)}{4\left(A\,a+\frac{B\,a}{4}\right)}\right)(4\,A+B)}{4\,d}-\frac{a\,(4\,A+B)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{4\,d}-\frac{\left(A\,a-\frac{B\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*sin(c + d*x))*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(4*A + B))/(4*(A*a + (B*a)/4)))*(4*A + B))/(4
*d) - (a*(4*A + B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4*d) - ((2*A*a)/3
 + (2*B*a)/3 - tan(c/2 + (d*x)/2)*(A*a - (B*a)/4) + tan(c/2 + (d*x)/2)^4*(2
*A*a + 2*B*a) + tan(c/2 + (d*x)/2)^6*(2*A*a + 2*B*a) + tan(c/2 + (d*x)/2)^2
*((2*A*a)/3 + (2*B*a)/3) + tan(c/2 + (d*x)/2)^7*(A*a - (B*a)/4) - tan(c/2 +
 (d*x)/2)^3*(A*a + (7*B*a)/4) + tan(c/2 + (d*x)/2)^5*(A*a + (7*B*a)/4))/(d*
(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 +
 tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [A] time = 1.01, size = 199, normalized size = 2.37

$$\begin{cases} \frac{Aax\sin^2(c+dx)}{2} + \frac{Aax\cos^2(c+dx)}{2} + \frac{Aa\sin(c+dx)\cos(c+dx)}{2d} - \frac{Aa\cos^3(c+dx)}{3d} + \frac{Bax\sin^4(c+dx)}{8} + \frac{Bax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{Bax\cos \\ x(A+B\sin(c))(a\sin(c)+a)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
d*x)*cos(c + d*x)/(2*d) - A*a*cos(c + d*x)**3/(3*d) + B*a*x*sin(c + d*x)**4
/8 + B*a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + B*a*x*cos(c + d*x)**4/8 + B*
a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - B*a*sin(c + d*x)*cos(c + d*x)**3/(8*
d) - B*a*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)
*cos(c)**2, True))
```

3.964 $\int \sec^2(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=29

$$\frac{(A+B)\sec(c+dx)(a\sin(c+dx)+a)}{d}-aBx$$

```
[Out] -a*B*x+(A+B)*sec(d*x+c)*(a+a*sin(d*x+c))/d
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2855, 8}

$$\frac{(A+B)\sec(c+dx)(a\sin(c+dx)+a)}{d}-aBx$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a*B*x) + ((A + B)*Sec[c + d*x]*(a + a*Sin[c + d*x]))/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{(A+B)\sec(c+dx)(a+a\sin(c+dx))}{d}-(aB)\int 1\,dx\\ &= -aBx+\frac{(A+B)\sec(c+dx)(a+a\sin(c+dx))}{d}\end{aligned}$$

Mathematica [B] time = 0.35, size = 85, normalized size = 2.93

$$\frac{a\left(2(A+B)\sin\left(\frac{dx}{2}\right)+Bdx\sin\left(c+\frac{dx}{2}\right)-Bdx\cos\left(\frac{dx}{2}\right)\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(-(B*d*x*Cos[(d*x)/2]) + 2*(A + B)*Sin[(d*x)/2] + B*d*x*Sin[c + (d*x)/2]
))/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
```

fricas [B] time = 0.52, size = 73, normalized size = 2.52

$$-\frac{Badx-(A+B)a+(Badx-(A+B)a)\cos(dx+c)-(Badx+(A+B)a)\sin(dx+c)}{d\cos(dx+c)-d\sin(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -(B*a*d*x - (A + B)*a + (B*a*d*x - (A + B)*a)*cos(d*x + c) - (B*a*d*x + (A
+ B)*a)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [A] time = 0.18, size = 36, normalized size = 1.24

$$-\frac{(dx+c)Ba+\frac{2(Aa+Ba)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -((d*x + c)*B*a + 2*(A*a + B*a)/(tan(1/2*d*x + 1/2*c) - 1))/d
```

maple [A] time = 0.46, size = 54, normalized size = 1.86

$$\frac{\frac{aA}{\cos(dx+c)}+aB(\tan(dx+c)-dx-c)+aA\tan(dx+c)+\frac{aB}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*A/cos(d*x+c)+a*B*(tan(d*x+c)-d*x-c)+a*A*tan(d*x+c)+a*B/cos(d*x+c))
```

maxima [A] time = 0.43, size = 56, normalized size = 1.93

$$-\frac{(dx+c-\tan(dx+c))Ba - Aa\tan(dx+c) - \frac{Aa}{\cos(dx+c)} - \frac{Ba}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -((d*x + c - tan(d*x + c))*B*a - A*a*tan(d*x + c) - A*a/cos(d*x + c) - B*a/
cos(d*x + c))/d
```

mupad [B] time = 9.17, size = 33, normalized size = 1.14

$$-\frac{2\,A\,a+2\,B\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}-B\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] - (2*A*a + 2*B*a)/(d*(tan(c/2 + (d*x)/2) - 1)) - B*a*x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^2(c+dx)\,dx + \int A\sin(c+dx)\sec^2(c+dx)\,dx + \int B\sin(c+dx)\sec^2(c+dx)\,dx + \int B\sin^2(c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**2, x) + Integral(A*sin(c + d*x)*sec(c + d*x)**2
, x) + Integral(B*sin(c + d*x)*sec(c + d*x)**2, x) + Integral(B*sin(c + d*x
)**2*sec(c + d*x)**2, x))
```

3.965 $\int \sec^4(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=50

$$\frac{a(2A-B)\tan(c+dx)}{3d}+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)}{3d}$$

[Out] 1/3*(A+B)*sec(d*x+c)^3*(a+a*sin(d*x+c))/d+1/3*a*(2*A-B)*tan(d*x+c)/d

Rubi [A] time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2855, 3767, 8}

$$\frac{a(2A-B)\tan(c+dx)}{3d}+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]

[Out] ((A + B)*Sec[c + d*x]^3*(a + a*Sin[c + d*x]))/(3*d) + (a*(2*A - B)*Tan[c + d*x])/(3*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))}{3d} + \frac{1}{3}(a(2A-$$
$$= \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))}{3d} - \frac{(a(2A-B}{}$$
$$= \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))}{3d} + \frac{a(2A-B)}{}$$

Mathematica [A] time = 0.63, size = 97, normalized size = 1.94

$$\frac{a\sec(c)(\sin(c+dx)+1)\sec^3(c+dx)(-2(A+B)\cos(c+dx)+A\sin(2(c+dx))+4A\cos(c+2dx)+8A\sin(dx}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c]*Sec[c + d*x]^3*(1 + Sin[c + d*x])*(6*B*Cos[c] - 2*(A + B)*Cos[c +
 d*x] + 4*A*Cos[c + 2*d*x] - 2*B*Cos[c + 2*d*x] + 8*A*Sin[d*x] - 4*B*Sin[d*
x] + A*Sin[2*(c + d*x)] + B*Sin[2*(c + d*x)]))/(12*d)
```

fricas [A] time = 0.76, size = 69, normalized size = 1.38

$$-\frac{(2A-B)a\cos(dx+c)^2+(2A-B)a\sin(dx+c)-(A-2B)a}{3(d\cos(dx+c)\sin(dx+c)-d\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/3*((2*A - B)*a*cos(d*x + c)^2 + (2*A - B)*a*sin(d*x + c) - (A - 2*B)*a)/
(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

giac [B] time = 0.19, size = 94, normalized size = 1.88

$$-\frac{\frac{3(Aa-Ba)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}+\frac{9Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7Aa+Ba}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(3*(A*a - B*a)/(tan(1/2*d*x + 1/2*c) + 1) + (9*A*a*tan(1/2*d*x + 1/2*c
)^2 + 3*B*a*tan(1/2*d*x + 1/2*c)^2 - 12*A*a*tan(1/2*d*x + 1/2*c) + 7*A*a +
B*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

maple [A] time = 0.52, size = 72, normalized size = 1.44

$$\frac{\frac{aA}{3\cos(dx+c)^3}+\frac{aB\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}-aA\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{aB}{3\cos(dx+c)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*A/cos(d*x+c)^3+1/3*a*B*sin(d*x+c)^3/cos(d*x+c)^3-a*A*(-2/3-1/3*s
ec(d*x+c)^2)*tan(d*x+c)+1/3*a*B/cos(d*x+c)^3)
```

maxima [A] time = 0.32, size = 59, normalized size = 1.18

$$\frac{Ba\tan(dx+c)^3+\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa+\frac{Aa}{\cos(dx+c)^3}+\frac{Ba}{\cos(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*(B*a*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*A*a + A*a/cos(d
*x + c)^3 + B*a/cos(d*x + c)^3)/d
```

mupad [B] time = 9.25, size = 107, normalized size = 2.14

$$\frac{\frac{2a\left(\frac{3B}{2}+A\cos(c+dx)+B\cos(c+dx)+2A\sin(c+dx)-B\sin(c+dx)+A\cos(2c+2dx)-\frac{B\cos(2c+2dx)}{2}\right)}{3}-\frac{4a\cos(c+dx)\left(\frac{A}{2}+\frac{B}{2}\right)}{3}}{d\,\left(2\cos(c+dx)-\sin(2c+2dx)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] ((2*a*((3*B)/2 + A*cos(c + d*x) + B*cos(c + d*x) + 2*A*sin(c + d*x) - B*sin
(c + d*x) + A*cos(2*c + 2*d*x) - (B*cos(2*c + 2*d*x))/2))/3 - (4*a*cos(c +
d*x)*(A/2 + B/2))/3)/(d*(2*cos(c + d*x) - sin(2*c + 2*d*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^4(c+dx)\,dx+\int A\sin(c+dx)\sec^4(c+dx)\,dx+\int B\sin(c+dx)\sec^4(c+dx)\,dx+\int B\sin^2(c+d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**4, x) + Integral(A*sin(c + d*x)*sec(c + d*x)**4
, x) + Integral(B*sin(c + d*x)*sec(c + d*x)**4, x) + Integral(B*sin(c + d*x
)**2*sec(c + d*x)**4, x))
```

3.966 $\int \sec^6(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=73

$$\frac{a(4A-B)\tan^3(c+dx)}{15d}+\frac{a(4A-B)\tan(c+dx)}{5d}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+dx)+a)}{5d}$$

```
[Out] 1/5*(A+B)*sec(d*x+c)^5*(a+a*sin(d*x+c))/d+1/5*a*(4*A-B)*tan(d*x+c)/d+1/15*a
*(4*A-B)*tan(d*x+c)^3/d
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2855, 3767}

$$\frac{a(4A-B)\tan^3(c+dx)}{15d}+\frac{a(4A-B)\tan(c+dx)}{5d}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+dx)+a)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^5*(a + a*Sin[c + d*x]))/(5*d) + (a*(4*A - B)*Tan[c +
d*x])/(5*d) + (a*(4*A - B)*Tan[c + d*x]^3)/(15*d)
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))}{5d} + \frac{1}{5}(a(4A -$$
$$= \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))}{5d} - \frac{(a(4A - B}{}$$
$$= \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))}{5d} + \frac{a(4A - B)}{}$$

Mathematica [B] time = 1.27, size = 223, normalized size = 3.05

$$\frac{a\sec(c)(-54(A+B)\cos(c+dx)+18A\sin(2(c+dx))+9A\sin(4(c+dx))+128A\sin(2c+3dx)-18A\cos(3(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c]*(240*B*Cos[c] - 54*(A + B)*Cos[c + d*x] - 18*A*Cos[3*(c + d*x)] -
 18*B*Cos[3*(c + d*x)] + 128*A*Cos[c + 2*d*x] - 32*B*Cos[c + 2*d*x] + 64*A*
Cos[3*c + 4*d*x] - 16*B*Cos[3*c + 4*d*x] + 384*A*Sin[d*x] - 96*B*Sin[d*x] +
 18*A*Sin[2*(c + d*x)] + 18*B*Sin[2*(c + d*x)] + 9*A*Sin[4*(c + d*x)] + 9*B
*Sin[4*(c + d*x)] + 128*A*Sin[2*c + 3*d*x] - 32*B*Sin[2*c + 3*d*x]))/(960*d
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^3)
```

fricas [A] time = 0.74, size = 112, normalized size = 1.53

$$-\frac{2\,(4\,A-B)a\cos\left(dx+c\right)^4-(4\,A-B)a\cos\left(dx+c\right)^2-(A-4\,B)a+\left(2\,(4\,A-B)a\cos\left(dx+c\right)^2+(4\,A-B)a\right)\mathrm{s}}{15\left(d\cos\left(dx+c\right)^3\sin\left(dx+c\right)-d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/15*(2*(4*A - B)*a*cos(d*x + c)^4 - (4*A - B)*a*cos(d*x + c)^2 - (A - 4*B
)*a + (2*(4*A - B)*a*cos(d*x + c)^2 + (4*A - B)*a)*sin(d*x + c))/(d*cos(d*x
 + c)^3*sin(d*x + c) - d*cos(d*x + c)^3)
```

giac [B] time = 0.21, size = 225, normalized size = 3.08

$$-\frac{\frac{5\left(15\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\,Aa-7\,Ba\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}+\frac{165\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+45\,Ba\tan\left(\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/120*(5*(15*A*a*tan(1/2*d*x + 1/2*c)^2 - 9*B*a*tan(1/2*d*x + 1/2*c)^2 + 2
4*A*a*tan(1/2*d*x + 1/2*c) - 12*B*a*tan(1/2*d*x + 1/2*c) + 13*A*a - 7*B*a)/
(tan(1/2*d*x + 1/2*c) + 1)^3 + (165*A*a*tan(1/2*d*x + 1/2*c)^4 + 45*B*a*tan
(1/2*d*x + 1/2*c)^4 - 480*A*a*tan(1/2*d*x + 1/2*c)^3 - 60*B*a*tan(1/2*d*x +
 1/2*c)^3 + 650*A*a*tan(1/2*d*x + 1/2*c)^2 + 70*B*a*tan(1/2*d*x + 1/2*c)^2
- 400*A*a*tan(1/2*d*x + 1/2*c) - 20*B*a*tan(1/2*d*x + 1/2*c) + 113*A*a + 13
*B*a)/(tan(1/2*d*x + 1/2*c) - 1)^5)/d
```

maple [A] time = 0.55, size = 102, normalized size = 1.40

$$\frac{\frac{aA}{5\cos(dx+c)^5}+aB\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)-aA\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{aB}{5\cos(dx+c)^5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a*A/cos(d*x+c)^5+a*B*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c
)^3/cos(d*x+c)^3)-a*A*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)
+1/5*a*B/cos(d*x+c)^5)
```

maxima [A] time = 0.37, size = 86, normalized size = 1.18

$$\frac{\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa+\left(3\tan(dx+c)^5+5\tan(dx+c)^3\right)Ba+\frac{3\,Aa}{\cos(dx+c)^5}+\frac{}{c}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a + (3*tan
(d*x + c)^5 + 5*tan(d*x + c)^3)*B*a + 3*A*a/cos(d*x + c)^5 + 3*B*a/cos(d*x
+ c)^5)/d
```

mupad [B] time = 11.14, size = 224, normalized size = 3.07

$$-\frac{a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{A\cos\left(\frac{5c}{2}+\frac{5\,d\,x}{2}\right)}{4}-\frac{9\,A\cos\left(\frac{3c}{2}+\frac{3\,d\,x}{2}\right)}{4}-A\cos\left(\frac{7c}{2}+\frac{7\,d\,x}{2}\right)-\frac{15\,B\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{3\,B\cos\left(\frac{3c}{2}+\frac{3\,d\,x}{2}\right)}{2}-B\cos\left(\frac{5}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^6,x)
```

```
[Out] -(a*cos(c/2 + (d*x)/2)*((A*cos((5*c)/2 + (5*d*x)/2))/4 - (9*A*cos((3*c)/2 +
 (3*d*x)/2))/4 - A*cos((7*c)/2 + (7*d*x)/2) - (15*B*cos(c/2 + (d*x)/2))/4 +
 (3*B*cos((3*c)/2 + (3*d*x)/2))/2 - B*cos((5*c)/2 + (5*d*x)/2) + (B*cos((7*
c)/2 + (7*d*x)/2))/4 - (73*A*sin(c/2 + (d*x)/2))/8 + (25*A*sin((3*c)/2 + (3
*d*x)/2))/8 - (19*A*sin((5*c)/2 + (5*d*x)/2))/8 + (3*A*sin((7*c)/2 + (7*d*x
)/2))/8 + (7*B*sin(c/2 + (d*x)/2))/8 + (5*B*sin((3*c)/2 + (3*d*x)/2))/8 + (
B*sin((5*c)/2 + (5*d*x)/2))/8 + (3*B*sin((7*c)/2 + (7*d*x)/2))/8))/(120*d*c
os(c/2 - pi/4 + (d*x)/2)^3*cos(c/2 + pi/4 + (d*x)/2)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.967 $\int \sec^8(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=96

$$\frac{a(6A-B)\tan^5(c+dx)}{35d}+\frac{2a(6A-B)\tan^3(c+dx)}{21d}+\frac{a(6A-B)\tan(c+dx)}{7d}+\frac{(A+B)\sec^7(c+dx)(a\sin(c+dx)+$$

```
[Out] 1/7*(A+B)*sec(d*x+c)^7*(a+a*sin(d*x+c))/d+1/7*a*(6*A-B)*tan(d*x+c)/d+2/21*a
*(6*A-B)*tan(d*x+c)^3/d+1/35*a*(6*A-B)*tan(d*x+c)^5/d
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2855, 3767}

$$\frac{a(6A-B)\tan^5(c+dx)}{35d}+\frac{2a(6A-B)\tan^3(c+dx)}{21d}+\frac{a(6A-B)\tan(c+dx)}{7d}+\frac{(A+B)\sec^7(c+dx)(a\sin(c+dx)+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^7*(a + a*Sin[c + d*x]))/(7*d) + (a*(6*A - B)*Tan[c +
d*x])/(7*d) + (2*a*(6*A - B)*Tan[c + d*x]^3)/(21*d) + (a*(6*A - B)*Tan[c +
d*x]^5)/(35*d)
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))}{7d} + \frac{1}{7}(a(6A-$$
$$= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))}{7d} - \frac{(a(6A-B}{}$$
$$= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))}{7d} + \frac{a(6A-B)}{}$$

Mathematica [B] time = 2.03, size = 315, normalized size = 3.28

$$\frac{a\sec(c)(-1500(A+B)\cos(c+dx)+375A\sin(2(c+dx))+300A\sin(4(c+dx))+75A\sin(6(c+dx))+7680A}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c]*(8960*B*Cos[c] - 1500*(A + B)*Cos[c + d*x] - 750*A*Cos[3*(c + d*x
)] - 750*B*Cos[3*(c + d*x)] - 150*A*Cos[5*(c + d*x)] - 150*B*Cos[5*(c + d*x
)] + 3840*A*Cos[c + 2*d*x] - 640*B*Cos[c + 2*d*x] + 3072*A*Cos[3*c + 4*d*x]
 - 512*B*Cos[3*c + 4*d*x] + 768*A*Cos[5*c + 6*d*x] - 128*B*Cos[5*c + 6*d*x]
 + 15360*A*Sin[d*x] - 2560*B*Sin[d*x] + 375*A*Sin[2*(c + d*x)] + 375*B*Sin[
2*(c + d*x)] + 300*A*Sin[4*(c + d*x)] + 300*B*Sin[4*(c + d*x)] + 75*A*Sin[6
*(c + d*x)] + 75*B*Sin[6*(c + d*x)] + 7680*A*Sin[2*c + 3*d*x] - 1280*B*Sin[
2*c + 3*d*x] + 1536*A*Sin[4*c + 5*d*x] - 256*B*Sin[4*c + 5*d*x]))/(53760*d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^5)
```

fricas [A] time = 0.80, size = 149, normalized size = 1.55

$$-\frac{8\,(6\,A-B)a\cos\left(dx+c\right)^6-4\,(6\,A-B)a\cos\left(dx+c\right)^4-(6\,A-B)a\cos\left(dx+c\right)^2-3\,(A-6\,B)a+\left(8\,(6\,A-B)\right.}{105\,\left(d\cos\left(dx+c\right)^5\sin\left(dx+c\right)-d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/105*(8*(6*A - B)*a*cos(d*x + c)^6 - 4*(6*A - B)*a*cos(d*x + c)^4 - (6*A
- B)*a*cos(d*x + c)^2 - 3*(A - 6*B)*a + (8*(6*A - B)*a*cos(d*x + c)^4 + 4*(
6*A - B)*a*cos(d*x + c)^2 + 3*(6*A - B)*a)*sin(d*x + c))/(d*cos(d*x + c)^5*
sin(d*x + c) - d*cos(d*x + c)^5)
```

giac [B] time = 0.21, size = 345, normalized size = 3.59

$$-\frac{\frac{7\left(165\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-75\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+540\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-210\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+750\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-280\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\cdots}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/1680*(7*(165*A*a*tan(1/2*d*x + 1/2*c)^4 - 75*B*a*tan(1/2*d*x + 1/2*c)^4
+ 540*A*a*tan(1/2*d*x + 1/2*c)^3 - 210*B*a*tan(1/2*d*x + 1/2*c)^3 + 750*A*a
*tan(1/2*d*x + 1/2*c)^2 - 280*B*a*tan(1/2*d*x + 1/2*c)^2 + 480*A*a*tan(1/2*
d*x + 1/2*c) - 170*B*a*tan(1/2*d*x + 1/2*c) + 129*A*a - 49*B*a)/(tan(1/2*d*
x + 1/2*c) + 1)^5 + (2205*A*a*tan(1/2*d*x + 1/2*c)^6 + 525*B*a*tan(1/2*d*x
+ 1/2*c)^6 - 10080*A*a*tan(1/2*d*x + 1/2*c)^5 - 1470*B*a*tan(1/2*d*x + 1/2*
c)^5 + 21945*A*a*tan(1/2*d*x + 1/2*c)^4 + 2555*B*a*tan(1/2*d*x + 1/2*c)^4 -
 26460*A*a*tan(1/2*d*x + 1/2*c)^3 - 2240*B*a*tan(1/2*d*x + 1/2*c)^3 + 18963
*A*a*tan(1/2*d*x + 1/2*c)^2 + 1407*B*a*tan(1/2*d*x + 1/2*c)^2 - 7476*A*a*ta
n(1/2*d*x + 1/2*c) - 434*B*a*tan(1/2*d*x + 1/2*c) + 1383*A*a + 137*B*a)/(ta
n(1/2*d*x + 1/2*c) - 1)^7)/d
```

maple [A] time = 0.60, size = 130, normalized size = 1.35

$$\frac{\frac{aA}{7\cos(dx+c)^{7}}+aB\left(\frac{\sin^{3}(dx+c)}{7\cos(dx+c)^{7}}+\frac{4\left(\sin^{3}(dx+c)\right)}{35\cos(dx+c)^{5}}+\frac{8\left(\sin^{3}(dx+c)\right)}{105\cos(dx+c)^{3}}\right)-aA\left(-\frac{16}{35}-\frac{\left(\sec^{6}(dx+c)\right)}{7}-\frac{6\left(\sec^{4}(dx+c)\right)}{35}-\frac{8\left(\sec^{2}(dx+c)\right)}{35}\right)\tan\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/7*a*A/cos(d*x+c)^7+a*B*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c
)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)-a*A*(-16/35-1/7*sec(d*x+c
)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c)+1/7*a*B/cos(d*x+c)^7)
```

maxima [A] time = 0.33, size = 107, normalized size = 1.11

$$\frac{3\left(5\,\tan\left(dx+c\right)^{7}+21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+35\,\tan\left(dx+c\right)\right)Aa+\left(15\,\tan\left(dx+c\right)^{7}+42\,\tan\left(dx+\cdots\right.\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/105*(3*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan
(d*x + c))*A*a + (15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3
)*B*a + 15*A*a/cos(d*x + c)^7 + 15*B*a/cos(d*x + c)^7)/d
```

mupad [B] time = 12.64, size = 320, normalized size = 3.33

$$-\frac{a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{15\,A\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}-\frac{75\,A\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}-\frac{105\,A\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}+\frac{9\,A\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}-\frac{3\,A\,\cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{2}-\frac{35}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^8,x)
```

```
[Out] -(a*cos(c/2 + (d*x)/2)*((15*A*cos((5*c)/2 + (5*d*x)/2))/8 - (75*A*cos((3*c)
/2 + (3*d*x)/2))/8 - (105*A*cos((7*c)/2 + (7*d*x)/2))/16 + (9*A*cos((9*c)/2
 + (9*d*x)/2))/16 - (3*A*cos((11*c)/2 + (11*d*x)/2))/2 - (35*B*cos(c/2 + (d
*x)/2))/2 + (65*B*cos((3*c)/2 + (3*d*x)/2))/8 - (55*B*cos((5*c)/2 + (5*d*x)
/2))/8 + (35*B*cos((7*c)/2 + (7*d*x)/2))/16 - (19*B*cos((9*c)/2 + (9*d*x)/2
))/16 + (B*cos((11*c)/2 + (11*d*x)/2))/4 - (843*A*sin(c/2 + (d*x)/2))/16 +
(363*A*sin((3*c)/2 + (3*d*x)/2))/16 - (651*A*sin((5*c)/2 + (5*d*x)/2))/32 +
 (171*A*sin((7*c)/2 + (7*d*x)/2))/32 - (111*A*sin((9*c)/2 + (9*d*x)/2))/32
+ (15*A*sin((11*c)/2 + (11*d*x)/2))/32 + (53*B*sin(c/2 + (d*x)/2))/16 + (27
*B*sin((3*c)/2 + (3*d*x)/2))/16 + (21*B*sin((5*c)/2 + (5*d*x)/2))/32 + (59*
B*sin((7*c)/2 + (7*d*x)/2))/32 + (B*sin((9*c)/2 + (9*d*x)/2))/32 + (15*B*si
n((11*c)/2 + (11*d*x)/2))/32))/(3360*d*cos(c/2 - pi/4 + (d*x)/2)^5*cos(c/2
+ pi/4 + (d*x)/2)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.968 $\int \sec^{10}(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=119

$$\frac{a(8A-B)\tan^7(c+dx)}{63d}+\frac{a(8A-B)\tan^5(c+dx)}{15d}+\frac{a(8A-B)\tan^3(c+dx)}{9d}+\frac{a(8A-B)\tan(c+dx)}{9d}+\frac{(A+B)\sec^9(c+dx)$$

```
[Out] 1/9*(A+B)*sec(d*x+c)^9*(a+a*sin(d*x+c))/d+1/9*a*(8*A-B)*tan(d*x+c)/d+1/9*a*
(8*A-B)*tan(d*x+c)^3/d+1/15*a*(8*A-B)*tan(d*x+c)^5/d+1/63*a*(8*A-B)*tan(d*x
+c)^7/d
```

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2855, 3767}

$$\frac{a(8A-B)\tan^7(c+dx)}{63d}+\frac{a(8A-B)\tan^5(c+dx)}{15d}+\frac{a(8A-B)\tan^3(c+dx)}{9d}+\frac{a(8A-B)\tan(c+dx)}{9d}+\frac{(A+B)\sec^9(c+dx)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^9*(a + a*Sin[c + d*x]))/(9*d) + (a*(8*A - B)*Tan[c +
d*x])/(9*d) + (a*(8*A - B)*Tan[c + d*x]^3)/(9*d) + (a*(8*A - B)*Tan[c + d*x
]^5)/(15*d) + (a*(8*A - B)*Tan[c + d*x]^7)/(63*d)
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))}{9d} + \frac{1}{9}(a(8A-$$
$$= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))}{9d} - \frac{(a(8A-}{}$$
$$= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))}{9d} + \frac{a(8A-B}{}$$

Mathematica [B] time = 4.34, size = 407, normalized size = 3.42

$$\frac{a\sec(c)(-85750(A+B)\cos(c+dx)+17150A\sin(2(c+dx))+17150A\sin(4(c+dx))+7350A\sin(6(c+dx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c]*(645120*B*Cos[c] - 85750*(A + B)*Cos[c + d*x] - 51450*A*Cos[3*(c
+ d*x)] - 51450*B*Cos[3*(c + d*x)] - 17150*A*Cos[5*(c + d*x)] - 17150*B*Cos
[5*(c + d*x)] - 2450*A*Cos[7*(c + d*x)] - 2450*B*Cos[7*(c + d*x)] + 229376*
A*Cos[c + 2*d*x] - 28672*B*Cos[c + 2*d*x] + 229376*A*Cos[3*c + 4*d*x] - 286
72*B*Cos[3*c + 4*d*x] + 98304*A*Cos[5*c + 6*d*x] - 12288*B*Cos[5*c + 6*d*x]
 + 16384*A*Cos[7*c + 8*d*x] - 2048*B*Cos[7*c + 8*d*x] + 1146880*A*Sin[d*x]
- 143360*B*Sin[d*x] + 17150*A*Sin[2*(c + d*x)] + 17150*B*Sin[2*(c + d*x)] +
 17150*A*Sin[4*(c + d*x)] + 17150*B*Sin[4*(c + d*x)] + 7350*A*Sin[6*(c + d*
x)] + 7350*B*Sin[6*(c + d*x)] + 1225*A*Sin[8*(c + d*x)] + 1225*B*Sin[8*(c +
 d*x)] + 688128*A*Sin[2*c + 3*d*x] - 86016*B*Sin[2*c + 3*d*x] + 229376*A*Si
n[4*c + 5*d*x] - 28672*B*Sin[4*c + 5*d*x] + 32768*A*Sin[6*c + 7*d*x] - 4096
*B*Sin[6*c + 7*d*x]))/(5160960*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^9*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

fricas [A] time = 0.64, size = 185, normalized size = 1.55

$$-\frac{16\,(8\,A-B)a\cos\left(dx+c\right)^{8}-8\,(8\,A-B)a\cos\left(dx+c\right)^{6}-2\,(8\,A-B)a\cos\left(dx+c\right)^{4}-(8\,A-B)a\cos\left(dx+c\right)^{2}}{315\left(d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fri
cas")
```

```
[Out] -1/315*(16*(8*A - B)*a*cos(d*x + c)^8 - 8*(8*A - B)*a*cos(d*x + c)^6 - 2*(8
*A - B)*a*cos(d*x + c)^4 - (8*A - B)*a*cos(d*x + c)^2 - 5*(A - 8*B)*a + (16
```

```
*(8*A - B)*a*cos(d*x + c)^6 + 8*(8*A - B)*a*cos(d*x + c)^4 + 6*(8*A - B)*a*
cos(d*x + c)^2 + 5*(8*A - B)*a)*sin(d*x + c))/(d*cos(d*x + c)^7*sin(d*x + c
) - d*cos(d*x + c)^7)
```

giac [B] time = 0.23, size = 465, normalized size = 3.91

$$-\frac{\frac{3\left(9765\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-3675\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+48720\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15960\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+109865\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-33775\,Ba\,\mathrm{ta}\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="gia
c")
```

```
[Out] -1/40320*(3*(9765*A*a*tan(1/2*d*x + 1/2*c)^6 - 3675*B*a*tan(1/2*d*x + 1/2*c
)^6 + 48720*A*a*tan(1/2*d*x + 1/2*c)^5 - 15960*B*a*tan(1/2*d*x + 1/2*c)^5 +
 109865*A*a*tan(1/2*d*x + 1/2*c)^4 - 33775*B*a*tan(1/2*d*x + 1/2*c)^4 + 136
640*A*a*tan(1/2*d*x + 1/2*c)^3 - 39760*B*a*tan(1/2*d*x + 1/2*c)^3 + 99183*A
*a*tan(1/2*d*x + 1/2*c)^2 - 28161*B*a*tan(1/2*d*x + 1/2*c)^2 + 39536*A*a*ta
n(1/2*d*x + 1/2*c) - 11032*B*a*tan(1/2*d*x + 1/2*c) + 7043*A*a - 2101*B*a)/
(tan(1/2*d*x + 1/2*c) + 1)^7 + (51345*A*a*tan(1/2*d*x + 1/2*c)^8 + 11025*B*
a*tan(1/2*d*x + 1/2*c)^8 - 322560*A*a*tan(1/2*d*x + 1/2*c)^7 - 47880*B*a*ta
n(1/2*d*x + 1/2*c)^7 + 976500*A*a*tan(1/2*d*x + 1/2*c)^6 + 117180*B*a*tan(1
/2*d*x + 1/2*c)^6 - 1753920*A*a*tan(1/2*d*x + 1/2*c)^5 - 168840*B*a*tan(1/2
*d*x + 1/2*c)^5 + 2037294*A*a*tan(1/2*d*x + 1/2*c)^4 + 165942*B*a*tan(1/2*d
*x + 1/2*c)^4 - 1550976*A*a*tan(1/2*d*x + 1/2*c)^3 - 106008*B*a*tan(1/2*d*x
 + 1/2*c)^3 + 760644*A*a*tan(1/2*d*x + 1/2*c)^2 + 47772*B*a*tan(1/2*d*x + 1
/2*c)^2 - 219456*A*a*tan(1/2*d*x + 1/2*c) - 12888*B*a*tan(1/2*d*x + 1/2*c)
+ 30089*A*a + 2657*B*a)/(tan(1/2*d*x + 1/2*c) - 1)^9)/d
```

maple [A] time = 0.63, size = 158, normalized size = 1.33

$$\frac{\frac{aA}{9\cos(dx+c)^{9}}+aB\left(\frac{\sin^{3}(dx+c)}{9\cos(dx+c)^{9}}+\frac{2\left(\sin^{3}(dx+c)\right)}{21\cos(dx+c)^{7}}+\frac{8\left(\sin^{3}(dx+c)\right)}{105\cos(dx+c)^{5}}+\frac{16\left(\sin^{3}(dx+c)\right)}{315\cos(dx+c)^{3}}\right)-aA\left(-\frac{128}{315}-\frac{\left(\sec^{8}(dx+c)\right)}{9}-\frac{8\left(\sec^{6}(dx+c)\right)}{63}-\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/9*a*A/cos(d*x+c)^9+a*B*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c
)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*
x+c)^3)-a*A*(-128/315-1/9*sec(d*x+c)^8-8/63*sec(d*x+c)^6-16/105*sec(d*x+c)^
4-64/315*sec(d*x+c)^2)*tan(d*x+c)+1/9*a*B/cos(d*x+c)^9)
```

maxima [A] time = 0.33, size = 126, normalized size = 1.06

$$\frac{\left(35\,\tan\left(dx+c\right)^{9}+180\,\tan\left(dx+c\right)^{7}+378\,\tan\left(dx+c\right)^{5}+420\,\tan\left(dx+c\right)^{3}+315\,\tan\left(dx+c\right)\right)Aa+\left(35\,\tan\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="max
ima")
```

```
[Out] 1/315*((35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 378*tan(d*x + c)^5 + 420*t
an(d*x + c)^3 + 315*tan(d*x + c))*A*a + (35*tan(d*x + c)^9 + 135*tan(d*x +
c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*B*a + 35*A*a/cos(d*x + c)^9
 + 35*B*a/cos(d*x + c)^9)/d
```

mupad [B] time = 13.30, size = 416, normalized size = 3.50

$$-\frac{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{329\,A\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{64}-\frac{1225\,A\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{64}-\frac{133\,A\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{8}+\frac{21\,A\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{8}-\frac{413\,A\,\cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{64}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^10,x)
```

```
[Out] -(a*cos(c/2 + (d*x)/2)*((329*A*cos((5*c)/2 + (5*d*x)/2))/64 - (1225*A*cos((
3*c)/2 + (3*d*x)/2))/64 - (133*A*cos((7*c)/2 + (7*d*x)/2))/8 + (21*A*cos((9
*c)/2 + (9*d*x)/2))/8 - (413*A*cos((11*c)/2 + (11*d*x)/2))/64 + (29*A*cos((
13*c)/2 + (13*d*x)/2))/64 - A*cos((15*c)/2 + (15*d*x)/2) - (315*B*cos(c/2 +
 (d*x)/2))/8 + (1295*B*cos((3*c)/2 + (3*d*x)/2))/64 - (1183*B*cos((5*c)/2 +
 (5*d*x)/2))/64 + 7*B*cos((7*c)/2 + (7*d*x)/2) - (21*B*cos((9*c)/2 + (9*d*x
)/2))/4 + (91*B*cos((11*c)/2 + (11*d*x)/2))/64 - (43*B*cos((13*c)/2 + (13*d
*x)/2))/64 + (B*cos((15*c)/2 + (15*d*x)/2))/8 - (17609*A*sin(c/2 + (d*x)/2)
)/128 + (8649*A*sin((3*c)/2 + (3*d*x)/2))/128 - (8159*A*sin((5*c)/2 + (5*d*
x)/2))/128 + (2783*A*sin((7*c)/2 + (7*d*x)/2))/128 - (2293*A*sin((9*c)/2 +
(9*d*x)/2))/128 + (501*A*sin((11*c)/2 + (11*d*x)/2))/128 - (291*A*sin((13*c
)/2 + (13*d*x)/2))/128 + (35*A*sin((15*c)/2 + (15*d*x)/2))/128 + (823*B*sin
(c/2 + (d*x)/2))/128 + (297*B*sin((3*c)/2 + (3*d*x)/2))/128 + (193*B*sin((5
*c)/2 + (5*d*x)/2))/128 + (479*B*sin((7*c)/2 + (7*d*x)/2))/128 + (11*B*sin(
(9*c)/2 + (9*d*x)/2))/128 + (213*B*sin((11*c)/2 + (11*d*x)/2))/128 - (3*B*s
in((13*c)/2 + (13*d*x)/2))/128 + (35*B*sin((15*c)/2 + (15*d*x)/2))/128))/(4
0320*d*cos(c/2 - pi/4 + (d*x)/2)^7*cos(c/2 + pi/4 + (d*x)/2)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.969 $\int \cos^7(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$-\frac{B(a\sin(c+dx)+a)^{10}}{10a^8d}-\frac{(A-7B)(a\sin(c+dx)+a)^9}{9a^7d}+\frac{3(A-3B)(a\sin(c+dx)+a)^8}{4a^6d}-\frac{4(3A-5B)(a\sin(c+dx}{7a^5d}$$

```
[Out] 4/3*(A-B)*(a+a*sin(d*x+c))^6/a^4/d-4/7*(3*A-5*B)*(a+a*sin(d*x+c))^7/a^5/d+3
/4*(A-3*B)*(a+a*sin(d*x+c))^8/a^6/d-1/9*(A-7*B)*(a+a*sin(d*x+c))^9/a^7/d-1/
10*B*(a+a*sin(d*x+c))^10/a^8/d
```

Rubi [A] time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$-\frac{(A-7B)(a\sin(c+dx)+a)^9}{9a^7d}+\frac{3(A-3B)(a\sin(c+dx)+a)^8}{4a^6d}-\frac{4(3A-5B)(a\sin(c+dx)+a)^7}{7a^5d}+\frac{4(A-B)(a\sin}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (4*(A - B)*(a + a*Sin[c + d*x])^6)/(3*a^4*d) - (4*(3*A - 5*B)*(a + a*Sin[c
+ d*x])^7)/(7*a^5*d) + (3*(A - 3*B)*(a + a*Sin[c + d*x])^8)/(4*a^6*d) - ((A
 - 7*B)*(a + a*Sin[c + d*x])^9)/(9*a^7*d) - (B*(a + a*Sin[c + d*x])^10)/(10
*a^8*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^5\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+d\right.}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(8a^3(A-B)(a+x)^5 - 4a^2(3A-5B)(a+x\right.\right.}{}$$

$$= \frac{4(A-B)(a+a\sin(c+dx))^6}{3a^4 d} - \frac{4(3A-5B)(a+a\sin}{7a^5 d}$$

Mathematica [A] time = 1.19, size = 86, normalized size = 0.64

$$-\frac{a^2(\sin(c+dx)+1)^6\left(28(5A-17B)\sin^3(c+dx)+(651B-525A)\sin^2(c+dx)+6(115A-61B)\sin(c+dx)-3\right.}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/1260*(a^2*(1 + Sin[c + d*x])^6*(-325*A + 61*B + 6*(115*A - 61*B)*Sin[c +
 d*x] + (-525*A + 651*B)*Sin[c + d*x]^2 + 28*(5*A - 17*B)*Sin[c + d*x]^3 +
126*B*Sin[c + d*x]^4))/d
```

fricas [A] time = 0.85, size = 127, normalized size = 0.95

$$\frac{126\,Ba^2\cos(dx+c)^{10} - 315\,(A+B)a^2\cos(dx+c)^8 - 4\left(35\,(A+2\,B)a^2\cos(dx+c)^8 - 10\,(5\,A+B)a^2\cos(dx+\right.}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/1260*(126*B*a^2*cos(d*x + c)^10 - 315*(A + B)*a^2*cos(d*x + c)^8 - 4*(35*
(A + 2*B)*a^2*cos(d*x + c)^8 - 10*(5*A + B)*a^2*cos(d*x + c)^6 - 12*(5*A +
B)*a^2*cos(d*x + c)^4 - 16*(5*A + B)*a^2*cos(d*x + c)^2 - 32*(5*A + B)*a^2)
*sin(d*x + c))/d
```

giac [A] time = 0.44, size = 239, normalized size = 1.78

$$\frac{Ba^2\cos(10\,dx+10\,c)}{5120\,d} - \frac{Aa^2\cos(8\,dx+8\,c)}{512\,d} + \frac{7\,Aa^2\sin(3\,dx+3\,c)}{64\,d} - \frac{\left(16\,Aa^2+7\,Ba^2\right)\cos(6\,dx+6\,c)}{1024\,d} - \frac{\left(7\,Aa^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/5120*B*a^2*cos(10*d*x + 10*c)/d - 1/512*A*a^2*cos(8*d*x + 8*c)/d + 7/64*A
*a^2*sin(3*d*x + 3*c)/d - 1/1024*(16*A*a^2 + 7*B*a^2)*cos(6*d*x + 6*c)/d -
1/128*(7*A*a^2 + 4*B*a^2)*cos(4*d*x + 4*c)/d - 7/512*(8*A*a^2 + 5*B*a^2)*co
s(2*d*x + 2*c)/d - 1/2304*(A*a^2 + 2*B*a^2)*sin(9*d*x + 9*c)/d - 1/1792*(A*
a^2 + 10*B*a^2)*sin(7*d*x + 7*c)/d + 1/320*(5*A*a^2 - 4*B*a^2)*sin(5*d*x +
5*c)/d + 7/128*(11*A*a^2 + 2*B*a^2)*sin(d*x + c)/d
```

maple [A] time = 0.48, size = 231, normalized size = 1.72

$$a^2A\left(-\frac{\left(\cos^8(dx+c)\right)\sin(dx+c)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)+B\,a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^8(dx+c)\right)}{10}-\frac{\left(\cos^8(dx\right.}{40}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(-1/9*cos(d*x+c)^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*
x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+B*a^2*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8
-1/40*cos(d*x+c)^8)-1/4*a^2*A*cos(d*x+c)^8+2*B*a^2*(-1/9*cos(d*x+c)^8*sin(d
*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
)+1/7*a^2*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c
)-1/8*B*a^2*cos(d*x+c)^8)
```

maxima [A] time = 0.31, size = 168, normalized size = 1.25

$$-\frac{126\,Ba^2\sin\left(dx+c\right)^{10}+140\,(A+2\,B)a^2\sin\left(dx+c\right)^9+315\,(A-B)a^2\sin\left(dx+c\right)^8-360\,(A+3\,B)a^2\sin\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/1260*(126*B*a^2*sin(d*x + c)^10 + 140*(A + 2*B)*a^2*sin(d*x + c)^9 + 315
*(A - B)*a^2*sin(d*x + c)^8 - 360*(A + 3*B)*a^2*sin(d*x + c)^7 - 1260*A*a^2
*sin(d*x + c)^6 + 1512*B*a^2*sin(d*x + c)^5 + 630*(3*A + B)*a^2*sin(d*x + c
)^4 + 840*(A - B)*a^2*sin(d*x + c)^3 - 630*(2*A + B)*a^2*sin(d*x + c)^2 - 1
260*A*a^2*sin(d*x + c))/d
```

mupad [B] time = 9.20, size = 168, normalized size = 1.25

$$-\frac{\frac{2\,a^2\,\sin(c+d\,x)^3\,(A-B)}{3}-\frac{a^2\,\sin(c+d\,x)^2\,(2\,A+B)}{2}-A\,a^2\,\sin\left(c+d\,x\right)^6+\frac{a^2\,\sin(c+d\,x)^4\,(3\,A+B)}{2}+\frac{a^2\,\sin(c+d\,x)^8\,(A-B)}{4}-\frac{2\,a^2\,\sin(c+d\,x)^7}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2,x)
```

```
[Out] -((2*a^2*sin(c + d*x)^3*(A - B))/3 - (a^2*sin(c + d*x)^2*(2*A + B))/2 - A*a
^2*sin(c + d*x)^6 + (a^2*sin(c + d*x)^4*(3*A + B))/2 + (a^2*sin(c + d*x)^8*
(A - B))/4 - (2*a^2*sin(c + d*x)^7*(A + 3*B))/7 + (a^2*sin(c + d*x)^9*(A +
2*B))/9 + (6*B*a^2*sin(c + d*x)^5)/5 + (B*a^2*sin(c + d*x)^10)/10 - A*a^2*s
in(c + d*x))/d
```

sympy [A] time = 24.90, size = 440, normalized size = 3.28

$$\begin{cases} \frac{16Aa^2\sin^9(c+dx)}{315d}+\frac{8Aa^2\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{16Aa^2\sin^7(c+dx)}{35d}+\frac{2Aa^2\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{8Aa^2\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{Aa}{} \\ x\left(A+B\sin(c)\right)\left(a\sin(c)+a\right)^2\cos^7(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*A*a**2*sin(c + d*x)**9/(315*d) + 8*A*a**2*sin(c + d*x)**7*cos
(c + d*x)**2/(35*d) + 16*A*a**2*sin(c + d*x)**7/(35*d) + 2*A*a**2*sin(c + d
*x)**5*cos(c + d*x)**4/(5*d) + 8*A*a**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*
d) + A*a**2*sin(c + d*x)**3*cos(c + d*x)**6/(3*d) + 2*A*a**2*sin(c + d*x)**
3*cos(c + d*x)**4/d + A*a**2*sin(c + d*x)*cos(c + d*x)**6/d - A*a**2*cos(c
+ d*x)**8/(4*d) + B*a**2*sin(c + d*x)**10/(40*d) + 32*B*a**2*sin(c + d*x)**
9/(315*d) + B*a**2*sin(c + d*x)**8*cos(c + d*x)**2/(8*d) + 16*B*a**2*sin(c
+ d*x)**7*cos(c + d*x)**2/(35*d) + B*a**2*sin(c + d*x)**6*cos(c + d*x)**4/(
4*d) + 4*B*a**2*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + B*a**2*sin(c + d*x)
**4*cos(c + d*x)**6/(4*d) + 2*B*a**2*sin(c + d*x)**3*cos(c + d*x)**6/(3*d)
- B*a**2*cos(c + d*x)**8/(8*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)
**2*cos(c)**7, True))
```

3.970 $\int \cos^5(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{B(a\sin(c+dx)+a)^8}{8a^6d}+\frac{(A-5B)(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(A-2B)(a\sin(c+dx)+a)^6}{3a^4d}+\frac{4(A-B)(a\sin(c+dx)+a)^5}{5a^3d}$$

```
[Out] 4/5*(A-B)*(a+a*sin(d*x+c))^5/a^3/d-2/3*(A-2*B)*(a+a*sin(d*x+c))^6/a^4/d+1/7
*(A-5*B)*(a+a*sin(d*x+c))^7/a^5/d+1/8*B*(a+a*sin(d*x+c))^8/a^6/d
```

Rubi [A] time = 0.15, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{(A-5B)(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(A-2B)(a\sin(c+dx)+a)^6}{3a^4d}+\frac{4(A-B)(a\sin(c+dx)+a)^5}{5a^3d}+\frac{B(a\sin(c+dx)+a)^8}{8a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (4*(A - B)*(a + a*Sin[c + d*x])^5)/(5*a^3*d) - (2*(A - 2*B)*(a + a*Sin[c +
d*x])^6)/(3*a^4*d) + ((A - 5*B)*(a + a*Sin[c + d*x])^7)/(7*a^5*d) + (B*(a +
 a*Sin[c + d*x])^8)/(8*a^6*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^4\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+d\right.}{a^5d}$$

$$= \frac{\text{Subst}\left(\int \left(4a^2(A-B)(a+x)^4 - 4a(A-2B)(a+x)^5\right.\right.}{a^5d}$$

$$= \frac{4(A-B)(a+a\sin(c+dx))^5}{5a^3d} - \frac{2(A-2B)(a+a\sin(}{3a^4d}$$

Mathematica [A] time = 0.36, size = 70, normalized size = 0.67

$$\frac{a^2(\sin(c+dx)+1)^5\left(15(8A-19B)\sin^2(c+dx)-5(64A-47B)\sin(c+dx)+232A+105B\sin^3(c+dx)-47B\right)}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^5*(232*A - 47*B - 5*(64*A - 47*B)*Sin[c + d*x] + 15
*(8*A - 19*B)*Sin[c + d*x]^2 + 105*B*Sin[c + d*x]^3))/(840*d)
```

fricas [A] time = 0.77, size = 109, normalized size = 1.04

$$\frac{105\,Ba^2\cos(dx+c)^8 - 280\,(A+B)a^2\cos(dx+c)^6 - 8\left(15\,(A+2\,B)a^2\cos(dx+c)^6 - 6\,(4\,A+B)a^2\cos(dx+c)\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/840*(105*B*a^2*cos(d*x + c)^8 - 280*(A + B)*a^2*cos(d*x + c)^6 - 8*(15*(A
 + 2*B)*a^2*cos(d*x + c)^6 - 6*(4*A + B)*a^2*cos(d*x + c)^4 - 8*(4*A + B)*a
^2*cos(d*x + c)^2 - 16*(4*A + B)*a^2)*sin(d*x + c))/d
```

giac [B] time = 0.33, size = 202, normalized size = 1.92

$$\frac{Ba^2\cos(8\,dx+8\,c)}{1024\,d} - \frac{\left(4\,Aa^2+Ba^2\right)\cos(6\,dx+6\,c)}{384\,d} - \frac{\left(16\,Aa^2+9\,Ba^2\right)\cos(4\,dx+4\,c)}{256\,d} - \frac{\left(20\,Aa^2+13\,Ba^2\right)\cos(2}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/1024*B*a^2*cos(8*d*x + 8*c)/d - 1/384*(4*A*a^2 + B*a^2)*cos(6*d*x + 6*c)/
d - 1/256*(16*A*a^2 + 9*B*a^2)*cos(4*d*x + 4*c)/d - 1/128*(20*A*a^2 + 13*B*
a^2)*cos(2*d*x + 2*c)/d - 1/448*(A*a^2 + 2*B*a^2)*sin(7*d*x + 7*c)/d + 1/32
0*(A*a^2 - 6*B*a^2)*sin(5*d*x + 5*c)/d + 1/192*(19*A*a^2 - 2*B*a^2)*sin(3*d
*x + 3*c)/d + 5/64*(9*A*a^2 + 2*B*a^2)*sin(d*x + c)/d
```

maple [B] time = 0.47, size = 201, normalized size = 1.91

$$\frac{a^2 A\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+B a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)-\frac{a^2 A\left(c\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c))+B*a^2*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)
-1/3*a^2*A*cos(d*x+c)^6+2*B*a^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos
(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos
(d*x+c)^2)*sin(d*x+c)-1/6*B*a^2*cos(d*x+c)^6)
```

maxima [A] time = 0.31, size = 142, normalized size = 1.35

$$\frac{105\,Ba^2\sin\left(dx+c\right)^8+120\,(A+2\,B)a^2\sin\left(dx+c\right)^7+140\,(2\,A-B)a^2\sin\left(dx+c\right)^6-168\,(A+4\,B)a^2\sin\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/840*(105*B*a^2*sin(d*x + c)^8 + 120*(A + 2*B)*a^2*sin(d*x + c)^7 + 140*(2
*A - B)*a^2*sin(d*x + c)^6 - 168*(A + 4*B)*a^2*sin(d*x + c)^5 - 210*(4*A +
B)*a^2*sin(d*x + c)^4 - 280*(A - 2*B)*a^2*sin(d*x + c)^3 + 420*(2*A + B)*a^
2*sin(d*x + c)^2 + 840*A*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.12, size = 140, normalized size = 1.33

$$\frac{\frac{a^2\sin(c+d\,x)^2\,(2\,A+B)}{2}-\frac{a^2\sin(c+d\,x)^3\,(A-2\,B)}{3}-\frac{a^2\sin(c+d\,x)^4\,(4\,A+B)}{4}-\frac{a^2\sin(c+d\,x)^5\,(A+4\,B)}{5}+\frac{a^2\sin(c+d\,x)^7\,(A+2\,B)}{7}+\frac{B\,a^2\sin(c+d\,x}{8}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((a^2*sin(c + d*x)^2*(2*A + B))/2 - (a^2*sin(c + d*x)^3*(A - 2*B))/3 - (a^2
*sin(c + d*x)^4*(4*A + B))/4 - (a^2*sin(c + d*x)^5*(A + 4*B))/5 + (a^2*sin(
c + d*x)^7*(A + 2*B))/7 + (B*a^2*sin(c + d*x)^8)/8 + (a^2*sin(c + d*x)^6*(2
*A - B))/6 + A*a^2*sin(c + d*x))/d
```

sympy [A] time = 10.17, size = 335, normalized size = 3.19

$$\begin{cases} \frac{8Aa^2\sin^7(c+dx)}{105d} + \frac{4Aa^2\sin^5(c+dx)\cos^2(c+dx)}{15d} + \frac{8Aa^2\sin^5(c+dx)}{15d} + \frac{Aa^2\sin^3(c+dx)\cos^4(c+dx)}{3d} + \frac{4Aa^2\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{Aa^2\sin \\ x(A+B\sin(c))(a\sin(c)+a)^2\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*A*a**2*sin(c + d*x)**7/(105*d) + 4*A*a**2*sin(c + d*x)**5*cos(
c + d*x)**2/(15*d) + 8*A*a**2*sin(c + d*x)**5/(15*d) + A*a**2*sin(c + d*x)*
*3*cos(c + d*x)**4/(3*d) + 4*A*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) +
 A*a**2*sin(c + d*x)*cos(c + d*x)**4/d - A*a**2*cos(c + d*x)**6/(3*d) + B*a
**2*sin(c + d*x)**8/(24*d) + 16*B*a**2*sin(c + d*x)**7/(105*d) + B*a**2*sin
(c + d*x)**6*cos(c + d*x)**2/(6*d) + 8*B*a**2*sin(c + d*x)**5*cos(c + d*x)*
*2/(15*d) + B*a**2*sin(c + d*x)**4*cos(c + d*x)**4/(4*d) + 2*B*a**2*sin(c +
 d*x)**3*cos(c + d*x)**4/(3*d) - B*a**2*cos(c + d*x)**6/(6*d), Ne(d, 0)), (
x*(A + B*sin(c))*(a*sin(c) + a)**2*cos(c)**5, True))
```

3.971 $\int \cos^3(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$-\frac{B(a\sin(c+dx)+a)^6}{6a^4d}-\frac{(A-3B)(a\sin(c+dx)+a)^5}{5a^3d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{2a^2d}$$

```
[Out] 1/2*(A-B)*(a+a*sin(d*x+c))^4/a^2/d-1/5*(A-3*B)*(a+a*sin(d*x+c))^5/a^3/d-1/6
*B*(a+a*sin(d*x+c))^6/a^4/d
```

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$-\frac{(A-3B)(a\sin(c+dx)+a)^5}{5a^3d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{2a^2d}-\frac{B(a\sin(c+dx)+a)^6}{6a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^4)/(2*a^2*d) - ((A - 3*B)*(a + a*Sin[c + d*x]
)^5)/(5*a^3*d) - (B*(a + a*Sin[c + d*x])^6)/(6*a^4*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int(a-x)(a+x)^3\left(A+\frac{Bx}{a}\right)dx,x,a\sin(c+dx\right.}{a^3d}$$
$$= \frac{\text{Subst}\left(\int\left(2a(A-B)(a+x)^3+(-A+3B)(a+x)^4-\right.\right.}{a^3d}$$
$$= \frac{(A-B)(a+a\sin(c+dx))^4}{2a^2d} - \frac{(A-3B)(a+a\sin(c+\right.}{5a^3d}$$

Mathematica [A] time = 0.43, size = 66, normalized size = 0.85

$$\frac{a^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^8(-4(3A-4B)\sin(c+dx)+18A+5B\cos(2(c+dx))-9B)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*(18*A - 9*B + 5*B*Cos[2*(c + d
*x)] - 4*(3*A - 4*B)*Sin[c + d*x]))/(60*d)
```

fricas [A] time = 0.76, size = 91, normalized size = 1.17

$$\frac{5\,Ba^2\cos\left(dx+c\right)^6-15\,(A+B)a^2\cos\left(dx+c\right)^4-2\left(3\,(A+2\,B)a^2\cos\left(dx+c\right)^4-2\,(3\,A+B)a^2\cos\left(dx+c\right)^2-4\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/30*(5*B*a^2*cos(d*x + c)^6 - 15*(A + B)*a^2*cos(d*x + c)^4 - 2*(3*(A + 2*
B)*a^2*cos(d*x + c)^4 - 2*(3*A + B)*a^2*cos(d*x + c)^2 - 4*(3*A + B)*a^2)*s
in(d*x + c))/d
```

giac [A] time = 0.25, size = 116, normalized size = 1.49

$$-\frac{5\,Ba^2\sin\left(dx+c\right)^6+6\,Aa^2\sin\left(dx+c\right)^5+12\,Ba^2\sin\left(dx+c\right)^5+15\,Aa^2\sin\left(dx+c\right)^4-20\,Ba^2\sin\left(dx+c\right)^3-3}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/30*(5*B*a^2*sin(d*x + c)^6 + 6*A*a^2*sin(d*x + c)^5 + 12*B*a^2*sin(d*x +
 c)^5 + 15*A*a^2*sin(d*x + c)^4 - 20*B*a^2*sin(d*x + c)^3 - 30*A*a^2*sin(d*
x + c)^2 - 15*B*a^2*sin(d*x + c)^2 - 30*A*a^2*sin(d*x + c))/d
```

maple [B] time = 0.48, size = 171, normalized size = 2.19

$$\frac{a^2A\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)+B\,a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)-\frac{a^2A\left(\cos^4(dx+c)\right)}{2}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))+
B*a^2*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)-1/2*a^2*A*cos(d*x+
c)^4+2*B*a^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c)
)+1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)-1/4*B*a^2*cos(d*x+c)^4)
```

maxima [A] time = 0.31, size = 96, normalized size = 1.23

$$-\frac{5\,Ba^2\sin\left(dx+c\right)^6+6\,(A+2\,B)a^2\sin\left(dx+c\right)^5+15\,Aa^2\sin\left(dx+c\right)^4-20\,Ba^2\sin\left(dx+c\right)^3-15\,(2\,A+B)a^2}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/30*(5*B*a^2*sin(d*x + c)^6 + 6*(A + 2*B)*a^2*sin(d*x + c)^5 + 15*A*a^2*s
in(d*x + c)^4 - 20*B*a^2*sin(d*x + c)^3 - 15*(2*A + B)*a^2*sin(d*x + c)^2 -
 30*A*a^2*sin(d*x + c))/d
```

mupad [B] time = 9.08, size = 96, normalized size = 1.23

$$-\frac{\frac{A\,a^2\,\sin(c+d\,x)^4}{2}-\frac{a^2\,\sin(c+d\,x)^2\,(2\,A+B)}{2}+\frac{a^2\,\sin(c+d\,x)^5\,(A+2\,B)}{5}-\frac{2\,B\,a^2\,\sin(c+d\,x)^3}{3}+\frac{B\,a^2\,\sin(c+d\,x)^6}{6}-A\,a^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2,x)
```

```
[Out] -((A*a^2*sin(c + d*x)^4)/2 - (a^2*sin(c + d*x)^2*(2*A + B))/2 + (a^2*sin(c
+ d*x)^5*(A + 2*B))/5 - (2*B*a^2*sin(c + d*x)^3)/3 + (B*a^2*sin(c + d*x)^6)
/6 - A*a^2*sin(c + d*x))/d
```

sympy [A] time = 3.51, size = 228, normalized size = 2.92

$$\begin{cases} \frac{2Aa^2\sin^5(c+dx)}{15d} + \frac{Aa^2\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{2Aa^2\sin^3(c+dx)}{3d} + \frac{Aa^2\sin(c+dx)\cos^2(c+dx)}{d} - \frac{Aa^2\cos^4(c+dx)}{2d} + \frac{Ba^2\sin^6(c+dx)}{12d} + \frac{4B \\ x(A+B\sin(c))(a\sin(c)+a)^2\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*A*a**2*sin(c + d*x)**5/(15*d) + A*a**2*sin(c + d*x)**3*cos(c +
 d*x)**2/(3*d) + 2*A*a**2*sin(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)*cos(c
 + d*x)**2/d - A*a**2*cos(c + d*x)**4/(2*d) + B*a**2*sin(c + d*x)**6/(12*d)
 + 4*B*a**2*sin(c + d*x)**5/(15*d) + B*a**2*sin(c + d*x)**4*cos(c + d*x)**2
/(4*d) + 2*B*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - B*a**2*cos(c + d*
x)**4/(4*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**2*cos(c)**3, True
))
```

3.972 $\int \cos(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=51

$$\frac{B(a\sin(c+dx)+a)^4}{4a^2d}+\frac{(A-B)(a\sin(c+dx)+a)^3}{3ad}$$

```
[Out] 1/3*(A-B)*(a+a*sin(d*x+c))^3/a/d+1/4*B*(a+a*sin(d*x+c))^4/a^2/d
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{B(a\sin(c+dx)+a)^4}{4a^2d}+\frac{(A-B)(a\sin(c+dx)+a)^3}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^3)/(3*a*d) + (B*(a + a*Sin[c + d*x])^4)/(4*a^
2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{\text{Subst}\left(\int \left((A-B)(a+x)^2+\frac{B(a+x)^3}{a}\right)dx, x, a\sin(c+ \right.}{ad}$$

$$= \frac{(A-B)(a+a\sin(c+dx))^3}{3ad}+\frac{B(a+a\sin(c+dx))^4}{4a^2d}$$

Mathematica [**A**] time = 0.10, size = 49, normalized size = 0.96

$$\frac{\frac{1}{3}(A-B)(a\sin(c+dx)+a)^3+\frac{B(a\sin(c+dx)+a)^4}{4a}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (((A - B)*(a + a*Sin[c + d*x])^3)/3 + (B*(a + a*Sin[c + d*x])^4)/(4*a))/(a*
d)
```

fricas [**A**] time = 0.88, size = 72, normalized size = 1.41

$$\frac{3\,Ba^2\cos\left(dx+c\right)^4-12\,(A+B)a^2\cos\left(dx+c\right)^2-4\left((A+2\,B)a^2\cos\left(dx+c\right)^2-2\,(2\,A+B)a^2\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*B*a^2*cos(d*x + c)^4 - 12*(A + B)*a^2*cos(d*x + c)^2 - 4*((A + 2*B)
*a^2*cos(d*x + c)^2 - 2*(2*A + B)*a^2)*sin(d*x + c))/d
```

giac [**A**] time = 0.16, size = 88, normalized size = 1.73

$$\frac{3\,Ba^2\sin\left(dx+c\right)^4+4\,Aa^2\sin\left(dx+c\right)^3+8\,Ba^2\sin\left(dx+c\right)^3+12\,Aa^2\sin\left(dx+c\right)^2+6\,Ba^2\sin\left(dx+c\right)^2+12\,A}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/12*(3*B*a^2*sin(d*x + c)^4 + 4*A*a^2*sin(d*x + c)^3 + 8*B*a^2*sin(d*x + c
)^3 + 12*A*a^2*sin(d*x + c)^2 + 6*B*a^2*sin(d*x + c)^2 + 12*A*a^2*sin(d*x +
 c))/d
```

maple [A] time = 0.24, size = 75, normalized size = 1.47

$$\frac{\frac{Ba^2(\sin^4(dx+c))}{4} + \frac{(a^2A+2Ba^2)(\sin^3(dx+c))}{3} + \frac{(2a^2A+Ba^2)(\sin^2(dx+c))}{2} + a^2A\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/4*B*a^2*sin(d*x+c)^4+1/3*(A*a^2+2*B*a^2)*sin(d*x+c)^3+1/2*(2*A*a^2+B
*a^2)*sin(d*x+c)^2+a^2*A*sin(d*x+c))
```

maxima [A] time = 0.31, size = 68, normalized size = 1.33

$$\frac{3\,Ba^2\sin(dx+c)^4 + 4\,(A+2\,B)a^2\sin(dx+c)^3 + 6\,(2\,A+B)a^2\sin(dx+c)^2 + 12\,Aa^2\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(3*B*a^2*sin(d*x + c)^4 + 4*(A + 2*B)*a^2*sin(d*x + c)^3 + 6*(2*A + B)
*a^2*sin(d*x + c)^2 + 12*A*a^2*sin(d*x + c))/d
```

mupad [B] time = 9.10, size = 66, normalized size = 1.29

$$\frac{\frac{a^2\sin(c+d\,x)^2\,(2\,A+B)}{2} + \frac{a^2\sin(c+d\,x)^3\,(A+2\,B)}{3} + \frac{B\,a^2\sin(c+d\,x)^4}{4} + A\,a^2\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((a^2*sin(c + d*x)^2*(2*A + B))/2 + (a^2*sin(c + d*x)^3*(A + 2*B))/3 + (B*a
^2*sin(c + d*x)^4)/4 + A*a^2*sin(c + d*x))/d
```

sympy [A] time = 0.99, size = 117, normalized size = 2.29

$$\begin{cases} \frac{Aa^2\sin^3(c+dx)}{3d} + \frac{Aa^2\sin(c+dx)}{d} - \frac{Aa^2\cos^2(c+dx)}{d} + \frac{Ba^2\sin^4(c+dx)}{4d} + \frac{2Ba^2\sin^3(c+dx)}{3d} - \frac{Ba^2\cos^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(A+B\sin(c))(a\sin(c)+a)^2\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*sin(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)/d - A*a**2*co
s(c + d*x)**2/d + B*a**2*sin(c + d*x)**4/(4*d) + 2*B*a**2*sin(c + d*x)**3/(
3*d) - B*a**2*cos(c + d*x)**2/(2*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c)
 + a)**2*cos(c), True))
```

3.973 $\int \sec(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$-\frac{a^2(A+B)\sin(c+dx)}{d}-\frac{2a^2(A+B)\log(1-\sin(c+dx))}{d}-\frac{B(a\sin(c+dx)+a)^2}{2d}$$

```
[Out] -2*a^2*(A+B)*ln(1-sin(d*x+c))/d-a^2*(A+B)*sin(d*x+c)/d-1/2*B*(a+a*sin(d*x+c
))^2/d
```

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 77}

$$-\frac{a^2(A+B)\sin(c+dx)}{d}-\frac{2a^2(A+B)\log(1-\sin(c+dx))}{d}-\frac{B(a\sin(c+dx)+a)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*(A + B)*Log[1 - Sin[c + d*x]])/d - (a^2*(A + B)*Sin[c + d*x])/d - (
B*(a + a*Sin[c + d*x])^2)/(2*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{a\,\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{a}\right)}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(-A-B+\frac{2a(A+B)}{a-x}-\frac{B(a+x)}{a}\right)\,dx, x, a\sin(c+\right.}{d}$$
$$= -\frac{2a^2(A+B)\log(1-\sin(c+dx))}{d} - \frac{a^2(A+B)\sin(c+}{d}$$

Mathematica [A] time = 0.08, size = 51, normalized size = 0.85

$$\frac{a\left(-a(A+2B)\sin(c+dx)-2a(A+B)\log(1-\sin(c+dx))-\frac{1}{2}aB\sin^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
[Out] (a*(-2*a*(A + B)*Log[1 - Sin[c + d*x]] - a*(A + 2*B)*Sin[c + d*x] - (a*B*Si
n[c + d*x]^2)/2))/d
```

fricas [A] time = 0.79, size = 54, normalized size = 0.90

$$\frac{Ba^2\cos(dx+c)^2-4(A+B)a^2\log(-\sin(dx+c)+1)-2(A+2B)a^2\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fric
as")
[Out] 1/2*(B*a^2*cos(d*x + c)^2 - 4*(A + B)*a^2*log(-sin(d*x + c) + 1) - 2*(A + 2
*B)*a^2*sin(d*x + c))/d
```

giac [B] time = 0.20, size = 220, normalized size = 3.67

$$\frac{2\left(Aa^2+Ba^2\right)\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)-4\left(Aa^2+Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{3Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+3Ba^2\tan}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] (2*(A*a^2 + B*a^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 4*(A*a^2 + B*a^2)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) - (3*A*a^2*tan(1/2*d*x + 1/2*c)^4 + 3*B*a^2*
tan(1/2*d*x + 1/2*c)^4 + 2*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 4*B*a^2*tan(1/2*d
*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c)^2 + 8*B*a^2*tan(1/2*d*x + 1/2*
c)^2 + 2*A*a^2*tan(1/2*d*x + 1/2*c) + 4*B*a^2*tan(1/2*d*x + 1/2*c) + 3*A*a^
2 + 3*B*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [B] time = 0.37, size = 127, normalized size = 2.12

$$-\frac{a^2 A \sin(dx+c)}{d}+\frac{2a^2 A \ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{B a^2 \left(\sin^2(dx+c)\right)}{2d}-\frac{2B a^2 \ln(\cos(dx+c))}{d}-\frac{2a^2 A \ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] -1/d*a^2*A*sin(d*x+c)+2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))-1/2/d*B*a^2*sin(d
*x+c)^2-2/d*B*a^2*ln(cos(d*x+c))-2/d*a^2*A*ln(cos(d*x+c))-2/d*B*a^2*sin(d*x
+c)+2/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.31, size = 52, normalized size = 0.87

$$-\frac{Ba^2 \sin(dx+c)^2 + 4(A+B)a^2 \log(\sin(dx+c)-1) + 2(A+2B)a^2 \sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(B*a^2*sin(d*x + c)^2 + 4*(A + B)*a^2*log(sin(d*x + c) - 1) + 2*(A + 2
*B)*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.08, size = 63, normalized size = 1.05

$$-\frac{\sin(c+d\,x)\,\left(a^2\,(A+B)+B\,a^2\right)+\ln(\sin(c+d\,x)-1)\,\left(2\,A\,a^2+2\,B\,a^2\right)+\frac{B\,a^2\,\sin(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x),x)
```

```
[Out] -(sin(c + d*x)*(a^2*(A + B) + B*a^2) + log(sin(c + d*x) - 1)*(2*A*a^2 + 2*B
*a^2) + (B*a^2*sin(c + d*x)^2)/2)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sec(c+dx)\,dx+\int 2A\sin(c+dx)\sec(c+dx)\,dx+\int A\sin^2(c+dx)\sec(c+dx)\,dx+\int B\sin(c+dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x), x) + Integral(2*A*sin(c + d*x)*sec(c + d*x),
 x) + Integral(A*sin(c + d*x)**2*sec(c + d*x), x) + Integral(B*sin(c + d*x)
*sec(c + d*x), x) + Integral(2*B*sin(c + d*x)**2*sec(c + d*x), x) + Integra
l(B*sin(c + d*x)**3*sec(c + d*x), x))
```

3.974 $\int \sec^3(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{a^3(A+B)}{d(a-a\sin(c+dx))}+\frac{a^2B\log(1-\sin(c+dx))}{d}$$

```
[Out] a^2*B*ln(1-sin(d*x+c))/d+a^3*(A+B)/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 43}

$$\frac{a^3(A+B)}{d(a-a\sin(c+dx))}+\frac{a^2B\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*B*Log[1 - Sin[c + d*x]])/d + (a^3*(A + B))/(d*(a - a*Sin[c + d*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{a^3 \operatorname{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3 \operatorname{Subst}\left(\int \left(\frac{A+B}{(a-x)^2}-\frac{B}{a(a-x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2 B\log(1-\sin(c+dx))}{d} + \frac{a^3(A+B)}{d(a-a\sin(c+dx))}$$

Mathematica [A] time = 0.08, size = 41, normalized size = 0.95

$$\frac{a^3\left(\frac{A+B}{a-a\sin(c+dx)}+\frac{B\log(1-\sin(c+dx))}{a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
[Out] (a^3*((B*Log[1 - Sin[c + d*x]])/a + (A + B)/(a - a*Sin[c + d*x])))/d
```

fricas [A] time = 0.81, size = 55, normalized size = 1.28

$$-\frac{(A+B)a^2-\left(Ba^2\sin(dx+c)-Ba^2\right)\log(-\sin(dx+c)+1)}{d\sin(dx+c)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
[Out] -((A + B)*a^2 - (B*a^2*sin(d*x + c) - B*a^2)*log(-sin(d*x + c) + 1))/(d*sin
(d*x + c) - d)
```

giac [B] time = 0.21, size = 112, normalized size = 2.60

$$-\frac{Ba^2\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)-2Ba^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{3Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8Ba^2\tan(}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -(B*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*B*a^2*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + (3*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 2*A*a^2*tan(1/2*d*x + 1/2*c)
- 8*B*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^2)/d
```

maple [B] time = 0.60, size = 189, normalized size = 4.40

$$\frac{a^2A\left(\sin^3(dx+c)\right)}{2d\cos(dx+c)^2}+\frac{a^2A\sin(dx+c)}{2d}+\frac{B\,a^2\left(\tan^2(dx+c)\right)}{2d}+\frac{B\,a^2\ln(\cos(dx+c))}{d}+\frac{a^2A}{d\cos(dx+c)^2}+\frac{B\,a^2\left(\sin^3\right.}{d\cos(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/2/d*a^2*A*sin(d*x+c)^3/cos(d*x+c)^2+1/2/d*a^2*A*sin(d*x+c)+1/2/d*B*a^2*ta
n(d*x+c)^2+1/d*B*a^2*ln(cos(d*x+c))+1/d*a^2*A/cos(d*x+c)^2+1/d*B*a^2*sin(d*
x+c)^3/cos(d*x+c)^2+1/d*B*a^2*sin(d*x+c)-1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c)
)+1/2/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a^2/cos(d*x+c)^2
```

maxima [A] time = 0.35, size = 37, normalized size = 0.86

$$\frac{Ba^2\log(\sin(dx+c)-1)-\frac{(A+B)a^2}{\sin(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] (B*a^2*log(sin(d*x + c) - 1) - (A + B)*a^2/(sin(d*x + c) - 1))/d
```

mupad [B] time = 0.06, size = 44, normalized size = 1.02

$$\frac{B\,a^2\,\ln(\sin(c+d\,x)-1)}{d}-\frac{A\,a^2+B\,a^2}{d\,(\sin(c+d\,x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^3,x)
```

```
[Out] (B*a^2*log(sin(c + d*x) - 1))/d - (A*a^2 + B*a^2)/(d*(sin(c + d*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sec^3(c+dx)\,dx+\int 2A\sin(c+dx)\sec^3(c+dx)\,dx+\int A\sin^2(c+dx)\sec^3(c+dx)\,dx+\int B\sin(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**3, x) + Integral(2*A*sin(c + d*x)*sec(c + d*
x)**3, x) + Integral(A*sin(c + d*x)**2*sec(c + d*x)**3, x) + Integral(B*sin
(c + d*x)*sec(c + d*x)**3, x) + Integral(2*B*sin(c + d*x)**2*sec(c + d*x)**
3, x) + Integral(B*sin(c + d*x)**3*sec(c + d*x)**3, x))
```

3.975 $\int \sec^5(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=77

$$\frac{a^4(A+B)}{4d(a-a\sin(c+dx))^2}+\frac{a^3(A-B)}{4d(a-a\sin(c+dx))}+\frac{a^2(A-B)\tanh^{-1}(\sin(c+dx))}{4d}$$

```
[Out] 1/4*a^2*(A-B)*arctanh(sin(d*x+c))/d+1/4*a^4*(A+B)/d/(a-a*sin(d*x+c))^2+1/4*
a^3*(A-B)/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.12, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$\frac{a^4(A+B)}{4d(a-a\sin(c+dx))^2}+\frac{a^3(A-B)}{4d(a-a\sin(c+dx))}+\frac{a^2(A-B)\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(A - B)*ArcTanh[Sin[c + d*x]])/(4*d) + (a^4*(A + B))/(4*d*(a - a*Sin[c
 + d*x])^2) + (a^3*(A - B))/(4*d*(a - a*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
```

```
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{A+B}{2a(a-x)^3}+\frac{A-B}{4a^2(a-x)^2}+\frac{A-B}{4a^2\left(a^2-x^2\right)}\right)dx, x, a\right.}{d}$$

$$= \frac{a^4(A+B)}{4d(a-a\sin(c+dx))^2}+\frac{a^3(A-B)}{4d(a-a\sin(c+dx))}+\frac{\left(a^3\right.}{}$$

$$= \frac{a^2(A-B)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4(A+B)}{4d(a-a\sin(c+dx}$$

Mathematica [A] time = 0.14, size = 75, normalized size = 0.97

$$\frac{a^5\left(\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{4a^3}+\frac{A-B}{4a^2(a-a\sin(c+dx))}+\frac{A+B}{4a(a-a\sin(c+dx))^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^5*(((A - B)*ArcTanh[Sin[c + d*x]])/(4*a^3) + (A + B)/(4*a*(a - a*Sin[c +
 d*x])^2) + (A - B)/(4*a^2*(a - a*Sin[c + d*x]))))/d
```

fricas [B] time = 0.69, size = 161, normalized size = 2.09

$$\frac{2\,(A-B)a^2\sin(dx+c)-4\,Aa^2+\left((A-B)a^2\cos(dx+c)^2+2\,(A-B)a^2\sin(dx+c)-2\,(A-B)a^2\right)\log(\sin(dx}{8\left(d\cos(dx+c)^2+2\,d\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/8*(2*(A - B)*a^2*sin(d*x + c) - 4*A*a^2 + ((A - B)*a^2*cos(d*x + c)^2 + 2
*(A - B)*a^2*sin(d*x + c) - 2*(A - B)*a^2)*log(sin(d*x + c) + 1) - ((A - B)
```

```
*a^2*cos(d*x + c)^2 + 2*(A - B)*a^2*sin(d*x + c) - 2*(A - B)*a^2)*log(-sin(
d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

giac [A] time = 0.24, size = 130, normalized size = 1.69

$$\frac{2\left(Aa^2 - Ba^2\right)\log\left(|\sin\left(dx+c\right)+1|\right) - 2\left(Aa^2 - Ba^2\right)\log\left(|\sin\left(dx+c\right)-1|\right) + \frac{3\,Aa^2\sin(dx+c)^2 - 3\,Ba^2\sin(dx+c)^2 - 10\,Aa^2}{(\sin(dx}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/16*(2*(A*a^2 - B*a^2)*log(abs(sin(d*x + c) + 1)) - 2*(A*a^2 - B*a^2)*log(
abs(sin(d*x + c) - 1)) + (3*A*a^2*sin(d*x + c)^2 - 3*B*a^2*sin(d*x + c)^2 -
 10*A*a^2*sin(d*x + c) + 10*B*a^2*sin(d*x + c) + 11*A*a^2 - 3*B*a^2)/(sin(d
*x + c) - 1)^2)/d
```

maple [B] time = 0.57, size = 281, normalized size = 3.65

$$\frac{a^2A\left(\sin^3\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4}+\frac{a^2A\left(\sin^3\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^2}+\frac{a^2A\sin\left(dx+c\right)}{8d}+\frac{a^2A\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{4d}+\frac{B\,a^2\left(\sin^4\left(dx\right.\right.}{4d\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a^2*A*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a^2*A*sin(d*x+c)^3/cos(d*x+c)^2
+1/8/d*a^2*A*sin(d*x+c)+1/4/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*B*a^2*s
in(d*x+c)^4/cos(d*x+c)^4+1/2/d*a^2*A/cos(d*x+c)^4+1/2/d*B*a^2*sin(d*x+c)^3/
cos(d*x+c)^4+1/4/d*B*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/4/d*B*a^2*sin(d*x+c)-1
/4/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a^2*A*tan(d*x+c)*sec(d*x+c)^3+3/
8/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/4/d*B*a^2/cos(d*x+c)^4
```

maxima [A] time = 0.49, size = 87, normalized size = 1.13

$$\frac{(A-B)a^2\log\left(\sin\left(dx+c\right)+1\right) - (A-B)a^2\log\left(\sin\left(dx+c\right)-1\right) - \frac{2\left((A-B)a^2\sin(dx+c) - 2\,Aa^2\right)}{\sin(dx+c)^2 - 2\,\sin(dx+c) + 1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/8*((A - B)*a^2*log(sin(d*x + c) + 1) - (A - B)*a^2*log(sin(d*x + c) - 1)
- 2*((A - B)*a^2*sin(d*x + c) - 2*A*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) +
 1))/d
```

mupad [B] time = 0.11, size = 73, normalized size = 0.95

$$\frac{\frac{A\,a^2}{2}-\sin\left(c+d\,x\right)\,\left(\frac{A\,a^2}{4}-\frac{B\,a^2}{4}\right)}{d\,\left({\sin\left(c+d\,x\right)}^2-2\,\sin\left(c+d\,x\right)+1\right)}+\frac{a^2\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(A-B\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] ((A*a^2)/2 - sin(c + d*x)*((A*a^2)/4 - (B*a^2)/4))/(d*(sin(c + d*x)^2 - 2*s
in(c + d*x) + 1)) + (a^2*atanh(sin(c + d*x))*(A - B))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.976 $\int \sec^7(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=132

$$\frac{a^5(A+B)}{12d(a-a\sin(c+dx))^3}+\frac{a^4A}{8d(a-a\sin(c+dx))^2}+\frac{a^3(3A-B)}{16d(a-a\sin(c+dx))}-\frac{a^3(A-B)}{16d(a\sin(c+dx)+a)}+\frac{a^2(2A-B)\tan}{}$$

```
[Out] 1/8*a^2*(2*A-B)*arctanh(sin(d*x+c))/d+1/12*a^5*(A+B)/d/(a-a*sin(d*x+c))^3+1
/8*a^4*A/d/(a-a*sin(d*x+c))^2+1/16*a^3*(3*A-B)/d/(a-a*sin(d*x+c))-1/16*a^3*
(A-B)/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$\frac{a^5(A+B)}{12d(a-a\sin(c+dx))^3}+\frac{a^3(3A-B)}{16d(a-a\sin(c+dx))}-\frac{a^3(A-B)}{16d(a\sin(c+dx)+a)}+\frac{a^2(2A-B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{}{8d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(2*A - B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^5*(A + B))/(12*d*(a - a*Si
n[c + d*x])^3) + (a^4*A)/(8*d*(a - a*Sin[c + d*x])^2) + (a^3*(3*A - B))/(16
*d*(a - a*Sin[c + d*x])) - (a^3*(A - B))/(16*d*(a + a*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{A+B}{4a^2(a-x)^4} + \frac{A}{4a^3(a-x)^3} + \frac{3A-B}{16a^4(a-x)^2} + \frac{A-}{16a^4(a-}\right.\right.}{d}$$

$$= \frac{a^5(A+B)}{12d(a-a\sin(c+dx))^3} + \frac{a^4A}{8d(a-a\sin(c+dx))^2} + \frac{}{1}$$

$$= \frac{a^2(2A-B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^5(A+B)}{12d(a-a\sin(c+}$$

Mathematica [A] time = 0.71, size = 90, normalized size = 0.68

$$\frac{a^2\left(\frac{3B-9A}{\sin(c+dx)-1} - \frac{3(A-B)}{\sin(c+dx)+1} - \frac{4(A+B)}{(\sin(c+dx)-1)^3} + 6(2A-B)\tanh^{-1}(\sin(c+dx)) + \frac{6A}{(\sin(c+dx)-1)^2}\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(6*(2*A - B)*ArcTanh[Sin[c + d*x]] - (4*(A + B))/(-1 + Sin[c + d*x])^3
 + (6*A)/(-1 + Sin[c + d*x])^2 + (-9*A + 3*B)/(-1 + Sin[c + d*x]) - (3*(A -
 B))/(1 + Sin[c + d*x])))/(48*d)
```

fricas [B] time = 0.78, size = 271, normalized size = 2.05

$$-\frac{12\,(2\,A-B)a^2\cos(dx+c)^2 - 8\,(A-2\,B)a^2 - 3\left((2\,A-B)a^2\cos(dx+c)^4 + 2\,(2\,A-B)a^2\cos(dx+c)^2\sin(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/48*(12*(2*A - B)*a^2*cos(d*x + c)^2 - 8*(A - 2*B)*a^2 - 3*((2*A - B)*a^2
*cos(d*x + c)^4 + 2*(2*A - B)*a^2*cos(d*x + c)^2*sin(d*x + c) - 2*(2*A - B)
*a^2*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + 3*((2*A - B)*a^2*cos(d*x + c)^
4 + 2*(2*A - B)*a^2*cos(d*x + c)^2*sin(d*x + c) - 2*(2*A - B)*a^2*cos(d*x +
 c)^2)*log(-sin(d*x + c) + 1) - 2*(3*(2*A - B)*a^2*cos(d*x + c)^2 - 4*(2*A
- B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2*sin(d*x + c)
 - 2*d*cos(d*x + c)^2)
```

giac [A] time = 0.27, size = 209, normalized size = 1.58

$$\frac{6\left(2\,Aa^2 - Ba^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right) - 6\left(2\,Aa^2 - Ba^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right) - \frac{6\left(2\,Aa^2\sin(dx+c) - Ba^2\sin(dx+c) + 3\,A}{\sin(dx+c)+1}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/96*(6*(2*A*a^2 - B*a^2)*log(abs(sin(d*x + c) + 1)) - 6*(2*A*a^2 - B*a^2)*
log(abs(sin(d*x + c) - 1)) - 6*(2*A*a^2*sin(d*x + c) - B*a^2*sin(d*x + c) +
 3*A*a^2 - 2*B*a^2)/(sin(d*x + c) + 1) + (22*A*a^2*sin(d*x + c)^3 - 11*B*a^
2*sin(d*x + c)^3 - 84*A*a^2*sin(d*x + c)^2 + 39*B*a^2*sin(d*x + c)^2 + 114*
A*a^2*sin(d*x + c) - 45*B*a^2*sin(d*x + c) - 60*A*a^2 + 9*B*a^2)/(sin(d*x +
 c) - 1)^3)/d
```

maple [B] time = 0.73, size = 379, normalized size = 2.87

$$\frac{a^2A\left(\sin^3\left(dx+c\right)\right)}{6d\cos\left(dx+c\right)^6} + \frac{a^2A\left(\sin^3\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^4} + \frac{a^2A\left(\sin^3\left(dx+c\right)\right)}{16d\cos\left(dx+c\right)^2} + \frac{a^2A\sin\left(dx+c\right)}{16d} + \frac{a^2A\ln\left(\sec\left(dx+c\right)+\tan\left(dx\right.\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/6/d*a^2*A*sin(d*x+c)^3/cos(d*x+c)^6+1/8/d*a^2*A*sin(d*x+c)^3/cos(d*x+c)^4
+1/16/d*a^2*A*sin(d*x+c)^3/cos(d*x+c)^2+1/16/d*a^2*A*sin(d*x+c)+1/4/d*a^2*A
*ln(sec(d*x+c)+tan(d*x+c))+1/6/d*B*a^2*sin(d*x+c)^4/cos(d*x+c)^6+1/12/d*B*a
^2*sin(d*x+c)^4/cos(d*x+c)^4+1/3/d*a^2*A/cos(d*x+c)^6+1/3/d*B*a^2*sin(d*x+c
)^3/cos(d*x+c)^6+1/4/d*B*a^2*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*B*a^2*sin(d*x+
c)^3/cos(d*x+c)^2+1/8/d*B*a^2*sin(d*x+c)-1/8/d*B*a^2*ln(sec(d*x+c)+tan(d*x+
c))+1/6/d*a^2*A*tan(d*x+c)*sec(d*x+c)^5+5/24/d*a^2*A*tan(d*x+c)*sec(d*x+c)^
3+5/16/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/6/d*B*a^2/cos(d*x+c)^6
```

maxima [A] time = 0.43, size = 148, normalized size = 1.12

$$\frac{3(2A-B)a^2\log(\sin(dx+c)+1)-3(2A-B)a^2\log(\sin(dx+c)-1)-\frac{2\left(3(2A-B)a^2\sin(dx+c)^3-6(2A-B)a^2\sin(dx+c)^2\right.}{\sin(dx+c)^4-2\sin(dx+c)^3+2}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(3*(2*A - B)*a^2*log(sin(d*x + c) + 1) - 3*(2*A - B)*a^2*log(sin(d*x +
 c) - 1) - 2*(3*(2*A - B)*a^2*sin(d*x + c)^3 - 6*(2*A - B)*a^2*sin(d*x + c)
^2 + (2*A - B)*a^2*sin(d*x + c) + 2*(4*A + B)*a^2)/(sin(d*x + c)^4 - 2*sin(
d*x + c)^3 + 2*sin(d*x + c) - 1))/d
```

mupad [B] time = 9.22, size = 136, normalized size = 1.03

$$\frac{a^2\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(2\,A-B\right)}{8\,d}-\frac{\sin\left(c+d\,x\right)^3\,\left(\frac{A\,a^2}{4}-\frac{B\,a^2}{8}\right)-\sin\left(c+d\,x\right)^2\,\left(\frac{A\,a^2}{2}-\frac{B\,a^2}{4}\right)+\frac{A\,a^2}{3}+\frac{B\,a^2}{12}+\sin\left(c+\right.}{d\,\left(\sin\left(c+d\,x\right)^4-2\,\sin\left(c+d\,x\right)^3+2\,\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^7,x)
```

```
[Out] (a^2*atanh(sin(c + d*x))*(2*A - B))/(8*d) - (sin(c + d*x)^3*((A*a^2)/4 - (B
*a^2)/8) - sin(c + d*x)^2*((A*a^2)/2 - (B*a^2)/4) + (A*a^2)/3 + (B*a^2)/12
+ sin(c + d*x)*((A*a^2)/12 - (B*a^2)/24))/(d*(2*sin(c + d*x) - 2*sin(c + d*
x)^3 + sin(c + d*x)^4 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.977 $\int \cos^6(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=196

$$-\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}-\frac{(9A+2B)\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{72d}+\frac{a^2(9A+2B)\sin(c+dx)\cos^5(c+dx)}{48d}$$

```
[Out] 5/128*a^2*(9*A+2*B)*x-1/56*a^2*(9*A+2*B)*cos(d*x+c)^7/d+5/128*a^2*(9*A+2*B)
*cos(d*x+c)*sin(d*x+c)/d+5/192*a^2*(9*A+2*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/48
*a^2*(9*A+2*B)*cos(d*x+c)^5*sin(d*x+c)/d-1/9*B*cos(d*x+c)^7*(a+a*sin(d*x+c)
)^2/d-1/72*(9*A+2*B)*cos(d*x+c)^7*(a^2+a^2*sin(d*x+c))/d
```

Rubi [A] time = 0.21, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}-\frac{(9A+2B)\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{72d}+\frac{a^2(9A+2B)\sin(c+dx)\cos^5(c+dx)}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (5*a^2*(9*A + 2*B)*x)/128 - (a^2*(9*A + 2*B)*Cos[c + d*x]^7)/(56*d) + (5*a^
2*(9*A + 2*B)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (5*a^2*(9*A + 2*B)*Cos[c
 + d*x]^3*Sin[c + d*x])/(192*d) + (a^2*(9*A + 2*B)*Cos[c + d*x]^5*Sin[c + d
*x])/(48*d) - (B*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2)/(9*d) - ((9*A + 2*B
)*Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x]))/(72*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
```

```
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = -\frac{B\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}+\frac{1}{9}(9A+2B)\int$$

$$= -\frac{B\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}-\frac{(9A+2B)\cos}{}$$

$$= -\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}-\frac{B\cos^7(c+dx)(a+a\sin}{9d}$$

$$= -\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}+\frac{a^2(9A+2B)\cos^5(c+}{48d}$$

$$= -\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}+\frac{5a^2(9A+2B)\cos^3(c+}{192d}$$

$$= -\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}+\frac{5a^2(9A+2B)\cos(c+}{128d}$$

$$= \frac{5}{128}a^2(9A+2B)x-\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}+\frac{5a^2}{}$$

Mathematica [A] time = 5.10, size = 216, normalized size = 1.10

$$-\frac{a^2\cos(c+dx)\left(32(135A+86B)\cos(2(c+dx))+16(108A+59B)\cos(4(c+dx))+\frac{2520(9A+2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/32256*(a^2*Cos[c + d*x]*(2880*A + 1900*B + (2520*(9*A + 2*B)*ArcSin[Sqrt
[1 - Sin[c + d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + 32*(135*A + 86*B)*Cos[2
*(c + d*x)] + 16*(108*A + 59*B)*Cos[4*(c + d*x)] + 288*A*Cos[6*(c + d*x)] +
 64*B*Cos[6*(c + d*x)] - 28*B*Cos[8*(c + d*x)] - 13671*A*Sin[c + d*x] - 247
8*B*Sin[c + d*x] - 2457*A*Sin[3*(c + d*x)] + 462*B*Sin[3*(c + d*x)] - 63*A*
Sin[5*(c + d*x)] + 546*B*Sin[5*(c + d*x)] + 63*A*Sin[7*(c + d*x)] + 126*B*S
in[7*(c + d*x)]))/d
```

fricas [A] time = 0.70, size = 135, normalized size = 0.69

$$\frac{896\,Ba^2\cos(dx+c)^9-2304\,(A+B)a^2\cos(dx+c)^7+315\,(9\,A+2\,B)a^2dx-21\left(48\,(A+2\,B)a^2\cos(dx+c)^7-\right.}{8064\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/8064*(896*B*a^2*cos(d*x + c)^9 - 2304*(A + B)*a^2*cos(d*x + c)^7 + 315*(9
*A + 2*B)*a^2*d*x - 21*(48*(A + 2*B)*a^2*cos(d*x + c)^7 - 8*(9*A + 2*B)*a^2
*cos(d*x + c)^5 - 10*(9*A + 2*B)*a^2*cos(d*x + c)^3 - 15*(9*A + 2*B)*a^2*co
s(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.38, size = 235, normalized size = 1.20

$$\frac{Ba^2\cos(9\,dx+9\,c)}{2304\,d}-\frac{Ba^2\sin(6\,dx+6\,c)}{96\,d}+\frac{5}{128}\left(9\,Aa^2+2\,Ba^2\right)x-\frac{\left(8\,Aa^2+Ba^2\right)\cos(7\,dx+7\,c)}{1792\,d}-\frac{\left(2\,Aa^2+Ba^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2304*B*a^2*cos(9*d*x + 9*c)/d - 1/96*B*a^2*sin(6*d*x + 6*c)/d + 5/128*(9*
A*a^2 + 2*B*a^2)*x - 1/1792*(8*A*a^2 + B*a^2)*cos(7*d*x + 7*c)/d - 1/64*(2*
A*a^2 + B*a^2)*cos(5*d*x + 5*c)/d - 1/192*(18*A*a^2 + 11*B*a^2)*cos(3*d*x +
```

```
3*c)/d - 1/128*(20*A*a^2 + 13*B*a^2)*cos(d*x + c)/d - 1/1024*(A*a^2 + 2*B*
a^2)*sin(8*d*x + 8*c)/d + 1/128*(5*A*a^2 - 2*B*a^2)*sin(4*d*x + 4*c)/d + 1/
32*(8*A*a^2 + B*a^2)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.50, size = 245, normalized size = 1.25

$$\frac{a^2A\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+B\,a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{9}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^
3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)+B*a^2*(-1/9*sin(d*x+c)^2*c
os(d*x+c)^7-2/63*cos(d*x+c)^7)-2/7*a^2*A*cos(d*x+c)^7+2*B*a^2*(-1/8*cos(d*x
+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d
*x+c)+5/128*d*x+5/128*c)+a^2*A*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos
(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)-1/7*B*a^2*cos(d*x+c)^7)
```

maxima [A] time = 0.34, size = 208, normalized size = 1.06

$$-\frac{18432\,Aa^2\cos\left(dx+c\right)^7+9216\,Ba^2\cos\left(dx+c\right)^7-21\left(64\,\sin\left(2\,dx+2\,c\right)^3+120\,dx+120\,c-3\,\sin\left(8\,dx+8\,c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/64512*(18432*A*a^2*cos(d*x + c)^7 + 9216*B*a^2*cos(d*x + c)^7 - 21*(64*s
in(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4
*c))*A*a^2 + 336*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c)
 - 48*sin(2*d*x + 2*c))*A*a^2 - 1024*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*
B*a^2 - 42*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) -
24*sin(4*d*x + 4*c))*B*a^2)/d
```

mupad [B] time = 10.81, size = 622, normalized size = 3.17

$$\frac{5\,a^2\,\mathrm{atan}\left(\frac{5\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(9\,A+2\,B\right)}{64\,\left(\frac{45\,A\,a^2}{64}+\frac{5\,B\,a^2}{32}\right)}\right)\,\left(9\,A+2\,B\right)}{64\,d}-\frac{5\,a^2\,\left(9\,A+2\,B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{64\,d}-\frac{\frac{4\,A\,a^2}{7}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (5*a^2*atan((5*a^2*tan(c/2 + (d*x)/2)*(9*A + 2*B))/(64*((45*A*a^2)/64 + (5*
B*a^2)/32)))*(9*A + 2*B))/(64*d) - (5*a^2*(9*A + 2*B)*(atan(tan(c/2 + (d*x)
/2)) - (d*x)/2))/(64*d) - ((4*A*a^2)/7 - tan(c/2 + (d*x)/2)*((83*A*a^2)/64
- (5*B*a^2)/32) + (22*B*a^2)/63 + tan(c/2 + (d*x)/2)^16*(4*A*a^2 + 2*B*a^2)
 + tan(c/2 + (d*x)/2)^14*(8*A*a^2 + 8*B*a^2) + tan(c/2 + (d*x)/2)^2*((8*A*a
^2)/7 + (8*B*a^2)/7) + tan(c/2 + (d*x)/2)^8*(32*A*a^2 + 4*B*a^2) + tan(c/2
+ (d*x)/2)^6*(24*A*a^2 + 24*B*a^2) + tan(c/2 + (d*x)/2)^12*(24*A*a^2 + (16*
B*a^2)/3) + tan(c/2 + (d*x)/2)^10*(40*A*a^2 + 40*B*a^2) + tan(c/2 + (d*x)/2
)^4*((88*A*a^2)/7 + (32*B*a^2)/7) + tan(c/2 + (d*x)/2)^17*((83*A*a^2)/64 -
(5*B*a^2)/32) - tan(c/2 + (d*x)/2)^5*((149*A*a^2)/32 - (83*B*a^2)/16) + tan
(c/2 + (d*x)/2)^13*((149*A*a^2)/32 - (83*B*a^2)/16) - tan(c/2 + (d*x)/2)^3*
((189*A*a^2)/32 + (191*B*a^2)/48) + tan(c/2 + (d*x)/2)^15*((189*A*a^2)/32 +
 (191*B*a^2)/48) - tan(c/2 + (d*x)/2)^7*((409*A*a^2)/32 + (145*B*a^2)/16) +
 tan(c/2 + (d*x)/2)^11*((409*A*a^2)/32 + (145*B*a^2)/16))/(d*(9*tan(c/2 + (
d*x)/2)^2 + 36*tan(c/2 + (d*x)/2)^4 + 84*tan(c/2 + (d*x)/2)^6 + 126*tan(c/2
 + (d*x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 + 84*tan(c/2 + (d*x)/2)^12 + 36*t
an(c/2 + (d*x)/2)^14 + 9*tan(c/2 + (d*x)/2)^16 + tan(c/2 + (d*x)/2)^18 + 1)
)
```

sympy [A] time = 17.40, size = 719, normalized size = 3.67

$$\begin{cases} \frac{5Aa^2x\sin^8(c+dx)}{128} + \frac{5Aa^2x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{5Aa^2x\sin^6(c+dx)}{16} + \frac{15Aa^2x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{15Aa^2x\sin^4(c+dx)\cos^2(c+dx}{16} \\ x(A+B\sin(c))(a\sin(c)+a)^2\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*A*a**2*x*sin(c + d*x)**8/128 + 5*A*a**2*x*sin(c + d*x)**6*cos(
c + d*x)**2/32 + 5*A*a**2*x*sin(c + d*x)**6/16 + 15*A*a**2*x*sin(c + d*x)**
4*cos(c + d*x)**4/64 + 15*A*a**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 5*A
*a**2*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 15*A*a**2*x*sin(c + d*x)**2*co
s(c + d*x)**4/16 + 5*A*a**2*x*cos(c + d*x)**8/128 + 5*A*a**2*x*cos(c + d*x)
**6/16 + 5*A*a**2*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*A*a**2*sin(c +
d*x)**5*cos(c + d*x)**3/(384*d) + 5*A*a**2*sin(c + d*x)**5*cos(c + d*x)/(16
*d) + 73*A*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 5*A*a**2*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) - 5*A*a**2*sin(c + d*x)*cos(c + d*x)**7/(128*
d) + 11*A*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 2*A*a**2*cos(c + d*x)*
*7/(7*d) + 5*B*a**2*x*sin(c + d*x)**8/64 + 5*B*a**2*x*sin(c + d*x)**6*cos(c
 + d*x)**2/16 + 15*B*a**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 5*B*a**2*x
```

```
*sin(c + d*x)**2*cos(c + d*x)**6/16 + 5*B*a**2*x*cos(c + d*x)**8/64 + 5*B*a
**2*sin(c + d*x)**7*cos(c + d*x)/(64*d) + 55*B*a**2*sin(c + d*x)**5*cos(c +
 d*x)**3/(192*d) + 73*B*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(192*d) - B*a*
*2*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 5*B*a**2*sin(c + d*x)*cos(c + d*
x)**7/(64*d) - 2*B*a**2*cos(c + d*x)**9/(63*d) - B*a**2*cos(c + d*x)**7/(7*
d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**2*cos(c)**6, True))
```

3.978 $\int \cos^4(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=165

$$-\frac{a^2(7A+2B)\cos^5(c+dx)}{30d}-\frac{(7A+2B)\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{42d}+\frac{a^2(7A+2B)\sin(c+dx)\cos^3(c+dx)}{24d}$$

```
[Out] 1/16*a^2*(7*A+2*B)*x-1/30*a^2*(7*A+2*B)*cos(d*x+c)^5/d+1/16*a^2*(7*A+2*B)*c
os(d*x+c)*sin(d*x+c)/d+1/24*a^2*(7*A+2*B)*cos(d*x+c)^3*sin(d*x+c)/d-1/7*B*c
os(d*x+c)^5*(a+a*sin(d*x+c))^2/d-1/42*(7*A+2*B)*cos(d*x+c)^5*(a^2+a^2*sin(d
*x+c))/d
```

Rubi [A] time = 0.19, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{a^2(7A+2B)\cos^5(c+dx)}{30d}-\frac{(7A+2B)\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{42d}+\frac{a^2(7A+2B)\sin(c+dx)\cos^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(7*A + 2*B)*x)/16 - (a^2*(7*A + 2*B)*Cos[c + d*x]^5)/(30*d) + (a^2*(7*
A + 2*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(7*A + 2*B)*Cos[c + d*x]^
3*Sin[c + d*x])/(24*d) - (B*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2)/(7*d) -
((7*A + 2*B)*Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x]))/(42*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = -\frac{B\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{1}{7}(7A+2B)\int$$

$$= -\frac{B\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}-\frac{(7A+2B)\cos}{}$$

$$= -\frac{a^2(7A+2B)\cos^5(c+dx)}{30d}-\frac{B\cos^5(c+dx)(a+a\,\mathrm{si}}{7d}$$

$$= -\frac{a^2(7A+2B)\cos^5(c+dx)}{30d}+\frac{a^2(7A+2B)\cos^3(c+}{24d}$$

$$= -\frac{a^2(7A+2B)\cos^5(c+dx)}{30d}+\frac{a^2(7A+2B)\cos(c+d}{16d}$$

$$= \frac{1}{16}a^2(7A+2B)x-\frac{a^2(7A+2B)\cos^5(c+dx)}{30d}+\frac{a^2(7}{}$$

Mathematica [A] time = 1.73, size = 171, normalized size = 1.04

$$-\frac{a^2\cos(c+dx)\left((672A+447B)\cos(2(c+dx))+6(28A+13B)\cos(4(c+dx))+\frac{420(7A+2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}-164\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/3360*(a^2*Cos[c + d*x]*(504*A + 354*B + (420*(7*A + 2*B)*ArcSin[Sqrt[1 -
 Sin[c + d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + (672*A + 447*B)*Cos[2*(c +
d*x)] + 6*(28*A + 13*B)*Cos[4*(c + d*x)] - 15*B*Cos[6*(c + d*x)] - 1645*A*S
in[c + d*x] - 350*B*Sin[c + d*x] - 140*A*Sin[3*(c + d*x)] + 140*B*Sin[3*(c
+ d*x)] + 35*A*Sin[5*(c + d*x)] + 70*B*Sin[5*(c + d*x)]))/d
```

fricas [A] time = 0.74, size = 115, normalized size = 0.70

$$\frac{240\,Ba^2\cos(dx+c)^7-672\,(A+B)a^2\cos(dx+c)^5+105\,(7\,A+2\,B)a^2dx-35\left(8\,(A+2\,B)a^2\cos(dx+c)^5-2\right.}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/1680*(240*B*a^2*cos(d*x + c)^7 - 672*(A + B)*a^2*cos(d*x + c)^5 + 105*(7*
A + 2*B)*a^2*d*x - 35*(8*(A + 2*B)*a^2*cos(d*x + c)^5 - 2*(7*A + 2*B)*a^2*c
os(d*x + c)^3 - 3*(7*A + 2*B)*a^2*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.30, size = 192, normalized size = 1.16

$$\frac{Ba^2\cos(7\,dx+7\,c)}{448\,d}+\frac{1}{16}\left(7\,Aa^2+2\,Ba^2\right)x-\frac{\left(8\,Aa^2+3\,Ba^2\right)\cos(5\,dx+5\,c)}{320\,d}-\frac{\left(8\,Aa^2+5\,Ba^2\right)\cos(3\,dx+3\,c)}{64\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/448*B*a^2*cos(7*d*x + 7*c)/d + 1/16*(7*A*a^2 + 2*B*a^2)*x - 1/320*(8*A*a^
2 + 3*B*a^2)*cos(5*d*x + 5*c)/d - 1/64*(8*A*a^2 + 5*B*a^2)*cos(3*d*x + 3*c)
/d - 1/64*(16*A*a^2 + 11*B*a^2)*cos(d*x + c)/d - 1/192*(A*a^2 + 2*B*a^2)*si
n(6*d*x + 6*c)/d + 1/64*(A*a^2 - 2*B*a^2)*sin(4*d*x + 4*c)/d + 1/64*(17*A*a
^2 + 2*B*a^2)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.49, size = 215, normalized size = 1.30

$$\frac{a^2A\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+B\,a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))
*sin(d*x+c)+1/16*d*x+1/16*c)+B*a^2*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos
(d*x+c)^5)-2/5*a^2*A*cos(d*x+c)^5+2*B*a^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/2
4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+a^2*A*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-1/5*B*a^2*cos(d*x+c)^5)
```

maxima [A] time = 0.45, size = 171, normalized size = 1.04

$$-\frac{2688\,Aa^2\cos\left(dx+c\right)^5+1344\,Ba^2\cos\left(dx+c\right)^5-35\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)Aa^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/6720*(2688*A*a^2*cos(d*x + c)^5 + 1344*B*a^2*cos(d*x + c)^5 - 35*(4*sin(
2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*A*a^2 - 210*(12*d*x +
12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 192*(5*cos(d*x + c)^7
 - 7*cos(d*x + c)^5)*B*a^2 - 70*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*s
in(4*d*x + 4*c))*B*a^2)/d
```

mupad [B] time = 11.00, size = 494, normalized size = 2.99

$$\frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(7\,A+2\,B\right)}{8\,\left(\frac{7\,A\,a^2}{8}+\frac{B\,a^2}{4}\right)}\right)\,\left(7\,A+2\,B\right)}{8\,d}-\frac{a^2\,\left(7\,A+2\,B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{8\,d}-\frac{\frac{4\,A\,a^2}{5}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{9\,A}{8}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*atan((a^2*tan(c/2 + (d*x)/2)*(7*A + 2*B))/(8*((7*A*a^2)/8 + (B*a^2)/4)
))*(7*A + 2*B))/(8*d) - (a^2*(7*A + 2*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/
2))/(8*d) - ((4*A*a^2)/5 - tan(c/2 + (d*x)/2)*((9*A*a^2)/8 - (B*a^2)/4) + (
18*B*a^2)/35 + tan(c/2 + (d*x)/2)^12*(4*A*a^2 + 2*B*a^2) + tan(c/2 + (d*x)/
2)^8*(12*A*a^2 + 2*B*a^2) + tan(c/2 + (d*x)/2)^10*(8*A*a^2 + 8*B*a^2) + tan
(c/2 + (d*x)/2)^2*((8*A*a^2)/5 + (8*B*a^2)/5) + tan(c/2 + (d*x)/2)^13*((9*A
*a^2)/8 - (B*a^2)/4) + tan(c/2 + (d*x)/2)^6*(16*A*a^2 + 16*B*a^2) - tan(c/2
 + (d*x)/2)^3*((29*A*a^2)/6 + (11*B*a^2)/3) + tan(c/2 + (d*x)/2)^11*((29*A*
a^2)/6 + (11*B*a^2)/3) + tan(c/2 + (d*x)/2)^4*((44*A*a^2)/5 + (14*B*a^2)/5)
 - tan(c/2 + (d*x)/2)^5*((23*A*a^2)/24 - (31*B*a^2)/12) + tan(c/2 + (d*x)/2
```

```
)^9*((23*A*a^2)/24 - (31*B*a^2)/12))/(d*(7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/
2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan
(c/2 + (d*x)/2)^10 + 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 + 1))
```

sympy [A] time = 6.70, size = 539, normalized size = 3.27

$$\begin{cases} \frac{Aa^2x\sin^6(c+dx)}{16} + \frac{3Aa^2x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3Aa^2x\sin^4(c+dx)}{8} + \frac{3Aa^2x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3Aa^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \\ x(A+B\sin(c))(a\sin(c)+a)^2\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**6/16 + 3*A*a**2*x*sin(c + d*x)**4*cos(c +
 d*x)**2/16 + 3*A*a**2*x*sin(c + d*x)**4/8 + 3*A*a**2*x*sin(c + d*x)**2*cos
(c + d*x)**4/16 + 3*A*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + A*a**2*x*c
os(c + d*x)**6/16 + 3*A*a**2*x*cos(c + d*x)**4/8 + A*a**2*sin(c + d*x)**5*c
os(c + d*x)/(16*d) + A*a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 3*A*a**
2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - A*a**2*sin(c + d*x)*cos(c + d*x)**5/
(16*d) + 5*A*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*A*a**2*cos(c + d*x
)**5/(5*d) + B*a**2*x*sin(c + d*x)**6/8 + 3*B*a**2*x*sin(c + d*x)**4*cos(c
+ d*x)**2/8 + 3*B*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + B*a**2*x*cos(c
 + d*x)**6/8 + B*a**2*sin(c + d*x)**5*cos(c + d*x)/(8*d) + B*a**2*sin(c + d
*x)**3*cos(c + d*x)**3/(3*d) - B*a**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d)
 - B*a**2*sin(c + d*x)*cos(c + d*x)**5/(8*d) - 2*B*a**2*cos(c + d*x)**7/(35
*d) - B*a**2*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c)
+ a)**2*cos(c)**4, True))
```

3.979 $\int \cos^2(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$-\frac{a^2(5A+2B)\cos^3(c+dx)}{12d}-\frac{(5A+2B)\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{20d}+\frac{a^2(5A+2B)\sin(c+dx)\cos(c+dx)}{8d}$$

```
[Out] 1/8*a^2*(5*A+2*B)*x-1/12*a^2*(5*A+2*B)*cos(d*x+c)^3/d+1/8*a^2*(5*A+2*B)*cos
(d*x+c)*sin(d*x+c)/d-1/5*B*cos(d*x+c)^3*(a+a*sin(d*x+c))^2/d-1/20*(5*A+2*B)
*cos(d*x+c)^3*(a^2+a^2*sin(d*x+c))/d
```

Rubi [A] time = 0.17, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{a^2(5A+2B)\cos^3(c+dx)}{12d}-\frac{(5A+2B)\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{20d}+\frac{a^2(5A+2B)\sin(c+dx)\cos(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(5*A + 2*B)*x)/8 - (a^2*(5*A + 2*B)*Cos[c + d*x]^3)/(12*d) + (a^2*(5*A
 + 2*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (B*Cos[c + d*x]^3*(a + a*Sin[c +
 d*x])^2)/(5*d) - ((5*A + 2*B)*Cos[c + d*x]^3*(a^2 + a^2*Sin[c + d*x]))/(20
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = -\frac{B\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}+\frac{1}{5}(5A+2B)$$

$$= -\frac{B\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}-\frac{(5A+2B)\,\mathrm{c}}{}$$

$$= -\frac{a^2(5A+2B)\cos^3(c+dx)}{12d}-\frac{B\cos^3(c+dx)(a+a}{5d}$$

$$= -\frac{a^2(5A+2B)\cos^3(c+dx)}{12d}+\frac{a^2(5A+2B)\cos(c+}{8d}$$

$$= \frac{1}{8}a^2(5A+2B)x-\frac{a^2(5A+2B)\cos^3(c+dx)}{12d}+\frac{a^2(5}{}$$

Mathematica [A] time = 0.98, size = 133, normalized size = 0.99

$$-\frac{a^2\cos(c+dx)\left(8(10A+7B)\cos(2(c+dx))+\frac{60(5A+2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}-135A\sin(c+dx)+15A\sin(3(c+dx\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/240*(a^2*Cos[c + d*x]*(80*A + 62*B + (60*(5*A + 2*B)*ArcSin[Sqrt[1 - Sin
[c + d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + 8*(10*A + 7*B)*Cos[2*(c + d*x)]
 - 6*B*Cos[4*(c + d*x)] - 135*A*Sin[c + d*x] - 30*B*Sin[c + d*x] + 15*A*Sin
[3*(c + d*x)] + 30*B*Sin[3*(c + d*x)]))/d
```

fricas [A] time = 0.68, size = 95, normalized size = 0.71

$$\frac{24\,Ba^2\cos\left(dx+c\right)^5-80\,(A+B)a^2\cos\left(dx+c\right)^3+15\,(5\,A+2\,B)a^2dx-15\,\Big(2\,(A+2\,B)a^2\cos\left(dx+c\right)^3-(5\,A+}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(24*B*a^2*cos(d*x + c)^5 - 80*(A + B)*a^2*cos(d*x + c)^3 + 15*(5*A +
2*B)*a^2*d*x - 15*(2*(A + 2*B)*a^2*cos(d*x + c)^3 - (5*A + 2*B)*a^2*cos(d*x
 + c))*sin(d*x + c))/d
```

giac [A] time = 0.23, size = 130, normalized size = 0.97

$$\frac{Ba^2\cos\left(5\,dx+5\,c\right)}{80\,d}+\frac{Aa^2\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{1}{8}\left(5\,Aa^2+2\,Ba^2\right)x-\frac{\left(8\,Aa^2+5\,Ba^2\right)\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{\left(4\,Aa^2+3\,Ba^2\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/80*B*a^2*cos(5*d*x + 5*c)/d + 1/4*A*a^2*sin(2*d*x + 2*c)/d + 1/8*(5*A*a^2
 + 2*B*a^2)*x - 1/48*(8*A*a^2 + 5*B*a^2)*cos(3*d*x + 3*c)/d - 1/8*(4*A*a^2
+ 3*B*a^2)*cos(d*x + c)/d - 1/32*(A*a^2 + 2*B*a^2)*sin(4*d*x + 4*c)/d
```

maple [A] time = 0.39, size = 182, normalized size = 1.36

$$\frac{a^2A\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+B\,a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)-\frac{2a^2A\left(\cos^3(dx\right.}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+
1/8*c)+B*a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)-2/3*a^2*A*c
```

```
os(d*x+c)^3+2*B*a^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)
+1/8*d*x+1/8*c)+a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-1/3*B*a^2*c
os(d*x+c)^3)
```

maxima [A] time = 0.33, size = 134, normalized size = 1.00

$$-\frac{320\,Aa^2\cos(dx+c)^3+160\,Ba^2\cos(dx+c)^3-15\,(4\,dx+4\,c-\sin(4\,dx+4\,c))Aa^2-120\,(2\,dx+2\,c+\sin(2}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/480*(320*A*a^2*cos(d*x + c)^3 + 160*B*a^2*cos(d*x + c)^3 - 15*(4*d*x + 4
*c - sin(4*d*x + 4*c))*A*a^2 - 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 -
 32*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*B*a^2 - 30*(4*d*x + 4*c - sin(4*d
*x + 4*c))*B*a^2)/d
```

mupad [B] time = 10.47, size = 367, normalized size = 2.74

$$\frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(5\,A+2\,B)}{4\left(\frac{5\,A\,a^2}{4}+\frac{B\,a^2}{2}\right)}\right)\,(5\,A+2\,B)}{4\,d}-\frac{a^2\,(5\,A+2\,B)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{4\,d}-\frac{\frac{4\,A\,a^2}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*atan((a^2*tan(c/2 + (d*x)/2)*(5*A + 2*B))/(4*((5*A*a^2)/4 + (B*a^2)/2)
))*(5*A + 2*B))/(4*d) - (a^2*(5*A + 2*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/
2))/(4*d) - ((4*A*a^2)/3 - tan(c/2 + (d*x)/2)*((3*A*a^2)/4 - (B*a^2)/2) + (
14*B*a^2)/15 + tan(c/2 + (d*x)/2)^8*(4*A*a^2 + 2*B*a^2) - tan(c/2 + (d*x)/2
)^3*((7*A*a^2)/2 + 3*B*a^2) + tan(c/2 + (d*x)/2)^7*((7*A*a^2)/2 + 3*B*a^2)
+ tan(c/2 + (d*x)/2)^9*((3*A*a^2)/4 - (B*a^2)/2) + tan(c/2 + (d*x)/2)^6*(8*
A*a^2 + 8*B*a^2) + tan(c/2 + (d*x)/2)^2*((8*A*a^2)/3 + (8*B*a^2)/3) + tan(c
/2 + (d*x)/2)^4*((16*A*a^2)/3 + (4*B*a^2)/3))/(d*(5*tan(c/2 + (d*x)/2)^2 +
10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8
+ tan(c/2 + (d*x)/2)^10 + 1))
```

sympy [A] time = 2.43, size = 371, normalized size = 2.77

$$\begin{cases} \frac{Aa^2x\sin^4(c+dx)}{8}+\frac{Aa^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{Aa^2x\sin^2(c+dx)}{2}+\frac{Aa^2x\cos^4(c+dx)}{8}+\frac{Aa^2x\cos^2(c+dx)}{2}+\frac{Aa^2\sin^3(c+dx)\cos(c+dx)}{8d} \\ x\,(A+B\sin(c))\,(a\sin(c)+a)^2\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**4/8 + A*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + A*a**2*x*sin(c + d*x)**2/2 + A*a**2*x*cos(c + d*x)**4/8 + A*a**2*
x*cos(c + d*x)**2/2 + A*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - A*a**2*si
n(c + d*x)*cos(c + d*x)**3/(8*d) + A*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) -
 2*A*a**2*cos(c + d*x)**3/(3*d) + B*a**2*x*sin(c + d*x)**4/4 + B*a**2*x*sin
(c + d*x)**2*cos(c + d*x)**2/2 + B*a**2*x*cos(c + d*x)**4/4 + B*a**2*sin(c
+ d*x)**3*cos(c + d*x)/(4*d) - B*a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*d)
 - B*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) - 2*B*a**2*cos(c + d*x)**5/(15
*d) - B*a**2*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c)
+ a)**2*cos(c)**2, True))
```

3.980 $\int \sec^2(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=55

$$\frac{a^2(A+2B)\cos(c+dx)}{d} - \left(a^2x(A+2B)\right) + \frac{(A+B)\sec(c+dx)(a\sin(c+dx)+a)^2}{d}$$

```
[Out] -a^2*(A+2*B)*x+a^2*(A+2*B)*cos(d*x+c)/d+(A+B)*sec(d*x+c)*(a+a*sin(d*x+c))^2
/d
```

Rubi [A] time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2855, 2638}

$$\frac{a^2(A+2B)\cos(c+dx)}{d} + a^2x(-(A+2B)) + \frac{(A+B)\sec(c+dx)(a\sin(c+dx)+a)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^2*(A + 2*B)*x) + (a^2*(A + 2*B)*Cos[c + d*x])/d + ((A + B)*Sec[c + d*x]
*(a + a*Sin[c + d*x])^2)/d
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec(c+dx)(a+a\sin(c+dx))^2}{d} - (a(A+2B)$$
$$= -a^2(A+2B)x + \frac{(A+B)\sec(c+dx)(a+a\sin(c+dx)}{d}$$
$$= -a^2(A+2B)x + \frac{a^2(A+2B)\cos(c+dx)}{d} + \frac{(A+B)s}{}$$

Mathematica [A] time = 0.24, size = 91, normalized size = 1.65

$$\frac{a^2\sec(c+dx)\left(4(A+2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{\cos^2(c+dx)}+4A\sin(c+dx)+4A+4B\sin(c+dx)+B\cos(2(\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*Sec[c + d*x]*(4*A + 5*B + 4*(A + 2*B)*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sq
rt[2]]*Sqrt[Cos[c + d*x]^2] + B*Cos[2*(c + d*x)] + 4*A*Sin[c + d*x] + 4*B*S
in[c + d*x]))/(2*d)
```

fricas [B] time = 0.68, size = 128, normalized size = 2.33

$$-\frac{(A+2B)a^2dx - Ba^2\cos(dx+c)^2 - 2(A+B)a^2 + \left((A+2B)a^2dx - (2A+3B)a^2\right)\cos(dx+c) - \left((A+2B)a^2\right.}{d\cos(dx+c) - d\sin(dx+c) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -((A + 2*B)*a^2*d*x - B*a^2*cos(d*x + c)^2 - 2*(A + B)*a^2 + ((A + 2*B)*a^2
*d*x - (2*A + 3*B)*a^2)*cos(d*x + c) - ((A + 2*B)*a^2*d*x - B*a^2*cos(d*x +
 c) + 2*(A + B)*a^2)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [B] time = 0.20, size = 125, normalized size = 2.27

$$-\frac{\left(Aa^2+2Ba^2\right)(dx+c)+\frac{2\left(2Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2Aa^2+3Ba^2\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -((A*a^2 + 2*B*a^2)*(d*x + c) + 2*(2*A*a^2*tan(1/2*d*x + 1/2*c)^2 + 2*B*a^2
*tan(1/2*d*x + 1/2*c)^2 - B*a^2*tan(1/2*d*x + 1/2*c) + 2*A*a^2 + 3*B*a^2)/(
tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) - 1)
)/d
```

maple [B] time = 0.61, size = 123, normalized size = 2.24

$$\frac{a^2 A \left(\tan \left(dx+c\right)-dx-c\right)+B\,a^2 \left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2\left(dx+c\right)\right)\cos\left(dx+c\right)\right)+\frac{2a^2 A}{\cos(dx+c)}+2B\,a^2\left(\tan\left(dx+c\right)\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(tan(d*x+c)-d*x-c)+B*a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^
2)*cos(d*x+c))+2*a^2*A/cos(d*x+c)+2*B*a^2*(tan(d*x+c)-d*x-c)+a^2*A*tan(d*x+
c)+B*a^2/cos(d*x+c))
```

maxima [A] time = 0.50, size = 104, normalized size = 1.89

$$-\frac{(dx+c-\tan\left(dx+c\right))Aa^2+2\left(dx+c-\tan\left(dx+c\right)\right)Ba^2-Ba^2\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)-Aa^2\tan\left(dx+c\right)-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -((d*x + c - tan(d*x + c))*A*a^2 + 2*(d*x + c - tan(d*x + c))*B*a^2 - B*a^2
*(1/cos(d*x + c) + cos(d*x + c)) - A*a^2*tan(d*x + c) - 2*A*a^2/cos(d*x + c
) - B*a^2/cos(d*x + c))/d
```

mupad [B] time = 9.30, size = 110, normalized size = 2.00

$$-\frac{4\,A\,a^2+6\,B\,a^2+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(4\,A\,a^2+4\,B\,a^2\right)-2\,B\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}-A\,a^2\,x-2\,B\,a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] - (4*A*a^2 + 6*B*a^2 + tan(c/2 + (d*x)/2)^2*(4*A*a^2 + 4*B*a^2) - 2*B*a^2*t
an(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2 + tan(c/2
+ (d*x)/2)^3 - 1)) - A*a^2*x - 2*B*a^2*x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sec^2(c+dx)\,dx+\int 2A\sin(c+dx)\sec^2(c+dx)\,dx+\int A\sin^2(c+dx)\sec^2(c+dx)\,dx+\int B\sin(c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**2, x) + Integral(2*A*sin(c + d*x)*sec(c + d*
x)**2, x) + Integral(A*sin(c + d*x)**2*sec(c + d*x)**2, x) + Integral(B*sin
(c + d*x)*sec(c + d*x)**2, x) + Integral(2*B*sin(c + d*x)**2*sec(c + d*x)**
2, x) + Integral(B*sin(c + d*x)**3*sec(c + d*x)**2, x))
```

3.981 $\int \sec^4(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=73

$$\frac{a^2(A-2B)\tan(c+dx)}{3d}+\frac{a^2(A-2B)\sec(c+dx)}{3d}+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)^2}{3d}$$

```
[Out] 1/3*a^2*(A-2*B)*sec(d*x+c)/d+1/3*(A+B)*sec(d*x+c)^3*(a+a*sin(d*x+c))^2/d+1/
3*a^2*(A-2*B)*tan(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2855, 2669, 3767, 8}

$$\frac{a^2(A-2B)\tan(c+dx)}{3d}+\frac{a^2(A-2B)\sec(c+dx)}{3d}+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(A - 2*B)*Sec[c + d*x])/(3*d) + ((A + B)*Sec[c + d*x]^3*(a + a*Sin[c +
 d*x])^2)/(3*d) + (a^2*(A - 2*B)*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} + \frac{1}{3}(a(A-$$
$$= \frac{a^2(A-2B)\sec(c+dx)}{3d} + \frac{(A+B)\sec^3(c+dx)(a+a}{3d}$$
$$= \frac{a^2(A-2B)\sec(c+dx)}{3d} + \frac{(A+B)\sec^3(c+dx)(a+a}{3d}$$
$$= \frac{a^2(A-2B)\sec(c+dx)}{3d} + \frac{(A+B)\sec^3(c+dx)(a+a}{3d}$$

Mathematica [A] time = 0.02, size = 121, normalized size = 1.66

$$-\frac{a^2A\tan^3(c+dx)}{3d}+\frac{2a^2A\sec^3(c+dx)}{3d}+\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{d}+\frac{2a^2B\tan^3(c+dx)}{3d}-\frac{a^2B\sec^3(c+dx)}{3d}+\frac{a^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (2*a^2*A*Sec[c + d*x]^3)/(3*d) - (a^2*B*Sec[c + d*x]^3)/(3*d) + (a^2*A*Sec[
c + d*x]^2*Tan[c + d*x])/d + (a^2*B*Sec[c + d*x]*Tan[c + d*x]^2)/d - (a^2*A
*Tan[c + d*x]^3)/(3*d) + (2*a^2*B*Tan[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.49, size = 120, normalized size = 1.64

$$-\frac{(A-2B)a^2\cos(dx+c)^2+(2A-B)a^2\cos(dx+c)+(A+B)a^2-\left((A-2B)a^2\cos(dx+c)-(A+B)a^2\right)\sin(}{3\left(d\cos(dx+c)^2-d\cos(dx+c)+(d\cos(dx+c)+2d)\sin(dx+c)-2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/3*((A - 2*B)*a^2*cos(d*x + c)^2 + (2*A - B)*a^2*cos(d*x + c) + (A + B)*a
^2 - ((A - 2*B)*a^2*cos(d*x + c) - (A + B)*a^2)*sin(d*x + c))/(d*cos(d*x +
c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.20, size = 78, normalized size = 1.07

$$-\frac{2 \left(3 \, A a^{2} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - 3 \, A a^{2} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 3 \, B a^{2} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 2 \, A a^{2} - B a^{2}\right)}{3 \, d {\left(\tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 1\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -2/3*(3*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a
^2*tan(1/2*d*x + 1/2*c) + 2*A*a^2 - B*a^2)/(d*(tan(1/2*d*x + 1/2*c) - 1)^3)
```

maple [B] time = 0.66, size = 162, normalized size = 2.22

$$\frac{\frac{a^{2} A \left(\sin^{3}\left(d x +c \right)\right)}{3 \cos \left(d x +c \right)^{3}}+B \,a^{2} \left(\frac{\sin^{4}\left(d x +c \right)}{3 \cos \left(d x +c \right)^{3}}-\frac{\sin^{4}\left(d x +c \right)}{3 \cos \left(d x +c \right)}-\frac{\left(2+\sin^{2}\left(d x +c \right)\right) \cos \left(d x +c \right)}{3}\right)+\frac{2 a^{2} A}{3 \cos \left(d x +c \right)^{3}}+\frac{2 B \,a^{2} \left(\sin^{3}\left(d x +c \right)\right)}{3 \cos \left(d x +c \right)^{3}}-a^{2} A \left(-\frac{2}{3}-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a^2*A*sin(d*x+c)^3/cos(d*x+c)^3+B*a^2*(1/3*sin(d*x+c)^4/cos(d*x+c)
^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+2/3*a^2*A/c
os(d*x+c)^3+2/3*B*a^2*sin(d*x+c)^3/cos(d*x+c)^3-a^2*A*(-2/3-1/3*sec(d*x+c)^
2)*tan(d*x+c)+1/3*B*a^2/cos(d*x+c)^3)
```

maxima [A] time = 0.44, size = 108, normalized size = 1.48

$$\frac{A a^{2} \tan \left(d x + c\right)^{3} + 2 \, B a^{2} \tan \left(d x + c\right)^{3} + {\left(\tan \left(d x + c\right)^{3} + 3 \, \tan \left(d x + c\right)\right)} A a^{2} - \frac{{\left(3 \, \cos \left(d x + c\right)^{2} - 1\right)} B a^{2}}{\cos \left(d x + c\right)^{3}} + \frac{2 \, A a^{2}}{\cos \left(d x + c\right)^{3}} +}{3 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/3*(A*a^2*tan(d*x + c)^3 + 2*B*a^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*ta
n(d*x + c))*A*a^2 - (3*cos(d*x + c)^2 - 1)*B*a^2/cos(d*x + c)^3 + 2*A*a^2/c
os(d*x + c)^3 + B*a^2/cos(d*x + c)^3)/d
```

mupad [B] time = 9.15, size = 77, normalized size = 1.05

$$-\frac{\sqrt{2}\,a^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{B}{2}-\frac{5\,A}{2}+\frac{A\,\cos(c+d\,x)}{2}+\frac{B\,\cos(c+d\,x)}{2}+\frac{3\,A\,\sin(c+d\,x)}{2}-\frac{3\,B\,\sin(c+d\,x)}{2}\right)}{6\,d\,\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^4,x)
```

```
[Out] -(2^(1/2)*a^2*cos(c/2 + (d*x)/2)*(B/2 - (5*A)/2 + (A*cos(c + d*x))/2 + (B*c
os(c + d*x))/2 + (3*A*sin(c + d*x))/2 - (3*B*sin(c + d*x))/2))/(6*d*cos(c/2
 + pi/4 + (d*x)/2)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.982 $\int \sec^6(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=104

$$\frac{a^2(3A-2B)\tan^3(c+dx)}{15d}+\frac{a^2(3A-2B)\tan(c+dx)}{5d}+\frac{a^2(3A-2B)\sec^3(c+dx)}{15d}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+$$

```
[Out] 1/15*a^2*(3*A-2*B)*sec(d*x+c)^3/d+1/5*(A+B)*sec(d*x+c)^5*(a+a*sin(d*x+c))^2
/d+1/5*a^2*(3*A-2*B)*tan(d*x+c)/d+1/15*a^2*(3*A-2*B)*tan(d*x+c)^3/d
```

Rubi [A] time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2855, 2669, 3767}

$$\frac{a^2(3A-2B)\tan^3(c+dx)}{15d}+\frac{a^2(3A-2B)\tan(c+dx)}{5d}+\frac{a^2(3A-2B)\sec^3(c+dx)}{15d}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(3*A - 2*B)*Sec[c + d*x]^3)/(15*d) + ((A + B)*Sec[c + d*x]^5*(a + a*Si
n[c + d*x])^2)/(5*d) + (a^2*(3*A - 2*B)*Tan[c + d*x])/(5*d) + (a^2*(3*A - 2
*B)*Tan[c + d*x]^3)/(15*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))^2}{5d} + \frac{1}{5}(a(3A-$$
$$= \frac{a^2(3A-2B)\sec^3(c+dx)}{15d} + \frac{(A+B)\sec^5(c+dx)(a+}{5d}$$
$$= \frac{a^2(3A-2B)\sec^3(c+dx)}{15d} + \frac{(A+B)\sec^5(c+dx)(a+}{5d}$$
$$= \frac{a^2(3A-2B)\sec^3(c+dx)}{15d} + \frac{(A+B)\sec^5(c+dx)(a+}{5d}$$

Mathematica [A] time = 0.02, size = 178, normalized size = 1.71

$$\frac{2a^2A\tan^5(c+dx)}{5d}+\frac{2a^2A\sec^5(c+dx)}{5d}+\frac{a^2A\tan(c+dx)\sec^4(c+dx)}{d}-\frac{a^2A\tan^3(c+dx)\sec^2(c+dx)}{d}-\frac{4a^2B\tan^5}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (2*a^2*A*Sec[c + d*x]^5)/(5*d) + (a^2*B*Sec[c + d*x]^5)/(15*d) + (a^2*A*Sec
[c + d*x]^4*Tan[c + d*x])/d + (a^2*B*Sec[c + d*x]^3*Tan[c + d*x]^2)/(3*d) -
 (a^2*A*Sec[c + d*x]^2*Tan[c + d*x]^3)/d + (2*a^2*B*Sec[c + d*x]^2*Tan[c +
d*x]^3)/(3*d) + (2*a^2*A*Tan[c + d*x]^5)/(5*d) - (4*a^2*B*Tan[c + d*x]^5)/(
15*d)
```

fricas [A] time = 0.68, size = 113, normalized size = 1.09

$$-\frac{4\,(3\,A-2\,B)a^2\cos{(dx+c)}^2-3\,(2\,A-3\,B)a^2-\left(2\,(3\,A-2\,B)a^2\cos{(dx+c)}^2-3\,(3\,A-2\,B)a^2\right)\sin{(dx+c)}}{15\left(d\cos{(dx+c)}^3+2\,d\cos{(dx+c)}\sin{(dx+c)}-2\,d\cos{(dx+c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/15*(4*(3*A - 2*B)*a^2*cos(d*x + c)^2 - 3*(2*A - 3*B)*a^2 - (2*(3*A - 2*B
)*a^2*cos(d*x + c)^2 - 3*(3*A - 2*B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^3 +
 2*d*cos(d*x + c)*sin(d*x + c) - 2*d*cos(d*x + c))
```

giac [A] time = 0.21, size = 192, normalized size = 1.85

$$-\frac{\frac{15\left(Aa^2-Ba^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{105\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+15\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-270\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+30\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+360\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/60*(15*(A*a^2 - B*a^2)/(tan(1/2*d*x + 1/2*c) + 1) + (105*A*a^2*tan(1/2*d
*x + 1/2*c)^4 + 15*B*a^2*tan(1/2*d*x + 1/2*c)^4 - 270*A*a^2*tan(1/2*d*x + 1
/2*c)^3 + 30*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 360*A*a^2*tan(1/2*d*x + 1/2*c)^
2 - 40*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 210*A*a^2*tan(1/2*d*x + 1/2*c) + 50*B
*a^2*tan(1/2*d*x + 1/2*c) + 63*A*a^2 - 7*B*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^
5)/d
```

maple [B] time = 0.64, size = 231, normalized size = 2.22

$$\frac{a^2A\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+B\,a^2\left(\frac{\sin^4(dx+c)}{5\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{15\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{15\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{15}\right)+\frac{2a^2A}{5\cos(dx+c)^5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+B
*a^2*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/15*sin
(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+2/5*a^2*A/cos(d*x+c)
^5+2*B*a^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)-a
^2*A*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/5*B*a^2/cos(d*
x+c)^5)
```

maxima [A] time = 0.48, size = 147, normalized size = 1.41

$$\frac{\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)Aa^2+\left(3\,\tan\left(dx+c\right)^5+5\,\tan\left(dx+c\right)^3\right)Aa^2+2\left(3\,\tan\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^2 + (3*t
an(d*x + c)^5 + 5*tan(d*x + c)^3)*A*a^2 + 2*(3*tan(d*x + c)^5 + 5*tan(d*x +
 c)^3)*B*a^2 - (5*cos(d*x + c)^2 - 3)*B*a^2/cos(d*x + c)^5 + 6*A*a^2/cos(d*
x + c)^5 + 3*B*a^2/cos(d*x + c)^5)/d
```

mupad [B] time = 9.36, size = 175, normalized size = 1.68

$$\frac{2\,a^2\,\left(\frac{5\,B\,\sin(c+d\,x)}{2}-\frac{15\,A\,\cos(c+d\,x)}{4}-\frac{5\,B\,\cos(c+d\,x)}{8}-\frac{15\,A\,\sin(c+d\,x)}{4}-\frac{5\,B}{2}-3\,A\,\cos\left(2\,c+2\,d\,x\right)+\frac{3\,A\,\cos(3\,c+3\,d\,x)}{4}+2\,B\,\cdots\right.}{15\,d\,\left(\frac{\cos(3\,c+3\,d\,x)}{4}-\frac{5\,\cos(c+d\,x)}{4}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^6,x)
```

```
[Out] (2*a^2*((5*B*sin(c + d*x))/2 - (15*A*cos(c + d*x))/4 - (5*B*cos(c + d*x))/8
 - (15*A*sin(c + d*x))/4 - (5*B)/2 - 3*A*cos(2*c + 2*d*x) + (3*A*cos(3*c +
3*d*x))/4 + 2*B*cos(2*c + 2*d*x) + (B*cos(3*c + 3*d*x))/8 + 3*A*sin(2*c + 2
*d*x) + (3*A*sin(3*c + 3*d*x))/4 + (B*sin(2*c + 2*d*x))/2 - (B*sin(3*c + 3*
d*x))/2))/(15*d*(cos(3*c + 3*d*x)/4 - (5*cos(c + d*x))/4 + sin(2*c + 2*d*x)
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.983 $\int \sec^8(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a^2(5A-2B)\tan^5(c+dx)}{35d}+\frac{2a^2(5A-2B)\tan^3(c+dx)}{21d}+\frac{a^2(5A-2B)\tan(c+dx)}{7d}+\frac{a^2(5A-2B)\sec^5(c+dx)}{35d}+\frac{(A$$

```
[Out] 1/35*a^2*(5*A-2*B)*sec(d*x+c)^5/d+1/7*(A+B)*sec(d*x+c)^7*(a+a*sin(d*x+c))^2
/d+1/7*a^2*(5*A-2*B)*tan(d*x+c)/d+2/21*a^2*(5*A-2*B)*tan(d*x+c)^3/d+1/35*a^
2*(5*A-2*B)*tan(d*x+c)^5/d
```

Rubi [A] time = 0.13, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2855, 2669, 3767}

$$\frac{a^2(5A-2B)\tan^5(c+dx)}{35d}+\frac{2a^2(5A-2B)\tan^3(c+dx)}{21d}+\frac{a^2(5A-2B)\tan(c+dx)}{7d}+\frac{a^2(5A-2B)\sec^5(c+dx)}{35d}+\frac{(A$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(5*A - 2*B)*Sec[c + d*x]^5)/(35*d) + ((A + B)*Sec[c + d*x]^7*(a + a*Si
n[c + d*x])^2)/(7*d) + (a^2*(5*A - 2*B)*Tan[c + d*x])/(7*d) + (2*a^2*(5*A -
 2*B)*Tan[c + d*x]^3)/(21*d) + (a^2*(5*A - 2*B)*Tan[c + d*x]^5)/(35*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{1}{7}(a(5A-$$
$$=\frac{a^2(5A-2B)\sec^5(c+dx)}{35d}+\frac{(A+B)\sec^7(c+dx)(a+$$
$$=\frac{a^2(5A-2B)\sec^5(c+dx)}{35d}+\frac{(A+B)\sec^7(c+dx)(a+$$
$$=\frac{a^2(5A-2B)\sec^5(c+dx)}{35d}+\frac{(A+B)\sec^7(c+dx)(a+$$

Mathematica [A] time = 0.34, size = 130, normalized size = 1.01

$$\frac{a^2\left(8(2B-5A)\tan^7(c+dx)+(30A+9B)\sec^7(c+dx)-35(5A-2B)\tan^3(c+dx)\sec^4(c+dx)+28(5A-2B)\,\mathrm{t}\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*((30*A + 9*B)*Sec[c + d*x]^7 + 105*A*Sec[c + d*x]^6*Tan[c + d*x] + 21*
B*Sec[c + d*x]^5*Tan[c + d*x]^2 - 35*(5*A - 2*B)*Sec[c + d*x]^4*Tan[c + d*x
]^3 + 28*(5*A - 2*B)*Sec[c + d*x]^2*Tan[c + d*x]^5 + 8*(-5*A + 2*B)*Tan[c +
 d*x]^7))/(105*d)
```

fricas [A] time = 0.68, size = 157, normalized size = 1.22

$$-\frac{16\,(5\,A-2\,B)a^2\cos{(dx+c)}^4-8\,(5\,A-2\,B)a^2\cos{(dx+c)}^2-5\,(2\,A-5\,B)a^2-\left(8\,(5\,A-2\,B)a^2\cos{(dx+c)}^4-\right.}{105\left(d\cos{(dx+c)}^5+2\,d\cos{(dx+c)}^3\sin{(dx+c)}-2\,d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/105*(16*(5*A - 2*B)*a^2*cos(d*x + c)^4 - 8*(5*A - 2*B)*a^2*cos(d*x + c)^
2 - 5*(2*A - 5*B)*a^2 - (8*(5*A - 2*B)*a^2*cos(d*x + c)^4 - 12*(5*A - 2*B)*
```

```
a^2*cos(d*x + c)^2 - 5*(5*A - 2*B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5 + 2
*d*cos(d*x + c)^3*sin(d*x + c) - 2*d*cos(d*x + c)^3)
```

giac [B] time = 0.23, size = 325, normalized size = 2.52

$$-\frac{\frac{35\left(9\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,Aa^2-5\,Ba^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}+\frac{1365\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+210\,B}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/840*(35*(9*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*B*a^2*tan(1/2*d*x + 1/2*c)^2
 + 15*A*a^2*tan(1/2*d*x + 1/2*c) - 9*B*a^2*tan(1/2*d*x + 1/2*c) + 8*A*a^2 -
 5*B*a^2)/(tan(1/2*d*x + 1/2*c) + 1)^3 + (1365*A*a^2*tan(1/2*d*x + 1/2*c)^6
 + 210*B*a^2*tan(1/2*d*x + 1/2*c)^6 - 5775*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 1
05*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 12250*A*a^2*tan(1/2*d*x + 1/2*c)^4 - 175*
B*a^2*tan(1/2*d*x + 1/2*c)^4 - 14350*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 910*B*a
^2*tan(1/2*d*x + 1/2*c)^3 + 10185*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 756*B*a^2*
tan(1/2*d*x + 1/2*c)^2 - 3955*A*a^2*tan(1/2*d*x + 1/2*c) + 427*B*a^2*tan(1/
2*d*x + 1/2*c) + 760*A*a^2 - 31*B*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^7)/d
```

maple [B] time = 0.71, size = 295, normalized size = 2.29

$$\frac{a^2A\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+B\,a^2\left(\frac{\sin^4(dx+c)}{7\cos(dx+c)^7}+\frac{3\left(\sin^4(dx+c)\right)}{35\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{35\cos(dx+c)}-\frac{(2+\sin}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/
105*sin(d*x+c)^3/cos(d*x+c)^3)+B*a^2*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*si
n(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/co
s(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c))+2/7*a^2*A/cos(d*x+c)^7+2*B*a^2*(
1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+
c)^3/cos(d*x+c)^3)-a^2*A*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*se
c(d*x+c)^2)*tan(d*x+c)+1/7*B*a^2/cos(d*x+c)^7)
```

maxima [A] time = 0.34, size = 178, normalized size = 1.38

$$\frac{\left(15\,\tan\left(dx+c\right)^7+42\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3\right)Aa^2+3\left(5\,\tan\left(dx+c\right)^7+21\,\tan\left(dx+c\right)^5+35\,\tan\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/105*((15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*A*a^2 +
3*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x +
c))*A*a^2 + 2*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*B
*a^2 - 3*(7*cos(d*x + c)^2 - 5)*B*a^2/cos(d*x + c)^7 + 30*A*a^2/cos(d*x + c
)^7 + 15*B*a^2/cos(d*x + c)^7)/d
```

mupad [B] time = 12.23, size = 274, normalized size = 2.12

$$-\frac{a^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{25A\cos\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{4}-\frac{105A\cos\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{4}-\frac{95A\cos\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{8}+\frac{15A\cos\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{8}-21B\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^8,x)
```

```
[Out] -(a^2*cos(c/2 + (d*x)/2)*((25*A*cos((5*c)/2 + (5*d*x)/2))/4 - (105*A*cos((3
*c)/2 + (3*d*x)/2))/4 - (95*A*cos((7*c)/2 + (7*d*x)/2))/8 + (15*A*cos((9*c)
/2 + (9*d*x)/2))/8 - 21*B*cos(c/2 + (d*x)/2) + (105*B*cos((3*c)/2 + (3*d*x)
/2))/8 - (41*B*cos((5*c)/2 + (5*d*x)/2))/8 + (55*B*cos((7*c)/2 + (7*d*x)/2)
)/16 + (9*B*cos((9*c)/2 + (9*d*x)/2))/16 - (125*A*sin(c/2 + (d*x)/2))/2 + (
55*A*sin((3*c)/2 + (3*d*x)/2))/2 - (25*A*sin((5*c)/2 + (5*d*x)/2))/2 + 5*A*
sin((7*c)/2 + (7*d*x)/2) + (5*A*sin((9*c)/2 + (9*d*x)/2))/2 + (37*B*sin(c/2
 + (d*x)/2))/4 + (19*B*sin((3*c)/2 + (3*d*x)/2))/4 - (B*sin((5*c)/2 + (5*d*
x)/2))/4 + (13*B*sin((7*c)/2 + (7*d*x)/2))/4 - B*sin((9*c)/2 + (9*d*x)/2)))
/(1680*d*cos(c/2 - pi/4 + (d*x)/2)^3*cos(c/2 + pi/4 + (d*x)/2)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.984 $\int \sec^{10}(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=154

$$\frac{a^2(7A-2B)\tan^7(c+dx)}{63d}+\frac{a^2(7A-2B)\tan^5(c+dx)}{15d}+\frac{a^2(7A-2B)\tan^3(c+dx)}{9d}+\frac{a^2(7A-2B)\tan(c+dx)}{9d}+\frac{a^2}{}$$

```
[Out] 1/63*a^2*(7*A-2*B)*sec(d*x+c)^7/d+1/9*(A+B)*sec(d*x+c)^9*(a+a*sin(d*x+c))^2
/d+1/9*a^2*(7*A-2*B)*tan(d*x+c)/d+1/9*a^2*(7*A-2*B)*tan(d*x+c)^3/d+1/15*a^2
*(7*A-2*B)*tan(d*x+c)^5/d+1/63*a^2*(7*A-2*B)*tan(d*x+c)^7/d
```

Rubi [A] time = 0.14, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2855, 2669, 3767}

$$\frac{a^2(7A-2B)\tan^7(c+dx)}{63d}+\frac{a^2(7A-2B)\tan^5(c+dx)}{15d}+\frac{a^2(7A-2B)\tan^3(c+dx)}{9d}+\frac{a^2(7A-2B)\tan(c+dx)}{9d}+\frac{a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(7*A - 2*B)*Sec[c + d*x]^7)/(63*d) + ((A + B)*Sec[c + d*x]^9*(a + a*Si
n[c + d*x])^2)/(9*d) + (a^2*(7*A - 2*B)*Tan[c + d*x])/(9*d) + (a^2*(7*A - 2
*B)*Tan[c + d*x]^3)/(9*d) + (a^2*(7*A - 2*B)*Tan[c + d*x]^5)/(15*d) + (a^2*
(7*A - 2*B)*Tan[c + d*x]^7)/(63*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^2}{9d}+\frac{1}{9}(a(7A$$
$$= \frac{a^2(7A-2B)\sec^7(c+dx)}{63d}+\frac{(A+B)\sec^9(c+dx)(a}{9d}$$
$$= \frac{a^2(7A-2B)\sec^7(c+dx)}{63d}+\frac{(A+B)\sec^9(c+dx)(a}{9d}$$
$$= \frac{a^2(7A-2B)\sec^7(c+dx)}{63d}+\frac{(A+B)\sec^9(c+dx)(a}{9d}$$

Mathematica [A] time = 0.45, size = 156, normalized size = 1.01

$$\frac{a^2\left(16(7A-2B)\tan^9(c+dx)+5(14A+5B)\sec^9(c+dx)-105(7A-2B)\tan^3(c+dx)\sec^6(c+dx)+126(7A-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(5*(14*A + 5*B)*Sec[c + d*x]^9 + 315*A*Sec[c + d*x]^8*Tan[c + d*x] + 4
5*B*Sec[c + d*x]^7*Tan[c + d*x]^2 - 105*(7*A - 2*B)*Sec[c + d*x]^6*Tan[c +
d*x]^3 + 126*(7*A - 2*B)*Sec[c + d*x]^4*Tan[c + d*x]^5 - 72*(7*A - 2*B)*Sec
[c + d*x]^2*Tan[c + d*x]^7 + 16*(7*A - 2*B)*Tan[c + d*x]^9))/(315*d)
```

fricas [A] time = 0.69, size = 197, normalized size = 1.28

$$-\frac{32\,(7\,A-2\,B)a^2\cos{(dx+c)}^6-16\,(7\,A-2\,B)a^2\cos{(dx+c)}^4-4\,(7\,A-2\,B)a^2\cos{(dx+c)}^2-7\,(2\,A-7\,B)a^2-}{315\left(d\cos{(dx+c)}^7+2\,d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] -1/315*(32*(7*A - 2*B)*a^2*cos(d*x + c)^6 - 16*(7*A - 2*B)*a^2*cos(d*x + c)
^4 - 4*(7*A - 2*B)*a^2*cos(d*x + c)^2 - 7*(2*A - 7*B)*a^2 - (16*(7*A - 2*B)
```

```
*a^2*cos(d*x + c)^6 - 24*(7*A - 2*B)*a^2*cos(d*x + c)^4 - 10*(7*A - 2*B)*a^
2*cos(d*x + c)^2 - 7*(7*A - 2*B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^7 + 2*d
*cos(d*x + c)^5*sin(d*x + c) - 2*d*cos(d*x + c)^5)
```

giac [B] time = 0.25, size = 461, normalized size = 2.99

$$-\frac{\frac{21\left(435\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-225\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1470\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-690\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2060\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-940\,Ba^2\tan}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="g
iac")
```

```
[Out] -1/20160*(21*(435*A*a^2*tan(1/2*d*x + 1/2*c)^4 - 225*B*a^2*tan(1/2*d*x + 1/
2*c)^4 + 1470*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 690*B*a^2*tan(1/2*d*x + 1/2*c)
^3 + 2060*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 940*B*a^2*tan(1/2*d*x + 1/2*c)^2 +
 1330*A*a^2*tan(1/2*d*x + 1/2*c) - 590*B*a^2*tan(1/2*d*x + 1/2*c) + 353*A*a
^2 - 163*B*a^2)/(tan(1/2*d*x + 1/2*c) + 1)^5 + (31185*A*a^2*tan(1/2*d*x + 1
/2*c)^8 + 4725*B*a^2*tan(1/2*d*x + 1/2*c)^8 - 185220*A*a^2*tan(1/2*d*x + 1/
2*c)^7 - 11340*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 546840*A*a^2*tan(1/2*d*x + 1/
2*c)^6 + 15120*B*a^2*tan(1/2*d*x + 1/2*c)^6 - 961380*A*a^2*tan(1/2*d*x + 1/
2*c)^5 + 3780*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 1101618*A*a^2*tan(1/2*d*x + 1/
2*c)^4 - 24318*B*a^2*tan(1/2*d*x + 1/2*c)^4 - 828492*A*a^2*tan(1/2*d*x + 1/
2*c)^3 + 33852*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 404208*A*a^2*tan(1/2*d*x + 1/
2*c)^2 - 19368*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 116172*A*a^2*tan(1/2*d*x + 1/
2*c) + 6732*B*a^2*tan(1/2*d*x + 1/2*c) + 16373*A*a^2 - 223*B*a^2)/(tan(1/2*
d*x + 1/2*c) - 1)^9)/d
```

maple [B] time = 0.71, size = 359, normalized size = 2.33

$$\frac{a^2A\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^5}+\frac{16\left(\sin^3(dx+c)\right)}{315\cos(dx+c)^3}\right)+B\,a^2\left(\frac{\sin^4(dx+c)}{9\cos(dx+c)^9}+\frac{5\left(\sin^4(dx+c)\right)}{63\cos(dx+c)^7}+\frac{\sin^4(dx+c)}{21\cos(dx+c)^5}+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/
105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+B*a^2*(1/9*
sin(d*x+c)^4/cos(d*x+c)^9+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/
cos(d*x+c)^5+1/63*sin(d*x+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/
63*(2+sin(d*x+c)^2)*cos(d*x+c))+2/9*a^2*A/cos(d*x+c)^9+2*B*a^2*(1/9*sin(d*x
```

```
+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*
x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)-a^2*A*(-128/315-1/9*sec(d*x+c)^8-8
/63*sec(d*x+c)^6-16/105*sec(d*x+c)^4-64/315*sec(d*x+c)^2)*tan(d*x+c)+1/9*B*
a^2/cos(d*x+c)^9)
```

maxima [A] time = 0.47, size = 207, normalized size = 1.34

$$\frac{\left(35\,\tan\left(dx+c\right)^{9}+180\,\tan\left(dx+c\right)^{7}+378\,\tan\left(dx+c\right)^{5}+420\,\tan\left(dx+c\right)^{3}+315\,\tan\left(dx+c\right)\right)Aa^{2}+\left(35\,\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="m
axima")
```

```
[Out] 1/315*((35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 378*tan(d*x + c)^5 + 420*t
an(d*x + c)^3 + 315*tan(d*x + c))*A*a^2 + (35*tan(d*x + c)^9 + 135*tan(d*x
+ c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*A*a^2 + 2*(35*tan(d*x + c
)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*B*a^2 -
 5*(9*cos(d*x + c)^2 - 7)*B*a^2/cos(d*x + c)^9 + 70*A*a^2/cos(d*x + c)^9 +
35*B*a^2/cos(d*x + c)^9)/d
```

mupad [B] time = 12.95, size = 370, normalized size = 2.40

$$-\frac{a^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{455\,A\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{32}-\frac{1575\,A\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{32}-35\,A\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)+7\,A\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)-\frac{259\,A\,\cos\left(\right.}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^10,x)
```

```
[Out] -(a^2*cos(c/2 + (d*x)/2)*((455*A*cos((5*c)/2 + (5*d*x)/2))/32 - (1575*A*cos
((3*c)/2 + (3*d*x)/2))/32 - 35*A*cos((7*c)/2 + (7*d*x)/2) + 7*A*cos((9*c)/2
 + (9*d*x)/2) - (259*A*cos((11*c)/2 + (11*d*x)/2))/32 + (35*A*cos((13*c)/2
+ (13*d*x)/2))/32 - 45*B*cos(c/2 + (d*x)/2) + (1755*B*cos((3*c)/2 + (3*d*x)
/2))/64 - (1115*B*cos((5*c)/2 + (5*d*x)/2))/64 + 10*B*cos((7*c)/2 + (7*d*x)
/2) - 2*B*cos((9*c)/2 + (9*d*x)/2) + (103*B*cos((11*c)/2 + (11*d*x)/2))/64
+ (25*B*cos((13*c)/2 + (13*d*x)/2))/64 - (623*A*sin(c/2 + (d*x)/2))/4 + 77*
A*sin((3*c)/2 + (3*d*x)/2) - (441*A*sin((5*c)/2 + (5*d*x)/2))/8 + (175*A*si
n((7*c)/2 + (7*d*x)/2))/8 - (35*A*sin((9*c)/2 + (9*d*x)/2))/8 + (21*A*sin((
11*c)/2 + (11*d*x)/2))/8 + (7*A*sin((13*c)/2 + (13*d*x)/2))/4 + (131*B*sin(
c/2 + (d*x)/2))/8 + (49*B*sin((3*c)/2 + (3*d*x)/2))/8 + (27*B*sin((5*c)/2 +
 (5*d*x)/2))/16 + (125*B*sin((7*c)/2 + (7*d*x)/2))/16 - (25*B*sin((9*c)/2 +
```

```
(9*d*x)/2))/16 + (33*B*sin((11*c)/2 + (11*d*x)/2))/16 - (B*sin((13*c)/2 +
(13*d*x)/2))/2))/(20160*d*cos(c/2 - pi/4 + (d*x)/2)^5*cos(c/2 + pi/4 + (d*x
)/2)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.985 $\int \sec^{12}(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=179

$$\frac{a^2(9A-2B)\tan^9(c+dx)}{99d}+\frac{4a^2(9A-2B)\tan^7(c+dx)}{77d}+\frac{6a^2(9A-2B)\tan^5(c+dx)}{55d}+\frac{4a^2(9A-2B)\tan^3(c+dx)}{33d}+$$

```
[Out] 1/99*a^2*(9*A-2*B)*sec(d*x+c)^9/d+1/11*(A+B)*sec(d*x+c)^11*(a+a*sin(d*x+c))
^2/d+1/11*a^2*(9*A-2*B)*tan(d*x+c)/d+4/33*a^2*(9*A-2*B)*tan(d*x+c)^3/d+6/55
*a^2*(9*A-2*B)*tan(d*x+c)^5/d+4/77*a^2*(9*A-2*B)*tan(d*x+c)^7/d+1/99*a^2*(9
*A-2*B)*tan(d*x+c)^9/d
```

Rubi [A] time = 0.15, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2855, 2669, 3767}

$$\frac{a^2(9A-2B)\tan^9(c+dx)}{99d}+\frac{4a^2(9A-2B)\tan^7(c+dx)}{77d}+\frac{6a^2(9A-2B)\tan^5(c+dx)}{55d}+\frac{4a^2(9A-2B)\tan^3(c+dx)}{33d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(9*A - 2*B)*Sec[c + d*x]^9)/(99*d) + ((A + B)*Sec[c + d*x]^11*(a + a*S
in[c + d*x])^2)/(11*d) + (a^2*(9*A - 2*B)*Tan[c + d*x])/(11*d) + (4*a^2*(9*
A - 2*B)*Tan[c + d*x]^3)/(33*d) + (6*a^2*(9*A - 2*B)*Tan[c + d*x]^5)/(55*d)
 + (4*a^2*(9*A - 2*B)*Tan[c + d*x]^7)/(77*d) + (a^2*(9*A - 2*B)*Tan[c + d*x
]^9)/(99*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^{12}(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^{11}(c+dx)(a+a\sin(c+dx))^2}{11d} + \frac{1}{11}(a(9$$
$$= \frac{a^2(9A-2B)\sec^9(c+dx)}{99d} + \frac{(A+B)\sec^{11}(c+dx)(}{11a}$$
$$= \frac{a^2(9A-2B)\sec^9(c+dx)}{99d} + \frac{(A+B)\sec^{11}(c+dx)(}{11a}$$
$$= \frac{a^2(9A-2B)\sec^9(c+dx)}{99d} + \frac{(A+B)\sec^{11}(c+dx)(}{11a}$$

Mathematica [A] time = 0.92, size = 181, normalized size = 1.01

$$\frac{a^2\left(128(2B-9A)\tan^{11}(c+dx)+35(18A+7B)\sec^{11}(c+dx)-1155(9A-2B)\tan^3(c+dx)\sec^8(c+dx)+184\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(35*(18*A + 7*B)*Sec[c + d*x]^11 + 3465*A*Sec[c + d*x]^10*Tan[c + d*x]
 + 385*B*Sec[c + d*x]^9*Tan[c + d*x]^2 - 1155*(9*A - 2*B)*Sec[c + d*x]^8*Ta
n[c + d*x]^3 + 1848*(9*A - 2*B)*Sec[c + d*x]^6*Tan[c + d*x]^5 - 1584*(9*A -
 2*B)*Sec[c + d*x]^4*Tan[c + d*x]^7 + 704*(9*A - 2*B)*Sec[c + d*x]^2*Tan[c
+ d*x]^9 + 128*(-9*A + 2*B)*Tan[c + d*x]^11))/(3465*d)
```

fricas [A] time = 0.77, size = 237, normalized size = 1.32

$$-\frac{256\,(9\,A-2\,B)a^2\cos(dx+c)^8-128\,(9\,A-2\,B)a^2\cos(dx+c)^6-32\,(9\,A-2\,B)a^2\cos(dx+c)^4-16\,(9\,A-2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] -1/3465*(256*(9*A - 2*B)*a^2*cos(d*x + c)^8 - 128*(9*A - 2*B)*a^2*cos(d*x +
 c)^6 - 32*(9*A - 2*B)*a^2*cos(d*x + c)^4 - 16*(9*A - 2*B)*a^2*cos(d*x + c)
^2 - 45*(2*A - 9*B)*a^2 - (128*(9*A - 2*B)*a^2*cos(d*x + c)^8 - 192*(9*A -
2*B)*a^2*cos(d*x + c)^6 - 80*(9*A - 2*B)*a^2*cos(d*x + c)^4 - 56*(9*A - 2*B
)*a^2*cos(d*x + c)^2 - 45*(9*A - 2*B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^9
+ 2*d*cos(d*x + c)^7*sin(d*x + c) - 2*d*cos(d*x + c)^7)
```

giac [B] time = 0.30, size = 597, normalized size = 3.34

$$-\frac{\frac{33\left(6825\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-2940\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+34965\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-13755\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+79800\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30065}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="g
iac")
```

```
[Out] -1/443520*(33*(6825*A*a^2*tan(1/2*d*x + 1/2*c)^6 - 2940*B*a^2*tan(1/2*d*x +
 1/2*c)^6 + 34965*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 13755*B*a^2*tan(1/2*d*x +
1/2*c)^5 + 79800*A*a^2*tan(1/2*d*x + 1/2*c)^4 - 30065*B*a^2*tan(1/2*d*x + 1
/2*c)^4 + 100170*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 36470*B*a^2*tan(1/2*d*x + 1
/2*c)^3 + 73017*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 26166*B*a^2*tan(1/2*d*x + 1/
2*c)^2 + 29169*A*a^2*tan(1/2*d*x + 1/2*c) - 10367*B*a^2*tan(1/2*d*x + 1/2*c
) + 5142*A*a^2 - 1901*B*a^2)/(tan(1/2*d*x + 1/2*c) + 1)^7 + (661815*A*a^2*t
an(1/2*d*x + 1/2*c)^10 + 97020*B*a^2*tan(1/2*d*x + 1/2*c)^10 - 5083155*A*a^
2*tan(1/2*d*x + 1/2*c)^9 - 405405*B*a^2*tan(1/2*d*x + 1/2*c)^9 + 19355490*A
*a^2*tan(1/2*d*x + 1/2*c)^8 + 952875*B*a^2*tan(1/2*d*x + 1/2*c)^8 - 4544694
0*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 1122660*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 722
95146*A*a^2*tan(1/2*d*x + 1/2*c)^6 + 557172*B*a^2*tan(1/2*d*x + 1/2*c)^6 -
80611146*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 563178*B*a^2*tan(1/2*d*x + 1/2*c)^5
 + 63771840*A*a^2*tan(1/2*d*x + 1/2*c)^4 - 1126950*B*a^2*tan(1/2*d*x + 1/2*
c)^4 - 35253900*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 955020*B*a^2*tan(1/2*d*x + 1
/2*c)^3 + 13119975*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 406120*B*a^2*tan(1/2*d*x
+ 1/2*c)^2 - 2978811*A*a^2*tan(1/2*d*x + 1/2*c) + 97163*B*a^2*tan(1/2*d*x +
 1/2*c) + 330966*A*a^2 - 13*B*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^11)/d
```

maple [B] time = 0.81, size = 423, normalized size = 2.36

$$\frac{a^2A\left(\frac{\sin^3(dx+c)}{11\cos(dx+c)^{11}}+\frac{8\left(\sin^3(dx+c)\right)}{99\cos(dx+c)^9}+\frac{16\left(\sin^3(dx+c)\right)}{231\cos(dx+c)^7}+\frac{64\left(\sin^3(dx+c)\right)}{1155\cos(dx+c)^5}+\frac{128\left(\sin^3(dx+c)\right)}{3465\cos(dx+c)^3}\right)+B\,a^2\left(\frac{\sin^4(dx+c)}{11\cos(dx+c)^{11}}+\frac{7\left(\sin^4(dx+c)\right)}{99\cos(dx+c)^9}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(1/11*sin(d*x+c)^3/cos(d*x+c)^11+8/99*sin(d*x+c)^3/cos(d*x+c)^9+
16/231*sin(d*x+c)^3/cos(d*x+c)^7+64/1155*sin(d*x+c)^3/cos(d*x+c)^5+128/3465
*sin(d*x+c)^3/cos(d*x+c)^3)+B*a^2*(1/11*sin(d*x+c)^4/cos(d*x+c)^11+7/99*sin
(d*x+c)^4/cos(d*x+c)^9+5/99*sin(d*x+c)^4/cos(d*x+c)^7+1/33*sin(d*x+c)^4/cos
(d*x+c)^5+1/99*sin(d*x+c)^4/cos(d*x+c)^3-1/99*sin(d*x+c)^4/cos(d*x+c)-1/99*
(2+sin(d*x+c)^2)*cos(d*x+c))+2/11*a^2*A/cos(d*x+c)^11+2*B*a^2*(1/11*sin(d*x
+c)^3/cos(d*x+c)^11+8/99*sin(d*x+c)^3/cos(d*x+c)^9+16/231*sin(d*x+c)^3/cos(
d*x+c)^7+64/1155*sin(d*x+c)^3/cos(d*x+c)^5+128/3465*sin(d*x+c)^3/cos(d*x+c)
^3)-a^2*A*(-256/693-1/11*sec(d*x+c)^10-10/99*sec(d*x+c)^8-80/693*sec(d*x+c)
^6-32/231*sec(d*x+c)^4-128/693*sec(d*x+c)^2)*tan(d*x+c)+1/11*B*a^2/cos(d*x+
c)^11)
```

maxima [A] time = 0.71, size = 238, normalized size = 1.33

$$\frac{\left(315\,\tan\left(dx+c\right)^{11}+1540\,\tan\left(dx+c\right)^{9}+2970\,\tan\left(dx+c\right)^{7}+2772\,\tan\left(dx+c\right)^{5}+1155\,\tan\left(dx+c\right)^{3}\right)Aa^{2}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="m
axima")
```

```
[Out] 1/3465*((315*tan(d*x + c)^11 + 1540*tan(d*x + c)^9 + 2970*tan(d*x + c)^7 +
2772*tan(d*x + c)^5 + 1155*tan(d*x + c)^3)*A*a^2 + 5*(63*tan(d*x + c)^11 +
385*tan(d*x + c)^9 + 990*tan(d*x + c)^7 + 1386*tan(d*x + c)^5 + 1155*tan(d*
x + c)^3 + 693*tan(d*x + c))*A*a^2 + 2*(315*tan(d*x + c)^11 + 1540*tan(d*x
+ c)^9 + 2970*tan(d*x + c)^7 + 2772*tan(d*x + c)^5 + 1155*tan(d*x + c)^3)*B
*a^2 - 35*(11*cos(d*x + c)^2 - 9)*B*a^2/cos(d*x + c)^11 + 630*A*a^2/cos(d*x
 + c)^11 + 315*B*a^2/cos(d*x + c)^11)/d
```

mupad [B] time = 13.96, size = 466, normalized size = 2.60

$$-\frac{a^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{8127\,A\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{64}-\frac{24255\,A\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{64}-\frac{21357\,A\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{64}+\frac{5229\,A\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{64}-\frac{8379\,A\,\cos\left(\cdots\right)}{64}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^12,x)
```

```
[Out] -(a^2*cos(c/2 + (d*x)/2)*((8127*A*cos((5*c)/2 + (5*d*x)/2))/64 - (24255*A*c
os((3*c)/2 + (3*d*x)/2))/64 - (21357*A*cos((7*c)/2 + (7*d*x)/2))/64 + (5229
```

```
*A*cos((9*c)/2 + (9*d*x)/2))/64 - (8379*A*cos((11*c)/2 + (11*d*x)/2))/64 +
(1467*A*cos((13*c)/2 + (13*d*x)/2))/64 - (2619*A*cos((15*c)/2 + (15*d*x)/2)
)/128 + (315*A*cos((17*c)/2 + (17*d*x)/2))/128 - 385*B*cos(c/2 + (d*x)/2) +
 (30415*B*cos((3*c)/2 + (3*d*x)/2))/128 - (23247*B*cos((5*c)/2 + (5*d*x)/2)
)/128 + (12957*B*cos((7*c)/2 + (7*d*x)/2))/128 - (5789*B*cos((9*c)/2 + (9*d
*x)/2))/128 + (3339*B*cos((11*c)/2 + (11*d*x)/2))/128 - (267*B*cos((13*c)/2
 + (13*d*x)/2))/128 + (779*B*cos((15*c)/2 + (15*d*x)/2))/256 + (245*B*cos((
17*c)/2 + (17*d*x)/2))/256 - (47889*A*sin(c/2 + (d*x)/2))/32 + (25713*A*sin
((3*c)/2 + (3*d*x)/2))/32 - (21303*A*sin((5*c)/2 + (5*d*x)/2))/32 + (9207*A
*sin((7*c)/2 + (7*d*x)/2))/32 - (4797*A*sin((9*c)/2 + (9*d*x)/2))/32 + (191
7*A*sin((11*c)/2 + (11*d*x)/2))/32 - (27*A*sin((13*c)/2 + (13*d*x)/2))/32 +
 (171*A*sin((15*c)/2 + (15*d*x)/2))/32 + (9*A*sin((17*c)/2 + (17*d*x)/2))/2
 + (7809*B*sin(c/2 + (d*x)/2))/64 + (2047*B*sin((3*c)/2 + (3*d*x)/2))/64 +
(1383*B*sin((5*c)/2 + (5*d*x)/2))/64 + (3993*B*sin((7*c)/2 + (7*d*x)/2))/64
 - (563*B*sin((9*c)/2 + (9*d*x)/2))/64 + (1843*B*sin((11*c)/2 + (11*d*x)/2)
)/64 - (373*B*sin((13*c)/2 + (13*d*x)/2))/64 + (309*B*sin((15*c)/2 + (15*d*
x)/2))/64 - B*sin((17*c)/2 + (17*d*x)/2)))/(887040*d*cos(c/2 - pi/4 + (d*x)
/2)^7*cos(c/2 + pi/4 + (d*x)/2)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**12*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.986 $\int \cos^7(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$-\frac{B(a\sin(c+dx)+a)^{11}}{11a^8d}-\frac{(A-7B)(a\sin(c+dx)+a)^{10}}{10a^7d}+\frac{2(A-3B)(a\sin(c+dx)+a)^9}{3a^6d}-\frac{(3A-5B)(a\sin(c+dx}{2a^5d}$$

```
[Out] 8/7*(A-B)*(a+a*sin(d*x+c))^7/a^4/d-1/2*(3*A-5*B)*(a+a*sin(d*x+c))^8/a^5/d+2
/3*(A-3*B)*(a+a*sin(d*x+c))^9/a^6/d-1/10*(A-7*B)*(a+a*sin(d*x+c))^10/a^7/d-
1/11*B*(a+a*sin(d*x+c))^11/a^8/d
```

Rubi [A] time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$-\frac{(A-7B)(a\sin(c+dx)+a)^{10}}{10a^7d}+\frac{2(A-3B)(a\sin(c+dx)+a)^9}{3a^6d}-\frac{(3A-5B)(a\sin(c+dx)+a)^8}{2a^5d}+\frac{8(A-B)(a\sin}{7a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (8*(A - B)*(a + a*Sin[c + d*x])^7)/(7*a^4*d) - ((3*A - 5*B)*(a + a*Sin[c +
d*x])^8)/(2*a^5*d) + (2*(A - 3*B)*(a + a*Sin[c + d*x])^9)/(3*a^6*d) - ((A -
 7*B)*(a + a*Sin[c + d*x])^10)/(10*a^7*d) - (B*(a + a*Sin[c + d*x])^11)/(11
*a^8*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^6\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+d\right.}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(8a^3(A-B)(a+x)^6 - 4a^2(3A-5B)(a+x\right.\right.}{}$$

$$= \frac{8(A-B)(a+a\sin(c+dx))^7}{7a^4 d} - \frac{(3A-5B)(a+a\sin(}{2a^5 d}$$

Mathematica [A] time = 1.53, size = 86, normalized size = 0.64

$$-\frac{a^3(\sin(c+dx)+1)^7\left(21(11A-37B)\sin^3(c+dx)+(1029B-847A)\sin^2(c+dx)+14(77A-39B)\sin(c+dx)-\right.}{2310d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/2310*(a^3*(1 + Sin[c + d*x])^7*(-484*A + 78*B + 14*(77*A - 39*B)*Sin[c +
 d*x] + (-847*A + 1029*B)*Sin[c + d*x]^2 + 21*(11*A - 37*B)*Sin[c + d*x]^3
+ 210*B*Sin[c + d*x]^4))/d
```

fricas [A] time = 0.75, size = 155, normalized size = 1.16

$$\frac{231\,(A+3\,B)a^3\cos(dx+c)^{10} - 1155\,(A+B)a^3\cos(dx+c)^8 + 2\left(105\,Ba^3\cos(dx+c)^{10} - 35\,(11\,A+15\,B)a^3\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2310*(231*(A + 3*B)*a^3*cos(d*x + c)^10 - 1155*(A + B)*a^3*cos(d*x + c)^8
 + 2*(105*B*a^3*cos(d*x + c)^10 - 35*(11*A + 15*B)*a^3*cos(d*x + c)^8 + 20*
(11*A + 3*B)*a^3*cos(d*x + c)^6 + 24*(11*A + 3*B)*a^3*cos(d*x + c)^4 + 32*(
11*A + 3*B)*a^3*cos(d*x + c)^2 + 64*(11*A + 3*B)*a^3)*sin(d*x + c))/d
```

giac [B] time = 0.89, size = 283, normalized size = 2.11

$$\frac{Ba^3\sin(11\,dx+11\,c)}{11264\,d} + \frac{\left(Aa^3+3\,Ba^3\right)\cos(10\,dx+10\,c)}{5120\,d} - \frac{\left(Aa^3-Ba^3\right)\cos(8\,dx+8\,c)}{512\,d} - \frac{\left(23\,Aa^3+5\,Ba^3\right)\cos(6\,d}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/11264*B*a^3*sin(11*d*x + 11*c)/d + 1/5120*(A*a^3 + 3*B*a^3)*cos(10*d*x +
10*c)/d - 1/512*(A*a^3 - B*a^3)*cos(8*d*x + 8*c)/d - 1/1024*(23*A*a^3 + 5*B
*a^3)*cos(6*d*x + 6*c)/d - 1/128*(11*A*a^3 + 5*B*a^3)*cos(4*d*x + 4*c)/d -
7/512*(13*A*a^3 + 7*B*a^3)*cos(2*d*x + 2*c)/d - 1/3072*(4*A*a^3 + 3*B*a^3)*
sin(9*d*x + 9*c)/d - 1/7168*(44*A*a^3 + 61*B*a^3)*sin(7*d*x + 7*c)/d + 1/51
20*(16*A*a^3 - 107*B*a^3)*sin(5*d*x + 5*c)/d + 1/512*(56*A*a^3 - B*a^3)*sin
(3*d*x + 3*c)/d + 91/512*(4*A*a^3 + B*a^3)*sin(d*x + c)/d
```

maple [B] time = 0.49, size = 345, normalized size = 2.57

$$\frac{a^3 A\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^8(dx+c)\right)}{10}-\frac{\left(\cos^8(dx+c)\right)}{40}\right)+B\,a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^8(dx+c)\right)}{11}-\frac{\left(\cos^8(dx+c)\right)\sin(dx+c)}{33}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6(\mathrm{co}}{}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8-1/40*cos(d*x+c)^8)+B*a^3*(-1/11
*sin(d*x+c)^3*cos(d*x+c)^8-1/33*cos(d*x+c)^8*sin(d*x+c)+1/231*(16/5+cos(d*x
+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+3*a^3*A*(-1/9*cos(d*x+
c)^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*
sin(d*x+c))+3*B*a^3*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8-1/40*cos(d*x+c)^8)-3/8
*a^3*A*cos(d*x+c)^8+3*B*a^3*(-1/9*cos(d*x+c)^8*sin(d*x+c)+1/63*(16/5+cos(d*
x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+1/7*a^3*A*(16/5+cos(d
*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)-1/8*B*a^3*cos(d*x+c)^
8)
```

maxima [A] time = 0.51, size = 182, normalized size = 1.36

$$-\frac{210\,Ba^3\sin(dx+c)^{11}+231\,(A+3\,B)a^3\sin(dx+c)^{10}+770\,Aa^3\sin(dx+c)^9-2310\,Ba^3\sin(dx+c)^8-660}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/2310*(210*B*a^3*sin(d*x + c)^11 + 231*(A + 3*B)*a^3*sin(d*x + c)^10 + 77
0*A*a^3*sin(d*x + c)^9 - 2310*B*a^3*sin(d*x + c)^8 - 660*(4*A + 3*B)*a^3*si
n(d*x + c)^7 - 2310*(A - B)*a^3*sin(d*x + c)^6 + 924*(3*A + 4*B)*a^3*sin(d*
```

```
x + c)^5 + 4620*A*a^3*sin(d*x + c)^4 - 2310*B*a^3*sin(d*x + c)^3 - 1155*(3*
A + B)*a^3*sin(d*x + c)^2 - 2310*A*a^3*sin(d*x + c))/d
```

mupad [B] time = 0.20, size = 177, normalized size = 1.32

$$\frac{\frac{a^3\sin(c+dx)^2(3A+B)}{2}-\frac{Aa^3\sin(c+dx)^9}{3}-2Aa^3\sin(c+dx)^4+a^3\sin(c+dx)^6(A-B)-\frac{a^3\sin(c+dx)^{10}(A+3B)}{10}+Ba^3\sin}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^2*(3*A + B))/2 - (A*a^3*sin(c + d*x)^9)/3 - 2*A*a^3*sin(
c + d*x)^4 + a^3*sin(c + d*x)^6*(A - B) - (a^3*sin(c + d*x)^10*(A + 3*B))/1
0 + B*a^3*sin(c + d*x)^3 + B*a^3*sin(c + d*x)^8 - (B*a^3*sin(c + d*x)^11)/1
1 - (2*a^3*sin(c + d*x)^5*(3*A + 4*B))/5 + (2*a^3*sin(c + d*x)^7*(4*A + 3*B
))/7 + A*a^3*sin(c + d*x))/d
```

sympy [A] time = 38.22, size = 636, normalized size = 4.75

$$\begin{cases}\frac{Aa^3\sin^{10}(c+dx)}{40d}+\frac{16Aa^3\sin^9(c+dx)}{105d}+\frac{Aa^3\sin^8(c+dx)\cos^2(c+dx)}{8d}+\frac{24Aa^3\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{16Aa^3\sin^7(c+dx)}{35d}+\frac{Aa^3\sin^6(c+dx}{4d} \\ x(A+B\sin(c))(a\sin(c)+a)^3\cos^7(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*sin(c + d*x)**10/(40*d) + 16*A*a**3*sin(c + d*x)**9/(105*
d) + A*a**3*sin(c + d*x)**8*cos(c + d*x)**2/(8*d) + 24*A*a**3*sin(c + d*x)*
*7*cos(c + d*x)**2/(35*d) + 16*A*a**3*sin(c + d*x)**7/(35*d) + A*a**3*sin(c
 + d*x)**6*cos(c + d*x)**4/(4*d) + 6*A*a**3*sin(c + d*x)**5*cos(c + d*x)**4
/(5*d) + 8*A*a**3*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + A*a**3*sin(c + d*
x)**4*cos(c + d*x)**6/(4*d) + A*a**3*sin(c + d*x)**3*cos(c + d*x)**6/d + 2*
A*a**3*sin(c + d*x)**3*cos(c + d*x)**4/d + A*a**3*sin(c + d*x)*cos(c + d*x)
**6/d - 3*A*a**3*cos(c + d*x)**8/(8*d) + 16*B*a**3*sin(c + d*x)**11/(1155*d
) + 3*B*a**3*sin(c + d*x)**10/(40*d) + 8*B*a**3*sin(c + d*x)**9*cos(c + d*x
)**2/(105*d) + 16*B*a**3*sin(c + d*x)**9/(105*d) + 3*B*a**3*sin(c + d*x)**8
*cos(c + d*x)**2/(8*d) + 6*B*a**3*sin(c + d*x)**7*cos(c + d*x)**4/(35*d) +
24*B*a**3*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 3*B*a**3*sin(c + d*x)**6
*cos(c + d*x)**4/(4*d) + B*a**3*sin(c + d*x)**5*cos(c + d*x)**6/(5*d) + 6*B
*a**3*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 3*B*a**3*sin(c + d*x)**4*cos(
c + d*x)**6/(4*d) + B*a**3*sin(c + d*x)**3*cos(c + d*x)**6/d - B*a**3*cos(c
 + d*x)**8/(8*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**3*cos(c)**7,
 True))
```

3.987 $\int \cos^5(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{B(a\sin(c+dx)+a)^9}{9a^6d}+\frac{(A-5B)(a\sin(c+dx)+a)^8}{8a^5d}-\frac{4(A-2B)(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(A-B)(a\sin(c+dx)+a)^6}{3a^3d}$$

```
[Out] 2/3*(A-B)*(a+a*sin(d*x+c))^6/a^3/d-4/7*(A-2*B)*(a+a*sin(d*x+c))^7/a^4/d+1/8
*(A-5*B)*(a+a*sin(d*x+c))^8/a^5/d+1/9*B*(a+a*sin(d*x+c))^9/a^6/d
```

Rubi [A] time = 0.15, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{(A-5B)(a\sin(c+dx)+a)^8}{8a^5d}-\frac{4(A-2B)(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(A-B)(a\sin(c+dx)+a)^6}{3a^3d}+\frac{B(a\sin(c+dx)+a)^9}{9a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[c + d*x])^6)/(3*a^3*d) - (4*(A - 2*B)*(a + a*Sin[c +
d*x])^7)/(7*a^4*d) + ((A - 5*B)*(a + a*Sin[c + d*x])^8)/(8*a^5*d) + (B*(a +
 a*Sin[c + d*x])^9)/(9*a^6*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^5\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+d\right.}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^2(A-B)(a+x)^5 - 4a(A-2B)(a+x)^6\right.\right.}{a^5d}$$
$$= \frac{2(A-B)(a+a\sin(c+dx))^6}{3a^3d} - \frac{4(A-2B)(a+a\sin(}{7a^4d}$$

Mathematica [A] time = 0.44, size = 70, normalized size = 0.67

$$\frac{a^3(\sin(c+dx)+1)^6\left(21(3A-7B)\sin^2(c+dx)-6(27A-19B)\sin(c+dx)+111A+56B\sin^3(c+dx)-19B\right)}{504d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^6*(111*A - 19*B - 6*(27*A - 19*B)*Sin[c + d*x] + 21
*(3*A - 7*B)*Sin[c + d*x]^2 + 56*B*Sin[c + d*x]^3))/(504*d)
```

fricas [A] time = 0.73, size = 129, normalized size = 1.23

$$\frac{63\,(A+3\,B)a^3\cos(dx+c)^8 - 336\,(A+B)a^3\cos(dx+c)^6 + 8\left(7\,Ba^3\cos(dx+c)^8 - (27\,A+37\,B)a^3\cos(dx+c\right.}{504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/504*(63*(A + 3*B)*a^3*cos(d*x + c)^8 - 336*(A + B)*a^3*cos(d*x + c)^6 + 8
*(7*B*a^3*cos(d*x + c)^8 - (27*A + 37*B)*a^3*cos(d*x + c)^6 + 6*(3*A + B)*a
^3*cos(d*x + c)^4 + 8*(3*A + B)*a^3*cos(d*x + c)^2 + 16*(3*A + B)*a^3)*sin(
d*x + c))/d
```

giac [B] time = 0.60, size = 230, normalized size = 2.19

$$\frac{Ba^3\sin(9\,dx+9\,c)}{2304\,d} + \frac{\left(Aa^3+3\,Ba^3\right)\cos(8\,dx+8\,c)}{1024\,d} - \frac{\left(5\,Aa^3-Ba^3\right)\cos(6\,dx+6\,c)}{384\,d} - \frac{\left(25\,Aa^3+11\,Ba^3\right)\cos(4\,dx}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2304*B*a^3*sin(9*d*x + 9*c)/d + 1/1024*(A*a^3 + 3*B*a^3)*cos(8*d*x + 8*c)
/d - 1/384*(5*A*a^3 - B*a^3)*cos(6*d*x + 6*c)/d - 1/256*(25*A*a^3 + 11*B*a^
3)*cos(4*d*x + 4*c)/d - 1/128*(33*A*a^3 + 19*B*a^3)*cos(2*d*x + 2*c)/d - 1/
1792*(12*A*a^3 + 11*B*a^3)*sin(7*d*x + 7*c)/d - 1/64*(A*a^3 + 2*B*a^3)*sin(
5*d*x + 5*c)/d + 1/192*(17*A*a^3 - 4*B*a^3)*sin(3*d*x + 3*c)/d + 11/128*(10
*A*a^3 + 3*B*a^3)*sin(d*x + c)/d
```

maple [B] time = 0.49, size = 305, normalized size = 2.90

$$a^3 A\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+B\,a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos}{}\right.}{105}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+B*a^3*(-1/9*s
in(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)
^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+3*a^3*A*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+3*B*a^3*(-1/8*sin(d*x+c)^2
*cos(d*x+c)^6-1/24*cos(d*x+c)^6)-1/2*a^3*A*cos(d*x+c)^6+3*B*a^3*(-1/7*sin(d
*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+1/5
*a^3*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/6*B*a^3*cos(d*x+c)^
6)
```

maxima [A] time = 0.37, size = 158, normalized size = 1.50

$$\frac{56\,Ba^3\sin(dx+c)^9+63\,(A+3\,B)a^3\sin(dx+c)^8+72\,(3\,A+B)a^3\sin(dx+c)^7+84\,(A-5\,B)a^3\sin(dx+c)^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/504*(56*B*a^3*sin(d*x + c)^9 + 63*(A + 3*B)*a^3*sin(d*x + c)^8 + 72*(3*A
+ B)*a^3*sin(d*x + c)^7 + 84*(A - 5*B)*a^3*sin(d*x + c)^6 - 504*(A + B)*a^3
*sin(d*x + c)^5 - 126*(5*A - B)*a^3*sin(d*x + c)^4 + 168*(A + 3*B)*a^3*sin(
d*x + c)^3 + 252*(3*A + B)*a^3*sin(d*x + c)^2 + 504*A*a^3*sin(d*x + c))/d
```

mupad [B] time = 0.14, size = 156, normalized size = 1.49

$$\frac{\frac{a^3\sin(c+d\,x)^2\,(3\,A+B)}{2}+\frac{a^3\sin(c+d\,x)^3\,(A+3\,B)}{3}+\frac{a^3\sin(c+d\,x)^7\,(3\,A+B)}{7}+\frac{a^3\sin(c+d\,x)^6\,(A-5\,B)}{6}+\frac{a^3\sin(c+d\,x)^8\,(A+3\,B)}{8}+\frac{B\,a^3\sin(c+d\,x}{9}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^2*(3*A + B))/2 + (a^3*sin(c + d*x)^3*(A + 3*B))/3 + (a^3
*sin(c + d*x)^7*(3*A + B))/7 + (a^3*sin(c + d*x)^6*(A - 5*B))/6 + (a^3*sin(
c + d*x)^8*(A + 3*B))/8 + (B*a^3*sin(c + d*x)^9)/9 - (a^3*sin(c + d*x)^4*(5
*A - B))/4 + A*a^3*sin(c + d*x) - a^3*sin(c + d*x)^5*(A + B))/d
```

sympy [A] time = 16.66, size = 471, normalized size = 4.49

$$\begin{cases} \frac{Aa^3\sin^8(c+dx)}{24d} + \frac{8Aa^3\sin^7(c+dx)}{35d} + \frac{Aa^3\sin^6(c+dx)\cos^2(c+dx)}{6d} + \frac{4Aa^3\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{8Aa^3\sin^5(c+dx)}{15d} + \frac{Aa^3\sin^4(c+dx)\cos}{4d} \\ x(A+B\sin(c))(a\sin(c)+a)^3\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*sin(c + d*x)**8/(24*d) + 8*A*a**3*sin(c + d*x)**7/(35*d)
+ A*a**3*sin(c + d*x)**6*cos(c + d*x)**2/(6*d) + 4*A*a**3*sin(c + d*x)**5*c
os(c + d*x)**2/(5*d) + 8*A*a**3*sin(c + d*x)**5/(15*d) + A*a**3*sin(c + d*x
)**4*cos(c + d*x)**4/(4*d) + A*a**3*sin(c + d*x)**3*cos(c + d*x)**4/d + 4*A
*a**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + A*a**3*sin(c + d*x)*cos(c + d
*x)**4/d - A*a**3*cos(c + d*x)**6/(2*d) + 8*B*a**3*sin(c + d*x)**9/(315*d)
+ B*a**3*sin(c + d*x)**8/(8*d) + 4*B*a**3*sin(c + d*x)**7*cos(c + d*x)**2/(
35*d) + 8*B*a**3*sin(c + d*x)**7/(35*d) + B*a**3*sin(c + d*x)**6*cos(c + d*
x)**2/(2*d) + B*a**3*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 4*B*a**3*sin(c
 + d*x)**5*cos(c + d*x)**2/(5*d) + 3*B*a**3*sin(c + d*x)**4*cos(c + d*x)**4
/(4*d) + B*a**3*sin(c + d*x)**3*cos(c + d*x)**4/d - B*a**3*cos(c + d*x)**6/
(6*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**3*cos(c)**5, True))
```

3.988 $\int \cos^3(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$-\frac{B(a\sin(c+dx)+a)^7}{7a^4d}-\frac{(A-3B)(a\sin(c+dx)+a)^6}{6a^3d}+\frac{2(A-B)(a\sin(c+dx)+a)^5}{5a^2d}$$

```
[Out] 2/5*(A-B)*(a+a*sin(d*x+c))^5/a^2/d-1/6*(A-3*B)*(a+a*sin(d*x+c))^6/a^3/d-1/7
*B*(a+a*sin(d*x+c))^7/a^4/d
```

Rubi [A] time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$-\frac{(A-3B)(a\sin(c+dx)+a)^6}{6a^3d}+\frac{2(A-B)(a\sin(c+dx)+a)^5}{5a^2d}-\frac{B(a\sin(c+dx)+a)^7}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[c + d*x])^5)/(5*a^2*d) - ((A - 3*B)*(a + a*Sin[c + d*
x])^6)/(6*a^3*d) - (B*(a + a*Sin[c + d*x])^7)/(7*a^4*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^4\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+dx\right.}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(2a(A-B)(a+x)^4+(-A+3B)(a+x)^5-\right.\right.}{a^3d}$$
$$= \frac{2(A-B)(a+a\sin(c+dx))^5}{5a^2d} - \frac{(A-3B)(a+a\sin(c}{6a^3d}$$

Mathematica [A] time = 0.24, size = 53, normalized size = 0.68

$$-\frac{a^3(\sin(c+dx)+1)^5\left(5(7A-9B)\sin(c+dx)-49A+30B\sin^2(c+dx)+9B\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/210*(a^3*(1 + Sin[c + d*x])^5*(-49*A + 9*B + 5*(7*A - 9*B)*Sin[c + d*x]
+ 30*B*Sin[c + d*x]^2))/d
```

fricas [A] time = 0.73, size = 115, normalized size = 1.47

$$\frac{35\,(A+3\,B)a^3\cos(dx+c)^6-210\,(A+B)a^3\cos(dx+c)^4+2\left(15\,Ba^3\cos(dx+c)^6-3\,(21\,A+29\,B)a^3\cos(dx+\right.}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/210*(35*(A + 3*B)*a^3*cos(d*x + c)^6 - 210*(A + B)*a^3*cos(d*x + c)^4 + 2
*(15*B*a^3*cos(d*x + c)^6 - 3*(21*A + 29*B)*a^3*cos(d*x + c)^4 + 8*(7*A + 3
*B)*a^3*cos(d*x + c)^2 + 16*(7*A + 3*B)*a^3)*sin(d*x + c))/d
```

giac [B] time = 0.35, size = 172, normalized size = 2.21

$$-\frac{30\,Ba^3\sin(dx+c)^7+35\,Aa^3\sin(dx+c)^6+105\,Ba^3\sin(dx+c)^6+126\,Aa^3\sin(dx+c)^5+84\,Ba^3\sin(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/210*(30*B*a^3*sin(d*x + c)^7 + 35*A*a^3*sin(d*x + c)^6 + 105*B*a^3*sin(d
*x + c)^6 + 126*A*a^3*sin(d*x + c)^5 + 84*B*a^3*sin(d*x + c)^5 + 105*A*a^3*
sin(d*x + c)^4 - 105*B*a^3*sin(d*x + c)^4 - 140*A*a^3*sin(d*x + c)^3 - 210*
B*a^3*sin(d*x + c)^3 - 315*A*a^3*sin(d*x + c)^2 - 105*B*a^3*sin(d*x + c)^2
- 210*A*a^3*sin(d*x + c))/d
```

maple [B] time = 0.48, size = 265, normalized size = 3.40

$$a^3 A\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)+B\,a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7}-\frac{3\sin(dx+c)\left(\cos^4(dx+c)\right)}{35}+\frac{\left(2+\cos^2(dx+c)\right)\sin(\cdots}{35}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+B*a^3*(-1/7*s
in(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*(2+cos(d*x+c)^2)
*sin(d*x+c))+3*a^3*A*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*si
n(d*x+c))+3*B*a^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)-3/4*a^
3*A*cos(d*x+c)^4+3*B*a^3*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2
)*sin(d*x+c))+1/3*a^3*A*(2+cos(d*x+c)^2)*sin(d*x+c)-1/4*B*a^3*cos(d*x+c)^4)
```

maxima [A] time = 0.44, size = 126, normalized size = 1.62

$$-\frac{30\,Ba^3\sin(dx+c)^7+35\,(A+3\,B)a^3\sin(dx+c)^6+42\,(3\,A+2\,B)a^3\sin(dx+c)^5+105\,(A-B)a^3\sin(dx+c\cdots}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/210*(30*B*a^3*sin(d*x + c)^7 + 35*(A + 3*B)*a^3*sin(d*x + c)^6 + 42*(3*A
 + 2*B)*a^3*sin(d*x + c)^5 + 105*(A - B)*a^3*sin(d*x + c)^4 - 70*(2*A + 3*B
)*a^3*sin(d*x + c)^3 - 105*(3*A + B)*a^3*sin(d*x + c)^2 - 210*A*a^3*sin(d*x
 + c))/d
```

mupad [B] time = 9.14, size = 126, normalized size = 1.62

$$-\frac{\frac{a^3\sin(c+d\,x)^4\,(A-B)}{2}-\frac{a^3\sin(c+d\,x)^2\,(3\,A+B)}{2}+\frac{a^3\sin(c+d\,x)^6\,(A+3\,B)}{6}+\frac{B\,a^3\sin(c+d\,x)^7}{7}-\frac{a^3\sin(c+d\,x)^3\,(2\,A+3\,B)}{3}+\frac{a^3\sin(c+d\,x)^5\,(3\,A\cdots}{5}\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] -((a^3*sin(c + d*x)^4*(A - B))/2 - (a^3*sin(c + d*x)^2*(3*A + B))/2 + (a^3*
sin(c + d*x)^6*(A + 3*B))/6 + (B*a^3*sin(c + d*x)^7)/7 - (a^3*sin(c + d*x)^
3*(2*A + 3*B))/3 + (a^3*sin(c + d*x)^5*(3*A + 2*B))/5 - A*a^3*sin(c + d*x))
/d
```

sympy [A] time = 6.34, size = 313, normalized size = 4.01

$$\begin{cases} \frac{Aa^3\sin^6(c+dx)}{12d} + \frac{2Aa^3\sin^5(c+dx)}{5d} + \frac{Aa^3\sin^4(c+dx)\cos^2(c+dx)}{4d} + \frac{Aa^3\sin^3(c+dx)\cos^2(c+dx)}{d} + \frac{2Aa^3\sin^3(c+dx)}{3d} + \frac{Aa^3\sin(c+dx)\cos^2(c+dx)}{d} \\ x(A+B\sin(c))(a\sin(c)+a)^3\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*sin(c + d*x)**6/(12*d) + 2*A*a**3*sin(c + d*x)**5/(5*d) +
 A*a**3*sin(c + d*x)**4*cos(c + d*x)**2/(4*d) + A*a**3*sin(c + d*x)**3*cos(
c + d*x)**2/d + 2*A*a**3*sin(c + d*x)**3/(3*d) + A*a**3*sin(c + d*x)*cos(c
+ d*x)**2/d - 3*A*a**3*cos(c + d*x)**4/(4*d) + 2*B*a**3*sin(c + d*x)**7/(35
*d) + B*a**3*sin(c + d*x)**6/(4*d) + B*a**3*sin(c + d*x)**5*cos(c + d*x)**2
/(5*d) + 2*B*a**3*sin(c + d*x)**5/(5*d) + 3*B*a**3*sin(c + d*x)**4*cos(c +
d*x)**2/(4*d) + B*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d - B*a**3*cos(c + d
*x)**4/(4*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**3*cos(c)**3, Tru
e))
```

3.989 $\int \cos(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=51

$$\frac{B(a\sin(c+dx)+a)^5}{5a^2d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{4ad}$$

```
[Out] 1/4*(A-B)*(a+a*sin(d*x+c))^4/a/d+1/5*B*(a+a*sin(d*x+c))^5/a^2/d
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{B(a\sin(c+dx)+a)^5}{5a^2d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^4)/(4*a*d) + (B*(a + a*Sin[c + d*x])^5)/(5*a^
2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)^3\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left((A-B)(a+x)^3+\frac{B(a+x)^4}{a}\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{(A-B)(a+a\sin(c+dx))^4}{4ad}+\frac{B(a+a\sin(c+dx))^5}{5a^2d}$$

Mathematica [A] time = 0.09, size = 36, normalized size = 0.71

$$\frac{a^3(\sin(c+dx)+1)^4(5A+4B\sin(c+dx)-B)}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^4*(5*A - B + 4*B*Sin[c + d*x]))/(20*d)
```

fricas [A] time = 0.56, size = 94, normalized size = 1.84

$$\frac{5\,(A+3\,B)a^3\cos\left(dx+c\right)^4-40\,(A+B)a^3\cos\left(dx+c\right)^2+4\left(Ba^3\cos\left(dx+c\right)^4-(5\,A+7\,B)a^3\cos\left(dx+c\right)^2+2\,(\right.}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/20*(5*(A + 3*B)*a^3*cos(d*x + c)^4 - 40*(A + B)*a^3*cos(d*x + c)^2 + 4*(B
*a^3*cos(d*x + c)^4 - (5*A + 7*B)*a^3*cos(d*x + c)^2 + 2*(5*A + 3*B)*a^3)*s
in(d*x + c))/d
```

giac [B] time = 0.23, size = 116, normalized size = 2.27

$$\frac{4\,Ba^3\sin\left(dx+c\right)^5+5\,Aa^3\sin\left(dx+c\right)^4+15\,Ba^3\sin\left(dx+c\right)^4+20\,Aa^3\sin\left(dx+c\right)^3+20\,Ba^3\sin\left(dx+c\right)^3+30}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/20*(4*B*a^3*sin(d*x + c)^5 + 5*A*a^3*sin(d*x + c)^4 + 15*B*a^3*sin(d*x +
c)^4 + 20*A*a^3*sin(d*x + c)^3 + 20*B*a^3*sin(d*x + c)^3 + 30*A*a^3*sin(d*x
 + c)^2 + 10*B*a^3*sin(d*x + c)^2 + 20*A*a^3*sin(d*x + c))/d
```

maple [B] time = 0.23, size = 98, normalized size = 1.92

$$\frac{\frac{B a^3\left(\sin^5(dx+c)\right)}{5}+\frac{\left(a^3A+3B a^3\right)\left(\sin^4(dx+c)\right)}{4}+\frac{\left(3a^3A+3B a^3\right)\left(\sin^3(dx+c)\right)}{3}+\frac{\left(3a^3A+B a^3\right)\left(\sin^2(dx+c)\right)}{2}+a^3A\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/5*B*a^3*sin(d*x+c)^5+1/4*(A*a^3+3*B*a^3)*sin(d*x+c)^4+1/3*(3*A*a^3+3
*B*a^3)*sin(d*x+c)^3+1/2*(3*A*a^3+B*a^3)*sin(d*x+c)^2+a^3*A*sin(d*x+c))
```

maxima [A] time = 0.31, size = 84, normalized size = 1.65

$$\frac{4\,Ba^3\sin\left(dx+c\right)^5+5\left(A+3\,B\right)a^3\sin\left(dx+c\right)^4+20\left(A+B\right)a^3\sin\left(dx+c\right)^3+10\left(3\,A+B\right)a^3\sin\left(dx+c\right)^2+20}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/20*(4*B*a^3*sin(d*x + c)^5 + 5*(A + 3*B)*a^3*sin(d*x + c)^4 + 20*(A + B)*
a^3*sin(d*x + c)^3 + 10*(3*A + B)*a^3*sin(d*x + c)^2 + 20*A*a^3*sin(d*x + c
))/d
```

mupad [B] time = 9.07, size = 81, normalized size = 1.59

$$\frac{\frac{a^3\sin(c+d\,x)^2\,(3\,A+B)}{2}+\frac{a^3\sin(c+d\,x)^4\,(A+3\,B)}{4}+\frac{B\,a^3\sin(c+d\,x)^5}{5}+A\,a^3\,\sin\left(c+d\,x\right)+a^3\sin\left(c+d\,x\right)^3\,\left(A+B\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^2*(3*A + B))/2 + (a^3*sin(c + d*x)^4*(A + 3*B))/4 + (B*a
^3*sin(c + d*x)^5)/5 + A*a^3*sin(c + d*x) + a^3*sin(c + d*x)^3*(A + B))/d
```

sympy [A] time = 2.02, size = 151, normalized size = 2.96

$$\begin{cases}\frac{Aa^3\sin^4(c+dx)}{4d}+\frac{Aa^3\sin^3(c+dx)}{d}+\frac{Aa^3\sin(c+dx)}{d}-\frac{3Aa^3\cos^2(c+dx)}{2d}+\frac{Ba^3\sin^5(c+dx)}{5d}+\frac{3Ba^3\sin^4(c+dx)}{4d}+\frac{Ba^3\sin^3(c+dx)}{d}-\frac{Ba^3 c}{} \\ x\left(A+B\sin\left(c\right)\right)\left(a\sin\left(c\right)+a\right)^3\cos\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*sin(c + d*x)**4/(4*d) + A*a**3*sin(c + d*x)**3/d + A*a**3
*sin(c + d*x)/d - 3*A*a**3*cos(c + d*x)**2/(2*d) + B*a**3*sin(c + d*x)**5/(
5*d) + 3*B*a**3*sin(c + d*x)**4/(4*d) + B*a**3*sin(c + d*x)**3/d - B*a**3*c
os(c + d*x)**2/(2*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**3*cos(c)
, True))
```

3.990 $\int \sec(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a^3(A+B)\sin^2(c+dx)}{2d}-\frac{3a^3(A+B)\sin(c+dx)}{d}-\frac{4a^3(A+B)\log(1-\sin(c+dx))}{d}-\frac{B(a\sin(c+dx)+a)^3}{3d}$$

```
[Out] -4*a^3*(A+B)*ln(1-sin(d*x+c))/d-3*a^3*(A+B)*sin(d*x+c)/d-1/2*a^3*(A+B)*sin(
d*x+c)^2/d-1/3*B*(a+a*sin(d*x+c))^3/d
```

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 77}

$$-\frac{a^3(A+B)\sin^2(c+dx)}{2d}-\frac{3a^3(A+B)\sin(c+dx)}{d}-\frac{4a^3(A+B)\log(1-\sin(c+dx))}{d}-\frac{B(a\sin(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (-4*a^3*(A + B)*Log[1 - Sin[c + d*x]])/d - (3*a^3*(A + B)*Sin[c + d*x])/d -
 (a^3*(A + B)*Sin[c + d*x]^2)/(2*d) - (B*(a + a*Sin[c + d*x])^3)/(3*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{a\,\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{a}\right)}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(-3a(A+B)+\frac{4a^2(A+B)}{a-x}-(A+B)x-\frac{B(a+\dots}{a}\right.\right.}{d}$$
$$= -\frac{4a^3(A+B)\log(1-\sin(c+dx))}{d} - \frac{3a^3(A+B)\sin(c\dots}{d}$$

Mathematica [A] time = 0.13, size = 68, normalized size = 0.84

$$-\frac{a^3\left(3(A+3B)\sin^2(c+dx)+6(3A+4B)\sin(c+dx)+24(A+B)\log(1-\sin(c+dx))+2B\sin^3(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a^3*(24*(A + B)*Log[1 - Sin[c + d*x]] + 6*(3*A + 4*B)*Sin[c + d*x] +
3*(A + 3*B)*Sin[c + d*x]^2 + 2*B*Sin[c + d*x]^3))/d
```

fricas [A] time = 0.69, size = 77, normalized size = 0.95

$$\frac{3\,(A+3\,B)a^3\cos(dx+c)^2-24\,(A+B)a^3\log(-\sin(dx+c)+1)+2\left(Ba^3\cos(dx+c)^2-(9\,A+13\,B)a^3\right)\sin(d\dots}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(A + 3*B)*a^3*cos(d*x + c)^2 - 24*(A + B)*a^3*log(-sin(d*x + c) + 1)
 + 2*(B*a^3*cos(d*x + c)^2 - (9*A + 13*B)*a^3)*sin(d*x + c))/d
```

giac [B] time = 0.21, size = 289, normalized size = 3.57

$$\frac{2\left(6\left(Aa^3+Ba^3\right)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-12\left(Aa^3+Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{11\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+11\dots}{\dots}\right.}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 2/3*(6*(A*a^3 + B*a^3)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*(A*a^3 + B*a^3)
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - (11*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 11
*B*a^3*tan(1/2*d*x + 1/2*c)^6 + 9*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^3*t
an(1/2*d*x + 1/2*c)^5 + 36*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 42*B*a^3*tan(1/2*
d*x + 1/2*c)^4 + 18*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 28*B*a^3*tan(1/2*d*x + 1
/2*c)^3 + 36*A*a^3*tan(1/2*d*x + 1/2*c)^2 + 42*B*a^3*tan(1/2*d*x + 1/2*c)^2
 + 9*A*a^3*tan(1/2*d*x + 1/2*c) + 12*B*a^3*tan(1/2*d*x + 1/2*c) + 11*A*a^3
+ 11*B*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [B] time = 0.36, size = 161, normalized size = 1.99

$$-\frac{a^3 A \left(\sin^2(dx+c)\right)}{2d}-\frac{4a^3 A \ln(\cos(dx+c))}{d}-\frac{B a^3 \left(\sin^3(dx+c)\right)}{3d}-\frac{4a^3 B \sin(dx+c)}{d}+\frac{4B a^3 \ln(\sec(dx+c)+t}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a^3*A*sin(d*x+c)^2-4/d*a^3*A*ln(cos(d*x+c))-1/3/d*B*a^3*sin(d*x+c)^3
-4*a^3*B*sin(d*x+c)/d+4/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))-3/d*a^3*A*sin(d*x
+c)+4/d*a^3*A*ln(sec(d*x+c)+tan(d*x+c))-3/2/d*B*a^3*sin(d*x+c)^2-4/d*B*a^3*
ln(cos(d*x+c))
```

maxima [A] time = 0.35, size = 73, normalized size = 0.90

$$-\frac{2 Ba^3 \sin(dx+c)^3 + 3(A+3B)a^3 \sin(dx+c)^2 + 24(A+B)a^3 \log(\sin(dx+c)-1) + 6(3A+4B)a^3 \sin(dx}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/6*(2*B*a^3*sin(d*x + c)^3 + 3*(A + 3*B)*a^3*sin(d*x + c)^2 + 24*(A + B)*
a^3*log(sin(d*x + c) - 1) + 6*(3*A + 4*B)*a^3*sin(d*x + c))/d
```

mupad [B] time = 0.09, size = 100, normalized size = 1.23

$$-\frac{\sin(c+d\,x)^2\,\left(\frac{a^3\,(A+2\,B)}{2}+\frac{B\,a^3}{2}\right)+\sin(c+d\,x)\,\left(a^3\,\left(A+2\,B\right)+a^3\,\left(2\,A+B\right)+B\,a^3\right)+\ln(\sin(c+d\,x)-1)\,\left(4\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x),x)
```

```
[Out] -(sin(c + d*x)^2*((a^3*(A + 2*B))/2 + (B*a^3)/2) + sin(c + d*x)*(a^3*(A + 2
*B) + a^3*(2*A + B) + B*a^3) + log(sin(c + d*x) - 1)*(4*A*a^3 + 4*B*a^3) +
(B*a^3*sin(c + d*x)^3)/3)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int A\sec(c+dx)\,dx+\int 3A\sin(c+dx)\sec(c+dx)\,dx+\int 3A\sin^2(c+dx)\sec(c+dx)\,dx+\int A\sin^3(c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x), x) + Integral(3*A*sin(c + d*x)*sec(c + d*x),
 x) + Integral(3*A*sin(c + d*x)**2*sec(c + d*x), x) + Integral(A*sin(c + d*
x)**3*sec(c + d*x), x) + Integral(B*sin(c + d*x)*sec(c + d*x), x) + Integra
l(3*B*sin(c + d*x)**2*sec(c + d*x), x) + Integral(3*B*sin(c + d*x)**3*sec(c
 + d*x), x) + Integral(B*sin(c + d*x)**4*sec(c + d*x), x))
```

3.991 $\int \sec^3(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=62

$$\frac{2a^4(A+B)}{d(a-a\sin(c+dx))}+\frac{a^3(A+3B)\log(1-\sin(c+dx))}{d}+\frac{a^3B\sin(c+dx)}{d}$$

```
[Out] a^3*(A+3*B)*ln(1-sin(d*x+c))/d+a^3*B*sin(d*x+c)/d+2*a^4*(A+B)/d/(a-a*sin(d*
x+c))
```

Rubi [A] time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{2a^4(A+B)}{d(a-a\sin(c+dx))}+\frac{a^3(A+3B)\log(1-\sin(c+dx))}{d}+\frac{a^3B\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(A + 3*B)*Log[1 - Sin[c + d*x]])/d + (a^3*B*Sin[c + d*x])/d + (2*a^4*(
A + B))/(d*(a - a*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{a^3 \operatorname{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{a}\right)}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3 \operatorname{Subst}\left(\int \left(\frac{B}{a}+\frac{2a(A+B)}{(a-x)^2}+\frac{-A-3B}{a-x}\right)dx, x, a\sin(c+d$$
$$= \frac{a^3(A+3B)\log(1-\sin(c+dx))}{d}+\frac{a^3B\sin(c+dx)}{d}+$$

Mathematica [A] time = 0.12, size = 48, normalized size = 0.77

$$\frac{a^3\left(-\frac{2(A+B)}{\sin(c+dx)-1}+(A+3B)\log(1-\sin(c+dx))+B\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*((A + 3*B)*Log[1 - Sin[c + d*x]] - (2*(A + B))/(-1 + Sin[c + d*x]) + B
*Sin[c + d*x]))/d
```

fricas [A] time = 0.54, size = 89, normalized size = 1.44

$$-\frac{Ba^3\cos(dx+c)^2+Ba^3\sin(dx+c)+(2A+B)a^3-\left((A+3B)a^3\sin(dx+c)-(A+3B)a^3\right)\log(-\sin(dx+c)}{d\sin(dx+c)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -(B*a^3*cos(d*x + c)^2 + B*a^3*sin(d*x + c) + (2*A + B)*a^3 - ((A + 3*B)*a^
3*sin(d*x + c) - (A + 3*B)*a^3)*log(-sin(d*x + c) + 1))/(d*sin(d*x + c) - d
)
```

giac [B] time = 0.26, size = 228, normalized size = 3.68

$$-\frac{\left(Aa^3+3Ba^3\right)\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)-2\left(Aa^3+3Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3Ba^3}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -((A*a^3 + 3*B*a^3)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*(A*a^3 + 3*B*a^3)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - (A*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*B*a^3
*tan(1/2*d*x + 1/2*c)^2 + 2*B*a^3*tan(1/2*d*x + 1/2*c) + A*a^3 + 3*B*a^3)/(
tan(1/2*d*x + 1/2*c)^2 + 1) + (3*A*a^3*tan(1/2*d*x + 1/2*c)^2 + 9*B*a^3*tan
(1/2*d*x + 1/2*c)^2 - 10*A*a^3*tan(1/2*d*x + 1/2*c) - 22*B*a^3*tan(1/2*d*x
+ 1/2*c) + 3*A*a^3 + 9*B*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^2)/d
```

maple [B] time = 0.58, size = 290, normalized size = 4.68

$$\frac{a^3 A\left(\tan^2(dx+c)\right)}{2d}+\frac{a^3 A\ln(\cos(dx+c))}{d}+\frac{B a^3\left(\sin^5(dx+c)\right)}{2d\cos(dx+c)^2}+\frac{B a^3\left(\sin^3(dx+c)\right)}{2d}+\frac{3a^3B\sin(dx+c)}{d}-\frac{3B a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/2/d*a^3*A*tan(d*x+c)^2+1/d*a^3*A*ln(cos(d*x+c))+1/2/d*B*a^3*sin(d*x+c)^5/
cos(d*x+c)^2+1/2/d*B*a^3*sin(d*x+c)^3+3*a^3*B*sin(d*x+c)/d-3/d*B*a^3*ln(sec
(d*x+c)+tan(d*x+c))+3/2/d*a^3*A*sin(d*x+c)^3/cos(d*x+c)^2+3/2/d*a^3*A*sin(d
*x+c)-1/d*a^3*A*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*B*a^3*tan(d*x+c)^2+3/d*B*a^
3*ln(cos(d*x+c))+3/2/d*a^3*A/cos(d*x+c)^2+3/2/d*B*a^3*sin(d*x+c)^3/cos(d*x+
c)^2+1/2/d*a^3*A*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a^3/cos(d*x+c)^2
```

maxima [A] time = 0.31, size = 52, normalized size = 0.84

$$\frac{(A+3\,B)a^3\log(\sin(dx+c)-1)+Ba^3\sin(dx+c)-\frac{2\,(A+B)a^3}{\sin(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] ((A + 3*B)*a^3*log(sin(d*x + c) - 1) + B*a^3*sin(d*x + c) - 2*(A + B)*a^3/(
sin(d*x + c) - 1))/d
```

mupad [B] time = 0.08, size = 63, normalized size = 1.02

$$\frac{\ln(\sin(c+d\,x)-1)\,\left(A\,a^3+3\,B\,a^3\right)-\frac{2\,A\,a^3+2\,B\,a^3}{\sin(c+d\,x)-1}+B\,a^3\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^3,x)
```

```
[Out] (log(sin(c + d*x) - 1)*(A*a^3 + 3*B*a^3) - (2*A*a^3 + 2*B*a^3)/(sin(c + d*x
) - 1) + B*a^3*sin(c + d*x))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.992 $\int \sec^5(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{a^3(aA+aB\sin(c+dx))^2}{2d(A+B)(a-a\sin(c+dx))^2}$$

[Out] 1/2*a^3*(a*A+a*B*sin(d*x+c))^2/(A+B)/d/(a-a*sin(d*x+c))^2

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 37}

$$\frac{a^3(aA+aB\sin(c+dx))^2}{2d(A+B)(a-a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]

[Out] (a^3*(a*A + a*B*Sin[c + d*x])^2)/(2*(A + B)*d*(a - a*Sin[c + d*x])^2)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3(aA+aB\sin(c+dx))^2}{2(A+B)d(a-a\sin(c+dx))^2}$$

Mathematica [A] time = 0.05, size = 37, normalized size = 0.86

$$\frac{a^3(A+B\sin(c+dx))^2}{2d(A+B)(\sin(c+dx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(A + B*Sin[c + d*x])^2)/(2*(A + B)*d*(-1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.75, size = 49, normalized size = 1.14

$$-\frac{2\,Ba^3\sin(dx+c)+(A-B)a^3}{2\left(d\cos(dx+c)^2+2\,d\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*B*a^3*sin(d*x + c) + (A - B)*a^3)/(d*cos(d*x + c)^2 + 2*d*sin(d*x +
 c) - 2*d)
```

giac [A] time = 0.26, size = 82, normalized size = 1.91

$$\frac{2\left(Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^3*tan(1/2*d*x + 1/2*c)^2 + B*a^3*tan(
1/2*d*x + 1/2*c)^2 + A*a^3*tan(1/2*d*x + 1/2*c))/(d*(tan(1/2*d*x + 1/2*c) -
 1)^4)
```

maple [B] time = 0.58, size = 312, normalized size = 7.26

$$\frac{a^{3} A \left(\sin^{4}\left(dx+c\right)\right)}{4 d \cos\left(dx+c\right)^{4}}+\frac{B a^{3} \left(\sin^{5}\left(dx+c\right)\right)}{4 d \cos\left(dx+c\right)^{4}}-\frac{B a^{3} \left(\sin^{5}\left(dx+c\right)\right)}{8 d \cos\left(dx+c\right)^{2}}-\frac{B a^{3} \left(\sin^{3}\left(dx+c\right)\right)}{8 d}+\frac{3 a^{3} A \left(\sin^{3}\left(dx+c\right)\right)}{4 d \cos\left(dx+c\right)^{4}}+\frac{3 a^{3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a^3*A*sin(d*x+c)^4/cos(d*x+c)^4+1/4/d*B*a^3*sin(d*x+c)^5/cos(d*x+c)^4
-1/8/d*B*a^3*sin(d*x+c)^5/cos(d*x+c)^2-1/8/d*B*a^3*sin(d*x+c)^3+3/4/d*a^3*A
*sin(d*x+c)^3/cos(d*x+c)^4+3/8/d*a^3*A*sin(d*x+c)^3/cos(d*x+c)^2+3/8/d*a^3*
A*sin(d*x+c)+3/4/d*B*a^3*sin(d*x+c)^4/cos(d*x+c)^4+3/4/d*a^3*A/cos(d*x+c)^4
+3/4/d*B*a^3*sin(d*x+c)^3/cos(d*x+c)^4+3/8/d*B*a^3*sin(d*x+c)^3/cos(d*x+c)^
2+1/4/d*a^3*A*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^3*A*sec(d*x+c)*tan(d*x+c)+1/4
/d*B*a^3/cos(d*x+c)^4
```

maxima [A] time = 0.30, size = 47, normalized size = 1.09

$$\frac{2 \, B a^{3} \sin\left(dx+c\right) + (A-B) a^{3}}{2 \left(\sin\left(dx+c\right)^{2} - 2 \, \sin\left(dx+c\right) + 1\right) d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/2*(2*B*a^3*sin(d*x + c) + (A - B)*a^3)/((sin(d*x + c)^2 - 2*sin(d*x + c)
+ 1)*d)
```

mupad [B] time = 9.15, size = 36, normalized size = 0.84

$$\frac{\frac{a^3\,\left(A-B\right)}{2}+B\,a^3\,\sin\left(c+d\,x\right)}{d\,{\left(\sin\left(c+d\,x\right)-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] ((a^3*(A - B))/2 + B*a^3*sin(c + d*x))/(d*(sin(c + d*x) - 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
[Out] Timed out
```

3.993 $\int \sec^7(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{a^6(A+B)}{6d(a-a\sin(c+dx))^3}+\frac{a^5(A-B)}{8d(a-a\sin(c+dx))^2}+\frac{a^4(A-B)}{8d(a-a\sin(c+dx))}+\frac{a^3(A-B)\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] 1/8*a^3*(A-B)*arctanh(sin(d*x+c))/d+1/6*a^6*(A+B)/d/(a-a*sin(d*x+c))^3+1/8*
a^5*(A-B)/d/(a-a*sin(d*x+c))^2+1/8*a^4*(A-B)/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$\frac{a^6(A+B)}{6d(a-a\sin(c+dx))^3}+\frac{a^5(A-B)}{8d(a-a\sin(c+dx))^2}+\frac{a^4(A-B)}{8d(a-a\sin(c+dx))}+\frac{a^3(A-B)\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(A - B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^6*(A + B))/(6*d*(a - a*Sin[c
 + d*x])^3) + (a^5*(A - B))/(8*d*(a - a*Sin[c + d*x])^2) + (a^4*(A - B))/(8
*d*(a - a*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{A+B}{2a(a-x)^4} + \frac{A-B}{4a^2(a-x)^3} + \frac{A-B}{8a^3(a-x)^2} + \frac{A-B}{8a^3(a^2-x^2)}\right.\right.}{d}$$

$$= \frac{a^6(A+B)}{6d(a-a\sin(c+dx))^3} + \frac{a^5(A-B)}{8d(a-a\sin(c+dx))^2} + \frac{}{8d}$$

$$= \frac{a^3(A-B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^6(A+B)}{6d(a-a\sin(c+dx}$$

Mathematica [A] time = 0.26, size = 95, normalized size = 0.90

$$\frac{a^3\left(-3(A-B)\sin^2(c+dx)+9(A-B)\sin(c+dx)-3(A-B)\tanh^{-1}(\sin(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.\right.}{24d(\sin(c+dx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(2*(-5*A + B) - 3*(A - B)*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^6 + 9*(A - B)*Sin[c + d*x] - 3*(A - B)*Sin[c + d*x]^2))/(24
*d*(-1 + Sin[c + d*x])^3)
```

fricas [B] time = 0.72, size = 242, normalized size = 2.30

$$\frac{6\,(A-B)a^3\cos(dx+c)^2+18\,(A-B)a^3\sin(dx+c)-2\,(13\,A-5\,B)a^3+3\left(3\,(A-B)a^3\cos(dx+c)^2-4\,(A-B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/48*(6*(A - B)*a^3*cos(d*x + c)^2 + 18*(A - B)*a^3*sin(d*x + c) - 2*(13*A
- 5*B)*a^3 + 3*(3*(A - B)*a^3*cos(d*x + c)^2 - 4*(A - B)*a^3 - ((A - B)*a^3
*cos(d*x + c)^2 - 4*(A - B)*a^3)*sin(d*x + c))*log(sin(d*x + c) + 1) - 3*(3
*(A - B)*a^3*cos(d*x + c)^2 - 4*(A - B)*a^3 - ((A - B)*a^3*cos(d*x + c)^2 -
 4*(A - B)*a^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/(3*d*cos(d*x + c)^2 -
 (d*cos(d*x + c)^2 - 4*d)*sin(d*x + c) - 4*d)
```

giac [A] time = 0.29, size = 158, normalized size = 1.50

$$\frac{6\left(Aa^3 - Ba^3\right)\log\left(|\sin\left(dx+c\right)+1|\right) - 6\left(Aa^3 - Ba^3\right)\log\left(|\sin\left(dx+c\right)-1|\right) + \frac{11\,Aa^3\sin(dx+c)^3 - 11\,Ba^3\sin(dx+c)^3 - 45\,A}{}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/96*(6*(A*a^3 - B*a^3)*log(abs(sin(d*x + c) + 1)) - 6*(A*a^3 - B*a^3)*log(
abs(sin(d*x + c) - 1)) + (11*A*a^3*sin(d*x + c)^3 - 11*B*a^3*sin(d*x + c)^3
 - 45*A*a^3*sin(d*x + c)^2 + 45*B*a^3*sin(d*x + c)^2 + 69*A*a^3*sin(d*x + c
) - 69*B*a^3*sin(d*x + c) - 51*A*a^3 + 19*B*a^3)/(sin(d*x + c) - 1)^3)/d
```

maple [B] time = 0.57, size = 521, normalized size = 4.96

$$\frac{3a^3A\sin\left(dx+c\right)}{16d}+\frac{a^3A}{2d\cos\left(dx+c\right)^6}+\frac{B\,a^3}{6d\cos\left(dx+c\right)^6}-\frac{B\,a^3\left(\sin^3\left(dx+c\right)\right)}{48d}+\frac{a^3B\sin\left(dx+c\right)}{8d}-\frac{B\,a^3\ln\left(\sec\left(dx+\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 3/16/d*a^3*A*sin(d*x+c)+1/2/d*a^3*A/cos(d*x+c)^6+1/6/d*B*a^3/cos(d*x+c)^6-1
/48/d*B*a^3*sin(d*x+c)^3+1/8*a^3*B*sin(d*x+c)/d-1/8/d*B*a^3*ln(sec(d*x+c)+t
an(d*x+c))+1/6/d*a^3*A*sin(d*x+c)^4/cos(d*x+c)^6+1/6/d*B*a^3*sin(d*x+c)^5/c
os(d*x+c)^6+1/24/d*B*a^3*sin(d*x+c)^5/cos(d*x+c)^4-1/48/d*B*a^3*sin(d*x+c)^
5/cos(d*x+c)^2+1/2/d*a^3*A*sin(d*x+c)^3/cos(d*x+c)^6+3/8/d*a^3*A*sin(d*x+c)
^3/cos(d*x+c)^4+3/16/d*a^3*A*sin(d*x+c)^3/cos(d*x+c)^2+1/2/d*B*a^3*sin(d*x+
c)^4/cos(d*x+c)^6+1/2/d*B*a^3*sin(d*x+c)^3/cos(d*x+c)^6+3/8/d*B*a^3*sin(d*x
+c)^3/cos(d*x+c)^4+3/16/d*B*a^3*sin(d*x+c)^3/cos(d*x+c)^2+1/6/d*a^3*A*tan(d
*x+c)*sec(d*x+c)^5+5/24/d*a^3*A*tan(d*x+c)*sec(d*x+c)^3+5/16/d*a^3*A*sec(d*
x+c)*tan(d*x+c)+1/12/d*a^3*A*sin(d*x+c)^4/cos(d*x+c)^4+1/4/d*B*a^3*sin(d*x+
c)^4/cos(d*x+c)^4+1/8/d*a^3*A*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.31, size = 123, normalized size = 1.17

$$\frac{3\left(A-B\right)a^3\log\left(\sin\left(dx+c\right)+1\right) - 3\left(A-B\right)a^3\log\left(\sin\left(dx+c\right)-1\right) - \frac{2\left(3\left(A-B\right)a^3\sin(dx+c)^2 - 9\left(A-B\right)a^3\sin(dx+c) + 2\left(5A\right.\right.}{\sin(dx+c)^3 - 3\sin(dx+c)^2 + 3\sin(dx+c) - 1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(3*(A - B)*a^3*log(sin(d*x + c) + 1) - 3*(A - B)*a^3*log(sin(d*x + c)
 - 1) - 2*(3*(A - B)*a^3*sin(d*x + c)^2 - 9*(A - B)*a^3*sin(d*x + c) + 2*(5*
A - B)*a^3)/(sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 3*sin(d*x + c) - 1))/d
```

mupad [B] time = 9.14, size = 112, normalized size = 1.07

$$\frac{a^3 \operatorname{atanh}(\sin(c+d\,x))\,(A-B)}{8\,d}-\frac{\sin(c+d\,x)^2\left(\frac{A\,a^3}{8}-\frac{B\,a^3}{8}\right)+\frac{5\,A\,a^3}{12}-\frac{B\,a^3}{12}-\sin(c+d\,x)\left(\frac{3\,A\,a^3}{8}-\frac{3\,B\,a^3}{8}\right)}{d\left(\sin(c+d\,x)^3-3\sin(c+d\,x)^2+3\,\sin(c+d\,x)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^7,x)
```

```
[Out] (a^3*atanh(sin(c + d*x))*(A - B))/(8*d) - (sin(c + d*x)^2*((A*a^3)/8 - (B*a
^3)/8) + (5*A*a^3)/12 - (B*a^3)/12 - sin(c + d*x)*((3*A*a^3)/8 - (3*B*a^3)/
8))/(d*(3*sin(c + d*x) - 3*sin(c + d*x)^2 + sin(c + d*x)^3 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.994 $\int \sec^9(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=162

$$\frac{a^7(A+B)}{16d(a-a\sin(c+dx))^4}+\frac{a^6A}{12d(a-a\sin(c+dx))^3}+\frac{a^5(3A-B)}{32d(a-a\sin(c+dx))^2}+\frac{a^4(2A-B)}{16d(a-a\sin(c+dx))}-\frac{a^4(A-}{32d(a\sin(c}$$

```
[Out] 1/32*a^3*(5*A-3*B)*arctanh(sin(d*x+c))/d+1/16*a^7*(A+B)/d/(a-a*sin(d*x+c))^
4+1/12*a^6*A/d/(a-a*sin(d*x+c))^3+1/32*a^5*(3*A-B)/d/(a-a*sin(d*x+c))^2+1/1
6*a^4*(2*A-B)/d/(a-a*sin(d*x+c))-1/32*a^4*(A-B)/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.19, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$\frac{a^7(A+B)}{16d(a-a\sin(c+dx))^4}+\frac{a^5(3A-B)}{32d(a-a\sin(c+dx))^2}+\frac{a^4(2A-B)}{16d(a-a\sin(c+dx))}-\frac{a^4(A-B)}{32d(a\sin(c+dx)+a)}+\frac{a^3(5A-3B)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(5*A - 3*B)*ArcTanh[Sin[c + d*x]])/(32*d) + (a^7*(A + B))/(16*d*(a - a
*Sin[c + d*x])^4) + (a^6*A)/(12*d*(a - a*Sin[c + d*x])^3) + (a^5*(3*A - B))
/(32*d*(a - a*Sin[c + d*x])^2) + (a^4*(2*A - B))/(16*d*(a - a*Sin[c + d*x])
) - (a^4*(A - B))/(32*d*(a + a*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^9(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{a^9\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^5(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^9\,\text{Subst}\left(\int \left(\frac{A+B}{4a^2(a-x)^5}+\frac{A}{4a^3(a-x)^4}+\frac{3A-B}{16a^4(a-x)^3}+\frac{2A-}{16a^5(a-}\right.\right.}{d}$$

$$= \frac{a^7(A+B)}{16d(a-a\sin(c+dx))^4}+\frac{a^6A}{12d(a-a\sin(c+dx))^3}+$$

$$= \frac{a^3(5A-3B)\tanh^{-1}(\sin(c+dx))}{32d}+\frac{a^7(A+B)}{16d(a-a\sin(c+}$$

Mathematica [A] time = 0.66, size = 151, normalized size = 0.93

$$\frac{a^9\left(\frac{(5A-3B)\tanh^{-1}(\sin(c+dx))}{32a^6}+\frac{2A-B}{16a^5(a-a\sin(c+dx))}-\frac{A-B}{32a^5(a\sin(c+dx)+a)}+\frac{3A-B}{32a^4(a-a\sin(c+dx))^2}+\frac{A}{12a^3(a-a\sin(c+dx))^3}+\frac{A+}{16a^2(a-a\sin}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^9*(((5*A - 3*B)*ArcTanh[Sin[c + d*x]])/(32*a^6) + (A + B)/(16*a^2*(a - a
*Sin[c + d*x])^4) + A/(12*a^3*(a - a*Sin[c + d*x])^3) + (3*A - B)/(32*a^4*(
a - a*Sin[c + d*x])^2) + (2*A - B)/(16*a^5*(a - a*Sin[c + d*x])) - (A - B)/
(32*a^5*(a + a*Sin[c + d*x]))))/d
```

fricas [B] time = 0.75, size = 353, normalized size = 2.18

$$\frac{6\,(5\,A-3\,B)a^3\cos(dx+c)^4-26\,(5\,A-3\,B)a^3\cos(dx+c)^2+12\,(3\,A-5\,B)a^3+3\left(3\,(5\,A-3\,B)a^3\cos(dx+c)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/192*(6*(5*A - 3*B)*a^3*cos(d*x + c)^4 - 26*(5*A - 3*B)*a^3*cos(d*x + c)^2
 + 12*(3*A - 5*B)*a^3 + 3*(3*(5*A - 3*B)*a^3*cos(d*x + c)^4 - 4*(5*A - 3*B)
*a^3*cos(d*x + c)^2 - ((5*A - 3*B)*a^3*cos(d*x + c)^4 - 4*(5*A - 3*B)*a^3*c
os(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 3*(3*(5*A - 3*B)*a^3*c
os(d*x + c)^4 - 4*(5*A - 3*B)*a^3*cos(d*x + c)^2 - ((5*A - 3*B)*a^3*cos(d*x
 + c)^4 - 4*(5*A - 3*B)*a^3*cos(d*x + c)^2)*sin(d*x + c))*log(-sin(d*x + c)
 + 1) + 6*(3*(5*A - 3*B)*a^3*cos(d*x + c)^2 - 2*(5*A - 3*B)*a^3)*sin(d*x +
c))/(3*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 - (d*cos(d*x + c)^4 - 4*d*cos(
d*x + c)^2)*sin(d*x + c))
```

giac [A] time = 0.32, size = 237, normalized size = 1.46

$$\frac{12\left(5\,Aa^3-3\,Ba^3\right)\log\left(|\sin\left(dx+c\right)+1|\right)-12\left(5\,Aa^3-3\,Ba^3\right)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{12\left(5\,Aa^3\sin(dx+c)-3\,Ba^3\sin(\cdots}{\sin(dx+c)+1\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/768*(12*(5*A*a^3 - 3*B*a^3)*log(abs(sin(d*x + c) + 1)) - 12*(5*A*a^3 - 3*
B*a^3)*log(abs(sin(d*x + c) - 1)) - 12*(5*A*a^3*sin(d*x + c) - 3*B*a^3*sin(
d*x + c) + 7*A*a^3 - 5*B*a^3)/(sin(d*x + c) + 1) + (125*A*a^3*sin(d*x + c)^
4 - 75*B*a^3*sin(d*x + c)^4 - 596*A*a^3*sin(d*x + c)^3 + 348*B*a^3*sin(d*x
+ c)^3 + 1110*A*a^3*sin(d*x + c)^2 - 618*B*a^3*sin(d*x + c)^2 - 996*A*a^3*s
in(d*x + c) + 492*B*a^3*sin(d*x + c) + 405*A*a^3 - 99*B*a^3)/(sin(d*x + c)
- 1)^4)/d
```

maple [B] time = 0.65, size = 669, normalized size = 4.13

$$-\frac{B\,a^3\left(\sin^3\left(dx+c\right)\right)}{128d}-\frac{3B\,a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{32d}+\frac{15a^3A\sin\left(dx+c\right)}{128d}+\frac{5a^3A\ln\left(\sec\left(dx+c\right)+\tan\left(\cdots\right.\right.}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/64/d*B*a^3*sin(d*x+c)^5/cos(d*x+c)^4+15/64/d*a^3*A*sin(d*x+c)^3/cos(d*x+c
)^4+1/8/d*B*a^3*sin(d*x+c)^4/cos(d*x+c)^4+15/64/d*B*a^3*sin(d*x+c)^3/cos(d*
x+c)^4+35/192/d*a^3*A*tan(d*x+c)*sec(d*x+c)^3-1/128/d*B*a^3*sin(d*x+c)^3-3/
32/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))+15/128/d*a^3*A*sin(d*x+c)+5/32/d*a^3*A
*ln(sec(d*x+c)+tan(d*x+c))+1/16/d*B*a^3*sin(d*x+c)^5/cos(d*x+c)^6+5/16/d*a^
```

```
3*A*sin(d*x+c)^3/cos(d*x+c)^6+1/4/d*B*a^3*sin(d*x+c)^4/cos(d*x+c)^6+5/16/d*
B*a^3*sin(d*x+c)^3/cos(d*x+c)^6+7/48/d*a^3*A*tan(d*x+c)*sec(d*x+c)^5+1/8/d*
B*a^3/cos(d*x+c)^8+3/8/d*a^3*A/cos(d*x+c)^8-1/128/d*B*a^3*sin(d*x+c)^5/cos(
d*x+c)^2+15/128/d*a^3*A*sin(d*x+c)^3/cos(d*x+c)^2+15/128/d*B*a^3*sin(d*x+c)
^3/cos(d*x+c)^2+35/128/d*a^3*A*sec(d*x+c)*tan(d*x+c)+1/24/d*a^3*A*sin(d*x+c
)^4/cos(d*x+c)^4+1/8/d*a^3*A*sin(d*x+c)^4/cos(d*x+c)^8+3/8/d*B*a^3*sin(d*x+
c)^3/cos(d*x+c)^8+1/8/d*a^3*A*tan(d*x+c)*sec(d*x+c)^7+1/8/d*B*a^3*sin(d*x+c
)^5/cos(d*x+c)^8+3/8/d*a^3*A*sin(d*x+c)^3/cos(d*x+c)^8+3/8/d*B*a^3*sin(d*x+
c)^4/cos(d*x+c)^8+1/12/d*a^3*A*sin(d*x+c)^4/cos(d*x+c)^6+3/32*a^3*B*sin(d*x
+c)/d
```

maxima [A] time = 0.32, size = 185, normalized size = 1.14

$$\frac{3\,(5\,A-3\,B)a^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(5\,A-3\,B)a^3\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,(5\,A-3\,B)a^3\sin(dx+c)^4-9\,(5\,A-3\,B)a^3\sin\cdots\right.}{\sin(dx+c)^5-3\,\sin(dx+c)\cdots}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/192*(3*(5*A - 3*B)*a^3*log(sin(d*x + c) + 1) - 3*(5*A - 3*B)*a^3*log(sin(
d*x + c) - 1) - 2*(3*(5*A - 3*B)*a^3*sin(d*x + c)^4 - 9*(5*A - 3*B)*a^3*sin
(d*x + c)^3 + 7*(5*A - 3*B)*a^3*sin(d*x + c)^2 + 3*(5*A - 3*B)*a^3*sin(d*x
+ c) - 32*A*a^3)/(sin(d*x + c)^5 - 3*sin(d*x + c)^4 + 2*sin(d*x + c)^3 + 2*
sin(d*x + c)^2 - 3*sin(d*x + c) + 1))/d
```

mupad [B] time = 9.18, size = 172, normalized size = 1.06

$$\frac{a^3\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(5\,A-3\,B\right)}{32\,d}-\frac{{\sin\left(c+d\,x\right)}^4\,\left(\frac{5\,A\,a^3}{32}-\frac{3\,B\,a^3}{32}\right)-{\sin\left(c+d\,x\right)}^3\,\left(\frac{15\,A\,a^3}{32}-\frac{9\,B\,a^3}{32}\right)+{\sin\left(c+d\,x\right)}^2\cdots}{d\,\left({\sin\left(c+d\,x\right)}^5-3\,{\sin\left(c+d\,x\right)}^4+2\,{\sin\left(c+d\,x\right)}^3+2\,\mathrm{s}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^9,x)
```

```
[Out] (a^3*atanh(sin(c + d*x))*(5*A - 3*B))/(32*d) - (sin(c + d*x)^4*((5*A*a^3)/3
2 - (3*B*a^3)/32) - sin(c + d*x)^3*((15*A*a^3)/32 - (9*B*a^3)/32) + sin(c +
 d*x)^2*((35*A*a^3)/96 - (7*B*a^3)/32) - (A*a^3)/3 + sin(c + d*x)*((5*A*a^3
)/32 - (3*B*a^3)/32))/(d*(2*sin(c + d*x)^2 - 3*sin(c + d*x) + 2*sin(c + d*x
)^3 - 3*sin(c + d*x)^4 + sin(c + d*x)^5 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.995 $\int \cos^6(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=231

$$-\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}-\frac{11(10A+3B)\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{720d}+\frac{11a^3(10A+3B)\sin(c+dx)\cos}{480d}$$

[Out] 11/256*a^3*(10*A+3*B)*x-11/560*a^3*(10*A+3*B)*cos(d*x+c)^7/d+11/256*a^3*(10*A+3*B)*cos(d*x+c)*sin(d*x+c)/d+11/384*a^3*(10*A+3*B)*cos(d*x+c)^3*sin(d*x+c)/d+11/480*a^3*(10*A+3*B)*cos(d*x+c)^5*sin(d*x+c)/d-1/90*a*(10*A+3*B)*cos(d*x+c)^7*(a+a*sin(d*x+c))^2/d-1/10*B*cos(d*x+c)^7*(a+a*sin(d*x+c))^3/d-11/720*(10*A+3*B)*cos(d*x+c)^7*(a^3+a^3*sin(d*x+c))/d

Rubi [A] time = 0.27, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}-\frac{11(10A+3B)\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{720d}+\frac{11a^3(10A+3B)\sin(c+dx)\cos}{480d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]

[Out] (11*a^3*(10*A + 3*B)*x)/256 - (11*a^3*(10*A + 3*B)*Cos[c + d*x]^7)/(560*d) + (11*a^3*(10*A + 3*B)*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (11*a^3*(10*A + 3*B)*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + (11*a^3*(10*A + 3*B)*Cos[c + d*x]^5*Sin[c + d*x])/(480*d) - (a*(10*A + 3*B)*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2)/(90*d) - (B*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3)/(10*d) - (11*(10*A + 3*B)*Cos[c + d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(720*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = -\frac{B\cos^7(c+dx)(a+a\sin(c+dx))^3}{10d} + \frac{1}{10}(10A+3B)$$
$$= -\frac{a(10A+3B)\cos^7(c+dx)(a+a\sin(c+dx))^2}{90d} - \frac{B\text{c}}{}$$
$$= -\frac{a(10A+3B)\cos^7(c+dx)(a+a\sin(c+dx))^2}{90d} - \frac{B\text{c}}{}$$
$$= -\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d} - \frac{a(10A+3B)\cos^7(}{}$$
$$= -\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d} + \frac{11a^3(10A+3B)\text{co}}{4}$$
$$= -\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d} + \frac{11a^3(10A+3B)\text{co}}{3}$$
$$= -\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d} + \frac{11a^3(10A+3B)\text{co}}{25}$$
$$= \frac{11}{256}a^3(10A+3B)x - \frac{11a^3(10A+3B)\cos^7(c+dx)}{560d} +$$

Mathematica [A] time = 6.05, size = 344, normalized size = 1.49

$$-\frac{32\sqrt{2}\,a^2(10aA+3aB)\left(\frac{1}{2}(\sin(c+dx)-1)+1\right)^{13/2}\left(\frac{385\left(\frac{\sqrt{2}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}}{\sqrt{\frac{1}{2}(\sin(c+dx)-1)+1}}-\frac{2}{15}(1-\sin(c+dx))^3-\frac{1}{3}(1-\sin(c+dx))^2+\right.}{8192\left(\frac{1}{2}(\sin(c+dx)-1)+1\right)^6(1-\sin(c+dx))^4}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/10*(B*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3)/d - (32*Sqrt[2]*a^2*(10*a*A
 + 3*a*B)*Cos[c + d*x]^7*(1 + (-1 + Sin[c + d*x])/2)^(13/2)*((7*(99/(2048*(
1 + (-1 + Sin[c + d*x])/2)^6) + 33/(256*(1 + (-1 + Sin[c + d*x])/2)^5) + 33
/(128*(1 + (-1 + Sin[c + d*x])/2)^4) + 99/(224*(1 + (-1 + Sin[c + d*x])/2)^
3) + 11/(16*(1 + (-1 + Sin[c + d*x])/2)^2) + (1 + (-1 + Sin[c + d*x])/2)^(-
1)))/18 + (385*(-1 + (Sqrt[2]*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1
 - Sin[c + d*x]])/Sqrt[1 + (-1 + Sin[c + d*x])/2] - (1 - Sin[c + d*x])^2/3
- (2*(1 - Sin[c + d*x])^3)/15 + Sin[c + d*x]))/(8192*(1 + (-1 + Sin[c + d*x
])/2)^6*(1 - Sin[c + d*x])^4)))/(35*d*(1 + Sin[c + d*x])^(7/2))
```

fricas [A] time = 0.93, size = 155, normalized size = 0.67

$$\frac{8960\,(A+3\,B)a^3\cos(dx+c)^9-46080\,(A+B)a^3\cos(dx+c)^7+3465\,(10\,A+3\,B)a^3dx+21\,\Big(384\,Ba^3\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/80640*(8960*(A + 3*B)*a^3*cos(d*x + c)^9 - 46080*(A + B)*a^3*cos(d*x + c)
^7 + 3465*(10*A + 3*B)*a^3*d*x + 21*(384*B*a^3*cos(d*x + c)^9 - 48*(30*A +
41*B)*a^3*cos(d*x + c)^7 + 88*(10*A + 3*B)*a^3*cos(d*x + c)^5 + 110*(10*A +
 3*B)*a^3*cos(d*x + c)^3 + 165*(10*A + 3*B)*a^3*cos(d*x + c))*sin(d*x + c))
/d
```

giac [A] time = 0.74, size = 273, normalized size = 1.18

$$\frac{Ba^3\sin(10\,dx+10\,c)}{5120\,d}+\frac{11}{256}\left(10\,Aa^3+3\,Ba^3\right)x+\frac{\left(Aa^3+3\,Ba^3\right)\cos(9\,dx+9\,c)}{2304\,d}-\frac{\left(9\,Aa^3-5\,Ba^3\right)\cos(7\,dx+7\,c)}{1792\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/5120*B*a^3*sin(10*d*x + 10*c)/d + 11/256*(10*A*a^3 + 3*B*a^3)*x + 1/2304*
(A*a^3 + 3*B*a^3)*cos(9*d*x + 9*c)/d - 1/1792*(9*A*a^3 - 5*B*a^3)*cos(7*d*x
 + 7*c)/d - 1/64*(3*A*a^3 + B*a^3)*cos(5*d*x + 5*c)/d - 1/192*(29*A*a^3 + 1
5*B*a^3)*cos(3*d*x + 3*c)/d - 1/128*(33*A*a^3 + 19*B*a^3)*cos(d*x + c)/d -
1/2048*(6*A*a^3 + 5*B*a^3)*sin(8*d*x + 8*c)/d - 1/3072*(32*A*a^3 + 51*B*a^3
)*sin(6*d*x + 6*c)/d + 1/256*(6*A*a^3 - 7*B*a^3)*sin(4*d*x + 4*c)/d + 1/512
*(144*A*a^3 + 25*B*a^3)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.46, size = 363, normalized size = 1.57

$$\frac{a^3A\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{9}-\frac{2\left(\cos^7(dx+c)\right)}{63}\right)+B\,a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(dx+c)+\frac{5(\cos^3}{}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+B*a^3*(-1/10*
sin(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/160*(cos(d*x+c)^5+
```

```
5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)+3*a^3*A*(-1
/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x
+c))*sin(d*x+c)+5/128*d*x+5/128*c)+3*B*a^3*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-
2/63*cos(d*x+c)^7)-3/7*a^3*A*cos(d*x+c)^7+3*B*a^3*(-1/8*cos(d*x+c)^7*sin(d*
x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*
d*x+5/128*c)+a^3*A*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin
(d*x+c)+5/16*d*x+5/16*c)-1/7*B*a^3*cos(d*x+c)^7)
```

maxima [A] time = 0.34, size = 284, normalized size = 1.23

$$-\frac{276480\,Aa^3\cos(dx+c)^7+92160\,Ba^3\cos(dx+c)^7-10240\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)Aa^3-630\left(64\,\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/645120*(276480*A*a^3*cos(d*x + c)^7 + 92160*B*a^3*cos(d*x + c)^7 - 10240
*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*A*a^3 - 630*(64*sin(2*d*x + 2*c)^3 +
 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*A*a^3 + 3360*(
4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x +
2*c))*A*a^3 - 30720*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*B*a^3 - 63*(32*si
n(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*
c))*B*a^3 - 630*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*
c) - 24*sin(4*d*x + 4*c))*B*a^3)/d
```

mupad [B] time = 10.83, size = 711, normalized size = 3.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (11*a^3*atan((11*a^3*tan(c/2 + (d*x)/2)*(10*A + 3*B))/(128*((55*A*a^3)/64 +
 (33*B*a^3)/128)))*(10*A + 3*B))/(128*d) - (11*a^3*(10*A + 3*B)*(atan(tan(c
/2 + (d*x)/2)) - (d*x)/2))/(128*d) - ((58*A*a^3)/63 - tan(c/2 + (d*x)/2)*((
73*A*a^3)/64 - (33*B*a^3)/128) + (10*B*a^3)/21 + tan(c/2 + (d*x)/2)^18*(6*A
*a^3 + 2*B*a^3) + tan(c/2 + (d*x)/2)^16*(22*A*a^3 + 18*B*a^3) + tan(c/2 + (
d*x)/2)^8*(84*A*a^3 + 28*B*a^3) + tan(c/2 + (d*x)/2)^14*((136*A*a^3)/3 + 8*
B*a^3) + tan(c/2 + (d*x)/2)^4*((136*A*a^3)/7 + (24*B*a^3)/7) + tan(c/2 + (d
*x)/2)^10*(116*A*a^3 + 60*B*a^3) + tan(c/2 + (d*x)/2)^19*((73*A*a^3)/64 - (
33*B*a^3)/128) + tan(c/2 + (d*x)/2)^2*((202*A*a^3)/63 + (58*B*a^3)/21) + ta
n(c/2 + (d*x)/2)^12*((328*A*a^3)/3 + 72*B*a^3) - tan(c/2 + (d*x)/2)^7*((341
*A*a^3)/16 - (333*B*a^3)/32) + tan(c/2 + (d*x)/2)^13*((341*A*a^3)/16 - (333
```

```
*B*a^3)/32) + tan(c/2 + (d*x)/2)^6*((456*A*a^3)/7 + (344*B*a^3)/7) - tan(c/
2 + (d*x)/2)^5*((449*A*a^3)/48 + (577*B*a^3)/160) + tan(c/2 + (d*x)/2)^15*(
(449*A*a^3)/48 + (577*B*a^3)/160) - tan(c/2 + (d*x)/2)^3*((2117*A*a^3)/192
+ (705*B*a^3)/128) + tan(c/2 + (d*x)/2)^17*((2117*A*a^3)/192 + (705*B*a^3)/
128) - tan(c/2 + (d*x)/2)^9*((699*A*a^3)/32 + (2749*B*a^3)/64) + tan(c/2 +
(d*x)/2)^11*((699*A*a^3)/32 + (2749*B*a^3)/64))/(d*(10*tan(c/2 + (d*x)/2)^2
 + 45*tan(c/2 + (d*x)/2)^4 + 120*tan(c/2 + (d*x)/2)^6 + 210*tan(c/2 + (d*x)
/2)^8 + 252*tan(c/2 + (d*x)/2)^10 + 210*tan(c/2 + (d*x)/2)^12 + 120*tan(c/2
 + (d*x)/2)^14 + 45*tan(c/2 + (d*x)/2)^16 + 10*tan(c/2 + (d*x)/2)^18 + tan(
c/2 + (d*x)/2)^20 + 1))
```

sympy [A] time = 27.75, size = 1042, normalized size = 4.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((15*A*a**3*x*sin(c + d*x)**8/128 + 15*A*a**3*x*sin(c + d*x)**6*co
s(c + d*x)**2/32 + 5*A*a**3*x*sin(c + d*x)**6/16 + 45*A*a**3*x*sin(c + d*x)
**4*cos(c + d*x)**4/64 + 15*A*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 1
5*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 15*A*a**3*x*sin(c + d*x)**2
*cos(c + d*x)**4/16 + 15*A*a**3*x*cos(c + d*x)**8/128 + 5*A*a**3*x*cos(c +
d*x)**6/16 + 15*A*a**3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*A*a**3*sin
(c + d*x)**5*cos(c + d*x)**3/(128*d) + 5*A*a**3*sin(c + d*x)**5*cos(c + d*x
)/(16*d) + 73*A*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) + 5*A*a**3*sin
(c + d*x)**3*cos(c + d*x)**3/(6*d) - A*a**3*sin(c + d*x)**2*cos(c + d*x)**7
/(7*d) - 15*A*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) + 11*A*a**3*sin(c +
 d*x)*cos(c + d*x)**5/(16*d) - 2*A*a**3*cos(c + d*x)**9/(63*d) - 3*A*a**3*c
os(c + d*x)**7/(7*d) + 3*B*a**3*x*sin(c + d*x)**10/256 + 15*B*a**3*x*sin(c
+ d*x)**8*cos(c + d*x)**2/256 + 15*B*a**3*x*sin(c + d*x)**8/128 + 15*B*a**3
*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 15*B*a**3*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 15*B*a**3*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 45*B*a**3*
x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 15*B*a**3*x*sin(c + d*x)**2*cos(c +
d*x)**8/256 + 15*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 3*B*a**3*x*c
os(c + d*x)**10/256 + 15*B*a**3*x*cos(c + d*x)**8/128 + 3*B*a**3*sin(c + d*
x)**9*cos(c + d*x)/(256*d) + 7*B*a**3*sin(c + d*x)**7*cos(c + d*x)**3/(128*
d) + 15*B*a**3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + B*a**3*sin(c + d*x)**
5*cos(c + d*x)**5/(10*d) + 55*B*a**3*sin(c + d*x)**5*cos(c + d*x)**3/(128*d
) - 7*B*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) + 73*B*a**3*sin(c + d*
x)**3*cos(c + d*x)**5/(128*d) - 3*B*a**3*sin(c + d*x)**2*cos(c + d*x)**7/(7
*d) - 3*B*a**3*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 15*B*a**3*sin(c + d*x
)*cos(c + d*x)**7/(128*d) - 2*B*a**3*cos(c + d*x)**9/(21*d) - B*a**3*cos(c
+ d*x)**7/(7*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**3*cos(c)**6,
True))
```

3.996 $\int \cos^4(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=200

$$-\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}-\frac{3(8A+3B)\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{112d}+\frac{3a^3(8A+3B)\sin(c+dx)\cos^3(c+}{64d}$$

```
[Out] 9/128*a^3*(8*A+3*B)*x-3/80*a^3*(8*A+3*B)*cos(d*x+c)^5/d+9/128*a^3*(8*A+3*B)
*cos(d*x+c)*sin(d*x+c)/d+3/64*a^3*(8*A+3*B)*cos(d*x+c)^3*sin(d*x+c)/d-1/56*
a*(8*A+3*B)*cos(d*x+c)^5*(a+a*sin(d*x+c))^2/d-1/8*B*cos(d*x+c)^5*(a+a*sin(d
*x+c))^3/d-3/112*(8*A+3*B)*cos(d*x+c)^5*(a^3+a^3*sin(d*x+c))/d
```

Rubi [A] time = 0.24, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}-\frac{3(8A+3B)\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{112d}+\frac{3a^3(8A+3B)\sin(c+dx)\cos^3(c+}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (9*a^3*(8*A + 3*B)*x)/128 - (3*a^3*(8*A + 3*B)*Cos[c + d*x]^5)/(80*d) + (9*
a^3*(8*A + 3*B)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (3*a^3*(8*A + 3*B)*Cos
[c + d*x]^3*Sin[c + d*x])/(64*d) - (a*(8*A + 3*B)*Cos[c + d*x]^5*(a + a*Sin
[c + d*x])^2)/(56*d) - (B*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3)/(8*d) - (3
*(8*A + 3*B)*Cos[c + d*x]^5*(a^3 + a^3*Sin[c + d*x]))/(112*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
```

```
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = -\frac{B\cos^5(c+dx)(a+a\sin(c+dx))^3}{8d}+\frac{1}{8}(8A+3B)$$

$$= -\frac{a(8A+3B)\cos^5(c+dx)(a+a\sin(c+dx))^2}{56d}-\frac{B}{}$$

$$= -\frac{a(8A+3B)\cos^5(c+dx)(a+a\sin(c+dx))^2}{56d}-\frac{B}{}$$

$$= -\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}-\frac{a(8A+3B)\cos^5(c}{}$$

$$= -\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}+\frac{3a^3(8A+3B)\cos^3(}{64}$$

$$= -\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}+\frac{9a^3(8A+3B)\cos(c}{128}$$

$$= \frac{9}{128}a^3(8A+3B)x-\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}+$$

Mathematica [A] time = 2.25, size = 183, normalized size = 0.92

$$-\frac{a^3\cos(c+dx)\left(16(373A+223B)\cos(2(c+dx))+32(41A+11B)\cos(4(c+dx))+\frac{2520(8A+3B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/17920*(a^3*Cos[c + d*x]*(4576*A + 2976*B + (2520*(8*A + 3*B)*ArcSin[Sqrt
[1 - Sin[c + d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + 16*(373*A + 223*B)*Cos[
2*(c + d*x)] + 32*(41*A + 11*B)*Cos[4*(c + d*x)] - 80*A*Cos[6*(c + d*x)] -
240*B*Cos[6*(c + d*x)] - 10640*A*Sin[c + d*x] - 3045*B*Sin[c + d*x] + 1365*
B*Sin[3*(c + d*x)] + 560*A*Sin[5*(c + d*x)] + 595*B*Sin[5*(c + d*x)] - 35*B
*Sin[7*(c + d*x)]))/d
```

fricas [A] time = 0.71, size = 135, normalized size = 0.68

$$\frac{640\,(A+3\,B)a^3\cos\left(dx+c\right)^7-3584\,(A+B)a^3\cos\left(dx+c\right)^5+315\,(8\,A+3\,B)a^3dx+35\,\left(16\,Ba^3\cos\left(dx+c\right)^7-\right.}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/4480*(640*(A + 3*B)*a^3*cos(d*x + c)^7 - 3584*(A + B)*a^3*cos(d*x + c)^5
+ 315*(8*A + 3*B)*a^3*d*x + 35*(16*B*a^3*cos(d*x + c)^7 - 8*(8*A + 11*B)*a^
3*cos(d*x + c)^5 + 6*(8*A + 3*B)*a^3*cos(d*x + c)^3 + 9*(8*A + 3*B)*a^3*cos
(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.38, size = 217, normalized size = 1.08

$$\frac{Ba^3\sin\left(8\,dx+8\,c\right)}{1024\,d}+\frac{9}{128}\left(8\,Aa^3+3\,Ba^3\right)x+\frac{\left(Aa^3+3\,Ba^3\right)\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{\left(11\,Aa^3+Ba^3\right)\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{(13}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/1024*B*a^3*sin(8*d*x + 8*c)/d + 9/128*(8*A*a^3 + 3*B*a^3)*x + 1/448*(A*a^
3 + 3*B*a^3)*cos(7*d*x + 7*c)/d - 1/320*(11*A*a^3 + B*a^3)*cos(5*d*x + 5*c)
/d - 1/64*(13*A*a^3 + 7*B*a^3)*cos(3*d*x + 3*c)/d - 1/64*(27*A*a^3 + 17*B*a
^3)*cos(d*x + c)/d - 1/64*(A*a^3 + B*a^3)*sin(6*d*x + 6*c)/d - 1/128*(2*A*a
```

```
^3 + 7*B*a^3)*sin(4*d*x + 4*c)/d + 1/64*(19*A*a^3 + 3*B*a^3)*sin(2*d*x + 2*
c)/d
```

maple [A] time = 0.45, size = 323, normalized size = 1.62

$$\frac{a^3 A\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+B a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx}{2}\right.}{64}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+B*a^3*(-1/8*s
in(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+3*a^3*A*(-1/6*sin(d*x+c)*cos(d*
x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+3*B*a
^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)-3/5*a^3*A*cos(d*x+c)^
5+3*B*a^3*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*
sin(d*x+c)+1/16*d*x+1/16*c)+a^3*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*
x+c)+3/8*d*x+3/8*c)-1/5*B*a^3*cos(d*x+c)^5)
```

maxima [A] time = 0.32, size = 232, normalized size = 1.16

$$-\frac{21504\,Aa^3\cos\left(dx+c\right)^5+7168\,Ba^3\cos\left(dx+c\right)^5-1024\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)Aa^3-560\left(4\,\sin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/35840*(21504*A*a^3*cos(d*x + c)^5 + 7168*B*a^3*cos(d*x + c)^5 - 1024*(5*
cos(d*x + c)^7 - 7*cos(d*x + c)^5)*A*a^3 - 560*(4*sin(2*d*x + 2*c)^3 + 12*d
*x + 12*c - 3*sin(4*d*x + 4*c))*A*a^3 - 1120*(12*d*x + 12*c + sin(4*d*x + 4
*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 3072*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5
)*B*a^3 - 560*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*B
*a^3 - 35*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*B*a^3)/d
```

mupad [B] time = 10.69, size = 584, normalized size = 2.92

$$\frac{9\,a^3\,\mathrm{atan}\left(\frac{9\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(8\,A+3\,B\right)}{64\,\left(\frac{9\,A\,a^3}{8}+\frac{27\,B\,a^3}{64}\right)}\right)\,\left(8\,A+3\,B\right)}{64\,d}-\frac{9\,a^3\,\left(8\,A+3\,B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{64\,d}-\frac{\frac{46\,A\,a^3}{35}-\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (9*a^3*atan((9*a^3*tan(c/2 + (d*x)/2)*(8*A + 3*B))/(64*((9*A*a^3)/8 + (27*B
*a^3)/64)))*(8*A + 3*B))/(64*d) - (9*a^3*(8*A + 3*B)*(atan(tan(c/2 + (d*x)/
2)) - (d*x)/2))/(64*d) - ((46*A*a^3)/35 - tan(c/2 + (d*x)/2)*((7*A*a^3)/8 -
 (27*B*a^3)/64) + (26*B*a^3)/35 + tan(c/2 + (d*x)/2)^14*(6*A*a^3 + 2*B*a^3)
 + tan(c/2 + (d*x)/2)^10*(30*A*a^3 + 10*B*a^3) + tan(c/2 + (d*x)/2)^12*(22*
A*a^3 + 18*B*a^3) + tan(c/2 + (d*x)/2)^8*(46*A*a^3 + 26*B*a^3) + tan(c/2 +
(d*x)/2)^4*((74*A*a^3)/5 + (14*B*a^3)/5) + tan(c/2 + (d*x)/2)^15*((7*A*a^3)
/8 - (27*B*a^3)/64) + tan(c/2 + (d*x)/2)^2*((158*A*a^3)/35 + (138*B*a^3)/35
) + tan(c/2 + (d*x)/2)^6*((218*A*a^3)/5 + (158*B*a^3)/5) - tan(c/2 + (d*x)/
2)^3*((75*A*a^3)/8 + (305*B*a^3)/64) + tan(c/2 + (d*x)/2)^13*((75*A*a^3)/8
+ (305*B*a^3)/64) - tan(c/2 + (d*x)/2)^5*((55*A*a^3)/8 + (437*B*a^3)/64) +
tan(c/2 + (d*x)/2)^11*((55*A*a^3)/8 + (437*B*a^3)/64) + tan(c/2 + (d*x)/2)^
7*((13*A*a^3)/8 + (919*B*a^3)/64) - tan(c/2 + (d*x)/2)^9*((13*A*a^3)/8 + (9
19*B*a^3)/64))/(d*(8*tan(c/2 + (d*x)/2)^2 + 28*tan(c/2 + (d*x)/2)^4 + 56*ta
n(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 + 56*tan(c/2 + (d*x)/2)^10 + 2
8*tan(c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 +
 1))
```

sympy [A] time = 11.67, size = 823, normalized size = 4.12

$$\begin{cases} \frac{3Aa^3x\sin^6(c+dx)}{16} + \frac{9Aa^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3Aa^3x\sin^4(c+dx)}{8} + \frac{9Aa^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \\ x(A+B\sin(c))(a\sin(c)+a)^3\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**6/16 + 9*A*a**3*x*sin(c + d*x)**4*cos(c
 + d*x)**2/16 + 3*A*a**3*x*sin(c + d*x)**4/8 + 9*A*a**3*x*sin(c + d*x)**2*c
os(c + d*x)**4/16 + 3*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*A*a**3
*x*cos(c + d*x)**6/16 + 3*A*a**3*x*cos(c + d*x)**4/8 + 3*A*a**3*sin(c + d*x
)**5*cos(c + d*x)/(16*d) + A*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) + 3
*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - A*a**3*sin(c + d*x)**2*cos(c +
 d*x)**5/(5*d) - 3*A*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 5*A*a**3*si
n(c + d*x)*cos(c + d*x)**3/(8*d) - 2*A*a**3*cos(c + d*x)**7/(35*d) - 3*A*a*
*3*cos(c + d*x)**5/(5*d) + 3*B*a**3*x*sin(c + d*x)**8/128 + 3*B*a**3*x*sin(
c + d*x)**6*cos(c + d*x)**2/32 + 3*B*a**3*x*sin(c + d*x)**6/16 + 9*B*a**3*x
*sin(c + d*x)**4*cos(c + d*x)**4/64 + 9*B*a**3*x*sin(c + d*x)**4*cos(c + d*
x)**2/16 + 3*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 9*B*a**3*x*sin(c
 + d*x)**2*cos(c + d*x)**4/16 + 3*B*a**3*x*cos(c + d*x)**8/128 + 3*B*a**3*x
```

```
*cos(c + d*x)**6/16 + 3*B*a**3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 11*B*
a**3*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) + 3*B*a**3*sin(c + d*x)**5*cos
(c + d*x)/(16*d) - 11*B*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) + B*a*
*3*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*B*a**3*sin(c + d*x)**2*cos(c +
 d*x)**5/(5*d) - 3*B*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 3*B*a**3*s
in(c + d*x)*cos(c + d*x)**5/(16*d) - 6*B*a**3*cos(c + d*x)**7/(35*d) - B*a*
*3*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**3*co
s(c)**4, True))
```

3.997 $\int \cos^2(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=159

$$-\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}-\frac{7(2A+B)\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{40d}+\frac{7a^3(2A+B)\sin(c+dx)\cos(c+dx)}{16d}$$

```
[Out] 7/16*a^3*(2*A+B)*x-7/24*a^3*(2*A+B)*cos(d*x+c)^3/d+7/16*a^3*(2*A+B)*cos(d*x
+c)*sin(d*x+c)/d-1/10*a*(2*A+B)*cos(d*x+c)^3*(a+a*sin(d*x+c))^2/d-1/6*B*cos
(d*x+c)^3*(a+a*sin(d*x+c))^3/d-7/40*(2*A+B)*cos(d*x+c)^3*(a^3+a^3*sin(d*x+c
))/d
```

Rubi [A] time = 0.22, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}-\frac{7(2A+B)\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{40d}+\frac{7a^3(2A+B)\sin(c+dx)\cos(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (7*a^3*(2*A + B)*x)/16 - (7*a^3*(2*A + B)*Cos[c + d*x]^3)/(24*d) + (7*a^3*(
2*A + B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (a*(2*A + B)*Cos[c + d*x]^3*(a
 + a*Sin[c + d*x])^2)/(10*d) - (B*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(6
*d) - (7*(2*A + B)*Cos[c + d*x]^3*(a^3 + a^3*Sin[c + d*x]))/(40*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = -\frac{B\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d}+\frac{1}{2}(2A+B)\int$$
$$= -\frac{a(2A+B)\cos^3(c+dx)(a+a\sin(c+dx))^2}{10d}-\frac{B\,\mathrm{co}}{}$$
$$= -\frac{a(2A+B)\cos^3(c+dx)(a+a\sin(c+dx))^2}{10d}-\frac{B\,\mathrm{co}}{}$$
$$= -\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}-\frac{a(2A+B)\cos^3(c+d}{1}$$
$$= -\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}+\frac{7a^3(2A+B)\cos(c+}{16d}$$
$$= \frac{7}{16}a^3(2A+B)x-\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}+\frac{7a^3(}{}$$

Mathematica [A] time = 1.42, size = 146, normalized size = 0.92

$$-\frac{a^3\cos(c+dx)\left(16(17A+11B)\cos(2(c+dx))-12(A+3B)\cos(4(c+dx))+\frac{420(2A+B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}-330A\right.}{48}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/480*(a^3*Cos[c + d*x]*(284*A + 212*B + (420*(2*A + B)*ArcSin[Sqrt[1 - Si
n[c + d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + 16*(17*A + 11*B)*Cos[2*(c + d*
x)] - 12*(A + 3*B)*Cos[4*(c + d*x)] - 330*A*Sin[c + d*x] - 95*B*Sin[c + d*x
] + 90*A*Sin[3*(c + d*x)] + 110*B*Sin[3*(c + d*x)] - 5*B*Sin[5*(c + d*x)]))
/d
```

fricas [A] time = 0.61, size = 111, normalized size = 0.70

$$\frac{48\,(A+3\,B)a^3\cos{(dx+c)}^5-320\,(A+B)a^3\cos{(dx+c)}^3+105\,(2\,A+B)a^3dx+5\left(8\,Ba^3\cos{(dx+c)}^5-2\,(18\,A\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(48*(A + 3*B)*a^3*cos(d*x + c)^5 - 320*(A + B)*a^3*cos(d*x + c)^3 + 1
05*(2*A + B)*a^3*d*x + 5*(8*B*a^3*cos(d*x + c)^5 - 2*(18*A + 25*B)*a^3*cos(
d*x + c)^3 + 21*(2*A + B)*a^3*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.29, size = 165, normalized size = 1.04

$$\frac{Ba^3\sin{(6\,dx+6\,c)}}{192\,d}+\frac{7}{16}\left(2\,Aa^3+Ba^3\right)x+\frac{\left(Aa^3+3\,Ba^3\right)\cos{(5\,dx+5\,c)}}{80\,d}-\frac{\left(13\,Aa^3+7\,Ba^3\right)\cos{(3\,dx+3\,c)}}{48\,d}-\frac{\left(7\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*B*a^3*sin(6*d*x + 6*c)/d + 7/16*(2*A*a^3 + B*a^3)*x + 1/80*(A*a^3 + 3
*B*a^3)*cos(5*d*x + 5*c)/d - 1/48*(13*A*a^3 + 7*B*a^3)*cos(3*d*x + 3*c)/d -
 1/8*(7*A*a^3 + 5*B*a^3)*cos(d*x + c)/d - 1/64*(6*A*a^3 + 7*B*a^3)*sin(4*d*
x + 4*c)/d + 1/64*(16*A*a^3 - B*a^3)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.35, size = 279, normalized size = 1.75

$$\frac{a^3A\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+B\,a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{8}+\frac{\cos(dx+c)\sin(dx+c)}{16}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)

[Out] 1/d*(a^3*A*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+B*a^3*(-1/6*s in(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/16*cos(d*x+c)*sin(d* x+c)+1/16*d*x+1/16*c)+3*a^3*A*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)* sin(d*x+c)+1/8*d*x+1/8*c)+3*B*a^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)-a^3*A*cos(d*x+c)^3+3*B*a^3*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)+a^3*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1 /2*c)-1/3*B*a^3*cos(d*x+c)^3)

maxima [A] time = 0.32, size = 199, normalized size = 1.25

$$-\frac{960\,Aa^3\cos(dx+c)^3+320\,Ba^3\cos(dx+c)^3-64\left(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3\right)Aa^3-90\,(4\,dx+4\,c-\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma xima")

[Out] -1/960*(960*A*a^3*cos(d*x + c)^3 + 320*B*a^3*cos(d*x + c)^3 - 64*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*A*a^3 - 90*(4*d*x + 4*c - sin(4*d*x + 4*c))*A*a ^3 - 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 - 192*(3*cos(d*x + c)^5 - 5 *cos(d*x + c)^3)*B*a^3 + 5*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4* d*x + 4*c))*B*a^3 - 90*(4*d*x + 4*c - sin(4*d*x + 4*c))*B*a^3)/d

mupad [B] time = 10.73, size = 451, normalized size = 2.84

$$\frac{7\,a^3\,\mathrm{atan}\left(\frac{7\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A+B\right)}{8\,\left(\frac{7\,A\,a^3}{4}+\frac{7\,B\,a^3}{8}\right)}\right)\,\left(2\,A+B\right)}{8\,d}-\frac{\frac{34\,A\,a^3}{15}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A\,a^3}{4}-\frac{7\,B\,a^3}{8}\right)+\frac{22\,B\,a^3}{15}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(6\,A\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)

[Out] (7*a^3*atan((7*a^3*tan(c/2 + (d*x)/2)*(2*A + B))/(8*((7*A*a^3)/4 + (7*B*a^3)/8)))*(2*A + B))/(8*d) - ((34*A*a^3)/15 - tan(c/2 + (d*x)/2)*((A*a^3)/4 - (7*B*a^3)/8) + (22*B*a^3)/15 + tan(c/2 + (d*x)/2)^10*(6*A*a^3 + 2*B*a^3) + tan(c/2 + (d*x)/2)^4*(12*A*a^3 + 4*B*a^3) + tan(c/2 + (d*x)/2)^11*((A*a^3)/ 4 - (7*B*a^3)/8) + tan(c/2 + (d*x)/2)^8*(22*A*a^3 + 18*B*a^3) - tan(c/2 + (d*x)/2)^5*((13*A*a^3)/2 + (37*B*a^3)/4) + tan(c/2 + (d*x)/2)^7*((13*A*a^3)/ 2 + (37*B*a^3)/4) + tan(c/2 + (d*x)/2)^2*((38*A*a^3)/5 + (34*B*a^3)/5) + ta n(c/2 + (d*x)/2)^6*((68*A*a^3)/3 + (44*B*a^3)/3) - tan(c/2 + (d*x)/2)^3*((2

```
7*A*a^3)/4 + (73*B*a^3)/24) + tan(c/2 + (d*x)/2)^9*((27*A*a^3)/4 + (73*B*a^
3)/24))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 +
 (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 +
 (d*x)/2)^12 + 1)) - (7*a^3*(2*A + B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))
/(8*d)
```

sympy [A] time = 4.38, size = 588, normalized size = 3.70

$$\begin{cases} \frac{3Aa^3x\sin^4(c+dx)}{8} + \frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{Aa^3x\sin^2(c+dx)}{2} + \frac{3Aa^3x\cos^4(c+dx)}{8} + \frac{Aa^3x\cos^2(c+dx)}{2} + \frac{3Aa^3\sin^3(c+dx)\cos(c+}{8d} \\ x(A+B\sin(c))(a\sin(c)+a)^3\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**4/8 + A
*a**3*x*cos(c + d*x)**2/2 + 3*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - A
*a**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 3*A*a**3*sin(c + d*x)*cos(c +
 d*x)**3/(8*d) + A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*A*a**3*cos(c +
d*x)**5/(15*d) - A*a**3*cos(c + d*x)**3/d + B*a**3*x*sin(c + d*x)**6/16 + 3
*B*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*B*a**3*x*sin(c + d*x)**4/8
 + 3*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3*B*a**3*x*sin(c + d*x)*
*2*cos(c + d*x)**2/4 + B*a**3*x*cos(c + d*x)**6/16 + 3*B*a**3*x*cos(c + d*x
)**4/8 + B*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) - B*a**3*sin(c + d*x)**
3*cos(c + d*x)**3/(6*d) + 3*B*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - B*a
**3*sin(c + d*x)**2*cos(c + d*x)**3/d - B*a**3*sin(c + d*x)*cos(c + d*x)**5
/(16*d) - 3*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*B*a**3*cos(c + d*
x)**5/(5*d) - B*a**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a
*sin(c) + a)**3*cos(c)**2, True))
```

3.998 $\int \sec^2(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=91

$$\frac{2a^3(2A+3B)\cos(c+dx)}{d}+\frac{a^3(2A+3B)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3}{2}a^3x(2A+3B)+\frac{(A+B)\sec(c+dx)(a\sin(c+$$

```
[Out] -3/2*a^3*(2*A+3*B)*x+2*a^3*(2*A+3*B)*cos(d*x+c)/d+1/2*a^3*(2*A+3*B)*cos(d*x
+c)*sin(d*x+c)/d+(A+B)*sec(d*x+c)*(a+a*sin(d*x+c))^3/d
```

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2855, 2644}

$$\frac{2a^3(2A+3B)\cos(c+dx)}{d}+\frac{a^3(2A+3B)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3}{2}a^3x(2A+3B)+\frac{(A+B)\sec(c+dx)(a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (-3*a^3*(2*A + 3*B)*x)/2 + (2*a^3*(2*A + 3*B)*Cos[c + d*x])/d + (a^3*(2*A +
 3*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + ((A + B)*Sec[c + d*x]*(a + a*Sin[c
 + d*x])^3)/d
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec(c+dx)(a+a\sin(c+dx))^3}{d} - (a(2A+3B$$
$$= -\frac{3}{2}a^3(2A+3B)x + \frac{2a^3(2A+3B)\cos(c+dx)}{d} + \frac{a^3(2$$

Mathematica [C] time = 0.26, size = 82, normalized size = 0.90

$$\frac{\sec(c+dx)\left(4\sqrt{2}\,a^3(2A+3B)\sqrt{\sin(c+dx)+1}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)-B(a\sin(c+dx)+a)^3\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(4*Sqrt[2]*a^3*(2*A + 3*B)*Hypergeometric2F1[-3/2, -1/2, 1/2,
 (1 - Sin[c + d*x])/2]*Sqrt[1 + Sin[c + d*x]] - B*(a + a*Sin[c + d*x])^3))/
(2*d)
```

fricas [A] time = 0.68, size = 173, normalized size = 1.90

$$\frac{Ba^3\cos(dx+c)^3 - 3(2A+3B)a^3dx + 2(A+3B)a^3\cos(dx+c)^2 + 8(A+B)a^3 - \left(3(2A+3B)a^3dx - (10A+\right.}{2(d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*(B*a^3*cos(d*x + c)^3 - 3*(2*A + 3*B)*a^3*d*x + 2*(A + 3*B)*a^3*cos(d*x
 + c)^2 + 8*(A + B)*a^3 - (3*(2*A + 3*B)*a^3*d*x - (10*A + 13*B)*a^3)*cos(d
*x + c) + (3*(2*A + 3*B)*a^3*d*x + B*a^3*cos(d*x + c)^2 - (2*A + 5*B)*a^3*c
os(d*x + c) + 8*(A + B)*a^3)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c)
 + d)
```

giac [A] time = 0.20, size = 147, normalized size = 1.62

$$-\frac{3\left(2Aa^3+3Ba^3\right)(dx+c)+\frac{16\left(Aa^3+Ba^3\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}+\frac{2\left(Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-6Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/2*(3*(2*A*a^3 + 3*B*a^3)*(d*x + c) + 16*(A*a^3 + B*a^3)/(tan(1/2*d*x + 1
/2*c) - 1) + 2*(B*a^3*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^3*tan(1/2*d*x + 1/2*c)
^2 - 6*B*a^3*tan(1/2*d*x + 1/2*c)^2 - B*a^3*tan(1/2*d*x + 1/2*c) - 2*A*a^3
- 6*B*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [B] time = 0.68, size = 219, normalized size = 2.41

$$a^3 A\left(\frac{\sin^4(dx+c)}{\cos(dx+c)} + \left(2+\sin^2(dx+c)\right)\cos(dx+c)\right) + B a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)} + \left(\sin^3(dx+c) + \frac{3\sin(dx+c)}{2}\right)\cos(dx+c) - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+B*a^3*(sin
(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c
)+3*a^3*A*(tan(d*x+c)-d*x-c)+3*B*a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)
^2)*cos(d*x+c))+3*a^3*A/cos(d*x+c)+3*B*a^3*(tan(d*x+c)-d*x-c)+a^3*A*tan(d*x
+c)+B*a^3/cos(d*x+c))
```

maxima [A] time = 0.41, size = 167, normalized size = 1.84

$$-\frac{6\,(dx+c-\tan(dx+c))Aa^3 + \left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1} - 2\,\tan(dx+c)\right)Ba^3 + 6\,(dx+c-\tan(dx+c))Ba^3 - }{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/2*(6*(d*x + c - tan(d*x + c))*A*a^3 + (3*d*x + 3*c - tan(d*x + c)/(tan(d
*x + c)^2 + 1) - 2*tan(d*x + c))*B*a^3 + 6*(d*x + c - tan(d*x + c))*B*a^3 -
 2*A*a^3*(1/cos(d*x + c) + cos(d*x + c)) - 6*B*a^3*(1/cos(d*x + c) + cos(d*
x + c)) - 2*A*a^3*tan(d*x + c) - 6*A*a^3/cos(d*x + c) - 2*B*a^3/cos(d*x + c
))/d
```

mupad [B] time = 11.51, size = 234, normalized size = 2.57

$$-\frac{10\,A\,a^3 - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(2\,A\,a^3+5\,B\,a^3\right) + 14\,B\,a^3 - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(2\,A\,a^3+7\,B\,a^3\right) + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(8\,A\,a^3\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + \tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] - (10*A*a^3 - tan(c/2 + (d*x)/2)*(2*A*a^3 + 5*B*a^3) + 14*B*a^3 - tan(c/2 +
 (d*x)/2)^3*(2*A*a^3 + 7*B*a^3) + tan(c/2 + (d*x)/2)^4*(8*A*a^3 + 9*B*a^3)
+ tan(c/2 + (d*x)/2)^2*(18*A*a^3 + 21*B*a^3))/(d*(tan(c/2 + (d*x)/2) - 2*ta
n(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^4 + tan(c/
2 + (d*x)/2)^5 - 1)) - (3*a^3*atan((3*a^3*tan(c/2 + (d*x)/2)*(2*A + 3*B))/(
6*A*a^3 + 9*B*a^3))*(2*A + 3*B))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int A\sec^2(c+dx)\,dx+\int 3A\sin(c+dx)\sec^2(c+dx)\,dx+\int 3A\sin^2(c+dx)\sec^2(c+dx)\,dx+\int A\sin^3(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x)**2, x) + Integral(3*A*sin(c + d*x)*sec(c + d*
x)**2, x) + Integral(3*A*sin(c + d*x)**2*sec(c + d*x)**2, x) + Integral(A*s
in(c + d*x)**3*sec(c + d*x)**2, x) + Integral(B*sin(c + d*x)*sec(c + d*x)**
2, x) + Integral(3*B*sin(c + d*x)**2*sec(c + d*x)**2, x) + Integral(3*B*sin
(c + d*x)**3*sec(c + d*x)**2, x) + Integral(B*sin(c + d*x)**4*sec(c + d*x)*
*2, x))
```

3.999 $\int \sec^4(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=69

$$a^3Bx-\frac{2a^5B\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

```
[Out] a^3*B*x+1/3*(A+B)*sec(d*x+c)^3*(a+a*sin(d*x+c))^3/d-2*a^5*B*cos(d*x+c)/d/(a
^2-a^2*sin(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2855, 2670, 2680, 8}

$$-\frac{2a^5B\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}+a^3Bx+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] a^3*B*x + ((A + B)*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*d) - (2*a^5*B*
Cos[c + d*x])/(d*(a^2 - a^2*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - (aB)\int \text{se}$$

$$= \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \left(a^5B\right)\int$$

$$= \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \frac{2a^5B\,\text{c}}{d\left(a^2-a^2\right.}$$

$$= a^3Bx + \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \frac{}{d}$$

Mathematica [A] time = 1.11, size = 121, normalized size = 1.75

$$-\frac{a^3\left(-3\cos\left(\frac{1}{2}(c+dx)\right)(2A+3B(c+dx+2))+\cos\left(\frac{3}{2}(c+dx)\right)(2A+B(3c+3dx+14))+6B\sin\left(\frac{1}{2}(c+dx)\right)(2\right.}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a^3*(-3*(2*A + 3*B*(2 + c + d*x))*Cos[(c + d*x)/2] + (2*A + B*(14 + 3
*c + 3*d*x))*Cos[(3*(c + d*x))/2] + 6*B*(2*(2 + c + d*x) + (c + d*x)*Cos[c
+ d*x])*Sin[(c + d*x)/2]))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)
```

fricas [B] time = 0.70, size = 167, normalized size = 2.42

$$-\frac{6\,Ba^3dx+2\,(A+B)a^3-\left(3\,Ba^3dx+(A+7\,B)a^3\right)\cos\left(dx+c\right)^2+\left(3\,Ba^3dx+(A-5\,B)a^3\right)\cos\left(dx+c\right)-\left(6\,Ba^3\right.}{3\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\text{si}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/3*(6*B*a^3*d*x + 2*(A + B)*a^3 - (3*B*a^3*d*x + (A + 7*B)*a^3)*cos(d*x +
 c)^2 + (3*B*a^3*d*x + (A - 5*B)*a^3)*cos(d*x + c) - (6*B*a^3*d*x - 2*(A +
B)*a^3 + (3*B*a^3*d*x - (A + 7*B)*a^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.25, size = 93, normalized size = 1.35

$$\frac{3\,(dx+c)Ba^3 - \frac{2\left(3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Aa^3-5\,Ba^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(d*x + c)*B*a^3 - 2*(3*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 3*B*a^3*tan(1/
2*d*x + 1/2*c)^2 + 12*B*a^3*tan(1/2*d*x + 1/2*c) + A*a^3 - 5*B*a^3)/(tan(1/
2*d*x + 1/2*c) - 1)^3)/d
```

maple [B] time = 0.67, size = 248, normalized size = 3.59

$$a^3A\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+B\,a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan\left(dx+c\right)+dx+c\right)+\frac{a^3A\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(
2+sin(d*x+c)^2)*cos(d*x+c))+B*a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+a^3*A
*sin(d*x+c)^3/cos(d*x+c)^3+3*B*a^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d
*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+a^3*A/cos(d*x+c)^3+B*a^
3*sin(d*x+c)^3/cos(d*x+c)^3-a^3*A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/3*B*
a^3/cos(d*x+c)^3)
```

maxima [B] time = 0.42, size = 164, normalized size = 2.38

$$\frac{3\,Aa^3\tan\left(dx+c\right)^3+3\,Ba^3\tan\left(dx+c\right)^3+\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa^3+\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/3*(3*A*a^3*tan(d*x + c)^3 + 3*B*a^3*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*
tan(d*x + c))*A*a^3 + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*B*a^3
 - (3*cos(d*x + c)^2 - 1)*A*a^3/cos(d*x + c)^3 - 3*(3*cos(d*x + c)^2 - 1)*B
*a^3/cos(d*x + c)^3 + 3*A*a^3/cos(d*x + c)^3 + B*a^3/cos(d*x + c)^3)/d
```

mupad [B] time = 9.81, size = 140, normalized size = 2.03

$$B\,a^3\,x-\frac{\frac{a^3\,\left(2\,A-10\,B+3\,B\,\left(c+d\,x\right)\right)}{3}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{a^3\,\left(6\,A-6\,B+9\,B\,\left(c+d\,x\right)\right)}{3}-3\,B\,a^3\,\left(c+d\,x\right)\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^3\,\left(24\,B-9\,B\,\left(c+\right.\right.}{3}\right.}{d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^4,x)
```

```
[Out] B*a^3*x - ((a^3*(2*A - 10*B + 3*B*(c + d*x)))/3 + tan(c/2 + (d*x)/2)^2*((a^
3*(6*A - 6*B + 9*B*(c + d*x)))/3 - 3*B*a^3*(c + d*x)) + tan(c/2 + (d*x)/2)*
((a^3*(24*B - 9*B*(c + d*x)))/3 + 3*B*a^3*(c + d*x)) - B*a^3*(c + d*x))/(d*
(tan(c/2 + (d*x)/2) - 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1000 $\int \sec^6(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=107

$$\frac{a^5(2A-3B)\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{a^5(2A-3B)\cos(c+dx)}{15d\left(a^2-a^2\sin(c+dx)\right)}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+dx)+a)^3}{5d}$$

```
[Out] 1/15*a^5*(2*A-3*B)*cos(d*x+c)/d/(a-a*sin(d*x+c))^2+1/5*(A+B)*sec(d*x+c)^5*(
a+a*sin(d*x+c))^3/d+1/15*a^5*(2*A-3*B)*cos(d*x+c)/d/(a^2-a^2*sin(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2855, 2670, 2650, 2648}

$$\frac{a^5(2A-3B)\cos(c+dx)}{15d\left(a^2-a^2\sin(c+dx)\right)}+\frac{a^5(2A-3B)\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+dx)+a)^3}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^5*(2*A - 3*B)*Cos[c + d*x])/(15*d*(a - a*Sin[c + d*x])^2) + ((A + B)*Sec
[c + d*x]^5*(a + a*Sin[c + d*x])^3)/(5*d) + (a^5*(2*A - 3*B)*Cos[c + d*x])/
(15*d*(a^2 - a^2*Sin[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
```

```
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx &= \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))^3}{5d}+\frac{1}{5}(a(2A-\\&= \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))^3}{5d}+\frac{1}{5}\Big(a^5(2A\\&= \frac{a^5(2A-3B)\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{(A+B)\sec^5(c+dx)(a+}{5d}\\&= \frac{a^5(2A-3B)\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{a^4(2A-3B)\cos(c+dx)}{15d(a-a\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.18, size = 94, normalized size = 0.88

$$-\frac{a^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(6(2A-3B)\sin(c+dx)+(2A-3B)\cos(2(c+dx))-16A+9B)}{30d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/30*(a^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-16*A + 9*B + (2*A - 3*B)
*Cos[2*(c + d*x)] + 6*(2*A - 3*B)*Sin[c + d*x]))/(d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^5)
```

fricas [A] time = 0.69, size = 188, normalized size = 1.76

$$\frac{(2A-3B)a^3\cos(dx+c)^3-2(2A-3B)a^3\cos(dx+c)^2-3(3A-2B)a^3\cos(dx+c)-3(A+B)a^3+\Big((2A-}{15\Big(d\cos(dx+c)^3+3d\cos(dx+c)^2-2d\cos(dx+c)-\Big(d\cos(dx+c)^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/15*((2*A - 3*B)*a^3*cos(d*x + c)^3 - 2*(2*A - 3*B)*a^3*cos(d*x + c)^2 - 3
*(3*A - 2*B)*a^3*cos(d*x + c) - 3*(A + B)*a^3 + ((2*A - 3*B)*a^3*cos(d*x +
c)^2 + 3*(2*A - 3*B)*a^3*cos(d*x + c) - 3*(A + B)*a^3)*sin(d*x + c))/(d*cos
(d*x + c)^3 + 3*d*cos(d*x + c)^2 - 2*d*cos(d*x + c) - (d*cos(d*x + c)^2 - 2
*d*cos(d*x + c) - 4*d)*sin(d*x + c) - 4*d)
```

giac [A] time = 0.25, size = 146, normalized size = 1.36

$$-\frac{2\left(15\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\right.}{15\,d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -2/15*(15*A*a^3*tan(1/2*d*x + 1/2*c)^4 - 30*A*a^3*tan(1/2*d*x + 1/2*c)^3 +
15*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*B*a^
3*tan(1/2*d*x + 1/2*c)^2 - 20*A*a^3*tan(1/2*d*x + 1/2*c) + 15*B*a^3*tan(1/2
*d*x + 1/2*c) + 7*A*a^3 - 3*B*a^3)/(d*(tan(1/2*d*x + 1/2*c) - 1)^5)
```

maple [B] time = 0.66, size = 333, normalized size = 3.11

$$\frac{a^3A\left(\frac{\sin^4(dx+c)}{5\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{15\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{15\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{15}\right)+\frac{B\,a^3\left(\sin^5(dx+c)\right)}{5\cos(dx+c)^5}+3a^3A\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(d\right.}{15\cos(d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/
15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+1/5*B*a^3*sin(
d*x+c)^5/cos(d*x+c)^5+3*a^3*A*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c
)^3/cos(d*x+c)^3)+3*B*a^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/
cos(d*x+c)^3-1/15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))
+3/5*a^3*A/cos(d*x+c)^5+3*B*a^3*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x
+c)^3/cos(d*x+c)^3)-a^3*A*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*
x+c)+1/5*B*a^3/cos(d*x+c)^5)
```

maxima [A] time = 0.33, size = 188, normalized size = 1.76

$$\frac{3\,Ba^3\tan\left(dx+c\right)^5+\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)Aa^3+3\left(3\,\tan\left(dx+c\right)^5+5\,\tan\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/15*(3*B*a^3*tan(d*x + c)^5 + (3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*t
an(d*x + c))*A*a^3 + 3*(3*tan(d*x + c)^5 + 5*tan(d*x + c)^3)*A*a^3 + 3*(3*t
an(d*x + c)^5 + 5*tan(d*x + c)^3)*B*a^3 - (5*cos(d*x + c)^2 - 3)*A*a^3/cos(
d*x + c)^5 - 3*(5*cos(d*x + c)^2 - 3)*B*a^3/cos(d*x + c)^5 + 9*A*a^3/cos(d*
x + c)^5 + 3*B*a^3/cos(d*x + c)^5)/d
```

mupad [B] time = 11.31, size = 113, normalized size = 1.06

$$-\frac{\sqrt{2}\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,B-\frac{53\,A}{4}+4\,A\,\cos\left(c+d\,x\right)+\frac{3\,B\,\cos(c+d\,x)}{2}+\frac{25\,A\,\sin(c+d\,x)}{2}-\frac{15\,B\,\sin(c+d\,x)}{2}+\frac{9\,A\,\cos(2\,c+2\,d\,x)}{4}\right.}{60\,d\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^6,x)
```

```
[Out] -(2^(1/2)*a^3*cos(c/2 + (d*x)/2)*(3*B - (53*A)/4 + 4*A*cos(c + d*x) + (3*B*
cos(c + d*x))/2 + (25*A*sin(c + d*x))/2 - (15*B*sin(c + d*x))/2 + (9*A*cos(
2*c + 2*d*x))/4 - (3*B*cos(2*c + 2*d*x))/2 - (5*A*sin(2*c + 2*d*x))/4))/(60
*d*cos(c/2 + pi/4 + (d*x)/2)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1001 $\int \sec^8(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$\frac{a^3(4A-3B)\tan^3(c+dx)}{35d}+\frac{3a^3(4A-3B)\tan(c+dx)}{35d}+\frac{2(4A-3B)\sec^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{35d}+\frac{(A+B)\text{:}}{}$$

```
[Out] 1/7*(A+B)*sec(d*x+c)^7*(a+a*sin(d*x+c))^3/d+2/35*(4*A-3*B)*sec(d*x+c)^5*(a^
3+a^3*sin(d*x+c))/d+3/35*a^3*(4*A-3*B)*tan(d*x+c)/d+1/35*a^3*(4*A-3*B)*tan(
d*x+c)^3/d
```

Rubi [A] time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2855, 2676, 3767}

$$\frac{a^3(4A-3B)\tan^3(c+dx)}{35d}+\frac{3a^3(4A-3B)\tan(c+dx)}{35d}+\frac{2(4A-3B)\sec^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{35d}+\frac{(A+B)\text{:}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3)/(7*d) + (2*(4*A - 3*B)*Sec[
c + d*x]^5*(a^3 + a^3*Sin[c + d*x]))/(35*d) + (3*a^3*(4*A - 3*B)*Tan[c + d*
x])/(35*d) + (a^3*(4*A - 3*B)*Tan[c + d*x]^3)/(35*d)
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^3}{7d} + \frac{1}{7}(a(4A-$$

$$= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^3}{7d} + \frac{2(4A-3}{}$$

$$= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^3}{7d} + \frac{2(4A-3}{}$$

$$= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^3}{7d} + \frac{2(4A-3}{}$$

Mathematica [A] time = 0.49, size = 135, normalized size = 1.17

$$\frac{a^3(14(4A-3B)\cos(2(c+dx))+(3B-4A)\cos(4(c+dx))+56A\sin(c+dx)-24A\sin(3(c+dx))-42B\sin(c+}{140d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^7\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(35*B + 14*(4*A - 3*B)*Cos[2*(c + d*x)] + (-4*A + 3*B)*Cos[4*(c + d*x)
] + 56*A*Sin[c + d*x] - 42*B*Sin[c + d*x] - 24*A*Sin[3*(c + d*x)] + 18*B*Si
n[3*(c + d*x)]))/(140*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.86, size = 146, normalized size = 1.27

$$\frac{2\,(4\,A-3\,B)a^3\cos{(dx+c)}^4-9\,(4\,A-3\,B)a^3\cos{(dx+c)}^2+5\,(3\,A-4\,B)a^3+\left(6\,(4\,A-3\,B)a^3\cos{(dx+c)}^2-5\right.}{35\left(3\,d\cos{(dx+c)}^3-4\,d\cos{(dx+c)}-\left(d\cos{(dx+c)}^3-4\,d\cos{(dx+c)}\right)\sin{(dx+c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/35*(2*(4*A - 3*B)*a^3*cos(d*x + c)^4 - 9*(4*A - 3*B)*a^3*cos(d*x + c)^2 +
 5*(3*A - 4*B)*a^3 + (6*(4*A - 3*B)*a^3*cos(d*x + c)^2 - 5*(4*A - 3*B)*a^3)
*sin(d*x + c))/(3*d*cos(d*x + c)^3 - 4*d*cos(d*x + c) - (d*cos(d*x + c)^3 -
 4*d*cos(d*x + c))*sin(d*x + c))
```

giac [B] time = 0.29, size = 260, normalized size = 2.26

$$-\frac{\frac{35\left(Aa^3-Ba^3\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}+\frac{525\,Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+35\,Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-1960\,Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+280\,Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+4025\,Aa^3\tan\left(\frac{1}{2}dx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/280*(35*(A*a^3 - B*a^3)/(tan(1/2*d*x + 1/2*c) + 1) + (525*A*a^3*tan(1/2*
d*x + 1/2*c)^6 + 35*B*a^3*tan(1/2*d*x + 1/2*c)^6 - 1960*A*a^3*tan(1/2*d*x +
 1/2*c)^5 + 280*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 4025*A*a^3*tan(1/2*d*x + 1/2
*c)^4 - 665*B*a^3*tan(1/2*d*x + 1/2*c)^4 - 4480*A*a^3*tan(1/2*d*x + 1/2*c)^
3 + 1120*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 3143*A*a^3*tan(1/2*d*x + 1/2*c)^2 -
 791*B*a^3*tan(1/2*d*x + 1/2*c)^2 - 1176*A*a^3*tan(1/2*d*x + 1/2*c) + 392*B
*a^3*tan(1/2*d*x + 1/2*c) + 243*A*a^3 - 51*B*a^3)/(tan(1/2*d*x + 1/2*c) - 1
)^7)/d
```

maple [B] time = 0.67, size = 435, normalized size = 3.78

$$\frac{a^3A\left(\frac{\sin^4(dx+c)}{7\cos(dx+c)^7}+\frac{3\left(\sin^4(dx+c)\right)}{35\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{35\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{35}\right)+Ba^3\left(\frac{\sin^5(dx+c)}{7\cos(dx+c)^7}+\frac{2\left(\sin^5(dx+c\right.}{35\cos(dx+c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/
35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c
)^2)*cos(d*x+c))+B*a^3*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos
(d*x+c)^5)+3*a^3*A*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x
+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+3*B*a^3*(1/7*sin(d*x+c)^4/cos(d*x+c)
^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d
*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c))+3/7*a^3*A/cos(d*x+c)^7
+3*B*a^3*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/10
5*sin(d*x+c)^3/cos(d*x+c)^3)-a^3*A*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)
^4-8/35*sec(d*x+c)^2)*tan(d*x+c)+1/7*B*a^3/cos(d*x+c)^7)
```

maxima [B] time = 0.37, size = 228, normalized size = 1.98

$$\frac{\left(15\,\tan\left(dx+c\right)^{7}+42\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}\right)Aa^{3}+\left(5\,\tan\left(dx+c\right)^{7}+21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/35*((15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*A*a^3 + (
5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))
*A*a^3 + (15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*B*a^3
+ (5*tan(d*x + c)^7 + 7*tan(d*x + c)^5)*B*a^3 - (7*cos(d*x + c)^2 - 5)*A*a^
3/cos(d*x + c)^7 - 3*(7*cos(d*x + c)^2 - 5)*B*a^3/cos(d*x + c)^7 + 15*A*a^3
/cos(d*x + c)^7 + 5*B*a^3/cos(d*x + c)^7)/d
```

mupad [B] time = 11.18, size = 213, normalized size = 1.85

$$-\frac{a^{3}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{35\,A\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{4}-\frac{91\,A\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{4}+A\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)-\frac{35\,B\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{21\,B\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{2}-\frac{3\,B}{}\right.}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^8,x)
```

```
[Out] -(a^3*cos(c/2 + (d*x)/2)*((35*A*cos((5*c)/2 + (5*d*x)/2))/4 - (91*A*cos((3*
c)/2 + (3*d*x)/2))/4 + A*cos((7*c)/2 + (7*d*x)/2) - (35*B*cos(c/2 + (d*x)/2
))/4 + (21*B*cos((3*c)/2 + (3*d*x)/2))/2 - (3*B*cos((7*c)/2 + (7*d*x)/2))/4
 - (233*A*sin(c/2 + (d*x)/2))/8 + (121*A*sin((3*c)/2 + (3*d*x)/2))/8 + (61*
A*sin((5*c)/2 + (5*d*x)/2))/8 - (13*A*sin((7*c)/2 + (7*d*x)/2))/8 + (61*B*s
in(c/2 + (d*x)/2))/8 + (23*B*sin((3*c)/2 + (3*d*x)/2))/8 - (37*B*sin((5*c)/
2 + (5*d*x)/2))/8 + (B*sin((7*c)/2 + (7*d*x)/2))/8))/(280*d*cos(c/2 - pi/4
+ (d*x)/2)*cos(c/2 + pi/4 + (d*x)/2)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1002 $\int \sec^{10}(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{a^3(2A-B)\tan^5(c+dx)}{21d}+\frac{10a^3(2A-B)\tan^3(c+dx)}{63d}+\frac{5a^3(2A-B)\tan(c+dx)}{21d}+\frac{2(2A-B)\sec^7(c+dx)\left(a^3\sin\right.}{21d}$$

```
[Out] 1/9*(A+B)*sec(d*x+c)^9*(a+a*sin(d*x+c))^3/d+2/21*(2*A-B)*sec(d*x+c)^7*(a^3+
a^3*sin(d*x+c))/d+5/21*a^3*(2*A-B)*tan(d*x+c)/d+10/63*a^3*(2*A-B)*tan(d*x+c
)^3/d+1/21*a^3*(2*A-B)*tan(d*x+c)^5/d
```

Rubi [A] time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2855, 2676, 3767}

$$\frac{a^3(2A-B)\tan^5(c+dx)}{21d}+\frac{10a^3(2A-B)\tan^3(c+dx)}{63d}+\frac{5a^3(2A-B)\tan(c+dx)}{21d}+\frac{2(2A-B)\sec^7(c+dx)\left(a^3\sin\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^9*(a + a*Sin[c + d*x])^3)/(9*d) + (2*(2*A - B)*Sec[c
+ d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(21*d) + (5*a^3*(2*A - B)*Tan[c + d*x])/
(21*d) + (10*a^3*(2*A - B)*Tan[c + d*x]^3)/(63*d) + (a^3*(2*A - B)*Tan[c +
d*x]^5)/(21*d)
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^3}{9d} + \frac{1}{3}(a(2A\cdots$$

$$= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^3}{9d} + \frac{2(2A-\cdots}{\cdots}$$

$$= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^3}{9d} + \frac{2(2A-\cdots}{\cdots}$$

$$= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^3}{9d} + \frac{2(2A-\cdots}{\cdots}$$

Mathematica [A] time = 0.61, size = 176, normalized size = 1.26

$$-\frac{a^3(27(B-2A)\cos(2(c+dx))+12(B-2A)\cos(4(c+dx))-72A\sin(c+dx)-4A\sin(3(c+dx))+12A\sin(5(\cdots}{252d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+d\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/252*(a^3*(-42*B + 27*(-2*A + B)*Cos[2*(c + d*x)] + 12*(-2*A + B)*Cos[4*(
c + d*x)] + 2*A*Cos[6*(c + d*x)] - B*Cos[6*(c + d*x)] - 72*A*Sin[c + d*x] +
 36*B*Sin[c + d*x] - 4*A*Sin[3*(c + d*x)] + 2*B*Sin[3*(c + d*x)] + 12*A*Sin
[5*(c + d*x)] - 6*B*Sin[5*(c + d*x)]))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^9*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

fricas [A] time = 0.68, size = 188, normalized size = 1.34

$$\frac{8(2A-B)a^3\cos(dx+c)^6-36(2A-B)a^3\cos(dx+c)^4+15(2A-B)a^3\cos(dx+c)^2+7(A-2B)a^3+\left(24(2\cdots\right.}{63\left(3d\cos(dx+c)^5-4d\cos(dx+c)^3-\left(d\cos(dx+c)^5-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] 1/63*(8*(2*A - B)*a^3*cos(d*x + c)^6 - 36*(2*A - B)*a^3*cos(d*x + c)^4 + 15
*(2*A - B)*a^3*cos(d*x + c)^2 + 7*(A - 2*B)*a^3 + (24*(2*A - B)*a^3*cos(d*x
 + c)^4 - 20*(2*A - B)*a^3*cos(d*x + c)^2 - 7*(2*A - B)*a^3)*sin(d*x + c))/
(3*d*cos(d*x + c)^5 - 4*d*cos(d*x + c)^3 - (d*cos(d*x + c)^5 - 4*d*cos(d*x
+ c)^3)*sin(d*x + c))
```

giac [B] time = 0.30, size = 393, normalized size = 2.81

$$-\frac{\frac{21\left(21\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+19\,Aa^3-13\,Ba^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}+\frac{3591\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="g
iac")
```

```
[Out] -1/2016*(21*(21*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*B*a^3*tan(1/2*d*x + 1/2*c
)^2 + 36*A*a^3*tan(1/2*d*x + 1/2*c) - 24*B*a^3*tan(1/2*d*x + 1/2*c) + 19*A*
a^3 - 13*B*a^3)/(tan(1/2*d*x + 1/2*c) + 1)^3 + (3591*A*a^3*tan(1/2*d*x + 1/
2*c)^8 + 315*B*a^3*tan(1/2*d*x + 1/2*c)^8 - 19656*A*a^3*tan(1/2*d*x + 1/2*c
)^7 + 756*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 56196*A*a^3*tan(1/2*d*x + 1/2*c)^6
 - 4200*B*a^3*tan(1/2*d*x + 1/2*c)^6 - 95760*A*a^3*tan(1/2*d*x + 1/2*c)^5 +
 11340*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 107730*A*a^3*tan(1/2*d*x + 1/2*c)^4 -
 14994*B*a^3*tan(1/2*d*x + 1/2*c)^4 - 79464*A*a^3*tan(1/2*d*x + 1/2*c)^3 +
13356*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 38484*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 6
768*B*a^3*tan(1/2*d*x + 1/2*c)^2 - 10944*A*a^3*tan(1/2*d*x + 1/2*c) + 2196*
B*a^3*tan(1/2*d*x + 1/2*c) + 1615*A*a^3 - 209*B*a^3)/(tan(1/2*d*x + 1/2*c)
- 1)^9)/d
```

maple [B] time = 0.69, size = 535, normalized size = 3.82

$$\frac{a^3 A\left(\frac{\sin^4(dx+c)}{9\cos(dx+c)^9}+\frac{5\left(\sin^4(dx+c)\right)}{63\cos(dx+c)^7}+\frac{\sin^4(dx+c)}{21\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{63\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{63\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{63}\right)+B\,a^3\left(\frac{\sin^5(dx+c)}{9\cos(dx+c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(1/9*sin(d*x+c)^4/cos(d*x+c)^9+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/
21*sin(d*x+c)^4/cos(d*x+c)^5+1/63*sin(d*x+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)
```

```
^4/cos(d*x+c)-1/63*(2+sin(d*x+c)^2)*cos(d*x+c))+B*a^3*(1/9*sin(d*x+c)^5/cos
(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*sin(d*x+c)^5/cos(d*x+c)^5)+3
*a^3*A*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*
sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+3*B*a^3*(1/9*si
n(d*x+c)^4/cos(d*x+c)^9+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/co
s(d*x+c)^5+1/63*sin(d*x+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/63
*(2+sin(d*x+c)^2)*cos(d*x+c))+1/3*a^3*A/cos(d*x+c)^9+3*B*a^3*(1/9*sin(d*x+c
)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+
c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)-a^3*A*(-128/315-1/9*sec(d*x+c)^8-8/6
3*sec(d*x+c)^6-16/105*sec(d*x+c)^4-64/315*sec(d*x+c)^2)*tan(d*x+c)+1/9*B*a^
3/cos(d*x+c)^9)
```

maxima [B] time = 0.35, size = 270, normalized size = 1.93

$$\frac{\left(35 \tan\left(dx+c\right)^{9} + 180 \tan\left(dx+c\right)^{7} + 378 \tan\left(dx+c\right)^{5} + 420 \tan\left(dx+c\right)^{3} + 315 \tan\left(dx+c\right)\right)Aa^{3} + 3\left(35 \tan \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="m
axima")
```

```
[Out] 1/315*((35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 378*tan(d*x + c)^5 + 420*t
an(d*x + c)^3 + 315*tan(d*x + c))*A*a^3 + 3*(35*tan(d*x + c)^9 + 135*tan(d*
x + c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*A*a^3 + 3*(35*tan(d*x +
 c)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*B*a^3
 + (35*tan(d*x + c)^9 + 90*tan(d*x + c)^7 + 63*tan(d*x + c)^5)*B*a^3 - 5*(9
*cos(d*x + c)^2 - 7)*A*a^3/cos(d*x + c)^9 - 15*(9*cos(d*x + c)^2 - 7)*B*a^3
/cos(d*x + c)^9 + 105*A*a^3/cos(d*x + c)^9 + 35*B*a^3/cos(d*x + c)^9)/d
```

mupad [B] time = 12.21, size = 322, normalized size = 2.30

$$-\frac{a^{3} \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{63\,A \cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8} - \frac{171\,A \cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8} - \frac{145\,A \cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16} + \frac{49\,A \cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16} + \frac{A \cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{2} - \frac{21 \cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^10,x)
```

```
[Out] -(a^3*cos(c/2 + (d*x)/2)*((63*A*cos((5*c)/2 + (5*d*x)/2))/8 - (171*A*cos((3
*c)/2 + (3*d*x)/2))/8 - (145*A*cos((7*c)/2 + (7*d*x)/2))/16 + (49*A*cos((9*
c)/2 + (9*d*x)/2))/16 + (A*cos((11*c)/2 + (11*d*x)/2))/2 - (21*B*cos(c/2 +
(d*x)/2))/2 + (75*B*cos((3*c)/2 + (3*d*x)/2))/8 - (21*B*cos((5*c)/2 + (5*d*
```

```
x)/2))/8 + (41*B*cos((7*c)/2 + (7*d*x)/2))/16 + (7*B*cos((9*c)/2 + (9*d*x)/
2))/16 - (B*cos((11*c)/2 + (11*d*x)/2))/4 - (617*A*sin(c/2 + (d*x)/2))/16 +
 (329*A*sin((3*c)/2 + (3*d*x)/2))/16 - (145*A*sin((5*c)/2 + (5*d*x)/2))/32
+ (113*A*sin((7*c)/2 + (7*d*x)/2))/32 + (115*A*sin((9*c)/2 + (9*d*x)/2))/32
 - (19*A*sin((11*c)/2 + (11*d*x)/2))/32 + (109*B*sin(c/2 + (d*x)/2))/16 + (
35*B*sin((3*c)/2 + (3*d*x)/2))/16 - (43*B*sin((5*c)/2 + (5*d*x)/2))/32 + (5
9*B*sin((7*c)/2 + (7*d*x)/2))/32 - (47*B*sin((9*c)/2 + (9*d*x)/2))/32 - (B*
sin((11*c)/2 + (11*d*x)/2))/32))/(2016*d*cos(c/2 - pi/4 + (d*x)/2)^3*cos(c/
2 + pi/4 + (d*x)/2)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
[Out] Timed out
```

3.1003 $\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=105

$$\frac{B(a-a\sin(c+dx))^7}{7a^8d}-\frac{(A+5B)(a-a\sin(c+dx))^6}{6a^7d}+\frac{4(A+2B)(a-a\sin(c+dx))^5}{5a^6d}-\frac{(A+B)(a-a\sin(c+dx))^4}{a^5d}$$

```
[Out] -(A+B)*(a-a*sin(d*x+c))^4/a^5/d+4/5*(A+2*B)*(a-a*sin(d*x+c))^5/a^6/d-1/6*(A
+5*B)*(a-a*sin(d*x+c))^6/a^7/d+1/7*B*(a-a*sin(d*x+c))^7/a^8/d
```

Rubi [A] time = 0.15, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$-\frac{(A+5B)(a-a\sin(c+dx))^6}{6a^7d}+\frac{4(A+2B)(a-a\sin(c+dx))^5}{5a^6d}-\frac{(A+B)(a-a\sin(c+dx))^4}{a^5d}+\frac{B(a-a\sin(c+dx))^7}{7a^8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(((A + B)*(a - a*Sin[c + d*x])^4)/(a^5*d)) + (4*(A + 2*B)*(a - a*Sin[c + d
*x])^5)/(5*a^6*d) - ((A + 5*B)*(a - a*Sin[c + d*x])^6)/(6*a^7*d) + (B*(a -
a*Sin[c + d*x])^7)/(7*a^8*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^2\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^2(A+B)(a-x)^3 - 4a(A+2B)(a-x)^4 + (A+5B)(a-x\right.\right.}{a^7d}$$
$$= -\frac{(A+B)(a-a\sin(c+dx))^4}{a^5d} + \frac{4(A+2B)(a-a\sin(c+dx))^5}{5a^6d} - \frac{(A+}{}$$

Mathematica [A] time = 0.23, size = 69, normalized size = 0.66

$$-\frac{(\sin(c+dx)-1)^4\left(5(7A+17B)\sin^2(c+dx)+(98A+76B)\sin(c+dx)+77A+30B\sin^3(c+dx)+19B\right)}{210ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/210*((-1 + Sin[c + d*x])^4*(77*A + 19*B + (98*A + 76*B)*Sin[c + d*x] + 5
*(7*A + 17*B)*Sin[c + d*x]^2 + 30*B*Sin[c + d*x]^3))/(a*d)
```

fricas [A] time = 0.75, size = 84, normalized size = 0.80

$$\frac{35\,(A-B)\cos\left(dx+c\right)^6+2\left(15\,B\cos\left(dx+c\right)^6+3\,(7\,A-B)\cos\left(dx+c\right)^4+4\,(7\,A-B)\cos\left(dx+c\right)^2+56\,A-\right.}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/210*(35*(A - B)*cos(d*x + c)^6 + 2*(15*B*cos(d*x + c)^6 + 3*(7*A - B)*cos
(d*x + c)^4 + 4*(7*A - B)*cos(d*x + c)^2 + 56*A - 8*B)*sin(d*x + c))/(a*d)
```

giac [A] time = 0.21, size = 139, normalized size = 1.32

$$-\frac{30\,B\sin\left(dx+c\right)^7+35\,A\sin\left(dx+c\right)^6-35\,B\sin\left(dx+c\right)^6-42\,A\sin\left(dx+c\right)^5-84\,B\sin\left(dx+c\right)^5-105\,A\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/210*(30*B*sin(d*x + c)^7 + 35*A*sin(d*x + c)^6 - 35*B*sin(d*x + c)^6 - 4
2*A*sin(d*x + c)^5 - 84*B*sin(d*x + c)^5 - 105*A*sin(d*x + c)^4 + 105*B*sin
(d*x + c)^4 + 140*A*sin(d*x + c)^3 + 70*B*sin(d*x + c)^3 + 105*A*sin(d*x +
c)^2 - 105*B*sin(d*x + c)^2 - 210*A*sin(d*x + c))/(a*d)
```

maple [A] time = 0.56, size = 107, normalized size = 1.02

$$\frac{-\frac{B\left(\sin^{7}(dx+c)\right)}{7}+\frac{(-A+B)\left(\sin^{6}(dx+c)\right)}{6}+\frac{(A+2B)\left(\sin^{5}(dx+c)\right)}{5}+\frac{(2A-2B)\left(\sin^{4}(dx+c)\right)}{4}+\frac{(-2A-B)\left(\sin^{3}(dx+c)\right)}{3}+\frac{(-A+B)\left(\sin^{2}(dx+c)\right)}{2}+A}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/7*B*sin(d*x+c)^7+1/6*(-A+B)*sin(d*x+c)^6+1/5*(A+2*B)*sin(d*x+c)^5
+1/4*(2*A-2*B)*sin(d*x+c)^4+1/3*(-2*A-B)*sin(d*x+c)^3+1/2*(-A+B)*sin(d*x+c)
^2+A*sin(d*x+c))
```

maxima [A] time = 0.32, size = 104, normalized size = 0.99

$$-\frac{30\,B\sin\left(dx+c\right)^{7}+35\,(A-B)\sin\left(dx+c\right)^{6}-42\,(A+2\,B)\sin\left(dx+c\right)^{5}-105\,(A-B)\sin\left(dx+c\right)^{4}+70\,(2\,A+}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/210*(30*B*sin(d*x + c)^7 + 35*(A - B)*sin(d*x + c)^6 - 42*(A + 2*B)*sin(
d*x + c)^5 - 105*(A - B)*sin(d*x + c)^4 + 70*(2*A + B)*sin(d*x + c)^3 + 105
*(A - B)*sin(d*x + c)^2 - 210*A*sin(d*x + c))/(a*d)
```

mupad [B] time = 0.11, size = 124, normalized size = 1.18

$$-\frac{\frac{\sin(c+d\,x)^{2}\,(A-B)}{2\,a}+\frac{\sin(c+d\,x)^{3}\,(2\,A+B)}{3\,a}-\frac{\sin(c+d\,x)^{5}\,(A+2\,B)}{5\,a}+\frac{\sin(c+d\,x)^{6}\,(A-B)}{6\,a}+\frac{B\sin(c+d\,x)^{7}}{7\,a}-\frac{\sin(c+d\,x)^{4}\,(2\,A-2\,B)}{4\,a}-\frac{A\,\sin(c+d\,x}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x)),x)
```

```
[Out] -((sin(c + d*x)^2*(A - B))/(2*a) + (sin(c + d*x)^3*(2*A + B))/(3*a) - (sin(
c + d*x)^5*(A + 2*B))/(5*a) + (sin(c + d*x)^6*(A - B))/(6*a) + (B*sin(c + d
*x)^7)/(7*a) - (sin(c + d*x)^4*(2*A - 2*B))/(4*a) - (A*sin(c + d*x))/a)/d
```

sympy [A] time = 83.08, size = 3363, normalized size = 32.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((210*A*tan(c/2 + d*x/2)**13/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a
*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2
+ d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 +
 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 210*A*tan(c/2 + d*x/2)**12/(105*a
*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 +
 d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 +
 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 70
0*A*tan(c/2 + d*x/2)**11/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 +
d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 +
 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(
c/2 + d*x/2)**2 + 105*a*d) - 210*A*tan(c/2 + d*x/2)**10/(105*a*d*tan(c/2 +
d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 +
 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan
(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 1582*A*tan(c/2
+ d*x/2)**9/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 +
2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan
(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)*
*2 + 105*a*d) - 700*A*tan(c/2 + d*x/2)**8/(105*a*d*tan(c/2 + d*x/2)**14 + 7
35*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(
c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)*
*4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 2184*A*tan(c/2 + d*x/2)**7/(1
05*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c
/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)*
*6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d)
- 700*A*tan(c/2 + d*x/2)**6/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2
 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**
8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*t
an(c/2 + d*x/2)**2 + 105*a*d) + 1582*A*tan(c/2 + d*x/2)**5/(105*a*d*tan(c/2
 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**1
0 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*
tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 210*A*tan(c/
2 + d*x/2)**4/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12
+ 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*t
an(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2
)**2 + 105*a*d) + 700*A*tan(c/2 + d*x/2)**3/(105*a*d*tan(c/2 + d*x/2)**14 +
 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*ta
n(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2
)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 210*A*tan(c/2 + d*x/2)**2/(
105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(
c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)
**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d)
```

```
 + 210*A*tan(c/2 + d*x/2)/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 +
 d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8
+ 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan
(c/2 + d*x/2)**2 + 105*a*d) + 210*B*tan(c/2 + d*x/2)**12/(105*a*d*tan(c/2 +
 d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10
+ 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*ta
n(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 280*B*tan(c/2
+ d*x/2)**11/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 +
 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*ta
n(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)
**2 + 105*a*d) + 210*B*tan(c/2 + d*x/2)**10/(105*a*d*tan(c/2 + d*x/2)**14 +
 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*ta
n(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2
)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 224*B*tan(c/2 + d*x/2)**9/(
105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(
c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)
**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d)
 + 700*B*tan(c/2 + d*x/2)**8/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/
2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)*
*8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*
tan(c/2 + d*x/2)**2 + 105*a*d) - 912*B*tan(c/2 + d*x/2)**7/(105*a*d*tan(c/2
 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**1
0 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*
tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 700*B*tan(c/
2 + d*x/2)**6/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12
+ 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*t
an(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2
)**2 + 105*a*d) + 224*B*tan(c/2 + d*x/2)**5/(105*a*d*tan(c/2 + d*x/2)**14 +
 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*ta
n(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2
)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 210*B*tan(c/2 + d*x/2)**4/(
105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(
c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)
**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d)
 - 280*B*tan(c/2 + d*x/2)**3/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/
2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)*
*8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*
tan(c/2 + d*x/2)**2 + 105*a*d) + 210*B*tan(c/2 + d*x/2)**2/(105*a*d*tan(c/2
 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**1
0 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*
tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d), Ne(d, 0)), (x
*(A + B*sin(c))*cos(c)**7/(a*sin(c) + a), True))
```

3.1004 $\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=79

$$-\frac{B(a-a\sin(c+dx))^5}{5a^6d}+\frac{(A+3B)(a-a\sin(c+dx))^4}{4a^5d}-\frac{2(A+B)(a-a\sin(c+dx))^3}{3a^4d}$$

```
[Out] -2/3*(A+B)*(a-a*sin(d*x+c))^3/a^4/d+1/4*(A+3*B)*(a-a*sin(d*x+c))^4/a^5/d-1/
5*B*(a-a*sin(d*x+c))^5/a^6/d
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{(A+3B)(a-a\sin(c+dx))^4}{4a^5d}-\frac{2(A+B)(a-a\sin(c+dx))^3}{3a^4d}-\frac{B(a-a\sin(c+dx))^5}{5a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*(A + B)*(a - a*Sin[c + d*x])^3)/(3*a^4*d) + ((A + 3*B)*(a - a*Sin[c + d
*x])^4)/(4*a^5*d) - (B*(a - a*Sin[c + d*x])^5)/(5*a^6*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(2a(A+B)(a-x)^2+(-A-3B)(a-x)^3+\frac{B(a-x)^4}{a}\right)dx, x, a\sin\right.}{a^5 d}$$
$$= -\frac{2(A+B)(a-a\sin(c+dx))^3}{3a^4 d}+\frac{(A+3B)(a-a\sin(c+dx))^4}{4a^5 d}-\frac{B(a-}{}$$

Mathematica [**A**] time = 0.15, size = 72, normalized size = 0.91

$$\frac{\sin(c+dx)\left(15(A-B)\sin^3(c+dx)-20(A+B)\sin^2(c+dx)-30(A-B)\sin(c+dx)+60A+12B\sin^4(c+dx)\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(60*A - 30*(A - B)*Sin[c + d*x] - 20*(A + B)*Sin[c + d*x]^2 +
 15*(A - B)*Sin[c + d*x]^3 + 12*B*Sin[c + d*x]^4))/(60*a*d)
```

fricas [**A**] time = 0.72, size = 66, normalized size = 0.84

$$\frac{15\,(A-B)\cos(dx+c)^4+4\left(3\,B\cos(dx+c)^4+(5\,A-B)\cos(dx+c)^2+10\,A-2\,B\right)\sin(dx+c)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/60*(15*(A - B)*cos(d*x + c)^4 + 4*(3*B*cos(d*x + c)^4 + (5*A - B)*cos(d*x
 + c)^2 + 10*A - 2*B)*sin(d*x + c))/(a*d)
```

giac [**A**] time = 0.19, size = 95, normalized size = 1.20

$$\frac{12\,B\sin(dx+c)^5+15\,A\sin(dx+c)^4-15\,B\sin(dx+c)^4-20\,A\sin(dx+c)^3-20\,B\sin(dx+c)^3-30\,A\sin(dx}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/60*(12*B*sin(d*x + c)^5 + 15*A*sin(d*x + c)^4 - 15*B*sin(d*x + c)^4 - 20*
A*sin(d*x + c)^3 - 20*B*sin(d*x + c)^3 - 30*A*sin(d*x + c)^2 + 30*B*sin(d*x
 + c)^2 + 60*A*sin(d*x + c))/(a*d)
```

maple [A] time = 0.45, size = 75, normalized size = 0.95

$$\frac{\frac{B\left(\sin^5(dx+c)\right)}{5}+\frac{(A-B)\left(\sin^4(dx+c)\right)}{4}+\frac{(-A-B)\left(\sin^3(dx+c)\right)}{3}+\frac{(-A+B)\left(\sin^2(dx+c)\right)}{2}+A\sin\left(dx+c\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/5*B*sin(d*x+c)^5+1/4*(A-B)*sin(d*x+c)^4+1/3*(-A-B)*sin(d*x+c)^3+1/
2*(-A+B)*sin(d*x+c)^2+A*sin(d*x+c))
```

maxima [A] time = 0.32, size = 72, normalized size = 0.91

$$\frac{12\,B\sin\left(dx+c\right)^5+15\,(A-B)\sin\left(dx+c\right)^4-20\,(A+B)\sin\left(dx+c\right)^3-30\,(A-B)\sin\left(dx+c\right)^2+60\,A\sin\left(dx\right.}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/60*(12*B*sin(d*x + c)^5 + 15*(A - B)*sin(d*x + c)^4 - 20*(A + B)*sin(d*x
+ c)^3 - 30*(A - B)*sin(d*x + c)^2 + 60*A*sin(d*x + c))/(a*d)
```

mupad [B] time = 9.20, size = 82, normalized size = 1.04

$$\frac{\frac{\sin(c+d\,x)^4\,(A-B)}{4\,a}-\frac{\sin(c+d\,x)^2\,(A-B)}{2\,a}+\frac{B\sin(c+d\,x)^5}{5\,a}+\frac{A\,\sin(c+d\,x)}{a}-\frac{\sin(c+d\,x)^3\,(A+B)}{3\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x)),x)
```

```
[Out] ((sin(c + d*x)^4*(A - B))/(4*a) - (sin(c + d*x)^2*(A - B))/(2*a) + (B*sin(c
 + d*x)^5)/(5*a) + (A*sin(c + d*x))/a - (sin(c + d*x)^3*(A + B))/(3*a))/d
```

sympy [A] time = 30.97, size = 1703, normalized size = 21.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((30*A*tan(c/2 + d*x/2)**9/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*t
an(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)
**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 30*A*tan(c/2 + d*x/2)**8/(15*a
*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*
x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d
) + 80*A*tan(c/2 + d*x/2)**7/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2
+ d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 7
5*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 30*A*tan(c/2 + d*x/2)**6/(15*a*d*tan(
c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6
 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 100
*A*tan(c/2 + d*x/2)**5/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/
2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*
tan(c/2 + d*x/2)**2 + 15*a*d) - 30*A*tan(c/2 + d*x/2)**4/(15*a*d*tan(c/2 +
d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150
*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 80*A*tan(
c/2 + d*x/2)**3/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 +
 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2
 + d*x/2)**2 + 15*a*d) - 30*A*tan(c/2 + d*x/2)**2/(15*a*d*tan(c/2 + d*x/2)*
*10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*ta
n(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*A*tan(c/2 + d
*x/2)/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*t
an(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)*
*2 + 15*a*d) + 30*B*tan(c/2 + d*x/2)**8/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a
*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*
x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 40*B*tan(c/2 + d*x/2)**7/(
15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2
+ d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15
*a*d) + 30*B*tan(c/2 + d*x/2)**6/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(
c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4
 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 16*B*tan(c/2 + d*x/2)**5/(15*a*d*
tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2
)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) +
 30*B*tan(c/2 + d*x/2)**4/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d
*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a
*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 40*B*tan(c/2 + d*x/2)**3/(15*a*d*tan(c/2
 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 +
150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*B*t
an(c/2 + d*x/2)**2/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**
8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(
c/2 + d*x/2)**2 + 15*a*d), Ne(d, 0)), (x*(A + B*sin(c))*cos(c)**5/(a*sin(c)
 + a), True))
```

3.1005 $\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=57

$$-\frac{(A-B)\sin^2(c+dx)}{2ad}+\frac{A\sin(c+dx)}{ad}-\frac{B\sin^3(c+dx)}{3ad}$$

```
[Out] A*sin(d*x+c)/a/d-1/2*(A-B)*sin(d*x+c)^2/a/d-1/3*B*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.10, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 43}

$$-\frac{(A-B)\sin^2(c+dx)}{2ad}+\frac{A\sin(c+dx)}{ad}-\frac{B\sin^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (A*Sin[c + d*x])/(a*d) - ((A - B)*Sin[c + d*x]^2)/(2*a*d) - (B*Sin[c + d*x]
^3)/(3*a*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int (a-x)\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(aA-(A-B)x-\frac{Bx^2}{a}\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{A\sin(c+dx)}{ad} - \frac{(A-B)\sin^2(c+dx)}{2ad} - \frac{B\sin^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.10, size = 44, normalized size = 0.77

$$\frac{\sin(c+dx)\left(-3(A-B)\sin(c+dx)+6A-2B\sin^2(c+dx)\right)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
[Out] (Sin[c + d*x]*(6*A - 3*(A - B)*Sin[c + d*x] - 2*B*Sin[c + d*x]^2))/(6*a*d)
```

fricas [A] time = 0.78, size = 49, normalized size = 0.86

$$\frac{3\,(A-B)\cos\left(dx+c\right)^2+2\left(B\cos\left(dx+c\right)^2+3\,A-B\right)\sin\left(dx+c\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
[Out] 1/6*(3*(A - B)*cos(d*x + c)^2 + 2*(B*cos(d*x + c)^2 + 3*A - B)*sin(d*x + c)
)/(a*d)
```

giac [A] time = 0.18, size = 51, normalized size = 0.89

$$-\frac{2\,B\sin\left(dx+c\right)^3+3\,A\sin\left(dx+c\right)^2-3\,B\sin\left(dx+c\right)^2-6\,A\sin\left(dx+c\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(2*B*sin(d*x + c)^3 + 3*A*sin(d*x + c)^2 - 3*B*sin(d*x + c)^2 - 6*A*si
n(d*x + c))/(a*d)
```

maple [A] time = 0.44, size = 43, normalized size = 0.75

$$\frac{-\frac{B\left(\sin^3(dx+c)\right)}{3}+\frac{(-A+B)\left(\sin^2(dx+c)\right)}{2}+A\sin(dx+c)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
[Out] 1/d/a*(-1/3*B*sin(d*x+c)^3+1/2*(-A+B)*sin(d*x+c)^2+A*sin(d*x+c))
```

maxima [A] time = 0.33, size = 44, normalized size = 0.77

$$-\frac{2\,B\sin\left(dx+c\right)^3+3\,(A-B)\sin\left(dx+c\right)^2-6\,A\sin\left(dx+c\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
[Out] -1/6*(2*B*sin(d*x + c)^3 + 3*(A - B)*sin(d*x + c)^2 - 6*A*sin(d*x + c))/(a*
d)
```

mupad [B] time = 0.08, size = 47, normalized size = 0.82

$$\frac{\sin\left(c+d\,x\right)\,\left(6\,A-3\,A\,\sin\left(c+d\,x\right)+3\,B\,\sin\left(c+d\,x\right)-2\,B\sin\left(c+d\,x\right)^2\right)}{6\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x)),x)
[Out] (sin(c + d*x)*(6*A - 3*A*sin(c + d*x) + 3*B*sin(c + d*x) - 2*B*sin(c + d*x)
^2))/(6*a*d)
```

sympy [A] time = 9.86, size = 588, normalized size = 10.32

$$\begin{cases} \frac{6A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad}-\frac{6A\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad}+\frac{}{3ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9} \\ \frac{x(A+B\sin(c))\cos^3(c)}{a\sin(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((6*A*tan(c/2 + d*x/2)**5/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c
/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) - 6*A*tan(c/2 + d*x/2)*
*4/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 +
 d*x/2)**2 + 3*a*d) + 12*A*tan(c/2 + d*x/2)**3/(3*a*d*tan(c/2 + d*x/2)**6 +
 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) - 6*A*tan(c
/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a
*d*tan(c/2 + d*x/2)**2 + 3*a*d) + 6*A*tan(c/2 + d*x/2)/(3*a*d*tan(c/2 + d*x
/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) + 6
*B*tan(c/2 + d*x/2)**4/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)*
*4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) - 8*B*tan(c/2 + d*x/2)**3/(3*a*d*ta
n(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 +
 3*a*d) + 6*B*tan(c/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/
2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d), Ne(d, 0)), (x*(A + B*si
n(c))*cos(c)**3/(a*sin(c) + a), True))
```

3.1006 $\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{(A-B)\log(\sin(c+dx)+1)}{ad}+\frac{B\sin(c+dx)}{ad}$$

```
[Out] (A-B)*ln(1+sin(d*x+c))/a/d+B*sin(d*x+c)/a/d
```

Rubi [A] time = 0.08, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{(A-B)\log(\sin(c+dx)+1)}{ad}+\frac{B\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*Log[1 + Sin[c + d*x]])/(a*d) + (B*Sin[c + d*x])/(a*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{B}{a}+\frac{A-B}{a+x}\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{(A-B)\log(1+\sin(c+dx))}{ad} + \frac{B\sin(c+dx)}{ad}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 0.86

$$\frac{(A-B)\log(\sin(c+dx)+1)+B\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
[Out] ((A - B)*Log[1 + Sin[c + d*x]] + B*Sin[c + d*x])/(a*d)
```

fricas [A] time = 0.56, size = 31, normalized size = 0.86

$$\frac{(A-B)\log(\sin(dx+c)+1)+B\sin(dx+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fricas
")
[Out] ((A - B)*log(sin(d*x + c) + 1) + B*sin(d*x + c))/(a*d)
```

giac [A] time = 0.17, size = 35, normalized size = 0.97

$$\frac{\frac{(A-B)\log(|\sin(dx+c)+1|)}{a}+\frac{B\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] ((A - B)*log(abs(sin(d*x + c) + 1))/a + B*sin(d*x + c)/a)/d
```

maple [A] time = 0.26, size = 51, normalized size = 1.42

$$\frac{\ln\left(1+\sin\left(dx+c\right)\right)A}{da}-\frac{\ln\left(1+\sin\left(dx+c\right)\right)B}{da}+\frac{B\sin\left(dx+c\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*ln(1+sin(d*x+c))*A-1/d/a*ln(1+sin(d*x+c))*B+B*sin(d*x+c)/d/a
```

maxima [A] time = 0.32, size = 34, normalized size = 0.94

$$\frac{\frac{(A-B)\log(\sin(dx+c)+1)}{a}+\frac{B\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] ((A - B)*log(sin(d*x + c) + 1)/a + B*sin(d*x + c)/a)/d
```

mupad [B] time = 9.23, size = 36, normalized size = 1.00

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(A-B\right)}{a\,d}+\frac{B\,\sin\left(c+d\,x\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x)),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(A - B))/(a*d) + (B*sin(c + d*x))/(a*d)
```

sympy [A] time = 0.58, size = 60, normalized size = 1.67

$$\begin{cases} \frac{A\log\left(\sin\left(c+dx\right)+1\right)}{ad}-\frac{B\log\left(\sin\left(c+dx\right)+1\right)}{ad}+\frac{B\sin\left(c+dx\right)}{ad} & \text{for}\ d\neq 0\\ \frac{x(A+B\sin\left(c\right))\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*log(sin(c + d*x) + 1)/(a*d) - B*log(sin(c + d*x) + 1)/(a*d) +
B*sin(c + d*x)/(a*d), Ne(d, 0)), (x*(A + B*sin(c))*cos(c)/(a*sin(c) + a), T
rue))
```

3.1007 $\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=45

$$\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{A-B}{2d(a\sin(c+dx)+a)}$$

```
[Out] 1/2*(A+B)*arctanh(sin(d*x+c))/a/d+1/2*(-A+B)/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 77, 206}

$$\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{A-B}{2d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*ArcTanh[Sin[c + d*x]])/(2*a*d) - (A - B)/(2*d*(a + a*Sin[c + d*x])
)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{a\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{A-B}{2a(a+x)^2}+\frac{A+B}{2a\left(a^2-x^2\right)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{A-B}{2d(a+a\sin(c+dx))}+\frac{(A+B)\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin(c+dx)\right)}{2d}$$
$$= \frac{(A+B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{A-B}{2d(a+a\sin(c+dx))}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 0.98

$$\frac{(A+B)(\sin(c+dx)+1)\tanh^{-1}(\sin(c+dx))-A+B}{2ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-A + B + (A + B)*ArcTanh[Sin[c + d*x]]*(1 + Sin[c + d*x]))/(2*a*d*(1 + Sin
[c + d*x]))
```

fricas [A] time = 0.67, size = 73, normalized size = 1.62

$$\frac{((A+B)\sin(dx+c)+A+B)\log(\sin(dx+c)+1)-((A+B)\sin(dx+c)+A+B)\log(-\sin(dx+c)+1)-}{4\,(ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/4*(((A + B)*sin(d*x + c) + A + B)*log(sin(d*x + c) + 1) - ((A + B)*sin(d*
x + c) + A + B)*log(-sin(d*x + c) + 1) - 2*A + 2*B)/(a*d*sin(d*x + c) + a*d
)
```

giac [A] time = 0.19, size = 79, normalized size = 1.76

$$\frac{\frac{(A+B)\log(|\sin(dx+c)+1|)}{a}-\frac{(A+B)\log(|\sin(dx+c)-1|)}{a}-\frac{A\sin(dx+c)+B\sin(dx+c)+3A-B}{a(\sin(dx+c)+1)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*((A + B)*log(abs(sin(d*x + c) + 1))/a - (A + B)*log(abs(sin(d*x + c) -
1))/a - (A*sin(d*x + c) + B*sin(d*x + c) + 3*A - B)/(a*(sin(d*x + c) + 1)))
/d
```

maple [B] time = 0.46, size = 112, normalized size = 2.49

$$-\frac{\ln(\sin(dx+c)-1)A}{4ad}-\frac{\ln(\sin(dx+c)-1)B}{4ad}-\frac{A}{2ad(1+\sin(dx+c))}+\frac{B}{2ad(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+}{4da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/a/d*ln(sin(d*x+c)-1)*A-1/4/a/d*ln(sin(d*x+c)-1)*B-1/2/a/d/(1+sin(d*x+c
))*A+1/2/a/d/(1+sin(d*x+c))*B+1/4/d/a*ln(1+sin(d*x+c))*B+1/4/d/a*ln(1+sin(d
*x+c))*A
```

maxima [A] time = 0.32, size = 58, normalized size = 1.29

$$\frac{\frac{(A+B)\log(\sin(dx+c)+1)}{a}-\frac{(A+B)\log(\sin(dx+c)-1)}{a}-\frac{2(A-B)}{a\sin(dx+c)+a}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/4*((A + B)*log(sin(d*x + c) + 1)/a - (A + B)*log(sin(d*x + c) - 1)/a - 2*
(A - B)/(a*sin(d*x + c) + a))/d
```

mupad [B] time = 0.10, size = 43, normalized size = 0.96

$$\frac{\operatorname{atanh}(\sin(c+d\,x))\,(A+B)}{2\,a\,d}-\frac{\frac{A}{2}-\frac{B}{2}}{d\,(a+a\,\sin(c+d\,x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] (atanh(sin(c + d*x))*(A + B))/(2*a*d) - (A/2 - B/2)/(d*(a + a*sin(c + d*x))
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{B\sin(c+dx)\sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sin(c + d*x) + 1), x) + Integral(B*sin(c + d*x)*s
ec(c + d*x)/(sin(c + d*x) + 1), x))/a
```

3.1008 $\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{A+B}{8d(a-a\sin(c+dx))}-\frac{a(A-B)}{8d(a\sin(c+dx)+a)^2}+\frac{(3A+B)\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{A}{4d(a\sin(c+dx)+a)}$$

```
[Out] 1/8*(3*A+B)*arctanh(sin(d*x+c))/a/d+1/8*(A+B)/d/(a-a*sin(d*x+c))-1/8*a*(A-B
)/d/(a+a*sin(d*x+c))^2-1/4*A/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$\frac{A+B}{8d(a-a\sin(c+dx))}-\frac{a(A-B)}{8d(a\sin(c+dx)+a)^2}+\frac{(3A+B)\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{A}{4d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((3*A + B)*ArcTanh[Sin[c + d*x]])/(8*a*d) + (A + B)/(8*d*(a - a*Sin[c + d*x
])) - (a*(A - B))/(8*d*(a + a*Sin[c + d*x])^2) - A/(4*d*(a + a*Sin[c + d*x]
))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
```

```
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{a^3\operatorname{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\operatorname{Subst}\left(\int \left(\frac{A+B}{8a^3(a-x)^2}+\frac{A-B}{4a^2(a+x)^3}+\frac{A}{4a^3(a+x)^2}+\frac{3A+B}{8a^3\left(a^2-x^2\right)}\right) dx, x, a\sin(c\right.}{d}$$

$$= \frac{A+B}{8d(a-a\sin(c+dx))}-\frac{a(A-B)}{8d(a+a\sin(c+dx))^2}-\frac{A}{4d(a+a\sin(c+dx))}$$

$$= \frac{(3A+B)\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{A+B}{8d(a-a\sin(c+dx))}-\frac{a(A-}{8d(a+a\sin(}$$

Mathematica [A] time = 0.29, size = 75, normalized size = 0.82

$$\frac{\frac{A+B}{a-a\sin(c+dx)}+\frac{B-A}{a(\sin(c+dx)+1)^2}+\frac{(3A+B)\tanh^{-1}(\sin(c+dx))}{a}-\frac{2A}{a\sin(c+dx)+a}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (((3*A + B)*ArcTanh[Sin[c + d*x]])/a + (-A + B)/(a*(1 + Sin[c + d*x])^2) +
(A + B)/(a - a*Sin[c + d*x]) - (2*A)/(a + a*Sin[c + d*x]))/(8*d)
```

fricas [A] time = 0.61, size = 161, normalized size = 1.77

$$-\frac{2\,(3\,A+B)\cos\left(dx+c\right)^2-\left((3\,A+B)\cos\left(dx+c\right)^2\sin\left(dx+c\right)+(3\,A+B)\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right.}{16\left(ad\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/16*(2*(3*A + B)*cos(d*x + c)^2 - ((3*A + B)*cos(d*x + c)^2*sin(d*x + c)
+ (3*A + B)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((3*A + B)*cos(d*x + c)
```

```
^2*sin(d*x + c) + (3*A + B)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(3*A
 + B)*sin(d*x + c) - 2*A - 6*B)/(a*d*cos(d*x + c)^2*sin(d*x + c) + a*d*cos(
d*x + c)^2)
```

giac [A] time = 0.23, size = 147, normalized size = 1.62

$$\frac{\frac{2(3A+B)\log(|\sin(dx+c)+1|)}{a} - \frac{2(3A+B)\log(|\sin(dx+c)-1|)}{a} + \frac{2(3A\sin(dx+c)+B\sin(dx+c)-5A-3B)}{a(\sin(dx+c)-1)} - \frac{9A\sin(dx+c)^2+3B\sin(dx+c)^2+26A\sin}{a(\sin(dx+c)-}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/32*(2*(3*A + B)*log(abs(sin(d*x + c) + 1))/a - 2*(3*A + B)*log(abs(sin(d*
x + c) - 1))/a + 2*(3*A*sin(d*x + c) + B*sin(d*x + c) - 5*A - 3*B)/(a*(sin(
d*x + c) - 1)) - (9*A*sin(d*x + c)^2 + 3*B*sin(d*x + c)^2 + 26*A*sin(d*x +
c) + 6*B*sin(d*x + c) + 21*A - B)/(a*(sin(d*x + c) + 1)^2))/d
```

maple [B] time = 0.56, size = 169, normalized size = 1.86

$$-\frac{3\ln(\sin(dx+c)-1)A}{16ad}-\frac{\ln(\sin(dx+c)-1)B}{16ad}-\frac{A}{8ad(\sin(dx+c)-1)}-\frac{B}{8ad(\sin(dx+c)-1)}-\frac{A}{4ad(1+\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] -3/16/a/d*ln(sin(d*x+c)-1)*A-1/16/a/d*ln(sin(d*x+c)-1)*B-1/8/a/d/(sin(d*x+c
)-1)*A-1/8/a/d/(sin(d*x+c)-1)*B-1/4/a/d/(1+sin(d*x+c))*A-1/8/a/d/(1+sin(d*x
+c))^2*A+1/8/a/d/(1+sin(d*x+c))^2*B+3/16/d/a*ln(1+sin(d*x+c))*A+1/16/d/a*ln
(1+sin(d*x+c))*B
```

maxima [A] time = 0.33, size = 113, normalized size = 1.24

$$\frac{\frac{(3A+B)\log(\sin(dx+c)+1)}{a} - \frac{(3A+B)\log(\sin(dx+c)-1)}{a} - \frac{2\left((3A+B)\sin(dx+c)^2+(3A+B)\sin(dx+c)-2A+2B\right)}{a\sin(dx+c)^3+a\sin(dx+c)^2-a\sin(dx+c)-a}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/16*((3*A + B)*log(sin(d*x + c) + 1)/a - (3*A + B)*log(sin(d*x + c) - 1)/a
 - 2*((3*A + B)*sin(d*x + c)^2 + (3*A + B)*sin(d*x + c) - 2*A + 2*B)/(a*sin
(d*x + c)^3 + a*sin(d*x + c)^2 - a*sin(d*x + c) - a))/d
```

mupad [B] time = 0.13, size = 96, normalized size = 1.05

$$\frac{\left(\frac{3A}{8}+\frac{B}{8}\right)\sin\left(c+d\,x\right)^2+\left(\frac{3A}{8}+\frac{B}{8}\right)\sin\left(c+d\,x\right)-\frac{A}{4}+\frac{B}{4}}{d\,\left(-a\sin\left(c+d\,x\right)^3-a\sin\left(c+d\,x\right)^2+a\,\sin\left(c+d\,x\right)+a\right)}+\frac{\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(3\,A+B\right)}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (B/4 - A/4 + sin(c + d*x)*((3*A)/8 + B/8) + sin(c + d*x)^2*((3*A)/8 + B/8))
/(d*(a + a*sin(c + d*x) - a*sin(c + d*x)^2 - a*sin(c + d*x)^3)) + (atanh(si
n(c + d*x))*(3*A + B))/(8*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A\sec^3\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx+\int\frac{B\sin\left(c+dx\right)\sec^3\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(B*sin(c + d*x
)*sec(c + d*x)**3/(sin(c + d*x) + 1), x))/a
```

3.1009 $\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=146

$$-\frac{a^2(A-B)}{24d(a\sin(c+dx)+a)^3}+\frac{a(A+B)}{32d(a-a\sin(c+dx))^2}-\frac{a(3A-B)}{32d(a\sin(c+dx)+a)^2}+\frac{2A+B}{16d(a-a\sin(c+dx))}+\frac{(5A+B)\,\text{ta}}{}$$

```
[Out] 1/16*(5*A+B)*arctanh(sin(d*x+c))/a/d+1/32*a*(A+B)/d/(a-a*sin(d*x+c))^2+1/16
*(2*A+B)/d/(a-a*sin(d*x+c))-1/24*a^2*(A-B)/d/(a+a*sin(d*x+c))^3-1/32*a*(3*A
-B)/d/(a+a*sin(d*x+c))^2-3/16*A/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.19, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$-\frac{a^2(A-B)}{24d(a\sin(c+dx)+a)^3}+\frac{a(A+B)}{32d(a-a\sin(c+dx))^2}-\frac{a(3A-B)}{32d(a\sin(c+dx)+a)^2}+\frac{2A+B}{16d(a-a\sin(c+dx))}+\frac{(5A+B)\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((5*A + B)*ArcTanh[Sin[c + d*x]])/(16*a*d) + (a*(A + B))/(32*d*(a - a*Sin[c
 + d*x])^2) + (2*A + B)/(16*d*(a - a*Sin[c + d*x])) - (a^2*(A - B))/(24*d*(
a + a*Sin[c + d*x])^3) - (a*(3*A - B))/(32*d*(a + a*Sin[c + d*x])^2) - (3*A
)/(16*d*(a + a*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{A+B}{16a^4(a-x)^3}+\frac{2A+B}{16a^5(a-x)^2}+\frac{A-B}{8a^3(a+x)^4}+\frac{3A-B}{16a^4(a+x)^3}+\frac{3A}{16a^5(a+x)^2}\right.\right.}{d}$$

$$= \frac{a(A+B)}{32d(a-a\sin(c+dx))^2}+\frac{2A+B}{16d(a-a\sin(c+dx))}-\frac{a^2(A-B)}{24d(a+a\sin(c+d}$$

$$= \frac{(5A+B)\tanh^{-1}(\sin(c+dx))}{16ad}+\frac{a(A+B)}{32d(a-a\sin(c+dx))^2}+\frac{2A}{16d(a-a\,\mathrm{s}}$$

Mathematica [A] time = 0.57, size = 105, normalized size = 0.72

$$\frac{-\frac{6(2A+B)}{\sin(c+dx)-1}+\frac{3(A+B)}{(\sin(c+dx)-1)^2}+\frac{3B-9A}{(\sin(c+dx)+1)^2}-\frac{4(A-B)}{(\sin(c+dx)+1)^3}+6(5A+B)\tanh^{-1}(\sin(c+dx))-\frac{18A}{\sin(c+dx)+1}}{96ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6*(5*A + B)*ArcTanh[Sin[c + d*x]] + (3*(A + B))/(-1 + Sin[c + d*x])^2 - (6
*(2*A + B))/(-1 + Sin[c + d*x]) - (4*(A - B))/(1 + Sin[c + d*x])^3 + (-9*A
+ 3*B)/(1 + Sin[c + d*x])^2 - (18*A)/(1 + Sin[c + d*x]))/(96*a*d)
```

fricas [A] time = 0.72, size = 194, normalized size = 1.33

$$-\frac{6\,(5\,A+B)\cos\left(dx+c\right)^4-2\,(5\,A+B)\cos\left(dx+c\right)^2-3\left((5\,A+B)\cos\left(dx+c\right)^4\sin\left(dx+c\right)+(5\,A+B)\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/96*(6*(5*A + B)*cos(d*x + c)^4 - 2*(5*A + B)*cos(d*x + c)^2 - 3*((5*A +
B)*cos(d*x + c)^4*sin(d*x + c) + (5*A + B)*cos(d*x + c)^4)*log(sin(d*x + c)
 + 1) + 3*((5*A + B)*cos(d*x + c)^4*sin(d*x + c) + (5*A + B)*cos(d*x + c)^4
)*log(-sin(d*x + c) + 1) - 2*(3*(5*A + B)*cos(d*x + c)^2 + 10*A + 2*B)*sin(
d*x + c) - 4*A - 20*B)/(a*d*cos(d*x + c)^4*sin(d*x + c) + a*d*cos(d*x + c)^
4)
```

giac [A] time = 0.26, size = 192, normalized size = 1.32

$$\frac{\frac{6\,(5\,A+B)\log(|\sin(dx+c)+1|)}{a}-\frac{6\,(5\,A+B)\log(|\sin(dx+c)-1|)}{a}+\frac{3\left(15\,A\sin(dx+c)^2+3\,B\sin(dx+c)^2-38\,A\sin(dx+c)-10\,B\sin(dx+c)+25\,A+9\,B\right)}{a(\sin(dx+c)-1)^2}-5}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/192*(6*(5*A + B)*log(abs(sin(d*x + c) + 1))/a - 6*(5*A + B)*log(abs(sin(d
*x + c) - 1))/a + 3*(15*A*sin(d*x + c)^2 + 3*B*sin(d*x + c)^2 - 38*A*sin(d*
x + c) - 10*B*sin(d*x + c) + 25*A + 9*B)/(a*(sin(d*x + c) - 1)^2) - (55*A*s
in(d*x + c)^3 + 11*B*sin(d*x + c)^3 + 201*A*sin(d*x + c)^2 + 33*B*sin(d*x +
 c)^2 + 255*A*sin(d*x + c) + 27*B*sin(d*x + c) + 117*A - 3*B)/(a*(sin(d*x +
 c) + 1)^3))/d
```

maple [A] time = 0.53, size = 245, normalized size = 1.68

$$-\frac{5\ln(\sin(dx+c)-1)\,A}{32ad}-\frac{\ln(\sin(dx+c)-1)\,B}{32ad}+\frac{A}{32ad\,(\sin(dx+c)-1)^2}+\frac{B}{32ad\,(\sin(dx+c)-1)^2}-\frac{A}{8ad\,(\sin(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] -5/32/a/d*ln(sin(d*x+c)-1)*A-1/32/a/d*ln(sin(d*x+c)-1)*B+1/32/a/d/(sin(d*x+
c)-1)^2*A+1/32/a/d/(sin(d*x+c)-1)^2*B-1/8/a/d/(sin(d*x+c)-1)*A-1/16/a/d/(si
n(d*x+c)-1)*B-3/16/a/d/(1+sin(d*x+c))*A-1/24/a/d/(1+sin(d*x+c))^3*A+1/24/a/
d/(1+sin(d*x+c))^3*B-3/32/a/d/(1+sin(d*x+c))^2*A+1/32/a/d/(1+sin(d*x+c))^2*
B+5/32/d/a*ln(1+sin(d*x+c))*A+1/32/d/a*ln(1+sin(d*x+c))*B
```

maxima [A] time = 0.35, size = 165, normalized size = 1.13

$$\frac{\frac{3\,(5\,A+B)\log(\sin(dx+c)+1)}{a}-\frac{3\,(5\,A+B)\log(\sin(dx+c)-1)}{a}-\frac{2\left(3\,(5\,A+B)\sin(dx+c)^4+3\,(5\,A+B)\sin(dx+c)^3-5\,(5\,A+B)\sin(dx+c)^2-5\,(5\,A+B)\sin(d}{a\sin(dx+c)^5+a\sin(dx+c)^4-2\,a\sin(dx+c)^3-2\,a\sin(dx+c)^2+a\sin(dx+c)+}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(3*(5*A + B)*log(sin(d*x + c) + 1)/a - 3*(5*A + B)*log(sin(d*x + c) -
1)/a - 2*(3*(5*A + B)*sin(d*x + c)^4 + 3*(5*A + B)*sin(d*x + c)^3 - 5*(5*A
+ B)*sin(d*x + c)^2 - 5*(5*A + B)*sin(d*x + c) + 8*A - 8*B)/(a*sin(d*x + c)
^5 + a*sin(d*x + c)^4 - 2*a*sin(d*x + c)^3 - 2*a*sin(d*x + c)^2 + a*sin(d*x
 + c) + a))/d
```

mupad [B] time = 9.18, size = 151, normalized size = 1.03

$$\frac{\operatorname{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(5\,A+B\right)}{16\,a\,d}-\frac{\left(\frac{5\,A}{16}+\frac{B}{16}\right)\,\sin\left(c+d\,x\right)^4+\left(\frac{5\,A}{16}+\frac{B}{16}\right)\,\sin\left(c+d\,x\right)^3+\left(-\frac{25\,A}{48}-\frac{5\,B}{48}\right)\,\sin\left(c+d\,x\right.}{d\,\left(a\,\sin\left(c+d\,x\right)^5+a\,\sin\left(c+d\,x\right)^4-2\,a\,\sin\left(c+d\,x\right)^3-2\,a\,\sin\left(c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^5*(a + a*sin(c + d*x))),x)
```

```
[Out] (atanh(sin(c + d*x))*(5*A + B))/(16*a*d) - (A/6 - B/6 - sin(c + d*x)*((25*A
)/48 + (5*B)/48) + sin(c + d*x)^3*((5*A)/16 + B/16) + sin(c + d*x)^4*((5*A)
/16 + B/16) - sin(c + d*x)^2*((25*A)/48 + (5*B)/48))/(d*(a + a*sin(c + d*x)
 - 2*a*sin(c + d*x)^2 - 2*a*sin(c + d*x)^3 + a*sin(c + d*x)^4 + a*sin(c + d
*x)^5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^{5}{\left(c+dx\right)}}{\sin{\left(c+dx\right)}+1}\, dx+\int \frac{B\sin{\left(c+dx\right)}\sec^{5}{\left(c+dx\right)}}{\sin{\left(c+dx\right)}+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**5/(sin(c + d*x) + 1), x) + Integral(B*sin(c + d*x
)*sec(c + d*x)**5/(sin(c + d*x) + 1), x))/a
```

3.1010 $\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=205

$$-\frac{a^3(A-B)}{64d(a\sin(c+dx)+a)^4}+\frac{a^2(A+B)}{96d(a-a\sin(c+dx))^3}-\frac{a^2(2A-B)}{48d(a\sin(c+dx)+a)^3}+\frac{a(5A+3B)}{128d(a-a\sin(c+dx))^2}-\frac{a(5}{64d(a\sin}$$

```
[Out] 5/128*(7*A+B)*arctanh(sin(d*x+c))/a/d+1/96*a^2*(A+B)/d/(a-a*sin(d*x+c))^3+1
/128*a*(5*A+3*B)/d/(a-a*sin(d*x+c))^2+5/128*(3*A+B)/d/(a-a*sin(d*x+c))-1/64
*a^3*(A-B)/d/(a+a*sin(d*x+c))^4-1/48*a^2*(2*A-B)/d/(a+a*sin(d*x+c))^3-1/64*
a*(5*A-B)/d/(a+a*sin(d*x+c))^2-5/32*A/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.25, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$-\frac{a^3(A-B)}{64d(a\sin(c+dx)+a)^4}+\frac{a^2(A+B)}{96d(a-a\sin(c+dx))^3}-\frac{a^2(2A-B)}{48d(a\sin(c+dx)+a)^3}+\frac{a(5A+3B)}{128d(a-a\sin(c+dx))^2}-\frac{a(5}{64d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*(7*A + B)*ArcTanh[Sin[c + d*x]])/(128*a*d) + (a^2*(A + B))/(96*d*(a - a*
Sin[c + d*x])^3) + (a*(5*A + 3*B))/(128*d*(a - a*Sin[c + d*x])^2) + (5*(3*A
 + B))/(128*d*(a - a*Sin[c + d*x])) - (a^3*(A - B))/(64*d*(a + a*Sin[c + d*
x])^4) - (a^2*(2*A - B))/(48*d*(a + a*Sin[c + d*x])^3) - (a*(5*A - B))/(64*
d*(a + a*Sin[c + d*x])^2) - (5*A)/(32*d*(a + a*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{A+B}{32a^5(a-x)^4}+\frac{5A+3B}{64a^6(a-x)^3}+\frac{5(3A+B)}{128a^7(a-x)^2}+\frac{A-B}{16a^4(a+x)^5}+\frac{2A-B}{16a^5(a+x)^4}\right.\right.}{d}$$

$$= \frac{a^2(A+B)}{96d(a-a\sin(c+dx))^3}+\frac{a(5A+3B)}{128d(a-a\sin(c+dx))^2}+\frac{5(3A+B)}{128d(a-a\sin(c}$$

$$= \frac{5(7A+B)\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{a^2(A+B)}{96d(a-a\sin(c+dx))^3}+\frac{a(5}{128d(a-}$$

Mathematica [A] time = 0.88, size = 142, normalized size = 0.69

$$\frac{\frac{-15(7A+B)\sin^6(c+dx)-15(7A+B)\sin^5(c+dx)+40(7A+B)\sin^4(c+dx)+40(7A+B)\sin^3(c+dx)-33(7A+B)\sin^2(c+dx)-33(7A+B)\sin(c+dx)+48(A-B)}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^4}}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (15*(7*A + B)*ArcTanh[Sin[c + d*x]] + (48*(A - B) - 33*(7*A + B)*Sin[c + d*
x] - 33*(7*A + B)*Sin[c + d*x]^2 + 40*(7*A + B)*Sin[c + d*x]^3 + 40*(7*A +
B)*Sin[c + d*x]^4 - 15*(7*A + B)*Sin[c + d*x]^5 - 15*(7*A + B)*Sin[c + d*x]
^6)/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(384*a*d)
```

fricas [A] time = 0.61, size = 224, normalized size = 1.09

$$-\frac{30\,(7\,A+B)\cos(dx+c)^6-10\,(7\,A+B)\cos(dx+c)^4-4\,(7\,A+B)\cos(dx+c)^2-15\,\left((7\,A+B)\cos(dx+c)^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/768*(30*(7*A + B)*cos(d*x + c)^6 - 10*(7*A + B)*cos(d*x + c)^4 - 4*(7*A
+ B)*cos(d*x + c)^2 - 15*((7*A + B)*cos(d*x + c)^6*sin(d*x + c) + (7*A + B)
*cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*((7*A + B)*cos(d*x + c)^6*sin(d
*x + c) + (7*A + B)*cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(15*(7*A + B
)*cos(d*x + c)^4 + 10*(7*A + B)*cos(d*x + c)^2 + 56*A + 8*B)*sin(d*x + c) -
 16*A - 112*B)/(a*d*cos(d*x + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

giac [A] time = 0.31, size = 236, normalized size = 1.15

$$\frac{\frac{60\,(7\,A+B)\log(|\sin(dx+c)+1|)}{a} - \frac{60\,(7\,A+B)\log(|\sin(dx+c)-1|)}{a} + \frac{2\left(385\,A\sin(dx+c)^3+55\,B\sin(dx+c)^3-1335\,A\sin(dx+c)^2-225\,B\sin(dx+c)^2+1575\right.}{a(\sin(dx+c)-1)^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/3072*(60*(7*A + B)*log(abs(sin(d*x + c) + 1))/a - 60*(7*A + B)*log(abs(si
n(d*x + c) - 1))/a + 2*(385*A*sin(d*x + c)^3 + 55*B*sin(d*x + c)^3 - 1335*A
*sin(d*x + c)^2 - 225*B*sin(d*x + c)^2 + 1575*A*sin(d*x + c) + 321*B*sin(d*
x + c) - 641*A - 167*B)/(a*(sin(d*x + c) - 1)^3) - (875*A*sin(d*x + c)^4 +
125*B*sin(d*x + c)^4 + 3980*A*sin(d*x + c)^3 + 500*B*sin(d*x + c)^3 + 6930*
A*sin(d*x + c)^2 + 702*B*sin(d*x + c)^2 + 5548*A*sin(d*x + c) + 340*B*sin(d
*x + c) + 1771*A - 35*B)/(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.50, size = 321, normalized size = 1.57

$$-\frac{35\ln(\sin(dx+c)-1)\,A}{256ad}-\frac{5\ln(\sin(dx+c)-1)\,B}{256ad}+\frac{5A}{128ad\,(\sin(dx+c)-1)^2}+\frac{3B}{128ad\,(\sin(dx+c)-1)^2}-\frac{}{96ad\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] -35/256/a/d*ln(sin(d*x+c)-1)*A-5/256/a/d*ln(sin(d*x+c)-1)*B+5/128/a/d/(sin(
d*x+c)-1)^2*A+3/128/a/d/(sin(d*x+c)-1)^2*B-1/96/a/d/(sin(d*x+c)-1)^3*A-1/96
/a/d/(sin(d*x+c)-1)^3*B-15/128/a/d/(sin(d*x+c)-1)*A-5/128/a/d/(sin(d*x+c)-1
)*B-5/32/a/d/(1+sin(d*x+c))*A-1/64/a/d/(1+sin(d*x+c))^4*A+1/64/a/d/(1+sin(d
*x+c))^4*B-1/24/a/d/(1+sin(d*x+c))^3*A+1/48/a/d/(1+sin(d*x+c))^3*B-5/64/a/d
/(1+sin(d*x+c))^2*A+1/64/a/d/(1+sin(d*x+c))^2*B+35/256/d/a*ln(1+sin(d*x+c))
*A+5/256/d/a*ln(1+sin(d*x+c))*B
```

maxima [A] time = 0.34, size = 220, normalized size = 1.07

$$\frac{\frac{15\,(7\,A+B)\log(\sin(dx+c)+1)}{a}-\frac{15\,(7\,A+B)\log(\sin(dx+c)-1)}{a}-\frac{2\left(15\,(7\,A+B)\sin(dx+c)^{6}+15\,(7\,A+B)\sin(dx+c)^{5}-40\,(7\,A+B)\sin(dx+c)^{4}-40\,(7\,A\cdots\right.}{a\sin(dx+c)^{7}+a\sin(dx+c)^{6}-3\,a\sin(dx+c)^{5}-3\,a\sin(dx+c\cdots}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/768*(15*(7*A + B)*log(sin(d*x + c) + 1)/a - 15*(7*A + B)*log(sin(d*x + c)
 - 1)/a - 2*(15*(7*A + B)*sin(d*x + c)^6 + 15*(7*A + B)*sin(d*x + c)^5 - 40
*(7*A + B)*sin(d*x + c)^4 - 40*(7*A + B)*sin(d*x + c)^3 + 33*(7*A + B)*sin(
d*x + c)^2 + 33*(7*A + B)*sin(d*x + c) - 48*A + 48*B)/(a*sin(d*x + c)^7 + a
*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x + c
)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a))/d
```

mupad [B] time = 9.35, size = 206, normalized size = 1.00

$$\frac{\left(\frac{35\,A}{128}+\frac{5\,B}{128}\right)\sin\left(c+d\,x\right)^6+\left(\frac{35\,A}{128}+\frac{5\,B}{128}\right)\sin\left(c+d\,x\right)^5+\left(-\frac{35\,A}{48}-\frac{5\,B}{48}\right)\sin\left(c+d\,x\right)^4+\left(-\frac{35\,A}{48}-\frac{5\,B}{48}\right)\sin\left(c+d\cdots\right.}{d\,\left(-a\sin\left(c+d\,x\right)^7-a\sin\left(c+d\,x\right)^6+3\,a\sin\left(c+d\,x\right)^5+3\,a\sin\left(c+d\,x\right)^4-3\,a\sin\left(c+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (B/8 - A/8 + sin(c + d*x)*((77*A)/128 + (11*B)/128) - sin(c + d*x)^3*((35*A
)/48 + (5*B)/48) - sin(c + d*x)^4*((35*A)/48 + (5*B)/48) + sin(c + d*x)^5*(
(35*A)/128 + (5*B)/128) + sin(c + d*x)^6*((35*A)/128 + (5*B)/128) + sin(c +
 d*x)^2*((77*A)/128 + (11*B)/128))/(d*(a + a*sin(c + d*x) - 3*a*sin(c + d*x
)^2 - 3*a*sin(c + d*x)^3 + 3*a*sin(c + d*x)^4 + 3*a*sin(c + d*x)^5 - a*sin(
c + d*x)^6 - a*sin(c + d*x)^7)) + (5*atanh(sin(c + d*x))*(7*A + B))/(128*a*
d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1011 $\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{B(a-a\sin(c+dx))^6}{6a^8d}+\frac{(A+3B)(a-a\sin(c+dx))^5}{5a^7d}-\frac{(A+B)(a-a\sin(c+dx))^4}{2a^6d}$$

```
[Out] -1/2*(A+B)*(a-a*sin(d*x+c))^4/a^6/d+1/5*(A+3*B)*(a-a*sin(d*x+c))^5/a^7/d-1/
6*B*(a-a*sin(d*x+c))^6/a^8/d
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{(A+3B)(a-a\sin(c+dx))^5}{5a^7d}-\frac{(A+B)(a-a\sin(c+dx))^4}{2a^6d}-\frac{B(a-a\sin(c+dx))^6}{6a^8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((A + B)*(a - a*Sin[c + d*x])^4)/(2*a^6*d) + ((A + 3*B)*(a - a*Sin[c + d*x
])^5)/(5*a^7*d) - (B*(a - a*Sin[c + d*x])^6)/(6*a^8*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(2a(A+B)(a-x)^3+(-A-3B)(a-x)^4+\frac{B(a-x)^5}{a}\right)dx, x, a\,\text{si}\right.}{a^7 d}$$

$$= -\frac{(A+B)(a-a\sin(c+dx))^4}{2a^6 d}+\frac{(A+3B)(a-a\sin(c+dx))^5}{5a^7 d}-\frac{B(a-}{}$$

Mathematica [A] time = 0.17, size = 52, normalized size = 0.66

$$-\frac{(\sin(c+dx)-1)^4\left((6A+8B)\sin(c+dx)+9A+5B\sin^2(c+dx)+2B\right)}{30a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/30*((-1 + Sin[c + d*x])^4*(9*A + 2*B + (6*A + 8*B)*Sin[c + d*x] + 5*B*Si
n[c + d*x]^2))/(a^2*d)
```

fricas [A] time = 0.81, size = 82, normalized size = 1.04

$$\frac{5\,B\cos(dx+c)^6+15\,(A-B)\cos(dx+c)^4-2\left(3\,(A-2\,B)\cos(dx+c)^4-2\,(3\,A-B)\cos(dx+c)^2-12\,A+4\right.}{30\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/30*(5*B*cos(d*x + c)^6 + 15*(A - B)*cos(d*x + c)^4 - 2*(3*(A - 2*B)*cos(d
*x + c)^4 - 2*(3*A - B)*cos(d*x + c)^2 - 12*A + 4*B)*sin(d*x + c))/(a^2*d)
```

giac [A] time = 0.23, size = 95, normalized size = 1.20

$$-\frac{5\,B\sin(dx+c)^6+6\,A\sin(dx+c)^5-12\,B\sin(dx+c)^5-15\,A\sin(dx+c)^4+20\,B\sin(dx+c)^3+30\,A\sin(d}{30\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/30*(5*B*sin(d*x + c)^6 + 6*A*sin(d*x + c)^5 - 12*B*sin(d*x + c)^5 - 15*A
*sin(d*x + c)^4 + 20*B*sin(d*x + c)^3 + 30*A*sin(d*x + c)^2 - 15*B*sin(d*x
+ c)^2 - 30*A*sin(d*x + c))/(a^2*d)
```

maple [A] time = 0.61, size = 82, normalized size = 1.04

$$-\frac{-\frac{B\left(\sin^{6}(dx+c)\right)}{6}+\frac{(-A+2B)\left(\sin^{5}(dx+c)\right)}{5}+\frac{A\left(\sin^{4}(dx+c)\right)}{2}-\frac{2B\left(\sin^{3}(dx+c)\right)}{3}+\frac{(-2A+B)\left(\sin^{2}(dx+c)\right)}{2}+A\sin\left(dx+c\right)}{d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/6*B*sin(d*x+c)^6+1/5*(-A+2*B)*sin(d*x+c)^5+1/2*A*sin(d*x+c)^4-2
/3*B*sin(d*x+c)^3+1/2*(-2*A+B)*sin(d*x+c)^2+A*sin(d*x+c))
```

maxima [A] time = 0.34, size = 83, normalized size = 1.05

$$-\frac{5\,B\sin\left(dx+c\right)^{6}+6\,(A-2\,B)\sin\left(dx+c\right)^{5}-15\,A\sin\left(dx+c\right)^{4}+20\,B\sin\left(dx+c\right)^{3}+15\,(2\,A-B)\sin\left(dx+c\right)^{2}}{30\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/30*(5*B*sin(d*x + c)^6 + 6*(A - 2*B)*sin(d*x + c)^5 - 15*A*sin(d*x + c)^
4 + 20*B*sin(d*x + c)^3 + 15*(2*A - B)*sin(d*x + c)^2 - 30*A*sin(d*x + c))/
(a^2*d)
```

mupad [B] time = 0.08, size = 98, normalized size = 1.24

$$-\frac{\frac{\sin(c+d\,x)^{5}\,(A-2\,B)}{5\,a^{2}}-\frac{A\sin(c+d\,x)^{4}}{2\,a^{2}}+\frac{2\,B\sin(c+d\,x)^{3}}{3\,a^{2}}+\frac{B\sin(c+d\,x)^{6}}{6\,a^{2}}+\frac{\sin(c+d\,x)^{2}\,(2\,A-B)}{2\,a^{2}}-\frac{A\,\sin(c+d\,x)}{a^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] -((sin(c + d*x)^5*(A - 2*B))/(5*a^2) - (A*sin(c + d*x)^4)/(2*a^2) + (2*B*si
n(c + d*x)^3)/(3*a^2) + (B*sin(c + d*x)^6)/(6*a^2) + (sin(c + d*x)^2*(2*A -
 B))/(2*a^2) - (A*sin(c + d*x))/a^2)/d
```

sympy [A] time = 132.04, size = 2705, normalized size = 34.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((30*A*tan(c/2 + d*x/2)**11/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a
**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*ta
n(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*
x/2)**2 + 15*a**2*d) - 60*A*tan(c/2 + d*x/2)**10/(15*a**2*d*tan(c/2 + d*x/2
)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 3
00*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*
tan(c/2 + d*x/2)**2 + 15*a**2*d) + 150*A*tan(c/2 + d*x/2)**9/(15*a**2*d*tan
(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d
*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4
+ 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 120*A*tan(c/2 + d*x/2)**8/(1
5*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d
*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2
+ d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 204*A*tan(c/2 +
d*x/2)**7/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10
+ 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**
2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 120*
A*tan(c/2 + d*x/2)**6/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 +
 d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)*
*6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a*
*2*d) + 204*A*tan(c/2 + d*x/2)**5/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2
*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c
/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2
)**2 + 15*a**2*d) - 120*A*tan(c/2 + d*x/2)**4/(15*a**2*d*tan(c/2 + d*x/2)**
12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*
a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan
(c/2 + d*x/2)**2 + 15*a**2*d) + 150*A*tan(c/2 + d*x/2)**3/(15*a**2*d*tan(c/
2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/
2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 9
0*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 60*A*tan(c/2 + d*x/2)**2/(15*a*
*2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan
(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*
x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 30*A*tan(c/2 + d*x/2
)/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a*
*2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(
c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 30*B*tan(c/2
 + d*x/2)**10/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)*
*10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225
*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) -
80*B*tan(c/2 + d*x/2)**9/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/
2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/
2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15
```

```
*a**2*d) + 120*B*tan(c/2 + d*x/2)**8/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a
**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*ta
n(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*
x/2)**2 + 15*a**2*d) - 48*B*tan(c/2 + d*x/2)**7/(15*a**2*d*tan(c/2 + d*x/2)
**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 30
0*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*t
an(c/2 + d*x/2)**2 + 15*a**2*d) + 20*B*tan(c/2 + d*x/2)**6/(15*a**2*d*tan(c
/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x
/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 +
90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 48*B*tan(c/2 + d*x/2)**5/(15*a
**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*ta
n(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d
*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 120*B*tan(c/2 + d*x
/2)**4/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 2
25*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d
*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 80*B*ta
n(c/2 + d*x/2)**3/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x
/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 +
 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d
) + 30*B*tan(c/2 + d*x/2)**2/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*ta
n(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 +
d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2
+ 15*a**2*d), Ne(d, 0)), (x*(A + B*sin(c))*cos(c)**7/(a*sin(c) + a)**2, Tru
e))
```

3.1012 $\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=51

$$\frac{B(a-a\sin(c+dx))^4}{4a^6d}-\frac{(A+B)(a-a\sin(c+dx))^3}{3a^5d}$$

```
[Out] -1/3*(A+B)*(a-a*sin(d*x+c))^3/a^5/d+1/4*B*(a-a*sin(d*x+c))^4/a^6/d
```

Rubi [A] time = 0.10, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 43}

$$\frac{B(a-a\sin(c+dx))^4}{4a^6d}-\frac{(A+B)(a-a\sin(c+dx))^3}{3a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((A + B)*(a - a*Sin[c + d*x])^3)/(3*a^5*d) + (B*(a - a*Sin[c + d*x])^4)/(4
*a^6*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int (a-x)^2\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left((A+B)(a-x)^2-\frac{B(a-x)^3}{a}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= -\frac{(A+B)(a-a\sin(c+dx))^3}{3a^5 d}+\frac{B(a-a\sin(c+dx))^4}{4a^6 d}$$

Mathematica [**A**] time = 0.06, size = 34, normalized size = 0.67

$$\frac{(\sin(c+dx)-1)^3(4A+3B\sin(c+dx)+B)}{12a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((-1 + Sin[c + d*x])^3*(4*A + B + 3*B*Sin[c + d*x]))/(12*a^2*d)
```

fricas [**A**] time = 0.82, size = 64, normalized size = 1.25

$$\frac{3\,B\cos\left(dx+c\right)^4+12\,(A-B)\cos\left(dx+c\right)^2-4\left((A-2\,B)\cos\left(dx+c\right)^2-4\,A+2\,B\right)\sin\left(dx+c\right)}{12\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*B*cos(d*x + c)^4 + 12*(A - B)*cos(d*x + c)^2 - 4*((A - 2*B)*cos(d*x
 + c)^2 - 4*A + 2*B)*sin(d*x + c))/(a^2*d)
```

giac [**A**] time = 0.22, size = 73, normalized size = 1.43

$$\frac{3\,B\sin\left(dx+c\right)^4+4\,A\sin\left(dx+c\right)^3-8\,B\sin\left(dx+c\right)^3-12\,A\sin\left(dx+c\right)^2+6\,B\sin\left(dx+c\right)^2+12\,A\sin\left(dx+c\right.}{12\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/12*(3*B*sin(d*x + c)^4 + 4*A*sin(d*x + c)^3 - 8*B*sin(d*x + c)^3 - 12*A*s
in(d*x + c)^2 + 6*B*sin(d*x + c)^2 + 12*A*sin(d*x + c))/(a^2*d)
```

maple [A] time = 0.59, size = 58, normalized size = 1.14

$$\frac{\frac{B\left(\sin^4(dx+c)\right)}{4}+\frac{(A-2B)\left(\sin^3(dx+c)\right)}{3}+\frac{(-2A+B)\left(\sin^2(dx+c)\right)}{2}+A\sin\left(dx+c\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(1/4*B*sin(d*x+c)^4+1/3*(A-2*B)*sin(d*x+c)^3+1/2*(-2*A+B)*sin(d*x+c
)^2+A*sin(d*x+c))
```

maxima [A] time = 0.32, size = 61, normalized size = 1.20

$$\frac{3\,B\sin\left(dx+c\right)^4+4\,(A-2\,B)\sin\left(dx+c\right)^3-6\,(2\,A-B)\sin\left(dx+c\right)^2+12\,A\sin\left(dx+c\right)}{12\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/12*(3*B*sin(d*x + c)^4 + 4*(A - 2*B)*sin(d*x + c)^3 - 6*(2*A - B)*sin(d*x
 + c)^2 + 12*A*sin(d*x + c))/(a^2*d)
```

mupad [B] time = 9.12, size = 68, normalized size = 1.33

$$\frac{\frac{\sin(c+d\,x)^3\,(A-2\,B)}{3\,a^2}+\frac{B\sin(c+d\,x)^4}{4\,a^2}-\frac{\sin(c+d\,x)^2\,(2\,A-B)}{2\,a^2}+\frac{A\,\sin(c+d\,x)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((sin(c + d*x)^3*(A - 2*B))/(3*a^2) + (B*sin(c + d*x)^4)/(4*a^2) - (sin(c +
 d*x)^2*(2*A - B))/(2*a^2) + (A*sin(c + d*x))/a^2)/d
```

sympy [A] time = 53.39, size = 1182, normalized size = 23.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((6*A*tan(c/2 + d*x/2)**7/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*
d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
 d*x/2)**2 + 3*a**2*d) - 12*A*tan(c/2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2
)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a
**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 26*A*tan(c/2 + d*x/2)**5/(3*a**2*d*
tan(c/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d
*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 24*A*tan(c/2 + d*x/2
)**4/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**
2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 26*A*
tan(c/2 + d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x
/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*
a**2*d) - 12*A*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*
d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
 d*x/2)**2 + 3*a**2*d) + 6*A*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 + d*x/2)**8
 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*
d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 6*B*tan(c/2 + d*x/2)**6/(3*a**2*d*tan(c
/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)
**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 16*B*tan(c/2 + d*x/2)**5/
(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*t
an(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 24*B*tan(c
/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**
6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*
d) - 16*B*tan(c/2 + d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*tan
(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/
2)**2 + 3*a**2*d) + 6*B*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**8 +
 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*
tan(c/2 + d*x/2)**2 + 3*a**2*d), Ne(d, 0)), (x*(A + B*sin(c))*cos(c)**5/(a*
sin(c) + a)**2, True))
```

3.1013 $\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$-\frac{B(a-a\sin(c+dx))^2}{2a^4d}-\frac{(A-B)\sin(c+dx)}{a^2d}+\frac{2(A-B)\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] 2*(A-B)*ln(1+sin(d*x+c))/a^2/d-(A-B)*sin(d*x+c)/a^2/d-1/2*B*(a-a*sin(d*x+c)
)^2/a^4/d
```

Rubi [A] time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$-\frac{(A-B)\sin(c+dx)}{a^2d}+\frac{2(A-B)\log(\sin(c+dx)+1)}{a^2d}-\frac{B(a-a\sin(c+dx))^2}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*(A - B)*Log[1 + Sin[c + d*x]])/(a^2*d) - ((A - B)*Sin[c + d*x])/(a^2*d)
- (B*(a - a*Sin[c + d*x])^2)/(2*a^4*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)\left(A+\frac{Bx}{a}\right)}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(-A+B+\frac{B(a-x)}{a}+\frac{2a(A-B)}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{2(A-B)\log(1+\sin(c+dx))}{a^2d} - \frac{(A-B)\sin(c+dx)}{a^2d} - \frac{B(a-a\sin(c+d}{2a^4d}$$

Mathematica [A] time = 0.10, size = 51, normalized size = 0.77

$$-\frac{2(A-2B)\sin(c+dx)-4(A-B)\log(\sin(c+dx)+1)+B\sin^2(c+dx)+B}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(B - 4*(A - B)*Log[1 + Sin[c + d*x]] + 2*(A - 2*B)*Sin[c + d*x] + B*Si
n[c + d*x]^2)/(a^2*d)
```

fricas [A] time = 0.69, size = 48, normalized size = 0.73

$$\frac{B\cos(dx+c)^2+4(A-B)\log(\sin(dx+c)+1)-2(A-2B)\sin(dx+c)}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*(B*cos(d*x + c)^2 + 4*(A - B)*log(sin(d*x + c) + 1) - 2*(A - 2*B)*sin(d
*x + c))/(a^2*d)
```

giac [A] time = 0.20, size = 92, normalized size = 1.39

$$-\frac{\frac{4(A-B)\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a^2}+\frac{(a\sin(dx+c)+a)^2\left(B+\frac{2(Aa^2-3Ba^2)}{(a\sin(dx+c)+a)a}\right)}{a^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*(A - B)*log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a))
)/a^2 + (a*sin(d*x + c) + a)^2*(B + 2*(A*a^2 - 3*B*a^2)/((a*sin(d*x + c) +
a)*a))/a^4)/d
```

maple [A] time = 0.61, size = 85, normalized size = 1.29

$$-\frac{B\left(\sin^2(dx+c)\right)}{2d\,a^2}-\frac{A\sin(dx+c)}{d\,a^2}+\frac{2B\sin(dx+c)}{d\,a^2}+\frac{2\ln(1+\sin(dx+c))\,A}{d\,a^2}-\frac{2B\ln(1+\sin(dx+c))}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d/a^2*B*sin(d*x+c)^2-1/d/a^2*A*sin(d*x+c)+2/d/a^2*B*sin(d*x+c)+2/d/a^2
*ln(1+sin(d*x+c))*A-2*B*ln(1+sin(d*x+c))/a^2/d
```

maxima [A] time = 0.32, size = 54, normalized size = 0.82

$$\frac{\frac{4\,(A-B)\log(\sin(dx+c)+1)}{a^2}-\frac{B\sin(dx+c)^2+2\,(A-2\,B)\sin(dx+c)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*(4*(A - B)*log(sin(d*x + c) + 1)/a^2 - (B*sin(d*x + c)^2 + 2*(A - 2*B)*
sin(d*x + c))/a^2)/d
```

mupad [B] time = 0.07, size = 61, normalized size = 0.92

$$-\frac{2\,A\,\sin(c+d\,x)-4\,B\,\sin(c+d\,x)+B\sin(c+d\,x)^2-4\,A\,\ln(\sin(c+d\,x)+1)+4\,B\,\ln(\sin(c+d\,x)+1)}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] -(2*A*sin(c + d*x) - 4*B*sin(c + d*x) + B*sin(c + d*x)^2 - 4*A*log(sin(c +
d*x) + 1) + 4*B*log(sin(c + d*x) + 1))/(2*a^2*d)
```

sympy [A] time = 19.01, size = 1096, normalized size = 16.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((4*A*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(a**2*d*tan(c/
2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d) + 8*A*log(tan(c/2 +
d*x/2) + 1)*tan(c/2 + d*x/2)**2/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(
c/2 + d*x/2)**2 + a**2*d) + 4*A*log(tan(c/2 + d*x/2) + 1)/(a**2*d*tan(c/2 +
 d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d) - 2*A*log(tan(c/2 + d*x
/2)**2 + 1)*tan(c/2 + d*x/2)**4/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(
c/2 + d*x/2)**2 + a**2*d) - 4*A*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/
2)**2/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d)
- 2*A*log(tan(c/2 + d*x/2)**2 + 1)/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*t
an(c/2 + d*x/2)**2 + a**2*d) - 2*A*tan(c/2 + d*x/2)**3/(a**2*d*tan(c/2 + d*
x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d) - 2*A*tan(c/2 + d*x/2)/(a*
*2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d) - 4*B*log
(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(a**2*d*tan(c/2 + d*x/2)**4 + 2*
a**2*d*tan(c/2 + d*x/2)**2 + a**2*d) - 8*B*log(tan(c/2 + d*x/2) + 1)*tan(c/
2 + d*x/2)**2/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 +
a**2*d) - 4*B*log(tan(c/2 + d*x/2) + 1)/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**
2*d*tan(c/2 + d*x/2)**2 + a**2*d) + 2*B*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/
2 + d*x/2)**4/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 +
a**2*d) + 4*B*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**2/(a**2*d*tan(
c/2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d) + 2*B*log(tan(c/2
+ d*x/2)**2 + 1)/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2
 + a**2*d) + 4*B*tan(c/2 + d*x/2)**3/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d
*tan(c/2 + d*x/2)**2 + a**2*d) - 2*B*tan(c/2 + d*x/2)**2/(a**2*d*tan(c/2 +
d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d) + 4*B*tan(c/2 + d*x/2)/(
a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d), Ne(d,
0)), (x*(A + B*sin(c))*cos(c)**3/(a*sin(c) + a)**2, True))
```

3.1014 $\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=44

$$\frac{B\log(\sin(c+dx)+1)}{a^2d}-\frac{A-B}{d\left(a^2\sin(c+dx)+a^2\right)}$$

```
[Out] B*ln(1+sin(d*x+c))/a^2/d+(-A+B)/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{B\log(\sin(c+dx)+1)}{a^2d}-\frac{A-B}{d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (B*Log[1 + Sin[c + d*x]])/(a^2*d) - (A - B)/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{A-B}{(a+x)^2} + \frac{B}{a(a+x)}\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{B\log(1+\sin(c+dx))}{a^2 d} - \frac{A-B}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [**A**] time = 0.07, size = 41, normalized size = 0.93

$$\frac{\frac{B\log(\sin(c+dx)+1)}{a} - \frac{A-B}{a\sin(c+dx)+a}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((B*Log[1 + Sin[c + d*x]])/a - (A - B)/(a + a*Sin[c + d*x]))/(a*d)
```

fricas [**A**] time = 0.63, size = 45, normalized size = 1.02

$$\frac{(B\sin(dx+c)+B)\log(\sin(dx+c)+1) - A + B}{a^2 d\sin(dx+c) + a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] ((B*sin(d*x + c) + B)*log(sin(d*x + c) + 1) - A + B)/(a^2*d*sin(d*x + c) +
a^2*d)
```

giac [**A**] time = 0.18, size = 76, normalized size = 1.73

$$-\frac{\frac{B\left(\frac{\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a} - \frac{1}{a\sin(dx+c)+a}\right)}{a} + \frac{A}{(a\sin(dx+c)+a)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] -(B*(log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a - 1/(a*
sin(d*x + c) + a))/a + A/((a*sin(d*x + c) + a)*a))/d
```

maple [A] time = 0.45, size = 56, normalized size = 1.27

$$\frac{B\ln\left(1+\sin\left(dx+c\right)\right)}{a^{2}d}-\frac{A}{d\,a^{2}\left(1+\sin\left(dx+c\right)\right)}+\frac{B}{d\,a^{2}\left(1+\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] B*ln(1+sin(d*x+c))/a^2/d-1/d/a^2/(1+sin(d*x+c))*A+1/d/a^2/(1+sin(d*x+c))*B
```

maxima [A] time = 0.40, size = 43, normalized size = 0.98

$$-\frac{\frac{A-B}{a^{2}\sin(dx+c)+a^{2}}-\frac{B\log(\sin(dx+c)+1)}{a^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] -((A - B)/(a^2*sin(d*x + c) + a^2) - B*log(sin(d*x + c) + 1)/a^2)/d
```

mupad [B] time = 0.06, size = 41, normalized size = 0.93

$$\frac{B\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^2\,d}-\frac{A-B}{a^2\,d\,\left(\sin\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (B*log(sin(c + d*x) + 1))/(a^2*d) - (A - B)/(a^2*d*(sin(c + d*x) + 1))
```

sympy [A] time = 0.84, size = 121, normalized size = 2.75

$$\begin{cases} -\frac{A}{a^{2}d\sin\left(c+dx\right)+a^{2}d}+\frac{B\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{a^{2}d\sin\left(c+dx\right)+a^{2}d}+\frac{B\log\left(\sin\left(c+dx\right)+1\right)}{a^{2}d\sin\left(c+dx\right)+a^{2}d}+\frac{B}{a^{2}d\sin\left(c+dx\right)+a^{2}d} & \text{for}\: d\neq 0 \\ \frac{x\left(A+B\sin\left(c\right)\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-A/(a**2*d*sin(c + d*x) + a**2*d) + B*log(sin(c + d*x) + 1)*sin(
c + d*x)/(a**2*d*sin(c + d*x) + a**2*d) + B*log(sin(c + d*x) + 1)/(a**2*d*s
in(c + d*x) + a**2*d) + B/(a**2*d*sin(c + d*x) + a**2*d), Ne(d, 0)), (x*(A
+ B*sin(c))*cos(c)/(a*sin(c) + a)**2, True))
```

3.1015 $\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=71

$$-\frac{A+B}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{A-B}{4d(a\sin(c+dx)+a)^2}$$

```
[Out] 1/4*(A+B)*arctanh(sin(d*x+c))/a^2/d+1/4*(-A+B)/d/(a+a*sin(d*x+c))^2+1/4*(-A
-B)/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 77, 206}

$$-\frac{A+B}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{A-B}{4d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((A + B)*ArcTanh[Sin[c + d*x]])/(4*a^2*d) - (A - B)/(4*d*(a + a*Sin[c + d*x
])^2) - (A + B)/(4*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
```

```
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{a\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{A-B}{2a(a+x)^3} + \frac{A+B}{4a^2(a+x)^2} + \frac{A+B}{4a^2(a^2-x^2)}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{A-B}{4d(a+a\sin(c+dx))^2} - \frac{A+B}{4d\left(a^2+a^2\sin(c+dx)\right)} + \frac{(A+B)\,\text{Subst}\left(\int \frac{\ }{a}\right.}{}$$
$$= \frac{(A+B)\tanh^{-1}(\sin(c+dx))}{4a^2d} - \frac{A-B}{4d(a+a\sin(c+dx))^2} - \frac{A+B}{4d\left(a^2+a^2\sin(c\right.}$$

Mathematica [A] time = 0.12, size = 69, normalized size = 0.97

$$\frac{a\left(\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{4a^3} - \frac{A+B}{4a^2(a\sin(c+dx)+a)} - \frac{A-B}{4a(a\sin(c+dx)+a)^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a*(((A + B)*ArcTanh[Sin[c + d*x]])/(4*a^3) - (A - B)/(4*a*(a + a*Sin[c + d
*x])^2) - (A + B)/(4*a^2*(a + a*Sin[c + d*x]))))/d
```

fricas [B] time = 0.65, size = 134, normalized size = 1.89

$$\frac{\left((A+B)\cos(dx+c)^2 - 2(A+B)\sin(dx+c) - 2A - 2B\right)\log(\sin(dx+c)+1) - \left((A+B)\cos(dx+c)^2 - 2(A\right.}{8\left(a^2d\cos(dx+c)^2 - 2a^2d\sin(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] 1/8*(((A + B)*cos(d*x + c)^2 - 2*(A + B)*sin(d*x + c) - 2*A - 2*B)*log(sin(
d*x + c) + 1) - ((A + B)*cos(d*x + c)^2 - 2*(A + B)*sin(d*x + c) - 2*A - 2*
```

```
B)*log(-sin(d*x + c) + 1) + 2*(A + B)*sin(d*x + c) + 4*A)/(a^2*d*cos(d*x +
c)^2 - 2*a^2*d*sin(d*x + c) - 2*a^2*d)
```

giac [A] time = 0.20, size = 104, normalized size = 1.46

$$\frac{\frac{2(A+B)\log(|\sin(dx+c)+1|)}{a^2}-\frac{2(A+B)\log(|\sin(dx+c)-1|)}{a^2}-\frac{3A\sin(dx+c)^2+3B\sin(dx+c)^2+10A\sin(dx+c)+10B\sin(dx+c)+11A+3B}{a^2(\sin(dx+c)+1)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] 1/16*(2*(A + B)*log(abs(sin(d*x + c) + 1))/a^2 - 2*(A + B)*log(abs(sin(d*x
+ c) - 1))/a^2 - (3*A*sin(d*x + c)^2 + 3*B*sin(d*x + c)^2 + 10*A*sin(d*x +
c) + 10*B*sin(d*x + c) + 11*A + 3*B)/(a^2*(sin(d*x + c) + 1)^2))/d
```

maple [B] time = 0.62, size = 150, normalized size = 2.11

$$-\frac{\ln(\sin(dx+c)-1)A}{8d\,a^2}-\frac{\ln(\sin(dx+c)-1)B}{8d\,a^2}-\frac{A}{4d\,a^2(1+\sin(dx+c))^2}+\frac{B}{4d\,a^2(1+\sin(dx+c))^2}-\frac{}{4d\,a^2(1+\text{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/8/d/a^2*ln(sin(d*x+c)-1)*A-1/8/d/a^2*ln(sin(d*x+c)-1)*B-1/4/d/a^2/(1+sin
(d*x+c))^2*A+1/4/d/a^2/(1+sin(d*x+c))^2*B-1/4/d/a^2/(1+sin(d*x+c))*A-1/4/d/
a^2/(1+sin(d*x+c))*B+1/8/d/a^2*ln(1+sin(d*x+c))*A+1/8*B*ln(1+sin(d*x+c))/a^
2/d
```

maxima [A] time = 0.43, size = 84, normalized size = 1.18

$$-\frac{\frac{2((A+B)\sin(dx+c)+2A)}{a^2\sin(dx+c)^2+2a^2\sin(dx+c)+a^2}-\frac{(A+B)\log(\sin(dx+c)+1)}{a^2}+\frac{(A+B)\log(\sin(dx+c)-1)}{a^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/8*(2*((A + B)*sin(d*x + c) + 2*A)/(a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x +
c) + a^2) - (A + B)*log(sin(d*x + c) + 1)/a^2 + (A + B)*log(sin(d*x + c) -
1)/a^2)/d
```

mupad [B] time = 9.25, size = 71, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(A+B\right)}{4\,a^2\,d}-\frac{\frac{A}{2}+\sin\left(c+d\,x\right)\,\left(\frac{A}{4}+\frac{B}{4}\right)}{d\,\left(a^2\,{\sin\left(c+d\,x\right)}^2+2\,a^2\,\sin\left(c+d\,x\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (atanh(sin(c + d*x))*(A + B))/(4*a^2*d) - (A/2 + sin(c + d*x)*(A/4 + B/4))/
(d*(2*a^2*sin(c + d*x) + a^2 + a^2*sin(c + d*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec\left(c+dx\right)}{\sin^2\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx+\int \frac{B\sin\left(c+dx\right)\sec\left(c+dx\right)}{\sin^2\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x) + Integ
ral(B*sin(c + d*x)*sec(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x))
/a**2
```

3.1016 $\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=123

$$\frac{A+B}{16d\left(a^2-a^2\sin(c+dx)\right)}-\frac{3A+B}{16d\left(a^2\sin(c+dx)+a^2\right)}+\frac{(2A+B)\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{a(A-B)}{12d(a\sin(c+dx)+a)^3}-\frac{}{8}$$

```
[Out] 1/8*(2*A+B)*arctanh(sin(d*x+c))/a^2/d-1/12*a*(A-B)/d/(a+a*sin(d*x+c))^3-1/8
*A/d/(a+a*sin(d*x+c))^2+1/16*(A+B)/d/(a^2-a^2*sin(d*x+c))+1/16*(-3*A-B)/d/(
a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$\frac{A+B}{16d\left(a^2-a^2\sin(c+dx)\right)}-\frac{3A+B}{16d\left(a^2\sin(c+dx)+a^2\right)}+\frac{(2A+B)\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{a(A-B)}{12d(a\sin(c+dx)+a)^3}-\frac{}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((2*A + B)*ArcTanh[Sin[c + d*x]])/(8*a^2*d) - (a*(A - B))/(12*d*(a + a*Sin[
c + d*x])^3) - A/(8*d*(a + a*Sin[c + d*x])^2) + (A + B)/(16*d*(a^2 - a^2*Si
n[c + d*x])) - (3*A + B)/(16*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{a^3\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{A+B}{16a^4(a-x)^2} + \frac{A-B}{4a^2(a+x)^4} + \frac{A}{4a^3(a+x)^3} + \frac{3A+B}{16a^4(a+x)^2} + \frac{2A+B}{8a^4\left(a^2-x^2\right)}\right) dx\right.}{d}$$

$$= -\frac{a(A-B)}{12d(a+a\sin(c+dx))^3} - \frac{A}{8d(a+a\sin(c+dx))^2} + \frac{A+B}{16d\left(a^2-a^2\sin(c\right.}$$

$$= \frac{(2A+B)\tanh^{-1}(\sin(c+dx))}{8a^2d} - \frac{a(A-B)}{12d(a+a\sin(c+dx))^3} - \frac{A}{8d(a+a\sin(}$$

Mathematica [A] time = 0.74, size = 87, normalized size = 0.71

$$-\frac{\frac{3(A+B)}{\sin(c+dx)-1} + \frac{3(3A+B)}{\sin(c+dx)+1} + \frac{4(A-B)}{(\sin(c+dx)+1)^3} - 6(2A+B)\tanh^{-1}(\sin(c+dx)) + \frac{6A}{(\sin(c+dx)+1)^2}}{48a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/48*(-6*(2*A + B)*ArcTanh[Sin[c + d*x]] + (3*(A + B))/(-1 + Sin[c + d*x])
 + (4*(A - B))/(1 + Sin[c + d*x])^3 + (6*A)/(1 + Sin[c + d*x])^2 + (3*(3*A
+ B))/(1 + Sin[c + d*x]))/(a^2*d)
```

fricas [B] time = 0.86, size = 230, normalized size = 1.87

$$\frac{12\,(2\,A+B)\cos\,(dx+c)^2 + 3\left((2\,A+B)\cos\,(dx+c)^4 - 2\,(2\,A+B)\cos\,(dx+c)^2\sin\,(dx+c) - 2\,(2\,A+B)\cos\,(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/48*(12*(2*A + B)*cos(d*x + c)^2 + 3*((2*A + B)*cos(d*x + c)^4 - 2*(2*A +
B)*cos(d*x + c)^2*sin(d*x + c) - 2*(2*A + B)*cos(d*x + c)^2)*log(sin(d*x +
c) + 1) - 3*((2*A + B)*cos(d*x + c)^4 - 2*(2*A + B)*cos(d*x + c)^2*sin(d*x
+ c) - 2*(2*A + B)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(3*(2*A + B)*
cos(d*x + c)^2 - 8*A - 4*B)*sin(d*x + c) - 8*A - 16*B)/(a^2*d*cos(d*x + c)^
4 - 2*a^2*d*cos(d*x + c)^2*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2)
```

giac [A] time = 0.32, size = 169, normalized size = 1.37

$$\frac{\frac{6\,(2\,A+B)\log(|\sin(dx+c)+1|)}{a^2} - \frac{6\,(2\,A+B)\log(|\sin(dx+c)-1|)}{a^2} + \frac{6\,(2\,A\sin(dx+c)+B\sin(dx+c)-3\,A-2\,B)}{a^2(\sin(dx+c)-1)} - \frac{22\,A\sin(dx+c)^3+11\,B\sin(dx+c)^3+84\,\ldots}{\ldots}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/96*(6*(2*A + B)*log(abs(sin(d*x + c) + 1))/a^2 - 6*(2*A + B)*log(abs(sin(
d*x + c) - 1))/a^2 + 6*(2*A*sin(d*x + c) + B*sin(d*x + c) - 3*A - 2*B)/(a^2
*(sin(d*x + c) - 1)) - (22*A*sin(d*x + c)^3 + 11*B*sin(d*x + c)^3 + 84*A*si
n(d*x + c)^2 + 39*B*sin(d*x + c)^2 + 114*A*sin(d*x + c) + 45*B*sin(d*x + c)
 + 60*A + 9*B)/(a^2*(sin(d*x + c) + 1)^3))/d
```

maple [A] time = 0.76, size = 207, normalized size = 1.68

$$-\frac{\ln(\sin(dx+c)-1)\,A}{8d\,a^2} - \frac{\ln(\sin(dx+c)-1)\,B}{16d\,a^2} - \frac{A}{16d\,a^2(\sin(dx+c)-1)} - \frac{B}{16d\,a^2(\sin(dx+c)-1)} - \frac{\ldots}{8d\,a^2(1+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/8/d/a^2*ln(sin(d*x+c)-1)*A-1/16/d/a^2*ln(sin(d*x+c)-1)*B-1/16/d/a^2/(sin
(d*x+c)-1)*A-1/16/d/a^2/(sin(d*x+c)-1)*B-1/8/d/a^2/(1+sin(d*x+c))^2*A-1/12/
d/a^2/(1+sin(d*x+c))^3*A+1/12/d/a^2/(1+sin(d*x+c))^3*B+1/8/d/a^2*ln(1+sin(d
*x+c))*A+1/16*B*ln(1+sin(d*x+c))/a^2/d-3/16/d/a^2/(1+sin(d*x+c))*A-1/16/d/a
^2/(1+sin(d*x+c))*B
```

maxima [A] time = 0.42, size = 139, normalized size = 1.13

$$-\frac{\frac{2\left(3\,(2\,A+B)\sin(dx+c)^3+6\,(2\,A+B)\sin(dx+c)^2+(2\,A+B)\sin(dx+c)-8\,A+2\,B\right)}{a^2\sin(dx+c)^4+2\,a^2\sin(dx+c)^3-2\,a^2\sin(dx+c)-a^2} - \frac{3\,(2\,A+B)\log(\sin(dx+c)+1)}{a^2} + \frac{3\,(2\,A+B)\log(\sin(dx+c)-1)}{a^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/48*(2*(3*(2*A + B)*sin(d*x + c)^3 + 6*(2*A + B)*sin(d*x + c)^2 + (2*A +
B)*sin(d*x + c) - 8*A + 2*B)/(a^2*sin(d*x + c)^4 + 2*a^2*sin(d*x + c)^3 - 2
*a^2*sin(d*x + c) - a^2) - 3*(2*A + B)*log(sin(d*x + c) + 1)/a^2 + 3*(2*A +
 B)*log(sin(d*x + c) - 1)/a^2)/d
```

mupad [B] time = 0.15, size = 121, normalized size = 0.98

$$\frac{\left(\frac{A}{4}+\frac{B}{8}\right)\sin\left(c+d\,x\right)^3+\left(\frac{A}{2}+\frac{B}{4}\right)\sin\left(c+d\,x\right)^2+\left(\frac{A}{12}+\frac{B}{24}\right)\sin\left(c+d\,x\right)-\frac{A}{3}+\frac{B}{12}}{d\,\left(-a^2\sin\left(c+d\,x\right)^4-2\,a^2\sin\left(c+d\,x\right)^3+2\,a^2\,\sin\left(c+d\,x\right)+a^2\right)}+\frac{\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(2\,A+\right.}{8\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (B/12 - A/3 + sin(c + d*x)*(A/12 + B/24) + sin(c + d*x)^2*(A/2 + B/4) + sin
(c + d*x)^3*(A/4 + B/8))/(d*(2*a^2*sin(c + d*x) + a^2 - 2*a^2*sin(c + d*x)^
3 - a^2*sin(c + d*x)^4)) + (atanh(sin(c + d*x))*(2*A + B))/(8*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx+\int \frac{B\sin(c+dx)\sec^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x) + In
tegral(B*sin(c + d*x)*sec(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x))/a**2
```

3.1017 $\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$-\frac{a^2(A-B)}{32d(a\sin(c+dx)+a)^4}+\frac{5A+3B}{64d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5A+B}{32d\left(a^2\sin(c+dx)+a^2\right)}+\frac{5(3A+B)\tanh^{-1}(\sin(c+dx))}{64a^2d}$$

```
[Out] 5/64*(3*A+B)*arctanh(sin(d*x+c))/a^2/d+1/64*(A+B)/d/(a-a*sin(d*x+c))^2-1/32
*a^2*(A-B)/d/(a+a*sin(d*x+c))^4-1/48*a*(3*A-B)/d/(a+a*sin(d*x+c))^3-3/32*A/
d/(a+a*sin(d*x+c))^2+1/64*(5*A+3*B)/d/(a^2-a^2*sin(d*x+c))+1/32*(-5*A-B)/d/
(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.21, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$-\frac{a^2(A-B)}{32d(a\sin(c+dx)+a)^4}+\frac{5A+3B}{64d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5A+B}{32d\left(a^2\sin(c+dx)+a^2\right)}+\frac{5(3A+B)\tanh^{-1}(\sin(c+dx))}{64a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*(3*A + B)*ArcTanh[Sin[c + d*x]])/(64*a^2*d) + (A + B)/(64*d*(a - a*Sin[c
 + d*x])^2) - (a^2*(A - B))/(32*d*(a + a*Sin[c + d*x])^4) - (a*(3*A - B))/(
48*d*(a + a*Sin[c + d*x])^3) - (3*A)/(32*d*(a + a*Sin[c + d*x])^2) + (5*A +
 3*B)/(64*d*(a^2 - a^2*Sin[c + d*x])) - (5*A + B)/(32*d*(a^2 + a^2*Sin[c +
d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{A+B}{32a^5(a-x)^3} + \frac{5A+3B}{64a^6(a-x)^2} + \frac{A-B}{8a^3(a+x)^5} + \frac{3A-B}{16a^4(a+x)^4} + \frac{3A}{16a^5(a+x)^3} + \right.\right.}{d}$$

$$= \frac{A+B}{64d(a-a\sin(c+dx))^2} - \frac{a^2(A-B)}{32d(a+a\sin(c+dx))^4} - \frac{a(3A-B)}{48d(a+a\sin(c+d}$$

$$= \frac{5(3A+B)\tanh^{-1}(\sin(c+dx))}{64a^2d} + \frac{A+B}{64d(a-a\sin(c+dx))^2} - \frac{a^2(A}{32d(a+a\,\mathrm{s}}$$

Mathematica [A] time = 0.69, size = 123, normalized size = 0.69

$$\frac{-\frac{3(5A+3B)}{\sin(c+dx)-1} - \frac{6(5A+B)}{\sin(c+dx)+1} + \frac{3(A+B)}{(\sin(c+dx)-1)^2} + \frac{4(B-3A)}{(\sin(c+dx)+1)^3} - \frac{6(A-B)}{(\sin(c+dx)+1)^4} + 15(3A+B)\tanh^{-1}(\sin(c+dx)) - \frac{18A}{(\sin(c+dx}}{192a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (15*(3*A + B)*ArcTanh[Sin[c + d*x]] + (3*(A + B))/(-1 + Sin[c + d*x])^2 - (
3*(5*A + 3*B))/(-1 + Sin[c + d*x]) - (6*(A - B))/(1 + Sin[c + d*x])^4 + (4*
(-3*A + B))/(1 + Sin[c + d*x])^3 - (18*A)/(1 + Sin[c + d*x])^2 - (6*(5*A +
B))/(1 + Sin[c + d*x]))/(192*a^2*d)
```

fricas [A] time = 0.60, size = 260, normalized size = 1.45

$$\frac{60\,(3\,A+B)\cos(dx+c)^4 - 20\,(3\,A+B)\cos(dx+c)^2 + 15\left((3\,A+B)\cos(dx+c)^6 - 2\,(3\,A+B)\cos(dx+c)^4\,\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/384*(60*(3*A + B)*cos(d*x + c)^4 - 20*(3*A + B)*cos(d*x + c)^2 + 15*((3*A
 + B)*cos(d*x + c)^6 - 2*(3*A + B)*cos(d*x + c)^4*sin(d*x + c) - 2*(3*A + B
)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - 15*((3*A + B)*cos(d*x + c)^6 - 2*
(3*A + B)*cos(d*x + c)^4*sin(d*x + c) - 2*(3*A + B)*cos(d*x + c)^4)*log(-si
n(d*x + c) + 1) + 2*(15*(3*A + B)*cos(d*x + c)^4 - 20*(3*A + B)*cos(d*x + c
)^2 - 36*A - 12*B)*sin(d*x + c) - 24*A - 72*B)/(a^2*d*cos(d*x + c)^6 - 2*a^
2*d*cos(d*x + c)^4*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^4)
```

giac [A] time = 0.34, size = 214, normalized size = 1.20

$$\frac{\frac{60\,(3\,A+B)\log(|\sin(dx+c)+1|)}{a^2}-\frac{60\,(3\,A+B)\log(|\sin(dx+c)-1|)}{a^2}+\frac{6\left(45\,A\sin(dx+c)^2+15\,B\sin(dx+c)^2-110\,A\sin(dx+c)-42\,B\sin(dx+c)+69\,A+31}{a^2(\sin(dx+c)-1)^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/1536*(60*(3*A + B)*log(abs(sin(d*x + c) + 1))/a^2 - 60*(3*A + B)*log(abs(
sin(d*x + c) - 1))/a^2 + 6*(45*A*sin(d*x + c)^2 + 15*B*sin(d*x + c)^2 - 110
*A*sin(d*x + c) - 42*B*sin(d*x + c) + 69*A + 31*B)/(a^2*(sin(d*x + c) - 1)^
2) - (375*A*sin(d*x + c)^4 + 125*B*sin(d*x + c)^4 + 1740*A*sin(d*x + c)^3 +
 548*B*sin(d*x + c)^3 + 3114*A*sin(d*x + c)^2 + 894*B*sin(d*x + c)^2 + 2604
*A*sin(d*x + c) + 612*B*sin(d*x + c) + 903*A + 93*B)/(a^2*(sin(d*x + c) + 1
)^4))/d
```

maple [A] time = 0.76, size = 283, normalized size = 1.58

$$-\frac{15\ln(\sin(dx+c)-1)\,A}{128d\,a^2}-\frac{5\ln(\sin(dx+c)-1)\,B}{128d\,a^2}+\frac{A}{64d\,a^2\,(\sin(dx+c)-1)^2}+\frac{B}{64d\,a^2\,(\sin(dx+c)-1)^2}-\frac{}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -15/128/d/a^2*ln(sin(d*x+c)-1)*A-5/128/d/a^2*ln(sin(d*x+c)-1)*B+1/64/d/a^2/
(sin(d*x+c)-1)^2*A+1/64/d/a^2/(sin(d*x+c)-1)^2*B-5/64/d/a^2/(sin(d*x+c)-1)*
A-3/64/d/a^2/(sin(d*x+c)-1)*B-3/32/d/a^2/(1+sin(d*x+c))^2*A-1/32/d/a^2/(1+s
in(d*x+c))^4*A+1/32/d/a^2/(1+sin(d*x+c))^4*B-1/16/d/a^2/(1+sin(d*x+c))^3*A+
1/48/d/a^2/(1+sin(d*x+c))^3*B-5/32/d/a^2/(1+sin(d*x+c))*A-1/32/d/a^2/(1+sin
(d*x+c))*B+15/128/d/a^2*ln(1+sin(d*x+c))*A+5/128*B*ln(1+sin(d*x+c))/a^2/d
```

maxima [A] time = 0.34, size = 207, normalized size = 1.16

$$-\frac{\frac{2\left(15\,(3\,A+B)\sin(dx+c)^5+30\,(3\,A+B)\sin(dx+c)^4-10\,(3\,A+B)\sin(dx+c)^3-50\,(3\,A+B)\sin(dx+c)^2-17\,(3\,A+B)\sin(dx+c)+48\,A-16\,B\right)}{a^2\sin(dx+c)^6+2\,a^2\sin(dx+c)^5-a^2\sin(dx+c)^4-4\,a^2\sin(dx+c)^3-a^2\sin(dx+c)^2+2\,a^2\sin(dx+c)+a^2}-\frac{15\,(3\,A+B)}{}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/384*(2*(15*(3*A + B)*sin(d*x + c)^5 + 30*(3*A + B)*sin(d*x + c)^4 - 10*(
3*A + B)*sin(d*x + c)^3 - 50*(3*A + B)*sin(d*x + c)^2 - 17*(3*A + B)*sin(d*
x + c) + 48*A - 16*B)/(a^2*sin(d*x + c)^6 + 2*a^2*sin(d*x + c)^5 - a^2*sin(
d*x + c)^4 - 4*a^2*sin(d*x + c)^3 - a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x + c)
 + a^2) - 15*(3*A + B)*log(sin(d*x + c) + 1)/a^2 + 15*(3*A + B)*log(sin(d*x
 + c) - 1)/a^2)/d
```

mupad [B] time = 9.50, size = 193, normalized size = 1.08

$$\frac{\left(-\frac{15\,A}{64}-\frac{5\,B}{64}\right)\sin\left(c+d\,x\right)^5+\left(-\frac{15\,A}{32}-\frac{5\,B}{32}\right)\sin\left(c+d\,x\right)^4+\left(\frac{5\,A}{32}+\frac{5\,B}{96}\right)\sin\left(c+d\,x\right)^3+\left(\frac{25\,A}{32}+\frac{25\,B}{96}\right)\sin\left(c+d\,x\right)^2}{d\,\left(a^2\sin\left(c+d\,x\right)^6+2\,a^2\sin\left(c+d\,x\right)^5-a^2\sin\left(c+d\,x\right)^4-4\,a^2\sin\left(c+d\,x\right)^3-a^2\sin\left(c+d\,x\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^5*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (B/12 - A/4 + sin(c + d*x)*((17*A)/64 + (17*B)/192) - sin(c + d*x)^4*((15*A
)/32 + (5*B)/32) + sin(c + d*x)^3*((5*A)/32 + (5*B)/96) - sin(c + d*x)^5*((
15*A)/64 + (5*B)/64) + sin(c + d*x)^2*((25*A)/32 + (25*B)/96))/(d*(2*a^2*si
n(c + d*x) + a^2 - a^2*sin(c + d*x)^2 - 4*a^2*sin(c + d*x)^3 - a^2*sin(c +
d*x)^4 + 2*a^2*sin(c + d*x)^5 + a^2*sin(c + d*x)^6)) + (5*atanh(sin(c + d*x
))*(3*A + B))/(64*a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1018 $\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=236

$$-\frac{a^3(A-B)}{80d(a\sin(c+dx)+a)^5}-\frac{a^2(2A-B)}{64d(a\sin(c+dx)+a)^4}+\frac{3(7A+3B)}{256d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5(7A+B)}{256d\left(a^2\sin(c+dx)+a^2\right)}+\frac{7(4}{}$$

```
[Out] 7/128*(4*A+B)*arctanh(sin(d*x+c))/a^2/d+1/192*a*(A+B)/d/(a-a*sin(d*x+c))^3+
1/128*(3*A+2*B)/d/(a-a*sin(d*x+c))^2-1/80*a^3*(A-B)/d/(a+a*sin(d*x+c))^5-1/
64*a^2*(2*A-B)/d/(a+a*sin(d*x+c))^4-1/96*a*(5*A-B)/d/(a+a*sin(d*x+c))^3-5/6
4*A/d/(a+a*sin(d*x+c))^2+3/256*(7*A+3*B)/d/(a^2-a^2*sin(d*x+c))-5/256*(7*A+
B)/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.28, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$-\frac{a^3(A-B)}{80d(a\sin(c+dx)+a)^5}-\frac{a^2(2A-B)}{64d(a\sin(c+dx)+a)^4}+\frac{3(7A+3B)}{256d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5(7A+B)}{256d\left(a^2\sin(c+dx)+a^2\right)}+\frac{7(4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (7*(4*A + B)*ArcTanh[Sin[c + d*x]])/(128*a^2*d) + (a*(A + B))/(192*d*(a - a
*Sin[c + d*x])^3) + (3*A + 2*B)/(128*d*(a - a*Sin[c + d*x])^2) - (a^3*(A -
B))/(80*d*(a + a*Sin[c + d*x])^5) - (a^2*(2*A - B))/(64*d*(a + a*Sin[c + d*
x])^4) - (a*(5*A - B))/(96*d*(a + a*Sin[c + d*x])^3) - (5*A)/(64*d*(a + a*S
in[c + d*x])^2) + (3*(7*A + 3*B))/(256*d*(a^2 - a^2*Sin[c + d*x])) - (5*(7*
A + B))/(256*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)^6}\,dx,x,a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{A+B}{64a^6(a-x)^4}+\frac{3A+2B}{64a^7(a-x)^3}+\frac{3(7A+3B)}{256a^8(a-x)^2}+\frac{A-B}{16a^4(a+x)^6}+\frac{2A-B}{16a^5(a+x)^5}\right.\right.}{d}$$

$$= \frac{a(A+B)}{192d(a-a\sin(c+dx))^3}+\frac{3A+2B}{128d(a-a\sin(c+dx))^2}-\frac{a^3(A-B)}{80d(a+a\sin(c+}$$

$$= \frac{7(4A+B)\tanh^{-1}(\sin(c+dx))}{128a^2d}+\frac{a(A+B)}{192d(a-a\sin(c+dx))^3}+\frac{3A}{128d(a-a}$$

Mathematica [A] time = 1.48, size = 160, normalized size = 0.68

$$\frac{210(4A+B)\tanh^{-1}(\sin(c+dx))-\frac{2\left(105(4A+B)\sin^7(c+dx)+210(4A+B)\sin^6(c+dx)-175(4A+B)\sin^5(c+dx)-560(4A+B)\sin^4(c+dx)-49(\right.}{(\sin(c+dx)-1)^3(\sin(c+dx)+}}{3840a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (210*(4*A + B)*ArcTanh[Sin[c + d*x]] - (2*(48*(-8*A + 3*B) + 183*(4*A + B)*
Sin[c + d*x] + 462*(4*A + B)*Sin[c + d*x]^2 - 49*(4*A + B)*Sin[c + d*x]^3 -
 560*(4*A + B)*Sin[c + d*x]^4 - 175*(4*A + B)*Sin[c + d*x]^5 + 210*(4*A + B
)*Sin[c + d*x]^6 + 105*(4*A + B)*Sin[c + d*x]^7))/((-1 + Sin[c + d*x])^3*(1
 + Sin[c + d*x])^5))/(3840*a^2*d)
```

fricas [A] time = 0.77, size = 290, normalized size = 1.23

$$\frac{420\,(4\,A+B)\cos(dx+c)^6 - 140\,(4\,A+B)\cos(dx+c)^4 - 56\,(4\,A+B)\cos(dx+c)^2 + 105\left((4\,A+B)\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/3840*(420*(4*A + B)*cos(d*x + c)^6 - 140*(4*A + B)*cos(d*x + c)^4 - 56*(4
*A + B)*cos(d*x + c)^2 + 105*((4*A + B)*cos(d*x + c)^8 - 2*(4*A + B)*cos(d*
x + c)^6*sin(d*x + c) - 2*(4*A + B)*cos(d*x + c)^6)*log(sin(d*x + c) + 1) -
 105*((4*A + B)*cos(d*x + c)^8 - 2*(4*A + B)*cos(d*x + c)^6*sin(d*x + c) -
2*(4*A + B)*cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(105*(4*A + B)*cos(d
*x + c)^6 - 140*(4*A + B)*cos(d*x + c)^4 - 84*(4*A + B)*cos(d*x + c)^2 - 25
6*A - 64*B)*sin(d*x + c) - 128*A - 512*B)/(a^2*d*cos(d*x + c)^8 - 2*a^2*d*c
os(d*x + c)^6*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^6)
```

giac [A] time = 0.39, size = 258, normalized size = 1.09

$$\frac{\frac{420\,(4\,A+B)\log(|\sin(dx+c)+1|)}{a^2} - \frac{420\,(4\,A+B)\log(|\sin(dx+c)-1|)}{a^2} + \frac{10\left(308\,A\sin(dx+c)^3+77\,B\sin(dx+c)^3-1050\,A\sin(dx+c)^2-285\,B\sin(dx+c)^2\right.}{a^2(\sin(dx+c)-1)^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/15360*(420*(4*A + B)*log(abs(sin(d*x + c) + 1))/a^2 - 420*(4*A + B)*log(a
bs(sin(d*x + c) - 1))/a^2 + 10*(308*A*sin(d*x + c)^3 + 77*B*sin(d*x + c)^3
- 1050*A*sin(d*x + c)^2 - 285*B*sin(d*x + c)^2 + 1212*A*sin(d*x + c) + 363*
B*sin(d*x + c) - 478*A - 163*B)/(a^2*(sin(d*x + c) - 1)^3) - (3836*A*sin(d*
x + c)^5 + 959*B*sin(d*x + c)^5 + 21280*A*sin(d*x + c)^4 + 5095*B*sin(d*x +
 c)^4 + 47960*A*sin(d*x + c)^3 + 10790*B*sin(d*x + c)^3 + 55360*A*sin(d*x +
 c)^2 + 11230*B*sin(d*x + c)^2 + 33260*A*sin(d*x + c) + 5435*B*sin(d*x + c)
 + 8608*A + 667*B)/(a^2*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.76, size = 359, normalized size = 1.52

$$-\frac{7\ln(\sin(dx+c)-1)\,A}{64d\,a^2} - \frac{7\ln(\sin(dx+c)-1)\,B}{256d\,a^2} + \frac{3A}{128d\,a^2(\sin(dx+c)-1)^2} + \frac{B}{64d\,a^2(\sin(dx+c)-1)^2} - \frac{}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)

[Out] -7/64/d/a^2*ln(sin(d*x+c)-1)*A-7/256/d/a^2*ln(sin(d*x+c)-1)*B+3/128/d/a^2/(sin(d*x+c)-1)^2*A+1/64/d/a^2/(sin(d*x+c)-1)^2*B-1/192/d/a^2/(sin(d*x+c)-1)^3*A-1/192/d/a^2/(sin(d*x+c)-1)^3*B-21/256/d/a^2/(sin(d*x+c)-1)*A-9/256/d/a^2/(sin(d*x+c)-1)*B-5/64/d/a^2/(1+sin(d*x+c))^2*A-1/80/d/a^2/(1+sin(d*x+c))^5*A+1/80/d/a^2/(1+sin(d*x+c))^5*B-1/32/d/a^2/(1+sin(d*x+c))^4*A+1/64/d/a^2/(1+sin(d*x+c))^4*B-5/96/d/a^2/(1+sin(d*x+c))^3*A+1/96/d/a^2/(1+sin(d*x+c))^3*B+7/64/d/a^2*ln(1+sin(d*x+c))*A+7/256*B*ln(1+sin(d*x+c))/a^2/d-35/256/d/a^2/(1+sin(d*x+c))*A-5/256/d/a^2/(1+sin(d*x+c))*B

maxima [A] time = 0.35, size = 252, normalized size = 1.07

$$-\frac{\frac{2\left(105\,(4\,A+B)\sin(dx+c)^{7}+210\,(4\,A+B)\sin(dx+c)^{6}-175\,(4\,A+B)\sin(dx+c)^{5}-560\,(4\,A+B)\sin(dx+c)^{4}-49\,(4\,A+B)\sin(dx+c)^{3}+462\,(4\,A+B)\sin(dx+c\cdots}{a^{2}\sin(dx+c)^{8}+2\,a^{2}\sin(dx+c)^{7}-2\,a^{2}\sin(dx+c)^{6}-6\,a^{2}\sin(dx+c)^{5}+6\,a^{2}\sin(dx+c)^{3}+2\,a^{2}\sin(dx+c)^{2}-2\,a^{2}\sin(dx+\cdots}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/3840*(2*(105*(4*A + B)*sin(d*x + c)^7 + 210*(4*A + B)*sin(d*x + c)^6 - 175*(4*A + B)*sin(d*x + c)^5 - 560*(4*A + B)*sin(d*x + c)^4 - 49*(4*A + B)*sin(d*x + c)^3 + 462*(4*A + B)*sin(d*x + c)^2 + 183*(4*A + B)*sin(d*x + c) - 384*A + 144*B)/(a^2*sin(d*x + c)^8 + 2*a^2*sin(d*x + c)^7 - 2*a^2*sin(d*x + c)^6 - 6*a^2*sin(d*x + c)^5 + 6*a^2*sin(d*x + c)^3 + 2*a^2*sin(d*x + c)^2 - 2*a^2*sin(d*x + c) - a^2) - 105*(4*A + B)*log(sin(d*x + c) + 1)/a^2 + 105*(4*A + B)*log(sin(d*x + c) - 1)/a^2)/d

mupad [B] time = 9.60, size = 240, normalized size = 1.02

$$\frac{\left(\frac{7\,A}{32}+\frac{7\,B}{128}\right)\,\sin\left(c+d\,x\right)^7+\left(\frac{7\,A}{16}+\frac{7\,B}{64}\right)\,\sin\left(c+d\,x\right)^6+\left(-\frac{35\,A}{96}-\frac{35\,B}{384}\right)\,\sin\left(c+d\,x\right)^5+\left(-\frac{7\,A}{6}-\frac{7\,B}{24}\right)\,\sin\left(c+d\,x\right)^4\cdots}{d\,\left(-a^2\,\sin\left(c+d\,x\right)^8-2\,a^2\,\sin\left(c+d\,x\right)^7+2\,a^2\,\sin\left(c+d\,x\right)^6+6\,a^2\,\sin\left(c+d\,x\right)^5\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^7*(a + a*sin(c + d*x))^2),x)

[Out] ((3*B)/40 - A/5 + sin(c + d*x)*((61*A)/160 + (61*B)/640) - sin(c + d*x)^4*((7*A)/6 + (7*B)/24) + sin(c + d*x)^6*((7*A)/16 + (7*B)/64) + sin(c + d*x)^7*((7*A)/32 + (7*B)/128) - sin(c + d*x)^5*((35*A)/96 + (35*B)/384) + sin(c + d*x)^2*((77*A)/80 + (77*B)/320) - sin(c + d*x)^3*((49*A)/480 + (49*B)/1920))/(d*(2*a^2*sin(c + d*x) + a^2 - 2*a^2*sin(c + d*x)^2 - 6*a^2*sin(c + d*x)^3 + 6*a^2*sin(c + d*x)^5 + 2*a^2*sin(c + d*x)^6 - 2*a^2*sin(c + d*x)^7 - a^2*sin(c + d*x)^8)) + (7*atanh(sin(c + d*x))*(4*A + B))/(128*a^2*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
[Out] Timed out
```

3.1019 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=170

$$-\frac{a2^{\frac{1}{2}(2m+p+1)}(A(m+p+1)+Bm)(a\sin(e+fx)+a)^{m-1}(g\cos(e+fx))^{p+1}(\sin(e+fx)+1)^{\frac{1}{2}(-2m-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2r\right.}{fg(p+1)(m+p+1)}$$

```
[Out] -2^(1/2+m+1/2*p)*a*(B*m+A*(1+m+p))*(g*cos(f*x+e))^(1+p)*hypergeom([1/2+1/2*
p, 1/2-m-1/2*p],[3/2+1/2*p],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m-1/2*p
)*(a+a*sin(f*x+e))^(-1+m)/f/g/(1+p)/(1+m+p)-B*(g*cos(f*x+e))^(1+p)*(a+a*sin
(f*x+e))^m/f/g/(1+m+p)
```

Rubi [A] time = 0.27, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2860, 2689, 70, 69}

$$-\frac{a2^{\frac{1}{2}(2m+p+1)}(A(m+p+1)+Bm)(a\sin(e+fx)+a)^{m-1}(g\cos(e+fx))^{p+1}(\sin(e+fx)+1)^{\frac{1}{2}(-2m-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2r\right.}{fg(p+1)(m+p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((2^((1 + 2*m + p)/2)*a*(B*m + A*(1 + m + p))*(g*Cos[e + f*x])^(1 + p)*Hyp
ergeometric2F1[(1 - 2*m - p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[e + f*x])/2]
*(1 + Sin[e + f*x])^((1 - 2*m - p)/2)*(a + a*Sin[e + f*x])^(-1 + m))/(f*g*(
1 + p)*(1 + m + p))) - (B*(g*Cos[e + f*x])^(1 + p)*(a + a*Sin[e + f*x])^m)/
(f*g*(1 + m + p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
```

```
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))^m}{fg(1+m+p)} + \Big(A$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))^m}{fg(1+m+p)} + \frac{\Big(a^2}{}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))^m}{fg(1+m+p)} + \frac{\Big(2}{}$$

$$= -\frac{2^{\frac{1}{2}(1+2m+p)}a\left(A+\frac{Bm}{1+m+p}\right)(g\cos(e+fx))^{1+p}\,{}_2F}{}$$

Mathematica [A] time = 0.46, size = 154, normalized size = 0.91

$$-\frac{\cos(e+fx)(a(\sin(e+fx)+1))^m(g\cos(e+fx))^p(\sin(e+fx)+1)^{\frac{1}{2}(-2m-p-1)}\left(2^{\frac{1}{2}(2m+p+1)}(A(m+p+1)+Bm)\,{}_2\right.}{f(p+1)(m+p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(g*Cos[e + f*x])^p*(1 + Sin[e + f*x])^((-1 - 2*m - p)/2)*(a
*(1 + Sin[e + f*x]))^m*(2^((1 + 2*m + p)/2)*(B*m + A*(1 + m + p))*Hypergeom
etric2F1[(1 - 2*m - p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[e + f*x])/2] + B*(
1 + p)*(1 + Sin[e + f*x])^((1 + 2*m + p)/2)))/(f*(1 + p)*(1 + m + p)))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m, x
)
```

maple [F] time = 7.63, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{p}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(g\,\cos\left(e+f\,x\right)\right)^p\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int((g*cos(e + f*x))^p*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(g\cos\left(e+fx\right)\right)^{p}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(g*cos(e + f*x))**p*(A + B*sin(e + f*x))
, x)
```

3.1020 $\int \cos^7(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=159

$$-\frac{B(a\sin(e+fx)+a)^{m+8}}{a^8f(m+8)}-\frac{(A-7B)(a\sin(e+fx)+a)^{m+7}}{a^7f(m+7)}+\frac{6(A-3B)(a\sin(e+fx)+a)^{m+6}}{a^6f(m+6)}-\frac{4(3A-5B)(a\sin(e+fx)+a)^{m+5}}{a^5f(m+5)}$$

```
[Out] 8*(A-B)*(a+a*sin(f*x+e))^(4+m)/a^4/f/(4+m)-4*(3*A-5*B)*(a+a*sin(f*x+e))^(5+
m)/a^5/f/(5+m)+6*(A-3*B)*(a+a*sin(f*x+e))^(6+m)/a^6/f/(6+m)-(A-7*B)*(a+a*si
n(f*x+e))^(7+m)/a^7/f/(7+m)-B*(a+a*sin(f*x+e))^(8+m)/a^8/f/(8+m)
```

Rubi [A] time = 0.17, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{8(A-B)(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}-\frac{4(3A-5B)(a\sin(e+fx)+a)^{m+5}}{a^5f(m+5)}+\frac{6(A-3B)(a\sin(e+fx)+a)^{m+6}}{a^6f(m+6)}-\frac{(A-7B)(a\sin(e+fx)+a)^{m+7}}{a^7f(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^7*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (8*(A - B)*(a + a*Sin[e + f*x])^(4 + m))/(a^4*f*(4 + m)) - (4*(3*A - 5*B)*(
a + a*Sin[e + f*x])^(5 + m))/(a^5*f*(5 + m)) + (6*(A - 3*B)*(a + a*Sin[e +
f*x])^(6 + m))/(a^6*f*(6 + m)) - ((A - 7*B)*(a + a*Sin[e + f*x])^(7 + m))/(
a^7*f*(7 + m)) - (B*(a + a*Sin[e + f*x])^(8 + m))/(a^8*f*(8 + m))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{3+m}\left(A+\frac{Bx}{a}\right)dx, x, a\sin(e\right.}{a^7 f}$$

$$= \frac{\text{Subst}\left(\int \left(8a^3(A-B)(a+x)^{3+m} - 4a^2(3A-5B)(\right.\right.}{}$$

$$= \frac{8(A-B)(a+a\sin(e+fx))^{4+m}}{a^4 f(4+m)} - \frac{4(3A-5B)(a+}{a^5 f(}$$

Mathematica [A] time = 0.80, size = 132, normalized size = 0.83

$$\frac{(a(\sin(e+fx)+1))^{m+4}\left(-\frac{a^4(A-7B)(\sin(e+fx)+1)^3}{m+7} + \frac{6a^4(A-3B)(\sin(e+fx)+1)^2}{m+6} - \frac{4a^4(3A-5B)(\sin(e+fx)+1)}{m+5} + \frac{8a^4(A-B)}{m+4} - \frac{B(a\sin}{}\right.}{a^8 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^7*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(4 + m)*((8*a^4*(A - B))/(4 + m) - (4*a^4*(3*A - 5*
B)*(1 + Sin[e + f*x]))/(5 + m) + (6*a^4*(A - 3*B)*(1 + Sin[e + f*x])^2)/(6
+ m) - (a^4*(A - 7*B)*(1 + Sin[e + f*x])^3)/(7 + m) - (B*(a + a*Sin[e + f*x
])^4)/(8 + m)))/(a^8*f)
```

fricas [B] time = 0.79, size = 333, normalized size = 2.09

$$-\frac{\left(\left(Bm^4 + 22\,Bm^3 + 179\,Bm^2 + 638\,Bm + 840\,B\right)\cos\left(fx+e\right)^8 - \left((A+B)m^4 + (17\,A + 9\,B)m^3 + 4\,(23\,A + 5\,B\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] -((B*m^4 + 22*B*m^3 + 179*B*m^2 + 638*B*m + 840*B)*cos(f*x + e)^8 - ((A + B
)*m^4 + (17*A + 9*B)*m^3 + 4*(23*A + 5*B)*m^2 + 160*A*m)*cos(f*x + e)^6 - 1
2*((A + B)*m^3 + (11*A + 3*B)*m^2 + 24*A*m)*cos(f*x + e)^4 - 96*((A + B)*m^
2 + 8*A*m)*cos(f*x + e)^2 - 384*(A + B)*m - (((A + B)*m^4 + (23*A + 15*B)*m
^3 + 2*(97*A + 37*B)*m^2 + 8*(89*A + 15*B)*m + 960*A)*cos(f*x + e)^6 + 12*(
(A + B)*m^3 + (15*A + 7*B)*m^2 + 4*(17*A + 3*B)*m + 96*A)*cos(f*x + e)^4 +
```

```
96*((A + B)*m^2 + 2*(5*A + B)*m + 16*A)*cos(f*x + e)^2 + 384*(A + B)*m + 30
72*A)*sin(f*x + e) - 3072*A)*(a*sin(f*x + e) + a)^m/(f*m^5 + 30*f*m^4 + 355
*f*m^3 + 2070*f*m^2 + 5944*f*m + 6720*f)
```

giac [B] time = 0.29, size = 1402, normalized size = 8.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] -(((a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*m^3 - 6*(a*sin(f*x + e) +
a)^6*(a*sin(f*x + e) + a)^m*a*m^3 + 12*(a*sin(f*x + e) + a)^5*(a*sin(f*x +
e) + a)^m*a^2*m^3 - 8*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a^3*m^3
 + 15*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*m^2 - 96*(a*sin(f*x + e
) + a)^6*(a*sin(f*x + e) + a)^m*a*m^2 + 204*(a*sin(f*x + e) + a)^5*(a*sin(f
*x + e) + a)^m*a^2*m^2 - 144*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*
a^3*m^2 + 74*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*m - 498*(a*sin(f
*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a*m + 1128*(a*sin(f*x + e) + a)^5*(a*
sin(f*x + e) + a)^m*a^2*m - 856*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)
^m*a^3*m + 120*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m - 840*(a*sin(f
*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a + 2016*(a*sin(f*x + e) + a)^5*(a*si
n(f*x + e) + a)^m*a^2 - 1680*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*
a^3)*A/(a^6*m^4 + 22*a^6*m^3 + 179*a^6*m^2 + 638*a^6*m + 840*a^6) + ((a*sin
(f*x + e) + a)^8*(a*sin(f*x + e) + a)^m*m^4 - 7*(a*sin(f*x + e) + a)^7*(a*s
in(f*x + e) + a)^m*a*m^4 + 18*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m
*a^2*m^4 - 20*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a^3*m^4 + 8*(a*
sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a^4*m^4 + 22*(a*sin(f*x + e) + a
)^8*(a*sin(f*x + e) + a)^m*m^3 - 161*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e)
 + a)^m*a*m^3 + 432*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a^2*m^3 -
 500*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a^3*m^3 + 208*(a*sin(f*x
 + e) + a)^4*(a*sin(f*x + e) + a)^m*a^4*m^3 + 179*(a*sin(f*x + e) + a)^8*(a
*sin(f*x + e) + a)^m*m^2 - 1358*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)
^m*a*m^2 + 3798*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a^2*m^2 - 460
0*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a^3*m^2 + 2008*(a*sin(f*x +
 e) + a)^4*(a*sin(f*x + e) + a)^m*a^4*m^2 + 638*(a*sin(f*x + e) + a)^8*(a*s
in(f*x + e) + a)^m*m - 4984*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*a
*m + 14472*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a^2*m - 18400*(a*s
in(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a^3*m + 8528*(a*sin(f*x + e) + a)
^4*(a*sin(f*x + e) + a)^m*a^4*m + 840*(a*sin(f*x + e) + a)^8*(a*sin(f*x + e
) + a)^m - 6720*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*a + 20160*(a*
sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a^2 - 26880*(a*sin(f*x + e) + a)
^5*(a*sin(f*x + e) + a)^m*a^3 + 13440*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e
```

```
) + a)^m*a^4)*B/((a^6*m^5 + 30*a^6*m^4 + 355*a^6*m^3 + 2070*a^6*m^2 + 5944*
a^6*m + 6720*a^6)*a))/(a*f)
```

maple [F] time = 29.03, size = 0, normalized size = 0.00

$$\int \left(\cos^{7}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [B] time = 0.43, size = 1207, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -((((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*a^m*sin(f*x
 + e)^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*a^m*sin(f*x +
 e)^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*a^m*sin(f*x + e)^5 + 30*(
m^4 + 6*m^3 + 11*m^2 + 6*m)*a^m*sin(f*x + e)^4 - 120*(m^3 + 3*m^2 + 2*m)*a^
m*sin(f*x + e)^3 + 360*(m^2 + m)*a^m*sin(f*x + e)^2 - 720*a^m*m*sin(f*x + e
) + 720*a^m)*A*(sin(f*x + e) + 1)^m/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 67
69*m^3 + 13132*m^2 + 13068*m + 5040) - 3*((m^4 + 10*m^3 + 35*m^2 + 50*m + 2
4)*a^m*sin(f*x + e)^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*a^m*sin(f*x + e)^4 - 4
*(m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^3 + 12*(m^2 + m)*a^m*sin(f*x + e)^2 -
 24*a^m*m*sin(f*x + e) + 24*a^m)*A*(sin(f*x + e) + 1)^m/(m^5 + 15*m^4 + 85*
m^3 + 225*m^2 + 274*m + 120) + 3*((m^2 + 3*m + 2)*a^m*sin(f*x + e)^3 + (m^2
 + m)*a^m*sin(f*x + e)^2 - 2*a^m*m*sin(f*x + e) + 2*a^m)*A*(sin(f*x + e) +
1)^m/(m^3 + 6*m^2 + 11*m + 6) + ((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*
m^3 + 13132*m^2 + 13068*m + 5040)*a^m*sin(f*x + e)^8 + (m^7 + 21*m^6 + 175*
m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*a^m*sin(f*x + e)^7 - 7*(m^6 +
15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*a^m*sin(f*x + e)^6 + 42*(m^5 +
 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*a^m*sin(f*x + e)^5 - 210*(m^4 + 6*m^3 + 1
1*m^2 + 6*m)*a^m*sin(f*x + e)^4 + 840*(m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^
3 - 2520*(m^2 + m)*a^m*sin(f*x + e)^2 + 5040*a^m*m*sin(f*x + e) - 5040*a^m)
*B*(sin(f*x + e) + 1)^m/(m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67
284*m^3 + 118124*m^2 + 109584*m + 40320) - 3*((m^5 + 15*m^4 + 85*m^3 + 225*
m^2 + 274*m + 120)*a^m*sin(f*x + e)^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 2
4*m)*a^m*sin(f*x + e)^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*a^m*sin(f*x + e)^4
 + 20*(m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^3 - 60*(m^2 + m)*a^m*sin(f*x + e
```

```
)^2 + 120*a^m*m*sin(f*x + e) - 120*a^m)*B*(sin(f*x + e) + 1)^m/(m^6 + 21*m^
5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720) + 3*((m^3 + 6*m^2 + 11*m +
 6)*a^m*sin(f*x + e)^4 + (m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^3 - 3*(m^2 +
m)*a^m*sin(f*x + e)^2 + 6*a^m*m*sin(f*x + e) - 6*a^m)*B*(sin(f*x + e) + 1)^
m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) - (a^m*(m + 1)*sin(f*x + e)^2 + a^m*m
*sin(f*x + e) - a^m)*B*(sin(f*x + e) + 1)^m/(m^2 + 3*m + 2) - (a*sin(f*x +
e) + a)^(m + 1)*A/(a*(m + 1)))/f
```

mupad [B] time = 17.90, size = 783, normalized size = 4.92

$$-\mathrm{e}^{-e\,8\mathrm{i}-f\,x\,8\mathrm{i}}\left(a+a\,\sin\left(e+f\,x\right)\right)^m\left(-\frac{\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\left(786432\,A-58800\,B+237056\,A\,m+53644\,B\,m+32320\,A\,m^2 -\right.}{256\,f\,\left(m^5+30\,m^4+355\,m^3+2070\,\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^7*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] -exp(- e*8i - f*x*8i)*(a + a*sin(e + f*x))^m*((exp(e*8i + f*x*8i)*cos(4*e +
 4*f*x)*(23520*B - 8448*A*m + 17864*B*m - 4320*A*m^2 - 600*A*m^3 - 24*A*m^4
 + 3956*B*m^2 + 208*B*m^3 + 4*B*m^4))/(128*f*(5944*m + 2070*m^2 + 355*m^3 +
 30*m^4 + m^5 + 6720)) - (exp(e*8i + f*x*8i)*(786432*A - 58800*B + 237056*A
*m + 53644*B*m + 32320*A*m^2 + 2512*A*m^3 + 80*A*m^4 + 4814*B*m^2 + 332*B*m
^3 + 10*B*m^4))/(256*f*(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720))
 - (exp(e*8i + f*x*8i)*cos(2*e + 2*f*x)*(77184*A*m - 47040*B - 35728*B*m +
20112*A*m^2 + 1788*A*m^3 + 60*A*m^4 - 376*B*m^2 + 76*B*m^3 + 4*B*m^4))/(128
*f*(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720)) + (B*exp(e*8i + f*x
*8i)*cos(8*e + 8*f*x)*(638*m + 179*m^2 + 22*m^3 + m^4 + 840))/(128*f*(5944*
m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720)) + (exp(e*8i + f*x*8i)*sin(5*
e + 5*f*x)*(A*8i + A*m*1i + B*m*1i)*(706*m + 123*m^2 + 5*m^3 + 1176)*1i)/(6
4*f*(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720)) + (exp(e*8i + f*x*
8i)*sin(3*e + 3*f*x)*(A*8i + A*m*1i + B*m*1i)*(1070*m + 93*m^2 + 3*m^3 + 19
60)*3i)/(64*f*(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720)) + (exp(e
*8i + f*x*8i)*cos(6*e + 6*f*x)*(9*m + m^2 + 20)*(84*B - 8*A*m + 26*B*m - A*
m^2 + B*m^2))/(32*f*(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720)) +
(exp(e*8i + f*x*8i)*sin(7*e + 7*f*x)*(A*8i + A*m*1i + B*m*1i)*(74*m + 15*m^
2 + m^3 + 120)*1i)/(64*f*(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720
)) + (exp(e*8i + f*x*8i)*sin(e + f*x)*(A*8i + A*m*1i + B*m*1i)*(2578*m + 17
1*m^2 + 5*m^3 + 29400)*1i)/(64*f*(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^
5 + 6720)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**7*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
[Out] Timed out
```

3.1021 $\int \cos^5(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=123

$$\frac{B(a\sin(e+fx)+a)^{m+6}}{a^6f(m+6)}+\frac{(A-5B)(a\sin(e+fx)+a)^{m+5}}{a^5f(m+5)}-\frac{4(A-2B)(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{4(A-B)(a\sin(e+ \ldots}{a^3f(m+ \ldots}$$

```
[Out] 4*(A-B)*(a+a*sin(f*x+e))^(3+m)/a^3/f/(3+m)-4*(A-2*B)*(a+a*sin(f*x+e))^(4+m)
/a^4/f/(4+m)+(A-5*B)*(a+a*sin(f*x+e))^(5+m)/a^5/f/(5+m)+B*(a+a*sin(f*x+e))^
(6+m)/a^6/f/(6+m)
```

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{4(A-B)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}-\frac{4(A-2B)(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{(A-5B)(a\sin(e+fx)+a)^{m+5}}{a^5f(m+5)}+\frac{B(a\sin(e+ \ldots}{a^6f(n \ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (4*(A - B)*(a + a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m)) - (4*(A - 2*B)*(a
+ a*Sin[e + f*x])^(4 + m))/(a^4*f*(4 + m)) + ((A - 5*B)*(a + a*Sin[e + f*x]
)^(5 + m))/(a^5*f*(5 + m)) + (B*(a + a*Sin[e + f*x])^(6 + m))/(a^6*f*(6 + m
))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{2+m}\left(A+\frac{Bx}{a}\right)dx, x, a\sin(e$$

$$= \frac{\text{Subst}\left(\int \left(4a^2(A-B)(a+x)^{2+m}-4a(A-2B)(a+\right.\right.}{}$$

$$= \frac{4(A-B)(a+a\sin(e+fx))^{3+m}}{a^3f(3+m)} - \frac{4(A-2B)(a+a}{a^4f(4}$$

Mathematica [A] time = 0.40, size = 103, normalized size = 0.84

$$\frac{(a(\sin(e+fx)+1))^{m+3}\left(\frac{a^3(A-5B)(\sin(e+fx)+1)^2}{m+5}-\frac{4a^3(A-2B)(\sin(e+fx)+1)}{m+4}+\frac{4a^3(A-B)}{m+3}+\frac{B(a\sin(e+fx)+a)^3}{m+6}\right)}{a^6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3 + m)*((4*a^3*(A - B))/(3 + m) - (4*a^3*(A - 2*B)
*(1 + Sin[e + f*x]))/(4 + m) + (a^3*(A - 5*B)*(1 + Sin[e + f*x])^2)/(5 + m)
 + (B*(a + a*Sin[e + f*x])^3)/(6 + m)))/(a^6*f)
```

fricas [A] time = 0.66, size = 221, normalized size = 1.80

$$-\frac{\left(\left(Bm^3+12\,Bm^2+47\,Bm+60\,B\right)\cos\left(fx+e\right)^6-\left((A+B)m^3+3\,(3\,A+B)m^2+18\,Am\right)\cos\left(fx+e\right)^4-8\left((\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] -((B*m^3 + 12*B*m^2 + 47*B*m + 60*B)*cos(f*x + e)^6 - ((A + B)*m^3 + 3*(3*A
 + B)*m^2 + 18*A*m)*cos(f*x + e)^4 - 8*((A + B)*m^2 + 6*A*m)*cos(f*x + e)^2
 - 32*(A + B)*m - (((A + B)*m^3 + (13*A + 7*B)*m^2 + 6*(9*A + 2*B)*m + 72*A
)*cos(f*x + e)^4 + 8*((A + B)*m^2 + 2*(4*A + B)*m + 12*A)*cos(f*x + e)^2 +
32*(A + B)*m + 192*A)*sin(f*x + e) - 192*A)*(a*sin(f*x + e) + a)^m/(f*m^4 +
 18*f*m^3 + 119*f*m^2 + 342*f*m + 360*f)
```

giac [B] time = 0.23, size = 861, normalized size = 7.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] (((a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*m^2 - 4*(a*sin(f*x + e) + a
)^4*(a*sin(f*x + e) + a)^m*a*m^2 + 4*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e)
 + a)^m*a^2*m^2 + 7*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*m - 32*(a
*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a*m + 36*(a*sin(f*x + e) + a)^3
*(a*sin(f*x + e) + a)^m*a^2*m + 12*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) +
 a)^m - 60*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a + 80*(a*sin(f*x
+ e) + a)^3*(a*sin(f*x + e) + a)^m*a^2)*A/(a^4*m^3 + 12*a^4*m^2 + 47*a^4*m
+ 60*a^4) + ((a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*m^3 - 5*(a*sin(f
*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a*m^3 + 8*(a*sin(f*x + e) + a)^4*(a*s
in(f*x + e) + a)^m*a^2*m^3 - 4*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^
m*a^3*m^3 + 12*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*m^2 - 65*(a*si
n(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a*m^2 + 112*(a*sin(f*x + e) + a)^4
*(a*sin(f*x + e) + a)^m*a^2*m^2 - 60*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e)
 + a)^m*a^3*m^2 + 47*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*m - 270*
(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a*m + 504*(a*sin(f*x + e) + a
)^4*(a*sin(f*x + e) + a)^m*a^2*m - 296*(a*sin(f*x + e) + a)^3*(a*sin(f*x +
e) + a)^m*a^3*m + 60*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m - 360*(a
*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a + 720*(a*sin(f*x + e) + a)^4*
(a*sin(f*x + e) + a)^m*a^2 - 480*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a
)^m*a^3)*B/((a^4*m^4 + 18*a^4*m^3 + 119*a^4*m^2 + 342*a^4*m + 360*a^4)*a))/
(a*f)
```

maple [F] time = 13.37, size = 0, normalized size = 0.00

$$\int \left(\cos^5\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [B] time = 0.42, size = 643, normalized size = 5.23

$$\frac{\left(\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)a^m\sin\left(fx+e\right)^5+\left(m^4+6\,m^3+11\,m^2+6\,m\right)a^m\sin\left(fx+e\right)^4-4\left(m^3+3\,m^2+2\,m\right)a^m\sin\left(fx+e\right)^3+12\left(m^2+m\right)a^m\sin\left(fx+e\right)^2-2}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] (((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*a^m*sin(f*x + e)^5 + (m^4 + 6*m^3 + 1
1*m^2 + 6*m)*a^m*sin(f*x + e)^4 - 4*(m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^3
+ 12*(m^2 + m)*a^m*sin(f*x + e)^2 - 24*a^m*m*sin(f*x + e) + 24*a^m)*A*(sin(
f*x + e) + 1)^m/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120) - 2*((m^2 +
 3*m + 2)*a^m*sin(f*x + e)^3 + (m^2 + m)*a^m*sin(f*x + e)^2 - 2*a^m*m*sin(f
*x + e) + 2*a^m)*A*(sin(f*x + e) + 1)^m/(m^3 + 6*m^2 + 11*m + 6) + ((m^5 +
15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*a^m*sin(f*x + e)^6 + (m^5 + 10*m^4
 + 35*m^3 + 50*m^2 + 24*m)*a^m*sin(f*x + e)^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6
*m)*a^m*sin(f*x + e)^4 + 20*(m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^3 - 60*(m^
2 + m)*a^m*sin(f*x + e)^2 + 120*a^m*m*sin(f*x + e) - 120*a^m)*B*(sin(f*x +
e) + 1)^m/(m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720) - 2*
((m^3 + 6*m^2 + 11*m + 6)*a^m*sin(f*x + e)^4 + (m^3 + 3*m^2 + 2*m)*a^m*sin(
f*x + e)^3 - 3*(m^2 + m)*a^m*sin(f*x + e)^2 + 6*a^m*m*sin(f*x + e) - 6*a^m)
*B*(sin(f*x + e) + 1)^m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) + (a^m*(m + 1)*
sin(f*x + e)^2 + a^m*m*sin(f*x + e) - a^m)*B*(sin(f*x + e) + 1)^m/(m^2 + 3*
m + 2) + (a*sin(f*x + e) + a)^(m + 1)*A/(a*(m + 1)))/f
```

mupad [B] time = 15.82, size = 517, normalized size = 4.20

$$-e^{-e\,6i-f\,x\,6i}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(-\frac{e^{e\,6i+f\,x\,6i}\,\left(12288\,A-1200\,B+4016\,A\,m+1108\,B\,m+472\,A\,m^2+24\,A\,m\right.}{64\,f\,\left(m^4+18\,m^3+119\,m^2+342\,m+360\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] -exp(- e*6i - f*x*6i)*(a + a*sin(e + f*x))^m*((exp(e*6i + f*x*6i)*cos(4*e +
 4*f*x)*(m + 3)*(60*B - 12*A*m + 27*B*m - 2*A*m^2 + B*m^2))/(16*f*(342*m +
119*m^2 + 18*m^3 + m^4 + 360)) - (exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(1056
*A*m - 900*B - 705*B*m + 272*A*m^2 + 16*A*m^3 - 4*B*m^2 + B*m^3))/(32*f*(34
2*m + 119*m^2 + 18*m^3 + m^4 + 360)) - (exp(e*6i + f*x*6i)*(12288*A - 1200*
B + 4016*A*m + 1108*B*m + 472*A*m^2 + 24*A*m^3 + 88*B*m^2 + 4*B*m^3))/(64*f
*(342*m + 119*m^2 + 18*m^3 + m^4 + 360)) + (exp(e*6i + f*x*6i)*sin(e + f*x)
*(A*6i + A*m*1i + B*m*1i)*(23*m + m^2 + 300)*1i)/(8*f*(342*m + 119*m^2 + 18
*m^3 + m^4 + 360)) + (exp(e*6i + f*x*6i)*sin(5*e + 5*f*x)*(A*6i + A*m*1i +
B*m*1i)*(7*m + m^2 + 12)*1i)/(16*f*(342*m + 119*m^2 + 18*m^3 + m^4 + 360))
+ (B*exp(e*6i + f*x*6i)*cos(6*e + 6*f*x)*(47*m + 12*m^2 + m^3 + 60))/(32*f*
(342*m + 119*m^2 + 18*m^3 + m^4 + 360)) + (exp(e*6i + f*x*6i)*sin(3*e + 3*f
*x)*(A*6i + A*m*1i + B*m*1i)*(53*m + 3*m^2 + 100)*1i)/(16*f*(342*m + 119*m^
2 + 18*m^3 + m^4 + 360)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1022 $\int \cos^3(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=93

$$-\frac{B(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}-\frac{(A-3B)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{2(A-B)(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}$$

```
[Out] 2*(A-B)*(a+a*sin(f*x+e))^(2+m)/a^2/f/(2+m)-(A-3*B)*(a+a*sin(f*x+e))^(3+m)/a
^3/f/(3+m)-B*(a+a*sin(f*x+e))^(4+m)/a^4/f/(4+m)
```

Rubi [A] time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{2(A-B)(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}-\frac{(A-3B)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}-\frac{B(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) - ((A - 3*B)*(a +
a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m)) - (B*(a + a*Sin[e + f*x])^(4 + m))
/(a^4*f*(4 + m))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^{1+m}\left(A+\frac{Bx}{a}\right)dx, x, a\sin(e+\right.}{a^3 f}$$

$$= \frac{\text{Subst}\left(\int \left(2a(A-B)(a+x)^{1+m}+(-A+3B)(a+x)\right.\right.}{a^3 f}$$

$$= \frac{2(A-B)(a+a\sin(e+fx))^{2+m}}{a^2 f(2+m)} - \frac{(A-3B)(a+a\sin}{a^3 f(3+}$$

Mathematica [A] time = 0.31, size = 93, normalized size = 1.00

$$-\frac{B(a\sin(e+fx)+a)^{m+4}}{a^4 f(m+4)} - \frac{(A-3B)(a\sin(e+fx)+a)^{m+3}}{a^3 f(m+3)} + \frac{2(A-B)(a\sin(e+fx)+a)^{m+2}}{a^2 f(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) - ((A - 3*B)*(a +
a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m)) - (B*(a + a*Sin[e + f*x])^(4 + m))
/(a^4*f*(4 + m))
```

fricas [A] time = 0.71, size = 135, normalized size = 1.45

$$-\frac{\left(\left(Bm^2+5\,Bm+6\,B\right)\cos\left(fx+e\right)^4-\left((A+B)m^2+4\,Am\right)\cos\left(fx+e\right)^2-4\,(A+B)m-\left(\left(\left((A+B)m^2+2\,(3\,A\right.\right.\right.\right.}{fm^3+9\,fm^2+26\,fm+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] -((B*m^2 + 5*B*m + 6*B)*cos(f*x + e)^4 - ((A + B)*m^2 + 4*A*m)*cos(f*x + e)
^2 - 4*(A + B)*m - (((A + B)*m^2 + 2*(3*A + B)*m + 8*A)*cos(f*x + e)^2 + 4*
(A + B)*m + 16*A)*sin(f*x + e) - 16*A)*(a*sin(f*x + e) + a)^m/(f*m^3 + 9*f*
m^2 + 26*f*m + 24*f)
```

giac [B] time = 0.19, size = 458, normalized size = 4.92

$$-\frac{\frac{\left(\left(a\sin\left(fx+e\right)+a\right)^3\left(a\sin\left(fx+e\right)+a\right)^m m-2\left(a\sin\left(fx+e\right)+a\right)^2\left(a\sin\left(fx+e\right)+a\right)^m am+2\left(a\sin\left(fx+e\right)+a\right)^3\left(a\sin\left(fx+e\right)+a\right)^m-6\left(a\sin\left(fx+e\right)+a\right)^2\left(a\sin\left(\right.\right.\right.}{a^2m^2+5\,a^2m+6\,a^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] -(((a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m - 2*(a*sin(f*x + e) + a)
^2*(a*sin(f*x + e) + a)^m*a*m + 2*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) +
a)^m - 6*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a)*A/(a^2*m^2 + 5*a^
2*m + 6*a^2) + ((a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^2 - 3*(a*si
n(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a*m^2 + 2*(a*sin(f*x + e) + a)^2*(
a*sin(f*x + e) + a)^m*a^2*m^2 + 5*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) +
a)^m*m - 18*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a*m + 14*(a*sin(f
*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^2*m + 6*(a*sin(f*x + e) + a)^4*(a*s
in(f*x + e) + a)^m - 24*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a + 2
4*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^2)*B/((a^2*m^3 + 9*a^2*m^
2 + 26*a^2*m + 24*a^2)*a))/(a*f)
```

maple [F] time = 6.83, size = 0, normalized size = 0.00

$$\int \left(\cos^3\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [B] time = 0.40, size = 285, normalized size = 3.06

$$-\frac{\frac{\left(\left(m^2+3m+2\right)a^m\sin\left(fx+e\right)^3+\left(m^2+m\right)a^m\sin\left(fx+e\right)^2-2a^m m\sin\left(fx+e\right)+2a^m\right)A\left(\sin\left(fx+e\right)+1\right)^m}{m^3+6m^2+11m+6}+\frac{\left(\left(m^3+6m^2+11m+6\right)a^m\sin\left(fx+e\right)^4+\left(m^3+3\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -((((m^2 + 3*m + 2)*a^m*sin(f*x + e)^3 + (m^2 + m)*a^m*sin(f*x + e)^2 - 2*a^
m*m*sin(f*x + e) + 2*a^m)*A*(sin(f*x + e) + 1)^m/(m^3 + 6*m^2 + 11*m + 6) +
 ((m^3 + 6*m^2 + 11*m + 6)*a^m*sin(f*x + e)^4 + (m^3 + 3*m^2 + 2*m)*a^m*sin
(f*x + e)^3 - 3*(m^2 + m)*a^m*sin(f*x + e)^2 + 6*a^m*m*sin(f*x + e) - 6*a^m
)*B*(sin(f*x + e) + 1)^m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) - (a^m*(m + 1)
*sin(f*x + e)^2 + a^m*m*sin(f*x + e) - a^m)*B*(sin(f*x + e) + 1)^m/(m^2 + 3
*m + 2) - (a*sin(f*x + e) + a)^(m + 1)*A/(a*(m + 1)))/f
```

mupad [B] time = 11.74, size = 272, normalized size = 2.92

$$\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(128\,A-18\,B+48\,A\,m+17\,B\,m+144\,A\,\sin\left(e+f\,x\right)-24\,B\,\cos\left(2\,e+2\,f\,x\right)-6\,B\,\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(128*A - 18*B + 48*A*m + 17*B*m + 144*A*sin(e + f
*x) - 24*B*cos(2*e + 2*f*x) - 6*B*cos(4*e + 4*f*x) + 4*A*m^2 + B*m^2 + 16*A
*sin(3*e + 3*f*x) + 4*A*m^2*cos(2*e + 2*f*x) - B*m^2*cos(4*e + 4*f*x) + 2*A
*m^2*sin(3*e + 3*f*x) + 2*B*m^2*sin(3*e + 3*f*x) + 44*A*m*sin(e + f*x) + 36
*B*m*sin(e + f*x) + 16*A*m*cos(2*e + 2*f*x) - 20*B*m*cos(2*e + 2*f*x) - 5*B
*m*cos(4*e + 4*f*x) + 12*A*m*sin(3*e + 3*f*x) + 2*A*m^2*sin(e + f*x) + 4*B*
m*sin(3*e + 3*f*x) + 2*B*m^2*sin(e + f*x)))/(8*f*(26*m + 9*m^2 + m^3 + 24))
```

sympy [A] time = 54.48, size = 5243, normalized size = 56.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((x*(A + B*sin(e))*(a*sin(e) + a)**m*cos(e)**3, Eq(f, 0)), (4*A*si
n(e + f*x)**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a*
*4*f*sin(e + f*x) + 6*a**4*f) + 6*A*sin(e + f*x)/(6*a**4*f*sin(e + f*x)**3
+ 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 2*A*cos(
e + f*x)**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4
*f*sin(e + f*x) + 6*a**4*f) + 2*A/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin
(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 6*B*log(sin(e + f*x) +
1)*sin(e + f*x)**3/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 +
18*a**4*f*sin(e + f*x) + 6*a**4*f) - 18*B*log(sin(e + f*x) + 1)*sin(e + f*x
)**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(
e + f*x) + 6*a**4*f) - 18*B*log(sin(e + f*x) + 1)*sin(e + f*x)/(6*a**4*f*si
n(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4
*f) - 6*B*log(sin(e + f*x) + 1)/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e
 + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 10*B*sin(e + f*x)**2/(6*a
**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x)
+ 6*a**4*f) - 3*B*sin(e + f*x)*cos(e + f*x)**2/(6*a**4*f*sin(e + f*x)**3 +
18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 18*B*sin(e
 + f*x)/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*s
in(e + f*x) + 6*a**4*f) - B*cos(e + f*x)**2/(6*a**4*f*sin(e + f*x)**3 + 18*
a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 8*B/(6*a**4*f
```

*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a **4*f), Eq(m, -4)), (-2*A*log(sin(e + f*x) + 1)*sin(e + f*x)**2/(2*a**3*f*s in(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 4*A*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3 *f) - 2*A*log(sin(e + f*x) + 1)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 2*A*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f* sin(e + f*x) + 2*a**3*f) - A*cos(e + f*x)**2/(2*a**3*f*sin(e + f*x)**2 + 4* a**3*f*sin(e + f*x) + 2*a**3*f) - 2*A/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f* sin(e + f*x) + 2*a**3*f) + 6*B*log(sin(e + f*x) + 1)*sin(e + f*x)**2/(2*a** 3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 12*B*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 6*B*log(sin(e + f*x) + 1)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f* sin(e + f*x) + 2*a**3*f) - 4*B*sin(e + f*x)**3/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 2*B*sin(e + f*x)*cos(e + f*x)**2/(2*a** 3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 14*B*sin(e + f*x) /(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - B*cos(e + f*x)**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 10* B/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f), Eq(m, -3)) , (4*A*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*x/2)**4/(a**2*f*tan(e/2 + f*x/ 2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) + 8*A*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*x/2)**2/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f* x/2)**2 + a**2*f) + 4*A*log(tan(e/2 + f*x/2) + 1)/(a**2*f*tan(e/2 + f*x/2)* *4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) - 2*A*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/2)**4/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f* x/2)**2 + a**2*f) - 4*A*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/2)**2/(a **2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) - 2*A*lo g(tan(e/2 + f*x/2)**2 + 1)/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) - 2*A*tan(e/2 + f*x/2)**3/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) - 2*A*tan(e/2 + f*x/2)/(a**2*f*tan (e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) - 4*B*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*x/2)**4/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*t an(e/2 + f*x/2)**2 + a**2*f) - 8*B*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*x/ 2)**2/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) - 4*B*log(tan(e/2 + f*x/2) + 1)/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) + 2*B*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/ 2)**4/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) + 4*B*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/2)**2/(a**2*f*tan(e/2 + f* x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) + 2*B*log(tan(e/2 + f*x/2) **2 + 1)/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2* f) + 4*B*tan(e/2 + f*x/2)**3/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) - 2*B*tan(e/2 + f*x/2)**2/(a**2*f*tan(e/2 + f*x/2)** 4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) + 4*B*tan(e/2 + f*x/2)/(a**2*f*t an(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f), Eq(m, -2)), (6 *A*tan(e/2 + f*x/2)**5/(3*a*f*tan(e/2 + f*x/2)**6 + 9*a*f*tan(e/2 + f*x/2)* *4 + 9*a*f*tan(e/2 + f*x/2)**2 + 3*a*f) - 6*A*tan(e/2 + f*x/2)**4/(3*a*f*ta

```
n(e/2 + f*x/2)**6 + 9*a*f*tan(e/2 + f*x/2)**4 + 9*a*f*tan(e/2 + f*x/2)**2 +
 3*a*f) + 12*A*tan(e/2 + f*x/2)**3/(3*a*f*tan(e/2 + f*x/2)**6 + 9*a*f*tan(e
/2 + f*x/2)**4 + 9*a*f*tan(e/2 + f*x/2)**2 + 3*a*f) - 6*A*tan(e/2 + f*x/2)*
*2/(3*a*f*tan(e/2 + f*x/2)**6 + 9*a*f*tan(e/2 + f*x/2)**4 + 9*a*f*tan(e/2 +
 f*x/2)**2 + 3*a*f) + 6*A*tan(e/2 + f*x/2)/(3*a*f*tan(e/2 + f*x/2)**6 + 9*a
*f*tan(e/2 + f*x/2)**4 + 9*a*f*tan(e/2 + f*x/2)**2 + 3*a*f) + 6*B*tan(e/2 +
 f*x/2)**4/(3*a*f*tan(e/2 + f*x/2)**6 + 9*a*f*tan(e/2 + f*x/2)**4 + 9*a*f*t
an(e/2 + f*x/2)**2 + 3*a*f) - 8*B*tan(e/2 + f*x/2)**3/(3*a*f*tan(e/2 + f*x/
2)**6 + 9*a*f*tan(e/2 + f*x/2)**4 + 9*a*f*tan(e/2 + f*x/2)**2 + 3*a*f) + 6*
B*tan(e/2 + f*x/2)**2/(3*a*f*tan(e/2 + f*x/2)**6 + 9*a*f*tan(e/2 + f*x/2)**
4 + 9*a*f*tan(e/2 + f*x/2)**2 + 3*a*f), Eq(m, -1)), (A*m**3*(a*sin(e + f*x)
 + a)**m*sin(e + f*x)*cos(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*
f*m + 24*f) + A*m**3*(a*sin(e + f*x) + a)**m*cos(e + f*x)**2/(f*m**4 + 10*f
*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 2*A*m**2*(a*sin(e + f*x) + a)**m*sin(e
 + f*x)**3/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 4*A*m**2*(a*s
in(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*
m + 24*f) + 9*A*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)*cos(e + f*x)**2/(
f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 2*A*m**2*(a*sin(e + f*x)
+ a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 9*A
*m**2*(a*sin(e + f*x) + a)**m*cos(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m*
*2 + 50*f*m + 24*f) + 12*A*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**
4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 22*A*m*(a*sin(e + f*x) + a)**m
*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 26*A*m*
(a*sin(e + f*x) + a)**m*sin(e + f*x)*cos(e + f*x)**2/(f*m**4 + 10*f*m**3 +
35*f*m**2 + 50*f*m + 24*f) + 8*A*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*
m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 26*A*m*(a*sin(e + f*x) + a)
**m*cos(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) - 2*A*
m*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f)
+ 16*A*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**4 + 10*f*m**3 + 35*f*m
**2 + 50*f*m + 24*f) + 24*A*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4
 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 24*A*(a*sin(e + f*x) + a)**m*si
n(e + f*x)*cos(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f)
 + 24*A*(a*sin(e + f*x) + a)**m*cos(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*
m**2 + 50*f*m + 24*f) - 8*A*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 + 3
5*f*m**2 + 50*f*m + 24*f) + B*m**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2*
cos(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + B*m**3*(
a*sin(e + f*x) + a)**m*sin(e + f*x)*cos(e + f*x)**2/(f*m**4 + 10*f*m**3 + 3
5*f*m**2 + 50*f*m + 24*f) + 2*B*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**
4/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 4*B*m**2*(a*sin(e + f*
x) + a)**m*sin(e + f*x)**3/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f)
 + 8*B*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2*cos(e + f*x)**2/(f*m**4
 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 2*B*m**2*(a*sin(e + f*x) + a)**
m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 7*B*m*
*2*(a*sin(e + f*x) + a)**m*sin(e + f*x)*cos(e + f*x)**2/(f*m**4 + 10*f*m**3
 + 35*f*m**2 + 50*f*m + 24*f) - B*m**2*(a*sin(e + f*x) + a)**m*cos(e + f*x)
```

```
**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 8*B*m*(a*sin(e + f*x
) + a)**m*sin(e + f*x)**4/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f)
+ 10*B*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**4 + 10*f*m**3 + 35*f
*m**2 + 50*f*m + 24*f) + 19*B*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2*cos
(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) - 4*B*m*(a*si
n(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m
 + 24*f) + 12*B*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)*cos(e + f*x)**2/(f*m
**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) - 6*B*m*(a*sin(e + f*x) + a)**
m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) - 7*B*m*(a*
sin(e + f*x) + a)**m*cos(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f
*m + 24*f) + 6*B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**4/(f*m**4 + 10*f*m**
3 + 35*f*m**2 + 50*f*m + 24*f) + 12*B*(a*sin(e + f*x) + a)**m*sin(e + f*x)*
*2*cos(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) - 12*B*
(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 5
0*f*m + 24*f) - 12*B*(a*sin(e + f*x) + a)**m*cos(e + f*x)**2/(f*m**4 + 10*f
*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 6*B*(a*sin(e + f*x) + a)**m/(f*m**4 +
10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f), True))
```

3.1023 $\int \cos(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=59

$$\frac{B(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(A-B)(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

```
[Out] (A-B)*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)+B*(a+a*sin(f*x+e))^(2+m)/a^2/f/(2+m)
```

Rubi [A] time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{B(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(A-B)(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (B*(a + a*Sin[e + f*
x])^(2 + m))/(a^2*f*(2 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{\text{Subst}\left(\int (a+x)^m\left(A+\frac{Bx}{a}\right)dx, x, a\sin(e+fx)\right)}{af}$$
$$= \frac{\text{Subst}\left(\int \left((A-B)(a+x)^m+\frac{B(a+x)^{1+m}}{a}\right)dx, x, a\sin$$
$$= \frac{(A-B)(a+a\sin(e+fx))^{1+m}}{af(1+m)} + \frac{B(a+a\sin(e+f$$
$$a^2f(2+m)$$

Mathematica [A] time = 0.13, size = 51, normalized size = 0.86

$$\frac{(a(\sin(e+fx)+1))^{m+1}(A(m+2)+B(m+1)\sin(e+fx)-B)}{af(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*(-B + A*(2 + m) + B*(1 + m)*Sin[e + f*x]))/
(a*f*(1 + m)*(2 + m))
```

fricas [A] time = 0.70, size = 70, normalized size = 1.19

$$-\frac{\left((Bm+B)\cos\left(fx+e\right)^2-(A+B)m-((A+B)m+2A)\sin\left(fx+e\right)-2A\right)\left(a\sin\left(fx+e\right)+a\right)^m}{fm^2+3fm+2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -((B*m + B)*cos(f*x + e)^2 - (A + B)*m - ((A + B)*m + 2*A)*sin(f*x + e) - 2
*A)*(a*sin(f*x + e) + a)^m/(f*m^2 + 3*f*m + 2*f)
```

giac [B] time = 0.16, size = 156, normalized size = 2.64

$$\frac{\frac{\left(a\sin\left(fx+e\right)+a\right)^{m+1}A}{m+1}+\frac{\left(\left(a\sin\left(fx+e\right)+a\right)^2\left(a\sin\left(fx+e\right)+a\right)^m m-\left(a\sin\left(fx+e\right)+a\right)\left(a\sin\left(fx+e\right)+a\right)^m am+\left(a\sin\left(fx+e\right)+a\right)^2\left(a\sin\left(fx+e\right)+a\right)^m-2\left(a\right.}{\left(m^2+3m+2\right)a}}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] ((a*sin(f*x + e) + a)^(m + 1)*A/(m + 1) + ((a*sin(f*x + e) + a)^2*(a*sin(f*
x + e) + a)^m*m - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a*m + (a*sin(
f*x + e) + a)^2*(a*sin(f*x + e) + a)^m - 2*(a*sin(f*x + e) + a)*(a*sin(f*x
+ e) + a)^m*a)*B/((m^2 + 3*m + 2)*a))/(a*f)
```

maple [F] time = 4.70, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [A] time = 0.40, size = 83, normalized size = 1.41

$$\frac{\frac{\left(a^{m}(m+1)\sin\left(fx+e\right)^{2}+a^{m}m\sin\left(fx+e\right)-a^{m}\right)B\left(\sin\left(fx+e\right)+1\right)^{m}}{m^{2}+3\,m+2}+\frac{\left(a\sin\left(fx+e\right)+a\right)^{m+1}A}{a(m+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] ((a^m*(m + 1)*sin(f*x + e)^2 + a^m*m*sin(f*x + e) - a^m)*B*(sin(f*x + e) +
1)^m/(m^2 + 3*m + 2) + (a*sin(f*x + e) + a)^(m + 1)*A/(a*(m + 1)))/f
```

mupad [B] time = 10.17, size = 99, normalized size = 1.68

$$\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(4\,A-B+2\,A\,m+B\,m+4\,A\,\sin\left(e+f\,x\right)+B\,\left(2\,\sin\left(e+f\,x\right)^2-1\right)+2\,A\,m\,\sin\left(e+\right.\right.}{2\,f\,\left(m^2+3\,m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(4*A - B + 2*A*m + B*m + 4*A*sin(e + f*x) + B*(2*
sin(e + f*x)^2 - 1) + 2*A*m*sin(e + f*x) + 2*B*m*sin(e + f*x) + B*m*(2*sin(
e + f*x)^2 - 1)))/(2*f*(3*m + m^2 + 2))
```

sympy [A] time = 5.82, size = 428, normalized size = 7.25

$$\begin{cases} x(A + B\sin(e))(a\sin(e) + a)^m \cos(e) \\ -\frac{A}{a^2 f\sin(e+fx)+a^2 f} + \frac{B\log(\sin(e+fx)+1)\sin(e+fx)}{a^2 f\sin(e+fx)+a^2 f} + \frac{B\log(\sin(e+fx)+1)}{a^2 f\sin(e+fx)+a^2 f} + \frac{B}{a^2 f\sin(e+fx)+a^2 f} \\ \frac{A\log(\sin(e+fx)+1)}{af} - \frac{B\log(\sin(e+fx)+1)}{af} + \frac{B\sin(e+fx)}{af} \\ \frac{Am(a\sin(e+fx)+a)^m\sin(e+fx)}{fm^2+3fm+2f} + \frac{Am(a\sin(e+fx)+a)^m}{fm^2+3fm+2f} + \frac{2A(a\sin(e+fx)+a)^m\sin(e+fx)}{fm^2+3fm+2f} + \frac{2A(a\sin(e+fx)+a)^m}{fm^2+3fm+2f} + \frac{Bm(a\sin(e+fx)+a}{fm^2+3f} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((x*(A + B*sin(e))*(a*sin(e) + a)**m*cos(e), Eq(f, 0)), (-A/(a**2*
f*sin(e + f*x) + a**2*f) + B*log(sin(e + f*x) + 1)*sin(e + f*x)/(a**2*f*sin
(e + f*x) + a**2*f) + B*log(sin(e + f*x) + 1)/(a**2*f*sin(e + f*x) + a**2*f
) + B/(a**2*f*sin(e + f*x) + a**2*f), Eq(m, -2)), (A*log(sin(e + f*x) + 1)/
(a*f) - B*log(sin(e + f*x) + 1)/(a*f) + B*sin(e + f*x)/(a*f), Eq(m, -1)), (
A*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**2 + 3*f*m + 2*f) + A*m*(a*si
n(e + f*x) + a)**m/(f*m**2 + 3*f*m + 2*f) + 2*A*(a*sin(e + f*x) + a)**m*sin
(e + f*x)/(f*m**2 + 3*f*m + 2*f) + 2*A*(a*sin(e + f*x) + a)**m/(f*m**2 + 3*
f*m + 2*f) + B*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**2 + 3*f*m +
2*f) + B*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**2 + 3*f*m + 2*f) + B*
(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**2 + 3*f*m + 2*f) - B*(a*sin(e
 + f*x) + a)**m/(f*m**2 + 3*f*m + 2*f), True))
```

3.1024 $\int \sec(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=80

$$\frac{(A+B)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(e+fx)+1)\right)}{4af(m+1)}+\frac{(A-B)(a\sin(e+fx)+a)^m}{2fm}$$

```
[Out] 1/2*(A-B)*(a+a*sin(f*x+e))^m/f/m+1/4*(A+B)*hypergeom([1, 1+m],[2+m],1/2+1/2
*sin(f*x+e))*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)
```

Rubi [A] time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 79, 68}

$$\frac{(A+B)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(e+fx)+1)\right)}{4af(m+1)}+\frac{(A-B)(a\sin(e+fx)+a)^m}{2fm}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[e + f*x])^m)/(2*f*m) + ((A + B)*Hypergeometric2F1[1, 1
+ m, 2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^(1 + m))/(4*a*f*(1 +
 m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{a\,\text{Subst}\left(\int \frac{(a+x)^{-1+m}\left(A+\frac{Bx}{a}\right)}{a-x}\,dx, x, a\sin(e+fx)\right)}{f}$$

$$= \frac{(A-B)(a+a\sin(e+fx))^m}{2fm} + \frac{(A+B)\,\text{Subst}\left(\int \frac{(a\cdots}{a\cdots}\right.}{\cdots}$$

$$= \frac{(A-B)(a+a\sin(e+fx))^m}{2fm} + \frac{(A+B)\,{}_2F_1\left(1,1+\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.12, size = 71, normalized size = 0.89

$$\frac{(a(\sin(e+fx)+1))^m\left(m(A+B)(\sin(e+fx)+1)\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(e+fx)+1)\right)+2(m+1)(A-B)\right)}{4fm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^m*(2*(A - B)*(1 + m) + (A + B)*m*Hypergeometric2F1[
1, 1 + m, 2 + m, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x])))/(4*f*m*(1 + m))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sec\left(fx+e\right)\sin\left(fx+e\right)+A\sec\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] integral((B*sec(f*x + e)*sin(f*x + e) + A*sec(f*x + e))*(a*sin(f*x + e) + a
)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e), x)
```

maple [F] time = 2.12, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*sec(e + f*x), x)
```

3.1025 $\int \sec^3(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=100

$$\frac{a^2(A+B)(a\sin(e+fx)+a)^{m-1}}{2f(a-a\sin(e+fx))}-\frac{a(A(2-m)-Bm)(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{1}{2}(\sin(e+fx)+1)\right)}{4f(1-m)}$$

```
[Out] -1/4*a*(A*(2-m)-B*m)*hypergeom([1, -1+m],[m],1/2+1/2*sin(f*x+e))*(a+a*sin(f
*x+e))^(-1+m)/f/(1-m)+1/2*a^2*(A+B)*(a+a*sin(f*x+e))^(-1+m)/f/(a-a*sin(f*x+
e))
```

Rubi [A] time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 78, 68}

$$\frac{a^2(A+B)(a\sin(e+fx)+a)^{m-1}}{2f(a-a\sin(e+fx))}-\frac{a(A(2-m)-Bm)(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{1}{2}(\sin(e+fx)+1)\right)}{4f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(a*(A*(2 - m) - B*m)*Hypergeometric2F1[1, -1 + m, m, (1 + Sin[e + f*x])/2]
*(a + a*Sin[e + f*x])^(-1 + m))/(4*f*(1 - m)) + (a^2*(A + B)*(a + a*Sin[e +
 f*x])^(-1 + m))/(2*f*(a - a*Sin[e + f*x]))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{a^3\,\text{Subst}\left(\int \frac{(a+x)^{-2+m}\left(A+\frac{Bx}{a}\right)}{(a-x)^2}\,dx, x, a\sin(e+fx)\right)}{f}$$

$$= \frac{a^2(A+B)(a+a\sin(e+fx))^{-1+m}}{2f(a-a\sin(e+fx))} + \frac{\left(a^2(A(2-m)\right.}{}$$

$$= -\frac{a(A(2-m)-Bm)\,{}_2F_1\left(1,-1+m;m;\frac{1}{2}(1+\sin(e\right.}{4f(1-m)}$$

Mathematica [A] time = 0.17, size = 82, normalized size = 0.82

$$-\frac{a(a(\sin(e+fx)+1))^{m-1}\left((A(m-2)+Bm)(\sin(e+fx)-1)\,{}_2F_1\left(1,m-1;m;\frac{1}{2}(\sin(e+fx)+1)\right)+2(m-1)(A\right.}{4f(m-1)(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/4*(a*(2*(A + B)*(-1 + m) + (A*(-2 + m) + B*m)*Hypergeometric2F1[1, -1 +
m, m, (1 + Sin[e + f*x])/2]*(-1 + Sin[e + f*x]))*(a*(1 + Sin[e + f*x]))^(-1
 + m))/(f*(-1 + m)*(-1 + Sin[e + f*x]))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sec\left(fx+e\right)^3\sin\left(fx+e\right)+A\sec\left(fx+e\right)^3\right)\left(a\sin\left(fx+e\right)+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^3*sin(f*x + e) + A*sec(f*x + e)^3)*(a*sin(f*x + e)
 + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

maple [F] time = 0.75, size = 0, normalized size = 0.00

$$\int \left(\sec^{3}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\cos\left(e+f\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1026 $\int \sec^5(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=104

$$\frac{a^4(A+B)(a\sin(e+fx)+a)^{m-2}}{4f(a-a\sin(e+fx))^2}-\frac{a^2(A(4-m)-Bm)(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(2,m-2;m-1;\frac{1}{2}(\sin(e+fx)+\right.}{16f(2-m)}$$

```
[Out] -1/16*a^2*(A*(4-m)-B*m)*hypergeom([2, -2+m],[-1+m],1/2+1/2*sin(f*x+e))*(a+a
*sin(f*x+e))^(-2+m)/f/(2-m)+1/4*a^4*(A+B)*(a+a*sin(f*x+e))^(-2+m)/f/(a-a*si
n(f*x+e))^2
```

Rubi [A] time = 0.14, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 78, 68}

$$\frac{a^4(A+B)(a\sin(e+fx)+a)^{m-2}}{4f(a-a\sin(e+fx))^2}-\frac{a^2(A(4-m)-Bm)(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(2,m-2;m-1;\frac{1}{2}(\sin(e+fx)+\right.}{16f(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(a^2*(A*(4 - m) - B*m)*Hypergeometric2F1[2, -2 + m, -1 + m, (1 + Sin[e + f
*x])/2]*(a + a*Sin[e + f*x])^(-2 + m))/(16*f*(2 - m)) + (a^4*(A + B)*(a + a
*Sin[e + f*x])^(-2 + m))/(4*f*(a - a*Sin[e + f*x])^2)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{(a+x)^{-3+m}\left(A+\frac{Bx}{a}\right)}{(a-x)^3}\,dx, x, a\sin(e+fx)\right)}{f}$$

$$= \frac{a^4(A+B)(a+a\sin(e+fx))^{-2+m}}{4f(a-a\sin(e+fx))^2} + \frac{\left(a^4(A(4-m)\right.}{}$$

$$= -\frac{a^2(A(4-m)-Bm)\,{}_2F_1\left(2,-2+m;-1+m;\frac{1}{2}(1+\right.}{16f(2-m)}$$

Mathematica [A] time = 0.18, size = 76, normalized size = 0.73

$$\frac{a^2(a(\sin(e+fx)+1))^{m-2}\left(\frac{4(A+B)}{(\sin(e+fx)-1)^2}-\frac{(A(m-4)+Bm)\,{}_2F_1\left(2,m-2;m-1;\frac{1}{2}(\sin(e+fx)+1)\right)}{m-2}\right)}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^2*(-(((A*(-4 + m) + B*m)*Hypergeometric2F1[2, -2 + m, -1 + m, (1 + Sin[e
 + f*x])/2])/(-2 + m)) + (4*(A + B))/(-1 + Sin[e + f*x])^2)*(a*(1 + Sin[e +
 f*x]))^(-2 + m))/(16*f)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sec\left(fx+e\right)^5\sin\left(fx+e\right)+A\sec\left(fx+e\right)^5\right)\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^5*sin(f*x + e) + A*sec(f*x + e)^5)*(a*sin(f*x + e)
 + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^5, x)
```

maple [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \left(\sec^{5}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{\cos\left(e+f\,x\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^5,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^5, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1027 $\int \cos^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=129

$$-\frac{a^3 2^{m+\frac{7}{2}}(A(m+7)+Bm)\cos^7(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{7}{2},-m-\frac{5}{2};\frac{9}{2};\frac{1}{2}(1-\sin(e
}{7f(m+7)}$$

```
[Out] -1/7*2^(7/2+m)*a^3*(B*m+A*(7+m))*cos(f*x+e)^7*hypergeom([7/2, -5/2-m],[9/2]
,1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-3+m)/f/(7+m
)-B*cos(f*x+e)^7*(a+a*sin(f*x+e))^m/f/(7+m)
```

Rubi [A] time = 0.20, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2860, 2689, 70, 69}

$$-\frac{a^3 2^{m+\frac{7}{2}}(A(m+7)+Bm)\cos^7(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{7}{2},-m-\frac{5}{2};\frac{9}{2};\frac{1}{2}(1-\sin(e
}{7f(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(2^(7/2 + m)*a^3*(B*m + A*(7 + m))*Cos[e + f*x]^7*Hypergeometric2F1[7/2, -
5/2 - m, 9/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Si
n[e + f*x])^(-3 + m))/(7*f*(7 + m)) - (B*Cos[e + f*x]^7*(a + a*Sin[e + f*x]
)^m)/(f*(7 + m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos^7(e+fx)(a+a\sin(e+fx))^m}{f(7+m)} + \left(A+\frac{Bm}{7+\ldots}\right.$$

$$= -\frac{B\cos^7(e+fx)(a+a\sin(e+fx))^m}{f(7+m)} + \frac{\left(a^2\left(A+\ldots\right.\right.}{\ldots}$$

$$= -\frac{B\cos^7(e+fx)(a+a\sin(e+fx))^m}{f(7+m)} + \frac{\left(2^{\frac{5}{2}+m}a^4\left(\ldots\right.\right.}{\ldots}$$

$$= -\frac{2^{\frac{7}{2}+m}a^3\left(A+\frac{Bm}{7+m}\right)\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2},-\frac{5}{2}-m;\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.89, size = 111, normalized size = 0.86

$$-\frac{\cos^7(e+fx)(\sin(e+fx)+1)^{-m-\frac{7}{2}}(a(\sin(e+fx)+1))^m\left(2^{m+\frac{7}{2}}(A(m+7)+Bm)\,{}_2F_1\left(\frac{7}{2},-m-\frac{5}{2};\frac{9}{2};\frac{1}{2}(1-\sin(e\ldots\right.\right.}{7f(m+7)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/7*(Cos[e + f*x]^7*(1 + Sin[e + f*x])^(-7/2 - m)*(a*(1 + Sin[e + f*x]))^m
*(2^(7/2 + m)*(B*m + A*(7 + m))*Hypergeometric2F1[7/2, -5/2 - m, 9/2, (1 -
Sin[e + f*x])/2] + 7*B*(1 + Sin[e + f*x])^(7/2 + m)))/(f*(7 + m))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(fx+e\right)^{6}\sin\left(fx+e\right)+A\cos\left(fx+e\right)^{6}\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(f*x + e)^6*sin(f*x + e) + A*cos(f*x + e)^6)*(a*sin(f*x + e)
 + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^6, x)
```

maple [F] time = 22.62, size = 0, normalized size = 0.00

$$\int\left(\cos^{6}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos \left(e + f\, x\right)^6\, \left(A + B\, \sin \left(e + f\, x\right)\right)\, \left(a + a\, \sin \left(e + f\, x\right)\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
[Out] int(cos(e + f*x)^6*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
[Out] Timed out
```

3.1028 $\int \cos^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=129

$$-\frac{a^2 2^{m+\frac{5}{2}}(A(m+5)+Bm)\cos^5(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-\sin(e\right.}{5f(m+5)}$$

```
[Out] -1/5*2^(5/2+m)*a^2*(B*m+A*(5+m))*cos(f*x+e)^5*hypergeom([5/2, -3/2-m],[7/2]
,1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-2+m)/f/(5+m
)-B*cos(f*x+e)^5*(a+a*sin(f*x+e))^m/f/(5+m)
```

Rubi [A] time = 0.20, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2860, 2689, 70, 69}

$$-\frac{a^2 2^{m+\frac{5}{2}}(A(m+5)+Bm)\cos^5(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-\sin(e\right.}{5f(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(2^(5/2 + m)*a^2*(B*m + A*(5 + m))*Cos[e + f*x]^5*Hypergeometric2F1[5/2, -
3/2 - m, 7/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Si
n[e + f*x])^(-2 + m))/(5*f*(5 + m)) - (B*Cos[e + f*x]^5*(a + a*Sin[e + f*x]
)^m)/(f*(5 + m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos^5(e+fx)(a+a\sin(e+fx))^m}{f(5+m)} + \left(A+\frac{Bm}{5+m}\right.$$

$$= -\frac{B\cos^5(e+fx)(a+a\sin(e+fx))^m}{f(5+m)} + \frac{\left(a^2\left(A+\right.\right.}{}$$

$$= -\frac{B\cos^5(e+fx)(a+a\sin(e+fx))^m}{f(5+m)} + \frac{\left(2^{\frac{3}{2}+m}a^3\left(\right.\right.}{}$$

$$= -\frac{2^{\frac{5}{2}+m}a^2\left(A+\frac{Bm}{5+m}\right)\cos^5(e+fx)\,{}_2F_1\left(\frac{5}{2},-\frac{3}{2}-m;\right.}{}$$

Mathematica [A] time = 0.49, size = 111, normalized size = 0.86

$$-\frac{\cos^5(e+fx)(\sin(e+fx)+1)^{-m-\frac{5}{2}}(a(\sin(e+fx)+1))^m\left(2^{m+\frac{5}{2}}(A(m+5)+Bm)\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-\sin(e\right.\right.}{5f(m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

[Out] -1/5*(Cos[e + f*x]^5*(1 + Sin[e + f*x])^(-5/2 - m)*(a*(1 + Sin[e + f*x]))^m *(2^(5/2 + m)*(B*m + A*(5 + m))*Hypergeometric2F1[5/2, -3/2 - m, 7/2, (1 - Sin[e + f*x])/2] + 5*B*(1 + Sin[e + f*x])^(5/2 + m)))/(f*(5 + m))

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(fx+e\right)^{4}\sin\left(fx+e\right)+A\cos\left(fx+e\right)^{4}\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")

[Out] integral((B*cos(f*x + e)^4*sin(f*x + e) + A*cos(f*x + e)^4)*(a*sin(f*x + e) + a)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^4, x)

maple [F] time = 9.51, size = 0, normalized size = 0.00

$$\int\left(\cos^{4}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)

[Out] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^4\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
[Out] int(cos(e + f*x)^4*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
[Out] Timed out
```

3.1029 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=127

$$-\frac{a2^{m+\frac{3}{2}}(A(m+3)+Bm)\cos^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(e\right.}{3f(m+3)}$$

```
[Out] -1/3*2^(3/2+m)*a*(B*m+A*(3+m))*cos(f*x+e)^3*hypergeom([3/2, -1/2-m],[5/2],1
/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-1+m)/f/(3+m)-
B*cos(f*x+e)^3*(a+a*sin(f*x+e))^m/f/(3+m)
```

Rubi [A] time = 0.19, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2860, 2689, 70, 69}

$$-\frac{a2^{m+\frac{3}{2}}(A(m+3)+Bm)\cos^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(e\right.}{3f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(2^(3/2 + m)*a*(B*m + A*(3 + m))*Cos[e + f*x]^3*Hypergeometric2F1[3/2, -1/
2 - m, 5/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[
e + f*x])^(-1 + m))/(3*f*(3 + m)) - (B*Cos[e + f*x]^3*(a + a*Sin[e + f*x])^
m)/(f*(3 + m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos^3(e+fx)(a+a\sin(e+fx))^m}{f(3+m)} + \left(A+\frac{Bm}{3+m}\right)\ldots$$

$$= -\frac{B\cos^3(e+fx)(a+a\sin(e+fx))^m}{f(3+m)} + \frac{\left(a^2\left(A+\ldots\right.\right.}{\ldots}$$

$$= -\frac{B\cos^3(e+fx)(a+a\sin(e+fx))^m}{f(3+m)} + \frac{\left(2^{\frac{1}{2}+m}a^2\left(\ldots\right.\right.}{\ldots}$$

$$= -\frac{2^{\frac{3}{2}+m}a\left(A+\frac{Bm}{3+m}\right)\cos^3(e+fx)\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2}\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.33, size = 111, normalized size = 0.87

$$-\frac{\cos^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{3}{2}}(a(\sin(e+fx)+1))^m\left(2^{m+\frac{3}{2}}(A(m+3)+Bm)\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(e\ldots\right.\right.}{3f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/3*(Cos[e + f*x]^3*(1 + Sin[e + f*x])^(-3/2 - m)*(a*(1 + Sin[e + f*x]))^m
*(2^(3/2 + m)*(B*m + A*(3 + m))*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 -
Sin[e + f*x])/2] + 3*B*(1 + Sin[e + f*x])^(3/2 + m)))/(f*(3 + m))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)+A\cos\left(fx+e\right)^{2}\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(f*x + e)^2*sin(f*x + e) + A*cos(f*x + e)^2)*(a*sin(f*x + e)
 + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

maple [F] time = 4.23, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^2\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
[Out] int(cos(e + f*x)^2*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)\cos^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*cos(e + f*x)**2, x)
```

3.1030 $\int \sec^2(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=123

$$\frac{2^{m-\frac{1}{2}}(A(1-m)-Bm)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(1-m)}$$

```
[Out] B*sec(f*x+e)*(a+a*sin(f*x+e))^m/f/(1-m)+2^(-1/2+m)*(A*(1-m)-B*m)*hypergeom(
[-1/2, 3/2-m],[1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(
a+a*sin(f*x+e))^m/f/(1-m)
```

Rubi [A] time = 0.19, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2860, 2689, 70, 69}

$$\frac{2^{m-\frac{1}{2}}(A(1-m)-Bm)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (B*Sec[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 - m)) + (2^(-1/2 + m)*(A*(1 -
 m) - B*m)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Sec[
e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 - m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \sec^2(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{B\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)} + \left(A-\frac{Bm}{1-m}\right)\ldots$$

$$= \frac{B\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)} + \frac{\left(a^2\left(A-\frac{Bm}{1-m}\right)\ldots\right.}{\ldots}$$

$$= \frac{B\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)} + \frac{\left(2^{-\frac{3}{2}+m}a\left(A-\ldots\right.\right.}{\ldots}$$

$$= \frac{B\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)} + \frac{2^{-\frac{1}{2}+m}\left(A-\ldots\right.}{\ldots}$$

Mathematica [C] time = 6.55, size = 3925, normalized size = 31.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

[Out] -1/4*((A + B)*(Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Cot[(-e + Pi/2 - f*x)/4]*(
a + a*Sin[e + f*x])^m*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)) +
 (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi
/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)
^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m,
 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + P
i/2 - f*x)/4]^2)))/(f*(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/
2 + f*x)/2])^2*(-1/2*(m*(Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(-(AppellF1[-1/2
, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*
(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[
(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/
4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/
2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1
[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)
/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2))) - ((Cos[(-e + Pi/2
- f*x)/4]^2)^(2*m)*Csc[(-e + Pi/2 - f*x)/4]^2*(-(AppellF1[-1/2, -2*m, 2*m,
1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi
/2 - f*x)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan
[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e +
Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m
, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + Appe
llF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
 f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2)))/8 + ((Cos[(-e + Pi/2 - f*x)/4]^2
)^(2*m)*Cot[(-e + Pi/2 - f*x)/4]*(-(m*AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-
e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4
]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]) - (Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(
m*AppellF1[1/2, 1 - 2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + P
i/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4] + m*Ap
pellF1[1/2, -2*m, 1 + 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) + (3*App
ellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*
x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e +
 Pi/2 - f*x)/4]^2)^(2*m))/(2*(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/
2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2
*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF
1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*
x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2)) + (3*Tan[(-e + Pi/2 - f*x)/4]^2*(-1/
3*(m*AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
+ Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) -
(m*AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +

```
 Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/3)*
(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Ta
n[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2,
 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2
] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
+ Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2) - (3*m*AppellF1[1/2, -2*m,
 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e
 + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^3*(1 - Tan[(-e + Pi/2 - f*x)/4
]^2)^(-1 + 2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^
2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan
[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m,
 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Ta
n[(-e + Pi/2 - f*x)/4]^2) - (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 -
Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(-2*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, T
an[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*
m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*
Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4] + 3*(-1/3*(m*AppellF1[3
/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4
]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) - (m*AppellF1[3/2
, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]
^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/3) - 4*m*Tan[(-e +
 Pi/2 - f*x)/4]^2*((-6*m*AppellF1[5/2, 1 - 2*m, 1 + 2*m, 7/2, Tan[(-e + Pi/
2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[
(-e + Pi/2 - f*x)/4])/5 + (3*(1 - 2*m)*AppellF1[5/2, 2 - 2*m, 2*m, 7/2, Tan
[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)
/4]^2*Tan[(-e + Pi/2 - f*x)/4])/10 - (3*(1 + 2*m)*AppellF1[5/2, -2*m, 2 + 2
*m, 7/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e +
 Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/10)))/(3*AppellF1[1/2, -2*m, 2*
m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(App
ellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
 f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2,
 -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2)^2))/2)) + ((A -
B)*Hypergeometric2F1[1/2, (-1 + 2*m)/2, (1 + 2*m)/2, Cos[(-e + Pi/2 - f*x)/
2]^2]*(a + a*Sin[e + f*x])^m*Tan[(-e + Pi/2 - f*x)/2])/(2*f*(-1 + 2*m)*Sqrt
[Sin[(-e + Pi/2 - f*x)/2]^2])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sec\left(fx+e\right)^{2}\sin\left(fx+e\right)+A\sec\left(fx+e\right)^{2}\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^2*sin(f*x + e) + A*sec(f*x + e)^2)*(a*sin(f*x + e)
 + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\sec(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

maple [F] time = 0.76, size = 0, normalized size = 0.00

$$\int \left(\sec^2(fx+e)\right)(a+a\sin(fx+e))^m(A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\sec(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{\cos(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
[Out] Timed out
```

3.1031 $\int \sec^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=135

$$\frac{2^{m-\frac{3}{2}}(A(3-m)-Bm)\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3af(3-m)}$$

```
[Out] B*sec(f*x+e)^3*(a+a*sin(f*x+e))^m/f/(3-m)+1/3*2^(-3/2+m)*(A*(3-m)-B*m)*hype
rgeom([-3/2, 5/2-m],[-1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)^3*(1+sin(f*x+e))^
(1/2-m)*(a+a*sin(f*x+e))^(1+m)/a/f/(3-m)
```

Rubi [A] time = 0.20, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2860, 2689, 70, 69}

$$\frac{2^{m-\frac{3}{2}}(A(3-m)-Bm)\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3af(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (B*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^m)/(f*(3 - m)) + (2^(-3/2 + m)*(A*(3
 - m) - B*m)*Hypergeometric2F1[-3/2, 5/2 - m, -1/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(1 + m))/(3
*a*f*(3 - m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \sec^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^m}{f(3-m)} + \left(A-\frac{Bm}{3-m}\right)$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^m}{f(3-m)} + \frac{\left(a^2\left(A-\frac{Bm}{3-m}\right.\right.}{}$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^m}{f(3-m)} + \frac{\left(2^{-\frac{5}{2}+m}\left(A-\right.\right.}{}$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^m}{f(3-m)} + \frac{2^{-\frac{3}{2}+m}\left(A-\right.}{}$$

Mathematica [F] time = 1.77, size = 0, normalized size = 0.00

$$\int \sec^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] Integrate[Sec[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]), x]
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sec\left(fx+e\right)^4\sin\left(fx+e\right)+A\sec\left(fx+e\right)^4\right)\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^4*sin(f*x + e) + A*sec(f*x + e)^4)*(a*sin(f*x + e)
 + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

maple [F] time = 0.82, size = 0, normalized size = 0.00

$$\int\left(\sec^4\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^4,x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
[Out] Timed out
```

3.1032 $\int \sec^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=135

$$\frac{2^{m-\frac{5}{2}}(A(5-m)-Bm)\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{7}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+f$$
$$5a^2f(5-m)$$

```
[Out] B*sec(f*x+e)^5*(a+a*sin(f*x+e))^m/f/(5-m)+1/5*2^(-5/2+m)*(A*(5-m)-B*m)*hype
rgeom([-5/2, 7/2-m],[-3/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)^5*(1+sin(f*x+e))^
(1/2-m)*(a+a*sin(f*x+e))^(2+m)/a^2/f/(5-m)
```

Rubi [A] time = 0.19, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2860, 2689, 70, 69}

$$\frac{2^{m-\frac{5}{2}}(A(5-m)-Bm)\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{7}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+f$$
$$5a^2f(5-m)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (B*Sec[e + f*x]^5*(a + a*Sin[e + f*x])^m)/(f*(5 - m)) + (2^(-5/2 + m)*(A*(5
 - m) - B*m)*Hypergeometric2F1[-5/2, 7/2 - m, -3/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(2 + m))/(5
*a^2*f*(5 - m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \sec^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^m}{f(5-m)} + \left(A-\frac{Bm}{5-m}\right.$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^m}{f(5-m)} + \frac{\left(a^2\left(A-\frac{Bm}{5-m}\right.\right.}{}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^m}{f(5-m)} + \frac{\left(2^{-\frac{7}{2}+m}\left(A-\right.\right.}{}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^m}{f(5-m)} + \frac{2^{-\frac{5}{2}+m}\left(A-\right.}{}$$

Mathematica [F] time = 3.91, size = 0, normalized size = 0.00

$$\int \sec^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] Integrate[Sec[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]), x]
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sec\left(fx+e\right)^{6}\sin\left(fx+e\right)+A\sec\left(fx+e\right)^{6}\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^6*sin(f*x + e) + A*sec(f*x + e)^6)*(a*sin(f*x + e)
 + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^6, x)
```

maple [F] time = 0.94, size = 0, normalized size = 0.00

$$\int\left(\sec^{6}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\cos\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^6,x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
[Out] Timed out
```

3.1033 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-p}\,dx$

Optimal. Leaf size=239

$$\frac{2(3A-B(p+4))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{c^3fg(p+1)(p+3)(p+5)(p+7)}+\frac{2(3A-B(p+4))(c-c\sin(e+fx))^{-p-2}(g\cos(e+fx)}{c^2fg(p+3)(p+5)(p+7)}$$

```
[Out] (A+B)*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-4-p)/f/g/(7+p)+(3*A-B*(4+p))*
(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-3-p)/c/f/g/(p^2+12*p+35)+2*(3*A-B*(
4+p))*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-2-p)/c^2/f/g/(5+p)/(p^2+10*p+
21)+2*(3*A-B*(4+p))*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-1-p)/c^3/f/g/(p
^2+6*p+5)/(p^2+10*p+21)
```

Rubi [A] time = 0.44, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2859, 2672, 2671}

$$\frac{2(3A-B(p+4))(c-c\sin(e+fx))^{-p-2}(g\cos(e+fx))^{p+1}}{c^2fg(p+3)(p+5)(p+7)}+\frac{2(3A-B(p+4))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx)}{c^3fg(p+1)(p+3)(p+5)(p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-4 - p),x
]
```

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-4 - p))/(f*g*(7 +
p)) + ((3*A - B*(4 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-3
- p))/(c*f*g*(5 + p)*(7 + p)) + (2*(3*A - B*(4 + p))*(g*Cos[e + f*x])^(1 +
p)*(c - c*Sin[e + f*x])^(-2 - p))/(c^2*f*g*(3 + p)*(5 + p)*(7 + p)) + (2*(3
*A - B*(4 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-1 - p))/(c^
3*f*g*(1 + p)*(3 + p)*(5 + p)*(7 + p))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
```

```
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-p}\,dx = \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-}}{fg(7+p)}$$

$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-}}{fg(7+p)}$$

$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-}}{fg(7+p)}$$

$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-}}{fg(7+p)}$$

Mathematica [A] time = 0.57, size = 160, normalized size = 0.67

$$\frac{\cos(e+fx)(c-c\sin(e+fx))^{-p}(g\cos(e+fx))^p\left(\left(p^2+8p+13\right)(B(p+4)-3A)\sin(e+fx)+(2B(p+4)-6A\right.}{c^4f(p+1)(p+3)(p+5)(p+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-4
- p),x]
```

```
[Out] (Cos[e + f*x]*(g*Cos[e + f*x])^p*(-(B*(13 + 8*p + p^2)) + A*(36 + 41*p + 12
*p^2 + p^3) + (13 + 8*p + p^2)*(-3*A + B*(4 + p))*Sin[e + f*x] - 2*(4 + p)*
(-3*A + B*(4 + p))*Sin[e + f*x]^2 + (-6*A + 2*B*(4 + p))*Sin[e + f*x]^3))/(
c^4*f*(1 + p)*(3 + p)*(5 + p)*(7 + p)*(-1 + Sin[e + f*x])^4*(c - c*Sin[e +
f*x])^p)
```

fricas [A] time = 0.80, size = 197, normalized size = 0.82

$$\frac{\left(2\left(Bp^2-(3A-8B)p-12A+16B\right)\cos\left(fx+e\right)^3+\left(Ap^3+3(4A-B)p^2+(47A-24B)p+60A-45B\right)\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x, algo
rithm="fricas")
```

```
[Out] (2*(B*p^2 - (3*A - 8*B)*p - 12*A + 16*B)*cos(f*x + e)^3 + (A*p^3 + 3*(4*A -
 B)*p^2 + (47*A - 24*B)*p + 60*A - 45*B)*cos(f*x + e) - (2*(B*p - 3*A + 4*B
)*cos(f*x + e)^3 - (B*p^3 - 3*(A - 4*B)*p^2 - (24*A - 47*B)*p - 45*A + 60*B
)*cos(f*x + e))*sin(f*x + e))*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
- 4)/(f*p^4 + 16*f*p^3 + 86*f*p^2 + 176*f*p + 105*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p-4}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x, algo
rithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 4), x)
```

maple [F] time = 9.04, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{p}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-4-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p-4}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x, algo
rithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 4), x)
```

mupad [B] time = 18.42, size = 441, normalized size = 1.85

$$\frac{\cos\left(e+f\,x\right)\,\left(g\,\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)\right)^p\,\left(A\,168\mathrm{i}-B\,84\mathrm{i}+A\,p\,170\mathrm{i}-B\,p\,48\mathrm{i}+A\,p^2\,48\mathrm{i}+A\,p^3\,4\mathrm{i}-B\,p^2\,6\mathrm{i}\right)}{4\,f\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{p+4}\,\left(p^4\,1\mathrm{i}+p^3\,16\mathrm{i}+p^2\,86\mathrm{i}+p\,176\mathrm{i}+105\mathrm{i}\right)}-\frac{}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(A + B*sin(e + f*x)))/(c - c*sin(e + f*x))^(p + 4),
x)
```

```
[Out] (cos(e + f*x)*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^p*(A*168i
 - B*84i + A*p*170i - B*p*48i + A*p^2*48i + A*p^3*4i - B*p^2*6i))/(4*f*(c -
 c*sin(e + f*x))^(p + 4)*(p*176i + p^2*86i + p^3*16i + p^4*1i + 105i)) - (s
in(4*e + 4*f*x)*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^p*(4*B
- 3*A + B*p)*1i)/(4*f*(c - c*sin(e + f*x))^(p + 4)*(p*176i + p^2*86i + p^3*
16i + p^4*1i + 105i)) + (cos(3*e + 3*f*x)*(g*(exp(- e*1i - f*x*1i)/2 + exp(
e*1i + f*x*1i)/2))^p*(p + 4)*(B*4i - A*3i + B*p*1i))/(2*f*(c - c*sin(e + f*
x))^(p + 4)*(p*176i + p^2*86i + p^3*16i + p^4*1i + 105i)) + (sin(2*e + 2*f*
x)*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^p*(4*B - 3*A + B*p)*
(8*p + p^2 + 14)*1i)/(2*f*(c - c*sin(e + f*x))^(p + 4)*(p*176i + p^2*86i +
p^3*16i + p^4*1i + 105i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-4-p),x)
```

```
[Out] Timed out
```

3.1034 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-p}\,dx$

Optimal. Leaf size=168

$$\frac{(2A-B(p+3))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{c^2fg(p+1)(p+3)(p+5)}+\frac{(A+B)(c-c\sin(e+fx))^{-p-3}(g\cos(e+fx))^{p+1}}{fg(p+5)}+\frac{(2A}{}$$

```
[Out] (A+B)*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-3-p)/f/g/(5+p)+(2*A-B*(3+p))*
(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-2-p)/c/f/g/(p^2+8*p+15)+(2*A-B*(3+p
))*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-1-p)/c^2/f/g/(3+p)/(p^2+6*p+5)
```

Rubi [A] time = 0.31, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2859, 2672, 2671}

$$\frac{(2A-B(p+3))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{c^2fg(p+1)(p+3)(p+5)}+\frac{(A+B)(c-c\sin(e+fx))^{-p-3}(g\cos(e+fx))^{p+1}}{fg(p+5)}+\frac{(2A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-3 - p),x
]
```

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-3 - p))/(f*g*(5 +
p)) + ((2*A - B*(3 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-2
- p))/(c*f*g*(3 + p)*(5 + p)) + ((2*A - B*(3 + p))*(g*Cos[e + f*x])^(1 + p)
*(c - c*Sin[e + f*x])^(-1 - p))/(c^2*f*g*(1 + p)*(3 + p)*(5 + p))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-p}\,dx = \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-}}{fg(5+p)}$$
$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-}}{fg(5+p)}$$
$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-}}{fg(5+p)}$$

Mathematica [A] time = 0.30, size = 119, normalized size = 0.71

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{-p}(g\cos(e+fx))^p\left((2A-B(p+3))\sin^2(e+fx)+(p+3)(B(p+3)-2A)\sin(e\right.}{c^3f(p+1)(p+3)(p+5)(\sin(e+fx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-3
- p),x]
```

```
[Out] -((Cos[e + f*x]*(g*Cos[e + f*x])^p*(-(B*(3 + p)) + A*(7 + 6*p + p^2) + (3 +
 p)*(-2*A + B*(3 + p))*Sin[e + f*x] + (2*A - B*(3 + p))*Sin[e + f*x]^2))/(c
^3*f*(1 + p)*(3 + p)*(5 + p)*(-1 + Sin[e + f*x])^3*(c - c*Sin[e + f*x])^p))
```

fricas [A] time = 0.75, size = 131, normalized size = 0.78

$$\frac{\left(\left(Bp-2A+3B\right)\cos\left(fx+e\right)^3+\left(Bp^2-2(A-3B)p-6A+9B\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(Ap^2+2(3A-\right.\right.}{fp^3+9fp^2+23fp+15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x, algo
rithm="fricas")
```

```
[Out] ((B*p - 2*A + 3*B)*cos(f*x + e)^3 + (B*p^2 - 2*(A - 3*B)*p - 6*A + 9*B)*cos
(f*x + e)*sin(f*x + e) + (A*p^2 + 2*(3*A - B)*p + 9*A - 6*B)*cos(f*x + e))*
(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p - 3)/(f*p^3 + 9*f*p^2 + 23*f*p
 + 15*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x, algo
rithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 3), x)
```

maple [F] time = 8.20, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-3-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x, algo
rithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 3), x)
```

mupad [B] time = 11.94, size = 234, normalized size = 1.39

$$-\frac{\left(g\,\cos\left(e+f\,x\right)\right)^p\,\left(30\,A\,\cos\left(e+f\,x\right)-15\,B\,\cos\left(e+f\,x\right)-2\,A\,\cos\left(3\,e+3\,f\,x\right)+3\,B\,\cos\left(3\,e+3\,f\,x\right)-12\,A}{c^3\,f\,\left(-c\,\left(s\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(A + B*sin(e + f*x)))/(c - c*sin(e + f*x))^(p + 3),
x)
```

```
[Out] -((g*cos(e + f*x))^p*(30*A*cos(e + f*x) - 15*B*cos(e + f*x) - 2*A*cos(3*e +
 3*f*x) + 3*B*cos(3*e + 3*f*x) - 12*A*sin(2*e + 2*f*x) + 18*B*sin(2*e + 2*f
*x) + 2*B*p^2*sin(2*e + 2*f*x) + 24*A*p*cos(e + f*x) - 5*B*p*cos(e + f*x) +
 4*A*p^2*cos(e + f*x) + B*p*cos(3*e + 3*f*x) - 4*A*p*sin(2*e + 2*f*x) + 12*
B*p*sin(2*e + 2*f*x)))/(c^3*f*(-c*(sin(e + f*x) - 1))^p*(23*p + 9*p^2 + p^3
 + 15)*(15*sin(e + f*x) + 6*cos(2*e + 2*f*x) - sin(3*e + 3*f*x) - 10))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-3-p),x)
```

```
[Out] Timed out
```

3.1035 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-p}\,dx$

Optimal. Leaf size=102

$$\frac{(A+B)(c-c\sin(e+fx))^{-p-2}(g\cos(e+fx))^{p+1}}{fg(p+3)}+\frac{(A-B(p+2))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{cfg(p+1)(p+3)}$$

```
[Out] (A+B)*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-2-p)/f/g/(3+p)+(A-B*(2+p))*(g
*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-1-p)/c/f/g/(p^2+4*p+3)
```

Rubi [A] time = 0.21, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2859, 2671}

$$\frac{(A+B)(c-c\sin(e+fx))^{-p-2}(g\cos(e+fx))^{p+1}}{fg(p+3)}+\frac{(A-B(p+2))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{cfg(p+1)(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-2 - p),x
]
```

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-2 - p))/(f*g*(3 +
p)) + ((A - B*(2 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-1 -
p))/(c*f*g*(1 + p)*(3 + p))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-p}\,dx = \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-}}{fg(3+p)}$$
$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-}}{fg(3+p)}$$

Mathematica [A] time = 0.18, size = 83, normalized size = 0.81

$$\frac{\cos(e+fx)(c-c\sin(e+fx))^{-p}(g\cos(e+fx))^p((B(p+2)-A)\sin(e+fx)+A(p+2)-B)}{c^2f(p+1)(p+3)(\sin(e+fx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-2
- p),x]
```

```
[Out] (Cos[e + f*x]*(g*Cos[e + f*x])^p*(-B + A*(2 + p) + (-A + B*(2 + p))*Sin[e +
 f*x]))/(c^2*f*(1 + p)*(3 + p)*(-1 + Sin[e + f*x])^2*(c - c*Sin[e + f*x])^p
)
```

fricas [A] time = 0.53, size = 84, normalized size = 0.82

$$\frac{\left(\left(Bp-A+2\,B\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(Ap+2\,A-B\right)\cos\left(fx+e\right)\right)\left(g\cos\left(fx+e\right)\right)^p\left(-c\sin\left(fx+e\right)+c\right.}{fp^2+4\,fp+3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x, algo
rithm="fricas")
```

```
[Out] ((B*p - A + 2*B)*cos(f*x + e)*sin(f*x + e) + (A*p + 2*A - B)*cos(f*x + e))*
(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p - 2)/(f*p^2 + 4*f*p + 3*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^p\left(-c\sin\left(fx+e\right)+c\right)^{-p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x, algo
rithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 2), x)
```

maple [F] time = 7.81, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-2-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x, algo
rithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 2), x)
```

mupad [B] time = 1.28, size = 129, normalized size = 1.26

$$-\frac{\left(g\ \cos\left(e+f\,x\right)\right)^p\ \left(4\,A\ \cos\left(e+f\,x\right)-2\,B\ \cos\left(e+f\,x\right)-A\ \sin\left(2\,e+2\,f\,x\right)+2\,B\ \sin\left(2\,e+2\,f\,x\right)+2\,A\,p\ \mathrm{co}\right.}{c^2\,f\,\left(-c\ \left(\sin\left(e+f\,x\right)-1\right)\right)^p\ \left(4\ \sin\left(e+f\,x\right)+\cos\left(2\,e+2\,f\,x\right)-3\right)\ \left(p^2+4\,p+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(A + B*sin(e + f*x)))/(c - c*sin(e + f*x))^(p + 2),
x)
```

```
[Out] -((g*cos(e + f*x))^p*(4*A*cos(e + f*x) - 2*B*cos(e + f*x) - A*sin(2*e + 2*f
*x) + 2*B*sin(2*e + 2*f*x) + 2*A*p*cos(e + f*x) + B*p*sin(2*e + 2*f*x)))/(c
^2*f*(-c*(sin(e + f*x) - 1))^p*(4*sin(e + f*x) + cos(2*e + 2*f*x) - 3)*(4*p
 + p^2 + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-2-p),x)
```

```
[Out] Timed out
```

3.1036 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-p}\,dx$

Optimal. Leaf size=151

$$\frac{(A+B)(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{fg(p+1)}-\frac{B2^{\frac{1}{2}-\frac{p}{2}}(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx}{fg(p+1)}$$

```
[Out] (A+B)*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-1-p)/f/g/(1+p)-2^(1/2-1/2*p)*
B*(g*cos(f*x+e))^(1+p)*hypergeom([1/2+1/2*p, 1/2+1/2*p],[3/2+1/2*p],1/2+1/2
*sin(f*x+e))*(1-sin(f*x+e))^(1/2+1/2*p)*(c-c*sin(f*x+e))^(-1-p)/f/g/(1+p)
```

Rubi [A] time = 0.26, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2859, 2689, 70, 69}

$$\frac{(A+B)(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{fg(p+1)}-\frac{B2^{\frac{1}{2}-\frac{p}{2}}(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx}{fg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 - p),x
]
```

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 +
p)) - (2^(1/2 - p/2)*B*(g*Cos[e + f*x])^(1 + p)*Hypergeometric2F1[(1 + p)/2
, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^((1 + p)/2
)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 + p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-p}\,dx=\frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-1-}}{fg(1+p)}$$

$$=\frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-1-}}{fg(1+p)}$$

$$=\frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-1-}}{fg(1+p)}$$

$$=\frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-1-}}{fg(1+p)}$$

Mathematica [C] time = 3.75, size = 300, normalized size = 1.99

$$-\frac{2^{-p}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^p\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-p-1)-2p}\left(\frac{1-\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{1}{\cos(e+fx)+1}}}\right)^{2p}\left(\frac{1-\tan}{\sqrt{\sec}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1
- p),x]
```

```
[Out] -(((g*Cos[e + f*x])^p*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(-2*(-1 - p) -
2*p)*(c - c*Sin[e + f*x])^(-1 - p)*((1 - Tan[(e + f*x)/2])/Sqrt[(1 + Cos[e
+ f*x])^(-1)])^(2*p)*((-I)*B*(1 + p)*Hypergeometric2F1[1, -p, 1 - p, ((-I)*
(-1 + Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])]*(-1 + Tan[(e + f*x)/2]) +
I*B*(1 + p)*Hypergeometric2F1[1, -p, 1 - p, (I*(-1 + Tan[(e + f*x)/2]))/(1
+ Tan[(e + f*x)/2])]*(-1 + Tan[(e + f*x)/2]) + (A + B)*p*(1 + Tan[(e + f*x)
/2])))/(2^p*f*p*(1 + p)*((1 - Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(
2*p)*(-1 + Tan[(e + f*x)/2])))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x, algo
rithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
- 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x, algo
rithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 1), x)
```

maple [F] time = 4.14, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{p}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-1-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sin(fx+e)+A)(g\cos(fx+e))^p(-c\sin(fx+e)+c)^{-p-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x, algo
rithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\cos(e+fx))^p\,(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{p+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(A + B*sin(e + f*x)))/(c - c*sin(e + f*x))^(p + 1),
x)
```

```
[Out] int(((g*cos(e + f*x))^p*(A + B*sin(e + f*x)))/(c - c*sin(e + f*x))^(p + 1),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-1-p),x)
```

```
[Out] Timed out
```

3.1037 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-p}\,dx$

Optimal. Leaf size=147

$$\frac{c2^{\frac{1}{2}-\frac{p}{2}}(A+Bp)(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+fx)+1\right.}{fg(p+1)}$$

```
[Out] 2^(1/2-1/2*p)*c*(B*p+A)*(g*cos(f*x+e))^(1+p)*hypergeom([1/2+1/2*p, 1/2+1/2*
p],[3/2+1/2*p],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+1/2*p)*(c-c*sin(f*x+
e))^(-1-p)/f/g/(1+p)-B*(g*cos(f*x+e))^(1+p)/f/g/((c-c*sin(f*x+e))^p)
```

Rubi [A] time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2860, 2689, 70, 69}

$$\frac{c2^{\frac{1}{2}-\frac{p}{2}}(A+Bp)(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+fx)+1\right.}{fg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^p,x]
```

```
[Out] (2^(1/2 - p/2)*c*(A + B*p)*(g*Cos[e + f*x])^(1 + p)*Hypergeometric2F1[(1 +
p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^((1 +
p)/2)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 + p)) - (B*(g*Cos[e + f*x])^(1
 + p))/(f*g*(c - c*Sin[e + f*x])^p)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-p}\,dx = -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-p}}{fg} + (A+$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-p}}{fg} + \frac{\Big(c^2(}{}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-p}}{fg} + \frac{\Big(2^{-}}{}$$

$$= \frac{2^{\frac{1}{2}-\frac{p}{2}}c(A+Bp)(g\cos(e+fx))^{1+p}\,{}_2F_1\Big(\frac{1+p}{2},\frac{1+p}{2};\frac{3}{}}{}$$

Mathematica [A] time = 0.56, size = 144, normalized size = 0.98

$$-\frac{2^{\frac{1}{2}(-p-1)}\cos(e+fx)(c-c\sin(e+fx))^{-p}(g\cos(e+fx))^p\Big(2(A+Bp)(1-\sin(e+fx))^{\frac{p+1}{2}}\,{}_2F_1\Big(\frac{p+1}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin}{f(p+1)(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^p*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^p,
x]
```

```
[Out] -((2^((-1 - p)/2)*Cos[e + f*x]*(g*Cos[e + f*x])^p*(2*(A + B*p)*Hypergeometr
ic2F1[(1 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f
*x])^((1 + p)/2) + 2^((1 + p)/2)*B*(1 + p)*(-1 + Sin[e + f*x])))/(f*(1 + p)
*(-1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^p))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}}{\left(-c\sin\left(fx+e\right)+c\right)^{p}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x, algorit
hm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p/(-c*sin(f*x + e) + c)^p, x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}}{\left(-c\sin\left(fx+e\right)+c\right)^{p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p/(-c*sin(f*x + e) + c)^p,
x)
```

maple [F] time = 4.82, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{p}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}}{\left(-c\sin\left(fx+e\right)+c\right)^{p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p/(-c*sin(f*x + e) + c)^p,
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^p\,\left(A+B\,\sin\left(e+f\,x\right)\right)}{\left(c-c\,\sin\left(e+f\,x\right)\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(A + B*sin(e + f*x)))/(c - c*sin(e + f*x))^p,x)
```

```
[Out] int(((g*cos(e + f*x))^p*(A + B*sin(e + f*x)))/(c - c*sin(e + f*x))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))**p),x)
```

```
[Out] Timed out
```

3.1038 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-p}\,dx$

Optimal. Leaf size=160

$$\frac{c^2 2^{\frac{1}{2}-\frac{p}{2}}(2A-B(1-p))(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p-1}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+\right.}{fg(p+1)}$$

```
[Out] 2^(1/2-1/2*p)*c^2*(2*A-B*(1-p))*(g*cos(f*x+e))^(1+p)*hypergeom([1/2+1/2*p,
-1/2+1/2*p],[3/2+1/2*p],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+1/2*p)*(c-c
*sin(f*x+e))^(-1-p)/f/g/(1+p)-1/2*B*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(
1-p)/f/g
```

Rubi [**A**] time = 0.27, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2860, 2689, 70, 69}

$$\frac{c^2 2^{\frac{1}{2}-\frac{p}{2}}(2A-B(1-p))(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p-1}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+\right.}{fg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(1 - p),x]
```

```
[Out] (2^(1/2 - p/2)*c^2*(2*A - B*(1 - p))*(g*Cos[e + f*x])^(1 + p)*Hypergeometri
c2F1[(-1 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f
*x])^((1 + p)/2)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 + p)) - (B*(g*Cos[e
 + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(1 - p))/(2*f*g)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-p}\,dx = -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{1-p}}{2fg} - \frac{(-\ldots}{\ldots}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{1-p}}{2fg} - \frac{\left(c\ldots\right.}{\ldots}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{1-p}}{2fg} - \frac{\left(2\ldots\right.}{\ldots}$$

$$= \frac{2^{\frac{1}{2}-\frac{p}{2}}c^2(2A-B(1-p))(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1}{2}\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.66, size = 150, normalized size = 0.94

$$\frac{c2^{\frac{1}{2}(-p-3)}\cos(e+fx)(c-c\sin(e+fx))^{-p}(g\cos(e+fx))^p\left(B2^{\frac{p+1}{2}}(p+1)(\sin(e+fx)-1)^2-4(2A+B(p-1))(1-\ldots\right.}{f(p+1)(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(1 -
 p),x]
```

```
[Out] (2^((-3 - p)/2)*c*Cos[e + f*x]*(g*Cos[e + f*x])^p*(-4*(2*A + B*(-1 + p))*Hy
pergeometric2F1[(-1 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1
- Sin[e + f*x])^((1 + p)/2) + 2^((1 + p)/2)*B*(1 + p)*(-1 + Sin[e + f*x])^2
))/(f*(1 + p)*(-1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^p)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x, algor
ithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
+ 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 + 1), x)
```

maple [F] time = 4.06, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{p}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{1-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^p\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{1-p} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1 - p),x)
```

```
[Out] int((g*cos(e + f*x))^p*(A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1 - p), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1-p),x)
```

```
[Out] Timed out
```

3.1039 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-p}\,dx$

Optimal. Leaf size=163

$$\frac{c^3 2^{\frac{5}{2}-\frac{p}{2}}(3A-B(2-p))(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p-3}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+\right.}{3fg(p+1)}$$

```
[Out] 1/3*2^(5/2-1/2*p)*c^3*(3*A-B*(2-p))*(g*cos(f*x+e))^(1+p)*hypergeom([1/2+1/2
*p, -3/2+1/2*p],[3/2+1/2*p],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+1/2*p)*
(c-c*sin(f*x+e))^(-1-p)/f/g/(1+p)-1/3*B*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e
))^(2-p)/f/g
```

Rubi [A] time = 0.28, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2860, 2689, 70, 69}

$$\frac{c^3 2^{\frac{5}{2}-\frac{p}{2}}(3A-B(2-p))(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p-3}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+\right.}{3fg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(2 - p),x]
```

```
[Out] (2^(5/2 - p/2)*c^3*(3*A - B*(2 - p))*(g*Cos[e + f*x])^(1 + p)*Hypergeometri
c2F1[(-3 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f
*x])^((1 + p)/2)*(c - c*Sin[e + f*x])^(-1 - p))/(3*f*g*(1 + p)) - (B*(g*Cos
[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(2 - p))/(3*f*g)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-p}\,dx = -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{2-p}}{3fg}-\frac{1}{3}($$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{2-p}}{3fg}-\frac{(c}{}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{2-p}}{3fg}-\frac{(2}{}$$

$$= \frac{2^{\frac{5}{2}-\frac{p}{2}}c^3(3A-B(2-p))(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1}{2}\right.}{}$$

Mathematica [A] time = 0.91, size = 155, normalized size = 0.95

$$-\frac{c^2 2^{\frac{1}{2}(-p-1)}\cos(e+fx)(c-c\sin(e+fx))^{-p}(g\cos(e+fx))^p\left(8(3A+B(p-2))(1-\sin(e+fx))^{\frac{p+1}{2}}\,{}_2F_1\left(\frac{p-3}{2},\frac{p+1}{2};\right.\right.}{3f(p+1)(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(2 -
 p),x]
```

```
[Out] -1/3*(2^((-1 - p)/2)*c^2*Cos[e + f*x]*(g*Cos[e + f*x])^p*(8*(3*A + B*(-2 +
p))*Hypergeometric2F1[(-3 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/
2]*(1 - Sin[e + f*x])^((1 + p)/2) + 2^((1 + p)/2)*B*(1 + p)*(-1 + Sin[e + f
*x])^3))/(f*(1 + p)*(-1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^p)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x, algor
ithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
+ 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p+2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 + 2), x)
```

maple [F] time = 4.14, size = 0, normalized size = 0.00

$$\int\left(g\cos\left(fx+e\right)\right)^{p}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{2-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p+2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(g\cos\left(e+f\,x\right)\right)^p\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{2-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(2 - p),x)
```

```
[Out] int((g*cos(e + f*x))^p*(A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(2 - p), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(2-p),x)
```

```
[Out] Timed out
```

3.1040 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(Am-A(1+m+p)\sin(e+fx))\,dx$

Optimal. Leaf size=32

$$\frac{A(a\sin(e+fx)+a)^m(g\cos(e+fx))^{p+1}}{fg}$$

```
[Out] A*(g*cos(f*x+e))^(1+p)*(a+a*sin(f*x+e))^m/f/g
```

Rubi [A] time = 0.12, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2854}

$$\frac{A(a\sin(e+fx)+a)^m(g\cos(e+fx))^{p+1}}{fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(A*m - A*(1 + m + p)*Sin[e + f*x]),x]
```

```
[Out] (A*(g*Cos[e + f*x])^(1 + p)*(a + a*Sin[e + f*x])^m)/(f*g)
```

Rule 2854

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + p + 1), 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(Am-A(1+m+p)\sin(e+fx))\,dx = \frac{A(g\cos(e+fx))^{1+p}(a+a\sin(e+$$

Mathematica [A] time = 0.17, size = 33, normalized size = 1.03

$$\frac{A\cos(e+fx)(a(\sin(e+fx)+1))^m(g\cos(e+fx))^p}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(A*m - A*(1 + m + p)*Si
n[e + f*x]),x]
```

```
[Out] (A*Cos[e + f*x]*(g*Cos[e + f*x])^p*(a*(1 + Sin[e + f*x]))^m)/f
```

fricas [A] time = 0.63, size = 33, normalized size = 1.03

$$\frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}A\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x,
 algorithm="fricas")
```

```
[Out] (g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*A*cos(f*x + e)/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8
*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*p
i/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
```

4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-(-A*exp(-m*ln(2)+2*m*ln(2*abs(tan((2*f*x-pi+2*exp(1))/8)^2-1)/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+m*ln(abs(a))-p*ln(2)+p*ln(2*abs(tan((2*f*x-pi+2*exp(1))/8)^2-1)/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+p*ln(4*abs(g)*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1)))*tan((4*m*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+3*pi+2*exp(1))*1/4/pi)+4*m*pi*floor((f*x+pi+exp(1))*1/2/pi)+4*m*pi*floor(-(sign(a)-2)/4)+m*pi*sign(a)-2*m*pi*sign(tan((f*x+exp(1))/2)^2-1)-3*m*pi+2*p*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+3*pi+2*exp(1))*1/4/pi)+2*p*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+4*p*pi*floor((f*x+pi+exp(1))*1/2/pi)+p*pi*sign(g)*sign(g*tan((f*x+exp(1))/2)^2-2*g*tan((f*x+exp(1))/2)+g)*sign(tan((f*x+exp(1))/2)^2-1)+p*pi*sign(g*tan((f*x+exp(1))/2)^2-2*g*tan((f*x+exp(1))/2)+g)-p*pi*sign(tan((f*x+exp(1))/2)^2-1)-p*pi)/4)^2*tan((f*x+exp(1))/2)^2+A*exp(-m*ln(2)+2*m*ln(2*abs(tan((2*f*x-pi+2*exp(1))/8)^2-1)/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+m*ln(abs(a))-p*ln(2)+p*ln(2*abs(tan((2*f*x-pi+2*exp(1))/8)^2-1)/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+p*ln(4*abs(g)*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1)))*tan((4*m*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+3*pi+2*exp(1))*1/4/pi)+4*m*pi*floor((f*x+pi+exp(1))*1/2/pi)+4*m*pi*floor(-(sign(a)-2)/4)+m*pi*sign(a)-2*m*pi*sign(tan((f*x+exp(1))/2)^2-1)-3*m*pi+2*p*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+3*pi+2*exp(1))*1/4/pi)+2*p*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+4*p*pi*floor((f*x+pi+exp(1))*1/2/pi)+p*pi*sign(g)*sign(g*tan((f*x+exp(1))/2)^2-2*g*tan((f*x+exp(1))/2)+g)*sign(tan((f*x+exp(1))/2)^2-1)+p*pi*sign(g*tan((f*x+exp(1))/2)^2-2*g*tan((f*x+exp(1))/2)+g)-p*pi*si

```
gn(tan((f*x+exp(1))/2)^2-1)-p*pi)/4)^2+A*exp(-m*ln(2)+2*m*ln(2*abs(tan((2*f
*x-pi+2*exp(1))/8)^2-1)/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+m*ln(abs(a))-p*ln
(2)+p*ln(2*abs(tan((2*f*x-pi+2*exp(1))/8)^2-1)/(tan((2*f*x-pi+2*exp(1))/8)^
2+1))+p*ln(4*abs(g)*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1)
)/8)^2+1)))*tan((f*x+exp(1))/2)^2-A*exp(-m*ln(2)+2*m*ln(2*abs(tan((2*f*x-pi
+2*exp(1))/8)^2-1)/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+m*ln(abs(a))-p*ln(2)+p
*ln(2*abs(tan((2*f*x-pi+2*exp(1))/8)^2-1)/(tan((2*f*x-pi+2*exp(1))/8)^2+1))
+p*ln(4*abs(g)*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^
2+1))))/(f*tan((4*m*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+3*pi
+2*exp(1))*1/4/pi)+4*m*pi*floor((f*x+pi+exp(1))*1/2/pi)+4*m*pi*floor(-(sign
(a)-2)/4)+m*pi*sign(a)-2*m*pi*sign(tan((f*x+exp(1))/2)^2-1)-3*m*pi+2*p*pi*f
loor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+3*pi+2*exp(1))*1/4/pi)+2*p*p
i*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+4*p*
pi*floor((f*x+pi+exp(1))*1/2/pi)+p*pi*sign(g)*sign(g*tan((f*x+exp(1))/2)^2-
2*g*tan((f*x+exp(1))/2)+g)*sign(tan((f*x+exp(1))/2)^2-1)+p*pi*sign(g*tan((f
*x+exp(1))/2)^2-2*g*tan((f*x+exp(1))/2)+g)-p*pi*sign(tan((f*x+exp(1))/2)^2-
1)-p*pi)/4)^2*tan((f*x+exp(1))/2)^2+f*tan((4*m*pi*floor((2*f*x-4*pi*floor((
f*x+pi+exp(1))*1/2/pi)+3*pi+2*exp(1))*1/4/pi)+4*m*pi*floor((f*x+pi+exp(1))*
1/2/pi)+4*m*pi*floor(-(sign(a)-2)/4)+m*pi*sign(a)-2*m*pi*sign(tan((f*x+exp(
1))/2)^2-1)-3*m*pi+2*p*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+3
*pi+2*exp(1))*1/4/pi)+2*p*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi
)+pi+2*exp(1))*1/4/pi)+4*p*pi*floor((f*x+pi+exp(1))*1/2/pi)+p*pi*sign(g)*si
gn(g*tan((f*x+exp(1))/2)^2-2*g*tan((f*x+exp(1))/2)+g)*sign(tan((f*x+exp(1))
/2)^2-1)+p*pi*sign(g*tan((f*x+exp(1))/2)^2-2*g*tan((f*x+exp(1))/2)+g)-p*pi*
sign(tan((f*x+exp(1))/2)^2-1)-p*pi)/4)^2+f*tan((f*x+exp(1))/2)^2+f)
```

maple [F] time = 10.90, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^p (a+a\sin(fx+e))^m (Am - A(1+m+p)\sin(fx+e))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x)
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (A(m+p+1)\sin(fx+e) - Am)(g\cos(fx+e))^p (a\sin(fx+e)+a)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x,
 algorithm="maxima")
```

```
[Out] -integrate((A*(m + p + 1)*sin(f*x + e) - A*m)*(g*cos(f*x + e))^p*(a*sin(f*x
 + e) + a)^m, x)
```

mupad [B] time = 9.70, size = 33, normalized size = 1.03

$$\frac{A\cos\left(e+f\,x\right)\,{\left(g\,\cos\left(e+f\,x\right)\right)}^p\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(A*m - A*sin(e + f*x)*(m + p + 1))*(a + a*sin(e + f*
x))^m,x)
```

```
[Out] (A*cos(e + f*x)*(g*cos(e + f*x))^p*(a*(sin(e + f*x) + 1))^m)/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**m*(A*m-A*(1+m+p)*sin(f*x+e)),
x)
```

```
[Out] Timed out
```

3.1041 $\int (g\cos(e+fx))^p(a-a\sin(e+fx))^m(Am+A(1+m+p)\sin(e+fx))\,dx$

Optimal. Leaf size=34

$$-\frac{A(a-a\sin(e+fx))^m(g\cos(e+fx))^{p+1}}{fg}$$

```
[Out] -A*(g*cos(f*x+e))^(1+p)*(a-a*sin(f*x+e))^m/f/g
```

Rubi [A] time = 0.12, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2854}

$$-\frac{A(a-a\sin(e+fx))^m(g\cos(e+fx))^{p+1}}{fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a - a*Sin[e + f*x])^m*(A*m + A*(1 + m + p)*Sin[e + f*x]),x]
```

```
[Out] -((A*(g*Cos[e + f*x])^(1 + p)*(a - a*Sin[e + f*x])^m)/(f*g))
```

Rule 2854

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + p + 1), 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(a-a\sin(e+fx))^m(Am+A(1+m+p)\sin(e+fx))\,dx=-\frac{A(g\cos(e+fx))^{1+p}(a-a\sin(e+$$

Mathematica [A] time = 0.07, size = 35, normalized size = 1.03

$$-\frac{A\cos(e+fx)(a-a\sin(e+fx))^m(g\cos(e+fx))^p}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a - a*Sin[e + f*x])^m*(A*m + A*(1 + m + p)*Si
n[e + f*x]),x]
```

```
[Out] -((A*Cos[e + f*x]*(g*Cos[e + f*x])^p*(a - a*Sin[e + f*x])^m)/f)
```

fricas [A] time = 0.83, size = 35, normalized size = 1.03

$$-\frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(-a\sin\left(fx+e\right)+a\right)^{m}A\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x,
 algorithm="fricas")
```

```
[Out] -(g*cos(f*x + e))^p*(-a*sin(f*x + e) + a)^m*A*cos(f*x + e)/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8
*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*p
i/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
```

4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/
x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi
/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to chec
k sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)(-A*exp(-m*ln
(2)+2*m*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+
1))+m*ln(abs(a))-p*ln(2)+p*ln(2*abs(g)*abs(tan((2*f*x-pi+2*exp(1))/8)^2-1)/
(tan((2*f*x-pi+2*exp(1))/8)^2+1))+p*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(t
an((2*f*x-pi+2*exp(1))/8)^2+1)))*tan((4*m*pi*floor((2*f*x-4*pi*floor((f*x+p
i+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+4*m*pi*floor((f*x+pi+exp(1))*1/2/pi)
+4*m*pi*floor(-(sign(a)-4)/4)+m*pi*sign(a)+2*m*pi*sign(tan((f*x+exp(1))/2)^
2-1)+m*pi+2*p*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+3*pi+2*exp
(1))*1/4/pi)+2*p*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*ex
p(1))*1/4/pi)+4*p*pi*floor((f*x+pi+exp(1))*1/2/pi)-p*pi*sign(g)*sign(g*tan(
(f*x+exp(1))/2)^2+2*g*tan((f*x+exp(1))/2)+g)*sign(tan((f*x+exp(1))/2)^2-1)-
p*pi*sign(g*tan((f*x+exp(1))/2)^2+2*g*tan((f*x+exp(1))/2)+g)+p*pi*sign(tan(
(f*x+exp(1))/2)^2-1)+p*pi)/4)^2*tan((f*x+exp(1))/2)^2+A*exp(-m*ln(2)+2*m*ln
(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+m*ln(a
bs(a))-p*ln(2)+p*ln(2*abs(g)*abs(tan((2*f*x-pi+2*exp(1))/8)^2-1)/(tan((2*f*
x-pi+2*exp(1))/8)^2+1))+p*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-
pi+2*exp(1))/8)^2+1)))*tan((4*m*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*
1/2/pi)+pi+2*exp(1))*1/4/pi)+4*m*pi*floor((f*x+pi+exp(1))*1/2/pi)+4*m*pi*fl
oor(-(sign(a)-4)/4)+m*pi*sign(a)+2*m*pi*sign(tan((f*x+exp(1))/2)^2-1)+m*pi+
2*p*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+3*pi+2*exp(1))*1/4/p
i)+2*p*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/
pi)+4*p*pi*floor((f*x+pi+exp(1))*1/2/pi)-p*pi*sign(g)*sign(g*tan((f*x+exp(1
))/2)^2+2*g*tan((f*x+exp(1))/2)+g)*sign(tan((f*x+exp(1))/2)^2-1)-p*pi*sign(
g*tan((f*x+exp(1))/2)^2+2*g*tan((f*x+exp(1))/2)+g)+p*pi*sign(tan((f*x+exp(1

```
))/2)^2-1)+p*pi)/4)^2+A*exp(-m*ln(2)+2*m*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8
))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+m*ln(abs(a))-p*ln(2)+p*ln(2*abs(g)*abs
(tan((2*f*x-pi+2*exp(1))/8)^2-1)/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+p*ln(4*a
bs(tan((2*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1)))*tan((f*x+
exp(1))/2)^2-A*exp(-m*ln(2)+2*m*ln(4*abs(tan((2*f*x-pi+2*exp(1))/8))/(tan((
2*f*x-pi+2*exp(1))/8)^2+1))+m*ln(abs(a))-p*ln(2)+p*ln(2*abs(g)*abs(tan((2*f
*x-pi+2*exp(1))/8)^2-1)/(tan((2*f*x-pi+2*exp(1))/8)^2+1))+p*ln(4*abs(tan((2
*f*x-pi+2*exp(1))/8))/(tan((2*f*x-pi+2*exp(1))/8)^2+1))))/(f*tan((4*m*pi*fl
oor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+4*m*pi*f
loor((f*x+pi+exp(1))*1/2/pi)+4*m*pi*floor(-(sign(a)-4)/4)+m*pi*sign(a)+2*m*
pi*sign(tan((f*x+exp(1))/2)^2-1)+m*pi+2*p*pi*floor((2*f*x-4*pi*floor((f*x+p
i+exp(1))*1/2/pi)+3*pi+2*exp(1))*1/4/pi)+2*p*pi*floor((2*f*x-4*pi*floor((f*
x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+4*p*pi*floor((f*x+pi+exp(1))*1/2/
pi)-p*pi*sign(g)*sign(g*tan((f*x+exp(1))/2)^2+2*g*tan((f*x+exp(1))/2)+g)*si
gn(tan((f*x+exp(1))/2)^2-1)-p*pi*sign(g*tan((f*x+exp(1))/2)^2+2*g*tan((f*x+
exp(1))/2)+g)+p*pi*sign(tan((f*x+exp(1))/2)^2-1)+p*pi)/4)^2*tan((f*x+exp(1)
)/2)^2+f*tan((4*m*pi*floor((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*e
xp(1))*1/4/pi)+4*m*pi*floor((f*x+pi+exp(1))*1/2/pi)+4*m*pi*floor(-(sign(a)-
4)/4)+m*pi*sign(a)+2*m*pi*sign(tan((f*x+exp(1))/2)^2-1)+m*pi+2*p*pi*floor((
2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+3*pi+2*exp(1))*1/4/pi)+2*p*pi*floo
r((2*f*x-4*pi*floor((f*x+pi+exp(1))*1/2/pi)+pi+2*exp(1))*1/4/pi)+4*p*pi*flo
or((f*x+pi+exp(1))*1/2/pi)-p*pi*sign(g)*sign(g*tan((f*x+exp(1))/2)^2+2*g*ta
n((f*x+exp(1))/2)+g)*sign(tan((f*x+exp(1))/2)^2-1)-p*pi*sign(g*tan((f*x+exp
(1))/2)^2+2*g*tan((f*x+exp(1))/2)+g)+p*pi*sign(tan((f*x+exp(1))/2)^2-1)+p*p
i)/4)^2+f*tan((f*x+exp(1))/2)^2+f)
```

maple [F] time = 10.27, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{p} \left(a-a\sin\left(fx+e\right)\right)^{m} \left(Am+A\left(1+m+p\right)\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A\left(m+p+1\right)\sin\left(fx+e\right)+Am\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x,
 algorithm="maxima")
```

```
[Out] integrate((A*(m + p + 1)*sin(f*x + e) + A*m)*(g*cos(f*x + e))^p*(-a*sin(f*x
 + e) + a)^m, x)
```

mupad [B] time = 0.52, size = 35, normalized size = 1.03

$$-\frac{A\cos\left(e+f\,x\right)\,{\left(g\,\cos\left(e+f\,x\right)\right)}^p\,{\left(-a\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^m}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(A*m + A*sin(e + f*x)*(m + p + 1))*(a - a*sin(e + f*
x))^m,x)
```

```
[Out] -(A*cos(e + f*x)*(g*cos(e + f*x))^p*(-a*(sin(e + f*x) - 1))^m)/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a-a*sin(f*x+e))**m*(A*m+A*(1+m+p)*sin(f*x+e)),
x)
```

```
[Out] Timed out
```

3.1042 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=168

$$\frac{g2^{\frac{p+1}{2}}(1-\sin(e+fx))^{\frac{1-p}{2}}(a\sin(e+fx)+a)^{m+1}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(\frac{1}{2}(2m\right.}{af(2m+p+1)}$$

```
[Out] 2^(1/2+1/2*p)*g*AppellF1(1/2+m+1/2*p,-n,1/2-1/2*p,3/2+m+1/2*p,-d*(1+sin(f*x
+e))/(c-d),1/2+1/2*sin(f*x+e))*(g*cos(f*x+e))^(-1+p)*(1-sin(f*x+e))^(1/2-1/
2*p)*(a+a*sin(f*x+e))^(1+m)*(c+d*sin(f*x+e))^n/a/f/(1+2*m+p)/(((c+d*sin(f*x
+e))/(c-d))^n)
```

Rubi [A] time = 0.28, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2921, 140, 139, 138}

$$\frac{g2^{\frac{p+1}{2}}(1-\sin(e+fx))^{\frac{1-p}{2}}(a\sin(e+fx)+a)^{m+1}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(\frac{1}{2}(2m\right.}{af(2m+p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (2^((1 + p)/2)*g*AppellF1[(1 + 2*m + p)/2, (1 - p)/2, -n, (3 + 2*m + p)/2,
(1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*(g*Cos[e + f*x])^(
-1 + p)*(1 - Sin[e + f*x])^((1 - p)/2)*(a + a*Sin[e + f*x])^(1 + m)*(c + d*
Sin[e + f*x])^n)/(a*f*(1 + 2*m + p)*((c + d*Sin[e + f*x])/(c - d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(g
*(g*Cos[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(a-a\sin(e+fx))^{\frac{1-p}{2}}(a+a\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}+\frac{p}{2}}g(g\cos(e+fx))^{-1+p}(a-a\sin(e+fx))^{-\frac{1}{2}}\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}+\frac{p}{2}}g(g\cos(e+fx))^{-1+p}(a-a\sin(e+fx))^{-\frac{1}{2}}\right.}{}$$

$$= \frac{2^{\frac{1+p}{2}}gF_1\left(\frac{1}{2}(1+2m+p);\frac{1-p}{2},-n;\frac{1}{2}(3+2m+p);\right.}{}$$

Mathematica [B] time = 10.40, size = 798, normalized size = 4.75

$$-\frac{}{f\left(-\frac{dnF_1\left(\frac{p+1}{2};m+n+p+1,-n;\frac{p+3}{2};-\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right),-\frac{(c-d)\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)\cos^2(e+fx)}{c+d\sin(e+fx)}+\frac{2(p+1)\left((c-d)nF_1\left(\frac{p+3}{2};m+n+p+1,1-n;\frac{p+5}{2};-\mathrm{t}\right.\right.}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] (-2*AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3 + p)/2, -Tan[(2*e - Pi + 2*f*
x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*(g*Cos[e + f*x])
^p*Cos[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^
n*Sin[(2*e + Pi + 2*f*x)/4])/(f*(AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3
+ p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^
2)/(c + d))] + (2*(1 + p)*((c - d)*n*AppellF1[(3 + p)/2, 1 + m + n + p, 1 -
 n, (5 + p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f
*x)/4]^2)/(c + d))] - (c + d)*(1 + m + n + p)*AppellF1[(3 + p)/2, 2 + m + n
 + p, -n, (5 + p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi
 + 2*f*x)/4]^2)/(c + d))])*Cot[(2*e + Pi + 2*f*x)/4]^2)/((c + d)*(3 + p)) +
 p*AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3 + p)/2, -Tan[(2*e - Pi + 2*f*x
)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Sin[e + f*x] - (d
*n*AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3 + p)/2, -Tan[(2*e - Pi + 2*f*x
)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Cos[e + f*x]^2)/(
c + d*Sin[e + f*x]) + 2*(n + p)*AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3 +
 p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2
)/(c + d))]*Sin[(2*e - Pi + 2*f*x)/4]^2 - (2*(c - d)*n*AppellF1[(1 + p)/2,
1 + m + n + p, -n, (3 + p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[
(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d*Sin[
e + f*x])))
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
 x)
```

maple [F] time = 6.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^p\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
[Out] Timed out
```

3.1043 $\int (g\cos(e+fx))^p (a+a\sin(e+fx))^2 (c+d\sin(e+fx))^n \, dx$

Optimal. Leaf size=149

$$-\frac{a^2 2^{\frac{p}{2}+\frac{5}{2}}(\sin(e+fx)+1)^{\frac{1}{2}(-p-5)+2}(g\cos(e+fx))^{p+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{1}{2}(-p-3),-n;\frac{p}{}\right.}{fg(p+1)}$$

```
[Out] -2^(5/2+1/2*p)*a^2*AppellF1(1/2+1/2*p,-n,-3/2-1/2*p,3/2+1/2*p,d*(1-sin(f*x+
e))/(c+d),1/2-1/2*sin(f*x+e))*(g*cos(f*x+e))^(1+p)*(1+sin(f*x+e))^(-1/2-1/2
*p)*(c+d*sin(f*x+e))^n/f/g/(1+p)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Rubi [A] time = 0.22, antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2920, 139, 138}

$$-\frac{a^2 g 2^{\frac{p+5}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{1}{2}(-\right.}{f(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -((2^((5 + p)/2)*a^2*g*AppellF1[(1 + p)/2, (-3 - p)/2, -n, (3 + p)/2, (1 -
Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*
(1 - Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(
f*(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \frac{\left(a^2g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\right.}{}$$

$$= \frac{\left(a^2g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\right.}{}$$

$$= -\frac{2^{\frac{5+p}{2}}a^2gF_1\left(\frac{1+p}{2};\frac{1}{2}(-3-p),-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e\right.}{}$$

Mathematica [F] time = 22.61, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,
 x]
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\left(g\cos\left(fx+e\right)\right)^p\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*(g*cos(f*x + e)
)^p*(d*sin(f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{2} \left(g \cos\left(fx+e\right)\right)^{p} \left(d \sin\left(fx+e\right)+c\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n,
 x)
```

maple [F] time = 4.08, size = 0, normalized size = 0.00

$$\int \left(g \cos\left(fx+e\right)\right)^{p} \left(a+a \sin\left(fx+e\right)\right)^{2} \left(c+d \sin\left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{2} \left(g \cos\left(fx+e\right)\right)^{p} \left(d \sin\left(fx+e\right)+c\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n,
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^p\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.1044 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=145

$$-\frac{a2^{\frac{p}{2}+\frac{3}{2}}(\sin(e+fx)+1)^{\frac{1}{2}(-p-1)}(g\cos(e+fx))^{p+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{1}{2}(-p-1),-n;\frac{p+3}{2};\right.}{fg(p+1)}$$

```
[Out] -2^(3/2+1/2*p)*a*AppellF1(1/2+1/2*p,-n,-1/2-1/2*p,3/2+1/2*p,d*(1-sin(f*x+e)
)/(c+d),1/2-1/2*sin(f*x+e))*(g*cos(f*x+e))^(1+p)*(1+sin(f*x+e))^(-1/2-1/2*p
)*(c+d*sin(f*x+e))^n/f/g/(1+p)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Rubi [A] time = 0.17, antiderivative size = 151, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2868, 139, 138}

$$-\frac{ag2^{\frac{p+3}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{1}{2}(-p\right.}{f(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -((2^((3 + p)/2)*a*g*AppellF1[(1 + p)/2, (-1 - p)/2, -n, (3 + p)/2, (1 - Si
n[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1
 - Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(f*
(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2868

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(c*g*(g
*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(1 + (d*x)/c)^((p + 1)/2)*(1 - (d*x)/c)^((p - 1)/
2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{\left(ag(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\mathrm{si}\right.}{}$$

$$= \frac{\left(ag(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\mathrm{si}\right.}{}$$

$$= -\frac{2^{\frac{3+p}{2}}agF_1\left(\frac{1+p}{2};\frac{1}{2}(-1-p),-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+\right.}{}$$

Mathematica [F] time = 4.28, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x
]
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n, x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n, x
)
```

maple **[F]** time = 1.54, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(a+a\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n, x
)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^p\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
[Out] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\left(g\cos\left(e+fx\right)\right)^p\left(c+d\sin\left(e+fx\right)\right)^n\,dx+\int\left(g\cos\left(e+fx\right)\right)^p\left(c+d\sin\left(e+fx\right)\right)^n\sin\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
[Out] a*(Integral((g*cos(e + f*x))**p*(c + d*sin(e + f*x))**n, x) + Integral((g*c
os(e + f*x))**p*(c + d*sin(e + f*x))**n*sin(e + f*x), x))
```

3.1045 $\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=149

$$-\frac{2^{\frac{p}{2}-\frac{1}{2}}(\sin(e+fx)+1)^{\frac{1-p}{2}-1}(g\cos(e+fx))^{p+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{3-p}{2},-n;\frac{p+3}{2};\frac{1}{2}(1-\sin\right.}{afg(p+1)}$$

```
[Out] -2^(-1/2+1/2*p)*AppellF1(1/2+1/2*p,-n,3/2-1/2*p,3/2+1/2*p,d*(1-sin(f*x+e))/
(c+d),1/2-1/2*sin(f*x+e))*(g*cos(f*x+e))^(1+p)*(1+sin(f*x+e))^(-1/2-1/2*p)*
(c+d*sin(f*x+e))^n/a/f/g/(1+p)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Rubi [A] time = 0.25, antiderivative size = 155, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2920, 139, 138}

$$-\frac{g2^{\frac{p}{2}-\frac{1}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{3-p}{2},-\right.}{af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((2^(-1/2 + p/2)*g*AppellF1[(1 + p)/2, (3 - p)/2, -n, (3 + p)/2, (1 - Sin[
e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1 -
 Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(a*f*
(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}\right)\text{Sub}}{af}$$

$$= \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}(c+}{}$$

$$= -\frac{2^{-\frac{1}{2}+\frac{p}{2}}gF_1\left(\frac{1+p}{2};\frac{3-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)(g}{}$$

Mathematica [F] time = 7.70, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),
x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),
 x]
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(g\cos\left(fx+e\right)\right)^p\left(d\sin\left(fx+e\right)+c\right)^n}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x
)
```

maple [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(c+d\sin\left(fx+e\right)\right)^{n}}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^p\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)),x)
[Out] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(g\cos\left(e+fx\right)\right)^{p}\left(c+d\sin\left(e+fx\right)\right)^{n}}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
[Out] Integral((g*cos(e + f*x))**p*(c + d*sin(e + f*x))**n/(sin(e + f*x) + 1), x)
/a
```

3.1046 $\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=149

$$-\frac{2^{\frac{p}{2}-\frac{3}{2}}(\sin(e+fx)+1)^{\frac{3-p}{2}-2}(g\cos(e+fx))^{p+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{5-p}{2},-n;\frac{p+3}{2};\frac{1}{2}(1-\sin\right.}{a^2fg(p+1)}$$

```
[Out] -2^(-3/2+1/2*p)*AppellF1(1/2+1/2*p,-n,5/2-1/2*p,3/2+1/2*p,d*(1-sin(f*x+e))/
(c+d),1/2-1/2*sin(f*x+e))*(g*cos(f*x+e))^(1+p)*(1+sin(f*x+e))^(-1/2-1/2*p)*
(c+d*sin(f*x+e))^n/a^2/f/g/(1+p)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Rubi [A] time = 0.23, antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2920, 139, 138}

$$-\frac{g2^{\frac{p-3}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{5-p}{2},-n\right.}{a^2f(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((2^((-3 + p)/2)*g*AppellF1[(1 + p)/2, (5 - p)/2, -n, (3 + p)/2, (1 - Sin[
e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1 -
 Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(a^2*
f*(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}\right)\text{Sub}}{a^2f}$$

$$= \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}(c+$$

$$= -\frac{2^{\frac{1}{2}(-3+p)}gF_1\left(\frac{1+p}{2};\frac{5-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)(g$$

Mathematica [F] time = 12.06, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
2, x]
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(g\cos\left(fx+e\right)\right)^p\left(d\sin\left(fx+e\right)+c\right)^n}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2 - 2
*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2,
 x)
```

maple [F] time = 4.01, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2,
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^p\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,x)
[Out] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(g\cos\left(e+fx\right)\right)^p\left(c+d\sin\left(e+fx\right)\right)^n}{\sin^2\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
[Out] Integral((g*cos(e + f*x))**p*(c + d*sin(e + f*x))**n/(sin(e + f*x)**2 + 2*s
in(e + f*x) + 1), x)/a**2
```

3.1047 $\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=149

$$-\frac{2^{\frac{p}{2}-\frac{5}{2}}(\sin(e+fx)+1)^{\frac{5-p}{2}-3}(g\cos(e+fx))^{p+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{7-p}{2},-n;\frac{p+3}{2};\frac{1}{2}(1-\sin$$

$$a^3fg(p+1)$$

```
[Out] -2^(-5/2+1/2*p)*AppellF1(1/2+1/2*p,-n,7/2-1/2*p,3/2+1/2*p,d*(1-sin(f*x+e))/
(c+d),1/2-1/2*sin(f*x+e))*(g*cos(f*x+e))^(1+p)*(1+sin(f*x+e))^(-1/2-1/2*p)*
(c+d*sin(f*x+e))^n/a^3/f/g/(1+p)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Rubi [A] time = 0.23, antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2920, 139, 138}

$$-\frac{g2^{\frac{p-5}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{7-p}{2},-n\right.}{a^3f(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((2^((-5 + p)/2)*g*AppellF1[(1 + p)/2, (7 - p)/2, -n, (3 + p)/2, (1 - Sin[
e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1 -
 Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(a^3*
f*(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}\right)\text{Sub}}{a^3 f}$$

$$= \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}(c+\right.}{}$$

$$= -\frac{2^{\frac{1}{2}(-5+p)}gF_1\left(\frac{1+p}{2};\frac{7-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)(g}{}$$

Mathematica [F] time = 15.57, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
3, x]
```

fricas [F] time = 1.38, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(g\cos\left(fx+e\right)\right)^p\left(d\sin\left(fx+e\right)+c\right)^n}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] integral(-(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(3*a^3*cos(f*x + e)^2 -
 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^3,
 x)
```

maple [F] time = 2.74, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^p\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3,x)
[Out] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
[Out] Timed out
```

3.1048 $\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx$

Optimal. Leaf size=149

$$-\frac{2^{\frac{p}{2}-\frac{7}{2}}(\sin(e+fx)+1)^{\frac{7-p}{2}-4}(g\cos(e+fx))^{p+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{9-p}{2},-n;\frac{p+3}{2};\frac{1}{2}(1-\sin\right.}{a^4fg(p+1)}$$

```
[Out] -2^(-7/2+1/2*p)*AppellF1(1/2+1/2*p,-n,9/2-1/2*p,3/2+1/2*p,d*(1-sin(f*x+e))/
(c+d),1/2-1/2*sin(f*x+e))*(g*cos(f*x+e))^(1+p)*(1+sin(f*x+e))^(-1/2-1/2*p)*
(c+d*sin(f*x+e))^n/a^4/f/g/(1+p)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Rubi [A] time = 0.23, antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2920, 139, 138}

$$-\frac{g2^{\frac{p-7}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{9-p}{2},-n\right.}{a^4f(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^4,x]
```

```
[Out] -((2^((-7 + p)/2)*g*AppellF1[(1 + p)/2, (9 - p)/2, -n, (3 + p)/2, (1 - Sin[
e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1 -
 Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(a^4*
f*(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}\right)\text{Sub}}{a^4 f}$$

$$= \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}(c+}{}$$

$$= -\frac{2^{\frac{1}{2}(-7+p)}gF_1\left(\frac{1+p}{2};\frac{9-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)(g}{}$$

Mathematica [F] time = 20.90, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
4,x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
4, x]
```

fricas [F] time = 1.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{a^4\cos\left(fx+e\right)^4-8\,a^4\cos\left(fx+e\right)^2+8\,a^4-4\left(a^4\cos\left(fx+e\right)^2-2\,a^4\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a^4*cos(f*x + e)^4 - 8*
a^4*cos(f*x + e)^2 + 8*a^4 - 4*(a^4*cos(f*x + e)^2 - 2*a^4)*sin(f*x + e)),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^4,
 x)
```

maple [F] time = 3.40, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x)
```

```
[Out] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\cos\left(e+f\,x\right)\right)^p\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^4,x)
[Out] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**4,x)
[Out] Timed out
```

3.1049 $\int (g\sec(e+fx))^p(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=175

$$\frac{2^{\frac{1}{2}-\frac{p}{2}}\sec(e+fx)(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^{m+1}(g\sec(e+fx))^p(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1}{af(2m-p+1)}$$

```
[Out] 2^(1/2-1/2*p)*AppellF1(1/2+m-1/2*p,-n,1/2+1/2*p,3/2+m-1/2*p,-d*(1+sin(f*x+e
))/(c-d),1/2+1/2*sin(f*x+e))*sec(f*x+e)*(g*sec(f*x+e))^p*(1-sin(f*x+e))^(1/
2+1/2*p)*(a+a*sin(f*x+e))^(1+m)*(c+d*sin(f*x+e))^n/a/f/(1+2*m-p)/(((c+d*sin
(f*x+e))/(c-d))^n)
```

Rubi [A] time = 0.43, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2926, 2921, 140, 139, 138}

$$\frac{2^{\frac{1}{2}-\frac{p}{2}}\sec(e+fx)(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^{m+1}(g\sec(e+fx))^p(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1}{af(2m-p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 - p/2)*AppellF1[(1 + 2*m - p)/2, (1 + p)/2, -n, (3 + 2*m - p)/2, (1
 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*(g*Sec[
e + f*x])^p*(1 - Sin[e + f*x])^((1 + p)/2)*(a + a*Sin[e + f*x])^(1 + m)*(c
+ d*Sin[e + f*x])^n)/(a*f*(1 + 2*m - p)*((c + d*Sin[e + f*x])/(c - d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(g
*(g*Cos[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 2926

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]*(g*Sec[e + f*x])^FracPart[p
], Int[((a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(g*Cos[e + f*x])^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int (g\sec(e+fx))^p(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \left((g\cos(e+fx))^p(g\sec(e+fx))^p\right)\int (g\cos(e+f$$

$$= \frac{\left(\sec(e+fx)(g\sec(e+fx))^p(a-a\sin(e+fx))^{\frac{1+}{2}}\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}-\frac{p}{2}}\sec(e+fx)(g\sec(e+fx))^p(a-a\sin(e+\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}-\frac{p}{2}}\sec(e+fx)(g\sec(e+fx))^p(a-a\sin(e+\right.}{}$$

$$= \frac{2^{\frac{1}{2}-\frac{p}{2}}F_1\left(\frac{1}{2}(1+2m-p);\frac{1+p}{2},-n;\frac{1}{2}(3+2m-p);\frac{1}{2}\right.}{}$$

Mathematica [B] time = 11.13, size = 893, normalized size = 5.10

$$\frac{}{f\left(\frac{dnF_1\left(\frac{1-p}{2};m+n-p+1,-n;\frac{3-p}{2};-\tan^2\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right),-\frac{(c-d)\tan^2\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)}{c+d}\right)\cos^2(e+fx)}{c+d\sin(e+fx)}-2(n-p)F_1\left(\frac{1-p}{2};m+n-p+1,-n;\frac{3-p}{2};\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] (2*AppellF1[(1 - p)/2, 1 + m + n - p, -n, (3 - p)/2, -Tan[(-e + Pi/2 - f*x)
/2]^2, ((-c + d)*Tan[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)
/2]*(g*Sec[e + f*x])^p*Sin[(-e + Pi/2 - f*x)/2]*(a + a*Sin[e + f*x])^m*(c +
 d*Sin[e + f*x])^n)/(f*(-AppellF1[(1 - p)/2, 1 + m + n - p, -n, (3 - p)/2,
-Tan[(-e + Pi/2 - f*x)/2]^2, ((-c + d)*Tan[(-e + Pi/2 - f*x)/2]^2)/(c + d)]
```

```
 - 2*(n - p)*AppellF1[(1 - p)/2, 1 + m + n - p, -n, (3 - p)/2, -Tan[(-e + P
i/2 - f*x)/2]^2, ((-c + d)*Tan[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Sin[(-e + P
i/2 - f*x)/2]^2 + p*AppellF1[(1 - p)/2, 1 + m + n - p, -n, (3 - p)/2, -Tan[
(-e + Pi/2 - f*x)/2]^2, ((-c + d)*Tan[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Sin[
e + f*x] + (d*n*AppellF1[(1 - p)/2, 1 + m + n - p, -n, (3 - p)/2, -Tan[(-e
+ Pi/2 - f*x)/2]^2, -(((c - d)*Tan[(-e + Pi/2 - f*x)/2]^2)/(c + d))]*Cos[e
+ f*x]^2)/(c + d*Sin[e + f*x]) + (2*(c - d)*n*AppellF1[(1 - p)/2, 1 + m + n
 - p, -n, (3 - p)/2, -Tan[(-e + Pi/2 - f*x)/2]^2, ((-c + d)*Tan[(-e + Pi/2
- f*x)/2]^2)/(c + d)]*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d*Sin[e + f*x]) + (2
*(1 - p)*(-(((c - d)*n*AppellF1[(3 - p)/2, 1 + m + n - p, 1 - n, (5 - p)/2,
 -Tan[(-e + Pi/2 - f*x)/2]^2, ((-c + d)*Tan[(-e + Pi/2 - f*x)/2]^2)/(c + d)
])/(c + d)) + (1 + m + n - p)*AppellF1[(3 - p)/2, 2 + m + n - p, -n, (5 - p
)/2, -Tan[(-e + Pi/2 - f*x)/2]^2, ((-c + d)*Tan[(-e + Pi/2 - f*x)/2]^2)/(c
+ d)])*Tan[(-e + Pi/2 - f*x)/2]^2)/(3 - p)))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(g\sec\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g\sec\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
 x)
```

maple [F] time = 6.29, size = 0, normalized size = 0.00

$$\int\left(g\sec\left(fx+e\right)\right)^{p}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\sec\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^p\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((g/cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.1050 $\int \cos^2(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{b\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{b\sin(c+dx)\cos(c+d$$

```
[Out] 1/16*b*x-1/3*a*cos(d*x+c)^3/d+1/5*a*cos(d*x+c)^5/d+1/16*b*cos(d*x+c)*sin(d*
x+c)/d-1/8*b*cos(d*x+c)^3*sin(d*x+c)/d-1/6*b*cos(d*x+c)^3*sin(d*x+c)^3/d
```

Rubi [A] time = 0.16, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2565, 14, 2568, 2635, 8}

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{b\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{b\sin(c+dx)\cos(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/16 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (b*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) - (b*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (b*Cos
[c + d*x]^3*Sin[c + d*x]^3)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
```

```
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^2(c+dx)\sin^3(c+dx)\,dx + b\int \cos^2(c+dx)\sin^4(c+$$
$$= -\frac{b\cos^3(c+dx)\sin^3(c+dx)}{6d} + \frac{1}{2}b\int \cos^2(c+dx)\sin^2(c+d$$
$$= -\frac{b\cos^3(c+dx)\sin(c+dx)}{8d} - \frac{b\cos^3(c+dx)\sin^3(c+dx)}{6d} +$$
$$= -\frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{b\cos(c+dx)\sin(c+dx)}{16d}$$
$$= \frac{bx}{16} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{b\cos(c+dx)\sin(c+}{16d}$$

Mathematica [A] time = 0.18, size = 77, normalized size = 0.73

$$\frac{-120a\cos(c+dx) - 20a\cos(3(c+dx)) + 12a\cos(5(c+dx)) - 15b\sin(2(c+dx)) - 15b\sin(4(c+dx)) + 5b\sin(6}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (60*b*d*x - 120*a*Cos[c + d*x] - 20*a*Cos[3*(c + d*x)] + 12*a*Cos[5*(c + d*
x)] - 15*b*Sin[2*(c + d*x)] - 15*b*Sin[4*(c + d*x)] + 5*b*Sin[6*(c + d*x)])
/(960*d)
```

fricas [A] time = 0.54, size = 73, normalized size = 0.70

$$\frac{48\,a\cos\left(dx+c\right)^{5}-80\,a\cos\left(dx+c\right)^{3}+15\,bdx+5\left(8\,b\cos\left(dx+c\right)^{5}-14\,b\cos\left(dx+c\right)^{3}+3\,b\cos\left(dx+c\right)\right)\sin}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 15*b*d*x + 5*(8*b*cos(d*
x + c)^5 - 14*b*cos(d*x + c)^3 + 3*b*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.20, size = 92, normalized size = 0.88

$$\frac{1}{16}\,bx+\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}+\frac{b\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{b\sin\left(4\,dx+4\,c\right)}{64\,d}-\frac{b\sin\left(2\,dx+2\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*b*x + 1/80*a*cos(5*d*x + 5*c)/d - 1/48*a*cos(3*d*x + 3*c)/d - 1/8*a*co
s(d*x + c)/d + 1/192*b*sin(6*d*x + 6*c)/d - 1/64*b*sin(4*d*x + 4*c)/d - 1/6
4*b*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.13, size = 95, normalized size = 0.90

$$\frac{a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{3}(dx+c)\right)}{5}-\frac{2\left(\cos^{3}(dx+c)\right)}{15}\right)+b\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{3}(dx+c)\right)}{6}-\frac{\sin(dx+c)\left(\cos^{3}(dx+c)\right)}{8}+\frac{\cos(dx+c)\sin(dx+c)}{16}+\frac{dx}{16}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+b*(-1/6*sin(d*x+c
)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*cos(d*x+c)*sin(d*x+c)+1/1
6*d*x+1/16*c))
```

maxima [A] time = 0.30, size = 65, normalized size = 0.62

$$\frac{64\left(3\,\cos\left(dx+c\right)^{5}-5\,\cos\left(dx+c\right)^{3}\right)a-5\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-12\,dx-12\,c+3\,\sin\left(4\,dx+4\,c\right)\right)b}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*(64*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a - 5*(4*sin(2*d*x + 2*c)^3
 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*b)/d
```

mupad [B] time = 12.98, size = 153, normalized size = 1.46

$$\frac{b\,x}{16}-\frac{-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\frac{17\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{19\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}-\frac{19\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+\frac{17\,b\tan\left(\frac{c}{2}+\cdots\right.}{24}\cdots}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + b*sin(c + d*x)),x)
```

```
[Out] (b*x)/16 - ((4*a)/15 + (b*tan(c/2 + (d*x)/2))/8 + (8*a*tan(c/2 + (d*x)/2)^2
)/5 + (8*a*tan(c/2 + (d*x)/2)^6)/3 + 4*a*tan(c/2 + (d*x)/2)^8 + (17*b*tan(c
/2 + (d*x)/2)^3)/24 - (19*b*tan(c/2 + (d*x)/2)^5)/4 + (19*b*tan(c/2 + (d*x)
/2)^7)/4 - (17*b*tan(c/2 + (d*x)/2)^9)/24 - (b*tan(c/2 + (d*x)/2)^11)/8)/(d
*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [A] time = 3.57, size = 192, normalized size = 1.83

$$\begin{cases}-\frac{a\sin^{2}(c+dx)\cos^{3}(c+dx)}{3d}-\frac{2a\cos^{5}(c+dx)}{15d}+\frac{bx\sin^{6}(c+dx)}{16}+\frac{3bx\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{3bx\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{bx\cos^{6}(c+dx)}{16}+\cdots\\ x\left(a+b\sin(c)\right)\sin^{3}(c)\cos^{2}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*a*cos(c + d*x)**5/(
15*d) + b*x*sin(c + d*x)**6/16 + 3*b*x*sin(c + d*x)**4*cos(c + d*x)**2/16 +
 3*b*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + b*x*cos(c + d*x)**6/16 + b*sin(
c + d*x)**5*cos(c + d*x)/(16*d) - b*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) -
 b*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)
**3*cos(c)**2, True))
```

3.1051 $\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}+\frac{b\cos^5(c+dx)}{5d}-\frac{b\cos^3(c+dx)}{3d}$$

```
[Out] 1/8*a*x-1/3*b*cos(d*x+c)^3/d+1/5*b*cos(d*x+c)^5/d+1/8*a*cos(d*x+c)*sin(d*x+
c)/d-1/4*a*cos(d*x+c)^3*sin(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 14}

$$-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}+\frac{b\cos^5(c+dx)}{5d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/8 - (b*Cos[c + d*x]^3)/(3*d) + (b*Cos[c + d*x]^5)/(5*d) + (a*Cos[c +
d*x]*Sin[c + d*x])/(8*d) - (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
```

```
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^2(c+dx)\sin^2(c+dx)\,dx + b\int \cos^2(c+dx)\sin^3(c+$$

$$= -\frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}a\int \cos^2(c+dx)\,dx - \frac{b\,\text{Sub}}{}$$

$$= \frac{a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{8}a$$

$$= \frac{ax}{8} - \frac{b\cos^3(c+dx)}{3d} + \frac{b\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c+}{8d}$$

Mathematica [A] time = 0.11, size = 59, normalized size = 0.73

$$\frac{-15a\sin(4(c+dx)) + 60ac + 60adx - 60b\cos(c+dx) - 10b\cos(3(c+dx)) + 6b\cos(5(c+dx))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (60*a*c + 60*a*d*x - 60*b*Cos[c + d*x] - 10*b*Cos[3*(c + d*x)] + 6*b*Cos[5*
(c + d*x)] - 15*a*Sin[4*(c + d*x)])/(480*d)
```

fricas [A] time = 0.68, size = 62, normalized size = 0.77

$$\frac{24\,b\cos\left(dx+c\right)^5 - 40\,b\cos\left(dx+c\right)^3 + 15\,adx - 15\left(2\,a\cos\left(dx+c\right)^3 - a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*b*cos(d*x + c)^5 - 40*b*cos(d*x + c)^3 + 15*a*d*x - 15*(2*a*cos(d
*x + c)^3 - a*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.18, size = 62, normalized size = 0.77

$$\frac{1}{8}\,ax + \frac{b\cos\left(5\,dx + 5\,c\right)}{80\,d} - \frac{b\cos\left(3\,dx + 3\,c\right)}{48\,d} - \frac{b\cos\left(dx + c\right)}{8\,d} - \frac{a\sin\left(4\,dx + 4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*a*x + 1/80*b*cos(5*d*x + 5*c)/d - 1/48*b*cos(3*d*x + 3*c)/d - 1/8*b*cos
(d*x + c)/d - 1/32*a*sin(4*d*x + 4*c)/d
```

maple [A] time = 0.12, size = 77, normalized size = 0.95

$$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^{3}(dx+c)\right)}{4} + \frac{\cos(dx+c)\sin(dx+c)}{8} + \frac{dx}{8} + \frac{c}{8}\right) + b\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{3}(dx+c)\right)}{5} - \frac{2\left(\cos^{3}(dx+c)\right)}{15}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*
c)+b*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3))
```

maxima [A] time = 0.30, size = 52, normalized size = 0.64

$$\frac{15\left(4\,dx + 4\,c - \sin\left(4\,dx + 4\,c\right)\right)a + 32\left(3\,\cos\left(dx + c\right)^{5} - 5\,\cos\left(dx + c\right)^{3}\right)b}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a + 32*(3*cos(d*x + c)^5 - 5*cos
(d*x + c)^3)*b)/d
```

mupad [B] time = 12.89, size = 125, normalized size = 1.54

$$\frac{a\,x}{8} - \frac{-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{4} + \frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2} + 4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} - \frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{3} - \frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{2} + \frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{3} + \frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*x)/8 - ((4*b)/15 + (a*tan(c/2 + (d*x)/2))/4 - (3*a*tan(c/2 + (d*x)/2)^3)
/2 + (3*a*tan(c/2 + (d*x)/2)^7)/2 - (a*tan(c/2 + (d*x)/2)^9)/4 + (4*b*tan(c
/2 + (d*x)/2)^2)/3 - (4*b*tan(c/2 + (d*x)/2)^4)/3 + 4*b*tan(c/2 + (d*x)/2)^
6)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

sympy [A] time = 1.81, size = 144, normalized size = 1.78

$$\begin{cases} \frac{ax\sin^4(c+dx)}{8} + \frac{ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{ax\cos^4(c+dx)}{8} + \frac{a\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{a\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{b\sin^2(c+dx)\cos^3(c+dx)}{3d} \\ x(a+b\sin(c))\sin^2(c)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**4/8 + a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
a*x*cos(c + d*x)**4/8 + a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a*sin(c + d*
x)*cos(c + d*x)**3/(8*d) - b*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*b*co
s(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)**2*cos(c)**2, Tru
e))
```

3.1052 $\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{b\sin(c+dx)\cos(c+dx)}{8d}+\frac{bx}{8}$$

```
[Out] 1/8*b*x-1/3*a*cos(d*x+c)^3/d+1/8*b*cos(d*x+c)*sin(d*x+c)/d-1/4*b*cos(d*x+c)
^3*sin(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2838, 2565, 30, 2568, 2635, 8}

$$-\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{b\sin(c+dx)\cos(c+dx)}{8d}+\frac{bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(8*d) -
(b*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^2(c+dx)\sin(c+dx)\,dx + b\int \cos^2(c+dx)\sin^2(c+d$$
$$= -\frac{b\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}b\int \cos^2(c+dx)\,dx - \frac{a\,\text{Subs}}{}$$
$$= -\frac{a\cos^3(c+dx)}{3d} + \frac{b\cos(c+dx)\sin(c+dx)}{8d} - \frac{b\cos^3(c+dx)}{4d}$$
$$= \frac{bx}{8} - \frac{a\cos^3(c+dx)}{3d} + \frac{b\cos(c+dx)\sin(c+dx)}{8d} - \frac{b\cos^3(c+}{}$$

Mathematica [A] time = 0.12, size = 61, normalized size = 0.94

$$-\frac{a\cos^3(c+dx)}{3d} + \frac{1}{8}b\left(-\frac{\sin(4c)\cos(4dx)}{4d} - \frac{\cos(4c)\sin(4dx)}{4d}\right) + \frac{bx}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (b*(-1/4*(Cos[4*d*x]*Sin[4*c])/d - (Co
s[4*c]*Sin[4*d*x])/(4*d)))/8
```

fricas [A] time = 0.69, size = 51, normalized size = 0.78

$$-\frac{8\,a\cos\left(dx+c\right)^3 - 3\,bdx + 3\left(2\,b\cos\left(dx+c\right)^3 - b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*a*cos(d*x + c)^3 - 3*b*d*x + 3*(2*b*cos(d*x + c)^3 - b*cos(d*x + c
))*sin(d*x + c))/d
```

giac [A] time = 0.15, size = 47, normalized size = 0.72

$$\frac{1}{8}\,bx - \frac{a\cos\left(3\,dx + 3\,c\right)}{12\,d} - \frac{a\cos\left(dx + c\right)}{4\,d} - \frac{b\sin\left(4\,dx + 4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*b*x - 1/12*a*cos(3*d*x + 3*c)/d - 1/4*a*cos(d*x + c)/d - 1/32*b*sin(4*d
*x + 4*c)/d
```

maple [A] time = 0.09, size = 57, normalized size = 0.88

$$\frac{-\frac{\left(\cos^{3}(dx+c)\right)a}{3} + b\left(-\frac{\sin(dx+c)\left(\cos^{3}(dx+c)\right)}{4} + \frac{\cos(dx+c)\sin(dx+c)}{8} + \frac{dx}{8} + \frac{c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/3*cos(d*x+c)^3*a+b*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin
(d*x+c)+1/8*d*x+1/8*c))
```

maxima [A] time = 0.37, size = 39, normalized size = 0.60

$$-\frac{32\,a\cos\left(dx + c\right)^{3} - 3\left(4\,dx + 4\,c - \sin\left(4\,dx + 4\,c\right)\right)b}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*a*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*b)/d
```

mupad [B] time = 12.66, size = 125, normalized size = 1.92

$$\frac{b\,x}{8} - \frac{-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4} + 2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} + \frac{7\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4} + 2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4} - \frac{7\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{4} + \frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{3} + \frac{b\tan(\cdots}{\cdots}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + b*sin(c + d*x)),x)
```

```
[Out] (b*x)/8 - ((2*a)/3 + (b*tan(c/2 + (d*x)/2))/4 + (2*a*tan(c/2 + (d*x)/2)^2)/
3 + 2*a*tan(c/2 + (d*x)/2)^4 + 2*a*tan(c/2 + (d*x)/2)^6 - (7*b*tan(c/2 + (d
*x)/2)^3)/4 + (7*b*tan(c/2 + (d*x)/2)^5)/4 - (b*tan(c/2 + (d*x)/2)^7)/4)/(d
*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [A] time = 0.90, size = 119, normalized size = 1.83

$$\begin{cases} -\frac{a\cos^3(c+dx)}{3d} + \frac{bx\sin^4(c+dx)}{8} + \frac{bx\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{bx\cos^4(c+dx)}{8} + \frac{b\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{b\sin(c+dx)\cos^3(c+dx)}{8d} & \text{for } a \\ x(a+b\sin(c))\sin(c)\cos^2(c) & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)**3/(3*d) + b*x*sin(c + d*x)**4/8 + b*x*sin(c + d
*x)**2*cos(c + d*x)**2/4 + b*x*cos(c + d*x)**4/8 + b*sin(c + d*x)**3*cos(c
+ d*x)/(8*d) - b*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*s
in(c))*sin(c)*cos(c)**2, True))
```

3.1053 $\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=51

$$\frac{a\cos(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

```
[Out] 1/2*b*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d+1/2*b*cos(d*x+c)*sin(d*x+c)/
d
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2838, 2592, 321, 206, 2635, 8}

$$\frac{a\cos(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/2 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (b*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cos(c+dx)\cot(c+dx)\,dx + b\int \cos^2(c+dx)\,dx\\
&= \frac{b\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}b\int 1\,dx - \frac{a\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x,\right.}{d}\\
&= \frac{bx}{2} + \frac{a\cos(c+dx)}{d} + \frac{b\cos(c+dx)\sin(c+dx)}{2d} - \frac{a\,\mathrm{Subst}\left(\int \frac{}{1}\right.}{}\\
&= \frac{bx}{2} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{b\cos(c+dx)\,\mathrm{si}}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 74, normalized size = 1.45

$$
\frac{a\cos(c+dx)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{b(c+dx)}{2d} + \frac{b\sin(2(c+dx))}{4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*(c + d*x))/(2*d) + (a*Cos[c + d*x])/d - (a*Log[Cos[(c + d*x)/2]])/d + (a
*Log[Sin[(c + d*x)/2]])/d + (b*Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.62, size = 60, normalized size = 1.18

$$\frac{bdx + b\cos(dx+c)\sin(dx+c) + 2\,a\cos(dx+c) - a\log\left(\frac{1}{2}\,\cos(dx+c) + \frac{1}{2}\right) + a\log\left(-\frac{1}{2}\,\cos(dx+c) + \frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x + b*cos(d*x + c)*sin(d*x + c) + 2*a*cos(d*x + c) - a*log(1/2*cos
(d*x + c) + 1/2) + a*log(-1/2*cos(d*x + c) + 1/2))/d
```

giac [A] time = 0.15, size = 87, normalized size = 1.71

$$\frac{(dx+c)b + 2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - \frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*b + 2*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(b*tan(1/2*d*x +
1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.28, size = 63, normalized size = 1.24

$$\frac{a\cos(dx+c)}{d} + \frac{a\ln(\csc(dx+c) - \cot(dx+c))}{d} + \frac{b\cos(dx+c)\sin(dx+c)}{2d} + \frac{bx}{2} + \frac{bc}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] a*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))+1/2*b*cos(d*x+c)*sin(d*x+c)/
d+1/2*b*x+1/2*b*c/d
```

maxima [A] time = 0.36, size = 57, normalized size = 1.12

$$\frac{(2\,dx + 2\,c + \sin(2\,dx + 2\,c))b + 2\,a\left(2\,\cos(dx+c) - \log(\cos(dx+c) + 1) + \log(\cos(dx+c) - 1)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*b + 2*a*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d

mupad [B] time = 9.87, size = 157, normalized size = 3.08

$$\frac{b\,\mathrm{atan}\left(\frac{b^2}{2\,a\,b-b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{2\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,b-b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{-b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{a\,\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x)))/sin(c + d*x),x)

[Out] (b*atan(b^2/(2*a*b - b^2*tan(c/2 + (d*x)/2)) + (2*a*b*tan(c/2 + (d*x)/2))/(2*a*b - b^2*tan(c/2 + (d*x)/2))))/d + (2*a + b*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^2 - b*tan(c/2 + (d*x)/2)^3)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 1)) + (a*log(tan(c/2 + (d*x)/2)))/d

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))\cos^2(c+dx)\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+b*sin(d*x+c)),x)

[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**2*csc(c + d*x), x)

3.1054 $\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$-\frac{a\cot(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -a*x-b*arctanh(cos(d*x+c))/d+b*cos(d*x+c)/d-a*cot(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2722, 2592, 321, 206, 3473, 8}

$$-\frac{a\cot(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (b*ArcTanh[Cos[c + d*x]])/d + (b*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx &= \int \left(b\cos(c+dx)\cot(c+dx)+a\cot^2(c+dx)\right)\,dx\\ &= a\int \cot^2(c+dx)\,dx+b\int \cos(c+dx)\cot(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)}{d}-a\int 1\,dx-\frac{b\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -ax+\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{b\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -ax-\frac{b\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}\end{aligned}$$

Mathematica [C] time = 0.04, size = 75, normalized size = 1.83

$$-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}+\frac{b\cos(c+dx)}{d}+\frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c
 + d*x]^2])/d - (b*Log[Cos[(c + d*x)/2]])/d + (b*Log[Sin[(c + d*x)/2]])/d
```

fricas [B] time = 0.84, size = 84, normalized size = 2.05

$$-\frac{b\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-b\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\,a\cos\left(dx+c\right)+2\,(adx-}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + 2*a*cos(d*x + c) + 2*(a*d*x - b*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

giac [B] time = 0.17, size = 108, normalized size = 2.63

$$-\frac{6\left(dx+c\right)a-6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a - 6*b*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + (2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 10*b*
tan(1/2*d*x + 1/2*c) + 3*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))
)/d
```

maple [A] time = 0.24, size = 57, normalized size = 1.39

$$-ax-\frac{a\cot\left(dx+c\right)}{d}+\frac{b\cos\left(dx+c\right)}{d}+\frac{b\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}-\frac{ca}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] -a*x-a*cot(d*x+c)/d+b*cos(d*x+c)/d+1/d*b*ln(csc(d*x+c)-cot(d*x+c))-1/d*c*a
```

maxima [A] time = 0.50, size = 54, normalized size = 1.32

$$-\frac{2\left(dx+c+\frac{1}{\tan\left(dx+c\right)}\right)a-b\left(2\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a - b*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d

mupad [B] time = 9.54, size = 158, normalized size = 3.85

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{2\,a\,\mathrm{atan}\left(\frac{4\,a^2}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+4\,b\,a}-\frac{4\,a}{4\tan(}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x)))/sin(c + d*x)^2,x)

[Out] (a*tan(c/2 + (d*x)/2))/(2*d) + (b*log(tan(c/2 + (d*x)/2)))/d - (a - 4*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)/(d*(2*tan(c/2 + (d*x)/2) + 2*tan(c/2 + (d*x)/2)^3)) + (2*a*atan((4*a^2)/(4*a*b + 4*a^2*tan(c/2 + (d*x)/2)) - (4*a*b*tan(c/2 + (d*x)/2))/(4*a*b + 4*a^2*tan(c/2 + (d*x)/2))))/d

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))\cos^2(c+dx)\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+b*sin(d*x+c)),x)

[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**2*csc(c + d*x)**2, x)

3.1055 $\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\cot(c+dx)}{d}-bx$$

```
[Out] -b*x+1/2*a*arctanh(cos(d*x+c))/d-b*cot(d*x+c)/d-1/2*a*cot(d*x+c)*csc(d*x+c)
/d
```

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2838, 2611, 3770, 3473, 8}

$$\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\cot(c+dx)}{d}-bx$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*x) + (a*ArcTanh[Cos[c + d*x]])/(2*d) - (b*Cot[c + d*x])/d - (a*Cot[c +
d*x]*Csc[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc(c+dx)\,dx + b\int \cot^2(c+dx)\,dx$$
$$= -\frac{b\cot(c+dx)}{d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d} - \frac{1}{2}a\int \csc(c+dx$$
$$= -bx + \frac{a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{b\cot(c+dx)}{d} - \frac{a\cot(c+dx)\,c}{2d}$$

Mathematica [C] time = 0.05, size = 109, normalized size = 2.10

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{b\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (b*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1,
1/2, -Tan[c + d*x]^2])/d + (a*Log[Cos[(c + d*x)/2]])/(2*d) - (a*Log[Sin[(c
+ d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

fricas [B] time = 0.81, size = 114, normalized size = 2.19

$$-\frac{4\,bdx\cos\left(dx+c\right)^2-4\,bdx-4\,b\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a\cos\left(dx+c\right)-\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\cos\left(dx\right.\right.}{4\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

[Out] -1/4*(4*b*d*x*cos(d*x + c)^2 - 4*b*d*x - 4*b*cos(d*x + c)*sin(d*x + c) - 2*a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + (a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)

giac [A] time = 0.17, size = 95, normalized size = 1.83

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,(dx+c)b-4\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 - 8*(d*x + c)*b - 4*a*log(abs(tan(1/2*d*x + 1/2*c))) + 4*b*tan(1/2*d*x + 1/2*c) + (6*a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^2)/d
```

maple [A] time = 0.33, size = 81, normalized size = 1.56

$$-\frac{a\left(\cos^3(dx+c)\right)}{2d\sin(dx+c)^2}-\frac{a\cos(dx+c)}{2d}-\frac{a\ln(\csc(dx+c)-\cot(dx+c))}{2d}-bx-\frac{b\cot(dx+c)}{d}-\frac{bc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^3-1/2*a*cos(d*x+c)/d-1/2/d*a*ln(csc(d*x+c)-cot(d*x+c))-b*x-b*cot(d*x+c)/d-b*c/d
```

maxima [A] time = 0.50, size = 66, normalized size = 1.27

$$-\frac{4\left(dx+c+\frac{1}{\tan(dx+c)}\right)b-a\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c + 1/tan(d*x + c))*b - a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.36, size = 151, normalized size = 2.90

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{b\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2\,d}-\frac{2\,b\,\mathrm{atan}\left(\frac{2\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a\tan\left(\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(2*d) - (b*cot(c/2 + (d*x)/2))/(2*d) - (a*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*d) - (2*b*atan((2*b*cos(c/2 + (d*x)/2)
 + a*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2) - 2*b*sin(c/2 + (d*x)/2))))/
d - (a*cot(c/2 + (d*x)/2)^2)/(8*d) + (a*tan(c/2 + (d*x)/2)^2)/(8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))\cos^2(c+dx)\csc^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**2*csc(c + d*x)**3, x)
```

3.1056 $\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] 1/2*b*arctanh(cos(d*x+c))/d-1/3*a*cot(d*x+c)^3/d-1/2*b*cot(d*x+c)*csc(d*x+c
)/d
```

Rubi [A] time = 0.11, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2838, 2607, 30, 2611, 3770}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x]^3)/(3*d) - (b*Cot[c + d*x
]*Csc[c + d*x])/(2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^2(c+dx)\,dx + b\int \cot^2(c+dx)\csc(c+
$$
$$= -\frac{b\cot(c+dx)\csc(c+dx)}{2d} - \frac{1}{2}b\int \csc(c+dx)\,dx + \frac{a\,\mathrm{Subst}}{}$$
$$= \frac{b\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\cot(c+dx)\csc(c}{2d}$$

Mathematica [A] time = 0.04, size = 95, normalized size = 1.83

$$-\frac{a\cot^3(c+dx)}{3d} - \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/3*(a*Cot[c + d*x]^3)/d - (b*Csc[(c + d*x)/2]^2)/(8*d) + (b*Log[Cos[(c +
d*x)/2]])/(2*d) - (b*Log[Sin[(c + d*x)/2]])/(2*d) + (b*Sec[(c + d*x)/2]^2)/
(8*d)
```

fricas [B] time = 0.67, size = 119, normalized size = 2.29

$$\frac{4\,a\cos\left(dx+c\right)^3 + 6\,b\cos\left(dx+c\right)\sin\left(dx+c\right) + 3\left(b\cos\left(dx+c\right)^2 - b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right) + \frac{1}{2}\right)\sin\left(dx+c\right) - 3\,(}{12\left(d\cos\left(dx+c\right)^2 - d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*cos(d*x + c)^3 + 6*b*cos(d*x + c)*sin(d*x + c) + 3*(b*cos(d*x + c
)^2 - b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(b*cos(d*x + c)^2 - b
)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*
x + c))
```

giac [B] time = 0.19, size = 115, normalized size = 2.21

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{22\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*b*tan(1/2*d*x + 1/2*c)^2 - 12*b*log(abs(
tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x + 1/2*c) + (22*b*tan(1/2*d*x + 1/2
*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 3*b*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*
d*x + 1/2*c)^3)/d
```

maple [A] time = 0.40, size = 80, normalized size = 1.54

$$-\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)^{3}}-\frac{b\left(\cos^{3}\left(dx+c\right)\right)}{2d\sin\left(dx+c\right)^{2}}-\frac{b\cos\left(dx+c\right)}{2d}-\frac{b\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^3-1/2/d*b/sin(d*x+c)^2*cos(d*x+c)^3-1/2*b*
cos(d*x+c)/d-1/2/d*b*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.38, size = 61, normalized size = 1.17

$$\frac{3\,b\left(\frac{2\,\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{2}-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{4\,a}{\tan\left(dx+c\right)^{3}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - lo
g(cos(d*x + c) - 1)) - 4*a/tan(d*x + c)^3)/d
```

mupad [B] time = 9.31, size = 111, normalized size = 2.13

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2))/(8*d) + (b*tan(c/2
 + (d*x)/2)^2)/(8*d) - (b*log(tan(c/2 + (d*x)/2)))/(2*d) - (cot(c/2 + (d*x)
/2)^3*(a/3 + b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))\cos^2(c+dx)\csc^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**2*csc(c + d*x)**4, x)
```

3.1057 $\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^3(c+dx)}{3d}$$

```
[Out] 1/8*a*arctanh(cos(d*x+c))/d-1/3*b*cot(d*x+c)^3/d+1/8*a*cot(d*x+c)*csc(d*x+c
)/d-1/4*a*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A] time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 30}

$$\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(8*d) - (b*Cot[c + d*x]^3)/(3*d) + (a*Cot[c + d*x
]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^3(c+dx)\,dx + b\int \cot^2(c+dx)\csc^2(c+$$

$$= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}a\int \csc^3(c+dx)\,dx + \frac{b\,\mathrm{Subs}}{}$$

$$= -\frac{b\cot^3(c+dx)}{3d} + \frac{a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)\,\mathrm{c}}{4d}$$

$$= \frac{a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b\cot^3(c+dx)}{3d} + \frac{a\cot(c+dx)\csc(c}{8d}$$

Mathematica [A] time = 0.04, size = 135, normalized size = 1.82

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{a\log\Big(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/3*(b*Cot[c + d*x]^3)/d + (a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x
)/2]^4)/(64*d) + (a*Log[Cos[(c + d*x)/2]])/(8*d) - (a*Log[Sin[(c + d*x)/2]]
)/(8*d) - (a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

fricas [B] time = 0.74, size = 137, normalized size = 1.85

$$-\frac{16\,b\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+6\,a\cos\left(dx+c\right)^{3}+6\,a\cos\left(dx+c\right)-3\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)}{48\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(16*b*cos(d*x + c)^3*sin(d*x + c) + 6*a*cos(d*x + c)^3 + 6*a*cos(d*x
+ c) - 3*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) +
 1/2) + 3*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c)
 + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.19, size = 116, normalized size = 1.57

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{50\,a\tan\left(\frac{1}{2}\,dx+\right.}{}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*log(a
bs(tan(1/2*d*x + 1/2*c))) - 24*b*tan(1/2*d*x + 1/2*c) + (50*a*tan(1/2*d*x +
 1/2*c)^4 + 24*b*tan(1/2*d*x + 1/2*c)^3 - 8*b*tan(1/2*d*x + 1/2*c) - 3*a)/t
an(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.32, size = 102, normalized size = 1.38

$$-\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{4d\sin\left(dx+c\right)^{4}}-\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^{2}}-\frac{a\cos\left(dx+c\right)}{8d}-\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{8d}-\frac{b\left(\cos^{3}\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^3-1/8/d*a/sin(d*x+c)^2*cos(d*x+c)^3-1/8*a*
cos(d*x+c)/d-1/8/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*b/sin(d*x+c)^3*cos(d*x
+c)^3
```

maxima [A] time = 0.31, size = 80, normalized size = 1.08

$$-\frac{3\,a\left(\frac{2\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\,b}{\tan\left(dx+c\right)^{3}}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/48*(3*a*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x +
 c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 16*b/tan(d*x
+ c)^3)/d
```

mupad [B] time = 9.33, size = 112, normalized size = 1.51

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{2\,b}{\cdots}\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^4)/(64*d) - (b*tan(c/2 + (d*x)/2))/(8*d) + (b*tan(c/2
 + (d*x)/2)^3)/(24*d) - (a*log(tan(c/2 + (d*x)/2)))/(8*d) - (cot(c/2 + (d*x
)/2)^4*(a/4 + (2*b*tan(c/2 + (d*x)/2))/3 - 2*b*tan(c/2 + (d*x)/2)^3))/(16*d
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1058 $\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=90

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{3d}+\frac{b\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{b\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] 1/8*b*arctanh(cos(d*x+c))/d-1/3*a*cot(d*x+c)^3/d-1/5*a*cot(d*x+c)^5/d+1/8*b
*cot(d*x+c)*csc(d*x+c)/d-1/4*b*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A] time = 0.13, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2607, 14, 2611, 3768, 3770}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{3d}+\frac{b\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{b\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^3)/(3*d) - (a*Cot[c + d*x
]^5)/(5*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (b*Cot[c + d*x]*Csc[c +
d*x]^3)/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^4(c+dx)\,dx + b\int \cot^2(c+dx)\csc^3(c+$$

$$= -\frac{b\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}b\int \csc^3(c+dx)\,dx + \frac{a\,\mathrm{Subs}}{}$$

$$= \frac{b\cot(c+dx)\csc(c+dx)}{8d} - \frac{b\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{8}b\int$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} + \frac{b}{}$$

Mathematica [A] time = 0.07, size = 177, normalized size = 1.97

$$\frac{2a\cot(c+dx)}{15d} - \frac{a\cot(c+dx)\csc^4(c+dx)}{5d} + \frac{a\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a*Cot[c + d*x])/(15*d) + (b*Csc[(c + d*x)/2]^2)/(32*d) - (b*Csc[(c + d*x
)/2]^4)/(64*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*Cot[c + d*x]*C
```

```
sc[c + d*x]^4)/(5*d) + (b*Log[Cos[(c + d*x)/2]])/(8*d) - (b*Log[Sin[(c + d*
x)/2]])/(8*d) - (b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)/(64*
d)
```

fricas [B] time = 0.74, size = 169, normalized size = 1.88

$$\frac{32\,a\cos\left(dx+c\right)^{5}-80\,a\cos\left(dx+c\right)^{3}+15\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+b\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin}{240\left(d\cos\left(d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(32*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 15*(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(b*cos
(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 30*(b*cos(d*x + c)^3 + b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.18, size = 144, normalized size = 1.60

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-120\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-60\,a\tan}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*b*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(
1/2*d*x + 1/2*c)^3 - 120*b*log(abs(tan(1/2*d*x + 1/2*c))) - 60*a*tan(1/2*d*
x + 1/2*c) + (274*b*tan(1/2*d*x + 1/2*c)^5 + 60*a*tan(1/2*d*x + 1/2*c)^4 -
10*a*tan(1/2*d*x + 1/2*c)^2 - 15*b*tan(1/2*d*x + 1/2*c) - 6*a)/tan(1/2*d*x
+ 1/2*c)^5)/d
```

maple [A] time = 0.32, size = 124, normalized size = 1.38

$$-\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)^{5}}-\frac{2a\left(\cos^{3}\left(dx+c\right)\right)}{15d\sin\left(dx+c\right)^{3}}-\frac{b\left(\cos^{3}\left(dx+c\right)\right)}{4d\sin\left(dx+c\right)^{4}}-\frac{b\left(\cos^{3}\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^{2}}-\frac{b\cos\left(dx+c\right)}{8d}-\frac{b\ln\left(\csc\left(dx+c\right)-\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^3-2/15/d*a/sin(d*x+c)^3*cos(d*x+c)^3-1/4/d
*b/sin(d*x+c)^4*cos(d*x+c)^3-1/8/d*b/sin(d*x+c)^2*cos(d*x+c)^3-1/8*b*cos(d*
x+c)/d-1/8/d*b*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.41, size = 92, normalized size = 1.02

$$-\frac{15\,b\left(\frac{2\,\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\,\left(5\,\tan\left(dx+c\right)^2+3\right)a}{\tan\left(dx+c\right)^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(15*b*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x
 + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 16*(5*tan(d
*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

mupad [B] time = 9.33, size = 143, normalized size = 1.59

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(-2\,a\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^3)/(96*d) - (a*tan(c/2 + (d*x)/2))/(16*d) + (a*tan(c/
2 + (d*x)/2)^5)/(160*d) + (b*tan(c/2 + (d*x)/2)^4)/(64*d) - (b*log(tan(c/2
+ (d*x)/2)))/(8*d) - (cot(c/2 + (d*x)/2)^5*(a/5 + (b*tan(c/2 + (d*x)/2))/2
+ (a*tan(c/2 + (d*x)/2)^2)/3 - 2*a*tan(c/2 + (d*x)/2)^4))/(32*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1059 $\int \cos^2(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=190

$$\frac{(7a^2+4b^2)\cos^3(c+dx)}{105d}-\frac{(7a^2+4b^2)\cos(c+dx)}{35d}+\frac{(2a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{35d}+\frac{ab\sin^5(c+dx)\cos(c-}{21d}$$

```
[Out] 1/8*a*b*x-1/35*(7*a^2+4*b^2)*cos(d*x+c)/d+1/105*(7*a^2+4*b^2)*cos(d*x+c)^3/
d-1/8*a*b*cos(d*x+c)*sin(d*x+c)/d-1/12*a*b*cos(d*x+c)*sin(d*x+c)^3/d+1/35*(
2*a^2-b^2)*cos(d*x+c)*sin(d*x+c)^4/d+1/21*a*b*cos(d*x+c)*sin(d*x+c)^5/d+1/7
*cos(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c))^2/d
```

Rubi [A] time = 0.38, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2889, 3050, 3033, 3023, 2748, 2633, 2635, 8}

$$\frac{(7a^2+4b^2)\cos^3(c+dx)}{105d}-\frac{(7a^2+4b^2)\cos(c+dx)}{35d}+\frac{(2a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{35d}+\frac{ab\sin^5(c+dx)\cos(c-}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*x)/8 - ((7*a^2 + 4*b^2)*Cos[c + d*x])/(35*d) + ((7*a^2 + 4*b^2)*Cos[c
+ d*x]^3)/(105*d) - (a*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a*b*Cos[c + d*
x]*Sin[c + d*x]^3)/(12*d) + ((2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(35
*d) + (a*b*Cos[c + d*x]*Sin[c + d*x]^5)/(21*d) + (Cos[c + d*x]*Sin[c + d*x]
^4*(a + b*Sin[c + d*x])^2)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rubi steps

$$\int \cos^2(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx = \int \sin^3(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\,dx$$
$$= \frac{\cos(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))^2}{7d} + \frac{1}{7}\int \sin^3(c$$
$$= \frac{ab\cos(c+dx)\sin^5(c+dx)}{21d} + \frac{\cos(c+dx)\sin^4(c+dx)(a+}{7d}$$
$$= \frac{\left(2a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{35d} + \frac{ab\cos(c+dx)\sin^5(}{21d}$$
$$= \frac{\left(2a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{35d} + \frac{ab\cos(c+dx)\sin^5(}{21d}$$
$$= -\frac{ab\cos(c+dx)\sin^3(c+dx)}{12d} + \frac{\left(2a^2-b^2\right)\cos(c+dx)\sin}{35d}$$
$$= -\frac{\left(7a^2+4b^2\right)\cos(c+dx)}{35d} + \frac{\left(7a^2+4b^2\right)\cos^3(c+dx)}{105d} - \frac{ab}{}$$
$$= \frac{abx}{8} - \frac{\left(7a^2+4b^2\right)\cos(c+dx)}{35d} + \frac{\left(7a^2+4b^2\right)\cos^3(c+dx)}{105d}$$

Mathematica [A] time = 0.55, size = 132, normalized size = 0.69

$$\frac{-105\left(8a^2+5b^2\right)\cos(c+dx) - 35\left(4a^2+b^2\right)\cos(3(c+dx)) + 84a^2\cos(5(c+dx)) - 210ab\sin(2(c+dx)) - 210}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (840*a*b*c + 840*a*b*d*x - 105*(8*a^2 + 5*b^2)*Cos[c + d*x] - 35*(4*a^2 + b
^2)*Cos[3*(c + d*x)] + 84*a^2*Cos[5*(c + d*x)] + 63*b^2*Cos[5*(c + d*x)] -
15*b^2*Cos[7*(c + d*x)] - 210*a*b*Sin[2*(c + d*x)] - 210*a*b*Sin[4*(c + d*x
)] + 70*a*b*Sin[6*(c + d*x)])/(6720*d)
```

fricas [A] time = 0.76, size = 104, normalized size = 0.55

$$-\frac{120\,b^2\cos(dx+c)^7 - 168\left(a^2+2\,b^2\right)\cos(dx+c)^5 - 105\,abdx + 280\left(a^2+b^2\right)\cos(dx+c)^3 - 35\left(8\,ab\cos(dx\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/840*(120*b^2*cos(d*x + c)^7 - 168*(a^2 + 2*b^2)*cos(d*x + c)^5 - 105*a*b
*d*x + 280*(a^2 + b^2)*cos(d*x + c)^3 - 35*(8*a*b*cos(d*x + c)^5 - 14*a*b*c
os(d*x + c)^3 + 3*a*b*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.24, size = 141, normalized size = 0.74

$$\frac{1}{8}abx - \frac{b^2\cos(7dx+7c)}{448d} + \frac{ab\sin(6dx+6c)}{96d} - \frac{ab\sin(4dx+4c)}{32d} - \frac{ab\sin(2dx+2c)}{32d} + \frac{(4a^2+3b^2)\cos(5dx+5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*a*b*x - 1/448*b^2*cos(7*d*x + 7*c)/d + 1/96*a*b*sin(6*d*x + 6*c)/d - 1/
32*a*b*sin(4*d*x + 4*c)/d - 1/32*a*b*sin(2*d*x + 2*c)/d + 1/320*(4*a^2 + 3*
b^2)*cos(5*d*x + 5*c)/d - 1/192*(4*a^2 + b^2)*cos(3*d*x + 3*c)/d - 1/64*(8*
a^2 + 5*b^2)*cos(d*x + c)/d
```

maple [A] time = 0.20, size = 150, normalized size = 0.79

$$\frac{a^2\left(-\frac{(\sin^2(dx+c))(\cos^3(dx+c))}{5} - \frac{2(\cos^3(dx+c))}{15}\right) + 2ab\left(-\frac{(\sin^3(dx+c))(\cos^3(dx+c))}{6} - \frac{\sin(dx+c)(\cos^3(dx+c))}{8} + \frac{\cos(dx+c)\sin(dx+c)}{16} + \frac{d}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+2*a*b*(-1/6*sin
(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*cos(d*x+c)*sin(d*x+
c)+1/16*d*x+1/16*c)+b^2*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*c
os(d*x+c)^3-8/105*cos(d*x+c)^3))
```

maxima [A] time = 0.40, size = 104, normalized size = 0.55

$$\frac{224\left(3\cos(dx+c)^5 - 5\cos(dx+c)^3\right)a^2 - 35\left(4\sin(2dx+2c)^3 - 12dx - 12c + 3\sin(4dx+4c)\right)ab - 32(15}{3360d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3360*(224*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2 - 35*(4*sin(2*d*x + 2
*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a*b - 32*(15*cos(d*x + c)^7 - 4
2*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*b^2)/d
```

mupad [B] time = 12.97, size = 233, normalized size = 1.23

$$\frac{a\,b\,x}{8}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{8\,a^2}{3}-\frac{16\,b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{8\,a^2}{5}+\frac{16\,b^2}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(\frac{20\,a^2}{3}+\frac{32\,b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (a*b*x)/8 - (tan(c/2 + (d*x)/2)^6*((8*a^2)/3 - (16*b^2)/3) + tan(c/2 + (d*x
)/2)^4*((8*a^2)/5 + (16*b^2)/5) + tan(c/2 + (d*x)/2)^8*((20*a^2)/3 + (32*b^
2)/3) + tan(c/2 + (d*x)/2)^2*((28*a^2)/15 + (16*b^2)/15) + 4*a^2*tan(c/2 +
(d*x)/2)^10 + (4*a^2)/15 + (16*b^2)/105 + (5*a*b*tan(c/2 + (d*x)/2)^3)/3 -
(97*a*b*tan(c/2 + (d*x)/2)^5)/12 + (97*a*b*tan(c/2 + (d*x)/2)^9)/12 - (5*a*
b*tan(c/2 + (d*x)/2)^11)/3 - (a*b*tan(c/2 + (d*x)/2)^13)/4 + (a*b*tan(c/2 +
 (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [A] time = 5.91, size = 275, normalized size = 1.45

$$\begin{cases} -\frac{a^2\sin^2(c+dx)\cos^3(c+dx)}{3d}-\frac{2a^2\cos^5(c+dx)}{15d}+\frac{abx\sin^6(c+dx)}{8}+\frac{3abx\sin^4(c+dx)\cos^2(c+dx)}{8}+\frac{3abx\sin^2(c+dx)\cos^4(c+dx)}{8}+\frac{abx\cos^6(}{8} \\ x\left(a+b\sin(c)\right)^2\sin^3(c)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*a**2*cos(c + d*x
)**5/(15*d) + a*b*x*sin(c + d*x)**6/8 + 3*a*b*x*sin(c + d*x)**4*cos(c + d*x
)**2/8 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + a*b*x*cos(c + d*x)**6/
8 + a*b*sin(c + d*x)**5*cos(c + d*x)/(8*d) - a*b*sin(c + d*x)**3*cos(c + d*
x)**3/(3*d) - a*b*sin(c + d*x)*cos(c + d*x)**5/(8*d) - b**2*sin(c + d*x)**4
*cos(c + d*x)**3/(3*d) - 4*b**2*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) - 8*
b**2*cos(c + d*x)**7/(105*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**3*cos
(c)**2, True))
```

3.1060 $\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=163

$$\frac{(2a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{(2a^2+b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(2a^2+b^2\right)+\frac{2ab\cos^3(c+dx)}{15d}-\frac{2ab\,c}{}$$

```
[Out] 1/16*(2*a^2+b^2)*x-2/5*a*b*cos(d*x+c)/d+2/15*a*b*cos(d*x+c)^3/d-1/16*(2*a^2
+b^2)*cos(d*x+c)*sin(d*x+c)/d+1/24*(2*a^2-b^2)*cos(d*x+c)*sin(d*x+c)^3/d+1/
15*a*b*cos(d*x+c)*sin(d*x+c)^4/d+1/6*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c
))^2/d
```

Rubi [A] time = 0.38, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3050, 3033, 3023, 2748, 2635, 8, 2633}

$$\frac{(2a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{(2a^2+b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(2a^2+b^2\right)+\frac{2ab\cos^3(c+dx)}{15d}-\frac{2ab\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/16 - (2*a*b*Cos[c + d*x])/(5*d) + (2*a*b*Cos[c + d*x]^3)/
(15*d) - ((2*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((2*a^2 - b^2)*
Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) + (a*b*Cos[c + d*x]*Sin[c + d*x]^4)/(15
*d) + (Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx = \int \sin^2(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\,dx$$
$$= \frac{\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2}{6d} + \frac{1}{6}\int \sin^2(c+$$
$$= \frac{ab\cos(c+dx)\sin^4(c+dx)}{15d} + \frac{\cos(c+dx)\sin^3(c+dx)(a+}{6d}$$
$$= \frac{\left(2a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{24d} + \frac{ab\cos(c+dx)\sin^4(c}{15d}$$
$$= \frac{\left(2a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{24d} + \frac{ab\cos(c+dx)\sin^4(c}{15d}$$
$$= -\frac{\left(2a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{16d} + \frac{\left(2a^2-b^2\right)\cos(c+d}{24d}$$
$$= \frac{1}{16}\left(2a^2+b^2\right)x - \frac{2ab\cos(c+dx)}{5d} + \frac{2ab\cos^3(c+dx)}{15d} - \frac{(2a}{}$$

Mathematica [A] time = 0.22, size = 120, normalized size = 0.74

$$\frac{-30a^2\sin(4(c+dx))+120a^2c+120a^2dx-240ab\cos(c+dx)-40ab\cos(3(c+dx))+24ab\cos(5(c+dx))-15b}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (120*a^2*c + 60*b^2*c + 120*a^2*d*x + 60*b^2*d*x - 240*a*b*Cos[c + d*x] - 4
0*a*b*Cos[3*(c + d*x)] + 24*a*b*Cos[5*(c + d*x)] - 15*b^2*Sin[2*(c + d*x)]
- 30*a^2*Sin[4*(c + d*x)] - 15*b^2*Sin[4*(c + d*x)] + 5*b^2*Sin[6*(c + d*x)
])/(960*d)
```

fricas [A] time = 0.64, size = 103, normalized size = 0.63

$$\frac{96\,ab\cos(dx+c)^5-160\,ab\cos(dx+c)^3+15\left(2\,a^2+b^2\right)dx+5\left(8\,b^2\cos(dx+c)^5-2\left(6\,a^2+7\,b^2\right)\cos(dx+c)^3\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

[Out] 1/240*(96*a*b*cos(d*x + c)^5 - 160*a*b*cos(d*x + c)^3 + 15*(2*a^2 + b^2)*d*x + 5*(8*b^2*cos(d*x + c)^5 - 2*(6*a^2 + 7*b^2)*cos(d*x + c)^3 + 3*(2*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d

giac [A] time = 0.24, size = 115, normalized size = 0.71

$$\frac{1}{16}\left(2\,a^2+b^2\right)x+\frac{ab\cos\left(5\,dx+5\,c\right)}{40\,d}-\frac{ab\cos\left(3\,dx+3\,c\right)}{24\,d}-\frac{ab\cos\left(dx+c\right)}{4\,d}+\frac{b^2\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{b^2\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/16*(2*a^2 + b^2)*x + 1/40*a*b*cos(5*d*x + 5*c)/d - 1/24*a*b*cos(3*d*x + 3*c)/d - 1/4*a*b*cos(d*x + c)/d + 1/192*b^2*sin(6*d*x + 6*c)/d - 1/64*b^2*sin(2*d*x + 2*c)/d - 1/64*(2*a^2 + b^2)*sin(4*d*x + 4*c)/d

maple [A] time = 0.22, size = 141, normalized size = 0.87

$$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+b^2\left(-\frac{\left(\sin^3(dx\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x)

[Out] 1/d*(a^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)+2*a*b*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+b^2*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c))

maxima [A] time = 0.39, size = 92, normalized size = 0.56

$$\frac{30\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^2+128\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)ab-5\left(4\,\sin\left(2\,dx+2\,c\right)^3-12\,dx-\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/960*(30*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2 + 128*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a*b - 5*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*b^2)/d

mupad [B] time = 9.62, size = 112, normalized size = 0.69

$$-\frac{\frac{15\,a^2\,\sin(4\,c+4\,d\,x)}{2}+\frac{15\,b^2\,\sin(2\,c+2\,d\,x)}{4}+\frac{15\,b^2\,\sin(4\,c+4\,d\,x)}{4}-\frac{5\,b^2\,\sin(6\,c+6\,d\,x)}{4}+60\,a\,b\,\cos\left(c+d\,x\right)+10\,a\,b\,\cos\left(3\,c+3\,d\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

```
[Out] -((15*a^2*sin(4*c + 4*d*x))/2 + (15*b^2*sin(2*c + 2*d*x))/4 + (15*b^2*sin(4
*c + 4*d*x))/4 - (5*b^2*sin(6*c + 6*d*x))/4 + 60*a*b*cos(c + d*x) + 10*a*b*
cos(3*c + 3*d*x) - 6*a*b*cos(5*c + 5*d*x) - 30*a^2*d*x - 15*b^2*d*x)/(240*d
)
```

sympy [A] time = 3.57, size = 309, normalized size = 1.90

$$\begin{cases} \frac{a^2x\sin^4(c+dx)}{8}+\frac{a^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{a^2x\cos^4(c+dx)}{8}+\frac{a^2\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{2ab\sin^2(c+dx)\cos^3}{3d} \\ x\left(a+b\sin(c)\right)^2\sin^2(c)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**4/8 + a**2*x*sin(c + d*x)**2*cos(c + d*x)**
2/4 + a**2*x*cos(c + d*x)**4/8 + a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) -
a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*a*b*sin(c + d*x)**2*cos(c + d*x
)**3/(3*d) - 4*a*b*cos(c + d*x)**5/(15*d) + b**2*x*sin(c + d*x)**6/16 + 3*b
**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*b**2*x*sin(c + d*x)**2*cos(c +
 d*x)**4/16 + b**2*x*cos(c + d*x)**6/16 + b**2*sin(c + d*x)**5*cos(c + d*x)
/(16*d) - b**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - b**2*sin(c + d*x)*co
s(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**2*cos(c)**2,
True))
```

3.1061 $\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=106

$$-\frac{\left(a^2+4b^2\right)\cos^3(c+dx)}{30d}-\frac{\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d}-\frac{a\cos^3(c+dx)(a+b\sin(c+dx))}{10d}+\frac{ab\sin(c+dx)\,c}{4d}$$

```
[Out] 1/4*a*b*x-1/30*(a^2+4*b^2)*cos(d*x+c)^3/d+1/4*a*b*cos(d*x+c)*sin(d*x+c)/d-1
/10*a*cos(d*x+c)^3*(a+b*sin(d*x+c))/d-1/5*cos(d*x+c)^3*(a+b*sin(d*x+c))^2/d
```

Rubi [A] time = 0.16, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2862, 2669, 2635, 8}

$$-\frac{\left(a^2+4b^2\right)\cos^3(c+dx)}{30d}-\frac{\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d}-\frac{a\cos^3(c+dx)(a+b\sin(c+dx))}{10d}+\frac{ab\sin(c+dx)\,c}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*x)/4 - ((a^2 + 4*b^2)*Cos[c + d*x]^3)/(30*d) + (a*b*Cos[c + d*x]*Sin[c
 + d*x])/(4*d) - (a*Cos[c + d*x]^3*(a + b*Sin[c + d*x]))/(10*d) - (Cos[c +
d*x]^3*(a + b*Sin[c + d*x])^2)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d} + \frac{1}{5}\int \cos^2(c+dx)(2b+$$
$$= -\frac{a\cos^3(c+dx)(a+b\sin(c+dx))}{10d} - \frac{\cos^3(c+dx)(a+b\sin(c+dx)}{5d}$$
$$= -\frac{\left(a^2+4b^2\right)\cos^3(c+dx)}{30d} - \frac{a\cos^3(c+dx)(a+b\sin(c+dx))}{10d}$$
$$= -\frac{\left(a^2+4b^2\right)\cos^3(c+dx)}{30d} + \frac{ab\cos(c+dx)\sin(c+dx)}{4d} - \frac{a\,c}{}$$
$$= \frac{abx}{4} - \frac{\left(a^2+4b^2\right)\cos^3(c+dx)}{30d} + \frac{ab\cos(c+dx)\sin(c+dx)}{4d}$$

Mathematica [A] time = 0.37, size = 77, normalized size = 0.73

$$\frac{-30\left(2a^2+b^2\right)\cos(c+dx)-5\left(4a^2+b^2\right)\cos(3(c+dx))+3b(20a(c+dx)-5a\sin(4(c+dx))+b\cos(5(c+dx)))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-30*(2*a^2 + b^2)*Cos[c + d*x] - 5*(4*a^2 + b^2)*Cos[3*(c + d*x)] + 3*b*(2
0*a*(c + d*x) + b*Cos[5*(c + d*x)] - 5*a*Sin[4*(c + d*x)]))/(240*d)
```

fricas [A] time = 0.86, size = 73, normalized size = 0.69

$$\frac{12\,b^2\cos\left(dx+c\right)^5+15\,abdx-20\left(a^2+b^2\right)\cos\left(dx+c\right)^3-15\left(2\,ab\cos\left(dx+c\right)^3-ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^2*cos(d*x + c)^5 + 15*a*b*d*x - 20*(a^2 + b^2)*cos(d*x + c)^3 -
15*(2*a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.18, size = 82, normalized size = 0.77

$$\frac{1}{4}\,abx+\frac{b^2\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{ab\sin\left(4\,dx+4\,c\right)}{16\,d}-\frac{\left(4\,a^2+b^2\right)\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{\left(2\,a^2+b^2\right)\cos\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*a*b*x + 1/80*b^2*cos(5*d*x + 5*c)/d - 1/16*a*b*sin(4*d*x + 4*c)/d - 1/4
8*(4*a^2 + b^2)*cos(3*d*x + 3*c)/d - 1/8*(2*a^2 + b^2)*cos(d*x + c)/d
```

maple [A] time = 0.19, size = 94, normalized size = 0.89

$$\frac{-\frac{a^2\left(\cos^3(dx+c)\right)}{3}+2ab\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+b^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/3*a^2*cos(d*x+c)^3+2*a*b*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+
c)*sin(d*x+c)+1/8*d*x+1/8*c)+b^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d
*x+c)^3))
```

maxima [A] time = 0.38, size = 68, normalized size = 0.64

$$-\frac{80\,a^2\cos\left(dx+c\right)^3-15\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)ab-16\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)b^2}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(80*a^2*cos(d*x + c)^3 - 15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a*b - 1
6*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*b^2)/d
```

mupad [B] time = 12.77, size = 180, normalized size = 1.70

$$\frac{a\,b\,x}{4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(4\,a^2+4\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{4\,a^2}{3}+\frac{4\,b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{8\,a^2}{3}-\frac{4\,b^2}{3}\right)+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (a*b*x)/4 - (tan(c/2 + (d*x)/2)^6*(4*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^2*((
4*a^2)/3 + (4*b^2)/3) + tan(c/2 + (d*x)/2)^4*((8*a^2)/3 - (4*b^2)/3) + 2*a^
2*tan(c/2 + (d*x)/2)^8 + (2*a^2)/3 + (4*b^2)/15 - 3*a*b*tan(c/2 + (d*x)/2)^
3 + 3*a*b*tan(c/2 + (d*x)/2)^7 - (a*b*tan(c/2 + (d*x)/2)^9)/2 + (a*b*tan(c/
2 + (d*x)/2))/2)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

sympy [A] time = 1.96, size = 172, normalized size = 1.62

$$\begin{cases} -\frac{a^2\cos^3(c+dx)}{3d} + \frac{abx\sin^4(c+dx)}{4} + \frac{abx\sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{abx\cos^4(c+dx)}{4} + \frac{ab\sin^3(c+dx)\cos(c+dx)}{4d} - \frac{ab\sin(c+dx)\cos^3(c+dx)}{4d} - \\ x\,(a+b\sin(c))^2\sin(c)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)**3/(3*d) + a*b*x*sin(c + d*x)**4/4 + a*b*x*si
n(c + d*x)**2*cos(c + d*x)**2/2 + a*b*x*cos(c + d*x)**4/4 + a*b*sin(c + d*x
)**3*cos(c + d*x)/(4*d) - a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) - b**2*sin
(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*b**2*cos(c + d*x)**5/(15*d), Ne(d, 0
)), (x*(a + b*sin(c))**2*sin(c)*cos(c)**2, True))
```

3.1062 $\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=90

$$\frac{\left(2a^2-b^2\right)\cos(c+dx)}{3d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{ab\sin(c+dx)\cos(c+dx)}{3d}+\frac{\cos(c+dx)(a+b\sin(c+dx))^2}{3d}+a$$

```
[Out] a*b*x-a^2*arctanh(cos(d*x+c))/d+1/3*(2*a^2-b^2)*cos(d*x+c)/d+1/3*a*b*cos(d*
x+c)*sin(d*x+c)/d+1/3*cos(d*x+c)*(a+b*sin(d*x+c))^2/d
```

Rubi [A] time = 0.25, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2889, 3050, 3033, 3023, 2735, 3770}

$$\frac{\left(2a^2-b^2\right)\cos(c+dx)}{3d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{ab\sin(c+dx)\cos(c+dx)}{3d}+\frac{\cos(c+dx)(a+b\sin(c+dx))^2}{3d}+a$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] a*b*x - (a^2*ArcTanh[Cos[c + d*x]])/d + ((2*a^2 - b^2)*Cos[c + d*x])/(3*d)
+ (a*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (Cos[c + d*x]*(a + b*Sin[c + d*x]
)^2)/(3*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx = \int \csc(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\,dx$$
$$= \frac{\cos(c+dx)(a+b\sin(c+dx))^2}{3d} + \frac{1}{3}\int \csc(c+dx)(a+b\sin$$
$$= \frac{ab\cos(c+dx)\sin(c+dx)}{3d} + \frac{\cos(c+dx)(a+b\sin(c+dx))^2}{3d}$$
$$= \frac{\left(2a^2-b^2\right)\cos(c+dx)}{3d} + \frac{ab\cos(c+dx)\sin(c+dx)}{3d} + \frac{\cos(c$$
$$= abx + \frac{\left(2a^2-b^2\right)\cos(c+dx)}{3d} + \frac{ab\cos(c+dx)\sin(c+dx)}{3d} +$$
$$= abx - \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{\left(2a^2-b^2\right)\cos(c+dx)}{3d} + \frac{ab$$

Mathematica [A] time = 0.22, size = 91, normalized size = 1.01

$$\frac{3\left(4a^2-b^2\right)\cos(c+dx) + 6a\left(2a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + b\sin(2(c+dx)) + 2bc + 2b}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*(4*a^2 - b^2)*Cos[c + d*x] - b^2*Cos[3*(c + d*x)] + 6*a*(2*b*c + 2*b*d*x
 - 2*a*Log[Cos[(c + d*x)/2]] + 2*a*Log[Sin[(c + d*x)/2]] + b*Sin[2*(c + d*x
)]))/(12*d)
```

fricas [A] time = 0.77, size = 84, normalized size = 0.93

$$-\frac{2\,b^2\cos\left(dx+c\right)^3 - 6\,abdx - 6\,ab\cos\left(dx+c\right)\sin\left(dx+c\right) - 6\,a^2\cos\left(dx+c\right) + 3\,a^2\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right) -}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^2*cos(d*x + c)^3 - 6*a*b*d*x - 6*a*b*cos(d*x + c)*sin(d*x + c) -
6*a^2*cos(d*x + c) + 3*a^2*log(1/2*cos(d*x + c) + 1/2) - 3*a^2*log(-1/2*cos
(d*x + c) + 1/2))/d
```

giac [A] time = 0.20, size = 133, normalized size = 1.48

$$\frac{3\,(dx+c)ab+3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a*b + 3*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(3*a*b*tan(
1/2*d*x + 1/2*c)^5 - 3*a^2*tan(1/2*d*x + 1/2*c)^4 + 3*b^2*tan(1/2*d*x + 1/2
*c)^4 - 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2 +
 b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.40, size = 83, normalized size = 0.92

$$\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{a^2\cos(dx+c)}{d}+\frac{ab\cos(dx+c)\sin(dx+c)}{d}+abx+\frac{abc}{d}-\frac{b^2\left(\cos^3(dx+c)\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+a^2*cos(d*x+c)/d+a*b*cos(d*x+c)*sin(d*x+c
)/d+a*b*x+1/d*a*b*c-1/3*b^2*cos(d*x+c)^3/d
```

maxima [A] time = 0.31, size = 74, normalized size = 0.82

$$-\frac{2\,b^2\cos(dx+c)^3-3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))ab-3\,a^2\left(2\,\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(d\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^2*cos(d*x + c)^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b - 3*a^2
*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.87, size = 225, normalized size = 2.50

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(2\,a^2-2\,b^2\right)+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2-\frac{2\,b^2}{3}-2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\,a\,b}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^2)/sin(c + d*x),x)

[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)^4*(2*a^2 - 2*b^2) + 4*a^2*tan(c/2 + (d*x)/2)^2 + 2*a^2 - (2*b^2)/3 - 2*a*b*tan(c/2 + (d*x)/2)^5 + 2*a*b*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (2*a*b*atan((4*a^2*b^2)/(4*a^3*b - 4*a^2*b^2*tan(c/2 + (d*x)/2)) + (4*a^3*b*tan(c/2 + (d*x)/2))/(4*a^3*b - 4*a^2*b^2*tan(c/2 + (d*x)/2))))/d

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^2 \cos^2(c + dx)\csc(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+b*sin(d*x+c))**2,x)

[Out] Integral((a + b*sin(c + d*x))**2*cos(c + d*x)**2*csc(c + d*x), x)

3.1063 $\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=78

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{b^2x}{2}$$

```
[Out] -a^2*x+1/2*b^2*x-2*a*b*arctanh(cos(d*x+c))/d+2*a*b*cos(d*x+c)/d-a^2*cot(d*x
+c)/d+1/2*b^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2722, 2635, 8, 2592, 321, 206, 3473}

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (b^2*x)/2 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c + d*x
])/d - (a^2*Cot[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx &= \int \left(b^2\cos^2(c+dx)+2ab\cos(c+dx)\cot(c+dx)+a^2\cot^2(c+dx)\right)dx\\ &= a^2\int \cot^2(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot(c+dx)\,dx+b^2\int \cos^2\\ &= -\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{2d}-a^2\int 1\,dx+\frac{1}{2}b^2\int 1\,dx\\ &= -a^2x+\frac{b^2x}{2}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c\\ &= -a^2x+\frac{b^2x}{2}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\cot(c+\end{aligned}$$

Mathematica [A] time = 0.41, size = 116, normalized size = 1.49

$$\frac{2a^2\tan\left(\frac{1}{2}(c+dx)\right)-2a^2\cot\left(\frac{1}{2}(c+dx)\right)-4a^2c-4a^2dx+8ab\cos(c+dx)+8ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-8ab\,\mathrm{lo}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*a^2*c + 2*b^2*c - 4*a^2*d*x + 2*b^2*d*x + 8*a*b*Cos[c + d*x] - 2*a^2*Co
t[(c + d*x)/2] - 8*a*b*Log[Cos[(c + d*x)/2]] + 8*a*b*Log[Sin[(c + d*x)/2]]
+ b^2*Sin[2*(c + d*x)] + 2*a^2*Tan[(c + d*x)/2])/(4*d)
```

fricas [A] time = 0.78, size = 118, normalized size = 1.51

$$-\frac{b^2\cos(dx+c)^3 + 2\,ab\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 2\,ab\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) + \left(2\,a^2 \cdots\right.}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(b^2*cos(d*x + c)^3 + 2*a*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 2*a*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c) + ((2*a^2 - b^2)*d*x - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c)
)
```

giac [A] time = 0.19, size = 148, normalized size = 1.90

$$\frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) + a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - \left(2\,a^2-b^2\right)(dx+c) - \frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} - \frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a \cdots\right.}{\left(\tan \cdots\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + a^2*tan(1/2*d*x + 1/2*c) - (2*a
^2 - b^2)*(d*x + c) - (4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*
c) - 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan
(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.38, size = 102, normalized size = 1.31

$$-a^2x - \frac{a^2\cot(dx+c)}{d} - \frac{a^2c}{d} + \frac{2ab\cos(dx+c)}{d} + \frac{2ab\ln(\csc(dx+c)-\cot(dx+c))}{d} + \frac{b^2\cos(dx+c)\sin(dx+c)}{2d} + \frac{b^2 \cdots}{2 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -a^2*x-a^2*cot(d*x+c)/d-1/d*a^2*c+2*a*b*cos(d*x+c)/d+2/d*a*b*ln(csc(d*x+c)-
cot(d*x+c))+1/2*b^2*cos(d*x+c)*sin(d*x+c)/d+1/2*b^2*x+1/2/d*b^2*c
```

maxima [A] time = 0.42, size = 79, normalized size = 1.01

$$-\frac{4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2-(2\,dx+2\,c+\sin(2\,dx+2\,c))b^2-4\,ab\left(2\,\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/4*(4*(d*x + c + 1/tan(d*x + c))*a^2 - (2*d*x + 2*c + sin(2*d*x + 2*c))*b
^2 - 4*a*b*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1))
)/d
```

mupad [B] time = 9.71, size = 277, normalized size = 3.55

$$\frac{b^2\,\mathrm{atan}\left(\frac{-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}\right)-2\,a^2\,\mathrm{atan}\left(\frac{-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}\right)+2\,a\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (b^2*atan((b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) + 4*a*b*sin(c/
2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - b^2*sin(c/2 + (d*x)/2) + 4*a*b*co
s(c/2 + (d*x)/2))) - 2*a^2*atan((b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (
d*x)/2) + 4*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - b^2*sin(c/2
 + (d*x)/2) + 4*a*b*cos(c/2 + (d*x)/2))) + 2*a*b*log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/d - (a^2*cos(c + d*x) - (b^2*cos(c + d*x))/8 + (b^2*cos(3
*c + 3*d*x))/8 - a*b*sin(2*c + 2*d*x))/(d*sin(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^2\cos^2(c+dx)\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cos(c + d*x)**2*csc(c + d*x)**2, x)
```

3.1064 $\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=89

$$\frac{(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{ab\cot(c+dx)}{d}-\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{2d}-2abx+\frac{3b^2\cos(c+d}{2d}$$

```
[Out] -2*a*b*x+1/2*(a^2-2*b^2)*arctanh(cos(d*x+c))/d+3/2*b^2*cos(d*x+c)/d-a*b*cot
(d*x+c)/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^2/d
```

Rubi [A] time = 0.31, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2889, 3048, 3031, 3023, 2735, 3770}

$$\frac{(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{ab\cot(c+dx)}{d}-\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{2d}-2abx+\frac{3b^2\cos(c+d}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -2*a*b*x + ((a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) + (3*b^2*Cos[c + d*x
])/(2*d) - (a*b*Cot[c + d*x])/d - (Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c +
 d*x])^2)/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx = \int \csc^3(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\,dx$$

$$= -\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{2d} + \frac{1}{2}\int \csc^2(c+$$

$$= -\frac{ab\cot(c+dx)}{d} - \frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{2d}$$

$$= \frac{3b^2\cos(c+dx)}{2d} - \frac{ab\cot(c+dx)}{d} - \frac{\cot(c+dx)\csc(c+dx)(a}{2d}$$

$$= -2abx + \frac{3b^2\cos(c+dx)}{2d} - \frac{ab\cot(c+dx)}{d} - \frac{\cot(c+dx)\csc(}{}$$

$$= -2abx + \frac{(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d} + \frac{3b^2\cos(c+dx)}{2d} -$$

Mathematica [A] time = 0.89, size = 155, normalized size = 1.74

$$\frac{a^2\left(-\csc^2\left(\frac{1}{2}(c+dx)\right)\right) + a^2\sec^2\left(\frac{1}{2}(c+dx)\right) - 4a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 8ab\tan\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-16*a*b*c - 16*a*b*d*x + 8*b^2*Cos[c + d*x] - 8*a*b*Cot[(c + d*x)/2] - a^2
*Csc[(c + d*x)/2]^2 + 4*a^2*Log[Cos[(c + d*x)/2]] - 8*b^2*Log[Cos[(c + d*x)
/2]] - 4*a^2*Log[Sin[(c + d*x)/2]] + 8*b^2*Log[Sin[(c + d*x)/2]] + a^2*Sec[
(c + d*x)/2]^2 + 8*a*b*Tan[(c + d*x)/2])/(8*d)
```

fricas [B] time = 0.65, size = 168, normalized size = 1.89

$$-\frac{8\,abdx\cos(dx+c)^2 - 4\,b^2\cos(dx+c)^3 - 8\,abdx - 8\,ab\cos(dx+c)\sin(dx+c) - 2\left(a^2-2\,b^2\right)\cos(dx+c) - \left(\left(\right.\right.}{4\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/4*(8*a*b*d*x*cos(d*x + c)^2 - 4*b^2*cos(d*x + c)^3 - 8*a*b*d*x - 8*a*b*c
os(d*x + c)*sin(d*x + c) - 2*(a^2 - 2*b^2)*cos(d*x + c) - ((a^2 - 2*b^2)*co
s(d*x + c)^2 - a^2 + 2*b^2)*log(1/2*cos(d*x + c) + 1/2) + ((a^2 - 2*b^2)*co
```

```
s(d*x + c)^2 - a^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2
 - d)
```

giac [A] time = 0.20, size = 148, normalized size = 1.66

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,(dx+c)ab+8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\left(a^2-2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{16\,b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 - 16*(d*x + c)*a*b + 8*a*b*tan(1/2*d*x + 1/
2*c) - 4*(a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 16*b^2/(tan(1/2*d*x
 + 1/2*c)^2 + 1) + (6*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2
*c)^2 - 8*a*b*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2)/d
```

maple [A] time = 0.51, size = 126, normalized size = 1.42

$$-\frac{a^2\left(\cos^3(dx+c)\right)}{2d\sin(dx+c)^2}-\frac{a^2\cos(dx+c)}{2d}-\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d}-2abx-\frac{2ab\cot(dx+c)}{d}-\frac{2abc}{d}+\frac{b^2\cos(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*a^2*cos(d*x+c)/d-1/2/d*a^2*ln(csc(
d*x+c)-cot(d*x+c))-2*a*b*x-2*a*b*cot(d*x+c)/d-2/d*a*b*c+b^2*cos(d*x+c)/d+1/
d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.42, size = 103, normalized size = 1.16

$$-\frac{8\left(dx+c+\frac{1}{\tan(dx+c)}\right)ab-a^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)-2\,b^2\left(2\cos(dx+c\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/4*(8*(d*x + c + 1/tan(d*x + c))*a*b - a^2*(2*cos(d*x + c)/(cos(d*x + c)^
2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) - 2*b^2*(2*cos(d*x
+ c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 10.20, size = 397, normalized size = 4.46

$$\cos(c+dx)\left(\frac{a^2}{2}-\frac{b^2}{4}\right)-\frac{b^2}{2}+\frac{a^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{4}-\frac{b^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2}+\frac{b^2\cos(2c+2dx)}{2}+\frac{b^2\cos(3c+3dx)}{4}-2ab\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\right.}{-\cos\left(\frac{c}{2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (cos(c + d*x)*(a^2/2 - b^2/4) - b^2/2 + (a^2*log(sin(c/2 + (d*x)/2)/cos(c/2
 + (d*x)/2)))/4 - (b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (b^2
*cos(2*c + 2*d*x))/2 + (b^2*cos(3*c + 3*d*x))/4 - 2*a*b*atan((a^2*sin(c/2 +
 (d*x)/2) - 2*b^2*sin(c/2 + (d*x)/2) + 4*a*b*cos(c/2 + (d*x)/2))/(2*b^2*cos
(c/2 + (d*x)/2) - a^2*cos(c/2 + (d*x)/2) + 4*a*b*sin(c/2 + (d*x)/2))) - (a^
2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/4 + (b^2*log
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2 + a*b*sin(2*c +
 2*d*x) + 2*a*b*atan((a^2*sin(c/2 + (d*x)/2) - 2*b^2*sin(c/2 + (d*x)/2) + 4
*a*b*cos(c/2 + (d*x)/2))/(2*b^2*cos(c/2 + (d*x)/2) - a^2*cos(c/2 + (d*x)/2)
 + 4*a*b*sin(c/2 + (d*x)/2)))*cos(2*c + 2*d*x))/(d*(cos(c + d*x)^2 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1065 $\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=96

$$\frac{(a^2-2b^2)\cot(c+dx)}{3d}+\frac{ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{ab\cot(c+dx)\csc(c+dx)}{3d}-\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{3d}$$

```
[Out] -b^2*x+a*b*arctanh(cos(d*x+c))/d+1/3*(a^2-2*b^2)*cot(d*x+c)/d-1/3*a*b*cot(d
*x+c)*csc(d*x+c)/d-1/3*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^2/d
```

Rubi [A] time = 0.39, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2889, 3048, 3031, 3021, 2735, 3770}

$$\frac{(a^2-2b^2)\cot(c+dx)}{3d}+\frac{ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{ab\cot(c+dx)\csc(c+dx)}{3d}-\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(b^2*x) + (a*b*ArcTanh[Cos[c + d*x]])/d + ((a^2 - 2*b^2)*Cot[c + d*x])/(3*
d) - (a*b*Cot[c + d*x]*Csc[c + d*x])/(3*d) - (Cot[c + d*x]*Csc[c + d*x]^2*(
a + b*Sin[c + d*x])^2)/(3*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx = \int \csc^4(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\,dx$$
$$= -\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{3d} + \frac{1}{3}\int \csc^3($$
$$= -\frac{ab\cot(c+dx)\csc(c+dx)}{3d} - \frac{\cot(c+dx)\csc^2(c+dx)(a+}{3d}$$
$$= \frac{\left(a^2-2b^2\right)\cot(c+dx)}{3d} - \frac{ab\cot(c+dx)\csc(c+dx)}{3d} - \frac{\cot(}{}$$
$$= -b^2x + \frac{\left(a^2-2b^2\right)\cot(c+dx)}{3d} - \frac{ab\cot(c+dx)\csc(c+dx)}{3d}$$
$$= -b^2x + \frac{ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{\left(a^2-2b^2\right)\cot(c+dx)}{3d} -$$

Mathematica [B] time = 6.19, size = 538, normalized size = 5.60

$$\frac{\sin^2(c+dx)\csc\left(\frac{1}{2}(c+dx)\right)\left(a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)(a\csc(c+dx)+b)^2}{6d(a+b\sin(c+dx))^2} + \frac{\sin^2(c+dx)\sec\left(\frac{1}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((b^2*(c + d*x)*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(d*(a + b*Sin[c + d
*x])^2)) + ((a^2*Cos[(c + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2
]*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(6*d*(a + b*Sin[c + d*x])^2) - (a*
b*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(4*d*(a + b*Sin
[c + d*x])^2) - (a^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x
])^2*Sin[c + d*x]^2)/(24*d*(a + b*Sin[c + d*x])^2) + (a*b*(b + a*Csc[c + d*
x])^2*Log[Cos[(c + d*x)/2]]*Sin[c + d*x]^2)/(d*(a + b*Sin[c + d*x])^2) - (a
*b*(b + a*Csc[c + d*x])^2*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]^2)/(d*(a + b*S
in[c + d*x])^2) + (a*b*(b + a*Csc[c + d*x])^2*Sec[(c + d*x)/2]^2*Sin[c + d*
x]^2)/(4*d*(a + b*Sin[c + d*x])^2) + ((b + a*Csc[c + d*x])^2*Sec[(c + d*x)/
2]*(-(a^2*Sin[(c + d*x)/2]) + 3*b^2*Sin[(c + d*x)/2])*Sin[c + d*x]^2)/(6*d*
(a + b*Sin[c + d*x])^2) + (a^2*(b + a*Csc[c + d*x])^2*Sec[(c + d*x)/2]^2*Si
n[c + d*x]^2*Tan[(c + d*x)/2])/(24*d*(a + b*Sin[c + d*x])^2)
```

fricas [A] time = 0.75, size = 167, normalized size = 1.74

$$\frac{2\left(a^2-3\,b^2\right)\cos\left(dx+c\right)^3+6\,b^2\cos\left(dx+c\right)+3\left(ab\cos\left(dx+c\right)^2-ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-}{6\left(d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*(a^2 - 3*b^2)*cos(d*x + c)^3 + 6*b^2*cos(d*x + c) + 3*(a*b*cos(d*x +
 c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(a*b*cos(d*x + c)
^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*(b^2*d*x*cos(d*x +
c)^2 - b^2*d*x - a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*si
n(d*x + c))
```

giac [A] time = 0.21, size = 167, normalized size = 1.74

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,(dx+c)b^2-24\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 - 24*(d*x +
 c)*b^2 - 24*a*b*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a^2*tan(1/2*d*x + 1/2*c
) + 12*b^2*tan(1/2*d*x + 1/2*c) + (44*a*b*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*ta
n(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b*tan(1/2*d*x +
1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.42, size = 114, normalized size = 1.19

$$-\frac{a^2\left(\cos^3(dx+c)\right)}{3d\sin(dx+c)^3}-\frac{ab\left(\cos^3(dx+c)\right)}{d\sin(dx+c)^2}-\frac{ab\cos(dx+c)}{d}-\frac{ab\ln(\csc(dx+c)-\cot(dx+c))}{d}-b^2x-\frac{b^2\cot(dx+c)}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d*a^2/sin(d*x+c)^3*cos(d*x+c)^3-1/d*a*b/sin(d*x+c)^2*cos(d*x+c)^3-a*b*
cos(d*x+c)/d-1/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-b^2*x-b^2*cot(d*x+c)/d-1/d*b
^2*c
```

maxima [A] time = 0.52, size = 82, normalized size = 0.85

$$-\frac{6\left(dx+c+\frac{1}{\tan(dx+c)}\right)b^2-3\,ab\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)+\frac{2\,a^2}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/6*(6*(d*x + c + 1/tan(d*x + c))*b^2 - 3*a*b*(2*cos(d*x + c)/(cos(d*x + c
)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 2*a^2/tan(d*x +
 c)^3)/d
```

mupad [B] time = 9.48, size = 231, normalized size = 2.41

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}+\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{b^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{2\,b^2\,\dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (a^2*cot(c/2 + (d*x)/2)^3)/(24*d) + (a^
2*cot(c/2 + (d*x)/2))/(8*d) - (b^2*cot(c/2 + (d*x)/2))/(2*d) - (a^2*tan(c/2
 + (d*x)/2))/(8*d) + (b^2*tan(c/2 + (d*x)/2))/(2*d) - (2*b^2*atan((b*cos(c/
2 + (d*x)/2) + a*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2) - b*sin(c/2 + (d
*x)/2))))/d - (a*b*cot(c/2 + (d*x)/2)^2)/(4*d) + (a*b*tan(c/2 + (d*x)/2)^2)
/(4*d) - (a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1066 $\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=123

$$\frac{(a^2+4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{8d}+\frac{2ab\cot(c+dx)}{3d}-\frac{ab\cot(c+dx)\csc^2(c+dx)}{6d}$$

```
[Out] 1/8*(a^2+4*b^2)*arctanh(cos(d*x+c))/d+2/3*a*b*cot(d*x+c)/d+1/8*(a^2-2*b^2)*
cot(d*x+c)*csc(d*x+c)/d-1/6*a*b*cot(d*x+c)*csc(d*x+c)^2/d-1/4*cot(d*x+c)*cs
c(d*x+c)^3*(a+b*sin(d*x+c))^2/d
```

Rubi [A] time = 0.36, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3048, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{(a^2+4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{8d}+\frac{2ab\cot(c+dx)}{3d}-\frac{ab\cot(c+dx)\csc^2(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + (2*a*b*Cot[c + d*x])/(3*d) +
((a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*b*Cot[c + d*x]*Csc[c +
 d*x]^2)/(6*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx = \int \csc^5(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\,dx$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2}{4d} + \frac{1}{4}\int \csc^4(c$$

$$= -\frac{ab\cot(c+dx)\csc^2(c+dx)}{6d} - \frac{\cot(c+dx)\csc^3(c+dx)(a+}{4d}$$

$$= \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{8d} - \frac{ab\cot(c+dx)\csc^2(c+}{6d}$$

$$= \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{8d} - \frac{ab\cot(c+dx)\csc^2(c+}{6d}$$

$$= \frac{\left(a^2+4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} + \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc}{8d}$$

$$= \frac{\left(a^2+4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} + \frac{2ab\cot(c+dx)}{3d} + \frac{\left(a^2-2\right.}{}$$

Mathematica [B] time = 6.17, size = 579, normalized size = 4.71

$$\frac{\left(a^2-4b^2\right)\sin^2(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a\csc(c+dx)+b)^2}{32d(a+b\sin(c+dx))^2} + \frac{\left(-a^2-4b^2\right)\sin^2(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\csc}{8d(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*Cot[(c + d*x)/2]*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(3*d*(a + b*Si
n[c + d*x])^2) + ((a^2 - 4*b^2)*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x])^2*S
in[c + d*x]^2)/(32*d*(a + b*Sin[c + d*x])^2) - (a*b*Cot[(c + d*x)/2]*Csc[(c
 + d*x)/2]^2*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(12*d*(a + b*Sin[c + d*
x])^2) - (a^2*Csc[(c + d*x)/2]^4*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(64
*d*(a + b*Sin[c + d*x])^2) + ((a^2 + 4*b^2)*(b + a*Csc[c + d*x])^2*Log[Cos[
(c + d*x)/2]]*Sin[c + d*x]^2)/(8*d*(a + b*Sin[c + d*x])^2) + ((-a^2 - 4*b^2
)*(b + a*Csc[c + d*x])^2*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]^2)/(8*d*(a + b*
Sin[c + d*x])^2) + ((-a^2 + 4*b^2)*(b + a*Csc[c + d*x])^2*Sec[(c + d*x)/2]^
2*Sin[c + d*x]^2)/(32*d*(a + b*Sin[c + d*x])^2) + (a^2*(b + a*Csc[c + d*x])
^2*Sec[(c + d*x)/2]^4*Sin[c + d*x]^2)/(64*d*(a + b*Sin[c + d*x])^2) - (a*b*
(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2*Tan[(c + d*x)/2])/(3*d*(a + b*Sin[c +
 d*x])^2) + (a*b*(b + a*Csc[c + d*x])^2*Sec[(c + d*x)/2]^2*Sin[c + d*x]^2*T
an[(c + d*x)/2])/(12*d*(a + b*Sin[c + d*x])^2)
```

fricas [A] time = 0.79, size = 200, normalized size = 1.63

$$-\frac{32\,ab\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+6\left(a^{2}-4\,b^{2}\right)\cos\left(dx+c\right)^{3}+6\left(a^{2}+4\,b^{2}\right)\cos\left(dx+c\right)-3\left(\left(a^{2}+4\,b^{2}\right)\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(32*a*b*cos(d*x + c)^3*sin(d*x + c) + 6*(a^2 - 4*b^2)*cos(d*x + c)^3
+ 6*(a^2 + 4*b^2)*cos(d*x + c) - 3*((a^2 + 4*b^2)*cos(d*x + c)^4 - 2*(a^2 +
 4*b^2)*cos(d*x + c)^2 + a^2 + 4*b^2)*log(1/2*cos(d*x + c) + 1/2) + 3*((a^2
 + 4*b^2)*cos(d*x + c)^4 - 2*(a^2 + 4*b^2)*cos(d*x + c)^2 + a^2 + 4*b^2)*lo
g(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.22, size = 182, normalized size = 1.48

$$\frac{3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+24\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\left(a^{2}+4\,b^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a*b*tan(1/2*d*x + 1/2*c)^3 + 24*b^
2*tan(1/2*d*x + 1/2*c)^2 - 48*a*b*tan(1/2*d*x + 1/2*c) - 24*(a^2 + 4*b^2)*l
og(abs(tan(1/2*d*x + 1/2*c))) + (50*a^2*tan(1/2*d*x + 1/2*c)^4 + 200*b^2*ta
n(1/2*d*x + 1/2*c)^4 + 48*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*b^2*tan(1/2*d*x +
 1/2*c)^2 - 16*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.46, size = 173, normalized size = 1.41

$$-\frac{a^{2}\left(\cos^{3}\left(dx+c\right)\right)}{4d\sin\left(dx+c\right)^{4}}-\frac{a^{2}\left(\cos^{3}\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^{2}}-\frac{a^{2}\cos\left(dx+c\right)}{8d}-\frac{a^{2}\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{8d}-\frac{2ab\left(\cos^{3}\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2/sin(d*x+c)^4*cos(d*x+c)^3-1/8/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3-1/
8*a^2*cos(d*x+c)/d-1/8/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/3/d*a*b/sin(d*x+c)
```

```
^3*cos(d*x+c)^3-1/2/d*b^2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*b^2*cos(d*x+c)/d-1/
2/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.41, size = 129, normalized size = 1.05

$$-\frac{3\,a^2\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-12\,b^2\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx\right.\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/48*(3*a^2*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x
 + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 12*b^2*(2*c
os(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c)
 - 1)) + 32*a*b/tan(d*x + c)^3)/d
```

mupad [B] time = 9.33, size = 165, normalized size = 1.34

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(\frac{a^2}{8}+\frac{b^2}{2}\right)}{d}+\frac{b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{a^2}{4}-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{4\,a\,b}{\cdots}\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^4)/(64*d) - (log(tan(c/2 + (d*x)/2))*(a^2/8 + b^2/2
))/d + (b^2*tan(c/2 + (d*x)/2)^2)/(8*d) - (cot(c/2 + (d*x)/2)^4*(2*b^2*tan(
c/2 + (d*x)/2)^2 + a^2/4 - 4*a*b*tan(c/2 + (d*x)/2)^3 + (4*a*b*tan(c/2 + (d
*x)/2))/3))/(16*d) + (a*b*tan(c/2 + (d*x)/2)^3)/(12*d) - (a*b*tan(c/2 + (d*
x)/2))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1067 $\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=148

$$\frac{(2a^2+5b^2)\cot(c+dx)}{15d}+\frac{(a^2-2b^2)\cot(c+dx)\csc^2(c+dx)}{15d}+\frac{ab\tanh^{-1}(\cos(c+dx))}{4d}-\frac{ab\cot(c+dx)\csc^3(c+a}{10d}$$

```
[Out] 1/4*a*b*arctanh(cos(d*x+c))/d+1/15*(2*a^2+5*b^2)*cot(d*x+c)/d+1/4*a*b*cot(d
*x+c)*csc(d*x+c)/d+1/15*(a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)^2/d-1/10*a*b*cot(
d*x+c)*csc(d*x+c)^3/d-1/5*cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^2/d
```

Rubi [A] time = 0.39, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2889, 3048, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{(2a^2+5b^2)\cot(c+dx)}{15d}+\frac{(a^2-2b^2)\cot(c+dx)\csc^2(c+dx)}{15d}+\frac{ab\tanh^{-1}(\cos(c+dx))}{4d}-\frac{ab\cot(c+dx)\csc^3(c+a}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*ArcTanh[Cos[c + d*x]])/(4*d) + ((2*a^2 + 5*b^2)*Cot[c + d*x])/(15*d) +
 (a*b*Cot[c + d*x]*Csc[c + d*x])/(4*d) + ((a^2 - 2*b^2)*Cot[c + d*x]*Csc[c
+ d*x]^2)/(15*d) - (a*b*Cot[c + d*x]*Csc[c + d*x]^3)/(10*d) - (Cot[c + d*x]
*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx = \int \csc^6(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\,dx$$
$$= -\frac{\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2}{5d} + \frac{1}{5}\int \csc^5($$
$$= -\frac{ab\cot(c+dx)\csc^3(c+dx)}{10d} - \frac{\cot(c+dx)\csc^4(c+dx)(a}{5d}$$
$$= \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{ab\cot(c+dx)\csc^3(c}{10d}$$
$$= \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{ab\cot(c+dx)\csc^3(c}{10d}$$
$$= \frac{ab\cot(c+dx)\csc(c+dx)}{4d} + \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc^2(c}{15d}$$
$$= \frac{ab\tanh^{-1}(\cos(c+dx))}{4d} + \frac{\left(2a^2+5b^2\right)\cot(c+dx)}{15d} + \frac{ab\,\mathrm{co}}{}$$

Mathematica [A] time = 0.82, size = 236, normalized size = 1.59

$$\frac{\csc^5(c+dx)\left(-40\left(4a^2+b^2\right)\cos(c+dx)+20\left(b^2-2a^2\right)\cos(3(c+dx))+8a^2\cos(5(c+dx))-180ab\sin(2(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^5*(-40*(4*a^2 + b^2)*Cos[c + d*x] + 20*(-2*a^2 + b^2)*Cos[3*(
c + d*x)] + 8*a^2*Cos[5*(c + d*x)] + 20*b^2*Cos[5*(c + d*x)] + 150*a*b*Log[
Cos[(c + d*x)/2]]*Sin[c + d*x] - 150*a*b*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]
 - 180*a*b*Sin[2*(c + d*x)] - 75*a*b*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)]
 + 75*a*b*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 30*a*b*Sin[4*(c + d*x)]
+ 15*a*b*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*a*b*Log[Sin[(c + d*x)/
2]]*Sin[5*(c + d*x)]))/(960*d)
```

fricas [A] time = 0.69, size = 195, normalized size = 1.32

$$\frac{8\left(2\,a^{2}+5\,b^{2}\right)\cos\left(dx+c\right)^{5}-40\left(a^{2}+b^{2}\right)\cos\left(dx+c\right)^{3}+15\left(ab\cos\left(dx+c\right)^{4}-2\,ab\cos\left(dx+c\right)^{2}+ab\right)\log\left(\frac{1}{2}\,c\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(8*(2*a^2 + 5*b^2)*cos(d*x + c)^5 - 40*(a^2 + b^2)*cos(d*x + c)^3 + 1
5*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2 + a*b)*log(1/2*cos(d*x + c) +
1/2)*sin(d*x + c) - 15*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2 + a*b)*lo
g(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*(a*b*cos(d*x + c)^3 + a*b*cos(
d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*
x + c))
```

giac [A] time = 0.22, size = 222, normalized size = 1.50

$$\frac{3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+5\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+20\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-120\,ab\log\left(\left|\mathrm{ta}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a*b*tan(1/2*d*x + 1/2*c)^4 + 5*a^2
*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a*b*log(abs(t
an(1/2*d*x + 1/2*c))) - 30*a^2*tan(1/2*d*x + 1/2*c) - 60*b^2*tan(1/2*d*x +
1/2*c) + (274*a*b*tan(1/2*d*x + 1/2*c)^5 + 30*a^2*tan(1/2*d*x + 1/2*c)^4 +
60*b^2*tan(1/2*d*x + 1/2*c)^4 - 5*a^2*tan(1/2*d*x + 1/2*c)^2 - 20*b^2*tan(1
/2*d*x + 1/2*c)^2 - 15*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/tan(1/2*d*x + 1/2*
c)^5)/d
```

maple [A] time = 0.49, size = 156, normalized size = 1.05

$$-\frac{a^{2}\left(\cos^{3}\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)^{5}}-\frac{2a^{2}\left(\cos^{3}\left(dx+c\right)\right)}{15d\sin\left(dx+c\right)^{3}}-\frac{ab\left(\cos^{3}\left(dx+c\right)\right)}{2d\sin\left(dx+c\right)^{4}}-\frac{ab\left(\cos^{3}\left(dx+c\right)\right)}{4d\sin\left(dx+c\right)^{2}}-\frac{ab\cos\left(dx+c\right)}{4d}-\frac{ab\ln\left(\csc\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/5/d*a^2/sin(d*x+c)^5*cos(d*x+c)^3-2/15/d*a^2/sin(d*x+c)^3*cos(d*x+c)^3-1
/2/d*a*b/sin(d*x+c)^4*cos(d*x+c)^3-1/4/d*a*b/sin(d*x+c)^2*cos(d*x+c)^3-1/4*
a*b*cos(d*x+c)/d-1/4/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*b^2/sin(d*x+c)^3
*cos(d*x+c)^3
```

maxima [A] time = 0.38, size = 108, normalized size = 0.73

$$-\frac{15\,ab\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{40\,b^2}{\tan(dx+c)^3}+\frac{8\left(5\tan(dx+c)^2+3\right)a^2}{\tan(dx+c)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*(15*a*b*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 40*b^2/ta
n(d*x + c)^3 + 8*(5*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

mupad [B] time = 9.36, size = 187, normalized size = 1.26

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{a^2}{3}+\frac{4\,b^2}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(2\,a^2+4\,b^2\right)+\frac{a^2}{5}+a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^5)/(160*d) - (cot(c/2 + (d*x)/2)^5*(tan(c/2 + (d*x)
/2)^2*(a^2/3 + (4*b^2)/3) - tan(c/2 + (d*x)/2)^4*(2*a^2 + 4*b^2) + a^2/5 +
a*b*tan(c/2 + (d*x)/2)))/(32*d) - (tan(c/2 + (d*x)/2)*(a^2/16 + b^2/8))/d +
 (tan(c/2 + (d*x)/2)^3*(a^2/96 + b^2/24))/d + (a*b*tan(c/2 + (d*x)/2)^4)/(3
2*d) - (a*b*log(tan(c/2 + (d*x)/2)))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1068 $\int \cot^2(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=170

$$\frac{(a^2+2b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{(a^2-2b^2)\cot(c+dx)\csc^3(c+dx)}{24d}+\frac{(a^2+2b^2)\cot(c+dx)\csc(c+dx)}{16d}+\frac{2ab\cot}{1}$$

```
[Out] 1/16*(a^2+2*b^2)*arctanh(cos(d*x+c))/d+2/5*a*b*cot(d*x+c)/d+2/15*a*b*cot(d*
x+c)^3/d+1/16*(a^2+2*b^2)*cot(d*x+c)*csc(d*x+c)/d+1/24*(a^2-2*b^2)*cot(d*x+
c)*csc(d*x+c)^3/d-1/15*a*b*cot(d*x+c)*csc(d*x+c)^4/d-1/6*cot(d*x+c)*csc(d*x
+c)^5*(a+b*sin(d*x+c))^2/d
```

Rubi [A] time = 0.40, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3048, 3031, 3021, 2748, 3767, 3768, 3770}

$$\frac{(a^2+2b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{(a^2-2b^2)\cot(c+dx)\csc^3(c+dx)}{24d}+\frac{(a^2+2b^2)\cot(c+dx)\csc(c+dx)}{16d}+\frac{2ab\cot}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) + (2*a*b*Cot[c + d*x])/(5*d) +
 (2*a*b*Cot[c + d*x]^3)/(15*d) + ((a^2 + 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/
(16*d) + ((a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a*b*Cot[c +
d*x]*Csc[c + d*x]^4)/(15*d) - (Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin[c + d
*x])^2)/(6*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2\,dx = \int \csc^7(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)dx$$
$$= -\frac{\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2}{6d} + \frac{1}{6}\int \csc^6(c$$
$$= -\frac{ab\cot(c+dx)\csc^4(c+dx)}{15d} - \frac{\cot(c+dx)\csc^5(c+dx)(a+}{6d}$$
$$= \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{ab\cot(c+dx)\csc^4(c+}{15d}$$
$$= \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{ab\cot(c+dx)\csc^4(c+}{15d}$$
$$= \frac{\left(a^2+2b^2\right)\cot(c+dx)\csc(c+dx)}{16d} + \frac{\left(a^2-2b^2\right)\cot(c+dx)}{24d}$$
$$= \frac{\left(a^2+2b^2\right)\tanh^{-1}(\cos(c+dx))}{16d} + \frac{2ab\cot(c+dx)}{5d} + \frac{2ab\cot}{1}$$

Mathematica [A] time = 0.80, size = 296, normalized size = 1.74

$$30\left(a^2+2b^2\right)\csc^2\left(\tfrac{1}{2}(c+dx)\right)+5a^2\sec^6\left(\tfrac{1}{2}(c+dx)\right)-30a^2\sec^2\left(\tfrac{1}{2}(c+dx)\right)-120a^2\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+120$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (256*a*b*Cot[(c + d*x)/2] + 30*(a^2 + 2*b^2)*Csc[(c + d*x)/2]^2 + 120*a^2*L
og[Cos[(c + d*x)/2]] + 240*b^2*Log[Cos[(c + d*x)/2]] - 120*a^2*Log[Sin[(c +
 d*x)/2]] - 240*b^2*Log[Sin[(c + d*x)/2]] - 30*a^2*Sec[(c + d*x)/2]^2 - 60*
b^2*Sec[(c + d*x)/2]^2 + 30*b^2*Sec[(c + d*x)/2]^4 + 5*a^2*Sec[(c + d*x)/2]
^6 - 64*a*b*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 768*a*b*Csc[c + d*x]^5*Sin[
(c + d*x)/2]^6 - a*Csc[(c + d*x)/2]^6*(5*a + 12*b*Sin[c + d*x]) + Csc[(c +
d*x)/2]^4*(-30*b^2 + 4*a*b*Sin[c + d*x]) - 256*a*b*Tan[(c + d*x)/2])/(1920*
d)
```

fricas [A] time = 0.80, size = 283, normalized size = 1.66

$$-\frac{30\left(a^{2}+2\,b^{2}\right)\cos\left(dx+c\right)^{5}-80\,a^{2}\cos\left(dx+c\right)^{3}-30\left(a^{2}+2\,b^{2}\right)\cos\left(dx+c\right)-15\left(\left(a^{2}+2\,b^{2}\right)\cos\left(dx+c\right)^{6}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(30*(a^2 + 2*b^2)*cos(d*x + c)^5 - 80*a^2*cos(d*x + c)^3 - 30*(a^2 +
 2*b^2)*cos(d*x + c) - 15*((a^2 + 2*b^2)*cos(d*x + c)^6 - 3*(a^2 + 2*b^2)*c
os(d*x + c)^4 + 3*(a^2 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(1/2*cos(d
*x + c) + 1/2) + 15*((a^2 + 2*b^2)*cos(d*x + c)^6 - 3*(a^2 + 2*b^2)*cos(d*x
 + c)^4 + 3*(a^2 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(-1/2*cos(d*x +
c) + 1/2) + 64*(2*a*b*cos(d*x + c)^5 - 5*a*b*cos(d*x + c)^3)*sin(d*x + c))/
(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.23, size = 276, normalized size = 1.62

$$\frac{5\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+30\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+40\,ab\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a*b*tan(1/2*d*x + 1/2*c)^5 + 15*a
^2*tan(1/2*d*x + 1/2*c)^4 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 + 40*a*b*tan(1/2*
d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c)^2 - 240*a*b*tan(1/2*d*x + 1/2*
c) - 120*(a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (294*a^2*tan(1/2*d*
x + 1/2*c)^6 + 588*b^2*tan(1/2*d*x + 1/2*c)^6 + 240*a*b*tan(1/2*d*x + 1/2*c
)^5 + 15*a^2*tan(1/2*d*x + 1/2*c)^4 - 40*a*b*tan(1/2*d*x + 1/2*c)^3 - 15*a^
2*tan(1/2*d*x + 1/2*c)^2 - 30*b^2*tan(1/2*d*x + 1/2*c)^2 - 24*a*b*tan(1/2*d
*x + 1/2*c) - 5*a^2)/tan(1/2*d*x + 1/2*c)^6)/d
```

maple [A] time = 0.47, size = 244, normalized size = 1.44

$$-\frac{a^{2}\left(\cos^{3}\left(dx+c\right)\right)}{6d\sin\left(dx+c\right)^{6}}-\frac{a^{2}\left(\cos^{3}\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^{4}}-\frac{a^{2}\left(\cos^{3}\left(dx+c\right)\right)}{16d\sin\left(dx+c\right)^{2}}-\frac{a^{2}\cos\left(dx+c\right)}{16d}-\frac{a^{2}\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{16d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/6/d*a^2/sin(d*x+c)^6*cos(d*x+c)^3-1/8/d*a^2/sin(d*x+c)^4*cos(d*x+c)^3-1/
16/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3-1/16*a^2*cos(d*x+c)/d-1/16/d*a^2*ln(csc(
d*x+c)-cot(d*x+c))-2/5/d*a*b/sin(d*x+c)^5*cos(d*x+c)^3-4/15/d*a*b/sin(d*x+c
)^3*cos(d*x+c)^3-1/4/d*b^2/sin(d*x+c)^4*cos(d*x+c)^3-1/8/d*b^2/sin(d*x+c)^2
*cos(d*x+c)^3-1/8*b^2*cos(d*x+c)/d-1/8/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.33, size = 186, normalized size = 1.09

$$-\frac{5\,a^2\left(\frac{2\left(3\,\cos(dx+c)^5-8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+30\,b^2\left(\frac{2\left(\cos(dx+c \ldots}{\cos(dx+c)^4- \ldots} \ldots}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*(5*a^2*(2*(3*cos(d*x + c)^5 - 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(co
s(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c
) + 1) + 3*log(cos(d*x + c) - 1)) + 30*b^2*(2*(cos(d*x + c)^3 + cos(d*x + c
))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(co
s(d*x + c) - 1)) + 64*(5*tan(d*x + c)^2 + 3)*a*b/tan(d*x + c)^5)/d
```

mupad [B] time = 9.44, size = 245, normalized size = 1.44

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{a^2}{16}+\frac{b^2}{8}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{a^2}{6}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+\tan\left(\frac{c}{2}+\frac{d \ldots}{} \ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^6)/(384*d) - (a^2*tan(c/2 + (d*x)/2)^2)/(128*d) - (
log(tan(c/2 + (d*x)/2))*(a^2/16 + b^2/8))/d - (cot(c/2 + (d*x)/2)^6*(a^2/6
- (a^2*tan(c/2 + (d*x)/2)^4)/2 + tan(c/2 + (d*x)/2)^2*(a^2/2 + b^2) + (4*a*
b*tan(c/2 + (d*x)/2)^3)/3 - 8*a*b*tan(c/2 + (d*x)/2)^5 + (4*a*b*tan(c/2 + (
d*x)/2))/5))/(64*d) + (tan(c/2 + (d*x)/2)^4*(a^2/128 + b^2/64))/d + (a*b*ta
n(c/2 + (d*x)/2)^3)/(48*d) + (a*b*tan(c/2 + (d*x)/2)^5)/(80*d) - (a*b*tan(c
/2 + (d*x)/2))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**7*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1069 $\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=232

$$\frac{b\left(21a^2+4b^2\right)\cos^3(c+dx)}{105d}-\frac{b\left(21a^2+4b^2\right)\cos(c+dx)}{35d}+\frac{b\left(a^2-b^2\right)\sin^4(c+dx)\cos(c+dx)}{35d}+\frac{a\left(2a^2-7b^2\right)\sin^3}{5}$$

```
[Out] 1/16*a*(2*a^2+3*b^2)*x-1/35*b*(21*a^2+4*b^2)*cos(d*x+c)/d+1/105*b*(21*a^2+4
*b^2)*cos(d*x+c)^3/d-1/16*a*(2*a^2+3*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/56*a*(2
*a^2-7*b^2)*cos(d*x+c)*sin(d*x+c)^3/d+1/35*b*(a^2-b^2)*cos(d*x+c)*sin(d*x+c
)^4/d+1/14*a*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^2/d+1/7*cos(d*x+c)*si
n(d*x+c)^3*(a+b*sin(d*x+c))^3/d
```

Rubi [A] time = 0.57, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2889, 3050, 3049, 3033, 3023, 2748, 2635, 8, 2633}

$$\frac{b\left(21a^2+4b^2\right)\cos^3(c+dx)}{105d}-\frac{b\left(21a^2+4b^2\right)\cos(c+dx)}{35d}+\frac{b\left(a^2-b^2\right)\sin^4(c+dx)\cos(c+dx)}{35d}+\frac{a\left(2a^2-7b^2\right)\sin^3}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*x)/16 - (b*(21*a^2 + 4*b^2)*Cos[c + d*x])/(35*d) + (b*(2
1*a^2 + 4*b^2)*Cos[c + d*x]^3)/(105*d) - (a*(2*a^2 + 3*b^2)*Cos[c + d*x]*Si
n[c + d*x])/(16*d) + (a*(2*a^2 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(56*d)
 + (b*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(35*d) + (a*Cos[c + d*x]*Sin
[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(14*d) + (Cos[c + d*x]*Sin[c + d*x]^3*(
a + b*Sin[c + d*x])^3)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
```

```
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \sin^2(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)\,dx$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^3}{7d} + \frac{1}{7}\int \sin^2(c+$$

$$= \frac{a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2}{14d} + \frac{\cos(c+dx)}{}$$

$$= \frac{b\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{35d} + \frac{a\cos(c+dx)\sin^3(c-}{}$$

$$= \frac{a\left(2a^2-7b^2\right)\cos(c+dx)\sin^3(c+dx)}{56d} + \frac{b\left(a^2-b^2\right)\cos(c+}{35}$$

$$= \frac{a\left(2a^2-7b^2\right)\cos(c+dx)\sin^3(c+dx)}{56d} + \frac{b\left(a^2-b^2\right)\cos(c+}{35}$$

$$= -\frac{a\left(2a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a\left(2a^2-7b^2\right)\cos(}{5}$$

$$= \frac{1}{16}a\left(2a^2+3b^2\right)x - \frac{b\left(21a^2+4b^2\right)\cos(c+dx)}{35d} + \frac{b\left(21a^2+\right.}{}$$

Mathematica [A] time = 0.82, size = 157, normalized size = 0.68

$$\frac{-35\left(12a^2b+b^3\right)\cos(3(c+dx))+63\left(4a^2b+b^3\right)\cos(5(c+dx))+105a\left(-\left(2a^2+3b^2\right)\sin(4(c+dx))+8a^2c+8a\right.}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-105*b*(24*a^2 + 5*b^2)*Cos[c + d*x] - 35*(12*a^2*b + b^3)*Cos[3*(c + d*x)
] + 63*(4*a^2*b + b^3)*Cos[5*(c + d*x)] - 15*b^3*Cos[7*(c + d*x)] + 105*a*(
8*a^2*c + 12*b^2*c + 8*a^2*d*x + 12*b^2*d*x - 3*b^2*Sin[2*(c + d*x)] - (2*a
^2 + 3*b^2)*Sin[4*(c + d*x)] + b^2*Sin[6*(c + d*x)]))/(6720*d)
```

fricas [A] time = 0.72, size = 141, normalized size = 0.61

$$-\frac{240\,b^3\cos(dx+c)^7-336\left(3\,a^2b+2\,b^3\right)\cos(dx+c)^5+560\left(3\,a^2b+b^3\right)\cos(dx+c)^3-105\left(2\,a^3+3\,ab^2\right)dx-}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/1680*(240*b^3*cos(d*x + c)^7 - 336*(3*a^2*b + 2*b^3)*cos(d*x + c)^5 + 56
0*(3*a^2*b + b^3)*cos(d*x + c)^3 - 105*(2*a^3 + 3*a*b^2)*d*x - 105*(8*a*b^2
*cos(d*x + c)^5 - 2*(2*a^3 + 7*a*b^2)*cos(d*x + c)^3 + (2*a^3 + 3*a*b^2)*co
s(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.28, size = 166, normalized size = 0.72

$$-\frac{b^3\cos(7\,dx+7\,c)}{448\,d}+\frac{ab^2\sin(6\,dx+6\,c)}{64\,d}-\frac{3\,ab^2\sin(2\,dx+2\,c)}{64\,d}+\frac{1}{16}\left(2\,a^3+3\,ab^2\right)x+\frac{3\left(4\,a^2b+b^3\right)\cos(5\,dx+5}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/448*b^3*cos(7*d*x + 7*c)/d + 1/64*a*b^2*sin(6*d*x + 6*c)/d - 3/64*a*b^2*
sin(2*d*x + 2*c)/d + 1/16*(2*a^3 + 3*a*b^2)*x + 3/320*(4*a^2*b + b^3)*cos(5
*d*x + 5*c)/d - 1/192*(12*a^2*b + b^3)*cos(3*d*x + 3*c)/d - 1/64*(24*a^2*b
+ 5*b^3)*cos(d*x + c)/d - 1/64*(2*a^3 + 3*a*b^2)*sin(4*d*x + 4*c)/d
```

maple [A] time = 0.22, size = 196, normalized size = 0.84

$$a^3\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+3a^2b\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+3a\,b^2\left(-\frac{(\sin}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)+3*a^2*b*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3*a*b^2*(-1
/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*cos(d*x+c)*si
n(d*x+c)+1/16*d*x+1/16*c)+b^3*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+
c)^2*cos(d*x+c)^3-8/105*cos(d*x+c)^3))
```

maxima [A] time = 0.33, size = 131, normalized size = 0.56

$$\frac{210\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^3+1344\left(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3\right)a^2b-105\left(4\,\sin(2\,dx+2\,c)^3-12\,\right.}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6720*(210*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^3 + 1344*(3*cos(d*x + c)^5 -
 5*cos(d*x + c)^3)*a^2*b - 105*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*si
n(4*d*x + 4*c))*a*b^2 - 64*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(
d*x + c)^3)*b^3)/d
```

mupad [B] time = 10.66, size = 455, normalized size = 1.96

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(2\,a^2+3\,b^2\right)}{8\left(\frac{a^3}{4}+\frac{3\,a\,b^2}{8}\right)}\right)\left(2\,a^2+3\,b^2\right)}{8\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{a^3}{4}+\frac{3\,a\,b^2}{8}\right)+\frac{4\,a^2\,b}{5}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{5\,a\,b^2}{2}-a^3\right)-\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2))/(8*((3*a*b^2)/8 + a^3/4)))*(
2*a^2 + 3*b^2))/(8*d) - (tan(c/2 + (d*x)/2)*((3*a*b^2)/8 + a^3/4) + (4*a^2*
b)/5 + tan(c/2 + (d*x)/2)^3*((5*a*b^2)/2 - a^3) - tan(c/2 + (d*x)/2)^11*((5
*a*b^2)/2 - a^3) - tan(c/2 + (d*x)/2)^13*((3*a*b^2)/8 + a^3/4) - tan(c/2 +
(d*x)/2)^5*((97*a*b^2)/8 + (11*a^3)/4) + tan(c/2 + (d*x)/2)^9*((97*a*b^2)/8
 + (11*a^3)/4) + tan(c/2 + (d*x)/2)^6*(8*a^2*b - (16*b^3)/3) + tan(c/2 + (d
*x)/2)^4*((24*a^2*b)/5 + (16*b^3)/5) + tan(c/2 + (d*x)/2)^8*(20*a^2*b + (32
*b^3)/3) + tan(c/2 + (d*x)/2)^2*((28*a^2*b)/5 + (16*b^3)/15) + (16*b^3)/105
 + 12*a^2*b*tan(c/2 + (d*x)/2)^10)/(d*(7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2
+ (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c
/2 + (d*x)/2)^10 + 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 + 1)) -
(a*(2*a^2 + 3*b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(8*d)
```

sympy [A] time = 6.29, size = 394, normalized size = 1.70

$$\begin{cases} \frac{a^3x\sin^4(c+dx)}{8} + \frac{a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{a^3x\cos^4(c+dx)}{8} + \frac{a^3\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{a^3\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{a^2b\sin^2(c+dx)\mathrm{co}}{d} \\ x(a+b\sin(c))^3\sin^2(c)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(c + d*x)**4/8 + a**3*x*sin(c + d*x)**2*cos(c + d*x)**
2/4 + a**3*x*cos(c + d*x)**4/8 + a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) -
a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a**2*b*sin(c + d*x)**2*cos(c + d*
x)**3/d - 2*a**2*b*cos(c + d*x)**5/(5*d) + 3*a*b**2*x*sin(c + d*x)**6/16 +
9*a*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a*b**2*x*sin(c + d*x)**2*
cos(c + d*x)**4/16 + 3*a*b**2*x*cos(c + d*x)**6/16 + 3*a*b**2*sin(c + d*x)*
*5*cos(c + d*x)/(16*d) - a*b**2*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*a
*b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - b**3*sin(c + d*x)**4*cos(c + d*
x)**3/(3*d) - 4*b**3*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) - 8*b**3*cos(c
+ d*x)**7/(105*d), Ne(d, 0)), (x*(a + b*sin(c))**3*sin(c)**2*cos(c)**2, Tru
e))
```

3.1070 $\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=163

$$-\frac{a(2a^2+33b^2)\cos^3(c+dx)}{120d}-\frac{(2a^2+5b^2)\cos^3(c+dx)(a+b\sin(c+dx))}{40d}+\frac{b(6a^2+b^2)\sin(c+dx)\cos(c+dx)}{16d}+$$

```
[Out] 1/16*b*(6*a^2+b^2)*x-1/120*a*(2*a^2+33*b^2)*cos(d*x+c)^3/d+1/16*b*(6*a^2+b^
2)*cos(d*x+c)*sin(d*x+c)/d-1/40*(2*a^2+5*b^2)*cos(d*x+c)^3*(a+b*sin(d*x+c))
/d-1/10*a*cos(d*x+c)^3*(a+b*sin(d*x+c))^2/d-1/6*cos(d*x+c)^3*(a+b*sin(d*x+c
))^3/d
```

Rubi [A] time = 0.29, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2862, 2669, 2635, 8}

$$-\frac{a(2a^2+33b^2)\cos^3(c+dx)}{120d}-\frac{(2a^2+5b^2)\cos^3(c+dx)(a+b\sin(c+dx))}{40d}+\frac{b(6a^2+b^2)\sin(c+dx)\cos(c+dx)}{16d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(6*a^2 + b^2)*x)/16 - (a*(2*a^2 + 33*b^2)*Cos[c + d*x]^3)/(120*d) + (b*(
6*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - ((2*a^2 + 5*b^2)*Cos[c + d
*x]^3*(a + b*Sin[c + d*x]))/(40*d) - (a*Cos[c + d*x]^3*(a + b*Sin[c + d*x])
^2)/(10*d) - (Cos[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{\cos^3(c+dx)(a+b\sin(c+dx))^3}{6d} + \frac{1}{6}\int \cos^2(c+dx)(3b+$$
$$= -\frac{a\cos^3(c+dx)(a+b\sin(c+dx))^2}{10d} - \frac{\cos^3(c+dx)(a+b\,\mathrm{si}}{6d}$$
$$= -\frac{\left(2a^2+5b^2\right)\cos^3(c+dx)(a+b\sin(c+dx))}{40d} - \frac{a\cos^3(c+}{}$$
$$= -\frac{a\left(2a^2+33b^2\right)\cos^3(c+dx)}{120d} - \frac{\left(2a^2+5b^2\right)\cos^3(c+dx)(a}{40d}$$
$$= -\frac{a\left(2a^2+33b^2\right)\cos^3(c+dx)}{120d} + \frac{b\left(6a^2+b^2\right)\cos(c+dx)\,\mathrm{si}}{16d}$$
$$= \frac{1}{16}b\left(6a^2+b^2\right)x - \frac{a\left(2a^2+33b^2\right)\cos^3(c+dx)}{120d} + \frac{b\left(6a^2+\right.}{}$$

Mathematica [A] time = 0.76, size = 138, normalized size = 0.85

$$\frac{-20\left(4a^3+3ab^2\right)\cos(3(c+dx))-120a\left(2a^2+3b^2\right)\cos(c+dx)+b\left(5\left(-3\left(6a^2+b^2\right)\sin(4(c+dx))+72a^2c+\right.\right.}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-120*a*(2*a^2 + 3*b^2)*Cos[c + d*x] - 20*(4*a^3 + 3*a*b^2)*Cos[3*(c + d*x)
] + b*(36*a*b*Cos[5*(c + d*x)] + 5*(72*a^2*c + 18*b^2*c + 72*a^2*d*x + 12*b
^2*d*x - 3*b^2*Sin[2*(c + d*x)] - 3*(6*a^2 + b^2)*Sin[4*(c + d*x)] + b^2*Si
n[6*(c + d*x)])))/(960*d)
```

fricas [A] time = 0.70, size = 116, normalized size = 0.71

$$\frac{144\,ab^2\cos(dx+c)^5-80\left(a^3+3\,ab^2\right)\cos(dx+c)^3+15\left(6\,a^2b+b^3\right)dx+5\left(8\,b^3\cos(dx+c)^5-2\left(18\,a^2b+7\,b^3\right)\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(144*a*b^2*cos(d*x + c)^5 - 80*(a^3 + 3*a*b^2)*cos(d*x + c)^3 + 15*(6
*a^2*b + b^3)*d*x + 5*(8*b^3*cos(d*x + c)^5 - 2*(18*a^2*b + 7*b^3)*cos(d*x
+ c)^3 + 3*(6*a^2*b + b^3)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.25, size = 139, normalized size = 0.85

$$\frac{3\,ab^2\cos(5\,dx+5\,c)}{80\,d}+\frac{b^3\sin(6\,dx+6\,c)}{192\,d}-\frac{b^3\sin(2\,dx+2\,c)}{64\,d}+\frac{1}{16}\left(6\,a^2b+b^3\right)x-\frac{\left(4\,a^3+3\,ab^2\right)\cos(3\,dx+3\,c)}{48\,d}-\frac{(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 3/80*a*b^2*cos(5*d*x + 5*c)/d + 1/192*b^3*sin(6*d*x + 6*c)/d - 1/64*b^3*sin
(2*d*x + 2*c)/d + 1/16*(6*a^2*b + b^3)*x - 1/48*(4*a^3 + 3*a*b^2)*cos(3*d*x
 + 3*c)/d - 1/8*(2*a^3 + 3*a*b^2)*cos(d*x + c)/d - 1/64*(6*a^2*b + b^3)*sin
(4*d*x + 4*c)/d
```

maple [A] time = 0.21, size = 158, normalized size = 0.97

$$\frac{-\frac{a^3\left(\cos^3(dx+c)\right)}{3}+3a^2b\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+3a\,b^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+\right.}{15}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/3*a^3*cos(d*x+c)^3+3*a^2*b*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*
x+c)*sin(d*x+c)+1/8*d*x+1/8*c)+3*a*b^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15
*cos(d*x+c)^3)+b^3*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x
+c)+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c))
```

maxima [A] time = 0.32, size = 108, normalized size = 0.66

$$-\frac{320\,a^3\cos(dx+c)^3-90\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^2b-192\left(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3\right)ab^2+5\left(4\,\mathrm{si}\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(320*a^3*cos(d*x + c)^3 - 90*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2*b
- 192*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a*b^2 + 5*(4*sin(2*d*x + 2*c)^3
 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*b^3)/d
```

mupad [B] time = 10.75, size = 425, normalized size = 2.61

$$\frac{b\,\mathrm{atan}\left(\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(6\,a^2+b^2\right)}{8\left(\frac{3\,a^2\,b}{4}+\frac{b^3}{8}\right)}\right)\left(6\,a^2+b^2\right)}{8\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a^2\,b}{4}+\frac{b^3}{8}\right)+4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (b*atan((b*tan(c/2 + (d*x)/2)*(6*a^2 + b^2))/(8*((3*a^2*b)/4 + b^3/8)))*(6*
a^2 + b^2))/(8*d) - (tan(c/2 + (d*x)/2)*((3*a^2*b)/4 + b^3/8) + 4*a^3*tan(c
/2 + (d*x)/2)^4 + 2*a^3*tan(c/2 + (d*x)/2)^10 + (4*a*b^2)/5 + tan(c/2 + (d*
x)/2)^8*(12*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/2)^2*((24*a*b^2)/5 + 2*a^3) +
tan(c/2 + (d*x)/2)^6*(8*a*b^2 + (20*a^3)/3) - tan(c/2 + (d*x)/2)^11*((3*a^2
*b)/4 + b^3/8) - tan(c/2 + (d*x)/2)^5*((9*a^2*b)/2 + (19*b^3)/4) + tan(c/2
+ (d*x)/2)^7*((9*a^2*b)/2 + (19*b^3)/4) - tan(c/2 + (d*x)/2)^3*((15*a^2*b)/
4 - (17*b^3)/24) + tan(c/2 + (d*x)/2)^9*((15*a^2*b)/4 - (17*b^3)/24) + (2*a
^3)/3)/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 +
(d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 +
(d*x)/2)^12 + 1)) - (b*(6*a^2 + b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/
(8*d)
```

sympy [A] time = 3.61, size = 340, normalized size = 2.09

$$\begin{cases} -\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^2bx\sin^4(c+dx)}{8}+\frac{3a^2bx\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^2bx\cos^4(c+dx)}{8}+\frac{3a^2b\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{3a^2b\sin(c+dx)\,\mathrm{c}}{8d} \\ x\left(a+b\sin(c)\right)^3\sin(c)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-a**3*cos(c + d*x)**3/(3*d) + 3*a**2*b*x*sin(c + d*x)**4/8 + 3*a
**2*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**2*b*x*cos(c + d*x)**4/8 +
```

```
3*a**2*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*a**2*b*sin(c + d*x)*cos(c +
 d*x)**3/(8*d) - a*b**2*sin(c + d*x)**2*cos(c + d*x)**3/d - 2*a*b**2*cos(c
+ d*x)**5/(5*d) + b**3*x*sin(c + d*x)**6/16 + 3*b**3*x*sin(c + d*x)**4*cos(
c + d*x)**2/16 + 3*b**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + b**3*x*cos(c
 + d*x)**6/16 + b**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) - b**3*sin(c + d*x
)**3*cos(c + d*x)**3/(6*d) - b**3*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d
, 0)), (x*(a + b*sin(c))**3*sin(c)*cos(c)**2, True))
```

3.1071 $\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=136

$$-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\left(a^2-2b^2\right)\cos(c+dx)}{2d}+\frac{b\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}bx\left(12a^2+b^2\right)+\frac{a\,\mathrm{co}}{}$$

```
[Out] 1/8*b*(12*a^2+b^2)*x-a^3*arctanh(cos(d*x+c))/d+1/2*a*(a^2-2*b^2)*cos(d*x+c)
/d+1/8*b*(2*a^2-b^2)*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)*(a+b*sin(d*x+
c))^2/d+1/4*cos(d*x+c)*(a+b*sin(d*x+c))^3/d
```

Rubi [**A**] time = 0.41, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2889, 3050, 3049, 3033, 3023, 2735, 3770}

$$\frac{a\left(a^2-2b^2\right)\cos(c+dx)}{2d}+\frac{b\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}bx\left(12a^2+b^2\right)-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(12*a^2 + b^2)*x)/8 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a*(a^2 - 2*b^2)*C
os[c + d*x])/(2*d) + (b*(2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(4*d) + (Cos[c + d*x]*(a + b*Sin[c +
d*x])^3)/(4*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)\,dx$$
$$= \frac{\cos(c+dx)(a+b\sin(c+dx))^3}{4d}+\frac{1}{4}\int \csc(c+dx)(a+b\sin$$
$$= \frac{a\cos(c+dx)(a+b\sin(c+dx))^2}{4d}+\frac{\cos(c+dx)(a+b\sin(c}{4d}$$
$$= \frac{b\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos(c+dx)(a+b\sin}{4d}$$
$$= \frac{a\left(a^2-2b^2\right)\cos(c+dx)}{2d}+\frac{b\left(2a^2-b^2\right)\cos(c+dx)\sin(c+}{8d}$$
$$= \frac{1}{8}b\left(12a^2+b^2\right)x+\frac{a\left(a^2-2b^2\right)\cos(c+dx)}{2d}+\frac{b\left(2a^2-b^2\right)c}{}$$
$$= \frac{1}{8}b\left(12a^2+b^2\right)x-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\left(a^2-2b^2\right)co}{2d}$$

Mathematica [A] time = 0.29, size = 129, normalized size = 0.95

$$\frac{32a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-32a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+8a\left(4a^2-3b^2\right)\cos(c+dx)+24a^2b\sin(2(c+dx))+4}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (48*a^2*b*c + 4*b^3*c + 48*a^2*b*d*x + 4*b^3*d*x + 8*a*(4*a^2 - 3*b^2)*Cos[
c + d*x] - 8*a*b^2*Cos[3*(c + d*x)] - 32*a^3*Log[Cos[(c + d*x)/2]] + 32*a^3
*Log[Sin[(c + d*x)/2]] + 24*a^2*b*Sin[2*(c + d*x)] - b^3*Sin[4*(c + d*x)])/
(32*d)
```

fricas [A] time = 0.75, size = 116, normalized size = 0.85

$$-\frac{8\,ab^2\cos\left(dx+c\right)^3-8\,a^3\cos\left(dx+c\right)+4\,a^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-4\,a^3\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(12\,a^2\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a*b^2*cos(d*x + c)^3 - 8*a^3*cos(d*x + c) + 4*a^3*log(1/2*cos(d*x +
 c) + 1/2) - 4*a^3*log(-1/2*cos(d*x + c) + 1/2) - (12*a^2*b + b^3)*d*x + (2
*b^3*cos(d*x + c)^3 - (12*a^2*b + b^3)*cos(d*x + c))*sin(d*x + c))/d
```

giac [B] time = 0.23, size = 293, normalized size = 2.15

$$8\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\left(12\,a^{2}b+b^{3}\right)(dx+c)-\frac{2\left(12\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-8\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+24\,ab^{2}\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + (12*a^2*b + b^3)*(d*x + c) - 2*
(12*a^2*b*tan(1/2*d*x + 1/2*c)^7 - b^3*tan(1/2*d*x + 1/2*c)^7 - 8*a^3*tan(1
/2*d*x + 1/2*c)^6 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x
+ 1/2*c)^5 + 7*b^3*tan(1/2*d*x + 1/2*c)^5 - 24*a^3*tan(1/2*d*x + 1/2*c)^4 +
 24*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 7*b^3*
tan(1/2*d*x + 1/2*c)^3 - 24*a^3*tan(1/2*d*x + 1/2*c)^2 + 8*a*b^2*tan(1/2*d*
x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c) - 8
*a^3 + 8*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.43, size = 150, normalized size = 1.10

$$\frac{a^{3}\cos(dx+c)}{d}+\frac{a^{3}\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{3a^{2}b\cos(dx+c)\sin(dx+c)}{2d}+\frac{3a^{2}bx}{2}+\frac{3a^{2}bc}{2d}-\frac{a\,b^{2}\left(\cos^{3}(dx\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] a^3*cos(d*x+c)/d+1/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+3/2/d*a^2*b*cos(d*x+c)*s
in(d*x+c)+3/2*a^2*b*x+3/2/d*a^2*b*c-a*b^2*cos(d*x+c)^3/d-1/4/d*b^3*cos(d*x+
c)^3*sin(d*x+c)+1/8*b^3*cos(d*x+c)*sin(d*x+c)/d+1/8*b^3*x+1/8/d*b^3*c
```

maxima [A] time = 0.40, size = 101, normalized size = 0.74

$$-\frac{32\,ab^{2}\cos(dx+c)^{3}-24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^{2}b-(4\,dx+4\,c-\sin(4\,dx+4\,c))b^{3}-16\,a^{3}\left(2\,\cos(dx+\right.}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(32*a*b^2*cos(d*x + c)^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2*b
- (4*d*x + 4*c - sin(4*d*x + 4*c))*b^3 - 16*a^3*(2*cos(d*x + c) - log(cos(d
*x + c) + 1) + log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 11.12, size = 567, normalized size = 4.17

$$\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^2\,b-\frac{b^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a\,b^2-6\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(6\,a\,b\cdots\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\cdots}{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d - (2*a*b^2 - tan(c/2 + (d*x)/2)*(3*a^2*b -
b^3/4) + tan(c/2 + (d*x)/2)^2*(2*a*b^2 - 6*a^3) + tan(c/2 + (d*x)/2)^6*(6*a
*b^2 - 2*a^3) + tan(c/2 + (d*x)/2)^4*(6*a*b^2 - 6*a^3) + tan(c/2 + (d*x)/2)
^7*(3*a^2*b - b^3/4) - tan(c/2 + (d*x)/2)^3*(3*a^2*b + (7*b^3)/4) + tan(c/2
 + (d*x)/2)^5*(3*a^2*b + (7*b^3)/4) - 2*a^3)/(d*(4*tan(c/2 + (d*x)/2)^2 + 6
*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
 + (b*atan(((b*(12*a^2 + b^2)*(3*a^2*b + b^3/4 + 2*a^3*tan(c/2 + (d*x)/2) -
 (b*tan(c/2 + (d*x)/2)*(12*a^2 + b^2)*3i)/4))/8 + (b*(12*a^2 + b^2)*(3*a^2*
b + b^3/4 + 2*a^3*tan(c/2 + (d*x)/2) + (b*tan(c/2 + (d*x)/2)*(12*a^2 + b^2)
*3i)/4))/8)/(2*tan(c/2 + (d*x)/2)*(b^6/16 + (3*a^2*b^4)/2 + 9*a^4*b^2) + 6*
a^5*b + (a^3*b^3)/2 - (b*(12*a^2 + b^2)*(3*a^2*b + b^3/4 + 2*a^3*tan(c/2 +
(d*x)/2) - (b*tan(c/2 + (d*x)/2)*(12*a^2 + b^2)*3i)/4)*1i)/8 + (b*(12*a^2 +
 b^2)*(3*a^2*b + b^3/4 + 2*a^3*tan(c/2 + (d*x)/2) + (b*tan(c/2 + (d*x)/2)*(
12*a^2 + b^2)*3i)/4)*1i)/8))*(12*a^2 + b^2))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^3\cos^2(c+dx)\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cos(c + d*x)**2*csc(c + d*x), x)
```

3.1072 $\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=102

$$-\frac{a^3\cot(c+dx)}{d}+a^3(-x)+\frac{3a^2b\cos(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}ab^2x-\frac{b^3}{}$$

```
[Out] -a^3*x+3/2*a*b^2*x-3*a^2*b*arctanh(cos(d*x+c))/d+3*a^2*b*cos(d*x+c)/d-1/3*b
^3*cos(d*x+c)^3/d-a^3*cot(d*x+c)/d+3/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.14, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 2635, 8, 2592, 321, 206, 3473, 2565, 30}

$$\frac{3a^2b\cos(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+a^3(-x)+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}ab^2x-\frac{b^3\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cos[c + d*x]])/d + (3*a^2*b*Cos
[c + d*x])/d - (b^3*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a*b^2
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(3ab^2\cos^2(c+dx)+3a^2b\cos(c+dx)\cot(c+dx)+a^3\cot^2(c+dx)+\right.$$
$$= a^3\int \cot^2(c+dx)\,dx + \left(3a^2b\right)\int \cos(c+dx)\cot(c+dx)\,dx + \left(3ab^2\right)\int$$
$$= -\frac{a^3\cot(c+dx)}{d} + \frac{3ab^2\cos(c+dx)\sin(c+dx)}{2d} - a^3\int 1\,dx + \frac{1}{2}\left(3ab^2\right)$$
$$= -a^3x + \frac{3}{2}ab^2x + \frac{3a^2b\cos(c+dx)}{d} - \frac{b^3\cos^3(c+dx)}{3d} - \frac{a^3\cot(c+dx)}{d} +$$
$$= -a^3x + \frac{3}{2}ab^2x - \frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^2b\cos(c+dx)}{d} - \frac{b^3\cos^3}{3}$$

Mathematica [A] time = 1.32, size = 143, normalized size = 1.40

$$\frac{-6a^3\cot\left(\frac{1}{2}(c+dx)\right)+\left(36a^2b-3b^3\right)\cos(c+dx)+6a\left(a^2\tan\left(\frac{1}{2}(c+dx)\right)-2a^2c-2a^2dx+6ab\log\left(\sin\left(\frac{1}{2}(c+\right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((36*a^2*b - 3*b^3)*Cos[c + d*x] - b^3*Cos[3*(c + d*x)] - 6*a^3*Cot[(c + d*
x)/2] + 9*a*b^2*Sin[2*(c + d*x)] + 6*a*(-2*a^2*c + 3*b^2*c - 2*a^2*d*x + 3*
b^2*d*x - 6*a*b*Log[Cos[(c + d*x)/2]] + 6*a*b*Log[Sin[(c + d*x)/2]] + a^2*T
an[(c + d*x)/2]))/(12*d)
```

fricas [A] time = 0.71, size = 143, normalized size = 1.40

$$-\frac{9\,ab^2\cos(dx+c)^3+9\,a^2b\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-9\,a^2b\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+}{6\,d\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(9*a*b^2*cos(d*x + c)^3 + 9*a^2*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 9*a^2*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(2*a^3 - 3*a*b
^2)*cos(d*x + c) + (2*b^3*cos(d*x + c)^3 - 18*a^2*b*cos(d*x + c) + 3*(2*a^3
 - 3*a*b^2)*d*x)*sin(d*x + c))/(d*sin(d*x + c))
```

giac [B] time = 0.24, size = 199, normalized size = 1.95

$$\frac{18\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\left(2\,a^3-3\,ab^2\right)(dx+c)-\frac{3\left(6\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(9\,ab^2\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(18*a^2*b*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/2*d*x + 1/2*c) -
 3*(2*a^3 - 3*a*b^2)*(d*x + c) - 3*(6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/tan
(1/2*d*x + 1/2*c) - 2*(9*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b*tan(1/2*d*
x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)
^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 2*b^3)/(tan(1/2*d*x + 1/2*c)
^2 + 1)^3)/d
```

maple [A] time = 0.40, size = 125, normalized size = 1.23

$$-a^3x-\frac{a^3\cot(dx+c)}{d}-\frac{a^3c}{d}+\frac{3a^2b\cos(dx+c)}{d}+\frac{3a^2b\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{3a\,b^2\cos(dx+c)\sin(dx+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -a^3*x-a^3*cot(d*x+c)/d-1/d*a^3*c+3*a^2*b*cos(d*x+c)/d+3/d*a^2*b*ln(csc(d*x
+c)-cot(d*x+c))+3/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d+3/2*a*b^2*x+3/2/d*a*b^2*c
-1/3*b^3*cos(d*x+c)^3/d
```

maxima [A] time = 0.41, size = 95, normalized size = 0.93

$$-\frac{4\,b^3\cos(dx+c)^3+12\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3-9\,(2\,dx+2\,c+\sin(2\,dx+2\,c))ab^2-18\,a^2b\left(2\,\cos(dx+c)-\mathrm{lo}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/12*(4*b^3*cos(d*x + c)^3 + 12*(d*x + c + 1/tan(d*x + c))*a^3 - 9*(2*d*x
+ 2*c + sin(2*d*x + 2*c))*a*b^2 - 18*a^2*b*(2*cos(d*x + c) - log(cos(d*x +
c) + 1) + log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.47, size = 289, normalized size = 2.83

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)\left(\frac{a\,b^2\,3\mathrm{i}}{2}-a^3\,1\mathrm{i}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(12\,a^2\,b-\frac{4\,b^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,\left(a^3+6\,a\,b^2\right.}{d\,\left(2\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2))/(2*d) - (log(tan(c/2 + (d*x)/2) - 1i)*((a*b^2*3i)/
2 - a^3*1i))/d + (tan(c/2 + (d*x)/2)*(12*a^2*b - (4*b^3)/3) - tan(c/2 + (d*
x)/2)^6*(6*a*b^2 + a^3) - 3*a^3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^2
*(6*a*b^2 - 3*a^3) + tan(c/2 + (d*x)/2)^5*(12*a^2*b - 4*b^3) - a^3 + 24*a^2
*b*tan(c/2 + (d*x)/2)^3)/(d*(2*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3
+ 6*tan(c/2 + (d*x)/2)^5 + 2*tan(c/2 + (d*x)/2)^7)) + (3*a^2*b*log(tan(c/2
+ (d*x)/2)))/d - (a*log(tan(c/2 + (d*x)/2) + 1i)*(2*a^2 - 3*b^2)*1i)/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1073 $\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=138

$$\frac{a\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{1}{2}bx\left(6a^2-b^2\right)+\frac{15ab^2\cos(c+dx)}{2d}-\frac{3b\cot(c+dx)(a+b\sin(c+dx))^2}{2d}-\frac{\cot(c$$

```
[Out] -1/2*b*(6*a^2-b^2)*x+1/2*a*(a^2-6*b^2)*arctanh(cos(d*x+c))/d+15/2*a*b^2*cos
(d*x+c)/d+5/2*b^3*cos(d*x+c)*sin(d*x+c)/d-3/2*b*cot(d*x+c)*(a+b*sin(d*x+c))
^2/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^3/d
```

Rubi [A] time = 0.47, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2889, 3048, 3047, 3033, 3023, 2735, 3770}

$$\frac{a\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{1}{2}bx\left(6a^2-b^2\right)+\frac{15ab^2\cos(c+dx)}{2d}-\frac{3b\cot(c+dx)(a+b\sin(c+dx))^2}{2d}-\frac{\cot(c$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(b*(6*a^2 - b^2)*x)/2 + (a*(a^2 - 6*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) + (1
5*a*b^2*Cos[c + d*x])/(2*d) + (5*b^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (3*
b*Cot[c + d*x]*(a + b*Sin[c + d*x])^2)/(2*d) - (Cot[c + d*x]*Csc[c + d*x]*(
a + b*Sin[c + d*x])^3)/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc^3(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)dx$$
$$= -\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3}{2d} + \frac{1}{2}\int \csc^2(c$$
$$= -\frac{3b\cot(c+dx)(a+b\sin(c+dx))^2}{2d} - \frac{\cot(c+dx)\csc(c+d}{2}$$
$$= \frac{5b^3\cos(c+dx)\sin(c+dx)}{2d} - \frac{3b\cot(c+dx)(a+b\sin(c+}{2d}$$
$$= \frac{15ab^2\cos(c+dx)}{2d} + \frac{5b^3\cos(c+dx)\sin(c+dx)}{2d} - \frac{3b\cot(c}{}$$
$$= -\frac{1}{2}b\left(6a^2-b^2\right)x + \frac{15ab^2\cos(c+dx)}{2d} + \frac{5b^3\cos(c+dx)\sin}{2d}$$
$$= -\frac{1}{2}b\left(6a^2-b^2\right)x + \frac{a\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2d} + \frac{15a}{}$$

Mathematica [A] time = 1.31, size = 192, normalized size = 1.39

$$\frac{a^3\left(-\csc^2\left(\frac{1}{2}(c+dx)\right)\right) + a^3\sec^2\left(\frac{1}{2}(c+dx)\right) - 4a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 12a^2b\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-24*a^2*b*c + 4*b^3*c - 24*a^2*b*d*x + 4*b^3*d*x + 24*a*b^2*Cos[c + d*x] -
 12*a^2*b*Cot[(c + d*x)/2] - a^3*Csc[(c + d*x)/2]^2 + 4*a^3*Log[Cos[(c + d*
x)/2]] - 24*a*b^2*Log[Cos[(c + d*x)/2]] - 4*a^3*Log[Sin[(c + d*x)/2]] + 24*
a*b^2*Log[Sin[(c + d*x)/2]] + a^3*Sec[(c + d*x)/2]^2 + 2*b^3*Sin[2*(c + d*x
)] + 12*a^2*b*Tan[(c + d*x)/2])/(8*d)
```

fricas [A] time = 0.75, size = 216, normalized size = 1.57

$$\frac{12\,ab^2\cos(dx+c)^3 - 2\left(6\,a^2b-b^3\right)dx\cos(dx+c)^2 + 2\left(6\,a^2b-b^3\right)dx + 2\left(a^3-6\,ab^2\right)\cos(dx+c) - \left(a^3-6\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(12*a*b^2*cos(d*x + c)^3 - 2*(6*a^2*b - b^3)*d*x*cos(d*x + c)^2 + 2*(6*
a^2*b - b^3)*d*x + 2*(a^3 - 6*a*b^2)*cos(d*x + c) - (a^3 - 6*a*b^2 - (a^3 -
 6*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (a^3 - 6*a*b^2 - (a
^3 - 6*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) + 2*(b^3*cos(d*x
 + c)^3 + (6*a^2*b - b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d
)
```

giac [B] time = 0.30, size = 272, normalized size = 1.97

$$a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 12\,a^2b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 4\left(6\,a^2b - b^3\right)(dx + c) - 4\left(a^3 - 6\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*b*tan(1/2*d*x + 1/2*c) - 4*(6*a^2*
b - b^3)*(d*x + c) - 4*(a^3 - 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (2*
a^3*tan(1/2*d*x + 1/2*c)^6 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 12*a^2*b*tan
(1/2*d*x + 1/2*c)^5 - 8*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/
2*c)^4 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x + 1/2*c)^3
+ 8*b^3*tan(1/2*d*x + 1/2*c)^3 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b
*tan(1/2*d*x + 1/2*c) - a^3)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)
)^2)/d
```

maple [A] time = 0.50, size = 171, normalized size = 1.24

$$-\frac{a^3\left(\cos^3(dx+c)\right)}{2d\sin(dx+c)^2} - \frac{a^3\cos(dx+c)}{2d} - \frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d} - 3a^2bx - \frac{3a^2b\cot(dx+c)}{d} - \frac{3a^2bc}{d} + \frac{3a\,b^2\,c\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3/sin(d*x+c)^2*cos(d*x+c)^3-1/2*a^3*cos(d*x+c)/d-1/2/d*a^3*ln(csc(
d*x+c)-cot(d*x+c))-3*a^2*b*x-3*a^2*b*cot(d*x+c)/d-3/d*a^2*b*c+3*a*b^2*cos(d
*x+c)/d+3/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))+1/2*b^3*cos(d*x+c)*sin(d*x+c)/d
+1/2*b^3*x+1/2/d*b^3*c
```

maxima [A] time = 0.50, size = 128, normalized size = 0.93

$$-\frac{12\left(dx + c + \frac{1}{\tan(dx+c)}\right)a^2b - (2\,dx + 2\,c + \sin(2\,dx + 2\,c))b^3 - a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} + \log(\cos(dx+c)+1) - \log(\cos\cdots\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/4*(12*(d*x + c + 1/tan(d*x + c))*a^2*b - (2*d*x + 2*c + sin(2*d*x + 2*c)
)*b^3 - a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) -
log(cos(d*x + c) - 1)) - 6*a*b^2*(2*cos(d*x + c) - log(cos(d*x + c) + 1) +
log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.46, size = 585, normalized size = 4.24

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(3\,a\,b^2-\frac{a^3}{2}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(6\,a^2\,b+4\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(24\,a\,b^2-\frac{a^3}{2}\right)}{d\,\left(4\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (log(tan(c/2 + (d*x)/2))*(3*a*b^2 - a^3/
2))/d - (tan(c/2 + (d*x)/2)^5*(6*a^2*b + 4*b^3) - tan(c/2 + (d*x)/2)^4*(24*
a*b^2 - a^3/2) - tan(c/2 + (d*x)/2)^2*(24*a*b^2 - a^3) + tan(c/2 + (d*x)/2)
^3*(12*a^2*b - 4*b^3) + a^3/2 + 6*a^2*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 +
 (d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6)) + (3*a^2*b*
tan(c/2 + (d*x)/2))/(2*d) + (b*atan(((b*(6*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(
6*a*b^2 - a^3) - 6*a^2*b + b^3 - b*tan(c/2 + (d*x)/2)*(6*a^2 - b^2)*3i))/2
+ (b*(6*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(6*a*b^2 - a^3) - 6*a^2*b + b^3 + b*
tan(c/2 + (d*x)/2)*(6*a^2 - b^2)*3i))/2)/(2*tan(c/2 + (d*x)/2)*(b^6 - 12*a^
2*b^4 + 36*a^4*b^2) + 6*a*b^5 + 6*a^5*b - 37*a^3*b^3 - (b*(6*a^2 - b^2)*(ta
n(c/2 + (d*x)/2)*(6*a*b^2 - a^3) - 6*a^2*b + b^3 - b*tan(c/2 + (d*x)/2)*(6*
a^2 - b^2)*3i)*1i)/2 + (b*(6*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(6*a*b^2 - a^3)
 - 6*a^2*b + b^3 + b*tan(c/2 + (d*x)/2)*(6*a^2 - b^2)*3i)*1i)/2))*(6*a^2 -
b^2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1074 $\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=138

$$\frac{a(a^2-3b^2)\cot(c+dx)}{3d}+\frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-3ab^2x-\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{3d}$$

```
[Out] -3*a*b^2*x+1/2*b*(3*a^2-2*b^2)*arctanh(cos(d*x+c))/d+11/6*b^3*cos(d*x+c)/d+
1/3*a*(a^2-3*b^2)*cot(d*x+c)/d-1/2*b*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))
^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^3/d
```

Rubi [A] time = 0.48, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2889, 3048, 3047, 3031, 3023, 2735, 3770}

$$\frac{a(a^2-3b^2)\cot(c+dx)}{3d}+\frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-3ab^2x-\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -3*a*b^2*x + (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) + (11*b^3*Cos[
c + d*x])/(6*d) + (a*(a^2 - 3*b^2)*Cot[c + d*x])/(3*d) - (b*Cot[c + d*x]*Cs
c[c + d*x]*(a + b*Sin[c + d*x])^2)/(2*d) - (Cot[c + d*x]*Csc[c + d*x]^2*(a
+ b*Sin[c + d*x])^3)/(3*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc^4(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)\,dx$$
$$= -\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{3d} + \frac{1}{3}\int \csc^3(c$$
$$= -\frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{2d} - \frac{\cot(c+dx)}{}$$
$$= \frac{a\left(a^2-3b^2\right)\cot(c+dx)}{3d} - \frac{b\cot(c+dx)\csc(c+dx)(a+b\sin}{2d}$$
$$= \frac{11b^3\cos(c+dx)}{6d} + \frac{a\left(a^2-3b^2\right)\cot(c+dx)}{3d} - \frac{b\cot(c+dx)}{}$$
$$= -3ab^2x + \frac{11b^3\cos(c+dx)}{6d} + \frac{a\left(a^2-3b^2\right)\cot(c+dx)}{3d} - \frac{b\,c}{}$$
$$= -3ab^2x + \frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2d} + \frac{11b^3\cos(c+}{6d}$$

Mathematica [B] time = 6.19, size = 615, normalized size = 4.46

$$\frac{\sin^3(c+dx)\csc\left(\frac{1}{2}(c+dx)\right)\left(a^3\cos\left(\frac{1}{2}(c+dx)\right)-9ab^2\cos\left(\frac{1}{2}(c+dx)\right)\right)(a\csc(c+dx)+b)^3}{6d(a+b\sin(c+dx))^3}+\frac{\sin^3(c+dx)\sec\left(\frac{1}{2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a*b^2*(c + d*x)*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(d*(a + b*Sin[c
+ d*x])^3) + (b^3*Cos[c + d*x]*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(d*(a
 + b*Sin[c + d*x])^3) + ((a^3*Cos[(c + d*x)/2] - 9*a*b^2*Cos[(c + d*x)/2])*
Csc[(c + d*x)/2]*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(6*d*(a + b*Sin[c +
 d*x])^3) - (3*a^2*b*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x])^3*Sin[c + d*x]
^3)/(8*d*(a + b*Sin[c + d*x])^3) - (a^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2
*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(24*d*(a + b*Sin[c + d*x])^3) + ((3
*a^2*b - 2*b^3)*(b + a*Csc[c + d*x])^3*Log[Cos[(c + d*x)/2]]*Sin[c + d*x]^3
)/(2*d*(a + b*Sin[c + d*x])^3) + ((-3*a^2*b + 2*b^3)*(b + a*Csc[c + d*x])^3
*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]^3)/(2*d*(a + b*Sin[c + d*x])^3) + (3*a^
2*b*(b + a*Csc[c + d*x])^3*Sec[(c + d*x)/2]^2*Sin[c + d*x]^3)/(8*d*(a + b*S
```

```
in[c + d*x])^3) + ((b + a*Csc[c + d*x])^3*Sec[(c + d*x)/2]*(-(a^3*Sin[(c +
d*x)/2]) + 9*a*b^2*Sin[(c + d*x)/2])*Sin[c + d*x]^3)/(6*d*(a + b*Sin[c + d*
x])^3) + (a^3*(b + a*Csc[c + d*x])^3*Sec[(c + d*x)/2]^2*Sin[c + d*x]^3*Tan[
(c + d*x)/2])/(24*d*(a + b*Sin[c + d*x])^3)
```

fricas [A] time = 0.74, size = 231, normalized size = 1.67

$$\frac{36\,ab^2\cos\left(dx+c\right)+4\left(a^3-9\,ab^2\right)\cos\left(dx+c\right)^3-3\left(3\,a^2b-2\,b^3-\left(3\,a^2b-2\,b^3\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\cos\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(36*a*b^2*cos(d*x + c) + 4*(a^3 - 9*a*b^2)*cos(d*x + c)^3 - 3*(3*a^2*b
 - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*si
n(d*x + c) + 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/
2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*(6*a*b^2*d*x*cos(d*x + c)^2 - 2*b^3*
cos(d*x + c)^3 - 6*a*b^2*d*x - (3*a^2*b - 2*b^3)*cos(d*x + c))*sin(d*x + c)
)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.25, size = 222, normalized size = 1.61

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,(dx+c)ab^2-3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 72*(d*x
 + c)*a*b^2 - 3*a^3*tan(1/2*d*x + 1/2*c) + 36*a*b^2*tan(1/2*d*x + 1/2*c) +
48*b^3/(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*(3*a^2*b - 2*b^3)*log(abs(tan(1/2*
d*x + 1/2*c))) + (66*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 44*b^3*tan(1/2*d*x + 1/
2*c)^3 + 3*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 9
*a^2*b*tan(1/2*d*x + 1/2*c) - a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.46, size = 159, normalized size = 1.15

$$-\frac{a^3\left(\cos^3\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)^3}-\frac{3a^2b\left(\cos^3\left(dx+c\right)\right)}{2d\sin\left(dx+c\right)^2}-\frac{3a^2b\cos\left(dx+c\right)}{2d}-\frac{3a^2b\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{2d}-3a\,b^2x-\frac{3a\,b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/3/d*a^3/sin(d*x+c)^3*cos(d*x+c)^3-3/2/d*a^2*b/sin(d*x+c)^2*cos(d*x+c)^3-
3/2*a^2*b*cos(d*x+c)/d-3/2/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))-3*a*b^2*x-3*a*
b^2*cot(d*x+c)/d-3/d*a*b^2*c+b^3*cos(d*x+c)/d+1/d*b^3*ln(csc(d*x+c)-cot(d*x
+c))
```

maxima [A] time = 0.42, size = 119, normalized size = 0.86

$$-\frac{36\left(dx+c+\frac{1}{\tan(dx+c)}\right)ab^2-9\,a^2b\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)-6\,b^3\left(2\,\cos\left(dx\right.\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/12*(36*(d*x + c + 1/tan(d*x + c))*a*b^2 - 9*a^2*b*(2*cos(d*x + c)/(cos(d
*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) - 6*b^3*(2*
cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 4*a^3/tan(d
*x + c)^3)/d
```

mupad [B] time = 10.71, size = 477, normalized size = 3.46

$$-\frac{\frac{a^3\,\cos(c+d\,x)}{4}-\frac{3\,b^3\,\sin(c+d\,x)}{4}+\frac{a^3\,\cos(3\,c+3\,d\,x)}{12}-\frac{b^3\,\sin(2\,c+2\,d\,x)}{4}+\frac{b^3\,\sin(3\,c+3\,d\,x)}{4}+\frac{b^3\,\sin(4\,c+4\,d\,x)}{8}-\frac{3\,a\,b^2\,\cos(3\,c+3\,d\,x)}{4}-\frac{3\,b^3}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] -((a^3*cos(c + d*x))/4 - (3*b^3*sin(c + d*x))/4 + (a^3*cos(3*c + 3*d*x))/12
 - (b^3*sin(2*c + 2*d*x))/4 + (b^3*sin(3*c + 3*d*x))/4 + (b^3*sin(4*c + 4*d
*x))/8 - (3*a*b^2*cos(3*c + 3*d*x))/4 - (3*b^3*sin(c + d*x)*log(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (3*a^2*b*sin(2*c + 2*d*x))/4 + (b^3*log(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x))/4 + (3*a*b^2*cos(c +
 d*x))/4 - (3*a^2*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*
d*x))/8 - (9*a*b^2*atan((3*a^2*sin(c/2 + (d*x)/2) - 2*b^2*sin(c/2 + (d*x)/2
) + 6*a*b*cos(c/2 + (d*x)/2))/(2*b^2*cos(c/2 + (d*x)/2) - 3*a^2*cos(c/2 + (
d*x)/2) + 6*a*b*sin(c/2 + (d*x)/2)))*sin(c + d*x))/2 + (3*a*b^2*atan((3*a^2
*sin(c/2 + (d*x)/2) - 2*b^2*sin(c/2 + (d*x)/2) + 6*a*b*cos(c/2 + (d*x)/2))/
(2*b^2*cos(c/2 + (d*x)/2) - 3*a^2*cos(c/2 + (d*x)/2) + 6*a*b*sin(c/2 + (d*x
)/2)))*sin(3*c + 3*d*x))/2 + (9*a^2*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/c
os(c/2 + (d*x)/2)))/8)/(d*sin(c + d*x)^3)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1075 $\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=152

$$\frac{b\left(2a^2-b^2\right)\cot(c+dx)}{2d}+\frac{a\left(a^2+12b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{8d}-\frac{\cot(c+dx)\csc}{}$$

```
[Out] -b^3*x+1/8*a*(a^2+12*b^2)*arctanh(cos(d*x+c))/d+1/2*b*(2*a^2-b^2)*cot(d*x+c
)/d+1/8*a*(a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)/d-1/4*b*cot(d*x+c)*csc(d*x+c)^2
*(a+b*sin(d*x+c))^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^3/d
```

Rubi [A] time = 0.51, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2889, 3048, 3047, 3031, 3021, 2735, 3770}

$$\frac{b\left(2a^2-b^2\right)\cot(c+dx)}{2d}+\frac{a\left(a^2+12b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(b^3*x) + (a*(a^2 + 12*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + (b*(2*a^2 - b^2
)*Cot[c + d*x])/(2*d) + (a*(a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*d) -
 (b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2)/(4*d) - (Cot[c + d*
x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(4*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
```

```
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc^5(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)\,dx$$
$$= -\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3}{4d} + \frac{1}{4}\int \csc^4(c$$
$$= -\frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\cot(c+dx}{}$$
$$= \frac{a\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{8d} - \frac{b\cot(c+dx)\csc^2(c+}{}$$
$$= \frac{b\left(2a^2-b^2\right)\cot(c+dx)}{2d} + \frac{a\left(a^2-2b^2\right)\cot(c+dx)\csc(c+d}{8d}$$
$$= -b^3x + \frac{b\left(2a^2-b^2\right)\cot(c+dx)}{2d} + \frac{a\left(a^2-2b^2\right)\cot(c+dx)\,\mathrm{c}}{8d}$$
$$= -b^3x + \frac{a\left(a^2+12b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} + \frac{b\left(2a^2-b^2\right)\mathrm{co}}{2d}$$

Mathematica [B] time = 6.23, size = 690, normalized size = 4.54

$$\frac{\left(a^3-12ab^2\right)\sin^3(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a\csc(c+dx)+b)^3}{32d(a+b\sin(c+dx))^3}+\frac{\left(-a^3-12ab^2\right)\sin^3(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d(a+b\sin(c+dx))^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((b^3*(c + d*x)*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(d*(a + b*Sin[c + d
*x])^3)) + ((a^2*b*Cos[(c + d*x)/2] - b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2
]*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(2*d*(a + b*Sin[c + d*x])^3) + ((a
^3 - 12*a*b^2)*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(3
2*d*(a + b*Sin[c + d*x])^3) - (a^2*b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(b
 + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(8*d*(a + b*Sin[c + d*x])^3) - (a^3*Cs
c[(c + d*x)/2]^4*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(64*d*(a + b*Sin[c
+ d*x])^3) + ((a^3 + 12*a*b^2)*(b + a*Csc[c + d*x])^3*Log[Cos[(c + d*x)/2]]
*Sin[c + d*x]^3)/(8*d*(a + b*Sin[c + d*x])^3) + ((-a^3 - 12*a*b^2)*(b + a*C
```

```
sc[c + d*x])^3*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]^3)/(8*d*(a + b*Sin[c + d*
x])^3) + ((-a^3 + 12*a*b^2)*(b + a*Csc[c + d*x])^3*Sec[(c + d*x)/2]^2*Sin[c
 + d*x]^3)/(32*d*(a + b*Sin[c + d*x])^3) + (a^3*(b + a*Csc[c + d*x])^3*Sec[
(c + d*x)/2]^4*Sin[c + d*x]^3)/(64*d*(a + b*Sin[c + d*x])^3) + ((b + a*Csc[
c + d*x])^3*Sec[(c + d*x)/2]*(-(a^2*b*Sin[(c + d*x)/2]) + b^3*Sin[(c + d*x)
/2])*Sin[c + d*x]^3)/(2*d*(a + b*Sin[c + d*x])^3) + (a^2*b*(b + a*Csc[c + d
*x])^3*Sec[(c + d*x)/2]^2*Sin[c + d*x]^3*Tan[(c + d*x)/2])/(8*d*(a + b*Sin[
c + d*x])^3)
```

fricas [A] time = 0.63, size = 265, normalized size = 1.74

$$-\frac{16 b^3 dx \cos(dx+c)^4 - 32 b^3 dx \cos(dx+c)^2 + 16 b^3 dx + 2\left(a^3 - 12 ab^2\right)\cos(dx+c)^3 + 2\left(a^3 + 12 ab^2\right)\cos(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*(16*b^3*d*x*cos(d*x + c)^4 - 32*b^3*d*x*cos(d*x + c)^2 + 16*b^3*d*x +
 2*(a^3 - 12*a*b^2)*cos(d*x + c)^3 + 2*(a^3 + 12*a*b^2)*cos(d*x + c) - ((a^
3 + 12*a*b^2)*cos(d*x + c)^4 + a^3 + 12*a*b^2 - 2*(a^3 + 12*a*b^2)*cos(d*x
+ c)^2)*log(1/2*cos(d*x + c) + 1/2) + ((a^3 + 12*a*b^2)*cos(d*x + c)^4 + a^
3 + 12*a*b^2 - 2*(a^3 + 12*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1
/2) + 16*(b^3*cos(d*x + c) + (a^2*b - b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.26, size = 234, normalized size = 1.54

$$\frac{3 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 24 a^2 b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 72 ab^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 192 (dx+c) b^3 - 72 a^2 b \tan\left(\frac{1}{2} dx +\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^3*tan(1/2*d*x + 1/2*c)^4 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 72*
a*b^2*tan(1/2*d*x + 1/2*c)^2 - 192*(d*x + c)*b^3 - 72*a^2*b*tan(1/2*d*x + 1
/2*c) + 96*b^3*tan(1/2*d*x + 1/2*c) - 24*(a^3 + 12*a*b^2)*log(abs(tan(1/2*d
*x + 1/2*c))) + (50*a^3*tan(1/2*d*x + 1/2*c)^4 + 600*a*b^2*tan(1/2*d*x + 1/
2*c)^4 + 72*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 96*b^3*tan(1/2*d*x + 1/2*c)^3 -
72*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*b*tan(1/2*d*x + 1/2*c) - 3*a^3)/ta
n(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.48, size = 207, normalized size = 1.36

$$-\frac{a^3\left(\cos^3(dx+c)\right)}{4d\sin(dx+c)^4}-\frac{a^3\left(\cos^3(dx+c)\right)}{8d\sin(dx+c)^2}-\frac{a^3\cos(dx+c)}{8d}-\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{8d}-\frac{a^2b\left(\cos^3(dx+c)\right)}{d\sin(dx+c)^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/4/d*a^3/sin(d*x+c)^4*cos(d*x+c)^3-1/8/d*a^3/sin(d*x+c)^2*cos(d*x+c)^3-1/
8*a^3*cos(d*x+c)/d-1/8/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-1/d*a^2*b/sin(d*x+c)
^3*cos(d*x+c)^3-3/2/d*a*b^2/sin(d*x+c)^2*cos(d*x+c)^3-3/2*a*b^2*cos(d*x+c)/
d-3/2/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))-b^3*x-1/d*cot(d*x+c)*b^3-1/d*b^3*c
```

maxima [A] time = 0.42, size = 149, normalized size = 0.98

$$-\frac{16\left(dx+c+\frac{1}{\tan(dx+c)}\right)b^3+a^3\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-12\,ab^2\Big(}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(16*(d*x + c + 1/tan(d*x + c))*b^3 + a^3*(2*(cos(d*x + c)^3 + cos(d*x
 + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + lo
g(cos(d*x + c) - 1)) - 12*a*b^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(
cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 16*a^2*b/tan(d*x + c)^3)/d
```

mupad [B] time = 9.87, size = 348, normalized size = 2.29

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{8\,d}-\frac{b^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{2\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^3\cdots}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^3\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^4)/(64*d) - (a^3*cot(c/2 + (d*x)/2)^4)/(64*d) - (a^
3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(8*d) - (b^3*cot(c/2 + (d*x)/
2))/(2*d) + (b^3*tan(c/2 + (d*x)/2))/(2*d) - (2*b^3*atan((8*b^3*cos(c/2 + (
d*x)/2) + a^3*sin(c/2 + (d*x)/2) + 12*a*b^2*sin(c/2 + (d*x)/2))/(a^3*cos(c/
2 + (d*x)/2) - 8*b^3*sin(c/2 + (d*x)/2) + 12*a*b^2*cos(c/2 + (d*x)/2))))/d
```

```
- (3*a*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*d) + (3*a^2*b*cot
(c/2 + (d*x)/2))/(8*d) - (3*a^2*b*tan(c/2 + (d*x)/2))/(8*d) - (3*a*b^2*cot(
c/2 + (d*x)/2)^2)/(8*d) - (a^2*b*cot(c/2 + (d*x)/2)^3)/(8*d) + (3*a*b^2*tan
(c/2 + (d*x)/2)^2)/(8*d) + (a^2*b*tan(c/2 + (d*x)/2)^3)/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1076 $\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=183

$$\frac{a\left(2a^2+15b^2\right)\cot(c+dx)}{15d}+\frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a\left(2a^2-3b^2\right)\cot(c+dx)\csc^2(c+dx)}{30d}+\frac{3b\left(5a^2-\right.}{}$$

```
[Out] 1/8*b*(3*a^2+4*b^2)*arctanh(cos(d*x+c))/d+1/15*a*(2*a^2+15*b^2)*cot(d*x+c)/
d+3/40*b*(5*a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)/d+1/30*a*(2*a^2-3*b^2)*cot(d*x
+c)*csc(d*x+c)^2/d-3/20*b*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^2/d-1/5*
cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^3/d
```

Rubi [A] time = 0.57, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2889, 3048, 3047, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{a\left(2a^2+15b^2\right)\cot(c+dx)}{15d}+\frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a\left(2a^2-3b^2\right)\cot(c+dx)\csc^2(c+dx)}{30d}+\frac{3b\left(5a^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(3*a^2 + 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + (a*(2*a^2 + 15*b^2)*Cot[c
 + d*x])/(15*d) + (3*b*(5*a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(40*d) +
(a*(2*a^2 - 3*b^2)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*d) - (3*b*Cot[c + d*x]*
Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(20*d) - (Cot[c + d*x]*Csc[c + d*x]^
4*(a + b*Sin[c + d*x])^3)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
```

```
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc^6(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)\,dx$$

$$= -\frac{\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3}{5d} + \frac{1}{5}\int \csc^5(c$$

$$= -\frac{3b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2}{20d} - \frac{\cot(c+d}{}$$

$$= \frac{a\left(2a^2-3b^2\right)\cot(c+dx)\csc^2(c+dx)}{30d} - \frac{3b\cot(c+dx)\csc^3(}{}$$

$$= \frac{3b\left(5a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{40d} + \frac{a\left(2a^2-3b^2\right)\cot(c}{30}$$

$$= \frac{3b\left(5a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{40d} + \frac{a\left(2a^2-3b^2\right)\cot(c}{30}$$

$$= \frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} + \frac{3b\left(5a^2-2b^2\right)\cot(c+}{40d}$$

$$= \frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} + \frac{a\left(2a^2+15b^2\right)\cot(c+}{15d}$$

Mathematica [A] time = 1.29, size = 344, normalized size = 1.88

$$\frac{32\left(2a^3+15ab^2\right)\cot\left(\frac{1}{2}(c+dx)\right) - 64a^3\tan\left(\frac{1}{2}(c+dx)\right) - 3a^3\sin(c+dx)\csc^6\left(\frac{1}{2}(c+dx)\right) - 16a^3\sin^4\left(\frac{1}{2}(c+dx)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (32*(2*a^3 + 15*a*b^2)*Cot[(c + d*x)/2] + 30*(3*a^2*b - 4*b^3)*Csc[(c + d*x
)/2]^2 + 360*a^2*b*Log[Cos[(c + d*x)/2]] + 480*b^3*Log[Cos[(c + d*x)/2]] -
360*a^2*b*Log[Sin[(c + d*x)/2]] - 480*b^3*Log[Sin[(c + d*x)/2]] - 90*a^2*b*
Sec[(c + d*x)/2]^2 + 120*b^3*Sec[(c + d*x)/2]^2 + 45*a^2*b*Sec[(c + d*x)/2]
^4 - 16*a^3*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 960*a*b^2*Csc[c + d*x]^3*Si
n[(c + d*x)/2]^4 - 3*a^3*Csc[(c + d*x)/2]^6*Sin[c + d*x] + a*Csc[(c + d*x)/
2]^4*(-45*a*b + (a^2 - 60*b^2)*Sin[c + d*x]) - 64*a^3*Tan[(c + d*x)/2] - 48
0*a*b^2*Tan[(c + d*x)/2] + 6*a^3*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*
d)
```

fricas [A] time = 0.72, size = 275, normalized size = 1.50

$$16\left(2a^3+15ab^2\right)\cos\left(dx+c\right)^5-80\left(a^3+3ab^2\right)\cos\left(dx+c\right)^3+15\left(\left(3a^2b+4b^3\right)\cos\left(dx+c\right)^4+3a^2b+4b^3-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/240*(16*(2*a^3 + 15*a*b^2)*cos(d*x + c)^5 - 80*(a^3 + 3*a*b^2)*cos(d*x +
c)^3 + 15*((3*a^2*b + 4*b^3)*cos(d*x + c)^4 + 3*a^2*b + 4*b^3 - 2*(3*a^2*b
+ 4*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*((3*
a^2*b + 4*b^3)*cos(d*x + c)^4 + 3*a^2*b + 4*b^3 - 2*(3*a^2*b + 4*b^3)*cos(d
*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*((3*a^2*b - 4*b^3
)*cos(d*x + c)^3 + (3*a^2*b + 4*b^3)*cos(d*x + c))*sin(d*x + c))/((d*cos(d*
x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.27, size = 290, normalized size = 1.58

$$6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+45a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+10a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+120ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+120b^3\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 10*
a^3*tan(1/2*d*x + 1/2*c)^3 + 120*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*b^3*tan
(1/2*d*x + 1/2*c)^2 - 60*a^3*tan(1/2*d*x + 1/2*c) - 360*a*b^2*tan(1/2*d*x +
```

```
 1/2*c) - 120*(3*a^2*b + 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (822*a^2*b
*tan(1/2*d*x + 1/2*c)^5 + 1096*b^3*tan(1/2*d*x + 1/2*c)^5 + 60*a^3*tan(1/2*
d*x + 1/2*c)^4 + 360*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*b^3*tan(1/2*d*x + 1
/2*c)^3 - 10*a^3*tan(1/2*d*x + 1/2*c)^2 - 120*a*b^2*tan(1/2*d*x + 1/2*c)^2
- 45*a^2*b*tan(1/2*d*x + 1/2*c) - 6*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.49, size = 227, normalized size = 1.24

$$-\frac{a^3\left(\cos^3(dx+c)\right)}{5d\sin(dx+c)^5}-\frac{2a^3\left(\cos^3(dx+c)\right)}{15d\sin(dx+c)^3}-\frac{3a^2b\left(\cos^3(dx+c)\right)}{4d\sin(dx+c)^4}-\frac{3a^2b\left(\cos^3(dx+c)\right)}{8d\sin(dx+c)^2}-\frac{3a^2b\cos(dx+c)}{8d}-\frac{3a^2b\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/5/d*a^3/sin(d*x+c)^5*cos(d*x+c)^3-2/15/d*a^3/sin(d*x+c)^3*cos(d*x+c)^3-3
/4/d*a^2*b/sin(d*x+c)^4*cos(d*x+c)^3-3/8/d*a^2*b/sin(d*x+c)^2*cos(d*x+c)^3-
3/8*a^2*b*cos(d*x+c)/d-3/8/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))-1/d*a*b^2/sin(
d*x+c)^3*cos(d*x+c)^3-1/2/d*b^3/sin(d*x+c)^2*cos(d*x+c)^3-1/2*b^3*cos(d*x+c
)/d-1/2/d*b^3*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.33, size = 157, normalized size = 0.86

$$-\frac{45\,a^2b\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-60\,b^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/240*(45*a^2*b*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos
(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 60*b^3*
(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x
+ c) - 1)) + 240*a*b^2/tan(d*x + c)^3 + 16*(5*tan(d*x + c)^2 + 3)*a^3/tan(d
*x + c)^5)/d
```

mupad [B] time = 9.44, size = 241, normalized size = 1.32

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{a^3}{96}+\frac{a\,b^2}{8}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(\frac{3\,a^2\,b}{8}+\frac{b^3}{2}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^5)/(160*d) + (b^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (ta
n(c/2 + (d*x)/2)^3*((a*b^2)/8 + a^3/96))/d - (log(tan(c/2 + (d*x)/2))*((3*a
^2*b)/8 + b^3/2))/d - (cot(c/2 + (d*x)/2)^5*(4*b^3*tan(c/2 + (d*x)/2)^3 + t
an(c/2 + (d*x)/2)^2*(4*a*b^2 + a^3/3) - tan(c/2 + (d*x)/2)^4*(12*a*b^2 + 2*
a^3) + a^3/5 + (3*a^2*b*tan(c/2 + (d*x)/2))/2))/(32*d) - (tan(c/2 + (d*x)/2
)*((3*a*b^2)/8 + a^3/16))/d + (3*a^2*b*tan(c/2 + (d*x)/2)^4)/(64*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1077 $\int \cot^2(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=212

$$\frac{b(6a^2+5b^2)\cot(c+dx)}{15d}+\frac{a(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{a(5a^2-6b^2)\cot(c+dx)\csc^3(c+dx)}{120d}+\frac{b(3a^2-b^2)}{}$$

```
[Out] 1/16*a*(a^2+6*b^2)*arctanh(cos(d*x+c))/d+1/15*b*(6*a^2+5*b^2)*cot(d*x+c)/d+
1/16*a*(a^2+6*b^2)*cot(d*x+c)*csc(d*x+c)/d+1/15*b*(3*a^2-b^2)*cot(d*x+c)*cs
c(d*x+c)^2/d+1/120*a*(5*a^2-6*b^2)*cot(d*x+c)*csc(d*x+c)^3/d-1/10*b*cot(d*x
+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^2/d-1/6*cot(d*x+c)*csc(d*x+c)^5*(a+b*sin(
d*x+c))^3/d
```

Rubi [A] time = 0.60, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2889, 3048, 3047, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{b(6a^2+5b^2)\cot(c+dx)}{15d}+\frac{a(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{a(5a^2-6b^2)\cot(c+dx)\csc^3(c+dx)}{120d}+\frac{b(3a^2-b^2)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 6*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) + (b*(6*a^2 + 5*b^2)*Cot[c +
 d*x])/(15*d) + (a*(a^2 + 6*b^2)*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (b*(3*
a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) + (a*(5*a^2 - 6*b^2)*Cot[c +
 d*x]*Csc[c + d*x]^3)/(120*d) - (b*Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c
 + d*x])^2)/(10*d) - (Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3)/(
6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
```

```
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc^7(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)\,dx$$
$$= -\frac{\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3}{6d} + \frac{1}{6}\int \csc^6($$
$$= -\frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2}{10d} - \frac{\cot(c+}{}$$
$$= \frac{a\left(5a^2-6b^2\right)\cot(c+dx)\csc^3(c+dx)}{120d} - \frac{b\cot(c+dx)\csc^4}{}$$
$$= \frac{b\left(3a^2-b^2\right)\cot(c+dx)\csc^2(c+dx)}{15d} + \frac{a\left(5a^2-6b^2\right)\cot(c}{12}$$
$$= \frac{b\left(3a^2-b^2\right)\cot(c+dx)\csc^2(c+dx)}{15d} + \frac{a\left(5a^2-6b^2\right)\cot(c}{12}$$
$$= \frac{a\left(a^2+6b^2\right)\cot(c+dx)\csc(c+dx)}{16d} + \frac{b\left(3a^2-b^2\right)\cot(c+}{15d}$$
$$= \frac{a\left(a^2+6b^2\right)\tanh^{-1}(\cos(c+dx))}{16d} + \frac{b\left(6a^2+5b^2\right)\cot(c+}{15d}$$

Mathematica [A] time = 2.10, size = 369, normalized size = 1.74

$$-\frac{-30\left(a^3+6ab^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right) - 5a^3\sec^6\left(\frac{1}{2}(c+dx)\right) + 30a^3\sec^2\left(\frac{1}{2}(c+dx)\right) + 120a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/1920*(-64*(6*a^2*b + 5*b^3)*Cot[(c + d*x)/2] - 30*(a^3 + 6*a*b^2)*Csc[(c
 + d*x)/2]^2 - 120*a^3*Log[Cos[(c + d*x)/2]] - 720*a*b^2*Log[Cos[(c + d*x)/
2]] + 120*a^3*Log[Sin[(c + d*x)/2]] + 720*a*b^2*Log[Sin[(c + d*x)/2]] + 30*
a^3*Sec[(c + d*x)/2]^2 + 180*a*b^2*Sec[(c + d*x)/2]^2 - 90*a*b^2*Sec[(c + d
*x)/2]^4 - 5*a^3*Sec[(c + d*x)/2]^6 + 96*a^2*b*Csc[c + d*x]^3*Sin[(c + d*x)
/2]^4 - 640*b^3*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + a^2*Csc[(c + d*x)/2]^6*
(5*a + 18*b*Sin[c + d*x]) + 2*b*Csc[(c + d*x)/2]^4*(45*a*b + (-3*a^2 + 20*b
^2)*Sin[c + d*x]) + 384*a^2*b*Tan[(c + d*x)/2] + 320*b^3*Tan[(c + d*x)/2] -
 36*a^2*b*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/d
```

fricas [A] time = 0.78, size = 310, normalized size = 1.46

$$\frac{80\,a^3\cos(dx+c)^3 - 30\left(a^3+6\,ab^2\right)\cos(dx+c)^5 + 30\left(a^3+6\,ab^2\right)\cos(dx+c) + 15\left(\left(a^3+6\,ab^2\right)\cos(dx+c)^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/480*(80*a^3*cos(d*x + c)^3 - 30*(a^3 + 6*a*b^2)*cos(d*x + c)^5 + 30*(a^3
+ 6*a*b^2)*cos(d*x + c) + 15*((a^3 + 6*a*b^2)*cos(d*x + c)^6 - 3*(a^3 + 6*a
*b^2)*cos(d*x + c)^4 - a^3 - 6*a*b^2 + 3*(a^3 + 6*a*b^2)*cos(d*x + c)^2)*lo
g(1/2*cos(d*x + c) + 1/2) - 15*((a^3 + 6*a*b^2)*cos(d*x + c)^6 - 3*(a^3 + 6
*a*b^2)*cos(d*x + c)^4 - a^3 - 6*a*b^2 + 3*(a^3 + 6*a*b^2)*cos(d*x + c)^2)*
log(-1/2*cos(d*x + c) + 1/2) - 32*((6*a^2*b + 5*b^3)*cos(d*x + c)^5 - 5*(3*
a^2*b + b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x
+ c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.29, size = 354, normalized size = 1.67

$$\frac{5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+90\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+60\,a^2b\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 15
*a^3*tan(1/2*d*x + 1/2*c)^4 + 90*a*b^2*tan(1/2*d*x + 1/2*c)^4 + 60*a^2*b*ta
n(1/2*d*x + 1/2*c)^3 + 80*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x +
 1/2*c)^2 - 360*a^2*b*tan(1/2*d*x + 1/2*c) - 240*b^3*tan(1/2*d*x + 1/2*c) -
 120*(a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (294*a^3*tan(1/2*d*x
+ 1/2*c)^6 + 1764*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 360*a^2*b*tan(1/2*d*x + 1/
2*c)^5 + 240*b^3*tan(1/2*d*x + 1/2*c)^5 + 15*a^3*tan(1/2*d*x + 1/2*c)^4 - 6
0*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 80*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan
(1/2*d*x + 1/2*c)^2 - 90*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 36*a^2*b*tan(1/2*d*
x + 1/2*c) - 5*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

maple [A] time = 0.48, size = 276, normalized size = 1.30

$$-\frac{a^3\left(\cos^3\left(dx+c\right)\right)}{6d\sin\left(dx+c\right)^6}-\frac{a^3\left(\cos^3\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^4}-\frac{a^3\left(\cos^3\left(dx+c\right)\right)}{16d\sin\left(dx+c\right)^2}-\frac{a^3\cos\left(dx+c\right)}{16d}-\frac{a^3\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{16d}-\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/6/d*a^3/sin(d*x+c)^6*cos(d*x+c)^3-1/8/d*a^3/sin(d*x+c)^4*cos(d*x+c)^3-1/
16/d*a^3/sin(d*x+c)^2*cos(d*x+c)^3-1/16*a^3*cos(d*x+c)/d-1/16/d*a^3*ln(csc(
```

```
d*x+c)-cot(d*x+c))-3/5/d*a^2*b/sin(d*x+c)^5*cos(d*x+c)^3-2/5/d*a^2*b/sin(d*
x+c)^3*cos(d*x+c)^3-3/4/d*a*b^2/sin(d*x+c)^4*cos(d*x+c)^3-3/8/d*a*b^2/sin(d
*x+c)^2*cos(d*x+c)^3-3/8*a*b^2*cos(d*x+c)/d-3/8/d*a*b^2*ln(csc(d*x+c)-cot(d
*x+c))-1/3/d*b^3/sin(d*x+c)^3*cos(d*x+c)^3
```

maxima [A] time = 0.40, size = 202, normalized size = 0.95

$$-\frac{5\,a^3\left(\frac{2\left(3\,\cos(dx+c)^5-8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+90\,ab^2\left(\frac{2\left(\cos(d\right.}{\cos(dx+c}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/480*(5*a^3*(2*(3*cos(d*x + c)^5 - 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(co
s(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c
) + 1) + 3*log(cos(d*x + c) - 1)) + 90*a*b^2*(2*(cos(d*x + c)^3 + cos(d*x +
 c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(
cos(d*x + c) - 1)) + 160*b^3/tan(d*x + c)^3 + 96*(5*tan(d*x + c)^2 + 3)*a^2
*b/tan(d*x + c)^5)/d
```

mupad [B] time = 9.69, size = 292, normalized size = 1.38

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^3}{2}+3\,a\,b^2\right)-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^6)/(384*d) - (a^3*tan(c/2 + (d*x)/2)^2)/(128*d) - (
cot(c/2 + (d*x)/2)^6*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 + a^3/2) - (a^3*tan(c/2
 + (d*x)/2)^4)/2 + tan(c/2 + (d*x)/2)^3*(2*a^2*b + (8*b^3)/3) - tan(c/2 + (
d*x)/2)^5*(12*a^2*b + 8*b^3) + a^3/6 + (6*a^2*b*tan(c/2 + (d*x)/2))/5))/(64
*d) + (tan(c/2 + (d*x)/2)^4*((3*a*b^2)/64 + a^3/128))/d + (tan(c/2 + (d*x)/
2)^3*((a^2*b)/32 + b^3/24))/d - (log(tan(c/2 + (d*x)/2))*((3*a*b^2)/8 + a^3
/16))/d - (tan(c/2 + (d*x)/2)*((3*a^2*b)/16 + b^3/8))/d + (3*a^2*b*tan(c/2
+ (d*x)/2)^5)/(160*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**7*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1078 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=188

$$-\frac{2a^2\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}+\frac{ax\left(4a^2-b^2\right)}{b^5}+\frac{\left(12a^2-b^2\right)\cos(c+dx)}{3b^4d}-\frac{2a\sin(c+dx)\cos(c+dx)}{b^3d}-$$

```
[Out] a*(4*a^2-b^2)*x/b^5+1/3*(12*a^2-b^2)*cos(d*x+c)/b^4/d-2*a*cos(d*x+c)*sin(d*
x+c)/b^3/d+4/3*cos(d*x+c)*sin(d*x+c)^2/b^2/d-cos(d*x+c)*sin(d*x+c)^3/b/d/(a
+b*sin(d*x+c))-2*a^2*(4*a^2-3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2
)^(1/2))/b^5/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2889, 3048, 3050, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{\left(12a^2-b^2\right)\cos(c+dx)}{3b^4d}-\frac{2a^2\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}+\frac{ax\left(4a^2-b^2\right)}{b^5}-\frac{2a\sin(c+dx)\cos(c+dx)}{b^3d}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*(4*a^2 - b^2)*x)/b^5 - (2*a^2*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*d) + ((12*a^2 - b^2)*Cos[c + d
*x])/(3*b^4*d) - (2*a*Cos[c + d*x]*Sin[c + d*x])/(b^3*d) + (4*Cos[c + d*x]*
Sin[c + d*x]^2)/(3*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(b*d*(a + b*Sin[c
 + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
```

```
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\sin^3(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\int \frac{\sin^2(c+dx)\left(-3\left(a^2-b^2\right)+4\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{4\cos(c+dx)\sin^2(c+dx)}{3b^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\int \frac{\sin(c+dx)\left(8a\left(a^2-b^2\right)-b\left(a^2\right.\right.}{}}{3}$$

$$= -\frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4\cos(c+dx)\sin^2(c+dx)}{3b^2d} - \frac{\cos(c+dx)\sin^3(c}{bd(a+b\sin(c+}$$

$$= \frac{\left(12a^2-b^2\right)\cos(c+dx)}{3b^4d} - \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4\cos(c+dx)\sin^2(c+}{3b^2d}$$

$$= \frac{a\left(4a^2-b^2\right)x}{b^5} + \frac{\left(12a^2-b^2\right)\cos(c+dx)}{3b^4d} - \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4\cos(}{}$$

$$= \frac{a\left(4a^2-b^2\right)x}{b^5} + \frac{\left(12a^2-b^2\right)\cos(c+dx)}{3b^4d} - \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4\cos(}{}$$

$$= \frac{a\left(4a^2-b^2\right)x}{b^5} + \frac{\left(12a^2-b^2\right)\cos(c+dx)}{3b^4d} - \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4\cos(}{}$$

$$= \frac{a\left(4a^2-b^2\right)x}{b^5} - \frac{2a^2\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}\,d} + \frac{\left(12a^2-b^2\right)\cos(c+}{3b^4d}$$

Mathematica [A] time = 2.53, size = 246, normalized size = 1.31

$$\frac{\frac{96a^4c+96a^4dx+96a^3bc\sin(c+dx)+96a^3bdx\sin(c+dx)+24a^2b^2\sin(2(c+dx))+12ab\left(8a^2-b^2\right)\cos(c+dx)-24a^2b^2c-24a^2b^2dx-24ab^3c\sin(c+dx)-24ab^3dx\,s}{a+b\sin(c+dx)}}{24b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-48*a^2*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/Sqrt[a^2 - b^2] + (96*a^4*c - 24*a^2*b^2*c + 96*a^4*d*x - 24*a^2*b^2*d*x +
 12*a*b*(8*a^2 - b^2)*Cos[c + d*x] + 4*a*b^3*Cos[3*(c + d*x)] + 96*a^3*b*c*
Sin[c + d*x] - 24*a*b^3*c*Sin[c + d*x] + 96*a^3*b*d*x*Sin[c + d*x] - 24*a*b
^3*d*x*Sin[c + d*x] + 24*a^2*b^2*Sin[2*(c + d*x)] - 2*b^4*Sin[2*(c + d*x)]
- b^4*Sin[4*(c + d*x)])/(a + b*Sin[c + d*x]))/(24*b^5*d)
```

fricas [A] time = 0.90, size = 643, normalized size = 3.42

$$\left[\frac{4\left(a^{3}b^{3}-ab^{5}\right)\cos\left(dx+c\right)^{3}+6\left(4\,a^{6}-5\,a^{4}b^{2}+a^{2}b^{4}\right)dx+3\left(4\,a^{5}-3\,a^{3}b^{2}+\left(4\,a^{4}b-3\,a^{2}b^{3}\right)\sin\left(dx+c\right)\right)\sqrt{-a^{2}}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/6*(4*(a^3*b^3 - a*b^5)*cos(d*x + c)^3 + 6*(4*a^6 - 5*a^4*b^2 + a^2*b^4)*
d*x + 3*(4*a^5 - 3*a^3*b^2 + (4*a^4*b - 3*a^2*b^3)*sin(d*x + c))*sqrt(-a^2
+ b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 +
 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(4*a^5*b - 5*a^3*b^3 +
a*b^5)*cos(d*x + c) - 2*((a^2*b^4 - b^6)*cos(d*x + c)^3 - 3*(4*a^5*b - 5*a^
3*b^3 + a*b^5)*d*x - 6*(a^4*b^2 - a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^
2*b^6 - b^8)*d*sin(d*x + c) + (a^3*b^5 - a*b^7)*d), 1/3*(2*(a^3*b^3 - a*b^5
)*cos(d*x + c)^3 + 3*(4*a^6 - 5*a^4*b^2 + a^2*b^4)*d*x + 3*(4*a^5 - 3*a^3*b
^2 + (4*a^4*b - 3*a^2*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 3*(4*a^5*b - 5*a^3*b^3 + a*b^5
)*cos(d*x + c) - ((a^2*b^4 - b^6)*cos(d*x + c)^3 - 3*(4*a^5*b - 5*a^3*b^3 +
 a*b^5)*d*x - 6*(a^4*b^2 - a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^6 -
 b^8)*d*sin(d*x + c) + (a^3*b^5 - a*b^7)*d)]
```

giac [A] time = 0.20, size = 261, normalized size = 1.39

$$\frac{\frac{3\left(4\,a^{3}-ab^{2}\right)(dx+c)}{b^{5}}-\frac{6\left(4\,a^{4}-3\,a^{2}b^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,b^{5}}+\frac{6\left(a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{3}\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)b^{4}}+\frac{2\left(3\,ab\tan\cdots\right.}{\cdots}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(4*a^3 - a*b^2)*(d*x + c)/b^5 - 6*(4*a^4 - 3*a^2*b^2)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 -
 b^2)))/(sqrt(a^2 - b^2)*b^5) + 6*(a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/((a*ta
n(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^4) + 2*(3*a*b*tan(1/
2*d*x + 1/2*c)^5 + 9*a^2*tan(1/2*d*x + 1/2*c)^4 - 3*b^2*tan(1/2*d*x + 1/2*c
)^4 + 18*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*a^2 -
b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^4))/d
```

maple [B] time = 0.55, size = 460, normalized size = 2.45

$$\frac{2a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{6\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{d\,b^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{12a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{}{d\,b^3(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)^5+6/d/b^4/(1+tan(1/
2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)
^3*tan(1/2*d*x+1/2*c)^4+12/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^3*a^2*tan(1/2*d*x
+1/2*c)^2-2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)+6/d/b^4/(
1+tan(1/2*d*x+1/2*c)^2)^3*a^2-2/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3+8/d/b^5*
arctan(tan(1/2*d*x+1/2*c))*a^3-2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a+2/d*a^2
/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2
/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-8/d*a^4/b^5/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d*
a^2/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 11.85, size = 1688, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + b*sin(c + d*x))^2,x)

[Out] ((28*a^2*tan(c/2 + (d*x)/2)^3)/b^3 - (2*(a*b^2 - 12*a^3))/(3*b^4) + (4*a^2*tan(c/2 + (d*x)/2)^7)/b^3 + (2*tan(c/2 + (d*x)/2)^6*(a*b^2 + 4*a^3))/b^4 - (2*tan(c/2 + (d*x)/2)^4*(a*b^2 - 12*a^3))/b^4 - (2*tan(c/2 + (d*x)/2)^2*(7*a*b^2 - 36*a^3))/(3*b^4) + (4*tan(c/2 + (d*x)/2)*(9*a^2 - b^2))/(3*b^3) + (4*tan(c/2 + (d*x)/2)^5*(5*a^2 - b^2))/b^3)/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 4*a*tan(c/2 + (d*x)/2)^2 + 6*a*tan(c/2 + (d*x)/2)^4 + 4*a*tan(c/2 + (d*x)/2)^6 + a*tan(c/2 + (d*x)/2)^8 + 6*b*tan(c/2 + (d*x)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^7)) + (atan((64*a^4*tan(c/2 + (d*x)/2))/(64*a^4 - (256*a^6)/b^2) + (256*a^6*tan(c/2 + (d*x)/2))/(256*a^6 - 64*a^4*b^2))*(a*b^2*1i - a^3*4i)*2i)/(b^5*d) + (a^2*atan(((a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((32*(a^4*b^8 - 8*a^6*b^6 + 16*a^8*b^4))/b^11 + (32*tan(c/2 + (d*x)/2)*(2*a^3*b^10 - 26*a^5*b^8 + 64*a^7*b^6 - 32*a^9*b^4))/b^12 + (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((32*(a^2*b^12 - 2*a^4*b^10))/b^11 + (32*tan(c/2 + (d*x)/2)*(6*a^3*b^12 - 8*a^5*b^10))/b^12 + (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*(32*a^2*b^3 + (32*tan(c/2 + (d*x)/2)*(3*a*b^16 - 2*a^3*b^14))/b^12))/(b^7 - a^2*b^5)))/(b^7 - a^2*b^5))*1i)/(b^7 - a^2*b^5) + (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((32*(a^4*b^8 - 8*a^6*b^6 + 16*a^8*b^4))/b^11 + (32*tan(c/2 + (d*x)/2)*(2*a^3*b^10 - 26*a^5*b^8 + 64*a^7*b^6 - 32*a^9*b^4))/b^12 - (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((32*(a^2*b^12 - 2*a^4*b^10))/b^11 + (32*tan(c/2 + (d*x)/2)*(6*a^3*b^12 - 8*a^5*b^10))/b^12 - (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*(32*a^2*b^3 + (32*tan(c/2 + (d*x)/2)*(3*a*b^16 - 2*a^3*b^14))/b^12))/(b^7 - a^2*b^5)))/(b^7 - a^2*b^5))*1i)/(b^7 - a^2*b^5))/((64*(32*a^10 + 6*a^6*b^4 - 32*a^8*b^2))/b^11 + (64*tan(c/2 + (d*x)/2)*(128*a^11 - 6*a^5*b^6 + 56*a^7*b^4 - 160*a^9*b^2))/b^12 - (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((32*(a^4*b^8 - 8*a^6*b^6 + 16*a^8*b^4))/b^11 + (32*tan(c/2 + (d*x)/2)*(2*a^3*b^10 - 26*a^5*b^8 + 64*a^7*b^6 - 32*a^9*b^4))/b^12 + (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((32*(a^2*b^12 - 2*a^4*b^10))/b^11 + (32*tan(c/2 + (d*x)/2)*(6*a^3*b^12 - 8*a^5*b^10))/b^12 + (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*(32*a^2*b^3 + (32*tan(c/2 + (d*x)/2)*(3*a*b^16 - 2*a^3*b^14))/b^12))/(b^7 - a^2*b^5)))/(b^7 - a^2*b^5)))/(b^7 - a^2*b^5) + (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((32*(a^4*b^8 - 8*a^6*b^6 + 16*a^8*b^4))/b^11 + (32*tan(c/2 + (d*x)/2)*(2*a^3*b^10 - 26*a^5*b^8 + 64*a^7*b^6 - 32*a^9*b^4))/b^12 - (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((32*(a^2*b^12 - 2*a^4*b^10))/b^11 + (32*tan(c/2 + (d*x)/2)*(6*a^3*b^12 - 8*a^5*b^10))/b^12 - (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*(32*a^2*b^3 + (32*tan(c/2 + (d*x)/2)*(3*a*b^16 - 2*a^3*b^14))/b^12))/(b^7 - a^2*b^5)))/(b^7 - a^2*b^5)))/(b^7 - a^2*b^5)))*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*2i)/(d*(b^7 - a^2*b^5))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1079 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=153

$$\frac{2a\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d\sqrt{a^2-b^2}}-\frac{x\left(6a^2-b^2\right)}{2b^4}-\frac{3a\cos(c+dx)}{b^3d}-\frac{\sin^2(c+dx)\cos(c+dx)}{bd(a+b\sin(c+dx))}+\frac{3\sin(c+dx)\,\mathrm{co}}{2b^2d}$$

```
[Out] -1/2*(6*a^2-b^2)*x/b^4-3*a*cos(d*x+c)/b^3/d+3/2*cos(d*x+c)*sin(d*x+c)/b^2/d
-cos(d*x+c)*sin(d*x+c)^2/b/d/(a+b*sin(d*x+c))+2*a*(3*a^2-2*b^2)*arctan((b+a
*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^4/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3048, 3050, 3023, 2735, 2660, 618, 204}

$$\frac{2a\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d\sqrt{a^2-b^2}}-\frac{x\left(6a^2-b^2\right)}{2b^4}-\frac{3a\cos(c+dx)}{b^3d}-\frac{\sin^2(c+dx)\cos(c+dx)}{bd(a+b\sin(c+dx))}+\frac{3\sin(c+dx)\,\mathrm{co}}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((6*a^2 - b^2)*x)/(2*b^4) + (2*a*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(b^4*Sqrt[a^2 - b^2]*d) - (3*a*Cos[c + d*x])/(b^3*
d) + (3*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^2
)/(b*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
```

```
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\sin^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= -\frac{\cos(c+dx)\sin^2(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\int \frac{\sin(c+dx)\left(-2\left(a^2-b^2\right)+3\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx)\sin^2(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\int \frac{3a\left(a^2-b^2\right)-b\left(a^2-b^2\right)\sin(\cdots}{a+b\,\mathrm{si}\cdots}}{2b^2\left(\cdots\right.}$$

$$= -\frac{3a\cos(c+dx)}{b^3d} + \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx)\sin^2(c+dx)}{bd(a+b\sin(c+dx))} - \int\cdots$$

$$= -\frac{\left(6a^2-b^2\right)x}{2b^4} - \frac{3a\cos(c+dx)}{b^3d} + \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx)\sin\cdots}{bd(a+b\sin(\cdots}$$

$$= -\frac{\left(6a^2-b^2\right)x}{2b^4} - \frac{3a\cos(c+dx)}{b^3d} + \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx)\sin\cdots}{bd(a+b\sin(\cdots}$$

$$= -\frac{\left(6a^2-b^2\right)x}{2b^4} - \frac{3a\cos(c+dx)}{b^3d} + \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx)\sin\cdots}{bd(a+b\sin(\cdots}$$

$$= -\frac{\left(6a^2-b^2\right)x}{2b^4} + \frac{2a\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}\,d} - \frac{3a\cos(c+dx)}{b^3d} + \frac{3\,\mathrm{c}\cdots}{\cdots}$$

Mathematica [A] time = 0.42, size = 129, normalized size = 0.84

$$\frac{2\left(b^2-6a^2\right)(c+dx) + \frac{8a\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{4a^2b\cos(c+dx)}{a+b\sin(c+dx)} - 8ab\cos(c+dx) + b^2\sin(2(c+dx))}{4b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(-6*a^2 + b^2)*(c + d*x) + (8*a*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 8*a*b*Cos[c + d*x] - (4*a^2*b*C
os[c + d*x])/(a + b*Sin[c + d*x]) + b^2*Sin[2*(c + d*x)])/(4*b^4*d)
```

fricas [A] time = 0.69, size = 568, normalized size = 3.71

$$\left[-\frac{\left(a^2b^3 - b^5\right)\cos\left(dx + c\right)^3 + \left(6\,a^5 - 7\,a^3b^2 + ab^4\right)dx - \left(3\,a^4 - 2\,a^2b^2 + \left(3\,a^3b - 2\,ab^3\right)\sin\left(dx + c\right)\right)\sqrt{-a^2 + b^2}\ \mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*((a^2*b^3 - b^5)*cos(d*x + c)^3 + (6*a^5 - 7*a^3*b^2 + a*b^4)*d*x - (
3*a^4 - 2*a^2*b^2 + (3*a^3*b - 2*a*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(
-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(
d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + (6*a^4*b - 7*a^2*b^3 + b^5)*cos(d*x
 + c) + ((6*a^4*b - 7*a^2*b^3 + b^5)*d*x + 3*(a^3*b^2 - a*b^4)*cos(d*x + c)
)*sin(d*x + c))/((a^2*b^5 - b^7)*d*sin(d*x + c) + (a^3*b^4 - a*b^6)*d), -1/
2*((a^2*b^3 - b^5)*cos(d*x + c)^3 + (6*a^5 - 7*a^3*b^2 + a*b^4)*d*x + 2*(3*
a^4 - 2*a^2*b^2 + (3*a^3*b - 2*a*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(
-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (6*a^4*b - 7*a^2*b^
3 + b^5)*cos(d*x + c) + ((6*a^4*b - 7*a^2*b^3 + b^5)*d*x + 3*(a^3*b^2 - a*b
^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^5 - b^7)*d*sin(d*x + c) + (a^3*b^4
- a*b^6)*d)]
```

giac [A] time = 0.19, size = 211, normalized size = 1.38

$$-\frac{\frac{\left(6\,a^2-b^2\right)\left(dx+c\right)}{b^4} - \frac{4\left(3\,a^3-2\,ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4} + \frac{4\left(ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)b^3} + \frac{2\left(b\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((6*a^2 - b^2)*(d*x + c)/b^4 - 4*(3*a^3 - 2*a*b^2)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*b^4) + 4*(a*b*tan(1/2*d*x + 1/2*c) + a^2)/((a*tan(1/2*d
*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^3) + 2*(b*tan(1/2*d*x + 1/2
*c)^3 + 4*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c) + 4*a)/((tan(1/
2*d*x + 1/2*c)^2 + 1)^2*b^3))/d
```

maple [B] time = 0.51, size = 353, normalized size = 2.31

$$-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{4a}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{6\,}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-4/d/b^3/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a+1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)-4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*a-6/d/b^4*arctan(tan(
1/2*d*x+1/2*c))*a^2+1/d/b^2*arctan(tan(1/2*d*x+1/2*c))-2/d/b^2/(tan(1/2*d*x
+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a*tan(1/2*d*x+1/2*c)-2/d/b^3/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^2+6/d/b^4*a^3/(a^2-b^2)^(1/2)*a
rctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-4/d/b^2*a/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 10.40, size = 479, normalized size = 3.13

$$-\frac{\frac{6\,a^2}{b^3}+\frac{9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(3\,a^2+b^2\right)}{b^3}+\frac{12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{b^2}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{b^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(6\,a^2-b^2\right)}{b^3}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (atan((24*a^3*tan(c/2 + (d*x)/2))/(24*a^3 - 8*a*b^2 + (144*a^5)/b^2) + (144
*a^5*tan(c/2 + (d*x)/2))/(144*a^5 - 8*a*b^4 + 24*a^3*b^2) - (8*a*tan(c/2 +
(d*x)/2))/((24*a^3)/b^2 - 8*a + (144*a^5)/b^4))*(a^2*6i - b^2*1i)*1i)/(b^4*
d) - ((6*a^2)/b^3 + (9*a*tan(c/2 + (d*x)/2))/b^2 + (2*tan(c/2 + (d*x)/2)^4*
(3*a^2 + b^2))/b^3 + (12*a*tan(c/2 + (d*x)/2)^3)/b^2 + (3*a*tan(c/2 + (d*x)
/2)^5)/b^2 + (2*tan(c/2 + (d*x)/2)^2*(6*a^2 - b^2))/b^3)/(d*(a + 2*b*tan(c/
2 + (d*x)/2) + 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 + a*tan(
c/2 + (d*x)/2)^6 + 4*b*tan(c/2 + (d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/2)^5)) -
(log(b + a*tan(c/2 + (d*x)/2) - (b^2 - a^2)^(1/2))*(3*a^3*(b^2 - a^2)^(1/2)
 - 2*a*b^2*(b^2 - a^2)^(1/2)))/(b^4*d*(a^2 - b^2)) - (a*log(b + a*tan(c/2 +
 (d*x)/2) + (b^2 - a^2)^(1/2))*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 2*b^2))/(d
*(b^6 - a^2*b^4))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1080 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=106

$$-\frac{2\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\sqrt{a^2-b^2}}+\frac{2ax}{b^3}+\frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))}$$

```
[Out] 2*a*x/b^3+cos(d*x+c)*(2*a+b*sin(d*x+c))/b^2/d/(a+b*sin(d*x+c))-2*(2*a^2-b^2
)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^3/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2863, 2735, 2660, 618, 204}

$$-\frac{2\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\sqrt{a^2-b^2}}+\frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))}+\frac{2ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*x)/b^3 - (2*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(b^3*Sqrt[a^2 - b^2]*d) + (Cos[c + d*x]*(2*a + b*Sin[c + d*x]))/(b^2*
d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))} - \frac{\int \frac{-b-2a\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b^2} \\
&= \frac{2ax}{b^3} + \frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))} - \frac{\left(2a^2-b^2\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^3} \\
&= \frac{2ax}{b^3} + \frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))} - \frac{\left(2\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x\right.}{b^3d} \\
&= \frac{2ax}{b^3} + \frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))} + \frac{\left(4\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx\right.}{b^3d} \\
&= \frac{2ax}{b^3} - \frac{2\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d} + \frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))}
\end{aligned}
$$

Mathematica [A] time = 1.07, size = 130, normalized size = 1.23

$$\frac{\frac{4a^2c+4a^2dx+4ab(c+dx)\sin(c+dx)+4ab\cos(c+dx)+b^2\sin(2(c+dx))}{a+b\sin(c+dx)}-\frac{4(2a^2-b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-4*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a
^2 - b^2] + (4*a^2*c + 4*a^2*d*x + 4*a*b*Cos[c + d*x] + 4*a*b*(c + d*x)*Sin
[c + d*x] + b^2*Sin[2*(c + d*x)])/(a + b*Sin[c + d*x]))/(2*b^3*d)
```

fricas [A] time = 0.78, size = 479, normalized size = 4.52

$$\left[\frac{4\left(a^4-a^2b^2\right)dx+\left(2a^3-ab^2+\left(2a^2b-b^3\right)\sin\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a}{b^2\cos(dx+c)^2-2ab\,s}\right.}{2\left(\left(a^2b^4-b^6\right)d\,s\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*(a^4 - a^2*b^2)*d*x + (2*a^3 - a*b^2 + (2*a^2*b - b^3)*sin(d*x + c)
)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b
^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 4*(a^3*b - a*
b^3)*cos(d*x + c) + 2*(2*(a^3*b - a*b^3)*d*x + (a^2*b^2 - b^4)*cos(d*x + c)
)*sin(d*x + c))/((a^2*b^4 - b^6)*d*sin(d*x + c) + (a^3*b^3 - a*b^5)*d), (2*
(a^4 - a^2*b^2)*d*x + (2*a^3 - a*b^2 + (2*a^2*b - b^3)*sin(d*x + c))*sqrt(a
^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 2*
(a^3*b - a*b^3)*cos(d*x + c) + (2*(a^3*b - a*b^3)*d*x + (a^2*b^2 - b^4)*cos
(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d*sin(d*x + c) + (a^3*b^3 - a*b^5
)*d)]
```

giac [A] time = 0.19, size = 191, normalized size = 1.80

$$\frac{2\left(\frac{(dx+c)a}{b^3}-\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\left(2a^2-b^2\right)}{\sqrt{a^2-b^2}\,b^3}+\frac{b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}\right.\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((d*x + c)*a/b^3 - (pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*(2*a^2 - b^2)/(sqrt(a^2 - b^2)*b
^3) + (b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 3*b*tan(1/2*
d*x + 1/2*c) + 2*a)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3
 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^2))/d
```

maple [B] time = 0.45, size = 229, normalized size = 2.16

$$\frac{2}{d\,b^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^{3}}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)}+\frac{}{d\,b^{2}\left(\left(\tan^{2}\left(\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)+4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a+2/d/b
/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/b
^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a-4/d*a^2/b^3/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/b/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 10.02, size = 269, normalized size = 2.54

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{b}+\frac{4\,a}{b^{2}}+\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}+\frac{4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{b^{2}}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}+\frac{2\,a\,x}{b^{3}}-\frac{\ln\left(b+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^3)/b + (4*a)/b^2 + (6*tan(c/2 + (d*x)/2))/b + (4*a*t
an(c/2 + (d*x)/2)^2)/b^2)/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d
*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^3)) + (2*a*x)/b^
3 - (log(b + a*tan(c/2 + (d*x)/2) - (b^2 - a^2)^(1/2))*(-(a + b)*(a - b))^(
1/2)*(2*a^2 - b^2))/(d*(b^5 - a^2*b^3)) - (log(b + a*tan(c/2 + (d*x)/2) + (
b^2 - a^2)^(1/2))*(2*a^2*(b^2 - a^2)^(1/2) - b^2*(b^2 - a^2)^(1/2)))/(b^3*d
*(a^2 - b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1081 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=92

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{ad(a+b\sin(c+dx))}$$

```
[Out] -arctanh(cos(d*x+c))/a^2/d+cos(d*x+c)/a/d/(a+b*sin(d*x+c))-2*b*arctan((b+a*
tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2889, 3056, 12, 2747, 3770, 2660, 618, 204}

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*Sqrt[a^2 - b^2
]*d) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(a*d*(a + b*Sin[c + d*x
]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\csc(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$
$$= \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\left(a^2-b^2\right)\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$
$$= \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$
$$= \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \csc(c+dx)\,dx}{a^2} - \frac{b\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \text{ta}\right.}{a^2d}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} + \frac{(4b)\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x,\right.}{a^2d}$$
$$= -\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))}$$

Mathematica [A] time = 0.22, size = 97, normalized size = 1.05

$$\frac{-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{a\cos(c+dx)}{a+b\sin(c+dx)} + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] -
Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]] + (a*Cos[c + d*x])/(a + b*Sin
[c + d*x]))/(a^2*d)
```

fricas [B] time = 1.02, size = 483, normalized size = 5.25

$$\left[-\frac{\left(b^2\sin\left(dx+c\right)+ab\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2-2\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*sin(d*x + c) + a*b)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c)
+ b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c
) - a^2 - b^2)) - 2*(a^3 - a*b^2)*cos(d*x + c) + (a^3 - a*b^2 + (a^2*b - b^
3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (a^3 - a*b^2 + (a^2*b - b^3)
*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^4*b - a^2*b^3)*d*sin(d*x +
 c) + (a^5 - a^3*b^2)*d), 1/2*(2*(b^2*sin(d*x + c) + a*b)*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 2*(a^3 - a*b^
2)*cos(d*x + c) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x + c))*log(1/2*cos(d*
x + c) + 1/2) + (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x + c))*log(-1/2*cos(d*x
 + c) + 1/2))/((a^4*b - a^2*b^3)*d*sin(d*x + c) + (a^5 - a^3*b^2)*d)]
```

giac [A] time = 0.21, size = 130, normalized size = 1.41

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}\,a^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*b/(sqrt(a^2 - b^2)*a^2) - log(abs(tan(1/2*d*x + 1
/2*c)))/a^2 - 2*(b*tan(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 + 2
*b*tan(1/2*d*x + 1/2*c) + a)*a^2))/d
```

maple [A] time = 0.68, size = 153, normalized size = 1.66

$$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{d\,a^2\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)}+\frac{2}{da\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*ln(tan(1/2*d*x+1/2*c))+2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)*b+2/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
```

```
d*x+1/2*c)*b+a)-2/d/a^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 10.16, size = 523, normalized size = 5.68

$$\frac{a^3\,\cos\left(c+d\,x\right)-a\,b^2-b^3\,\sin\left(c+d\,x\right)+a^3+a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-a\,b^2\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-b^3\,\sin\left(c+d\,x\right)\,\ln\left(\frac{\sin\left(\frac{c}{2}\right.}{\cos\left(\frac{c}{2}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (a^3*cos(c + d*x) - a*b^2 - b^3*sin(c + d*x) + a^3 + a^3*log(sin(c/2 + (d*x
)/2)/cos(c/2 + (d*x)/2)) - a*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
 - b^3*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - a*b^2*cos(
c + d*x) + a^2*b*sin(c + d*x) + b^2*sin(c + d*x)*atan((b^2*sin(c/2 + (d*x)/
2)*(b^2 - a^2)^(1/2)*4i - a^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i + a*b
*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*2i)/(a^3*cos(c/2 + (d*x)/2) - 4*b^3*s
in(c/2 + (d*x)/2) - 2*a*b^2*cos(c/2 + (d*x)/2) + 3*a^2*b*sin(c/2 + (d*x)/2)
))*(b^2 - a^2)^(1/2)*2i + a*b*atan((b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
)*4i - a^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i + a*b*cos(c/2 + (d*x)/2)
*(b^2 - a^2)^(1/2)*2i)/(a^3*cos(c/2 + (d*x)/2) - 4*b^3*sin(c/2 + (d*x)/2) -
 2*a*b^2*cos(c/2 + (d*x)/2) + 3*a^2*b*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2
)*2i + a^2*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^2*
d*(a^2 - b^2)*(a + b*sin(c + d*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+b*sin(d*x+c))**2,x)
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

3.1082 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=115

$$\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\cot(c+dx)}{a^2d}-\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\sqrt{a^2-b^2}}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

```
[Out] 2*b*arctanh(cos(d*x+c))/a^3/d-2*cot(d*x+c)/a^2/d+cot(d*x+c)/a/d/(a+b*sin(d*
x+c))-2*(a^2-2*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/d/
(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 204}

$$-\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\sqrt{a^2-b^2}}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\cot(c+dx)}{a^2d}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*Sq
rt[a^2 - b^2]*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - (2*Cot[c + d*x])/(
a^2*d) + Cot[c + d*x]/(a*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)\left(2\left(a^2-b^2\right)-\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)\left(-2b\left(a^2-b^2\right)-a\left(a^2-b^2\right)\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2b)\int \csc(c+dx)\,dx}{a^3} - \frac{\left(a^2-2b^2\right)\int \frac{1}{a+b\sin}}{a^3}$$

$$= \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{\left(2\left(a^2-2b^2\right)\right)\mathrm{Sub}}{}$$

$$= \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\left(4\left(a^2-2b^2\right)\right)\mathrm{Sub}}{}$$

$$= -\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}\,d} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{}{ad}$$

Mathematica [A] time = 0.76, size = 139, normalized size = 1.21

$$-\frac{\frac{4\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{2ab\cos(c+dx)}{a+b\sin(c+dx)} - a\tan\left(\frac{1}{2}(c+dx)\right) + a\cot\left(\frac{1}{2}(c+dx)\right) + 4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 4b}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*((4*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + a*Cot[(c + d*x)/2] - 4*b*Log[Cos[(c + d*x)/2]] + 4*b*Log[Si
n[(c + d*x)/2]] + (2*a*b*Cos[c + d*x])/(a + b*Sin[c + d*x]) - a*Tan[(c + d*
x)/2])/(a^3*d)
```

fricas [B] time = 0.98, size = 768, normalized size = 6.68

$$\left[\frac{4\left(a^{3}b-ab^{3}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(a^{2}b-2\,b^{3}-\left(a^{2}b-2\,b^{3}\right)\cos\left(dx+c\right)^{2}+\left(a^{3}-2\,ab^{2}\right)\sin\left(dx+c\right)\right)\sqrt{-a}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(4*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (a^2*b - 2*b^3 - (a^2*b
 - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*l
og(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*co
s(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(a^4 - a^2*b^2)*cos(d*x + c) -
2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x
 + c))*log(1/2*cos(d*x + c) + 1/2) + 2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos
(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(
(a^5*b - a^3*b^3)*d*cos(d*x + c)^2 - (a^6 - a^4*b^2)*d*sin(d*x + c) - (a^5*
b - a^3*b^3)*d), (2*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (a^2*b - 2*
b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x + c))*sqrt(a
^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (a
^4 - a^2*b^2)*cos(d*x + c) - (a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^
2 + (a^3*b - a*b^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^2*b^2 -
b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(-1
/2*cos(d*x + c) + 1/2))/((a^5*b - a^3*b^3)*d*cos(d*x + c)^2 - (a^6 - a^4*b^
2)*d*sin(d*x + c) - (a^5*b - a^3*b^3)*d)]
```

giac [A] time = 0.20, size = 218, normalized size = 1.90

$$-\frac{\frac{12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{3}}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}}+\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)\left(a^{2}-2\,b^{2}\right)}{\sqrt{a^{2}-b^{2}}\,a^{3}}-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,a^{2}\tan\left(\cdots\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\cdots\right.\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*tan(1/2*d*x + 1/2*c)/a^2
+ 12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))*(a^2 - 2*b^2)/(sqrt(a^2 - b^2)*a^3) - (4*a*b*tan
(1/2*d*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 4*b^2*tan(1/2*d*x + 1/
```

```
2*c)^2 - 14*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/((a*tan(1/2*d*x + 1/2*c)^3 +
2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))*a^3))/d
```

maple [B] time = 0.75, size = 245, normalized size = 2.13

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{1}{2d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}-\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)}-\frac{}{d\,a^2\left(\left(\mathrm{t}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/2/d/a^2/tan(1/2*d*x+1/2*c)-2/d/a^3*b*ln(tan(
1/2*d*x+1/2*c))-2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b
^2*tan(1/2*d*x+1/2*c)-2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)*b-2/d/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))+4/d/a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 11.41, size = 1616, normalized size = 14.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] -(a^4*cos(c + d*x) - b^4/2 - b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
 + (a^2*b^2)/2 + (b^4*cos(2*c + 2*d*x))/2 - a^2*b^2*cos(c + d*x) - a*b^3*si
n(2*c + 2*d*x) + a^3*b*sin(2*c + 2*d*x) + a^2*b^2*log(sin(c/2 + (d*x)/2)/co
s(c/2 + (d*x)/2)) + b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c
```

```
+ 2*d*x) + b^3*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(
c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^
(1/2)*4i + a^2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d
*x)/2) + 8*b^4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*co
s(c/2 + (d*x)/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i - (a
^2*b^2*cos(2*c + 2*d*x))/2 - a*b^3*sin(c + d*x) + a^3*b*sin(c + d*x) - a^2*
b*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^
2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^
4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)
/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i - a^3*sin(c + d*x
)*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^
2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^
4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)
/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i - a^2*b^2*log(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x) - b^3*cos(2*c + 2*d*x)
*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2)
*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2
*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^4
*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)/
2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i - 2*a*b^3*sin(c +
d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 2*a^3*b*sin(c + d*x)*log(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + a*b^2*sin(c + d*x)*atan((a^3*cos(c
/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/
2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2*b*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^4*sin(c/2 + (d*x)
/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)/2) - 8*a^2*b^2*s
in(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*4i + a^2*b*cos(2*c + 2*d*x)*atan((a^3
*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2)*(b^2 - a^
2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2*b*sin(c/2
 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^4*sin(c/2 +
 (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)/2) - 8*a^2
*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i)/(2*a^3*d*(a^2 - b^2)*(b/4 +
 (a*sin(c + d*x))/2 - (b*cos(2*c + 2*d*x))/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

3.1083 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=157

$$\frac{3b\cot(c+dx)}{a^3d}-\frac{3\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2b\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\sqrt{a^2-b^2}}+\frac{\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}$$

```
[Out] 1/2*(a^2-6*b^2)*arctanh(cos(d*x+c))/a^4/d+3*b*cot(d*x+c)/a^3/d-3/2*cot(d*x+
c)*csc(d*x+c)/a^2/d+cot(d*x+c)*csc(d*x+c)/a/d/(a+b*sin(d*x+c))+2*b*(2*a^2-3
*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^4/d/(a^2-b^2)^(1/2
)
```

Rubi [A] time = 0.77, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\sqrt{a^2-b^2}}+\frac{\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{3b\cot(c+dx)}{a^3d}-\frac{3\cot(c+dx)\csc(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*Sqrt[a^2 - b^2]*d) + ((a^2 - 6*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + (3*
b*Cot[c + d*x])/(a^3*d) - (3*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) + (Cot[c
+ d*x]*Csc[c + d*x])/(a*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
```

```
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\csc^3(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^3(c+dx)\left(3\left(a^2-b^2\right)-2\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{3\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)\left(-6b\left(a^2-b^2\right)-a\left(a^2 \cdots\right.\right.}{a \cdots}}{2a \cdots}$$

$$= \frac{3b\cot(c+dx)}{a^3d} - \frac{3\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+d \cdots}{\cdots}}{\cdots}$$

$$= \frac{3b\cot(c+dx)}{a^3d} - \frac{3\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))} - \frac{\left(a^2-6b^2 \cdots\right.}{\cdots}$$

$$= \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{3b\cot(c+dx)}{a^3d} - \frac{3\cot(c+dx)\csc(c+dx)}{2a^2d} + \cdots$$

$$= \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{3b\cot(c+dx)}{a^3d} - \frac{3\cot(c+dx)\csc(c+dx)}{2a^2d} + \cdots$$

$$= \frac{2b\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}\,d} + \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{3b\cot \cdots}{\cdots}$$

Mathematica [A] time = 3.09, size = 196, normalized size = 1.25

$$\frac{-\frac{16b\left(3b^2-2a^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-4\left(a^2-6b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\left(a^2-6b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-a^2\csc}{8a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-16*b*(-2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
Sqrt[a^2 - b^2] + 8*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 + 4*(a^2
- 6*b^2)*Log[Cos[(c + d*x)/2]] - 4*(a^2 - 6*b^2)*Log[Sin[(c + d*x)/2]] + a^
2*Sec[(c + d*x)/2]^2 + (8*a*b^2*Cos[c + d*x])/(a + b*Sin[c + d*x]) - 8*a*b*
Tan[(c + d*x)/2])/(8*a^4*d)
```

fricas [B] time = 1.29, size = 1130, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/4*(12*(a^3*b^2 - a*b^4)*cos(d*x + c)^3 - 6*(a^4*b - a^2*b^3)*cos(d*x + c
)*sin(d*x + c) - 2*(2*a^3*b - 3*a*b^3 - (2*a^3*b - 3*a*b^3)*cos(d*x + c)^2
+ (2*a^2*b^2 - 3*b^4 - (2*a^2*b^2 - 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sq
rt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^
2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2)
)/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(a^5 - 7*a^3*b
^2 + 6*a*b^4)*cos(d*x + c) - (a^5 - 7*a^3*b^2 + 6*a*b^4 - (a^5 - 7*a^3*b^2
+ 6*a*b^4)*cos(d*x + c)^2 + (a^4*b - 7*a^2*b^3 + 6*b^5 - (a^4*b - 7*a^2*b^3
 + 6*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^5
- 7*a^3*b^2 + 6*a*b^4 - (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^2 + (a^4*b
 - 7*a^2*b^3 + 6*b^5 - (a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7 - a^5*b^2)*d*cos(d*x + c)^2 - (a^
7 - a^5*b^2)*d + ((a^6*b - a^4*b^3)*d*cos(d*x + c)^2 - (a^6*b - a^4*b^3)*d)
*sin(d*x + c)), 1/4*(12*(a^3*b^2 - a*b^4)*cos(d*x + c)^3 - 6*(a^4*b - a^2*b
^3)*cos(d*x + c)*sin(d*x + c) + 4*(2*a^3*b - 3*a*b^3 - (2*a^3*b - 3*a*b^3)*
cos(d*x + c)^2 + (2*a^2*b^2 - 3*b^4 - (2*a^2*b^2 - 3*b^4)*cos(d*x + c)^2)*s
in(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*
cos(d*x + c))) + 2*(a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c) - (a^5 - 7*a^3*
b^2 + 6*a*b^4 - (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^2 + (a^4*b - 7*a^2
*b^3 + 6*b^5 - (a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2)*sin(d*x + c))*lo
```

```
g(1/2*cos(d*x + c) + 1/2) + (a^5 - 7*a^3*b^2 + 6*a*b^4 - (a^5 - 7*a^3*b^2 +
 6*a*b^4)*cos(d*x + c)^2 + (a^4*b - 7*a^2*b^3 + 6*b^5 - (a^4*b - 7*a^2*b^3
+ 6*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7
- a^5*b^2)*d*cos(d*x + c)^2 - (a^7 - a^5*b^2)*d + ((a^6*b - a^4*b^3)*d*cos(
d*x + c)^2 - (a^6*b - a^4*b^3)*d)*sin(d*x + c))]
```

giac [A] time = 0.21, size = 257, normalized size = 1.64

$$-\frac{\frac{4\left(a^2-6b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}-\frac{16\left(2a^2b-3b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^4}-\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}-}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*(a^2 - 6*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - 16*(2*a^2*b - 3*
b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (a^2*tan(1/2*d*x + 1/2*c
)^2 - 8*a*b*tan(1/2*d*x + 1/2*c))/a^4 - 16*(b^3*tan(1/2*d*x + 1/2*c) + a*b^
2)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^4) - (6*a^2
*tan(1/2*d*x + 1/2*c)^2 - 36*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a*b*tan(1/2*d*x
 + 1/2*c) - a^2)/(a^4*tan(1/2*d*x + 1/2*c)^2))/d
```

maple [B] time = 0.81, size = 307, normalized size = 1.96

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2d}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{d\,a^3}-\frac{1}{8a^2d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{d\,a^4}+\frac{b}{d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^3*tan(1/2*d*x+1/2*c)*b-1/8/a^2/d/tan(1
/2*d*x+1/2*c)^2-1/2/d/a^2*ln(tan(1/2*d*x+1/2*c))+3/d/a^4*ln(tan(1/2*d*x+1/2
*c))*b^2+1/d*b/a^3/tan(1/2*d*x+1/2*c)+2/d*b^3/a^4/(tan(1/2*d*x+1/2*c)^2*a+2
*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d*b^2/a^3/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+4/d/a^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-6/d*b^3/a^4/(a^2-b^2)^(1/2)*arctan(1
/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.89, size = 966, normalized size = 6.15

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8a^2d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^2}{2}-16b^2\right)+\frac{a^2}{2}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a^2b+2b^3)}{a}-3ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(4a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+8ba^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d}-\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (tan(c/2 + (d*x)/2)^2*(a^2/2 - 16*b^2) + a
^2/2 - (4*tan(c/2 + (d*x)/2)^3*(a^2*b + 2*b^3))/a - 3*a*b*tan(c/2 + (d*x)/2
))/(d*(4*a^4*tan(c/2 + (d*x)/2)^2 + 4*a^4*tan(c/2 + (d*x)/2)^4 + 8*a^3*b*ta
n(c/2 + (d*x)/2)^3)) - (b*tan(c/2 + (d*x)/2))/(a^3*d) - (log(tan(c/2 + (d*x
)/2))*(a^2 - 6*b^2))/(2*a^4*d) - (b*atan(((b*(-(a + b)*(a - b))^(1/2)*(2*a^
2 - 3*b^2)*((5*a^6*b - 12*a^4*b^3)/a^6 - (tan(c/2 + (d*x)/2)*(a^6 + 24*a^2*
b^4 - 16*a^4*b^2))/a^5 + (b*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 +
(d*x)/2)*(6*a^8 - 8*a^6*b^2))/a^5)*(2*a^2 - 3*b^2))/(a^6 - a^4*b^2))*1i)/(a
^6 - a^4*b^2) - (b*(-(a + b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((tan(c/2 + (d*
x)/2)*(a^6 + 24*a^2*b^4 - 16*a^4*b^2))/a^5 - (5*a^6*b - 12*a^4*b^3)/a^6 + (
b*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^8 - 8*a^6*b^
2))/a^5)*(2*a^2 - 3*b^2))/(a^6 - a^4*b^2))*1i)/(a^6 - a^4*b^2))/((2*(2*a^4*
b + 18*b^5 - 15*a^2*b^3))/a^6 + (2*tan(c/2 + (d*x)/2)*(18*b^4 - 12*a^2*b^2)
)/a^5 + (b*(-(a + b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((5*a^6*b - 12*a^4*b^3)
/a^6 - (tan(c/2 + (d*x)/2)*(a^6 + 24*a^2*b^4 - 16*a^4*b^2))/a^5 + (b*(-(a +
 b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^8 - 8*a^6*b^2))/a^5)
```

```
*(2*a^2 - 3*b^2))/(a^6 - a^4*b^2)))/(a^6 - a^4*b^2) + (b*(-(a + b)*(a - b))
^(1/2)*(2*a^2 - 3*b^2)*((tan(c/2 + (d*x)/2)*(a^6 + 24*a^2*b^4 - 16*a^4*b^2)
)/a^5 - (5*a^6*b - 12*a^4*b^3)/a^6 + (b*(-(a + b)*(a - b))^(1/2)*(2*a^2*b -
 (tan(c/2 + (d*x)/2)*(6*a^8 - 8*a^6*b^2))/a^5)*(2*a^2 - 3*b^2))/(a^6 - a^4*
b^2)))/(a^6 - a^4*b^2)))*(-(a + b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*2i)/(d*(a
^6 - a^4*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

3.1084 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=193

$$\frac{2b\cot(c+dx)\csc(c+dx)}{a^3d}-\frac{4\cot(c+dx)\csc^2(c+dx)}{3a^2d}-\frac{2b^2\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}-\frac{b\left(a^2-4b^2\right)\tan$$

[Out] -b*(a^2-4*b^2)*arctanh(cos(d*x+c))/a^5/d+1/3*(a^2-12*b^2)*cot(d*x+c)/a^4/d+2*b*cot(d*x+c)*csc(d*x+c)/a^3/d-4/3*cot(d*x+c)*csc(d*x+c)^2/a^2/d+cot(d*x+c)*csc(d*x+c)^2/a/d/(a+b*sin(d*x+c))-2*b^2*(3*a^2-4*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^5/d/(a^2-b^2)^(1/2)

Rubi [A] time = 1.04, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b^2\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}+\frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d}-\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{2b\cot(c+$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]

[Out] (-2*b^2*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) - (b*(a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(a^5*d) + ((a^2 - 12*b^2)*Cot[c + d*x])/(3*a^4*d) + (2*b*Cot[c + d*x]*Csc[c + d*x])/(a^3*d) - (4*Cot[c + d*x]*Csc[c + d*x]^2)/(3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^2)/(a*d*(a + b*Sin[c + d*x]))

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
```

```
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\csc^4(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{\cot(c+dx)\csc^2(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^4(c+dx)\left(4\left(a^2-b^2\right)-3\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{4\cot(c+dx)\csc^2(c+dx)}{3a^2d} + \frac{\cot(c+dx)\csc^2(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^3(c+dx)\left(-12b\left(a^2-b^2\right)\right.}{}}{}$$

$$= \frac{2b\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{4\cot(c+dx)\csc^2(c+dx)}{3a^2d} + \frac{\cot(c+dx)\csc^2(c+}{ad(a+b\sin(c+dx}$$

$$= \frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d} + \frac{2b\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{4\cot(c+dx)\csc^2(c+d}{3a^2d}$$

$$= \frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d} + \frac{2b\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{4\cot(c+dx)\csc^2(c+d}{3a^2d}$$

$$= -\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d} + \frac{2b\cot(c+dx)}{a^3d}$$

$$= -\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d} + \frac{2b\cot(c+dx)}{a^3d}$$

$$= -\frac{2b^2\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} - \frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} +$$

Mathematica [A] time = 6.34, size = 385, normalized size = 1.99

$$-\frac{b^3\cos(c+dx)}{a^4d(a+b\sin(c+dx))} + \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d} + \frac{\tan\left(\frac{1}{2}(c+dx\right.}{2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b^2*(3*a^2 - 4*b^2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Si
n[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) + ((a^2*Cos[(c +
 d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + (b*Csc[(c
+ d*x)/2]^2)/(4*a^3*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a^2*d) +
 ((-(a^2*b) + 4*b^3)*Log[Cos[(c + d*x)/2]])/(a^5*d) + ((a^2*b - 4*b^3)*Log[
Sin[(c + d*x)/2]])/(a^5*d) - (b*Sec[(c + d*x)/2]^2)/(4*a^3*d) + (Sec[(c + d
*x)/2]*(-(a^2*Sin[(c + d*x)/2]) + 9*b^2*Sin[(c + d*x)/2]))/(6*a^4*d) - (b^3
*Cos[c + d*x])/(a^4*d*(a + b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/(24*a^2*d)
```

fricas [B] time = 0.84, size = 1471, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/6*(2*(a^6 - 7*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c)^3 - 3*(3*a^2*b^3 - 4*b^
5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - 2*(3*a^2*b^3 - 4*b^5)*cos(d*x + c)
^2 + (3*a^3*b^2 - 4*a*b^4 - (3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2)*sin(d*x +
 c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c
) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2
+ b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 12*(a^4*b^
2 - a^2*b^4)*cos(d*x + c) + 3*(a^4*b^2 - 5*a^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a
^2*b^4 + 4*b^6)*cos(d*x + c)^4 - 2*(a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x +
c)^2 + (a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x
 + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(a^4*b^2 - 5*a^2*b^4
 + 4*b^6 + (a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - 2*(a^4*b^2 - 5*a^
2*b^4 + 4*b^6)*cos(d*x + c)^2 + (a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a
^3*b^3 + 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2
) + 2*((a^5*b - 13*a^3*b^3 + 12*a*b^5)*cos(d*x + c)^3 - 3*(a^5*b - 5*a^3*b^
3 + 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - a^5*b^3)*d*cos(d*x + c)^
4 - 2*(a^7*b - a^5*b^3)*d*cos(d*x + c)^2 + (a^7*b - a^5*b^3)*d - ((a^8 - a^
6*b^2)*d*cos(d*x + c)^2 - (a^8 - a^6*b^2)*d)*sin(d*x + c)), -1/6*(2*(a^6 -
7*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c)^3 - 6*(3*a^2*b^3 - 4*b^5 + (3*a^2*b^3 -
 4*b^5)*cos(d*x + c)^4 - 2*(3*a^2*b^3 - 4*b^5)*cos(d*x + c)^2 + (3*a^3*b^2
- 4*a*b^4 - (3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 -
b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 12*(a^4
*b^2 - a^2*b^4)*cos(d*x + c) + 3*(a^4*b^2 - 5*a^2*b^4 + 4*b^6 + (a^4*b^2 -
5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - 2*(a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x
 + c)^2 + (a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(
d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(a^4*b^2 - 5*a^2*
b^4 + 4*b^6 + (a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - 2*(a^4*b^2 - 5
*a^2*b^4 + 4*b^6)*cos(d*x + c)^2 + (a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b -
```

```
5*a^3*b^3 + 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) +
1/2) + 2*((a^5*b - 13*a^3*b^3 + 12*a*b^5)*cos(d*x + c)^3 - 3*(a^5*b - 5*a^3
*b^3 + 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - a^5*b^3)*d*cos(d*x +
c)^4 - 2*(a^7*b - a^5*b^3)*d*cos(d*x + c)^2 + (a^7*b - a^5*b^3)*d - ((a^8 -
 a^6*b^2)*d*cos(d*x + c)^2 - (a^8 - a^6*b^2)*d)*sin(d*x + c))]
```

giac [A] time = 0.22, size = 329, normalized size = 1.70

$$\frac{\frac{24\left(a^{2}b-4b^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{5}}-\frac{48\left(3a^{2}b^{2}-4b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,a^{5}}+\frac{a^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-6a^{3}b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\cdots}{a\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 - 48*(3*a^2*b^2
 - 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
+ 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) + (a^4*tan(1/2*d*x +
1/2*c)^3 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 3*a^4*tan(1/2*d*x + 1/2*c) + 36
*a^2*b^2*tan(1/2*d*x + 1/2*c))/a^6 - 48*(b^4*tan(1/2*d*x + 1/2*c) + a*b^3)/
((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^5) - (44*a^2*b
*tan(1/2*d*x + 1/2*c)^3 - 176*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*a^3*tan(1/2*d*
x + 1/2*c)^2 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*b*tan(1/2*d*x + 1/2*
c) + a^3)/(a^5*tan(1/2*d*x + 1/2*c)^3))/d
```

maple [B] time = 0.82, size = 390, normalized size = 2.02

$$\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^{2}}-\frac{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{4d\,a^{3}}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{2}}+\frac{3b^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{4}}-\frac{1}{24a^{2}d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{1}{8d\,a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{\cdots}{2d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^3*tan(1/2*d*x+1/2*c)^2*b-1/8/d/a^2*
tan(1/2*d*x+1/2*c)+3/2/d/a^4*b^2*tan(1/2*d*x+1/2*c)-1/24/a^2/d/tan(1/2*d*x+
1/2*c)^3+1/8/d/a^2/tan(1/2*d*x+1/2*c)-3/2/d/a^4/tan(1/2*d*x+1/2*c)*b^2+1/4/
d/a^3*b/tan(1/2*d*x+1/2*c)^2+1/d/a^3*b*ln(tan(1/2*d*x+1/2*c))-4/d/a^5*b^3*l
n(tan(1/2*d*x+1/2*c))-2/d*b^4/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)*tan(1/2*d*x+1/2*c)-2/d*b^3/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)-6/d/a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2
```

```
*b)/(a^2-b^2)^(1/2))*b^2+8/d*b^4/a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.99, size = 1089, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^2*d) - (tan(c/2 + (d*x)/2)^2*(8*a*b^2 - (2*a^3)/
3) - tan(c/2 + (d*x)/2)^3*(4*a^2*b - 40*b^3) + a^3/3 - (4*a^2*b*tan(c/2 + (
d*x)/2))/3 + (tan(c/2 + (d*x)/2)^4*(16*b^4 - a^4 + 12*a^2*b^2))/a)/(d*(8*a^
5*tan(c/2 + (d*x)/2)^3 + 8*a^5*tan(c/2 + (d*x)/2)^5 + 16*a^4*b*tan(c/2 + (d
*x)/2)^4)) + (tan(c/2 + (d*x)/2)*(1/(8*a^2) - (16*a^2 + 32*b^2)/(64*a^4) +
(2*b^2)/a^4))/d - (b*tan(c/2 + (d*x)/2)^2)/(4*a^3*d) + (b*log(tan(c/2 + (d*
x)/2))*(a^2 - 4*b^2))/(a^5*d) - (b^2*atan(((b^2*(-(a + b)*(a - b))^(1/2)*(3
*a^2 - 4*b^2)*((2*(8*a^5*b^4 - 4*a^7*b^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(a^7
*b + 16*a^3*b^5 - 12*a^5*b^3))/a^7 + (b^2*(-(a + b)*(a - b))^(1/2)*(2*a^2*b
 - (2*tan(c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(3*a^2 - 4*b^2))/(a^7 -
 a^5*b^2))*1i)/(a^7 - a^5*b^2) + (b^2*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 4*b
^2)*((2*(8*a^5*b^4 - 4*a^7*b^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(a^7*b + 16*a^
3*b^5 - 12*a^5*b^3))/a^7 - (b^2*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (2*tan(
c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(3*a^2 - 4*b^2))/(a^7 - a^5*b^2))
*1i)/(a^7 - a^5*b^2))/((4*(16*b^7 - 16*a^2*b^5 + 3*a^4*b^3))/a^8 + (4*tan(c
/2 + (d*x)/2)*(16*b^6 - 12*a^2*b^4))/a^7 + (b^2*(-(a + b)*(a - b))^(1/2)*(3
*a^2 - 4*b^2)*((2*(8*a^5*b^4 - 4*a^7*b^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(a^7
*b + 16*a^3*b^5 - 12*a^5*b^3))/a^7 + (b^2*(-(a + b)*(a - b))^(1/2)*(2*a^2*b
 - (2*tan(c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(3*a^2 - 4*b^2))/(a^7 -
 a^5*b^2)))/(a^7 - a^5*b^2) - (b^2*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)
*((2*(8*a^5*b^4 - 4*a^7*b^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(a^7*b + 16*a^3*b
^5 - 12*a^5*b^3))/a^7 - (b^2*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (2*tan(c/2
```

```
 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(3*a^2 - 4*b^2))/(a^7 - a^5*b^2)))/(
a^7 - a^5*b^2)))*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)*2i)/(d*(a^7 - a^5
*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**4/(a + b*sin(c + d*x))**2, x)
```

3.1085 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=266

$$-\frac{\left(4a^2-3b^2\right)\sin^2(c+dx)\cos(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{x\left(12a^2-b^2\right)}{2b^5}-\frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4d\left(a^2-b^2\right)}+\frac{\left(6a^2-5b^2\right)\sin(c+dx)\cos(c+dx)}{2b^3d\left(a^2-b^2\right)}$$

```
[Out] -1/2*(12*a^2-b^2)*x/b^5+a*(12*a^4-19*a^2*b^2+6*b^4)*arctan((b+a*tan(1/2*d*x
+1/2*c))/(a^2-b^2)^(1/2))/b^5/(a^2-b^2)^(3/2)/d-1/2*a*(12*a^2-11*b^2)*cos(d
*x+c)/b^4/(a^2-b^2)/d+1/2*(6*a^2-5*b^2)*cos(d*x+c)*sin(d*x+c)/b^3/(a^2-b^2)
/d-1/2*cos(d*x+c)*sin(d*x+c)^3/b/d/(a+b*sin(d*x+c))^2-1/2*(4*a^2-3*b^2)*cos
(d*x+c)*sin(d*x+c)^2/b^2/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.88, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3048, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4d\left(a^2-b^2\right)}+\frac{a\left(-19a^2b^2+12a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\left(a^2-b^2\right)^{3/2}}-\frac{\left(4a^2-3b^2\right)\sin^2(c+dx)\cos(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((12*a^2 - b^2)*x)/(2*b^5) + (a*(12*a^4 - 19*a^2*b^2 + 6*b^4)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*(a^2 - b^2)^(3/2)*d) - (a*(12*a^
2 - 11*b^2)*Cos[c + d*x])/(2*b^4*(a^2 - b^2)*d) + ((6*a^2 - 5*b^2)*Cos[c +
d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(2
*b*d*(a + b*Sin[c + d*x])^2) - ((4*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]^2
)/(2*b^2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\sin^3(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\int \frac{\sin^2(c+dx)\left(-3\left(a^2-b^2\right)+4\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\left(4a^2-3b^2\right)\cos(c+dx)\sin^2(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\sin(c+dx)(}{}}{}$$

$$= \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\cos(c+dx)\sin^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\left(4a^2-3b^2\right)c}{2b^2\left(a^2-b^2\right.}$$

$$= -\frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4\left(a^2-b^2\right)d} + \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\cos(c+}{2bd(a+}$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} - \frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4\left(a^2-b^2\right)d} + \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c-}{2b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} - \frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4\left(a^2-b^2\right)d} + \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c-}{2b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} - \frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4\left(a^2-b^2\right)d} + \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c-}{2b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} + \frac{a\left(12a^4-19a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\left(a^2-b^2\right)^{3/2}d} - \frac{a\left(12a^2-1\right.}{2b^4\left(\right.}$$

Mathematica [A] time = 5.88, size = 288, normalized size = 1.08

$$\frac{\frac{4a\left(12a^4-19a^2b^2+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{-2b^2\left(12a^4-13a^2b^2+b^4\right)(c+dx)\sin^2(c+dx)-\left(a^2\left(\left(18a^2b^2-17b^4\right)\sin(2(c+dx))+2\left(12a^4-13a^2b^2+b^4\right)(c\right.\right.}{}}{4b^5d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((4*a*(12*a^4 - 19*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/Sqrt[a^2 - b^2] + (-4*a*b*(12*a^4 - 13*a^2*b^2 + b^4)*(c + d*x)*
Sin[c + d*x] - 2*b^2*(12*a^4 - 13*a^2*b^2 + b^4)*(c + d*x)*Sin[c + d*x]^2 +
 Cos[c + d*x]*(-24*a^5*b + 22*a^3*b^3 - 8*a*b^3*(a^2 - b^2)*Sin[c + d*x]^2
+ 2*b^4*(a^2 - b^2)*Sin[c + d*x]^3) - a^2*(2*(12*a^4 - 13*a^2*b^2 + b^4)*(c
 + d*x) + (18*a^2*b^2 - 17*b^4)*Sin[2*(c + d*x)]))/(a + b*Sin[c + d*x])^2)/
(4*(a - b)*b^5*(a + b)*d)
```

fricas [A] time = 1.15, size = 1058, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(2*(12*a^6*b^2 - 25*a^4*b^4 + 14*a^2*b^6 - b^8)*d*x*cos(d*x + c)^2 +
8*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c)^3 - 2*(12*a^8 - 13*a^6*b^2 - 1
1*a^4*b^4 + 13*a^2*b^6 - b^8)*d*x + (12*a^7 - 7*a^5*b^2 - 13*a^3*b^4 + 6*a*
b^6 - (12*a^5*b^2 - 19*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2 + 2*(12*a^6*b - 19
*a^4*b^3 + 6*a^2*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x +
 c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x
 + c) - a^2 - b^2)) - 2*(12*a^7*b - 19*a^5*b^3 + 3*a^3*b^5 + 4*a*b^7)*cos(d
*x + c) - 2*((a^4*b^4 - 2*a^2*b^6 + b^8)*cos(d*x + c)^3 + 2*(12*a^7*b - 25*
a^5*b^3 + 14*a^3*b^5 - a*b^7)*d*x + (18*a^6*b^2 - 36*a^4*b^4 + 19*a^2*b^6 -
 b^8)*cos(d*x + c))*sin(d*x + c))/((a^4*b^7 - 2*a^2*b^9 + b^11)*d*cos(d*x +
 c)^2 - 2*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*d*sin(d*x + c) - (a^6*b^5 - a^4*b^
7 - a^2*b^9 + b^11)*d), -1/2*((12*a^6*b^2 - 25*a^4*b^4 + 14*a^2*b^6 - b^8)*
d*x*cos(d*x + c)^2 + 4*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c)^3 - (12*a
^8 - 13*a^6*b^2 - 11*a^4*b^4 + 13*a^2*b^6 - b^8)*d*x - (12*a^7 - 7*a^5*b^2
- 13*a^3*b^4 + 6*a*b^6 - (12*a^5*b^2 - 19*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2
 + 2*(12*a^6*b - 19*a^4*b^3 + 6*a^2*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arct
an(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (12*a^7*b - 19*a
^5*b^3 + 3*a^3*b^5 + 4*a*b^7)*cos(d*x + c) - ((a^4*b^4 - 2*a^2*b^6 + b^8)*c
os(d*x + c)^3 + 2*(12*a^7*b - 25*a^5*b^3 + 14*a^3*b^5 - a*b^7)*d*x + (18*a^
6*b^2 - 36*a^4*b^4 + 19*a^2*b^6 - b^8)*cos(d*x + c))*sin(d*x + c))/((a^4*b^
7 - 2*a^2*b^9 + b^11)*d*cos(d*x + c)^2 - 2*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*d
*sin(d*x + c) - (a^6*b^5 - a^4*b^7 - a^2*b^9 + b^11)*d)]
```

giac [B] time = 0.24, size = 535, normalized size = 2.01

$$\frac{\frac{2\left(12a^5-19a^3b^2+6ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^5-b^7\right)\sqrt{a^2-b^2}}-\frac{2\left(6a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-5a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+5a\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(12*a^5 - 19*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^2*b^5 - b^
7)*sqrt(a^2 - b^2)) - 2*(6*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 5*a^2*b^3*tan(1/2
*d*x + 1/2*c)^7 + 12*a^5*tan(1/2*d*x + 1/2*c)^6 + 5*a^3*b^2*tan(1/2*d*x + 1
/2*c)^6 - 14*a*b^4*tan(1/2*d*x + 1/2*c)^6 + 54*a^4*b*tan(1/2*d*x + 1/2*c)^5
 - 45*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 4*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a^
5*tan(1/2*d*x + 1/2*c)^4 + 15*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 44*a*b^4*tan
(1/2*d*x + 1/2*c)^4 + 90*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 87*a^2*b^3*tan(1/2*
d*x + 1/2*c)^3 + 4*b^5*tan(1/2*d*x + 1/2*c)^3 + 36*a^5*tan(1/2*d*x + 1/2*c)
^2 - a^3*b^2*tan(1/2*d*x + 1/2*c)^2 - 30*a*b^4*tan(1/2*d*x + 1/2*c)^2 + 42*
a^4*b*tan(1/2*d*x + 1/2*c) - 39*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*a^5 - 11*
a^3*b^2)/((a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2
*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) - (12
*a^2 - b^2)*(d*x + c)/b^5)/d
```

maple [B] time = 0.58, size = 845, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-6/d/b^4/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)-6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^2*a-12/d/b^5*arctan(tan
(1/2*d*x+1/2*c))*a^2+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))-5/d*a^4/b^3/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3
+4/d*a^2/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*ta
n(1/2*d*x+1/2*c)^3-6/d*a^5/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)
*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-7/d*a^3/b^2/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2+10/d*a/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-1
9/d*a^4/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*t
```

```
an(1/2*d*x+1/2*c)+16/d*a^2/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b
+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)-6/d*a^5/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*ta
n(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)+5/d*a^3/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan
(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)+12/d*a^5/b^5/(a^2-b^2)^(3/2)*arctan(1/2*(2
*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-19/d*a^3/b^3/(a^2-b^2)^(3/2)*ar
ctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d*a/b/(a^2-b^2)^(3
/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 17.72, size = 4943, normalized size = 18.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] - ((12*a^5 - 11*a^3*b^2)/(b^4*(a^2 - b^2)) + (tan(c/2 + (d*x)/2)^7*(6*a^4 -
 5*a^2*b^2))/(b^3*(a^2 - b^2)) - (tan(c/2 + (d*x)/2)^5*(4*b^4 - 54*a^4 + 45
*a^2*b^2))/(b^3*(a^2 - b^2)) + (tan(c/2 + (d*x)/2)^3*(90*a^4 + 4*b^4 - 87*a
^2*b^2))/(b^3*(a^2 - b^2)) + (tan(c/2 + (d*x)/2)^6*(12*a^5 - 14*a*b^4 + 5*a
^3*b^2))/(b^4*(a^2 - b^2)) - (tan(c/2 + (d*x)/2)^2*(30*a*b^4 - 36*a^5 + a^3
*b^2))/(b^4*(a^2 - b^2)) + (3*tan(c/2 + (d*x)/2)*(14*a^4 - 13*a^2*b^2))/(b^
3*(a^2 - b^2)) - (tan(c/2 + (d*x)/2)^4*(11*a*b^2 - 12*a^3)*(3*a^2 + 4*b^2))
/(b^4*(a^2 - b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(4*a^2 + 4*b^2) + tan(c/2 + (d
*x)/2)^6*(4*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(6*a^2 + 8*b^2) + a^2*tan(c
/2 + (d*x)/2)^8 + a^2 + 12*a*b*tan(c/2 + (d*x)/2)^3 + 12*a*b*tan(c/2 + (d*x
)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2))) - (atan(((
(a^2*12i - b^2*1i)*((4*(2*a^2*b^12 - 52*a^4*b^10 + 386*a^6*b^8 - 624*a^8*b^
6 + 288*a^10*b^4))/(b^15 - 2*a^2*b^13 + a^4*b^11) - ((a^2*12i - b^2*1i)*((4
*(4*a*b^16 - 36*a^3*b^14 + 56*a^5*b^12 - 24*a^7*b^10))/(b^15 - 2*a^2*b^13 +
 a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(24*a^2*b^16 - 100*a^4*b^14 + 124*a^6*b^
12 - 48*a^8*b^10))/(b^16 - 2*a^2*b^14 + a^4*b^12) - ((a^2*12i - b^2*1i)*((4
*(8*a^2*b^18 - 16*a^4*b^16 + 8*a^6*b^14))/(b^15 - 2*a^2*b^13 + a^4*b^11) +
```

(8*tan(c/2 + (d*x)/2)*(12*a*b^20 - 32*a^3*b^18 + 28*a^5*b^16 - 8*a^7*b^14))
/(b^16 - 2*a^2*b^14 + a^4*b^12)))/(2*b^5)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)
*(2*a*b^14 - 89*a^3*b^12 + 640*a^5*b^10 - 1322*a^7*b^8 + 1056*a^9*b^6 - 288
*a^11*b^4))/(b^16 - 2*a^2*b^14 + a^4*b^12))*1i)/(2*b^5) + ((a^2*12i - b^2*1
i)*((4*(2*a^2*b^12 - 52*a^4*b^10 + 386*a^6*b^8 - 624*a^8*b^6 + 288*a^10*b^4
))/(b^15 - 2*a^2*b^13 + a^4*b^11) + ((a^2*12i - b^2*1i)*((4*(4*a*b^16 - 36*
a^3*b^14 + 56*a^5*b^12 - 24*a^7*b^10))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*
tan(c/2 + (d*x)/2)*(24*a^2*b^16 - 100*a^4*b^14 + 124*a^6*b^12 - 48*a^8*b^10
))/(b^16 - 2*a^2*b^14 + a^4*b^12) + ((a^2*12i - b^2*1i)*((4*(8*a^2*b^18 - 1
6*a^4*b^16 + 8*a^6*b^14))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*
x)/2)*(12*a*b^20 - 32*a^3*b^18 + 28*a^5*b^16 - 8*a^7*b^14))/(b^16 - 2*a^2*b
^14 + a^4*b^12)))/(2*b^5)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(2*a*b^14 - 89*
a^3*b^12 + 640*a^5*b^10 - 1322*a^7*b^8 + 1056*a^9*b^6 - 288*a^11*b^4))/(b^1
6 - 2*a^2*b^14 + a^4*b^12))*1i)/(2*b^5))/((8*(864*a^11 + 30*a^3*b^8 - 491*a
^5*b^6 + 1746*a^7*b^4 - 2160*a^9*b^2))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (16
*tan(c/2 + (d*x)/2)*(1728*a^12 - 6*a^2*b^10 + 169*a^4*b^8 - 1495*a^6*b^6 +
4356*a^8*b^4 - 4752*a^10*b^2))/(b^16 - 2*a^2*b^14 + a^4*b^12) + ((a^2*12i -
b^2*1i)*((4*(2*a^2*b^12 - 52*a^4*b^10 + 386*a^6*b^8 - 624*a^8*b^6 + 288*a^
10*b^4))/(b^15 - 2*a^2*b^13 + a^4*b^11) - ((a^2*12i - b^2*1i)*((4*(4*a*b^16
- 36*a^3*b^14 + 56*a^5*b^12 - 24*a^7*b^10))/(b^15 - 2*a^2*b^13 + a^4*b^11)
+ (8*tan(c/2 + (d*x)/2)*(24*a^2*b^16 - 100*a^4*b^14 + 124*a^6*b^12 - 48*a^
8*b^10))/(b^16 - 2*a^2*b^14 + a^4*b^12) - ((a^2*12i - b^2*1i)*((4*(8*a^2*b^
18 - 16*a^4*b^16 + 8*a^6*b^14))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2
+ (d*x)/2)*(12*a*b^20 - 32*a^3*b^18 + 28*a^5*b^16 - 8*a^7*b^14))/(b^16 - 2
*a^2*b^14 + a^4*b^12)))/(2*b^5)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(2*a*b^14
- 89*a^3*b^12 + 640*a^5*b^10 - 1322*a^7*b^8 + 1056*a^9*b^6 - 288*a^11*b^4)
)/(b^16 - 2*a^2*b^14 + a^4*b^12)))/(2*b^5) - ((a^2*12i - b^2*1i)*((4*(2*a^2
*b^12 - 52*a^4*b^10 + 386*a^6*b^8 - 624*a^8*b^6 + 288*a^10*b^4))/(b^15 - 2*
a^2*b^13 + a^4*b^11) + ((a^2*12i - b^2*1i)*((4*(4*a*b^16 - 36*a^3*b^14 + 56
*a^5*b^12 - 24*a^7*b^10))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*
x)/2)*(24*a^2*b^16 - 100*a^4*b^14 + 124*a^6*b^12 - 48*a^8*b^10))/(b^16 - 2*
a^2*b^14 + a^4*b^12) + ((a^2*12i - b^2*1i)*((4*(8*a^2*b^18 - 16*a^4*b^16 +
8*a^6*b^14))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(12*a*b
^20 - 32*a^3*b^18 + 28*a^5*b^16 - 8*a^7*b^14))/(b^16 - 2*a^2*b^14 + a^4*b^1
2)))/(2*b^5)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(2*a*b^14 - 89*a^3*b^12 + 64
0*a^5*b^10 - 1322*a^7*b^8 + 1056*a^9*b^6 - 288*a^11*b^4))/(b^16 - 2*a^2*b^1
4 + a^4*b^12)))/(2*b^5)))*(a^2*12i - b^2*1i)*1i)/(b^5*d) - (a*atan(((a*(-(a
+ b)^3*(a - b)^3)^(1/2)*((4*(2*a^2*b^12 - 52*a^4*b^10 + 386*a^6*b^8 - 624*
a^8*b^6 + 288*a^10*b^4))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x
)/2)*(2*a*b^14 - 89*a^3*b^12 + 640*a^5*b^10 - 1322*a^7*b^8 + 1056*a^9*b^6 -
288*a^11*b^4))/(b^16 - 2*a^2*b^14 + a^4*b^12) - (a*(-(a + b)^3*(a - b)^3)^
(1/2)*((4*(4*a*b^16 - 36*a^3*b^14 + 56*a^5*b^12 - 24*a^7*b^10))/(b^15 - 2*a
^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(24*a^2*b^16 - 100*a^4*b^14 + 1
24*a^6*b^12 - 48*a^8*b^10))/(b^16 - 2*a^2*b^14 + a^4*b^12) - (a*(-(a + b)^3
(a - b)^3)^(1/2)((4*(8*a^2*b^18 - 16*a^4*b^16 + 8*a^6*b^14))/(b^15 - 2*a^

2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(12*a*b^20 - 32*a^3*b^18 + 28*a^5*b^16 - 8*a^7*b^14))/(b^16 - 2*a^2*b^14 + a^4*b^12))*(12*a^4 + 6*b^4 - 19*a^2*b^2))/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(12*a^4 + 6*b^4 - 19*a^2*b^2))/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(12*a^4 + 6*b^4 - 19*a^2*b^2)*1i)/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((4*(2*a^2*b^12 - 52*a^4*b^10 + 386*a^6*b^8 - 624*a^8*b^6 + 288*a^10*b^4))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(2*a*b^14 - 89*a^3*b^12 + 640*a^5*b^10 - 1322*a^7*b^8 + 1056*a^9*b^6 - 288*a^11*b^4))/(b^16 - 2*a^2*b^14 + a^4*b^12) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((4*(4*a*b^16 - 36*a^3*b^14 + 56*a^5*b^12 - 24*a^7*b^10))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(24*a^2*b^16 - 100*a^4*b^14 + 124*a^6*b^12 - 48*a^8*b^10))/(b^16 - 2*a^2*b^14 + a^4*b^12) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((4*(8*a^2*b^18 - 16*a^4*b^16 + 8*a^6*b^14))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(12*a*b^20 - 32*a^3*b^18 + 28*a^5*b^16 - 8*a^7*b^14))/(b^16 - 2*a^2*b^14 + a^4*b^12))*(12*a^4 + 6*b^4 - 19*a^2*b^2))/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(12*a^4 + 6*b^4 - 19*a^2*b^2))/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(12*a^4 + 6*b^4 - 19*a^2*b^2)*1i)/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))/((8*(864*a^11 + 30*a^3*b^8 - 491*a^5*b^6 + 1746*a^7*b^4 - 2160*a^9*b^2))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (16*tan(c/2 + (d*x)/2)*(1728*a^12 - 6*a^2*b^10 + 169*a^4*b^8 - 1495*a^6*b^6 + 4356*a^8*b^4 - 4752*a^10*b^2))/(b^16 - 2*a^2*b^14 + a^4*b^12) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((4*(2*a^2*b^12 - 52*a^4*b^10 + 386*a^6*b^8 - 624*a^8*b^6 + 288*a^10*b^4))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(2*a*b^14 - 89*a^3*b^12 + 640*a^5*b^10 - 1322*a^7*b^8 + 1056*a^9*b^6 - 288*a^11*b^4))/(b^16 - 2*a^2*b^14 + a^4*b^12) - (a*(-(a + b)^3*(a - b)^3)^(1/2)*((4*(4*a*b^16 - 36*a^3*b^14 + 56*a^5*b^12 - 24*a^7*b^10))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(24*a^2*b^16 - 100*a^4*b^14 + 124*a^6*b^12 - 48*a^8*b^10))/(b^16 - 2*a^2*b^14 + a^4*b^12) - (a*(-(a + b)^3*(a - b)^3)^(1/2)*((4*(8*a^2*b^18 - 16*a^4*b^16 + 8*a^6*b^14))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(12*a*b^20 - 32*a^3*b^18 + 28*a^5*b^16 - 8*a^7*b^14))/(b^16 - 2*a^2*b^14 + a^4*b^12))*(12*a^4 + 6*b^4 - 19*a^2*b^2))/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(12*a^4 + 6*b^4 - 19*a^2*b^2))/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(12*a^4 + 6*b^4 - 19*a^2*b^2))/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)) - (a*(-(a + b)^3*(a - b)^3)^(1/2)*((4*(2*a^2*b^12 - 52*a^4*b^10 + 386*a^6*b^8 - 624*a^8*b^6 + 288*a^10*b^4))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(2*a*b^14 - 89*a^3*b^12 + 640*a^5*b^10 - 1322*a^7*b^8 + 1056*a^9*b^6 - 288*a^11*b^4))/(b^16 - 2*a^2*b^14 + a^4*b^12) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((4*(4*a*b^16 - 36*a^3*b^14 + 56*a^5*b^12 - 24*a^7*b^10))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(24*a^2*b^16 - 100*a^4*b^14 + 124*a^6*b^12 - 48*a^8*b^10))/(b^16 - 2*a^2*b^14 + a^4*b^12) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((4*(8*a^2*b^18 - 16*a^4*b^16 + 8*a^6*b^14))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(12*a*b^20 - 32*a^3*b^18 + 28*a^5*b^16 - 8*a^7*b^14))/(b^16 - 2*a^2*b^14 + a^4*b^12))*(12*a^4 + 6*b^4 - 19*a^2*b^2))/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(12*a^4 + 6*

```
b^4 - 19*a^2*b^2))/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(12*a^4 +
6*b^4 - 19*a^2*b^2))/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))))*(-(a +
b)^3*(a - b)^3)^(1/2)*(12*a^4 + 6*b^4 - 19*a^2*b^2)*1i)/(d*(b^11 - 3*a^2*b^
9 + 3*a^4*b^7 - a^6*b^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1086 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$\frac{a(3a^2-2b^2)\cos(c+dx)}{2b^3d(a^2-b^2)(a+b\sin(c+dx))}-\frac{(6a^4-9a^2b^2+2b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d(a^2-b^2)^{3/2}}+\frac{3ax}{b^4}-\frac{\sin^2(c+dx)\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}+$$

```
[Out] 3*a*x/b^4-(6*a^4-9*a^2*b^2+2*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)
^(1/2))/b^4/(a^2-b^2)^(3/2)/d+3/2*cos(d*x+c)/b^3/d-1/2*cos(d*x+c)*sin(d*x+c
)^2/b/d/(a+b*sin(d*x+c))^2+1/2*a*(3*a^2-2*b^2)*cos(d*x+c)/b^3/(a^2-b^2)/d/(
a+b*sin(d*x+c))
```

Rubi [A] time = 0.56, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3048, 3032, 3023, 2735, 2660, 618, 204}

$$-\frac{(-9a^2b^2+6a^4+2b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d(a^2-b^2)^{3/2}}+\frac{a(3a^2-2b^2)\cos(c+dx)}{2b^3d(a^2-b^2)(a+b\sin(c+dx))}+\frac{3ax}{b^4}-\frac{\sin^2(c+dx)\cos(c+dx)}{2bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*x)/b^4 - ((6*a^4 - 9*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(3/2)*d) + (3*Cos[c + d*x])/(2*b^3*d) -
(Cos[c + d*x]*Sin[c + d*x]^2)/(2*b*d*(a + b*Sin[c + d*x])^2) + (a*(3*a^2 -
2*b^2)*Cos[c + d*x])/(2*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\sin^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= -\frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\int \frac{\sin(c+dx)\left(-2\left(a^2-b^2\right)+3\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\left(3a^2-2b^2\right)\cos(c+dx)}{2b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{b\left(3a^4-5a^2b^2 \right.}{}}{}$$

$$= \frac{3\cos(c+dx)}{2b^3d} - \frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\left(3a^2-2b^2\right)\cos(c+dx)}{2b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))}$$

$$= \frac{3ax}{b^4} + \frac{3\cos(c+dx)}{2b^3d} - \frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\left(3a^2-2b^2\right)\cos(c+\right.}{2b^3\left(a^2-b^2\right)d(a+b\sin(c}$$

$$= \frac{3ax}{b^4} + \frac{3\cos(c+dx)}{2b^3d} - \frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\left(3a^2-2b^2\right)\cos(c+\right.}{2b^3\left(a^2-b^2\right)d(a+b\sin(c}$$

$$= \frac{3ax}{b^4} + \frac{3\cos(c+dx)}{2b^3d} - \frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\left(3a^2-2b^2\right)\cos(c+\right.}{2b^3\left(a^2-b^2\right)d(a+b\sin(c}$$

$$= \frac{3ax}{b^4} - \frac{\left(6a^4-9a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}d} + \frac{3\cos(c+dx)}{2b^3d} - \frac{\cos(c+}{2bd(a+}$$

Mathematica [A] time = 1.22, size = 159, normalized size = 0.88

$$\frac{\frac{ab(5a^2-4b^2)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))}-\frac{a^2b\cos(c+dx)}{(a+b\sin(c+dx))^2}-\frac{2(6a^4-9a^2b^2+2b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+6a(c+dx)+2b\cos(c+dx)}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (6*a*(c + d*x) - (2*(6*a^4 - 9*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 2*b*Cos[c + d*x] - (a^2*b*Cos[c
+ d*x])/(a + b*Sin[c + d*x])^2 + (a*b*(5*a^2 - 4*b^2)*Cos[c + d*x])/((a - b
)*(a + b)*(a + b*Sin[c + d*x])))/(2*b^4*d)
```

fricas [B] time = 0.76, size = 919, normalized size = 5.11

$$\left[\frac{12\left(a^5b^2-2a^3b^4+ab^6\right)dx\cos(dx+c)^2+4\left(a^4b^3-2a^2b^5+b^7\right)\cos(dx+c)^3-12\left(a^7-a^5b^2-a^3b^4+ab^6\right)dx-\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(12*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*x*cos(d*x + c)^2 + 4*(a^4*b^3 - 2*
a^2*b^5 + b^7)*cos(d*x + c)^3 - 12*(a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*x -
(6*a^6 - 3*a^4*b^2 - 7*a^2*b^4 + 2*b^6 - (6*a^4*b^2 - 9*a^2*b^4 + 2*b^6)*co
s(d*x + c)^2 + 2*(6*a^5*b - 9*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 +
b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2
*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(6*a^6*b - 9*a^4*b^3 + a^
2*b^5 + 2*b^7)*cos(d*x + c) - 2*(12*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*x + (9*
a^5*b^2 - 17*a^3*b^4 + 8*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a
^2*b^8 + b^10)*d*cos(d*x + c)^2 - 2*(a^5*b^5 - 2*a^3*b^7 + a*b^9)*d*sin(d*x
 + c) - (a^6*b^4 - a^4*b^6 - a^2*b^8 + b^10)*d), 1/2*(6*(a^5*b^2 - 2*a^3*b^
4 + a*b^6)*d*x*cos(d*x + c)^2 + 2*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^
3 - 6*(a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*x - (6*a^6 - 3*a^4*b^2 - 7*a^2*b^
4 + 2*b^6 - (6*a^4*b^2 - 9*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(6*a^5*b - 9
*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
 b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (6*a^6*b - 9*a^4*b^3 + a^2*b^5 + 2*b^
7)*cos(d*x + c) - (12*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*x + (9*a^5*b^2 - 17*a
```

```
^3*b^4 + 8*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a^2*b^8 + b^10)
*d*cos(d*x + c)^2 - 2*(a^5*b^5 - 2*a^3*b^7 + a*b^9)*d*sin(d*x + c) - (a^6*b
^4 - a^4*b^6 - a^2*b^8 + b^10)*d)]
```

giac [A] time = 0.22, size = 302, normalized size = 1.68

$$-\frac{\frac{\left(6\,a^4-9\,a^2b^2+2\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{a^2-b^2}}-\frac{3\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5\,a^2b^2\mathrm{ta}}{\left(a^2b^3-b^5\right)\left(a\tan\left(\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((6*a^4 - 9*a^2*b^2 + 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + ar
ctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a
^2 - b^2)) - (3*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)
^3 + 4*a^4*tan(1/2*d*x + 1/2*c)^2 + 5*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*b^
4*tan(1/2*d*x + 1/2*c)^2 + 13*a^3*b*tan(1/2*d*x + 1/2*c) - 10*a*b^3*tan(1/2
*d*x + 1/2*c) + 4*a^4 - 3*a^2*b^2)/((a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1/2*c)
^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) - 3*(d*x + c)*a/b^4 - 2/((tan(1/2*d*x
 + 1/2*c)^2 + 1)*b^3))/d
```

maple [B] time = 0.54, size = 711, normalized size = 3.95

$$\frac{2}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{6a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^4}+\frac{3a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)^2\left(a^2-b^2\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)+6/d/b^4*a*arctan(tan(1/2*d*x+1/2*c))+3/d/b
^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^3/(a^2-b^2)*tan(1/
2*d*x+1/2*c)^3-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a/(a
^2-b^2)*tan(1/2*d*x+1/2*c)^3+4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2*a^4+5/d/b/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2*a^2-6/d*b/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2
*c)^2+13/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^3/(a^2
-b^2)*tan(1/2*d*x+1/2*c)-10/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
```

```
b+a)^2*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)+4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan
(1/2*d*x+1/2*c)*b+a)^2*a^4/(a^2-b^2)-3/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)^2*a^2/(a^2-b^2)-6/d/b^4/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*t
an(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^4+9/d/b^2/(a^2-b^2)^(3/2)*arctan(
1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2-2/d/(a^2-b^2)^(3/2)*a
rctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 15.79, size = 3031, normalized size = 16.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] ((6*a^4 - 5*a^2*b^2)/(b^3*(a^2 - b^2)) + (3*tan(c/2 + (d*x)/2)^4*(2*a^4 - 2
*b^4 + a^2*b^2))/(b^3*(a^2 - b^2)) + (2*tan(c/2 + (d*x)/2)^2*(6*a^4 - 7*b^4
 + 3*a^2*b^2))/(b^3*(a^2 - b^2)) - (3*tan(c/2 + (d*x)/2)*(6*a*b^2 - 7*a^3))
/(b^2*(a^2 - b^2)) - (tan(c/2 + (d*x)/2)^5*(2*a*b^2 - 3*a^3))/(b^2*(a^2 - b
^2)) - (4*tan(c/2 + (d*x)/2)^3*(5*a*b^2 - 6*a^3))/(b^2*(a^2 - b^2)))/(d*(ta
n(c/2 + (d*x)/2)^2*(3*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(3*a^2 + 4*b^2) +
 a^2*tan(c/2 + (d*x)/2)^6 + a^2 + 8*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/
2 + (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2))) + (6*a*atan((192*a^2*b^7*tan(c/
2 + (d*x)/2))/((192*a^2*b^19)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (384*a^4*b^17
)/(b^12 - 2*a^2*b^10 + a^4*b^8) + (48*a^6*b^15)/(b^12 - 2*a^2*b^10 + a^4*b^
8) + (288*a^8*b^13)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (144*a^10*b^11)/(b^12 -
 2*a^2*b^10 + a^4*b^8)) - (144*a^6*b^3*tan(c/2 + (d*x)/2))/((192*a^2*b^19)/
(b^12 - 2*a^2*b^10 + a^4*b^8) - (384*a^4*b^17)/(b^12 - 2*a^2*b^10 + a^4*b^8
) + (48*a^6*b^15)/(b^12 - 2*a^2*b^10 + a^4*b^8) + (288*a^8*b^13)/(b^12 - 2*
a^2*b^10 + a^4*b^8) - (144*a^10*b^11)/(b^12 - 2*a^2*b^10 + a^4*b^8))))/(b^4
*d) + (atan((((-(a + b)^3*(a - b)^3)^(1/2)*(3*a^4 + b^4 - (9*a^2*b^2)/2)*((
8*(36*a^4*b^7 - 72*a^6*b^5 + 36*a^8*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (
8*tan(c/2 + (d*x)/2)*(4*a*b^11 - 108*a^3*b^9 + 285*a^5*b^7 - 252*a^7*b^5 +
```

72*a^9*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) + ((-(a + b)^3*(a - b)^3)^(1/2)*
(3*a^4 + b^4 - (9*a^2*b^2)/2)*((8*tan(c/2 + (d*x)/2)*(8*a*b^14 - 44*a^3*b^1
2 + 60*a^5*b^10 - 24*a^7*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) - (8*(8*a^2*b^
12 - 14*a^4*b^10 + 6*a^6*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + ((-(a + b)^3
(a - b)^3)^(1/2)((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2
*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*
b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(3*a^4 + b^4 - (9*a^2*b^
2)/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4
*b^6 - a^6*b^4))*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + ((-(a + b)^
3*(a - b)^3)^(1/2)*(3*a^4 + b^4 - (9*a^2*b^2)/2)*((8*(36*a^4*b^7 - 72*a^6*b
^5 + 36*a^8*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (8*tan(c/2 + (d*x)/2)*(4*
a*b^11 - 108*a^3*b^9 + 285*a^5*b^7 - 252*a^7*b^5 + 72*a^9*b^3))/(b^13 - 2*a
^2*b^11 + a^4*b^9) + ((-(a + b)^3*(a - b)^3)^(1/2)*(3*a^4 + b^4 - (9*a^2*b^
2)/2)*((8*(8*a^2*b^12 - 14*a^4*b^10 + 6*a^6*b^8))/(b^12 - 2*a^2*b^10 + a^4*
b^8) - (8*tan(c/2 + (d*x)/2)*(8*a*b^14 - 44*a^3*b^12 + 60*a^5*b^10 - 24*a^7
*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) + ((-(a + b)^3*(a - b)^3)^(1/2)*((8*(4
*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*ta
n(c/2 + (d*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^1
3 - 2*a^2*b^11 + a^4*b^9))*(3*a^4 + b^4 - (9*a^2*b^2)/2))/(b^10 - 3*a^2*b^8
 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*1i)/(b^
10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))/((16*(54*a^8 - 12*a^2*b^6 + 72*a^4*b
^4 - 117*a^6*b^2))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (16*tan(c/2 + (d*x)/2)*(
216*a^9 - 72*a^3*b^6 + 396*a^5*b^4 - 540*a^7*b^2))/(b^13 - 2*a^2*b^11 + a^4
b^9) - ((-(a + b)^3(a - b)^3)^(1/2)*(3*a^4 + b^4 - (9*a^2*b^2)/2)*((8*(36
*a^4*b^7 - 72*a^6*b^5 + 36*a^8*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (8*tan
(c/2 + (d*x)/2)*(4*a*b^11 - 108*a^3*b^9 + 285*a^5*b^7 - 252*a^7*b^5 + 72*a^
9*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) + ((-(a + b)^3*(a - b)^3)^(1/2)*(3*a^
4 + b^4 - (9*a^2*b^2)/2)*((8*tan(c/2 + (d*x)/2)*(8*a*b^14 - 44*a^3*b^12 + 6
0*a^5*b^10 - 24*a^7*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) - (8*(8*a^2*b^12 -
14*a^4*b^10 + 6*a^6*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + ((-(a + b)^3*(a -
 b)^3)^(1/2)*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10
 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13
- 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(3*a^4 + b^4 - (9*a^2*b^2)/2)
)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6
- a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + ((-(a + b)^3*(a - b
)^3)^(1/2)*(3*a^4 + b^4 - (9*a^2*b^2)/2)*((8*(36*a^4*b^7 - 72*a^6*b^5 + 36*
a^8*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (8*tan(c/2 + (d*x)/2)*(4*a*b^11 -
 108*a^3*b^9 + 285*a^5*b^7 - 252*a^7*b^5 + 72*a^9*b^3))/(b^13 - 2*a^2*b^11
+ a^4*b^9) + ((-(a + b)^3*(a - b)^3)^(1/2)*(3*a^4 + b^4 - (9*a^2*b^2)/2)*((
8*(8*a^2*b^12 - 14*a^4*b^10 + 6*a^6*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (
8*tan(c/2 + (d*x)/2)*(8*a*b^14 - 44*a^3*b^12 + 60*a^5*b^10 - 24*a^7*b^8))/(
b^13 - 2*a^2*b^11 + a^4*b^9) + ((-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^1
5 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 +
(d*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^
2*b^11 + a^4*b^9))*(3*a^4 + b^4 - (9*a^2*b^2)/2))/(b^10 - 3*a^2*b^8 + 3*a^4

```
*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*
b^8 + 3*a^4*b^6 - a^6*b^4)))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*a^4 + b^4 - (9
*a^2*b^2)/2)*2i)/(d*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1087 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=167

$$-\frac{a\cos^3(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{\cos(c+dx)\left(2\left(a^2-b^2\right)+ab\sin(c+dx)\right)}{2b^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right.}{b^3d\left(a^2-b^2\right)^{3/2}}$$

```
[Out] -x/b^3+a*(2*a^2-3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^3
/(a^2-b^2)^(3/2)/d-1/2*a*cos(d*x+c)^3/(a^2-b^2)/d/(a+b*sin(d*x+c))^2-1/2*co
s(d*x+c)*(2*a^2-2*b^2+a*b*sin(d*x+c))/b^2/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.28, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2864, 2863, 2735, 2660, 618, 204}

$$\frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\left(a^2-b^2\right)^{3/2}}-\frac{a\cos^3(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{\cos(c+dx)\left(2\left(a^2-b^2\right)+ab\sin(c+dx\right.}{2b^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(x/b^3) + (a*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(b^3*(a^2 - b^2)^(3/2)*d) - (a*Cos[c + d*x]^3)/(2*(a^2 - b^2)*d*(a + b
*Sin[c + d*x])^2) - (Cos[c + d*x]*(2*(a^2 - b^2) + a*b*Sin[c + d*x]))/(2*b^
2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{a\cos^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)(2b+a\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{a\cos^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)+ab\sin(c+dx)\right)}{2b^2\left(a^2-b^2\right)d(a+b\sin(c+dx))}$$

$$= -\frac{x}{b^3} - \frac{a\cos^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)+ab\sin(c+\right.}{2b^2\left(a^2-b^2\right)d(a+b\sin(c+dx)}$$

$$= -\frac{x}{b^3} - \frac{a\cos^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)+ab\sin(c+\right.}{2b^2\left(a^2-b^2\right)d(a+b\sin(c+dx)}$$

$$= -\frac{x}{b^3} - \frac{a\cos^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)+ab\sin(c+\right.}{2b^2\left(a^2-b^2\right)d(a+b\sin(c+dx)}$$

$$= -\frac{x}{b^3} + \frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}d} - \frac{a\cos^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2}$$

Mathematica [A] time = 2.06, size = 289, normalized size = 1.73

$$\frac{\frac{\frac{ab\left(4a^2-3b^2\right)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))^2}+\frac{2a\left(8a^4-20a^2b^2+15b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{3b\left(4a^4-7a^2b^2+2b^4\right)\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}-8(c+dx)}{b^3} - \frac{\frac{6ab\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{\cos(c+dx)(b}{}}{(a-b)^2(a+b)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-8*(c + d*x) + (2*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[(b + a*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(4*a^2 - 3*b^2)*Cos[c +
 d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x])^2) - (3*b*(4*a^4 - 7*a^2*b^2 +
 2*b^4)*Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/b^3 - ((6
*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (C
os[c + d*x]*(a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Sin[c + d*x]))/(a + b*Sin[c
+ d*x])^2)/((a - b)^2*(a + b)^2))/(8*d)
```

fricas [B] time = 0.79, size = 793, normalized size = 4.75

$$\left[-\frac{4\left(a^4b^2-2a^2b^4+b^6\right)dx\cos\left(dx+c\right)^2-4\left(a^6-a^4b^2-a^2b^4+b^6\right)dx-\left(2a^5-a^3b^2-3ab^4-\left(2a^3b^2-3ab^4\right)\text{co}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x*cos(d*x + c)^2 - 4*(a^6 - a^4*b^2
- a^2*b^4 + b^6)*d*x - (2*a^5 - a^3*b^2 - 3*a*b^4 - (2*a^3*b^2 - 3*a*b^4)*c
os(d*x + c)^2 + 2*(2*a^4*b - 3*a^2*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(
d*x + c) - 2*(4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x + (3*a^4*b^2 - 5*a^2*b^4 +
2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x +
 c)^2 - 2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*sin(d*x + c) - (a^6*b^3 - a^4*b^5
 - a^2*b^7 + b^9)*d), -1/2*(2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x*cos(d*x + c)^
2 - 2*(a^6 - a^4*b^2 - a^2*b^4 + b^6)*d*x - (2*a^5 - a^3*b^2 - 3*a*b^4 - (2
*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + 2*(2*a^4*b - 3*a^2*b^3)*sin(d*x + c))*
sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))
) - (2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c) - (4*(a^5*b - 2*a^3*b^3 + a*
b^5)*d*x + (3*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^
4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2 - 2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)
*d*sin(d*x + c) - (a^6*b^3 - a^4*b^5 - a^2*b^7 + b^9)*d)]
```

giac [A] time = 0.23, size = 256, normalized size = 1.53

$$\frac{\frac{\left(2a^3-3ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{a^2-b^2}}-\frac{a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\cdots}{\left(a^3b^2-ab^4\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}d\cdots\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*a^3 - 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan
(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^2*b^3 - b^5)*sqrt(a^2 - b^2))
- (a^3*b*tan(1/2*d*x + 1/2*c)^3 + 2*a^4*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b^2*
tan(1/2*d*x + 1/2*c)^2 - 2*b^4*tan(1/2*d*x + 1/2*c)^2 + 7*a^3*b*tan(1/2*d*x
 + 1/2*c) - 4*a*b^3*tan(1/2*d*x + 1/2*c) + 2*a^4 - a^2*b^2)/((a^3*b^2 - a*b
```

```
^4)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) - (d*x + c
)/b^3)/d
```

maple [B] time = 0.51, size = 576, normalized size = 3.45

$$-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^3}-\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)^2\left(a^2-b^2\right)}-\frac{2a^3\left(\tan\right.}{d\,b^2\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -2/d/b^3*arctan(tan(1/2*d*x+1/2*c))-1/d*a^2/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan
(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3-2/d*a^3/b^2/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-3
/d*a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*
d*x+1/2*c)^2+2/d*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a/
(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-7/d*a^2/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)+4/d*b/(tan(1/2*d*x+1/2*c)^2*
a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)-2/d*a^3/b^2/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)+1/d/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a/(a^2-b^2)+2/d*a^3/b^3/(a^2-b^2)^(3
/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-3/d*a/b/(a^2-b
^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.93, size = 2709, normalized size = 16.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + b*sin(c + d*x))^3,x)
```

```
[Out] (2*atan((64*a*b^5*tan(c/2 + (d*x)/2))/((176*a^3*b^12)/(b^9 - 2*a^2*b^7 + a^
4*b^5) - (160*a^5*b^10)/(b^9 - 2*a^2*b^7 + a^4*b^5) + (48*a^7*b^8)/(b^9 - 2
*a^2*b^7 + a^4*b^5) - (64*a*b^14)/(b^9 - 2*a^2*b^7 + a^4*b^5)) - (48*a^3*b^
3*tan(c/2 + (d*x)/2))/((176*a^3*b^12)/(b^9 - 2*a^2*b^7 + a^4*b^5) - (160*a^
5*b^10)/(b^9 - 2*a^2*b^7 + a^4*b^5) + (48*a^7*b^8)/(b^9 - 2*a^2*b^7 + a^4*b
^5) - (64*a*b^14)/(b^9 - 2*a^2*b^7 + a^4*b^5))))/(b^3*d) - ((tan(c/2 + (d*x
)/2)*(7*a^2 - 4*b^2))/(b*(a^2 - b^2)) - (a*b^2 - 2*a^3)/(b^2*(a^2 - b^2)) +
 (a^2*tan(c/2 + (d*x)/2)^3)/(b*(a^2 - b^2)) + (tan(c/2 + (d*x)/2)^2*(a^2 +
2*b^2)*(2*a^2 - b^2))/(a*b^2*(a^2 - b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(2*a^2
+ 4*b^2) + a^2*tan(c/2 + (d*x)/2)^4 + a^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*
a*b*tan(c/2 + (d*x)/2))) - (a*atan(((a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^
3)^(1/2)*((8*(4*a^2*b^6 - 8*a^4*b^4 + 4*a^6*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^
5) + (8*tan(c/2 + (d*x)/2)*(8*a*b^8 - 29*a^3*b^6 + 28*a^5*b^4 - 8*a^7*b^2))
/(b^10 - 2*a^2*b^8 + a^4*b^6) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(
1/2)*((8*(4*a*b^10 - 6*a^3*b^8 + 2*a^5*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) +
(8*tan(c/2 + (d*x)/2)*(12*a^2*b^10 - 20*a^4*b^8 + 8*a^6*b^6))/(b^10 - 2*a^2
*b^8 + a^4*b^6) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^
2*b^12 - 8*a^4*b^10 + 4*a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2
+ (d*x)/2)*(12*a*b^14 - 32*a^3*b^12 + 28*a^5*b^10 - 8*a^7*b^8))/(b^10 - 2*a
^2*b^8 + a^4*b^6)))/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))))/(2*(b^9 -
 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*1i)/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a
^6*b^3)) + (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^6 -
 8*a^4*b^4 + 4*a^6*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2
)*(8*a*b^8 - 29*a^3*b^6 + 28*a^5*b^4 - 8*a^7*b^2))/(b^10 - 2*a^2*b^8 + a^4*
b^6) + (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a*b^10 - 6*a^
3*b^8 + 2*a^5*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2)*(12
*a^2*b^10 - 20*a^4*b^8 + 8*a^6*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(2*a
^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^12 - 8*a^4*b^10 + 4*a
^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2)*(12*a*b^14 - 3
2*a^3*b^12 + 28*a^5*b^10 - 8*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6)))/(2*(b
^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))))/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 -
a^6*b^3)))*1i)/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))/((16*(2*a^5 - 3
*a^3*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (16*tan(c/2 + (d*x)/2)*(8*a^6 + 12
*a^2*b^4 - 20*a^4*b^2))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (a*(2*a^2 - 3*b^2)*(
-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^6 - 8*a^4*b^4 + 4*a^6*b^2))/(b^9 -
 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2)*(8*a*b^8 - 29*a^3*b^6 + 28*a^
5*b^4 - 8*a^7*b^2))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (a*(2*a^2 - 3*b^2)*(-(a
+ b)^3*(a - b)^3)^(1/2)*((8*(4*a*b^10 - 6*a^3*b^8 + 2*a^5*b^6))/(b^9 - 2*a^
2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2)*(12*a^2*b^10 - 20*a^4*b^8 + 8*a^6*
b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)
^3)^(1/2)*((8*(4*a^2*b^12 - 8*a^4*b^10 + 4*a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4
*b^5) + (8*tan(c/2 + (d*x)/2)*(12*a*b^14 - 32*a^3*b^12 + 28*a^5*b^10 - 8*a^
7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6)))/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^
6*b^3))))/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))))/(2*(b^9 - 3*a^2*b^7
 + 3*a^4*b^5 - a^6*b^3)) + (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*
```

```
((8*(4*a^2*b^6 - 8*a^4*b^4 + 4*a^6*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*t
an(c/2 + (d*x)/2)*(8*a*b^8 - 29*a^3*b^6 + 28*a^5*b^4 - 8*a^7*b^2))/(b^10 -
2*a^2*b^8 + a^4*b^6) + (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*
(4*a*b^10 - 6*a^3*b^8 + 2*a^5*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/
2 + (d*x)/2)*(12*a^2*b^10 - 20*a^4*b^8 + 8*a^6*b^6))/(b^10 - 2*a^2*b^8 + a^
4*b^6) + (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^12 -
8*a^4*b^10 + 4*a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2
)*(12*a*b^14 - 32*a^3*b^12 + 28*a^5*b^10 - 8*a^7*b^8))/(b^10 - 2*a^2*b^8 +
a^4*b^6)))/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))))/(2*(b^9 - 3*a^2*b^
7 + 3*a^4*b^5 - a^6*b^3))))/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))))*(
2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*1i)/(d*(b^9 - 3*a^2*b^7 + 3*a^4
*b^5 - a^6*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1088 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=154

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\left(a^2-2b^2\right)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{3/2}}+\frac{\cos(c+dx}{2ad(a+b\sin(c}$$

```
[Out] -b*(3*a^2-2*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/(a^2-
b^2)^(3/2)/d-arctanh(cos(d*x+c))/a^3/d+1/2*cos(d*x+c)/a/d/(a+b*sin(d*x+c))^
2+1/2*(a^2-2*b^2)*cos(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.48, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2889, 3056, 3001, 3770, 2660, 618, 204}

$$-\frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{3/2}}+\frac{\left(a^2-2b^2\right)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx}{2ad(a+b\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((b*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3
*(a^2 - b^2)^(3/2)*d)) - ArcTanh[Cos[c + d*x]]/(a^3*d) + Cos[c + d*x]/(2*a*
d*(a + b*Sin[c + d*x])^2) + ((a^2 - 2*b^2)*Cos[c + d*x])/(2*a^2*(a^2 - b^2)
*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\csc(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc(c+dx)\left(2\left(a^2-b^2\right)-\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)\left(2\left(a^2-b^2\right)^{\cdots}\right.}{a+b\,\mathrm{s}\cdots}}{2a^2\left(\cdots\right.}$$

$$= \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \csc(c+dx)\,dx}{a^3} \cdots$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+\cdots}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+\cdots}$$

$$= -\frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx}{2ad(a+b\sin(c\cdots}$$

Mathematica [A] time = 1.18, size = 154, normalized size = 1.00

$$\frac{\frac{2b\left(2b^2-3a^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{a\cos(c+dx)\left(2a^3+b\left(a^2-2b^2\right)\sin(c+dx)-3ab^2\right)}{(a-b)(a+b)(a+b\sin(c+dx))^2} + 2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2\log\left(\cos\left(\frac{1}{2}(c+d\cdots\right.\right.}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*b*(-3*a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(3/2) - 2*Log[Cos[(c + d*x)/2]] + 2*Log[Sin[(c + d*x)/2]] + (a*Co
s[c + d*x]*(2*a^3 - 3*a*b^2 + b*(a^2 - 2*b^2)*Sin[c + d*x]))/((a - b)*(a +
b)*(a + b*Sin[c + d*x])^2))/(2*a^3*d)
```

fricas [B] time = 0.93, size = 996, normalized size = 6.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c) - (3*a^4*b
 + a^2*b^3 - 2*b^5 - (3*a^2*b^3 - 2*b^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 2*
a*b^4)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2)) + 2*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c) - 2*(a^6 - a^4*b^2 -
a^2*b^4 + b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(a^5*b - 2*a
^3*b^3 + a*b^5)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 2*(a^6 - a^4*b^
2 - a^2*b^4 + b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(a^5*b -
 2*a^3*b^3 + a*b^5)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7*b^2 -
 2*a^5*b^4 + a^3*b^6)*d*cos(d*x + c)^2 - 2*(a^8*b - 2*a^6*b^3 + a^4*b^5)*d*
sin(d*x + c) - (a^9 - a^7*b^2 - a^5*b^4 + a^3*b^6)*d), -1/2*((a^5*b - 3*a^3
*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c) + (3*a^4*b + a^2*b^3 - 2*b^5 - (3
*a^2*b^3 - 2*b^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 2*a*b^4)*sin(d*x + c))*sq
rt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))
+ (2*a^6 - 5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c) - (a^6 - a^4*b^2 - a^2*b^4 +
 b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3 +
a*b^5)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^6 - a^4*b^2 - a^2*b^4
 + b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3
+ a*b^5)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7*b^2 - 2*a^5*b^4
+ a^3*b^6)*d*cos(d*x + c)^2 - 2*(a^8*b - 2*a^6*b^3 + a^4*b^5)*d*sin(d*x + c
) - (a^9 - a^7*b^2 - a^5*b^4 + a^3*b^6)*d)]
```

giac [A] time = 0.26, size = 277, normalized size = 1.80

$$-\frac{\frac{\left(3\,a^{2}b-2\,b^{3}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{5}-a^{3}b^{2}\right)\sqrt{a^{2}-b^{2}}}-\frac{3\,a^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4\,ab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\cdots\right.}{\left(a^{5}-a^{3}b^{2}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,\cdots\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((3*a^2*b - 2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*ta
n(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2))
 - (3*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 4*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*a^4
```

```
*tan(1/2*d*x + 1/2*c)^2 + a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*b^4*tan(1/2*d*
x + 1/2*c)^2 + 5*a^3*b*tan(1/2*d*x + 1/2*c) - 8*a*b^3*tan(1/2*d*x + 1/2*c)
+ 2*a^4 - 3*a^2*b^2)/((a^5 - a^3*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1
/2*d*x + 1/2*c) + a)^2) - log(abs(tan(1/2*d*x + 1/2*c)))/a^3)/d
```

maple [B] time = 0.72, size = 632, normalized size = 4.10

$$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}+\frac{3b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)^2\left(a^2-b^2\right)}-\frac{4b^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*ln(tan(1/2*d*x+1/2*c))+3/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3-4/d/a^2/(tan(1/2*d*x+1/2*c)^2*
a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3+2/d*a/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c
)^2+1/d*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a/(a^2-b^2)
*tan(1/2*d*x+1/2*c)^2-6/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2*b^4+5/d*b/(tan(1/2*d*x+1/2*c)^2*a+2*t
an(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)-8/d/a^2/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)+2/d
/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a/(a^2-b^2)-3/d/a/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^2/(a^2-b^2)-3/d/a*b/(a^2-
b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/a^3
*b^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2
))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.44, size = 1610, normalized size = 10.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + b*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) + ((2*a^2 - 3*b^2)/(a*(a^2 - b^2)) + (tan(c
/2 + (d*x)/2)^2*(2*a^4 - 6*b^4 + a^2*b^2))/(a^3*(a^2 - b^2)) + (tan(c/2 + (
d*x)/2)*(5*a^2*b - 8*b^3))/(a^2*(a^2 - b^2)) + (b*tan(c/2 + (d*x)/2)^3*(3*a
^2 - 4*b^2))/(a^2*(a^2 - b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(2*a^2 + 4*b^2) +
a^2*tan(c/2 + (d*x)/2)^4 + a^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2
 + (d*x)/2))) + (b*atan(((b*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((
5*a^5*b - 4*a^3*b^3)/(a^6 - a^4*b^2) + (tan(c/2 + (d*x)/2)*(8*a*b^6 - 2*a^7
 - 20*a^3*b^4 + 14*a^5*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (b*((2*a^8*b - 2
*a^6*b^3)/(a^6 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(6*a^10 - 8*a^4*b^6 + 22*a^
6*b^4 - 20*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(3*a^2 - 2*b^2)*(-(a + b)
^3*(a - b)^3)^(1/2))/(2*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*1i)/(2*(a
^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)) + (b*(3*a^2 - 2*b^2)*(-(a + b)^3*(a
- b)^3)^(1/2)*((5*a^5*b - 4*a^3*b^3)/(a^6 - a^4*b^2) + (tan(c/2 + (d*x)/2)*
(8*a*b^6 - 2*a^7 - 20*a^3*b^4 + 14*a^5*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) +
(b*((2*a^8*b - 2*a^6*b^3)/(a^6 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(6*a^10 - 8
*a^4*b^6 + 22*a^6*b^4 - 20*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(3*a^2 -
2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(2*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*
b^2)))*1i)/(2*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))/((2*(3*a^2*b - 2*b^
3))/(a^6 - a^4*b^2) + (2*tan(c/2 + (d*x)/2)*(2*b^4 - 3*a^2*b^2))/(a^7 + a^3
*b^4 - 2*a^5*b^2) - (b*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((5*a^5
*b - 4*a^3*b^3)/(a^6 - a^4*b^2) + (tan(c/2 + (d*x)/2)*(8*a*b^6 - 2*a^7 - 20
*a^3*b^4 + 14*a^5*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (b*((2*a^8*b - 2*a^6*
b^3)/(a^6 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(6*a^10 - 8*a^4*b^6 + 22*a^6*b^4
 - 20*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(3*a^2 - 2*b^2)*(-(a + b)^3*(a
 - b)^3)^(1/2))/(2*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))))/(2*(a^9 - a^3
*b^6 + 3*a^5*b^4 - 3*a^7*b^2)) + (b*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^
(1/2)*((5*a^5*b - 4*a^3*b^3)/(a^6 - a^4*b^2) + (tan(c/2 + (d*x)/2)*(8*a*b^6
 - 2*a^7 - 20*a^3*b^4 + 14*a^5*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b*((2*a
^8*b - 2*a^6*b^3)/(a^6 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(6*a^10 - 8*a^4*b^6
 + 22*a^6*b^4 - 20*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(3*a^2 - 2*b^2)*(
-(a + b)^3*(a - b)^3)^(1/2))/(2*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))))/
(2*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))))*(3*a^2 - 2*b^2)*(-(a + b)^3*(
a - b)^3)^(1/2)*1i)/(d*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

3.1089 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=202

$$\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{(2a^2-3b^2)\cot(c+dx)}{2a^2d(a^2-b^2)(a+b\sin(c+dx))}-\frac{(2a^4-9a^2b^2+6b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d(a^2-b^2)^{3/2}}-\frac{(5a^2-}{2a}$$

```
[Out] -(2*a^4-9*a^2*b^2+6*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a
^4/(a^2-b^2)^(3/2)/d+3*b*arctanh(cos(d*x+c))/a^4/d-1/2*(5*a^2-6*b^2)*cot(d*
x+c)/a^3/(a^2-b^2)/d+1/2*cot(d*x+c)/a/d/(a+b*sin(d*x+c))^2+1/2*(2*a^2-3*b^2
)*cot(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.79, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2723, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{(-9a^2b^2+2a^4+6b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d(a^2-b^2)^{3/2}}-\frac{(5a^2-6b^2)\cot(c+dx)}{2a^3d(a^2-b^2)}+\frac{(2a^2-3b^2)\cot(c+dx)}{2a^2d(a^2-b^2)(a+b\sin(c+dx))}+\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^4*(a^2 - b^2)^(3/2)*d)) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) - ((
5*a^2 - 6*b^2)*Cot[c + d*x])/(2*a^3*(a^2 - b^2)*d) + Cot[c + d*x]/(2*a*d*(a
 + b*Sin[c + d*x])^2) + ((2*a^2 - 3*b^2)*Cot[c + d*x])/(2*a^2*(a^2 - b^2)*d
*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
```

```
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^2(c+dx)\left(3\left(a^2-b^2\right)-2\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)\left(5a^4-11a^2b^2+\right.}{}}{}$$

$$= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+d}$$

$$= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+d}$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{}{2a^2(}$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{}{2a^2(}$$

$$= -\frac{\left(2a^4-9a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right.}{2a^3\left(a\right.}$$

Mathematica [A] time = 5.78, size = 195, normalized size = 0.97

$$\frac{\frac{ab\left(4b^2-3a^2\right)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))} - \frac{a^2b\cos(c+dx)}{(a+b\sin(c+dx))^2} - \frac{2\left(2a^4-9a^2b^2+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + a\tan\left(\frac{1}{2}(c+dx)\right) - a\cot\left(\frac{1}{2}(c+dx)\right) -}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^2 - b^2)^(3/2) - a*Cot[(c + d*x)/2] + 6*b*Log[Cos[(c + d*x)/2]]
```

```
- 6*b*Log[Sin[(c + d*x)/2]] - (a^2*b*Cos[c + d*x])/(a + b*Sin[c + d*x])^2 +
 (a*b*(-3*a^2 + 4*b^2)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x]))
 + a*Tan[(c + d*x)/2])/(2*a^4*d)
```

fricas [B] time = 1.39, size = 1394, normalized size = 6.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(2*(5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - 2*(8*a^6*b - 1
7*a^4*b^3 + 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3 +
12*a*b^5 - 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7*a^
4*b^2 - 3*a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^2)
*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*s
in(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*
sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) -
2*(2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 6*(2*a^5*b^2 - 4*a
^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b
- a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin
(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 6*(2*a^5*b^2 - 4*a^3*b^4 + 2*a*b^6
 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3 - a^2*
b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-
1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)^2
- 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4 + a^4*b^6)*d*co
s(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(d*x + c)), -1/2*
((5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - (8*a^6*b - 17*a^4*b^3
+ 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3 + 12*a*b^5 -
 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7*a^4*b^2 - 3*
a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^2)*sin(d*x +
 c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x
+ c))) - (2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 3*(2*a^5*b^
2 - 4*a^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 +
(a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)
^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(2*a^5*b^2 - 4*a^3*b^4 +
2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3
 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c)
)*log(-1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x
+ c)^2 - 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4 + a^4*b^
6)*d*cos(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(d*x + c))
]
```

giac [A] time = 0.27, size = 339, normalized size = 1.68

$$-\frac{\frac{2\left(2\,a^{4}-9\,a^{2}b^{2}+6\,b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{6}-a^{4}b^{2}\right)\sqrt{a^{2}-b^{2}}}+\frac{2\left(5\,a^{3}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6\,ab^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,a^{2}\cdots}{\left(a^{6}-a^{4}b^{2}\right)\left(a\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*
sqrt(a^2 - b^2)) + 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^4*tan(1/2*d*
x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b^3*tan(1/2*d*x + 1/2
*c)^2 - 10*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 1
4*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b - 5*a^2*b^3)/((a^6 - a^4*b^2)*(a*tan
(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 6*b*log(abs(tan(1/
2*d*x + 1/2*c)))/a^4 - tan(1/2*d*x + 1/2*c)/a^3 - (6*b*tan(1/2*d*x + 1/2*c)
 - a)/(a^4*tan(1/2*d*x + 1/2*c)))/d
```

maple [B] time = 0.74, size = 729, normalized size = 3.61

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{3}}-\frac{1}{2d\,a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{4}}-\frac{5b^{2}\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)^{2}\left(a^{2}-b^{2}\right)}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^4*b*ln(tan(
1/2*d*x+1/2*c))-5/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b
^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3+6/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)^2*b^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3-4/d*b/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-3/d/a^2
/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)*tan(1/2*
d*x+1/2*c)^2+10/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b
^5/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-11/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)+14/d/a^3/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)-4/d/(
tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b/(a^2-b^2)+5/d/a^2/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)-2/d/(a^2-b^2)
```

```
^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+9/d/a^2/(a^
2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-6
/d/a^4/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1
/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 10.63, size = 1762, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + b*sin(c + d*x))^3),x)

[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - (a^2 - (2*tan(c/2 + (d*x)/2)*(7*a*b^3 - 6*a^
3*b))/(a^2 - b^2) + (tan(c/2 + (d*x)/2)^4*(a^4 - 12*b^4 + 9*a^2*b^2))/(a^2
- b^2) + (2*tan(c/2 + (d*x)/2)^2*(a^4 - 16*b^4 + 12*a^2*b^2))/(a^2 - b^2) +
 (2*tan(c/2 + (d*x)/2)^3*(6*a^4*b - 10*b^5 + a^2*b^3))/(a*(a^2 - b^2)))/(d*
(2*a^5*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^3*(4*a^5 + 8*a^3*b^2) + 2*
a^5*tan(c/2 + (d*x)/2) + 8*a^4*b*tan(c/2 + (d*x)/2)^2 + 8*a^4*b*tan(c/2 + (
d*x)/2)^4)) - (3*b*log(tan(c/2 + (d*x)/2)))/(a^4*d) - (atan((((-(a + b)^3*(
a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4 - 15*a^6
*b^2)/(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7 + 60*a^4
*b^5 - 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7*b^2) + (((2*a^10*b - 2*a^8*b^3)/
(a^8 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20
*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^4
+ 3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^
10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + ((-(a + b)^3*(a - b)^3)^(1/2)*(a^4
+ 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4 - 15*a^6*b^2)/(a^8 - a^6*b^2)
 + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7 + 60*a^4*b^5 - 46*a^6*b^3))/(
a^9 + a^5*b^4 - 2*a^7*b^2) - (((2*a^10*b - 2*a^8*b^3)/(a^8 - a^6*b^2) - (ta
n(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/(a^9 + a^
5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)
/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^10 - a^4*b^6 + 3*a^6*
```

```
b^4 - 3*a^8*b^2))/((2*(6*a^4*b + 18*b^5 - 27*a^2*b^3))/(a^8 - a^6*b^2) + (2
*tan(c/2 + (d*x)/2)*(4*a^6 - 18*b^6 + 39*a^2*b^4 - 24*a^4*b^2))/(a^9 + a^5*
b^4 - 2*a^7*b^2) - ((-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)
/2)*((2*a^8 + 12*a^4*b^4 - 15*a^6*b^2)/(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2
)*(10*a^8*b - 24*a^2*b^7 + 60*a^4*b^5 - 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7
*b^2) + (((2*a^10*b - 2*a^8*b^3)/(a^8 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a
^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(
-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6
+ 3*a^6*b^4 - 3*a^8*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + ((-(a
 + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4
 - 15*a^6*b^2)/(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7
 + 60*a^4*b^5 - 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7*b^2) - (((2*a^10*b - 2*
a^8*b^3)/(a^8 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8
*b^4 - 20*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1
/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)
))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*
(a^4 + 3*b^4 - (9*a^2*b^2)/2)*2i)/(d*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

3.1090 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=269

$$\frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{2a^2d(a^2-b^2)(a+b\sin(c+dx))}+\frac{(a^2-12b^2)\tanh^{-1}(\cos(c+dx))}{2a^5d}+\frac{b(11a^2-12b^2)\cot(c+dx)}{2a^4d(a^2-b^2)}-\frac{(5a^2-6b^2)}{2}$$

```
[Out] b*(6*a^4-19*a^2*b^2+12*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2)
)/a^5/(a^2-b^2)^(3/2)/d+1/2*(a^2-12*b^2)*arctanh(cos(d*x+c))/a^5/d+1/2*b*(1
1*a^2-12*b^2)*cot(d*x+c)/a^4/(a^2-b^2)/d-1/2*(5*a^2-6*b^2)*cot(d*x+c)*csc(d
*x+c)/a^3/(a^2-b^2)/d+1/2*cot(d*x+c)*csc(d*x+c)/a/d/(a+b*sin(d*x+c))^2+1/2*
(3*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 1.15, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{b(-19a^2b^2+6a^4+12b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d(a^2-b^2)^{3/2}}+\frac{b(11a^2-12b^2)\cot(c+dx)}{2a^4d(a^2-b^2)}+\frac{(a^2-12b^2)\tanh^{-1}(\cos(c+dx))}{2a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^5*(a^2 - b^2)^(3/2)*d) + ((a^2 - 12*b^2)*ArcTanh[Cos[c + d*x]])/
(2*a^5*d) + (b*(11*a^2 - 12*b^2)*Cot[c + d*x])/(2*a^4*(a^2 - b^2)*d) - ((5*
a^2 - 6*b^2)*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*(a^2 - b^2)*d) + (Cot[c + d*
x]*Csc[c + d*x])/(2*a*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 - 4*b^2)*Cot[c +
d*x]*Csc[c + d*x])/(2*a^2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
```

```
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\csc^3(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^3(c+dx)\left(4\left(a^2-b^2\right)-3\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\csc^3(c+dx)\left(2\left(5a\right.\right.}{}}{}$$

$$= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)\csc(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(3a^2-4b^2\right)\mathrm{c}}{2a^2\left(a^2-b^2\right)}$$

$$= \frac{b\left(11a^2-12b^2\right)\cot(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)\csc(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\,\mathrm{si}}$$

$$= \frac{b\left(11a^2-12b^2\right)\cot(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)\csc(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\,\mathrm{si}}$$

$$= \frac{\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d} + \frac{b\left(11a^2-12b^2\right)\cot(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\mathrm{co}}{2a^3}$$

$$= \frac{\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d} + \frac{b\left(11a^2-12b^2\right)\cot(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\mathrm{co}}{2a^3}$$

$$= \frac{b\left(6a^4-19a^2b^2+12b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{3/2}d} + \frac{\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx}{2a^5d}$$

Mathematica [A] time = 6.33, size = 330, normalized size = 1.23

$$-\frac{3b\tan\left(\frac{1}{2}(c+dx)\right)}{2a^4d} + \frac{3b\cot\left(\frac{1}{2}(c+dx)\right)}{2a^4d} + \frac{b^2\cos(c+dx)}{2a^3d(a+b\sin(c+dx))^2} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d} + \frac{\left(12b^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/
2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*(a^2 - b^2)^(3/2)*d) + (3*
b*Cot[(c + d*x)/2])/(2*a^4*d) - Csc[(c + d*x)/2]^2/(8*a^3*d) + ((a^2 - 12*b
^2)*Log[Cos[(c + d*x)/2]])/(2*a^5*d) + ((-a^2 + 12*b^2)*Log[Sin[(c + d*x)/2
]])/(2*a^5*d) + Sec[(c + d*x)/2]^2/(8*a^3*d) + (b^2*Cos[c + d*x])/(2*a^3*d*
(a + b*Sin[c + d*x])^2) + (5*a^2*b^2*Cos[c + d*x] - 6*b^4*Cos[c + d*x])/(2*
a^4*(a - b)*(a + b)*d*(a + b*Sin[c + d*x])) - (3*b*Tan[(c + d*x)/2])/(2*a^4
*d)
```

fricas [B] time = 1.62, size = 1922, normalized size = 7.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(2*(17*a^6*b^2 - 35*a^4*b^4 + 18*a^2*b^6)*cos(d*x + c)^3 - (6*a^6*b -
 13*a^4*b^3 - 7*a^2*b^5 + 12*b^7 + (6*a^4*b^3 - 19*a^2*b^5 + 12*b^7)*cos(d*
x + c)^4 - (6*a^6*b - 7*a^4*b^3 - 26*a^2*b^5 + 24*b^7)*cos(d*x + c)^2 + 2*(
6*a^5*b^2 - 19*a^3*b^4 + 12*a*b^6 - (6*a^5*b^2 - 19*a^3*b^4 + 12*a*b^6)*cos
(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 2*(a^8 - 19*a^6*b^2 + 36*a^4*b^4 - 18*a^2*b^6)*cos(d*x + c) -
(a^8 - 13*a^6*b^2 + 11*a^4*b^4 + 13*a^2*b^6 - 12*b^8 + (a^6*b^2 - 14*a^4*b^
4 + 25*a^2*b^6 - 12*b^8)*cos(d*x + c)^4 - (a^8 - 12*a^6*b^2 - 3*a^4*b^4 + 3
8*a^2*b^6 - 24*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 1
2*a*b^7 - (a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7)*cos(d*x + c)^2)*sin(
d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^8 - 13*a^6*b^2 + 11*a^4*b^4 + 13
*a^2*b^6 - 12*b^8 + (a^6*b^2 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*cos(d*x +
c)^4 - (a^8 - 12*a^6*b^2 - 3*a^4*b^4 + 38*a^2*b^6 - 24*b^8)*cos(d*x + c)^2
+ 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7 - (a^7*b - 14*a^5*b^3 + 25*
a^3*b^5 - 12*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1
/2) + 2*((11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7)*cos(d*x + c)^3 - (4*a^7*b + 3
*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 2
*a^7*b^4 + a^5*b^6)*d*cos(d*x + c)^4 - (a^11 - 3*a^7*b^4 + 2*a^5*b^6)*d*cos
(d*x + c)^2 + (a^11 - a^9*b^2 - a^7*b^4 + a^5*b^6)*d - 2*((a^10*b - 2*a^8*b
^3 + a^6*b^5)*d*cos(d*x + c)^2 - (a^10*b - 2*a^8*b^3 + a^6*b^5)*d)*sin(d*x
+ c)), -1/4*(2*(17*a^6*b^2 - 35*a^4*b^4 + 18*a^2*b^6)*cos(d*x + c)^3 + 2*(6
*a^6*b - 13*a^4*b^3 - 7*a^2*b^5 + 12*b^7 + (6*a^4*b^3 - 19*a^2*b^5 + 12*b^7
)*cos(d*x + c)^4 - (6*a^6*b - 7*a^4*b^3 - 26*a^2*b^5 + 24*b^7)*cos(d*x + c)
^2 + 2*(6*a^5*b^2 - 19*a^3*b^4 + 12*a*b^6 - (6*a^5*b^2 - 19*a^3*b^4 + 12*a*
```

b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c)
+ b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 2*(a^8 - 19*a^6*b^2 + 36*a^4*b^4 - 1
8*a^2*b^6)*cos(d*x + c) - (a^8 - 13*a^6*b^2 + 11*a^4*b^4 + 13*a^2*b^6 - 12*
b^8 + (a^6*b^2 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*cos(d*x + c)^4 - (a^8 -
12*a^6*b^2 - 3*a^4*b^4 + 38*a^2*b^6 - 24*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 1
4*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7 - (a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a
*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^8 - 13
*a^6*b^2 + 11*a^4*b^4 + 13*a^2*b^6 - 12*b^8 + (a^6*b^2 - 14*a^4*b^4 + 25*a^
2*b^6 - 12*b^8)*cos(d*x + c)^4 - (a^8 - 12*a^6*b^2 - 3*a^4*b^4 + 38*a^2*b^6
 - 24*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7 -
 (a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))
*log(-1/2*cos(d*x + c) + 1/2) + 2*((11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7)*cos
(d*x + c)^3 - (4*a^7*b + 3*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7)*cos(d*x + c))*s
in(d*x + c))/((a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d*cos(d*x + c)^4 - (a^11 - 3*
a^7*b^4 + 2*a^5*b^6)*d*cos(d*x + c)^2 + (a^11 - a^9*b^2 - a^7*b^4 + a^5*b^6
)*d - 2*((a^10*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d*x + c)^2 - (a^10*b - 2*a^8*
b^3 + a^6*b^5)*d)*sin(d*x + c))]

giac [B] time = 0.28, size = 526, normalized size = 1.96

$$\frac{\frac{8\left(6a^4b-19a^2b^3+12b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^7-a^5b^2\right)\sqrt{a^2-b^2}}+\frac{2a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-26a^4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+24a^2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+20}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/8*(8*(6*a^4*b - 19*a^2*b^3 + 12*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^7 - a^5*b^
2)*sqrt(a^2 - b^2)) + (2*a^6*tan(1/2*d*x + 1/2*c)^6 - 26*a^4*b^2*tan(1/2*d*
x + 1/2*c)^6 + 24*a^2*b^4*tan(1/2*d*x + 1/2*c)^6 + 20*a^5*b*tan(1/2*d*x + 1
/2*c)^5 - 60*a^3*b^3*tan(1/2*d*x + 1/2*c)^5 + 32*a*b^5*tan(1/2*d*x + 1/2*c)
^5 + 3*a^6*tan(1/2*d*x + 1/2*c)^4 + 53*a^4*b^2*tan(1/2*d*x + 1/2*c)^4 - 64*
a^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 16*b^6*tan(1/2*d*x + 1/2*c)^4 + 28*a^5*b*t
an(1/2*d*x + 1/2*c)^3 + 60*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 112*a*b^5*tan(1
/2*d*x + 1/2*c)^3 + 68*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 76*a^2*b^4*tan(1/2*
d*x + 1/2*c)^2 + 8*a^5*b*tan(1/2*d*x + 1/2*c) - 8*a^3*b^3*tan(1/2*d*x + 1/2
*c) - a^6 + a^4*b^2)/((a^7 - a^5*b^2)*(a*tan(1/2*d*x + 1/2*c)^3 + 2*b*tan(1
/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))^2) - 4*(a^2 - 12*b^2)*log(abs(t
an(1/2*d*x + 1/2*c)))/a^5 + (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*
d*x + 1/2*c))/a^6)/d

maple [B] time = 0.93, size = 803, normalized size = 2.99

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{2d\,a^4}-\frac{1}{8d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^3}+\frac{6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{d\,a^5}+\frac{3b}{2d\,a^4\tan\left(\frac{dx}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^4*tan(1/2*d*x+1/2*c)*b-1/8/d/a^3/tan
(1/2*d*x+1/2*c)^2-1/2/d/a^3*ln(tan(1/2*d*x+1/2*c))+6/d/a^5*ln(tan(1/2*d*x+1
/2*c))*b^2+3/2/d*b/a^4/tan(1/2*d*x+1/2*c)+7/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2
*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3-8/d*b^5/a^4/(
tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/
2*c)^3+6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a/(a^2-b
^2)*tan(1/2*d*x+1/2*c)^2+5/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*
c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2*b^4-14/d*b^6/a^5/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2+17/d/a^2/
(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)*tan(1/2*d
*x+1/2*c)-20/d*b^5/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/
(a^2-b^2)*tan(1/2*d*x+1/2*c)+6/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*b^2/(a^2-b^2)-7/d*b^4/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x
+1/2*c)*b+a)^2/(a^2-b^2)+6/d/a*b/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*
x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-19/d/a^3*b^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a
*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+12/d*b^5/a^5/(a^2-b^2)^(3/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 11.01, size = 1906, normalized size = 7.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + b*sin(c + d*x))^3),x)

[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) - (a^3/2 - (2*tan(c/2 + (d*x)/2)^5*(3*a^4*b - 16*b^5 + 11*a^2*b^3))/(a^2 - b^2) - (2*tan(c/2 + (d*x)/2)^3*(5*a^4*b - 52*b^5 + 41*a^2*b^3))/(a^2 - b^2) - 4*a^2*b*tan(c/2 + (d*x)/2) + (tan(c/2 + (d*x)/2)^2*(50*a*b^4 + a^5 - 47*a^3*b^2))/(a^2 - b^2) + (tan(c/2 + (d*x)/2)^4*(a^6 + 112*b^6 + 8*a^2*b^4 - 97*a^4*b^2))/(2*a*(a^2 - b^2)))/(d*(4*a^6*tan(c/2 + (d*x)/2)^2 + 4*a^6*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^4*(8*a^6 + 16*a^4*b^2) + 16*a^5*b*tan(c/2 + (d*x)/2)^3 + 16*a^5*b*tan(c/2 + (d*x)/2)^5)) - (3*b*tan(c/2 + (d*x)/2))/(2*a^4*d) - (log(tan(c/2 + (d*x)/2))*(a^2 - 12*b^2))/(2*a^5*d) - (b*atan(((b*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*((7*a^9*b + 24*a^5*b^5 - 32*a^7*b^3)/(a^10 - a^8*b^2) - (tan(c/2 + (d*x)/2)*(a^11 + 48*a^3*b^8 - 124*a^5*b^6 + 103*a^7*b^4 - 28*a^9*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2) + (b*((2*a^12*b - 2*a^10*b^3)/(a^10 - a^8*b^2) - (tan(c/2 + (d*x)/2)*(6*a^14 - 8*a^8*b^6 + 22*a^10*b^4 - 20*a^12*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*1i)/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)) - (b*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*((tan(c/2 + (d*x)/2)*(a^11 + 48*a^3*b^8 - 124*a^5*b^6 + 103*a^7*b^4 - 28*a^9*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2) - (7*a^9*b + 24*a^5*b^5 - 32*a^7*b^3)/(a^10 - a^8*b^2) + (b*((2*a^12*b - 2*a^10*b^3)/(a^10 - a^8*b^2) - (tan(c/2 + (d*x)/2)*(6*a^14 - 8*a^8*b^6 + 22*a^10*b^4 - 20*a^12*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*1i)/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))/((6*a^6*b - 144*b^7 + 240*a^2*b^5 - 91*a^4*b^3)/(a^10 - a^8*b^2) + (2*tan(c/2 + (d*x)/2)*(72*b^8 - 174*a^2*b^6 + 131*a^4*b^4 - 30*a^6*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2) + (b*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*((7*a^9*b + 24*a^5*b^5 - 32*a^7*b^3)/(a^10 - a^8*b^2) - (tan(c/2 + (d*x)/2)*(a^11 + 48*a^3*b^8 - 124*a^5*b^6 + 103*a^7*b^4 - 28*a^9*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2) + (b*((2*a^12*b - 2*a^10*b^3)/(a^10 - a^8*b^2) - (tan(c/2 + (d*x)/2)*(6*a^14 - 8*a^8*b^6 + 22*a^10*b^4 - 20*a^12*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)) + (b*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*((tan(c/2 + (d*x)/2)*(a^11 + 48*a^3*b^8 - 124*a^5*b^6 + 103*a^7*b^4 - 28*a^9*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2) - (7*a^9*b + 24*a^5*b^5 - 32*a^7*b^3)/(a^10 - a^8*b^2) + (b*((2*a^12*b - 2*a^10*b^3)/(a^10 - a^8*b^2) - (tan(c/2 + (d*x)/2)*(6*a^14 - 8*a^8*b^6 + 22*a^10*b^4 - 20*a^12*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))))*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*1i)/(d*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(a + b*sin(c + d*x))**3, x)
```

3.1091 $\int \frac{\cos^2(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=347

$$-\frac{4b\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{d\sin(e+fx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3\sqrt{d}\,f\sqrt{a+b}}+\frac{4b\cos(e+fx}{3af\left(a^2-b^2\right)\sqrt{d\sin(e+fx)}\,\surd}$$

```
[Out] 2/3*cos(f*x+e)*(d*sin(f*x+e))^(1/2)/a/d/f/(a+b*sin(f*x+e))^(3/2)+4/3*b*cos(
f*x+e)/a/(a^2-b^2)/f/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2)-4/3*b*Elli
pticE(d^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2),((-a-
b)/(a-b))^(1/2))*(a*(1-csc(f*x+e))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1
/2)*tan(f*x+e)/a^3/f/(a+b)^(1/2)/d^(1/2)-4/3*EllipticF(d^(1/2)*(a+b*sin(f*x
+e))^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-csc
(f*x+e))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/a^2/f/(a+b)
^(1/2)/d^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2887, 2800, 2998, 2816, 2994}

$$\frac{4b\cos(e+fx)}{3af\left(a^2-b^2\right)\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}-\frac{4\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\sin}}{\sqrt{a+b}\sqrt{d\sin}}\right.\right.}{3a^2\sqrt{d}\,f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (2*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(3*a*d*f*(a + b*Sin[e + f*x])^(3/2))
+ (4*b*Cos[e + f*x])/(3*a*(a^2 - b^2)*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin
[e + f*x]]) - (4*b*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e
+ f*x]))/(a - b)]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt
[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(3*a^3*Sq
rt[a + b]*Sqrt[d]*f) - (4*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 +
 Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]]
)/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(3
*a^2*Sqrt[a + b]*Sqrt[d]*f)
```

Rule 2800

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(3/2)), x_Symbol] :> Simp[(2*b*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
 b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(b + a
```

```
*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2887

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> -Simp[(g*(g*C
os[e + f*x])^(p - 1)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(m + 1))/(a*
d*f*(m + 1)), x] + Dist[(g^2*(2*m + 3))/(2*a*(m + 1)), Int[((g*Cos[e + f*x]
)^(p - 2)*(a + b*Sin[e + f*x])^(m + 1))/Sqrt[d*Sin[e + f*x]], x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && EqQ[m + p +
 1/2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{5/2}}\,dx = \frac{2\cos(e+fx)\sqrt{d\sin(e+fx)}}{3adf(a+b\sin(e+fx))^{3/2}} + \frac{2\int \frac{1}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{3/2}}\,dx}{3a}$$

$$= \frac{2\cos(e+fx)\sqrt{d\sin(e+fx)}}{3adf(a+b\sin(e+fx))^{3/2}} + \frac{4b\cos(e+fx)}{3a\left(a^2-b^2\right)f\sqrt{d\sin(e+fx)}\,\sqrt{a+\ldots}}$$

$$= \frac{2\cos(e+fx)\sqrt{d\sin(e+fx)}}{3adf(a+b\sin(e+fx))^{3/2}} + \frac{4b\cos(e+fx)}{3a\left(a^2-b^2\right)f\sqrt{d\sin(e+fx)}\,\sqrt{a+\ldots}}$$

$$= \frac{2\cos(e+fx)\sqrt{d\sin(e+fx)}}{3adf(a+b\sin(e+fx))^{3/2}} + \frac{4b\cos(e+fx)}{3a\left(a^2-b^2\right)f\sqrt{d\sin(e+fx)}\,\sqrt{a+\ldots}}$$

Mathematica [B] time = 23.01, size = 3348, normalized size = 9.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(5/2)),
x]
```

```
[Out] (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((2*Cos[e + f*x])/(3*a*(a + b*Sin[e
+ f*x])^2) - (4*b^2*Cos[e + f*x])/(3*a^2*(a^2 - b^2)*(a + b*Sin[e + f*x])))
)/(f*Sqrt[d*Sin[e + f*x]]) + (4*Sqrt[a + b*Sin[e + f*x]]*((2*Sqrt[a + b*Sin
[e + f*x]])/(3*a*(a^2 - b^2)*Sqrt[Sin[e + f*x]]) - (4*b*Sqrt[Sin[e + f*x]]*
Sqrt[a + b*Sin[e + f*x]])/(3*a^2*(a^2 - b^2)))*(-2*b*Sin[(e + f*x)/2]^2 - (
2*a*(-(b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])
/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*
Tan[(e + f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(
e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^
2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan
[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]))/(Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e
+ f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*Sqrt[(a*Tan[(e + f*x)/2])/(-
b + Sqrt[-a^2 + b^2])])))/(3*a^2*(a^2 - b^2)*f*Sqrt[d*Sin[e + f*x]]*((2*b*C
os[e + f*x]*(-2*b*Sin[(e + f*x)/2]^2 - (2*a*(-(b*EllipticE[ArcSin[Sqrt[(-b
+ Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqr
t[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*EllipticF[Arc
Sin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt
[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2]
)/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2
```

]))]))/(Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(
a^2 - b^2)]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/(3*a^2*(a
^2 - b^2)*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (2*Cos[e + f*x]*Sq
rt[a + b*Sin[e + f*x]]*(-2*b*Sin[(e + f*x)/2]^2 - (2*a*(-(b*EllipticE[ArcSi
n[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[
2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*El
lipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 +
 b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(
e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt
[-a^2 + b^2]))]))/(Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e
 + f*x]))/(a^2 - b^2)]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]))
)/(3*a^2*(a^2 - b^2)*Sin[e + f*x]^(3/2)) + (4*Sqrt[a + b*Sin[e + f*x]]*(-2*
b*Cos[(e + f*x)/2]*Sin[(e + f*x)/2] + (a^2*Sec[(e + f*x)/2]^2*(-(b*Elliptic
E[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]
]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2])
 + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt
[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(
a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b
 + Sqrt[-a^2 + b^2]))]))/(2*Sqrt[-a^2 + b^2]*(-b + Sqrt[-a^2 + b^2])*Sqrt[(
a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*((a*Tan[(e + f*x)/2
])/(-b + Sqrt[-a^2 + b^2]))^(3/2)) + (a*((a*b*Cos[e + f*x]*Sec[(e + f*x)/2]
^2)/(a^2 - b^2) + (a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x])*Tan[(e + f*x)/
2])/(a^2 - b^2))*(-(b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[
(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-
a^2 + b^2])]*Tan[(e + f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b
^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/
(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*
Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]))/(Sqrt[-a^2 + b^2]*((
a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2))^(3/2)*Sqrt[(a*Tan[(
e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]) - (2*a*(-1/2*(b*EllipticE[ArcSin[Sqr
t[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]],
(2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Sec[(e + f*x)/2]^2) - (a^2*El
lipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 +
 b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sec[(e + f*x)
/2]^2*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])/(4*(b + Sqrt[-a^2
 + b^2])*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]) + (a^2*Ellip
ticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^
2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sec[(e + f*x)/2]
^2*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))])/(4*(-b + Sqrt[-a^2
 + b^2])*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]) + (a*b*Sec[(e
+ f*x)/2]^2*Tan[(e + f*x)/2]*Sqrt[1 - (-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f
*x)/2])/(-b + Sqrt[-a^2 + b^2])])/(4*Sqrt[2]*Sqrt[-a^2 + b^2]*Sqrt[(-b + Sq
rt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]*Sqrt[1 - (-b + Sqrt[
-a^2 + b^2] - a*Tan[(e + f*x)/2])/(2*Sqrt[-a^2 + b^2])]) + (a^2*Sec[(e + f*
x)/2]^2*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e

```
 + f*x)/2])/(b + Sqrt[-a^2 + b^2]))])/(4*Sqrt[2]*Sqrt[-a^2 + b^2]*Sqrt[(b +
 Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]*Sqrt[1 - (b + Sqr
t[-a^2 + b^2] + a*Tan[(e + f*x)/2])/(2*Sqrt[-a^2 + b^2])]*Sqrt[1 - (b + Sqr
t[-a^2 + b^2] + a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2])]))/(Sqrt[-a^2 +
 b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*Sqrt[(a
*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/(3*a^2*(a^2 - b^2)*Sqrt[Sin[
e + f*x]])))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)}\,\cos\left(fx+e\right)^{2}}{b^{3}d\cos\left(fx+e\right)^{4}-\left(3\,a^{2}b+2\,b^{3}\right)d\cos\left(fx+e\right)^{2}+\left(3\,a^{2}b+b^{3}\right)d-\left(3\,ab^{2}d\cos\left(fx+e\right)^{2}-\left(a^{3}+3\,ab^{2}\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))*cos(f*x + e)^2/(b^3*
d*cos(f*x + e)^4 - (3*a^2*b + 2*b^3)*d*cos(f*x + e)^2 + (3*a^2*b + b^3)*d -
 (3*a*b^2*d*cos(f*x + e)^2 - (a^3 + 3*a*b^2)*d)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e))),
 x)
```

maple [B] time = 0.66, size = 4675, normalized size = 13.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*(-4*cos(f*x+e)*sin(f*x+e)*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+
e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2
```

)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2
)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(-(
-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2)
)/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/
2))*(-a^2+b^2)^(1/2)*b^3+4*cos(f*x+e)*sin(f*x+e)*(-(-(-a^2+b^2)^(1/2)*sin(f
*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(
((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/
sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)
*EllipticE((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+
(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2
+b^2)^(1/2))^(1/2))*a^2*b^2-4*cos(f*x+e)*sin(f*x+e)*(-(-(-a^2+b^2)^(1/2)*si
n(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2
)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/
2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1
/2)*EllipticE((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/
(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-
a^2+b^2)^(1/2))^(1/2))*b^4+2*cos(f*x+e)*sin(f*x+e)*(-(-(-a^2+b^2)^(1/2)*sin
(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)
*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2
)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/
2)*EllipticF((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(
b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a
^2+b^2)^(1/2))^(1/2))*(-a^2+b^2)^(1/2)*a^2*b-4*cos(f*x+e)*(-(-(-a^2+b^2)^(1
/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e)
)^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^
2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+
e))^(1/2)*EllipticE((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)
*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/
2))/(-a^2+b^2)^(1/2))^(1/2))*(-a^2+b^2)^(1/2)*a*b^2+4*cos(f*x+e)*(-(-(-a^2+
b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin
(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(
-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/s
in(f*x+e))^(1/2)*EllipticE((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos
(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b
^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^3*b-4*cos(f*x+e)*(-(-(-a^2+b^2)^(1/2)
*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(
1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^
(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))
^(1/2)*EllipticE((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-
a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))
/(-a^2+b^2)^(1/2))^(1/2))*a*b^3+2*cos(f*x+e)*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e
)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a
^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(
f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*Ell
ipticF((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^

2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2
)^(1/2))^(1/2))*(-a^2+b^2)^(1/2)*a^3-4*sin(f*x+e)*(-(-(-a^2+b^2)^(1/2)*sin(
f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*
(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)
/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2
)*EllipticE((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b
+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^
2+b^2)^(1/2))^(1/2))*(-a^2+b^2)^(1/2)*b^3+4*sin(f*x+e)*(-(-(-a^2+b^2)^(1/2)
*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(
1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^
(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))
^(1/2)*EllipticE((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-
a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))
/(-a^2+b^2)^(1/2))^(1/2))*a^2*b^2-4*sin(f*x+e)*(-(-(-a^2+b^2)^(1/2)*sin(f*x
+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((
-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/si
n(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*E
llipticE((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-
a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b
^2)^(1/2))^(1/2))*b^4+2*sin(f*x+e)*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*
x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1
/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1
/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((-(
-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/
2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(
1/2))*(-a^2+b^2)^(1/2)*a^2*b-4*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)
+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*
sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*
(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(-(-a
^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/
sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2)
)*(-a^2+b^2)^(1/2)*a*b^2+4*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos
(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(
f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(
-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(-(-a^2+b
^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(
f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^
3*b-4*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2
+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+c
os(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a
^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-
b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(
1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b^3+2*(-(-(-a^2+b^2)^
(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+
e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+

```
b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*
x+e))^(1/2)*EllipticF((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+
e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(
1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-a^2+b^2)^(1/2)*a^3+2*cos(f*x+e)^2*2^(1/2)*
a*b^3+cos(f*x+e)*sin(f*x+e)*2^(1/2)*a^4+cos(f*x+e)*sin(f*x+e)*2^(1/2)*a^2*b
^2+2*cos(f*x+e)*2^(1/2)*a^3*b-4*sin(f*x+e)*2^(1/2)*a^2*b^2-2*2^(1/2)*a^3*b-
2*2^(1/2)*a*b^3)*(a+b*sin(f*x+e))^(1/2)/(b^2*cos(f*x+e)^2-2*a*b*sin(f*x+e)-
a^2-b^2)/(d*sin(f*x+e))^(1/2)*2^(1/2)/(a^2-b^2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e))),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2}{\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(cos(e + f*x)^2/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sin(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.1092 $\int \cos^4(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=143

$$-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3a\sin(c+dx)\cos(c+dx)}{128d}$$

```
[Out] 3/128*a*x-1/5*b*cos(d*x+c)^5/d+2/7*b*cos(d*x+c)^7/d-1/9*b*cos(d*x+c)^9/d+3/
128*a*cos(d*x+c)*sin(d*x+c)/d+1/64*a*cos(d*x+c)^3*sin(d*x+c)/d-1/16*a*cos(d
*x+c)^5*sin(d*x+c)/d-1/8*a*cos(d*x+c)^5*sin(d*x+c)^3/d
```

Rubi [A] time = 0.19, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 270}

$$-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3a\sin(c+dx)\cos(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/128 - (b*Cos[c + d*x]^5)/(5*d) + (2*b*Cos[c + d*x]^7)/(7*d) - (b*Co
s[c + d*x]^9)/(9*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (a*Cos[c +
d*x]^3*Sin[c + d*x])/(64*d) - (a*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (a*C
os[c + d*x]^5*Sin[c + d*x]^3)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^4(c+dx)\,dx + b\int \cos^4(c+dx)\sin^5(c$$

$$= -\frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{8}(3a)\int \cos^4(c+dx)\sin^2($$

$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} +$$

$$= -\frac{b\cos^5(c+dx)}{5d} + \frac{2b\cos^7(c+dx)}{7d} - \frac{b\cos^9(c+dx)}{9d} + \frac{a\cos$$

$$= -\frac{b\cos^5(c+dx)}{5d} + \frac{2b\cos^7(c+dx)}{7d} - \frac{b\cos^9(c+dx)}{9d} + \frac{3a\cos$$

$$= \frac{3ax}{128} - \frac{b\cos^5(c+dx)}{5d} + \frac{2b\cos^7(c+dx)}{7d} - \frac{b\cos^9(c+dx)}{9d} +$$

Mathematica [A] time = 0.25, size = 92, normalized size = 0.64

$$\frac{-2520a\sin(4(c+dx)) + 315a\sin(8(c+dx)) + 7560ac + 7560adx - 7560b\cos(c+dx) - 1680b\cos(3(c+dx)) +}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (7560*a*c + 7560*a*d*x - 7560*b*Cos[c + d*x] - 1680*b*Cos[3*(c + d*x)] + 10
08*b*Cos[5*(c + d*x)] + 180*b*Cos[7*(c + d*x)] - 140*b*Cos[9*(c + d*x)] - 2
520*a*Sin[4*(c + d*x)] + 315*a*Sin[8*(c + d*x)])/(322560*d)
```

fricas [A] time = 0.69, size = 95, normalized size = 0.66

$$-\frac{4480\,b\cos\left(dx+c\right)^{9}-11520\,b\cos\left(dx+c\right)^{7}+8064\,b\cos\left(dx+c\right)^{5}-945\,adx-315\left(16\,a\cos\left(dx+c\right)^{7}-24\,a\cos\right.}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/40320*(4480*b*cos(d*x + c)^9 - 11520*b*cos(d*x + c)^7 + 8064*b*cos(d*x +
 c)^5 - 945*a*d*x - 315*(16*a*cos(d*x + c)^7 - 24*a*cos(d*x + c)^5 + 2*a*co
s(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.30, size = 107, normalized size = 0.75

$$\frac{3}{128}\,ax-\frac{b\cos\left(9\,dx+9\,c\right)}{2304\,d}+\frac{b\cos\left(7\,dx+7\,c\right)}{1792\,d}+\frac{b\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{b\cos\left(3\,dx+3\,c\right)}{192\,d}-\frac{3\,b\cos\left(dx+c\right)}{128\,d}+\frac{a\sin\left(8\,dx\right.}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/128*a*x - 1/2304*b*cos(9*d*x + 9*c)/d + 1/1792*b*cos(7*d*x + 7*c)/d + 1/3
20*b*cos(5*d*x + 5*c)/d - 1/192*b*cos(3*d*x + 3*c)/d - 3/128*b*cos(d*x + c)
/d + 1/1024*a*sin(8*d*x + 8*c)/d - 1/128*a*sin(4*d*x + 4*c)/d
```

maple [A] time = 0.30, size = 124, normalized size = 0.87

$$\frac{a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^{5}(dx+c)\right)}{16}+\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{64}+\frac{3dx}{128}+\frac{3c}{128}\right)+b\left(-\frac{\left(\sin^{4}(dx+c)\right)\left(\cos^{5}(dx+c\right.\right.}{9}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+b*(-1/9*sin(d*x+c
)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5))
```

maxima [A] time = 0.31, size = 71, normalized size = 0.50

$$\frac{315\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\,\sin\left(4\,dx+4\,c\right)\right)a-1024\left(35\,\cos\left(dx+c\right)^{9}-90\,\cos\left(dx+c\right)^{7}+63\,\cos\left(\right.\right.}{322560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/322560*(315*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a - 1
024*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*b)/d
```

mupad [B] time = 13.32, size = 223, normalized size = 1.56

$$\frac{3\,a\,x}{128}-\frac{-\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{64}-\frac{13\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{32}+\frac{155\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32}+\frac{32\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}-\frac{169\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{32}-16\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^4*(a + b*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/128 - ((16*b)/315 + (3*a*tan(c/2 + (d*x)/2))/64 + (13*a*tan(c/2 + (
d*x)/2)^3)/32 - (155*a*tan(c/2 + (d*x)/2)^5)/32 + (169*a*tan(c/2 + (d*x)/2)
^7)/32 - (169*a*tan(c/2 + (d*x)/2)^11)/32 + (155*a*tan(c/2 + (d*x)/2)^13)/3
2 - (13*a*tan(c/2 + (d*x)/2)^15)/32 - (3*a*tan(c/2 + (d*x)/2)^17)/64 + (16*
b*tan(c/2 + (d*x)/2)^2)/35 + (64*b*tan(c/2 + (d*x)/2)^4)/35 - (32*b*tan(c/2
 + (d*x)/2)^6)/5 + (112*b*tan(c/2 + (d*x)/2)^8)/5 - 16*b*tan(c/2 + (d*x)/2)
^10 + (32*b*tan(c/2 + (d*x)/2)^12)/3)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

sympy [A] time = 15.15, size = 272, normalized size = 1.90

$$\begin{cases}\frac{3ax\sin^{8}(c+dx)}{128}+\frac{3ax\sin^{6}(c+dx)\cos^{2}(c+dx)}{32}+\frac{9ax\sin^{4}(c+dx)\cos^{4}(c+dx)}{64}+\frac{3ax\sin^{2}(c+dx)\cos^{6}(c+dx)}{32}+\frac{3ax\cos^{8}(c+dx)}{128}+\frac{3a\sin^{7}(c+}{12} \\ x\left(a+b\sin(c)\right)\sin^{4}(c)\cos^{4}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**8/128 + 3*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 9*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*a*x*sin(c + d*x)**2*cos
(c + d*x)**6/32 + 3*a*x*cos(c + d*x)**8/128 + 3*a*sin(c + d*x)**7*cos(c + d
*x)/(128*d) + 11*a*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*a*sin(c + d
```

```
*x)**3*cos(c + d*x)**5/(128*d) - 3*a*sin(c + d*x)*cos(c + d*x)**7/(128*d) -
 b*sin(c + d*x)**4*cos(c + d*x)**5/(5*d) - 4*b*sin(c + d*x)**2*cos(c + d*x)
**7/(35*d) - 8*b*cos(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(
c)**4*cos(c)**4, True))
```

3.1093 $\int \cos^4(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=127

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{b\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{b\sin(c+dx)\cos^3(c+$$

```
[Out] 3/128*b*x-1/5*a*cos(d*x+c)^5/d+1/7*a*cos(d*x+c)^7/d+3/128*b*cos(d*x+c)*sin(
d*x+c)/d+1/64*b*cos(d*x+c)^3*sin(d*x+c)/d-1/16*b*cos(d*x+c)^5*sin(d*x+c)/d-
1/8*b*cos(d*x+c)^5*sin(d*x+c)^3/d
```

Rubi [A] time = 0.19, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2565, 14, 2568, 2635, 8}

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{b\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{b\sin(c+dx)\cos^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*x)/128 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^7)/(7*d) + (3*b*Co
s[c + d*x]*Sin[c + d*x])/(128*d) + (b*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) -
 (b*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (b*Cos[c + d*x]^5*Sin[c + d*x]^3)
/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^3(c+dx)\,dx + b\int \cos^4(c+dx)\sin^4(c+$$
$$= -\frac{b\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{8}(3b)\int \cos^4(c+dx)\sin^2(c$$
$$= -\frac{b\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{b\cos^5(c+dx)\sin^3(c+dx)}{8d} +$$
$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{b\cos^3(c+dx)\sin(c+d}{64d}$$
$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{3b\cos(c+dx)\sin(c+d}{128d}$$
$$= \frac{3bx}{128} - \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{3b\cos(c+dx)\sin(}{128d}$$

Mathematica [A] time = 0.19, size = 77, normalized size = 0.61

$$\frac{-1680a\cos(c+dx) - 560a\cos(3(c+dx)) + 112a\cos(5(c+dx)) + 80a\cos(7(c+dx)) - 280b\sin(4(c+dx)) + 35}{35840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (840*b*d*x - 1680*a*Cos[c + d*x] - 560*a*Cos[3*(c + d*x)] + 112*a*Cos[5*(c
+ d*x)] + 80*a*Cos[7*(c + d*x)] - 280*b*Sin[4*(c + d*x)] + 35*b*Sin[8*(c +
d*x)])/(35840*d)
```

fricas [A] time = 0.75, size = 84, normalized size = 0.66

$$\frac{640\,a\cos\left(dx+c\right)^{7}-896\,a\cos\left(dx+c\right)^{5}+105\,bdx+35\left(16\,b\cos\left(dx+c\right)^{7}-24\,b\cos\left(dx+c\right)^{5}+2\,b\cos\left(dx+c\right.\right.}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4480*(640*a*cos(d*x + c)^7 - 896*a*cos(d*x + c)^5 + 105*b*d*x + 35*(16*b*
cos(d*x + c)^7 - 24*b*cos(d*x + c)^5 + 2*b*cos(d*x + c)^3 + 3*b*cos(d*x + c
))*sin(d*x + c))/d
```

giac [A] time = 0.22, size = 92, normalized size = 0.72

$$\frac{3}{128}\,bx+\frac{a\cos\left(7\,dx+7\,c\right)}{448\,d}+\frac{a\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{3\,a\cos\left(dx+c\right)}{64\,d}+\frac{b\sin\left(8\,dx+8\,c\right)}{1024\,d}-\frac{b\sin\left(4\,dx\right.}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/128*b*x + 1/448*a*cos(7*d*x + 7*c)/d + 1/320*a*cos(5*d*x + 5*c)/d - 1/64*
a*cos(3*d*x + 3*c)/d - 3/64*a*cos(d*x + c)/d + 1/1024*b*sin(8*d*x + 8*c)/d
- 1/128*b*sin(4*d*x + 4*c)/d
```

maple [A] time = 0.28, size = 106, normalized size = 0.83

$$\frac{a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{7}-\frac{2\left(\cos^{5}(dx+c)\right)}{35}\right)+b\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^{5}(dx+c)\right)}{16}+\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin\ldots}{64}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+b*(-1/8*sin(d*x+c
)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*
x+c))*sin(d*x+c)+3/128*d*x+3/128*c))
```

maxima [A] time = 0.38, size = 61, normalized size = 0.48

$$\frac{1024\left(5\cos\left(dx+c\right)^{7}-7\cos\left(dx+c\right)^{5}\right)a+35\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\sin\left(4\,dx+4\,c\right)\right)b}{35840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/35840*(1024*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a + 35*(24*d*x + 24*c +
 sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*b)/d
```

mupad [B] time = 13.17, size = 209, normalized size = 1.65

$$\frac{3\,b\,x}{128}-\frac{-\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}-\frac{23\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\frac{333\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64}-\frac{671\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{64}+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^3*(a + b*sin(c + d*x)),x)
```

```
[Out] (3*b*x)/128 - ((4*a)/35 + (3*b*tan(c/2 + (d*x)/2))/64 + (32*a*tan(c/2 + (d*
x)/2)^2)/35 - (4*a*tan(c/2 + (d*x)/2)^4)/5 + (32*a*tan(c/2 + (d*x)/2)^6)/5
+ 4*a*tan(c/2 + (d*x)/2)^8 + 4*a*tan(c/2 + (d*x)/2)^12 + (23*b*tan(c/2 + (d
*x)/2)^3)/64 - (333*b*tan(c/2 + (d*x)/2)^5)/64 + (671*b*tan(c/2 + (d*x)/2)^
7)/64 - (671*b*tan(c/2 + (d*x)/2)^9)/64 + (333*b*tan(c/2 + (d*x)/2)^11)/64
- (23*b*tan(c/2 + (d*x)/2)^13)/64 - (3*b*tan(c/2 + (d*x)/2)^15)/64)/(d*(tan
(c/2 + (d*x)/2)^2 + 1)^8)
```

sympy [A] time = 9.16, size = 248, normalized size = 1.95

$$\begin{cases} -\frac{a\sin^{2}\left(c+dx\right)\cos^{5}\left(c+dx\right)}{5d}-\frac{2a\cos^{7}\left(c+dx\right)}{35d}+\frac{3bx\sin^{8}\left(c+dx\right)}{128}+\frac{3bx\sin^{6}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{32}+\frac{9bx\sin^{4}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{64}+\frac{3bx\sin^{2}\left(c+dx\right)}{32} \\ x\left(a+b\sin\left(c\right)\right)\sin^{3}\left(c\right)\cos^{4}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 2*a*cos(c + d*x)**7/(
35*d) + 3*b*x*sin(c + d*x)**8/128 + 3*b*x*sin(c + d*x)**6*cos(c + d*x)**2/3
2 + 9*b*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*b*x*sin(c + d*x)**2*cos(c
+ d*x)**6/32 + 3*b*x*cos(c + d*x)**8/128 + 3*b*sin(c + d*x)**7*cos(c + d*x)
```

```
/(128*d) + 11*b*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*b*sin(c + d*x)
**3*cos(c + d*x)**5/(128*d) - 3*b*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(
d, 0)), (x*(a + b*sin(c))*sin(c)**3*cos(c)**4, True))
```

3.1094 $\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=103

$$-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}+\frac{ax}{16}+\frac{b\cos^7(c+dx)}{7d}-\frac{b\cos^5(c}{5d}$$

```
[Out] 1/16*a*x-1/5*b*cos(d*x+c)^5/d+1/7*b*cos(d*x+c)^7/d+1/16*a*cos(d*x+c)*sin(d*
x+c)/d+1/24*a*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 14}

$$-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}+\frac{ax}{16}+\frac{b\cos^7(c+dx)}{7d}-\frac{b\cos^5(c}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/16 - (b*Cos[c + d*x]^5)/(5*d) + (b*Cos[c + d*x]^7)/(7*d) + (a*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a*Co
s[c + d*x]^5*Sin[c + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
```

```
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^2(c+dx)\,dx + b\int \cos^4(c+dx)\sin^3(c$$

$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}a\int \cos^4(c+dx)\,dx - \frac{b\,\mathrm{Su}}{}$$

$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{8}$$

$$= -\frac{b\cos^5(c+dx)}{5d} + \frac{b\cos^7(c+dx)}{7d} + \frac{a\cos(c+dx)\sin(c+d}{16d}$$

$$= \frac{ax}{16} - \frac{b\cos^5(c+dx)}{5d} + \frac{b\cos^7(c+dx)}{7d} + \frac{a\cos(c+dx)\sin(c}{16d}$$

Mathematica [A] time = 0.19, size = 88, normalized size = 0.85

$$\frac{105a\sin(2(c+dx)) - 105a\sin(4(c+dx)) - 35a\sin(6(c+dx)) + 420adx - 315b\cos(c+dx) - 105b\cos(3(c+d}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (420*a*d*x - 315*b*Cos[c + d*x] - 105*b*Cos[3*(c + d*x)] + 21*b*Cos[5*(c +
d*x)] + 15*b*Cos[7*(c + d*x)] + 105*a*Sin[2*(c + d*x)] - 105*a*Sin[4*(c + d
*x)] - 35*a*Sin[6*(c + d*x)])/(6720*d)
```

fricas [A] time = 0.83, size = 73, normalized size = 0.71

$$\frac{240\,b\cos\left(dx+c\right)^{7}-336\,b\cos\left(dx+c\right)^{5}+105\,adx-35\left(8\,a\cos\left(dx+c\right)^{5}-2\,a\cos\left(dx+c\right)^{3}-3\,a\cos\left(dx+c\right)\right)\mathrm{si}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*b*cos(d*x + c)^7 - 336*b*cos(d*x + c)^5 + 105*a*d*x - 35*(8*a*c
os(d*x + c)^5 - 2*a*cos(d*x + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.20, size = 107, normalized size = 1.04

$$\frac{1}{16}\,ax+\frac{b\cos\left(7\,dx+7\,c\right)}{448\,d}+\frac{b\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{b\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{3\,b\cos\left(dx+c\right)}{64\,d}-\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\sin\left(4\,dx+\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*a*x + 1/448*b*cos(7*d*x + 7*c)/d + 1/320*b*cos(5*d*x + 5*c)/d - 1/64*b
*cos(3*d*x + 3*c)/d - 3/64*b*cos(d*x + c)/d - 1/192*a*sin(6*d*x + 6*c)/d -
1/64*a*sin(4*d*x + 4*c)/d + 1/64*a*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.29, size = 88, normalized size = 0.85

$$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^{5}(dx+c)\right)}{6}+\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+b\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{7}-\frac{2\left(\cos^{5}(dx+c)\right)}{35}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin
(d*x+c)+1/16*d*x+1/16*c)+b*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^
5))
```

maxima [A] time = 0.39, size = 65, normalized size = 0.63

$$\frac{35\left(4\,\sin\left(2\,dx+2\,c\right)^{3}+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a+192\left(5\,\cos\left(dx+c\right)^{7}-7\,\cos\left(dx+c\right)^{5}\right)b}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6720*(35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a +
192*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b)/d
```

mupad [B] time = 13.02, size = 181, normalized size = 1.76

$$\frac{a\,x}{16}-\frac{-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{6}+4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{31\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}-4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+8\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{\cdots}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*x)/16 - ((4*b)/35 + (a*tan(c/2 + (d*x)/2))/8 - (11*a*tan(c/2 + (d*x)/2)^
3)/6 + (31*a*tan(c/2 + (d*x)/2)^5)/24 - (31*a*tan(c/2 + (d*x)/2)^9)/24 + (1
1*a*tan(c/2 + (d*x)/2)^11)/6 - (a*tan(c/2 + (d*x)/2)^13)/8 + (4*b*tan(c/2 +
 (d*x)/2)^2)/5 - (8*b*tan(c/2 + (d*x)/2)^4)/5 + 8*b*tan(c/2 + (d*x)/2)^6 -
4*b*tan(c/2 + (d*x)/2)^8 + 4*b*tan(c/2 + (d*x)/2)^10)/(d*(tan(c/2 + (d*x)/2
)^2 + 1)^7)
```

sympy [A] time = 5.49, size = 192, normalized size = 1.86

$$\begin{cases} \frac{ax\sin^6(c+dx)}{16}+\frac{3ax\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3ax\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{ax\cos^6(c+dx)}{16}+\frac{a\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{a\sin^3(c+dx)\cos^3\cdots}{6d} \\ x(a+b\sin(c))\sin^2(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**6/16 + 3*a*x*sin(c + d*x)**4*cos(c + d*x)**2/1
6 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a*x*cos(c + d*x)**6/16 + a*s
in(c + d*x)**5*cos(c + d*x)/(16*d) + a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - b*sin(c + d*x)**2*cos(c + d*x)*
*5/(5*d) - 2*b*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)*
*2*cos(c)**4, True))
```

3.1095 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{b\sin(c+dx)\cos(c+dx)}{16d}+\frac{bx}{16}$$

```
[Out] 1/16*b*x-1/5*a*cos(d*x+c)^5/d+1/16*b*cos(d*x+c)*sin(d*x+c)/d+1/24*b*cos(d*x
+c)^3*sin(d*x+c)/d-1/6*b*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2838, 2565, 30, 2568, 2635, 8}

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{b\sin(c+dx)\cos(c+dx)}{16d}+\frac{bx}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/16 - (a*Cos[c + d*x]^5)/(5*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(16*d)
+ (b*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (b*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
```

```
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cos^4(c+dx)\sin(c+dx)\,dx+b\int \cos^4(c+dx)\sin^2(c+\\ &= -\frac{b\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{6}b\int \cos^4(c+dx)\,dx-\frac{a\,\mathrm{Sub}}{}\\ &= -\frac{a\cos^5(c+dx)}{5d}+\frac{b\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{b\cos^5(c+d}{6}\\ &= -\frac{a\cos^5(c+dx)}{5d}+\frac{b\cos(c+dx)\sin(c+dx)}{16d}+\frac{b\cos^3(c+dx}{24}\\ &= \frac{bx}{16}-\frac{a\cos^5(c+dx)}{5d}+\frac{b\cos(c+dx)\sin(c+dx)}{16d}+\frac{b\cos^3(c}{}\end{aligned}$$

Mathematica [A] time = 0.17, size = 77, normalized size = 0.89

$$-\frac{120a\cos(c+dx)+60a\cos(3(c+dx))+12a\cos(5(c+dx))-15b\sin(2(c+dx))+15b\sin(4(c+dx))+5b\sin}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/960*(-60*b*d*x + 120*a*Cos[c + d*x] + 60*a*Cos[3*(c + d*x)] + 12*a*Cos[5
*(c + d*x)] - 15*b*Sin[2*(c + d*x)] + 15*b*Sin[4*(c + d*x)] + 5*b*Sin[6*(c
+ d*x)])/d
```

fricas [A] time = 0.72, size = 62, normalized size = 0.71

$$-\frac{48\,a\cos\left(dx+c\right)^{5}-15\,bdx+5\,\left(8\,b\cos\left(dx+c\right)^{5}-2\,b\cos\left(dx+c\right)^{3}-3\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(48*a*cos(d*x + c)^5 - 15*b*d*x + 5*(8*b*cos(d*x + c)^5 - 2*b*cos(d*
x + c)^3 - 3*b*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.17, size = 92, normalized size = 1.06

$$\frac{1}{16}\,bx-\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}-\frac{b\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{b\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{b\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*b*x - 1/80*a*cos(5*d*x + 5*c)/d - 1/16*a*cos(3*d*x + 3*c)/d - 1/8*a*co
s(d*x + c)/d - 1/192*b*sin(6*d*x + 6*c)/d - 1/64*b*sin(4*d*x + 4*c)/d + 1/6
4*b*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.28, size = 68, normalized size = 0.78

$$\frac{-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{5}+b\left(-\frac{\sin\left(dx+c\right)\left(\cos^{5}\left(dx+c\right)\right)}{6}+\frac{\left(\cos^{3}\left(dx+c\right)+\frac{3\cos\left(dx+c\right)}{2}\right)\sin\left(dx+c\right)}{24}+\frac{dx}{16}+\frac{c}{16}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/5*a*cos(d*x+c)^5+b*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3
+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c))
```

maxima [A] time = 0.30, size = 52, normalized size = 0.60

$$-\frac{192\,a\cos\left(dx+c\right)^{5}-5\,\left(4\,\sin\left(2\,dx+2\,c\right)^{3}+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)b}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/960*(192*a*cos(d*x + c)^5 - 5*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*
sin(4*d*x + 4*c))*b)/d
```

mupad [B] time = 12.86, size = 181, normalized size = 2.08

$$\frac{b\,x}{16}-\frac{-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{47\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-\frac{13\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x)),x)
```

```
[Out] (b*x)/16 - ((2*a)/5 + (b*tan(c/2 + (d*x)/2))/8 + (2*a*tan(c/2 + (d*x)/2)^2)
/5 + 4*a*tan(c/2 + (d*x)/2)^4 + 4*a*tan(c/2 + (d*x)/2)^6 + 2*a*tan(c/2 + (d
*x)/2)^8 + 2*a*tan(c/2 + (d*x)/2)^10 - (47*b*tan(c/2 + (d*x)/2)^3)/24 + (13
*b*tan(c/2 + (d*x)/2)^5)/4 - (13*b*tan(c/2 + (d*x)/2)^7)/4 + (47*b*tan(c/2
+ (d*x)/2)^9)/24 - (b*tan(c/2 + (d*x)/2)^11)/8)/(d*(tan(c/2 + (d*x)/2)^2 +
1)^6)
```

sympy [A] time = 3.24, size = 167, normalized size = 1.92

$$\begin{cases} -\frac{a\cos^{5}(c+dx)}{5d}+\frac{bx\sin^{6}(c+dx)}{16}+\frac{3bx\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{3bx\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{bx\cos^{6}(c+dx)}{16}+\frac{b\sin^{5}(c+dx)\cos(c+dx)}{16d}+ \\ x(a+b\sin(c))\sin(c)\cos^{4}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)**5/(5*d) + b*x*sin(c + d*x)**6/16 + 3*b*x*sin(c
+ d*x)**4*cos(c + d*x)**2/16 + 3*b*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + b
*x*cos(c + d*x)**6/16 + b*sin(c + d*x)**5*cos(c + d*x)/(16*d) + b*sin(c + d
*x)**3*cos(c + d*x)**3/(6*d) - b*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d,
 0)), (x*(a + b*sin(c))*sin(c)*cos(c)**4, True))
```

3.1096 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=89

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}$$

```
[Out] 3/8*b*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d+3/8*b*c
os(d*x+c)*sin(d*x+c)/d+1/4*b*cos(d*x+c)^3*sin(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2838, 2592, 302, 206, 2635, 8}

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*x)/8 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c + d
*x]^3)/(3*d) + (3*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (b*Cos[c + d*x]^3*Si
n[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^3(c+dx)\cot(c+dx)\,dx + b\int \cos^4(c+dx)\,dx$$

$$= \frac{b\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3b)\int \cos^2(c+dx)\,dx - \frac{a\,\mathrm{Su}}{}$$

$$= \frac{3b\cos(c+dx)\sin(c+dx)}{8d} + \frac{b\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{8}$$

$$= \frac{3bx}{8} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{3b\cos(c+dx)\sin(}{8d}$$

$$= \frac{3bx}{8} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+}{3d}$$

Mathematica [A] time = 0.12, size = 109, normalized size = 1.22

$$\frac{5a\cos(c+dx)}{4d} + \frac{a\cos(3(c+dx))}{12d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{3b(c+dx)}{8d} + \frac{b\sin(2(c+dx}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

[Out] (3*b*(c + d*x))/(8*d) + (5*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12*d) - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/2]])/d + (b*Sin[2*(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)])/(32*d)

fricas [A] time = 0.81, size = 88, normalized size = 0.99

$$\frac{8\,a\cos\left(dx+c\right)^{3}+9\,bdx+24\,a\cos\left(dx+c\right)-12\,a\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+12\,a\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+3\,(}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/24*(8*a*cos(d*x + c)^3 + 9*b*d*x + 24*a*cos(d*x + c) - 12*a*log(1/2*cos(d*x + c) + 1/2) + 12*a*log(-1/2*cos(d*x + c) + 1/2) + 3*(2*b*cos(d*x + c)^3 + 3*b*cos(d*x + c))*sin(d*x + c))/d

giac [A] time = 0.17, size = 145, normalized size = 1.63

$$\frac{9\,(dx+c)b+24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-96\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+9\,b}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] 1/24*(9*(d*x + c)*b + 24*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(15*b*tan(1/2*d*x + 1/2*c)^7 - 48*a*tan(1/2*d*x + 1/2*c)^6 - 9*b*tan(1/2*d*x + 1/2*c)^5 - 96*a*tan(1/2*d*x + 1/2*c)^4 + 9*b*tan(1/2*d*x + 1/2*c)^3 - 80*a*tan(1/2*d*x + 1/2*c)^2 - 15*b*tan(1/2*d*x + 1/2*c) - 32*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d

maple [A] time = 0.42, size = 97, normalized size = 1.09

$$\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{3d}+\frac{a\cos\left(dx+c\right)}{d}+\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}+\frac{b\left(\cos^{3}\left(dx+c\right)\right)\sin\left(dx+c\right)}{4d}+\frac{3b\cos\left(dx+c\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c)),x)

[Out] 1/3*a*cos(d*x+c)^3/d+a*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))+1/4*b*cos(d*x+c)^3*sin(d*x+c)/d+3/8*b*cos(d*x+c)*sin(d*x+c)/d+3/8*b*x+3/8*b*c/d

maxima [A] time = 0.40, size = 81, normalized size = 0.91

$$\frac{16\left(2\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)a+3\left(12\,dx+12\,c+\sin\ldots\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(16*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*b)/d
```

mupad [B] time = 10.82, size = 242, normalized size = 2.72

$$\frac{3\,b\,\mathrm{atan}\left(\frac{9\,b^2}{16\,\left(\frac{3\,a\,b}{2}-\frac{9\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{3\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,\left(\frac{3\,a\,b}{2}-\frac{9\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}+\frac{-\frac{5\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{4}+4\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+\frac{3\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{4}+8\,a\,\mathrm{ta}\ldots}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] (3*b*atan((9*b^2)/(16*((3*a*b)/2 - (9*b^2*tan(c/2 + (d*x)/2))/16)) + (3*a*b
*tan(c/2 + (d*x)/2))/(2*((3*a*b)/2 - (9*b^2*tan(c/2 + (d*x)/2))/16))))/(4*d
) + ((8*a)/3 + (5*b*tan(c/2 + (d*x)/2))/4 + (20*a*tan(c/2 + (d*x)/2)^2)/3 +
 8*a*tan(c/2 + (d*x)/2)^4 + 4*a*tan(c/2 + (d*x)/2)^6 - (3*b*tan(c/2 + (d*x)
/2)^3)/4 + (3*b*tan(c/2 + (d*x)/2)^5)/4 - (5*b*tan(c/2 + (d*x)/2)^7)/4)/(d*
(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 +
 tan(c/2 + (d*x)/2)^8 + 1)) + (a*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(c+dx\right)\right)\cos^{4}\left(c+dx\right)\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**4*csc(c + d*x), x)
```

3.1097 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=83

$$-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{3ax}{2}+\frac{b\cos^3(c+dx)}{3d}+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -3/2*a*x-b*arctanh(cos(d*x+c))/d+b*cos(d*x+c)/d+1/3*b*cos(d*x+c)^3/d-3/2*a*
cot(d*x+c)/d+1/2*a*cos(d*x+c)^2*cot(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2838, 2591, 288, 321, 203, 2592, 302, 206}

$$-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{3ax}{2}+\frac{b\cos^3(c+dx)}{3d}+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*a*x)/2 - (b*ArcTanh[Cos[c + d*x]])/d + (b*Cos[c + d*x])/d + (b*Cos[c +
d*x]^3)/(3*d) - (3*a*Cot[c + d*x])/(2*d) + (a*Cos[c + d*x]^2*Cot[c + d*x])/
(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^2(c+dx)\cot^2(c+dx)\,dx + b\int \cos^3(c+dx)\cot(c+$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,\right.}{d}$$

$$= \frac{a\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{(3a)\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \cot(c+\right.}{2d}$$

$$= \frac{b\cos(c+dx)}{d} + \frac{b\cos^3(c+dx)}{3d} - \frac{3a\cot(c+dx)}{2d} + \frac{a\cos^2(c+}{}$$

$$= -\frac{3ax}{2} - \frac{b\tanh^{-1}(\cos(c+dx))}{d} + \frac{b\cos(c+dx)}{d} + \frac{b\cos^3(c+}{3d}$$

Mathematica [A] time = 0.33, size = 105, normalized size = 1.27

$$-\frac{3a(c+dx)}{2d} - \frac{a\sin(2(c+dx))}{4d} - \frac{a\cot(c+dx)}{d} + \frac{5b\cos(c+dx)}{4d} + \frac{b\cos(3(c+dx))}{12d} + \frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{b\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*a*(c + d*x))/(2*d) + (5*b*Cos[c + d*x])/(4*d) + (b*Cos[3*(c + d*x)])/(1
2*d) - (a*Cot[c + d*x])/d - (b*Log[Cos[(c + d*x)/2]])/d + (b*Log[Sin[(c + d
*x)/2]])/d - (a*Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.76, size = 107, normalized size = 1.29

$$\frac{3\,a\cos(dx+c)^3 - 3\,b\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) + 3\,b\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 9\,a\cos(}{6\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*cos(d*x + c)^3 - 3*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*
b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*a*cos(d*x + c) + (2*b*cos(d
*x + c)^3 - 9*a*d*x + 6*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.21, size = 142, normalized size = 1.71

$$-\frac{9\,(dx+c)a-6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+12\,b\tan\cdots\right.}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)*a - 6*b*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + 3*(2*b*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c) - 2*(3*a*t
an(1/2*d*x + 1/2*c)^5 + 12*b*tan(1/2*d*x + 1/2*c)^4 + 12*b*tan(1/2*d*x + 1/
2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) + 8*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.39, size = 119, normalized size = 1.43

$$-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}-\frac{a\left(\cos^{3}\left(dx+c\right)\right)\sin\left(dx+c\right)}{d}-\frac{3a\cos\left(dx+c\right)\sin\left(dx+c\right)}{2d}-\frac{3ax}{2}-\frac{3ca}{2d}+\frac{b\left(\cos^{3}\left(dx+c\right)\right)}{3d}+\frac{b\,\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^5-a*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a*cos(d*x+c)
*sin(d*x+c)/d-3/2*a*x-3/2/d*c*a+1/3*b*cos(d*x+c)^3/d+b*cos(d*x+c)/d+1/d*b*l
n(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.50, size = 91, normalized size = 1.10

$$-\frac{3\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^{2}+2}{\tan\left(dx+c\right)^{3}+\tan\left(dx+c\right)}\right)a-\left(2\,\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\cdots\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*a - (2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*lo
g(cos(d*x + c) - 1))*b)/d
```

mupad [B] time = 9.39, size = 241, normalized size = 2.90

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{16\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-}{d\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] ((16*b*tan(c/2 + (d*x)/2))/3 - a - 5*a*tan(c/2 + (d*x)/2)^2 - 3*a*tan(c/2 +
 (d*x)/2)^4 + a*tan(c/2 + (d*x)/2)^6 + 8*b*tan(c/2 + (d*x)/2)^3 + 8*b*tan(c
/2 + (d*x)/2)^5)/(d*(2*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3 + 6*tan(
c/2 + (d*x)/2)^5 + 2*tan(c/2 + (d*x)/2)^7)) + (a*tan(c/2 + (d*x)/2))/(2*d)
+ (b*log(tan(c/2 + (d*x)/2)))/d + (3*a*atan((9*a^2)/(6*a*b + 9*a^2*tan(c/2
+ (d*x)/2)) - (6*a*b*tan(c/2 + (d*x)/2))/(6*a*b + 9*a^2*tan(c/2 + (d*x)/2))
))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))\cos^4(c+dx)\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**4*csc(c + d*x)**2, x)
```

3.1098 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=94

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3b\cot(c+dx)}{2d}+\frac{b\cos^2(c+dx)\cot(c+dx)}{2d}$$

```
[Out] -3/2*b*x+3/2*a*arctanh(cos(d*x+c))/d-3/2*a*cos(d*x+c)/d-3/2*b*cot(d*x+c)/d+
1/2*b*cos(d*x+c)^2*cot(d*x+c)/d-1/2*a*cos(d*x+c)*cot(d*x+c)^2/d
```

Rubi [A] time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2838, 2592, 288, 321, 206, 2591, 203}

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3b\cot(c+dx)}{2d}+\frac{b\cos^2(c+dx)\cot(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*b*x)/2 + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) -
 (3*b*Cot[c + d*x])/(2*d) + (b*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (a*Cos[
c + d*x]*Cot[c + d*x]^2)/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos(c+dx)\cot^3(c+dx)\,dx + b\int \cos^2(c+dx)\cot^2(c+d$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx\right.}{d}$$

$$= \frac{b\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d} + \frac{(3a}{}$$

$$= -\frac{3a\cos(c+dx)}{2d} - \frac{3b\cot(c+dx)}{2d} + \frac{b\cos^2(c+dx)\cot(c+dx}{2d}$$

$$= -\frac{3bx}{2} + \frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3a\cos(c+dx)}{2d} - \frac{3b\cot(c-}{2d}$$

Mathematica [A] time = 1.48, size = 132, normalized size = 1.40

$$-\frac{a\cos(c+dx)}{d}-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{3b$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*b*(c + d*x))/(2*d) - (a*Cos[c + d*x])/d - (b*Cot[c + d*x])/d - (a*Csc[(
c + d*x)/2]^2)/(8*d) + (3*a*Log[Cos[(c + d*x)/2]])/(2*d) - (3*a*Log[Sin[(c
+ d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d) - (b*Sin[2*(c + d*x)])/(4*
d)
```

fricas [A] time = 0.73, size = 139, normalized size = 1.48

$$-\frac{6\,bdx\cos\left(dx+c\right)^2+4\,a\cos\left(dx+c\right)^3-6\,bdx-6\,a\cos\left(dx+c\right)-3\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\right.}{4\left(d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*b*d*x*cos(d*x + c)^2 + 4*a*cos(d*x + c)^3 - 6*b*d*x - 6*a*cos(d*x +
 c) - 3*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 3*(a*cos(d*x +
 c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 2*(b*cos(d*x + c)^3 - 3*b*cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.19, size = 163, normalized size = 1.73

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,(dx+c)b-12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+4\,b\tan\left(\frac{1}{2}\right.}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 - 12*(d*x + c)*b - 12*a*log(abs(tan(1/2*d*x +
 1/2*c))) + 4*b*tan(1/2*d*x + 1/2*c) + (6*a*tan(1/2*d*x + 1/2*c)^6 + 4*b*ta
n(1/2*d*x + 1/2*c)^5 - 5*a*tan(1/2*d*x + 1/2*c)^4 - 16*b*tan(1/2*d*x + 1/2*
c)^3 - 12*a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c) - a)/(tan(1/2
*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2)/d
```

maple [A] time = 0.46, size = 143, normalized size = 1.52

$$-\frac{a\left(\cos^5(dx+c)\right)}{2d\sin(dx+c)^2}-\frac{a\left(\cos^3(dx+c)\right)}{2d}-\frac{3a\cos(dx+c)}{2d}-\frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{b\left(\cos^5(dx+c)\right)}{d\sin(dx+c)}-\frac{b\left(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^5-1/2*a*cos(d*x+c)^3/d-3/2*a*cos(d*x+c)/d-
3/2/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/d*b/sin(d*x+c)*cos(d*x+c)^5-b*cos(d*x+c
)^3*sin(d*x+c)/d-3/2*b*cos(d*x+c)*sin(d*x+c)/d-3/2*b*x-3/2*b*c/d
```

maxima [A] time = 0.45, size = 101, normalized size = 1.07

$$-\frac{2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)b-a\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos(dx+c)+3\,\log(\cos(dx+c)+1)-3\,\log(\cos(dx\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*b - a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos
(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.37, size = 236, normalized size = 2.51

$$\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d}-\frac{-2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{17\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2}+8\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+9\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+}{d\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(2*d) - (a/2 + 2*b*tan(c/2 + (d*x)/2) + 9*a*tan(c/2
+ (d*x)/2)^2 + (17*a*tan(c/2 + (d*x)/2)^4)/2 + 8*b*tan(c/2 + (d*x)/2)^3 - 2
*b*tan(c/2 + (d*x)/2)^5)/(d*(4*tan(c/2 + (d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^
4 + 4*tan(c/2 + (d*x)/2)^6)) - (3*b*atan((9*b^2)/(9*a*b - 9*b^2*tan(c/2 + (
d*x)/2)) + (9*a*b*tan(c/2 + (d*x)/2))/(9*a*b - 9*b^2*tan(c/2 + (d*x)/2))))/
d + (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*a*log(tan(c/2 + (d*x)/2)))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c)),x)
[Out] Timed out
```

3.1099 $\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=82

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}+ax-\frac{3b\cos(c+dx)}{2d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}$$

```
[Out] a*x+3/2*b*arctanh(cos(d*x+c))/d-3/2*b*cos(d*x+c)/d+a*cot(d*x+c)/d-1/2*b*cos
(d*x+c)*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2722, 2592, 288, 321, 206, 3473, 8}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}+ax-\frac{3b\cos(c+dx)}{2d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] a*x + (3*b*ArcTanh[Cos[c + d*x]])/(2*d) - (3*b*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (b*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx = \int \left(b\cos(c+dx)\cot^3(c+dx)+a\cot^4(c+dx)\right)\,dx$$
$$= a\int \cot^4(c+dx)\,dx + b\int \cos(c+dx)\cot^3(c+dx)\,dx$$
$$= -\frac{a\cot^3(c+dx)}{3d} - a\int \cot^2(c+dx)\,dx - \frac{b\,\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx, x, \cos(c+\right.}{d}$$
$$= \frac{a\cot(c+dx)}{d} - \frac{b\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} + a\int 1\,dx +$$
$$= ax - \frac{3b\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{b\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3}{}$$
$$= ax + \frac{3b\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3b\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{b\cos(c-}{}$$

Mathematica [C] time = 0.04, size = 125, normalized size = 1.52

$$-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} - \frac{b\cos(c+dx)}{d} - \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{3b\log\left(\text{si}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x])/d) - (b*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*Hy
pergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*b*Log[Cos[(c +
d*x)/2]])/(2*d) - (3*b*Log[Sin[(c + d*x)/2]])/(2*d) + (b*Sec[(c + d*x)/2]^2
)/(8*d)
```

fricas [B] time = 0.69, size = 160, normalized size = 1.95

$$\frac{16\,a\cos(dx+c)^3 + 9\left(b\cos(dx+c)^2 - b\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 9\left(b\cos(dx+c)^2 - b\right)\log\left(-\frac{1}{2}\right.}{12\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(16*a*cos(d*x + c)^3 + 9*(b*cos(d*x + c)^2 - b)*log(1/2*cos(d*x + c) +
 1/2)*sin(d*x + c) - 9*(b*cos(d*x + c)^2 - b)*log(-1/2*cos(d*x + c) + 1/2)*
```

```
sin(d*x + c) - 12*a*cos(d*x + c) + 6*(2*a*d*x*cos(d*x + c)^2 - 2*b*cos(d*x
+ c)^3 - 2*a*d*x + 3*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*
sin(d*x + c))
```

giac [A] time = 0.20, size = 141, normalized size = 1.72

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,(dx+c)a-36\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*b*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*
a - 36*b*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*b/
(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*b*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*
d*x + 1/2*c)^2 - 3*b*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.31, size = 106, normalized size = 1.29

$$-\frac{a\left(\cot^{3}(dx+c)\right)}{3d}+\frac{a\cot(dx+c)}{d}+ax+\frac{ca}{d}-\frac{b\left(\cos^{5}(dx+c)\right)}{2d\sin(dx+c)^{2}}-\frac{b\left(\cos^{3}(dx+c)\right)}{2d}-\frac{3b\cos(dx+c)}{2d}-\frac{3b\ln(\csc(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/3*a*cot(d*x+c)^3/d+a*cot(d*x+c)/d+a*x+1/d*c*a-1/2/d*b/sin(d*x+c)^2*cos(d
*x+c)^5-1/2*b*cos(d*x+c)^3/d-3/2*b*cos(d*x+c)/d-3/2/d*b*ln(csc(d*x+c)-cot(d
*x+c))
```

maxima [A] time = 0.48, size = 92, normalized size = 1.12

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^{2}-1}{\tan(dx+c)^{3}}\right)a+3\,b\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^{2}-1}-4\,\cos(dx+c)+3\,\log(\cos(dx+c)+1)-3\,\log(\cos(dx+c)\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + 3*b*(2*co
s(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
- 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.45, size = 225, normalized size = 2.74

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+17\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{3}}{d\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a/3 + b*tan(c/2 + (d*x)/2) - (14*a*tan(c
/2 + (d*x)/2)^2)/3 - 5*a*tan(c/2 + (d*x)/2)^4 + 17*b*tan(c/2 + (d*x)/2)^3)/
(d*(8*tan(c/2 + (d*x)/2)^3 + 8*tan(c/2 + (d*x)/2)^5)) - (5*a*tan(c/2 + (d*x
)/2))/(8*d) + (b*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*b*log(tan(c/2 + (d*x)/2))
)/(2*d) - (2*a*atan((4*a^2)/(6*a*b + 4*a^2*tan(c/2 + (d*x)/2)) - (6*a*b*tan
(c/2 + (d*x)/2))/(6*a*b + 4*a^2*tan(c/2 + (d*x)/2))))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1100 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=88

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^3(c+dx)}{3d}+\frac{b\cot(c+dx)}{d}$$

```
[Out] b*x-3/8*a*arctanh(cos(d*x+c))/d+b*cot(d*x+c)/d-1/3*b*cot(d*x+c)^3/d+3/8*a*c
ot(d*x+c)*csc(d*x+c)/d-1/4*a*cot(d*x+c)^3*csc(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2838, 2611, 3770, 3473, 8}

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^3(c+dx)}{3d}+\frac{b\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] b*x - (3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (b*Cot[c + d*x])/d - (b*Cot[c + d
*x]^3)/(3*d) + (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]^3*Cs
c[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc(c+dx)\,dx + b\int \cot^4(c+dx)\,dx$$
$$= -\frac{b\cot^3(c+dx)}{3d} - \frac{a\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{1}{4}(3a)\int \cot^2(c$$
$$= \frac{b\cot(c+dx)}{d} - \frac{b\cot^3(c+dx)}{3d} + \frac{3a\cot(c+dx)\csc(c+dx)}{8d} -$$
$$= bx - \frac{3a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{b\cot(c+dx)}{d} - \frac{b\cot^3(c+dx)}{3d}$$

Mathematica [C] time = 0.05, size = 153, normalized size = 1.74

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{3a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (5*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (b*Cot[c
+ d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) - (3*a*Lo
g[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) - (5*a*Sec[(
c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

fricas [B] time = 0.74, size = 180, normalized size = 2.05

$$\frac{48\,bdx\cos\left(dx+c\right)^4 - 96\,bdx\cos\left(dx+c\right)^2 - 30\,a\cos\left(dx+c\right)^3 + 48\,bdx + 18\,a\cos\left(dx+c\right) - 9\left(a\cos\left(dx+c\right)^4 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(48*b*d*x*cos(d*x + c)^4 - 96*b*d*x*cos(d*x + c)^2 - 30*a*cos(d*x + c)
^3 + 48*b*d*x + 18*a*cos(d*x + c) - 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^
2 + a)*log(1/2*cos(d*x + c) + 1/2) + 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)
^2 + a)*log(-1/2*cos(d*x + c) + 1/2) - 16*(4*b*cos(d*x + c)^3 - 3*b*cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.21, size = 153, normalized size = 1.74

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+192\,(dx+c)b+72\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1
/2*d*x + 1/2*c)^2 + 192*(d*x + c)*b + 72*a*log(abs(tan(1/2*d*x + 1/2*c))) -
 120*b*tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 - 120*b*tan(1/2
*d*x + 1/2*c)^3 - 24*a*tan(1/2*d*x + 1/2*c)^2 + 8*b*tan(1/2*d*x + 1/2*c) +
3*a)/tan(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.34, size = 128, normalized size = 1.45

$$-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{4d\sin\left(dx+c\right)^{4}}+\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^{2}}+\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{8d}+\frac{3a\cos\left(dx+c\right)}{8d}+\frac{3a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{8d}-\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^5+1/8/d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/8*a*
cos(d*x+c)^3/d+3/8*a*cos(d*x+c)/d+3/8/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/3*b*c
ot(d*x+c)^3/d+b*cot(d*x+c)/d+b*x+b*c/d
```

maxima [A] time = 0.46, size = 107, normalized size = 1.22

$$\frac{16\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^{2}-1}{\tan\left(dx+c\right)^{3}}\right)b-3\,a\left(\frac{2\left(5\,\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*b - 3*a*(2*(
5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)
+ 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.67, size = 221, normalized size = 2.51

$$\frac{3\,a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{8\,d}+\frac{5\,b\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{5\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{2\,b\,\mathrm{atan}\left(\frac{8\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3\,a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3\,a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-8\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{a\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (3*a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(8*d) + (5*b*cot(c/2 + (d*
x)/2))/(8*d) - (5*b*tan(c/2 + (d*x)/2))/(8*d) + (2*b*atan((8*b*cos(c/2 + (d
*x)/2) + 3*a*sin(c/2 + (d*x)/2))/(3*a*cos(c/2 + (d*x)/2) - 8*b*sin(c/2 + (d
*x)/2))))/d + (a*cot(c/2 + (d*x)/2)^2)/(8*d) - (a*cot(c/2 + (d*x)/2)^4)/(64
*d) - (b*cot(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2)^2)/(8*d) + (a
*tan(c/2 + (d*x)/2)^4)/(64*d) + (b*tan(c/2 + (d*x)/2)^3)/(24*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1101 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{3b\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3b\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] -3/8*b*arctanh(cos(d*x+c))/d-1/5*a*cot(d*x+c)^5/d+3/8*b*cot(d*x+c)*csc(d*x+
c)/d-1/4*b*cot(d*x+c)^3*csc(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2838, 2607, 30, 2611, 3770}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{3b\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3b\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*b*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^5)/(5*d) + (3*b*Cot[c
+ d*x]*Csc[c + d*x])/(8*d) - (b*Cot[c + d*x]^3*Csc[c + d*x])/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^2(c+dx)\,dx + b\int \cot^4(c+dx)\csc(c+$$

$$= -\frac{b\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{1}{4}(3b)\int \cot^2(c+dx)\csc(c+$$

$$= -\frac{a\cot^5(c+dx)}{5d} + \frac{3b\cot(c+dx)\csc(c+dx)}{8d} - \frac{b\cot^3(c+dx)}{4}$$

$$= -\frac{3b\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^5(c+dx)}{5d} + \frac{3b\cot(c+dx)c}{8d}$$

Mathematica [A] time = 0.04, size = 135, normalized size = 1.82

$$-\frac{a\cot^5(c+dx)}{5d} - \frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{5b\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{b\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5b\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3b\log\left(\sin\left(\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/5*(a*Cot[c + d*x]^5)/d + (5*b*Csc[(c + d*x)/2]^2)/(32*d) - (b*Csc[(c + d
*x)/2]^4)/(64*d) - (3*b*Log[Cos[(c + d*x)/2]])/(8*d) + (3*b*Log[Sin[(c + d*
x)/2]])/(8*d) - (5*b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)/(6
4*d)
```

fricas [B] time = 0.80, size = 160, normalized size = 2.16

$$-\frac{16\,a\cos(dx+c)^5 + 15\left(b\cos(dx+c)^4 - 2\,b\cos(dx+c)^2 + b\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - 15\left(b\cos\right.}{80\left(d\cos(dx+c)^4 - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/80*(16*a*cos(d*x + c)^5 + 15*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)
*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(b*cos(d*x + c)^4 - 2*b*cos(
d*x + c)^2 + b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 10*(5*b*cos(d*x
 + c)^3 - 3*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x
+ c)^2 + d)*sin(d*x + c))
```

giac [B] time = 0.22, size = 173, normalized size = 2.34

$$2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+5\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+120\,b\log\left(\left|\tan\left(\frac{1}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/320*(2*a*tan(1/2*d*x + 1/2*c)^5 + 5*b*tan(1/2*d*x + 1/2*c)^4 - 10*a*tan(1
/2*d*x + 1/2*c)^3 - 40*b*tan(1/2*d*x + 1/2*c)^2 + 120*b*log(abs(tan(1/2*d*x
 + 1/2*c))) + 20*a*tan(1/2*d*x + 1/2*c) - (274*b*tan(1/2*d*x + 1/2*c)^5 + 2
0*a*tan(1/2*d*x + 1/2*c)^4 - 40*b*tan(1/2*d*x + 1/2*c)^3 - 10*a*tan(1/2*d*x
 + 1/2*c)^2 + 5*b*tan(1/2*d*x + 1/2*c) + 2*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.34, size = 116, normalized size = 1.57

$$-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)^{5}}-\frac{b\left(\cos^{5}\left(dx+c\right)\right)}{4d\sin\left(dx+c\right)^{4}}+\frac{b\left(\cos^{5}\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^{2}}+\frac{b\left(\cos^{3}\left(dx+c\right)\right)}{8d}+\frac{3b\cos\left(dx+c\right)}{8d}+\frac{3b\ln\left(\csc\left(dx+c\right)\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^5-1/4/d*b/sin(d*x+c)^4*cos(d*x+c)^5+1/8/d*
b/sin(d*x+c)^2*cos(d*x+c)^5+1/8*b*cos(d*x+c)^3/d+3/8*b*cos(d*x+c)/d+3/8/d*b
*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.32, size = 86, normalized size = 1.16

$$-\frac{5\,b\left(\frac{2\left(5\,\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\,a}{\tan\left(dx+c\right)^{5}}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80*(5*b*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 16*a/
tan(d*x + c)^5)/d
```

mupad [B] time = 9.56, size = 174, normalized size = 2.35

$$\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{32\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,d}+\frac{3\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,d}-\frac{\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(16*d) - (a*tan(c/2 + (d*x)/2)^3)/(32*d) + (a*tan(c/
2 + (d*x)/2)^5)/(160*d) - (b*tan(c/2 + (d*x)/2)^2)/(8*d) + (b*tan(c/2 + (d*
x)/2)^4)/(64*d) + (3*b*log(tan(c/2 + (d*x)/2)))/(8*d) - (cot(c/2 + (d*x)/2)
^5*(a/5 + (b*tan(c/2 + (d*x)/2))/2 - a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 +
 (d*x)/2)^4 - 4*b*tan(c/2 + (d*x)/2)^3))/(32*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1102 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=98

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{a\cot(c+dx)\csc(c+dx)}{16d}-\frac{b\,\mathrm{co}}{}$$

```
[Out] -1/16*a*arctanh(cos(d*x+c))/d-1/5*b*cot(d*x+c)^5/d-1/16*a*cot(d*x+c)*csc(d*
x+c)/d+1/8*a*cot(d*x+c)*csc(d*x+c)^3/d-1/6*a*cot(d*x+c)^3*csc(d*x+c)^3/d
```

Rubi [A] time = 0.17, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 30}

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{a\cot(c+dx)\csc(c+dx)}{16d}-\frac{b\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(16*d) - (b*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]*Csc[c + d*x])/(16*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^3)/(6*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cot^4(c+dx)\csc^3(c+dx)\,dx+b\int \cot^4(c+dx)\csc^2(c+\\ &= -\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}-\frac{1}{2}a\int \cot^2(c+dx)\csc^3(c+dx\\ &= -\frac{b\cot^5(c+dx)}{5d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{a\cot^3(c+dx)}{6}\\ &= -\frac{b\cot^5(c+dx)}{5d}-\frac{a\cot(c+dx)\csc(c+dx)}{16d}+\frac{a\cot(c+dx)\,c}{8d}\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{b\cot^5(c+dx)}{5d}-\frac{a\cot(c+dx)\csc(}{16d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 175, normalized size = 1.79

$$-\frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d}+\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{a\sec^2\left(\frac{1}{2}(c\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/5*(b*Cot[c + d*x]^5)/d - (a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c + d*x
)/2]^4)/(64*d) - (a*Csc[(c + d*x)/2]^6)/(384*d) - (a*Log[Cos[(c + d*x)/2]])
```

```
/(16*d) + (a*Log[Sin[(c + d*x)/2]])/(16*d) + (a*Sec[(c + d*x)/2]^2)/(64*d)
- (a*Sec[(c + d*x)/2]^4)/(64*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

fricas [B] time = 0.72, size = 187, normalized size = 1.91

$$\frac{96\,b\cos\left(dx+c\right)^{5}\sin\left(dx+c\right)+30\,a\cos\left(dx+c\right)^{5}+80\,a\cos\left(dx+c\right)^{3}-30\,a\cos\left(dx+c\right)-15\left(a\cos\left(dx+c\right)^{6}-\right.}{480\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/480*(96*b*cos(d*x + c)^5*sin(d*x + c) + 30*a*cos(d*x + c)^5 + 80*a*cos(d*
x + c)^3 - 30*a*cos(d*x + c) - 15*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 +
3*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 15*(a*cos(d*x + c)^6
- 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2)
)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [B] time = 0.24, size = 201, normalized size = 2.05

$$5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+12\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-60\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a\tan\left(\frac{1}{2}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*a*tan(1/2*d*x + 1/2*c)^6 + 12*b*tan(1/2*d*x + 1/2*c)^5 - 15*a*tan
(1/2*d*x + 1/2*c)^4 - 60*b*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2*
c)^2 + 120*a*log(abs(tan(1/2*d*x + 1/2*c))) + 120*b*tan(1/2*d*x + 1/2*c) -
(294*a*tan(1/2*d*x + 1/2*c)^6 + 120*b*tan(1/2*d*x + 1/2*c)^5 - 15*a*tan(1/2
*d*x + 1/2*c)^4 - 60*b*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2*c)^2
 + 12*b*tan(1/2*d*x + 1/2*c) + 5*a)/tan(1/2*d*x + 1/2*c)^6)/d
```

maple [A] time = 0.34, size = 138, normalized size = 1.41

$$-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{6d\sin\left(dx+c\right)^{6}}-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{24d\sin\left(dx+c\right)^{4}}+\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{48d\sin\left(dx+c\right)^{2}}+\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{48d}+\frac{a\cos\left(dx+c\right)}{16d}+\frac{a\ln\left(\csc\left(dx+c\right)-\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c)),x)
```

[Out] -1/6/d*a/sin(d*x+c)^6*cos(d*x+c)^5-1/24/d*a/sin(d*x+c)^4*cos(d*x+c)^5+1/48/
d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/48*a*cos(d*x+c)^3/d+1/16*a*cos(d*x+c)/d+1/1
6/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/5/d*b/sin(d*x+c)^5*cos(d*x+c)^5

maxima [A] time = 0.32, size = 106, normalized size = 1.08

$$\frac{5\,a\left(\frac{2\left(3\,\cos\left(dx+c\right)^{5}+8\,\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{6}-3\,\cos\left(dx+c\right)^{4}+3\,\cos\left(dx+c\right)^{2}-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{96\,b}{\tan\left(dx+c\right)^{5}}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/480*(5*a*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d
*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) +
 1) + 3*log(cos(d*x + c) - 1)) - 96*b/tan(d*x + c)^5)/d

mupad [B] time = 9.52, size = 205, normalized size = 2.09

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{128\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{128\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{384\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{32\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{160\,d}-\frac{\cot\left(\frac{c}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^7,x)

[Out] (b*tan(c/2 + (d*x)/2))/(16*d) - (a*tan(c/2 + (d*x)/2)^2)/(128*d) - (a*tan(c
/2 + (d*x)/2)^4)/(128*d) + (a*tan(c/2 + (d*x)/2)^6)/(384*d) - (b*tan(c/2 +
(d*x)/2)^3)/(32*d) + (b*tan(c/2 + (d*x)/2)^5)/(160*d) - (cot(c/2 + (d*x)/2)
^6*(a/6 + (2*b*tan(c/2 + (d*x)/2))/5 - (a*tan(c/2 + (d*x)/2)^2)/2 - (a*tan(
c/2 + (d*x)/2)^4)/2 - 2*b*tan(c/2 + (d*x)/2)^3 + 4*b*tan(c/2 + (d*x)/2)^5))
/(64*d) + (a*log(tan(c/2 + (d*x)/2)))/(16*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+b*sin(d*x+c)),x)

[Out] Timed out

3.1103 $\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=114

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{b\tanh^{-1}(\cos(c+dx))}{16d}-\frac{b\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{b\cot(c+dx)\csc^3(c+dx)}{8d}$$

```
[Out] -1/16*b*arctanh(cos(d*x+c))/d-1/5*a*cot(d*x+c)^5/d-1/7*a*cot(d*x+c)^7/d-1/1
6*b*cot(d*x+c)*csc(d*x+c)/d+1/8*b*cot(d*x+c)*csc(d*x+c)^3/d-1/6*b*cot(d*x+c
)^3*csc(d*x+c)^3/d
```

Rubi [A] time = 0.17, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2607, 14, 2611, 3768, 3770}

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{b\tanh^{-1}(\cos(c+dx))}{16d}-\frac{b\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{b\cot(c+dx)\csc^3(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]^7)/(7*d) - (b*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (b*Cot[c + d*x]*Csc[c
 + d*x]^3)/(8*d) - (b*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cot^4(c+dx)\csc^4(c+dx)\,dx + b\int \cot^4(c+dx)\csc^3(c+\\
&= -\frac{b\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{1}{2}b\int \cot^2(c+dx)\csc^3(c+dx\\
&= \frac{b\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{b\cot^3(c+dx)\csc^3(c+dx)}{6d} + \frac{1}{8}b\\
&= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{b\cot(c+dx)\csc(c+dx)}{16d}\\
&= -\frac{b\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} -\end{aligned}$$

Mathematica [B] time = 0.08, size = 239, normalized size = 2.10

$$-\frac{2a\cot(c+dx)}{35d}-\frac{a\cot(c+dx)\csc^6(c+dx)}{7d}+\frac{8a\cot(c+dx)\csc^4(c+dx)}{35d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{35d}-\frac{b\csc^6\left(\frac{1}{2}(c\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(35*d) - (b*Csc[(c + d*x)/2]^2)/(64*d) + (b*Csc[(c + d*
x)/2]^4)/(64*d) - (b*Csc[(c + d*x)/2]^6)/(384*d) - (a*Cot[c + d*x]*Csc[c +
d*x]^2)/(35*d) + (8*a*Cot[c + d*x]*Csc[c + d*x]^4)/(35*d) - (a*Cot[c + d*x]
*Csc[c + d*x]^6)/(7*d) - (b*Log[Cos[(c + d*x)/2]])/(16*d) + (b*Log[Sin[(c +
 d*x)/2]])/(16*d) + (b*Sec[(c + d*x)/2]^2)/(64*d) - (b*Sec[(c + d*x)/2]^4)/
(64*d) + (b*Sec[(c + d*x)/2]^6)/(384*d)
```

fricas [B] time = 0.89, size = 221, normalized size = 1.94

$$-\frac{192\,a\cos\left(dx+c\right)^{7}-672\,a\cos\left(dx+c\right)^{5}+105\left(b\cos\left(dx+c\right)^{6}-3\,b\cos\left(dx+c\right)^{4}+3\,b\cos\left(dx+c\right)^{2}-b\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3360*(192*a*cos(d*x + c)^7 - 672*a*cos(d*x + c)^5 + 105*(b*cos(d*x + c)^
6 - 3*b*cos(d*x + c)^4 + 3*b*cos(d*x + c)^2 - b)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) - 105*(b*cos(d*x + c)^6 - 3*b*cos(d*x + c)^4 + 3*b*cos(d*x +
 c)^2 - b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 70*(3*b*cos(d*x + c)
^5 + 8*b*cos(d*x + c)^3 - 3*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^
6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [B] time = 0.23, size = 229, normalized size = 2.01

$$\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+35\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-105\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-105\,a\tan\left(\frac{1}{2}\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*a*tan(1/2*d*x + 1/2*c)^7 + 35*b*tan(1/2*d*x + 1/2*c)^6 - 21*a*t
an(1/2*d*x + 1/2*c)^5 - 105*b*tan(1/2*d*x + 1/2*c)^4 - 105*a*tan(1/2*d*x +
1/2*c)^3 - 105*b*tan(1/2*d*x + 1/2*c)^2 + 840*b*log(abs(tan(1/2*d*x + 1/2*c
))) + 315*a*tan(1/2*d*x + 1/2*c) - (2178*b*tan(1/2*d*x + 1/2*c)^7 + 315*a*t
an(1/2*d*x + 1/2*c)^6 - 105*b*tan(1/2*d*x + 1/2*c)^5 - 105*a*tan(1/2*d*x +
1/2*c)^4 - 105*b*tan(1/2*d*x + 1/2*c)^3 - 21*a*tan(1/2*d*x + 1/2*c)^2 + 35*
b*tan(1/2*d*x + 1/2*c) + 15*a)/tan(1/2*d*x + 1/2*c)^7)/d
```

maple [A] time = 0.34, size = 160, normalized size = 1.40

$$-\frac{a\left(\cos^{5}\left(dx+c\right)\right)}{7d\sin\left(dx+c\right)^{7}}-\frac{2a\left(\cos^{5}\left(dx+c\right)\right)}{35d\sin\left(dx+c\right)^{5}}-\frac{b\left(\cos^{5}\left(dx+c\right)\right)}{6d\sin\left(dx+c\right)^{6}}-\frac{b\left(\cos^{5}\left(dx+c\right)\right)}{24d\sin\left(dx+c\right)^{4}}+\frac{b\left(\cos^{5}\left(dx+c\right)\right)}{48d\sin\left(dx+c\right)^{2}}+\frac{b\left(\cos^{3}\left(dx+c\right.\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/7/d*a/sin(d*x+c)^7*cos(d*x+c)^5-2/35/d*a/sin(d*x+c)^5*cos(d*x+c)^5-1/6/d
*b/sin(d*x+c)^6*cos(d*x+c)^5-1/24/d*b/sin(d*x+c)^4*cos(d*x+c)^5+1/48/d*b/si
n(d*x+c)^2*cos(d*x+c)^5+1/48*b*cos(d*x+c)^3/d+1/16*b*cos(d*x+c)/d+1/16/d*b*
ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.39, size = 118, normalized size = 1.04

$$\frac{35\,b\left(\frac{2\left(3\,\cos(dx+c)^{5}+8\,\cos(dx+c)^{3}-3\,\cos(dx+c)\right)}{\cos(dx+c)^{6}-3\,\cos(dx+c)^{4}+3\,\cos(dx+c)^{2}-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{96\left(7\,\tan(dx+c)^{2}+5\right)a}{\tan(dx+c)^{7}}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3360*(35*b*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
 + 1) + 3*log(cos(d*x + c) - 1)) - 96*(7*tan(d*x + c)^2 + 5)*a/tan(d*x + c)
^7)/d
```

mupad [B] time = 10.99, size = 399, normalized size = 3.50

$$15\,a\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-15\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-21\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-105\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^8,x)
```

```
[Out] (15*a*sin(c/2 + (d*x)/2)^14 - 15*a*cos(c/2 + (d*x)/2)^14 - 21*a*cos(c/2 + (
d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 105*a*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x
)/2)^10 + 315*a*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 - 315*a*cos(c/2 +
 (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 105*a*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d
*x)/2)^4 + 21*a*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2 - 105*b*cos(c/2
+ (d*x)/2)^3*sin(c/2 + (d*x)/2)^11 - 105*b*cos(c/2 + (d*x)/2)^5*sin(c/2 + (
d*x)/2)^9 + 105*b*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5 + 105*b*cos(c/2
 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 35*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*
x)/2)^13 - 35*b*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) + 840*b*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)
/(13440*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+b*sin(d*x+c)),x)
[Out] Timed out
```

3.1104 $\int \cot^4(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=136

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d}+\frac{a\cot(c+dx)\csc^5(c+dx)}{16d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{64d}-\frac{3a}{}$$

```
[Out] -3/128*a*arctanh(cos(d*x+c))/d-1/5*b*cot(d*x+c)^5/d-1/7*b*cot(d*x+c)^7/d-3/
128*a*cot(d*x+c)*csc(d*x+c)/d-1/64*a*cot(d*x+c)*csc(d*x+c)^3/d+1/16*a*cot(d
*x+c)*csc(d*x+c)^5/d-1/8*a*cot(d*x+c)^3*csc(d*x+c)^5/d
```

Rubi [A] time = 0.18, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 14}

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d}+\frac{a\cot(c+dx)\csc^5(c+dx)}{16d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{64d}-\frac{3a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(128*d) - (b*Cot[c + d*x]^5)/(5*d) - (b*Cot[c
+ d*x]^7)/(7*d) - (3*a*Cot[c + d*x]*Csc[c + d*x])/(128*d) - (a*Cot[c + d*x]
*Csc[c + d*x]^3)/(64*d) + (a*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^5)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^5(c+dx)\,dx + b\int \cot^4(c+dx)\csc^4(c$$

$$= -\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{1}{8}(3a)\int \cot^2(c+dx)\csc^5(c$$

$$= \frac{a\cot(c+dx)\csc^5(c+dx)}{16d} - \frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d} +$$

$$= -\frac{b\cot^5(c+dx)}{5d} - \frac{b\cot^7(c+dx)}{7d} - \frac{a\cot(c+dx)\csc^3(c+d}{64d}$$

$$= -\frac{b\cot^5(c+dx)}{5d} - \frac{b\cot^7(c+dx)}{7d} - \frac{3a\cot(c+dx)\csc(c+d}{128d}$$

$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{128d} - \frac{b\cot^5(c+dx)}{5d} - \frac{b\cot^7(c+dx)}{7d}$$

Mathematica [B] time = 0.07, size = 279, normalized size = 2.05

$$-\frac{a\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d}+\frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{512d}+\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d}-\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{512d}+\frac{a\sec^8\left(\frac{1}{2}(c+dx)\right)}{2048d}-\frac{a\sec^6\left(\frac{1}{2}\right.}{51}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*Cot[c + d*x])/(35*d) - (3*a*Csc[(c + d*x)/2]^2)/(512*d) + (a*Csc[(c +
 d*x)/2]^4)/(1024*d) + (a*Csc[(c + d*x)/2]^6)/(512*d) - (a*Csc[(c + d*x)/2]
^8)/(2048*d) - (b*Cot[c + d*x]*Csc[c + d*x]^2)/(35*d) + (8*b*Cot[c + d*x]*C
sc[c + d*x]^4)/(35*d) - (b*Cot[c + d*x]*Csc[c + d*x]^6)/(7*d) - (3*a*Log[Co
s[(c + d*x)/2]])/(128*d) + (3*a*Log[Sin[(c + d*x)/2]])/(128*d) + (3*a*Sec[(
c + d*x)/2]^2)/(512*d) - (a*Sec[(c + d*x)/2]^4)/(1024*d) - (a*Sec[(c + d*x)
/2]^6)/(512*d) + (a*Sec[(c + d*x)/2]^8)/(2048*d)
```

fricas [A] time = 0.79, size = 239, normalized size = 1.76

$$210\,a\cos\left(dx+c\right)^{7}-770\,a\cos\left(dx+c\right)^{5}-770\,a\cos\left(dx+c\right)^{3}+210\,a\cos\left(dx+c\right)-105\left(a\cos\left(dx+c\right)^{8}-4\,a\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8960*(210*a*cos(d*x + c)^7 - 770*a*cos(d*x + c)^5 - 770*a*cos(d*x + c)^3
+ 210*a*cos(d*x + c) - 105*(a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos
(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 105*(a*
cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)
^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 256*(2*b*cos(d*x + c)^7 - 7*b*cos(d*
x + c)^5)*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*
x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.26, size = 201, normalized size = 1.48

$$35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+80\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-112\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-560\,b\tan\left(\frac{1}{2}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/71680*(35*a*tan(1/2*d*x + 1/2*c)^8 + 80*b*tan(1/2*d*x + 1/2*c)^7 - 112*b*
tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^4 - 560*b*tan(1/2*d*x +
 1/2*c)^3 + 1680*a*log(abs(tan(1/2*d*x + 1/2*c))) + 1680*b*tan(1/2*d*x + 1/
2*c) - (4566*a*tan(1/2*d*x + 1/2*c)^8 + 1680*b*tan(1/2*d*x + 1/2*c)^7 - 560
*b*tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^4 - 112*b*tan(1/2*d*
x + 1/2*c)^3 + 80*b*tan(1/2*d*x + 1/2*c) + 35*a)/tan(1/2*d*x + 1/2*c)^8)/d
```

maple [A] time = 0.34, size = 182, normalized size = 1.34

$$-\frac{a\left(\cos^5(dx+c)\right)}{8d\sin(dx+c)^8}-\frac{a\left(\cos^5(dx+c)\right)}{16d\sin(dx+c)^6}-\frac{a\left(\cos^5(dx+c)\right)}{64d\sin(dx+c)^4}+\frac{a\left(\cos^5(dx+c)\right)}{128d\sin(dx+c)^2}+\frac{a\left(\cos^3(dx+c)\right)}{128d}+\frac{3a\cos(dx+c)}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/8/d*a/sin(d*x+c)^8*cos(d*x+c)^5-1/16/d*a/sin(d*x+c)^6*cos(d*x+c)^5-1/64/
d*a/sin(d*x+c)^4*cos(d*x+c)^5+1/128/d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/128*a*c
os(d*x+c)^3/d+3/128*a*cos(d*x+c)/d+3/128/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/7/
d*b/sin(d*x+c)^7*cos(d*x+c)^5-2/35/d*b/sin(d*x+c)^5*cos(d*x+c)^5
```

maxima [A] time = 0.39, size = 138, normalized size = 1.01

$$\frac{35\,a\left(\frac{2\left(3\cos(dx+c)^7-11\cos(dx+c)^5-11\cos(dx+c)^3+3\cos(dx+c)\right)}{\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1}-3\log\left(\cos(dx+c)+1\right)+3\log\left(\cos(dx+c)-1\right)\right)-\frac{256}{}}{8960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8960*(35*a*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3 +
 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*
cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) -
256*(7*tan(d*x + c)^2 + 5)*b/tan(d*x + c)^7)/d
```

mupad [B] time = 9.88, size = 205, normalized size = 1.51

$$\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2048\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{128\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{896\,d}-\frac{\cot\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^9,x)
```

```
[Out] (3*b*tan(c/2 + (d*x)/2))/(128*d) - (a*tan(c/2 + (d*x)/2)^4)/(256*d) + (a*ta
n(c/2 + (d*x)/2)^8)/(2048*d) - (b*tan(c/2 + (d*x)/2)^3)/(128*d) - (b*tan(c/
2 + (d*x)/2)^5)/(640*d) + (b*tan(c/2 + (d*x)/2)^7)/(896*d) - (cot(c/2 + (d*
x)/2)^8*(a/8 + (2*b*tan(c/2 + (d*x)/2))/7 - a*tan(c/2 + (d*x)/2)^4 - (2*b*t
an(c/2 + (d*x)/2)^3)/5 - 2*b*tan(c/2 + (d*x)/2)^5 + 6*b*tan(c/2 + (d*x)/2)^
7))/(256*d) + (3*a*log(tan(c/2 + (d*x)/2)))/(128*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1105 $\int \cos^4(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=301

$$\frac{(9a^2+4b^2)\cos^3(c+dx)}{315d}-\frac{(9a^2+4b^2)\cos(c+dx)}{105d}-\frac{a(10a^2-29b^2)\sin^5(c+dx)\cos(c+dx)}{504bd}-\frac{5(3a^2-8b^2)\sin}{}$$

```
[Out] 3/64*a*b*x-1/105*(9*a^2+4*b^2)*cos(d*x+c)/d+1/315*(9*a^2+4*b^2)*cos(d*x+c)^
3/d-3/64*a*b*cos(d*x+c)*sin(d*x+c)/d-1/32*a*b*cos(d*x+c)*sin(d*x+c)^3/d-1/6
30*(15*a^4-44*a^2*b^2+6*b^4)*cos(d*x+c)*sin(d*x+c)^4/b^2/d-1/504*a*(10*a^2-
29*b^2)*cos(d*x+c)*sin(d*x+c)^5/b/d-5/252*(3*a^2-8*b^2)*cos(d*x+c)*sin(d*x+
c)^4*(a+b*sin(d*x+c))^2/b^2/d+1/12*a*cos(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c
))^3/b^2/d-1/9*cos(d*x+c)*sin(d*x+c)^5*(a+b*sin(d*x+c))^3/b/d
```

Rubi [A] time = 0.65, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2895, 3049, 3033, 3023, 2748, 2633, 2635, 8}

$$\frac{(9a^2+4b^2)\cos^3(c+dx)}{315d}-\frac{(9a^2+4b^2)\cos(c+dx)}{105d}-\frac{a(10a^2-29b^2)\sin^5(c+dx)\cos(c+dx)}{504bd}-\frac{5(3a^2-8b^2)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*b*x)/64 - ((9*a^2 + 4*b^2)*Cos[c + d*x])/(105*d) + ((9*a^2 + 4*b^2)*Co
s[c + d*x]^3)/(315*d) - (3*a*b*Cos[c + d*x]*Sin[c + d*x])/(64*d) - (a*b*Cos
[c + d*x]*Sin[c + d*x]^3)/(32*d) - ((15*a^4 - 44*a^2*b^2 + 6*b^4)*Cos[c + d
*x]*Sin[c + d*x]^4)/(630*b^2*d) - (a*(10*a^2 - 29*b^2)*Cos[c + d*x]*Sin[c +
 d*x]^5)/(504*b*d) - (5*(3*a^2 - 8*b^2)*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*
Sin[c + d*x])^2)/(252*b^2*d) + (a*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*Sin[c
+ d*x])^3)/(12*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^5*(a + b*Sin[c + d*x])^3
)/(9*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^4(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{a\cos(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))^3}{12b^2d} - \frac{\cos(c+d}{}$$

$$= -\frac{5\left(3a^2-8b^2\right)\cos(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))^2}{252b^2d}$$

$$= -\frac{a\left(10a^2-29b^2\right)\cos(c+dx)\sin^5(c+dx)}{504bd} - \frac{5\left(3a^2-8b^2\right)}{}$$

$$= -\frac{\left(15a^4-44a^2b^2+6b^4\right)\cos(c+dx)\sin^4(c+dx)}{630b^2d} - \frac{a\left(10a\right.}{}$$

$$= -\frac{\left(15a^4-44a^2b^2+6b^4\right)\cos(c+dx)\sin^4(c+dx)}{630b^2d} - \frac{a\left(10a\right.}{}$$

$$= -\frac{ab\cos(c+dx)\sin^3(c+dx)}{32d} - \frac{\left(15a^4-44a^2b^2+6b^4\right)\cos}{630b^2d}$$

$$= -\frac{\left(9a^2+4b^2\right)\cos(c+dx)}{105d} + \frac{\left(9a^2+4b^2\right)\cos^3(c+dx)}{315d} - \frac{3a}{}$$

$$= \frac{3abx}{64} - \frac{\left(9a^2+4b^2\right)\cos(c+dx)}{105d} + \frac{\left(9a^2+4b^2\right)\cos^3(c+dx}{315d}$$

Mathematica [A] time = 0.96, size = 144, normalized size = 0.48

$$\frac{-3780\left(2a^2+b^2\right)\cos(c+dx) - 840\left(3a^2+b^2\right)\cos(3(c+dx)) + 504a^2\cos(5(c+dx)) + 360a^2\cos(7(c+dx)) -}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (7560*a*b*c + 7560*a*b*d*x - 3780*(2*a^2 + b^2)*Cos[c + d*x] - 840*(3*a^2 +
 b^2)*Cos[3*(c + d*x)] + 504*a^2*Cos[5*(c + d*x)] + 504*b^2*Cos[5*(c + d*x)
] + 360*a^2*Cos[7*(c + d*x)] + 90*b^2*Cos[7*(c + d*x)] - 70*b^2*Cos[9*(c +
d*x)] - 2520*a*b*Sin[4*(c + d*x)] + 315*a*b*Sin[8*(c + d*x)])/(161280*d)
```

fricas [A] time = 0.91, size = 116, normalized size = 0.39

$$-\frac{2240\,b^2\cos\left(dx+c\right)^9-2880\left(a^2+2\,b^2\right)\cos\left(dx+c\right)^7+4032\left(a^2+b^2\right)\cos\left(dx+c\right)^5-945\,abdx-315\left(16\,ab\cos\right.}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20160*(2240*b^2*cos(d*x + c)^9 - 2880*(a^2 + 2*b^2)*cos(d*x + c)^7 + 403
2*(a^2 + b^2)*cos(d*x + c)^5 - 945*a*b*d*x - 315*(16*a*b*cos(d*x + c)^7 - 2
4*a*b*cos(d*x + c)^5 + 2*a*b*cos(d*x + c)^3 + 3*a*b*cos(d*x + c))*sin(d*x +
 c))/d
```

giac [A] time = 0.37, size = 142, normalized size = 0.47

$$\frac{3}{64}\,abx-\frac{b^2\cos\left(9\,dx+9\,c\right)}{2304\,d}+\frac{ab\sin\left(8\,dx+8\,c\right)}{512\,d}-\frac{ab\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{\left(4\,a^2+b^2\right)\cos\left(7\,dx+7\,c\right)}{1792\,d}+\frac{\left(a^2+b^2\right)\cos\left(5\right.}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/64*a*b*x - 1/2304*b^2*cos(9*d*x + 9*c)/d + 1/512*a*b*sin(8*d*x + 8*c)/d -
 1/64*a*b*sin(4*d*x + 4*c)/d + 1/1792*(4*a^2 + b^2)*cos(7*d*x + 7*c)/d + 1/
320*(a^2 + b^2)*cos(5*d*x + 5*c)/d - 1/192*(3*a^2 + b^2)*cos(3*d*x + 3*c)/d
 - 3/128*(2*a^2 + b^2)*cos(d*x + c)/d
```

maple [A] time = 0.33, size = 161, normalized size = 0.53

$$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+2ab\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\mathrm{s}}{64}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+2*a*b*(-1/8*sin
(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*
```

```
cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+b^2*(-1/9*sin(d*x+c)^4*cos(d*x+c)
^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5))
```

maxima [A] time = 0.32, size = 100, normalized size = 0.33

$$\frac{4608\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^2+315\left(24\,dx+24\,c+\sin(8\,dx+8\,c)-8\sin(4\,dx+4\,c)\right)ab-512\left(3\right.}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/161280*(4608*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^2 + 315*(24*d*x + 24
*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a*b - 512*(35*cos(d*x + c)^9 -
90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*b^2)/d
```

mupad [B] time = 12.92, size = 309, normalized size = 1.03

$$\frac{3\,a\,b\,x}{64}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(4\,a^2-16\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\,\left(4\,a^2+\frac{32\,b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\,\left(\frac{28\,a^2}{5}-\frac{32\,b^2}{5}\right)+\tan\left(\frac{c}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^3*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (3*a*b*x)/64 - (tan(c/2 + (d*x)/2)^10*(4*a^2 - 16*b^2) + tan(c/2 + (d*x)/2)
^12*(4*a^2 + (32*b^2)/3) + tan(c/2 + (d*x)/2)^6*((28*a^2)/5 - (32*b^2)/5) +
 tan(c/2 + (d*x)/2)^2*((36*a^2)/35 + (16*b^2)/35) + tan(c/2 + (d*x)/2)^4*((
4*a^2)/35 + (64*b^2)/35) + tan(c/2 + (d*x)/2)^8*((52*a^2)/5 + (112*b^2)/5)
+ 4*a^2*tan(c/2 + (d*x)/2)^14 + (4*a^2)/35 + (16*b^2)/315 + (13*a*b*tan(c/2
 + (d*x)/2)^3)/16 - (155*a*b*tan(c/2 + (d*x)/2)^5)/16 + (169*a*b*tan(c/2 +
(d*x)/2)^7)/16 - (169*a*b*tan(c/2 + (d*x)/2)^11)/16 + (155*a*b*tan(c/2 + (d
*x)/2)^13)/16 - (13*a*b*tan(c/2 + (d*x)/2)^15)/16 - (3*a*b*tan(c/2 + (d*x)/
2)^17)/32 + (3*a*b*tan(c/2 + (d*x)/2))/32)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

sympy [A] time = 15.58, size = 335, normalized size = 1.11

$$\begin{cases} -\frac{a^2\sin^2(c+dx)\cos^5(c+dx)}{5d}-\frac{2a^2\cos^7(c+dx)}{35d}+\frac{3abx\sin^8(c+dx)}{64}+\frac{3abx\sin^6(c+dx)\cos^2(c+dx)}{16}+\frac{9abx\sin^4(c+dx)\cos^4(c+dx)}{32}+\frac{3abx\sin}{} \\ x\,(a+b\sin(c))^2\sin^3(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 2*a**2*cos(c + d*x
)**7/(35*d) + 3*a*b*x*sin(c + d*x)**8/64 + 3*a*b*x*sin(c + d*x)**6*cos(c +
d*x)**2/16 + 9*a*b*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 3*a*b*x*sin(c + d
*x)**2*cos(c + d*x)**6/16 + 3*a*b*x*cos(c + d*x)**8/64 + 3*a*b*sin(c + d*x)
**7*cos(c + d*x)/(64*d) + 11*a*b*sin(c + d*x)**5*cos(c + d*x)**3/(64*d) - 1
1*a*b*sin(c + d*x)**3*cos(c + d*x)**5/(64*d) - 3*a*b*sin(c + d*x)*cos(c + d
*x)**7/(64*d) - b**2*sin(c + d*x)**4*cos(c + d*x)**5/(5*d) - 4*b**2*sin(c +
 d*x)**2*cos(c + d*x)**7/(35*d) - 8*b**2*cos(c + d*x)**9/(315*d), Ne(d, 0))
, (x*(a + b*sin(c))**2*sin(c)**3*cos(c)**4, True))
```

3.1106 $\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=278

$$-\frac{a\left(20a^2-69b^2\right)\sin^4(c+dx)\cos(c+dx)}{840bd}-\frac{\left(20a^2-63b^2\right)\sin^3(c+dx)\cos(c+dx)(a+b\sin(c+dx))^2}{336b^2d}-\frac{\left(8a^2+3\right.}{}$$

```
[Out] 1/128*(8*a^2+3*b^2)*x-6/35*a*b*cos(d*x+c)/d+2/35*a*b*cos(d*x+c)^3/d-1/128*(
8*a^2+3*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/1344*(40*a^4-140*a^2*b^2+21*b^4)*cos
(d*x+c)*sin(d*x+c)^3/b^2/d-1/840*a*(20*a^2-69*b^2)*cos(d*x+c)*sin(d*x+c)^4/
b/d-1/336*(20*a^2-63*b^2)*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^2/b^2/d+
5/56*a*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^3/b^2/d-1/8*cos(d*x+c)*sin(
d*x+c)^4*(a+b*sin(d*x+c))^3/b/d
```

Rubi [A] time = 0.63, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2895, 3049, 3033, 3023, 2748, 2635, 8, 2633}

$$-\frac{a\left(20a^2-69b^2\right)\sin^4(c+dx)\cos(c+dx)}{840bd}-\frac{\left(20a^2-63b^2\right)\sin^3(c+dx)\cos(c+dx)(a+b\sin(c+dx))^2}{336b^2d}-\frac{\left(-140a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((8*a^2 + 3*b^2)*x)/128 - (6*a*b*Cos[c + d*x])/(35*d) + (2*a*b*Cos[c + d*x]
^3)/(35*d) - ((8*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(128*d) - ((40*a^4
 - 140*a^2*b^2 + 21*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/(1344*b^2*d) - (a*(20
*a^2 - 69*b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(840*b*d) - ((20*a^2 - 63*b^2)*
Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(336*b^2*d) + (5*a*Cos[
c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(56*b^2*d) - (Cos[c + d*x]*
Sin[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(8*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{5a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^3}{56b^2d} - \frac{\cos(c+\ldots}{}$$

$$= -\frac{\left(20a^2-63b^2\right)\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx)\ldots}{336b^2d}$$

$$= -\frac{a\left(20a^2-69b^2\right)\cos(c+dx)\sin^4(c+dx)}{840bd} - \frac{\left(20a^2-63b^2\ldots\right.}{}$$

$$= -\frac{\left(40a^4-140a^2b^2+21b^4\right)\cos(c+dx)\sin^3(c+dx)}{1344b^2d} - \frac{a\left(2\ldots\right.}{}$$

$$= -\frac{\left(40a^4-140a^2b^2+21b^4\right)\cos(c+dx)\sin^3(c+dx)}{1344b^2d} - \frac{a\left(2\ldots\right.}{}$$

$$= -\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{128d} - \frac{\left(40a^4-140a^2b^2+\ldots\right.}{}$$

$$= \frac{1}{128}\left(8a^2+3b^2\right)x - \frac{6ab\cos(c+dx)}{35d} + \frac{2ab\cos^3(c+dx)}{35d} - \ldots$$

Mathematica [A] time = 0.62, size = 141, normalized size = 0.51

$$\frac{840a^2\sin(2(c+dx)) - 840a^2\sin(4(c+dx)) - 280a^2\sin(6(c+dx)) + 3360a^2dx - 5040ab\cos(c+dx) - 1680ab\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (1680*b^2*c + 3360*a^2*d*x + 1260*b^2*d*x - 5040*a*b*Cos[c + d*x] - 1680*a*
b*Cos[3*(c + d*x)] + 336*a*b*Cos[5*(c + d*x)] + 240*a*b*Cos[7*(c + d*x)] +
840*a^2*Sin[2*(c + d*x)] - 840*a^2*Sin[4*(c + d*x)] - 420*b^2*Sin[4*(c + d*
x)] - 280*a^2*Sin[6*(c + d*x)] + (105*b^2*Sin[8*(c + d*x)])/2)/(53760*d)
```

fricas [A] time = 0.75, size = 128, normalized size = 0.46

$$\frac{3840\,ab\cos\left(dx+c\right)^{7}-5376\,ab\cos\left(dx+c\right)^{5}+105\left(8\,a^{2}+3\,b^{2}\right)dx+35\left(48\,b^{2}\cos\left(dx+c\right)^{7}-8\left(8\,a^{2}+9\,b^{2}\right)\cos\right.}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/13440*(3840*a*b*cos(d*x + c)^7 - 5376*a*b*cos(d*x + c)^5 + 105*(8*a^2 + 3
*b^2)*d*x + 35*(48*b^2*cos(d*x + c)^7 - 8*(8*a^2 + 9*b^2)*cos(d*x + c)^5 +
2*(8*a^2 + 3*b^2)*cos(d*x + c)^3 + 3*(8*a^2 + 3*b^2)*cos(d*x + c))*sin(d*x
+ c))/d
```

giac [A] time = 0.31, size = 150, normalized size = 0.54

$$\frac{1}{128}\left(8\,a^{2}+3\,b^{2}\right)x+\frac{ab\cos\left(7\,dx+7\,c\right)}{224\,d}+\frac{ab\cos\left(5\,dx+5\,c\right)}{160\,d}-\frac{ab\cos\left(3\,dx+3\,c\right)}{32\,d}-\frac{3\,ab\cos\left(dx+c\right)}{32\,d}+\frac{b^{2}\sin\left(8\,dx+\right.}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/128*(8*a^2 + 3*b^2)*x + 1/224*a*b*cos(7*d*x + 7*c)/d + 1/160*a*b*cos(5*d*
x + 5*c)/d - 1/32*a*b*cos(3*d*x + 3*c)/d - 3/32*a*b*cos(d*x + c)/d + 1/1024
*b^2*sin(8*d*x + 8*c)/d - 1/192*a^2*sin(6*d*x + 6*c)/d + 1/64*a^2*sin(2*d*x
 + 2*c)/d - 1/128*(2*a^2 + b^2)*sin(4*d*x + 4*c)/d
```

maple [A] time = 0.33, size = 163, normalized size = 0.59

$$\frac{a^{2}\left(-\frac{\sin(dx+c)\left(\cos^{5}(dx+c)\right)}{6}+\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+2ab\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{7}-\frac{2\left(\cos^{5}(dx+c)\right)}{35}\right)+b^{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)+2*a*b*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d
```

```
*x+c)^5)+b^2*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1
/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c))
```

maxima [A] time = 0.35, size = 101, normalized size = 0.36

$$\frac{560\left(4\sin\left(2\,dx+2\,c\right)^{3}+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a^{2}+6144\left(5\,\cos\left(dx+c\right)^{7}-7\,\cos\left(dx+c\right)^{5}\right)ab+105}{107520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/107520*(560*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a
^2 + 6144*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a*b + 105*(24*d*x + 24*c +
sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*b^2)/d
```

mupad [B] time = 9.94, size = 139, normalized size = 0.50

$$210\,a^{2}\,\sin\left(2\,c+2\,d\,x\right)-210\,a^{2}\,\sin\left(4\,c+4\,d\,x\right)-70\,a^{2}\,\sin\left(6\,c+6\,d\,x\right)-105\,b^{2}\,\sin\left(4\,c+4\,d\,x\right)+\frac{105\,b^{2}\,\sin(8\,c+8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (210*a^2*sin(2*c + 2*d*x) - 210*a^2*sin(4*c + 4*d*x) - 70*a^2*sin(6*c + 6*d
*x) - 105*b^2*sin(4*c + 4*d*x) + (105*b^2*sin(8*c + 8*d*x))/8 - 1260*a*b*co
s(c + d*x) - 420*a*b*cos(3*c + 3*d*x) + 84*a*b*cos(5*c + 5*d*x) + 60*a*b*co
s(7*c + 7*d*x) + 840*a^2*d*x + 315*b^2*d*x)/(13440*d)
```

sympy [A] time = 9.87, size = 420, normalized size = 1.51

$$\begin{cases} \frac{a^{2}x\sin^{6}\left(c+dx\right)}{16}+\frac{3a^{2}x\sin^{4}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{16}+\frac{3a^{2}x\sin^{2}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{16}+\frac{a^{2}x\cos^{6}\left(c+dx\right)}{16}+\frac{a^{2}\sin^{5}\left(c+dx\right)\cos\left(c+dx\right)}{16d}+\frac{a^{2}\sin^{3}\left(c+d\right.}{} \\ x\left(a+b\sin\left(c\right)\right)^{2}\sin^{2}\left(c\right)\cos^{4}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/16 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a**2*x*cos(c + d*x)
**6/16 + a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + a**2*sin(c + d*x)**3*co
s(c + d*x)**3/(6*d) - a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 2*a*b*sin(
c + d*x)**2*cos(c + d*x)**5/(5*d) - 4*a*b*cos(c + d*x)**7/(35*d) + 3*b**2*x
```

```
*sin(c + d*x)**8/128 + 3*b**2*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 9*b**2
*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*b**2*x*sin(c + d*x)**2*cos(c + d*
x)**6/32 + 3*b**2*x*cos(c + d*x)**8/128 + 3*b**2*sin(c + d*x)**7*cos(c + d*
x)/(128*d) + 11*b**2*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*b**2*sin(
c + d*x)**3*cos(c + d*x)**5/(128*d) - 3*b**2*sin(c + d*x)*cos(c + d*x)**7/(
128*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**2*cos(c)**4, True))
```

3.1107 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=129

$$-\frac{(a^2+6b^2)\cos^5(c+dx)}{105d}-\frac{\cos^5(c+dx)(a+b\sin(c+dx))^2}{7d}-\frac{a\cos^5(c+dx)(a+b\sin(c+dx))}{21d}+\frac{ab\sin(c+dx)\,\mathrm{c}}{12d}$$

```
[Out] 1/8*a*b*x-1/105*(a^2+6*b^2)*cos(d*x+c)^5/d+1/8*a*b*cos(d*x+c)*sin(d*x+c)/d+
1/12*a*b*cos(d*x+c)^3*sin(d*x+c)/d-1/21*a*cos(d*x+c)^5*(a+b*sin(d*x+c))/d-1
/7*cos(d*x+c)^5*(a+b*sin(d*x+c))^2/d
```

Rubi [A] time = 0.18, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2862, 2669, 2635, 8}

$$-\frac{(a^2+6b^2)\cos^5(c+dx)}{105d}-\frac{\cos^5(c+dx)(a+b\sin(c+dx))^2}{7d}-\frac{a\cos^5(c+dx)(a+b\sin(c+dx))}{21d}+\frac{ab\sin(c+dx)\,\mathrm{c}}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*x)/8 - ((a^2 + 6*b^2)*Cos[c + d*x]^5)/(105*d) + (a*b*Cos[c + d*x]*Sin[
c + d*x])/(8*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(12*d) - (a*Cos[c + d*x
]^5*(a + b*Sin[c + d*x]))/(21*d) - (Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2)/
(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{\cos^5(c+dx)(a+b\sin(c+dx))^2}{7d}+\frac{1}{7}\int \cos^4(c+dx)(2b+$$
$$= -\frac{a\cos^5(c+dx)(a+b\sin(c+dx))}{21d}-\frac{\cos^5(c+dx)(a+b\sin(c+dx)}{7d}$$
$$= -\frac{\left(a^2+6b^2\right)\cos^5(c+dx)}{105d}-\frac{a\cos^5(c+dx)(a+b\sin(c+dx))}{21d}$$
$$= -\frac{\left(a^2+6b^2\right)\cos^5(c+dx)}{105d}+\frac{ab\cos^3(c+dx)\sin(c+dx)}{12d}-\frac{a}{}$$
$$= -\frac{\left(a^2+6b^2\right)\cos^5(c+dx)}{105d}+\frac{ab\cos(c+dx)\sin(c+dx)}{8d}+\frac{ab}{}$$
$$= \frac{abx}{8}-\frac{\left(a^2+6b^2\right)\cos^5(c+dx)}{105d}+\frac{ab\cos(c+dx)\sin(c+dx)}{8d}$$

Mathematica [A] time = 0.43, size = 132, normalized size = 1.02

$$\frac{-105\left(8a^2+3b^2\right)\cos(c+dx)-105\left(4a^2+b^2\right)\cos(3(c+dx))-84a^2\cos(5(c+dx))+210ab\sin(2(c+dx))-210}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (840*a*b*c + 840*a*b*d*x - 105*(8*a^2 + 3*b^2)*Cos[c + d*x] - 105*(4*a^2 +
b^2)*Cos[3*(c + d*x)] - 84*a^2*Cos[5*(c + d*x)] + 21*b^2*Cos[5*(c + d*x)] +
 15*b^2*Cos[7*(c + d*x)] + 210*a*b*Sin[2*(c + d*x)] - 210*a*b*Sin[4*(c + d*
x)] - 70*a*b*Sin[6*(c + d*x)])/(6720*d)
```

fricas [A] time = 0.74, size = 85, normalized size = 0.66

$$\frac{120\,b^2\cos\left(dx+c\right)^7 - 168\left(a^2+b^2\right)\cos\left(dx+c\right)^5 + 105\,abdx - 35\left(8\,ab\cos\left(dx+c\right)^5 - 2\,ab\cos\left(dx+c\right)^3 - 3\,ab$$

$$840\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*b^2*cos(d*x + c)^7 - 168*(a^2 + b^2)*cos(d*x + c)^5 + 105*a*b*d*
x - 35*(8*a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c))*s
in(d*x + c))/d
```

giac [A] time = 0.22, size = 141, normalized size = 1.09

$$\frac{1}{8}\,abx + \frac{b^2\cos\left(7\,dx+7\,c\right)}{448\,d} - \frac{ab\sin\left(6\,dx+6\,c\right)}{96\,d} - \frac{ab\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{ab\sin\left(2\,dx+2\,c\right)}{32\,d} - \frac{\left(4\,a^2-b^2\right)\cos\left(5\,dx+5\right.}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*a*b*x + 1/448*b^2*cos(7*d*x + 7*c)/d - 1/96*a*b*sin(6*d*x + 6*c)/d - 1/
32*a*b*sin(4*d*x + 4*c)/d + 1/32*a*b*sin(2*d*x + 2*c)/d - 1/320*(4*a^2 - b^
2)*cos(5*d*x + 5*c)/d - 1/64*(4*a^2 + b^2)*cos(3*d*x + 3*c)/d - 1/64*(8*a^2
 + 3*b^2)*cos(d*x + c)/d
```

maple [A] time = 0.33, size = 105, normalized size = 0.81

$$\frac{-\frac{a^2\left(\cos^5\left(dx+c\right)\right)}{5} + 2ab\left(-\frac{\sin\left(dx+c\right)\left(\cos^5\left(dx+c\right)\right)}{6} + \frac{\left(\cos^3\left(dx+c\right)+\frac{3\cos\left(dx+c\right)}{2}\right)\sin\left(dx+c\right)}{24} + \frac{dx}{16} + \frac{c}{16}\right) + b^2\left(-\frac{\left(\sin^2\left(dx+c\right)\right)\left(\cos^5\left(dx+c\right)\right)}{7} -\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/5*a^2*cos(d*x+c)^5+2*a*b*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*
x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+b^2*(-1/7*sin(d*x+c)^2*c
os(d*x+c)^5-2/35*cos(d*x+c)^5))
```

maxima [A] time = 0.38, size = 81, normalized size = 0.63

$$-\frac{672\,a^2\cos\left(dx+c\right)^5 - 35\left(4\,\sin\left(2\,dx+2\,c\right)^3 + 12\,dx + 12\,c - 3\,\sin\left(4\,dx+4\,c\right)\right)ab - 96\left(5\,\cos\left(dx+c\right)^7 - 7\,\mathrm{co}\right.}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*(672*a^2*cos(d*x + c)^5 - 35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c
- 3*sin(4*d*x + 4*c))*a*b - 96*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b^2)/d
```

mupad [B] time = 13.01, size = 256, normalized size = 1.98

$$\frac{a\,b\,x}{8} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(6\,a^2-4\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(4\,a^2+4\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{4\,a^2}{5}+\frac{4\,b^2}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (a*b*x)/8 - (tan(c/2 + (d*x)/2)^8*(6*a^2 - 4*b^2) + tan(c/2 + (d*x)/2)^10*(
4*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^2*((4*a^2)/5 + (4*b^2)/5) + tan(c/2 + (
d*x)/2)^6*(8*a^2 + 8*b^2) + tan(c/2 + (d*x)/2)^4*((22*a^2)/5 - (8*b^2)/5) +
 2*a^2*tan(c/2 + (d*x)/2)^12 + (2*a^2)/5 + (4*b^2)/35 - (11*a*b*tan(c/2 + (
d*x)/2)^3)/3 + (31*a*b*tan(c/2 + (d*x)/2)^5)/12 - (31*a*b*tan(c/2 + (d*x)/2
)^9)/12 + (11*a*b*tan(c/2 + (d*x)/2)^11)/3 - (a*b*tan(c/2 + (d*x)/2)^13)/4
+ (a*b*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [A] time = 5.77, size = 223, normalized size = 1.73

$$\begin{cases} -\frac{a^2\cos^5(c+dx)}{5d}+\frac{abx\sin^6(c+dx)}{8}+\frac{3abx\sin^4(c+dx)\cos^2(c+dx)}{8}+\frac{3abx\sin^2(c+dx)\cos^4(c+dx)}{8}+\frac{abx\cos^6(c+dx)}{8}+\frac{ab\sin^5(c+dx)\cos(c+d}{8d} \\ x\left(a+b\sin(c)\right)^2\sin(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)**5/(5*d) + a*b*x*sin(c + d*x)**6/8 + 3*a*b*x*
sin(c + d*x)**4*cos(c + d*x)**2/8 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**4
/8 + a*b*x*cos(c + d*x)**6/8 + a*b*sin(c + d*x)**5*cos(c + d*x)/(8*d) + a*b
*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - a*b*sin(c + d*x)*cos(c + d*x)**5/(
8*d) - b**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 2*b**2*cos(c + d*x)**7/
(35*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)*cos(c)**4, True))
```

3.1108 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{ab\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3ab\sin(c+dx)\cos(c+}{4d}$$

```
[Out] 3/4*a*b*x-a^2*arctanh(cos(d*x+c))/d+a^2*cos(d*x+c)/d+1/3*a^2*cos(d*x+c)^3/d
-1/5*b^2*cos(d*x+c)^5/d+3/4*a*b*cos(d*x+c)*sin(d*x+c)/d+1/2*a*b*cos(d*x+c)^
3*sin(d*x+c)/d
```

Rubi [A] time = 0.45, antiderivative size = 190, normalized size of antiderivative = 1.64, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2895, 3049, 3033, 3023, 2735, 3770}

$$-\frac{(-14a^2b^2+a^4+3b^4)\cos(c+dx)}{15b^2d}-\frac{(a^2-12b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{30b^2d}-\frac{a(2a^2-27b^2)\sin(c+dx)\cos}{60bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*b*x)/4 - (a^2*ArcTanh[Cos[c + d*x]])/d - ((a^4 - 14*a^2*b^2 + 3*b^4)*C
os[c + d*x])/(15*b^2*d) - (a*(2*a^2 - 27*b^2)*Cos[c + d*x]*Sin[c + d*x])/(6
0*b*d) - ((a^2 - 12*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(30*b^2*d) +
(a*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(10*b^2*d) - (Cos[c + d*x]*Sin[c +
d*x]*(a + b*Sin[c + d*x])^3)/(5*b*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
```

n + 4, 0]

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{a\cos(c+dx)(a+b\sin(c+dx))^3}{10b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{5bd}$$
$$= -\frac{\left(a^2-12b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{30b^2d} + \frac{a\cos(c+d}{}$$
$$= -\frac{a\left(2a^2-27b^2\right)\cos(c+dx)\sin(c+dx)}{60bd} - \frac{\left(a^2-12b^2\right)\cos(}{}$$
$$= -\frac{\left(a^4-14a^2b^2+3b^4\right)\cos(c+dx)}{15b^2d} - \frac{a\left(2a^2-27b^2\right)\cos(c+}{60bd}$$
$$= \frac{3abx}{4} - \frac{\left(a^4-14a^2b^2+3b^4\right)\cos(c+dx)}{15b^2d} - \frac{a\left(2a^2-27b^2\right)c}{}$$
$$= \frac{3abx}{4} - \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{\left(a^4-14a^2b^2+3b^4\right)\cos(}{15b^2d}$$

Mathematica [A] time = 0.50, size = 125, normalized size = 1.08

$$\frac{30\left(10a^2-b^2\right)\cos(c+dx)+5\left(4a^2-3b^2\right)\cos(3(c+dx))+15a\left(4\left(4a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4a\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (30*(10*a^2 - b^2)*Cos[c + d*x] + 5*(4*a^2 - 3*b^2)*Cos[3*(c + d*x)] - 3*b^
2*Cos[5*(c + d*x)] + 15*a*(4*(3*b*(c + d*x) - 4*a*Log[Cos[(c + d*x)/2]] + 4
*a*Log[Sin[(c + d*x)/2]]) + 8*b*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)]))/(24
0*d)
```

fricas [A] time = 0.64, size = 112, normalized size = 0.97

$$-\frac{12\,b^2\cos\left(dx+c\right)^5-20\,a^2\cos\left(dx+c\right)^3-45\,abdx-60\,a^2\cos\left(dx+c\right)+30\,a^2\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-30\,a^2}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(12*b^2*cos(d*x + c)^5 - 20*a^2*cos(d*x + c)^3 - 45*a*b*d*x - 60*a^2*
cos(d*x + c) + 30*a^2*log(1/2*cos(d*x + c) + 1/2) - 30*a^2*log(-1/2*cos(d*x
 + c) + 1/2) - 15*(2*a*b*cos(d*x + c)^3 + 3*a*b*cos(d*x + c))*sin(d*x + c))
/d
```

giac [B] time = 0.21, size = 213, normalized size = 1.84

$$\frac{45\,(dx+c)ab+60\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-120\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+60\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+30\,ab\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(45*(d*x + c)*a*b + 60*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(75*a*b*
tan(1/2*d*x + 1/2*c)^9 - 120*a^2*tan(1/2*d*x + 1/2*c)^8 + 60*b^2*tan(1/2*d*
x + 1/2*c)^8 + 30*a*b*tan(1/2*d*x + 1/2*c)^7 - 360*a^2*tan(1/2*d*x + 1/2*c)
^6 - 440*a^2*tan(1/2*d*x + 1/2*c)^4 + 120*b^2*tan(1/2*d*x + 1/2*c)^4 - 30*a
*b*tan(1/2*d*x + 1/2*c)^3 - 280*a^2*tan(1/2*d*x + 1/2*c)^2 - 75*a*b*tan(1/2
*d*x + 1/2*c) - 80*a^2 + 12*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

maple [A] time = 0.54, size = 123, normalized size = 1.06

$$\frac{a^2\left(\cos^3(dx+c)\right)}{3d}+\frac{a^2\cos(dx+c)}{d}+\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{ab\left(\cos^3(dx+c)\right)\sin(dx+c)}{2d}+\frac{3ab\cos(\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/3*a^2*cos(d*x+c)^3/d+a^2*cos(d*x+c)/d+1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+1
/2*a*b*cos(d*x+c)^3*sin(d*x+c)/d+3/4*a*b*cos(d*x+c)*sin(d*x+c)/d+3/4*a*b*x+
3/4/d*a*b*c-1/5*b^2*cos(d*x+c)^5/d
```

maxima [A] time = 0.39, size = 97, normalized size = 0.84

$$-\frac{48\,b^2\cos(dx+c)^5-40\left(2\,\cos(dx+c)^3+6\,\cos(dx+c)-3\,\log(\cos(dx+c)+1)+3\,\log(\cos(dx+c)-1)\right)a^2\ldots}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(48*b^2*cos(d*x + c)^5 - 40*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*l
og(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^2 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b)/d
```

mupad [B] time = 11.12, size = 319, normalized size = 2.75

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(4\,a^2-2\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{44\,a^2}{3}-4\,b^2\right)+\frac{28\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+12\,a^2\,\tan\left(\cdots\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)^8*(4*a^2 - 2*b^2) + t
an(c/2 + (d*x)/2)^4*((44*a^2)/3 - 4*b^2) + (28*a^2*tan(c/2 + (d*x)/2)^2)/3
+ 12*a^2*tan(c/2 + (d*x)/2)^6 + (8*a^2)/3 - (2*b^2)/5 + a*b*tan(c/2 + (d*x)
/2)^3 - a*b*tan(c/2 + (d*x)/2)^7 - (5*a*b*tan(c/2 + (d*x)/2)^9)/2 + (5*a*b*
tan(c/2 + (d*x)/2))/2)/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4
 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10
 + 1)) + (3*a*b*atan((9*a^2*b^2)/(4*(3*a^3*b - (9*a^2*b^2*tan(c/2 + (d*x)/2
))/4)) + (3*a^3*b*tan(c/2 + (d*x)/2))/(3*a^3*b - (9*a^2*b^2*tan(c/2 + (d*x)
/2))/4)))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sin \left(c + dx\right)\right)^{2} \cos^{4}\left(c + dx\right) \csc \left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cos(c + d*x)**4*csc(c + d*x), x)
```

3.1109 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=181

$$\frac{a\left(a^2+28b^2\right)\cos(c+dx)}{6bd}+\frac{\left(a^2+12b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{12abd}+\frac{\left(2a^2+39b^2\right)\sin(c+dx)\cos(c+dx)}{24d}-\frac{3}{8}$$

```
[Out] -3/8*(4*a^2-b^2)*x-2*a*b*arctanh(cos(d*x+c))/d+1/6*a*(a^2+28*b^2)*cos(d*x+c
)/b/d+1/24*(2*a^2+39*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/12*(a^2+12*b^2)*cos(d*x
+c)*(a+b*sin(d*x+c))^2/a/b/d-1/4*cos(d*x+c)*(a+b*sin(d*x+c))^3/b/d-cot(d*x+
c)*(a+b*sin(d*x+c))^3/a/d
```

Rubi [A] time = 0.52, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2894, 3049, 3033, 3023, 2735, 3770}

$$\frac{a\left(a^2+28b^2\right)\cos(c+dx)}{6bd}+\frac{\left(a^2+12b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{12abd}+\frac{\left(2a^2+39b^2\right)\sin(c+dx)\cos(c+dx)}{24d}-\frac{3}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*(4*a^2 - b^2)*x)/8 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (a*(a^2 + 28*b^2
)*Cos[c + d*x])/(6*b*d) + ((2*a^2 + 39*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*
d) + ((a^2 + 12*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(12*a*b*d) - (Cos
[c + d*x]*(a + b*Sin[c + d*x])^3)/(4*b*d) - (Cot[c + d*x]*(a + b*Sin[c + d*
x])^3)/(a*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
```

```
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{\cos(c+dx)(a+b\sin(c+dx))^3}{4bd} - \frac{\cot(c+dx)(a+b\sin(c+\cdots}{ad}$$
$$= \frac{\left(a^2+12b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{12abd} - \frac{\cos(c+dx)(a\cdots}{}$$
$$= \frac{\left(2a^2+39b^2\right)\cos(c+dx)\sin(c+dx)}{24d} + \frac{\left(a^2+12b^2\right)\cos(c+\cdots}{1\cdots}$$
$$= \frac{a\left(a^2+28b^2\right)\cos(c+dx)}{6bd} + \frac{\left(2a^2+39b^2\right)\cos(c+dx)\sin(c\cdots}{24d}$$
$$= -\frac{3}{8}\left(4a^2-b^2\right)x + \frac{a\left(a^2+28b^2\right)\cos(c+dx)}{6bd} + \frac{\left(2a^2+39b^2\right)\cdots}{}$$
$$= -\frac{3}{8}\left(4a^2-b^2\right)x - \frac{2ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\left(a^2+28b^2\right)\cdots}{6b\cdots}$$

Mathematica [A] time = 0.70, size = 167, normalized size = 0.92

$$-\frac{3a^2(c+dx)}{2d}-\frac{a^2\sin(2(c+dx))}{4d}-\frac{a^2\cot(c+dx)}{d}+\frac{5ab\cos(c+dx)}{2d}+\frac{ab\cos(3(c+dx))}{6d}+\frac{2ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*a^2*(c + d*x))/(2*d) + (3*b^2*(c + d*x))/(8*d) + (5*a*b*Cos[c + d*x])/(
2*d) + (a*b*Cos[3*(c + d*x)])/(6*d) - (a^2*Cot[c + d*x])/d - (2*a*b*Log[Cos
[(c + d*x)/2]])/d + (2*a*b*Log[Sin[(c + d*x)/2]])/d - (a^2*Sin[2*(c + d*x)]
)/(4*d) + (b^2*Sin[2*(c + d*x)])/(4*d) + (b^2*Sin[4*(c + d*x)])/(32*d)
```

fricas [A] time = 0.80, size = 155, normalized size = 0.86

$$-\frac{6\,b^2\cos\left(dx+c\right)^5-3\left(4\,a^2-b^2\right)\cos\left(dx+c\right)^3+24\,ab\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-24\,ab\log\left(-\frac{1}{2}\,\cos\cdots\right.}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*b^2*cos(d*x + c)^5 - 3*(4*a^2 - b^2)*cos(d*x + c)^3 + 24*a*b*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 24*a*b*log(-1/2*cos(d*x + c) + 1/2)*s
```

```
in(d*x + c) + 9*(4*a^2 - b^2)*cos(d*x + c) - (16*a*b*cos(d*x + c)^3 - 9*(4*
a^2 - b^2)*d*x + 48*a*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.22, size = 274, normalized size = 1.51

$$48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\left(4\,a^2-b^2\right)(dx+c)-\frac{12\left(4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(12\,a^2\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(48*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 12*a^2*tan(1/2*d*x + 1/2*c) -
 9*(4*a^2 - b^2)*(d*x + c) - 12*(4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*
d*x + 1/2*c) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 - 15*b^2*tan(1/2*d*x + 1/2*
c)^7 + 96*a*b*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*tan(1/2*d*x + 1/2*c)^5 + 9*b^
2*tan(1/2*d*x + 1/2*c)^5 + 192*a*b*tan(1/2*d*x + 1/2*c)^4 - 12*a^2*tan(1/2*
d*x + 1/2*c)^3 - 9*b^2*tan(1/2*d*x + 1/2*c)^3 + 160*a*b*tan(1/2*d*x + 1/2*c
)^2 - 12*a^2*tan(1/2*d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/2*c) + 64*a*b)/(
tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.54, size = 191, normalized size = 1.06

$$-\frac{a^2\left(\cos^5(dx+c)\right)}{d\sin(dx+c)}-\frac{a^2\left(\cos^3(dx+c)\right)\sin(dx+c)}{d}-\frac{3a^2\cos(dx+c)\sin(dx+c)}{2d}-\frac{3a^2x}{2}-\frac{3a^2c}{2d}+\frac{2ab\left(\cos^3(dx+\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/sin(d*x+c)*cos(d*x+c)^5-a^2*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a^2*cos(
d*x+c)*sin(d*x+c)/d-3/2*a^2*x-3/2/d*a^2*c+2/3*a*b*cos(d*x+c)^3/d+2*a*b*cos(
d*x+c)/d+2/d*a*b*ln(csc(d*x+c)-cot(d*x+c))+1/4*b^2*cos(d*x+c)^3*sin(d*x+c)/
d+3/8*b^2*cos(d*x+c)*sin(d*x+c)/d+3/8*b^2*x+3/8/d*b^2*c
```

maxima [A] time = 0.49, size = 127, normalized size = 0.70

$$-\frac{48\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^2-32\left(2\,\cos(dx+c)^3+6\,\cos(dx+c)-3\,\log(\cos(dx+c)+1)+3\,\log\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

[Out] -1/96*(48*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*a^2 - 32*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a*b - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*b^2)/d

mupad [B] time = 9.54, size = 578, normalized size = 3.19

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(a^2-\frac{5\,b^2}{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(6\,a^2-\frac{5\,b^2}{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(8\,a^2+\frac{3\,b^2}{2}\right)-a^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(2\,a^2-\right.}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x)^2,x)

[Out] (tan(c/2 + (d*x)/2)^8*(a^2 - (5*b^2)/2) - tan(c/2 + (d*x)/2)^2*(6*a^2 - (5*b^2)/2) - tan(c/2 + (d*x)/2)^4*(8*a^2 + (3*b^2)/2) - a^2 - tan(c/2 + (d*x)/2)^6*(2*a^2 - (3*b^2)/2) + (80*a*b*tan(c/2 + (d*x)/2)^3)/3 + 32*a*b*tan(c/2 + (d*x)/2)^5 + 16*a*b*tan(c/2 + (d*x)/2)^7 + (32*a*b*tan(c/2 + (d*x)/2))/3)/(d*(2*tan(c/2 + (d*x)/2) + 8*tan(c/2 + (d*x)/2)^3 + 12*tan(c/2 + (d*x)/2)^5 + 8*tan(c/2 + (d*x)/2)^7 + 2*tan(c/2 + (d*x)/2)^9)) + (a^2*tan(c/2 + (d*x)/2))/(2*d) - (atan((((a^2*3i)/2 - (b^2*3i)/8)*((3*b^2)/4 - 3*a^2 + 6*tan(c/2 + (d*x)/2)*((a^2*3i)/2 - (b^2*3i)/8) + 4*a*b*tan(c/2 + (d*x)/2))*1i - ((a^2*3i)/2 - (b^2*3i)/8)*(3*a^2 - (3*b^2)/4 + 6*tan(c/2 + (d*x)/2)*((a^2*3i)/2 - (b^2*3i)/8) - 4*a*b*tan(c/2 + (d*x)/2))*1i)/(((a^2*3i)/2 - (b^2*3i)/8)*((3*b^2)/4 - 3*a^2 + 6*tan(c/2 + (d*x)/2)*((a^2*3i)/2 - (b^2*3i)/8) + 4*a*b*tan(c/2 + (d*x)/2)) + ((a^2*3i)/2 - (b^2*3i)/8)*(3*a^2 - (3*b^2)/4 + 6*tan(c/2 + (d*x)/2)*((a^2*3i)/2 - (b^2*3i)/8) - 4*a*b*tan(c/2 + (d*x)/2)) + 3*a*b^3 - 12*a^3*b + 2*tan(c/2 + (d*x)/2)*(9*a^4 + (9*b^4)/16 - (9*a^2*b^2)/2)))*(3*a^2 - (3*b^2)/4))/d + (2*a*b*log(tan(c/2 + (d*x)/2)))/d

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**2,x)

[Out] Timed out

3.1110 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=189

$$-\frac{\left(4a^2-23b^2\right)\cos(c+dx)}{6d}-\frac{\left(2a^2-3b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{6a^2d}-\frac{b\left(a^2-3b^2\right)\sin(c+dx)\cos(c+dx)}{3ad}+$$

```
[Out] -3*a*b*x+1/2*(3*a^2-2*b^2)*arctanh(cos(d*x+c))/d-1/6*(4*a^2-23*b^2)*cos(d*x
+c)/d-1/3*b*(a^2-3*b^2)*cos(d*x+c)*sin(d*x+c)/a/d-1/6*(2*a^2-3*b^2)*cos(d*x
+c)*(a+b*sin(d*x+c))^2/a^2/d-1/2*b*cot(d*x+c)*(a+b*sin(d*x+c))^3/a^2/d-1/2*
cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^3/a/d
```

Rubi [A] time = 0.48, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2893, 3049, 3033, 3023, 2735, 3770}

$$-\frac{\left(4a^2-23b^2\right)\cos(c+dx)}{6d}-\frac{\left(2a^2-3b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{6a^2d}-\frac{b\left(a^2-3b^2\right)\sin(c+dx)\cos(c+dx)}{3ad}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -3*a*b*x + ((3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) - ((4*a^2 - 23*b^2
)*Cos[c + d*x])/(6*d) - (b*(a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(3*a*d)
 - ((2*a^2 - 3*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(6*a^2*d) - (b*Cot
[c + d*x]*(a + b*Sin[c + d*x])^3)/(2*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]*(a
 + b*Sin[c + d*x])^3)/(2*a*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

```
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{b\cot(c+dx)(a+b\sin(c+dx))^3}{2a^2d} - \frac{\cot(c+dx)\csc(c+dx}{2a}$$
$$= -\frac{\left(2a^2-3b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{6a^2d} - \frac{b\cot(c+dx}{}$$
$$= -\frac{b\left(a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{3ad} - \frac{\left(2a^2-3b^2\right)\cos(c+}{}$$
$$= -\frac{\left(4a^2-23b^2\right)\cos(c+dx)}{6d} - \frac{b\left(a^2-3b^2\right)\cos(c+dx)\sin(c}{3ad}$$
$$= -3abx - \frac{\left(4a^2-23b^2\right)\cos(c+dx)}{6d} - \frac{b\left(a^2-3b^2\right)\cos(c+dx}{3ad}$$
$$= -3abx + \frac{\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2d} - \frac{\left(4a^2-23b^2\right)c}{6d}$$

Mathematica [A] time = 3.36, size = 191, normalized size = 1.01

$$\frac{-6\left(4a^2-5b^2\right)\cos(c+dx)+3\left(a^2\left(-\csc^2\left(\frac{1}{2}(c+dx)\right)\right)+a^2\sec^2\left(\frac{1}{2}(c+dx)\right)-12a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-6*(4*a^2 - 5*b^2)*Cos[c + d*x] + 2*b^2*Cos[3*(c + d*x)] + 3*(-24*a*b*c -
24*a*b*d*x - 8*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 + 12*a^2*Log[C
os[(c + d*x)/2]] - 8*b^2*Log[Cos[(c + d*x)/2]] - 12*a^2*Log[Sin[(c + d*x)/2
]] + 8*b^2*Log[Sin[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 - 4*a*b*Sin[2*(c
+ d*x)] + 8*a*b*Tan[(c + d*x)/2]))/(24*d)
```

fricas [A] time = 0.75, size = 210, normalized size = 1.11

$$\frac{4\,b^2\cos\left(dx+c\right)^5-36\,abdx\cos\left(dx+c\right)^2+36\,abdx-4\left(3\,a^2-2\,b^2\right)\cos\left(dx+c\right)^3+6\left(3\,a^2-2\,b^2\right)\cos\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(4*b^2*cos(d*x + c)^5 - 36*a*b*d*x*cos(d*x + c)^2 + 36*a*b*d*x - 4*(3*
a^2 - 2*b^2)*cos(d*x + c)^3 + 6*(3*a^2 - 2*b^2)*cos(d*x + c) + 3*((3*a^2 -
```

```
2*b^2)*cos(d*x + c)^2 - 3*a^2 + 2*b^2)*log(1/2*cos(d*x + c) + 1/2) - 3*((3*
a^2 - 2*b^2)*cos(d*x + c)^2 - 3*a^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2) -
 12*(a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
^2 - d)
```

giac [A] time = 0.24, size = 252, normalized size = 1.33

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,(dx+c)ab+24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\left(3\,a^2-2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{3\left(18\,a^2\,t\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*tan(1/2*d*x + 1/2*c)^2 - 72*(d*x + c)*a*b + 24*a*b*tan(1/2*d*x
+ 1/2*c) - 12*(3*a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 3*(18*a^2*ta
n(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d*x +
1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2 + 16*(3*a*b*tan(1/2*d*x + 1/2*c)^5 - 3
*a^2*tan(1/2*d*x + 1/2*c)^4 + 6*b^2*tan(1/2*d*x + 1/2*c)^4 - 6*a^2*tan(1/2*
d*x + 1/2*c)^2 + 6*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c)
- 3*a^2 + 4*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.65, size = 208, normalized size = 1.10

$$-\frac{a^2\left(\cos^5(dx+c)\right)}{2d\sin(dx+c)^2}-\frac{a^2\left(\cos^3(dx+c)\right)}{2d}-\frac{3a^2\cos(dx+c)}{2d}-\frac{3a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{2ab\left(\cos^5(dx+c)\right)}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*a^2*cos(d*x+c)^3/d-3/2*a^2*cos(d*x
+c)/d-3/2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/d*a*b/sin(d*x+c)*cos(d*x+c)^5-2
*a*b*cos(d*x+c)^3*sin(d*x+c)/d-3*a*b*cos(d*x+c)*sin(d*x+c)/d-3*a*b*x-3/d*a*
b*c+1/3*b^2*cos(d*x+c)^3/d+b^2*cos(d*x+c)/d+1/d*b^2*ln(csc(d*x+c)-cot(d*x+c
))
```

maxima [A] time = 0.48, size = 150, normalized size = 0.79

$$-\frac{12\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)ab-2\left(2\,\cos(dx+c)^3+6\,\cos(dx+c)-3\,\log(\cos(dx+c)+1)+3\,\log(c\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*(12*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x +
 c)))*a*b - 2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1)
+ 3*log(cos(d*x + c) - 1))*b^2 - 3*a^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1)
 - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [**B**] time = 9.44, size = 397, normalized size = 2.10

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{2}-b^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{17\,a^2}{2}-16\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{35\,a^2}{2}-16\,b^2\right)+}{d\,\left(4\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) - (log(tan(c/2 + (d*x)/2))*((3*a^2)/2 - b^
2))/d - (tan(c/2 + (d*x)/2)^6*((17*a^2)/2 - 16*b^2) + tan(c/2 + (d*x)/2)^4*
((35*a^2)/2 - 16*b^2) + tan(c/2 + (d*x)/2)^2*((19*a^2)/2 - (32*b^2)/3) + a^
2/2 + 20*a*b*tan(c/2 + (d*x)/2)^3 + 12*a*b*tan(c/2 + (d*x)/2)^5 - 4*a*b*tan
(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 +
12*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 + 4*tan(c/2 + (d*x)/2)^8)
) + (6*a*b*atan((36*a^2*b^2)/(12*a*b^3 - 18*a^3*b + 36*a^2*b^2*tan(c/2 + (d
*x)/2)) - (12*a*b^3*tan(c/2 + (d*x)/2))/(12*a*b^3 - 18*a^3*b + 36*a^2*b^2*t
an(c/2 + (d*x)/2)) + (18*a^3*b*tan(c/2 + (d*x)/2))/(12*a*b^3 - 18*a^3*b + 3
6*a^2*b^2*tan(c/2 + (d*x)/2))))/d + (a*b*tan(c/2 + (d*x)/2))/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1111 $\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=133

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{3ab\cos(c+dx)}{d}-\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-$$

```
[Out] a^2*x-3/2*b^2*x+3*a*b*arctanh(cos(d*x+c))/d-3*a*b*cos(d*x+c)/d+a^2*cot(d*x+
c)/d-3/2*b^2*cot(d*x+c)/d+1/2*b^2*cos(d*x+c)^2*cot(d*x+c)/d-a*b*cos(d*x+c)*
cot(d*x+c)^2/d-1/3*a^2*cot(d*x+c)^3/d
```

Rubi [A] time = 0.16, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 2591, 288, 321, 203, 2592, 206, 3473, 8}

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{3ab\cos(c+dx)}{d}-\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] a^2*x - (3*b^2*x)/2 + (3*a*b*ArcTanh[Cos[c + d*x]])/d - (3*a*b*Cos[c + d*x]
)/d + (a^2*Cot[c + d*x])/d - (3*b^2*Cot[c + d*x])/(2*d) + (b^2*Cos[c + d*x]
^2*Cot[c + d*x])/(2*d) - (a*b*Cos[c + d*x]*Cot[c + d*x]^2)/d - (a^2*Cot[c +
 d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx = \int \left(b^2\cos^2(c+dx)\cot^2(c+dx)+2ab\cos(c+dx)\cot^3(c+dx)+a^2\cot^4(\right.$$
$$= a^2\int \cot^4(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot^3(c+dx)\,dx+b^2\int \cos^2($$
$$= -\frac{a^2\cot^3(c+dx)}{3d}-a^2\int \cot^2(c+dx)\,dx-\frac{(2ab)\,\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx,x,c\right.}{d}$$
$$= \frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}$$
$$= a^2x-\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(c+dx)}{d}-\frac{3b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2(c+}{}$$
$$= a^2x-\frac{3b^2x}{2}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(c+d}{d}$$

Mathematica [B] time = 6.19, size = 293, normalized size = 2.20

$$\frac{(2a^2-3b^2)(c+dx)}{2d}+\frac{\csc\left(\frac{1}{2}(c+dx)\right)\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{6d}+\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(3b^2\sin\left(\frac{1}{2}(c\right.\right.}{6d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - 3*b^2)*(c + d*x))/(2*d) - (2*a*b*Cos[c + d*x])/d + ((4*a^2*Cos[(c
 + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (a*b*Csc[(c
+ d*x)/2]^2)/(4*d) - (a^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (3*
a*b*Log[Cos[(c + d*x)/2]])/d - (3*a*b*Log[Sin[(c + d*x)/2]])/d + (a*b*Sec[(
c + d*x)/2]^2)/(4*d) + (Sec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 3*b^2*S
in[(c + d*x)/2]))/(6*d) - (b^2*Sin[2*(c + d*x)])/(4*d) + (a^2*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/(24*d)
```

fricas [A] time = 0.56, size = 218, normalized size = 1.64

$$\frac{3b^2\cos(dx+c)^5+4\left(2a^2-3b^2\right)\cos(dx+c)^3+9\left(ab\cos(dx+c)^2-ab\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*b^2*cos(d*x + c)^5 + 4*(2*a^2 - 3*b^2)*cos(d*x + c)^3 + 9*(a*b*cos(d
*x + c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a*b*cos(d*x
+ c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(2*a^2 - 3*b^2)
*cos(d*x + c) + 3*((2*a^2 - 3*b^2)*d*x*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)^
3 - (2*a^2 - 3*b^2)*d*x + 6*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c
)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.26, size = 241, normalized size = 1.81

$$a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,b^2\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 - 72*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c))) - 15*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2
*d*x + 1/2*c) + 12*(2*a^2 - 3*b^2)*(d*x + c) + 24*(b^2*tan(1/2*d*x + 1/2*c)
^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan(1/2*d*x + 1/2*c) - 4*a*b)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a*b*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/
2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b*tan(1/2*d*x + 1/2*
c) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.45, size = 199, normalized size = 1.50

$$-\frac{a^2\left(\cot^3(dx+c)\right)}{3d}+\frac{a^2\cot(dx+c)}{d}+a^2x+\frac{a^2c}{d}-\frac{ab\left(\cos^5(dx+c)\right)}{d\sin(dx+c)^2}-\frac{ab\left(\cos^3(dx+c)\right)}{d}-\frac{3ab\cos(dx+c)}{d}-\frac{3ab\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3*a^2*cot(d*x+c)^3/d+a^2*cot(d*x+c)/d+a^2*x+1/d*a^2*c-1/d*a*b/sin(d*x+c)
^2*cos(d*x+c)^5-a*b*cos(d*x+c)^3/d-3*a*b*cos(d*x+c)/d-3/d*a*b*ln(csc(d*x+c)
-cot(d*x+c))-1/d*b^2/sin(d*x+c)*cos(d*x+c)^5-b^2*cos(d*x+c)^3*sin(d*x+c)/d-
3/2*b^2*cos(d*x+c)*sin(d*x+c)/d-3/2*b^2*x-3/2/d*b^2*c
```

maxima [A] time = 0.56, size = 138, normalized size = 1.04

$$\frac{2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-3\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)b^2+3\,ab\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos(dx+c)+3\,\mathrm{lo}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*(3*d*x
 + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*b^2 + 3*a*
b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c
) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 11.75, size = 584, normalized size = 4.39

$$-\frac{\frac{5\,b^2\,\cos(c+d\,x)}{16}+\frac{a^2\,\cos(3\,c+3\,d\,x)}{3}-\frac{11\,b^2\,\cos(3\,c+3\,d\,x)}{32}+\frac{b^2\,\cos(5\,c+5\,d\,x)}{32}+\frac{a^2\,\mathrm{atan}\left(\frac{-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+6\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b+3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+6\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b-3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}\right)\,\sin(}{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] -((5*b^2*cos(c + d*x))/16 + (a^2*cos(3*c + 3*d*x))/3 - (11*b^2*cos(3*c + 3*
d*x))/32 + (b^2*cos(5*c + 5*d*x))/32 + (a^2*atan((3*b^2*cos(c/2 + (d*x)/2)
- 2*a^2*cos(c/2 + (d*x)/2) + 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*
x)/2) - 3*b^2*sin(c/2 + (d*x)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d
*x))/2 - (3*b^2*atan((3*b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) +
 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - 3*b^2*sin(c/2 + (d*x
)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d*x))/4 + (3*a*b*sin(c + d*x)
)/2 - (3*a^2*atan((3*b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) + 6*
a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - 3*b^2*sin(c/2 + (d*x)/2
) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(c + d*x))/2 + (9*b^2*atan((3*b^2*cos(c/2
 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) + 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*s
in(c/2 + (d*x)/2) - 3*b^2*sin(c/2 + (d*x)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*s
in(c + d*x))/4 + a*b*sin(2*c + 2*d*x) - (a*b*sin(3*c + 3*d*x))/2 - (a*b*sin
(4*c + 4*d*x))/4 + (9*a*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2)))/4 - (3*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d
*x))/4)/(d*sin(c + d*x)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1112 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=178

$$-\frac{b^2\left(39a^2+2b^2\right)\cos(c+dx)}{24a^2d}-\frac{3\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{12a^2d}$$

```
[Out] 2*a*b*x-3/8*(a^2-4*b^2)*arctanh(cos(d*x+c))/d-1/24*b^2*(39*a^2+2*b^2)*cos(d
*x+c)/a^2/d+17/12*a*b*cot(d*x+c)/d+5/8*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c
))^2/d+1/12*b*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^3/a^2/d-1/4*cot(d*x+
c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^3/a/d
```

Rubi [A] time = 0.46, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2893, 3047, 3031, 3023, 2735, 3770}

$$-\frac{b^2\left(39a^2+2b^2\right)\cos(c+dx)}{24a^2d}-\frac{3\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] 2*a*b*x - (3*(a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) - (b^2*(39*a^2 + 2*
b^2)*Cos[c + d*x])/(24*a^2*d) + (17*a*b*Cot[c + d*x])/(12*d) + (5*Cot[c + d
*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^2)/(8*d) + (b*Cot[c + d*x]*Csc[c + d*
x]^2*(a + b*Sin[c + d*x])^3)/(12*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a +
 b*Sin[c + d*x])^3)/(4*a*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

```
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{12a^2d} - \frac{\cot(c+dx)}{}$$
$$= \frac{5\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{8d} + \frac{b\cot(c+dx}{}$$
$$= \frac{17ab\cot(c+dx)}{12d} + \frac{5\cot(c+dx)\csc(c+dx)(a+b\sin(c+}{8d}$$
$$= -\frac{b^2(39a^2+2b^2)\cos(c+dx)}{24a^2d} + \frac{17ab\cot(c+dx)}{12d} + \frac{5\cot(c}{}$$
$$= 2abx - \frac{b^2(39a^2+2b^2)\cos(c+dx)}{24a^2d} + \frac{17ab\cot(c+dx)}{12d} + \frac{5}{}$$
$$= 2abx - \frac{3(a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b^2(39a^2+2b^2)}{24a^2d}$$

Mathematica [A] time = 2.74, size = 270, normalized size = 1.52

$$\frac{-3a^2\csc^4\left(\frac{1}{2}(c+dx)\right)+30a^2\csc^2\left(\frac{1}{2}(c+dx)\right)+3a^2\sec^4\left(\frac{1}{2}(c+dx)\right)-30a^2\sec^2\left(\frac{1}{2}(c+dx)\right)+72a^2\log\left(\sin\left(\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (384*a*b*c + 384*a*b*d*x - 192*b^2*Cos[c + d*x] + 256*a*b*Cot[(c + d*x)/2]
+ 30*a^2*Csc[(c + d*x)/2]^2 - 24*b^2*Csc[(c + d*x)/2]^2 - 3*a^2*Csc[(c + d*
x)/2]^4 - 72*a^2*Log[Cos[(c + d*x)/2]] + 288*b^2*Log[Cos[(c + d*x)/2]] + 72
*a^2*Log[Sin[(c + d*x)/2]] - 288*b^2*Log[Sin[(c + d*x)/2]] - 30*a^2*Sec[(c
+ d*x)/2]^2 + 24*b^2*Sec[(c + d*x)/2]^2 + 3*a^2*Sec[(c + d*x)/2]^4 + 128*a*
b*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 8*a*b*Csc[(c + d*x)/2]^4*Sin[c + d*x]
 - 256*a*b*Tan[(c + d*x)/2])/(192*d)
```

fricas [A] time = 0.71, size = 260, normalized size = 1.46

$$\frac{96\,abdx\cos(dx+c)^4 - 48\,b^2\cos(dx+c)^5 - 192\,abdx\cos(dx+c)^2 + 96\,abdx - 30\left(a^2-4\,b^2\right)\cos(dx+c)^3 + 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/48*(96*a*b*d*x*cos(d*x + c)^4 - 48*b^2*cos(d*x + c)^5 - 192*a*b*d*x*cos(d
*x + c)^2 + 96*a*b*d*x - 30*(a^2 - 4*b^2)*cos(d*x + c)^3 + 18*(a^2 - 4*b^2)
*cos(d*x + c) - 9*((a^2 - 4*b^2)*cos(d*x + c)^4 - 2*(a^2 - 4*b^2)*cos(d*x +
 c)^2 + a^2 - 4*b^2)*log(1/2*cos(d*x + c) + 1/2) + 9*((a^2 - 4*b^2)*cos(d*x
 + c)^4 - 2*(a^2 - 4*b^2)*cos(d*x + c)^2 + a^2 - 4*b^2)*log(-1/2*cos(d*x +
c) + 1/2) - 32*(4*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.28, size = 244, normalized size = 1.37

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+384\,(dx+c)a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*a^
2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 384*(d*x + c)*a*
b - 240*a*b*tan(1/2*d*x + 1/2*c) + 72*(a^2 - 4*b^2)*log(abs(tan(1/2*d*x + 1
/2*c))) - 384*b^2/(tan(1/2*d*x + 1/2*c)^2 + 1) - (150*a^2*tan(1/2*d*x + 1/2
*c)^4 - 600*b^2*tan(1/2*d*x + 1/2*c)^4 - 240*a*b*tan(1/2*d*x + 1/2*c)^3 - 2
4*a^2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*tan(1
/2*d*x + 1/2*c) + 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.48, size = 223, normalized size = 1.25

$$-\frac{a^2\left(\cos^5(dx+c)\right)}{4d\sin(dx+c)^4}+\frac{a^2\left(\cos^5(dx+c)\right)}{8d\sin(dx+c)^2}+\frac{a^2\left(\cos^3(dx+c)\right)}{8d}+\frac{3a^2\cos(dx+c)}{8d}+\frac{3a^2\ln(\csc(dx+c)-\cot(dx+c))}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2/sin(d*x+c)^4*cos(d*x+c)^5+1/8/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5+1/
8*a^2*cos(d*x+c)^3/d+3/8*a^2*cos(d*x+c)/d+3/8/d*a^2*ln(csc(d*x+c)-cot(d*x+c
))-2/3*a*b*cot(d*x+c)^3/d+2*a*b*cot(d*x+c)/d+2*a*b*x+2/d*a*b*c-1/2/d*b^2/si
n(d*x+c)^2*cos(d*x+c)^5-1/2*b^2*cos(d*x+c)^3/d-3/2*b^2*cos(d*x+c)/d-3/2/d*b
^2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.41, size = 166, normalized size = 0.93

$$\frac{32\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)ab-3\,a^2\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos(dx+c)+1\right)-3\,\log\left(\cos(dx+c)-\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/48*(32*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a*b - 3*a^2*
(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 +
 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 12*b^2*(2*cos(d*
x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*
log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 10.89, size = 825, normalized size = 4.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] -(3*a^2*cos(c/2 + (d*x)/2)^10 - 3*a^2*sin(c/2 + (d*x)/2)^10 + 21*a^2*cos(c/
2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 24*a^2*cos(c/2 + (d*x)/2)^4*sin(c/2 +
 (d*x)/2)^6 - 24*a^2*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 - 21*a^2*cos
(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 - 24*b^2*cos(c/2 + (d*x)/2)^2*sin(c/
2 + (d*x)/2)^8 - 24*b^2*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 + 408*b^2
*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 24*b^2*cos(c/2 + (d*x)/2)^8*si
n(c/2 + (d*x)/2)^2 - 16*a*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^9 + 16*a*
b*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) - 72*a^2*log(sin(c/2 + (d*x)/2)/c
os(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 - 72*a^2*log(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/
2)^4 + 288*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2
)^4*sin(c/2 + (d*x)/2)^6 + 288*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
))*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 224*a*b*cos(c/2 + (d*x)/2)^3
*sin(c/2 + (d*x)/2)^7 - 224*a*b*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3 +
 768*a*b*atan((3*a^2*sin(c/2 + (d*x)/2) - 12*b^2*sin(c/2 + (d*x)/2) + 16*a*
b*cos(c/2 + (d*x)/2))/(12*b^2*cos(c/2 + (d*x)/2) - 3*a^2*cos(c/2 + (d*x)/2)
 + 16*a*b*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 +
768*a*b*atan((3*a^2*sin(c/2 + (d*x)/2) - 12*b^2*sin(c/2 + (d*x)/2) + 16*a*b
*cos(c/2 + (d*x)/2))/(12*b^2*cos(c/2 + (d*x)/2) - 3*a^2*cos(c/2 + (d*x)/2)
+ 16*a*b*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4)/(1
92*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4*(cos(c/2 + (d*x)/2)^2 + sin(
c/2 + (d*x)/2)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1113 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=209

$$\frac{b\left(27a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{60ad}+\frac{\left(12a^2-b^2\right)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{30a^2d}+\frac{b\cot(c+dx)\text{c}}{}$$

```
[Out] b^2*x-3/4*a*b*arctanh(cos(d*x+c))/d-1/15*(3*a^4-14*a^2*b^2+b^4)*cot(d*x+c)/
a^2/d+1/60*b*(27*a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)/a/d+1/30*(12*a^2-b^2)*cot
(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^2/a^2/d+1/10*b*cot(d*x+c)*csc(d*x+c)^
3*(a+b*sin(d*x+c))^3/a^2/d-1/5*cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^3/a
/d
```

Rubi [A] time = 0.52, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2893, 3047, 3031, 3021, 2735, 3770}

$$-\frac{\left(-14a^2b^2+3a^4+b^4\right)\cot(c+dx)}{15a^2d}+\frac{b\left(27a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{60ad}+\frac{\left(12a^2-b^2\right)\cot(c+dx)\csc^2(c+dx}{30a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] b^2*x - (3*a*b*ArcTanh[Cos[c + d*x]])/(4*d) - ((3*a^4 - 14*a^2*b^2 + b^4)*C
ot[c + d*x])/(15*a^2*d) + (b*(27*a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(6
0*a*d) + ((12*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2
)/(30*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(10*a
^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(5*a*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
```

```
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3}{10a^2d} - \frac{\cot(c+dx}{}$$

$$= \frac{\left(12a^2-b^2\right)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{30a^2d} +$$

$$= \frac{b\left(27a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{60ad} + \frac{\left(12a^2-b^2\right)\cot(c}{}$$

$$= -\frac{\left(3a^4-14a^2b^2+b^4\right)\cot(c+dx)}{15a^2d} + \frac{b\left(27a^2-2b^2\right)\cot(c+}{60ad}$$

$$= b^2x - \frac{\left(3a^4-14a^2b^2+b^4\right)\cot(c+dx)}{15a^2d} + \frac{b\left(27a^2-2b^2\right)\mathrm{co}}{6}$$

$$= b^2x - \frac{3ab\tanh^{-1}(\cos(c+dx))}{4d} - \frac{\left(3a^4-14a^2b^2+b^4\right)\cot(}{15a^2d}$$

Mathematica [A] time = 1.54, size = 285, normalized size = 1.36

$$\frac{\left(640b^2-96a^2\right)\cot\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)\left(\left(21a^2-20b^2\right)\sin(c+dx)-30ab\right)+96a^2\tan\left(\frac{1}{2}(c+dx)\right)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (960*b^2*c + 960*b^2*d*x + (-96*a^2 + 640*b^2)*Cot[(c + d*x)/2] + 300*a*b*C
sc[(c + d*x)/2]^2 - 720*a*b*Log[Cos[(c + d*x)/2]] + 720*a*b*Log[Sin[(c + d*
x)/2]] - 300*a*b*Sec[(c + d*x)/2]^2 + 30*a*b*Sec[(c + d*x)/2]^4 - 336*a^2*C
sc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*b^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^
4 + 192*a^2*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 - 3*a^2*Csc[(c + d*x)/2]^6*Si
n[c + d*x] + Csc[(c + d*x)/2]^4*(-30*a*b + (21*a^2 - 20*b^2)*Sin[c + d*x])
+ 96*a^2*Tan[(c + d*x)/2] - 640*b^2*Tan[(c + d*x)/2])/(960*d)
```

fricas [A] time = 0.68, size = 241, normalized size = 1.15

$$-\frac{8\left(3\,a^2-20\,b^2\right)\cos\left(dx+c\right)^5+280\,b^2\cos\left(dx+c\right)^3-120\,b^2\cos\left(dx+c\right)+45\left(ab\cos\left(dx+c\right)^4-2\,ab\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/120*(8*(3*a^2 - 20*b^2)*cos(d*x + c)^5 + 280*b^2*cos(d*x + c)^3 - 120*b^
2*cos(d*x + c) + 45*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2 + a*b)*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 45*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*
x + c)^2 + a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*(4*b^2*d*x*c
os(d*x + c)^4 - 8*b^2*d*x*cos(d*x + c)^2 - 5*a*b*cos(d*x + c)^3 + 4*b^2*d*x
 + 3*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^
2 + d)*sin(d*x + c))
```

giac [A] time = 0.25, size = 263, normalized size = 1.26

$$3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,ab\tan\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a*b*tan(1/2*d*x + 1/2*c)^4 - 15*a^
2*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a*b*tan(1/2*
d*x + 1/2*c)^2 + 480*(d*x + c)*b^2 + 360*a*b*log(abs(tan(1/2*d*x + 1/2*c)))
 + 30*a^2*tan(1/2*d*x + 1/2*c) - 300*b^2*tan(1/2*d*x + 1/2*c) - (822*a*b*ta
n(1/2*d*x + 1/2*c)^5 + 30*a^2*tan(1/2*d*x + 1/2*c)^4 - 300*b^2*tan(1/2*d*x
+ 1/2*c)^4 - 120*a*b*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c)^2
 + 20*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a*b*tan(1/2*d*x + 1/2*c) + 3*a^2)/tan
(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.50, size = 165, normalized size = 0.79

$$-\frac{a^2\left(\cos^5\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)^5}-\frac{ab\left(\cos^5\left(dx+c\right)\right)}{2d\sin\left(dx+c\right)^4}+\frac{ab\left(\cos^5\left(dx+c\right)\right)}{4d\sin\left(dx+c\right)^2}+\frac{ab\left(\cos^3\left(dx+c\right)\right)}{4d}+\frac{3ab\cos\left(dx+c\right)}{4d}+\frac{3ab\ln\left(\csc\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/5/d*a^2/sin(d*x+c)^5*cos(d*x+c)^5-1/2/d*a*b/sin(d*x+c)^4*cos(d*x+c)^5+1/
4/d*a*b/sin(d*x+c)^2*cos(d*x+c)^5+1/4*a*b*cos(d*x+c)^3/d+3/4*a*b*cos(d*x+c)
/d+3/4/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/3*b^2*cot(d*x+c)^3/d+b^2*cot(d*x+c
)/d+b^2*x+1/d*b^2*c
```

maxima [A] time = 0.41, size = 123, normalized size = 0.59

$$\frac{40\left(3\,dx + 3\,c + \frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)b^2 - 15\,ab\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1} + 3\,\log\left(\cos\left(dx+c\right)+1\right) - 3\,\log\left(\cos\left(dx+c\right.\right.\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/120*(40*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*b^2 - 15*a*
b*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2
 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) - 24*a^2/tan(d*x
 + c)^5)/d
```

mupad [B] time = 10.10, size = 346, normalized size = 1.66

$$\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{b^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] (a^2*cot(c/2 + (d*x)/2)^3)/(32*d) - (a^2*cot(c/2 + (d*x)/2)^5)/(160*d) - (b
^2*cot(c/2 + (d*x)/2)^3)/(24*d) - (a^2*tan(c/2 + (d*x)/2)^3)/(32*d) + (a^2*
tan(c/2 + (d*x)/2)^5)/(160*d) + (b^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (a^2*co
t(c/2 + (d*x)/2))/(16*d) + (5*b^2*cot(c/2 + (d*x)/2))/(8*d) + (a^2*tan(c/2
+ (d*x)/2))/(16*d) - (5*b^2*tan(c/2 + (d*x)/2))/(8*d) + (2*b^2*atan((4*b*co
s(c/2 + (d*x)/2) + 3*a*sin(c/2 + (d*x)/2))/(3*a*cos(c/2 + (d*x)/2) - 4*b*si
n(c/2 + (d*x)/2))))/d + (a*b*cot(c/2 + (d*x)/2)^2)/(4*d) - (a*b*cot(c/2 + (
d*x)/2)^4)/(32*d) - (a*b*tan(c/2 + (d*x)/2)^2)/(4*d) + (a*b*tan(c/2 + (d*x)
/2)^4)/(32*d) + (3*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1114 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=236

$$-\frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{b\left(13a^2-2b^2\right)\cot(c+dx)\csc^2(c+dx)}{60ad}+\frac{\left(35a^2-6b^2\right)\cot(c+dx)\csc^3(c+dx)(}{120a^2d}$$

```
[Out] -1/16*(a^2+6*b^2)*arctanh(cos(d*x+c))/d-2/5*a*b*cot(d*x+c)/d-1/240*(15*a^4-
80*a^2*b^2+12*b^4)*cot(d*x+c)*csc(d*x+c)/a^2/d+1/60*b*(13*a^2-2*b^2)*cot(d*
x+c)*csc(d*x+c)^2/a/d+1/120*(35*a^2-6*b^2)*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin
(d*x+c))^2/a^2/d+1/10*b*cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^3/a^2/d-1/
6*cot(d*x+c)*csc(d*x+c)^5*(a+b*sin(d*x+c))^3/a/d
```

Rubi [A] time = 0.60, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2893, 3047, 3031, 3021, 2748, 3767, 8, 3770}

$$-\frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{b\left(13a^2-2b^2\right)\cot(c+dx)\csc^2(c+dx)}{60ad}-\frac{\left(-80a^2b^2+15a^4+12b^4\right)\cot(c+dx)\,}{240a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2 + 6*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a*b*Cot[c + d*x])/(5*d)
- ((15*a^4 - 80*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c + d*x])/(240*a^2*d) +
(b*(13*a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x]^2)/(60*a*d) + ((35*a^2 - 6*b^
2)*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(120*a^2*d) + (b*Cot
[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(10*a^2*d) - (Cot[c + d*x]
*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3)/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
```

```
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3}{10a^2d} - \frac{\cot(c+dx)}{}$$

$$= \frac{\left(35a^2-6b^2\right)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2}{120a^2d} +$$

$$= \frac{b\left(13a^2-2b^2\right)\cot(c+dx)\csc^2(c+dx)}{60ad} + \frac{\left(35a^2-6b^2\right)\cot(}{}$$

$$= -\frac{\left(15a^4-80a^2b^2+12b^4\right)\cot(c+dx)\csc(c+dx)}{240a^2d} + \frac{b\left(13a^2\right.}{}$$

$$= -\frac{\left(15a^4-80a^2b^2+12b^4\right)\cot(c+dx)\csc(c+dx)}{240a^2d} + \frac{b\left(13a^2\right.}{}$$

$$= -\frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{\left(15a^4-80a^2b^2+12b^4\right)}{240a}$$

$$= -\frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{2ab\cot(c+dx)}{5d} - \frac{\left(15a^4\right.}{}$$

Mathematica [A] time = 0.89, size = 319, normalized size = 1.35

$$\frac{-30\left(a^2-10b^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)+6\csc^4\left(\frac{1}{2}(c+dx)\right)\left(5a^2+14ab\sin(c+dx)-5b^2\right)+5a^2\sec^6\left(\frac{1}{2}(c+dx)\right)-30a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-384*a*b*Cot[(c + d*x)/2] - 30*(a^2 - 10*b^2)*Csc[(c + d*x)/2]^2 - 120*a^2
*Log[Cos[(c + d*x)/2]] - 720*b^2*Log[Cos[(c + d*x)/2]] + 120*a^2*Log[Sin[(c
```

```
 + d*x)/2]] + 720*b^2*Log[Sin[(c + d*x)/2]] + 30*a^2*Sec[(c + d*x)/2]^2 - 3
00*b^2*Sec[(c + d*x)/2]^2 - 30*a^2*Sec[(c + d*x)/2]^4 + 30*b^2*Sec[(c + d*x
)/2]^4 + 5*a^2*Sec[(c + d*x)/2]^6 - 1344*a*b*Csc[c + d*x]^3*Sin[(c + d*x)/2
]^4 + 768*a*b*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 - a*Csc[(c + d*x)/2]^6*(5*a
 + 12*b*Sin[c + d*x]) + 6*Csc[(c + d*x)/2]^4*(5*a^2 - 5*b^2 + 14*a*b*Sin[c
+ d*x]) + 384*a*b*Tan[(c + d*x)/2])/(1920*d)
```

fricas [A] time = 0.80, size = 274, normalized size = 1.16

$$\frac{192\,ab\cos\left(dx+c\right)^{5}\sin\left(dx+c\right)+30\left(a^{2}-10\,b^{2}\right)\cos\left(dx+c\right)^{5}+80\left(a^{2}+6\,b^{2}\right)\cos\left(dx+c\right)^{3}-30\left(a^{2}+6\,b^{2}\right)\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/480*(192*a*b*cos(d*x + c)^5*sin(d*x + c) + 30*(a^2 - 10*b^2)*cos(d*x + c)
^5 + 80*(a^2 + 6*b^2)*cos(d*x + c)^3 - 30*(a^2 + 6*b^2)*cos(d*x + c) - 15*(
(a^2 + 6*b^2)*cos(d*x + c)^6 - 3*(a^2 + 6*b^2)*cos(d*x + c)^4 + 3*(a^2 + 6*
b^2)*cos(d*x + c)^2 - a^2 - 6*b^2)*log(1/2*cos(d*x + c) + 1/2) + 15*((a^2 +
 6*b^2)*cos(d*x + c)^6 - 3*(a^2 + 6*b^2)*cos(d*x + c)^4 + 3*(a^2 + 6*b^2)*c
os(d*x + c)^2 - a^2 - 6*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^
6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.27, size = 309, normalized size = 1.31

$$\frac{5\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+30\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-120\,ab\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a*b*tan(1/2*d*x + 1/2*c)^5 - 15*a
^2*tan(1/2*d*x + 1/2*c)^4 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*a*b*tan(1/2
*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c)^2 - 240*b^2*tan(1/2*d*x + 1/2
*c)^2 + 240*a*b*tan(1/2*d*x + 1/2*c) + 120*(a^2 + 6*b^2)*log(abs(tan(1/2*d*
x + 1/2*c))) - (294*a^2*tan(1/2*d*x + 1/2*c)^6 + 1764*b^2*tan(1/2*d*x + 1/2
*c)^6 + 240*a*b*tan(1/2*d*x + 1/2*c)^5 - 15*a^2*tan(1/2*d*x + 1/2*c)^4 - 24
0*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*a*b*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(
1/2*d*x + 1/2*c)^2 + 30*b^2*tan(1/2*d*x + 1/2*c)^2 + 24*a*b*tan(1/2*d*x + 1
/2*c) + 5*a^2)/tan(1/2*d*x + 1/2*c)^6)/d
```

maple [A] time = 0.49, size = 253, normalized size = 1.07

$$-\frac{a^2\left(\cos^5(dx+c)\right)}{6d\sin(dx+c)^6}-\frac{a^2\left(\cos^5(dx+c)\right)}{24d\sin(dx+c)^4}+\frac{a^2\left(\cos^5(dx+c)\right)}{48d\sin(dx+c)^2}+\frac{a^2\left(\cos^3(dx+c)\right)}{48d}+\frac{a^2\cos(dx+c)}{16d}+\frac{a^2\ln(\csc(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/6/d*a^2/sin(d*x+c)^6*cos(d*x+c)^5-1/24/d*a^2/sin(d*x+c)^4*cos(d*x+c)^5+1
/48/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5+1/48*a^2*cos(d*x+c)^3/d+1/16*a^2*cos(d*
x+c)/d+1/16/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/5/d*a*b/sin(d*x+c)^5*cos(d*x+
c)^5-1/4/d*b^2/sin(d*x+c)^4*cos(d*x+c)^5+1/8/d*b^2/sin(d*x+c)^2*cos(d*x+c)^
5+1/8*b^2*cos(d*x+c)^3/d+3/8*b^2*cos(d*x+c)/d+3/8/d*b^2*ln(csc(d*x+c)-cot(d
*x+c))
```

maxima [A] time = 0.41, size = 180, normalized size = 0.76

$$\frac{5\,a^2\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos(dx+c)+1\right)+3\,\log\left(\cos(dx+c)-1\right)\right)-30\,b^2\left(\frac{2\left(5\,\cos(dx+c)\right.}{\cos(dx+c)^4-2}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/480*(5*a^2*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
 + 1) + 3*log(cos(d*x + c) - 1)) - 30*b^2*(2*(5*cos(d*x + c)^3 - 3*cos(d*x
+ c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3
*log(cos(d*x + c) - 1)) - 192*a*b/tan(d*x + c)^5)/d
```

mupad [B] time = 9.67, size = 262, normalized size = 1.11

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(\frac{a^2}{16}+\frac{3\,b^2}{8}\right)}{d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{a^2}{2}-b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{a^2}{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))*(a^2/16 + (3*b^2)/8))/d + (a^2*tan(c/2 + (d*x)/2)^
6)/(384*d) + (cot(c/2 + (d*x)/2)^6*(tan(c/2 + (d*x)/2)^2*(a^2/2 - b^2) + ta
n(c/2 + (d*x)/2)^4*(a^2/2 + 8*b^2) - a^2/6 + 4*a*b*tan(c/2 + (d*x)/2)^3 - 8
```

```
*a*b*tan(c/2 + (d*x)/2)^5 - (4*a*b*tan(c/2 + (d*x)/2))/5))/(64*d) - (tan(c/
2 + (d*x)/2)^2*(a^2/128 + b^2/8))/d - (tan(c/2 + (d*x)/2)^4*(a^2/128 - b^2/
64))/d - (a*b*tan(c/2 + (d*x)/2)^3)/(16*d) + (a*b*tan(c/2 + (d*x)/2)^5)/(80
*d) + (a*b*tan(c/2 + (d*x)/2))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1115 $\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=261

$$-\frac{(2a^2+7b^2)\cot(c+dx)}{35d}+\frac{b(53a^2-12b^2)\cot(c+dx)\csc^3(c+dx)}{420ad}+\frac{2(4a^2-b^2)\cot(c+dx)\csc^4(c+dx)(a+b\sin}{35a^2d}$$

```
[Out] -1/8*a*b*arctanh(cos(d*x+c))/d-1/35*(2*a^2+7*b^2)*cot(d*x+c)/d-1/8*a*b*cot(
d*x+c)*csc(d*x+c)/d-1/105*(3*a^4-18*a^2*b^2+4*b^4)*cot(d*x+c)*csc(d*x+c)^2/
a^2/d+1/420*b*(53*a^2-12*b^2)*cot(d*x+c)*csc(d*x+c)^3/a/d+2/35*(4*a^2-b^2)*
cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^2/a^2/d+2/21*b*cot(d*x+c)*csc(d*x+
c)^5*(a+b*sin(d*x+c))^3/a^2/d-1/7*cot(d*x+c)*csc(d*x+c)^6*(a+b*sin(d*x+c))^
3/a/d
```

Rubi [A] time = 0.63, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2893, 3047, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$-\frac{(2a^2+7b^2)\cot(c+dx)}{35d}+\frac{b(53a^2-12b^2)\cot(c+dx)\csc^3(c+dx)}{420ad}-\frac{(-18a^2b^2+3a^4+4b^4)\cot(c+dx)\csc^2(c+}{105a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a*b*ArcTanh[Cos[c + d*x]])/(8*d) - ((2*a^2 + 7*b^2)*Cot[c + d*x])/(35*d)
- (a*b*Cot[c + d*x]*Csc[c + d*x])/(8*d) - ((3*a^4 - 18*a^2*b^2 + 4*b^4)*Cot
[c + d*x]*Csc[c + d*x]^2)/(105*a^2*d) + (b*(53*a^2 - 12*b^2)*Cot[c + d*x]*C
sc[c + d*x]^3)/(420*a*d) + (2*(4*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^4*(a
+ b*Sin[c + d*x])^2)/(35*a^2*d) + (2*b*Cot[c + d*x]*Csc[c + d*x]^5*(a + b*S
in[c + d*x])^3)/(21*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^6*(a + b*Sin[c + d*
x])^3)/(7*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{2b\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3}{21a^2d} - \frac{\cot(c+dx}{}$$

$$= \frac{2\left(4a^2-b^2\right)\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2}{35a^2d} + \frac{2}{}$$

$$= \frac{b\left(53a^2-12b^2\right)\cot(c+dx)\csc^3(c+dx)}{420ad} + \frac{2\left(4a^2-b^2\right)\cot(}{}$$

$$= -\frac{\left(3a^4-18a^2b^2+4b^4\right)\cot(c+dx)\csc^2(c+dx)}{105a^2d} + \frac{b\left(53a^2-\right.}{}$$

$$= -\frac{\left(3a^4-18a^2b^2+4b^4\right)\cot(c+dx)\csc^2(c+dx)}{105a^2d} + \frac{b\left(53a^2-\right.}{}$$

$$= -\frac{ab\cot(c+dx)\csc(c+dx)}{8d} - \frac{\left(3a^4-18a^2b^2+4b^4\right)\cot(c+}{105a^2d}$$

$$= -\frac{ab\tanh^{-1}(\cos(c+dx))}{8d} - \frac{\left(2a^2+7b^2\right)\cot(c+dx)}{35d} - \frac{ab\cot}{}$$

Mathematica [A] time = 1.33, size = 322, normalized size = 1.23

$$-\frac{\csc^7(c+dx)\left(840\left(6a^2+b^2\right)\cos(c+dx)+168\left(14a^2-b^2\right)\cos(3(c+dx))+336a^2\cos(5(c+dx))-48a^2\cos(7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/53760*(Csc[c + d*x]^7*(840*(6*a^2 + b^2)*Cos[c + d*x] + 168*(14*a^2 - b^
2)*Cos[3*(c + d*x)] + 336*a^2*Cos[5*(c + d*x)] - 504*b^2*Cos[5*(c + d*x)] -
 48*a^2*Cos[7*(c + d*x)] - 168*b^2*Cos[7*(c + d*x)] + 3675*a*b*Log[Cos[(c +
 d*x)/2]]*Sin[c + d*x] - 3675*a*b*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] + 2170
*a*b*Sin[2*(c + d*x)] - 2205*a*b*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] + 2
205*a*b*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 3080*a*b*Sin[4*(c + d*x)]
+ 735*a*b*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 735*a*b*Log[Sin[(c + d*x
)/2]]*Sin[5*(c + d*x)] + 210*a*b*Sin[6*(c + d*x)] - 105*a*b*Log[Cos[(c + d*
x)/2]]*Sin[7*(c + d*x)] + 105*a*b*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)]))/
d
```

fricas [A] time = 0.61, size = 248, normalized size = 0.95

$$-\frac{48\left(2a^2+7b^2\right)\cos(dx+c)^7-336\left(a^2+b^2\right)\cos(dx+c)^5+105\left(ab\cos(dx+c)^6-3ab\cos(dx+c)^4+3ab\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/1680*(48*(2*a^2 + 7*b^2)*cos(d*x + c)^7 - 336*(a^2 + b^2)*cos(d*x + c)^5
 + 105*(a*b*cos(d*x + c)^6 - 3*a*b*cos(d*x + c)^4 + 3*a*b*cos(d*x + c)^2 -
a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 105*(a*b*cos(d*x + c)^6 - 3
*a*b*cos(d*x + c)^4 + 3*a*b*cos(d*x + c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) - 70*(3*a*b*cos(d*x + c)^5 + 8*a*b*cos(d*x + c)^3 - 3*a*b*
cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*c
os(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.29, size = 347, normalized size = 1.33

$$\frac{15a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+70ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-21a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+84b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-210ab\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*a^2*tan(1/2*d*x + 1/2*c)^7 + 70*a*b*tan(1/2*d*x + 1/2*c)^6 - 21
*a^2*tan(1/2*d*x + 1/2*c)^5 + 84*b^2*tan(1/2*d*x + 1/2*c)^5 - 210*a*b*tan(1
/2*d*x + 1/2*c)^4 - 105*a^2*tan(1/2*d*x + 1/2*c)^3 - 420*b^2*tan(1/2*d*x +
1/2*c)^3 - 210*a*b*tan(1/2*d*x + 1/2*c)^2 + 1680*a*b*log(abs(tan(1/2*d*x +
1/2*c))) + 315*a^2*tan(1/2*d*x + 1/2*c) + 840*b^2*tan(1/2*d*x + 1/2*c) - (4
356*a*b*tan(1/2*d*x + 1/2*c)^7 + 315*a^2*tan(1/2*d*x + 1/2*c)^6 + 840*b^2*t
an(1/2*d*x + 1/2*c)^6 - 210*a*b*tan(1/2*d*x + 1/2*c)^5 - 105*a^2*tan(1/2*d*
x + 1/2*c)^4 - 420*b^2*tan(1/2*d*x + 1/2*c)^4 - 210*a*b*tan(1/2*d*x + 1/2*c
)^3 - 21*a^2*tan(1/2*d*x + 1/2*c)^2 + 84*b^2*tan(1/2*d*x + 1/2*c)^2 + 70*a*
b*tan(1/2*d*x + 1/2*c) + 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/d
```

maple [A] time = 0.47, size = 194, normalized size = 0.74

$$-\frac{a^2\left(\cos^5(dx+c)\right)}{7d\sin(dx+c)^7}-\frac{2a^2\left(\cos^5(dx+c)\right)}{35d\sin(dx+c)^5}-\frac{ab\left(\cos^5(dx+c)\right)}{3d\sin(dx+c)^6}-\frac{ab\left(\cos^5(dx+c)\right)}{12d\sin(dx+c)^4}+\frac{ab\left(\cos^5(dx+c)\right)}{24d\sin(dx+c)^2}+\frac{ab\left(\cos^3(dx\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/7/d*a^2/sin(d*x+c)^7*cos(d*x+c)^5-2/35/d*a^2/sin(d*x+c)^5*cos(d*x+c)^5-1
/3/d*a*b/sin(d*x+c)^6*cos(d*x+c)^5-1/12/d*a*b/sin(d*x+c)^4*cos(d*x+c)^5+1/2
4/d*a*b/sin(d*x+c)^2*cos(d*x+c)^5+1/24*a*b*cos(d*x+c)^3/d+1/8*a*b*cos(d*x+c
)/d+1/8/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/5/d*b^2/sin(d*x+c)^5*cos(d*x+c)^5
```

maxima [A] time = 0.33, size = 134, normalized size = 0.51

$$\frac{35\,ab\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos(dx+c)+1\right)+3\,\log\left(\cos(dx+c)-1\right)\right)-\frac{336\,b^2}{\tan(dx+c)^5}-\frac{48\left(7\right.}{}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/1680*(35*a*b*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(c
os(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x +
c) + 1) + 3*log(cos(d*x + c) - 1)) - 336*b^2/tan(d*x + c)^5 - 48*(7*tan(d*x
 + c)^2 + 5)*a^2/tan(d*x + c)^7)/d
```

mupad [B] time = 9.92, size = 302, normalized size = 1.16

$$\frac{a^2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{896\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 \left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 \left(\frac{a^2}{5}-\frac{4\,b^2}{5}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 \left(3\,a^2+8\,b^2\right)-\frac{a^2}{7}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 \right.}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x)^8,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^7)/(896*d) + (cot(c/2 + (d*x)/2)^7*(tan(c/2 + (d*x)
/2)^2*(a^2/5 - (4*b^2)/5) - tan(c/2 + (d*x)/2)^6*(3*a^2 + 8*b^2) - a^2/7 +
tan(c/2 + (d*x)/2)^4*(a^2 + 4*b^2) + 2*a*b*tan(c/2 + (d*x)/2)^3 + 2*a*b*tan
(c/2 + (d*x)/2)^5 - (2*a*b*tan(c/2 + (d*x)/2))/3))/(128*d) + (tan(c/2 + (d*
x)/2)*((3*a^2)/128 + b^2/16))/d - (tan(c/2 + (d*x)/2)^3*(a^2/128 + b^2/32))
/d - (tan(c/2 + (d*x)/2)^5*(a^2/640 - b^2/160))/d - (a*b*tan(c/2 + (d*x)/2)
^2)/(64*d) - (a*b*tan(c/2 + (d*x)/2)^4)/(64*d) + (a*b*tan(c/2 + (d*x)/2)^6)
/(192*d) + (a*b*log(tan(c/2 + (d*x)/2)))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1116 $\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=354

$$\frac{b(27a^2+4b^2)\cos^3(c+dx)}{315d}-\frac{b(27a^2+4b^2)\cos(c+dx)}{105d}-\frac{5(a^2-4b^2)\sin^3(c+dx)\cos(c+dx)(a+b\sin(c+dx))^3}{126b^2d}$$

```
[Out] 1/128*a*(8*a^2+9*b^2)*x-1/105*b*(27*a^2+4*b^2)*cos(d*x+c)/d+1/315*b*(27*a^2
+4*b^2)*cos(d*x+c)^3/d-1/128*a*(8*a^2+9*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/4032
*a*(40*a^4-188*a^2*b^2+189*b^4)*cos(d*x+c)*sin(d*x+c)^3/b^2/d-1/2520*(20*a^
4-93*a^2*b^2+24*b^4)*cos(d*x+c)*sin(d*x+c)^4/b/d-1/1008*a*(20*a^2-87*b^2)*c
os(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^2/b^2/d-5/126*(a^2-4*b^2)*cos(d*x+c
)*sin(d*x+c)^3*(a+b*sin(d*x+c))^3/b^2/d+5/72*a*cos(d*x+c)*sin(d*x+c)^3*(a+b
*sin(d*x+c))^4/b^2/d-1/9*cos(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c))^4/b/d
```

Rubi [A] time = 0.93, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2895, 3049, 3033, 3023, 2748, 2635, 8, 2633}

$$\frac{b(27a^2+4b^2)\cos^3(c+dx)}{315d}-\frac{b(27a^2+4b^2)\cos(c+dx)}{105d}-\frac{(-93a^2b^2+20a^4+24b^4)\sin^4(c+dx)\cos(c+dx)}{2520bd}-\frac{5(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(8*a^2 + 9*b^2)*x)/128 - (b*(27*a^2 + 4*b^2)*Cos[c + d*x])/(105*d) + (b*
(27*a^2 + 4*b^2)*Cos[c + d*x]^3)/(315*d) - (a*(8*a^2 + 9*b^2)*Cos[c + d*x]*
Sin[c + d*x])/(128*d) - (a*(40*a^4 - 188*a^2*b^2 + 189*b^4)*Cos[c + d*x]*Si
n[c + d*x]^3)/(4032*b^2*d) - ((20*a^4 - 93*a^2*b^2 + 24*b^4)*Cos[c + d*x]*S
in[c + d*x]^4)/(2520*b*d) - (a*(20*a^2 - 87*b^2)*Cos[c + d*x]*Sin[c + d*x]^
3*(a + b*Sin[c + d*x])^2)/(1008*b^2*d) - (5*(a^2 - 4*b^2)*Cos[c + d*x]*Sin[
c + d*x]^3*(a + b*Sin[c + d*x])^3)/(126*b^2*d) + (5*a*Cos[c + d*x]*Sin[c +
d*x]^3*(a + b*Sin[c + d*x])^4)/(72*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^4*(a
 + b*Sin[c + d*x])^4)/(9*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
```

```
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{5a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^4}{72b^2d} - \frac{\cos(c+d}{}$$

$$= -\frac{5\left(a^2-4b^2\right)\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^3}{126b^2d} +$$

$$= -\frac{a\left(20a^2-87b^2\right)\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx)}{1008b^2d}$$

$$= -\frac{\left(20a^4-93a^2b^2+24b^4\right)\cos(c+dx)\sin^4(c+dx)}{2520bd} - \frac{a\left(20a\right.}{}$$

$$= -\frac{a\left(40a^4-188a^2b^2+189b^4\right)\cos(c+dx)\sin^3(c+dx)}{4032b^2d} - \frac{(2}{}$$

$$= -\frac{a\left(40a^4-188a^2b^2+189b^4\right)\cos(c+dx)\sin^3(c+dx)}{4032b^2d} - \frac{(2}{}$$

$$= -\frac{a\left(8a^2+9b^2\right)\cos(c+dx)\sin(c+dx)}{128d} - \frac{a\left(40a^4-188a^2b^2\right.}{}$$

$$= \frac{1}{128}a\left(8a^2+9b^2\right)x - \frac{b\left(27a^2+4b^2\right)\cos(c+dx)}{105d} + \frac{b\left(27a^2\right.}{}$$

Mathematica [A] time = 1.28, size = 204, normalized size = 0.58

$$\frac{2520a^3\sin(2(c+dx)) - 2520a^3\sin(4(c+dx)) - 840a^3\sin(6(c+dx)) + 10080a^3dx - 840\left(9a^2b+b^3\right)\cos(3(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (15120*a*b^2*c + 10080*a^3*d*x + 11340*a*b^2*d*x - 3780*b*(6*a^2 + b^2)*Cos
[c + d*x] - 840*(9*a^2*b + b^3)*Cos[3*(c + d*x)] + 1512*a^2*b*Cos[5*(c + d*
x)] + 504*b^3*Cos[5*(c + d*x)] + 1080*a^2*b*Cos[7*(c + d*x)] + 90*b^3*Cos[7
*(c + d*x)] - 70*b^3*Cos[9*(c + d*x)] + 2520*a^3*Sin[2*(c + d*x)] - 2520*a^
3*Sin[4*(c + d*x)] - 3780*a*b^2*Sin[4*(c + d*x)] - 840*a^3*Sin[6*(c + d*x)]
 + (945*a*b^2*Sin[8*(c + d*x)])/2)/(161280*d)
```

fricas [A] time = 0.98, size = 164, normalized size = 0.46

$$-\frac{4480\,b^3\cos\left(dx+c\right)^9-5760\left(3\,a^2b+2\,b^3\right)\cos\left(dx+c\right)^7+8064\left(3\,a^2b+b^3\right)\cos\left(dx+c\right)^5-315\left(8\,a^3+9\,ab^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/40320*(4480*b^3*cos(d*x + c)^9 - 5760*(3*a^2*b + 2*b^3)*cos(d*x + c)^7 +
 8064*(3*a^2*b + b^3)*cos(d*x + c)^5 - 315*(8*a^3 + 9*a*b^2)*d*x - 105*(144
*a*b^2*cos(d*x + c)^7 - 8*(8*a^3 + 27*a*b^2)*cos(d*x + c)^5 + 2*(8*a^3 + 9*
a*b^2)*cos(d*x + c)^3 + 3*(8*a^3 + 9*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.46, size = 204, normalized size = 0.58

$$-\frac{b^3\cos\left(9\,dx+9\,c\right)}{2304\,d}+\frac{3\,ab^2\sin\left(8\,dx+8\,c\right)}{1024\,d}-\frac{a^3\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{a^3\sin\left(2\,dx+2\,c\right)}{64\,d}+\frac{1}{128}\left(8\,a^3+9\,ab^2\right)x+\frac{\left(12\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2304*b^3*cos(9*d*x + 9*c)/d + 3/1024*a*b^2*sin(8*d*x + 8*c)/d - 1/192*a^
3*sin(6*d*x + 6*c)/d + 1/64*a^3*sin(2*d*x + 2*c)/d + 1/128*(8*a^3 + 9*a*b^2
)*x + 1/1792*(12*a^2*b + b^3)*cos(7*d*x + 7*c)/d + 1/320*(3*a^2*b + b^3)*co
s(5*d*x + 5*c)/d - 1/192*(9*a^2*b + b^3)*cos(3*d*x + 3*c)/d - 3/128*(6*a^2*
b + b^3)*cos(d*x + c)/d - 1/128*(2*a^3 + 3*a*b^2)*sin(4*d*x + 4*c)/d
```

maple [A] time = 0.34, size = 218, normalized size = 0.62

$$\frac{a^3\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+3a^2b\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)+3*a^2*b*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos
(d*x+c)^5)+3*a*b^2*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+
c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+b^3*(
-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x
+c)^5))
```

maxima [A] time = 0.45, size = 140, normalized size = 0.40

$$\frac{1680\left(4\sin\left(2\,dx+2\,c\right)^{3}+12\,dx+12\,c-3\sin\left(4\,dx+4\,c\right)\right)a^{3}+27648\left(5\cos\left(dx+c\right)^{7}-7\cos\left(dx+c\right)^{5}\right)a^{2}b+9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/322560*(1680*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*
a^3 + 27648*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^2*b + 945*(24*d*x + 24*
c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a*b^2 - 1024*(35*cos(d*x + c)^9
- 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*b^3)/d
```

mupad [B] time = 10.77, size = 578, normalized size = 1.63

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(8\,a^2+9\,b^2\right)}{64\left(\frac{a^3}{8}+\frac{9\,a\,b^2}{64}\right)}\right)\left(8\,a^2+9\,b^2\right)}{64\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{a^3}{8}+\frac{9\,a\,b^2}{64}\right)+\frac{12\,a^2\,b}{35}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}\left(\frac{a^3}{8}+\frac{9\,a\,b^2}{64}\right)+\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(8*a^2 + 9*b^2))/(64*((9*a*b^2)/64 + a^3/8)))
*(8*a^2 + 9*b^2))/(64*d) - (tan(c/2 + (d*x)/2)*((9*a*b^2)/64 + a^3/8) + (12
*a^2*b)/35 - tan(c/2 + (d*x)/2)^17*((9*a*b^2)/64 + a^3/8) + tan(c/2 + (d*x)
/2)^3*((39*a*b^2)/32 - (19*a^3)/12) - tan(c/2 + (d*x)/2)^15*((39*a*b^2)/32
- (19*a^3)/12) - tan(c/2 + (d*x)/2)^5*((465*a*b^2)/32 + (9*a^3)/4) + tan(c/
2 + (d*x)/2)^13*((465*a*b^2)/32 + (9*a^3)/4) + tan(c/2 + (d*x)/2)^7*((507*a
*b^2)/32 + (3*a^3)/4) - tan(c/2 + (d*x)/2)^11*((507*a*b^2)/32 + (3*a^3)/4)
+ tan(c/2 + (d*x)/2)^10*(12*a^2*b - 16*b^3) + tan(c/2 + (d*x)/2)^12*(12*a^2
```

```
*b + (32*b^3)/3) + tan(c/2 + (d*x)/2)^6*((84*a^2*b)/5 - (32*b^3)/5) + tan(c
/2 + (d*x)/2)^4*((12*a^2*b)/35 + (64*b^3)/35) + tan(c/2 + (d*x)/2)^2*((108*
a^2*b)/35 + (16*b^3)/35) + tan(c/2 + (d*x)/2)^8*((156*a^2*b)/5 + (112*b^3)/
5) + (16*b^3)/315 + 12*a^2*b*tan(c/2 + (d*x)/2)^14)/(d*(9*tan(c/2 + (d*x)/2
)^2 + 36*tan(c/2 + (d*x)/2)^4 + 84*tan(c/2 + (d*x)/2)^6 + 126*tan(c/2 + (d*
x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 + 84*tan(c/2 + (d*x)/2)^12 + 36*tan(c/2
 + (d*x)/2)^14 + 9*tan(c/2 + (d*x)/2)^16 + tan(c/2 + (d*x)/2)^18 + 1)) - (a
*(8*a^2 + 9*b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(64*d)
```

sympy [A] time = 16.18, size = 505, normalized size = 1.43

$$\begin{cases} \frac{a^3x\sin^6(c+dx)}{16} + \frac{3a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3a^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{a^3x\cos^6(c+dx)}{16} + \frac{a^3\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{a^3\sin^3(c+d \\ x(a+b\sin(c))^3\sin^2(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(c + d*x)**6/16 + 3*a**3*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a**3*x*cos(c + d*x)
**6/16 + a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + a**3*sin(c + d*x)**3*co
s(c + d*x)**3/(6*d) - a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 3*a**2*b*s
in(c + d*x)**2*cos(c + d*x)**5/(5*d) - 6*a**2*b*cos(c + d*x)**7/(35*d) + 9*
a*b**2*x*sin(c + d*x)**8/128 + 9*a*b**2*x*sin(c + d*x)**6*cos(c + d*x)**2/3
2 + 27*a*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 9*a*b**2*x*sin(c + d*x
)**2*cos(c + d*x)**6/32 + 9*a*b**2*x*cos(c + d*x)**8/128 + 9*a*b**2*sin(c +
 d*x)**7*cos(c + d*x)/(128*d) + 33*a*b**2*sin(c + d*x)**5*cos(c + d*x)**3/(
128*d) - 33*a*b**2*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 9*a*b**2*sin(c
 + d*x)*cos(c + d*x)**7/(128*d) - b**3*sin(c + d*x)**4*cos(c + d*x)**5/(5*d
) - 4*b**3*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - 8*b**3*cos(c + d*x)**9/
(315*d), Ne(d, 0)), (x*(a + b*sin(c))**3*sin(c)**2*cos(c)**4, True))
```

3.1117 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=194

$$-\frac{a\left(2a^2+61b^2\right)\cos^5(c+dx)}{560d}-\frac{\left(2a^2+7b^2\right)\cos^5(c+dx)(a+b\sin(c+dx))}{112d}+\frac{b\left(8a^2+b^2\right)\sin(c+dx)\cos^3(c+dx)}{64d}$$

```
[Out] 3/128*b*(8*a^2+b^2)*x-1/560*a*(2*a^2+61*b^2)*cos(d*x+c)^5/d+3/128*b*(8*a^2+
b^2)*cos(d*x+c)*sin(d*x+c)/d+1/64*b*(8*a^2+b^2)*cos(d*x+c)^3*sin(d*x+c)/d-1
/112*(2*a^2+7*b^2)*cos(d*x+c)^5*(a+b*sin(d*x+c))/d-3/56*a*cos(d*x+c)^5*(a+b
*sin(d*x+c))^2/d-1/8*cos(d*x+c)^5*(a+b*sin(d*x+c))^3/d
```

Rubi [A] time = 0.33, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2862, 2669, 2635, 8}

$$-\frac{a\left(2a^2+61b^2\right)\cos^5(c+dx)}{560d}-\frac{\left(2a^2+7b^2\right)\cos^5(c+dx)(a+b\sin(c+dx))}{112d}+\frac{b\left(8a^2+b^2\right)\sin(c+dx)\cos^3(c+dx)}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*(8*a^2 + b^2)*x)/128 - (a*(2*a^2 + 61*b^2)*Cos[c + d*x]^5)/(560*d) + (
3*b*(8*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (b*(8*a^2 + b^2)*Cos
[c + d*x]^3*Sin[c + d*x])/(64*d) - ((2*a^2 + 7*b^2)*Cos[c + d*x]^5*(a + b*S
in[c + d*x]))/(112*d) - (3*a*Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2)/(56*d)
- (Cos[c + d*x]^5*(a + b*Sin[c + d*x])^3)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{\cos^5(c+dx)(a+b\sin(c+dx))^3}{8d}+\frac{1}{8}\int \cos^4(c+dx)(3b+$$
$$= -\frac{3a\cos^5(c+dx)(a+b\sin(c+dx))^2}{56d}-\frac{\cos^5(c+dx)(a+b\,\mathrm{s}}{8d}$$
$$= -\frac{\left(2a^2+7b^2\right)\cos^5(c+dx)(a+b\sin(c+dx))}{112d}-\frac{3a\cos^5(c+}{}$$
$$= -\frac{a\left(2a^2+61b^2\right)\cos^5(c+dx)}{560d}-\frac{\left(2a^2+7b^2\right)\cos^5(c+dx)(a}{112d}$$
$$= -\frac{a\left(2a^2+61b^2\right)\cos^5(c+dx)}{560d}+\frac{b\left(8a^2+b^2\right)\cos^3(c+dx)\,\mathrm{si}}{64d}$$
$$= -\frac{a\left(2a^2+61b^2\right)\cos^5(c+dx)}{560d}+\frac{3b\left(8a^2+b^2\right)\cos(c+dx)\,\mathrm{s}}{128d}$$
$$= \frac{3}{128}b\left(8a^2+b^2\right)x-\frac{a\left(2a^2+61b^2\right)\cos^5(c+dx)}{560d}+\frac{3b\left(8a^2\right.}{}$$

Mathematica [A] time = 0.87, size = 189, normalized size = 0.97

$$\frac{-280\left(4a^3+3ab^2\right)\cos(3(c+dx))-224a^3\cos(5(c+dx))-280a\left(8a^2+9b^2\right)\cos(c+dx)+840a^2b\sin(2(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3360*a^2*b*c + 840*b^3*c + 3360*a^2*b*d*x + 420*b^3*d*x - 280*a*(8*a^2 + 9
*b^2)*Cos[c + d*x] - 280*(4*a^3 + 3*a*b^2)*Cos[3*(c + d*x)] - 224*a^3*Cos[5
*(c + d*x)] + 168*a*b^2*Cos[5*(c + d*x)] + 120*a*b^2*Cos[7*(c + d*x)] + 840
*a^2*b*Sin[2*(c + d*x)] - 840*a^2*b*Sin[4*(c + d*x)] - 140*b^3*Sin[4*(c + d
*x)] - 280*a^2*b*Sin[6*(c + d*x)] + (35*b^3*Sin[8*(c + d*x)])/2)/(17920*d)
```

fricas [A] time = 0.86, size = 136, normalized size = 0.70

$$\frac{1920\,ab^2\cos(dx+c)^7 - 896\left(a^3+3\,ab^2\right)\cos(dx+c)^5 + 105\left(8\,a^2b+b^3\right)dx + 35\left(16\,b^3\cos(dx+c)^7 - 8\left(8\,a^2b+\right.\right.}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4480*(1920*a*b^2*cos(d*x + c)^7 - 896*(a^3 + 3*a*b^2)*cos(d*x + c)^5 + 10
5*(8*a^2*b + b^3)*d*x + 35*(16*b^3*cos(d*x + c)^7 - 8*(8*a^2*b + 3*b^3)*cos
(d*x + c)^5 + 2*(8*a^2*b + b^3)*cos(d*x + c)^3 + 3*(8*a^2*b + b^3)*cos(d*x
+ c))*sin(d*x + c))/d
```

giac [A] time = 0.36, size = 184, normalized size = 0.95

$$\frac{3\,ab^2\cos(7\,dx+7\,c)}{448\,d}+\frac{b^3\sin(8\,dx+8\,c)}{1024\,d}-\frac{a^2b\sin(6\,dx+6\,c)}{64\,d}+\frac{3\,a^2b\sin(2\,dx+2\,c)}{64\,d}+\frac{3}{128}\left(8\,a^2b+b^3\right)x-\frac{\left(4\,a^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 3/448*a*b^2*cos(7*d*x + 7*c)/d + 1/1024*b^3*sin(8*d*x + 8*c)/d - 1/64*a^2*b
*sin(6*d*x + 6*c)/d + 3/64*a^2*b*sin(2*d*x + 2*c)/d + 3/128*(8*a^2*b + b^3)
*x - 1/320*(4*a^3 - 3*a*b^2)*cos(5*d*x + 5*c)/d - 1/64*(4*a^3 + 3*a*b^2)*co
s(3*d*x + 3*c)/d - 1/64*(8*a^3 + 9*a*b^2)*cos(d*x + c)/d - 1/128*(6*a^2*b +
 b^3)*sin(4*d*x + 4*c)/d
```

maple [A] time = 0.33, size = 180, normalized size = 0.93

$$\frac{-\frac{a^3\left(\cos^5(dx+c)\right)}{5}+3a^2b\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+3a\,b^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/5*a^3*cos(d*x+c)^5+3*a^2*b*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(
d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+3*a*b^2*(-1/7*sin(d*x+
```

```
c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+b^3*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/
16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/
128*d*x+3/128*c))
```

maxima [A] time = 0.40, size = 117, normalized size = 0.60

$$-\frac{7168\,a^3\cos\left(dx+c\right)^5-560\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a^2b-3072\left(5\,\cos\left(dx+c\right)^7\right.}{35840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/35840*(7168*a^3*cos(d*x + c)^5 - 560*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12
*c - 3*sin(4*d*x + 4*c))*a^2*b - 3072*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)
*a*b^2 - 35*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*b^3)/d
```

mupad [B] time = 10.92, size = 552, normalized size = 2.85

$$\frac{3\,b\,\mathrm{atan}\left(\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(8\,a^2+b^2\right)}{64\left(\frac{3\,a^2\,b}{8}+\frac{3\,b^3}{64}\right)}\right)\left(8\,a^2+b^2\right)}{64\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a^2\,b}{8}+\frac{3\,b^3}{64}\right)+10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (3*b*atan((3*b*tan(c/2 + (d*x)/2)*(8*a^2 + b^2))/(64*((3*a^2*b)/8 + (3*b^3)
/64)))*(8*a^2 + b^2))/(64*d) - (tan(c/2 + (d*x)/2)*((3*a^2*b)/8 + (3*b^3)/6
4) + 10*a^3*tan(c/2 + (d*x)/2)^10 + 2*a^3*tan(c/2 + (d*x)/2)^14 + (12*a*b^2
)/35 + tan(c/2 + (d*x)/2)^12*(12*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/2)^8*(12*
a*b^2 + 14*a^3) - tan(c/2 + (d*x)/2)^4*((12*a*b^2)/5 - (26*a^3)/5) + tan(c/
2 + (d*x)/2)^2*((96*a*b^2)/35 + (6*a^3)/5) + tan(c/2 + (d*x)/2)^6*((96*a*b^
2)/5 + (62*a^3)/5) - tan(c/2 + (d*x)/2)^15*((3*a^2*b)/8 + (3*b^3)/64) - tan
(c/2 + (d*x)/2)^3*((41*a^2*b)/8 - (23*b^3)/64) + tan(c/2 + (d*x)/2)^13*((41
*a^2*b)/8 - (23*b^3)/64) - tan(c/2 + (d*x)/2)^5*((13*a^2*b)/8 + (333*b^3)/6
4) + tan(c/2 + (d*x)/2)^11*((13*a^2*b)/8 + (333*b^3)/64) + tan(c/2 + (d*x)/
2)^7*((31*a^2*b)/8 + (671*b^3)/64) - tan(c/2 + (d*x)/2)^9*((31*a^2*b)/8 + (
671*b^3)/64) + (2*a^3)/5)/(d*(8*tan(c/2 + (d*x)/2)^2 + 28*tan(c/2 + (d*x)/2
)^4 + 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 + 56*tan(c/2 + (d*x
)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d
*x)/2)^16 + 1)) - (3*b*(8*a^2 + b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/
(64*d)
```

sympy [A] time = 10.06, size = 456, normalized size = 2.35

$$\begin{cases} -\frac{a^3\cos^5(c+dx)}{5d} + \frac{3a^2bx\sin^6(c+dx)}{16} + \frac{9a^2bx\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{9a^2bx\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3a^2bx\cos^6(c+dx)}{16} + \frac{3a^2b\sin^5(c+dx)}{16d} \\ x(a+b\sin(c))^3\sin(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-a**3*cos(c + d*x)**5/(5*d) + 3*a**2*b*x*sin(c + d*x)**6/16 + 9*
a**2*b*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a**2*b*x*sin(c + d*x)**2*co
s(c + d*x)**4/16 + 3*a**2*b*x*cos(c + d*x)**6/16 + 3*a**2*b*sin(c + d*x)**5
*cos(c + d*x)/(16*d) + a**2*b*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*a**
2*b*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 3*a*b**2*sin(c + d*x)**2*cos(c +
d*x)**5/(5*d) - 6*a*b**2*cos(c + d*x)**7/(35*d) + 3*b**3*x*sin(c + d*x)**8/
128 + 3*b**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 9*b**3*x*sin(c + d*x)**
4*cos(c + d*x)**4/64 + 3*b**3*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 3*b**3
*x*cos(c + d*x)**8/128 + 3*b**3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 11*b
**3*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*b**3*sin(c + d*x)**3*cos(c
 + d*x)**5/(128*d) - 3*b**3*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(d, 0))
, (x*(a + b*sin(c))**3*sin(c)*cos(c)**4, True))
```

3.1118 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=250

$$-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{\left(2a^2-35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{120b^2d}-\frac{a\left(2a^2-39b^2\right)\cos(c+dx)(a+b\sin(c+dx$$

```
[Out] 1/16*b*(18*a^2+b^2)*x-a^3*arctanh(cos(d*x+c))/d-1/60*a*(2*a^4-43*a^2*b^2+36
*b^4)*cos(d*x+c)/b^2/d-1/240*(4*a^4-84*a^2*b^2+15*b^4)*cos(d*x+c)*sin(d*x+c
)/b/d-1/120*a*(2*a^2-39*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^2/b^2/d-1/120*(2*a
^2-35*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^3/b^2/d+1/15*a*cos(d*x+c)*(a+b*sin(d
*x+c))^4/b^2/d-1/6*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^4/b/d
```

Rubi [A] time = 0.66, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2895, 3049, 3033, 3023, 2735, 3770}

$$-\frac{a\left(-43a^2b^2+2a^4+36b^4\right)\cos(c+dx)}{60b^2d}-\frac{\left(2a^2-35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{120b^2d}-\frac{a\left(2a^2-39b^2\right)\cos(c+d}{120b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(18*a^2 + b^2)*x)/16 - (a^3*ArcTanh[Cos[c + d*x]])/d - (a*(2*a^4 - 43*a^
2*b^2 + 36*b^4)*Cos[c + d*x])/(60*b^2*d) - ((4*a^4 - 84*a^2*b^2 + 15*b^4)*C
os[c + d*x]*Sin[c + d*x])/(240*b*d) - (a*(2*a^2 - 39*b^2)*Cos[c + d*x]*(a +
 b*Sin[c + d*x])^2)/(120*b^2*d) - ((2*a^2 - 35*b^2)*Cos[c + d*x]*(a + b*Sin
[c + d*x])^3)/(120*b^2*d) + (a*Cos[c + d*x]*(a + b*Sin[c + d*x])^4)/(15*b^2
*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^4)/(6*b*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
```

```
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{a\cos(c+dx)(a+b\sin(c+dx))^4}{15b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{6bd}$$
$$= -\frac{\left(2a^2-35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{120b^2d} + \frac{a\cos(c+}{}$$
$$= -\frac{a\left(2a^2-39b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{120b^2d} - \frac{\left(2a^2-3\right.}{}$$
$$= -\frac{\left(4a^4-84a^2b^2+15b^4\right)\cos(c+dx)\sin(c+dx)}{240bd} - \frac{a\left(2a^2-\right.}{}$$
$$= -\frac{a\left(2a^4-43a^2b^2+36b^4\right)\cos(c+dx)}{60b^2d} - \frac{\left(4a^4-84a^2b^2+15\right.}{}$$
$$= \frac{1}{16}b\left(18a^2+b^2\right)x - \frac{a\left(2a^4-43a^2b^2+36b^4\right)\cos(c+dx)}{60b^2d} -$$
$$= \frac{1}{16}b\left(18a^2+b^2\right)x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{a\left(2a^4-43a\right.}{}$$

Mathematica [A] time = 0.53, size = 191, normalized size = 0.76

$$\frac{20\left(4a^3-9ab^2\right)\cos(3(c+dx))+960a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-960a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+120a\left(10a^2-3b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (1080*a^2*b*c + 60*b^3*c + 1080*a^2*b*d*x + 60*b^3*d*x + 120*a*(10*a^2 - 3*
b^2)*Cos[c + d*x] + 20*(4*a^3 - 9*a*b^2)*Cos[3*(c + d*x)] - 36*a*b^2*Cos[5*
(c + d*x)] - 960*a^3*Log[Cos[(c + d*x)/2]] + 960*a^3*Log[Sin[(c + d*x)/2]]
+ 720*a^2*b*Sin[2*(c + d*x)] + 15*b^3*Sin[2*(c + d*x)] + 90*a^2*b*Sin[4*(c
+ d*x)] - 15*b^3*Sin[4*(c + d*x)] - 5*b^3*Sin[6*(c + d*x)])/(960*d)
```

fricas [A] time = 0.98, size = 150, normalized size = 0.60

$$-\frac{144\,ab^2\cos\left(dx+c\right)^5-80\,a^3\cos\left(dx+c\right)^3-240\,a^3\cos\left(dx+c\right)+120\,a^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-120\,a^3\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*a*b^2*cos(d*x + c)^5 - 80*a^3*cos(d*x + c)^3 - 240*a^3*cos(d*x
+ c) + 120*a^3*log(1/2*cos(d*x + c) + 1/2) - 120*a^3*log(-1/2*cos(d*x + c)
+ 1/2) - 15*(18*a^2*b + b^3)*d*x + 5*(8*b^3*cos(d*x + c)^5 - 2*(18*a^2*b +
b^3)*cos(d*x + c)^3 - 3*(18*a^2*b + b^3)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.27, size = 427, normalized size = 1.71

$$240\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+15\left(18\,a^2b+b^3\right)(dx+c)-\frac{2\left(450\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-15\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-480\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(240*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 15*(18*a^2*b + b^3)*(d*x +
c) - 2*(450*a^2*b*tan(1/2*d*x + 1/2*c)^11 - 15*b^3*tan(1/2*d*x + 1/2*c)^11
- 480*a^3*tan(1/2*d*x + 1/2*c)^10 + 720*a*b^2*tan(1/2*d*x + 1/2*c)^10 + 630
*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 235*b^3*tan(1/2*d*x + 1/2*c)^9 - 1920*a^3*t
an(1/2*d*x + 1/2*c)^8 + 720*a*b^2*tan(1/2*d*x + 1/2*c)^8 + 180*a^2*b*tan(1/
2*d*x + 1/2*c)^7 - 390*b^3*tan(1/2*d*x + 1/2*c)^7 - 3200*a^3*tan(1/2*d*x +
1/2*c)^6 + 1440*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 180*a^2*b*tan(1/2*d*x + 1/2*
c)^5 + 390*b^3*tan(1/2*d*x + 1/2*c)^5 - 2880*a^3*tan(1/2*d*x + 1/2*c)^4 + 1
440*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 630*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 235*b
^3*tan(1/2*d*x + 1/2*c)^3 - 1440*a^3*tan(1/2*d*x + 1/2*c)^2 + 144*a*b^2*tan
(1/2*d*x + 1/2*c)^2 - 450*a^2*b*tan(1/2*d*x + 1/2*c) + 15*b^3*tan(1/2*d*x +
 1/2*c) - 320*a^3 + 144*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 0.58, size = 211, normalized size = 0.84

$$\frac{a^3\left(\cos^3(dx+c)\right)}{3d}+\frac{a^3\cos(dx+c)}{d}+\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{3a^2b\sin(dx+c)\left(\cos^3(dx+c)\right)}{4d}+\frac{9a^2b\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/3*a^3*cos(d*x+c)^3/d+a^3*cos(d*x+c)/d+1/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+3
/4/d*a^2*b*sin(d*x+c)*cos(d*x+c)^3+9/8/d*a^2*b*cos(d*x+c)*sin(d*x+c)+9/8*a^
2*b*x+9/8/d*a^2*b*c-3/5/d*cos(d*x+c)^5*a*b^2-1/6/d*b^3*sin(d*x+c)*cos(d*x+c
)^5+1/24/d*b^3*cos(d*x+c)^3*sin(d*x+c)+1/16*b^3*cos(d*x+c)*sin(d*x+c)/d+1/1
6*b^3*x+1/16/d*b^3*c
```

maxima [A] time = 0.43, size = 137, normalized size = 0.55

$$-\frac{576\,ab^2\cos(dx+c)^5-160\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(576*a*b^2*cos(d*x + c)^5 - 160*(2*cos(d*x + c)^3 + 6*cos(d*x + c) -
 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^3 - 90*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2*b - 5*(4*sin(2*d*x + 2*c)^3 +
 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*b^3)/d
```

mupad [B] time = 11.57, size = 690, normalized size = 2.76

$$\frac{a^3\ \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\frac{6\,a\,b^2}{5}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{15\,a^2\,b}{4}-\frac{b^3}{8}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(6\,a\,b^2-4\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{6\,a\,b^2}{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d - ((6*a*b^2)/5 - tan(c/2 + (d*x)/2)*((15*a^
2*b)/4 - b^3/8) + tan(c/2 + (d*x)/2)^10*(6*a*b^2 - 4*a^3) + tan(c/2 + (d*x)
/2)^2*((6*a*b^2)/5 - 12*a^3) + tan(c/2 + (d*x)/2)^8*(6*a*b^2 - 16*a^3) + ta
n(c/2 + (d*x)/2)^4*(12*a*b^2 - 24*a^3) + tan(c/2 + (d*x)/2)^6*(12*a*b^2 - (
80*a^3)/3) - tan(c/2 + (d*x)/2)^5*((3*a^2*b)/2 - (13*b^3)/4) + tan(c/2 + (d
*x)/2)^7*((3*a^2*b)/2 - (13*b^3)/4) + tan(c/2 + (d*x)/2)^11*((15*a^2*b)/4 -
 b^3/8) - tan(c/2 + (d*x)/2)^3*((21*a^2*b)/4 + (47*b^3)/24) + tan(c/2 + (d*
x)/2)^9*((21*a^2*b)/4 + (47*b^3)/24) - (8*a^3)/3)/(d*(6*tan(c/2 + (d*x)/2)^
2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/
2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (b*atan(((b*
(18*a^2 + b^2)*((9*a^2*b)/4 + b^3/8 + 2*a^3*tan(c/2 + (d*x)/2) - (b*tan(c/2
 + (d*x)/2)*(18*a^2 + b^2)*3i)/8))/16 + (b*(18*a^2 + b^2)*((9*a^2*b)/4 + b^
3/8 + 2*a^3*tan(c/2 + (d*x)/2) + (b*tan(c/2 + (d*x)/2)*(18*a^2 + b^2)*3i)/8
))/16)/(2*tan(c/2 + (d*x)/2)*(b^6/64 + (9*a^2*b^4)/16 + (81*a^4*b^2)/16) +
(9*a^5*b)/2 + (a^3*b^3)/4 - (b*(18*a^2 + b^2)*((9*a^2*b)/4 + b^3/8 + 2*a^3*
tan(c/2 + (d*x)/2) - (b*tan(c/2 + (d*x)/2)*(18*a^2 + b^2)*3i)/8)*1i)/16 + (
b*(18*a^2 + b^2)*((9*a^2*b)/4 + b^3/8 + 2*a^3*tan(c/2 + (d*x)/2) + (b*tan(c
/2 + (d*x)/2)*(18*a^2 + b^2)*3i)/8)*1i)/16))*(18*a^2 + b^2))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1119 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=229

$$\frac{(a^2+20b^2)\cos(c+dx)(a+b\sin(c+dx))^3}{20abd}+\frac{(a^2+28b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{20bd}+\frac{a(2a^2+83b^2)\sin(c+}{40d}$$

```
[Out] -3/8*a*(4*a^2-3*b^2)*x-3*a^2*b*arctanh(cos(d*x+c))/d+1/10*(a^4+56*a^2*b^2-2
*b^4)*cos(d*x+c)/b/d+1/40*a*(2*a^2+83*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/20*(a^
2+28*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^2/b/d+1/20*(a^2+20*b^2)*cos(d*x+c)*(a
+b*sin(d*x+c))^3/a/b/d-1/5*cos(d*x+c)*(a+b*sin(d*x+c))^4/b/d-cot(d*x+c)*(a+
b*sin(d*x+c))^4/a/d
```

Rubi [A] time = 0.68, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2894, 3049, 3033, 3023, 2735, 3770}

$$\frac{(56a^2b^2+a^4-2b^4)\cos(c+dx)}{10bd}+\frac{(a^2+20b^2)\cos(c+dx)(a+b\sin(c+dx))^3}{20abd}+\frac{(a^2+28b^2)\cos(c+dx)(a+b\sin}{20bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a*(4*a^2 - 3*b^2)*x)/8 - (3*a^2*b*ArcTanh[Cos[c + d*x]])/d + ((a^4 + 56
*a^2*b^2 - 2*b^4)*Cos[c + d*x])/(10*b*d) + (a*(2*a^2 + 83*b^2)*Cos[c + d*x]
*Sin[c + d*x])/(40*d) + ((a^2 + 28*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2
)/(20*b*d) + ((a^2 + 20*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(20*a*b*d
) - (Cos[c + d*x]*(a + b*Sin[c + d*x])^4)/(5*b*d) - (Cot[c + d*x]*(a + b*Si
n[c + d*x])^4)/(a*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
```

```
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{\cos(c+dx)(a+b\sin(c+dx))^4}{5bd} - \frac{\cot(c+dx)(a+b\sin(c}{ad}$$
$$= \frac{\left(a^2+20b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{20abd} - \frac{\cos(c+dx)}{}$$
$$= \frac{\left(a^2+28b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{20bd} + \frac{\left(a^2+20b^2\right)}{}$$
$$= \frac{a\left(2a^2+83b^2\right)\cos(c+dx)\sin(c+dx)}{40d} + \frac{\left(a^2+28b^2\right)\cos(}{}$$
$$= \frac{\left(a^4+56a^2b^2-2b^4\right)\cos(c+dx)}{10bd} + \frac{a\left(2a^2+83b^2\right)\cos(c+}{40d}$$
$$= -\frac{3}{8}a\left(4a^2-3b^2\right)x + \frac{\left(a^4+56a^2b^2-2b^4\right)\cos(c+dx)}{10bd} + \frac{a}{}$$
$$= -\frac{3}{8}a\left(4a^2-3b^2\right)x - \frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d} + \frac{\left(a^4+56\right.}{}$$

Mathematica [A] time = 3.04, size = 194, normalized size = 0.85

$$\frac{-40a^3\sin(2(c+dx)) + 80a^3\tan\left(\frac{1}{2}(c+dx)\right) - 80a^3\cot\left(\frac{1}{2}(c+dx)\right) - 240a^3c - 240a^3dx + 10\left(4a^2b-b^3\right)\cos(3(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-240*a^3*c + 180*a*b^2*c - 240*a^3*d*x + 180*a*b^2*d*x - 20*b*(-30*a^2 + b
^2)*Cos[c + d*x] + 10*(4*a^2*b - b^3)*Cos[3*(c + d*x)] - 2*b^3*Cos[5*(c + d
*x)] - 80*a^3*Cot[(c + d*x)/2] - 480*a^2*b*Log[Cos[(c + d*x)/2]] + 480*a^2*
b*Log[Sin[(c + d*x)/2]] - 40*a^3*Sin[2*(c + d*x)] + 120*a*b^2*Sin[2*(c + d*
x)] + 15*a*b^2*Sin[4*(c + d*x)] + 80*a^3*Tan[(c + d*x)/2])/(160*d)
```

fricas [A] time = 0.78, size = 179, normalized size = 0.78

$$-\frac{30\,ab^2\cos\left(dx+c\right)^5 + 60\,a^2b\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right) - 60\,a^2b\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/40*(30*a*b^2*cos(d*x + c)^5 + 60*a^2*b*log(1/2*cos(d*x + c) + 1/2)*sin(d
*x + c) - 60*a^2*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 5*(4*a^3 - 3
*a*b^2)*cos(d*x + c)^3 + 15*(4*a^3 - 3*a*b^2)*cos(d*x + c) + (8*b^3*cos(d*x
 + c)^5 - 40*a^2*b*cos(d*x + c)^3 - 120*a^2*b*cos(d*x + c) + 15*(4*a^3 - 3*
a*b^2)*d*x)*sin(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.30, size = 345, normalized size = 1.51

$$120\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\left(4\,a^3-3\,ab^2\right)(dx+c)-\frac{20\left(6\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/40*(120*a^2*b*log(abs(tan(1/2*d*x + 1/2*c))) + 20*a^3*tan(1/2*d*x + 1/2*c
) - 15*(4*a^3 - 3*a*b^2)*(d*x + c) - 20*(6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3
)/tan(1/2*d*x + 1/2*c) + 2*(20*a^3*tan(1/2*d*x + 1/2*c)^9 - 75*a*b^2*tan(1/
2*d*x + 1/2*c)^9 + 240*a^2*b*tan(1/2*d*x + 1/2*c)^8 - 40*b^3*tan(1/2*d*x +
1/2*c)^8 + 40*a^3*tan(1/2*d*x + 1/2*c)^7 - 30*a*b^2*tan(1/2*d*x + 1/2*c)^7
+ 720*a^2*b*tan(1/2*d*x + 1/2*c)^6 + 880*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 80*
b^3*tan(1/2*d*x + 1/2*c)^4 - 40*a^3*tan(1/2*d*x + 1/2*c)^3 + 30*a*b^2*tan(1
/2*d*x + 1/2*c)^3 + 560*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 20*a^3*tan(1/2*d*x +
 1/2*c) + 75*a*b^2*tan(1/2*d*x + 1/2*c) + 160*a^2*b - 8*b^3)/(tan(1/2*d*x +
 1/2*c)^2 + 1)^5)/d
```

maple [A] time = 0.54, size = 216, normalized size = 0.94

$$-\frac{a^3\left(\cos^5(dx+c)\right)}{d\sin(dx+c)}-\frac{a^3\left(\cos^3(dx+c)\right)\sin(dx+c)}{d}-\frac{3a^3\cos(dx+c)\sin(dx+c)}{2d}-\frac{3a^3x}{2}-\frac{3a^3c}{2d}+\frac{a^2b\left(\cos^3(dx+c\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d*a^3/sin(d*x+c)*cos(d*x+c)^5-a^3*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a^3*cos(
d*x+c)*sin(d*x+c)/d-3/2*a^3*x-3/2/d*a^3*c+1/d*a^2*b*cos(d*x+c)^3+3*a^2*b*co
s(d*x+c)/d+3/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))+3/4/d*a*b^2*sin(d*x+c)*cos(d
*x+c)^3+9/8*a*b^2*cos(d*x+c)*sin(d*x+c)/d+9/8*a*b^2*x+9/8/d*a*b^2*c-1/5/d*c
os(d*x+c)^5*b^3
```

maxima [A] time = 0.51, size = 143, normalized size = 0.62

$$-\frac{32\,b^3\cos(dx+c)^5+80\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^3-80\left(2\,\cos(dx+c)^3+6\,\cos(dx+c)-3\,\log(\cos(\right.}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/160*(32*b^3*cos(d*x + c)^5 + 80*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(t
an(d*x + c)^3 + tan(d*x + c)))*a^3 - 80*(2*cos(d*x + c)^3 + 6*cos(d*x + c)
- 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^2*b - 15*(12*d*x + 1
2*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b^2)/d
```

mupad [B] time = 9.59, size = 674, normalized size = 2.94

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(16\,a^2\,b-\frac{4\,b^3}{5}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(a^3+3\,a\,b^2\right)-10\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(3\,a\,b^2-14\,a^3\right.}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(16*a^2*b - (4*b^3)/5) - tan(c/2 + (d*x)/2)^8*(3*a*b^2
+ a^3) - 10*a^3*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^4*(3*a*b^2 - 14*a
^3) + tan(c/2 + (d*x)/2)^2*((15*a*b^2)/2 - 7*a^3) - tan(c/2 + (d*x)/2)^10*(
(15*a*b^2)/2 - a^3) + tan(c/2 + (d*x)/2)^9*(24*a^2*b - 4*b^3) + tan(c/2 + (
d*x)/2)^5*(88*a^2*b - 8*b^3) - a^3 + 56*a^2*b*tan(c/2 + (d*x)/2)^3 + 72*a^2
*b*tan(c/2 + (d*x)/2)^7)/(d*(2*tan(c/2 + (d*x)/2) + 10*tan(c/2 + (d*x)/2)^3
 + 20*tan(c/2 + (d*x)/2)^5 + 20*tan(c/2 + (d*x)/2)^7 + 10*tan(c/2 + (d*x)/2
)^9 + 2*tan(c/2 + (d*x)/2)^11)) + (a^3*tan(c/2 + (d*x)/2))/(2*d) + (3*a*ata
n(((3*a*(4*a^2 - 3*b^2)*((9*a*b^2)/4 - 3*a^3 - (a*tan(c/2 + (d*x)/2)*(4*a^2
 - 3*b^2)*9i)/4 + 6*a^2*b*tan(c/2 + (d*x)/2)))/8 + (3*a*(4*a^2 - 3*b^2)*((9
*a*b^2)/4 - 3*a^3 + (a*tan(c/2 + (d*x)/2)*(4*a^2 - 3*b^2)*9i)/4 + 6*a^2*b*t
an(c/2 + (d*x)/2)))/8)/(2*tan(c/2 + (d*x)/2)*(9*a^6 + (81*a^2*b^4)/16 - (27
*a^4*b^2)/2) - 18*a^5*b + (27*a^3*b^3)/2 - (a*(4*a^2 - 3*b^2)*((9*a*b^2)/4
- 3*a^3 - (a*tan(c/2 + (d*x)/2)*(4*a^2 - 3*b^2)*9i)/4 + 6*a^2*b*tan(c/2 + (
d*x)/2))*3i)/8 + (a*(4*a^2 - 3*b^2)*((9*a*b^2)/4 - 3*a^3 + (a*tan(c/2 + (d*
x)/2)*(4*a^2 - 3*b^2)*9i)/4 + 6*a^2*b*tan(c/2 + (d*x)/2))*3i)/8))*(4*a^2 -
3*b^2))/(4*d) + (3*a^2*b*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1120 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=231

$$-\frac{a\left(a^2-17b^2\right)\cos(c+dx)}{2d}-\frac{\left(a^2-4b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{4a^2d}-\frac{\left(a^2-6b^2\right)\cos(c+dx)(a+b\sin(c+dx)}{4ad}$$

```
[Out] -3/8*b*(12*a^2-b^2)*x+3/2*a*(a^2-2*b^2)*arctanh(cos(d*x+c))/d-1/2*a*(a^2-17
*b^2)*cos(d*x+c)/d-1/8*b*(2*a^2-21*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/4*(a^2-6*
b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^2/a/d-1/4*(a^2-4*b^2)*cos(d*x+c)*(a+b*sin(
d*x+c))^3/a^2/d-b*cot(d*x+c)*(a+b*sin(d*x+c))^4/a^2/d-1/2*cot(d*x+c)*csc(d*
x+c)*(a+b*sin(d*x+c))^4/a/d
```

Rubi [A] time = 0.69, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2893, 3049, 3033, 3023, 2735, 3770}

$$-\frac{a\left(a^2-17b^2\right)\cos(c+dx)}{2d}-\frac{\left(a^2-4b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{4a^2d}-\frac{\left(a^2-6b^2\right)\cos(c+dx)(a+b\sin(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b*(12*a^2 - b^2)*x)/8 + (3*a*(a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d)
 - (a*(a^2 - 17*b^2)*Cos[c + d*x])/(2*d) - (b*(2*a^2 - 21*b^2)*Cos[c + d*x]
*Sin[c + d*x])/(8*d) - ((a^2 - 6*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/
(4*a*d) - ((a^2 - 4*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(4*a^2*d) - (
b*Cot[c + d*x]*(a + b*Sin[c + d*x])^4)/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x]
*(a + b*Sin[c + d*x])^4)/(2*a*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
```

```
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{b\cot(c+dx)(a+b\sin(c+dx))^4}{a^2d} - \frac{\cot(c+dx)\csc(c+dx}{2a}$$
$$= -\frac{(a^2-4b^2)\cos(c+dx)(a+b\sin(c+dx))^3}{4a^2d} - \frac{b\cot(c+dx)}{}$$
$$= -\frac{(a^2-6b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{4ad} - \frac{(a^2-4b^2)\,\mathrm{co}}{}$$
$$= -\frac{b(2a^2-21b^2)\cos(c+dx)\sin(c+dx)}{8d} - \frac{(a^2-6b^2)\cos(c}{}$$
$$= -\frac{a(a^2-17b^2)\cos(c+dx)}{2d} - \frac{b(2a^2-21b^2)\cos(c+dx)\sin}{8d}$$
$$= -\frac{3}{8}b(12a^2-b^2)x - \frac{a(a^2-17b^2)\cos(c+dx)}{2d} - \frac{b(2a^2-21}{}$$
$$= -\frac{3}{8}b(12a^2-b^2)x + \frac{3a(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a}{}$$

Mathematica [A] time = 6.16, size = 252, normalized size = 1.09

$$-\frac{3(a^3-2ab^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{3(a^3-2ab^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^3\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a^3\sec^2\left(\frac{1}{2}(c+\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*(-12*a^2 + b^2)*(c + d*x))/(8*d) - (a*(4*a^2 - 15*b^2)*Cos[c + d*x])/(
4*d) + (a*b^2*Cos[3*(c + d*x)])/(4*d) - (3*a^2*b*Cot[(c + d*x)/2])/(2*d) -
(a^3*Csc[(c + d*x)/2]^2)/(8*d) + (3*(a^3 - 2*a*b^2)*Log[Cos[(c + d*x)/2]])/
(2*d) - (3*(a^3 - 2*a*b^2)*Log[Sin[(c + d*x)/2]])/(2*d) + (a^3*Sec[(c + d*x
)/2]^2)/(8*d) + (b*(-3*a^2 + b^2)*Sin[2*(c + d*x)])/(4*d) + (b^3*Sin[4*(c +
 d*x)])/(32*d) + (3*a^2*b*Tan[(c + d*x)/2])/(2*d)
```

fricas [A] time = 0.64, size = 260, normalized size = 1.13

$$\frac{8\,ab^2\cos(dx+c)^5 - 3\left(12\,a^2b-b^3\right)dx\cos(dx+c)^2 - 8\left(a^3-2\,ab^2\right)\cos(dx+c)^3 + 3\left(12\,a^2b-b^3\right)dx + 12\left(a^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/8*(8*a*b^2*cos(d*x + c)^5 - 3*(12*a^2*b - b^3)*d*x*cos(d*x + c)^2 - 8*(a^
3 - 2*a*b^2)*cos(d*x + c)^3 + 3*(12*a^2*b - b^3)*d*x + 12*(a^3 - 2*a*b^2)*c
os(d*x + c) - 6*(a^3 - 2*a*b^2 - (a^3 - 2*a*b^2)*cos(d*x + c)^2)*log(1/2*co
s(d*x + c) + 1/2) + 6*(a^3 - 2*a*b^2 - (a^3 - 2*a*b^2)*cos(d*x + c)^2)*log(
-1/2*cos(d*x + c) + 1/2) + (2*b^3*cos(d*x + c)^5 - (12*a^2*b - b^3)*cos(d*x
 + c)^3 + 3*(12*a^2*b - b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2
- d)
```

giac [A] time = 0.31, size = 400, normalized size = 1.73

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\left(12\,a^2b-b^3\right)(dx+c)-12\left(a^3-2\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*b*tan(1/2*d*x + 1/2*c) - 3*(12*a^2
*b - b^3)*(d*x + c) - 12*(a^3 - 2*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (
18*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*
tan(1/2*d*x + 1/2*c) - a^3)/tan(1/2*d*x + 1/2*c)^2 + 2*(12*a^2*b*tan(1/2*d*
x + 1/2*c)^7 - 5*b^3*tan(1/2*d*x + 1/2*c)^7 - 8*a^3*tan(1/2*d*x + 1/2*c)^6
+ 48*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^3
*tan(1/2*d*x + 1/2*c)^5 - 24*a^3*tan(1/2*d*x + 1/2*c)^4 + 96*a*b^2*tan(1/2*
d*x + 1/2*c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*b^3*tan(1/2*d*x + 1/2*
c)^3 - 24*a^3*tan(1/2*d*x + 1/2*c)^2 + 80*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12
*a^2*b*tan(1/2*d*x + 1/2*c) + 5*b^3*tan(1/2*d*x + 1/2*c) - 8*a^3 + 32*a*b^2
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.66, size = 279, normalized size = 1.21

$$-\frac{a^3\left(\cos^5(dx+c)\right)}{2d\sin(dx+c)^2}-\frac{a^3\left(\cos^3(dx+c)\right)}{2d}-\frac{3a^3\cos(dx+c)}{2d}-\frac{3a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{3a^2b\left(\cos^5(dx+c\right.}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5-1/2*a^3*cos(d*x+c)^3/d-3/2*a^3*cos(d*x
+c)/d-3/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3/d*a^2*b/sin(d*x+c)*cos(d*x+c)^5
-3/d*a^2*b*sin(d*x+c)*cos(d*x+c)^3-9/2/d*a^2*b*cos(d*x+c)*sin(d*x+c)-9/2*a^
```

```
2*b*x-9/2/d*a^2*b*c+a*b^2*cos(d*x+c)^3/d+3*a*b^2*cos(d*x+c)/d+3/d*a*b^2*ln(
csc(d*x+c)-cot(d*x+c))+1/4/d*b^3*cos(d*x+c)^3*sin(d*x+c)+3/8*b^3*cos(d*x+c)
*sin(d*x+c)/d+3/8*b^3*x+3/8/d*b^3*c
```

maxima [A] time = 0.46, size = 186, normalized size = 0.81

$$-\frac{48\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^2b-16\left(2\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\mathrm{lo}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/32*(48*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x +
 c)))*a^2*b - 16*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) +
1) + 3*log(cos(d*x + c) - 1))*a*b^2 - (12*d*x + 12*c + sin(4*d*x + 4*c) + 8
*sin(2*d*x + 2*c))*b^3 - 8*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos
(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.59, size = 718, normalized size = 3.11

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(32\,a\,b^2-10\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(48\,a\,b^2-\frac{17\,a^3}{2}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(80\,a\,b^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (tan(c/2 + (d*x)/2)^2*(32*a*b^2 - 10*a^3
) + tan(c/2 + (d*x)/2)^8*(48*a*b^2 - (17*a^3)/2) + tan(c/2 + (d*x)/2)^4*(80
*a*b^2 - 27*a^3) + tan(c/2 + (d*x)/2)^6*(96*a*b^2 - 26*a^3) + tan(c/2 + (d*
x)/2)^9*(6*a^2*b - 5*b^3) - tan(c/2 + (d*x)/2)^7*(12*a^2*b - 3*b^3) - tan(c
/2 + (d*x)/2)^3*(36*a^2*b - 5*b^3) - tan(c/2 + (d*x)/2)^5*(48*a^2*b + 3*b^3
) - a^3/2 - 6*a^2*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 16*tan
(c/2 + (d*x)/2)^4 + 24*tan(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2)^8 + 4*t
an(c/2 + (d*x)/2)^10)) + (log(tan(c/2 + (d*x)/2))*(3*a*b^2 - (3*a^3)/2))/d
+ (3*a^2*b*tan(c/2 + (d*x)/2))/(2*d) + (3*b*atan(((3*b*(12*a^2 - b^2)*(tan(
c/2 + (d*x)/2)*(6*a*b^2 - 3*a^3) - 9*a^2*b + (3*b^3)/4 - (b*tan(c/2 + (d*x)
```

```
/2)*(12*a^2 - b^2)*9i)/4))/8 + (3*b*(12*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(6*a
*b^2 - 3*a^3) - 9*a^2*b + (3*b^3)/4 + (b*tan(c/2 + (d*x)/2)*(12*a^2 - b^2)*
9i)/4))/8)/(2*tan(c/2 + (d*x)/2)*((9*b^6)/16 - (27*a^2*b^4)/2 + 81*a^4*b^2)
 + (9*a*b^5)/2 + 27*a^5*b - (225*a^3*b^3)/4 - (b*(12*a^2 - b^2)*(tan(c/2 +
(d*x)/2)*(6*a*b^2 - 3*a^3) - 9*a^2*b + (3*b^3)/4 - (b*tan(c/2 + (d*x)/2)*(1
2*a^2 - b^2)*9i)/4)*3i)/8 + (b*(12*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(6*a*b^2
- 3*a^3) - 9*a^2*b + (3*b^3)/4 + (b*tan(c/2 + (d*x)/2)*(12*a^2 - b^2)*9i)/4
)*3i)/8))*(12*a^2 - b^2))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1121 $\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=194

$$-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}+a^3x-\frac{9a^2b\cos(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{9a^2b\tanh^{-1}(\cos(c+a$$

```
[Out] a^3*x-9/2*a*b^2*x+9/2*a^2*b*arctanh(cos(d*x+c))/d-b^3*arctanh(cos(d*x+c))/d
-9/2*a^2*b*cos(d*x+c)/d+b^3*cos(d*x+c)/d+1/3*b^3*cos(d*x+c)^3/d+a^3*cot(d*x
+c)/d-9/2*a*b^2*cot(d*x+c)/d+3/2*a*b^2*cos(d*x+c)^2*cot(d*x+c)/d-3/2*a^2*b*
cos(d*x+c)*cot(d*x+c)^2/d-1/3*a^3*cot(d*x+c)^3/d
```

Rubi [A] time = 0.22, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2722, 2592, 302, 206, 2591, 288, 321, 203, 3473, 8}

$$-\frac{9a^2b\cos(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] a^3*x - (9*a*b^2*x)/2 + (9*a^2*b*ArcTanh[Cos[c + d*x]])/(2*d) - (b^3*ArcTan
h[Cos[c + d*x]])/d - (9*a^2*b*Cos[c + d*x])/(2*d) + (b^3*Cos[c + d*x])/d +
(b^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x])/
(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (3*a^2*b*Cos[c + d*x]
*Cot[c + d*x]^2)/(2*d) - (a^3*Cot[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(b^3\cos^3(c+dx)\cot(c+dx)+3ab^2\cos^2(c+dx)\cot^2(c+dx)+3a^2b\right.$$
$$= a^3\int \cot^4(c+dx)\,dx+\left(3a^2b\right)\int \cos(c+dx)\cot^3(c+dx)\,dx+\left(3ab^2\right)$$
$$= -\frac{a^3\cot^3(c+dx)}{3d}-a^3\int \cot^2(c+dx)\,dx-\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx,x\right.}{d}$$
$$= \frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2}{2d}$$
$$= a^3x-\frac{9a^2b\cos(c+dx)}{2d}+\frac{b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}+\frac{a^3\cot(c+}{d}$$
$$= a^3x-\frac{9}{2}ab^2x+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{9}{}$$

Mathematica [A] time = 6.25, size = 355, normalized size = 1.83

$$\frac{\csc\left(\frac{1}{2}(c+dx)\right)\left(4a^3\cos\left(\frac{1}{2}(c+dx)\right)-9ab^2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{6d}+\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(9ab^2\sin\left(\frac{1}{2}(c+dx)\right)-4a^3\sin\left(\frac{1}{2}\right.\right.}{6d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 - 9*b^2)*(c + d*x))/(2*d) + (b*(-12*a^2 + 5*b^2)*Cos[c + d*x])/(4
*d) + (b^3*Cos[3*(c + d*x)])/(12*d) + ((4*a^3*Cos[(c + d*x)/2] - 9*a*b^2*Co
s[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (3*a^2*b*Csc[(c + d*x)/2]^2)/(8*d
) - (a^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + ((9*a^2*b - 2*b^3)*L
og[Cos[(c + d*x)/2]])/(2*d) + ((-9*a^2*b + 2*b^3)*Log[Sin[(c + d*x)/2]])/(2
*d) + (3*a^2*b*Sec[(c + d*x)/2]^2)/(8*d) + (Sec[(c + d*x)/2]*(-4*a^3*Sin[(c
 + d*x)/2] + 9*a*b^2*Sin[(c + d*x)/2]))/(6*d) - (3*a*b^2*Sin[2*(c + d*x)])/
(4*d) + (a^3*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)
```

fricas [A] time = 0.84, size = 293, normalized size = 1.51

$$\frac{18\,ab^2\cos(dx+c)^5+8\left(2\,a^3-9\,ab^2\right)\cos(dx+c)^3-3\left(9\,a^2b-2\,b^3-\left(9\,a^2b-2\,b^3\right)\cos(dx+c)^2\right)\log\left(\frac{1}{2}\,\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(18*a*b^2*cos(d*x + c)^5 + 8*(2*a^3 - 9*a*b^2)*cos(d*x + c)^3 - 3*(9*a
^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 3*(9*a^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log
(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*(2*a^3 - 9*a*b^2)*cos(d*x + c) +
 2*(2*b^3*cos(d*x + c)^5 + 3*(2*a^3 - 9*a*b^2)*d*x*cos(d*x + c)^2 - 2*(9*a^
2*b - 2*b^3)*cos(d*x + c)^3 - 3*(2*a^3 - 9*a*b^2)*d*x + 3*(9*a^2*b - 2*b^3)
*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [B] time = 0.32, size = 421, normalized size = 2.17

$$\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+27\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+108\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\left(2\,a^3-9\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/72*(3*a^3*tan(1/2*d*x + 1/2*c)^3 + 27*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 45*a
^3*tan(1/2*d*x + 1/2*c) + 108*a*b^2*tan(1/2*d*x + 1/2*c) + 36*(2*a^3 - 9*a*
b^2)*(d*x + c) - 36*(9*a^2*b - 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (198
*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 44*b^3*tan(1/2*d*x + 1/2*c)^9 + 45*a^3*tan(
1/2*d*x + 1/2*c)^8 + 108*a*b^2*tan(1/2*d*x + 1/2*c)^8 + 135*a^2*b*tan(1/2*d
*x + 1/2*c)^7 + 156*b^3*tan(1/2*d*x + 1/2*c)^7 + 132*a^3*tan(1/2*d*x + 1/2*
c)^6 - 324*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 351*a^2*b*tan(1/2*d*x + 1/2*c)^5
+ 156*b^3*tan(1/2*d*x + 1/2*c)^5 + 126*a^3*tan(1/2*d*x + 1/2*c)^4 - 540*a*b
^2*tan(1/2*d*x + 1/2*c)^4 - 315*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 148*b^3*tan(
1/2*d*x + 1/2*c)^3 + 36*a^3*tan(1/2*d*x + 1/2*c)^2 - 108*a*b^2*tan(1/2*d*x
+ 1/2*c)^2 - 27*a^2*b*tan(1/2*d*x + 1/2*c) - 3*a^3)/(tan(1/2*d*x + 1/2*c)^3
 + tan(1/2*d*x + 1/2*c))^3)/d
```

maple [A] time = 0.60, size = 264, normalized size = 1.36

$$-\frac{a^3\left(\cot^3(dx+c)\right)}{3d}+\frac{a^3\cot(dx+c)}{d}+a^3x+\frac{a^3c}{d}-\frac{3a^2b\left(\cos^5(dx+c)\right)}{2d\sin(dx+c)^2}-\frac{3a^2b\left(\cos^3(dx+c)\right)}{2d}-\frac{9a^2b\cos(dx+c)}{2d}-\frac{9a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/3*a^3*cot(d*x+c)^3/d+a^3*cot(d*x+c)/d+a^3*x+1/d*a^3*c-3/2/d*a^2*b/sin(d*
x+c)^2*cos(d*x+c)^5-3/2/d*a^2*b*cos(d*x+c)^3-9/2*a^2*b*cos(d*x+c)/d-9/2/d*a
^2*b*ln(csc(d*x+c)-cot(d*x+c))-3/d*a*b^2/sin(d*x+c)*cos(d*x+c)^5-3/d*a*b^2*
sin(d*x+c)*cos(d*x+c)^3-9/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d-9/2*a*b^2*x-9/2/d
*a*b^2*c+1/3*b^3*cos(d*x+c)^3/d+b^3*cos(d*x+c)/d+1/d*b^3*ln(csc(d*x+c)-cot(
d*x+c))
```

maxima [A] time = 0.47, size = 187, normalized size = 0.96

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3-18\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)ab^2+2\left(2\,\cos(dx+c)^3+6\,\cos(dx+c)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 - 18*(3*d
*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*a*b^2 +
2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*log(cos(
d*x + c) - 1))*b^3 + 9*a^2*b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d
*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.65, size = 405, normalized size = 2.09

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(\frac{9\,a^2\,b}{2}-b^3\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)\left(\frac{a\,b^2\,9{}\mathrm{i}}{2}-a^3\,1{}\mathrm{i}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a\,b^2}{2}-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^3)/(24*d) - (log(tan(c/2 + (d*x)/2))*((9*a^2*b)/2 -
 b^3))/d - (log(tan(c/2 + (d*x)/2) + 1i)*((a*b^2*9i)/2 - a^3*1i))/d + (tan(
c/2 + (d*x)/2)*((3*a*b^2)/2 - (5*a^3)/8))/d - (tan(c/2 + (d*x)/2)^2*(12*a*b
^2 - 4*a^3) - tan(c/2 + (d*x)/2)^8*(12*a*b^2 + 5*a^3) + tan(c/2 + (d*x)/2)^
4*(60*a*b^2 - 14*a^3) + tan(c/2 + (d*x)/2)^6*(36*a*b^2 - (44*a^3)/3) + tan(
c/2 + (d*x)/2)^7*(51*a^2*b - 32*b^3) + tan(c/2 + (d*x)/2)^3*(57*a^2*b - (64
*b^3)/3) + tan(c/2 + (d*x)/2)^5*(105*a^2*b - 32*b^3) + a^3/3 + 3*a^2*b*tan(
c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 24*tan(c/2 + (d*x)/2)^5 + 24*t
an(c/2 + (d*x)/2)^7 + 8*tan(c/2 + (d*x)/2)^9)) + (3*a^2*b*tan(c/2 + (d*x)/2
)^2)/(8*d) - (a*log(tan(c/2 + (d*x)/2) - 1i)*(2*a^2 - 9*b^2)*1i)/(2*d)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1122 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=187

$$-\frac{b^2\left(73a^2-2b^2\right)\cos(c+dx)}{8ad}-\frac{3a\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{3}{2}bx\left(2a^2-b^2\right)+\frac{17b\cot(c+dx)(a+b\sin(c}{8d}$$

```
[Out] 3/2*b*(2*a^2-b^2)*x-3/8*a*(a^2-12*b^2)*arctanh(cos(d*x+c))/d-1/8*b^2*(73*a^
2-2*b^2)*cos(d*x+c)/a/d-13/4*b^3*cos(d*x+c)*sin(d*x+c)/d+17/8*b*cot(d*x+c)*
(a+b*sin(d*x+c))^2/d+5/8*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^3/d-1/4*cot
(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^4/a/d
```

Rubi [A] time = 0.66, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2893, 3047, 3033, 3023, 2735, 3770}

$$-\frac{b^2\left(73a^2-2b^2\right)\cos(c+dx)}{8ad}-\frac{3a\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{3}{2}bx\left(2a^2-b^2\right)+\frac{17b\cot(c+dx)(a+b\sin(c}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*(2*a^2 - b^2)*x)/2 - (3*a*(a^2 - 12*b^2)*ArcTanh[Cos[c + d*x]])/(8*d)
- (b^2*(73*a^2 - 2*b^2)*Cos[c + d*x])/(8*a*d) - (13*b^3*Cos[c + d*x]*Sin[c
+ d*x])/(4*d) + (17*b*Cot[c + d*x]*(a + b*Sin[c + d*x])^2)/(8*d) + (5*Cot[c
 + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^3)/(8*d) - (Cot[c + d*x]*Csc[c +
d*x]^3*(a + b*Sin[c + d*x])^4)/(4*a*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

```
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^4}{4ad} - \frac{\int \csc^3(c+$$

$$= \frac{5\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3}{8d} - \frac{\cot(c+dx)}{}$$

$$= \frac{17b\cot(c+dx)(a+b\sin(c+dx))^2}{8d} + \frac{5\cot(c+dx)\csc(c+}{}$$

$$= -\frac{13b^3\cos(c+dx)\sin(c+dx)}{4d} + \frac{17b\cot(c+dx)(a+b\sin(}{8d}$$

$$= -\frac{b^2\left(73a^2-2b^2\right)\cos(c+dx)}{8ad} - \frac{13b^3\cos(c+dx)\sin(c+dx}{4d}$$

$$= \frac{3}{2}b\left(2a^2-b^2\right)x - \frac{b^2\left(73a^2-2b^2\right)\cos(c+dx)}{8ad} - \frac{13b^3\cos(c}{}$$

$$= \frac{3}{2}b\left(2a^2-b^2\right)x - \frac{3a\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b^2(}{}$$

Mathematica [B] time = 6.29, size = 381, normalized size = 2.04

$$\frac{\left(5a^3-12ab^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{\left(12ab^2-5a^3\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{3\left(a^3-12ab^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{3\left(a^3-1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b*(-2*a^2 + b^2)*(c + d*x))/(2*d) - (3*a*b^2*Cos[c + d*x])/d + ((4*a^2*
b*Cos[(c + d*x)/2] - b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(2*d) + ((5*a^
3 - 12*a*b^2)*Csc[(c + d*x)/2]^2)/(32*d) - (a^2*b*Cot[(c + d*x)/2]*Csc[(c +
 d*x)/2]^2)/(8*d) - (a^3*Csc[(c + d*x)/2]^4)/(64*d) - (3*(a^3 - 12*a*b^2)*L
og[Cos[(c + d*x)/2]])/(8*d) + (3*(a^3 - 12*a*b^2)*Log[Sin[(c + d*x)/2]])/(8
*d) + ((-5*a^3 + 12*a*b^2)*Sec[(c + d*x)/2]^2)/(32*d) + (a^3*Sec[(c + d*x)/
2]^4)/(64*d) + (Sec[(c + d*x)/2]*(-4*a^2*b*Sin[(c + d*x)/2] + b^3*Sin[(c +
d*x)/2]))/(2*d) - (b^3*Sin[2*(c + d*x)])/(4*d) + (a^2*b*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2])/(8*d)
```

fricas [A] time = 0.89, size = 331, normalized size = 1.77

$$-\frac{48\,ab^2\cos(dx+c)^5-24\left(2\,a^2b-b^3\right)dx\cos(dx+c)^4+48\left(2\,a^2b-b^3\right)dx\cos(dx+c)^2+10\left(a^3-12\,ab^2\right)\cos(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*(48*a*b^2*cos(d*x + c)^5 - 24*(2*a^2*b - b^3)*d*x*cos(d*x + c)^4 + 48
*(2*a^2*b - b^3)*d*x*cos(d*x + c)^2 + 10*(a^3 - 12*a*b^2)*cos(d*x + c)^3 -
24*(2*a^2*b - b^3)*d*x - 6*(a^3 - 12*a*b^2)*cos(d*x + c) + 3*((a^3 - 12*a*b
^2)*cos(d*x + c)^4 + a^3 - 12*a*b^2 - 2*(a^3 - 12*a*b^2)*cos(d*x + c)^2)*lo
g(1/2*cos(d*x + c) + 1/2) - 3*((a^3 - 12*a*b^2)*cos(d*x + c)^4 + a^3 - 12*a
*b^2 - 2*(a^3 - 12*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) + 8*
(b^3*cos(d*x + c)^5 + 4*(2*a^2*b - b^3)*cos(d*x + c)^3 - 3*(2*a^2*b - b^3)*
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.33, size = 343, normalized size = 1.83

$$a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^4 + 8\,a^2 b \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 8\,a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 24\,ab^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 120\,a^2 b \tan\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*tan(1/2*d*x + 1/2*c)^4 + 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*t
an(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 120*a^2*b*tan(1/2
*d*x + 1/2*c) + 32*b^3*tan(1/2*d*x + 1/2*c) + 96*(2*a^2*b - b^3)*(d*x + c)
+ 24*(a^3 - 12*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 64*(b^3*tan(1/2*d*x
+ 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c) - 6*
a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - (50*a^3*tan(1/2*d*x + 1/2*c)^4 - 60
0*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 32*b^3*
tan(1/2*d*x + 1/2*c)^3 - 8*a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*
x + 1/2*c)^2 + 8*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^4)/
d
```

maple [A] time = 0.64, size = 316, normalized size = 1.69

$$-\frac{a^3\left(\cos^5(dx+c)\right)}{4d\sin(dx+c)^4}+\frac{a^3\left(\cos^5(dx+c)\right)}{8d\sin(dx+c)^2}+\frac{a^3\left(\cos^3(dx+c)\right)}{8d}+\frac{3a^3\cos(dx+c)}{8d}+\frac{3a^3\ln(\csc(dx+c)-\cot(dx+c))}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/4/d*a^3/sin(d*x+c)^4*cos(d*x+c)^5+1/8/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5+1/
8*a^3*cos(d*x+c)^3/d+3/8*a^3*cos(d*x+c)/d+3/8/d*a^3*ln(csc(d*x+c)-cot(d*x+c
))-1/d*a^2*b*cot(d*x+c)^3+3*a^2*b*x+3*a^2*b*cot(d*x+c)/d+3/d*a^2*b*c-3/2/d*
a*b^2/sin(d*x+c)^2*cos(d*x+c)^5-3/2*a*b^2*cos(d*x+c)^3/d-9/2*a*b^2*cos(d*x+
c)/d-9/2/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))-1/d*b^3/sin(d*x+c)*cos(d*x+c)^5-
1/d*b^3*cos(d*x+c)^3*sin(d*x+c)-3/2*b^3*cos(d*x+c)*sin(d*x+c)/d-3/2*b^3*x-3
/2/d*b^3*c
```

maxima [A] time = 0.65, size = 212, normalized size = 1.13

$$\frac{16\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2b-8\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)b^3-a^3\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log(c\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/16*(16*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2*b - 8*(3
*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*b^3 -
a^3*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)
^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 12*a*b^2*(2*
cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1
) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.50, size = 699, normalized size = 3.74

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{3\,a\,b^2}{8}-\frac{a^3}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(6\,a\,b^2-\frac{3\,a^3}{2}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(102\,a\,b^2-2\,a^3\right)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^4)/(64*d) + (tan(c/2 + (d*x)/2)^2*((3*a*b^2)/8 - a^
3/8))/d - (tan(c/2 + (d*x)/2)^2*(6*a*b^2 - (3*a^3)/2) + tan(c/2 + (d*x)/2)^
6*(102*a*b^2 - 2*a^3) + tan(c/2 + (d*x)/2)^4*(108*a*b^2 - (15*a^3)/4) - tan
(c/2 + (d*x)/2)^3*(26*a^2*b - 8*b^3) - tan(c/2 + (d*x)/2)^7*(30*a^2*b + 8*b
^3) - tan(c/2 + (d*x)/2)^5*(58*a^2*b - 32*b^3) + a^3/4 + 2*a^2*b*tan(c/2 +
(d*x)/2))/(d*(16*tan(c/2 + (d*x)/2)^4 + 32*tan(c/2 + (d*x)/2)^6 + 16*tan(c/
2 + (d*x)/2)^8)) - (tan(c/2 + (d*x)/2)*((15*a^2*b)/8 - b^3/2))/d + (3*a*log
```

```
(tan(c/2 + (d*x)/2))*(a^2 - 12*b^2))/(8*d) + (a^2*b*tan(c/2 + (d*x)/2)^3)/(
8*d) - (3*b*atan(((3*b*(2*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 - (3*a^3)
/4) - 6*a^2*b + 3*b^3 - b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)*9i))/2 + (3*b*(2
*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 - (3*a^3)/4) - 6*a^2*b + 3*b^3 + b
*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)*9i))/2)/(2*tan(c/2 + (d*x)/2)*(9*b^6 - 36
*a^2*b^4 + 36*a^4*b^2) + 27*a*b^5 + (9*a^5*b)/2 - (225*a^3*b^3)/4 - (b*(2*a
^2 - b^2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 - (3*a^3)/4) - 6*a^2*b + 3*b^3 - b*t
an(c/2 + (d*x)/2)*(2*a^2 - b^2)*9i)*3i)/2 + (b*(2*a^2 - b^2)*(tan(c/2 + (d*
x)/2)*(9*a*b^2 - (3*a^3)/4) - 6*a^2*b + 3*b^3 + b*tan(c/2 + (d*x)/2)*(2*a^2
 - b^2)*9i)*3i)/2))*(2*a^2 - b^2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1123 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=227

$$-\frac{a(4a^2-29b^2)\cot(c+dx)}{20d}-\frac{3b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b^3(83a^2+2b^2)\cos(c+dx)}{40a^2d}+\frac{b\cot(c+dx)\text{cs}}{}$$

```
[Out] 3*a*b^2*x-3/8*b*(3*a^2-4*b^2)*arctanh(cos(d*x+c))/d-1/40*b^3*(83*a^2+2*b^2)
*cos(d*x+c)/a^2/d-1/20*a*(4*a^2-29*b^2)*cot(d*x+c)/d+27/40*b*cot(d*x+c)*csc
(d*x+c)*(a+b*sin(d*x+c))^2/d+2/5*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^3
/d+1/20*b*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^4/a^2/d-1/5*cot(d*x+c)*c
sc(d*x+c)^4*(a+b*sin(d*x+c))^4/a/d
```

Rubi [A] time = 0.71, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2893, 3047, 3031, 3023, 2735, 3770}

$$-\frac{b^3(83a^2+2b^2)\cos(c+dx)}{40a^2d}-\frac{a(4a^2-29b^2)\cot(c+dx)}{20d}-\frac{3b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{b\cot(c+dx)\text{cs}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] 3*a*b^2*x - (3*b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) - (b^3*(83*a^
2 + 2*b^2)*Cos[c + d*x])/(40*a^2*d) - (a*(4*a^2 - 29*b^2)*Cot[c + d*x])/(20
*d) + (27*b*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^2)/(40*d) + (2*C
ot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3)/(5*d) + (b*Cot[c + d*x]*
Csc[c + d*x]^3*(a + b*Sin[c + d*x])^4)/(20*a^2*d) - (Cot[c + d*x]*Csc[c + d
*x]^4*(a + b*Sin[c + d*x])^4)/(5*a*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
```

```
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^4}{20a^2d} - \frac{\cot(c+dx}{}$$

$$= \frac{2\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{5d} + \frac{b\cot(c+}{}$$

$$= \frac{27b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{40d} + \frac{2\cot(c+}{}$$

$$= -\frac{a\left(4a^2-29b^2\right)\cot(c+dx)}{20d} + \frac{27b\cot(c+dx)\csc(c+dx)(}{40d}$$

$$= -\frac{b^3\left(83a^2+2b^2\right)\cos(c+dx)}{40a^2d} - \frac{a\left(4a^2-29b^2\right)\cot(c+dx)}{20d}$$

$$= 3ab^2x - \frac{b^3\left(83a^2+2b^2\right)\cos(c+dx)}{40a^2d} - \frac{a\left(4a^2-29b^2\right)\cot(}{20d}$$

$$= 3ab^2x - \frac{3b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b^3\left(83a^2+\right.}{4}$$

Mathematica [A] time = 1.30, size = 405, normalized size = 1.78

$$\frac{-32\left(a^3-20ab^2\right)\cot\left(\frac{1}{2}(c+dx)\right)+32a^3\tan\left(\frac{1}{2}(c+dx)\right)-a^3\sin(c+dx)\csc^6\left(\frac{1}{2}(c+dx)\right)+64a^3\sin^6\left(\frac{1}{2}(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (960*a*b^2*c + 960*a*b^2*d*x - 320*b^3*Cos[c + d*x] - 32*(a^3 - 20*a*b^2)*C
ot[(c + d*x)/2] + 150*a^2*b*Csc[(c + d*x)/2]^2 - 40*b^3*Csc[(c + d*x)/2]^2
- 15*a^2*b*Csc[(c + d*x)/2]^4 - 360*a^2*b*Log[Cos[(c + d*x)/2]] + 480*b^3*L
og[Cos[(c + d*x)/2]] + 360*a^2*b*Log[Sin[(c + d*x)/2]] - 480*b^3*Log[Sin[(c
 + d*x)/2]] - 150*a^2*b*Sec[(c + d*x)/2]^2 + 40*b^3*Sec[(c + d*x)/2]^2 + 15
*a^2*b*Sec[(c + d*x)/2]^4 - 112*a^3*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320
*a*b^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 64*a^3*Csc[c + d*x]^5*Sin[(c + d
*x)/2]^6 + 7*a^3*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a*b^2*Csc[(c + d*x)/2
]^4*Sin[c + d*x] - a^3*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 32*a^3*Tan[(c + d*
x)/2] - 640*a*b^2*Tan[(c + d*x)/2])/(320*d)
```

fricas [A] time = 0.91, size = 334, normalized size = 1.47

$$-\frac{560\,ab^2\cos(dx+c)^3+16\left(a^3-20\,ab^2\right)\cos(dx+c)^5-240\,ab^2\cos(dx+c)+15\left(\left(3\,a^2b-4\,b^3\right)\cos(dx+c)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/80*(560*a*b^2*cos(d*x + c)^3 + 16*(a^3 - 20*a*b^2)*cos(d*x + c)^5 - 240*
a*b^2*cos(d*x + c) + 15*((3*a^2*b - 4*b^3)*cos(d*x + c)^4 + 3*a^2*b - 4*b^3
 - 2*(3*a^2*b - 4*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 15*((3*a^2*b - 4*b^3)*cos(d*x + c)^4 + 3*a^2*b - 4*b^3 - 2*(3*a^2*b
- 4*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 10*(24
*a*b^2*d*x*cos(d*x + c)^4 - 8*b^3*cos(d*x + c)^5 - 48*a*b^2*d*x*cos(d*x + c
)^2 + 24*a*b^2*d*x - 5*(3*a^2*b - 4*b^3)*cos(d*x + c)^3 + 3*(3*a^2*b - 4*b^
3)*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
*sin(d*x + c))
```

giac [A] time = 0.35, size = 356, normalized size = 1.57

$$\frac{2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,a^2b\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/320*(2*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 10*
a^3*tan(1/2*d*x + 1/2*c)^3 + 40*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a^2*b*ta
n(1/2*d*x + 1/2*c)^2 + 40*b^3*tan(1/2*d*x + 1/2*c)^2 + 960*(d*x + c)*a*b^2
+ 20*a^3*tan(1/2*d*x + 1/2*c) - 600*a*b^2*tan(1/2*d*x + 1/2*c) - 640*b^3/(t
an(1/2*d*x + 1/2*c)^2 + 1) + 120*(3*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/
2*c))) - (822*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 1096*b^3*tan(1/2*d*x + 1/2*c)^
5 + 20*a^3*tan(1/2*d*x + 1/2*c)^4 - 600*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*
a^2*b*tan(1/2*d*x + 1/2*c)^3 + 40*b^3*tan(1/2*d*x + 1/2*c)^3 - 10*a^3*tan(1
/2*d*x + 1/2*c)^2 + 40*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^2*b*tan(1/2*d*x
+ 1/2*c) + 2*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.51, size = 260, normalized size = 1.15

$$-\frac{a^3\left(\cos^5(dx+c)\right)}{5d\sin(dx+c)^5}-\frac{3a^2b\left(\cos^5(dx+c)\right)}{4d\sin(dx+c)^4}+\frac{3a^2b\left(\cos^5(dx+c)\right)}{8d\sin(dx+c)^2}+\frac{3a^2b\left(\cos^3(dx+c)\right)}{8d}+\frac{9a^2b\cos(dx+c)}{8d}+\frac{9a^2b\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/5/d*a^3/sin(d*x+c)^5*cos(d*x+c)^5-3/4/d*a^2*b/sin(d*x+c)^4*cos(d*x+c)^5+
3/8/d*a^2*b/sin(d*x+c)^2*cos(d*x+c)^5+3/8/d*a^2*b*cos(d*x+c)^3+9/8*a^2*b*co
s(d*x+c)/d+9/8/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))-a*b^2*cot(d*x+c)^3/d+3*a*b
^2*x+3*a*b^2*cot(d*x+c)/d+3/d*a*b^2*c-1/2/d*b^3/sin(d*x+c)^2*cos(d*x+c)^5-1
/2*b^3*cos(d*x+c)^3/d-3/2*b^3*cos(d*x+c)/d-3/2/d*b^3*ln(csc(d*x+c)-cot(d*x+
c))
```

maxima [A] time = 0.49, size = 182, normalized size = 0.80

$$\frac{80\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)ab^2-15\,a^2b\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+\right.\right.\right.}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/80*(80*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a*b^2 - 15*a
^2*b*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c
)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 20*b^3*(2*c
os(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
 - 3*log(cos(d*x + c) - 1)) - 16*a^3/tan(d*x + c)^5)/d
```

mupad [B] time = 12.28, size = 1007, normalized size = 4.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] -(2*a^3*cos(c/2 + (d*x)/2)^12 - 2*a^3*sin(c/2 + (d*x)/2)^12 + 8*a^3*cos(c/2
 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 10*a^3*cos(c/2 + (d*x)/2)^4*sin(c/2 +
 (d*x)/2)^8 + 10*a^3*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 8*a^3*cos(
c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2 - 40*b^3*cos(c/2 + (d*x)/2)^3*sin(c/
2 + (d*x)/2)^9 - 40*b^3*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 680*b^3
*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5 + 40*b^3*cos(c/2 + (d*x)/2)^9*si
n(c/2 + (d*x)/2)^3 + 480*b^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos
(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 480*b^3*log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5 - 15*a^2*b*cos(c
/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^11 + 15*a^2*b*cos(c/2 + (d*x)/2)^11*sin(c/
2 + (d*x)/2) - 40*a*b^2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 560*a*
b^2*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 - 560*a*b^2*cos(c/2 + (d*x)/2
)^8*sin(c/2 + (d*x)/2)^4 + 40*a*b^2*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2
)^2 + 105*a^2*b*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^9 + 120*a^2*b*cos(c
/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 120*a^2*b*cos(c/2 + (d*x)/2)^7*sin(c
```

```
/2 + (d*x)/2)^5 - 105*a^2*b*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3 + 192
0*a*b^2*atan((3*a^2*sin(c/2 + (d*x)/2) - 4*b^2*sin(c/2 + (d*x)/2) + 8*a*b*c
os(c/2 + (d*x)/2))/(4*b^2*cos(c/2 + (d*x)/2) - 3*a^2*cos(c/2 + (d*x)/2) + 8
*a*b*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 1920*
a*b^2*atan((3*a^2*sin(c/2 + (d*x)/2) - 4*b^2*sin(c/2 + (d*x)/2) + 8*a*b*cos
(c/2 + (d*x)/2))/(4*b^2*cos(c/2 + (d*x)/2) - 3*a^2*cos(c/2 + (d*x)/2) + 8*a
*b*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5 - 360*a^2
*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2
+ (d*x)/2)^7 - 360*a^2*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2
 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)/(320*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (
d*x)/2)^5*(cos(c/2 + (d*x)/2)^2 + sin(c/2 + (d*x)/2)^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1124 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=275

$$-\frac{a\left(a^2+18b^2\right)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{\left(35a^2-2b^2\right)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3}{120a^2d}+\frac{b\left(39a^2-2b^2\right)\,\mathrm{c}}{}$$

```
[Out] b^3*x-1/16*a*(a^2+18*b^2)*arctanh(cos(d*x+c))/d-1/60*b*(36*a^4-43*a^2*b^2+2
*b^4)*cot(d*x+c)/a^2/d-1/240*(15*a^4-84*a^2*b^2+4*b^4)*cot(d*x+c)*csc(d*x+c
)/a/d+1/120*b*(39*a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^2/a^2
/d+1/120*(35*a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^3/a^2/d+1/
15*b*cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^4/a^2/d-1/6*cot(d*x+c)*csc(d*
x+c)^5*(a+b*sin(d*x+c))^4/a/d
```

Rubi [A] time = 0.76, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2893, 3047, 3031, 3021, 2735, 3770}

$$-\frac{b\left(-43a^2b^2+36a^4+2b^4\right)\cot(c+dx)}{60a^2d}-\frac{a\left(a^2+18b^2\right)\tanh^{-1}(\cos(c+dx))}{16d}-\frac{\left(-84a^2b^2+15a^4+4b^4\right)\cot(c+dx}{240ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] b^3*x - (a*(a^2 + 18*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) - (b*(36*a^4 - 43*a
^2*b^2 + 2*b^4)*Cot[c + d*x])/(60*a^2*d) - ((15*a^4 - 84*a^2*b^2 + 4*b^4)*C
ot[c + d*x]*Csc[c + d*x])/(240*a*d) + (b*(39*a^2 - 2*b^2)*Cot[c + d*x]*Csc[
c + d*x]^2*(a + b*Sin[c + d*x])^2)/(120*a^2*d) + ((35*a^2 - 2*b^2)*Cot[c +
d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(120*a^2*d) + (b*Cot[c + d*x]*C
sc[c + d*x]^4*(a + b*Sin[c + d*x])^4)/(15*a^2*d) - (Cot[c + d*x]*Csc[c + d*
x]^5*(a + b*Sin[c + d*x])^4)/(6*a*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
```

```
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^4}{15a^2d} - \frac{\cot(c+dx}{}$$
$$= \frac{\left(35a^2-2b^2\right)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3}{120a^2d}$$
$$= \frac{b\left(39a^2-2b^2\right)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{120a^2d}$$
$$= -\frac{\left(15a^4-84a^2b^2+4b^4\right)\cot(c+dx)\csc(c+dx)}{240ad} + \frac{b\left(39a^2\right.}{}$$
$$= -\frac{b\left(36a^4-43a^2b^2+2b^4\right)\cot(c+dx)}{60a^2d} - \frac{\left(15a^4-84a^2b^2+\right.}{}$$
$$= b^3x - \frac{b\left(36a^4-43a^2b^2+2b^4\right)\cot(c+dx)}{60a^2d} - \frac{\left(15a^4-84a^2\right.}{}$$
$$= b^3x - \frac{a\left(a^2+18b^2\right)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{b\left(36a^4-43a^2\right.}{}$$

Mathematica [A] time = 1.87, size = 408, normalized size = 1.48

$$\frac{-30\left(a^3-30ab^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)+5a^3\sec^6\left(\frac{1}{2}(c+dx)\right)-30a^3\sec^4\left(\frac{1}{2}(c+dx)\right)+30a^3\sec^2\left(\frac{1}{2}(c+dx)\right)+120}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (1920*b^3*c + 1920*b^3*d*x - 64*(9*a^2*b - 20*b^3)*Cot[(c + d*x)/2] - 30*(a
^3 - 30*a*b^2)*Csc[(c + d*x)/2]^2 - 120*a^3*Log[Cos[(c + d*x)/2]] - 2160*a*
b^2*Log[Cos[(c + d*x)/2]] + 120*a^3*Log[Sin[(c + d*x)/2]] + 2160*a*b^2*Log[
Sin[(c + d*x)/2]] + 30*a^3*Sec[(c + d*x)/2]^2 - 900*a*b^2*Sec[(c + d*x)/2]^
2 - 30*a^3*Sec[(c + d*x)/2]^4 + 90*a*b^2*Sec[(c + d*x)/2]^4 + 5*a^3*Sec[(c
+ d*x)/2]^6 - 2016*a^2*b*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 640*b^3*Csc[c
+ d*x]^3*Sin[(c + d*x)/2]^4 - a^2*Csc[(c + d*x)/2]^6*(5*a + 18*b*Sin[c + d*
x]) + 2*Csc[(c + d*x)/2]^4*(15*(a^3 - 3*a*b^2) + b*(63*a^2 - 20*b^2)*Sin[c
+ d*x]) + 576*a^2*b*Tan[(c + d*x)/2] - 1280*b^3*Tan[(c + d*x)/2] + 36*a^2*b
*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(1920*d)
```

fricas [A] time = 0.85, size = 373, normalized size = 1.36

$$480\,b^3dx\cos\left(dx+c\right)^6-1440\,b^3dx\cos\left(dx+c\right)^4+1440\,b^3dx\cos\left(dx+c\right)^2+30\left(a^3-30\,ab^2\right)\cos\left(dx+c\right)^5-480$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/480*(480*b^3*d*x*cos(d*x + c)^6 - 1440*b^3*d*x*cos(d*x + c)^4 + 1440*b^3*
d*x*cos(d*x + c)^2 + 30*(a^3 - 30*a*b^2)*cos(d*x + c)^5 - 480*b^3*d*x + 80*
(a^3 + 18*a*b^2)*cos(d*x + c)^3 - 30*(a^3 + 18*a*b^2)*cos(d*x + c) - 15*((a
^3 + 18*a*b^2)*cos(d*x + c)^6 - 3*(a^3 + 18*a*b^2)*cos(d*x + c)^4 - a^3 - 1
8*a*b^2 + 3*(a^3 + 18*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) +
15*((a^3 + 18*a*b^2)*cos(d*x + c)^6 - 3*(a^3 + 18*a*b^2)*cos(d*x + c)^4 - a
^3 - 18*a*b^2 + 3*(a^3 + 18*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) +
1/2) + 32*(35*b^3*cos(d*x + c)^3 + (9*a^2*b - 20*b^3)*cos(d*x + c)^5 - 15*b
^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d
*cos(d*x + c)^2 - d)
```

giac [A] time = 0.36, size = 399, normalized size = 1.45

$$5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+90\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-180\,a^2b\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 15
*a^3*tan(1/2*d*x + 1/2*c)^4 + 90*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 180*a^2*b*t
an(1/2*d*x + 1/2*c)^3 + 80*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x
+ 1/2*c)^2 - 720*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 1920*(d*x + c)*b^3 + 360*a^
2*b*tan(1/2*d*x + 1/2*c) - 1200*b^3*tan(1/2*d*x + 1/2*c) + 120*(a^3 + 18*a*
b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - (294*a^3*tan(1/2*d*x + 1/2*c)^6 + 529
2*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 360*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 1200*b^
3*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*tan(1/2*d*x + 1/2*c)^4 - 720*a*b^2*tan(1/
2*d*x + 1/2*c)^4 - 180*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 80*b^3*tan(1/2*d*x +
1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 90*a*b^2*tan(1/2*d*x + 1/2*c)^2
+ 36*a^2*b*tan(1/2*d*x + 1/2*c) + 5*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

maple [A] time = 0.51, size = 302, normalized size = 1.10

$$-\frac{a^3\left(\cos^5(dx+c)\right)}{6d\sin(dx+c)^6}-\frac{a^3\left(\cos^5(dx+c)\right)}{24d\sin(dx+c)^4}+\frac{a^3\left(\cos^5(dx+c)\right)}{48d\sin(dx+c)^2}+\frac{a^3\left(\cos^3(dx+c)\right)}{48d}+\frac{a^3\cos(dx+c)}{16d}+\frac{a^3\ln(\csc(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/6/d*a^3/sin(d*x+c)^6*cos(d*x+c)^5-1/24/d*a^3/sin(d*x+c)^4*cos(d*x+c)^5+1
/48/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5+1/48*a^3*cos(d*x+c)^3/d+1/16*a^3*cos(d*
x+c)/d+1/16/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3/5/d*a^2*b/sin(d*x+c)^5*cos(d*
x+c)^5-3/4/d*a*b^2/sin(d*x+c)^4*cos(d*x+c)^5+3/8/d*a*b^2/sin(d*x+c)^2*cos(d
*x+c)^5+3/8*a*b^2*cos(d*x+c)^3/d+9/8*a*b^2*cos(d*x+c)/d+9/8/d*a*b^2*ln(csc(
d*x+c)-cot(d*x+c))-1/3/d*b^3*cot(d*x+c)^3+1/d*cot(d*x+c)*b^3+b^3*x+1/d*b^3*
c
```

maxima [A] time = 0.42, size = 217, normalized size = 0.79

$$\frac{160\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)b^3+5\,a^3\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log(\cdots}{4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/480*(160*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*b^3 + 5*a^
3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^6
 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1)) - 90*a*b^2*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(co
s(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(
d*x + c) - 1)) - 288*a^2*b/tan(d*x + c)^5)/d
```

mupad [B] time = 9.93, size = 446, normalized size = 1.62

$$\frac{2\,b^3\,\mathrm{atan}\left(\frac{4\,b^6}{\frac{a^3\,b^3}{4}+\frac{9\,a\,b^5}{2}-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^6}+\frac{9\,a\,b^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{a^3\,b^3}{4}+\frac{9\,a\,b^5}{2}-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^6\right)}+\frac{a^3\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\left(\frac{a^3\,b^3}{4}+\frac{9\,a\,b^5}{2}-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^6\right)}\right)}{d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] (2*b^3*atan((4*b^6)/((9*a*b^5)/2 + (a^3*b^3)/4 - 4*b^6*tan(c/2 + (d*x)/2))
+ (9*a*b^5*tan(c/2 + (d*x)/2))/(2*((9*a*b^5)/2 + (a^3*b^3)/4 - 4*b^6*tan(c/
2 + (d*x)/2))) + (a^3*b^3*tan(c/2 + (d*x)/2))/(4*((9*a*b^5)/2 + (a^3*b^3)/4
 - 4*b^6*tan(c/2 + (d*x)/2)))))/d + (a^3*tan(c/2 + (d*x)/2)^6)/(384*d) - (t
an(c/2 + (d*x)/2)^2*((3*a*b^2)/8 + a^3/128))/d + (tan(c/2 + (d*x)/2)^4*((3*
a*b^2)/64 - a^3/128))/d - (tan(c/2 + (d*x)/2)^3*((3*a^2*b)/32 - b^3/24))/d
- (tan(c/2 + (d*x)/2)^2*(3*a*b^2 - a^3/2) - tan(c/2 + (d*x)/2)^4*(24*a*b^2
+ a^3/2) - tan(c/2 + (d*x)/2)^3*(6*a^2*b - (8*b^3)/3) + tan(c/2 + (d*x)/2)^
5*(12*a^2*b - 40*b^3) + a^3/6 + (6*a^2*b*tan(c/2 + (d*x)/2))/5)/(64*d*tan(c
/2 + (d*x)/2)^6) + (tan(c/2 + (d*x)/2)*((3*a^2*b)/16 - (5*b^3)/8))/d + (a*l
og(tan(c/2 + (d*x)/2))*(a^2 + 18*b^2))/(16*d) + (3*a^2*b*tan(c/2 + (d*x)/2)
^5)/(160*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1125 $\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=303

$$-\frac{a\left(2a^2+21b^2\right)\cot(c+dx)}{35d}-\frac{3b\left(a^2+2b^2\right)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{\left(8a^2-b^2\right)\cot(c+dx)\csc^4(c+dx)(a+b\sin(c}{35a^2d}$$

```
[Out] -3/16*b*(a^2+2*b^2)*arctanh(cos(d*x+c))/d-1/35*a*(2*a^2+21*b^2)*cot(d*x+c)/
d-1/560*b*(105*a^4-116*a^2*b^2+12*b^4)*cot(d*x+c)*csc(d*x+c)/a^2/d-1/140*(4
*a^4-19*a^2*b^2+2*b^4)*cot(d*x+c)*csc(d*x+c)^2/a/d+1/280*b*(53*a^2-6*b^2)*c
ot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^2/a^2/d+1/35*(8*a^2-b^2)*cot(d*x+c)
*csc(d*x+c)^4*(a+b*sin(d*x+c))^3/a^2/d+1/14*b*cot(d*x+c)*csc(d*x+c)^5*(a+b*
sin(d*x+c))^4/a^2/d-1/7*cot(d*x+c)*csc(d*x+c)^6*(a+b*sin(d*x+c))^4/a/d
```

Rubi [A] time = 0.86, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2893, 3047, 3031, 3021, 2748, 3767, 8, 3770}

$$-\frac{a\left(2a^2+21b^2\right)\cot(c+dx)}{35d}-\frac{3b\left(a^2+2b^2\right)\tanh^{-1}(\cos(c+dx))}{16d}-\frac{\left(-19a^2b^2+4a^4+2b^4\right)\cot(c+dx)\csc^2(c+dx}{140ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b*(a^2 + 2*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) - (a*(2*a^2 + 21*b^2)*Cot
[c + d*x])/(35*d) - (b*(105*a^4 - 116*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c
+ d*x])/(560*a^2*d) - ((4*a^4 - 19*a^2*b^2 + 2*b^4)*Cot[c + d*x]*Csc[c + d*
x]^2)/(140*a*d) + (b*(53*a^2 - 6*b^2)*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Si
n[c + d*x])^2)/(280*a^2*d) + ((8*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^4*(a
+ b*Sin[c + d*x])^3)/(35*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin
[c + d*x])^4)/(14*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^6*(a + b*Sin[c + d*x]
)^4)/(7*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^4}{14a^2d} - \frac{\cot(c+dx}{}$$

$$= \frac{\left(8a^2-b^2\right)\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3}{35a^2d} + \frac{b}{}$$

$$= \frac{b\left(53a^2-6b^2\right)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2}{280a^2d}$$

$$= -\frac{\left(4a^4-19a^2b^2+2b^4\right)\cot(c+dx)\csc^2(c+dx)}{140ad} + \frac{b\left(53a^2\right.}{}$$

$$= -\frac{b\left(105a^4-116a^2b^2+12b^4\right)\cot(c+dx)\csc(c+dx)}{560a^2d} - \frac{\left(4a\right.}{}$$

$$= -\frac{b\left(105a^4-116a^2b^2+12b^4\right)\cot(c+dx)\csc(c+dx)}{560a^2d} - \frac{\left(4a\right.}{}$$

$$= -\frac{3b\left(a^2+2b^2\right)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{b\left(105a^4-116a^2b^2\right.}{}$$

$$= -\frac{3b\left(a^2+2b^2\right)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\left(2a^2+21b^2\right)\cot(c}{35d}$$

Mathematica [A] time = 0.97, size = 324, normalized size = 1.07

$$-\frac{112a^3\cos(5(c+dx))\csc^7(c+dx)-16a^3\cos(7(c+dx))\csc^7(c+dx)+56a\left(14a^2-3b^2\right)\cos(3(c+dx))\csc^7(c-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/17920*(56*a*(14*a^2 - 3*b^2)*Cos[3*(c + d*x)]*Csc[c + d*x]^7 + 112*a^3*C
os[5*(c + d*x)]*Csc[c + d*x]^7 - 504*a*b^2*Cos[5*(c + d*x)]*Csc[c + d*x]^7
- 16*a^3*Cos[7*(c + d*x)]*Csc[c + d*x]^7 - 168*a*b^2*Cos[7*(c + d*x)]*Csc[c
 + d*x]^7 + 3360*a^2*b*Log[Cos[(c + d*x)/2]] + 6720*b^3*Log[Cos[(c + d*x)/2
]] - 3360*a^2*b*Log[Sin[(c + d*x)/2]] - 6720*b^3*Log[Sin[(c + d*x)/2]] + 70
*Cot[c + d*x]*Csc[c + d*x]^6*(12*a*(2*a^2 + b^2) + b*(31*a^2 - 18*b^2)*Sin[
c + d*x]) + 1540*a^2*b*Csc[c + d*x]^7*Sin[4*(c + d*x)] + 840*b^3*Csc[c + d*
x]^7*Sin[4*(c + d*x)] + 105*a^2*b*Csc[c + d*x]^7*Sin[6*(c + d*x)] - 350*b^3
*Csc[c + d*x]^7*Sin[6*(c + d*x)])/d
```

fricas [A] time = 0.63, size = 347, normalized size = 1.15

$$-\frac{32\left(2\,a^3+21\,ab^2\right)\cos\left(dx+c\right)^7-224\left(a^3+3\,ab^2\right)\cos\left(dx+c\right)^5+105\left(\left(a^2b+2\,b^3\right)\cos\left(dx+c\right)^6-3\left(a^2b+2\,b^3 \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/1120*(32*(2*a^3 + 21*a*b^2)*cos(d*x + c)^7 - 224*(a^3 + 3*a*b^2)*cos(d*x
 + c)^5 + 105*((a^2*b + 2*b^3)*cos(d*x + c)^6 - 3*(a^2*b + 2*b^3)*cos(d*x +
 c)^4 - a^2*b - 2*b^3 + 3*(a^2*b + 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 105*((a^2*b + 2*b^3)*cos(d*x + c)^6 - 3*(a^2*b +
2*b^3)*cos(d*x + c)^4 - a^2*b - 2*b^3 + 3*(a^2*b + 2*b^3)*cos(d*x + c)^2)*l
og(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 70*((3*a^2*b - 10*b^3)*cos(d*x +
 c)^5 + 8*(a^2*b + 2*b^3)*cos(d*x + c)^3 - 3*(a^2*b + 2*b^3)*cos(d*x + c))*
sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2
- d)*sin(d*x + c))
```

giac [A] time = 0.39, size = 456, normalized size = 1.50

$$\frac{5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+35\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+84\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-105\,a^2b\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5*a^3*tan(1/2*d*x + 1/2*c)^7 + 35*a^2*b*tan(1/2*d*x + 1/2*c)^6 - 7*
a^3*tan(1/2*d*x + 1/2*c)^5 + 84*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 105*a^2*b*ta
n(1/2*d*x + 1/2*c)^4 + 70*b^3*tan(1/2*d*x + 1/2*c)^4 - 35*a^3*tan(1/2*d*x +
 1/2*c)^3 - 420*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 105*a^2*b*tan(1/2*d*x + 1/2*
c)^2 - 560*b^3*tan(1/2*d*x + 1/2*c)^2 + 105*a^3*tan(1/2*d*x + 1/2*c) + 840*
a*b^2*tan(1/2*d*x + 1/2*c) + 840*(a^2*b + 2*b^3)*log(abs(tan(1/2*d*x + 1/2*
c))) - (2178*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 4356*b^3*tan(1/2*d*x + 1/2*c)^7
 + 105*a^3*tan(1/2*d*x + 1/2*c)^6 + 840*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 105*
a^2*b*tan(1/2*d*x + 1/2*c)^5 - 560*b^3*tan(1/2*d*x + 1/2*c)^5 - 35*a^3*tan(
1/2*d*x + 1/2*c)^4 - 420*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 105*a^2*b*tan(1/2*d
*x + 1/2*c)^3 + 70*b^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c)^
2 + 84*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 35*a^2*b*tan(1/2*d*x + 1/2*c) + 5*a^3
)/tan(1/2*d*x + 1/2*c)^7)/d
```

maple [A] time = 0.50, size = 309, normalized size = 1.02

$$-\frac{a^3\left(\cos^5(dx+c)\right)}{7d\sin(dx+c)^7}-\frac{2a^3\left(\cos^5(dx+c)\right)}{35d\sin(dx+c)^5}-\frac{a^2b\left(\cos^5(dx+c)\right)}{2d\sin(dx+c)^6}-\frac{a^2b\left(\cos^5(dx+c)\right)}{8d\sin(dx+c)^4}+\frac{a^2b\left(\cos^5(dx+c)\right)}{16d\sin(dx+c)^2}+\frac{a^2b\left(\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/7/d*a^3/sin(d*x+c)^7*cos(d*x+c)^5-2/35/d*a^3/sin(d*x+c)^5*cos(d*x+c)^5-1
/2/d*a^2*b/sin(d*x+c)^6*cos(d*x+c)^5-1/8/d*a^2*b/sin(d*x+c)^4*cos(d*x+c)^5+
1/16/d*a^2*b/sin(d*x+c)^2*cos(d*x+c)^5+1/16/d*a^2*b*cos(d*x+c)^3+3/16*a^2*b
*cos(d*x+c)/d+3/16/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))-3/5/d*a*b^2/sin(d*x+c)
^5*cos(d*x+c)^5-1/4/d*b^3/sin(d*x+c)^4*cos(d*x+c)^5+1/8/d*b^3/sin(d*x+c)^2*
cos(d*x+c)^5+1/8*b^3*cos(d*x+c)^3/d+3/8*b^3*cos(d*x+c)/d+3/8/d*b^3*ln(csc(d
*x+c)-cot(d*x+c))
```

maxima [A] time = 0.33, size = 208, normalized size = 0.69

$$\frac{35\,a^2b\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-70\,b^3\left(\frac{2\left(5\,\cos(\right.}{\cos(dx+}\right.}{1120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/1120*(35*a^2*b*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/
(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x
+ c) + 1) + 3*log(cos(d*x + c) - 1)) - 70*b^3*(2*(5*cos(d*x + c)^3 - 3*cos(
d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1)
```

```
 - 3*log(cos(d*x + c) - 1)) - 672*a*b^2/tan(d*x + c)^5 - 32*(7*tan(d*x + c)
^2 + 5)*a^3/tan(d*x + c)^7)/d
```

mupad [B] time = 9.91, size = 359, normalized size = 1.18

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{896\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{a^3}{128}+\frac{3\,a\,b^2}{32}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{3\,a\,b^2}{160}-\frac{a^3}{640}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3\,a^2\,b}{128}+\frac{b^3}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^8,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^7)/(896*d) - (tan(c/2 + (d*x)/2)^3*((3*a*b^2)/32 +
a^3/128))/d + (tan(c/2 + (d*x)/2)^5*((3*a*b^2)/160 - a^3/640))/d - (tan(c/2
 + (d*x)/2)^2*((3*a^2*b)/128 + b^3/8))/d - (tan(c/2 + (d*x)/2)^4*((3*a^2*b)
/128 - b^3/64))/d + (log(tan(c/2 + (d*x)/2))*((3*a^2*b)/16 + (3*b^3)/8))/d
- (tan(c/2 + (d*x)/2)^2*((12*a*b^2)/5 - a^3/5) - tan(c/2 + (d*x)/2)^4*(12*a
*b^2 + a^3) + tan(c/2 + (d*x)/2)^6*(24*a*b^2 + 3*a^3) - tan(c/2 + (d*x)/2)^
3*(3*a^2*b - 2*b^3) - tan(c/2 + (d*x)/2)^5*(3*a^2*b + 16*b^3) + a^3/7 + a^2
*b*tan(c/2 + (d*x)/2))/(128*d*tan(c/2 + (d*x)/2)^7) + (tan(c/2 + (d*x)/2)*(
(3*a*b^2)/16 + (3*a^3)/128))/d + (a^2*b*tan(c/2 + (d*x)/2)^6)/(128*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1126 $\int \cot^4(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=334

$$-\frac{b\left(6a^2+7b^2\right)\cot(c+dx)}{35d}-\frac{3a\left(a^2+8b^2\right)\tanh^{-1}(\cos(c+dx))}{128d}-\frac{3a\left(a^2+8b^2\right)\cot(c+dx)\csc(c+dx)}{128d}+\frac{\left(21a^2-\right.}{}$$

```
[Out] -3/128*a*(a^2+8*b^2)*arctanh(cos(d*x+c))/d-1/35*b*(6*a^2+7*b^2)*cot(d*x+c)/
d-3/128*a*(a^2+8*b^2)*cot(d*x+c)*csc(d*x+c)/d-1/280*b*(24*a^4-25*a^2*b^2+4*
b^4)*cot(d*x+c)*csc(d*x+c)^2/a^2/d-1/2240*(35*a^4-148*a^2*b^2+24*b^4)*cot(d
*x+c)*csc(d*x+c)^3/a/d+3/560*b*(23*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)^4*(a+b*
sin(d*x+c))^2/a^2/d+1/112*(21*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)^5*(a+b*sin(d
*x+c))^3/a^2/d+1/14*b*cot(d*x+c)*csc(d*x+c)^6*(a+b*sin(d*x+c))^4/a^2/d-1/8*
cot(d*x+c)*csc(d*x+c)^7*(a+b*sin(d*x+c))^4/a/d
```

Rubi [A] time = 0.91, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2893, 3047, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$-\frac{b\left(6a^2+7b^2\right)\cot(c+dx)}{35d}-\frac{3a\left(a^2+8b^2\right)\tanh^{-1}(\cos(c+dx))}{128d}-\frac{\left(-148a^2b^2+35a^4+24b^4\right)\cot(c+dx)\csc^3(c+}{2240ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a*(a^2 + 8*b^2)*ArcTanh[Cos[c + d*x]])/(128*d) - (b*(6*a^2 + 7*b^2)*Cot
[c + d*x])/(35*d) - (3*a*(a^2 + 8*b^2)*Cot[c + d*x]*Csc[c + d*x])/(128*d) -
 (b*(24*a^4 - 25*a^2*b^2 + 4*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(280*a^2*d)
- ((35*a^4 - 148*a^2*b^2 + 24*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(2240*a*d)
+ (3*b*(23*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2)
/(560*a^2*d) + ((21*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin[c +
 d*x])^3)/(112*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^6*(a + b*Sin[c + d*x])
^4)/(14*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^7*(a + b*Sin[c + d*x])^4)/(8*a*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
```

```
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b\cot(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))^4}{14a^2d} - \frac{\cot(c+dx)}{}$$

$$= \frac{\left(21a^2-4b^2\right)\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3}{112a^2d} +$$

$$= \frac{3b\left(23a^2-4b^2\right)\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2}{560a^2d}$$

$$= -\frac{\left(35a^4-148a^2b^2+24b^4\right)\cot(c+dx)\csc^3(c+dx)}{2240ad} + \frac{3b\left(23\right.}{}$$

$$= -\frac{b\left(24a^4-25a^2b^2+4b^4\right)\cot(c+dx)\csc^2(c+dx)}{280a^2d} - \frac{\left(35a^4\right.}{}$$

$$= -\frac{b\left(24a^4-25a^2b^2+4b^4\right)\cot(c+dx)\csc^2(c+dx)}{280a^2d} - \frac{\left(35a^4\right.}{}$$

$$= -\frac{3a\left(a^2+8b^2\right)\cot(c+dx)\csc(c+dx)}{128d} - \frac{b\left(24a^4-25a^2b^2+\right.}{}$$

$$= -\frac{3a\left(a^2+8b^2\right)\tanh^{-1}(\cos(c+dx))}{128d} - \frac{b\left(6a^2+7b^2\right)\cot(c+}{35d}$$

Mathematica [A] time = 1.60, size = 268, normalized size = 0.80

$$-\frac{-6720a\left(a^2+8b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+6720a\left(a^2+8b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\csc^8(c+dx)\left(35\left(333a^3+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/286720*(6720*a*(a^2 + 8*b^2)*Log[Cos[(c + d*x)/2]] - 6720*a*(a^2 + 8*b^2
)*Log[Sin[(c + d*x)/2]] + Csc[c + d*x]^8*(35*a*(671*a^2 + 248*b^2)*Cos[c +
d*x] + 35*(333*a^3 + 104*a*b^2)*Cos[3*(c + d*x)] + 805*a^3*Cos[5*(c + d*x)]
 - 11480*a*b^2*Cos[5*(c + d*x)] - 105*a^3*Cos[7*(c + d*x)] - 840*a*b^2*Cos[
7*(c + d*x)] + 21504*a^2*b*Sin[2*(c + d*x)] + 2688*b^3*Sin[2*(c + d*x)] + 1
6128*a^2*b*Sin[4*(c + d*x)] + 896*b^3*Sin[4*(c + d*x)] + 3072*a^2*b*Sin[6*(
c + d*x)] - 896*b^3*Sin[6*(c + d*x)] - 384*a^2*b*Sin[8*(c + d*x)] - 448*b^3
*Sin[8*(c + d*x)]))/d
```

fricas [A] time = 0.68, size = 384, normalized size = 1.15

$$\frac{210\left(a^3+8\,ab^2\right)\cos\left(dx+c\right)^7-70\left(11\,a^3-40\,ab^2\right)\cos\left(dx+c\right)^5-770\left(a^3+8\,ab^2\right)\cos\left(dx+c\right)^3+210\left(a^3+8\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/8960*(210*(a^3 + 8*a*b^2)*cos(d*x + c)^7 - 70*(11*a^3 - 40*a*b^2)*cos(d*x
 + c)^5 - 770*(a^3 + 8*a*b^2)*cos(d*x + c)^3 + 210*(a^3 + 8*a*b^2)*cos(d*x
+ c) - 105*((a^3 + 8*a*b^2)*cos(d*x + c)^8 - 4*(a^3 + 8*a*b^2)*cos(d*x + c)
^6 + 6*(a^3 + 8*a*b^2)*cos(d*x + c)^4 + a^3 + 8*a*b^2 - 4*(a^3 + 8*a*b^2)*c
os(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 105*((a^3 + 8*a*b^2)*cos(d*x +
 c)^8 - 4*(a^3 + 8*a*b^2)*cos(d*x + c)^6 + 6*(a^3 + 8*a*b^2)*cos(d*x + c)^4
 + a^3 + 8*a*b^2 - 4*(a^3 + 8*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c)
+ 1/2) + 256*((6*a^2*b + 7*b^3)*cos(d*x + c)^7 - 7*(3*a^2*b + b^3)*cos(d*x
+ c)^5)*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x
+ c)^4 - 4*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.39, size = 457, normalized size = 1.37

$$35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+240\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+560\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-336\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+44$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/71680*(35*a^3*tan(1/2*d*x + 1/2*c)^8 + 240*a^2*b*tan(1/2*d*x + 1/2*c)^7 +
 560*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 336*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 448*
b^3*tan(1/2*d*x + 1/2*c)^5 - 280*a^3*tan(1/2*d*x + 1/2*c)^4 - 1680*a*b^2*ta
n(1/2*d*x + 1/2*c)^4 - 1680*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 2240*b^3*tan(1/2
*d*x + 1/2*c)^3 - 1680*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 5040*a^2*b*tan(1/2*d*
x + 1/2*c) + 4480*b^3*tan(1/2*d*x + 1/2*c) + 1680*(a^3 + 8*a*b^2)*log(abs(t
an(1/2*d*x + 1/2*c))) - (4566*a^3*tan(1/2*d*x + 1/2*c)^8 + 36528*a*b^2*tan(
1/2*d*x + 1/2*c)^8 + 5040*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 4480*b^3*tan(1/2*d
*x + 1/2*c)^7 - 1680*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 1680*a^2*b*tan(1/2*d*x
+ 1/2*c)^5 - 2240*b^3*tan(1/2*d*x + 1/2*c)^5 - 280*a^3*tan(1/2*d*x + 1/2*c)
^4 - 1680*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 336*a^2*b*tan(1/2*d*x + 1/2*c)^3 +
 448*b^3*tan(1/2*d*x + 1/2*c)^3 + 560*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*a^
2*b*tan(1/2*d*x + 1/2*c) + 35*a^3)/tan(1/2*d*x + 1/2*c)^8)/d
```

maple [A] time = 0.50, size = 358, normalized size = 1.07

$$-\frac{a^3\left(\cos^5\left(dx+c\right)\right)}{8d\sin\left(dx+c\right)^8}-\frac{a^3\left(\cos^5\left(dx+c\right)\right)}{16d\sin\left(dx+c\right)^6}-\frac{a^3\left(\cos^5\left(dx+c\right)\right)}{64d\sin\left(dx+c\right)^4}+\frac{a^3\left(\cos^5\left(dx+c\right)\right)}{128d\sin\left(dx+c\right)^2}+\frac{a^3\left(\cos^3\left(dx+c\right)\right)}{128d}+\frac{3a^3\cos\left(dx\right.}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/8/d*a^3/sin(d*x+c)^8*cos(d*x+c)^5-1/16/d*a^3/sin(d*x+c)^6*cos(d*x+c)^5-1
/64/d*a^3/sin(d*x+c)^4*cos(d*x+c)^5+1/128/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5+1
/128*a^3*cos(d*x+c)^3/d+3/128*a^3*cos(d*x+c)/d+3/128/d*a^3*ln(csc(d*x+c)-co
t(d*x+c))-3/7/d*a^2*b/sin(d*x+c)^7*cos(d*x+c)^5-6/35/d*a^2*b/sin(d*x+c)^5*c
os(d*x+c)^5-1/2/d*a*b^2/sin(d*x+c)^6*cos(d*x+c)^5-1/8/d*a*b^2/sin(d*x+c)^4*
cos(d*x+c)^5+1/16/d*a*b^2/sin(d*x+c)^2*cos(d*x+c)^5+1/16*a*b^2*cos(d*x+c)^3
/d+3/16*a*b^2*cos(d*x+c)/d+3/16/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))-1/5/d*b^3
/sin(d*x+c)^5*cos(d*x+c)^5
```

maxima [A] time = 0.33, size = 248, normalized size = 0.74

$$35\,a^3\left(\frac{2\left(3\,\cos(dx+c)^7-11\,\cos(dx+c)^5-11\,\cos(dx+c)^3+3\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+280$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8960*(35*a^3*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3
 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 -
4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))
+ 280*a*b^2*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(
d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
+ 1) + 3*log(cos(d*x + c) - 1)) - 1792*b^3/tan(d*x + c)^5 - 768*(7*tan(d*x
+ c)^2 + 5)*a^2*b/tan(d*x + c)^7)/d
```

mupad [B] time = 9.95, size = 381, normalized size = 1.14

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2048\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a^3}{256}+\frac{3\,a\,b^2}{128}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{3\,a^2\,b}{128}+\frac{b^3}{32}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{3\,a^2\,b}{640}-\frac{b^3}{160}\right)}{d}+\frac{\ln\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^9,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^8)/(2048*d) - (tan(c/2 + (d*x)/2)^4*((3*a*b^2)/128
+ a^3/256))/d - (tan(c/2 + (d*x)/2)^3*((3*a^2*b)/128 + b^3/32))/d - (tan(c/
2 + (d*x)/2)^5*((3*a^2*b)/640 - b^3/160))/d + (log(tan(c/2 + (d*x)/2))*((3*
a*b^2)/16 + (3*a^3)/128))/d + (tan(c/2 + (d*x)/2)^4*(6*a*b^2 + a^3) + tan(c
/2 + (d*x)/2)^5*(6*a^2*b + 8*b^3) + tan(c/2 + (d*x)/2)^3*((6*a^2*b)/5 - (8*
b^3)/5) - tan(c/2 + (d*x)/2)^7*(18*a^2*b + 16*b^3) - a^3/8 - (6*a^2*b*tan(c
```

```
/2 + (d*x)/2))/7 - 2*a*b^2*tan(c/2 + (d*x)/2)^2 + 6*a*b^2*tan(c/2 + (d*x)/2
)^6)/(256*d*tan(c/2 + (d*x)/2)^8) + (tan(c/2 + (d*x)/2)*((9*a^2*b)/128 + b^
3/16))/d - (3*a*b^2*tan(c/2 + (d*x)/2)^2)/(128*d) + (a*b^2*tan(c/2 + (d*x)/
2)^6)/(128*d) + (3*a^2*b*tan(c/2 + (d*x)/2)^7)/(896*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1127 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=307

$$-\frac{\left(a^2-b^2\right)\sin^4(c+dx)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))}+\frac{3a\left(4a^2-3b^2\right)\sin(c+dx)\cos(c+dx)}{4b^5d}-\frac{\left(10a^2-7b^2\right)\sin^2(c+dx)\cos(c+dx)}{5b^4d}$$

```
[Out] -3/4*a*(8*a^4-8*a^2*b^2+b^4)*x/b^7-1/5*(30*a^4-25*a^2*b^2+b^4)*cos(d*x+c)/b
^6/d+3/4*a*(4*a^2-3*b^2)*cos(d*x+c)*sin(d*x+c)/b^5/d-1/5*(10*a^2-7*b^2)*cos
(d*x+c)*sin(d*x+c)^2/b^4/d+1/2*(3*a^2-2*b^2)*cos(d*x+c)*sin(d*x+c)^3/a/b^3/
d-1/5*cos(d*x+c)*sin(d*x+c)^4/b^2/d-(a^2-b^2)*cos(d*x+c)*sin(d*x+c)^4/a/b^2
/d/(a+b*sin(d*x+c))+6*a^2*(2*a^4-3*a^2*b^2+b^4)*arctan((b+a*tan(1/2*d*x+1/2
*c))/(a^2-b^2)^(1/2))/b^7/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 1.02, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2892, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{\left(-25a^2b^2+30a^4+b^4\right)\cos(c+dx)}{5b^6d}+\frac{6a^2\left(-3a^2b^2+2a^4+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d\sqrt{a^2-b^2}}-\frac{\left(a^2-b^2\right)\sin^4(c+dx)\cos}{ab^2d(a+b\sin(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*a*(8*a^4 - 8*a^2*b^2 + b^4)*x)/(4*b^7) + (6*a^2*(2*a^4 - 3*a^2*b^2 + b^
4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*Sqrt[a^2 - b^2]*d
) - ((30*a^4 - 25*a^2*b^2 + b^4)*Cos[c + d*x])/(5*b^6*d) + (3*a*(4*a^2 - 3*
b^2)*Cos[c + d*x]*Sin[c + d*x])/(4*b^5*d) - ((10*a^2 - 7*b^2)*Cos[c + d*x]*
Sin[c + d*x]^2)/(5*b^4*d) + ((3*a^2 - 2*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(
2*a*b^3*d) - (Cos[c + d*x]*Sin[c + d*x]^4)/(5*b^2*d) - ((a^2 - b^2)*Cos[c +
 d*x]*Sin[c + d*x]^4)/(a*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2892

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/(b^2*d*f*(m + n + 4)), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cos(c+dx)\sin^4(c+dx)}{5b^2d} - \frac{\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d(a+b\sin(c+dx))} + \frac{\int \frac{\sin^3(c+dx)(3(}{}}{}$$

$$= \frac{\left(3a^2-2b^2\right)\cos(c+dx)\sin^3(c+dx)}{2ab^3d} - \frac{\cos(c+dx)\sin^4(c+dx)}{5b^2d} - \frac{\left(a^2-b^2\right)\text{co}}{ab^2d(a}$$

$$= -\frac{\left(10a^2-7b^2\right)\cos(c+dx)\sin^2(c+dx)}{5b^4d} + \frac{\left(3a^2-2b^2\right)\cos(c+dx)\sin^3(c+dx)}{2ab^3d}$$

$$= \frac{3a\left(4a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{4b^5d} - \frac{\left(10a^2-7b^2\right)\cos(c+dx)\sin^2(c+dx)}{5b^4d}$$

$$= -\frac{\left(30a^4-25a^2b^2+b^4\right)\cos(c+dx)}{5b^6d} + \frac{3a\left(4a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{4b^5d} -$$

$$= -\frac{3a\left(8a^4-8a^2b^2+b^4\right)x}{4b^7} - \frac{\left(30a^4-25a^2b^2+b^4\right)\cos(c+dx)}{5b^6d} + \frac{3a\left(4a^2-3b^2\right)}{}$$

$$= -\frac{3a\left(8a^4-8a^2b^2+b^4\right)x}{4b^7} - \frac{\left(30a^4-25a^2b^2+b^4\right)\cos(c+dx)}{5b^6d} + \frac{3a\left(4a^2-3b^2\right)}{}$$

$$= -\frac{3a\left(8a^4-8a^2b^2+b^4\right)x}{4b^7} - \frac{\left(30a^4-25a^2b^2+b^4\right)\cos(c+dx)}{5b^6d} + \frac{3a\left(4a^2-3b^2\right)}{}$$

$$= -\frac{3a\left(8a^4-8a^2b^2+b^4\right)x}{4b^7} + \frac{6a^2\left(2a^4-3a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7\sqrt{a^2-b^2}\,d} - \frac{(3}{}$$

Mathematica [A] time = 4.57, size = 378, normalized size = 1.23

$$\frac{\frac{960a^2\left(2a^4-3a^2b^2+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{960a^6c+960a^6dx+960a^5bc\sin(c+dx)+960a^5bdx\sin(c+dx)+240a^4b^2\sin(2(c+dx))-960a^4b^2c-960a}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((960*a^2*(2*a^4 - 3*a^2*b^2 + b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/Sqrt[a^2 - b^2] - (960*a^6*c - 960*a^4*b^2*c + 120*a^2*b^4*c + 9
60*a^6*d*x - 960*a^4*b^2*d*x + 120*a^2*b^4*d*x + 60*a*b*(16*a^4 - 14*a^2*b^
2 + b^4)*Cos[c + d*x] + 5*(8*a^3*b^3 - 5*a*b^5)*Cos[3*(c + d*x)] - 3*a*b^5*
Cos[5*(c + d*x)] + 960*a^5*b*c*Sin[c + d*x] - 960*a^3*b^3*c*Sin[c + d*x] +
120*a*b^5*c*Sin[c + d*x] + 960*a^5*b*d*x*Sin[c + d*x] - 960*a^3*b^3*d*x*Sin
[c + d*x] + 120*a*b^5*d*x*Sin[c + d*x] + 240*a^4*b^2*Sin[2*(c + d*x)] - 200
*a^2*b^4*Sin[2*(c + d*x)] + 5*b^6*Sin[2*(c + d*x)] - 10*a^2*b^4*Sin[4*(c +
d*x)] + 4*b^6*Sin[4*(c + d*x)] + b^6*Sin[6*(c + d*x)])/(a + b*Sin[c + d*x])
)/(160*b^7*d)
```

fricas [A] time = 0.83, size = 649, normalized size = 2.11

$$\left[\frac{6\,ab^5\cos\left(dx+c\right)^5-5\left(4\,a^3b^3-ab^5\right)\cos\left(dx+c\right)^3-15\left(8\,a^6-8\,a^4b^2+a^2b^4\right)dx-30\left(2\,a^5-a^3b^2+\left(2\,a^4b-a^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/20*(6*a*b^5*cos(d*x + c)^5 - 5*(4*a^3*b^3 - a*b^5)*cos(d*x + c)^3 - 15*(
8*a^6 - 8*a^4*b^2 + a^2*b^4)*d*x - 30*(2*a^5 - a^3*b^2 + (2*a^4*b - a^2*b^3
)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*
sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))
*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) -
 15*(8*a^5*b - 8*a^3*b^3 + a*b^5)*cos(d*x + c) - (4*b^6*cos(d*x + c)^5 - 10
*a^2*b^4*cos(d*x + c)^3 + 15*(8*a^5*b - 8*a^3*b^3 + a*b^5)*d*x + 15*(4*a^4*
b^2 - 3*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d)
, 1/20*(6*a*b^5*cos(d*x + c)^5 - 5*(4*a^3*b^3 - a*b^5)*cos(d*x + c)^3 - 15*
(8*a^6 - 8*a^4*b^2 + a^2*b^4)*d*x - 60*(2*a^5 - a^3*b^2 + (2*a^4*b - a^2*b^
3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b
^2)*cos(d*x + c))) - 15*(8*a^5*b - 8*a^3*b^3 + a*b^5)*cos(d*x + c) - (4*b^6
```

```
*cos(d*x + c)^5 - 10*a^2*b^4*cos(d*x + c)^3 + 15*(8*a^5*b - 8*a^3*b^3 + a*b
^5)*d*x + 15*(4*a^4*b^2 - 3*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(b^8*d*sin
(d*x + c) + a*b^7*d)]
```

giac [A] time = 0.23, size = 536, normalized size = 1.75

$$-\frac{\frac{15\left(8a^5-8a^3b^2+ab^4\right)(dx+c)}{b^7}-\frac{120\left(2a^6-3a^4b^2+a^2b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^7}+\frac{40\left(a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(15*(8*a^5 - 8*a^3*b^2 + a*b^4)*(d*x + c)/b^7 - 120*(2*a^6 - 3*a^4*b^
2 + a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d
*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 40*(a^4*b*tan(1/
2*d*x + 1/2*c) - a^2*b^3*tan(1/2*d*x + 1/2*c) + a^5 - a^3*b^2)/((a*tan(1/2*
d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^6) + 2*(40*a^3*b*tan(1/2*d
*x + 1/2*c)^9 - 25*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 100*a^4*tan(1/2*d*x + 1/2
*c)^8 - 120*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 20*b^4*tan(1/2*d*x + 1/2*c)^8
+ 80*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 10*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 400*a
^4*tan(1/2*d*x + 1/2*c)^6 - 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 + 600*a^4*ta
n(1/2*d*x + 1/2*c)^4 - 440*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 40*b^4*tan(1/2*
d*x + 1/2*c)^4 - 80*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 10*a*b^3*tan(1/2*d*x + 1
/2*c)^3 + 400*a^4*tan(1/2*d*x + 1/2*c)^2 - 280*a^2*b^2*tan(1/2*d*x + 1/2*c)
^2 - 40*a^3*b*tan(1/2*d*x + 1/2*c) + 25*a*b^3*tan(1/2*d*x + 1/2*c) + 100*a^
4 - 80*a^2*b^2 + 4*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*b^6))/d
```

maple [B] time = 0.58, size = 1119, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a^3+2/d*a^2/b^3/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+1/d/b^3/
(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a+12/d/b^5*arctan(tan(1/2*d
*x+1/2*c))*a^3-3/2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a+2/d*a^3/b^4/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^
5*a^4+8/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*a^2-12/d/b^7*arctan(tan(1/2*d*x+1/
2*c))*a^5-2/d*a^5/b^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-2/d
```

```
/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8-4/d/b^2/(1+tan(1/2*d*x
+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4-2/5/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5-40/d
/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^4+28/d/b^4/(1+tan(1/
2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^2+4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)
^5*tan(1/2*d*x+1/2*c)*a^3-5/2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+
1/2*c)*a-2/d*a^4/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(
1/2*d*x+1/2*c)+12/d*a^6/b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2
*c)+2*b)/(a^2-b^2)^(1/2))-4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/
2*c)^9*a^3+5/2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9*a-10/d
/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8*a^4+12/d/b^4/(1+tan(1/
2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8*a^2-40/d/b^6/(1+tan(1/2*d*x+1/2*c)^2
)^5*tan(1/2*d*x+1/2*c)^6*a^4+36/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*
x+1/2*c)^6*a^2-60/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^4
+44/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^2+8/d/b^5/(1+ta
n(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a^3-1/d/b^3/(1+tan(1/2*d*x+1/2*c
)^2)^5*tan(1/2*d*x+1/2*c)^3*a-18/d*a^4/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d*a^2/b^3/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.32, size = 2390, normalized size = 7.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] - ((2*(a*b^4 + 30*a^5 - 25*a^3*b^2))/(5*b^6) - (3*tan(c/2 + (d*x)/2)^10*(a*
b^4 - 4*a^5 + 2*a^3*b^2))/b^6 + (6*tan(c/2 + (d*x)/2)^4*(a*b^4 + 20*a^5 - 1
8*a^3*b^2))/b^6 + (4*tan(c/2 + (d*x)/2)^6*(a*b^4 + 30*a^5 - 25*a^3*b^2))/b^
6 + (3*tan(c/2 + (d*x)/2)^2*(9*a*b^4 + 100*a^5 - 90*a^3*b^2))/(5*b^6) + (ta
n(c/2 + (d*x)/2)*(180*a^4 + 8*b^4 - 155*a^2*b^2))/(10*b^5) + (3*tan(c/2 + (
d*x)/2)^11*(4*a^4 - 3*a^2*b^2))/(2*b^5) + (6*tan(c/2 + (d*x)/2)^8*(10*a^5 -
```

7*a^3*b^2))/b^6 + (3*tan(c/2 + (d*x)/2)^7*(36*a^4 - 31*a^2*b^2))/b^5 + (ta n(c/2 + (d*x)/2)^3*(156*a^4 - 125*a^2*b^2))/(2*b^5) + (tan(c/2 + (d*x)/2)^9 *(84*a^4 + 8*b^4 - 75*a^2*b^2))/(2*b^5) + (tan(c/2 + (d*x)/2)^5*(132*a^4 + 8*b^4 - 107*a^2*b^2))/b^5)/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 6*a*tan(c/2 + (d*x)/2)^2 + 15*a*tan(c/2 + (d*x)/2)^4 + 20*a*tan(c/2 + (d*x)/2)^6 + 15*a*ta n(c/2 + (d*x)/2)^8 + 6*a*tan(c/2 + (d*x)/2)^10 + a*tan(c/2 + (d*x)/2)^12 + 10*b*tan(c/2 + (d*x)/2)^3 + 20*b*tan(c/2 + (d*x)/2)^5 + 20*b*tan(c/2 + (d*x)/2)^7 + 10*b*tan(c/2 + (d*x)/2)^9 + 2*b*tan(c/2 + (d*x)/2)^11)) - (2*atanh ((108*a^6*(b^2 - a^2)^(1/2))/(108*a^6*b - (324*a^8)/b + (216*a^10)/b^3 - 64 8*a^7*tan(c/2 + (d*x)/2) + 216*a^5*b^2*tan(c/2 + (d*x)/2) + (432*a^9*tan(c/ 2 + (d*x)/2))/b^2) - (216*a^8*(b^2 - a^2)^(1/2))/(108*a^6*b^3 - 324*a^8*b + (216*a^10)/b + 432*a^9*tan(c/2 + (d*x)/2) + 216*a^5*b^4*tan(c/2 + (d*x)/2) - 648*a^7*b^2*tan(c/2 + (d*x)/2)) + (216*a^5*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(108*a^6 - (324*a^8)/b^2 + (216*a^10)/b^4 + 216*a^5*b*tan(c/2 + (d *x)/2) - (648*a^7*tan(c/2 + (d*x)/2))/b + (432*a^9*tan(c/2 + (d*x)/2))/b^3) - (540*a^7*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(108*a^6*b^2 - 324*a^8 + (216*a^10)/b^2 - 648*a^7*b*tan(c/2 + (d*x)/2) + 216*a^5*b^3*tan(c/2 + (d*x) /2) + (432*a^9*tan(c/2 + (d*x)/2))/b) + (216*a^9*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(216*a^10 + 108*a^6*b^4 - 324*a^8*b^2 + 432*a^9*b*tan(c/2 + (d* x)/2) + 216*a^5*b^5*tan(c/2 + (d*x)/2) - 648*a^7*b^3*tan(c/2 + (d*x)/2)))*(6*a^4*(b^2 - a^2)^(1/2) - 3*a^2*b^2*(b^2 - a^2)^(1/2)))/(b^7*d) - (3*a*atan (((3*a*(8*a^4 + b^4 - 8*a^2*b^2)*((2*(9*a^4*b^14 - 144*a^6*b^12 + 720*a^8*b ^10 - 1152*a^10*b^8 + 576*a^12*b^6))/b^17 + (2*tan(c/2 + (d*x)/2)*(18*a^3*b ^16 - 441*a^5*b^14 + 2448*a^7*b^12 - 4896*a^9*b^10 + 4032*a^11*b^8 - 1152*a ^13*b^6))/b^18 - (a*((2*(12*a^2*b^18 - 60*a^4*b^16 + 48*a^6*b^14))/b^17 + (2*tan(c/2 + (d*x)/2)*(96*a^3*b^18 - 288*a^5*b^16 + 192*a^7*b^14))/b^18 - (a *(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*b^22 - 32*a^3*b^20))/b^18)*(8*a^ 4 + b^4 - 8*a^2*b^2)*3i)/(4*b^7))*(8*a^4 + b^4 - 8*a^2*b^2)*3i)/(4*b^7)))/(4*b^7) + (3*a*(8*a^4 + b^4 - 8*a^2*b^2)*((2*(9*a^4*b^14 - 144*a^6*b^12 + 72 0*a^8*b^10 - 1152*a^10*b^8 + 576*a^12*b^6))/b^17 + (2*tan(c/2 + (d*x)/2)*(1 8*a^3*b^16 - 441*a^5*b^14 + 2448*a^7*b^12 - 4896*a^9*b^10 + 4032*a^11*b^8 - 1152*a^13*b^6))/b^18 + (a*((2*(12*a^2*b^18 - 60*a^4*b^16 + 48*a^6*b^14))/b ^17 + (2*tan(c/2 + (d*x)/2)*(96*a^3*b^18 - 288*a^5*b^16 + 192*a^7*b^14))/b^ 18 + (a*(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*b^22 - 32*a^3*b^20))/b^18)*(8*a^4 + b^4 - 8*a^2*b^2)*3i)/(4*b^7))*(8*a^4 + b^4 - 8*a^2*b^2)*3i)/(4*b ^7)))/(4*b^7))/((4*(1728*a^16 - 81*a^6*b^10 + 999*a^8*b^8 - 3942*a^10*b^6 + 6912*a^12*b^4 - 5616*a^14*b^2))/b^17 + (4*tan(c/2 + (d*x)/2)*(6912*a^17 + 54*a^5*b^12 - 1026*a^7*b^10 + 7020*a^9*b^8 - 21600*a^11*b^6 + 32832*a^13*b^ 4 - 24192*a^15*b^2))/b^18 - (a*(8*a^4 + b^4 - 8*a^2*b^2)*((2*(9*a^4*b^14 - 144*a^6*b^12 + 720*a^8*b^10 - 1152*a^10*b^8 + 576*a^12*b^6))/b^17 + (2*tan(c/2 + (d*x)/2)*(18*a^3*b^16 - 441*a^5*b^14 + 2448*a^7*b^12 - 4896*a^9*b^10 + 4032*a^11*b^8 - 1152*a^13*b^6))/b^18 - (a*((2*(12*a^2*b^18 - 60*a^4*b^16 + 48*a^6*b^14))/b^17 + (2*tan(c/2 + (d*x)/2)*(96*a^3*b^18 - 288*a^5*b^16 + 192*a^7*b^14))/b^18 - (a*(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*b^22 - 3 2*a^3*b^20))/b^18)*(8*a^4 + b^4 - 8*a^2*b^2)*3i)/(4*b^7))*(8*a^4 + b^4 - 8*

```
a^2*b^2)*3i)/(4*b^7))*3i)/(4*b^7) + (a*(8*a^4 + b^4 - 8*a^2*b^2)*((2*(9*a^4
*b^14 - 144*a^6*b^12 + 720*a^8*b^10 - 1152*a^10*b^8 + 576*a^12*b^6))/b^17 +
 (2*tan(c/2 + (d*x)/2)*(18*a^3*b^16 - 441*a^5*b^14 + 2448*a^7*b^12 - 4896*a
^9*b^10 + 4032*a^11*b^8 - 1152*a^13*b^6))/b^18 + (a*((2*(12*a^2*b^18 - 60*a
^4*b^16 + 48*a^6*b^14))/b^17 + (2*tan(c/2 + (d*x)/2)*(96*a^3*b^18 - 288*a^5
*b^16 + 192*a^7*b^14))/b^18 + (a*(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*
b^22 - 32*a^3*b^20))/b^18)*(8*a^4 + b^4 - 8*a^2*b^2)*3i)/(4*b^7))*(8*a^4 +
b^4 - 8*a^2*b^2)*3i)/(4*b^7))*3i)/(4*b^7)))*(8*a^4 + b^4 - 8*a^2*b^2))/(2*b
^7*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1128 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=267

$$-\frac{(a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))}+\frac{a(15a^2-11b^2)\cos(c+dx)}{3b^5d}-\frac{(20a^2-13b^2)\sin(c+dx)\cos(c+dx)}{8b^4d}+\frac{(5a^2-}{}$$

```
[Out] 1/8*(40*a^4-36*a^2*b^2+3*b^4)*x/b^6+1/3*a*(15*a^2-11*b^2)*cos(d*x+c)/b^5/d-
1/8*(20*a^2-13*b^2)*cos(d*x+c)*sin(d*x+c)/b^4/d+1/3*(5*a^2-3*b^2)*cos(d*x+c
)*sin(d*x+c)^2/a/b^3/d-1/4*cos(d*x+c)*sin(d*x+c)^3/b^2/d-(a^2-b^2)*cos(d*x+
c)*sin(d*x+c)^3/a/b^2/d/(a+b*sin(d*x+c))-2*a*(5*a^4-7*a^2*b^2+2*b^4)*arctan
((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^6/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2892, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{a(15a^2-11b^2)\cos(c+dx)}{3b^5d}-\frac{2a(-7a^2b^2+5a^4+2b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d\sqrt{a^2-b^2}}-\frac{(a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((40*a^4 - 36*a^2*b^2 + 3*b^4)*x)/(8*b^6) - (2*a*(5*a^4 - 7*a^2*b^2 + 2*b^4
)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*Sqrt[a^2 - b^2]*d)
 + (a*(15*a^2 - 11*b^2)*Cos[c + d*x])/(3*b^5*d) - ((20*a^2 - 13*b^2)*Cos[c
+ d*x]*Sin[c + d*x])/(8*b^4*d) + ((5*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]
^2)/(3*a*b^3*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b^2*d) - ((a^2 - b^2)*Co
s[c + d*x]*Sin[c + d*x]^3)/(a*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2892

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/(b^2*d*f*(m + n + 4)), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
```

```
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cos(c+dx)\sin^3(c+dx)}{4b^2d} - \frac{\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d(a+b\sin(c+dx))} + \frac{\int \frac{\sin^2(c+dx)(15}{}}{}$$

$$= \frac{\left(5a^2-3b^2\right)\cos(c+dx)\sin^2(c+dx)}{3ab^3d} - \frac{\cos(c+dx)\sin^3(c+dx)}{4b^2d} - \frac{\left(a^2-b^2\right)\text{co}}{ab^2d(a}$$

$$= -\frac{\left(20a^2-13b^2\right)\cos(c+dx)\sin(c+dx)}{8b^4d} + \frac{\left(5a^2-3b^2\right)\cos(c+dx)\sin^2(c+dx)}{3ab^3d}$$

$$= \frac{a\left(15a^2-11b^2\right)\cos(c+dx)}{3b^5d} - \frac{\left(20a^2-13b^2\right)\cos(c+dx)\sin(c+dx)}{8b^4d} + \frac{\left(5a^2-\right.}{}$$

$$= \frac{\left(40a^4-36a^2b^2+3b^4\right)x}{8b^6} + \frac{a\left(15a^2-11b^2\right)\cos(c+dx)}{3b^5d} - \frac{\left(20a^2-13b^2\right)\cos(c}{8b^4d}$$

$$= \frac{\left(40a^4-36a^2b^2+3b^4\right)x}{8b^6} + \frac{a\left(15a^2-11b^2\right)\cos(c+dx)}{3b^5d} - \frac{\left(20a^2-13b^2\right)\cos(c}{8b^4d}$$

$$= \frac{\left(40a^4-36a^2b^2+3b^4\right)x}{8b^6} + \frac{a\left(15a^2-11b^2\right)\cos(c+dx)}{3b^5d} - \frac{\left(20a^2-13b^2\right)\cos(c}{8b^4d}$$

$$= \frac{\left(40a^4-36a^2b^2+3b^4\right)x}{8b^6} - \frac{2a\left(5a^4-7a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}\,d} + \frac{a\left(\right.}{}$$

Mathematica [A] time = 3.67, size = 325, normalized size = 1.22

$$\frac{960a^5c+960a^5dx+960a^4bc\sin(c+dx)+960a^4bdx\sin(c+dx)+240a^3b^2\sin(2(c+dx))-864a^3b^2c-864a^3b^2dx-864a^2b^3c\sin(c+dx)-864a^2b^3dx\sin(c+dx)+\left(4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-384*a*(5*a^4 - 7*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]])/Sqrt[a^2 - b^2] + (960*a^5*c - 864*a^3*b^2*c + 72*a*b^4*c + 960
*a^5*d*x - 864*a^3*b^2*d*x + 72*a*b^4*d*x + 24*b*(40*a^4 - 31*a^2*b^2 + b^4
)*Cos[c + d*x] + (40*a^2*b^3 - 21*b^5)*Cos[3*(c + d*x)] - 3*b^5*Cos[5*(c +
d*x)] + 960*a^4*b*c*Sin[c + d*x] - 864*a^2*b^3*c*Sin[c + d*x] + 72*b^5*c*Si
n[c + d*x] + 960*a^4*b*d*x*Sin[c + d*x] - 864*a^2*b^3*d*x*Sin[c + d*x] + 72
*b^5*d*x*Sin[c + d*x] + 240*a^3*b^2*Sin[2*(c + d*x)] - 176*a*b^4*Sin[2*(c +
 d*x)] - 10*a*b^4*Sin[4*(c + d*x)])/(a + b*Sin[c + d*x]))/(192*b^6*d)
```

fricas [A] time = 0.94, size = 604, normalized size = 2.26

$$\left[-\frac{6\,b^5\cos\left(dx+c\right)^5-\left(20\,a^2b^3-3\,b^5\right)\cos\left(dx+c\right)^3-3\left(40\,a^5-36\,a^3b^2+3\,ab^4\right)dx+12\left(5\,a^4-2\,a^2b^2+\left(5\,a^3b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/24*(6*b^5*cos(d*x + c)^5 - (20*a^2*b^3 - 3*b^5)*cos(d*x + c)^3 - 3*(40*
a^5 - 36*a^3*b^2 + 3*a*b^4)*d*x + 12*(5*a^4 - 2*a^2*b^2 + (5*a^3*b - 2*a*b^
3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*
b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c
))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))
 - 3*(40*a^4*b - 36*a^2*b^3 + 3*b^5)*cos(d*x + c) + (10*a*b^4*cos(d*x + c)^
3 - 3*(40*a^4*b - 36*a^2*b^3 + 3*b^5)*d*x - 3*(20*a^3*b^2 - 13*a*b^4)*cos(d
*x + c))*sin(d*x + c))/(b^7*d*sin(d*x + c) + a*b^6*d), -1/24*(6*b^5*cos(d*x
 + c)^5 - (20*a^2*b^3 - 3*b^5)*cos(d*x + c)^3 - 3*(40*a^5 - 36*a^3*b^2 + 3*
a*b^4)*d*x - 24*(5*a^4 - 2*a^2*b^2 + (5*a^3*b - 2*a*b^3)*sin(d*x + c))*sqrt
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) -
3*(40*a^4*b - 36*a^2*b^3 + 3*b^5)*cos(d*x + c) + (10*a*b^4*cos(d*x + c)^3 -
 3*(40*a^4*b - 36*a^2*b^3 + 3*b^5)*d*x - 3*(20*a^3*b^2 - 13*a*b^4)*cos(d*x
+ c))*sin(d*x + c))/(b^7*d*sin(d*x + c) + a*b^6*d)]
```

giac [A] time = 0.21, size = 449, normalized size = 1.68

$$\frac{\frac{3\left(40\,a^4-36\,a^2b^2+3\,b^4\right)(dx+c)}{b^6}-\frac{48\left(5\,a^5-7\,a^3b^2+2\,ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6}+\frac{48\left(a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(40*a^4 - 36*a^2*b^2 + 3*b^4)*(d*x + c)/b^6 - 48*(5*a^5 - 7*a^3*b^2
 + 2*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*
x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 48*(a^3*b*tan(1/2
*d*x + 1/2*c) - a*b^3*tan(1/2*d*x + 1/2*c) + a^4 - a^2*b^2)/((a*tan(1/2*d*x
 + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^5) + 2*(36*a^2*b*tan(1/2*d*x
+ 1/2*c)^7 - 15*b^3*tan(1/2*d*x + 1/2*c)^7 + 96*a^3*tan(1/2*d*x + 1/2*c)^6
- 96*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^3
*tan(1/2*d*x + 1/2*c)^5 + 288*a^3*tan(1/2*d*x + 1/2*c)^4 - 192*a*b^2*tan(1/
2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 9*b^3*tan(1/2*d*x + 1/
2*c)^3 + 288*a^3*tan(1/2*d*x + 1/2*c)^2 - 160*a*b^2*tan(1/2*d*x + 1/2*c)^2
- 36*a^2*b*tan(1/2*d*x + 1/2*c) + 15*b^3*tan(1/2*d*x + 1/2*c) + 96*a^3 - 64
*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^5))/d
```

maple [B] time = 0.50, size = 938, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -10/d/b^6*a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-
b^2)^(1/2))+14/d/b^4*a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2))-4/d/b^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*
x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d
*x+1/2*c)^7*a^2-5/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+3
/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-3/4/d/b^2/(1+tan(1
/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3+5/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^
4*tan(1/2*d*x+1/2*c)+8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*a^3-16/3/d/b^3/(1+t
an(1/2*d*x+1/2*c)^2)^4*a+10/d/b^6*arctan(tan(1/2*d*x+1/2*c))*a^4-9/d/b^4*ar
ctan(tan(1/2*d*x+1/2*c))*a^2+2/d/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)*a^4-2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a
^2+3/4/d/b^2*arctan(tan(1/2*d*x+1/2*c))+8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*
tan(1/2*d*x+1/2*c)^6*a^3-8/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2
*c)^6*a+3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a^2+24/d/b^
5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a^3-16/d/b^3/(1+tan(1/2*d
*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a-3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*ta
n(1/2*d*x+1/2*c)^3*a^2+24/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*
c)^2*a^3-40/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a-3/d/b
^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*a^2+2/d/b^4/(tan(1/2*d*x+1
/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^3*tan(1/2*d*x+1/2*c)-2/d/b^2/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a*tan(1/2*d*x+1/2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 11.79, size = 1003, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (60*a^4*b + (3*b^5*cos(c + d*x))/2 + 120*a^5*atan(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)) - 44*a^2*b^3 - (21*b^5*cos(3*c + 3*d*x))/16 - (3*b^5*cos(5*c
+ 5*d*x))/16 + 9*a*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - (93*a^
2*b^3*cos(c + d*x))/2 + 9*b^5*sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2
+ (d*x)/2)) - 11*a*b^4*sin(2*c + 2*d*x) - (5*a*b^4*sin(4*c + 4*d*x))/8 + 60
*a^3*b^2*sin(c + d*x) - 108*a^3*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2)) + (5*a^2*b^3*cos(3*c + 3*d*x))/2 - 120*a^4*atanh((2*b^2*sin(c/2 + (d*x
)/2)*(b^2 - a^2)^(1/2) - a^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + a*b*cos
(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(a^3*cos(c/2 + (d*x)/2) - 2*b^3*sin(c/2
+ (d*x)/2) - a*b^2*cos(c/2 + (d*x)/2) + 2*a^2*b*sin(c/2 + (d*x)/2)))*(b^2 -
 a^2)^(1/2) + 15*a^3*b^2*sin(2*c + 2*d*x) + 60*a^4*b*cos(c + d*x) - 44*a*b^
4*sin(c + d*x) - 108*a^2*b^3*sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2)) + 48*a^2*b^2*atanh((2*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) -
 a^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(b^2 - a
^2)^(1/2))/(a^3*cos(c/2 + (d*x)/2) - 2*b^3*sin(c/2 + (d*x)/2) - a*b^2*cos(c
/2 + (d*x)/2) + 2*a^2*b*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2) + 120*a^4*b*
sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 48*a*b^3*sin(c +
 d*x)*atanh((2*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^2*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(a^3*co
s(c/2 + (d*x)/2) - 2*b^3*sin(c/2 + (d*x)/2) - a*b^2*cos(c/2 + (d*x)/2) + 2*
a^2*b*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2) - 120*a^3*b*sin(c + d*x)*atanh
((2*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^2*sin(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(a^3*cos(c/2 + (d*
x)/2) - 2*b^3*sin(c/2 + (d*x)/2) - a*b^2*cos(c/2 + (d*x)/2) + 2*a^2*b*sin(c
/2 + (d*x)/2)))*(b^2 - a^2)^(1/2))/(12*b^6*d*(a + b*sin(c + d*x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1129 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=163

$$-\frac{ax\left(4a^2-3b^2\right)}{b^5}-\frac{\cos(c+dx)\left(4a^2-2ab\sin(c+dx)-b^2\right)}{b^4d}+\frac{2\left(4a^4-5a^2b^2+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}+\frac{\cos^3(c}{3}$$

```
[Out] -a*(4*a^2-3*b^2)*x/b^5+1/3*cos(d*x+c)^3*(4*a+b*sin(d*x+c))/b^2/d/(a+b*sin(d
*x+c))-cos(d*x+c)*(4*a^2-b^2-2*a*b*sin(d*x+c))/b^4/d+2*(4*a^4-5*a^2*b^2+b^4
)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^5/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2863, 2865, 2735, 2660, 618, 204}

$$\frac{2\left(-5a^2b^2+4a^4+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}-\frac{\cos(c+dx)\left(4a^2-2ab\sin(c+dx)-b^2\right)}{b^4d}-\frac{ax\left(4a^2-3b^2\right)}{b^5}+\frac{\cos^3(c}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a*(4*a^2 - 3*b^2)*x)/b^5) + (2*(4*a^4 - 5*a^2*b^2 + b^4)*ArcTan[(b + a*T
an[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*d) + (Cos[c + d*x]^
3*(4*a + b*Sin[c + d*x]))/(3*b^2*d*(a + b*Sin[c + d*x])) - (Cos[c + d*x]*(4
*a^2 - b^2 - 2*a*b*Sin[c + d*x]))/(b^4*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\int \frac{\cos^2(c+dx)(-b-4a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\cos(c+dx)\left(4a^2-b^2-2ab\sin(c+dx)\right)}{b^4d} -$$

$$= -\frac{a\left(4a^2-3b^2\right)x}{b^5} + \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\cos(c+dx)\left(4a^2-b^2-\right.}{b^4d}$$

$$= -\frac{a\left(4a^2-3b^2\right)x}{b^5} + \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\cos(c+dx)\left(4a^2-b^2-\right.}{b^4d}$$

$$= -\frac{a\left(4a^2-3b^2\right)x}{b^5} + \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\cos(c+dx)\left(4a^2-b^2-\right.}{b^4d}$$

$$= -\frac{a\left(4a^2-3b^2\right)x}{b^5} + \frac{2\left(4a^4-5a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}\,d} + \frac{\cos^3(c+dx}{3b^2d(a}$$

Mathematica [A] time = 2.51, size = 247, normalized size = 1.52

$$\frac{\frac{48\left(4a^4-5a^2b^2+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{-96a^4c-96a^4dx+\left(60ab^3-96a^3b\right)\cos(c+dx)-96a^3bc\sin(c+dx)-96a^3bdx\sin(c+dx)-24a^2b^2\sin(2(c+dx}{}}{24b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((48*(4*a^4 - 5*a^2*b^2 + b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/Sqrt[a^2 - b^2] + (-96*a^4*c + 72*a^2*b^2*c - 96*a^4*d*x + 72*a^2*b^2
*d*x + (-96*a^3*b + 60*a*b^3)*Cos[c + d*x] - 4*a*b^3*Cos[3*(c + d*x)] - 96*
a^3*b*c*Sin[c + d*x] + 72*a*b^3*c*Sin[c + d*x] - 96*a^3*b*d*x*Sin[c + d*x]
+ 72*a*b^3*d*x*Sin[c + d*x] - 24*a^2*b^2*Sin[2*(c + d*x)] + 14*b^4*Sin[2*(c
 + d*x)] + b^4*Sin[4*(c + d*x)])/(a + b*Sin[c + d*x]))/(24*b^5*d)
```

fricas [A] time = 0.69, size = 507, normalized size = 3.11

$$\left[-\frac{4\,ab^3\cos(dx+c)^3 + 6\left(4\,a^4-3\,a^2b^2\right)dx + 3\left(4\,a^3-ab^2+\left(4\,a^2b-b^3\right)\sin(dx+c)\right)\sqrt{-a^2+b^2}\,\log\left(\frac{\left(2\,a^2-b^2\right)\text{co}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(4*a*b^3*cos(d*x + c)^3 + 6*(4*a^4 - 3*a^2*b^2)*d*x + 3*(4*a^3 - a*b^
2 + (4*a^2*b - b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d
*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c)
 + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
c) - a^2 - b^2)) + 6*(4*a^3*b - 3*a*b^3)*cos(d*x + c) - 2*(b^4*cos(d*x + c)
^3 - 3*(4*a^3*b - 3*a*b^3)*d*x - 3*(2*a^2*b^2 - b^4)*cos(d*x + c))*sin(d*x
+ c))/(b^6*d*sin(d*x + c) + a*b^5*d), -1/3*(2*a*b^3*cos(d*x + c)^3 + 3*(4*a
^4 - 3*a^2*b^2)*d*x + 3*(4*a^3 - a*b^2 + (4*a^2*b - b^3)*sin(d*x + c))*sqrt
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) +
3*(4*a^3*b - 3*a*b^3)*cos(d*x + c) - (b^4*cos(d*x + c)^3 - 3*(4*a^3*b - 3*a
*b^3)*d*x - 3*(2*a^2*b^2 - b^4)*cos(d*x + c))*sin(d*x + c))/(b^6*d*sin(d*x
+ c) + a*b^5*d)]
```

giac [A] time = 0.20, size = 300, normalized size = 1.84

$$-\frac{\frac{3\left(4a^3-3ab^2\right)(dx+c)}{b^5}-\frac{6\left(4a^4-5a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^5}+\frac{6\left(a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^3-ab^2\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)b^4}+\cdots}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(4*a^3 - 3*a*b^2)*(d*x + c)/b^5 - 6*(4*a^4 - 5*a^2*b^2 + b^4)*(pi*f
loor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/s
qrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^5) + 6*(a^2*b*tan(1/2*d*x + 1/2*c) - b^
3*tan(1/2*d*x + 1/2*c) + a^3 - a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(
1/2*d*x + 1/2*c) + a)*b^4) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*tan(1/
2*d*x + 1/2*c)^4 - 6*b^2*tan(1/2*d*x + 1/2*c)^4 + 18*a^2*tan(1/2*d*x + 1/2*
c)^2 - 6*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*a^2 -
4*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^4))/d
```

maple [B] time = 0.47, size = 627, normalized size = 3.85

$$-\frac{2a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{6\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{d\,b^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{12a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)^5-6/d/b^4/(1+tan(1
/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2+4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2
)^3*tan(1/2*d*x+1/2*c)^4-12/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^3*a^2*tan(1/2*d*
x+1/2*c)^2+4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2+2/d/b^3/
(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)-6/d/b^4/(1+tan(1/2*d*x+1/2*
c)^2)^3*a^2+8/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3-8/d/b^5*arctan(tan(1/2*d*x
+1/2*c))*a^3+6/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a-2/d*a^2/b^3/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/b/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-2/d*a^3/b^4/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)*a+8/d*a^4/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan
(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-10/d*a^2/b^3/(a^2-b^2)^(1/2)*arctan(1
/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/b/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 11.53, size = 964, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((2*(7*a*b^2 - 12*a^3))/(3*b^4) + (2*tan(c/2 + (d*x)/2)^6*(a*b^2 - 4*a^3))/
b^4 + (2*tan(c/2 + (d*x)/2)^4*(7*a*b^2 - 12*a^3))/b^4 + (2*tan(c/2 + (d*x)/
2)^2*(25*a*b^2 - 36*a^3))/(3*b^4) - (2*tan(c/2 + (d*x)/2)*(18*a^2 - 11*b^2)
)/(3*b^3) - (14*tan(c/2 + (d*x)/2)^3*(2*a^2 - b^2))/b^3 - (2*tan(c/2 + (d*x
)/2)^7*(2*a^2 - b^2))/b^3 - (2*tan(c/2 + (d*x)/2)^5*(10*a^2 - 7*b^2))/b^3)/
(d*(a + 2*b*tan(c/2 + (d*x)/2) + 4*a*tan(c/2 + (d*x)/2)^2 + 6*a*tan(c/2 + (
d*x)/2)^4 + 4*a*tan(c/2 + (d*x)/2)^6 + a*tan(c/2 + (d*x)/2)^8 + 6*b*tan(c/2
 + (d*x)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^7)) - (2*
atanh((64*a^2*(b^2 - a^2)^(1/2))/(64*a^2*b - (320*a^4)/b + (256*a^6)/b^3 -
```

```
640*a^3*tan(c/2 + (d*x)/2) + 128*a*b^2*tan(c/2 + (d*x)/2) + (512*a^5*tan(c/
2 + (d*x)/2))/b^2) - (256*a^4*(b^2 - a^2)^(1/2))/(64*a^2*b^3 - 320*a^4*b +
(256*a^6)/b + 512*a^5*tan(c/2 + (d*x)/2) + 128*a*b^4*tan(c/2 + (d*x)/2) - 6
40*a^3*b^2*tan(c/2 + (d*x)/2)) + (128*a*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
))/(64*a^2 - (320*a^4)/b^2 + (256*a^6)/b^4 - (640*a^3*tan(c/2 + (d*x)/2))/b
 + (512*a^5*tan(c/2 + (d*x)/2))/b^3 + 128*a*b*tan(c/2 + (d*x)/2)) - (576*a^
3*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(64*a^2*b^2 - 320*a^4 + (256*a^6)/b
^2 + 128*a*b^3*tan(c/2 + (d*x)/2) - 640*a^3*b*tan(c/2 + (d*x)/2) + (512*a^5
*tan(c/2 + (d*x)/2))/b) + (256*a^5*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(2
56*a^6 + 64*a^2*b^4 - 320*a^4*b^2 + 128*a*b^5*tan(c/2 + (d*x)/2) + 512*a^5*
b*tan(c/2 + (d*x)/2) - 640*a^3*b^3*tan(c/2 + (d*x)/2)))*(4*a^2*(b^2 - a^2)^
(1/2) - b^2*(b^2 - a^2)^(1/2)))/(b^5*d) - (2*a*atan((192*a^2*tan(c/2 + (d*x
)/2))/(192*a^2 - (448*a^4)/b^2 + (256*a^6)/b^4) - (448*a^4*tan(c/2 + (d*x)/
2))/(192*a^2*b^2 - 448*a^4 + (256*a^6)/b^2) + (256*a^6*tan(c/2 + (d*x)/2))/
(256*a^6 + 192*a^2*b^4 - 448*a^4*b^2))*(4*a^2 - 3*b^2))/(b^5*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
[Out] Timed out
```

3.1130 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=137

$$-\frac{\left(a^2-b^2\right)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))}+\frac{2\sqrt{a^2-b^2}\,\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2ax}{b^3}-\frac{\cos(c+dx)}{b^2d}$$

```
[Out] -2*a*x/b^3-arctanh(cos(d*x+c))/a^2/d-cos(d*x+c)/b^2/d-(a^2-b^2)*cos(d*x+c)/
a/b^2/d/(a+b*sin(d*x+c))+2*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2
-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^2/b^3/d
```

Rubi [A] time = 0.27, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2892, 3057, 2660, 618, 204, 3770}

$$\frac{2\sqrt{a^2-b^2}\,\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^3d}-\frac{\left(a^2-b^2\right)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2ax}{b^3}-\frac{\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*x)/b^3 + (2*Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]])/(a^2*b^3*d) - ArcTanh[Cos[c + d*x]]/(a^2*d) - Cos[c
+ d*x]/(b^2*d) - ((a^2 - b^2)*Cos[c + d*x])/(a*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2892

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/(b^2*d*f*(m + n + 4)), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cos(c+dx)}{b^2d} - \frac{\left(a^2-b^2\right)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)\left(b^2-ab\sin(c+dx)-2a^2\sin^2(c+dx)\right)}{a+b\sin(c+dx)}}{ab^2}$$

$$= -\frac{2ax}{b^3} - \frac{\cos(c+dx)}{b^2d} - \frac{\left(a^2-b^2\right)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))} + \frac{\int \csc(c+dx)\,dx}{a^2} - \frac{\left(-2a^4+\right.}{}$$

$$= -\frac{2ax}{b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{b^2d} - \frac{\left(a^2-b^2\right)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))} - \frac{\left(2\left(-\right.\right.}{}$$

$$= -\frac{2ax}{b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{b^2d} - \frac{\left(a^2-b^2\right)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))} + \frac{\left(4\left(-\right.\right.}{}$$

$$= -\frac{2ax}{b^3} + \frac{2\left(2a^4-a^2b^2-b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^3\sqrt{a^2-b^2}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \mathrm{c}$$

Mathematica [A] time = 0.71, size = 161, normalized size = 1.18

$$\frac{\frac{\left(b^2-a^2\right)\cos(c+dx)}{ab^2(a+b\sin(c+dx))} + \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2} + \frac{2\left(2a^4-a^2b^2-b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^3\sqrt{a^2-b^2}} - \frac{2a(c+dx)}{b^3} - \frac{\cos(c+dx)}{b^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*a*(c + d*x))/b^3 + (2*(2*a^4 - a^2*b^2 - b^4)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a^2*b^3*Sqrt[a^2 - b^2]) - Cos[c + d*x]/b^2 - Lo
g[Cos[(c + d*x)/2]]/a^2 + Log[Sin[(c + d*x)/2]]/a^2 + ((-a^2 + b^2)*Cos[c +
 d*x])/(a*b^2*(a + b*Sin[c + d*x])))/d
```

fricas [A] time = 0.92, size = 516, normalized size = 3.77

$$\left[-\frac{4\,a^4dx - \left(2\,a^3+ab^2+\left(2\,a^2b+b^3\right)\sin(dx+c)\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-2\,(a\cos(dx+\right.}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*a^4*d*x - (2*a^3 + a*b^2 + (2*a^2*b + b^3)*sin(d*x + c))*sqrt(-a^2
 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(2*a^3*b - a*b^3)*cos
(d*x + c) + (b^4*sin(d*x + c) + a*b^3)*log(1/2*cos(d*x + c) + 1/2) - (b^4*s
in(d*x + c) + a*b^3)*log(-1/2*cos(d*x + c) + 1/2) + 2*(2*a^3*b*d*x + a^2*b^
2*cos(d*x + c))*sin(d*x + c))/(a^2*b^4*d*sin(d*x + c) + a^3*b^3*d), -1/2*(4
*a^4*d*x + 2*(2*a^3 + a*b^2 + (2*a^2*b + b^3)*sin(d*x + c))*sqrt(a^2 - b^2)
*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 2*(2*a^3*b
- a*b^3)*cos(d*x + c) + (b^4*sin(d*x + c) + a*b^3)*log(1/2*cos(d*x + c) + 1
/2) - (b^4*sin(d*x + c) + a*b^3)*log(-1/2*cos(d*x + c) + 1/2) + 2*(2*a^3*b*
d*x + a^2*b^2*cos(d*x + c))*sin(d*x + c))/(a^2*b^4*d*sin(d*x + c) + a^3*b^3
*d)]
```

giac [B] time = 0.22, size = 286, normalized size = 2.09

$$-\frac{\frac{2\,(dx+c)a}{b^3}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{2\left(2\,a^4-a^2b^2-b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^2b^3}+\frac{2\left(a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*(d*x + c)*a/b^3 - log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 2*(2*a^4 - a^2*b
^2 - b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
+ 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^2*b^3) + 2*(a^2*b*tan(1/
2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 2*a^3*tan(1/2*d*x + 1/2*c)^
2 - a*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b*tan(1/2*d*x + 1/2*c) - b^3*tan(1
/2*d*x + 1/2*c) + 2*a^3 - a*b^2)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d
*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*
a^2*b^2))/d
```

maple [B] time = 0.71, size = 380, normalized size = 2.77

$$-\frac{2}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^3}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)-4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a+1/d/
a^2*ln(tan(1/2*d*x+1/2*c))-2/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*
c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)*a+2/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+4/d
*a^2/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^
(1/2))-2/d/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))-2/d/a^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b
)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 11.15, size = 2773, normalized size = 20.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) + (atan((((2*(b^2 - a^2)^(1/2))/b^3 + (b^2
- a^2)^(1/2)/(a^2*b))*(((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*
b))*(((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*(((32*(4*a^4*b
^10 - 3*a^6*b^8))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^14 - 17*a^6*
b^12 + 2*a^8*b^10))/(a^3*b^8))*((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/
2)/(a^2*b)) + (32*(8*a^2*b^10 - 7*a^6*b^6))/(a^2*b^5) + (32*tan(c/2 + (d*x)
/2)*(16*a^2*b^14 - 5*a^4*b^12 - 18*a^6*b^10 + 8*a^8*b^8))/(a^3*b^8)) + (32*
(4*b^10 + 3*a^2*b^8 - 7*a^4*b^6 + 16*a^6*b^4 - 12*a^8*b^2))/(a^2*b^5) + (32
*tan(c/2 + (d*x)/2)*(6*a^4*b^10 - 11*a^2*b^12 + 101*a^6*b^8 - 100*a^8*b^6 +
 8*a^10*b^4))/(a^3*b^8)) - (32*(28*a^8 + 2*a^2*b^6 - 28*a^4*b^4 - 6*a^6*b^2
))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(b^12 + 2*a^2*b^10 + 61*a^4*b^8 - 20*
a^6*b^6 - 72*a^8*b^4 + 32*a^10*b^2))/(a^3*b^8))*1i - ((2*(b^2 - a^2)^(1/2))
/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*(((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)
^(1/2)/(a^2*b))*((32*(4*b^10 + 3*a^2*b^8 - 7*a^4*b^6 + 16*a^6*b^4 - 12*a^8*
```

b^2))/(a^2*b^5) - ((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*((32*(8*a^2*b^10 - 7*a^6*b^6))/(a^2*b^5) - ((32*(4*a^4*b^10 - 3*a^6*b^8))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^14 - 17*a^6*b^12 + 2*a^8*b^10))/(a^3*b^8))*((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b)) + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^14 - 5*a^4*b^12 - 18*a^6*b^10 + 8*a^8*b^8))/(a^3*b^8)) + (32*tan(c/2 + (d*x)/2)*(6*a^4*b^10 - 11*a^2*b^12 + 101*a^6*b^8 - 100*a^8*b^6 + 8*a^10*b^4))/(a^3*b^8)) + (32*(28*a^8 + 2*a^2*b^6 - 28*a^4*b^4 - 6*a^6*b^2))/(a^2*b^5) - (32*tan(c/2 + (d*x)/2)*(b^12 + 2*a^2*b^10 + 61*a^4*b^8 - 20*a^6*b^6 - 72*a^8*b^4 + 32*a^10*b^2))/(a^3*b^8))*1i)/(((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*(((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*(((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*(((32*(4*a^4*b^10 - 3*a^6*b^8))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^14 - 17*a^6*b^12 + 2*a^8*b^10))/(a^3*b^8))*((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b)) + (32*(8*a^2*b^10 - 7*a^6*b^6))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^14 - 5*a^4*b^12 - 18*a^6*b^10 + 8*a^8*b^8))/(a^3*b^8)) + (32*(4*b^10 + 3*a^2*b^8 - 7*a^4*b^6 + 16*a^6*b^4 - 12*a^8*b^2))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(6*a^4*b^10 - 11*a^2*b^12 + 101*a^6*b^8 - 100*a^8*b^6 + 8*a^10*b^4))/(a^3*b^8)) - (32*(28*a^8 + 2*a^2*b^6 - 28*a^4*b^4 - 6*a^6*b^2))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(b^12 + 2*a^2*b^10 + 61*a^4*b^8 - 20*a^6*b^6 - 72*a^8*b^4 + 32*a^10*b^2))/(a^3*b^8)) + ((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*(((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*((32*(4*b^10 + 3*a^2*b^8 - 7*a^4*b^6 + 16*a^6*b^4 - 12*a^8*b^2))/(a^2*b^5) - ((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*((32*(8*a^2*b^10 - 7*a^6*b^6))/(a^2*b^5) - ((32*(4*a^4*b^10 - 3*a^6*b^8))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^14 - 17*a^6*b^12 + 2*a^8*b^10))/(a^3*b^8))*((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b)) + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^14 - 5*a^4*b^12 - 18*a^6*b^10 + 8*a^8*b^8))/(a^3*b^8)) + (32*tan(c/2 + (d*x)/2)*(6*a^4*b^10 - 11*a^2*b^12 + 101*a^6*b^8 - 100*a^8*b^6 + 8*a^10*b^4))/(a^3*b^8)) + (32*(28*a^8 + 2*a^2*b^6 - 28*a^4*b^4 - 6*a^6*b^2))/(a^2*b^5) - (32*tan(c/2 + (d*x)/2)*(b^12 + 2*a^2*b^10 + 61*a^4*b^8 - 20*a^6*b^6 - 72*a^8*b^4 + 32*a^10*b^2))/(a^3*b^8)) - (64*(28*a^6 + 2*b^6 - 12*a^2*b^4 - 18*a^4*b^2))/(a^2*b^5) - (64*tan(c/2 + (d*x)/2)*(128*a^10 + 48*a^4*b^6 - 16*a^6*b^4 - 160*a^8*b^2))/(a^3*b^8)))*(((b^2 - a^2)^(1/2)*4i)/b^3 + ((b^2 - a^2)^(1/2)*2i)/(a^2*b)))/d - ((2*(2*a^2 - b^2))/(a*b^2) + (2*tan(c/2 + (d*x)/2)*(3*a^2 - b^2))/(a^2*b) + (2*tan(c/2 + (d*x)/2)^3*(a^2 - b^2))/(a^2*b) + (2*tan(c/2 + (d*x)/2)^2*(2*a^2 - b^2))/(a*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^3)) - (4*a*atan((256*tan(c/2 + (d*x)/2))/((128*b^2)/a^2 - (384*a^2)/b^2 + (256*a*tan(c/2 + (d*x)/2))/b + (512*a^3*tan(c/2 + (d*x)/2))/b^3 - (768*a^5*tan(c/2 + (d*x)/2))/b^5 + 256) - (512*a^3)/(256*b^3 - 384*a^2*b + (128*b^5)/a^2 + 512*a^3*tan(c/2 + (d*x)/2) + 256*a*b^2*tan(c/2 + (d*x)/2) - (768*a^5*tan(c/2 + (d*x)/2))/b^2) + (768*a^5)/(256*b^5 - 384*a^2*b^3 + (128*b^7)/a^2 - 768*a^5*tan(c/2 + (d*x)/2) + 256*a*b^4*tan(c/2 + (d*x)/2) + 512*a^3*b^2*tan(c/2 + (d*x)/2)) - (256*a)/(256*b + 256*a*tan(c/2 + (d*x)/2) - (384*a^2)/b + (128*b^3)/a^2 + (51

```
2*a^3*tan(c/2 + (d*x)/2))/b^2 - (768*a^5*tan(c/2 + (d*x)/2))/b^4) - (384*a^
2*tan(c/2 + (d*x)/2))/(256*b^2 - 384*a^2 + (128*b^4)/a^2 + (512*a^3*tan(c/2
 + (d*x)/2))/b - (768*a^5*tan(c/2 + (d*x)/2))/b^3 + 256*a*b*tan(c/2 + (d*x)
/2)) + (128*b*tan(c/2 + (d*x)/2))/(128*b + (256*a^2)/b - (384*a^4)/b^3 + (2
56*a^3*tan(c/2 + (d*x)/2))/b^2 + (512*a^5*tan(c/2 + (d*x)/2))/b^4 - (768*a^
7*tan(c/2 + (d*x)/2))/b^6)))/(b^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

3.1131 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=154

$$\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\left(a^2-2b^2\right)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))}-\frac{2\left(a^4+a^2b^2-2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^2d\sqrt{a^2-b^2}}-\frac{\cot(c+dx)}{ad(a+b\sin(c+dx}$$

```
[Out] x/b^2+2*b*arctanh(cos(d*x+c))/a^3/d+(a^2-2*b^2)*cos(d*x+c)/a^2/b/d/(a+b*sin
(d*x+c))-cot(d*x+c)/a/d/(a+b*sin(d*x+c))-2*(a^4+a^2*b^2-2*b^4)*arctan((b+a*
tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/b^2/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2890, 3057, 2660, 618, 204, 3770}

$$-\frac{2\left(a^2b^2+a^4-2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^2d\sqrt{a^2-b^2}}+\frac{\left(a^2-2b^2\right)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{ad(a+b\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] x/b^2 - (2*(a^4 + a^2*b^2 - 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(a^3*b^2*Sqrt[a^2 - b^2]*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d)
 + ((a^2 - 2*b^2)*Cos[c + d*x])/(a^2*b*d*(a + b*Sin[c + d*x])) - Cot[c + d*
x]/(a*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\left(a^2-2b^2\right)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)\left(-2b^2-ab\sin(c+dx)+a^2\right.}{a+b\sin(c+dx)}}{a^2b}$$

$$= \frac{x}{b^2} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2b)\int \csc(c+dx)\,dx}{a^3} -$$

$$= \frac{x}{b^2} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx)}$$

$$= \frac{x}{b^2} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx)}$$

$$= \frac{x}{b^2} - \frac{2\left(a^4+a^2b^2-2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^2\sqrt{a^2-b^2}\,d} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{(}{a^2}$$

Mathematica [A] time = 1.79, size = 182, normalized size = 1.18

$$\frac{-\frac{4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{4b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{2\left(a^2-b^2\right)\cos(c+dx)}{a^2b(a+b\sin(c+dx))} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{a^2} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{a^2} - \frac{4\left(a^4+a^2b^2-2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{a^2-b^2}}\right.}{a^3b^2\sqrt{a^2-b^2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*(c + d*x))/b^2 - (4*(a^4 + a^2*b^2 - 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]])/(a^3*b^2*Sqrt[a^2 - b^2]) - Cot[(c + d*x)/2]/a^2 + (
4*b*Log[Cos[(c + d*x)/2]])/a^3 - (4*b*Log[Sin[(c + d*x)/2]])/a^3 + (2*(a^2
- b^2)*Cos[c + d*x])/(a^2*b*(a + b*Sin[c + d*x])) + Tan[(c + d*x)/2]/a^2)/(
2*d)
```

fricas [A] time = 0.86, size = 675, normalized size = 4.38

$$\left[\frac{2a^3bdx\cos(dx+c)^2 - 2a^3bdx + 2a^2b^2\cos(dx+c) - \left(a^2b+2b^3-\left(a^2b+2b^3\right)\cos(dx+c)^2+\left(a^3+2ab^2\right)\sin\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(2*a^3*b*d*x*cos(d*x + c)^2 - 2*a^3*b*d*x + 2*a^2*b^2*cos(d*x + c) - (
a^2*b + 2*b^3 - (a^2*b + 2*b^3)*cos(d*x + c)^2 + (a^3 + 2*a*b^2)*sin(d*x +
c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 +
 b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(b^4*cos(
d*x + c)^2 - a*b^3*sin(d*x + c) - b^4)*log(1/2*cos(d*x + c) + 1/2) - 2*(b^4
*cos(d*x + c)^2 - a*b^3*sin(d*x + c) - b^4)*log(-1/2*cos(d*x + c) + 1/2) -
2*(a^4*d*x + (a^3*b - 2*a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*cos(d
*x + c)^2 - a^4*b^2*d*sin(d*x + c) - a^3*b^3*d), (a^3*b*d*x*cos(d*x + c)^2
- a^3*b*d*x + a^2*b^2*cos(d*x + c) - (a^2*b + 2*b^3 - (a^2*b + 2*b^3)*cos(d
*x + c)^2 + (a^3 + 2*a*b^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*
x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (b^4*cos(d*x + c)^2 - a*b^3*s
in(d*x + c) - b^4)*log(1/2*cos(d*x + c) + 1/2) - (b^4*cos(d*x + c)^2 - a*b^
3*sin(d*x + c) - b^4)*log(-1/2*cos(d*x + c) + 1/2) - (a^4*d*x + (a^3*b - 2*
a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*cos(d*x + c)^2 - a^4*b^2*d*si
n(d*x + c) - a^3*b^3*d)]
```

giac [A] time = 0.24, size = 260, normalized size = 1.69

$$\frac{\frac{6\,(dx+c)}{b^2} - \frac{12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} + \frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2} - \frac{12\,\left(a^4+a^2b^2-2\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^3b^2} + \frac{4\,ab^2\tan\left(\frac{1}{2}\,dx\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)/b^2 - 12*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 3*tan(1/2*
d*x + 1/2*c)/a^2 - 12*(a^4 + a^2*b^2 - 2*b^4)*(pi*floor(1/2*(d*x + c)/pi +
1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a
^2 - b^2)*a^3*b^2) + (4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*b*tan(1/2*d*x
+ 1/2*c)^2 - 4*b^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) - 1
4*a*b^2*tan(1/2*d*x + 1/2*c) - 3*a^2*b)/((a*tan(1/2*d*x + 1/2*c)^3 + 2*b*ta
n(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))*a^3*b))/d
```

maple [B] time = 0.75, size = 396, normalized size = 2.57

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2}-\frac{1}{2d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{d\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)+2/d/b^2*arctan(tan(1/2*d*x+1/2*c))-1/2/d/a^2/t
an(1/2*d*x+1/2*c)-2/d/a^3*b*ln(tan(1/2*d*x+1/2*c))+2/d/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)/a*tan(1/2*d*x+1/2*c)-2/d/a^3/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^2*tan(1/2*d*x+1/2*c)+2/d/b/(tan(1/2*d*x
+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan
(1/2*d*x+1/2*c)*b+a)*b-2/d/b^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*
x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d/a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.09, size = 6377, normalized size = 41.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (2*cos(c/2 + (d*x)/2)^2)/(b*d*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2
)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) + sin(c/2 + (d*x)/2)^3/(2
*a*d*cos(c/2 + (d*x)/2)*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 +
2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) - cos(c/2 + (d*x)/2)^3/(2*a*d*s
in(c/2 + (d*x)/2)*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*co
s(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) - (3*b*cos(c/2 + (d*x)/2)^2)/(a^2*d*(
a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*si
n(c/2 + (d*x)/2))) + (2*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))/(a*d*(a*cos(
c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2
+ (d*x)/2))) + (b*sin(c/2 + (d*x)/2)^2)/(a^2*d*(a*cos(c/2 + (d*x)/2)^2 + a*
sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) - (2*b*l
```

og(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^2)/(a^2*d*(a*c
os(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c
/2 + (d*x)/2))) + (atan((40*a^5*b^5*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) -
4*a^12*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 128*b^10*sin(c/2 + (d*x)/2)*(
b^2 - a^2)^(3/2) - 16*a^3*b^7*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 4*a^10
*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 4*a^5*b^7*cos(c/2 + (d*x)/2)*(b^2 -
a^2)^(1/2) + 20*a^7*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 20*a^7*b^5*
cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 9*a^9*b^3*cos(c/2 + (d*x)/2)*(b^2 -
a^2)^(1/2) + 8*a^2*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*a^2*b^10*si
n(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 104*a^4*b^6*sin(c/2 + (d*x)/2)*(b^2 -
a^2)^(3/2) + 8*a^4*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 18*a^6*b^4*si
n(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 26*a^6*b^6*sin(c/2 + (d*x)/2)*(b^2 - a
^2)^(1/2) - 16*a^8*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 28*a^8*b^4*si
n(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 64*a*b^9*cos(c/2 + (d*x)/2)*(b^2 - a^2
)^(3/2) + 2*a^9*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 5*a^11*b*cos(c/2 +
(d*x)/2)*(b^2 - a^2)^(1/2))/(a^3*b^10*cos(c/2 + (d*x)/2)*80i - a*b^12*cos(
c/2 + (d*x)/2)*64i - a^12*b*sin(c/2 + (d*x)/2)*3i - b^13*sin(c/2 + (d*x)/2)
*128i + a^5*b^8*cos(c/2 + (d*x)/2)*44i - a^7*b^6*cos(c/2 + (d*x)/2)*72i + a
^9*b^4*cos(c/2 + (d*x)/2)*3i + a^11*b^2*cos(c/2 + (d*x)/2)*9i + a^2*b^11*si
n(c/2 + (d*x)/2)*192i + a^4*b^9*sin(c/2 + (d*x)/2)*56i - a^6*b^7*sin(c/2 +
(d*x)/2)*172i + a^8*b^5*sin(c/2 + (d*x)/2)*34i + a^10*b^3*sin(c/2 + (d*x)/2
)*21i))*cos(c/2 + (d*x)/2)^2*(b^2 - a^2)^(1/2)*4i)/(a^2*d*(a*cos(c/2 + (d*x
)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)
)) + (atan((40*a^5*b^5*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 4*a^12*sin(c/
2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 128*b^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3
/2) - 16*a^3*b^7*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 4*a^10*sin(c/2 + (d
x)/2)(b^2 - a^2)^(3/2) + 4*a^5*b^7*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) +
20*a^7*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 20*a^7*b^5*cos(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2) + 9*a^9*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) +
8*a^2*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*a^2*b^10*sin(c/2 + (d*x)
/2)*(b^2 - a^2)^(1/2) + 104*a^4*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) +
8*a^4*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 18*a^6*b^4*sin(c/2 + (d*x)
/2)*(b^2 - a^2)^(3/2) - 26*a^6*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 1
6*a^8*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 28*a^8*b^4*sin(c/2 + (d*x)
/2)*(b^2 - a^2)^(1/2) - 64*a*b^9*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 2*a
^9*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 5*a^11*b*cos(c/2 + (d*x)/2)*(b^
2 - a^2)^(1/2))/(a^3*b^10*cos(c/2 + (d*x)/2)*80i - a*b^12*cos(c/2 + (d*x)/2
)*64i - a^12*b*sin(c/2 + (d*x)/2)*3i - b^13*sin(c/2 + (d*x)/2)*128i + a^5*b
^8*cos(c/2 + (d*x)/2)*44i - a^7*b^6*cos(c/2 + (d*x)/2)*72i + a^9*b^4*cos(c/
2 + (d*x)/2)*3i + a^11*b^2*cos(c/2 + (d*x)/2)*9i + a^2*b^11*sin(c/2 + (d*x)
/2)*192i + a^4*b^9*sin(c/2 + (d*x)/2)*56i - a^6*b^7*sin(c/2 + (d*x)/2)*172i
+ a^8*b^5*sin(c/2 + (d*x)/2)*34i + a^10*b^3*sin(c/2 + (d*x)/2)*21i))*cos(c
/2 + (d*x)/2)^2*(b^2 - a^2)^(1/2)*2i)/(b^2*d*(a*cos(c/2 + (d*x)/2)^2 + a*si
n(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) - (2*b*log
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(c/2 + (d*x)/2)^2)/(a^2*d*(a*cos

(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) - (2*b^2*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))/(a^3*d*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) + (atan((40*a^5*b^5*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 4*a^12*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 128*b^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 16*a^3*b^7*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 4*a^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 4*a^5*b^7*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 20*a^7*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 20*a^7*b^5*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 9*a^9*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*a^2*b^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 104*a^4*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 8*a^4*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 18*a^6*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 26*a^6*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 16*a^8*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 28*a^8*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 64*a*b^9*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 2*a^9*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 5*a^11*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(a^3*b^10*cos(c/2 + (d*x)/2)*80i - a*b^12*cos(c/2 + (d*x)/2)*64i - a^12*b*sin(c/2 + (d*x)/2)*3i - b^13*sin(c/2 + (d*x)/2)*128i + a^5*b^8*cos(c/2 + (d*x)/2)*44i - a^7*b^6*cos(c/2 + (d*x)/2)*72i + a^9*b^4*cos(c/2 + (d*x)/2)*3i + a^11*b^2*cos(c/2 + (d*x)/2)*9i + a^2*b^11*sin(c/2 + (d*x)/2)*192i + a^4*b^9*sin(c/2 + (d*x)/2)*56i - a^6*b^7*sin(c/2 + (d*x)/2)*172i + a^8*b^5*sin(c/2 + (d*x)/2)*34i + a^10*b^3*sin(c/2 + (d*x)/2)*21i))*sin(c/2 + (d*x)/2)^2*(b^2 - a^2)^(1/2)*4i)/(a^2*d*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) + (atan((40*a^5*b^5*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 4*a^12*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 128*b^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 16*a^3*b^7*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 4*a^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 4*a^5*b^7*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 20*a^7*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 20*a^7*b^5*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 9*a^9*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*a^2*b^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 104*a^4*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 8*a^4*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 18*a^6*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 26*a^6*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 16*a^8*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 28*a^8*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 64*a*b^9*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 2*a^9*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 5*a^11*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(a^3*b^10*cos(c/2 + (d*x)/2)*80i - a*b^12*cos(c/2 + (d*x)/2)*64i - a^12*b*sin(c/2 + (d*x)/2)*3i - b^13*sin(c/2 + (d*x)/2)*128i + a^5*b^8*cos(c/2 + (d*x)/2)*44i - a^7*b^6*cos(c/2 + (d*x)/2)*72i + a^9*b^4*cos(c/2 + (d*x)/2)*3i + a^11*b^2*cos(c/2 + (d*x)/2)*9i + a^2*b^11*sin(c/2 + (d*x)/2)*192i + a^4*b^9*sin(c/2 + (d*x)/2)*56i - a^6*b^7*sin(c/2 + (d*x)/2)*172i + a^8*b^5*sin(c/2 + (d*x)/2)*34i + a^10*b^3*sin(c/2 + (d*x)/2)*21i))*sin(c/2 + (d*x)/2)^2*(b^2 - a^2)^(1/2)*2i)/(b^2*d*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) - (2*a*atan(

(a^3*cos(c/2 + (d*x)/2) - 2*b^3*sin(c/2 + (d*x)/2))/(2*b^3*cos(c/2 + (d*x)/
2) + a^3*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^2)/(b^2*d*(a*cos(c/2 + (d*
x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2
))) - (2*a*atan((a^3*cos(c/2 + (d*x)/2) - 2*b^3*sin(c/2 + (d*x)/2))/(2*b^3*
cos(c/2 + (d*x)/2) + a^3*sin(c/2 + (d*x)/2)))*sin(c/2 + (d*x)/2)^2)/(b^2*d*
(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*s
in(c/2 + (d*x)/2))) - (4*atan((a^3*cos(c/2 + (d*x)/2) - 2*b^3*sin(c/2 + (d*
x)/2))/(2*b^3*cos(c/2 + (d*x)/2) + a^3*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)
/2)*sin(c/2 + (d*x)/2))/(b*d*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)
^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) - (4*b^2*log(sin(c/2 + (d*
x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))/(a^3*d*(a*
cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(
c/2 + (d*x)/2))) + (atan((40*a^5*b^5*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) -
 4*a^12*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 128*b^10*sin(c/2 + (d*x)/2)*
(b^2 - a^2)^(3/2) - 16*a^3*b^7*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 4*a^1
0*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 4*a^5*b^7*cos(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2) + 20*a^7*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 20*a^7*b^5
*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 9*a^9*b^3*cos(c/2 + (d*x)/2)*(b^2 -
 a^2)^(1/2) + 8*a^2*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*a^2*b^10*s
in(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 104*a^4*b^6*sin(c/2 + (d*x)/2)*(b^2 -
 a^2)^(3/2) + 8*a^4*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 18*a^6*b^4*s
in(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 26*a^6*b^6*sin(c/2 + (d*x)/2)*(b^2 -
a^2)^(1/2) - 16*a^8*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 28*a^8*b^4*s
in(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 64*a*b^9*cos(c/2 + (d*x)/2)*(b^2 - a^
2)^(3/2) + 2*a^9*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 5*a^11*b*cos(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2))/(a^3*b^10*cos(c/2 + (d*x)/2)*80i - a*b^12*cos
(c/2 + (d*x)/2)*64i - a^12*b*sin(c/2 + (d*x)/2)*3i - b^13*sin(c/2 + (d*x)/2
)*128i + a^5*b^8*cos(c/2 + (d*x)/2)*44i - a^7*b^6*cos(c/2 + (d*x)/2)*72i +
a^9*b^4*cos(c/2 + (d*x)/2)*3i + a^11*b^2*cos(c/2 + (d*x)/2)*9i + a^2*b^11*s
in(c/2 + (d*x)/2)*192i + a^4*b^9*sin(c/2 + (d*x)/2)*56i - a^6*b^7*sin(c/2 +
 (d*x)/2)*172i + a^8*b^5*sin(c/2 + (d*x)/2)*34i + a^10*b^3*sin(c/2 + (d*x)/
2)*21i))*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a*b*d
*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*
sin(c/2 + (d*x)/2))) + (b*atan((40*a^5*b^5*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(
3/2) - 4*a^12*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 128*b^10*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(3/2) - 16*a^3*b^7*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) -
 4*a^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 4*a^5*b^7*cos(c/2 + (d*x)/2)
*(b^2 - a^2)^(1/2) + 20*a^7*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 20*a
^7*b^5*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 9*a^9*b^3*cos(c/2 + (d*x)/2)*
(b^2 - a^2)^(1/2) + 8*a^2*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*a^2*
b^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 104*a^4*b^6*sin(c/2 + (d*x)/2)*
(b^2 - a^2)^(3/2) + 8*a^4*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 18*a^6
*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 26*a^6*b^6*sin(c/2 + (d*x)/2)*(
b^2 - a^2)^(1/2) - 16*a^8*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 28*a^8
*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 64*a*b^9*cos(c/2 + (d*x)/2)*(b^

```
2 - a^2)^(3/2) + 2*a^9*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 5*a^11*b*co
s(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(a^3*b^10*cos(c/2 + (d*x)/2)*80i - a*b^
12*cos(c/2 + (d*x)/2)*64i - a^12*b*sin(c/2 + (d*x)/2)*3i - b^13*sin(c/2 + (
d*x)/2)*128i + a^5*b^8*cos(c/2 + (d*x)/2)*44i - a^7*b^6*cos(c/2 + (d*x)/2)*
72i + a^9*b^4*cos(c/2 + (d*x)/2)*3i + a^11*b^2*cos(c/2 + (d*x)/2)*9i + a^2*
b^11*sin(c/2 + (d*x)/2)*192i + a^4*b^9*sin(c/2 + (d*x)/2)*56i - a^6*b^7*sin
(c/2 + (d*x)/2)*172i + a^8*b^5*sin(c/2 + (d*x)/2)*34i + a^10*b^3*sin(c/2 +
(d*x)/2)*21i))*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i)/
(a^3*d*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

3.1132 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=158

$$\frac{2b\cot(c+dx)}{a^3d}-\frac{\cos(c+dx)}{2a^2d\left(1-\cos^2(c+dx)\right)}+\frac{6b\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}+\frac{3\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}$$

```
[Out] 3/2*(a^2-2*b^2)*arctanh(cos(d*x+c))/a^4/d-1/2*cos(d*x+c)/a^2/d/(1-cos(d*x+c
)^2)+2*b*cot(d*x+c)/a^3/d-(a^2-b^2)*cos(d*x+c)/a^3/d/(a+b*sin(d*x+c))+6*b*a
rctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^4/d
```

Rubi [A] time = 0.45, antiderivative size = 180, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2890, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{6b\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}-\frac{\left(a^2-3b^2\right)\cot(c+dx)}{a^3bd}+\frac{3\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (6*b*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*d) + (3*(a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) - ((a^2 - 3*b^2)*Co
t[c + d*x])/(a^3*b*d) + ((2*a^2 - 3*b^2)*Cot[c + d*x])/(2*a^2*b*d*(a + b*Si
n[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)\left(2\left(a^2-3b^2\right)-ab\sin(c}{a+b\sin(c}}{2a^2}$$

$$= -\frac{\left(a^2-3b^2\right)\cot(c+dx)}{a^3bd} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))}$$

$$= -\frac{\left(a^2-3b^2\right)\cot(c+dx)}{a^3bd} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))}$$

$$= \frac{3\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} - \frac{\left(a^2-3b^2\right)\cot(c+dx)}{a^3bd} + \frac{\left(2a^2-3b^2\right)\cot(}{2a^2bd(a+b\sin(}$$

$$= \frac{3\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} - \frac{\left(a^2-3b^2\right)\cot(c+dx)}{a^3bd} + \frac{\left(2a^2-3b^2\right)\cot(}{2a^2bd(a+b\sin(}$$

$$= \frac{6b\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} + \frac{3\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} - \frac{\left(a^2\right.}{}$$

Mathematica [A] time = 4.30, size = 191, normalized size = 1.21

$$\frac{48b\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) - 12\left(a^2-2b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 12\left(a^2-2b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (48*b*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 8*
a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 + 12*(a^2 - 2*b^2)*Log[Cos[(c
 + d*x)/2]] - 12*(a^2 - 2*b^2)*Log[Sin[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]
^2 + (8*a*(-a^2 + b^2)*Cos[c + d*x])/(a + b*Sin[c + d*x]) - 8*a*b*Tan[(c +
d*x)/2])/(8*a^4*d)
```

fricas [B] time = 0.99, size = 804, normalized size = 5.09

$$\left[-\frac{6\,a^{2}b\cos\left(dx+c\right)\sin\left(dx+c\right)+4\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{3}-6\left(ab\cos\left(dx+c\right)^{2}-ab+\left(b^{2}\cos\left(dx+c\right)^{2}-b^{2}\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/4*(6*a^2*b*cos(d*x + c)*sin(d*x + c) + 4*(a^3 - 3*a*b^2)*cos(d*x + c)^3
 - 6*(a*b*cos(d*x + c)^2 - a*b + (b^2*cos(d*x + c)^2 - b^2)*sin(d*x + c))*s
qrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(a^3 - 2*a*b^
2)*cos(d*x + c) + 3*(a^3 - 2*a*b^2 - (a^3 - 2*a*b^2)*cos(d*x + c)^2 + (a^2*
b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x +
 c) + 1/2) - 3*(a^3 - 2*a*b^2 - (a^3 - 2*a*b^2)*cos(d*x + c)^2 + (a^2*b - 2
*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c)
+ 1/2))/(a^5*d*cos(d*x + c)^2 - a^5*d + (a^4*b*d*cos(d*x + c)^2 - a^4*b*d)*
sin(d*x + c)), -1/4*(6*a^2*b*cos(d*x + c)*sin(d*x + c) + 4*(a^3 - 3*a*b^2)*
cos(d*x + c)^3 + 12*(a*b*cos(d*x + c)^2 - a*b + (b^2*cos(d*x + c)^2 - b^2)*
sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)
*cos(d*x + c))) - 6*(a^3 - 2*a*b^2)*cos(d*x + c) + 3*(a^3 - 2*a*b^2 - (a^3
- 2*a*b^2)*cos(d*x + c)^2 + (a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2
)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(a^3 - 2*a*b^2 - (a^3 - 2*a
*b^2)*cos(d*x + c)^2 + (a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*sin
(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(a^5*d*cos(d*x + c)^2 - a^5*d + (a
^4*b*d*cos(d*x + c)^2 - a^4*b*d)*sin(d*x + c))]
```

giac [A] time = 0.23, size = 275, normalized size = 1.74

$$-\frac{\frac{12\left(a^{2}-2\,b^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{4}}-\frac{48\left(a^{2}b-b^{3}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,a^{4}}-\frac{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{4}}+\frac{16}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(12*(a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - 48*(a^2*b - b^3
)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c)
```

```
 + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (a^2*tan(1/2*d*x + 1/2*c)^2
 - 8*a*b*tan(1/2*d*x + 1/2*c))/a^4 + 16*(a^2*b*tan(1/2*d*x + 1/2*c) - b^3*t
an(1/2*d*x + 1/2*c) + a^3 - a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2
*d*x + 1/2*c) + a)*a^4) - (18*a^2*tan(1/2*d*x + 1/2*c)^2 - 36*b^2*tan(1/2*d
*x + 1/2*c)^2 + 8*a*b*tan(1/2*d*x + 1/2*c) - a^2)/(a^4*tan(1/2*d*x + 1/2*c)
^2))/d
```

maple [B] time = 0.87, size = 339, normalized size = 2.15

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2d}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{d\,a^3}-\frac{1}{8a^2d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{d\,a^4}+\frac{b}{d\,a^3\tan\left(\frac{dx}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^3*tan(1/2*d*x+1/2*c)*b-1/8/a^2/d/tan(1
/2*d*x+1/2*c)^2-3/2/d/a^2*ln(tan(1/2*d*x+1/2*c))+3/d/a^4*ln(tan(1/2*d*x+1/2
*c))*b^2+1/d*b/a^3/tan(1/2*d*x+1/2*c)-2/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*t
an(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d*b^3/a^4/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-2/d/a/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)+2/d*b^2/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)+6/d/a^4*b*(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.73, size = 675, normalized size = 4.27

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8a^2d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{17a^2}{2}-16b^2\right)+\frac{a^2}{2}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a^2b-2b^3)}{a}-3ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(4a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+8ba^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d}-\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (tan(c/2 + (d*x)/2)^2*((17*a^2)/2 - 16*b^2
) + a^2/2 + (4*tan(c/2 + (d*x)/2)^3*(a^2*b - 2*b^3))/a - 3*a*b*tan(c/2 + (d
*x)/2))/(d*(4*a^4*tan(c/2 + (d*x)/2)^2 + 4*a^4*tan(c/2 + (d*x)/2)^4 + 8*a^3
*b*tan(c/2 + (d*x)/2)^3)) - (b*tan(c/2 + (d*x)/2))/(a^3*d) - (log(tan(c/2 +
 (d*x)/2))*(3*a^2 - 6*b^2))/(2*a^4*d) - (6*b*atanh((72*b^4*(b^2 - a^2)^(1/2
))/(18*a^4*b + 72*b^5 - 90*a^2*b^3 - 216*a*b^4*tan(c/2 + (d*x)/2) + 72*a^3*
b^2*tan(c/2 + (d*x)/2) + (144*b^6*tan(c/2 + (d*x)/2))/a) - (54*b^2*(b^2 - a
^2)^(1/2))/(18*a^2*b - 90*b^3 + (72*b^5)/a^2 + 72*a*b^2*tan(c/2 + (d*x)/2)
- (216*b^4*tan(c/2 + (d*x)/2))/a + (144*b^6*tan(c/2 + (d*x)/2))/a^3) + (18*
b*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(18*a*b - (90*b^3)/a + (72*b^5)/a^3
 + 72*b^2*tan(c/2 + (d*x)/2) - (216*b^4*tan(c/2 + (d*x)/2))/a^2 + (144*b^6*
tan(c/2 + (d*x)/2))/a^4) - (144*b^3*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(
18*a^3*b - 90*a*b^3 + (72*b^5)/a - 216*b^4*tan(c/2 + (d*x)/2) + 72*a^2*b^2*
tan(c/2 + (d*x)/2) + (144*b^6*tan(c/2 + (d*x)/2))/a^2) + (144*b^5*tan(c/2 +
 (d*x)/2)*(b^2 - a^2)^(1/2))/(72*a*b^5 + 18*a^5*b - 90*a^3*b^3 + 144*b^6*ta
n(c/2 + (d*x)/2) - 216*a^2*b^4*tan(c/2 + (d*x)/2) + 72*a^4*b^2*tan(c/2 + (d
*x)/2)))*(b^2 - a^2)^(1/2))/(a^4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1133 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=238

$$\frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d}-\frac{(a^2-2b^2)\text{co}}{}$$

```
[Out] -b*(3*a^2-4*b^2)*arctanh(cos(d*x+c))/a^5/d+1/3*(7*a^2-12*b^2)*cot(d*x+c)/a^
4/d-(a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/b/d+1/3*(3*a^2-4*b^2)*cot(d*x+c)*
csc(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d/(a+b*si
n(d*x+c))+2*(a^4-5*a^2*b^2+4*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)
^(1/2))/a^5/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2724, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2(-5a^2b^2+a^4+4b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}+\frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d}-\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}-\frac{(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^5*Sqrt[a^2 - b^2]*d) - (b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(a^
5*d) + ((7*a^2 - 12*b^2)*Cot[c + d*x])/(3*a^4*d) - ((a^2 - 2*b^2)*Cot[c + d
*x]*Csc[c + d*x])/(a^3*b*d) + ((3*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(
3*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a +
 b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m +
 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 -
b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 +
b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*
Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
m, -1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))} + \frac{\int \csc^3(c+dx)\left(6\left(a^2-2b\right.\right.}{}$$

$$= -\frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\cot(c}{3ad(a}$$

$$= \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{(3a^2-4b^2)\cot(c}{3a^2bd(a+b\,s}$$

$$= \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{(3a^2-4b^2)\cot(c}{3a^2bd(a+b\,s}$$

$$= -\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot(c}{a^3}$$

$$= -\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot(c}{a^3}$$

$$= \frac{2(a^4-5a^2b^2+4b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} - \frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d} +$$

Mathematica [A] time = 6.21, size = 403, normalized size = 1.69

$$\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d} + \frac{(3a}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] +
 a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) + ((4*a^2*C
```

```
os[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + (b*
Csc[(c + d*x)/2]^2)/(4*a^3*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a
^2*d) + ((-3*a^2*b + 4*b^3)*Log[Cos[(c + d*x)/2]])/(a^5*d) + ((3*a^2*b - 4*
b^3)*Log[Sin[(c + d*x)/2]])/(a^5*d) - (b*Sec[(c + d*x)/2]^2)/(4*a^3*d) + (S
ec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2]))/(6*a^4*
d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(a^4*d*(a + b*Sin[c + d*x])) +
 (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^2*d)
```

fricas [B] time = 0.76, size = 1149, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/6*(4*(2*a^4 - 3*a^2*b^2)*cos(d*x + c)^3 + 3*((a^2*b - 4*b^3)*cos(d*x +
c)^4 + a^2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*cos(d*x + c)^2 + (a^3 - 4*a*b^2 -
(a^3 - 4*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2
- b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*
sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*
b*sin(d*x + c) - a^2 - b^2)) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c) + 3*((3*a^2
*b^2 - 4*b^4)*cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*co
s(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin
(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*cos(d*x + c
)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^2 + (3*a^3*b -
 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*cos(d*x + c)^3 - 3*(3*a^3*b - 4*a*
b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*
x + c)^2 + a^5*b*d - (a^6*d*cos(d*x + c)^2 - a^6*d)*sin(d*x + c)), -1/6*(4*
(2*a^4 - 3*a^2*b^2)*cos(d*x + c)^3 + 6*((a^2*b - 4*b^3)*cos(d*x + c)^4 + a^
2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*cos(d*x + c)^2 + (a^3 - 4*a*b^2 - (a^3 - 4*
a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c
) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c) +
 3*((3*a^2*b^2 - 4*b^4)*cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 -
 4*b^4)*cos(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x +
 c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*c
os(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^2 +
(3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(
-1/2*cos(d*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*cos(d*x + c)^3 - 3*(3*a^
3*b - 4*a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*
b*d*cos(d*x + c)^2 + a^5*b*d - (a^6*d*cos(d*x + c)^2 - a^6*d)*sin(d*x + c))
]
```

giac [A] time = 0.25, size = 356, normalized size = 1.50

$$\frac{\frac{24\left(3a^{2}b-4b^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{5}}+\frac{48\left(a^{4}-5a^{2}b^{2}+4b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,a^{5}}+\frac{a^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-6a^{3}b\tan\left(\frac{1}{2}dx\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(3*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 + 48*(a^4 - 5
*a^2*b^2 + 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(
1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) + (a^4*tan(1/
2*d*x + 1/2*c)^3 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^4*tan(1/2*d*x + 1/
2*c) + 36*a^2*b^2*tan(1/2*d*x + 1/2*c))/a^6 + 48*(a^2*b^2*tan(1/2*d*x + 1/2
*c) - b^4*tan(1/2*d*x + 1/2*c) + a^3*b - a*b^3)/((a*tan(1/2*d*x + 1/2*c)^2
+ 2*b*tan(1/2*d*x + 1/2*c) + a)*a^5) - (132*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
176*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a*b^2*t
an(1/2*d*x + 1/2*c)^2 - 6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^5*tan(1/2*d*
x + 1/2*c)^3))/d
```

maple [B] time = 0.79, size = 527, normalized size = 2.21

$$\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^{2}}-\frac{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{4d\,a^{3}}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{2}}+\frac{3b^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{4}}-\frac{1}{24a^{2}d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{5}{8d\,a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^3*tan(1/2*d*x+1/2*c)^2*b-5/8/d/a^2*
tan(1/2*d*x+1/2*c)+3/2/d/a^4*b^2*tan(1/2*d*x+1/2*c)-1/24/a^2/d/tan(1/2*d*x+
1/2*c)^3+5/8/d/a^2/tan(1/2*d*x+1/2*c)-3/2/d/a^4/tan(1/2*d*x+1/2*c)*b^2+1/4/
d/a^3*b/tan(1/2*d*x+1/2*c)^2+3/d/a^3*b*ln(tan(1/2*d*x+1/2*c))-4/d/a^5*b^3*l
n(tan(1/2*d*x+1/2*c))+2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)*b^2*tan(1/2*d*x+1/2*c)-2/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)*tan(1/2*d*x+1/2*c)*b^4+2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)*b-2/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a
)*b^3+2/d/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^
2)^(1/2))-10/d/a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))*b^2+8/d/a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/
2*c)+2*b)/(a^2-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.80, size = 973, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(28*a^2*b - 40*b^3) - tan(c/2 + (d*x)/2)^2*(8*a*b^2 -
 (14*a^3)/3) - a^3/3 + (4*a^2*b*tan(c/2 + (d*x)/2))/3 + (tan(c/2 + (d*x)/2)
^4*(5*a^4 - 16*b^4 + 4*a^2*b^2))/a)/(d*(8*a^5*tan(c/2 + (d*x)/2)^3 + 8*a^5*
tan(c/2 + (d*x)/2)^5 + 16*a^4*b*tan(c/2 + (d*x)/2)^4)) + tan(c/2 + (d*x)/2)
^3/(24*a^2*d) - (tan(c/2 + (d*x)/2)*((16*a^2 + 32*b^2)/(64*a^4) + 3/(8*a^2)
 - (2*b^2)/a^4))/d - (b*tan(c/2 + (d*x)/2)^2)/(4*a^3*d) + (b*log(tan(c/2 +
(d*x)/2))*(3*a^2 - 4*b^2))/(a^5*d) + (atan((((b^2 - a^2)^(1/2)*(a^2 - 4*b^2
)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(5*a^7*b +
 16*a^3*b^5 - 20*a^5*b^3))/a^7 + ((2*a^2*b - (2*tan(c/2 + (d*x)/2)*(3*a^10
- 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/a^5)*1i)/a^5 + ((b^2 -
a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))/a^8 + (2*tan(c/
2 + (d*x)/2)*(5*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 - ((2*a^2*b - (2*tan(
c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/
a^5)*1i)/a^5)/((4*(3*a^6*b - 16*b^7 + 32*a^2*b^5 - 19*a^4*b^3))/a^8 + (4*ta
n(c/2 + (d*x)/2)*(2*a^6 - 16*b^6 + 28*a^2*b^4 - 14*a^4*b^2))/a^7 - ((b^2 -
a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))/a^8 + (2*tan(c/
2 + (d*x)/2)*(5*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 + ((2*a^2*b - (2*tan(
c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/
a^5))/a^5 + ((b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b
^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(5*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 -
 ((2*a^2*b - (2*tan(c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(
1/2)*(a^2 - 4*b^2))/a^5))/a^5))*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*2i)/(a^5*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1134 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=292

$$\frac{\left(4a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))}-\frac{b\left(11a^2-15b^2\right)\cot(c+dx)}{3a^5d}+\frac{\left(13a^2-20b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d}-\frac{\left(3a^2-\right.}{}$$

```
[Out] -1/8*(3*a^4-36*a^2*b^2+40*b^4)*arctanh(cos(d*x+c))/a^6/d-1/3*b*(11*a^2-15*b
^2)*cot(d*x+c)/a^5/d+1/8*(13*a^2-20*b^2)*cot(d*x+c)*csc(d*x+c)/a^4/d-1/3*(3
*a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)^2/a^3/b/d+1/4*(4*a^2-5*b^2)*cot(d*x+c)*cs
c(d*x+c)^2/a^2/b/d/(a+b*sin(d*x+c))-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d/(a+b*si
n(d*x+c))-2*b*(2*a^4-7*a^2*b^2+5*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-
b^2)^(1/2))/a^6/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 1.05, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2890, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b\left(-7a^2b^2+2a^4+5b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d\sqrt{a^2-b^2}}-\frac{b\left(11a^2-15b^2\right)\cot(c+dx)}{3a^5d}-\frac{\left(-36a^2b^2+3a^4+40b^4\right)\tanh^{-1}(c}{8a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b*(2*a^4 - 7*a^2*b^2 + 5*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/(a^6*Sqrt[a^2 - b^2]*d) - ((3*a^4 - 36*a^2*b^2 + 40*b^4)*ArcTanh[C
os[c + d*x]])/(8*a^6*d) - (b*(11*a^2 - 15*b^2)*Cot[c + d*x])/(3*a^5*d) + ((
13*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*d) - ((3*a^2 - 5*b^2)*Co
t[c + d*x]*Csc[c + d*x]^2)/(3*a^3*b*d) + ((4*a^2 - 5*b^2)*Cot[c + d*x]*Csc[
c + d*x]^2)/(4*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^3
)/(4*a*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\left(4a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^4(c+dx)(4(3}{}}{}$$

$$= -\frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{3a^3bd} + \frac{\left(4a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))} -$$

$$= \frac{\left(13a^2-20b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{3a^3bd} +$$

$$= -\frac{b\left(11a^2-15b^2\right)\cot(c+dx)}{3a^5d} + \frac{\left(13a^2-20b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{\left(3a^2-5\right.}{}$$

$$= -\frac{b\left(11a^2-15b^2\right)\cot(c+dx)}{3a^5d} + \frac{\left(13a^2-20b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{\left(3a^2-5\right.}{}$$

$$= -\frac{\left(3a^4-36a^2b^2+40b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{b\left(11a^2-15b^2\right)\cot(c+dx)}{3a^5d} + \frac{\left(1\right.}{}$$

$$= -\frac{\left(3a^4-36a^2b^2+40b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{b\left(11a^2-15b^2\right)\cot(c+dx)}{3a^5d} + \frac{\left(1\right.}{}$$

$$= -\frac{2b\left(2a^4-7a^2b^2+5b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d} - \frac{\left(3a^4-36a^2b^2+40b^4\right)\tanh^{-1}}{8a^6d}$$

Mathematica [A] time = 6.29, size = 496, normalized size = 1.70

$$\frac{b\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{12a^3d} - \frac{b\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{12a^3d} - \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64a^2d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64a^2d} - \frac{2\csc}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b*(2*a^4 - 7*a^2*b^2 + 5*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)
/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) - (2*(
2*a^2*b*Cos[(c + d*x)/2] - 3*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(3*a^5
*d) + ((5*a^2 - 12*b^2)*Csc[(c + d*x)/2]^2)/(32*a^4*d) + (b*Cot[(c + d*x)/2
]*Csc[(c + d*x)/2]^2)/(12*a^3*d) - Csc[(c + d*x)/2]^4/(64*a^2*d) + ((-3*a^4
 + 36*a^2*b^2 - 40*b^4)*Log[Cos[(c + d*x)/2]])/(8*a^6*d) + ((3*a^4 - 36*a^2
*b^2 + 40*b^4)*Log[Sin[(c + d*x)/2]])/(8*a^6*d) + ((-5*a^2 + 12*b^2)*Sec[(c
 + d*x)/2]^2)/(32*a^4*d) + Sec[(c + d*x)/2]^4/(64*a^2*d) + (2*Sec[(c + d*x)
/2]*(2*a^2*b*Sin[(c + d*x)/2] - 3*b^3*Sin[(c + d*x)/2]))/(3*a^5*d) + (-(a^2
*b^2*Cos[c + d*x]) + b^4*Cos[c + d*x])/(a^5*d*(a + b*Sin[c + d*x])) - (b*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(12*a^3*d)
```

fricas [B] time = 1.15, size = 1578, normalized size = 5.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/48*(16*(11*a^3*b^2 - 15*a*b^4)*cos(d*x + c)^5 + 2*(15*a^5 - 196*a^3*b^2
 + 240*a*b^4)*cos(d*x + c)^3 + 24*((2*a^3*b - 5*a*b^3)*cos(d*x + c)^4 + 2*a
^3*b - 5*a*b^3 - 2*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^2 + ((2*a^2*b^2 - 5*b^4
)*cos(d*x + c)^4 + 2*a^2*b^2 - 5*b^4 - 2*(2*a^2*b^2 - 5*b^4)*cos(d*x + c)^2
)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c)
)*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))
- 6*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c) + 3*(3*a^5 - 36*a^3*b^2 +
40*a*b^4 + (3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 36*a
^3*b^2 + 40*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - 36*a^2*b^3 + 40*b^5 + (3*a^4
*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 36*a^2*b^3 + 40*b^5
)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(3*a^5 - 36
*a^3*b^2 + 40*a*b^4 + (3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^4 - 2*(3
*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - 36*a^2*b^3 + 40*b
^5 + (3*a^4*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 36*a^2*b
^3 + 40*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2
*((49*a^4*b - 60*a^2*b^3)*cos(d*x + c)^3 - 3*(13*a^4*b - 20*a^2*b^3)*cos(d*
x + c))*sin(d*x + c))/(a^7*d*cos(d*x + c)^4 - 2*a^7*d*cos(d*x + c)^2 + a^7*
d + (a^6*b*d*cos(d*x + c)^4 - 2*a^6*b*d*cos(d*x + c)^2 + a^6*b*d)*sin(d*x +
 c)), -1/48*(16*(11*a^3*b^2 - 15*a*b^4)*cos(d*x + c)^5 + 2*(15*a^5 - 196*a^
3*b^2 + 240*a*b^4)*cos(d*x + c)^3 - 48*((2*a^3*b - 5*a*b^3)*cos(d*x + c)^4
```

+ 2*a^3*b - 5*a*b^3 - 2*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^2 + ((2*a^2*b^2 - 5*b^4)*cos(d*x + c)^4 + 2*a^2*b^2 - 5*b^4 - 2*(2*a^2*b^2 - 5*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c) + 3*(3*a^5 - 36*a^3*b^2 + 40*a*b^4 + (3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - 36*a^2*b^3 + 40*b^5 + (3*a^4*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(3*a^5 - 36*a^3*b^2 + 40*a*b^4 + (3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - 36*a^2*b^3 + 40*b^5 + (3*a^4*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((49*a^4*b - 60*a^2*b^3)*cos(d*x + c)^3 - 3*(13*a^4*b - 20*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d*cos(d*x + c)^4 - 2*a^7*d*cos(d*x + c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4 - 2*a^6*b*d*cos(d*x + c)^2 + a^6*b*d)*sin(d*x + c))]

giac [A] time = 0.28, size = 461, normalized size = 1.58

$$\frac{\frac{24\left(3a^4-36a^2b^2+40b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^6}-\frac{384\left(2a^4b-7a^2b^3+5b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^6}-\frac{384\left(a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/192*(24*(3*a^4 - 36*a^2*b^2 + 40*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 - 384*(2*a^4*b - 7*a^2*b^3 + 5*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) - 384*(a^2*b^3*tan(1/2*d*x + 1/2*c) - b^5*tan(1/2*d*x + 1/2*c) + a^3*b^2 - a*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^6) + (3*a^6*tan(1/2*d*x + 1/2*c)^4 - 16*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 24*a^6*tan(1/2*d*x + 1/2*c)^2 + 72*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*a^5*b*tan(1/2*d*x + 1/2*c) - 384*a^3*b^3*tan(1/2*d*x + 1/2*c))/a^8 - (150*a^4*tan(1/2*d*x + 1/2*c)^4 - 1800*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 2000*b^4*tan(1/2*d*x + 1/2*c)^4 + 240*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 384*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 24*a^4*tan(1/2*d*x + 1/2*c)^2 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 16*a^3*b*tan(1/2*d*x + 1/2*c) + 3*a^4)/(a^6*tan(1/2*d*x + 1/2*c)^4))/d

maple [B] time = 0.84, size = 634, normalized size = 2.17

$$\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a^2d}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{12d\,a^3}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2d}+\frac{3b^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{4d\,a^3}-\frac{2b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^5}-\frac{}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/64/d/a^2*tan(1/2*d*x+1/2*c)^4-1/12/d/a^3*tan(1/2*d*x+1/2*c)^3*b-1/8/d/a^2
*tan(1/2*d*x+1/2*c)^2+3/8/d/a^4*b^2*tan(1/2*d*x+1/2*c)^2+5/4/d/a^3*tan(1/2*
d*x+1/2*c)*b-2/d/a^5*b^3*tan(1/2*d*x+1/2*c)-1/64/a^2/d/tan(1/2*d*x+1/2*c)^4
+1/8/a^2/d/tan(1/2*d*x+1/2*c)^2-3/8/d/a^4/tan(1/2*d*x+1/2*c)^2*b^2+3/8/d/a^
2*ln(tan(1/2*d*x+1/2*c))-9/2/d/a^4*ln(tan(1/2*d*x+1/2*c))*b^2+5/d/a^6*ln(ta
n(1/2*d*x+1/2*c))*b^4+1/12/d/a^3*b/tan(1/2*d*x+1/2*c)^3-5/4/d*b/a^3/tan(1/2
*d*x+1/2*c)+2/d*b^3/a^5/tan(1/2*d*x+1/2*c)-2/d*b^3/a^4/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/a^6*b^5/(tan(1/2*d*x+1
/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-2/d*b^2/a^3/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+2/d/a^5*b^4/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)-4/d/a^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+14/d/a^4*b^3/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-10/d/a^6*b^5/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.83, size = 1158, normalized size = 3.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^4/(64*a^2*d) + (tan(c/2 + (d*x)/2)^4*(2*a^4 + 96*b^4 - 7
8*a^2*b^2) - a^4/4 + tan(c/2 + (d*x)/2)^2*((7*a^4)/4 - (10*a^2*b^2)/3) + ta
n(c/2 + (d*x)/2)^3*(20*a*b^3 - (44*a^3*b)/3) - (4*tan(c/2 + (d*x)/2)^5*(5*a
^4*b - 8*b^5))/a + (5*a^3*b*tan(c/2 + (d*x)/2))/6)/(d*(16*a^6*tan(c/2 + (d*
x)/2)^4 + 16*a^6*tan(c/2 + (d*x)/2)^6 + 32*a^5*b*tan(c/2 + (d*x)/2)^5)) - (
tan(c/2 + (d*x)/2)^2*((32*a^2 + 64*b^2)/(512*a^4) + 1/(16*a^2) - b^2/(2*a^4
)))/d + (tan(c/2 + (d*x)/2)*((b*(32*a^2 + 64*b^2))/(64*a^5) - b/(4*a^3) + (
4*b*((32*a^2 + 64*b^2)/(256*a^4) + 1/(8*a^2) - b^2/a^4))/a))/d - (b*tan(c/2
 + (d*x)/2)^3)/(12*a^3*d) + (log(tan(c/2 + (d*x)/2))*(3*a^4 + 40*b^4 - 36*a
^2*b^2))/(8*a^6*d) - (b*atan(((b*(b^2 - a^2)^(1/2)*(2*a^2 - 5*b^2)*((tan(c/
2 + (d*x)/2)*(3*a^10 - 160*a^4*b^6 + 224*a^6*b^4 - 74*a^8*b^2))/(4*a^9) - (
19*a^10*b + 80*a^6*b^5 - 92*a^8*b^3)/(4*a^10) + (b*(2*a^2*b - (tan(c/2 + (d
*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(b^2 - a^2)^(1/2)*(2*a^2 - 5*b^2))
/a^6)*1i)/a^6 - (b*(b^2 - a^2)^(1/2)*(2*a^2 - 5*b^2)*((19*a^10*b + 80*a^6*b
^5 - 92*a^8*b^3)/(4*a^10) - (tan(c/2 + (d*x)/2)*(3*a^10 - 160*a^4*b^6 + 224
*a^6*b^4 - 74*a^8*b^2))/(4*a^9) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^1
2 - 32*a^10*b^2))/(4*a^9))*(b^2 - a^2)^(1/2)*(2*a^2 - 5*b^2))/a^6)*1i)/a^6)
/((6*a^8*b + 200*b^9 - 460*a^2*b^7 + 347*a^4*b^5 - 93*a^6*b^3)/(2*a^10) + (
tan(c/2 + (d*x)/2)*(200*b^8 - 410*a^2*b^6 + 262*a^4*b^4 - 52*a^6*b^2))/(2*a
^9) + (b*(b^2 - a^2)^(1/2)*(2*a^2 - 5*b^2)*((tan(c/2 + (d*x)/2)*(3*a^10 - 1
60*a^4*b^6 + 224*a^6*b^4 - 74*a^8*b^2))/(4*a^9) - (19*a^10*b + 80*a^6*b^5 -
 92*a^8*b^3)/(4*a^10) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^1
0*b^2))/(4*a^9))*(b^2 - a^2)^(1/2)*(2*a^2 - 5*b^2))/a^6))/a^6 + (b*(b^2 - a
^2)^(1/2)*(2*a^2 - 5*b^2)*((19*a^10*b + 80*a^6*b^5 - 92*a^8*b^3)/(4*a^10) -
 (tan(c/2 + (d*x)/2)*(3*a^10 - 160*a^4*b^6 + 224*a^6*b^4 - 74*a^8*b^2))/(4*
a^9) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*
(b^2 - a^2)^(1/2)*(2*a^2 - 5*b^2))/a^6))/a^6))*(b^2 - a^2)^(1/2)*(2*a^2 - 5
*b^2)*2i)/(a^6*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1135 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=331

$$\frac{(7a^2-2b^2)\sin^4(c+dx)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}-\frac{(a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}+\frac{a(30a^2-13b^2)\cos(c+dx)}{2b^6d}-\frac{3(20a^2$$

```
[Out] 3/8*(40*a^4-24*a^2*b^2+b^4)*x/b^7+1/2*a*(30*a^2-13*b^2)*cos(d*x+c)/b^6/d-3/
8*(20*a^2-7*b^2)*cos(d*x+c)*sin(d*x+c)/b^5/d+1/2*(10*a^2-3*b^2)*cos(d*x+c)*
sin(d*x+c)^2/a/b^4/d-1/4*(15*a^2-4*b^2)*cos(d*x+c)*sin(d*x+c)^3/a^2/b^3/d-1
/2*(a^2-b^2)*cos(d*x+c)*sin(d*x+c)^4/a/b^2/d/(a+b*sin(d*x+c))^2+1/2*(7*a^2-
2*b^2)*cos(d*x+c)*sin(d*x+c)^4/a^2/b^2/d/(a+b*sin(d*x+c))-3*a*(10*a^4-11*a^
2*b^2+2*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^7/d/(a^2-b^
2)^(1/2)
```

Rubi [A] time = 1.01, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2891, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{a(30a^2-13b^2)\cos(c+dx)}{2b^6d}-\frac{3a(-11a^2b^2+10a^4+2b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d\sqrt{a^2-b^2}}+\frac{(7a^2-2b^2)\sin^4(c+dx)\cos(c}{2a^2b^2d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*(40*a^4 - 24*a^2*b^2 + b^4)*x)/(8*b^7) - (3*a*(10*a^4 - 11*a^2*b^2 + 2*b
^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*Sqrt[a^2 - b^2]*
d) + (a*(30*a^2 - 13*b^2)*Cos[c + d*x])/(2*b^6*d) - (3*(20*a^2 - 7*b^2)*Cos
[c + d*x]*Sin[c + d*x])/(8*b^5*d) + ((10*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c +
d*x]^2)/(2*a*b^4*d) - ((15*a^2 - 4*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(4*a^2
*b^3*d) - ((a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(2*a*b^2*d*(a + b*Sin[c
 + d*x])^2) + ((7*a^2 - 2*b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(2*a^2*b^2*d*(a
 + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
```

```
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{\left(7a^2-2b^2\right)\cos(c+dx)\sin^4(c+dx)}{2a^2b^2d(a+b\sin(c+dx))} -$$

$$= -\frac{\left(15a^2-4b^2\right)\cos(c+dx)\sin^3(c+dx)}{4a^2b^3d} - \frac{\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}$$

$$= \frac{\left(10a^2-3b^2\right)\cos(c+dx)\sin^2(c+dx)}{2ab^4d} - \frac{\left(15a^2-4b^2\right)\cos(c+dx)\sin^3(c+dx)}{4a^2b^3d}$$

$$= -\frac{3\left(20a^2-7b^2\right)\cos(c+dx)\sin(c+dx)}{8b^5d} + \frac{\left(10a^2-3b^2\right)\cos(c+dx)\sin^2(c+}{2ab^4d}$$

$$= \frac{a\left(30a^2-13b^2\right)\cos(c+dx)}{2b^6d} - \frac{3\left(20a^2-7b^2\right)\cos(c+dx)\sin(c+dx)}{8b^5d} + \frac{(10a^2}{}$$

$$= \frac{3\left(40a^4-24a^2b^2+b^4\right)x}{8b^7} + \frac{a\left(30a^2-13b^2\right)\cos(c+dx)}{2b^6d} - \frac{3\left(20a^2-7b^2\right)\cos}{8b}$$

$$= \frac{3\left(40a^4-24a^2b^2+b^4\right)x}{8b^7} + \frac{a\left(30a^2-13b^2\right)\cos(c+dx)}{2b^6d} - \frac{3\left(20a^2-7b^2\right)\cos}{8b}$$

$$= \frac{3\left(40a^4-24a^2b^2+b^4\right)x}{8b^7} + \frac{a\left(30a^2-13b^2\right)\cos(c+dx)}{2b^6d} - \frac{3\left(20a^2-7b^2\right)\cos}{8b}$$

$$= \frac{3\left(40a^4-24a^2b^2+b^4\right)x}{8b^7} - \frac{3a\left(10a^4-11a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7\sqrt{a^2-b^2}\,d} -$$

Mathematica [B] time = 10.56, size = 1250, normalized size = 3.78

$$-\frac{-\frac{6\left(-8(c+dx)+\frac{2a\left(8a^4-20b^2a^2+15b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{3b\left(4a^4-7b^2a^2+2b^4\right)\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\frac{ab\left(4a^2-3b^2\right)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))^2}\right)}{b^3}+\frac{6\left(\frac{6ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\cdots\right)}{(a-\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]

[Out] -1/256*((-6*(-8*(c + d*x) + (2*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(4*a^2 - 3*b
^2)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x])^2) - (3*b*(4*a^4 -
7*a^2*b^2 + 2*b^4)*Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x]))
))/b^3 + (6*((6*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[
a^2 - b^2] + (Cos[c + d*x]*(a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Sin[c + d*x])
)/(a + b*Sin[c + d*x])^2))/((a - b)^2*(a + b)^2) + (2*(-24*(-8*a^2 + b^2)*(
c + d*x) - (6*a*(64*a^6 - 168*a^4*b^2 + 140*a^2*b^4 - 35*b^6)*ArcTan[(b + a
*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + 96*a*b*Cos[c + d*x
] + (a*b*(-16*a^4 + 20*a^2*b^2 - 5*b^4)*Cos[c + d*x])/((a - b)*(a + b)*(a +
 b*Sin[c + d*x])^2) + (b*(112*a^6 - 220*a^4*b^2 + 115*a^2*b^4 - 10*b^6)*Cos
[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])) - 8*b^2*Sin[2*(c + d*
x)]))/b^5 + ((12*a*(640*a^8 - 1920*a^6*b^2 + 2016*a^4*b^4 - 840*a^2*b^6 + 1
05*b^8)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + (-3840*a^10*(c + d*x) + 7680*a^8*b^2*(c + d*x) - 2976*a^6*b^4*(c + d*x)
- 1776*a^4*b^6*(c + d*x) + 960*a^2*b^8*(c + d*x) - 48*b^10*(c + d*x) - 3840
*a^9*b*Cos[c + d*x] + 8640*a^7*b^3*Cos[c + d*x] - 5696*a^5*b^5*Cos[c + d*x]
 + 788*a^3*b^7*Cos[c + d*x] + 114*a*b^9*Cos[c + d*x] + 1920*a^8*b^2*(c + d*
x)*Cos[2*(c + d*x)] - 4800*a^6*b^4*(c + d*x)*Cos[2*(c + d*x)] + 3888*a^4*b^
6*(c + d*x)*Cos[2*(c + d*x)] - 1056*a^2*b^8*(c + d*x)*Cos[2*(c + d*x)] + 48
*b^10*(c + d*x)*Cos[2*(c + d*x)] + 320*a^7*b^3*Cos[3*(c + d*x)] - 760*a^5*b
^5*Cos[3*(c + d*x)] + 560*a^3*b^7*Cos[3*(c + d*x)] - 120*a*b^9*Cos[3*(c + d
*x)] - 8*a^5*b^5*Cos[5*(c + d*x)] + 16*a^3*b^7*Cos[5*(c + d*x)] - 8*a*b^9*C
os[5*(c + d*x)] - 7680*a^9*b*(c + d*x)*Sin[c + d*x] + 19200*a^7*b^3*(c + d*
x)*Sin[c + d*x] - 15552*a^5*b^5*(c + d*x)*Sin[c + d*x] + 4224*a^3*b^7*(c +
d*x)*Sin[c + d*x] - 192*a*b^9*(c + d*x)*Sin[c + d*x] - 2880*a^8*b^2*Sin[2*(
c + d*x)] + 6880*a^6*b^4*Sin[2*(c + d*x)] - 5182*a^4*b^6*Sin[2*(c + d*x)] +
 1221*a^2*b^8*Sin[2*(c + d*x)] - 36*b^10*Sin[2*(c + d*x)] - 40*a^6*b^4*Sin[
4*(c + d*x)] + 88*a^4*b^6*Sin[4*(c + d*x)] - 56*a^2*b^8*Sin[4*(c + d*x)] +
8*b^10*Sin[4*(c + d*x)] + 2*a^4*b^6*Sin[6*(c + d*x)] - 4*a^2*b^8*Sin[6*(c +
 d*x)] + 2*b^10*Sin[6*(c + d*x)])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2))/b
^7)/d
```

fricas [A] time = 0.83, size = 1110, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")

[Out] [-1/8*(4*(a^3*b^5 - a*b^7)*cos(d*x + c)^5 - 3*(40*a^6*b^2 - 64*a^4*b^4 + 25
*a^2*b^6 - b^8)*d*x*cos(d*x + c)^2 - 2*(20*a^5*b^3 - 27*a^3*b^5 + 7*a*b^7)*
cos(d*x + c)^3 + 3*(40*a^8 - 24*a^6*b^2 - 39*a^4*b^4 + 24*a^2*b^6 - b^8)*d*
x - 6*(10*a^7 - a^5*b^2 - 9*a^3*b^4 + 2*a*b^6 - (10*a^5*b^2 - 11*a^3*b^4 +
2*a*b^6)*cos(d*x + c)^2 + 2*(10*a^6*b - 11*a^4*b^3 + 2*a^2*b^5)*sin(d*x + c
))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 +
 b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(20*a^7*b
 - 22*a^5*b^3 - a^3*b^5 + 3*a*b^7)*cos(d*x + c) - (2*(a^2*b^6 - b^8)*cos(d*
x + c)^5 - (10*a^4*b^4 - 11*a^2*b^6 + b^8)*cos(d*x + c)^3 - 6*(40*a^7*b - 6
4*a^5*b^3 + 25*a^3*b^5 - a*b^7)*d*x - 3*(60*a^6*b^2 - 91*a^4*b^4 + 32*a^2*b
^6 - b^8)*cos(d*x + c))*sin(d*x + c))/((a^2*b^9 - b^11)*d*cos(d*x + c)^2 -
2*(a^3*b^8 - a*b^10)*d*sin(d*x + c) - (a^4*b^7 - b^11)*d), -1/8*(4*(a^3*b^5
 - a*b^7)*cos(d*x + c)^5 - 3*(40*a^6*b^2 - 64*a^4*b^4 + 25*a^2*b^6 - b^8)*d
*x*cos(d*x + c)^2 - 2*(20*a^5*b^3 - 27*a^3*b^5 + 7*a*b^7)*cos(d*x + c)^3 +
3*(40*a^8 - 24*a^6*b^2 - 39*a^4*b^4 + 24*a^2*b^6 - b^8)*d*x + 12*(10*a^7 -
a^5*b^2 - 9*a^3*b^4 + 2*a*b^6 - (10*a^5*b^2 - 11*a^3*b^4 + 2*a*b^6)*cos(d*x
 + c)^2 + 2*(10*a^6*b - 11*a^4*b^3 + 2*a^2*b^5)*sin(d*x + c))*sqrt(a^2 - b^
2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 6*(20*a^7
*b - 22*a^5*b^3 - a^3*b^5 + 3*a*b^7)*cos(d*x + c) - (2*(a^2*b^6 - b^8)*cos(
d*x + c)^5 - (10*a^4*b^4 - 11*a^2*b^6 + b^8)*cos(d*x + c)^3 - 6*(40*a^7*b -
 64*a^5*b^3 + 25*a^3*b^5 - a*b^7)*d*x - 3*(60*a^6*b^2 - 91*a^4*b^4 + 32*a^2
*b^6 - b^8)*cos(d*x + c))*sin(d*x + c))/((a^2*b^9 - b^11)*d*cos(d*x + c)^2
- 2*(a^3*b^8 - a*b^10)*d*sin(d*x + c) - (a^4*b^7 - b^11)*d)]
```

giac [A] time = 0.30, size = 540, normalized size = 1.63

$$\frac{3\left(40\,a^4-24\,a^2b^2+b^4\right)(dx+c)}{b^7}-\frac{24\left(10\,a^5-11\,a^3b^2+2\,ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^7}+\frac{8\left(9\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a^2b^3\tan\left(\frac{1}{2}\,\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(40*a^4 - 24*a^2*b^2 + b^4)*(d*x + c)/b^7 - 24*(10*a^5 - 11*a^3*b^2
+ 2*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
 + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 8*(9*a^4*b*tan(1/2
*d*x + 1/2*c)^3 - 4*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 10*a^5*tan(1/2*d*x + 1
/2*c)^2 + 15*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 - 10*a*b^4*tan(1/2*d*x + 1/2*c)
^2 + 31*a^4*b*tan(1/2*d*x + 1/2*c) - 16*a^2*b^3*tan(1/2*d*x + 1/2*c) + 10*a
^5 - 5*a^3*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^
2*b^6) + 2*(24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 5*b^3*tan(1/2*d*x + 1/2*c)^7
+ 80*a^3*tan(1/2*d*x + 1/2*c)^6 - 48*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*
b*tan(1/2*d*x + 1/2*c)^5 + 3*b^3*tan(1/2*d*x + 1/2*c)^5 + 240*a^3*tan(1/2*d
*x + 1/2*c)^4 - 96*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x + 1/
2*c)^3 - 3*b^3*tan(1/2*d*x + 1/2*c)^3 + 240*a^3*tan(1/2*d*x + 1/2*c)^2 - 80
*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*b*tan(1/2*d*x + 1/2*c) + 5*b^3*tan(1
/2*d*x + 1/2*c) + 80*a^3 - 32*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^6))/
d
```

maple [B] time = 0.61, size = 1193, normalized size = 3.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 15/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*
x+1/2*c)^2-10/d*a/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*t
an(1/2*d*x+1/2*c)^2+31/d*a^4/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*
c)*b+a)^2*tan(1/2*d*x+1/2*c)-16/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-30/d*a^5/b^7/(a^2-b^2)^(1/2)*arctan(1
/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+3/4/d/b^3*arctan(tan(1/2*d
*x+1/2*c))+10/d*a^5/b^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2
*tan(1/2*d*x+1/2*c)^2+10/d*a^5/b^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2-5/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2
-5/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+3/4/d/b^3/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-3/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2
)^4*tan(1/2*d*x+1/2*c)^3+5/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)+20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^4*a^3-8/d/b^4/(1+tan(1/2*d*x+1/2*c)
^2)^4*a+30/d/b^7*arctan(tan(1/2*d*x+1/2*c))*a^4-18/d/b^5*arctan(tan(1/2*d*x
+1/2*c))*a^2+60/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a^3-2
4/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a-6/d/b^5/(1+tan(1/
2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*a^2+60/d/b^6/(1+tan(1/2*d*x+1/2*c)^2
)^4*tan(1/2*d*x+1/2*c)^2*a^3-20/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*
x+1/2*c)^2*a-6/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*a^2+9/d*
a^4/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2
*c)^3-4/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1
/2*d*x+1/2*c)^3+6/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a^2
```

```
+33/d*a^3/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-
b^2)^(1/2))-6/d*a/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*
b)/(a^2-b^2)^(1/2))+6/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7
*a^2+20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6*a^3-12/d/b^4/
(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 16.60, size = 3581, normalized size = 10.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] ((30*a^5 - 13*a^3*b^2)/b^6 + (15*tan(c/2 + (d*x)/2)^8*(10*a^5 - 6*a*b^4 + 9
*a^3*b^2))/b^6 + (3*tan(c/2 + (d*x)/2)^10*(10*a^5 - 5*a*b^4 + 9*a^3*b^2))/b
^6 + (tan(c/2 + (d*x)/2)^2*(150*a^5 - 37*a*b^4 + 15*a^3*b^2))/b^6 + (2*tan(
c/2 + (d*x)/2)^4*(150*a^5 - 59*a*b^4 + 75*a^3*b^2))/b^6 + (tan(c/2 + (d*x)/
2)*(420*a^4 - 187*a^2*b^2))/(4*b^5) + (3*tan(c/2 + (d*x)/2)^11*(20*a^4 - 7*
a^2*b^2))/(4*b^5) + (3*tan(c/2 + (d*x)/2)^7*(340*a^4 + 2*b^4 - 139*a^2*b^2)
)/(2*b^5) - (tan(c/2 + (d*x)/2)^9*(20*b^4 - 660*a^4 + 231*a^2*b^2))/(4*b^5)
 - (tan(c/2 + (d*x)/2)^5*(6*b^4 - 1380*a^4 + 623*a^2*b^2))/(2*b^5) + (tan(c
/2 + (d*x)/2)^3*(1740*a^4 + 20*b^4 - 809*a^2*b^2))/(4*b^5) - (2*tan(c/2 + (
d*x)/2)^6*(13*a*b^2 - 30*a^3)*(5*a^2 + 6*b^2))/b^6)/(d*(tan(c/2 + (d*x)/2)^
2*(6*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^10*(6*a^2 + 4*b^2) + tan(c/2 + (d*x)
/2)^4*(15*a^2 + 16*b^2) + tan(c/2 + (d*x)/2)^8*(15*a^2 + 16*b^2) + tan(c/2
+ (d*x)/2)^6*(20*a^2 + 24*b^2) + a^2*tan(c/2 + (d*x)/2)^12 + a^2 + 20*a*b*t
an(c/2 + (d*x)/2)^3 + 40*a*b*tan(c/2 + (d*x)/2)^5 + 40*a*b*tan(c/2 + (d*x)/
2)^7 + 20*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2)^11 + 4*a*b*ta
n(c/2 + (d*x)/2))) + (atan((((a^4*40i + b^4*1i - a^2*b^2*24i)*(((9*a^2*b^14
)/2 - 216*a^4*b^12 + 2952*a^6*b^10 - 8640*a^8*b^8 + 7200*a^10*b^6)/b^17 + (
tan(c/2 + (d*x)/2)*(18*a*b^16 - 1449*a^3*b^14 + 18576*a^5*b^12 - 63648*a^7*
b^10 + 77760*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) - (3*(a^4*40i + b^4*1i - a
```

^2*b^2*24i)*((12*a*b^18 - 204*a^3*b^16 + 240*a^5*b^14)/b^17 - (3*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b^18))*(a^4*40i + b^4*1i - a^2*b^2*24i))/(8*b^7) + (tan(c/2 + (d*x)/2)*(384*a^2*b^18 - 2112*a^4*b^16 + 1920*a^6*b^14))/(2*b^18)))/(8*b^7))*3i)/(8*b^7) + ((a^4*40i + b^4*1i - a^2*b^2*24i)*(((9*a^2*b^14)/2 - 216*a^4*b^12 + 2952*a^6*b^10 - 8640*a^8*b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(18*a*b^16 - 1449*a^3*b^14 + 18576*a^5*b^12 - 63648*a^7*b^10 + 77760*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) + (3*(a^4*40i + b^4*1i - a^2*b^2*24i)*((12*a*b^18 - 204*a^3*b^16 + 240*a^5*b^14)/b^17 + (3*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b^18))*(a^4*40i + b^4*1i - a^2*b^2*24i))/(8*b^7) + (tan(c/2 + (d*x)/2)*(384*a^2*b^18 - 2112*a^4*b^16 + 1920*a^6*b^14))/(2*b^18)))/(8*b^7))*3i)/(8*b^7))/((108000*a^13 - 189*a^3*b^10 + (12231*a^5*b^8)/2 - 49383*a^7*b^6 + 159840*a^9*b^4 - 221400*a^11*b^2)/b^17 - (3*(a^4*40i + b^4*1i - a^2*b^2*24i)*(((9*a^2*b^14)/2 - 216*a^4*b^12 + 2952*a^6*b^10 - 8640*a^8*b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(18*a*b^16 - 1449*a^3*b^14 + 18576*a^5*b^12 - 63648*a^7*b^10 + 77760*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) - (3*(a^4*40i + b^4*1i - a^2*b^2*24i)*((12*a*b^18 - 204*a^3*b^16 + 240*a^5*b^14)/b^17 - (3*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b^18))*(a^4*40i + b^4*1i - a^2*b^2*24i))/(8*b^7) + (tan(c/2 + (d*x)/2)*(384*a^2*b^18 - 2112*a^4*b^16 + 1920*a^6*b^14))/(2*b^18)))/(8*b^7)))/(8*b^7) + (3*(a^4*40i + b^4*1i - a^2*b^2*24i)*(((9*a^2*b^14)/2 - 216*a^4*b^12 + 2952*a^6*b^10 - 8640*a^8*b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(18*a*b^16 - 1449*a^3*b^14 + 18576*a^5*b^12 - 63648*a^7*b^10 + 77760*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) + (3*(a^4*40i + b^4*1i - a^2*b^2*24i)*((12*a*b^18 - 204*a^3*b^16 + 240*a^5*b^14)/b^17 + (3*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b^18))*(a^4*40i + b^4*1i - a^2*b^2*24i))/(8*b^7) + (tan(c/2 + (d*x)/2)*(384*a^2*b^18 - 2112*a^4*b^16 + 1920*a^6*b^14))/(2*b^18)))/(8*b^7)))/(8*b^7) + (tan(c/2 + (d*x)/2)*(432000*a^14 + 54*a^2*b^12 - 2889*a^4*b^10 + 49950*a^6*b^8 - 311472*a^8*b^6 + 833760*a^10*b^4 - 993600*a^12*b^2))/b^18))*(a^4*40i + b^4*1i - a^2*b^2*24i)*3i)/(4*b^7*d) + (a*atan(((a*(-(a + b)*(a - b))^(1/2)*(10*a^4 + 2*b^4 - 11*a^2*b^2)*(((9*a^2*b^14)/2 - 216*a^4*b^12 + 2952*a^6*b^10 - 8640*a^8*b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(18*a*b^16 - 1449*a^3*b^14 + 18576*a^5*b^12 - 63648*a^7*b^10 + 77760*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) - (3*a*(-(a + b)*(a - b))^(1/2)*((12*a*b^18 - 204*a^3*b^16 + 240*a^5*b^14)/b^17 + (tan(c/2 + (d*x)/2)*(384*a^2*b^18 - 2112*a^4*b^16 + 1920*a^6*b^14))/(2*b^18) - (3*a*(-(a + b)*(a - b))^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b^18))*(10*a^4 + 2*b^4 - 11*a^2*b^2))/(2*(b^9 - a^2*b^7)))*(10*a^4 + 2*b^4 - 11*a^2*b^2))/(2*(b^9 - a^2*b^7)))*3i)/(2*(b^9 - a^2*b^7)) + (a*(-(a + b)*(a - b))^(1/2)*(10*a^4 + 2*b^4 - 11*a^2*b^2)*(((9*a^2*b^14)/2 - 216*a^4*b^12 + 2952*a^6*b^10 - 8640*a^8*b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(18*a*b^16 - 1449*a^3*b^14 + 18576*a^5*b^12 - 63648*a^7*b^10 + 77760*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) + (3*a*(-(a + b)*(a - b))^(1/2)*((12*a*b^18 - 204*a^3*b^16 + 240*a^5*b^14)/b^17 + (tan(c/2 + (d*x)/2)*(384*a^2*b^18 - 2112*a^4*b^16 + 1920*a^6*b^14))/(2*b^18) + (3*a*(-(a + b)*(a - b))^

```
(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b^1
8))*(10*a^4 + 2*b^4 - 11*a^2*b^2))/(2*(b^9 - a^2*b^7)))*(10*a^4 + 2*b^4 - 1
1*a^2*b^2))/(2*(b^9 - a^2*b^7)))*3i)/(2*(b^9 - a^2*b^7)))/((108000*a^13 - 1
89*a^3*b^10 + (12231*a^5*b^8)/2 - 49383*a^7*b^6 + 159840*a^9*b^4 - 221400*a
^11*b^2)/b^17 + (tan(c/2 + (d*x)/2)*(432000*a^14 + 54*a^2*b^12 - 2889*a^4*b
^10 + 49950*a^6*b^8 - 311472*a^8*b^6 + 833760*a^10*b^4 - 993600*a^12*b^2))/
b^18 - (3*a*(-(a + b)*(a - b))^(1/2)*(10*a^4 + 2*b^4 - 11*a^2*b^2)*(((9*a^2
*b^14)/2 - 216*a^4*b^12 + 2952*a^6*b^10 - 8640*a^8*b^8 + 7200*a^10*b^6)/b^1
7 + (tan(c/2 + (d*x)/2)*(18*a*b^16 - 1449*a^3*b^14 + 18576*a^5*b^12 - 63648
*a^7*b^10 + 77760*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) - (3*a*(-(a + b)*(a -
 b))^(1/2)*((12*a*b^18 - 204*a^3*b^16 + 240*a^5*b^14)/b^17 + (tan(c/2 + (d*
x)/2)*(384*a^2*b^18 - 2112*a^4*b^16 + 1920*a^6*b^14))/(2*b^18) - (3*a*(-(a
+ b)*(a - b))^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3
*b^20))/(2*b^18))*(10*a^4 + 2*b^4 - 11*a^2*b^2))/(2*(b^9 - a^2*b^7)))*(10*a
^4 + 2*b^4 - 11*a^2*b^2))/(2*(b^9 - a^2*b^7))))/(2*(b^9 - a^2*b^7)) + (3*a*
(-(a + b)*(a - b))^(1/2)*(10*a^4 + 2*b^4 - 11*a^2*b^2)*(((9*a^2*b^14)/2 - 2
16*a^4*b^12 + 2952*a^6*b^10 - 8640*a^8*b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2
 + (d*x)/2)*(18*a*b^16 - 1449*a^3*b^14 + 18576*a^5*b^12 - 63648*a^7*b^10 +
77760*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) + (3*a*(-(a + b)*(a - b))^(1/2)*(
(12*a*b^18 - 204*a^3*b^16 + 240*a^5*b^14)/b^17 + (tan(c/2 + (d*x)/2)*(384*a
^2*b^18 - 2112*a^4*b^16 + 1920*a^6*b^14))/(2*b^18) + (3*a*(-(a + b)*(a - b)
)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b
^18))*(10*a^4 + 2*b^4 - 11*a^2*b^2))/(2*(b^9 - a^2*b^7)))*(10*a^4 + 2*b^4 -
 11*a^2*b^2))/(2*(b^9 - a^2*b^7))))/(2*(b^9 - a^2*b^7))))*(-(a + b)*(a - b)
)^(1/2)*(10*a^4 + 2*b^4 - 11*a^2*b^2)*3i)/(d*(b^9 - a^2*b^7))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1136 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=284

$$\frac{(6a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}-\frac{(a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}-\frac{(60a^2-17b^2)\cos(c+dx)}{6b^5d}+\frac{(5a^2-b^2)\,\text{si}}{}$$

```
[Out] 1/2*a*(9-20/b^2*a^2)*x/b^4-1/6*(60*a^2-17*b^2)*cos(d*x+c)/b^5/d+(5*a^2-b^2)
*cos(d*x+c)*sin(d*x+c)/a/b^4/d-1/6*(20*a^2-3*b^2)*cos(d*x+c)*sin(d*x+c)^2/a
^2/b^3/d-1/2*(a^2-b^2)*cos(d*x+c)*sin(d*x+c)^3/a/b^2/d/(a+b*sin(d*x+c))^2+1
/2*(6*a^2-b^2)*cos(d*x+c)*sin(d*x+c)^3/a^2/b^2/d/(a+b*sin(d*x+c))+(20*a^4-1
9*a^2*b^2+2*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^6/d/(a^
2-b^2)^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2891, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{(60a^2-17b^2)\cos(c+dx)}{6b^5d}+\frac{(-19a^2b^2+20a^4+2b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d\sqrt{a^2-b^2}}+\frac{(6a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(9 - (20*a^2)/b^2)*x)/(2*b^4) + ((20*a^4 - 19*a^2*b^2 + 2*b^4)*ArcTan[(b
 + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*Sqrt[a^2 - b^2]*d) - ((60*a^2
 - 17*b^2)*Cos[c + d*x])/(6*b^5*d) + ((5*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*
x])/(a*b^4*d) - ((20*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]^2)/(6*a^2*b^3*d
) - ((a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(2*a*b^2*d*(a + b*Sin[c + d*x
])^2) + ((6*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(2*a^2*b^2*d*(a + b*Sin
[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{\left(6a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{2a^2b^2d(a+b\sin(c+dx))} - \int$$

$$= -\frac{\left(20a^2-3b^2\right)\cos(c+dx)\sin^2(c+dx)}{6a^2b^3d} - \frac{\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} +$$

$$= \frac{\left(5a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{ab^4d} - \frac{\left(20a^2-3b^2\right)\cos(c+dx)\sin^2(c+dx)}{6a^2b^3d} -$$

$$= -\frac{\left(60a^2-17b^2\right)\cos(c+dx)}{6b^5d} + \frac{\left(5a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{ab^4d} - \frac{\left(20a^2-3b\right.}{}$$

$$= -\frac{a\left(20a^2-9b^2\right)x}{2b^6} - \frac{\left(60a^2-17b^2\right)\cos(c+dx)}{6b^5d} + \frac{\left(5a^2-b^2\right)\cos(c+dx)\sin(c}{ab^4d}$$

$$= -\frac{a\left(20a^2-9b^2\right)x}{2b^6} - \frac{\left(60a^2-17b^2\right)\cos(c+dx)}{6b^5d} + \frac{\left(5a^2-b^2\right)\cos(c+dx)\sin(c}{ab^4d}$$

$$= -\frac{a\left(20a^2-9b^2\right)x}{2b^6} - \frac{\left(60a^2-17b^2\right)\cos(c+dx)}{6b^5d} + \frac{\left(5a^2-b^2\right)\cos(c+dx)\sin(c}{ab^4d}$$

$$= -\frac{a\left(20a^2-9b^2\right)x}{2b^6} + \frac{\left(20a^4-19a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}\,d} - \frac{\left(60a^2-1\right.}{}$$

Mathematica [B] time = 6.44, size = 1030, normalized size = 3.63

$$-\frac{12\left(-48a(c+dx)+\frac{6\left(16a^6-40b^2a^4+30b^4a^2-5b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-16b\cos(c+dx)+\frac{ab\left(-40a^4+72b^2a^2-29b^4\right)\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\frac{b\left(8a^4-8b^2a^2+b^4\right)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))^2}\right)}{b^4}+1$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-12*(-48*a*(c + d*x) + (6*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*ArcT
an[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 16*b*Cos[
c + d*x] + (b*(8*a^4 - 8*a^2*b^2 + b^4)*Cos[c + d*x])/((a - b)*(a + b)*(a +
 b*Sin[c + d*x])^2) + (a*b*(-40*a^4 + 72*a^2*b^2 - 29*b^4)*Cos[c + d*x])/((
a - b)^2*(a + b)^2*(a + b*Sin[c + d*x]))))/b^4 + 12*((2*(2*a^2 + b^2)*ArcTa
n[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (b*Cos[c +
 d*x]*(4*a^2 - b^2 + 3*a*b*Sin[c + d*x]))/((a - b)^2*(a + b)^2*(a + b*Sin[c
 + d*x])^2)) + (6*((-6*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/Sqrt[a^2 - b^2] + (Cos[c + d*x]*(-(b*(2*a^2 + b^2)) + a*(2*a^2 - 5*b^2)*S
in[c + d*x]))/(a + b*Sin[c + d*x])^2))/((a - b)^2*(a + b)^2) - ((-12*(640*a
^8 - 1792*a^6*b^2 + 1680*a^4*b^4 - 560*a^2*b^6 + 35*b^8)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (3840*a^9*(c + d*x) - 6
912*a^7*b^2*(c + d*x) + 1728*a^5*b^4*(c + d*x) + 1920*a^3*b^6*(c + d*x) - 5
76*a*b^8*(c + d*x) + 3840*a^8*b*Cos[c + d*x] - 7872*a^6*b^3*Cos[c + d*x] +
4256*a^4*b^5*Cos[c + d*x] - 172*a^2*b^7*Cos[c + d*x] - 70*b^9*Cos[c + d*x]
- 1920*a^7*b^2*(c + d*x)*Cos[2*(c + d*x)] + 4416*a^5*b^4*(c + d*x)*Cos[2*(c
 + d*x)] - 3072*a^3*b^6*(c + d*x)*Cos[2*(c + d*x)] + 576*a*b^8*(c + d*x)*Co
s[2*(c + d*x)] - 320*a^6*b^3*Cos[3*(c + d*x)] + 696*a^4*b^5*Cos[3*(c + d*x)
] - 432*a^2*b^7*Cos[3*(c + d*x)] + 56*b^9*Cos[3*(c + d*x)] + 8*a^4*b^5*Cos[
5*(c + d*x)] - 16*a^2*b^7*Cos[5*(c + d*x)] + 8*b^9*Cos[5*(c + d*x)] + 7680*
a^8*b*(c + d*x)*Sin[c + d*x] - 17664*a^6*b^3*(c + d*x)*Sin[c + d*x] + 12288
*a^4*b^5*(c + d*x)*Sin[c + d*x] - 2304*a^2*b^7*(c + d*x)*Sin[c + d*x] + 288
0*a^7*b^2*Sin[2*(c + d*x)] - 6304*a^5*b^4*Sin[2*(c + d*x)] + 4022*a^3*b^6*S
in[2*(c + d*x)] - 607*a*b^8*Sin[2*(c + d*x)] + 40*a^5*b^4*Sin[4*(c + d*x)]
- 80*a^3*b^6*Sin[4*(c + d*x)] + 40*a*b^8*Sin[4*(c + d*x)])/((a^2 - b^2)^2*(
a + b*Sin[c + d*x])^2))/b^6)/(384*d)
```

fricas [A] time = 0.85, size = 976, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")

[Out] [1/12*(4*(a^2*b^5 - b^7)*cos(d*x + c)^5 - 6*(20*a^5*b^2 - 29*a^3*b^4 + 9*a*
b^6)*d*x*cos(d*x + c)^2 - 8*(5*a^4*b^3 - 6*a^2*b^5 + b^7)*cos(d*x + c)^3 +
6*(20*a^7 - 9*a^5*b^2 - 20*a^3*b^4 + 9*a*b^6)*d*x + 3*(20*a^6 + a^4*b^2 - 1
7*a^2*b^4 + 2*b^6 - (20*a^4*b^2 - 19*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(2
0*a^5*b - 19*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2
- b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*
sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*
b*sin(d*x + c) - a^2 - b^2)) + 6*(20*a^6*b - 19*a^4*b^3 - 3*a^2*b^5 + 2*b^7
)*cos(d*x + c) + 2*(5*(a^3*b^4 - a*b^6)*cos(d*x + c)^3 + 6*(20*a^6*b - 29*a
^4*b^3 + 9*a^2*b^5)*d*x + 3*(30*a^5*b^2 - 41*a^3*b^4 + 11*a*b^6)*cos(d*x +
c))*sin(d*x + c))/((a^2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^3*b^7 - a*b^9)*
d*sin(d*x + c) - (a^4*b^6 - b^10)*d), 1/6*(2*(a^2*b^5 - b^7)*cos(d*x + c)^5
 - 3*(20*a^5*b^2 - 29*a^3*b^4 + 9*a*b^6)*d*x*cos(d*x + c)^2 - 4*(5*a^4*b^3
- 6*a^2*b^5 + b^7)*cos(d*x + c)^3 + 3*(20*a^7 - 9*a^5*b^2 - 20*a^3*b^4 + 9*
a*b^6)*d*x + 3*(20*a^6 + a^4*b^2 - 17*a^2*b^4 + 2*b^6 - (20*a^4*b^2 - 19*a^
2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(20*a^5*b - 19*a^3*b^3 + 2*a*b^5)*sin(d*x
 + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*
x + c))) + 3*(20*a^6*b - 19*a^4*b^3 - 3*a^2*b^5 + 2*b^7)*cos(d*x + c) + (5*
(a^3*b^4 - a*b^6)*cos(d*x + c)^3 + 6*(20*a^6*b - 29*a^4*b^3 + 9*a^2*b^5)*d*
x + 3*(30*a^5*b^2 - 41*a^3*b^4 + 11*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^
2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^3*b^7 - a*b^9)*d*sin(d*x + c) - (a^4*
b^6 - b^10)*d)]

giac [A] time = 0.27, size = 393, normalized size = 1.38

$$-\frac{\frac{3\left(20a^3-9ab^2\right)(dx+c)}{b^6}-\frac{6\left(20a^4-19a^2b^2+2b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6}+\frac{6\left(7a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/6*(3*(20*a^3 - 9*a*b^2)*(d*x + c)/b^6 - 6*(20*a^4 - 19*a^2*b^2 + 2*b^4)*
(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) +
 b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 6*(7*a^3*b*tan(1/2*d*x + 1/2*
c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^4*tan(1/2*d*x + 1/2*c)^2 + 13*a
^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*b^4*tan(1/2*d*x + 1/2*c)^2 + 25*a^3*b*tan
(1/2*d*x + 1/2*c) - 10*a*b^3*tan(1/2*d*x + 1/2*c) + 8*a^4 - 3*a^2*b^2)/((a*
tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*b^5) + 2*(9*a*b*ta
n(1/2*d*x + 1/2*c)^5 + 36*a^2*tan(1/2*d*x + 1/2*c)^4 - 12*b^2*tan(1/2*d*x +

```
 1/2*c)^4 + 72*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 -
 9*a*b*tan(1/2*d*x + 1/2*c) + 36*a^2 - 8*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)
^3*b^5))/d
```

maple [B] time = 0.53, size = 880, normalized size = 3.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)^5-12/d/b^5/(1+tan(
1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2+4/d/b^3/(1+tan(1/2*d*x+1/2*c)^
2)^3*tan(1/2*d*x+1/2*c)^4-24/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1
/2*c)^2*a^2+4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2+3/d/b^4
/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)-12/d/b^5/(1+tan(1/2*d*x+1/
2*c)^2)^3*a^2+8/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3-20/d/b^6*arctan(tan(1/2*
d*x+1/2*c))*a^3+9/d/b^4*a*arctan(tan(1/2*d*x+1/2*c))-7/d/b^4/(tan(1/2*d*x+1
/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*a^3+2/d/b^2/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*a-8/d/
b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^
2*a^4-13/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*
d*x+1/2*c)^2*a^2+6/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*
tan(1/2*d*x+1/2*c)^2-25/d/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)^2*tan(1/2*d*x+1/2*c)*a^3+10/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*a-8/d/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(
1/2*d*x+1/2*c)*b+a)^2*a^4+3/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*a^2+20/d/b^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+
2*b)/(a^2-b^2)^(1/2))*a^4-19/d/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*
d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2+2/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 14.34, size = 2034, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^3,x)

[Out] - ((60*a^4 - 17*a^2*b^2)/(3*b^5) - (20*tan(c/2 + (d*x)/2)^7*(a*b^2 - 5*a^3)
)/b^4 - (2*tan(c/2 + (d*x)/2)^9*(a*b^2 - 5*a^3))/b^4 - (4*tan(c/2 + (d*x)/2
)^5*(17*a*b^2 - 60*a^3))/b^4 - (4*tan(c/2 + (d*x)/2)^3*(53*a*b^2 - 165*a^3)
)/(3*b^4) + (tan(c/2 + (d*x)/2)^8*(20*a^4 - 6*b^4 + 21*a^2*b^2))/b^5 + (2*t
an(c/2 + (d*x)/2)^6*(40*a^4 - 17*b^4 + 42*a^2*b^2))/b^5 + (2*tan(c/2 + (d*x
)/2)^2*(120*a^4 - 25*b^4 + 46*a^2*b^2))/(3*b^5) + (2*tan(c/2 + (d*x)/2)^4*(
180*a^4 - 51*b^4 + 149*a^2*b^2))/(3*b^5) - (2*tan(c/2 + (d*x)/2)*(31*a*b^2
 - 105*a^3))/(3*b^4))/(d*(tan(c/2 + (d*x)/2)^2*(5*a^2 + 4*b^2) + tan(c/2 + (
d*x)/2)^8*(5*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(10*a^2 + 12*b^2) + tan(c/
2 + (d*x)/2)^6*(10*a^2 + 12*b^2) + a^2*tan(c/2 + (d*x)/2)^10 + a^2 + 16*a*b
*tan(c/2 + (d*x)/2)^3 + 24*a*b*tan(c/2 + (d*x)/2)^5 + 16*a*b*tan(c/2 + (d*x
)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2))) - (a*atan(
(280*a^4*tan(c/2 + (d*x)/2))/(280*a^4 - 288*a^2*b^2 + (800*a^6)/b^2) - (288
*a^2*tan(c/2 + (d*x)/2))/((280*a^4)/b^2 - 288*a^2 + (800*a^6)/b^4) + (800*a
^6*tan(c/2 + (d*x)/2))/(800*a^6 - 288*a^2*b^4 + 280*a^4*b^2))*(20*a^2 - 9*b
^2))/(b^6*d) - (atan((((-(a + b)*(a - b))^(1/2)*(10*a^4 + b^4 - (19*a^2*b^2
)/2)*((8*(81*a^4*b^9 - 360*a^6*b^7 + 400*a^8*b^5))/b^14 - (8*tan(c/2 + (d*x
)/2)*(4*a*b^13 - 238*a^3*b^11 + 1242*a^5*b^9 - 1920*a^7*b^7 + 800*a^9*b^5))
/b^15 + ((-(a + b)*(a - b))^(1/2)*(10*a^4 + b^4 - (19*a^2*b^2)/2)*((8*tan(c
/2 + (d*x)/2)*(8*a*b^16 - 76*a^3*b^14 + 80*a^5*b^12))/b^15 - (8*(14*a^2*b^1
4 - 20*a^4*b^12))/b^14 + ((-(a + b)*(a - b))^(1/2)*(32*a^2*b^3 + (8*tan(c/2
 + (d*x)/2)*(12*a*b^19 - 8*a^3*b^17))/b^15)*(10*a^4 + b^4 - (19*a^2*b^2)/2)
)/(b^8 - a^2*b^6)))/(b^8 - a^2*b^6))*1i)/(b^8 - a^2*b^6) + ((-(a + b)*(a -
b))^(1/2)*(10*a^4 + b^4 - (19*a^2*b^2)/2)*((8*(81*a^4*b^9 - 360*a^6*b^7 + 4
00*a^8*b^5))/b^14 - (8*tan(c/2 + (d*x)/2)*(4*a*b^13 - 238*a^3*b^11 + 1242*a
^5*b^9 - 1920*a^7*b^7 + 800*a^9*b^5))/b^15 + ((-(a + b)*(a - b))^(1/2)*(10*
a^4 + b^4 - (19*a^2*b^2)/2)*((8*(14*a^2*b^14 - 20*a^4*b^12))/b^14 - (8*tan(
c/2 + (d*x)/2)*(8*a*b^16 - 76*a^3*b^14 + 80*a^5*b^12))/b^15 + ((-(a + b)*(a
 - b))^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^19 - 8*a^3*b^17))/
b^15)*(10*a^4 + b^4 - (19*a^2*b^2)/2))/(b^8 - a^2*b^6)))/(b^8 - a^2*b^6))*1
i)/(b^8 - a^2*b^6))/((16*(2000*a^10 + 18*a^2*b^8 - 301*a^4*b^6 + 1615*a^6*b
^4 - 3200*a^8*b^2))/b^14 + (16*tan(c/2 + (d*x)/2)*(8000*a^11 + 162*a^3*b^8
 - 2259*a^5*b^6 + 9260*a^7*b^4 - 14800*a^9*b^2))/b^15 + ((-(a + b)*(a - b))^
(1/2)*(10*a^4 + b^4 - (19*a^2*b^2)/2)*((8*(81*a^4*b^9 - 360*a^6*b^7 + 400*a
^8*b^5))/b^14 - (8*tan(c/2 + (d*x)/2)*(4*a*b^13 - 238*a^3*b^11 + 1242*a^5*b
^9 - 1920*a^7*b^7 + 800*a^9*b^5))/b^15 + ((-(a + b)*(a - b))^(1/2)*(10*a^4
 + b^4 - (19*a^2*b^2)/2)*((8*tan(c/2 + (d*x)/2)*(8*a*b^16 - 76*a^3*b^14 + 80
```

```
*a^5*b^12))/b^15 - (8*(14*a^2*b^14 - 20*a^4*b^12))/b^14 + ((-(a + b)*(a - b
))^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^19 - 8*a^3*b^17))/b^15
)*(10*a^4 + b^4 - (19*a^2*b^2)/2))/(b^8 - a^2*b^6)))/(b^8 - a^2*b^6)))/(b^8
 - a^2*b^6) - ((-(a + b)*(a - b))^(1/2)*(10*a^4 + b^4 - (19*a^2*b^2)/2)*((8
*(81*a^4*b^9 - 360*a^6*b^7 + 400*a^8*b^5))/b^14 - (8*tan(c/2 + (d*x)/2)*(4*
a*b^13 - 238*a^3*b^11 + 1242*a^5*b^9 - 1920*a^7*b^7 + 800*a^9*b^5))/b^15 +
((-(a + b)*(a - b))^(1/2)*(10*a^4 + b^4 - (19*a^2*b^2)/2)*((8*(14*a^2*b^14
- 20*a^4*b^12))/b^14 - (8*tan(c/2 + (d*x)/2)*(8*a*b^16 - 76*a^3*b^14 + 80*a
^5*b^12))/b^15 + ((-(a + b)*(a - b))^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)
/2)*(12*a*b^19 - 8*a^3*b^17))/b^15)*(10*a^4 + b^4 - (19*a^2*b^2)/2))/(b^8 -
 a^2*b^6)))/(b^8 - a^2*b^6)))/(b^8 - a^2*b^6)))*(-(a + b)*(a - b))^(1/2)*(1
0*a^4 + b^4 - (19*a^2*b^2)/2)*2i)/(d*(b^8 - a^2*b^6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1137 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=173

$$-\frac{3a\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}+\frac{3x\left(4a^2-b^2\right)}{2b^5}+\frac{3\cos(c+dx)\left(4a^2+2ab\sin(c+dx)-b^2\right)}{2b^4d(a+b\sin(c+dx))}+\frac{\cos^3(c+dx)(}{2b^2d(a+}$$

```
[Out] 3/2*(4*a^2-b^2)*x/b^5+1/2*cos(d*x+c)^3*(2*a+b*sin(d*x+c))/b^2/d/(a+b*sin(d*
x+c))^2+3/2*cos(d*x+c)*(4*a^2-b^2+2*a*b*sin(d*x+c))/b^4/d/(a+b*sin(d*x+c))-
3*a*(4*a^2-3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^5/d/(a
^2-b^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2863, 2735, 2660, 618, 204}

$$-\frac{3a\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}+\frac{3\cos(c+dx)\left(4a^2+2ab\sin(c+dx)-b^2\right)}{2b^4d(a+b\sin(c+dx))}+\frac{3x\left(4a^2-b^2\right)}{2b^5}+\frac{\cos^3(c+dx)(}{2b^2d(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*(4*a^2 - b^2)*x)/(2*b^5) - (3*a*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*d) + (Cos[c + d*x]^3*(2*a +
b*Sin[c + d*x]))/(2*b^2*d*(a + b*Sin[c + d*x])^2) + (3*Cos[c + d*x]*(4*a^2
- b^2 + 2*a*b*Sin[c + d*x]))/(2*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} - \frac{3\int \frac{\cos^2(c+dx)(-2b-4a\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{4b^2}$$

$$= \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} + \frac{3\cos(c+dx)\left(4a^2-b^2+2ab\sin(c+dx)\right)}{2b^4d(a+b\sin(c+dx))} +$$

$$= \frac{3\left(4a^2-b^2\right)x}{2b^5} + \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} + \frac{3\cos(c+dx)\left(4a^2-b^2+2\right.}{2b^4d(a+b\sin(c+\right.}$$

$$= \frac{3\left(4a^2-b^2\right)x}{2b^5} + \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} + \frac{3\cos(c+dx)\left(4a^2-b^2+2\right.}{2b^4d(a+b\sin(c+\right.}$$

$$= \frac{3\left(4a^2-b^2\right)x}{2b^5} + \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} + \frac{3\cos(c+dx)\left(4a^2-b^2+2\right.}{2b^4d(a+b\sin(c+\right.}$$

$$= \frac{3\left(4a^2-b^2\right)x}{2b^5} - \frac{3a\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}\,d} + \frac{\cos^3(c+dx)(2a+b\sin\right.}{2b^2d(a+b\sin(c+\right.}$$

Mathematica [A] time = 3.46, size = 274, normalized size = 1.58

$$\frac{\frac{96a^4c+96a^4dx+192a^3bc\sin(c+dx)+192a^3bdx\sin(c+dx)+96a^3b\cos(c+dx)+72a^2b^2\sin(2(c+dx))+12b^2\left(b^2-4a^2\right)(c+dx)\cos(2(c+dx))+24a^2b^2c+24a^2b^2d}{(a+b\sin(c+dx))^2}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-48*a*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] + (96*a^4*c + 24*a^2*b^2*c - 12*b^4*c + 96*a^4*d*x + 24*a^2*
b^2*d*x - 12*b^4*d*x + 96*a^3*b*Cos[c + d*x] + 12*b^2*(-4*a^2 + b^2)*(c + d
*x)*Cos[2*(c + d*x)] - 8*a*b^3*Cos[3*(c + d*x)] + 192*a^3*b*c*Sin[c + d*x]
- 48*a*b^3*c*Sin[c + d*x] + 192*a^3*b*d*x*Sin[c + d*x] - 48*a*b^3*d*x*Sin[c
 + d*x] + 72*a^2*b^2*Sin[2*(c + d*x)] - 10*b^4*Sin[2*(c + d*x)] + b^4*Sin[4
*(c + d*x)])/(a + b*Sin[c + d*x])^2)/(16*b^5*d)
```

fricas [B] time = 0.80, size = 837, normalized size = 4.84

$$\left[\frac{6\left(4a^4b^2-5a^2b^4+b^6\right)dx\cos(dx+c)^2+8\left(a^3b^3-ab^5\right)\cos(dx+c)^3-6\left(4a^6-a^4b^2-4a^2b^4+b^6\right)dx-3\left(4a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(6*(4*a^4*b^2 - 5*a^2*b^4 + b^6)*d*x*cos(d*x + c)^2 + 8*(a^3*b^3 - a*b
^5)*cos(d*x + c)^3 - 6*(4*a^6 - a^4*b^2 - 4*a^2*b^4 + b^6)*d*x - 3*(4*a^5 +
 a^3*b^2 - 3*a*b^4 - (4*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + 2*(4*a^4*b - 3*
a^2*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 6*(4*a^5*b - 3*a^3*b^3 - a*b^5)*cos(d*x + c) - 2*((a^2*b^4 - b^6)*c
os(d*x + c)^3 + 6*(4*a^5*b - 5*a^3*b^3 + a*b^5)*d*x + 3*(6*a^4*b^2 - 7*a^2*
b^4 + b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^7 - b^9)*d*cos(d*x + c)^2 -
2*(a^3*b^6 - a*b^8)*d*sin(d*x + c) - (a^4*b^5 - b^9)*d), 1/2*(3*(4*a^4*b^2
- 5*a^2*b^4 + b^6)*d*x*cos(d*x + c)^2 + 4*(a^3*b^3 - a*b^5)*cos(d*x + c)^3
- 3*(4*a^6 - a^4*b^2 - 4*a^2*b^4 + b^6)*d*x - 3*(4*a^5 + a^3*b^2 - 3*a*b^4
- (4*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + 2*(4*a^4*b - 3*a^2*b^3)*sin(d*x +
c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x +
 c))) - 3*(4*a^5*b - 3*a^3*b^3 - a*b^5)*cos(d*x + c) - ((a^2*b^4 - b^6)*cos
(d*x + c)^3 + 6*(4*a^5*b - 5*a^3*b^3 + a*b^5)*d*x + 3*(6*a^4*b^2 - 7*a^2*b^
4 + b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^7 - b^9)*d*cos(d*x + c)^2 - 2*
(a^3*b^6 - a*b^8)*d*sin(d*x + c) - (a^4*b^5 - b^9)*d)]
```

giac [B] time = 0.24, size = 429, normalized size = 2.48

$$\frac{\frac{3\left(4a^2-b^2\right)(dx+c)}{b^5}-\frac{6\left(4a^3-3ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^5}+\frac{2\left(6a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+15a^2b^2\tan\left(\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*a^2 - b^2)*(d*x + c)/b^5 - 6*(4*a^3 - 3*a*b^2)*(pi*floor(1/2*(d*x
 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2
)))/(sqrt(a^2 - b^2)*b^5) + 2*(6*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 12*a^4*tan(
1/2*d*x + 1/2*c)^6 + 15*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 - 2*b^4*tan(1/2*d*x
```

```
+ 1/2*c)^6 + 54*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 36*a^4*tan(1/2*d*x + 1/2*c)^
4 + 45*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 4*b^4*tan(1/2*d*x + 1/2*c)^4 + 90*a
^3*b*tan(1/2*d*x + 1/2*c)^3 - 12*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 36*a^4*tan(
1/2*d*x + 1/2*c)^2 + 29*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 2*b^4*tan(1/2*d*x
+ 1/2*c)^2 + 42*a^3*b*tan(1/2*d*x + 1/2*c) - 4*a*b^3*tan(1/2*d*x + 1/2*c) +
 12*a^4 - a^2*b^2)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3
+ 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a*b^4))/d
```

maple [B] time = 0.54, size = 639, normalized size = 3.69

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{6a}{d\,b^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{12\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+6/d/b^4/(1+tan(1/2*
d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a-1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*t
an(1/2*d*x+1/2*c)+6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^2*a+12/d/b^5*arctan(tan(
1/2*d*x+1/2*c))*a^2-3/d/b^3*arctan(tan(1/2*d*x+1/2*c))+5/d*a^2/b^3/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+6/d*a^3/b^
4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+
11/d*a/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+
1/2*c)^2-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*
d*x+1/2*c)^2+19/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)
^2*tan(1/2*d*x+1/2*c)-4/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2*tan(1/2*d*x+1/2*c)+6/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)^2-1/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a-
12/d*a^3/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))+9/d*a/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b
)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.41, size = 1743, normalized size = 10.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^3,x)
```

```
[Out] ((54*a^2*tan(c/2 + (d*x)/2)^5)/b^3 - (a*b^2 - 12*a^3)/b^4 + (6*a^2*tan(c/2
+ (d*x)/2)^7)/b^3 + (2*tan(c/2 + (d*x)/2)*(21*a^2 - 2*b^2))/b^3 + (6*tan(c/
2 + (d*x)/2)^3*(15*a^2 - 2*b^2))/b^3 + (tan(c/2 + (d*x)/2)^6*(12*a^4 - 2*b^
4 + 15*a^2*b^2))/(a*b^4) + (tan(c/2 + (d*x)/2)^2*(36*a^4 - 2*b^4 + 29*a^2*b
^2))/(a*b^4) + (tan(c/2 + (d*x)/2)^4*(3*a^2 + 4*b^2)*(12*a^2 - b^2))/(a*b^4
))/(d*(tan(c/2 + (d*x)/2)^2*(4*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^6*(4*a^2 +
 4*b^2) + tan(c/2 + (d*x)/2)^4*(6*a^2 + 8*b^2) + a^2*tan(c/2 + (d*x)/2)^8 +
 a^2 + 12*a*b*tan(c/2 + (d*x)/2)^3 + 12*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*ta
n(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2))) + (atan((864*a^3*tan(c/2 +
(d*x)/2))/(216*a*b^2 - 864*a^3) - (216*a*tan(c/2 + (d*x)/2))/(216*a - (864*
a^3)/b^2))*(a^2*6i - (b^2*3i)/2)*2i)/(b^5*d) + (a*atan(((a*(-(a + b)*(a - b
))^(1/2)*(4*a^2 - 3*b^2)*((8*(9*a^2*b^8 - 72*a^4*b^6 + 144*a^6*b^4))/b^11 +
 (8*tan(c/2 + (d*x)/2)*(18*a*b^10 - 234*a^3*b^8 + 576*a^5*b^6 - 288*a^7*b^4
))/b^12 - (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((8*(6*a*b^12 - 12*
a^3*b^10))/b^11 + (8*tan(c/2 + (d*x)/2)*(36*a^2*b^12 - 48*a^4*b^10))/b^12 -
 (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*(32*a^2*b^3 + (8*tan(c/2 + (
d*x)/2)*(12*a*b^16 - 8*a^3*b^14))/b^12))/(2*(b^7 - a^2*b^5))))/(2*(b^7 - a^
2*b^5)))*3i)/(2*(b^7 - a^2*b^5)) + (a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b
^2)*((8*(9*a^2*b^8 - 72*a^4*b^6 + 144*a^6*b^4))/b^11 + (8*tan(c/2 + (d*x)/2
)*(18*a*b^10 - 234*a^3*b^8 + 576*a^5*b^6 - 288*a^7*b^4))/b^12 + (3*a*(-(a +
 b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((8*(6*a*b^12 - 12*a^3*b^10))/b^11 + (8*
tan(c/2 + (d*x)/2)*(36*a^2*b^12 - 48*a^4*b^10))/b^12 + (3*a*(-(a + b)*(a -
b))^(1/2)*(4*a^2 - 3*b^2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^16 -
8*a^3*b^14))/b^12))/(2*(b^7 - a^2*b^5))))/(2*(b^7 - a^2*b^5)))*3i)/(2*(b^7
- a^2*b^5)))/((16*(432*a^7 + 81*a^3*b^4 - 432*a^5*b^2))/b^11 + (16*tan(c/2
+ (d*x)/2)*(1728*a^8 - 81*a^2*b^6 + 756*a^4*b^4 - 2160*a^6*b^2))/b^12 + (3*
a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((8*(9*a^2*b^8 - 72*a^4*b^6 + 14
4*a^6*b^4))/b^11 + (8*tan(c/2 + (d*x)/2)*(18*a*b^10 - 234*a^3*b^8 + 576*a^5
*b^6 - 288*a^7*b^4))/b^12 - (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*(
(8*(6*a*b^12 - 12*a^3*b^10))/b^11 + (8*tan(c/2 + (d*x)/2)*(36*a^2*b^12 - 48
*a^4*b^10))/b^12 - (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*(32*a^2*b^
3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^16 - 8*a^3*b^14))/b^12))/(2*(b^7 - a^2*b^
5))))/(2*(b^7 - a^2*b^5))))/(2*(b^7 - a^2*b^5)) - (3*a*(-(a + b)*(a - b))^(
1/2)*(4*a^2 - 3*b^2)*((8*(9*a^2*b^8 - 72*a^4*b^6 + 144*a^6*b^4))/b^11 + (8*
tan(c/2 + (d*x)/2)*(18*a*b^10 - 234*a^3*b^8 + 576*a^5*b^6 - 288*a^7*b^4))/b
```

```
^12 + (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((8*(6*a*b^12 - 12*a^3*
b^10))/b^11 + (8*tan(c/2 + (d*x)/2)*(36*a^2*b^12 - 48*a^4*b^10))/b^12 + (3*
a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)
/2)*(12*a*b^16 - 8*a^3*b^14))/b^12))/(2*(b^7 - a^2*b^5))))/(2*(b^7 - a^2*b^
5))))/(2*(b^7 - a^2*b^5))))*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*3i)/(d
*(b^7 - a^2*b^5))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1138 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=175

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\left(3a^2+2b^2\right)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}-\frac{\left(a^2-b^2\right)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}-\frac{\left(2a^4-a^2b^2+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(\cdots}{\sqrt{a^{\cdots}}}\right.}{a^3b^3d\sqrt{a^2-b^2}}$$

```
[Out] x/b^3-arctanh(cos(d*x+c))/a^3/d-1/2*(a^2-b^2)*cos(d*x+c)/a/b^2/d/(a+b*sin(d
*x+c))^2+1/2*(3*a^2+2*b^2)*cos(d*x+c)/a^2/b^2/d/(a+b*sin(d*x+c))-(2*a^4-a^2
*b^2+2*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/b^3/d/(a^2
-b^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2891, 3057, 2660, 618, 204, 3770}

$$-\frac{\left(-a^2b^2+2a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d\sqrt{a^2-b^2}}+\frac{\left(3a^2+2b^2\right)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}-\frac{\left(a^2-b^2\right)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}-\frac{\tanh^{-1}(\mathrm{co}\cdots}{a^{3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] x/b^3 - ((2*a^4 - a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(a^3*b^3*Sqrt[a^2 - b^2]*d) - ArcTanh[Cos[c + d*x]]/(a^3*d) - ((a
^2 - b^2)*Cos[c + d*x])/(2*a*b^2*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 + 2*b^
2)*Cos[c + d*x])/(2*a^2*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\left(a^2-b^2\right)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{\left(3a^2+2b^2\right)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))} - \frac{\int \frac{\csc(c+dx)\left(-2b^2-ab\sin\right.}{a+b\sin}}{2a}$$

$$= \frac{x}{b^3} - \frac{\left(a^2-b^2\right)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{\left(3a^2+2b^2\right)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))} + \frac{\int \csc(c+dx)\,d}{a^3}$$

$$= \frac{x}{b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{\left(a^2-b^2\right)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{\left(3a^2+2b^2\right)\cos(c+}{2a^2b^2d(a+b\sin(c+}$$

$$= \frac{x}{b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{\left(a^2-b^2\right)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{\left(3a^2+2b^2\right)\cos(c+}{2a^2b^2d(a+b\sin(c+}$$

$$= \frac{x}{b^3} - \frac{\left(2a^4-a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^3\sqrt{a^2-b^2}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{(a}{2ab^2}$$

Mathematica [A] time = 1.78, size = 176, normalized size = 1.01

$$\frac{2\left(\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{c+dx}{b^3}\right) + \frac{\cos(c+dx)\left(2a^3+b\left(3a^2+2b^2\right)\sin(c+dx)+3ab^2\right)}{a^2b^2(a+b\sin(c+dx))^2} - \frac{2\left(2a^4-a^2b^2+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right.}{\sqrt{a^2-b^2}}\right.}{a^3b^3\sqrt{a^2-b^2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 - a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^3*b^3*Sqrt[a^2 - b^2]) + 2*((c + d*x)/b^3 - Log[Cos[(c + d*x)/2]]/
a^3 + Log[Sin[(c + d*x)/2]]/a^3) + (Cos[c + d*x]*(2*a^3 + 3*a*b^2 + b*(3*a^
2 + 2*b^2)*Sin[c + d*x]))/(a^2*b^2*(a + b*Sin[c + d*x])^2))/(2*d)
```

fricas [B] time = 1.42, size = 1042, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^5*b^2 - a^3*b^4)*d*x*cos(d*x + c)^2 - 4*(a^7 - a^3*b^4)*d*x + (2
*a^6 + a^4*b^2 + a^2*b^4 + 2*b^6 - (2*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x +
```

```
c)^2 + 2*(2*a^5*b - a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*co
s(d*x + c) + 2*(a^4*b^3 - b^7 - (a^2*b^5 - b^7)*cos(d*x + c)^2 + 2*(a^3*b^4
 - a*b^6)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 2*(a^4*b^3 - b^7 - (a
^2*b^5 - b^7)*cos(d*x + c)^2 + 2*(a^3*b^4 - a*b^6)*sin(d*x + c))*log(-1/2*c
os(d*x + c) + 1/2) - 2*(4*(a^6*b - a^4*b^3)*d*x + (3*a^5*b^2 - a^3*b^4 - 2*
a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^5 - a^3*b^7)*d*cos(d*x + c)^2 -
2*(a^6*b^4 - a^4*b^6)*d*sin(d*x + c) - (a^7*b^3 - a^3*b^7)*d), 1/2*(2*(a^5*
b^2 - a^3*b^4)*d*x*cos(d*x + c)^2 - 2*(a^7 - a^3*b^4)*d*x - (2*a^6 + a^4*b^
2 + a^2*b^4 + 2*b^6 - (2*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(2*a
^5*b - a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
+ c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^6*b + a^4*b^3 - 3*a^2*b^5)
*cos(d*x + c) + (a^4*b^3 - b^7 - (a^2*b^5 - b^7)*cos(d*x + c)^2 + 2*(a^3*b^
4 - a*b^6)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (a^4*b^3 - b^7 - (a^
2*b^5 - b^7)*cos(d*x + c)^2 + 2*(a^3*b^4 - a*b^6)*sin(d*x + c))*log(-1/2*co
s(d*x + c) + 1/2) - (4*(a^6*b - a^4*b^3)*d*x + (3*a^5*b^2 - a^3*b^4 - 2*a*b
^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^5 - a^3*b^7)*d*cos(d*x + c)^2 - 2*(
a^6*b^4 - a^4*b^6)*d*sin(d*x + c) - (a^7*b^3 - a^3*b^7)*d)]
```

giac [A] time = 0.23, size = 275, normalized size = 1.57

$$\frac{\frac{dx+c}{b^3} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} - \frac{\left(2\,a^4-a^2b^2+2\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^3b^3} + \frac{a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((d*x + c)/b^3 + log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (2*a^4 - a^2*b^2 + 2*
b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^3*b^3) + (a^3*b*tan(1/2*d*x +
 1/2*c)^3 + 4*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*a^4*tan(1/2*d*x + 1/2*c)^2 +
 7*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 6*b^4*tan(1/2*d*x + 1/2*c)^2 + 7*a^3*b*
tan(1/2*d*x + 1/2*c) + 8*a*b^3*tan(1/2*d*x + 1/2*c) + 2*a^4 + 3*a^2*b^2)/((
a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^3*b^2))/d
```

maple [B] time = 0.83, size = 600, normalized size = 3.43

$$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^3}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)^2}+\frac{4b\left(\right.}{d\,a^2\left(\left(\tan^2\left(\frac{dx}{2}+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 2/d/b^3*arctan(tan(1/2*d*x+1/2*c))+1/d/a^3*ln(tan(1/2*d*x+1/2*c))+1/d/b/(ta
n(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+4/d/a
^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)
^3+2/d*a/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*
x+1/2*c)^2+7/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/
2*d*x+1/2*c)^2+6/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a
)^2*tan(1/2*d*x+1/2*c)^2+7/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)
*b+a)^2*tan(1/2*d*x+1/2*c)+8/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2*a+3/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)
^2-2/d*a/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))+1/d/a/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))-2/d/a^3*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.50, size = 3001, normalized size = 17.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + b*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) + ((2*a^2 + 3*b^2)/(a*b^2) + (tan(c/2 + (d*
x)/2)*(7*a^2 + 8*b^2))/(a^2*b) + (tan(c/2 + (d*x)/2)^3*(a^2 + 4*b^2))/(a^2*
b) + (tan(c/2 + (d*x)/2)^2*(2*a^4 + 6*b^4 + 7*a^2*b^2))/(a^3*b^2))/(d*(tan(
c/2 + (d*x)/2)^2*(2*a^2 + 4*b^2) + a^2*tan(c/2 + (d*x)/2)^4 + a^2 + 4*a*b*t
an(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2))) - (2*atan((144*tan(c/2 + (
d*x)/2))/((64*b^2)/a^2 - (64*b^4)/a^4 - (64*a*tan(c/2 + (d*x)/2))/b + (64*b
*tan(c/2 + (d*x)/2))/a + (144*a^3*tan(c/2 + (d*x)/2))/b^3 - 144) + (144*a)/
(144*a*tan(c/2 + (d*x)/2) - (144*b^3)/a^2 + (64*b^5)/a^4 - (64*b^7)/a^6 - (
64*b^2*tan(c/2 + (d*x)/2))/a + (64*b^4*tan(c/2 + (d*x)/2))/a^3) + 64/(64*ta
n(c/2 + (d*x)/2) - (144*a)/b + (64*b)/a - (64*b^3)/a^3 - (64*a^2*tan(c/2 +
(d*x)/2))/b^2 + (144*a^4*tan(c/2 + (d*x)/2))/b^4) + (64*tan(c/2 + (d*x)/2))
/((64*b^2)/a^2 + (144*a^2)/b^2 - (64*a*tan(c/2 + (d*x)/2))/b + (64*a^3*tan(
c/2 + (d*x)/2))/b^3 - (144*a^5*tan(c/2 + (d*x)/2))/b^5 - 64) + 64/(64*tan(c
/2 + (d*x)/2) + (144*b)/a - (64*b^3)/a^3 + (64*b^5)/a^5 - (64*b^2*tan(c/2 +
 (d*x)/2))/a^2 - (144*a^2*tan(c/2 + (d*x)/2))/b^2) - (64*b*tan(c/2 + (d*x)/
2))/(64*b - (64*a^2)/b + (144*a^4)/b^3 - (64*a^3*tan(c/2 + (d*x)/2))/b^2 +
(64*a^5*tan(c/2 + (d*x)/2))/b^4 - (144*a^7*tan(c/2 + (d*x)/2))/b^6)))/(b^3*
d) + (atan((((-(a + b)*(a - b))^(1/2)*((8*(14*a^9 + 4*a^3*b^6 + 28*a^5*b^4
- 15*a^7*b^2))/(a^6*b^5) + ((-(a + b)*(a - b))^(1/2)*((8*(16*a^2*b^10 - 12*
a^4*b^8 + 14*a^6*b^6 + 16*a^8*b^4 - 12*a^10*b^2))/(a^6*b^5) + ((-(a + b)*(a
 - b))^(1/2)*(a^4 + b^4 - (a^2*b^2)/2)*((8*(32*a^5*b^10 - 24*a^7*b^8 + 14*a
^9*b^6))/(a^6*b^5) + ((-(a + b)*(a - b))^(1/2)*((8*(16*a^8*b^10 - 12*a^10*b
^8))/(a^6*b^5) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^14 - 68*a^9*b^12 + 8*a^11*
b^10))/(a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2))/(a^3*b^5 - a^5*b^3) + (8*tan(c
/2 + (d*x)/2)*(64*a^4*b^14 - 68*a^6*b^12 + 48*a^8*b^10 - 16*a^10*b^8))/(a^6
*b^8)))/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*x)/2)*(4*a^3*b^12 - 36*a^5*b^
10 + 89*a^7*b^8 - 100*a^9*b^6 + 8*a^11*b^4))/(a^6*b^8))*(a^4 + b^4 - (a^2*b
^2)/2))/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*x)/2)*(4*b^12 - 4*a^2*b^10 +
73*a^4*b^8 - 68*a^6*b^6 + 48*a^8*b^4 - 16*a^10*b^2))/(a^6*b^8))*(a^4 + b^4
- (a^2*b^2)/2)*1i)/(a^3*b^5 - a^5*b^3) + ((-(a + b)*(a - b))^(1/2)*((8*(14*
a^9 + 4*a^3*b^6 + 28*a^5*b^4 - 15*a^7*b^2))/(a^6*b^5) - ((-(a + b)*(a - b))
^(1/2)*((8*(16*a^2*b^10 - 12*a^4*b^8 + 14*a^6*b^6 + 16*a^8*b^4 - 12*a^10*b^
2))/(a^6*b^5) - ((-(a + b)*(a - b))^(1/2)*(a^4 + b^4 - (a^2*b^2)/2)*((8*(32
*a^5*b^10 - 24*a^7*b^8 + 14*a^9*b^6))/(a^6*b^5) - ((-(a + b)*(a - b))^(1/2)
*((8*(16*a^8*b^10 - 12*a^10*b^8))/(a^6*b^5) + (8*tan(c/2 + (d*x)/2)*(64*a^7
*b^14 - 68*a^9*b^12 + 8*a^11*b^10))/(a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2))/(
a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^14 - 68*a^6*b^12 + 48*
a^8*b^10 - 16*a^10*b^8))/(a^6*b^8)))/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*
x)/2)*(4*a^3*b^12 - 36*a^5*b^10 + 89*a^7*b^8 - 100*a^9*b^6 + 8*a^11*b^4))/(
a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2))/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*
x)/2)*(4*b^12 - 4*a^2*b^10 + 73*a^4*b^8 - 68*a^6*b^6 + 48*a^8*b^4 - 16*a^10
*b^2))/(a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2)*1i)/(a^3*b^5 - a^5*b^3))/((16*(
14*a^6 + 4*b^6 + 12*a^2*b^4 - 3*a^4*b^2))/(a^6*b^5) + ((-(a + b)*(a - b))^(
1/2)*((8*(14*a^9 + 4*a^3*b^6 + 28*a^5*b^4 - 15*a^7*b^2))/(a^6*b^5) + ((-(a
+ b)*(a - b))^(1/2)*((8*(16*a^2*b^10 - 12*a^4*b^8 + 14*a^6*b^6 + 16*a^8*b^4
```

```
 - 12*a^10*b^2))/(a^6*b^5) + ((-(a + b)*(a - b))^(1/2)*(a^4 + b^4 - (a^2*b^
2)/2)*((8*(32*a^5*b^10 - 24*a^7*b^8 + 14*a^9*b^6))/(a^6*b^5) + ((-(a + b)*(
a - b))^(1/2)*((8*(16*a^8*b^10 - 12*a^10*b^8))/(a^6*b^5) + (8*tan(c/2 + (d*
x)/2)*(64*a^7*b^14 - 68*a^9*b^12 + 8*a^11*b^10))/(a^6*b^8))*(a^4 + b^4 - (a
^2*b^2)/2))/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^14 - 68*a
^6*b^12 + 48*a^8*b^10 - 16*a^10*b^8))/(a^6*b^8)))/(a^3*b^5 - a^5*b^3) + (8*
tan(c/2 + (d*x)/2)*(4*a^3*b^12 - 36*a^5*b^10 + 89*a^7*b^8 - 100*a^9*b^6 + 8
*a^11*b^4))/(a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2))/(a^3*b^5 - a^5*b^3) + (8*
tan(c/2 + (d*x)/2)*(4*b^12 - 4*a^2*b^10 + 73*a^4*b^8 - 68*a^6*b^6 + 48*a^8*
b^4 - 16*a^10*b^2))/(a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2))/(a^3*b^5 - a^5*b^
3) - ((-(a + b)*(a - b))^(1/2)*((8*(14*a^9 + 4*a^3*b^6 + 28*a^5*b^4 - 15*a^
7*b^2))/(a^6*b^5) - ((-(a + b)*(a - b))^(1/2)*((8*(16*a^2*b^10 - 12*a^4*b^8
 + 14*a^6*b^6 + 16*a^8*b^4 - 12*a^10*b^2))/(a^6*b^5) - ((-(a + b)*(a - b))^
(1/2)*(a^4 + b^4 - (a^2*b^2)/2)*((8*(32*a^5*b^10 - 24*a^7*b^8 + 14*a^9*b^6)
)/(a^6*b^5) - ((-(a + b)*(a - b))^(1/2)*((8*(16*a^8*b^10 - 12*a^10*b^8))/(a
^6*b^5) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^14 - 68*a^9*b^12 + 8*a^11*b^10))/
(a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2))/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d
*x)/2)*(64*a^4*b^14 - 68*a^6*b^12 + 48*a^8*b^10 - 16*a^10*b^8))/(a^6*b^8)))
/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*x)/2)*(4*a^3*b^12 - 36*a^5*b^10 + 89
*a^7*b^8 - 100*a^9*b^6 + 8*a^11*b^4))/(a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2))
/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*x)/2)*(4*b^12 - 4*a^2*b^10 + 73*a^4*
b^8 - 68*a^6*b^6 + 48*a^8*b^4 - 16*a^10*b^2))/(a^6*b^8))*(a^4 + b^4 - (a^2*
b^2)/2))/(a^3*b^5 - a^5*b^3) - (16*tan(c/2 + (d*x)/2)*(32*a^9 + 32*a^5*b^4
- 16*a^7*b^2))/(a^6*b^8)))*(-(a + b)*(a - b))^(1/2)*(a^4 + b^4 - (a^2*b^2)/
2)*2i)/(d*(a^3*b^5 - a^5*b^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

3.1139 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=182

$$\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{\left(a^2-3b^2\right)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2}-\frac{3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\sqrt{a^2-b^2}}-\frac{\left(a^2+6b^2\right)\cos(c+dx)}{2a^3bd(a+b\sin(c+dx))}$$

```
[Out] 3*b*arctanh(cos(d*x+c))/a^4/d+1/2*(a^2-3*b^2)*cos(d*x+c)/a^2/b/d/(a+b*sin(d
*x+c))^2-cot(d*x+c)/a/d/(a+b*sin(d*x+c))^2-1/2*(a^2+6*b^2)*cos(d*x+c)/a^3/b
/d/(a+b*sin(d*x+c))-3*(a^2-2*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)
^(1/2))/a^4/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2890, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\sqrt{a^2-b^2}}-\frac{\left(a^2+6b^2\right)\cos(c+dx)}{2a^3bd(a+b\sin(c+dx))}+\frac{\left(a^2-3b^2\right)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2}+\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*Sq
rt[a^2 - b^2]*d) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) + ((a^2 - 3*b^2)*Cos
[c + d*x])/(2*a^2*b*d*(a + b*Sin[c + d*x])^2) - Cot[c + d*x]/(a*d*(a + b*Si
n[c + d*x])^2) - ((a^2 + 6*b^2)*Cos[c + d*x])/(2*a^3*b*d*(a + b*Sin[c + d*x
]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\left(a^2-3b^2\right)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc(c+dx)\left(-6b^2-2ab\sin(c+dx\ldots\right.}{(a+b\sin(c+\ldots}}{2a^2b\ldots}$$

$$= \frac{\left(a^2-3b^2\right)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^2} - \frac{\left(a^2+6b^2\right)\cos(c+dx)}{2a^3bd(a+b\sin(c+dx))}\ldots$$

$$= \frac{\left(a^2-3b^2\right)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^2} - \frac{\left(a^2+6b^2\right)\cos(c+dx)}{2a^3bd(a+b\sin(c+dx))}\ldots$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\left(a^2-3b^2\right)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^2} - \ldots$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\left(a^2-3b^2\right)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^2} - \ldots$$

$$= -\frac{3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}\,d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\left(a^2-3b^2\right)\ldots}{2a^2bd(a+b\ldots}$$

Mathematica [A] time = 2.55, size = 184, normalized size = 1.01

$$\frac{-\frac{6\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{a^2\left(a^2-b^2\right)\cos(c+dx)}{b(a+b\sin(c+dx))^2} - \frac{a\left(a^2+4b^2\right)\cos(c+dx)}{b(a+b\sin(c+dx))} + a\tan\left(\frac{1}{2}(c+dx)\right) - a\cot\left(\frac{1}{2}(c+dx)\right) - 6b\log\ldots}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-6*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a
^2 - b^2] - a*Cot[(c + d*x)/2] + 6*b*Log[Cos[(c + d*x)/2]] - 6*b*Log[Sin[(c
 + d*x)/2]] + (a^2*(a^2 - b^2)*Cos[c + d*x])/(b*(a + b*Sin[c + d*x])^2) - (
a*(a^2 + 4*b^2)*Cos[c + d*x])/(b*(a + b*Sin[c + d*x])) + a*Tan[(c + d*x)/2]
)/(2*a^4*d)
```

fricas [B] time = 0.94, size = 1064, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(2*(a^5 + 5*a^3*b^2 - 6*a*b^4)*cos(d*x + c)^3 - 18*(a^4*b - a^2*b^3)*
cos(d*x + c)*sin(d*x + c) + 3*(2*a^3*b - 4*a*b^3 - 2*(a^3*b - 2*a*b^3)*cos(
d*x + c)^2 + (a^4 - a^2*b^2 - 2*b^4 - (a^2*b^2 - 2*b^4)*cos(d*x + c)^2)*sin
(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d
*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt
(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(a
^5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c) + 6*(2*a^3*b^2 - 2*a*b^4 - 2*(a^3*b^2
- a*b^4)*cos(d*x + c)^2 + (a^4*b - b^5 - (a^2*b^3 - b^5)*cos(d*x + c)^2)*si
n(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 6*(2*a^3*b^2 - 2*a*b^4 - 2*(a^3*b
^2 - a*b^4)*cos(d*x + c)^2 + (a^4*b - b^5 - (a^2*b^3 - b^5)*cos(d*x + c)^2)
*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(2*(a^7*b - a^5*b^3)*d*cos(d*x
 + c)^2 - 2*(a^7*b - a^5*b^3)*d + ((a^6*b^2 - a^4*b^4)*d*cos(d*x + c)^2 - (
a^8 - a^4*b^4)*d)*sin(d*x + c)), -1/2*((a^5 + 5*a^3*b^2 - 6*a*b^4)*cos(d*x
+ c)^3 - 9*(a^4*b - a^2*b^3)*cos(d*x + c)*sin(d*x + c) + 3*(2*a^3*b - 4*a*b
^3 - 2*(a^3*b - 2*a*b^3)*cos(d*x + c)^2 + (a^4 - a^2*b^2 - 2*b^4 - (a^2*b^2
 - 2*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
+ c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 3*(a^5 + a^3*b^2 - 2*a*b^4)*cos
(d*x + c) + 3*(2*a^3*b^2 - 2*a*b^4 - 2*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 + (
a^4*b - b^5 - (a^2*b^3 - b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x
 + c) + 1/2) - 3*(2*a^3*b^2 - 2*a*b^4 - 2*(a^3*b^2 - a*b^4)*cos(d*x + c)^2
+ (a^4*b - b^5 - (a^2*b^3 - b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos
(d*x + c) + 1/2))/(2*(a^7*b - a^5*b^3)*d*cos(d*x + c)^2 - 2*(a^7*b - a^5*b^
3)*d + ((a^6*b^2 - a^4*b^4)*d*cos(d*x + c)^2 - (a^8 - a^4*b^4)*d)*sin(d*x +
 c))]
```

giac [A] time = 0.26, size = 273, normalized size = 1.50

$$-\frac{\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}+\frac{6\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\left(a^2-2\,b^2\right)}{\sqrt{a^2-b^2}\,a^4}-\frac{6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(a^3\tan\left(\frac{1}{2}\cdots\right.\right.}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - tan(1/2*d*x + 1/2*c)/a^3 + 6
*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c)
+ b)/sqrt(a^2 - b^2)))*(a^2 - 2*b^2)/(sqrt(a^2 - b^2)*a^4) - (6*b*tan(1/2*d
*x + 1/2*c) - a)/(a^4*tan(1/2*d*x + 1/2*c)) - 2*(a^3*tan(1/2*d*x + 1/2*c)^3
 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 10*b^3
*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 14*a*b^2*tan(1/2*d*x +
 1/2*c) - 5*a^2*b)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) +
a)^2*a^4))/d
```

maple [B] time = 0.79, size = 489, normalized size = 2.69

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}-\frac{1}{2d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^4}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)^2}-\frac{}{d\,a^3\left(\left(\mathrm{ta}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^4*b*ln(tan(
1/2*d*x+1/2*c))+1/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*t
an(1/2*d*x+1/2*c)^3-6/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2*tan(1/2*d*x+1/2*c)^3*b^2-5/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x
+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b-10/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*ta
n(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^3-1/d/a/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-14/d/a^3/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^2-5/d/a^2/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b-3/d/a^2/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d/a^4/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.88, size = 956, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - (tan(c/2 + (d*x)/2)^2*(4*a^2 + 32*b^2) + a^2
 - tan(c/2 + (d*x)/2)^4*(a^2 - 12*b^2) + (2*tan(c/2 + (d*x)/2)^3*(7*a^2*b +
 10*b^3))/a + 14*a*b*tan(c/2 + (d*x)/2))/(d*(2*a^5*tan(c/2 + (d*x)/2)^5 + t
an(c/2 + (d*x)/2)^3*(4*a^5 + 8*a^3*b^2) + 2*a^5*tan(c/2 + (d*x)/2) + 8*a^4*
b*tan(c/2 + (d*x)/2)^2 + 8*a^4*b*tan(c/2 + (d*x)/2)^4)) - (3*b*log(tan(c/2
+ (d*x)/2)))/(a^4*d) - (atan((((-(a + b)*(a - b))^(1/2)*(a^2 - 2*b^2)*((3*a
^6 - 12*a^4*b^2)/a^6 + (tan(c/2 + (d*x)/2)*(12*a^4*b - 24*a^2*b^3))/a^5 - (
3*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^8 - 8*a^6*b^
2))/a^5)*(a^2 - 2*b^2))/(2*(a^6 - a^4*b^2)))*3i)/(2*(a^6 - a^4*b^2)) + ((-(
a + b)*(a - b))^(1/2)*(a^2 - 2*b^2)*((3*a^6 - 12*a^4*b^2)/a^6 + (tan(c/2 +
(d*x)/2)*(12*a^4*b - 24*a^2*b^3))/a^5 + (3*(-(a + b)*(a - b))^(1/2)*(2*a^2*
b - (tan(c/2 + (d*x)/2)*(6*a^8 - 8*a^6*b^2))/a^5)*(a^2 - 2*b^2))/(2*(a^6 -
a^4*b^2)))*3i)/(2*(a^6 - a^4*b^2)))/((2*(9*a^2*b - 18*b^3))/a^6 + (2*tan(c/
2 + (d*x)/2)*(9*a^2 - 18*b^2))/a^5 + (3*(-(a + b)*(a - b))^(1/2)*(a^2 - 2*b
^2)*((3*a^6 - 12*a^4*b^2)/a^6 + (tan(c/2 + (d*x)/2)*(12*a^4*b - 24*a^2*b^3)
)/a^5 - (3*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^8 -
 8*a^6*b^2))/a^5)*(a^2 - 2*b^2))/(2*(a^6 - a^4*b^2))))/(2*(a^6 - a^4*b^2))
- (3*(-(a + b)*(a - b))^(1/2)*(a^2 - 2*b^2)*((3*a^6 - 12*a^4*b^2)/a^6 + (ta
n(c/2 + (d*x)/2)*(12*a^4*b - 24*a^2*b^3))/a^5 + (3*(-(a + b)*(a - b))^(1/2)
*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^8 - 8*a^6*b^2))/a^5)*(a^2 - 2*b^2))/(2
*(a^6 - a^4*b^2))))/(2*(a^6 - a^4*b^2))))*(-(a + b)*(a - b))^(1/2)*(a^2 - 2
*b^2)*3i)/(d*(a^6 - a^4*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

3.1140 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=218

$$-\frac{3b\cot(c+dx)}{a^3d(a+b\sin(c+dx))}+\frac{(a^2-2b^2)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^2}+\frac{3b(3a^2-4b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}+\frac{3(a^2-4b^2)\tanh^{-1}(\cdots}{2a^5d}$$

```
[Out] 3/2*(a^2-4*b^2)*arctanh(cos(d*x+c))/a^5/d-1/2*(a^2-12*b^2)*cot(d*x+c)/a^4/b
/d+1/2*(a^2-2*b^2)*cot(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^2-1/2*cot(d*x+c)*csc
(d*x+c)/a/d/(a+b*sin(d*x+c))^2-3*b*cot(d*x+c)/a^3/d/(a+b*sin(d*x+c))+3*b*(3
*a^2-4*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^5/d/(a^2-b^2
)^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2890, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{3b(3a^2-4b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}-\frac{(a^2-12b^2)\cot(c+dx)}{2a^4bd}+\frac{3(a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{2a^5d}+\frac{(a^2-2b^2)\cdots}{2a^2bd(a+b\sin\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5
*Sqrt[a^2 - b^2]*d) + (3*(a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^5*d) - (
(a^2 - 12*b^2)*Cot[c + d*x])/(2*a^4*b*d) + ((a^2 - 2*b^2)*Cot[c + d*x])/(2*
a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d*(a + b
*Sin[c + d*x])^2) - (3*b*Cot[c + d*x])/(a^3*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\left(a^2-2b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^2(c+dx)\left(2\left(a^2-6b^2\right)-2ab\sin\right.}{(a+b\sin(c}}{4a^2}$$

$$= \frac{\left(a^2-2b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} - \frac{3b\cot(c+dx)}{a^3d(a+b\sin(c+dx))} +$$

$$= -\frac{\left(a^2-12b^2\right)\cot(c+dx)}{2a^4bd} + \frac{\left(a^2-2b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

$$= -\frac{\left(a^2-12b^2\right)\cot(c+dx)}{2a^4bd} + \frac{\left(a^2-2b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

$$= \frac{3\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d} - \frac{\left(a^2-12b^2\right)\cot(c+dx)}{2a^4bd} + \frac{\left(a^2-2b^2\right)\cot(c}{2a^2bd(a+b\sin(c}$$

$$= \frac{3\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d} - \frac{\left(a^2-12b^2\right)\cot(c+dx)}{2a^4bd} + \frac{\left(a^2-2b^2\right)\cot(c}{2a^2bd(a+b\sin(c}$$

$$= \frac{3b\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} + \frac{3\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d} - \frac{\left(a^2\right.}{}$$

Mathematica [A] time = 6.18, size = 319, normalized size = 1.46

$$-\frac{3b\tan\left(\frac{1}{2}(c+dx)\right)}{2a^4d} + \frac{3b\cot\left(\frac{1}{2}(c+dx)\right)}{2a^4d} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d} - \frac{3\left(a^2-4b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*(3*a^2 - 4*b^2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(
c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) + (3*b*Cot[(c + d*x
```

```
)/2])/(2*a^4*d) - Csc[(c + d*x)/2]^2/(8*a^3*d) + (3*(a^2 - 4*b^2)*Log[Cos[(
c + d*x)/2]])/(2*a^5*d) - (3*(a^2 - 4*b^2)*Log[Sin[(c + d*x)/2]])/(2*a^5*d)
 + Sec[(c + d*x)/2]^2/(8*a^3*d) + (-(a^2*Cos[c + d*x]) + b^2*Cos[c + d*x])/
(2*a^3*d*(a + b*Sin[c + d*x])^2) + (-(a^2*Cos[c + d*x]) + 6*b^2*Cos[c + d*x
])/(2*a^4*d*(a + b*Sin[c + d*x])) - (3*b*Tan[(c + d*x)/2])/(2*a^4*d)
```

fricas [B] time = 1.20, size = 1560, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(4*(a^6 - 10*a^4*b^2 + 9*a^2*b^4)*cos(d*x + c)^3 + 3*(3*a^4*b - a^2*b^
3 - 4*b^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b
^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 4*a*b^4 - (3*a^3*b^2 - 4*a*b^4)*cos(d*x
 + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) - 6*(a^6 - 7*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c) + 3*(a^6 - 4*a^4*b^2
- a^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - (a^6 - 3
*a^4*b^2 - 6*a^2*b^4 + 8*b^6)*cos(d*x + c)^2 + 2*(a^5*b - 5*a^3*b^3 + 4*a*b
^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*co
s(d*x + c) + 1/2) - 3*(a^6 - 4*a^4*b^2 - a^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a^2
*b^4 + 4*b^6)*cos(d*x + c)^4 - (a^6 - 3*a^4*b^2 - 6*a^2*b^4 + 8*b^6)*cos(d*
x + c)^2 + 2*(a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*c
os(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*((a^5*b - 13*
a^3*b^3 + 12*a*b^5)*cos(d*x + c)^3 + 3*(a^5*b + 3*a^3*b^3 - 4*a*b^5)*cos(d*
x + c))*sin(d*x + c))/((a^7*b^2 - a^5*b^4)*d*cos(d*x + c)^4 - (a^9 + a^7*b^
2 - 2*a^5*b^4)*d*cos(d*x + c)^2 + (a^9 - a^5*b^4)*d - 2*((a^8*b - a^6*b^3)*
d*cos(d*x + c)^2 - (a^8*b - a^6*b^3)*d)*sin(d*x + c)), 1/4*(4*(a^6 - 10*a^4
*b^2 + 9*a^2*b^4)*cos(d*x + c)^3 - 6*(3*a^4*b - a^2*b^3 - 4*b^5 + (3*a^2*b^
3 - 4*b^5)*cos(d*x + c)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos(d*x + c)^2 +
2*(3*a^3*b^2 - 4*a*b^4 - (3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2)*sin(d*x + c)
)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c
))) - 6*(a^6 - 7*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c) + 3*(a^6 - 4*a^4*b^2 - a
^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - (a^6 - 3*a^
4*b^2 - 6*a^2*b^4 + 8*b^6)*cos(d*x + c)^2 + 2*(a^5*b - 5*a^3*b^3 + 4*a*b^5
- (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d
*x + c) + 1/2) - 3*(a^6 - 4*a^4*b^2 - a^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a^2*b^
4 + 4*b^6)*cos(d*x + c)^4 - (a^6 - 3*a^4*b^2 - 6*a^2*b^4 + 8*b^6)*cos(d*x +
 c)^2 + 2*(a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(
d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*((a^5*b - 13*a^3
*b^3 + 12*a*b^5)*cos(d*x + c)^3 + 3*(a^5*b + 3*a^3*b^3 - 4*a*b^5)*cos(d*x +
```

```
c))*sin(d*x + c))/((a^7*b^2 - a^5*b^4)*d*cos(d*x + c)^4 - (a^9 + a^7*b^2 -
2*a^5*b^4)*d*cos(d*x + c)^2 + (a^9 - a^5*b^4)*d - 2*((a^8*b - a^6*b^3)*d*c
os(d*x + c)^2 - (a^8*b - a^6*b^3)*d)*sin(d*x + c))]
```

giac [A] time = 0.29, size = 395, normalized size = 1.81

$$-\frac{\frac{12\left(a^2-4b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^5}-\frac{24\left(3a^2b-4b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^5}-\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(12*(a^2 - 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 - 24*(3*a^2*b - 4
*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/
2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) - (a^3*tan(1/2*d*x + 1/2*
c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c))/a^6 - (6*a^4*tan(1/2*d*x + 1/2*c)^6 -
24*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 32*a
*b^3*tan(1/2*d*x + 1/2*c)^5 - 5*a^4*tan(1/2*d*x + 1/2*c)^4 + 48*a^2*b^2*tan
(1/2*d*x + 1/2*c)^4 + 16*b^4*tan(1/2*d*x + 1/2*c)^4 + 4*a^3*b*tan(1/2*d*x +
1/2*c)^3 + 112*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 12*a^4*tan(1/2*d*x + 1/2*c)^
2 + 76*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a^3*b*tan(1/2*d*x + 1/2*c) - a^4)
/((a*tan(1/2*d*x + 1/2*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x +
1/2*c))^2*a^5))/d
```

maple [B] time = 0.91, size = 642, normalized size = 2.94

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{2d\,a^4}-\frac{1}{8d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^3}+\frac{6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{d\,a^5}+\frac{3b}{2d\,a^4\tan\left(\frac{dx}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^4*tan(1/2*d*x+1/2*c)*b-1/8/d/a^3/tan
(1/2*d*x+1/2*c)^2-3/2/d/a^3*ln(tan(1/2*d*x+1/2*c))+6/d/a^5*ln(tan(1/2*d*x+1
/2*c))*b^2+3/2/d*b/a^4/tan(1/2*d*x+1/2*c)-3/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a
+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+8/d/a^4/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^3-2/d/(tan(1/2*d*
x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^2+3/d/a^3*b^2
```

```
/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+1
4/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2
*c)^2*b^4-5/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan
(1/2*d*x+1/2*c)+20/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^
2*tan(1/2*d*x+1/2*c)*b^3-2/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)
*b+a)^2+7/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^2+9/d
/a^3*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1
/2))-12/d/a^5*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(
a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.90, size = 1100, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (tan(c/2 + (d*x)/2)^2*(50*a*b^2 - 9*a^3) -
 tan(c/2 + (d*x)/2)^5*(6*a^2*b - 32*b^3) - tan(c/2 + (d*x)/2)^3*(10*a^2*b -
 104*b^3) - a^3/2 + 4*a^2*b*tan(c/2 + (d*x)/2) + (tan(c/2 + (d*x)/2)^4*(112
*b^4 - 17*a^4 + 72*a^2*b^2))/(2*a))/(d*(4*a^6*tan(c/2 + (d*x)/2)^2 + 4*a^6*
tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^4*(8*a^6 + 16*a^4*b^2) + 16*a^5*b
*tan(c/2 + (d*x)/2)^3 + 16*a^5*b*tan(c/2 + (d*x)/2)^5)) - (3*b*tan(c/2 + (d
*x)/2))/(2*a^4*d) - (log(tan(c/2 + (d*x)/2))*(3*a^2 - 12*b^2))/(2*a^5*d) -
(b*atan(((b*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)*((12*a^7*b - 24*a^5*b^
3)/a^8 - (tan(c/2 + (d*x)/2)*(3*a^7 + 48*a^3*b^4 - 36*a^5*b^2))/a^7 + (3*b*
(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^10 - 8*a^8*b^2
))/a^7)*(3*a^2 - 4*b^2))/(2*(a^7 - a^5*b^2)))*3i)/(2*(a^7 - a^5*b^2)) - (b*
(-(a + b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)*((tan(c/2 + (d*x)/2)*(3*a^7 + 48*a
^3*b^4 - 36*a^5*b^2))/a^7 - (12*a^7*b - 24*a^5*b^3)/a^8 + (3*b*(-(a + b)*(a
 - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^10 - 8*a^8*b^2))/a^7)*(3*a
^2 - 4*b^2))/(2*(a^7 - a^5*b^2)))*3i)/(2*(a^7 - a^5*b^2)))/((27*a^4*b + 144
```

```
*b^5 - 144*a^2*b^3)/a^8 + (2*tan(c/2 + (d*x)/2)*(72*b^4 - 54*a^2*b^2))/a^7
+ (3*b*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)*((12*a^7*b - 24*a^5*b^3)/a^
8 - (tan(c/2 + (d*x)/2)*(3*a^7 + 48*a^3*b^4 - 36*a^5*b^2))/a^7 + (3*b*(-(a
+ b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^10 - 8*a^8*b^2))/a^
7)*(3*a^2 - 4*b^2))/(2*(a^7 - a^5*b^2))))/(2*(a^7 - a^5*b^2)) + (3*b*(-(a +
 b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)*((tan(c/2 + (d*x)/2)*(3*a^7 + 48*a^3*b^4
 - 36*a^5*b^2))/a^7 - (12*a^7*b - 24*a^5*b^3)/a^8 + (3*b*(-(a + b)*(a - b))
^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^10 - 8*a^8*b^2))/a^7)*(3*a^2 - 4
*b^2))/(2*(a^7 - a^5*b^2))))/(2*(a^7 - a^5*b^2))))*(-(a + b)*(a - b))^(1/2)
*(3*a^2 - 4*b^2)*3i)/(d*(a^7 - a^5*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1141 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=289

$$\frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d}+\frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d}-\frac{(a^2-5b^2)}{}$$

```
[Out] -1/2*b*(9*a^2-20*b^2)*arctanh(cos(d*x+c))/a^6/d+1/6*(17*a^2-60*b^2)*cot(d*x
+c)/a^5/d-(a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)/a^4/b/d+1/6*(3*a^2-5*b^2)*cot(d
*x+c)*csc(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^2-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d
/(a+b*sin(d*x+c))^2+1/6*(3*a^2-20*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/b/d/(a+b*s
in(d*x+c))+(2*a^4-19*a^2*b^2+20*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b
^2)^(1/2))/a^6/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2724, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{(-19a^2b^2+2a^4+20b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d\sqrt{a^2-b^2}}+\frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d}-\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^6*Sqrt[a^2 - b^2]*d) - (b*(9*a^2 - 20*b^2)*ArcTanh[Cos[c + d*x]])/
(2*a^6*d) + ((17*a^2 - 60*b^2)*Cot[c + d*x])/(6*a^5*d) - ((a^2 - 5*b^2)*Cot
[c + d*x]*Csc[c + d*x])/(a^4*b*d) + ((3*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d
*x])/(6*a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*
a*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/
(6*a^3*b*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m +
 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 -
b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 +
b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*
Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
m, -1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^3(c+dx)\left(2\left(3a^2-1\right.\right.}{}}{}$$

$$= \frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} + \frac{\left(3a^2-20b^2\right)\cot(}{6a^3bd(a+b}$$

$$= -\frac{\left(a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c}{3ad(a}$$

$$= \frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d} - \frac{\left(a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{\left(3a^2-5b^2\right)\cot(c}{6a^2bd(a+b}$$

$$= \frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d} - \frac{\left(a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{\left(3a^2-5b^2\right)\cot(c}{6a^2bd(a+b}$$

$$= -\frac{b\left(9a^2-20b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d} - \frac{\left(a^2-5b^2\right)\cot}{}$$

$$= -\frac{b\left(9a^2-20b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d} - \frac{\left(a^2-5b^2\right)\cot}{}$$

$$= \frac{\left(2a^4-19a^2b^2+20b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d} - \frac{b\left(9a^2-20b^2\right)\tanh^{-1}(\cos(c+dx)}{2a^6d}$$

Mathematica [A] time = 6.20, size = 459, normalized size = 1.59

$$\frac{3b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d} - \frac{3b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^3d} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{24a^3d} + \cdots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2]
 + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) + ((2*a^2
*Cos[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(3*a^5*d) + (
3*b*Csc[(c + d*x)/2]^2)/(8*a^4*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(
24*a^3*d) + ((-9*a^2*b + 20*b^3)*Log[Cos[(c + d*x)/2]])/(2*a^6*d) + ((9*a^2
*b - 20*b^3)*Log[Sin[(c + d*x)/2]])/(2*a^6*d) - (3*b*Sec[(c + d*x)/2]^2)/(8
*a^4*d) + (Sec[(c + d*x)/2]*(-2*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/
2]))/(3*a^5*d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(2*a^4*d*(a + b*Si
n[c + d*x])^2) + (3*a^2*b*Cos[c + d*x] - 8*b^3*Cos[c + d*x])/(2*a^5*d*(a +
b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^3*d)
```

fricas [B] time = 1.68, size = 2027, normalized size = 7.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^5 - 4*(4*a^7 + 3
*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^3 - 3*(4*a^5*b - 38*a^3*b^3
+ 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^4 - 4*(2*a^5*
b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 17*a^4*b^2 + a^2*b^4 +
 20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^6)*cos(d*x + c)^4 - (2*a^6 - 15*a^
4*b^2 - 18*a^2*b^4 + 40*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)
*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*
cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(2*a^7 - 3*a^5*b^2 - 19*a^3*b
^4 + 20*a*b^6)*cos(d*x + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*
a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4
 + 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 +
(9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 -
38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1
/2) + 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 2
0*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c
)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5
+ 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*cos
(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*(14*a^6*b -
59*a^4*b^3 + 45*a^2*b^5)*cos(d*x + c)^3 - 3*(11*a^6*b - 41*a^4*b^3 + 30*a^2
*b^5)*cos(d*x + c))*sin(d*x + c))/(2*(a^9*b - a^7*b^3)*d*cos(d*x + c)^4 - 4
*(a^9*b - a^7*b^3)*d*cos(d*x + c)^2 + 2*(a^9*b - a^7*b^3)*d + ((a^8*b^2 - a
```

^6*b^4)*d*cos(d*x + c)^4 - (a^10 + a^8*b^2 - 2*a^6*b^4)*d*cos(d*x + c)^2 + (a^10 - a^6*b^4)*d)*sin(d*x + c)), 1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^5 - 4*(4*a^7 + 3*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^3 - 6*(4*a^5*b - 38*a^3*b^3 + 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^4 - 4*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 17*a^4*b^2 + a^2*b^4 + 20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^6)*cos(d*x + c)^4 - (2*a^6 - 15*a^4*b^2 - 18*a^2*b^4 + 40*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 6*(2*a^7 - 3*a^5*b^2 - 19*a^3*b^4 + 20*a*b^6)*cos(d*x + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*(14*a^6*b - 59*a^4*b^3 + 45*a^2*b^5)*cos(d*x + c)^3 - 3*(11*a^6*b - 41*a^4*b^3 + 30*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(2*(a^9*b - a^7*b^3)*d*cos(d*x + c)^4 - 4*(a^9*b - a^7*b^3)*d*cos(d*x + c)^2 + 2*(a^9*b - a^7*b^3)*d + ((a^8*b^2 - a^6*b^4)*d*cos(d*x + c)^4 - (a^10 + a^8*b^2 - 2*a^6*b^4)*d*cos(d*x + c)^2 + (a^10 - a^6*b^4)*d)*sin(d*x + c))]

giac [A] time = 0.29, size = 451, normalized size = 1.56

$$\frac{12\left(9a^2b-20b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^6}+\frac{24\left(2a^4-19a^2b^2+20b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^6}+\frac{24\left(5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-10\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/24*(12*(9*a^2*b - 20*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 + 24*(2*a^4 - 19*a^2*b^2 + 20*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) + 24*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 - a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 18*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 26*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b - 9*a^2*b^3)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^6) + (a^6*tan(1/2*d*x + 1/2*c)^3 - 9*a^5*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^6*tan(1/2*d*x + 1/2*c) + 72*a^4*b^2*tan(1/2*d*x + 1/2*c))/a^9

```
- (198*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 440*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a
^3*tan(1/2*d*x + 1/2*c)^2 + 72*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*b*tan(1
/2*d*x + 1/2*c) + a^3)/(a^6*tan(1/2*d*x + 1/2*c)^3))/d
```

maple [B] time = 0.88, size = 780, normalized size = 2.70

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^3}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{8d\,a^4}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^3}+\frac{3b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^5}-\frac{1}{24d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{5}{8d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d/a^3*tan(1/2*d*x+1/2*c)^3-3/8/d/a^4*tan(1/2*d*x+1/2*c)^2*b-5/8/d/a^3*
tan(1/2*d*x+1/2*c)+3/d/a^5*b^2*tan(1/2*d*x+1/2*c)-1/24/d/a^3/tan(1/2*d*x+1/
2*c)^3+5/8/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^5/tan(1/2*d*x+1/2*c)*b^2+3/8/d/a^
4*b/tan(1/2*d*x+1/2*c)^2+9/2/d/a^4*b*ln(tan(1/2*d*x+1/2*c))-10/d/a^6*b^3*ln
(tan(1/2*d*x+1/2*c))+5/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b
+a)^2*tan(1/2*d*x+1/2*c)^3*b^2-10/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^4+4/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2
*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b-1/d/a^4/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^3-18/d/a^6/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^5+11/d
/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)
*b^2-26/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d
*x+1/2*c)*b^4+4/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b
-9/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3+2/d/a^2/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-19/d
/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2
))*b^2+20/d/a^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^
2-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 10.04, size = 1261, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (tan(c/2 + (d*x)/2)*((3*(a^2 + 4*b^2))/(8
*a^5) + 1/(4*a^3) - (9*b^2)/(2*a^5)))/d + (tan(c/2 + (d*x)/2)^6*(5*a^4 - 80
*b^4 + 16*a^2*b^2) + tan(c/2 + (d*x)/2)^4*((29*a^4)/3 - 304*b^4 + 72*a^2*b^
2) - a^4/3 + tan(c/2 + (d*x)/2)^2*((13*a^4)/3 - (40*a^2*b^2)/3) - tan(c/2 +
 (d*x)/2)^3*(156*a*b^3 - (170*a^3*b)/3) - (tan(c/2 + (d*x)/2)^5*(144*b^5 -
55*a^4*b + 104*a^2*b^3))/a + (5*a^3*b*tan(c/2 + (d*x)/2))/3)/(d*(8*a^7*tan(
c/2 + (d*x)/2)^3 + 8*a^7*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^5*(16*a^
7 + 32*a^5*b^2) + 32*a^6*b*tan(c/2 + (d*x)/2)^4 + 32*a^6*b*tan(c/2 + (d*x)/
2)^6)) + (log(tan(c/2 + (d*x)/2))*(9*a^2*b - 20*b^3))/(2*a^6*d) - (3*b*tan(
c/2 + (d*x)/2)^2)/(8*a^4*d) + (atan((((-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^
4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*b^2)/a^10 + (tan(c/2 + (
d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 + ((-(a + b)*(a - b))^(1/
2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*(a^4 + 10*b^4
 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2))*1i)/(a^8 - a^6*b^2) + ((-(a + b)*(a -
b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*b^
2)/a^10 + (tan(c/2 + (d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 - (
(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^
2))/a^9)*(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2))*1i)/(a^8 - a^6*b
^2))/((18*a^6*b - 400*b^7 + 560*a^2*b^5 - 211*a^4*b^3)/a^10 + (2*tan(c/2 +
(d*x)/2)*(4*a^6 - 200*b^6 + 230*a^2*b^4 - 58*a^4*b^2))/a^9 - ((-(a + b)*(a
- b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*
b^2)/a^10 + (tan(c/2 + (d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 +
 ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*
b^2))/a^9)*(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2)))/(a^8 - a^6*b^
2) + ((-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*((2*a^10 + 4
0*a^6*b^4 - 28*a^8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(13*a^8*b + 80*a^4*b^5 -
 76*a^6*b^3))/a^9 - ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2
)*(6*a^12 - 8*a^10*b^2))/a^9)*(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b
^2)))/(a^8 - a^6*b^2)))*(-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^
2)/2)*2i)/(d*(a^8 - a^6*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1142 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=340

$$\frac{(2a^2-3b^2)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))^2}-\frac{b(13a^2-30b^2)\cot(c+dx)}{2a^6d}+\frac{3(7a^2-20b^2)\cot(c+dx)\csc(c+dx)}{8a^5d}-\frac{(3a^2\cdots}{}$$

```
[Out] -3/8*(a^4-24*a^2*b^2+40*b^4)*arctanh(cos(d*x+c))/a^7/d-1/2*b*(13*a^2-30*b^2
)*cot(d*x+c)/a^6/d+3/8*(7*a^2-20*b^2)*cot(d*x+c)*csc(d*x+c)/a^5/d-1/2*(3*a^
2-10*b^2)*cot(d*x+c)*csc(d*x+c)^2/a^4/b/d+1/4*(2*a^2-3*b^2)*cot(d*x+c)*csc(
d*x+c)^2/a^2/b/d/(a+b*sin(d*x+c))^2-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d/(a+b*si
n(d*x+c))^2+1/4*(4*a^2-15*b^2)*cot(d*x+c)*csc(d*x+c)^2/a^3/b/d/(a+b*sin(d*x
+c))-3*b*(2*a^4-11*a^2*b^2+10*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2
)^(1/2))/a^7/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 1.46, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2890, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{3b(-11a^2b^2+2a^4+10b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d\sqrt{a^2-b^2}}-\frac{b(13a^2-30b^2)\cot(c+dx)}{2a^6d}-\frac{3(-24a^2b^2+a^4+40b^4)\tanh\cdots}{8a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b*(2*a^4 - 11*a^2*b^2 + 10*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^7*Sqrt[a^2 - b^2]*d) - (3*(a^4 - 24*a^2*b^2 + 40*b^4)*ArcTanh
[Cos[c + d*x]])/(8*a^7*d) - (b*(13*a^2 - 30*b^2)*Cot[c + d*x])/(2*a^6*d) +
(3*(7*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^5*d) - ((3*a^2 - 10*b^2
)*Cot[c + d*x]*Csc[c + d*x]^2)/(2*a^4*b*d) + ((2*a^2 - 3*b^2)*Cot[c + d*x]*
Csc[c + d*x]^2)/(4*a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c +
d*x]^3)/(4*a*d*(a + b*Sin[c + d*x])^2) + ((4*a^2 - 15*b^2)*Cot[c + d*x]*Csc
[c + d*x]^2)/(4*a^3*b*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^4(c+dx)(6}{}}{}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))^2} + \frac{\left(4a^2-15b^2\right)}{4a^3bd}$$

$$= -\frac{\left(3a^2-10b^2\right)\cot(c+dx)\csc^2(c+dx)}{2a^4bd} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))^2}$$

$$= \frac{3\left(7a^2-20b^2\right)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{\left(3a^2-10b^2\right)\cot(c+dx)\csc^2(c+dx)}{2a^4bd}$$

$$= -\frac{b\left(13a^2-30b^2\right)\cot(c+dx)}{2a^6d} + \frac{3\left(7a^2-20b^2\right)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{\left(3a^2\right.}{}$$

$$= -\frac{b\left(13a^2-30b^2\right)\cot(c+dx)}{2a^6d} + \frac{3\left(7a^2-20b^2\right)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{\left(3a^2\right.}{}$$

$$= -\frac{3\left(a^4-24a^2b^2+40b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^7d} - \frac{b\left(13a^2-30b^2\right)\cot(c+dx)}{2a^6d} +$$

$$= -\frac{3\left(a^4-24a^2b^2+40b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^7d} - \frac{b\left(13a^2-30b^2\right)\cot(c+dx)}{2a^6d} +$$

$$= -\frac{3b\left(2a^4-11a^2b^2+10b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7\sqrt{a^2-b^2}\,d} - \frac{3\left(a^4-24a^2b^2+40b^4\right)\tan}{8a^7d}$$

Mathematica [A] time = 4.76, size = 347, normalized size = 1.02

$$-\frac{\frac{384b\left(2a^4-11a^2b^2+10b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-48\left(a^4-24a^2b^2+40b^4\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+48\left(a^4-24a^2b^2+40b^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/128*((384*b*(2*a^4 - 11*a^2*b^2 + 10*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2]
)/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 48*(a^4 - 24*a^2*b^2 + 40*b^4)*Log[Co
s[(c + d*x)/2]] - 48*(a^4 - 24*a^2*b^2 + 40*b^4)*Log[Sin[(c + d*x)/2]] + (2
*a*Cot[c + d*x]*Csc[c + d*x]^5*(-4*a^5 + 289*a^3*b^2 - 540*a*b^4 + 4*(5*a^5
 - 93*a^3*b^2 + 180*a*b^4)*Cos[2*(c + d*x)] + (83*a^3*b^2 - 180*a*b^4)*Cos[
4*(c + d*x)] + 100*a^4*b*Sin[c + d*x] + 20*a^2*b^3*Sin[c + d*x] - 600*b^5*S
in[c + d*x] - 44*a^4*b*Sin[3*(c + d*x)] - 50*a^2*b^3*Sin[3*(c + d*x)] + 300
*b^5*Sin[3*(c + d*x)] + 26*a^2*b^3*Sin[5*(c + d*x)] - 60*b^5*Sin[5*(c + d*x
)]))/(b + a*Csc[c + d*x])^2)/(a^7*d)
```

fricas [B] time = 1.88, size = 2592, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/16*(2*(83*a^6*b^2 - 263*a^4*b^4 + 180*a^2*b^6)*cos(d*x + c)^5 + 2*(5*a^8
 - 181*a^6*b^2 + 536*a^4*b^4 - 360*a^2*b^6)*cos(d*x + c)^3 + 12*(2*a^6*b -
9*a^4*b^3 - a^2*b^5 + 10*b^7 - (2*a^4*b^3 - 11*a^2*b^5 + 10*b^7)*cos(d*x +
c)^6 + (2*a^6*b - 5*a^4*b^3 - 23*a^2*b^5 + 30*b^7)*cos(d*x + c)^4 - (4*a^6*
b - 16*a^4*b^3 - 13*a^2*b^5 + 30*b^7)*cos(d*x + c)^2 + 2*(2*a^5*b^2 - 11*a^
3*b^4 + 10*a*b^6 + (2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^4 - 2*(
2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2
+ b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
- 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*c
os(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(a^8 - 32*a^6*b^2 + 91
*a^4*b^4 - 60*a^2*b^6)*cos(d*x + c) + 3*(a^8 - 24*a^6*b^2 + 39*a^4*b^4 + 24
*a^2*b^6 - 40*b^8 - (a^6*b^2 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*cos(d*x +
c)^6 + (a^8 - 22*a^6*b^2 - 11*a^4*b^4 + 152*a^2*b^6 - 120*b^8)*cos(d*x + c)
^4 - (2*a^8 - 47*a^6*b^2 + 53*a^4*b^4 + 112*a^2*b^6 - 120*b^8)*cos(d*x + c)
^2 + 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7 + (a^7*b - 25*a^5*b^3 +
64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^4 - 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5
```

```
- 40*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(
a^8 - 24*a^6*b^2 + 39*a^4*b^4 + 24*a^2*b^6 - 40*b^8 - (a^6*b^2 - 25*a^4*b^4
 + 64*a^2*b^6 - 40*b^8)*cos(d*x + c)^6 + (a^8 - 22*a^6*b^2 - 11*a^4*b^4 + 1
52*a^2*b^6 - 120*b^8)*cos(d*x + c)^4 - (2*a^8 - 47*a^6*b^2 + 53*a^4*b^4 + 1
12*a^2*b^6 - 120*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 -
 40*a*b^7 + (a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^4 - 2
*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))
*log(-1/2*cos(d*x + c) + 1/2) + 4*(2*(13*a^5*b^3 - 43*a^3*b^5 + 30*a*b^7)*c
os(d*x + c)^5 - (11*a^7*b + 21*a^5*b^3 - 152*a^3*b^5 + 120*a*b^7)*cos(d*x +
 c)^3 + 3*(3*a^7*b - a^5*b^3 - 22*a^3*b^5 + 20*a*b^7)*cos(d*x + c))*sin(d*x
 + c))/((a^9*b^2 - a^7*b^4)*d*cos(d*x + c)^6 - (a^11 + 2*a^9*b^2 - 3*a^7*b^
4)*d*cos(d*x + c)^4 + (2*a^11 + a^9*b^2 - 3*a^7*b^4)*d*cos(d*x + c)^2 - (a^
11 - a^7*b^4)*d - 2*((a^10*b - a^8*b^3)*d*cos(d*x + c)^4 - 2*(a^10*b - a^8*
b^3)*d*cos(d*x + c)^2 + (a^10*b - a^8*b^3)*d)*sin(d*x + c)), 1/16*(2*(83*a^
6*b^2 - 263*a^4*b^4 + 180*a^2*b^6)*cos(d*x + c)^5 + 2*(5*a^8 - 181*a^6*b^2
+ 536*a^4*b^4 - 360*a^2*b^6)*cos(d*x + c)^3 - 24*(2*a^6*b - 9*a^4*b^3 - a^2
*b^5 + 10*b^7 - (2*a^4*b^3 - 11*a^2*b^5 + 10*b^7)*cos(d*x + c)^6 + (2*a^6*b
 - 5*a^4*b^3 - 23*a^2*b^5 + 30*b^7)*cos(d*x + c)^4 - (4*a^6*b - 16*a^4*b^3
- 13*a^2*b^5 + 30*b^7)*cos(d*x + c)^2 + 2*(2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^
6 + (2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^4 - 2*(2*a^5*b^2 - 11*
a^3*b^4 + 10*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(
a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(a^8 - 32*a^6*b^2 +
 91*a^4*b^4 - 60*a^2*b^6)*cos(d*x + c) + 3*(a^8 - 24*a^6*b^2 + 39*a^4*b^4 +
 24*a^2*b^6 - 40*b^8 - (a^6*b^2 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*cos(d*x
 + c)^6 + (a^8 - 22*a^6*b^2 - 11*a^4*b^4 + 152*a^2*b^6 - 120*b^8)*cos(d*x +
 c)^4 - (2*a^8 - 47*a^6*b^2 + 53*a^4*b^4 + 112*a^2*b^6 - 120*b^8)*cos(d*x +
 c)^2 + 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7 + (a^7*b - 25*a^5*b^3
 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^4 - 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b
^5 - 40*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) -
3*(a^8 - 24*a^6*b^2 + 39*a^4*b^4 + 24*a^2*b^6 - 40*b^8 - (a^6*b^2 - 25*a^4*
b^4 + 64*a^2*b^6 - 40*b^8)*cos(d*x + c)^6 + (a^8 - 22*a^6*b^2 - 11*a^4*b^4
+ 152*a^2*b^6 - 120*b^8)*cos(d*x + c)^4 - (2*a^8 - 47*a^6*b^2 + 53*a^4*b^4
+ 112*a^2*b^6 - 120*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^
5 - 40*a*b^7 + (a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^4
- 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^2)*sin(d*x +
c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(2*(13*a^5*b^3 - 43*a^3*b^5 + 30*a*b^7
)*cos(d*x + c)^5 - (11*a^7*b + 21*a^5*b^3 - 152*a^3*b^5 + 120*a*b^7)*cos(d*
x + c)^3 + 3*(3*a^7*b - a^5*b^3 - 22*a^3*b^5 + 20*a*b^7)*cos(d*x + c))*sin(
d*x + c))/((a^9*b^2 - a^7*b^4)*d*cos(d*x + c)^6 - (a^11 + 2*a^9*b^2 - 3*a^7
*b^4)*d*cos(d*x + c)^4 + (2*a^11 + a^9*b^2 - 3*a^7*b^4)*d*cos(d*x + c)^2 -
(a^11 - a^7*b^4)*d - 2*((a^10*b - a^8*b^3)*d*cos(d*x + c)^4 - 2*(a^10*b - a
^8*b^3)*d*cos(d*x + c)^2 + (a^10*b - a^8*b^3)*d)*sin(d*x + c))]
```

giac [A] time = 0.35, size = 550, normalized size = 1.62

$$\frac{\frac{24\left(a^{4}-24\,a^{2}b^{2}+40\,b^{4}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{7}}-\frac{192\left(2\,a^{4}b-11\,a^{2}b^{3}+10\,b^{5}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,a^{7}}-\frac{64\left(7\,a^{3}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(24*(a^4 - 24*a^2*b^2 + 40*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^7 - 1
92*(2*a^4*b - 11*a^2*b^3 + 10*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a)
 + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a
^7) - 64*(7*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 12*a*b^5*tan(1/2*d*x + 1/2*c)^
3 + 6*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + a^2*b^4*tan(1/2*d*x + 1/2*c)^2 - 22*
b^6*tan(1/2*d*x + 1/2*c)^2 + 17*a^3*b^3*tan(1/2*d*x + 1/2*c) - 32*a*b^5*tan
(1/2*d*x + 1/2*c) + 6*a^4*b^2 - 11*a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*
b*tan(1/2*d*x + 1/2*c) + a)^2*a^7) - (50*a^4*tan(1/2*d*x + 1/2*c)^4 - 1200*
a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 2000*b^4*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*
b*tan(1/2*d*x + 1/2*c)^3 - 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 8*a^4*tan(1/2
*d*x + 1/2*c)^2 + 48*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a^3*b*tan(1/2*d*x +
 1/2*c) + a^4)/(a^7*tan(1/2*d*x + 1/2*c)^4) + (a^9*tan(1/2*d*x + 1/2*c)^4 -
 8*a^8*b*tan(1/2*d*x + 1/2*c)^3 - 8*a^9*tan(1/2*d*x + 1/2*c)^2 + 48*a^7*b^2
*tan(1/2*d*x + 1/2*c)^2 + 120*a^8*b*tan(1/2*d*x + 1/2*c) - 320*a^6*b^3*tan(
1/2*d*x + 1/2*c))/a^12)/d
```

maple [B] time = 0.89, size = 889, normalized size = 2.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -6/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^2+15/8/d/a^4
*tan(1/2*d*x+1/2*c)*b-9/d/a^5*ln(tan(1/2*d*x+1/2*c))*b^2-15/8/d*b/a^4/tan(1
/2*d*x+1/2*c)+5/d*b^3/a^6/tan(1/2*d*x+1/2*c)+11/d/a^5*b^4/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-1/8/d/a^4*tan(1/2*d*x+1/2*c)^3*b+3/4/d/a
^5*b^2*tan(1/2*d*x+1/2*c)^2-5/d/a^6*b^3*tan(1/2*d*x+1/2*c)-3/4/d/a^5/tan(1/
2*d*x+1/2*c)^2*b^2+15/d/a^7*ln(tan(1/2*d*x+1/2*c))*b^4+1/8/d/a^4*b/tan(1/2*
d*x+1/2*c)^3+1/8/d/a^3/tan(1/2*d*x+1/2*c)^2+1/64/d/a^3*tan(1/2*d*x+1/2*c)^4
-1/64/d/a^3/tan(1/2*d*x+1/2*c)^4+32/d/a^6*b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan
(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-30/d/a^7*b^5/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+22/d/a^7*b^6/(tan(1/2*
```

```
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+12/d/a^6*b^
5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-
6/d/a^3*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)
^(1/2))+3/8/d/a^3*ln(tan(1/2*d*x+1/2*c))-7/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*
tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^3-1/d/a^5/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^4-17/d/a^4/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^3+33/d/
a^5*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2))-6/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1
/2*d*x+1/2*c)^2-1/8/d/a^3*tan(1/2*d*x+1/2*c)^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 10.21, size = 1487, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^4/(64*a^3*d) - (tan(c/2 + (d*x)/2)^2*((3*(a^2 + 4*b^2))/
(32*a^5) + 1/(32*a^3) - (9*b^2)/(8*a^5)))/d + (tan(c/2 + (d*x)/2)*((6*b*((3
*(a^2 + 4*b^2))/(16*a^5) + 1/(16*a^3) - (9*b^2)/(4*a^5)))/a - (192*a^2*b +
128*b^3)/(256*a^6) + (9*b*(a^2 + 4*b^2))/(8*a^6)))/d - (tan(c/2 + (d*x)/2)^
3*(19*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^4*(448*a*b^4 + (15*a^5)/4 -
224*a^3*b^2) + tan(c/2 + (d*x)/2)^7*(30*a^4*b - 192*b^5 + 32*a^2*b^3) + tan
(c/2 + (d*x)/2)^5*(50*a^4*b - 832*b^5 + 280*a^2*b^3) + a^5/4 - tan(c/2 + (d
*x)/2)^2*((3*a^5)/2 - 5*a^3*b^2) - a^4*b*tan(c/2 + (d*x)/2) - (2*tan(c/2 +
(d*x)/2)^6*(a^6 + 176*b^6 + 152*a^2*b^4 - 114*a^4*b^2))/a)/(d*(16*a^8*tan(c
/2 + (d*x)/2)^4 + 16*a^8*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^6*(32*a^
8 + 64*a^6*b^2) + 64*a^7*b*tan(c/2 + (d*x)/2)^5 + 64*a^7*b*tan(c/2 + (d*x)/
2)^7)) - (b*tan(c/2 + (d*x)/2)^3)/(8*a^4*d) + (log(tan(c/2 + (d*x)/2))*((3*
a^4)/8 + 15*b^4 - 9*a^2*b^2))/(a^7*d) + (b*atan(((b*(-(a + b)*(a - b))^(1/2
)*(((27*a^11*b)/4 + 60*a^7*b^5 - 51*a^9*b^3)/a^12 - (tan(c/2 + (d*x)/2)*(3*
```

```
a^11 - 480*a^5*b^6 + 528*a^7*b^4 - 126*a^9*b^2))/(4*a^11) + (3*b*(-(a + b)*
(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^2))/(4*a
^11))*(2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4 -
 11*a^2*b^2)*3i)/(2*(a^9 - a^7*b^2)) - (b*(-(a + b)*(a - b))^(1/2)*((tan(c/
2 + (d*x)/2)*(3*a^11 - 480*a^5*b^6 + 528*a^7*b^4 - 126*a^9*b^2))/(4*a^11) -
 ((27*a^11*b)/4 + 60*a^7*b^5 - 51*a^9*b^3)/a^12 + (3*b*(-(a + b)*(a - b))^(
1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^2))/(4*a^11))*(2*a
^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b^
2)*3i)/(2*(a^9 - a^7*b^2)))/(((9*a^8*b)/2 + 900*b^9 - 1530*a^2*b^7 + (1593*
a^4*b^5)/2 - (531*a^6*b^3)/4)/a^12 + (tan(c/2 + (d*x)/2)*(1800*b^8 - 2610*a
^2*b^6 + 1053*a^4*b^4 - 126*a^6*b^2))/(2*a^11) + (3*b*(-(a + b)*(a - b))^(1
/2)*(((27*a^11*b)/4 + 60*a^7*b^5 - 51*a^9*b^3)/a^12 - (tan(c/2 + (d*x)/2)*(
3*a^11 - 480*a^5*b^6 + 528*a^7*b^4 - 126*a^9*b^2))/(4*a^11) + (3*b*(-(a + b
)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^2))/(4
*a^11))*(2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4
 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)) + (3*b*(-(a + b)*(a - b))^(1/2)*((tan(c
/2 + (d*x)/2)*(3*a^11 - 480*a^5*b^6 + 528*a^7*b^4 - 126*a^9*b^2))/(4*a^11)
- ((27*a^11*b)/4 + 60*a^7*b^5 - 51*a^9*b^3)/a^12 + (3*b*(-(a + b)*(a - b))^
(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^2))/(4*a^11))*(2*
a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b
^2))/(2*(a^9 - a^7*b^2))))*(-(a + b)*(a - b))^(1/2)*(2*a^4 + 10*b^4 - 11*a^
2*b^2)*3i)/(d*(a^9 - a^7*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1143 $\int \cos^4(c+dx)\sin^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=463

$$\frac{8a\left(40a^2-81b^2\right)\sin(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{3003b^4d}-\frac{10\left(16a^2-33b^2\right)\sin^2(c+dx)\cos(c+dx)(a+b\sin}{1287b^3d}$$

```
[Out] -8/45045*(480*a^4-937*a^2*b^2+231*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b^
5/d+8/3003*a*(40*a^2-81*b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b
^4/d-10/1287*(16*a^2-33*b^2)*cos(d*x+c)*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2)
/b^3/d+20/143*a*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^(3/2)/b^2/d-2/13*c
os(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c))^(3/2)/b/d+16/45045*a*(160*a^4-279*a
^2*b^2+27*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^5/d+8/45045*(320*a^6-798
*a^4*b^2+435*a^2*b^4-693*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2
))*(a+b*sin(d*x+c))^(1/2)/b^6/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-16/45045*a*(
160*a^6-439*a^4*b^2+306*a^2*b^4-27*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(
a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^6/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.05, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2895, 3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{10\left(16a^2-33b^2\right)\sin^2(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{1287b^3d}+\frac{8a\left(40a^2-81b^2\right)\sin(c+dx)\cos(c+dx)(a+b}{3003b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (16*a*(160*a^4 - 279*a^2*b^2 + 27*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]
])/(45045*b^5*d) - (8*(480*a^4 - 937*a^2*b^2 + 231*b^4)*Cos[c + d*x]*(a + b
*Sin[c + d*x])^(3/2))/(45045*b^5*d) + (8*a*(40*a^2 - 81*b^2)*Cos[c + d*x]*S
in[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(3003*b^4*d) - (10*(16*a^2 - 33*b^2
)*Cos[c + d*x]*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(1287*b^3*d) + (2
0*a*Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2))/(143*b^2*d) - (
2*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2))/(13*b*d) - (8*(32
0*a^6 - 798*a^4*b^2 + 435*a^2*b^4 - 693*b^6)*EllipticE[(c - Pi/2 + d*x)/2,
(2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(45045*b^6*d*Sqrt[(a + b*Sin[c + d
*x])/(a + b)]) + (16*a*(160*a^6 - 439*a^4*b^2 + 306*a^2*b^4 - 27*b^6)*Ellip
```

```
ticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
/(45045*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{20a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^{3/2}}{143b^2d} - \frac{2\cos(c}{}$$

$$= -\frac{10\left(16a^2-33b^2\right)\cos(c+dx)\sin^2(c+dx)(a+b\sin(c+dx)}{1287b^3d}$$

$$= \frac{8a\left(40a^2-81b^2\right)\cos(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{3}}{3003b^4d}$$

$$= -\frac{8\left(480a^4-937a^2b^2+231b^4\right)\cos(c+dx)(a+b\sin(c+dx)}{45045b^5d}$$

$$= \frac{16a\left(160a^4-279a^2b^2+27b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx}}{45045b^5d}$$

$$= \frac{16a\left(160a^4-279a^2b^2+27b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx}}{45045b^5d}$$

$$= \frac{16a\left(160a^4-279a^2b^2+27b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx}}{45045b^5d}$$

$$= \frac{16a\left(160a^4-279a^2b^2+27b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx}}{45045b^5d}$$

Mathematica [A] time = 5.10, size = 327, normalized size = 0.71

$$\frac{\sqrt{a+b\sin(c+dx)}\left(128\left(320a^6-798a^4b^2+435a^2b^4-693b^6\right)E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-256a\left(160a^5-160a^4b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*(128*(320*a^6 - 798*a^4*b^2 + 435*a^2*b^4 - 693*b
^6)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - 256*a*(160*a^5 - 160*
a^4*b - 279*a^3*b^2 + 279*a^2*b^3 + 27*a*b^4 - 27*b^5)*EllipticF[(-2*c + Pi
 - 2*d*x)/4, (2*b)/(a + b)] - 2*b*Cos[c + d*x]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)]*(10240*a^5 - 21056*a^3*b^2 + 5898*a*b^4 - 1600*(2*a^3*b^2 - 3*a*b^4)
```

```
*Cos[2*(c + d*x)] + 630*a*b^4*Cos[4*(c + d*x)] - 7680*a^4*b*Sin[c + d*x] +
13592*a^2*b^3*Sin[c + d*x] - 19866*b^5*Sin[c + d*x] + 1400*a^2*b^3*Sin[3*(c
 + d*x)] + 5775*b^5*Sin[3*(c + d*x)] + 3465*b^5*Sin[5*(c + d*x)])))/(720720
*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(dx+c\right)^{6}-\cos\left(dx+c\right)^{4}\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(cos(d*x + c)^6 - cos(d*x + c)^4)*sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

maple [B] time = 2.17, size = 1619, normalized size = 3.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/45045*(-50*a^3*b^5*sin(d*x+c)^5+10470*a*b^7*sin(d*x+c)^5+80*a^4*b^4*sin(
d*x+c)^4-232*a^2*b^6*sin(d*x+c)^4-160*a^5*b^3*sin(d*x+c)^3+454*a^3*b^5*sin(
d*x+c)^3-8322*a*b^7*sin(d*x+c)^3-640*a^6*b^2*sin(d*x+c)^2+1436*a^4*b^4*sin(
d*x+c)^2-511*a^2*b^6*sin(d*x+c)^2+160*a^5*b^3*sin(d*x+c)-404*a^3*b^5*sin(d*
x+c)+1632*a*b^7*sin(d*x+c)-3780*a*b^7*sin(d*x+c)^7+640*a^6*b^2+4512*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
a^2*b^6+35*a^2*b^6*sin(d*x+c)^6-6699*b^8*sin(d*x+c)^4+924*b^8*sin(d*x+c)^2-
3465*b^8*sin(d*x+c)^8+9240*b^8*sin(d*x+c)^6-1516*a^4*b^4+708*a^2*b^6-2772*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
```

```
1/2))*b^8+2772*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))*b^8-1280*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^8-4296*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6-216*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*
b^7+4472*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*a^6*b^2-4932*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4+1280*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*b-960*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
6*b^2-3512*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^5*b^3+2484*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4+2448*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^5)/b^7/cos(
d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin(dx+c)+a}\,\cos(dx+c)^4\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^4\sin(c+d\,x)^2\sqrt{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sin(c + dx)} \sin^2(c + dx) \cos^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(c + d*x))*sin(c + d*x)**2*cos(c + d*x)**4, x)
```

3.1144 $\int \cos^4(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=332

$$-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(8a^2-7ab\sin(c+dx)-9b^2\right)}{693b^2d}+\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(32a^4-24ab\right.}{3465b^4}$$

```
[Out] -2/11*cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2)/d-2/693*cos(d*x+c)^3*(8*a^2-9*b^2
-7*a*b*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^2/d+4/3465*cos(d*x+c)*(32*a^4-6
9*a^2*b^2+45*b^4-24*a*b*(a^2-2*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^4/
d-8/3465*a*(32*a^4-93*a^2*b^2+93*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/s
in(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+
b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^5/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+8/34
65*(32*a^6-101*a^4*b^2+114*a^2*b^4-45*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(
b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^5/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2862, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(8a^2-7ab\sin(c+dx)-9b^2\right)}{693b^2d}+\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(-24ab\left(a^2-\right.\right.}{3465b^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(11*d) + (8*a*(32*a^4 - 93*a^2
*b^2 + 93*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[
c + d*x]])/(3465*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*(32*a^6 - 1
01*a^4*b^2 + 114*a^2*b^4 - 45*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a +
 b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3465*b^5*d*Sqrt[a + b*Sin[c + d*x
]]) - (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(8*a^2 - 9*b^2 - 7*a*b*Sin
[c + d*x]))/(693*b^2*d) + (4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^4
- 69*a^2*b^2 + 45*b^4 - 24*a*b*(a^2 - 2*b^2)*Sin[c + d*x]))/(3465*b^4*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
```

```
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{2}{11}\int \frac{\cos^4(c+dx)\left(\frac{b}{2}\right.}{\sqrt{a+b\sin}}$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} - \frac{2\cos^3(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} - \frac{2\cos^3(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} - \frac{2\cos^3(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} - \frac{2\cos^3(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{8a\left(32a^4-93a^2b^2+93\right.}{}$$

Mathematica [A] time = 4.12, size = 326, normalized size = 0.98

$$\frac{64\left(32a^6-101a^4b^2+114a^2b^4-45b^6\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+b\cos(c+dx)\left(1024a^5+256a^4b\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-64*a*(32*a^5 + 32*a^4*b - 93*a^3*b^2 - 93*a^2*b^3 + 93*a*b^4 + 93*b^5)*El
lipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
+ b)] + 64*(32*a^6 - 101*a^4*b^2 + 114*a^2*b^4 - 45*b^6)*EllipticF[(-2*c +
Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*Cos[c
```

```
+ d*x]*(1024*a^5 - 2912*a^3*b^2 + 748*a*b^4 + 16*(4*a^3*b^2 - 183*a*b^4)*Co
s[2*(c + d*x)] - 700*a*b^4*Cos[4*(c + d*x)] + 256*a^4*b*Sin[c + d*x] - 692*
a^2*b^3*Sin[c + d*x] + 990*b^5*Sin[c + d*x] - 20*a^2*b^3*Sin[3*(c + d*x)] -
 765*b^5*Sin[3*(c + d*x)] - 315*b^5*Sin[5*(c + d*x)]))/(27720*b^5*d*Sqrt[a
+ b*Sin[c + d*x]])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{4}\sin\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c), x)
```

maple [B] time = 1.94, size = 1356, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3465*(-96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a^5*b^2+350*a*b^6*sin(d*x+c)^6-180*a*b^6+64*a^5*b^2-178*
a^3*b^4-5*a^2*b^5*sin(d*x+c)^5+8*a^3*b^4*sin(d*x+c)^4-1066*a*b^6*sin(d*x+c)
^4-16*a^4*b^3*sin(d*x+c)^3+52*a^2*b^5*sin(d*x+c)^3-64*a^5*b^2*sin(d*x+c)^2+
170*a^3*b^4*sin(d*x+c)^2+896*a*b^6*sin(d*x+c)^2+16*a^4*b^3*sin(d*x+c)-47*a^
2*b^5*sin(d*x+c)+315*b^7*sin(d*x+c)^7-900*b^7*sin(d*x+c)^5+765*b^7*sin(d*x+
c)^3-180*b^7*sin(d*x+c)-180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^7-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
```

```
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((
a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7-404*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3+37
2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*a*b^6+288*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+456*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5+128*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b-192*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a*b^6+500*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a^5*b^2-744*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4)/b^6/cos(d*x+c)/(a+b*sin(
d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin(dx+c) + a} \cos(dx+c)^4 \sin(dx+c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)^4 \sin(c+dx) \sqrt{a+b\sin(c+dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.1145 $\int \cos^3(c+dx)\cot(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=338

$$-\frac{2(8a^2-45b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d}+\frac{2a(8a^2-51b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2(8$$

```
[Out] 8/35*a*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b^2/d-2/7*cos(d*x+c)*sin(d*x+c)*(a
+b*sin(d*x+c))^(3/2)/b/d-2/105*(8*a^2-45*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(
1/2)/b^2/d-2/105*a*(8*a^2-51*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^
(1/2))*(a+b*sin(d*x+c))^(1/2)/b^3/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+2/105*(8
*a^4-53*a^2*b^2-60*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a
+b*sin(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*sin(d*x+c))^(1/2)-2*a*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi
+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*
sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2895, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2(8a^2-45b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d}-\frac{2(-53a^2b^2+8a^4-60b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{a+b\sin(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*(8*a^2 - 45*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(105*b^2*d) + (
8*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(35*b^2*d) - (2*Cos[c + d*x]*S
in[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(7*b*d) + (2*a*(8*a^2 - 51*b^2)*Ell
ipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(105*b^
3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (2*(8*a^4 - 53*a^2*b^2 - 60*b^4)*
EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
 b)])/(105*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (2*a*EllipticPi[2, (c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*S
in[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
```

```
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{8a\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35b^2d} - \frac{2\cos(c+dx)\sin(c+}{}$$

$$= -\frac{2\left(8a^2-45b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\cos(c}{}$$

$$= -\frac{2\left(8a^2-45b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\cos(c}{}$$

$$= -\frac{2\left(8a^2-45b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\cos(c}{}$$

$$= -\frac{2\left(8a^2-45b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\cos(c}{}$$

$$= -\frac{2\left(8a^2-45b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\cos(c}{}$$

Mathematica [C] time = 3.51, size = 435, normalized size = 1.29

$$\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(8a^2-6ab\sin(c+dx)+15b^2\cos(2(c+dx))+75b^2\right) - \frac{8b\left(a^2+30b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\Big(}{\sqrt{a+b\sin(c+}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(-8*a^2 + 51*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
qrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(b^2*Sqrt[-(a + b)^(-1)]) - (8*b*(a^2
 + 30*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[
c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*a*(8*a^2 + 159*b^2)*Ellip
```

```
ticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/Sqrt[a + b*Sin[c + d*x]] + 2*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*
(8*a^2 + 75*b^2 + 15*b^2*Cos[2*(c + d*x)] - 6*a*b*Sin[c + d*x]))/(210*b^2*d
)
```

fricas [F] time = 61.37, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{3}\cot\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^3*cot(d*x + c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [B] time = 2.19, size = 1155, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/105*(15*b^5*sin(d*x+c)^5+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-53*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^
3+111*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a*b^4-60*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*b^5-8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
```

```
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5+59*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-51*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a
*b^4-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*b^4*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),
(a-b)/a,((a-b)/(a+b))^(1/2))*a+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^5*EllipticPi(((a+b
*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))+18*a*b^4*sin(d*x+c)^
4-a^2*b^3*sin(d*x+c)^3-60*b^5*sin(d*x+c)^3-4*a^3*b^2*sin(d*x+c)^2-63*a*b^4*
sin(d*x+c)^2+a^2*b^3*sin(d*x+c)+45*b^5*sin(d*x+c)+4*a^3*b^2+45*a*b^4)/b^4/c
os(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin(dx+c) + a} \cos(dx+c)^3 \cot(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^3*cot(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^3 \cot(c+d\,x)\, \sqrt{a+b\,\sin(c+d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*cot(c + d*x)*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*cot(c + d*x)*(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.1146 $\int \cos^2(c+dx)\cot^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=323

$$\frac{\left(4a^2+15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd}+\frac{a\left(4a^2+11b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\sin(c+dx)}}-\frac{\left(4a^2+57b^2\right)}{}$$

```
[Out] -2/5*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b/d-cot(d*x+c)*(a+b*sin(d*x+c))^(3/2
)/a/d+1/15*(4*a^2+15*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a/b/d+1/15*(4*a
^2+57*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))
^(1/2)/b^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/15*a*(4*a^2+11*b^2)*(sin(1/2*
c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/
4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^2/d
/(a+b*sin(d*x+c))^(1/2)-b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2)
)*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.88, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2894, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2+15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd}+\frac{a\left(4a^2+11b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\sin(c+dx)}}-\frac{\left(4a^2+57b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((4*a^2 + 15*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(15*a*b*d) - (2*Co
s[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(5*b*d) - (Cot[c + d*x]*(a + b*Sin[c
 + d*x])^(3/2))/(a*d) - ((4*a^2 + 57*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15*b^2*d*Sqrt[(a + b*Sin[c + d*x])/(
a + b)]) + (a*(4*a^2 + 11*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]
*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15*b^2*d*Sqrt[a + b*Sin[c + d*x]]) +
(b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
```

```
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = -\frac{2\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5bd} - \frac{\cot(c+dx)(a+b\,\mathrm{si}}{ad}$$

$$= \frac{\left(4a^2+15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(c+d}{}$$

$$= \frac{\left(4a^2+15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(c+d}{}$$

$$= \frac{\left(4a^2+15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(c+d}{}$$

$$= \frac{\left(4a^2+15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(c+d}{}$$

$$= \frac{\left(4a^2+15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(c+d}{}$$

Mathematica [C] time = 3.65, size = 422, normalized size = 1.31

$$\frac{\frac{2\left(4a^2+27b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{b\sqrt{a+b\sin(c+dx)}} + \frac{2i\left(4a^2+57b^2\right)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sin(c+dx)+1)}{a-b}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a}\right.\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(4*a^2 + 57*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Elli
pticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
(a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b^3*Sqrt[-(a + b)^(-1)]) + (184*a
*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/
(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(4*a^2 + 27*b^2)*EllipticPi[2, (-2*
```

```
c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(b*Sq
rt[a + b*Sin[c + d*x]]) - (4*Sqrt[a + b*Sin[c + d*x]]*(2*a*Cos[c + d*x] + 3
*b*(5*Cot[c + d*x] + Sin[2*(c + d*x)])))/b)/(60*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.90, size = 656, normalized size = 2.03

$$-\frac{-6a\,b^4\sin(dx+c)\left(\cos^4(dx+c)\right)+\left(2a^3b^2+21a\,b^4\right)\left(\cos^2(dx+c)\right)\sin(dx+c)+\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\,\sqrt{-\frac{b\sin(\cdots}{a+\cdots}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/15*(-6*a*b^4*sin(d*x+c)*cos(d*x+c)^4+(2*a^3*b^2+21*a*b^4)*cos(d*x+c)^2*s
in(d*x+c)+(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))
^(1/2)*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(15*EllipticPi((b/(a-b)*sin(d*x+c
)+a/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4-15*EllipticPi((b/(a-b)*
sin(d*x+c)+a/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5+4*EllipticF((b/(
a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b+42*EllipticF((b/(
a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+11*EllipticF((b
/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3-57*EllipticF(
(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-4*EllipticE((
b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5-53*EllipticE((b/
(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+57*EllipticE((
```

```
b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4)*sin(d*x+c)-8*
a^2*b^3*cos(d*x+c)^4+23*a^2*b^3*cos(d*x+c)^2)/a/b^3/sin(d*x+c)/cos(d*x+c)/(
a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{2}\cot\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2*cot(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^2\,\cot\left(c+d\,x\right)^2\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*cot(c + d*x)^2*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*cot(c + d*x)^2*(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin{\left(c+dx \right)}}\cos^{2}{\left(c+dx \right)}\cot^{2}{\left(c+dx \right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*cos(c + d*x)**2*cot(c + d*x)**2, x)
```

3.1147 $\int \cos(c+dx)\cot^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=345

$$-\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d}-\frac{\left(8a^2+31b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{12bd\sqrt{a+b\sin(c+dx)}}+\frac{\left(8a^2-3b^2\right)\sqrt{a}}{}$$

```
[Out] 1/4*b*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+
b*sin(d*x+c))^(3/2)/a/d-1/12*(8*a^2+3*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2
)/a^2/d-1/12*(8*a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/
4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*
(a+b*sin(d*x+c))^(1/2)/a/b/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+1/12*(8*a^2+31*
b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
F(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b
))^(1/2)/b/d/(a+b*sin(d*x+c))^(1/2)+1/4*(12*a^2+b^2)*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x
),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a/d/(a+b*sin(d*
x+c))^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2893, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d}-\frac{\left(8a^2+31b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{12bd\sqrt{a+b\sin(c+dx)}}+\frac{\left(8a^2-3b^2\right)\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((8*a^2 + 3*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(12*a^2*d) + (b*Co
t[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*
x]*(a + b*Sin[c + d*x])^(3/2))/(2*a*d) + ((8*a^2 - 3*b^2)*EllipticE[(c - Pi
/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(12*a*b*d*Sqrt[(a + b
*Sin[c + d*x])/(a + b)]) - ((8*a^2 + 31*b^2)*EllipticF[(c - Pi/2 + d*x)/2,
(2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(12*b*d*Sqrt[a + b*Sin[c
 + d*x]]) - ((12*a^2 + b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
```

```
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2a}$$

$$= -\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c+dx}{}$$

$$= -\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c+dx}{}$$

$$= -\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c+dx}{}$$

$$= -\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c+dx}{}$$

$$= -\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c+dx}{}$$

Mathematica [C] time = 3.58, size = 450, normalized size = 1.30

$$\frac{\frac{2\left(64a^2+9b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{a\sqrt{a+b\sin(c+dx)}} + \frac{2i\left(8a^2-3b^2\right)\cos(2(c+dx))\csc^2(c+dx)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sin(c+dx)+1)}{a-b}}\left(2a(a-b\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(8*a^2 - 3*b^2)*Cos[2*(c + d*x)]*Csc[c + d*x]^2*(2*a*(a - b)*Ellipt
icE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b
)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]
]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)
]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1
+ Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a^2*b^
2*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*x]^2)) - (4*(8*a*Cos[c + d*x] + 3*Cot
[c + d*x]*(b + 2*a*Csc[c + d*x]))*Sqrt[a + b*Sin[c + d*x]])/a + (136*b*Elli
```

```
pticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(64*a^2 + 9*b^2)*EllipticPi[2, (-2*c + P
i - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*Sqrt[a
+ b*Sin[c + d*x]]))/(48*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [B] time = 2.07, size = 1364, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/12*(8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b*sin(d*x+c)^2-42*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)^2+31*b^3*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^2*sin(d*x+c)^2+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-8*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^2+
```

```
11*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^3*b^2*sin(d*x+c)^2-3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2+36*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*b^2*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^
(1/2))*a^3*sin(d*x+c)^2-36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^3*EllipticPi(((a+b*sin(d*x
+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^2+3*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^
(1/2))*a*b^4*sin(d*x+c)^2-3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c)
)/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^2+8*a^2*b^3*sin(
d*x+c)^5+8*a^3*b^2*sin(d*x+c)^4+3*a*b^4*sin(d*x+c)^4+a^2*b^3*sin(d*x+c)^3-2
*a^3*b^2*sin(d*x+c)^2-3*a*b^4*sin(d*x+c)^2-9*a^2*b^3*sin(d*x+c)-6*a^3*b^2)/
a^2/b^2/sin(d*x+c)^2/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)\cot\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)*cot(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)\,{\cot\left(c+d\,x\right)}^3\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*cot(c + d*x)^3*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)*cot(c + d*x)^3*(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin\left(c+dx\right)}\cos\left(c+dx\right)\cot^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3*(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(c + d*x))*cos(c + d*x)*cot(c + d*x)**3, x)
```

3.1148 $\int \cot^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=351

$$\frac{\left(32a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d}-\frac{\left(32a^2+b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{24ad\sqrt{a+b\sin(c+dx)}}+\frac{\left(80a^2+3b^2\right)\sqrt{}}{}$$

```
[Out] 1/4*b*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d-1/3*cot(d*x+c)*csc
(d*x+c)^2*(a+b*sin(d*x+c))^(3/2)/a/d+1/24*(32*a^2-3*b^2)*cot(d*x+c)*(a+b*si
n(d*x+c))^(1/2)/a^2/d-1/24*(80*a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b
/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+
1/24*(32*a^2+b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin
(d*x+c))/(a+b))^(1/2)/a/d/(a+b*sin(d*x+c))^(1/2)+1/8*b*(12*a^2-b^2)*(sin(1/
2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c
+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/
a^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2725, 3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(32a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d}-\frac{\left(32a^2+b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{24ad\sqrt{a+b\sin(c+dx)}}+\frac{\left(80a^2+3b^2\right)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((32*a^2 - 3*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(24*a^2*d) + (b*Co
t[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(4*a^2*d) - (Cot[c + d*
x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(3*a*d) + ((80*a^2 + 3*b^2)*E
llipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(24*a
^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - ((32*a^2 + b^2)*EllipticF[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(24*a*d*Sq
rt[a + b*Sin[c + d*x]]) - (b*(12*a^2 - b^2)*EllipticPi[2, (c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*a^2*d*Sqrt[a + b*S
in[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}}{4a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)(}{3ad}$$

$$= \frac{\left(32a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c+dx)(a}{4a^2d}$$

$$= \frac{\left(32a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c+dx)(a}{4a^2d}$$

$$= \frac{\left(32a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c+dx)(a}{4a^2d}$$

$$= \frac{\left(32a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c+dx)(a}{4a^2d}$$

$$= \frac{\left(32a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c+dx)(a}{4a^2d}$$

Mathematica [C] time = 5.71, size = 473, normalized size = 1.35

$$-\frac{4\cot(c+dx)\sqrt{a+b\sin(c+dx)}\left(8a^2\csc^2(c+dx)-32a^2+2ab\csc(c+dx)-3b^2\right)}{a^2} + \frac{-\frac{8a\left(24a^2+b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} - \frac{2b\left(8a^2+9b^2\right)\sqrt{\frac{a+b\sin(c+}{a+b}}}{\sqrt{a+b\,s}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((-4*Cot[c + d*x]*(-32*a^2 - 3*b^2 + 2*a*b*Csc[c + d*x] + 8*a^2*Csc[c + d*x
]^2)*Sqrt[a + b*Sin[c + d*x]])/a^2 + (((2*I)*(80*a^2 + 3*b^2)*Cos[2*(c + d*
x)]*Csc[c + d*x]^2*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqr
t[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[
-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(
a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/
(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[-((b*
(1 + Sin[c + d*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*x]^
```

```
2)) - (8*a*(24*a^2 + b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*b*(8*a^2 +
 9*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin
[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/a^2)/(96*d)
```

fricas [F] time = 76.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sin\left(dx+c\right)+a}\,\cot\left(dx+c\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*cot(d*x + c)^4, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.13, size = 1495, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/24*(80*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^5*sin(d*x+c)^3-77*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-3*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a
*b^4*sin(d*x+c)^3-48*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*sin(d*x+c)^3-32*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^3+7
8*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
```

```
a+b))^(1/2))*a^3*sin(d*x+c)^3-b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3+3*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*si
n(d*x+c)^3-36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2)
,(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3+36*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b
^3*sin(d*x+c)^3+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(
1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3-3*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5*
sin(d*x+c)^3+32*a^3*b^2*sin(d*x+c)^5+3*a*b^4*sin(d*x+c)^5+32*a^4*b*sin(d*x+
c)^4+a^2*b^3*sin(d*x+c)^4-42*a^3*b^2*sin(d*x+c)^3-3*a*b^4*sin(d*x+c)^3-40*a
^4*b*sin(d*x+c)^2-a^2*b^3*sin(d*x+c)^2+10*a^3*b^2*sin(d*x+c)+8*a^4*b)/a^3/b
/sin(d*x+c)^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin(dx + c) + a} \cot(dx + c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cot(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + d\,x)^4 \sqrt{a + b\,\sin(c + d\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sin(c + dx)} \cot^4(c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(c + d*x))*cot(c + d*x)**4, x)
```

3.1149 $\int \cot^4(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=412

$$\frac{b\left(196a^2+5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{192a^2d\sqrt{a+b\sin(c+dx)}}+\frac{5\left(4a^2-b^2\right)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{32a^2d}+\frac{5b}{}$$

```
[Out] 5/24*b*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2)/a^2/d-1/4*cot(d*x+c)*
csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2)/a/d+1/192*b*(68*a^2-15*b^2)*cot(d*x+c)*
(a+b*sin(d*x+c))^(1/2)/a^3/d+5/32*(4*a^2-b^2)*cot(d*x+c)*csc(d*x+c)*(a+b*si
n(d*x+c))^(1/2)/a^2/d-1/192*b*(68*a^2-15*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^3/d/((a+b*sin(d*x+c))/(a+b))^(1
/2)-1/192*b*(196*a^2+5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))
*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*sin(d*x+c))^(1/2)-1/64*(48*a^4+2
4*a^2*b^2-5*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*s
in(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.24, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2893, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(68a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d}+\frac{b\left(196a^2+5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{192a^2d\sqrt{a+b\sin(c+dx)}}+\frac{b\left(68a^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (b*(68*a^2 - 15*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(192*a^3*d) + (
5*(4*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(32*a^2
*d) + (5*b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(24*a^2*
d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2))/(4*a*d) + (b*
(68*a^2 - 15*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(192*a^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(196*a^2
+ 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d
*x])/(a + b)])/(192*a^2*d*Sqrt[a + b*Sin[c + d*x]]) + ((48*a^4 + 24*a^2*b^2
 - 5*b^4)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[
c + d*x])/(a + b)])/(64*a^3*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
```

```
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{5b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}}{24a^2d} - \frac{\cot(c+a}{}$$

$$= \frac{5\left(4a^2-b^2\right)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{32a^2d} + \frac{5b}{}$$

$$= \frac{b\left(68a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5\left(4a^2-b\right.}{}$$

$$= \frac{b\left(68a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5\left(4a^2-b\right.}{}$$

$$= \frac{b\left(68a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5\left(4a^2-b\right.}{}$$

$$= \frac{b\left(68a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5\left(4a^2-b\right.}{}$$

$$= \frac{b\left(68a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5\left(4a^2-b\right.}{}$$

Mathematica [C] time = 6.62, size = 643, normalized size = 1.56

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{5\csc^2(c+dx)\left(12a^2\cos(c+dx)+b^2\cos(c+dx)\right)}{96a^2}+\frac{\csc(c+dx)\left(68a^2b\cos(c+dx)-15b^3\cos(c+dx)\right)}{192a^3}-\frac{b\cot(c+dx)\csc^2(c+dx)}{24a}-\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((((68*a^2*b*Cos[c + d*x] - 15*b^3*Cos[c + d*x])*Csc[c + d*x])/(192*a^3) +
(5*(12*a^2*Cos[c + d*x] + b^2*Cos[c + d*x])*Csc[c + d*x]^2)/(96*a^2) - (b*C
ot[c + d*x]*Csc[c + d*x]^2)/(24*a) - (Cot[c + d*x]*Csc[c + d*x]^3)/4)*Sqrt[
a + b*Sin[c + d*x]])/d + ((-2*(528*a^3*b - 20*a*b^3)*EllipticF[(-c + Pi/2 -
 d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[
c + d*x]] - (2*(288*a^4 + 212*a^2*b^2 - 45*b^4)*EllipticPi[2, (-c + Pi/2 -
d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c
 + d*x]] - ((2*I)*(-68*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a
*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
 b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqr
t[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*
Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a +
 b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x])
 - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) +
 (a + b*Sin[c + d*x])^2)/b^2)]))/(768*a^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

[Out] Timed out

maple [B] time = 2.46, size = 1761, normalized size = 4.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2),x)

[Out] 1/192*(-196*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4-78*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4-5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)^4+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4+144*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4-72*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4+72*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)^4+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4+83*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4+264*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4-48*a^5-68*a^3*b^2*sin(d*x+c)^6+15*a*b^4*sin(d*x+c)^6-188*a^4*b*sin(d*x+c)^5+244*a^4*b*sin(d*x+c)^3-56*a^4*b*sin(d*x+c)-120*a^5*sin(d*x+c)^4+168*a^5*sin(d*x+c)^2+5*a^2*b^3*sin(d*x+c)^5+66*a^3*b^2*sin(d*x+c)^4-15*a*b^4*sin(d*x+c)^4-5*a^2*b^3*sin(d*x+c)^3+2*a^3*b^2*sin(d*x+c)^2-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4-144*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^4-68*((a+b*sin(d*x+c))/(a-b))^

```
(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellip
ticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4)/
a^4/sin(d*x+c)^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(dx+c\right)+a}\,\cot\left(dx+c\right)^{4}\csc\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cot(d*x + c)^4*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\sin\left(c+d\,x\right)^2-1\right)^2\,\sqrt{a+b\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(1/2))/sin(c + d*x),x)
```

```
[Out] int(((sin(c + d*x)^2 - 1)^2*(a + b*sin(c + d*x))^(1/2))/sin(c + d*x)^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin\left(c+dx\right)}\,\cot^{4}\left(c+dx\right)\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*cot(c + d*x)**4*csc(c + d*x), x)
```

3.1150 $\int \cot^4(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=484

$$\frac{(96a^2-35b^2)\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}}{240a^2d}+\frac{7b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}}{40a^2d}-\frac{(3$$

```
[Out] 7/40*b*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2)/a^2/d-1/5*cot(d*x+c)*
csc(d*x+c)^4*(a+b*sin(d*x+c))^(3/2)/a/d-1/1920*(384*a^4+332*a^2*b^2-105*b^4
)*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^4/d+1/960*b*(108*a^2-35*b^2)*cot(d*x+
c)*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^3/d+1/240*(96*a^2-35*b^2)*cot(d*x+c)
*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/a^2/d+1/1920*(384*a^4+332*a^2*b^2-105*
b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
E(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)
/a^4/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/1920*(384*a^4+116*a^2*b^2-35*b^4)*(
sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(
1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/
2)/a^3/d/(a+b*sin(d*x+c))^(1/2)-1/128*b*(48*a^4-24*a^2*b^2+7*b^4)*(sin(1/2*
c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1
/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^
4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.62, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2893, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{(332a^2b^2+384a^4-105b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1920a^4d}+\frac{(116a^2b^2+384a^4-35b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+\right.\right.}{1920a^3d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((384*a^4 + 332*a^2*b^2 - 105*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/
(1920*a^4*d) + (b*(108*a^2 - 35*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*S
in[c + d*x]])/(960*a^3*d) + ((96*a^2 - 35*b^2)*Cot[c + d*x]*Csc[c + d*x]^2*
Sqrt[a + b*Sin[c + d*x]])/(240*a^2*d) + (7*b*Cot[c + d*x]*Csc[c + d*x]^3*(a
 + b*Sin[c + d*x])^(3/2))/(40*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*
Sin[c + d*x])^(3/2))/(5*a*d) - ((384*a^4 + 332*a^2*b^2 - 105*b^4)*EllipticE
[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(1920*a^4*d*S
qrt[(a + b*Sin[c + d*x])/(a + b)]) + ((384*a^4 + 116*a^2*b^2 - 35*b^4)*Elli
```

```
pticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]
)/(1920*a^3*d*Sqrt[a + b*Sin[c + d*x]]) + (b*(48*a^4 - 24*a^2*b^2 + 7*b^4)*
EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/
(a + b)])/(128*a^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
```

```
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{7b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}}{40a^2d} - \frac{\cot(c+}{}$$

$$= \frac{\left(96a^2-35b^2\right)\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}}{240a^2d}$$

$$= \frac{b\left(108a^2-35b^2\right)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{960a^3d}$$

$$= -\frac{\left(384a^4+332a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx}}{1920a^4d}$$

$$= -\frac{\left(384a^4+332a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx}}{1920a^4d}$$

$$= -\frac{\left(384a^4+332a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx}}{1920a^4d}$$

$$= -\frac{\left(384a^4+332a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx}}{1920a^4d}$$

$$= -\frac{\left(384a^4+332a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx}}{1920a^4d}$$

Mathematica [C] time = 6.70, size = 702, normalized size = 1.45

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{\csc^3(c+dx)\left(96a^2\cos(c+dx)+7b^2\cos(c+dx)\right)}{240a^2}+\frac{\csc(c+dx)\left(-384a^4\cos(c+dx)-332a^2b^2\cos(c+dx)+105b^4\cos(c+dx)\right)}{1920a^4}+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((((-384*a^4*Cos[c + d*x] - 332*a^2*b^2*Cos[c + d*x] + 105*b^4*Cos[c + d*x]
)*Csc[c + d*x])/(1920*a^4) + ((108*a^2*b*Cos[c + d*x] - 35*b^3*Cos[c + d*x]
```

```
)*Csc[c + d*x]^2)/(960*a^3) + ((96*a^2*Cos[c + d*x] + 7*b^2*Cos[c + d*x])*C
sc[c + d*x]^3)/(240*a^2) - (b*Cot[c + d*x]*Csc[c + d*x]^3)/(40*a) - (Cot[c
+ d*x]*Csc[c + d*x]^4)/5)*Sqrt[a + b*Sin[c + d*x]])/d + (b*((-2*(-432*a^3*b
 + 140*a*b^3)*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin
[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(1056*a^4 - 1052*a^2*b^2
 + 315*b^4)*EllipticPi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*S
in[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(384*a^4 + 332*a^2
*b^2 - 105*b^4)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*
a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b
)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
 b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sq
rt[-((b + b*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c
+ d*x]^2]*(-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])
^2)*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/
b^2)])))/(7680*a^4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [B] time = 2.50, size = 2075, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x)
```

[Out] -1/1920*(384*a^6*b-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5+384*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b*sin(d*x+c)^5-168*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5+116*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d*x+c)^5-402*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5-35*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^5+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5+720*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5-720*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d*x+c)^5-360*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5+360*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^5+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5+52*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5+437*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^7*sin(d*x+c)^5-384*a^5*b^2*sin(d*x+c)^7-332*a^3*b^4*sin(d*x+c)^7+105*a*b^6*sin(d*x+c)^7-384*a^6*b*sin(d*x+c)^6-116*a^4*b^3*sin(d*x+c)^6+35*a^2*b^5*sin(d*x+c)^6+1152*a^6*b*sin(d*x+c)^4+124*a^4*b^3*sin(d*x+c)^4-35*a^2*b^5*sin(d*x+c)^4-1416*a^5*b^2*sin(d*x+c)^3+1368*a^5*b^2*sin(d*x+c)^5-105*a*b^6*sin(d*x+c)^5+318*a^3*b^4*sin(d*x+c)^5+14*a^3*b^4*sin(d*x+c)^3-1152*a^6*b*sin(d*x+c)^2-8*a^4*b^3*sin(d*x+c)^2+432*a^5*b^2*sin(d*x+c)-384*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c

```
)^5)/a^5/b/sin(d*x+c)^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin(dx+c)+a}\,\cot(dx+c)^4\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cot(d*x + c)^4*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\sin(c+d\,x)^2-1\right)^2\sqrt{a+b\,\sin(c+d\,x)}}{\sin(c+d\,x)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(1/2))/sin(c + d*x)^2,x)
```

```
[Out] int(((sin(c + d*x)^2 - 1)^2*(a + b*sin(c + d*x))^(1/2))/sin(c + d*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.1151 $\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=528

$$\frac{8a\left(8a^2-21b^2\right)\sin(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{1287b^4d}-\frac{2\left(80a^2-221b^2\right)\sin^2(c+dx)\cos(c+dx)(a+b\sin}{2145b^3d}$$

```
[Out] 16/45045*a*(32*a^4-47*a^2*b^2-27*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b^5
/d-8/45045*(160*a^4-375*a^2*b^2+117*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(5/2)/
b^5/d+8/1287*a*(8*a^2-21*b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2)/
b^4/d-2/2145*(80*a^2-221*b^2)*cos(d*x+c)*sin(d*x+c)^2*(a+b*sin(d*x+c))^(5/2
)/b^3/d+4/39*a*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^(5/2)/b^2/d-2/15*co
s(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c))^(5/2)/b/d+8/45045*(64*a^6-174*a^4*b^
2+81*a^2*b^4-195*b^6)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^5/d+16/45045*a*(3
2*a^6-111*a^4*b^2+102*a^2*b^4-471*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a
+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^6/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-8/4
5045*(64*a^8-238*a^6*b^2+255*a^4*b^4-276*a^2*b^6+195*b^8)*(sin(1/2*c+1/4*Pi
+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2
*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^6/d/(a+b*si
n(d*x+c))^(1/2)
```

Rubi [A] time = 1.21, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2895, 3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(80a^2-221b^2\right)\sin^2(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{2145b^3d}+\frac{8a\left(8a^2-21b^2\right)\sin(c+dx)\cos(c+dx)(a+b}{1287b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (8*(64*a^6 - 174*a^4*b^2 + 81*a^2*b^4 - 195*b^6)*Cos[c + d*x]*Sqrt[a + b*Si
n[c + d*x]])/(45045*b^5*d) + (16*a*(32*a^4 - 47*a^2*b^2 - 27*b^4)*Cos[c + d
*x]*(a + b*Sin[c + d*x])^(3/2))/(45045*b^5*d) - (8*(160*a^4 - 375*a^2*b^2 +
 117*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(45045*b^5*d) + (8*a*(8*
a^2 - 21*b^2)*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(1287*b
^4*d) - (2*(80*a^2 - 221*b^2)*Cos[c + d*x]*Sin[c + d*x]^2*(a + b*Sin[c + d*
x])^(5/2))/(2145*b^3*d) + (4*a*Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d
*x])^(5/2))/(39*b^2*d) - (2*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*Sin[c + d*x]
)^(5/2))/(15*b*d) - (16*a*(32*a^6 - 111*a^4*b^2 + 102*a^2*b^4 - 471*b^6)*El
```

```
lipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(45045
*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(64*a^8 - 238*a^6*b^2 + 255
*a^4*b^4 - 276*a^2*b^6 + 195*b^8)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(45045*b^6*d*Sqrt[a + b*Sin[c + d*x
]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
&& IntegerQ[2*m]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{4a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^{5/2}}{39b^2d} - \frac{2\cos(c}{}$$

$$= -\frac{2\left(80a^2-221b^2\right)\cos(c+dx)\sin^2(c+dx)(a+b\sin(c+a}{2145b^3d}$$

$$= \frac{8a\left(8a^2-21b^2\right)\cos(c+dx)\sin(c+dx)(a+b\sin(c+dx))^5}{1287b^4d}$$

$$= -\frac{8\left(160a^4-375a^2b^2+117b^4\right)\cos(c+dx)(a+b\sin(c+d}{45045b^5d}$$

$$= \frac{16a\left(32a^4-47a^2b^2-27b^4\right)\cos(c+dx)(a+b\sin(c+dx))}{45045b^5d}$$

$$= \frac{8\left(64a^6-174a^4b^2+81a^2b^4-195b^6\right)\cos(c+dx)\sqrt{a+b\,\mathrm{si}}}{45045b^5d}$$

$$= \frac{8\left(64a^6-174a^4b^2+81a^2b^4-195b^6\right)\cos(c+dx)\sqrt{a+b\,\mathrm{si}}}{45045b^5d}$$

$$= \frac{8\left(64a^6-174a^4b^2+81a^2b^4-195b^6\right)\cos(c+dx)\sqrt{a+b\,\mathrm{si}}}{45045b^5d}$$

$$= \frac{8\left(64a^6-174a^4b^2+81a^2b^4-195b^6\right)\cos(c+dx)\sqrt{a+b\,\mathrm{si}}}{45045b^5d}$$

Mathematica [A] time = 15.66, size = 382, normalized size = 0.72

$$\frac{\sqrt{a+b\sin(c+dx)}\left(512\left(32a^7-111a^5b^2+102a^3b^4-471ab^6\right)E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-2b\cos(c+dx)\sqrt{\frac{a+b\sin}{a+}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*(512*(32*a^7 - 111*a^5*b^2 + 102*a^3*b^4 - 471*a*
b^6)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - 256*(64*a^7 - 64*a^6
```

```
*b - 174*a^5*b^2 + 174*a^4*b^3 + 81*a^3*b^4 - 81*a^2*b^5 - 195*a*b^6 + 195*
b^7)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - 2*b*Cos[c + d*x]*Sqr
t[(a + b*Sin[c + d*x])/(a + b)]*(4096*a^6 - 12416*a^4*b^2 + 8100*a^2*b^4 +
6786*b^6 + (-1280*a^4*b^2 + 3168*a^2*b^4 + 21723*b^6)*Cos[2*(c + d*x)] + 42
*(6*a^2*b^4 - 13*b^6)*Cos[4*(c + d*x)] - 3003*b^6*Cos[6*(c + d*x)] - 3072*a
^5*b*Sin[c + d*x] + 8432*a^3*b^3*Sin[c + d*x] - 41424*a*b^5*Sin[c + d*x] +
560*a^3*b^3*Sin[3*(c + d*x)] + 13776*a*b^5*Sin[3*(c + d*x)] + 7392*a*b^5*Si
n[5*(c + d*x)])))/(1441440*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
```

fricas [F] time = 1.35, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a\cos\left(dx+c\right)^{6}-a\cos\left(dx+c\right)^{4}+\left(b\cos\left(dx+c\right)^{6}-b\cos\left(dx+c\right)^{4}\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(a*cos(d*x + c)^6 - a*cos(d*x + c)^4 + (b*cos(d*x + c)^6 - b*cos(
d*x + c)^4)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

maple [B] time = 2.03, size = 1801, normalized size = 3.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/45045*(7*a^3*b^6*sin(d*x+c)^6+17682*a*b^8*sin(d*x+c)^6-10*a^4*b^5*sin(d*
x+c)^5+10362*a^2*b^7*sin(d*x+c)^5+16*a^5*b^4*sin(d*x+c)^4-62*a^3*b^6*sin(d*
x+c)^4-12603*a*b^8*sin(d*x+c)^4-32*a^6*b^3*sin(d*x+c)^3+122*a^4*b^5*sin(d*x
+c)^3-8115*a^2*b^7*sin(d*x+c)^3-128*a^7*b^2*sin(d*x+c)^2+412*a^5*b^4*sin(d*
x+c)^2-305*a^3*b^6*sin(d*x+c)^2+840*a*b^8*sin(d*x+c)^2+32*a^6*b^3*sin(d*x+c
)-112*a^4*b^5*sin(d*x+c)+1512*a^2*b^7*sin(d*x+c)-6699*a*b^8*sin(d*x+c)^8+78
```

```
0*a*b^8+128*a^7*b^2+360*a^3*b^6-428*a^5*b^4-3759*a^2*b^7*sin(d*x+c)^7-256*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^9-952*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^6*b^3-3003*b^9*sin(d*x+c)^9+7644*b^9*sin(d*x+c)^7-51
09*b^9*sin(d*x+c)^5-312*b^9*sin(d*x+c)^3+780*b^9*sin(d*x+c)+684*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*
b^4+2988*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*a*b^8+1020*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^5-3480*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^6-1104*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a
^2*b^7+1144*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a^7*b^2+780*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^9-1704*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
E(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^4+4584*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a
^3*b^6-3768*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a*b^8+256*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^8*b-192*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*b^2)/b^7/cos(d*x+c
)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^2\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.1152 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=394

$$-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a\left(2a^2-5b^2\right)-7b\left(a^2+11b^2\right)\sin(c+dx)\right)}{3003b^2d}+\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx}}{}$$

```
[Out] -2/13*cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2)/d-6/143*a*cos(d*x+c)^5*(a+b*sin(d
*x+c))^(1/2)/d-2/3003*cos(d*x+c)^3*(4*a*(2*a^2-5*b^2)-7*b*(a^2+11*b^2)*sin(
d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^2/d+4/15015*cos(d*x+c)*(a*(32*a^4-113*a^2*
b^2+177*b^4)-3*b*(8*a^4-27*a^2*b^2-77*b^4)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/
2)/b^4/d-8/15015*(32*a^6-137*a^4*b^2+258*a^2*b^4+231*b^6)*(sin(1/2*c+1/4*Pi
+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2
*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^5/d/((a+b*sin(d*x+c
))/(a+b))^(1/2)+8/15015*a*(32*a^6-145*a^4*b^2+290*a^2*b^4-177*b^6)*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1
/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^5/
d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2862, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a\left(2a^2-5b^2\right)-7b\left(a^2+11b^2\right)\sin(c+dx)\right)}{3003b^2d}+\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-6*a*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(143*d) - (2*Cos[c + d*x]^5*
(a + b*Sin[c + d*x])^(3/2))/(13*d) + (8*(32*a^6 - 137*a^4*b^2 + 258*a^2*b^4
 + 231*b^6)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c +
 d*x]])/(15015*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*a*(32*a^6 - 1
45*a^4*b^2 + 290*a^2*b^4 - 177*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15015*b^5*d*Sqrt[a + b*Sin[c + d
*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(4*a*(2*a^2 - 5*b^2) - 7
*b*(a^2 + 11*b^2)*Sin[c + d*x]))/(3003*b^2*d) + (4*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]]*(a*(32*a^4 - 113*a^2*b^2 + 177*b^4) - 3*b*(8*a^4 - 27*a^2*b^2
 - 77*b^4)*Sin[c + d*x]))/(15015*b^4*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{2\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d} + \frac{2}{13}\int \cos^4(c+dx)$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx)(a+b}{13d}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx)(a+b}{13d}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx)(a+b}{13d}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx)(a+b}{13d}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx)(a+b}{13d}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx)(a+b}{13d}$$

Mathematica [A] time = 12.07, size = 382, normalized size = 0.97

$$384a\left(32a^6-145a^4b^2+290a^2b^4-177b^6\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-3b\cos(c+dx)\left(-2048a^6-5\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-384*(32*a^7 + 32*a^6*b - 137*a^5*b^2 - 137*a^4*b^3 + 258*a^3*b^4 + 258*a^
2*b^5 + 231*a*b^6 + 231*b^7)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 384*a*(32*a^6 - 145*a^4*b^2 + 290*a^
2*b^4 - 177*b^6)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a +
b*Sin[c + d*x])/(a + b)] - 3*b*Cos[c + d*x]*(-2048*a^6 + 8640*a^4*b^2 + 198
0*a^2*b^4 - 6622*b^6 + (-128*a^4*b^2 + 24512*a^2*b^4 + 8547*b^6)*Cos[2*(c +
 d*x)] + 70*(86*a^2*b^4 - 11*b^6)*Cos[4*(c + d*x)] - 1155*b^6*Cos[6*(c + d*
x)] - 512*a^5*b*Sin[c + d*x] + 2088*a^3*b^3*Sin[c + d*x] - 19492*a*b^5*Sin[
c + d*x] + 40*a^3*b^3*Sin[3*(c + d*x)] + 11870*a*b^5*Sin[3*(c + d*x)] + 525
0*a*b^5*Sin[5*(c + d*x)]))/(720720*b^5*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b\cos\left(dx+c\right)^{6}-a\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)-b\cos\left(dx+c\right)^{4}\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(-(b*cos(d*x + c)^6 - a*cos(d*x + c)^4*sin(d*x + c) - b*cos(d*x + c
)^4)*sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

maple [B] time = 1.96, size = 1619, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/15015*(-5*a^3*b^5*sin(d*x+c)^5-7390*a*b^7*sin(d*x+c)^5+8*a^4*b^4*sin(d*x+
c)^4-4542*a^2*b^6*sin(d*x+c)^4-16*a^5*b^3*sin(d*x+c)^3+74*a^3*b^5*sin(d*x+c
)^3+6089*a*b^7*sin(d*x+c)^3-64*a^6*b^2*sin(d*x+c)^2+258*a^4*b^4*sin(d*x+c)^
```

```
2+4053*a^2*b^6*sin(d*x+c)^2+16*a^5*b^3*sin(d*x+c)-69*a^3*b^5*sin(d*x+c)-132
4*a*b^7*sin(d*x+c)+2625*a*b^7*sin(d*x+c)^7+64*a^6*b^2+108*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6+15
05*a^2*b^6*sin(d*x+c)^6+2233*b^8*sin(d*x+c)^4-308*b^8*sin(d*x+c)^2+1155*b^8
*sin(d*x+c)^8-3080*b^8*sin(d*x+c)^6-266*a^4*b^4-1016*a^2*b^6+924*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^8
-924*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*b^8-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b)
)^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^
(1/2),((a-b)/(a+b))^(1/2))*a^8+600*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6-708*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^7+676*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*a^6*b^2-1580*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a^4*b^4+128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*b-96*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^2-580*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^5*b^3+420*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^4*b^4+1160*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^5)/b^6/cos(d*x+c)/(a+b*sin(d
*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)^4 \sin(c+dx) \left(a+b\sin(c+dx)\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.1153 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=390

$$-\frac{2\left(8a^2-77b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315b^2d}-\frac{2a\left(8a^2-87b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d}+\frac{2a^2\sqrt{\frac{a+b\sin(c}{a+b}}}{d}$$

```
[Out] -2/315*(8*a^2-77*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b^2/d+8/63*a*cos(d*
x+c)*(a+b*sin(d*x+c))^(5/2)/b^2/d-2/9*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c)
)^(5/2)/b/d-2/315*a*(8*a^2-87*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^2/d-
2/315*(8*a^4-93*a^2*b^2+84*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1
/2))*(a+b*sin(d*x+c))^(1/2)/b^3/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+2/315*a*(8
*a^4-95*a^2*b^2-228*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((
a+b*sin(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*sin(d*x+c))^(1/2)-2*a^2*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4
*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a
+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.16, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2895, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2\left(8a^2-77b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315b^2d}-\frac{2a\left(8a^2-87b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d}-\frac{2a\left(-95a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*(8*a^2 - 87*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(315*b^2*d) -
 (2*(8*a^2 - 77*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(315*b^2*d) +
 (8*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(63*b^2*d) - (2*Cos[c + d*x]
*Sin[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(9*b*d) + (2*(8*a^4 - 93*a^2*b^2
+ 84*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d
*x]])/(315*b^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (2*a*(8*a^4 - 95*a^2
*b^2 - 228*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Si
n[c + d*x])/(a + b)])/(315*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (2*a^2*Ellipti
cPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)
])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
```

```
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{8a\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{63b^2d} - \frac{2\cos(c+dx)\sin(c$$

$$= -\frac{2\left(8a^2-77b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315b^2d} + \frac{8a\,\mathrm{co}}{}$$

$$= -\frac{2a\left(8a^2-87b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \frac{2\left(8a^2\right.}{}$$

$$= -\frac{2a\left(8a^2-87b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \frac{2\left(8a^2\right.}{}$$

$$= -\frac{2a\left(8a^2-87b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \frac{2\left(8a^2\right.}{}$$

$$= -\frac{2a\left(8a^2-87b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \frac{2\left(8a^2\right.}{}$$

$$= -\frac{2a\left(8a^2-87b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \frac{2\left(8a^2\right.}{}$$

Mathematica [C] time = 3.82, size = 477, normalized size = 1.22

$$\frac{-\frac{8ab\left(a^2+156b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} - \frac{2\left(8a^4+537a^2b^2+84b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} - \frac{2i\left(8a^4-93a^2b^2+84b^4\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)*(8*a^4 - 93*a^2*b^2 + 84*b^4)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Ell
ipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a
- b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Si
n[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x])
```

```
)/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(a*b^2*Sqrt[-(a + b)^(-1
)]) - (8*a*b*(a^2 + 156*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(8*a^4
+ 537*a^2*b^2 + 84*b^4)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]
*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + Cos[c + d*x
]*Sqrt[a + b*Sin[c + d*x]]*(16*a^3 + 556*a*b^2 + 100*a*b^2*Cos[2*(c + d*x)]
 + (-12*a^2*b + 203*b^3)*Sin[c + d*x] + 35*b^3*Sin[3*(c + d*x)]))/(630*b^2*
d)
```

fricas [F] time = 3.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^{3}\cot\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)^{3}\cot\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] integral((b*cos(d*x + c)^3*cot(d*x + c)*sin(d*x + c) + a*cos(d*x + c)^3*cot
(d*x + c))*sqrt(b*sin(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [B] time = 1.92, size = 1405, normalized size = 3.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/315*(35*b^6*sin(d*x+c)^6-112*b^6*sin(d*x+c)^4+77*b^6*sin(d*x+c)^2+85*a*b^
5*sin(d*x+c)^5-315*a^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^4*EllipticPi(((a+b*sin(d*x+c))
/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))+53*a^2*b^4*sin(d*x+c)^4-a^3*b^3*
sin(d*x+c)^3-326*a*b^5*sin(d*x+c)^3-4*a^4*b^2*sin(d*x+c)^2-217*a^2*b^4*sin(
d*x+c)^2+a^3*b^3*sin(d*x+c)+241*a*b^5*sin(d*x+c)+405*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
```

```
lipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+315*a*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*b^5*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((
a-b)/(a+b))^(1/2))+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^5*b-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-95*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3-228*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a*b^5+101*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))*a^4*b^2+4*a^4*b^2+164*a^2*b^4-177*((a+b*sin(d*x+c))/(
a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)
*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+84*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*b^6-84*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*b^6-8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6)/b^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)
/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^3*cot(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^3\cot(c+d\,x)\,\left(a+b\,\sin(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*cot(c + d*x)*(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3*cot(c + d*x)*(a + b*sin(c + d*x))^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.1154 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=374

$$\frac{\left(4a^2+35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35abd}+\frac{\left(4a^2+65b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd}-\frac{a\left(4a^2+167b^2\right)\sqrt{}}{}$$

```
[Out] 1/35*(4*a^2+35*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a/b/d-2/7*cos(d*x+c)*
(a+b*sin(d*x+c))^(5/2)/b/d-cot(d*x+c)*(a+b*sin(d*x+c))^(5/2)/a/d+1/35*(4*a^
2+65*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b/d+1/35*a*(4*a^2+167*b^2)*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^2/d/((
a+b*sin(d*x+c))/(a+b))^(1/2)-1/35*(4*a^4+61*a^2*b^2+40*b^4)*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*
sin(d*x+c))^(1/2)-3*a*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*
((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.17, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2894, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2+35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35abd}+\frac{\left(4a^2+65b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd}+\frac{\left(61a^2b^2+4a^4+4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((4*a^2 + 65*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(35*b*d) + ((4*a^2
 + 35*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(35*a*b*d) - (2*Cos[c +
 d*x]*(a + b*Sin[c + d*x])^(5/2))/(7*b*d) - (Cot[c + d*x]*(a + b*Sin[c + d*
x])^(5/2))/(a*d) - (a*(4*a^2 + 167*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)
/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(35*b^2*d*Sqrt[(a + b*Sin[c + d*x])/(a
+ b)]) + ((4*a^4 + 61*a^2*b^2 + 40*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)
/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(35*b^2*d*Sqrt[a + b*Sin[c +
d*x]]) + (3*a*b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a +
b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
```

```
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{2\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7bd} - \frac{\cot(c+dx)(a+b\sin}{ad}$$

$$= \frac{\left(4a^2+35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35abd} - \frac{2\cos(c+}{}$$

$$= \frac{\left(4a^2+65b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{\left(4a^2+35b\right.}{}$$

$$= \frac{\left(4a^2+65b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{\left(4a^2+35b\right.}{}$$

$$= \frac{\left(4a^2+65b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{\left(4a^2+35b\right.}{}$$

$$= \frac{\left(4a^2+65b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{\left(4a^2+35b\right.}{}$$

$$= \frac{\left(4a^2+65b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{\left(4a^2+35b\right.}{}$$

Mathematica [C] time = 4.23, size = 452, normalized size = 1.21

$$\frac{\frac{8\left(53a^2-20b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2a\left(4a^2-43b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{b\sqrt{a+b\sin(c+dx)}} - \frac{2\sqrt{a+b\sin(c+dx)}\left(\left(4a^2-55b^2\right)\cos(\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)*(4*a^2 + 167*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
```

```
qrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(b^3*Sqrt[-(a + b)^(-1)]) + (8*(53*a^
2 - 20*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin
[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*a*(4*a^2 - 43*b^2)*Ellip
ticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/(b*Sqrt[a + b*Sin[c + d*x]]) - (2*Sqrt[a + b*Sin[c + d*x]]*((4*a^2
- 55*b^2)*Cos[c + d*x] + b*(-5*b*Cos[3*(c + d*x)] + 70*a*Cot[c + d*x] + 16*
a*Sin[2*(c + d*x)]))))/b)/(140*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.77, size = 726, normalized size = 1.94

$$-\frac{-26a\,b^4\sin(dx+c)\left(\cos^4(dx+c)\right)+\left(2a^3b^2+31a\,b^4\right)\left(\cos^2(dx+c)\right)\sin(dx+c)+\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\,\sqrt{\frac{b\sin}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/35*(-26*a*b^4*sin(d*x+c)*cos(d*x+c)^4+(2*a^3*b^2+31*a*b^4)*cos(d*x+c)^2*
sin(d*x+c)+(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+a/(a-b))
^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(105*EllipticPi((b/(a-b)*sin(d*x
+c)+a/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4-105*EllipticPi((b/(a-
b)*sin(d*x+c)+a/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5-4*EllipticE((
b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5-163*EllipticE((b
/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+167*EllipticE
```

```
((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+4*EllipticF(
(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b+102*EllipticF
((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+61*Ellipti
cF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3-207*Elli
pticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+40*Elli
pticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^5)*sin(d*x+
c)+10*b^5*cos(d*x+c)^6+(-18*a^2*b^3+10*b^5)*cos(d*x+c)^4+(53*a^2*b^3-20*b^5
)*cos(d*x+c)^2)/sin(d*x+c)/b^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*cot(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2\cot(c+d\,x)^2\,(a+b\,\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*cot(c + d*x)^2*(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*cot(c + d*x)^2*(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.1155 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=383

$$-\frac{\left(8a^2-5b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{20a^2d}-\frac{\left(8a^2-15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad}-\frac{a\left(8a^2+37b^2\right)\sqrt{}}{20b}$$

```
[Out] -1/20*(8*a^2-5*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d-1/4*b*cot(d*x+c
)*(a+b*sin(d*x+c))^(5/2)/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(
5/2)/a/d-1/20*(8*a^2-15*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a/d-1/20*(8*
a^2-81*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*E
llipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c)
)^(1/2)/b/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+1/20*a*(8*a^2+37*b^2)*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4
*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b/d/(a
+b*sin(d*x+c))^(1/2)+3/4*(4*a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/si
n(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(
a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.16, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2893, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(8a^2-5b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{20a^2d}-\frac{\left(8a^2-15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad}-\frac{a\left(8a^2+37b^2\right)\sqrt{}}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((8*a^2 - 15*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(20*a*d) - ((8*a^
2 - 5*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(20*a^2*d) - (b*Cot[c +
 d*x]*(a + b*Sin[c + d*x])^(5/2))/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]*(a
 + b*Sin[c + d*x])^(5/2))/(2*a*d) + ((8*a^2 - 81*b^2)*EllipticE[(c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(20*b*d*Sqrt[(a + b*Sin[c
 + d*x])/(a + b)]) - (a*(8*a^2 + 37*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(20*b*d*Sqrt[a + b*Sin[c + d
*x]]) - (3*(4*a^2 - b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)])/(4*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
```

```
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{b\cot(c+dx)(a+b\sin(c+dx))^{5/2}}{4a^2d} - \frac{\cot(c+dx)\csc(c+d\ldots}{2\ldots}$$

$$= -\frac{\left(8a^2-5b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{20a^2d} - \frac{b\cot(c+\ldots}{\ldots}$$

$$= -\frac{\left(8a^2-15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{\left(8a^2-5b^2\ldots\right.}{\ldots}$$

$$= -\frac{\left(8a^2-15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{\left(8a^2-5b^2\ldots\right.}{\ldots}$$

$$= -\frac{\left(8a^2-15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{\left(8a^2-5b^2\ldots\right.}{\ldots}$$

$$= -\frac{\left(8a^2-15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{\left(8a^2-5b^2\ldots\right.}{\ldots}$$

$$= -\frac{\left(8a^2-15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{\left(8a^2-5b^2\ldots\right.}{\ldots}$$

Mathematica [C] time = 3.41, size = 434, normalized size = 1.13

$$\frac{\frac{2\left(112a^2+51b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2i\left(81b^2-8a^2\right)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sin(c+dx)+1)}{a-b}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{\ldots}\right.\right.\right.\right.}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)*(-8*a^2 + 81*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
qrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b^2*Sqrt[-(a + b)^(-1)]) + (472*
```

```
a*b*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(112*a^2 + 51*b^2)*EllipticPi[2,
 (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/
Sqrt[a + b*Sin[c + d*x]] + 4*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d
*x]]*(-18*a + 8*a*Cos[2*(c + d*x)] - 31*b*Sin[c + d*x] + 2*b*Sin[3*(c + d*x
)]))/(80*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [B] time = 2.09, size = 1379, normalized size = 3.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/20*(8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b*sin(d*x+c)^2-126*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)^2+37*b^3*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a^2*sin(d*x+c)^2+81*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-8*((a+b*sin(d*x+c))/(a-b
```

```
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
lipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^
2+89*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^3*b^2*sin(d*x+c)^2-81*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2+60*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
b))^(1/2)*b^2*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b
))^(1/2))*a^3*sin(d*x+c)^2-60*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^3*EllipticPi(((a+b*sin(
d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^2-15*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+
b))^(1/2))*a*b^4*sin(d*x+c)^2+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d
*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^2+8*a*b^4*s
in(d*x+c)^6+24*a^2*b^3*sin(d*x+c)^5+16*a^3*b^2*sin(d*x+c)^4+17*a*b^4*sin(d*
x+c)^4+11*a^2*b^3*sin(d*x+c)^3-6*a^3*b^2*sin(d*x+c)^2-25*a*b^4*sin(d*x+c)^2
-35*a^2*b^3*sin(d*x+c)-10*a^3*b^2)/a/b^2/sin(d*x+c)^2/cos(d*x+c)/(a+b*sin(d
*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\cot(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)*cot(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)\,\cot(c+d\,x)^3\,(a+b\,\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*cot(c + d*x)^3*(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)*cot(c + d*x)^3*(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.1156 $\int \cot^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=386

$$-\frac{b\left(16a^2+b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d}+\frac{\left(32a^2+b^2\right)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{24a^2d}-\frac{\left(16a^2+21b^2\right)\sqrt{\frac{a+}{}}}{8d\sqrt{}}$$

```
[Out] 1/24*(32*a^2+b^2)*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d+1/12*b*cot(d*x+c)
*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2)/a^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2*(a+b*s
in(d*x+c))^(5/2)/a/d-1/8*b*(16*a^2+b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a
^2/d-1/8*(32*a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+
1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*
sin(d*x+c))^(1/2)/a/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+1/8*(16*a^2+21*b^2)*(s
in(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1
/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2
)/d/(a+b*sin(d*x+c))^(1/2)+1/8*b*(36*a^2+b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(
1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a/d/(a+b*sin(d*x+c))^(
1/2)
```

Rubi [A] time = 1.10, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2725, 3047, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(16a^2+b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d}+\frac{\left(32a^2+b^2\right)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{24a^2d}-\frac{\left(16a^2+21b^2\right)\sqrt{\frac{a+}{}}}{8d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(b*(16*a^2 + b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(8*a^2*d) + ((32*
a^2 + b^2)*Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(24*a^2*d) + (b*Cot[c +
 d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(12*a^2*d) - (Cot[c + d*x]*C
sc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2))/(3*a*d) + ((32*a^2 - b^2)*Ellipti
cE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(8*a*d*Sqrt
[(a + b*Sin[c + d*x])/(a + b)]) - ((16*a^2 + 21*b^2)*EllipticF[(c - Pi/2 +
d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*d*Sqrt[a + b*
Sin[c + d*x]]) - (b*(36*a^2 + b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/
(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*a*d*Sqrt[a + b*Sin[c + d*x]
])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{5/2}}{12a^2d} - \frac{\cot(c+dx)\csc^2(c+ }{3}$$

$$= \frac{\left(32a^2+b^2\right)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{24a^2d} + \frac{b\cot(c+dx)\csc(c+}{12}$$

$$= -\frac{b\left(16a^2+b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{\left(32a^2+b^2\right)\cot(c+}{2}$$

$$= -\frac{b\left(16a^2+b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{\left(32a^2+b^2\right)\cot(c+}{2}$$

$$= -\frac{b\left(16a^2+b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{\left(32a^2+b^2\right)\cot(c+}{2}$$

$$= -\frac{b\left(16a^2+b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{\left(32a^2+b^2\right)\cot(c+}{2}$$

$$= -\frac{b\left(16a^2+b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{\left(32a^2+b^2\right)\cot(c+}{2}$$

Mathematica [C] time = 6.62, size = 600, normalized size = 1.55

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{\csc(c+dx)\left(32a^2\cos(c+dx)-3b^2\cos(c+dx)\right)}{24a} - \frac{1}{3}a\cot(c+dx)\csc^2(c+dx) - \frac{2}{3}b\cos(c+dx) - \frac{7}{12}b\cot(c\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-2*b*Cos[c + d*x])/3 + ((32*a^2*Cos[c + d*x] - 3*b^2*Cos[c + d*x])*Csc[c
 + d*x])/(24*a) - (7*b*Cot[c + d*x]*Csc[c + d*x])/12 - (a*Cot[c + d*x]*Csc[
c + d*x]^2)/3)*Sqrt[a + b*Sin[c + d*x]])/d + ((-2*(32*a^3 - 44*a*b^2)*Ellip
ticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]
)/Sqrt[a + b*Sin[c + d*x]] - (2*(-40*a^2*b - 3*b^3)*EllipticPi[2, (-c + Pi/
2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*S
in[c + d*x]] - ((2*I)*(-32*a^2*b + b^3)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*
(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
(a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt
[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*S
in[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a +
b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x])
- 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) +
(a + b*Sin[c + d*x])^2)/b^2)])))/(32*a*d)
```

fricas [F] time = 146.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cot\left(dx+c\right)^{4}\sin\left(dx+c\right)+a\cot\left(dx+c\right)^{4}\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cot(d*x + c)^4*sin(d*x + c) + a*cot(d*x + c)^4)*sqrt(b*sin(d*x
+ c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.16, size = 1511, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/24*(48*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)^3+48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^3-162*b^2*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^3*sin(d*x+c)^3+63*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3+3*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)
^3-96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^5*sin(d*x+c)^3+99*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-3*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4
*sin(d*x+c)^3+108*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(
1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-108*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*
a^2*b^3*sin(d*x+c)^3+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-
b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3-3*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))
*b^5*sin(d*x+c)^3+16*a^2*b^3*sin(d*x+c)^6-16*a^3*b^2*sin(d*x+c)^5+3*a*b^4*s
in(d*x+c)^5-32*a^4*b*sin(d*x+c)^4+a^2*b^3*sin(d*x+c)^4+38*a^3*b^2*sin(d*x+c
)^3-3*a*b^4*sin(d*x+c)^3+40*a^4*b*sin(d*x+c)^2-17*a^2*b^3*sin(d*x+c)^2-22*a
^3*b^2*sin(d*x+c)-8*a^4*b)/a^2/b/sin(d*x+c)^3/cos(d*x+c)/(a+b*sin(d*x+c))^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cot(d*x + c)^4, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cot (c+d\,x)^4\,{\left(a+b\,\sin (c+d\,x)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.1157 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=408

$$\frac{b(68a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d}-\frac{b(20a^2+b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{64ad\sqrt{a+b\sin(c+dx)}}+\frac{b(236a^2+3}{}$$

```
[Out] 1/32*(20*a^2-b^2)*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d+1/8*b*
cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2)/a^2/d-1/4*cot(d*x+c)*csc(d*x
+c)^3*(a+b*sin(d*x+c))^(5/2)/a/d+1/64*b*(68*a^2-3*b^2)*cot(d*x+c)*(a+b*sin(
d*x+c))^(1/2)/a^2/d-1/64*b*(236*a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(
b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)
+1/64*b*(20*a^2+b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1
/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*
sin(d*x+c))/(a+b))^(1/2)/a/d/(a+b*sin(d*x+c))^(1/2)-3/64*(16*a^4-24*a^2*b^2
+b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/
(a+b))^(1/2)/a^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.24, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2893, 3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b(68a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d}-\frac{b(20a^2+b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{64ad\sqrt{a+b\sin(c+dx)}}+\frac{b(236a^2+3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (b*(68*a^2 - 3*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(64*a^2*d) + ((2
0*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(32*a^2*
d) + (b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2))/(8*a^2*d) -
 (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2))/(4*a*d) + (b*(236
*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c
 + d*x]])/(64*a^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (b*(20*a^2 + b^2)
*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
+ b)])/(64*a*d*Sqrt[a + b*Sin[c + d*x]]) + (3*(16*a^4 - 24*a^2*b^2 + b^4)*E
llipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(64*a^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
```

```
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}}{8a^2d} - \frac{\cot(c+dx}{}$$

$$= \frac{\left(20a^2-b^2\right)\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}}{32a^2d} +$$

$$= \frac{b\left(68a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{\left(20a^2-b^2\right)}{}$$

$$= \frac{b\left(68a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{\left(20a^2-b^2\right)}{}$$

$$= \frac{b\left(68a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{\left(20a^2-b^2\right)}{}$$

$$= \frac{b\left(68a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{\left(20a^2-b^2\right)}{}$$

$$= \frac{b\left(68a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{\left(20a^2-b^2\right)}{}$$

<u>Mathematica</u> [C] time = 6.70, size = 641, normalized size = 1.57

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{3\csc(c+dx)\left(36a^2b\cos(c+dx)+b^3\cos(c+dx)\right)}{64a^2} + \frac{\csc^2(c+dx)\left(20a^2\cos(c+dx)-b^2\cos(c+dx)\right)}{32a} - \frac{1}{4}a\cot(c+dx)\csc^3(c\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((3*(36*a^2*b*Cos[c + d*x] + b^3*Cos[c + d*x])*Csc[c + d*x])/(64*a^2) + ((
20*a^2*Cos[c + d*x] - b^2*Cos[c + d*x])*Csc[c + d*x]^2)/(32*a) - (3*b*Cot[c
 + d*x]*Csc[c + d*x]^2)/8 - (a*Cot[c + d*x]*Csc[c + d*x]^3)/4)*Sqrt[a + b*S
in[c + d*x]])/d + ((-2*(432*a^3*b + 4*a*b^3)*EllipticF[(-c + Pi/2 - d*x)/2,
 (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]
```

```
] - (2*(96*a^4 + 92*a^2*b^2 + 9*b^4)*EllipticPi[2, (-c + Pi/2 - d*x)/2, (2*
b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] -
((2*I)*(-236*a^2*b^2 - 3*b^4)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*El
lipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a
 - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c +
d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x
])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a + b)^(-1)]*S
qrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b
*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin
[c + d*x])^2)/b^2)]))/(256*a^2*d)
```

fricas [F] time = 123.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cot\left(dx+c\right)^{4}\csc\left(dx+c\right)\sin\left(dx+c\right)+a\cot\left(dx+c\right)^{4}\csc\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] integral((b*cot(d*x + c)^4*csc(d*x + c)*sin(d*x + c) + a*cot(d*x + c)^4*csc
(d*x + c))*sqrt(b*sin(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [B] time = 2.35, size = 1760, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/64*(-20*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4+234*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
```

```
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4-((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^2*b^3*sin(d*x+c)^4+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4-48*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
lipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b*
sin(d*x+c)^4-72*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/
2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4+72*((a+b*sin(d*x+c))/(
a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)
*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2
*b^3*sin(d*x+c)^4+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b)
)^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))
^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4-233*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*s
in(d*x+c)^4-216*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4+16*a^5+108*a^3*b^2*sin(d*x+c)^6+3*a
*b^4*sin(d*x+c)^6+148*a^4*b*sin(d*x+c)^5-188*a^4*b*sin(d*x+c)^3+40*a^4*b*si
n(d*x+c)+40*a^5*sin(d*x+c)^4-56*a^5*sin(d*x+c)^2+a^2*b^3*sin(d*x+c)^5-134*a
^3*b^2*sin(d*x+c)^4-3*a*b^4*sin(d*x+c)^4-a^2*b^3*sin(d*x+c)^3+26*a^3*b^2*si
n(d*x+c)^2-3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4+48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((
a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4-
3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a
-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^4+236*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(si
n(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*
sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4)/a^3/sin(d*x
+c)^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^4\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cot(d*x + c)^4*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\sin\left(c+d\,x\right)^2-1\right)^2\,\left(a+b\,\sin\left(c+d\,x\right)\right)^{3/2}}{\sin\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(3/2))/sin(c + d*x),x)
[Out] int(((sin(c + d*x)^2 - 1)^2*(a + b*sin(c + d*x))^(3/2))/sin(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.1158 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=484

$$\frac{(32a^2-5b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}}{80a^2d}+\frac{3b(36a^2-5b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c-}}{320a^2d}$$

```
[Out] 1/80*(32*a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2)/a^2/d+1/
8*b*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2)/a^2/d-1/5*cot(d*x+c)*csc
(d*x+c)^4*(a+b*sin(d*x+c))^(5/2)/a/d-1/640*(128*a^4-116*a^2*b^2+15*b^4)*cot
(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^3/d+3/320*b*(36*a^2-5*b^2)*cot(d*x+c)*csc(
d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^2/d+1/640*(128*a^4-116*a^2*b^2+15*b^4)*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^3/d/((
a+b*sin(d*x+c))/(a+b))^(1/2)-1/640*(128*a^4+692*a^2*b^2+5*b^4)*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*P
i+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^2/d/(a
+b*sin(d*x+c))^(1/2)-3/128*b*(48*a^4+8*a^2*b^2-b^4)*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x)
,2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d
*x+c))^(1/2)
```

Rubi [A] time = 1.65, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2893, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{(-116a^2b^2+128a^4+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d}+\frac{(692a^2b^2+128a^4+5b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx\right.\right.}{640a^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((128*a^4 - 116*a^2*b^2 + 15*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(
640*a^3*d) + (3*b*(36*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin
[c + d*x]])/(320*a^2*d) + ((32*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x]^2*(a
+ b*Sin[c + d*x])^(3/2))/(80*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3*(a + b
*Sin[c + d*x])^(5/2))/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c
 + d*x])^(5/2))/(5*a*d) - ((128*a^4 - 116*a^2*b^2 + 15*b^4)*EllipticE[(c -
Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(640*a^3*d*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]) + ((128*a^4 + 692*a^2*b^2 + 5*b^4)*EllipticF[(c
```

```
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(640*a
^2*d*Sqrt[a + b*Sin[c + d*x]]) + (3*b*(48*a^4 + 8*a^2*b^2 - b^4)*EllipticPi
[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/
(128*a^3*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
```

```
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/2}}{8a^2d} - \frac{\cot(c+a}{}$$

$$= \frac{(32a^2-5b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}}{80a^2d}$$

$$= \frac{3b(36a^2-5b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{320a^2d}$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d}$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d}$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d}$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d}$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d}$$

Mathematica [C] time = 6.76, size = 700, normalized size = 1.45

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{\csc^2(c+dx)(236a^2b\cos(c+dx)+5b^3\cos(c+dx))}{320a^2} + \frac{\csc^3(c+dx)(32a^2\cos(c+dx)-b^2\cos(c+dx))}{80a} + \frac{\csc(c+dx)(-128a^4\cos(c+}{}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((((-128*a^4*Cos[c + d*x] + 116*a^2*b^2*Cos[c + d*x] - 15*b^4*Cos[c + d*x])
*Csc[c + d*x])/(640*a^3) + ((236*a^2*b*Cos[c + d*x] + 5*b^3*Cos[c + d*x])*C
```

```
sc[c + d*x]^2)/(320*a^2) + ((32*a^2*Cos[c + d*x] - b^2*Cos[c + d*x])*Csc[c
+ d*x]^3)/(80*a) - (11*b*Cot[c + d*x]*Csc[c + d*x]^3)/40 - (a*Cot[c + d*x]*
Csc[c + d*x]^4)/5)*Sqrt[a + b*Sin[c + d*x]])/d + (b*((-2*(1616*a^3*b - 20*a
*b^3)*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(1312*a^4 + 356*a^2*b^2 - 45*b^4
)*EllipticPi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(128*a^4 - 116*a^2*b^2 + 15*
b^4)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(
a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF
[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]
- b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c +
d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b
*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(
-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-(
(a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)])))/(2
560*a^3*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(SimpleAlgebraicExtension(InnerPr
imeField(7),SparseUnivariatePolynomial(InnerPrimeField(7)),?^2+1)),failed)
cannot be coerced to mode SparseUnivariatePolynomial(SimpleAlgebraicExtensi
on(InnerPrimeField(7),SparseUnivariatePolynomial(InnerPrimeField(7)),?^2+1)
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [B] time = 2.40, size = 2075, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x)

[Out] -1/640*(128*a^6*b+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5+128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b*sin(d*x+c)^5-936*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5+692*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d*x+c)^5+126*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5+5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^5-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5+720*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5-720*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d*x+c)^5+120*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5-120*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^5-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5+244*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5-131*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^7*sin(d*x+c)^5-128*a^5*b^2*sin(d*x+c)^7+116*a^3*b^4*sin(d*x+c)^7-15*a*b^6*sin(d*x+c)^7-128*a^6*b*sin(d*x+c)^6+588*a^4*b^3*sin(d*x+c)^6-5*a^2*b^5*sin(d*x+c)^6+384*a^6*b*sin(d*x+c)^4-772*a^4*b^3*sin(d*x+c)^4+5*a^2*b^5*sin(d*x+c)^4-1032*a^5*b^2*sin(d*x+c)^3+856*a^5*b^2*sin(d*x+c)^5+15*a*b^6*sin(d*x+c)^5-114*

```
a^3*b^4*sin(d*x+c)^5-2*a^3*b^4*sin(d*x+c)^3-384*a^6*b*sin(d*x+c)^2+184*a^4*
b^3*sin(d*x+c)^2+304*a^5*b^2*sin(d*x+c)-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^5)/a^4/b/
sin(d*x+c)^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^4\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cot(d*x + c)^4*csc(d*x + c)^2, x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(3/2))/sin(c + d*x)^2,x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.1159 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=551

$$\frac{7\left(4a^2-b^2\right)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}}{96a^2d}+\frac{b\left(156a^2-35b^2\right)\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c}}{960a^2d}$$

```
[Out] 7/96*(4*a^2-b^2)*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2)/a^2/d+7/60*
b*cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^(5/2)/a^2/d-1/6*cot(d*x+c)*csc(d
*x+c)^5*(a+b*sin(d*x+c))^(5/2)/a/d-1/7680*b*(2064*a^4+512*a^2*b^2-105*b^4)*
cot(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^4/d-1/3840*(240*a^4-168*a^2*b^2+35*b^4)
*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^3/d+1/960*b*(156*a^2-35*b^2
)*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/a^2/d+1/7680*b*(2064*a^4+5
12*a^2*b^2-105*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/
2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*si
n(d*x+c))^(1/2)/a^4/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/7680*b*(2544*a^4+176
*a^2*b^2-35*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(
d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)-1/512*(64*a^6+144*a^4*b^2
-36*a^2*b^4+7*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2
*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b
*sin(d*x+c))/(a+b))^(1/2)/a^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.98, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2893, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(512a^2b^2+2064a^4-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{7680a^4d}+\frac{b\left(176a^2b^2+2544a^4-35b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\right(}{7680a^3d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(b*(2064*a^4 + 512*a^2*b^2 - 105*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]
])/(7680*a^4*d) - ((240*a^4 - 168*a^2*b^2 + 35*b^4)*Cot[c + d*x]*Csc[c + d*
x]*Sqrt[a + b*Sin[c + d*x]])/(3840*a^3*d) + (b*(156*a^2 - 35*b^2)*Cot[c + d
*x]*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(960*a^2*d) + (7*(4*a^2 - b^2)
*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2))/(96*a^2*d) + (7*b*
Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2))/(60*a^2*d) - (Cot[c
 + d*x]*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2))/(6*a*d) - (b*(2064*a^4 +
```

```
 512*a^2*b^2 - 105*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a
 + b*Sin[c + d*x]])/(7680*a^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(2
544*a^4 + 176*a^2*b^2 - 35*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(7680*a^3*d*Sqrt[a + b*Sin[c + d*x]])
 + ((64*a^6 + 144*a^4*b^2 - 36*a^2*b^4 + 7*b^6)*EllipticPi[2, (c - Pi/2 + d
*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(512*a^4*d*Sqrt[a
 + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{7b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^{5/2}}{60a^2d} - \frac{\cot(c+}{}$$

$$= \frac{7\left(4a^2-b^2\right)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}}{96a^2d}$$

$$= \frac{b\left(156a^2-35b^2\right)\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx}}{960a^2d}$$

$$= -\frac{\left(240a^4-168a^2b^2+35b^4\right)\cot(c+dx)\csc(c+dx)\sqrt{a+b}}{3840a^3d}$$

$$= -\frac{b\left(2064a^4+512a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+}}{7680a^4d}$$

$$= -\frac{b\left(2064a^4+512a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+}}{7680a^4d}$$

$$= -\frac{b\left(2064a^4+512a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+}}{7680a^4d}$$

$$= -\frac{b\left(2064a^4+512a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+}}{7680a^4d}$$

$$= -\frac{b\left(2064a^4+512a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+}}{7680a^4d}$$

Mathematica [C] time = 6.78, size = 771, normalized size = 1.40

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{\csc^3(c+dx)\left(436a^2b\cos(c+dx)+7b^3\cos(c+dx)\right)}{960a^2}+\frac{\csc^4(c+dx)\left(140a^2\cos(c+dx)-3b^2\cos(c+dx)\right)}{480a}+\frac{\csc(c+dx)\left(-2064a^4b\,\mathrm{co}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((((-2064*a^4*b*Cos[c + d*x] - 512*a^2*b^3*Cos[c + d*x] + 105*b^5*Cos[c + d
*x])*Csc[c + d*x])/(7680*a^4) + ((-240*a^4*Cos[c + d*x] + 168*a^2*b^2*Cos[c
 + d*x] - 35*b^4*Cos[c + d*x])*Csc[c + d*x]^2)/(3840*a^3) + ((436*a^2*b*Cos
[c + d*x] + 7*b^3*Cos[c + d*x])*Csc[c + d*x]^3)/(960*a^2) + ((140*a^2*Cos[c
 + d*x] - 3*b^2*Cos[c + d*x])*Csc[c + d*x]^4)/(480*a) - (13*b*Cot[c + d*x]*
Csc[c + d*x]^4)/60 - (a*Cot[c + d*x]*Csc[c + d*x]^5)/6)*Sqrt[a + b*Sin[c +
d*x]])/d + ((-2*(960*a^5*b - 672*a^3*b^3 + 140*a*b^5)*EllipticF[(-c + Pi/2
- d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin
[c + d*x]] - (2*(1920*a^6 + 2256*a^4*b^2 - 1592*a^2*b^4 + 315*b^6)*Elliptic
Pi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)
])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(2064*a^4*b^2 + 512*a^2*b^4 - 105*b^6)
*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*A
rcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*
EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]
]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin
[c + d*x])/(a - b))])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a
^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2
 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(30720*
a^4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [B] time = 3.03, size = 2458, normalized size = 4.46

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x)

[Out] -1/7680*(-960*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^6*b*sin(d*x+c)^6+2160*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^6-2160*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d*x+c)^6-540*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6+540*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^6+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6+1552*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^6+617*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6+2544*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b*sin(d*x+c)^6-1728*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^6-105*a*b^6*sin(d*x+c)^6-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^7*sin(d*x+c)^6-35*a^2*b^5*sin(d*x+c)^5+14*a^3*b^4*sin(d*x+c)^4-8*a^4*b^3*sin(d*x+c)^3+1712*a^5*b^2*sin(d*x+c)^2+1280*a^7-2064*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^6-480*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^6+960*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^6-2064*a^5*b^2*sin(d*x+c)^8-512*a^3*b^4*sin(d*x+c)^8+105*a*b^6*sin(d*x+c)^8-2544*a^6*b*sin(d*x+c)^7-176*a^4*b^3*sin(d*x+c)^7+184*a^4*b^3*sin(d*x+c)^5+498*a^3*b^4*sin(d*x+c)^6+5888*a^5*b^2*sin(d*x+c)^6+35*a^2*b^5*sin(d*x+c)^7+8272*a^6*b*sin(d*x+c)^5-5536*a^5*b^2*sin(d*x+c)^4-8672*a^6*b*sin(d*x+c)^3+2944*

```
a^6*b*sin(d*x+c)-480*a^7*sin(d*x+c)^6+2720*a^7*sin(d*x+c)^4-3520*a^7*sin(d*
x+c)^2+176*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^4*b^3*sin(d*x+c)^6-582*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((
a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6-35*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^2*b^5*sin(d*x+c)^6+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6)/a^5/sin(d*x+c)^6/
cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^4\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cot(d*x + c)^4*csc(d*x + c)^3, x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(3/2))/sin(c + d*x)^3,x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.1160 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=451

$$-\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(8a^4-7ab\left(a^2+63b^2\right)\sin(c+\right.}{9009b^2d}$$

```
[Out] -2/39*a*cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2)/d-2/15*cos(d*x+c)^5*(a+b*sin(d*
x+c))^(5/2)/d-2/429*(3*a^2+13*b^2)*cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2)/d-2/
9009*cos(d*x+c)^3*(8*a^4-33*a^2*b^2-39*b^4-7*a*b*(a^2+63*b^2)*sin(d*x+c))*(
a+b*sin(d*x+c))^(1/2)/b^2/d+4/45045*cos(d*x+c)*(32*a^6-165*a^4*b^2+450*a^2*
b^4+195*b^6-24*a*b*(a^4-5*a^2*b^2-60*b^4)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2
)/b^4/d-8/45045*a*(32*a^6-189*a^4*b^2+570*a^2*b^4+1635*b^6)*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^5/d/((a+b*sin(d*x
+c))/(a+b))^(1/2)+8/45045*(32*a^8-197*a^6*b^2+615*a^4*b^4-255*a^2*b^6-195*b
^8)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF
(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b)
)^(1/2)/b^5/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.07, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2862, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(-7ab\left(a^2+63b^2\right)\sin(c+dx\right.}{9009b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(3*a^2 + 13*b^2)*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(429*d) - (2*
a*Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2))/(39*d) - (2*Cos[c + d*x]^5*(a
+ b*Sin[c + d*x])^(5/2))/(15*d) + (8*a*(32*a^6 - 189*a^4*b^2 + 570*a^2*b^4
+ 1635*b^6)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c +
 d*x]])/(45045*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*(32*a^8 - 197
*a^6*b^2 + 615*a^4*b^4 - 255*a^2*b^6 - 195*b^8)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(45045*b^5*d*Sqrt[a +
 b*Sin[c + d*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(8*a^4 - 33*
a^2*b^2 - 39*b^4 - 7*a*b*(a^2 + 63*b^2)*Sin[c + d*x]))/(9009*b^2*d) + (4*Co
s[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^6 - 165*a^4*b^2 + 450*a^2*b^4 + 1
95*b^6 - 24*a*b*(a^4 - 5*a^2*b^2 - 60*b^4)*Sin[c + d*x]))/(45045*b^4*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{2\cos^5(c+dx)(a+b\sin(c+dx))^{5/2}}{15d}+\frac{2}{15}\int \cos^4(c+dx)$$

$$= -\frac{2a\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{39d}-\frac{2\cos^5(c+dx)(a+}{15}$$

$$= -\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2a\cos^5(}{}$$

$$= -\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2a\cos^5(}{}$$

$$= -\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2a\cos^5(}{}$$

$$= -\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2a\cos^5(}{}$$

$$= -\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2a\cos^5(}{}$$

$$= -\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2a\cos^5(}{}$$

Mathematica [A] time = 21.65, size = 450, normalized size = 1.00

$$\frac{256\left(32a^8-197a^6b^2+615a^4b^4-255a^2b^6-195b^8\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+b\cos(c+dx)\left(409\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-256*a*(32*a^7 + 32*a^6*b - 189*a^5*b^2 - 189*a^4*b^3 + 570*a^3*b^4 + 570*
a^2*b^5 + 1635*a*b^6 + 1635*b^7)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 256*(32*a^8 - 197*a^6*b^2 + 615*
a^4*b^4 - 255*a^2*b^6 - 195*b^8)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*Cos[c + d*x]*(4096*a^7 - 23936
*a^5*b^2 - 36512*a^3*b^4 + 67584*a*b^6 + 8*(32*a^5*b^2 - 18192*a^3*b^4 - 18
741*a*b^6)*Cos[2*(c + d*x)] - 224*(161*a^3*b^4 - 54*a*b^6)*Cos[4*(c + d*x)]
 + 20328*a*b^6*Cos[6*(c + d*x)] + 1024*a^6*b*Sin[c + d*x] - 5840*a^4*b^3*Si
n[c + d*x] + 186768*a^2*b^5*Sin[c + d*x] + 8151*b^7*Sin[c + d*x] - 80*a^4*b
^3*Sin[3*(c + d*x)] - 101688*a^2*b^5*Sin[3*(c + d*x)] - 22269*b^7*Sin[3*(c
+ d*x)] - 46536*a^2*b^5*Sin[5*(c + d*x)] - 2457*b^7*Sin[5*(c + d*x)] + 3003
*b^7*Sin[7*(c + d*x)]))/(1441440*b^5*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 1.24, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(2\,ab\cos\left(dx+c\right)^6-2\,ab\cos\left(dx+c\right)^4+\left(b^2\cos\left(dx+c\right)^6-\left(a^2+b^2\right)\cos\left(dx+c\right)^4\right)\sin\left(dx+c\right)\right)\sqrt{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] integral(-(2*a*b*cos(d*x + c)^6 - 2*a*b*cos(d*x + c)^4 + (b^2*cos(d*x + c)^
6 - (a^2 + b^2)*cos(d*x + c)^4)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^4\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

maple [B] time = 1.86, size = 1801, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/45045*(4508*a^3*b^6*sin(d*x+c)^6-26922*a*b^8*sin(d*x+c)^6-5*a^4*b^5*sin(d
*x+c)^5-32532*a^2*b^7*sin(d*x+c)^5+8*a^5*b^4*sin(d*x+c)^4-13564*a^3*b^6*sin
(d*x+c)^4+19302*a*b^8*sin(d*x+c)^4-16*a^6*b^3*sin(d*x+c)^3+100*a^4*b^5*sin(
d*x+c)^3+26382*a^2*b^7*sin(d*x+c)^3-64*a^7*b^2*sin(d*x+c)^2+362*a^5*b^4*sin
(d*x+c)^2+12464*a^3*b^6*sin(d*x+c)^2-1764*a*b^8*sin(d*x+c)^2+16*a^6*b^3*sin
(d*x+c)-95*a^4*b^5*sin(d*x+c)-5484*a^2*b^7*sin(d*x+c)+10164*a*b^8*sin(d*x+c
)^8-780*a*b^8+64*a^7*b^2-3408*a^3*b^6-370*a^5*b^4+11634*a^2*b^7*sin(d*x+c)^
7-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^9-788*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^3+3003*b^9*sin(d*x+c)^9-7644*b^9*sin(d*x+
c)^7+5109*b^9*sin(d*x+c)^5+312*b^9*sin(d*x+c)^3-780*b^9*sin(d*x+c)+576*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*a^5*b^4-5760*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a*b^8+2460*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^5+5280*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^6-1020*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^2*b^7+884*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a^7*b^2-780*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^9-3036*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
lipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^4-4260*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a^3*b^6+6540*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a*b^8+128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(
```

```
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^8*b-96*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*b^2)/b^6/cos(
d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^4\,\sin(c+d\,x)\,\left(a+b\,\sin(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.1161 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=447

$$\frac{2a^3\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(8a^2-117b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{693b^2d}-\frac{2a\left(8a^2-131b^2\right)c}{}$$

```
[Out] -2/693*a*(8*a^2-131*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b^2/d-2/693*(8*a
^2-117*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(5/2)/b^2/d+8/99*a*cos(d*x+c)*(a+b*
sin(d*x+c))^(7/2)/b^2/d-2/11*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(7/2)/b
/d-2/693*(8*a^4-141*a^2*b^2+36*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^2/d
-2/693*a*(8*a^4-147*a^2*b^2+444*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/si
n(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b
))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^3/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+2/693
*(8*a^6-149*a^4*b^2-516*a^2*b^4-36*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(
a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*sin(d*x+c))^(1/2)-2*
a^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticP
i(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a
+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.42, antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2895, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2\left(8a^2-117b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{693b^2d}-\frac{2a\left(8a^2-131b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{693b^2d}-\frac{2\left(-141a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(8*a^4 - 141*a^2*b^2 + 36*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(
693*b^2*d) - (2*a*(8*a^2 - 131*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2)
)/(693*b^2*d) - (2*(8*a^2 - 117*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2
))/(693*b^2*d) + (8*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(99*b^2*d) -
 (2*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(11*b*d) + (2*a*(
8*a^4 - 147*a^2*b^2 + 444*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]
*Sqrt[a + b*Sin[c + d*x]])/(693*b^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) -
 (2*(8*a^6 - 149*a^4*b^2 - 516*a^2*b^4 - 36*b^6)*EllipticF[(c - Pi/2 + d*x)
/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(693*b^3*d*Sqrt[a +
```

```
b*Sin[c + d*x]]) + (2*a^3*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{8a\cos(c+dx)(a+b\sin(c+dx))^{7/2}}{99b^2d} - \frac{2\cos(c+dx)\sin(c$$

$$= -\frac{2\left(8a^2-117b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{693b^2d} + \frac{8a\,\mathrm{co}$$

$$= -\frac{2a\left(8a^2-131b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{693b^2d} - \frac{2\left(8\right.}{}$$

$$= -\frac{2\left(8a^4-141a^2b^2+36b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^2d}$$

$$= -\frac{2\left(8a^4-141a^2b^2+36b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^2d}$$

$$= -\frac{2\left(8a^4-141a^2b^2+36b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^2d}$$

$$= -\frac{2\left(8a^4-141a^2b^2+36b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^2d}$$

$$= -\frac{2\left(8a^4-141a^2b^2+36b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^2d}$$

Mathematica [C] time = 4.02, size = 521, normalized size = 1.17

$$\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(32a^4-24a^3b\sin(c+dx)+2660a^2b^2+4\left(113a^2b^2-54b^4\right)\cos(2(c+dx))+195\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(((2*I)*(8*a^4 - 147*a^2*b^2 + 444*b^4)*(-2*a*(a - b)*EllipticE[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*
```

```
a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b
)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
 b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c +
d*x]))/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(b^2*Sqrt[-(a + b)^
(-1)]) + (8*b*(a^4 + 480*a^2*b^2 + 18*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4,
 (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]
] + (2*a*(8*a^4 + 1239*a^2*b^2 + 444*b^4)*EllipticPi[2, (-2*c + Pi - 2*d*x)
/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d
*x]]) + Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^4 + 2660*a^2*b^2 - 9*b^
4 + 4*(113*a^2*b^2 - 54*b^4)*Cos[2*(c + d*x)] - 63*b^4*Cos[4*(c + d*x)] - 2
4*a^3*b*Sin[c + d*x] + 1954*a*b^3*Sin[c + d*x] + 322*a*b^3*Sin[3*(c + d*x)]
))/(2772*b^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [B] time = 2.04, size = 1573, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/693*(-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^5*b^2+224*a*b^6*sin(d*x+c)^6-36*a*b^6+4*a^5*b^2-693*a^3*
((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x
```

```
+c))*b/(a-b))^(1/2)*b^4*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,(
(a-b)/(a+b))^(1/2))+389*a^3*b^4+274*a^2*b^5*sin(d*x+c)^5+116*a^3*b^4*sin(d*
x+c)^4-706*a*b^6*sin(d*x+c)^4-a^4*b^3*sin(d*x+c)^3-1028*a^2*b^5*sin(d*x+c)^
3-4*a^5*b^2*sin(d*x+c)^2-505*a^3*b^4*sin(d*x+c)^2+518*a*b^6*sin(d*x+c)^2+a^
4*b^3*sin(d*x+c)+754*a^2*b^5*sin(d*x+c)+63*b^7*sin(d*x+c)^7-180*b^7*sin(d*x
+c)^5+153*b^7*sin(d*x+c)^3-36*b^7*sin(d*x+c)-36*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^7-8*((a+b*sin(d*x+
c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^
(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7-149
*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*
x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))*a^4*b^3+444*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b)
)^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^
(1/2),((a-b)/(a+b))^(1/2))*a*b^6+1107*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4-516*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5+8*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^6*b-408*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a*b^6+155*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-591*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+693*a^2*
((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x
+c))*b/(a-b))^(1/2)*b^5*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,(
(a-b)/(a+b))^(1/2)))/b^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^3*cot(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^3\cot(c+d\,x)\,\left(a+b\,\sin(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*cot(c + d*x)*(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^3*cot(c + d*x)*(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.1162 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=426

$$\frac{(4a^2+63b^2)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{63abd}+\frac{(20a^2+469b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315bd}+\frac{a(20a^2+759}{}$$

```
[Out] 1/315*(20*a^2+469*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b/d+1/63*(4*a^2+63
*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(5/2)/a/b/d-2/9*cos(d*x+c)*(a+b*sin(d*x+c
))^(7/2)/b/d-cot(d*x+c)*(a+b*sin(d*x+c))^(7/2)/a/d+1/315*a*(20*a^2+759*b^2)
*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b/d+1/315*(20*a^4+1689*a^2*b^2-168*b^4)*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos
(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^2/
d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/315*a*(20*a^4+739*a^2*b^2+816*b^4)*(sin(
1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*
c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b
^2/d/(a+b*sin(d*x+c))^(1/2)-5*a^2*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a
+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.42, antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2894, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(4a^2+63b^2)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{63abd}+\frac{(20a^2+469b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315bd}+\frac{a(20a^2+759}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a*(20*a^2 + 759*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(315*b*d) + ((
20*a^2 + 469*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(315*b*d) + ((4*
a^2 + 63*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(63*a*b*d) - (2*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(9*b*d) - (Cot[c + d*x]*(a + b*Sin[c +
 d*x])^(7/2))/(a*d) - ((20*a^4 + 1689*a^2*b^2 - 168*b^4)*EllipticE[(c - Pi/
2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^2*d*Sqrt[(a + b
*Sin[c + d*x])/(a + b)]) + (a*(20*a^4 + 739*a^2*b^2 + 816*b^4)*EllipticF[(c
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(315*b
^2*d*Sqrt[a + b*Sin[c + d*x]]) + (5*a^2*b*EllipticPi[2, (c - Pi/2 + d*x)/2,
```

```
(2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d
*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{2\cos(c+dx)(a+b\sin(c+dx))^{7/2}}{9bd} - \frac{\cot(c+dx)(a+b\sin}{ad}$$

$$= \frac{\left(4a^2+63b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{63abd} - \frac{2\cos(c+}{}$$

$$= \frac{\left(20a^2+469b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315bd} + \frac{\left(4a^2+\right.}{}$$

$$= \frac{a\left(20a^2+759b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + \frac{\left(20a^2\right.}{}$$

$$= \frac{a\left(20a^2+759b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + \frac{\left(20a^2\right.}{}$$

$$= \frac{a\left(20a^2+759b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + \frac{\left(20a^2\right.}{}$$

$$= \frac{a\left(20a^2+759b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + \frac{\left(20a^2\right.}{}$$

$$= \frac{a\left(20a^2+759b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + \frac{\left(20a^2\right.}{}$$

Mathematica [C] time = 4.79, size = 496, normalized size = 1.16

$$\frac{\frac{8ab\left(475a^2-492b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2\left(20a^4-1461a^2b^2-168b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} - \frac{2i\left(-20a^4-1689a^2b\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)*(-20*a^4 - 1689*a^2*b^2 + 168*b^4)*(-2*a*(a - b)*EllipticE[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*
```

```
a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b
)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
 b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c +
d*x]))/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(a*b^2*Sqrt[-(a + b
)^(-1)]) + (8*a*b*(475*a^2 - 492*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (
2*(20*a^4 - 1461*a^2*b^2 - 168*b^4)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] -
 Sqrt[a + b*Sin[c + d*x]]*((40*a^3 - 2202*a*b^2)*Cos[c + d*x] + 2*b*(-95*a*
b*Cos[3*(c + d*x)] + 630*a^2*Cot[c + d*x] + (150*a^2 - 119*b^2 - 35*b^2*Cos
[2*(c + d*x)])*Sin[2*(c + d*x)])))/(1260*b*d)
```

fricas [F] time = 100.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(2\,ab\cos\left(dx+c\right)^{2}\cot\left(dx+c\right)^{2}\sin\left(dx+c\right)-\left(b^{2}\cos\left(dx+c\right)^{4}-\left(a^{2}+b^{2}\right)\cos\left(dx+c\right)^{2}\right)\cot\left(dx+c\right)^{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral((2*a*b*cos(d*x + c)^2*cot(d*x + c)^2*sin(d*x + c) - (b^2*cos(d*x +
 c)^4 - (a^2 + b^2)*cos(d*x + c)^2)*cot(d*x + c)^2)*sqrt(b*sin(d*x + c) + a
), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [A] time = 1.95, size = 865, normalized size = 2.03

$$-\frac{70b^{6}\sin\left(dx+c\right)\left(\cos^{6}\left(dx+c\right)\right)+\left(-340a^{2}b^{4}+14b^{6}\right)\left(\cos^{4}\left(dx+c\right)\right)\sin\left(dx+c\right)+\left(10a^{4}b^{2}+57a^{2}b^{4}-84b^{6}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/315*(70*b^6*sin(d*x+c)*cos(d*x+c)^6+(-340*a^2*b^4+14*b^6)*cos(d*x+c)^4*s
in(d*x+c)+(10*a^4*b^2+57*a^2*b^4-84*b^6)*cos(d*x+c)^2*sin(d*x+c)-(b/(a-b)*s
```

```
in(d*x+c)+a/(a-b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(
d*x+c)+b/(a+b))^(1/2)*(20*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^6+1669*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^4*b^2-1857*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^2*b^4+168*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))*b^6-1575*EllipticPi((b/(a-b)*sin(d*x+c)+a/(a-b))^(1
/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^4+1575*EllipticPi((b/(a-b)*sin(d*x+c
)+a/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^5-20*EllipticF((b/(a-b)*s
in(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b-930*EllipticF((b/(a-b)*
sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-739*EllipticF((b/(a-
b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3+2673*EllipticF((b
/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-816*EllipticF
((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5-168*Elliptic
F((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6)*sin(d*x+c)+2
60*a*b^5*cos(d*x+c)^6+(-160*a^3*b^3+232*a*b^5)*cos(d*x+c)^4+(475*a^3*b^3-49
2*a*b^5)*cos(d*x+c)^2)/sin(d*x+c)/b^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^2*cot(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2\cot(c+d\,x)^2\,(a+b\,\sin(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*cot(c + d*x)^2*(a + b*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^2*cot(c + d*x)^2*(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2*(a+b*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.1163 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=430

$$-\frac{\left(8a^2-21b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{28a^2d}-\frac{\left(8a^2-35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{28ad}-\frac{\left(8a^2-73b^2\right)\mathrm{co}}{}$$

```
[Out] -1/28*(8*a^2-35*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a/d-1/28*(8*a^2-21*b
^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(5/2)/a^2/d-3/4*b*cot(d*x+c)*(a+b*sin(d*x+c
))^(7/2)/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(7/2)/a/d-1/28*(8
*a^2-73*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/d-1/28*a*(8*a^2-247*b^2)*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/
2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b/d/((a
+b*sin(d*x+c))/(a+b))^(1/2)+1/28*(8*a^4+3*a^2*b^2-32*b^4)*(sin(1/2*c+1/4*Pi
+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2
*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b/d/(a+b*sin(
d*x+c))^(1/2)+3/4*a*(4*a^2-5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b
))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.42, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2893, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(8a^2-21b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{28a^2d}-\frac{\left(8a^2-35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{28ad}-\frac{\left(8a^2-73b^2\right)\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((8*a^2 - 73*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(28*d) - ((8*a^2
 - 35*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(28*a*d) - ((8*a^2 - 21*
b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(28*a^2*d) - (3*b*Cot[c + d*x
]*(a + b*Sin[c + d*x])^(7/2))/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]*(a + b
*Sin[c + d*x])^(7/2))/(2*a*d) + (a*(8*a^2 - 247*b^2)*EllipticE[(c - Pi/2 +
d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(28*b*d*Sqrt[(a + b*Sin[c
 + d*x])/(a + b)]) - ((8*a^4 + 3*a^2*b^2 - 32*b^4)*EllipticF[(c - Pi/2 + d*x
)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(28*b*d*Sqrt[a + b*
Sin[c + d*x]]) - (3*a*(4*a^2 - 5*b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*
```

```
b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*d*Sqrt[a + b*Sin[c + d*x
]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{3b\cot(c+dx)(a+b\sin(c+dx))^{7/2}}{4a^2d} - \frac{\cot(c+dx)\csc(c+$$

$$= -\frac{\left(8a^2-21b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{28a^2d} - \frac{3b\cot(c$$

$$= -\frac{\left(8a^2-35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{28ad} - \frac{\left(8a^2-2\right.}{}$$

$$= -\frac{\left(8a^2-73b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{\left(8a^2-35b\right.}{}$$

$$= -\frac{\left(8a^2-73b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{\left(8a^2-35b\right.}{}$$

$$= -\frac{\left(8a^2-73b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{\left(8a^2-35b\right.}{}$$

$$= -\frac{\left(8a^2-73b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{\left(8a^2-35b\right.}{}$$

$$= -\frac{\left(8a^2-73b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{\left(8a^2-35b\right.}{}$$

Mathematica [C] time = 5.54, size = 460, normalized size = 1.07

$$\frac{\frac{8b\left(125a^2-16b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2a\left(160a^2+37b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + 4\sqrt{a+b\sin(c+dx)}\,\left(\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)*(-8*a^2 + 247*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*Arc
```

```
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*El
lipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]]
, (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*
Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(b^2*Sqrt[-(a + b)^(-1)]) + (8*b*(12
5*a^2 - 16*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b
*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*a*(160*a^2 + 37*b^2)
*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*
x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + 4*Sqrt[a + b*Sin[c + d*x]]*((-24*a
^2 + 22*b^2)*Cos[c + d*x] + 2*b^2*Cos[3*(c + d*x)] - 7*a*Cot[c + d*x]*(9*b
+ 2*a*Csc[c + d*x]) - 12*a*b*Sin[2*(c + d*x)]))/(112*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [B] time = 2.24, size = 1520, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/28*(8*b^5*sin(d*x+c)^7+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^2-258*b^2*((a+b*sin(d*x+
c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^
(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*sin
(d*x+c)^2+3*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
```

```
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^2+279*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-32*b^
5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)^2-8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^2+255*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+
c)^2-247*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2+84*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^2*EllipticPi(((
a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)^2-
84*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*b^3*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/
a,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^2-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(
((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c
)^2+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)
/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^2+32*a*b^4*sin(d*x+c)^6+48*a^2*b^3*s
in(d*x+c)^5-32*b^5*sin(d*x+c)^5+24*a^3*b^2*sin(d*x+c)^4+7*a*b^4*sin(d*x+c)^
4+29*a^2*b^3*sin(d*x+c)^3+24*b^5*sin(d*x+c)^3-10*a^3*b^2*sin(d*x+c)^2-39*a*
b^4*sin(d*x+c)^2-77*a^2*b^3*sin(d*x+c)-14*a^3*b^2)/b^2/sin(d*x+c)^2/cos(d*x
+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\cot(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)*cot(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)\,\cot(c+d\,x)^3\,(a+b\,\sin(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*cot(c + d*x)^3*(a + b*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)*cot(c + d*x)^3*(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3*(a+b*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.1164 $\int \cot^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=429

$$-\frac{b\left(208a^2-25b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{120a^2d}-\frac{b\left(96a^2-25b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad}+\frac{\left(32a^2-3b^2\right.}{}$$

```
[Out] -1/120*b*(208*a^2-25*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d+1/24*(32*
a^2-3*b^2)*cot(d*x+c)*(a+b*sin(d*x+c))^(5/2)/a^2/d-1/12*b*cot(d*x+c)*csc(d*
x+c)*(a+b*sin(d*x+c))^(7/2)/a^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+
c))^(7/2)/a/d-1/40*b*(96*a^2-25*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a/d-
1/40*(176*a^2-167*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi
+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b
*sin(d*x+c))^(1/2)/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+1/40*a*(96*a^2+179*b^2)
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(
1/2)/d/(a+b*sin(d*x+c))^(1/2)+5/8*b*(12*a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,
2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^
(1/2)
```

Rubi [A] time = 1.36, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {2725, 3047, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(208a^2-25b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{120a^2d}-\frac{b\left(96a^2-25b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad}+\frac{\left(32a^2-3b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(b*(96*a^2 - 25*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(40*a*d) - (b*
(208*a^2 - 25*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(120*a^2*d) + (
(32*a^2 - 3*b^2)*Cot[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(24*a^2*d) - (b*C
ot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(12*a^2*d) - (Cot[c +
d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(7/2))/(3*a*d) + ((176*a^2 - 167*b
^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/
(40*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (a*(96*a^2 + 179*b^2)*EllipticF
[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(40
*d*Sqrt[a + b*Sin[c + d*x]]) - (5*b*(12*a^2 - b^2)*EllipticPi[2, (c - Pi/2
```

```
+ d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*d*Sqrt[a +
b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{7/2}}{12a^2d} - \frac{\cot(c+dx)\csc^2(c+$$

$$= \frac{\left(32a^2-3b^2\right)\cot(c+dx)(a+b\sin(c+dx))^{5/2}}{24a^2d} - \frac{b\cot(c+dx)\csc(c+}{1}$$

$$= -\frac{b\left(208a^2-25b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{120a^2d} + \frac{\left(32a^2-3b^2\right)\mathrm{co}}{}$$

$$= -\frac{b\left(96a^2-25b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b\left(208a^2-25b^2\right)\mathrm{c}}{}$$

$$= -\frac{b\left(96a^2-25b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b\left(208a^2-25b^2\right)\mathrm{c}}{}$$

$$= -\frac{b\left(96a^2-25b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b\left(208a^2-25b^2\right)\mathrm{c}}{}$$

$$= -\frac{b\left(96a^2-25b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b\left(208a^2-25b^2\right)\mathrm{c}}{}$$

$$= -\frac{b\left(96a^2-25b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b\left(208a^2-25b^2\right)\mathrm{c}}{}$$

Mathematica [C] time = 3.65, size = 466, normalized size = 1.09

$$-\frac{8a(40a^2-173b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}}+\frac{2b(424a^2+117b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}}-\frac{4}{3}\sqrt{a+b\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)*(-176*a^2 + 167*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b
)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*A
rcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*
EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]
]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))
]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]) - (8*a
*(40*a^2 - 173*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*b*(424*a^2 + 117
*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
 + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (4*Sqrt[a + b*Sin[c + d*x]]*(
176*a*b*Cos[c + d*x] + 5*Cot[c + d*x]*(-32*a^2 + 33*b^2 + 26*a*b*Csc[c + d*
x] + 8*a^2*Csc[c + d*x]^2) + 24*b^2*Sin[2*(c + d*x)]))/3)/(160*d)
```

fricas [F] time = 124.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(2\,ab\cot\left(dx+c\right)^4\sin\left(dx+c\right)-\left(b^2\cos\left(dx+c\right)^2-a^2-b^2\right)\cot\left(dx+c\right)^4\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((2*a*b*cot(d*x + c)^4*sin(d*x + c) - (b^2*cos(d*x + c)^2 - a^2 - b
^2)*cot(d*x + c)^4)*sqrt(b*sin(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.06, size = 1526, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x)

[Out] 1/120*(240*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*sin(d*x+c)^3+288*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^3-1566*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)^3+537*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3+501*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3-528*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^3+1029*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-501*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3+900*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-900*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)^3-75*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3+75*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^3+48*a*b^4*sin(d*x+c)^7+224*a^2*b^3*sin(d*x+c)^6+16*a^3*b^2*sin(d*x+c)^5+117*a*b^4*sin(d*x+c)^5-160*a^4*b*sin(d*x+c)^4+71*a^2*b^3*sin(d*x+c)^4+154*a^3*b^2*sin(d*x+c)^3-165*a*b^4*sin(d*x+c)^3+200*a^4*b*sin(d*x+c)^2-295*a^2*b^3*sin(d*x+c)^2-170*a^3*b^2*sin(d*x+c)-40*a^4*b)/a/b/sin(d*x+c)^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cot(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^4\,(a+b\,\sin(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.1165 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=449

$$-\frac{b^2\left(196a^2+5b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d}+\frac{5b\left(68a^2+b^2\right)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{192a^2d}-\frac{b\left(148a^2+\right.}{}$$

```
[Out] 5/192*b*(68*a^2+b^2)*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d+1/96*(60*a^2+b
^2)*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2)/a^2/d+1/24*b*cot(d*x+c)*cs
c(d*x+c)^2*(a+b*sin(d*x+c))^(7/2)/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3*(a+b*si
n(d*x+c))^(7/2)/a/d-1/64*b^2*(196*a^2+5*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1
/2)/a^2/d-1/64*b*(492*a^2-5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(
1/2))*(a+b*sin(d*x+c))^(1/2)/a/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+1/64*b*(148
*a^2+169*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x
+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)-1/64*(48*a^4-360*a^2*b^2-5*b^4)*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(co
s(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))
^(1/2)/a/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.51, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2893, 3047, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b^2\left(196a^2+5b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d}+\frac{5b\left(68a^2+b^2\right)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{192a^2d}-\frac{b\left(148a^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(b^2*(196*a^2 + 5*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(64*a^2*d) +
 (5*b*(68*a^2 + b^2)*Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(192*a^2*d) +
 ((60*a^2 + b^2)*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(96*
a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(7/2))/(24*a^2
*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(7/2))/(4*a*d) + (b
*(492*a^2 - 5*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*
Sin[c + d*x]])/(64*a*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (b*(148*a^2 +
169*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d
*x])/(a + b)])/(64*d*Sqrt[a + b*Sin[c + d*x]]) + ((48*a^4 - 360*a^2*b^2 - 5
```

*b^4)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(64*a*d*Sqrt[a + b*Sin[c + d*x]])

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
```

```
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{7/2}}{24a^2d} - \frac{\cot(c+dx}{}$$

$$= \frac{\left(60a^2+b^2\right)\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{5/2}}{96a^2d} +$$

$$= \frac{5b\left(68a^2+b^2\right)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{192a^2d} + \frac{\left(60a^2+b\right.}{}$$

$$= -\frac{b^2\left(196a^2+5b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{5b\left(68a\right.}{}$$

$$= -\frac{b^2\left(196a^2+5b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{5b\left(68a\right.}{}$$

$$= -\frac{b^2\left(196a^2+5b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{5b\left(68a\right.}{}$$

$$= -\frac{b^2\left(196a^2+5b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{5b\left(68a\right.}{}$$

$$= -\frac{b^2\left(196a^2+5b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{5b\left(68a\right.}{}$$

Mathematica [C] time = 6.70, size = 655, normalized size = 1.46

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{5\csc(c+dx)\left(116a^2b\cos(c+dx)-3b^3\cos(c+dx)\right)}{192a}+\frac{1}{96}\csc^2(c+dx)\left(60a^2\cos(c+dx)-59b^2\cos(c+dx)\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((-2*b^2*Cos[c + d*x])/3 + (5*(116*a^2*b*Cos[c + d*x] - 3*b^3*Cos[c + d*x]
)*Csc[c + d*x])/(192*a) + ((60*a^2*Cos[c + d*x] - 59*b^2*Cos[c + d*x])*Csc[
c + d*x]^2)/96 - (17*a*b*Cot[c + d*x]*Csc[c + d*x]^2)/24 - (a^2*Cot[c + d*x
]*Csc[c + d*x]^3)/4)*Sqrt[a + b*Sin[c + d*x]])/d + ((-2*(688*a^3*b - 348*a*
b^3)*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x]
)/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(96*a^4 - 228*a^2*b^2 - 15*b^4)*E
llipticPi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/
(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(-492*a^2*b^2 + 5*b^4)*Cos[c +
d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticP
i[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a +
b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x]
)/(a - b))]])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2
 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 -
2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(256*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

[Out] Timed out

maple [B] time = 2.09, size = 1777, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2),x)

[Out] 1/192*(444*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4-1998*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4+507*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)^4+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4+144*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4+1080*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4-1080*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)^4+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4+1491*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4+1032*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4-48*a^5-452*a^3*b^2*sin(d*x+c)^6+15*a*b^4*sin(d*x+c)^6-700*a^4*b*sin(d*x+c)^5+884*a^4*b*sin(d*x+c)^3-184*a^4*b*sin(d*x+c)-120*a^5*sin(d*x+c)^4+168*a^5*sin(d*x+c)^2+128*a^2*b^3*sin(d*x+c)^7+5*a^2*b^3*sin(d*x+c)^5+706*a^3*b^2*sin(d*x+c)^4-15*a*b^4*sin(d*x+c)^4-133*a^2*b^3*sin(d*x+c)^3-254*a^3*b^2*sin(d*x+c)^2-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4-144*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5

```
*sin(d*x+c)^4-1476*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4)/a^2/sin(d*x+c)^4/cos(d*x+c)/(a+
b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^4\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cot(d*x + c)^4*csc(d*x + c), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(5/2))/sin(c + d*x),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.1166 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=482

$$\frac{(32a^2-b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}}{80a^2d}+\frac{b(36a^2-b^2)\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx)}{64a^2d}$$

```
[Out] 1/64*b*(36*a^2-b^2)*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d+1/80
*(32*a^2-b^2)*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2)/a^2/d+3/40*b*c
ot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^(7/2)/a^2/d-1/5*cot(d*x+c)*csc(d*x+
c)^4*(a+b*sin(d*x+c))^(7/2)/a/d-1/640*(128*a^4-580*a^2*b^2+15*b^4)*cot(d*x+
c)*(a+b*sin(d*x+c))^(1/2)/a^2/d+1/640*(128*a^4-2476*a^2*b^2-15*b^4)*(sin(1/
2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+
1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^2/d/((a+b
*sin(d*x+c))/(a+b))^(1/2)-1/640*(128*a^4+492*a^2*b^2-5*b^4)*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a/d/(a+b*si
n(d*x+c))^(1/2)-3/128*b*(80*a^4-40*a^2*b^2+b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^
2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2
^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*sin(d*x+c
))^(1/2)
```

Rubi [A] time = 1.62, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2893, 3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{(-580a^2b^2+128a^4+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^2d}+\frac{(492a^2b^2+128a^4-5b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx\right.\right.}{640ad\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((128*a^4 - 580*a^2*b^2 + 15*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(
640*a^2*d) + (b*(36*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x
])^(3/2))/(64*a^2*d) + ((32*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*S
in[c + d*x])^(5/2))/(80*a^2*d) + (3*b*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Si
n[c + d*x])^(7/2))/(40*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c +
 d*x])^(7/2))/(5*a*d) - ((128*a^4 - 2476*a^2*b^2 - 15*b^4)*EllipticE[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(640*a^2*d*Sqrt[(a +
 b*Sin[c + d*x])/(a + b)]) + ((128*a^4 + 492*a^2*b^2 - 5*b^4)*EllipticF[(c
```

```
- Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(640*a*
d*Sqrt[a + b*Sin[c + d*x]]) + (3*b*(80*a^4 - 40*a^2*b^2 + b^4)*EllipticPi[2
, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(1
28*a^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{3b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{7/2}}{40a^2d} - \frac{\cot(c+}{}$$

$$= \frac{\left(32a^2-b^2\right)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}}{80a^2d}$$

$$= \frac{b\left(36a^2-b^2\right)\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}}{64a^2d}$$

$$= -\frac{\left(128a^4-580a^2b^2+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx}}{640a^2d}$$

$$= -\frac{\left(128a^4-580a^2b^2+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx}}{640a^2d}$$

$$= -\frac{\left(128a^4-580a^2b^2+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx}}{640a^2d}$$

$$= -\frac{\left(128a^4-580a^2b^2+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx}}{640a^2d}$$

$$= -\frac{\left(128a^4-580a^2b^2+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx}}{640a^2d}$$

Mathematica [C] time = 6.84, size = 700, normalized size = 1.45

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{\csc^2(c+dx)\left(436a^2b\cos(c+dx)-5b^3\cos(c+dx)\right)}{320a}+\frac{1}{80}\csc^3(c+dx)\left(32a^2\cos(c+dx)-31b^2\cos(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((((-128*a^4*Cos[c + d*x] + 1196*a^2*b^2*Cos[c + d*x] + 15*b^4*Cos[c + d*x]
)*Csc[c + d*x])/(640*a^2) + ((436*a^2*b*Cos[c + d*x] - 5*b^3*Cos[c + d*x])*
Csc[c + d*x]^2)/(320*a) + ((32*a^2*Cos[c + d*x] - 31*b^2*Cos[c + d*x])*Csc[
c + d*x]^3)/80 - (21*a*b*Cot[c + d*x]*Csc[c + d*x]^3)/40 - (a^2*Cot[c + d*x
]*Csc[c + d*x]^4)/5)*Sqrt[a + b*Sin[c + d*x]])/d + (b*((-2*(5936*a^3*b + 20
*a*b^3)*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d
*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(2272*a^4 + 1276*a^2*b^2 + 45*
b^4)*EllipticPi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(128*a^4 - 2476*a^2*b^2 -
 15*b^4)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqr
t[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*Ellip
ticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a -
b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[
c + d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b
 + b*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^
2]*(-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqr
t[-((a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))
)/(2560*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [B] time = 2.42, size = 2075, normalized size = 4.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x)

[Out] 1/640*(-128*a^6*b+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5-128*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b*sin(d*x+c)^
5+3096*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5-492*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(si
n(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*
sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d*x+c)^5-2466*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^3*b^4*sin(d*x+c)^5+5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^5-15*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(
d*x+c)^5-1200*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2)
,(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5+1200*((a+b*sin(d*x+c))/(
a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)
*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4
*b^3*sin(d*x+c)^5+600*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b
))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5-600*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/
2))*a^2*b^5*sin(d*x+c)^5-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c)
)/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5-2604*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
a^5*b^2*sin(d*x+c)^5+2461*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5+15*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^
7*sin(d*x+c)^5+128*a^5*b^2*sin(d*x+c)^7-1196*a^3*b^4*sin(d*x+c)^7-15*a*b^6*
sin(d*x+c)^7+128*a^6*b*sin(d*x+c)^6-2068*a^4*b^3*sin(d*x+c)^6-5*a^2*b^5*sin
(d*x+c)^6-384*a^6*b*sin(d*x+c)^4+2652*a^4*b^3*sin(d*x+c)^4+5*a^2*b^5*sin(d*
x+c)^4+1592*a^5*b^2*sin(d*x+c)^3-1256*a^5*b^2*sin(d*x+c)^5+15*a*b^6*sin(d*x
+c)^5+1454*a^3*b^4*sin(d*x+c)^5-258*a^3*b^4*sin(d*x+c)^3+384*a^6*b*sin(d*x+
c)^2-584*a^4*b^3*sin(d*x+c)^2-464*a^5*b^2*sin(d*x+c)+128*((a+b*sin(d*x+c))/
```

```
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*sin(d*x
+c)^5)/a^3/b/sin(d*x+c)^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(5/2))/sin(c + d*x)^2,x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.1167 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=551

$$\frac{\left(28a^2-3b^2\right)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/2}}{96a^2d}+\frac{b\left(52a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+$$

```
[Out] 1/192*b*(52*a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2)/a^2/d
+1/96*(28*a^2-3*b^2)*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2)/a^2/d+1
/12*b*cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^(7/2)/a^2/d-1/6*cot(d*x+c)*c
sc(d*x+c)^5*(a+b*sin(d*x+c))^(7/2)/a/d-1/1536*b*(720*a^4-176*a^2*b^2+15*b^4
)*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^3/d-1/256*(16*a^4-56*a^2*b^2+5*b^4)*c
ot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^2/d+1/1536*b*(720*a^4-176*a^2
*b^2+15*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c
))^(1/2)/a^3/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/1536*b*(816*a^4+1696*a^2*b^
2+5*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elli
pticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/
(a+b))^(1/2)/a^2/d/(a+b*sin(d*x+c))^(1/2)-1/512*(64*a^6+720*a^4*b^2+60*a^2*
b^4-5*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x
+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.97, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2893, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(-176a^2b^2+720a^4+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1536a^3d}+\frac{b\left(1696a^2b^2+816a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c\right.\right.}{1536a^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(b*(720*a^4 - 176*a^2*b^2 + 15*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])
/(1536*a^3*d) - ((16*a^4 - 56*a^2*b^2 + 5*b^4)*Cot[c + d*x]*Csc[c + d*x]*Sq
rt[a + b*Sin[c + d*x]])/(256*a^2*d) + (b*(52*a^2 - 5*b^2)*Cot[c + d*x]*Csc[
c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(192*a^2*d) + ((28*a^2 - 3*b^2)*Cot[
c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2))/(96*a^2*d) + (b*Cot[c +
 d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^(7/2))/(12*a^2*d) - (Cot[c + d*x]
*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^(7/2))/(6*a*d) - (b*(720*a^4 - 176*a^2
```

```
*b^2 + 15*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[
c + d*x]])/(1536*a^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(816*a^4 +
1696*a^2*b^2 + 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)])/(1536*a^2*d*Sqrt[a + b*Sin[c + d*x]]) + ((64*a^
6 + 720*a^4*b^2 + 60*a^2*b^4 - 5*b^6)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(512*a^3*d*Sqrt[a + b*Sin[c
 + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^{7/2}}{12a^2d} - \frac{\cot(c+}{}$$

$$= \frac{\left(28a^2-3b^2\right)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/}}{96a^2d}$$

$$= \frac{b\left(52a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))}{192a^2d}$$

$$= -\frac{\left(16a^4-56a^2b^2+5b^4\right)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin}}{256a^2d}$$

$$= -\frac{b\left(720a^4-176a^2b^2+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+d}}{1536a^3d}$$

$$= -\frac{b\left(720a^4-176a^2b^2+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+d}}{1536a^3d}$$

$$= -\frac{b\left(720a^4-176a^2b^2+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+d}}{1536a^3d}$$

$$= -\frac{b\left(720a^4-176a^2b^2+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+d}}{1536a^3d}$$

$$= -\frac{b\left(720a^4-176a^2b^2+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+d}}{1536a^3d}$$

Mathematica [C] time = 6.81, size = 771, normalized size = 1.40

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{\csc^3(c+dx)\left(164a^2b\cos(c+dx)-b^3\cos(c+dx)\right)}{192a}+\frac{1}{96}\csc^4(c+dx)\left(28a^2\cos(c+dx)-27b^2\cos(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((((-720*a^4*b*Cos[c + d*x] + 176*a^2*b^3*Cos[c + d*x] - 15*b^5*Cos[c + d*x
])*Csc[c + d*x])/(1536*a^3) + ((-48*a^4*Cos[c + d*x] + 600*a^2*b^2*Cos[c +
d*x] + 5*b^4*Cos[c + d*x])*Csc[c + d*x]^2)/(768*a^2) + ((164*a^2*b*Cos[c +
d*x] - b^3*Cos[c + d*x])*Csc[c + d*x]^3)/(192*a) + ((28*a^2*Cos[c + d*x] -
27*b^2*Cos[c + d*x])*Csc[c + d*x]^4)/96 - (5*a*b*Cot[c + d*x]*Csc[c + d*x]^
4)/12 - (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/6)*Sqrt[a + b*Sin[c + d*x]])/d +
((-2*(192*a^5*b + 3744*a^3*b^3 - 20*a*b^5)*EllipticF[(-c + Pi/2 - d*x)/2, (
2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]]
- (2*(384*a^6 + 3600*a^4*b^2 + 536*a^2*b^4 - 45*b^6)*EllipticPi[2, (-c + Pi
/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*
Sin[c + d*x]] - ((2*I)*(720*a^4*b^2 - 176*a^2*b^4 + 15*b^6)*Cos[c + d*x]*Co
s[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a +
b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a -
 b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a -
b))])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(
a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a
+ b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(6144*a^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [B] time = 2.85, size = 2458, normalized size = 4.46

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x)

[Out] 1/1536*(192*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(
a-b)/a,((a-b)/(a+b))^(1/2))*a^6*b*sin(d*x+c)^6-2160*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*b^2
*sin(d*x+c)^6+2160*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^
(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d*x+c)^6-180*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))
*a^3*b^4*sin(d*x+c)^6+180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/
(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^6+15*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^
(1/2))*a*b^6*sin(d*x+c)^6-896*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^6+191*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^
4*sin(d*x+c)^6-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6-816*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b*sin(d*x+c)^6+2
592*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+
b))^(1/2))*a^5*b^2*sin(d*x+c)^6-15*a*b^6*sin(d*x+c)^6-15*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^
7*sin(d*x+c)^6-5*a^2*b^5*sin(d*x+c)^5+2*a^3*b^4*sin(d*x+c)^4-440*a^4*b^3*si
n(d*x+c)^3-1072*a^5*b^2*sin(d*x+c)^2-256*a^7+720*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^6+96
*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*
x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))*a^7*sin(d*x+c)^6-192*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c
))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^6+720*a^5*b^2*s
in(d*x+c)^8-176*a^3*b^4*sin(d*x+c)^8+15*a*b^6*sin(d*x+c)^8+816*a^6*b*sin(d*
x+c)^7-1376*a^4*b^3*sin(d*x+c)^7+1816*a^4*b^3*sin(d*x+c)^5+174*a^3*b^4*sin(
d*x+c)^6-3232*a^5*b^2*sin(d*x+c)^6+5*a^2*b^5*sin(d*x+c)^7-2576*a^6*b*sin(d*
x+c)^5+3584*a^5*b^2*sin(d*x+c)^4+2656*a^6*b*sin(d*x+c)^3-896*a^6*b*sin(d*x+
```

```
c)+96*a^7*sin(d*x+c)^6-544*a^7*sin(d*x+c)^4+704*a^7*sin(d*x+c)^2-1696*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^4*b^3*sin(d*x+c)^6-186*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6-5*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5*sin
(d*x+c)^6+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6)/a^4/sin(d*x+c)^6/cos(d*x+c)/(a+b*si
n(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(5/2))/sin(c + d*x)^3,x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.1168 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=471

$$\frac{4a\left(160a^2-223b^2\right)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3003b^4d}-\frac{10\left(8a^2-11b^2\right)\sin^3(c+dx)\cos(c+dx)\sqrt{a+b}}{429b^3d}$$

```
[Out] 64/15015*a*(80*a^4-118*a^2*b^2+17*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^
6/d-8/15015*(480*a^4-683*a^2*b^2+77*b^4)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x
+c))^(1/2)/b^5/d+4/3003*a*(160*a^2-223*b^2)*cos(d*x+c)*sin(d*x+c)^2*(a+b*si
n(d*x+c))^(1/2)/b^4/d-10/429*(8*a^2-11*b^2)*cos(d*x+c)*sin(d*x+c)^3*(a+b*si
n(d*x+c))^(1/2)/b^3/d+24/143*a*cos(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c))^(1/
2)/b^2/d-2/13*cos(d*x+c)*sin(d*x+c)^5*(a+b*sin(d*x+c))^(1/2)/b/d-8/15015*(1
280*a^6-2048*a^4*b^2+453*a^2*b^4+231*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b
/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^7/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+
8/15015*a*(1280*a^6-2368*a^4*b^2+875*a^2*b^4+213*b^6)*(sin(1/2*c+1/4*Pi+1/2
*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x
),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^7/d/(a+b*sin(d*
x+c))^(1/2)
```

Rubi [A] time = 1.18, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2895, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{10\left(8a^2-11b^2\right)\sin^3(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{429b^3d}+\frac{4a\left(160a^2-223b^2\right)\sin^2(c+dx)\cos(c+dx)\sqrt{a+}}{3003b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (64*a*(80*a^4 - 118*a^2*b^2 + 17*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]
)/(15015*b^6*d) - (8*(480*a^4 - 683*a^2*b^2 + 77*b^4)*Cos[c + d*x]*Sin[c +
d*x]*Sqrt[a + b*Sin[c + d*x]])/(15015*b^5*d) + (4*a*(160*a^2 - 223*b^2)*Cos
[c + d*x]*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(3003*b^4*d) - (10*(8*a^
2 - 11*b^2)*Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(429*b^3*
d) + (24*a*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]])/(143*b^2*d
) - (2*Cos[c + d*x]*Sin[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(13*b*d) + (8*
(1280*a^6 - 2048*a^4*b^2 + 453*a^2*b^4 + 231*b^6)*EllipticE[(c - Pi/2 + d*x
)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15015*b^7*d*Sqrt[(a + b*Sin[
```

```
c + d*x])/(a + b)]) - (8*a*(1280*a^6 - 2368*a^4*b^2 + 875*a^2*b^4 + 213*b^6
)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/(15015*b^7*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
```

```
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{24a\cos(c+dx)\sin^4(c+dx)\sqrt{a+b\sin(c+dx)}}{143b^2d} - \frac{2\cos(c+dx)\sin^5(c+dx)\sqrt{}}{13bd}$$

$$= -\frac{10\left(8a^2-11b^2\right)\cos(c+dx)\sin^3(c+dx)\sqrt{a+b\sin(c+dx)}}{429b^3d} + \frac{24a\cos(c+dx}{}$$

$$= \frac{4a\left(160a^2-223b^2\right)\cos(c+dx)\sin^2(c+dx)\sqrt{a+b\sin(c+dx)}}{3003b^4d} - \frac{10\left(8a^2-11\right.}{}$$

$$= -\frac{8\left(480a^4-683a^2b^2+77b^4\right)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^5d} + \frac{4a}{}$$

$$= \frac{64a\left(80a^4-118a^2b^2+17b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^6d} - \frac{8\left(480a^4-683a\right.}{}$$

$$= \frac{64a\left(80a^4-118a^2b^2+17b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^6d} - \frac{8\left(480a^4-683a\right.}{}$$

$$= \frac{64a\left(80a^4-118a^2b^2+17b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^6d} - \frac{8\left(480a^4-683a\right.}{}$$

$$= \frac{64a\left(80a^4-118a^2b^2+17b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^6d} - \frac{8\left(480a^4-683a\right.}{}$$

Mathematica [A] time = 5.59, size = 382, normalized size = 0.81

$$\frac{384a\left(1280a^6-2368a^4b^2+875a^2b^4+213b^6\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+3b\cos(c+dx)\left(81920a^6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-384*(1280*a^7 + 1280*a^6*b - 2048*a^5*b^2 - 2048*a^4*b^3 + 453*a^3*b^4 +
453*a^2*b^5 + 231*a*b^6 + 231*b^7)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(
a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 384*a*(1280*a^6 - 2368*a^4*b^2
```

```
 + 875*a^2*b^4 + 213*b^6)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)] + 3*b*Cos[c + d*x]*(81920*a^6 - 125952*a^
4*b^2 + 23760*a^2*b^4 + 6622*b^6 + (5120*a^4*b^2 - 5792*a^2*b^4 - 8547*b^6)
*Cos[2*(c + d*x)] - 70*(8*a^2*b^4 - 11*b^6)*Cos[4*(c + d*x)] + 1155*b^6*Cos
[6*(c + d*x)] + 20480*a^5*b*Sin[c + d*x] - 28608*a^3*b^3*Sin[c + d*x] + 233
2*a*b^5*Sin[c + d*x] - 1600*a^3*b^3*Sin[3*(c + d*x)] + 1390*a*b^5*Sin[3*(c
+ d*x)] + 210*a*b^5*Sin[5*(c + d*x)]))/(720720*b^7*d*Sqrt[a + b*Sin[c + d*x
]])
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(dx+c\right)^{6}-\cos\left(dx+c\right)^{4}\right)\sin\left(dx+c\right)}{\sqrt{b\sin\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(cos(d*x + c)^6 - cos(d*x + c)^4)*sin(d*x + c)/sqrt(b*sin(d*x + c
) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{3}}{\sqrt{b\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/sqrt(b*sin(d*x + c) + a), x)
```

maple [B] time = 1.75, size = 1619, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/15015*(-200*a^3*b^5*sin(d*x+c)^5+410*a*b^7*sin(d*x+c)^5+320*a^4*b^4*sin(d
*x+c)^4-642*a^2*b^6*sin(d*x+c)^4-640*a^5*b^3*sin(d*x+c)^3+1244*a^3*b^5*sin(
d*x+c)^3-541*a*b^7*sin(d*x+c)^3-2560*a^6*b^2*sin(d*x+c)^2+3456*a^4*b^4*sin(
d*x+c)^2-42*a^2*b^6*sin(d*x+c)^2+640*a^5*b^3*sin(d*x+c)-1044*a^3*b^5*sin(d*
```

```
x+c)+236*a*b^7*sin(d*x+c)-105*a*b^7*sin(d*x+c)^7+2560*a^6*b^2+888*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
2*b^6+140*a^2*b^6*sin(d*x+c)^6+2233*b^8*sin(d*x+c)^4-308*b^8*sin(d*x+c)^2+1
155*b^8*sin(d*x+c)^8-3080*b^8*sin(d*x+c)^6-3776*a^4*b^4+544*a^2*b^6+924*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*b^8-924*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*b^8-5120*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^8-1740*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6+852*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^7
+13312*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a^6*b^2-10004*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4+5120*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*b-3840*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
6*b^2-9472*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^5*b^3+6504*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4+3500*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^5)/b^8/cos(
d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^3}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/sqrt(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^3}{\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.1169 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=405

$$\frac{8a\left(120a^2-179b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^4d}-\frac{2\left(80a^2-117b^2\right)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^3d}$$

```
[Out] -8/3465*(160*a^4-247*a^2*b^2+45*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^5/
d+8/3465*a*(120*a^2-179*b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b
^4/d-2/693*(80*a^2-117*b^2)*cos(d*x+c)*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/
b^3/d+20/99*a*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^(1/2)/b^2/d-2/11*cos
(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c))^(1/2)/b/d+16/3465*a*(160*a^4-267*a^2*
b^2+69*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*E
llipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c)
)^(1/2)/b^6/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-8/3465*(320*a^6-614*a^4*b^2+24
9*a^2*b^4+45*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin
(d*x+c))/(a+b))^(1/2)/b^6/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2895, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(80a^2-117b^2\right)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^3d}+\frac{8a\left(120a^2-179b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-8*(160*a^4 - 247*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])
/(3465*b^5*d) + (8*a*(120*a^2 - 179*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]])/(3465*b^4*d) - (2*(80*a^2 - 117*b^2)*Cos[c + d*x]*Sin[c +
 d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(693*b^3*d) + (20*a*Cos[c + d*x]*Sin[c +
d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(99*b^2*d) - (2*Cos[c + d*x]*Sin[c + d*x]^
4*Sqrt[a + b*Sin[c + d*x]])/(11*b*d) - (16*a*(160*a^4 - 267*a^2*b^2 + 69*b^
4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(
3465*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(320*a^6 - 614*a^4*b^2
+ 249*a^2*b^4 + 45*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(
a + b*Sin[c + d*x])/(a + b)])/(3465*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{20a\cos(c+dx)\sin^3(c+dx)\sqrt{a+b\sin(c+dx)}}{99b^2d} - \frac{2\cos(c+dx)\sin^4(c+dx)\cdot}{11bd}$$

$$= -\frac{2\left(80a^2-117b^2\right)\cos(c+dx)\sin^2(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^3d} + \frac{20a\cos(c+}{}$$

$$= \frac{8a\left(120a^2-179b^2\right)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^4d} - \frac{2\left(80a^2-11\right.}{}$$

$$= -\frac{8\left(160a^4-247a^2b^2+45b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^5d} + \frac{8a\left(120a^2-1\right.}{}$$

$$= -\frac{8\left(160a^4-247a^2b^2+45b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^5d} + \frac{8a\left(120a^2-1\right.}{}$$

$$= -\frac{8\left(160a^4-247a^2b^2+45b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^5d} + \frac{8a\left(120a^2-1\right.}{}$$

$$= -\frac{8\left(160a^4-247a^2b^2+45b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^5d} + \frac{8a\left(120a^2-1\right.}{}$$

Mathematica [A] time = 4.36, size = 326, normalized size = 0.80

$$\frac{-64\left(320a^6-614a^4b^2+249a^2b^4+45b^6\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+b\cos(c+dx)\left(-10240a^5-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (128*a*(160*a^5 + 160*a^4*b - 267*a^3*b^2 - 267*a^2*b^3 + 69*a*b^4 + 69*b^5
)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
/(a + b)] - 64*(320*a^6 - 614*a^4*b^2 + 249*a^2*b^4 + 45*b^6)*EllipticF[(-2
*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*C
os[c + d*x]*(-10240*a^5 + 16448*a^3*b^2 - 3718*a*b^4 - 128*(5*a^3*b^2 - 6*a
*b^4)*Cos[2*(c + d*x)] + 70*a*b^4*Cos[4*(c + d*x)] - 2560*a^4*b*Sin[c + d*x
] + 3752*a^2*b^3*Sin[c + d*x] + 990*b^5*Sin[c + d*x] + 200*a^2*b^3*Sin[3*(c
```

```
 + d*x)] - 765*b^5*Sin[3*(c + d*x)] - 315*b^5*Sin[5*(c + d*x)]))/(27720*b^6
*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 1.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\cos(dx+c)^6-\cos(dx+c)^4}{\sqrt{b\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(cos(d*x + c)^6 - cos(d*x + c)^4)/sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4\sin(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

maple [B] time = 1.70, size = 1356, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/3465*(-960*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^5*b^2+35*a*b^6*sin(d*x+c)^6+180*a*b^6+640*a^5*b^2-98
8*a^3*b^4-50*a^2*b^5*sin(d*x+c)^5+80*a^3*b^4*sin(d*x+c)^4-166*a*b^6*sin(d*x
+c)^4-160*a^4*b^3*sin(d*x+c)^3+322*a^2*b^5*sin(d*x+c)^3-640*a^5*b^2*sin(d*x
+c)^2+908*a^3*b^4*sin(d*x+c)^2-49*a*b^6*sin(d*x+c)^2+160*a^4*b^3*sin(d*x+c)
-272*a^2*b^5*sin(d*x+c)-315*b^7*sin(d*x+c)^7+900*b^7*sin(d*x+c)^5-765*b^7*s
in(d*x+c)^3+180*b^7*sin(d*x+c)+180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^7-1280*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7-2456*((a+b*si
```

```
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a
^4*b^3+552*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a*b^6+1692*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+996*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5+1280*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
a^6*b-732*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a*b^6+3416*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-2688*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4)/b^7/cos(d*x
+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{2}}{\sqrt{b\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{4}\sin\left(c+d\,x\right)^{2}}{\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{\sqrt{a+b\sin\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(sin(c + d*x)**2*cos(c + d*x)**4/sqrt(a + b*sin(c + d*x)), x)
```

3.1170 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=283

$$\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(a\left(32a^2-33b^2\right)-3b\left(8a^2-7b^2\right)\sin(c+dx)\right)}{315b^4d}-\frac{8a\left(32a^4-65a^2b^2+33b^4\right)\sqrt{\frac{a+}{}}}{315b^5d\sqrt{a+}}$$

```
[Out] -2/63*cos(d*x+c)^3*(8*a-7*b*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^2/d+4/315*
cos(d*x+c)*(a*(32*a^2-33*b^2)-3*b*(8*a^2-7*b^2)*sin(d*x+c))*(a+b*sin(d*x+c)
)^(1/2)/b^4/d-8/315*(32*a^4-57*a^2*b^2+21*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2
)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/
2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^5/d/((a+b*sin(d*x+c))/(a+b))^(
1/2)+8/315*a*(32*a^4-65*a^2*b^2+33*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(
a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^5/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2865, 2752, 2663, 2661, 2655, 2653}

$$\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(a\left(32a^2-33b^2\right)-3b\left(8a^2-7b^2\right)\sin(c+dx)\right)}{315b^4d}-\frac{8a\left(-65a^2b^2+32a^4+33b^4\right)\sqrt{}}{315b^5d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]^3*(8*a - 7*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(63*b
^2*d) + (8*(32*a^4 - 57*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^5*d*Sqrt[(a + b*Sin[c + d*x])/
(a + b)]) - (8*a*(32*a^4 - 65*a^2*b^2 + 33*b^4)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(315*b^5*d*Sqrt[a + b
*Sin[c + d*x]]) + (4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(a*(32*a^2 - 33*
b^2) - 3*b*(8*a^2 - 7*b^2)*Sin[c + d*x]))/(315*b^4*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{4\int \frac{\cos^2(c+dx)\left(-\frac{ab}{2}-\frac{1}{2}(\right.}{\sqrt{a+b\sin}}}{21}$$

$$= -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{4\cos(c+dx)\sqrt{a+}}{}$$

$$= -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{4\cos(c+dx)\sqrt{a+}}{}$$

$$= -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{4\cos(c+dx)\sqrt{a+}}{}$$

$$= -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{8\left(32a^4-57a^2b^2+\right.}{}$$

Mathematica [A] time = 3.14, size = 275, normalized size = 0.97

$$\frac{32a\left(32a^4-65a^2b^2+33b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-b\cos(c+dx)\left(-512a^4-128a^3b\sin(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-32*(32*a^5 + 32*a^4*b - 57*a^3*b^2 - 57*a^2*b^3 + 21*a*b^4 + 21*b^5)*Elli
pticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)] + 32*a*(32*a^4 - 65*a^2*b^2 + 33*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4,
(2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - b*Cos[c + d*x]*(-512*a^
4 + 880*a^2*b^2 - 203*b^4 - 8*(4*a^2*b^2 - 21*b^4)*Cos[2*(c + d*x)] + 35*b^
4*Cos[4*(c + d*x)] - 128*a^3*b*Sin[c + d*x] + 202*a*b^3*Sin[c + d*x] + 10*a
*b^3*Sin[3*(c + d*x)]))/(1260*b^5*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^4\sin\left(dx+c\right)}{\sqrt{b\sin\left(dx+c\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fric as")

[Out] integral(cos(d*x + c)^4*sin(d*x + c)/sqrt(b*sin(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac ")

[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/sqrt(b*sin(d*x + c) + a), x)

maple [B] time = 1.62, size = 1190, normalized size = 4.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(1/2),x)

[Out] 2/315*(35*b^6*sin(d*x+c)^6-5*a*b^5*sin(d*x+c)^5+128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b-96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-260*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3+180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+132*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5-84*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6+356*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-312*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+84*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/

```
2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6+8*a^2*
b^4*sin(d*x+c)^4-112*b^6*sin(d*x+c)^4-16*a^3*b^3*sin(d*x+c)^3+34*a*b^5*sin(
d*x+c)^3-64*a^4*b^2*sin(d*x+c)^2+98*a^2*b^4*sin(d*x+c)^2+77*b^6*sin(d*x+c)^
2+16*a^3*b^3*sin(d*x+c)-29*a*b^5*sin(d*x+c)+64*a^4*b^2-106*a^2*b^4)/b^6/cos
(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)}{\sqrt{b\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^4\,\sin\left(c+d\,x\right)}{\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.1171 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=288

$$-\frac{2a\left(8a^2-23b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(8a^2-21b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] 8/15*a*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^2/d-2/5*cos(d*x+c)*sin(d*x+c)*(a
+b*sin(d*x+c))^(1/2)/b/d-2/15*(8*a^2-21*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^
(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)
*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^3/d/((a+b*sin(d*x+c))/(a+b))^(1/
2)+2/15*a*(8*a^2-23*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((
a+b*sin(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*sin(d*x+c))^(1/2)-2*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+
1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*s
in(d*x+c))^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2895, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2a\left(8a^2-23b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(8a^2-21b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (8*a*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(15*b^2*d) - (2*Cos[c + d*x]*Si
n[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(5*b*d) + (2*(8*a^2 - 21*b^2)*Elliptic
E[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15*b^3*d*Sq
rt[(a + b*Sin[c + d*x])/(a + b)]) - (2*a*(8*a^2 - 23*b^2)*EllipticF[(c - Pi
/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15*b^3*d*S
qrt[a + b*Sin[c + d*x]]) + (2*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
```

```
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+d}}{5bd}$$

$$= \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+d}}{5bd}$$

$$= \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+d}}{5bd}$$

$$= \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+d}}{5bd}$$

$$= \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+d}}{5bd}$$

Mathematica [C] time = 3.75, size = 408, normalized size = 1.42

$$-\frac{2\left(8a^2+9b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2i\left(21b^2-8a^2\right)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{\frac{b(\sin(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(-8*a^2 + 21*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
qrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(a*b^2*Sqrt[-(a + b)^(-1)]) + 4*Cos[c
 + d*x]*(4*a - 3*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]] - (8*a*b*Elliptic
F[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
/Sqrt[a + b*Sin[c + d*x]] - (2*(8*a^2 + 9*b^2)*EllipticPi[2, (-2*c + Pi - 2
*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[
c + d*x]])/(30*b^2*d)
```

fricas [F] time = 3.33, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{3}\cot\left(dx+c\right)}{\sqrt{b\sin\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(cos(d*x + c)^3*cot(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{3}\cot\left(dx+c\right)}{\sqrt{b\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)^3*cot(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

maple [B] time = 1.86, size = 1018, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/15*(8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-23*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+21*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-8*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^5+29*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a^3*b^2-21*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
```

```
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-15*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^4*Elli
pticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a+15*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*b^5*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-
b)/(a+b))^(1/2))+3*a*b^4*sin(d*x+c)^4-a^2*b^3*sin(d*x+c)^3-4*a^3*b^2*sin(d*
x+c)^2-3*a*b^4*sin(d*x+c)^2+a^2*b^3*sin(d*x+c)+4*a^3*b^2)/a/b^4/cos(d*x+c)/
(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^3 \cot(dx+c)}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^3*cot(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3\cot(c+d\,x)}{\sqrt{a+b\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*cot(c + d*x))/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^3*cot(c + d*x))/(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.1172 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=285

$$\frac{\left(4a^2-7b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\sin(c+dx)}}-\frac{\left(4a^2+3b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3ab^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos(}{}$$

```
[Out] -2/3*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b/d-cot(d*x+c)*(a+b*sin(d*x+c))^(1/2
)/a/d+1/3*(4*a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+
b*sin(d*x+c))^(1/2)/a/b^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/3*(4*a^2-7*b^2
)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(c
os(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^
(1/2)/b^2/d/(a+b*sin(d*x+c))^(1/2)+b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/si
n(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(
a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2894, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2-7b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\sin(c+dx)}}-\frac{\left(4a^2+3b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3ab^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(3*b*d) - (Cot[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]])/(a*d) - ((4*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2
*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*a*b^2*d*Sqrt[(a + b*Sin[c + d*x])
/(a + b)]) + ((4*a^2 - 7*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d*x]]) - (b
*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
/(a + b)])/(a*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
```

```
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} - \frac{2\int \frac{\csc}{\ }}{\ }$$

$$= -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} + \frac{2\int \frac{\csc}{\ }}{\ }$$

$$= -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} + \frac{1}{6}\Big(-7$$

$$= -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} - \frac{(4a^2+}{\ }$$

$$= -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} - \frac{(4a^2+}{\ }$$

Mathematica [C] time = 3.64, size = 416, normalized size = 1.46

$$\frac{\frac{2(4a^2+9b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)\middle|\frac{2b}{a+b}\right)}{ab\sqrt{a+b\sin(c+dx)}} + \frac{2i(4a^2+3b^2)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sin(c+dx)+1)}{a-b}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+}\right.\right.\right.\right.}{\ }}{\ }$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(4*a^2 + 3*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1
)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSin
h[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ellip
ticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (
a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqr
t[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a^2*b^3*Sqrt[-(a + b)^(-1)]) - (4*Co
t[c + d*x]*(3*b + 2*a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(a*b) + (40*E
llipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(4*a^2 + 9*b^2)*EllipticPi[2, (-2*c +
 Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*b*Sqr
t[a + b*Sin[c + d*x]]))/(12*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2\cot(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^2*cot(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

maple [A] time = 3.08, size = 704, normalized size = 2.47

$$\sqrt{-(-a-b\sin(dx+c))\left(\cos^2(dx+c)\right)}\,\left(\left(-2a^3b^2-3a\,b^4\right)\sin(dx+c)\left(\cos^2(dx+c)\right)+\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\,\sqrt{\frac{b\,s}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/3*(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*((-2*a^3*b^2-3*a*b^4)*sin(d*x+c
)*cos(d*x+c)^2+(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+a/(a
-b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(3*EllipticPi((b/(a-b)*sin(d
*x+c)+a/(a-b))^(1/2),(a*b-b^2)/a/b,((a-b)/(a+b))^(1/2))*a*b^4-3*EllipticPi(
(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),(a*b-b^2)/a/b,((a-b)/(a+b))^(1/2))*b^5-4
*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-6*
EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+7
*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+
3*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+4
*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5-Elli
pticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-3*Ell
ipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4)*sin(d
*x+c)+2*a^2*b^3*cos(d*x+c)^4-5*a^2*b^3*cos(d*x+c)^2)/b^3/(cos(d*x+c)^2*sin(
```

```
d*x+c)*b+cos(d*x+c)^2*a)^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)/(a+b*sin(d*x+c))^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2 \cot(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*cot(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2 \cot(c+d\,x)^2}{\sqrt{a+b\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*cot(c + d*x)^2)/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*cot(c + d*x)^2)/(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*cot(c + d*x)**2/sqrt(a + b*sin(c + d*x)), x)
```

3.1173 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=307

$$-\frac{\left(8a^2+b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4abd\sqrt{a+b\sin(c+dx)}}+\frac{\left(8a^2+3b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4a^2bd\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{3\left(4a^2\right.}{}$$

```
[Out] 3/4*b*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+
b*sin(d*x+c))^(1/2)/a/d-1/4*(8*a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b
/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^2/b/d/((a+b*sin(d*x+c))/(a+b))^(1/2
)+1/4*(8*a^2+b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin
(d*x+c))/(a+b))^(1/2)/a/b/d/(a+b*sin(d*x+c))^(1/2)+3/4*(4*a^2-b^2)*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+
1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a
^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2893, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(8a^2+b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4abd\sqrt{a+b\sin(c+dx)}}+\frac{\left(8a^2+3b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4a^2bd\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{3\left(4a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (3*b*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(4*a^2*d) - (Cot[c + d*x]*Csc[c
 + d*x]*Sqrt[a + b*Sin[c + d*x]])/(2*a*d) + ((8*a^2 + 3*b^2)*EllipticE[(c -
 Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(4*a^2*b*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) - ((8*a^2 + b^2)*EllipticF[(c - Pi/2 + d*x)/2,
 (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a*b*d*Sqrt[a + b*Sin
[c + d*x]]) - (3*(4*a^2 - b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a +
 b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
```

```
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{2ad}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{2ad}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{2ad}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{2ad}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{2ad}$$

Mathematica [C] time = 3.35, size = 443, normalized size = 1.44

$$\frac{\frac{2(16a^2-9b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{a^2\sqrt{a+b\sin(c+dx)}} - \frac{4\cot(c+dx)\sqrt{a+b\sin(c+dx)}\,(2a\csc(c+dx)-3b)}{a^2} + \frac{2i(8a^2+3b^2)\cos(2(c+dx))\csc^2(c+dx)\sec}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(8*a^2 + 3*b^2)*Cos[2*(c + d*x)]*Csc[c + d*x]^2*(2*a*(a - b)*Ellipt
icE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b
)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]
]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)
]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1
+ Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a^3*b^
2*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*x]^2)) - (4*Cot[c + d*x]*(-3*b + 2*a*
Csc[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/a^2 - (8*b*EllipticF[(-2*c + Pi - 2
*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*Sqrt[a + b*S
in[c + d*x]]) + (2*(16*a^2 - 9*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a^2*Sqrt[a + b*Sin[c + d*
x]]))/(16*d)
```

fricas [F] time = 3.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)\cot\left(dx+c\right)^{3}}{\sqrt{b\sin\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(cos(d*x + c)*cot(d*x + c)^3/sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)\cot\left(dx+c\right)^{3}}{\sqrt{b\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)*cot(d*x + c)^3/sqrt(b*sin(d*x + c) + a), x)
```

maple [B] time = 3.31, size = 913, normalized size = 2.97

$$\sqrt{-\left(-a-b\sin\left(dx+c\right)\right)\left(\cos^{2}\left(dx+c\right)\right)}\;\left(\frac{2\left(\frac{a}{b}-1\right)\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\;\sqrt{\frac{b(1-\sin(dx+c))}{a+b}}\;\sqrt{\frac{(-1-\sin(dx+c))b}{a-b}}\;\left(\left(-\frac{a}{b}-1\right)\text{EllipticE}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\right.\right.}{\sqrt{-(-a-b\sin(dx+c))\left(\cos^{2}(dx+c)\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] (-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*(2*(a/b-1)*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a
-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2)))+4*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+
c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*
x+c)^2)^(1/2)/a*b*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),-(-a/b+1)/a*b,(
(a-b)/(a+b))^(1/2))-1/2/a*(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)/sin(d*x+c
)^2+3/4/a^2*b*(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)/sin(d*x+c)+1/2/a*b*(a
```

```
/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-si
n(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*EllipticF(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+3/4*b^2/a^2*(a/b-1)*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))
*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)))-1/4*(4*a^2+3*b^2)/a^3*(a/b-1)*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*
b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*b*EllipticPi(((a+b*s
in(d*x+c))/(a-b))^(1/2),-(-a/b+1)/a*b,((a-b)/(a+b))^(1/2)))/cos(d*x+c)/(a+b
*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)\cot(dx+c)^3}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)*cot(d*x + c)^3/sqrt(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\cot(c+d\,x)^3}{\sqrt{a+b\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*cot(c + d*x)^3)/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)*cot(c + d*x)^3)/(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)*cot(c + d*x)**3/sqrt(a + b*sin(c + d*x)), x)
```

3.1174 $\int \frac{\cot^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=353

$$\frac{\left(16a^2+5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{24a^2d\sqrt{a+b\sin(c+dx)}}+\frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d}+\frac{\left(32a^2-15b^2\right)}{}$$

[Out] 1/24*(32*a^2-15*b^2)*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^3/d+5/12*b*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/a/d-1/24*(32*a^2-15*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^3/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/24*(16*a^2+5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*sin(d*x+c))^(1/2)-1/8*b*(12*a^2-5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)

Rubi [A] time = 0.88, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2725, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(32a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d}+\frac{\left(16a^2+5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{24a^2d\sqrt{a+b\sin(c+dx)}}+\frac{\left(32a^2-15b^2\right)}{}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]

[Out] ((32*a^2 - 15*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(24*a^3*d) + (5*b*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(12*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(3*a*d) + ((32*a^2 - 15*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(24*a^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((16*a^2 + 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(24*a^2*d*Sqrt[a + b*Sin[c + d*x]]) + (b*(12*a^2 - 5*b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*a^3*d*Sqrt[a + b*Sin[c + d*x]])

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(}}{3ad}$$

$$= \frac{\left(32a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin}}{12a^2d}$$

$$= \frac{\left(32a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin}}{12a^2d}$$

$$= \frac{\left(32a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin}}{12a^2d}$$

$$= \frac{\left(32a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin}}{12a^2d}$$

$$= \frac{\left(32a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin}}{12a^2d}$$

Mathematica [C] time = 5.72, size = 475, normalized size = 1.35

$$\frac{-\frac{4\cot(c+dx)\sqrt{a+b\sin(c+dx)}\left(8a^2\csc^2(c+dx)-32a^2-10ab\csc(c+dx)+15b^2\right)}{a^3} + \frac{-\frac{8a\left(24a^2-5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2b\left(45b^2-104a^2\right)\sqrt{\frac{a}{}}}{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((-4*Cot[c + d*x]*(-32*a^2 + 15*b^2 - 10*a*b*Csc[c + d*x] + 8*a^2*Csc[c + d
*x]^2)*Sqrt[a + b*Sin[c + d*x]])/a^3 + (((2*I)*(32*a^2 - 15*b^2)*Cos[2*(c +
 d*x)]*Csc[c + d*x]^2*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticP
i[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a +
b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[-(
```

```
(b*(1 + Sin[c + d*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*
x]^2)) - (8*a*(24*a^2 - 5*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*b*(-1
04*a^2 + 45*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(
a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/a^3)/(96*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^4}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

maple [B] time = 2.13, size = 1496, normalized size = 4.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/24*(48*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)^3-16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^3-42*b^2*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^3*sin(d*x+c)^3-5*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3+15*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^
```

```
3-32*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^5*sin(d*x+c)^3+47*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-15*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4
*sin(d*x+c)^3-36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1
/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3+36*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^
2*b^3*sin(d*x+c)^3+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b
))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3-15*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))
*b^5*sin(d*x+c)^3-32*a^3*b^2*sin(d*x+c)^5+15*a*b^4*sin(d*x+c)^5-32*a^4*b*si
n(d*x+c)^4+5*a^2*b^3*sin(d*x+c)^4+30*a^3*b^2*sin(d*x+c)^3-15*a*b^4*sin(d*x+
c)^3+40*a^4*b*sin(d*x+c)^2-5*a^2*b^3*sin(d*x+c)^2+2*a^3*b^2*sin(d*x+c)-8*a^
4*b)/a^4/sin(d*x+c)^3/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^4}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^4}{\sqrt{a+b\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b*sin(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^4/(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**4/sqrt(a + b*sin(c + d*x)), x)
```

3.1175 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=412

$$\frac{7b\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d}-\frac{b\left(188a^2-105b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d}-\frac{b\left(188a^2-10\right.}{}$$

```
[Out] -1/192*b*(188*a^2-105*b^2)*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^4/d+5/96*(12
*a^2-7*b^2)*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^3/d+7/24*b*cot(d
*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3
*(a+b*sin(d*x+c))^(1/2)/a/d+1/192*b*(188*a^2-105*b^2)*(sin(1/2*c+1/4*Pi+1/2
*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x
),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^4/d/((a+b*sin(d*x+c))/(
a+b))^(1/2)-1/192*b*(68*a^2-35*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b)
)^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)-1/64*(
48*a^4-72*a^2*b^2+35*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2)
)*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.24, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2893, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(188a^2-105b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d}+\frac{b\left(68a^2-35b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{192a^3d\sqrt{a+b\sin(c+dx)}}-\frac{b\left(18\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -(b*(188*a^2 - 105*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(192*a^4*d)
+ (5*(12*a^2 - 7*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(
96*a^3*d) + (7*b*Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(24*
a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(4*a*d) - (
b*(188*a^2 - 105*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a +
 b*Sin[c + d*x]])/(192*a^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(68*a
^2 - 35*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
 + d*x])/(a + b)])/(192*a^3*d*Sqrt[a + b*Sin[c + d*x]]) + ((48*a^4 - 72*a^2
```

```
*b^2 + 35*b^4)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b
*Sin[c + d*x])/(a + b)])/(64*a^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{7b\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a+b}}{4ad}$$

$$= \frac{5\left(12a^2-7b^2\right)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{96a^3d} + \frac{7b\cot(c+dx)\csc^2(}{}$$

$$= -\frac{b\left(188a^2-105b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5\left(12a^2-7b^2\right)\cot(c+dx)}{9}$$

$$= -\frac{b\left(188a^2-105b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5\left(12a^2-7b^2\right)\cot(c+dx)}{9}$$

$$= -\frac{b\left(188a^2-105b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5\left(12a^2-7b^2\right)\cot(c+dx)}{9}$$

$$= -\frac{b\left(188a^2-105b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5\left(12a^2-7b^2\right)\cot(c+dx)}{9}$$

$$= -\frac{b\left(188a^2-105b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5\left(12a^2-7b^2\right)\cot(c+dx)}{9}$$

Mathematica [C] time = 6.70, size = 647, normalized size = 1.57

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{7b\cot(c+dx)\csc^2(c+dx)}{24a^2} + \frac{\csc(c+dx)\left(105b^3\cos(c+dx)-188a^2b\cos(c+dx)\right)}{192a^4} + \frac{5\csc^2(c+dx)\left(12a^2\cos(c+dx)-7b^2\cos(c+dx\right.}{96a^3}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((((-188*a^2*b*Cos[c + d*x] + 105*b^3*Cos[c + d*x])*Csc[c + d*x])/(192*a^4)
 + (5*(12*a^2*Cos[c + d*x] - 7*b^2*Cos[c + d*x])*Csc[c + d*x]^2)/(96*a^3) +
 (7*b*Cot[c + d*x]*Csc[c + d*x]^2)/(24*a^2) - (Cot[c + d*x]*Csc[c + d*x]^3)
/(4*a))*Sqrt[a + b*Sin[c + d*x]])/d + ((-2*(-240*a^3*b + 140*a*b^3)*Ellipti
cF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/
Sqrt[a + b*Sin[c + d*x]] - (2*(288*a^4 - 620*a^2*b^2 + 315*b^4)*EllipticPi[
2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/
Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(188*a^2*b^2 - 105*b^4)*Cos[c + d*x]*Cos[
2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)
/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b
)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a - b)
)])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(a
+ b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a +
b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(768*a^4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^4 \csc(dx+c)}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(cot(d*x + c)^4*csc(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

maple [B] time = 2.34, size = 1761, normalized size = 4.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/192*(68*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4-258*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4-35*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^2*b^3*sin(d*x+c)^4+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4-144*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*
a^4*b*sin(d*x+c)^4-216*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-
b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4+216*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1
/2))*a^2*b^3*sin(d*x+c)^4+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+
c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4+293*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^3*b^2*sin(d*x+c)^4+120*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4+48*a^5-188*a^3*b^2*sin(d*
x+c)^6+105*a*b^4*sin(d*x+c)^6-68*a^4*b*sin(d*x+c)^5+76*a^4*b*sin(d*x+c)^3-8
*a^4*b*sin(d*x+c)+120*a^5*sin(d*x+c)^4-168*a^5*sin(d*x+c)^2+35*a^2*b^3*sin(
d*x+c)^5+174*a^3*b^2*sin(d*x+c)^4-105*a*b^4*sin(d*x+c)^4-35*a^2*b^3*sin(d*x
+c)^3+14*a^3*b^2*sin(d*x+c)^2-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4+144*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1
/2))*a^5*sin(d*x+c)^4-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/
(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^4-188*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*s
in(d*x+c)^4)/a^5/sin(d*x+c)^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^4 \csc(dx+c)}{\sqrt{b\sin(dx+c)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cot(d*x + c)^4*csc(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\sin\left(c+d\,x\right)^2-1\right)^2}{\sin\left(c+d\,x\right)^5\,\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(sin(c + d*x)*(a + b*sin(c + d*x))^(1/2)),x)
```

```
[Out] int((sin(c + d*x)^2 - 1)^2/(sin(c + d*x)^5*(a + b*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{4}{\left(c+dx\right)}\csc{\left(c+dx\right)}}{\sqrt{a+b\sin{\left(c+dx\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**4*csc(c + d*x)/sqrt(a + b*sin(c + d*x)), x)
```

3.1176 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=466

$$-\frac{2\left(a^2-b^2\right)\sin^4(c+dx)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{8a\left(480a^2-419b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{1155b^5d}-\frac{20\left(32a^2-\right.}{}$$

```
[Out] -2*(a^2-b^2)*cos(d*x+c)*sin(d*x+c)^4/a/b^2/d/(a+b*sin(d*x+c))^(1/2)-8/1155*
(640*a^4-592*a^2*b^2+15*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^6/d+8/1155
*a*(480*a^2-419*b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^5/d-20/
231*(32*a^2-27*b^2)*cos(d*x+c)*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/b^4/d+2/
33*(40*a^2-33*b^2)*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^(1/2)/a/b^3/d-2
/11*cos(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c))^(1/2)/b^2/d+8/1155*a*(1280*a^4
-1344*a^2*b^2+123*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi
+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b
*sin(d*x+c))^(1/2)/b^7/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-8/1155*(1280*a^6-16
64*a^4*b^2+369*a^2*b^4+15*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/
2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^7/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.21, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2892, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)\sin^4(c+dx)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(40a^2-33b^2\right)\sin^3(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{33ab^3d}-\frac{20\left(32a^2-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(a*b^2*d*Sqrt[a + b*Sin[c + d*
x]]) - (8*(640*a^4 - 592*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c +
d*x]])/(1155*b^6*d) + (8*a*(480*a^2 - 419*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sq
rt[a + b*Sin[c + d*x]])/(1155*b^5*d) - (20*(32*a^2 - 27*b^2)*Cos[c + d*x]*S
in[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(231*b^4*d) + (2*(40*a^2 - 33*b^2)*
Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(33*a*b^3*d) - (2*Cos
[c + d*x]*Sin[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]])/(11*b^2*d) - (8*a*(1280*
a^4 - 1344*a^2*b^2 + 123*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[a + b*Sin[c + d*x]])/(1155*b^7*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) +
 (8*(1280*a^6 - 1664*a^4*b^2 + 369*a^2*b^4 + 15*b^6)*EllipticF[(c - Pi/2 +
```

d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(1155*b^7*d*Sqrt[a + b*Sin[c + d*x]])

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2892

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/(b^2*d*f*(m + n + 4)), x]) /; Free
```

```
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4(c+dx)\sqrt{a+b\sin(}{11b^2d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{2\left(40a^2-33b^2\right)\cos(c+dx)\sin^3(c+}{33ab^3d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{20\left(32a^2-27b^2\right)\cos(c+dx)\sin^2(c+}{231b^4d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a\left(480a^2-419b^2\right)\cos(c+dx)\sin(c}{1155b^5d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{8\left(640a^4-592a^2b^2+15b^4\right)\cos(c+}{1155b^6d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{8\left(640a^4-592a^2b^2+15b^4\right)\cos(c+}{1155b^6d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{8\left(640a^4-592a^2b^2+15b^4\right)\cos(c+}{1155b^6d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{8\left(640a^4-592a^2b^2+15b^4\right)\cos(c+}{1155b^6d}$$

Mathematica [A] time = 6.88, size = 326, normalized size = 0.70

$$\frac{-64\left(1280a^6-1664a^4b^2+369a^2b^4+15b^6\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+b\cos(c+dx)\left(-40960a^5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (64*a*(1280*a^5 + 1280*a^4*b - 1344*a^3*b^2 - 1344*a^2*b^3 + 123*a*b^4 + 12
3*b^5)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
```

```
d*x])/(a + b)] - 64*(1280*a^6 - 1664*a^4*b^2 + 369*a^2*b^4 + 15*b^6)*Ellipt
icF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)
] + b*Cos[c + d*x]*(-40960*a^5 + 40448*a^3*b^2 - 2728*a*b^4 - 16*(160*a^3*b
^2 - 93*a*b^4)*Cos[2*(c + d*x)] + 280*a*b^4*Cos[4*(c + d*x)] - 10240*a^4*b*
Sin[c + d*x] + 8672*a^2*b^3*Sin[c + d*x] + 330*b^5*Sin[c + d*x] + 800*a^2*b
^3*Sin[3*(c + d*x)] - 255*b^5*Sin[3*(c + d*x)] - 105*b^5*Sin[5*(c + d*x)]))
/(9240*b^7*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos\left(dx+c\right)^{6}-\cos\left(dx+c\right)^{4}\right)\sqrt{b\sin\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral((cos(d*x + c)^6 - cos(d*x + c)^4)*sqrt(b*sin(d*x + c) + a)*sin(d*x
 + c)/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{3}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.92, size = 1356, normalized size = 2.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/1155*(-3840*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))*a^5*b^2+140*a*b^6*sin(d*x+c)^6+60*a*b^6+2560*a^5*b^2-
2368*a^3*b^4-200*a^2*b^5*sin(d*x+c)^5+320*a^3*b^4*sin(d*x+c)^4-466*a*b^6*si
n(d*x+c)^4-640*a^4*b^3*sin(d*x+c)^3+892*a^2*b^5*sin(d*x+c)^3-2560*a^5*b^2*s
```

```
in(d*x+c)^2+2048*a^3*b^4*sin(d*x+c)^2+266*a*b^6*sin(d*x+c)^2+640*a^4*b^3*si
n(d*x+c)-692*a^2*b^5*sin(d*x+c)-105*b^7*sin(d*x+c)^7+300*b^7*sin(d*x+c)^5-2
55*b^7*sin(d*x+c)^3+60*b^7*sin(d*x+c)+60*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^7-5120*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7-6656*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a^4*b^3+492*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))*a*b^6+4392*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+1476*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5+5120*
((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x
+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^
(1/2))*a^6*b-552*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))*a*b^6+10496*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-5868*((a+b*sin(d*x+c))/(
a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)
*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4)/b^8
/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^3}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(b*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\,\sin(c+d\,x)^3}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.1177 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=401

$$-\frac{2\left(a^2-b^2\right)\sin^3(c+dx)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{8a\left(160a^2-139b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^5d}-\frac{16\left(60a^2-49b^2\right)\sin}{}$$

```
[Out] -2*(a^2-b^2)*cos(d*x+c)*sin(d*x+c)^3/a/b^2/d/(a+b*sin(d*x+c))^(1/2)+8/315*a
*(160*a^2-139*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^5/d-16/315*(60*a^2-4
9*b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^4/d+2/63*(80*a^2-63*b
^2)*cos(d*x+c)*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/a/b^3/d-2/9*cos(d*x+c)*s
in(d*x+c)^3*(a+b*sin(d*x+c))^(1/2)/b^2/d-8/315*(320*a^4-318*a^2*b^2+21*b^4)
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^6
/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+16/315*a*(160*a^4-199*a^2*b^2+39*b^4)*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/
2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)
/b^6/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2892, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)\sin^3(c+dx)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(80a^2-63b^2\right)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{63ab^3d}-\frac{16\left(60a^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(a*b^2*d*Sqrt[a + b*Sin[c + d*
x]]) + (8*a*(160*a^2 - 139*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(315
*b^5*d) - (16*(60*a^2 - 49*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[a + b*Sin[c
+ d*x]])/(315*b^4*d) + (2*(80*a^2 - 63*b^2)*Cos[c + d*x]*Sin[c + d*x]^2*Sqr
t[a + b*Sin[c + d*x]])/(63*a*b^3*d) - (2*Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a
 + b*Sin[c + d*x]])/(9*b^2*d) + (8*(320*a^4 - 318*a^2*b^2 + 21*b^4)*Ellipti
cE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^6*d*
Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (16*a*(160*a^4 - 199*a^2*b^2 + 39*b^4
)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/(315*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2892

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/(b^2*d*f*(m + n + 4)), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^3(c+dx)\sqrt{a+b\sin(c}{9b^2d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{2\left(80a^2-63b^2\right)\cos(c+dx)\sin^2(c+d}{63ab^3d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{16\left(60a^2-49b^2\right)\cos(c+dx)\sin(c+d}{315b^4d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a\left(160a^2-139b^2\right)\cos(c+dx)\sqrt{a+b}}{315b^5d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a\left(160a^2-139b^2\right)\cos(c+dx)\sqrt{a+b}}{315b^5d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a\left(160a^2-139b^2\right)\cos(c+dx)\sqrt{a+b}}{315b^5d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a\left(160a^2-139b^2\right)\cos(c+dx)\sqrt{a+b}}{315b^5d}$$

Mathematica [A] time = 5.30, size = 275, normalized size = 0.69

$$\frac{64a\left(160a^4-199a^2b^2+39b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-b\cos(c+dx)\left(-5120a^4-1280a^3b\sin(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-32*(320*a^5 + 320*a^4*b - 318*a^3*b^2 - 318*a^2*b^3 + 21*a*b^4 + 21*b^5)*
EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)] + 64*a*(160*a^4 - 199*a^2*b^2 + 39*b^4)*EllipticF[(-2*c + Pi - 2*d*
x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - b*Cos[c + d*x]*(-
5120*a^4 + 4768*a^2*b^2 - 203*b^4 - 8*(40*a^2*b^2 - 21*b^4)*Cos[2*(c + d*x)
```

```
] + 35*b^4*Cos[4*(c + d*x)] - 1280*a^3*b*Sin[c + d*x] + 1012*a*b^3*Sin[c +
d*x] + 100*a*b^3*Sin[3*(c + d*x)]))/(1260*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 1.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos\left(dx+c\right)^{6}-\cos\left(dx+c\right)^{4}\right)\sqrt{b\sin\left(dx+c\right)+a}}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral((cos(d*x + c)^6 - cos(d*x + c)^4)*sqrt(b*sin(d*x + c) + a)/(b^2*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{2}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.85, size = 1190, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/315*(35*b^6*sin(d*x+c)^6-50*a*b^5*sin(d*x+c)^5+1280*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b-960*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*a^4*b^2-1592*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a^3*b^3+1044*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+312*((a+b*sin(d*x+c))/(a
```

```
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5-84*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*b^6-1280*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a^6+2552*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-1356*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+84*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^
6+80*a^2*b^4*sin(d*x+c)^4-112*b^6*sin(d*x+c)^4-160*a^3*b^3*sin(d*x+c)^3+214
*a*b^5*sin(d*x+c)^3-640*a^4*b^2*sin(d*x+c)^2+476*a^2*b^4*sin(d*x+c)^2+77*b^
6*sin(d*x+c)^2+160*a^3*b^3*sin(d*x+c)-164*a*b^5*sin(d*x+c)+640*a^4*b^2-556*
a^2*b^4)/b^7/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{2}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^2}{\left(a+b\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.1178 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=261

$$-\frac{8a\left(32a^2-29b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{35b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(32a^2-24ab\sin(\right.}{35b^4d}$$

```
[Out] 2/7*cos(d*x+c)^3*(8*a+b*sin(d*x+c))/b^2/d/(a+b*sin(d*x+c))^(1/2)-4/35*cos(d
*x+c)*(32*a^2-5*b^2-24*a*b*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^4/d+8/35*a*
(32*a^2-29*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*
x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*
x+c))^(1/2)/b^5/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-8/35*(32*a^4-37*a^2*b^2+5*
b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
F(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b
))^(1/2)/b^5/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2863, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(32a^2-24ab\sin(c+dx)-5b^2\right)}{35b^4d}+\frac{8\left(-37a^2b^2+32a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\right.}{35b^5d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*a*(32*a^2 - 29*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a
 + b*Sin[c + d*x]])/(35*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(32*
a^4 - 37*a^2*b^2 + 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt
[(a + b*Sin[c + d*x])/(a + b)])/(35*b^5*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Co
s[c + d*x]^3*(8*a + b*Sin[c + d*x]))/(7*b^2*d*Sqrt[a + b*Sin[c + d*x]]) - (
4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^2 - 5*b^2 - 24*a*b*Sin[c + d*
x]))/(35*b^4*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{2\cos^3(c+dx)(8a+b\sin(c+dx))}{7b^2d\sqrt{a+b\sin(c+dx)}} - \frac{12\int \frac{\cos^2(c+dx)\left(-\frac{b}{2}-4a\sin(c+dx)\right)}{\sqrt{a+b\sin(c+dx)}}\,dx}{7b^2}$$

$$= \frac{2\cos^3(c+dx)(8a+b\sin(c+dx))}{7b^2d\sqrt{a+b\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(32a^2-5\right.}{35b^4d}$$

$$= \frac{2\cos^3(c+dx)(8a+b\sin(c+dx))}{7b^2d\sqrt{a+b\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(32a^2-5\right.}{35b^4d}$$

$$= \frac{2\cos^3(c+dx)(8a+b\sin(c+dx))}{7b^2d\sqrt{a+b\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(32a^2-5\right.}{35b^4d}$$

$$= -\frac{8a\left(32a^2-29b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{35b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{8\left(32a^4-37a^2\right.}{}$$

Mathematica [A] time = 4.16, size = 222, normalized size = 0.85

$$\frac{-16\left(32a^4-37a^2b^2+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+b\cos(c+dx)\left(-256a^3+\left(45b^3-64a^2b\right)\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*a*(32*a^3 + 32*a^2*b - 29*a*b^2 - 29*b^3)*EllipticE[(-2*c + Pi - 2*d*x)
/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 16*(32*a^4 - 37*a^2
*b^2 + 5*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*S
in[c + d*x])/(a + b)] + b*Cos[c + d*x]*(-256*a^3 + 216*a*b^2 - 16*a*b^2*Cos
[2*(c + d*x)] + (-64*a^2*b + 45*b^3)*Sin[c + d*x] + 5*b^3*Sin[3*(c + d*x)])
)/(70*b^5*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 2.18, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,\cos(dx+c)^4\sin(dx+c)}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c)/(b^2*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4\sin(dx+c)}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.65, size = 943, normalized size = 3.61

$$-\frac{2\left(-5b^5\left(\sin^5(dx+c)\right)+128\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\,\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\text{EllipticF}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/35*(-5*b^5*sin(d*x+c)^5+128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-96*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-148*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
2*b^3+96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*a*b^4+20*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a
```

```
-b))^(1/2),((a-b)/(a+b))^(1/2))*b^5-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5+244*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-116*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a*b^4+8*a*b^4*sin(d*x+c)^4-16*a^2*b^3*sin(d*x+c)^3+20*b^5*sin(d*x+c)^
3-64*a^3*b^2*sin(d*x+c)^2+42*a*b^4*sin(d*x+c)^2+16*a^2*b^3*sin(d*x+c)-15*b^
5*sin(d*x+c)+64*a^3*b^2-50*a*b^4)/b^6/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(b*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\,\sin(c+d\,x)}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.1179 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=296

$$-\frac{2\left(a^2-b^2\right)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(8a^2-5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(8a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}}{3ab^3d\sqrt{\frac{a+b\sin}{a+}}}$$

```
[Out] -2*(a^2-b^2)*cos(d*x+c)/a/b^2/d/(a+b*sin(d*x+c))^(1/2)-2/3*cos(d*x+c)*(a+b*
sin(d*x+c))^(1/2)/b^2/d+2/3*(8*a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b
/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a/b^3/d/((a+b*sin(d*x+c))/(a+b))^(1/2
)-2/3*(8*a^2-5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/
2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*s
in(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*sin(d*x+c))^(1/2)-2*(sin(1/2*c+1/4*Pi+1/
2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d
*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a/d/(a+b*sin(
d*x+c))^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2892, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2\left(a^2-b^2\right)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(8a^2-5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(8a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}}{3ab^3d\sqrt{\frac{a+b\sin}{a+}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x])/(a*b^2*d*Sqrt[a + b*Sin[c + d*x]]) - (2*Cos[c
 + d*x]*Sqrt[a + b*Sin[c + d*x]])/(3*b^2*d) - (2*(8*a^2 - 3*b^2)*EllipticE[
(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*a*b^3*d*Sqr
t[(a + b*Sin[c + d*x])/(a + b)]) + (2*(8*a^2 - 5*b^2)*EllipticF[(c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^3*d*Sqrt[a
 + b*Sin[c + d*x]]) + (2*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)])/(a*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2892

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
```

```
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/(b^2*d*f*(m + n + 4)), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} + \frac{4\int \frac{\csc(c+dx)\left(\frac{3b^2}{4}\right.}{}}{}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} - \frac{4\int \frac{\csc(c+dx)\left(-\frac{3b^3}{4}\right.}{\sqrt{a}}}{}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} + \frac{\int \frac{\csc(c+dx)}{\sqrt{a+b\sin(c+dx)}}}{a}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} - \frac{2\left(8a^2-3b^2\right)E}{}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} - \frac{2\left(8a^2-3b^2\right)E}{}$$

Mathematica [C] time = 3.87, size = 419, normalized size = 1.42

$$-\frac{4\cos(c+dx)\left(4a^2+ab\sin(c+dx)-3b^2\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2\left(8a^2-9b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} - \frac{2i\left(3b^2-8a^2\right)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sin(c+}{a-}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)*(-8*a^2 + 3*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
qrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b^2*Sqrt[-(a + b)^(-1)]) + (8*a*
b*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(8*a^2 - 9*b^2)*EllipticPi[2, (-2*
c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[
a + b*Sin[c + d*x]] - (4*Cos[c + d*x]*(4*a^2 - 3*b^2 + a*b*Sin[c + d*x]))/S
qrt[a + b*Sin[c + d*x]])/(6*a*b^2*d)
```

fricas [F] time = 2.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{3}\cot\left(dx+c\right)}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^3*cot(d*x + c)/(b^2*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{3}\cot\left(dx+c\right)}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)^3*cot(d*x + c)/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.88, size = 1010, normalized size = 3.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/3*(8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+3*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-8*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^5+11*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
```

```
-b)/(a+b))^(1/2))*a^3*b^2-3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^4*Elliptic
Pi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a-3*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*b^5*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a
+b))^(1/2))-a^2*b^3*sin(d*x+c)^3-4*a^3*b^2*sin(d*x+c)^2+3*a*b^4*sin(d*x+c)^
2+a^2*b^3*sin(d*x+c)+4*a^3*b^2-3*a*b^4)/a^2/b^4/cos(d*x+c)/(a+b*sin(d*x+c))
^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3\,\cot(c+d\,x)}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*cot(c + d*x))/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^3*cot(c + d*x))/(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.1180 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=294

$$\frac{(2a^2-3b^2)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}}-\frac{(4a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{(4a^2-3b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\right.}{a^2b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] (2*a^2-3*b^2)*cos(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^(1/2)-cot(d*x+c)/a/d/(a+b
*sin(d*x+c))^(1/2)-(4*a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(
1/2))*(a+b*sin(d*x+c))^(1/2)/a^2/b^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+(4*a^
2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a
+b))^(1/2)/a/b^2/d/(a+b*sin(d*x+c))^(1/2)+3*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(
1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*sin(d*x+c))
^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2890, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(2a^2-3b^2)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}}-\frac{(4a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{(4a^2-3b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\right.}{a^2b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((2*a^2 - 3*b^2)*Cos[c + d*x])/(a^2*b*d*Sqrt[a + b*Sin[c + d*x]]) - Cot[c +
 d*x]/(a*d*Sqrt[a + b*Sin[c + d*x]]) + ((4*a^2 - 3*b^2)*EllipticE[(c - Pi/2
 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(a^2*b^2*d*Sqrt[(a + b*
Sin[c + d*x])/(a + b)]) - ((4*a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*b^2*d*Sqrt[a + b*Sin[c +
d*x]]) - (3*b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*
Sin[c + d*x])/(a + b)])/(a^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
```

```
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{\left(2a^2-3b^2\right)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} + \frac{2\int \frac{\csc(c+dx)\left(-\frac{3b^2}{4}-\frac{1}{2}ab\sin(c+dx)\right.}{\sqrt{a+b\sin}}}{a^2b}$$

$$= \frac{\left(2a^2-3b^2\right)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} + \frac{1}{2}\left(-\frac{3}{a^2}+\frac{4}{b^2}\right)\int\sqrt{a+b\sin}$$

$$= \frac{\left(2a^2-3b^2\right)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} - \frac{(3b)\int \frac{\csc(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{2a^2} - $$

$$= \frac{\left(2a^2-3b^2\right)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} - \frac{\left(\frac{3}{a^2}-\frac{4}{b^2}\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\right.}{d\sqrt{\frac{a+b\sin}{a}}}$$

$$= \frac{\left(2a^2-3b^2\right)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} - \frac{\left(\frac{3}{a^2}-\frac{4}{b^2}\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\right.}{d\sqrt{\frac{a+b\sin}{a}}}$$

Mathematica [C] time = 3.60, size = 433, normalized size = 1.47

$$\frac{\frac{4a\left(a^2-b^2\right)\cos(c+dx)}{b\sqrt{a+b\sin(c+dx)}} - \frac{a\left(4a^2-9b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{b\sqrt{a+b\sin(c+dx)}} + \frac{i\left(3b^2-4a^2\right)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{\frac{b(\sin(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\,\mathrm{s}\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((I*(-4*a^2 + 3*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[S
qrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Elliptic
Pi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a +
 b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[(
b*(1 + Sin[c + d*x]))/(-a + b)])/(b^3*Sqrt[-(a + b)^(-1)]) + (4*a*(a^2 - b^
2)*Cos[c + d*x])/(b*Sqrt[a + b*Sin[c + d*x]]) - 2*a*Cot[c + d*x]*Sqrt[a + b
*Sin[c + d*x]] + (4*a^2*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqr
t[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (a*(4*a^2 - 9*b
^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
 d*x])/(a + b)])/(b*Sqrt[a + b*Sin[c + d*x]]))/(2*a^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2\cot(dx+c)^2}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^2*cot(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [A] time = 1.69, size = 618, normalized size = 2.10

$$-\frac{(-2a^3b^2+3ab^4)\sin(dx+c)(\cos^2(dx+c))+\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\,\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\,\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\,(4\,\mathrm{El}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -((-2*a^3*b^2+3*a*b^4)*sin(d*x+c)*cos(d*x+c)^2+(b/(a-b)*sin(d*x+c)+a/(a-b))
^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1
/2)*(4*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
5-7*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b
^2+3*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^
4-3*EllipticPi((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/
2))*a*b^4+3*EllipticPi((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),(a-b)/a,((a-b)/(a
+b))^(1/2))*b^5-4*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*a^4*b+6*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))*a^3*b^2+EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^
(1/2))*a^2*b^3-3*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))*a*b^4)*sin(d*x+c)+a^2*b^3*cos(d*x+c)^2)/b^3/sin(d*x+c)/a^3/cos(d*x+
c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\cot(c+d\,x)^2}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*cot(c + d*x)^2)/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*cot(c + d*x)^2)/(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x))**(3/2), x)
```

3.1181 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=366

$$\frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}}+\frac{(8a^2-5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4a^2bd\sqrt{a+b\sin(c+dx)}}-\frac{(8a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin}}{4a^3bd}$$

```
[Out] 1/2*(4*a^2-5*b^2)*cot(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^(1/2)-1/2*cot(d*x+c)*
csc(d*x+c)/a/d/(a+b*sin(d*x+c))^(1/2)-1/4*(8*a^2-15*b^2)*cot(d*x+c)*(a+b*si
n(d*x+c))^(1/2)/a^3/b/d+1/4*(8*a^2-15*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(
b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^3/b/d/((a+b*sin(d*x+c))/(a+b))^(1/
2)-1/4*(8*a^2-5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1
/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*
sin(d*x+c))/(a+b))^(1/2)/a^2/b/d/(a+b*sin(d*x+c))^(1/2)+3/4*(4*a^2-5*b^2)*(
sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos
(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^
(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2890, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{(8a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^3bd}+\frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}}+\frac{(8a^2-5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx\right.\right.}{4a^2bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((4*a^2 - 5*b^2)*Cot[c + d*x])/(2*a^2*b*d*Sqrt[a + b*Sin[c + d*x]]) - (Cot[
c + d*x]*Csc[c + d*x])/(2*a*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a^2 - 15*b^2)
*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(4*a^3*b*d) - ((8*a^2 - 15*b^2)*Ell
ipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(4*a^3*
b*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((8*a^2 - 5*b^2)*EllipticF[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^2*b*d
*Sqrt[a + b*Sin[c + d*x]]) - (3*(4*a^2 - 5*b^2)*EllipticPi[2, (c - Pi/2 + d
*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^3*d*Sqrt[a +
 b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
```

```
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(\frac{1}{4}(8a^2-15b^2)-\frac{1}{2}ab \ldots\right.}{\sqrt{a+b\sin \ldots}}}{a^2 \ldots}$$

$$= \frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{(8a^2-15b^2)\cot(c+dx)\sqrt{\ldots}}{4a^3bd}$$

$$= \frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{(8a^2-15b^2)\cot(c+dx)\sqrt{\ldots}}{4a^3bd}$$

$$= \frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{(8a^2-15b^2)\cot(c+dx)\sqrt{\ldots}}{4a^3bd}$$

$$= \frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{(8a^2-15b^2)\cot(c+dx)\sqrt{\ldots}}{4a^3bd}$$

$$= \frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{(8a^2-15b^2)\cot(c+dx)\sqrt{\ldots}}{4a^3bd}$$

Mathematica [C] time = 5.03, size = 435, normalized size = 1.19

$$\frac{\frac{(60b^2-32a^2)\cos(c+dx)+4a\cot(c+dx)(5b-2a\csc(c+dx))}{a^3\sqrt{a+b\sin(c+dx)}} + \frac{\frac{2(32a^2-45b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} - \frac{2i(15b^2-8a^2)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{\ldots}}{\ldots}}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-32*a^2 + 60*b^2)*Cos[c + d*x] + 4*a*Cot[c + d*x]*(5*b - 2*a*Csc[c + d*x
]))/(a^3*Sqrt[a + b*Sin[c + d*x]]) + (((-2*I)*(-8*a^2 + 15*b^2)*(-2*a*(a -
b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a +
b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Si
n[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a
 + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt
```

```
[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))
])/(a*b^2*Sqrt[-(a + b)^(-1)]) - (40*a*b*EllipticF[(-2*c + Pi - 2*d*x)/4, (
2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]]
+ (2*(32*a^2 - 45*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/a^3)/(16*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(SimpleAlgebraicExtension(InnerPr
imeField(7),SparseUnivariatePolynomial(InnerPrimeField(7)),?^2+2)),failed)
cannot be coerced to mode SparseUnivariatePolynomial(SimpleAlgebraicExtensi
on(InnerPrimeField(7),SparseUnivariatePolynomial(InnerPrimeField(7)),?^2+2)
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)\cot(dx+c)^3}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)*cot(d*x + c)^3/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [B] time = 2.02, size = 1349, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/4*(8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b*sin(d*x+c)^2-18*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)^2-5*b^3*((a+
```

```
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*a^2*sin(d*x+c)^2+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-8*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^2+
23*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^3*b^2*sin(d*x+c)^2-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-12*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*b^2*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))
^(1/2))*a^3*sin(d*x+c)^2+12*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^3*EllipticPi(((a+b*sin(d*
x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^2+15*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b)
)^(1/2))*a*b^4*sin(d*x+c)^2-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x
+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^2-8*a^3*b^2*s
in(d*x+c)^4+15*a*b^4*sin(d*x+c)^4+5*a^2*b^3*sin(d*x+c)^3+6*a^3*b^2*sin(d*x+
c)^2-15*a*b^4*sin(d*x+c)^2-5*a^2*b^3*sin(d*x+c)+2*a^3*b^2)/b^2/sin(d*x+c)^2
/a^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\cot(c+d\,x)^3}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*cot(c + d*x)^3)/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)*cot(c + d*x)^3)/(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)*cot(c + d*x)**3/(a + b*sin(c + d*x))**(3/2), x)
```

3.1182 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=416

$$\frac{(6a^2-7b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}}+\frac{5(16a^2-21b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^4d}+\frac{5(16a^2-21b^2)\sqrt{a+b\sin(\ldots}}{24a^4d\sqrt{\ldots}}$$

[Out] 1/3*(6*a^2-7*b^2)*cot(d*x+c)*csc(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^(1/2)-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d/(a+b*sin(d*x+c))^(1/2)+5/24*(16*a^2-21*b^2)*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^4/d-1/12*(24*a^2-35*b^2)*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^3/b/d-5/24*(16*a^2-21*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^4/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+1/24*(32*a^2-35*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)-1/8*b*(36*a^2-35*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^4/d/(a+b*sin(d*x+c))^(1/2)

Rubi [A] time = 1.15, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2724, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{5(16a^2-21b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^4d}-\frac{(32a^2-35b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{24a^3d\sqrt{a+b\sin(c+dx)}}+\frac{5(16a^2-21\ldots}{\ldots}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]

[Out] ((6*a^2 - 7*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*b*d*Sqrt[a + b*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*Sqrt[a + b*Sin[c + d*x]]) + (5*(16*a^2 - 21*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(24*a^4*d) - ((24*a^2 - 35*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(12*a^3*b*d) + (5*(16*a^2 - 21*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(24*a^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - ((32*a^2 - 35*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(24*a^3*d*Sqrt[a + b*Sin[c + d*x]]) + (b*(36*a^2 - 35

```
*b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
d*x])/(a + b)])/(8*a^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m +
 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 -
b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 +
b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*
Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
m, -1] && IntegerQ[2*m]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{\left(6a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + 2\int \frac{\csc^3(c+dx)\left(\frac{1}{4}(2\right.}{}$$

$$= \frac{\left(6a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} - \frac{\left(24a^2-35b^2\right)\mathrm{c}}{}$$

$$= \frac{\left(6a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5\left(16a^2-21b^2\right)}{}$$

$$= \frac{\left(6a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5\left(16a^2-21b^2\right)}{}$$

$$= \frac{\left(6a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5\left(16a^2-21b^2\right)}{}$$

$$= \frac{\left(6a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5\left(16a^2-21b^2\right)}{}$$

$$= \frac{\left(6a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5\left(16a^2-21b^2\right)}{}$$

Mathematica [C] time = 5.85, size = 468, normalized size = 1.12

$$\frac{-\frac{4\left(\left(105b^3-80a^2b\right)\cos(c+dx)+a\cot(c+dx)\left(8a^2\csc^2(c+dx)-32a^2-14ab\csc(c+dx)+35b^2\right)\right)}{a^4\sqrt{a+b\sin(c+dx)}} + \frac{-\frac{8a\left(24a^2-35b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2b(}{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((-4*((-80*a^2*b + 105*b^3)*Cos[c + d*x] + a*Cot[c + d*x]*(-32*a^2 + 35*b^2
 - 14*a*b*Csc[c + d*x] + 8*a^2*Csc[c + d*x]^2)))/(a^4*Sqrt[a + b*Sin[c + d*
```

```
x]]) + (((10*I)*(-16*a^2 + 21*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-
(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Ellipti
cF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)
] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c
+ d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a
 + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))]])/(a*b*Sqrt[-(a + b)^(-1)])
- (8*a*(24*a^2 - 35*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sq
rt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*b*(-296*a^2
 + 315*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b
*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/a^4)/(96*d)
```

fricas [F] time = 1.39, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\,\cot\left(dx+c\right)^{4}}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cot(d*x + c)^4/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(dx+c\right)^{4}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^4/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [B] time = 2.18, size = 1496, normalized size = 3.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/24*(48*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)^3+32*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
```

```
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^3-150*b^2*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^3*sin(d*x+c)^3-35*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3+105*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+
c)^3-80*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^5*sin(d*x+c)^3+185*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-105*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
a*b^4*sin(d*x+c)^3-108*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-
b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3+108*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1
/2))*a^2*b^3*sin(d*x+c)^3+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+
c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3-105*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b
))^(1/2))*b^5*sin(d*x+c)^3-80*a^3*b^2*sin(d*x+c)^5+105*a*b^4*sin(d*x+c)^5-3
2*a^4*b*sin(d*x+c)^4+35*a^2*b^3*sin(d*x+c)^4+66*a^3*b^2*sin(d*x+c)^3-105*a*
b^4*sin(d*x+c)^3+40*a^4*b*sin(d*x+c)^2-35*a^2*b^3*sin(d*x+c)^2+14*a^3*b^2*s
in(d*x+c)-8*a^4*b)/b/a^5/sin(d*x+c)^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^4}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b*sin(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^4/(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**(3/2), x)
```

3.1183 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=469

$$\frac{2(13a^2-5b^2)\sin^4(c+dx)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2(a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}+\frac{128a(40a^2-19b^2)\cos(c+dx)\sqrt{}}{315b^6d}$$

```
[Out] -2/3*(a^2-b^2)*cos(d*x+c)*sin(d*x+c)^4/a/b^2/d/(a+b*sin(d*x+c))^(3/2)+2/3*(
13*a^2-5*b^2)*cos(d*x+c)*sin(d*x+c)^4/a^2/b^2/d/(a+b*sin(d*x+c))^(1/2)+128/
315*a*(40*a^2-19*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^6/d-8/315*(480*a^
2-203*b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^5/d+4/63*(160*a^2
-63*b^2)*cos(d*x+c)*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/a/b^4/d-10/9*(8*a^2
-3*b^2)*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^(1/2)/a^2/b^3/d-8/315*(128
0*a^4-768*a^2*b^2+21*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(
a+b*sin(d*x+c))^(1/2)/b^7/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+8/315*a*(1280*a^
4-1088*a^2*b^2+123*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a
+b*sin(d*x+c))/(a+b))^(1/2)/b^7/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.18, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2891, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2(13a^2-5b^2)\sin^4(c+dx)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2(a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}-\frac{10(8a^2-3b^2)\sin^3(c+dx)\cos(c}{9a^2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(3*a*b^2*d*(a + b*Sin[c + d*x]
)^(3/2)) + (2*(13*a^2 - 5*b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(3*a^2*b^2*d*Sq
rt[a + b*Sin[c + d*x]]) + (128*a*(40*a^2 - 19*b^2)*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]])/(315*b^6*d) - (8*(480*a^2 - 203*b^2)*Cos[c + d*x]*Sin[c + d*
x]*Sqrt[a + b*Sin[c + d*x]])/(315*b^5*d) + (4*(160*a^2 - 63*b^2)*Cos[c + d*
x]*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(63*a*b^4*d) - (10*(8*a^2 - 3*b
^2)*Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(9*a^2*b^3*d) + (
8*(1280*a^4 - 768*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a
+ b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^7*d*Sqrt[(a + b*Sin[c + d*x])/(a + b
)]) - (8*a*(1280*a^4 - 1088*a^2*b^2 + 123*b^4)*EllipticF[(c - Pi/2 + d*x)/2
```

```
, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(315*b^7*d*Sqrt[a + b*
Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
```

```
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

Mathematica [B] time = 9.83, size = 1044, normalized size = 2.23

$$\frac{315\left(\frac{\left(\left(a^2+3b^2\right)E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+a(b-a)F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}{(a-b)^2b} - \frac{\cos(c+dx)\left(2a\left(a^2+b^2\right)+b\left(a^2+3b^2\right)\sin(c+dx)\right)}{\left(a^2-b^2\right)^2}\right) + \frac{315\Big(\Big(}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (315*((((a^2 + 3*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] + a*(
-a + b)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((a + b*Sin[c + d*
x])/(a + b))^(3/2))/((a - b)^2*b) - (Cos[c + d*x]*(2*a*(a^2 + b^2) + b*(a^2
 + 3*b^2)*Sin[c + d*x]))/(a^2 - b^2)^2) + (315*((((32*a^4 - 57*a^2*b^2 + 21
*b^4)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] + a*(-32*a^3 + 32*a^2
*b + 33*a*b^2 - 33*b^3)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((
a + b*Sin[c + d*x])/(a + b))^(3/2))/(a - b)^2 - (b*(4*a*(8*a^4 - 13*a^2*b^2
 + 3*b^4)*Cos[c + d*x] + b*(20*a^4 - 33*a^2*b^2 + 9*b^4)*Sin[2*(c + d*x)]))
/(2*(a^2 - b^2)^2)))/b^3 - (21*((((-2048*a^6 + 4192*a^4*b^2 - 2355*a^2*b^4
+ 231*b^6)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] + a*(2048*a^5 -
2048*a^4*b - 2656*a^3*b^2 + 2656*a^2*b^3 + 603*a*b^4 - 603*b^5)*EllipticF[(
-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((a + b*Sin[c + d*x])/(a + b))^(3/2))
/(a - b)^2 + (b*Cos[c + d*x]*(-64*a*b^2*(a^2 - b^2)^2*Cos[2*(c + d*x)] + b*
(1280*a^6 - 2536*a^4*b^2 + 1347*a^2*b^4 - 111*b^6)*Sin[c + d*x] + 2*(512*a^
7 - 952*a^5*b^2 + 423*a^3*b^4 + 7*a*b^6 + 6*b^3*(a^2 - b^2)^2*Sin[3*(c + d*
x)])))/(a^2 - b^2)^2))/b^5 - (5*(a + b*Sin[c + d*x])*(((-4*b*(-4096*a^7*b +
 8960*a^5*b^3 - 5884*a^3*b^5 + 1041*a*b^7)*EllipticF[(-2*c + Pi - 2*d*x)/4,
 (2*b)/(a + b)] + (65536*a^8 - 161792*a^6*b^2 + 129664*a^4*b^4 - 35109*a^2*
b^6 + 1617*b^8)*((a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] -
a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]))*Sqrt[(a + b*Sin[c + d*x
])/(a + b)])/((a - b)^2*(a + b)^2) + b*(a + b*Sin[c + d*x])*(-128*a*(88*a^2
 - 27*b^2)*Cos[c + d*x] + 416*a*b^2*Cos[3*(c + d*x)] + (21*a*(64*a^6 - 112*
a^4*b^2 + 56*a^2*b^4 - 7*b^6)*Cos[c + d*x])/((a^2 - b^2)*(a + b*Sin[c + d*x
])^2) - (21*(1088*a^8 - 2576*a^6*b^2 + 1960*a^4*b^4 - 497*a^2*b^6 + 21*b^8)
*Cos[c + d*x])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])) - 8*b*(-276*a^2 + 35*b^
2)*Sin[2*(c + d*x)] - 56*b^3*Sin[4*(c + d*x)])))/b^7)/(10080*d*(a + b*Sin[c
 + d*x])^(3/2))
```

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos\left(dx+c\right)^{6}-\cos\left(dx+c\right)^{4}\right)\sqrt{b\sin\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{3\,ab^{2}\cos\left(dx+c\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(dx+c\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral((cos(d*x + c)^6 - cos(d*x + c)^4)*sqrt(b*sin(d*x + c) + a)*sin(d*x
 + c)/(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2
*b - b^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{3}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [B] time = 2.10, size = 2033, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/315*(-35*b^7*sin(d*x+c)*cos(d*x+c)^6+(120*a^2*b^5-7*b^7)*cos(d*x+c)^4*sin
(d*x+c)+(3200*a^4*b^3-1740*a^2*b^5+42*b^7)*cos(d*x+c)^2*sin(d*x+c)-4*(-b/(a
+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a-b)*
sin(d*x+c)-b/(a-b))^(1/2)*b*(1280*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a^6-2048*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a^4*b^2+789*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-21*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*b^6-1280*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b+960*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2+1088*EllipticF((b/(a-b)*sin(d*x+c)+a
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3-666*EllipticF((b/(a-b)*sin(d*x+c
)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-123*EllipticF((b/(a-b)*sin(d*
x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5+21*EllipticF((b/(a-b)*sin(d*
x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6)*sin(d*x+c)+60*a*b^6*cos(d*x+c
)^6+(-320*a^3*b^4+102*a*b^6)*cos(d*x+c)^4+(2560*a^5*b^2-896*a^3*b^4-162*a*b
^6)*cos(d*x+c)^2-5120*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+
c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*si
n(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7+8192*(b/(a-b)*sin(d*x+c)+a
/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a
-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^5*b^2-3156*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a
+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c
)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+84*(b/(a-b)*sin(d*x+c)+a/(a-b
))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^
(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b
^6+5120*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1
/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b-3840*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2
)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*E
llipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-43
52*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(
```

```
-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*a^4*b^3+2664*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(
-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*Elli
pticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+492*(
b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(
a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))*a^2*b^5-84*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+
b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF(
(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6)/(a+b*sin(d*x
+c))^(3/2)/b^8/cos(d*x+c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^3}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(b*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\,\sin(c+d\,x)^3}{(a+b\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.1184 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=411

$$\frac{2\left(11a^2-3b^2\right)\sin^3(c+dx)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(a^2-b^2\right)\sin^3(c+dx)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}-\frac{16a\left(32a^2-15b^2\right)\sqrt{a+b\sin(c+d}{21b^6d\sqrt{\frac{a+b\sin(}{a+l}}}$$

```
[Out] -2/3*(a^2-b^2)*cos(d*x+c)*sin(d*x+c)^3/a/b^2/d/(a+b*sin(d*x+c))^(3/2)+2/3*(
11*a^2-3*b^2)*cos(d*x+c)*sin(d*x+c)^3/a^2/b^2/d/(a+b*sin(d*x+c))^(1/2)-8/21
*(32*a^2-11*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^5/d+8/21*(24*a^2-7*b^2
)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a/b^4/d-2/21*(80*a^2-21*b^2)
*cos(d*x+c)*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/a^2/b^3/d+16/21*a*(32*a^2-1
5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/
2)/b^6/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-8/21*(64*a^4-46*a^2*b^2+3*b^4)*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/
b^6/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.92, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2891, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(11a^2-3b^2\right)\sin^3(c+dx)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(a^2-b^2\right)\sin^3(c+dx)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}-\frac{2\left(80a^2-21b^2\right)\sin^2(c+dx)\cos(c}{21a^2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(3*a*b^2*d*(a + b*Sin[c + d*x]
)^(3/2)) + (2*(11*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(3*a^2*b^2*d*Sq
rt[a + b*Sin[c + d*x]]) - (8*(32*a^2 - 11*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[
c + d*x]])/(21*b^5*d) + (8*(24*a^2 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[
a + b*Sin[c + d*x]])/(21*a*b^4*d) - (2*(80*a^2 - 21*b^2)*Cos[c + d*x]*Sin[c
 + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(21*a^2*b^3*d) - (16*a*(32*a^2 - 15*b^2
)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(2
1*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(64*a^4 - 46*a^2*b^2 + 3*b
^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/
(a + b)])/(21*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(11a^2-3b^2\right)\cos(c+dx)\sin^3(c+d}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(11a^2-3b^2\right)\cos(c+dx)\sin^3(c+d}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(11a^2-3b^2\right)\cos(c+dx)\sin^3(c+d}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(11a^2-3b^2\right)\cos(c+dx)\sin^3(c+d}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(11a^2-3b^2\right)\cos(c+dx)\sin^3(c+d}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(11a^2-3b^2\right)\cos(c+dx)\sin^3(c+d}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(11a^2-3b^2\right)\cos(c+dx)\sin^3(c+d}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

Mathematica [A] time = 7.96, size = 257, normalized size = 0.63

$$\frac{32a\left(32a^2-15b^2\right)(a+b)^2\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-16\left(64a^4-46a^2b^2+3b^4\right)(a+b)\left(\frac{a+b\sin(}{a+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (32*a*(a + b)^2*(32*a^2 - 15*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
 + b)]*((a + b*Sin[c + d*x])/(a + b))^(3/2) - 16*(a + b)*(64*a^4 - 46*a^2*b
^2 + 3*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin[c +
 d*x])/(a + b))^(3/2) - (b*Cos[c + d*x]*(1024*a^4 - 288*a^2*b^2 - 27*b^4 -
8*(8*a^2*b^2 - 3*b^4)*Cos[2*(c + d*x)] + 3*b^4*Cos[4*(c + d*x)] + 1280*a^3*
```

```
b*Sin[c + d*x] - 516*a*b^3*Sin[c + d*x] + 12*a*b^3*Sin[3*(c + d*x)]))/2)/(4
2*b^6*d*(a + b*Sin[c + d*x])^(3/2))
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos\left(dx+c\right)^{6}-\cos\left(dx+c\right)^{4}\right)\sqrt{b\sin\left(dx+c\right)+a}}{3\,ab^{2}\cos\left(dx+c\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(dx+c\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral((cos(d*x + c)^6 - cos(d*x + c)^4)*sqrt(b*sin(d*x + c) + a)/(3*a*b^
2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin
(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{2}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.90, size = 1642, normalized size = 4.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/21*(6*a*b^5*sin(d*x+c)*cos(d*x+c)^4+(160*a^3*b^3-66*a*b^5)*cos(d*x+c)^2*
sin(d*x+c)+4*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b
))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b*(64*EllipticF((b/(a-b)*sin(d
*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-48*EllipticF((b/(a-b)*sin(d
*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-46*EllipticF((b/(a-b)*sin
(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+27*EllipticF((b/(a-b)*s
in(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+3*EllipticF((b/(a-b)*si
n(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^5-64*EllipticE((b/(a-b)*sin(
d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5+94*EllipticE((b/(a-b)*sin(d*
```

```
x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-30*EllipticE((b/(a-b)*sin(
d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4)*sin(d*x+c)+3*b^6*cos(d*x+
c)^6+(-16*a^2*b^4+3*b^6)*cos(d*x+c)^4+(128*a^4*b^2-28*a^2*b^4-6*b^6)*cos(d*
x+c)^2+256*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))
^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b-192*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1
/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)
*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-
184*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*
(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^
(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3+108*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(
-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*Elli
pticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+12*(b
/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a
-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a*b^5-256*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)
*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b
/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6+376*(b/(a-b)*sin(
d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x
+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^4*b^2-120*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+
c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*si
n(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4)/(a+b*sin(d*x+c))^(3/2
)/b^7/cos(d*x+c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{2}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^2}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.1185 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=254

$$-\frac{8a\left(32a^2-17b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^5d\sqrt{a+b\sin(c+dx)}}+\frac{8\left(32a^2-9b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2}{a+}\right.}{15b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] 2/15*cos(d*x+c)^3*(8*a+3*b*sin(d*x+c))/b^2/d/(a+b*sin(d*x+c))^(3/2)+4/15*co
s(d*x+c)*(32*a^2-9*b^2+8*a*b*sin(d*x+c))/b^4/d/(a+b*sin(d*x+c))^(1/2)-8/15*
(32*a^2-9*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x
)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x
+c))^(1/2)/b^5/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+8/15*a*(32*a^2-17*b^2)*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/
b^5/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2863, 2752, 2663, 2661, 2655, 2653}

$$\frac{4\cos(c+dx)\left(32a^2+8ab\sin(c+dx)-9b^2\right)}{15b^4d\sqrt{a+b\sin(c+dx)}}-\frac{8a\left(32a^2-17b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^5d\sqrt{a+b\sin(c+dx)}}+\frac{8\left(32a^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (8*(32*a^2 - 9*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b
*Sin[c + d*x]])/(15*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*a*(32*a^
2 - 17*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/(15*b^5*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Cos[c + d*x]^3*(
8*a + 3*b*Sin[c + d*x]))/(15*b^2*d*(a + b*Sin[c + d*x])^(3/2)) + (4*Cos[c +
 d*x]*(32*a^2 - 9*b^2 + 8*a*b*Sin[c + d*x]))/(15*b^4*d*Sqrt[a + b*Sin[c + d
*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{2\cos^3(c+dx)(8a+3b\sin(c+dx))}{15b^2d(a+b\sin(c+dx))^{3/2}} - \frac{4\int \frac{\cos^2(c+dx)\left(-\frac{3b}{2}-4a\sin(c+dx)\right)}{(a+b\sin(c+dx))^{3/2}}dx}{5b^2}$$

$$= \frac{2\cos^3(c+dx)(8a+3b\sin(c+dx))}{15b^2d(a+b\sin(c+dx))^{3/2}} + \frac{4\cos(c+dx)\left(32a^2-9b^2+8ab\sin(c+\right.}{15b^4d\sqrt{a+b\sin(c+dx)}}$$

$$= \frac{2\cos^3(c+dx)(8a+3b\sin(c+dx))}{15b^2d(a+b\sin(c+dx))^{3/2}} + \frac{4\cos(c+dx)\left(32a^2-9b^2+8ab\sin(c+\right.}{15b^4d\sqrt{a+b\sin(c+dx)}}$$

$$= \frac{2\cos^3(c+dx)(8a+3b\sin(c+dx))}{15b^2d(a+b\sin(c+dx))^{3/2}} + \frac{4\cos(c+dx)\left(32a^2-9b^2+8ab\sin(c+\right.}{15b^4d\sqrt{a+b\sin(c+dx)}}$$

$$= \frac{8\left(32a^2-9b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{15b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} - \frac{8a\left(32a^2-17b^2\right)}{15b}$$

Mathematica [A] time = 6.28, size = 211, normalized size = 0.83

$$\frac{32a\left(32a^2-17b^2\right)(a+b)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-32\left(32a^2-9b^2\right)(a+b)^2\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}E}{60}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-32*(a + b)^2*(32*a^2 - 9*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a +
 b)]*((a + b*Sin[c + d*x])/(a + b))^(3/2) + 32*a*(a + b)*(32*a^2 - 17*b^2)*
EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin[c + d*x])/(a +
b))^(3/2) + 2*b*Cos[c + d*x]*(256*a^3 - 24*a*b^2 - 16*a*b^2*Cos[2*(c + d*x)
] + b*(320*a^2 - 69*b^2)*Sin[c + d*x] + 3*b^3*Sin[3*(c + d*x)]))/(60*b^5*d*
(a + b*Sin[c + d*x])^(3/2))
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,\cos(dx+c)^4\sin(dx+c)}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")

[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c)/(3*a*b^2*cos
(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x
+ c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")

[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(b*sin(d*x + c) + a)^(5/2), x)

maple [B] time = 1.92, size = 1430, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(5/2),x)

[Out] 2/15*(3*b^5*sin(d*x+c)*cos(d*x+c)^4+(80*a^2*b^3-18*b^5)*cos(d*x+c)^2*sin(d*
x+c)+4*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/
2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b*(32*EllipticF((b/(a-b)*sin(d*x+c)+
a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b-24*EllipticF((b/(a-b)*sin(d*x+c)+
a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^2-17*EllipticF((b/(a-b)*sin(d*x+c
)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^3+9*EllipticF((b/(a-b)*sin(d*x+c)
+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^4-32*EllipticE((b/(a-b)*sin(d*x+c)+a
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4+41*EllipticE((b/(a-b)*sin(d*x+c)+a/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^2-9*EllipticE((b/(a-b)*sin(d*x+c)+a/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^4)*sin(d*x+c)-8*a*b^4*cos(d*x+c)^4+(64*
a^3*b^2-2*a*b^4)*cos(d*x+c)^2+128*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a
+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF
((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-96*(b/(a-b)*
sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin
(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a^3*b^2-68*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d
x+c)+b/(a+b))^(1/2)(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)
*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+36*(b/(a-b)*sin(d*x
+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)

```
-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^
(1/2))*a*b^4-128*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/
(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x
+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5+164*(b/(a-b)*sin(d*x+c)+a/(a-b)
)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(
1/2)*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*
b^2-36*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/
2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4)/(a+b*sin(d*x+c))^(3/2)/b^6/cos(d*x+c)/
d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(b*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\,\sin(c+d\,x)}{(a+b\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.1186 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=313

$$\frac{2\left(5a^2+3b^2\right)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(a^2-b^2\right)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}-\frac{2\left(8a^2+b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3ab^3d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(8a\right.}{}$$

```
[Out] -2/3*(a^2-b^2)*cos(d*x+c)/a/b^2/d/(a+b*sin(d*x+c))^(3/2)+2/3*(5*a^2+3*b^2)*
cos(d*x+c)/a^2/b^2/d/(a+b*sin(d*x+c))^(1/2)-2/3*(8*a^2+3*b^2)*(sin(1/2*c+1/
4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi
+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^2/b^3/d/((a+b*s
in(d*x+c))/(a+b))^(1/2)+2/3*(8*a^2+b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(
a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a/b^3/d/(a+b*sin(d*x+c))^(1/2)-
2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(
cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b
))^(1/2)/a^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2891, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2\left(5a^2+3b^2\right)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(a^2-b^2\right)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}-\frac{2\left(8a^2+b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3ab^3d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(8a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x])/(3*a*b^2*d*(a + b*Sin[c + d*x])^(3/2)) + (2*(
5*a^2 + 3*b^2)*Cos[c + d*x])/(3*a^2*b^2*d*Sqrt[a + b*Sin[c + d*x]]) + (2*(8
*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c
 + d*x]])/(3*a^2*b^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (2*(8*a^2 + b^
2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(3*a*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (2*EllipticPi[2, (c - Pi/2
+ d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a^2*d*Sqrt[a
+ b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
```

```
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(5a^2+3b^2\right)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} - \frac{4\int \frac{\csc(c+dx)\left(-\frac{3b^2}{4}-\right.}{}}{}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(5a^2+3b^2\right)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} + \frac{4\int \frac{\csc(c+dx)\left(\frac{3b^3}{4}-\frac{1}{4}\right.}{\sqrt{a+b}}}{3}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(5a^2+3b^2\right)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)}{\sqrt{a+b\sin(c+dx)}}dx}{a^2}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(5a^2+3b^2\right)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} + \frac{2\left(8a^2+3b^2\right)E\left(\right.}{}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(5a^2+3b^2\right)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} + \frac{2\left(8a^2+3b^2\right)E\left(\right.}{}$$

Mathematica [C] time = 5.18, size = 443, normalized size = 1.42

$$-\frac{\frac{2a^2\left(a^2-b^2\right)\cos(c+dx)}{(a+b\sin(c+dx))^{3/2}} - \frac{2a\left(5a^2+3b^2\right)\cos(c+dx)}{\sqrt{a+b\sin(c+dx)}} + \frac{a\left(8a^2+9b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{i\left(8a^2+3b^2\right)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-}{a+b}}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/3*((I*(8*a^2 + 3*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Elli
pticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
(a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(b^2*Sqrt[-(a + b)^(-1)]) + (2*a^2*(a^
2 - b^2)*Cos[c + d*x])/(a + b*Sin[c + d*x])^(3/2) - (2*a*(5*a^2 + 3*b^2)*Co
s[c + d*x])/Sqrt[a + b*Sin[c + d*x]] + (4*a^2*b*EllipticF[(-2*c + Pi - 2*d*
x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c +
 d*x]] + (a*(8*a^2 + 9*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a +
```

```
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/(a^3*b^2
*d)
```

fricas [F] time = 73.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b \sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{3}\cot\left(dx+c\right)}{3\,ab^{2}\cos\left(dx+c\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(dx+c\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^3*cot(d*x + c)/(3*a*b^2*cos
(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x
+ c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{3}\cot\left(dx+c\right)}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)^3*cot(d*x + c)/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [B] time = 6.70, size = 1375, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] (-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*(1/b^3*(2*b*(a/b-1)*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1
/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2)))-4*a*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*
(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x
+c))*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2)))+1/b^3*(3*a^4-2*a^2*b^2-b^4)/a^2*(2*b*cos(d*x+c)^2/(a^2-b^2)/(-
(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)+2*a/(a^2-b^2)*(a/b-1)*((a+b*sin(d*x+c
```

```
))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1
/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))+2*b/(a^2-b^2)*(a/b-1)*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(
-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))))-2/a^3*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-s
in(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))
*cos(d*x+c)^2)^(1/2)*b*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),-(-a/b+1)/
a*b,((a-b)/(a+b))^(1/2))+(-a^4+2*a^2*b^2-b^4)/a/b^3*(2/3/b/(a^2-b^2)*(-(-a-
b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)/(sin(d*x+c)+a/b)^2+8/3*b*cos(d*x+c)^2/(a^
2-b^2)^2*a/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)+2*(3*a^2+b^2)/(3*a^4-6*a
^2*b^2+3*b^4)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b
))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^
(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+8/3*a*b
/(a^2-b^2)^2*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b)
)^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(
1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)))))/cos(d*x+
c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^3 \cot(dx+c)}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^3*cot(d*x + c)/(b*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3\,\cot(c+d\,x)}{{(a+b\,\sin(c+d\,x))}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*cot(c + d*x))/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^3*cot(c + d*x))/(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.1187 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=346

$$-\frac{5b\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{a^3d\sqrt{a+b\sin(c+dx)}}+\frac{\left(2a^2-5b^2\right)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}}+\frac{\left(4a^2+5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx\right.\right.}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] 1/3*(2*a^2-5*b^2)*cos(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^(3/2)-cot(d*x+c)/a/d/
(a+b*sin(d*x+c))^(3/2)-1/3*(4*a^2+15*b^2)*cos(d*x+c)/a^3/b/d/(a+b*sin(d*x+c
))^(1/2)+1/3*(4*a^2+15*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))
*(a+b*sin(d*x+c))^(1/2)/a^3/b^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/3*(4*a^2
+5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellip
ticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(
a+b))^(1/2)/a^2/b^2/d/(a+b*sin(d*x+c))^(1/2)+5*b*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,
2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d*x+
c))^(1/2)
```

Rubi [A] time = 0.99, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2890, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(4a^2+15b^2\right)\cos(c+dx)}{3a^3bd\sqrt{a+b\sin(c+dx)}}+\frac{\left(2a^2-5b^2\right)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}}+\frac{\left(4a^2+5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{(4a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((2*a^2 - 5*b^2)*Cos[c + d*x])/(3*a^2*b*d*(a + b*Sin[c + d*x])^(3/2)) - Cot
[c + d*x]/(a*d*(a + b*Sin[c + d*x])^(3/2)) - ((4*a^2 + 15*b^2)*Cos[c + d*x]
)/(3*a^3*b*d*Sqrt[a + b*Sin[c + d*x]]) - ((4*a^2 + 15*b^2)*EllipticE[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*a^3*b^2*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) + ((4*a^2 + 5*b^2)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*a^2*b^2*d*Sqrt[a +
 b*Sin[c + d*x]]) - (5*b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)])/(a^3*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
```

```
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{(2a^2-5b^2)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} + \frac{2\int \frac{\csc(c+dx)\left(-\frac{15b^2}{4}-\frac{3}{2}ab\,\mathrm{s}\cdots\right)}{(a+\cdots}}{\cdots}$$

$$= \frac{(2a^2-5b^2)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{(4a^2+15b^2)\cos(c+\cdots}{3a^3bd\sqrt{a+b\sin(c+\cdots}}$$

$$= \frac{(2a^2-5b^2)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{(4a^2+15b^2)\cos(c+\cdots}{3a^3bd\sqrt{a+b\sin(c+\cdots}}$$

$$= \frac{(2a^2-5b^2)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{(4a^2+15b^2)\cos(c+\cdots}{3a^3bd\sqrt{a+b\sin(c+\cdots}}$$

$$= \frac{(2a^2-5b^2)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{(4a^2+15b^2)\cos(c+\cdots}{3a^3bd\sqrt{a+b\sin(c+\cdots}}$$

$$= \frac{(2a^2-5b^2)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{(4a^2+15b^2)\cos(c+\cdots}{3a^3bd\sqrt{a+b\sin(c+\cdots}}$$

Mathematica [C] time = 5.46, size = 445, normalized size = 1.29

$$\frac{\frac{2(4a^2+45b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} - \frac{2\left(b(4a^2+15b^2)\sin(2(c+dx))+4a(a^2+10b^2)\cos(c+dx)+6a^2b\cot(c+dx)\right)}{(a+b\sin(c+dx))^{3/2}} + \frac{2i(4a^2+15b^2)\sec\cdots}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)*(4*a^2 + 15*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Elli
pticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
(a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b^2*Sqrt[-(a + b)^(-1)]) + (40*a*
b*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
```

```
/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(4*a^2 + 45*b^2)*EllipticPi[2, (-2
*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt
[a + b*Sin[c + d*x]] - (2*(4*a*(a^2 + 10*b^2)*Cos[c + d*x] + 6*a^2*b*Cot[c
+ d*x] + b*(4*a^2 + 15*b^2)*Sin[2*(c + d*x)]))/(a + b*Sin[c + d*x])^(3/2))/
(12*a^3*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2\cot(dx+c)^2}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^2*cot(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [B] time = 2.19, size = 2112, normalized size = 6.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/3*(15*a*b^5*sin(d*x+c)^2-4*a^3*b^3*sin(d*x+c)^4-15*a*b^5*sin(d*x+c)^4-2*
a^4*b^2*sin(d*x+c)^3-20*a^2*b^4*sin(d*x+c)^3+a^3*b^3*sin(d*x+c)^2+2*a^4*b^2
*sin(d*x+c)+6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^2+5*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^2-15*
((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x
+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^
(1/2))*a*b^5*sin(d*x+c)^2-4*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
```

```
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)^2-11*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3*sin
(d*x+c)^2+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^2+20*a^2*b^4*sin(d*x+c)+15*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1
/2))*b^6*sin(d*x+c)^2-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(
a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^2+4*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b*
sin(d*x+c)+6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)+5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)-15*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^2*b^4*sin(d*x+c)-11*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)+15*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
lipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x
+c)-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/
a,((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)+15*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi
(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+
c)+4*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^4*b^2*sin(d*x+c)^2+3*a^3*b^3-4*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*sin(d*x+c))/a^4/sin
(d*x+c)/(a+b*sin(d*x+c))^(3/2)/b^3/cos(d*x+c)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (c+d\,x)^2\cot (c+d\,x)^2}{{\left (a+b\,\sin (c+d\,x)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*cot(c + d*x)^2)/(a + b*sin(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^2*cot(c + d*x)^2)/(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(c + dx\right)\cot^{2}\left(c + dx\right)}{\left(a + b\sin\left(c + dx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c))**(5/2),x)
[Out] Integral(cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x))**(5/2), x)
```

3.1188 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=407

$$\frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}}-\frac{(8a^2-105b^2)\cos(c+dx)}{12a^4d\sqrt{a+b\sin(c+dx)}}-\frac{(8a^2-105b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{12a^4bd\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] 1/6*(4*a^2-7*b^2)*cot(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^(3/2)-1/2*cot(d*x+c)*
csc(d*x+c)/a/d/(a+b*sin(d*x+c))^(3/2)-1/12*(8*a^2-105*b^2)*cos(d*x+c)/a^4/d
/(a+b*sin(d*x+c))^(1/2)-1/12*(8*a^2-35*b^2)*cot(d*x+c)/a^3/b/d/(a+b*sin(d*x
+c))^(1/2)+1/12*(8*a^2-105*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1
/2))*(a+b*sin(d*x+c))^(1/2)/a^4/b/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/12*(8*
a^2-35*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*E
llipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c
))/(a+b))^(1/2)/a^3/b/d/(a+b*sin(d*x+c))^(1/2)+1/4*(12*a^2-35*b^2)*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+
1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a
^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.27, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2890, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{(8a^2-105b^2)\cos(c+dx)}{12a^4d\sqrt{a+b\sin(c+dx)}}-\frac{(8a^2-35b^2)\cot(c+dx)}{12a^3bd\sqrt{a+b\sin(c+dx)}}+\frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}}+\frac{(8a^2-35b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{12a^3bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((4*a^2 - 7*b^2)*Cot[c + d*x])/(6*a^2*b*d*(a + b*Sin[c + d*x])^(3/2)) - (Co
t[c + d*x]*Csc[c + d*x])/(2*a*d*(a + b*Sin[c + d*x])^(3/2)) - ((8*a^2 - 105
*b^2)*Cos[c + d*x])/(12*a^4*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a^2 - 35*b^2)
*Cot[c + d*x])/(12*a^3*b*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a^2 - 105*b^2)*E
llipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(12*a
^4*b*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((8*a^2 - 35*b^2)*EllipticF[(c
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(12*a^
3*b*d*Sqrt[a + b*Sin[c + d*x]]) - ((12*a^2 - 35*b^2)*EllipticPi[2, (c - Pi/
```

```
2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^4*d*Sqr
t[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{\left(4a^2-7b^2\right)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc^2(c+dx)\left(\frac{1}{4}\left(8a^2-35b^2\right.\right.}{(a+}}{}$$

$$= \frac{\left(4a^2-7b^2\right)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{\left(8a^2-35b^2\right)\cot(c}{12a^3bd\sqrt{a+b\sin(c}}$$

$$= \frac{\left(4a^2-7b^2\right)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{\left(8a^2-105b^2\right)\cos(c}{12a^4d\sqrt{a+b\sin(c+}}$$

$$= \frac{\left(4a^2-7b^2\right)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{\left(8a^2-105b^2\right)\cos(c}{12a^4d\sqrt{a+b\sin(c+}}$$

$$= \frac{\left(4a^2-7b^2\right)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{\left(8a^2-105b^2\right)\cos(c}{12a^4d\sqrt{a+b\sin(c+}}$$

$$= \frac{\left(4a^2-7b^2\right)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{\left(8a^2-105b^2\right)\cos(c}{12a^4d\sqrt{a+b\sin(c+}}$$

$$= \frac{\left(4a^2-7b^2\right)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{\left(8a^2-105b^2\right)\cos(c}{12a^4d\sqrt{a+b\sin(c+}}$$

Mathematica [C] time = 6.71, size = 622, normalized size = 1.53

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{11b\cot(c+dx)}{4a^4} - \frac{\cot(c+dx)\csc(c+dx)}{2a^3} - \frac{2\left(a^2\cos(c+dx)-9b^2\cos(c+dx)\right)}{3a^4(a+b\sin(c+dx))} - \frac{2\left(a^2\cos(c+dx)-b^2\cos(c+dx)\right)}{3a^3(a+b\sin(c+dx))^2}\right)}{d} + \frac{-\frac{2(315}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*((11*b*Cot[c + d*x])/(4*a^4) - (Cot[c + d*x]*Csc[
c + d*x])/(2*a^3) - (2*(a^2*Cos[c + d*x] - b^2*Cos[c + d*x]))/(3*a^3*(a + b
*Sin[c + d*x])^2) - (2*(a^2*Cos[c + d*x] - 9*b^2*Cos[c + d*x]))/(3*a^4*(a +
 b*Sin[c + d*x]))))/d + ((-280*a*b*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(-8
0*a^2 + 315*b^2)*EllipticPi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(8*a^2 - 105*
b^2)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(
a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF
[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]
- b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c +
d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b
*Sin[c + d*x])/(a - b))]])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(
-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-(
(a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(48
*a^4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)\cot(dx+c)^3}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)*cot(d*x + c)^3/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [B] time = 2.43, size = 2617, normalized size = 6.43

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x)
```

[Out] -1/12*(-35*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^2+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((
a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^2+105*
a*b^5*sin(d*x+c)^5-8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^6*sin(d*x+c)^2-105*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^5*sin(
d*x+c)^2-78*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^2-21*a^3*b^3*sin(d*x+c)+140*a^2*b^4*s
in(d*x+c)^4+29*a^3*b^3*sin(d*x+c)^3-105*a*b^5*sin(d*x+c)^3+10*a^4*b^2*sin(d
*x+c)^2-140*a^2*b^4*sin(d*x+c)^2-8*a^3*b^3*sin(d*x+c)^5-16*a^4*b^2*sin(d*x+
c)^4+6*a^4*b^2-78*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^3-35*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^
3+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a*b^5*sin(d*x+c)^3-8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)^3+113*((a+b*sin(d
x+c))/(a-b))^(1/2)(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*
b^3*sin(d*x+c)^3-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^3-36*((a+b*sin(d*x+c))/(a-b))^
(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellip
ticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^3*s
in(d*x+c)^3+36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2
),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^3+105*((a+b*sin(d*x+c))/(
a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)
*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b
^5*sin(d*x+c)^3+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)^2+113*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^2
-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^2*b^4*sin(d*x+c)^2-36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d

```
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*si
n(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^2+36
*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*
x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-
b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^2-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((
a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^6*sin(d*x+c)^3+
105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,(
(a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^2+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((
a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^3)/a^5
/sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2)/b^2/cos(d*x+c)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\cot(c+d\,x)^3}{(a+b\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*cot(c + d*x)^3)/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)*cot(c + d*x)^3)/(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)*cot(c + d*x)**3/(a + b*sin(c + d*x))**(5/2), x)
```

3.1189 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=458

$$\frac{(2a^2-3b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}}+\frac{b(32a^2-105b^2)\cos(c+dx)}{8a^5d\sqrt{a+b\sin(c+dx)}}+\frac{(32a^2-105b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+\right.\right.}{8a^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

[Out] 1/3*(2*a^2-3*b^2)*cot(d*x+c)*csc(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^(3/2)-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d/(a+b*sin(d*x+c))^(3/2)+1/8*b*(32*a^2-105*b^2)*cos(d*x+c)/a^5/d/(a+b*sin(d*x+c))^(1/2)+1/8*(16*a^2-35*b^2)*cot(d*x+c)/a^4/d/(a+b*sin(d*x+c))^(1/2)-1/12*(8*a^2-21*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/b/d/(a+b*sin(d*x+c))^(1/2)-1/8*(32*a^2-105*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^5/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+1/8*(16*a^2-35*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^4/d/(a+b*sin(d*x+c))^(1/2)-15/8*b*(4*a^2-7*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^5/d/(a+b*sin(d*x+c))^(1/2)

Rubi [A] time = 1.56, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2724, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b(32a^2-105b^2)\cos(c+dx)}{8a^5d\sqrt{a+b\sin(c+dx)}}+\frac{(16a^2-35b^2)\cot(c+dx)}{8a^4d\sqrt{a+b\sin(c+dx)}}-\frac{(16a^2-35b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{8a^4d\sqrt{a+b\sin(c+dx)}}+\cdots$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]

[Out] ((2*a^2 - 3*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*b*d*(a + b*Sin[c + d*x])^(3/2)) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a + b*Sin[c + d*x])^(3/2)) + (b*(32*a^2 - 105*b^2)*Cos[c + d*x])/(8*a^5*d*Sqrt[a + b*Sin[c + d*x]]) + ((16*a^2 - 35*b^2)*Cot[c + d*x])/(8*a^4*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a^2 - 21*b^2)*Cot[c + d*x]*Csc[c + d*x])/(12*a^3*b*d*Sqrt[a + b*Sin[c + d*x]]) + ((32*a^2 - 105*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(8*a^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - ((16*a^2 - 35*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c

```
+ d*x])/(a + b)])/(8*a^4*d*Sqrt[a + b*Sin[c + d*x]]) + (15*b*(4*a^2 - 7*b^2
)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x]
)/(a + b)])/(8*a^5*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m +
 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 -
b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 +
b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*
Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
m, -1] && IntegerQ[2*m]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{2\int \frac{\csc^3(c+dx)\left(\frac{3}{4}\left(8a\right.\right.}{}}{}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} - \frac{\left(8a^2-21b^2\right)\cot(}{12a^3bd\sqrt{a+}}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{\left(16a^2-35b^2\right)\text{co}}{8a^4d\sqrt{a+b\sin(}}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b\left(32a^2-105b^2\right)}{8a^5d\sqrt{a+b\,\text{si}}}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b\left(32a^2-105b^2\right)}{8a^5d\sqrt{a+b\,\text{si}}}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b\left(32a^2-105b^2\right)}{8a^5d\sqrt{a+b\,\text{si}}}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b\left(32a^2-105b^2\right)}{8a^5d\sqrt{a+b\,\text{si}}}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b\left(32a^2-105b^2\right)}{8a^5d\sqrt{a+b\,\text{si}}}$$

Mathematica [C] time = 6.97, size = 680, normalized size = 1.48

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{17b\cot(c+dx)\csc(c+dx)}{12a^4} - \frac{\cot(c+dx)\csc^2(c+dx)}{3a^3} + \frac{8\left(a^2b\cos(c+dx)-3b^3\cos(c+dx)\right)}{3a^5(a+b\sin(c+dx))} + \frac{\csc(c+dx)\left(32a^2\cos(c+dx)-123b\right.}{24a^5}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*(((32*a^2*Cos[c + d*x] - 123*b^2*Cos[c + d*x])*Cs
c[c + d*x])/(24*a^5) + (17*b*Cot[c + d*x]*Csc[c + d*x])/(12*a^4) - (Cot[c +
 d*x]*Csc[c + d*x]^2)/(3*a^3) + (2*(a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x]))
/(3*a^4*(a + b*Sin[c + d*x])^2) + (8*(a^2*b*Cos[c + d*x] - 3*b^3*Cos[c + d*
x]))/(3*a^5*(a + b*Sin[c + d*x]))))/d + ((-2*(32*a^3 - 140*a*b^2)*EllipticF
[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sq
rt[a + b*Sin[c + d*x]] - (2*(152*a^2*b - 315*b^3)*EllipticPi[2, (-c + Pi/2
- d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin
[c + d*x]] - ((2*I)*(-32*a^2*b + 105*b^3)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*
a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]]
, (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b
*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a - b))]])/(a*Sqrt[-(a
+ b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]
) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Sin[c + d*x])
+ (a + b*Sin[c + d*x])^2)/b^2)]))/(32*a^5*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^4}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^4/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [B] time = 2.44, size = 2870, normalized size = 6.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x)

[Out] 1/24*(159*a^3*b^3*sin(d*x+c)^4-315*a*b^5*sin(d*x+c)^4+126*a^4*b^2*sin(d*x+c
)^3-420*a^2*b^4*sin(d*x+c)^3-63*a^3*b^3*sin(d*x+c)^2+18*a^4*b^2*sin(d*x+c)+
48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^6*sin(d*x+c)^3+411*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^4-315*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5
*sin(d*x+c)^4-180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(
1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^4-8*a^5*b-96*a^3*b^3*s
in(d*x+c)^6+315*a*b^5*sin(d*x+c)^6-144*a^4*b^2*sin(d*x+c)^5+420*a^2*b^4*sin
(d*x+c)^5-32*a^5*b*sin(d*x+c)^4+40*a^5*b*sin(d*x+c)^2+180*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a
^2*b^4*sin(d*x+c)^4+315*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a
-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^4+48*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b*
sin(d*x+c)^3+48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)^4+411*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^3-3
15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^2*b^4*sin(d*x+c)^3-180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin
(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^3+48*
((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x
+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^
(1/2))*a^4*b^2*sin(d*x+c)^4-306*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^4-105*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*
b^4*sin(d*x+c)^4+315*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^4-96*((a+b*sin(d*x+c))/(a-b))^
(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellip
ticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)^4
```

```
-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^3*b^3*sin(d*x+c)^3+315*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^3+180*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b
))^(1/2))*a^3*b^3*sin(d*x+c)^3+315*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin
(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^3-315
*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*
x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-
b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^3-306*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^3-315*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(
a+b))^(1/2))*b^6*sin(d*x+c)^4-96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*sin(d*x+c)^3)/sin(d*x+c)^3/a^6/
(a+b*sin(d*x+c))^(3/2)/b/cos(d*x+c)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{4}\left(c + dx\right)}{\left(a + b\sin\left(c + dx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**(5/2), x)
```

3.1190 $\int \frac{\cos^4(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=510

$$\frac{12\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^2df(a+b\sin(e+fx))^{5/2}}-\frac{32b\left(2a^2-b^2\right)\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\sin(e+f}}{\sqrt{a+b}\sqrt{d\sin(e+f}}\right.\right.}{35a^5\sqrt{d}f(a-b)(a+b)^{3/2}}$$

```
[Out] 2/7*cos(f*x+e)^3*(d*sin(f*x+e))^(1/2)/a/d/f/(a+b*sin(f*x+e))^(7/2)+12/35*co
s(f*x+e)*(d*sin(f*x+e))^(1/2)/a^2/d/f/(a+b*sin(f*x+e))^(5/2)+8/35*(a^2-2*b^
2)*cos(f*x+e)*(d*sin(f*x+e))^(1/2)/a^3/(a^2-b^2)/d/f/(a+b*sin(f*x+e))^(3/2)
+32/35*b*(2*a^2-b^2)*cos(f*x+e)/a^3/(a^2-b^2)^2/f/(d*sin(f*x+e))^(1/2)/(a+b
*sin(f*x+e))^(1/2)-32/35*b*(2*a^2-b^2)*EllipticE(d^(1/2)*(a+b*sin(f*x+e))^(
1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-csc(f*x+e
))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/a^5/(a-b)/(a+b)^(
3/2)/f/d^(1/2)-8/35*(5*a^2-3*a*b-4*b^2)*EllipticF(d^(1/2)*(a+b*sin(f*x+e))^
(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-csc(f*x+
e))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/a^4/(a-b)/(a+b)^
(3/2)/f/d^(1/2)
```

Rubi [A] time = 1.91, antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2887, 2889, 3056, 2993, 2998, 2816, 2994}

$$\frac{32b\left(2a^2-b^2\right)\cos(e+fx)}{35a^3f\left(a^2-b^2\right)^2\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}+\frac{8\left(a^2-2b^2\right)\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^3df\left(a^2-b^2\right)(a+b\sin(e+fx))^{3/2}}-\frac{8\left(5a^2-3ab-4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(9/2)),x]
```

```
[Out] (2*Cos[e + f*x]^3*Sqrt[d*Sin[e + f*x]])/(7*a*d*f*(a + b*Sin[e + f*x])^(7/2)
) + (12*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(35*a^2*d*f*(a + b*Sin[e + f*x])
^(5/2)) + (8*(a^2 - 2*b^2)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(35*a^3*(a^2
- b^2)*d*f*(a + b*Sin[e + f*x])^(3/2)) + (32*b*(2*a^2 - b^2)*Cos[e + f*x])/
(35*a^3*(a^2 - b^2)^2*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (3
2*b*(2*a^2 - b^2)*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e +
 f*x]))/(a - b)]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[
a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(35*a^5*(a
 - b)*(a + b)^(3/2)*Sqrt[d]*f) - (8*(5*a^2 - 3*a*b - 4*b^2)*Sqrt[(a*(1 - Cs
c[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin
```

```
[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -(
(a + b)/(a - b))]*Tan[e + f*x])/(35*a^4*(a - b)*(a + b)^(3/2)*Sqrt[d]*f)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2887

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> -Simp[(g*(g*C
os[e + f*x])^(p - 1)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(m + 1))/(a*
d*f*(m + 1)), x] + Dist[(g^2*(2*m + 3))/(2*a*(m + 1)), Int[((g*Cos[e + f*x]
)^(p - 2)*(a + b*Sin[e + f*x])^(m + 1))/Sqrt[d*Sin[e + f*x]], x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && EqQ[m + p +
 1/2, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{9/2}}\,dx = \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{6\int \frac{\cos^2(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{7/2}}dx}{7a}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{6\int \frac{1-\sin^2(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{7/2}}dx}{7a}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^2df(a+b\sin(e+fx))^{5/2}} + \frac{1}{}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^2df(a+b\sin(e+fx))^{5/2}} + \frac{8}{3}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^2df(a+b\sin(e+fx))^{5/2}} + \frac{8}{3}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^2df(a+b\sin(e+fx))^{5/2}} + \frac{8}{3}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^2df(a+b\sin(e+fx))^{5/2}} + \frac{8}{3}$$

Mathematica [C] time = 6.55, size = 1670, normalized size = 3.27

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(9/2)),
x]
```

```
[Out] (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((-2*(a^2*Cos[e + f*x] - b^2*Cos[e +
 f*x]))/(7*a*b^2*(a + b*Sin[e + f*x])^4) + (4*(5*a^2*Cos[e + f*x] + 3*b^2*C
os[e + f*x]))/(35*a^2*b^2*(a + b*Sin[e + f*x])^3) - (2*(5*a^4*Cos[e + f*x]
- 9*a^2*b^2*Cos[e + f*x] + 8*b^4*Cos[e + f*x]))/(35*a^3*b^2*(a^2 - b^2)*(a
+ b*Sin[e + f*x])^2) - (32*(2*a^2*b^2*Cos[e + f*x] - b^4*Cos[e + f*x]))/(35
*a^4*(a^2 - b^2)^2*(a + b*Sin[e + f*x]))))/(f*Sqrt[d*Sin[e + f*x]]) + (4*Sq
```

```
rt[Sin[e + f*x]]*((4*a*(5*a^4 - 9*a^2*b^2 + 4*b^4)*Sqrt[((a + b)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]
^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x]
)/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*S
qrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) + 4*a*(-8*a^3*b + 4*a*b^3)*((Sq
rt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Cs
c[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 -
f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/a])/((a + b)*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (Sqrt[(
(a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqr
t[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi
/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Si
n[e + f*x]))/a])/(b*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])) + 2*(8*a^
2*b^2 - 4*b^4)*((Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(b*Sqrt[Sin[e + f*x
]]) + (I*Cos[(-e + Pi/2 - f*x)/2]*Csc[e + f*x]*EllipticE[I*ArcSinh[Sin[(-e
+ Pi/2 - f*x)/2]/Sqrt[Sin[e + f*x]]], (-2*a)/(-a - b)]*Sqrt[a + b*Sin[e + f
*x]])/(b*Sqrt[Cos[(-e + Pi/2 - f*x)/2]^2*Csc[e + f*x]]*Sqrt[(Csc[e + f*x]*(
a + b*Sin[e + f*x]))/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*
x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b
*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 -
 f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqr
t[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqrt[Sin[e
 + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (a*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)
/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2
*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/
a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/(b*Sqrt[Sin[
e + f*x]]*Sqrt[a + b*Sin[e + f*x]]))))/b)))/(35*a^4*(a - b)^2*(a + b)^2*f*Sq
rt[d*Sin[e + f*x]])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(fx+e\right)+}}{b^5 d\cos\left(fx+e\right)^6-\left(10\,a^2b^3+3\,b^5\right)d\cos\left(fx+e\right)^4+\left(5\,a^4b+20\,a^2b^3+3\,b^5\right)d\cos\left(fx+e\right)^2-\left(5\,a^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))*cos(f*x + e)^4/(b^5
*d*cos(f*x + e)^6 - (10*a^2*b^3 + 3*b^5)*d*cos(f*x + e)^4 + (5*a^4*b + 20*a
```

```
^2*b^3 + 3*b^5)*d*cos(f*x + e)^2 - (5*a^4*b + 10*a^2*b^3 + b^5)*d - (5*a*b^
4*d*cos(f*x + e)^4 - 10*(a^3*b^2 + a*b^4)*d*cos(f*x + e)^2 + (a^5 + 10*a^3*
b^2 + 5*a*b^4)*d)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{9}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/((b*sin(f*x + e) + a)^(9/2)*sqrt(d*sin(f*x + e))),
 x)
```

maple [B] time = 1.16, size = 24365, normalized size = 47.77

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{9}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/((b*sin(f*x + e) + a)^(9/2)*sqrt(d*sin(f*x + e))),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(e+f\,x\right)}^4}{\sqrt{d\,\sin\left(e+f\,x\right)}\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(9/2)),x)
[Out] int(cos(e + f*x)^4/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(9/2)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sin(f*x+e))**(9/2)/(d*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

3.1191 $\int \frac{\cos^4(c+dx) \sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\, dx$

Optimal. Leaf size=36

$$\operatorname{Int}\left(\frac{\sqrt[3]{\sin(c+dx)}\,\cos^4(c+dx)}{\sqrt{a+b\sin(c+dx)}},x\right)$$

[Out] Unintegrable(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x)

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^4(c+dx)\sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^(1/3))/Sqrt[a + b*Sin[c + d*x]],x]

[Out] Defer[Int][(Cos[c + d*x]^4*Sin[c + d*x]^(1/3))/Sqrt[a + b*Sin[c + d*x]], x]

Rubi steps

$$\int \frac{\cos^4(c+dx)\sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx = \int \frac{\cos^4(c+dx)\sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Mathematica [A] time = 24.52, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^(1/3))/Sqrt[a + b*Sin[c + d*x]],x]

[Out] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^(1/3))/Sqrt[a + b*Sin[c + d*x]], x]

fricas [A] time = 28.00, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{\frac{1}{3}}}{\sqrt{b\sin\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(cos(d*x + c)^4*sin(d*x + c)^(1/3)/sqrt(b*sin(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{4}\left(dx+c\right)\right)\left(\sin^{\frac{1}{3}}\left(dx+c\right)\right)}{\sqrt{a+b\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{\frac{1}{3}}}{\sqrt{b\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^(1/3)/sqrt(b*sin(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\cos\left(c+d\,x\right)}^4\,{\sin\left(c+d\,x\right)}^{1/3}}{\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^(1/3))/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^(1/3))/(a + b*sin(c + d*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sin (c + dx)} \cos^4 (c + dx)}{\sqrt{a + b \sin (c + dx)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**(1/3)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)**(1/3)*cos(c + d*x)**4/sqrt(a + b*sin(c + d*x)), x)
```

3.1192 $\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p,x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p,x]
[Out] Defer[Int][Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx=\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$$

Mathematica [A] time = 6.50, size = 0, normalized size = 0.00

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p,x]
[Out] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p, x]
```

fricas [A] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^p\sin(dx+c)^n\cos(dx+c)^4\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="fricas
")
```

```
[Out] integral((b*sin(d*x + c) + a)^p*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.06, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)\left(\sin^n(dx+c)\right)\left(a+b\sin(dx+c)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x)
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^p\sin(dx+c)^n\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="maxima
")
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \cos(c+d\,x)^4\sin(c+d\,x)^n\left(a+b\,\sin(c+d\,x)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x))^p,x)
[Out] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+b*sin(d*x+c))**p,x)
```

```
[Out] Timed out
```

3.1193 $\int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p\,dx$

Optimal. Leaf size=36

$$\text{Int}\left(\cos^4(c+dx)\sin^{-p-3}(c+dx)(a+b\sin(c+dx))^p,x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^(-3 - p)*(a + b*Sin[c + d*x])^p,x]
[Out] Defer[Int][Cos[c + d*x]^4*Sin[c + d*x]^(-3 - p)*(a + b*Sin[c + d*x])^p, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p\,dx = \int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Mathematica [A] time = 4.99, size = 0, normalized size = 0.00

$$\int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^(-3 - p)*(a + b*Sin[c + d*x])^p,x]
[Out] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^(-3 - p)*(a + b*Sin[c + d*x])^p, x]
```

fricas [A] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^p\sin(dx+c)^{-p-3}\cos(dx+c)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x, algorithm="f
ricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 3)*cos(d*x + c)^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^p \sin(dx+c)^{-p-3}\cos(dx+c)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x, algorithm="g
iac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 3)*cos(d*x + c)^4, x)
```

maple [A] time = 1.60, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)\left(\sin^{-3-p}(dx+c)\right)(a+b\sin(dx+c))^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^p \sin(dx+c)^{-p-3}\cos(dx+c)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x, algorithm="m
axima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 3)*cos(d*x + c)^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(c+d\,x)^4\,(a+b\,\sin(c+d\,x))^p}{\sin(c+d\,x)^{p+3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^p)/sin(c + d*x)^(p + 3),x)
```

```
[Out] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^p)/sin(c + d*x)^(p + 3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**(-3-p)*(a+b*sin(d*x+c))**p,x)
```

```
[Out] Timed out
```

3.1194 $\int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,dx$

Optimal. Leaf size=36

$$\text{Int}\left(\cos^4(c+dx)\sin^{-p-4}(c+dx)(a+b\sin(c+dx))^p,x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^(-4 - p)*(a + b*Sin[c + d*x])^p,x]
[Out] Defer[Int][Cos[c + d*x]^4*Sin[c + d*x]^(-4 - p)*(a + b*Sin[c + d*x])^p, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,dx = \int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Mathematica [A] time = 5.65, size = 0, normalized size = 0.00

$$\int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^(-4 - p)*(a + b*Sin[c + d*x])^p,x]
[Out] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^(-4 - p)*(a + b*Sin[c + d*x])^p, x]
```

fricas [A] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^p\sin(dx+c)^{-p-4}\cos(dx+c)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x, algorithm="f
ricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 4)*cos(d*x + c)^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^p \sin(dx+c)^{-p-4}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x, algorithm="g
iac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 4)*cos(d*x + c)^4, x)
```

maple [A] time = 1.40, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)\left(\sin^{-4-p}(dx+c)\right)(a+b\sin(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^p \sin(dx+c)^{-p-4}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x, algorithm="m
axima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 4)*cos(d*x + c)^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(c+d\,x)^4\,(a+b\,\sin(c+d\,x))^p}{\sin(c+d\,x)^{p+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^p)/sin(c + d*x)^(p + 4),x)
```

```
[Out] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^p)/sin(c + d*x)^(p + 4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**(-4-p)*(a+b*sin(d*x+c))**p,x)
```

```
[Out] Timed out
```

3.1195 $\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=623

$$\frac{3a\left(a^2(n+6)+3b^2(n+1)\right)\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)(n+2)(n+4)(n+6)\sqrt{\cos^2(c+dx)}}+\frac{3b\left(3a^2(n+7)+b^2(n+2)\right.}{d(n+}$$

```
[Out] -3*a*(2*a^4*(n^2+5*n+6)+3*b^4*(n^2+12*n+35)-2*a^2*b^2*(n^2+16*n+58))*cos(d*
x+c)*sin(d*x+c)^(1+n)/b^2/d/(5+n)/(6+n)/(7+n)/(n^2+6*n+8)-3*(2*a^4*(n^2+5*n
+6)+b^4*(n^2+10*n+24)-2*a^2*b^2*(n^2+16*n+57))*cos(d*x+c)*sin(d*x+c)^(2+n)/
b/d/(3+n)/(4+n)/(5+n)/(6+n)/(7+n)-3*a*(a^2*(n^2+5*n+6)-b^2*(n^2+15*n+53))*c
os(d*x+c)*sin(d*x+c)^(1+n)*(a+b*sin(d*x+c))^2/b^2/d/(4+n)/(5+n)/(6+n)/(7+n)
-(a^2*(2+n)*(3+n)-b^2*(6+n)*(8+n))*cos(d*x+c)*sin(d*x+c)^(1+n)*(a+b*sin(d*x
+c))^3/b^2/d/(5+n)/(6+n)/(7+n)+a*(3+n)*cos(d*x+c)*sin(d*x+c)^(1+n)*(a+b*sin
(d*x+c))^4/b^2/d/(6+n)/(7+n)-cos(d*x+c)*sin(d*x+c)^(2+n)*(a+b*sin(d*x+c))^4
/b/d/(7+n)+3*a*(3*b^2*(1+n)+a^2*(6+n))*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*n
],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^(1+n)/d/(6+n)/(n^3+7*n^2+14*n+8)/(co
s(d*x+c)^2)^(1/2)+3*b*(b^2*(2+n)+3*a^2*(7+n))*cos(d*x+c)*hypergeom([1/2, 1+
1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(5+n)/(7+n)/(n^2+5*n+6)/(
cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 1.76, antiderivative size = 623, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2895, 3049, 3033, 3023, 2748, 2643}

$$\frac{3a\left(a^2(n+6)+3b^2(n+1)\right)\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)(n+2)(n+4)(n+6)\sqrt{\cos^2(c+dx)}}+\frac{3b\left(3a^2(n+7)+b^2(n+2)\right.}{d(n+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a*(2*a^4*(6 + 5*n + n^2) + 3*b^4*(35 + 12*n + n^2) - 2*a^2*b^2*(58 + 16
*n + n^2))*Cos[c + d*x]*Sin[c + d*x]^(1 + n))/(b^2*d*(2 + n)*(4 + n)*(5 + n
)*(6 + n)*(7 + n)) + (3*a*(3*b^2*(1 + n) + a^2*(6 + n))*Cos[c + d*x]*Hyperg
eometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n)
)/(d*(1 + n)*(2 + n)*(4 + n)*(6 + n)*Sqrt[Cos[c + d*x]^2]) - (3*(2*a^4*(6 +
 5*n + n^2) + b^4*(24 + 10*n + n^2) - 2*a^2*b^2*(57 + 16*n + n^2))*Cos[c +
d*x]*Sin[c + d*x]^(2 + n))/(b*d*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7 + n)) +
(3*b*(b^2*(2 + n) + 3*a^2*(7 + n))*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 +
 n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(d*(2 + n)*(3 + n)*
(5 + n)*(7 + n)*Sqrt[Cos[c + d*x]^2]) - (3*a*(a^2*(6 + 5*n + n^2) - b^2*(53
 + 15*n + n^2))*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^2)/(
```

```
b^2*d*(4 + n)*(5 + n)*(6 + n)*(7 + n)) - ((a^2*(2 + n)*(3 + n) - b^2*(6 + n
)*(8 + n))*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^3)/(b^2*d
*(5 + n)*(6 + n)*(7 + n)) + (a*(3 + n)*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a
 + b*Sin[c + d*x])^4)/(b^2*d*(6 + n)*(7 + n)) - (Cos[c + d*x]*Sin[c + d*x]^
(2 + n)*(a + b*Sin[c + d*x])^4)/(b*d*(7 + n))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{a(3+n)\cos(c+dx)\sin^{1+n}(c+dx)(a+b\sin(c+dx))^4}{b^2d(6+n)(7+n)} - \frac{\text{co}}{}$$

$$= -\frac{\left(a^2(2+n)(3+n)-b^2(6+n)(8+n)\right)\cos(c+dx)\sin^{1+n}(c}{b^2d(5+n)(6+n)(7+n)}$$

$$= -\frac{3a\left(a^2\left(6+5n+n^2\right)-b^2\left(53+15n+n^2\right)\right)\cos(c+dx)\sin}{b^2d(4+n)(5+n)(6+n)(7+}$$

$$= -\frac{3\left(2a^4\left(6+5n+n^2\right)+b^4\left(24+10n+n^2\right)-2a^2b^2\left(57+1\right.\right.}{bd(3+n)(4+n)(5+n)(6+n}$$

$$= -\frac{3a\left(2a^4\left(6+5n+n^2\right)+3b^4\left(35+12n+n^2\right)-2a^2b^2\left(58\right.\right.}{b^2d(2+n)(4+n)(5+n)(6+}$$

$$= -\frac{3a\left(2a^4\left(6+5n+n^2\right)+3b^4\left(35+12n+n^2\right)-2a^2b^2\left(58\right.\right.}{b^2d(2+n)(4+n)(5+n)(6+}$$

$$= -\frac{3a\left(2a^4\left(6+5n+n^2\right)+3b^4\left(35+12n+n^2\right)-2a^2b^2\left(58\right.\right.}{b^2d(2+n)(4+n)(5+n)(6+}$$

Mathematica [A] time = 0.85, size = 195, normalized size = 0.31

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{n+1}(c+dx)\left(\frac{a^3\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{n+1}+b\sin(c+dx)\left(\frac{3a^2\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{n+2}\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*((a^3*Hypergeometri
c2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2])/(1 + n) + b*Sin[c + d*x]*
((3*a^2*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2])/(2 +
 n) + b*Sin[c + d*x]*((3*a*Hypergeometric2F1[-3/2, (3 + n)/2, (5 + n)/2, Si
n[c + d*x]^2])/(3 + n) + (b*Hypergeometric2F1[-3/2, (4 + n)/2, (6 + n)/2, S
in[c + d*x]^2]*Sin[c + d*x])/(4 + n)))))/d
```

fricas [F] time = 1.21, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,ab^2\cos(dx+c)^6-\left(a^3+3\,ab^2\right)\cos(dx+c)^4+\left(b^3\cos(dx+c)^6-\left(3\,a^2b+b^3\right)\cos(dx+c)^4\right)\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^6 - (a^3 + 3*a*b^2)*cos(d*x + c)^4 + (b^3*c
os(d*x + c)^6 - (3*a^2*b + b^3)*cos(d*x + c)^4)*sin(d*x + c))*sin(d*x + c)^
n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^3\sin(dx+c)^n\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

maple [F] time = 28.10, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)\left(\sin^n(dx+c)\right)(a+b\sin(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^3 \sin(dx+c)^n \cos(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^4 \sin(c+d\,x)^n \left(a+b\,\sin(c+d\,x)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1196 $\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=487

$$\frac{3\left(a^2(n+6)+b^2(n+1)\right)\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)(n+2)(n+4)(n+6)\sqrt{\cos^2(c+dx)}}-\frac{2a\left(a^2\left(n^2+5n+6\right)-b^2\left(n\right.\right.}{bd(n+3}$$

```
[Out] -(3*b^4*(5+n)+2*a^4*(n^2+5*n+6)-2*a^2*b^2*(n^2+13*n+40))*cos(d*x+c)*sin(d*x
+c)^(1+n)/b^2/d/(5+n)/(6+n)/(n^2+6*n+8)-2*a*(a^2*(n^2+5*n+6)-b^2*(n^2+13*n+
39))*cos(d*x+c)*sin(d*x+c)^(2+n)/b/d/(3+n)/(4+n)/(5+n)/(6+n)-(a^2*(2+n)*(3+
n)-b^2*(5+n)*(7+n))*cos(d*x+c)*sin(d*x+c)^(1+n)*(a+b*sin(d*x+c))^2/b^2/d/(4
+n)/(5+n)/(6+n)+a*(3+n)*cos(d*x+c)*sin(d*x+c)^(1+n)*(a+b*sin(d*x+c))^3/b^2/
d/(5+n)/(6+n)-cos(d*x+c)*sin(d*x+c)^(2+n)*(a+b*sin(d*x+c))^3/b/d/(6+n)+3*(b
^2*(1+n)+a^2*(6+n))*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(d
*x+c)^2)*sin(d*x+c)^(1+n)/d/(4+n)/(6+n)/(n^2+3*n+2)/(cos(d*x+c)^2)^(1/2)+6*
a*b*cos(d*x+c)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^
(2+n)/d/(5+n)/(n^2+5*n+6)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 1.12, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2895, 3049, 3033, 3023, 2748, 2643}

$$\frac{3\left(a^2(n+6)+b^2(n+1)\right)\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)(n+2)(n+4)(n+6)\sqrt{\cos^2(c+dx)}}-\frac{\left(-2a^2b^2\left(n^2+13n+40\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((3*b^4*(5 + n) + 2*a^4*(6 + 5*n + n^2) - 2*a^2*b^2*(40 + 13*n + n^2))*Co
s[c + d*x]*Sin[c + d*x]^(1 + n))/(b^2*d*(2 + n)*(4 + n)*(5 + n)*(6 + n))) +
 (3*(b^2*(1 + n) + a^2*(6 + n))*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n)
/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*(2 + n)*(4
+ n)*(6 + n)*Sqrt[Cos[c + d*x]^2]) - (2*a*(a^2*(6 + 5*n + n^2) - b^2*(39 +
13*n + n^2))*Cos[c + d*x]*Sin[c + d*x]^(2 + n))/(b*d*(3 + n)*(4 + n)*(5 + n
)*(6 + n)) + (6*a*b*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/
2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(d*(2 + n)*(3 + n)*(5 + n)*Sqrt[Co
s[c + d*x]^2]) - ((a^2*(2 + n)*(3 + n) - b^2*(5 + n)*(7 + n))*Cos[c + d*x]*
Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^2)/(b^2*d*(4 + n)*(5 + n)*(6 + n)
) + (a*(3 + n)*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^3)/(b
^2*d*(5 + n)*(6 + n)) - (Cos[c + d*x]*Sin[c + d*x]^(2 + n)*(a + b*Sin[c + d
*x])^3)/(b*d*(6 + n))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{a(3+n)\cos(c+dx)\sin^{1+n}(c+dx)(a+b\sin(c+dx))^3}{b^2d(5+n)(6+n)} - $$

$$= -\frac{\left(a^2(2+n)(3+n)-b^2(5+n)(7+n)\right)\cos(c+dx)\sin^{1+n}}{b^2d(4+n)(5+n)(6+n)}$$

$$= -\frac{2a\left(a^2\left(6+5n+n^2\right)-b^2\left(39+13n+n^2\right)\right)\cos(c+dx)\,\mathrm{s}}{bd(3+n)(4+n)(5+n)(6+n)}$$

$$= -\frac{\left(3b^4(5+n)+2a^4\left(6+5n+n^2\right)-2a^2b^2\left(40+13n+n^2\right)\right.}{b^2d(2+n)(4+n)(5+n)(6+}$$

$$= -\frac{\left(3b^4(5+n)+2a^4\left(6+5n+n^2\right)-2a^2b^2\left(40+13n+n^2\right)\right.}{b^2d(2+n)(4+n)(5+n)(6+}$$

$$= -\frac{\left(3b^4(5+n)+2a^4\left(6+5n+n^2\right)-2a^2b^2\left(40+13n+n^2\right)\right.}{b^2d(2+n)(4+n)(5+n)(6+}$$

Mathematica [A] time = 0.34, size = 167, normalized size = 0.34

$$\frac{\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\sin^{n+1}(c+dx)\left(a^2\left(n^2+5n+6\right)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+b(n+1)\sin(c+\right.}{d(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*(a^2*(6 + 5*n + n^2
)*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2] + b*(1 + n)
*Sin[c + d*x]*(2*a*(3 + n)*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Si
n[c + d*x]^2] + b*(2 + n)*Hypergeometric2F1[-3/2, (3 + n)/2, (5 + n)/2, Sin
[c + d*x]^2]*Sin[c + d*x])))/(d*(1 + n)*(2 + n)*(3 + n))
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos\left(dx+c\right)^6-2\,ab\cos\left(dx+c\right)^4\sin\left(dx+c\right)-\left(a^2+b^2\right)\cos\left(dx+c\right)^4\right)\sin\left(dx+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral(-(b^2*cos(d*x + c)^6 - 2*a*b*cos(d*x + c)^4*sin(d*x + c) - (a^2 +
b^2)*cos(d*x + c)^4)*sin(d*x + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)+a\right)^2\sin\left(dx+c\right)^n\cos\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

maple [F] time = 17.91, size = 0, normalized size = 0.00

$$\int\left(\cos^4\left(dx+c\right)\right)\left(\sin^n\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)+a\right)^2\sin\left(dx+c\right)^n\cos\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^n\,\left(a+b\,\sin\left(c+d\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x))^2,x)
[Out] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+b*sin(d*x+c))**2,x)
[Out] Timed out
```

3.1197 $\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{b\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] a*cos(d*x+c)*hypergeom([-3/2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+
c)^(1+n)/d/(1+n)/(cos(d*x+c)^2)^(1/2)+b*cos(d*x+c)*hypergeom([-3/2, 1+1/2*n
],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2838, 2577}

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{b\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^
2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (b*Cos[c + d*x]
*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^n(c+dx)\,dx + b\int \cos^4(c+dx)\sin^{1+n}$$
$$= \frac{a\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx}{d(1+n)\sqrt{\cos^2(c+dx)}}$$

Mathematica [A] time = 0.16, size = 111, normalized size = 0.86

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{n+1}(c+dx)\left(a(n+2)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+b(n+1)\sin(c+dx)\,{}_2F_1\left(\right.\right.}{d(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*(a*(2 + n)*Hypergeo
metric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2] + b*(1 + n)*Hypergeom
etric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]))/(d*(1 +
 n)*(2 + n))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^4\sin\left(dx+c\right)+a\cos\left(dx+c\right)^4\right)\sin\left(dx+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4*sin(d*x + c) + a*cos(d*x + c)^4)*sin(d*x + c)^n,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)^n\cos\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

maple [F] time = 7.33, size = 0, normalized size = 0.00

$$\int \left(\cos^4\left(dx+c\right)\right)\left(\sin^n\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)\sin(dx+c)^n\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^4\sin(c+d\,x)^n\,\left(a+b\,\sin(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1198 $\int \cos^5(c+dx)\sin^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{a\sin^{10}(c+dx)}{10d}-\frac{a\sin^8(c+dx)}{4d}+\frac{a\sin^6(c+dx)}{6d}+\frac{b\sin^{11}(c+dx)}{11d}-\frac{2b\sin^9(c+dx)}{9d}+\frac{b\sin^7(c+dx)}{7d}$$

```
[Out] 1/6*a*sin(d*x+c)^6/d+1/7*b*sin(d*x+c)^7/d-1/4*a*sin(d*x+c)^8/d-2/9*b*sin(d*
x+c)^9/d+1/10*a*sin(d*x+c)^10/d+1/11*b*sin(d*x+c)^11/d
```

Rubi [A] time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 766}

$$\frac{a\sin^{10}(c+dx)}{10d}-\frac{a\sin^8(c+dx)}{4d}+\frac{a\sin^6(c+dx)}{6d}+\frac{b\sin^{11}(c+dx)}{11d}-\frac{2b\sin^9(c+dx)}{9d}+\frac{b\sin^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^6)/(6*d) + (b*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^8)/(4
*d) - (2*b*Sin[c + d*x]^9)/(9*d) + (a*Sin[c + d*x]^10)/(10*d) + (b*Sin[c +
d*x]^11)/(11*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^5(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+x)\left(b^2-x^2\right)^2}{b^5}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$

$$= \frac{\text{Subst}\left(\int x^5(a+x)\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^{10}d}$$

$$= \frac{\text{Subst}\left(\int \left(ab^4x^5+b^4x^6-2ab^2x^7-2b^2x^8+ax^9+x^{10}\right)dx, x,\right.}{b^{10}d}$$

$$= \frac{a\sin^6(c+dx)}{6d}+\frac{b\sin^7(c+dx)}{7d}-\frac{a\sin^8(c+dx)}{4d}-\frac{2b\sin^9(c}{9d}$$

Mathematica [A] time = 0.40, size = 105, normalized size = 1.08

$$\frac{-34650a\cos(2(c+dx))+5775a\cos(6(c+dx))-693a\cos(10(c+dx))+34650b\sin(c+dx)-11550b\sin(3(c+}{3548160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-34650*a*Cos[2*(c + d*x)] + 5775*a*Cos[6*(c + d*x)] - 693*a*Cos[10*(c + d*
x)] + 34650*b*Sin[c + d*x] - 11550*b*Sin[3*(c + d*x)] - 3465*b*Sin[5*(c + d
*x)] + 2475*b*Sin[7*(c + d*x)] + 385*b*Sin[9*(c + d*x)] - 315*b*Sin[11*(c +
 d*x)])/(3548160*d)
```

fricas [A] time = 0.91, size = 106, normalized size = 1.09

$$-\frac{1386\,a\cos\left(dx+c\right)^{10}-3465\,a\cos\left(dx+c\right)^{8}+2310\,a\cos\left(dx+c\right)^{6}+20\left(63\,b\cos\left(dx+c\right)^{10}-161\,b\cos\left(dx+c\right)^{8}\right.}{13860\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/13860*(1386*a*cos(d*x + c)^10 - 3465*a*cos(d*x + c)^8 + 2310*a*cos(d*x +
 c)^6 + 20*(63*b*cos(d*x + c)^10 - 161*b*cos(d*x + c)^8 + 113*b*cos(d*x + c
)^6 - 3*b*cos(d*x + c)^4 - 4*b*cos(d*x + c)^2 - 8*b)*sin(d*x + c))/d
```

giac [A] time = 0.32, size = 133, normalized size = 1.37

$$-\frac{a\cos\left(10\,dx+10\,c\right)}{5120\,d}+\frac{5\,a\cos\left(6\,dx+6\,c\right)}{3072\,d}-\frac{5\,a\cos\left(2\,dx+2\,c\right)}{512\,d}-\frac{b\sin\left(11\,dx+11\,c\right)}{11264\,d}+\frac{b\sin\left(9\,dx+9\,c\right)}{9216\,d}+\frac{5\,b\sin\left(7\right.}{716}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] -1/5120*a*cos(10*d*x + 10*c)/d + 5/3072*a*cos(6*d*x + 6*c)/d - 5/512*a*cos(2*d*x + 2*c)/d - 1/11264*b*sin(11*d*x + 11*c)/d + 1/9216*b*sin(9*d*x + 9*c)/d + 5/7168*b*sin(7*d*x + 7*c)/d - 1/1024*b*sin(5*d*x + 5*c)/d - 5/1536*b*sin(3*d*x + 3*c)/d + 5/512*b*sin(d*x + c)/d

maple [A] time = 0.26, size = 138, normalized size = 1.42

$$\frac{a\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^6(dx+c)\right)}{10}-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{20}-\frac{\left(\cos^6(dx+c)\right)}{60}\right)+b\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^6(dx+c)\right)}{11}-\frac{5\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{99}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x)

[Out] 1/d*(a*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/60*cos(d*x+c)^6)+b*(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^3*cos(d*x+c)^6-5/231*sin(d*x+c)*cos(d*x+c)^6+1/231*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))

maxima [A] time = 0.32, size = 72, normalized size = 0.74

$$\frac{1260\,b\sin(dx+c)^{11}+1386\,a\sin(dx+c)^{10}-3080\,b\sin(dx+c)^{9}-3465\,a\sin(dx+c)^{8}+1980\,b\sin(dx+c)^{7}}{13860\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/13860*(1260*b*sin(d*x + c)^11 + 1386*a*sin(d*x + c)^10 - 3080*b*sin(d*x + c)^9 - 3465*a*sin(d*x + c)^8 + 1980*b*sin(d*x + c)^7 + 2310*a*sin(d*x + c)^6)/d

mupad [B] time = 0.08, size = 71, normalized size = 0.73

$$\frac{\frac{b\sin(c+d\,x)^{11}}{11}+\frac{a\sin(c+d\,x)^{10}}{10}-\frac{2\,b\sin(c+d\,x)^{9}}{9}-\frac{a\sin(c+d\,x)^{8}}{4}+\frac{b\sin(c+d\,x)^{7}}{7}+\frac{a\sin(c+d\,x)^{6}}{6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^5*sin(c + d*x)^5*(a + b*sin(c + d*x)),x)

```
[Out] ((a*sin(c + d*x)^6)/6 - (a*sin(c + d*x)^8)/4 + (a*sin(c + d*x)^10)/10 + (b*
sin(c + d*x)^7)/7 - (2*b*sin(c + d*x)^9)/9 + (b*sin(c + d*x)^11)/11)/d
```

sympy [A] time = 32.62, size = 136, normalized size = 1.40

$$\begin{cases} \frac{a\sin^{10}(c+dx)}{60d} + \frac{a\sin^{8}(c+dx)\cos^{2}(c+dx)}{12d} + \frac{a\sin^{6}(c+dx)\cos^{4}(c+dx)}{6d} + \frac{8b\sin^{11}(c+dx)}{693d} + \frac{4b\sin^{9}(c+dx)\cos^{2}(c+dx)}{63d} + \frac{b\sin^{7}(c+dx)\cos^{4}(c+ \\ x\left(a+b\sin\left(c\right)\right)\sin^{5}\left(c\right)\cos^{5}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)**10/(60*d) + a*sin(c + d*x)**8*cos(c + d*x)**2/(1
2*d) + a*sin(c + d*x)**6*cos(c + d*x)**4/(6*d) + 8*b*sin(c + d*x)**11/(693*
d) + 4*b*sin(c + d*x)**9*cos(c + d*x)**2/(63*d) + b*sin(c + d*x)**7*cos(c +
 d*x)**4/(7*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)**5*cos(c)**5, True))
```

3.1199 $\int \cos^5(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{a\sin^9(c+dx)}{9d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{b\sin^{10}(c+dx)}{10d}-\frac{b\sin^8(c+dx)}{4d}+\frac{b\sin^6(c+dx)}{6d}$$

```
[Out] 1/5*a*sin(d*x+c)^5/d+1/6*b*sin(d*x+c)^6/d-2/7*a*sin(d*x+c)^7/d-1/4*b*sin(d*
x+c)^8/d+1/9*a*sin(d*x+c)^9/d+1/10*b*sin(d*x+c)^10/d
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 766}

$$\frac{a\sin^9(c+dx)}{9d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{b\sin^{10}(c+dx)}{10d}-\frac{b\sin^8(c+dx)}{4d}+\frac{b\sin^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^5)/(5*d) + (b*Sin[c + d*x]^6)/(6*d) - (2*a*Sin[c + d*x]^7)/
(7*d) - (b*Sin[c + d*x]^8)/(4*d) + (a*Sin[c + d*x]^9)/(9*d) + (b*Sin[c + d*
x]^10)/(10*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+x)\left(b^2-x^2\right)^2}{b^4}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int x^4(a+x)\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^9 d}$$
$$= \frac{\text{Subst}\left(\int \left(ab^4x^4+b^4x^5-2ab^2x^6-2b^2x^7+ax^8+x^9\right)dx, x,\right.}{b^9 d}$$
$$= \frac{a\sin^5(c+dx)}{5d}+\frac{b\sin^6(c+dx)}{6d}-\frac{2a\sin^7(c+dx)}{7d}-\frac{b\sin^8(c}{4d}$$

Mathematica [A] time = 0.31, size = 94, normalized size = 0.97

$$\frac{7560a\sin(c+dx)-1680a\sin(3(c+dx))-1008a\sin(5(c+dx))+180a\sin(7(c+dx))+140a\sin(9(c+dx))-31}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3150*b*Cos[2*(c + d*x)] + 525*b*Cos[6*(c + d*x)] - 63*b*Cos[10*(c + d*x)]
 + 7560*a*Sin[c + d*x] - 1680*a*Sin[3*(c + d*x)] - 1008*a*Sin[5*(c + d*x)]
+ 180*a*Sin[7*(c + d*x)] + 140*a*Sin[9*(c + d*x)])/(322560*d)
```

fricas [A] time = 0.66, size = 95, normalized size = 0.98

$$-\frac{126\,b\cos(dx+c)^{10}-315\,b\cos(dx+c)^{8}+210\,b\cos(dx+c)^{6}-4\left(35\,a\cos(dx+c)^{8}-50\,a\cos(dx+c)^{6}+3\,a\,c\right.}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/1260*(126*b*cos(d*x + c)^10 - 315*b*cos(d*x + c)^8 + 210*b*cos(d*x + c)^
6 - 4*(35*a*cos(d*x + c)^8 - 50*a*cos(d*x + c)^6 + 3*a*cos(d*x + c)^4 + 4*a
*cos(d*x + c)^2 + 8*a)*sin(d*x + c))/d
```

giac [A] time = 0.32, size = 118, normalized size = 1.22

$$-\frac{b\cos(10\,dx+10\,c)}{5120\,d}+\frac{5\,b\cos(6\,dx+6\,c)}{3072\,d}-\frac{5\,b\cos(2\,dx+2\,c)}{512\,d}+\frac{a\sin(9\,dx+9\,c)}{2304\,d}+\frac{a\sin(7\,dx+7\,c)}{1792\,d}-\frac{a\sin(5\,dx+}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] -1/5120*b*cos(10*d*x + 10*c)/d + 5/3072*b*cos(6*d*x + 6*c)/d - 5/512*b*cos(2*d*x + 2*c)/d + 1/2304*a*sin(9*d*x + 9*c)/d + 1/1792*a*sin(7*d*x + 7*c)/d - 1/320*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x + 3*c)/d + 3/128*a*sin(d*x + c)/d

maple [A] time = 0.27, size = 120, normalized size = 1.24

$$\frac{a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{105}\right)+b\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^6(dx+c)\right)}{10}-\frac{(\mathrm{si}}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x)

[Out] 1/d*(a*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+b*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/60*cos(d*x+c)^6))

maxima [A] time = 0.32, size = 72, normalized size = 0.74

$$\frac{126\,b\sin\left(dx+c\right)^{10}+140\,a\sin\left(dx+c\right)^{9}-315\,b\sin\left(dx+c\right)^{8}-360\,a\sin\left(dx+c\right)^{7}+210\,b\sin\left(dx+c\right)^{6}+252\,a}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/1260*(126*b*sin(d*x + c)^10 + 140*a*sin(d*x + c)^9 - 315*b*sin(d*x + c)^8 - 360*a*sin(d*x + c)^7 + 210*b*sin(d*x + c)^6 + 252*a*sin(d*x + c)^5)/d

mupad [B] time = 0.06, size = 71, normalized size = 0.73

$$\frac{\frac{b\sin(c+d\,x)^{10}}{10}+\frac{a\sin(c+d\,x)^{9}}{9}-\frac{b\sin(c+d\,x)^{8}}{4}-\frac{2\,a\sin(c+d\,x)^{7}}{7}+\frac{b\sin(c+d\,x)^{6}}{6}+\frac{a\sin(c+d\,x)^{5}}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^5*sin(c + d*x)^4*(a + b*sin(c + d*x)),x)

[Out] ((a*sin(c + d*x)^5)/5 - (2*a*sin(c + d*x)^7)/7 + (a*sin(c + d*x)^9)/9 + (b*sin(c + d*x)^6)/6 - (b*sin(c + d*x)^8)/4 + (b*sin(c + d*x)^10)/10)/d

sympy [A] time = 20.99, size = 136, normalized size = 1.40

$$\begin{cases} \frac{8a\sin^9(c+dx)}{315d} + \frac{4a\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{a\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{b\sin^{10}(c+dx)}{60d} + \frac{b\sin^8(c+dx)\cos^2(c+dx)}{12d} + \frac{b\sin^6(c+dx)\cos^4(c+d}{6d} \\ x(a+b\sin(c))\sin^4(c)\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**9/(315*d) + 4*a*sin(c + d*x)**7*cos(c + d*x)**
2/(35*d) + a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + b*sin(c + d*x)**10/(60
*d) + b*sin(c + d*x)**8*cos(c + d*x)**2/(12*d) + b*sin(c + d*x)**6*cos(c +
d*x)**4/(6*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)**4*cos(c)**5, True))
```

3.1200 $\int \cos^5(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}+\frac{b\sin^9(c+dx)}{9d}-\frac{2b\sin^7(c+dx)}{7d}+\frac{b\sin^5(c+dx)}{5d}$$

```
[Out] -1/6*a*cos(d*x+c)^6/d+1/8*a*cos(d*x+c)^8/d+1/5*b*sin(d*x+c)^5/d-2/7*b*sin(d
*x+c)^7/d+1/9*b*sin(d*x+c)^9/d
```

Rubi [A] time = 0.14, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2565, 14, 2564, 270}

$$\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}+\frac{b\sin^9(c+dx)}{9d}-\frac{2b\sin^7(c+dx)}{7d}+\frac{b\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (a*Cos[c + d*x]^8)/(8*d) + (b*Sin[c + d*x]^5)/(
5*d) - (2*b*Sin[c + d*x]^7)/(7*d) + (b*Sin[c + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^5(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin^3(c+dx)\,dx + b\int \cos^5(c+dx)\sin^4(c+$$

$$= -\frac{a\,\mathrm{Subst}\left(\int x^5\left(1-x^2\right)dx, x, \cos(c+dx)\right)}{d} + \frac{b\,\mathrm{Subst}\left(\int x^4\left(\right.\right.}{}$$

$$= -\frac{a\,\mathrm{Subst}\left(\int \left(x^5-x^7\right)dx, x, \cos(c+dx)\right)}{d} + \frac{b\,\mathrm{Subst}\left(\int \left(x^4-\right.\right.}{}$$

$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\cos^8(c+dx)}{8d} + \frac{b\sin^5(c+dx)}{5d} - \frac{2b\sin^7}{7}$$

Mathematica [A] time = 0.29, size = 105, normalized size = 1.30

$$\frac{-7560a\cos(2(c+dx)) - 1260a\cos(4(c+dx)) + 840a\cos(6(c+dx)) + 315a\cos(8(c+dx)) + 7560b\sin(c+dx) -}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-7560*a*Cos[2*(c + d*x)] - 1260*a*Cos[4*(c + d*x)] + 840*a*Cos[6*(c + d*x)
] + 315*a*Cos[8*(c + d*x)] + 7560*b*Sin[c + d*x] - 1680*b*Sin[3*(c + d*x)]
- 1008*b*Sin[5*(c + d*x)] + 180*b*Sin[7*(c + d*x)] + 140*b*Sin[9*(c + d*x)]
)/(322560*d)
```

fricas [A] time = 0.83, size = 84, normalized size = 1.04

$$\frac{315\,a\cos\left(dx+c\right)^8 - 420\,a\cos\left(dx+c\right)^6 + 8\left(35\,b\cos\left(dx+c\right)^8 - 50\,b\cos\left(dx+c\right)^6 + 3\,b\cos\left(dx+c\right)^4 + 4\,b\cos\left(\right.\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*cos(d*x + c)^8 - 420*a*cos(d*x + c)^6 + 8*(35*b*cos(d*x + c)^
8 - 50*b*cos(d*x + c)^6 + 3*b*cos(d*x + c)^4 + 4*b*cos(d*x + c)^2 + 8*b)*si
n(d*x + c))/d
```

giac [A] time = 0.25, size = 133, normalized size = 1.64

$$\frac{a\cos\left(8\,dx+8\,c\right)}{1024\,d}+\frac{a\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{a\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{3\,a\cos\left(2\,dx+2\,c\right)}{128\,d}+\frac{b\sin\left(9\,dx+9\,c\right)}{2304\,d}+\frac{b\sin\left(7\,dx+7\,c\right)}{1792\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1024*a*cos(8*d*x + 8*c)/d + 1/384*a*cos(6*d*x + 6*c)/d - 1/256*a*cos(4*d*
x + 4*c)/d - 3/128*a*cos(2*d*x + 2*c)/d + 1/2304*b*sin(9*d*x + 9*c)/d + 1/1
792*b*sin(7*d*x + 7*c)/d - 1/320*b*sin(5*d*x + 5*c)/d - 1/192*b*sin(3*d*x +
 3*c)/d + 3/128*b*sin(d*x + c)/d
```

maple [A] time = 0.26, size = 102, normalized size = 1.26

$$\frac{a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{6}(dx+c)\right)}{8}-\frac{\left(\cos^{6}(dx+c)\right)}{24}\right)+b\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{6}(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)}{105}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+b*(-1/9*sin(d*x+c
)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*c
os(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.31, size = 72, normalized size = 0.89

$$\frac{280\,b\sin\left(dx+c\right)^{9}+315\,a\sin\left(dx+c\right)^{8}-720\,b\sin\left(dx+c\right)^{7}-840\,a\sin\left(dx+c\right)^{6}+504\,b\sin\left(dx+c\right)^{5}+630\,a}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2520*(280*b*sin(d*x + c)^9 + 315*a*sin(d*x + c)^8 - 720*b*sin(d*x + c)^7
- 840*a*sin(d*x + c)^6 + 504*b*sin(d*x + c)^5 + 630*a*sin(d*x + c)^4)/d
```

mupad [B] time = 0.06, size = 71, normalized size = 0.88

$$\frac{\frac{b\sin(c+dx)^9}{9}+\frac{a\sin(c+dx)^8}{8}-\frac{2b\sin(c+dx)^7}{7}-\frac{a\sin(c+dx)^6}{3}+\frac{b\sin(c+dx)^5}{5}+\frac{a\sin(c+dx)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^3*(a + b*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^4)/4 - (a*sin(c + d*x)^6)/3 + (a*sin(c + d*x)^8)/8 + (b*si
n(c + d*x)^5)/5 - (2*b*sin(c + d*x)^7)/7 + (b*sin(c + d*x)^9)/9)/d
```

sympy [A] time = 13.68, size = 136, normalized size = 1.68

$$\begin{cases}\frac{a\sin^8(c+dx)}{24d}+\frac{a\sin^6(c+dx)\cos^2(c+dx)}{6d}+\frac{a\sin^4(c+dx)\cos^4(c+dx)}{4d}+\frac{8b\sin^9(c+dx)}{315d}+\frac{4b\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{b\sin^5(c+dx)\cos^4(c+dx)}{5d} \\ x(a+b\sin(c))\sin^3(c)\cos^5(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)**8/(24*d) + a*sin(c + d*x)**6*cos(c + d*x)**2/(6*
d) + a*sin(c + d*x)**4*cos(c + d*x)**4/(4*d) + 8*b*sin(c + d*x)**9/(315*d)
+ 4*b*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + b*sin(c + d*x)**5*cos(c + d*
x)**4/(5*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)**3*cos(c)**5, True))
```

3.1201 $\int \cos^5(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{b\cos^8(c+dx)}{8d}-\frac{b\cos^6(c+dx)}{6d}$$

```
[Out] -1/6*b*cos(d*x+c)^6/d+1/8*b*cos(d*x+c)^8/d+1/3*a*sin(d*x+c)^3/d-2/5*a*sin(d
*x+c)^5/d+1/7*a*sin(d*x+c)^7/d
```

Rubi [A] time = 0.14, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2564, 270, 2565, 14}

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{b\cos^8(c+dx)}{8d}-\frac{b\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^6)/(6*d) + (b*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^3)/(
3*d) - (2*a*Sin[c + d*x]^5)/(5*d) + (a*Sin[c + d*x]^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^5(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin^2(c+dx)\,dx + b\int \cos^5(c+dx)\sin^3(c+$$

$$= \frac{a\,\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx, x, \sin(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int x^5\left(1\right.\right.}{}$$

$$= \frac{a\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \sin(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int\left(\right.\right.}{}$$

$$= -\frac{b\cos^6(c+dx)}{6d} + \frac{b\cos^8(c+dx)}{8d} + \frac{a\sin^3(c+dx)}{3d} - \frac{2a\sin^5}{5}$$

Mathematica [A] time = 0.30, size = 94, normalized size = 1.16

$$-\frac{-8400a\sin(c+dx)+560a\sin(3(c+dx))+1008a\sin(5(c+dx))+240a\sin(7(c+dx))+2520b\cos(2(c+dx))}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/107520*(2520*b*Cos[2*(c + d*x)] + 420*b*Cos[4*(c + d*x)] - 280*b*Cos[6*(
c + d*x)] - 105*b*Cos[8*(c + d*x)] - 8400*a*Sin[c + d*x] + 560*a*Sin[3*(c +
 d*x)] + 1008*a*Sin[5*(c + d*x)] + 240*a*Sin[7*(c + d*x)])/d
```

fricas [A] time = 0.58, size = 73, normalized size = 0.90

$$\frac{105\,b\cos\left(dx+c\right)^8-140\,b\cos\left(dx+c\right)^6-8\left(15\,a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4-4\,a\cos\left(dx+c\right)^2-8\,a\right)\sin\left(d\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/840*(105*b*cos(d*x + c)^8 - 140*b*cos(d*x + c)^6 - 8*(15*a*cos(d*x + c)^6
 - 3*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 - 8*a)*sin(d*x + c))/d
```

giac [A] time = 0.24, size = 118, normalized size = 1.46

$$\frac{b\cos(8\,dx+8\,c)}{1024\,d}+\frac{b\cos(6\,dx+6\,c)}{384\,d}-\frac{b\cos(4\,dx+4\,c)}{256\,d}-\frac{3\,b\cos(2\,dx+2\,c)}{128\,d}-\frac{a\sin(7\,dx+7\,c)}{448\,d}-\frac{3\,a\sin(5\,dx+5}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1024*b*cos(8*d*x + 8*c)/d + 1/384*b*cos(6*d*x + 6*c)/d - 1/256*b*cos(4*d*
x + 4*c)/d - 3/128*b*cos(2*d*x + 2*c)/d - 1/448*a*sin(7*d*x + 7*c)/d - 3/32
0*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x + 3*c)/d + 5/64*a*sin(d*x + c)/d
```

maple [A] time = 0.27, size = 84, normalized size = 1.04

$$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+b\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c))+b*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6))
```

maxima [A] time = 0.32, size = 72, normalized size = 0.89

$$\frac{105\,b\sin(dx+c)^8+120\,a\sin(dx+c)^7-280\,b\sin(dx+c)^6-336\,a\sin(dx+c)^5+210\,b\sin(dx+c)^4+280\,a\,s}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/840*(105*b*sin(d*x + c)^8 + 120*a*sin(d*x + c)^7 - 280*b*sin(d*x + c)^6 -
 336*a*sin(d*x + c)^5 + 210*b*sin(d*x + c)^4 + 280*a*sin(d*x + c)^3)/d
```

mupad [B] time = 0.06, size = 71, normalized size = 0.88

$$\frac{\frac{b\sin(c+d\,x)^8}{8}+\frac{a\sin(c+d\,x)^7}{7}-\frac{b\sin(c+d\,x)^6}{3}-\frac{2\,a\sin(c+d\,x)^5}{5}+\frac{b\sin(c+d\,x)^4}{4}+\frac{a\sin(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^2*(a + b*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^3)/3 - (2*a*sin(c + d*x)^5)/5 + (a*sin(c + d*x)^7)/7 + (b*
sin(c + d*x)^4)/4 - (b*sin(c + d*x)^6)/3 + (b*sin(c + d*x)^8)/8)/d
```

sympy [A] time = 8.39, size = 136, normalized size = 1.68

$$\begin{cases} \frac{8a\sin^7(c+dx)}{105d} + \frac{4a\sin^5(c+dx)\cos^2(c+dx)}{15d} + \frac{a\sin^3(c+dx)\cos^4(c+dx)}{3d} + \frac{b\sin^8(c+dx)}{24d} + \frac{b\sin^6(c+dx)\cos^2(c+dx)}{6d} + \frac{b\sin^4(c+dx)\cos^4(c+dx)}{4d} \\ x(a+b\sin(c))\sin^2(c)\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**7/(105*d) + 4*a*sin(c + d*x)**5*cos(c + d*x)**
2/(15*d) + a*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) + b*sin(c + d*x)**8/(24*
d) + b*sin(c + d*x)**6*cos(c + d*x)**2/(6*d) + b*sin(c + d*x)**4*cos(c + d*
x)**4/(4*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)**2*cos(c)**5, True))
```

3.1202 $\int \cos^5(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\cos^6(c+dx)}{6d}+\frac{b\sin^7(c+dx)}{7d}-\frac{2b\sin^5(c+dx)}{5d}+\frac{b\sin^3(c+dx)}{3d}$$

```
[Out] -1/6*a*cos(d*x+c)^6/d+1/3*b*sin(d*x+c)^3/d-2/5*b*sin(d*x+c)^5/d+1/7*b*sin(d
*x+c)^7/d
```

Rubi [A] time = 0.10, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2834, 2565, 30, 2564, 270}

$$-\frac{a\cos^6(c+dx)}{6d}+\frac{b\sin^7(c+dx)}{7d}-\frac{2b\sin^5(c+dx)}{5d}+\frac{b\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (b*Sin[c + d*x]^3)/(3*d) - (2*b*Sin[c + d*x]^5)
/(5*d) + (b*Sin[c + d*x]^7)/(7*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cos^5(c+dx)\sin(c+dx)\,dx + b\int \cos^5(c+dx)\sin^2(c+d\\ &= -\frac{a\,\mathrm{Subst}\left(\int x^5\,dx, x, \cos(c+dx)\right)}{d} + \frac{b\,\mathrm{Subst}\left(\int x^2\left(1-x^2\right)^2\,d\right.}{d}\\ &= -\frac{a\cos^6(c+dx)}{6d} + \frac{b\,\mathrm{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \sin(c+d\right.}{d}\\ &= -\frac{a\cos^6(c+dx)}{6d} + \frac{b\sin^3(c+dx)}{3d} - \frac{2b\sin^5(c+dx)}{5d} + \frac{b\sin^7(c}{7d}\end{aligned}$$

Mathematica [A] time = 0.22, size = 86, normalized size = 1.32

$$-\frac{525a\cos(2(c+dx))+210a\cos(4(c+dx))+35a\cos(6(c+dx))+350a-525b\sin(c+dx)+35b\sin(3(c+dx))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6720*(350*a + 525*a*Cos[2*(c + d*x)] + 210*a*Cos[4*(c + d*x)] + 35*a*Cos
[6*(c + d*x)] - 525*b*Sin[c + d*x] + 35*b*Sin[3*(c + d*x)] + 63*b*Sin[5*(c
+ d*x)] + 15*b*Sin[7*(c + d*x)])/d
```

fricas [A] time = 0.58, size = 62, normalized size = 0.95

$$-\frac{35\,a\cos\left(dx+c\right)^{6}+2\left(15\,b\cos\left(dx+c\right)^{6}-3\,b\cos\left(dx+c\right)^{4}-4\,b\cos\left(dx+c\right)^{2}-8\,b\right)\sin\left(dx+c\right)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/210*(35*a*cos(d*x + c)^6 + 2*(15*b*cos(d*x + c)^6 - 3*b*cos(d*x + c)^4 - 4*b*cos(d*x + c)^2 - 8*b)*sin(d*x + c))/d

giac [A] time = 0.21, size = 103, normalized size = 1.58

$$-\frac{a\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\,a\cos\left(2\,dx+2\,c\right)}{64\,d}-\frac{b\sin\left(7\,dx+7\,c\right)}{448\,d}-\frac{3\,b\sin\left(5\,dx+5\,c\right)}{320\,d}-\frac{b\sin\left(3\,dx+\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] -1/192*a*cos(6*d*x + 6*c)/d - 1/32*a*cos(4*d*x + 4*c)/d - 5/64*a*cos(2*d*x + 2*c)/d - 1/448*b*sin(7*d*x + 7*c)/d - 3/320*b*sin(5*d*x + 5*c)/d - 1/192*b*sin(3*d*x + 3*c)/d + 5/64*b*sin(d*x + c)/d

maple [A] time = 0.27, size = 64, normalized size = 0.98

$$\frac{-\frac{a\left(\cos^{6}(dx+c)\right)}{6}+b\left(-\frac{\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x)

[Out] 1/d*(-1/6*a*cos(d*x+c)^6+b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))

maxima [A] time = 0.31, size = 72, normalized size = 1.11

$$\frac{30\,b\sin\left(dx+c\right)^{7}+35\,a\sin\left(dx+c\right)^{6}-84\,b\sin\left(dx+c\right)^{5}-105\,a\sin\left(dx+c\right)^{4}+70\,b\sin\left(dx+c\right)^{3}+105\,a\sin\left(d\right.}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/210*(30*b*sin(d*x + c)^7 + 35*a*sin(d*x + c)^6 - 84*b*sin(d*x + c)^5 - 105*a*sin(d*x + c)^4 + 70*b*sin(d*x + c)^3 + 105*a*sin(d*x + c)^2)/d

mupad [B] time = 11.59, size = 71, normalized size = 1.09

$$\frac{\frac{b\sin(c+d\,x)^7}{7}+\frac{a\sin(c+d\,x)^6}{6}-\frac{2\,b\sin(c+d\,x)^5}{5}-\frac{a\sin(c+d\,x)^4}{2}+\frac{b\sin(c+d\,x)^3}{3}+\frac{a\sin(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)*(a + b*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^2)/2 - (a*sin(c + d*x)^4)/2 + (a*sin(c + d*x)^6)/6 + (b*si
n(c + d*x)^3)/3 - (2*b*sin(c + d*x)^5)/5 + (b*sin(c + d*x)^7)/7)/d
```

sympy [**A**] time = 5.07, size = 90, normalized size = 1.38

$$\begin{cases} -\frac{a\cos^6(c+dx)}{6d} + \frac{8b\sin^7(c+dx)}{105d} + \frac{4b\sin^5(c+dx)\cos^2(c+dx)}{15d} + \frac{b\sin^3(c+dx)\cos^4(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a+b\sin(c))\sin(c)\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)**6/(6*d) + 8*b*sin(c + d*x)**7/(105*d) + 4*b*sin
(c + d*x)**5*cos(c + d*x)**2/(15*d) + b*sin(c + d*x)**3*cos(c + d*x)**4/(3*
d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)*cos(c)**5, True))
```

3.1203 $\int \cos^4(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin^5(c+dx)}{5d}-\frac{2b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

```
[Out] a*ln(sin(d*x+c))/d+b*sin(d*x+c)/d-a*sin(d*x+c)^2/d-2/3*b*sin(d*x+c)^3/d+1/4
*a*sin(d*x+c)^4/d+1/5*b*sin(d*x+c)^5/d
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2837, 12, 766}

$$\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin^5(c+dx)}{5d}-\frac{2b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/d - (2*b*
Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) + (b*Sin[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b(a+x)\left(b^2-x^2\right)^2}{x}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x}\,dx, x, b\sin(c+dx)\right)}{b^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(b^4+\frac{ab^4}{x}-2ab^2x-2b^2x^2+ax^3+x^4\right)dx, x, b\sin(c$$

$$= \frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{d}-\frac{2b\sin^3}{3}$$

Mathematica [A] time = 0.03, size = 86, normalized size = 1.00

$$\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin^5(c+dx)}{5d}-\frac{2b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/d - (2*b*
Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) + (b*Sin[c + d*x]^5)/(5*d)
```

fricas [A] time = 0.75, size = 74, normalized size = 0.86

$$\frac{15\,a\cos(dx+c)^4+30\,a\cos(dx+c)^2+60\,a\log\left(\frac{1}{2}\sin(dx+c)\right)+4\left(3\,b\cos(dx+c)^4+4\,b\cos(dx+c)^2+8\,b\right)s}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*a*cos(d*x + c)^4 + 30*a*cos(d*x + c)^2 + 60*a*log(1/2*sin(d*x + c)
) + 4*(3*b*cos(d*x + c)^4 + 4*b*cos(d*x + c)^2 + 8*b)*sin(d*x + c))/d
```

giac [A] time = 0.21, size = 70, normalized size = 0.81

$$\frac{12\,b\sin(dx+c)^5+15\,a\sin(dx+c)^4-40\,b\sin(dx+c)^3-60\,a\sin(dx+c)^2+60\,a\log(|\sin(dx+c)|)+60\,b\sin}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(12*b*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 40*b*sin(d*x + c)^3 - 60*
a*sin(d*x + c)^2 + 60*a*log(abs(sin(d*x + c))) + 60*b*sin(d*x + c))/d
```

maple [A] time = 0.38, size = 94, normalized size = 1.09

$$\frac{a\left(\cos^4(dx+c)\right)}{4d}+\frac{a\left(\cos^2(dx+c)\right)}{2d}+\frac{a\ln(\sin(dx+c))}{d}+\frac{8b\sin(dx+c)}{15d}+\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)\,b}{5d}+\frac{4b\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4*a*cos(d*x+c)^4/d+1/2*a*cos(d*x+c)^2/d+a*ln(sin(d*x+c))/d+8/15*b*sin(d*x
+c)/d+1/5/d*cos(d*x+c)^4*sin(d*x+c)*b+4/15/d*b*sin(d*x+c)*cos(d*x+c)^2
```

maxima [A] time = 0.32, size = 69, normalized size = 0.80

$$\frac{12\,b\sin(dx+c)^5+15\,a\sin(dx+c)^4-40\,b\sin(dx+c)^3-60\,a\sin(dx+c)^2+60\,a\log(\sin(dx+c))+60\,b\sin}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(12*b*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 40*b*sin(d*x + c)^3 - 60*
a*sin(d*x + c)^2 + 60*a*log(sin(d*x + c)) + 60*b*sin(d*x + c))/d
```

mupad [B] time = 12.00, size = 126, normalized size = 1.47

$$\frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}+\frac{a\cos(c+d\,x)^2}{2\,d}+\frac{a\cos(c+d\,x)^4}{4\,d}+\frac{8\,b\,\sin(c+d\,x)}{15\,d}+\frac{4\,b\cos(c+d\,x)^2\,\sin(}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*log(1/cos(c/2 + (d*x)
/2)^2))/d + (a*cos(c + d*x)^2)/(2*d) + (a*cos(c + d*x)^4)/(4*d) + (8*b*sin(
c + d*x))/(15*d) + (4*b*cos(c + d*x)^2*sin(c + d*x))/(15*d) + (b*cos(c + d*
x)^4*sin(c + d*x))/(5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))\cos^5(c+dx)\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**5*csc(c + d*x), x)
```

3.1204 $\int \cos^3(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=83

$$\frac{a\sin^3(c+dx)}{3d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{b\sin^4(c+dx)}{4d}-\frac{b\sin^2(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}$$

```
[Out] -a*csc(d*x+c)/d+b*ln(sin(d*x+c))/d-2*a*sin(d*x+c)/d-b*sin(d*x+c)^2/d+1/3*a*
sin(d*x+c)^3/d+1/4*b*sin(d*x+c)^4/d
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 766}

$$\frac{a\sin^3(c+dx)}{3d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{b\sin^4(c+dx)}{4d}-\frac{b\sin^2(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (b*Log[Sin[c + d*x]])/d - (2*a*Sin[c + d*x])/d - (b
*Sin[c + d*x]^2)/d + (a*Sin[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^2(a+x)\left(b^2-x^2\right)^2}{x^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^2}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2ab^2+\frac{ab^4}{x^2}+\frac{b^4}{x}-2b^2x+ax^2+x^3\right)dx, x, b\sin(c\right.}{b^3 d}$$
$$= -\frac{a\csc(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}-\frac{2a\sin(c+dx)}{d}-\frac{b\sin}{}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 1.00

$$\frac{a\sin^3(c+dx)}{3d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{b\sin^4(c+dx)}{4d}-\frac{b\sin^2(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (b*Log[Sin[c + d*x]])/d - (2*a*Sin[c + d*x])/d - (b
*Sin[c + d*x]^2)/d + (a*Sin[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^4)/(4*d)
```

fricas [A] time = 0.81, size = 91, normalized size = 1.10

$$\frac{32\,a\cos\left(dx+c\right)^4+128\,a\cos\left(dx+c\right)^2+96\,b\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+3\left(8\,b\cos\left(dx+c\right)^4+16\,b\cos\left(d\right.\right.}{96\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(32*a*cos(d*x + c)^4 + 128*a*cos(d*x + c)^2 + 96*b*log(1/2*sin(d*x + c
))*sin(d*x + c) + 3*(8*b*cos(d*x + c)^4 + 16*b*cos(d*x + c)^2 - 11*b)*sin(d
*x + c) - 256*a)/(d*sin(d*x + c))
```

giac [A] time = 0.19, size = 79, normalized size = 0.95

$$\frac{3\,b\sin\left(dx+c\right)^4+4\,a\sin\left(dx+c\right)^3-12\,b\sin\left(dx+c\right)^2+12\,b\log\left(\left|\sin\left(dx+c\right)\right|\right)-24\,a\sin\left(dx+c\right)-\frac{12\,(b\sin(dx+c)}{\sin(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*b*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 12*b*sin(d*x + c)^2 + 12*b*
log(abs(sin(d*x + c))) - 24*a*sin(d*x + c) - 12*(b*sin(d*x + c) + a)/sin(d*
x + c))/d
```

maple [A] time = 0.34, size = 116, normalized size = 1.40

$$-\frac{a\left(\cos^6(dx+c)\right)}{d\sin(dx+c)}-\frac{8a\sin(dx+c)}{3d}-\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)\,a}{d}-\frac{4a\sin(dx+c)\left(\cos^2(dx+c)\right)}{3d}+\frac{b\left(\cos^4(dx+\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^6-8/3*a*sin(d*x+c)/d-1/d*cos(d*x+c)^4*sin(d*x+
c)*a-4/3/d*a*sin(d*x+c)*cos(d*x+c)^2+1/4*b*cos(d*x+c)^4/d+1/2*b*cos(d*x+c)^
2/d+b*ln(sin(d*x+c))/d
```

maxima [A] time = 0.31, size = 69, normalized size = 0.83

$$\frac{3\,b\sin(dx+c)^4+4\,a\sin(dx+c)^3-12\,b\sin(dx+c)^2+12\,b\log(\sin(dx+c))-24\,a\sin(dx+c)-\frac{12\,a}{\sin(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 12*b*sin(d*x + c)^2 + 12*b*
log(sin(d*x + c)) - 24*a*sin(d*x + c) - 12*a/sin(d*x + c))/d
```

mupad [B] time = 11.93, size = 250, normalized size = 3.01

$$\frac{b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{b\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}-\frac{4\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d}+\frac{8\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}-\frac{8\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{d}+\frac{4\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] (b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (b*log(1/cos(c/2 + (d*x)
/2)^2))/d - (4*b*cos(c/2 + (d*x)/2)^2)/d + (8*b*cos(c/2 + (d*x)/2)^4)/d - (
```

```
8*b*cos(c/2 + (d*x)/2)^6)/d + (4*b*cos(c/2 + (d*x)/2)^8)/d - (9*a*cos(c/2 +
 (d*x)/2))/(2*d*sin(c/2 + (d*x)/2)) - (a*sin(c/2 + (d*x)/2))/(2*d*cos(c/2 +
 (d*x)/2)) + (20*a*cos(c/2 + (d*x)/2)^3)/(3*d*sin(c/2 + (d*x)/2)) - (16*a*c
os(c/2 + (d*x)/2)^5)/(3*d*sin(c/2 + (d*x)/2)) + (8*a*cos(c/2 + (d*x)/2)^7)/
(3*d*sin(c/2 + (d*x)/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1205 $\int \cos^2(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$\frac{a\sin^2(c+dx)}{2d}-\frac{a\csc^2(c+dx)}{2d}-\frac{2a\log(\sin(c+dx))}{d}+\frac{b\sin^3(c+dx)}{3d}-\frac{2b\sin(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

```
[Out] -b*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-2*a*ln(sin(d*x+c))/d-2*b*sin(d*x+c)/d+
1/2*a*sin(d*x+c)^2/d+1/3*b*sin(d*x+c)^3/d
```

Rubi [A] time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 766}

$$\frac{a\sin^2(c+dx)}{2d}-\frac{a\csc^2(c+dx)}{2d}-\frac{2a\log(\sin(c+dx))}{d}+\frac{b\sin^3(c+dx)}{3d}-\frac{2b\sin(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (2*a*Log[Sin[c + d*x]])/
d - (2*b*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d) + (b*Sin[c + d*x]^3)/(3
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^3(a+x)\left(b^2-x^2\right)^2}{x^3}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^3}\,dx, x, b\sin(c+dx)\right)}{b^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2b^2+\frac{ab^4}{x^3}+\frac{b^4}{x^2}-\frac{2ab^2}{x}+ax+x^2\right)dx, x, b\sin(c+\right.}{b^2 d}$$
$$= -\frac{b\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{2a\log(\sin(c+dx))}{d}-\frac{2b\,\mathrm{s}}{}$$

Mathematica [A] time = 0.23, size = 77, normalized size = 0.90

$$-\frac{a\left(-\sin^2(c+dx)+\csc^2(c+dx)+4\log(\sin(c+dx))\right)}{2d}+\frac{b\sin^3(c+dx)}{3d}-\frac{2b\sin(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) - (2*b*Sin[c + d*x])/d + (b*Sin[c + d*x]^3)/(3*d) - (
a*(Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - Sin[c + d*x]^2))/(2*d)
```

fricas [A] time = 1.26, size = 102, normalized size = 1.19

$$-\frac{6\,a\cos\left(dx+c\right)^4-9\,a\cos\left(dx+c\right)^2+24\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+4\left(b\cos\left(dx+c\right)^4+4\,b\cos\right.}{12\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*cos(d*x + c)^4 - 9*a*cos(d*x + c)^2 + 24*(a*cos(d*x + c)^2 - a)*
log(1/2*sin(d*x + c)) + 4*(b*cos(d*x + c)^4 + 4*b*cos(d*x + c)^2 - 8*b)*sin
(d*x + c) - 3*a)/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.23, size = 82, normalized size = 0.95

$$\frac{2\,b\sin\left(dx+c\right)^3+3\,a\sin\left(dx+c\right)^2-12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-12\,b\sin\left(dx+c\right)+\frac{3\left(6\,a\sin\left(dx+c\right)^2-2\,b\sin\left(dx+c\right)-a\right)}{\sin\left(dx+c\right)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 12*a*log(abs(sin(d*x + c)))
- 12*b*sin(d*x + c) + 3*(6*a*sin(d*x + c)^2 - 2*b*sin(d*x + c) - a)/sin(d*x
 + c)^2)/d
```

maple [A] time = 0.38, size = 139, normalized size = 1.62

$$-\frac{a\left(\cos^{6}\left(dx+c\right)\right)}{2d\sin\left(dx+c\right)^{2}}-\frac{a\left(\cos^{4}\left(dx+c\right)\right)}{2d}-\frac{a\left(\cos^{2}\left(dx+c\right)\right)}{d}-\frac{2a\ln\left(\sin\left(dx+c\right)\right)}{d}-\frac{b\left(\cos^{6}\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}-\frac{8b\sin\left(dx+c\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^6-1/2*a*cos(d*x+c)^4/d-a*cos(d*x+c)^2/d-2*
a*ln(sin(d*x+c))/d-1/d*b/sin(d*x+c)*cos(d*x+c)^6-8/3*b*sin(d*x+c)/d-1/d*cos
(d*x+c)^4*sin(d*x+c)*b-4/3/d*b*sin(d*x+c)*cos(d*x+c)^2
```

maxima [A] time = 0.32, size = 68, normalized size = 0.79

$$\frac{2\,b\sin\left(dx+c\right)^{3}+3\,a\sin\left(dx+c\right)^{2}-12\,a\log\left(\sin\left(dx+c\right)\right)-12\,b\sin\left(dx+c\right)-\frac{3\,(2\,b\sin(dx+c)+a)}{\sin(dx+c)^{2}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 12*a*log(sin(d*x + c)) - 12*
b*sin(d*x + c) - 3*(2*b*sin(d*x + c) + a)/sin(d*x + c)^2)/d
```

mupad [B] time = 11.89, size = 229, normalized size = 2.66

$$\frac{2\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}{d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{18\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}-\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{2}+\frac{82\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{3}-\frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{2}}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (2*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (b*tan(c/2 + (d*x)/2))/(2*d) - (a/2
 + 2*b*tan(c/2 + (d*x)/2) + (3*a*tan(c/2 + (d*x)/2)^2)/2 - (13*a*tan(c/2 +
```

```
(d*x)/2)^4)/2 - (15*a*tan(c/2 + (d*x)/2)^6)/2 + 22*b*tan(c/2 + (d*x)/2)^3 +
 (82*b*tan(c/2 + (d*x)/2)^5)/3 + 18*b*tan(c/2 + (d*x)/2)^7)/(d*(4*tan(c/2 +
 (d*x)/2)^2 + 12*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 + 4*tan(c/2
 + (d*x)/2)^8)) - (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (2*a*log(tan(c/2 + (d*x)
/2)))/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1206 $\int \cos(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=85

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc(c+dx)}{d}+\frac{b\sin^2(c+dx)}{2d}-\frac{b\csc^2(c+dx)}{2d}-\frac{2b\log(\sin(c+dx))}{d}$$

```
[Out] 2*a*csc(d*x+c)/d-1/2*b*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-2*b*ln(sin(d*x+c
))/d+a*sin(d*x+c)/d+1/2*b*sin(d*x+c)^2/d
```

Rubi [A] time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2837, 12, 766}

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc(c+dx)}{d}+\frac{b\sin^2(c+dx)}{2d}-\frac{b\csc^2(c+dx)}{2d}-\frac{2b\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (b*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
- (2*b*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d + (b*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^4(a+x)\left(b^2-x^2\right)^2}{x^4}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^4}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(a+\frac{ab^4}{x^4}+\frac{b^4}{x^3}-\frac{2ab^2}{x^2}-\frac{2b^2}{x}+x\right)dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{2a\csc(c+dx)}{d}-\frac{b\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{2b\log(\sin}{d}$$

Mathematica [A] time = 0.15, size = 76, normalized size = 0.89

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc(c+dx)}{d}-\frac{b\left(-\sin^2(c+dx)+\csc^2(c+dx)+4\log(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (a*Sin[c + d*x])/d - (b*(
Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - Sin[c + d*x]^2))/(2*d)
```

fricas [A] time = 0.67, size = 117, normalized size = 1.38

$$-\frac{12\,a\cos\left(dx+c\right)^4-48\,a\cos\left(dx+c\right)^2+24\left(b\cos\left(dx+c\right)^2-b\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+3\left(2\,b\cos\left(dx\right.\right.}{12\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^4 - 48*a*cos(d*x + c)^2 + 24*(b*cos(d*x + c)^2 - b
)*log(1/2*sin(d*x + c))*sin(d*x + c) + 3*(2*b*cos(d*x + c)^4 - 3*b*cos(d*x
+ c)^2 - b)*sin(d*x + c) + 32*a)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.21, size = 81, normalized size = 0.95

$$\frac{3\,b\sin\left(dx+c\right)^2-12\,b\log\left(\left|\sin\left(dx+c\right)\right|\right)+6\,a\sin\left(dx+c\right)+\frac{22\,b\sin(dx+c)^3+12\,a\sin(dx+c)^2-3\,b\sin(dx+c)-2\,a}{\sin(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*b*sin(d*x + c)^2 - 12*b*log(abs(sin(d*x + c))) + 6*a*sin(d*x + c) +
(22*b*sin(d*x + c)^3 + 12*a*sin(d*x + c)^2 - 3*b*sin(d*x + c) - 2*a)/sin(d*
x + c)^3)/d
```

maple [A] time = 0.38, size = 159, normalized size = 1.87

$$-\frac{a\left(\cos^{6}\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)^{3}}+\frac{a\left(\cos^{6}\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}+\frac{8a\sin\left(dx+c\right)}{3d}+\frac{\left(\cos^{4}\left(dx+c\right)\right)\sin\left(dx+c\right)a}{d}+\frac{4a\sin\left(dx+c\right)\left(\cos^{2}\left(dx+\right.\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^6+1/d*a/sin(d*x+c)*cos(d*x+c)^6+8/3*a*sin(
d*x+c)/d+1/d*cos(d*x+c)^4*sin(d*x+c)*a+4/3/d*a*sin(d*x+c)*cos(d*x+c)^2-1/2/
d*b/sin(d*x+c)^2*cos(d*x+c)^6-1/2*b*cos(d*x+c)^4/d-b*cos(d*x+c)^2/d-2*b*ln(
sin(d*x+c))/d
```

maxima [A] time = 0.33, size = 69, normalized size = 0.81

$$\frac{3\,b\sin\left(dx+c\right)^{2}-12\,b\log\left(\sin\left(dx+c\right)\right)+6\,a\sin\left(dx+c\right)+\frac{12\,a\sin\left(dx+c\right)^{2}-3\,b\sin\left(dx+c\right)-2\,a}{\sin\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*b*sin(d*x + c)^2 - 12*b*log(sin(d*x + c)) + 6*a*sin(d*x + c) + (12*a
*sin(d*x + c)^2 - 3*b*sin(d*x + c) - 2*a)/sin(d*x + c)^3)/d
```

mupad [B] time = 11.91, size = 218, normalized size = 2.56

$$\frac{23\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+\frac{89\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{3}-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+\frac{19\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{3}-b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-}{d\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] ((19*a*tan(c/2 + (d*x)/2)^2)/3 - b*tan(c/2 + (d*x)/2) - a/3 + (89*a*tan(c/2
 + (d*x)/2)^4)/3 + 23*a*tan(c/2 + (d*x)/2)^6 - 2*b*tan(c/2 + (d*x)/2)^3 + 1
5*b*tan(c/2 + (d*x)/2)^5)/(d*(8*tan(c/2 + (d*x)/2)^3 + 16*tan(c/2 + (d*x)/2
)^5 + 8*tan(c/2 + (d*x)/2)^7)) + (7*a*tan(c/2 + (d*x)/2))/(8*d) + (2*b*log(
tan(c/2 + (d*x)/2)^2 + 1))/d - (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (b*tan(c/2
 + (d*x)/2)^2)/(8*d) - (2*b*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1207 $\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b\csc(c+dx)}{d}$$

```
[Out] 2*b*csc(d*x+c)/d+a*csc(d*x+c)^2/d-1/3*b*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/d
+a*ln(sin(d*x+c))/d+b*sin(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 766}

$$-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (b*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^5}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{ab^4}{x^5}+\frac{b^4}{x^4}-\frac{2ab^2}{x^3}-\frac{2b^2}{x^2}+\frac{a}{x}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{2b\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\log}{}$$

Mathematica [A] time = 0.26, size = 87, normalized size = 1.07

$$\frac{a\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\cos(c+dx))\right)}{4d}+\frac{b\sin(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d) + (a*(2*Cot[c + d*x]^2 - Co
t[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*d) + (b*Sin[c
 + d*x])/d
```

fricas [A] time = 0.57, size = 110, normalized size = 1.36

$$-\frac{12\,a\cos(dx+c)^2-12\left(a\cos(dx+c)^4-2\,a\cos(dx+c)^2+a\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-4\left(3\,b\cos(dx+c)^4-12\,b\right.}{12\left(d\cos(dx+c)^4-2\,d\cos(dx+c)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)
*log(1/2*sin(d*x + c)) - 4*(3*b*cos(d*x + c)^4 - 12*b*cos(d*x + c)^2 + 8*b)
*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.22, size = 82, normalized size = 1.01

$$\frac{12\,a\log\left(|\sin(dx+c)|\right)+12\,b\sin(dx+c)-\frac{25\,a\sin(dx+c)^4-24\,b\sin(dx+c)^3-12\,a\sin(dx+c)^2+4\,b\sin(dx+c)+3\,a}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(sin(d*x + c))) + 12*b*sin(d*x + c) - (25*a*sin(d*x + c)^
4 - 24*b*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*b*sin(d*x + c) + 3*a)/sin
(d*x + c)^4)/d
```

maple [A] time = 0.41, size = 136, normalized size = 1.68

$$-\frac{a\left(\cot^4(dx+c)\right)}{4d}+\frac{a\left(\cot^2(dx+c)\right)}{2d}+\frac{a\ln(\sin(dx+c))}{d}-\frac{b\left(\cos^6(dx+c)\right)}{3d\sin(dx+c)^3}+\frac{b\left(\cos^6(dx+c)\right)}{d\sin(dx+c)}+\frac{8b\sin(dx+c)}{3d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a*cot(d*x+c)^4+1/2*a*cot(d*x+c)^2/d+a*ln(sin(d*x+c))/d-1/3/d*b/sin(d
*x+c)^3*cos(d*x+c)^6+1/d*b/sin(d*x+c)*cos(d*x+c)^6+8/3*b*sin(d*x+c)/d+1/d*c
os(d*x+c)^4*sin(d*x+c)*b+4/3/d*b*sin(d*x+c)*cos(d*x+c)^2
```

maxima [A] time = 0.32, size = 69, normalized size = 0.85

$$\frac{12\,a\log(\sin(dx+c))+12\,b\sin(dx+c)+\frac{24\,b\sin(dx+c)^3+12\,a\sin(dx+c)^2-4\,b\sin(dx+c)-3\,a}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*a*log(sin(d*x + c)) + 12*b*sin(d*x + c) + (24*b*sin(d*x + c)^3 + 1
2*a*sin(d*x + c)^2 - 4*b*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d
```

mupad [B] time = 12.26, size = 207, normalized size = 2.56

$$\frac{7\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{46\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{40\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{a}{4}}{d\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}-\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (7*b*tan(c/2 + (d*x)/2))/(8*d) + ((11*a*tan(c/2 + (d*x)/2)^2)/4 - (2*b*tan(
c/2 + (d*x)/2))/3 - a/4 + 3*a*tan(c/2 + (d*x)/2)^4 + (40*b*tan(c/2 + (d*x)/
2)^3)/3 + 46*b*tan(c/2 + (d*x)/2)^5)/(d*(16*tan(c/2 + (d*x)/2)^4 + 16*tan(c
/2 + (d*x)/2)^6)) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (3*a*tan(c/2 + (d
```

```
*x)/2)^2)/(16*d) - (a*tan(c/2 + (d*x)/2)^4)/(64*d) - (b*tan(c/2 + (d*x)/2)^
3)/(24*d) + (a*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1208 $\int \cot^5(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}-\frac{b\csc^4(c+dx)}{4d}+\frac{b\csc^2(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}$$

```
[Out] -a*csc(d*x+c)/d+b*csc(d*x+c)^2/d+2/3*a*csc(d*x+c)^3/d-1/4*b*csc(d*x+c)^4/d-
1/5*a*csc(d*x+c)^5/d+b*ln(sin(d*x+c))/d
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2837, 12, 766}

$$-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}-\frac{b\csc^4(c+dx)}{4d}+\frac{b\csc^2(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (b*Csc[c + d*x]^2)/d + (2*a*Csc[c + d*x]^3)/(3*d) -
 (b*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) + (b*Log[Sin[c + d*x]]
)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^6(a+x)\left(b^2-x^2\right)^2}{x^6}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{b\,\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^6}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(\frac{ab^4}{x^6}+\frac{b^4}{x^5}-\frac{2ab^2}{x^4}-\frac{2b^2}{x^3}+\frac{a}{x^2}+\frac{1}{x}\right)dx, x, b\sin(c+dx\right.}{d}$$
$$= -\frac{a\csc(c+dx)}{d}+\frac{b\csc^2(c+dx)}{d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{b\csc^4(c\cdot}{4d}$$

Mathematica [A] time = 0.18, size = 92, normalized size = 1.07

$$-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{b\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (2*a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*
d) + (b*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Ta
n[c + d*x]]))/(4*d)
```

fricas [A] time = 0.80, size = 124, normalized size = 1.44

$$-\frac{60\,a\cos(dx+c)^4-80\,a\cos(dx+c)^2-60\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2+b\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx\cdot}{60\left(d\cos(dx+c)^4-2\,d\cos(dx+c)^2+d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(60*a*cos(d*x + c)^4 - 80*a*cos(d*x + c)^2 - 60*(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + b)*log(1/2*sin(d*x + c))*sin(d*x + c) + 15*(4*b*cos(d*x
 + c)^2 - 3*b)*sin(d*x + c) + 32*a)/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2
 + d)*sin(d*x + c))
```

giac [A] time = 0.22, size = 84, normalized size = 0.98

$$\frac{60\,b\log\left(|\sin(dx+c)|\right)-\frac{137\,b\sin(dx+c)^5+60\,a\sin(dx+c)^4-60\,b\sin(dx+c)^3-40\,a\sin(dx+c)^2+15\,b\sin(dx+c)+12\,a}{\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*b*log(abs(sin(d*x + c))) - (137*b*sin(d*x + c)^5 + 60*a*sin(d*x +
c)^4 - 60*b*sin(d*x + c)^3 - 40*a*sin(d*x + c)^2 + 15*b*sin(d*x + c) + 12*a
)/sin(d*x + c)^5)/d
```

maple [A] time = 0.40, size = 160, normalized size = 1.86

$$-\frac{a\left(\cos^{6}(dx+c)\right)}{5d\sin(dx+c)^{5}}+\frac{a\left(\cos^{6}(dx+c)\right)}{15d\sin(dx+c)^{3}}-\frac{a\left(\cos^{6}(dx+c)\right)}{5d\sin(dx+c)}-\frac{8a\sin(dx+c)}{15d}-\frac{\left(\cos^{4}(dx+c)\right)\sin(dx+c)\,a}{5d}-\frac{4a\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^6+1/15/d*a/sin(d*x+c)^3*cos(d*x+c)^6-1/5/d
*a/sin(d*x+c)*cos(d*x+c)^6-8/15*a*sin(d*x+c)/d-1/5/d*cos(d*x+c)^4*sin(d*x+c
)*a-4/15/d*a*sin(d*x+c)*cos(d*x+c)^2-1/4/d*b*cot(d*x+c)^4+1/2*b*cot(d*x+c)^
2/d+b*ln(sin(d*x+c))/d
```

maxima [A] time = 0.32, size = 72, normalized size = 0.84

$$\frac{60\,b\log\left(\sin\left(dx+c\right)\right)-\frac{60\,a\sin(dx+c)^{4}-60\,b\sin(dx+c)^{3}-40\,a\sin(dx+c)^{2}+15\,b\sin(dx+c)+12\,a}{\sin(dx+c)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*b*log(sin(d*x + c)) - (60*a*sin(d*x + c)^4 - 60*b*sin(d*x + c)^3 -
 40*a*sin(d*x + c)^2 + 15*b*sin(d*x + c) + 12*a)/sin(d*x + c)^5)/d
```

mupad [B] time = 11.80, size = 193, normalized size = 2.24

$$\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{96\,d}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}{d}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{160\,d}+\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{16\,d}-\frac{b\tan\left(\frac{c}{2}+\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (5*a*tan(c/2 + (d*x)/2)^3)/(96*d) - (b*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (
5*a*tan(c/2 + (d*x)/2))/(16*d) - (a*tan(c/2 + (d*x)/2)^5)/(160*d) + (3*b*ta
```

```
n(c/2 + (d*x)/2)^2)/(16*d) - (b*tan(c/2 + (d*x)/2)^4)/(64*d) + (b*log(tan(c
/2 + (d*x)/2)))/d - (cot(c/2 + (d*x)/2)^5*(a/5 + (b*tan(c/2 + (d*x)/2))/2 -
 (5*a*tan(c/2 + (d*x)/2)^2)/3 + 10*a*tan(c/2 + (d*x)/2)^4 - 6*b*tan(c/2 + (
d*x)/2)^3))/(32*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1209 $\int \cot^5(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=61

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{b\csc^5(c+dx)}{5d}+\frac{2b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}$$

```
[Out] -1/6*a*cot(d*x+c)^6/d-b*csc(d*x+c)/d+2/3*b*csc(d*x+c)^3/d-1/5*b*csc(d*x+c)^
5/d
```

Rubi [A] time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2607, 30, 2606, 194}

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{b\csc^5(c+dx)}{5d}+\frac{2b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (b*Csc[c + d*x])/d + (2*b*Csc[c + d*x]^3)/(3*d)
 - (b*Csc[c + d*x]^5)/(5*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^2(c+dx)\,dx + b\int \cot^5(c+dx)\csc(c+$$

$$= -\frac{a\,\text{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \left(-1+x^2\right)^2}{d}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{b\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(c+dx}{d}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{b\csc(c+dx)}{d} + \frac{2b\csc^3(c+dx)}{3d} - \frac{b\csc^5(c}{5d}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.00

$$-\frac{a\cot^6(c+dx)}{6d} - \frac{b\csc^5(c+dx)}{5d} + \frac{2b\csc^3(c+dx)}{3d} - \frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cot[c + d*x]^6)/d - (b*Csc[c + d*x])/d + (2*b*Csc[c + d*x]^3)/(3*d)
 - (b*Csc[c + d*x]^5)/(5*d)
```

fricas [A] time = 0.73, size = 100, normalized size = 1.64

$$\frac{15\,a\cos\left(dx+c\right)^4 - 15\,a\cos\left(dx+c\right)^2 + 2\left(15\,b\cos\left(dx+c\right)^4 - 20\,b\cos\left(dx+c\right)^2 + 8\,b\right)\sin\left(dx+c\right) + 5\,a}{30\left(d\cos\left(dx+c\right)^6 - 3\,d\cos\left(dx+c\right)^4 + 3\,d\cos\left(dx+c\right)^2 - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*a*cos(d*x + c)^4 - 15*a*cos(d*x + c)^2 + 2*(15*b*cos(d*x + c)^4 -
20*b*cos(d*x + c)^2 + 8*b)*sin(d*x + c) + 5*a)/(d*cos(d*x + c)^6 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.24, size = 70, normalized size = 1.15

$$-\frac{30\,b\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-20\,b\sin\left(dx+c\right)^{3}-15\,a\sin\left(dx+c\right)^{2}+6\,b\sin\left(dx+c\right)+5\,a}{30\,d\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(30*b*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*b*sin(d*x + c)^3 - 15
*a*sin(d*x + c)^2 + 6*b*sin(d*x + c) + 5*a)/(d*sin(d*x + c)^6)
```

maple [A] time = 0.43, size = 110, normalized size = 1.80

$$\frac{-\frac{a\left(\cos^{6}\left(dx+c\right)\right)}{6\sin\left(dx+c\right)^{6}}+b\left(-\frac{\cos^{6}\left(dx+c\right)}{5\sin\left(dx+c\right)^{5}}+\frac{\cos^{6}\left(dx+c\right)}{15\sin\left(dx+c\right)^{3}}-\frac{\cos^{6}\left(dx+c\right)}{5\sin\left(dx+c\right)}-\frac{\left(\frac{8}{3}+\cos^{4}\left(dx+c\right)+\frac{4\left(\cos^{2}\left(dx+c\right)\right)}{3}\right)\sin\left(dx+c\right)}{5}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/6*a/sin(d*x+c)^6*cos(d*x+c)^6+b*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/1
5/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)
^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.32, size = 70, normalized size = 1.15

$$-\frac{30\,b\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-20\,b\sin\left(dx+c\right)^{3}-15\,a\sin\left(dx+c\right)^{2}+6\,b\sin\left(dx+c\right)+5\,a}{30\,d\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(30*b*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*b*sin(d*x + c)^3 - 15
*a*sin(d*x + c)^2 + 6*b*sin(d*x + c) + 5*a)/(d*sin(d*x + c)^6)
```

mupad [B] time = 11.79, size = 69, normalized size = 1.13

$$-\frac{b\sin\left(c+d\,x\right)^{5}+\frac{a\sin\left(c+d\,x\right)^{4}}{2}-\frac{2\,b\sin\left(c+d\,x\right)^{3}}{3}-\frac{a\sin\left(c+d\,x\right)^{2}}{2}+\frac{b\,\sin\left(c+d\,x\right)}{5}+\frac{a}{6}}{d\sin\left(c+d\,x\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^7,x)
```

```
[Out] -(a/6 + (b*sin(c + d*x))/5 - (a*sin(c + d*x)^2)/2 + (a*sin(c + d*x)^4)/2 -
(2*b*sin(c + d*x)^3)/3 + b*sin(c + d*x)^5)/(d*sin(c + d*x)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1210 $\int \cot^5(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-\frac{b\cot^6(c+dx)}{6d}$$

```
[Out] -1/6*b*cot(d*x+c)^6/d-1/3*a*csc(d*x+c)^3/d+2/5*a*csc(d*x+c)^5/d-1/7*a*csc(d
*x+c)^7/d
```

Rubi [A] time = 0.12, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2606, 270, 2607, 30}

$$-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-\frac{b\cot^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cot[c + d*x]^6)/(6*d) - (a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)
/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^3(c+dx)\,dx + b\int \cot^5(c+dx)\csc^2(c+$$

$$= -\frac{a\,\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \csc(c+dx)\right)}{d} - \frac{b\,\mathrm{Subst}\left(\int x\right.}{}$$

$$= -\frac{b\cot^6(c+dx)}{6d} - \frac{a\,\mathrm{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \csc(c+d\right.}{d}$$

$$= -\frac{b\cot^6(c+dx)}{6d} - \frac{a\csc^3(c+dx)}{3d} + \frac{2a\csc^5(c+dx)}{5d} - \frac{a\csc^7(}{7}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.00

$$-\frac{a\csc^7(c+dx)}{7d} + \frac{2a\csc^5(c+dx)}{5d} - \frac{a\csc^3(c+dx)}{3d} - \frac{b\cot^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(b*Cot[c + d*x]^6)/d - (a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)
/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

fricas [A] time = 0.93, size = 106, normalized size = 1.63

$$\frac{70\,a\cos\left(dx+c\right)^4 - 56\,a\cos\left(dx+c\right)^2 + 35\left(3\,b\cos\left(dx+c\right)^4 - 3\,b\cos\left(dx+c\right)^2 + b\right)\sin\left(dx+c\right) + 16\,a}{210\left(d\cos\left(dx+c\right)^6 - 3\,d\cos\left(dx+c\right)^4 + 3\,d\cos\left(dx+c\right)^2 - d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/210*(70*a*cos(d*x + c)^4 - 56*a*cos(d*x + c)^2 + 35*(3*b*cos(d*x + c)^4 -
 3*b*cos(d*x + c)^2 + b)*sin(d*x + c) + 16*a)/((d*cos(d*x + c)^6 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.23, size = 70, normalized size = 1.08

$$-\frac{105\,b\sin\left(dx+c\right)^{5}+70\,a\sin\left(dx+c\right)^{4}-105\,b\sin\left(dx+c\right)^{3}-84\,a\sin\left(dx+c\right)^{2}+35\,b\sin\left(dx+c\right)+30\,a}{210\,d\sin\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/210*(105*b*sin(d*x + c)^5 + 70*a*sin(d*x + c)^4 - 105*b*sin(d*x + c)^3 -
 84*a*sin(d*x + c)^2 + 35*b*sin(d*x + c) + 30*a)/(d*sin(d*x + c)^7)
```

maple [B] time = 0.45, size = 128, normalized size = 1.97

$$\frac{a\left(-\frac{\cos^{6}(dx+c)}{7\sin(dx+c)^{7}}-\frac{\cos^{6}(dx+c)}{35\sin(dx+c)^{5}}+\frac{\cos^{6}(dx+c)}{105\sin(dx+c)^{3}}-\frac{\cos^{6}(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{b\left(\cos^{6}(dx+c)\right)}{6\sin(dx+c)^{6}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/105
/sin(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c
)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/6*b/sin(d*x+c)^6*cos(d*x+c)^6)
```

maxima [A] time = 0.34, size = 70, normalized size = 1.08

$$-\frac{105\,b\sin\left(dx+c\right)^{5}+70\,a\sin\left(dx+c\right)^{4}-105\,b\sin\left(dx+c\right)^{3}-84\,a\sin\left(dx+c\right)^{2}+35\,b\sin\left(dx+c\right)+30\,a}{210\,d\sin\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/210*(105*b*sin(d*x + c)^5 + 70*a*sin(d*x + c)^4 - 105*b*sin(d*x + c)^3 -
 84*a*sin(d*x + c)^2 + 35*b*sin(d*x + c) + 30*a)/(d*sin(d*x + c)^7)
```

mupad [B] time = 11.61, size = 70, normalized size = 1.08

$$-\frac{105\,b\,{\sin\left(c+d\,x\right)}^5+70\,a\,{\sin\left(c+d\,x\right)}^4-105\,b\,{\sin\left(c+d\,x\right)}^3-84\,a\,{\sin\left(c+d\,x\right)}^2+35\,b\,\sin\left(c+d\,x\right)+30\,a}{210\,d\,{\sin\left(c+d\,x\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^8,x)
```

```
[Out] -(30*a + 35*b*sin(c + d*x) - 84*a*sin(c + d*x)^2 + 70*a*sin(c + d*x)^4 - 10
5*b*sin(c + d*x)^3 + 105*b*sin(c + d*x)^5)/(210*d*sin(c + d*x)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**8*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1211 $\int \cot^5(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{b\csc^7(c+dx)}{7d}+\frac{2b\csc^5(c+dx)}{5d}-\frac{b\csc^3(c+dx)}{3d}$$

```
[Out] -1/6*a*cot(d*x+c)^6/d-1/8*a*cot(d*x+c)^8/d-1/3*b*csc(d*x+c)^3/d+2/5*b*csc(d
*x+c)^5/d-1/7*b*csc(d*x+c)^7/d
```

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2607, 14, 2606, 270}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{b\csc^7(c+dx)}{7d}+\frac{2b\csc^5(c+dx)}{5d}-\frac{b\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Cot[c + d*x]^8)/(8*d) - (b*Csc[c + d*x]^3)/(
3*d) + (2*b*Csc[c + d*x]^5)/(5*d) - (b*Csc[c + d*x]^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^4(c+dx)\,dx + b\int \cot^5(c+dx)\csc^3(c+$$

$$= -\frac{a\,\text{Subst}\left(\int x^5\left(1+x^2\right)\,dx, x, -\cot(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int x^2\right.}{}$$

$$= -\frac{a\,\text{Subst}\left(\int \left(x^5+x^7\right)\,dx, x, -\cot(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \left(x^2\right.\right.}{}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\cot^8(c+dx)}{8d} - \frac{b\csc^3(c+dx)}{3d} + \frac{2b\csc^5(}{5d}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 1.09

$$-\frac{a\left(3\csc^8(c+dx)-8\csc^6(c+dx)+6\csc^4(c+dx)\right)}{24d}-\frac{b\csc^7(c+dx)}{7d}+\frac{2b\csc^5(c+dx)}{5d}-\frac{b\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/3*(b*Csc[c + d*x]^3)/d + (2*b*Csc[c + d*x]^5)/(5*d) - (b*Csc[c + d*x]^7)
/(7*d) - (a*(6*Csc[c + d*x]^4 - 8*Csc[c + d*x]^6 + 3*Csc[c + d*x]^8))/(24*d
)
```

fricas [A] time = 0.50, size = 109, normalized size = 1.35

$$-\frac{210\,a\cos\left(dx+c\right)^4-140\,a\cos\left(dx+c\right)^2+8\left(35\,b\cos\left(dx+c\right)^4-28\,b\cos\left(dx+c\right)^2+8\,b\right)\sin\left(dx+c\right)+35\,a}{840\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/840*(210*a*cos(d*x + c)^4 - 140*a*cos(d*x + c)^2 + 8*(35*b*cos(d*x + c)^
4 - 28*b*cos(d*x + c)^2 + 8*b)*sin(d*x + c) + 35*a)/(d*cos(d*x + c)^8 - 4*d
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.24, size = 70, normalized size = 0.86

$$-\frac{280\,b\sin(dx+c)^5+210\,a\sin(dx+c)^4-336\,b\sin(dx+c)^3-280\,a\sin(dx+c)^2+120\,b\sin(dx+c)+105\,a}{840\,d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/840*(280*b*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 336*b*sin(d*x + c)^3
- 280*a*sin(d*x + c)^2 + 120*b*sin(d*x + c) + 105*a)/(d*sin(d*x + c)^8)
```

maple [B] time = 0.35, size = 148, normalized size = 1.83

$$\frac{a\left(-\frac{\cos^6(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^6(dx+c)}{24\sin(dx+c)^6}\right)+b\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)}{35}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/8/sin(d*x+c)^8*cos(d*x+c)^6-1/24/sin(d*x+c)^6*cos(d*x+c)^6)+b*(-
1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/105/sin(d*x+
c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c)^4+4/3*c
os(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.35, size = 70, normalized size = 0.86

$$-\frac{280\,b\sin(dx+c)^5+210\,a\sin(dx+c)^4-336\,b\sin(dx+c)^3-280\,a\sin(dx+c)^2+120\,b\sin(dx+c)+105\,a}{840\,d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*(280*b*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 336*b*sin(d*x + c)^3
- 280*a*sin(d*x + c)^2 + 120*b*sin(d*x + c) + 105*a)/(d*sin(d*x + c)^8)
```

mupad [B] time = 11.64, size = 70, normalized size = 0.86

$$-\frac{280\,b\sin\left(c+d\,x\right)^5+210\,a\sin\left(c+d\,x\right)^4-336\,b\sin\left(c+d\,x\right)^3-280\,a\sin\left(c+d\,x\right)^2+120\,b\,\sin\left(c+d\,x\right)+105\,a}{840\,d\sin\left(c+d\,x\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^9,x)
```

```
[Out] -(105*a + 120*b*sin(c + d*x) - 280*a*sin(c + d*x)^2 + 210*a*sin(c + d*x)^4
- 336*b*sin(c + d*x)^3 + 280*b*sin(c + d*x)^5)/(840*d*sin(c + d*x)^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**9*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1212 $\int \cot^5(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\csc^9(c+dx)}{9d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{b\cot^8(c+dx)}{8d}-\frac{b\cot^6(c+dx)}{6d}$$

```
[Out] -1/6*b*cot(d*x+c)^6/d-1/8*b*cot(d*x+c)^8/d-1/5*a*csc(d*x+c)^5/d+2/7*a*csc(d
*x+c)^7/d-1/9*a*csc(d*x+c)^9/d
```

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2606, 270, 2607, 14}

$$-\frac{a\csc^9(c+dx)}{9d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{b\cot^8(c+dx)}{8d}-\frac{b\cot^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cot[c + d*x]^6)/(6*d) - (b*Cot[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^5)/(
5*d) + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^5(c+dx)\,dx + b\int \cot^5(c+dx)\csc^4(c+$$

$$= -\frac{a\,\text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx, x, \csc(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int x\right.}{}$$

$$= -\frac{a\,\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)dx, x, \csc(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{b\cot^6(c+dx)}{6d} - \frac{b\cot^8(c+dx)}{8d} - \frac{a\csc^5(c+dx)}{5d} + \frac{2a\csc^7(}{7a}$$

Mathematica [A] time = 0.12, size = 88, normalized size = 1.09

$$-\frac{a\csc^9(c+dx)}{9d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{b\left(3\csc^8(c+dx)-8\csc^6(c+dx)+6\csc^4(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/5*(a*Csc[c + d*x]^5)/d + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)
/(9*d) - (b*(6*Csc[c + d*x]^4 - 8*Csc[c + d*x]^6 + 3*Csc[c + d*x]^8))/(24*d
)
```

fricas [A] time = 0.62, size = 115, normalized size = 1.42

$$-\frac{504\,a\cos(dx+c)^4-288\,a\cos(dx+c)^2+105\left(6\,b\cos(dx+c)^4-4\,b\cos(dx+c)^2+b\right)\sin(dx+c)+64\,a}{2520\left(d\cos(dx+c)^8-4\,d\cos(dx+c)^6+6\,d\cos(dx+c)^4-4\,d\cos(dx+c)^2+d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+b*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2520*(504*a*cos(d*x + c)^4 - 288*a*cos(d*x + c)^2 + 105*(6*b*cos(d*x + c
)^4 - 4*b*cos(d*x + c)^2 + b)*sin(d*x + c) + 64*a)/((d*cos(d*x + c)^8 - 4*d
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c)
)
```

giac [A] time = 0.27, size = 70, normalized size = 0.86

$$-\frac{630\,b\sin\left(dx+c\right)^{5}+504\,a\sin\left(dx+c\right)^{4}-840\,b\sin\left(dx+c\right)^{3}-720\,a\sin\left(dx+c\right)^{2}+315\,b\sin\left(dx+c\right)+280\,a}{2520\,d\sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2520*(630*b*sin(d*x + c)^5 + 504*a*sin(d*x + c)^4 - 840*b*sin(d*x + c)^3
 - 720*a*sin(d*x + c)^2 + 315*b*sin(d*x + c) + 280*a)/(d*sin(d*x + c)^9)
```

maple [B] time = 0.46, size = 166, normalized size = 2.05

$$\frac{a\left(-\frac{\cos^{6}(dx+c)}{9\sin(dx+c)^{9}}-\frac{\cos^{6}(dx+c)}{21\sin(dx+c)^{7}}-\frac{\cos^{6}(dx+c)}{105\sin(dx+c)^{5}}+\frac{\cos^{6}(dx+c)}{315\sin(dx+c)^{3}}-\frac{\cos^{6}(dx+c)}{105\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{105}\right)+b\left(-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^10*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^6-1/21/sin(d*x+c)^7*cos(d*x+c)^6-1/105
/sin(d*x+c)^5*cos(d*x+c)^6+1/315/sin(d*x+c)^3*cos(d*x+c)^6-1/105/sin(d*x+c)
*cos(d*x+c)^6-1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+b*(-1/8
/sin(d*x+c)^8*cos(d*x+c)^6-1/24/sin(d*x+c)^6*cos(d*x+c)^6))
```

maxima [A] time = 0.32, size = 70, normalized size = 0.86

$$-\frac{630\,b\sin\left(dx+c\right)^{5}+504\,a\sin\left(dx+c\right)^{4}-840\,b\sin\left(dx+c\right)^{3}-720\,a\sin\left(dx+c\right)^{2}+315\,b\sin\left(dx+c\right)+280\,a}{2520\,d\sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+b*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/2520*(630*b*sin(d*x + c)^5 + 504*a*sin(d*x + c)^4 - 840*b*sin(d*x + c)^3
 - 720*a*sin(d*x + c)^2 + 315*b*sin(d*x + c) + 280*a)/(d*sin(d*x + c)^9)
```

mupad [B] time = 11.67, size = 70, normalized size = 0.86

$$-\frac{\frac{b\sin(c+d\,x)^5}{4}+\frac{a\sin(c+d\,x)^4}{5}-\frac{b\sin(c+d\,x)^3}{3}-\frac{2\,a\sin(c+d\,x)^2}{7}+\frac{b\,\sin(c+d\,x)}{8}+\frac{a}{9}}{d\sin\left(c+d\,x\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^10,x)
```

```
[Out] -(a/9 + (b*sin(c + d*x))/8 - (2*a*sin(c + d*x)^2)/7 + (a*sin(c + d*x)^4)/5
- (b*sin(c + d*x)^3)/3 + (b*sin(c + d*x)^5)/4)/(d*sin(c + d*x)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**10*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1213 $\int \cot^5(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\csc^{10}(c+dx)}{10d}+\frac{a\csc^8(c+dx)}{4d}-\frac{a\csc^6(c+dx)}{6d}-\frac{b\csc^9(c+dx)}{9d}+\frac{2b\csc^7(c+dx)}{7d}-\frac{b\csc^5(c+dx)}{5d}$$

```
[Out] -1/5*b*csc(d*x+c)^5/d-1/6*a*csc(d*x+c)^6/d+2/7*b*csc(d*x+c)^7/d+1/4*a*csc(d
*x+c)^8/d-1/9*b*csc(d*x+c)^9/d-1/10*a*csc(d*x+c)^10/d
```

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 766}

$$-\frac{a\csc^{10}(c+dx)}{10d}+\frac{a\csc^8(c+dx)}{4d}-\frac{a\csc^6(c+dx)}{6d}-\frac{b\csc^9(c+dx)}{9d}+\frac{2b\csc^7(c+dx)}{7d}-\frac{b\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)/(6*d) + (2*b*Csc[c + d*x]^7)
/(7*d) + (a*Csc[c + d*x]^8)/(4*d) - (b*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d
*x]^10)/(10*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^{11}(a+x)\left(b^2-x^2\right)^2}{x^{11}}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{b^6\,\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^{11}}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^6\,\text{Subst}\left(\int \left(\frac{ab^4}{x^{11}}+\frac{b^4}{x^{10}}-\frac{2ab^2}{x^9}-\frac{2b^2}{x^8}+\frac{a}{x^7}+\frac{1}{x^6}\right)dx, x, b\sin(c+\right.}{d}$$
$$= -\frac{b\csc^5(c+dx)}{5d}-\frac{a\csc^6(c+dx)}{6d}+\frac{2b\csc^7(c+dx)}{7d}+\frac{a\csc^8(}{4}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 0.91

$$-\frac{a\left(6\csc^{10}(c+dx)-15\csc^8(c+dx)+10\csc^6(c+dx)\right)}{60d}-\frac{b\csc^9(c+dx)}{9d}+\frac{2b\csc^7(c+dx)}{7d}-\frac{b\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/5*(b*Csc[c + d*x]^5)/d + (2*b*Csc[c + d*x]^7)/(7*d) - (b*Csc[c + d*x]^9)
/(9*d) - (a*(10*Csc[c + d*x]^6 - 15*Csc[c + d*x]^8 + 6*Csc[c + d*x]^10))/(6
0*d)
```

fricas [A] time = 1.23, size = 122, normalized size = 1.26

$$\frac{210\,a\cos(dx+c)^4-105\,a\cos(dx+c)^2+4\left(63\,b\cos(dx+c)^4-36\,b\cos(dx+c)^2+8\,b\right)\sin(dx+c)+21\,a}{1260\left(d\cos(dx+c)^{10}-5\,d\cos(dx+c)^8+10\,d\cos(dx+c)^6-10\,d\cos(dx+c)^4+5\,d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+b*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1260*(210*a*cos(d*x + c)^4 - 105*a*cos(d*x + c)^2 + 4*(63*b*cos(d*x + c)^
4 - 36*b*cos(d*x + c)^2 + 8*b)*sin(d*x + c) + 21*a)/(d*cos(d*x + c)^10 - 5*
d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x
+ c)^2 - d)
```

giac [A] time = 0.26, size = 70, normalized size = 0.72

$$-\frac{252\,b\sin(dx+c)^5+210\,a\sin(dx+c)^4-360\,b\sin(dx+c)^3-315\,a\sin(dx+c)^2+140\,b\sin(dx+c)+126\,a}{1260\,d\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1260*(252*b*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 360*b*sin(d*x + c)^3
 - 315*a*sin(d*x + c)^2 + 140*b*sin(d*x + c) + 126*a)/(d*sin(d*x + c)^10)
```

maple [B] time = 0.33, size = 184, normalized size = 1.90

$$\frac{a\left(-\frac{\cos^6(dx+c)}{10\sin(dx+c)^{10}}-\frac{\cos^6(dx+c)}{20\sin(dx+c)^8}-\frac{\cos^6(dx+c)}{60\sin(dx+c)^6}\right)+b\left(-\frac{\cos^6(dx+c)}{9\sin(dx+c)^9}-\frac{\cos^6(dx+c)}{21\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{105\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{315\sin(dx+c)^3}-\frac{\cos^6(}{105\sin}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^11*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/10/sin(d*x+c)^10*cos(d*x+c)^6-1/20/sin(d*x+c)^8*cos(d*x+c)^6-1/6
0/sin(d*x+c)^6*cos(d*x+c)^6)+b*(-1/9/sin(d*x+c)^9*cos(d*x+c)^6-1/21/sin(d*x
+c)^7*cos(d*x+c)^6-1/105/sin(d*x+c)^5*cos(d*x+c)^6+1/315/sin(d*x+c)^3*cos(d
*x+c)^6-1/105/sin(d*x+c)*cos(d*x+c)^6-1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c)))
```

maxima [A] time = 0.33, size = 70, normalized size = 0.72

$$-\frac{252\,b\sin\left(dx+c\right)^5+210\,a\sin\left(dx+c\right)^4-360\,b\sin\left(dx+c\right)^3-315\,a\sin\left(dx+c\right)^2+140\,b\sin\left(dx+c\right)+126\,a}{1260\,d\sin\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+b*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/1260*(252*b*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 360*b*sin(d*x + c)^3
 - 315*a*sin(d*x + c)^2 + 140*b*sin(d*x + c) + 126*a)/(d*sin(d*x + c)^10)
```

mupad [B] time = 11.64, size = 70, normalized size = 0.72

$$-\frac{252\,b\sin\left(c+d\,x\right)^5+210\,a\sin\left(c+d\,x\right)^4-360\,b\sin\left(c+d\,x\right)^3-315\,a\sin\left(c+d\,x\right)^2+140\,b\,\sin\left(c+d\,x\right)+126}{1260\,d\sin\left(c+d\,x\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^11,x)
```

```
[Out] -(126*a + 140*b*sin(c + d*x) - 315*a*sin(c + d*x)^2 + 210*a*sin(c + d*x)^4
- 360*b*sin(c + d*x)^3 + 252*b*sin(c + d*x)^5)/(1260*d*sin(c + d*x)^10)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**11*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1214 $\int \cot^5(c+dx)\csc^7(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\csc^{11}(c+dx)}{11d}+\frac{2a\csc^9(c+dx)}{9d}-\frac{a\csc^7(c+dx)}{7d}-\frac{b\csc^{10}(c+dx)}{10d}+\frac{b\csc^8(c+dx)}{4d}-\frac{b\csc^6(c+dx)}{6d}$$

```
[Out] -1/6*b*csc(d*x+c)^6/d-1/7*a*csc(d*x+c)^7/d+1/4*b*csc(d*x+c)^8/d+2/9*a*csc(d
*x+c)^9/d-1/10*b*csc(d*x+c)^10/d-1/11*a*csc(d*x+c)^11/d
```

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 766}

$$-\frac{a\csc^{11}(c+dx)}{11d}+\frac{2a\csc^9(c+dx)}{9d}-\frac{a\csc^7(c+dx)}{7d}-\frac{b\csc^{10}(c+dx)}{10d}+\frac{b\csc^8(c+dx)}{4d}-\frac{b\csc^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^7*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Csc[c + d*x]^6)/(6*d) - (a*Csc[c + d*x]^7)/(7*d) + (b*Csc[c + d*x]^8)/(
4*d) + (2*a*Csc[c + d*x]^9)/(9*d) - (b*Csc[c + d*x]^10)/(10*d) - (a*Csc[c +
 d*x]^11)/(11*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^7(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^{12}(a+x)\left(b^2-x^2\right)^2}{x^{12}}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{b^7\,\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^{12}}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^7\,\text{Subst}\left(\int \left(\frac{ab^4}{x^{12}}+\frac{b^4}{x^{11}}-\frac{2ab^2}{x^{10}}-\frac{2b^2}{x^9}+\frac{a}{x^8}+\frac{1}{x^7}\right)dx, x, b\sin(c+\right.}{d}$$
$$= -\frac{b\csc^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{b\csc^8(c+dx)}{4d}+\frac{2a\csc^9(}{9}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 0.91

$$-\frac{a\csc^{11}(c+dx)}{11d}+\frac{2a\csc^9(c+dx)}{9d}-\frac{a\csc^7(c+dx)}{7d}-\frac{b\left(6\csc^{10}(c+dx)-15\csc^8(c+dx)+10\csc^6(c+dx)\right)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^7*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/7*(a*Csc[c + d*x]^7)/d + (2*a*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d*x]^11
)/(11*d) - (b*(10*Csc[c + d*x]^6 - 15*Csc[c + d*x]^8 + 6*Csc[c + d*x]^10))/
(60*d)
```

fricas [A] time = 0.70, size = 128, normalized size = 1.32

$$\frac{1980\,a\cos(dx+c)^4-880\,a\cos(dx+c)^2+231\left(10\,b\cos(dx+c)^4-5\,b\cos(dx+c)^2+b\right)\sin(dx+c)+160}{13860\left(d\cos(dx+c)^{10}-5\,d\cos(dx+c)^8+10\,d\cos(dx+c)^6-10\,d\cos(dx+c)^4+5\,d\cos(dx+c)^2-d\right)\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+b*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/13860*(1980*a*cos(d*x + c)^4 - 880*a*cos(d*x + c)^2 + 231*(10*b*cos(d*x +
 c)^4 - 5*b*cos(d*x + c)^2 + b)*sin(d*x + c) + 160*a)/((d*cos(d*x + c)^10 -
 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d
*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.24, size = 70, normalized size = 0.72

$$-\frac{2310\,b\sin(dx+c)^5+1980\,a\sin(dx+c)^4-3465\,b\sin(dx+c)^3-3080\,a\sin(dx+c)^2+1386\,b\sin(dx+c)+1}{13860\,d\sin(dx+c)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/13860*(2310*b*sin(d*x + c)^5 + 1980*a*sin(d*x + c)^4 - 3465*b*sin(d*x +
c)^3 - 3080*a*sin(d*x + c)^2 + 1386*b*sin(d*x + c) + 1260*a)/(d*sin(d*x + c
)^11)
```

maple [B] time = 0.44, size = 202, normalized size = 2.08

$$\frac{a\left(-\frac{\cos^6(dx+c)}{11\sin(dx+c)^{11}}-\frac{5\left(\cos^6(dx+c)\right)}{99\sin(dx+c)^9}-\frac{5\left(\cos^6(dx+c)\right)}{231\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{231\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{693\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{231\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.}{231}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^12*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/11/sin(d*x+c)^11*cos(d*x+c)^6-5/99/sin(d*x+c)^9*cos(d*x+c)^6-5/2
31/sin(d*x+c)^7*cos(d*x+c)^6-1/231/sin(d*x+c)^5*cos(d*x+c)^6+1/693/sin(d*x+
c)^3*cos(d*x+c)^6-1/231/sin(d*x+c)*cos(d*x+c)^6-1/231*(8/3+cos(d*x+c)^4+4/3
*cos(d*x+c)^2)*sin(d*x+c))+b*(-1/10/sin(d*x+c)^10*cos(d*x+c)^6-1/20/sin(d*x
+c)^8*cos(d*x+c)^6-1/60/sin(d*x+c)^6*cos(d*x+c)^6))
```

maxima [A] time = 0.34, size = 70, normalized size = 0.72

$$-\frac{2310\,b\sin\left(dx+c\right)^5+1980\,a\sin\left(dx+c\right)^4-3465\,b\sin\left(dx+c\right)^3-3080\,a\sin\left(dx+c\right)^2+1386\,b\sin\left(dx+c\right)+}{13860\,d\sin\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+b*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/13860*(2310*b*sin(d*x + c)^5 + 1980*a*sin(d*x + c)^4 - 3465*b*sin(d*x +
c)^3 - 3080*a*sin(d*x + c)^2 + 1386*b*sin(d*x + c) + 1260*a)/(d*sin(d*x + c
)^11)
```

mupad [B] time = 11.69, size = 70, normalized size = 0.72

$$-\frac{\frac{b\sin(c+d\,x)^5}{6}+\frac{a\sin(c+d\,x)^4}{7}-\frac{b\sin(c+d\,x)^3}{4}-\frac{2\,a\sin(c+d\,x)^2}{9}+\frac{b\,\sin(c+d\,x)}{10}+\frac{a}{11}}{d\sin\left(c+d\,x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^12,x)
```

```
[Out] -(a/11 + (b*sin(c + d*x))/10 - (2*a*sin(c + d*x)^2)/9 + (a*sin(c + d*x)^4)/
7 - (b*sin(c + d*x)^3)/4 + (b*sin(c + d*x)^5)/6)/(d*sin(c + d*x)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**12*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1215 $\int \cos^5(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=138

$$\frac{(a^2-2b^2)\sin^7(c+dx)}{7d}-\frac{(2a^2-b^2)\sin^5(c+dx)}{5d}+\frac{a^2\sin^3(c+dx)}{3d}+\frac{ab\sin^8(c+dx)}{4d}-\frac{2ab\sin^6(c+dx)}{3d}+\frac{ab\sin^4(c}{2d}$$

```
[Out] 1/3*a^2*sin(d*x+c)^3/d+1/2*a*b*sin(d*x+c)^4/d-1/5*(2*a^2-b^2)*sin(d*x+c)^5/
d-2/3*a*b*sin(d*x+c)^6/d+1/7*(a^2-2*b^2)*sin(d*x+c)^7/d+1/4*a*b*sin(d*x+c)^
8/d+1/9*b^2*sin(d*x+c)^9/d
```

Rubi [A] time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{(a^2-2b^2)\sin^7(c+dx)}{7d}-\frac{(2a^2-b^2)\sin^5(c+dx)}{5d}+\frac{a^2\sin^3(c+dx)}{3d}+\frac{ab\sin^8(c+dx)}{4d}-\frac{2ab\sin^6(c+dx)}{3d}+\frac{ab\sin^4(c}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^3)/(3*d) + (a*b*Sin[c + d*x]^4)/(2*d) - ((2*a^2 - b^2)*Si
n[c + d*x]^5)/(5*d) - (2*a*b*Sin[c + d*x]^6)/(3*d) + ((a^2 - 2*b^2)*Sin[c +
 d*x]^7)/(7*d) + (a*b*Sin[c + d*x]^8)/(4*d) + (b^2*Sin[c + d*x]^9)/(9*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+x)^2\left(b^2-x^2\right)^2}{b^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int x^2(a+x)^2\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2b^4x^2+2ab^4x^3+b^2\left(-2a^2+b^2\right)x^4-4ab^2x^5+\right.\right.}{b^7 d}$$
$$= \frac{a^2\sin^3(c+dx)}{3d}+\frac{ab\sin^4(c+dx)}{2d}-\frac{\left(2a^2-b^2\right)\sin^5(c+dx)}{5d}$$

Mathematica [A] time = 0.78, size = 169, normalized size = 1.22

$$\frac{12600a^2\sin(c+dx)-840a^2\sin(3(c+dx))-1512a^2\sin(5(c+dx))-360a^2\sin(7(c+dx))-7560ab\cos(2(c+dx$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-7560*a*b*Cos[2*(c + d*x)] - 1260*a*b*Cos[4*(c + d*x)] + 840*a*b*Cos[6*(c
+ d*x)] + 315*a*b*Cos[8*(c + d*x)] + 12600*a^2*Sin[c + d*x] + 3780*b^2*Sin[
c + d*x] - 840*a^2*Sin[3*(c + d*x)] - 840*b^2*Sin[3*(c + d*x)] - 1512*a^2*S
in[5*(c + d*x)] - 504*b^2*Sin[5*(c + d*x)] - 360*a^2*Sin[7*(c + d*x)] + 90*
b^2*Sin[7*(c + d*x)] + 70*b^2*Sin[9*(c + d*x)])/(161280*d)
```

fricas [A] time = 0.71, size = 121, normalized size = 0.88

$$\frac{315\,ab\cos\left(dx+c\right)^8-420\,ab\cos\left(dx+c\right)^6+4\left(35\,b^2\cos\left(dx+c\right)^8-5\left(9\,a^2+10\,b^2\right)\cos\left(dx+c\right)^6+3\left(3\,a^2+b^2\right)}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/1260*(315*a*b*cos(d*x + c)^8 - 420*a*b*cos(d*x + c)^6 + 4*(35*b^2*cos(d*x
 + c)^8 - 5*(9*a^2 + 10*b^2)*cos(d*x + c)^6 + 3*(3*a^2 + b^2)*cos(d*x + c)^
4 + 4*(3*a^2 + b^2)*cos(d*x + c)^2 + 24*a^2 + 8*b^2)*sin(d*x + c))/d
```

giac [A] time = 0.33, size = 173, normalized size = 1.25

$$\frac{ab\cos(8\,dx+8\,c)}{512\,d}+\frac{ab\cos(6\,dx+6\,c)}{192\,d}-\frac{ab\cos(4\,dx+4\,c)}{128\,d}-\frac{3\,ab\cos(2\,dx+2\,c)}{64\,d}+\frac{b^2\sin(9\,dx+9\,c)}{2304\,d}-\frac{(4\,a^2-b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/512*a*b*cos(8*d*x + 8*c)/d + 1/192*a*b*cos(6*d*x + 6*c)/d - 1/128*a*b*cos
(4*d*x + 4*c)/d - 3/64*a*b*cos(2*d*x + 2*c)/d + 1/2304*b^2*sin(9*d*x + 9*c)
/d - 1/1792*(4*a^2 - b^2)*sin(7*d*x + 7*c)/d - 1/320*(3*a^2 + b^2)*sin(5*d*
x + 5*c)/d - 1/192*(a^2 + b^2)*sin(3*d*x + 3*c)/d + 1/128*(10*a^2 + 3*b^2)*
sin(d*x + c)/d
```

maple [A] time = 0.31, size = 155, normalized size = 1.12

$$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+b^2\left(-\frac{(s}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))+2*a*b*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+b
^2*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+
cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.33, size = 108, normalized size = 0.78

$$\frac{140\,b^2\sin(dx+c)^9+315\,ab\sin(dx+c)^8-840\,ab\sin(dx+c)^6+180\left(a^2-2\,b^2\right)\sin(dx+c)^7+630\,ab\sin(dx}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/1260*(140*b^2*sin(d*x + c)^9 + 315*a*b*sin(d*x + c)^8 - 840*a*b*sin(d*x +
 c)^6 + 180*(a^2 - 2*b^2)*sin(d*x + c)^7 + 630*a*b*sin(d*x + c)^4 - 252*(2*
a^2 - b^2)*sin(d*x + c)^5 + 420*a^2*sin(d*x + c)^3)/d
```

mupad [B] time = 11.44, size = 108, normalized size = 0.78

$$\frac{\sin(c+dx)^7\left(\frac{a^2}{7}-\frac{2b^2}{7}\right)-\sin(c+dx)^5\left(\frac{2a^2}{5}-\frac{b^2}{5}\right)+\frac{a^2\sin(c+dx)^3}{3}+\frac{b^2\sin(c+dx)^9}{9}+\frac{ab\sin(c+dx)^4}{2}-\frac{2ab\sin(c+dx)^6}{3}+\frac{ab}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^7*(a^2/7 - (2*b^2)/7) - sin(c + d*x)^5*((2*a^2)/5 - b^2/5) +
(a^2*sin(c + d*x)^3)/3 + (b^2*sin(c + d*x)^9)/9 + (a*b*sin(c + d*x)^4)/2 -
(2*a*b*sin(c + d*x)^6)/3 + (a*b*sin(c + d*x)^8)/4)/d
```

sympy [A] time = 14.27, size = 214, normalized size = 1.55

$$\begin{cases}\frac{8a^2\sin^7(c+dx)}{105d}+\frac{4a^2\sin^5(c+dx)\cos^2(c+dx)}{15d}+\frac{a^2\sin^3(c+dx)\cos^4(c+dx)}{3d}+\frac{ab\sin^8(c+dx)}{12d}+\frac{ab\sin^6(c+dx)\cos^2(c+dx)}{3d}+\frac{ab\sin^4(c+dx)\cos}{2d}\\ x(a+b\sin(c))^2\sin^2(c)\cos^5(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((8*a**2*sin(c + d*x)**7/(105*d) + 4*a**2*sin(c + d*x)**5*cos(c +
d*x)**2/(15*d) + a**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) + a*b*sin(c + d
*x)**8/(12*d) + a*b*sin(c + d*x)**6*cos(c + d*x)**2/(3*d) + a*b*sin(c + d*x
)**4*cos(c + d*x)**4/(2*d) + 8*b**2*sin(c + d*x)**9/(315*d) + 4*b**2*sin(c
+ d*x)**7*cos(c + d*x)**2/(35*d) + b**2*sin(c + d*x)**5*cos(c + d*x)**4/(5*
d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**2*cos(c)**5, True))
```

3.1216 $\int \cos^5(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=138

$$\frac{(a^2-2b^2)\sin^6(c+dx)}{6d}-\frac{(2a^2-b^2)\sin^4(c+dx)}{4d}+\frac{a^2\sin^2(c+dx)}{2d}+\frac{2ab\sin^7(c+dx)}{7d}-\frac{4ab\sin^5(c+dx)}{5d}+\frac{2ab\sin^3}{3}$$

```
[Out] 1/2*a^2*sin(d*x+c)^2/d+2/3*a*b*sin(d*x+c)^3/d-1/4*(2*a^2-b^2)*sin(d*x+c)^4/
d-4/5*a*b*sin(d*x+c)^5/d+1/6*(a^2-2*b^2)*sin(d*x+c)^6/d+2/7*a*b*sin(d*x+c)^
7/d+1/8*b^2*sin(d*x+c)^8/d
```

Rubi [A] time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 772}

$$\frac{(a^2-2b^2)\sin^6(c+dx)}{6d}-\frac{(2a^2-b^2)\sin^4(c+dx)}{4d}+\frac{a^2\sin^2(c+dx)}{2d}+\frac{2ab\sin^7(c+dx)}{7d}-\frac{4ab\sin^5(c+dx)}{5d}+\frac{2ab\sin^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^2)/(2*d) + (2*a*b*Sin[c + d*x]^3)/(3*d) - ((2*a^2 - b^2)*
Sin[c + d*x]^4)/(4*d) - (4*a*b*Sin[c + d*x]^5)/(5*d) + ((a^2 - 2*b^2)*Sin[c
 + d*x]^6)/(6*d) + (2*a*b*Sin[c + d*x]^7)/(7*d) + (b^2*Sin[c + d*x]^8)/(8*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^2\left(b^2-x^2\right)^2}{b}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int x(a+x)^2\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^6 d}$$

$$= \frac{\text{Subst}\left(\int \left(a^2b^4x + 2ab^4x^2 + b^2\left(-2a^2+b^2\right)x^3 - 4ab^2x^4 + \left(a\right.\right.\right.}{b^6 d}$$

$$= \frac{a^2\sin^2(c+dx)}{2d} + \frac{2ab\sin^3(c+dx)}{3d} - \frac{\left(2a^2-b^2\right)\sin^4(c+dx)}{4d}$$

Mathematica [A] time = 0.68, size = 138, normalized size = 1.00

$$-\frac{840\left(10a^2+3b^2\right)\cos(2(c+dx)) + 420\left(8a^2+b^2\right)\cos(4(c+dx)) + 560a^2\cos(6(c+dx)) - 16800ab\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/107520*(-2590*b^2 + 840*(10*a^2 + 3*b^2)*Cos[2*(c + d*x)] + 420*(8*a^2 +
 b^2)*Cos[4*(c + d*x)] + 560*a^2*Cos[6*(c + d*x)] - 280*b^2*Cos[6*(c + d*x)
] - 105*b^2*Cos[8*(c + d*x)] - 16800*a*b*Sin[c + d*x] + 1120*a*b*Sin[3*(c +
 d*x)] + 2016*a*b*Sin[5*(c + d*x)] + 480*a*b*Sin[7*(c + d*x)])/d
```

fricas [A] time = 1.05, size = 85, normalized size = 0.62

$$\frac{105\,b^2\cos(dx+c)^8 - 140\left(a^2+b^2\right)\cos(dx+c)^6 - 16\left(15\,ab\cos(dx+c)^6 - 3\,ab\cos(dx+c)^4 - 4\,ab\cos(dx+c)^2\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*b^2*cos(d*x + c)^8 - 140*(a^2 + b^2)*cos(d*x + c)^6 - 16*(15*a*b
*cos(d*x + c)^6 - 3*a*b*cos(d*x + c)^4 - 4*a*b*cos(d*x + c)^2 - 8*a*b)*sin(
d*x + c))/d
```

giac [A] time = 0.27, size = 152, normalized size = 1.10

$$\frac{b^2\cos(8\,dx+8\,c)}{1024\,d} - \frac{ab\sin(7\,dx+7\,c)}{224\,d} - \frac{3\,ab\sin(5\,dx+5\,c)}{160\,d} - \frac{ab\sin(3\,dx+3\,c)}{96\,d} + \frac{5\,ab\sin(dx+c)}{32\,d} - \frac{\left(2\,a^2-b^2\right)\cos}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/1024*b^2*cos(8*d*x + 8*c)/d - 1/224*a*b*sin(7*d*x + 7*c)/d - 3/160*a*b*sin(5*d*x + 5*c)/d - 1/96*a*b*sin(3*d*x + 3*c)/d + 5/32*a*b*sin(d*x + c)/d - 1/384*(2*a^2 - b^2)*cos(6*d*x + 6*c)/d - 1/256*(8*a^2 + b^2)*cos(4*d*x + 4*c)/d - 1/128*(10*a^2 + 3*b^2)*cos(2*d*x + 2*c)/d

maple [A] time = 0.31, size = 101, normalized size = 0.73

$$\frac{-\frac{a^2\left(\cos^6(dx+c)\right)}{6}+2ab\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+b^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(\right.}{2}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x)

[Out] 1/d*(-1/6*a^2*cos(d*x+c)^6+2*a*b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+b^2*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6))

maxima [A] time = 0.31, size = 108, normalized size = 0.78

$$\frac{105\,b^2\sin(dx+c)^8+240\,ab\sin(dx+c)^7-672\,ab\sin(dx+c)^5+140\left(a^2-2\,b^2\right)\sin(dx+c)^6+560\,ab\sin(dx}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/840*(105*b^2*sin(d*x + c)^8 + 240*a*b*sin(d*x + c)^7 - 672*a*b*sin(d*x + c)^5 + 140*(a^2 - 2*b^2)*sin(d*x + c)^6 + 560*a*b*sin(d*x + c)^3 - 210*(2*a^2 - b^2)*sin(d*x + c)^4 + 420*a^2*sin(d*x + c)^2)/d

mupad [B] time = 11.47, size = 108, normalized size = 0.78

$$\frac{\sin(c+d\,x)^6\,\left(\frac{a^2}{6}-\frac{b^2}{3}\right)-\sin(c+d\,x)^4\,\left(\frac{a^2}{2}-\frac{b^2}{4}\right)+\frac{a^2\sin(c+d\,x)^2}{2}+\frac{b^2\sin(c+d\,x)^8}{8}+\frac{2\,a\,b\sin(c+d\,x)^3}{3}-\frac{4\,a\,b\sin(c+d\,x)^5}{5}+\frac{2\,a}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^5*sin(c + d*x)*(a + b*sin(c + d*x))^2,x)

```
[Out] (sin(c + d*x)^6*(a^2/6 - b^2/3) - sin(c + d*x)^4*(a^2/2 - b^2/4) + (a^2*sin
(c + d*x)^2)/2 + (b^2*sin(c + d*x)^8)/8 + (2*a*b*sin(c + d*x)^3)/3 - (4*a*b
*sin(c + d*x)^5)/5 + (2*a*b*sin(c + d*x)^7)/7)/d
```

sympy [A] time = 8.70, size = 163, normalized size = 1.18

$$\begin{cases} -\frac{a^2\cos^6(c+dx)}{6d} + \frac{16ab\sin^7(c+dx)}{105d} + \frac{8ab\sin^5(c+dx)\cos^2(c+dx)}{15d} + \frac{2ab\sin^3(c+dx)\cos^4(c+dx)}{3d} + \frac{b^2\sin^8(c+dx)}{24d} + \frac{b^2\sin^6(c+dx)\cos^2(c+dx)}{6d} \\ x(a+b\sin(c))^2\sin(c)\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)**6/(6*d) + 16*a*b*sin(c + d*x)**7/(105*d) + 8
*a*b*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) + 2*a*b*sin(c + d*x)**3*cos(c +
 d*x)**4/(3*d) + b**2*sin(c + d*x)**8/(24*d) + b**2*sin(c + d*x)**6*cos(c +
 d*x)**2/(6*d) + b**2*sin(c + d*x)**4*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*
(a + b*sin(c))**2*sin(c)*cos(c)**5, True))
```

3.1217 $\int \cos^4(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=130

$$\frac{(a^2-2b^2)\sin^4(c+dx)}{4d}-\frac{(2a^2-b^2)\sin^2(c+dx)}{2d}+\frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin^5(c+dx)}{5d}-\frac{4ab\sin^3(c+dx)}{3d}+\frac{2ab}{}$$

```
[Out] a^2*ln(sin(d*x+c))/d+2*a*b*sin(d*x+c)/d-1/2*(2*a^2-b^2)*sin(d*x+c)^2/d-4/3*
a*b*sin(d*x+c)^3/d+1/4*(a^2-2*b^2)*sin(d*x+c)^4/d+2/5*a*b*sin(d*x+c)^5/d+1/
6*b^2*sin(d*x+c)^6/d
```

Rubi [A] time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 948}

$$\frac{(a^2-2b^2)\sin^4(c+dx)}{4d}-\frac{(2a^2-b^2)\sin^2(c+dx)}{2d}+\frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin^5(c+dx)}{5d}-\frac{4ab\sin^3(c+dx)}{3d}+\frac{2ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d - ((2*a^2 - b^2)*Sin[c +
 d*x]^2)/(2*d) - (4*a*b*Sin[c + d*x]^3)/(3*d) + ((a^2 - 2*b^2)*Sin[c + d*x]
^4)/(4*d) + (2*a*b*Sin[c + d*x]^5)/(5*d) + (b^2*Sin[c + d*x]^6)/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b(a+x)^2\left(b^2-x^2\right)^2}{x}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^2}{x}\,dx, x, b\sin(c+dx)\right)}{b^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(2ab^4+\frac{a^2b^4}{x}-b^2\left(2a^2-b^2\right)x-4ab^2x^2+\left(a^2-2b^2\right.\right.\right.}{b^4 d}$$
$$= \frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin(c+dx)}{d}-\frac{\left(2a^2-b^2\right)\sin^2(c+}{2d}$$

Mathematica [A] time = 0.15, size = 105, normalized size = 0.81

$$\frac{15\left(a^2-2b^2\right)\sin^4(c+dx)+30\left(b^2-2a^2\right)\sin^2(c+dx)+60a^2\log(\sin(c+dx))+24ab\sin^5(c+dx)-80ab\sin^3(c}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (60*a^2*Log[Sin[c + d*x]] + 120*a*b*Sin[c + d*x] + 30*(-2*a^2 + b^2)*Sin[c
+ d*x]^2 - 80*a*b*Sin[c + d*x]^3 + 15*(a^2 - 2*b^2)*Sin[c + d*x]^4 + 24*a*b
*Sin[c + d*x]^5 + 10*b^2*Sin[c + d*x]^6)/(60*d)
```

fricas [A] time = 0.89, size = 96, normalized size = 0.74

$$-\frac{10\,b^2\cos\left(dx+c\right)^6-15\,a^2\cos\left(dx+c\right)^4-30\,a^2\cos\left(dx+c\right)^2-60\,a^2\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-8\left(3\,ab\cos\left(dx+c\right)^4\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^2*cos(d*x + c)^6 - 15*a^2*cos(d*x + c)^4 - 30*a^2*cos(d*x + c)^
2 - 60*a^2*log(1/2*sin(d*x + c)) - 8*(3*a*b*cos(d*x + c)^4 + 4*a*b*cos(d*x
+ c)^2 + 8*a*b)*sin(d*x + c))/d
```

giac [A] time = 0.24, size = 118, normalized size = 0.91

$$\frac{10\,b^2\sin(dx+c)^6+24\,ab\sin(dx+c)^5+15\,a^2\sin(dx+c)^4-30\,b^2\sin(dx+c)^4-80\,ab\sin(dx+c)^3-60\,a^2\sin}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*b^2*sin(d*x + c)^6 + 24*a*b*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4
 - 30*b^2*sin(d*x + c)^4 - 80*a*b*sin(d*x + c)^3 - 60*a^2*sin(d*x + c)^2 +
30*b^2*sin(d*x + c)^2 + 60*a^2*log(abs(sin(d*x + c))) + 120*a*b*sin(d*x + c
))/d
```

maple [A] time = 0.53, size = 119, normalized size = 0.92

$$\frac{a^2\left(\cos^4(dx+c)\right)}{4d}+\frac{a^2\left(\cos^2(dx+c)\right)}{2d}+\frac{a^2\ln(\sin(dx+c))}{d}+\frac{16ab\sin(dx+c)}{15d}+\frac{2ab\sin(dx+c)\left(\cos^4(dx+c)\right)}{5d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*cos(d*x+c)^4+1/2/d*a^2*cos(d*x+c)^2+a^2*ln(sin(d*x+c))/d+16/15*a*
b*sin(d*x+c)/d+2/5/d*a*b*sin(d*x+c)*cos(d*x+c)^4+8/15/d*a*b*sin(d*x+c)*cos(
d*x+c)^2-1/6/d*cos(d*x+c)^6*b^2
```

maxima [A] time = 0.32, size = 105, normalized size = 0.81

$$\frac{10\,b^2\sin(dx+c)^6+24\,ab\sin(dx+c)^5-80\,ab\sin(dx+c)^3+15\left(a^2-2\,b^2\right)\sin(dx+c)^4+60\,a^2\log(\sin(dx+}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/60*(10*b^2*sin(d*x + c)^6 + 24*a*b*sin(d*x + c)^5 - 80*a*b*sin(d*x + c)^3
 + 15*(a^2 - 2*b^2)*sin(d*x + c)^4 + 60*a^2*log(sin(d*x + c)) + 120*a*b*sin
(d*x + c) - 30*(2*a^2 - b^2)*sin(d*x + c)^2)/d
```

mupad [B] time = 11.74, size = 153, normalized size = 1.18

$$\frac{a^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a^2\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}+\frac{a^2\cos(c+d\,x)^2}{2\,d}+\frac{a^2\cos(c+d\,x)^4}{4\,d}-\frac{b^2\cos(c+d\,x)^6}{6\,d}+\frac{16\,a\,b\,\sin(c+d\,x}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a^2*log(1/cos(c/2 + (
d*x)/2)^2))/d + (a^2*cos(c + d*x)^2)/(2*d) + (a^2*cos(c + d*x)^4)/(4*d) - (
b^2*cos(c + d*x)^6)/(6*d) + (16*a*b*sin(c + d*x))/(15*d) + (8*a*b*cos(c + d
*x)^2*sin(c + d*x))/(15*d) + (2*a*b*cos(c + d*x)^4*sin(c + d*x))/(5*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1218 $\int \cos^3(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=125

$$\frac{(a^2-2b^2)\sin^3(c+dx)}{3d}-\frac{(2a^2-b^2)\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{ab\sin^4(c+dx)}{2d}-\frac{2ab\sin^2(c+dx)}{d}+\frac{2ab\log(\sin}{d}$$

```
[Out] -a^2*csc(d*x+c)/d+2*a*b*ln(sin(d*x+c))/d-(2*a^2-b^2)*sin(d*x+c)/d-2*a*b*sin
(d*x+c)^2/d+1/3*(a^2-2*b^2)*sin(d*x+c)^3/d+1/2*a*b*sin(d*x+c)^4/d+1/5*b^2*s
in(d*x+c)^5/d
```

Rubi [A] time = 0.16, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{(a^2-2b^2)\sin^3(c+dx)}{3d}-\frac{(2a^2-b^2)\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{ab\sin^4(c+dx)}{2d}-\frac{2ab\sin^2(c+dx)}{d}+\frac{2ab\log(\sin}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a*b*Log[Sin[c + d*x]])/d - ((2*a^2 - b^2)*Sin[
c + d*x])/d - (2*a*b*Sin[c + d*x]^2)/d + ((a^2 - 2*b^2)*Sin[c + d*x]^3)/(3*
d) + (a*b*Sin[c + d*x]^4)/(2*d) + (b^2*Sin[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^2(a+x)^2\left(b^2-x^2\right)^2}{x^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^2}{x^2}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^2b^2\left(1-\frac{b^2}{2a^2}\right)+\frac{a^2b^4}{x^2}+\frac{2ab^4}{x}-4ab^2x+\left(a^2-2\right.\right.\right.}{b^3 d}$$
$$= -\frac{a^2\csc(c+dx)}{d}+\frac{2ab\log(\sin(c+dx))}{d}-\frac{\left(2a^2-b^2\right)\sin(c+}{d}$$

Mathematica [A] time = 0.05, size = 142, normalized size = 1.14

$$\frac{a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{ab\sin^4(c+dx)}{2d}-\frac{2ab\sin^2(c+dx)}{d}+\frac{2ab\log(\sin(c+dx))}{d}+\frac{b^2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a*b*Log[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x]
)/d + (b^2*Sin[c + d*x])/d - (2*a*b*Sin[c + d*x]^2)/d + (a^2*Sin[c + d*x]^3
)/(3*d) - (2*b^2*Sin[c + d*x]^3)/(3*d) + (a*b*Sin[c + d*x]^4)/(2*d) + (b^2*
Sin[c + d*x]^5)/(5*d)
```

fricas [A] time = 0.66, size = 135, normalized size = 1.08

$$-\frac{48\,b^2\cos(dx+c)^6-16\left(5\,a^2-b^2\right)\cos(dx+c)^4-480\,ab\log\left(\frac{1}{2}\,\sin(dx+c)\right)\sin(dx+c)-64\left(5\,a^2-b^2\right)\cos(}{240\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(48*b^2*cos(d*x + c)^6 - 16*(5*a^2 - b^2)*cos(d*x + c)^4 - 480*a*b*l
og(1/2*sin(d*x + c))*sin(d*x + c) - 64*(5*a^2 - b^2)*cos(d*x + c)^2 + 640*a
```

```
^2 - 128*b^2 - 15*(8*a*b*cos(d*x + c)^4 + 16*a*b*cos(d*x + c)^2 - 11*a*b)*s
in(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.24, size = 127, normalized size = 1.02

$$\frac{6\,b^2\sin\left(dx+c\right)^5+15\,ab\sin\left(dx+c\right)^4+10\,a^2\sin\left(dx+c\right)^3-20\,b^2\sin\left(dx+c\right)^3-60\,ab\sin\left(dx+c\right)^2+60\,ab\log}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*sin(d*x + c)^5 + 15*a*b*sin(d*x + c)^4 + 10*a^2*sin(d*x + c)^3
- 20*b^2*sin(d*x + c)^3 - 60*a*b*sin(d*x + c)^2 + 60*a*b*log(abs(sin(d*x +
c))) - 60*a^2*sin(d*x + c) + 30*b^2*sin(d*x + c) - 30*(2*a*b*sin(d*x + c) +
 a^2)/sin(d*x + c))/d
```

maple [A] time = 0.45, size = 185, normalized size = 1.48

$$-\frac{a^2\left(\cos^6\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}-\frac{8a^2\sin\left(dx+c\right)}{3d}-\frac{a^2\sin\left(dx+c\right)\left(\cos^4\left(dx+c\right)\right)}{d}-\frac{4a^2\sin\left(dx+c\right)\left(\cos^2\left(dx+c\right)\right)}{3d}+\frac{ab\left(\cos^4\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/sin(d*x+c)*cos(d*x+c)^6-8/3*a^2*sin(d*x+c)/d-1/d*a^2*sin(d*x+c)*co
s(d*x+c)^4-4/3/d*a^2*sin(d*x+c)*cos(d*x+c)^2+1/2/d*a*b*cos(d*x+c)^4+1/d*a*b
*cos(d*x+c)^2+2*a*b*ln(sin(d*x+c))/d+8/15*b^2*sin(d*x+c)/d+1/5/d*sin(d*x+c)
*b^2*cos(d*x+c)^4+4/15/d*sin(d*x+c)*b^2*cos(d*x+c)^2
```

maxima [A] time = 0.33, size = 105, normalized size = 0.84

$$\frac{6\,b^2\sin\left(dx+c\right)^5+15\,ab\sin\left(dx+c\right)^4-60\,ab\sin\left(dx+c\right)^2+10\left(a^2-2\,b^2\right)\sin\left(dx+c\right)^3+60\,ab\log\left(\sin\left(dx+\right.\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*(6*b^2*sin(d*x + c)^5 + 15*a*b*sin(d*x + c)^4 - 60*a*b*sin(d*x + c)^2
+ 10*(a^2 - 2*b^2)*sin(d*x + c)^3 + 60*a*b*log(sin(d*x + c)) - 30*(2*a^2 -
b^2)*sin(d*x + c) - 30*a^2/sin(d*x + c))/d
```

mupad [B] time = 12.02, size = 445, normalized size = 3.56

$$\frac{16\,a\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}-\frac{8\,a\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d}-\frac{16\,a\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{d}+\frac{8\,a\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{d}+\frac{20\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{16\,a^2}{3\,d\,s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (16*a*b*cos(c/2 + (d*x)/2)^4)/d - (8*a*b*cos(c/2 + (d*x)/2)^2)/d - (16*a*b*
cos(c/2 + (d*x)/2)^6)/d + (8*a*b*cos(c/2 + (d*x)/2)^8)/d + (20*a^2*cos(c/2
+ (d*x)/2)^3)/(3*d*sin(c/2 + (d*x)/2)) - (16*a^2*cos(c/2 + (d*x)/2)^5)/(3*d
*sin(c/2 + (d*x)/2)) + (8*a^2*cos(c/2 + (d*x)/2)^7)/(3*d*sin(c/2 + (d*x)/2)
) - (22*b^2*cos(c/2 + (d*x)/2)^3)/(3*d*sin(c/2 + (d*x)/2)) + (256*b^2*cos(c
/2 + (d*x)/2)^5)/(15*d*sin(c/2 + (d*x)/2)) - (368*b^2*cos(c/2 + (d*x)/2)^7)
/(15*d*sin(c/2 + (d*x)/2)) + (96*b^2*cos(c/2 + (d*x)/2)^9)/(5*d*sin(c/2 + (
d*x)/2)) - (32*b^2*cos(c/2 + (d*x)/2)^11)/(5*d*sin(c/2 + (d*x)/2)) - (2*a*b
*log(1/cos(c/2 + (d*x)/2)^2))/d + (2*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2)))/d - (9*a^2*cos(c/2 + (d*x)/2))/(2*d*sin(c/2 + (d*x)/2)) - (a^2*si
n(c/2 + (d*x)/2))/(2*d*cos(c/2 + (d*x)/2)) + (2*b^2*cos(c/2 + (d*x)/2))/(d*
sin(c/2 + (d*x)/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1219 $\int \cos^2(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=127

$$\frac{(a^2-2b^2)\sin^2(c+dx)}{2d}-\frac{(2a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^2(c+dx)}{2d}+\frac{2ab\sin^3(c+dx)}{3d}-\frac{4ab\sin(c+dx)}{d}-\frac{2ab\,c}{}$$

```
[Out] -2*a*b*csc(d*x+c)/d-1/2*a^2*csc(d*x+c)^2/d-(2*a^2-b^2)*ln(sin(d*x+c))/d-4*a
*b*sin(d*x+c)/d+1/2*(a^2-2*b^2)*sin(d*x+c)^2/d+2/3*a*b*sin(d*x+c)^3/d+1/4*b
^2*sin(d*x+c)^4/d
```

Rubi [A] time = 0.16, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{(a^2-2b^2)\sin^2(c+dx)}{2d}-\frac{(2a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^2(c+dx)}{2d}+\frac{2ab\sin^3(c+dx)}{3d}-\frac{4ab\sin(c+dx)}{d}-\frac{2ab\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - ((2*a^2 - b^2)*Log[S
in[c + d*x]])/d - (4*a*b*Sin[c + d*x])/d + ((a^2 - 2*b^2)*Sin[c + d*x]^2)/(
2*d) + (2*a*b*Sin[c + d*x]^3)/(3*d) + (b^2*Sin[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^3(a+x)^2\left(b^2-x^2\right)^2}{x^3}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^2}{x^3}\,dx, x, b\sin(c+dx)\right)}{b^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-4ab^2 + \frac{a^2b^4}{x^3} + \frac{2ab^4}{x^2} + \frac{-2a^2b^2+b^4}{x} + \left(a^2-2b^2\right)x + \right.\right.}{b^2 d}$$
$$= -\frac{2ab\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{2d} - \frac{\left(2a^2-b^2\right)\log(\sin(c+}{d}$$

Mathematica [A] time = 0.30, size = 103, normalized size = 0.81

$$\frac{6\left(a^2-2b^2\right)\sin^2(c+dx)+12\left(b^2-2a^2\right)\log(\sin(c+dx))-6a^2\csc^2(c+dx)+8ab\sin^3(c+dx)-48ab\sin(c+dx}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-24*a*b*Csc[c + d*x] - 6*a^2*Csc[c + d*x]^2 + 12*(-2*a^2 + b^2)*Log[Sin[c
+ d*x]] - 48*a*b*Sin[c + d*x] + 6*(a^2 - 2*b^2)*Sin[c + d*x]^2 + 8*a*b*Sin[
c + d*x]^3 + 3*b^2*Sin[c + d*x]^4)/(12*d)
```

fricas [A] time = 1.35, size = 160, normalized size = 1.26

$$\frac{24\,b^2\cos(dx+c)^6-24\left(2\,a^2-b^2\right)\cos(dx+c)^4+9\left(8\,a^2-9\,b^2\right)\cos(dx+c)^2+24\,a^2+33\,b^2-96\left(\left(2\,a^2-b^2\right)c\right.}{96\left(d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/96*(24*b^2*cos(d*x + c)^6 - 24*(2*a^2 - b^2)*cos(d*x + c)^4 + 9*(8*a^2 -
9*b^2)*cos(d*x + c)^2 + 24*a^2 + 33*b^2 - 96*((2*a^2 - b^2)*cos(d*x + c)^2
```

```
- 2*a^2 + b^2)*log(1/2*sin(d*x + c)) - 64*(a*b*cos(d*x + c)^4 + 4*a*b*cos(d
*x + c)^2 - 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.26, size = 140, normalized size = 1.10

$$\frac{3\,b^2\sin\left(dx+c\right)^4+8\,ab\sin\left(dx+c\right)^3+6\,a^2\sin\left(dx+c\right)^2-12\,b^2\sin\left(dx+c\right)^2-48\,ab\sin\left(dx+c\right)-12\left(2\,a^2-b^2\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*sin(d*x + c)^4 + 8*a*b*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)^2 -
12*b^2*sin(d*x + c)^2 - 48*a*b*sin(d*x + c) - 12*(2*a^2 - b^2)*log(abs(sin(
d*x + c))) + 6*(6*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 - 4*a*b*sin(d*x
 + c) - a^2)/sin(d*x + c)^2)/d
```

maple [A] time = 0.54, size = 197, normalized size = 1.55

$$-\frac{a^2\left(\cos^6\left(dx+c\right)\right)}{2d\sin\left(dx+c\right)^2}-\frac{a^2\left(\cos^4\left(dx+c\right)\right)}{2d}-\frac{a^2\left(\cos^2\left(dx+c\right)\right)}{d}-\frac{2a^2\ln\left(\sin\left(dx+c\right)\right)}{d}-\frac{2ab\left(\cos^6\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}-\frac{16ab\sin}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^6-1/2/d*a^2*cos(d*x+c)^4-1/d*a^2*cos(d*x
+c)^2-2*a^2*ln(sin(d*x+c))/d-2/d*a*b/sin(d*x+c)*cos(d*x+c)^6-16/3*a*b*sin(d
*x+c)/d-2/d*a*b*sin(d*x+c)*cos(d*x+c)^4-8/3/d*a*b*sin(d*x+c)*cos(d*x+c)^2+1
/4/d*b^2*cos(d*x+c)^4+1/2/d*b^2*cos(d*x+c)^2+b^2*ln(sin(d*x+c))/d
```

maxima [A] time = 0.33, size = 104, normalized size = 0.82

$$\frac{3\,b^2\sin\left(dx+c\right)^4+8\,ab\sin\left(dx+c\right)^3-48\,ab\sin\left(dx+c\right)+6\left(a^2-2\,b^2\right)\sin\left(dx+c\right)^2-12\left(2\,a^2-b^2\right)\log\left(\sin\left(d\right.\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(3*b^2*sin(d*x + c)^4 + 8*a*b*sin(d*x + c)^3 - 48*a*b*sin(d*x + c) + 6
*(a^2 - 2*b^2)*sin(d*x + c)^2 - 12*(2*a^2 - b^2)*log(sin(d*x + c)) - 6*(4*a
*b*sin(d*x + c) + a^2)/sin(d*x + c)^2)/d
```

mupad [B] time = 11.68, size = 331, normalized size = 2.61

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,\left(2\,a^2-b^2\right)}{d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(14\,a^2-16\,b^2\right)-\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(2*a^2 - b^2))/d - (a^2*tan(c/2 + (d*x)/2)^2
)/(8*d) - (2*a^2*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^6*(14*a^2 - 16*b
^2) - tan(c/2 + (d*x)/2)^8*((15*a^2)/2 - 16*b^2) - tan(c/2 + (d*x)/2)^4*(5*
a^2 - 16*b^2) + a^2/2 + 48*a*b*tan(c/2 + (d*x)/2)^3 + (296*a*b*tan(c/2 + (d
*x)/2)^5)/3 + (272*a*b*tan(c/2 + (d*x)/2)^7)/3 + 36*a*b*tan(c/2 + (d*x)/2)^
9 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 16*tan(c/2 + (d*
x)/2)^4 + 24*tan(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2)^8 + 4*tan(c/2 + (
d*x)/2)^10)) - (log(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/d - (a*b*tan(c/2 + (
d*x)/2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1220 $\int \cos(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=120

$$\frac{(a^2-2b^2)\sin(c+dx)}{d}+\frac{(2a^2-b^2)\csc(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}+\frac{ab\sin^2(c+dx)}{d}-\frac{ab\csc^2(c+dx)}{d}-\frac{4ab\log(\sin($$

```
[Out] (2*a^2-b^2)*csc(d*x+c)/d-a*b*csc(d*x+c)^2/d-1/3*a^2*csc(d*x+c)^3/d-4*a*b*ln
(sin(d*x+c))/d+(a^2-2*b^2)*sin(d*x+c)/d+a*b*sin(d*x+c)^2/d+1/3*b^2*sin(d*x+
c)^3/d
```

Rubi [A] time = 0.14, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 948}

$$\frac{(a^2-2b^2)\sin(c+dx)}{d}+\frac{(2a^2-b^2)\csc(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}+\frac{ab\sin^2(c+dx)}{d}-\frac{ab\csc^2(c+dx)}{d}-\frac{4ab\log(\sin($$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - b^2)*Csc[c + d*x])/d - (a*b*Csc[c + d*x]^2)/d - (a^2*Csc[c + d*x]
^3)/(3*d) - (4*a*b*Log[Sin[c + d*x]])/d + ((a^2 - 2*b^2)*Sin[c + d*x])/d +
(a*b*Sin[c + d*x]^2)/d + (b^2*Sin[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^4(a+x)^2\left(b^2-x^2\right)^2}{x^4}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^2}{x^4}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{2b^2}{a^2}\right)+\frac{a^2b^4}{x^4}+\frac{2ab^4}{x^3}+\frac{-2a^2b^2+b^4}{x^2}-\frac{4ab^2}{x}+2ax\right.\right.}{bd}$$
$$= \frac{\left(2a^2-b^2\right)\csc(c+dx)}{d}-\frac{ab\csc^2(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}-$$

Mathematica [A] time = 0.28, size = 103, normalized size = 0.86

$$\frac{3\left(a^2-2b^2\right)\sin(c+dx)+\left(6a^2-3b^2\right)\csc(c+dx)-a^2\csc^3(c+dx)+3ab\sin^2(c+dx)-3ab\csc^2(c+dx)-12ab}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((6*a^2 - 3*b^2)*Csc[c + d*x] - 3*a*b*Csc[c + d*x]^2 - a^2*Csc[c + d*x]^3 -
 12*a*b*Log[Sin[c + d*x]] + 3*(a^2 - 2*b^2)*Sin[c + d*x] + 3*a*b*Sin[c + d*
x]^2 + b^2*Sin[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.99, size = 158, normalized size = 1.32

$$\frac{2\,b^2\cos\left(dx+c\right)^6-6\left(a^2-b^2\right)\cos\left(dx+c\right)^4+24\left(a^2-b^2\right)\cos\left(dx+c\right)^2-24\left(ab\cos\left(dx+c\right)^2-ab\right)\log\left(\frac{1}{2}\sin\left(\right.\right.}{6\left(d\cos\left(dx+c\right)^2-d\right)\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*b^2*cos(d*x + c)^6 - 6*(a^2 - b^2)*cos(d*x + c)^4 + 24*(a^2 - b^2)*c
os(d*x + c)^2 - 24*(a*b*cos(d*x + c)^2 - a*b)*log(1/2*sin(d*x + c))*sin(d*x
```

```
 + c) - 16*a^2 + 16*b^2 - 3*(2*a*b*cos(d*x + c)^4 - 3*a*b*cos(d*x + c)^2 -
a*b)*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.28, size = 127, normalized size = 1.06

$$\frac{b^2\sin(dx+c)^3+3\,ab\sin(dx+c)^2-12\,ab\log(|\sin(dx+c)|)+3\,a^2\sin(dx+c)-6\,b^2\sin(dx+c)+\frac{22\,ab\sin(dx+}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*sin(d*x + c)^3 + 3*a*b*sin(d*x + c)^2 - 12*a*b*log(abs(sin(d*x + c
))) + 3*a^2*sin(d*x + c) - 6*b^2*sin(d*x + c) + (22*a*b*sin(d*x + c)^3 + 6*
a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 - 3*a*b*sin(d*x + c) - a^2)/sin(d
*x + c)^3)/d
```

maple [B] time = 0.50, size = 255, normalized size = 2.12

$$-\frac{a^2\left(\cos^6(dx+c)\right)}{3d\sin(dx+c)^3}+\frac{a^2\left(\cos^6(dx+c)\right)}{d\sin(dx+c)}+\frac{8a^2\sin(dx+c)}{3d}+\frac{a^2\sin(dx+c)\left(\cos^4(dx+c)\right)}{d}+\frac{4a^2\sin(dx+c)\left(\cos^2\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d*a^2/sin(d*x+c)^3*cos(d*x+c)^6+1/d*a^2/sin(d*x+c)*cos(d*x+c)^6+8/3*a^
2*sin(d*x+c)/d+1/d*a^2*sin(d*x+c)*cos(d*x+c)^4+4/3/d*a^2*sin(d*x+c)*cos(d*x
+c)^2-1/d*a*b/sin(d*x+c)^2*cos(d*x+c)^6-1/d*a*b*cos(d*x+c)^4-2/d*a*b*cos(d*
x+c)^2-4*a*b*ln(sin(d*x+c))/d-1/d*b^2/sin(d*x+c)*cos(d*x+c)^6-8/3*b^2*sin(d
*x+c)/d-1/d*sin(d*x+c)*b^2*cos(d*x+c)^4-4/3/d*sin(d*x+c)*b^2*cos(d*x+c)^2
```

maxima [A] time = 0.33, size = 103, normalized size = 0.86

$$\frac{b^2\sin(dx+c)^3+3\,ab\sin(dx+c)^2-12\,ab\log(\sin(dx+c))+3\left(a^2-2\,b^2\right)\sin(dx+c)-\frac{3\,ab\sin(dx+c)-3\left(2\,a^2-b^2\right)s}{\sin(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(b^2*sin(d*x + c)^3 + 3*a*b*sin(d*x + c)^2 - 12*a*b*log(sin(d*x + c)) +
 3*(a^2 - 2*b^2)*sin(d*x + c) - (3*a*b*sin(d*x + c) - 3*(2*a^2 - b^2)*sin(d
*x + c)^2 + a^2)/sin(d*x + c)^3)/d
```

mupad [B] time = 11.71, size = 315, normalized size = 2.62

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(6\,a^2-4\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(23\,a^2-36\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(36\,a^2-44\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{158}{3}\right.}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+24\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(6*a^2 - 4*b^2) + tan(c/2 + (d*x)/2)^8*(23*a^2 - 36*b
^2) + tan(c/2 + (d*x)/2)^4*(36*a^2 - 44*b^2) + tan(c/2 + (d*x)/2)^6*((158*a
^2)/3 - (164*b^2)/3) - a^2/3 - 6*a*b*tan(c/2 + (d*x)/2)^3 + 26*a*b*tan(c/2
+ (d*x)/2)^5 + 30*a*b*tan(c/2 + (d*x)/2)^7 - 2*a*b*tan(c/2 + (d*x)/2))/(d*(
8*tan(c/2 + (d*x)/2)^3 + 24*tan(c/2 + (d*x)/2)^5 + 24*tan(c/2 + (d*x)/2)^7
+ 8*tan(c/2 + (d*x)/2)^9)) - (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) + (tan(c/2 +
 (d*x)/2)*((7*a^2)/8 - b^2/2))/d - (a*b*tan(c/2 + (d*x)/2)^2)/(4*d) - (4*a*
b*log(tan(c/2 + (d*x)/2)))/d + (4*a*b*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1221 $\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}+\frac{(a^2-2b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^4(c+dx)}{4d}+\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}+\frac{4ab\,c}{}$$

```
[Out] 4*a*b*csc(d*x+c)/d+1/2*(2*a^2-b^2)*csc(d*x+c)^2/d-2/3*a*b*csc(d*x+c)^3/d-1/
4*a^2*csc(d*x+c)^4/d+(a^2-2*b^2)*ln(sin(d*x+c))/d+2*a*b*sin(d*x+c)/d+1/2*b^
2*sin(d*x+c)^2/d
```

Rubi [A] time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 948}

$$\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}+\frac{(a^2-2b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^4(c+dx)}{4d}+\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}+\frac{4ab\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (4*a*b*Csc[c + d*x])/d + ((2*a^2 - b^2)*Csc[c + d*x]^2)/(2*d) - (2*a*b*Csc[
c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) + ((a^2 - 2*b^2)*Log[Sin[c +
 d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d)
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^5}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(2a+\frac{a^2b^4}{x^5}+\frac{2ab^4}{x^4}+\frac{-2a^2b^2+b^4}{x^3}-\frac{4ab^2}{x^2}+\frac{a^2-2b^2}{x}+x\right)dx, x, b\sin(c\right.}{d}$$
$$= \frac{4ab\csc(c+dx)}{d}+\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c}{4d}$$

Mathematica [A] time = 0.74, size = 107, normalized size = 0.85

$$\frac{6(2a^2-b^2)\csc^2(c+dx)+6\left(2(a^2-2b^2)\log(\sin(c+dx))+4ab\sin(c+dx)+b^2\sin^2(c+dx)\right)-3a^2\csc^4(c+dx}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (48*a*b*Csc[c + d*x] + 6*(2*a^2 - b^2)*Csc[c + d*x]^2 - 8*a*b*Csc[c + d*x]^
3 - 3*a^2*Csc[c + d*x]^4 + 6*(2*(a^2 - 2*b^2)*Log[Sin[c + d*x]] + 4*a*b*Sin
[c + d*x] + b^2*Sin[c + d*x]^2))/(12*d)
```

fricas [A] time = 0.74, size = 177, normalized size = 1.40

$$-\frac{6b^2\cos(dx+c)^6-15b^2\cos(dx+c)^4+6(2a^2+b^2)\cos(dx+c)^2-9a^2+3b^2-12\left((a^2-2b^2)\cos(dx+c)^4\right.}{12\left(d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/12*(6*b^2*cos(d*x + c)^6 - 15*b^2*cos(d*x + c)^4 + 6*(2*a^2 + b^2)*cos(d
*x + c)^2 - 9*a^2 + 3*b^2 - 12*((a^2 - 2*b^2)*cos(d*x + c)^4 - 2*(a^2 - 2*b
^2)*cos(d*x + c)^2 + a^2 - 2*b^2)*log(1/2*sin(d*x + c)) - 8*(3*a*b*cos(d*x
+ c)^4 - 12*a*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^4 - 2
*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.29, size = 138, normalized size = 1.10

$$\frac{6b^2\sin(dx+c)^2+24ab\sin(dx+c)+12(a^2-2b^2)\log(|\sin(dx+c)|)-\frac{25a^2\sin(dx+c)^4-50b^2\sin(dx+c)^4-48ab\sin(dx+c)}{\sin}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(abs
(sin(d*x + c))) - (25*a^2*sin(d*x + c)^4 - 50*b^2*sin(d*x + c)^4 - 48*a*b*s
in(d*x + c)^3 - 12*a^2*sin(d*x + c)^2 + 6*b^2*sin(d*x + c)^2 + 8*a*b*sin(d*
x + c) + 3*a^2)/sin(d*x + c)^4)/d
```

maple [A] time = 0.53, size = 220, normalized size = 1.75

$$-\frac{a^2\left(\cot^4(dx+c)\right)}{4d}+\frac{a^2\left(\cot^2(dx+c)\right)}{2d}+\frac{a^2\ln(\sin(dx+c))}{d}-\frac{2ab\left(\cos^6(dx+c)\right)}{3d\sin(dx+c)^3}+\frac{2ab\left(\cos^6(dx+c)\right)}{d\sin(dx+c)}+\frac{16ab\sin}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2*cot(d*x+c)^4+1/2/d*a^2*cot(d*x+c)^2+a^2*ln(sin(d*x+c))/d-2/3/d*a
*b/sin(d*x+c)^3*cos(d*x+c)^6+2/d*a*b/sin(d*x+c)*cos(d*x+c)^6+16/3*a*b*sin(d
*x+c)/d+2/d*a*b*sin(d*x+c)*cos(d*x+c)^4+8/3/d*a*b*sin(d*x+c)*cos(d*x+c)^2-1
/2/d*b^2/sin(d*x+c)^2*cos(d*x+c)^6-1/2/d*b^2*cos(d*x+c)^4-1/d*b^2*cos(d*x+c
)^2-2*b^2*ln(sin(d*x+c))/d
```

maxima [A] time = 0.32, size = 105, normalized size = 0.83

$$\frac{6\,b^2\sin(dx+c)^2+24\,ab\sin(dx+c)+12\left(a^2-2\,b^2\right)\log(\sin(dx+c))+\frac{48\,ab\sin(dx+c)^3-8\,ab\sin(dx+c)+6\left(2\,a^2-b^2\right)\sin(}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(sin
(d*x + c)) + (48*a*b*sin(d*x + c)^3 - 8*a*b*sin(d*x + c) + 6*(2*a^2 - b^2)*
sin(d*x + c)^2 - 3*a^2)/sin(d*x + c)^4)/d
```

mupad [B] time = 11.69, size = 310, normalized size = 2.46

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{5\,a^2}{2}-2\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{23\,a^2}{4}-4\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(3\,a^2+30\,b^2\right)-\frac{a^2}{4}+\frac{76\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}}{d\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*((5*a^2)/2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*((23*a^2)/
4 - 4*b^2) + tan(c/2 + (d*x)/2)^6*(3*a^2 + 30*b^2) - a^2/4 + (76*a*b*tan(c/
2 + (d*x)/2)^3)/3 + (356*a*b*tan(c/2 + (d*x)/2)^5)/3 + 92*a*b*tan(c/2 + (d*
x)/2)^7 - (4*a*b*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2)^4 + 32*ta
n(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2)^8)) - (log(tan(c/2 + (d*x)/2)^2
+ 1)*(a^2 - 2*b^2))/d - (a^2*tan(c/2 + (d*x)/2)^4)/(64*d) + (log(tan(c/2 +
(d*x)/2))*(a^2 - 2*b^2))/d + (tan(c/2 + (d*x)/2)^2*((3*a^2)/16 - b^2/8))/d
- (a*b*tan(c/2 + (d*x)/2)^3)/(12*d) + (7*a*b*tan(c/2 + (d*x)/2))/(4*d)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1222 $\int \cot^5(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=124

$$\frac{\left(2a^2-b^2\right)\csc^3(c+dx)}{3d}-\frac{\left(a^2-2b^2\right)\csc(c+dx)}{d}-\frac{a^2\csc^5(c+dx)}{5d}-\frac{ab\csc^4(c+dx)}{2d}+\frac{2ab\csc^2(c+dx)}{d}+\frac{2ab\log(\mathrm{si}}{}$$

```
[Out] -(a^2-2*b^2)*csc(d*x+c)/d+2*a*b*csc(d*x+c)^2/d+1/3*(2*a^2-b^2)*csc(d*x+c)^3
/d-1/2*a*b*csc(d*x+c)^4/d-1/5*a^2*csc(d*x+c)^5/d+2*a*b*ln(sin(d*x+c))/d+b^2
*sin(d*x+c)/d
```

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 948}

$$\frac{\left(2a^2-b^2\right)\csc^3(c+dx)}{3d}-\frac{\left(a^2-2b^2\right)\csc(c+dx)}{d}-\frac{a^2\csc^5(c+dx)}{5d}-\frac{ab\csc^4(c+dx)}{2d}+\frac{2ab\csc^2(c+dx)}{d}+\frac{2ab\log(\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((a^2 - 2*b^2)*Csc[c + d*x])/d) + (2*a*b*Csc[c + d*x]^2)/d + ((2*a^2 - b^
2)*Csc[c + d*x]^3)/(3*d) - (a*b*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x]^5
)/(5*d) + (2*a*b*Log[Sin[c + d*x]])/d + (b^2*Sin[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^6(a+x)^2(b^2-x^2)^2}{x^6}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{b\,\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^6}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(1+\frac{a^2b^4}{x^6}+\frac{2ab^4}{x^5}+\frac{-2a^2b^2+b^4}{x^4}-\frac{4ab^2}{x^3}+\frac{a^2-2b^2}{x^2}+\frac{2a}{x}\right)\right.}{d}$$
$$= -\frac{(a^2-2b^2)\csc(c+dx)}{d}+\frac{2ab\csc^2(c+dx)}{d}+\frac{(2a^2-b^2)\csc^3}{3d}$$

Mathematica [A] time = 0.19, size = 105, normalized size = 0.85

$$\frac{10(2a^2-b^2)\csc^3(c+dx)-30(a^2-2b^2)\csc(c+dx)-6a^2\csc^5(c+dx)-15ab\csc^4(c+dx)+60ab\csc^2(c+dx)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-30*(a^2 - 2*b^2)*Csc[c + d*x] + 60*a*b*Csc[c + d*x]^2 + 10*(2*a^2 - b^2)*
Csc[c + d*x]^3 - 15*a*b*Csc[c + d*x]^4 - 6*a^2*Csc[c + d*x]^5 + 30*b*(2*a*L
og[Sin[c + d*x]] + b*Sin[c + d*x]))/(30*d)
```

fricas [A] time = 0.64, size = 166, normalized size = 1.34

$$-\frac{30\,b^2\cos(dx+c)^6+30(a^2-5\,b^2)\cos(dx+c)^4-40(a^2-5\,b^2)\cos(dx+c)^2-60(ab\cos(dx+c)^4-2\,ab\cos}{30(d\cos(dx+c)^4-2\,d\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/30*(30*b^2*cos(d*x + c)^6 + 30*(a^2 - 5*b^2)*cos(d*x + c)^4 - 40*(a^2 -
5*b^2)*cos(d*x + c)^2 - 60*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2 + a*b
```

```
)*log(1/2*sin(d*x + c))*sin(d*x + c) + 16*a^2 - 80*b^2 + 15*(4*a*b*cos(d*x
+ c)^2 - 3*a*b)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*
sin(d*x + c))
```

giac [A] time = 0.27, size = 131, normalized size = 1.06

$$\frac{60\,ab\log\left(\left|\sin\left(dx+c\right)\right|\right)+30\,b^{2}\sin\left(dx+c\right)-\frac{137\,ab\sin(dx+c)^{5}+30\,a^{2}\sin(dx+c)^{4}-60\,b^{2}\sin(dx+c)^{4}-60\,ab\sin(dx+c)^{3}-20\,a^{2}\sin(dx+c)^{2}}{\sin(dx+c)^{5}}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(60*a*b*log(abs(sin(d*x + c))) + 30*b^2*sin(d*x + c) - (137*a*b*sin(d*
x + c)^5 + 30*a^2*sin(d*x + c)^4 - 60*b^2*sin(d*x + c)^4 - 60*a*b*sin(d*x +
 c)^3 - 20*a^2*sin(d*x + c)^2 + 10*b^2*sin(d*x + c)^2 + 15*a*b*sin(d*x + c)
 + 6*a^2)/sin(d*x + c)^5)/d
```

maple [B] time = 0.54, size = 279, normalized size = 2.25

$$-\frac{a^{2}\left(\cos^{6}\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)^{5}}+\frac{a^{2}\left(\cos^{6}\left(dx+c\right)\right)}{15d\sin\left(dx+c\right)^{3}}-\frac{a^{2}\left(\cos^{6}\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)}-\frac{8a^{2}\sin\left(dx+c\right)}{15d}-\frac{a^{2}\sin\left(dx+c\right)\left(\cos^{4}\left(dx+c\right)\right)}{5d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/5/d*a^2/sin(d*x+c)^5*cos(d*x+c)^6+1/15/d*a^2/sin(d*x+c)^3*cos(d*x+c)^6-1
/5/d*a^2/sin(d*x+c)*cos(d*x+c)^6-8/15*a^2*sin(d*x+c)/d-1/5/d*a^2*sin(d*x+c)
*cos(d*x+c)^4-4/15/d*a^2*sin(d*x+c)*cos(d*x+c)^2-1/2/d*a*b*cot(d*x+c)^4+1/d
*a*b*cot(d*x+c)^2+2*a*b*ln(sin(d*x+c))/d-1/3/d*b^2/sin(d*x+c)^3*cos(d*x+c)^
6+1/d*b^2/sin(d*x+c)*cos(d*x+c)^6+8/3*b^2*sin(d*x+c)/d+1/d*sin(d*x+c)*b^2*c
os(d*x+c)^4+4/3/d*sin(d*x+c)*b^2*cos(d*x+c)^2
```

maxima [A] time = 0.32, size = 105, normalized size = 0.85

$$\frac{60\,ab\log\left(\sin\left(dx+c\right)\right)+30\,b^{2}\sin\left(dx+c\right)+\frac{60\,ab\sin(dx+c)^{3}-30\left(a^{2}-2\,b^{2}\right)\sin(dx+c)^{4}-15\,ab\sin(dx+c)+10\left(2\,a^{2}-b^{2}\right)\sin(dx+c)^{2}-6}{\sin(dx+c)^{5}}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*(60*a*b*log(sin(d*x + c)) + 30*b^2*sin(d*x + c) + (60*a*b*sin(d*x + c)
^3 - 30*(a^2 - 2*b^2)*sin(d*x + c)^4 - 15*a*b*sin(d*x + c) + 10*(2*a^2 - b^
2)*sin(d*x + c)^2 - 6*a^2)/sin(d*x + c)^5)/d
```

mupad [B] time = 11.71, size = 297, normalized size = 2.40

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{5\,a^2}{96}-\frac{b^2}{24}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a^2}{16}-\frac{7\,b^2}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(10\,a^2-92\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{22\,a^2}{15}-\frac{4\,b^2}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((5*a^2)/96 - b^2/24))/d - (tan(c/2 + (d*x)/2)*((5*a^
2)/16 - (7*b^2)/8))/d - (tan(c/2 + (d*x)/2)^6*(10*a^2 - 92*b^2) - tan(c/2 +
 (d*x)/2)^2*((22*a^2)/15 - (4*b^2)/3) + tan(c/2 + (d*x)/2)^4*((25*a^2)/3 -
(80*b^2)/3) + a^2/5 - 11*a*b*tan(c/2 + (d*x)/2)^3 - 12*a*b*tan(c/2 + (d*x)/
2)^5 + a*b*tan(c/2 + (d*x)/2))/(d*(32*tan(c/2 + (d*x)/2)^5 + 32*tan(c/2 + (
d*x)/2)^7)) - (a^2*tan(c/2 + (d*x)/2)^5)/(160*d) + (3*a*b*tan(c/2 + (d*x)/2
)^2)/(8*d) - (a*b*tan(c/2 + (d*x)/2)^4)/(32*d) + (2*a*b*log(tan(c/2 + (d*x)
/2)))/d - (2*a*b*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1223 $\int \cot^5(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=130

$$\frac{(2a^2-b^2)\csc^4(c+dx)}{4d}-\frac{(a^2-2b^2)\csc^2(c+dx)}{2d}-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2ab\csc^5(c+dx)}{5d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(}{d}$$

```
[Out] -2*a*b*csc(d*x+c)/d-1/2*(a^2-2*b^2)*csc(d*x+c)^2/d+4/3*a*b*csc(d*x+c)^3/d+1
/4*(2*a^2-b^2)*csc(d*x+c)^4/d-2/5*a*b*csc(d*x+c)^5/d-1/6*a^2*csc(d*x+c)^6/d
+b^2*ln(sin(d*x+c))/d
```

Rubi [A] time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{(2a^2-b^2)\csc^4(c+dx)}{4d}-\frac{(a^2-2b^2)\csc^2(c+dx)}{2d}-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2ab\csc^5(c+dx)}{5d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Csc[c + d*x])/d - ((a^2 - 2*b^2)*Csc[c + d*x]^2)/(2*d) + (4*a*b*Csc
[c + d*x]^3)/(3*d) + ((2*a^2 - b^2)*Csc[c + d*x]^4)/(4*d) - (2*a*b*Csc[c +
d*x]^5)/(5*d) - (a^2*Csc[c + d*x]^6)/(6*d) + (b^2*Log[Sin[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^7(a+x)^2\left(b^2-x^2\right)^2}{x^7}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{b^2\,\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^2}{x^7}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^2\,\text{Subst}\left(\int \left(\frac{a^2b^4}{x^7}+\frac{2ab^4}{x^6}+\frac{-2a^2b^2+b^4}{x^5}-\frac{4ab^2}{x^4}+\frac{a^2-2b^2}{x^3}+\frac{2a}{x^2}+\frac{1}{x}\right.$$
$$= -\frac{2ab\csc(c+dx)}{d}-\frac{\left(a^2-2b^2\right)\csc^2(c+dx)}{2d}+\frac{4ab\csc^3(c+d}{3d}$$

Mathematica [A] time = 0.17, size = 107, normalized size = 0.82

$$\frac{15\left(2a^2-b^2\right)\csc^4(c+dx)-30\left(a^2-2b^2\right)\csc^2(c+dx)-10a^2\csc^6(c+dx)-24ab\csc^5(c+dx)+80ab\csc^3(c+dx}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-120*a*b*Csc[c + d*x] - 30*(a^2 - 2*b^2)*Csc[c + d*x]^2 + 80*a*b*Csc[c + d
*x]^3 + 15*(2*a^2 - b^2)*Csc[c + d*x]^4 - 24*a*b*Csc[c + d*x]^5 - 10*a^2*Cs
c[c + d*x]^6 + 60*b^2*Log[Sin[c + d*x]])/(60*d)
```

fricas [A] time = 1.02, size = 183, normalized size = 1.41

$$\frac{30\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^4-15\left(2\,a^2-7\,b^2\right)\cos\left(dx+c\right)^2+10\,a^2-45\,b^2+60\left(b^2\cos\left(dx+c\right)^6-3\,b^2\cos\left(dx+\right.\right.}{60\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/60*(30*(a^2 - 2*b^2)*cos(d*x + c)^4 - 15*(2*a^2 - 7*b^2)*cos(d*x + c)^2 +
 10*a^2 - 45*b^2 + 60*(b^2*cos(d*x + c)^6 - 3*b^2*cos(d*x + c)^4 + 3*b^2*co
```

```
s(d*x + c)^2 - b^2)*log(1/2*sin(d*x + c)) + 8*(15*a*b*cos(d*x + c)^4 - 20*a
*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x +
c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.30, size = 134, normalized size = 1.03

$$\frac{60\,b^2\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{147\,b^2\sin(dx+c)^6+120\,ab\sin(dx+c)^5+30\,a^2\sin(dx+c)^4-60\,b^2\sin(dx+c)^4-80\,ab\sin(dx+c)^3-30\,a^2\sin(dx+c)^2+15}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*b^2*log(abs(sin(d*x + c))) - (147*b^2*sin(d*x + c)^6 + 120*a*b*sin
(d*x + c)^5 + 30*a^2*sin(d*x + c)^4 - 60*b^2*sin(d*x + c)^4 - 80*a*b*sin(d*
x + c)^3 - 30*a^2*sin(d*x + c)^2 + 15*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x +
 c) + 10*a^2)/sin(d*x + c)^6)/d
```

maple [A] time = 0.56, size = 196, normalized size = 1.51

$$-\frac{a^2\left(\cos^6\left(dx+c\right)\right)}{6d\sin\left(dx+c\right)^6}-\frac{2ab\left(\cos^6\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)^5}+\frac{2ab\left(\cos^6\left(dx+c\right)\right)}{15d\sin\left(dx+c\right)^3}-\frac{2ab\left(\cos^6\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)}-\frac{16ab\sin\left(dx+c\right)}{15d}-\frac{2ab\sin\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/6/d*a^2/sin(d*x+c)^6*cos(d*x+c)^6-2/5/d*a*b/sin(d*x+c)^5*cos(d*x+c)^6+2/
15/d*a*b/sin(d*x+c)^3*cos(d*x+c)^6-2/5/d*a*b/sin(d*x+c)*cos(d*x+c)^6-16/15*
a*b*sin(d*x+c)/d-2/5/d*a*b*sin(d*x+c)*cos(d*x+c)^4-8/15/d*a*b*sin(d*x+c)*co
s(d*x+c)^2-1/4/d*b^2*cot(d*x+c)^4+1/2/d*b^2*cot(d*x+c)^2+b^2*ln(sin(d*x+c))
/d
```

maxima [A] time = 0.34, size = 108, normalized size = 0.83

$$\frac{60\,b^2\log\left(\sin\left(dx+c\right)\right)-\frac{120\,ab\sin(dx+c)^5-80\,ab\sin(dx+c)^3+30\left(a^2-2\,b^2\right)\sin(dx+c)^4+24\,ab\sin(dx+c)-15\left(2\,a^2-b^2\right)\sin(dx+c)^2+10\,a^2}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/60*(60*b^2*log(sin(d*x + c)) - (120*a*b*sin(d*x + c)^5 - 80*a*b*sin(d*x +
 c)^3 + 30*(a^2 - 2*b^2)*sin(d*x + c)^4 + 24*a*b*sin(d*x + c) - 15*(2*a^2 -
 b^2)*sin(d*x + c)^2 + 10*a^2)/sin(d*x + c)^6)/d
```

mupad [B] time = 11.87, size = 274, normalized size = 2.11

$$\frac{b^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{5\,a^2}{2}-12\,b^2\right)+\frac{a^2}{6}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(a^2-\right.\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] (b^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*tan(c/2 + (d*x)/2)^6)/(384*d) - (cot
(c/2 + (d*x)/2)^6*(tan(c/2 + (d*x)/2)^4*((5*a^2)/2 - 12*b^2) + a^2/6 - tan(
c/2 + (d*x)/2)^2*(a^2 - b^2) - (20*a*b*tan(c/2 + (d*x)/2)^3)/3 + 40*a*b*tan
(c/2 + (d*x)/2)^5 + (4*a*b*tan(c/2 + (d*x)/2))/5))/(64*d) - (b^2*log(tan(c/
2 + (d*x)/2)^2 + 1))/d + (tan(c/2 + (d*x)/2)^4*(a^2/64 - b^2/64))/d - (tan(
c/2 + (d*x)/2)^2*((5*a^2)/128 - (3*b^2)/16))/d + (5*a*b*tan(c/2 + (d*x)/2)^
3)/(48*d) - (a*b*tan(c/2 + (d*x)/2)^5)/(80*d) - (5*a*b*tan(c/2 + (d*x)/2))/
(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1224 $\int \cot^5(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=129

$$\frac{(2a^2-b^2)\csc^5(c+dx)}{5d}-\frac{(a^2-2b^2)\csc^3(c+dx)}{3d}-\frac{a^2\csc^7(c+dx)}{7d}-\frac{ab\csc^6(c+dx)}{3d}+\frac{ab\csc^4(c+dx)}{d}-\frac{ab\csc^2(c+}{d}$$

```
[Out] -b^2*csc(d*x+c)/d-a*b*csc(d*x+c)^2/d-1/3*(a^2-2*b^2)*csc(d*x+c)^3/d+a*b*csc
(d*x+c)^4/d+1/5*(2*a^2-b^2)*csc(d*x+c)^5/d-1/3*a*b*csc(d*x+c)^6/d-1/7*a^2*c
sc(d*x+c)^7/d
```

Rubi [A] time = 0.16, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{(2a^2-b^2)\csc^5(c+dx)}{5d}-\frac{(a^2-2b^2)\csc^3(c+dx)}{3d}-\frac{a^2\csc^7(c+dx)}{7d}-\frac{ab\csc^6(c+dx)}{3d}+\frac{ab\csc^4(c+dx)}{d}-\frac{ab\csc^2(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((b^2*Csc[c + d*x])/d) - (a*b*Csc[c + d*x]^2)/d - ((a^2 - 2*b^2)*Csc[c + d
*x]^3)/(3*d) + (a*b*Csc[c + d*x]^4)/d + ((2*a^2 - b^2)*Csc[c + d*x]^5)/(5*d
) - (a*b*Csc[c + d*x]^6)/(3*d) - (a^2*Csc[c + d*x]^7)/(7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^8(a+x)^2\left(b^2-x^2\right)^2}{x^8}\,dx,x,b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{b^3\,\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^2}{x^8}\,dx,x,b\sin(c+dx)\right)}{d}$$
$$= \frac{b^3\,\text{Subst}\left(\int \left(\frac{a^2b^4}{x^8}+\frac{2ab^4}{x^7}+\frac{-2a^2b^2+b^4}{x^6}-\frac{4ab^2}{x^5}+\frac{a^2-2b^2}{x^4}+\frac{2a}{x^3}+\frac{1}{x}\right.\right.}{d}$$
$$= -\frac{b^2\csc(c+dx)}{d}-\frac{ab\csc^2(c+dx)}{d}-\frac{\left(a^2-2b^2\right)\csc^3(c+dx)}{3d}$$

Mathematica [A] time = 0.20, size = 104, normalized size = 0.81

$$-\frac{\csc(c+dx)\left(21\left(b^2-2a^2\right)\csc^4(c+dx)+35\left(a^2-2b^2\right)\csc^2(c+dx)+15a^2\csc^6(c+dx)+35ab\csc^5(c+dx)-1\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/105*(Csc[c + d*x]*(105*b^2 + 105*a*b*Csc[c + d*x] + 35*(a^2 - 2*b^2)*Csc
[c + d*x]^2 - 105*a*b*Csc[c + d*x]^3 + 21*(-2*a^2 + b^2)*Csc[c + d*x]^4 + 3
5*a*b*Csc[c + d*x]^5 + 15*a^2*Csc[c + d*x]^6))/d
```

fricas [A] time = 0.86, size = 146, normalized size = 1.13

$$-\frac{105\,b^2\cos(dx+c)^6-35\left(a^2+7\,b^2\right)\cos(dx+c)^4+28\left(a^2+7\,b^2\right)\cos(dx+c)^2-8\,a^2-56\,b^2-35\left(3\,ab\cos(dx\right.}{105\left(d\cos(dx+c)^6-3\,d\cos(dx+c)^4+3\,d\cos(dx+c)^2-d\right)\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(105*b^2*cos(d*x + c)^6 - 35*(a^2 + 7*b^2)*cos(d*x + c)^4 + 28*(a^2
+ 7*b^2)*cos(d*x + c)^2 - 8*a^2 - 56*b^2 - 35*(3*a*b*cos(d*x + c)^4 - 3*a*b
```

```
*cos(d*x + c)^2 + a*b)*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^
4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.28, size = 118, normalized size = 0.91

$$-\frac{105\,b^2\sin(dx+c)^6+105\,ab\sin(dx+c)^5+35\,a^2\sin(dx+c)^4-70\,b^2\sin(dx+c)^4-105\,ab\sin(dx+c)^3-42}{105\,d\sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/105*(105*b^2*sin(d*x + c)^6 + 105*a*b*sin(d*x + c)^5 + 35*a^2*sin(d*x +
c)^4 - 70*b^2*sin(d*x + c)^4 - 105*a*b*sin(d*x + c)^3 - 42*a^2*sin(d*x + c)
^2 + 21*b^2*sin(d*x + c)^2 + 35*a*b*sin(d*x + c) + 15*a^2)/(d*sin(d*x + c)^
7)
```

maple [A] time = 0.56, size = 218, normalized size = 1.69

$$\frac{a^2\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{ab\left(\cos^6(dx+c)\right)}{3\sin(dx+c)^6}+b^2\left(\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/1
05/sin(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x
+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/3*a*b/sin(d*x+c)^6*cos(d*x+c)^6+b^2*(
-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c
)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.33, size = 106, normalized size = 0.82

$$-\frac{105\,b^2\sin(dx+c)^6+105\,ab\sin(dx+c)^5-105\,ab\sin(dx+c)^3+35\left(a^2-2\,b^2\right)\sin(dx+c)^4+35\,ab\sin(dx+}{105\,d\sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/105*(105*b^2*sin(d*x + c)^6 + 105*a*b*sin(d*x + c)^5 - 105*a*b*sin(d*x +
 c)^3 + 35*(a^2 - 2*b^2)*sin(d*x + c)^4 + 35*a*b*sin(d*x + c) - 21*(2*a^2 -
 b^2)*sin(d*x + c)^2 + 15*a^2)/(d*sin(d*x + c)^7)
```

mupad [B] time = 11.79, size = 105, normalized size = 0.81

$$-\frac{\frac{a^2}{7}+\sin\left(c+d\,x\right)^4\,\left(\frac{a^2}{3}-\frac{2\,b^2}{3}\right)-\sin\left(c+d\,x\right)^2\,\left(\frac{2\,a^2}{5}-\frac{b^2}{5}\right)+b^2\,\sin\left(c+d\,x\right)^6+\frac{a\,b\,\sin(c+d\,x)}{3}-a\,b\,\sin\left(c+d\,x\right)^3+a}{d\,\sin\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^8,x)
```

```
[Out] -(a^2/7 + sin(c + d*x)^4*(a^2/3 - (2*b^2)/3) - sin(c + d*x)^2*((2*a^2)/5 -
b^2/5) + b^2*sin(c + d*x)^6 + (a*b*sin(c + d*x))/3 - a*b*sin(c + d*x)^3 + a
*b*sin(c + d*x)^5)/(d*sin(c + d*x)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**8*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1225 $\int \cot^5(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=138

$$\frac{(2a^2-b^2)\csc^6(c+dx)}{6d}-\frac{(a^2-2b^2)\csc^4(c+dx)}{4d}-\frac{a^2\csc^8(c+dx)}{8d}-\frac{2ab\csc^7(c+dx)}{7d}+\frac{4ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^3}{3}$$

```
[Out] -1/2*b^2*csc(d*x+c)^2/d-2/3*a*b*csc(d*x+c)^3/d-1/4*(a^2-2*b^2)*csc(d*x+c)^4
/d+4/5*a*b*csc(d*x+c)^5/d+1/6*(2*a^2-b^2)*csc(d*x+c)^6/d-2/7*a*b*csc(d*x+c)
^7/d-1/8*a^2*csc(d*x+c)^8/d
```

Rubi [A] time = 0.16, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{(2a^2-b^2)\csc^6(c+dx)}{6d}-\frac{(a^2-2b^2)\csc^4(c+dx)}{4d}-\frac{a^2\csc^8(c+dx)}{8d}-\frac{2ab\csc^7(c+dx)}{7d}+\frac{4ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(b^2*Csc[c + d*x]^2)/(2*d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - ((a^2 - 2*b^2)
*Csc[c + d*x]^4)/(4*d) + (4*a*b*Csc[c + d*x]^5)/(5*d) + ((2*a^2 - b^2)*Csc[
c + d*x]^6)/(6*d) - (2*a*b*Csc[c + d*x]^7)/(7*d) - (a^2*Csc[c + d*x]^8)/(8*
d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^9(a+x)^2\left(b^2-x^2\right)^2}{x^9}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{b^4\,\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^2}{x^9}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^4\,\text{Subst}\left(\int \left(\frac{a^2b^4}{x^9}+\frac{2ab^4}{x^8}+\frac{-2a^2b^2+b^4}{x^7}-\frac{4ab^2}{x^6}+\frac{a^2-2b^2}{x^5}+\frac{2a}{x^4}+\frac{1}{x}\right.\right.}{d}$$
$$= -\frac{b^2\csc^2(c+dx)}{2d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{\left(a^2-2b^2\right)\csc^4(c+d}{4d}$$

Mathematica [A] time = 0.25, size = 108, normalized size = 0.78

$$-\frac{\csc^2(c+dx)\left(-140\left(2a^2-b^2\right)\csc^4(c+dx)+210\left(a^2-2b^2\right)\csc^2(c+dx)+105a^2\csc^6(c+dx)+240ab\csc^5(c+\right.}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/840*(Csc[c + d*x]^2*(420*b^2 + 560*a*b*Csc[c + d*x] + 210*(a^2 - 2*b^2)*
Csc[c + d*x]^2 - 672*a*b*Csc[c + d*x]^3 - 140*(2*a^2 - b^2)*Csc[c + d*x]^4
+ 240*a*b*Csc[c + d*x]^5 + 105*a^2*Csc[c + d*x]^6))/d
```

fricas [A] time = 0.74, size = 148, normalized size = 1.07

$$\frac{420\,b^2\cos(dx+c)^6-210\left(a^2+4\,b^2\right)\cos(dx+c)^4+140\left(a^2+4\,b^2\right)\cos(dx+c)^2-35\,a^2-140\,b^2-16\left(35\,ab\cos\right.}{840\left(d\cos(dx+c)^8-4\,d\cos(dx+c)^6+6\,d\cos(dx+c)^4-4\,d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/840*(420*b^2*cos(d*x + c)^6 - 210*(a^2 + 4*b^2)*cos(d*x + c)^4 + 140*(a^2
 + 4*b^2)*cos(d*x + c)^2 - 35*a^2 - 140*b^2 - 16*(35*a*b*cos(d*x + c)^4 - 2
```

```
8*a*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x
 + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.33, size = 118, normalized size = 0.86

$$-\frac{420\,b^2\sin\left(dx+c\right)^6+560\,ab\sin\left(dx+c\right)^5+210\,a^2\sin\left(dx+c\right)^4-420\,b^2\sin\left(dx+c\right)^4-672\,ab\sin\left(dx+c\right)^3-}{840\,d\sin\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(420*b^2*sin(d*x + c)^6 + 560*a*b*sin(d*x + c)^5 + 210*a^2*sin(d*x +
 c)^4 - 420*b^2*sin(d*x + c)^4 - 672*a*b*sin(d*x + c)^3 - 280*a^2*sin(d*x +
 c)^2 + 140*b^2*sin(d*x + c)^2 + 240*a*b*sin(d*x + c) + 105*a^2)/(d*sin(d*x
 + c)^8)
```

maple [A] time = 0.56, size = 173, normalized size = 1.25

$$\frac{a^2\left(-\frac{\cos^6(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^6(dx+c)}{24\sin(dx+c)^6}\right)+2ab\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(d}{3}\right.}{35}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/8/sin(d*x+c)^8*cos(d*x+c)^6-1/24/sin(d*x+c)^6*cos(d*x+c)^6)+2*
a*b*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/105/si
n(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c)^4
+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/6*b^2/sin(d*x+c)^6*cos(d*x+c)^6)
```

maxima [A] time = 0.33, size = 106, normalized size = 0.77

$$-\frac{420\,b^2\sin\left(dx+c\right)^6+560\,ab\sin\left(dx+c\right)^5-672\,ab\sin\left(dx+c\right)^3+210\left(a^2-2\,b^2\right)\sin\left(dx+c\right)^4+240\,ab\sin\left(d\right.}{840\,d\sin\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/840*(420*b^2*sin(d*x + c)^6 + 560*a*b*sin(d*x + c)^5 - 672*a*b*sin(d*x +
 c)^3 + 210*(a^2 - 2*b^2)*sin(d*x + c)^4 + 240*a*b*sin(d*x + c) - 140*(2*a^
2 - b^2)*sin(d*x + c)^2 + 105*a^2)/(d*sin(d*x + c)^8)
```

mupad [B] time = 11.78, size = 107, normalized size = 0.78

$$-\frac{\frac{a^2}{8}+\sin\left(c+d\,x\right)^4\,\left(\frac{a^2}{4}-\frac{b^2}{2}\right)-\sin\left(c+d\,x\right)^2\,\left(\frac{a^2}{3}-\frac{b^2}{6}\right)+\frac{b^2\,\sin\left(c+d\,x\right)^6}{2}+\frac{2\,a\,b\,\sin\left(c+d\,x\right)}{7}-\frac{4\,a\,b\,\sin\left(c+d\,x\right)^3}{5}+\frac{2\,a\,b\,\sin\left(c+d\,x\right)}{3}}{d\,\sin\left(c+d\,x\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^9,x)
```

```
[Out] -(a^2/8 + sin(c + d*x)^4*(a^2/4 - b^2/2) - sin(c + d*x)^2*(a^2/3 - b^2/6) +
 (b^2*sin(c + d*x)^6)/2 + (2*a*b*sin(c + d*x))/7 - (4*a*b*sin(c + d*x)^3)/5
 + (2*a*b*sin(c + d*x)^5)/3)/(d*sin(c + d*x)^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**9*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1226 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=235

$$-\frac{4a\left(a^2-b^2\right)\sin^3(c+dx)}{3b^5d}+\frac{\left(3a^2-2b^2\right)\sin^4(c+dx)}{4b^4d}+\frac{a^2\left(7a^4-10a^2b^2+3b^4\right)\log(a+b\sin(c+dx))}{b^8d}-\frac{2a\left(3a^4-\right.}{}$$

```
[Out] a^2*(7*a^4-10*a^2*b^2+3*b^4)*ln(a+b*sin(d*x+c))/b^8/d-2*a*(3*a^4-4*a^2*b^2+
b^4)*sin(d*x+c)/b^7/d+1/2*(5*a^4-6*a^2*b^2+b^4)*sin(d*x+c)^2/b^6/d-4/3*a*(a
^2-b^2)*sin(d*x+c)^3/b^5/d+1/4*(3*a^2-2*b^2)*sin(d*x+c)^4/b^4/d-2/5*a*sin(d
*x+c)^5/b^3/d+1/6*sin(d*x+c)^6/b^2/d+a^3*(a^2-b^2)^2/b^8/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.28, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{\left(3a^2-2b^2\right)\sin^4(c+dx)}{4b^4d}-\frac{4a\left(a^2-b^2\right)\sin^3(c+dx)}{3b^5d}+\frac{\left(-6a^2b^2+5a^4+b^4\right)\sin^2(c+dx)}{2b^6d}-\frac{2a\left(-4a^2b^2+3a^4+b^4\right)}{b^7d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(7*a^4 - 10*a^2*b^2 + 3*b^4)*Log[a + b*Sin[c + d*x]])/(b^8*d) - (2*a*(
3*a^4 - 4*a^2*b^2 + b^4)*Sin[c + d*x])/(b^7*d) + ((5*a^4 - 6*a^2*b^2 + b^4)
*Sin[c + d*x]^2)/(2*b^6*d) - (4*a*(a^2 - b^2)*Sin[c + d*x]^3)/(3*b^5*d) + (
(3*a^2 - 2*b^2)*Sin[c + d*x]^4)/(4*b^4*d) - (2*a*Sin[c + d*x]^5)/(5*b^3*d)
+ Sin[c + d*x]^6/(6*b^2*d) + (a^3*(a^2 - b^2)^2)/(b^8*d*(a + b*Sin[c + d*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)^2}{b^3(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^8 d}$$

$$= \frac{\text{Subst}\left(\int \left(-2a\left(3a^4-4a^2b^2+b^4\right)+\left(5a^4-6a^2b^2+b^4\right)x-4a\left(a^2-b^2\right)x^2+\left(3\right.\right.\right.}{b^8 d}$$

$$= \frac{a^2\left(7a^4-10a^2b^2+3b^4\right)\log(a+b\sin(c+dx))}{b^8 d} - \frac{2a\left(3a^4-4a^2b^2+b^4\right)\sin(c+}{b^7 d}$$

Mathematica [A] time = 2.19, size = 264, normalized size = 1.12

$$\frac{\left(50ab^6-35a^3b^4\right)\sin^4(c+dx)+60a^2b\left(a^2-b^2\right)\sin(c+dx)\left(\left(7a^2-3b^2\right)\log(a+b\sin(c+dx))-6a^2+2b^2\right)+3b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (60*a^3*(a^2 - b^2)*(a^2 - b^2 + (7*a^2 - 3*b^2)*Log[a + b*Sin[c + d*x]]) +
 60*a^2*b*(a^2 - b^2)*(-6*a^2 + 2*b^2 + (7*a^2 - 3*b^2)*Log[a + b*Sin[c + d
*x]])*Sin[c + d*x] - 30*a*b^2*(7*a^4 - 10*a^2*b^2 + 3*b^4)*Sin[c + d*x]^2 +
 10*b^3*(7*a^4 - 10*a^2*b^2 + 3*b^4)*Sin[c + d*x]^3 + (-35*a^3*b^4 + 50*a*b
^6)*Sin[c + d*x]^4 + 3*b^5*(7*a^2 - 10*b^2)*Sin[c + d*x]^5 - 14*a*b^6*Sin[c
 + d*x]^6 + 10*b^7*Sin[c + d*x]^7)/(60*b^8*d*(a + b*Sin[c + d*x]))
```

fricas [A] time = 0.86, size = 279, normalized size = 1.19

$$\frac{112\,ab^6\cos(dx+c)^6+480\,a^7-3240\,a^5b^2+3185\,a^3b^4-487\,ab^6-8\left(35\,a^3b^4-8\,ab^6\right)\cos(dx+c)^4+16\left(105\,a^5b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/480*(112*a*b^6*cos(d*x + c)^6 + 480*a^7 - 3240*a^5*b^2 + 3185*a^3*b^4 - 4
87*a*b^6 - 8*(35*a^3*b^4 - 8*a*b^6)*cos(d*x + c)^4 + 16*(105*a^5*b^2 - 115*
a^3*b^4 + 16*a*b^6)*cos(d*x + c)^2 + 480*(7*a^7 - 10*a^5*b^2 + 3*a^3*b^4 +
(7*a^6*b - 10*a^4*b^3 + 3*a^2*b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) -
(80*b^7*cos(d*x + c)^6 - 168*a^2*b^5*cos(d*x + c)^4 + 2880*a^6*b - 3800*a^4
*b^3 + 1007*a^2*b^5 - 25*b^7 + 16*(35*a^4*b^3 - 29*a^2*b^5)*cos(d*x + c)^2)
*sin(d*x + c))/(b^9*d*sin(d*x + c) + a*b^8*d)
```

giac [A] time = 0.23, size = 300, normalized size = 1.28

$$\frac{\frac{60\left(7a^6-10a^4b^2+3a^2b^4\right)\log(|b\sin(dx+c)+a|)}{b^8}-\frac{60\left(7a^6b\sin(dx+c)-10a^4b^3\sin(dx+c)+3a^2b^5\sin(dx+c)+6a^7-8a^5b^2+2a^3b^4\right)}{(b\sin(dx+c)+a)b^8}+\frac{10b^{10}\sin(dx+c)}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*(7*a^6 - 10*a^4*b^2 + 3*a^2*b^4)*log(abs(b*sin(d*x + c) + a))/b^8
- 60*(7*a^6*b*sin(d*x + c) - 10*a^4*b^3*sin(d*x + c) + 3*a^2*b^5*sin(d*x +
c) + 6*a^7 - 8*a^5*b^2 + 2*a^3*b^4)/((b*sin(d*x + c) + a)*b^8) + (10*b^10*s
in(d*x + c)^6 - 24*a*b^9*sin(d*x + c)^5 + 45*a^2*b^8*sin(d*x + c)^4 - 30*b^
10*sin(d*x + c)^4 - 80*a^3*b^7*sin(d*x + c)^3 + 80*a*b^9*sin(d*x + c)^3 + 1
50*a^4*b^6*sin(d*x + c)^2 - 180*a^2*b^8*sin(d*x + c)^2 + 30*b^10*sin(d*x +
c)^2 - 360*a^5*b^5*sin(d*x + c) + 480*a^3*b^7*sin(d*x + c) - 120*a*b^9*sin(
d*x + c))/b^12)/d
```

maple [A] time = 0.55, size = 342, normalized size = 1.46

$$\frac{\sin^6(dx+c)}{6b^2d}-\frac{2a\left(\sin^5(dx+c)\right)}{5b^3d}+\frac{3\left(\sin^4(dx+c)\right)a^2}{4d\,b^4}-\frac{\sin^4(dx+c)}{2b^2d}-\frac{4\left(\sin^3(dx+c)\right)a^3}{3d\,b^5}+\frac{4a\left(\sin^3(dx+c)\right)}{3b^3d}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/6*sin(d*x+c)^6/b^2/d-2/5*a*sin(d*x+c)^5/b^3/d+3/4/d/b^4*sin(d*x+c)^4*a^2-
1/2*sin(d*x+c)^4/b^2/d-4/3/d/b^5*sin(d*x+c)^3*a^3+4/3*a*sin(d*x+c)^3/b^3/d+
5/2/d/b^6*sin(d*x+c)^2*a^4-3/d/b^4*sin(d*x+c)^2*a^2+1/2*sin(d*x+c)^2/b^2/d-
6/d/b^7*a^5*sin(d*x+c)+8/d/b^5*a^3*sin(d*x+c)-2*a*sin(d*x+c)/b^3/d+7/d*a^6/
b^8*ln(a+b*sin(d*x+c))-10/d*a^4/b^6*ln(a+b*sin(d*x+c))+3/d*a^2/b^4*ln(a+b*s
```

```
in(d*x+c))+1/d*a^7/b^8/(a+b*sin(d*x+c))-2/d*a^5/b^6/(a+b*sin(d*x+c))+1/d*a^
3/b^4/(a+b*sin(d*x+c))
```

maxima [A] time = 0.32, size = 218, normalized size = 0.93

$$\frac{\frac{60\left(a^{7}-2\,a^{5}b^{2}+a^{3}b^{4}\right)}{b^{9}\sin(dx+c)+ab^{8}}+\frac{10\,b^{5}\sin(dx+c)^{6}-24\,ab^{4}\sin(dx+c)^{5}+15\left(3\,a^{2}b^{3}-2\,b^{5}\right)\sin(dx+c)^{4}-80\left(a^{3}b^{2}-ab^{4}\right)\sin(dx+c)^{3}+30\left(5\,a^{4}b-6\,a^{2}b^{3}+b^{5}\right)\sin(dx+c)^{2}\cdots}{b^{7}}\cdots}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/60*(60*(a^7 - 2*a^5*b^2 + a^3*b^4)/(b^9*sin(d*x + c) + a*b^8) + (10*b^5*s
in(d*x + c)^6 - 24*a*b^4*sin(d*x + c)^5 + 15*(3*a^2*b^3 - 2*b^5)*sin(d*x +
c)^4 - 80*(a^3*b^2 - a*b^4)*sin(d*x + c)^3 + 30*(5*a^4*b - 6*a^2*b^3 + b^5)
*sin(d*x + c)^2 - 120*(3*a^5 - 4*a^3*b^2 + a*b^4)*sin(d*x + c))/b^7 + 60*(7
*a^6 - 10*a^4*b^2 + 3*a^2*b^4)*log(b*sin(d*x + c) + a)/b^8)/d
```

mupad [B] time = 0.14, size = 375, normalized size = 1.60

$$\frac{\sin(c+d\,x)^{3}\left(\frac{2\,a^{3}}{3\,b^{5}}+\frac{2\,a\left(\frac{2}{b^{2}}-\frac{3\,a^{2}}{b^{4}}\right)}{3\,b}\right)}{d}-\frac{\sin(c+d\,x)^{4}\left(\frac{1}{2\,b^{2}}-\frac{3\,a^{2}}{4\,b^{4}}\right)}{d}+\frac{\sin(c+d\,x)^{2}\left(\frac{1}{2\,b^{2}}+\frac{a^{2}\left(\frac{2}{b^{2}}-\frac{3\,a^{2}}{b^{4}}\right)}{2\,b^{2}}-\frac{a\left(\frac{2\,a^{3}}{b^{5}}+\frac{2\,a\left(\frac{2}{b^{2}}-\frac{3\,a^{2}}{b^{4}}\right)}{b}\right)}{b}\cdots\right)}{d}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^3)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^3*((2*a^3)/(3*b^5) + (2*a*(2/b^2 - (3*a^2)/b^4))/(3*b)))/d -
(sin(c + d*x)^4*(1/(2*b^2) - (3*a^2)/(4*b^4)))/d + (sin(c + d*x)^2*(1/(2*b^
2) + (a^2*(2/b^2 - (3*a^2)/b^4))/(2*b^2) - (a*((2*a^3)/b^5 + (2*a*(2/b^2 -
(3*a^2)/b^4))/b))/b))/d - (sin(c + d*x)*((a^2*((2*a^3)/b^5 + (2*a*(2/b^2 -
(3*a^2)/b^4))/b))/b^2 + (2*a*(1/b^2 + (a^2*(2/b^2 - (3*a^2)/b^4))/b^2 - (2*
a*((2*a^3)/b^5 + (2*a*(2/b^2 - (3*a^2)/b^4))/b))/b))/b))/d + sin(c + d*x)^6
/(6*b^2*d) - (2*a*sin(c + d*x)^5)/(5*b^3*d) + (log(a + b*sin(c + d*x))*(7*a
```

```
^6 + 3*a^2*b^4 - 10*a^4*b^2))/(b^8*d) + (a^7 + a^3*b^4 - 2*a^5*b^2)/(b*d*(a
*b^7 + b^8*sin(c + d*x)))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1227 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=193

$$-\frac{\left(2-\frac{3a^2}{b^2}\right)\sin^3(c+dx)}{3b^2d}-\frac{a^2\left(a^2-b^2\right)^2}{b^7d(a+b\sin(c+dx))}-\frac{2a\left(a^2-b^2\right)\sin^2(c+dx)}{b^5d}-\frac{2a\left(3a^4-4a^2b^2+b^4\right)\log(a+b\sin(c}{b^7d}$$

```
[Out] -2*a*(3*a^4-4*a^2*b^2+b^4)*ln(a+b*sin(d*x+c))/b^7/d+(5*a^4-6*a^2*b^2+b^4)*s
in(d*x+c)/b^6/d-2*a*(a^2-b^2)*sin(d*x+c)^2/b^5/d-1/3*(2-3/b^2*a^2)*sin(d*x+
c)^3/b^2/d-1/2*a*sin(d*x+c)^4/b^3/d+1/5*sin(d*x+c)^5/b^2/d-a^2*(a^2-b^2)^2/
b^7/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.24, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$-\frac{\left(2-\frac{3a^2}{b^2}\right)\sin^3(c+dx)}{3b^2d}-\frac{2a\left(a^2-b^2\right)\sin^2(c+dx)}{b^5d}+\frac{\left(-6a^2b^2+5a^4+b^4\right)\sin(c+dx)}{b^6d}-\frac{a^2\left(a^2-b^2\right)^2}{b^7d(a+b\sin(c+dx))}-\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*(3*a^4 - 4*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(b^7*d) + ((5*a^4
- 6*a^2*b^2 + b^4)*Sin[c + d*x])/(b^6*d) - (2*a*(a^2 - b^2)*Sin[c + d*x]^2)
/(b^5*d) - ((2 - (3*a^2)/b^2)*Sin[c + d*x]^3)/(3*b^2*d) - (a*Sin[c + d*x]^4
)/(2*b^3*d) + Sin[c + d*x]^5/(5*b^2*d) - (a^2*(a^2 - b^2)^2)/(b^7*d*(a + b*
Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(b^2-x^2\right)^2}{b^2(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(b^2-x^2\right)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(5a^4\left(1+\frac{-6a^2b^2+b^4}{5a^4}\right)-4a\left(a^2-b^2\right)x+\left(3a^2-2b^2\right)x^2-2ax^3+x^4+\right.\right.}{b^7 d}$$

$$= -\frac{2a\left(3a^4-4a^2b^2+b^4\right)\log(a+b\sin(c+dx))}{b^7 d}+\frac{\left(5a^4-6a^2b^2+b^4\right)\sin(c+d}{b^6 d}$$

Mathematica [A] time = 1.49, size = 225, normalized size = 1.17

$$\frac{\left(40ab^5-30a^3b^3\right)\sin^3(c+dx)-30ab\left(a^2-b^2\right)\sin(c+dx)\left(\left(6a^2-2b^2\right)\log(a+b\sin(c+dx))-5a^2+b^2\right)-30a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-30*a^2*(a^2 - b^2)*(a^2 - b^2 + (6*a^2 - 2*b^2)*Log[a + b*Sin[c + d*x]])
- 30*a*b*(a^2 - b^2)*(-5*a^2 + b^2 + (6*a^2 - 2*b^2)*Log[a + b*Sin[c + d*x]
])*Sin[c + d*x] + 30*b^2*(3*a^4 - 4*a^2*b^2 + b^4)*Sin[c + d*x]^2 + (-30*a^
3*b^3 + 40*a*b^5)*Sin[c + d*x]^3 + 5*b^4*(3*a^2 - 4*b^2)*Sin[c + d*x]^4 - 9
*a*b^5*Sin[c + d*x]^5 + 6*b^6*Sin[c + d*x]^6)/(30*b^7*d*(a + b*Sin[c + d*x]
))
```

fricas [A] time = 1.05, size = 246, normalized size = 1.27

$$-\frac{48\,b^6\cos\left(dx+c\right)^6+240\,a^6-1440\,a^4b^2+1275\,a^2b^4-128\,b^6-8\left(15\,a^2b^4-2\,b^6\right)\cos\left(dx+c\right)^4+16\left(45\,a^4b^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(48*b^6*cos(d*x + c)^6 + 240*a^6 - 1440*a^4*b^2 + 1275*a^2*b^4 - 128
*b^6 - 8*(15*a^2*b^4 - 2*b^6)*cos(d*x + c)^4 + 16*(45*a^4*b^2 - 45*a^2*b^4
+ 4*b^6)*cos(d*x + c)^2 + 480*(3*a^6 - 4*a^4*b^2 + a^2*b^4 + (3*a^5*b - 4*a
^3*b^3 + a*b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) + (72*a*b^5*cos(d*x +
 c)^4 - 1200*a^5*b + 1440*a^3*b^3 - 293*a*b^5 - 16*(15*a^3*b^3 - 11*a*b^5)*
cos(d*x + c)^2)*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d)
```

giac [A] time = 0.20, size = 249, normalized size = 1.29

$$-\frac{\frac{60\left(3a^{5}-4a^{3}b^{2}+ab^{4}\right)\log(|b\sin(dx+c)+a|)}{b^{7}}-\frac{30\left(6a^{5}b\sin(dx+c)-8a^{3}b^{3}\sin(dx+c)+2ab^{5}\sin(dx+c)+5a^{6}-6a^{4}b^{2}+a^{2}b^{4}\right)}{(b\sin(dx+c)+a)b^{7}}-\frac{6b^{8}\sin(dx+c)^{5}-15ab^{7}\text{s}}{}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(60*(3*a^5 - 4*a^3*b^2 + a*b^4)*log(abs(b*sin(d*x + c) + a))/b^7 - 30
*(6*a^5*b*sin(d*x + c) - 8*a^3*b^3*sin(d*x + c) + 2*a*b^5*sin(d*x + c) + 5*
a^6 - 6*a^4*b^2 + a^2*b^4)/((b*sin(d*x + c) + a)*b^7) - (6*b^8*sin(d*x + c)
^5 - 15*a*b^7*sin(d*x + c)^4 + 30*a^2*b^6*sin(d*x + c)^3 - 20*b^8*sin(d*x +
 c)^3 - 60*a^3*b^5*sin(d*x + c)^2 + 60*a*b^7*sin(d*x + c)^2 + 150*a^4*b^4*s
in(d*x + c) - 180*a^2*b^6*sin(d*x + c) + 30*b^8*sin(d*x + c))/b^10)/d
```

maple [A] time = 0.52, size = 285, normalized size = 1.48

$$\frac{\sin^{5}(dx+c)}{5b^{2}d}-\frac{a\left(\sin^{4}(dx+c)\right)}{2b^{3}d}+\frac{\left(\sin^{3}(dx+c)\right)a^{2}}{d\,b^{4}}-\frac{2\left(\sin^{3}(dx+c)\right)}{3b^{2}d}-\frac{2\left(\sin^{2}(dx+c)\right)a^{3}}{d\,b^{5}}+\frac{2a\left(\sin^{2}(dx+c)\right)}{b^{3}d}+\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/5*sin(d*x+c)^5/b^2/d-1/2*a*sin(d*x+c)^4/b^3/d+1/d/b^4*sin(d*x+c)^3*a^2-2/
3*sin(d*x+c)^3/b^2/d-2/d/b^5*sin(d*x+c)^2*a^3+2*a*sin(d*x+c)^2/b^3/d+5/d/b^
6*a^4*sin(d*x+c)-6/d/b^4*sin(d*x+c)*a^2+sin(d*x+c)/b^2/d-6/d*a^5/b^7*ln(a+b
*sin(d*x+c))+8/d*a^3/b^5*ln(a+b*sin(d*x+c))-2*a*ln(a+b*sin(d*x+c))/b^3/d-1/
d*a^6/b^7/(a+b*sin(d*x+c))+2/d*a^4/b^5/(a+b*sin(d*x+c))-1/d*a^2/b^3/(a+b*si
n(d*x+c))
```

maxima [A] time = 0.33, size = 184, normalized size = 0.95

$$-\frac{\frac{30\left(a^{6}-2a^{4}b^{2}+a^{2}b^{4}\right)}{b^{8}\sin(dx+c)+ab^{7}}-\frac{6b^{4}\sin(dx+c)^{5}-15ab^{3}\sin(dx+c)^{4}+10\left(3a^{2}b^{2}-2b^{4}\right)\sin(dx+c)^{3}-60\left(a^{3}b-ab^{3}\right)\sin(dx+c)^{2}+30\left(5a^{4}-6a^{2}b^{2}+b^{4}\right)\sin(dx+c)}{b^{6}}-}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/30*(30*(a^6 - 2*a^4*b^2 + a^2*b^4)/(b^8*sin(d*x + c) + a*b^7) - (6*b^4*s
in(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 10*(3*a^2*b^2 - 2*b^4)*sin(d*x +
c)^3 - 60*(a^3*b - a*b^3)*sin(d*x + c)^2 + 30*(5*a^4 - 6*a^2*b^2 + b^4)*sin
(d*x + c))/b^6 + 60*(3*a^5 - 4*a^3*b^2 + a*b^4)*log(b*sin(d*x + c) + a)/b^7
)/d
```

mupad [B] time = 11.46, size = 254, normalized size = 1.32

$$\frac{\sin\left(c+d\,x\right)^2\,\left(\frac{a^3}{b^5}+\frac{a\,\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)}{b}\right)}{d}-\frac{\sin\left(c+d\,x\right)^3\,\left(\frac{2}{3\,b^2}-\frac{a^2}{b^4}\right)}{d}+\frac{\sin\left(c+d\,x\right)\,\left(\frac{1}{b^2}+\frac{a^2\,\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)}{b^2}-\frac{2\,a\,\left(\frac{2\,a^3}{b^5}+\frac{2\,a\,\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)}{b}\right)}{b}\right)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^2*(a^3/b^5 + (a*(2/b^2 - (3*a^2)/b^4))/b))/d - (sin(c + d*x)^
3*(2/(3*b^2) - a^2/b^4))/d + (sin(c + d*x)*(1/b^2 + (a^2*(2/b^2 - (3*a^2)/b
^4))/b^2 - (2*a*((2*a^3)/b^5 + (2*a*(2/b^2 - (3*a^2)/b^4))/b))/b))/d + sin(
c + d*x)^5/(5*b^2*d) - (a*sin(c + d*x)^4)/(2*b^3*d) - (log(a + b*sin(c + d*
x))*(2*a*b^4 + 6*a^5 - 8*a^3*b^2))/(b^7*d) - (a^6 + a^2*b^4 - 2*a^4*b^2)/(b
*d*(a*b^6 + b^7*sin(c + d*x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1228 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=157

$$\frac{a\left(a^2-b^2\right)^2}{b^6d(a+b\sin(c+dx))}-\frac{4a\left(a^2-b^2\right)\sin(c+dx)}{b^5d}+\frac{\left(3a^2-2b^2\right)\sin^2(c+dx)}{2b^4d}+\frac{\left(5a^4-6a^2b^2+b^4\right)\log(a+b\sin(c+}{b^6d}$$

```
[Out] (5*a^4-6*a^2*b^2+b^4)*ln(a+b*sin(d*x+c))/b^6/d-4*a*(a^2-b^2)*sin(d*x+c)/b^5
/d+1/2*(3*a^2-2*b^2)*sin(d*x+c)^2/b^4/d-2/3*a*sin(d*x+c)^3/b^3/d+1/4*sin(d*
x+c)^4/b^2/d+a*(a^2-b^2)^2/b^6/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 772}

$$\frac{\left(3a^2-2b^2\right)\sin^2(c+dx)}{2b^4d}-\frac{4a\left(a^2-b^2\right)\sin(c+dx)}{b^5d}+\frac{a\left(a^2-b^2\right)^2}{b^6d(a+b\sin(c+dx))}+\frac{\left(-6a^2b^2+5a^4+b^4\right)\log(a+b\sin(c-}{b^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((5*a^4 - 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(b^6*d) - (4*a*(a^2 - b
^2)*Sin[c + d*x])/(b^5*d) + ((3*a^2 - 2*b^2)*Sin[c + d*x]^2)/(2*b^4*d) - (2
*a*Sin[c + d*x]^3)/(3*b^3*d) + Sin[c + d*x]^4/(4*b^2*d) + (a*(a^2 - b^2)^2)
/(b^6*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)^2}{b(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(-4\left(a^3-ab^2\right)+\left(3a^2-2b^2\right)x-2ax^2+x^3-\frac{a(a^2-b^2)^2}{(a+x)^2}+\frac{5a^4-6a^2b^2+b^4}{a+x}\right.\right.}{b^6 d}$$
$$= \frac{\left(5a^4-6a^2b^2+b^4\right)\log(a+b\sin(c+dx))}{b^6 d}-\frac{4a\left(a^2-b^2\right)\sin(c+dx)}{b^5 d}+\frac{\left(3a^2-\right.}{}$$

Mathematica [A] time = 1.04, size = 188, normalized size = 1.20

$$\frac{-6ab^2\left(5a^2-6b^2\right)\sin^2(c+dx)+12b\left(b^2-a^2\right)\sin(c+dx)\left(\left(b^2-5a^2\right)\log(a+b\sin(c+dx))+4a^2\right)+12a\left(a^2-\right.}{12b^6 d(a\ }$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (12*a*(a^2 - b^2)*(a^2 - b^2 + (5*a^2 - b^2)*Log[a + b*Sin[c + d*x]]) + 12*
b*(-a^2 + b^2)*(4*a^2 + (-5*a^2 + b^2)*Log[a + b*Sin[c + d*x]])*Sin[c + d*x
] - 6*a*b^2*(5*a^2 - 6*b^2)*Sin[c + d*x]^2 + 2*b^3*(5*a^2 - 6*b^2)*Sin[c +
d*x]^3 - 5*a*b^4*Sin[c + d*x]^4 + 3*b^5*Sin[c + d*x]^5)/(12*b^6*d*(a + b*Si
n[c + d*x]))
```

fricas [A] time = 1.04, size = 203, normalized size = 1.29

$$-\frac{40\,ab^4\cos(dx+c)^4-96\,a^5+504\,a^3b^2-383\,ab^4-16\left(15\,a^3b^2-13\,ab^4\right)\cos(dx+c)^2-96\left(5\,a^5-6\,a^3b^2+ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/96*(40*a*b^4*cos(d*x + c)^4 - 96*a^5 + 504*a^3*b^2 - 383*a*b^4 - 16*(15*
a^3*b^2 - 13*a*b^4)*cos(d*x + c)^2 - 96*(5*a^5 - 6*a^3*b^2 + a*b^4 + (5*a^4
*b - 6*a^2*b^3 + b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) - (24*b^5*cos(d
*x + c)^4 - 384*a^4*b + 392*a^2*b^3 - 33*b^5 - 16*(5*a^2*b^3 - 3*b^5)*cos(d
*x + c)^2)*sin(d*x + c))/(b^7*d*sin(d*x + c) + a*b^6*d)
```

giac [A] time = 0.19, size = 194, normalized size = 1.24

$$\frac{\frac{12\left(5a^4-6a^2b^2+b^4\right)\log(|b\sin(dx+c)+a|)}{b^6}-\frac{12\left(5a^4b\sin(dx+c)-6a^2b^3\sin(dx+c)+b^5\sin(dx+c)+4a^5-4a^3b^2\right)}{(b\sin(dx+c)+a)b^6}+\frac{3b^6\sin(dx+c)^4-8ab^5\sin(dx+c)^3+1}{}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(12*(5*a^4 - 6*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/b^6 - 12*(5
*a^4*b*sin(d*x + c) - 6*a^2*b^3*sin(d*x + c) + b^5*sin(d*x + c) + 4*a^5 - 4
*a^3*b^2)/((b*sin(d*x + c) + a)*b^6) + (3*b^6*sin(d*x + c)^4 - 8*a*b^5*sin(
d*x + c)^3 + 18*a^2*b^4*sin(d*x + c)^2 - 12*b^6*sin(d*x + c)^2 - 48*a^3*b^3
*sin(d*x + c) + 48*a*b^5*sin(d*x + c))/b^8)/d
```

maple [A] time = 0.55, size = 229, normalized size = 1.46

$$\frac{\sin^4(dx+c)}{4b^2d}-\frac{2a\left(\sin^3(dx+c)\right)}{3b^3d}+\frac{3\left(\sin^2(dx+c)\right)a^2}{2d\,b^4}-\frac{\sin^2(dx+c)}{b^2d}-\frac{4a^3\sin(dx+c)}{d\,b^5}+\frac{4a\sin(dx+c)}{b^3d}+\frac{5a^4\ln(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4*sin(d*x+c)^4/b^2/d-2/3*a*sin(d*x+c)^3/b^3/d+3/2/d/b^4*sin(d*x+c)^2*a^2-
sin(d*x+c)^2/b^2/d-4/d/b^5*a^3*sin(d*x+c)+4*a*sin(d*x+c)/b^3/d+5/d*a^4/b^6*
ln(a+b*sin(d*x+c))-6/d*a^2/b^4*ln(a+b*sin(d*x+c))+1/d/b^2*ln(a+b*sin(d*x+c)
)+1/d*a^5/b^6/(a+b*sin(d*x+c))-2/d*a^3/b^4/(a+b*sin(d*x+c))+1/d*a/b^2/(a+b*
sin(d*x+c))
```

maxima [A] time = 0.33, size = 148, normalized size = 0.94

$$\frac{\frac{12\left(a^5-2a^3b^2+ab^4\right)}{b^7\sin(dx+c)+ab^6}+\frac{3b^3\sin(dx+c)^4-8ab^2\sin(dx+c)^3+6\left(3a^2b-2b^3\right)\sin(dx+c)^2-48\left(a^3-ab^2\right)\sin(dx+c)}{b^5}+\frac{12\left(5a^4-6a^2b^2+b^4\right)\log(b\sin(dx+c)+a}{b^6}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

[Out] 1/12*(12*(a^5 - 2*a^3*b^2 + a*b^4)/(b^7*sin(d*x + c) + a*b^6) + (3*b^3*sin(
d*x + c)^4 - 8*a*b^2*sin(d*x + c)^3 + 6*(3*a^2*b - 2*b^3)*sin(d*x + c)^2 -
48*(a^3 - a*b^2)*sin(d*x + c))/b^5 + 12*(5*a^4 - 6*a^2*b^2 + b^4)*log(b*sin
(d*x + c) + a)/b^6)/d

mupad [B] time = 0.09, size = 161, normalized size = 1.03

$$\frac{\frac{\sin(c+d\,x)^4}{4\,b^2}-\sin\left(c+d\,x\right)^2\,\left(\frac{1}{b^2}-\frac{3\,a^2}{2\,b^4}\right)+\sin\left(c+d\,x\right)\,\left(\frac{2\,a^3}{b^5}+\frac{2\,a\,\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)}{b}\right)-\frac{2\,a\,\sin(c+d\,x)^3}{3\,b^3}+\frac{a^5-2\,a^3\,b^2+a\,b^4}{b\,\left(\sin(c+d\,x)\,b^6+a\,b^5\right)}+\frac{\ln(a+}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^5*sin(c + d*x))/(a + b*sin(c + d*x))^2,x)

[Out] (sin(c + d*x)^4/(4*b^2) - sin(c + d*x)^2*(1/b^2 - (3*a^2)/(2*b^4)) + sin(c
+ d*x)*((2*a^3)/b^5 + (2*a*(2/b^2 - (3*a^2)/b^4))/b) - (2*a*sin(c + d*x)^3)
/(3*b^3) + (a*b^4 + a^5 - 2*a^3*b^2)/(b*(a*b^5 + b^6*sin(c + d*x))) + (log(
a + b*sin(c + d*x))*(5*a^4 + b^4 - 6*a^2*b^2))/b^6)/d

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)

[Out] Timed out

3.1229 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{(a^2-b^2)^2}{ab^4d(a+b\sin(c+dx))}+\frac{(3a^2+b^2)(a^2-b^2)\log(a+b\sin(c+dx))}{a^2b^4d}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{2a\sin(c+dx)}{b^3d}+\frac{\sin^2(c+}{2b^2d}$$

```
[Out] ln(sin(d*x+c))/a^2/d+(a^2-b^2)*(3*a^2+b^2)*ln(a+b*sin(d*x+c))/a^2/b^4/d-2*a
*sin(d*x+c)/b^3/d+1/2*sin(d*x+c)^2/b^2/d+(a^2-b^2)^2/a/b^4/d/(a+b*sin(d*x+c
))
```

Rubi [A] time = 0.16, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{(a^2-b^2)^2}{ab^4d(a+b\sin(c+dx))}+\frac{(3a^2+b^2)(a^2-b^2)\log(a+b\sin(c+dx))}{a^2b^4d}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{2a\sin(c+dx)}{b^3d}+\frac{\sin^2(c+}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^2*d) + ((a^2 - b^2)*(3*a^2 + b^2)*Log[a + b*Sin[c + d*
x]])/(a^2*b^4*d) - (2*a*Sin[c + d*x])/(b^3*d) + Sin[c + d*x]^2/(2*b^2*d) +
(a^2 - b^2)^2/(a*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b\left(b^2-x^2\right)^2}{x(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{b^4}{a^2x}+x-\frac{\left(a^2-b^2\right)^2}{a(a+x)^2}+\frac{\left(a^2-b^2\right)\left(3a^2+b^2\right)}{a^2(a+x)}\right)dx, x, b\sin(c+dx)\right)}{b^4 d}$$
$$= \frac{\log(\sin(c+dx))}{a^2 d}+\frac{\left(a^2-b^2\right)\left(3a^2+b^2\right)\log(a+b\sin(c+dx))}{a^2b^4d}-\frac{2a\sin(c+dx}{b^3d}$$

Mathematica [A] time = 0.52, size = 111, normalized size = 0.92

$$\frac{\frac{2\left(a^2-b^2\right)^2}{ab^4(a+b\sin(c+dx))}+\frac{2(a-b)(a+b)\left(3a^2+b^2\right)\log(a+b\sin(c+dx))}{a^2b^4}+\frac{2\log(\sin(c+dx))}{a^2}-\frac{4a\sin(c+dx)}{b^3}+\frac{\sin^2(c+dx)}{b^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*Log[Sin[c + d*x]])/a^2 + (2*(a - b)*(a + b)*(3*a^2 + b^2)*Log[a + b*Sin
[c + d*x]])/(a^2*b^4) - (4*a*Sin[c + d*x])/b^3 + Sin[c + d*x]^2/b^2 + (2*(a
^2 - b^2)^2)/(a*b^4*(a + b*Sin[c + d*x])))/(2*d)
```

fricas [A] time = 0.81, size = 189, normalized size = 1.58

$$\frac{6\,a^3b^2\cos\left(dx+c\right)^2+4\,a^5-15\,a^3b^2+4\,ab^4+4\left(3\,a^5-2\,a^3b^2-ab^4+\left(3\,a^4b-2\,a^2b^3-b^5\right)\sin\left(dx+c\right)\right)\log\left(b\,s\right.}{4\left(a^2b^5d\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*a^3*b^2*cos(d*x + c)^2 + 4*a^5 - 15*a^3*b^2 + 4*a*b^4 + 4*(3*a^5 - 2
*a^3*b^2 - a*b^4 + (3*a^4*b - 2*a^2*b^3 - b^5)*sin(d*x + c))*log(b*sin(d*x
+ c) + a) + 4*(b^5*sin(d*x + c) + a*b^4)*log(-1/2*sin(d*x + c)) - (2*a^2*b^
3*cos(d*x + c)^2 + 8*a^4*b - a^2*b^3)*sin(d*x + c))/(a^2*b^5*d*sin(d*x + c)
 + a^3*b^4*d)
```

giac [A] time = 0.19, size = 154, normalized size = 1.28

$$\frac{\frac{2\log(|\sin(dx+c)|)}{a^2}+\frac{b^2\sin(dx+c)^2-4ab\sin(dx+c)}{b^4}+\frac{2\left(3a^4-2a^2b^2-b^4\right)\log(|b\sin(dx+c)+a|)}{a^2b^4}-\frac{2\left(3a^4b\sin(dx+c)-2a^2b^3\sin(dx+c)-b^5\sin(dx+c)+\cdots\right.}{(b\sin(dx+c)+a)a^2b^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(sin(d*x + c)))/a^2 + (b^2*sin(d*x + c)^2 - 4*a*b*sin(d*x + c
))/b^4 + 2*(3*a^4 - 2*a^2*b^2 - b^4)*log(abs(b*sin(d*x + c) + a))/(a^2*b^4)
 - 2*(3*a^4*b*sin(d*x + c) - 2*a^2*b^3*sin(d*x + c) - b^5*sin(d*x + c) + 2*
a^5 - 2*a*b^4)/((b*sin(d*x + c) + a)*a^2*b^4))/d
```

maple [A] time = 0.76, size = 169, normalized size = 1.41

$$\frac{\sin^2(dx+c)}{2b^2d}-\frac{2a\sin(dx+c)}{b^3d}+\frac{a^3}{d\,b^4\,(a+b\sin(dx+c))}-\frac{2a}{d\,b^2\,(a+b\sin(dx+c))}+\frac{1}{ad\,(a+b\sin(dx+c))}+\frac{3a^2\ln(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2*sin(d*x+c)^2/b^2/d-2*a*sin(d*x+c)/b^3/d+1/d*a^3/b^4/(a+b*sin(d*x+c))-2/
d*a/b^2/(a+b*sin(d*x+c))+1/a/d/(a+b*sin(d*x+c))+3/d*a^2/b^4*ln(a+b*sin(d*x+
c))-2/d/b^2*ln(a+b*sin(d*x+c))-ln(a+b*sin(d*x+c))/a^2/d+ln(sin(d*x+c))/a^2/
d
```

maxima [A] time = 0.33, size = 118, normalized size = 0.98

$$\frac{\frac{2\left(a^4-2a^2b^2+b^4\right)}{ab^5\sin(dx+c)+a^2b^4}+\frac{2\log(\sin(dx+c))}{a^2}+\frac{b\sin(dx+c)^2-4a\sin(dx+c)}{b^3}+\frac{2\left(3a^4-2a^2b^2-b^4\right)\log(b\sin(dx+c)+a)}{a^2b^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^4 - 2*a^2*b^2 + b^4)/(a*b^5*sin(d*x + c) + a^2*b^4) + 2*log(sin(d
*x + c))/a^2 + (b*sin(d*x + c)^2 - 4*a*sin(d*x + c))/b^3 + 2*(3*a^4 - 2*a^2
*b^2 - b^4)*log(b*sin(d*x + c) + a)/(a^2*b^4))/d
```

mupad [B] time = 12.08, size = 338, normalized size = 2.82

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\frac{6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b^2}+\frac{6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{b^2}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(3\,a^4-3\,a^2\,b^2+b^4\right)}{a^2\,b^3}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(3\,a^4-2\,a^2\,b^2+b^4\right)}{a^2\,b^3}+}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\,a\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - ((6*a*tan(c/2 + (d*x)/2)^2)/b^2 + (6*a*ta
n(c/2 + (d*x)/2)^4)/b^2 + (4*tan(c/2 + (d*x)/2)^3*(3*a^4 + b^4 - 3*a^2*b^2)
)/(a^2*b^3) + (2*tan(c/2 + (d*x)/2)^5*(3*a^4 + b^4 - 2*a^2*b^2))/(a^2*b^3)
+ (2*tan(c/2 + (d*x)/2)*(3*a^4 + b^4 - 2*a^2*b^2))/(a^2*b^3))/(d*(a + 2*b*t
an(c/2 + (d*x)/2) + 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 + a
*tan(c/2 + (d*x)/2)^6 + 4*b*tan(c/2 + (d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/2)^5
)) - (log(tan(c/2 + (d*x)/2)^2 + 1)*(3*a^2 - 2*b^2))/(b^4*d) - (log(a + 2*b
*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(b^4 - 3*a^4 + 2*a^2*b^2))/(a
^2*b^4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1230 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{2b\log(\sin(c+dx))}{a^3d}-\frac{\left(a^2-b^2\right)^2}{a^2b^3d(a+b\sin(c+dx))}-\frac{\csc(c+dx)}{a^2d}-\frac{2\left(a^4-b^4\right)\log(a+b\sin(c+dx))}{a^3b^3d}+\frac{\sin(c+dx)}{b^2d}$$

```
[Out] -csc(d*x+c)/a^2/d-2*b*ln(sin(d*x+c))/a^3/d-2*(a^4-b^4)*ln(a+b*sin(d*x+c))/a
^3/b^3/d+sin(d*x+c)/b^2/d-(a^2-b^2)^2/a^2/b^3/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$-\frac{\left(a^2-b^2\right)^2}{a^2b^3d(a+b\sin(c+dx))}-\frac{2\left(a^4-b^4\right)\log(a+b\sin(c+dx))}{a^3b^3d}-\frac{2b\log(\sin(c+dx))}{a^3d}-\frac{\csc(c+dx)}{a^2d}+\frac{\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]/(a^2*d)) - (2*b*Log[Sin[c + d*x]])/(a^3*d) - (2*(a^4 - b^4)*
Log[a + b*Sin[c + d*x]])/(a^3*b^3*d) + Sin[c + d*x]/(b^2*d) - (a^2 - b^2)^2
/(a^2*b^3*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^2\left(b^2-x^2\right)^2}{x^2(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^2(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{b^4}{a^2x^2}-\frac{2b^4}{a^3x}+\frac{\left(a^2-b^2\right)^2}{a^2(a+x)^2}-\frac{2\left(a^4-b^4\right)}{a^3(a+x)}\right) dx, x, b\sin(c+dx)\right)}{b^3 d}$$

$$= -\frac{\csc(c+dx)}{a^2 d}-\frac{2b\log(\sin(c+dx))}{a^3 d}-\frac{2\left(a^4-b^4\right)\log(a+b\sin(c+dx))}{a^3b^3 d}+\frac{\text{si}}{}$$

Mathematica [A] time = 0.62, size = 95, normalized size = 0.87

$$-\frac{2\left(\frac{a}{b^3}-\frac{b}{a^3}\right)\log(a+b\sin(c+dx))+\frac{2b\log(\sin(c+dx))}{a^3}+\frac{\left(a^2-b^2\right)^2}{a^2b^3(a+b\sin(c+dx))}+\frac{\csc(c+dx)}{a^2}-\frac{\sin(c+dx)}{b^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((Csc[c + d*x]/a^2 + (2*b*Log[Sin[c + d*x]])/a^3 + 2*(a/b^3 - b/a^3)*Log[a
 + b*Sin[c + d*x]] - Sin[c + d*x]/b^2 + (a^2 - b^2)^2/(a^2*b^3*(a + b*Sin[c
 + d*x])))/d)
```

fricas [A] time = 0.80, size = 214, normalized size = 1.96

$$\frac{a^4b\cos(dx+c)^2-a^4b+a^2b^3+2\left(a^4b-b^5-\left(a^4b-b^5\right)\cos(dx+c)^2+\left(a^5-ab^4\right)\sin(dx+c)\right)\log(b\sin(dx+}{a^3b^4d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (a^4*b*cos(d*x + c)^2 - a^4*b + a^2*b^3 + 2*(a^4*b - b^5 - (a^4*b - b^5)*co
s(d*x + c)^2 + (a^5 - a*b^4)*sin(d*x + c))*log(b*sin(d*x + c) + a) - 2*(b^5
*cos(d*x + c)^2 - a*b^4*sin(d*x + c) - b^5)*log(1/2*sin(d*x + c)) + (a^3*b^
2*cos(d*x + c)^2 + a^5 - 3*a^3*b^2 + 2*a*b^4)*sin(d*x + c))/(a^3*b^4*d*cos(
d*x + c)^2 - a^4*b^3*d*sin(d*x + c) - a^3*b^4*d)
```

giac [A] time = 0.22, size = 131, normalized size = 1.20

$$-\frac{\frac{2\,b\log\left(|\sin\left(dx+c\right)|\right)}{a^{3}}-\frac{\sin\left(dx+c\right)}{b^{2}}-\frac{a^{3}\sin\left(dx+c\right)^{2}+2\,a^{2}b\sin\left(dx+c\right)-2\,b^{3}\sin\left(dx+c\right)-ab^{2}}{\left(b\sin\left(dx+c\right)^{2}+a\sin\left(dx+c\right)\right)a^{2}b^{2}}+\frac{2\left(a^{4}-b^{4}\right)\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^{3}b^{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*b*log(abs(sin(d*x + c)))/a^3 - sin(d*x + c)/b^2 - (a^3*sin(d*x + c)^2 +
 2*a^2*b*sin(d*x + c) - 2*b^3*sin(d*x + c) - a*b^2)/((b*sin(d*x + c)^2 + a*
sin(d*x + c))*a^2*b^2) + 2*(a^4 - b^4)*log(abs(b*sin(d*x + c) + a))/(a^3*b^
3))/d
```

maple [A] time = 0.80, size = 151, normalized size = 1.39

$$\frac{\sin\left(dx+c\right)}{b^{2}d}-\frac{2a\ln\left(a+b\sin\left(dx+c\right)\right)}{b^{3}d}+\frac{2b\ln\left(a+b\sin\left(dx+c\right)\right)}{d\,a^{3}}-\frac{a^{2}}{d\,b^{3}\left(a+b\sin\left(dx+c\right)\right)}+\frac{2}{bd\left(a+b\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] sin(d*x+c)/b^2/d-2*a*ln(a+b*sin(d*x+c))/b^3/d+2/d/a^3*b*ln(a+b*sin(d*x+c))-
1/d*a^2/b^3/(a+b*sin(d*x+c))+2/b/d/(a+b*sin(d*x+c))-1/d*b/a^2/(a+b*sin(d*x+
c))-1/d/a^2/sin(d*x+c)-2*b*ln(sin(d*x+c))/a^3/d
```

maxima [A] time = 0.33, size = 120, normalized size = 1.10

$$-\frac{\frac{ab^{3}+\left(a^{4}-2\,a^{2}b^{2}+2\,b^{4}\right)\sin\left(dx+c\right)}{a^{2}b^{4}\sin\left(dx+c\right)^{2}+a^{3}b^{3}\sin\left(dx+c\right)}+\frac{2\,b\log\left(\sin\left(dx+c\right)\right)}{a^{3}}-\frac{\sin\left(dx+c\right)}{b^{2}}+\frac{2\left(a^{4}-b^{4}\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^{3}b^{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -((a*b^3 + (a^4 - 2*a^2*b^2 + 2*b^4)*sin(d*x + c))/(a^2*b^4*sin(d*x + c)^2
+ a^3*b^3*sin(d*x + c)) + 2*b*log(sin(d*x + c))/a^3 - sin(d*x + c)/b^2 + 2*
(a^4 - b^4)*log(b*sin(d*x + c) + a)/(a^3*b^3))/d
```

mupad [B] time = 12.10, size = 313, normalized size = 2.87

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(4\,a^2-b^2\right)}{b}-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(4\,a^4-5\,a^2\,b^2+2\,b^4\right)}{a\,b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(8\,a^4-9\,a^2\,b^2+4\,b^4\right)}{a\,b^2}}{d\left(2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,b\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,b\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{t}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^3*(4*a^2 - b^2))/b - 2*b*tan(c/2 + (d*x)/2) - a + (2
*tan(c/2 + (d*x)/2)^2*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(a*b^2) + (tan(c/2 + (d*
x)/2)^4*(8*a^4 + 4*b^4 - 9*a^2*b^2))/(a*b^2))/(d*(4*a^3*tan(c/2 + (d*x)/2)^
3 + 2*a^3*tan(c/2 + (d*x)/2)^5 + 2*a^3*tan(c/2 + (d*x)/2) + 4*a^2*b*tan(c/2
 + (d*x)/2)^2 + 4*a^2*b*tan(c/2 + (d*x)/2)^4)) - tan(c/2 + (d*x)/2)/(2*a^2*
d) + (2*a*log(tan(c/2 + (d*x)/2)^2 + 1))/(b^3*d) - (2*b*log(tan(c/2 + (d*x)
/2)))/(a^3*d) - (2*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)
*(a^4 - b^4))/(a^3*b^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1231 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=131

$$\frac{2b\csc(c+dx)}{a^3d}-\frac{\csc^2(c+dx)}{2a^2d}-\frac{\left(2a^2-3b^2\right)\log(\sin(c+dx))}{a^4d}+\frac{\left(a^4+2a^2b^2-3b^4\right)\log(a+b\sin(c+dx))}{a^4b^2d}+\frac{(a\cdots}{a^3b^2d(a\cdots}$$

```
[Out] 2*b*csc(d*x+c)/a^3/d-1/2*csc(d*x+c)^2/a^2/d-(2*a^2-3*b^2)*ln(sin(d*x+c))/a^
4/d+(a^4+2*a^2*b^2-3*b^4)*ln(a+b*sin(d*x+c))/a^4/b^2/d+(a^2-b^2)^2/a^3/b^2/
d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.20, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$\frac{\left(a^2-b^2\right)^2}{a^3b^2d(a+b\sin(c+dx))}-\frac{\left(2a^2-3b^2\right)\log(\sin(c+dx))}{a^4d}+\frac{\left(2a^2b^2+a^4-3b^4\right)\log(a+b\sin(c+dx))}{a^4b^2d}+\frac{2b\csc(c+dx\cdots}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^2*d) - ((2*a^2 - 3*b^2)*Lo
g[Sin[c + d*x]])/(a^4*d) + ((a^4 + 2*a^2*b^2 - 3*b^4)*Log[a + b*Sin[c + d*x
]])/(a^4*b^2*d) + (a^2 - b^2)^2/(a^3*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^3\left(b^2-x^2\right)^2}{x^3(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^3(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^2d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^2x^3} - \frac{2b^4}{a^3x^2} + \frac{-2a^2b^2+3b^4}{a^4x} - \frac{\left(a^2-b^2\right)^2}{a^3(a+x)^2} + \frac{a^4+2a^2b^2-3b^4}{a^4(a+x)}\right) dx, x, b\sin(c+dx\right.}{b^2d}$$

$$= \frac{2b\csc(c+dx)}{a^3d} - \frac{\csc^2(c+dx)}{2a^2d} - \frac{\left(2a^2-3b^2\right)\log(\sin(c+dx))}{a^4d} + \frac{\left(a^4+2a^2b^2\right.}{}$$

Mathematica [A] time = 0.76, size = 116, normalized size = 0.89

$$\frac{\frac{2a\left(a^2-b^2\right)^2}{b^2(a+b\sin(c+dx))} - 2\left(2a^2-3b^2\right)\log(\sin(c+dx)) - a^2\csc^2(c+dx) + \frac{2\left(a^4+2a^2b^2-3b^4\right)\log(a+b\sin(c+dx))}{b^2} + 4ab\csc(c+d}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (4*a*b*Csc[c + d*x] - a^2*Csc[c + d*x]^2 - 2*(2*a^2 - 3*b^2)*Log[Sin[c + d*
x]] + (2*(a^4 + 2*a^2*b^2 - 3*b^4)*Log[a + b*Sin[c + d*x]])/b^2 + (2*a*(a^2
 - b^2)^2)/(b^2*(a + b*Sin[c + d*x])))/(2*a^4*d)
```

fricas [B] time = 1.16, size = 335, normalized size = 2.56

$$-\frac{3\,a^2b^3\sin(dx+c) + 2\,a^5 - 5\,a^3b^2 + 6\,ab^4 - 2\left(a^5 - 2\,a^3b^2 + 3\,ab^4\right)\cos(dx+c)^2 + 2\left(a^5 + 2\,a^3b^2 - 3\,ab^4 - \left(a^5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(3*a^2*b^3*sin(d*x + c) + 2*a^5 - 5*a^3*b^2 + 6*a*b^4 - 2*(a^5 - 2*a^3
*b^2 + 3*a*b^4)*cos(d*x + c)^2 + 2*(a^5 + 2*a^3*b^2 - 3*a*b^4 - (a^5 + 2*a^
3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + (a^4*b + 2*a^2*b^3 - 3*b^5 - (a^4*b + 2*a
^2*b^3 - 3*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) - 2*(
2*a^3*b^2 - 3*a*b^4 - (2*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + (2*a^2*b^3 - 3
*b^5 - (2*a^2*b^3 - 3*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*sin(d*x +
 c)))/(a^5*b^2*d*cos(d*x + c)^2 - a^5*b^2*d + (a^4*b^3*d*cos(d*x + c)^2 - a
^4*b^3*d)*sin(d*x + c))
```

giac [A] time = 0.23, size = 190, normalized size = 1.45

$$-\frac{\frac{2\left(2a^2-3b^2\right)\log(|\sin(dx+c)|)}{a^4}-\frac{2\left(a^4+2a^2b^2-3b^4\right)\log(|b\sin(dx+c)+a|)}{a^4b^2}+\frac{2\left(a^4\sin(dx+c)+2a^2b^2\sin(dx+c)-3b^4\sin(dx+c)+4a^3b-4ab^3\right)}{(b\sin(dx+c)+a)a^4b}-\frac{6a^2}{}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(2*a^2 - 3*b^2)*log(abs(sin(d*x + c)))/a^4 - 2*(a^4 + 2*a^2*b^2 - 3
*b^4)*log(abs(b*sin(d*x + c) + a))/(a^4*b^2) + 2*(a^4*sin(d*x + c) + 2*a^2*
b^2*sin(d*x + c) - 3*b^4*sin(d*x + c) + 4*a^3*b - 4*a*b^3)/((b*sin(d*x + c)
 + a)*a^4*b) - (6*a^2*sin(d*x + c)^2 - 9*b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x
 + c) - a^2)/(a^4*sin(d*x + c)^2))/d
```

maple [A] time = 0.88, size = 189, normalized size = 1.44

$$\frac{a}{d\,b^2\,(a+b\sin(dx+c))}-\frac{2}{ad\,(a+b\sin(dx+c))}+\frac{b^2}{d\,a^3\,(a+b\sin(dx+c))}+\frac{\ln(a+b\sin(dx+c))}{d\,b^2}+\frac{2\ln(a+b\sin(}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a/b^2/(a+b*sin(d*x+c))-2/a/d/(a+b*sin(d*x+c))+1/d/a^3*b^2/(a+b*sin(d*x+
c))+1/d/b^2*ln(a+b*sin(d*x+c))+2*ln(a+b*sin(d*x+c))/a^2/d-3/d/a^4*b^2*ln(a+
b*sin(d*x+c))-1/2/d/a^2/sin(d*x+c)^2-2*ln(sin(d*x+c))/a^2/d+3/d/a^4*ln(sin(
d*x+c))*b^2+2/d/a^3*b/sin(d*x+c)
```

maxima [A] time = 0.34, size = 147, normalized size = 1.12

$$\frac{\frac{3ab^3\sin(dx+c)-a^2b^2+2\left(a^4-2a^2b^2+3b^4\right)\sin(dx+c)^2}{a^3b^3\sin(dx+c)^3+a^4b^2\sin(dx+c)^2}-\frac{2\left(2a^2-3b^2\right)\log(\sin(dx+c))}{a^4}+\frac{2\left(a^4+2a^2b^2-3b^4\right)\log(b\sin(dx+c)+a)}{a^4b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/2*((3*a*b^3*sin(d*x + c) - a^2*b^2 + 2*(a^4 - 2*a^2*b^2 + 3*b^4)*sin(d*x
+ c)^2)/(a^3*b^3*sin(d*x + c)^3 + a^4*b^2*sin(d*x + c)^2) - 2*(2*a^2 - 3*b^
2)*log(sin(d*x + c))/a^4 + 2*(a^4 + 2*a^2*b^2 - 3*b^4)*log(b*sin(d*x + c) +
 a)/(a^4*b^2))/d
```

mupad [B] time = 11.92, size = 280, normalized size = 2.14

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{2}-8\,b^2\right)+\frac{a^2}{2}-3\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^4-5\,a^2\,b^2+2\,b^4\right)}{a\,b}}{d\,\left(4\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+8\,b\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\right.\right.}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^3*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(a^3*d) - (tan(c/2 + (d*x)/2)^2*(a^2/2 - 8*b^2) + a^
2/2 - 3*a*b*tan(c/2 + (d*x)/2) + (4*tan(c/2 + (d*x)/2)^3*(2*a^4 + 2*b^4 - 5
*a^2*b^2))/(a*b))/(d*(4*a^4*tan(c/2 + (d*x)/2)^2 + 4*a^4*tan(c/2 + (d*x)/2)
^4 + 8*a^3*b*tan(c/2 + (d*x)/2)^3)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(b^2*d)
 - tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (log(tan(c/2 + (d*x)/2))*(2*a^2 - 3*b^2
))/(a^4*d) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^4
 - 3*b^4 + 2*a^2*b^2))/(a^4*b^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1232 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=147

$$\frac{b\csc^2(c+dx)}{a^3d}-\frac{\csc^3(c+dx)}{3a^2d}+\frac{4b\left(a^2-b^2\right)\log(\sin(c+dx))}{a^5d}-\frac{4b\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{a^5d}-\frac{\left(a^2-b^2\right)^2}{a^4bd(a+b\sin(c+dx))}$$

```
[Out] (2*a^2-3*b^2)*csc(d*x+c)/a^4/d+b*csc(d*x+c)^2/a^3/d-1/3*csc(d*x+c)^3/a^2/d+
4*b*(a^2-b^2)*ln(sin(d*x+c))/a^5/d-4*b*(a^2-b^2)*ln(a+b*sin(d*x+c))/a^5/d-(
a^2-b^2)^2/a^4/b/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$-\frac{\left(a^2-b^2\right)^2}{a^4bd(a+b\sin(c+dx))}+\frac{\left(2a^2-3b^2\right)\csc(c+dx)}{a^4d}+\frac{4b\left(a^2-b^2\right)\log(\sin(c+dx))}{a^5d}-\frac{4b\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - 3*b^2)*Csc[c + d*x])/(a^4*d) + (b*Csc[c + d*x]^2)/(a^3*d) - Csc[c
 + d*x]^3/(3*a^2*d) + (4*b*(a^2 - b^2)*Log[Sin[c + d*x]])/(a^5*d) - (4*b*(a
^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a^5*d) - (a^2 - b^2)^2/(a^4*b*d*(a + b*
Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^4\left(b^2-x^2\right)^2}{x^4(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^4(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^2x^4} - \frac{2b^4}{a^3x^3} + \frac{-2a^2b^2+3b^4}{a^4x^2} + \frac{4b^2\left(a^2-b^2\right)}{a^5x} + \frac{\left(a^2-b^2\right)^2}{a^4(a+x)^2} + \frac{4b^2\left(-a^2+b^2\right)}{a^5(a+x)}\right) dx, x, b$$

$$= \frac{\left(2a^2-3b^2\right)\csc(c+dx)}{a^4 d} + \frac{b\csc^2(c+dx)}{a^3 d} - \frac{\csc^3(c+dx)}{3a^2 d} + \frac{4b\left(a^2-b^2\right)\log(\sin}{a^5 d}$$

Mathematica [A] time = 1.90, size = 127, normalized size = 0.86

$$\frac{-a^3\csc^3(c+dx) - \frac{3a\left(a^2-b^2\right)^2}{b(a+b\sin(c+dx))} + 3a\left(2a^2-3b^2\right)\csc(c+dx) + 3a^2b\csc^2(c+dx) + 12b(a-b)(a+b)\log(\sin(c}{3a^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*(2*a^2 - 3*b^2)*Csc[c + d*x] + 3*a^2*b*Csc[c + d*x]^2 - a^3*Csc[c + d*
x]^3 + 12*(a - b)*b*(a + b)*Log[Sin[c + d*x]] - 12*(a - b)*b*(a + b)*Log[a
+ b*Sin[c + d*x]] - (3*a*(a^2 - b^2)^2)/(b*(a + b*Sin[c + d*x])))/(3*a^5*d)
```

fricas [B] time = 0.85, size = 401, normalized size = 2.73

$$\frac{5\,a^4b - 6\,a^2b^3 - 6\left(a^4b - a^2b^3\right)\cos\left(dx+c\right)^2 - 12\left(a^2b^3 - b^5 + \left(a^2b^3 - b^5\right)\cos\left(dx+c\right)^4 - 2\left(a^2b^3 - b^5\right)\cos\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/3*(5*a^4*b - 6*a^2*b^3 - 6*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 12*(a^2*b^3
 - b^5 + (a^2*b^3 - b^5)*cos(d*x + c)^4 - 2*(a^2*b^3 - b^5)*cos(d*x + c)^2
+ (a^3*b^2 - a*b^4 - (a^3*b^2 - a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*log(b*
sin(d*x + c) + a) + 12*(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cos(d*x + c)^4 - 2*
(a^2*b^3 - b^5)*cos(d*x + c)^2 + (a^3*b^2 - a*b^4 - (a^3*b^2 - a*b^4)*cos(d
*x + c)^2)*sin(d*x + c))*log(1/2*sin(d*x + c)) - (3*a^5 - 14*a^3*b^2 + 12*a
*b^4 - 3*(a^5 - 4*a^3*b^2 + 4*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(a^5*b^2
*d*cos(d*x + c)^4 - 2*a^5*b^2*d*cos(d*x + c)^2 + a^5*b^2*d - (a^6*b*d*cos(d
*x + c)^2 - a^6*b*d)*sin(d*x + c))
```

giac [A] time = 0.25, size = 211, normalized size = 1.44

$$\frac{\frac{12\left(a^2b-b^3\right)\log(|\sin(dx+c)|)}{a^5}-\frac{12\left(a^2b^2-b^4\right)\log(|b\sin(dx+c)+a|)}{a^5b}+\frac{3\left(4a^2b^3\sin(dx+c)-4b^5\sin(dx+c)-a^5+6a^3b^2-5ab^4\right)}{(b\sin(dx+c)+a)a^5b}-\frac{22a^2b\sin(dx+c)^3-22b}{}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(12*(a^2*b - b^3)*log(abs(sin(d*x + c)))/a^5 - 12*(a^2*b^2 - b^4)*log(a
bs(b*sin(d*x + c) + a))/(a^5*b) + 3*(4*a^2*b^3*sin(d*x + c) - 4*b^5*sin(d*x
 + c) - a^5 + 6*a^3*b^2 - 5*a*b^4)/((b*sin(d*x + c) + a)*a^5*b) - (22*a^2*b
*sin(d*x + c)^3 - 22*b^3*sin(d*x + c)^3 - 6*a^3*sin(d*x + c)^2 + 9*a*b^2*si
n(d*x + c)^2 - 3*a^2*b*sin(d*x + c) + a^3)/(a^5*sin(d*x + c)^3))/d
```

maple [A] time = 0.84, size = 209, normalized size = 1.42

$$-\frac{1}{bd\left(a+b\sin\left(dx+c\right)\right)}+\frac{2b}{d\,a^2\left(a+b\sin\left(dx+c\right)\right)}-\frac{b^3}{d\,a^4\left(a+b\sin\left(dx+c\right)\right)}-\frac{4b\ln\left(a+b\sin\left(dx+c\right)\right)}{d\,a^3}+\frac{4b^3\ln\left(a+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/b/d/(a+b*sin(d*x+c))+2/d*b/a^2/(a+b*sin(d*x+c))-1/d/a^4*b^3/(a+b*sin(d*x
+c))-4/d/a^3*b*ln(a+b*sin(d*x+c))+4/d*b^3/a^5*ln(a+b*sin(d*x+c))-1/3/d/a^2/
sin(d*x+c)^3+2/d/a^2/sin(d*x+c)-3/d/a^4/sin(d*x+c)*b^2+1/d/a^3*b/sin(d*x+c)
^2+4*b*ln(sin(d*x+c))/a^3/d-4/d*b^3/a^5*ln(sin(d*x+c))
```

maxima [A] time = 0.32, size = 158, normalized size = 1.07

$$\frac{\frac{2a^2b^2\sin(dx+c)-a^3b-3\left(a^4-4a^2b^2+4b^4\right)\sin(dx+c)^3+6\left(a^3b-ab^3\right)\sin(dx+c)^2}{a^4b^2\sin(dx+c)^4+a^5b\sin(dx+c)^3}-\frac{12\left(a^2b-b^3\right)\log(b\sin(dx+c)+a)}{a^5}+\frac{12\left(a^2b-b^3\right)\log(\sin(dx+c))}{a^5}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*((2*a^2*b^2*sin(d*x + c) - a^3*b - 3*(a^4 - 4*a^2*b^2 + 4*b^4)*sin(d*x
+ c)^3 + 6*(a^3*b - a*b^3)*sin(d*x + c)^2)/(a^4*b^2*sin(d*x + c)^4 + a^5*b*
sin(d*x + c)^3) - 12*(a^2*b - b^3)*log(b*sin(d*x + c) + a)/a^5 + 12*(a^2*b
- b^3)*log(sin(d*x + c))/a^5)/d
```

mupad [B] time = 11.72, size = 319, normalized size = 2.17

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(16\,a^2\,b-24\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(8\,a\,b^2-\frac{20\,a^3}{3}\right)-\frac{a^3}{3}+\frac{4\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(23\,a^4-44\,a^2\,b^2+16\right.}{a}}{d\,\left(8\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+16\,b\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^4*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(16*a^2*b - 24*b^3) - tan(c/2 + (d*x)/2)^2*(8*a*b^2 -
 (20*a^3)/3) - a^3/3 + (4*a^2*b*tan(c/2 + (d*x)/2))/3 + (tan(c/2 + (d*x)/2)
^4*(23*a^4 + 16*b^4 - 44*a^2*b^2))/a)/(d*(8*a^5*tan(c/2 + (d*x)/2)^3 + 8*a^
5*tan(c/2 + (d*x)/2)^5 + 16*a^4*b*tan(c/2 + (d*x)/2)^4)) - tan(c/2 + (d*x)/
2)^3/(24*a^2*d) + (tan(c/2 + (d*x)/2)*((a^2/4 + b^2/2)/a^4 + 5/(8*a^2) - (2
*b^2)/a^4))/d + (log(tan(c/2 + (d*x)/2))*(4*a^2*b - 4*b^3))/(a^5*d) + (b*ta
n(c/2 + (d*x)/2)^2)/(4*a^3*d) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2
 + (d*x)/2)^2)*(4*a^2*b - 4*b^3))/(a^5*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1233 $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=188

$$\frac{2b\csc^3(c+dx)}{3a^3d}-\frac{\csc^4(c+dx)}{4a^2d}+\frac{\left(a^2-b^2\right)^2}{a^5d(a+b\sin(c+dx))}-\frac{4b\left(a^2-b^2\right)\csc(c+dx)}{a^5d}+\frac{\left(2a^2-3b^2\right)\csc^2(c+dx)}{2a^4d}+\frac{\left(a^4-\right.}{}$$

```
[Out] -4*b*(a^2-b^2)*csc(d*x+c)/a^5/d+1/2*(2*a^2-3*b^2)*csc(d*x+c)^2/a^4/d+2/3*b*
csc(d*x+c)^3/a^3/d-1/4*csc(d*x+c)^4/a^2/d+(a^4-6*a^2*b^2+5*b^4)*ln(sin(d*x+
c))/a^6/d-(a^4-6*a^2*b^2+5*b^4)*ln(a+b*sin(d*x+c))/a^6/d+(a^2-b^2)^2/a^5/d/
(a+b*sin(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$\frac{\left(a^2-b^2\right)^2}{a^5d(a+b\sin(c+dx))}+\frac{\left(2a^2-3b^2\right)\csc^2(c+dx)}{2a^4d}-\frac{4b\left(a^2-b^2\right)\csc(c+dx)}{a^5d}+\frac{\left(-6a^2b^2+a^4+5b^4\right)\log(\sin(c+dx))}{a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*b*(a^2 - b^2)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) + ((a
^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2 + 5*
b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin[c +
 d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^5(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^2x^5} - \frac{2b^4}{a^3x^4} + \frac{-2a^2b^2+3b^4}{a^4x^3} + \frac{4b^2\left(a^2-b^2\right)}{a^5x^2} + \frac{a^4-6a^2b^2+5b^4}{a^6x} - \frac{\left(a^2-b^2\right)^2}{a^5(a+x)^2} + \frac{-a^4+6a^2b^2-5}{a^6(a+x)}\right.\right.}{d}$$

$$= -\frac{4b\left(a^2-b^2\right)\csc(c+dx)}{a^5d} + \frac{\left(2a^2-3b^2\right)\csc^2(c+dx)}{2a^4d} + \frac{2b\csc^3(c+dx)}{3a^3d} - \frac{\csc^4(c+}{4a^2d}$$

Mathematica [A] time = 6.14, size = 187, normalized size = 0.99

$$-\frac{4b(a-b)(a+b)\csc(c+dx)}{a^5d} + \frac{2b\csc^3(c+dx)}{3a^3d} - \frac{\csc^4(c+dx)}{4a^2d} + \frac{\left(a^2-b^2\right)^2}{a^5d(a+b\sin(c+dx))} + \frac{\left(2a^2-3b^2\right)\csc^2(c+dx)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*(a - b)*b*(a + b)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]
^2)/(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) +
 ((a^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2
+ 5*b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin
[c + d*x]))
```

fricas [B] time = 0.85, size = 542, normalized size = 2.88

$$\frac{21\,a^5 - 82\,a^3b^2 + 60\,ab^4 + 12\left(a^5 - 6\,a^3b^2 + 5\,ab^4\right)\cos(dx+c)^4 - 2\left(18\,a^5 - 77\,a^3b^2 + 60\,ab^4\right)\cos(dx+c)^2 - 12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(21*a^5 - 82*a^3*b^2 + 60*a*b^4 + 12*(a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d
*x + c)^4 - 2*(18*a^5 - 77*a^3*b^2 + 60*a*b^4)*cos(d*x + c)^2 - 12*(a^5 - 6
*a^3*b^2 + 5*a*b^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 -
6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b -
```

```
 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x
+ c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 12*(a^5 - 6*a^3*b^2 + 5*a*b
^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 6*a^3*b^2 + 5*a*
b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b - 6*a^2*b^3 + 5*b
^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-1/2*sin(d*x + c)) - (31*a^4*b - 30*a^2*b^3 - 6*(6*a^4*b - 5*a^2*
b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^7*d*cos(d*x + c)^4 - 2*a^7*d*cos(d*x
+ c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4 - 2*a^6*b*d*cos(d*x + c)^2 + a^6*b
*d)*sin(d*x + c))
```

giac [A] time = 0.26, size = 278, normalized size = 1.48

$$\frac{\frac{12\left(a^{4}-6a^{2}b^{2}+5b^{4}\right)\log(|\sin(dx+c)|)}{a^{6}}-\frac{12\left(a^{4}b-6a^{2}b^{3}+5b^{5}\right)\log(|b\sin(dx+c)+a|)}{a^{6}b}+\frac{12\left(a^{4}b\sin(dx+c)-6a^{2}b^{3}\sin(dx+c)+5b^{5}\sin(dx+c)+2a^{5}-8a^{3}b^{2}\right.}{(b\sin(dx+c)+a)a^{6}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(abs(sin(d*x + c)))/a^6 - 12*(a^4*b -
 6*a^2*b^3 + 5*b^5)*log(abs(b*sin(d*x + c) + a))/(a^6*b) + 12*(a^4*b*sin(d*
x + c) - 6*a^2*b^3*sin(d*x + c) + 5*b^5*sin(d*x + c) + 2*a^5 - 8*a^3*b^2 +
6*a*b^4)/((b*sin(d*x + c) + a)*a^6) - (25*a^4*sin(d*x + c)^4 - 150*a^2*b^2*
sin(d*x + c)^4 + 125*b^4*sin(d*x + c)^4 + 48*a^3*b*sin(d*x + c)^3 - 48*a*b^
3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 18*a^2*b^2*sin(d*x + c)^2 - 8*a^
3*b*sin(d*x + c) + 3*a^4)/(a^6*sin(d*x + c)^4))/d
```

maple [A] time = 0.88, size = 282, normalized size = 1.50

$$-\frac{\ln\left(a+b\sin\left(dx+c\right)\right)}{a^{2}d}+\frac{6b^{2}\ln\left(a+b\sin\left(dx+c\right)\right)}{d\,a^{4}}-\frac{5\ln\left(a+b\sin\left(dx+c\right)\right)b^{4}}{d\,a^{6}}+\frac{1}{ad\left(a+b\sin\left(dx+c\right)\right)}-\frac{}{d\,a^{3}\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -ln(a+b*sin(d*x+c))/a^2/d+6/d/a^4*b^2*ln(a+b*sin(d*x+c))-5/d/a^6*ln(a+b*sin
(d*x+c))*b^4+1/a/d/(a+b*sin(d*x+c))-2/d/a^3*b^2/(a+b*sin(d*x+c))+1/d/a^5/(a
+b*sin(d*x+c))*b^4-1/4/d/a^2/sin(d*x+c)^4+1/d/a^2/sin(d*x+c)^2-3/2/d/a^4/si
n(d*x+c)^2*b^2+ln(sin(d*x+c))/a^2/d-6/d/a^4*ln(sin(d*x+c))*b^2+5/d/a^6*ln(s
in(d*x+c))*b^4+2/3/d/a^3*b/sin(d*x+c)^3-4/d/a^3*b/sin(d*x+c)+4/d*b^3/a^5/si
n(d*x+c)
```

maxima [A] time = 0.37, size = 189, normalized size = 1.01

$$\frac{\frac{5a^{3}b\sin(dx+c)+12\left(a^{4}-6a^{2}b^{2}+5b^{4}\right)\sin(dx+c)^{4}-3a^{4}-6\left(6a^{3}b-5ab^{3}\right)\sin(dx+c)^{3}+2\left(6a^{4}-5a^{2}b^{2}\right)\sin(dx+c)^{2}}{a^{5}b\sin(dx+c)^{5}+a^{6}\sin(dx+c)^{4}}-\frac{12\left(a^{4}-6a^{2}b^{2}+5b^{4}\right)\log(b\sin(dx+c)\right.}{a^{6}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*((5*a^3*b*sin(d*x + c) + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*sin(d*x + c)^4 -
 3*a^4 - 6*(6*a^3*b - 5*a*b^3)*sin(d*x + c)^3 + 2*(6*a^4 - 5*a^2*b^2)*sin(d
*x + c)^2)/(a^5*b*sin(d*x + c)^5 + a^6*sin(d*x + c)^4) - 12*(a^4 - 6*a^2*b^
2 + 5*b^4)*log(b*sin(d*x + c) + a)/a^6 + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(s
in(d*x + c))/a^6)/d
```

mupad [B] time = 11.81, size = 439, normalized size = 2.34

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(3\,a^4-62\,a^2\,b^2+64\,b^4\right)-\frac{a^4}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{11\,a^4}{4}-\frac{10\,a^2\,b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(20\,a\,b^3-\frac{62\,a^3\,b}{3}\right)}{d\,\left(16\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+32\,b\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^5*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4*(3*a^4 + 64*b^4 - 62*a^2*b^2) - a^4/4 + tan(c/2 + (d*
x)/2)^2*((11*a^4)/4 - (10*a^2*b^2)/3) + tan(c/2 + (d*x)/2)^3*(20*a*b^3 - (6
2*a^3*b)/3) - (tan(c/2 + (d*x)/2)^5*(60*a^4*b + 32*b^5 - 96*a^2*b^3))/a + (
5*a^3*b*tan(c/2 + (d*x)/2))/6)/(d*(16*a^6*tan(c/2 + (d*x)/2)^4 + 16*a^6*tan
(c/2 + (d*x)/2)^6 + 32*a^5*b*tan(c/2 + (d*x)/2)^5)) - tan(c/2 + (d*x)/2)^4/
(64*a^2*d) + (tan(c/2 + (d*x)/2)^2*((a^2/16 + b^2/8)/a^4 + 1/(8*a^2) - b^2/
(2*a^4)))/d - (tan(c/2 + (d*x)/2)*((b*(32*a^2 + 64*b^2))/(64*a^5) - b/(4*a^
3) + (4*b*((a^2/8 + b^2/4)/a^4 + 1/(4*a^2) - b^2/a^4))/a))/d + (log(tan(c/2
 + (d*x)/2))*(a^4 + 5*b^4 - 6*a^2*b^2))/(a^6*d) + (b*tan(c/2 + (d*x)/2)^3)/
(12*a^3*d) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^4
 + 5*b^4 - 6*a^2*b^2))/(a^6*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1234 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=226

$$\frac{b\csc^4(c+dx)}{2a^3d}-\frac{\csc^5(c+dx)}{5a^2d}-\frac{b\left(a^2-b^2\right)^2}{a^6d(a+b\sin(c+dx))}-\frac{2b\left(a^2-b^2\right)\csc^2(c+dx)}{a^5d}+\frac{\left(2a^2-3b^2\right)\csc^3(c+dx)}{3a^4d}-\frac{2b\left(a^4\right.}{}$$

```
[Out] -(a^4-6*a^2*b^2+5*b^4)*csc(d*x+c)/a^6/d-2*b*(a^2-b^2)*csc(d*x+c)^2/a^5/d+1/
3*(2*a^2-3*b^2)*csc(d*x+c)^3/a^4/d+1/2*b*csc(d*x+c)^4/a^3/d-1/5*csc(d*x+c)^
5/a^2/d-2*b*(a^4-4*a^2*b^2+3*b^4)*ln(sin(d*x+c))/a^7/d+2*b*(a^4-4*a^2*b^2+3
*b^4)*ln(a+b*sin(d*x+c))/a^7/d-b*(a^2-b^2)^2/a^6/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.26, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$-\frac{b\left(a^2-b^2\right)^2}{a^6d(a+b\sin(c+dx))}+\frac{\left(2a^2-3b^2\right)\csc^3(c+dx)}{3a^4d}-\frac{2b\left(a^2-b^2\right)\csc^2(c+dx)}{a^5d}-\frac{\left(-6a^2b^2+a^4+5b^4\right)\csc(c+dx)}{a^6d}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((a^4 - 6*a^2*b^2 + 5*b^4)*Csc[c + d*x])/(a^6*d)) - (2*b*(a^2 - b^2)*Csc[
c + d*x]^2)/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^3)/(3*a^4*d) + (b*Csc[c
 + d*x]^4)/(2*a^3*d) - Csc[c + d*x]^5/(5*a^2*d) - (2*b*(a^4 - 4*a^2*b^2 + 3
*b^4)*Log[Sin[c + d*x]])/(a^7*d) + (2*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Log[a + b
*Sin[c + d*x]])/(a^7*d) - (b*(a^2 - b^2)^2)/(a^6*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^6\left(b^2-x^2\right)^2}{x^6(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^6(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \left(\frac{b^4}{a^2x^6} - \frac{2b^4}{a^3x^5} + \frac{-2a^2b^2+3b^4}{a^4x^4} + \frac{4b^2\left(a^2-b^2\right)}{a^5x^3} + \frac{a^4-6a^2b^2+5b^4}{a^6x^2} - \frac{2\left(a^4-4a^2b^2+3b^4\right)}{a^7x}\right.\right.}{d}$$

$$= -\frac{\left(a^4-6a^2b^2+5b^4\right)\csc(c+dx)}{a^6 d} - \frac{2b\left(a^2-b^2\right)\csc^2(c+dx)}{a^5 d} + \frac{\left(2a^2-3b^2\right)\csc}{3a^4 d}$$

Mathematica [A] time = 3.17, size = 220, normalized size = 0.97

$$-6a^6\csc^6(c+dx) + 9a^5b\csc^5(c+dx) + \left(30a^3b^3-40a^5b\right)\csc^3(c+dx) + 5a^4\left(4a^2-3b^2\right)\csc^4(c+dx) - 30a^2\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-30*a^2*(a^4 - 4*a^2*b^2 + 3*b^4)*Csc[c + d*x]^2 + (-40*a^5*b + 30*a^3*b^3
)*Csc[c + d*x]^3 + 5*a^4*(4*a^2 - 3*b^2)*Csc[c + d*x]^4 + 9*a^5*b*Csc[c + d
*x]^5 - 6*a^6*Csc[c + d*x]^6 - 60*b^2*(a^4 - 4*a^2*b^2 + 3*b^4)*(Log[Sin[c
+ d*x]] - Log[a + b*Sin[c + d*x]]) - 60*a*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Csc[c
 + d*x]*(1 + Log[Sin[c + d*x]] - Log[a + b*Sin[c + d*x]]))/(30*a^7*d*(b + a
*Csc[c + d*x]))
```

fricas [B] time = 1.00, size = 696, normalized size = 3.08

$$16\,a^6 - 105\,a^4b^2 + 90\,a^2b^4 + 30\left(a^6 - 4\,a^4b^2 + 3\,a^2b^4\right)\cos(dx+c)^4 - 5\left(8\,a^6 - 45\,a^4b^2 + 36\,a^2b^4\right)\cos(dx+c)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/30*(16*a^6 - 105*a^4*b^2 + 90*a^2*b^4 + 30*(a^6 - 4*a^4*b^2 + 3*a^2*b^4)*
cos(d*x + c)^4 - 5*(8*a^6 - 45*a^4*b^2 + 36*a^2*b^4)*cos(d*x + c)^2 + 60*((
a^4*b^2 - 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^6 - a^4*b^2 + 4*a^2*b^4 - 3*b^6 -
 3*(a^4*b^2 - 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 - 4*a^2*b^4 +
3*b^6)*cos(d*x + c)^2 - (a^5*b - 4*a^3*b^3 + 3*a*b^5 + (a^5*b - 4*a^3*b^3 +
 3*a*b^5)*cos(d*x + c)^4 - 2*(a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^2)*
sin(d*x + c))*log(b*sin(d*x + c) + a) - 60*((a^4*b^2 - 4*a^2*b^4 + 3*b^6)*c
os(d*x + c)^6 - a^4*b^2 + 4*a^2*b^4 - 3*b^6 - 3*(a^4*b^2 - 4*a^2*b^4 + 3*b^
6)*cos(d*x + c)^4 + 3*(a^4*b^2 - 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 - (a^5*b
 - 4*a^3*b^3 + 3*a*b^5 + (a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^4 - 2*(
a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*sin(d*x
+ c)) + (91*a^5*b - 270*a^3*b^3 + 180*a*b^5 + 60*(a^5*b - 4*a^3*b^3 + 3*a*b
^5)*cos(d*x + c)^4 - 10*(16*a^5*b - 51*a^3*b^3 + 36*a*b^5)*cos(d*x + c)^2)*
sin(d*x + c))/(a^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*
d*cos(d*x + c)^2 - a^7*b*d - (a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2
 + a^8*d)*sin(d*x + c))
```

giac [A] time = 0.24, size = 332, normalized size = 1.47

$$-\frac{\frac{60\left(a^{4}b-4a^{2}b^{3}+3b^{5}\right)\log(|\sin(dx+c)|)}{a^{7}}-\frac{60\left(a^{4}b^{2}-4a^{2}b^{4}+3b^{6}\right)\log(|b\sin(dx+c)+a|)}{a^{7}b}+\frac{30\left(2a^{4}b^{2}\sin(dx+c)-8a^{2}b^{4}\sin(dx+c)+6b^{6}\sin(dx+c)+3a^{5}\right.}{(b\sin(dx+c)+a)a^{7}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(abs(sin(d*x + c)))/a^7 - 60*(a^4*
b^2 - 4*a^2*b^4 + 3*b^6)*log(abs(b*sin(d*x + c) + a))/(a^7*b) + 30*(2*a^4*b
^2*sin(d*x + c) - 8*a^2*b^4*sin(d*x + c) + 6*b^6*sin(d*x + c) + 3*a^5*b - 1
0*a^3*b^3 + 7*a*b^5)/((b*sin(d*x + c) + a)*a^7) - (137*a^4*b*sin(d*x + c)^5
 - 548*a^2*b^3*sin(d*x + c)^5 + 411*b^5*sin(d*x + c)^5 - 30*a^5*sin(d*x + c
)^4 + 180*a^3*b^2*sin(d*x + c)^4 - 150*a*b^4*sin(d*x + c)^4 - 60*a^4*b*sin(
d*x + c)^3 + 60*a^2*b^3*sin(d*x + c)^3 + 20*a^5*sin(d*x + c)^2 - 30*a^3*b^2
*sin(d*x + c)^2 + 15*a^4*b*sin(d*x + c) - 6*a^5)/(a^7*sin(d*x + c)^5))/d
```

maple [A] time = 0.90, size = 343, normalized size = 1.52

$$-\frac{b}{d\,a^{2}\left(a+b\sin\left(dx+c\right)\right)}+\frac{2b^{3}}{d\,a^{4}\left(a+b\sin\left(dx+c\right)\right)}-\frac{b^{5}}{d\,a^{6}\left(a+b\sin\left(dx+c\right)\right)}+\frac{2b\ln\left(a+b\sin\left(dx+c\right)\right)}{d\,a^{3}}-\frac{8b^{3}\ln\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d*b/a^2/(a+b*sin(d*x+c))+2/d/a^4*b^3/(a+b*sin(d*x+c))-1/d*b^5/a^6/(a+b*s
in(d*x+c))+2/d/a^3*b*ln(a+b*sin(d*x+c))-8/d*b^3/a^5*ln(a+b*sin(d*x+c))+6/d*
b^5/a^7*ln(a+b*sin(d*x+c))-1/5/d/a^2/sin(d*x+c)^5+2/3/d/a^2/sin(d*x+c)^3-1/
d/a^4/sin(d*x+c)^3*b^2-1/d/a^2/sin(d*x+c)+6/d/a^4/sin(d*x+c)*b^2-5/d/a^6/si
n(d*x+c)*b^4+1/2/d/a^3*b/sin(d*x+c)^4-2/d/a^3*b/sin(d*x+c)^2+2/d*b^3/a^5/si
n(d*x+c)^2-2*b*ln(sin(d*x+c))/a^3/d+8/d*b^3/a^5*ln(sin(d*x+c))-6/d*b^5/a^7*
ln(sin(d*x+c))
```

maxima [A] time = 0.35, size = 225, normalized size = 1.00

$$\frac{\frac{9\,a^4b\sin(dx+c)-60\left(a^4b-4\,a^2b^3+3\,b^5\right)\sin(dx+c)^5-6\,a^5-30\left(a^5-4\,a^3b^2+3\,ab^4\right)\sin(dx+c)^4-10\left(4\,a^4b-3\,a^2b^3\right)\sin(dx+c)^3+5\left(4\,a^5-3\,a^3b^2\right)\sin(dx+c)^2}{a^6b\sin(dx+c)^6+a^7\sin(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*((9*a^4*b*sin(d*x + c) - 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*sin(d*x + c)^5
 - 6*a^5 - 30*(a^5 - 4*a^3*b^2 + 3*a*b^4)*sin(d*x + c)^4 - 10*(4*a^4*b - 3*
a^2*b^3)*sin(d*x + c)^3 + 5*(4*a^5 - 3*a^3*b^2)*sin(d*x + c)^2)/(a^6*b*sin(
d*x + c)^6 + a^7*sin(d*x + c)^5) + 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(b*sin
(d*x + c) + a)/a^7 - 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(sin(d*x + c))/a^7)/
d
```

mupad [B] time = 11.90, size = 628, normalized size = 2.78

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{64\,a^2+128\,b^2}{3072\,a^4}+\frac{1}{32\,a^2}-\frac{b^2}{6\,a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{b^2}{2\,a^4}-\frac{4\,b\left(\frac{b\left(64\,a^2+128\,b^2\right)}{256\,a^5}-\frac{b}{8\,a^3}+\frac{4\,b\left(\frac{64\,a^2+128\,b^2}{1024\,a^4}+\frac{3}{32\,a^2}-\frac{b^2}{2\,a^4}\right)}{a}\right)}{a}+\frac{(64\,a}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^6*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((64*a^2 + 128*b^2)/(3072*a^4) + 1/(32*a^2) - b^2/(6*
a^4)))/d - (tan(c/2 + (d*x)/2)*(b^2/(2*a^4) - (4*b*((b*(64*a^2 + 128*b^2))/
(256*a^5) - b/(8*a^3) + (4*b*((64*a^2 + 128*b^2)/(1024*a^4) + 3/(32*a^2) -
b^2/(2*a^4)))/a))/a + ((64*a^2 + 128*b^2)*((64*a^2 + 128*b^2)/(1024*a^4) +
```

```
3/(32*a^2) - b^2/(2*a^4)))/(32*a^2)))/d - tan(c/2 + (d*x)/2)^5/(160*a^2*d)
- (tan(c/2 + (d*x)/2)^3*((23*a^4*b)/3 - 8*a^2*b^3) + tan(c/2 + (d*x)/2)^4*(
48*a*b^4 + (25*a^5)/3 - 56*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(32*a^4*b + 160*
b^5 - 184*a^2*b^3) + a^5/5 - tan(c/2 + (d*x)/2)^2*((22*a^5)/15 - 2*a^3*b^2)
 - (3*a^4*b*tan(c/2 + (d*x)/2))/5 + (2*tan(c/2 + (d*x)/2)^6*(5*a^6 - 32*b^6
 + 104*a^2*b^4 - 74*a^4*b^2))/a)/(d*(32*a^7*tan(c/2 + (d*x)/2)^5 + 32*a^7*t
an(c/2 + (d*x)/2)^7 + 64*a^6*b*tan(c/2 + (d*x)/2)^6)) - (tan(c/2 + (d*x)/2)
^2*((b*(64*a^2 + 128*b^2))/(512*a^5) - b/(16*a^3) + (2*b*((64*a^2 + 128*b^2
)/(1024*a^4) + 3/(32*a^2) - b^2/(2*a^4)))/a))/d - (log(tan(c/2 + (d*x)/2))*
(2*a^4*b + 6*b^5 - 8*a^2*b^3))/(a^7*d) + (b*tan(c/2 + (d*x)/2)^4)/(32*a^3*d
) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(2*a^4*b + 6*
b^5 - 8*a^2*b^3))/(a^7*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1235 $\int \cos^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=170

$$-\frac{(2a^2-b^2)\sin^{n+3}(c+dx)}{d(n+3)}+\frac{(a^2-2b^2)\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2ab\sin^{n+2}(c+dx)}{d(n+2)}-\frac{4ab\sin^{n+4}(c+dx)}{d(n+4)}$$

```
[Out] a^2*sin(d*x+c)^(1+n)/d/(1+n)+2*a*b*sin(d*x+c)^(2+n)/d/(2+n)-(2*a^2-b^2)*sin
(d*x+c)^(3+n)/d/(3+n)-4*a*b*sin(d*x+c)^(4+n)/d/(4+n)+(a^2-2*b^2)*sin(d*x+c)
^(5+n)/d/(5+n)+2*a*b*sin(d*x+c)^(6+n)/d/(6+n)+b^2*sin(d*x+c)^(7+n)/d/(7+n)
```

Rubi [A] time = 0.21, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2837, 948}

$$-\frac{(2a^2-b^2)\sin^{n+3}(c+dx)}{d(n+3)}+\frac{(a^2-2b^2)\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2ab\sin^{n+2}(c+dx)}{d(n+2)}-\frac{4ab\sin^{n+4}(c+dx)}{d(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a*b*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - ((2*a^2 - b^2)*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (4*a*b*Sin[c +
d*x]^(4 + n))/(d*(4 + n)) + ((a^2 - 2*b^2)*Sin[c + d*x]^(5 + n))/(d*(5 + n)
) + (2*a*b*Sin[c + d*x]^(6 + n))/(d*(6 + n)) + (b^2*Sin[c + d*x]^(7 + n))/(
d*(7 + n))
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \left(\frac{x}{b}\right)^n (a+x)^2 \left(b^2-x^2\right)^2 dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2b^4\left(\frac{x}{b}\right)^n + 2ab^5\left(\frac{x}{b}\right)^{1+n} - b^4\left(2a^2-b^2\right)\left(\frac{x}{b}\right)^{2+n} -\right.\right.}{}$$
$$= \frac{a^2\sin^{1+n}(c+dx)}{d(1+n)} + \frac{2ab\sin^{2+n}(c+dx)}{d(2+n)} - \frac{\left(2a^2-b^2\right)\sin^{3+n}}{d(3+n)}$$

Mathematica [A] time = 0.76, size = 139, normalized size = 0.82

$$\frac{\sin^{n+1}(c+dx)\left(\frac{(a^2-2b^2)\sin^4(c+dx)}{n+5} - \frac{(2a^2-b^2)\sin^2(c+dx)}{n+3} + \frac{a^2}{n+1} + \frac{2ab\sin^5(c+dx)}{n+6} - \frac{4ab\sin^3(c+dx)}{n+4} + \frac{2ab\sin(c+dx)}{n+2} + \frac{b^2\sin^6(c+dx)}{n+7}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(a^2/(1 + n) + (2*a*b*Sin[c + d*x])/(2 + n) - ((2*a^2
 - b^2)*Sin[c + d*x]^2)/(3 + n) - (4*a*b*Sin[c + d*x]^3)/(4 + n) + ((a^2 -
2*b^2)*Sin[c + d*x]^4)/(5 + n) + (2*a*b*Sin[c + d*x]^5)/(6 + n) + (b^2*Sin[
c + d*x]^6)/(7 + n)))/d
```

fricas [B] time = 1.00, size = 572, normalized size = 3.36

$$-\frac{\left(2\left(abn^6 + 22\,abn^5 + 190\,abn^4 + 820\,abn^3 + 1849\,abn^2 + 2038\,abn + 840\,ab\right)\cos(dx+c)^6 - 16\,abn^4 - 256\,abn\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(2*(a*b*n^6 + 22*a*b*n^5 + 190*a*b*n^4 + 820*a*b*n^3 + 1849*a*b*n^2 + 2038
*a*b*n + 840*a*b)*cos(d*x + c)^6 - 16*a*b*n^4 - 256*a*b*n^3 - 2*(a*b*n^6 +
18*a*b*n^5 + 118*a*b*n^4 + 348*a*b*n^3 + 457*a*b*n^2 + 210*a*b*n)*cos(d*x +
 c)^4 - 1376*a*b*n^2 - 2816*a*b*n - 8*(a*b*n^5 + 16*a*b*n^4 + 86*a*b*n^3 +
176*a*b*n^2 + 105*a*b*n)*cos(d*x + c)^2 - 1680*a*b + ((b^2*n^6 + 21*b^2*n^5
 + 175*b^2*n^4 + 735*b^2*n^3 + 1624*b^2*n^2 + 1764*b^2*n + 720*b^2)*cos(d*x
 + c)^6 - 8*(a^2 + b^2)*n^4 - ((a^2 + b^2)*n^6 + (23*a^2 + 17*b^2)*n^5 + 3*
(69*a^2 + 37*b^2)*n^4 + 5*(185*a^2 + 71*b^2)*n^3 + 8*(268*a^2 + 73*b^2)*n^2
 + 1008*a^2 + 144*b^2 + 36*(67*a^2 + 13*b^2)*n)*cos(d*x + c)^4 - 8*(19*a^2
```

```
+ 13*b^2)*n^3 - 64*(16*a^2 + 7*b^2)*n^2 - 4*((a^2 + b^2)*n^5 + 2*(10*a^2 +
7*b^2)*n^4 + 3*(49*a^2 + 23*b^2)*n^3 + 4*(121*a^2 + 37*b^2)*n^2 + 336*a^2 +
 48*b^2 + 4*(173*a^2 + 35*b^2)*n)*cos(d*x + c)^2 - 2688*a^2 - 384*b^2 - 32*
(89*a^2 + 23*b^2)*n)*sin(d*x + c))*sin(d*x + c)^n/(d*n^7 + 28*d*n^6 + 322*d
*n^5 + 1960*d*n^4 + 6769*d*n^3 + 13132*d*n^2 + 13068*d*n + 5040*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 26.34, size = 0, normalized size = 0.00

$$\int \left(\cos^{5}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

maxima [A] time = 0.37, size = 178, normalized size = 1.05

$$\frac{\frac{b^{2}\sin(dx+c)^{n+7}}{n+7}+\frac{2\,ab\sin(dx+c)^{n+6}}{n+6}+\frac{a^{2}\sin(dx+c)^{n+5}}{n+5}-\frac{2\,b^{2}\sin(dx+c)^{n+5}}{n+5}-\frac{4\,ab\sin(dx+c)^{n+4}}{n+4}-\frac{2\,a^{2}\sin(dx+c)^{n+3}}{n+3}+\frac{b^{2}\sin(dx+c)^{n+3}}{n+3}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] (b^2*sin(d*x + c)^(n + 7)/(n + 7) + 2*a*b*sin(d*x + c)^(n + 6)/(n + 6) + a^
2*sin(d*x + c)^(n + 5)/(n + 5) - 2*b^2*sin(d*x + c)^(n + 5)/(n + 5) - 4*a*b
*sin(d*x + c)^(n + 4)/(n + 4) - 2*a^2*sin(d*x + c)^(n + 3)/(n + 3) + b^2*si
n(d*x + c)^(n + 3)/(n + 3) + 2*a*b*sin(d*x + c)^(n + 2)/(n + 2) + a^2*sin(d
*x + c)^(n + 1)/(n + 1))/d
```

mupad [B] time = 18.92, size = 887, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^n*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)*sin(c + d*x)^n*(44*n + 12*n^2 + n^3 + 48)*(1272*a^2*n + 581*b
^2*n + 4200*a^2 + 525*b^2 + 152*a^2*n^2 + 8*a^2*n^3 + 59*b^2*n^2 + 3*b^2*n^
3)*1i)/(64*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^
6*28i + n^7*1i + 5040i)) + (sin(c + d*x)^n*sin(5*c + 5*d*x)*(4*a^2*n - b^2*
n + 28*a^2 - 21*b^2)*(324*n + 260*n^2 + 95*n^3 + 16*n^4 + n^5 + 144)*1i)/(6
4*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n
^7*1i + 5040i)) - (b^2*sin(c + d*x)^n*sin(7*c + 7*d*x)*(1764*n + 1624*n^2 +
 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)*1i)/(64*d*(n*13068i + n^2*13132i +
 n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i)) + (sin(c + d
*x)^n*sin(3*c + 3*d*x)*(92*n + 56*n^2 + 13*n^3 + n^4 + 48)*(184*a^2*n + 40*
b^2*n + 700*a^2 - 35*b^2 + 12*a^2*n^2 + 3*b^2*n^2)*1i)/(64*d*(n*13068i + n^
2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i)) +
(a*b*sin(c + d*x)^n*(n*16958i + n^2*10137i + n^3*2788i + n^4*398i + n^5*30i
 + n^6*1i + 9240i))/(8*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n
^5*322i + n^6*28i + n^7*1i + 5040i)) - (a*b*sin(c + d*x)^n*cos(6*c + 6*d*x)
*(n*2038i + n^2*1849i + n^3*820i + n^4*190i + n^5*22i + n^6*1i + 840i))/(16
*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^
7*1i + 5040i)) - (a*b*sin(c + d*x)^n*cos(4*c + 4*d*x)*(n*5694i + n^2*4633i
+ n^3*1764i + n^4*334i + n^5*30i + n^6*1i + 2520i))/(8*d*(n*13068i + n^2*13
132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i)) - (a*b
*sin(c + d*x)^n*cos(2*c + 2*d*x)*(n*20490i + n^2*9159i + n^3*1228i - n^4*62
i - n^5*22i - n^6*1i + 12600i))/(16*d*(n*13068i + n^2*13132i + n^3*6769i +
n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1236 $\int \cos^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=123

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}-\frac{2a\sin^{n+3}(c+dx)}{d(n+3)}+\frac{a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{b\sin^{n+2}(c+dx)}{d(n+2)}-\frac{2b\sin^{n+4}(c+dx)}{d(n+4)}+\frac{b\sin^{n+6}(c+dx)}{d(n+6)}$$

```
[Out] a*sin(d*x+c)^(1+n)/d/(1+n)+b*sin(d*x+c)^(2+n)/d/(2+n)-2*a*sin(d*x+c)^(3+n)/
d/(3+n)-2*b*sin(d*x+c)^(4+n)/d/(4+n)+a*sin(d*x+c)^(5+n)/d/(5+n)+b*sin(d*x+c
)^(6+n)/d/(6+n)
```

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2837, 766}

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}-\frac{2a\sin^{n+3}(c+dx)}{d(n+3)}+\frac{a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{b\sin^{n+2}(c+dx)}{d(n+2)}-\frac{2b\sin^{n+4}(c+dx)}{d(n+4)}+\frac{b\sin^{n+6}(c+dx)}{d(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (b*Sin[c + d*x]^(2 + n))/(d*(2 + n))
 - (2*a*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (2*b*Sin[c + d*x]^(4 + n))/(d*(
4 + n)) + (a*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (b*Sin[c + d*x]^(6 + n))/(
d*(6 + n))
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \left(\frac{x}{b}\right)^n (a+x)\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(ab^4\left(\frac{x}{b}\right)^n + b^5\left(\frac{x}{b}\right)^{1+n} - 2ab^4\left(\frac{x}{b}\right)^{2+n} - 2b^5\left(\frac{x}{b}\right)^{3+n} +\right.\right.}{b^5 d}$$

$$= \frac{a\sin^{1+n}(c+dx)}{d(1+n)} + \frac{b\sin^{2+n}(c+dx)}{d(2+n)} - \frac{2a\sin^{3+n}(c+dx)}{d(3+n)} - \frac{2}{}$$

Mathematica [A] time = 0.21, size = 97, normalized size = 0.79

$$\frac{\sin^{n+1}(c+dx)\left(\frac{a\sin^4(c+dx)}{n+5} - \frac{2a\sin^2(c+dx)}{n+3} + \frac{a}{n+1} + \frac{b\sin^5(c+dx)}{n+6} - \frac{2b\sin^3(c+dx)}{n+4} + \frac{b\sin(c+dx)}{n+2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(a/(1 + n) + (b*Sin[c + d*x])/(2 + n) - (2*a*Sin[c +
d*x]^2)/(3 + n) - (2*b*Sin[c + d*x]^3)/(4 + n) + (a*Sin[c + d*x]^4)/(5 + n)
 + (b*Sin[c + d*x]^5)/(6 + n)))/d
```

fricas [B] time = 0.84, size = 282, normalized size = 2.29

$$-\frac{\left(\left(bn^5 + 15\,bn^4 + 85\,bn^3 + 225\,bn^2 + 274\,bn + 120\,b\right)\cos(dx+c)^6 - \left(bn^5 + 11\,bn^4 + 41\,bn^3 + 61\,bn^2 + 30\,bn\right)\text{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((b*n^5 + 15*b*n^4 + 85*b*n^3 + 225*b*n^2 + 274*b*n + 120*b)*cos(d*x + c)^
6 - (b*n^5 + 11*b*n^4 + 41*b*n^3 + 61*b*n^2 + 30*b*n)*cos(d*x + c)^4 - 8*b*
n^3 - 72*b*n^2 - 4*(b*n^4 + 9*b*n^3 + 23*b*n^2 + 15*b*n)*cos(d*x + c)^2 - 1
84*b*n - ((a*n^5 + 16*a*n^4 + 95*a*n^3 + 260*a*n^2 + 324*a*n + 144*a)*cos(d
*x + c)^4 + 8*a*n^3 + 96*a*n^2 + 4*(a*n^4 + 13*a*n^3 + 56*a*n^2 + 92*a*n +
48*a)*cos(d*x + c)^2 + 352*a*n + 384*a)*sin(d*x + c) - 120*b)*sin(d*x + c)^
n/(d*n^6 + 21*d*n^5 + 175*d*n^4 + 735*d*n^3 + 1624*d*n^2 + 1764*d*n + 720*d
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 11.38, size = 0, normalized size = 0.00

$$\int \left(\cos^5(dx+c)\right)(\sin^n(dx+c))(a+b\sin(dx+c))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

maxima [A] time = 0.33, size = 109, normalized size = 0.89

$$\frac{\frac{b\sin(dx+c)^{n+6}}{n+6}+\frac{a\sin(dx+c)^{n+5}}{n+5}-\frac{2\,b\sin(dx+c)^{n+4}}{n+4}-\frac{2\,a\sin(dx+c)^{n+3}}{n+3}+\frac{b\sin(dx+c)^{n+2}}{n+2}+\frac{a\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*sin(d*x + c)^(n + 6)/(n + 6) + a*sin(d*x + c)^(n + 5)/(n + 5) - 2*b*sin(
d*x + c)^(n + 4)/(n + 4) - 2*a*sin(d*x + c)^(n + 3)/(n + 3) + b*sin(d*x + c
)^(n + 2)/(n + 2) + a*sin(d*x + c)^(n + 1)/(n + 1))/d
```

mupad [B] time = 16.34, size = 550, normalized size = 4.47

$$\frac{b\sin(c+d\,x)^n\,\left(n^5+23\,n^4+237\,n^3+1129\,n^2+2234\,n+1320\right)}{16\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{b\sin(c+d\,x)^n\,\cos(6\,c+6\,d\,x)\,\left(n^5+15\,n^4\right.}{32\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^n*(a + b*sin(c + d*x)),x)
```

```
[Out] (b*sin(c + d*x)^n*(2234*n + 1129*n^2 + 237*n^3 + 23*n^4 + n^5 + 1320))/(16*
d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (b*sin(c
+ d*x)^n*cos(6*c + 6*d*x)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/
(32*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (b*si
n(c + d*x)^n*cos(4*c + 4*d*x)*(762*n + 553*n^2 + 173*n^3 + 23*n^4 + n^5 + 3
60))/(16*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) -
(a*sin(c + d*x)*sin(c + d*x)^n*(n*3876i + n^2*1476i + n^3*263i + n^4*24i +
n^5*1i + 3600i)*1i)/(8*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 +
```

```
n^6 + 720)) - (b*sin(c + d*x)^n*cos(2*c + 2*d*x)*(2670*n + 927*n^2 + 43*n^3
 - 15*n^4 - n^5 + 1800))/(32*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*
n^5 + n^6 + 720)) - (a*sin(c + d*x)^n*sin(5*c + 5*d*x)*(n*324i + n^2*260i +
 n^3*95i + n^4*16i + n^5*1i + 144i)*1i)/(16*d*(1764*n + 1624*n^2 + 735*n^3
+ 175*n^4 + 21*n^5 + n^6 + 720)) - (a*sin(c + d*x)^n*sin(3*c + 3*d*x)*(n*24
44i + n^2*1676i + n^3*493i + n^4*64i + n^5*3i + 1200i)*1i)/(16*d*(1764*n +
1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))
```

sympy [A] time = 100.87, size = 8675, normalized size = 70.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*(a + b*sin(c))*sin(c)**n*cos(c)**5, Eq(d, 0)), (-8*a/(15*d*sin
(c + d*x)) + 4*a*cos(c + d*x)**2/(15*d*sin(c + d*x)**3) - a*cos(c + d*x)**4
/(5*d*sin(c + d*x)**5) + b*log(sin(c + d*x))/d + b*cos(c + d*x)**2/(2*d*sin
(c + d*x)**2) - b*cos(c + d*x)**4/(4*d*sin(c + d*x)**4), Eq(n, -6)), (a*log
(sin(c + d*x))/d + a*cos(c + d*x)**2/(2*d*sin(c + d*x)**2) - a*cos(c + d*x)
**4/(4*d*sin(c + d*x)**4) + 8*b*sin(c + d*x)/(3*d) + 4*b*cos(c + d*x)**2/(3
*d*sin(c + d*x)) - b*cos(c + d*x)**4/(3*d*sin(c + d*x)**3), Eq(n, -5)), (-a
*tan(c/2 + d*x/2)**10/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5
+ 24*d*tan(c/2 + d*x/2)**3) + 19*a*tan(c/2 + d*x/2)**8/(24*d*tan(c/2 + d*x/
2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 110*a*tan(c/
2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*t
an(c/2 + d*x/2)**3) + 110*a*tan(c/2 + d*x/2)**4/(24*d*tan(c/2 + d*x/2)**7 +
 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 19*a*tan(c/2 + d*x/
2)**2/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 +
 d*x/2)**3) - a/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d
*tan(c/2 + d*x/2)**3) + 48*b*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)*
*7/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*
x/2)**3) + 96*b*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**5/(24*d*tan(
c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 48
*b*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**3/(24*d*tan(c/2 + d*x/2)*
*7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 48*b*log(tan(c/
2 + d*x/2))*tan(c/2 + d*x/2)**7/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 +
d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 96*b*log(tan(c/2 + d*x/2))*tan(c/2
+ d*x/2)**5/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan
(c/2 + d*x/2)**3) - 48*b*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**3/(24*d*ta
n(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) -
3*b*tan(c/2 + d*x/2)**9/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**
5 + 24*d*tan(c/2 + d*x/2)**3) + 54*b*tan(c/2 + d*x/2)**5/(24*d*tan(c/2 + d*
x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 3*b*tan(c/
2 + d*x/2)/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(
```

c/2 + d*x/2)**3), Eq(n, -4)), (48*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**8/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) + 144*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) + 144*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) + 48*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**2/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 48*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**8/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 144*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 144*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**4/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 48*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 3*a*tan(c/2 + d*x/2)**10/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) + 63*a*tan(c/2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) + 63*a*tan(c/2 + d*x/2)**4/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 3*a/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 12*b*tan(c/2 + d*x/2)**9/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 144*b*tan(c/2 + d*x/2)**7/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 200*b*tan(c/2 + d*x/2)**5/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 144*b*tan(c/2 + d*x/2)**3/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 12*b*tan(c/2 + d*x/2)/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2), Eq(n, -3)), (-3*a*tan(c/2 + d*x/2)**10/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 39*a*tan(c/2 + d*x/2)**8/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 86*a*tan(c/2 + d*x/2)**6/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 86*a*tan(c/2 + d*x/2)**4/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 39*a*tan(c/2 + d*x/2)**2/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 3*a/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*t

an(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 6*b
*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**9/(6*d*tan(c/2 + d*x/2)**9
+ 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/
2)**3 + 6*d*tan(c/2 + d*x/2)) - 24*b*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 +
d*x/2)**7/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c
/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 36*b*lo
g(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**5/(6*d*tan(c/2 + d*x/2)**9 + 2
4*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)*
*3 + 6*d*tan(c/2 + d*x/2)) - 24*b*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*
x/2)**3/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2
+ d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 6*b*log(ta
n(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan
(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*
d*tan(c/2 + d*x/2)) + 6*b*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**9/(6*d*ta
n(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 2
4*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) + 24*b*log(tan(c/2 + d*x/2)
)*tan(c/2 + d*x/2)**7/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 +
36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)
) + 36*b*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**5/(6*d*tan(c/2 + d*x/2)**9
+ 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x
/2)**3 + 6*d*tan(c/2 + d*x/2)) + 24*b*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2
)**3/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d
*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) + 6*b*log(tan(c
/2 + d*x/2))*tan(c/2 + d*x/2)/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x
/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2
+ d*x/2)) - 24*b*tan(c/2 + d*x/2)**7/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/
2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*t
an(c/2 + d*x/2)) - 24*b*tan(c/2 + d*x/2)**5/(6*d*tan(c/2 + d*x/2)**9 + 24*d
*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3
+ 6*d*tan(c/2 + d*x/2)) - 24*b*tan(c/2 + d*x/2)**3/(6*d*tan(c/2 + d*x/2)**9
+ 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x
/2)**3 + 6*d*tan(c/2 + d*x/2)), Eq(n, -2)), (-15*a*log(tan(c/2 + d*x/2)**2
+ 1)*tan(c/2 + d*x/2)**10/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2
)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2
+ d*x/2)**2 + 15*d) - 75*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**
8/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d
*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) - 1
50*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**6/(15*d*tan(c/2 + d*x/2
)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/
2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) - 150*a*log(tan(c/2 + d*x/
2)**2 + 1)*tan(c/2 + d*x/2)**4/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 +
d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*ta
n(c/2 + d*x/2)**2 + 15*d) - 75*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x
/2)**2/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/
2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d

) - 15*a*log(tan(c/2 + d*x/2)**2 + 1)/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan
(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 +
75*d*tan(c/2 + d*x/2)**2 + 15*d) + 15*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x
/2)**10/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c
/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*
d) + 75*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**8/(15*d*tan(c/2 + d*x/2)*
*10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2
+ d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) + 150*a*log(tan(c/2 + d*x/2)
)*tan(c/2 + d*x/2)**6/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8
 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d
*x/2)**2 + 15*d) + 150*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**4/(15*d*ta
n(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 +
 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) + 75*a*log(ta
n(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c
/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75
*d*tan(c/2 + d*x/2)**2 + 15*d) + 15*a*log(tan(c/2 + d*x/2))/(15*d*tan(c/2 +
 d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*
tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) - 60*a*tan(c/2 + d*x
/2)**8/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/
2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d
) - 120*a*tan(c/2 + d*x/2)**6/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d
*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan
(c/2 + d*x/2)**2 + 15*d) - 120*a*tan(c/2 + d*x/2)**4/(15*d*tan(c/2 + d*x/2)
10 + 75*d*tan(c/2 + d*x/2)8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2
 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) - 60*a*tan(c/2 + d*x/2)**2/
(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x
/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) + 30*
b*tan(c/2 + d*x/2)**9/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8
 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d
*x/2)**2 + 15*d) + 40*b*tan(c/2 + d*x/2)**7/(15*d*tan(c/2 + d*x/2)**10 + 75
*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)
4 + 75*d*tan(c/2 + d*x/2)2 + 15*d) + 116*b*tan(c/2 + d*x/2)**5/(15*d*ta
n(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 +
 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) + 40*b*tan(c/
2 + d*x/2)**3/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d
*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2
 + 15*d) + 30*b*tan(c/2 + d*x/2)/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2
+ d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*
tan(c/2 + d*x/2)**2 + 15*d), Eq(n, -1)), (a*n**5*sin(c + d*x)*sin(c + d*x)*
*n*cos(c + d*x)**4/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n
2 + 1764*d*n + 720*d) + 4*a*n4*sin(c + d*x)**3*sin(c + d*x)**n*cos(c +
d*x)**2/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*
d*n + 720*d) + 20*a*n**4*sin(c + d*x)*sin(c + d*x)**n*cos(c + d*x)**4/(d*n*
*6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d)
+ 8*a*n**3*sin(c + d*x)**5*sin(c + d*x)**n/(d*n**6 + 21*d*n**5 + 175*d*n**4

+ 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 68*a*n**3*sin(c + d*x)**3
*sin(c + d*x)**n*cos(c + d*x)**2/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n
3 + 1624*d*n2 + 1764*d*n + 720*d) + 155*a*n**3*sin(c + d*x)*sin(c + d*x
)**n*cos(c + d*x)**4/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d
*n**2 + 1764*d*n + 720*d) + 96*a*n**2*sin(c + d*x)**5*sin(c + d*x)**n/(d*n*
*6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d)
+ 416*a*n**2*sin(c + d*x)**3*sin(c + d*x)**n*cos(c + d*x)**2/(d*n**6 + 21*d
*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 580*a*n
2*sin(c + d*x)*sin(c + d*x)n*cos(c + d*x)**4/(d*n**6 + 21*d*n**5 + 175*
d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 352*a*n*sin(c + d*x
)**5*sin(c + d*x)**n/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d
*n**2 + 1764*d*n + 720*d) + 1072*a*n*sin(c + d*x)**3*sin(c + d*x)**n*cos(c
+ d*x)**2/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 176
4*d*n + 720*d) + 1044*a*n*sin(c + d*x)*sin(c + d*x)**n*cos(c + d*x)**4/(d*n
6 + 21*d*n5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d)
+ 384*a*sin(c + d*x)**5*sin(c + d*x)**n/(d*n**6 + 21*d*n**5 + 175*d*n**4 +
735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 960*a*sin(c + d*x)**3*sin(c
+ d*x)**n*cos(c + d*x)**2/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 +
1624*d*n**2 + 1764*d*n + 720*d) + 720*a*sin(c + d*x)*sin(c + d*x)**n*cos(c
+ d*x)**4/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 176
4*d*n + 720*d) + b*n**5*sin(c + d*x)**2*sin(c + d*x)**n*cos(c + d*x)**4/(d*
n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d
) + 4*b*n**4*sin(c + d*x)**4*sin(c + d*x)**n*cos(c + d*x)**2/(d*n**6 + 21*d
*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 19*b*n*
*4*sin(c + d*x)**2*sin(c + d*x)**n*cos(c + d*x)**4/(d*n**6 + 21*d*n**5 + 17
5*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 8*b*n**3*sin(c +
d*x)**6*sin(c + d*x)**n/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 162
4*d*n**2 + 1764*d*n + 720*d) + 60*b*n**3*sin(c + d*x)**4*sin(c + d*x)**n*co
s(c + d*x)**2/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 +
1764*d*n + 720*d) + 137*b*n**3*sin(c + d*x)**2*sin(c + d*x)**n*cos(c + d*x
)**4/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n
+ 720*d) + 72*b*n**2*sin(c + d*x)**6*sin(c + d*x)**n/(d*n**6 + 21*d*n**5 +
175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 308*b*n**2*sin
(c + d*x)**4*sin(c + d*x)**n*cos(c + d*x)**2/(d*n**6 + 21*d*n**5 + 175*d*n*
*4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 461*b*n**2*sin(c + d*x)
2*sin(c + d*x)n*cos(c + d*x)**4/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*
d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 184*b*n*sin(c + d*x)**6*sin(c +
d*x)**n/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*
d*n + 720*d) + 612*b*n*sin(c + d*x)**4*sin(c + d*x)**n*cos(c + d*x)**2/(d*n
6 + 21*d*n5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d)
+ 702*b*n*sin(c + d*x)**2*sin(c + d*x)**n*cos(c + d*x)**4/(d*n**6 + 21*d*n
5 + 175*d*n4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 120*b*sin
(c + d*x)**6*sin(c + d*x)**n/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3
+ 1624*d*n**2 + 1764*d*n + 720*d) + 360*b*sin(c + d*x)**4*sin(c + d*x)**n*c
os(c + d*x)**2/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2

```
+ 1764*d*n + 720*d) + 360*b*sin(c + d*x)**2*sin(c + d*x)**n*cos(c + d*x)**4
/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 7
20*d), True))
```

3.1237 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=167

$$\frac{\left(a^2-b^2\right)^2\sin^{n+1}(c+dx)\,{}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{ab^4d(n+1)}-\frac{a\left(a^2-2b^2\right)\sin^{n+1}(c+dx)}{b^4d(n+1)}+\frac{\left(a^2-2b^2\right)\sin^{n+2}(c+dx)}{b^3d(n+2)}$$

```
[Out] -a*(a^2-2*b^2)*sin(d*x+c)^(1+n)/b^4/d/(1+n)+(a^2-b^2)^2*hypergeom([1, 1+n],
[2+n],-b*sin(d*x+c)/a)*sin(d*x+c)^(1+n)/a/b^4/d/(1+n)+(a^2-2*b^2)*sin(d*x+c
)^(2+n)/b^3/d/(2+n)-a*sin(d*x+c)^(3+n)/b^2/d/(3+n)+sin(d*x+c)^(4+n)/b/d/(4+
n)
```

Rubi [A] time = 0.34, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 952, 1620, 64}

$$\frac{\left(a^2-b^2\right)^2\sin^{n+1}(c+dx)\,{}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{ab^4d(n+1)}-\frac{a\left(a^2-2b^2\right)\sin^{n+1}(c+dx)}{b^4d(n+1)}+\frac{\left(a^2-2b^2\right)\sin^{n+2}(c+dx)}{b^3d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*(a^2 - 2*b^2)*Sin[c + d*x]^(1 + n))/(b^4*d*(1 + n))) + ((a^2 - b^2)^2*
Hypergeometric2F1[1, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)]*Sin[c + d*x]^(1 +
 n))/(a*b^4*d*(1 + n)) + ((a^2 - 2*b^2)*Sin[c + d*x]^(2 + n))/(b^3*d*(2 + n
)) - (a*Sin[c + d*x]^(3 + n))/(b^2*d*(3 + n)) + Sin[c + d*x]^(4 + n)/(b*d*(
4 + n))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
```

```
x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n\left(b^2-x^2\right)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\sin^{4+n}(c+dx)}{bd(4+n)} + \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n\left(4+n-\frac{2(4+n)x^2}{b^2}-\frac{a(4+n)x^3}{b^4}\right)}{a+x}\,dx, x, b\sin(c+dx)\right)}{bd(4+n)}$$

$$= \frac{\sin^{4+n}(c+dx)}{bd(4+n)} + \frac{\text{Subst}\left(\int \left(-\frac{a\left(a^2-2b^2\right)(4+n)\left(\frac{x}{b}\right)^n}{b^4} - \frac{\left(-a^2+2b^2\right)(4+n)\left(\frac{x}{b}\right)^{1+n}}{b^3} - \frac{a(4+n)\left(\frac{x}{b}\right.}{b^2}\right.\right.}{bd(4+n)}$$

$$= -\frac{a\left(a^2-2b^2\right)\sin^{1+n}(c+dx)}{b^4 d(1+n)} + \frac{\left(a^2-2b^2\right)\sin^{2+n}(c+dx)}{b^3 d(2+n)} - \frac{a\sin^{3+n}(c+dx)}{b^2 d(3+n)}$$

$$= -\frac{a\left(a^2-2b^2\right)\sin^{1+n}(c+dx)}{b^4 d(1+n)} + \frac{\left(a^2-b^2\right)^2\, {}_2F_1\left(1,1+n;2+n;-\frac{b\sin(c+dx)}{a}\right)\sin}{ab^4 d(1+n)}$$

Mathematica [A] time = 0.58, size = 133, normalized size = 0.80

$$\frac{\sin^{n+1}(c+dx)\left(-\frac{a^3-2ab^2}{n+1}+\frac{\left(a^2-b^2\right)^2{}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{a(n+1)}+\frac{b\left(a^2-2b^2\right)\sin(c+dx)}{n+2}-\frac{ab^2\sin^2(c+dx)}{n+3}+\frac{b^3\sin^3(c+dx)}{n+4}\right)}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(-((a^3 - 2*a*b^2)/(1 + n)) + ((a^2 - b^2)^2*Hypergeo
metric2F1[1, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)])/(a*(1 + n)) + (b*(a^2 -
2*b^2)*Sin[c + d*x])/(2 + n) - (a*b^2*Sin[c + d*x]^2)/(3 + n) + (b^3*Sin[c
+ d*x]^3)/(4 + n)))/(b^4*d)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)^{5}}{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)^{5}}{b\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

maple [F] time = 3.27, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{5}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)}{a+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n \cos(dx+c)^5}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^5 \sin(c+d\,x)^n}{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + b*sin(c + d*x)),x)
[Out] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + b*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

3.1238 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=191

$$\frac{(a^2-b^2)\left(b^2n-a^2(n+4)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{a^2b^4d(n+1)}+\frac{(3a^2-2b^2)\sin^{n+1}(c+dx)}{b^4d(n+1)}+\frac{(a^2-b^2)}{ab^4d(a}$$

```
[Out] (3*a^2-2*b^2)*sin(d*x+c)^(1+n)/b^4/d/(1+n)+(a^2-b^2)*(b^2*n-a^2*(4+n))*hype
rgeom([1, 1+n],[2+n],-b*sin(d*x+c)/a)*sin(d*x+c)^(1+n)/a^2/b^4/d/(1+n)-2*a*
sin(d*x+c)^(2+n)/b^3/d/(2+n)+sin(d*x+c)^(3+n)/b^2/d/(3+n)+(a^2-b^2)^2*sin(d
*x+c)^(1+n)/a/b^4/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.36, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 950, 1620, 64}

$$\frac{(a^2-b^2)\left(b^2n-a^2(n+4)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{a^2b^4d(n+1)}+\frac{(3a^2-2b^2)\sin^{n+1}(c+dx)}{b^4d(n+1)}+\frac{(a^2-b^2)}{ab^4d(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((3*a^2 - 2*b^2)*Sin[c + d*x]^(1 + n))/(b^4*d*(1 + n)) + ((a^2 - b^2)*(b^2*
n - a^2*(4 + n))*Hypergeometric2F1[1, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)]*
Sin[c + d*x]^(1 + n))/(a^2*b^4*d*(1 + n)) - (2*a*Sin[c + d*x]^(2 + n))/(b^3
*d*(2 + n)) + Sin[c + d*x]^(3 + n)/(b^2*d*(3 + n)) + ((a^2 - b^2)^2*Sin[c +
 d*x]^(1 + n))/(a*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 950

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x
, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[(R*(d + e*x
)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)), x] + Dist[1/((m + 1)*(e
*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d*
```

```
g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n\left(b^2-x^2\right)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\left(a^2-b^2\right)^2\sin^{1+n}(c+dx)}{ab^4d(a+b\sin(c+dx))} + \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n\left(-b^3n-\frac{a^4(1+n)}{b}+2a^2b(1+n)+a\left(\frac{a^2}{b}-2b\right)x-\frac{a^2x^2}{b}\right.}{a+x}\right.}{ab^4d}$$

$$= \frac{\left(a^2-b^2\right)^2\sin^{1+n}(c+dx)}{ab^4d(a+b\sin(c+dx))} + \frac{\text{Subst}\left(\int\left(\frac{\left(3a^3-2ab^2\right)\left(\frac{x}{b}\right)^n}{b} - 2a^2\left(\frac{x}{b}\right)^{1+n} + ab\left(\frac{x}{b}\right)^{2+n}\right.\right.}{ab^4}$$

$$= \frac{\left(3a^2-2b^2\right)\sin^{1+n}(c+dx)}{b^4d(1+n)} - \frac{2a\sin^{2+n}(c+dx)}{b^3d(2+n)} + \frac{\sin^{3+n}(c+dx)}{b^2d(3+n)} + \frac{\left(a^2-b^2\right)}{ab^4d(a}$$

$$= \frac{\left(3a^2-2b^2\right)\sin^{1+n}(c+dx)}{b^4d(1+n)} + \frac{\left(a^2-b^2\right)\left(b^2n-a^2(4+n)\right){}_2F_1\left(1,1+n;2+n;\right.}{a^2b^4d(1+n)}$$

Mathematica [A] time = 0.44, size = 143, normalized size = 0.75

$$\frac{\sin^{n+1}(c+dx)\left(\frac{\left(a^2-b^2\right)^2 {}_2F_1\left(2,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{a^2(n+1)}-\frac{4\left(a^2-b^2\right) {}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{n+1}+\frac{3a^2-2b^2}{n+1}-\frac{2ab\sin(c+dx)}{n+2}+\frac{b^2\sin^2(c+dx)}{n+3}\right)}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*((3*a^2 - 2*b^2)/(1 + n) - (4*(a^2 - b^2)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)])/(1 + n) + ((a^2 - b^2)^2*Hy
pergeometric2F1[2, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)])/(a^2*(1 + n)) - (2
*a*b*Sin[c + d*x])/(2 + n) + (b^2*Sin[c + d*x]^2)/(3 + n)))/(b^4*d)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sin\left(dx+c\right)^n\cos\left(dx+c\right)^5}{b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)^5/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x
 + c) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(dx+c\right)^n\cos\left(dx+c\right)^5}{\left(b\sin\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a)^2, x)
```

maple [F] time = 8.67, size = 0, normalized size = 0.00

$$\int\frac{\left(\cos^5\left(dx+c\right)\right)\left(\sin^n\left(dx+c\right)\right)}{\left(a+b\sin\left(dx+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n \cos(dx+c)^5}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^5 \sin(c+d\,x)^n}{(a+b\,\sin(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + b*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1239 $\int \cos^6(c+dx)\sin^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=238

$$-\frac{(a^2+3b^2)\cos^{11}(c+dx)}{11d}+\frac{(2a^2+3b^2)\cos^9(c+dx)}{9d}-\frac{(a^2+b^2)\cos^7(c+dx)}{7d}-\frac{ab\sin^5(c+dx)\cos^7(c+dx)}{6d}-\frac{ab\,\mathrm{si}}{}$$

```
[Out] 5/512*a*b*x-1/7*(a^2+b^2)*cos(d*x+c)^7/d+1/9*(2*a^2+3*b^2)*cos(d*x+c)^9/d-1
/11*(a^2+3*b^2)*cos(d*x+c)^11/d+1/13*b^2*cos(d*x+c)^13/d+5/512*a*b*cos(d*x+
c)*sin(d*x+c)/d+5/768*a*b*cos(d*x+c)^3*sin(d*x+c)/d+1/192*a*b*cos(d*x+c)^5*
sin(d*x+c)/d-1/32*a*b*cos(d*x+c)^7*sin(d*x+c)/d-1/12*a*b*cos(d*x+c)^7*sin(d
*x+c)^3/d-1/6*a*b*cos(d*x+c)^7*sin(d*x+c)^5/d
```

Rubi [A] time = 0.37, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2911, 2568, 2635, 8, 3201, 446, 77}

$$-\frac{(a^2+3b^2)\cos^{11}(c+dx)}{11d}+\frac{(2a^2+3b^2)\cos^9(c+dx)}{9d}-\frac{(a^2+b^2)\cos^7(c+dx)}{7d}-\frac{ab\sin^5(c+dx)\cos^7(c+dx)}{6d}-\frac{ab\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a*b*x)/512 - ((a^2 + b^2)*Cos[c + d*x]^7)/(7*d) + ((2*a^2 + 3*b^2)*Cos[c
 + d*x]^9)/(9*d) - ((a^2 + 3*b^2)*Cos[c + d*x]^11)/(11*d) + (b^2*Cos[c + d*
x]^13)/(13*d) + (5*a*b*Cos[c + d*x]*Sin[c + d*x])/(512*d) + (5*a*b*Cos[c +
d*x]^3*Sin[c + d*x])/(768*d) + (a*b*Cos[c + d*x]^5*Sin[c + d*x])/(192*d) -
(a*b*Cos[c + d*x]^7*Sin[c + d*x])/(32*d) - (a*b*Cos[c + d*x]^7*Sin[c + d*x]
^3)/(12*d) - (a*b*Cos[c + d*x]^7*Sin[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3201

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFac
tors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Su
bst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Si
n[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^5(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\sin^6(c+dx)\,dx + \int \cos^6(c+dx)\sin^5$$

$$= -\frac{ab\cos^7(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}(5ab)\int \cos^6(c+dx)\sin^4$$

$$= -\frac{ab\cos^7(c+dx)\sin^3(c+dx)}{12d} - \frac{ab\cos^7(c+dx)\sin^5(c+dx}{6d}$$

$$= -\frac{ab\cos^7(c+dx)\sin(c+dx)}{32d} - \frac{ab\cos^7(c+dx)\sin^3(c+dx)}{12d}$$

$$= -\frac{(a^2+b^2)\cos^7(c+dx)}{7d} + \frac{(2a^2+3b^2)\cos^9(c+dx)}{9d} - \frac{(a^2+}{}$$

$$= -\frac{(a^2+b^2)\cos^7(c+dx)}{7d} + \frac{(2a^2+3b^2)\cos^9(c+dx)}{9d} - \frac{(a^2+}{}$$

$$= -\frac{(a^2+b^2)\cos^7(c+dx)}{7d} + \frac{(2a^2+3b^2)\cos^9(c+dx)}{9d} - \frac{(a^2+}{}$$

$$= \frac{5abx}{512} - \frac{(a^2+b^2)\cos^7(c+dx)}{7d} + \frac{(2a^2+3b^2)\cos^9(c+dx)}{9d} -$$

Mathematica [A] time = 2.23, size = 210, normalized size = 0.88

$$\frac{-180180(2a^2+b^2)\cos(c+dx) - 15015(8a^2+3b^2)\cos(3(c+dx)) + 36036a^2\cos(5(c+dx)) + 25740a^2\cos(7(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (360360*a*b*c + 360360*a*b*d*x - 180180*(2*a^2 + b^2)*Cos[c + d*x] - 15015*
(8*a^2 + 3*b^2)*Cos[3*(c + d*x)] + 36036*a^2*Cos[5*(c + d*x)] + 27027*b^2*C
os[5*(c + d*x)] + 25740*a^2*Cos[7*(c + d*x)] + 7722*b^2*Cos[7*(c + d*x)] -
4004*a^2*Cos[9*(c + d*x)] - 6006*b^2*Cos[9*(c + d*x)] - 3276*a^2*Cos[11*(c
+ d*x)] - 819*b^2*Cos[11*(c + d*x)] + 693*b^2*Cos[13*(c + d*x)] - 135135*a*
b*Sin[4*(c + d*x)] + 27027*a*b*Sin[8*(c + d*x)] - 3003*a*b*Sin[12*(c + d*x)
])/(36900864*d)
```

fricas [A] time = 0.89, size = 161, normalized size = 0.68

$$\frac{354816\,b^2\cos(dx+c)^{13} - 419328\,(a^2+3\,b^2)\cos(dx+c)^{11} + 512512\,(2\,a^2+3\,b^2)\cos(dx+c)^9 - 658944\,(a^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4612608*(354816*b^2*cos(d*x + c)^13 - 419328*(a^2 + 3*b^2)*cos(d*x + c)^1
1 + 512512*(2*a^2 + 3*b^2)*cos(d*x + c)^9 - 658944*(a^2 + b^2)*cos(d*x + c)
^7 + 45045*a*b*d*x - 3003*(256*a*b*cos(d*x + c)^11 - 640*a*b*cos(d*x + c)^9
 + 432*a*b*cos(d*x + c)^7 - 8*a*b*cos(d*x + c)^5 - 10*a*b*cos(d*x + c)^3 -
15*a*b*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.79, size = 214, normalized size = 0.90

$$\frac{5}{512}\,abx+\frac{b^2\cos(13\,dx+13\,c)}{53248\,d}-\frac{ab\sin(12\,dx+12\,c)}{12288\,d}+\frac{3\,ab\sin(8\,dx+8\,c)}{4096\,d}-\frac{15\,ab\sin(4\,dx+4\,c)}{4096\,d}-\frac{\left(4\,a^2+b^2\right)\mathrm{co}}{450}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/512*a*b*x + 1/53248*b^2*cos(13*d*x + 13*c)/d - 1/12288*a*b*sin(12*d*x + 1
2*c)/d + 3/4096*a*b*sin(8*d*x + 8*c)/d - 15/4096*a*b*sin(4*d*x + 4*c)/d - 1
/45056*(4*a^2 + b^2)*cos(11*d*x + 11*c)/d - 1/18432*(2*a^2 + 3*b^2)*cos(9*d
*x + 9*c)/d + 1/14336*(10*a^2 + 3*b^2)*cos(7*d*x + 7*c)/d + 1/4096*(4*a^2 +
 3*b^2)*cos(5*d*x + 5*c)/d - 5/12288*(8*a^2 + 3*b^2)*cos(3*d*x + 3*c)/d - 5
/1024*(2*a^2 + b^2)*cos(d*x + c)/d
```

maple [A] time = 0.33, size = 225, normalized size = 0.95

$$\frac{a^2\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^7(dx+c)\right)}{11}-\frac{4\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{99}-\frac{8\left(\cos^7(dx+c)\right)}{693}\right)+2ab\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^7(dx+c)\right)}{12}-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(d\right.}{24}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/
693*cos(d*x+c)^7)+2*a*b*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*
cos(d*x+c)^7-1/64*sin(d*x+c)*cos(d*x+c)^7+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c
)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c)+b^2*(-1/13*sin(d*x+c)^
6*cos(d*x+c)^7-6/143*sin(d*x+c)^4*cos(d*x+c)^7-8/429*sin(d*x+c)^2*cos(d*x+c
)^7-16/3003*cos(d*x+c)^7))
```

maxima [A] time = 0.34, size = 135, normalized size = 0.57

$$-\frac{53248\left(63\cos(dx+c)^{11}-154\cos(dx+c)^9+99\cos(dx+c)^7\right)a^2-3003\left(4\sin(4\,dx+4\,c)^3+120\,dx+120\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/36900864*(53248*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x +
c)^7)*a^2 - 3003*(4*sin(4*d*x + 4*c)^3 + 120*d*x + 120*c + 9*sin(8*d*x + 8*
c) - 48*sin(4*d*x + 4*c))*a*b - 12288*(231*cos(d*x + c)^13 - 819*cos(d*x +
c)^11 + 1001*cos(d*x + c)^9 - 429*cos(d*x + c)^7)*b^2)/d
```

mupad [B] time = 15.01, size = 441, normalized size = 1.85

$$\frac{5\,a\,b\,x}{512}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}\,\left(16\,a^2-96\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}\,\left(\frac{16\,a^2}{3}-32\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}\,\left(\frac{128\,a^2}{3}+192\,b^2\right)-\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^5*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (5*a*b*x)/512 - (tan(c/2 + (d*x)/2)^16*(16*a^2 - 96*b^2) - tan(c/2 + (d*x)/
2)^18*((16*a^2)/3 - 32*b^2) + tan(c/2 + (d*x)/2)^14*((128*a^2)/3 + 192*b^2)
 - tan(c/2 + (d*x)/2)^6*((256*a^2)/63 - (64*b^2)/21) + tan(c/2 + (d*x)/2)^4
*((416*a^2)/231 + (64*b^2)/77) + tan(c/2 + (d*x)/2)^10*((96*a^2)/7 + (768*b
^2)/7) + tan(c/2 + (d*x)/2)^2*((208*a^2)/693 + (32*b^2)/231) - tan(c/2 + (d
*x)/2)^12*((64*a^2)/21 + (1216*b^2)/7) + tan(c/2 + (d*x)/2)^8*((1376*a^2)/6
3 - (512*b^2)/21) + (32*a^2*tan(c/2 + (d*x)/2)^20)/3 + (16*a^2)/693 + (32*b
^2)/3003 + (95*a*b*tan(c/2 + (d*x)/2)^3)/384 + (277*a*b*tan(c/2 + (d*x)/2)^
5)/192 - (4025*a*b*tan(c/2 + (d*x)/2)^7)/128 + (59435*a*b*tan(c/2 + (d*x)/2
)^9)/768 - (16813*a*b*tan(c/2 + (d*x)/2)^11)/192 + (16813*a*b*tan(c/2 + (d*
x)/2)^15)/192 - (59435*a*b*tan(c/2 + (d*x)/2)^17)/768 + (4025*a*b*tan(c/2 +
 (d*x)/2)^19)/128 - (277*a*b*tan(c/2 + (d*x)/2)^21)/192 - (95*a*b*tan(c/2 +
 (d*x)/2)^23)/384 - (5*a*b*tan(c/2 + (d*x)/2)^25)/256 + (5*a*b*tan(c/2 + (d
*x)/2))/256)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^13)
```

sympy [A] time = 77.82, size = 488, normalized size = 2.05

$$\begin{cases} -\frac{a^2\sin^4(c+dx)\cos^7(c+dx)}{7d}-\frac{4a^2\sin^2(c+dx)\cos^9(c+dx)}{63d}-\frac{8a^2\cos^{11}(c+dx)}{693d}+\frac{5abx\sin^{12}(c+dx)}{512}+\frac{15abx\sin^{10}(c+dx)\cos^2(c+dx)}{256}+\frac{75abx\,\mathrm{s}}{} \\ x\,(a+b\sin(c))^2\sin^5(c)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

[Out] Piecewise((-a**2*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 4*a**2*sin(c + d*x
)**2*cos(c + d*x)**9/(63*d) - 8*a**2*cos(c + d*x)**11/(693*d) + 5*a*b*x*sin
(c + d*x)**12/512 + 15*a*b*x*sin(c + d*x)**10*cos(c + d*x)**2/256 + 75*a*b*
x*sin(c + d*x)**8*cos(c + d*x)**4/512 + 25*a*b*x*sin(c + d*x)**6*cos(c + d*
x)**6/128 + 75*a*b*x*sin(c + d*x)**4*cos(c + d*x)**8/512 + 15*a*b*x*sin(c +
 d*x)**2*cos(c + d*x)**10/256 + 5*a*b*x*cos(c + d*x)**12/512 + 5*a*b*sin(c
+ d*x)**11*cos(c + d*x)/(512*d) + 85*a*b*sin(c + d*x)**9*cos(c + d*x)**3/(1
536*d) + 33*a*b*sin(c + d*x)**7*cos(c + d*x)**5/(256*d) - 33*a*b*sin(c + d*
x)**5*cos(c + d*x)**7/(256*d) - 85*a*b*sin(c + d*x)**3*cos(c + d*x)**9/(153
6*d) - 5*a*b*sin(c + d*x)*cos(c + d*x)**11/(512*d) - b**2*sin(c + d*x)**6*c
os(c + d*x)**7/(7*d) - 2*b**2*sin(c + d*x)**4*cos(c + d*x)**9/(21*d) - 8*b*
*2*sin(c + d*x)**2*cos(c + d*x)**11/(231*d) - 16*b**2*cos(c + d*x)**13/(300
3*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**5*cos(c)**6, True))

3.1240 $\int \cos^6(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=250

$$\frac{(12a^2+25b^2)\sin(c+dx)\cos^9(c+dx)}{120d}-\frac{(44a^2+45b^2)\sin(c+dx)\cos^7(c+dx)}{320d}+\frac{(12a^2+5b^2)\sin(c+dx)\cos^5(c}{1920d}$$

```
[Out] 1/1024*(12*a^2+5*b^2)*x-2/7*a*b*cos(d*x+c)^7/d+4/9*a*b*cos(d*x+c)^9/d-2/11*
a*b*cos(d*x+c)^11/d+1/1024*(12*a^2+5*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/1536*(1
2*a^2+5*b^2)*cos(d*x+c)^3*sin(d*x+c)/d+1/1920*(12*a^2+5*b^2)*cos(d*x+c)^5*s
in(d*x+c)/d-1/320*(44*a^2+45*b^2)*cos(d*x+c)^7*sin(d*x+c)/d+1/120*(12*a^2+2
5*b^2)*cos(d*x+c)^9*sin(d*x+c)/d-1/12*b^2*cos(d*x+c)^11*sin(d*x+c)/d
```

Rubi [A] time = 0.35, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2911, 2565, 270, 3200, 455, 1157, 385, 199, 203}

$$\frac{(12a^2+25b^2)\sin(c+dx)\cos^9(c+dx)}{120d}-\frac{(44a^2+45b^2)\sin(c+dx)\cos^7(c+dx)}{320d}+\frac{(12a^2+5b^2)\sin(c+dx)\cos^5(c}{1920d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((12*a^2 + 5*b^2)*x)/1024 - (2*a*b*Cos[c + d*x]^7)/(7*d) + (4*a*b*Cos[c + d
*x]^9)/(9*d) - (2*a*b*Cos[c + d*x]^11)/(11*d) + ((12*a^2 + 5*b^2)*Cos[c + d
*x]*Sin[c + d*x])/(1024*d) + ((12*a^2 + 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])
/(1536*d) + ((12*a^2 + 5*b^2)*Cos[c + d*x]^5*Sin[c + d*x])/(1920*d) - ((44*
a^2 + 45*b^2)*Cos[c + d*x]^7*Sin[c + d*x])/(320*d) + ((12*a^2 + 25*b^2)*Cos
[c + d*x]^9*Sin[c + d*x])/(120*d) - (b^2*Cos[c + d*x]^11*Sin[c + d*x])/(12*
d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
```

```
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3200

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_)*((a_) + (b_.)
*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[
e + f*x], x]}, Dist[ff^(n + 1)/f, Subst[Int[(x^n*(a + (a + b)*ff^2*x^2)^p)/
(1 + ff^2*x^2)^((m + n)/2 + p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{
a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\sin^5(c+dx)\,dx + \int \cos^6(c+dx)\sin^4$$

$$= \frac{\text{Subst}\left(\int \frac{x^4\left(a^2+\left(a^2+b^2\right)x^2\right)}{\left(1+x^2\right)^7}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\Big(}{}$$

$$= -\frac{b^2\cos^{11}(c+dx)\sin(c+dx)}{12d} - \frac{\text{Subst}\left(\int \frac{-b^2+12b^2x^2-12\left(a^2+b^2\right.}{\left(1+x^2\right)^6}\right.}{12d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+dx)}{11d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+dx)}{11d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+dx)}{11d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+dx)}{11d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+dx)}{11d}$$

$$= \frac{\left(12a^2+5b^2\right)x}{1024} - \frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2a}{}$$

Mathematica [A] time = 1.47, size = 202, normalized size = 0.81

$$\frac{55440a^2\sin(2(c+dx))-110880a^2\sin(4(c+dx))-27720a^2\sin(6(c+dx))+13860a^2\sin(8(c+dx))+5544a^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (166320*b^2*c + 332640*a^2*d*x + 138600*b^2*d*x - 554400*a*b*Cos[c + d*x] -
 184800*a*b*Cos[3*(c + d*x)] + 55440*a*b*Cos[5*(c + d*x)] + 39600*a*b*Cos[7
*(c + d*x)] - 6160*a*b*Cos[9*(c + d*x)] - 5040*a*b*Cos[11*(c + d*x)] + 5544
0*a^2*Sin[2*(c + d*x)] - 110880*a^2*Sin[4*(c + d*x)] - 51975*b^2*Sin[4*(c +
 d*x)] - 27720*a^2*Sin[6*(c + d*x)] + 13860*a^2*Sin[8*(c + d*x)] + 10395*b^
2*Sin[8*(c + d*x)] + 5544*a^2*Sin[10*(c + d*x)] - 1155*b^2*Sin[12*(c + d*x)
])/(28385280*d)
```

fricas [A] time = 0.78, size = 182, normalized size = 0.73

$$-\frac{645120\,ab\cos\left(dx+c\right)^{11}-1576960\,ab\cos\left(dx+c\right)^{9}+1013760\,ab\cos\left(dx+c\right)^{7}-3465\left(12\,a^{2}+5\,b^{2}\right)dx+231}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3548160*(645120*a*b*cos(d*x + c)^11 - 1576960*a*b*cos(d*x + c)^9 + 10137
60*a*b*cos(d*x + c)^7 - 3465*(12*a^2 + 5*b^2)*d*x + 231*(1280*b^2*cos(d*x +
 c)^11 - 128*(12*a^2 + 25*b^2)*cos(d*x + c)^9 + 48*(44*a^2 + 45*b^2)*cos(d*
x + c)^7 - 8*(12*a^2 + 5*b^2)*cos(d*x + c)^5 - 10*(12*a^2 + 5*b^2)*cos(d*x
+ c)^3 - 15*(12*a^2 + 5*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.77, size = 226, normalized size = 0.90

$$\frac{1}{1024}\left(12\,a^{2}+5\,b^{2}\right)x-\frac{ab\cos\left(11\,dx+11\,c\right)}{5632\,d}-\frac{ab\cos\left(9\,dx+9\,c\right)}{4608\,d}+\frac{5\,ab\cos\left(7\,dx+7\,c\right)}{3584\,d}+\frac{ab\cos\left(5\,dx+5\,c\right)}{512\,d}-\frac{5\,ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1024*(12*a^2 + 5*b^2)*x - 1/5632*a*b*cos(11*d*x + 11*c)/d - 1/4608*a*b*co
s(9*d*x + 9*c)/d + 5/3584*a*b*cos(7*d*x + 7*c)/d + 1/512*a*b*cos(5*d*x + 5*
c)/d - 5/768*a*b*cos(3*d*x + 3*c)/d - 5/256*a*b*cos(d*x + c)/d - 1/24576*b^
2*sin(12*d*x + 12*c)/d + 1/5120*a^2*sin(10*d*x + 10*c)/d - 1/1024*a^2*sin(6
```

```
*d*x + 6*c)/d + 1/512*a^2*sin(2*d*x + 2*c)/d + 1/8192*(4*a^2 + 3*b^2)*sin(8
*d*x + 8*c)/d - 1/8192*(32*a^2 + 15*b^2)*sin(4*d*x + 4*c)/d
```

maple [A] time = 0.33, size = 237, normalized size = 0.95

$$a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\sin(dx+c)\left(\cos^7(dx+c)\right)}{80}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{160}+\frac{3dx}{256}+\frac{3c}{256}\right)+2ab\left(-\frac{(\mathrm{si}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*cos(d*x+c)^7+1/16
0*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/25
6*c)+2*a*b*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-
8/693*cos(d*x+c)^7)+b^2*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*
cos(d*x+c)^7-1/64*sin(d*x+c)*cos(d*x+c)^7+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c
)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c))
```

maxima [A] time = 0.43, size = 137, normalized size = 0.55

$$\frac{2772\left(32\sin(2\,dx+2\,c)^5+120\,dx+120\,c+5\sin(8\,dx+8\,c)-40\sin(4\,dx+4\,c)\right)a^2-81920\left(63\cos(dx+c)^{1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/28385280*(2772*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8
*c) - 40*sin(4*d*x + 4*c))*a^2 - 81920*(63*cos(d*x + c)^11 - 154*cos(d*x +
c)^9 + 99*cos(d*x + c)^7)*a*b + 1155*(4*sin(4*d*x + 4*c)^3 + 120*d*x + 120*
c + 9*sin(8*d*x + 8*c) - 48*sin(4*d*x + 4*c))*b^2)/d
```

mupad [B] time = 13.55, size = 207, normalized size = 0.83

$$\frac{6930\,a^2\sin(2\,c+2\,d\,x)-13860\,a^2\sin(4\,c+4\,d\,x)-3465\,a^2\sin(6\,c+6\,d\,x)+\frac{3465\,a^2\sin(8\,c+8\,d\,x)}{2}+693\,a^2\sin(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^4*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (6930*a^2*sin(2*c + 2*d*x) - 13860*a^2*sin(4*c + 4*d*x) - 3465*a^2*sin(6*c
+ 6*d*x) + (3465*a^2*sin(8*c + 8*d*x))/2 + 693*a^2*sin(10*c + 10*d*x) - (51
```

```
975*b^2*sin(4*c + 4*d*x))/8 + (10395*b^2*sin(8*c + 8*d*x))/8 - (1155*b^2*si
n(12*c + 12*d*x))/8 - 69300*a*b*cos(c + d*x) - 23100*a*b*cos(3*c + 3*d*x) +
 6930*a*b*cos(5*c + 5*d*x) + 4950*a*b*cos(7*c + 7*d*x) - 770*a*b*cos(9*c +
9*d*x) - 630*a*b*cos(11*c + 11*d*x) + 41580*a^2*d*x + 17325*b^2*d*x)/(35481
60*d)
```

sympy [A] time = 54.31, size = 656, normalized size = 2.62

$$\begin{cases} \frac{3a^2x\sin^{10}(c+dx)}{256} + \frac{15a^2x\sin^8(c+dx)\cos^2(c+dx)}{256} + \frac{15a^2x\sin^6(c+dx)\cos^4(c+dx)}{128} + \frac{15a^2x\sin^4(c+dx)\cos^6(c+dx)}{128} + \frac{15a^2x\sin^2(c+dx)\cos^8}{256} \\ x(a+b\sin(c))^2\sin^4(c)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**10/256 + 15*a**2*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 15*a**2*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 15*a**2*x*s
in(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)
**8/256 + 3*a**2*x*cos(c + d*x)**10/256 + 3*a**2*sin(c + d*x)**9*cos(c + d*
x)/(256*d) + 7*a**2*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) + a**2*sin(c +
d*x)**5*cos(c + d*x)**5/(10*d) - 7*a**2*sin(c + d*x)**3*cos(c + d*x)**7/(12
8*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 2*a*b*sin(c + d*x)**4*
cos(c + d*x)**7/(7*d) - 8*a*b*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) - 16*a
*b*cos(c + d*x)**11/(693*d) + 5*b**2*x*sin(c + d*x)**12/1024 + 15*b**2*x*si
n(c + d*x)**10*cos(c + d*x)**2/512 + 75*b**2*x*sin(c + d*x)**8*cos(c + d*x)
**4/1024 + 25*b**2*x*sin(c + d*x)**6*cos(c + d*x)**6/256 + 75*b**2*x*sin(c
+ d*x)**4*cos(c + d*x)**8/1024 + 15*b**2*x*sin(c + d*x)**2*cos(c + d*x)**10
/512 + 5*b**2*x*cos(c + d*x)**12/1024 + 5*b**2*sin(c + d*x)**11*cos(c + d*x
)/(1024*d) + 85*b**2*sin(c + d*x)**9*cos(c + d*x)**3/(3072*d) + 33*b**2*sin
(c + d*x)**7*cos(c + d*x)**5/(512*d) - 33*b**2*sin(c + d*x)**5*cos(c + d*x)
**7/(512*d) - 85*b**2*sin(c + d*x)**3*cos(c + d*x)**9/(3072*d) - 5*b**2*sin
(c + d*x)*cos(c + d*x)**11/(1024*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)
**4*cos(c)**6, True))
```

3.1241 $\int \cos^6(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=187

$$\frac{(a^2+2b^2)\cos^9(c+dx)}{9d}-\frac{(a^2+b^2)\cos^7(c+dx)}{7d}-\frac{ab\sin^3(c+dx)\cos^7(c+dx)}{5d}-\frac{3ab\sin(c+dx)\cos^7(c+dx)}{40d}+\frac{ab\,\text{s}}{}$$

```
[Out] 3/128*a*b*x-1/7*(a^2+b^2)*cos(d*x+c)^7/d+1/9*(a^2+2*b^2)*cos(d*x+c)^9/d-1/1
1*b^2*cos(d*x+c)^11/d+3/128*a*b*cos(d*x+c)*sin(d*x+c)/d+1/64*a*b*cos(d*x+c)
^3*sin(d*x+c)/d+1/80*a*b*cos(d*x+c)^5*sin(d*x+c)/d-3/40*a*b*cos(d*x+c)^7*si
n(d*x+c)/d-1/5*a*b*cos(d*x+c)^7*sin(d*x+c)^3/d
```

Rubi [A] time = 0.31, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2911, 2568, 2635, 8, 3201, 446, 77}

$$\frac{(a^2+2b^2)\cos^9(c+dx)}{9d}-\frac{(a^2+b^2)\cos^7(c+dx)}{7d}-\frac{ab\sin^3(c+dx)\cos^7(c+dx)}{5d}-\frac{3ab\sin(c+dx)\cos^7(c+dx)}{40d}+\frac{ab\,\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*b*x)/128 - ((a^2 + b^2)*Cos[c + d*x]^7)/(7*d) + ((a^2 + 2*b^2)*Cos[c +
 d*x]^9)/(9*d) - (b^2*Cos[c + d*x]^11)/(11*d) + (3*a*b*Cos[c + d*x]*Sin[c +
 d*x])/(128*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (a*b*Cos[c + d*
x]^5*Sin[c + d*x])/(80*d) - (3*a*b*Cos[c + d*x]^7*Sin[c + d*x])/(40*d) - (a
*b*Cos[c + d*x]^7*Sin[c + d*x]^3)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3201

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFac
tors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Su
bst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Si
n[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\sin^4(c+dx)\,dx + \int \cos^6(c+dx)\sin^3$$

$$= -\frac{ab\cos^7(c+dx)\sin^3(c+dx)}{5d} + \frac{1}{5}(3ab)\int \cos^6(c+dx)\sin$$

$$= -\frac{3ab\cos^7(c+dx)\sin(c+dx)}{40d} - \frac{ab\cos^7(c+dx)\sin^3(c+dx}{5d}$$

$$= \frac{ab\cos^5(c+dx)\sin(c+dx)}{80d} - \frac{3ab\cos^7(c+dx)\sin(c+dx)}{40d}$$

$$= -\frac{(a^2+b^2)\cos^7(c+dx)}{7d} + \frac{(a^2+2b^2)\cos^9(c+dx)}{9d} - \frac{b^2\cos}{}$$

$$= -\frac{(a^2+b^2)\cos^7(c+dx)}{7d} + \frac{(a^2+2b^2)\cos^9(c+dx)}{9d} - \frac{b^2\cos}{}$$

$$= \frac{3abx}{128} - \frac{(a^2+b^2)\cos^7(c+dx)}{7d} + \frac{(a^2+2b^2)\cos^9(c+dx)}{9d} -$$

Mathematica [A] time = 1.13, size = 197, normalized size = 1.05

$$\frac{-6930(12a^2+5b^2)\cos(c+dx) - 2310(16a^2+5b^2)\cos(3(c+dx)) + 5940a^2\cos(7(c+dx)) + 1540a^2\cos(9(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (83160*a*b*c + 83160*a*b*d*x - 6930*(12*a^2 + 5*b^2)*Cos[c + d*x] - 2310*(1
6*a^2 + 5*b^2)*Cos[3*(c + d*x)] + 3465*b^2*Cos[5*(c + d*x)] + 5940*a^2*Cos[
7*(c + d*x)] + 2475*b^2*Cos[7*(c + d*x)] + 1540*a^2*Cos[9*(c + d*x)] - 385*
b^2*Cos[9*(c + d*x)] - 315*b^2*Cos[11*(c + d*x)] + 13860*a*b*Sin[2*(c + d*x
)] - 27720*a*b*Sin[4*(c + d*x)] - 6930*a*b*Sin[6*(c + d*x)] + 3465*a*b*Sin[
8*(c + d*x)] + 1386*a*b*Sin[10*(c + d*x)])/(3548160*d)
```

fricas [A] time = 0.78, size = 128, normalized size = 0.68

$$-\frac{40320\,b^2\cos(dx+c)^{11} - 49280\,(a^2+2\,b^2)\cos(dx+c)^9 + 63360\,(a^2+b^2)\cos(dx+c)^7 - 10395\,abdx - 693\,(1}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/443520*(40320*b^2*cos(d*x + c)^11 - 49280*(a^2 + 2*b^2)*cos(d*x + c)^9 +
 63360*(a^2 + b^2)*cos(d*x + c)^7 - 10395*a*b*d*x - 693*(128*a*b*cos(d*x +
c)^9 - 176*a*b*cos(d*x + c)^7 + 8*a*b*cos(d*x + c)^5 + 10*a*b*cos(d*x + c)^
3 + 15*a*b*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.57, size = 217, normalized size = 1.16

$$\frac{3}{128}\,abx-\frac{b^2\cos(11\,dx+11\,c)}{11264\,d}+\frac{b^2\cos(5\,dx+5\,c)}{1024\,d}+\frac{ab\sin(10\,dx+10\,c)}{2560\,d}+\frac{ab\sin(8\,dx+8\,c)}{1024\,d}-\frac{ab\sin(6\,dx+6\,c)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/128*a*b*x - 1/11264*b^2*cos(11*d*x + 11*c)/d + 1/1024*b^2*cos(5*d*x + 5*c
)/d + 1/2560*a*b*sin(10*d*x + 10*c)/d + 1/1024*a*b*sin(8*d*x + 8*c)/d - 1/5
12*a*b*sin(6*d*x + 6*c)/d - 1/128*a*b*sin(4*d*x + 4*c)/d + 1/256*a*b*sin(2*
d*x + 2*c)/d + 1/9216*(4*a^2 - b^2)*cos(9*d*x + 9*c)/d + 1/7168*(12*a^2 + 5
*b^2)*cos(7*d*x + 7*c)/d - 1/1536*(16*a^2 + 5*b^2)*cos(3*d*x + 3*c)/d - 1/5
12*(12*a^2 + 5*b^2)*cos(d*x + c)/d
```

maple [A] time = 0.33, size = 171, normalized size = 0.91

$$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{9}-\frac{2\left(\cos^7(dx+c)\right)}{63}\right)+2ab\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\sin(dx+c)\left(\cos^7(dx+c)\right)}{80}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx\right.}{4}\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+2*a*b*(-1/10*si
n(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*cos(d*x+c)^7+1/160*(cos(d*x+c)^5+5/
4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)+b^2*(-1/11*si
n(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/693*cos(d*x+c)^7))
```

maxima [A] time = 0.34, size = 115, normalized size = 0.61

$$\frac{56320\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a^2+693\left(32\sin(2\,dx+2\,c)^5+120\,dx+120\,c+5\sin(8\,dx+8\,c)-4\right.}{3548160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3548160*(56320*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^2 + 693*(32*sin(2*
d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*
a*b - 5120*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c)^7)*b^
2)/d
```

mupad [B] time = 14.94, size = 386, normalized size = 2.06

$$\frac{3\,a\,b\,x}{128}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}\,\left(\frac{4\,a^2}{3}+\frac{32\,b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(8\,a^2-48\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\,\left(\frac{64\,a^2}{7}-\frac{48\,b^2}{7}\right)+\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^3*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (3*a*b*x)/128 - (tan(c/2 + (d*x)/2)^16*((4*a^2)/3 + (32*b^2)/3) + tan(c/2 +
 (d*x)/2)^10*(8*a^2 - 48*b^2) + tan(c/2 + (d*x)/2)^6*((64*a^2)/7 - (48*b^2)
/7) + tan(c/2 + (d*x)/2)^14*((32*a^2)/3 - (80*b^2)/3) + tan(c/2 + (d*x)/2)^
2*((44*a^2)/63 + (16*b^2)/63) - tan(c/2 + (d*x)/2)^4*((32*a^2)/63 - (80*b^2
)/63) + tan(c/2 + (d*x)/2)^12*((64*a^2)/3 + (176*b^2)/3) + tan(c/2 + (d*x)/
2)^8*((72*a^2)/7 + (240*b^2)/7) + 4*a^2*tan(c/2 + (d*x)/2)^18 + (4*a^2)/63
+ (16*b^2)/693 + (a*b*tan(c/2 + (d*x)/2)^3)/2 - (3323*a*b*tan(c/2 + (d*x)/2
)^5)/320 + (108*a*b*tan(c/2 + (d*x)/2)^7)/5 - (841*a*b*tan(c/2 + (d*x)/2)^9
)/32 + (841*a*b*tan(c/2 + (d*x)/2)^13)/32 - (108*a*b*tan(c/2 + (d*x)/2)^15)
/5 + (3323*a*b*tan(c/2 + (d*x)/2)^17)/320 - (a*b*tan(c/2 + (d*x)/2)^19)/2 -
 (3*a*b*tan(c/2 + (d*x)/2)^21)/64 + (3*a*b*tan(c/2 + (d*x)/2))/64)/(d*(tan(
c/2 + (d*x)/2)^2 + 1)^11)
```

sympy [A] time = 35.40, size = 384, normalized size = 2.05

$$\begin{cases} -\frac{a^2 \sin^2(c+dx)\cos^7(c+dx)}{7d} - \frac{2a^2\cos^9(c+dx)}{63d} + \frac{3abx\sin^{10}(c+dx)}{128} + \frac{15abx\sin^8(c+dx)\cos^2(c+dx)}{128} + \frac{15abx\sin^6(c+dx)\cos^4(c+dx)}{64} + \frac{15abx}{} \\ x\,(a+b\sin(c))^2\sin^3(c)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 2*a**2*cos(c + d*x
)**9/(63*d) + 3*a*b*x*sin(c + d*x)**10/128 + 15*a*b*x*sin(c + d*x)**8*cos(c
 + d*x)**2/128 + 15*a*b*x*sin(c + d*x)**6*cos(c + d*x)**4/64 + 15*a*b*x*sin
(c + d*x)**4*cos(c + d*x)**6/64 + 15*a*b*x*sin(c + d*x)**2*cos(c + d*x)**8/
128 + 3*a*b*x*cos(c + d*x)**10/128 + 3*a*b*sin(c + d*x)**9*cos(c + d*x)/(12
```

```
8*d) + 7*a*b*sin(c + d*x)**7*cos(c + d*x)**3/(64*d) + a*b*sin(c + d*x)**5*c
os(c + d*x)**5/(5*d) - 7*a*b*sin(c + d*x)**3*cos(c + d*x)**7/(64*d) - 3*a*b
*sin(c + d*x)*cos(c + d*x)**9/(128*d) - b**2*sin(c + d*x)**4*cos(c + d*x)**
7/(7*d) - 4*b**2*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) - 8*b**2*cos(c + d*
x)**11/(693*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**3*cos(c)**6, True))
```

3.1242 $\int \cos^6(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=201

$$-\frac{(10a^2+11b^2)\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{(10a^2+3b^2)\sin(c+dx)\cos^5(c+dx)}{480d}+\frac{(10a^2+3b^2)\sin(c+dx)\cos^3(c}{384d}$$

```
[Out] 1/256*(10*a^2+3*b^2)*x-2/7*a*b*cos(d*x+c)^7/d+2/9*a*b*cos(d*x+c)^9/d+1/256*
(10*a^2+3*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/384*(10*a^2+3*b^2)*cos(d*x+c)^3*si
n(d*x+c)/d+1/480*(10*a^2+3*b^2)*cos(d*x+c)^5*sin(d*x+c)/d-1/80*(10*a^2+11*b
^2)*cos(d*x+c)^7*sin(d*x+c)/d+1/10*b^2*cos(d*x+c)^9*sin(d*x+c)/d
```

Rubi [A] time = 0.27, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2911, 2565, 14, 3200, 455, 385, 199, 203}

$$-\frac{(10a^2+11b^2)\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{(10a^2+3b^2)\sin(c+dx)\cos^5(c+dx)}{480d}+\frac{(10a^2+3b^2)\sin(c+dx)\cos^3(c}{384d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((10*a^2 + 3*b^2)*x)/256 - (2*a*b*Cos[c + d*x]^7)/(7*d) + (2*a*b*Cos[c + d*
x]^9)/(9*d) + ((10*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(256*d) + ((10*a
^2 + 3*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + ((10*a^2 + 3*b^2)*Cos[c
+ d*x]^5*Sin[c + d*x])/(480*d) - ((10*a^2 + 11*b^2)*Cos[c + d*x]^7*Sin[c +
d*x])/(80*d) + (b^2*Cos[c + d*x]^9*Sin[c + d*x])/(10*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3200

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_)*((a_) + (b_.)
*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[
e + f*x], x]}, Dist[ff^(n + 1)/f, Subst[Int[(x^n*(a + (a + b)*ff^2*x^2)^p)/
(1 + ff^2*x^2)^((m + n)/2 + p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{
a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\sin^3(c+dx)\,dx + \int \cos^6(c+dx)\sin^2$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(a^2+\left(a^2+b^2\right)x^2\right)}{\left(1+x^2\right)^6}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\right.}{}$$

$$= \frac{b^2\cos^9(c+dx)\sin(c+dx)}{10d} - \frac{\text{Subst}\left(\int \frac{b^2-10\left(a^2+b^2\right)x^2}{\left(1+x^2\right)^5}\,dx, x, \text{t}\right.}{10d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+dx)}{9d} - \frac{\left(10a^2+11b^2\right)\cos^7}{8}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+dx)}{9d} + \frac{\left(10a^2+3b^2\right)\cos^5(}{48}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+dx)}{9d} + \frac{\left(10a^2+3b^2\right)\cos^3(}{38}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+dx)}{9d} + \frac{\left(10a^2+3b^2\right)\cos(c}{25}$$

$$= \frac{1}{256}\left(10a^2+3b^2\right)x - \frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+dx)}{9d}$$

Mathematica [A] time = 0.90, size = 193, normalized size = 0.96

$$\frac{5040a^2\sin(2(c+dx)) - 2520a^2\sin(4(c+dx)) - 1680a^2\sin(6(c+dx)) - 315a^2\sin(8(c+dx)) + 12600a^2dx - 151}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (6300*b^2*c + 12600*a^2*d*x + 3780*b^2*d*x - 15120*a*b*Cos[c + d*x] - 6720*
a*b*Cos[3*(c + d*x)] + 1080*a*b*Cos[7*(c + d*x)] + 280*a*b*Cos[9*(c + d*x)]
 + 5040*a^2*Sin[2*(c + d*x)] + 630*b^2*Sin[2*(c + d*x)] - 2520*a^2*Sin[4*(c
 + d*x)] - 1260*b^2*Sin[4*(c + d*x)] - 1680*a^2*Sin[6*(c + d*x)] - 315*b^2*
Sin[6*(c + d*x)] - 315*a^2*Sin[8*(c + d*x)] + (315*b^2*Sin[8*(c + d*x)])/2
+ 63*b^2*Sin[10*(c + d*x)])/(322560*d)
```

fricas [A] time = 0.89, size = 149, normalized size = 0.74

$$\frac{17920\,ab\cos\left(dx+c\right)^{9}-23040\,ab\cos\left(dx+c\right)^{7}+315\left(10\,a^{2}+3\,b^{2}\right)dx+21\left(384\,b^{2}\cos\left(dx+c\right)^{9}-48\left(10\,a^{2}+1\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/80640*(17920*a*b*cos(d*x + c)^9 - 23040*a*b*cos(d*x + c)^7 + 315*(10*a^2
+ 3*b^2)*d*x + 21*(384*b^2*cos(d*x + c)^9 - 48*(10*a^2 + 11*b^2)*cos(d*x +
c)^7 + 8*(10*a^2 + 3*b^2)*cos(d*x + c)^5 + 10*(10*a^2 + 3*b^2)*cos(d*x + c)
^3 + 15*(10*a^2 + 3*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.36, size = 189, normalized size = 0.94

$$\frac{1}{256}\left(10\,a^{2}+3\,b^{2}\right)x+\frac{ab\cos\left(9\,dx+9\,c\right)}{1152\,d}+\frac{3\,ab\cos\left(7\,dx+7\,c\right)}{896\,d}-\frac{ab\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{3\,ab\cos\left(dx+c\right)}{64\,d}+\frac{b^{2}\sin\left(10\right.}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/256*(10*a^2 + 3*b^2)*x + 1/1152*a*b*cos(9*d*x + 9*c)/d + 3/896*a*b*cos(7*
d*x + 7*c)/d - 1/48*a*b*cos(3*d*x + 3*c)/d - 3/64*a*b*cos(d*x + c)/d + 1/51
20*b^2*sin(10*d*x + 10*c)/d - 1/2048*(2*a^2 - b^2)*sin(8*d*x + 8*c)/d - 1/3
072*(16*a^2 + 3*b^2)*sin(6*d*x + 6*c)/d - 1/256*(2*a^2 + b^2)*sin(4*d*x + 4
*c)/d + 1/512*(8*a^2 + b^2)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.33, size = 183, normalized size = 0.91

$$\frac{a^{2}\left(-\frac{\sin(dx+c)\left(\cos^{7}(dx+c)\right)}{8}+\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+2ab\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{7}(dx+c)\right)}{9}-\frac{2(}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)+2*a*b*(-1/9*sin(d*x+c)^2*cos
(d*x+c)^7-2/63*cos(d*x+c)^7)+b^2*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(
d*x+c)*cos(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*s
in(d*x+c)+3/256*d*x+3/256*c))
```

maxima [A] time = 0.67, size = 127, normalized size = 0.63

$$\frac{210\left(64\sin\left(2\,dx+2\,c\right)^{3}+120\,dx+120\,c-3\sin\left(8\,dx+8\,c\right)-24\sin\left(4\,dx+4\,c\right)\right)a^{2}+20480\left(7\cos\left(dx+c\right)^{9}-\right.}{645120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/645120*(210*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c)
 - 24*sin(4*d*x + 4*c))*a^2 + 20480*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a
*b + 63*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*
sin(4*d*x + 4*c))*b^2)/d
```

mupad [B] time = 11.93, size = 237, normalized size = 1.18

$$\frac{5\,a^2\,x}{128}+\frac{3\,b^2\,x}{256}+\frac{5\,a^2\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{192\,d}+\frac{a^2\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)}{48\,d}-\frac{a^2\cos\left(c+d\,x\right)^7\,\sin\left(c+d\,x\right)}{8\,d}+\frac{b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (5*a^2*x)/128 + (3*b^2*x)/256 + (5*a^2*cos(c + d*x)^3*sin(c + d*x))/(192*d)
 + (a^2*cos(c + d*x)^5*sin(c + d*x))/(48*d) - (a^2*cos(c + d*x)^7*sin(c + d
*x))/(8*d) + (b^2*cos(c + d*x)^3*sin(c + d*x))/(128*d) + (b^2*cos(c + d*x)^
5*sin(c + d*x))/(160*d) - (11*b^2*cos(c + d*x)^7*sin(c + d*x))/(80*d) + (b^
2*cos(c + d*x)^9*sin(c + d*x))/(10*d) - (2*a*b*cos(c + d*x)^7)/(7*d) + (2*a
*b*cos(c + d*x)^9)/(9*d) + (5*a^2*cos(c + d*x)*sin(c + d*x))/(128*d) + (3*b
^2*cos(c + d*x)*sin(c + d*x))/(256*d)
```

sympy [A] time = 23.77, size = 529, normalized size = 2.63

$$\begin{cases} \frac{5a^2x\sin^8(c+dx)}{128}+\frac{5a^2x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{15a^2x\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{5a^2x\sin^2(c+dx)\cos^6(c+dx)}{32}+\frac{5a^2x\cos^8(c+dx)}{128}+\frac{5a^2\sin}{} \\ x\left(a+b\sin(c)\right)^2\sin^2(c)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**8/128 + 5*a**2*x*sin(c + d*x)**6*cos(c +
d*x)**2/32 + 15*a**2*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 5*a**2*x*sin(c
+ d*x)**2*cos(c + d*x)**6/32 + 5*a**2*x*cos(c + d*x)**8/128 + 5*a**2*sin(c
+ d*x)**7*cos(c + d*x)/(128*d) + 55*a**2*sin(c + d*x)**5*cos(c + d*x)**3/(3
```

```
84*d) + 73*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) - 5*a**2*sin(c + d*
x)*cos(c + d*x)**7/(128*d) - 2*a*b*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) -
4*a*b*cos(c + d*x)**9/(63*d) + 3*b**2*x*sin(c + d*x)**10/256 + 15*b**2*x*si
n(c + d*x)**8*cos(c + d*x)**2/256 + 15*b**2*x*sin(c + d*x)**6*cos(c + d*x)*
*4/128 + 15*b**2*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*b**2*x*sin(c +
d*x)**2*cos(c + d*x)**8/256 + 3*b**2*x*cos(c + d*x)**10/256 + 3*b**2*sin(c
+ d*x)**9*cos(c + d*x)/(256*d) + 7*b**2*sin(c + d*x)**7*cos(c + d*x)**3/(12
8*d) + b**2*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) - 7*b**2*sin(c + d*x)**3
*cos(c + d*x)**7/(128*d) - 3*b**2*sin(c + d*x)*cos(c + d*x)**9/(256*d), Ne(
d, 0)), (x*(a + b*sin(c))**2*sin(c)**2*cos(c)**6, True))
```

3.1243 $\int \cos^6(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=152

$$-\frac{\left(a^2+8b^2\right)\cos^7(c+dx)}{252d}-\frac{\cos^7(c+dx)(a+b\sin(c+dx))^2}{9d}-\frac{a\cos^7(c+dx)(a+b\sin(c+dx))}{36d}+\frac{ab\sin(c+dx)\cos}{24d}$$

```
[Out] 5/64*a*b*x-1/252*(a^2+8*b^2)*cos(d*x+c)^7/d+5/64*a*b*cos(d*x+c)*sin(d*x+c)/
d+5/96*a*b*cos(d*x+c)^3*sin(d*x+c)/d+1/24*a*b*cos(d*x+c)^5*sin(d*x+c)/d-1/3
6*a*cos(d*x+c)^7*(a+b*sin(d*x+c))/d-1/9*cos(d*x+c)^7*(a+b*sin(d*x+c))^2/d
```

Rubi [A] time = 0.21, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2862, 2669, 2635, 8}

$$-\frac{\left(a^2+8b^2\right)\cos^7(c+dx)}{252d}-\frac{\cos^7(c+dx)(a+b\sin(c+dx))^2}{9d}-\frac{a\cos^7(c+dx)(a+b\sin(c+dx))}{36d}+\frac{ab\sin(c+dx)\cos}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a*b*x)/64 - ((a^2 + 8*b^2)*Cos[c + d*x]^7)/(252*d) + (5*a*b*Cos[c + d*x]
*Sin[c + d*x])/(64*d) + (5*a*b*Cos[c + d*x]^3*Sin[c + d*x])/(96*d) + (a*b*C
os[c + d*x]^5*Sin[c + d*x])/(24*d) - (a*Cos[c + d*x]^7*(a + b*Sin[c + d*x])
)/(36*d) - (Cos[c + d*x]^7*(a + b*Sin[c + d*x])^2)/(9*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos^6(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{\cos^7(c+dx)(a+b\sin(c+dx))^2}{9d}+\frac{1}{9}\int \cos^6(c+dx)(2b+$$

$$= -\frac{a\cos^7(c+dx)(a+b\sin(c+dx))}{36d}-\frac{\cos^7(c+dx)(a+b\sin$$

$$= -\frac{\left(a^2+8b^2\right)\cos^7(c+dx)}{252d}-\frac{a\cos^7(c+dx)(a+b\sin(c+dx}{36d}$$

$$= -\frac{\left(a^2+8b^2\right)\cos^7(c+dx)}{252d}+\frac{ab\cos^5(c+dx)\sin(c+dx)}{24d}-$$

$$= -\frac{\left(a^2+8b^2\right)\cos^7(c+dx)}{252d}+\frac{5ab\cos^3(c+dx)\sin(c+dx)}{96d}+$$

$$= -\frac{\left(a^2+8b^2\right)\cos^7(c+dx)}{252d}+\frac{5ab\cos(c+dx)\sin(c+dx)}{64d}+$$

$$= \frac{5abx}{64}-\frac{\left(a^2+8b^2\right)\cos^7(c+dx)}{252d}+\frac{5ab\cos(c+dx)\sin(c+}{64d}$$

Mathematica [A] time = 1.02, size = 161, normalized size = 1.06

$$-\frac{126\left(10a^2+3b^2\right)\cos(c+dx)+84\left(9a^2+2b^2\right)\cos(3(c+dx))+252a^2\cos(5(c+dx))+36a^2\cos(7(c+dx))-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/16128*(-1260*a*b*c - 1260*a*b*d*x + 126*(10*a^2 + 3*b^2)*Cos[c + d*x] +
84*(9*a^2 + 2*b^2)*Cos[3*(c + d*x)] + 252*a^2*Cos[5*(c + d*x)] + 36*a^2*Cos
[7*(c + d*x)] - 27*b^2*Cos[7*(c + d*x)] - 7*b^2*Cos[9*(c + d*x)] - 504*a*b*
```

```
Sin[2*(c + d*x)] + 252*a*b*Sin[4*(c + d*x)] + 168*a*b*Sin[6*(c + d*x)] + (6
3*a*b*Sin[8*(c + d*x)])/2)/d
```

fricas [A] time = 0.73, size = 97, normalized size = 0.64

$$\frac{448\,b^2\cos(dx+c)^9 - 576\left(a^2+b^2\right)\cos(dx+c)^7 + 315\,abdx - 21\left(48\,ab\cos(dx+c)^7 - 8\,ab\cos(dx+c)^5 - 10\,ab\right.}{4032\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4032*(448*b^2*cos(d*x + c)^9 - 576*(a^2 + b^2)*cos(d*x + c)^7 + 315*a*b*d
*x - 21*(48*a*b*cos(d*x + c)^7 - 8*a*b*cos(d*x + c)^5 - 10*a*b*cos(d*x + c)
^3 - 15*a*b*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.28, size = 176, normalized size = 1.16

$$\frac{5}{64}\,abx + \frac{b^2\cos(9\,dx+9\,c)}{2304\,d} - \frac{a^2\cos(5\,dx+5\,c)}{64\,d} - \frac{ab\sin(8\,dx+8\,c)}{512\,d} - \frac{ab\sin(6\,dx+6\,c)}{96\,d} - \frac{ab\sin(4\,dx+4\,c)}{64\,d} + \frac{ab\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/64*a*b*x + 1/2304*b^2*cos(9*d*x + 9*c)/d - 1/64*a^2*cos(5*d*x + 5*c)/d -
1/512*a*b*sin(8*d*x + 8*c)/d - 1/96*a*b*sin(6*d*x + 6*c)/d - 1/64*a*b*sin(4
*d*x + 4*c)/d + 1/32*a*b*sin(2*d*x + 2*c)/d - 1/1792*(4*a^2 - 3*b^2)*cos(7*
d*x + 7*c)/d - 1/192*(9*a^2 + 2*b^2)*cos(3*d*x + 3*c)/d - 1/128*(10*a^2 + 3
*b^2)*cos(d*x + c)/d
```

maple [A] time = 0.32, size = 115, normalized size = 0.76

$$\frac{-\frac{a^2\left(\cos^7(dx+c)\right)}{7} + 2ab\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8} + \frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48} + \frac{5dx}{128} + \frac{5c}{128}\right) + b^2\left(-\frac{\left(\sin^2(dx+c)\right)}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/7*a^2*cos(d*x+c)^7+2*a*b*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*
x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)+b^2*
(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7))
```

maxima [A] time = 0.99, size = 92, normalized size = 0.61

$$-\frac{4608\,a^2\cos\left(dx+c\right)^7-21\left(64\,\sin\left(2\,dx+2\,c\right)^3+120\,dx+120\,c-3\,\sin\left(8\,dx+8\,c\right)-24\,\sin\left(4\,dx+4\,c\right)\right)ab-}{32256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/32256*(4608*a^2*cos(d*x + c)^7 - 21*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 1
20*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a*b - 512*(7*cos(d*x + c)^
9 - 9*cos(d*x + c)^7)*b^2)/d
```

mupad [B] time = 15.05, size = 332, normalized size = 2.18

$$\frac{5\,a\,b\,x}{64}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}\,\left(4\,a^2+4\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{4\,a^2}{7}+\frac{4\,b^2}{7}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(12\,a^2+12\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (5*a*b*x)/64 - (tan(c/2 + (d*x)/2)^14*(4*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^
2*((4*a^2)/7 + (4*b^2)/7) + tan(c/2 + (d*x)/2)^6*(12*a^2 + 12*b^2) + tan(c/
2 + (d*x)/2)^8*(16*a^2 - 12*b^2) + tan(c/2 + (d*x)/2)^12*(12*a^2 - (20*b^2)
/3) + tan(c/2 + (d*x)/2)^10*(20*a^2 + 20*b^2) + tan(c/2 + (d*x)/2)^4*((44*a
^2)/7 - (12*b^2)/7) + 2*a^2*tan(c/2 + (d*x)/2)^16 + (2*a^2)/7 + (4*b^2)/63
- (191*a*b*tan(c/2 + (d*x)/2)^3)/48 + (83*a*b*tan(c/2 + (d*x)/2)^5)/16 - (1
45*a*b*tan(c/2 + (d*x)/2)^7)/16 + (145*a*b*tan(c/2 + (d*x)/2)^11)/16 - (83*
a*b*tan(c/2 + (d*x)/2)^13)/16 + (191*a*b*tan(c/2 + (d*x)/2)^15)/48 - (5*a*b
*tan(c/2 + (d*x)/2)^17)/32 + (5*a*b*tan(c/2 + (d*x)/2))/32)/(d*(tan(c/2 + (
d*x)/2)^2 + 1)^9)
```

sympy [A] time = 14.94, size = 282, normalized size = 1.86

$$\begin{cases}-\frac{a^2\cos^7\left(c+dx\right)}{7d}+\frac{5abx\sin^8\left(c+dx\right)}{64}+\frac{5abx\sin^6\left(c+dx\right)\cos^2\left(c+dx\right)}{16}+\frac{15abx\sin^4\left(c+dx\right)\cos^4\left(c+dx\right)}{32}+\frac{5abx\sin^2\left(c+dx\right)\cos^6\left(c+dx\right)}{16}+\frac{5abx}{}\\ x\left(a+b\sin\left(c\right)\right)^2\sin\left(c\right)\cos^6\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)**7/(7*d) + 5*a*b*x*sin(c + d*x)**8/64 + 5*a*b
*x*sin(c + d*x)**6*cos(c + d*x)**2/16 + 15*a*b*x*sin(c + d*x)**4*cos(c + d*
x)**4/32 + 5*a*b*x*sin(c + d*x)**2*cos(c + d*x)**6/16 + 5*a*b*x*cos(c + d*x
)**8/64 + 5*a*b*sin(c + d*x)**7*cos(c + d*x)/(64*d) + 55*a*b*sin(c + d*x)**
5*cos(c + d*x)**3/(192*d) + 73*a*b*sin(c + d*x)**3*cos(c + d*x)**5/(192*d)
- 5*a*b*sin(c + d*x)*cos(c + d*x)**7/(64*d) - b**2*sin(c + d*x)**2*cos(c +
d*x)**7/(7*d) - 2*b**2*cos(c + d*x)**9/(63*d), Ne(d, 0)), (x*(a + b*sin(c))
**2*sin(c)*cos(c)**6, True))
```

3.1244 $\int \cos^5(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=157

$$\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{ab\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{5ab\sin$$

```
[Out] 5/8*a*b*x-a^2*arctanh(cos(d*x+c))/d+a^2*cos(d*x+c)/d+1/3*a^2*cos(d*x+c)^3/d
+1/5*a^2*cos(d*x+c)^5/d-1/7*b^2*cos(d*x+c)^7/d+5/8*a*b*cos(d*x+c)*sin(d*x+c
)/d+5/12*a*b*cos(d*x+c)^3*sin(d*x+c)/d+1/3*a*b*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] time = 0.21, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2911, 2635, 8, 14, 207}

$$\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{ab\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{5ab\sin$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a*b*x)/8 - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d + (a^2*C
os[c + d*x]^3)/(3*d) + (a^2*Cos[c + d*x]^5)/(5*d) - (b^2*Cos[c + d*x]^7)/(7
*d) + (5*a*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*a*b*Cos[c + d*x]^3*Sin[c
 + d*x])/(12*d) + (a*b*Cos[c + d*x]^5*Sin[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\,dx + \int \cos^5(c+dx)\cot(c+dx)\left(a^2+\right.$$

$$= \frac{ab\cos^5(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}(5ab)\int \cos^4(c+dx)\,dx - $$

$$= \frac{5ab\cos^3(c+dx)\sin(c+dx)}{12d} + \frac{ab\cos^5(c+dx)\sin(c+dx)}{3d} +$$

$$= \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} + \frac{a^2\cos^5(c+dx)}{5d} - \frac{b^2\cos}{}$$

$$= \frac{5abx}{8} - \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c}{3d}$$

Mathematica [A] time = 0.28, size = 166, normalized size = 1.06

$$105\left(88a^2-5b^2\right)\cos(c+dx)+35\left(28a^2-9b^2\right)\cos(3(c+dx))+84a^2\cos(5(c+dx))+6720a^2\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (4200*a*b*c + 4200*a*b*d*x + 105*(88*a^2 - 5*b^2)*Cos[c + d*x] + 35*(28*a^2
 - 9*b^2)*Cos[3*(c + d*x)] + 84*a^2*Cos[5*(c + d*x)] - 105*b^2*Cos[5*(c + d
*x)] - 15*b^2*Cos[7*(c + d*x)] - 6720*a^2*Log[Cos[(c + d*x)/2]] + 6720*a^2*
```

```
Log[Sin[(c + d*x)/2]] + 3150*a*b*Sin[2*(c + d*x)] + 630*a*b*Sin[4*(c + d*x)
] + 70*a*b*Sin[6*(c + d*x)])/(6720*d)
```

fricas [A] time = 1.08, size = 137, normalized size = 0.87

$$-\frac{120\,b^2\cos\left(dx+c\right)^7-168\,a^2\cos\left(dx+c\right)^5-280\,a^2\cos\left(dx+c\right)^3-525\,abdx-840\,a^2\cos\left(dx+c\right)+420\,a^2\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/840*(120*b^2*cos(d*x + c)^7 - 168*a^2*cos(d*x + c)^5 - 280*a^2*cos(d*x +
 c)^3 - 525*a*b*d*x - 840*a^2*cos(d*x + c) + 420*a^2*log(1/2*cos(d*x + c) +
 1/2) - 420*a^2*log(-1/2*cos(d*x + c) + 1/2) - 35*(8*a*b*cos(d*x + c)^5 + 1
0*a*b*cos(d*x + c)^3 + 15*a*b*cos(d*x + c))*sin(d*x + c))/d
```

giac [B] time = 0.27, size = 291, normalized size = 1.85

$$525\left(dx+c\right)ab+840\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(1155\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-2520\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+840\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(525*(d*x + c)*a*b + 840*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(1155
*a*b*tan(1/2*d*x + 1/2*c)^13 - 2520*a^2*tan(1/2*d*x + 1/2*c)^12 + 840*b^2*t
an(1/2*d*x + 1/2*c)^12 + 980*a*b*tan(1/2*d*x + 1/2*c)^11 - 10080*a^2*tan(1/
2*d*x + 1/2*c)^10 + 2975*a*b*tan(1/2*d*x + 1/2*c)^9 - 20440*a^2*tan(1/2*d*x
 + 1/2*c)^8 + 4200*b^2*tan(1/2*d*x + 1/2*c)^8 - 24640*a^2*tan(1/2*d*x + 1/2
*c)^6 - 2975*a*b*tan(1/2*d*x + 1/2*c)^5 - 16968*a^2*tan(1/2*d*x + 1/2*c)^4
+ 2520*b^2*tan(1/2*d*x + 1/2*c)^4 - 980*a*b*tan(1/2*d*x + 1/2*c)^3 - 6496*a
^2*tan(1/2*d*x + 1/2*c)^2 - 1155*a*b*tan(1/2*d*x + 1/2*c) - 1288*a^2 + 120*
b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/d
```

maple [A] time = 0.55, size = 160, normalized size = 1.02

$$\frac{a^2\left(\cos^5\left(dx+c\right)\right)}{5d}+\frac{a^2\left(\cos^3\left(dx+c\right)\right)}{3d}+\frac{a^2\cos\left(dx+c\right)}{d}+\frac{a^2\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}+\frac{ab\left(\cos^5\left(dx+c\right)\right)\sin}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/5*a^2*cos(d*x+c)^5/d+1/3*a^2*cos(d*x+c)^3/d+a^2*cos(d*x+c)/d+1/d*a^2*ln(c
sc(d*x+c)-cot(d*x+c))+1/3*a*b*cos(d*x+c)^5*sin(d*x+c)/d+5/12*a*b*cos(d*x+c)
^3*sin(d*x+c)/d+5/8*a*b*cos(d*x+c)*sin(d*x+c)/d+5/8*a*b*x+5/8/d*a*b*c-1/7*b
^2*cos(d*x+c)^7/d
```

maxima [A] time = 0.55, size = 122, normalized size = 0.78

$$-\frac{480\,b^2\cos\left(dx+c\right)^7-112\left(6\,\cos\left(dx+c\right)^5+10\,\cos\left(dx+c\right)^3+30\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx+c\right)+1\right)+15}{3360}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*(480*b^2*cos(d*x + c)^7 - 112*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3
 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a
^2 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin
(2*d*x + 2*c))*a*b)/d
```

mupad [B] time = 13.71, size = 415, normalized size = 2.64

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\,\left(6\,a^2-2\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\,\left(\frac{146\,a^2}{3}-10\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\,\left(\frac{202\,a^2}{5}-6\,b^2\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+7\,\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)^12*(6*a^2 - 2*b^2) +
tan(c/2 + (d*x)/2)^8*((146*a^2)/3 - 10*b^2) + tan(c/2 + (d*x)/2)^4*((202*a^
2)/5 - 6*b^2) + (232*a^2*tan(c/2 + (d*x)/2)^2)/15 + (176*a^2*tan(c/2 + (d*x
)/2)^6)/3 + 24*a^2*tan(c/2 + (d*x)/2)^10 + (46*a^2)/15 - (2*b^2)/7 + (7*a*b
*tan(c/2 + (d*x)/2)^3)/3 + (85*a*b*tan(c/2 + (d*x)/2)^5)/12 - (85*a*b*tan(c
/2 + (d*x)/2)^9)/12 - (7*a*b*tan(c/2 + (d*x)/2)^11)/3 - (11*a*b*tan(c/2 + (
d*x)/2)^13)/4 + (11*a*b*tan(c/2 + (d*x)/2))/4)/(d*(7*tan(c/2 + (d*x)/2)^2 +
 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^
8 + 21*tan(c/2 + (d*x)/2)^10 + 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)
^14 + 1)) + (5*a*b*atan((25*a^2*b^2)/(16*((5*a^3*b)/2 - (25*a^2*b^2*tan(c/2
 + (d*x)/2))/16)) + (5*a^3*b*tan(c/2 + (d*x)/2))/(2*((5*a^3*b)/2 - (25*a^2*
b^2*tan(c/2 + (d*x)/2))/16))))/(4*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1245 $\int \cos^4(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=178

$$-\frac{(6a^2-5b^2)\sin(c+dx)\cos^3(c+dx)}{24d}-\frac{(14a^2-5b^2)\sin(c+dx)\cos(c+dx)}{16d}-\frac{5}{16}x(6a^2-b^2)-\frac{a^2\cot(c+dx)}{d}+\frac{2ab}{}$$

```
[Out] -5/16*(6*a^2-b^2)*x-2*a*b*arctanh(cos(d*x+c))/d+2*a*b*cos(d*x+c)/d+2/3*a*b*
cos(d*x+c)^3/d+2/5*a*b*cos(d*x+c)^5/d-a^2*cot(d*x+c)/d-1/16*(14*a^2-5*b^2)*
cos(d*x+c)*sin(d*x+c)/d-1/24*(6*a^2-5*b^2)*cos(d*x+c)^3*sin(d*x+c)/d+1/6*b^
2*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] time = 0.46, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2911, 2592, 302, 206, 434, 456, 453, 203}

$$-\frac{(6a^2-5b^2)\sin(c+dx)\cos^3(c+dx)}{24d}-\frac{(14a^2-5b^2)\sin(c+dx)\cos(c+dx)}{16d}-\frac{5}{16}x(6a^2-b^2)-\frac{a^2\cot(c+dx)}{d}+\frac{2ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-5*(6*a^2 - b^2)*x)/16 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c +
d*x])/d + (2*a*b*Cos[c + d*x]^3)/(3*d) + (2*a*b*Cos[c + d*x]^5)/(5*d) - (a^
2*Cot[c + d*x])/d - ((14*a^2 - 5*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (
(6*a^2 - 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (b^2*Cos[c + d*x]^5*S
in[c + d*x])/(6*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^5(c+dx)\cot(c+dx)\,dx + \int \cos^4(c+dx)\cot^2($$

$$= \frac{\text{Subst}\left(\int \frac{a^2+b^2+\frac{a^2}{x^2}}{(1+x^2)^4}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int \frac{x^6}{1-x^2}\right.}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a^2+(a^2+b^2)x^2}{x^2(1+x^2)^4}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int \left(\right.\right.}{}$$

$$= \frac{2ab\cos(c+dx)}{d} + \frac{2ab\cos^3(c+dx)}{3d} + \frac{2ab\cos^5(c+dx)}{5d} + \frac{b}{}$$

$$= -\frac{2ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{2ab\cos(c+dx)}{d} + \frac{2ab\cos^3(c}{3d}$$

$$= -\frac{2ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{2ab\cos(c+dx)}{d} + \frac{2ab\cos^3(c}{3d}$$

$$= -\frac{2ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{2ab\cos(c+dx)}{d} + \frac{2ab\cos^3(c}{3d}$$

$$= -\frac{5}{16}\left(6a^2-b^2\right)x - \frac{2ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{2ab\cos(c+}{d}$$

Mathematica [A] time = 0.40, size = 220, normalized size = 1.24

$$-\frac{15a^2(c+dx)}{8d} - \frac{a^2\sin(2(c+dx))}{2d} - \frac{a^2\sin(4(c+dx))}{32d} - \frac{a^2\cot(c+dx)}{d} + \frac{11ab\cos(c+dx)}{4d} + \frac{7ab\cos(3(c+dx))}{24d} + \frac{ab\,c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-15*a^2*(c + d*x))/(8*d) + (5*b^2*(c + d*x))/(16*d) + (11*a*b*Cos[c + d*x]
)/(4*d) + (7*a*b*Cos[3*(c + d*x)])/(24*d) + (a*b*Cos[5*(c + d*x)])/(40*d) -
 (a^2*Cot[c + d*x])/d - (2*a*b*Log[Cos[(c + d*x)/2]])/d + (2*a*b*Log[Sin[(c
 + d*x)/2]])/d - (a^2*Sin[2*(c + d*x)])/(2*d) + (15*b^2*Sin[2*(c + d*x)])/(
64*d) - (a^2*Sin[4*(c + d*x)])/(32*d) + (3*b^2*Sin[4*(c + d*x)])/(64*d) + (
b^2*Sin[6*(c + d*x)])/(192*d)
```

fricas [A] time = 0.76, size = 188, normalized size = 1.06

$$-\frac{40\,b^2\cos\left(dx+c\right)^7-10\left(6\,a^2-b^2\right)\cos\left(dx+c\right)^5-25\left(6\,a^2-b^2\right)\cos\left(dx+c\right)^3+240\,ab\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(40*b^2*cos(d*x + c)^7 - 10*(6*a^2 - b^2)*cos(d*x + c)^5 - 25*(6*a^2
 - b^2)*cos(d*x + c)^3 + 240*a*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 240*a*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 75*(6*a^2 - b^2)*cos(d
*x + c) - (96*a*b*cos(d*x + c)^5 + 160*a*b*cos(d*x + c)^3 - 75*(6*a^2 - b^2
)*d*x + 480*a*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

giac [B] time = 0.27, size = 368, normalized size = 2.07

$$\frac{480\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+120\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-75\left(6\,a^2-b^2\right)(dx+c)-\frac{120\left(4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(27\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(480*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 120*a^2*tan(1/2*d*x + 1/2*c
) - 75*(6*a^2 - b^2)*(d*x + c) - 120*(4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan
(1/2*d*x + 1/2*c) + 2*(270*a^2*tan(1/2*d*x + 1/2*c)^11 - 165*b^2*tan(1/2*d*
x + 1/2*c)^11 + 1440*a*b*tan(1/2*d*x + 1/2*c)^10 + 570*a^2*tan(1/2*d*x + 1/
2*c)^9 + 25*b^2*tan(1/2*d*x + 1/2*c)^9 + 4320*a*b*tan(1/2*d*x + 1/2*c)^8 +
300*a^2*tan(1/2*d*x + 1/2*c)^7 - 450*b^2*tan(1/2*d*x + 1/2*c)^7 + 7360*a*b*
tan(1/2*d*x + 1/2*c)^6 - 300*a^2*tan(1/2*d*x + 1/2*c)^5 + 450*b^2*tan(1/2*d
*x + 1/2*c)^5 + 6720*a*b*tan(1/2*d*x + 1/2*c)^4 - 570*a^2*tan(1/2*d*x + 1/2
*c)^3 - 25*b^2*tan(1/2*d*x + 1/2*c)^3 + 2976*a*b*tan(1/2*d*x + 1/2*c)^2 - 2
70*a^2*tan(1/2*d*x + 1/2*c) + 165*b^2*tan(1/2*d*x + 1/2*c) + 736*a*b)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 0.46, size = 250, normalized size = 1.40

$$-\frac{a^2\left(\cos^7\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}-\frac{a^2\left(\cos^5\left(dx+c\right)\right)\sin\left(dx+c\right)}{d}-\frac{5a^2\left(\cos^3\left(dx+c\right)\right)\sin\left(dx+c\right)}{4d}-\frac{15a^2\cos\left(dx+c\right)\sin\left(dx+\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/sin(d*x+c)*cos(d*x+c)^7-a^2*cos(d*x+c)^5*sin(d*x+c)/d-5/4*a^2*cos(
d*x+c)^3*sin(d*x+c)/d-15/8*a^2*cos(d*x+c)*sin(d*x+c)/d-15/8*a^2*x-15/8/d*a^
2*c+2/5*a*b*cos(d*x+c)^5/d+2/3*a*b*cos(d*x+c)^3/d+2*a*b*cos(d*x+c)/d+2/d*a*
b*ln(csc(d*x+c)-cot(d*x+c))+1/6*b^2*cos(d*x+c)^5*sin(d*x+c)/d+5/24*b^2*cos(
d*x+c)^3*sin(d*x+c)/d+5/16*b^2*cos(d*x+c)*sin(d*x+c)/d+5/16*b^2*x+5/16/d*b^
2*c
```

maxima [A] time = 0.77, size = 172, normalized size = 0.97

$$-\frac{120\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+25\,\tan(dx+c)^2+8}{\tan(dx+c)^5+2\,\tan(dx+c)^3+\tan(dx+c)}\right)a^2-64\left(6\,\cos\left(dx+c\right)^5+10\,\cos\left(dx+c\right)^3+30\,\cos\left(dx+c\right)\cdot\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/960*(120*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(t
an(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x + c)))*a^2 - 64*(6*cos(d*x + c)^
5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log
(cos(d*x + c) - 1))*a*b + 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4
*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^2)/d
```

mupad [B] time = 11.89, size = 683, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^10*((7*a^2)/2 + (5*b^2)/12) - tan(c/2 + (d*x)/2)^8*(10*
a^2 + (15*b^2)/2) + tan(c/2 + (d*x)/2)^12*((7*a^2)/2 - (11*b^2)/4) - tan(c/
2 + (d*x)/2)^2*((21*a^2)/2 - (11*b^2)/4) - tan(c/2 + (d*x)/2)^6*(25*a^2 - (
15*b^2)/2) - tan(c/2 + (d*x)/2)^4*((49*a^2)/2 + (5*b^2)/12) - a^2 + (248*a*
b*tan(c/2 + (d*x)/2)^3)/5 + 112*a*b*tan(c/2 + (d*x)/2)^5 + (368*a*b*tan(c/2
 + (d*x)/2)^7)/3 + 72*a*b*tan(c/2 + (d*x)/2)^9 + 24*a*b*tan(c/2 + (d*x)/2)^
11 + (184*a*b*tan(c/2 + (d*x)/2))/15)/(d*(2*tan(c/2 + (d*x)/2) + 12*tan(c/2
 + (d*x)/2)^3 + 30*tan(c/2 + (d*x)/2)^5 + 40*tan(c/2 + (d*x)/2)^7 + 30*tan(
c/2 + (d*x)/2)^9 + 12*tan(c/2 + (d*x)/2)^11 + 2*tan(c/2 + (d*x)/2)^13)) + (
a^2*tan(c/2 + (d*x)/2))/(2*d) - (atan((((a^2*15i)/8 - (b^2*5i)/16)*((5*b^2)
/8 - (15*a^2)/4 + 6*tan(c/2 + (d*x)/2)*((a^2*15i)/8 - (b^2*5i)/16) + 4*a*b*
tan(c/2 + (d*x)/2))*1i - ((a^2*15i)/8 - (b^2*5i)/16)*((15*a^2)/4 - (5*b^2)/
8 + 6*tan(c/2 + (d*x)/2)*((a^2*15i)/8 - (b^2*5i)/16) - 4*a*b*tan(c/2 + (d*x
```

```
)/2))*1i)/(((a^2*15i)/8 - (b^2*5i)/16)*((5*b^2)/8 - (15*a^2)/4 + 6*tan(c/2
+ (d*x)/2)*((a^2*15i)/8 - (b^2*5i)/16) + 4*a*b*tan(c/2 + (d*x)/2)) + ((a^2*
15i)/8 - (b^2*5i)/16)*((15*a^2)/4 - (5*b^2)/8 + 6*tan(c/2 + (d*x)/2)*((a^2*
15i)/8 - (b^2*5i)/16) - 4*a*b*tan(c/2 + (d*x)/2)) + (5*a*b^3)/2 - 15*a^3*b
+ 2*tan(c/2 + (d*x)/2)*((225*a^4)/16 + (25*b^4)/64 - (75*a^2*b^2)/16)))*((1
5*a^2)/4 - (5*b^2)/8))/d + (2*a*b*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1246 $\int \cos^3(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=180

$$-\frac{(a^2-b^2)\cos^3(c+dx)}{3d}-\frac{(2a^2-b^2)\cos(c+dx)}{d}+\frac{(5a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-\frac{1}{}$$

[Out] -15/4*a*b*x+1/2*(5*a^2-2*b^2)*arctanh(cos(d*x+c))/d-(2*a^2-b^2)*cos(d*x+c)/d-1/3*(a^2-b^2)*cos(d*x+c)^3/d+1/5*b^2*cos(d*x+c)^5/d-15/4*a*b*cot(d*x+c)/d+5/4*a*b*cos(d*x+c)^2*cot(d*x+c)/d+1/2*a*b*cos(d*x+c)^4*cot(d*x+c)/d-1/2*a^2*cot(d*x+c)*csc(d*x+c)/d

Rubi [A] time = 0.30, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2911, 2591, 288, 321, 203, 455, 1810, 206}

$$-\frac{(a^2-b^2)\cos^3(c+dx)}{3d}-\frac{(2a^2-b^2)\cos(c+dx)}{d}+\frac{(5a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-\frac{1}{}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]

[Out] (-15*a*b*x)/4 + ((5*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) - ((2*a^2 - b^2)*Cos[c + d*x])/d - ((a^2 - b^2)*Cos[c + d*x]^3)/(3*d) + (b^2*Cos[c + d*x]^5)/(5*d) - (15*a*b*Cot[c + d*x])/(4*d) + (5*a*b*Cos[c + d*x]^2*Cot[c + d*x])/(4*d) + (a*b*Cos[c + d*x]^4*Cot[c + d*x])/(2*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/(2*d)

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^4(c+dx)\cot^2(c+dx)\,dx + \int \cos^3(c+dx)\cot^3$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6\left(a^2+b^2-b^2x^2\right)}{\left(1-x^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\right.}{}$$

$$= \frac{ab\cos^4(c+dx)\cot(c+dx)}{2d} - \frac{a^2\cot(c+dx)\csc(c+dx)}{2d} + $$

$$= \frac{5ab\cos^2(c+dx)\cot(c+dx)}{4d} + \frac{ab\cos^4(c+dx)\cot(c+dx)}{2d}$$

$$= -\frac{\left(2a^2-b^2\right)\cos(c+dx)}{d} - \frac{\left(a^2-b^2\right)\cos^3(c+dx)}{3d} + \frac{b^2\cos^5}{5}$$

$$= -\frac{15}{4}abx + \frac{\left(5a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2d} - \frac{\left(2a^2-b^2\right)\cos}{d}$$

Mathematica [A] time = 6.19, size = 250, normalized size = 1.39

$$-\frac{\left(18a^2-11b^2\right)\cos(c+dx)}{8d} - \frac{\left(4a^2-7b^2\right)\cos(3(c+dx))}{48d} + \frac{\left(2b^2-5a^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{\left(5a^2-2b^2\right)\log\left(\cos\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-15*a*b*(c + d*x))/(4*d) - ((18*a^2 - 11*b^2)*Cos[c + d*x])/(8*d) - ((4*a^
2 - 7*b^2)*Cos[3*(c + d*x)])/(48*d) + (b^2*Cos[5*(c + d*x)])/(80*d) - (a*b*
Cot[(c + d*x)/2])/d - (a^2*Csc[(c + d*x)/2]^2)/(8*d) + ((5*a^2 - 2*b^2)*Log
[Cos[(c + d*x)/2]])/(2*d) + ((-5*a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]])/(2*d)
+ (a^2*Sec[(c + d*x)/2]^2)/(8*d) - (a*b*Sin[2*(c + d*x)])/d - (a*b*Sin[4*(c
 + d*x)])/(16*d) + (a*b*Tan[(c + d*x)/2])/d
```

fricas [A] time = 1.24, size = 244, normalized size = 1.36

$$\frac{12\,b^2\cos(dx+c)^7 - 225\,abdx\cos(dx+c)^2 - 4\left(5\,a^2-2\,b^2\right)\cos(dx+c)^5 + 225\,abdx - 20\left(5\,a^2-2\,b^2\right)\cos(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/60*(12*b^2*cos(d*x + c)^7 - 225*a*b*d*x*cos(d*x + c)^2 - 4*(5*a^2 - 2*b^2
)*cos(d*x + c)^5 + 225*a*b*d*x - 20*(5*a^2 - 2*b^2)*cos(d*x + c)^3 + 30*(5*
a^2 - 2*b^2)*cos(d*x + c) + 15*((5*a^2 - 2*b^2)*cos(d*x + c)^2 - 5*a^2 + 2*
b^2)*log(1/2*cos(d*x + c) + 1/2) - 15*((5*a^2 - 2*b^2)*cos(d*x + c)^2 - 5*a
^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2) - 15*(2*a*b*cos(d*x + c)^5 + 5*a*b
*cos(d*x + c)^3 - 15*a*b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

giac [B] time = 0.33, size = 346, normalized size = 1.92

$$\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-450\,(dx+c)ab+120\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-60\left(5\,a^2-2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+-\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/120*(15*a^2*tan(1/2*d*x + 1/2*c)^2 - 450*(d*x + c)*a*b + 120*a*b*tan(1/2*
d*x + 1/2*c) - 60*(5*a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 15*(30*a
^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d
*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2 + 4*(135*a*b*tan(1/2*d*x + 1/2*c)
^9 - 180*a^2*tan(1/2*d*x + 1/2*c)^8 + 180*b^2*tan(1/2*d*x + 1/2*c)^8 + 150*
a*b*tan(1/2*d*x + 1/2*c)^7 - 600*a^2*tan(1/2*d*x + 1/2*c)^6 + 360*b^2*tan(1
/2*d*x + 1/2*c)^6 - 800*a^2*tan(1/2*d*x + 1/2*c)^4 + 560*b^2*tan(1/2*d*x +
1/2*c)^4 - 150*a*b*tan(1/2*d*x + 1/2*c)^3 - 520*a^2*tan(1/2*d*x + 1/2*c)^2
+ 280*b^2*tan(1/2*d*x + 1/2*c)^2 - 135*a*b*tan(1/2*d*x + 1/2*c) - 140*a^2 +
 92*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

maple [A] time = 0.55, size = 261, normalized size = 1.45

$$-\frac{a^2\left(\cos^7(dx+c)\right)}{2d\sin(dx+c)^2}-\frac{a^2\left(\cos^5(dx+c)\right)}{2d}-\frac{5a^2\left(\cos^3(dx+c)\right)}{6d}-\frac{5a^2\cos(dx+c)}{2d}-\frac{5a^2\ln(\csc(dx+c)-\cot(dx+c\cdots}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-1/2*a^2*cos(d*x+c)^5/d-5/6*a^2*cos(d*x
+c)^3/d-5/2*a^2*cos(d*x+c)/d-5/2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/d*a*b/si
n(d*x+c)*cos(d*x+c)^7-2*a*b*cos(d*x+c)^5*sin(d*x+c)/d-5/2*a*b*cos(d*x+c)^3*
sin(d*x+c)/d-15/4*a*b*cos(d*x+c)*sin(d*x+c)/d-15/4*a*b*x-15/4/d*a*b*c+1/5*b
```

^2*cos(d*x+c)^5/d+1/3*b^2*cos(d*x+c)^3/d+b^2*cos(d*x+c)/d+1/d*b^2*ln(csc(d*x+c)-cot(d*x+c))

maxima [A] time = 0.59, size = 190, normalized size = 1.06

$$-\frac{5\left(4\,\cos\left(dx+c\right)^{3}-\frac{6\,\cos(dx+c)}{\cos(dx+c)^{2}-1}+24\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^{2}+15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/60*(5*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 + 15*(15
*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 +
 2*tan(d*x + c)^3 + tan(d*x + c)))*a*b - 2*(6*cos(d*x + c)^5 + 10*cos(d*x +
 c)^3 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
1))*b^2)/d
```

mupad [B] time = 11.76, size = 484, normalized size = 2.69

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{5\,a^2}{2}-b^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(\frac{49\,a^2}{2}-24\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(\frac{165\,a^2}{2}-48\,b^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) - (log(tan(c/2 + (d*x)/2))*((5*a^2)/2 - b^
2))/d - (tan(c/2 + (d*x)/2)^10*((49*a^2)/2 - 24*b^2) + tan(c/2 + (d*x)/2)^8
*((165*a^2)/2 - 48*b^2) + tan(c/2 + (d*x)/2)^2*((127*a^2)/6 - (184*b^2)/15)
 + tan(c/2 + (d*x)/2)^4*((223*a^2)/3 - (112*b^2)/3) + tan(c/2 + (d*x)/2)^6*
((335*a^2)/3 - (224*b^2)/3) + a^2/2 + 38*a*b*tan(c/2 + (d*x)/2)^3 + 60*a*b*
tan(c/2 + (d*x)/2)^5 + 40*a*b*tan(c/2 + (d*x)/2)^7 - 14*a*b*tan(c/2 + (d*x)
/2)^11 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 20*tan(c/2
+ (d*x)/2)^4 + 40*tan(c/2 + (d*x)/2)^6 + 40*tan(c/2 + (d*x)/2)^8 + 20*tan(c
/2 + (d*x)/2)^10 + 4*tan(c/2 + (d*x)/2)^12)) + (15*a*b*atan((225*a^2*b^2)/(
4*(15*a*b^3 - (75*a^3*b)/2 + (225*a^2*b^2*tan(c/2 + (d*x)/2))/4)) - (15*a*b
^3*tan(c/2 + (d*x)/2))/(15*a*b^3 - (75*a^3*b)/2 + (225*a^2*b^2*tan(c/2 + (d
```

```
*x)/2))/4) + (75*a^3*b*tan(c/2 + (d*x)/2))/(2*(15*a*b^3 - (75*a^3*b)/2 + (2
25*a^2*b^2*tan(c/2 + (d*x)/2))/4))))/(2*d) + (a*b*tan(c/2 + (d*x)/2))/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1247 $\int \cos^2(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=177

$$\frac{(2a^2-b^2)\cot(c+dx)}{d}+\frac{(4a^2-7b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}x(4a^2-3b^2)-\frac{a^2\cot^3(c+dx)}{3d}-\frac{5ab\cos^3(c+dx)}{3d}-$$

```
[Out] 5/8*(4*a^2-3*b^2)*x+5*a*b*arctanh(cos(d*x+c))/d-5*a*b*cos(d*x+c)/d-5/3*a*b*
cos(d*x+c)^3/d+(2*a^2-b^2)*cot(d*x+c)/d-a*b*cos(d*x+c)^3*cot(d*x+c)^2/d-1/3
*a^2*cot(d*x+c)^3/d+1/8*(4*a^2-7*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/4*b^2*cos(d
*x+c)^3*sin(d*x+c)/d
```

Rubi [A] time = 0.44, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2911, 2592, 288, 302, 206, 456, 1259, 1261, 203}

$$\frac{(2a^2-b^2)\cot(c+dx)}{d}+\frac{(4a^2-7b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}x(4a^2-3b^2)-\frac{a^2\cot^3(c+dx)}{3d}-\frac{5ab\cos^3(c+dx)}{3d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*(4*a^2 - 3*b^2)*x)/8 + (5*a*b*ArcTanh[Cos[c + d*x]])/d - (5*a*b*Cos[c +
d*x])/d - (5*a*b*Cos[c + d*x]^3)/(3*d) + ((2*a^2 - b^2)*Cot[c + d*x])/d - (
a*b*Cos[c + d*x]^3*Cot[c + d*x]^2)/d - (a^2*Cot[c + d*x]^3)/(3*d) + ((4*a^2
 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (b^2*Cos[c + d*x]^3*Sin[c + d*
x])/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^3(c+dx)\cot^3(c+dx)\,dx + \int \cos^2(c+dx)\cot^4$$

$$= \frac{\text{Subst}\left(\int \frac{a^2+\left(a^2+b^2\right)x^2}{x^4\left(1+x^2\right)^3}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int \frac{}{(}\right.}{}$$

$$= -\frac{ab\cos^3(c+dx)\cot^2(c+dx)}{d} - \frac{b^2\cos^3(c+dx)\sin(c+dx)}{4d}$$

$$= -\frac{ab\cos^3(c+dx)\cot^2(c+dx)}{d} + \frac{\left(4a^2-7b^2\right)\cos(c+dx)\sin}{8d}$$

$$= -\frac{5ab\cos(c+dx)}{d} - \frac{5ab\cos^3(c+dx)}{3d} - \frac{ab\cos^3(c+dx)\cot^2}{d}$$

$$= \frac{5ab\tanh^{-1}(\cos(c+dx))}{d} - \frac{5ab\cos(c+dx)}{d} - \frac{5ab\cos^3(c+}{3d}$$

$$= \frac{5}{8}\left(4a^2-3b^2\right)x + \frac{5ab\tanh^{-1}(\cos(c+dx))}{d} - \frac{5ab\cos(c+d}{d}$$

Mathematica [A] time = 6.28, size = 336, normalized size = 1.90

$$\frac{5\left(4a^2-3b^2\right)(c+dx)}{8d} + \frac{\left(a^2-2b^2\right)\sin(2(c+dx))}{4d} + \frac{\csc\left(\frac{1}{2}(c+dx)\right)\left(7a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{6d} + \frac{\text{s}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*(4*a^2 - 3*b^2)*(c + d*x))/(8*d) - (9*a*b*Cos[c + d*x])/(2*d) - (a*b*Cos
[3*(c + d*x)])/(6*d) + ((7*a^2*Cos[(c + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*C
```

```
sc[(c + d*x)/2])/(6*d) - (a*b*Csc[(c + d*x)/2]^2)/(4*d) - (a^2*Cot[(c + d*x
)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (5*a*b*Log[Cos[(c + d*x)/2]])/d - (5*a*b*
Log[Sin[(c + d*x)/2]])/d + (a*b*Sec[(c + d*x)/2]^2)/(4*d) + (Sec[(c + d*x)/
2]*(-7*a^2*Sin[(c + d*x)/2] + 3*b^2*Sin[(c + d*x)/2]))/(6*d) + ((a^2 - 2*b^
2)*Sin[2*(c + d*x)])/(4*d) - (b^2*Sin[4*(c + d*x)])/(32*d) + (a^2*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)
```

fricas [A] time = 0.68, size = 252, normalized size = 1.42

$$6\,b^2\cos\left(dx+c\right)^7-3\left(4\,a^2-3\,b^2\right)\cos\left(dx+c\right)^5+20\left(4\,a^2-3\,b^2\right)\cos\left(dx+c\right)^3+60\left(ab\cos\left(dx+c\right)^2-ab\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(6*b^2*cos(d*x + c)^7 - 3*(4*a^2 - 3*b^2)*cos(d*x + c)^5 + 20*(4*a^2 -
 3*b^2)*cos(d*x + c)^3 + 60*(a*b*cos(d*x + c)^2 - a*b)*log(1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) - 60*(a*b*cos(d*x + c)^2 - a*b)*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) - 15*(4*a^2 - 3*b^2)*cos(d*x + c) - (16*a*b*cos(d*x + c
)^5 - 15*(4*a^2 - 3*b^2)*d*x*cos(d*x + c)^2 + 80*a*b*cos(d*x + c)^3 + 15*(4
*a^2 - 3*b^2)*d*x - 120*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2
- d)*sin(d*x + c))
```

giac [B] time = 0.31, size = 366, normalized size = 2.07

$$a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-27\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,b^2\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 - 120*a*b*l
og(abs(tan(1/2*d*x + 1/2*c))) - 27*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/
2*d*x + 1/2*c) + 15*(4*a^2 - 3*b^2)*(d*x + c) + (220*a*b*tan(1/2*d*x + 1/2*
c)^3 + 27*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*
b*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^3 - 2*(12*a^2*tan(1/2*d*
x + 1/2*c)^7 - 27*b^2*tan(1/2*d*x + 1/2*c)^7 + 144*a*b*tan(1/2*d*x + 1/2*c)
^6 + 12*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^2*tan(1/2*d*x + 1/2*c)^5 + 336*a*b
*tan(1/2*d*x + 1/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 + 3*b^2*tan(1/2*d*x
```

```
 + 1/2*c)^3 + 304*a*b*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*tan(1/2*d*x + 1/2*c)
+ 27*b^2*tan(1/2*d*x + 1/2*c) + 112*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.52, size = 321, normalized size = 1.81

$$-\frac{a^2\left(\cos^7(dx+c)\right)}{3d\sin(dx+c)^3}+\frac{4a^2\left(\cos^7(dx+c)\right)}{3d\sin(dx+c)}+\frac{4a^2\left(\cos^5(dx+c)\right)\sin(dx+c)}{3d}+\frac{5a^2\left(\cos^3(dx+c)\right)\sin(dx+c)}{3d}+\frac{5a^2 c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d*a^2/sin(d*x+c)^3*cos(d*x+c)^7+4/3/d*a^2/sin(d*x+c)*cos(d*x+c)^7+4/3*
a^2*cos(d*x+c)^5*sin(d*x+c)/d+5/3*a^2*cos(d*x+c)^3*sin(d*x+c)/d+5/2*a^2*cos
(d*x+c)*sin(d*x+c)/d+5/2*a^2*x+5/2/d*a^2*c-1/d*a*b/sin(d*x+c)^2*cos(d*x+c)^
7-a*b*cos(d*x+c)^5/d-5/3*a*b*cos(d*x+c)^3/d-5*a*b*cos(d*x+c)/d-5/d*a*b*ln(c
sc(d*x+c)-cot(d*x+c))-1/d*b^2/sin(d*x+c)*cos(d*x+c)^7-b^2*cos(d*x+c)^5*sin(
d*x+c)/d-5/4*b^2*cos(d*x+c)^3*sin(d*x+c)/d-15/8*b^2*cos(d*x+c)*sin(d*x+c)/d
-15/8*b^2*x-15/8/d*b^2*c
```

maxima [A] time = 0.58, size = 189, normalized size = 1.07

$$\frac{4\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)a^2-4\left(4\,\cos(dx+c)^3-\frac{6\,\cos(dx+c)}{\cos(dx+c)^2-1}+24\,\cos(dx+c)-15\,\log(\cos(\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*(4*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d
*x + c)^5 + tan(d*x + c)^3))*a^2 - 4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(co
s(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos
(d*x + c) - 1))*a*b - 3*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x +
c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x + c)))*b^2)/d
```

mupad [B] time = 12.10, size = 665, normalized size = 3.76

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{\frac{a^2}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(42\,a^2-34\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(\frac{83\,a^2}{3}-14\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{182\,a^2}{3}-26\right.}{d\left(8\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (a^2/3 - tan(c/2 + (d*x)/2)^4*(42*a^2 -
 34*b^2) - tan(c/2 + (d*x)/2)^8*((83*a^2)/3 - 14*b^2) - tan(c/2 + (d*x)/2)^
6*((182*a^2)/3 - 26*b^2) - tan(c/2 + (d*x)/2)^2*((23*a^2)/3 - 4*b^2) - tan(
c/2 + (d*x)/2)^10*(a^2 + 14*b^2) + (248*a*b*tan(c/2 + (d*x)/2)^3)/3 + (644*
a*b*tan(c/2 + (d*x)/2)^5)/3 + 232*a*b*tan(c/2 + (d*x)/2)^7 + 98*a*b*tan(c/2
 + (d*x)/2)^9 + 2*a*b*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 32*t
an(c/2 + (d*x)/2)^5 + 48*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9 + 8
*tan(c/2 + (d*x)/2)^11)) - (tan(c/2 + (d*x)/2)*((9*a^2)/8 - b^2/2))/d + (at
an((((a^2*5i)/2 - (b^2*15i)/8)*((15*b^2)/4 - 5*a^2 + 6*tan(c/2 + (d*x)/2)*(
(a^2*5i)/2 - (b^2*15i)/8) + 10*a*b*tan(c/2 + (d*x)/2))*1i - ((a^2*5i)/2 - (
b^2*15i)/8)*(5*a^2 - (15*b^2)/4 + 6*tan(c/2 + (d*x)/2)*((a^2*5i)/2 - (b^2*1
5i)/8) - 10*a*b*tan(c/2 + (d*x)/2))*1i)/(((a^2*5i)/2 - (b^2*15i)/8)*((15*b^
2)/4 - 5*a^2 + 6*tan(c/2 + (d*x)/2)*((a^2*5i)/2 - (b^2*15i)/8) + 10*a*b*tan
(c/2 + (d*x)/2)) + ((a^2*5i)/2 - (b^2*15i)/8)*(5*a^2 - (15*b^2)/4 + 6*tan(c
/2 + (d*x)/2)*((a^2*5i)/2 - (b^2*15i)/8) - 10*a*b*tan(c/2 + (d*x)/2)) + (75
*a*b^3)/2 - 50*a^3*b + 2*tan(c/2 + (d*x)/2)*(25*a^4 + (225*b^4)/16 - (75*a^
2*b^2)/2)))*(5*a^2 - (15*b^2)/4))/d + (a*b*tan(c/2 + (d*x)/2)^2)/(4*d) - (5
*a*b*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1248 $\int \cos(c+dx)\cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=174

$$\frac{(a^2-2b^2)\cos(c+dx)}{d}-\frac{5(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{(9a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^2\cot(c+dx)\text{cs}}{4d}$$

```
[Out] 5*a*b*x-5/8*(3*a^2-4*b^2)*arctanh(cos(d*x+c))/d+(a^2-2*b^2)*cos(d*x+c)/d-1/
3*b^2*cos(d*x+c)^3/d+5*a*b*cot(d*x+c)/d-5/3*a*b*cot(d*x+c)^3/d+a*b*cos(d*x+
c)^2*cot(d*x+c)^3/d+1/8*(9*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/d-1/4*a^2*cot(d
*x+c)*csc(d*x+c)^3/d
```

Rubi [A] time = 0.28, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2911, 2591, 288, 302, 203, 455, 1814, 1153, 206}

$$\frac{(a^2-2b^2)\cos(c+dx)}{d}-\frac{5(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{(9a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^2\cot(c+dx)\text{cs}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] 5*a*b*x - (5*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + ((a^2 - 2*b^2)*
Cos[c + d*x])/d - (b^2*Cos[c + d*x]^3)/(3*d) + (5*a*b*Cot[c + d*x])/d - (5*
a*b*Cot[c + d*x]^3)/(3*d) + (a*b*Cos[c + d*x]^2*Cot[c + d*x]^3)/d + ((9*a^2
 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]
^3)/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^5(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^2(c+dx)\cot^4(c+dx)\,dx + \int \cos(c+dx)\cot^5(c$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6\left(a^2+b^2-b^2x^2\right)}{\left(1-x^2\right)^3}\,dx, x, \cos(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\Big(\int}{}$$

$$= \frac{ab\cos^2(c+dx)\cot^3(c+dx)}{d} - \frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} +$$

$$= \frac{ab\cos^2(c+dx)\cot^3(c+dx)}{d} + \frac{\left(9a^2-4b^2\right)\cot(c+dx)\csc(c}{8d}$$

$$= \frac{5ab\cot(c+dx)}{d} - \frac{5ab\cot^3(c+dx)}{3d} + \frac{ab\cos^2(c+dx)\cot^3(c}{d}$$

$$= 5abx + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{d} - \frac{b^2\cos^3(c+dx)}{3d} + \frac{5ab\cot(c}{d}$$

$$= 5abx - \frac{5\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} + \frac{\left(a^2-2b^2\right)\cos(c}{d}$$

Mathematica [A] time = 6.18, size = 337, normalized size = 1.94

$$\frac{\left(9a^2-4b^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{\left(4b^2-9a^2\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{5\left(3a^2-4b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{5\left(3a^2-4b^2\right)\text{lo}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a*b*(c + d*x))/d + ((2*a - 3*b)*(2*a + 3*b)*Cos[c + d*x])/(4*d) - (b^2*C
os[3*(c + d*x)])/(12*d) + (7*a*b*Cot[(c + d*x)/2])/(3*d) + ((9*a^2 - 4*b^2)
*Csc[(c + d*x)/2]^2)/(32*d) - (a*b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(12
```

```
*d) - (a^2*Csc[(c + d*x)/2]^4)/(64*d) - (5*(3*a^2 - 4*b^2)*Log[Cos[(c + d*x
)/2]])/(8*d) + (5*(3*a^2 - 4*b^2)*Log[Sin[(c + d*x)/2]])/(8*d) + ((-9*a^2 +
 4*b^2)*Sec[(c + d*x)/2]^2)/(32*d) + (a^2*Sec[(c + d*x)/2]^4)/(64*d) + (a*b
*Sin[2*(c + d*x)])/(2*d) - (7*a*b*Tan[(c + d*x)/2])/(3*d) + (a*b*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/(12*d)
```

fricas [A] time = 0.80, size = 309, normalized size = 1.78

$$-\frac{16\,b^2\cos\left(dx+c\right)^7-240\,abdx\cos\left(dx+c\right)^4+480\,abdx\cos\left(dx+c\right)^2-16\left(3\,a^2-4\,b^2\right)\cos\left(dx+c\right)^5-240\,abd}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(16*b^2*cos(d*x + c)^7 - 240*a*b*d*x*cos(d*x + c)^4 + 480*a*b*d*x*cos
(d*x + c)^2 - 16*(3*a^2 - 4*b^2)*cos(d*x + c)^5 - 240*a*b*d*x + 50*(3*a^2 -
 4*b^2)*cos(d*x + c)^3 - 30*(3*a^2 - 4*b^2)*cos(d*x + c) + 15*((3*a^2 - 4*b
^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*b^2)*cos(d*x + c)^2 + 3*a^2 - 4*b^2)*log(
1/2*cos(d*x + c) + 1/2) - 15*((3*a^2 - 4*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4
*b^2)*cos(d*x + c)^2 + 3*a^2 - 4*b^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*(3*
a*b*cos(d*x + c)^5 - 20*a*b*cos(d*x + c)^3 + 15*a*b*cos(d*x + c))*sin(d*x +
 c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [B] time = 0.33, size = 346, normalized size = 1.99

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-48\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+960\,(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a*b*tan(1/2*d*x + 1/2*c)^3 - 48*a^
2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 960*(d*x + c)*a*
b - 432*a*b*tan(1/2*d*x + 1/2*c) + 120*(3*a^2 - 4*b^2)*log(abs(tan(1/2*d*x
+ 1/2*c))) - 128*(3*a*b*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*tan(1/2*d*x + 1/2*c)
^4 + 9*b^2*tan(1/2*d*x + 1/2*c)^4 - 6*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*t
an(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2 + 7*b^2)/(tan(1/
2*d*x + 1/2*c)^2 + 1)^3 - (750*a^2*tan(1/2*d*x + 1/2*c)^4 - 1000*b^2*tan(1/
2*d*x + 1/2*c)^4 - 432*a*b*tan(1/2*d*x + 1/2*c)^3 - 48*a^2*tan(1/2*d*x + 1/
```

```
2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*tan(1/2*d*x + 1/2*c) + 3*a^
2)/tan(1/2*d*x + 1/2*c)^4)/d
```

maple [B] time = 0.54, size = 334, normalized size = 1.92

$$-\frac{a^2\left(\cos^7(dx+c)\right)}{4d\sin(dx+c)^4}+\frac{3a^2\left(\cos^7(dx+c)\right)}{8d\sin(dx+c)^2}+\frac{3a^2\left(\cos^5(dx+c)\right)}{8d}+\frac{5a^2\left(\cos^3(dx+c)\right)}{8d}+\frac{15a^2\cos(dx+c)}{8d}+\frac{15a^2\ln(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7+3/8/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7+3/
8*a^2*cos(d*x+c)^5/d+5/8*a^2*cos(d*x+c)^3/d+15/8*a^2*cos(d*x+c)/d+15/8/d*a^
2*ln(csc(d*x+c)-cot(d*x+c))-2/3/d*a*b/sin(d*x+c)^3*cos(d*x+c)^7+8/3/d*a*b/s
in(d*x+c)*cos(d*x+c)^7+8/3*a*b*cos(d*x+c)^5*sin(d*x+c)/d+10/3*a*b*cos(d*x+c
)^3*sin(d*x+c)/d+5*a*b*cos(d*x+c)*sin(d*x+c)/d+5*a*b*x+5/d*a*b*c-1/2/d*b^2/
sin(d*x+c)^2*cos(d*x+c)^7-1/2*b^2*cos(d*x+c)^5/d-5/6*b^2*cos(d*x+c)^3/d-5/2
*b^2*cos(d*x+c)/d-5/2/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.50, size = 205, normalized size = 1.18

$$\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)ab-4\left(4\,\cos(dx+c)^3-\frac{6\,\cos(dx+c)}{\cos(dx+c)^2-1}+24\,\cos(dx+c)-15\,\log(\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/48*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(
d*x + c)^5 + tan(d*x + c)^3))*a*b - 4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(c
os(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(co
s(d*x + c) - 1))*b^2 - 3*a^2*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*
x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) +
1) - 15*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 11.74, size = 479, normalized size = 2.75

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(\frac{15\,a^2}{8}-\frac{5\,b^2}{2}\right)}{d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{13\,a^2}{4}-2\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(36\,a^2-98\,b^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^5,x)

[Out] (log(tan(c/2 + (d*x)/2))*((15*a^2)/8 - (5*b^2)/2))/d + (a^2*tan(c/2 + (d*x)/2)^4)/(64*d) + (tan(c/2 + (d*x)/2)^2*((13*a^2)/4 - 2*b^2) + tan(c/2 + (d*x)/2)^8*(36*a^2 - 98*b^2) + tan(c/2 + (d*x)/2)^4*((173*a^2)/4 - (242*b^2)/3) + tan(c/2 + (d*x)/2)^6*((303*a^2)/4 - 134*b^2) - a^2/4 + 32*a*b*tan(c/2 + (d*x)/2)^3 + 136*a*b*tan(c/2 + (d*x)/2)^5 + (320*a*b*tan(c/2 + (d*x)/2)^7)/3 + 4*a*b*tan(c/2 + (d*x)/2)^9 - (4*a*b*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2)^4 + 48*tan(c/2 + (d*x)/2)^6 + 48*tan(c/2 + (d*x)/2)^8 + 16*tan(c/2 + (d*x)/2)^10)) - (tan(c/2 + (d*x)/2)^2*(a^2/4 - b^2/8))/d + (a*b*tan(c/2 + (d*x)/2)^3)/(12*d) - (10*a*b*atan((100*a^2*b^2)/(50*a*b^3 - (75*a^3*b)/2 + 100*a^2*b^2*tan(c/2 + (d*x)/2)) - (50*a*b^3*tan(c/2 + (d*x)/2))/(50*a*b^3 - (75*a^3*b)/2 + 100*a^2*b^2*tan(c/2 + (d*x)/2)) + (75*a^3*b*tan(c/2 + (d*x)/2))/(2*(50*a*b^3 - (75*a^3*b)/2 + 100*a^2*b^2*tan(c/2 + (d*x)/2)))))/d - (9*a*b*tan(c/2 + (d*x)/2))/(4*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5*(a+b*sin(d*x+c))**2,x)

[Out] Timed out

3.1249 $\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=202

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x+\frac{15ab\cos(c+dx)}{4d}-\frac{ab\cos(c+dx)\cot^4(c+dx)}{2d}+\frac{5ab\cos(}{}$$

```
[Out] -a^2*x+5/2*b^2*x-15/4*a*b*arctanh(cos(d*x+c))/d+15/4*a*b*cos(d*x+c)/d-a^2*c
ot(d*x+c)/d+5/2*b^2*cot(d*x+c)/d+5/4*a*b*cos(d*x+c)*cot(d*x+c)^2/d+1/3*a^2*
cot(d*x+c)^3/d-5/6*b^2*cot(d*x+c)^3/d+1/2*b^2*cos(d*x+c)^2*cot(d*x+c)^3/d-1
/2*a*b*cos(d*x+c)*cot(d*x+c)^4/d-1/5*a^2*cot(d*x+c)^5/d
```

Rubi [A] time = 0.19, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2722, 2591, 288, 302, 203, 2592, 321, 206, 3473, 8}

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x+\frac{15ab\cos(c+dx)}{4d}-\frac{ab\cos(c+dx)\cot^4(c+dx)}{2d}+\frac{5ab\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (5*b^2*x)/2 - (15*a*b*ArcTanh[Cos[c + d*x]])/(4*d) + (15*a*b*Cos
[c + d*x])/(4*d) - (a^2*Cot[c + d*x])/d + (5*b^2*Cot[c + d*x])/(2*d) + (5*a
*b*Cos[c + d*x]*Cot[c + d*x]^2)/(4*d) + (a^2*Cot[c + d*x]^3)/(3*d) - (5*b^2
*Cot[c + d*x]^3)/(6*d) + (b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a*b*C
os[c + d*x]*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx = \int \left(b^2\cos^2(c+dx)\cot^4(c+dx)+2ab\cos(c+dx)\cot^5(c+dx)+a^2\cot^6\right.$$

$$= a^2\int \cot^6(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot^5(c+dx)\,dx+b^2\int \cos^2($$

$$= -\frac{a^2\cot^5(c+dx)}{5d}-a^2\int \cot^4(c+dx)\,dx-\frac{(2ab)\,\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx,x,\text{c}\right.}{d}$$

$$= \frac{a^2\cot^3(c+dx)}{3d}+\frac{b^2\cos^2(c+dx)\cot^3(c+dx)}{2d}-\frac{ab\cos(c+dx)\cot^4(c+}{2d}$$

$$= -\frac{a^2\cot(c+dx)}{d}+\frac{5ab\cos(c+dx)\cot^2(c+dx)}{4d}+\frac{a^2\cot^3(c+dx)}{3d}+\frac{b^2\text{c}}{}$$

$$= -a^2x+\frac{15ab\cos(c+dx)}{4d}-\frac{a^2\cot(c+dx)}{d}+\frac{5b^2\cot(c+dx)}{2d}+\frac{5ab\cos(c}{}$$

$$= -a^2x+\frac{5b^2x}{2}-\frac{15ab\tanh^{-1}(\cos(c+dx))}{4d}+\frac{15ab\cos(c+dx)}{4d}-\frac{a^2\cot(c}{d}$$

Mathematica [A] time = 1.12, size = 351, normalized size = 1.74

$$\frac{\left(560b^2-368a^2\right)\cot\left(\frac{1}{2}(c+dx)\right)+368a^2\tan\left(\frac{1}{2}(c+dx)\right)-\frac{3}{2}a^2\sin(c+dx)\csc^6\left(\frac{1}{2}(c+dx)\right)+96a^2\sin^6\left(\frac{1}{2}(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-480*a^2*c + 1200*b^2*c - 480*a^2*d*x + 1200*b^2*d*x + 960*a*b*Cos[c + d*x
] + (-368*a^2 + 560*b^2)*Cot[(c + d*x)/2] + 270*a*b*Csc[(c + d*x)/2]^2 - 15
*a*b*Csc[(c + d*x)/2]^4 - 1800*a*b*Log[Cos[(c + d*x)/2]] + 1800*a*b*Log[Sin
[(c + d*x)/2]] - 270*a*b*Sec[(c + d*x)/2]^2 + 15*a*b*Sec[(c + d*x)/2]^4 - 3
28*a^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 160*b^2*Csc[c + d*x]^3*Sin[(c +
d*x)/2]^4 + 96*a^2*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + (41*a^2*Csc[(c + d*x
)/2]^4*Sin[c + d*x])/2 - 10*b^2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - (3*a^2*Cs
c[(c + d*x)/2]^6*Sin[c + d*x])/2 + 120*b^2*Sin[2*(c + d*x)] + 368*a^2*Tan[(
c + d*x)/2] - 560*b^2*Tan[(c + d*x)/2])/(480*d)
```

fricas [A] time = 0.82, size = 306, normalized size = 1.51

$$-\frac{60\,b^2\cos\left(dx+c\right)^7+92\left(2\,a^2-5\,b^2\right)\cos\left(dx+c\right)^5-140\left(2\,a^2-5\,b^2\right)\cos\left(dx+c\right)^3+225\left(ab\cos\left(dx+c\right)^4-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/120*(60*b^2*cos(d*x + c)^7 + 92*(2*a^2 - 5*b^2)*cos(d*x + c)^5 - 140*(2*
a^2 - 5*b^2)*cos(d*x + c)^3 + 225*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^
2 + a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(a*b*cos(d*x + c)^4
 - 2*a*b*cos(d*x + c)^2 + a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
60*(2*a^2 - 5*b^2)*cos(d*x + c) + 30*(2*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^4
- 8*a*b*cos(d*x + c)^5 - 4*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^2 + 25*a*b*cos(
d*x + c)^3 + 2*(2*a^2 - 5*b^2)*d*x - 15*a*b*cos(d*x + c))*sin(d*x + c))/((d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.31, size = 337, normalized size = 1.67

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,ab\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^
2*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 - 240*a*b*tan(1/2*
d*x + 1/2*c)^2 + 1800*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 330*a^2*tan(1/2*
d*x + 1/2*c) - 540*b^2*tan(1/2*d*x + 1/2*c) - 240*(2*a^2 - 5*b^2)*(d*x + c)
 - 480*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan
(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - (4110*a*b*tan(1
/2*d*x + 1/2*c)^5 + 330*a^2*tan(1/2*d*x + 1/2*c)^4 - 540*b^2*tan(1/2*d*x +
1/2*c)^4 - 240*a*b*tan(1/2*d*x + 1/2*c)^3 - 35*a^2*tan(1/2*d*x + 1/2*c)^2 +
 20*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a*b*tan(1/2*d*x + 1/2*c) + 3*a^2)/tan(1
/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.56, size = 302, normalized size = 1.50

$$-\frac{a^2\left(\cot^5\left(dx+c\right)\right)}{5d}+\frac{a^2\left(\cot^3\left(dx+c\right)\right)}{3d}-\frac{a^2\cot\left(dx+c\right)}{d}-a^2x-\frac{a^2c}{d}-\frac{ab\left(\cos^7\left(dx+c\right)\right)}{2d\sin\left(dx+c\right)^4}+\frac{3ab\left(\cos^7\left(dx+c\right)\right)}{4d\sin\left(dx+c\right)^2}+\frac{3ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/5*a^2*cot(d*x+c)^5/d+1/3*a^2*cot(d*x+c)^3/d-a^2*cot(d*x+c)/d-a^2*x-1/d*a
^2*c-1/2/d*a*b/sin(d*x+c)^4*cos(d*x+c)^7+3/4/d*a*b/sin(d*x+c)^2*cos(d*x+c)^
7+3/4*a*b*cos(d*x+c)^5/d+5/4*a*b*cos(d*x+c)^3/d+15/4*a*b*cos(d*x+c)/d+15/4/
d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*b^2/sin(d*x+c)^3*cos(d*x+c)^7+4/3/d*b
^2/sin(d*x+c)*cos(d*x+c)^7+4/3*b^2*cos(d*x+c)^5*sin(d*x+c)/d+5/3*b^2*cos(d*
x+c)^3*sin(d*x+c)/d+5/2*b^2*cos(d*x+c)*sin(d*x+c)/d+5/2*b^2*x+5/2/d*b^2*c
```

maxima [A] time = 0.56, size = 183, normalized size = 0.91

$$-\frac{8\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2-20\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)b^2+15\,ab\left(\frac{2\,(9\,\cos(\ldots}{\cos(dx+\ldots}\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*(8*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d
*x + c)^5)*a^2 - 20*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2
 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*b^2 + 15*a*b*(2*(9*cos(d*x + c)^3
- 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c)
 + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 16.04, size = 888, normalized size = 4.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] ((95*b^2*cos(c + d*x))/384 - (5*a^2*cos(c + d*x))/24 + (5*a^2*cos(3*c + 3*d
*x))/48 - (23*a^2*cos(5*c + 5*d*x))/240 - (163*b^2*cos(3*c + 3*d*x))/384 +
(71*b^2*cos(5*c + 5*d*x))/384 - (b^2*cos(7*c + 7*d*x))/128 + (5*a^2*atan((1
0*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x
)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/
2 + (d*x)/2)))*sin(3*c + 3*d*x))/8 - (a^2*atan((10*b^2*cos(c/2 + (d*x)/2) -
 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*
x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(5*c + 5
*d*x))/8 - (25*b^2*atan((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/
2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2
+ (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d*x))/16 + (5*b^2*atan
```

```
((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (
d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos
(c/2 + (d*x)/2)))*sin(5*c + 5*d*x))/16 + (5*a*b*sin(c + d*x))/4 - (5*a^2*at
an((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 +
 (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*c
os(c/2 + (d*x)/2)))*sin(c + d*x))/4 + (25*b^2*atan((10*b^2*cos(c/2 + (d*x)/
2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 +
 (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(c +
 d*x))/8 + (5*a*b*sin(2*c + 2*d*x))/8 - (5*a*b*sin(3*c + 3*d*x))/8 - (17*a*
b*sin(4*c + 4*d*x))/32 + (a*b*sin(5*c + 5*d*x))/8 + (a*b*sin(6*c + 6*d*x))/
16 + (75*a*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/32 -
(75*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x))/64 + (
15*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(5*c + 5*d*x))/64)/(d*
sin(c + d*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1250 $\int \cot^6(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=175

$$\frac{5\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{\left(13a^2-6b^2\right)\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{\left(11a^2-18b^2\right)\cot(c+dx)\csc(c+dx)}{16d}-$$

```
[Out] -2*a*b*x+5/16*(a^2-6*b^2)*arctanh(cos(d*x+c))/d+b^2*cos(d*x+c)/d-2*a*b*cot(
d*x+c)/d+2/3*a*b*cot(d*x+c)^3/d-2/5*a*b*cot(d*x+c)^5/d-1/16*(11*a^2-18*b^2)
*cot(d*x+c)*csc(d*x+c)/d+1/24*(13*a^2-6*b^2)*cot(d*x+c)*csc(d*x+c)^3/d-1/6*
a^2*cot(d*x+c)*csc(d*x+c)^5/d
```

Rubi [A] time = 0.26, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2911, 3473, 8, 4366, 455, 1814, 1157, 388, 206}

$$\frac{5\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{\left(13a^2-6b^2\right)\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{\left(11a^2-18b^2\right)\cot(c+dx)\csc(c+dx)}{16d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -2*a*b*x + (5*(a^2 - 6*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) + (b^2*Cos[c + d*
x])/d - (2*a*b*Cot[c + d*x])/d + (2*a*b*Cot[c + d*x]^3)/(3*d) - (2*a*b*Cot[
c + d*x]^5)/(5*d) - ((11*a^2 - 18*b^2)*Cot[c + d*x]*Csc[c + d*x])/(16*d) +
((13*a^2 - 6*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^2*Cot[c + d*x]*C
sc[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \cot^6(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\,dx + \int \cot^6(c+dx)\csc(c+dx)\left(a^2+b\right.$$

$$= -\frac{2ab\cot^5(c+dx)}{5d} - (2ab)\int \cot^4(c+dx)\,dx - \frac{\text{Subst}\left(\int \frac{x^6(a}{(}\right.}{}$$

$$= \frac{2ab\cot^3(c+dx)}{3d} - \frac{2ab\cot^5(c+dx)}{5d} - \frac{a^2\cot(c+dx)\csc^5(c+}{6d}$$

$$= -\frac{2ab\cot(c+dx)}{d} + \frac{2ab\cot^3(c+dx)}{3d} - \frac{2ab\cot^5(c+dx)}{5d} + \frac{(1}{}$$

$$= -2abx - \frac{2ab\cot(c+dx)}{d} + \frac{2ab\cot^3(c+dx)}{3d} - \frac{2ab\cot^5(c+d}{5d}$$

$$= -2abx + \frac{b^2\cos(c+dx)}{d} - \frac{2ab\cot(c+dx)}{d} + \frac{2ab\cot^3(c+dx)}{3d}$$

$$= -2abx + \frac{5\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{16d} + \frac{b^2\cos(c+dx)}{d}$$

Mathematica [B] time = 1.05, size = 384, normalized size = 2.19

$$\frac{-5a^2\csc^6\left(\frac{1}{2}(c+dx)\right) + 60a^2\csc^4\left(\frac{1}{2}(c+dx)\right) - 330a^2\csc^2\left(\frac{1}{2}(c+dx)\right) + 5a^2\sec^6\left(\frac{1}{2}(c+dx)\right) - 60a^2\sec^4\left(\frac{1}{2}(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3840*a*b*c - 3840*a*b*d*x + 1920*b^2*Cos[c + d*x] - 2944*a*b*Cot[(c + d*x
)/2] - 330*a^2*Csc[(c + d*x)/2]^2 + 540*b^2*Csc[(c + d*x)/2]^2 + 60*a^2*Csc
[(c + d*x)/2]^4 - 30*b^2*Csc[(c + d*x)/2]^4 - 5*a^2*Csc[(c + d*x)/2]^6 + 60
```

```
0*a^2*Log[Cos[(c + d*x)/2]] - 3600*b^2*Log[Cos[(c + d*x)/2]] - 600*a^2*Log[
Sin[(c + d*x)/2]] + 3600*b^2*Log[Sin[(c + d*x)/2]] + 330*a^2*Sec[(c + d*x)/
2]^2 - 540*b^2*Sec[(c + d*x)/2]^2 - 60*a^2*Sec[(c + d*x)/2]^4 + 30*b^2*Sec[
(c + d*x)/2]^4 + 5*a^2*Sec[(c + d*x)/2]^6 - 2624*a*b*Csc[c + d*x]^3*Sin[(c
+ d*x)/2]^4 + 768*a*b*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + 164*a*b*Csc[(c +
d*x)/2]^4*Sin[c + d*x] - 12*a*b*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 2944*a*b*
Tan[(c + d*x)/2])/(1920*d)
```

fricas [B] time = 0.57, size = 360, normalized size = 2.06

$$-\frac{960\,abdx\cos\left(dx+c\right)^{6}-480\,b^{2}\cos\left(dx+c\right)^{7}-2880\,abdx\cos\left(dx+c\right)^{4}+2880\,abdx\cos\left(dx+c\right)^{2}-330\,\left(a^{2}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(960*a*b*d*x*cos(d*x + c)^6 - 480*b^2*cos(d*x + c)^7 - 2880*a*b*d*x*
cos(d*x + c)^4 + 2880*a*b*d*x*cos(d*x + c)^2 - 330*(a^2 - 6*b^2)*cos(d*x +
c)^5 - 960*a*b*d*x + 400*(a^2 - 6*b^2)*cos(d*x + c)^3 - 150*(a^2 - 6*b^2)*c
os(d*x + c) - 75*((a^2 - 6*b^2)*cos(d*x + c)^6 - 3*(a^2 - 6*b^2)*cos(d*x +
c)^4 + 3*(a^2 - 6*b^2)*cos(d*x + c)^2 - a^2 + 6*b^2)*log(1/2*cos(d*x + c) +
 1/2) + 75*((a^2 - 6*b^2)*cos(d*x + c)^6 - 3*(a^2 - 6*b^2)*cos(d*x + c)^4 +
 3*(a^2 - 6*b^2)*cos(d*x + c)^2 - a^2 + 6*b^2)*log(-1/2*cos(d*x + c) + 1/2)
 - 64*(23*a*b*cos(d*x + c)^5 - 35*a*b*cos(d*x + c)^3 + 15*a*b*cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2
- d)
```

giac [B] time = 0.35, size = 337, normalized size = 1.93

$$\frac{5\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-45\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+30\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-280\,ab\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a*b*tan(1/2*d*x + 1/2*c)^5 - 45*a
^2*tan(1/2*d*x + 1/2*c)^4 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 - 280*a*b*tan(1/2
*d*x + 1/2*c)^3 + 225*a^2*tan(1/2*d*x + 1/2*c)^2 - 480*b^2*tan(1/2*d*x + 1/
2*c)^2 - 3840*(d*x + c)*a*b + 2640*a*b*tan(1/2*d*x + 1/2*c) - 600*(a^2 - 6*
b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 3840*b^2/(tan(1/2*d*x + 1/2*c)^2 + 1)
```

```
 + (1470*a^2*tan(1/2*d*x + 1/2*c)^6 - 8820*b^2*tan(1/2*d*x + 1/2*c)^6 - 264
0*a*b*tan(1/2*d*x + 1/2*c)^5 - 225*a^2*tan(1/2*d*x + 1/2*c)^4 + 480*b^2*tan
(1/2*d*x + 1/2*c)^4 + 280*a*b*tan(1/2*d*x + 1/2*c)^3 + 45*a^2*tan(1/2*d*x +
 1/2*c)^2 - 30*b^2*tan(1/2*d*x + 1/2*c)^2 - 24*a*b*tan(1/2*d*x + 1/2*c) - 5
*a^2)/tan(1/2*d*x + 1/2*c)^6)/d
```

maple [A] time = 0.46, size = 318, normalized size = 1.82

$$-\frac{a^2\left(\cos^7(dx+c)\right)}{6d\sin(dx+c)^6}+\frac{a^2\left(\cos^7(dx+c)\right)}{24d\sin(dx+c)^4}-\frac{a^2\left(\cos^7(dx+c)\right)}{16d\sin(dx+c)^2}-\frac{a^2\left(\cos^5(dx+c)\right)}{16d}-\frac{5a^2\left(\cos^3(dx+c)\right)}{48d}-\frac{5a^2\cos(dx+}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/6/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+1/24/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7-1
/16/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-1/16*a^2*cos(d*x+c)^5/d-5/48*a^2*cos(d*
x+c)^3/d-5/16*a^2*cos(d*x+c)/d-5/16/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/5*a*b
*cot(d*x+c)^5/d+2/3*a*b*cot(d*x+c)^3/d-2*a*b*cot(d*x+c)/d-2*a*b*x-2/d*a*b*c
-1/4/d*b^2/sin(d*x+c)^4*cos(d*x+c)^7+3/8/d*b^2/sin(d*x+c)^2*cos(d*x+c)^7+3/
8*b^2*cos(d*x+c)^5/d+5/8*b^2*cos(d*x+c)^3/d+15/8*b^2*cos(d*x+c)/d+15/8/d*b^
2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.42, size = 219, normalized size = 1.25

$$-\frac{64\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)ab-5\,a^2\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log(\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*(64*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a*b - 5*a^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(
d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*l
og(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 30*b^2*(2*(9*cos(d*x + c
)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x
+ c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 15.06, size = 985, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] (5*a^2*sin(c/2 + (d*x)/2)^14 - 5*a^2*cos(c/2 + (d*x)/2)^14 - 40*a^2*cos(c/2
 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 + 180*a^2*cos(c/2 + (d*x)/2)^4*sin(c/2
+ (d*x)/2)^10 + 225*a^2*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 - 225*a^2
*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 - 180*a^2*cos(c/2 + (d*x)/2)^10*
sin(c/2 + (d*x)/2)^4 + 40*a^2*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2 +
30*b^2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 450*b^2*cos(c/2 + (d*x)
/2)^4*sin(c/2 + (d*x)/2)^10 - 480*b^2*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/
2)^8 + 4320*b^2*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 450*b^2*cos(c/2
 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^4 - 30*b^2*cos(c/2 + (d*x)/2)^12*sin(c/2
+ (d*x)/2)^2 + 24*a*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^13 - 24*a*b*cos
(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) - 600*a^2*log(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 - 600*a^2*log(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)
^6 + 3600*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)
^6*sin(c/2 + (d*x)/2)^8 + 3600*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
))*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 - 256*a*b*cos(c/2 + (d*x)/2)^3
*sin(c/2 + (d*x)/2)^11 + 2360*a*b*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9
 - 2360*a*b*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5 + 256*a*b*cos(c/2 + (
d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 7680*a*b*atan((5*a^2*sin(c/2 + (d*x)/2) -
 30*b^2*sin(c/2 + (d*x)/2) + 32*a*b*cos(c/2 + (d*x)/2))/(30*b^2*cos(c/2 + (
d*x)/2) - 5*a^2*cos(c/2 + (d*x)/2) + 32*a*b*sin(c/2 + (d*x)/2)))*cos(c/2 +
(d*x)/2)^6*sin(c/2 + (d*x)/2)^8 + 7680*a*b*atan((5*a^2*sin(c/2 + (d*x)/2) -
 30*b^2*sin(c/2 + (d*x)/2) + 32*a*b*cos(c/2 + (d*x)/2))/(30*b^2*cos(c/2 + (
d*x)/2) - 5*a^2*cos(c/2 + (d*x)/2) + 32*a*b*sin(c/2 + (d*x)/2)))*cos(c/2 +
(d*x)/2)^8*sin(c/2 + (d*x)/2)^6)/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*
x)/2)^6*(cos(c/2 + (d*x)/2)^2 + sin(c/2 + (d*x)/2)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1251 $\int \cot^6(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=158

$$-\frac{a^2\cot^7(c+dx)}{7d}+\frac{5ab\tanh^{-1}(\cos(c+dx))}{8d}-\frac{ab\cot^5(c+dx)\csc(c+dx)}{3d}+\frac{5ab\cot^3(c+dx)\csc(c+dx)}{12d}-\frac{5ab\cot(c$$

```
[Out] -b^2*x+5/8*a*b*arctanh(cos(d*x+c))/d-b^2*cot(d*x+c)/d+1/3*b^2*cot(d*x+c)^3/
d-1/5*b^2*cot(d*x+c)^5/d-1/7*a^2*cot(d*x+c)^7/d-5/8*a*b*cot(d*x+c)*csc(d*x+
c)/d+5/12*a*b*cot(d*x+c)^3*csc(d*x+c)/d-1/3*a*b*cot(d*x+c)^5*csc(d*x+c)/d
```

Rubi [A] time = 0.41, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2911, 2611, 3770, 14, 203}

$$-\frac{a^2\cot^7(c+dx)}{7d}+\frac{5ab\tanh^{-1}(\cos(c+dx))}{8d}-\frac{ab\cot^5(c+dx)\csc(c+dx)}{3d}+\frac{5ab\cot^3(c+dx)\csc(c+dx)}{12d}-\frac{5ab\cot(c$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(b^2*x) + (5*a*b*ArcTanh[Cos[c + d*x]])/(8*d) - (b^2*Cot[c + d*x])/d + (b^
2*Cot[c + d*x]^3)/(3*d) - (b^2*Cot[c + d*x]^5)/(5*d) - (a^2*Cot[c + d*x]^7)
/(7*d) - (5*a*b*Cot[c + d*x]*Csc[c + d*x])/(8*d) + (5*a*b*Cot[c + d*x]^3*Cs
c[c + d*x])/(12*d) - (a*b*Cot[c + d*x]^5*Csc[c + d*x])/(3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc(c+dx)\,dx + \int \cot^6(c+dx)\csc^2($$

$$= -\frac{ab\cot^5(c+dx)\csc(c+dx)}{3d} - \frac{1}{3}(5ab)\int \cot^4(c+dx)\csc($$

$$= \frac{5ab\cot^3(c+dx)\csc(c+dx)}{12d} - \frac{ab\cot^5(c+dx)\csc(c+dx)}{3d}$$

$$= -\frac{b^2\cot(c+dx)}{d} + \frac{b^2\cot^3(c+dx)}{3d} - \frac{b^2\cot^5(c+dx)}{5d} - \frac{a^2 c}{}$$

$$= -b^2x + \frac{5ab\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b^2\cot(c+dx)}{d} + \frac{b^2\cot}{}$$

Mathematica [A] time = 1.40, size = 280, normalized size = 1.77

$$\frac{\csc^7(c+dx)\left(-84\left(15a^2-41b^2\right)\cos(3(c+dx)) - 28\left(15a^2+71b^2\right)\cos(5(c+dx)) - 60a^2\cos(7(c+dx)) + 980ab}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-14700*b^2*(c + d*x)*Csc[c + d*x]^6 + 16800*a*b*(Log[Cos[(c + d*x)/2]] - L
og[Sin[(c + d*x)/2]]) - 350*Cot[c + d*x]*Csc[c + d*x]^6*(6*(a^2 + b^2) + 17
*a*b*Sin[c + d*x]) + Csc[c + d*x]^7*(-84*(15*a^2 - 41*b^2)*Cos[3*(c + d*x)]
 - 28*(15*a^2 + 71*b^2)*Cos[5*(c + d*x)] - 60*a^2*Cos[7*(c + d*x)] + 644*b^
2*Cos[7*(c + d*x)] + 8820*b^2*c*Sin[3*(c + d*x)] + 8820*b^2*d*x*Sin[3*(c +
d*x)] + 980*a*b*Sin[4*(c + d*x)] - 2940*b^2*c*Sin[5*(c + d*x)] - 2940*b^2*d
*x*Sin[5*(c + d*x)] - 1155*a*b*Sin[6*(c + d*x)] + 420*b^2*c*Sin[7*(c + d*x)
] + 420*b^2*d*x*Sin[7*(c + d*x)]))/(26880*d)
```

fricas [B] time = 1.03, size = 320, normalized size = 2.03

$$\frac{16\left(15\,a^2-161\,b^2\right)\cos\left(dx+c\right)^7+6496\,b^2\cos\left(dx+c\right)^5-5600\,b^2\cos\left(dx+c\right)^3+1680\,b^2\cos\left(dx+c\right)+525\,\left(ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/1680*(16*(15*a^2 - 161*b^2)*cos(d*x + c)^7 + 6496*b^2*cos(d*x + c)^5 - 56
00*b^2*cos(d*x + c)^3 + 1680*b^2*cos(d*x + c) + 525*(a*b*cos(d*x + c)^6 - 3
*a*b*cos(d*x + c)^4 + 3*a*b*cos(d*x + c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/
2)*sin(d*x + c) - 525*(a*b*cos(d*x + c)^6 - 3*a*b*cos(d*x + c)^4 + 3*a*b*co
s(d*x + c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 70*(24*b^2*
d*x*cos(d*x + c)^6 - 72*b^2*d*x*cos(d*x + c)^4 - 33*a*b*cos(d*x + c)^5 + 72
*b^2*d*x*cos(d*x + c)^2 + 40*a*b*cos(d*x + c)^3 - 24*b^2*d*x - 15*a*b*cos(d
*x + c))*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*
x + c)^2 - d)*sin(d*x + c))
```

giac [B] time = 0.34, size = 356, normalized size = 2.25

$$\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+70\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+84\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-630\,ab\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*a^2*tan(1/2*d*x + 1/2*c)^7 + 70*a*b*tan(1/2*d*x + 1/2*c)^6 - 10
5*a^2*tan(1/2*d*x + 1/2*c)^5 + 84*b^2*tan(1/2*d*x + 1/2*c)^5 - 630*a*b*tan(
1/2*d*x + 1/2*c)^4 + 315*a^2*tan(1/2*d*x + 1/2*c)^3 - 980*b^2*tan(1/2*d*x +
 1/2*c)^3 + 3150*a*b*tan(1/2*d*x + 1/2*c)^2 - 13440*(d*x + c)*b^2 - 8400*a*
b*log(abs(tan(1/2*d*x + 1/2*c))) - 525*a^2*tan(1/2*d*x + 1/2*c) + 9240*b^2*
```

```
tan(1/2*d*x + 1/2*c) + (21780*a*b*tan(1/2*d*x + 1/2*c)^7 + 525*a^2*tan(1/2*
d*x + 1/2*c)^6 - 9240*b^2*tan(1/2*d*x + 1/2*c)^6 - 3150*a*b*tan(1/2*d*x + 1
/2*c)^5 - 315*a^2*tan(1/2*d*x + 1/2*c)^4 + 980*b^2*tan(1/2*d*x + 1/2*c)^4 +
 630*a*b*tan(1/2*d*x + 1/2*c)^3 + 105*a^2*tan(1/2*d*x + 1/2*c)^2 - 84*b^2*t
an(1/2*d*x + 1/2*c)^2 - 70*a*b*tan(1/2*d*x + 1/2*c) - 15*a^2)/tan(1/2*d*x +
 1/2*c)^7)/d
```

maple [A] time = 0.46, size = 222, normalized size = 1.41

$$-\frac{a^2\left(\cos^7(dx+c)\right)}{7d\sin(dx+c)^7}-\frac{ab\left(\cos^7(dx+c)\right)}{3d\sin(dx+c)^6}+\frac{ab\left(\cos^7(dx+c)\right)}{12d\sin(dx+c)^4}-\frac{ab\left(\cos^7(dx+c)\right)}{8d\sin(dx+c)^2}-\frac{ab\left(\cos^5(dx+c)\right)}{8d}-\frac{5ab\left(\cos^3(\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/7/d*a^2/sin(d*x+c)^7*cos(d*x+c)^7-1/3/d*a*b/sin(d*x+c)^6*cos(d*x+c)^7+1/
12/d*a*b/sin(d*x+c)^4*cos(d*x+c)^7-1/8/d*a*b/sin(d*x+c)^2*cos(d*x+c)^7-1/8*
a*b*cos(d*x+c)^5/d-5/24*a*b*cos(d*x+c)^3/d-5/8*a*b*cos(d*x+c)/d-5/8/d*a*b*l
n(csc(d*x+c)-cot(d*x+c))-1/5*b^2*cot(d*x+c)^5/d+1/3*b^2*cot(d*x+c)^3/d-b^2*
cot(d*x+c)/d-b^2*x-1/d*b^2*c
```

maxima [A] time = 0.42, size = 153, normalized size = 0.97

$$-\frac{112\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)b^2-35\,ab\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log(\cos(\right.}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/1680*(112*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/ta
n(d*x + c)^5)*b^2 - 35*a*b*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*c
os(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 1
5*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 240*a^2/tan(d*x + c)^
7)/d
```

mupad [B] time = 11.87, size = 379, normalized size = 2.40

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{896\,d}-\frac{2\,b^2\,\mathrm{atan}\left(\frac{4\,b^4}{\frac{5\,a\,b^3}{2}-4\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{5\,a\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{5\,a\,b^3}{2}-4\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{5\,a^2}{128}-\frac{11\,b^2}{16}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^8,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^7)/(896*d) - (2*b^2*atan((4*b^4)/((5*a*b^3)/2 - 4*b
^4*tan(c/2 + (d*x)/2)) + (5*a*b^3*tan(c/2 + (d*x)/2))/(2*((5*a*b^3)/2 - 4*b
^4*tan(c/2 + (d*x)/2)))))/d - (tan(c/2 + (d*x)/2)*((5*a^2)/128 - (11*b^2)/1
6))/d - (tan(c/2 + (d*x)/2)^4*(3*a^2 - (28*b^2)/3) - tan(c/2 + (d*x)/2)^6*(
5*a^2 - 88*b^2) + a^2/7 - tan(c/2 + (d*x)/2)^2*(a^2 - (4*b^2)/5) - 6*a*b*ta
n(c/2 + (d*x)/2)^3 + 30*a*b*tan(c/2 + (d*x)/2)^5 + (2*a*b*tan(c/2 + (d*x)/2
))/3)/(128*d*tan(c/2 + (d*x)/2)^7) + (tan(c/2 + (d*x)/2)^3*((3*a^2)/128 - (
7*b^2)/96))/d - (tan(c/2 + (d*x)/2)^5*(a^2/128 - b^2/160))/d + (15*a*b*tan(
c/2 + (d*x)/2)^2)/(64*d) - (3*a*b*tan(c/2 + (d*x)/2)^4)/(64*d) + (a*b*tan(c
/2 + (d*x)/2)^6)/(192*d) - (5*a*b*log(tan(c/2 + (d*x)/2)))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1252 $\int \cot^6(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=159

$$\frac{5\left(a^2+8b^2\right)\tanh^{-1}(\cos(c+dx))}{128d}+\frac{\left(17a^2-8b^2\right)\cot(c+dx)\csc^5(c+dx)}{48d}-\frac{\left(59a^2-104b^2\right)\cot(c+dx)\csc^3(c+d}{192d}$$

```
[Out] 5/128*(a^2+8*b^2)*arctanh(cos(d*x+c))/d-2/7*a*b*cot(d*x+c)^7/d+1/128*(5*a^2
-88*b^2)*cot(d*x+c)*csc(d*x+c)/d-1/192*(59*a^2-104*b^2)*cot(d*x+c)*csc(d*x+
c)^3/d+1/48*(17*a^2-8*b^2)*cot(d*x+c)*csc(d*x+c)^5/d-1/8*a^2*cot(d*x+c)*csc
(d*x+c)^7/d
```

Rubi [A] time = 0.32, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2911, 2607, 30, 4366, 455, 1814, 1157, 385, 206}

$$\frac{5\left(a^2+8b^2\right)\tanh^{-1}(\cos(c+dx))}{128d}+\frac{\left(17a^2-8b^2\right)\cot(c+dx)\csc^5(c+dx)}{48d}-\frac{\left(59a^2-104b^2\right)\cot(c+dx)\csc^3(c+d}{192d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*(a^2 + 8*b^2)*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a*b*Cot[c + d*x]^7)/(7
*d) + ((5*a^2 - 88*b^2)*Cot[c + d*x]*Csc[c + d*x])/(128*d) - ((59*a^2 - 104
*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(192*d) + ((17*a^2 - 8*b^2)*Cot[c + d*x]
*Csc[c + d*x]^5)/(48*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^7)/(8*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
```

```
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \cot^6(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc^2(c+dx)\,dx + \int \cot^6(c+dx)\csc^3$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6\left(a^2+b^2-b^2x^2\right)}{\left(1-x^2\right)^5}\,dx, x, \cos(c+dx)\right)}{d} + \frac{(2ab)\,\text{Subst}}{}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{a^2\cot(c+dx)\csc^7(c+dx)}{8d} + \frac{\text{Subst}}{}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} + \frac{\left(17a^2-8b^2\right)\cot(c+dx)\csc^5(c+dx)}{48d}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{\left(59a^2-104b^2\right)\cot(c+dx)\csc^3(c+d}{192d}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} + \frac{\left(5a^2-88b^2\right)\cot(c+dx)\csc(c+dx)}{128d}$$

$$= \frac{5\left(a^2+8b^2\right)\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2ab\cot^7(c+dx)}{7d} + \frac{(5a}{}$$

Mathematica [A] time = 0.81, size = 282, normalized size = 1.77

$$-\frac{7\left(895a^2-904b^2\right)\cos(3(c+dx))\csc^8(c+dx)+7\cot(c+dx)\csc^7(c+dx)\left(1765a^2+1536ab\sin(c+dx)+68\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/172032*(7*(895*a^2 - 904*b^2)*Cos[3*(c + d*x)]*Csc[c + d*x]^8 + 2779*a^2
*Cos[5*(c + d*x)]*Csc[c + d*x]^8 + 3416*b^2*Cos[5*(c + d*x)]*Csc[c + d*x]^8
 + 105*a^2*Cos[7*(c + d*x)]*Csc[c + d*x]^8 - 1848*b^2*Cos[7*(c + d*x)]*Csc[
c + d*x]^8 - 6720*a^2*Log[Cos[(c + d*x)/2]] - 53760*b^2*Log[Cos[(c + d*x)/2
]] + 6720*a^2*Log[Sin[(c + d*x)/2]] + 53760*b^2*Log[Sin[(c + d*x)/2]] + 7*C
ot[c + d*x]*Csc[c + d*x]^7*(1765*a^2 + 680*b^2 + 1536*a*b*Sin[c + d*x]) + 5
376*a*b*Csc[c + d*x]^8*Sin[4*(c + d*x)] + 2304*a*b*Csc[c + d*x]^8*Sin[6*(c
+ d*x)] + 384*a*b*Csc[c + d*x]^8*Sin[8*(c + d*x)])/d
```

fricas [B] time = 0.86, size = 338, normalized size = 2.13

$$-\frac{1536\,ab\cos(dx+c)^7\sin(dx+c)+42\left(5\,a^2-88\,b^2\right)\cos(dx+c)^7+1022\left(a^2+8\,b^2\right)\cos(dx+c)^5-770\left(a^2+8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/5376*(1536*a*b*cos(d*x + c)^7*sin(d*x + c) + 42*(5*a^2 - 88*b^2)*cos(d*x
 + c)^7 + 1022*(a^2 + 8*b^2)*cos(d*x + c)^5 - 770*(a^2 + 8*b^2)*cos(d*x + c
)^3 + 210*(a^2 + 8*b^2)*cos(d*x + c) - 105*((a^2 + 8*b^2)*cos(d*x + c)^8 -
4*(a^2 + 8*b^2)*cos(d*x + c)^6 + 6*(a^2 + 8*b^2)*cos(d*x + c)^4 - 4*(a^2 +
8*b^2)*cos(d*x + c)^2 + a^2 + 8*b^2)*log(1/2*cos(d*x + c) + 1/2) + 105*((a^
2 + 8*b^2)*cos(d*x + c)^8 - 4*(a^2 + 8*b^2)*cos(d*x + c)^6 + 6*(a^2 + 8*b^2
)*cos(d*x + c)^4 - 4*(a^2 + 8*b^2)*cos(d*x + c)^2 + a^2 + 8*b^2)*log(-1/2*c
os(d*x + c) + 1/2))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x +
c)^4 - 4*d*cos(d*x + c)^2 + d)
```

giac [B] time = 0.37, size = 402, normalized size = 2.53

$$\frac{21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+96\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-112\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+112\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-672\,ab\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/43008*(21*a^2*tan(1/2*d*x + 1/2*c)^8 + 96*a*b*tan(1/2*d*x + 1/2*c)^7 - 11
2*a^2*tan(1/2*d*x + 1/2*c)^6 + 112*b^2*tan(1/2*d*x + 1/2*c)^6 - 672*a*b*tan
(1/2*d*x + 1/2*c)^5 + 168*a^2*tan(1/2*d*x + 1/2*c)^4 - 1008*b^2*tan(1/2*d*x
 + 1/2*c)^4 + 2016*a*b*tan(1/2*d*x + 1/2*c)^3 + 336*a^2*tan(1/2*d*x + 1/2*c
```

```
)^2 + 5040*b^2*tan(1/2*d*x + 1/2*c)^2 - 3360*a*b*tan(1/2*d*x + 1/2*c) - 168
0*(a^2 + 8*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (4566*a^2*tan(1/2*d*x + 1/
2*c)^8 + 36528*b^2*tan(1/2*d*x + 1/2*c)^8 + 3360*a*b*tan(1/2*d*x + 1/2*c)^7
 - 336*a^2*tan(1/2*d*x + 1/2*c)^6 - 5040*b^2*tan(1/2*d*x + 1/2*c)^6 - 2016*
a*b*tan(1/2*d*x + 1/2*c)^5 - 168*a^2*tan(1/2*d*x + 1/2*c)^4 + 1008*b^2*tan(
1/2*d*x + 1/2*c)^4 + 672*a*b*tan(1/2*d*x + 1/2*c)^3 + 112*a^2*tan(1/2*d*x +
 1/2*c)^2 - 112*b^2*tan(1/2*d*x + 1/2*c)^2 - 96*a*b*tan(1/2*d*x + 1/2*c) -
21*a^2)/tan(1/2*d*x + 1/2*c)^8)/d
```

maple [B] time = 0.46, size = 333, normalized size = 2.09

$$-\frac{a^2\left(\cos^7(dx+c)\right)}{8d\sin(dx+c)^8}-\frac{a^2\left(\cos^7(dx+c)\right)}{48d\sin(dx+c)^6}+\frac{a^2\left(\cos^7(dx+c)\right)}{192d\sin(dx+c)^4}-\frac{a^2\left(\cos^7(dx+c)\right)}{128d\sin(dx+c)^2}-\frac{a^2\left(\cos^5(dx+c)\right)}{128d}-\frac{5a^2\left(\cos^3(\right.}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/8/d*a^2/sin(d*x+c)^8*cos(d*x+c)^7-1/48/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+1
/192/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7-1/128/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-
1/128*a^2*cos(d*x+c)^5/d-5/384*a^2*cos(d*x+c)^3/d-5/128*a^2*cos(d*x+c)/d-5/
128/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/7/d*a*b/sin(d*x+c)^7*cos(d*x+c)^7-1/6
/d*b^2/sin(d*x+c)^6*cos(d*x+c)^7+1/24/d*b^2/sin(d*x+c)^4*cos(d*x+c)^7-1/16/
d*b^2/sin(d*x+c)^2*cos(d*x+c)^7-1/16*b^2*cos(d*x+c)^5/d-5/48*b^2*cos(d*x+c)
^3/d-5/16*b^2*cos(d*x+c)/d-5/16/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.34, size = 220, normalized size = 1.38

$$-\frac{7\,a^2\left(\frac{2\left(15\,\cos(dx+c)^7+73\,\cos(dx+c)^5-55\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{53}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/5376*(7*a^2*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^
3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
1)) - 56*b^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))/(
cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(cos(d*x
+ c) + 1) - 15*log(cos(d*x + c) - 1)) + 1536*a*b/tan(d*x + c)^7)/d
```

mupad [B] time = 12.69, size = 343, normalized size = 2.16

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2048\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(\frac{5\,a^2}{128}+\frac{5\,b^2}{16}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(2\,a^2+30\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^9,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^8)/(2048*d) - (log(tan(c/2 + (d*x)/2))*((5*a^2)/128
 + (5*b^2)/16))/d - (cot(c/2 + (d*x)/2)^8*(tan(c/2 + (d*x)/2)^6*(2*a^2 + 30
*b^2) - tan(c/2 + (d*x)/2)^2*((2*a^2)/3 - (2*b^2)/3) + a^2/8 + tan(c/2 + (d
*x)/2)^4*(a^2 - 6*b^2) - 4*a*b*tan(c/2 + (d*x)/2)^3 + 12*a*b*tan(c/2 + (d*x
)/2)^5 - 20*a*b*tan(c/2 + (d*x)/2)^7 + (4*a*b*tan(c/2 + (d*x)/2))/7))/(256*
d) + (tan(c/2 + (d*x)/2)^2*(a^2/128 + (15*b^2)/128))/d + (tan(c/2 + (d*x)/2
)^4*(a^2/256 - (3*b^2)/128))/d - (tan(c/2 + (d*x)/2)^6*(a^2/384 - b^2/384))
/d + (3*a*b*tan(c/2 + (d*x)/2)^3)/(64*d) - (a*b*tan(c/2 + (d*x)/2)^5)/(64*d
) + (a*b*tan(c/2 + (d*x)/2)^7)/(448*d) - (5*a*b*tan(c/2 + (d*x)/2))/(64*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1253 $\int \cot^6(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=151

$$-\frac{\left(a^2+b^2\right)\cot^7(c+dx)}{7d}-\frac{a^2\cot^9(c+dx)}{9d}+\frac{5ab\tanh^{-1}(\cos(c+dx))}{64d}-\frac{ab\cot^5(c+dx)\csc^3(c+dx)}{4d}+\frac{5ab\cot^3(c+$$

```
[Out] 5/64*a*b*arctanh(cos(d*x+c))/d-1/7*(a^2+b^2)*cot(d*x+c)^7/d-1/9*a^2*cot(d*x
+c)^9/d+5/64*a*b*cot(d*x+c)*csc(d*x+c)/d-5/32*a*b*cot(d*x+c)*csc(d*x+c)^3/d
+5/24*a*b*cot(d*x+c)^3*csc(d*x+c)^3/d-1/4*a*b*cot(d*x+c)^5*csc(d*x+c)^3/d
```

Rubi [A] time = 0.41, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2911, 2611, 3768, 3770, 14}

$$-\frac{\left(a^2+b^2\right)\cot^7(c+dx)}{7d}-\frac{a^2\cot^9(c+dx)}{9d}+\frac{5ab\tanh^{-1}(\cos(c+dx))}{64d}-\frac{ab\cot^5(c+dx)\csc^3(c+dx)}{4d}+\frac{5ab\cot^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a*b*ArcTanh[Cos[c + d*x]])/(64*d) - ((a^2 + b^2)*Cot[c + d*x]^7)/(7*d) -
 (a^2*Cot[c + d*x]^9)/(9*d) + (5*a*b*Cot[c + d*x]*Csc[c + d*x])/(64*d) - (5
*a*b*Cot[c + d*x]*Csc[c + d*x]^3)/(32*d) + (5*a*b*Cot[c + d*x]^3*Csc[c + d*
x]^3)/(24*d) - (a*b*Cot[c + d*x]^5*Csc[c + d*x]^3)/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
```

```
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc^3(c+dx)\,dx + \int \cot^6(c+dx)\csc^4($$

$$= -\frac{ab\cot^5(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}(5ab)\int \cot^4(c+dx)\csc^3$$

$$= \frac{5ab\cot^3(c+dx)\csc^3(c+dx)}{24d} - \frac{ab\cot^5(c+dx)\csc^3(c+dx)}{4d}$$

$$= -\frac{\left(a^2+b^2\right)\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} - \frac{5ab\cot(c+dx)}{32d}$$

$$= -\frac{\left(a^2+b^2\right)\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} + \frac{5ab\cot(c+dx)}{64d}$$

$$= \frac{5ab\tanh^{-1}(\cos(c+dx))}{64d} - \frac{\left(a^2+b^2\right)\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(}{9}$$

Mathematica [A] time = 1.16, size = 204, normalized size = 1.35

$$-\frac{\csc^9(c+dx)\left(4032\left(8a^2+b^2\right)\cos(c+dx)+18816a^2\cos(3(c+dx))+5760a^2\cos(5(c+dx))+576a^2\cos(7(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/516096*(-40320*a*b*Log[Cos[(c + d*x)/2]] + 40320*a*b*Log[Sin[(c + d*x)/2
]] + Csc[c + d*x]^9*(4032*(8*a^2 + b^2)*Cos[c + d*x] + 18816*a^2*Cos[3*(c +
 d*x)] + 5760*a^2*Cos[5*(c + d*x)] - 2304*b^2*Cos[5*(c + d*x)] + 576*a^2*Co
s[7*(c + d*x)] - 1440*b^2*Cos[7*(c + d*x)] - 64*a^2*Cos[9*(c + d*x)] - 288*
b^2*Cos[9*(c + d*x)] + 18270*a*b*Sin[2*(c + d*x)] + 10458*a*b*Sin[4*(c + d*
x)] + 8022*a*b*Sin[6*(c + d*x)] + 315*a*b*Sin[8*(c + d*x)]))/d
```

fricas [B] time = 0.73, size = 291, normalized size = 1.93

$$128\left(2\,a^2+9\,b^2\right)\cos\left(dx+c\right)^9-1152\left(a^2+b^2\right)\cos\left(dx+c\right)^7+315\left(ab\cos\left(dx+c\right)^8-4\,ab\cos\left(dx+c\right)^6+6\,ab\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+b*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/8064*(128*(2*a^2 + 9*b^2)*cos(d*x + c)^9 - 1152*(a^2 + b^2)*cos(d*x + c)^
7 + 315*(a*b*cos(d*x + c)^8 - 4*a*b*cos(d*x + c)^6 + 6*a*b*cos(d*x + c)^4 -
 4*a*b*cos(d*x + c)^2 + a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 315
*(a*b*cos(d*x + c)^8 - 4*a*b*cos(d*x + c)^6 + 6*a*b*cos(d*x + c)^4 - 4*a*b*
cos(d*x + c)^2 + a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 42*(15*a*
b*cos(d*x + c)^7 + 73*a*b*cos(d*x + c)^5 - 55*a*b*cos(d*x + c)^3 + 15*a*b*c
os(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*co
s(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [B] time = 0.34, size = 408, normalized size = 2.70

$$14\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+63\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-54\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+72\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-336\,ab\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+b*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/64512*(14*a^2*tan(1/2*d*x + 1/2*c)^9 + 63*a*b*tan(1/2*d*x + 1/2*c)^8 - 54
*a^2*tan(1/2*d*x + 1/2*c)^7 + 72*b^2*tan(1/2*d*x + 1/2*c)^7 - 336*a*b*tan(1
/2*d*x + 1/2*c)^6 - 504*b^2*tan(1/2*d*x + 1/2*c)^5 + 504*a*b*tan(1/2*d*x +
1/2*c)^4 + 336*a^2*tan(1/2*d*x + 1/2*c)^3 + 1512*b^2*tan(1/2*d*x + 1/2*c)^3
 + 1008*a*b*tan(1/2*d*x + 1/2*c)^2 - 5040*a*b*log(abs(tan(1/2*d*x + 1/2*c))
) - 756*a^2*tan(1/2*d*x + 1/2*c) - 2520*b^2*tan(1/2*d*x + 1/2*c) + (14258*a
*b*tan(1/2*d*x + 1/2*c)^9 + 756*a^2*tan(1/2*d*x + 1/2*c)^8 + 2520*b^2*tan(1
```

```
/2*d*x + 1/2*c)^8 - 1008*a*b*tan(1/2*d*x + 1/2*c)^7 - 336*a^2*tan(1/2*d*x +
 1/2*c)^6 - 1512*b^2*tan(1/2*d*x + 1/2*c)^6 - 504*a*b*tan(1/2*d*x + 1/2*c)^
5 + 504*b^2*tan(1/2*d*x + 1/2*c)^4 + 336*a*b*tan(1/2*d*x + 1/2*c)^3 + 54*a^
2*tan(1/2*d*x + 1/2*c)^2 - 72*b^2*tan(1/2*d*x + 1/2*c)^2 - 63*a*b*tan(1/2*d
*x + 1/2*c) - 14*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

maple [A] time = 0.46, size = 232, normalized size = 1.54

$$-\frac{a^2\left(\cos^7(dx+c)\right)}{9d\sin(dx+c)^9}-\frac{2a^2\left(\cos^7(dx+c)\right)}{63d\sin(dx+c)^7}-\frac{ab\left(\cos^7(dx+c)\right)}{4d\sin(dx+c)^8}-\frac{ab\left(\cos^7(dx+c)\right)}{24d\sin(dx+c)^6}+\frac{ab\left(\cos^7(dx+c)\right)}{96d\sin(dx+c)^4}-\frac{ab\left(\cos^7(dx\right.}{64d\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^10*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/9/d*a^2/sin(d*x+c)^9*cos(d*x+c)^7-2/63/d*a^2/sin(d*x+c)^7*cos(d*x+c)^7-1
/4/d*a*b/sin(d*x+c)^8*cos(d*x+c)^7-1/24/d*a*b/sin(d*x+c)^6*cos(d*x+c)^7+1/9
6/d*a*b/sin(d*x+c)^4*cos(d*x+c)^7-1/64/d*a*b/sin(d*x+c)^2*cos(d*x+c)^7-1/64
*a*b*cos(d*x+c)^5/d-5/192*a*b*cos(d*x+c)^3/d-5/64*a*b*cos(d*x+c)/d-5/64/d*a
*b*ln(csc(d*x+c)-cot(d*x+c))-1/7/d*b^2/sin(d*x+c)^7*cos(d*x+c)^7
```

maxima [A] time = 0.33, size = 154, normalized size = 1.02

$$-\frac{21\,ab\left(\frac{2\left(15\,\cos(dx+c)^7+73\,\cos(dx+c)^5-55\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{8064\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+b*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/8064*(21*a*b*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)
^3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
 - 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
 1)) + 1152*b^2/tan(d*x + c)^7 + 128*(9*tan(d*x + c)^2 + 7)*a^2/tan(d*x + c
)^9)/d
```

mupad [B] time = 11.86, size = 373, normalized size = 2.47

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4608\,d}-\frac{b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{128\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a^2}{256}+\frac{5\,b^2}{128}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{8\,a^2}{3}+12\,b^2\right)-\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^10,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^9)/(4608*d) - (b^2*tan(c/2 + (d*x)/2)^5)/(128*d) -
(tan(c/2 + (d*x)/2)*((3*a^2)/256 + (5*b^2)/128))/d - (cot(c/2 + (d*x)/2)^9*
(tan(c/2 + (d*x)/2)^6*((8*a^2)/3 + 12*b^2) - tan(c/2 + (d*x)/2)^2*((3*a^2)/
7 - (4*b^2)/7) - tan(c/2 + (d*x)/2)^8*(6*a^2 + 20*b^2) - 4*b^2*tan(c/2 + (d
*x)/2)^4 + a^2/9 - (8*a*b*tan(c/2 + (d*x)/2)^3)/3 + 4*a*b*tan(c/2 + (d*x)/2
)^5 + 8*a*b*tan(c/2 + (d*x)/2)^7 + (a*b*tan(c/2 + (d*x)/2))/2))/(512*d) + (
tan(c/2 + (d*x)/2)^3*(a^2/192 + (3*b^2)/128))/d - (tan(c/2 + (d*x)/2)^7*((3
*a^2)/3584 - b^2/896))/d + (a*b*tan(c/2 + (d*x)/2)^2)/(64*d) + (a*b*tan(c/2
 + (d*x)/2)^4)/(128*d) - (a*b*tan(c/2 + (d*x)/2)^6)/(192*d) + (a*b*tan(c/2
+ (d*x)/2)^8)/(1024*d) - (5*a*b*log(tan(c/2 + (d*x)/2)))/(64*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**10*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1254 $\int \cot^6(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=210

$$\frac{(3a^2+10b^2)\tanh^{-1}(\cos(c+dx))}{256d}+\frac{(21a^2-10b^2)\cot(c+dx)\csc^7(c+dx)}{80d}-\frac{(93a^2-170b^2)\cot(c+dx)\csc^5(c+d}{480d}$$

```
[Out] 1/256*(3*a^2+10*b^2)*arctanh(cos(d*x+c))/d-2/7*a*b*cot(d*x+c)^7/d-2/9*a*b*c
ot(d*x+c)^9/d+1/256*(3*a^2+10*b^2)*cot(d*x+c)*csc(d*x+c)/d+1/384*(3*a^2-118
*b^2)*cot(d*x+c)*csc(d*x+c)^3/d-1/480*(93*a^2-170*b^2)*cot(d*x+c)*csc(d*x+c
)^5/d+1/80*(21*a^2-10*b^2)*cot(d*x+c)*csc(d*x+c)^7/d-1/10*a^2*cot(d*x+c)*cs
c(d*x+c)^9/d
```

Rubi [A] time = 0.34, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2911, 2607, 14, 4366, 455, 1814, 1157, 385, 199, 206}

$$\frac{(3a^2+10b^2)\tanh^{-1}(\cos(c+dx))}{256d}+\frac{(21a^2-10b^2)\cot(c+dx)\csc^7(c+dx)}{80d}-\frac{(93a^2-170b^2)\cot(c+dx)\csc^5(c+d}{480d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((3*a^2 + 10*b^2)*ArcTanh[Cos[c + d*x]])/(256*d) - (2*a*b*Cot[c + d*x]^7)/(
7*d) - (2*a*b*Cot[c + d*x]^9)/(9*d) + ((3*a^2 + 10*b^2)*Cot[c + d*x]*Csc[c
+ d*x])/(256*d) + ((3*a^2 - 118*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(384*d) -
 ((93*a^2 - 170*b^2)*Cot[c + d*x]*Csc[c + d*x]^5)/(480*d) + ((21*a^2 - 10*b
^2)*Cot[c + d*x]*Csc[c + d*x]^7)/(80*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^9)
/(10*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \cot^6(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc^4(c+dx)\,dx + \int \cot^6(c+dx)\csc^5$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6\left(a^2+b^2-b^2x^2\right)}{\left(1-x^2\right)^6}\,dx, x, \cos(c+dx)\right)}{d} + \frac{(2ab)\,\text{Subst}}{}$$

$$= -\frac{a^2\cot(c+dx)\csc^9(c+dx)}{10d} + \frac{\text{Subst}\left(\int \frac{a^2+10a^2x^2+10a^2x^4-1}{\left(1-x^2\right)^5}\right.}{10d}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{2ab\cot^9(c+dx)}{9d} + \frac{\left(21a^2-10b^2\right)\cot}{8}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{2ab\cot^9(c+dx)}{9d} - \frac{\left(93a^2-170b^2\right)\cot}{4}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{2ab\cot^9(c+dx)}{9d} + \frac{\left(3a^2-118b^2\right)\cot}{3}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{2ab\cot^9(c+dx)}{9d} + \frac{\left(3a^2+10b^2\right)\cot(}{25}$$

$$= \frac{\left(3a^2+10b^2\right)\tanh^{-1}(\cos(c+dx))}{256d} - \frac{2ab\cot^7(c+dx)}{7d} - \frac{2a}{}$$

Mathematica [A] time = 1.47, size = 244, normalized size = 1.16

$$-\frac{80640\left(3a^2+10b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 80640\left(3a^2+10b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \csc^{10}(c+dx)\left(630\left(18\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/20643840*(-80640*(3*a^2 + 10*b^2)*Log[Cos[(c + d*x)/2]] + 80640*(3*a^2 +
 10*b^2)*Log[Sin[(c + d*x)/2]] + Csc[c + d*x]^10*(630*(1879*a^2 + 290*b^2)*
Cos[c + d*x] + 1260*(519*a^2 - 62*b^2)*Cos[3*(c + d*x)] + 218484*a^2*Cos[5*
(c + d*x)] - 24360*b^2*Cos[5*(c + d*x)] + 9135*a^2*Cos[7*(c + d*x)] - 77070
*b^2*Cos[7*(c + d*x)] - 945*a^2*Cos[9*(c + d*x)] - 3150*b^2*Cos[9*(c + d*x)
] + 537600*a*b*Sin[2*(c + d*x)] + 522240*a*b*Sin[4*(c + d*x)] + 207360*a*b*
```

```
Sin[6*(c + d*x)] + 25600*a*b*Sin[8*(c + d*x)] - 2560*a*b*Sin[10*(c + d*x)])
)/d
```

fricas [B] time = 0.79, size = 455, normalized size = 2.17

$$-\frac{630\left(3a^2+10b^2\right)\cos\left(dx+c\right)^9-420\left(21a^2-58b^2\right)\cos\left(dx+c\right)^7-5376\left(3a^2+10b^2\right)\cos\left(dx+c\right)^5+2940\left(3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+b*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/161280*(630*(3*a^2 + 10*b^2)*cos(d*x + c)^9 - 420*(21*a^2 - 58*b^2)*cos(
d*x + c)^7 - 5376*(3*a^2 + 10*b^2)*cos(d*x + c)^5 + 2940*(3*a^2 + 10*b^2)*c
os(d*x + c)^3 - 630*(3*a^2 + 10*b^2)*cos(d*x + c) - 315*((3*a^2 + 10*b^2)*c
os(d*x + c)^10 - 5*(3*a^2 + 10*b^2)*cos(d*x + c)^8 + 10*(3*a^2 + 10*b^2)*co
s(d*x + c)^6 - 10*(3*a^2 + 10*b^2)*cos(d*x + c)^4 + 5*(3*a^2 + 10*b^2)*cos(
d*x + c)^2 - 3*a^2 - 10*b^2)*log(1/2*cos(d*x + c) + 1/2) + 315*((3*a^2 + 10
*b^2)*cos(d*x + c)^10 - 5*(3*a^2 + 10*b^2)*cos(d*x + c)^8 + 10*(3*a^2 + 10*
b^2)*cos(d*x + c)^6 - 10*(3*a^2 + 10*b^2)*cos(d*x + c)^4 + 5*(3*a^2 + 10*b^
2)*cos(d*x + c)^2 - 3*a^2 - 10*b^2)*log(-1/2*cos(d*x + c) + 1/2) + 5120*(2*
a*b*cos(d*x + c)^9 - 9*a*b*cos(d*x + c)^7)*sin(d*x + c))/(d*cos(d*x + c)^10
 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos
(d*x + c)^2 - d)
```

giac [B] time = 0.40, size = 468, normalized size = 2.23

$$\frac{126a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+560ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-315a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+630b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-2160\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+b*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/1290240*(126*a^2*tan(1/2*d*x + 1/2*c)^10 + 560*a*b*tan(1/2*d*x + 1/2*c)^9
 - 315*a^2*tan(1/2*d*x + 1/2*c)^8 + 630*b^2*tan(1/2*d*x + 1/2*c)^8 - 2160*a
*b*tan(1/2*d*x + 1/2*c)^7 - 630*a^2*tan(1/2*d*x + 1/2*c)^6 - 3360*b^2*tan(1
/2*d*x + 1/2*c)^6 + 2520*a^2*tan(1/2*d*x + 1/2*c)^4 + 5040*b^2*tan(1/2*d*x
+ 1/2*c)^4 + 13440*a*b*tan(1/2*d*x + 1/2*c)^3 + 1260*a^2*tan(1/2*d*x + 1/2*
c)^2 + 10080*b^2*tan(1/2*d*x + 1/2*c)^2 - 30240*a*b*tan(1/2*d*x + 1/2*c) -
5040*(3*a^2 + 10*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (44286*a^2*tan(1/2*d
```

```
*x + 1/2*c)^10 + 147620*b^2*tan(1/2*d*x + 1/2*c)^10 + 30240*a*b*tan(1/2*d*x
 + 1/2*c)^9 - 1260*a^2*tan(1/2*d*x + 1/2*c)^8 - 10080*b^2*tan(1/2*d*x + 1/2
*c)^8 - 13440*a*b*tan(1/2*d*x + 1/2*c)^7 - 2520*a^2*tan(1/2*d*x + 1/2*c)^6
- 5040*b^2*tan(1/2*d*x + 1/2*c)^6 + 630*a^2*tan(1/2*d*x + 1/2*c)^4 + 3360*b
^2*tan(1/2*d*x + 1/2*c)^4 + 2160*a*b*tan(1/2*d*x + 1/2*c)^3 + 315*a^2*tan(1
/2*d*x + 1/2*c)^2 - 630*b^2*tan(1/2*d*x + 1/2*c)^2 - 560*a*b*tan(1/2*d*x +
1/2*c) - 126*a^2)/tan(1/2*d*x + 1/2*c)^10)/d
```

maple [B] time = 0.46, size = 404, normalized size = 1.92

$$-\frac{a^2\left(\cos^7(dx+c)\right)}{10d\sin(dx+c)^{10}}-\frac{3a^2\left(\cos^7(dx+c)\right)}{80d\sin(dx+c)^8}-\frac{a^2\left(\cos^7(dx+c)\right)}{160d\sin(dx+c)^6}+\frac{a^2\left(\cos^7(dx+c)\right)}{640d\sin(dx+c)^4}-\frac{3a^2\left(\cos^7(dx+c)\right)}{1280d\sin(dx+c)^2}-\frac{3a^2\left(\cos\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^11*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/10/d*a^2/sin(d*x+c)^10*cos(d*x+c)^7-3/80/d*a^2/sin(d*x+c)^8*cos(d*x+c)^7
-1/160/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+1/640/d*a^2/sin(d*x+c)^4*cos(d*x+c)^
7-3/1280/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-3/1280*a^2*cos(d*x+c)^5/d-1/256*a^
2*cos(d*x+c)^3/d-3/256*a^2*cos(d*x+c)/d-3/256/d*a^2*ln(csc(d*x+c)-cot(d*x+c
))-2/9/d*a*b/sin(d*x+c)^9*cos(d*x+c)^7-4/63/d*a*b/sin(d*x+c)^7*cos(d*x+c)^7
-1/8/d*b^2/sin(d*x+c)^8*cos(d*x+c)^7-1/48/d*b^2/sin(d*x+c)^6*cos(d*x+c)^7+1
/192/d*b^2/sin(d*x+c)^4*cos(d*x+c)^7-1/128/d*b^2/sin(d*x+c)^2*cos(d*x+c)^7-
1/128*b^2*cos(d*x+c)^5/d-5/384*b^2*cos(d*x+c)^3/d-5/128*b^2*cos(d*x+c)/d-5/
128/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.33, size = 272, normalized size = 1.30

$$-\frac{63\,a^2\left(\frac{2\left(15\,\cos(dx+c)^9-70\,\cos(dx+c)^7-128\,\cos(dx+c)^5+70\,\cos(dx+c)^3-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^8+10\,\cos(dx+c)^6-10\,\cos(dx+c)^4+5\,\cos(dx+c)^2-1}-15\,\log\left(\cos(dx+c)+1\right)+15\,\log\left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+b*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/161280*(63*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x +
 c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x +
 c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*
log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 210*b^2*(2*(15*cos(d*x
+ c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x
+ c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*l
og(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 5120*(9*tan(d*x + c)^2 +
 7)*a*b/tan(d*x + c)^9)/d
```

mupad [B] time = 11.93, size = 394, normalized size = 1.88

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{10240\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(\frac{3\,a^2}{256}+\frac{5\,b^2}{128}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(2\,a^2+4\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^11,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^10)/(10240*d) - (log(tan(c/2 + (d*x)/2))*((3*a^2)/2
56 + (5*b^2)/128))/d - (cot(c/2 + (d*x)/2)^10*(tan(c/2 + (d*x)/2)^6*(2*a^2
+ 4*b^2) - tan(c/2 + (d*x)/2)^2*(a^2/4 - b^2/2) - tan(c/2 + (d*x)/2)^4*(a^2
/2 + (8*b^2)/3) + a^2/10 + tan(c/2 + (d*x)/2)^8*(a^2 + 8*b^2) - (12*a*b*tan
(c/2 + (d*x)/2)^3)/7 + (32*a*b*tan(c/2 + (d*x)/2)^7)/3 - 24*a*b*tan(c/2 + (
d*x)/2)^9 + (4*a*b*tan(c/2 + (d*x)/2))/9))/(1024*d) + (tan(c/2 + (d*x)/2)^4
*(a^2/512 + b^2/256))/d + (tan(c/2 + (d*x)/2)^2*(a^2/1024 + b^2/128))/d - (
tan(c/2 + (d*x)/2)^6*(a^2/2048 + b^2/384))/d - (tan(c/2 + (d*x)/2)^8*(a^2/4
096 - b^2/2048))/d + (a*b*tan(c/2 + (d*x)/2)^3)/(96*d) - (3*a*b*tan(c/2 + (
d*x)/2)^7)/(1792*d) + (a*b*tan(c/2 + (d*x)/2)^9)/(2304*d) - (3*a*b*tan(c/2
+ (d*x)/2))/(128*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**11*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1255 $\int \cot^6(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=198

$$-\frac{\left(2a^2+b^2\right)\cot^9(c+dx)}{9d}-\frac{\left(a^2+b^2\right)\cot^7(c+dx)}{7d}-\frac{a^2\cot^{11}(c+dx)}{11d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{128d}-\frac{ab\cot^5(c+dx)\,c}{5d}$$

```
[Out] 3/128*a*b*arctanh(cos(d*x+c))/d-1/7*(a^2+b^2)*cot(d*x+c)^7/d-1/9*(2*a^2+b^2
)*cot(d*x+c)^9/d-1/11*a^2*cot(d*x+c)^11/d+3/128*a*b*cot(d*x+c)*csc(d*x+c)/d
+1/64*a*b*cot(d*x+c)*csc(d*x+c)^3/d-1/16*a*b*cot(d*x+c)*csc(d*x+c)^5/d+1/8*
a*b*cot(d*x+c)^3*csc(d*x+c)^5/d-1/5*a*b*cot(d*x+c)^5*csc(d*x+c)^5/d
```

Rubi [A] time = 0.46, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2911, 2611, 3768, 3770, 448}

$$-\frac{\left(2a^2+b^2\right)\cot^9(c+dx)}{9d}-\frac{\left(a^2+b^2\right)\cot^7(c+dx)}{7d}-\frac{a^2\cot^{11}(c+dx)}{11d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{128d}-\frac{ab\cot^5(c+dx)\,c}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*b*ArcTanh[Cos[c + d*x]])/(128*d) - ((a^2 + b^2)*Cot[c + d*x]^7)/(7*d)
- ((2*a^2 + b^2)*Cot[c + d*x]^9)/(9*d) - (a^2*Cot[c + d*x]^11)/(11*d) + (3*
a*b*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (a*b*Cot[c + d*x]*Csc[c + d*x]^3)/
(64*d) - (a*b*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) + (a*b*Cot[c + d*x]^3*Csc
[c + d*x]^5)/(8*d) - (a*b*Cot[c + d*x]^5*Csc[c + d*x]^5)/(5*d)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc^5(c+dx)\,dx + \int \cot^6(c+dx)\csc^6($$

$$= -\frac{ab\cot^5(c+dx)\csc^5(c+dx)}{5d} - (ab)\int \cot^4(c+dx)\csc^5(c +$$

$$= \frac{ab\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{ab\cot^5(c+dx)\csc^5(c+dx)}{5d}$$

$$= -\frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{(2a^2+b^2)\cot^9(c+dx)}{9d} - \frac{a^2\cot^1}{1}$$

$$= -\frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{(2a^2+b^2)\cot^9(c+dx)}{9d} - \frac{a^2\cot^1}{1}$$

$$= -\frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{(2a^2+b^2)\cot^9(c+dx)}{9d} - \frac{a^2\cot^1}{1}$$

$$= \frac{3ab\tanh^{-1}(\cos(c+dx))}{128d} - \frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{(2a^2 +}{}$$

Mathematica [A] time = 1.67, size = 250, normalized size = 1.26

$$-\frac{\csc^{11}(c+dx)\left(1478400\left(8a^2+b^2\right)\cos(c+dx)+42240\left(160a^2-b^2\right)\cos(3(c+dx))+1943040a^2\cos(5(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/227082240*(-5322240*a*b*Log[Cos[(c + d*x)/2]] + 5322240*a*b*Log[Sin[(c +
 d*x)/2]] + Csc[c + d*x]^11*(1478400*(8*a^2 + b^2)*Cos[c + d*x] + 42240*(16
0*a^2 - b^2)*Cos[3*(c + d*x)] + 1943040*a^2*Cos[5*(c + d*x)] - 865920*b^2*C
os[5*(c + d*x)] + 140800*a^2*Cos[7*(c + d*x)] - 499840*b^2*Cos[7*(c + d*x)]
 - 28160*a^2*Cos[9*(c + d*x)] - 77440*b^2*Cos[9*(c + d*x)] + 2560*a^2*Cos[1
1*(c + d*x)] + 7040*b^2*Cos[11*(c + d*x)] + 5828130*a*b*Sin[2*(c + d*x)] +
4790016*a*b*Sin[4*(c + d*x)] + 2302839*a*b*Sin[6*(c + d*x)] + 110880*a*b*Si
n[8*(c + d*x)] - 10395*a*b*Sin[10*(c + d*x)]))/d
```

fricas [B] time = 0.82, size = 363, normalized size = 1.83

$$2560\left(4a^2+11b^2\right)\cos\left(dx+c\right)^{11}-14080\left(4a^2+11b^2\right)\cos\left(dx+c\right)^9+126720\left(a^2+b^2\right)\cos\left(dx+c\right)^7+10395$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+b*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/887040*(2560*(4*a^2 + 11*b^2)*cos(d*x + c)^11 - 14080*(4*a^2 + 11*b^2)*co
s(d*x + c)^9 + 126720*(a^2 + b^2)*cos(d*x + c)^7 + 10395*(a*b*cos(d*x + c)^
10 - 5*a*b*cos(d*x + c)^8 + 10*a*b*cos(d*x + c)^6 - 10*a*b*cos(d*x + c)^4 +
 5*a*b*cos(d*x + c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 103
95*(a*b*cos(d*x + c)^10 - 5*a*b*cos(d*x + c)^8 + 10*a*b*cos(d*x + c)^6 - 10
*a*b*cos(d*x + c)^4 + 5*a*b*cos(d*x + c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) - 1386*(15*a*b*cos(d*x + c)^9 - 70*a*b*cos(d*x + c)^7 - 12
8*a*b*cos(d*x + c)^5 + 70*a*b*cos(d*x + c)^3 - 15*a*b*cos(d*x + c))*sin(d*x
 + c))/((d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*
d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [B] time = 0.36, size = 502, normalized size = 2.54

$$315\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1386\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-385\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+1540\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+b*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/7096320*(315*a^2*tan(1/2*d*x + 1/2*c)^11 + 1386*a*b*tan(1/2*d*x + 1/2*c)^
10 - 385*a^2*tan(1/2*d*x + 1/2*c)^9 + 1540*b^2*tan(1/2*d*x + 1/2*c)^9 - 346
5*a*b*tan(1/2*d*x + 1/2*c)^8 - 2475*a^2*tan(1/2*d*x + 1/2*c)^7 - 5940*b^2*t
an(1/2*d*x + 1/2*c)^7 - 6930*a*b*tan(1/2*d*x + 1/2*c)^6 + 3465*a^2*tan(1/2*
d*x + 1/2*c)^5 + 27720*a*b*tan(1/2*d*x + 1/2*c)^4 + 11550*a^2*tan(1/2*d*x +
 1/2*c)^3 + 36960*b^2*tan(1/2*d*x + 1/2*c)^3 + 13860*a*b*tan(1/2*d*x + 1/2*
c)^2 - 166320*a*b*log(abs(tan(1/2*d*x + 1/2*c))) - 34650*a^2*tan(1/2*d*x +
1/2*c) - 83160*b^2*tan(1/2*d*x + 1/2*c) + (502266*a*b*tan(1/2*d*x + 1/2*c)^
11 + 34650*a^2*tan(1/2*d*x + 1/2*c)^10 + 83160*b^2*tan(1/2*d*x + 1/2*c)^10
- 13860*a*b*tan(1/2*d*x + 1/2*c)^9 - 11550*a^2*tan(1/2*d*x + 1/2*c)^8 - 369
60*b^2*tan(1/2*d*x + 1/2*c)^8 - 27720*a*b*tan(1/2*d*x + 1/2*c)^7 - 3465*a^2
*tan(1/2*d*x + 1/2*c)^6 + 6930*a*b*tan(1/2*d*x + 1/2*c)^5 + 2475*a^2*tan(1/
2*d*x + 1/2*c)^4 + 5940*b^2*tan(1/2*d*x + 1/2*c)^4 + 3465*a*b*tan(1/2*d*x +
 1/2*c)^3 + 385*a^2*tan(1/2*d*x + 1/2*c)^2 - 1540*b^2*tan(1/2*d*x + 1/2*c)^
2 - 1386*a*b*tan(1/2*d*x + 1/2*c) - 315*a^2)/tan(1/2*d*x + 1/2*c)^11)/d
```

maple [A] time = 0.46, size = 303, normalized size = 1.53

$$-\frac{a^{2}\left(\cos^{7}\left(dx+c\right)\right)}{11d\sin\left(dx+c\right)^{11}}-\frac{4a^{2}\left(\cos^{7}\left(dx+c\right)\right)}{99d\sin\left(dx+c\right)^{9}}-\frac{8a^{2}\left(\cos^{7}\left(dx+c\right)\right)}{693d\sin\left(dx+c\right)^{7}}-\frac{ab\left(\cos^{7}\left(dx+c\right)\right)}{5d\sin\left(dx+c\right)^{10}}-\frac{3ab\left(\cos^{7}\left(dx+c\right)\right)}{40d\sin\left(dx+c\right)^{8}}-\frac{ab\left(\cos^{7}\left(\right.\right.}{80d\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^12*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/11/d*a^2/sin(d*x+c)^11*cos(d*x+c)^7-4/99/d*a^2/sin(d*x+c)^9*cos(d*x+c)^7
-8/693/d*a^2/sin(d*x+c)^7*cos(d*x+c)^7-1/5/d*a*b/sin(d*x+c)^10*cos(d*x+c)^7
-3/40/d*a*b/sin(d*x+c)^8*cos(d*x+c)^7-1/80/d*a*b/sin(d*x+c)^6*cos(d*x+c)^7+
1/320/d*a*b/sin(d*x+c)^4*cos(d*x+c)^7-3/640/d*a*b/sin(d*x+c)^2*cos(d*x+c)^7
-3/640*a*b*cos(d*x+c)^5/d-1/128*a*b*cos(d*x+c)^3/d-3/128*a*b*cos(d*x+c)/d-3
/128/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/9/d*b^2/sin(d*x+c)^9*cos(d*x+c)^7-2/
63/d*b^2/sin(d*x+c)^7*cos(d*x+c)^7
```

maxima [A] time = 0.34, size = 196, normalized size = 0.99

$$-\frac{693\,ab\left(\frac{2\left(15\,\cos(dx+c)^{9}-70\,\cos(dx+c)^{7}-128\,\cos(dx+c)^{5}+70\,\cos(dx+c)^{3}-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^{8}+10\,\cos(dx+c)^{6}-10\,\cos(dx+c)^{4}+5\,\cos(dx+c)^{2}-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\right.\right.}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+b*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/887040*(693*a*b*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x
+ c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x
```

```
+ c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15
*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 14080*(9*tan(d*x + c)^
2 + 7)*b^2/tan(d*x + c)^9 + 1280*(99*tan(d*x + c)^4 + 154*tan(d*x + c)^2 +
63)*a^2/tan(d*x + c)^11)/d
```

mupad [B] time = 16.33, size = 448, normalized size = 2.26

$$-\frac{\frac{5\,a^2\,\cos(c+d\,x)}{96}+\frac{5\,b^2\,\cos(c+d\,x)}{768}+\frac{5\,a^2\,\cos(3\,c+3\,d\,x)}{168}+\frac{23\,a^2\,\cos(5\,c+5\,d\,x)}{2688}+\frac{5\,a^2\,\cos(7\,c+7\,d\,x)}{8064}-\frac{a^2\,\cos(9\,c+9\,d\,x)}{8064}+\frac{a^2\,\cos(11\,c+11\,d\,x)}{88704}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^12,x)
```

```
[Out] -((5*a^2*cos(c + d*x))/96 + (5*b^2*cos(c + d*x))/768 + (5*a^2*cos(3*c + 3*d
*x))/168 + (23*a^2*cos(5*c + 5*d*x))/2688 + (5*a^2*cos(7*c + 7*d*x))/8064 -
 (a^2*cos(9*c + 9*d*x))/8064 + (a^2*cos(11*c + 11*d*x))/88704 - (b^2*cos(3*
c + 3*d*x))/5376 - (41*b^2*cos(5*c + 5*d*x))/10752 - (71*b^2*cos(7*c + 7*d*
x))/32256 - (11*b^2*cos(9*c + 9*d*x))/32256 + (b^2*cos(11*c + 11*d*x))/3225
6 + (841*a*b*sin(2*c + 2*d*x))/32768 + (27*a*b*sin(4*c + 4*d*x))/1280 + (33
23*a*b*sin(6*c + 6*d*x))/327680 + (a*b*sin(8*c + 8*d*x))/2048 - (3*a*b*sin(
10*c + 10*d*x))/65536 + (693*a*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)))/65536 - (495*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*
sin(3*c + 3*d*x))/65536 + (495*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
))*sin(5*c + 5*d*x))/131072 - (165*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*sin(7*c + 7*d*x))/131072 + (33*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2))*sin(9*c + 9*d*x))/131072 - (3*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2
+ (d*x)/2))*sin(11*c + 11*d*x))/131072)/(d*sin(c + d*x)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**12*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1256 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=525

$$-\frac{2a^2\left(8a^2-3b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^9d}-\frac{3b\sin^5(c+dx)\cos(c+dx)}{20a^2d(a+b\sin(c+dx))}+\frac{\left(224a^4-340a^2b^2+105b^4\right)\sin}{140a^2b^4d}$$

```
[Out] 1/8*a*(64*a^6-120*a^4*b^2+60*a^2*b^4-5*b^6)*x/b^9-2*a^2*(8*a^2-3*b^2)*(a^2-
b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^9/d+1/105*(84
0*a^6-1435*a^4*b^2+588*a^2*b^4-15*b^6)*cos(d*x+c)/b^8/d-1/8*a*(32*a^4-52*a^
2*b^2+19*b^4)*cos(d*x+c)*sin(d*x+c)/b^7/d+1/105*(280*a^4-441*a^2*b^2+150*b^
4)*cos(d*x+c)*sin(d*x+c)^2/b^6/d-1/12*(24*a^4-37*a^2*b^2+12*b^4)*cos(d*x+c)
*sin(d*x+c)^3/a/b^5/d+1/140*(224*a^4-340*a^2*b^2+105*b^4)*cos(d*x+c)*sin(d*
x+c)^4/a^2/b^4/d+1/4*cos(d*x+c)*sin(d*x+c)^4/a/d/(a+b*sin(d*x+c))-3/20*b*co
s(d*x+c)*sin(d*x+c)^5/a^2/d/(a+b*sin(d*x+c))-1/15*(20*a^4-30*a^2*b^2+9*b^4)
*cos(d*x+c)*sin(d*x+c)^5/a^2/b^3/d/(a+b*sin(d*x+c))-4/21*a*cos(d*x+c)*sin(d
*x+c)^6/b^2/d/(a+b*sin(d*x+c))+1/7*cos(d*x+c)*sin(d*x+c)^7/b/d/(a+b*sin(d*x
+c))
```

Rubi [A] time = 1.91, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2896, 3047, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{\left(-1435a^4b^2+588a^2b^4+840a^6-15b^6\right)\cos(c+dx)}{105b^8d}-\frac{2a^2\left(8a^2-3b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^9d}-\left(-30a^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*(64*a^6 - 120*a^4*b^2 + 60*a^2*b^4 - 5*b^6)*x)/(8*b^9) - (2*a^2*(8*a^2 -
 3*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(b^9*d) + ((840*a^6 - 1435*a^4*b^2 + 588*a^2*b^4 - 15*b^6)*Cos[c + d*x])/(
105*b^8*d) - (a*(32*a^4 - 52*a^2*b^2 + 19*b^4)*Cos[c + d*x]*Sin[c + d*x])/(
8*b^7*d) + ((280*a^4 - 441*a^2*b^2 + 150*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/
(105*b^6*d) - ((24*a^4 - 37*a^2*b^2 + 12*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/
(12*a*b^5*d) + ((224*a^4 - 340*a^2*b^2 + 105*b^4)*Cos[c + d*x]*Sin[c + d*x]
^4)/(140*a^2*b^4*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*a*d*(a + b*Sin[c + d
*x])) - (3*b*Cos[c + d*x]*Sin[c + d*x]^5)/(20*a^2*d*(a + b*Sin[c + d*x])) -
 ((20*a^4 - 30*a^2*b^2 + 9*b^4)*Cos[c + d*x]*Sin[c + d*x]^5)/(15*a^2*b^3*d*
```

```
(a + b*Sin[c + d*x])) - (4*a*Cos[c + d*x]*Sin[c + d*x]^6)/(21*b^2*d*(a + b*
Sin[c + d*x])) + (Cos[c + d*x]*Sin[c + d*x]^7)/(7*b*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
```

```
n + 5, 0] && NeQ[m + n + 6, 0] && !IGtQ[m, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))} - \frac{3b\cos(c+dx)\sin^5(c+dx)}{20a^2d(a+b\sin(c+dx))} - \frac{4a\cos(c+dx)\sin^6(\ldots}{21b^2d(a+b\sin(c\ldots}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))} - \frac{3b\cos(c+dx)\sin^5(c+dx)}{20a^2d(a+b\sin(c+dx))} - \frac{\left(20a^4-30a^2b^2+9\ldots\right.}{15a^2b^3d(\ldots}$$

$$= \frac{\left(224a^4-340a^2b^2+105b^4\right)\cos(c+dx)\sin^4(c+dx)}{140a^2b^4d} + \frac{\cos(c+dx)\sin^4(c+\ldots}{4ad(a+b\sin(c+dx\ldots}$$

$$= -\frac{\left(24a^4-37a^2b^2+12b^4\right)\cos(c+dx)\sin^3(c+dx)}{12ab^5d} + \frac{\left(224a^4-340a^2b^2+105\ldots\right.}{14\ldots}$$

$$= \frac{\left(280a^4-441a^2b^2+150b^4\right)\cos(c+dx)\sin^2(c+dx)}{105b^6d} - \frac{\left(24a^4-37a^2b^2+12b^{\ldots}\right.}{1\ldots}$$

$$= -\frac{a\left(32a^4-52a^2b^2+19b^4\right)\cos(c+dx)\sin(c+dx)}{8b^7d} + \frac{\left(280a^4-441a^2b^2+150\ldots\right.}{1\ldots}$$

$$= \frac{\left(840a^6-1435a^4b^2+588a^2b^4-15b^6\right)\cos(c+dx)}{105b^8d} - \frac{a\left(32a^4-52a^2b^2+19b^{\ldots}\right.}{8\ldots}$$

$$= \frac{a\left(64a^6-120a^4b^2+60a^2b^4-5b^6\right)x}{8b^9} + \frac{\left(840a^6-1435a^4b^2+588a^2b^4-15b^6\right)\ldots}{105b^8d}$$

$$= \frac{a\left(64a^6-120a^4b^2+60a^2b^4-5b^6\right)x}{8b^9} + \frac{\left(840a^6-1435a^4b^2+588a^2b^4-15b^6\right)\ldots}{105b^8d}$$

$$= \frac{a\left(64a^6-120a^4b^2+60a^2b^4-5b^6\right)x}{8b^9} + \frac{\left(840a^6-1435a^4b^2+588a^2b^4-15b^6\right)\ldots}{105b^8d}$$

$$= \frac{a\left(64a^6-120a^4b^2+60a^2b^4-5b^6\right)x}{8b^9} - \frac{2a^2\left(8a^2-3b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+\ldots}{\ldots}\right.}{b^9d}$$

Mathematica [A] time = 9.18, size = 531, normalized size = 1.01

$$\frac{107520a^8c+107520a^8dx+107520a^7bc\sin(c+dx)+107520a^7bdx\sin(c+dx)+26880a^6b^2\sin(2(c+dx))-201600a^6b^2c-201600a^6b^2dx-201600a^5b^3c\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-26880*a^2*(8*a^2 - 3*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2
])/Sqrt[a^2 - b^2]] + (107520*a^8*c - 201600*a^6*b^2*c + 100800*a^4*b^4*c -
 8400*a^2*b^6*c + 107520*a^8*d*x - 201600*a^6*b^2*d*x + 100800*a^4*b^4*d*x
- 8400*a^2*b^6*d*x + 840*a*b*(128*a^6 - 224*a^4*b^2 + 98*a^2*b^4 - 5*b^6)*C
os[c + d*x] + 70*(64*a^5*b^3 - 96*a^3*b^5 + 27*a*b^7)*Cos[3*(c + d*x)] - 33
6*a^3*b^5*Cos[5*(c + d*x)] + 350*a*b^7*Cos[5*(c + d*x)] + 40*a*b^7*Cos[7*(c
 + d*x)] + 107520*a^7*b*c*Sin[c + d*x] - 201600*a^5*b^3*c*Sin[c + d*x] + 10
0800*a^3*b^5*c*Sin[c + d*x] - 8400*a*b^7*c*Sin[c + d*x] + 107520*a^7*b*d*x*
Sin[c + d*x] - 201600*a^5*b^3*d*x*Sin[c + d*x] + 100800*a^3*b^5*d*x*Sin[c +
 d*x] - 8400*a*b^7*d*x*Sin[c + d*x] + 26880*a^6*b^2*Sin[2*(c + d*x)] - 4592
0*a^4*b^4*Sin[2*(c + d*x)] + 18480*a^2*b^6*Sin[2*(c + d*x)] - 210*b^8*Sin[2
*(c + d*x)] - 1120*a^4*b^4*Sin[4*(c + d*x)] + 1428*a^2*b^6*Sin[4*(c + d*x)]
 - 210*b^8*Sin[4*(c + d*x)] + 112*a^2*b^6*Sin[6*(c + d*x)] - 90*b^8*Sin[6*(
c + d*x)] - 15*b^8*Sin[8*(c + d*x)])/(a + b*Sin[c + d*x]))/(13440*b^9*d)
```

fricas [A] time = 1.10, size = 871, normalized size = 1.66

$$\left[\frac{160\,ab^7\cos(dx+c)^7-14\left(24\,a^3b^5-5\,ab^7\right)\cos(dx+c)^5+35\left(32\,a^5b^3-36\,a^3b^5+5\,ab^7\right)\cos(dx+c)^3+105\left(6\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/840*(160*a*b^7*cos(d*x + c)^7 - 14*(24*a^3*b^5 - 5*a*b^7)*cos(d*x + c)^5
 + 35*(32*a^5*b^3 - 36*a^3*b^5 + 5*a*b^7)*cos(d*x + c)^3 + 105*(64*a^8 - 12
0*a^6*b^2 + 60*a^4*b^4 - 5*a^2*b^6)*d*x + 420*(8*a^7 - 11*a^5*b^2 + 3*a^3*b
^4 + (8*a^6*b - 11*a^4*b^3 + 3*a^2*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 105*(64*a^7*b - 120*a^5*b^3 + 60*a^3
*b^5 - 5*a*b^7)*cos(d*x + c) - (120*b^8*cos(d*x + c)^7 - 224*a^2*b^6*cos(d*
```

```
x + c)^5 + 70*(8*a^4*b^4 - 7*a^2*b^6)*cos(d*x + c)^3 - 105*(64*a^7*b - 120*
a^5*b^3 + 60*a^3*b^5 - 5*a*b^7)*d*x - 105*(32*a^6*b^2 - 52*a^4*b^4 + 19*a^2
*b^6)*cos(d*x + c))*sin(d*x + c))/(b^10*d*sin(d*x + c) + a*b^9*d), 1/840*(1
60*a*b^7*cos(d*x + c)^7 - 14*(24*a^3*b^5 - 5*a*b^7)*cos(d*x + c)^5 + 35*(32
*a^5*b^3 - 36*a^3*b^5 + 5*a*b^7)*cos(d*x + c)^3 + 105*(64*a^8 - 120*a^6*b^2
 + 60*a^4*b^4 - 5*a^2*b^6)*d*x + 840*(8*a^7 - 11*a^5*b^2 + 3*a^3*b^4 + (8*a
^6*b - 11*a^4*b^3 + 3*a^2*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin
(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 105*(64*a^7*b - 120*a^5*b^
3 + 60*a^3*b^5 - 5*a*b^7)*cos(d*x + c) - (120*b^8*cos(d*x + c)^7 - 224*a^2*
b^6*cos(d*x + c)^5 + 70*(8*a^4*b^4 - 7*a^2*b^6)*cos(d*x + c)^3 - 105*(64*a^
7*b - 120*a^5*b^3 + 60*a^3*b^5 - 5*a*b^7)*d*x - 105*(32*a^6*b^2 - 52*a^4*b^
4 + 19*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/(b^10*d*sin(d*x + c) + a*b^9*d)
]
```

giac [A] time = 0.27, size = 965, normalized size = 1.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(105*(64*a^7 - 120*a^5*b^2 + 60*a^3*b^4 - 5*a*b^6)*(d*x + c)/b^9 - 16
80*(8*a^8 - 19*a^6*b^2 + 14*a^4*b^4 - 3*a^2*b^6)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqr
t(a^2 - b^2)*b^9) + 1680*(a^6*b*tan(1/2*d*x + 1/2*c) - 2*a^4*b^3*tan(1/2*d*
x + 1/2*c) + a^2*b^5*tan(1/2*d*x + 1/2*c) + a^7 - 2*a^5*b^2 + a^3*b^4)/((a*
tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^8) + 2*(2520*a^5*b
*tan(1/2*d*x + 1/2*c)^13 - 3780*a^3*b^3*tan(1/2*d*x + 1/2*c)^13 + 1155*a*b^
5*tan(1/2*d*x + 1/2*c)^13 + 5880*a^6*tan(1/2*d*x + 1/2*c)^12 - 12600*a^4*b^
2*tan(1/2*d*x + 1/2*c)^12 + 7560*a^2*b^4*tan(1/2*d*x + 1/2*c)^12 - 840*b^6*
tan(1/2*d*x + 1/2*c)^12 + 10080*a^5*b*tan(1/2*d*x + 1/2*c)^11 - 11760*a^3*b
^3*tan(1/2*d*x + 1/2*c)^11 + 980*a*b^5*tan(1/2*d*x + 1/2*c)^11 + 35280*a^6*
tan(1/2*d*x + 1/2*c)^10 - 67200*a^4*b^2*tan(1/2*d*x + 1/2*c)^10 + 30240*a^2
*b^4*tan(1/2*d*x + 1/2*c)^10 + 12600*a^5*b*tan(1/2*d*x + 1/2*c)^9 - 12180*a
^3*b^3*tan(1/2*d*x + 1/2*c)^9 + 2975*a*b^5*tan(1/2*d*x + 1/2*c)^9 + 88200*a
^6*tan(1/2*d*x + 1/2*c)^8 - 152600*a^4*b^2*tan(1/2*d*x + 1/2*c)^8 + 61320*a
^2*b^4*tan(1/2*d*x + 1/2*c)^8 - 4200*b^6*tan(1/2*d*x + 1/2*c)^8 + 117600*a^
6*tan(1/2*d*x + 1/2*c)^6 - 190400*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 + 73920*a^
2*b^4*tan(1/2*d*x + 1/2*c)^6 - 12600*a^5*b*tan(1/2*d*x + 1/2*c)^5 + 12180*a
^3*b^3*tan(1/2*d*x + 1/2*c)^5 - 2975*a*b^5*tan(1/2*d*x + 1/2*c)^5 + 88200*a
^6*tan(1/2*d*x + 1/2*c)^4 - 138600*a^4*b^2*tan(1/2*d*x + 1/2*c)^4 + 50904*a
^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 2520*b^6*tan(1/2*d*x + 1/2*c)^4 - 10080*a^5
*b*tan(1/2*d*x + 1/2*c)^3 + 11760*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 980*a*b^
5*tan(1/2*d*x + 1/2*c)^3 + 35280*a^6*tan(1/2*d*x + 1/2*c)^2 - 56000*a^4*b^2
*tan(1/2*d*x + 1/2*c)^2 + 19488*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 - 2520*a^5*b
```

```
*tan(1/2*d*x + 1/2*c) + 3780*a^3*b^3*tan(1/2*d*x + 1/2*c) - 1155*a*b^5*tan(
1/2*d*x + 1/2*c) + 5880*a^6 - 9800*a^4*b^2 + 3864*a^2*b^4 - 120*b^6)/((tan(
1/2*d*x + 1/2*c)^2 + 1)^7*b^8))/d
```

maple [B] time = 0.57, size = 2076, normalized size = 3.95

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 15/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^3-5/4/d/b^3*arctan(tan(1/2*d*x+1/2*c)
)*a+2/d*a^7/b^8/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-4/d*a^5/b
^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+2/d*a^3/b^4/(tan(1/2*d
*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-30/d/b^7*arctan(tan(1/2*d*x+1/2*c))
*a^5+14/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^7*a^6-70/3/d/b^6/(1+tan(1/2*d*x+1/2*
c)^2)^7*a^4+46/5/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*a^2-2/d/b^2/(1+tan(1/2*d*
x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12-10/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan
(1/2*d*x+1/2*c)^8-6/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4+1
6/d/b^9*arctan(tan(1/2*d*x+1/2*c))*a^7-2/7/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^7
-28/d*a^4/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-
b^2)^(1/2))+6/d*a^2/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+
2*b)/(a^2-b^2)^(1/2))+28/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c
)^3*a^3+210/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8*a^6-1090/
3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8*a^4+146/d/b^4/(1+ta
n(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8*a^2+280/d/b^8/(1+tan(1/2*d*x+1/2
*c)^2)^7*tan(1/2*d*x+1/2*c)^6*a^6-1360/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*t
an(1/2*d*x+1/2*c)^6*a^4+84/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2
*c)^10*a^6-29/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^9*a^3-30/
d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^5*a^5+29/d/b^5/(1+tan(1
/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^5*a^3-85/12/d/b^3/(1+tan(1/2*d*x+1/2*
c)^2)^7*tan(1/2*d*x+1/2*c)^5*a+210/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2
*d*x+1/2*c)^4*a^6+2/d*a^6/b^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)*tan(1/2*d*x+1/2*c)-4/d*a^4/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+176/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*ta
n(1/2*d*x+1/2*c)^6*a^2-7/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2
*c)^3*a+84/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2*a^6-400/3/
d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2*a^4-160/d/b^6/(1+tan(
1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^10*a^4-16/d*a^8/b^9/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+38/d*a^6/b^7/(a^2-
b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d/b^7
/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13*a^5-9/d/b^5/(1+tan(1/2*d*
x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13*a^3+11/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)
^7*tan(1/2*d*x+1/2*c)^13*a+14/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+
```

```
1/2*c)^12*a^6-30/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12*a^4
+18/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12*a^2+24/d/b^7/(1+
tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11*a^5-28/d/b^5/(1+tan(1/2*d*x+1
/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11*a^3+232/5/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7
*tan(1/2*d*x+1/2*c)^2*a^2-11/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x
+1/2*c)*a+7/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11*a+30/d
/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^9*a^5-330/d/b^6/(1+tan(1
/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4*a^4-6/d/b^7/(1+tan(1/2*d*x+1/2*c)^2
)^7*tan(1/2*d*x+1/2*c)*a^5+9/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1
/2*c)*a^3+606/5/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4*a^2-2
4/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^3*a^5+85/12/d/b^3/(1+
tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^9*a+72/d/b^4/(1+tan(1/2*d*x+1/2*
c)^2)^7*tan(1/2*d*x+1/2*c)^10*a^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 18.11, size = 3724, normalized size = 7.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^3)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^14*(7*a*b^6 + 32*a^7 + 4*a^3*b^4 - 44*a^5*b^2))/(2*b^8
) - (2*(15*a*b^6 - 840*a^7 - 588*a^3*b^4 + 1435*a^5*b^2))/(105*b^8) + (2*ta
n(c/2 + (d*x)/2)^12*(4*a*b^6 + 168*a^7 + 72*a^3*b^4 - 255*a^5*b^2))/(3*b^8)
 - (2*tan(c/2 + (d*x)/2)^8*(15*a*b^6 - 840*a^7 - 588*a^3*b^4 + 1435*a^5*b^2
))/(3*b^8) + (tan(c/2 + (d*x)/2)^10*(25*a*b^6 + 2016*a^7 + 1212*a^3*b^4 - 3
284*a^5*b^2))/(6*b^8) - (2*tan(c/2 + (d*x)/2)^4*(80*a*b^6 - 2520*a^7 - 1992
*a^3*b^4 + 4465*a^5*b^2))/(15*b^8) - (tan(c/2 + (d*x)/2)^6*(605*a*b^6 - 168
00*a^7 - 12756*a^3*b^4 + 29500*a^5*b^2))/(30*b^8) - (tan(c/2 + (d*x)/2)^2*(
1215*a*b^6 - 23520*a^7 - 18396*a^3*b^4 + 41300*a^5*b^2))/(210*b^8) + (tan(c
/2 + (d*x)/2)*(10080*a^6 - 240*b^6 + 7413*a^2*b^4 - 17500*a^4*b^2))/(420*b^
7) + (tan(c/2 + (d*x)/2)^15*(32*a^6 + 19*a^2*b^4 - 52*a^4*b^2))/(4*b^7) + (
```

tan(c/2 + (d*x)/2)^11*(3168*a^6 + 2345*a^2*b^4 - 5532*a^4*b^2))/(12*b^7) + (tan(c/2 + (d*x)/2)^7*(7200*a^6 + 4979*a^2*b^4 - 12212*a^4*b^2))/(12*b^7) + (tan(c/2 + (d*x)/2)^3*(9120*a^6 + 6103*a^2*b^4 - 15460*a^4*b^2))/(60*b^7) + (tan(c/2 + (d*x)/2)^13*(864*a^6 - 48*b^6 + 661*a^2*b^4 - 1500*a^4*b^2))/(12*b^7) + (tan(c/2 + (d*x)/2)^9*(6240*a^6 - 240*b^6 + 4429*a^2*b^4 - 10748*a^4*b^2))/(12*b^7) + (tan(c/2 + (d*x)/2)^5*(24480*a^6 - 720*b^6 + 16499*a^2*b^4 - 41220*a^4*b^2))/(60*b^7))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 8*a*tan(c/2 + (d*x)/2)^2 + 28*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)/2)^6 + 70*a*tan(c/2 + (d*x)/2)^8 + 56*a*tan(c/2 + (d*x)/2)^10 + 28*a*tan(c/2 + (d*x)/2)^12 + 8*a*tan(c/2 + (d*x)/2)^14 + a*tan(c/2 + (d*x)/2)^16 + 14*b*tan(c/2 + (d*x)/2)^3 + 42*b*tan(c/2 + (d*x)/2)^5 + 70*b*tan(c/2 + (d*x)/2)^7 + 70*b*tan(c/2 + (d*x)/2)^9 + 42*b*tan(c/2 + (d*x)/2)^11 + 14*b*tan(c/2 + (d*x)/2)^13 + 2*b*tan(c/2 + (d*x)/2)^15)) + (a*atan(((a*(((25*a^4*b^20)/2 - 300*a^6*b^18 + 2400*a^8*b^16 - 7520*a^10*b^14 + 11040*a^12*b^12 - 7680*a^14*b^10 + 2048*a^16*b^8)/b^23 + (tan(c/2 + (d*x)/2)*(50*a^3*b^22 - 1801*a^5*b^20 + 15576*a^7*b^18 - 54720*a^9*b^16 + 96320*a^11*b^14 - 90240*a^13*b^12 + 43008*a^15*b^10 - 8192*a^17*b^8))/(2*b^24) - (a*((20*a^2*b^24 - 164*a^4*b^22 + 272*a^6*b^20 - 128*a^8*b^18)/b^23 + (tan(c/2 + (d*x)/2)*(384*a^3*b^24 - 1792*a^5*b^22 + 2432*a^7*b^20 - 1024*a^9*b^18))/(2*b^24) - (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^28 - 128*a^3*b^26))/(2*b^24))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b^9))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b^9))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2))/(8*b^9) + (a*(((25*a^4*b^20)/2 - 300*a^6*b^18 + 2400*a^8*b^16 - 7520*a^10*b^14 + 11040*a^12*b^12 - 7680*a^14*b^10 + 2048*a^16*b^8)/b^23 + (tan(c/2 + (d*x)/2)*(50*a^3*b^22 - 1801*a^5*b^20 + 15576*a^7*b^18 - 54720*a^9*b^16 + 96320*a^11*b^14 - 90240*a^13*b^12 + 43008*a^15*b^10 - 8192*a^17*b^8))/(2*b^24) + (a*((20*a^2*b^24 - 164*a^4*b^22 + 272*a^6*b^20 - 128*a^8*b^18)/b^23 + (tan(c/2 + (d*x)/2)*(384*a^3*b^24 - 1792*a^5*b^22 + 2432*a^7*b^20 - 1024*a^9*b^18))/(2*b^24) + (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^28 - 128*a^3*b^26))/(2*b^24))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b^9))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b^9))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2))/(8*b^9))/((16384*a^22 + 285*a^6*b^16 - 5530*a^8*b^14 + 38085*a^10*b^12 - 133328*a^12*b^10 + 269608*a^14*b^8 - 329120*a^16*b^6 + 239872*a^18*b^4 - 96256*a^20*b^2)/b^23 + (tan(c/2 + (d*x)/2)*(65536*a^23 - 150*a^5*b^18 + 4300*a^7*b^16 - 46550*a^9*b^14 + 247840*a^11*b^12 - 745600*a^13*b^10 + 1358720*a^15*b^8 - 1534336*a^17*b^6 + 1051648*a^19*b^4 - 401408*a^21*b^2))/b^24 + (a*(((25*a^4*b^20)/2 - 300*a^6*b^18 + 2400*a^8*b^16 - 7520*a^10*b^14 + 11040*a^12*b^12 - 7680*a^14*b^10 + 2048*a^16*b^8)/b^23 + (tan(c/2 + (d*x)/2)*(50*a^3*b^22 - 1801*a^5*b^20 + 15576*a^7*b^18 - 54720*a^9*b^16 + 96320*a^11*b^14 - 90240*a^13*b^12 + 43008*a^15*b^10 - 8192*a^17*b^8))/(2*b^24) - (a*((20*a^2*b^24 - 164*a^4*b^22 + 272*a^6*b^20 - 128*a^8*b^18)/b^23 + (tan(c/2 + (d*x)/2)*(384*a^3*b^24 - 1792*a^5*b^22 + 2432*a^7*b^20 - 1024*a^9*b^18))/(2*b^24) - (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^28 - 128*a^3*b^26))/(2*b^24))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b^9))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b^9))*(64*a^6 -

```
5*b^6 + 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b^9) - (a*(((25*a^4*b^20)/2 - 300
*a^6*b^18 + 2400*a^8*b^16 - 7520*a^10*b^14 + 11040*a^12*b^12 - 7680*a^14*b^
10 + 2048*a^16*b^8)/b^23 + (tan(c/2 + (d*x)/2)*(50*a^3*b^22 - 1801*a^5*b^20
 + 15576*a^7*b^18 - 54720*a^9*b^16 + 96320*a^11*b^14 - 90240*a^13*b^12 + 43
008*a^15*b^10 - 8192*a^17*b^8))/(2*b^24) + (a*((20*a^2*b^24 - 164*a^4*b^22
+ 272*a^6*b^20 - 128*a^8*b^18)/b^23 + (tan(c/2 + (d*x)/2)*(384*a^3*b^24 - 1
792*a^5*b^22 + 2432*a^7*b^20 - 1024*a^9*b^18))/(2*b^24) + (a*(32*a^2*b^3 +
(tan(c/2 + (d*x)/2)*(192*a*b^28 - 128*a^3*b^26))/(2*b^24))*(64*a^6 - 5*b^6
+ 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b^9))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120
*a^4*b^2)*1i)/(8*b^9))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b
^9)))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2))/(4*b^9*d) - (2*a^2*atanh
((75*a^6*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(75*a^6*b^3 - 422*a^8*b
 + (811*a^10)/b - (656*a^12)/b^3 + (192*a^14)/b^5 + 1622*a^9*tan(c/2 + (d*x
)/2) + 150*a^5*b^4*tan(c/2 + (d*x)/2) - 844*a^7*b^2*tan(c/2 + (d*x)/2) - (1
312*a^11*tan(c/2 + (d*x)/2))/b^2 + (384*a^13*tan(c/2 + (d*x)/2))/b^4) - (27
2*a^8*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(811*a^10*b + 75*a^6*b^5 -
 422*a^8*b^3 - (656*a^12)/b + (192*a^14)/b^3 - 1312*a^11*tan(c/2 + (d*x)/2)
 + 150*a^5*b^6*tan(c/2 + (d*x)/2) - 844*a^7*b^4*tan(c/2 + (d*x)/2) + 1622*a
^9*b^2*tan(c/2 + (d*x)/2) + (384*a^13*tan(c/2 + (d*x)/2))/b^2) + (192*a^10*
(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(75*a^6*b^7 - 656*a^12*b - 422*a
^8*b^5 + 811*a^10*b^3 + (192*a^14)/b + 384*a^13*tan(c/2 + (d*x)/2) + 150*a^
5*b^8*tan(c/2 + (d*x)/2) - 844*a^7*b^6*tan(c/2 + (d*x)/2) + 1622*a^9*b^4*ta
n(c/2 + (d*x)/2) - 1312*a^11*b^2*tan(c/2 + (d*x)/2)) + (150*a^5*tan(c/2 + (
d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(75*a^6*b^2 - 422*a^8 +
(811*a^10)/b^2 - (656*a^12)/b^4 + (192*a^14)/b^6 - 844*a^7*b*tan(c/2 + (d*x
)/2) + 150*a^5*b^3*tan(c/2 + (d*x)/2) + (1622*a^9*tan(c/2 + (d*x)/2))/b - (
1312*a^11*tan(c/2 + (d*x)/2))/b^3 + (384*a^13*tan(c/2 + (d*x)/2))/b^5) - (6
19*a^7*tan(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(811*a
^10 + 75*a^6*b^4 - 422*a^8*b^2 - (656*a^12)/b^2 + (192*a^14)/b^4 + 1622*a^9
*b*tan(c/2 + (d*x)/2) + 150*a^5*b^5*tan(c/2 + (d*x)/2) - 844*a^7*b^3*tan(c/
2 + (d*x)/2) - (1312*a^11*tan(c/2 + (d*x)/2))/b + (384*a^13*tan(c/2 + (d*x)
/2))/b^3) + (656*a^9*tan(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)
^(1/2))/(75*a^6*b^6 - 656*a^12 - 422*a^8*b^4 + 811*a^10*b^2 + (192*a^14)/b^
2 - 1312*a^11*b*tan(c/2 + (d*x)/2) + 150*a^5*b^7*tan(c/2 + (d*x)/2) - 844*a
^7*b^5*tan(c/2 + (d*x)/2) + 1622*a^9*b^3*tan(c/2 + (d*x)/2) + (384*a^13*tan
(c/2 + (d*x)/2))/b) - (192*a^11*tan(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 +
 3*a^4*b^2)^(1/2))/(192*a^14 + 75*a^6*b^8 - 422*a^8*b^6 + 811*a^10*b^4 - 65
6*a^12*b^2 + 384*a^13*b*tan(c/2 + (d*x)/2) + 150*a^5*b^9*tan(c/2 + (d*x)/2)
 - 844*a^7*b^7*tan(c/2 + (d*x)/2) + 1622*a^9*b^5*tan(c/2 + (d*x)/2) - 1312*
a^11*b^3*tan(c/2 + (d*x)/2)))*(8*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))
/(b^9*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1257 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=471

$$\frac{2a\left(7a^2-2b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^8d}-\frac{b\sin^4(c+dx)\cos(c+dx)}{6a^2d(a+b\sin(c+dx))}+\frac{\left(42a^4-61a^2b^2+16b^4\right)\sin^3(c+}{24a^2b^4d}$$

```
[Out] -1/16*(112*a^6-200*a^4*b^2+90*a^2*b^4-5*b^6)*x/b^8+2*a*(7*a^2-2*b^2)*(a^2-b
^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^8/d-1/15*a*(10
5*a^4-170*a^2*b^2+61*b^4)*cos(d*x+c)/b^7/d+1/16*(56*a^4-86*a^2*b^2+27*b^4)*
cos(d*x+c)*sin(d*x+c)/b^6/d-1/15*(35*a^4-52*a^2*b^2+15*b^4)*cos(d*x+c)*sin(
d*x+c)^2/a/b^5/d+1/24*(42*a^4-61*a^2*b^2+16*b^4)*cos(d*x+c)*sin(d*x+c)^3/a^
2/b^4/d+1/3*cos(d*x+c)*sin(d*x+c)^3/a/d/(a+b*sin(d*x+c))-1/6*b*cos(d*x+c)*s
in(d*x+c)^4/a^2/d/(a+b*sin(d*x+c))-1/10*(14*a^4-20*a^2*b^2+5*b^4)*cos(d*x+c
)*sin(d*x+c)^4/a^2/b^3/d/(a+b*sin(d*x+c))-7/30*a*cos(d*x+c)*sin(d*x+c)^5/b^
2/d/(a+b*sin(d*x+c))+1/6*cos(d*x+c)*sin(d*x+c)^6/b/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 1.55, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2896, 3047, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{a\left(-170a^2b^2+105a^4+61b^4\right)\cos(c+dx)}{15b^7d}+\frac{2a\left(7a^2-2b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^8d}-\frac{\left(-20a^2b^2+14a}{10a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((112*a^6 - 200*a^4*b^2 + 90*a^2*b^4 - 5*b^6)*x)/(16*b^8) + (2*a*(7*a^2 -
2*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(b^8*d) - (a*(105*a^4 - 170*a^2*b^2 + 61*b^4)*Cos[c + d*x])/(15*b^7*d) + ((
56*a^4 - 86*a^2*b^2 + 27*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*b^6*d) - ((35*
a^4 - 52*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(15*a*b^5*d) + ((42
*a^4 - 61*a^2*b^2 + 16*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/(24*a^2*b^4*d) + (
Cos[c + d*x]*Sin[c + d*x]^3)/(3*a*d*(a + b*Sin[c + d*x])) - (b*Cos[c + d*x]
*Sin[c + d*x]^4)/(6*a^2*d*(a + b*Sin[c + d*x])) - ((14*a^4 - 20*a^2*b^2 + 5
*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(10*a^2*b^3*d*(a + b*Sin[c + d*x])) - (7
*a*Cos[c + d*x]*Sin[c + d*x]^5)/(30*b^2*d*(a + b*Sin[c + d*x])) + (Cos[c +
d*x]*Sin[c + d*x]^6)/(6*b*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))} - \frac{b\cos(c+dx)\sin^4(c+dx)}{6a^2d(a+b\sin(c+dx))} - \frac{7a\cos(c+dx)\sin^5(c+}{30b^2d(a+b\sin(c+}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))} - \frac{b\cos(c+dx)\sin^4(c+dx)}{6a^2d(a+b\sin(c+dx))} - \frac{\left(14a^4-20a^2b^2+5b^4\right)}{10a^2b^3d(a+}$$

$$= \frac{\left(42a^4-61a^2b^2+16b^4\right)\cos(c+dx)\sin^3(c+dx)}{24a^2b^4d} + \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))} -$$

$$= -\frac{\left(35a^4-52a^2b^2+15b^4\right)\cos(c+dx)\sin^2(c+dx)}{15ab^5d} + \frac{\left(42a^4-61a^2b^2+16b^4\right)\text{co}}{24a^2b}$$

$$= \frac{\left(56a^4-86a^2b^2+27b^4\right)\cos(c+dx)\sin(c+dx)}{16b^6d} - \frac{\left(35a^4-52a^2b^2+15b^4\right)\cos(}{15ab^5d}$$

$$= -\frac{a\left(105a^4-170a^2b^2+61b^4\right)\cos(c+dx)}{15b^7d} + \frac{\left(56a^4-86a^2b^2+27b^4\right)\cos(c+dx}{16b^6d}$$

$$= -\frac{\left(112a^6-200a^4b^2+90a^2b^4-5b^6\right)x}{16b^8} - \frac{a\left(105a^4-170a^2b^2+61b^4\right)\cos(c+dx}{15b^7d}$$

$$= -\frac{\left(112a^6-200a^4b^2+90a^2b^4-5b^6\right)x}{16b^8} - \frac{a\left(105a^4-170a^2b^2+61b^4\right)\cos(c+dx}{15b^7d}$$

$$= -\frac{\left(112a^6-200a^4b^2+90a^2b^4-5b^6\right)x}{16b^8} - \frac{a\left(105a^4-170a^2b^2+61b^4\right)\cos(c+dx}{15b^7d}$$

$$= -\frac{\left(112a^6-200a^4b^2+90a^2b^4-5b^6\right)x}{16b^8} + \frac{2a\left(7a^2-2b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\,\mathrm{t}}{}\right.}{b^8d}$$

Mathematica [A] time = 8.17, size = 462, normalized size = 0.98

$$\frac{3840a\left(7a^2-2b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) - \frac{13440a^7c+13440a^7dx+13440a^6bc\sin(c+dx)+13440a^6bdx\sin(c+dx)+3360a^5}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3840*a*(7*a^2 - 2*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]] - (13440*a^7*c - 24000*a^5*b^2*c + 10800*a^3*b^4*c - 600*a*
b^6*c + 13440*a^7*d*x - 24000*a^5*b^2*d*x + 10800*a^3*b^4*d*x - 600*a*b^6*d
*x + 15*b*(896*a^6 - 1488*a^4*b^2 + 576*a^2*b^4 - 15*b^6)*Cos[c + d*x] + 10
*(56*a^4*b^3 - 79*a^2*b^5 + 18*b^7)*Cos[3*(c + d*x)] - 42*a^2*b^5*Cos[5*(c
+ d*x)] + 40*b^7*Cos[5*(c + d*x)] + 5*b^7*Cos[7*(c + d*x)] + 13440*a^6*b*c*
Sin[c + d*x] - 24000*a^4*b^3*c*Sin[c + d*x] + 10800*a^2*b^5*c*Sin[c + d*x]
- 600*b^7*c*Sin[c + d*x] + 13440*a^6*b*d*x*Sin[c + d*x] - 24000*a^4*b^3*d*x
*Sin[c + d*x] + 10800*a^2*b^5*d*x*Sin[c + d*x] - 600*b^7*d*x*Sin[c + d*x] +
 3360*a^5*b^2*Sin[2*(c + d*x)] - 5440*a^3*b^4*Sin[2*(c + d*x)] + 1910*a*b^6
*Sin[2*(c + d*x)] - 140*a^3*b^4*Sin[4*(c + d*x)] + 166*a*b^6*Sin[4*(c + d*x
)] + 14*a*b^6*Sin[6*(c + d*x)])/(a + b*Sin[c + d*x]))/(1920*b^8*d)
```

fricas [A] time = 1.01, size = 814, normalized size = 1.73

$$\left[-\frac{40\,b^7\cos\left(dx+c\right)^7-2\left(42\,a^2b^5-5\,b^7\right)\cos\left(dx+c\right)^5+5\left(56\,a^4b^3-58\,a^2b^5+5\,b^7\right)\cos\left(dx+c\right)^3+15\left(112\,a^7\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/240*(40*b^7*cos(d*x + c)^7 - 2*(42*a^2*b^5 - 5*b^7)*cos(d*x + c)^5 + 5*
(56*a^4*b^3 - 58*a^2*b^5 + 5*b^7)*cos(d*x + c)^3 + 15*(112*a^7 - 200*a^5*b^
2 + 90*a^3*b^4 - 5*a*b^6)*d*x - 120*(7*a^6 - 9*a^4*b^2 + 2*a^2*b^4 + (7*a^5
*b - 9*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2
)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d
*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2)) + 15*(112*a^6*b - 200*a^4*b^3 + 90*a^2*b^5 - 5*b^7)
*cos(d*x + c) + (56*a*b^6*cos(d*x + c)^5 - 10*(14*a^3*b^4 - 11*a*b^6)*cos(d
*x + c)^3 + 15*(112*a^6*b - 200*a^4*b^3 + 90*a^2*b^5 - 5*b^7)*d*x + 15*(56*
a^5*b^2 - 86*a^3*b^4 + 27*a*b^6)*cos(d*x + c))*sin(d*x + c))/(b^9*d*sin(d*x
 + c) + a*b^8*d), -1/240*(40*b^7*cos(d*x + c)^7 - 2*(42*a^2*b^5 - 5*b^7)*co
s(d*x + c)^5 + 5*(56*a^4*b^3 - 58*a^2*b^5 + 5*b^7)*cos(d*x + c)^3 + 15*(112
*a^7 - 200*a^5*b^2 + 90*a^3*b^4 - 5*a*b^6)*d*x + 240*(7*a^6 - 9*a^4*b^2 + 2
*a^2*b^4 + (7*a^5*b - 9*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*ar
ctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 15*(112*a^6*b
- 200*a^4*b^3 + 90*a^2*b^5 - 5*b^7)*cos(d*x + c) + (56*a*b^6*cos(d*x + c)^5
 - 10*(14*a^3*b^4 - 11*a*b^6)*cos(d*x + c)^3 + 15*(112*a^6*b - 200*a^4*b^3
+ 90*a^2*b^5 - 5*b^7)*d*x + 15*(56*a^5*b^2 - 86*a^3*b^4 + 27*a*b^6)*cos(d*x
 + c))*sin(d*x + c))/(b^9*d*sin(d*x + c) + a*b^8*d)]
```

giac [A] time = 0.24, size = 835, normalized size = 1.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/240*(15*(112*a^6 - 200*a^4*b^2 + 90*a^2*b^4 - 5*b^6)*(d*x + c)/b^8 - 480
*(7*a^7 - 16*a^5*b^2 + 11*a^3*b^4 - 2*a*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1
/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^
2 - b^2)*b^8) + 480*(a^5*b*tan(1/2*d*x + 1/2*c) - 2*a^3*b^3*tan(1/2*d*x + 1
/2*c) + a*b^5*tan(1/2*d*x + 1/2*c) + a^6 - 2*a^4*b^2 + a^2*b^4)/((a*tan(1/2
*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^7) + 2*(600*a^4*b*tan(1/2
*d*x + 1/2*c)^11 - 810*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 165*b^5*tan(1/2*d*
x + 1/2*c)^11 + 1440*a^5*tan(1/2*d*x + 1/2*c)^10 - 2880*a^3*b^2*tan(1/2*d*x
 + 1/2*c)^10 + 1440*a*b^4*tan(1/2*d*x + 1/2*c)^10 + 1800*a^4*b*tan(1/2*d*x
+ 1/2*c)^9 - 1710*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 - 25*b^5*tan(1/2*d*x + 1/2
*c)^9 + 7200*a^5*tan(1/2*d*x + 1/2*c)^8 - 12480*a^3*b^2*tan(1/2*d*x + 1/2*c
)^8 + 4320*a*b^4*tan(1/2*d*x + 1/2*c)^8 + 1200*a^4*b*tan(1/2*d*x + 1/2*c)^7
 - 900*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 450*b^5*tan(1/2*d*x + 1/2*c)^7 + 14
400*a^5*tan(1/2*d*x + 1/2*c)^6 - 22400*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 + 736
0*a*b^4*tan(1/2*d*x + 1/2*c)^6 - 1200*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 900*a^
2*b^3*tan(1/2*d*x + 1/2*c)^5 - 450*b^5*tan(1/2*d*x + 1/2*c)^5 + 14400*a^5*t
an(1/2*d*x + 1/2*c)^4 - 21120*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 6720*a*b^4*t
an(1/2*d*x + 1/2*c)^4 - 1800*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 1710*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^3 + 25*b^5*tan(1/2*d*x + 1/2*c)^3 + 7200*a^5*tan(1/2*d*x
 + 1/2*c)^2 - 10560*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 2976*a*b^4*tan(1/2*d*x
 + 1/2*c)^2 - 600*a^4*b*tan(1/2*d*x + 1/2*c) + 810*a^2*b^3*tan(1/2*d*x + 1/
2*c) - 165*b^5*tan(1/2*d*x + 1/2*c) + 1440*a^5 - 2240*a^3*b^2 + 736*a*b^4)/
((tan(1/2*d*x + 1/2*c)^2 + 1)^6*b^7))/d
```

maple [B] time = 0.54, size = 1817, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*a^4-12/d/b^7/(1+ta
n(1/2*d*x+1/2*c)^2)^6*a^5-11/8/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x
+1/2*c)^11+5/24/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9-15/4/
d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7+15/4/d/b^2/(1+tan(1/2
*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5-5/24/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6
*tan(1/2*d*x+1/2*c)^3+11/8/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2
```

```
*c)+56/3/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*a^3-92/15/d/b^3/(1+tan(1/2*d*x+1/
2*c)^2)^6*a-14/d/b^8*arctan(tan(1/2*d*x+1/2*c))*a^6+25/d/b^6*arctan(tan(1/2
*d*x+1/2*c))*a^4-45/4/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^2+4/d/b^5/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^4-2/d/b^3/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^2-2/d*a^6/b^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(
1/2*d*x+1/2*c)*b+a)+5/8/d/b^2*arctan(tan(1/2*d*x+1/2*c))-60/d/b^7/(1+tan(1/
2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8*a^5+104/d/b^5/(1+tan(1/2*d*x+1/2*c)^
2)^6*tan(1/2*d*x+1/2*c)^8*a^3-36/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d
*x+1/2*c)^8*a-5/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*a^4-
120/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a^5+176/d/b^5/(1+
tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a^3-56/d/b^3/(1+tan(1/2*d*x+1/
2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a-32/d/b^6*a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2
*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+22/d/b^4*a^3/(a^2-b^2)^(1/2)*ar
ctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-4/d/b^2*a/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d/b^4/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^3*tan(1/2*d*x+1/2*c)+14/d*a
^7/b^8/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1
/2))+27/4/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*a^2-15/d/b
^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*a^4+57/4/d/b^4/(1+tan(1/
2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*a^2-10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)^7*a^4+15/2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*
d*x+1/2*c)^7*a^2-120/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*
a^5+560/3/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a^3-184/3/d
/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a-15/2/d/b^4/(1+tan(1/
2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*a^2+15/d/b^6/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)^3*a^4-57/4/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*
d*x+1/2*c)^3*a^2-60/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a
^5+88/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a^3-124/5/d/b^3
/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a+5/d/b^6/(1+tan(1/2*d*x+1
/2*c)^2)^6*tan(1/2*d*x+1/2*c)*a^4-27/4/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan
(1/2*d*x+1/2*c)*a^2-12/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^
10*a^5+24/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a^3-12/d/b
^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a-2/d*a^5/b^6/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-2/d/b^2/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a*tan(1/2*d*x+1/2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
```

```
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 16.38, size = 4067, normalized size = 8.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] - ((2*(105*a^6 + 61*a^2*b^4 - 170*a^4*b^2))/(15*b^7) + (tan(c/2 + (d*x)/2)^
13*(27*a*b^4 + 56*a^5 - 86*a^3*b^2))/(8*b^6) + (8*tan(c/2 + (d*x)/2)^7*(61*
a*b^4 + 105*a^5 - 170*a^3*b^2))/(3*b^6) + (tan(c/2 + (d*x)/2)^11*(223*a*b^4
 + 336*a^5 - 558*a^3*b^2))/(6*b^6) + (tan(c/2 + (d*x)/2)^3*(1813*a*b^4 + 33
60*a^5 - 5370*a^3*b^2))/(30*b^6) + (tan(c/2 + (d*x)/2)^9*(2533*a*b^4 + 4200
*a^5 - 6954*a^3*b^2))/(24*b^6) + (tan(c/2 + (d*x)/2)^5*(3323*a*b^4 + 5880*a
^5 - 9366*a^3*b^2))/(24*b^6) + (tan(c/2 + (d*x)/2)^12*(56*a^6 + 11*b^6 + 2*
a^2*b^4 - 72*a^4*b^2))/(4*b^7) + (tan(c/2 + (d*x)/2)^10*(1008*a^6 - 5*b^6 +
 378*a^2*b^4 - 1464*a^4*b^2))/(12*b^7) + (tan(c/2 + (d*x)/2)^8*(1260*a^6 +
45*b^6 + 674*a^2*b^4 - 1984*a^4*b^2))/(6*b^7) + (tan(c/2 + (d*x)/2)^6*(1680
*a^6 - 45*b^6 + 1034*a^2*b^4 - 2776*a^4*b^2))/(6*b^7) + (tan(c/2 + (d*x)/2)
^2*(5040*a^6 - 165*b^6 + 3386*a^2*b^4 - 8440*a^4*b^2))/(60*b^7) + (tan(c/2
+ (d*x)/2)^4*(12600*a^6 + 25*b^6 + 8358*a^2*b^4 - 21240*a^4*b^2))/(60*b^7)
+ (tan(c/2 + (d*x)/2)*(1547*a*b^4 + 2520*a^5 - 4150*a^3*b^2))/(120*b^6))/(d
*(a + 2*b*tan(c/2 + (d*x)/2) + 7*a*tan(c/2 + (d*x)/2)^2 + 21*a*tan(c/2 + (d
*x)/2)^4 + 35*a*tan(c/2 + (d*x)/2)^6 + 35*a*tan(c/2 + (d*x)/2)^8 + 21*a*tan
(c/2 + (d*x)/2)^10 + 7*a*tan(c/2 + (d*x)/2)^12 + a*tan(c/2 + (d*x)/2)^14 +
12*b*tan(c/2 + (d*x)/2)^3 + 30*b*tan(c/2 + (d*x)/2)^5 + 40*b*tan(c/2 + (d*x
)/2)^7 + 30*b*tan(c/2 + (d*x)/2)^9 + 12*b*tan(c/2 + (d*x)/2)^11 + 2*b*tan(c
/2 + (d*x)/2)^13)) - (atan((((((25*a^2*b^19)/8 - (225*a^4*b^17)/2 + (2525*a
^6*b^15)/2 - 4640*a^8*b^13 + 7520*a^10*b^11 - 5600*a^12*b^9 + 1568*a^14*b^7
)/b^20 - (((10*a*b^22 - 126*a^3*b^20 + 228*a^5*b^18 - 112*a^7*b^16)/b^20 -
((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21))*(
a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8) + (tan(c/2 + (d*x
)/2)*(1024*a^2*b^22 - 5632*a^4*b^20 + 8192*a^6*b^18 - 3584*a^8*b^16))/(8*b^
21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8) + (tan(c/2
+ (d*x)/2)*(50*a*b^21 - 2849*a^3*b^19 + 32364*a^5*b^17 - 131700*a^7*b^15 +
254720*a^9*b^13 - 254720*a^11*b^11 + 127232*a^13*b^9 - 25088*a^15*b^7))/(8*
b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i)*1i)/(16*b^8) + ((((
25*a^2*b^19)/8 - (225*a^4*b^17)/2 + (2525*a^6*b^15)/2 - 4640*a^8*b^13 + 752
0*a^10*b^11 - 5600*a^12*b^9 + 1568*a^14*b^7)/b^20 + (((10*a*b^22 - 126*a^3*
b^20 + 228*a^5*b^18 - 112*a^7*b^16)/b^20 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/
2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i
- a^4*b^2*200i))/(16*b^8) + (tan(c/2 + (d*x)/2)*(1024*a^2*b^22 - 5632*a^4*b
```

^20 + 8192*a^6*b^18 - 3584*a^8*b^16))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8) + (tan(c/2 + (d*x)/2)*(50*a*b^21 - 2849*a^3*b^19 + 32364*a^5*b^17 - 131700*a^7*b^15 + 254720*a^9*b^13 - 254720*a^11*b^11 + 127232*a^13*b^9 - 25088*a^15*b^7))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i)*1i)/(16*b^8))/((10976*a^19 + (135*a^3*b^16)/2 - (7205*a^5*b^14)/4 + 15115*a^7*b^12 - (244853*a^9*b^10)/4 + 138577*a^11*b^8 - 184965*a^13*b^6 + 144788*a^15*b^4 - 61544*a^17*b^2)/b^20 + (tan(c/2 + (d*x)/2)*(175616*a^20 - 100*a^2*b^18 + 4150*a^4*b^16 - 61000*a^6*b^14 + 399830*a^8*b^12 - 1393080*a^10*b^10 + 2831960*a^12*b^8 - 3480576*a^14*b^6 + 2551808*a^16*b^4 - 1028608*a^18*b^2))/(4*b^21) + ((((25*a^2*b^19)/8 - (225*a^4*b^17)/2 + (2525*a^6*b^15)/2 - 4640*a^8*b^13 + 7520*a^10*b^11 - 5600*a^12*b^9 + 1568*a^14*b^7)/b^20 - (((10*a*b^22 - 126*a^3*b^20 + 228*a^5*b^18 - 112*a^7*b^16)/b^20 - ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8) + (tan(c/2 + (d*x)/2)*(1024*a^2*b^22 - 5632*a^4*b^20 + 8192*a^6*b^18 - 3584*a^8*b^16))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8) + (tan(c/2 + (d*x)/2)*(50*a*b^21 - 2849*a^3*b^19 + 32364*a^5*b^17 - 131700*a^7*b^15 + 254720*a^9*b^13 - 254720*a^11*b^11 + 127232*a^13*b^9 - 25088*a^15*b^7))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8) - ((((25*a^2*b^19)/8 - (225*a^4*b^17)/2 + (2525*a^6*b^15)/2 - 4640*a^8*b^13 + 7520*a^10*b^11 - 5600*a^12*b^9 + 1568*a^14*b^7)/b^20 + (((10*a*b^22 - 126*a^3*b^20 + 228*a^5*b^18 - 112*a^7*b^16)/b^20 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8) + (tan(c/2 + (d*x)/2)*(1024*a^2*b^22 - 5632*a^4*b^20 + 8192*a^6*b^18 - 3584*a^8*b^16))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8) + (tan(c/2 + (d*x)/2)*(50*a*b^21 - 2849*a^3*b^19 + 32364*a^5*b^17 - 131700*a^7*b^15 + 254720*a^9*b^13 - 254720*a^11*b^11 + 127232*a^13*b^9 - 25088*a^15*b^7))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8)))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i)*1i)/(8*b^8*d) - (a*atan(((a*(7*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(((25*a^2*b^19)/8 - (225*a^4*b^17)/2 + (2525*a^6*b^15)/2 - 4640*a^8*b^13 + 7520*a^10*b^11 - 5600*a^12*b^9 + 1568*a^14*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a*b^21 - 2849*a^3*b^19 + 32364*a^5*b^17 - 131700*a^7*b^15 + 254720*a^9*b^13 - 254720*a^11*b^11 + 127232*a^13*b^9 - 25088*a^15*b^7))/(8*b^21) + (a*(7*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((10*a*b^22 - 126*a^3*b^20 + 228*a^5*b^18 - 112*a^7*b^16)/b^20 + (tan(c/2 + (d*x)/2)*(1024*a^2*b^22 - 5632*a^4*b^20 + 8192*a^6*b^18 - 3584*a^8*b^16))/(8*b^21) + (a*(7*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21)))/b^8))/b^8)*1i)/b^8 + (a*(7*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(((25*a^2*b^19)/8 - (225*a^4*b^17)/2 + (2525*a^6*b^15)/2 - 4640*a^8*b^13 + 7520*a^10*b^11 - 5600*a^12*b^9 + 1568*a^14*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a*b^21 - 2849*a^3*b^19 + 32364*a^5*b^17 - 131700*a^7*b^15 + 254720*a^9*b^13 - 254720*a^11*b^11 + 127232*a^13*b^9 - 25088*a^15*b^7))/(8*b^21) - (a*(7*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((10*a*b^22 - 126*a^3*b^20 + 228*a^5*b^18 - 112*a^7*b^16)/b^20 +

```
(tan(c/2 + (d*x)/2)*(1024*a^2*b^22 - 5632*a^4*b^20 + 8192*a^6*b^18 - 3584*a
^8*b^16))/(8*b^21) - (a*(7*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(32*a^
2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21)))/b^8))/b
^8)*1i)/b^8)/((10976*a^19 + (135*a^3*b^16)/2 - (7205*a^5*b^14)/4 + 15115*a^
7*b^12 - (244853*a^9*b^10)/4 + 138577*a^11*b^8 - 184965*a^13*b^6 + 144788*a
^15*b^4 - 61544*a^17*b^2)/b^20 + (tan(c/2 + (d*x)/2)*(175616*a^20 - 100*a^2
*b^18 + 4150*a^4*b^16 - 61000*a^6*b^14 + 399830*a^8*b^12 - 1393080*a^10*b^1
0 + 2831960*a^12*b^8 - 3480576*a^14*b^6 + 2551808*a^16*b^4 - 1028608*a^18*b
^2))/(4*b^21) - (a*(7*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(((25*a^2*b
^19)/8 - (225*a^4*b^17)/2 + (2525*a^6*b^15)/2 - 4640*a^8*b^13 + 7520*a^10*b
^11 - 5600*a^12*b^9 + 1568*a^14*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a*b^21
- 2849*a^3*b^19 + 32364*a^5*b^17 - 131700*a^7*b^15 + 254720*a^9*b^13 - 2547
20*a^11*b^11 + 127232*a^13*b^9 - 25088*a^15*b^7))/(8*b^21) + (a*(7*a^2 - 2*
b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((10*a*b^22 - 126*a^3*b^20 + 228*a^5*b^18
 - 112*a^7*b^16)/b^20 + (tan(c/2 + (d*x)/2)*(1024*a^2*b^22 - 5632*a^4*b^20
+ 8192*a^6*b^18 - 3584*a^8*b^16))/(8*b^21) + (a*(7*a^2 - 2*b^2)*(-(a + b)^3
*(a - b)^3)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b
^23))/(8*b^21)))/b^8))/b^8))/b^8 + (a*(7*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3
)^(1/2)*(((25*a^2*b^19)/8 - (225*a^4*b^17)/2 + (2525*a^6*b^15)/2 - 4640*a^8
*b^13 + 7520*a^10*b^11 - 5600*a^12*b^9 + 1568*a^14*b^7)/b^20 + (tan(c/2 + (
d*x)/2)*(50*a*b^21 - 2849*a^3*b^19 + 32364*a^5*b^17 - 131700*a^7*b^15 + 254
720*a^9*b^13 - 254720*a^11*b^11 + 127232*a^13*b^9 - 25088*a^15*b^7))/(8*b^2
1) - (a*(7*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((10*a*b^22 - 126*a^3*
b^20 + 228*a^5*b^18 - 112*a^7*b^16)/b^20 + (tan(c/2 + (d*x)/2)*(1024*a^2*b^
22 - 5632*a^4*b^20 + 8192*a^6*b^18 - 3584*a^8*b^16))/(8*b^21) - (a*(7*a^2 -
 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768
*a*b^25 - 512*a^3*b^23))/(8*b^21)))/b^8))/b^8))/b^8))*(7*a^2 - 2*b^2)*(-(a
+ b)^3*(a - b)^3)^(1/2)*2i)/(b^8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1258 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=231

$$-\frac{2\left(a^2-b^2\right)^{3/2}\left(6a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d}-\frac{\cos^3(c+dx)\left(2\left(6a^2-b^2\right)-9ab\sin(c+dx)\right)}{6b^4d}+\frac{ax\left(24a^4-40a\right.}{4b^7}$$

```
[Out] 1/4*a*(24*a^4-40*a^2*b^2+15*b^4)*x/b^7-2*(a^2-b^2)^(3/2)*(6*a^2-b^2)*arctan
((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^7/d+1/5*cos(d*x+c)^5*(6*a+b*si
n(d*x+c))/b^2/d/(a+b*sin(d*x+c))-1/6*cos(d*x+c)^3*(12*a^2-2*b^2-9*a*b*sin(d
*x+c))/b^4/d+1/4*cos(d*x+c)*(24*a^4-28*a^2*b^2+4*b^4-a*b*(12*a^2-11*b^2)*si
n(d*x+c))/b^6/d
```

Rubi [A] time = 0.51, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2863, 2865, 2735, 2660, 618, 204}

$$-\frac{2\left(a^2-b^2\right)^{3/2}\left(6a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d}-\frac{\cos^3(c+dx)\left(2\left(6a^2-b^2\right)-9ab\sin(c+dx)\right)}{6b^4d}+\cos(c+dx)\left(4\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*(24*a^4 - 40*a^2*b^2 + 15*b^4)*x)/(4*b^7) - (2*(a^2 - b^2)^(3/2)*(6*a^2
- b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*d) + (Cos[c +
 d*x]^5*(6*a + b*Sin[c + d*x]))/(5*b^2*d*(a + b*Sin[c + d*x])) - (Cos[c + d
*x]^3*(2*(6*a^2 - b^2) - 9*a*b*Sin[c + d*x]))/(6*b^4*d) + (Cos[c + d*x]*(4*
(6*a^4 - 7*a^2*b^2 + b^4) - a*b*(12*a^2 - 11*b^2)*Sin[c + d*x]))/(4*b^6*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\int \frac{\cos^4(c+dx)(-b-6a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+dx)\left(2\left(6a^2-b^2\right)-9ab\sin(c+dx\right.}{6b^4d}$$

$$= \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+dx)\left(2\left(6a^2-b^2\right)-9ab\sin(c+dx\right.}{6b^4d}$$

$$= \frac{a\left(24a^4-40a^2b^2+15b^4\right)x}{4b^7} + \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+dx)}{}$$

$$= \frac{a\left(24a^4-40a^2b^2+15b^4\right)x}{4b^7} + \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+dx)}{}$$

$$= \frac{a\left(24a^4-40a^2b^2+15b^4\right)x}{4b^7} + \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+dx)}{}$$

$$= \frac{a\left(24a^4-40a^2b^2+15b^4\right)x}{4b^7} - \frac{2\left(a^2-b^2\right)^{3/2}\left(6a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7d}$$

Mathematica [A] time = 4.35, size = 371, normalized size = 1.61

$$\frac{2880a^6c+2880a^6dx+2880a^5bc\sin(c+dx)+2880a^5bdx\sin(c+dx)+720a^4b^2\sin(2(c+dx))-4800a^4b^2c-4800a^4b^2dx-4800a^3b^3c\sin(c+dx)-4800a^3b^3dx\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-960*(a^2 - b^2)^(3/2)*(6*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[
a^2 - b^2]] + (2880*a^6*c - 4800*a^4*b^2*c + 1800*a^2*b^4*c + 2880*a^6*d*x
- 4800*a^4*b^2*d*x + 1800*a^2*b^4*d*x + 60*a*b*(48*a^4 - 74*a^2*b^2 + 23*b^
4)*Cos[c + d*x] + 5*(24*a^3*b^3 - 31*a*b^5)*Cos[3*(c + d*x)] - 9*a*b^5*Cos[
5*(c + d*x)] + 2880*a^5*b*c*Sin[c + d*x] - 4800*a^3*b^3*c*Sin[c + d*x] + 18
00*a*b^5*c*Sin[c + d*x] + 2880*a^5*b*d*x*Sin[c + d*x] - 4800*a^3*b^3*d*x*Si
n[c + d*x] + 1800*a*b^5*d*x*Sin[c + d*x] + 720*a^4*b^2*Sin[2*(c + d*x)] - 1
080*a^2*b^4*Sin[2*(c + d*x)] + 295*b^6*Sin[2*(c + d*x)] - 30*a^2*b^4*Sin[4*
```

```
(c + d*x)] + 32*b^6*Sin[4*(c + d*x)] + 3*b^6*Sin[6*(c + d*x)])/(a + b*Sin[c
 + d*x]))/(480*b^7*d)
```

fricas [A] time = 1.24, size = 697, normalized size = 3.02

$$\left[-\frac{18\,ab^5\cos\left(dx+c\right)^5-5\left(12\,a^3b^3-11\,ab^5\right)\cos\left(dx+c\right)^3-15\left(24\,a^6-40\,a^4b^2+15\,a^2b^4\right)dx-30\left(6\,a^5-7\,a^3b^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(18*a*b^5*cos(d*x + c)^5 - 5*(12*a^3*b^3 - 11*a*b^5)*cos(d*x + c)^3
- 15*(24*a^6 - 40*a^4*b^2 + 15*a^2*b^4)*d*x - 30*(6*a^5 - 7*a^3*b^2 + a*b^4
 + (6*a^4*b - 7*a^2*b^3 + b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 -
 b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*s
in(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2)) - 15*(24*a^5*b - 40*a^3*b^3 + 15*a*b^5)*cos(d*x
 + c) - (12*b^6*cos(d*x + c)^5 - 10*(3*a^2*b^4 - 2*b^6)*cos(d*x + c)^3 + 15
*(24*a^5*b - 40*a^3*b^3 + 15*a*b^5)*d*x + 15*(12*a^4*b^2 - 17*a^2*b^4 + 4*b
^6)*cos(d*x + c))*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d), -1/60*(18*a
*b^5*cos(d*x + c)^5 - 5*(12*a^3*b^3 - 11*a*b^5)*cos(d*x + c)^3 - 15*(24*a^6
 - 40*a^4*b^2 + 15*a^2*b^4)*d*x - 60*(6*a^5 - 7*a^3*b^2 + a*b^4 + (6*a^4*b
- 7*a^2*b^3 + b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 15*(24*a^5*b - 40*a^3*b^3 + 15*a*b^5)*
cos(d*x + c) - (12*b^6*cos(d*x + c)^5 - 10*(3*a^2*b^4 - 2*b^6)*cos(d*x + c)
^3 + 15*(24*a^5*b - 40*a^3*b^3 + 15*a*b^5)*d*x + 15*(12*a^4*b^2 - 17*a^2*b^
4 + 4*b^6)*cos(d*x + c))*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d)]
```

giac [B] time = 0.23, size = 593, normalized size = 2.57

$$\frac{\frac{15\left(24\,a^5-40\,a^3b^2+15\,ab^4\right)(dx+c)}{b^7}-\frac{120\left(6\,a^6-13\,a^4b^2+8\,a^2b^4-b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^7}+\frac{120\left(a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^2b^3\right.}{\left(a\tan\left(\frac{1}{2}\,dx\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(15*(24*a^5 - 40*a^3*b^2 + 15*a*b^4)*(d*x + c)/b^7 - 120*(6*a^6 - 13*a
^4*b^2 + 8*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 120
```

```
*(a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) + b^5*tan(1/2
*d*x + 1/2*c) + a^5 - 2*a^3*b^2 + a*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*t
an(1/2*d*x + 1/2*c) + a)*b^6) + 2*(120*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 135*a
*b^3*tan(1/2*d*x + 1/2*c)^9 + 300*a^4*tan(1/2*d*x + 1/2*c)^8 - 540*a^2*b^2*
tan(1/2*d*x + 1/2*c)^8 + 180*b^4*tan(1/2*d*x + 1/2*c)^8 + 240*a^3*b*tan(1/2
*d*x + 1/2*c)^7 - 150*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 1200*a^4*tan(1/2*d*x +
 1/2*c)^6 - 1800*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 + 360*b^4*tan(1/2*d*x + 1/2
*c)^6 + 1800*a^4*tan(1/2*d*x + 1/2*c)^4 - 2400*a^2*b^2*tan(1/2*d*x + 1/2*c)
^4 + 560*b^4*tan(1/2*d*x + 1/2*c)^4 - 240*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 15
0*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 1200*a^4*tan(1/2*d*x + 1/2*c)^2 - 1560*a^2
*b^2*tan(1/2*d*x + 1/2*c)^2 + 280*b^4*tan(1/2*d*x + 1/2*c)^2 - 120*a^3*b*ta
n(1/2*d*x + 1/2*c) + 135*a*b^3*tan(1/2*d*x + 1/2*c) + 300*a^4 - 420*a^2*b^2
 + 92*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*b^6))/d
```

maple [B] time = 0.47, size = 1321, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a^3+9/2/d/b^3/(1+tan
(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a+4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^
5*tan(1/2*d*x+1/2*c)^9*a^3-20/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^3+15/2/d/b
^3*arctan(tan(1/2*d*x+1/2*c))*a+2/d*a^5/b^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1
/2*d*x+1/2*c)*b+a)-4/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)
*b+a)+12/d/b^7*arctan(tan(1/2*d*x+1/2*c))*a^5+2/d/b^2/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)*a+2/d/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1
/2*d*x+1/2*c)^8+12/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6+56
/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4+28/3/d/b^2/(1+tan(
1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2+10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^
5*a^4-14/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*a^2+2/d/b/(tan(1/2*d*x+1/2*c)^2*a
+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+46/15/d/b^2/(1+tan(1/2*d*x+1/
2*c)^2)^5-9/2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9*a+10/d/
b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8*a^4-18/d/b^4/(1+tan(1/2
*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8*a^2+8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^
5*tan(1/2*d*x+1/2*c)^7*a^3-5/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1
/2*c)^7*a+40/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6*a^4-60/d
/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6*a^2+60/d/b^6/(1+tan(1/
2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^4-80/d/b^4/(1+tan(1/2*d*x+1/2*c)^2
)^5*tan(1/2*d*x+1/2*c)^4*a^2-8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x
+1/2*c)^3*a^3+5/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a+40/
d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^4-52/d/b^4/(1+tan(1
/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^2+26/d*a^4/b^5/(a^2-b^2)^(1/2)*ar
```

```
ctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-16/d*a^2/b^3/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d*a^4/b
^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-4/d
*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*
c)-12/d*a^6/b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^
2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 14.53, size = 3135, normalized size = 13.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x))/(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((2*(38*a*b^4 + 90*a^5 - 135*a^3*b^2))/(15*b^6) - (tan(c/2 + (d*x)/2)^10*(a
*b^4 - 12*a^5 + 14*a^3*b^2))/b^6 + (2*tan(c/2 + (d*x)/2)^8*(9*a*b^4 + 30*a^
5 - 41*a^3*b^2))/b^6 + (2*tan(c/2 + (d*x)/2)^4*(29*a*b^4 + 60*a^5 - 94*a^3*
b^2))/b^6 + (4*tan(c/2 + (d*x)/2)^6*(38*a*b^4 + 90*a^5 - 135*a^3*b^2))/(3*b
^6) + (tan(c/2 + (d*x)/2)^2*(157*a*b^4 + 300*a^5 - 470*a^3*b^2))/(5*b^6) +
(tan(c/2 + (d*x)/2)*(540*a^4 + 244*b^4 - 825*a^2*b^2))/(30*b^5) + (tan(c/2
+ (d*x)/2)^11*(12*a^4 + 4*b^4 - 17*a^2*b^2))/(2*b^5) + (tan(c/2 + (d*x)/2)^
9*(84*a^4 + 44*b^4 - 131*a^2*b^2))/(2*b^5) + (tan(c/2 + (d*x)/2)^7*(108*a^4
 + 44*b^4 - 165*a^2*b^2))/b^5 + (tan(c/2 + (d*x)/2)^5*(396*a^4 + 172*b^4 -
585*a^2*b^2))/(3*b^5) + (tan(c/2 + (d*x)/2)^3*(468*a^4 + 172*b^4 - 687*a^2*
b^2))/(6*b^5))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 6*a*tan(c/2 + (d*x)/2)^2 +
15*a*tan(c/2 + (d*x)/2)^4 + 20*a*tan(c/2 + (d*x)/2)^6 + 15*a*tan(c/2 + (d*x
)/2)^8 + 6*a*tan(c/2 + (d*x)/2)^10 + a*tan(c/2 + (d*x)/2)^12 + 10*b*tan(c/2
 + (d*x)/2)^3 + 20*b*tan(c/2 + (d*x)/2)^5 + 20*b*tan(c/2 + (d*x)/2)^7 + 10*
b*tan(c/2 + (d*x)/2)^9 + 2*b*tan(c/2 + (d*x)/2)^11)) + (a*atan(((a*((2*(225
*a^4*b^14 - 1200*a^6*b^12 + 2320*a^8*b^10 - 1920*a^10*b^8 + 576*a^12*b^6))/
b^17 - (2*tan(c/2 + (d*x)/2)*(16*a*b^18 - 706*a^3*b^16 + 4065*a^5*b^14 - 93
60*a^7*b^12 + 10400*a^9*b^10 - 5568*a^11*b^8 + 1152*a^13*b^6))/b^18 + (a*(2
4*a^4 + 15*b^4 - 40*a^2*b^2)*((2*tan(c/2 + (d*x)/2)*(32*a*b^20 - 256*a^3*b^
```

18 + 416*a^5*b^16 - 192*a^7*b^14))/b^18 - (2*(44*a^2*b^18 - 92*a^4*b^16 + 4
8*a^6*b^14))/b^17 + (a*(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*b^22 - 32*
a^3*b^20))/b^18)*(24*a^4 + 15*b^4 - 40*a^2*b^2)*1i)/(4*b^7))*1i)/(4*b^7))*(
24*a^4 + 15*b^4 - 40*a^2*b^2))/(4*b^7) + (a*((2*(225*a^4*b^14 - 1200*a^6*b^
12 + 2320*a^8*b^10 - 1920*a^10*b^8 + 576*a^12*b^6))/b^17 - (2*tan(c/2 + (d*
x)/2)*(16*a*b^18 - 706*a^3*b^16 + 4065*a^5*b^14 - 9360*a^7*b^12 + 10400*a^9
*b^10 - 5568*a^11*b^8 + 1152*a^13*b^6))/b^18 + (a*(24*a^4 + 15*b^4 - 40*a^2
b^2)((2*(44*a^2*b^18 - 92*a^4*b^16 + 48*a^6*b^14))/b^17 - (2*tan(c/2 + (d
x)/2)(32*a*b^20 - 256*a^3*b^18 + 416*a^5*b^16 - 192*a^7*b^14))/b^18 + (a*
(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*b^22 - 32*a^3*b^20))/b^18)*(24*a^
4 + 15*b^4 - 40*a^2*b^2)*1i)/(4*b^7))*1i)/(4*b^7))*(24*a^4 + 15*b^4 - 40*a^
2*b^2))/(4*b^7))/((4*(1728*a^16 - 60*a^2*b^14 + 895*a^4*b^12 - 5056*a^6*b^1
0 + 14291*a^8*b^8 - 22310*a^10*b^6 + 19584*a^12*b^4 - 9072*a^14*b^2))/b^17
+ (4*tan(c/2 + (d*x)/2)*(6912*a^17 - 450*a^3*b^14 + 6000*a^5*b^12 - 29690*a
^7*b^10 + 74860*a^9*b^8 - 106592*a^11*b^6 + 86976*a^13*b^4 - 38016*a^15*b^2
))/b^18 - (a*((2*(225*a^4*b^14 - 1200*a^6*b^12 + 2320*a^8*b^10 - 1920*a^10*
b^8 + 576*a^12*b^6))/b^17 - (2*tan(c/2 + (d*x)/2)*(16*a*b^18 - 706*a^3*b^16
 + 4065*a^5*b^14 - 9360*a^7*b^12 + 10400*a^9*b^10 - 5568*a^11*b^8 + 1152*a^
13*b^6))/b^18 + (a*(24*a^4 + 15*b^4 - 40*a^2*b^2)*((2*tan(c/2 + (d*x)/2)*(3
2*a*b^20 - 256*a^3*b^18 + 416*a^5*b^16 - 192*a^7*b^14))/b^18 - (2*(44*a^2*b
^18 - 92*a^4*b^16 + 48*a^6*b^14))/b^17 + (a*(32*a^2*b^3 + (2*tan(c/2 + (d*x
)/2)*(48*a*b^22 - 32*a^3*b^20))/b^18)*(24*a^4 + 15*b^4 - 40*a^2*b^2)*1i)/(4
*b^7))*1i)/(4*b^7))*(24*a^4 + 15*b^4 - 40*a^2*b^2)*1i)/(4*b^7) + (a*((2*(22
5*a^4*b^14 - 1200*a^6*b^12 + 2320*a^8*b^10 - 1920*a^10*b^8 + 576*a^12*b^6))
/b^17 - (2*tan(c/2 + (d*x)/2)*(16*a*b^18 - 706*a^3*b^16 + 4065*a^5*b^14 - 9
360*a^7*b^12 + 10400*a^9*b^10 - 5568*a^11*b^8 + 1152*a^13*b^6))/b^18 + (a*(
24*a^4 + 15*b^4 - 40*a^2*b^2)*((2*(44*a^2*b^18 - 92*a^4*b^16 + 48*a^6*b^14)
)/b^17 - (2*tan(c/2 + (d*x)/2)*(32*a*b^20 - 256*a^3*b^18 + 416*a^5*b^16 - 1
92*a^7*b^14))/b^18 + (a*(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*b^22 - 32
*a^3*b^20))/b^18)*(24*a^4 + 15*b^4 - 40*a^2*b^2)*1i)/(4*b^7))*1i)/(4*b^7))*
(24*a^4 + 15*b^4 - 40*a^2*b^2)*1i)/(4*b^7)))*(24*a^4 + 15*b^4 - 40*a^2*b^2)
)/(2*b^7*d) - (2*atanh((64*a^2*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(
64*a^2*b^3 - 572*a^4*b + (1312*a^6)/b - (1164*a^8)/b^3 + (360*a^10)/b^5 + 2
624*a^5*tan(c/2 + (d*x)/2) + 128*a*b^4*tan(c/2 + (d*x)/2) - 1144*a^3*b^2*ta
n(c/2 + (d*x)/2) - (2328*a^7*tan(c/2 + (d*x)/2))/b^2 + (720*a^9*tan(c/2 + (
d*x)/2))/b^4) - (444*a^4*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(1312*a
^6*b + 64*a^2*b^5 - 572*a^4*b^3 - (1164*a^8)/b + (360*a^10)/b^3 - 2328*a^7*
tan(c/2 + (d*x)/2) + 128*a*b^6*tan(c/2 + (d*x)/2) - 1144*a^3*b^4*tan(c/2 +
(d*x)/2) + 2624*a^5*b^2*tan(c/2 + (d*x)/2) + (720*a^9*tan(c/2 + (d*x)/2))/b
^2) + (360*a^6*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(64*a^2*b^7 - 116
4*a^8*b - 572*a^4*b^5 + 1312*a^6*b^3 + (360*a^10)/b + 720*a^9*tan(c/2 + (d*
x)/2) + 128*a*b^8*tan(c/2 + (d*x)/2) - 1144*a^3*b^6*tan(c/2 + (d*x)/2) + 26
24*a^5*b^4*tan(c/2 + (d*x)/2) - 2328*a^7*b^2*tan(c/2 + (d*x)/2)) + (128*a*t
an(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(64*a^2*b^2 -
572*a^4 + (1312*a^6)/b^2 - (1164*a^8)/b^4 + (360*a^10)/b^6 + 128*a*b^3*tan(

```
c/2 + (d*x)/2) - 1144*a^3*b*tan(c/2 + (d*x)/2) + (2624*a^5*tan(c/2 + (d*x)/
2))/b - (2328*a^7*tan(c/2 + (d*x)/2))/b^3 + (720*a^9*tan(c/2 + (d*x)/2))/b^
5) - (952*a^3*tan(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))
/(1312*a^6 + 64*a^2*b^4 - 572*a^4*b^2 - (1164*a^8)/b^2 + (360*a^10)/b^4 + 1
28*a*b^5*tan(c/2 + (d*x)/2) + 2624*a^5*b*tan(c/2 + (d*x)/2) - 1144*a^3*b^3*
tan(c/2 + (d*x)/2) - (2328*a^7*tan(c/2 + (d*x)/2))/b + (720*a^9*tan(c/2 + (
d*x)/2))/b^3) + (1164*a^5*tan(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4
*b^2)^(1/2))/(64*a^2*b^6 - 1164*a^8 - 572*a^4*b^4 + 1312*a^6*b^2 + (360*a^1
0)/b^2 + 128*a*b^7*tan(c/2 + (d*x)/2) - 2328*a^7*b*tan(c/2 + (d*x)/2) - 114
4*a^3*b^5*tan(c/2 + (d*x)/2) + 2624*a^5*b^3*tan(c/2 + (d*x)/2) + (720*a^9*t
an(c/2 + (d*x)/2))/b) - (360*a^7*tan(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(1/2))/(360*a^10 + 64*a^2*b^8 - 572*a^4*b^6 + 1312*a^6*b^4 - 1
164*a^8*b^2 + 128*a*b^9*tan(c/2 + (d*x)/2) + 720*a^9*b*tan(c/2 + (d*x)/2) -
 1144*a^3*b^7*tan(c/2 + (d*x)/2) + 2624*a^5*b^5*tan(c/2 + (d*x)/2) - 2328*a
^7*b^3*tan(c/2 + (d*x)/2)))*(6*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^
7*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1259 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=266

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}-\frac{2\left(a^2-b^2\right)^{3/2}\left(5a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^5d}+\frac{2ax\left(2a^2-3b^2\right)}{b^5}+\frac{3\left(a^2-\right.}{}$$

```
[Out] a*x/b^3+2*a*(2*a^2-3*b^2)*x/b^5+2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1
/2*c))/(a^2-b^2)^(1/2))/b^5/d-2*(a^2-b^2)^(3/2)*(5*a^2+b^2)*arctan((b+a*tan
(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/b^5/d-arctanh(cos(d*x+c))/a^2/d+cos(d
*x+c)/b^2/d+3*(a^2-b^2)*cos(d*x+c)/b^4/d-1/3*cos(d*x+c)^3/b^2/d-a*cos(d*x+c
)*sin(d*x+c)/b^3/d+(a^2-b^2)^2*cos(d*x+c)/a/b^4/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.35, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2897, 3770, 2638, 2635, 8, 2633, 2664, 12, 2660, 618, 204}

$$\frac{3\left(a^2-b^2\right)\cos(c+dx)}{b^4d}+\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}-\frac{2\left(a^2-b^2\right)^{3/2}\left(5a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^5d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*x)/b^3 + (2*a*(2*a^2 - 3*b^2)*x)/b^5 + (2*(a^2 - b^2)^(3/2)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*d) - (2*(a^2 - b^2)^(3/2)*(5*a^2
 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*b^5*d) - Arc
Tanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(b^2*d) + (3*(a^2 - b^2)*Cos[c +
d*x])/(b^4*d) - Cos[c + d*x]^3/(3*b^2*d) - (a*Cos[c + d*x]*Sin[c + d*x])/(b
^3*d) + ((a^2 - b^2)^2*Cos[c + d*x])/(a*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(-\frac{2\left(-2a^3+3ab^2\right)}{b^5}+\frac{\csc(c+dx)}{a^2}+\frac{3\left(-a^2+b^2\right)\sin(c+dx)}{b^4}+\frac{2a\sin^2(c+}{b^3}\right.$$

$$=\frac{2a\left(2a^2-3b^2\right)x}{b^5}+\frac{\int\csc(c+dx)\,dx}{a^2}+\frac{(2a)\int\sin^2(c+dx)\,dx}{b^3}-\frac{\int\sin^3(c+dx)}{b^2}$$

$$=\frac{2a\left(2a^2-3b^2\right)x}{b^5}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{3\left(a^2-b^2\right)\cos(c+dx)}{b^4d}-\frac{a\cos(c+}{}$$

$$=\frac{ax}{b^3}+\frac{2a\left(2a^2-3b^2\right)x}{b^5}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{b^2d}+\frac{3\left(a^2-b^2\right)\cos}{b^4d}$$

$$=\frac{ax}{b^3}+\frac{2a\left(2a^2-3b^2\right)x}{b^5}-\frac{2\left(a^2-b^2\right)^{3/2}\left(5a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^5d}-\frac{\tan}{}$$

$$=\frac{ax}{b^3}+\frac{2a\left(2a^2-3b^2\right)x}{b^5}-\frac{2\left(a^2-b^2\right)^{3/2}\left(5a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^5d}-\frac{\tan}{}$$

$$=\frac{ax}{b^3}+\frac{2a\left(2a^2-3b^2\right)x}{b^5}+\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5d}-\frac{2\left(a^2-b^2\right)^{3/2}}{}$$

Mathematica [A] time = 1.74, size = 207, normalized size = 0.78

$$\frac{\frac{12a(4a^2-5b^2)(c+dx)}{b^5}-\frac{24(4a^2+b^2)(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^5}+\frac{9(4a^2-3b^2)\cos(c+dx)}{b^4}+\frac{12(a^2-b^2)^2\cos(c+dx)}{ab^4(a+b\sin(c+dx))}+\frac{12\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((12*a*(4*a^2 - 5*b^2)*(c + d*x))/b^5 - (24*(a^2 - b^2)^(3/2)*(4*a^2 + b^2)
*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*b^5) + (9*(4*a^2 -
3*b^2)*Cos[c + d*x])/b^4 - Cos[3*(c + d*x)]/b^2 - (12*Log[Cos[(c + d*x)/2]]
)/a^2 + (12*Log[Sin[(c + d*x)/2]])/a^2 + (12*(a^2 - b^2)^2*Cos[c + d*x])/(a
*b^4*(a + b*Sin[c + d*x])) - (6*a*Sin[2*(c + d*x)])/b^3)/(12*d)
```

fricas [A] time = 1.27, size = 698, normalized size = 2.62

$$\left[\frac{4\,a^3b^3\cos(dx+c)^3+6\left(4\,a^6-5\,a^4b^2\right)dx-3\left(4\,a^5-3\,a^3b^2-ab^4+\left(4\,a^4b-3\,a^2b^3-b^5\right)\sin(dx+c)\right)\sqrt{-a^2+b^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(4*a^3*b^3*cos(d*x + c)^3 + 6*(4*a^6 - 5*a^4*b^2)*d*x - 3*(4*a^5 - 3*a
^3*b^2 - a*b^4 + (4*a^4*b - 3*a^2*b^3 - b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)
*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a
*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(4*a^5*b - 5*a^3*b^3 + a*b^5
)*cos(d*x + c) - 3*(b^6*sin(d*x + c) + a*b^5)*log(1/2*cos(d*x + c) + 1/2) +
 3*(b^6*sin(d*x + c) + a*b^5)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^2*b^4*cos
(d*x + c)^3 - 3*(4*a^5*b - 5*a^3*b^3)*d*x - 6*(a^4*b^2 - a^2*b^4)*cos(d*x +
 c))*sin(d*x + c))/(a^2*b^6*d*sin(d*x + c) + a^3*b^5*d), 1/6*(4*a^3*b^3*cos
(d*x + c)^3 + 6*(4*a^6 - 5*a^4*b^2)*d*x + 6*(4*a^5 - 3*a^3*b^2 - a*b^4 + (4
*a^4*b - 3*a^2*b^3 - b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
+ c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 6*(4*a^5*b - 5*a^3*b^3 + a*b^5)
*cos(d*x + c) - 3*(b^6*sin(d*x + c) + a*b^5)*log(1/2*cos(d*x + c) + 1/2) +
3*(b^6*sin(d*x + c) + a*b^5)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^2*b^4*cos(
d*x + c)^3 - 3*(4*a^5*b - 5*a^3*b^3)*d*x - 6*(a^4*b^2 - a^2*b^4)*cos(d*x +
c))*sin(d*x + c))/(a^2*b^6*d*sin(d*x + c) + a^3*b^5*d)]
```

giac [A] time = 0.26, size = 353, normalized size = 1.33

$$\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{3\left(4\,a^3-5\,ab^2\right)(dx+c)}{b^5}-\frac{6\left(4\,a^6-7\,a^4b^2+2\,a^2b^4+b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^2b^5}+\frac{2\left(3\,ab\tan\left(\frac{1}{2}\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 3*(4*a^3 - 5*a*b^2)*(d*x + c)/b
^5 - 6*(4*a^6 - 7*a^4*b^2 + 2*a^2*b^4 + b^6)*(pi*floor(1/2*(d*x + c)/pi + 1
/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^
2 - b^2)*a^2*b^5) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*tan(1/2*d*x + 1
/2*c)^4 - 9*b^2*tan(1/2*d*x + 1/2*c)^4 + 18*a^2*tan(1/2*d*x + 1/2*c)^2 - 12
*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*a^2 - 7*b^2)/(
(tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^4) + 6*(a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2
*b^3*tan(1/2*d*x + 1/2*c) + b^5*tan(1/2*d*x + 1/2*c) + a^5 - 2*a^3*b^2 + a*
b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^2*b^4))/d
```

maple [B] time = 0.70, size = 778, normalized size = 2.92

$$\frac{2a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{6\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{d\,b^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{6\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{12a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{}{d\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)^5+6/d/b^4/(1+tan(1/
2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2-6/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)
^3*tan(1/2*d*x+1/2*c)^4+12/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2
*c)^2*a^2-8/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2-2/d/b^3/(
1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)+6/d/b^4/(1+tan(1/2*d*x+1/2*c
)^2)^3*a^2-14/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3+8/d/b^5*arctan(tan(1/2*d*x
+1/2*c))*a^3-10/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a+1/d/a^2*ln(tan(1/2*d*x+1
/2*c))+2/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/
2*d*x+1/2*c)-4/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/
2*d*x+1/2*c)+2/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*ta
n(1/2*d*x+1/2*c)+2/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b
+a)-4/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a+2/d/a/(tan(
```

```
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-8/d*a^4/b^5/(a^2-b^2)^(1/2)*ar
ctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+14/d*a^2/b^3/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-4/d/b/(a^
2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a
^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2
))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.60, size = 5197, normalized size = 19.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) + ((2*(12*a^4 + 3*b^4 - 13*a^2*b^2))/(3*a*b
^4) + (2*tan(c/2 + (d*x)/2)*(18*a^4 + 3*b^4 - 20*a^2*b^2))/(3*a^2*b^3) + (2
*tan(c/2 + (d*x)/2)^7*(2*a^4 + b^4 - 2*a^2*b^2))/(a^2*b^3) + (2*tan(c/2 + (
d*x)/2)^6*(4*a^4 + b^4 - 3*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^5*(10*
a^4 + 3*b^4 - 12*a^2*b^2))/(a^2*b^3) + (2*tan(c/2 + (d*x)/2)^4*(12*a^4 + 3*
b^4 - 13*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^3*(14*a^4 + 3*b^4 - 14*a
^2*b^2))/(a^2*b^3) + (2*tan(c/2 + (d*x)/2)^2*(36*a^4 + 9*b^4 - 43*a^2*b^2))
/(3*a*b^4))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 4*a*tan(c/2 + (d*x)/2)^2 + 6*a
*tan(c/2 + (d*x)/2)^4 + 4*a*tan(c/2 + (d*x)/2)^6 + a*tan(c/2 + (d*x)/2)^8 +
 6*b*tan(c/2 + (d*x)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/
2)^7)) + (2*a*atan(((a*(4*a^2 - 5*b^2)*((a*(4*a^2 - 5*b^2)*((32*(4*b^16 + 7
*a^2*b^14 - 25*a^4*b^12 + 114*a^6*b^10 - 235*a^8*b^8 + 184*a^10*b^6 - 48*a^
12*b^4))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(20*a^4*b^18 - 19*a^2*b^20 + 6
10*a^6*b^16 - 1596*a^8*b^14 + 1434*a^10*b^12 - 480*a^12*b^10 + 32*a^14*b^8)
)/(a^3*b^16) + (a*(4*a^2 - 5*b^2)*((32*(8*a^2*b^16 + 4*a^4*b^14 - 25*a^6*b^
12 + 14*a^8*b^10))/(a^2*b^11) + (a*((32*(4*a^4*b^16 - 3*a^6*b^14))/(a^2*b^1
1) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^22 - 17*a^6*b^20 + 2*a^8*b^18))/(a^3*
b^16))*(4*a^2 - 5*b^2)*1i)/b^5 + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^22 + 3*a^
4*b^20 - 62*a^6*b^18 + 60*a^8*b^16 - 16*a^10*b^14))/(a^3*b^16))*1i)/b^5)*1i
```

)/b^5 - (32*(5*a^2*b^12 - 224*a^14 - 184*a^4*b^10 + 154*a^6*b^8 + 722*a^8*b
^6 - 1434*a^10*b^4 + 960*a^12*b^2))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(b^
20 + 4*a^2*b^18 + 390*a^4*b^16 - 560*a^6*b^14 - 1414*a^8*b^12 + 4028*a^10*b
^10 - 3792*a^12*b^8 + 1600*a^14*b^6 - 256*a^16*b^4))/(a^3*b^16)))/b^5 - (a*
(4*a^2 - 5*b^2)*((32*(5*a^2*b^12 - 224*a^14 - 184*a^4*b^10 + 154*a^6*b^8 +
722*a^8*b^6 - 1434*a^10*b^4 + 960*a^12*b^2))/(a^2*b^11) + (a*(4*a^2 - 5*b^2
)*((32*(4*b^16 + 7*a^2*b^14 - 25*a^4*b^12 + 114*a^6*b^10 - 235*a^8*b^8 + 18
4*a^10*b^6 - 48*a^12*b^4))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(20*a^4*b^18
 - 19*a^2*b^20 + 610*a^6*b^16 - 1596*a^8*b^14 + 1434*a^10*b^12 - 480*a^12*b
^10 + 32*a^14*b^8))/(a^3*b^16) - (a*(4*a^2 - 5*b^2)*((32*(8*a^2*b^16 + 4*a^
4*b^14 - 25*a^6*b^12 + 14*a^8*b^10))/(a^2*b^11) - (a*((32*(4*a^4*b^16 - 3*a
^6*b^14))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^22 - 17*a^6*b^20 +
2*a^8*b^18))/(a^3*b^16))*(4*a^2 - 5*b^2)*1i)/b^5 + (32*tan(c/2 + (d*x)/2)*(
16*a^2*b^22 + 3*a^4*b^20 - 62*a^6*b^18 + 60*a^8*b^16 - 16*a^10*b^14))/(a^3*
b^16))*1i)/b^5)*1i)/b^5 - (32*tan(c/2 + (d*x)/2)*(b^20 + 4*a^2*b^18 + 390*a
^4*b^16 - 560*a^6*b^14 - 1414*a^8*b^12 + 4028*a^10*b^10 - 3792*a^12*b^8 + 1
600*a^14*b^6 - 256*a^16*b^4))/(a^3*b^16)))/b^5)/((64*(224*a^12 - 5*b^12 + 8
4*a^2*b^10 + 81*a^4*b^8 - 906*a^6*b^6 + 1482*a^8*b^4 - 960*a^10*b^2))/(a^2*
b^11) - (64*tan(c/2 + (d*x)/2)*(2048*a^18 + 500*a^4*b^14 - 1200*a^6*b^12 -
4540*a^8*b^10 + 21944*a^10*b^8 - 36160*a^12*b^6 + 29696*a^14*b^4 - 12288*a^
16*b^2))/(a^3*b^16) + (a*(4*a^2 - 5*b^2)*((a*(4*a^2 - 5*b^2)*((32*(4*b^16 +
 7*a^2*b^14 - 25*a^4*b^12 + 114*a^6*b^10 - 235*a^8*b^8 + 184*a^10*b^6 - 48*
a^12*b^4))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(20*a^4*b^18 - 19*a^2*b^20 +
 610*a^6*b^16 - 1596*a^8*b^14 + 1434*a^10*b^12 - 480*a^12*b^10 + 32*a^14*b^
8))/(a^3*b^16) + (a*(4*a^2 - 5*b^2)*((32*(8*a^2*b^16 + 4*a^4*b^14 - 25*a^6*
b^12 + 14*a^8*b^10))/(a^2*b^11) + (a*((32*(4*a^4*b^16 - 3*a^6*b^14))/(a^2*b
^11) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^22 - 17*a^6*b^20 + 2*a^8*b^18))/(a^
3*b^16))*(4*a^2 - 5*b^2)*1i)/b^5 + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^22 + 3*
a^4*b^20 - 62*a^6*b^18 + 60*a^8*b^16 - 16*a^10*b^14))/(a^3*b^16))*1i)/b^5)*
1i)/b^5 - (32*(5*a^2*b^12 - 224*a^14 - 184*a^4*b^10 + 154*a^6*b^8 + 722*a^8
*b^6 - 1434*a^10*b^4 + 960*a^12*b^2))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(
b^20 + 4*a^2*b^18 + 390*a^4*b^16 - 560*a^6*b^14 - 1414*a^8*b^12 + 4028*a^10
*b^10 - 3792*a^12*b^8 + 1600*a^14*b^6 - 256*a^16*b^4))/(a^3*b^16))*1i)/b^5
+ (a*(4*a^2 - 5*b^2)*((32*(5*a^2*b^12 - 224*a^14 - 184*a^4*b^10 + 154*a^6*b
^8 + 722*a^8*b^6 - 1434*a^10*b^4 + 960*a^12*b^2))/(a^2*b^11) + (a*(4*a^2 -
5*b^2)*((32*(4*b^16 + 7*a^2*b^14 - 25*a^4*b^12 + 114*a^6*b^10 - 235*a^8*b^8
 + 184*a^10*b^6 - 48*a^12*b^4))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(20*a^4
*b^18 - 19*a^2*b^20 + 610*a^6*b^16 - 1596*a^8*b^14 + 1434*a^10*b^12 - 480*a
^12*b^10 + 32*a^14*b^8))/(a^3*b^16) - (a*(4*a^2 - 5*b^2)*((32*(8*a^2*b^16 +
 4*a^4*b^14 - 25*a^6*b^12 + 14*a^8*b^10))/(a^2*b^11) - (a*((32*(4*a^4*b^16
 - 3*a^6*b^14))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^22 - 17*a^6*b^
20 + 2*a^8*b^18))/(a^3*b^16))*(4*a^2 - 5*b^2)*1i)/b^5 + (32*tan(c/2 + (d*x)
/2)*(16*a^2*b^22 + 3*a^4*b^20 - 62*a^6*b^18 + 60*a^8*b^16 - 16*a^10*b^14))/
(a^3*b^16))*1i)/b^5)*1i)/b^5 - (32*tan(c/2 + (d*x)/2)*(b^20 + 4*a^2*b^18 +
390*a^4*b^16 - 560*a^6*b^14 - 1414*a^8*b^12 + 4028*a^10*b^10 - 3792*a^12*b^

8 + 1600*a^14*b^6 - 256*a^16*b^4))/(a^3*b^16))*1i)/b^5))*(4*a^2 - 5*b^2))/(
b^5*d) + (atan((((4*a^2 + b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 +
(d*x)/2)*(b^20 + 4*a^2*b^18 + 390*a^4*b^16 - 560*a^6*b^14 - 1414*a^8*b^12 +
4028*a^10*b^10 - 3792*a^12*b^8 + 1600*a^14*b^6 - 256*a^16*b^4))/(a^3*b^16)
- (32*(5*a^2*b^12 - 224*a^14 - 184*a^4*b^10 + 154*a^6*b^8 + 722*a^8*b^6 -
1434*a^10*b^4 + 960*a^12*b^2))/(a^2*b^11) + ((4*a^2 + b^2)*(-(a + b)^3*(a -
b)^3)^(1/2)*((32*(4*b^16 + 7*a^2*b^14 - 25*a^4*b^12 + 114*a^6*b^10 - 235*a
^8*b^8 + 184*a^10*b^6 - 48*a^12*b^4))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(
20*a^4*b^18 - 19*a^2*b^20 + 610*a^6*b^16 - 1596*a^8*b^14 + 1434*a^10*b^12 -
480*a^12*b^10 + 32*a^14*b^8))/(a^3*b^16) + ((4*a^2 + b^2)*(-(a + b)^3*(a -
b)^3)^(1/2)*((32*(8*a^2*b^16 + 4*a^4*b^14 - 25*a^6*b^12 + 14*a^8*b^10))/(a
^2*b^11) + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^22 + 3*a^4*b^20 - 62*a^6*b^18 +
60*a^8*b^16 - 16*a^10*b^14))/(a^3*b^16) + (((32*(4*a^4*b^16 - 3*a^6*b^14))
/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^22 - 17*a^6*b^20 + 2*a^8*b^1
8))/(a^3*b^16))*(4*a^2 + b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^2*b^5)))/(a^
2*b^5)))/(a^2*b^5))*1i)/(a^2*b^5) - ((4*a^2 + b^2)*(-(a + b)^3*(a - b)^3)^(
1/2)*((32*(5*a^2*b^12 - 224*a^14 - 184*a^4*b^10 + 154*a^6*b^8 + 722*a^8*b^6
- 1434*a^10*b^4 + 960*a^12*b^2))/(a^2*b^11) - (32*tan(c/2 + (d*x)/2)*(b^20
+ 4*a^2*b^18 + 390*a^4*b^16 - 560*a^6*b^14 - 1414*a^8*b^12 + 4028*a^10*b^1
0 - 3792*a^12*b^8 + 1600*a^14*b^6 - 256*a^16*b^4))/(a^3*b^16) + ((4*a^2 + b
^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(4*b^16 + 7*a^2*b^14 - 25*a^4*b^12 +
114*a^6*b^10 - 235*a^8*b^8 + 184*a^10*b^6 - 48*a^12*b^4))/(a^2*b^11) + (32*
tan(c/2 + (d*x)/2)*(20*a^4*b^18 - 19*a^2*b^20 + 610*a^6*b^16 - 1596*a^8*b^1
4 + 1434*a^10*b^12 - 480*a^12*b^10 + 32*a^14*b^8))/(a^3*b^16) - ((4*a^2 + b
^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(8*a^2*b^16 + 4*a^4*b^14 - 25*a^6*b^1
2 + 14*a^8*b^10))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^22 + 3*a^4*
b^20 - 62*a^6*b^18 + 60*a^8*b^16 - 16*a^10*b^14))/(a^3*b^16) - (((32*(4*a^4
*b^16 - 3*a^6*b^14))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^22 - 17*
a^6*b^20 + 2*a^8*b^18))/(a^3*b^16))*(4*a^2 + b^2)*(-(a + b)^3*(a - b)^3)^(1
/2))/(a^2*b^5)))/(a^2*b^5)))/(a^2*b^5))*1i)/(a^2*b^5))/((64*(224*a^12 - 5*b
^12 + 84*a^2*b^10 + 81*a^4*b^8 - 906*a^6*b^6 + 1482*a^8*b^4 - 960*a^10*b^2)
)/(a^2*b^11) - (64*tan(c/2 + (d*x)/2)*(2048*a^18 + 500*a^4*b^14 - 1200*a^6*
b^12 - 4540*a^8*b^10 + 21944*a^10*b^8 - 36160*a^12*b^6 + 29696*a^14*b^4 - 1
2288*a^16*b^2))/(a^3*b^16) + ((4*a^2 + b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((
32*tan(c/2 + (d*x)/2)*(b^20 + 4*a^2*b^18 + 390*a^4*b^16 - 560*a^6*b^14 - 14
14*a^8*b^12 + 4028*a^10*b^10 - 3792*a^12*b^8 + 1600*a^14*b^6 - 256*a^16*b^4
))/(a^3*b^16) - (32*(5*a^2*b^12 - 224*a^14 - 184*a^4*b^10 + 154*a^6*b^8 + 7
22*a^8*b^6 - 1434*a^10*b^4 + 960*a^12*b^2))/(a^2*b^11) + ((4*a^2 + b^2)*(-(
a + b)^3*(a - b)^3)^(1/2)*((32*(4*b^16 + 7*a^2*b^14 - 25*a^4*b^12 + 114*a^6
*b^10 - 235*a^8*b^8 + 184*a^10*b^6 - 48*a^12*b^4))/(a^2*b^11) + (32*tan(c/2
+ (d*x)/2)*(20*a^4*b^18 - 19*a^2*b^20 + 610*a^6*b^16 - 1596*a^8*b^14 + 143
4*a^10*b^12 - 480*a^12*b^10 + 32*a^14*b^8))/(a^3*b^16) + ((4*a^2 + b^2)*(-(
a + b)^3*(a - b)^3)^(1/2)*((32*(8*a^2*b^16 + 4*a^4*b^14 - 25*a^6*b^12 + 14*
a^8*b^10))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^22 + 3*a^4*b^20 -
62*a^6*b^18 + 60*a^8*b^16 - 16*a^10*b^14))/(a^3*b^16) + (((32*(4*a^4*b^16 -

```
 3*a^6*b^14))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^22 - 17*a^6*b^2
0 + 2*a^8*b^18))/(a^3*b^16))*(4*a^2 + b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a
^2*b^5)))/(a^2*b^5)))/(a^2*b^5)))/(a^2*b^5) + ((4*a^2 + b^2)*(-(a + b)^3*(a
 - b)^3)^(1/2)*((32*(5*a^2*b^12 - 224*a^14 - 184*a^4*b^10 + 154*a^6*b^8 + 7
22*a^8*b^6 - 1434*a^10*b^4 + 960*a^12*b^2))/(a^2*b^11) - (32*tan(c/2 + (d*x
)/2)*(b^20 + 4*a^2*b^18 + 390*a^4*b^16 - 560*a^6*b^14 - 1414*a^8*b^12 + 402
8*a^10*b^10 - 3792*a^12*b^8 + 1600*a^14*b^6 - 256*a^16*b^4))/(a^3*b^16) + (
(4*a^2 + b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(4*b^16 + 7*a^2*b^14 - 25*a
^4*b^12 + 114*a^6*b^10 - 235*a^8*b^8 + 184*a^10*b^6 - 48*a^12*b^4))/(a^2*b^
11) + (32*tan(c/2 + (d*x)/2)*(20*a^4*b^18 - 19*a^2*b^20 + 610*a^6*b^16 - 15
96*a^8*b^14 + 1434*a^10*b^12 - 480*a^12*b^10 + 32*a^14*b^8))/(a^3*b^16) - (
(4*a^2 + b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(8*a^2*b^16 + 4*a^4*b^14 -
25*a^6*b^12 + 14*a^8*b^10))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^2
2 + 3*a^4*b^20 - 62*a^6*b^18 + 60*a^8*b^16 - 16*a^10*b^14))/(a^3*b^16) - ((
(32*(4*a^4*b^16 - 3*a^6*b^14))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(16*a^4*
b^22 - 17*a^6*b^20 + 2*a^8*b^18))/(a^3*b^16))*(4*a^2 + b^2)*(-(a + b)^3*(a
- b)^3)^(1/2))/(a^2*b^5)))/(a^2*b^5)))/(a^2*b^5)))/(a^2*b^5)))*(4*a^2 + b^2
)*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^2*b^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**6*csc(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

3.1260 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=254

$$\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^4d}-\frac{3x\left(a^2-b^2\right)}{b^4}-\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{a^2b^3d(a+b\sin(c+dx))}-\frac{\cot(c+dx)}{a^2d}$$

```
[Out] -1/2*x/b^2-3*(a^2-b^2)*x/b^4-2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*
c))/(a^2-b^2)^(1/2))/a/b^4/d+2*b*arctanh(cos(d*x+c))/a^3/d-2*a*cos(d*x+c)/b
^3/d-cot(d*x+c)/a^2/d+1/2*cos(d*x+c)*sin(d*x+c)/b^2/d-(a^2-b^2)^2*cos(d*x+c
)/a^2/b^3/d/(a+b*sin(d*x+c))+4*(2*a^6-3*a^4*b^2+b^6)*arctan((b+a*tan(1/2*d*
x+1/2*c))/(a^2-b^2)^(1/2))/a^3/b^4/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2897, 3770, 3767, 8, 2638, 2635, 2664, 12, 2660, 618, 204}

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^4d}+\frac{4\left(-3a^4b^2+2a^6+b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^4d\sqrt{a^2-b^2}}-\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{a^2b^3d(a+b\sin(c+dx))}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -x/(2*b^2) - (3*(a^2 - b^2)*x)/b^4 - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^4*d) + (4*(2*a^6 - 3*a^4*b^2 + b^6)*A
rcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^4*Sqrt[a^2 - b^2]*d
) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - (2*a*Cos[c + d*x])/(b^3*d) - Cot[
c + d*x]/(a^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - ((a^2 - b^2)^2*C
os[c + d*x])/(a^2*b^3*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
```

```
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(\frac{3\left(-a^2+b^2\right)}{b^4} - \frac{2b\csc(c+dx)}{a^3} + \frac{\csc^2(c+dx)}{a^2} + \frac{2a\sin(c+dx)}{b^3} - \frac{\sin^2(c+\right.$$

$$= -\frac{3\left(a^2-b^2\right)x}{b^4} + \frac{\int \csc^2(c+dx)\,dx}{a^2} + \frac{(2a)\int \sin(c+dx)\,dx}{b^3} - \frac{\int \sin^2(c+dx)\,dx}{b^2}$$

$$= -\frac{3\left(a^2-b^2\right)x}{b^4} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2a\cos(c+dx)}{b^3d} + \frac{\cos(c+dx)\sin(c}{2b^2d}$$

$$= -\frac{x}{2b^2} - \frac{3\left(a^2-b^2\right)x}{b^4} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2a\cos(c+dx)}{b^3d} - \frac{\cot(c+dx)}{a^2d}$$

$$= -\frac{x}{2b^2} - \frac{3\left(a^2-b^2\right)x}{b^4} + \frac{4\left(2a^6-3a^4b^2+b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^4\sqrt{a^2-b^2}\,d} + \frac{2b\tanh}{}$$

$$= -\frac{x}{2b^2} - \frac{3\left(a^2-b^2\right)x}{b^4} + \frac{4\left(2a^6-3a^4b^2+b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^4\sqrt{a^2-b^2}\,d} + \frac{2b\tanh}{}$$

$$= -\frac{x}{2b^2} - \frac{3\left(a^2-b^2\right)x}{b^4} - \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^4d} + \frac{4\left(2a^6-3a^4b^2\right.}{a^3}$$

Mathematica [A] time = 2.76, size = 215, normalized size = 0.85

$$\frac{-\frac{8b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}+\frac{8b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}+\frac{2(5b^2-6a^2)(c+dx)}{b^4}-\frac{4(a^2-b^2)^2\cos(c+dx)}{a^2b^3(a+b\sin(c+dx))}+\frac{2\tan\left(\frac{1}{2}(c+dx)\right)}{a^2}-\frac{2\cot\left(\frac{1}{2}(c+dx)\right)}{a^2}+\frac{8(3a^2+}{}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*(-6*a^2 + 5*b^2)*(c + d*x))/b^4 + (8*(a^2 - b^2)^(3/2)*(3*a^2 + 2*b^2)*
ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^4) - (8*a*Cos[c +
d*x])/b^3 - (2*Cot[(c + d*x)/2])/a^2 + (8*b*Log[Cos[(c + d*x)/2]])/a^3 - (8
*b*Log[Sin[(c + d*x)/2]])/a^3 - (4*(a^2 - b^2)^2*Cos[c + d*x])/(a^2*b^3*(a
+ b*Sin[c + d*x])) + Sin[2*(c + d*x)]/b^2 + (2*Tan[(c + d*x)/2])/a^2)/(4*d)
```

fricas [A] time = 1.50, size = 901, normalized size = 3.55

$$\left[-\frac{3a^4b^2\cos(dx+c)^3+\left(6a^5b-5a^3b^3\right)dx\cos(dx+c)^2-\left(6a^5b-5a^3b^3\right)dx-\left(3a^4b-a^2b^3-2b^5-\left(3a^4b-a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*(3*a^4*b^2*cos(d*x + c)^3 + (6*a^5*b - 5*a^3*b^3)*d*x*cos(d*x + c)^2
- (6*a^5*b - 5*a^3*b^3)*d*x - (3*a^4*b - a^2*b^3 - 2*b^5 - (3*a^4*b - a^2*b
^3 - 2*b^5)*cos(d*x + c)^2 + (3*a^5 - a^3*b^2 - 2*a*b^4)*sin(d*x + c))*sqrt
(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(
b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - (3*a^4*b^2 + 2*a^2*
b^4)*cos(d*x + c) - 2*(b^6*cos(d*x + c)^2 - a*b^5*sin(d*x + c) - b^6)*log(1
/2*cos(d*x + c) + 1/2) + 2*(b^6*cos(d*x + c)^2 - a*b^5*sin(d*x + c) - b^6)*
log(-1/2*cos(d*x + c) + 1/2) - (a^3*b^3*cos(d*x + c)^3 + (6*a^6 - 5*a^4*b^2
)*d*x + (6*a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^3*b^
5*d*cos(d*x + c)^2 - a^4*b^4*d*sin(d*x + c) - a^3*b^5*d), -1/2*(3*a^4*b^2*c
os(d*x + c)^3 + (6*a^5*b - 5*a^3*b^3)*d*x*cos(d*x + c)^2 - (6*a^5*b - 5*a^3
*b^3)*d*x - 2*(3*a^4*b - a^2*b^3 - 2*b^5 - (3*a^4*b - a^2*b^3 - 2*b^5)*cos(
d*x + c)^2 + (3*a^5 - a^3*b^2 - 2*a*b^4)*sin(d*x + c))*sqrt(a^2 - b^2)*arct
an(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (3*a^4*b^2 + 2*a
^2*b^4)*cos(d*x + c) - 2*(b^6*cos(d*x + c)^2 - a*b^5*sin(d*x + c) - b^6)*lo
g(1/2*cos(d*x + c) + 1/2) + 2*(b^6*cos(d*x + c)^2 - a*b^5*sin(d*x + c) - b^
```

```
6)*log(-1/2*cos(d*x + c) + 1/2) - (a^3*b^3*cos(d*x + c)^3 + (6*a^6 - 5*a^4*
b^2)*d*x + (6*a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^3
*b^5*d*cos(d*x + c)^2 - a^4*b^4*d*sin(d*x + c) - a^3*b^5*d)]
```

giac [A] time = 0.22, size = 384, normalized size = 1.51

$$-\frac{\frac{12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{3\left(6\,a^2-5\,b^2\right)(dx+c)}{b^4}+\frac{6\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2b^3}-\frac{12\,(3}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*tan(1/2*d*x + 1/2*c)/a^2
+ 3*(6*a^2 - 5*b^2)*(d*x + c)/b^4 + 6*(b*tan(1/2*d*x + 1/2*c)^3 + 4*a*tan(1
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c) + 4*a)/((tan(1/2*d*x + 1/2*c)^2
+ 1)^2*b^3) - 12*(3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2))
)/(sqrt(a^2 - b^2)*a^3*b^4) - (4*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 12*a^4*b*ta
n(1/2*d*x + 1/2*c)^2 + 21*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 4*b^5*tan(1/2*d*
x + 1/2*c)^2 - 12*a^5*tan(1/2*d*x + 1/2*c) + 24*a^3*b^2*tan(1/2*d*x + 1/2*c
) - 14*a*b^4*tan(1/2*d*x + 1/2*c) - 3*a^2*b^3)/((a*tan(1/2*d*x + 1/2*c)^3 +
 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))*a^3*b^3))/d
```

maple [B] time = 0.77, size = 680, normalized size = 2.68

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{4a}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x
+1/2*c)^3-4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a+1/d/b^2
/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-4/d/b^3/(1+tan(1/2*d*x+1/2*c
)^2)^2*a-6/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^2+5/d/b^2*arctan(tan(1/2*d*x+
1/2*c))-1/2/d/a^2/tan(1/2*d*x+1/2*c)-2/d/a^3*b*ln(tan(1/2*d*x+1/2*c))-2/d/b
^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a*tan(1/2*d*x+1/2*c)+4
/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)/a*tan(1/2*d*x+1/2*c)-2
```

```
/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/
2*c)-2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^2+4/d/b/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-2/d/a^2*b/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+6/d/b^4*a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-8/d/b^2*a/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a/(a^2-b^2)^(1/2)*ar
ctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d/a^3*b^2/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 13.48, size = 5214, normalized size = 20.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^2*d) - (a + (2*tan(c/2 + (d*x)/2)*(6*a^4 + 3*b^4 -
4*a^2*b^2))/b^3 + (2*tan(c/2 + (d*x)/2)^5*(6*a^4 + 3*b^4 - 2*a^2*b^2))/b^3
+ (4*tan(c/2 + (d*x)/2)^3*(6*a^4 + 3*b^4 - 5*a^2*b^2))/b^3 + (tan(c/2 + (d*
x)/2)^6*(6*a^4 + 4*b^4 - 7*a^2*b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^2*(18*a^
4 + 4*b^4 - 5*a^2*b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^4*(24*a^4 + 8*b^4 - 1
3*a^2*b^2))/(a*b^2))/(d*(6*a^3*tan(c/2 + (d*x)/2)^3 + 6*a^3*tan(c/2 + (d*x)
/2)^5 + 2*a^3*tan(c/2 + (d*x)/2)^7 + 2*a^3*tan(c/2 + (d*x)/2) + 4*a^2*b*tan
(c/2 + (d*x)/2)^2 + 8*a^2*b*tan(c/2 + (d*x)/2)^4 + 4*a^2*b*tan(c/2 + (d*x)/
2)^6)) + (atan((((a^2*6i - b^2*5i)*((8*(378*a^15 - 40*a^3*b^12 + 488*a^5*b^
10 - 1158*a^7*b^8 + 541*a^9*b^6 + 1008*a^11*b^4 - 1215*a^13*b^2))/(a^6*b^8)
 - ((a^2*6i - b^2*5i)*(((a^2*6i - b^2*5i)*((((8*(16*a^8*b^13 - 12*a^10*b^11
))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^18 - 68*a^9*b^16 + 8*a^11*b^
14))/(a^6*b^12))*(a^2*6i - b^2*5i))/(2*b^4) - (8*(64*a^5*b^14 - 48*a^7*b^12
 - 50*a^9*b^10 + 42*a^11*b^8))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(136*a^6*b
^17 - 128*a^4*b^19 + 96*a^8*b^15 - 160*a^10*b^13 + 48*a^12*b^11))/(a^6*b^12
)))/(2*b^4) - (8*(48*a^4*b^13 - 64*a^2*b^15 + 100*a^6*b^11 - 184*a^8*b^9 +
```

315*a^10*b^7 - 324*a^12*b^5 + 108*a^14*b^3))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(16*a^3*b^18 + 240*a^5*b^16 + 20*a^7*b^14 - 1696*a^9*b^12 + 2369*a^11*b^10 - 1020*a^13*b^8 + 72*a^15*b^6))/(a^6*b^12)))/(2*b^4) + (8*tan(c/2 + (d*x)/2)*(32*b^19 - 32*a^2*b^17 + 680*a^4*b^15 - 2560*a^6*b^13 + 2502*a^8*b^11 + 1216*a^10*b^9 - 3564*a^12*b^7 + 2160*a^14*b^5 - 432*a^16*b^3))/(a^6*b^12))*1i)/(2*b^4) + ((a^2*6i - b^2*5i)*((8*(378*a^15 - 40*a^3*b^12 + 488*a^5*b^10 - 1158*a^7*b^8 + 541*a^9*b^6 + 1008*a^11*b^4 - 1215*a^13*b^2))/(a^6*b^8) + ((a^2*6i - b^2*5i)*(((a^2*6i - b^2*5i)*((8*(64*a^5*b^14 - 48*a^7*b^12 - 50*a^9*b^10 + 42*a^11*b^8))/(a^6*b^8) + (((8*(16*a^8*b^13 - 12*a^10*b^11))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^18 - 68*a^9*b^16 + 8*a^11*b^14))/(a^6*b^12))*(a^2*6i - b^2*5i))/(2*b^4) - (8*tan(c/2 + (d*x)/2)*(136*a^6*b^17 - 128*a^4*b^19 + 96*a^8*b^15 - 160*a^10*b^13 + 48*a^12*b^11))/(a^6*b^12)))/(2*b^4) - (8*(48*a^4*b^13 - 64*a^2*b^15 + 100*a^6*b^11 - 184*a^8*b^9 + 315*a^10*b^7 - 324*a^12*b^5 + 108*a^14*b^3))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(16*a^3*b^18 + 240*a^5*b^16 + 20*a^7*b^14 - 1696*a^9*b^12 + 2369*a^11*b^10 - 1020*a^13*b^8 + 72*a^15*b^6))/(a^6*b^12)))/(2*b^4) + (8*tan(c/2 + (d*x)/2)*(32*b^19 - 32*a^2*b^17 + 680*a^4*b^15 - 2560*a^6*b^13 + 2502*a^8*b^11 + 1216*a^10*b^9 - 3564*a^12*b^7 + 2160*a^14*b^5 - 432*a^16*b^3))/(a^6*b^12))*1i)/(2*b^4))/(((a^2*6i - b^2*5i)*((8*(378*a^15 - 40*a^3*b^12 + 488*a^5*b^10 - 1158*a^7*b^8 + 541*a^9*b^6 + 1008*a^11*b^4 - 1215*a^13*b^2))/(a^6*b^8) - ((a^2*6i - b^2*5i)*(((a^2*6i - b^2*5i)*((((8*(16*a^8*b^13 - 12*a^10*b^11))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^18 - 68*a^9*b^16 + 8*a^11*b^14))/(a^6*b^12))*(a^2*6i - b^2*5i))/(2*b^4) - (8*(64*a^5*b^14 - 48*a^7*b^12 - 50*a^9*b^10 + 42*a^11*b^8))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(136*a^6*b^17 - 128*a^4*b^19 + 96*a^8*b^15 - 160*a^10*b^13 + 48*a^12*b^11))/(a^6*b^12)))/(2*b^4) - (8*(48*a^4*b^13 - 64*a^2*b^15 + 100*a^6*b^11 - 184*a^8*b^9 + 315*a^10*b^7 - 324*a^12*b^5 + 108*a^14*b^3))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(16*a^3*b^18 + 240*a^5*b^16 + 20*a^7*b^14 - 1696*a^9*b^12 + 2369*a^11*b^10 - 1020*a^13*b^8 + 72*a^15*b^6))/(a^6*b^12)))/(2*b^4) + (8*tan(c/2 + (d*x)/2)*(32*b^19 - 32*a^2*b^17 + 680*a^4*b^15 - 2560*a^6*b^13 + 2502*a^8*b^11 + 1216*a^10*b^9 - 3564*a^12*b^7 + 2160*a^14*b^5 - 432*a^16*b^3))/(a^6*b^12)))/(2*b^4) - ((a^2*6i - b^2*5i)*((8*(378*a^15 - 40*a^3*b^12 + 488*a^5*b^10 - 1158*a^7*b^8 + 541*a^9*b^6 + 1008*a^11*b^4 - 1215*a^13*b^2))/(a^6*b^8) + ((a^2*6i - b^2*5i)*(((a^2*6i - b^2*5i)*((8*(64*a^5*b^14 - 48*a^7*b^12 - 50*a^9*b^10 + 42*a^11*b^8))/(a^6*b^8) + (((8*(16*a^8*b^13 - 12*a^10*b^11))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^18 - 68*a^9*b^16 + 8*a^11*b^14))/(a^6*b^12))*(a^2*6i - b^2*5i))/(2*b^4) - (8*tan(c/2 + (d*x)/2)*(136*a^6*b^17 - 128*a^4*b^19 + 96*a^8*b^15 - 160*a^10*b^13 + 48*a^12*b^11))/(a^6*b^12)))/(2*b^4) - (8*(48*a^4*b^13 - 64*a^2*b^15 + 100*a^6*b^11 - 184*a^8*b^9 + 315*a^10*b^7 - 324*a^12*b^5 + 108*a^14*b^3))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(16*a^3*b^18 + 240*a^5*b^16 + 20*a^7*b^14 - 1696*a^9*b^12 + 2369*a^11*b^10 - 1020*a^13*b^8 + 72*a^15*b^6))/(a^6*b^12)))/(2*b^4) + (8*tan(c/2 + (d*x)/2)*(32*b^19 - 32*a^2*b^17 + 680*a^4*b^15 - 2560*a^6*b^13 + 2502*a^8*b^11 + 1216*a^10*b^9 - 3564*a^12*b^7 + 2160*a^14*b^5 - 432*a^16*b^3))/(a^6*b^12)))/(2*b^4) + (16*(80*b^13 - 756*a^12*b - 576*a^2*b^11 + 1056*a^4*b^9 + 214

*a^6*b^7 - 2448*a^8*b^5 + 2430*a^10*b^3))/(a^6*b^8) - (16*tan(c/2 + (d*x)/2
)*(1600*a^5*b^12 - 2592*a^17 - 5840*a^7*b^10 + 4504*a^9*b^8 + 8160*a^11*b^6
 - 17064*a^13*b^4 + 11232*a^15*b^2))/(a^6*b^12)))*(a^2*6i - b^2*5i)*1i)/(b^
4*d) - (2*b*log(tan(c/2 + (d*x)/2)))/(a^3*d) + (atan((((3*a^2 + 2*b^2)*(-(a
 + b)^3*(a - b)^3)^(1/2)*((8*(378*a^15 - 40*a^3*b^12 + 488*a^5*b^10 - 1158*
a^7*b^8 + 541*a^9*b^6 + 1008*a^11*b^4 - 1215*a^13*b^2))/(a^6*b^8) + (8*tan(
c/2 + (d*x)/2)*(32*b^19 - 32*a^2*b^17 + 680*a^4*b^15 - 2560*a^6*b^13 + 2502
*a^8*b^11 + 1216*a^10*b^9 - 3564*a^12*b^7 + 2160*a^14*b^5 - 432*a^16*b^3))/
(a^6*b^12) + ((3*a^2 + 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d
x)/2)(16*a^3*b^18 + 240*a^5*b^16 + 20*a^7*b^14 - 1696*a^9*b^12 + 2369*a^1
1*b^10 - 1020*a^13*b^8 + 72*a^15*b^6))/(a^6*b^12) - (8*(48*a^4*b^13 - 64*a^
2*b^15 + 100*a^6*b^11 - 184*a^8*b^9 + 315*a^10*b^7 - 324*a^12*b^5 + 108*a^1
4*b^3))/(a^6*b^8) + ((3*a^2 + 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(64*a
^5*b^14 - 48*a^7*b^12 - 50*a^9*b^10 + 42*a^11*b^8))/(a^6*b^8) - (8*tan(c/2
 + (d*x)/2)*(136*a^6*b^17 - 128*a^4*b^19 + 96*a^8*b^15 - 160*a^10*b^13 + 48*
a^12*b^11))/(a^6*b^12) + (((8*(16*a^8*b^13 - 12*a^10*b^11))/(a^6*b^8) + (8*
tan(c/2 + (d*x)/2)*(64*a^7*b^18 - 68*a^9*b^16 + 8*a^11*b^14))/(a^6*b^12))*(
3*a^2 + 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^3*b^4)))/(a^3*b^4)))/(a^3*b
^4))*1i)/(a^3*b^4) + ((3*a^2 + 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(378
*a^15 - 40*a^3*b^12 + 488*a^5*b^10 - 1158*a^7*b^8 + 541*a^9*b^6 + 1008*a^11
*b^4 - 1215*a^13*b^2))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(32*b^19 - 32*a^2*
b^17 + 680*a^4*b^15 - 2560*a^6*b^13 + 2502*a^8*b^11 + 1216*a^10*b^9 - 3564*
a^12*b^7 + 2160*a^14*b^5 - 432*a^16*b^3))/(a^6*b^12) - ((3*a^2 + 2*b^2)*(-(
a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(16*a^3*b^18 + 240*a^5*b^1
6 + 20*a^7*b^14 - 1696*a^9*b^12 + 2369*a^11*b^10 - 1020*a^13*b^8 + 72*a^15*
b^6))/(a^6*b^12) - (8*(48*a^4*b^13 - 64*a^2*b^15 + 100*a^6*b^11 - 184*a^8*b
^9 + 315*a^10*b^7 - 324*a^12*b^5 + 108*a^14*b^3))/(a^6*b^8) + ((3*a^2 + 2*b
^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(136*a^6*b^17 - 128
*a^4*b^19 + 96*a^8*b^15 - 160*a^10*b^13 + 48*a^12*b^11))/(a^6*b^12) - (8*(6
4*a^5*b^14 - 48*a^7*b^12 - 50*a^9*b^10 + 42*a^11*b^8))/(a^6*b^8) + (((8*(16
*a^8*b^13 - 12*a^10*b^11))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^18 -
 68*a^9*b^16 + 8*a^11*b^14))/(a^6*b^12))*(3*a^2 + 2*b^2)*(-(a + b)^3*(a - b
)^3)^(1/2))/(a^3*b^4)))/(a^3*b^4)))/(a^3*b^4))*1i)/(a^3*b^4))/((16*(80*b^13
 - 756*a^12*b - 576*a^2*b^11 + 1056*a^4*b^9 + 214*a^6*b^7 - 2448*a^8*b^5 +
2430*a^10*b^3))/(a^6*b^8) - (16*tan(c/2 + (d*x)/2)*(1600*a^5*b^12 - 2592*a^
17 - 5840*a^7*b^10 + 4504*a^9*b^8 + 8160*a^11*b^6 - 17064*a^13*b^4 + 11232*
a^15*b^2))/(a^6*b^12) - ((3*a^2 + 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(
378*a^15 - 40*a^3*b^12 + 488*a^5*b^10 - 1158*a^7*b^8 + 541*a^9*b^6 + 1008*a
^11*b^4 - 1215*a^13*b^2))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(32*b^19 - 32*a
^2*b^17 + 680*a^4*b^15 - 2560*a^6*b^13 + 2502*a^8*b^11 + 1216*a^10*b^9 - 35
64*a^12*b^7 + 2160*a^14*b^5 - 432*a^16*b^3))/(a^6*b^12) + ((3*a^2 + 2*b^2)*
(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(16*a^3*b^18 + 240*a^5*
b^16 + 20*a^7*b^14 - 1696*a^9*b^12 + 2369*a^11*b^10 - 1020*a^13*b^8 + 72*a^
15*b^6))/(a^6*b^12) - (8*(48*a^4*b^13 - 64*a^2*b^15 + 100*a^6*b^11 - 184*a^
8*b^9 + 315*a^10*b^7 - 324*a^12*b^5 + 108*a^14*b^3))/(a^6*b^8) + ((3*a^2 +

```
2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(64*a^5*b^14 - 48*a^7*b^12 - 50*a^9
*b^10 + 42*a^11*b^8))/(a^6*b^8) - (8*tan(c/2 + (d*x)/2)*(136*a^6*b^17 - 128
*a^4*b^19 + 96*a^8*b^15 - 160*a^10*b^13 + 48*a^12*b^11))/(a^6*b^12) + (((8*
(16*a^8*b^13 - 12*a^10*b^11))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^1
8 - 68*a^9*b^16 + 8*a^11*b^14))/(a^6*b^12))*(3*a^2 + 2*b^2)*(-(a + b)^3*(a
- b)^3)^(1/2))/(a^3*b^4)))/(a^3*b^4)))/(a^3*b^4)))/(a^3*b^4) + ((3*a^2 + 2*
b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(((8*(378*a^15 - 40*a^3*b^12 + 488*a^5*b^1
0 - 1158*a^7*b^8 + 541*a^9*b^6 + 1008*a^11*b^4 - 1215*a^13*b^2))/(a^6*b^8)
+ (8*tan(c/2 + (d*x)/2)*(32*b^19 - 32*a^2*b^17 + 680*a^4*b^15 - 2560*a^6*b^
13 + 2502*a^8*b^11 + 1216*a^10*b^9 - 3564*a^12*b^7 + 2160*a^14*b^5 - 432*a^
16*b^3))/(a^6*b^12) - ((3*a^2 + 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan
(c/2 + (d*x)/2)*(16*a^3*b^18 + 240*a^5*b^16 + 20*a^7*b^14 - 1696*a^9*b^12 +
 2369*a^11*b^10 - 1020*a^13*b^8 + 72*a^15*b^6))/(a^6*b^12) - (8*(48*a^4*b^1
3 - 64*a^2*b^15 + 100*a^6*b^11 - 184*a^8*b^9 + 315*a^10*b^7 - 324*a^12*b^5
+ 108*a^14*b^3))/(a^6*b^8) + ((3*a^2 + 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*
((8*tan(c/2 + (d*x)/2)*(136*a^6*b^17 - 128*a^4*b^19 + 96*a^8*b^15 - 160*a^1
0*b^13 + 48*a^12*b^11))/(a^6*b^12) - (8*(64*a^5*b^14 - 48*a^7*b^12 - 50*a^9
*b^10 + 42*a^11*b^8))/(a^6*b^8) + (((8*(16*a^8*b^13 - 12*a^10*b^11))/(a^6*b
^8) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^18 - 68*a^9*b^16 + 8*a^11*b^14))/(a^6
*b^12))*(3*a^2 + 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^3*b^4)))/(a^3*b^4)
))/(a^3*b^4)))/(a^3*b^4)))*(3*a^2 + 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*2i)
/(a^3*b^4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1261 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=251

$$\frac{2b\cot(c+dx)}{a^3d}+\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{3\left(a^2-b^2\right.}{}$$

```
[Out] 2*a*x/b^3+2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2)
)/a^2/b^3/d-6*(a^2-b^2)^(3/2)*(a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^
2-b^2)^(1/2))/a^4/b^3/d-1/2*arctanh(cos(d*x+c))/a^2/d+3*(a^2-b^2)*arctanh(c
os(d*x+c))/a^4/d+cos(d*x+c)/b^2/d+2*b*cot(d*x+c)/a^3/d-1/2*cot(d*x+c)*csc(d
*x+c)/a^2/d+(a^2-b^2)^2*cos(d*x+c)/a^3/b^2/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.34, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2897, 3770, 3767, 8, 3768, 2638, 2664, 12, 2660, 618, 204}

$$-\frac{6\left(a^2+b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4b^3d}+\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^3d}+\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{a^3b^2d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*x)/b^3 + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(a^2*b^3*d) - (6*(a^2 - b^2)^(3/2)*(a^2 + b^2)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*b^3*d) - ArcTanh[Cos[c + d*x]]/(2*a^2*
d) + (3*(a^2 - b^2)*ArcTanh[Cos[c + d*x]])/(a^4*d) + Cos[c + d*x]/(b^2*d) +
 (2*b*Cot[c + d*x])/(a^3*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) + ((a^2
 - b^2)^2*Cos[c + d*x])/(a^3*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(\frac{2a}{b^3} - \frac{3\left(a^2-b^2\right)\csc(c+dx)}{a^4} - \frac{2b\csc^2(c+dx)}{a^3} + \frac{\csc^3(c+dx)}{a^2} - \frac{\sin(c+\right.$$

$$= \frac{2ax}{b^3} + \frac{\int \csc^3(c+dx)\,dx}{a^2} - \frac{\int \sin(c+dx)\,dx}{b^2} - \frac{(2b)\int \csc^2(c+dx)\,dx}{a^3} - \frac{\left(3\left(a\right.\right.}{}$$

$$= \frac{2ax}{b^3} + \frac{3\left(a^2-b^2\right)\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{b^2d} - \frac{\cot(c+dx)\csc(c+d}{2a^2d}$$

$$= \frac{2ax}{b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} + \frac{3\left(a^2-b^2\right)\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{b^2d}$$

$$= \frac{2ax}{b^3} - \frac{6\left(a^2-b^2\right)^{3/2}\left(a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^3d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}$$

$$= \frac{2ax}{b^3} - \frac{6\left(a^2-b^2\right)^{3/2}\left(a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^3d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}$$

$$= \frac{2ax}{b^3} + \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^3d} - \frac{6\left(a^2-b^2\right)^{3/2}\left(a^2+b^2\right)\tan^{-1}\left(\right.}{a^4b^3d}$$

Mathematica [A] time = 6.19, size = 315, normalized size = 1.25

$$-\frac{b\tan\left(\frac{1}{2}(c+dx)\right)}{a^3d}+\frac{b\cot\left(\frac{1}{2}(c+dx)\right)}{a^3d}-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d}+\frac{\left(6b^2-5a^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*(c + d*x))/(b^3*d) - (2*(a^2 - b^2)^(3/2)*(2*a^2 + 3*b^2)*ArcTan[(Sec[
(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(
a^4*b^3*d) + Cos[c + d*x]/(b^2*d) + (b*Cot[(c + d*x)/2])/(a^3*d) - Csc[(c +
 d*x)/2]^2/(8*a^2*d) + ((5*a^2 - 6*b^2)*Log[Cos[(c + d*x)/2]])/(2*a^4*d) +
((-5*a^2 + 6*b^2)*Log[Sin[(c + d*x)/2]])/(2*a^4*d) + Sec[(c + d*x)/2]^2/(8*
a^2*d) + (a^4*Cos[c + d*x] - 2*a^2*b^2*Cos[c + d*x] + b^4*Cos[c + d*x])/(a^
3*b^2*d*(a + b*Sin[c + d*x])) - (b*Tan[(c + d*x)/2])/(a^3*d)
```

fricas [B] time = 1.74, size = 1210, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/4*(8*a^6*d*x*cos(d*x + c)^2 - 8*a^6*d*x + 4*(2*a^5*b - 2*a^3*b^3 + 3*a*b
^5)*cos(d*x + c)^3 + 2*(2*a^5 + a^3*b^2 - 3*a*b^4 - (2*a^5 + a^3*b^2 - 3*a*
b^4)*cos(d*x + c)^2 + (2*a^4*b + a^2*b^3 - 3*b^5 - (2*a^4*b + a^2*b^3 - 3*b
^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c
) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2)) - 2*(4*a^5*b - 5*a^3*b^3 + 6*a*b^5)*cos(d*x + c) - (5*a^3
*b^3 - 6*a*b^5 - (5*a^3*b^3 - 6*a*b^5)*cos(d*x + c)^2 + (5*a^2*b^4 - 6*b^6
- (5*a^2*b^4 - 6*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) +
1/2) + (5*a^3*b^3 - 6*a*b^5 - (5*a^3*b^3 - 6*a*b^5)*cos(d*x + c)^2 + (5*a^2
*b^4 - 6*b^6 - (5*a^2*b^4 - 6*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*c
os(d*x + c) + 1/2) + 2*(4*a^5*b*d*x*cos(d*x + c)^2 + 2*a^4*b^2*cos(d*x + c)
^3 - 4*a^5*b*d*x - (2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(a^5
*b^3*d*cos(d*x + c)^2 - a^5*b^3*d + (a^4*b^4*d*cos(d*x + c)^2 - a^4*b^4*d)*
sin(d*x + c)), 1/4*(8*a^6*d*x*cos(d*x + c)^2 - 8*a^6*d*x + 4*(2*a^5*b - 2*a
^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 - 4*(2*a^5 + a^3*b^2 - 3*a*b^4 - (2*a^5 +
a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + (2*a^4*b + a^2*b^3 - 3*b^5 - (2*a^4*b +
 a^2*b^3 - 3*b^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*
sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 2*(4*a^5*b - 5*a^3*b^3
```

```
+ 6*a*b^5)*cos(d*x + c) - (5*a^3*b^3 - 6*a*b^5 - (5*a^3*b^3 - 6*a*b^5)*cos(
d*x + c)^2 + (5*a^2*b^4 - 6*b^6 - (5*a^2*b^4 - 6*b^6)*cos(d*x + c)^2)*sin(d
*x + c))*log(1/2*cos(d*x + c) + 1/2) + (5*a^3*b^3 - 6*a*b^5 - (5*a^3*b^3 -
6*a*b^5)*cos(d*x + c)^2 + (5*a^2*b^4 - 6*b^6 - (5*a^2*b^4 - 6*b^6)*cos(d*x
+ c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(4*a^5*b*d*x*cos(d*x
 + c)^2 + 2*a^4*b^2*cos(d*x + c)^3 - 4*a^5*b*d*x - (2*a^4*b^2 + 3*a^2*b^4)*
cos(d*x + c))*sin(d*x + c))/(a^5*b^3*d*cos(d*x + c)^2 - a^5*b^3*d + (a^4*b^
4*d*cos(d*x + c)^2 - a^4*b^4*d)*sin(d*x + c))]
```

giac [A] time = 0.28, size = 463, normalized size = 1.84

$$\frac{\frac{16\,(dx+c)a}{b^3} - \frac{4\left(5\,a^2-6\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4} + \frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4} + \frac{30\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-36\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,ab\,\mathrm{t}}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(16*(d*x + c)*a/b^3 - 4*(5*a^2 - 6*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/
a^4 + (a^2*tan(1/2*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d*x + 1/2*c))/a^4 + (30*a
^2*tan(1/2*d*x + 1/2*c)^2 - 36*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a*b*tan(1/2*d
*x + 1/2*c) - a^2)/(a^4*tan(1/2*d*x + 1/2*c)^2) - 16*(2*a^6 - a^4*b^2 - 4*a
^2*b^4 + 3*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/
2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4*b^3) + 16*(a^4*b
*tan(1/2*d*x + 1/2*c)^3 - 2*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + b^5*tan(1/2*d*
x + 1/2*c)^3 + 2*a^5*tan(1/2*d*x + 1/2*c)^2 - 2*a^3*b^2*tan(1/2*d*x + 1/2*c
)^2 + a*b^4*tan(1/2*d*x + 1/2*c)^2 + 3*a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2*b
^3*tan(1/2*d*x + 1/2*c) + b^5*tan(1/2*d*x + 1/2*c) + 2*a^5 - 2*a^3*b^2 + a*
b^4)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*
d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^4*b^2))/d
```

maple [B] time = 0.81, size = 618, normalized size = 2.46

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2d} - \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{d\,a^3} + \frac{2}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} + \frac{4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^3} - \frac{1}{8a^2d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2} - \frac{5\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^3*tan(1/2*d*x+1/2*c)*b+2/d/b^2/(1+tan(
1/2*d*x+1/2*c)^2)+4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a-1/8/a^2/d/tan(1/2*d*
x+1/2*c)^2-5/2/d/a^2*ln(tan(1/2*d*x+1/2*c))+3/d/a^4*ln(tan(1/2*d*x+1/2*c))*
b^2+1/d*b/a^3/tan(1/2*d*x+1/2*c)+2/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-4/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/a^4*b^3/(tan(1/2*d*x+1/2*c)^2*a+2*
tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)*a-4/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)+2/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-4/d*
a^2/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2))+2/d/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^
2)^(1/2))+8/d/a^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))-6/d/a^4*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1
/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.90, size = 4294, normalized size = 17.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^3*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) + ((tan(c/2 + (d*x)/2)^2*(16*a^4 + 16*b^4 -
17*a^2*b^2))/b^2 - a^2/2 + (tan(c/2 + (d*x)/2)^4*(32*a^4 + 32*b^4 - 33*a^2*
b^2))/(2*b^2) + 3*a*b*tan(c/2 + (d*x)/2) + (4*tan(c/2 + (d*x)/2)^5*(2*a^4 +
 2*b^4 - 3*a^2*b^2))/(a*b) + (tan(c/2 + (d*x)/2)^3*(24*a^4 + 8*b^4 - 9*a^2*
b^2))/(a*b))/(d*(4*a^4*tan(c/2 + (d*x)/2)^2 + 8*a^4*tan(c/2 + (d*x)/2)^4 +
4*a^4*tan(c/2 + (d*x)/2)^6 + 8*a^3*b*tan(c/2 + (d*x)/2)^3 + 8*a^3*b*tan(c/2
 + (d*x)/2)^5)) - (b*tan(c/2 + (d*x)/2))/(a^3*d) - (log(tan(c/2 + (d*x)/2))
*(5*a^2 - 6*b^2))/(2*a^4*d) + (4*a*atan((2560*a)/((4800*a^2)/b - 2560*a*tan
(c/2 + (d*x)/2) - 12160*b + (4480*b^3)/a^2 + (11520*b^5)/a^4 - (12096*b^7)/
a^6 + (3456*b^9)/a^8 + (6144*b^2*tan(c/2 + (d*x)/2))/a - (5120*a^3*tan(c/2
```

+ (d*x)/2))/b^2 - (2304*b^4*tan(c/2 + (d*x)/2))/a^3 + (3840*a^5*tan(c/2 + (d*x)/2))/b^4) - (12160*tan(c/2 + (d*x)/2))/((4480*b^2)/a^2 + (4800*a^2)/b^2 + (11520*b^4)/a^4 - (12096*b^6)/a^6 + (3456*b^8)/a^8 - (2560*a*tan(c/2 + (d*x)/2))/b + (6144*b*tan(c/2 + (d*x)/2))/a - (2304*b^3*tan(c/2 + (d*x)/2))/a^3 - (5120*a^3*tan(c/2 + (d*x)/2))/b^3 + (3840*a^5*tan(c/2 + (d*x)/2))/b^5 - 12160) - 6144/(6144*tan(c/2 + (d*x)/2) - (12160*a)/b + (4480*b)/a + (11520*b^3)/a^3 + (4800*a^3)/b^3 - (12096*b^5)/a^5 + (3456*b^7)/a^7 - (2304*b^2*tan(c/2 + (d*x)/2))/a^2 - (2560*a^2*tan(c/2 + (d*x)/2))/b^2 - (5120*a^4*tan(c/2 + (d*x)/2))/b^4 + (3840*a^6*tan(c/2 + (d*x)/2))/b^6) + (5120*a^3)/(4800*a^2*b - 12160*b^3 + (4480*b^5)/a^2 + (11520*b^7)/a^4 - (12096*b^9)/a^6 + (3456*b^11)/a^8 - 5120*a^3*tan(c/2 + (d*x)/2) - 2560*a*b^2*tan(c/2 + (d*x)/2) + (6144*b^4*tan(c/2 + (d*x)/2))/a + (3840*a^5*tan(c/2 + (d*x)/2))/b^2 - (2304*b^6*tan(c/2 + (d*x)/2))/a^3) + (2304*b^2)/(4480*a*b + (11520*b^3)/a - (12160*a^3)/b - (12096*b^5)/a^3 + (4800*a^5)/b^3 + (3456*b^7)/a^5 + 6144*a^2*tan(c/2 + (d*x)/2) - 2304*b^2*tan(c/2 + (d*x)/2) - (2560*a^4*tan(c/2 + (d*x)/2))/b^2 - (5120*a^6*tan(c/2 + (d*x)/2))/b^4 + (3840*a^8*tan(c/2 + (d*x)/2))/b^6) - (3840*a^5)/(4800*a^2*b^3 - 12160*b^5 + (4480*b^7)/a^2 + (11520*b^9)/a^4 - (12096*b^11)/a^6 + (3456*b^13)/a^8 + 3840*a^5*tan(c/2 + (d*x)/2) - 2560*a*b^4*tan(c/2 + (d*x)/2) - 5120*a^3*b^2*tan(c/2 + (d*x)/2) + (6144*b^6*tan(c/2 + (d*x)/2))/a - (2304*b^8*tan(c/2 + (d*x)/2))/a^3) + (4800*a^2*tan(c/2 + (d*x)/2))/(4800*a^2 - 12160*b^2 + (4480*b^4)/a^2 + (11520*b^6)/a^4 - (12096*b^8)/a^6 + (3456*b^10)/a^8 + (6144*b^3*tan(c/2 + (d*x)/2))/a - (5120*a^3*tan(c/2 + (d*x)/2))/b - (2304*b^5*tan(c/2 + (d*x)/2))/a^3 + (3840*a^5*tan(c/2 + (d*x)/2))/b^3 - 2560*a*b*tan(c/2 + (d*x)/2)) + (11520*b^3*tan(c/2 + (d*x)/2))/(4480*a^2*b + 11520*b^3 - (12160*a^4)/b - (12096*b^5)/a^2 + (4800*a^6)/b^3 + (3456*b^7)/a^4 + 6144*a^3*tan(c/2 + (d*x)/2) - 2304*a*b^2*tan(c/2 + (d*x)/2) - (2560*a^5*tan(c/2 + (d*x)/2))/b^2 - (5120*a^7*tan(c/2 + (d*x)/2))/b^4 + (3840*a^9*tan(c/2 + (d*x)/2))/b^6) - (12096*b^5*tan(c/2 + (d*x)/2))/(4480*a^4*b - 12096*b^5 + 11520*a^2*b^3 - (12160*a^6)/b + (3456*b^7)/a^2 + (4800*a^8)/b^3 + 6144*a^5*tan(c/2 + (d*x)/2) - 2304*a^3*b^2*tan(c/2 + (d*x)/2) - (2560*a^7*tan(c/2 + (d*x)/2))/b^2 - (5120*a^9*tan(c/2 + (d*x)/2))/b^4 + (3840*a^11*tan(c/2 + (d*x)/2))/b^6) + (3456*b^7*tan(c/2 + (d*x)/2))/(4480*a^6*b + 3456*b^7 - 12096*a^2*b^5 + 11520*a^4*b^3 - (12160*a^8)/b + (4800*a^10)/b^3 + 6144*a^7*tan(c/2 + (d*x)/2) - 2304*a^5*b^2*tan(c/2 + (d*x)/2) - (2560*a^9*tan(c/2 + (d*x)/2))/b^2 - (5120*a^11*tan(c/2 + (d*x)/2))/b^4 + (3840*a^13*tan(c/2 + (d*x)/2))/b^6) + (4480*b*tan(c/2 + (d*x)/2))/(4480*b + 6144*a*tan(c/2 + (d*x)/2) - (12160*a^2)/b + (11520*b^3)/a^2 + (4800*a^4)/b^3 - (12096*b^5)/a^4 + (3456*b^7)/a^6 - (2304*b^2*tan(c/2 + (d*x)/2))/a - (2560*a^3*tan(c/2 + (d*x)/2))/b^2 - (5120*a^5*tan(c/2 + (d*x)/2))/b^4 + (3840*a^7*tan(c/2 + (d*x)/2))/b^6)))/(b^3*d) - (atan((((2*a^2 + 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((16*(56*a^14 + 36*a^4*b^10 - 96*a^6*b^8 + 232*a^8*b^6 - 224*a^10*b^4))/(a^8*b^5) + (16*tan(c/2 + (d*x)/2)*(54*b^18 - 189*a^2*b^16 + 180*a^4*b^14 + 70*a^6*b^12 + 194*a^8*b^10 - 653*a^10*b^8 + 292*a^12*b^6 + 120*a^14*b^4 - 64*a^16*b^2))/(a^9*b^8) + ((2*a^2 + 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((16*(72*a^2*b^14 - 174*a^4*b^12 + 95*a^6*b^10

+ 42*a^8*b^8 - 35*a^10*b^6 + 32*a^12*b^4 - 24*a^14*b^2))/(a^8*b^5) + (16*t
an(c/2 + (d*x)/2)*(18*a^4*b^16 + 60*a^6*b^14 - 220*a^8*b^12 + 132*a^10*b^10
 + 202*a^12*b^8 - 200*a^14*b^6 + 16*a^16*b^4))/(a^9*b^8) + ((2*a^2 + 3*b^2)
(-(a + b)^3(a - b)^3)^(1/2)*((16*(48*a^6*b^12 - 76*a^8*b^10 + 15*a^10*b^8
 + 14*a^12*b^6))/(a^8*b^5) + (16*tan(c/2 + (d*x)/2)*(96*a^6*b^16 - 182*a^8*
b^14 + 73*a^10*b^12 + 30*a^12*b^10 - 16*a^14*b^8))/(a^9*b^8) + (((16*(8*a^1
0*b^10 - 6*a^12*b^8))/(a^8*b^5) + (16*tan(c/2 + (d*x)/2)*(32*a^10*b^14 - 34
*a^12*b^12 + 4*a^14*b^10))/(a^9*b^8))*(2*a^2 + 3*b^2)*(-(a + b)^3*(a - b)^3
)^(1/2))/(a^4*b^3)))/(a^4*b^3)))/(a^4*b^3))*1i)/(a^4*b^3) + ((2*a^2 + 3*b^2
)*(-(a + b)^3*(a - b)^3)^(1/2)*((16*(56*a^14 + 36*a^4*b^10 - 96*a^6*b^8 + 2
32*a^8*b^6 - 224*a^10*b^4))/(a^8*b^5) + (16*tan(c/2 + (d*x)/2)*(54*b^18 - 1
89*a^2*b^16 + 180*a^4*b^14 + 70*a^6*b^12 + 194*a^8*b^10 - 653*a^10*b^8 + 29
2*a^12*b^6 + 120*a^14*b^4 - 64*a^16*b^2))/(a^9*b^8) - ((2*a^2 + 3*b^2)*(-(a
 + b)^3*(a - b)^3)^(1/2)*((16*(72*a^2*b^14 - 174*a^4*b^12 + 95*a^6*b^10 + 4
2*a^8*b^8 - 35*a^10*b^6 + 32*a^12*b^4 - 24*a^14*b^2))/(a^8*b^5) + (16*tan(c
/2 + (d*x)/2)*(18*a^4*b^16 + 60*a^6*b^14 - 220*a^8*b^12 + 132*a^10*b^10 + 2
02*a^12*b^8 - 200*a^14*b^6 + 16*a^16*b^4))/(a^9*b^8) - ((2*a^2 + 3*b^2)*(-(
a + b)^3*(a - b)^3)^(1/2)*((16*(48*a^6*b^12 - 76*a^8*b^10 + 15*a^10*b^8 + 1
4*a^12*b^6))/(a^8*b^5) + (16*tan(c/2 + (d*x)/2)*(96*a^6*b^16 - 182*a^8*b^14
 + 73*a^10*b^12 + 30*a^12*b^10 - 16*a^14*b^8))/(a^9*b^8) - (((16*(8*a^10*b^
10 - 6*a^12*b^8))/(a^8*b^5) + (16*tan(c/2 + (d*x)/2)*(32*a^10*b^14 - 34*a^1
2*b^12 + 4*a^14*b^10))/(a^9*b^8))*(2*a^2 + 3*b^2)*(-(a + b)^3*(a - b)^3)^(1
/2))/(a^4*b^3)))/(a^4*b^3)))/(a^4*b^3))*1i)/(a^4*b^3))/((32*(140*a^12 - 108
*b^12 + 378*a^2*b^10 - 648*a^4*b^8 + 556*a^6*b^6 - 318*a^10*b^2))/(a^8*b^5)
 - (32*tan(c/2 + (d*x)/2)*(432*a^8*b^8 - 256*a^16 - 960*a^10*b^6 + 368*a^12
*b^4 + 416*a^14*b^2))/(a^9*b^8) - ((2*a^2 + 3*b^2)*(-(a + b)^3*(a - b)^3)^(
1/2)*((16*(56*a^14 + 36*a^4*b^10 - 96*a^6*b^8 + 232*a^8*b^6 - 224*a^10*b^4)
)/(a^8*b^5) + (16*tan(c/2 + (d*x)/2)*(54*b^18 - 189*a^2*b^16 + 180*a^4*b^14
 + 70*a^6*b^12 + 194*a^8*b^10 - 653*a^10*b^8 + 292*a^12*b^6 + 120*a^14*b^4
 - 64*a^16*b^2))/(a^9*b^8) + ((2*a^2 + 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(
(16*(72*a^2*b^14 - 174*a^4*b^12 + 95*a^6*b^10 + 42*a^8*b^8 - 35*a^10*b^6 +
32*a^12*b^4 - 24*a^14*b^2))/(a^8*b^5) + (16*tan(c/2 + (d*x)/2)*(18*a^4*b^16
 + 60*a^6*b^14 - 220*a^8*b^12 + 132*a^10*b^10 + 202*a^12*b^8 - 200*a^14*b^6
 + 16*a^16*b^4))/(a^9*b^8) + ((2*a^2 + 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*
((16*(48*a^6*b^12 - 76*a^8*b^10 + 15*a^10*b^8 + 14*a^12*b^6))/(a^8*b^5) + (
16*tan(c/2 + (d*x)/2)*(96*a^6*b^16 - 182*a^8*b^14 + 73*a^10*b^12 + 30*a^12*
b^10 - 16*a^14*b^8))/(a^9*b^8) + (((16*(8*a^10*b^10 - 6*a^12*b^8))/(a^8*b^5
) + (16*tan(c/2 + (d*x)/2)*(32*a^10*b^14 - 34*a^12*b^12 + 4*a^14*b^10))/(a^
9*b^8))*(2*a^2 + 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^4*b^3)))/(a^4*b^3)
))/(a^4*b^3)))/(a^4*b^3) + ((2*a^2 + 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((
16*(56*a^14 + 36*a^4*b^10 - 96*a^6*b^8 + 232*a^8*b^6 - 224*a^10*b^4))/(a^8*
b^5) + (16*tan(c/2 + (d*x)/2)*(54*b^18 - 189*a^2*b^16 + 180*a^4*b^14 + 70*a
^6*b^12 + 194*a^8*b^10 - 653*a^10*b^8 + 292*a^12*b^6 + 120*a^14*b^4 - 64*a^
16*b^2))/(a^9*b^8) - ((2*a^2 + 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((16*(72
*a^2*b^14 - 174*a^4*b^12 + 95*a^6*b^10 + 42*a^8*b^8 - 35*a^10*b^6 + 32*a^12

```
*b^4 - 24*a^14*b^2))/(a^8*b^5) + (16*tan(c/2 + (d*x)/2)*(18*a^4*b^16 + 60*a
^6*b^14 - 220*a^8*b^12 + 132*a^10*b^10 + 202*a^12*b^8 - 200*a^14*b^6 + 16*a
^16*b^4))/(a^9*b^8) - ((2*a^2 + 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((16*(4
8*a^6*b^12 - 76*a^8*b^10 + 15*a^10*b^8 + 14*a^12*b^6))/(a^8*b^5) + (16*tan(
c/2 + (d*x)/2)*(96*a^6*b^16 - 182*a^8*b^14 + 73*a^10*b^12 + 30*a^12*b^10 -
16*a^14*b^8))/(a^9*b^8) - (((16*(8*a^10*b^10 - 6*a^12*b^8))/(a^8*b^5) + (16
*tan(c/2 + (d*x)/2)*(32*a^10*b^14 - 34*a^12*b^12 + 4*a^14*b^10))/(a^9*b^8))
*(2*a^2 + 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^4*b^3)))/(a^4*b^3)))/(a^4
*b^3)))/(a^4*b^3)))*(2*a^2 + 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^4*b
^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1262 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=287

$$\frac{b\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b\cot(c+dx)\csc(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d}$$

```
[Out] -x/b^2-2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a
^3/b^2/d+b*arctanh(cos(d*x+c))/a^3/d-2*b*(3*a^2-2*b^2)*arctanh(cos(d*x+c))/
a^5/d-cot(d*x+c)/a^2/d+3*(a^2-b^2)*cot(d*x+c)/a^4/d-1/3*cot(d*x+c)^3/a^2/d+
b*cot(d*x+c)*csc(d*x+c)/a^3/d-(a^2-b^2)^2*cos(d*x+c)/a^4/b/d/(a+b*sin(d*x+c
))+4*(a^6-3*a^2*b^4+2*b^6)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))
/a^5/b^2/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2897, 3770, 3767, 8, 3768, 2664, 12, 2660, 618, 204}

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^2d}+\frac{4\left(-3a^2b^4+a^6+2b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5b^2d\sqrt{a^2-b^2}}+\frac{3\left(a^2-b^2\right)\cot(c+dx)}{a^4d}-\frac{(a}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(x/b^2) - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(a^3*b^2*d) + (4*(a^6 - 3*a^2*b^4 + 2*b^6)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(a^5*b^2*Sqrt[a^2 - b^2]*d) + (b*ArcTanh[Cos[c + d
*x]])/(a^3*d) - (2*b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(a^5*d) - Cot[c
 + d*x]/(a^2*d) + (3*(a^2 - b^2)*Cot[c + d*x])/(a^4*d) - Cot[c + d*x]^3/(3*
a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(a^3*d) - ((a^2 - b^2)^2*Cos[c + d*x
])/(a^4*b*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int\left(-\frac{1}{b^2}+\frac{2\left(3a^2b-2b^3\right)\csc(c+dx)}{a^5}-\frac{3\left(a^2-b^2\right)\csc^2(c+dx)}{a^4}-\frac{2b\csc^3(c+\right.$$

$$= -\frac{x}{b^2}+\frac{\int\csc^4(c+dx)\,dx}{a^2}-\frac{(2b)\int\csc^3(c+dx)\,dx}{a^3}+\frac{\left(2b\left(3a^2-2b^2\right)\right)\int\csc(c+}{a^5}$$

$$= -\frac{x}{b^2}-\frac{2b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{b\cot(c+dx)\csc(c+dx)}{a^3d}-\frac{\left(a^2-\right.}{a^4bd(}$$

$$= -\frac{x}{b^2}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d}-\frac{\cot(c+}{a^2d}$$

$$= -\frac{x}{b^2}+\frac{4\left(a^6-3a^2b^4+2b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^2\sqrt{a^2-b^2}\,d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^3d}-$$

$$= -\frac{x}{b^2}+\frac{4\left(a^6-3a^2b^4+2b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^2\sqrt{a^2-b^2}\,d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^3d}-$$

$$= -\frac{x}{b^2}-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^2d}+\frac{4\left(a^6-3a^2b^4+2b^6\right)\tan^{-1}\left(\frac{b+a\,\mathrm{ta}}{}\right.}{a^5b^2\sqrt{a^2-b^2}\,d}$$

Mathematica [A] time = 6.30, size = 428, normalized size = 1.49

$$\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d}-\frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d}+\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d}+\frac{(5a^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((c + d*x)/(b^2*d)) + (2*(a^2 - b^2)^(3/2)*(a^2 + 4*b^2)*ArcTan[(Sec[(c +
d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*b
^2*d) + ((7*a^2*Cos[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2]
)/(6*a^4*d) + (b*Csc[(c + d*x)/2]^2)/(4*a^3*d) - (Cot[(c + d*x)/2]*Csc[(c +
 d*x)/2]^2)/(24*a^2*d) + ((-5*a^2*b + 4*b^3)*Log[Cos[(c + d*x)/2]])/(a^5*d)
 + ((5*a^2*b - 4*b^3)*Log[Sin[(c + d*x)/2]])/(a^5*d) - (b*Sec[(c + d*x)/2]^
2)/(4*a^3*d) + (Sec[(c + d*x)/2]*(-7*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c +
d*x)/2]))/(6*a^4*d) + (-(a^4*Cos[c + d*x]) + 2*a^2*b^2*Cos[c + d*x] - b^4*C
os[c + d*x])/(a^4*b*d*(a + b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/(24*a^2*d)
```

fricas [B] time = 1.53, size = 1437, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/6*(6*a^5*b*d*x*cos(d*x + c)^4 - 12*a^5*b*d*x*cos(d*x + c)^2 + 6*a^5*b*d
*x + 2*(7*a^4*b^2 - 6*a^2*b^4)*cos(d*x + c)^3 + 3*(a^4*b + 3*a^2*b^3 - 4*b^
5 + (a^4*b + 3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 3*a^2*b^3 - 4*b
^5)*cos(d*x + c)^2 + (a^5 + 3*a^3*b^2 - 4*a*b^4 - (a^5 + 3*a^3*b^2 - 4*a*b^
4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c)
+ b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c
) - a^2 - b^2)) - 12*(a^4*b^2 - a^2*b^4)*cos(d*x + c) + 3*(5*a^2*b^4 - 4*b^
6 + (5*a^2*b^4 - 4*b^6)*cos(d*x + c)^4 - 2*(5*a^2*b^4 - 4*b^6)*cos(d*x + c)
^2 + (5*a^3*b^3 - 4*a*b^5 - (5*a^3*b^3 - 4*a*b^5)*cos(d*x + c)^2)*sin(d*x +
 c))*log(1/2*cos(d*x + c) + 1/2) - 3*(5*a^2*b^4 - 4*b^6 + (5*a^2*b^4 - 4*b^
6)*cos(d*x + c)^4 - 2*(5*a^2*b^4 - 4*b^6)*cos(d*x + c)^2 + (5*a^3*b^3 - 4*a
*b^5 - (5*a^3*b^3 - 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x
 + c) + 1/2) - 2*(3*a^6*d*x*cos(d*x + c)^2 - 3*a^6*d*x + (3*a^5*b - 13*a^3*
b^3 + 12*a*b^5)*cos(d*x + c)^3 - 3*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x +
c))*sin(d*x + c))/(a^5*b^3*d*cos(d*x + c)^4 - 2*a^5*b^3*d*cos(d*x + c)^2 +
a^5*b^3*d - (a^6*b^2*d*cos(d*x + c)^2 - a^6*b^2*d)*sin(d*x + c)), -1/6*(6*a
^5*b*d*x*cos(d*x + c)^4 - 12*a^5*b*d*x*cos(d*x + c)^2 + 6*a^5*b*d*x + 2*(7*
a^4*b^2 - 6*a^2*b^4)*cos(d*x + c)^3 + 6*(a^4*b + 3*a^2*b^3 - 4*b^5 + (a^4*b
 + 3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 3*a^2*b^3 - 4*b^5)*cos(d*
x + c)^2 + (a^5 + 3*a^3*b^2 - 4*a*b^4 - (a^5 + 3*a^3*b^2 - 4*a*b^4)*cos(d*x
 + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a
```

```
^2 - b^2)*cos(d*x + c))) - 12*(a^4*b^2 - a^2*b^4)*cos(d*x + c) + 3*(5*a^2*b
^4 - 4*b^6 + (5*a^2*b^4 - 4*b^6)*cos(d*x + c)^4 - 2*(5*a^2*b^4 - 4*b^6)*cos
(d*x + c)^2 + (5*a^3*b^3 - 4*a*b^5 - (5*a^3*b^3 - 4*a*b^5)*cos(d*x + c)^2)*
sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(5*a^2*b^4 - 4*b^6 + (5*a^2*b
^4 - 4*b^6)*cos(d*x + c)^4 - 2*(5*a^2*b^4 - 4*b^6)*cos(d*x + c)^2 + (5*a^3*
b^3 - 4*a*b^5 - (5*a^3*b^3 - 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/
2*cos(d*x + c) + 1/2) - 2*(3*a^6*d*x*cos(d*x + c)^2 - 3*a^6*d*x + (3*a^5*b
- 13*a^3*b^3 + 12*a*b^5)*cos(d*x + c)^3 - 3*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*c
os(d*x + c))*sin(d*x + c))/(a^5*b^3*d*cos(d*x + c)^4 - 2*a^5*b^3*d*cos(d*x
+ c)^2 + a^5*b^3*d - (a^6*b^2*d*cos(d*x + c)^2 - a^6*b^2*d)*sin(d*x + c))]
```

giac [A] time = 0.28, size = 399, normalized size = 1.39

$$-\frac{\frac{24\,(dx+c)}{b^2}-\frac{24\left(5\,a^2b-4\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^5}-\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-27\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}-\frac{48}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(24*(d*x + c)/b^2 - 24*(5*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)
))/a^5 - (a^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 27*
a^4*tan(1/2*d*x + 1/2*c) + 36*a^2*b^2*tan(1/2*d*x + 1/2*c))/a^6 - 48*(a^6 +
 2*a^4*b^2 - 7*a^2*b^4 + 4*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) +
arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5*
b^2) + 48*(a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) + b^
5*tan(1/2*d*x + 1/2*c) + a^5 - 2*a^3*b^2 + a*b^4)/((a*tan(1/2*d*x + 1/2*c)^
2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^5*b) + (220*a^2*b*tan(1/2*d*x + 1/2*c)^
3 - 176*b^3*tan(1/2*d*x + 1/2*c)^3 - 27*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a*b
^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^5*tan(1/
2*d*x + 1/2*c)^3))/d
```

maple [B] time = 0.77, size = 678, normalized size = 2.36

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^2}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{4d\,a^3}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^2}+\frac{3b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^4}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2}-\frac{1}{24a^2d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^3*tan(1/2*d*x+1/2*c)^2*b-9/8/d/a^2*
tan(1/2*d*x+1/2*c)+3/2/d/a^4*b^2*tan(1/2*d*x+1/2*c)-2/d/b^2*arctan(tan(1/2*
d*x+1/2*c))-1/24/a^2/d/tan(1/2*d*x+1/2*c)^3+9/8/d/a^2/tan(1/2*d*x+1/2*c)-3/
2/d/a^4/tan(1/2*d*x+1/2*c)*b^2+1/4/d/a^3*b/tan(1/2*d*x+1/2*c)^2+5/d/a^3*b*l
n(tan(1/2*d*x+1/2*c))-4/d/a^5*b^3*ln(tan(1/2*d*x+1/2*c))-2/d/(tan(1/2*d*x+1
/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)/a*tan(1/2*d*x+1/2*c)+4/d/a^3*b^2/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-2/d/a^5*b^4/
(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-2/d/b/
(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+4/d/a^2*b/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-2/d/a^4*b^3/(tan(1/2*d*x+1/2*c)^2*a+2*ta
n(1/2*d*x+1/2*c)*b+a)+2/d/b^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x
+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-14/d/a^3*b^2/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+8/d/a^5*b^4/(a^2-b^2)^(1/2)*ar
ctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.78, size = 4740, normalized size = 16.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^4*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^2*d) - (tan(c/2 + (d*x)/2)*((24*a^2 + 32*b^2)/(6
4*a^4) + 3/(4*a^2) - (2*b^2)/a^4))/d - (tan(c/2 + (d*x)/2)^2*(8*a*b^2 - (26
*a^3)/3) + a^3/3 - (4*a^2*b*tan(c/2 + (d*x)/2))/3 + (tan(c/2 + (d*x)/2)^4*(
7*a^4 + 16*b^4 - 20*a^2*b^2))/a + (4*tan(c/2 + (d*x)/2)^3*(4*a^4 + 10*b^4 -
 13*a^2*b^2))/b)/(d*(8*a^5*tan(c/2 + (d*x)/2)^3 + 8*a^5*tan(c/2 + (d*x)/2)^
5 + 16*a^4*b*tan(c/2 + (d*x)/2)^4)) + (2*atan((((((((((32*(4*a^14*b^7 - 3*a
^16*b^5))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(16*a^13*b^10 - 17*a^15*b^8 +
 2*a^17*b^6))/(a^12*b^4))*1i)/b^2 - (32*(32*a^9*b^10 - 64*a^11*b^8 + 30*a^1
3*b^6 + a^15*b^4))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(148*a^10*b^11 - 64*
```

a^8*b^13 - 97*a^12*b^9 + 10*a^14*b^7 + 4*a^16*b^5))/(a^12*b^4))*1i)/b^2 - (
32*(3*a^16*b - 64*a^4*b^13 + 208*a^6*b^11 - 220*a^8*b^9 + 71*a^10*b^7 + 5*a
^12*b^5 - 4*a^14*b^3))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(16*a^5*b^14 - 4
0*a^7*b^12 + 5*a^9*b^10 + 44*a^11*b^8 - 6*a^13*b^6 - 20*a^15*b^4 + 2*a^17*b
^2))/(a^12*b^4))*1i)/b^2 - (32*(6*a^15 + 16*a^5*b^10 - 56*a^7*b^8 + 97*a^9*
b^6 - 84*a^11*b^4 + 20*a^13*b^2))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(4*a^
16*b - 64*b^17 + 304*a^2*b^15 - 540*a^4*b^13 + 405*a^6*b^11 - 124*a^8*b^9 +
78*a^10*b^7 - 72*a^12*b^5 + 10*a^14*b^3))/(a^12*b^4))/b^2 - ((((((((32*(4*
a^14*b^7 - 3*a^16*b^5))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(16*a^13*b^10 -
17*a^15*b^8 + 2*a^17*b^6))/(a^12*b^4))*1i)/b^2 + (32*(32*a^9*b^10 - 64*a^1
1*b^8 + 30*a^13*b^6 + a^15*b^4))/(a^12*b^2) - (32*tan(c/2 + (d*x)/2)*(148*a
^10*b^11 - 64*a^8*b^13 - 97*a^12*b^9 + 10*a^14*b^7 + 4*a^16*b^5))/(a^12*b^4
))*1i)/b^2 - (32*(3*a^16*b - 64*a^4*b^13 + 208*a^6*b^11 - 220*a^8*b^9 + 71*
a^10*b^7 + 5*a^12*b^5 - 4*a^14*b^3))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(1
6*a^5*b^14 - 40*a^7*b^12 + 5*a^9*b^10 + 44*a^11*b^8 - 6*a^13*b^6 - 20*a^15*
b^4 + 2*a^17*b^2))/(a^12*b^4))*1i)/b^2 + (32*(6*a^15 + 16*a^5*b^10 - 56*a^7
*b^8 + 97*a^9*b^6 - 84*a^11*b^4 + 20*a^13*b^2))/(a^12*b^2) - (32*tan(c/2 +
(d*x)/2)*(4*a^16*b - 64*b^17 + 304*a^2*b^15 - 540*a^4*b^13 + 405*a^6*b^11 -
124*a^8*b^9 + 78*a^10*b^7 - 72*a^12*b^5 + 10*a^14*b^3))/(a^12*b^4))/b^2)/(
((((((((32*(4*a^14*b^7 - 3*a^16*b^5))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*
(16*a^13*b^10 - 17*a^15*b^8 + 2*a^17*b^6))/(a^12*b^4))*1i)/b^2 - (32*(32*a^
9*b^10 - 64*a^11*b^8 + 30*a^13*b^6 + a^15*b^4))/(a^12*b^2) + (32*tan(c/2 +
(d*x)/2)*(148*a^10*b^11 - 64*a^8*b^13 - 97*a^12*b^9 + 10*a^14*b^7 + 4*a^16*
b^5))/(a^12*b^4))*1i)/b^2 - (32*(3*a^16*b - 64*a^4*b^13 + 208*a^6*b^11 - 22
0*a^8*b^9 + 71*a^10*b^7 + 5*a^12*b^5 - 4*a^14*b^3))/(a^12*b^2) + (32*tan(c/
2 + (d*x)/2)*(16*a^5*b^14 - 40*a^7*b^12 + 5*a^9*b^10 + 44*a^11*b^8 - 6*a^13
*b^6 - 20*a^15*b^4 + 2*a^17*b^2))/(a^12*b^4))*1i)/b^2 - (32*(6*a^15 + 16*a^
5*b^10 - 56*a^7*b^8 + 97*a^9*b^6 - 84*a^11*b^4 + 20*a^13*b^2))/(a^12*b^2) +
(32*tan(c/2 + (d*x)/2)*(4*a^16*b - 64*b^17 + 304*a^2*b^15 - 540*a^4*b^13 +
405*a^6*b^11 - 124*a^8*b^9 + 78*a^10*b^7 - 72*a^12*b^5 + 10*a^14*b^3))/(a^
12*b^4))*1i)/b^2 + (((((((((32*(4*a^14*b^7 - 3*a^16*b^5))/(a^12*b^2) + (32*
tan(c/2 + (d*x)/2)*(16*a^13*b^10 - 17*a^15*b^8 + 2*a^17*b^6))/(a^12*b^4))*1
i)/b^2 + (32*(32*a^9*b^10 - 64*a^11*b^8 + 30*a^13*b^6 + a^15*b^4))/(a^12*b^
2) - (32*tan(c/2 + (d*x)/2)*(148*a^10*b^11 - 64*a^8*b^13 - 97*a^12*b^9 + 10
*a^14*b^7 + 4*a^16*b^5))/(a^12*b^4))*1i)/b^2 - (32*(3*a^16*b - 64*a^4*b^13
+ 208*a^6*b^11 - 220*a^8*b^9 + 71*a^10*b^7 + 5*a^12*b^5 - 4*a^14*b^3))/(a^1
2*b^2) + (32*tan(c/2 + (d*x)/2)*(16*a^5*b^14 - 40*a^7*b^12 + 5*a^9*b^10 + 4
4*a^11*b^8 - 6*a^13*b^6 - 20*a^15*b^4 + 2*a^17*b^2))/(a^12*b^4))*1i)/b^2 +
(32*(6*a^15 + 16*a^5*b^10 - 56*a^7*b^8 + 97*a^9*b^6 - 84*a^11*b^4 + 20*a^13
*b^2))/(a^12*b^2) - (32*tan(c/2 + (d*x)/2)*(4*a^16*b - 64*b^17 + 304*a^2*b^
15 - 540*a^4*b^13 + 405*a^6*b^11 - 124*a^8*b^9 + 78*a^10*b^7 - 72*a^12*b^5
+ 10*a^14*b^3))/(a^12*b^4))*1i)/b^2 - (64*(30*a^12*b - 64*b^13 + 304*a^2*b^
11 - 604*a^4*b^9 + 613*a^6*b^7 - 280*a^8*b^5 + a^10*b^3))/(a^12*b^2) - (64*
tan(c/2 + (d*x)/2)*(8*a^15 - 16*a^7*b^8 + 60*a^9*b^6 - 64*a^11*b^4 + 12*a^1
3*b^2))/(a^12*b^4))))/(b^2*d) - (b*tan(c/2 + (d*x)/2)^2)/(4*a^3*d) + (b*log

(tan(c/2 + (d*x)/2))*(5*a^2 - 4*b^2))/(a^5*d) - (atan((((a^2 + 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(4*a^16*b - 64*b^17 + 304*a ^2*b^15 - 540*a^4*b^13 + 405*a^6*b^11 - 124*a^8*b^9 + 78*a^10*b^7 - 72*a^12 *b^5 + 10*a^14*b^3))/(a^12*b^4) - (32*(6*a^15 + 16*a^5*b^10 - 56*a^7*b^8 + 97*a^9*b^6 - 84*a^11*b^4 + 20*a^13*b^2))/(a^12*b^2) + ((a^2 + 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(16*a^5*b^14 - 40*a^7*b^12 + 5*a^9*b^10 + 44*a^11*b^8 - 6*a^13*b^6 - 20*a^15*b^4 + 2*a^17*b^2))/(a^12*b ^4) - (32*(3*a^16*b - 64*a^4*b^13 + 208*a^6*b^11 - 220*a^8*b^9 + 71*a^10*b^ 7 + 5*a^12*b^5 - 4*a^14*b^3))/(a^12*b^2) + ((a^2 + 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(148*a^10*b^11 - 64*a^8*b^13 - 97*a^12* b^9 + 10*a^14*b^7 + 4*a^16*b^5))/(a^12*b^4) - (32*(32*a^9*b^10 - 64*a^11*b^ 8 + 30*a^13*b^6 + a^15*b^4))/(a^12*b^2) + (((32*(4*a^14*b^7 - 3*a^16*b^5))/ (a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(16*a^13*b^10 - 17*a^15*b^8 + 2*a^17*b^ 6))/(a^12*b^4))*(a^2 + 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^5*b^2)))/(a^ 5*b^2)))/(a^5*b^2))*1i)/(a^5*b^2) - ((a^2 + 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(6*a^15 + 16*a^5*b^10 - 56*a^7*b^8 + 97*a^9*b^6 - 84*a^11*b^4 + 2 0*a^13*b^2))/(a^12*b^2) - (32*tan(c/2 + (d*x)/2)*(4*a^16*b - 64*b^17 + 304* a^2*b^15 - 540*a^4*b^13 + 405*a^6*b^11 - 124*a^8*b^9 + 78*a^10*b^7 - 72*a^1 2*b^5 + 10*a^14*b^3))/(a^12*b^4) + ((a^2 + 4*b^2)*(-(a + b)^3*(a - b)^3)^(1 /2)*((32*tan(c/2 + (d*x)/2)*(16*a^5*b^14 - 40*a^7*b^12 + 5*a^9*b^10 + 44*a^ 11*b^8 - 6*a^13*b^6 - 20*a^15*b^4 + 2*a^17*b^2))/(a^12*b^4) - (32*(3*a^16*b - 64*a^4*b^13 + 208*a^6*b^11 - 220*a^8*b^9 + 71*a^10*b^7 + 5*a^12*b^5 - 4* a^14*b^3))/(a^12*b^2) + ((a^2 + 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(3 2*a^9*b^10 - 64*a^11*b^8 + 30*a^13*b^6 + a^15*b^4))/(a^12*b^2) - (32*tan(c/ 2 + (d*x)/2)*(148*a^10*b^11 - 64*a^8*b^13 - 97*a^12*b^9 + 10*a^14*b^7 + 4*a ^16*b^5))/(a^12*b^4) + (((32*(4*a^14*b^7 - 3*a^16*b^5))/(a^12*b^2) + (32*ta n(c/2 + (d*x)/2)*(16*a^13*b^10 - 17*a^15*b^8 + 2*a^17*b^6))/(a^12*b^4))*(a^ 2 + 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^5*b^2)))/(a^5*b^2)))/(a^5*b^2)) *1i)/(a^5*b^2))/((64*(30*a^12*b - 64*b^13 + 304*a^2*b^11 - 604*a^4*b^9 + 61 3*a^6*b^7 - 280*a^8*b^5 + a^10*b^3))/(a^12*b^2) + (64*tan(c/2 + (d*x)/2)*(8 *a^15 - 16*a^7*b^8 + 60*a^9*b^6 - 64*a^11*b^4 + 12*a^13*b^2))/(a^12*b^4) - ((a^2 + 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(4*a^16 *b - 64*b^17 + 304*a^2*b^15 - 540*a^4*b^13 + 405*a^6*b^11 - 124*a^8*b^9 + 7 8*a^10*b^7 - 72*a^12*b^5 + 10*a^14*b^3))/(a^12*b^4) - (32*(6*a^15 + 16*a^5* b^10 - 56*a^7*b^8 + 97*a^9*b^6 - 84*a^11*b^4 + 20*a^13*b^2))/(a^12*b^2) + ((a^2 + 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(16*a^5* b^14 - 40*a^7*b^12 + 5*a^9*b^10 + 44*a^11*b^8 - 6*a^13*b^6 - 20*a^15*b^4 + 2*a^17*b^2))/(a^12*b^4) - (32*(3*a^16*b - 64*a^4*b^13 + 208*a^6*b^11 - 220* a^8*b^9 + 71*a^10*b^7 + 5*a^12*b^5 - 4*a^14*b^3))/(a^12*b^2) + ((a^2 + 4*b^ 2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(148*a^10*b^11 - 64 *a^8*b^13 - 97*a^12*b^9 + 10*a^14*b^7 + 4*a^16*b^5))/(a^12*b^4) - (32*(32*a ^9*b^10 - 64*a^11*b^8 + 30*a^13*b^6 + a^15*b^4))/(a^12*b^2) + (((32*(4*a^14 *b^7 - 3*a^16*b^5))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(16*a^13*b^10 - 17* a^15*b^8 + 2*a^17*b^6))/(a^12*b^4))*(a^2 + 4*b^2)*(-(a + b)^3*(a - b)^3)^(1 /2))/(a^5*b^2)))/(a^5*b^2)))/(a^5*b^2)))/(a^5*b^2) - ((a^2 + 4*b^2)*(-(a +

```
b)^3*(a - b)^3)^(1/2)*((32*(6*a^15 + 16*a^5*b^10 - 56*a^7*b^8 + 97*a^9*b^6
- 84*a^11*b^4 + 20*a^13*b^2))/(a^12*b^2) - (32*tan(c/2 + (d*x)/2)*(4*a^16*b
 - 64*b^17 + 304*a^2*b^15 - 540*a^4*b^13 + 405*a^6*b^11 - 124*a^8*b^9 + 78*
a^10*b^7 - 72*a^12*b^5 + 10*a^14*b^3))/(a^12*b^4) + ((a^2 + 4*b^2)*(-(a + b
)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(16*a^5*b^14 - 40*a^7*b^12 + 5
*a^9*b^10 + 44*a^11*b^8 - 6*a^13*b^6 - 20*a^15*b^4 + 2*a^17*b^2))/(a^12*b^4
) - (32*(3*a^16*b - 64*a^4*b^13 + 208*a^6*b^11 - 220*a^8*b^9 + 71*a^10*b^7
+ 5*a^12*b^5 - 4*a^14*b^3))/(a^12*b^2) + ((a^2 + 4*b^2)*(-(a + b)^3*(a - b)
^3)^(1/2)*((32*(32*a^9*b^10 - 64*a^11*b^8 + 30*a^13*b^6 + a^15*b^4))/(a^12*
b^2) - (32*tan(c/2 + (d*x)/2)*(148*a^10*b^11 - 64*a^8*b^13 - 97*a^12*b^9 +
10*a^14*b^7 + 4*a^16*b^5))/(a^12*b^4) + (((32*(4*a^14*b^7 - 3*a^16*b^5))/(a
^12*b^2) + (32*tan(c/2 + (d*x)/2)*(16*a^13*b^10 - 17*a^15*b^8 + 2*a^17*b^6)
)/(a^12*b^4))*(a^2 + 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^5*b^2)))/(a^5*
b^2)))/(a^5*b^2)))/(a^5*b^2)))*(a^2 + 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*2
i)/(a^5*b^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1263 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=303

$$\frac{5b\cot(c+dx)\csc^2(c+dx)}{12a^2d(a+b\sin(c+dx))}-\frac{10b\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{5\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d}-\frac{(6a^2$$

[Out] -10*b*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^6/d-5/8*(3*a^4-12*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^6/d+1/3*(3*a^4-20*a^2*b^2+15*b^4)*cot(d*x+c)/a^5/b/d+5/8*(5*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/a^4/d-cot(d*x+c)/b/d/(a+b*sin(d*x+c))-1/3*(6*a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/d/(a+b*sin(d*x+c))+5/12*b*cot(d*x+c)*csc(d*x+c)^2/a^2/d/(a+b*sin(d*x+c))-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d/(a+b*sin(d*x+c))

Rubi [A] time = 1.21, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{10b\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{\left(-20a^2b^2+3a^4+15b^4\right)\cot(c+dx)}{3a^5bd}-\frac{5\left(-12a^2b^2+3a^4+8b^4\right)\tanh^{-1}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x])^2,x]

[Out] (-10*b*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^6*d) - (5*(3*a^4 - 12*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) + ((3*a^4 - 20*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(3*a^5*b*d) + (5*(5*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*d) - Cot[c + d*x]/(b*d*(a + b*Sin[c + d*x])) - ((6*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^3*d*(a + b*Sin[c + d*x])) + (5*b*Cot[c + d*x]*Csc[c + d*x]^2)/(12*a^2*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d*(a + b*Sin[c + d*x]))

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cot(c+dx)}{bd(a+b\sin(c+dx))} + \frac{5b\cot(c+dx)\csc^2(c+dx)}{12a^2d(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))}$$

$$= -\frac{\cot(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\left(6a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{3a^3d(a+b\sin(c+dx))} + \frac{5b\cot(c+dx)\,\mathrm{c}}{12a^2d(a+b\sin}$$

$$= \frac{5\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{\cot(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\left(6a^2-5b^2\right)\cot(}{3a^3d(a+b}$$

$$= \frac{\left(3a^4-20a^2b^2+15b^4\right)\cot(c+dx)}{3a^5bd} + \frac{5\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{}{bd}$$

$$= \frac{\left(3a^4-20a^2b^2+15b^4\right)\cot(c+dx)}{3a^5bd} + \frac{5\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{}{bd}$$

$$= -\frac{5\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} + \frac{\left(3a^4-20a^2b^2+15b^4\right)\cot(c+}{3a^5bd}$$

$$= -\frac{5\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} + \frac{\left(3a^4-20a^2b^2+15b^4\right)\cot(c+}{3a^5bd}$$

$$= -\frac{10b\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} - \frac{5\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c}{8a^6d}$$

Mathematica [A] time = 6.23, size = 487, normalized size = 1.61

$$\frac{b\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{12a^3d} - \frac{b\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{12a^3d} - \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64a^2d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64a^2d} - \frac{10b\,(}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-10*b*(a^2 - b^2)^(3/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*S
in[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*d) + ((-7*a^2*b*Cos[(c + d*x)/2] +
 6*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(3*a^5*d) + (3*(3*a^2 - 4*b^2)*C
```

```
sc[(c + d*x)/2]^2)/(32*a^4*d) + (b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(12
*a^3*d) - Csc[(c + d*x)/2]^4/(64*a^2*d) - (5*(3*a^4 - 12*a^2*b^2 + 8*b^4)*L
og[Cos[(c + d*x)/2]])/(8*a^6*d) + (5*(3*a^4 - 12*a^2*b^2 + 8*b^4)*Log[Sin[(
c + d*x)/2]])/(8*a^6*d) - (3*(3*a^2 - 4*b^2)*Sec[(c + d*x)/2]^2)/(32*a^4*d)
 + Sec[(c + d*x)/2]^4/(64*a^2*d) + (Sec[(c + d*x)/2]*(7*a^2*b*Sin[(c + d*x)
/2] - 6*b^3*Sin[(c + d*x)/2]))/(3*a^5*d) + (a^4*Cos[c + d*x] - 2*a^2*b^2*Co
s[c + d*x] + b^4*Cos[c + d*x])/(a^5*d*(a + b*Sin[c + d*x])) - (b*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/(12*a^3*d)
```

fricas [B] time = 1.54, size = 1576, normalized size = 5.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/48*(16*(3*a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 10*(15*a^5 - 68*
a^3*b^2 + 48*a*b^4)*cos(d*x + c)^3 - 120*((a^3*b - a*b^3)*cos(d*x + c)^4 +
a^3*b - a*b^3 - 2*(a^3*b - a*b^3)*cos(d*x + c)^2 + ((a^2*b^2 - b^4)*cos(d*x
 + c)^4 + a^2*b^2 - b^4 - 2*(a^2*b^2 - b^4)*cos(d*x + c)^2)*sin(d*x + c))*s
qrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 30*(3*a^5 - 12*
a^3*b^2 + 8*a*b^4)*cos(d*x + c) - 15*(3*a^5 - 12*a^3*b^2 + 8*a*b^4 + (3*a^5
 - 12*a^3*b^2 + 8*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 12*a^3*b^2 + 8*a*b^4)*
cos(d*x + c)^2 + (3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*
b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*sin(
d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(3*a^5 - 12*a^3*b^2 + 8*a*b^4 +
(3*a^5 - 12*a^3*b^2 + 8*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 12*a^3*b^2 + 8*a
*b^4)*cos(d*x + c)^2 + (3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^
3 + 8*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2
)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 10*((17*a^4*b - 12*a^2*b^3)*
cos(d*x + c)^3 - 3*(5*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d
*cos(d*x + c)^4 - 2*a^7*d*cos(d*x + c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4
- 2*a^6*b*d*cos(d*x + c)^2 + a^6*b*d)*sin(d*x + c)), 1/48*(16*(3*a^5 - 20*a
^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 10*(15*a^5 - 68*a^3*b^2 + 48*a*b^4)*cos
(d*x + c)^3 + 240*((a^3*b - a*b^3)*cos(d*x + c)^4 + a^3*b - a*b^3 - 2*(a^3*
b - a*b^3)*cos(d*x + c)^2 + ((a^2*b^2 - b^4)*cos(d*x + c)^4 + a^2*b^2 - b^4
 - 2*(a^2*b^2 - b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-
(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(3*a^5 - 12*a^3*b
^2 + 8*a*b^4)*cos(d*x + c) - 15*(3*a^5 - 12*a^3*b^2 + 8*a*b^4 + (3*a^5 - 12
*a^3*b^2 + 8*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 12*a^3*b^2 + 8*a*b^4)*cos(d
*x + c)^2 + (3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*b^5)*
cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*sin(d*x +
```

```
c))*log(1/2*cos(d*x + c) + 1/2) + 15*(3*a^5 - 12*a^3*b^2 + 8*a*b^4 + (3*a^
5 - 12*a^3*b^2 + 8*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 12*a^3*b^2 + 8*a*b^4)
*cos(d*x + c)^2 + (3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8
*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*sin
(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 10*((17*a^4*b - 12*a^2*b^3)*cos(d
*x + c)^3 - 3*(5*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d*cos(
d*x + c)^4 - 2*a^7*d*cos(d*x + c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4 - 2*a
^6*b*d*cos(d*x + c)^2 + a^6*b*d)*sin(d*x + c))]
```

giac [A] time = 0.29, size = 475, normalized size = 1.57

$$\frac{\frac{120\left(3a^4-12a^2b^2+8b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^6}-\frac{1920\left(a^4b-2a^2b^3+b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^6}+\frac{384\left(a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\cdots\right.}{\left(a\tan\left(\cdots\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(120*(3*a^4 - 12*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6
- 1920*(a^4*b - 2*a^2*b^3 + b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) +
 arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6
) + 384*(a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) + b^5*
tan(1/2*d*x + 1/2*c) + a^5 - 2*a^3*b^2 + a*b^4)/((a*tan(1/2*d*x + 1/2*c)^2
+ 2*b*tan(1/2*d*x + 1/2*c) + a)*a^6) + (3*a^6*tan(1/2*d*x + 1/2*c)^4 - 16*a
^5*b*tan(1/2*d*x + 1/2*c)^3 - 48*a^6*tan(1/2*d*x + 1/2*c)^2 + 72*a^4*b^2*ta
n(1/2*d*x + 1/2*c)^2 + 432*a^5*b*tan(1/2*d*x + 1/2*c) - 384*a^3*b^3*tan(1/2
*d*x + 1/2*c))/a^8 - (750*a^4*tan(1/2*d*x + 1/2*c)^4 - 3000*a^2*b^2*tan(1/2
*d*x + 1/2*c)^4 + 2000*b^4*tan(1/2*d*x + 1/2*c)^4 + 432*a^3*b*tan(1/2*d*x +
 1/2*c)^3 - 384*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 48*a^4*tan(1/2*d*x + 1/2*c)^
2 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 16*a^3*b*tan(1/2*d*x + 1/2*c) + 3*a
^4)/(a^6*tan(1/2*d*x + 1/2*c)^4))/d
```

maple [B] time = 0.81, size = 718, normalized size = 2.37

$$\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a^2d}-\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d\,a^3}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2d}+\frac{3b^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{4d\,a^3}-\frac{2b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^5}-\frac{\cdots}{64a^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/64/d/a^2*tan(1/2*d*x+1/2*c)^4-1/12/d/a^3*b*tan(1/2*d*x+1/2*c)^3-1/4/d/a^2
*tan(1/2*d*x+1/2*c)^2+3/8/d/a^4*b^2*tan(1/2*d*x+1/2*c)^2+9/4/d/a^3*tan(1/2*
d*x+1/2*c)*b-2/d/a^5*b^3*tan(1/2*d*x+1/2*c)-1/64/a^2/d/tan(1/2*d*x+1/2*c)^4
+1/4/a^2/d/tan(1/2*d*x+1/2*c)^2-3/8/d/a^4/tan(1/2*d*x+1/2*c)^2*b^2+15/8/d/a
^2*ln(tan(1/2*d*x+1/2*c))-15/2/d/a^4*ln(tan(1/2*d*x+1/2*c))*b^2+5/d/a^6*ln(
tan(1/2*d*x+1/2*c))*b^4+1/12/d/a^3*b/tan(1/2*d*x+1/2*c)^3-9/4/d*b/a^3/tan(1
/2*d*x+1/2*c)+2/d*b^3/a^5/tan(1/2*d*x+1/2*c)+2/d/a^2*b/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-4/d/a^4*b^3/(tan(1/2*d*x+1
/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/a^6/(tan(1/2*d*x
+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)*b^5+2/d/a/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-4/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)+2/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)*b^4-10/d/a^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2))+20/d/a^4*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1
/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-10/d/a^6*b^5/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.03, size = 1117, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^5*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^4/(64*a^2*d) + (tan(c/2 + (d*x)/2)^4*(36*a^4 + 96*b^4 -
142*a^2*b^2) - a^4/4 + tan(c/2 + (d*x)/2)^2*((15*a^4)/4 - (10*a^2*b^2)/3) +
 tan(c/2 + (d*x)/2)^3*(20*a*b^3 - (80*a^3*b)/3) - (4*tan(c/2 + (d*x)/2)^5*(
a^4*b - 8*b^5 + 8*a^2*b^3))/a + (5*a^3*b*tan(c/2 + (d*x)/2))/6)/(d*(16*a^6*
tan(c/2 + (d*x)/2)^4 + 16*a^6*tan(c/2 + (d*x)/2)^6 + 32*a^5*b*tan(c/2 + (d*
x)/2)^5)) - (tan(c/2 + (d*x)/2)^2*((32*a^2 + 64*b^2)/(512*a^4) + 3/(16*a^2)
 - b^2/(2*a^4)))/d + (tan(c/2 + (d*x)/2)*((b*(32*a^2 + 64*b^2))/(64*a^5) -
b/(4*a^3) + (4*b*((32*a^2 + 64*b^2)/(256*a^4) + 3/(8*a^2) - b^2/a^4))/a))/d
```

```
 - (b*tan(c/2 + (d*x)/2)^3)/(12*a^3*d) + (log(tan(c/2 + (d*x)/2))*(15*a^4 +
 40*b^4 - 60*a^2*b^2))/(8*a^6*d) - (b*atan(((b*(-(a + b)^3*(a - b)^3)^(1/2)
*((tan(c/2 + (d*x)/2)*(15*a^10 - 160*a^4*b^6 + 320*a^6*b^4 - 170*a^8*b^2))/
(4*a^9) - (55*a^10*b + 80*a^6*b^5 - 140*a^8*b^3)/(4*a^10) + (5*b*(2*a^2*b -
 (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(-(a + b)^3*(a - b)^
3)^(1/2))/a^6)*5i)/a^6 - (b*(-(a + b)^3*(a - b)^3)^(1/2)*((55*a^10*b + 80*a
^6*b^5 - 140*a^8*b^3)/(4*a^10) - (tan(c/2 + (d*x)/2)*(15*a^10 - 160*a^4*b^6
 + 320*a^6*b^4 - 170*a^8*b^2))/(4*a^9) + (5*b*(2*a^2*b - (tan(c/2 + (d*x)/2
)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(-(a + b)^3*(a - b)^3)^(1/2))/a^6)*5i)/
a^6)/((75*a^8*b + 200*b^9 - 700*a^2*b^7 + 875*a^4*b^5 - 450*a^6*b^3)/(2*a^1
0) + (tan(c/2 + (d*x)/2)*(200*b^8 - 650*a^2*b^6 + 700*a^4*b^4 - 250*a^6*b^2
))/(2*a^9) + (5*b*(-(a + b)^3*(a - b)^3)^(1/2)*((tan(c/2 + (d*x)/2)*(15*a^1
0 - 160*a^4*b^6 + 320*a^6*b^4 - 170*a^8*b^2))/(4*a^9) - (55*a^10*b + 80*a^6
*b^5 - 140*a^8*b^3)/(4*a^10) + (5*b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12
 - 32*a^10*b^2))/(4*a^9))*(-(a + b)^3*(a - b)^3)^(1/2))/a^6))/a^6 + (5*b*(-
(a + b)^3*(a - b)^3)^(1/2)*((55*a^10*b + 80*a^6*b^5 - 140*a^8*b^3)/(4*a^10)
 - (tan(c/2 + (d*x)/2)*(15*a^10 - 160*a^4*b^6 + 320*a^6*b^4 - 170*a^8*b^2))
/(4*a^9) + (5*b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*
a^9))*(-(a + b)^3*(a - b)^3)^(1/2))/a^6))/a^6))*(-(a + b)^3*(a - b)^3)^(1/2
)*10i)/(a^6*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1264 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=424

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}-\frac{2\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}-\frac{\left(15a^4-82a^2b^2+60b^4\right)\cot(c+dx}{30a^4b^2d}$$

```
[Out] -2*(a^2-6*b^2)*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1
/2))/a^7/d+1/4*b*(15*a^4-40*a^2*b^2+24*b^4)*arctanh(cos(d*x+c))/a^7/d-1/15*
(38*a^4-135*a^2*b^2+90*b^4)*cot(d*x+c)/a^6/d+1/4*(4*a^4-17*a^2*b^2+12*b^4)*
cot(d*x+c)*csc(d*x+c)/a^5/b/d-1/30*(15*a^4-82*a^2*b^2+60*b^4)*cot(d*x+c)*cs
c(d*x+c)^2/a^4/b^2/d-1/2*cot(d*x+c)*csc(d*x+c)/b/d/(a+b*sin(d*x+c))+1/6*a*c
ot(d*x+c)*csc(d*x+c)^2/b^2/d/(a+b*sin(d*x+c))+1/6*(2*a^4-12*a^2*b^2+9*b^4)*
cot(d*x+c)*csc(d*x+c)^2/a^3/b^2/d/(a+b*sin(d*x+c))+3/10*b*cot(d*x+c)*csc(d*
x+c)^3/a^2/d/(a+b*sin(d*x+c))-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d/(a+b*sin(d*x+
c))
```

Rubi [A] time = 1.52, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}-\frac{\left(-135a^2b^2+38a^4+90b^4\right)\cot(c+dx)}{15a^6d}+\frac{b\left(-40a^2b^2+15a^4+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^7*d) + (b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*ArcTanh[Cos[c + d*x]
])/(4*a^7*d) - ((38*a^4 - 135*a^2*b^2 + 90*b^4)*Cot[c + d*x])/(15*a^6*d) +
((4*a^4 - 17*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c + d*x])/(4*a^5*b*d) - ((1
5*a^4 - 82*a^2*b^2 + 60*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^4*b^2*d) -
(Cot[c + d*x]*Csc[c + d*x])/(2*b*d*(a + b*Sin[c + d*x])) + (a*Cot[c + d*x]*
Csc[c + d*x]^2)/(6*b^2*d*(a + b*Sin[c + d*x])) + ((2*a^4 - 12*a^2*b^2 + 9*b
^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(6*a^3*b^2*d*(a + b*Sin[c + d*x])) + (3*b*
Cot[c + d*x]*Csc[c + d*x]^3)/(10*a^2*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x
]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e
 + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^
m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*
m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(
m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x])/Sin[e + f*x]^4,
x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*
x]^2), x] + Simp[(a*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*m*(m
- 1)*Sin[e + f*x]^3), x] - Simp[(b*(m - 4)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1))/(20*a^2*f*Sin[e + f*x]^4), x]) /; FreeQ[{a, b, e, f, m}, x] && N
eQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{\left(2a^4-12a^2b^2+9b^4\right)\cot(c}{6a^3b^2d(a+b\sin(}$$

$$= -\frac{\left(15a^4-82a^2b^2+60b^4\right)\cot(c+dx)\csc^2(c+dx)}{30a^4b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(}{6b^2}$$

$$= \frac{\left(4a^4-17a^2b^2+12b^4\right)\cot(c+dx)\csc(c+dx)}{4a^5bd} - \frac{\left(15a^4-82a^2b^2+60b^4\right)\cot(c+dx)}{30a^4b^2d}$$

$$= -\frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+dx)}{15a^6d} + \frac{\left(4a^4-17a^2b^2+12b^4\right)\cot(c+dx)\csc(c+}{4a^5bd}$$

$$= -\frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+dx)}{15a^6d} + \frac{\left(4a^4-17a^2b^2+12b^4\right)\cot(c+dx)\csc(c+}{4a^5bd}$$

$$= \frac{b\left(15a^4-40a^2b^2+24b^4\right)\tanh^{-1}(\cos(c+dx))}{4a^7d} - \frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+dx}{15a^6d}$$

$$= \frac{b\left(15a^4-40a^2b^2+24b^4\right)\tanh^{-1}(\cos(c+dx))}{4a^7d} - \frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+dx}{15a^6d}$$

$$= -\frac{2\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d} + \frac{b\left(15a^4-40a^2b^2+24b^4\right)\tanh^{-1}}{4a^7d}$$

Mathematica [A] time = 1.61, size = 361, normalized size = 0.85

$$-\frac{1920\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)+240b\left(15a^4-40a^2b^2+24b^4\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-240b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/960*(1920*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2]
)/Sqrt[a^2 - b^2]] - 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Cos[(c + d*x)
/2]] + 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Sin[(c + d*x)/2]] + (2*a*Co
t[c + d*x]*Csc[c + d*x]^5*(196*a^5 - 735*a^3*b^2 + 540*a*b^4 - 12*(16*a^5 -
 85*a^3*b^2 + 60*a*b^4)*Cos[2*(c + d*x)] + (92*a^5 - 285*a^3*b^2 + 180*a*b^
4)*Cos[4*(c + d*x)] + 1162*a^4*b*Sin[c + d*x] - 3060*a^2*b^3*Sin[c + d*x] +
 1800*b^5*Sin[c + d*x] - 562*a^4*b*Sin[3*(c + d*x)] + 1470*a^2*b^3*Sin[3*(c
 + d*x)] - 900*b^5*Sin[3*(c + d*x)] + 76*a^4*b*Sin[5*(c + d*x)] - 270*a^2*b
^3*Sin[5*(c + d*x)] + 180*b^5*Sin[5*(c + d*x)]))/(b + a*Csc[c + d*x]))/(a^7
*d)
```

fricas [B] time = 2.05, size = 2011, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/120*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 40*(7*a^6 -
 27*a^4*b^2 + 18*a^2*b^4)*cos(d*x + c)^3 + 60*((a^4*b - 7*a^2*b^3 + 6*b^5)*
cos(d*x + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*
cos(d*x + c)^4 + 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2 - (a^5 - 7*a^
3*b^2 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 7*a
^3*b^2 + 6*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^
2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c
)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2)) + 30*(4*a^6 - 17*a^4*b^2 + 12*a^2*b^4)*cos(d
*x + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^6 - 15*a^4*b^
2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)
^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (15*a^5*b - 40*a
^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^4 - 2*(
15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos
(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^6 -
15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos
(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (15*a^5
*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c
)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*lo
g(-1/2*cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^3*b^3 + 90*a*b^5)*cos(d
*x + c)^5 - 5*(79*a^5*b - 228*a^3*b^3 + 144*a*b^5)*cos(d*x + c)^3 + 15*(15*
a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^7*b*d*cos(d*x
 + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*cos(d*x + c)^2 - a^7*b*d - (
a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2 + a^8*d)*sin(d*x + c)), 1/120
*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 40*(7*a^6 - 27*a^
4*b^2 + 18*a^2*b^4)*cos(d*x + c)^3 + 120*((a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d
```

*x + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d
x + c)^4 + 3(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2 - (a^5 - 7*a^3*b^2
 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 7*a^3*b^
2 + 6*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d
*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(4*a^6 - 17*a^4*b^2 + 12*
a^2*b^4)*cos(d*x + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)
^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6
)*cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (1
5*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*
x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c
))*log(1/2*cos(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos
(d*x + c)^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4
 + 24*b^6)*cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x +
c)^2 - (15*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^
5)*cos(d*x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*si
n(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^3*b^3 + 9
0*a*b^5)*cos(d*x + c)^5 - 5*(79*a^5*b - 228*a^3*b^3 + 144*a*b^5)*cos(d*x +
c)^3 + 15*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a
^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*cos(d*x + c)^2
 - a^7*b*d - (a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2 + a^8*d)*sin(d*
x + c))]

giac [A] time = 0.29, size = 596, normalized size = 1.41

$$-\frac{\frac{120\left(15a^4b-40a^2b^3+24b^5\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^7}+\frac{960\left(a^6-8a^4b^2+13a^2b^4-6b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^7}+\frac{960\left(a^4b^2\,\mathrm{ta}\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/480*(120*(15*a^4*b - 40*a^2*b^3 + 24*b^5)*log(abs(tan(1/2*d*x + 1/2*c)))
/a^7 + 960*(a^6 - 8*a^4*b^2 + 13*a^2*b^4 - 6*b^6)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sq
rt(a^2 - b^2)*a^7) + 960*(a^4*b^2*tan(1/2*d*x + 1/2*c) - 2*a^2*b^4*tan(1/2*
d*x + 1/2*c) + b^6*tan(1/2*d*x + 1/2*c) + a^5*b - 2*a^3*b^3 + a*b^5)/((a*ta
n(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^7) - (3*a^8*tan(1/2*
d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^8*tan(1/2*d*x + 1/2
*c)^3 + 60*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^7*b*tan(1/2*d*x + 1/2*c)^
2 - 240*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 330*a^8*tan(1/2*d*x + 1/2*c) - 162
0*a^6*b^2*tan(1/2*d*x + 1/2*c) + 1200*a^4*b^4*tan(1/2*d*x + 1/2*c))/a^10 -
(4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 10960*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 +
 6576*b^5*tan(1/2*d*x + 1/2*c)^5 - 330*a^5*tan(1/2*d*x + 1/2*c)^4 + 1620*a^

```
3*b^2*tan(1/2*d*x + 1/2*c)^4 - 1200*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 240*a^4*
b*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 35*a^5*tan(
1/2*d*x + 1/2*c)^2 - 60*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*b*tan(1/2*d
*x + 1/2*c) - 3*a^5)/(a^7*tan(1/2*d*x + 1/2*c)^5))/d
```

maple [B] time = 0.82, size = 897, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/160/a^2/d/tan(1/2*d*x+1/2*c)^5+11/16/d/a^2*tan(1/2*d*x+1/2*c)-11/16/d/a^
2/tan(1/2*d*x+1/2*c)-15/4/d/a^3*b*ln(tan(1/2*d*x+1/2*c))-2/d/a^2*b/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-2/d/a/(a^2-b^2)^(1/2)*arctan(1/2*
(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+7/96/a^2/d/tan(1/2*d*x+1/2*c)
^3-7/96/d/a^2*tan(1/2*d*x+1/2*c)^3+1/160/d/a^2*tan(1/2*d*x+1/2*c)^5+1/2/d/a
^3*tan(1/2*d*x+1/2*c)^2*b-27/8/d/a^4*b^2*tan(1/2*d*x+1/2*c)+27/8/d/a^4/tan(
1/2*d*x+1/2*c)*b^2-1/2/d/a^3*b/tan(1/2*d*x+1/2*c)^2+10/d/a^5*b^3*ln(tan(1/2
*d*x+1/2*c))+4/d/a^4*b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-
5/2/d/a^6/tan(1/2*d*x+1/2*c)*b^4+1/32/d/a^3*b/tan(1/2*d*x+1/2*c)^4+1/2/d/a^
5*b^3/tan(1/2*d*x+1/2*c)^2-1/32/d/a^3*tan(1/2*d*x+1/2*c)^4*b+1/8/d/a^4*tan(
1/2*d*x+1/2*c)^3*b^2-1/2/d/a^5*tan(1/2*d*x+1/2*c)^2*b^3+5/2/d/a^6*b^4*tan(1
/2*d*x+1/2*c)-2/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^5
-2/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+
1/2*c)+16/d/a^3*b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))-6/d/a^7*b^5*ln(tan(1/2*d*x+1/2*c))+12/d/a^7/(a^2-b^2)^(1/
2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^6-2/d/a^7/(ta
n(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)*b^6-1/8/d
/a^4/tan(1/2*d*x+1/2*c)^3*b^2+4/d/a^5*b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-26/d/a^5*b^4/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.16, size = 1424, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^6*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a^2*d) + (tan(c/2 + (d*x)/2)*(1/(4*a^2) + b^2/(2*
a^4) - (4*b*((b*(64*a^2 + 128*b^2))/(256*a^5) - b/(8*a^3) + (4*b*((64*a^2 +
 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/a))/a + ((64*a^2 + 128*b^
2)*((64*a^2 + 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/(32*a^2)))/d
 - (tan(c/2 + (d*x)/2)^3*((64*a^2 + 128*b^2)/(3072*a^4) + 5/(96*a^2) - b^2/
(6*a^4)))/d - (tan(c/2 + (d*x)/2)^3*((31*a^4*b)/3 - 8*a^2*b^3) + tan(c/2 +
(d*x)/2)^4*(48*a*b^4 + (59*a^5)/3 - 72*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(124
*a^4*b + 224*b^5 - 360*a^2*b^3) + a^5/5 - tan(c/2 + (d*x)/2)^2*((32*a^5)/15
 - 2*a^3*b^2) - (3*a^4*b*tan(c/2 + (d*x)/2))/5 + (2*tan(c/2 + (d*x)/2)^6*(1
1*a^6 + 32*b^6 - 24*a^2*b^4 - 22*a^4*b^2))/a)/(d*(32*a^7*tan(c/2 + (d*x)/2)
^5 + 32*a^7*tan(c/2 + (d*x)/2)^7 + 64*a^6*b*tan(c/2 + (d*x)/2)^6)) + (tan(c
/2 + (d*x)/2)^2*((b*(64*a^2 + 128*b^2))/(512*a^5) - b/(16*a^3) + (2*b*((64*
a^2 + 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/a))/d - (log(tan(c/2
 + (d*x)/2))*(15*a^4*b + 24*b^5 - 40*a^2*b^3))/(4*a^7*d) - (b*tan(c/2 + (d*
x)/2)^4)/(32*a^3*d) - (atan((((a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((
4*a^13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) + (tan(c/2 + (d*x)
/2)*(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(2*a^11) + ((2*a^
2*b - (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11))*(a^2 - 6*b^2)*
(-(a + b)^3*(a - b)^3)^(1/2))/a^7)*1i)/a^7 + ((a^2 - 6*b^2)*(-(a + b)^3*(a
- b)^3)^(1/2)*((4*a^13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) +
(tan(c/2 + (d*x)/2)*(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(
2*a^11) - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11)
)*(a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/a^7)*1i)/a^7)/((15*a^10*b - 1
44*b^11 + 552*a^2*b^9 - 802*a^4*b^7 + 539*a^6*b^5 - 160*a^8*b^3)/a^12 + (ta
n(c/2 + (d*x)/2)*(8*a^10 - 144*b^10 + 516*a^2*b^8 - 682*a^4*b^6 + 400*a^6*b
^4 - 98*a^8*b^2))/a^11 - ((a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((4*a^
13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) + (tan(c/2 + (d*x)/2)*
(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(2*a^11) + ((2*a^2*b
- (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11))*(a^2 - 6*b^2)*(-(a
 + b)^3*(a - b)^3)^(1/2))/a^7))/a^7 + ((a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)
^(1/2)*((4*a^13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) + (tan(c/
2 + (d*x)/2)*(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(2*a^11)
 - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11))*(a^2
- 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/a^7))/a^7))*(a^2 - 6*b^2)*(-(a + b)^
3*(a - b)^3)^(1/2)*2i)/(a^7*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1265 $\int \frac{\cot^6(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=480

$$\frac{7b\cot(c+dx)\csc^4(c+dx)}{30a^2d(a+b\sin(c+dx))}+\frac{2b\left(2a^2-7b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}-\frac{\left(16a^4-61a^2b^2+42b^4\right)\cot(c+d}{24a^4b^2d}$$

[Out] 2*b*(2*a^2-7*b^2)*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^8/d+1/16*(5*a^6-90*a^4*b^2+200*a^2*b^4-112*b^6)*arctanh(cos(d*x+c))/a^8/d+1/15*b*(61*a^4-170*a^2*b^2+105*b^4)*cot(d*x+c)/a^7/d-1/16*(27*a^4-86*a^2*b^2+56*b^4)*cot(d*x+c)*csc(d*x+c)/a^6/d+1/15*(15*a^4-52*a^2*b^2+35*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^5/b/d-1/24*(16*a^4-61*a^2*b^2+42*b^4)*cot(d*x+c)*csc(d*x+c)^3/a^4/b^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2/b/d/(a+b*sin(d*x+c))+1/6*a*cot(d*x+c)*csc(d*x+c)^3/b^2/d/(a+b*sin(d*x+c))+1/10*(5*a^4-20*a^2*b^2+14*b^4)*cot(d*x+c)*csc(d*x+c)^3/a^3/b^2/d/(a+b*sin(d*x+c))+7/30*b*cot(d*x+c)*csc(d*x+c)^4/a^2/d/(a+b*sin(d*x+c))-1/6*cot(d*x+c)*csc(d*x+c)^5/a/d/(a+b*sin(d*x+c))

Rubi [A] time = 1.96, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b\left(2a^2-7b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}+\frac{b\left(-170a^2b^2+61a^4+105b^4\right)\cot(c+dx)}{15a^7d}+\frac{\left(-90a^4b^2+200a^2b\right.}{}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d*x]^6*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]

[Out] (2*b*(2*a^2 - 7*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^8*d) + ((5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*ArcTanh[Cos[c + d*x]])/(16*a^8*d) + (b*(61*a^4 - 170*a^2*b^2 + 105*b^4)*Cot[c + d*x])/(15*a^7*d) - ((27*a^4 - 86*a^2*b^2 + 56*b^4)*Cot[c + d*x]*Csc[c + d*x])/(16*a^6*d) + ((15*a^4 - 52*a^2*b^2 + 35*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(15*a^5*b*d) - ((16*a^4 - 61*a^2*b^2 + 42*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*a^4*b^2*d) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*b*d*(a + b*Sin[c + d*x])) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(6*b^2*d*(a + b*Sin[c + d*x])) + ((5*a^4 - 20*a^2*b^2 + 14*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(10*a^3*b^2*d*(a + b*Sin[c + d*x])) + (7*b*Cot[c + d*x]*Csc[c + d*x]^4)/(30*a^2*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a*d*(a + b*Sin[c + d*x]))

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cot(c+dx)\csc^2(c+dx)}{3bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^3(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{7b\cot(c+dx)\csc^4(c}{30a^2d(a+b\sin(c+}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{3bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^3(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{(5a^4-20a^2b^2+14b}{10a^3b^2d(a}$$

$$= -\frac{(16a^4-61a^2b^2+42b^4)\cot(c+dx)\csc^3(c+dx)}{24a^4b^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3bd(a+b\sin(c+dx))} +$$

$$= \frac{(15a^4-52a^2b^2+35b^4)\cot(c+dx)\csc^2(c+dx)}{15a^5bd} - \frac{(16a^4-61a^2b^2+42b^4)\cot}{24a^4b^2}$$

$$= -\frac{(27a^4-86a^2b^2+56b^4)\cot(c+dx)\csc(c+dx)}{16a^6d} + \frac{(15a^4-52a^2b^2+35b^4)\cot}{15a^5b}$$

$$= \frac{b(61a^4-170a^2b^2+105b^4)\cot(c+dx)}{15a^7d} - \frac{(27a^4-86a^2b^2+56b^4)\cot(c+dx)}{16a^6d}$$

$$= \frac{b(61a^4-170a^2b^2+105b^4)\cot(c+dx)}{15a^7d} - \frac{(27a^4-86a^2b^2+56b^4)\cot(c+dx)}{16a^6d}$$

$$= \frac{(5a^6-90a^4b^2+200a^2b^4-112b^6)\tanh^{-1}(\cos(c+dx))}{16a^8d} + \frac{b(61a^4-170a^2b^2+}{15a}$$

$$= \frac{(5a^6-90a^4b^2+200a^2b^4-112b^6)\tanh^{-1}(\cos(c+dx))}{16a^8d} + \frac{b(61a^4-170a^2b^2+}{15a}$$

$$= \frac{2b(2a^2-7b^2)(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d} + \frac{(5a^6-90a^4b^2+200a^2b^4}{1}$$

Mathematica [A] time = 1.72, size = 447, normalized size = 0.93

$$15360b(2a^2-7b^2)(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + 480(-5a^6+90a^4b^2-200a^2b^4+112b^6)\log\left(\sin\left(\frac{1}{2}(c+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (15360*b*(2*a^2 - 7*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]] + 480*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*Log[Cos
[(c + d*x)/2]] + 480*(-5*a^6 + 90*a^4*b^2 - 200*a^2*b^4 + 112*b^6)*Log[Sin[
(c + d*x)/2]] - (2*a*Cot[c + d*x]*Csc[c + d*x]^6*(590*a^6 - 6956*a^4*b^2 +
15280*a^2*b^4 - 8400*b^6 - 8*(35*a^6 - 1289*a^4*b^2 + 2830*a^2*b^4 - 1575*b
^6)*Cos[2*(c + d*x)] + (330*a^6 - 3844*a^4*b^2 + 8720*a^2*b^4 - 5040*b^6)*C
os[4*(c + d*x)] + 488*a^4*b^2*Cos[6*(c + d*x)] - 1360*a^2*b^4*Cos[6*(c + d*
x)] + 840*b^6*Cos[6*(c + d*x)] - 3942*a^5*b*Sin[c + d*x] + 12620*a^3*b^3*Si
n[c + d*x] - 8400*a*b^5*Sin[c + d*x] + 1967*a^5*b*Sin[3*(c + d*x)] - 6590*a
^3*b^3*Sin[3*(c + d*x)] + 4200*a*b^5*Sin[3*(c + d*x)] - 571*a^5*b*Sin[5*(c
+ d*x)] + 1430*a^3*b^3*Sin[5*(c + d*x)] - 840*a*b^5*Sin[5*(c + d*x)]))/(b +
 a*Csc[c + d*x]))/(7680*a^8*d)
```

fricas [B] time = 2.35, size = 2588, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/480*(32*(61*a^5*b^2 - 170*a^3*b^4 + 105*a*b^6)*cos(d*x + c)^7 + 2*(165*a
^7 - 3386*a^5*b^2 + 8440*a^3*b^4 - 5040*a*b^6)*cos(d*x + c)^5 - 80*(5*a^7 -
 94*a^5*b^2 + 218*a^3*b^4 - 126*a*b^6)*cos(d*x + c)^3 + 240*((2*a^5*b - 9*a
^3*b^3 + 7*a*b^5)*cos(d*x + c)^6 - 2*a^5*b + 9*a^3*b^3 - 7*a*b^5 - 3*(2*a^5
*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^4 + 3*(2*a^5*b - 9*a^3*b^3 + 7*a*b^5
)*cos(d*x + c)^2 + ((2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^6 - 2*a^4*
b^2 + 9*a^2*b^4 - 7*b^6 - 3*(2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^4
+ 3*(2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2
 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 30*(5*a^7 - 90*a^5*b^2
+ 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c) - 15*(5*a^7 - 90*a^5*b^2 + 200*a^3*
b^4 - 112*a*b^6 - (5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x +
c)^6 + 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^4 - 3*
(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^2 + (5*a^6*b -
90*a^4*b^3 + 200*a^2*b^5 - 112*b^7 - (5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 -
112*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*
cos(d*x + c)^4 - 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*cos(d*x +
 c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(5*a^7 - 90*a^5*b^2 +
 200*a^3*b^4 - 112*a*b^6 - (5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*c
os(d*x + c)^6 + 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x +
c)^4 - 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^2 + (5
```

```
*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7 - (5*a^6*b - 90*a^4*b^3 + 200*a
^2*b^5 - 112*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 -
112*b^7)*cos(d*x + c)^4 - 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*
cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((571*a^6*b
- 1430*a^4*b^3 + 840*a^2*b^5)*cos(d*x + c)^5 - 40*(23*a^6*b - 68*a^4*b^3 +
42*a^2*b^5)*cos(d*x + c)^3 + 15*(27*a^6*b - 86*a^4*b^3 + 56*a^2*b^5)*cos(d*
x + c))*sin(d*x + c))/(a^9*d*cos(d*x + c)^6 - 3*a^9*d*cos(d*x + c)^4 + 3*a^
9*d*cos(d*x + c)^2 - a^9*d + (a^8*b*d*cos(d*x + c)^6 - 3*a^8*b*d*cos(d*x +
c)^4 + 3*a^8*b*d*cos(d*x + c)^2 - a^8*b*d)*sin(d*x + c)), 1/480*(32*(61*a^5
*b^2 - 170*a^3*b^4 + 105*a*b^6)*cos(d*x + c)^7 + 2*(165*a^7 - 3386*a^5*b^2
+ 8440*a^3*b^4 - 5040*a*b^6)*cos(d*x + c)^5 - 80*(5*a^7 - 94*a^5*b^2 + 218*
a^3*b^4 - 126*a*b^6)*cos(d*x + c)^3 - 480*((2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*
cos(d*x + c)^6 - 2*a^5*b + 9*a^3*b^3 - 7*a*b^5 - 3*(2*a^5*b - 9*a^3*b^3 + 7
*a*b^5)*cos(d*x + c)^4 + 3*(2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^2 +
 ((2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^6 - 2*a^4*b^2 + 9*a^2*b^4 -
7*b^6 - 3*(2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^4 + 3*(2*a^4*b^2 - 9
*a^2*b^4 + 7*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*
sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(5*a^7 - 90*a^5*b^2
+ 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c) - 15*(5*a^7 - 90*a^5*b^2 + 200*a^3*
b^4 - 112*a*b^6 - (5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x +
c)^6 + 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^4 - 3*
(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^2 + (5*a^6*b -
90*a^4*b^3 + 200*a^2*b^5 - 112*b^7 - (5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 -
112*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*
cos(d*x + c)^4 - 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*cos(d*x +
 c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(5*a^7 - 90*a^5*b^2 +
 200*a^3*b^4 - 112*a*b^6 - (5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*c
os(d*x + c)^6 + 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x +
c)^4 - 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^2 + (5
*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7 - (5*a^6*b - 90*a^4*b^3 + 200*a
^2*b^5 - 112*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 -
112*b^7)*cos(d*x + c)^4 - 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*
cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((571*a^6*b
- 1430*a^4*b^3 + 840*a^2*b^5)*cos(d*x + c)^5 - 40*(23*a^6*b - 68*a^4*b^3 +
42*a^2*b^5)*cos(d*x + c)^3 + 15*(27*a^6*b - 86*a^4*b^3 + 56*a^2*b^5)*cos(d*
x + c))*sin(d*x + c))/(a^9*d*cos(d*x + c)^6 - 3*a^9*d*cos(d*x + c)^4 + 3*a^
9*d*cos(d*x + c)^2 - a^9*d + (a^8*b*d*cos(d*x + c)^6 - 3*a^8*b*d*cos(d*x +
c)^4 + 3*a^8*b*d*cos(d*x + c)^2 - a^8*b*d)*sin(d*x + c))]
```

giac [A] time = 0.30, size = 736, normalized size = 1.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/1920*(120*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*log(abs(tan(1/2*d
*x + 1/2*c)))/a^8 - 3840*(2*a^6*b - 11*a^4*b^3 + 16*a^2*b^5 - 7*b^7)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sq
rt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^8) - 3840*(a^4*b^3*tan(1/2*d*x + 1/2*c)
- 2*a^2*b^5*tan(1/2*d*x + 1/2*c) + b^7*tan(1/2*d*x + 1/2*c) + a^5*b^2 - 2*a
^3*b^4 + a*b^6)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*
a^8) - (5*a^10*tan(1/2*d*x + 1/2*c)^6 - 24*a^9*b*tan(1/2*d*x + 1/2*c)^5 - 4
5*a^10*tan(1/2*d*x + 1/2*c)^4 + 90*a^8*b^2*tan(1/2*d*x + 1/2*c)^4 + 280*a^9
*b*tan(1/2*d*x + 1/2*c)^3 - 320*a^7*b^3*tan(1/2*d*x + 1/2*c)^3 + 225*a^10*t
an(1/2*d*x + 1/2*c)^2 - 1440*a^8*b^2*tan(1/2*d*x + 1/2*c)^2 + 1200*a^6*b^4*
tan(1/2*d*x + 1/2*c)^2 - 2640*a^9*b*tan(1/2*d*x + 1/2*c) + 8640*a^7*b^3*tan
(1/2*d*x + 1/2*c) - 5760*a^5*b^5*tan(1/2*d*x + 1/2*c))/a^12 - (1470*a^6*tan
(1/2*d*x + 1/2*c)^6 - 26460*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 + 58800*a^2*b^4*
tan(1/2*d*x + 1/2*c)^6 - 32928*b^6*tan(1/2*d*x + 1/2*c)^6 + 2640*a^5*b*tan(
1/2*d*x + 1/2*c)^5 - 8640*a^3*b^3*tan(1/2*d*x + 1/2*c)^5 + 5760*a*b^5*tan(1
/2*d*x + 1/2*c)^5 - 225*a^6*tan(1/2*d*x + 1/2*c)^4 + 1440*a^4*b^2*tan(1/2*d
*x + 1/2*c)^4 - 1200*a^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 280*a^5*b*tan(1/2*d*x
 + 1/2*c)^3 + 320*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 45*a^6*tan(1/2*d*x + 1/2
*c)^2 - 90*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 24*a^5*b*tan(1/2*d*x + 1/2*c) -
 5*a^6)/(a^8*tan(1/2*d*x + 1/2*c)^6))/d
```

maple [B] time = 0.82, size = 1048, normalized size = 2.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -11/8/d/a^3*tan(1/2*d*x+1/2*c)*b+45/8/d/a^4*ln(tan(1/2*d*x+1/2*c))*b^2+11/8
/d*b/a^3/tan(1/2*d*x+1/2*c)+2/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)-5/16/d/a^2*ln(tan(1/2*d*x+1/2*c))-1/384/d/a^2/tan(1/2*d*x+1/
2*c)^6+1/384/d/a^2*tan(1/2*d*x+1/2*c)^6-3/128/d/a^2*tan(1/2*d*x+1/2*c)^4+3/
128/a^2/d/tan(1/2*d*x+1/2*c)^4+3/4/d/a^4/tan(1/2*d*x+1/2*c)^2*b^2-25/2/d/a^
6*ln(tan(1/2*d*x+1/2*c))*b^4-7/48/d/a^3*b/tan(1/2*d*x+1/2*c)^3-9/2/d*b^3/a^
5/tan(1/2*d*x+1/2*c)+15/128/d/a^2*tan(1/2*d*x+1/2*c)^2+32/d/a^6*b^5/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-15/128/a^
2/d/tan(1/2*d*x+1/2*c)^2+1/6/d/a^5*b^3/tan(1/2*d*x+1/2*c)^3+3/d*b^5/a^7/tan
(1/2*d*x+1/2*c)-1/80/d/a^3*b*tan(1/2*d*x+1/2*c)^5+3/64/d/a^4*tan(1/2*d*x+1/
2*c)^4*b^2-1/6/d/a^5*tan(1/2*d*x+1/2*c)^3*b^3+5/8/d/a^6*tan(1/2*d*x+1/2*c)^
2*b^4-3/d/a^7*b^5*tan(1/2*d*x+1/2*c)+2/d/a^7*b^6/(tan(1/2*d*x+1/2*c)^2*a+2*
tan(1/2*d*x+1/2*c)*b+a)-3/64/d/a^4/tan(1/2*d*x+1/2*c)^4*b^2-5/8/d/a^6/tan(1
/2*d*x+1/2*c)^2*b^4+7/d/a^8*ln(tan(1/2*d*x+1/2*c))*b^6+1/80/d/a^3*b/tan(1/2
*d*x+1/2*c)^5-4/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^4
-4/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*
c)*b^5+4/d/a^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a
```

```
^2-b^2)^(1/2))+2/d/a^4*b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a
)*tan(1/2*d*x+1/2*c)-22/d/a^4*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-14/d/a^8*b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+7/48/d/a^3*b*tan(1/2*d*x+1/2*c)^
3-3/4/d/a^4*b^2*tan(1/2*d*x+1/2*c)^2+9/2/d/a^5*b^3*tan(1/2*d*x+1/2*c)+2/d/a
^8*b^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.58, size = 1810, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^7*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^6/(384*a^2*d) - (tan(c/2 + (d*x)/2)^4*((128*a^2 + 256*b^
2)/(16384*a^4) + 1/(64*a^2) - b^2/(16*a^4)))/d + (tan(c/2 + (d*x)/2)^2*(3/(
128*a^2) + b^2/(8*a^4) - (2*b*((b*(128*a^2 + 256*b^2))/(1024*a^5) - b/(16*a
^3) + (4*b*((128*a^2 + 256*b^2)/(4096*a^4) + 1/(16*a^2) - b^2/(4*a^4)))/a))
/a + ((128*a^2 + 256*b^2)*((128*a^2 + 256*b^2)/(4096*a^4) + 1/(16*a^2) - b^
2/(4*a^4)))/(128*a^2)))/d + (tan(c/2 + (d*x)/2)*(b/(16*a^3) - ((128*a^2 + 2
56*b^2)*((b*(128*a^2 + 256*b^2))/(1024*a^5) - b/(16*a^3) + (4*b*((128*a^2 +
 256*b^2)/(4096*a^4) + 1/(16*a^2) - b^2/(4*a^4)))/a))/(64*a^2) + (4*b*((128
*a^2 + 256*b^2)/(4096*a^4) + 1/(16*a^2) - b^2/(4*a^4)))/a - (4*b*(3/(64*a^2
) + b^2/(4*a^4) - (4*b*((b*(128*a^2 + 256*b^2))/(1024*a^5) - b/(16*a^3) + (
4*b*((128*a^2 + 256*b^2)/(4096*a^4) + 1/(16*a^2) - b^2/(4*a^4)))/a))/a + ((
128*a^2 + 256*b^2)*((128*a^2 + 256*b^2)/(4096*a^4) + 1/(16*a^2) - b^2/(4*a^
4)))/(64*a^2)))/a))/d + (tan(c/2 + (d*x)/2)^3*((b*(128*a^2 + 256*b^2))/(307
2*a^5) - b/(48*a^3) + (4*b*((128*a^2 + 256*b^2)/(4096*a^4) + 1/(16*a^2) - b
^2/(4*a^4)))/(3*a)))/d - (tan(c/2 + (d*x)/2)^3*((83*a^5*b)/15 - (14*a^3*b^3
)/3) + a^6/6 + tan(c/2 + (d*x)/2)^4*(6*a^6 + (56*a^2*b^4)/3 - (79*a^4*b^2)/
3) - tan(c/2 + (d*x)/2)^5*(112*a*b^5 + (191*a^5*b)/3 - (544*a^3*b^3)/3) - t
an(c/2 + (d*x)/2)^2*((4*a^6)/3 - (7*a^4*b^2)/5) + tan(c/2 + (d*x)/2)^6*((15
```

```
*a^6)/2 - 512*b^6 + 872*a^2*b^4 - 352*a^4*b^2) - (8*tan(c/2 + (d*x)/2)^7*(1
1*a^6*b + 16*b^7 - 8*a^2*b^5 - 20*a^4*b^3))/a - (7*a^5*b*tan(c/2 + (d*x)/2)
)/15)/(d*(64*a^8*tan(c/2 + (d*x)/2)^6 + 64*a^8*tan(c/2 + (d*x)/2)^8 + 128*a
^7*b*tan(c/2 + (d*x)/2)^7)) - (b*tan(c/2 + (d*x)/2)^5)/(80*a^3*d) - (log(ta
n(c/2 + (d*x)/2))*(5*a^6 - 112*b^6 + 200*a^2*b^4 - 90*a^4*b^2))/(16*a^8*d)
+ (b*atan(((b*(2*a^2 - 7*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((tan(c/2 + (d*x
)/2)*(5*a^14 + 448*a^6*b^8 - 1024*a^8*b^6 + 732*a^10*b^4 - 164*a^12*b^2))/(
8*a^13) - (37*a^14*b - 224*a^8*b^7 + 456*a^10*b^5 - 266*a^12*b^3)/(8*a^14)
+ (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^16 - 64*a^14*b^2))/(8*a^13))*(2*a
^2 - 7*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/a^8)*1i)/a^8 - (b*(2*a^2 - 7*b^2)
*(-(a + b)^3*(a - b)^3)^(1/2)*((37*a^14*b - 224*a^8*b^7 + 456*a^10*b^5 - 26
6*a^12*b^3)/(8*a^14) - (tan(c/2 + (d*x)/2)*(5*a^14 + 448*a^6*b^8 - 1024*a^8
*b^6 + 732*a^10*b^4 - 164*a^12*b^2))/(8*a^13) + (b*(2*a^2*b - (tan(c/2 + (d
*x)/2)*(48*a^16 - 64*a^14*b^2))/(8*a^13))*(2*a^2 - 7*b^2)*(-(a + b)^3*(a -
b)^3)^(1/2))/a^8)*1i)/a^8)/((10*a^12*b + 784*b^13 - 3192*a^2*b^11 + 5062*a^
4*b^9 - 3899*a^6*b^7 + 1470*a^8*b^5 - 235*a^10*b^3)/(4*a^14) + (tan(c/2 + (
d*x)/2)*(784*b^12 - 2996*a^2*b^10 + 4362*a^4*b^8 - 2980*a^6*b^6 + 938*a^8*b
^4 - 108*a^10*b^2))/(4*a^13) + (b*(2*a^2 - 7*b^2)*(-(a + b)^3*(a - b)^3)^(1
/2)*((tan(c/2 + (d*x)/2)*(5*a^14 + 448*a^6*b^8 - 1024*a^8*b^6 + 732*a^10*b^
4 - 164*a^12*b^2))/(8*a^13) - (37*a^14*b - 224*a^8*b^7 + 456*a^10*b^5 - 266
*a^12*b^3)/(8*a^14) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^16 - 64*a^14*
b^2))/(8*a^13))*(2*a^2 - 7*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/a^8))/a^8 + (
b*(2*a^2 - 7*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((37*a^14*b - 224*a^8*b^7 +
456*a^10*b^5 - 266*a^12*b^3)/(8*a^14) - (tan(c/2 + (d*x)/2)*(5*a^14 + 448*a
^6*b^8 - 1024*a^8*b^6 + 732*a^10*b^4 - 164*a^12*b^2))/(8*a^13) + (b*(2*a^2*
b - (tan(c/2 + (d*x)/2)*(48*a^16 - 64*a^14*b^2))/(8*a^13))*(2*a^2 - 7*b^2)*
(-(a + b)^3*(a - b)^3)^(1/2))/a^8))/a^8))*(2*a^2 - 7*b^2)*(-(a + b)^3*(a -
b)^3)^(1/2)*2i)/(a^8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1266 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=536

$$-\frac{b\sin^5(c+dx)\cos(c+dx)}{10a^2d(a+b\sin(c+dx))^2}-\frac{\left(112a^4-110a^2b^2+15b^4\right)\sin^4(c+dx)\cos(c+dx)}{20a^2b^4d(a+b\sin(c+dx))}+\frac{a\sqrt{a^2-b^2}\left(56a^4-47a^2b^2+6\right.}{b^9d}$$

```
[Out] -1/16*(448*a^6-600*a^4*b^2+180*a^2*b^4-5*b^6)*x/b^9-1/30*a*(840*a^4-985*a^2
*b^2+213*b^4)*cos(d*x+c)/b^8/d+1/16*(224*a^4-244*a^2*b^2+43*b^4)*cos(d*x+c)
*sin(d*x+c)/b^7/d-1/30*(280*a^4-291*a^2*b^2+45*b^4)*cos(d*x+c)*sin(d*x+c)^2
/a/b^6/d+1/24*(168*a^4-169*a^2*b^2+24*b^4)*cos(d*x+c)*sin(d*x+c)^3/a^2/b^5/
d+1/4*cos(d*x+c)*sin(d*x+c)^4/a/d/(a+b*sin(d*x+c))^2-1/10*b*cos(d*x+c)*sin(
d*x+c)^5/a^2/d/(a+b*sin(d*x+c))^2-1/60*(56*a^4-60*a^2*b^2+9*b^4)*cos(d*x+c)
*sin(d*x+c)^5/a^2/b^3/d/(a+b*sin(d*x+c))^2-4/15*a*cos(d*x+c)*sin(d*x+c)^6/b
^2/d/(a+b*sin(d*x+c))^2+1/6*cos(d*x+c)*sin(d*x+c)^7/b/d/(a+b*sin(d*x+c))^2-
1/20*(112*a^4-110*a^2*b^2+15*b^4)*cos(d*x+c)*sin(d*x+c)^4/a^2/b^4/d/(a+b*si
n(d*x+c))+a*(56*a^4-47*a^2*b^2+6*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-
b^2)^(1/2))*(a^2-b^2)^(1/2)/b^9/d
```

Rubi [A] time = 2.20, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2896, 3047, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{a\left(-985a^2b^2+840a^4+213b^4\right)\cos(c+dx)}{30b^8d}+\frac{a\sqrt{a^2-b^2}\left(-47a^2b^2+56a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^9d}-\frac{\left(-60a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((448*a^6 - 600*a^4*b^2 + 180*a^2*b^4 - 5*b^6)*x)/(16*b^9) + (a*Sqrt[a^2 -
 b^2]*(56*a^4 - 47*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(b^9*d) - (a*(840*a^4 - 985*a^2*b^2 + 213*b^4)*Cos[c + d*x])/(30
*b^8*d) + ((224*a^4 - 244*a^2*b^2 + 43*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*
b^7*d) - ((280*a^4 - 291*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(30
*a*b^6*d) + ((168*a^4 - 169*a^2*b^2 + 24*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/
(24*a^2*b^5*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*a*d*(a + b*Sin[c + d*x])^
2) - (b*Cos[c + d*x]*Sin[c + d*x]^5)/(10*a^2*d*(a + b*Sin[c + d*x])^2) - ((
56*a^4 - 60*a^2*b^2 + 9*b^4)*Cos[c + d*x]*Sin[c + d*x]^5)/(60*a^2*b^3*d*(a
+ b*Sin[c + d*x])^2) - (4*a*Cos[c + d*x]*Sin[c + d*x]^6)/(15*b^2*d*(a + b*S
in[c + d*x])^2) + (Cos[c + d*x]*Sin[c + d*x]^7)/(6*b*d*(a + b*Sin[c + d*x])
```

```
^2) - ((112*a^4 - 110*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(20*a^
2*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
```

```
n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^5(c+dx)}{10a^2d(a+b\sin(c+dx))^2} - \frac{4a\cos(c+dx)\sin^6(c-}{15b^2d(a+b\sin(c+d}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^5(c+dx)}{10a^2d(a+b\sin(c+dx))^2} - \frac{\left(56a^4-60a^2b^2+9b^4\right)}{60a^2b^3d(a+}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^5(c+dx)}{10a^2d(a+b\sin(c+dx))^2} - \frac{\left(56a^4-60a^2b^2+9b^4\right)}{60a^2b^3d(a+}$$

$$= \frac{\left(168a^4-169a^2b^2+24b^4\right)\cos(c+dx)\sin^3(c+dx)}{24a^2b^5d} + \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))^2}$$

$$= -\frac{\left(280a^4-291a^2b^2+45b^4\right)\cos(c+dx)\sin^2(c+dx)}{30ab^6d} + \frac{\left(168a^4-169a^2b^2+24\right.}{24}$$

$$= \frac{\left(224a^4-244a^2b^2+43b^4\right)\cos(c+dx)\sin(c+dx)}{16b^7d} - \frac{\left(280a^4-291a^2b^2+45b^4\right)}{30a}$$

$$= -\frac{a\left(840a^4-985a^2b^2+213b^4\right)\cos(c+dx)}{30b^8d} + \frac{\left(224a^4-244a^2b^2+43b^4\right)\cos(c}{16b^7d}$$

$$= -\frac{\left(448a^6-600a^4b^2+180a^2b^4-5b^6\right)x}{16b^9} - \frac{a\left(840a^4-985a^2b^2+213b^4\right)\cos(c+}{30b^8d}$$

$$= -\frac{\left(448a^6-600a^4b^2+180a^2b^4-5b^6\right)x}{16b^9} - \frac{a\left(840a^4-985a^2b^2+213b^4\right)\cos(c+}{30b^8d}$$

$$= -\frac{\left(448a^6-600a^4b^2+180a^2b^4-5b^6\right)x}{16b^9} - \frac{a\left(840a^4-985a^2b^2+213b^4\right)\cos(c+}{30b^8d}$$

$$= -\frac{\left(448a^6-600a^4b^2+180a^2b^4-5b^6\right)x}{16b^9} + \frac{a\sqrt{a^2-b^2}\left(56a^4-47a^2b^2+6b^4\right)\tan}{b^9d}$$

Mathematica [B] time = 15.31, size = 2015, normalized size = 3.76

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-8*(c + d*x) + (2*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(4*a^2 - 3*b^2)*Cos[c +
d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x])^2) - (3*b*(4*a^4 - 7*a^2*b^2 +
2*b^4)*Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/(64*b^3*d)
 - (3*((6*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
b^2] + (Cos[c + d*x]*(a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Sin[c + d*x]))/(a +
 b*Sin[c + d*x])^2))/(256*(a - b)^2*(a + b)^2*d) - (3*((12*a*(640*a^8 - 192
0*a^6*b^2 + 2016*a^4*b^4 - 840*a^2*b^6 + 105*b^8)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (-3840*a^10*(c + d*x) + 7680*a
^8*b^2*(c + d*x) - 2976*a^6*b^4*(c + d*x) - 1776*a^4*b^6*(c + d*x) + 960*a^
2*b^8*(c + d*x) - 48*b^10*(c + d*x) - 3840*a^9*b*Cos[c + d*x] + 8640*a^7*b^
3*Cos[c + d*x] - 5696*a^5*b^5*Cos[c + d*x] + 788*a^3*b^7*Cos[c + d*x] + 114
*a*b^9*Cos[c + d*x] + 1920*a^8*b^2*(c + d*x)*Cos[2*(c + d*x)] - 4800*a^6*b^
4*(c + d*x)*Cos[2*(c + d*x)] + 3888*a^4*b^6*(c + d*x)*Cos[2*(c + d*x)] - 10
56*a^2*b^8*(c + d*x)*Cos[2*(c + d*x)] + 48*b^10*(c + d*x)*Cos[2*(c + d*x)]
+ 320*a^7*b^3*Cos[3*(c + d*x)] - 760*a^5*b^5*Cos[3*(c + d*x)] + 560*a^3*b^7
*Cos[3*(c + d*x)] - 120*a*b^9*Cos[3*(c + d*x)] - 8*a^5*b^5*Cos[5*(c + d*x)]
 + 16*a^3*b^7*Cos[5*(c + d*x)] - 8*a*b^9*Cos[5*(c + d*x)] - 7680*a^9*b*(c +
 d*x)*Sin[c + d*x] + 19200*a^7*b^3*(c + d*x)*Sin[c + d*x] - 15552*a^5*b^5*(
c + d*x)*Sin[c + d*x] + 4224*a^3*b^7*(c + d*x)*Sin[c + d*x] - 192*a*b^9*(c
+ d*x)*Sin[c + d*x] - 2880*a^8*b^2*Sin[2*(c + d*x)] + 6880*a^6*b^4*Sin[2*(c
 + d*x)] - 5182*a^4*b^6*Sin[2*(c + d*x)] + 1221*a^2*b^8*Sin[2*(c + d*x)] -
36*b^10*Sin[2*(c + d*x)] - 40*a^6*b^4*Sin[4*(c + d*x)] + 88*a^4*b^6*Sin[4*(
c + d*x)] - 56*a^2*b^8*Sin[4*(c + d*x)] + 8*b^10*Sin[4*(c + d*x)] + 2*a^4*b
^6*Sin[6*(c + d*x)] - 4*a^2*b^8*Sin[6*(c + d*x)] + 2*b^10*Sin[6*(c + d*x)])
/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2)))/(1024*b^7*d) - ((-60*a*(14336*a^1
0 - 49280*a^8*b^2 + 63360*a^6*b^4 - 36960*a^4*b^6 + 9240*a^2*b^8 - 693*b^10
)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (43
0080*a^12*(c + d*x) - 1048320*a^10*b^2*(c + d*x) + 691200*a^8*b^4*(c + d*x)
 + 83040*a^6*b^6*(c + d*x) - 198000*a^4*b^8*(c + d*x) + 43200*a^2*b^10*(c +
 d*x) - 1200*b^12*(c + d*x) + 430080*a^11*b*Cos[c + d*x] - 1155840*a^9*b^3*
Cos[c + d*x] + 1042880*a^7*b^5*Cos[c + d*x] - 332800*a^5*b^7*Cos[c + d*x] +
 11060*a^3*b^9*Cos[c + d*x] + 4530*a*b^11*Cos[c + d*x] - 215040*a^10*b^2*(c
 + d*x)*Cos[2*(c + d*x)] + 631680*a^8*b^4*(c + d*x)*Cos[2*(c + d*x)] - 6614
40*a^6*b^6*(c + d*x)*Cos[2*(c + d*x)] + 289200*a^4*b^8*(c + d*x)*Cos[2*(c +
 d*x)] - 45600*a^2*b^10*(c + d*x)*Cos[2*(c + d*x)] + 1200*b^12*(c + d*x)*Co
s[2*(c + d*x)] - 35840*a^9*b^3*Cos[3*(c + d*x)] + 100800*a^7*b^5*Cos[3*(c +
```

```
d*x)] - 98424*a^5*b^7*Cos[3*(c + d*x)] + 37808*a^3*b^9*Cos[3*(c + d*x)] -
4344*a*b^11*Cos[3*(c + d*x)] + 896*a^7*b^5*Cos[5*(c + d*x)] - 2184*a^5*b^7*
Cos[5*(c + d*x)] + 1680*a^3*b^9*Cos[5*(c + d*x)] - 392*a*b^11*Cos[5*(c + d*
x)] - 64*a^5*b^7*Cos[7*(c + d*x)] + 128*a^3*b^9*Cos[7*(c + d*x)] - 64*a*b^1
1*Cos[7*(c + d*x)] + 860160*a^11*b*(c + d*x)*Sin[c + d*x] - 2526720*a^9*b^3
*(c + d*x)*Sin[c + d*x] + 2645760*a^7*b^5*(c + d*x)*Sin[c + d*x] - 1156800*
a^5*b^7*(c + d*x)*Sin[c + d*x] + 182400*a^3*b^9*(c + d*x)*Sin[c + d*x] - 48
00*a*b^11*(c + d*x)*Sin[c + d*x] + 322560*a^10*b^2*Sin[2*(c + d*x)] - 91168
0*a^8*b^4*Sin[2*(c + d*x)] + 903680*a^6*b^6*Sin[2*(c + d*x)] - 362830*a^4*b
^8*Sin[2*(c + d*x)] + 49125*a^2*b^10*Sin[2*(c + d*x)] - 900*b^12*Sin[2*(c +
 d*x)] + 4480*a^8*b^4*Sin[4*(c + d*x)] - 11816*a^6*b^6*Sin[4*(c + d*x)] + 1
0392*a^4*b^8*Sin[4*(c + d*x)] - 3256*a^2*b^10*Sin[4*(c + d*x)] + 200*b^12*S
in[4*(c + d*x)] - 224*a^6*b^6*Sin[6*(c + d*x)] + 498*a^4*b^8*Sin[6*(c + d*x
)] - 324*a^2*b^10*Sin[6*(c + d*x)] + 50*b^12*Sin[6*(c + d*x)] + 20*a^4*b^8*
Sin[8*(c + d*x)] - 40*a^2*b^10*Sin[8*(c + d*x)] + 20*b^12*Sin[8*(c + d*x)])
/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2))/(15360*b^9*d)
```

fricas [A] time = 1.30, size = 1128, normalized size = 2.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/240*(64*a*b^7*cos(d*x + c)^7 - 4*(56*a^3*b^5 - 19*a*b^7)*cos(d*x + c)^5
 + 15*(448*a^6*b^2 - 600*a^4*b^4 + 180*a^2*b^6 - 5*b^8)*d*x*cos(d*x + c)^2
+ 10*(224*a^5*b^3 - 244*a^3*b^5 + 43*a*b^7)*cos(d*x + c)^3 - 15*(448*a^8 -
152*a^6*b^2 - 420*a^4*b^4 + 175*a^2*b^6 - 5*b^8)*d*x + 60*(56*a^7 + 9*a^5*b
^2 - 41*a^3*b^4 + 6*a*b^6 - (56*a^5*b^2 - 47*a^3*b^4 + 6*a*b^6)*cos(d*x + c
)^2 + 2*(56*a^6*b - 47*a^4*b^3 + 6*a^2*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*
log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*
cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 30*(224*a^7*b - 188*a^5*b^3 - 3
2*a^3*b^5 + 19*a*b^7)*cos(d*x + c) - (40*b^8*cos(d*x + c)^7 - 2*(56*a^2*b^6
 - 5*b^8)*cos(d*x + c)^5 + 5*(112*a^4*b^4 - 94*a^2*b^6 + 5*b^8)*cos(d*x + c
)^3 + 30*(448*a^7*b - 600*a^5*b^3 + 180*a^3*b^5 - 5*a*b^7)*d*x + 15*(672*a^
6*b^2 - 844*a^4*b^4 + 223*a^2*b^6 - 5*b^8)*cos(d*x + c))*sin(d*x + c))/(b^1
1*d*cos(d*x + c)^2 - 2*a*b^10*d*sin(d*x + c) - (a^2*b^9 + b^11)*d), -1/240*
(64*a*b^7*cos(d*x + c)^7 - 4*(56*a^3*b^5 - 19*a*b^7)*cos(d*x + c)^5 + 15*(4
48*a^6*b^2 - 600*a^4*b^4 + 180*a^2*b^6 - 5*b^8)*d*x*cos(d*x + c)^2 + 10*(22
4*a^5*b^3 - 244*a^3*b^5 + 43*a*b^7)*cos(d*x + c)^3 - 15*(448*a^8 - 152*a^6*
b^2 - 420*a^4*b^4 + 175*a^2*b^6 - 5*b^8)*d*x - 120*(56*a^7 + 9*a^5*b^2 - 41
*a^3*b^4 + 6*a*b^6 - (56*a^5*b^2 - 47*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2 + 2
*(56*a^6*b - 47*a^4*b^3 + 6*a^2*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-
```

```
(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 30*(224*a^7*b - 188*
a^5*b^3 - 32*a^3*b^5 + 19*a*b^7)*cos(d*x + c) - (40*b^8*cos(d*x + c)^7 - 2*
(56*a^2*b^6 - 5*b^8)*cos(d*x + c)^5 + 5*(112*a^4*b^4 - 94*a^2*b^6 + 5*b^8)*
cos(d*x + c)^3 + 30*(448*a^7*b - 600*a^5*b^3 + 180*a^3*b^5 - 5*a*b^7)*d*x +
 15*(672*a^6*b^2 - 844*a^4*b^4 + 223*a^2*b^6 - 5*b^8)*cos(d*x + c))*sin(d*x
 + c))/(b^11*d*cos(d*x + c)^2 - 2*a*b^10*d*sin(d*x + c) - (a^2*b^9 + b^11)*
d)]
```

giac [A] time = 0.34, size = 968, normalized size = 1.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(15*(448*a^6 - 600*a^4*b^2 + 180*a^2*b^4 - 5*b^6)*(d*x + c)/b^9 - 24
0*(56*a^7 - 103*a^5*b^2 + 53*a^3*b^4 - 6*a*b^6)*(pi*floor(1/2*(d*x + c)/pi
+ 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt
(a^2 - b^2)*b^9) + 240*(13*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 17*a^4*b^3*tan(1/
2*d*x + 1/2*c)^3 + 4*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 14*a^7*tan(1/2*d*x +
1/2*c)^2 + 9*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 - 33*a^3*b^4*tan(1/2*d*x + 1/2*
c)^2 + 10*a*b^6*tan(1/2*d*x + 1/2*c)^2 + 43*a^6*b*tan(1/2*d*x + 1/2*c) - 59
*a^4*b^3*tan(1/2*d*x + 1/2*c) + 16*a^2*b^5*tan(1/2*d*x + 1/2*c) + 14*a^7 -
19*a^5*b^2 + 5*a^3*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*
c) + a)^2*b^8) + 2*(1800*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 1620*a^2*b^3*tan(1
/2*d*x + 1/2*c)^11 + 165*b^5*tan(1/2*d*x + 1/2*c)^11 + 5040*a^5*tan(1/2*d*x
 + 1/2*c)^10 - 7200*a^3*b^2*tan(1/2*d*x + 1/2*c)^10 + 2160*a*b^4*tan(1/2*d*
x + 1/2*c)^10 + 5400*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 3420*a^2*b^3*tan(1/2*d*
x + 1/2*c)^9 - 25*b^5*tan(1/2*d*x + 1/2*c)^9 + 25200*a^5*tan(1/2*d*x + 1/2*
c)^8 - 31200*a^3*b^2*tan(1/2*d*x + 1/2*c)^8 + 6480*a*b^4*tan(1/2*d*x + 1/2*
c)^8 + 3600*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 1800*a^2*b^3*tan(1/2*d*x + 1/2*c
)^7 + 450*b^5*tan(1/2*d*x + 1/2*c)^7 + 50400*a^5*tan(1/2*d*x + 1/2*c)^6 - 5
6000*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 + 11040*a*b^4*tan(1/2*d*x + 1/2*c)^6 -
3600*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 1800*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 4
50*b^5*tan(1/2*d*x + 1/2*c)^5 + 50400*a^5*tan(1/2*d*x + 1/2*c)^4 - 52800*a^
3*b^2*tan(1/2*d*x + 1/2*c)^4 + 10080*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 5400*a^
4*b*tan(1/2*d*x + 1/2*c)^3 + 3420*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 25*b^5*t
an(1/2*d*x + 1/2*c)^3 + 25200*a^5*tan(1/2*d*x + 1/2*c)^2 - 26400*a^3*b^2*ta
n(1/2*d*x + 1/2*c)^2 + 4464*a*b^4*tan(1/2*d*x + 1/2*c)^2 - 1800*a^4*b*tan(1
/2*d*x + 1/2*c) + 1620*a^2*b^3*tan(1/2*d*x + 1/2*c) - 165*b^5*tan(1/2*d*x +
 1/2*c) + 5040*a^5 - 5600*a^3*b^2 + 1104*a*b^4)/((tan(1/2*d*x + 1/2*c)^2 +
1)^6*b^8))/d
```

maple [B] time = 0.59, size = 2174, normalized size = 4.06

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x)

[Out] -84/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a-10/d*a/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+5/8/d/b^3*arctan(tan(1/2*d*x+1/2*c))-42/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^6*a^5+140/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*a^3-46/5/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*a+11/8/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-11/8/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11+5/24/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9-15/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7+15/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5-5/24/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+75/d/b^7*arctan(tan(1/2*d*x+1/2*c))*a^4-45/2/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^2-56/d/b^9*arctan(tan(1/2*d*x+1/2*c))*a^6-14/d*a^7/b^8/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+19/d*a^5/b^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-5/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+45/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*a^4-57/2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*a^2-210/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a^5+220/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a^3-186/5/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a+15/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*a^4-27/2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*a^2-42/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a^5+60/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a^3-6/d*a/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+440/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a^3-15/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*a^4+27/2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*a^2-45/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*a^4+57/2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*a^2-16/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-9/d*a^5/b^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+33/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-43/d*a^6/b^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+59/d*a^4/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-30/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*a^4+15/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*a^2-420/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a^5+1400/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a^3-92/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a+30/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*a^4-15/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*a^2-13/d*a^6/b^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+17/d*a^4/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-4/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-14/d*a^7/b^8/(tan(1/2*d*x+1/2*c)^2*a+2*tan(

```
1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+56/d*a^7/b^9/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-103/d*a^5/b^7/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+53/d*a^3/
b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)
)-18/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a-210/d/b^8/(1+
tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8*a^5+260/d/b^6/(1+tan(1/2*d*x+1
/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8*a^3-54/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan
(1/2*d*x+1/2*c)^8*a-420/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)
^4*a^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 48.18, size = 4362, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^3)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] - ((840*a^7 + 213*a^3*b^4 - 985*a^5*b^2)/(15*b^8) + (tan(c/2 + (d*x)/2)^14*
(31*a*b^6 + 112*a^7 - 138*a^3*b^4 + 18*a^5*b^2))/(2*b^8) + (tan(c/2 + (d*x)
/2)^12*(410*a*b^6 + 1176*a^7 - 1533*a^3*b^4 + 189*a^5*b^2))/(3*b^8) + (tan(
c/2 + (d*x)/2)^10*(2281*a*b^6 + 7056*a^7 - 8766*a^3*b^4 + 686*a^5*b^2))/(6*
b^8) + (tan(c/2 + (d*x)/2)^2*(1239*a*b^6 + 11760*a^7 - 2402*a^3*b^4 - 9310*
a^5*b^2))/(30*b^8) + (tan(c/2 + (d*x)/2)^4*(3062*a*b^6 + 17640*a^7 - 10011*
a^3*b^4 - 8365*a^5*b^2))/(15*b^8) + (tan(c/2 + (d*x)/2)^6*(14155*a*b^6 + 58
800*a^7 - 50514*a^3*b^4 - 12950*a^5*b^2))/(30*b^8) + (tan(c/2 + (d*x)/2)*(2
3520*a^6 + 6171*a^2*b^4 - 27860*a^4*b^2))/(120*b^7) + (tan(c/2 + (d*x)/2)^1
5*(224*a^6 + 43*a^2*b^4 - 244*a^4*b^2))/(8*b^7) + (tan(c/2 + (d*x)/2)^13*(8
736*a^6 + 132*b^6 + 1453*a^2*b^4 - 9516*a^4*b^2))/(24*b^7) + (tan(c/2 + (d*
x)/2)^11*(38304*a^6 - 20*b^6 + 8033*a^2*b^4 - 43068*a^4*b^2))/(24*b^7) + (t
an(c/2 + (d*x)/2)^9*(84000*a^6 + 360*b^6 + 20341*a^2*b^4 - 97324*a^4*b^2))/
(24*b^7) + (tan(c/2 + (d*x)/2)^7*(104160*a^6 - 360*b^6 + 27371*a^2*b^4 - 12
3316*a^4*b^2))/(24*b^7) + (tan(c/2 + (d*x)/2)^3*(144480*a^6 - 660*b^6 + 404
```

47*a^2*b^4 - 173060*a^4*b^2))/(120*b^7) + (tan(c/2 + (d*x)/2)^5*(372960*a^6 + 100*b^6 + 102971*a^2*b^4 - 446580*a^4*b^2))/(120*b^7) + (tan(c/2 + (d*x)/2)^8*(7*a^2 + 8*b^2)*(213*a*b^4 + 840*a^5 - 985*a^3*b^2))/(3*b^8))/(d*(tan(c/2 + (d*x)/2)^2*(8*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^14*(8*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(28*a^2 + 24*b^2) + tan(c/2 + (d*x)/2)^12*(28*a^2 + 24*b^2) + tan(c/2 + (d*x)/2)^6*(56*a^2 + 60*b^2) + tan(c/2 + (d*x)/2)^10*(56*a^2 + 60*b^2) + tan(c/2 + (d*x)/2)^8*(70*a^2 + 80*b^2) + a^2*tan(c/2 + (d*x)/2)^16 + a^2 + 28*a*b*tan(c/2 + (d*x)/2)^3 + 84*a*b*tan(c/2 + (d*x)/2)^5 + 140*a*b*tan(c/2 + (d*x)/2)^7 + 140*a*b*tan(c/2 + (d*x)/2)^9 + 84*a*b*tan(c/2 + (d*x)/2)^11 + 28*a*b*tan(c/2 + (d*x)/2)^13 + 4*a*b*tan(c/2 + (d*x)/2)^15 + 4*a*b*tan(c/2 + (d*x)/2))) - (atan((((((25*a^2*b^20)/8 - 225*a^4*b^18 + 4800*a^6*b^16 - 27560*a^8*b^14 + 65160*a^10*b^12 - 67200*a^12*b^10 + 25088*a^14*b^8)/b^23 - (((10*a*b^24 - 274*a^3*b^22 + 712*a^5*b^20 - 448*a^7*b^18)/b^23 - ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^28 - 512*a^3*b^26))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(1536*a^2*b^24 - 13568*a^4*b^22 + 26368*a^6*b^20 - 14336*a^8*b^18))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^22 - 5929*a^3*b^20 + 119304*a^5*b^18 - 738240*a^7*b^16 + 2004800*a^9*b^14 - 2655360*a^11*b^12 + 1677312*a^13*b^10 - 401408*a^15*b^8))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i)*1i)/(16*b^9) + ((((25*a^2*b^20)/8 - 225*a^4*b^18 + 4800*a^6*b^16 - 27560*a^8*b^14 + 65160*a^10*b^12 - 67200*a^12*b^10 + 25088*a^14*b^8)/b^23 + (((10*a*b^24 - 274*a^3*b^22 + 712*a^5*b^20 - 448*a^7*b^18)/b^23 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^28 - 512*a^3*b^26))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(1536*a^2*b^24 - 13568*a^4*b^22 + 26368*a^6*b^20 - 14336*a^8*b^18))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^22 - 5929*a^3*b^20 + 119304*a^5*b^18 - 738240*a^7*b^16 + 2004800*a^9*b^14 - 2655360*a^11*b^12 + 1677312*a^13*b^10 - 401408*a^15*b^8))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i)*1i)/(16*b^9))/((702464*a^19 + (645*a^3*b^16)/4 - (65155*a^5*b^14)/8 + (922065*a^7*b^12)/8 - 740668*a^9*b^10 + 2522213*a^11*b^8 - 4837780*a^13*b^6 + 5244512*a^15*b^4 - 2998016*a^17*b^2)/b^23 + ((((25*a^2*b^20)/8 - 225*a^4*b^18 + 4800*a^6*b^16 - 27560*a^8*b^14 + 65160*a^10*b^12 - 67200*a^12*b^10 + 25088*a^14*b^8)/b^23 - (((10*a*b^24 - 274*a^3*b^22 + 712*a^5*b^20 - 448*a^7*b^18)/b^23 - ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^28 - 512*a^3*b^26))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(1536*a^2*b^24 - 13568*a^4*b^22 + 26368*a^6*b^20 - 14336*a^8*b^18))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^22 - 5929*a^3*b^20 + 119304*a^5*b^18 - 738240*a^7*b^16 + 2004800*a^9*b^14 - 2655360*a^11*b^12 + 1677312*a^13*b^10 - 401408*a^15*b^8))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) - ((((25*a^2*b^20)/8 - 225*a^4*b^18 + 4800*a^6*b^16 - 27560*a^8*b^14 + 65160*a^10*b^12 - 67200*a^12*b^10 + 25088*a^14*b^8)/b^23 + (((10*a*b^24 - 274*a^3*b^22 + 712*a^5*b^20 - 448*a^7*b^18)/b^23 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)

*(768*a*b^28 - 512*a^3*b^26))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(1536*a^2*b^24 - 13568*a^4*b^22 + 26368*a^6*b^20 - 14336*a^8*b^18))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^22 - 5929*a^3*b^20 + 119304*a^5*b^18 - 738240*a^7*b^16 + 2004800*a^9*b^14 - 2655360*a^11*b^12 + 1677312*a^13*b^10 - 401408*a^15*b^8))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(11239424*a^20 - 150*a^2*b^18 + 12125*a^4*b^16 - 328375*a^6*b^14 + 3544880*a^8*b^12 - 18869120*a^10*b^10 + 55713280*a^12*b^8 - 95735744*a^14*b^6 + 95201792*a^16*b^4 - 50778112*a^18*b^2))/(4*b^24)))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i)*1i)/(8*b^9*d) - (a*atan(((a*(-(a + b)*(a - b))^(1/2)*(56*a^4 + 6*b^4 - 47*a^2*b^2)*(((25*a^2*b^20)/8 - 225*a^4*b^18 + 4800*a^6*b^16 - 27560*a^8*b^14 + 65160*a^10*b^12 - 67200*a^12*b^10 + 25088*a^14*b^8)/b^23 + (tan(c/2 + (d*x)/2)*(50*a*b^22 - 5929*a^3*b^20 + 119304*a^5*b^18 - 738240*a^7*b^16 + 2004800*a^9*b^14 - 2655360*a^11*b^12 + 1677312*a^13*b^10 - 401408*a^15*b^8))/(8*b^24) - (a*(-(a + b)*(a - b))^(1/2)*((10*a*b^24 - 274*a^3*b^22 + 712*a^5*b^20 - 448*a^7*b^18)/b^23 + (tan(c/2 + (d*x)/2)*(1536*a^2*b^24 - 13568*a^4*b^22 + 26368*a^6*b^20 - 14336*a^8*b^18))/(8*b^24) - (a*(-(a + b)*(a - b))^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^28 - 512*a^3*b^26))/(8*b^24))*(56*a^4 + 6*b^4 - 47*a^2*b^2))/(2*b^9))*(56*a^4 + 6*b^4 - 47*a^2*b^2))/(2*b^9))*1i)/(2*b^9) + (a*(-(a + b)*(a - b))^(1/2)*(56*a^4 + 6*b^4 - 47*a^2*b^2)*(((25*a^2*b^20)/8 - 225*a^4*b^18 + 4800*a^6*b^16 - 27560*a^8*b^14 + 65160*a^10*b^12 - 67200*a^12*b^10 + 25088*a^14*b^8)/b^23 + (tan(c/2 + (d*x)/2)*(50*a*b^22 - 5929*a^3*b^20 + 119304*a^5*b^18 - 738240*a^7*b^16 + 2004800*a^9*b^14 - 2655360*a^11*b^12 + 1677312*a^13*b^10 - 401408*a^15*b^8))/(8*b^24) + (a*(-(a + b)*(a - b))^(1/2)*((10*a*b^24 - 274*a^3*b^22 + 712*a^5*b^20 - 448*a^7*b^18)/b^23 + (tan(c/2 + (d*x)/2)*(1536*a^2*b^24 - 13568*a^4*b^22 + 26368*a^6*b^20 - 14336*a^8*b^18))/(8*b^24) + (a*(-(a + b)*(a - b))^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^28 - 512*a^3*b^26))/(8*b^24))*(56*a^4 + 6*b^4 - 47*a^2*b^2))/(2*b^9))*(56*a^4 + 6*b^4 - 47*a^2*b^2))/(2*b^9))*1i)/(2*b^9))/((702464*a^19 + (645*a^3*b^16)/4 - (65155*a^5*b^14)/8 + (922065*a^7*b^12)/8 - 740668*a^9*b^10 + 2522213*a^11*b^8 - 4837780*a^13*b^6 + 5244512*a^15*b^4 - 2998016*a^17*b^2)/b^23 + (tan(c/2 + (d*x)/2)*(11239424*a^20 - 150*a^2*b^18 + 12125*a^4*b^16 - 328375*a^6*b^14 + 3544880*a^8*b^12 - 18869120*a^10*b^10 + 55713280*a^12*b^8 - 95735744*a^14*b^6 + 95201792*a^16*b^4 - 50778112*a^18*b^2))/(4*b^24) + (a*(-(a + b)*(a - b))^(1/2)*(56*a^4 + 6*b^4 - 47*a^2*b^2)*(((25*a^2*b^20)/8 - 225*a^4*b^18 + 4800*a^6*b^16 - 27560*a^8*b^14 + 65160*a^10*b^12 - 67200*a^12*b^10 + 25088*a^14*b^8)/b^23 + (tan(c/2 + (d*x)/2)*(50*a*b^22 - 5929*a^3*b^20 + 119304*a^5*b^18 - 738240*a^7*b^16 + 2004800*a^9*b^14 - 2655360*a^11*b^12 + 1677312*a^13*b^10 - 401408*a^15*b^8))/(8*b^24) - (a*(-(a + b)*(a - b))^(1/2)*((10*a*b^24 - 274*a^3*b^22 + 712*a^5*b^20 - 448*a^7*b^18)/b^23 + (tan(c/2 + (d*x)/2)*(1536*a^2*b^24 - 13568*a^4*b^22 + 26368*a^6*b^20 - 14336*a^8*b^18))/(8*b^24) - (a*(-(a + b)*(a - b))^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^28 - 512*a^3*b^26))/(8*b^24))*(56*a^4 + 6*b^4 - 47*a^2*b^2))/(2*b^9))*(56*a^4 +

```
6*b^4 - 47*a^2*b^2))/(2*b^9)))/(2*b^9) - (a*(-(a + b)*(a - b))^(1/2)*(56*a^
4 + 6*b^4 - 47*a^2*b^2)*(((25*a^2*b^20)/8 - 225*a^4*b^18 + 4800*a^6*b^16 -
27560*a^8*b^14 + 65160*a^10*b^12 - 67200*a^12*b^10 + 25088*a^14*b^8)/b^23 +
 (tan(c/2 + (d*x)/2)*(50*a*b^22 - 5929*a^3*b^20 + 119304*a^5*b^18 - 738240*
a^7*b^16 + 2004800*a^9*b^14 - 2655360*a^11*b^12 + 1677312*a^13*b^10 - 40140
8*a^15*b^8))/(8*b^24) + (a*(-(a + b)*(a - b))^(1/2)*((10*a*b^24 - 274*a^3*b
^22 + 712*a^5*b^20 - 448*a^7*b^18)/b^23 + (tan(c/2 + (d*x)/2)*(1536*a^2*b^2
4 - 13568*a^4*b^22 + 26368*a^6*b^20 - 14336*a^8*b^18))/(8*b^24) + (a*(-(a +
 b)*(a - b))^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^28 - 512*a^3*
b^26))/(8*b^24))*(56*a^4 + 6*b^4 - 47*a^2*b^2))/(2*b^9))*(56*a^4 + 6*b^4 -
47*a^2*b^2))/(2*b^9)))/(2*b^9)))*(-(a + b)*(a - b))^(1/2)*(56*a^4 + 6*b^4 -
 47*a^2*b^2)*1i)/(b^9*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1267 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=485

$$-\frac{b\sin^4(c+dx)\cos(c+dx)}{12a^2d(a+b\sin(c+dx))^2}-\frac{\left(63a^4-54a^2b^2+4b^4\right)\sin^3(c+dx)\cos(c+dx)}{12a^2b^4d(a+b\sin(c+dx))}-\frac{\sqrt{a^2-b^2}\left(42a^4-29a^2b^2+2b^4\right)\mathrm{t}}{b^8d}$$

```
[Out] 1/8*a*(168*a^4-200*a^2*b^2+45*b^4)*x/b^8+1/30*(630*a^4-645*a^2*b^2+91*b^4)*
cos(d*x+c)/b^7/d-1/8*(84*a^4-79*a^2*b^2+8*b^4)*cos(d*x+c)*sin(d*x+c)/a/b^6/
d+1/30*(210*a^4-187*a^2*b^2+15*b^4)*cos(d*x+c)*sin(d*x+c)^2/a^2/b^5/d+1/3*c
os(d*x+c)*sin(d*x+c)^3/a/d/(a+b*sin(d*x+c))^2-1/12*b*cos(d*x+c)*sin(d*x+c)^
4/a^2/d/(a+b*sin(d*x+c))^2-1/60*(63*a^4-60*a^2*b^2+5*b^4)*cos(d*x+c)*sin(d*
x+c)^4/a^2/b^3/d/(a+b*sin(d*x+c))^2-7/20*a*cos(d*x+c)*sin(d*x+c)^5/b^2/d/(a
+b*sin(d*x+c))^2+1/5*cos(d*x+c)*sin(d*x+c)^6/b/d/(a+b*sin(d*x+c))^2-1/12*(6
3*a^4-54*a^2*b^2+4*b^4)*cos(d*x+c)*sin(d*x+c)^3/a^2/b^4/d/(a+b*sin(d*x+c))-
(42*a^4-29*a^2*b^2+2*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*
(a^2-b^2)^(1/2)/b^8/d
```

Rubi [A] time = 1.72, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2896, 3047, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{\left(-645a^2b^2+630a^4+91b^4\right)\cos(c+dx)}{30b^7d}-\frac{\sqrt{a^2-b^2}\left(-29a^2b^2+42a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^8d}-\frac{\left(-60a^2b^2+\right.}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(168*a^4 - 200*a^2*b^2 + 45*b^4)*x)/(8*b^8) - (Sqrt[a^2 - b^2]*(42*a^4 -
 29*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^8
*d) + ((630*a^4 - 645*a^2*b^2 + 91*b^4)*Cos[c + d*x])/(30*b^7*d) - ((84*a^4
 - 79*a^2*b^2 + 8*b^4)*Cos[c + d*x]*Sin[c + d*x])/(8*a*b^6*d) + ((210*a^4 -
 187*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(30*a^2*b^5*d) + (Cos[c
 + d*x]*Sin[c + d*x]^3)/(3*a*d*(a + b*Sin[c + d*x])^2) - (b*Cos[c + d*x]*Si
n[c + d*x]^4)/(12*a^2*d*(a + b*Sin[c + d*x])^2) - ((63*a^4 - 60*a^2*b^2 + 5
*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(60*a^2*b^3*d*(a + b*Sin[c + d*x])^2) -
(7*a*Cos[c + d*x]*Sin[c + d*x]^5)/(20*b^2*d*(a + b*Sin[c + d*x])^2) + (Cos[
c + d*x]*Sin[c + d*x]^6)/(5*b*d*(a + b*Sin[c + d*x])^2) - ((63*a^4 - 54*a^2
*b^2 + 4*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/(12*a^2*b^4*d*(a + b*Sin[c + d*x
]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^4(c+dx)}{12a^2d(a+b\sin(c+dx))^2} - \frac{7a\cos(c+dx)\sin^5(c-}{20b^2d(a+b\sin(c+d}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^4(c+dx)}{12a^2d(a+b\sin(c+dx))^2} - \frac{\left(63a^4-60a^2b^2+5b^4\right)}{60a^2b^3d(a+}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^4(c+dx)}{12a^2d(a+b\sin(c+dx))^2} - \frac{\left(63a^4-60a^2b^2+5b^4\right)}{60a^2b^3d(a+}$$

$$= \frac{\left(210a^4-187a^2b^2+15b^4\right)\cos(c+dx)\sin^2(c+dx)}{30a^2b^5d} + \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$$= -\frac{\left(84a^4-79a^2b^2+8b^4\right)\cos(c+dx)\sin(c+dx)}{8ab^6d} + \frac{\left(210a^4-187a^2b^2+15b^4\right)\mathrm{c}}{30a^2b^5}$$

$$= \frac{\left(630a^4-645a^2b^2+91b^4\right)\cos(c+dx)}{30b^7d} - \frac{\left(84a^4-79a^2b^2+8b^4\right)\cos(c+dx)\sin}{8ab^6d}$$

$$= \frac{a\left(168a^4-200a^2b^2+45b^4\right)x}{8b^8} + \frac{\left(630a^4-645a^2b^2+91b^4\right)\cos(c+dx)}{30b^7d} - \frac{\left(84a\right.}{}$$

$$= \frac{a\left(168a^4-200a^2b^2+45b^4\right)x}{8b^8} + \frac{\left(630a^4-645a^2b^2+91b^4\right)\cos(c+dx)}{30b^7d} - \frac{\left(84a\right.}{}$$

$$= \frac{a\left(168a^4-200a^2b^2+45b^4\right)x}{8b^8} + \frac{\left(630a^4-645a^2b^2+91b^4\right)\cos(c+dx)}{30b^7d} - \frac{\left(84a\right.}{}$$

$$= \frac{a\left(168a^4-200a^2b^2+45b^4\right)x}{8b^8} - \frac{\sqrt{a^2-b^2}\left(42a^4-29a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan}{\sqrt{a}}\right.}{b^8d}$$

Mathematica [A] time = 12.87, size = 517, normalized size = 1.07

$$\frac{\left(a^2-b^2\right)^2\left(40320a^7c+40320a^7dx+80640a^6bc\sin(c+dx)+80640a^6bdx\sin(c+dx)+30240a^5b^2\sin(2(c+dx))-27840a^5b^2c-27840a^5b^2dx-96000a^4b^3c\sin(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-1920*(a^2 - b^2)^(5/2)*(42*a^4 - 29*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]] + ((a^2 - b^2)^2*(40320*a^7*c - 27840*a^5*b^2*
c - 13200*a^3*b^4*c + 5400*a*b^6*c + 40320*a^7*d*x - 27840*a^5*b^2*d*x - 13
200*a^3*b^4*d*x + 5400*a*b^6*d*x + 10*b*(4032*a^6 - 3792*a^4*b^2 + 216*a^2*
b^4 + 59*b^6)*Cos[c + d*x] - 120*a*b^2*(168*a^4 - 200*a^2*b^2 + 45*b^4)*(c
+ d*x)*Cos[2*(c + d*x)] - 3360*a^4*b^3*Cos[3*(c + d*x)] + 3580*a^2*b^5*Cos[
3*(c + d*x)] - 526*b^7*Cos[3*(c + d*x)] + 84*a^2*b^5*Cos[5*(c + d*x)] - 58*
b^7*Cos[5*(c + d*x)] - 6*b^7*Cos[7*(c + d*x)] + 80640*a^6*b*c*Sin[c + d*x]
- 96000*a^4*b^3*c*Sin[c + d*x] + 21600*a^2*b^5*c*Sin[c + d*x] + 80640*a^6*b
*d*x*Sin[c + d*x] - 96000*a^4*b^3*d*x*Sin[c + d*x] + 21600*a^2*b^5*d*x*Sin[
c + d*x] + 30240*a^5*b^2*Sin[2*(c + d*x)] - 32640*a^3*b^4*Sin[2*(c + d*x)]
+ 5675*a*b^6*Sin[2*(c + d*x)] + 420*a^3*b^4*Sin[4*(c + d*x)] - 374*a*b^6*Si
n[4*(c + d*x)] - 21*a*b^6*Sin[6*(c + d*x)]))/(a + b*Sin[c + d*x])^2)/(1920*
(a - b)^2*b^8*(a + b)^2*d)
```

fricas [A] time = 1.25, size = 995, normalized size = 2.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/120*(24*b^7*cos(d*x + c)^7 - 4*(21*a^2*b^5 - 4*b^7)*cos(d*x + c)^5 + 15*
(168*a^5*b^2 - 200*a^3*b^4 + 45*a*b^6)*d*x*cos(d*x + c)^2 + 10*(84*a^4*b^3
- 79*a^2*b^5 + 8*b^7)*cos(d*x + c)^3 - 15*(168*a^7 - 32*a^5*b^2 - 155*a^3*b
^4 + 45*a*b^6)*d*x - 30*(42*a^6 + 13*a^4*b^2 - 27*a^2*b^4 + 2*b^6 - (42*a^4
*b^2 - 29*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(42*a^5*b - 29*a^3*b^3 + 2*a*
b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x +
c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)
) - 30*(84*a^6*b - 58*a^4*b^3 - 17*a^2*b^5 + 4*b^7)*cos(d*x + c) + (42*a*b^
6*cos(d*x + c)^5 - 5*(42*a^3*b^4 - 29*a*b^6)*cos(d*x + c)^3 - 30*(168*a^6*b
 - 200*a^4*b^3 + 45*a^2*b^5)*d*x - 15*(252*a^5*b^2 - 279*a^3*b^4 + 53*a*b^6
)*cos(d*x + c))*sin(d*x + c))/(b^10*d*cos(d*x + c)^2 - 2*a*b^9*d*sin(d*x +
c) - (a^2*b^8 + b^10)*d), 1/120*(24*b^7*cos(d*x + c)^7 - 4*(21*a^2*b^5 - 4*
b^7)*cos(d*x + c)^5 + 15*(168*a^5*b^2 - 200*a^3*b^4 + 45*a*b^6)*d*x*cos(d*x
 + c)^2 + 10*(84*a^4*b^3 - 79*a^2*b^5 + 8*b^7)*cos(d*x + c)^3 - 15*(168*a^7
 - 32*a^5*b^2 - 155*a^3*b^4 + 45*a*b^6)*d*x - 60*(42*a^6 + 13*a^4*b^2 - 27*
a^2*b^4 + 2*b^6 - (42*a^4*b^2 - 29*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(42*
a^5*b - 29*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(
```

```
d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 30*(84*a^6*b - 58*a^4*b^3 -
 17*a^2*b^5 + 4*b^7)*cos(d*x + c) + (42*a*b^6*cos(d*x + c)^5 - 5*(42*a^3*b^
4 - 29*a*b^6)*cos(d*x + c)^3 - 30*(168*a^6*b - 200*a^4*b^3 + 45*a^2*b^5)*d*
x - 15*(252*a^5*b^2 - 279*a^3*b^4 + 53*a*b^6)*cos(d*x + c))*sin(d*x + c))/(
b^10*d*cos(d*x + c)^2 - 2*a*b^9*d*sin(d*x + c) - (a^2*b^8 + b^10)*d)]
```

giac [A] time = 0.28, size = 724, normalized size = 1.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(168*a^5 - 200*a^3*b^2 + 45*a*b^4)*(d*x + c)/b^8 - 120*(42*a^6 -
71*a^4*b^2 + 31*a^2*b^4 - 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) +
 arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^8
) + 120*(11*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 13*a^3*b^3*tan(1/2*d*x + 1/2*c)^
3 + 2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 12*a^6*tan(1/2*d*x + 1/2*c)^2 + 9*a^4*
b^2*tan(1/2*d*x + 1/2*c)^2 - 27*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 + 6*b^6*tan(
1/2*d*x + 1/2*c)^2 + 37*a^5*b*tan(1/2*d*x + 1/2*c) - 47*a^3*b^3*tan(1/2*d*x
 + 1/2*c) + 10*a*b^5*tan(1/2*d*x + 1/2*c) + 12*a^6 - 15*a^4*b^2 + 3*a^2*b^4
)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*b^7) + 2*(60
0*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 405*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 1800*a^
4*tan(1/2*d*x + 1/2*c)^8 - 2160*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 360*b^4*ta
n(1/2*d*x + 1/2*c)^8 + 1200*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 450*a*b^3*tan(1/
2*d*x + 1/2*c)^7 + 7200*a^4*tan(1/2*d*x + 1/2*c)^6 - 7200*a^2*b^2*tan(1/2*d
*x + 1/2*c)^6 + 720*b^4*tan(1/2*d*x + 1/2*c)^6 + 10800*a^4*tan(1/2*d*x + 1/
2*c)^4 - 9600*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 1120*b^4*tan(1/2*d*x + 1/2*c
)^4 - 1200*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 450*a*b^3*tan(1/2*d*x + 1/2*c)^3
+ 7200*a^4*tan(1/2*d*x + 1/2*c)^2 - 6240*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 5
60*b^4*tan(1/2*d*x + 1/2*c)^2 - 600*a^3*b*tan(1/2*d*x + 1/2*c) + 405*a*b^3*
tan(1/2*d*x + 1/2*c) + 1800*a^4 - 1680*a^2*b^2 + 184*b^4)/((tan(1/2*d*x + 1
/2*c)^2 + 1)^5*b^7))/d
```

maple [B] time = 0.64, size = 1676, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -36/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8*a^2+46/15/d/b^3/(
1+tan(1/2*d*x+1/2*c)^2)^5+20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1
/2*c)^7*a^3-15/2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a+12
```

```
0/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6*a^4-120/d/b^5/(1+ta
n(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6*a^2+180/d/b^7/(1+tan(1/2*d*x+1/2
*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^4-160/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(
1/2*d*x+1/2*c)^4*a^2-20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)
^3*a^3+15/2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a-13/d/b^
4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*
a^3+2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x
+1/2*c)^3*a+10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9*a^3-27
/4/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9*a+42/d/b^8*arctan(
tan(1/2*d*x+1/2*c))*a^5-15/d/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*
c)*b+a)^2*a^4+30/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^5*a^4+28/3/d/b^3/(1+tan(1/2
*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2+56/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5
*tan(1/2*d*x+1/2*c)^4+6/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)
^8+12/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6-28/d/b^5/(1+tan
(1/2*d*x+1/2*c)^2)^5*a^2+3/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*
c)*b+a)^2*a^2-42/d/b^8/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2
*b)/(a^2-b^2)^(1/2))*a^6+71/d/b^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^4-31/d/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2+11/d/b^6/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*a^5+30/d/b^7/(1+tan(1
/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8*a^4+12/d/b^7/(tan(1/2*d*x+1/2*c)^2*
a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*a^6+9/d/b^5/(tan(1/2*d*x
+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*a^4-27/d/b^3/(
tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*a^2
+37/d/b^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1
/2*c)*a^5-47/d/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(
1/2*d*x+1/2*c)*a^3+10/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2*tan(1/2*d*x+1/2*c)*a+120/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+
1/2*c)^2*a^4-104/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^2-
10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a^3+27/4/d/b^4/(1+ta
n(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a+45/4/d/b^4*arctan(tan(1/2*d*x+1/
2*c))*a+6/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d
*x+1/2*c)^2+12/d/b^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^
6+2/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)
^(1/2))-50/d/b^6*arctan(tan(1/2*d*x+1/2*c))*a^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
```

```
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 23.85, size = 3700, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^2)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] ((630*a^6 + 91*a^2*b^4 - 645*a^4*b^2)/(15*b^7) + (tan(c/2 + (d*x)/2)^13*(8*
a*b^4 + 84*a^5 - 79*a^3*b^2))/(4*b^6) + (tan(c/2 + (d*x)/2)^11*(17*a*b^4 +
252*a^5 - 237*a^3*b^2))/b^6 + (8*tan(c/2 + (d*x)/2)^7*(91*a*b^4 + 630*a^5 -
 645*a^3*b^2))/(3*b^6) + (9*tan(c/2 + (d*x)/2)^5*(112*a*b^4 + 700*a^5 - 733
*a^3*b^2))/(4*b^6) + (tan(c/2 + (d*x)/2)^9*(448*a*b^4 + 3780*a^5 - 3723*a^3
*b^2))/(4*b^6) + (tan(c/2 + (d*x)/2)^3*(643*a*b^4 + 3780*a^5 - 3975*a^3*b^2
))/(5*b^6) + (tan(c/2 + (d*x)/2)^12*(42*a^6 + 6*b^6 - 48*a^2*b^4 + 13*a^4*b
^2))/b^7 + (3*tan(c/2 + (d*x)/2)^10*(84*a^6 + 18*b^6 - 103*a^2*b^4 + 26*a^4
*b^2))/b^7 + (2*tan(c/2 + (d*x)/2)^6*(1260*a^6 + 202*b^6 - 1187*a^2*b^4 + 5
4*a^4*b^2))/(3*b^7) + (tan(c/2 + (d*x)/2)^8*(1890*a^6 + 324*b^6 - 2149*a^2*
b^4 + 417*a^4*b^2))/(3*b^7) + (tan(c/2 + (d*x)/2)^2*(3780*a^6 + 274*b^6 - 1
223*a^2*b^4 - 2190*a^4*b^2))/(15*b^7) + (tan(c/2 + (d*x)/2)^4*(9450*a^6 + 1
010*b^6 - 6354*a^2*b^4 - 2115*a^4*b^2))/(15*b^7) + (tan(c/2 + (d*x)/2)*(133
6*a*b^4 + 8820*a^5 - 9135*a^3*b^2))/(60*b^6))/(d*(tan(c/2 + (d*x)/2)^2*(7*a
^2 + 4*b^2) + tan(c/2 + (d*x)/2)^12*(7*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*
(21*a^2 + 20*b^2) + tan(c/2 + (d*x)/2)^10*(21*a^2 + 20*b^2) + tan(c/2 + (d*
x)/2)^6*(35*a^2 + 40*b^2) + tan(c/2 + (d*x)/2)^8*(35*a^2 + 40*b^2) + a^2*ta
n(c/2 + (d*x)/2)^14 + a^2 + 24*a*b*tan(c/2 + (d*x)/2)^3 + 60*a*b*tan(c/2 +
(d*x)/2)^5 + 80*a*b*tan(c/2 + (d*x)/2)^7 + 60*a*b*tan(c/2 + (d*x)/2)^9 + 24
*a*b*tan(c/2 + (d*x)/2)^11 + 4*a*b*tan(c/2 + (d*x)/2)^13 + 4*a*b*tan(c/2 +
(d*x)/2))) + (a*atan(((a*(((2025*a^4*b^15)/2 - 9000*a^6*b^13 + 27560*a^8*b^
11 - 33600*a^10*b^9 + 14112*a^12*b^7)/b^20 - (tan(c/2 + (d*x)/2)*(64*a*b^19
 - 6034*a^3*b^17 + 57945*a^5*b^15 - 201360*a^7*b^13 + 311840*a^9*b^11 - 219
072*a^11*b^9 + 56448*a^13*b^7))/(2*b^21) + (a*(168*a^4 + 45*b^4 - 200*a^2*b
^2)*((148*a^2*b^20 - 484*a^4*b^18 + 336*a^6*b^16)/b^20 - (tan(c/2 + (d*x)/2
)*(128*a*b^22 - 1984*a^3*b^20 + 4544*a^5*b^18 - 2688*a^7*b^16))/(2*b^21) +
(a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^25 - 128*a^3*b^23))/(2*b^21))
*(168*a^4 + 45*b^4 - 200*a^2*b^2)*1i)/(8*b^8))*1i)/(8*b^8))*(168*a^4 + 45*b
^4 - 200*a^2*b^2))/(8*b^8) + (a*(((2025*a^4*b^15)/2 - 9000*a^6*b^13 + 27560
*a^8*b^11 - 33600*a^10*b^9 + 14112*a^12*b^7)/b^20 - (tan(c/2 + (d*x)/2)*(64
*a*b^19 - 6034*a^3*b^17 + 57945*a^5*b^15 - 201360*a^7*b^13 + 311840*a^9*b^1
1 - 219072*a^11*b^9 + 56448*a^13*b^7))/(2*b^21) + (a*(168*a^4 + 45*b^4 - 20
0*a^2*b^2)*((tan(c/2 + (d*x)/2)*(128*a*b^22 - 1984*a^3*b^20 + 4544*a^5*b^18
 - 2688*a^7*b^16))/(2*b^21) - (148*a^2*b^20 - 484*a^4*b^18 + 336*a^6*b^16)/
```

b^20 + (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^25 - 128*a^3*b^23))/(2*b^21))*(168*a^4 + 45*b^4 - 200*a^2*b^2)*1i)/(8*b^8))*1i)/(8*b^8))*(168*a^4 + 45*b^4 - 200*a^2*b^2))/(8*b^8))/((296352*a^16 - 360*a^2*b^14 + 10735*a^4*b^12 - (227213*a^6*b^10)/2 + (1089913*a^8*b^8)/2 - 1331285*a^10*b^6 + 1725696*a^12*b^4 - 1132488*a^14*b^2)/b^20 + (tan(c/2 + (d*x)/2)*(1185408*a^17 - 4050*a^3*b^14 + 98775*a^5*b^12 - 812015*a^7*b^10 + 3206170*a^9*b^8 - 6809168*a^11*b^6 + 7961184*a^13*b^4 - 4826304*a^15*b^2))/b^21 + (a*(((2025*a^4*b^15)/2 - 9000*a^6*b^13 + 27560*a^8*b^11 - 33600*a^10*b^9 + 14112*a^12*b^7)/b^20 - (tan(c/2 + (d*x)/2)*(64*a*b^19 - 6034*a^3*b^17 + 57945*a^5*b^15 - 201360*a^7*b^13 + 311840*a^9*b^11 - 219072*a^11*b^9 + 56448*a^13*b^7))/(2*b^21) + (a*(168*a^4 + 45*b^4 - 200*a^2*b^2)*((148*a^2*b^20 - 484*a^4*b^18 + 336*a^6*b^16)/b^20 - (tan(c/2 + (d*x)/2)*(128*a*b^22 - 1984*a^3*b^20 + 4544*a^5*b^18 - 2688*a^7*b^16))/(2*b^21) + (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^25 - 128*a^3*b^23))/(2*b^21))*(168*a^4 + 45*b^4 - 200*a^2*b^2)*1i)/(8*b^8))*1i)/(8*b^8))*(168*a^4 + 45*b^4 - 200*a^2*b^2)*1i)/(8*b^8) - (a*(((2025*a^4*b^15)/2 - 9000*a^6*b^13 + 27560*a^8*b^11 - 33600*a^10*b^9 + 14112*a^12*b^7)/b^20 - (tan(c/2 + (d*x)/2)*(64*a*b^19 - 6034*a^3*b^17 + 57945*a^5*b^15 - 201360*a^7*b^13 + 311840*a^9*b^11 - 219072*a^11*b^9 + 56448*a^13*b^7))/(2*b^21) + (a*(168*a^4 + 45*b^4 - 200*a^2*b^2)*((tan(c/2 + (d*x)/2)*(128*a*b^22 - 1984*a^3*b^20 + 4544*a^5*b^18 - 2688*a^7*b^16))/(2*b^21) - (148*a^2*b^20 - 484*a^4*b^18 + 336*a^6*b^16)/b^20 + (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^25 - 128*a^3*b^23))/(2*b^21))*(168*a^4 + 45*b^4 - 200*a^2*b^2)*1i)/(8*b^8))*1i)/(8*b^8))*(168*a^4 + 45*b^4 - 200*a^2*b^2)*1i)/(8*b^8)))*(168*a^4 + 45*b^4 - 200*a^2*b^2))/(4*b^8*d) + (atan((((-(a + b)*(a - b))^(1/2)*(21*a^4 + b^4 - (29*a^2*b^2)/2)*(((2025*a^4*b^15)/2 - 9000*a^6*b^13 + 27560*a^8*b^11 - 33600*a^10*b^9 + 14112*a^12*b^7)/b^20 - (tan(c/2 + (d*x)/2)*(64*a*b^19 - 6034*a^3*b^17 + 57945*a^5*b^15 - 201360*a^7*b^13 + 311840*a^9*b^11 - 219072*a^11*b^9 + 56448*a^13*b^7))/(2*b^21) + ((-(a + b)*(a - b))^(1/2)*(21*a^4 + b^4 - (29*a^2*b^2)/2)*((148*a^2*b^20 - 484*a^4*b^18 + 336*a^6*b^16)/b^20 - (tan(c/2 + (d*x)/2)*(128*a*b^22 - 1984*a^3*b^20 + 4544*a^5*b^18 - 2688*a^7*b^16))/(2*b^21) + ((-(a + b)*(a - b))^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^25 - 128*a^3*b^23))/(2*b^21))*(21*a^4 + b^4 - (29*a^2*b^2)/2))/b^8))/b^8)*1i)/b^8 + ((-(a + b)*(a - b))^(1/2)*(21*a^4 + b^4 - (29*a^2*b^2)/2)*(((2025*a^4*b^15)/2 - 9000*a^6*b^13 + 27560*a^8*b^11 - 33600*a^10*b^9 + 14112*a^12*b^7)/b^20 - (tan(c/2 + (d*x)/2)*(64*a*b^19 - 6034*a^3*b^17 + 57945*a^5*b^15 - 201360*a^7*b^13 + 311840*a^9*b^11 - 219072*a^11*b^9 + 56448*a^13*b^7))/(2*b^21) + ((-(a + b)*(a - b))^(1/2)*(21*a^4 + b^4 - (29*a^2*b^2)/2)*((tan(c/2 + (d*x)/2)*(128*a*b^22 - 1984*a^3*b^20 + 4544*a^5*b^18 - 2688*a^7*b^16))/(2*b^21) - (148*a^2*b^20 - 484*a^4*b^18 + 336*a^6*b^16)/b^20 + ((-(a + b)*(a - b))^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^25 - 128*a^3*b^23))/(2*b^21))*(21*a^4 + b^4 - (29*a^2*b^2)/2))/b^8))/b^8)*1i)/b^8)/((296352*a^16 - 360*a^2*b^14 + 10735*a^4*b^12 - (227213*a^6*b^10)/2 + (1089913*a^8*b^8)/2 - 1331285*a^10*b^6 + 1725696*a^12*b^4 - 1132488*a^14*b^2)/b^20 + (tan(c/2 + (d*x)/2)*(1185408*a^17 - 4050*a^3*b^14 + 98775*a^5*b^12 - 812015*a^7*b^10 + 3206170*a^9*b^8 - 6809168*a^11*b^6 +

```
 7961184*a^13*b^4 - 4826304*a^15*b^2))/b^21 + ((-(a + b)*(a - b))^(1/2)*(21
*a^4 + b^4 - (29*a^2*b^2)/2)*(((2025*a^4*b^15)/2 - 9000*a^6*b^13 + 27560*a^
8*b^11 - 33600*a^10*b^9 + 14112*a^12*b^7)/b^20 - (tan(c/2 + (d*x)/2)*(64*a*
b^19 - 6034*a^3*b^17 + 57945*a^5*b^15 - 201360*a^7*b^13 + 311840*a^9*b^11 -
 219072*a^11*b^9 + 56448*a^13*b^7))/(2*b^21) + ((-(a + b)*(a - b))^(1/2)*(2
1*a^4 + b^4 - (29*a^2*b^2)/2)*((148*a^2*b^20 - 484*a^4*b^18 + 336*a^6*b^16)
/b^20 - (tan(c/2 + (d*x)/2)*(128*a*b^22 - 1984*a^3*b^20 + 4544*a^5*b^18 - 2
688*a^7*b^16))/(2*b^21) + ((-(a + b)*(a - b))^(1/2)*(32*a^2*b^3 + (tan(c/2
+ (d*x)/2)*(192*a*b^25 - 128*a^3*b^23))/(2*b^21))*(21*a^4 + b^4 - (29*a^2*b
^2)/2))/b^8))/b^8))/b^8 - ((-(a + b)*(a - b))^(1/2)*(21*a^4 + b^4 - (29*a^2
*b^2)/2)*(((2025*a^4*b^15)/2 - 9000*a^6*b^13 + 27560*a^8*b^11 - 33600*a^10*
b^9 + 14112*a^12*b^7)/b^20 - (tan(c/2 + (d*x)/2)*(64*a*b^19 - 6034*a^3*b^17
 + 57945*a^5*b^15 - 201360*a^7*b^13 + 311840*a^9*b^11 - 219072*a^11*b^9 + 5
6448*a^13*b^7))/(2*b^21) + ((-(a + b)*(a - b))^(1/2)*(21*a^4 + b^4 - (29*a^
2*b^2)/2)*((tan(c/2 + (d*x)/2)*(128*a*b^22 - 1984*a^3*b^20 + 4544*a^5*b^18
- 2688*a^7*b^16))/(2*b^21) - (148*a^2*b^20 - 484*a^4*b^18 + 336*a^6*b^16)/b
^20 + ((-(a + b)*(a - b))^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^
25 - 128*a^3*b^23))/(2*b^21))*(21*a^4 + b^4 - (29*a^2*b^2)/2))/b^8))/b^8))/
b^8))*(-(a + b)*(a - b))^(1/2)*(21*a^4 + b^4 - (29*a^2*b^2)/2)*2i)/(b^8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1268 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=237

$$-\frac{15\cos(c+dx)\left(4a\left(2a^2-b^2\right)-b\left(4a^2-b^2\right)\sin(c+dx)\right)}{8b^6d}+\frac{5\cos^3(c+dx)\left(4a^2+ab\sin(c+dx)-b^2\right)}{4b^4d(a+b\sin(c+dx))}+\frac{15a\left(2a^4\right.}{}$$

```
[Out] -15/8*(8*a^4-8*a^2*b^2+b^4)*x/b^7+1/4*cos(d*x+c)^5*(3*a+b*sin(d*x+c))/b^2/d
/(a+b*sin(d*x+c))^2+5/4*cos(d*x+c)^3*(4*a^2-b^2+a*b*sin(d*x+c))/b^4/d/(a+b*
sin(d*x+c))-15/8*cos(d*x+c)*(4*a*(2*a^2-b^2)-b*(4*a^2-b^2)*sin(d*x+c))/b^6/
d+15*a*(2*a^4-3*a^2*b^2+b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2
))/b^7/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2863, 2865, 2735, 2660, 618, 204}

$$\frac{15a\left(-3a^2b^2+2a^4+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d\sqrt{a^2-b^2}}+\frac{5\cos^3(c+dx)\left(4a^2+ab\sin(c+dx)-b^2\right)}{4b^4d(a+b\sin(c+dx))}-\frac{15\cos(c+dx)\left(4a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-15*(8*a^4 - 8*a^2*b^2 + b^4)*x)/(8*b^7) + (15*a*(2*a^4 - 3*a^2*b^2 + b^4)
*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*Sqrt[a^2 - b^2]*d)
+ (Cos[c + d*x]^5*(3*a + b*Sin[c + d*x]))/(4*b^2*d*(a + b*Sin[c + d*x])^2)
+ (5*Cos[c + d*x]^3*(4*a^2 - b^2 + a*b*Sin[c + d*x]))/(4*b^4*d*(a + b*Sin[c
 + d*x])) - (15*Cos[c + d*x]*(4*a*(2*a^2 - b^2) - b*(4*a^2 - b^2)*Sin[c + d
*x]))/(8*b^6*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} - \frac{5\int \frac{\cos^4(c+dx)(-2b-6a\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{8b^2}$$

$$= \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+dx)\left(4a^2-b^2+ab\sin(c+dx)\right)}{4b^4d(a+b\sin(c+dx))}$$

$$= \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+dx)\left(4a^2-b^2+ab\sin(c+dx)\right)}{4b^4d(a+b\sin(c+dx))}$$

$$= -\frac{15\left(8a^4-8a^2b^2+b^4\right)x}{8b^7} + \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+dx)}{4b^4d(}$$

$$= -\frac{15\left(8a^4-8a^2b^2+b^4\right)x}{8b^7} + \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+dx)}{4b^4d(}$$

$$= -\frac{15\left(8a^4-8a^2b^2+b^4\right)x}{8b^7} + \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+dx)}{4b^4d(}$$

$$= -\frac{15\left(8a^4-8a^2b^2+b^4\right)x}{8b^7} + \frac{15a\left(2a^4-3a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7\sqrt{a^2-b^2}\,d} + \frac{c}{}$$

Mathematica [B] time = 8.05, size = 1250, normalized size = 5.27

$$\frac{\frac{18\left(-8(c+dx)+\frac{2a\left(8a^4-20b^2a^2+15b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{3b\left(4a^4-7b^2a^2+2b^4\right)\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\frac{ab\left(4a^2-3b^2\right)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))^2}\right)}{b^3} - \frac{10\left(\frac{6ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\cdots\right)}{(a-\cdots}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((18*(-8*(c + d*x) + (2*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[(b + a*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(4*a^2 - 3*b^2)*Cos
[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x])^2) - (3*b*(4*a^4 - 7*a^2*b
^2 + 2*b^4)*Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x]))))/b^3
- (10*((6*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
```

```
b^2] + (Cos[c + d*x]*(a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Sin[c + d*x]))/(a +
 b*Sin[c + d*x])^2))/((a - b)^2*(a + b)^2) + (10*(-24*(-8*a^2 + b^2)*(c + d
*x) - (6*a*(64*a^6 - 168*a^4*b^2 + 140*a^2*b^4 - 35*b^6)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + 96*a*b*Cos[c + d*x] + (
a*b*(-16*a^4 + 20*a^2*b^2 - 5*b^4)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Si
n[c + d*x])^2) + (b*(112*a^6 - 220*a^4*b^2 + 115*a^2*b^4 - 10*b^6)*Cos[c +
d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])) - 8*b^2*Sin[2*(c + d*x)]))
/b^5 + ((12*a*(640*a^8 - 1920*a^6*b^2 + 2016*a^4*b^4 - 840*a^2*b^6 + 105*b^
8)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (-
3840*a^10*(c + d*x) + 7680*a^8*b^2*(c + d*x) - 2976*a^6*b^4*(c + d*x) - 177
6*a^4*b^6*(c + d*x) + 960*a^2*b^8*(c + d*x) - 48*b^10*(c + d*x) - 3840*a^9*
b*Cos[c + d*x] + 8640*a^7*b^3*Cos[c + d*x] - 5696*a^5*b^5*Cos[c + d*x] + 78
8*a^3*b^7*Cos[c + d*x] + 114*a*b^9*Cos[c + d*x] + 1920*a^8*b^2*(c + d*x)*Co
s[2*(c + d*x)] - 4800*a^6*b^4*(c + d*x)*Cos[2*(c + d*x)] + 3888*a^4*b^6*(c
+ d*x)*Cos[2*(c + d*x)] - 1056*a^2*b^8*(c + d*x)*Cos[2*(c + d*x)] + 48*b^10
*(c + d*x)*Cos[2*(c + d*x)] + 320*a^7*b^3*Cos[3*(c + d*x)] - 760*a^5*b^5*Co
s[3*(c + d*x)] + 560*a^3*b^7*Cos[3*(c + d*x)] - 120*a*b^9*Cos[3*(c + d*x)]
- 8*a^5*b^5*Cos[5*(c + d*x)] + 16*a^3*b^7*Cos[5*(c + d*x)] - 8*a*b^9*Cos[5*
(c + d*x)] - 7680*a^9*b*(c + d*x)*Sin[c + d*x] + 19200*a^7*b^3*(c + d*x)*Si
n[c + d*x] - 15552*a^5*b^5*(c + d*x)*Sin[c + d*x] + 4224*a^3*b^7*(c + d*x)*
Sin[c + d*x] - 192*a*b^9*(c + d*x)*Sin[c + d*x] - 2880*a^8*b^2*Sin[2*(c + d
*x)] + 6880*a^6*b^4*Sin[2*(c + d*x)] - 5182*a^4*b^6*Sin[2*(c + d*x)] + 1221
*a^2*b^8*Sin[2*(c + d*x)] - 36*b^10*Sin[2*(c + d*x)] - 40*a^6*b^4*Sin[4*(c
+ d*x)] + 88*a^4*b^6*Sin[4*(c + d*x)] - 56*a^2*b^8*Sin[4*(c + d*x)] + 8*b^1
0*Sin[4*(c + d*x)] + 2*a^4*b^6*Sin[6*(c + d*x)] - 4*a^2*b^8*Sin[6*(c + d*x)
] + 2*b^10*Sin[6*(c + d*x)])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2))/b^7)/(
256*d)
```

fricas [A] time = 0.91, size = 837, normalized size = 3.53

$$\left[\frac{4\,ab^5\cos(dx+c)^5 - 15\left(8\,a^4b^2 - 8\,a^2b^4 + b^6\right)dx\cos(dx+c)^2 - 10\left(4\,a^3b^3 - 3\,ab^5\right)\cos(dx+c)^3 + 15\left(8\,a^6 - 7\,a^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/8*(4*a*b^5*cos(d*x + c)^5 - 15*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*d*x*cos(d*x
 + c)^2 - 10*(4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^3 + 15*(8*a^6 - 7*a^2*b^4 +
 b^6)*d*x + 30*(2*a^5 + a^3*b^2 - a*b^4 - (2*a^3*b^2 - a*b^4)*cos(d*x + c)^
2 + 2*(2*a^4*b - a^2*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)
*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*
x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(
```

```
d*x + c) - a^2 - b^2)) + 30*(4*a^5*b - 2*a^3*b^3 - a*b^5)*cos(d*x + c) - (2
*b^6*cos(d*x + c)^5 - 5*(2*a^2*b^4 - b^6)*cos(d*x + c)^3 - 30*(8*a^5*b - 8*
a^3*b^3 + a*b^5)*d*x - 15*(12*a^4*b^2 - 11*a^2*b^4 + b^6)*cos(d*x + c))*sin
(d*x + c))/(b^9*d*cos(d*x + c)^2 - 2*a*b^8*d*sin(d*x + c) - (a^2*b^7 + b^9)
*d), 1/8*(4*a*b^5*cos(d*x + c)^5 - 15*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*d*x*cos
(d*x + c)^2 - 10*(4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^3 + 15*(8*a^6 - 7*a^2*b
^4 + b^6)*d*x + 60*(2*a^5 + a^3*b^2 - a*b^4 - (2*a^3*b^2 - a*b^4)*cos(d*x +
 c)^2 + 2*(2*a^4*b - a^2*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(
d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(4*a^5*b - 2*a^3*b^3 - a
*b^5)*cos(d*x + c) - (2*b^6*cos(d*x + c)^5 - 5*(2*a^2*b^4 - b^6)*cos(d*x +
c)^3 - 30*(8*a^5*b - 8*a^3*b^3 + a*b^5)*d*x - 15*(12*a^4*b^2 - 11*a^2*b^4 +
 b^6)*cos(d*x + c))*sin(d*x + c))/(b^9*d*cos(d*x + c)^2 - 2*a*b^8*d*sin(d*x
 + c) - (a^2*b^7 + b^9)*d)]
```

giac [B] time = 0.25, size = 581, normalized size = 2.45

$$-\frac{\frac{15\left(8a^4-8a^2b^2+b^4\right)(dx+c)}{b^7}-\frac{120\left(2a^5-3a^3b^2+ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^7}+\frac{8\left(9a^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-9a^3b^3\tan\left(\frac{1}{2}dx\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*(8*a^4 - 8*a^2*b^2 + b^4)*(d*x + c)/b^7 - 120*(2*a^5 - 3*a^3*b^2 +
 a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 8*(9*a^5*b*tan(1/2*d*
x + 1/2*c)^3 - 9*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 10*a^6*tan(1/2*d*x + 1/2*
c)^2 + 9*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 21*a^2*b^4*tan(1/2*d*x + 1/2*c)^2
 + 2*b^6*tan(1/2*d*x + 1/2*c)^2 + 31*a^5*b*tan(1/2*d*x + 1/2*c) - 35*a^3*b^
3*tan(1/2*d*x + 1/2*c) + 4*a*b^5*tan(1/2*d*x + 1/2*c) + 10*a^6 - 11*a^4*b^2
 + a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a*
b^6) + 2*(24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 9*b^3*tan(1/2*d*x + 1/2*c)^7 +
80*a^3*tan(1/2*d*x + 1/2*c)^6 - 72*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*b*
tan(1/2*d*x + 1/2*c)^5 - b^3*tan(1/2*d*x + 1/2*c)^5 + 240*a^3*tan(1/2*d*x +
 1/2*c)^4 - 168*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x + 1/2*c
)^3 + b^3*tan(1/2*d*x + 1/2*c)^3 + 240*a^3*tan(1/2*d*x + 1/2*c)^2 - 152*a*b
^2*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*b*tan(1/2*d*x + 1/2*c) + 9*b^3*tan(1/2*d
*x + 1/2*c) + 80*a^3 - 56*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^6))/d
```

maple [B] time = 0.57, size = 1325, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -6/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*a^2-20/d/b^6/(1+ta
n(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6*a^3+18/d/b^4/(1+tan(1/2*d*x+1/2*
c)^2)^4*tan(1/2*d*x+1/2*c)^6*a-6/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d
*x+1/2*c)^5*a^2-60/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a^
3+42/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a+21/d*a/b^2/(ta
n(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-15/4/
d/b^3*arctan(tan(1/2*d*x+1/2*c))-1/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2*a-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2
/a*tan(1/2*d*x+1/2*c)^2-4/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)^2*tan(1/2*d*x+1/2*c)+9/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+
1/2*c)^7+1/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-1/4/d/b^
3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-9/4/d/b^3/(1+tan(1/2*d*x+
1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^4*a^3+14/d
/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*a-30/d/b^7*arctan(tan(1/2*d*x+1/2*c))*a^4+3
0/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^2-10/d*a^5/b^6/(tan(1/2*d*x+1/2*c)^2*a
+2*tan(1/2*d*x+1/2*c)*b+a)^2+11/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)^2+6/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*a^
2+6/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*a^2-60/d/b^6/(1+t
an(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a^3+38/d/b^4/(1+tan(1/2*d*x+1/2
*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a+15/d*a/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+35/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-10/d*a^5/b^6/(tan(1/2*d*x
+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-9/d*a^3/b^4/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-31/d
*a^4/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/
2*c)-9/d*a^4/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/
2*d*x+1/2*c)^3+9/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a
)^2*tan(1/2*d*x+1/2*c)^3+30/d*a^5/b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1
/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-45/d*a^3/b^5/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 16.79, size = 2529, normalized size = 10.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x))/(a + b*sin(c + d*x))^3,x)
```

```
[Out] - ((a*b^4 + 30*a^5 - 25*a^3*b^2)/b^6 + (tan(c/2 + (d*x)/2)*(420*a^4 + 16*b^
4 - 355*a^2*b^2))/(4*b^5) + (15*tan(c/2 + (d*x)/2)^11*(4*a^4 - 3*a^2*b^2))/
(4*b^5) + (15*tan(c/2 + (d*x)/2)^7*(68*a^4 + 2*b^4 - 55*a^2*b^2))/(2*b^5) -
 (5*tan(c/2 + (d*x)/2)^9*(4*b^4 - 132*a^4 + 99*a^2*b^2))/(4*b^5) + (5*tan(c
/2 + (d*x)/2)^5*(276*a^4 + 10*b^4 - 235*a^2*b^2))/(2*b^5) + (5*tan(c/2 + (d
*x)/2)^3*(348*a^4 + 20*b^4 - 301*a^2*b^2))/(4*b^5) + (tan(c/2 + (d*x)/2)^10
*(30*a^6 + 2*b^6 - 30*a^2*b^4 + 15*a^4*b^2))/(a*b^6) + (tan(c/2 + (d*x)/2)^
2*(150*a^6 + 2*b^6 - 64*a^2*b^4 - 45*a^4*b^2))/(a*b^6) + (2*tan(c/2 + (d*x)
/2)^4*(150*a^6 + 4*b^6 - 110*a^2*b^4 + 15*a^4*b^2))/(a*b^6) + (tan(c/2 + (d
*x)/2)^8*(150*a^6 + 8*b^6 - 165*a^2*b^4 + 75*a^4*b^2))/(a*b^6) + (2*tan(c/2
 + (d*x)/2)^6*(5*a^2 + 6*b^2)*(30*a^4 + b^4 - 25*a^2*b^2))/(a*b^6))/(d*(tan
(c/2 + (d*x)/2)^2*(6*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^10*(6*a^2 + 4*b^2) +
 tan(c/2 + (d*x)/2)^4*(15*a^2 + 16*b^2) + tan(c/2 + (d*x)/2)^8*(15*a^2 + 16
*b^2) + tan(c/2 + (d*x)/2)^6*(20*a^2 + 24*b^2) + a^2*tan(c/2 + (d*x)/2)^12
+ a^2 + 20*a*b*tan(c/2 + (d*x)/2)^3 + 40*a*b*tan(c/2 + (d*x)/2)^5 + 40*a*b*
tan(c/2 + (d*x)/2)^7 + 20*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/
2)^11 + 4*a*b*tan(c/2 + (d*x)/2))) - (atanh((3375*a^3*(b^2 - a^2)^(1/2))/(2
*((3375*a^3*b)/2 - (10125*a^5)/(2*b) + (3375*a^7)/b^3 - 10125*a^4*tan(c/2 +
 (d*x)/2) + 3375*a^2*b^2*tan(c/2 + (d*x)/2) + (6750*a^6*tan(c/2 + (d*x)/2))
/b^2)) - (3375*a^5*(b^2 - a^2)^(1/2))/((3375*a^3*b^3)/2 - (10125*a^5*b)/2 +
 (3375*a^7)/b + 6750*a^6*tan(c/2 + (d*x)/2) + 3375*a^2*b^4*tan(c/2 + (d*x)/
2) - 10125*a^4*b^2*tan(c/2 + (d*x)/2)) + (3375*a^2*tan(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2))/((3375*a^3)/2 - (10125*a^5)/(2*b^2) + (3375*a^7)/b^4 + 3375*a
^2*b*tan(c/2 + (d*x)/2) - (10125*a^4*tan(c/2 + (d*x)/2))/b + (6750*a^6*tan(
c/2 + (d*x)/2))/b^3) - (16875*a^4*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(2*
((3375*a^3*b^2)/2 - (10125*a^5)/2 + (3375*a^7)/b^2 - 10125*a^4*b*tan(c/2 +
(d*x)/2) + 3375*a^2*b^3*tan(c/2 + (d*x)/2) + (6750*a^6*tan(c/2 + (d*x)/2))/
b)) + (3375*a^6*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(3375*a^7 + (3375*a^3
*b^4)/2 - (10125*a^5*b^2)/2 + 6750*a^6*b*tan(c/2 + (d*x)/2) + 3375*a^2*b^5*
tan(c/2 + (d*x)/2) - 10125*a^4*b^3*tan(c/2 + (d*x)/2)))*(30*a^3*(b^2 - a^2)
^(1/2) - 15*a*b^2*(b^2 - a^2)^(1/2)))/(b^7*d) - (atan((((a^4*8i + b^4*1i -
a^2*b^2*8i)*(((225*a^2*b^14)/2 - 1800*a^4*b^12 + 9000*a^6*b^10 - 14400*a^8*
b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(450*a*b^16 - 11025*a^3*b^1
4 + 61200*a^5*b^12 - 122400*a^7*b^10 + 100800*a^9*b^8 - 28800*a^11*b^6))/(2
*b^18) - (15*(a^4*8i + b^4*1i - a^2*b^2*8i)*((60*a*b^18 - 300*a^3*b^16 + 24
0*a^5*b^14)/b^17 - (15*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*
a^3*b^20))/(2*b^18))*(a^4*8i + b^4*1i - a^2*b^2*8i))/(8*b^7) + (tan(c/2 + (
```

```
d*x)/2)*(960*a^2*b^18 - 2880*a^4*b^16 + 1920*a^6*b^14))/(2*b^18)))/(8*b^7))
*15i)/(8*b^7) + ((a^4*8i + b^4*1i - a^2*b^2*8i)*(((225*a^2*b^14)/2 - 1800*a
^4*b^12 + 9000*a^6*b^10 - 14400*a^8*b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 +
(d*x)/2)*(450*a*b^16 - 11025*a^3*b^14 + 61200*a^5*b^12 - 122400*a^7*b^10 +
100800*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) + (15*(a^4*8i + b^4*1i - a^2*b^2
*8i)*((60*a*b^18 - 300*a^3*b^16 + 240*a^5*b^14)/b^17 + (15*(32*a^2*b^3 + (t
an(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b^18))*(a^4*8i + b^4*1i -
 a^2*b^2*8i))/(8*b^7) + (tan(c/2 + (d*x)/2)*(960*a^2*b^18 - 2880*a^4*b^16 +
 1920*a^6*b^14))/(2*b^18)))/(8*b^7))*15i)/(8*b^7))/((108000*a^13 - (10125*a
^3*b^10)/2 + (124875*a^5*b^8)/2 - 246375*a^7*b^6 + 432000*a^9*b^4 - 351000*
a^11*b^2)/b^17 - (15*(a^4*8i + b^4*1i - a^2*b^2*8i)*(((225*a^2*b^14)/2 - 18
00*a^4*b^12 + 9000*a^6*b^10 - 14400*a^8*b^8 + 7200*a^10*b^6)/b^17 + (tan(c/
2 + (d*x)/2)*(450*a*b^16 - 11025*a^3*b^14 + 61200*a^5*b^12 - 122400*a^7*b^1
0 + 100800*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) - (15*(a^4*8i + b^4*1i - a^2
*b^2*8i)*((60*a*b^18 - 300*a^3*b^16 + 240*a^5*b^14)/b^17 - (15*(32*a^2*b^3
+ (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b^18))*(a^4*8i + b^4*
1i - a^2*b^2*8i))/(8*b^7) + (tan(c/2 + (d*x)/2)*(960*a^2*b^18 - 2880*a^4*b^
16 + 1920*a^6*b^14))/(2*b^18)))/(8*b^7)))/(8*b^7) + (15*(a^4*8i + b^4*1i -
a^2*b^2*8i)*(((225*a^2*b^14)/2 - 1800*a^4*b^12 + 9000*a^6*b^10 - 14400*a^8*
b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(450*a*b^16 - 11025*a^3*b^1
4 + 61200*a^5*b^12 - 122400*a^7*b^10 + 100800*a^9*b^8 - 28800*a^11*b^6))/(2
*b^18) + (15*(a^4*8i + b^4*1i - a^2*b^2*8i)*((60*a*b^18 - 300*a^3*b^16 + 24
0*a^5*b^14)/b^17 + (15*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*
a^3*b^20))/(2*b^18))*(a^4*8i + b^4*1i - a^2*b^2*8i))/(8*b^7) + (tan(c/2 + (
d*x)/2)*(960*a^2*b^18 - 2880*a^4*b^16 + 1920*a^6*b^14))/(2*b^18)))/(8*b^7))
)/(8*b^7) + (tan(c/2 + (d*x)/2)*(432000*a^14 + 3375*a^2*b^12 - 64125*a^4*b^
10 + 438750*a^6*b^8 - 1350000*a^8*b^6 + 2052000*a^10*b^4 - 1512000*a^12*b^2
))/b^18))*(a^4*8i + b^4*1i - a^2*b^2*8i)*15i)/(4*b^7*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1269 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=399

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{(2a^2+b^2)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}-\frac{2(5a^2+b^2)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}$$

```
[Out] -1/2*x/b^3-3*(2*a^2-b^2)*x/b^5-arctanh(cos(d*x+c))/a^3/d-3*a*cos(d*x+c)/b^4
/d+1/2*cos(d*x+c)*sin(d*x+c)/b^3/d+1/2*(a^2-b^2)^2*cos(d*x+c)/a/b^4/d/(a+b*
sin(d*x+c))^2+3/2*(a^2-b^2)*cos(d*x+c)/b^4/d/(a+b*sin(d*x+c))-(a^2-b^2)*(5*
a^2+b^2)*cos(d*x+c)/a^2/b^4/d/(a+b*sin(d*x+c))+2*(10*a^6-9*a^4*b^2-b^6)*arc
tan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/b^5/d/(a^2-b^2)^(1/2)+(2*
a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a
/b^5/d-2*(5*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-
b^2)^(1/2)/a/b^5/d
```

Rubi [A] time = 0.52, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2897, 3770, 2638, 2635, 8, 2664, 2754, 12, 2660, 618, 204}

$$\frac{(2a^2+b^2)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}-\frac{2(5a^2+b^2)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}+\frac{2(-9a^4b^2+10a^6-}{a^3b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -x/(2*b^3) - (3*(2*a^2 - b^2)*x)/b^5 + (Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTa
n[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^5*d) - (2*Sqrt[a^2 - b^2]
*(5*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^5*d)
+ (2*(10*a^6 - 9*a^4*b^2 - b^6)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(a^3*b^5*Sqrt[a^2 - b^2]*d) - ArcTanh[Cos[c + d*x]]/(a^3*d) - (3*a*C
os[c + d*x])/(b^4*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*b^3*d) + ((a^2 - b^2)
^2*Cos[c + d*x])/(2*a*b^4*d*(a + b*Sin[c + d*x])^2) + (3*(a^2 - b^2)*Cos[c
+ d*x])/(2*b^4*d*(a + b*Sin[c + d*x])) - ((a^2 - b^2)*(5*a^2 + b^2)*Cos[c +
 d*x])/(a^2*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(\frac{3\left(-2a^2+b^2\right)}{b^5}+\frac{\csc(c+dx)}{a^3}+\frac{3a\sin(c+dx)}{b^4}-\frac{\sin^2(c+dx)}{b^3}+\frac{\left(a^2-\right.}{ab^5(a+b\,\mathrm{si}}\right.$$

$$=-\frac{3\left(2a^2-b^2\right)x}{b^5}+\frac{\int\csc(c+dx)\,dx}{a^3}+\frac{(3a)\int\sin(c+dx)\,dx}{b^4}-\frac{\int\sin^2(c+dx)\,dx}{b^3}$$

$$=-\frac{3\left(2a^2-b^2\right)x}{b^5}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{3a\cos(c+dx)}{b^4d}+\frac{\cos(c+dx)\sin(c+}{2b^3d}$$

$$=-\frac{x}{2b^3}-\frac{3\left(2a^2-b^2\right)x}{b^5}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{3a\cos(c+dx)}{b^4d}+\frac{\cos(c+dx)\,\mathrm{si}}{2b^3d}$$

$$=-\frac{x}{2b^3}-\frac{3\left(2a^2-b^2\right)x}{b^5}+\frac{2\left(10a^6-9a^4b^2-b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^5\sqrt{a^2-b^2}\,d}-\tanh^{-1}$$

$$=-\frac{x}{2b^3}-\frac{3\left(2a^2-b^2\right)x}{b^5}+\frac{2\left(10a^6-9a^4b^2-b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^5\sqrt{a^2-b^2}\,d}-\tanh^{-1}$$

$$=-\frac{x}{2b^3}-\frac{3\left(2a^2-b^2\right)x}{b^5}-\frac{2\sqrt{a^2-b^2}\left(5a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^5d}+\frac{2\left(10a^6\right.}{}$$

$$=-\frac{x}{2b^3}-\frac{3\left(2a^2-b^2\right)x}{b^5}+\frac{\sqrt{a^2-b^2}\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^5d}-\frac{2\sqrt{a^2-}}{}$$

Mathematica [A] time = 1.95, size = 243, normalized size = 0.61

$$\frac{\frac{4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}-\frac{4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}+\frac{2\left(5b^2-12a^2\right)(c+dx)}{b^5}+\frac{2\left(a^2-b^2\right)^2\cos(c+dx)}{ab^4(a+b\sin(c+dx))^2}+\frac{2\left(-7a^4+5a^2b^2+2b^4\right)\cos(c+dx)}{a^2b^4(a+b\sin(c+dx))}+\frac{4\left(12a^6-11a^4b^2\right.}{}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*(-12*a^2 + 5*b^2)*(c + d*x))/b^5 + (4*(12*a^6 - 11*a^4*b^2 + a^2*b^4 -
2*b^6)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^5*Sqrt[a^2
- b^2]) - (12*a*Cos[c + d*x])/b^4 - (4*Log[Cos[(c + d*x)/2]])/a^3 + (4*Log[
Sin[(c + d*x)/2]])/a^3 + (2*(a^2 - b^2)^2*Cos[c + d*x])/(a*b^4*(a + b*Sin[c
 + d*x])^2) + (2*(-7*a^4 + 5*a^2*b^2 + 2*b^4)*Cos[c + d*x])/(a^2*b^4*(a + b
*Sin[c + d*x])) + Sin[2*(c + d*x)]/b^3)/(4*d)
```

fricas [A] time = 1.44, size = 1007, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(8*a^4*b^3*cos(d*x + c)^3 + 2*(12*a^5*b^2 - 5*a^3*b^4)*d*x*cos(d*x +
c)^2 - 2*(12*a^7 + 7*a^5*b^2 - 5*a^3*b^4)*d*x + (12*a^6 + 13*a^4*b^2 + 3*a^
2*b^4 + 2*b^6 - (12*a^4*b^2 + a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(12*a^5*b
 + a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x +
 c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x
 + c) - a^2 - b^2)) - 2*(12*a^6*b + a^4*b^3 - 3*a^2*b^5)*cos(d*x + c) + 2*(
b^7*cos(d*x + c)^2 - 2*a*b^6*sin(d*x + c) - a^2*b^5 - b^7)*log(1/2*cos(d*x
+ c) + 1/2) - 2*(b^7*cos(d*x + c)^2 - 2*a*b^6*sin(d*x + c) - a^2*b^5 - b^7)
*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*b^4*cos(d*x + c)^3 + 2*(12*a^6*b - 5
*a^4*b^3)*d*x + 2*(9*a^5*b^2 - 3*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c
))/(a^3*b^7*d*cos(d*x + c)^2 - 2*a^4*b^6*d*sin(d*x + c) - (a^5*b^5 + a^3*b^
7)*d), -1/2*(4*a^4*b^3*cos(d*x + c)^3 + (12*a^5*b^2 - 5*a^3*b^4)*d*x*cos(d*
x + c)^2 - (12*a^7 + 7*a^5*b^2 - 5*a^3*b^4)*d*x - (12*a^6 + 13*a^4*b^2 + 3*
a^2*b^4 + 2*b^6 - (12*a^4*b^2 + a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(12*a^5
*b + a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (12*a^6*b + a^4*b^3 - 3*a^2*b^5)*
cos(d*x + c) + (b^7*cos(d*x + c)^2 - 2*a*b^6*sin(d*x + c) - a^2*b^5 - b^7)*
log(1/2*cos(d*x + c) + 1/2) - (b^7*cos(d*x + c)^2 - 2*a*b^6*sin(d*x + c) -
a^2*b^5 - b^7)*log(-1/2*cos(d*x + c) + 1/2) - (a^3*b^4*cos(d*x + c)^3 + 2*(
12*a^6*b - 5*a^4*b^3)*d*x + 2*(9*a^5*b^2 - 3*a^3*b^4 - a*b^6)*cos(d*x + c))
*sin(d*x + c))/(a^3*b^7*d*cos(d*x + c)^2 - 2*a^4*b^6*d*sin(d*x + c) - (a^5*
b^5 + a^3*b^7)*d)]
```

giac [A] time = 0.26, size = 635, normalized size = 1.59

$$\frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{\left(12a^2-5b^2\right)(dx+c)}{b^5}+\frac{2\left(12a^6-11a^4b^2+a^2b^4-2b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^3b^5}-\frac{2\left(6a^5b\tan\left(\frac{1}{2}dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (12*a^2 - 5*b^2)*(d*x + c)/b^5 + 2*(12*a^6 - 11*a^4*b^2 + a^2*b^4 - 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^3*b^5) - 2*(6*a^5*b*tan(1/2*d*x + 1/2*c)^7 - a^3*b^3*tan(1/2*d*x + 1/2*c)^7 - 4*a*b^5*tan(1/2*d*x + 1/2*c)^7 + 12*a^6*tan(1/2*d*x + 1/2*c)^6 + 13*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 - 9*a^2*b^4*tan(1/2*d*x + 1/2*c)^6 - 6*b^6*tan(1/2*d*x + 1/2*c)^6 + 54*a^5*b*tan(1/2*d*x + 1/2*c)^5 - 9*a^3*b^3*tan(1/2*d*x + 1/2*c)^5 - 16*a*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a^6*tan(1/2*d*x + 1/2*c)^4 + 39*a^4*b^2*tan(1/2*d*x + 1/2*c)^4 - 21*a^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 12*b^6*tan(1/2*d*x + 1/2*c)^4 + 90*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 27*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 20*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 36*a^6*tan(1/2*d*x + 1/2*c)^2 + 23*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 - 6*b^6*tan(1/2*d*x + 1/2*c)^2 + 42*a^5*b*tan(1/2*d*x + 1/2*c) - 11*a^3*b^3*tan(1/2*d*x + 1/2*c) - 8*a*b^5*tan(1/2*d*x + 1/2*c) + 12*a^6 - 3*a^4*b^2 - 3*a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^3*b^4))/d

maple [B] time = 0.80, size = 988, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^3,x)

[Out] -1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^2*a-12/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^2+5/d/b^3*arctan(tan(1/2*d*x+1/2*c))+1/d/a^3*ln(tan(1/2*d*x+1/2*c))-5/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+1/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+4/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-6/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-9/d*a/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+9/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^2+6/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-19/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+11/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+8/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-6/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+3/

```
d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a+3/d/a/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+12/d*a^3/b^5/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-11/d*a/b^3/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/a/b/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a^
3*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 14.55, size = 5354, normalized size = 13.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)*(a + b*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) + ((3*(b^4 - 4*a^4 + a^2*b^2))/(a*b^4) + (t
an(c/2 + (d*x)/2)*(8*b^4 - 42*a^4 + 11*a^2*b^2))/(a^2*b^3) - (3*tan(c/2 + (
d*x)/2)^4*(12*a^6 - 4*b^6 - 7*a^2*b^4 + 13*a^4*b^2))/(a^3*b^4) - (tan(c/2 +
 (d*x)/2)^6*(12*a^6 - 6*b^6 - 9*a^2*b^4 + 13*a^4*b^2))/(a^3*b^4) - (tan(c/2
 + (d*x)/2)^2*(36*a^6 - 6*b^6 - 15*a^2*b^4 + 23*a^4*b^2))/(a^3*b^4) + (tan(
c/2 + (d*x)/2)^7*(4*b^4 - 6*a^4 + a^2*b^2))/(a^2*b^3) + (tan(c/2 + (d*x)/2)
^5*(16*b^4 - 54*a^4 + 9*a^2*b^2))/(a^2*b^3) + (tan(c/2 + (d*x)/2)^3*(20*b^4
 - 90*a^4 + 27*a^2*b^2))/(a^2*b^3))/(d*(tan(c/2 + (d*x)/2)^2*(4*a^2 + 4*b^2
) + tan(c/2 + (d*x)/2)^6*(4*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(6*a^2 + 8*
b^2) + a^2*tan(c/2 + (d*x)/2)^8 + a^2 + 12*a*b*tan(c/2 + (d*x)/2)^3 + 12*a*
b*tan(c/2 + (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)
/2))) - (atan((((a^2*12i - b^2*5i)*((4*(20*a^3*b^12 - 6048*a^15 + 332*a^5*b
^10 - 1947*a^7*b^8 + 3979*a^9*b^6 - 7038*a^11*b^4 + 10800*a^13*b^2))/(a^6*b
^11) - ((a^2*12i - b^2*5i)*((4*(32*a^2*b^16 - 24*a^4*b^14 + 160*a^6*b^12 +
32*a^8*b^10 - 1110*a^10*b^8 + 1872*a^12*b^6 - 864*a^14*b^4))/(a^6*b^11) - (
(a^2*12i - b^2*5i)*((4*(64*a^5*b^16 - 48*a^7*b^14 + 160*a^9*b^12 - 168*a^11
*b^10))/(a^6*b^11) - (((4*(32*a^8*b^16 - 24*a^10*b^14))/(a^6*b^11) + (8*tan
(c/2 + (d*x)/2)*(64*a^7*b^22 - 68*a^9*b^20 + 8*a^11*b^18))/(a^6*b^16))*(a^2
```

*12i - b^2*5i))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^22 - 68*a^6*b^20 + 192*a^8*b^18 - 280*a^10*b^16 + 96*a^12*b^14))/(a^6*b^16)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(4*a^3*b^20 - 180*a^5*b^18 + 725*a^7*b^16 - 3454*a^9*b^14 + 6506*a^11*b^12 - 3840*a^13*b^10 + 288*a^15*b^8))/(a^6*b^16)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(4*b^20 - 4*a^2*b^18 + 445*a^4*b^16 - 2390*a^6*b^14 + 5544*a^8*b^12 - 9958*a^10*b^10 + 15912*a^12*b^8 - 12960*a^14*b^6 + 3456*a^16*b^4))/(a^6*b^16))*1i)/(2*b^5) + ((a^2*12i - b^2*5i)*((4*(20*a^3*b^12 - 6048*a^15 + 332*a^5*b^10 - 1947*a^7*b^8 + 3979*a^9*b^6 - 7038*a^11*b^4 + 10800*a^13*b^2))/(a^6*b^11) + ((a^2*12i - b^2*5i)*(((a^2*12i - b^2*5i)*((4*(64*a^5*b^16 - 48*a^7*b^14 + 160*a^9*b^12 - 168*a^11*b^10))/(a^6*b^11) + (((4*(32*a^8*b^16 - 24*a^10*b^14))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^22 - 68*a^9*b^20 + 8*a^11*b^18))/(a^6*b^16))*(a^2*12i - b^2*5i))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^22 - 68*a^6*b^20 + 192*a^8*b^18 - 280*a^10*b^16 + 96*a^12*b^14))/(a^6*b^16)))/(2*b^5) + (4*(32*a^2*b^16 - 24*a^4*b^14 + 160*a^6*b^12 + 32*a^8*b^10 - 1110*a^10*b^8 + 1872*a^12*b^6 - 864*a^14*b^4))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(4*a^3*b^20 - 180*a^5*b^18 + 725*a^7*b^16 - 3454*a^9*b^14 + 6506*a^11*b^12 - 3840*a^13*b^10 + 288*a^15*b^8))/(a^6*b^16)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(4*b^20 - 4*a^2*b^18 + 445*a^4*b^16 - 2390*a^6*b^14 + 5544*a^8*b^12 - 9958*a^10*b^10 + 15912*a^12*b^8 - 12960*a^14*b^6 + 3456*a^16*b^4))/(a^6*b^16))*1i)/(2*b^5))/(((a^2*12i - b^2*5i)*((4*(20*a^3*b^12 - 6048*a^15 + 332*a^5*b^10 - 1947*a^7*b^8 + 3979*a^9*b^6 - 7038*a^11*b^4 + 10800*a^13*b^2))/(a^6*b^11) - ((a^2*12i - b^2*5i)*((4*(32*a^2*b^16 - 24*a^4*b^14 + 160*a^6*b^12 + 32*a^8*b^10 - 1110*a^10*b^8 + 1872*a^12*b^6 - 864*a^14*b^4))/(a^6*b^11) - ((a^2*12i - b^2*5i)*((4*(64*a^5*b^16 - 48*a^7*b^14 + 160*a^9*b^12 - 168*a^11*b^10))/(a^6*b^11) - (((4*(32*a^8*b^16 - 24*a^10*b^14))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^22 - 68*a^9*b^20 + 8*a^11*b^18))/(a^6*b^16))*(a^2*12i - b^2*5i))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^22 - 68*a^6*b^20 + 192*a^8*b^18 - 280*a^10*b^16 + 96*a^12*b^14))/(a^6*b^16)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(4*a^3*b^20 - 180*a^5*b^18 + 725*a^7*b^16 - 3454*a^9*b^14 + 6506*a^11*b^12 - 3840*a^13*b^10 + 288*a^15*b^8))/(a^6*b^16)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(4*b^20 - 4*a^2*b^18 + 445*a^4*b^16 - 2390*a^6*b^14 + 5544*a^8*b^12 - 9958*a^10*b^10 + 15912*a^12*b^8 - 12960*a^14*b^6 + 3456*a^16*b^4))/(a^6*b^16)))/(2*b^5) - ((a^2*12i - b^2*5i)*((4*(20*a^3*b^12 - 6048*a^15 + 332*a^5*b^10 - 1947*a^7*b^8 + 3979*a^9*b^6 - 7038*a^11*b^4 + 10800*a^13*b^2))/(a^6*b^11) + ((a^2*12i - b^2*5i)*(((a^2*12i - b^2*5i)*((4*(64*a^5*b^16 - 48*a^7*b^14 + 160*a^9*b^12 - 168*a^11*b^10))/(a^6*b^11) + (((4*(32*a^8*b^16 - 24*a^10*b^14))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^22 - 68*a^9*b^20 + 8*a^11*b^18))/(a^6*b^16))*(a^2*12i - b^2*5i))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^22 - 68*a^6*b^20 + 192*a^8*b^18 - 280*a^10*b^16 + 96*a^12*b^14))/(a^6*b^16)))/(2*b^5) + (4*(32*a^2*b^16 - 24*a^4*b^14 + 160*a^6*b^12 + 32*a^8*b^10 - 1110*a^10*b^8 + 1872*a^12*b^6 - 864*a^14*b^4))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(4*a^3*b^20 - 180*a^5*b^18 + 725*a^7*b^16 - 3454*a^9*b^14 + 6506*a^11*b^12 - 3840*a^13*b^10 + 288*a^15*b^8))/(a^6*b^16)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(4*b^20 - 4*a^2*b^18 + 445*a^4*b^16 - 2390*a^6*b^14 + 5544*a^8*b^12 - 9

958*a^10*b^10 + 15912*a^12*b^8 - 12960*a^14*b^6 + 3456*a^16*b^4))/(a^6*b^16
)))/(2*b^5) - (8*(20*b^12 - 6048*a^12 + 132*a^2*b^10 - 837*a^4*b^8 + 2107*a
^6*b^6 - 6174*a^8*b^4 + 10800*a^10*b^2))/(a^6*b^11) + (16*tan(c/2 + (d*x)/2
)*(41472*a^17 + 1100*a^5*b^12 - 7030*a^7*b^10 + 21386*a^9*b^8 - 55200*a^11*
b^6 + 108864*a^13*b^4 - 110592*a^15*b^2))/(a^6*b^16)))*(a^2*12i - b^2*5i)*1
i)/(b^5*d) + (atan((((-(a + b)*(a - b))^(1/2)*(6*a^4 + b^4 + (a^2*b^2)/2)*(
(4*(20*a^3*b^12 - 6048*a^15 + 332*a^5*b^10 - 1947*a^7*b^8 + 3979*a^9*b^6 -
7038*a^11*b^4 + 10800*a^13*b^2))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(4*b^20
 - 4*a^2*b^18 + 445*a^4*b^16 - 2390*a^6*b^14 + 5544*a^8*b^12 - 9958*a^10*b^
10 + 15912*a^12*b^8 - 12960*a^14*b^6 + 3456*a^16*b^4))/(a^6*b^16) + ((-(a +
 b)*(a - b))^(1/2)*(6*a^4 + b^4 + (a^2*b^2)/2)*((4*(32*a^2*b^16 - 24*a^4*b^
14 + 160*a^6*b^12 + 32*a^8*b^10 - 1110*a^10*b^8 + 1872*a^12*b^6 - 864*a^14*
b^4))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(4*a^3*b^20 - 180*a^5*b^18 + 725*a
^7*b^16 - 3454*a^9*b^14 + 6506*a^11*b^12 - 3840*a^13*b^10 + 288*a^15*b^8))/
(a^6*b^16) + ((-(a + b)*(a - b))^(1/2)*((4*(64*a^5*b^16 - 48*a^7*b^14 + 160
*a^9*b^12 - 168*a^11*b^10))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^22
 - 68*a^6*b^20 + 192*a^8*b^18 - 280*a^10*b^16 + 96*a^12*b^14))/(a^6*b^16) +
 ((-(a + b)*(a - b))^(1/2)*((4*(32*a^8*b^16 - 24*a^10*b^14))/(a^6*b^11) + (
8*tan(c/2 + (d*x)/2)*(64*a^7*b^22 - 68*a^9*b^20 + 8*a^11*b^18))/(a^6*b^16))
*(6*a^4 + b^4 + (a^2*b^2)/2))/(a^3*b^5))*(6*a^4 + b^4 + (a^2*b^2)/2))/(a^3*
b^5)))/(a^3*b^5))*1i)/(a^3*b^5) + ((-(a + b)*(a - b))^(1/2)*(6*a^4 + b^4 +
(a^2*b^2)/2)*((4*(20*a^3*b^12 - 6048*a^15 + 332*a^5*b^10 - 1947*a^7*b^8 + 3
979*a^9*b^6 - 7038*a^11*b^4 + 10800*a^13*b^2))/(a^6*b^11) + (8*tan(c/2 + (d
x)/2)(4*b^20 - 4*a^2*b^18 + 445*a^4*b^16 - 2390*a^6*b^14 + 5544*a^8*b^12
- 9958*a^10*b^10 + 15912*a^12*b^8 - 12960*a^14*b^6 + 3456*a^16*b^4))/(a^6*b
^16) - ((-(a + b)*(a - b))^(1/2)*(6*a^4 + b^4 + (a^2*b^2)/2)*((4*(32*a^2*b^
16 - 24*a^4*b^14 + 160*a^6*b^12 + 32*a^8*b^10 - 1110*a^10*b^8 + 1872*a^12*b
^6 - 864*a^14*b^4))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(4*a^3*b^20 - 180*a^
5*b^18 + 725*a^7*b^16 - 3454*a^9*b^14 + 6506*a^11*b^12 - 3840*a^13*b^10 + 2
88*a^15*b^8))/(a^6*b^16) - ((-(a + b)*(a - b))^(1/2)*((4*(64*a^5*b^16 - 48*
a^7*b^14 + 160*a^9*b^12 - 168*a^11*b^10))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2
)*(64*a^4*b^22 - 68*a^6*b^20 + 192*a^8*b^18 - 280*a^10*b^16 + 96*a^12*b^14)
)/(a^6*b^16) - ((-(a + b)*(a - b))^(1/2)*((4*(32*a^8*b^16 - 24*a^10*b^14))/
(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^22 - 68*a^9*b^20 + 8*a^11*b^18
))/(a^6*b^16))*(6*a^4 + b^4 + (a^2*b^2)/2))/(a^3*b^5))*(6*a^4 + b^4 + (a^2*
b^2)/2))/(a^3*b^5)))/(a^3*b^5))*1i)/(a^3*b^5))/((8*(20*b^12 - 6048*a^12 + 1
32*a^2*b^10 - 837*a^4*b^8 + 2107*a^6*b^6 - 6174*a^8*b^4 + 10800*a^10*b^2))/
(a^6*b^11) - (16*tan(c/2 + (d*x)/2)*(41472*a^17 + 1100*a^5*b^12 - 7030*a^7*
b^10 + 21386*a^9*b^8 - 55200*a^11*b^6 + 108864*a^13*b^4 - 110592*a^15*b^2))
/(a^6*b^16) + ((-(a + b)*(a - b))^(1/2)*(6*a^4 + b^4 + (a^2*b^2)/2)*((4*(20
*a^3*b^12 - 6048*a^15 + 332*a^5*b^10 - 1947*a^7*b^8 + 3979*a^9*b^6 - 7038*a
^11*b^4 + 10800*a^13*b^2))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(4*b^20 - 4*a
^2*b^18 + 445*a^4*b^16 - 2390*a^6*b^14 + 5544*a^8*b^12 - 9958*a^10*b^10 + 1
5912*a^12*b^8 - 12960*a^14*b^6 + 3456*a^16*b^4))/(a^6*b^16) + ((-(a + b)*(a
 - b))^(1/2)*(6*a^4 + b^4 + (a^2*b^2)/2)*((4*(32*a^2*b^16 - 24*a^4*b^14 + 1

```
60*a^6*b^12 + 32*a^8*b^10 - 1110*a^10*b^8 + 1872*a^12*b^6 - 864*a^14*b^4))/
(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(4*a^3*b^20 - 180*a^5*b^18 + 725*a^7*b^1
6 - 3454*a^9*b^14 + 6506*a^11*b^12 - 3840*a^13*b^10 + 288*a^15*b^8))/(a^6*b
^16) + ((-(a + b)*(a - b))^(1/2)*((4*(64*a^5*b^16 - 48*a^7*b^14 + 160*a^9*b
^12 - 168*a^11*b^10))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^22 - 68*
a^6*b^20 + 192*a^8*b^18 - 280*a^10*b^16 + 96*a^12*b^14))/(a^6*b^16) + ((-(a
 + b)*(a - b))^(1/2)*((4*(32*a^8*b^16 - 24*a^10*b^14))/(a^6*b^11) + (8*tan(
c/2 + (d*x)/2)*(64*a^7*b^22 - 68*a^9*b^20 + 8*a^11*b^18))/(a^6*b^16))*(6*a^
4 + b^4 + (a^2*b^2)/2))/(a^3*b^5))*(6*a^4 + b^4 + (a^2*b^2)/2))/(a^3*b^5)))
/(a^3*b^5)))/(a^3*b^5) - ((-(a + b)*(a - b))^(1/2)*(6*a^4 + b^4 + (a^2*b^2)
/2)*((4*(20*a^3*b^12 - 6048*a^15 + 332*a^5*b^10 - 1947*a^7*b^8 + 3979*a^9*b
^6 - 7038*a^11*b^4 + 10800*a^13*b^2))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(4
*b^20 - 4*a^2*b^18 + 445*a^4*b^16 - 2390*a^6*b^14 + 5544*a^8*b^12 - 9958*a^
10*b^10 + 15912*a^12*b^8 - 12960*a^14*b^6 + 3456*a^16*b^4))/(a^6*b^16) - ((
-(a + b)*(a - b))^(1/2)*(6*a^4 + b^4 + (a^2*b^2)/2)*((4*(32*a^2*b^16 - 24*a
^4*b^14 + 160*a^6*b^12 + 32*a^8*b^10 - 1110*a^10*b^8 + 1872*a^12*b^6 - 864*
a^14*b^4))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(4*a^3*b^20 - 180*a^5*b^18 +
725*a^7*b^16 - 3454*a^9*b^14 + 6506*a^11*b^12 - 3840*a^13*b^10 + 288*a^15*b
^8))/(a^6*b^16) - ((-(a + b)*(a - b))^(1/2)*((4*(64*a^5*b^16 - 48*a^7*b^14
+ 160*a^9*b^12 - 168*a^11*b^10))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(64*a^4
*b^22 - 68*a^6*b^20 + 192*a^8*b^18 - 280*a^10*b^16 + 96*a^12*b^14))/(a^6*b^
16) - ((-(a + b)*(a - b))^(1/2)*((4*(32*a^8*b^16 - 24*a^10*b^14))/(a^6*b^11
) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^22 - 68*a^9*b^20 + 8*a^11*b^18))/(a^6*b
^16))*(6*a^4 + b^4 + (a^2*b^2)/2))/(a^3*b^5))*(6*a^4 + b^4 + (a^2*b^2)/2))/
(a^3*b^5)))/(a^3*b^5)))/(a^3*b^5)))*(-(a + b)*(a - b))^(1/2)*(6*a^4 + b^4 +
 (a^2*b^2)/2)*2i)/(a^3*b^5*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1270 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=314

$$\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{\cot(c+dx)}{a^3d}+\frac{3\left(2a^2+b^2\right)\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^4d}-\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{2a^2b^3d(a+b\sin(c+dx))^2}$$

```
[Out] 3*a*x/b^4+3*b*arctanh(cos(d*x+c))/a^4/d+cos(d*x+c)/b^3/d-cot(d*x+c)/a^3/d-1
/2*(a^2-b^2)^2*cos(d*x+c)/a^2/b^3/d/(a+b*sin(d*x+c))^2-3/2*(a^2-b^2)*cos(d*
x+c)/a/b^3/d/(a+b*sin(d*x+c))+2*(a^2-b^2)*(2*a^2+b^2)*cos(d*x+c)/a^3/b^3/d/
(a+b*sin(d*x+c))-6*(2*a^6-a^4*b^2-b^6)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2
-b^2)^(1/2))/a^4/b^4/d/(a^2-b^2)^(1/2)+3*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*
x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^2/b^4/d
```

Rubi [A] time = 0.49, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2897, 3770, 3767, 8, 2638, 2664, 2754, 12, 2660, 618, 204}

$$\frac{3\left(2a^2+b^2\right)\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^4d}-\frac{6\left(-a^4b^2+2a^6-b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4b^4d\sqrt{a^2-b^2}}-\frac{\left(a^2-b^2\right)^2\cos(c+}{2a^2b^3d(a+b\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*x)/b^4 + (3*Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/
2])/Sqrt[a^2 - b^2]])/(a^2*b^4*d) - (6*(2*a^6 - a^4*b^2 - b^6)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*b^4*Sqrt[a^2 - b^2]*d) + (3*b*Ar
cTanh[Cos[c + d*x]])/(a^4*d) + Cos[c + d*x]/(b^3*d) - Cot[c + d*x]/(a^3*d)
- ((a^2 - b^2)^2*Cos[c + d*x])/(2*a^2*b^3*d*(a + b*Sin[c + d*x])^2) - (3*(a
^2 - b^2)*Cos[c + d*x])/(2*a*b^3*d*(a + b*Sin[c + d*x])) + (2*(a^2 - b^2)*(
2*a^2 + b^2)*Cos[c + d*x])/(a^3*b^3*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
```

```
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(\frac{3a}{b^4} - \frac{3b\csc(c+dx)}{a^4} + \frac{\csc^2(c+dx)}{a^3} - \frac{\sin(c+dx)}{b^3} - \frac{\left(a^2-b^2\right)^3}{a^2b^4(a+b\sin(c+dx)}\right.$$

$$= \frac{3ax}{b^4} + \frac{\int\csc^2(c+dx)\,dx}{a^3} - \frac{\int\sin(c+dx)\,dx}{b^3} - \frac{(3b)\int\csc(c+dx)\,dx}{a^4} - \frac{\left(a^2-b^2\right.}{}$$

$$= \frac{3ax}{b^4} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{b^3d} - \frac{\left(a^2-b^2\right)^2\cos(c+dx)}{2a^2b^3d(a+b\sin(c+dx))^2} + $$

$$= \frac{3ax}{b^4} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{b^3d} - \frac{\cot(c+dx)}{a^3d} - \frac{\left(a^2-b^2\right)^2\cos}{2a^2b^3d(a+b\sin}$$

$$= \frac{3ax}{b^4} - \frac{6\left(2a^6-a^4b^2-b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^4\sqrt{a^2-b^2}\,d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + $$

$$= \frac{3ax}{b^4} - \frac{6\left(2a^6-a^4b^2-b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^4\sqrt{a^2-b^2}\,d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + $$

$$= \frac{3ax}{b^4} - \frac{4\left(\frac{1}{a^2}-\frac{2a^2}{b^4}+\frac{1}{b^2}\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} - \frac{6\left(2a^6-a^4b^2-b^6\right)\tan^{-1}\left(\frac{b+}{}\right.}{a^4b^4\sqrt{a^2-b^2}\,d}$$

$$= \frac{3ax}{b^4} - \frac{4\left(\frac{1}{a^2}-\frac{2a^2}{b^4}+\frac{1}{b^2}\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} - \frac{\sqrt{a^2-b^2}\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b}{}\right.}{a^2b^4d}$$

Mathematica [A] time = 6.21, size = 332, normalized size = 1.06

$$-\frac{3b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^4d} + \frac{3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^4d} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{2a^3d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{2a^3d} + \frac{a^4(-\cos(c+dx))+2a^2}{2a^2b^3d(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*(c + d*x))/(b^4*d) - (3*(2*a^6 - a^4*b^2 + a^2*b^4 - 2*b^6)*ArcTan[(Se
c[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])
```

```
/(a^4*b^4*Sqrt[a^2 - b^2]*d) + Cos[c + d*x]/(b^3*d) - Cot[(c + d*x)/2]/(2*a
^3*d) + (3*b*Log[Cos[(c + d*x)/2]])/(a^4*d) - (3*b*Log[Sin[(c + d*x)/2]])/(
a^4*d) + (-(a^4*Cos[c + d*x]) + 2*a^2*b^2*Cos[c + d*x] - b^4*Cos[c + d*x])/
(2*a^2*b^3*d*(a + b*Sin[c + d*x])^2) + (5*a^4*Cos[c + d*x] - a^2*b^2*Cos[c
+ d*x] - 4*b^4*Cos[c + d*x])/(2*a^3*b^3*d*(a + b*Sin[c + d*x])) + Tan[(c +
d*x)/2]/(2*a^3*d)
```

fricas [A] time = 1.41, size = 1171, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")

[Out] [1/4*(24*a^6*b*d*x*cos(d*x + c)^2 - 24*a^6*b*d*x + 2*(9*a^5*b^2 - a^3*b^4 -
 6*a*b^6)*cos(d*x + c)^3 - 3*(4*a^5*b + 2*a^3*b^3 + 4*a*b^5 - 2*(2*a^5*b +
a^3*b^3 + 2*a*b^5)*cos(d*x + c)^2 + (2*a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 2*b^6
- (2*a^4*b^2 + a^2*b^4 + 2*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b
^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*
(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d
*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(3*a^5*b^2 - a^3*b^4 - 2*a
*b^6)*cos(d*x + c) + 6*(2*a*b^6*cos(d*x + c)^2 - 2*a*b^6 + (b^7*cos(d*x + c
)^2 - a^2*b^5 - b^7)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 6*(2*a*b^6
*cos(d*x + c)^2 - 2*a*b^6 + (b^7*cos(d*x + c)^2 - a^2*b^5 - b^7)*sin(d*x +
c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(6*a^5*b^2*d*x*cos(d*x + c)^2 + 2*a^4*
b^3*cos(d*x + c)^3 - 6*(a^7 + a^5*b^2)*d*x - 3*(2*a^6*b + a^4*b^3 - 3*a^2*b
^5)*cos(d*x + c))*sin(d*x + c))/(2*a^5*b^5*d*cos(d*x + c)^2 - 2*a^5*b^5*d +
 (a^4*b^6*d*cos(d*x + c)^2 - (a^6*b^4 + a^4*b^6)*d)*sin(d*x + c)), 1/2*(12*
a^6*b*d*x*cos(d*x + c)^2 - 12*a^6*b*d*x + (9*a^5*b^2 - a^3*b^4 - 6*a*b^6)*c
os(d*x + c)^3 - 3*(4*a^5*b + 2*a^3*b^3 + 4*a*b^5 - 2*(2*a^5*b + a^3*b^3 + 2
*a*b^5)*cos(d*x + c)^2 + (2*a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 2*b^6 - (2*a^4*b^
2 + a^2*b^4 + 2*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-
(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 3*(3*a^5*b^2 - a^3*b
^4 - 2*a*b^6)*cos(d*x + c) + 3*(2*a*b^6*cos(d*x + c)^2 - 2*a*b^6 + (b^7*cos
(d*x + c)^2 - a^2*b^5 - b^7)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*
(2*a*b^6*cos(d*x + c)^2 - 2*a*b^6 + (b^7*cos(d*x + c)^2 - a^2*b^5 - b^7)*si
n(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + (6*a^5*b^2*d*x*cos(d*x + c)^2 +
2*a^4*b^3*cos(d*x + c)^3 - 6*(a^7 + a^5*b^2)*d*x - 3*(2*a^6*b + a^4*b^3 - 3
*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(2*a^5*b^5*d*cos(d*x + c)^2 - 2*a^5*b
^5*d + (a^4*b^6*d*cos(d*x + c)^2 - (a^6*b^4 + a^4*b^6)*d)*sin(d*x + c))]
```

giac [A] time = 0.28, size = 461, normalized size = 1.47

$$\frac{\frac{6\,(dx+c)a}{b^4}-\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{6\left(2\,a^6-a^4b^2+a^2b^4-2\,b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^4b^4}+\frac{2\,b^4\tan\left(\frac{1}{2}\,d\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*(d*x + c)*a/b^4 - 6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + tan(1/2*d
*x + 1/2*c)/a^3 - 6*(2*a^6 - a^4*b^2 + a^2*b^4 - 2*b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*a^4*b^4) + (2*b^4*tan(1/2*d*x + 1/2*c)^3 - a*b^3*tan(1/
2*d*x + 1/2*c)^2 + 4*a^4*tan(1/2*d*x + 1/2*c) + 2*b^4*tan(1/2*d*x + 1/2*c)
- a*b^3)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a^4*b^3) + 2*(3*a
^5*b*tan(1/2*d*x + 1/2*c)^3 + 3*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^5*ta
n(1/2*d*x + 1/2*c)^3 + 4*a^6*tan(1/2*d*x + 1/2*c)^2 + 9*a^4*b^2*tan(1/2*d*x
 + 1/2*c)^2 - 3*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 - 10*b^6*tan(1/2*d*x + 1/2*c
)^2 + 13*a^5*b*tan(1/2*d*x + 1/2*c) + a^3*b^3*tan(1/2*d*x + 1/2*c) - 14*a*b
^5*tan(1/2*d*x + 1/2*c) + 4*a^6 + a^4*b^2 - 5*a^2*b^4)/((a*tan(1/2*d*x + 1/
2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^4*b^3))/d
```

maple [B] time = 0.82, size = 903, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)+2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)+6/d/b^4*arcta
n(tan(1/2*d*x+1/2*c))*a-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^4*b*ln(tan(1/2*d
*x+1/2*c))+3/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(
1/2*d*x+1/2*c)^3*a+3/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^
2*tan(1/2*d*x+1/2*c)^3-6/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)
*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^2+4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*a^2+9/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*
tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-3/d/a^2/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b-10/d/a^4/(tan(1/2*d*
x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^3+13/d/b^2/
(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*a+1/
d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-
14/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/
```

2*c)*b^2+4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2+1/
d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-5/d/a^2/(tan(1/2*d*
x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b-6/d/b^4/(a^2-b^2)^(1/2)*arctan(1
/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2+3/d/b^2/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-3/d/a^2/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d/a^4/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.69, size = 4223, normalized size = 13.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) + ((tan(c/2 + (d*x)/2)^6*(6*a^4 - 12*b^4 + 5*a
^2*b^2))/b^2 - a^2 - (tan(c/2 + (d*x)/2)^2*(32*b^4 - 42*a^4 + a^2*b^2))/b^2
 + (tan(c/2 + (d*x)/2)^4*(48*a^4 - 44*b^4 + 5*a^2*b^2))/b^2 + (2*tan(c/2 +
(d*x)/2)*(6*a^5 - 7*a*b^4 + a^3*b^2))/b^3 + (4*tan(c/2 + (d*x)/2)^3*(6*a^6
- 5*b^6 - 6*a^2*b^4 + 9*a^4*b^2))/(a*b^3) + (2*tan(c/2 + (d*x)/2)^5*(6*a^6
- 10*b^6 - 5*a^2*b^4 + 9*a^4*b^2))/(a*b^3))/(d*(2*a^5*tan(c/2 + (d*x)/2)^7
+ tan(c/2 + (d*x)/2)^3*(6*a^5 + 8*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(6*a^5 +
8*a^3*b^2) + 2*a^5*tan(c/2 + (d*x)/2) + 8*a^4*b*tan(c/2 + (d*x)/2)^2 + 16*a
^4*b*tan(c/2 + (d*x)/2)^4 + 8*a^4*b*tan(c/2 + (d*x)/2)^6)) + (6*a*atan((648
0*tan(c/2 + (d*x)/2))/((6480*b^4)/a^4 - (12960*b^2)/a^2 - (5184*b^6)/a^6 +
(5184*b^8)/a^8 + (6480*a*tan(c/2 + (d*x)/2))/b - (5184*b*tan(c/2 + (d*x)/2)
)/a + (5184*b^3*tan(c/2 + (d*x)/2))/a^3 - (12960*a^3*tan(c/2 + (d*x)/2))/b^
3 + (6480*a^5*tan(c/2 + (d*x)/2))/b^5 + 6480) - (12960*tan(c/2 + (d*x)/2))/
((6480*b^2)/a^2 + (6480*a^2)/b^2 - (5184*b^4)/a^4 + (5184*b^6)/a^6 - (5184*
a*tan(c/2 + (d*x)/2))/b + (5184*b*tan(c/2 + (d*x)/2))/a + (6480*a^3*tan(c/2
 + (d*x)/2))/b^3 - (12960*a^5*tan(c/2 + (d*x)/2))/b^5 + (6480*a^7*tan(c/2 +
 (d*x)/2))/b^7 - 12960) - (6480*a)/(6480*b + 6480*a*tan(c/2 + (d*x)/2) - (1
```

2960*b^3)/a^2 + (6480*b^5)/a^4 - (5184*b^7)/a^6 + (5184*b^9)/a^8 - (5184*b^2*tan(c/2 + (d*x)/2))/a - (12960*a^3*tan(c/2 + (d*x)/2))/b^2 + (5184*b^4*tan(c/2 + (d*x)/2))/a^3 + (6480*a^5*tan(c/2 + (d*x)/2))/b^4) + 5184/((6480*a)/b - 5184*tan(c/2 + (d*x)/2) - (12960*b)/a + (6480*b^3)/a^3 - (5184*b^5)/a^5 + (5184*b^7)/a^7 + (5184*b^2*tan(c/2 + (d*x)/2))/a^2 + (6480*a^2*tan(c/2 + (d*x)/2))/b^2 - (12960*a^4*tan(c/2 + (d*x)/2))/b^4 + (6480*a^6*tan(c/2 + (d*x)/2))/b^6) - 5184/(5184*tan(c/2 + (d*x)/2) - (12960*a)/b + (6480*b)/a - (5184*b^3)/a^3 + (6480*a^3)/b^3 + (5184*b^5)/a^5 - (5184*a^2*tan(c/2 + (d*x)/2))/b^2 + (6480*a^4*tan(c/2 + (d*x)/2))/b^4 - (12960*a^6*tan(c/2 + (d*x)/2))/b^6 + (6480*a^8*tan(c/2 + (d*x)/2))/b^8) + (6480*tan(c/2 + (d*x)/2))/((5184*b^4)/a^4 - (12960*a^2)/b^2 - (5184*b^2)/a^2 + (6480*a^4)/b^4 + (5184*a*tan(c/2 + (d*x)/2))/b - (5184*a^3*tan(c/2 + (d*x)/2))/b^3 + (6480*a^5*tan(c/2 + (d*x)/2))/b^5 - (12960*a^7*tan(c/2 + (d*x)/2))/b^7 + (6480*a^9*tan(c/2 + (d*x)/2))/b^9 + 6480) + (12960*a^3)/(6480*b^3 - (12960*b^5)/a^2 + (6480*b^7)/a^4 - (5184*b^9)/a^6 + (5184*b^11)/a^8 - 12960*a^3*tan(c/2 + (d*x)/2) + 6480*a*b^2*tan(c/2 + (d*x)/2) - (5184*b^4*tan(c/2 + (d*x)/2))/a + (6480*a^5*tan(c/2 + (d*x)/2))/b^2 + (5184*b^6*tan(c/2 + (d*x)/2))/a^3) - (6480*a^5)/(6480*b^5 - (12960*b^7)/a^2 + (6480*b^9)/a^4 - (5184*b^11)/a^6 + (5184*b^13)/a^8 + 6480*a^5*tan(c/2 + (d*x)/2) + 6480*a*b^4*tan(c/2 + (d*x)/2) - 12960*a^3*b^2*tan(c/2 + (d*x)/2) - (5184*b^6*tan(c/2 + (d*x)/2))/a + (5184*b^8*tan(c/2 + (d*x)/2))/a^3) - (5184*b*tan(c/2 + (d*x)/2))/((6480*a^2)/b - 5184*b + (5184*b^3)/a^2 - (12960*a^4)/b^3 + (6480*a^6)/b^5 + (5184*a^3*tan(c/2 + (d*x)/2))/b^2 - (5184*a^5*tan(c/2 + (d*x)/2))/b^4 + (6480*a^7*tan(c/2 + (d*x)/2))/b^6 - (12960*a^9*tan(c/2 + (d*x)/2))/b^8 + (6480*a^11*tan(c/2 + (d*x)/2))/b^10) + (5184*b^3*tan(c/2 + (d*x)/2))/(5184*b^3 - 5184*a^2*b + (6480*a^4)/b - (12960*a^6)/b^3 + (6480*a^8)/b^5 + (5184*a^5*tan(c/2 + (d*x)/2))/b^2 - (5184*a^7*tan(c/2 + (d*x)/2))/b^4 + (6480*a^9*tan(c/2 + (d*x)/2))/b^6 - (12960*a^11*tan(c/2 + (d*x)/2))/b^8 + (6480*a^13*tan(c/2 + (d*x)/2))/b^10)))/(b^4*d) - (3*b*log(tan(c/2 + (d*x)/2)))/(a^4*d) + (atan((((-(a + b)*(a - b))^(1/2)*(2*a^4 + 2*b^4 + a^2*b^2)*((8*tan(c/2 + (d*x)/2)*(108*b^19 - 108*a^2*b^17 + 135*a^4*b^15 - 270*a^6*b^13 + 1863*a^8*b^11 - 1836*a^10*b^9 + 864*a^12*b^7 - 1080*a^14*b^5 + 432*a^16*b^3))/(a^9*b^12) - (8*(378*a^14 + 108*a^4*b^10 - 108*a^6*b^8 - 729*a^8*b^6 + 378*a^10*b^4 - 135*a^12*b^2))/(a^8*b^8) + (3*(-(a + b)*(a - b))^(1/2)*((8*(144*a^2*b^15 - 108*a^4*b^13 + 90*a^6*b^11 - 126*a^8*b^9 + 144*a^12*b^5 - 108*a^14*b^3))/(a^8*b^8) + (8*tan(c/2 + (d*x)/2)*(36*a^4*b^18 - 180*a^6*b^16 + 405*a^8*b^14 - 306*a^10*b^12 + 909*a^12*b^10 - 900*a^14*b^8 + 72*a^16*b^6))/(a^9*b^12) + (3*(-(a + b)*(a - b))^(1/2)*((8*(96*a^6*b^14 - 72*a^8*b^12 + 30*a^10*b^10 - 42*a^12*b^8))/(a^8*b^8) + (8*tan(c/2 + (d*x)/2)*(192*a^6*b^19 - 204*a^8*b^17 + 96*a^10*b^15 - 120*a^12*b^13 + 48*a^14*b^11))/(a^9*b^12) + (3*(-(a + b)*(a - b))^(1/2)*((8*(16*a^10*b^13 - 12*a^12*b^11))/(a^8*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^10*b^18 - 68*a^12*b^16 + 8*a^14*b^14))/(a^9*b^12))*(2*a^4 + 2*b^4 + a^2*b^2))/(2*a^4*b^4))*(2*a^4 + 2*b^4 + a^2*b^2))/(2*a^4*b^4))*(2*a^4 + 2*b^4 + a^2*b^2))/(2*a^4*b^4))*3i)/(2*a^4*b^4) - ((-(a + b)*(a - b))^(1/2)*(2*a^4 + 2*b^4 + a^2*b^2)*((8*(378*a^14 + 108*a^4*b^10 - 108*a^6*b^8 - 729*a^8*b

^6 + 378*a^10*b^4 - 135*a^12*b^2))/(a^8*b^8) - (8*tan(c/2 + (d*x)/2)*(108*b
^19 - 108*a^2*b^17 + 135*a^4*b^15 - 270*a^6*b^13 + 1863*a^8*b^11 - 1836*a^1
0*b^9 + 864*a^12*b^7 - 1080*a^14*b^5 + 432*a^16*b^3))/(a^9*b^12) + (3*(-(a
+ b)*(a - b))^(1/2)*((8*(144*a^2*b^15 - 108*a^4*b^13 + 90*a^6*b^11 - 126*a^
8*b^9 + 144*a^12*b^5 - 108*a^14*b^3))/(a^8*b^8) + (8*tan(c/2 + (d*x)/2)*(36
*a^4*b^18 - 180*a^6*b^16 + 405*a^8*b^14 - 306*a^10*b^12 + 909*a^12*b^10 - 9
00*a^14*b^8 + 72*a^16*b^6))/(a^9*b^12) - (3*(-(a + b)*(a - b))^(1/2)*((8*(9
6*a^6*b^14 - 72*a^8*b^12 + 30*a^10*b^10 - 42*a^12*b^8))/(a^8*b^8) + (8*tan(
c/2 + (d*x)/2)*(192*a^6*b^19 - 204*a^8*b^17 + 96*a^10*b^15 - 120*a^12*b^13
+ 48*a^14*b^11))/(a^9*b^12) - (3*(-(a + b)*(a - b))^(1/2)*((8*(16*a^10*b^13
 - 12*a^12*b^11))/(a^8*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^10*b^18 - 68*a^12
*b^16 + 8*a^14*b^14))/(a^9*b^12))*(2*a^4 + 2*b^4 + a^2*b^2))/(2*a^4*b^4))*(
2*a^4 + 2*b^4 + a^2*b^2))/(2*a^4*b^4))*(2*a^4 + 2*b^4 + a^2*b^2))/(2*a^4*b^
4))*3i)/(2*a^4*b^4))/((16*(1134*a^10*b + 324*b^11 - 324*a^2*b^9 - 891*a^4*b
^7 + 162*a^6*b^5 - 405*a^8*b^3))/(a^8*b^8) + (16*tan(c/2 + (d*x)/2)*(2592*a
^16 + 1296*a^8*b^8 - 3240*a^10*b^6 + 1944*a^12*b^4 - 2592*a^14*b^2))/(a^9*b
^12) - (3*(-(a + b)*(a - b))^(1/2)*(2*a^4 + 2*b^4 + a^2*b^2)*((8*tan(c/2 +
(d*x)/2)*(108*b^19 - 108*a^2*b^17 + 135*a^4*b^15 - 270*a^6*b^13 + 1863*a^8*
b^11 - 1836*a^10*b^9 + 864*a^12*b^7 - 1080*a^14*b^5 + 432*a^16*b^3))/(a^9*b
^12) - (8*(378*a^14 + 108*a^4*b^10 - 108*a^6*b^8 - 729*a^8*b^6 + 378*a^10*b
^4 - 135*a^12*b^2))/(a^8*b^8) + (3*(-(a + b)*(a - b))^(1/2)*((8*(144*a^2*b^
15 - 108*a^4*b^13 + 90*a^6*b^11 - 126*a^8*b^9 + 144*a^12*b^5 - 108*a^14*b^3
))/(a^8*b^8) + (8*tan(c/2 + (d*x)/2)*(36*a^4*b^18 - 180*a^6*b^16 + 405*a^8*
b^14 - 306*a^10*b^12 + 909*a^12*b^10 - 900*a^14*b^8 + 72*a^16*b^6))/(a^9*b^
12) + (3*(-(a + b)*(a - b))^(1/2)*((8*(96*a^6*b^14 - 72*a^8*b^12 + 30*a^10*
b^10 - 42*a^12*b^8))/(a^8*b^8) + (8*tan(c/2 + (d*x)/2)*(192*a^6*b^19 - 204*
a^8*b^17 + 96*a^10*b^15 - 120*a^12*b^13 + 48*a^14*b^11))/(a^9*b^12) + (3*(-
(a + b)*(a - b))^(1/2)*((8*(16*a^10*b^13 - 12*a^12*b^11))/(a^8*b^8) + (8*ta
n(c/2 + (d*x)/2)*(64*a^10*b^18 - 68*a^12*b^16 + 8*a^14*b^14))/(a^9*b^12))*(
2*a^4 + 2*b^4 + a^2*b^2))/(2*a^4*b^4))*(2*a^4 + 2*b^4 + a^2*b^2))/(2*a^4*b^
4))*(2*a^4 + 2*b^4 + a^2*b^2))/(2*a^4*b^4)))/(2*a^4*b^4) - (3*(-(a + b)*(a
- b))^(1/2)*(2*a^4 + 2*b^4 + a^2*b^2)*((8*(378*a^14 + 108*a^4*b^10 - 108*a^
6*b^8 - 729*a^8*b^6 + 378*a^10*b^4 - 135*a^12*b^2))/(a^8*b^8) - (8*tan(c/2
+ (d*x)/2)*(108*b^19 - 108*a^2*b^17 + 135*a^4*b^15 - 270*a^6*b^13 + 1863*a^
8*b^11 - 1836*a^10*b^9 + 864*a^12*b^7 - 1080*a^14*b^5 + 432*a^16*b^3))/(a^9
b^12) + (3(-(a + b)*(a - b))^(1/2)*((8*(144*a^2*b^15 - 108*a^4*b^13 + 90*
a^6*b^11 - 126*a^8*b^9 + 144*a^12*b^5 - 108*a^14*b^3))/(a^8*b^8) + (8*tan(c
/2 + (d*x)/2)*(36*a^4*b^18 - 180*a^6*b^16 + 405*a^8*b^14 - 306*a^10*b^12 +
909*a^12*b^10 - 900*a^14*b^8 + 72*a^16*b^6))/(a^9*b^12) - (3*(-(a + b)*(a -
 b))^(1/2)*((8*(96*a^6*b^14 - 72*a^8*b^12 + 30*a^10*b^10 - 42*a^12*b^8))/(a
^8*b^8) + (8*tan(c/2 + (d*x)/2)*(192*a^6*b^19 - 204*a^8*b^17 + 96*a^10*b^15
 - 120*a^12*b^13 + 48*a^14*b^11))/(a^9*b^12) - (3*(-(a + b)*(a - b))^(1/2)*
((8*(16*a^10*b^13 - 12*a^12*b^11))/(a^8*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^
10*b^18 - 68*a^12*b^16 + 8*a^14*b^14))/(a^9*b^12))*(2*a^4 + 2*b^4 + a^2*b^2
))/(2*a^4*b^4))*(2*a^4 + 2*b^4 + a^2*b^2))/(2*a^4*b^4))*(2*a^4 + 2*b^4 + a^

```
2*b^2))/(2*a^4*b^4)))/(2*a^4*b^4)))*(-(a + b)*(a - b))^(1/2)*(2*a^4 + 2*b^4
 + a^2*b^2)*3i)/(a^4*b^4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1271 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=395

$$-\frac{3\left(a^4-b^4\right)\cos(c+dx)}{a^4b^2d(a+b\sin(c+dx))}+\frac{3b\cot(c+dx)}{a^4d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{3\left(a^2-b^2\right)\cos(c+}{2a^2b^2d(a+b\sin(c+}$$

```
[Out] -x/b^3-1/2*arctanh(cos(d*x+c))/a^3/d+3*(a^2-2*b^2)*arctanh(cos(d*x+c))/a^5/
d+3*b*cot(d*x+c)/a^4/d-1/2*cot(d*x+c)*csc(d*x+c)/a^3/d+1/2*(a^2-b^2)^2*cos(
d*x+c)/a^3/b^2/d/(a+b*sin(d*x+c))^2+3/2*(a^2-b^2)*cos(d*x+c)/a^2/b^2/d/(a+b
*sin(d*x+c))-3*(a^4-b^4)*cos(d*x+c)/a^4/b^2/d/(a+b*sin(d*x+c))+6*(a^6+a^2*b
^4-2*b^6)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^5/b^3/d/(a^2-b
^2)^(1/2)-6*(a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2
-b^2)^(1/2)/a^3/b^3/d+(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)
^(1/2))*(a^2-b^2)^(1/2)/a^3/b^3/d
```

Rubi [A] time = 0.52, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2897, 3770, 3767, 8, 3768, 2664, 2754, 12, 2660, 618, 204}

$$-\frac{6\left(a^2+b^2\right)\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{\left(2a^2+b^2\right)\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{6\left(a^2b^4+a^6-2b^6\right)}{a^5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(x/b^3) - (6*Sqrt[a^2 - b^2]*(a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/(a^3*b^3*d) + (Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTan[(b + a
*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^3*d) + (6*(a^6 + a^2*b^4 - 2*b^
6)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5*b^3*Sqrt[a^2 - b^
2]*d) - ArcTanh[Cos[c + d*x]]/(2*a^3*d) + (3*(a^2 - 2*b^2)*ArcTanh[Cos[c +
d*x]])/(a^5*d) + (3*b*Cot[c + d*x])/(a^4*d) - (Cot[c + d*x]*Csc[c + d*x])/(
2*a^3*d) + ((a^2 - b^2)^2*Cos[c + d*x])/(2*a^3*b^2*d*(a + b*Sin[c + d*x])^2
) + (3*(a^2 - b^2)*Cos[c + d*x])/(2*a^2*b^2*d*(a + b*Sin[c + d*x])) - (3*(a
^4 - b^4)*Cos[c + d*x])/(a^4*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
```

```
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(-\frac{1}{b^3}-\frac{3\left(a^2-2b^2\right)\csc(c+dx)}{a^5}-\frac{3b\csc^2(c+dx)}{a^4}+\frac{\csc^3(c+dx)}{a^3}+\frac{}{a^3b^3(a}\right.$$

$$= -\frac{x}{b^3}+\frac{\int\csc^3(c+dx)\,dx}{a^3}-\frac{(3b)\int\csc^2(c+dx)\,dx}{a^4}-\frac{\left(3\left(a^2-2b^2\right)\right)\int\csc(c+d}{a^5}$$

$$= -\frac{x}{b^3}+\frac{3\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{\left(a^2-b\right.}{2a^3b^2d(a}$$

$$= -\frac{x}{b^3}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{3b\cot(c+d}{a^4d}$$

$$= -\frac{x}{b^3}+\frac{6\left(a^6+a^2b^4-2b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^3\sqrt{a^2-b^2}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3(}{}$$

$$= -\frac{x}{b^3}+\frac{6\left(a^6+a^2b^4-2b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^3\sqrt{a^2-b^2}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3(}{}$$

$$= -\frac{x}{b^3}-\frac{6\left(\frac{a}{b^3}-\frac{b}{a^3}\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}+\frac{6\left(a^6+a^2b^4-2b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\right.}{\sqrt{a^2-}}\right.}{a^5b^3\sqrt{a^2-b^2}\,d}$$

$$= -\frac{x}{b^3}-\frac{6\left(\frac{a}{b^3}-\frac{b}{a^3}\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}+\frac{\sqrt{a^2-b^2}\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan(}{\sqrt{a^2}}\right.}{a^3b^3d}$$

Mathematica [A] time = 6.26, size = 384, normalized size = 0.97

$$-\frac{3b\tan\left(\frac{1}{2}(c+dx)\right)}{2a^4d}+\frac{3b\cot\left(\frac{1}{2}(c+dx)\right)}{2a^4d}-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d}+\frac{\left(12b^2-5a^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((c + d*x)/(b^3*d)) + ((2*a^6 - a^4*b^2 + 11*a^2*b^4 - 12*b^6)*ArcTan[(Sec
[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/
```

```
(a^5*b^3*Sqrt[a^2 - b^2]*d) + (3*b*Cot[(c + d*x)/2])/(2*a^4*d) - Csc[(c + d
*x)/2]^2/(8*a^3*d) + ((5*a^2 - 12*b^2)*Log[Cos[(c + d*x)/2]])/(2*a^5*d) + (
(-5*a^2 + 12*b^2)*Log[Sin[(c + d*x)/2]])/(2*a^5*d) + Sec[(c + d*x)/2]^2/(8*
a^3*d) + (a^4*Cos[c + d*x] - 2*a^2*b^2*Cos[c + d*x] + b^4*Cos[c + d*x])/(2*
a^3*b^2*d*(a + b*Sin[c + d*x])^2) - (3*(a^4*Cos[c + d*x] + a^2*b^2*Cos[c +
d*x] - 2*b^4*Cos[c + d*x]))/(2*a^4*b^2*d*(a + b*Sin[c + d*x])) - (3*b*Tan[(
c + d*x)/2])/(2*a^4*d)
```

fricas [B] time = 1.72, size = 1658, normalized size = 4.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(4*a^5*b^2*d*x*cos(d*x + c)^4 - 4*(a^7 + 2*a^5*b^2)*d*x*cos(d*x + c)^
2 - 2*(2*a^6*b + 5*a^4*b^3 - 18*a^2*b^5)*cos(d*x + c)^3 + 4*(a^7 + a^5*b^2)
*d*x - (2*a^6 + 3*a^4*b^2 + 13*a^2*b^4 + 12*b^6 + (2*a^4*b^2 + a^2*b^4 + 12
*b^6)*cos(d*x + c)^4 - (2*a^6 + 5*a^4*b^2 + 14*a^2*b^4 + 24*b^6)*cos(d*x +
c)^2 + 2*(2*a^5*b + a^3*b^3 + 12*a*b^5 - (2*a^5*b + a^3*b^3 + 12*a*b^5)*cos
(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 4*(a^6*b + 3*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c) - (5*a^4*b^3 -
7*a^2*b^5 - 12*b^7 + (5*a^2*b^5 - 12*b^7)*cos(d*x + c)^4 - (5*a^4*b^3 - 2*a
^2*b^5 - 24*b^7)*cos(d*x + c)^2 + 2*(5*a^3*b^4 - 12*a*b^6 - (5*a^3*b^4 - 12
*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (5*a^4*
b^3 - 7*a^2*b^5 - 12*b^7 + (5*a^2*b^5 - 12*b^7)*cos(d*x + c)^4 - (5*a^4*b^3
 - 2*a^2*b^5 - 24*b^7)*cos(d*x + c)^2 + 2*(5*a^3*b^4 - 12*a*b^6 - (5*a^3*b^
4 - 12*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) -
2*(4*a^6*b*d*x*cos(d*x + c)^2 - 4*a^6*b*d*x + 3*(a^5*b^2 + a^3*b^4 - 4*a*b^
6)*cos(d*x + c)^3 - (3*a^5*b^2 - a^3*b^4 - 12*a*b^6)*cos(d*x + c))*sin(d*x
+ c))/(a^5*b^5*d*cos(d*x + c)^4 - (a^7*b^3 + 2*a^5*b^5)*d*cos(d*x + c)^2 +
(a^7*b^3 + a^5*b^5)*d - 2*(a^6*b^4*d*cos(d*x + c)^2 - a^6*b^4*d)*sin(d*x +
c)), -1/4*(4*a^5*b^2*d*x*cos(d*x + c)^4 - 4*(a^7 + 2*a^5*b^2)*d*x*cos(d*x +
 c)^2 - 2*(2*a^6*b + 5*a^4*b^3 - 18*a^2*b^5)*cos(d*x + c)^3 + 4*(a^7 + a^5*
b^2)*d*x + 2*(2*a^6 + 3*a^4*b^2 + 13*a^2*b^4 + 12*b^6 + (2*a^4*b^2 + a^2*b^
4 + 12*b^6)*cos(d*x + c)^4 - (2*a^6 + 5*a^4*b^2 + 14*a^2*b^4 + 24*b^6)*cos(
d*x + c)^2 + 2*(2*a^5*b + a^3*b^3 + 12*a*b^5 - (2*a^5*b + a^3*b^3 + 12*a*b^
5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 4*(a^6*b + 3*a^4*b^3 - 9*a^2*b^5)*cos(
d*x + c) - (5*a^4*b^3 - 7*a^2*b^5 - 12*b^7 + (5*a^2*b^5 - 12*b^7)*cos(d*x +
 c)^4 - (5*a^4*b^3 - 2*a^2*b^5 - 24*b^7)*cos(d*x + c)^2 + 2*(5*a^3*b^4 - 12
*a*b^6 - (5*a^3*b^4 - 12*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d
```

```
*x + c) + 1/2) + (5*a^4*b^3 - 7*a^2*b^5 - 12*b^7 + (5*a^2*b^5 - 12*b^7)*cos
(d*x + c)^4 - (5*a^4*b^3 - 2*a^2*b^5 - 24*b^7)*cos(d*x + c)^2 + 2*(5*a^3*b^
4 - 12*a*b^6 - (5*a^3*b^4 - 12*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/
2*cos(d*x + c) + 1/2) - 2*(4*a^6*b*d*x*cos(d*x + c)^2 - 4*a^6*b*d*x + 3*(a^
5*b^2 + a^3*b^4 - 4*a*b^6)*cos(d*x + c)^3 - (3*a^5*b^2 - a^3*b^4 - 12*a*b^6
)*cos(d*x + c))*sin(d*x + c))/(a^5*b^5*d*cos(d*x + c)^4 - (a^7*b^3 + 2*a^5*
b^5)*d*cos(d*x + c)^2 + (a^7*b^3 + a^5*b^5)*d - 2*(a^6*b^4*d*cos(d*x + c)^2
 - a^6*b^4*d)*sin(d*x + c))]
```

giac [A] time = 0.29, size = 512, normalized size = 1.30

$$-\frac{\frac{8(dx+c)}{b^3} + \frac{4\left(5a^2-12b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^5} - \frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6} - \frac{8\left(2a^6-a^4b^2+11a^2b^4-12b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\cdots\right.}{\sqrt{a^2-b^2}\,a^5b^3}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(8*(d*x + c)/b^3 + 4*(5*a^2 - 12*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a
^5 - (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c))/a^6 - 8*(
2*a^6 - a^4*b^2 + 11*a^2*b^4 - 12*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^
2)*a^5*b^3) - (10*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 - 24*a^2*b^4*tan(1/2*d*x +
 1/2*c)^6 - 8*a^5*b*tan(1/2*d*x + 1/2*c)^5 - 4*a^3*b^3*tan(1/2*d*x + 1/2*c)
^5 - 32*a*b^5*tan(1/2*d*x + 1/2*c)^5 - 16*a^6*tan(1/2*d*x + 1/2*c)^4 - 53*a
^4*b^2*tan(1/2*d*x + 1/2*c)^4 + 16*a^2*b^4*tan(1/2*d*x + 1/2*c)^4 + 16*b^6*
tan(1/2*d*x + 1/2*c)^4 - 56*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 44*a^3*b^3*tan(1
/2*d*x + 1/2*c)^3 + 112*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 16*a^6*tan(1/2*d*x +
 1/2*c)^2 - 32*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 76*a^2*b^4*tan(1/2*d*x + 1/
2*c)^2 + 8*a^3*b^3*tan(1/2*d*x + 1/2*c) - a^4*b^2)/((a*tan(1/2*d*x + 1/2*c)
^3 + 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))^2*a^5*b^2))/d
```

maple [B] time = 0.87, size = 943, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^4*tan(1/2*d*x+1/2*c)*b-2/d/b^3*arcta
n(tan(1/2*d*x+1/2*c))-1/8/d/a^3/tan(1/2*d*x+1/2*c)^2-5/2/d/a^3*ln(tan(1/2*d
*x+1/2*c))+6/d/a^5*ln(tan(1/2*d*x+1/2*c))*b^2+3/2/d*b/a^4/tan(1/2*d*x+1/2*c
```

```
)-1/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2
*c)^3-7/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2
*d*x+1/2*c)^3+8/d/a^4*b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)
^2*tan(1/2*d*x+1/2*c)^3-2/d*a/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-9/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^2-3/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*t
an(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+14/d/a^5*b^4/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-7/d/b/(tan(1/2*d*x
+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-13/d/a^2*b/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+20/d/a^4*
b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-
2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a-5/d/a/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+7/d/a^3*b^2/(tan(1/2*d*x+1/2*c
)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+2/d*a/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/d/a/b/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+11/d/a^3*b/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-12/d/a^5*b^3/(a^2-
b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.02, size = 4381, normalized size = 11.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^3*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (2*atan((1800*tan(c/2 + (d*x)/2))/((7120*b
^2)/a^2 - (12520*b^4)/a^4 + (24480*b^6)/a^6 - (31104*b^8)/a^8 + (13824*b^10
)/a^10 - (1120*a*tan(c/2 + (d*x)/2))/b + (2320*b*tan(c/2 + (d*x)/2))/a - (4
224*b^3*tan(c/2 + (d*x)/2))/a^3 + (720*a^3*tan(c/2 + (d*x)/2))/b^3 + (2304*
b^5*tan(c/2 + (d*x)/2))/a^5 - 1800) - (7120*tan(c/2 + (d*x)/2))/((24480*b^4
)/a^4 - (1800*a^2)/b^2 - (12520*b^2)/a^2 - (31104*b^6)/a^6 + (13824*b^8)/a^
```

8 + (2320*a*tan(c/2 + (d*x)/2))/b - (4224*b*tan(c/2 + (d*x)/2))/a + (2304*b
^3*tan(c/2 + (d*x)/2))/a^3 - (1120*a^3*tan(c/2 + (d*x)/2))/b^3 + (720*a^5*t
an(c/2 + (d*x)/2))/b^5 + 7120) + (720*a)/(720*a*tan(c/2 + (d*x)/2) - (1800*
b^3)/a^2 + (7120*b^5)/a^4 - (12520*b^7)/a^6 + (24480*b^9)/a^8 - (31104*b^11
)/a^10 + (13824*b^13)/a^12 - (1120*b^2*tan(c/2 + (d*x)/2))/a + (2320*b^4*ta
n(c/2 + (d*x)/2))/a^3 - (4224*b^6*tan(c/2 + (d*x)/2))/a^5 + (2304*b^8*tan(c
/2 + (d*x)/2))/a^7) - 4224/((7120*a)/b - 4224*tan(c/2 + (d*x)/2) - (12520*b
)/a + (24480*b^3)/a^3 - (1800*a^3)/b^3 - (31104*b^5)/a^5 + (13824*b^7)/a^7
 + (2304*b^2*tan(c/2 + (d*x)/2))/a^2 + (2320*a^2*tan(c/2 + (d*x)/2))/b^2 - (
1120*a^4*tan(c/2 + (d*x)/2))/b^4 + (720*a^6*tan(c/2 + (d*x)/2))/b^6) + 2320
/(2320*tan(c/2 + (d*x)/2) - (1800*a)/b + (7120*b)/a - (12520*b^3)/a^3 + (24
480*b^5)/a^5 - (31104*b^7)/a^7 + (13824*b^9)/a^9 - (4224*b^2*tan(c/2 + (d*x
)/2))/a^2 - (1120*a^2*tan(c/2 + (d*x)/2))/b^2 + (2304*b^4*tan(c/2 + (d*x)/2
))/a^4 + (720*a^4*tan(c/2 + (d*x)/2))/b^4) + (2304*b^2)/((24480*b^3)/a - 12
520*a*b + (7120*a^3)/b - (31104*b^5)/a^3 - (1800*a^5)/b^3 + (13824*b^7)/a^5
 - 4224*a^2*tan(c/2 + (d*x)/2) + 2304*b^2*tan(c/2 + (d*x)/2) + (2320*a^4*ta
n(c/2 + (d*x)/2))/b^2 - (1120*a^6*tan(c/2 + (d*x)/2))/b^4 + (720*a^8*tan(c/
2 + (d*x)/2))/b^6) - 1120/((7120*b^3)/a^3 - (1800*b)/a - 1120*tan(c/2 + (d*
x)/2) - (12520*b^5)/a^5 + (24480*b^7)/a^7 - (31104*b^9)/a^9 + (13824*b^11)/
a^11 + (2320*b^2*tan(c/2 + (d*x)/2))/a^2 + (720*a^2*tan(c/2 + (d*x)/2))/b^2
 - (4224*b^4*tan(c/2 + (d*x)/2))/a^4 + (2304*b^6*tan(c/2 + (d*x)/2))/a^6) -
 (24480*b^3*tan(c/2 + (d*x)/2))/(24480*b^3 - 12520*a^2*b + (7120*a^4)/b - (
31104*b^5)/a^2 - (1800*a^6)/b^3 + (13824*b^7)/a^4 - 4224*a^3*tan(c/2 + (d*x
)/2) + 2304*a*b^2*tan(c/2 + (d*x)/2) + (2320*a^5*tan(c/2 + (d*x)/2))/b^2 -
(1120*a^7*tan(c/2 + (d*x)/2))/b^4 + (720*a^9*tan(c/2 + (d*x)/2))/b^6) + (31
104*b^5*tan(c/2 + (d*x)/2))/(24480*a^2*b^3 - 31104*b^5 - 12520*a^4*b + (712
0*a^6)/b + (13824*b^7)/a^2 - (1800*a^8)/b^3 - 4224*a^5*tan(c/2 + (d*x)/2) +
 2304*a^3*b^2*tan(c/2 + (d*x)/2) + (2320*a^7*tan(c/2 + (d*x)/2))/b^2 - (112
0*a^9*tan(c/2 + (d*x)/2))/b^4 + (720*a^11*tan(c/2 + (d*x)/2))/b^6) - (13824
*b^7*tan(c/2 + (d*x)/2))/(13824*b^7 - 12520*a^6*b - 31104*a^2*b^5 + 24480*a
^4*b^3 + (7120*a^8)/b - (1800*a^10)/b^3 - 4224*a^7*tan(c/2 + (d*x)/2) + 230
4*a^5*b^2*tan(c/2 + (d*x)/2) + (2320*a^9*tan(c/2 + (d*x)/2))/b^2 - (1120*a^
11*tan(c/2 + (d*x)/2))/b^4 + (720*a^13*tan(c/2 + (d*x)/2))/b^6) + (12520*b*
tan(c/2 + (d*x)/2))/((7120*a^2)/b - 4224*a*tan(c/2 + (d*x)/2) - 12520*b + (
24480*b^3)/a^2 - (1800*a^4)/b^3 - (31104*b^5)/a^4 + (13824*b^7)/a^6 + (2304
*b^2*tan(c/2 + (d*x)/2))/a + (2320*a^3*tan(c/2 + (d*x)/2))/b^2 - (1120*a^5*
tan(c/2 + (d*x)/2))/b^4 + (720*a^7*tan(c/2 + (d*x)/2))/b^6)))/(b^3*d) - (a^
3/2 + (tan(c/2 + (d*x)/2)^2*(8*a^5 - 50*a*b^4 + 21*a^3*b^2))/b^2 - 4*a^2*b*
tan(c/2 + (d*x)/2) + (2*tan(c/2 + (d*x)/2)^5*(2*a^4 - 16*b^4 + 11*a^2*b^2))
/b + (2*tan(c/2 + (d*x)/2)^3*(14*a^4 - 52*b^4 + 21*a^2*b^2))/b + (tan(c/2 +
 (d*x)/2)^4*(16*a^6 - 112*b^6 - 24*a^2*b^4 + 73*a^4*b^2))/(2*a*b^2))/(d*(4*
a^6*tan(c/2 + (d*x)/2)^2 + 4*a^6*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^
4*(8*a^6 + 16*a^4*b^2) + 16*a^5*b*tan(c/2 + (d*x)/2)^3 + 16*a^5*b*tan(c/2 +
 (d*x)/2)^5)) - (3*b*tan(c/2 + (d*x)/2))/(2*a^4*d) - (log(tan(c/2 + (d*x)/2
))*(5*a^2 - 12*b^2))/(2*a^5*d) - (atan((((-(a + b)*(a - b))^(1/2)*(a^4 + 6*

b^4 + (a^2*b^2)/2)*((4*tan(c/2 + (d*x)/2)*(1728*b^18 - 3888*a^2*b^16 + 3060*a^4*b^14 - 1565*a^6*b^12 + 1658*a^8*b^10 - 1361*a^10*b^8 + 484*a^12*b^6 - 120*a^14*b^4 + 32*a^16*b^2))/(a^12*b^8) - (4*(28*a^15 + 288*a^5*b^10 - 528*a^7*b^8 - 94*a^9*b^6 + 308*a^11*b^4 - 30*a^13*b^2))/(a^12*b^5) + ((-(a + b)*(a - b))^(1/2)*(a^4 + 6*b^4 + (a^2*b^2)/2)*((4*(1152*a^4*b^14 - 1824*a^6*b^12 + 920*a^8*b^10 - 318*a^10*b^8 + 70*a^12*b^6 + 32*a^14*b^4 - 24*a^16*b^2))/(a^12*b^5) + (4*tan(c/2 + (d*x)/2)*(288*a^5*b^16 - 240*a^7*b^14 + 410*a^9*b^12 - 468*a^11*b^10 + 202*a^13*b^8 - 200*a^15*b^6 + 16*a^17*b^4))/(a^12*b^8) + ((-(a + b)*(a - b))^(1/2)*((4*(384*a^9*b^12 - 448*a^11*b^10 + 120*a^13*b^8 - 28*a^15*b^6))/(a^12*b^5) + (4*tan(c/2 + (d*x)/2)*(768*a^8*b^16 - 1136*a^10*b^14 + 484*a^12*b^12 - 120*a^14*b^10 + 32*a^16*b^8))/(a^12*b^8) + ((-(a + b)*(a - b))^(1/2)*((4*(32*a^14*b^10 - 24*a^16*b^8))/(a^12*b^5) + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^14 - 136*a^15*b^12 + 16*a^17*b^10))/(a^12*b^8))*(a^4 + 6*b^4 + (a^2*b^2)/2))/(a^5*b^3))*(a^4 + 6*b^4 + (a^2*b^2)/2))/(a^5*b^3)))/(a^5*b^3))*1i)/(a^5*b^3) - ((-(a + b)*(a - b))^(1/2)*(a^4 + 6*b^4 + (a^2*b^2)/2)*((4*(28*a^15 + 288*a^5*b^10 - 528*a^7*b^8 - 94*a^9*b^6 + 308*a^11*b^4 - 30*a^13*b^2))/(a^12*b^5) - (4*tan(c/2 + (d*x)/2)*(1728*b^18 - 3888*a^2*b^16 + 3060*a^4*b^14 - 1565*a^6*b^12 + 1658*a^8*b^10 - 1361*a^10*b^8 + 484*a^12*b^6 - 120*a^14*b^4 + 32*a^16*b^2))/(a^12*b^8) + ((-(a + b)*(a - b))^(1/2)*(a^4 + 6*b^4 + (a^2*b^2)/2)*((4*(1152*a^4*b^14 - 1824*a^6*b^12 + 920*a^8*b^10 - 318*a^10*b^8 + 70*a^12*b^6 + 32*a^14*b^4 - 24*a^16*b^2))/(a^12*b^5) + (4*tan(c/2 + (d*x)/2)*(288*a^5*b^16 - 240*a^7*b^14 + 410*a^9*b^12 - 468*a^11*b^10 + 202*a^13*b^8 - 200*a^15*b^6 + 16*a^17*b^4))/(a^12*b^8) - ((-(a + b)*(a - b))^(1/2)*((4*(384*a^9*b^12 - 448*a^11*b^10 + 120*a^13*b^8 - 28*a^15*b^6))/(a^12*b^5) + (4*tan(c/2 + (d*x)/2)*(768*a^8*b^16 - 1136*a^10*b^14 + 484*a^12*b^12 - 120*a^14*b^10 + 32*a^16*b^8))/(a^12*b^8) - ((-(a + b)*(a - b))^(1/2)*((4*(32*a^14*b^10 - 24*a^16*b^8))/(a^12*b^5) + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^14 - 136*a^15*b^12 + 16*a^17*b^10))/(a^12*b^8))*(a^4 + 6*b^4 + (a^2*b^2)/2))/(a^5*b^3))*(a^4 + 6*b^4 + (a^2*b^2)/2))/(a^5*b^3)))/(a^5*b^3))*1i)/(a^5*b^3))/((8*(1728*b^12 - 70*a^12 - 3888*a^2*b^10 + 1908*a^4*b^8 + 259*a^6*b^6 - 30*a^8*b^4 + 93*a^10*b^2))/(a^12*b^5) + (8*tan(c/2 + (d*x)/2)*(64*a^15 - 288*a^7*b^8 - 120*a^9*b^6 + 328*a^11*b^4 + 16*a^13*b^2))/(a^12*b^8) - ((-(a + b)*(a - b))^(1/2)*(a^4 + 6*b^4 + (a^2*b^2)/2)*((4*tan(c/2 + (d*x)/2)*(1728*b^18 - 3888*a^2*b^16 + 3060*a^4*b^14 - 1565*a^6*b^12 + 1658*a^8*b^10 - 1361*a^10*b^8 + 484*a^12*b^6 - 120*a^14*b^4 + 32*a^16*b^2))/(a^12*b^8) - (4*(28*a^15 + 288*a^5*b^10 - 528*a^7*b^8 - 94*a^9*b^6 + 308*a^11*b^4 - 30*a^13*b^2))/(a^12*b^5) + ((-(a + b)*(a - b))^(1/2)*(a^4 + 6*b^4 + (a^2*b^2)/2)*((4*(1152*a^4*b^14 - 1824*a^6*b^12 + 920*a^8*b^10 - 318*a^10*b^8 + 70*a^12*b^6 + 32*a^14*b^4 - 24*a^16*b^2))/(a^12*b^5) + (4*tan(c/2 + (d*x)/2)*(288*a^5*b^16 - 240*a^7*b^14 + 410*a^9*b^12 - 468*a^11*b^10 + 202*a^13*b^8 - 200*a^15*b^6 + 16*a^17*b^4))/(a^12*b^8) + ((-(a + b)*(a - b))^(1/2)*((4*(384*a^9*b^12 - 448*a^11*b^10 + 120*a^13*b^8 - 28*a^15*b^6))/(a^12*b^5) + (4*tan(c/2 + (d*x)/2)*(768*a^8*b^16 - 1136*a^10*b^14 + 484*a^12*b^12 - 120*a^14*b^10 + 32*a^16*b^8))/(a^12*b^8) + ((-(a + b)*(a - b))^(1/2)*((4*(32*a^14*b^10 - 24*a^16*b^8))/(a^12*b^5) + (4*tan(c/2 + (d*x

```
)/2)*(128*a^13*b^14 - 136*a^15*b^12 + 16*a^17*b^10))/(a^12*b^8))*(a^4 + 6*b
^4 + (a^2*b^2)/2))/(a^5*b^3))*(a^4 + 6*b^4 + (a^2*b^2)/2))/(a^5*b^3)))/(a^5
*b^3)))/(a^5*b^3) - ((-(a + b)*(a - b))^(1/2)*(a^4 + 6*b^4 + (a^2*b^2)/2)*(
(4*(28*a^15 + 288*a^5*b^10 - 528*a^7*b^8 - 94*a^9*b^6 + 308*a^11*b^4 - 30*a
^13*b^2))/(a^12*b^5) - (4*tan(c/2 + (d*x)/2)*(1728*b^18 - 3888*a^2*b^16 + 3
060*a^4*b^14 - 1565*a^6*b^12 + 1658*a^8*b^10 - 1361*a^10*b^8 + 484*a^12*b^6
 - 120*a^14*b^4 + 32*a^16*b^2))/(a^12*b^8) + ((-(a + b)*(a - b))^(1/2)*(a^4
 + 6*b^4 + (a^2*b^2)/2)*((4*(1152*a^4*b^14 - 1824*a^6*b^12 + 920*a^8*b^10 -
 318*a^10*b^8 + 70*a^12*b^6 + 32*a^14*b^4 - 24*a^16*b^2))/(a^12*b^5) + (4*t
an(c/2 + (d*x)/2)*(288*a^5*b^16 - 240*a^7*b^14 + 410*a^9*b^12 - 468*a^11*b^
10 + 202*a^13*b^8 - 200*a^15*b^6 + 16*a^17*b^4))/(a^12*b^8) - ((-(a + b)*(a
 - b))^(1/2)*((4*(384*a^9*b^12 - 448*a^11*b^10 + 120*a^13*b^8 - 28*a^15*b^6
))/(a^12*b^5) + (4*tan(c/2 + (d*x)/2)*(768*a^8*b^16 - 1136*a^10*b^14 + 484*
a^12*b^12 - 120*a^14*b^10 + 32*a^16*b^8))/(a^12*b^8) - ((-(a + b)*(a - b))^
(1/2)*((4*(32*a^14*b^10 - 24*a^16*b^8))/(a^12*b^5) + (4*tan(c/2 + (d*x)/2)*
(128*a^13*b^14 - 136*a^15*b^12 + 16*a^17*b^10))/(a^12*b^8))*(a^4 + 6*b^4 +
(a^2*b^2)/2))/(a^5*b^3))*(a^4 + 6*b^4 + (a^2*b^2)/2))/(a^5*b^3)))/(a^5*b^3)
))/(a^5*b^3)))*(-(a + b)*(a - b))^(1/2)*(a^4 + 6*b^4 + (a^2*b^2)/2)*2i)/(a^
5*b^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1272 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=329

$$\frac{5b\cot(c+dx)\csc(c+dx)}{6a^2d(a+b\sin(c+dx))^2}+\frac{5\left(a^2-4b^2\right)\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{5b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d}-$$

```
[Out] -5/2*b*(3*a^2-4*b^2)*arctanh(cos(d*x+c))/a^6/d+1/6*(3*a^4+35*a^2*b^2-60*b^4
)*cot(d*x+c)/a^5/b^2/d-cos(d*x+c)/b/d/(a+b*sin(d*x+c))^2-1/2*a*cot(d*x+c)/b
^2/d/(a+b*sin(d*x+c))^2-1/3*(3*a^2-5*b^2)*cot(d*x+c)/a^3/d/(a+b*sin(d*x+c))
^2+5/6*b*cot(d*x+c)*csc(d*x+c)/a^2/d/(a+b*sin(d*x+c))^2-1/3*cot(d*x+c)*csc(
d*x+c)^2/a/d/(a+b*sin(d*x+c))^2-5/2*(a^2-2*b^2)*cot(d*x+c)/a^4/d/(a+b*sin(d
*x+c))+5*(a^2-4*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-
b^2)^(1/2)/a^6/d
```

Rubi [A] time = 1.33, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{5\left(a^2-4b^2\right)\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{\left(35a^2b^2+3a^4-60b^4\right)\cot(c+dx)}{6a^5b^2d}-\frac{5b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*(a^2 - 4*b^2)*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^6*d) - (5*b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^6*d) + (
(3*a^4 + 35*a^2*b^2 - 60*b^4)*Cot[c + d*x])/(6*a^5*b^2*d) - Cos[c + d*x]/(b
*d*(a + b*Sin[c + d*x])^2) - (a*Cot[c + d*x])/(2*b^2*d*(a + b*Sin[c + d*x])
^2) - ((3*a^2 - 5*b^2)*Cot[c + d*x])/(3*a^3*d*(a + b*Sin[c + d*x])^2) + (5*
b*Cot[c + d*x]*Csc[c + d*x])/(6*a^2*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*
x]*Csc[c + d*x]^2)/(3*a*d*(a + b*Sin[c + d*x])^2) - (5*(a^2 - 2*b^2)*Cot[c
+ d*x])/(2*a^4*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(c+dx)}{2b^2d(a+b\sin(c+dx))^2} + \frac{5b\cot(c+dx)\csc(c+dx}{6a^2d(a+b\sin(c+dx))^2}$$

$$= -\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(c+dx)}{2b^2d(a+b\sin(c+dx))^2} - \frac{\left(3a^2-5b^2\right)\cot(c+dx)}{3a^3d(a+b\sin(c+dx))^2}$$

$$= -\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(c+dx)}{2b^2d(a+b\sin(c+dx))^2} - \frac{\left(3a^2-5b^2\right)\cot(c+dx)}{3a^3d(a+b\sin(c+dx))^2}$$

$$= \frac{\left(3a^4+35a^2b^2-60b^4\right)\cot(c+dx)}{6a^5b^2d} - \frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(c+d}{2b^2d(a+b\sin(c}$$

$$= \frac{\left(3a^4+35a^2b^2-60b^4\right)\cot(c+dx)}{6a^5b^2d} - \frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(c+d}{2b^2d(a+b\sin(c}$$

$$= -\frac{5b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{\left(3a^4+35a^2b^2-60b^4\right)\cot(c+dx)}{6a^5b^2d} - \frac{}{b}$$

$$= -\frac{5b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{\left(3a^4+35a^2b^2-60b^4\right)\cot(c+dx)}{6a^5b^2d} - \frac{}{b}$$

$$= \frac{5\left(a^2-4b^2\right)\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} - \frac{5b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+}{2a^6d}$$

Mathematica [A] time = 6.27, size = 490, normalized size = 1.49

$$\frac{3b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d} - \frac{3b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^3d} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{24a^3d} + \frac{5\,(}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] +
 a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) + ((7*a^2*C
```

```
os[(c + d*x)/2] - 18*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^5*d) + (3
*b*Csc[(c + d*x)/2]^2)/(8*a^4*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(2
4*a^3*d) - (5*(3*a^2*b - 4*b^3)*Log[Cos[(c + d*x)/2]])/(2*a^6*d) + (5*(3*a^
2*b - 4*b^3)*Log[Sin[(c + d*x)/2]])/(2*a^6*d) - (3*b*Sec[(c + d*x)/2]^2)/(8
*a^4*d) + (Sec[(c + d*x)/2]*(-7*a^2*Sin[(c + d*x)/2] + 18*b^2*Sin[(c + d*x)
/2]))/(6*a^5*d) + (-(a^4*Cos[c + d*x]) + 2*a^2*b^2*Cos[c + d*x] - b^4*Cos[c
 + d*x])/(2*a^4*b*d*(a + b*Sin[c + d*x])^2) + (a^4*Cos[c + d*x] + 7*a^2*b^2
*Cos[c + d*x] - 8*b^4*Cos[c + d*x])/(2*a^5*b*d*(a + b*Sin[c + d*x])) + (Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^3*d)
```

fricas [B] time = 1.46, size = 1553, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/12*(2*(3*a^5 + 35*a^3*b^2 - 60*a*b^4)*cos(d*x + c)^5 - 20*(2*a^5 + 5*a^3
*b^2 - 12*a*b^4)*cos(d*x + c)^3 - 15*(2*(a^3*b - 4*a*b^3)*cos(d*x + c)^4 +
2*a^3*b - 8*a*b^3 - 4*(a^3*b - 4*a*b^3)*cos(d*x + c)^2 + ((a^2*b^2 - 4*b^4)
*cos(d*x + c)^4 + a^4 - 3*a^2*b^2 - 4*b^4 - (a^4 - 2*a^2*b^2 - 8*b^4)*cos(d
*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2
 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(
d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
- b^2)) + 30*(a^5 + a^3*b^2 - 4*a*b^4)*cos(d*x + c) - 15*(6*a^3*b^2 - 8*a*b
^4 + 2*(3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^4 - 4*(3*a^3*b^2 - 4*a*b^4)*cos(d
*x + c)^2 + (3*a^4*b - a^2*b^3 - 4*b^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4
 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(
d*x + c) + 1/2) + 15*(6*a^3*b^2 - 8*a*b^4 + 2*(3*a^3*b^2 - 4*a*b^4)*cos(d*x
 + c)^4 - 4*(3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - a^2*b^3 - 4*b
^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos
(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 10*((11*a^4*b - 1
8*a^2*b^3)*cos(d*x + c)^3 - 6*(2*a^4*b - 3*a^2*b^3)*cos(d*x + c))*sin(d*x +
 c))/(2*a^7*b*d*cos(d*x + c)^4 - 4*a^7*b*d*cos(d*x + c)^2 + 2*a^7*b*d + (a^
6*b^2*d*cos(d*x + c)^4 - (a^8 + 2*a^6*b^2)*d*cos(d*x + c)^2 + (a^8 + a^6*b^
2)*d)*sin(d*x + c)), 1/12*(2*(3*a^5 + 35*a^3*b^2 - 60*a*b^4)*cos(d*x + c)^5
 - 20*(2*a^5 + 5*a^3*b^2 - 12*a*b^4)*cos(d*x + c)^3 - 30*(2*(a^3*b - 4*a*b^
3)*cos(d*x + c)^4 + 2*a^3*b - 8*a*b^3 - 4*(a^3*b - 4*a*b^3)*cos(d*x + c)^2
+ ((a^2*b^2 - 4*b^4)*cos(d*x + c)^4 + a^4 - 3*a^2*b^2 - 4*b^4 - (a^4 - 2*a^
2*b^2 - 8*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin
(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(a^5 + a^3*b^2 - 4*a*b^
4)*cos(d*x + c) - 15*(6*a^3*b^2 - 8*a*b^4 + 2*(3*a^3*b^2 - 4*a*b^4)*cos(d*x
 + c)^4 - 4*(3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - a^2*b^3 - 4*b
^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos
```

(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(6*a^3*b^2 - 8*a*b^4 + 2*(3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^4 - 4*(3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - a^2*b^3 - 4*b^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 10*((11*a^4*b - 18*a^2*b^3)*cos(d*x + c)^3 - 6*(2*a^4*b - 3*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(2*a^7*b*d*cos(d*x + c)^4 - 4*a^7*b*d*cos(d*x + c)^2 + 2*a^7*b*d + (a^6*b^2*d*cos(d*x + c)^4 - (a^8 + 2*a^6*b^2)*d*cos(d*x + c)^2 + (a^8 + a^6*b^2)*d)*sin(d*x + c))]

giac [A] time = 0.29, size = 478, normalized size = 1.45

$$\frac{60\,(3\,a^2b-4\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^6}+\frac{120\,(a^4-5\,a^2b^2+4\,b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^6}-\frac{24\left(a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-11\,a^3b^2\tan(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/24*(60*(3*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 + 120*(a^4 - 5*a^2*b^2 + 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) - 24*(a^5*tan(1/2*d*x + 1/2*c)^3 - 11*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 10*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 18*b^5*tan(1/2*d*x + 1/2*c)^2 - a^5*tan(1/2*d*x + 1/2*c) - 25*a^3*b^2*tan(1/2*d*x + 1/2*c) + 26*a*b^4*tan(1/2*d*x + 1/2*c) - 9*a^4*b + 9*a^2*b^3)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^6) + (a^6*tan(1/2*d*x + 1/2*c)^3 - 9*a^5*b*tan(1/2*d*x + 1/2*c)^2 - 27*a^6*tan(1/2*d*x + 1/2*c) + 72*a^4*b^2*tan(1/2*d*x + 1/2*c))/a^9 - (330*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 440*b^3*tan(1/2*d*x + 1/2*c)^3 - 27*a^3*tan(1/2*d*x + 1/2*c)^2 + 72*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^6*tan(1/2*d*x + 1/2*c)^3))/d

maple [B] time = 0.86, size = 873, normalized size = 2.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x)

[Out] 1/24/d/a^3*tan(1/2*d*x+1/2*c)^3-3/8/d/a^4*tan(1/2*d*x+1/2*c)^2*b-9/8/d/a^3*tan(1/2*d*x+1/2*c)+3/d/a^5*b^2*tan(1/2*d*x+1/2*c)-1/24/d/a^3/tan(1/2*d*x+1/2*c)^3+9/8/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^5/tan(1/2*d*x+1/2*c)*b^2+3/8/d/a^

```
4*b/tan(1/2*d*x+1/2*c)^2+15/2/d/a^4*b*ln(tan(1/2*d*x+1/2*c))-10/d/a^6*b^3*l
n(tan(1/2*d*x+1/2*c))-1/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2*tan(1/2*d*x+1/2*c)^3+11/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^2-10/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan
(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^4+9/d/a^2/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b+9/d/a^4/(tan(1/2*d*
x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^3-18/d/a^6/
(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^
5+1/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2
*c)+25/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*
x+1/2*c)*b^2-26/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*t
an(1/2*d*x+1/2*c)*b^4+9/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)^2*b-9/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3+5/
d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/
2))-25/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))*b^2+20/d/a^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+
2*b)/(a^2-b^2)^(1/2))*b^4
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 12.41, size = 1082, normalized size = 3.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^4*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^3*d) + (tan(c/2 + (d*x)/2)^4*((77*a^4)/3 - 304*b
^4 + 200*a^2*b^2) + tan(c/2 + (d*x)/2)^6*(a^4 - 80*b^4 + 64*a^2*b^2) - a^4/
3 + tan(c/2 + (d*x)/2)^2*((25*a^4)/3 - (40*a^2*b^2)/3) - tan(c/2 + (d*x)/2)
^3*(156*a*b^3 - (338*a^3*b)/3) - (3*tan(c/2 + (d*x)/2)^5*(48*b^5 - 37*a^4*b
 + 8*a^2*b^3))/a + (5*a^3*b*tan(c/2 + (d*x)/2))/3)/(d*(8*a^7*tan(c/2 + (d*x
)/2)^3 + 8*a^7*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^5*(16*a^7 + 32*a^5
*b^2) + 32*a^6*b*tan(c/2 + (d*x)/2)^4 + 32*a^6*b*tan(c/2 + (d*x)/2)^6)) - (
```

```
tan(c/2 + (d*x)/2)*((3*(a^2 + 4*b^2))/(8*a^5) + 3/(4*a^3) - (9*b^2)/(2*a^5)
))/d + (log(tan(c/2 + (d*x)/2))*(15*a^2*b - 20*b^3))/(2*a^6*d) - (3*b*tan(c
/2 + (d*x)/2)^2)/(8*a^4*d) + (atan((((b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*((5*a^
10 + 40*a^6*b^4 - 40*a^8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(25*a^8*b + 80*a^4
*b^5 - 100*a^6*b^3))/a^9 - (5*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^
10*b^2))/a^9)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/(2*a^6))*5i)/(2*a^6) + ((b^2
 - a^2)^(1/2)*(a^2 - 4*b^2)*((5*a^10 + 40*a^6*b^4 - 40*a^8*b^2)/a^10 + (tan
(c/2 + (d*x)/2)*(25*a^8*b + 80*a^4*b^5 - 100*a^6*b^3))/a^9 + (5*(2*a^2*b -
(tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*(b^2 - a^2)^(1/2)*(a^2 - 4*
b^2))/(2*a^6))*5i)/(2*a^6))/((75*a^6*b - 400*b^7 + 800*a^2*b^5 - 475*a^4*b^
3)/a^10 + (2*tan(c/2 + (d*x)/2)*(25*a^6 - 200*b^6 + 350*a^2*b^4 - 175*a^4*b
^2))/a^9 + (5*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*((5*a^10 + 40*a^6*b^4 - 40*a^
8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(25*a^8*b + 80*a^4*b^5 - 100*a^6*b^3))/a^
9 - (5*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*(b^2 - a^
2)^(1/2)*(a^2 - 4*b^2))/(2*a^6)))/(2*a^6) - (5*(b^2 - a^2)^(1/2)*(a^2 - 4*b
^2)*((5*a^10 + 40*a^6*b^4 - 40*a^8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(25*a^8*
b + 80*a^4*b^5 - 100*a^6*b^3))/a^9 + (5*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a
^12 - 8*a^10*b^2))/a^9)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/(2*a^6)))/(2*a^6))
)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*5i)/(a^6*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1273 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=355

$$\frac{b\cot(c+dx)\csc^2(c+dx)}{2a^2d(a+b\sin(c+dx))^2}-\frac{15b\left(a^2-2b^2\right)\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}+\frac{15\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+a}{8a^5d}$$

```
[Out] -15/8*(a^4-8*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^7/d+1/2*(a^4-25*a^2*b^2+3
0*b^4)*cot(d*x+c)/a^6/b/d+15/8*(3*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/a^5/d-1/
2*cot(d*x+c)/b/d/(a+b*sin(d*x+c))^2-1/4*(4*a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)
/a^3/d/(a+b*sin(d*x+c))^2+1/2*b*cot(d*x+c)*csc(d*x+c)^2/a^2/d/(a+b*sin(d*x+
c))^2-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d/(a+b*sin(d*x+c))^2-1/2*(7*a^2-10*b^2)
*cot(d*x+c)*csc(d*x+c)/a^4/d/(a+b*sin(d*x+c))-15*b*(a^2-2*b^2)*arctan((b+a*
tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^7/d
```

Rubi [A] time = 1.68, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{15b\left(a^2-2b^2\right)\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}+\frac{\left(-25a^2b^2+a^4+30b^4\right)\cot(c+dx)}{2a^6bd}-\frac{15\left(-8a^2b^2+a^4+8b^4\right)t}{8a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-15*b*(a^2 - 2*b^2)*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]])/(a^7*d) - (15*(a^4 - 8*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/
(8*a^7*d) + ((a^4 - 25*a^2*b^2 + 30*b^4)*Cot[c + d*x])/(2*a^6*b*d) + (15*(3
*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^5*d) - Cot[c + d*x]/(2*b*d*(a
 + b*Sin[c + d*x])^2) - ((4*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x])/(4*a^3*
d*(a + b*Sin[c + d*x])^2) + (b*Cot[c + d*x]*Csc[c + d*x]^2)/(2*a^2*d*(a + b
*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d*(a + b*Sin[c + d*x
])^2) - ((7*a^2 - 10*b^2)*Cot[c + d*x]*Csc[c + d*x])/(2*a^4*d*(a + b*Sin[c
+ d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cot(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{b\cot(c+dx)\csc^2(c+dx)}{2a^2d(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^3(c+dx}{4ad(a+b\sin(c+dx))^2}$$

$$= -\frac{\cot(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\left(4a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{4a^3d(a+b\sin(c+dx))^2} + \frac{b\cot(c+dx)}{2a^2d(a+b\sin}$$

$$= -\frac{\cot(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\left(4a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{4a^3d(a+b\sin(c+dx))^2} + \frac{b\cot(c+dx)}{2a^2d(a+b\sin}$$

$$= \frac{15\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{\cot(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\left(4a^2-5b^2\right)}{4a^3d(a}$$

$$= \frac{\left(a^4-25a^2b^2+30b^4\right)\cot(c+dx)}{2a^6bd} + \frac{15\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{}{2b}$$

$$= \frac{\left(a^4-25a^2b^2+30b^4\right)\cot(c+dx)}{2a^6bd} + \frac{15\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{}{2b}$$

$$= -\frac{15\left(a^4-8a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^7d} + \frac{\left(a^4-25a^2b^2+30b^4\right)\cot(c+dx}{2a^6bd}$$

$$= -\frac{15\left(a^4-8a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^7d} + \frac{\left(a^4-25a^2b^2+30b^4\right)\cot(c+dx}{2a^6bd}$$

$$= -\frac{15b\left(a^2-2b^2\right)\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d} - \frac{15\left(a^4-8a^2b^2+8b^4\right)\tanh^{-}}{8a^7d}$$

Mathematica [A] time = 1.39, size = 363, normalized size = 1.02

$$\frac{-\frac{1920b\left(a^4-3a^2b^2+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 240\left(a^4-8a^2b^2+8b^4\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 240\left(a^4-8a^2b^2+8b^4\right)\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-1920*b*(a^4 - 3*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/Sqrt[a^2 - b^2] - 240*(a^4 - 8*a^2*b^2 + 8*b^4)*Log[Cos[(c + d*x
)/2]] + 240*(a^4 - 8*a^2*b^2 + 8*b^4)*Log[Sin[(c + d*x)/2]] + (2*a*Cot[c +
d*x]*Csc[c + d*x]^5*(44*a^5 - 505*a^3*b^2 + 540*a*b^4 + (-68*a^5 + 660*a^3*
b^2 - 720*a*b^4)*Cos[2*(c + d*x)] + (8*a^5 - 155*a^3*b^2 + 180*a*b^4)*Cos[4
*(c + d*x)] - 176*a^4*b*Sin[c + d*x] - 260*a^2*b^3*Sin[c + d*x] + 600*b^5*S
in[c + d*x] + 66*a^4*b*Sin[3*(c + d*x)] + 170*a^2*b^3*Sin[3*(c + d*x)] - 30
0*b^5*Sin[3*(c + d*x)] + 2*a^4*b*Sin[5*(c + d*x)] - 50*a^2*b^3*Sin[5*(c + d
*x)] + 60*b^5*Sin[5*(c + d*x)]))/(b + a*Csc[c + d*x])^2)/(128*a^7*d)
```

fricas [B] time = 1.30, size = 2022, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/16*(2*(8*a^6 - 155*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 10*(5*a^6 -
64*a^4*b^2 + 72*a^2*b^4)*cos(d*x + c)^3 + 60*((a^2*b^3 - 2*b^5)*cos(d*x + c
)^6 - a^4*b + a^2*b^3 + 2*b^5 - (a^4*b + a^2*b^3 - 6*b^5)*cos(d*x + c)^4 +
(2*a^4*b - a^2*b^3 - 6*b^5)*cos(d*x + c)^2 - 2*(a^3*b^2 - 2*a*b^4 + (a^3*b^
2 - 2*a*b^4)*cos(d*x + c)^4 - 2*(a^3*b^2 - 2*a*b^4)*cos(d*x + c)^2)*sin(d*x
 + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
+ c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a
^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 30*(a^6
 - 11*a^4*b^2 + 12*a^2*b^4)*cos(d*x + c) + 15*((a^4*b^2 - 8*a^2*b^4 + 8*b^6
)*cos(d*x + c)^6 - a^6 + 7*a^4*b^2 - 8*b^6 - (a^6 - 5*a^4*b^2 - 16*a^2*b^4
+ 24*b^6)*cos(d*x + c)^4 + (2*a^6 - 13*a^4*b^2 - 8*a^2*b^4 + 24*b^6)*cos(d*
x + c)^2 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5 + (a^5*b - 8*a^3*b^3 + 8*a*b^5)*c
os(d*x + c)^4 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c)^2)*sin(d*x + c
))*log(1/2*cos(d*x + c) + 1/2) - 15*((a^4*b^2 - 8*a^2*b^4 + 8*b^6)*cos(d*x
+ c)^6 - a^6 + 7*a^4*b^2 - 8*b^6 - (a^6 - 5*a^4*b^2 - 16*a^2*b^4 + 24*b^6)*
cos(d*x + c)^4 + (2*a^6 - 13*a^4*b^2 - 8*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 -
 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5 + (a^5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c
)^4 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/
2*cos(d*x + c) + 1/2) + 4*(2*(a^5*b - 25*a^3*b^3 + 30*a*b^5)*cos(d*x + c)^5
 + 5*(3*a^5*b + 16*a^3*b^3 - 24*a*b^5)*cos(d*x + c)^3 - 15*(a^5*b + 2*a^3*b
^3 - 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^7*b^2*d*cos(d*x + c)^6 - (a^9
+ 3*a^7*b^2)*d*cos(d*x + c)^4 + (2*a^9 + 3*a^7*b^2)*d*cos(d*x + c)^2 - (a^9
 + a^7*b^2)*d - 2*(a^8*b*d*cos(d*x + c)^4 - 2*a^8*b*d*cos(d*x + c)^2 + a^8*
b*d)*sin(d*x + c)), -1/16*(2*(8*a^6 - 155*a^4*b^2 + 180*a^2*b^4)*cos(d*x +
c)^5 - 10*(5*a^6 - 64*a^4*b^2 + 72*a^2*b^4)*cos(d*x + c)^3 - 120*((a^2*b^3
```

```
- 2*b^5)*cos(d*x + c)^6 - a^4*b + a^2*b^3 + 2*b^5 - (a^4*b + a^2*b^3 - 6*b^
5)*cos(d*x + c)^4 + (2*a^4*b - a^2*b^3 - 6*b^5)*cos(d*x + c)^2 - 2*(a^3*b^2
 - 2*a*b^4 + (a^3*b^2 - 2*a*b^4)*cos(d*x + c)^4 - 2*(a^3*b^2 - 2*a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sq
rt(a^2 - b^2)*cos(d*x + c))) + 30*(a^6 - 11*a^4*b^2 + 12*a^2*b^4)*cos(d*x +
 c) + 15*((a^4*b^2 - 8*a^2*b^4 + 8*b^6)*cos(d*x + c)^6 - a^6 + 7*a^4*b^2 -
8*b^6 - (a^6 - 5*a^4*b^2 - 16*a^2*b^4 + 24*b^6)*cos(d*x + c)^4 + (2*a^6 - 1
3*a^4*b^2 - 8*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 8*a^3*b^3 + 8*a
*b^5 + (a^5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c)^4 - 2*(a^5*b - 8*a^3*b^3
+ 8*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(
(a^4*b^2 - 8*a^2*b^4 + 8*b^6)*cos(d*x + c)^6 - a^6 + 7*a^4*b^2 - 8*b^6 - (a
^6 - 5*a^4*b^2 - 16*a^2*b^4 + 24*b^6)*cos(d*x + c)^4 + (2*a^6 - 13*a^4*b^2
- 8*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5 + (a^
5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c)^4 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5)
*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(2*(a^5*b -
 25*a^3*b^3 + 30*a*b^5)*cos(d*x + c)^5 + 5*(3*a^5*b + 16*a^3*b^3 - 24*a*b^5
)*cos(d*x + c)^3 - 15*(a^5*b + 2*a^3*b^3 - 4*a*b^5)*cos(d*x + c))*sin(d*x +
 c))/(a^7*b^2*d*cos(d*x + c)^6 - (a^9 + 3*a^7*b^2)*d*cos(d*x + c)^4 + (2*a^
9 + 3*a^7*b^2)*d*cos(d*x + c)^2 - (a^9 + a^7*b^2)*d - 2*(a^8*b*d*cos(d*x +
c)^4 - 2*a^8*b*d*cos(d*x + c)^2 + a^8*b*d)*sin(d*x + c))]
```

giac [A] time = 0.34, size = 603, normalized size = 1.70

$$\frac{120\left(a^4-8a^2b^2+8b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^7}-\frac{960\left(a^4b-3a^2b^3+2b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^7}+\frac{64\left(3a^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(120*(a^4 - 8*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^7 - 96
0*(a^4*b - 3*a^2*b^3 + 2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + ar
ctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^7) +
 64*(3*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 1
2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 2*a^6*tan(1/2*d*x + 1/2*c)^2 - 9*a^4*b^2*t
an(1/2*d*x + 1/2*c)^2 - 15*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 + 22*b^6*tan(1/2*
d*x + 1/2*c)^2 + 5*a^5*b*tan(1/2*d*x + 1/2*c) - 37*a^3*b^3*tan(1/2*d*x + 1/
2*c) + 32*a*b^5*tan(1/2*d*x + 1/2*c) + 2*a^6 - 13*a^4*b^2 + 11*a^2*b^4)/((a
*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^7) - (250*a^4*t
an(1/2*d*x + 1/2*c)^4 - 2000*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 2000*b^4*tan(
1/2*d*x + 1/2*c)^4 + 216*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 320*a*b^3*tan(1/2*d
*x + 1/2*c)^3 - 16*a^4*tan(1/2*d*x + 1/2*c)^2 + 48*a^2*b^2*tan(1/2*d*x + 1/
2*c)^2 - 8*a^3*b*tan(1/2*d*x + 1/2*c) + a^4)/(a^7*tan(1/2*d*x + 1/2*c)^4) +
```

```
 (a^9*tan(1/2*d*x + 1/2*c)^4 - 8*a^8*b*tan(1/2*d*x + 1/2*c)^3 - 16*a^9*tan(
1/2*d*x + 1/2*c)^2 + 48*a^7*b^2*tan(1/2*d*x + 1/2*c)^2 + 216*a^8*b*tan(1/2*
d*x + 1/2*c) - 320*a^6*b^3*tan(1/2*d*x + 1/2*c))/a^12)/d
```

maple [B] time = 0.91, size = 1070, normalized size = 3.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -5/d/a^6*b^3*tan(1/2*d*x+1/2*c)-3/4/d/a^5/tan(1/2*d*x+1/2*c)^2*b^2+15/d/a^7
*ln(tan(1/2*d*x+1/2*c))*b^4+1/8/d/a^4*b/tan(1/2*d*x+1/2*c)^3+1/64/d/a^3*tan
(1/2*d*x+1/2*c)^4-1/64/d/a^3/tan(1/2*d*x+1/2*c)^4+2/d/a/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^2+27/8/d/a^4*tan(1/2*d*x+1/2*c)*b-15/d/a
^5*ln(tan(1/2*d*x+1/2*c))*b^2-27/8/d*b/a^4/tan(1/2*d*x+1/2*c)-13/d/a^3*b^2/
(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-1/4/d/a^3*tan(1/2*d*x+1
/2*c)^2+1/4/d/a^3/tan(1/2*d*x+1/2*c)^2-15/d/a^3*b/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+3/d/a^2*b/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+5/d*b^3/a^6/tan(1/2
*d*x+1/2*c)-1/8/d/a^4*b*tan(1/2*d*x+1/2*c)^3+11/d/a^5/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^4+15/8/d/a^3*ln(tan(1/2*d*x+1/2*c))-9/d/a^
3*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c
)^2+5/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d
*x+1/2*c)+3/4/d/a^5*b^2*tan(1/2*d*x+1/2*c)^2-15/d/a^4*b^3/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-15/d/a^5*b^4/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-37/d/a^4
*b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)
+32/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1
/2*c)*b^5+22/d/a^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(
1/2*d*x+1/2*c)^2*b^6-30/d/a^7*b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+12/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^5+45/d/a^5*b^3/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.55, size = 1275, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^5*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^4/(64*a^3*d) - (tan(c/2 + (d*x)/2)^3*(35*a^4*b - 40*a^2*
b^3) - tan(c/2 + (d*x)/2)^4*(448*a*b^4 + (159*a^5)/4 - 424*a^3*b^2) + tan(c
/2 + (d*x)/2)^7*(6*a^4*b - 192*b^5 + 160*a^2*b^3) + tan(c/2 + (d*x)/2)^5*(1
0*a^4*b - 832*b^5 + 696*a^2*b^3) + a^5/4 - tan(c/2 + (d*x)/2)^2*((7*a^5)/2
- 5*a^3*b^2) - a^4*b*tan(c/2 + (d*x)/2) - (4*tan(c/2 + (d*x)/2)^6*(9*a^6 +
88*b^6 + 20*a^2*b^4 - 93*a^4*b^2))/a)/(d*(16*a^8*tan(c/2 + (d*x)/2)^4 + 16*
a^8*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^6*(32*a^8 + 64*a^6*b^2) + 64*
a^7*b*tan(c/2 + (d*x)/2)^5 + 64*a^7*b*tan(c/2 + (d*x)/2)^7)) - (tan(c/2 + (
d*x)/2)^2*((3*(a^2 + 4*b^2))/(32*a^5) + 5/(32*a^3) - (9*b^2)/(8*a^5)))/d +
(tan(c/2 + (d*x)/2)*((6*b*((3*(a^2 + 4*b^2))/(16*a^5) + 5/(16*a^3) - (9*b^2
)/(4*a^5)))/a - (192*a^2*b + 128*b^3)/(256*a^6) + (9*b*(a^2 + 4*b^2))/(8*a^
6)))/d - (b*tan(c/2 + (d*x)/2)^3)/(8*a^4*d) + (log(tan(c/2 + (d*x)/2))*((15
*a^4)/8 + 15*b^4 - 15*a^2*b^2))/(a^7*d) + (b*atan(((b*(b^2 - a^2)^(1/2)*(a^
2 - 2*b^2)*(((75*a^11*b)/4 + 60*a^7*b^5 - 75*a^9*b^3)/a^12 - (tan(c/2 + (d*
x)/2)*(15*a^11 - 480*a^5*b^6 + 720*a^7*b^4 - 270*a^9*b^2))/(4*a^11) + (15*b
*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^2))/(4*a^11))*(b^2 - a
^2)^(1/2)*(a^2 - 2*b^2))/(2*a^7))*15i)/(2*a^7) - (b*(b^2 - a^2)^(1/2)*(a^2
- 2*b^2)*((tan(c/2 + (d*x)/2)*(15*a^11 - 480*a^5*b^6 + 720*a^7*b^4 - 270*a^
9*b^2))/(4*a^11) - ((75*a^11*b)/4 + 60*a^7*b^5 - 75*a^9*b^3)/a^12 + (15*b*(
2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^2))/(4*a^11))*(b^2 - a^2
)^(1/2)*(a^2 - 2*b^2))/(2*a^7))*15i)/(2*a^7))/(((225*a^8*b)/4 + 900*b^9 - 2
250*a^2*b^7 + (3825*a^4*b^5)/2 - (2475*a^6*b^3)/4)/a^12 + (tan(c/2 + (d*x)/
2)*(1800*b^8 - 4050*a^2*b^6 + 2925*a^4*b^4 - 675*a^6*b^2))/(2*a^11) + (15*b
*(b^2 - a^2)^(1/2)*(a^2 - 2*b^2)*(((75*a^11*b)/4 + 60*a^7*b^5 - 75*a^9*b^3)
/a^12 - (tan(c/2 + (d*x)/2)*(15*a^11 - 480*a^5*b^6 + 720*a^7*b^4 - 270*a^9*
b^2))/(4*a^11) + (15*b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^
2))/(4*a^11))*(b^2 - a^2)^(1/2)*(a^2 - 2*b^2))/(2*a^7)))/(2*a^7) + (15*b*(b
^2 - a^2)^(1/2)*(a^2 - 2*b^2)*((tan(c/2 + (d*x)/2)*(15*a^11 - 480*a^5*b^6 +
 720*a^7*b^4 - 270*a^9*b^2))/(4*a^11) - ((75*a^11*b)/4 + 60*a^7*b^5 - 75*a^
9*b^3)/a^12 + (15*b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^2))
/(4*a^11))*(b^2 - a^2)^(1/2)*(a^2 - 2*b^2))/(2*a^7)))/(2*a^7)))*(b^2 - a^2)
^(1/2)*(a^2 - 2*b^2)*15i)/(a^7*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1274 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=492

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{\left(4a^4-54a^2b^2+63b^4\right)\cot(c+dx)\csc^2(c+dx)}{12a^4b^2d(a+b\sin(c+dx))}-\frac{\sqrt{a^2-b^2}\left(2a^4-29a^2b^2+42b^4\right)\tan}{a^8d}$$

```
[Out] 1/8*b*(45*a^4-200*a^2*b^2+168*b^4)*arctanh(cos(d*x+c))/a^8/d-1/30*(91*a^4-6
45*a^2*b^2+630*b^4)*cot(d*x+c)/a^7/d+1/8*(8*a^4-79*a^2*b^2+84*b^4)*cot(d*x+
c)*csc(d*x+c)/a^6/b/d-1/30*(15*a^4-187*a^2*b^2+210*b^4)*cot(d*x+c)*csc(d*x+
c)^2/a^5/b^2/d-1/3*cot(d*x+c)*csc(d*x+c)/b/d/(a+b*sin(d*x+c))^2+1/12*a*cot(
d*x+c)*csc(d*x+c)^2/b^2/d/(a+b*sin(d*x+c))^2+1/60*(5*a^4-60*a^2*b^2+63*b^4)
*cot(d*x+c)*csc(d*x+c)^2/a^3/b^2/d/(a+b*sin(d*x+c))^2+7/20*b*cot(d*x+c)*csc
(d*x+c)^3/a^2/d/(a+b*sin(d*x+c))^2-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d/(a+b*sin
(d*x+c))^2+1/12*(4*a^4-54*a^2*b^2+63*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^4/b^2/d
/(a+b*sin(d*x+c))-(2*a^4-29*a^2*b^2+42*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))
/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^8/d
```

Rubi [A] time = 2.16, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{\sqrt{a^2-b^2}\left(-29a^2b^2+2a^4+42b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}-\frac{\left(-645a^2b^2+91a^4+630b^4\right)\cot(c+dx)}{30a^7d}+\frac{b\left(-200a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((Sqrt[a^2 - b^2]*(2*a^4 - 29*a^2*b^2 + 42*b^4)*ArcTan[(b + a*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/(a^8*d)) + (b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*ArcT
anh[Cos[c + d*x]])/(8*a^8*d) - ((91*a^4 - 645*a^2*b^2 + 630*b^4)*Cot[c + d*
x])/(30*a^7*d) + ((8*a^4 - 79*a^2*b^2 + 84*b^4)*Cot[c + d*x]*Csc[c + d*x])/
(8*a^6*b*d) - ((15*a^4 - 187*a^2*b^2 + 210*b^4)*Cot[c + d*x]*Csc[c + d*x]^2
)/(30*a^5*b^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(3*b*d*(a + b*Sin[c + d*x])^
2) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(12*b^2*d*(a + b*Sin[c + d*x])^2) + ((
5*a^4 - 60*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(60*a^3*b^2*d*(a
+ b*Sin[c + d*x])^2) + (7*b*Cot[c + d*x]*Csc[c + d*x]^3)/(20*a^2*d*(a + b*S
in[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x])
^2) + ((4*a^4 - 54*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(12*a^4*b
^2*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e
 + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^
m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*
m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(
m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x])/Sin[e + f*x]^4,
x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*
x]^2), x] + Simp[(a*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*m*(m
- 1)*Sin[e + f*x]^3), x] - Simp[(b*(m - 4)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1))/(20*a^2*f*Sin[e + f*x]^4), x]) /; FreeQ[{a, b, e, f, m}, x] && N
eQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{7b\cot(c+dx)\csc^3(c+dx}{20a^2d(a+b\sin(c+dx))^2}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{\left(5a^4-60a^2b^2+63b^4\right)\cot}{60a^3b^2d(a+b\,\mathrm{si}}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{\left(5a^4-60a^2b^2+63b^4\right)\cot}{60a^3b^2d(a+b\,\mathrm{si}}$$

$$= -\frac{\left(15a^4-187a^2b^2+210b^4\right)\cot(c+dx)\csc^2(c+dx)}{30a^5b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a}{1}$$

$$= \frac{\left(8a^4-79a^2b^2+84b^4\right)\cot(c+dx)\csc(c+dx)}{8a^6bd} - \frac{\left(15a^4-187a^2b^2+210b^4\right)\cot(c+}{30a^5b^2d}$$

$$= -\frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c+dx)}{30a^7d} + \frac{\left(8a^4-79a^2b^2+84b^4\right)\cot(c+dx)\csc(c}{8a^6bd}$$

$$= -\frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c+dx)}{30a^7d} + \frac{\left(8a^4-79a^2b^2+84b^4\right)\cot(c+dx)\csc(c}{8a^6bd}$$

$$= \frac{b\left(45a^4-200a^2b^2+168b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^8d} - \frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c}{30a^7d}$$

$$= \frac{b\left(45a^4-200a^2b^2+168b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^8d} - \frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c}{30a^7d}$$

$$= -\frac{\sqrt{a^2-b^2}\,\left(2a^4-29a^2b^2+42b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d} + \frac{b\left(45a^4-200a^2b^2+16\right.}{8}$$

Mathematica [A] time = 1.79, size = 448, normalized size = 0.91

$$\frac{-480b\left(45a^4-200a^2b^2+168b^4\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+480b\left(45a^4-200a^2b^2+168b^4\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-3840*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 480*b*(45*a^4 - 200*a^2*b^2 + 1
68*b^4)*Log[Cos[(c + d*x)/2]] - 480*b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*Log[
Sin[(c + d*x)/2]] + (2*a*Cot[c + d*x]*Csc[c + d*x]^6*(-784*a^6 + 3256*a^4*b
^2 + 7860*a^2*b^4 - 12600*b^6 + 2*(384*a^6 - 2131*a^4*b^2 - 6315*a^2*b^4 +
9450*b^6)*Cos[2*(c + d*x)] + (-368*a^6 + 824*a^4*b^2 + 6060*a^2*b^4 - 7560*
b^6)*Cos[4*(c + d*x)] + 182*a^4*b^2*Cos[6*(c + d*x)] - 1290*a^2*b^4*Cos[6*(
c + d*x)] + 1260*b^6*Cos[6*(c + d*x)] - 8156*a^5*b*Sin[c + d*x] + 42270*a^3
*b^3*Sin[c + d*x] - 37800*a*b^5*Sin[c + d*x] + 3956*a^5*b*Sin[3*(c + d*x)]
- 20715*a^3*b^3*Sin[3*(c + d*x)] + 18900*a*b^5*Sin[3*(c + d*x)] - 608*a^5*b
*Sin[5*(c + d*x)] + 3975*a^3*b^3*Sin[5*(c + d*x)] - 3780*a*b^5*Sin[5*(c + d
*x)]))/(b + a*Csc[c + d*x])^2)/(3840*a^8*d)
```

fricas [B] time = 2.20, size = 2571, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/240*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*cos(d*x + c)^7 - 4*(92*a^
7 + 67*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*cos(d*x + c)^5 + 40*(14*a^7 - 3
7*a^5*b^2 - 303*a^3*b^4 + 378*a*b^6)*cos(d*x + c)^3 - 60*(2*(2*a^5*b - 29*a
^3*b^3 + 42*a*b^5)*cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6*(2*
a^5*b - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^3 + 4
2*a*b^5)*cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*cos(d*x + c)^6
 - 2*a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 - 45*a^2*
b^4 + 126*b^6)*cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 126*b^6)*
cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x +
 c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b
*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2)) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6)*cos(d*x + c)
 + 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 +
 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d
*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a
^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168
*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(
d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^
2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(90*a^5*b^2 - 400*a^3*b^4
 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*
```

```
(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200
*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5
 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*
b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^
4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2) - 2*((608*a^6*b - 3975*a^4*b^3 + 3780*a^2*b^5)*cos(d*x + c)^
5 - 5*(289*a^6*b - 1632*a^4*b^3 + 1512*a^2*b^5)*cos(d*x + c)^3 + 15*(53*a^6
*b - 279*a^4*b^3 + 252*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(2*a^9*b*d*cos(
d*x + c)^6 - 6*a^9*b*d*cos(d*x + c)^4 + 6*a^9*b*d*cos(d*x + c)^2 - 2*a^9*b*
d + (a^8*b^2*d*cos(d*x + c)^6 - (a^10 + 3*a^8*b^2)*d*cos(d*x + c)^4 + (2*a^
10 + 3*a^8*b^2)*d*cos(d*x + c)^2 - (a^10 + a^8*b^2)*d)*sin(d*x + c)), -1/24
0*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*cos(d*x + c)^7 - 4*(92*a^7 + 67
*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*cos(d*x + c)^5 + 40*(14*a^7 - 37*a^5*
b^2 - 303*a^3*b^4 + 378*a*b^6)*cos(d*x + c)^3 - 120*(2*(2*a^5*b - 29*a^3*b^
3 + 42*a*b^5)*cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6*(2*a^5*b
 - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^3 + 42*a*b
^5)*cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*cos(d*x + c)^6 - 2*
a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 - 45*a^2*b^4 +
 126*b^6)*cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 126*b^6)*cos(d
*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt
(a^2 - b^2)*cos(d*x + c))) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6
)*cos(d*x + c) + 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 -
 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 16
8*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x
+ c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200
*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 +
 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)
*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(90*a^5*b^2
 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d
*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45
*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^
3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c
)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*
a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*
log(-1/2*cos(d*x + c) + 1/2) - 2*((608*a^6*b - 3975*a^4*b^3 + 3780*a^2*b^5)
*cos(d*x + c)^5 - 5*(289*a^6*b - 1632*a^4*b^3 + 1512*a^2*b^5)*cos(d*x + c)^
3 + 15*(53*a^6*b - 279*a^4*b^3 + 252*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(
2*a^9*b*d*cos(d*x + c)^6 - 6*a^9*b*d*cos(d*x + c)^4 + 6*a^9*b*d*cos(d*x + c
)^2 - 2*a^9*b*d + (a^8*b^2*d*cos(d*x + c)^6 - (a^10 + 3*a^8*b^2)*d*cos(d*x
+ c)^4 + (2*a^10 + 3*a^8*b^2)*d*cos(d*x + c)^2 - (a^10 + a^8*b^2)*d)*sin(d*
x + c))]
```

giac [A] time = 0.34, size = 731, normalized size = 1.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/960*(120*(45*a^4*b - 200*a^2*b^3 + 168*b^5)*log(abs(tan(1/2*d*x + 1/2*c)
))/a^8 + 960*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*a^8) + 960*(5*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 19*a^3*b
^4*tan(1/2*d*x + 1/2*c)^3 + 14*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*b*tan(1
/2*d*x + 1/2*c)^2 - 9*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 - 21*a^2*b^5*tan(1/2*d
*x + 1/2*c)^2 + 26*b^7*tan(1/2*d*x + 1/2*c)^2 + 11*a^5*b^2*tan(1/2*d*x + 1/
2*c) - 49*a^3*b^4*tan(1/2*d*x + 1/2*c) + 38*a*b^6*tan(1/2*d*x + 1/2*c) + 4*
a^6*b - 17*a^4*b^3 + 13*a^2*b^5)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d
*x + 1/2*c) + a)^2*a^8) - (12330*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 54800*a^2*b
^3*tan(1/2*d*x + 1/2*c)^5 + 46032*b^5*tan(1/2*d*x + 1/2*c)^5 - 660*a^5*tan(
1/2*d*x + 1/2*c)^4 + 6480*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 7200*a*b^4*tan(1
/2*d*x + 1/2*c)^4 - 720*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 1200*a^2*b^3*tan(1/2
*d*x + 1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 240*a^3*b^2*tan(1/2*d*x +
 1/2*c)^2 + 45*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^8*tan(1/2*d*x + 1/2*c
)^5) - (6*a^12*tan(1/2*d*x + 1/2*c)^5 - 45*a^11*b*tan(1/2*d*x + 1/2*c)^4 -
70*a^12*tan(1/2*d*x + 1/2*c)^3 + 240*a^10*b^2*tan(1/2*d*x + 1/2*c)^3 + 720*
a^11*b*tan(1/2*d*x + 1/2*c)^2 - 1200*a^9*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a
^12*tan(1/2*d*x + 1/2*c) - 6480*a^10*b^2*tan(1/2*d*x + 1/2*c) + 7200*a^8*b^
4*tan(1/2*d*x + 1/2*c))/a^15)/d

maple [B] time = 0.90, size = 1252, normalized size = 2.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^3,x)

[Out] 11/16/d/a^3*tan(1/2*d*x+1/2*c)-27/4/d/a^5*b^2*tan(1/2*d*x+1/2*c)-7/96/d/a^3
*tan(1/2*d*x+1/2*c)^3+7/96/d/a^3/tan(1/2*d*x+1/2*c)^3+1/160/d/a^3*tan(1/2*d
*x+1/2*c)^5-1/160/d/a^3/tan(1/2*d*x+1/2*c)^5+17/d/a^4/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3-11/16/d/a^3/tan(1/2*d*x+1/2*c)-45/8/d/a^
4*b*ln(tan(1/2*d*x+1/2*c))-4/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*b-2/d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*
b)/(a^2-b^2)^(1/2))+5/4/d/a^6*b^3/tan(1/2*d*x+1/2*c)^2-21/d/a^8*b^5*ln(tan(
1/2*d*x+1/2*c))-15/2/d/a^7/tan(1/2*d*x+1/2*c)*b^4+3/64/d/a^4*b/tan(1/2*d*x+
1/2*c)^4-1/4/d/a^5/tan(1/2*d*x+1/2*c)^3*b^2-5/4/d/a^6*tan(1/2*d*x+1/2*c)^2*
b^3+15/2/d/a^7*b^4*tan(1/2*d*x+1/2*c)-13/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*ta
n(1/2*d*x+1/2*c)*b+a)^2*b^5-3/64/d/a^4*tan(1/2*d*x+1/2*c)^4*b+1/4/d/a^5*tan
(1/2*d*x+1/2*c)^3*b^2+25/d/a^6*b^3*ln(tan(1/2*d*x+1/2*c))-3/4/d/a^4*b/tan(1
/2*d*x+1/2*c)^2+19/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^

```
2*tan(1/2*d*x+1/2*c)^3*b^4+21/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^5+49/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*ta
n(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^4-71/d/a^6/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4-5/d/a^3/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^2-4/d/a^2
/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b
+9/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/
2*c)^2*b^3-11/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan
(1/2*d*x+1/2*c)*b^2+31/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/
2*c)+2*b)/(a^2-b^2)^(1/2))*b^2+27/4/d/a^5/tan(1/2*d*x+1/2*c)*b^2+3/4/d/a^4*
tan(1/2*d*x+1/2*c)^2*b-14/d/a^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c
)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^6-26/d/a^8/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^7-38/d/a^7/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^6+42/d/a^8/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^6
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.63, size = 1614, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^6*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a^3*d) - (tan(c/2 + (d*x)/2)^3*((a^2 + 4*b^2)/(32
*a^5) + 1/(24*a^3) - (3*b^2)/(8*a^5)))/d + (tan(c/2 + (d*x)/2)*(1/(8*a^3) -
 (3*(a^2 + 4*b^2))/(32*a^5) - (6*b*((6*b*((3*(a^2 + 4*b^2))/(32*a^5) + 1/(8
*a^3) - (9*b^2)/(8*a^5)))/a - (384*a^2*b + 256*b^3)/(1024*a^6) + (9*b*(a^2
+ 4*b^2))/(16*a^6)))/a + (3*(a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(32*a^5) + 1/(
8*a^3) - (9*b^2)/(8*a^5)))/a^2 + (3*b*(384*a^2*b + 256*b^3))/(512*a^7)))/d
- (tan(c/2 + (d*x)/2)^3*((187*a^5*b)/15 - 14*a^3*b^3) + a^6/5 + tan(c/2 + (
d*x)/2)^4*((263*a^6)/15 + 112*a^2*b^4 - (358*a^4*b^2)/3) + tan(c/2 + (d*x)/
2)^5*(1216*a*b^5 + (1519*a^5*b)/6 - 1360*a^3*b^3) - tan(c/2 + (d*x)/2)^2*((
```

```
29*a^6)/15 - (14*a^4*b^2)/5) + tan(c/2 + (d*x)/2)^8*(22*a^6 + 448*b^6 - 368
*a^2*b^4 - 56*a^4*b^2) + tan(c/2 + (d*x)/2)^6*((125*a^6)/3 + 2176*b^6 - 211
2*a^2*b^4 + 112*a^4*b^2) + (8*tan(c/2 + (d*x)/2)^7*(30*a^6*b + 104*b^7 + 36
*a^2*b^5 - 149*a^4*b^3))/a - (7*a^5*b*tan(c/2 + (d*x)/2))/10)/(d*(32*a^9*ta
n(c/2 + (d*x)/2)^5 + 32*a^9*tan(c/2 + (d*x)/2)^9 + tan(c/2 + (d*x)/2)^7*(64
*a^9 + 128*a^7*b^2) + 128*a^8*b*tan(c/2 + (d*x)/2)^6 + 128*a^8*b*tan(c/2 +
(d*x)/2)^8)) + (tan(c/2 + (d*x)/2)^2*((3*b*((3*(a^2 + 4*b^2))/(32*a^5) + 1/
(8*a^3) - (9*b^2)/(8*a^5)))/a - (384*a^2*b + 256*b^3)/(2048*a^6) + (9*b*(a^
2 + 4*b^2))/(32*a^6)))/d - (log(tan(c/2 + (d*x)/2))*(45*a^4*b + 168*b^5 - 2
00*a^2*b^3))/(8*a^8*d) - (3*b*tan(c/2 + (d*x)/2)^4)/(64*a^4*d) - (atan((((-
(a + b)*(a - b))^(1/2)*(a^4 + 21*b^4 - (29*a^2*b^2)/2)*((2*a^14 - 84*a^8*b^
6 + 121*a^10*b^4 - (169*a^12*b^2)/4)/a^14 + (tan(c/2 + (d*x)/2)*(61*a^12*b
- 672*a^6*b^7 + 1136*a^8*b^5 - 538*a^10*b^3))/(4*a^13) + ((-(a + b)*(a - b)
)^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^16 - 32*a^14*b^2))/(4*a^13))*(
a^4 + 21*b^4 - (29*a^2*b^2)/2))/a^8)*1i)/a^8 + ((-(a + b)*(a - b))^(1/2)*(a
^4 + 21*b^4 - (29*a^2*b^2)/2)*((2*a^14 - 84*a^8*b^6 + 121*a^10*b^4 - (169*a
^12*b^2)/4)/a^14 + (tan(c/2 + (d*x)/2)*(61*a^12*b - 672*a^6*b^7 + 1136*a^8*
b^5 - 538*a^10*b^3))/(4*a^13) - ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c
/2 + (d*x)/2)*(24*a^16 - 32*a^14*b^2))/(4*a^13))*(a^4 + 21*b^4 - (29*a^2*b^
2)/2))/a^8)*1i)/a^8)/(((45*a^10*b)/2 - 1764*b^11 + 5082*a^2*b^9 - (10649*a^
4*b^7)/2 + (9731*a^6*b^5)/4 - (1795*a^8*b^3)/4)/a^14 + (tan(c/2 + (d*x)/2)*
(16*a^10 - 3528*b^10 + 9282*a^2*b^8 - 8549*a^4*b^6 + 3185*a^6*b^4 - 406*a^8
*b^2))/(2*a^13) - ((-(a + b)*(a - b))^(1/2)*(a^4 + 21*b^4 - (29*a^2*b^2)/2)
*((2*a^14 - 84*a^8*b^6 + 121*a^10*b^4 - (169*a^12*b^2)/4)/a^14 + (tan(c/2 +
 (d*x)/2)*(61*a^12*b - 672*a^6*b^7 + 1136*a^8*b^5 - 538*a^10*b^3))/(4*a^13)
 + ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^16 - 32*a
^14*b^2))/(4*a^13))*(a^4 + 21*b^4 - (29*a^2*b^2)/2))/a^8))/a^8 + ((-(a + b)
*(a - b))^(1/2)*(a^4 + 21*b^4 - (29*a^2*b^2)/2)*((2*a^14 - 84*a^8*b^6 + 121
*a^10*b^4 - (169*a^12*b^2)/4)/a^14 + (tan(c/2 + (d*x)/2)*(61*a^12*b - 672*a
^6*b^7 + 1136*a^8*b^5 - 538*a^10*b^3))/(4*a^13) - ((-(a + b)*(a - b))^(1/2)
*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^16 - 32*a^14*b^2))/(4*a^13))*(a^4 + 2
1*b^4 - (29*a^2*b^2)/2))/a^8))/a^8))*(-(a + b)*(a - b))^(1/2)*(a^4 + 21*b^4
 - (29*a^2*b^2)/2)*2i)/(a^8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1275 $\int \frac{\cot^6(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=600

$$\frac{3b\cot(c+dx)\csc^5(c+dx)}{14a^2d(a+b\sin(c+dx))^2}+\frac{\left(12a^4-65a^2b^2+60b^4\right)\cot(c+dx)\csc^4(c+dx)}{10a^4b^2d(a+b\sin(c+dx))}-\frac{3b^2\sqrt{a^2-b^2}\,\left(4a^4-23a^2b^2+24\right.}{a^{10}d}$$

```
[Out] -3/16*b*(5*a^6-100*a^4*b^2+280*a^2*b^4-192*b^6)*arctanh(cos(d*x+c))/a^10/d+
1/70*(10*a^6-889*a^4*b^2+3255*a^2*b^4-2520*b^6)*cot(d*x+c)/a^9/d+3/16*b*(27
*a^4-116*a^2*b^2+96*b^4)*cot(d*x+c)*csc(d*x+c)/a^8/d-1/70*(205*a^4-973*a^2*
b^2+840*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^7/d+1/8*(16*a^4-81*a^2*b^2+72*b^4)*c
ot(d*x+c)*csc(d*x+c)^3/a^6/b/d-3/70*(35*a^4-185*a^2*b^2+168*b^4)*cot(d*x+c)
*csc(d*x+c)^4/a^5/b^2/d-1/5*cot(d*x+c)*csc(d*x+c)^3/b/d/(a+b*sin(d*x+c))^2+
1/10*a*cot(d*x+c)*csc(d*x+c)^4/b^2/d/(a+b*sin(d*x+c))^2+1/35*(7*a^4-35*a^2*
b^2+30*b^4)*cot(d*x+c)*csc(d*x+c)^4/a^3/b^2/d/(a+b*sin(d*x+c))^2+3/14*b*cot
(d*x+c)*csc(d*x+c)^5/a^2/d/(a+b*sin(d*x+c))^2-1/7*cot(d*x+c)*csc(d*x+c)^6/a
/d/(a+b*sin(d*x+c))^2+1/10*(12*a^4-65*a^2*b^2+60*b^4)*cot(d*x+c)*csc(d*x+c)
^4/a^4/b^2/d/(a+b*sin(d*x+c))-3*b^2*(4*a^4-23*a^2*b^2+24*b^4)*arctan((b+a*t
an(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^10/d
```

Rubi [A] time = 3.22, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{3b^2\sqrt{a^2-b^2}\,\left(-23a^2b^2+4a^4+24b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^{10}d}+\frac{\left(-889a^4b^2+3255a^2b^4+10a^6-2520b^6\right)\cot(c}{70a^9d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b^2*Sqrt[a^2 - b^2]*(4*a^4 - 23*a^2*b^2 + 24*b^4)*ArcTan[(b + a*Tan[(c
+ d*x)/2])/Sqrt[a^2 - b^2]])/(a^10*d) - (3*b*(5*a^6 - 100*a^4*b^2 + 280*a^2
*b^4 - 192*b^6)*ArcTanh[Cos[c + d*x]])/(16*a^10*d) + ((10*a^6 - 889*a^4*b^2
 + 3255*a^2*b^4 - 2520*b^6)*Cot[c + d*x])/(70*a^9*d) + (3*b*(27*a^4 - 116*a
^2*b^2 + 96*b^4)*Cot[c + d*x]*Csc[c + d*x])/(16*a^8*d) - ((205*a^4 - 973*a^
2*b^2 + 840*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(70*a^7*d) + ((16*a^4 - 81*a^
2*b^2 + 72*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(8*a^6*b*d) - (3*(35*a^4 - 185
*a^2*b^2 + 168*b^4)*Cot[c + d*x]*Csc[c + d*x]^4)/(70*a^5*b^2*d) - (Cot[c +
d*x]*Csc[c + d*x]^3)/(5*b*d*(a + b*Sin[c + d*x])^2) + (a*Cot[c + d*x]*Csc[c
 + d*x]^4)/(10*b^2*d*(a + b*Sin[c + d*x])^2) + ((7*a^4 - 35*a^2*b^2 + 30*b^
```

```
4)*Cot[c + d*x]*Csc[c + d*x]^4)/(35*a^3*b^2*d*(a + b*Sin[c + d*x])^2) + (3*
b*Cot[c + d*x]*Csc[c + d*x]^5)/(14*a^2*d*(a + b*Sin[c + d*x])^2) - (Cot[c +
 d*x]*Csc[c + d*x]^6)/(7*a*d*(a + b*Sin[c + d*x])^2) + ((12*a^4 - 65*a^2*b^
2 + 60*b^4)*Cot[c + d*x]*Csc[c + d*x]^4)/(10*a^4*b^2*d*(a + b*Sin[c + d*x])
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cot(c+dx)\csc^3(c+dx)}{5bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^4(c+dx)}{10b^2d(a+b\sin(c+dx))^2} + \frac{3b\cot(c+dx)\csc^5(c}{14a^2d(a+b\sin(c+}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{5bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^4(c+dx)}{10b^2d(a+b\sin(c+dx))^2} + \frac{\left(7a^4-35a^2b^2+30b^4\right.}{35a^3b^2d(a-}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{5bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^4(c+dx)}{10b^2d(a+b\sin(c+dx))^2} + \frac{\left(7a^4-35a^2b^2+30b^4\right.}{35a^3b^2d(a-}$$

$$= -\frac{3\left(35a^4-185a^2b^2+168b^4\right)\cot(c+dx)\csc^4(c+dx)}{70a^5b^2d} - \frac{\cot(c+dx)\csc^3(c+d}{5bd(a+b\sin(c+dx)}$$

$$= \frac{\left(16a^4-81a^2b^2+72b^4\right)\cot(c+dx)\csc^3(c+dx)}{8a^6bd} - \frac{3\left(35a^4-185a^2b^2+168b^4\right)}{70a^5b}$$

$$= -\frac{\left(205a^4-973a^2b^2+840b^4\right)\cot(c+dx)\csc^2(c+dx)}{70a^7d} + \frac{\left(16a^4-81a^2b^2+72b^4\right.}{8}$$

$$= \frac{3b\left(27a^4-116a^2b^2+96b^4\right)\cot(c+dx)\csc(c+dx)}{16a^8d} - \frac{\left(205a^4-973a^2b^2+840b\right.}{70}$$

$$= \frac{\left(10a^6-889a^4b^2+3255a^2b^4-2520b^6\right)\cot(c+dx)}{70a^9d} + \frac{3b\left(27a^4-116a^2b^2+96\right.}{16}$$

$$= \frac{\left(10a^6-889a^4b^2+3255a^2b^4-2520b^6\right)\cot(c+dx)}{70a^9d} + \frac{3b\left(27a^4-116a^2b^2+96\right.}{16}$$

$$= -\frac{3b\left(5a^6-100a^4b^2+280a^2b^4-192b^6\right)\tanh^{-1}(\cos(c+dx))}{16a^{10}d} + \frac{\left(10a^6-889a^4b\right.}{}$$

$$= -\frac{3b\left(5a^6-100a^4b^2+280a^2b^4-192b^6\right)\tanh^{-1}(\cos(c+dx))}{16a^{10}d} + \frac{\left(10a^6-889a^4b\right.}{}$$

$$= -\frac{3b^2\sqrt{a^2-b^2}\left(4a^4-23a^2b^2+24b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^{10}d} - \frac{3b\left(5a^6-100a^4\right.}{}$$

Mathematica [A] time = 3.15, size = 728, normalized size = 1.21

$$\frac{\frac{215040b^2\left(-4a^6+27a^4b^2-47a^2b^4+24b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+13440b\left(5a^6-100a^4b^2+280a^2b^4-192b^6\right)\log\left(\sin\left(\frac{1}{2}(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((215040*b^2*(-4*a^6 + 27*a^4*b^2 - 47*a^2*b^4 + 24*b^6)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 13440*b*(-5*a^6 + 100*a^4
*b^2 - 280*a^2*b^4 + 192*b^6)*Log[Cos[(c + d*x)/2]] + 13440*b*(5*a^6 - 100*
a^4*b^2 + 280*a^2*b^4 - 192*b^6)*Log[Sin[(c + d*x)/2]] - (a*Csc[c + d*x]^9*
(28*(200*a^8 + 795*a^6*b^2 - 1218*a^4*b^4 - 4110*a^2*b^6 + 5040*b^8)*Cos[c
+ d*x] + 28*(120*a^8 - 1403*a^6*b^2 + 1952*a^4*b^4 + 8700*a^2*b^6 - 10080*b
^8)*Cos[3*(c + d*x)] + 1120*a^8*Cos[5*(c + d*x)] + 22948*a^6*b^2*Cos[5*(c +
 d*x)] - 18144*a^4*b^4*Cos[5*(c + d*x)] - 193200*a^2*b^6*Cos[5*(c + d*x)] +
 201600*b^8*Cos[5*(c + d*x)] + 160*a^8*Cos[7*(c + d*x)] - 5884*a^6*b^2*Cos[
7*(c + d*x)] - 5964*a^4*b^4*Cos[7*(c + d*x)] + 77700*a^2*b^6*Cos[7*(c + d*x
)] - 70560*b^8*Cos[7*(c + d*x)] - 40*a^6*b^2*Cos[9*(c + d*x)] + 3556*a^4*b^
4*Cos[9*(c + d*x)] - 13020*a^2*b^6*Cos[9*(c + d*x)] + 10080*b^8*Cos[9*(c +
d*x)] - 9660*a^7*b*Sin[2*(c + d*x)] + 194334*a^5*b^3*Sin[2*(c + d*x)] - 592
200*a^3*b^5*Sin[2*(c + d*x)] + 423360*a*b^7*Sin[2*(c + d*x)] + 6160*a^7*b*S
in[4*(c + d*x)] - 190582*a^5*b^3*Sin[4*(c + d*x)] + 585480*a^3*b^5*Sin[4*(c
 + d*x)] - 423360*a*b^7*Sin[4*(c + d*x)] - 3660*a^7*b*Sin[6*(c + d*x)] + 77
462*a^5*b^3*Sin[6*(c + d*x)] - 246120*a^3*b^5*Sin[6*(c + d*x)] + 181440*a*b
^7*Sin[6*(c + d*x)] + 160*a^7*b*Sin[8*(c + d*x)] - 11389*a^5*b^3*Sin[8*(c +
 d*x)] + 39900*a^3*b^5*Sin[8*(c + d*x)] - 30240*a*b^7*Sin[8*(c + d*x)]))/(b
 + a*Csc[c + d*x])^2)/(71680*a^10*d)
```

fricas [B] time = 2.66, size = 3687, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/1120*(16*(10*a^7*b^2 - 889*a^5*b^4 + 3255*a^3*b^6 - 2520*a*b^8)*cos(d*x
+ c)^9 - 4*(40*a^9 - 1381*a^7*b^2 - 9492*a^5*b^4 + 48720*a^3*b^6 - 40320*a*
b^8)*cos(d*x + c)^7 - 28*(563*a^7*b^2 + 1068*a^5*b^4 - 9720*a^3*b^6 + 8640*
a*b^8)*cos(d*x + c)^5 + 140*(105*a^7*b^2 + 20*a^5*b^4 - 1200*a^3*b^6 + 1152
*a*b^8)*cos(d*x + c)^3 + 840*(2*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x
```

+ c)^8 + 8*a^5*b^3 - 46*a^3*b^5 + 48*a*b^7 - 8*(4*a^5*b^3 - 23*a^3*b^5 + 2
4*a*b^7)*cos(d*x + c)^6 + 12*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x +
c)^4 - 8*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x + c)^2 + ((4*a^4*b^4 -
23*a^2*b^6 + 24*b^8)*cos(d*x + c)^8 + 4*a^6*b^2 - 19*a^4*b^4 + a^2*b^6 + 2
4*b^8 - (4*a^6*b^2 - 7*a^4*b^4 - 68*a^2*b^6 + 96*b^8)*cos(d*x + c)^6 + 3*(4
*a^6*b^2 - 15*a^4*b^4 - 22*a^2*b^6 + 48*b^8)*cos(d*x + c)^4 - (12*a^6*b^2 -
53*a^4*b^4 - 20*a^2*b^6 + 96*b^8)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2
+ b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 +
2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 420*(11*a^7*b^2 - 8*a^5*b
^4 - 92*a^3*b^6 + 96*a*b^8)*cos(d*x + c) - 105*(10*a^7*b^2 - 200*a^5*b^4 +
560*a^3*b^6 - 384*a*b^8 + 2*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*
b^8)*cos(d*x + c)^8 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)
*cos(d*x + c)^6 + 12*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*co
s(d*x + c)^4 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*cos(d*
x + c)^2 + (5*a^8*b - 95*a^6*b^3 + 180*a^4*b^5 + 88*a^2*b^7 - 192*b^9 + (5*
a^6*b^3 - 100*a^4*b^5 + 280*a^2*b^7 - 192*b^9)*cos(d*x + c)^8 - (5*a^8*b -
80*a^6*b^3 - 120*a^4*b^5 + 928*a^2*b^7 - 768*b^9)*cos(d*x + c)^6 + 3*(5*a^8
*b - 90*a^6*b^3 + 80*a^4*b^5 + 368*a^2*b^7 - 384*b^9)*cos(d*x + c)^4 - (15*
a^8*b - 280*a^6*b^3 + 440*a^4*b^5 + 544*a^2*b^7 - 768*b^9)*cos(d*x + c)^2)*
sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 105*(10*a^7*b^2 - 200*a^5*b^4 +
560*a^3*b^6 - 384*a*b^8 + 2*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a
*b^8)*cos(d*x + c)^8 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8
)*cos(d*x + c)^6 + 12*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*c
os(d*x + c)^4 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*cos(d
*x + c)^2 + (5*a^8*b - 95*a^6*b^3 + 180*a^4*b^5 + 88*a^2*b^7 - 192*b^9 + (5
*a^6*b^3 - 100*a^4*b^5 + 280*a^2*b^7 - 192*b^9)*cos(d*x + c)^8 - (5*a^8*b -
80*a^6*b^3 - 120*a^4*b^5 + 928*a^2*b^7 - 768*b^9)*cos(d*x + c)^6 + 3*(5*a^
8*b - 90*a^6*b^3 + 80*a^4*b^5 + 368*a^2*b^7 - 384*b^9)*cos(d*x + c)^4 - (15
*a^8*b - 280*a^6*b^3 + 440*a^4*b^5 + 544*a^2*b^7 - 768*b^9)*cos(d*x + c)^2)
*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((160*a^8*b - 11389*a^6*b^3
+ 39900*a^4*b^5 - 30240*a^2*b^7)*cos(d*x + c)^7 - 7*(165*a^8*b - 5207*a^6*
b^3 + 17340*a^4*b^5 - 12960*a^2*b^7)*cos(d*x + c)^5 + 35*(40*a^8*b - 1097*a
^6*b^3 + 3516*a^4*b^5 - 2592*a^2*b^7)*cos(d*x + c)^3 - 105*(5*a^8*b - 127*a
^6*b^3 + 396*a^4*b^5 - 288*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/(2*a^11*b*d
*cos(d*x + c)^8 - 8*a^11*b*d*cos(d*x + c)^6 + 12*a^11*b*d*cos(d*x + c)^4 -
8*a^11*b*d*cos(d*x + c)^2 + 2*a^11*b*d + (a^10*b^2*d*cos(d*x + c)^8 - (a^12
+ 4*a^10*b^2)*d*cos(d*x + c)^6 + 3*(a^12 + 2*a^10*b^2)*d*cos(d*x + c)^4 -
(3*a^12 + 4*a^10*b^2)*d*cos(d*x + c)^2 + (a^12 + a^10*b^2)*d)*sin(d*x + c))
, 1/1120*(16*(10*a^7*b^2 - 889*a^5*b^4 + 3255*a^3*b^6 - 2520*a*b^8)*cos(d*x
+ c)^9 - 4*(40*a^9 - 1381*a^7*b^2 - 9492*a^5*b^4 + 48720*a^3*b^6 - 40320*a
*b^8)*cos(d*x + c)^7 - 28*(563*a^7*b^2 + 1068*a^5*b^4 - 9720*a^3*b^6 + 8640
*a*b^8)*cos(d*x + c)^5 + 140*(105*a^7*b^2 + 20*a^5*b^4 - 1200*a^3*b^6 + 115
2*a*b^8)*cos(d*x + c)^3 + 1680*(2*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d
*x + c)^8 + 8*a^5*b^3 - 46*a^3*b^5 + 48*a*b^7 - 8*(4*a^5*b^3 - 23*a^3*b^5 +

```
24*a*b^7)*cos(d*x + c)^6 + 12*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x
+ c)^4 - 8*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x + c)^2 + ((4*a^4*b^4
 - 23*a^2*b^6 + 24*b^8)*cos(d*x + c)^8 + 4*a^6*b^2 - 19*a^4*b^4 + a^2*b^6 +
 24*b^8 - (4*a^6*b^2 - 7*a^4*b^4 - 68*a^2*b^6 + 96*b^8)*cos(d*x + c)^6 + 3*
(4*a^6*b^2 - 15*a^4*b^4 - 22*a^2*b^6 + 48*b^8)*cos(d*x + c)^4 - (12*a^6*b^2
 - 53*a^4*b^4 - 20*a^2*b^6 + 96*b^8)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2
 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 420*
(11*a^7*b^2 - 8*a^5*b^4 - 92*a^3*b^6 + 96*a*b^8)*cos(d*x + c) - 105*(10*a^7
*b^2 - 200*a^5*b^4 + 560*a^3*b^6 - 384*a*b^8 + 2*(5*a^7*b^2 - 100*a^5*b^4 +
 280*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^8 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280
*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^6 + 12*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^
3*b^6 - 192*a*b^8)*cos(d*x + c)^4 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^
6 - 192*a*b^8)*cos(d*x + c)^2 + (5*a^8*b - 95*a^6*b^3 + 180*a^4*b^5 + 88*a^
2*b^7 - 192*b^9 + (5*a^6*b^3 - 100*a^4*b^5 + 280*a^2*b^7 - 192*b^9)*cos(d*x
 + c)^8 - (5*a^8*b - 80*a^6*b^3 - 120*a^4*b^5 + 928*a^2*b^7 - 768*b^9)*cos(
d*x + c)^6 + 3*(5*a^8*b - 90*a^6*b^3 + 80*a^4*b^5 + 368*a^2*b^7 - 384*b^9)*
cos(d*x + c)^4 - (15*a^8*b - 280*a^6*b^3 + 440*a^4*b^5 + 544*a^2*b^7 - 768*
b^9)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 105*(10*a^
7*b^2 - 200*a^5*b^4 + 560*a^3*b^6 - 384*a*b^8 + 2*(5*a^7*b^2 - 100*a^5*b^4
+ 280*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^8 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 28
0*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^6 + 12*(5*a^7*b^2 - 100*a^5*b^4 + 280*a
^3*b^6 - 192*a*b^8)*cos(d*x + c)^4 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b
^6 - 192*a*b^8)*cos(d*x + c)^2 + (5*a^8*b - 95*a^6*b^3 + 180*a^4*b^5 + 88*a
^2*b^7 - 192*b^9 + (5*a^6*b^3 - 100*a^4*b^5 + 280*a^2*b^7 - 192*b^9)*cos(d*
x + c)^8 - (5*a^8*b - 80*a^6*b^3 - 120*a^4*b^5 + 928*a^2*b^7 - 768*b^9)*cos
(d*x + c)^6 + 3*(5*a^8*b - 90*a^6*b^3 + 80*a^4*b^5 + 368*a^2*b^7 - 384*b^9)
*cos(d*x + c)^4 - (15*a^8*b - 280*a^6*b^3 + 440*a^4*b^5 + 544*a^2*b^7 - 768
*b^9)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((160*
a^8*b - 11389*a^6*b^3 + 39900*a^4*b^5 - 30240*a^2*b^7)*cos(d*x + c)^7 - 7*(
165*a^8*b - 5207*a^6*b^3 + 17340*a^4*b^5 - 12960*a^2*b^7)*cos(d*x + c)^5 +
35*(40*a^8*b - 1097*a^6*b^3 + 3516*a^4*b^5 - 2592*a^2*b^7)*cos(d*x + c)^3 -
 105*(5*a^8*b - 127*a^6*b^3 + 396*a^4*b^5 - 288*a^2*b^7)*cos(d*x + c))*sin(
d*x + c))/(2*a^11*b*d*cos(d*x + c)^8 - 8*a^11*b*d*cos(d*x + c)^6 + 12*a^11*
b*d*cos(d*x + c)^4 - 8*a^11*b*d*cos(d*x + c)^2 + 2*a^11*b*d + (a^10*b^2*d*c
os(d*x + c)^8 - (a^12 + 4*a^10*b^2)*d*cos(d*x + c)^6 + 3*(a^12 + 2*a^10*b^2
)*d*cos(d*x + c)^4 - (3*a^12 + 4*a^10*b^2)*d*cos(d*x + c)^2 + (a^12 + a^10*
b^2)*d)*sin(d*x + c))]
```

giac [A] time = 0.39, size = 1018, normalized size = 1.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4480*(840*(5*a^6*b - 100*a^4*b^3 + 280*a^2*b^5 - 192*b^7)*log(abs(tan(1/2
*d*x + 1/2*c)))/a^10 - 13440*(4*a^6*b^2 - 27*a^4*b^4 + 47*a^2*b^6 - 24*b^8)
*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c)
+ b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^10) - 4480*(9*a^5*b^4*tan(1/2*d*x
 + 1/2*c)^3 - 27*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 18*a*b^8*tan(1/2*d*x + 1/
2*c)^3 + 8*a^6*b^3*tan(1/2*d*x + 1/2*c)^2 - 9*a^4*b^5*tan(1/2*d*x + 1/2*c)^
2 - 33*a^2*b^7*tan(1/2*d*x + 1/2*c)^2 + 34*b^9*tan(1/2*d*x + 1/2*c)^2 + 23*
a^5*b^4*tan(1/2*d*x + 1/2*c) - 73*a^3*b^6*tan(1/2*d*x + 1/2*c) + 50*a*b^8*t
an(1/2*d*x + 1/2*c) + 8*a^6*b^3 - 25*a^4*b^5 + 17*a^2*b^7)/((a*tan(1/2*d*x
+ 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^10) - (10890*a^6*b*tan(1/2*d
*x + 1/2*c)^7 - 217800*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 609840*a^2*b^5*tan(
1/2*d*x + 1/2*c)^7 - 418176*b^7*tan(1/2*d*x + 1/2*c)^7 - 175*a^7*tan(1/2*d*
x + 1/2*c)^6 + 18480*a^5*b^2*tan(1/2*d*x + 1/2*c)^6 - 75600*a^3*b^4*tan(1/2
*d*x + 1/2*c)^6 + 62720*a*b^6*tan(1/2*d*x + 1/2*c)^6 - 1575*a^6*b*tan(1/2*d
*x + 1/2*c)^5 + 11200*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 11760*a^2*b^5*tan(1/
2*d*x + 1/2*c)^5 + 105*a^7*tan(1/2*d*x + 1/2*c)^4 - 1960*a^5*b^2*tan(1/2*d*
x + 1/2*c)^4 + 2800*a^3*b^4*tan(1/2*d*x + 1/2*c)^4 + 315*a^6*b*tan(1/2*d*x
+ 1/2*c)^3 - 700*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 35*a^7*tan(1/2*d*x + 1/2*
c)^2 + 168*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 - 35*a^6*b*tan(1/2*d*x + 1/2*c) +
 5*a^7)/(a^10*tan(1/2*d*x + 1/2*c)^7) + (5*a^18*tan(1/2*d*x + 1/2*c)^7 - 35
*a^17*b*tan(1/2*d*x + 1/2*c)^6 - 35*a^18*tan(1/2*d*x + 1/2*c)^5 + 168*a^16*
b^2*tan(1/2*d*x + 1/2*c)^5 + 315*a^17*b*tan(1/2*d*x + 1/2*c)^4 - 700*a^15*b
^3*tan(1/2*d*x + 1/2*c)^4 + 105*a^18*tan(1/2*d*x + 1/2*c)^3 - 1960*a^16*b^2
*tan(1/2*d*x + 1/2*c)^3 + 2800*a^14*b^4*tan(1/2*d*x + 1/2*c)^3 - 1575*a^17*
b*tan(1/2*d*x + 1/2*c)^2 + 11200*a^15*b^3*tan(1/2*d*x + 1/2*c)^2 - 11760*a^
13*b^5*tan(1/2*d*x + 1/2*c)^2 - 175*a^18*tan(1/2*d*x + 1/2*c) + 18480*a^16*
b^2*tan(1/2*d*x + 1/2*c) - 75600*a^14*b^4*tan(1/2*d*x + 1/2*c) + 62720*a^12
*b^6*tan(1/2*d*x + 1/2*c))/a^21)/d
```

maple [B] time = 0.92, size = 1576, normalized size = 2.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -5/128/d/a^3*tan(1/2*d*x+1/2*c)+33/8/d/a^5*b^2*tan(1/2*d*x+1/2*c)+1/896/d/a
^3*tan(1/2*d*x+1/2*c)^7-1/896/d/a^3/tan(1/2*d*x+1/2*c)^7+3/128/d/a^3*tan(1/
2*d*x+1/2*c)^3-3/128/d/a^3/tan(1/2*d*x+1/2*c)^3-1/128/d/a^3*tan(1/2*d*x+1/2
*c)^5+1/128/d/a^3/tan(1/2*d*x+1/2*c)^5-8/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*ta
n(1/2*d*x+1/2*c)*b+a)^2*b^3+5/128/d/a^3/tan(1/2*d*x+1/2*c)+15/16/d/a^4*b*ln
(tan(1/2*d*x+1/2*c))-5/2/d/a^6*b^3/tan(1/2*d*x+1/2*c)^2+105/2/d/a^8*b^5*ln(
tan(1/2*d*x+1/2*c))+135/8/d/a^7/tan(1/2*d*x+1/2*c)*b^4-9/128/d/a^4*b/tan(1/
2*d*x+1/2*c)^4+7/16/d/a^5/tan(1/2*d*x+1/2*c)^3*b^2+5/2/d/a^6*tan(1/2*d*x+1/
2*c)^2*b^3-135/8/d/a^7*b^4*tan(1/2*d*x+1/2*c)+25/d/a^6/(tan(1/2*d*x+1/2*c)^
```

```
2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^5+9/128/d/a^4*tan(1/2*d*x+1/2*c)^4*b-7/16
/d/a^5*tan(1/2*d*x+1/2*c)^3*b^2-75/4/d/a^6*b^3*ln(tan(1/2*d*x+1/2*c))-17/d/
a^8*b^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-1/128/d/a^4*b*t
an(1/2*d*x+1/2*c)^6+3/80/d/a^5*tan(1/2*d*x+1/2*c)^5*b^2-5/32/d/a^6*tan(1/2*
d*x+1/2*c)^4*b^3+5/8/d/a^7*tan(1/2*d*x+1/2*c)^3*b^4-21/8/d/a^8*tan(1/2*d*x+
1/2*c)^2*b^5+14/d/a^9*b^6*tan(1/2*d*x+1/2*c)-3/80/d/a^5/tan(1/2*d*x+1/2*c)^
5*b^2-5/8/d/a^7/tan(1/2*d*x+1/2*c)^3*b^4-14/d/a^9/tan(1/2*d*x+1/2*c)*b^6+1/
128/d/a^4*b/tan(1/2*d*x+1/2*c)^6+5/32/d/a^6*b^3/tan(1/2*d*x+1/2*c)^4+21/8/d
/a^8*b^5/tan(1/2*d*x+1/2*c)^2-36/d/a^10*b^7*ln(tan(1/2*d*x+1/2*c))+45/128/d
/a^4*b/tan(1/2*d*x+1/2*c)^2-9/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^4+9/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan
(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^5-23/d/a^5/(tan(1/2*d*x+1/2*c
)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^4+81/d/a^6/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4-8/d/a^4
/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b
^3-12/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^
2)^(1/2))*b^2-33/8/d/a^5/tan(1/2*d*x+1/2*c)*b^2-45/128/d/a^4*tan(1/2*d*x+1/
2*c)^2*b-18/d/a^9*b^8/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*t
an(1/2*d*x+1/2*c)^3-34/d/a^10*b^9/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-50/d/a^9*b^8/(tan(1/2*d*x+1/2*c)^2*a+2*tan(
1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+72/d/a^10*b^8/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+27/d/a^7/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^6+33/d/a^8/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^7+
73/d/a^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/
2*c)*b^6-141/d/a^8/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))*b^6
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 13.29, size = 2795, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6/(sin(c + d*x)^8*(a + b*sin(c + d*x))^3),x)

[Out] (tan(c/2 + (d*x)/2)*(7/(128*a^3) + (9*b^2)/(32*a^5) - (6*b*((6*b*((3*(a^2 + 4*b^2))/(128*a^5) + 3/(128*a^3) + (6*b*((6*b*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a - (1536*a^2*b + 1024*b^3)/(16384*a^6) + (9*b*(a^2 + 4*b^2))/(64*a^6)))/a - (3*(a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a^2 - (3*b*(1536*a^2*b + 1024*b^3))/(8192*a^7)))/a - (3*b)/(64*a^4) + (9*b*(a^2 + 4*b^2))/(64*a^6) + ((1536*a^2*b + 1024*b^3)*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/(128*a^3) - (3*(a^2 + 4*b^2)*((6*b*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a - (1536*a^2*b + 1024*b^3)/(16384*a^6) + (9*b*(a^2 + 4*b^2))/(64*a^6)))/a^2))/a + (3*(a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a^2 - ((1536*a^2*b + 1024*b^3)*((6*b*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a - (1536*a^2*b + 1024*b^3)/(16384*a^6) + (9*b*(a^2 + 4*b^2))/(64*a^6)))/(128*a^3) + (3*(a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(128*a^5) + 3/(128*a^3) + (6*b*((6*b*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a - (1536*a^2*b + 1024*b^3)/(16384*a^6) + (9*b*(a^2 + 4*b^2))/(64*a^6)))/a - (3*(a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a^2 - (3*b*(1536*a^2*b + 1024*b^3))/(8192*a^7)))/a^2))/d + tan(c/2 + (d*x)/2)^7/(896*a^3*d) - (tan(c/2 + (d*x)/2)^5*((3*(a^2 + 4*b^2))/(640*a^5) + 1/(320*a^3) - (9*b^2)/(160*a^5)))/d + (tan(c/2 + (d*x)/2)^2*((3*b*((3*(a^2 + 4*b^2))/(128*a^5) + 3/(128*a^3) + (6*b*((6*b*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a - (1536*a^2*b + 1024*b^3)/(16384*a^6) + (9*b*(a^2 + 4*b^2))/(64*a^6)))/a - (3*(a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a^2 - (3*b*(1536*a^2*b + 1024*b^3))/(8192*a^7)))/a - (3*b)/(128*a^4) + (9*b*(a^2 + 4*b^2))/(128*a^6) + ((1536*a^2*b + 1024*b^3)*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/(256*a^3) - (3*(a^2 + 4*b^2)*((6*b*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a - (1536*a^2*b + 1024*b^3)/(16384*a^6) + (9*b*(a^2 + 4*b^2))/(64*a^6)))/(2*a^2)))/d + (tan(c/2 + (d*x)/2)^4*((3*b*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/(2*a) - (1536*a^2*b + 1024*b^3)/(65536*a^6) + (9*b*(a^2 + 4*b^2))/(256*a^6)))/d - (tan(c/2 + (d*x)/2)^3*((a^2 + 4*b^2)/(128*a^5) + 1/(128*a^3) + (2*b*((6*b*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a - (1536*a^2*b + 1024*b^3)/(16384*a^6) + (9*b*(a^2 + 4*b^2))/(64*a^6)))/a - ((a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a^2 - (b*(1536*a^2*b + 1024*b^3))/(8192*a^7)))/d - (tan(c/2 + (d*x)/2)^3*((25*a^7*b)/7 - (24*a^5*b^3)/5) - tan(c/2 + (d*x)/2)^5*(20*a^7*b + 96*a^3*b^5 - (556*a^5*b^3)/5) + tan(c/2 + (d*x)/2)^6*(768*a^2*b^6 - 1024*a^4*b^4 + (1444*a^6*b^2)/5) + tan(c/2 + (d*x)/2)^7*(8000*a*b^7 - 89*a^7*b - 10912*a^3*b^5 + 3352*a^5*b^3) + a^8/7 + tan(c/2 + (d*x)/2)^4*((8*a^8)/7 + (96*a^4*b^4)/5 - (92*a^6*b^2)/5) - tan(c/2 + (d*x)/2)^10*(5*a^8 - 2304*b^8 + 1664*a^2*b^6 + 1008*a^4*b^4 - 528*a^6*b^2) + tan(c/2 + (d*x)/2)^8*(13568*b^8 - 7*a^8 - 15744*a^2*b^6 + 2096*a^4*b^4 + 800*a^6*b^2) - tan(c/2 + (d*x)/2)^2*((5*a^8)/7 - (48*a^6*b^2)/35) -

```
(3*a^7*b*tan(c/2 + (d*x)/2))/7 + (tan(c/2 + (d*x)/2)^9*(4352*b^9 - 65*a^8*b
 + 2944*a^2*b^7 - 10128*a^4*b^5 + 3456*a^6*b^3))/a)/(d*(128*a^11*tan(c/2 +
(d*x)/2)^7 + 128*a^11*tan(c/2 + (d*x)/2)^11 + tan(c/2 + (d*x)/2)^9*(256*a^1
1 + 512*a^9*b^2) + 512*a^10*b*tan(c/2 + (d*x)/2)^8 + 512*a^10*b*tan(c/2 + (
d*x)/2)^10)) - (b*tan(c/2 + (d*x)/2)^6)/(128*a^4*d) + (log(tan(c/2 + (d*x)/
2))*(15*a^6*b - 576*b^7 + 840*a^2*b^5 - 300*a^4*b^3))/(16*a^10*d) - (b^2*at
an(((b^2*(-(a + b)*(a - b))^(1/2)*(4*a^4 + 24*b^4 - 23*a^2*b^2)*((2304*a^10
*b^8 - 3936*a^12*b^6 + 1896*a^14*b^4 - 222*a^16*b^2)/(16*a^18) + (tan(c/2 +
 (d*x)/2)*(15*a^16*b + 2304*a^8*b^9 - 4512*a^10*b^7 + 2736*a^12*b^5 - 522*a
^14*b^3))/(8*a^17) - (3*b^2*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 +
(d*x)/2)*(48*a^20 - 64*a^18*b^2))/(8*a^17))*(4*a^4 + 24*b^4 - 23*a^2*b^2))/
(2*a^10))*3i)/(2*a^10) + (b^2*(-(a + b)*(a - b))^(1/2)*(4*a^4 + 24*b^4 - 23
*a^2*b^2)*((2304*a^10*b^8 - 3936*a^12*b^6 + 1896*a^14*b^4 - 222*a^16*b^2)/(
16*a^18) + (tan(c/2 + (d*x)/2)*(15*a^16*b + 2304*a^8*b^9 - 4512*a^10*b^7 +
2736*a^12*b^5 - 522*a^14*b^3))/(8*a^17) + (3*b^2*(-(a + b)*(a - b))^(1/2)*(
2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^20 - 64*a^18*b^2))/(8*a^17))*(4*a^4 + 2
4*b^4 - 23*a^2*b^2))/(2*a^10))*3i)/(2*a^10))/((41472*b^15 - 141696*a^2*b^13
 + 186696*a^4*b^11 - 118332*a^6*b^9 + 36495*a^8*b^7 - 4815*a^10*b^5 + 180*a
^12*b^3)/(8*a^18) + (tan(c/2 + (d*x)/2)*(20736*b^14 - 65664*a^2*b^12 + 7822
8*a^4*b^10 - 43065*a^6*b^8 + 10737*a^8*b^6 - 972*a^10*b^4))/(4*a^17) - (3*b
^2*(-(a + b)*(a - b))^(1/2)*(4*a^4 + 24*b^4 - 23*a^2*b^2)*((2304*a^10*b^8 -
 3936*a^12*b^6 + 1896*a^14*b^4 - 222*a^16*b^2)/(16*a^18) + (tan(c/2 + (d*x)
/2)*(15*a^16*b + 2304*a^8*b^9 - 4512*a^10*b^7 + 2736*a^12*b^5 - 522*a^14*b^
3))/(8*a^17) - (3*b^2*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/
2)*(48*a^20 - 64*a^18*b^2))/(8*a^17))*(4*a^4 + 24*b^4 - 23*a^2*b^2))/(2*a^1
0)))/(2*a^10) + (3*b^2*(-(a + b)*(a - b))^(1/2)*(4*a^4 + 24*b^4 - 23*a^2*b^
2)*((2304*a^10*b^8 - 3936*a^12*b^6 + 1896*a^14*b^4 - 222*a^16*b^2)/(16*a^18
) + (tan(c/2 + (d*x)/2)*(15*a^16*b + 2304*a^8*b^9 - 4512*a^10*b^7 + 2736*a^
12*b^5 - 522*a^14*b^3))/(8*a^17) + (3*b^2*(-(a + b)*(a - b))^(1/2)*(2*a^2*b
 - (tan(c/2 + (d*x)/2)*(48*a^20 - 64*a^18*b^2))/(8*a^17))*(4*a^4 + 24*b^4 -
 23*a^2*b^2))/(2*a^10)))/(2*a^10)))*(-(a + b)*(a - b))^(1/2)*(4*a^4 + 24*b^
4 - 23*a^2*b^2)*3i)/(a^10*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1276 $\int \frac{\cos^6(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=712

$$-\frac{20\left(a^2-b^2\right)\cos(e+fx)\sqrt{d\sin(e+fx)}}{99a^2b^2df(a+b\sin(e+fx))^{9/2}}-\frac{4\left(5a^4-17a^2b^2+16b^4\right)\cos(e+fx)\sqrt{d\sin(e+fx)}}{231a^4b^2df\left(a^2-b^2\right)(a+b\sin(e+fx))^{5/2}}+\frac{80\left(3a^2+2b^2\right)\mathrm{c}}{693a^3b^2df(}$$

```
[Out] 2/11*cos(f*x+e)^5*(d*sin(f*x+e))^(1/2)/a/d/f/(a+b*sin(f*x+e))^(11/2)-20/99*
(a^2-b^2)*cos(f*x+e)*(d*sin(f*x+e))^(1/2)/a^2/b^2/d/f/(a+b*sin(f*x+e))^(9/2
)+80/693*(3*a^2+2*b^2)*cos(f*x+e)*(d*sin(f*x+e))^(1/2)/a^3/b^2/d/f/(a+b*sin
(f*x+e))^(7/2)-4/231*(5*a^4-17*a^2*b^2+16*b^4)*cos(f*x+e)*(d*sin(f*x+e))^(1
/2)/a^4/b^2/(a^2-b^2)/d/f/(a+b*sin(f*x+e))^(5/2)-8/693*(5*a^6-22*a^4*b^2+65
*a^2*b^4-32*b^6)*cos(f*x+e)*(d*sin(f*x+e))^(1/2)/a^5/b^2/(a^2-b^2)^2/d/f/(a
+b*sin(f*x+e))^(3/2)+16/693*b*(93*a^4-93*a^2*b^2+32*b^4)*cos(f*x+e)/a^5/(a^
2-b^2)^3/f/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2)-16/693*b*(93*a^4-93*
a^2*b^2+32*b^4)*EllipticE(d^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(d*sin
(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-csc(f*x+e))/(a+b))^(1/2)*(a*(1+c
sc(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/a^7/(a-b)^2/(a+b)^(5/2)/f/d^(1/2)-16/693
*(45*a^4-48*a^3*b-69*a^2*b^2+24*a*b^3+32*b^4)*EllipticF(d^(1/2)*(a+b*sin(f*
x+e))^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-cs
c(f*x+e))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/a^6/(a-b)^
2/(a+b)^(5/2)/f/d^(1/2)
```

Rubi [A] time = 2.64, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2887, 2891, 3055, 2993, 2998, 2816, 2994}

$$\frac{16b\left(-93a^2b^2+93a^4+32b^4\right)\cos(e+fx)}{693a^5f\left(a^2-b^2\right)^3\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}-\frac{8\left(-22a^4b^2+65a^2b^4+5a^6-32b^6\right)\cos(e+fx)\sqrt{d\sin(e+}}{693a^5b^2df\left(a^2-b^2\right)^2(a+b\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(13/2)),x]
```

```
[Out] (2*Cos[e + f*x]^5*Sqrt[d*Sin[e + f*x]])/(11*a*d*f*(a + b*Sin[e + f*x])^(11/
2)) - (20*(a^2 - b^2)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(99*a^2*b^2*d*f*(a
 + b*Sin[e + f*x])^(9/2)) + (80*(3*a^2 + 2*b^2)*Cos[e + f*x]*Sqrt[d*Sin[e +
 f*x]])/(693*a^3*b^2*d*f*(a + b*Sin[e + f*x])^(7/2)) - (4*(5*a^4 - 17*a^2*b
^2 + 16*b^4)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(231*a^4*b^2*(a^2 - b^2)*d*
f*(a + b*Sin[e + f*x])^(5/2)) - (8*(5*a^6 - 22*a^4*b^2 + 65*a^2*b^4 - 32*b^
```

```
6)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(693*a^5*b^2*(a^2 - b^2)^2*d*f*(a + b
*Sin[e + f*x])^(3/2)) + (16*b*(93*a^4 - 93*a^2*b^2 + 32*b^4)*Cos[e + f*x])/
(693*a^5*(a^2 - b^2)^3*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (
16*b*(93*a^4 - 93*a^2*b^2 + 32*b^4)*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sq
rt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin
[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e
+ f*x])/(693*a^7*(a - b)^2*(a + b)^(5/2)*Sqrt[d]*f) - (16*(45*a^4 - 48*a^3*
b - 69*a^2*b^2 + 24*a*b^3 + 32*b^4)*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sq
rt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin
[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e
+ f*x])/(693*a^6*(a - b)^2*(a + b)^(5/2)*Sqrt[d]*f)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2887

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> -Simp[(g*(g*C
os[e + f*x])^(p - 1)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(m + 1))/(a*
d*f*(m + 1)), x] + Dist[(g^2*(2*m + 3))/(2*a*(m + 1)), Int[((g*Cos[e + f*x]
)^(p - 2)*(a + b*Sin[e + f*x])^(m + 1))/Sqrt[d*Sin[e + f*x]], x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && EqQ[m + p +
 1/2, 0]
```

Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{13/2}}\,dx = \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} + \frac{10\int \frac{\cos^4(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{11/2}}\,c}{11a}$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20\left(a^2-b^2\right)\cos(e+fx)\sqrt{d\sin(}}{99a^2b^2df(a+b\sin(e+fx)}$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20\left(a^2-b^2\right)\cos(e+fx)\sqrt{d\sin(}}{99a^2b^2df(a+b\sin(e+fx)}$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20\left(a^2-b^2\right)\cos(e+fx)\sqrt{d\sin(}}{99a^2b^2df(a+b\sin(e+fx)}$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20\left(a^2-b^2\right)\cos(e+fx)\sqrt{d\sin(}}{99a^2b^2df(a+b\sin(e+fx)}$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20\left(a^2-b^2\right)\cos(e+fx)\sqrt{d\sin(}}{99a^2b^2df(a+b\sin(e+fx)}$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20\left(a^2-b^2\right)\cos(e+fx)\sqrt{d\sin(}}{99a^2b^2df(a+b\sin(e+fx)}$$

Mathematica [C] time = 6.98, size = 1906, normalized size = 2.68

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(13/2))
,x]
```

```
[Out] (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((2*(a^4*Cos[e + f*x] - 2*a^2*b^2*Co
s[e + f*x] + b^4*Cos[e + f*x]))/(11*a*b^4*(a + b*Sin[e + f*x])^6) - (4*(18*
a^4*Cos[e + f*x] - 13*a^2*b^2*Cos[e + f*x] - 5*b^4*Cos[e + f*x]))/(99*a^2*b
^4*(a + b*Sin[e + f*x])^5) + (4*(189*a^4*Cos[e + f*x] - 3*a^2*b^2*Cos[e + f
```

```
*x] + 40*b^4*Cos[e + f*x]))/(693*a^3*b^4*(a + b*Sin[e + f*x])^4) - (4*(42*a
^6*Cos[e + f*x] - 37*a^4*b^2*Cos[e + f*x] - 17*a^2*b^4*Cos[e + f*x] + 16*b^
6*Cos[e + f*x]))/(231*a^4*b^4*(a^2 - b^2)*(a + b*Sin[e + f*x])^3) + (2*(63*
a^8*Cos[e + f*x] - 146*a^6*b^2*Cos[e + f*x] + 151*a^4*b^4*Cos[e + f*x] - 26
0*a^2*b^6*Cos[e + f*x] + 128*b^8*Cos[e + f*x]))/(693*a^5*b^4*(a^2 - b^2)^2*
(a + b*Sin[e + f*x])^2) - (16*(93*a^4*b^2*Cos[e + f*x] - 93*a^2*b^4*Cos[e +
 f*x] + 32*b^6*Cos[e + f*x]))/(693*a^6*(a^2 - b^2)^3*(a + b*Sin[e + f*x])))
)/(f*Sqrt[d*Sin[e + f*x]]) + (8*Sqrt[Sin[e + f*x]]*((4*a*(45*a^6 - 114*a^4*
b^2 + 101*a^2*b^4 - 32*b^6)*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a +
 b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x])
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + P
i/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqrt[Sin[e + f*x]]*Sqrt[
a + b*Sin[e + f*x]]) + 4*a*(-93*a^5*b + 93*a^3*b^3 - 32*a*b^5)*((Sqrt[((a +
 b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e +
Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e
+ f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^
2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
a])/((a + b)*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (Sqrt[((a + b)*
Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(
-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
ec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x
)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*
x]))/a])/(b*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])) + 2*(93*a^4*b^2 -
 93*a^2*b^4 + 32*b^6)*((Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(b*Sqrt[Sin[
e + f*x]]) + (I*Cos[(-e + Pi/2 - f*x)/2]*Csc[e + f*x]*EllipticE[I*ArcSinh[S
in[(-e + Pi/2 - f*x)/2]/Sqrt[Sin[e + f*x]]], (-2*a)/(-a - b)]*Sqrt[a + b*Si
n[e + f*x]])/(b*Sqrt[Cos[(-e + Pi/2 - f*x)/2]^2*Csc[e + f*x]]*Sqrt[(Csc[e +
 f*x]*(a + b*Sin[e + f*x]))/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(-e + Pi
/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2
*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/
a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqr
t[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (a*Sqrt[((a + b)*Cot[(-e + Pi/2
 - f*x)/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*
x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Si
n[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e +
 f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/(b*Sq
rt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])))/b)))/(693*a^6*(a - b)^3*(a + b
)^3*f*Sqrt[d*Sin[e + f*x]])
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{}{b^7 d\cos\left(fx+e\right)^8-\left(21\,a^2b^5+4\,b^7\right)d\cos\left(fx+e\right)^6+\left(35\,a^4b^3+63\,a^2b^5+6\,b^7\right)d\cos\left(fx+e\right)^4-\left(7\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e))^(13/2)/(d*sin(f*x+e))^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))*cos(f*x + e)^6/(b^7*
d*cos(f*x + e)^8 - (21*a^2*b^5 + 4*b^7)*d*cos(f*x + e)^6 + (35*a^4*b^3 + 63
*a^2*b^5 + 6*b^7)*d*cos(f*x + e)^4 - (7*a^6*b + 70*a^4*b^3 + 63*a^2*b^5 + 4
*b^7)*d*cos(f*x + e)^2 + (7*a^6*b + 35*a^4*b^3 + 21*a^2*b^5 + b^7)*d - (7*a
*b^6*d*cos(f*x + e)^6 - 7*(5*a^3*b^4 + 3*a*b^6)*d*cos(f*x + e)^4 + 7*(3*a^5
*b^2 + 10*a^3*b^4 + 3*a*b^6)*d*cos(f*x + e)^2 - (a^7 + 21*a^5*b^2 + 35*a^3*
b^4 + 7*a*b^6)*d)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^6}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{13}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e))^(13/2)/(d*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/((b*sin(f*x + e) + a)^(13/2)*sqrt(d*sin(f*x + e)))
, x)
```

maple [B] time = 2.76, size = 56846, normalized size = 79.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sin(f*x+e))^(13/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^6}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{13}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e))^(13/2)/(d*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/((b*sin(f*x + e) + a)^(13/2)*sqrt(d*sin(f*x + e)))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^6}{\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(13/2)),x)
```

```
[Out] int(cos(e + f*x)^6/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(13/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sin(f*x+e))**(13/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.1277 $\int \frac{(a+b\sin(e+fx))^2}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=159

$$\frac{\left(2a^2-b^2\right)\sqrt{\sin(2e+2fx)}\,F\left(\frac{1}{4}(4e-\pi)+fx\middle|2\right)}{3fg^2\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2\left(a^2+b^2\right)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}}+\frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}}$$

```
[Out] 4/3*a*b*(d*sin(f*x+e))^(3/2)/d^2/f/g/(g*cos(f*x+e))^(3/2)+2/3*(a^2+b^2)*(d*
sin(f*x+e))^(1/2)/d/f/g/(g*cos(f*x+e))^(3/2)-1/3*(2*a^2-b^2)*(sin(e+1/4*Pi+
f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*
f*x+2*e)^(1/2)/f/g^2/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 161, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2911, 2563, 3202, 457, 329, 237, 335, 275, 232}

$$-\frac{2\left(2a^2-b^2\right)\left(1-\csc^2(e+fx)\right)^{3/4}(d\sin(e+fx))^{3/2}F\left(\frac{1}{2}\csc^{-1}(\sin(e+fx))\middle|2\right)}{3d^2fg(g\cos(e+fx))^{3/2}}+\frac{2\left(a^2+b^2\right)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}}+\frac{4}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]]),x]
```

```
[Out] (2*(a^2 + b^2)*Sqrt[d*Sin[e + f*x]])/(3*d*f*g*(g*Cos[e + f*x])^(3/2)) + (4*
a*b*(d*Sin[e + f*x])^(3/2))/(3*d^2*f*g*(g*Cos[e + f*x])^(3/2)) - (2*(2*a^2
- b^2)*(1 - Csc[e + f*x]^2)^(3/4)*EllipticF[ArcCsc[Sin[e + f*x]]/2, 2]*(d*S
in[e + f*x])^(3/2))/(3*d^2*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3202

```
Int[(cos[(e_.) + (f_.)*(x_)]*(c_.))^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Dist[(ff*c^(2*IntPart[(m - 1)/2] + 1)*(c*Co
```

```
s[e + f*x])^(2*FracPart[(m - 1)/2]))/(f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2
]), Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x],
 x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !Int
egerQ[m]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx = \frac{(2ab)\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{5/2}}\,dx}{d} + \int \frac{a^2+b^2\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx$$

$$= \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} + \frac{\cos^2(e+fx)^{3/4}\,\text{Subst}\left(\int \frac{a^2+b^2x^2}{\sqrt{dx}\left(1-x^2\right)^{7/4}}\,dx, x,\right.}{fg(g\cos(e+fx))^{3/2}}$$

$$= \frac{2\left(a^2+b^2\right)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} - \frac{\left(\left(-2a^2+b^2\right)\right.}{}$$

$$= \frac{2\left(a^2+b^2\right)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} - \frac{\left(2\left(-2a^2+b\right.\right.}{}$$

$$= \frac{2\left(a^2+b^2\right)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} - \frac{\left(2\left(-2a^2+b\right.\right.}{}$$

$$= \frac{2\left(a^2+b^2\right)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} + \frac{\left(2\left(-2a^2+b\right.\right.}{}$$

$$= \frac{2\left(a^2+b^2\right)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} + \frac{\left(\left(-2a^2+b^2\right)\right.}{}$$

$$= \frac{2\left(a^2+b^2\right)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} - \frac{2\left(2a^2-b^2\right)}{}$$

Mathematica [C] time = 0.45, size = 127, normalized size = 0.80

$$\frac{2\tan(e+fx)\left(15a^2\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{5}{4};\sin^2(e+fx)\right)+b\sin(e+fx)\left(10a+3b\sin(e+fx)\cos^2(e+fx)^{3/4}\right.\right.}{15fg^2\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x
]]),x]
```

```
[Out] (2*(15*a^2*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[1/4, 7/4, 5/4, Sin[e +
f*x]^2] + b*Sin[e + f*x]*(10*a + 3*b*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2
F1[5/4, 7/4, 9/4, Sin[e + f*x]^2]*Sin[e + f*x]))*Tan[e + f*x])/(15*f*g^2*Sq
rt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{d\sin\left(fx+e\right)}}{dg^3\cos\left(fx+e\right)^3\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(g*cos(
f*x + e))*sqrt(d*sin(f*x + e))/(d*g^3*cos(f*x + e)^3*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e
))), x)
```

maple [B] time = 0.93, size = 371, normalized size = 2.33

$$\frac{\left(-2\sqrt{-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*(-2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*sin(f*
x+e)*a^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*sin(f*x
+e)*b^2+2*2^(1/2)*cos(f*x+e)*sin(f*x+e)*a*b+2^(1/2)*cos(f*x+e)*a^2+2^(1/2)*
cos(f*x+e)*b^2-2*2^(1/2)*sin(f*x+e)*a*b-2^(1/2)*a^2-2^(1/2)*b^2)*cos(f*x+e)
*sin(f*x+e)/(-1+cos(f*x+e))/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)*2^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{2}}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/((g*cos(e + f*x))^(5/2)*(d*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int((a + b*sin(e + f*x))^2/((g*cos(e + f*x))^(5/2)*(d*sin(e + f*x))^(1/2)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2),x
)
```

```
[Out] Timed out
```

3.1278 $\int \frac{(a+b\sin(e+fx))^2}{(g\cos(e+fx))^{7/2}\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=193

$$\frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}}+\frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}}-\frac{8abE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{5dfg^4\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{d\sin}}{5}$$

```
[Out] 8/5*a*b*(d*sin(f*x+e))^(3/2)/d^2/f/g^3/(g*cos(f*x+e))^(1/2)+2/5*(a+b*sin(f*
x+e))^2*(d*sin(f*x+e))^(1/2)/d/f/g/(g*cos(f*x+e))^(5/2)+8/5*a^2*(d*sin(f*x+
e))^(1/2)/d/f/g^3/(g*cos(f*x+e))^(1/2)+8/5*a*b*(sin(e+1/4*Pi+f*x)^2)^(1/2)/
sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)
*(d*sin(f*x+e))^(1/2)/d/f/g^4/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2888, 2838, 2563, 2571, 2572, 2639}

$$\frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}}+\frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}}-\frac{8abE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{5dfg^4\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{d\sin}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/((g*Cos[e + f*x])^(7/2)*Sqrt[d*Sin[e + f*x]]),x]
```

```
[Out] (8*a^2*Sqrt[d*Sin[e + f*x]])/(5*d*f*g^3*Sqrt[g*Cos[e + f*x]]) + (8*a*b*(d*S
in[e + f*x])^(3/2))/(5*d^2*f*g^3*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[d*Sin[e +
f*x]]*(a + b*Sin[e + f*x])^2)/(5*d*f*g*(g*Cos[e + f*x])^(5/2)) - (8*a*b*Sqr
t[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(5*d*f
*g^4*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
```

```
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2888

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(g*Co
s[e + f*x])^(p + 1)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^m)/(d*f*g*(2*
m + 1)), x] + Dist[(2*a*m)/(g^2*(2*m + 1)), Int[((g*Cos[e + f*x])^(p + 2)*(
a + b*Sin[e + f*x])^(m - 1))/Sqrt[d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, e
, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && EqQ[m + p + 3/2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(g\cos(e+fx))^{7/2}\sqrt{d\sin(e+fx)}}\,dx = \frac{2\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^2}{5dfg(g\cos(e+fx))^{5/2}} + \frac{(4a)\int \frac{a+b\sin(e+fx)}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}}{5g^2}$$

$$= \frac{2\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^2}{5dfg(g\cos(e+fx))^{5/2}} + \frac{\left(4a^2\right)\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}}{5g^2}$$

$$= \frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}} + \frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}} + \frac{2\sqrt{d\sin(e+fx)}\,(a}{5dfg(g\cos}$$

$$= \frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}} + \frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}} + \frac{2\sqrt{d\sin(e+fx)}\,(a}{5dfg(g\cos}$$

$$= \frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}} + \frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}} + \frac{2\sqrt{d\sin(e+fx)}\,(a}{5dfg(g\cos}$$

Mathematica [C] time = 0.66, size = 105, normalized size = 0.54

$$\frac{2\tan(e+fx)\left(3\left(b^2-4a^2\right)\sin^2(e+fx)+15a^2+10ab\sin(e+fx)\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};\sin^2(e+fx)\right)\right)}{15fg^2\sqrt{d\sin(e+fx)}\,(g\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/((g*Cos[e + f*x])^(7/2)*Sqrt[d*Sin[e + f*x
]]),x]
```

```
[Out] (2*(15*a^2 + 10*a*b*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[3/4, 9/4, 7/4,
 Sin[e + f*x]^2]*Sin[e + f*x] + 3*(-4*a^2 + b^2)*Sin[e + f*x]^2)*Tan[e + f*
x])/(15*f*g^2*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{d\sin\left(fx+e\right)}}{dg^4\cos\left(fx+e\right)^4\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(g*cos(
f*x + e))*sqrt(d*sin(f*x + e))/(d*g^4*cos(f*x + e)^4*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(g\cos\left(fx+e\right)\right)^{\frac{7}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/((g*cos(f*x + e))^(7/2)*sqrt(d*sin(f*x + e
))), x)
```

maple [B] time = 0.93, size = 615, normalized size = 3.19

$$\frac{\left(8\,\mathrm{EllipticE}\left(\sqrt{-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/5/f*(8*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/
2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^3*a*b-4
*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^3*a*b+8*Ellipti
cE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*a*b-4*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2*a*b-4*cos(f*x+e)^3*2^(1/2)*a*
b+4*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*a^2-cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*b^2
+2*cos(f*x+e)^2*2^(1/2)*a*b+sin(f*x+e)*2^(1/2)*a^2+sin(f*x+e)*2^(1/2)*b^2+2
*a*b*2^(1/2))*cos(f*x+e)/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(g\cos\left(fx+e\right)\right)^{\frac{7}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/((g*cos(f*x + e))^(7/2)*sqrt(d*sin(f*x + e
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sin\left(e+f\,x\right)\right)^2}{\left(g\,\cos\left(e+f\,x\right)\right)^{7/2}\,\sqrt{d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/((g*cos(e + f*x))^(7/2)*(d*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int((a + b*sin(e + f*x))^2/((g*cos(e + f*x))^(7/2)*(d*sin(e + f*x))^(1/2)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(g*cos(f*x+e))**(7/2)/(d*sin(f*x+e))**(1/2),x
)
```

```
[Out] Timed out
```

3.1279 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=76

$$-\frac{a^3\log(a+b\sin(c+dx))}{b^4d}+\frac{a^2\sin(c+dx)}{b^3d}-\frac{a\sin^2(c+dx)}{2b^2d}+\frac{\sin^3(c+dx)}{3bd}$$

```
[Out] -a^3*ln(a+b*sin(d*x+c))/b^4/d+a^2*sin(d*x+c)/b^3/d-1/2*a*sin(d*x+c)^2/b^2/d
+1/3*sin(d*x+c)^3/b/d
```

Rubi [A] time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^2\sin(c+dx)}{b^3d}-\frac{a^3\log(a+b\sin(c+dx))}{b^4d}-\frac{a\sin^2(c+dx)}{2b^2d}+\frac{\sin^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a^3*Log[a + b*Sin[c + d*x]])/(b^4*d)) + (a^2*Sin[c + d*x])/(b^3*d) - (a*
Sin[c + d*x]^2)/(2*b^2*d) + Sin[c + d*x]^3/(3*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{b^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^4d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2-ax+x^2-\frac{a^3}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^4d}$$
$$= -\frac{a^3\log(a+b\sin(c+dx))}{b^4d}+\frac{a^2\sin(c+dx)}{b^3d}-\frac{a\sin^2(c+dx)}{2b^2d}+\frac{\sin^3(c+dx)}{3bd}$$

Mathematica [A] time = 0.20, size = 66, normalized size = 0.87

$$\frac{-6a^3\log(a+b\sin(c+dx))+6a^2b\sin(c+dx)-3ab^2\sin^2(c+dx)+2b^3\sin^3(c+dx)}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-6*a^3*Log[a + b*Sin[c + d*x]] + 6*a^2*b*Sin[c + d*x] - 3*a*b^2*Sin[c + d*
x]^2 + 2*b^3*Sin[c + d*x]^3)/(6*b^4*d)
```

fricas [A] time = 1.00, size = 71, normalized size = 0.93

$$\frac{3\,ab^2\cos(dx+c)^2-6\,a^3\log(b\sin(dx+c)+a)-2\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)}{6\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*b^2*cos(d*x + c)^2 - 6*a^3*log(b*sin(d*x + c) + a) - 2*(b^3*cos(d*
x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c))/(b^4*d)
```

giac [A] time = 0.16, size = 68, normalized size = 0.89

$$-\frac{\frac{6\,a^3\log(|b\sin(dx+c)+a|)}{b^4}-\frac{2\,b^2\sin(dx+c)^3-3\,ab\sin(dx+c)^2+6\,a^2\sin(dx+c)}{b^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*a^3*log(abs(b*sin(d*x + c) + a))/b^4 - (2*b^2*sin(d*x + c)^3 - 3*a*
b*sin(d*x + c)^2 + 6*a^2*sin(d*x + c))/b^3)/d
```

maple [A] time = 0.18, size = 73, normalized size = 0.96

$$-\frac{a^3 \ln\left(a+b\sin\left(dx+c\right)\right)}{b^4 d}+\frac{a^2 \sin\left(dx+c\right)}{b^3 d}-\frac{a\left(\sin^2\left(dx+c\right)\right)}{2b^2 d}+\frac{\sin^3\left(dx+c\right)}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -a^3*ln(a+b*sin(d*x+c))/b^4/d+a^2*sin(d*x+c)/b^3/d-1/2*a*sin(d*x+c)^2/b^2/d
+1/3*sin(d*x+c)^3/b/d
```

maxima [A] time = 0.32, size = 67, normalized size = 0.88

$$-\frac{\frac{6\,a^3 \log\left(b\sin\left(dx+c\right)+a\right)}{b^4}-\frac{2\,b^2 \sin\left(dx+c\right)^3-3\,ab\sin\left(dx+c\right)^2+6\,a^2 \sin\left(dx+c\right)}{b^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(6*a^3*log(b*sin(d*x + c) + a)/b^4 - (2*b^2*sin(d*x + c)^3 - 3*a*b*sin
(d*x + c)^2 + 6*a^2*sin(d*x + c))/b^3)/d
```

mupad [B] time = 0.07, size = 64, normalized size = 0.84

$$\frac{\frac{\sin\left(c+d\,x\right)^3}{3\,b}-\frac{a^3\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{b^4}-\frac{a\,\sin\left(c+d\,x\right)^2}{2\,b^2}+\frac{a^2\,\sin\left(c+d\,x\right)}{b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^3/(3*b) - (a^3*log(a + b*sin(c + d*x)))/b^4 - (a*sin(c + d*x)
^2)/(2*b^2) + (a^2*sin(c + d*x))/b^3)/d
```

sympy [A] time = 1.71, size = 105, normalized size = 1.38

$$\begin{cases} \frac{x\sin^3(c)\cos(c)}{a} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sin^4(c+dx)}{4ad} & \text{for } b = 0 \\ \frac{x\sin^3(c)\cos(c)}{a+b\sin(c)} & \text{for } d = 0 \\ -\frac{a^3\log\left(\frac{a}{b}+\sin(c+dx)\right)}{b^4d} + \frac{a^2\sin(c+dx)}{b^3d} + \frac{a\cos^2(c+dx)}{2b^2d} + \frac{\sin^3(c+dx)}{3bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*sin(c)**3*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)**4/(4*
a*d), Eq(b, 0)), (x*sin(c)**3*cos(c)/(a + b*sin(c)), Eq(d, 0)), (-a**3*log(
a/b + sin(c + d*x))/(b**4*d) + a**2*sin(c + d*x)/(b**3*d) + a*cos(c + d*x)*
*2/(2*b**2*d) + sin(c + d*x)**3/(3*b*d), True))
```

3.1280 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{a^2\log(a+b\sin(c+dx))}{b^3d}-\frac{a\sin(c+dx)}{b^2d}+\frac{\sin^2(c+dx)}{2bd}$$

```
[Out] a^2*ln(a+b*sin(d*x+c))/b^3/d-a*sin(d*x+c)/b^2/d+1/2*sin(d*x+c)^2/b/d
```

Rubi [A] time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^2\log(a+b\sin(c+dx))}{b^3d}-\frac{a\sin(c+dx)}{b^2d}+\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a^2*Log[a + b*Sin[c + d*x]])/(b^3*d) - (a*Sin[c + d*x])/(b^2*d) + Sin[c +
d*x]^2/(2*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{b^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(-a+x+\frac{a^2}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{a^2\log(a+b\sin(c+dx))}{b^3d} - \frac{a\sin(c+dx)}{b^2d} + \frac{\sin^2(c+dx)}{2bd}$$

Mathematica [A] time = 0.11, size = 49, normalized size = 0.89

$$\frac{2a^2\log(a+b\sin(c+dx)) - 2ab\sin(c+dx) + b^2\sin^2(c+dx)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a^2*Log[a + b*Sin[c + d*x]] - 2*a*b*Sin[c + d*x] + b^2*Sin[c + d*x]^2)/(
2*b^3*d)
```

fricas [A] time = 0.81, size = 47, normalized size = 0.85

$$-\frac{b^2\cos(dx+c)^2 - 2a^2\log(b\sin(dx+c)+a) + 2ab\sin(dx+c)}{2b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(d*x + c)^2 - 2*a^2*log(b*sin(d*x + c) + a) + 2*a*b*sin(d*x +
c))/(b^3*d)
```

giac [A] time = 0.16, size = 50, normalized size = 0.91

$$\frac{\frac{2a^2\log(|b\sin(dx+c)+a|)}{b^3} + \frac{b\sin(dx+c)^2 - 2a\sin(dx+c)}{b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(b*sin(d*x + c) + a))/b^3 + (b*sin(d*x + c)^2 - 2*a*sin(d
*x + c))/b^2)/d
```

maple [A] time = 0.18, size = 54, normalized size = 0.98

$$\frac{\ln \left(a+b \sin \left(d x+c\right)\right) a^{2}}{d \,b^{3}}-\frac{a \sin \left(d x+c\right)}{b^{2} d}+\frac{\sin^{2}\left(d x+c\right)}{2 b d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/b^3*ln(a+b*sin(d*x+c))*a^2-a*sin(d*x+c)/b^2/d+1/2*sin(d*x+c)^2/b/d
```

maxima [A] time = 0.31, size = 49, normalized size = 0.89

$$\frac{\frac{2 \, a^{2} \log\left(b \sin\left(d x+c\right)+a\right)}{b^{3}}+\frac{b \sin\left(d x+c\right)^{2}-2 \, a \sin\left(d x+c\right)}{b^{2}}}{2 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*log(b*sin(d*x + c) + a)/b^3 + (b*sin(d*x + c)^2 - 2*a*sin(d*x +
c))/b^2)/d
```

mupad [B] time = 0.07, size = 47, normalized size = 0.85

$$\frac{2\,a^2\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)+b^2\,{\sin\left(c+d\,x\right)}^2-2\,a\,b\,\sin\left(c+d\,x\right)}{2\,b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] (2*a^2*log(a + b*sin(c + d*x)) + b^2*sin(c + d*x)^2 - 2*a*b*sin(c + d*x))/(
2*b^3*d)
```

sympy [A] time = 1.05, size = 87, normalized size = 1.58

$$\begin{cases} \frac{x\sin^2(c)\cos(c)}{a} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sin^3(c+dx)}{3ad} & \text{for } b = 0 \\ \frac{x\sin^2(c)\cos(c)}{a+b\sin(c)} & \text{for } d = 0 \\ \frac{a^2\log\left(\frac{a}{b}+\sin(c+dx)\right)}{b^3d} - \frac{a\sin(c+dx)}{b^2d} - \frac{\cos^2(c+dx)}{2bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*sin(c)**2*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)**3/(3*
a*d), Eq(b, 0)), (x*sin(c)**2*cos(c)/(a + b*sin(c)), Eq(d, 0)), (a**2*log(a
/b + sin(c + d*x))/(b**3*d) - a*sin(c + d*x)/(b**2*d) - cos(c + d*x)**2/(2*
b*d), True))
```

3.1281 $\int \frac{\cos(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\sin(c+dx)}{bd}-\frac{a\log(a+b\sin(c+dx))}{b^2d}$$

```
[Out] -a*ln(a+b*sin(d*x+c))/b^2/d+sin(d*x+c)/b/d
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 43}

$$\frac{\sin(c+dx)}{bd}-\frac{a\log(a+b\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Log[a + b*Sin[c + d*x]])/(b^2*d)) + Sin[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{b(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{a}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= -\frac{a\log(a+b\sin(c+dx))}{b^2d} + \frac{\sin(c+dx)}{bd}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.97

$$-\frac{\frac{a\log(a+b\sin(c+dx))}{b^2} - \frac{\sin(c+dx)}{b}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] -(((a*Log[a + b*Sin[c + d*x]])/b^2 - Sin[c + d*x]/b)/d)
```

fricas [A] time = 1.03, size = 31, normalized size = 0.91

$$-\frac{a\log\left(b\sin\left(dx+c\right)+a\right)-b\sin\left(dx+c\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] -(a*log(b*sin(d*x + c) + a) - b*sin(d*x + c))/(b^2*d)
```

giac [A] time = 0.16, size = 34, normalized size = 1.00

$$-\frac{\frac{a\log(|b\sin(dx+c)+a|)}{b^2} - \frac{\sin(dx+c)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] -(a*log(abs(b*sin(d*x + c) + a))/b^2 - sin(d*x + c)/b)/d
```

maple [A] time = 0.12, size = 35, normalized size = 1.03

$$-\frac{a\ln\left(a+b\sin\left(dx+c\right)\right)}{d\,b^{2}}+\frac{\sin\left(dx+c\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/b^2*a*ln(a+b*sin(d*x+c))+sin(d*x+c)/b/d
```

maxima [A] time = 0.34, size = 33, normalized size = 0.97

$$-\frac{\frac{a\log(b\sin(dx+c)+a)}{b^{2}}-\frac{\sin(dx+c)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*log(b*sin(d*x + c) + a)/b^2 - sin(d*x + c)/b)/d
```

mupad [B] time = 0.06, size = 31, normalized size = 0.91

$$-\frac{a\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)-b\,\sin\left(c+d\,x\right)}{b^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] -(a*log(a + b*sin(c + d*x)) - b*sin(c + d*x))/(b^2*d)
```

sympy [A] time = 0.67, size = 66, normalized size = 1.94

$$\begin{cases} \frac{x\sin\left(c\right)\cos\left(c\right)}{a} & \text{for}\: b=0 \wedge d=0 \\ -\frac{\cos^{2}\left(c+dx\right)}{2ad} & \text{for}\: b=0 \\ \frac{x\sin\left(c\right)\cos\left(c\right)}{a+b\sin\left(c\right)} & \text{for}\: d=0 \\ -\frac{a\log\left(\frac{a}{b}+\sin\left(c+dx\right)\right)}{b^{2}d}+\frac{\sin\left(c+dx\right)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*sin(c)*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (-cos(c + d*x)**2/(2*a*
d), Eq(b, 0)), (x*sin(c)*cos(c)/(a + b*sin(c)), Eq(d, 0)), (-a*log(a/b + si
n(c + d*x))/(b**2*d) + sin(c + d*x)/(b*d), True))
```

3.1282 $\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b\sin(c+dx))}{ad}$$

```
[Out] ln(sin(d*x+c))/a/d-ln(a+b*sin(d*x+c))/a/d
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2721, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx)\right)}{ad}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.00

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

fricas [A] time = 0.92, size = 31, normalized size = 0.91

$$-\frac{\log\left(b\sin\left(dx+c\right)+a\right) - \log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] -(log(b*sin(d*x + c) + a) - log(-1/2*sin(d*x + c)))/(a*d)
```

giac [A] time = 0.15, size = 35, normalized size = 1.03

$$-\frac{\frac{\log(|b\sin(dx+c)+a|)}{a} - \frac{\log(|\sin(dx+c)|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] -(log(abs(b*sin(d*x + c) + a))/a - log(abs(sin(d*x + c)))/a)/d
```

maple [A] time = 0.22, size = 35, normalized size = 1.03

$$\frac{\ln\left(\sin\left(dx+c\right)\right)}{ad} - \frac{\ln\left(a+b\sin\left(dx+c\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] ln(sin(d*x+c))/a/d-1/d/a*ln(a+b*sin(d*x+c))
```

maxima [A] time = 0.31, size = 33, normalized size = 0.97

$$-\frac{\frac{\log(b\sin(dx+c)+a)}{a}-\frac{\log(\sin(dx+c))}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(b*sin(d*x + c) + a)/a - log(sin(d*x + c))/a)/d
```

mupad [B] time = 11.83, size = 48, normalized size = 1.41

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)) - log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*
x)/2)^2))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.1283 $\int \frac{\cot(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=50

$$-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{b\log(a+b\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

```
[Out] -csc(d*x+c)/a/d-b*ln(sin(d*x+c))/a^2/d+b*ln(a+b*sin(d*x+c))/a^2/d
```

Rubi [A] time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2833, 12, 44}

$$-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{b\log(a+b\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) + (b*Log[a + b*Sin[c
+ d*x]])/(a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{b\,\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{b\log(a+b\sin(c+dx))}{a^2d}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 1.00

$$-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{b\log(a+b\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) + (b*Log[a + b*Sin[c
+ d*x]])/(a^2*d)
```

fricas [A] time = 0.78, size = 56, normalized size = 1.12

$$\frac{b\log\left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)-b\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-a}{a^2d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (b*log(b*sin(d*x + c) + a)*sin(d*x + c) - b*log(1/2*sin(d*x + c))*sin(d*x +
 c) - a)/(a^2*d*sin(d*x + c))
```

giac [A] time = 0.15, size = 49, normalized size = 0.98

$$\frac{\frac{b\log(|b\sin(dx+c)+a|)}{a^2}-\frac{b\log(|\sin(dx+c)|)}{a^2}-\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(b*sin(d*x + c) + a))/a^2 - b*log(abs(sin(d*x + c)))/a^2 - 1/(a*s
in(d*x + c)))/d
```

maple [A] time = 0.19, size = 35, normalized size = 0.70

$$-\frac{\csc \left(dx+c\right)}{ad}+\frac{b \ln \left(a \csc \left(dx+c\right)+b\right)}{d\, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -csc(d*x+c)/a/d+1/d/a^2*b*ln(a*csc(d*x+c)+b)
```

maxima [A] time = 0.34, size = 47, normalized size = 0.94

$$\frac{\frac{b \log\left(b \sin\left(dx+c\right)+a\right)}{a^{2}}-\frac{b \log\left(\sin\left(dx+c\right)\right)}{a^{2}}-\frac{1}{a \sin\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log(b*sin(d*x + c) + a)/a^2 - b*log(sin(d*x + c))/a^2 - 1/(a*sin(d*x + c
)))/d
```

mupad [B] time = 11.81, size = 89, normalized size = 1.78

$$\frac{b\,\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{a^2\,d}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{1}{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (b*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(a^2*d) - (b*l
og(tan(c/2 + (d*x)/2)))/(a^2*d) - (tan(c/2 + (d*x)/2)/2 + 1/(2*tan(c/2 + (d
*x)/2)))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos \left(c+dx\right) \csc ^{2}\left(c+dx\right)}{a+b \sin \left(c+dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

3.1284 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{b^2\log(\sin(c+dx))}{a^3d}-\frac{b^2\log(a+b\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

```
[Out] b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d+b^2*ln(sin(d*x+c))/a^3/d-b^2*ln(a+b
*sin(d*x+c))/a^3/d
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$\frac{b^2\log(\sin(c+dx))}{a^3d}-\frac{b^2\log(a+b\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) + (b^2*Log[Sin[c + d*x]])
/(a^3*d) - (b^2*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^3}{x^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{b^2\,\text{Subst}\left(\int \frac{1}{x^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^2\,\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{1}{a^2x^2}+\frac{1}{a^3x}-\frac{1}{a^3(a+x)}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}+\frac{b^2\log(\sin(c+dx))}{a^3d}-\frac{b^2\log(a+b\sin(c+dx))}{a^3d}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 1.00

$$\frac{b^2\log(\sin(c+dx))}{a^3d}-\frac{b^2\log(a+b\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) + (b^2*Log[Sin[c + d*x]])
/(a^3*d) - (b^2*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

fricas [A] time = 0.91, size = 100, normalized size = 1.39

$$-\frac{2\,ab\sin(dx+c)-a^2+2\left(b^2\cos(dx+c)^2-b^2\right)\log(b\sin(dx+c)+a)-2\left(b^2\cos(dx+c)^2-b^2\right)\log\left(-\frac{1}{2}\sin(dx+c)\right)}{2\left(a^3d\cos(dx+c)^2-a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*sin(d*x + c) - a^2 + 2*(b^2*cos(d*x + c)^2 - b^2)*log(b*sin(d*x
 + c) + a) - 2*(b^2*cos(d*x + c)^2 - b^2)*log(-1/2*sin(d*x + c)))/(a^3*d*co
s(d*x + c)^2 - a^3*d)
```

giac [A] time = 0.16, size = 71, normalized size = 0.99

$$-\frac{\frac{2\,b^2\log(|b\sin(dx+c)+a|)}{a^3}-\frac{2\,b^2\log(|\sin(dx+c)|)}{a^3}-\frac{2\,ab\sin(dx+c)-a^2}{a^3\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*log(abs(b*sin(d*x + c) + a))/a^3 - 2*b^2*log(abs(sin(d*x + c)))
/a^3 - (2*a*b*sin(d*x + c) - a^2)/(a^3*sin(d*x + c)^2))/d
```

maple [A] time = 0.25, size = 73, normalized size = 1.01

$$-\frac{b^2\ln(a+b\sin(dx+c))}{a^3d}-\frac{1}{2da\sin(dx+c)^2}+\frac{b^2\ln(\sin(dx+c))}{a^3d}+\frac{b}{d\,a^2\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -b^2*ln(a+b*sin(d*x+c))/a^3/d-1/2/d/a/sin(d*x+c)^2+b^2*ln(sin(d*x+c))/a^3/d
+1/d/a^2*b/sin(d*x+c)
```

maxima [A] time = 0.33, size = 66, normalized size = 0.92

$$-\frac{\frac{2\,b^2\log(b\sin(dx+c)+a)}{a^3}-\frac{2\,b^2\log(\sin(dx+c))}{a^3}-\frac{2\,b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*log(b*sin(d*x + c) + a)/a^3 - 2*b^2*log(sin(d*x + c))/a^3 - (2*
b*sin(d*x + c) - a)/(a^2*sin(d*x + c)^2))/d
```

mupad [B] time = 11.79, size = 132, normalized size = 1.83

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{b^2\,\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{a^3\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a}{2}-2\,b\tan\left(\frac{c}{2}+\right.\right.}{4\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(2*a^2*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - (b^2*log(
a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(a^3*d) - (cot(c/2 +
(d*x)/2)^2*(a/2 - 2*b*tan(c/2 + (d*x)/2)))/(4*a^2*d) + (b^2*log(tan(c/2 + (
d*x)/2)))/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

3.1285 $\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=235

$$-\frac{a\left(4a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8b^4d}+\frac{\left(5a^2-b^2\right)\sin^2(c+dx)\cos(c+dx)}{15b^3d}-\frac{2a^4\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}$$

```
[Out] 1/8*a*(8*a^4-4*a^2*b^2-b^4)*x/b^6+1/15*(15*a^4-5*a^2*b^2-2*b^4)*cos(d*x+c)/
b^5/d-1/8*a*(4*a^2-b^2)*cos(d*x+c)*sin(d*x+c)/b^4/d+1/15*(5*a^2-b^2)*cos(d*
x+c)*sin(d*x+c)^2/b^3/d-1/4*a*cos(d*x+c)*sin(d*x+c)^3/b^2/d+1/5*cos(d*x+c)*
sin(d*x+c)^4/b/d-2*a^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^
2-b^2)^(1/2)/b^6/d
```

Rubi [A] time = 0.91, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3050, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{\left(-5a^2b^2+15a^4-2b^4\right)\cos(c+dx)}{15b^5d}-\frac{2a^4\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}+\frac{\left(5a^2-b^2\right)\sin^2(c+dx)\cos(c+dx)}{15b^3d}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(8*a^4 - 4*a^2*b^2 - b^4)*x)/(8*b^6) - (2*a^4*Sqrt[a^2 - b^2]*ArcTan[(b
+ a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*d) + ((15*a^4 - 5*a^2*b^2 - 2*
b^4)*Cos[c + d*x])/(15*b^5*d) - (a*(4*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])
/(8*b^4*d) + ((5*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^2)/(15*b^3*d) - (a*Co
s[c + d*x]*Sin[c + d*x]^3)/(4*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(5*b*d
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sin^4(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{\cos(c+dx)\sin^4(c+dx)}{5bd} + \frac{\int \frac{\sin^3(c+dx)\left(-4a+b\sin(c+dx)+5a\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{5b}$$
$$= -\frac{a\cos(c+dx)\sin^3(c+dx)}{4b^2d} + \frac{\cos(c+dx)\sin^4(c+dx)}{5bd} + \frac{\int \frac{\sin^2(c+dx)\left(15a^2-ab\sin\right.}{a+b}$$
$$= \frac{\left(5a^2-b^2\right)\cos(c+dx)\sin^2(c+dx)}{15b^3d} - \frac{a\cos(c+dx)\sin^3(c+dx)}{4b^2d} + \frac{\cos(c+dx)}{5}$$
$$= -\frac{a\left(4a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^4d} + \frac{\left(5a^2-b^2\right)\cos(c+dx)\sin^2(c+dx)}{15b^3d} -$$
$$= \frac{\left(15a^4-5a^2b^2-2b^4\right)\cos(c+dx)}{15b^5d} - \frac{a\left(4a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^4d} + \frac{\left(5a\right.}{}$$
$$= \frac{a\left(8a^4-4a^2b^2-b^4\right)x}{8b^6} + \frac{\left(15a^4-5a^2b^2-2b^4\right)\cos(c+dx)}{15b^5d} - \frac{a\left(4a^2-b^2\right)\cos(c}{8b}$$
$$= \frac{a\left(8a^4-4a^2b^2-b^4\right)x}{8b^6} + \frac{\left(15a^4-5a^2b^2-2b^4\right)\cos(c+dx)}{15b^5d} - \frac{a\left(4a^2-b^2\right)\cos(c}{8b}$$
$$= \frac{a\left(8a^4-4a^2b^2-b^4\right)x}{8b^6} + \frac{\left(15a^4-5a^2b^2-2b^4\right)\cos(c+dx)}{15b^5d} - \frac{a\left(4a^2-b^2\right)\cos(c}{8b}$$
$$= \frac{a\left(8a^4-4a^2b^2-b^4\right)x}{8b^6} - \frac{2a^4\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d} + \frac{\left(15a^4-5a^2b^2\right.}{1}$$

Mathematica [A] time = 1.77, size = 177, normalized size = 0.75

$$\frac{-10\left(4a^2b^3+b^5\right)\cos(3(c+dx)) - 960a^4\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + 15a\left(-8a^2b^2\sin(2(c+dx)) + 4\left(8a^4-\right.\right.}{480b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-960*a^4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
- 60*b*(-8*a^4 + 2*a^2*b^2 + b^4)*Cos[c + d*x] - 10*(4*a^2*b^3 + b^5)*Cos[3
*(c + d*x)] + 6*b^5*Cos[5*(c + d*x)] + 15*a*(4*(8*a^4 - 4*a^2*b^2 - b^4)*(c
 + d*x) - 8*a^2*b^2*Sin[2*(c + d*x)] + b^4*Sin[4*(c + d*x)]))/(480*b^6*d)
```

fricas [A] time = 0.72, size = 427, normalized size = 1.82

$$\left[\frac{24\,b^5\cos(dx+c)^5+120\,a^4b\cos(dx+c)+60\,\sqrt{-a^2+b^2}\,a^4\log\left(\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/120*(24*b^5*cos(d*x + c)^5 + 120*a^4*b*cos(d*x + c) + 60*sqrt(-a^2 + b^2
)*a^4*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 +
2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos
(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 40*(a^2*b^3 + b^5)*cos(d*x
 + c)^3 + 15*(8*a^5 - 4*a^3*b^2 - a*b^4)*d*x + 15*(2*a*b^4*cos(d*x + c)^3 -
 (4*a^3*b^2 + a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^6*d), 1/120*(24*b^5*cos
(d*x + c)^5 + 120*a^4*b*cos(d*x + c) + 120*sqrt(a^2 - b^2)*a^4*arctan(-(a*s
in(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 40*(a^2*b^3 + b^5)*cos(d
*x + c)^3 + 15*(8*a^5 - 4*a^3*b^2 - a*b^4)*d*x + 15*(2*a*b^4*cos(d*x + c)^3
 - (4*a^3*b^2 + a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^6*d)]
```

giac [B] time = 0.17, size = 467, normalized size = 1.99

$$\frac{\frac{15\left(8\,a^5-4\,a^3b^2-ab^4\right)(dx+c)}{b^6}-\frac{240\left(a^6-a^4b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6}+\frac{2\left(60\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-15\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+\cdots\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(15*(8*a^5 - 4*a^3*b^2 - a*b^4)*(d*x + c)/b^6 - 240*(a^6 - a^4*b^2)*(
pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) +
b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 2*(60*a^3*b*tan(1/2*d*x + 1/2*
c)^9 - 15*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*a^4*tan(1/2*d*x + 1/2*c)^8 - 1
20*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 90*a
*b^3*tan(1/2*d*x + 1/2*c)^7 + 480*a^4*tan(1/2*d*x + 1/2*c)^6 - 240*a^2*b^2*
tan(1/2*d*x + 1/2*c)^6 - 240*b^4*tan(1/2*d*x + 1/2*c)^6 + 720*a^4*tan(1/2*d
```

```
*x + 1/2*c)^4 - 160*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 80*b^4*tan(1/2*d*x + 1
/2*c)^4 - 120*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 90*a*b^3*tan(1/2*d*x + 1/2*c)^
3 + 480*a^4*tan(1/2*d*x + 1/2*c)^2 - 80*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 80
*b^4*tan(1/2*d*x + 1/2*c)^2 - 60*a^3*b*tan(1/2*d*x + 1/2*c) + 15*a*b^3*tan(
1/2*d*x + 1/2*c) + 120*a^4 - 40*a^2*b^2 - 16*b^4)/((tan(1/2*d*x + 1/2*c)^2
+ 1)^5*b^5))/d
```

maple [B] time = 0.32, size = 871, normalized size = 3.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/4/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))+2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*
tan(1/2*d*x+1/2*c)^8*a^4-2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2
*c)^8*a^2+2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a^3-4/d/b
/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6+4/3/d/b/(1+tan(1/2*d*x+1/2
*c)^2)^5*tan(1/2*d*x+1/2*c)^4-4/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*
x+1/2*c)^2+2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*a^4-2/3/d/b^3/(1+tan(1/2*d*x+
1/2*c)^2)^5*a^2-1/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^3+2/d/b^6*arctan(tan(1
/2*d*x+1/2*c))*a^5-1/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^
9*a-4/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^2-1/d/b^4/(
1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a^3+1/4/d/b^2/(1+tan(1/2*d*x+1
/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a+1/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*
d*x+1/2*c)^9*a^3-4/15/d/b/(1+tan(1/2*d*x+1/2*c)^2)^5-2/d/b^4/(1+tan(1/2*d*x
+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a^3-3/2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*
tan(1/2*d*x+1/2*c)^3*a-8/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2
*c)^4*a^2-4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6*a^2+8/d/b
^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^4-2/d*a^4*(a^2-b^2)^(1
/2)/b^6*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+3/2/d/b^2/
(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a+8/d/b^5/(1+tan(1/2*d*x+1/
2*c)^2)^5*tan(1/2*d*x+1/2*c)^6*a^4+12/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(
1/2*d*x+1/2*c)^4*a^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.39, size = 376, normalized size = 1.60

$$\frac{a^4\cos(c+dx)}{b^5 d}-\frac{\frac{\cos(c+dx)}{8}+\frac{\cos(3c+3dx)}{48}-\frac{\cos(5c+5dx)}{80}}{bd}-\frac{\frac{a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{4}-\frac{a\sin(4c+4dx)}{32}}{b^2 d}-\frac{\frac{a^2\cos(c+dx)}{4}+\frac{a^2\cos(3c+3d}{12}}{b^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^4)/(a + b*sin(c + d*x)),x)
```

```
[Out] (a^4*cos(c + d*x))/(b^5*d) - (cos(c + d*x)/8 + cos(3*c + 3*d*x)/48 - cos(5*
c + 5*d*x)/80)/(b*d) - ((a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 -
 (a*sin(4*c + 4*d*x))/32)/(b^2*d) - ((a^2*cos(c + d*x))/4 + (a^2*cos(3*c +
3*d*x))/12)/(b^3*d) - (a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (a
^3*sin(2*c + 2*d*x))/4)/(b^4*d) + (2*a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/(b^6*d) - (2*a^4*atanh((2*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/
2) - a^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(b^2
 - a^2)^(1/2))/(a^3*cos(c/2 + (d*x)/2) - 2*b^3*sin(c/2 + (d*x)/2) - a*b^2*c
os(c/2 + (d*x)/2) + 2*a^2*b*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2))/(b^6*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1286 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=191

$$-\frac{a\left(3a^2-b^2\right)\cos(c+dx)}{3b^4d}+\frac{\left(4a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8b^3d}-\frac{x\left(8a^4-4a^2b^2-b^4\right)}{8b^5}+\frac{2a^3\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}$$

```
[Out] -1/8*(8*a^4-4*a^2*b^2-b^4)*x/b^5-1/3*a*(3*a^2-b^2)*cos(d*x+c)/b^4/d+1/8*(4*
a^2-b^2)*cos(d*x+c)*sin(d*x+c)/b^3/d-1/3*a*cos(d*x+c)*sin(d*x+c)^2/b^2/d+1/
4*cos(d*x+c)*sin(d*x+c)^3/b/d+2*a^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^
2)^(1/2))*(a^2-b^2)^(1/2)/b^5/d
```

Rubi [A] time = 0.66, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3050, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{a\left(3a^2-b^2\right)\cos(c+dx)}{3b^4d}+\frac{2a^3\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}+\frac{\left(4a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8b^3d}-\frac{x\left(-4a^2b^2+\dots\right)}{8b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((8*a^4 - 4*a^2*b^2 - b^4)*x)/(8*b^5) + (2*a^3*Sqrt[a^2 - b^2]*ArcTan[(b +
 a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*d) - (a*(3*a^2 - b^2)*Cos[c + d
*x])/(3*b^4*d) + ((4*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*b^3*d) - (a*C
os[c + d*x]*Sin[c + d*x]^2)/(3*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*
d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
```

```
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sin^3(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} + \frac{\int \frac{\sin^2(c+dx)\left(-3a+b\sin(c+dx)+4a\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{4b}$$

$$= -\frac{a\cos(c+dx)\sin^2(c+dx)}{3b^2d} + \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} + \frac{\int \frac{\sin(c+dx)\left(8a^2-ab\sin(c\right.}{a+b\,s}}{}$$

$$= \frac{\left(4a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d} - \frac{a\cos(c+dx)\sin^2(c+dx)}{3b^2d} + \frac{\cos(c+dx)\,s}{4b}$$

$$= -\frac{a\left(3a^2-b^2\right)\cos(c+dx)}{3b^4d} + \frac{\left(4a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d} - \frac{a\cos(c+dx)}{3b}$$

$$= -\frac{\left(8a^4-4a^2b^2-b^4\right)x}{8b^5} - \frac{a\left(3a^2-b^2\right)\cos(c+dx)}{3b^4d} + \frac{\left(4a^2-b^2\right)\cos(c+dx)\sin(}{8b^3d}$$

$$= -\frac{\left(8a^4-4a^2b^2-b^4\right)x}{8b^5} - \frac{a\left(3a^2-b^2\right)\cos(c+dx)}{3b^4d} + \frac{\left(4a^2-b^2\right)\cos(c+dx)\sin(}{8b^3d}$$

$$= -\frac{\left(8a^4-4a^2b^2-b^4\right)x}{8b^5} - \frac{a\left(3a^2-b^2\right)\cos(c+dx)}{3b^4d} + \frac{\left(4a^2-b^2\right)\cos(c+dx)\sin(}{8b^3d}$$

$$= -\frac{\left(8a^4-4a^2b^2-b^4\right)x}{8b^5} + \frac{2a^3\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5d} - \frac{a\left(3a^2-b^2\right)\mathrm{co}}{3b^4d}$$

Mathematica [A] time = 1.11, size = 146, normalized size = 0.76

$$\frac{24a^2b^2\sin(2(c+dx))+24ab\left(b^2-4a^2\right)\cos(c+dx)-12\left(8a^4-4a^2b^2-b^4\right)(c+dx)+192a^3\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\,\mathrm{t}}{-}\right.}{96b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-12*(8*a^4 - 4*a^2*b^2 - b^4)*(c + d*x) + 192*a^3*Sqrt[a^2 - b^2]*ArcTan[(
b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 24*a*b*(-4*a^2 + b^2)*Cos[c + d*
x] + 8*a*b^3*Cos[3*(c + d*x)] + 24*a^2*b^2*Sin[2*(c + d*x)] - 3*b^4*Sin[4*(
c + d*x)])/(96*b^5*d)
```

fricas [A] time = 0.86, size = 380, normalized size = 1.99

$$\left[\frac{8\,ab^3\cos\left(dx+c\right)^3-24\,a^3b\cos\left(dx+c\right)+12\,\sqrt{-a^2+b^2}\,a^3\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-2\,(a\cos(dx+c)}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/24*(8*a*b^3*cos(d*x + c)^3 - 24*a^3*b*cos(d*x + c) + 12*sqrt(-a^2 + b^2)
*a^3*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 -
2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos
(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 3*(8*a^4 - 4*a^2*b^2 - b^4
)*d*x - 3*(2*b^4*cos(d*x + c)^3 - (4*a^2*b^2 + b^4)*cos(d*x + c))*sin(d*x +
 c))/(b^5*d), 1/24*(8*a*b^3*cos(d*x + c)^3 - 24*a^3*b*cos(d*x + c) - 24*sqr
t(a^2 - b^2)*a^3*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)
)) - 3*(8*a^4 - 4*a^2*b^2 - b^4)*d*x - 3*(2*b^4*cos(d*x + c)^3 - (4*a^2*b^2
 + b^4)*cos(d*x + c))*sin(d*x + c))/(b^5*d)]
```

giac [B] time = 0.18, size = 366, normalized size = 1.92

$$-\frac{\frac{3\left(8\,a^4-4\,a^2b^2-b^4\right)(dx+c)}{b^5}-\frac{48\left(a^5-a^3b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^5}+\frac{2\left(12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-3\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+24\,a\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(3*(8*a^4 - 4*a^2*b^2 - b^4)*(d*x + c)/b^5 - 48*(a^5 - a^3*b^2)*(pi*f
loor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/s
qrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^5) + 2*(12*a^2*b*tan(1/2*d*x + 1/2*c)^7
 - 3*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*a^3*tan(1/2*d*x + 1/2*c)^6 - 24*a*b^2*
tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 21*b^3*tan(1/2*d
*x + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2*c)^4 - 24*a*b^2*tan(1/2*d*x + 1/2*
c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 21*b^3*tan(1/2*d*x + 1/2*c)^3 + 72
*a^3*tan(1/2*d*x + 1/2*c)^2 - 8*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan
(1/2*d*x + 1/2*c) + 3*b^3*tan(1/2*d*x + 1/2*c) + 24*a^3 - 8*a*b^2)/((tan(1/
2*d*x + 1/2*c)^2 + 1)^4*b^4))/d
```

maple [B] time = 0.28, size = 657, normalized size = 3.44

$$-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{4db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{d\,b^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{}{d\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*a^2+1/4/d/b/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^
4*tan(1/2*d*x+1/2*c)^6*a^3+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)^6*a-1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a^2-7/4/d
/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-6/d/b^4/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a^3+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(
1/2*d*x+1/2*c)^4*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*
a^2+7/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-6/d/b^4/(1+tan(
1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a^3+2/3/d/b^2/(1+tan(1/2*d*x+1/2*c
)^2)^4*tan(1/2*d*x+1/2*c)^2*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*
x+1/2*c)*a^2-1/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-2/d/b^4/
(1+tan(1/2*d*x+1/2*c)^2)^4*a^3+2/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*a-2/d/b
^5*arctan(tan(1/2*d*x+1/2*c))*a^4+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2+1/
4/d/b*arctan(tan(1/2*d*x+1/2*c))+2/d*a^3*(a^2-b^2)^(1/2)/b^5*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.35, size = 2616, normalized size = 13.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] (7*tan(c/2 + (d*x)/2)^3)/(4*d*(b + 4*b*tan(c/2 + (d*x)/2)^2 + 6*b*tan(c/2 +
 (d*x)/2)^4 + 4*b*tan(c/2 + (d*x)/2)^6 + b*tan(c/2 + (d*x)/2)^8)) - (7*tan(
c/2 + (d*x)/2)^5)/(4*d*(b + 4*b*tan(c/2 + (d*x)/2)^2 + 6*b*tan(c/2 + (d*x)/
2)^4 + 4*b*tan(c/2 + (d*x)/2)^6 + b*tan(c/2 + (d*x)/2)^8)) + tan(c/2 + (d*x
)/2)^7/(4*d*(b + 4*b*tan(c/2 + (d*x)/2)^2 + 6*b*tan(c/2 + (d*x)/2)^4 + 4*b*
tan(c/2 + (d*x)/2)^6 + b*tan(c/2 + (d*x)/2)^8)) + (2*a)/(3*d*(4*b^2*tan(c/2
 + (d*x)/2)^2 + 6*b^2*tan(c/2 + (d*x)/2)^4 + 4*b^2*tan(c/2 + (d*x)/2)^6 + b
^2*tan(c/2 + (d*x)/2)^8 + b^2)) - (2*a^3)/(d*(4*b^4*tan(c/2 + (d*x)/2)^2 +
6*b^4*tan(c/2 + (d*x)/2)^4 + 4*b^4*tan(c/2 + (d*x)/2)^6 + b^4*tan(c/2 + (d*
x)/2)^8 + b^4)) + atan((11*a^3*tan(c/2 + (d*x)/2))/(8*((a*b^2)/8 + (11*a^3)
/8 + (3*a^5)/(2*b^2) - (11*a^7)/b^4 + (8*a^9)/b^6)) + (3*a^5*tan(c/2 + (d*x
)/2))/(2*((a*b^4)/8 + (3*a^5)/2 + (11*a^3*b^2)/8 - (11*a^7)/b^2 + (8*a^9)/b
^4)) - (11*a^7*tan(c/2 + (d*x)/2))/((a*b^6)/8 - 11*a^7 + (11*a^3*b^4)/8 + (
3*a^5*b^2)/2 + (8*a^9)/b^2) + (8*a^9*tan(c/2 + (d*x)/2))/((a*b^8)/8 + 8*a^9
 + (11*a^3*b^6)/8 + (3*a^5*b^4)/2 - 11*a^7*b^2) + (a*b*tan(c/2 + (d*x)/2))/
(8*((a*b)/8 + (11*a^3)/(8*b) + (3*a^5)/(2*b^3) - (11*a^7)/b^5 + (8*a^9)/b^7
)))/(4*b*d) - tan(c/2 + (d*x)/2)/(4*d*(b + 4*b*tan(c/2 + (d*x)/2)^2 + 6*b*t
an(c/2 + (d*x)/2)^4 + 4*b*tan(c/2 + (d*x)/2)^6 + b*tan(c/2 + (d*x)/2)^8)) +
 (2*a*tan(c/2 + (d*x)/2)^2)/(3*d*(4*b^2*tan(c/2 + (d*x)/2)^2 + 6*b^2*tan(c/
2 + (d*x)/2)^4 + 4*b^2*tan(c/2 + (d*x)/2)^6 + b^2*tan(c/2 + (d*x)/2)^8 + b^
2)) + (2*a*tan(c/2 + (d*x)/2)^4)/(d*(4*b^2*tan(c/2 + (d*x)/2)^2 + 6*b^2*tan
(c/2 + (d*x)/2)^4 + 4*b^2*tan(c/2 + (d*x)/2)^6 + b^2*tan(c/2 + (d*x)/2)^8 +
 b^2)) + (2*a*tan(c/2 + (d*x)/2)^6)/(d*(4*b^2*tan(c/2 + (d*x)/2)^2 + 6*b^2*
tan(c/2 + (d*x)/2)^4 + 4*b^2*tan(c/2 + (d*x)/2)^6 + b^2*tan(c/2 + (d*x)/2)^
8 + b^2)) + (a^2*tan(c/2 + (d*x)/2))/(d*(4*b^3*tan(c/2 + (d*x)/2)^2 + 6*b^3
*tan(c/2 + (d*x)/2)^4 + 4*b^3*tan(c/2 + (d*x)/2)^6 + b^3*tan(c/2 + (d*x)/2)
^8 + b^3)) + (a^2*atan((11*a^3*tan(c/2 + (d*x)/2))/(8*((a*b^2)/8 + (11*a^3)
/8 + (3*a^5)/(2*b^2) - (11*a^7)/b^4 + (8*a^9)/b^6)) + (3*a^5*tan(c/2 + (d*x
)/2))/(2*((a*b^4)/8 + (3*a^5)/2 + (11*a^3*b^2)/8 - (11*a^7)/b^2 + (8*a^9)/b
^4)) - (11*a^7*tan(c/2 + (d*x)/2))/((a*b^6)/8 - 11*a^7 + (11*a^3*b^4)/8 + (
3*a^5*b^2)/2 + (8*a^9)/b^2) + (8*a^9*tan(c/2 + (d*x)/2))/((a*b^8)/8 + 8*a^9
```

```
 + (11*a^3*b^6)/8 + (3*a^5*b^4)/2 - 11*a^7*b^2) + (a*b*tan(c/2 + (d*x)/2))/
(8*((a*b)/8 + (11*a^3)/(8*b) + (3*a^5)/(2*b^3) - (11*a^7)/b^5 + (8*a^9)/b^7
))))/(b^3*d) - (2*a^4*atan((11*a^3*tan(c/2 + (d*x)/2))/(8*((a*b^2)/8 + (11*
a^3)/8 + (3*a^5)/(2*b^2) - (11*a^7)/b^4 + (8*a^9)/b^6)) + (3*a^5*tan(c/2 +
(d*x)/2))/(2*((a*b^4)/8 + (3*a^5)/2 + (11*a^3*b^2)/8 - (11*a^7)/b^2 + (8*a^
9)/b^4)) - (11*a^7*tan(c/2 + (d*x)/2))/((a*b^6)/8 - 11*a^7 + (11*a^3*b^4)/8
 + (3*a^5*b^2)/2 + (8*a^9)/b^2) + (8*a^9*tan(c/2 + (d*x)/2))/((a*b^8)/8 + 8
*a^9 + (11*a^3*b^6)/8 + (3*a^5*b^4)/2 - 11*a^7*b^2) + (a*b*tan(c/2 + (d*x)/
2))/(8*((a*b)/8 + (11*a^3)/(8*b) + (3*a^5)/(2*b^3) - (11*a^7)/b^5 + (8*a^9)
/b^7))))/(b^5*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(d*(4*b^3*tan(c/2 + (d*x)/2)^
2 + 6*b^3*tan(c/2 + (d*x)/2)^4 + 4*b^3*tan(c/2 + (d*x)/2)^6 + b^3*tan(c/2 +
 (d*x)/2)^8 + b^3)) - (a^2*tan(c/2 + (d*x)/2)^5)/(d*(4*b^3*tan(c/2 + (d*x)/
2)^2 + 6*b^3*tan(c/2 + (d*x)/2)^4 + 4*b^3*tan(c/2 + (d*x)/2)^6 + b^3*tan(c/
2 + (d*x)/2)^8 + b^3)) - (a^2*tan(c/2 + (d*x)/2)^7)/(d*(4*b^3*tan(c/2 + (d*
x)/2)^2 + 6*b^3*tan(c/2 + (d*x)/2)^4 + 4*b^3*tan(c/2 + (d*x)/2)^6 + b^3*tan
(c/2 + (d*x)/2)^8 + b^3)) - (6*a^3*tan(c/2 + (d*x)/2)^2)/(d*(4*b^4*tan(c/2
+ (d*x)/2)^2 + 6*b^4*tan(c/2 + (d*x)/2)^4 + 4*b^4*tan(c/2 + (d*x)/2)^6 + b^
4*tan(c/2 + (d*x)/2)^8 + b^4)) - (6*a^3*tan(c/2 + (d*x)/2)^4)/(d*(4*b^4*tan
(c/2 + (d*x)/2)^2 + 6*b^4*tan(c/2 + (d*x)/2)^4 + 4*b^4*tan(c/2 + (d*x)/2)^6
 + b^4*tan(c/2 + (d*x)/2)^8 + b^4)) - (2*a^3*tan(c/2 + (d*x)/2)^6)/(d*(4*b^
4*tan(c/2 + (d*x)/2)^2 + 6*b^4*tan(c/2 + (d*x)/2)^4 + 4*b^4*tan(c/2 + (d*x)
/2)^6 + b^4*tan(c/2 + (d*x)/2)^8 + b^4)) - (2*a^3*atanh((a^5*(b^2 - a^2)^(1
/2))/(a^5*b + (7*a^7)/b - (8*a^9)/b^3 + 14*a^6*tan(c/2 + (d*x)/2) + 2*a^4*b
^2*tan(c/2 + (d*x)/2) - (16*a^8*tan(c/2 + (d*x)/2))/b^2) + (8*a^7*(b^2 - a^
2)^(1/2))/(7*a^7*b + a^5*b^3 - (8*a^9)/b - 16*a^8*tan(c/2 + (d*x)/2) + 2*a^
4*b^4*tan(c/2 + (d*x)/2) + 14*a^6*b^2*tan(c/2 + (d*x)/2)) + (2*a^4*tan(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2))/(a^5 + (7*a^7)/b^2 - (8*a^9)/b^4 + 2*a^4*b*ta
n(c/2 + (d*x)/2) + (14*a^6*tan(c/2 + (d*x)/2))/b - (16*a^8*tan(c/2 + (d*x)/
2))/b^3) + (15*a^6*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(7*a^7 + a^5*b^2 -
 (8*a^9)/b^2 + 14*a^6*b*tan(c/2 + (d*x)/2) + 2*a^4*b^3*tan(c/2 + (d*x)/2) -
 (16*a^8*tan(c/2 + (d*x)/2))/b) - (8*a^8*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/
2))/(a^5*b^4 - 8*a^9 + 7*a^7*b^2 - 16*a^8*b*tan(c/2 + (d*x)/2) + 2*a^4*b^5*
tan(c/2 + (d*x)/2) + 14*a^6*b^3*tan(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2))/(b^
5*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1287 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{2a^2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}+\frac{ax\left(2a^2-b^2\right)}{2b^4}+\frac{\left(3a^2-b^2\right)\cos(c+dx)}{3b^3d}-\frac{a\sin(c+dx)\cos(c+dx)}{2b^2d}+\sin^2(c$$

```
[Out] 1/2*a*(2*a^2-b^2)*x/b^4+1/3*(3*a^2-b^2)*cos(d*x+c)/b^3/d-1/2*a*cos(d*x+c)*s
in(d*x+c)/b^2/d+1/3*cos(d*x+c)*sin(d*x+c)^2/b/d-2*a^2*arctan((b+a*tan(1/2*d
*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/b^4/d
```

Rubi [A] time = 0.46, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3050, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{\left(3a^2-b^2\right)\cos(c+dx)}{3b^3d}-\frac{2a^2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}+\frac{ax\left(2a^2-b^2\right)}{2b^4}-\frac{a\sin(c+dx)\cos(c+dx)}{2b^2d}+\sin^2(c$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(2*a^2 - b^2)*x)/(2*b^4) - (2*a^2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c +
 d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*d) + ((3*a^2 - b^2)*Cos[c + d*x])/(3*b^3*d
) - (a*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^2)
/(3*b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
```

```
1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sin^2(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{3bd} + \frac{\int \frac{\sin(c+dx)\left(-2a+b\sin(c+dx)+3a\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3b}$$

$$= -\frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(c+dx)\sin^2(c+dx)}{3bd} + \frac{\int \frac{3a^2-ab\sin(c+dx)-2(3a}{a+b\sin(c-}}{6b^2}$$

$$= \frac{\left(3a^2-b^2\right)\cos(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(c+dx)\sin^2(c+dx}{3bd}$$

$$= \frac{a\left(2a^2-b^2\right)x}{2b^4} + \frac{\left(3a^2-b^2\right)\cos(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(c-}{}$$

$$= \frac{a\left(2a^2-b^2\right)x}{2b^4} + \frac{\left(3a^2-b^2\right)\cos(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(c-}{}$$

$$= \frac{a\left(2a^2-b^2\right)x}{2b^4} + \frac{\left(3a^2-b^2\right)\cos(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(c-}{}$$

$$= \frac{a\left(2a^2-b^2\right)x}{2b^4} - \frac{2a^2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d} + \frac{\left(3a^2-b^2\right)\cos(c+dx}{3b^3d}$$

Mathematica [A] time = 0.26, size = 130, normalized size = 0.88

$$-\frac{-12a^3c-12a^3dx+3b\left(b^2-4a^2\right)\cos(c+dx)+24a^2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)+3ab^2\sin(2(c+dx))+6}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

[Out] -1/12*(-12*a^3*c + 6*a*b^2*c - 12*a^3*d*x + 6*a*b^2*d*x + 24*a^2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 3*b*(-4*a^2 + b^2)*Cos[c + d*x] + b^3*Cos[3*(c + d*x)] + 3*a*b^2*Sin[2*(c + d*x)])/(b^4*d)

fricas [A] time = 0.91, size = 315, normalized size = 2.13

$$\left[-\frac{2\,b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)\sin\left(dx+c\right)-6\,a^{2}b\cos\left(dx+c\right)-3\,\sqrt{-a^{2}+b^{2}}\,a^{2}\log\left(\frac{\left(2\,a^{2}-b^{2}\right)\cos(dx+c)^{2}-2}{\cdots}\right.}{6\,b^{4}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] [-1/6*(2*b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)*sin(d*x + c) - 6*a^2*b*cos(d*x + c) - 3*sqrt(-a^2 + b^2)*a^2*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 3*(2*a^3 - a*b^2)*d*x)/(b^4*d), -1/6*(2*b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)*sin(d*x + c) - 6*a^2*b*cos(d*x + c) - 6*sqrt(a^2 - b^2)*a^2*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 3*(2*a^3 - a*b^2)*d*x)/(b^4*d)]

giac [A] time = 0.16, size = 207, normalized size = 1.40

$$\frac{\frac{3\left(2\,a^{3}-ab^{2}\right)(dx+c)}{b^{4}}-\frac{12\left(a^{4}-a^{2}b^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,b^{4}}+\frac{2\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-6\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\cdots\right.\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] 1/6*(3*(2*a^3 - a*b^2)*(d*x + c)/b^4 - 12*(a^4 - a^2*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^4) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*tan(1/2*d*x + 1/2*c)^4 - 6*b^2*tan(1/2*d*x + 1/2*c)^4 + 12*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*a^2 - 2*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^3))/d

maple [B] time = 0.27, size = 318, normalized size = 2.15

$$\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{}{d\,b^2\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)^5+2/d/b^3/(1+tan(1/
2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2-2/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3
*tan(1/2*d*x+1/2*c)^4+4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*a^2*tan(1/2*d*x+1/
2*c)^2-1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)+2/d/b^3/(1+t
an(1/2*d*x+1/2*c)^2)^3*a^2-2/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3+2/d/b^4*arcta
n(tan(1/2*d*x+1/2*c))*a^3-1/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))-2/d*a^2*(a^2
-b^2)^(1/2)/b^4*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.48, size = 225, normalized size = 1.52

$$\frac{a^2\,\cos\left(c+d\,x\right)}{b^3\,d}-\frac{a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4}}{b^2\,d}-\frac{\frac{\cos\left(c+d\,x\right)}{4}+\frac{\cos\left(3\,c+3\,d\,x\right)}{12}}{b\,d}+\frac{2\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^4\,d}+\frac{2\,a^2\,\mathrm{atanh}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] (a^2*cos(c + d*x))/(b^3*d) - (a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
 + (a*sin(2*c + 2*d*x))/4)/(b^2*d) - (cos(c + d*x)/4 + cos(3*c + 3*d*x)/12)
/(b*d) + (2*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^4*d) + (2*a
```

```
^2*atanh((2*b^2*sin(c/2 + (d*x)/2) - a^2*sin(c/2 + (d*x)/2) + a*b*cos(c/2 +
 (d*x)/2))/((b^2 - a^2)^(1/2)*(a*cos(c/2 + (d*x)/2) + 2*b*sin(c/2 + (d*x)/2
))))*(b^2 - a^2)^(1/2))/(b^4*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1288 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=100

$$\frac{2a\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{x\left(2a^2-b^2\right)}{2b^3}-\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d}$$

```
[Out] -1/2*(2*a^2-b^2)*x/b^3-1/2*cos(d*x+c)*(2*a-b*sin(d*x+c))/b^2/d+2*a*arctan((
b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/b^3/d
```

Rubi [A] time = 0.17, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2865, 2735, 2660, 618, 204}

$$\frac{2a\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{x\left(2a^2-b^2\right)}{2b^3}-\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((2*a^2 - b^2)*x)/(2*b^3) + (2*a*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(b^3*d) - (Cos[c + d*x]*(2*a - b*Sin[c + d*x]))/(2
*b^2*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d} + \frac{\int \frac{-ab-\left(2a^2-b^2\right)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b^2}$$

$$= -\frac{\left(2a^2-b^2\right)x}{2b^3} - \frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d} + \frac{\left(a\left(a^2-b^2\right)\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^3}$$

$$= -\frac{\left(2a^2-b^2\right)x}{2b^3} - \frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d} + \frac{\left(2a\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx\ldots}\right.}{b^3\ldots}$$

$$= -\frac{\left(2a^2-b^2\right)x}{2b^3} - \frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d} - \frac{\left(4a\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{}{-4\left(a^2\ldots\right.}\right.}{\ldots}$$

$$= -\frac{\left(2a^2-b^2\right)x}{2b^3} + \frac{2a\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3d} - \frac{\cos(c+dx)(2a-b\sin(c\ldots}{2b^2d}$$

Mathematica [A] time = 0.23, size = 104, normalized size = 1.04

$$\frac{8a\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)-4a^2c-4a^2dx-4ab\cos(c+dx)+b^2\sin(2(c+dx))+2b^2c+2b^2dx}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-4*a^2*c + 2*b^2*c - 4*a^2*d*x + 2*b^2*d*x + 8*a*Sqrt[a^2 - b^2]*ArcTan[(b
 + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 4*a*b*Cos[c + d*x] + b^2*Sin[2*(c
 + d*x)])/(4*b^3*d)
```

fricas [A] time = 0.85, size = 275, normalized size = 2.75

$$\left[\frac{b^2\cos(dx+c)\sin(dx+c)-\left(2a^2-b^2\right)dx-2ab\cos(dx+c)+\sqrt{-a^2+b^2}\,a\log\left(-\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c}{b^2\cos(}\right.}{2b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b^2*cos(d*x + c)*sin(d*x + c) - (2*a^2 - b^2)*d*x - 2*a*b*cos(d*x + c
) + sqrt(-a^2 + b^2)*a*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^
2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(b^3*d),
1/2*(b^2*cos(d*x + c)*sin(d*x + c) - (2*a^2 - b^2)*d*x - 2*a*b*cos(d*x + c)
 - 2*sqrt(a^2 - b^2)*a*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*
x + c))))/(b^3*d)]
```

giac [A] time = 0.16, size = 159, normalized size = 1.59

$$-\frac{\frac{\left(2a^2-b^2\right)(dx+c)}{b^3}-\frac{4\left(a^3-ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^3}+\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2a\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((2*a^2 - b^2)*(d*x + c)/b^3 - 4*(a^3 - a*b^2)*(pi*floor(1/2*(d*x + c)
/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(
```

```
sqrt(a^2 - b^2)*b^3) + 2*(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*
c)^2 - b*tan(1/2*d*x + 1/2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/
d
```

maple [B] time = 0.24, size = 214, normalized size = 2.14

$$-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{2a}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{2\,\mathrm{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-2/d/b^2/(1+tan(1/2*d
*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a+1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(
1/2*d*x+1/2*c)-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*a-2/d/b^3*arctan(tan(1/2*
d*x+1/2*c))*a^2+1/d/b*arctan(tan(1/2*d*x+1/2*c))+2/d*a*(a^2-b^2)^(1/2)/b^3*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.07, size = 190, normalized size = 1.90

$$\frac{\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{\sin(2\,c+2\,d\,x)}{4}}{b\,d}-\frac{2\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^3\,d}-\frac{a\,\cos\left(c+d\,x\right)}{b^2\,d}-\frac{2\,a\,\mathrm{atanh}\left(\frac{-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b+2\,\sin\left(\right.}{\sqrt{b^2-a^2}\,\left(a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,b\,\sin\left(\frac{c}{2}+\right.\right.}\right.}{b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] (atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + sin(2*c + 2*d*x)/4)/(b*d) -
(2*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d) - (a*cos(c + d*
```

```
x))/(b^2*d) - (2*a*atanh((2*b^2*sin(c/2 + (d*x)/2) - a^2*sin(c/2 + (d*x)/2)
 + a*b*cos(c/2 + (d*x)/2))/((b^2 - a^2)^(1/2)*(a*cos(c/2 + (d*x)/2) + 2*b*s
in(c/2 + (d*x)/2))))*(b^2 - a^2)^(1/2))/(b^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1289 $\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{b}$$

```
[Out] -x/b-arctanh(cos(d*x+c))/a/d+2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1
/2))*(a^2-b^2)^(1/2)/a/b/d
```

Rubi [A] time = 0.19, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2889, 3058, 2660, 618, 204, 3770}

$$\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(x/b) + (2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/(a*b*d) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3058

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(C*x
)/(b*d), x] + (Dist[(A*b^2 + a^2*C)/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f
*x]), x], x] - Dist[(c^2*C + A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f
*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx &= \int \frac{\csc(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx \\
&= -\frac{x}{b}+\frac{\int \csc(c+dx)\,dx}{a}-\left(-\frac{a}{b}+\frac{b}{a}\right)\int \frac{1}{a+b\sin(c+dx)}\,dx \\
&= -\frac{x}{b}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+d\right.\right.}{d} \\
&= -\frac{x}{b}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\left(4\left(\frac{a}{b}-\frac{b}{a}\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\right.}{d} \\
&= -\frac{x}{b}+\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 90, normalized size = 1.20

$$-\frac{-2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)+ac+adx-b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*c + a*d*x - 2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]] + b*Log[Cos[(c + d*x)/2]] - b*Log[Sin[(c + d*x)/2]])/(a*b*d))
```

fricas [A] time = 1.07, size = 262, normalized size = 3.49

$$\left[-\frac{2\,adx+b\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-b\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-\sqrt{-a^2+b^2}\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx\cdots}{b^2\cos\cdots}\right.}{2\,abd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*d*x + b*log(1/2*cos(d*x + c) + 1/2) - b*log(-1/2*cos(d*x + c) +
1/2) - sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
+ c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a
^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(a*b*d),
 -1/2*(2*a*d*x + b*log(1/2*cos(d*x + c) + 1/2) - b*log(-1/2*cos(d*x + c) +
1/2) + 2*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(
d*x + c))))/(a*b*d)]
```

giac [A] time = 0.17, size = 94, normalized size = 1.25

$$-\frac{\frac{dx+c}{b}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/b - log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(pi*floor(1/2*(d*x + c
)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*
sqrt(a^2 - b^2)/(a*b))/d
```

maple [A] time = 0.43, size = 137, normalized size = 1.83

$$-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{db\sqrt{a^2-b^2}}-\frac{2b\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{da\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d/b*arctan(tan(1/2*d*x+1/2*c))+1/a/d*ln(tan(1/2*d*x+1/2*c))+2/d*a/b/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d*b/
a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 11.97, size = 896, normalized size = 11.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) + (2*atan((64*a^3)/(64*a^2*b - 64*b^3 + 64*a^
3*tan(c/2 + (d*x)/2) - 64*a*b^2*tan(c/2 + (d*x)/2)) - (64*a*b^2)/(64*a^2*b
- 64*b^3 + 64*a^3*tan(c/2 + (d*x)/2) - 64*a*b^2*tan(c/2 + (d*x)/2)) + (64*b
^3*tan(c/2 + (d*x)/2))/(64*a^2*b - 64*b^3 + 64*a^3*tan(c/2 + (d*x)/2) - 64*
a*b^2*tan(c/2 + (d*x)/2)) - (64*a^2*b*tan(c/2 + (d*x)/2))/(64*a^2*b - 64*b^
3 + 64*a^3*tan(c/2 + (d*x)/2) - 64*a*b^2*tan(c/2 + (d*x)/2))))/(b*d) - (2*a
tanh((64*a^2*(b^2 - a^2)^(1/2))/(256*a^2*b - 768*b^3 + (512*b^5)/a^2 - 64*a
^3*tan(c/2 + (d*x)/2) + 832*a*b^2*tan(c/2 + (d*x)/2) - (1792*b^4*tan(c/2 +
(d*x)/2))/a + (1024*b^6*tan(c/2 + (d*x)/2))/a^3) - (512*b^2*(b^2 - a^2)^(1/
2))/(256*a^2*b - 768*b^3 + (512*b^5)/a^2 - 64*a^3*tan(c/2 + (d*x)/2) + 832*
a*b^2*tan(c/2 + (d*x)/2) - (1792*b^4*tan(c/2 + (d*x)/2))/a + (1024*b^6*tan(
```

```
c/2 + (d*x)/2))/a^3) + (512*b^4*(b^2 - a^2)^(1/2))/(256*a^4*b + 512*b^5 - 7
68*a^2*b^3 - 64*a^5*tan(c/2 + (d*x)/2) - 1792*a*b^4*tan(c/2 + (d*x)/2) + 83
2*a^3*b^2*tan(c/2 + (d*x)/2) + (1024*b^6*tan(c/2 + (d*x)/2))/a) - (1280*b^3
*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(256*a^3*b - 768*a*b^3 + (512*b^5)/a
 - 64*a^4*tan(c/2 + (d*x)/2) - 1792*b^4*tan(c/2 + (d*x)/2) + 832*a^2*b^2*ta
n(c/2 + (d*x)/2) + (1024*b^6*tan(c/2 + (d*x)/2))/a^2) + (1024*b^5*tan(c/2 +
 (d*x)/2)*(b^2 - a^2)^(1/2))/(512*a*b^5 + 256*a^5*b - 768*a^3*b^3 - 64*a^6*
tan(c/2 + (d*x)/2) + 1024*b^6*tan(c/2 + (d*x)/2) - 1792*a^2*b^4*tan(c/2 + (
d*x)/2) + 832*a^4*b^2*tan(c/2 + (d*x)/2)) + (320*a*b*tan(c/2 + (d*x)/2)*(b^
2 - a^2)^(1/2))/(256*a^2*b - 768*b^3 + (512*b^5)/a^2 - 64*a^3*tan(c/2 + (d*
x)/2) + 832*a*b^2*tan(c/2 + (d*x)/2) - (1792*b^4*tan(c/2 + (d*x)/2))/a + (1
024*b^6*tan(c/2 + (d*x)/2))/a^3))*(b^2 - a^2)^(1/2))/(a*b*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.1290 $\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] b*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a/d-2*arctan((b+a*tan(1/2*d*x+1/2*c)
)/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^2/d
```

Rubi [A] time = 0.25, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 204}

$$-\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*
d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$
$$= -\frac{\cot(c+dx)}{ad} + \frac{\int \frac{\csc(c+dx)(-b-a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{a}$$
$$= -\frac{\cot(c+dx)}{ad} - \frac{b\int \csc(c+dx)\,dx}{a^2} + \frac{\left(-a^2+b^2\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2}$$
$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{\left(2\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c\right.\right.}{a^2d}$$
$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} + \frac{\left(4\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+\right.}{a^2d}$$
$$= -\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad}$$

Mathematica [A] time = 0.24, size = 108, normalized size = 1.35

$$\frac{-4\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + a\tan\left(\frac{1}{2}(c+dx)\right) - a\cot\left(\frac{1}{2}(c+dx)\right) - 2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2b\log\left(\right.}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - a*Co
t[(c + d*x)/2] + 2*b*Log[Cos[(c + d*x)/2]] - 2*b*Log[Sin[(c + d*x)/2]] + a*
Tan[(c + d*x)/2])/(2*a^2*d)
```

fricas [A] time = 0.86, size = 314, normalized size = 3.92

$$\left[\frac{b\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - b\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) + \sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos(\right.}{}\right.}{2a^2d\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c)), 1/2*(b*log(1/2*
cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 2*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d
*x + c)))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c))]
```

giac [A] time = 0.20, size = 129, normalized size = 1.61

$$-\frac{\frac{2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{a^2}-\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - tan(1/2*d*x + 1/2*c)/a + 4*(
pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) +
b)/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/a^2 - (2*b*tan(1/2*d*x + 1/2*c) - a)/(
a^2*tan(1/2*d*x + 1/2*c)))/d
```

maple [B] time = 0.44, size = 155, normalized size = 1.94

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{1}{2ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}-\frac{2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}+\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\,a^2\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)-1/2/a/d/tan(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tan(1/2*
d*x+1/2*c))-2/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^
2-b^2)^(1/2))+2/d*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 11.94, size = 204, normalized size = 2.55

$$-\frac{\cot(c+d\,x)}{a\,d}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{\mathrm{atan}\left(\frac{a^3\,\sqrt{b^2-a^2}\,1\mathrm{i}-a\,b^2\,\sqrt{b^2-a^2}\,2\mathrm{i}-b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}\,4\mathrm{i}+a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}\,3\mathrm{i}}{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4-2\,a^3\,b-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2\,b^2+2\,a\,b^3+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^4}\right)\sqrt{}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (atan((a^3*(b^2 - a^2)^(1/2)*1i - a*b^2*(b^2 - a^2)^(1/2)*2i - b^3*tan(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2*b*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
)*3i)/(2*a*b^3 - 2*a^3*b + a^4*tan(c/2 + (d*x)/2) + 4*b^4*tan(c/2 + (d*x)/2
) - 5*a^2*b^2*tan(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i)/(a^2*d) - cot(c +
d*x)/(a*d) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

3.1291 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=114

$$\frac{b\cot(c+dx)}{a^2d}+\frac{2b\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d}+\frac{\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] 1/2*(a^2-2*b^2)*arctanh(cos(d*x+c))/a^3/d+b*cot(d*x+c)/a^2/d-1/2*cot(d*x+c)
*csc(d*x+c)/a/d+2*b*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b
^2)^(1/2)/a^3/d
```

Rubi [A] time = 0.46, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d}+\frac{\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3
*d) + ((a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (b*Cot[c + d*x])/(a
^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
```

```
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx &= \int \frac{\csc^3(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx \\
&= -\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \frac{\csc^2(c+dx)\left(-2b-a\sin(c+dx)+b\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{2a} \\
&= \frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \frac{\csc(c+dx)\left(-a^2+2b^2+ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{2a^2} \\
&= \frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}-\frac{\left(a^2-2b^2\right)\int\csc(c+dx)\,dx}{2a^3}+\frac{\left(b\left(a^2-\right.\right.}{} \\
&= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{(2b}{} \\
&= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}-\frac{(4b}{} \\
&= \frac{2b\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3d}+\frac{\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+}{a^2d}
\end{aligned}$$

Mathematica [A] time = 0.85, size = 181, normalized size = 1.59

$$\frac{16b\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)+a^2\left(-\csc^2\left(\frac{1}{2}(c+dx)\right)\right)+a^2\sec^2\left(\frac{1}{2}(c+dx)\right)-4a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (16*b*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 4*
a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 + 4*a^2*Log[Cos[(c + d*x)/2]]
 - 8*b^2*Log[Cos[(c + d*x)/2]] - 4*a^2*Log[Sin[(c + d*x)/2]] + 8*b^2*Log[Si
n[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 - 4*a*b*Tan[(c + d*x)/2])/(8*a^3*d
)
```

fricas [A] time = 1.04, size = 472, normalized size = 4.14

$$\left[-\frac{4\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a^2\cos\left(dx+c\right)-2\left(b\cos\left(dx+c\right)^2-b\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,a\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b*cos(d*x + c)*sin(d*x + c) - 2*a^2*cos(d*x + c) - 2*(b*cos(d*x
+ c)^2 - b)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sq
rt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - ((
a^2 - 2*b^2)*cos(d*x + c)^2 - a^2 + 2*b^2)*log(1/2*cos(d*x + c) + 1/2) + ((
a^2 - 2*b^2)*cos(d*x + c)^2 - a^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a
^3*d*cos(d*x + c)^2 - a^3*d), -1/4*(4*a*b*cos(d*x + c)*sin(d*x + c) - 2*a^2
*cos(d*x + c) + 4*(b*cos(d*x + c)^2 - b)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - ((a^2 - 2*b^2)*cos(d*x + c)^2
- a^2 + 2*b^2)*log(1/2*cos(d*x + c) + 1/2) + ((a^2 - 2*b^2)*cos(d*x + c)^2
- a^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3*d*cos(d*x + c)^2 - a^3*d)
]
```

giac [A] time = 0.19, size = 198, normalized size = 1.74

$$\frac{\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}-\frac{4\left(a^2-2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{16\left(a^2b-b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^3}+\frac{6\,a^2\tan}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*((a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/a^2 - 4*(a^2 - 2
*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 16*(a^2*b - b^3)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 -
b^2)))/(sqrt(a^2 - b^2)*a^3) + (6*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1
```

```
/2*d*x + 1/2*c)^2 + 4*a*b*tan(1/2*d*x + 1/2*c) - a^2)/(a^3*tan(1/2*d*x + 1/
2*c)^2))/d
```

maple [A] time = 0.48, size = 166, normalized size = 1.46

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{2d\,a^2}-\frac{1}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{d\,a^3}+\frac{b}{2d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/8/a/d*tan(1/2*d*x+1/2*c)^2-1/2/d/a^2*tan(1/2*d*x+1/2*c)*b-1/8/a/d/tan(1/2
*d*x+1/2*c)^2-1/2/a/d*ln(tan(1/2*d*x+1/2*c))+1/d/a^3*ln(tan(1/2*d*x+1/2*c))
*b^2+1/2/d/a^2*b/tan(1/2*d*x+1/2*c)+2/d*(a^2-b^2)^(1/2)/a^3*b*arctan(1/2*(2
*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.45, size = 790, normalized size = 6.93

$$\frac{b^2\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2\,\left(\frac{a^3\,d}{2}-\frac{a^3\,d\,\cos(2\,c+2\,d\,x)}{2}\right)}-\frac{a^2\,\left(\frac{\cos(c+d\,x)}{2}+\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4}-\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(2\,c+2\,d\,x)}{4}\right)}{\frac{a^3\,d}{2}-\frac{a^3\,d\,\cos(2\,c+2\,d\,x)}{2}}-\frac{b^2\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(2\,c+2\,d\right.}{2\,\left(\frac{a^3\,d}{2}-\frac{a^3\,d\,\cos(2\,c+2\,d\,x)}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*((a^3*d)/2 - (a^3*d*cos
(2*c + 2*d*x))/2)) - (a^2*(cos(c + d*x)/2 + log(sin(c/2 + (d*x)/2)/cos(c/2
```

```
+ (d*x)/2))/4 - (log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x
))/4))/((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x))/2) - (b^2*log(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/(2*((a^3*d)/2 - (a^3*d*cos(2*c + 2
*d*x))/2)) + (a*b*sin(2*c + 2*d*x))/(2*((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x)
)/2)) + (b*atan((a^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i + b^4*sin(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2)*8i - a^2*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1
/2)*8i + a*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i - a^3*b*cos(c/2 + (d
*x)/2)*(b^2 - a^2)^(1/2)*3i)/(a^5*cos(c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x
)/2) + 4*a*b^4*cos(c/2 + (d*x)/2) + 4*a^4*b*sin(c/2 + (d*x)/2) - 5*a^3*b^2*
cos(c/2 + (d*x)/2) - 12*a^2*b^3*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i)/
((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x))/2) - (b*cos(2*c + 2*d*x)*atan((a^4*si
n(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^
(1/2)*8i - a^2*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i + a*b^3*cos(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2)*4i - a^3*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
)*3i)/(a^5*cos(c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x)/2) + 4*a*b^4*cos(c/2
+ (d*x)/2) + 4*a^4*b*sin(c/2 + (d*x)/2) - 5*a^3*b^2*cos(c/2 + (d*x)/2) - 12
*a^2*b^3*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i)/((a^3*d)/2 - (a^3*d*cos
(2*c + 2*d*x))/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

3.1292 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{2b^2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}-\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{\left(a^2-3b^2\right)\cot}{3a^3d}$$

```
[Out] -1/2*b*(a^2-2*b^2)*arctanh(cos(d*x+c))/a^4/d+1/3*(a^2-3*b^2)*cot(d*x+c)/a^3
/d+1/2*b*cot(d*x+c)*csc(d*x+c)/a^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d-2*b^2*
arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^4/d
```

Rubi [A] time = 0.67, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b^2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}+\frac{\left(a^2-3b^2\right)\cot(c+dx)}{3a^3d}-\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{b\cot(c+dx)\,\mathrm{cs}}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^4*d) - (b*(a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + ((a^2 - 3*b^2)
*Cot[c + d*x])/(3*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) - (Cot[c
 + d*x]*Csc[c + d*x]^2)/(3*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
```

```
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^4(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{3ad} + \frac{\int \frac{\csc^3(c+dx)\left(-3b-a\sin(c+dx)+2b\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3a}$$

$$= \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} + \frac{\int \frac{\csc^2(c+dx)\left(-2\left(a^2-3b^2\right)+ab\,\mathrm{s}\right.}{a+b\sin(}}{6a}$$

$$= \frac{\left(a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} +$$

$$= \frac{\left(a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} +$$

$$= -\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\,\mathrm{csc}}{2a^2d}$$

$$= -\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\,\mathrm{csc}}{2a^2d}$$

$$= -\frac{2b^2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} - \frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(a^2\right.}{}$$

Mathematica [B] time = 6.23, size = 351, normalized size = 2.29

$$\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} + \frac{\left(a^2b-2b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\left(2b^3-a^2b\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} - \frac{2b}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^2*Sqrt[a^2 - b^2]*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Si
n[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*d) + ((a^2*Cos[(c + d*x)/2] - 3*b^2
*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^3*d) + (b*Csc[(c + d*x)/2]^2)/(8*
a^2*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a*d) + ((-(a^2*b) + 2*b^
3)*Log[Cos[(c + d*x)/2]])/(2*a^4*d) + ((a^2*b - 2*b^3)*Log[Sin[(c + d*x)/2]
])/(2*a^4*d) - (b*Sec[(c + d*x)/2]^2)/(8*a^2*d) + (Sec[(c + d*x)/2]*(-(a^2*
Sin[(c + d*x)/2]) + 3*b^2*Sin[(c + d*x)/2]))/(6*a^3*d) + (Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2])/(24*a*d)
```

fricas [A] time = 1.13, size = 591, normalized size = 3.86

$$\left[-\frac{6\,a^{2}b\cos\left(dx+c\right)\sin\left(dx+c\right)-12\,ab^{2}\cos\left(dx+c\right)-4\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{3}-6\left(b^{2}\cos\left(dx+c\right)^{2}-b^{2}\right)\sqrt{\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 12*a*b^2*cos(d*x + c) - 4*(a^3
- 3*a*b^2)*cos(d*x + c)^3 - 6*(b^2*cos(d*x + c)^2 - b^2)*sqrt(-a^2 + b^2)*l
og(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*co
s(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 3*(a^2*b - 2*b^3 - (
a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3
*(a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1
/2)*sin(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c)), -1/12*(6*a
^2*b*cos(d*x + c)*sin(d*x + c) - 12*a*b^2*cos(d*x + c) - 4*(a^3 - 3*a*b^2)*
cos(d*x + c)^3 - 12*(b^2*cos(d*x + c)^2 - b^2)*sqrt(a^2 - b^2)*arctan(-(a*s
in(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 3*(a^2*b -
2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) + 3*(a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x
 + c) + 1/2)*sin(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))]
```

giac [A] time = 0.19, size = 270, normalized size = 1.76

$$\frac{\frac{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{3}}+\frac{12\left(a^{2}b-2\,b^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{4}}-\frac{48\left(a^{2}b^{2}-b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*((a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*ta
n(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/a^3 + 12*(a^2*b - 2*b^3)*
log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - 48*(a^2*b^2 - b^4)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*a^4) - (22*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 44*b^3*tan(1/
2*d*x + 1/2*c)^3 - 3*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan(1/2*d*x + 1/
2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^4*tan(1/2*d*x + 1/2*c)^3))/
d
```

maple [**A**] time = 0.46, size = 250, normalized size = 1.63

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{8d\,a^2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}-\frac{1}{24da\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{1}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{}{2d\,a^3\,\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a^2*tan(1/2*d*x+1/2*c)^2*b-1/8/a/d*tan(
1/2*d*x+1/2*c)+1/2/d/a^3*b^2*tan(1/2*d*x+1/2*c)-1/24/d/a/tan(1/2*d*x+1/2*c)
^3+1/8/a/d/tan(1/2*d*x+1/2*c)-1/2/d/a^3/tan(1/2*d*x+1/2*c)*b^2+1/8/d/a^2*b/
tan(1/2*d*x+1/2*c)^2+1/2/d/a^2*b*ln(tan(1/2*d*x+1/2*c))-1/d/a^4*b^3*ln(tan(
1/2*d*x+1/2*c))-2/d*(a^2-b^2)^(1/2)/a^4*b^2*arctan(1/2*(2*a*tan(1/2*d*x+1/2
*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.51, size = 749, normalized size = 4.90

$$a^3\left(\frac{\cos(c+d\,x)}{8}+\frac{\cos(3\,c+3\,d\,x)}{24}\right)-a^2\left(\frac{b\,\sin(2\,c+2\,d\,x)}{8}-\frac{b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\sin(3\,c+3\,d\,x)}{16}+\frac{3\,b\,\sin(c+d\,x)\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{16}\right)+a\,\left(\frac{b^2\,\cos(}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] (a^3*(cos(c + d*x)/8 + cos(3*c + 3*d*x)/24) - a^2*((b*sin(2*c + 2*d*x))/8 -
 (b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x))/16 + (3*b*
sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/16) + a*((b^2*cos(
c + d*x))/8 - (b^2*cos(3*c + 3*d*x))/8) + (3*b^3*sin(c + d*x)*log(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/8 - (b^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2))*sin(3*c + 3*d*x))/8 + (b^2*sin(c + d*x)*atan((a^4*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i - a^2*
b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i + a*b^3*cos(c/2 + (d*x)/2)*(b^2
 - a^2)^(1/2)*4i - a^3*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*3i)/(a^5*cos(
c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x)/2) + 4*a*b^4*cos(c/2 + (d*x)/2) + 4*
a^4*b*sin(c/2 + (d*x)/2) - 5*a^3*b^2*cos(c/2 + (d*x)/2) - 12*a^2*b^3*sin(c/
2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*3i)/4 - (b^2*sin(3*c + 3*d*x)*atan((a^4*si
n(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^
(1/2)*8i - a^2*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i + a*b^3*cos(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2)*4i - a^3*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
)*3i)/(a^5*cos(c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x)/2) + 4*a*b^4*cos(c/2
+ (d*x)/2) + 4*a^4*b*sin(c/2 + (d*x)/2) - 5*a^3*b^2*cos(c/2 + (d*x)/2) - 12
*a^2*b^3*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i)/4)/((a^4*d*sin(3*c + 3*
d*x))/8 - (3*a^4*d*sin(c + d*x))/8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^2(c+dx)\csc^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

3.1293 $\int \frac{\cot^2(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d}+\frac{2b^3\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d}-\frac{b\left(a^2-3b^2\right)\cot(c+dx)}{3a^4d}+\frac{\left(a^2-4b^2\right)\cot(c+dx)}{8a^3d}$$

```
[Out] 1/8*(a^4+4*a^2*b^2-8*b^4)*arctanh(cos(d*x+c))/a^5/d-1/3*b*(a^2-3*b^2)*cot(d
*x+c)/a^4/d+1/8*(a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/d+1/3*b*cot(d*x+c)*cs
c(d*x+c)^2/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d+2*b^3*arctan((b+a*tan(1/2*
d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^5/d
```

Rubi [A] time = 0.95, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b^3\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d}-\frac{b\left(a^2-3b^2\right)\cot(c+dx)}{3a^4d}+\frac{\left(4a^2b^2+a^4-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d}+\frac{\left(a^2-4b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^3*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^5*d) + ((a^4 + 4*a^2*b^2 - 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^5*d) - (b*(a
^2 - 3*b^2)*Cot[c + d*x])/(3*a^4*d) + ((a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d
*x])/(8*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^2)/(3*a^2*d) - (Cot[c + d*x]*
Csc[c + d*x]^3)/(4*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
```

```
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^5(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{\int \frac{\csc^4(c+dx)\left(-4b-a\sin(c+dx)+3b\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{4a}$$

$$= \frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{\int \frac{\csc^3(c+dx)\left(-3\left(a^2-4b^2\right)+ab\right.}{a+b\sin}}{12}$$

$$= \frac{\left(a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d} - \frac{\cot(c+dx)\,c}{4a}$$

$$= -\frac{b\left(a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+dx)}{3a^2}$$

$$= -\frac{b\left(a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+dx)}{3a^2}$$

$$= \frac{\left(a^4+4a^2b^2-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d} - \frac{b\left(a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(a^2-4b^2\right.}{}$$

$$= \frac{\left(a^4+4a^2b^2-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d} - \frac{b\left(a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(a^2-4b^2\right.}{}$$

$$= \frac{2b^3\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5d} + \frac{\left(a^4+4a^2b^2-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d}$$

Mathematica [B] time = 6.27, size = 430, normalized size = 2.22

$$\frac{b\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d}-\frac{b\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d}+\frac{2b^3\sqrt{a^2-b^2}\ \tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(a\sin\left(\frac{1}{2}(c+\right.\right.}{\sqrt{a^2-}}\right.}{a^5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^3*Sqrt[a^2 - b^2]*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin
[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*d) + ((-(a^2*b*Cos[(c + d*x)/2]) + 3
*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + ((a^2 - 4*b^2)*Csc[(c
+ d*x)/2]^2)/(32*a^3*d) + (b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a^2*d
) - Csc[(c + d*x)/2]^4/(64*a*d) + ((a^4 + 4*a^2*b^2 - 8*b^4)*Log[Cos[(c + d
*x)/2]])/(8*a^5*d) + ((-a^4 - 4*a^2*b^2 + 8*b^4)*Log[Sin[(c + d*x)/2]])/(8*
a^5*d) + ((-a^2 + 4*b^2)*Sec[(c + d*x)/2]^2)/(32*a^3*d) + Sec[(c + d*x)/2]^
4/(64*a*d) + (Sec[(c + d*x)/2]*(a^2*b*Sin[(c + d*x)/2] - 3*b^3*Sin[(c + d*x
)/2]))/(6*a^4*d) - (b*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^2*d)
```

fricas [B] time = 1.02, size = 808, normalized size = 4.16

$$\left[-\frac{6\left(a^4-4a^2b^2\right)\cos(dx+c)^3-24\left(b^3\cos(dx+c)^4-2b^3\cos(dx+c)^2+b^3\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2a^2-b^2\right)\cos(dx+c)}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/48*(6*(a^4 - 4*a^2*b^2)*cos(d*x + c)^3 - 24*(b^3*cos(d*x + c)^4 - 2*b^3
*cos(d*x + c)^2 + b^3)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 6*(a^4 + 4*a^2*b^2)*cos(d*x + c) - 3*((a^4 + 4*a^2*b^2 - 8*b^4)*co
s(d*x + c)^4 + a^4 + 4*a^2*b^2 - 8*b^4 - 2*(a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*
x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 3*((a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*
x + c)^4 + a^4 + 4*a^2*b^2 - 8*b^4 - 2*(a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x +
c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*(3*a*b^3*cos(d*x + c) + (a^3*b - 3*
a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*
x + c)^2 + a^5*d), -1/48*(6*(a^4 - 4*a^2*b^2)*cos(d*x + c)^3 + 48*(b^3*cos(
d*x + c)^4 - 2*b^3*cos(d*x + c)^2 + b^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 6*(a^4 + 4*a^2*b^2)*cos(d*x +
c) - 3*((a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x + c)^4 + a^4 + 4*a^2*b^2 - 8*b^4
```

```
- 2*(a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) +
 3*((a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x + c)^4 + a^4 + 4*a^2*b^2 - 8*b^4 - 2*
(a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 16
*(3*a*b^3*cos(d*x + c) + (a^3*b - 3*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a
^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*x + c)^2 + a^5*d)]
```

giac [A] time = 0.20, size = 336, normalized size = 1.73

$$\frac{\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-96\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}-\frac{24\left(a^4+4\,a^2b^2-8\,b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx\right.\right.\right.}{a^5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*((3*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 24*
a*b^2*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*b*tan(1/2*d*x + 1/2*c) - 96*b^3*tan(1
/2*d*x + 1/2*c))/a^4 - 24*(a^4 + 4*a^2*b^2 - 8*b^4)*log(abs(tan(1/2*d*x + 1
/2*c)))/a^5 + 384*(a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a)
+ arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^
5) + (50*a^4*tan(1/2*d*x + 1/2*c)^4 + 200*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 -
400*b^4*tan(1/2*d*x + 1/2*c)^4 - 24*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 96*a*b^3
*tan(1/2*d*x + 1/2*c)^3 - 24*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a^3*b*tan(1
/2*d*x + 1/2*c) - 3*a^4)/(a^5*tan(1/2*d*x + 1/2*c)^4))/d
```

maple [A] time = 0.49, size = 315, normalized size = 1.62

$$\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64da}-\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^2}+\frac{b^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^3}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8d\,a^2}-\frac{b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^4}-\frac{1}{64da\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}-\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/64/d/a*tan(1/2*d*x+1/2*c)^4-1/24/d/a^2*b*tan(1/2*d*x+1/2*c)^3+1/8/d/a^3*b
^2*tan(1/2*d*x+1/2*c)^2+1/8/d/a^2*tan(1/2*d*x+1/2*c)*b-1/2/d/a^4*b^3*tan(1/
2*d*x+1/2*c)-1/64/d/a/tan(1/2*d*x+1/2*c)^4-1/8/a/d*ln(tan(1/2*d*x+1/2*c))-1
/2/d/a^3*ln(tan(1/2*d*x+1/2*c))*b^2+1/d/a^5*ln(tan(1/2*d*x+1/2*c))*b^4+1/24
/d/a^2*b/tan(1/2*d*x+1/2*c)^3-1/8/d*b^2/a^3/tan(1/2*d*x+1/2*c)^2-1/8/d/a^2*
b/tan(1/2*d*x+1/2*c)+1/2/d*b^3/a^4/tan(1/2*d*x+1/2*c)+2/d*(a^2-b^2)^(1/2)/a
^5*b^3*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.07, size = 873, normalized size = 4.50

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{b}{8\,a^2}-\frac{b^3}{2\,a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(2\,a^2\,b-8\,b^3\right)+\frac{a^3}{4}-\frac{2\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+2\,a\,b^2\tan\left(\frac{c}{2}+\right.}{16\,a^4\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^4/(64*a*d) + (tan(c/2 + (d*x)/2)*(b/(8*a^2) - b^3/(2*a^4
)))/d - (tan(c/2 + (d*x)/2)^3*(2*a^2*b - 8*b^3) + a^3/4 - (2*a^2*b*tan(c/2
+ (d*x)/2))/3 + 2*a*b^2*tan(c/2 + (d*x)/2)^2)/(16*a^4*d*tan(c/2 + (d*x)/2)^
4) - (log(tan(c/2 + (d*x)/2))*(a^4 - 8*b^4 + 4*a^2*b^2))/(8*a^5*d) - (b*tan
(c/2 + (d*x)/2)^3)/(24*a^2*d) + (b^2*tan(c/2 + (d*x)/2)^2)/(8*a^3*d) + (b^3
*atan(((b^3*(b^2 - a^2)^(1/2)*((tan(c/2 + (d*x)/2)*(a^9 + 32*a^3*b^6 - 32*a
^5*b^4 + 2*a^7*b^2))/(4*a^7) - (a^9*b - 16*a^5*b^5 + 12*a^7*b^3)/(4*a^8) +
(b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^10 - 32*a^8*b^2))/(4*a^7))*(b^2 -
 a^2)^(1/2))/a^5)*1i)/a^5 - (b^3*(b^2 - a^2)^(1/2)*((a^9*b - 16*a^5*b^5 + 1
2*a^7*b^3)/(4*a^8) - (tan(c/2 + (d*x)/2)*(a^9 + 32*a^3*b^6 - 32*a^5*b^4 + 2
*a^7*b^2))/(4*a^7) + (b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^10 - 32*a^8*
b^2))/(4*a^7))*(b^2 - a^2)^(1/2))/a^5)*1i)/a^5)/((8*b^9 - 12*a^2*b^7 + 3*a^
4*b^5 + a^6*b^3)/(2*a^8) + (tan(c/2 + (d*x)/2)*(8*b^8 - 10*a^2*b^6 + 2*a^4*
b^4))/(2*a^7) + (b^3*(b^2 - a^2)^(1/2)*((tan(c/2 + (d*x)/2)*(a^9 + 32*a^3*b
```

```
^6 - 32*a^5*b^4 + 2*a^7*b^2))/(4*a^7) - (a^9*b - 16*a^5*b^5 + 12*a^7*b^3)/(
4*a^8) + (b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^10 - 32*a^8*b^2))/(4*a^7
))*(b^2 - a^2)^(1/2))/a^5))/a^5 + (b^3*(b^2 - a^2)^(1/2)*((a^9*b - 16*a^5*b
^5 + 12*a^7*b^3)/(4*a^8) - (tan(c/2 + (d*x)/2)*(a^9 + 32*a^3*b^6 - 32*a^5*b
^4 + 2*a^7*b^2))/(4*a^7) + (b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^10 - 3
2*a^8*b^2))/(4*a^7))*(b^2 - a^2)^(1/2))/a^5))/a^5))*(b^2 - a^2)^(1/2)*2i)/(
a^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^5(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

3.1294 $\int \frac{\cot^2(c+dx)\csc^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=238

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{2b^4\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{b\left(a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d}+\frac{\left(a^2-5b^2\right)\ldots}{}$$

```
[Out] -1/8*b*(a^4+4*a^2*b^2-8*b^4)*arctanh(cos(d*x+c))/a^6/d+1/15*(2*a^4+5*a^2*b^
2-15*b^4)*cot(d*x+c)/a^5/d-1/8*b*(a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/a^4/d+1/
15*(a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)^2/a^3/d+1/4*b*cot(d*x+c)*csc(d*x+c)^3/
a^2/d-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d-2*b^4*arctan((b+a*tan(1/2*d*x+1/2*c))
/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^6/d
```

Rubi [A] time = 1.23, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b^4\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{\left(5a^2b^2+2a^4-15b^4\right)\cot(c+dx)}{15a^5d}-\frac{b\left(4a^2b^2+a^4-8b^4\right)\tanh^{-1}(\cos(c+\ldots}{8a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^6*d) - (b*(a^4 + 4*a^2*b^2 - 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) + ((
2*a^4 + 5*a^2*b^2 - 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (b*(a^2 - 4*b^2)*Cot
[c + d*x]*Csc[c + d*x])/(8*a^4*d) + ((a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x
]^2)/(15*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]
*Csc[c + d*x]^4)/(5*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
```

```
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^6(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$

$$= -\frac{\cot(c+dx)\csc^4(c+dx)}{5ad} + \frac{\int \frac{\csc^5(c+dx)\left(-5b-a\sin(c+dx)+4b\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{5a}$$

$$= \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad} + \frac{\int \frac{\csc^4(c+dx)\left(-4\left(a^2-5b^2\right)+ab \ldots}{a+b\sin \ldots}}{2 \ldots}$$

$$= \frac{\left(a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{15a^3d} + \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+dx) \ldots}{5a \ldots}$$

$$= -\frac{b\left(a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} + \frac{\left(a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{15a^3d} + \ldots$$

$$= \frac{\left(2a^4+5a^2b^2-15b^4\right)\cot(c+dx)}{15a^5d} - \frac{b\left(a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} + \frac{\left(a^2 \ldots\right.}{\ldots}$$

$$= \frac{\left(2a^4+5a^2b^2-15b^4\right)\cot(c+dx)}{15a^5d} - \frac{b\left(a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} + \frac{\left(a^2 \ldots\right.}{\ldots}$$

$$= -\frac{b\left(a^4+4a^2b^2-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} + \frac{\left(2a^4+5a^2b^2-15b^4\right)\cot(c+dx)}{15a^5d} \ldots$$

$$= -\frac{b\left(a^4+4a^2b^2-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} + \frac{\left(2a^4+5a^2b^2-15b^4\right)\cot(c+dx)}{15a^5d} \ldots$$

$$= -\frac{2b^4\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} - \frac{b\left(a^4+4a^2b^2-8b^4\right)\tanh^{-1}(\cos(c+d \ldots}{8a^6d}$$

Mathematica [B] time = 1.85, size = 506, normalized size = 2.13

$$\frac{-64a^5\tan\left(\frac{1}{2}(c+dx)\right) - 3a^5\sin(c+dx)\csc^6\left(\frac{1}{2}(c+dx)\right) + a^5\sin(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right) - 16a^5\sin^4\left(\frac{1}{2}(c+dx)\right) \ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-1920*b^4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
 + 32*(2*a^5 + 5*a^3*b^2 - 15*a*b^4)*Cot[(c + d*x)/2] - 30*a^4*b*Csc[(c + d
*x)/2]^2 + 120*a^2*b^3*Csc[(c + d*x)/2]^2 + 15*a^4*b*Csc[(c + d*x)/2]^4 - 1
20*a^4*b*Log[Cos[(c + d*x)/2]] - 480*a^2*b^3*Log[Cos[(c + d*x)/2]] + 960*b^
5*Log[Cos[(c + d*x)/2]] + 120*a^4*b*Log[Sin[(c + d*x)/2]] + 480*a^2*b^3*Log
[Sin[(c + d*x)/2]] - 960*b^5*Log[Sin[(c + d*x)/2]] + 30*a^4*b*Sec[(c + d*x)
/2]^2 - 120*a^2*b^3*Sec[(c + d*x)/2]^2 - 15*a^4*b*Sec[(c + d*x)/2]^4 - 16*a
^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*a^3*b^2*Csc[c + d*x]^3*Sin[(c +
d*x)/2]^4 + a^5*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a^3*b^2*Csc[(c + d*x)/
2]^4*Sin[c + d*x] - 3*a^5*Csc[(c + d*x)/2]^6*Sin[c + d*x] - 64*a^5*Tan[(c +
 d*x)/2] - 160*a^3*b^2*Tan[(c + d*x)/2] + 480*a*b^4*Tan[(c + d*x)/2] + 6*a^
5*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*a^6*d)
```

fricas [A] time = 0.95, size = 959, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(240*a*b^4*cos(d*x + c) - 16*(2*a^5 + 5*a^3*b^2 - 15*a*b^4)*cos(d*x
 + c)^5 + 80*(a^5 + a^3*b^2 - 6*a*b^4)*cos(d*x + c)^3 - 120*(b^4*cos(d*x +
c)^4 - 2*b^4*cos(d*x + c)^2 + b^4)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c
) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2))*sin(d*x + c) + 15*(a^4*b + 4*a^2*b^3 - 8*b^5 + (a^4*b + 4
*a^2*b^3 - 8*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x +
c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(a^4*b + 4*a^2*b^3 - 8*
b^5 + (a^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 4*a^2*b^3 - 8
*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*((a^4*
b - 4*a^2*b^3)*cos(d*x + c)^3 + (a^4*b + 4*a^2*b^3)*cos(d*x + c))*sin(d*x +
 c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))
, -1/240*(240*a*b^4*cos(d*x + c) - 16*(2*a^5 + 5*a^3*b^2 - 15*a*b^4)*cos(d*
x + c)^5 + 80*(a^5 + a^3*b^2 - 6*a*b^4)*cos(d*x + c)^3 - 240*(b^4*cos(d*x +
 c)^4 - 2*b^4*cos(d*x + c)^2 + b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c)
 + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) + 15*(a^4*b + 4*a^2*b^3
- 8*b^5 + (a^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 4*a^2*b^3
 - 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(a^
4*b + 4*a^2*b^3 - 8*b^5 + (a^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x + c)^4 - 2*(a
^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(
d*x + c) - 30*((a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 + (a^4*b + 4*a^2*b^3)*cos
(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 +
a^6*d)*sin(d*x + c))]
```

giac [B] time = 0.20, size = 444, normalized size = 1.87

$$\frac{6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-60\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-120\,a^2b^2\,\mathrm{t}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*((6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*b*tan(1/2*d*x + 1/2*c)^4 + 10
*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a*b^3
*tan(1/2*d*x + 1/2*c)^2 - 60*a^4*tan(1/2*d*x + 1/2*c) - 120*a^2*b^2*tan(1/2
*d*x + 1/2*c) + 480*b^4*tan(1/2*d*x + 1/2*c))/a^5 + 120*(a^4*b + 4*a^2*b^3
- 8*b^5)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 - 1920*(a^2*b^4 - b^6)*(pi*floo
r(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt
(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) - (274*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 1
096*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 2192*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*a
^5*tan(1/2*d*x + 1/2*c)^4 - 120*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 480*a*b^4*
tan(1/2*d*x + 1/2*c)^4 - 120*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 10*a^5*tan(1/
2*d*x + 1/2*c)^2 + 40*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^4*b*tan(1/2*d*x
 + 1/2*c) + 6*a^5)/(a^6*tan(1/2*d*x + 1/2*c)^5))/d
```

maple [A] time = 0.50, size = 439, normalized size = 1.84

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{160da}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{64d\,a^2}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{96da}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{24d\,a^3}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3}{8d\,a^4}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{b^2\tan}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/160/d/a*tan(1/2*d*x+1/2*c)^5-1/64/d/a^2*tan(1/2*d*x+1/2*c)^4*b+1/96/d/a*t
an(1/2*d*x+1/2*c)^3+1/24/d/a^3*tan(1/2*d*x+1/2*c)^3*b^2-1/8/d/a^4*tan(1/2*d
*x+1/2*c)^2*b^3-1/16/a/d*tan(1/2*d*x+1/2*c)-1/8/d/a^3*b^2*tan(1/2*d*x+1/2*c
)+1/2/d/a^5*b^4*tan(1/2*d*x+1/2*c)-1/160/d/a/tan(1/2*d*x+1/2*c)^5-1/96/d/a/
tan(1/2*d*x+1/2*c)^3-1/24/d/a^3/tan(1/2*d*x+1/2*c)^3*b^2+1/16/a/d/tan(1/2*d
*x+1/2*c)+1/8/d/a^3/tan(1/2*d*x+1/2*c)*b^2-1/2/d/a^5/tan(1/2*d*x+1/2*c)*b^4
+1/64/d/a^2*b/tan(1/2*d*x+1/2*c)^4+1/8/d*b^3/a^4/tan(1/2*d*x+1/2*c)^2+1/8/d
/a^2*b*ln(tan(1/2*d*x+1/2*c))+1/2/d/a^4*b^3*ln(tan(1/2*d*x+1/2*c))-1/d/a^6*
b^5*ln(tan(1/2*d*x+1/2*c))-2/d*(a^2-b^2)^(1/2)/a^6*b^4*arctan(1/2*(2*a*tan(
1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.08, size = 1007, normalized size = 4.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^6*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a*d) + (tan(c/2 + (d*x)/2)^2*(b/(32*a^2) - (b*(1/
(32*a) + b^2/(8*a^3)))/a))/d - (tan(c/2 + (d*x)/2)*(1/(16*a) + b^2/(8*a^3)
+ (2*b*(b/(16*a^2) - (2*b*(1/(32*a) + b^2/(8*a^3)))/a))/a))/d + (tan(c/2 +
(d*x)/2)^3*(1/(96*a) + b^2/(24*a^3)))/d + (log(tan(c/2 + (d*x)/2))*((a^4*b)
/8 - b^5 + (a^2*b^3)/2))/(a^6*d) - (b*tan(c/2 + (d*x)/2)^4)/(64*a^2*d) + (t
an(c/2 + (d*x)/2)^4*(2*a^4 - 16*b^4 + 4*a^2*b^2) - a^4/5 - tan(c/2 + (d*x)/
2)^2*(a^4/3 + (4*a^2*b^2)/3) + (a^3*b*tan(c/2 + (d*x)/2))/2 + 4*a*b^3*tan(c
/2 + (d*x)/2)^3)/(32*a^5*d*tan(c/2 + (d*x)/2)^5) - (b^4*atan(((b^4*(b^2 - a
^2)^(1/2)*((tan(c/2 + (d*x)/2)*(a^10*b + 32*a^4*b^7 - 32*a^6*b^5 + 2*a^8*b^
3))/(4*a^9) - (12*a^8*b^4 - 16*a^6*b^6 + a^10*b^2)/(4*a^10) + (b^4*(2*a^2*b
 - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(b^2 - a^2)^(1/2))
/a^6)*1i)/a^6 - (b^4*(b^2 - a^2)^(1/2)*((12*a^8*b^4 - 16*a^6*b^6 + a^10*b^2
)/(4*a^10) - (tan(c/2 + (d*x)/2)*(a^10*b + 32*a^4*b^7 - 32*a^6*b^5 + 2*a^8*
b^3))/(4*a^9) + (b^4*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2)
)/(4*a^9))*(b^2 - a^2)^(1/2))/a^6)*1i)/a^6)/((8*b^11 - 12*a^2*b^9 + 3*a^4*b
^7 + a^6*b^5)/(2*a^10) + (tan(c/2 + (d*x)/2)*(8*b^10 - 10*a^2*b^8 + 2*a^4*b
^6))/(2*a^9) + (b^4*(b^2 - a^2)^(1/2)*((tan(c/2 + (d*x)/2)*(a^10*b + 32*a^4
*b^7 - 32*a^6*b^5 + 2*a^8*b^3))/(4*a^9) - (12*a^8*b^4 - 16*a^6*b^6 + a^10*b
^2)/(4*a^10) + (b^4*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))
/(4*a^9))*(b^2 - a^2)^(1/2))/a^6))/a^6 + (b^4*(b^2 - a^2)^(1/2)*((12*a^8*b^
4 - 16*a^6*b^6 + a^10*b^2)/(4*a^10) - (tan(c/2 + (d*x)/2)*(a^10*b + 32*a^4*
b^7 - 32*a^6*b^5 + 2*a^8*b^3))/(4*a^9) + (b^4*(2*a^2*b - (tan(c/2 + (d*x)/2
)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(b^2 - a^2)^(1/2))/a^6))/a^6))*(b^2 - a
^2)^(1/2)*2i)/(a^6*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1295 $\int \frac{\cos^3(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2\left(a^2-b^2\right)\sin(c+dx)}{b^5d}+\frac{a\left(a^2-b^2\right)\sin^2(c+dx)}{2b^4d}-\frac{\left(a^2-b^2\right)\sin^3(c+dx)}{3b^3d}+\frac{a^3\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^6d}+$$

```
[Out] a^3*(a^2-b^2)*ln(a+b*sin(d*x+c))/b^6/d-a^2*(a^2-b^2)*sin(d*x+c)/b^5/d+1/2*a
*(a^2-b^2)*sin(d*x+c)^2/b^4/d-1/3*(a^2-b^2)*sin(d*x+c)^3/b^3/d+1/4*a*sin(d*
x+c)^4/b^2/d-1/5*sin(d*x+c)^5/b/d
```

Rubi [A] time = 0.20, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$-\frac{\left(a^2-b^2\right)\sin^3(c+dx)}{3b^3d}+\frac{a\left(a^2-b^2\right)\sin^2(c+dx)}{2b^4d}-\frac{a^2\left(a^2-b^2\right)\sin(c+dx)}{b^5d}+\frac{a^3\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^6d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a^3*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^6*d) - (a^2*(a^2 - b^2)*Sin[c
+ d*x])/(b^5*d) + (a*(a^2 - b^2)*Sin[c + d*x]^2)/(2*b^4*d) - ((a^2 - b^2)*S
in[c + d*x]^3)/(3*b^3*d) + (a*Sin[c + d*x]^4)/(4*b^2*d) - Sin[c + d*x]^5/(5
*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)}{b^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^6 d}$$

$$= \frac{\text{Subst}\left(\int \left(-a^4\left(1-\frac{b^2}{a^2}\right) + a\left(a^2-b^2\right)x - \left(a^2-b^2\right)x^2 + ax^3 - x^4 + \frac{a^5-a^3b^2}{a+x}\right) dx,\right.}{b^6 d}$$

$$= \frac{a^3\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^6 d} - \frac{a^2\left(a^2-b^2\right)\sin(c+dx)}{b^5 d} + \frac{a\left(a^2-b^2\right)\sin^2}{2b^4 d}$$

Mathematica [A] time = 0.28, size = 127, normalized size = 0.85

$$\frac{\frac{60a^3(a-b)(a+b)\log(a+b\sin(c+dx))}{b^6} - \frac{60a^2(a-b)(a+b)\sin(c+dx)}{b^5} + \frac{30a(a-b)(a+b)\sin^2(c+dx)}{b^4} - \frac{20(a-b)(a+b)\sin^3(c+dx)}{b^3} + \frac{15a\sin^4(c+dx)}{b^2} - 12}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((60*a^3*(a - b)*(a + b)*Log[a + b*Sin[c + d*x]])/b^6 - (60*a^2*(a - b)*(a
+ b)*Sin[c + d*x])/b^5 + (30*a*(a - b)*(a + b)*Sin[c + d*x]^2)/b^4 - (20*(a
 - b)*(a + b)*Sin[c + d*x]^3)/b^3 + (15*a*Sin[c + d*x]^4)/b^2 - (12*Sin[c +
 d*x]^5)/b)/(60*d)
```

fricas [A] time = 0.79, size = 127, normalized size = 0.85

$$\frac{15\,ab^4\cos(dx+c)^4 - 30\,a^3b^2\cos(dx+c)^2 + 60\left(a^5-a^3b^2\right)\log(b\sin(dx+c)+a) - 4\left(3\,b^5\cos(dx+c)^4 + 15\,a^4\right.}{60\,b^6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*a*b^4*cos(d*x + c)^4 - 30*a^3*b^2*cos(d*x + c)^2 + 60*(a^5 - a^3*b
^2)*log(b*sin(d*x + c) + a) - 4*(3*b^5*cos(d*x + c)^4 + 15*a^4*b - 10*a^2*b
^3 - 2*b^5 - (5*a^2*b^3 + b^5)*cos(d*x + c)^2)*sin(d*x + c))/(b^6*d)
```

giac [A] time = 0.17, size = 149, normalized size = 1.00

$$-\frac{\frac{12\,b^4 \sin(dx+c)^5-15\,ab^3 \sin(dx+c)^4+20\,a^2b^2 \sin(dx+c)^3-20\,b^4 \sin(dx+c)^3-30\,a^3b \sin(dx+c)^2+30\,ab^3 \sin(dx+c)^2+60\,a^4 \sin(dx+c)-60\,a^2b^2 \sin(dx+c)}{b^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*((12*b^4*sin(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 20*a^2*b^2*sin(d*
x + c)^3 - 20*b^4*sin(d*x + c)^3 - 30*a^3*b*sin(d*x + c)^2 + 30*a*b^3*sin(d
*x + c)^2 + 60*a^4*sin(d*x + c) - 60*a^2*b^2*sin(d*x + c))/b^5 - 60*(a^5 -
a^3*b^2)*log(abs(b*sin(d*x + c) + a))/b^6)/d
```

maple [A] time = 0.30, size = 182, normalized size = 1.22

$$-\frac{\sin^5(dx+c)}{5bd}+\frac{a\left(\sin^4(dx+c)\right)}{4b^2d}-\frac{\left(\sin^3(dx+c)\right)a^2}{3d\,b^3}+\frac{\sin^3(dx+c)}{3bd}+\frac{\left(\sin^2(dx+c)\right)a^3}{2d\,b^4}-\frac{a\left(\sin^2(dx+c)\right)}{2b^2d}-\frac{a^4 \sin\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/5*sin(d*x+c)^5/b/d+1/4*a*sin(d*x+c)^4/b^2/d-1/3/d/b^3*sin(d*x+c)^3*a^2+1
/3*sin(d*x+c)^3/b/d+1/2/d/b^4*sin(d*x+c)^2*a^3-1/2*a*sin(d*x+c)^2/b^2/d-1/d
/b^5*a^4*sin(d*x+c)+a^2*sin(d*x+c)/b^3/d+1/d*a^5/b^6*ln(a+b*sin(d*x+c))-a^3
*ln(a+b*sin(d*x+c))/b^4/d
```

maxima [A] time = 0.33, size = 131, normalized size = 0.88

$$-\frac{\frac{12\,b^4 \sin(dx+c)^5-15\,ab^3 \sin(dx+c)^4+20\left(a^2b^2-b^4\right)\sin(dx+c)^3-30\left(a^3b-ab^3\right)\sin(dx+c)^2+60\left(a^4-a^2b^2\right)\sin(dx+c)}{b^5}-\frac{60\left(a^5-a^3b^2\right)\log(b\sin(dx+c)+\cdots}{b^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((12*b^4*sin(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 20*(a^2*b^2 - b^4
)*sin(d*x + c)^3 - 30*(a^3*b - a*b^3)*sin(d*x + c)^2 + 60*(a^4 - a^2*b^2)*s
in(d*x + c))/b^5 - 60*(a^5 - a^3*b^2)*log(b*sin(d*x + c) + a)/b^6)/d
```

mupad [B] time = 0.08, size = 133, normalized size = 0.89

$$\frac{\sin(c+d\,x)^3\left(\frac{1}{3\,b}-\frac{a^2}{3\,b^3}\right)-\frac{\sin(c+d\,x)^5}{5\,b}+\frac{a\sin(c+d\,x)^4}{4\,b^2}+\frac{\ln(a+b\,\sin(c+d\,x))\left(a^5-a^3\,b^2\right)}{b^6}-\frac{a\sin(c+d\,x)^2\left(\frac{1}{b}-\frac{a^2}{b^3}\right)}{2\,b}+\frac{a^2\,\sin(c+d\,x)\left(\frac{1}{b}-\cdots\right)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*sin(c + d*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^3*(1/(3*b) - a^2/(3*b^3)) - sin(c + d*x)^5/(5*b) + (a*sin(c +
 d*x)^4)/(4*b^2) + (log(a + b*sin(c + d*x))*(a^5 - a^3*b^2))/b^6 - (a*sin(c
 + d*x)^2*(1/b - a^2/b^3))/(2*b) + (a^2*sin(c + d*x)*(1/b - a^2/b^3))/b^2)/
d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1296 $\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=119

$$-\frac{a^2\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^5d}+\frac{a\left(a^2-b^2\right)\sin(c+dx)}{b^4d}-\frac{\left(a^2-b^2\right)\sin^2(c+dx)}{2b^3d}+\frac{a\sin^3(c+dx)}{3b^2d}-\frac{\sin^4(c+d}{4bd}$$

```
[Out] -a^2*(a^2-b^2)*ln(a+b*sin(d*x+c))/b^5/d+a*(a^2-b^2)*sin(d*x+c)/b^4/d-1/2*(a
^2-b^2)*sin(d*x+c)^2/b^3/d+1/3*a*sin(d*x+c)^3/b^2/d-1/4*sin(d*x+c)^4/b/d
```

Rubi [A] time = 0.17, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$-\frac{\left(a^2-b^2\right)\sin^2(c+dx)}{2b^3d}+\frac{a\left(a^2-b^2\right)\sin(c+dx)}{b^4d}-\frac{a^2\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^5d}+\frac{a\sin^3(c+dx)}{3b^2d}-\frac{\sin^4(c+d}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a^2*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^5*d)) + (a*(a^2 - b^2)*Sin[c
 + d*x])/(b^4*d) - ((a^2 - b^2)*Sin[c + d*x]^2)/(2*b^3*d) + (a*Sin[c + d*x]
^3)/(3*b^2*d) - Sin[c + d*x]^4/(4*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(b^2-x^2)}{b^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(b^2-x^2)}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3\left(1-\frac{b^2}{a^2}\right)-\left(a^2-b^2\right)x+ax^2-x^3+\frac{a^2(-a^2+b^2)}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= -\frac{a^2\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^5 d}+\frac{a\left(a^2-b^2\right)\sin(c+dx)}{b^4 d}-\frac{\left(a^2-b^2\right)\sin^2(}{2b^3 d}$$

Mathematica [A] time = 0.36, size = 104, normalized size = 0.87

$$\frac{6b^2\left(b^2-a^2\right)\sin^2(c+dx)+12ab\left(a^2-b^2\right)\sin(c+dx)+12a^2\left(b^2-a^2\right)\log(a+b\sin(c+dx))+4ab^3\sin^3(c+dx)}{12b^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (12*a^2*(-a^2 + b^2)*Log[a + b*Sin[c + d*x]] + 12*a*b*(a^2 - b^2)*Sin[c + d
*x] + 6*b^2*(-a^2 + b^2)*Sin[c + d*x]^2 + 4*a*b^3*Sin[c + d*x]^3 - 3*b^4*Si
n[c + d*x]^4)/(12*b^5*d)
```

fricas [A] time = 0.53, size = 97, normalized size = 0.82

$$-\frac{3\,b^4\cos(dx+c)^4-6\,a^2b^2\cos(dx+c)^2+12\left(a^4-a^2b^2\right)\log(b\sin(dx+c)+a)+4\left(ab^3\cos(dx+c)^2-3\,a^3b+\right.}{12\,b^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*cos(d*x + c)^4 - 6*a^2*b^2*cos(d*x + c)^2 + 12*(a^4 - a^2*b^2)
*log(b*sin(d*x + c) + a) + 4*(a*b^3*cos(d*x + c)^2 - 3*a^3*b + 2*a*b^3)*sin
(d*x + c))/(b^5*d)
```

giac [A] time = 0.15, size = 117, normalized size = 0.98

$$-\frac{\frac{3\,b^{3}\sin(dx+c)^{4}-4\,ab^{2}\sin(dx+c)^{3}+6\,a^{2}b\sin(dx+c)^{2}-6\,b^{3}\sin(dx+c)^{2}-12\,a^{3}\sin(dx+c)+12\,ab^{2}\sin(dx+c)}{b^{4}}+\frac{12\left(a^{4}-a^{2}b^{2}\right)\log(|b\sin(dx+c)+a|)}{b^{5}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*((3*b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*a^2*b*sin(d*x + c
)^2 - 6*b^3*sin(d*x + c)^2 - 12*a^3*sin(d*x + c) + 12*a*b^2*sin(d*x + c))/b
^4 + 12*(a^4 - a^2*b^2)*log(abs(b*sin(d*x + c) + a))/b^5)/d
```

maple [A] time = 0.29, size = 144, normalized size = 1.21

$$-\frac{\sin^{4}(dx+c)}{4bd}+\frac{a\left(\sin^{3}(dx+c)\right)}{3b^{2}d}-\frac{a^{2}\left(\sin^{2}(dx+c)\right)}{2d\,b^{3}}+\frac{\sin^{2}(dx+c)}{2bd}+\frac{\sin(dx+c)\,a^{3}}{d\,b^{4}}-\frac{a\sin(dx+c)}{b^{2}d}-\frac{a^{4}\ln(a+b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/4*sin(d*x+c)^4/b/d+1/3*a*sin(d*x+c)^3/b^2/d-1/2/d/b^3*a^2*sin(d*x+c)^2+1
/2*sin(d*x+c)^2/b/d+1/d/b^4*sin(d*x+c)*a^3-a*sin(d*x+c)/b^2/d-1/d*a^4/b^5*l
n(a+b*sin(d*x+c))+1/d/b^3*ln(a+b*sin(d*x+c))*a^2
```

maxima [A] time = 0.31, size = 105, normalized size = 0.88

$$-\frac{\frac{3\,b^{3}\sin(dx+c)^{4}-4\,ab^{2}\sin(dx+c)^{3}+6\left(a^{2}b-b^{3}\right)\sin(dx+c)^{2}-12\left(a^{3}-ab^{2}\right)\sin(dx+c)}{b^{4}}+\frac{12\left(a^{4}-a^{2}b^{2}\right)\log(b\sin(dx+c)+a)}{b^{5}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((3*b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*(a^2*b - b^3)*sin
(d*x + c)^2 - 12*(a^3 - a*b^2)*sin(d*x + c))/b^4 + 12*(a^4 - a^2*b^2)*log(b
*sin(d*x + c) + a)/b^5)/d
```

mupad [B] time = 11.62, size = 107, normalized size = 0.90

$$-\frac{\frac{\sin(c+d\,x)^{4}}{4\,b}-\sin\left(c+d\,x\right)^{2}\left(\frac{1}{2\,b}-\frac{a^{2}}{2\,b^{3}}\right)-\frac{a\sin(c+d\,x)^{3}}{3\,b^{2}}+\frac{\ln(a+b\,\sin(c+d\,x))\left(a^{4}-a^{2}\,b^{2}\right)}{b^{5}}+\frac{a\,\sin(c+d\,x)\left(\frac{1}{b}-\frac{a^{2}}{b^{3}}\right)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*sin(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)^4/(4*b) - sin(c + d*x)^2*(1/(2*b) - a^2/(2*b^3)) - (a*sin(c
+ d*x)^3)/(3*b^2) + (log(a + b*sin(c + d*x))*(a^4 - a^2*b^2))/b^5 + (a*sin(
c + d*x)*(1/b - a^2/b^3))/b)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1297 $\int \frac{\cos^3(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=89

$$\frac{a\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^4d}-\frac{\left(a^2-b^2\right)\sin(c+dx)}{b^3d}+\frac{a\sin^2(c+dx)}{2b^2d}-\frac{\sin^3(c+dx)}{3bd}$$

```
[Out] a*(a^2-b^2)*ln(a+b*sin(d*x+c))/b^4/d-(a^2-b^2)*sin(d*x+c)/b^3/d+1/2*a*sin(d
*x+c)^2/b^2/d-1/3*sin(d*x+c)^3/b/d
```

Rubi [A] time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 772}

$$-\frac{\left(a^2-b^2\right)\sin(c+dx)}{b^3d}+\frac{a\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^4d}+\frac{a\sin^2(c+dx)}{2b^2d}-\frac{\sin^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^4*d) - ((a^2 - b^2)*Sin[c + d*x]
)/(b^3*d) + (a*Sin[c + d*x]^2)/(2*b^2*d) - Sin[c + d*x]^3/(3*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)}{b(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^4d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2\left(1-\frac{b^2}{a^2}\right)+ax-x^2+\frac{a^3-ab^2}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^4d}$$
$$= \frac{a(a^2-b^2)\log(a+b\sin(c+dx))}{b^4d} - \frac{(a^2-b^2)\sin(c+dx)}{b^3d} + \frac{a\sin^2(c+dx)}{2b^2d} - \text{si}$$

Mathematica [A] time = 0.20, size = 79, normalized size = 0.89

$$\frac{6b(b^2-a^2)\sin(c+dx)+6a(a^2-b^2)\log(a+b\sin(c+dx))+3ab^2\sin^2(c+dx)-2b^3\sin^3(c+dx)}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (6*a*(a^2 - b^2)*Log[a + b*Sin[c + d*x]] + 6*b*(-a^2 + b^2)*Sin[c + d*x] +
3*a*b^2*Sin[c + d*x]^2 - 2*b^3*Sin[c + d*x]^3)/(6*b^4*d)
```

fricas [A] time = 0.69, size = 78, normalized size = 0.88

$$-\frac{3\,ab^2\cos(dx+c)^2-6\left(a^3-ab^2\right)\log\left(b\sin(dx+c)+a\right)-2\left(b^3\cos(dx+c)^2-3\,a^2b+2\,b^3\right)\sin(dx+c)}{6\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*a*b^2*cos(d*x + c)^2 - 6*(a^3 - a*b^2)*log(b*sin(d*x + c) + a) - 2*
(b^3*cos(d*x + c)^2 - 3*a^2*b + 2*b^3)*sin(d*x + c))/(b^4*d)
```

giac [A] time = 0.16, size = 85, normalized size = 0.96

$$-\frac{\frac{2\,b^2\sin(dx+c)^3-3\,ab\sin(dx+c)^2+6\,a^2\sin(dx+c)-6\,b^2\sin(dx+c)}{b^3}-\frac{6\left(a^3-ab^2\right)\log(|b\sin(dx+c)+a|)}{b^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*((2*b^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c)^2 + 6*a^2*sin(d*x + c) - 6
*b^2*sin(d*x + c))/b^3 - 6*(a^3 - a*b^2)*log(abs(b*sin(d*x + c) + a))/b^4)/
d
```

maple [A] time = 0.21, size = 106, normalized size = 1.19

$$-\frac{\sin^3(dx+c)}{3bd}+\frac{a\left(\sin^2(dx+c)\right)}{2b^2d}-\frac{a^2\sin(dx+c)}{b^3d}+\frac{\sin(dx+c)}{bd}+\frac{a^3\ln(a+b\sin(dx+c))}{b^4d}-\frac{a\ln(a+b\sin(dx+}{d\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/3*sin(d*x+c)^3/b/d+1/2*a*sin(d*x+c)^2/b^2/d-a^2*sin(d*x+c)/b^3/d+sin(d*x
+c)/b/d+a^3*ln(a+b*sin(d*x+c))/b^4/d-1/d/b^2*a*ln(a+b*sin(d*x+c))
```

maxima [A] time = 0.32, size = 79, normalized size = 0.89

$$-\frac{\frac{2\,b^2\sin(dx+c)^3-3\,ab\sin(dx+c)^2+6\left(a^2-b^2\right)\sin(dx+c)}{b^3}-\frac{6\left(a^3-ab^2\right)\log(b\sin(dx+c)+a)}{b^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*((2*b^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c)^2 + 6*(a^2 - b^2)*sin(d*x
+ c))/b^3 - 6*(a^3 - a*b^2)*log(b*sin(d*x + c) + a)/b^4)/d
```

mupad [B] time = 0.07, size = 78, normalized size = 0.88

$$\frac{\sin\left(c+d\,x\right)\left(\frac{1}{b}-\frac{a^2}{b^3}\right)-\frac{\sin(c+d\,x)^3}{3\,b}+\frac{a\sin(c+d\,x)^2}{2\,b^2}-\frac{\ln(a+b\,\sin(c+d\,x))\left(a\,b^2-a^3\right)}{b^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*sin(c + d*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)*(1/b - a^2/b^3) - sin(c + d*x)^3/(3*b) + (a*sin(c + d*x)^2)/(
2*b^2) - (log(a + b*sin(c + d*x))*(a*b^2 - a^3))/b^4)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1298 $\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d}+\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{bd}$$

```
[Out] ln(sin(d*x+c))/a/d+(a^2-b^2)*ln(a+b*sin(d*x+c))/a/b^2/d-sin(d*x+c)/b/d
```

Rubi [A] time = 0.12, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d}+\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a*b^2*d) -
 Sin[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b(b^2-x^2)}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{ax}+\frac{a^2-b^2}{a(a+x)}\right)dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= \frac{\log(\sin(c+dx))}{ad} + \frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d} - \frac{\sin(c+dx)}{bd}$$

Mathematica [A] time = 0.07, size = 53, normalized size = 0.90

$$\frac{(a^2-b^2)\log(a+b\sin(c+dx)) - ab\sin(c+dx) + b^2\log(\sin(c+dx))}{ab^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^2*Log[Sin[c + d*x]] + (a^2 - b^2)*Log[a + b*Sin[c + d*x]] - a*b*Sin[c +
d*x])/(a*b^2*d)
```

fricas [A] time = 0.71, size = 55, normalized size = 0.93

$$\frac{b^2\log\left(-\frac{1}{2}\sin(dx+c)\right) - ab\sin(dx+c) + (a^2-b^2)\log(b\sin(dx+c)+a)}{ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (b^2*log(-1/2*sin(d*x + c)) - a*b*sin(d*x + c) + (a^2 - b^2)*log(b*sin(d*x
+ c) + a))/(a*b^2*d)
```

giac [A] time = 0.18, size = 56, normalized size = 0.95

$$\frac{\frac{\log(|\sin(dx+c)|)}{a} - \frac{\sin(dx+c)}{b} + \frac{(a^2-b^2)\log(|b\sin(dx+c)+a|)}{ab^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(sin(d*x + c)))/a - sin(d*x + c)/b + (a^2 - b^2)*log(abs(b*sin(d*x
+ c) + a))/(a*b^2))/d
```

maple [A] time = 0.40, size = 68, normalized size = 1.15

$$-\frac{\sin \left(dx+c\right)}{bd}+\frac{a \ln \left(a+b \sin \left(dx+c\right)\right)}{d \,b^{2}}-\frac{\ln \left(a+b \sin \left(dx+c\right)\right)}{da}+\frac{\ln \left(\sin \left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -sin(d*x+c)/b/d+1/d/b^2*a*ln(a+b*sin(d*x+c))-1/d/a*ln(a+b*sin(d*x+c))+ln(si
n(d*x+c))/a/d
```

maxima [A] time = 0.32, size = 54, normalized size = 0.92

$$\frac{\frac{\log\left(\sin\left(dx+c\right)\right)}{a}-\frac{\sin\left(dx+c\right)}{b}+\frac{\left(a^{2}-b^{2}\right)\log\left(b\sin\left(dx+c\right)+a\right)}{ab^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c))/a - sin(d*x + c)/b + (a^2 - b^2)*log(b*sin(d*x + c) + a)
/(a*b^2))/d
```

mupad [B] time = 11.86, size = 98, normalized size = 1.66

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{\sin\left(c+d\,x\right)}{b\,d}+\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(\frac{a}{b^2}-\frac{1}{a}\right)}{d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) - sin(c + d*x)/(b*d) + (log(a + 2*b*tan(c/2 +
 (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a/b^2 - 1/a))/d - (a*log(tan(c/2 + (d*
x)/2)^2 + 1))/(b^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1299 $\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

```
[Out] -csc(d*x+c)/a/d-b*ln(sin(d*x+c))/a^2/d-(1-b^2/a^2)*ln(a+b*sin(d*x+c))/b/d
```

Rubi [A] time = 0.12, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$-\frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) - ((1 - b^2/a^2)*Log[
a + b*Sin[c + d*x]])/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2(b^2-x^2)}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{ax^2}-\frac{b^2}{a^2x}+\frac{-a^2+b^2}{a^2(a+x)}\right)dx, x, b\sin(c+dx)\right)}{bd}$$
$$= -\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}$$

Mathematica [A] time = 0.08, size = 54, normalized size = 0.90

$$\frac{\left(b^2-a^2\right)\log(a+b\sin(c+dx))-ab\csc(c+dx)+b^2(-\log(\sin(c+dx)))}{a^2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(a*b*Csc[c + d*x]) - b^2*Log[Sin[c + d*x]] + (-a^2 + b^2)*Log[a + b*Sin[c
 + d*x]])/(a^2*b*d)
```

fricas [A] time = 0.66, size = 69, normalized size = 1.15

$$-\frac{b^2\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+\left(a^2-b^2\right)\log(b\sin(dx+c)+a)\sin(dx+c)+ab}{a^2bd\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(b^2*log(1/2*sin(d*x + c))*sin(d*x + c) + (a^2 - b^2)*log(b*sin(d*x + c) +
 a)*sin(d*x + c) + a*b)/(a^2*b*d*sin(d*x + c))
```

giac [A] time = 0.17, size = 59, normalized size = 0.98

$$-\frac{\frac{b\log(|\sin(dx+c)|)}{a^2}+\frac{\left(a^2-b^2\right)\log(|b\sin(dx+c)+a|)}{a^2b}+\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*log(abs(sin(d*x + c)))/a^2 + (a^2 - b^2)*log(abs(b*sin(d*x + c) + a))/(
a^2*b) + 1/(a*sin(d*x + c)))/d
```

maple [A] time = 0.39, size = 72, normalized size = 1.20

$$-\frac{\ln\left(a+b\sin\left(dx+c\right)\right)}{bd}+\frac{b\ln\left(a+b\sin\left(dx+c\right)\right)}{a^{2}d}-\frac{1}{da\sin\left(dx+c\right)}-\frac{b\ln\left(\sin\left(dx+c\right)\right)}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -ln(a+b*sin(d*x+c))/b/d+b*ln(a+b*sin(d*x+c))/a^2/d-1/d/a/sin(d*x+c)-b*ln(si
n(d*x+c))/a^2/d
```

maxima [A] time = 0.31, size = 57, normalized size = 0.95

$$-\frac{\frac{b\log(\sin(dx+c))}{a^{2}}+\frac{\left(a^{2}-b^{2}\right)\log(b\sin(dx+c)+a)}{a^{2}b}+\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b*log(sin(d*x + c))/a^2 + (a^2 - b^2)*log(b*sin(d*x + c) + a)/(a^2*b) + 1
/(a*sin(d*x + c)))/d
```

mupad [B] time = 11.86, size = 118, normalized size = 1.97

$$\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\left(\frac{b}{a^2}-\frac{1}{b}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{b\,d}-\frac{b\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(b/a^2 - 1/b))/d
- tan(c/2 + (d*x)/2)/(2*a*d) - cot(c/2 + (d*x)/2)/(2*a*d) + log(tan(c/2 + (
d*x)/2)^2 + 1)/(b*d) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1300 $\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{b\csc(c+dx)}{a^2d}-\frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d}+\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{a^3d}-\frac{\csc^2(c+dx)}{2ad}$$

```
[Out] b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d-(a^2-b^2)*ln(sin(d*x+c))/a^3/d+(a^2
-b^2)*ln(a+b*sin(d*x+c))/a^3/d
```

Rubi **[A]** time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$-\frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d}+\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((a^2 - b^2)*Log[Sin[c
+ d*x]])/(a^3*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{ax^3} - \frac{b^2}{a^2x^2} + \frac{-a^2+b^2}{a^3x} + \frac{a^2-b^2}{a^3(a+x)}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2ad} - \frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d} + \frac{(a^2-b^2)\log(a+b\sin(c+}{a^3d}$$

Mathematica [A] time = 0.16, size = 65, normalized size = 0.77

$$-\frac{2(a^2-b^2)(\log(\sin(c+dx)) - \log(a+b\sin(c+dx))) + a^2\csc^2(c+dx) - 2ab\csc(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*(-2*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - b^2)*(Log[Sin[c +
 d*x]] - Log[a + b*Sin[c + d*x]]))/(a^3*d)
```

fricas [A] time = 0.79, size = 118, normalized size = 1.40

$$-\frac{2\,ab\sin(dx+c) - a^2 - 2\left((a^2-b^2)\cos(dx+c)^2 - a^2 + b^2\right)\log(b\sin(dx+c)+a) + 2\left((a^2-b^2)\cos(dx+c)^2 -}{2\left(a^3d\cos(dx+c)^2 - a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*sin(d*x + c) - a^2 - 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)
*log(b*sin(d*x + c) + a) + 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*log(-
1/2*sin(d*x + c)))/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

giac [A] time = 0.18, size = 88, normalized size = 1.05

$$-\frac{\frac{2(a^2-b^2)\log(|\sin(dx+c)|)}{a^3} - \frac{2(a^2b-b^3)\log(|b\sin(dx+c)+a|)}{a^3b} - \frac{2\,ab\sin(dx+c)-a^2}{a^3\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^2 - b^2)*log(abs(sin(d*x + c)))/a^3 - 2*(a^2*b - b^3)*log(abs(b*
sin(d*x + c) + a))/(a^3*b) - (2*a*b*sin(d*x + c) - a^2)/(a^3*sin(d*x + c)^2
))/d
```

maple [A] time = 0.45, size = 106, normalized size = 1.26

$$\frac{\ln\left(a+b\sin\left(dx+c\right)\right)}{da}-\frac{b^2\ln\left(a+b\sin\left(dx+c\right)\right)}{a^3d}-\frac{1}{2da\sin\left(dx+c\right)^2}-\frac{\ln\left(\sin\left(dx+c\right)\right)}{ad}+\frac{b^2\ln\left(\sin\left(dx+c\right)\right)}{a^3d}+\frac{}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/a*ln(a+b*sin(d*x+c))-b^2*ln(a+b*sin(d*x+c))/a^3/d-1/2/d/a/sin(d*x+c)^2-
ln(sin(d*x+c))/a/d+b^2*ln(sin(d*x+c))/a^3/d+1/d/a^2*b/sin(d*x+c)
```

maxima [A] time = 0.31, size = 77, normalized size = 0.92

$$\frac{\frac{2\left(a^2-b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^3}-\frac{2\left(a^2-b^2\right)\log\left(\sin\left(dx+c\right)\right)}{a^3}+\frac{2\,b\sin\left(dx+c\right)-a}{a^2\sin\left(dx+c\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^2 - b^2)*log(b*sin(d*x + c) + a)/a^3 - 2*(a^2 - b^2)*log(sin(d*x
+ c))/a^3 + (2*b*sin(d*x + c) - a)/(a^2*sin(d*x + c)^2))/d
```

mupad [B] time = 11.76, size = 144, normalized size = 1.71

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-b^2\right)}{a^3\,d}-\frac{\frac{a}{2}-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^2\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}+\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\,\mathrm{ta}\dots\right.}{a^3\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(2*a^2*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - (log(tan(
c/2 + (d*x)/2))*(a^2 - b^2))/(a^3*d) - (a/2 - 2*b*tan(c/2 + (d*x)/2))/(4*a^
2*d*tan(c/2 + (d*x)/2)^2) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (
d*x)/2)^2)*(a^2 - b^2))/(a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1301 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=282

$$\frac{a\left(5a^2-6b^2\right)\sin^2(c+dx)\cos(c+dx)}{15b^4d}-\frac{\left(6a^2-7b^2\right)\sin^3(c+dx)\cos(c+dx)}{24b^3d}+\frac{a\left(15a^4-20a^2b^2+3b^4\right)\cos(c+a}{15b^6d}$$

```
[Out] 1/16*(16*a^6-24*a^4*b^2+6*a^2*b^4+b^6)*x/b^7-2*a^3*(a^2-b^2)^(3/2)*arctan((
b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^7/d+1/15*a*(15*a^4-20*a^2*b^2+3*
b^4)*cos(d*x+c)/b^6/d-1/16*(8*a^4-10*a^2*b^2+b^4)*cos(d*x+c)*sin(d*x+c)/b^5
/d+1/15*a*(5*a^2-6*b^2)*cos(d*x+c)*sin(d*x+c)^2/b^4/d-1/24*(6*a^2-7*b^2)*co
s(d*x+c)*sin(d*x+c)^3/b^3/d+1/5*a*cos(d*x+c)*sin(d*x+c)^4/b^2/d-1/6*cos(d*x
+c)*sin(d*x+c)^5/b/d
```

Rubi [A] time = 1.00, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2895, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{a\left(-20a^2b^2+15a^4+3b^4\right)\cos(c+dx)}{15b^6d}-\frac{2a^3\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d}-\frac{\left(6a^2-7b^2\right)\sin^3(c+dx)\cos(c+}{24b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((16*a^6 - 24*a^4*b^2 + 6*a^2*b^4 + b^6)*x)/(16*b^7) - (2*a^3*(a^2 - b^2)^(
3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*d) + (a*(15*a^4
 - 20*a^2*b^2 + 3*b^4)*Cos[c + d*x])/(15*b^6*d) - ((8*a^4 - 10*a^2*b^2 + b^
4)*Cos[c + d*x]*Sin[c + d*x])/(16*b^5*d) + (a*(5*a^2 - 6*b^2)*Cos[c + d*x]*
Sin[c + d*x]^2)/(15*b^4*d) - ((6*a^2 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/
(24*b^3*d) + (a*Cos[c + d*x]*Sin[c + d*x]^4)/(5*b^2*d) - (Cos[c + d*x]*Sin[
c + d*x]^5)/(6*b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
```

```
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\cos(c+dx)\sin^4(c+dx)}{5b^2d} - \frac{\cos(c+dx)\sin^5(c+dx)}{6bd} - \frac{\int \frac{\sin^3(c+dx)\left(6\left(4a^2-5b^2\right)-\right.}{}}{}$$

$$= -\frac{\left(6a^2-7b^2\right)\cos(c+dx)\sin^3(c+dx)}{24b^3d} + \frac{a\cos(c+dx)\sin^4(c+dx)}{5b^2d} - \frac{\cos(c+}{}$$

$$= \frac{a\left(5a^2-6b^2\right)\cos(c+dx)\sin^2(c+dx)}{15b^4d} - \frac{\left(6a^2-7b^2\right)\cos(c+dx)\sin^3(c+dx)}{24b^3d}$$

$$= -\frac{\left(8a^4-10a^2b^2+b^4\right)\cos(c+dx)\sin(c+dx)}{16b^5d} + \frac{a\left(5a^2-6b^2\right)\cos(c+dx)\sin}{15b^4d}$$

$$= \frac{a\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^6d} - \frac{\left(8a^4-10a^2b^2+b^4\right)\cos(c+dx)\sin(c}{16b^5d}$$

$$= \frac{\left(16a^6-24a^4b^2+6a^2b^4+b^6\right)x}{16b^7} + \frac{a\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^6d} - \frac{(8a}{}$$

$$= \frac{\left(16a^6-24a^4b^2+6a^2b^4+b^6\right)x}{16b^7} + \frac{a\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^6d} - \frac{(8a}{}$$

$$= \frac{\left(16a^6-24a^4b^2+6a^2b^4+b^6\right)x}{16b^7} + \frac{a\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^6d} - \frac{(8a}{}$$

$$= \frac{\left(16a^6-24a^4b^2+6a^2b^4+b^6\right)x}{16b^7} - \frac{2a^3\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7d} + \frac{a}{}$$

Mathematica [A] time = 2.21, size = 274, normalized size = 0.97

$$960a^6c + 960a^6dx - 240a^4b^2\sin(2(c+dx)) - 1440a^4b^2c - 1440a^4b^2dx + \left(60ab^5-80a^3b^3\right)\cos(3(c+dx)) + 240$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (960*a^6*c - 1440*a^4*b^2*c + 360*a^2*b^4*c + 60*b^6*c + 960*a^6*d*x - 1440
*a^4*b^2*d*x + 360*a^2*b^4*d*x + 60*b^6*d*x - 1920*a^3*(a^2 - b^2)^(3/2)*Ar
cTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 120*a*b*(8*a^4 - 10*a^2*b^
2 + b^4)*Cos[c + d*x] + (-80*a^3*b^3 + 60*a*b^5)*Cos[3*(c + d*x)] + 12*a*b^
5*Cos[5*(c + d*x)] - 240*a^4*b^2*Sin[2*(c + d*x)] + 240*a^2*b^4*Sin[2*(c +
d*x)] + 15*b^6*Sin[2*(c + d*x)] + 30*a^2*b^4*Sin[4*(c + d*x)] - 15*b^6*Sin[
4*(c + d*x)] - 5*b^6*Sin[6*(c + d*x)])/(960*b^7*d)
```

fricas [A] time = 1.03, size = 526, normalized size = 1.87

$$\left[\frac{48\,ab^{5}\cos\left(dx+c\right)^{5}-80\,a^{3}b^{3}\cos\left(dx+c\right)^{3}+15\left(16\,a^{6}-24\,a^{4}b^{2}+6\,a^{2}b^{4}+b^{6}\right)dx-120\left(a^{5}-a^{3}b^{2}\right)\sqrt{-a^{2}+b^{2}}\ \mathrm{l}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/240*(48*a*b^5*cos(d*x + c)^5 - 80*a^3*b^3*cos(d*x + c)^3 + 15*(16*a^6 -
24*a^4*b^2 + 6*a^2*b^4 + b^6)*d*x - 120*(a^5 - a^3*b^2)*sqrt(-a^2 + b^2)*lo
g(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*co
s(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 240*(a^5*b - a^3*b^3)*cos(d*x + c
) - 5*(8*b^6*cos(d*x + c)^5 - 2*(6*a^2*b^4 + b^6)*cos(d*x + c)^3 + 3*(8*a^4
*b^2 - 6*a^2*b^4 - b^6)*cos(d*x + c))*sin(d*x + c))/(b^7*d), 1/240*(48*a*b^
5*cos(d*x + c)^5 - 80*a^3*b^3*cos(d*x + c)^3 + 15*(16*a^6 - 24*a^4*b^2 + 6*
a^2*b^4 + b^6)*d*x + 240*(a^5 - a^3*b^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 240*(a^5*b - a^3*b^3)*cos(d*x
+ c) - 5*(8*b^6*cos(d*x + c)^5 - 2*(6*a^2*b^4 + b^6)*cos(d*x + c)^3 + 3*(8*
a^4*b^2 - 6*a^2*b^4 - b^6)*cos(d*x + c))*sin(d*x + c))/(b^7*d)]
```

giac [B] time = 0.21, size = 726, normalized size = 2.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(15*(16*a^6 - 24*a^4*b^2 + 6*a^2*b^4 + b^6)*(d*x + c)/b^7 - 480*(a^7
- 2*a^5*b^2 + a^3*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 2*(120
```

```
*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 150*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 15*b
^5*tan(1/2*d*x + 1/2*c)^11 + 240*a^5*tan(1/2*d*x + 1/2*c)^10 - 480*a^3*b^2*
tan(1/2*d*x + 1/2*c)^10 + 240*a*b^4*tan(1/2*d*x + 1/2*c)^10 + 360*a^4*b*tan
(1/2*d*x + 1/2*c)^9 - 210*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 - 235*b^5*tan(1/2*
d*x + 1/2*c)^9 + 1200*a^5*tan(1/2*d*x + 1/2*c)^8 - 1920*a^3*b^2*tan(1/2*d*x
 + 1/2*c)^8 + 240*a*b^4*tan(1/2*d*x + 1/2*c)^8 + 240*a^4*b*tan(1/2*d*x + 1/
2*c)^7 - 60*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 390*b^5*tan(1/2*d*x + 1/2*c)^7
 + 2400*a^5*tan(1/2*d*x + 1/2*c)^6 - 3200*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 +
480*a*b^4*tan(1/2*d*x + 1/2*c)^6 - 240*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 60*a^
2*b^3*tan(1/2*d*x + 1/2*c)^5 - 390*b^5*tan(1/2*d*x + 1/2*c)^5 + 2400*a^5*ta
n(1/2*d*x + 1/2*c)^4 - 2880*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 480*a*b^4*tan(
1/2*d*x + 1/2*c)^4 - 360*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 210*a^2*b^3*tan(1/2
*d*x + 1/2*c)^3 + 235*b^5*tan(1/2*d*x + 1/2*c)^3 + 1200*a^5*tan(1/2*d*x + 1
/2*c)^2 - 1440*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 48*a*b^4*tan(1/2*d*x + 1/2*
c)^2 - 120*a^4*b*tan(1/2*d*x + 1/2*c) + 150*a^2*b^3*tan(1/2*d*x + 1/2*c) -
15*b^5*tan(1/2*d*x + 1/2*c) + 240*a^5 - 320*a^3*b^2 + 48*a*b^4)/((tan(1/2*d
*x + 1/2*c)^2 + 1)^6*b^6))/d
```

maple [B] time = 0.31, size = 1501, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 3/4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2+1/2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)
^6*tan(1/2*d*x+1/2*c)^5*a^2-3/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+
1/2*c)^3*a^4+2/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a^5+1
/8/d/b*arctan(tan(1/2*d*x+1/2*c))+5/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(
1/2*d*x+1/2*c)*a^2-4/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10
*a^3+20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a^5-24/d/b^4/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a^3-80/3/d/b^4/(1+tan(1/2*d
*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a^3+4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*
tan(1/2*d*x+1/2*c)^6*a-16/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*
c)^8*a^3+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8*a-2/d*a^3/
b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)
)-3/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^4+2/d/b^7*arctan(tan(1/2*d*x+1/2*c))
*a^6+2/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*a^5+1/8/d/b/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)^11-47/24/d/b/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+
1/2*c)^9+13/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7-13/4/d/b/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5+47/24/d/b/(1+tan(1/2*d*x+1/
2*c)^2)^6*tan(1/2*d*x+1/2*c)^3-1/8/d/b/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d
*x+1/2*c)-8/3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*a^3+2/5/d/b^2/(1+tan(1/2*d*x
+1/2*c)^2)^6*a+7/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*a^
2+10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a^5-2/d/b^5/(1+t
```

```
an(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*a^4+3/d/b^5/(1+tan(1/2*d*x+1/2*
c)^2)^6*tan(1/2*d*x+1/2*c)^9*a^4-7/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1
/2*d*x+1/2*c)^9*a^2+2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7
*a^4-1/2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*a^2+20/d/b^6
/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a^5-12/d/b^4/(1+tan(1/2*d*
x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a^3+2/5/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6
*tan(1/2*d*x+1/2*c)^2*a+10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2
*c)^8*a^5+4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a+2/d/b^2
/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a-5/4/d/b^3/(1+tan(1/2*d*
x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*a^2-1/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*
tan(1/2*d*x+1/2*c)*a^4-2/d*a^7/b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*
d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d*a^5/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)
^6*tan(1/2*d*x+1/2*c)^11*a^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 14.64, size = 600, normalized size = 2.13

$$\frac{\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{8\,b\,d}+\frac{\sin\left(2\,c+2\,d\,x\right)}{64\,b\,d}-\frac{\sin\left(4\,c+4\,d\,x\right)}{64\,b\,d}-\frac{\sin\left(6\,c+6\,d\,x\right)}{192\,b\,d}+\frac{a\,\cos\left(3\,c+3\,d\,x\right)}{16\,b^2\,d}+\frac{a\,\cos\left(5\,c+5\,d\,x\right)}{80\,b^2\,d}-\frac{5\,}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(8*b*d) + sin(2*c + 2*d*x)/(64*
b*d) - sin(4*c + 4*d*x)/(64*b*d) - sin(6*c + 6*d*x)/(192*b*d) + (a*cos(3*c
+ 3*d*x))/(16*b^2*d) + (a*cos(5*c + 5*d*x))/(80*b^2*d) - (5*a^3*cos(c + d*x
))/(4*b^4*d) + (a^5*cos(c + d*x))/(b^6*d) + (3*a^2*atan(sin(c/2 + (d*x)/2)/
cos(c/2 + (d*x)/2)))/(4*b^3*d) - (3*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2)))/(b^5*d) + (2*a^6*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^
7*d) - (a^3*cos(3*c + 3*d*x))/(12*b^4*d) + (a^2*sin(2*c + 2*d*x))/(4*b^3*d)
 + (a^2*sin(4*c + 4*d*x))/(32*b^3*d) - (a^4*sin(2*c + 2*d*x))/(4*b^5*d) + (
```

```
a*cos(c + d*x))/(8*b^2*d) - (2*a^3*atanh((2*b^2*sin(c/2 + (d*x)/2)*(b^6 - a
^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - a^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a
^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 +
 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b
^4*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) - 2*a^3*b^2*cos(c/2 + (d
*x)/2) - 4*a^2*b^3*sin(c/2 + (d*x)/2)))*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)
^(1/2))/(b^7*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1302 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=235

$$\frac{2a^2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}+\frac{a\left(4a^2-5b^2\right)\sin(c+dx)\cos(c+dx)}{8b^4d}-\frac{\left(5a^2-6b^2\right)\sin^2(c+dx)\cos(c+d}{15b^3d}$$

```
[Out] -1/8*a*(8*a^4-12*a^2*b^2+3*b^4)*x/b^6+2*a^2*(a^2-b^2)^(3/2)*arctan((b+a*tan
(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^6/d-1/15*(15*a^4-20*a^2*b^2+3*b^4)*cos(
d*x+c)/b^5/d+1/8*a*(4*a^2-5*b^2)*cos(d*x+c)*sin(d*x+c)/b^4/d-1/15*(5*a^2-6*
b^2)*cos(d*x+c)*sin(d*x+c)^2/b^3/d+1/4*a*cos(d*x+c)*sin(d*x+c)^3/b^2/d-1/5*
cos(d*x+c)*sin(d*x+c)^4/b/d
```

Rubi [A] time = 0.72, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2895, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{\left(-20a^2b^2+15a^4+3b^4\right)\cos(c+dx)}{15b^5d}+\frac{2a^2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}-\frac{\left(5a^2-6b^2\right)\sin^2(c+dx)\cos(c+}{15b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*(8*a^4 - 12*a^2*b^2 + 3*b^4)*x)/(8*b^6) + (2*a^2*(a^2 - b^2)^(3/2)*ArcT
an[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*d) - ((15*a^4 - 20*a^2*b
^2 + 3*b^4)*Cos[c + d*x])/(15*b^5*d) + (a*(4*a^2 - 5*b^2)*Cos[c + d*x]*Sin[
c + d*x])/(8*b^4*d) - ((5*a^2 - 6*b^2)*Cos[c + d*x]*Sin[c + d*x]^2)/(15*b^3
*d) + (a*Cos[c + d*x]*Sin[c + d*x]^3)/(4*b^2*d) - (Cos[c + d*x]*Sin[c + d*x
]^4)/(5*b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
```

```
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\cos(c+dx)\sin^3(c+dx)}{4b^2d} - \frac{\cos(c+dx)\sin^4(c+dx)}{5bd} - \frac{\int \frac{\sin^2(c+dx)\left(5\left(3a^2-4b^2\right)-a\right.}{a+}}{}$$

$$= -\frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin^2(c+dx)}{15b^3d} + \frac{a\cos(c+dx)\sin^3(c+dx)}{4b^2d} - \frac{\cos(c+}{}$$

$$= \frac{a\left(4a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{8b^4d} - \frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin^2(c+dx)}{15b^3d} +$$

$$= -\frac{\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^5d} + \frac{a\left(4a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{8b^4d} -$$

$$= -\frac{a\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} - \frac{\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^5d} + \frac{a\left(4a^2-5b^2\right.}{}$$

$$= -\frac{a\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} - \frac{\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^5d} + \frac{a\left(4a^2-5b^2\right.}{}$$

$$= -\frac{a\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} - \frac{\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^5d} + \frac{a\left(4a^2-5b^2\right.}{}$$

$$= -\frac{a\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} + \frac{2a^2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d} - \frac{\left(15a^4-\right.}{}$$

Mathematica [A] time = 2.04, size = 186, normalized size = 0.79

$$\frac{960a^2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + 10\left(4a^2b^3-3b^5\right)\cos(3(c+dx)) - 15a\left(\left(8b^4-8a^2b^2\right)\sin(2(c+dx)) + \right.}{480b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (960*a^2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
 - 60*b*(8*a^4 - 10*a^2*b^2 + b^4)*Cos[c + d*x] + 10*(4*a^2*b^3 - 3*b^5)*Co
s[3*(c + d*x)] - 6*b^5*Cos[5*(c + d*x)] - 15*a*(4*(8*a^4 - 12*a^2*b^2 + 3*b
^4)*(c + d*x) + (-8*a^2*b^2 + 8*b^4)*Sin[2*(c + d*x)] + b^4*Sin[4*(c + d*x)
]))/(480*b^6*d)
```

fricas [A] time = 0.79, size = 457, normalized size = 1.94

$$\left[-\frac{24\,b^5\cos\left(dx+c\right)^5-40\,a^2b^3\cos\left(dx+c\right)^3+15\left(8\,a^5-12\,a^3b^2+3\,ab^4\right)dx+60\left(a^4-a^2b^2\right)\sqrt{-a^2+b^2}\,\log\left(\frac{(2\cdots}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/120*(24*b^5*cos(d*x + c)^5 - 40*a^2*b^3*cos(d*x + c)^3 + 15*(8*a^5 - 12
*a^3*b^2 + 3*a*b^4)*d*x + 60*(a^4 - a^2*b^2)*sqrt(-a^2 + b^2)*log(((2*a^2 -
 b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*s
in(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2)) + 120*(a^4*b - a^2*b^3)*cos(d*x + c) + 15*(2*a*
b^4*cos(d*x + c)^3 - (4*a^3*b^2 - 3*a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^6
*d), -1/120*(24*b^5*cos(d*x + c)^5 - 40*a^2*b^3*cos(d*x + c)^3 + 15*(8*a^5
- 12*a^3*b^2 + 3*a*b^4)*d*x + 120*(a^4 - a^2*b^2)*sqrt(a^2 - b^2)*arctan(-(
a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 120*(a^4*b - a^2*b^3)
*cos(d*x + c) + 15*(2*a*b^4*cos(d*x + c)^3 - (4*a^3*b^2 - 3*a*b^4)*cos(d*x
+ c))*sin(d*x + c))/(b^6*d)]
```

giac [B] time = 0.17, size = 458, normalized size = 1.95

$$-\frac{\frac{15\left(8\,a^5-12\,a^3b^2+3\,ab^4\right)(dx+c)}{b^6}-\frac{240\left(a^6-2\,a^4b^2+a^2b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6}+\frac{2\left(60\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,ab^3\tan\left(\cdots\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(15*(8*a^5 - 12*a^3*b^2 + 3*a*b^4)*(d*x + c)/b^6 - 240*(a^6 - 2*a^4*
b^2 + a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2
*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 2*(60*a^3*b*ta
```

```
n(1/2*d*x + 1/2*c)^9 - 75*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*a^4*tan(1/2*d*
x + 1/2*c)^8 - 240*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 120*b^4*tan(1/2*d*x + 1
/2*c)^8 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 30*a*b^3*tan(1/2*d*x + 1/2*c)^
7 + 480*a^4*tan(1/2*d*x + 1/2*c)^6 - 720*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 + 7
20*a^4*tan(1/2*d*x + 1/2*c)^4 - 880*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 240*b^
4*tan(1/2*d*x + 1/2*c)^4 - 120*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 30*a*b^3*tan(
1/2*d*x + 1/2*c)^3 + 480*a^4*tan(1/2*d*x + 1/2*c)^2 - 560*a^2*b^2*tan(1/2*d
*x + 1/2*c)^2 - 60*a^3*b*tan(1/2*d*x + 1/2*c) + 75*a*b^3*tan(1/2*d*x + 1/2*
c) + 120*a^4 - 160*a^2*b^2 + 24*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*b^5))/
d
```

maple [B] time = 0.31, size = 941, normalized size = 4.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -3/4/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))-2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*
tan(1/2*d*x+1/2*c)^8*a^4+4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2
*c)^8*a^2-2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a^3-2/d/b
/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8-4/d/b/(1+tan(1/2*d*x+1/2*c
)^2)^5*tan(1/2*d*x+1/2*c)^4-2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*a^4+8/3/d/b^
3/(1+tan(1/2*d*x+1/2*c)^2)^5*a^2+3/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^3-2/d
/b^6*arctan(tan(1/2*d*x+1/2*c))*a^5+5/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*ta
n(1/2*d*x+1/2*c)^9*a+28/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*
c)^2*a^2+1/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a^3-5/4/d/b^
2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a-1/d/b^4/(1+tan(1/2*d*x+1/
2*c)^2)^5*tan(1/2*d*x+1/2*c)^9*a^3+2/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2-2/5/d/b/(1+tan(1/2*d*x+1/2*c)^
2)^5+2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a^3-1/2/d/b^2/
(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a+44/3/d/b^3/(1+tan(1/2*d*x
+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^2+12/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*t
an(1/2*d*x+1/2*c)^6*a^2-8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*
c)^2*a^4-4/d*a^4/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b
)/(a^2-b^2)^(1/2))+2/d*a^6/b^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+
1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d
*x+1/2*c)^7*a-8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6*a^4-1
2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.37, size = 511, normalized size = 2.17

$$\frac{5\,a^2\,\cos\left(c+d\,x\right)}{4\,b^3\,d}-\frac{\cos\left(3\,c+3\,d\,x\right)}{16\,b\,d}-\frac{\cos\left(5\,c+5\,d\,x\right)}{80\,b\,d}-\frac{\cos\left(c+d\,x\right)}{8\,b\,d}-\frac{a^4\,\cos\left(c+d\,x\right)}{b^5\,d}-\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4\,b^2\,d}-\frac{a\,\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] (5*a^2*cos(c + d*x))/(4*b^3*d) - cos(3*c + 3*d*x)/(16*b*d) - cos(5*c + 5*d*
x)/(80*b*d) - cos(c + d*x)/(8*b*d) - (a^4*cos(c + d*x))/(b^5*d) - (a*sin(2*
c + 2*d*x))/(4*b^2*d) - (a*sin(4*c + 4*d*x))/(32*b^2*d) + (3*a^3*atan(sin(c
/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^4*d) - (2*a^5*atan(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2)))/(b^6*d) + (a^2*cos(3*c + 3*d*x))/(12*b^3*d) + (a^3*si
n(2*c + 2*d*x))/(4*b^4*d) - (3*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)
))/(4*b^2*d) + (2*a^2*atanh((2*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^
4 + 3*a^4*b^2)^(1/2) - a^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^
4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(
1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 +
(d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) - 2*a^3*b^2*cos(c/2 + (d*x)/2) - 4*a^
2*b^3*sin(c/2 + (d*x)/2)))*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(b^6*
d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1303 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=159

$$-\frac{2a\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}+\frac{\cos(c+dx)\left(8a\left(a^2-b^2\right)-b\left(4a^2-3b^2\right)\sin(c+dx)\right)}{8b^4d}+\frac{x\left(8a^4-12a^2b^2+\right.}{8b^5}$$

[Out] 1/8*(8*a^4-12*a^2*b^2+3*b^4)*x/b^5-2*a*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^5/d-1/12*cos(d*x+c)^3*(4*a-3*b*sin(d*x+c))/b^2/d+1/8*cos(d*x+c)*(8*a*(a^2-b^2)-b*(4*a^2-3*b^2)*sin(d*x+c))/b^4/d

Rubi [A] time = 0.32, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2865, 2735, 2660, 618, 204}

$$-\frac{2a\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}+\frac{\cos(c+dx)\left(8a\left(a^2-b^2\right)-b\left(4a^2-3b^2\right)\sin(c+dx)\right)}{8b^4d}+\frac{x\left(-12a^2b^2+8a^4\right.}{8b^5}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]

[Out] ((8*a^4 - 12*a^2*b^2 + 3*b^4)*x)/(8*b^5) - (2*a*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*d) - (Cos[c + d*x]^3*(4*a - 3*b*Sin[c + d*x]))/(12*b^2*d) + (Cos[c + d*x]*(8*a*(a^2 - b^2) - b*(4*a^2 - 3*b^2)*Sin[c + d*x]))/(8*b^4*d)

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d}+\frac{\int \frac{\cos^2(c+dx)\left(-ab-\left(4a^2-3b^2\right)\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{4b^2}$$

$$= -\frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d}+\frac{\cos(c+dx)\left(8a\left(a^2-b^2\right)-b\left(4a^2-3b^2\right.\right.}{8b^4d}$$

$$= \frac{\left(8a^4-12a^2b^2+3b^4\right)x}{8b^5}-\frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d}+\frac{\cos(c+dx)\left(8a\right.}{}$$

$$= \frac{\left(8a^4-12a^2b^2+3b^4\right)x}{8b^5}-\frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d}+\frac{\cos(c+dx)\left(8a\right.}{}$$

$$= \frac{\left(8a^4-12a^2b^2+3b^4\right)x}{8b^5}-\frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d}+\frac{\cos(c+dx)\left(8a\right.}{}$$

$$= \frac{\left(8a^4-12a^2b^2+3b^4\right)x}{8b^5}-\frac{2a\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5d}-\frac{\cos^3(c+dx}{}$$

Mathematica [A] time = 1.08, size = 155, normalized size = 0.97

$$\frac{24ab\left(4a^2-5b^2\right)\cos(c+dx)-192a\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)+3\left(\left(8b^4-8a^2b^2\right)\sin(2(c+dx))+4\left(8a^4\right.\right.}{96b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-192*a*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
+ 24*a*b*(4*a^2 - 5*b^2)*Cos[c + d*x] - 8*a*b^3*Cos[3*(c + d*x)] + 3*(4*(8*
a^4 - 12*a^2*b^2 + 3*b^4)*(c + d*x) + (-8*a^2*b^2 + 8*b^4)*Sin[2*(c + d*x)]
 + b^4*Sin[4*(c + d*x)]))/(96*b^5*d)
```

fricas [A] time = 0.71, size = 414, normalized size = 2.60

$$\left[-\frac{8\,ab^3\cos\left(dx+c\right)^3-3\left(8\,a^4-12\,a^2b^2+3\,b^4\right)dx+12\left(a^3-ab^2\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-}{b^2\cos\left(dx\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/24*(8*a*b^3*cos(d*x + c)^3 - 3*(8*a^4 - 12*a^2*b^2 + 3*b^4)*d*x + 12*(a
^3 - a*b^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sq
rt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 24
*(a^3*b - a*b^3)*cos(d*x + c) - 3*(2*b^4*cos(d*x + c)^3 - (4*a^2*b^2 - 3*b^
4)*cos(d*x + c))*sin(d*x + c))/(b^5*d), -1/24*(8*a*b^3*cos(d*x + c)^3 - 3*(
8*a^4 - 12*a^2*b^2 + 3*b^4)*d*x - 24*(a^3 - a*b^2)*sqrt(a^2 - b^2)*arctan(-
(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 24*(a^3*b - a*b^3)*c
os(d*x + c) - 3*(2*b^4*cos(d*x + c)^3 - (4*a^2*b^2 - 3*b^4)*cos(d*x + c))*s
in(d*x + c))/(b^5*d)]
```

giac [B] time = 0.17, size = 371, normalized size = 2.33

$$\frac{\frac{3\left(8\,a^4-12\,a^2b^2+3\,b^4\right)(dx+c)}{b^5}-\frac{48\left(a^5-2\,a^3b^2+ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^5}+\frac{2\left(12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(3*(8*a^4 - 12*a^2*b^2 + 3*b^4)*(d*x + c)/b^5 - 48*(a^5 - 2*a^3*b^2 +
a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^5) + 2*(12*a^2*b*tan(1/2*d*
x + 1/2*c)^7 - 15*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*a^3*tan(1/2*d*x + 1/2*c)^
6 - 48*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 9*b
^3*tan(1/2*d*x + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2*c)^4 - 96*a*b^2*tan(1/
2*d*x + 1/2*c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 9*b^3*tan(1/2*d*x + 1/
2*c)^3 + 72*a^3*tan(1/2*d*x + 1/2*c)^2 - 80*a*b^2*tan(1/2*d*x + 1/2*c)^2 -
12*a^2*b*tan(1/2*d*x + 1/2*c) + 15*b^3*tan(1/2*d*x + 1/2*c) + 24*a^3 - 32*a
*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^4))/d
```

maple [B] time = 0.25, size = 760, normalized size = 4.78

$$\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{5\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{d\,b^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{}{d\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*a^2-5/4/d/b/(1+tan(
1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4
*tan(1/2*d*x+1/2*c)^6*a^3-4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/
2*c)^6*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a^2+3/4/d/
b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5+6/d/b^4/(1+tan(1/2*d*x+1/
2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a^3-8/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1
/2*d*x+1/2*c)^4*a-1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*a
^2-3/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3+6/d/b^4/(1+tan(1
/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a^3-20/3/d/b^2/(1+tan(1/2*d*x+1/2*c
)^2)^4*tan(1/2*d*x+1/2*c)^2*a-1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*
x+1/2*c)*a^2+5/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)+2/d/b^4/
(1+tan(1/2*d*x+1/2*c)^2)^4*a^3-8/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*a+2/d/b
^5*arctan(tan(1/2*d*x+1/2*c))*a^4-3/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2+3/
4/d/b*arctan(tan(1/2*d*x+1/2*c))-2/d*a^5/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d*a^3/b^3/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d*a/b/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.95, size = 453, normalized size = 2.85

$$\frac{3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,b\,d}+\frac{\sin\left(2\,c+2\,d\,x\right)}{4\,b\,d}+\frac{\sin\left(4\,c+4\,d\,x\right)}{32\,b\,d}-\frac{a\,\cos\left(3\,c+3\,d\,x\right)}{12\,b^2\,d}+\frac{a^3\,\cos\left(c+d\,x\right)}{b^4\,d}-\frac{3\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] (3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(4*b*d) + sin(2*c + 2*d*x)/
(4*b*d) + sin(4*c + 4*d*x)/(32*b*d) - (a*cos(3*c + 3*d*x))/(12*b^2*d) + (a^
3*cos(c + d*x))/(b^4*d) - (3*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)
))/(b^3*d) + (2*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^5*d) -
(a^2*sin(2*c + 2*d*x))/(4*b^3*d) - (5*a*cos(c + d*x))/(4*b^2*d) - (2*a*atan
h((2*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - a^2
*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2
 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)
/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2
 + (d*x)/2) - 2*a^3*b^2*cos(c/2 + (d*x)/2) - 4*a^2*b^3*sin(c/2 + (d*x)/2)))
*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(b^5*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1304 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^3d}+\frac{x\left(2a^2-3b^2\right)}{2b^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\sin(c+dx)\cos(c+$$

```
[Out] 1/2*(2*a^2-3*b^2)*x/b^3-2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(
a^2-b^2)^(1/2))/a/b^3/d-arctanh(cos(d*x+c))/a/d+a*cos(d*x+c)/b^2/d-1/2*cos(
d*x+c)*sin(d*x+c)/b/d
```

Rubi [A] time = 0.29, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2895, 3057, 2660, 618, 204, 3770}

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^3d}+\frac{x\left(2a^2-3b^2\right)}{2b^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\sin(c+dx)\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a^2 - 3*b^2)*x)/(2*b^3) - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a*b^3*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (a*Cos[
c + d*x])/(b^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{\int \frac{\csc(c+dx)\left(-2b^2-ab\sin(c+dx)-\left(2a^2-3b^2\right)\sin}{a+b\sin(c+dx)}}{2b^2}$$

$$= \frac{\left(2a^2-3b^2\right)x}{2b^3} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \csc(c+dx)\,dx}{a} -$$

$$= \frac{\left(2a^2-3b^2\right)x}{2b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx}{2bd}$$

$$= \frac{\left(2a^2-3b^2\right)x}{2b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx}{2bd}$$

$$= \frac{\left(2a^2-3b^2\right)x}{2b^3} - \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^3d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} +$$

Mathematica [A] time = 0.26, size = 143, normalized size = 1.15

$$-\frac{-4a^3c-4a^3dx+8\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)-4a^2b\cos(c+dx)+ab^2\sin(2(c+dx))+6ab^2c+6ab^2d}{4ab^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(-4*a^3*c + 6*a*b^2*c - 4*a^3*d*x + 6*a*b^2*d*x + 8*(a^2 - b^2)^(3/2)*
ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 4*a^2*b*Cos[c + d*x] + 4
*b^3*Log[Cos[(c + d*x)/2]] - 4*b^3*Log[Sin[(c + d*x)/2]] + a*b^2*Sin[2*(c +
 d*x)])/(a*b^3*d)
```

fricas [A] time = 0.87, size = 350, normalized size = 2.82

$$\left[-\frac{ab^2\cos(dx+c)\sin(dx+c)-2a^2b\cos(dx+c)+b^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-b^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^2*cos(d*x + c)*sin(d*x + c) - 2*a^2*b*cos(d*x + c) + b^3*log(1/2
*cos(d*x + c) + 1/2) - b^3*log(-1/2*cos(d*x + c) + 1/2) - (2*a^3 - 3*a*b^2)
*d*x - (-a^2 + b^2)^(3/2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*
x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(
-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(a*b^3
*d), -1/2*(a*b^2*cos(d*x + c)*sin(d*x + c) - 2*a^2*b*cos(d*x + c) + b^3*log
(1/2*cos(d*x + c) + 1/2) - b^3*log(-1/2*cos(d*x + c) + 1/2) - (2*a^3 - 3*a*
b^2)*d*x - 2*(a^2 - b^2)^(3/2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x + c))))/(a*b^3*d)]
```

giac [A] time = 0.19, size = 183, normalized size = 1.48

$$\frac{\frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{\left(2a^2-3b^2\right)(dx+c)}{b^3}-\frac{4\left(a^4-2a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,ab^3}+\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\right.\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a + (2*a^2 - 3*b^2)*(d*x + c)/b^3 - 4
*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan(
(a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a*b^3) + 2*
(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/
2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

maple [B] time = 0.44, size = 334, normalized size = 2.69

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2a}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2\,\mathrm{arctan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+2/d/b^2/(1+tan(1/2*d*
x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a-1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1
/2*d*x+1/2*c)+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*a+2/d/b^3*arctan(tan(1/2*d
*x+1/2*c))*a^2-3/d/b*arctan(tan(1/2*d*x+1/2*c))+1/a/d*ln(tan(1/2*d*x+1/2*c)
)-2/d*a^3/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-
b^2)^(1/2))+4/d*a/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
```

```
/(a^2-b^2)^(1/2))-2/d*b/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 14.03, size = 1320, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) - sin(2*c + 2*d*x)/(4*b*d)
 - (3*atan((2*a^3*cos(c/2 + (d*x)/2) + 2*b^3*sin(c/2 + (d*x)/2) - 3*a*b^2*c
os(c/2 + (d*x)/2))/(2*b^3*cos(c/2 + (d*x)/2) - 2*a^3*sin(c/2 + (d*x)/2) + 3
*a*b^2*sin(c/2 + (d*x)/2))))/(b*d) + (a*cos(c + d*x))/(b^2*d) + (2*a^2*atan
((2*a^3*cos(c/2 + (d*x)/2) + 2*b^3*sin(c/2 + (d*x)/2) - 3*a*b^2*cos(c/2 + (
d*x)/2))/(2*b^3*cos(c/2 + (d*x)/2) - 2*a^3*sin(c/2 + (d*x)/2) + 3*a*b^2*sin
(c/2 + (d*x)/2))))/(b^3*d) + (atan((b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a
^2*b^4 + 3*a^4*b^2)^(3/2)*64i - a^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*
b^4 + 3*a^4*b^2)^(1/2)*16i - a^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(3/2)*16i - a^3*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(3/2)*42i + a^3*b^9*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(1/2)*66i - a^5*b^7*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(1/2)*176i + a^7*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
 + 3*a^4*b^2)^(1/2)*178i - a^9*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^
4 + 3*a^4*b^2)^(1/2)*81i - a^2*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^
4 + 3*a^4*b^2)^(3/2)*116i + a^4*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b
^4 + 3*a^4*b^2)^(3/2)*72i + a^2*b^10*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*
b^4 + 3*a^4*b^2)^(1/2)*148i - a^4*b^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2
*b^4 + 3*a^4*b^2)^(1/2)*460i + a^6*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^
2*b^4 + 3*a^4*b^2)^(1/2)*577i - a^8*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a
^2*b^4 + 3*a^4*b^2)^(1/2)*368i + a^10*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3
*a^2*b^4 + 3*a^4*b^2)^(1/2)*120i + a*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*
a^2*b^4 + 3*a^4*b^2)^(3/2)*32i + a^5*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^
```

```
2*b^4 + 3*a^4*b^2)^(3/2)*14i + a^11*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2
*b^4 + 3*a^4*b^2)^(1/2)*14i)/(64*b^15*sin(c/2 + (d*x)/2) + 32*a*b^14*cos(c/
2 + (d*x)/2) - 120*a^3*b^12*cos(c/2 + (d*x)/2) + 180*a^5*b^10*cos(c/2 + (d*
x)/2) - 137*a^7*b^8*cos(c/2 + (d*x)/2) + 54*a^9*b^6*cos(c/2 + (d*x)/2) - 9*
a^11*b^4*cos(c/2 + (d*x)/2) - 256*a^2*b^13*sin(c/2 + (d*x)/2) + 416*a^4*b^1
1*sin(c/2 + (d*x)/2) - 351*a^6*b^9*sin(c/2 + (d*x)/2) + 161*a^8*b^7*sin(c/2
 + (d*x)/2) - 37*a^10*b^5*sin(c/2 + (d*x)/2) + 3*a^12*b^3*sin(c/2 + (d*x)/2
)))*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*2i)/(a*b^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.1305 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=104

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{ax}{b^2}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

```
[Out] -a*x/b^2+2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))
/a^2/b^2/d+b*arctanh(cos(d*x+c))/a^2/d-cos(d*x+c)/b/d-cot(d*x+c)/a/d
```

Rubi [A] time = 0.27, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2894, 3057, 2660, 618, 204, 3770}

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{ax}{b^2}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^2*b^2*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cos[c + d*x]/(
b*d) - Cot[c + d*x]/(a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} - \frac{\int \frac{\csc(c+dx)\left(b^2+2ab\sin(c+dx)+a^2\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{ab}$$

$$= -\frac{ax}{b^2} - \frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} - \frac{b\int \csc(c+dx)\,dx}{a^2} + \frac{\left(a^2-b^2\right)^2 \int \frac{1}{a+b\sin(c}}{a^2b^2}$$

$$= -\frac{ax}{b^2} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} + \frac{\left(2\left(a^2-b^2\right)^2\right)\mathrm{S}}{}$$

$$= -\frac{ax}{b^2} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} - \frac{\left(4\left(a^2-b^2\right)^2\right)\mathrm{S}}{}$$

$$= -\frac{ax}{b^2} + \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^2d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c}{b}$$

Mathematica [A] time = 0.78, size = 146, normalized size = 1.40

$$-\frac{2a^3c+2a^3dx-4\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)+2a^2b\cos(c+dx)-ab^2\tan\left(\frac{1}{2}(c+dx)\right)+ab^2\cot\left(\frac{1}{2}(c+\right.}{2a^2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*(2*a^3*c + 2*a^3*d*x - 4*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]] + 2*a^2*b*Cos[c + d*x] + a*b^2*Cot[(c + d*x)/2] - 2*b
^3*Log[Cos[(c + d*x)/2]] + 2*b^3*Log[Sin[(c + d*x)/2]] - a*b^2*Tan[(c + d*x
)/2])/(a^2*b^2*d)
```

fricas [A] time = 1.06, size = 396, normalized size = 3.81

$$\left[\frac{b^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - b^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 2ab^2\cos(dx+c) - \left(a^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b^3*log(-1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 2*a*b^2*cos(d*x + c) - (a^2 - b^2)*sqrt(-a^2 + b^
2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(
a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 2*(a^3*d*x + a^2
*b*cos(d*x + c))*sin(d*x + c))/(a^2*b^2*d*sin(d*x + c)), 1/2*(b^3*log(1/2*c
os(d*x + c) + 1/2)*sin(d*x + c) - b^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 2*a*b^2*cos(d*x + c) - 2*(a^2 - b^2)^(3/2)*arctan(-(a*sin(d*x + c) +
 b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 2*(a^3*d*x + a^2*b*cos(d
*x + c))*sin(d*x + c))/(a^2*b^2*d*sin(d*x + c))]
```

giac [B] time = 0.20, size = 221, normalized size = 2.12

$$-\frac{\frac{6\,(dx+c)a}{b^2}+\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{12\left(a^4-2\,a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^2b^2}-\frac{2\,b^2\tan\left(\frac{1}{2}\,dx+\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a/b^2 + 6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 3*tan(1/
2*d*x + 1/2*c)/a - 12*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi +
1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a
^2 - b^2)*a^2*b^2) - (2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/
2*c)^2 - 12*a^2*tan(1/2*d*x + 1/2*c) + 2*b^2*tan(1/2*d*x + 1/2*c) - 3*a*b)/
((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a^2*b))/d
```

maple [B] time = 0.44, size = 249, normalized size = 2.39

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{2}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2}-\frac{1}{2ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}+\frac{2\,\mathrm{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)-2/d/b/(1+tan(1/2*d*x+1/2*c)^2)-2/d/b^2*a*arctan(
tan(1/2*d*x+1/2*c))-1/2/a/d/tan(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tan(1/2*d*x+1/2
*c))+2/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))*a^2-4/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
```

(a^2-b^2)^(1/2))+2/d*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more details)Is 4*b^2-4*a^2 positive or negative?

mupad [B] time = 13.78, size = 1167, normalized size = 11.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)

[Out] (atan((16*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) - 4*a^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 4*a^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) - 12*a^3*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + a^5*b^7*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 4*a^7*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 6*a^9*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 29*a^2*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 18*a^4*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + a^2*b^10*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 4*a^4*b^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 22*a^6*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 32*a^8*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 18*a^10*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 8*a*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 5*a^5*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 2*a^11*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(b^15*sin(c/2 + (d*x)/2)*16i + a*b^14*cos(c/2 + (d*x)/2)*8i - a^14*b*sin(c/2 + (d*x)/2)*3i - a^3*b^12*cos(c/2 + (d*x)/2)*48i + a^5*b^10*cos(c/2 + (d*x)/2)*123i - a^7*b^8*cos(c/2 + (d*x)/2)*167i + a^9*b^6*cos(c/2 + (d*x)/2)*126i - a^11*b^4*cos(c/2 + (d*x)/2)*51i + a^13*b^2*cos(c/2 + (d*x)/2)*9i - a^2*b^13*sin(c/2 + (d*x)/2)*100i + a^4*b^11*sin(c/2 + (d*x)/2)*269i - a^6*b^9*sin(c/2 + (d*x)/2)*390i + a^8*b^7*sin(c/2 + (d*x)/2)*323i - a^10*b^5*sin(c/2 + (d*x)/2)*151i + a^12*b^3*sin(c/2 +

```
(d*x)/2)*36i))*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^2*b^2*d) - (b*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^2*d) - sin(2*c + 2*d*x)/(2*b*d*sin(c +
 d*x)) - (2*a*atan((a^3*cos(c/2 + (d*x)/2) + b^3*sin(c/2 + (d*x)/2))/(b^3*c
os(c/2 + (d*x)/2) - a^3*sin(c/2 + (d*x)/2))))/(b^2*d) - cot(c + d*x)/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

3.1306 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=123

$$\frac{b\cot(c+dx)}{a^2d}-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3bd}+\frac{\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+$$

```
[Out] x/b-2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/
b/d+1/2*(3*a^2-2*b^2)*arctanh(cos(d*x+c))/a^3/d+b*cot(d*x+c)/a^2/d-1/2*cot(
d*x+c)*csc(d*x+c)/a/d
```

Rubi [A] time = 0.30, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2893, 3057, 2660, 618, 204, 3770}

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3bd}+\frac{\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] x/b - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/(a^3*b*d) + ((3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (b*Cot[c
+ d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\int \frac{\csc(c+dx)\left(3a^2-2b^2-ab\sin(c+dx)-2a^2\sin^2(c}{a+b\sin(c+dx)}}{2a^2}$$

$$= \frac{x}{b} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\left(3a^2-2b^2\right)\int\csc(c+dx)\,dx}{2a^3} -$$

$$= \frac{x}{b} + \frac{\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx}{2ad}$$

$$= \frac{x}{b} + \frac{\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx}{2ad}$$

$$= \frac{x}{b} - \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3bd} + \frac{\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d} +$$

Mathematica [A] time = 1.69, size = 204, normalized size = 1.66

$$\frac{8a^3c + 8a^3dx - 16\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) - a^2b\csc^2\left(\frac{1}{2}(c+dx)\right) + a^2b\sec^2\left(\frac{1}{2}(c+dx)\right) - 12a^2b\log(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (8*a^3*c + 8*a^3*d*x - 16*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]] + 4*a*b^2*Cot[(c + d*x)/2] - a^2*b*Csc[(c + d*x)/2]^2 + 1
2*a^2*b*Log[Cos[(c + d*x)/2]] - 8*b^3*Log[Cos[(c + d*x)/2]] - 12*a^2*b*Log[
Sin[(c + d*x)/2]] + 8*b^3*Log[Sin[(c + d*x)/2]] + a^2*b*Sec[(c + d*x)/2]^2
- 4*a*b^2*Tan[(c + d*x)/2])/(8*a^3*b*d)
```

fricas [B] time = 0.96, size = 572, normalized size = 4.65

$$\left[\frac{4\,a^3dx\cos\left(dx+c\right)^2 - 4\,a^3dx - 4\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right) + 2\,a^2b\cos\left(dx+c\right) - 2\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2 -\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] [1/4*(4*a^3*d*x*cos(d*x + c)^2 - 4*a^3*d*x - 4*a*b^2*cos(d*x + c)*sin(d*x + c) + 2*a^2*b*cos(d*x + c) - 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - (3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3*b*d*cos(d*x + c)^2 - a^3*b*d), 1/4*(4*a^3*d*x*cos(d*x + c)^2 - 4*a^3*d*x - 4*a*b^2*cos(d*x + c)*sin(d*x + c) + 2*a^2*b*cos(d*x + c) + 4*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3*b*d*cos(d*x + c)^2 - a^3*b*d)]

giac [A] time = 0.20, size = 217, normalized size = 1.76

$$\frac{\frac{8\,(dx+c)}{b}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}}-\frac{4\left(3\,a^{2}-2\,b^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{3}}-\frac{16\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\cdots\right.}{\sqrt{a^{2}-b\cdots}}\right.\right.}{\sqrt{a^{2}-b^{2}}\,a^{3}b}\cdots}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] 1/8*(8*(d*x + c)/b + (a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/a^2 - 4*(3*a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 16*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^3*b) + (18*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 + 4*a*b*tan(1/2*d*x + 1/2*c) - a^2)/(a^3*tan(1/2*d*x + 1/2*c)^2))/d

maple [B] time = 0.48, size = 286, normalized size = 2.33

$$\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{2d\,a^{2}}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db}-\frac{1}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\cdots\right.\right.}{d\,a^{3}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c)),x)

```
[Out] 1/8/a/d*tan(1/2*d*x+1/2*c)^2-1/2/d/a^2*tan(1/2*d*x+1/2*c)*b+2/d/b*arctan(ta
n(1/2*d*x+1/2*c))-1/8/a/d/tan(1/2*d*x+1/2*c)^2-3/2/a/d*ln(tan(1/2*d*x+1/2*c
))+1/d/a^3*ln(tan(1/2*d*x+1/2*c))*b^2+1/2/d/a^2*b/tan(1/2*d*x+1/2*c)-2/d*a/
b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+
4/d*b/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2))-2/d/a^3*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(
a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.68, size = 2718, normalized size = 22.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (b^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*1i)/(a^3*b*d*1i - a^3*b*d*c
os(2*c + 2*d*x)*1i) - (a^3*atan((2*a^3*cos(c/2 + (d*x)/2) + 2*b^3*sin(c/2 +
 (d*x)/2) - 3*a^2*b*sin(c/2 + (d*x)/2))/(2*a^3*sin(c/2 + (d*x)/2) - 2*b^3*c
os(c/2 + (d*x)/2) + 3*a^2*b*cos(c/2 + (d*x)/2)))*2i)/(a^3*b*d*1i - a^3*b*d*
cos(2*c + 2*d*x)*1i) + (2*atan((32*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^
2*b^4 + 3*a^4*b^2)^(3/2) - 14*a^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^
4 + 3*a^4*b^2)^(1/2) - 14*a^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3
*a^4*b^2)^(3/2) - 36*a^3*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*
a^4*b^2)^(3/2) + 2*a^5*b^7*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^
4*b^2)^(1/2) + 8*a^7*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*
b^2)^(1/2) - 24*a^9*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b
^2)^(1/2) - 82*a^2*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^
2)^(3/2) + 63*a^4*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2
)^(3/2) + 2*a^2*b^10*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)
^(1/2) - 11*a^4*b^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^
(1/2) + 56*a^6*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(
1/2) - 106*a^8*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(
1/2) + 72*a^10*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(
```

1/2) + 16*a*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2
) + 19*a^5*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) +
 13*a^11*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(b
^15*sin(c/2 + (d*x)/2)*32i + a*b^14*cos(c/2 + (d*x)/2)*16i - a^14*b*sin(c/2
 + (d*x)/2)*6i - a^3*b^12*cos(c/2 + (d*x)/2)*108i + a^5*b^10*cos(c/2 + (d*x
)/2)*309i - a^7*b^8*cos(c/2 + (d*x)/2)*469i + a^9*b^6*cos(c/2 + (d*x)/2)*39
0i - a^11*b^4*cos(c/2 + (d*x)/2)*165i + a^13*b^2*cos(c/2 + (d*x)/2)*27i - a
^2*b^13*sin(c/2 + (d*x)/2)*224i + a^4*b^11*sin(c/2 + (d*x)/2)*670i - a^6*b^
9*sin(c/2 + (d*x)/2)*1080i + a^8*b^7*sin(c/2 + (d*x)/2)*982i - a^10*b^5*sin
(c/2 + (d*x)/2)*482i + a^12*b^3*sin(c/2 + (d*x)/2)*108i))*(b^6 - a^6 - 3*a^
2*b^4 + 3*a^4*b^2)^(1/2))/(a^3*b*d*1i - a^3*b*d*cos(2*c + 2*d*x)*1i) - (a^2
*b*cos(c + d*x)*1i)/(a^3*b*d*1i - a^3*b*d*cos(2*c + 2*d*x)*1i) + (a^3*atan(
(2*a^3*cos(c/2 + (d*x)/2) + 2*b^3*sin(c/2 + (d*x)/2) - 3*a^2*b*sin(c/2 + (d
*x)/2))/(2*a^3*sin(c/2 + (d*x)/2) - 2*b^3*cos(c/2 + (d*x)/2) + 3*a^2*b*cos(
c/2 + (d*x)/2)))*cos(2*c + 2*d*x)*2i)/(a^3*b*d*1i - a^3*b*d*cos(2*c + 2*d*x
)*1i) - (a^2*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*3i)/(2*(a^3*b*d*1
i - a^3*b*d*cos(2*c + 2*d*x)*1i)) + (a*b^2*sin(2*c + 2*d*x)*1i)/(a^3*b*d*1i
 - a^3*b*d*cos(2*c + 2*d*x)*1i) - (2*cos(2*c + 2*d*x)*atan((32*b^6*sin(c/2
+ (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) - 14*a^12*sin(c/2 + (d
x)/2)(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 14*a^6*sin(c/2 + (d*x)/2
)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) - 36*a^3*b^3*cos(c/2 + (d*x)/2)
*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 2*a^5*b^7*cos(c/2 + (d*x)/2)*(
b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 8*a^7*b^5*cos(c/2 + (d*x)/2)*(b^
6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 24*a^9*b^3*cos(c/2 + (d*x)/2)*(b^6
 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 82*a^2*b^4*sin(c/2 + (d*x)/2)*(b^6
- a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 63*a^4*b^2*sin(c/2 + (d*x)/2)*(b^6 -
 a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 2*a^2*b^10*sin(c/2 + (d*x)/2)*(b^6 -
a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 11*a^4*b^8*sin(c/2 + (d*x)/2)*(b^6 - a
^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 56*a^6*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^
6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 106*a^8*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^
6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 72*a^10*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^
6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 16*a*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 -
 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 19*a^5*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*
a^2*b^4 + 3*a^4*b^2)^(3/2) + 13*a^11*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^
2*b^4 + 3*a^4*b^2)^(1/2))/(b^15*sin(c/2 + (d*x)/2)*32i + a*b^14*cos(c/2 + (
d*x)/2)*16i - a^14*b*sin(c/2 + (d*x)/2)*6i - a^3*b^12*cos(c/2 + (d*x)/2)*10
8i + a^5*b^10*cos(c/2 + (d*x)/2)*309i - a^7*b^8*cos(c/2 + (d*x)/2)*469i + a
^9*b^6*cos(c/2 + (d*x)/2)*390i - a^11*b^4*cos(c/2 + (d*x)/2)*165i + a^13*b^
2*cos(c/2 + (d*x)/2)*27i - a^2*b^13*sin(c/2 + (d*x)/2)*224i + a^4*b^11*sin(
c/2 + (d*x)/2)*670i - a^6*b^9*sin(c/2 + (d*x)/2)*1080i + a^8*b^7*sin(c/2 +
(d*x)/2)*982i - a^10*b^5*sin(c/2 + (d*x)/2)*482i + a^12*b^3*sin(c/2 + (d*x)
/2)*108i))*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^3*b*d*1i - a^3*b*d
*cos(2*c + 2*d*x)*1i) - (b^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos
(2*c + 2*d*x)*1i)/(a^3*b*d*1i - a^3*b*d*cos(2*c + 2*d*x)*1i) + (a^2*b*log(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*3i)/(2*(a^3*b*d*1i -

```
a^3*b*d*cos(2*c + 2*d*x)*1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

3.1307 $\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}-\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{\left(4a^2-3b^2\right)c}{3a^3}$$

```
[Out] 2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^4/d-1/
2*b*(3*a^2-2*b^2)*arctanh(cos(d*x+c))/a^4/d+1/3*(4*a^2-3*b^2)*cot(d*x+c)/a^
3/d+1/2*b*cot(d*x+c)*csc(d*x+c)/a^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d
```

Rubi [A] time = 0.45, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2725, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}+\frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d}-\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{b\cot(c+dx)c}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*d) - (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + ((4*a^2 - 3*b^2
)*Cot[c + d*x])/(3*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(3*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} - \frac{\int \frac{\csc^2(c+dx)\left(2\left(4a^2-3b^2\right)-ab\sin(c+dx)-3}{a+b\sin(c+dx)}}{6a^2}$$

$$= \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} - \int \frac{\csc(}{}$$

$$= \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} + \frac{\left(b\left(3a\right.\right.}{}$$

$$= -\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+}{2a^2d}$$

$$= -\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+}{2a^2d}$$

$$= \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} - \frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(4a^2-3b^2\right.}{3}$$

Mathematica [B] time = 6.13, size = 350, normalized size = 2.27

$$\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} + \frac{\left(3a^2b-2b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\left(2b^3-3a^2b\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} +$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(
c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*d) + ((4*a^2*Cos[(c + d*x)/2] - 3*b^2*
Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^3*d) + (b*Csc[(c + d*x)/2]^2)/(8*a
^2*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a*d) + ((-3*a^2*b + 2*b^3
)*Log[Cos[(c + d*x)/2]])/(2*a^4*d) + ((3*a^2*b - 2*b^3)*Log[Sin[(c + d*x)/2
]])/(2*a^4*d) - (b*Sec[(c + d*x)/2]^2)/(8*a^2*d) + (Sec[(c + d*x)/2]*(-4*a^
2*Sin[(c + d*x)/2] + 3*b^2*Sin[(c + d*x)/2]))/(6*a^3*d) + (Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2])/(24*a*d)
```

fricas [A] time = 0.88, size = 633, normalized size = 4.11

$$\left[-\frac{6\,a^2b\cos\left(dx+c\right)\sin\left(dx+c\right)-4\left(4\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^3+6\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2-a^2+b^2\right)\sqrt{-a^2+b}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 4*(4*a^3 - 3*a*b^2)*cos(d*x + c
)^3 + 6*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(-a^2 + b^2)*log(((2*a
^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x +
c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2
*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 3*(3*a^2*b - 2*b^3 - (3*a^2*
b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(3*
a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) + 12*(a^3 - a*b^2)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 -
a^4*d)*sin(d*x + c)), -1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 4*(4*a^3 -
 3*a*b^2)*cos(d*x + c)^3 + 12*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*si
n(d*x + c) - 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2
*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*
cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 12*(a^3 - a*b^2
)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))]
```

giac [A] time = 0.20, size = 273, normalized size = 1.77

$$\frac{\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}+\frac{12\left(3\,a^2b-2\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}+\frac{48\left(a^4-2\,a^2b^2+b^4\right)}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*((a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*t
an(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/a^3 + 12*(3*a^2*b - 2*b^
3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + 48*(a^4 - 2*a^2*b^2 + b^4)*(pi*floo
r(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt
(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (66*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 44
*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan(
```

```
1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^4*tan(1/2*d*x +
 1/2*c)^3))/d
```

maple [B] time = 0.46, size = 348, normalized size = 2.26

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{8d\,a^2}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}-\frac{1}{24da\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{5}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{}{2d\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a^2*tan(1/2*d*x+1/2*c)^2*b-5/8/a/d*tan(
1/2*d*x+1/2*c)+1/2/d/a^3*b^2*tan(1/2*d*x+1/2*c)-1/24/d/a/tan(1/2*d*x+1/2*c)
^3+5/8/a/d/tan(1/2*d*x+1/2*c)-1/2/d/a^3/tan(1/2*d*x+1/2*c)*b^2+1/8/d/a^2*b/
tan(1/2*d*x+1/2*c)^2+3/2/d/a^2*b*ln(tan(1/2*d*x+1/2*c))-1/d/a^4*b^3*ln(tan(
1/2*d*x+1/2*c))+2/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))-4/d*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1
/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.42, size = 654, normalized size = 4.25

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}+\frac{5\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}+\frac{3\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2\,a^2\,d}-\frac{b^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^4\,d}+\frac{b\cot\left(\frac{c}{2}\right.}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a*d) - cot(c/2 + (d*x)/2)^3/(24*a*d) + (5*cot(c/2
+ (d*x)/2))/(8*a*d) - (5*tan(c/2 + (d*x)/2))/(8*a*d) + (3*b*log(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2)))/(2*a^2*d) - (b^3*log(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)))/(a^4*d) + (b*cot(c/2 + (d*x)/2)^2)/(8*a^2*d) - (b^2*cot(c/2
+ (d*x)/2))/(2*a^3*d) - (b*tan(c/2 + (d*x)/2)^2)/(8*a^2*d) + (b^2*tan(c/2 +
 (d*x)/2))/(2*a^3*d) + (atan((2*a^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b
^4 + 3*a^4*b^2)^(1/2) + 8*b^5*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3
*a^4*b^2)^(1/2) - 7*a^3*b^2*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a
^4*b^2)^(1/2) - 16*a^2*b^3*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^
4*b^2)^(1/2) + 4*a*b^4*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^
2)^(1/2) + 7*a^4*b*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(
1/2))/(a^8*sin(c/2 + (d*x)/2)*2i + b^8*sin(c/2 + (d*x)/2)*8i + a*b^7*cos(c/
2 + (d*x)/2)*4i - a^7*b*cos(c/2 + (d*x)/2)*5i - a^3*b^5*cos(c/2 + (d*x)/2)*
13i + a^5*b^3*cos(c/2 + (d*x)/2)*14i - a^2*b^6*sin(c/2 + (d*x)/2)*28i + a^4
*b^4*sin(c/2 + (d*x)/2)*34i - a^6*b^2*sin(c/2 + (d*x)/2)*16i))*(-(a + b)^3*
(a - b)^3)^(1/2)*2i)/(a^4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1308 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=198

$$\frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d}-\frac{2b\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d}-\frac{b\left(4a^2-3b^2\right)\cot(c+dx)}{3a^4d}+\frac{\left(5a^2-4b^2\right)\cot(c+}{8a^3d}$$

```
[Out] -2*b*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^5/d
-1/8*(3*a^4-12*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^5/d-1/3*b*(4*a^2-3*b^2)
*cot(d*x+c)/a^4/d+1/8*(5*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/d+1/3*b*cot(d
*x+c)*csc(d*x+c)^2/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d
```

Rubi [A] time = 0.76, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2893, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d}-\frac{b\left(4a^2-3b^2\right)\cot(c+dx)}{3a^4d}-\frac{\left(-12a^2b^2+3a^4+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^5*d) - ((3*a^4 - 12*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^5*d) - (
b*(4*a^2 - 3*b^2)*Cot[c + d*x])/(3*a^4*d) + ((5*a^2 - 4*b^2)*Cot[c + d*x]*C
sc[c + d*x])/(8*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^2)/(3*a^2*d) - (Cot[c
 + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} - \frac{\int \frac{\csc^3(c+dx)\left(3\left(5a^2-4b^2\right)-ab\sin\ldots}{a+b\ldots}}{\ldots}$$

$$= \frac{\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d} - \frac{\cot(c+dx)\csc\ldots}{4a\ldots}$$

$$= -\frac{b\left(4a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+dx\ldots}{3\ldots}$$

$$= -\frac{b\left(4a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+dx\ldots}{3\ldots}$$

$$= -\frac{\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d} - \frac{b\left(4a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(5a^2\ldots\right.}{\ldots}$$

$$= -\frac{\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d} - \frac{b\left(4a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(5a^2\ldots\right.}{\ldots}$$

$$= -\frac{2b\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5d} - \frac{\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+\ldots}{8a^5d}$$

Mathematica [B] time = 6.21, size = 433, normalized size = 2.19

$$\frac{b\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d} - \frac{b\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d} - \frac{2b\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(a\sin\left(\frac{1}{2}(c+\ldots\right.\right.}{\sqrt{a^2-\ldots}}\right)}{a^5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(a^2 - b^2)^(3/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Si
n[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*d) + ((-4*a^2*b*Cos[(c + d*x)/2] +
3*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + ((5*a^2 - 4*b^2)*Csc[
(c + d*x)/2]^2)/(32*a^3*d) + (b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a^
2*d) - Csc[(c + d*x)/2]^4/(64*a*d) + ((-3*a^4 + 12*a^2*b^2 - 8*b^4)*Log[Cos
```

```
[(c + d*x)/2]])/(8*a^5*d) + ((3*a^4 - 12*a^2*b^2 + 8*b^4)*Log[Sin[(c + d*x)
/2]])/(8*a^5*d) + ((-5*a^2 + 4*b^2)*Sec[(c + d*x)/2]^2)/(32*a^3*d) + Sec[(c
 + d*x)/2]^4/(64*a*d) + (Sec[(c + d*x)/2]*(4*a^2*b*Sin[(c + d*x)/2] - 3*b^3
*Sin[(c + d*x)/2]))/(6*a^4*d) - (b*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24
*a^2*d)
```

fricas [B] time = 1.35, size = 904, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/48*(6*(5*a^4 - 4*a^2*b^2)*cos(d*x + c)^3 + 24*((a^2*b - b^3)*cos(d*x +
c)^4 + a^2*b - b^3 - 2*(a^2*b - b^3)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(-
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(3*a^4 - 4*a^2*b^2)*cos(d*x + c) +
 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 3*a^4 - 12*a^2*b^2 + 8*b^
4 - 2*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1
/2) - 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 3*a^4 - 12*a^2*b^2 +
 8*b^4 - 2*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^2)*log(-1/2*cos(d*x +
c) + 1/2) - 16*((4*a^3*b - 3*a*b^3)*cos(d*x + c)^3 - 3*(a^3*b - a*b^3)*cos(
d*x + c))*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*x + c)^2 + a^
5*d), -1/48*(6*(5*a^4 - 4*a^2*b^2)*cos(d*x + c)^3 - 48*((a^2*b - b^3)*cos(d
*x + c)^4 + a^2*b - b^3 - 2*(a^2*b - b^3)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(3*a^4 - 4*
a^2*b^2)*cos(d*x + c) + 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 3*
a^4 - 12*a^2*b^2 + 8*b^4 - 2*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^2)*l
og(1/2*cos(d*x + c) + 1/2) - 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^4
 + 3*a^4 - 12*a^2*b^2 + 8*b^4 - 2*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)
^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*((4*a^3*b - 3*a*b^3)*cos(d*x + c)^3 -
 3*(a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^
5*d*cos(d*x + c)^2 + a^5*d)]
```

giac [B] time = 0.21, size = 375, normalized size = 1.89

$$\frac{\frac{3\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-8\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-24\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+120\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-96\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{4}}+\frac{24\,(3\,a^{4}\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*((3*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 24*
a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 120*a^2*b*ta
n(1/2*d*x + 1/2*c) - 96*b^3*tan(1/2*d*x + 1/2*c))/a^4 + 24*(3*a^4 - 12*a^2*
b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 - 384*(a^4*b - 2*a^2*b^3 +
b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) - (150*a^4*tan(1/2*d*x + 1
/2*c)^4 - 600*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 400*b^4*tan(1/2*d*x + 1/2*c)
^4 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 96*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 2
4*a^4*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a^3*b*
tan(1/2*d*x + 1/2*c) + 3*a^4)/(a^5*tan(1/2*d*x + 1/2*c)^4))/d
```

maple [B] time = 0.49, size = 455, normalized size = 2.30

$$\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64da}-\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^2}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{b^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^3}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8d\,a^2}-\frac{b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^4}-\frac{}{64da\,\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/64/d/a*tan(1/2*d*x+1/2*c)^4-1/24/d/a^2*b*tan(1/2*d*x+1/2*c)^3-1/8/a/d*tan
(1/2*d*x+1/2*c)^2+1/8/d/a^3*b^2*tan(1/2*d*x+1/2*c)^2+5/8/d/a^2*tan(1/2*d*x+
1/2*c)*b-1/2/d/a^4*b^3*tan(1/2*d*x+1/2*c)-1/64/d/a/tan(1/2*d*x+1/2*c)^4+1/8
/a/d/tan(1/2*d*x+1/2*c)^2-1/8/d*b^2/a^3/tan(1/2*d*x+1/2*c)^2+3/8/a/d*ln(tan
(1/2*d*x+1/2*c))-3/2/d/a^3*ln(tan(1/2*d*x+1/2*c))*b^2+1/d/a^5*ln(tan(1/2*d*
x+1/2*c))*b^4+1/24/d/a^2*b/tan(1/2*d*x+1/2*c)^3-5/8/d/a^2*b/tan(1/2*d*x+1/2
*c)+1/2/d*b^3/a^4/tan(1/2*d*x+1/2*c)-2/d*b/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d/a^3*b^3/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a^5*b^5/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.16, size = 953, normalized size = 4.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^4/(64*a*d) - (tan(c/2 + (d*x)/2)^2*(1/(8*a) - b^2/(8*a^3
)))/d + (tan(c/2 + (d*x)/2)*(b/(8*a^2) + (2*b*(1/(4*a) - b^2/(4*a^3)))/a))/
d - (tan(c/2 + (d*x)/2)^2*(2*a*b^2 - 2*a^3) + tan(c/2 + (d*x)/2)^3*(10*a^2*
b - 8*b^3) + a^3/4 - (2*a^2*b*tan(c/2 + (d*x)/2))/3)/(16*a^4*d*tan(c/2 + (d
*x)/2)^4) + (log(tan(c/2 + (d*x)/2))*((3*a^4)/8 + b^4 - (3*a^2*b^2)/2))/(a^
5*d) - (b*tan(c/2 + (d*x)/2)^3)/(24*a^2*d) - (b*atan(((b*(-(a + b)^3*(a - b
)^3)^(1/2)*((tan(c/2 + (d*x)/2)*(3*a^9 - 32*a^3*b^6 + 64*a^5*b^4 - 34*a^7*b
^2))/(4*a^7) - (11*a^9*b + 16*a^5*b^5 - 28*a^7*b^3)/(4*a^8) + (b*(2*a^2*b -
 (tan(c/2 + (d*x)/2)*(24*a^10 - 32*a^8*b^2))/(4*a^7))*(-(a + b)^3*(a - b)^3
)^(1/2))/a^5)*1i)/a^5 - (b*(-(a + b)^3*(a - b)^3)^(1/2)*((11*a^9*b + 16*a^5
*b^5 - 28*a^7*b^3)/(4*a^8) - (tan(c/2 + (d*x)/2)*(3*a^9 - 32*a^3*b^6 + 64*a
^5*b^4 - 34*a^7*b^2))/(4*a^7) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^10
- 32*a^8*b^2))/(4*a^7))*(-(a + b)^3*(a - b)^3)^(1/2))/a^5)*1i)/a^5)/((3*a^8
*b + 8*b^9 - 28*a^2*b^7 + 35*a^4*b^5 - 18*a^6*b^3)/(2*a^8) + (tan(c/2 + (d*
x)/2)*(8*b^8 - 26*a^2*b^6 + 28*a^4*b^4 - 10*a^6*b^2))/(2*a^7) + (b*(-(a + b
)^3*(a - b)^3)^(1/2)*((tan(c/2 + (d*x)/2)*(3*a^9 - 32*a^3*b^6 + 64*a^5*b^4
- 34*a^7*b^2))/(4*a^7) - (11*a^9*b + 16*a^5*b^5 - 28*a^7*b^3)/(4*a^8) + (b*
(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^10 - 32*a^8*b^2))/(4*a^7))*(-(a + b)^3
*(a - b)^3)^(1/2))/a^5))/a^5 + (b*(-(a + b)^3*(a - b)^3)^(1/2)*((11*a^9*b +
 16*a^5*b^5 - 28*a^7*b^3)/(4*a^8) - (tan(c/2 + (d*x)/2)*(3*a^9 - 32*a^3*b^6
 + 64*a^5*b^4 - 34*a^7*b^2))/(4*a^7) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(2
4*a^10 - 32*a^8*b^2))/(4*a^7))*(-(a + b)^3*(a - b)^3)^(1/2))/a^5))/a^5))*(-
(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^5*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1309 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=244

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{2b^2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{b\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d}+\frac{\left(6a^2-\right.}{}$$

```
[Out] 2*b^2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^6/
d+1/8*b*(3*a^4-12*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^6/d-1/15*(3*a^4-20*a
^2*b^2+15*b^4)*cot(d*x+c)/a^5/d-1/8*b*(5*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/a
^4/d+1/15*(6*a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)^2/a^3/d+1/4*b*cot(d*x+c)*csc(
d*x+c)^3/a^2/d-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d
```

Rubi [A] time = 1.05, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2893, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b^2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{\left(-20a^2b^2+3a^4+15b^4\right)\cot(c+dx)}{15a^5d}+\frac{b\left(-12a^2b^2+3a^4+8b^4\right)\tanh^{-1}(\mathrm{co}}{8a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^6*d) + (b*(3*a^4 - 12*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d)
- ((3*a^4 - 20*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (b*(5*a^2 - 4*b
^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*d) + ((6*a^2 - 5*b^2)*Cot[c + d*x]*Cs
c[c + d*x]^2)/(15*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot
[c + d*x]*Csc[c + d*x]^4)/(5*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad} - \frac{\int \frac{\csc^4(c+dx)\left(4\left(6a^2-5b^2\right)-ab}{a+}}{}$$

$$= \frac{\left(6a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{15a^3d} + \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+dx)}{5}$$

$$= -\frac{b\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} + \frac{\left(6a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{15a^3d}$$

$$= -\frac{\left(3a^4-20a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \frac{b\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} +$$

$$= -\frac{\left(3a^4-20a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \frac{b\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} +$$

$$= \frac{b\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{\left(3a^4-20a^2b^2+15b^4\right)\cot(c+d}{15a^5d}$$

$$= \frac{b\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{\left(3a^4-20a^2b^2+15b^4\right)\cot(c+d}{15a^5d}$$

$$= \frac{2b^2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} + \frac{b\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c}{8a^6d}$$

Mathematica [B] time = 1.82, size = 507, normalized size = 2.08

$$\frac{96a^5\tan\left(\frac{1}{2}(c+dx)\right) - 3a^5\sin(c+dx)\csc^6\left(\frac{1}{2}(c+dx)\right) + 21a^5\sin(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right) - 336a^5\sin^4\left(\frac{1}{2}(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (1920*b^2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
] - 32*(3*a^5 - 20*a^3*b^2 + 15*a*b^4)*Cot[(c + d*x)/2] - 150*a^4*b*Csc[(c
+ d*x)/2]^2 + 120*a^2*b^3*Csc[(c + d*x)/2]^2 + 15*a^4*b*Csc[(c + d*x)/2]^4
+ 360*a^4*b*Log[Cos[(c + d*x)/2]] - 1440*a^2*b^3*Log[Cos[(c + d*x)/2]] + 96
0*b^5*Log[Cos[(c + d*x)/2]] - 360*a^4*b*Log[Sin[(c + d*x)/2]] + 1440*a^2*b^
3*Log[Sin[(c + d*x)/2]] - 960*b^5*Log[Sin[(c + d*x)/2]] + 150*a^4*b*Sec[(c
+ d*x)/2]^2 - 120*a^2*b^3*Sec[(c + d*x)/2]^2 - 15*a^4*b*Sec[(c + d*x)/2]^4
- 336*a^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*a^3*b^2*Csc[c + d*x]^3*Si
n[(c + d*x)/2]^4 + 21*a^5*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a^3*b^2*Csc[
(c + d*x)/2]^4*Sin[c + d*x] - 3*a^5*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 96*a^
5*Tan[(c + d*x)/2] - 640*a^3*b^2*Tan[(c + d*x)/2] + 480*a*b^4*Tan[(c + d*x)
/2] + 6*a^5*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*a^6*d)
```

fricas [B] time = 1.83, size = 1051, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(16*(3*a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 + 80*(7*a^3*b^2
- 6*a*b^4)*cos(d*x + c)^3 + 120*((a^2*b^2 - b^4)*cos(d*x + c)^4 + a^2*b^2 -
 b^4 - 2*(a^2*b^2 - b^4)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2
)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d
*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2))*sin(d*x + c) - 15*(3*a^4*b - 12*a^2*b^3 + 8*b^5 + (
3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*
b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(3*a^4*b
 - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(
3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 240*(a^3*b^2 - a*b^4)*cos(d*x + c) - 30*((5*a^4*b - 4*a^2*b^
3)*cos(d*x + c)^3 - (3*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6
*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c)), -1/240*(
16*(3*a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 + 80*(7*a^3*b^2 - 6*a*b^4
)*cos(d*x + c)^3 + 240*((a^2*b^2 - b^4)*cos(d*x + c)^4 + a^2*b^2 - b^4 - 2*
(a^2*b^2 - b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b
)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 15*(3*a^4*b - 12*a^2*b^3 +
 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^
2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 1
5*(3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x +
c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x +
c) + 1/2)*sin(d*x + c) - 240*(a^3*b^2 - a*b^4)*cos(d*x + c) - 30*((5*a^4*b
- 4*a^2*b^3)*cos(d*x + c)^3 - (3*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x +
```

```
 c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))
]
```

giac [B] time = 0.22, size = 484, normalized size = 1.98

$$\frac{6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+60\,a^4\,\mathrm{t}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*((6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 30
*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*a^3*b
*tan(1/2*d*x + 1/2*c)^2 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 60*a^4*tan(1/2
*d*x + 1/2*c) - 600*a^2*b^2*tan(1/2*d*x + 1/2*c) + 480*b^4*tan(1/2*d*x + 1/
2*c))/a^5 - 120*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*d*x + 1/2*c)
))/a^6 + 1920*(a^4*b^2 - 2*a^2*b^4 + b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 -
 b^2)*a^6) + (822*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 3288*a^2*b^3*tan(1/2*d*x +
 1/2*c)^5 + 2192*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*a^5*tan(1/2*d*x + 1/2*c)^4
 + 600*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 480*a*b^4*tan(1/2*d*x + 1/2*c)^4 -
120*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 30*
a^5*tan(1/2*d*x + 1/2*c)^2 - 40*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*b*t
an(1/2*d*x + 1/2*c) - 6*a^5)/(a^6*tan(1/2*d*x + 1/2*c)^5))/d
```

maple [B] time = 0.49, size = 583, normalized size = 2.39

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{160da}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{64d\,a^2}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{32da}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{24d\,a^3}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{8d\,a^2}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3}{8d\,a^4}+\mathrm{ta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/160/d/a*tan(1/2*d*x+1/2*c)^5-1/64/d/a^2*tan(1/2*d*x+1/2*c)^4*b-1/32/d/a*t
an(1/2*d*x+1/2*c)^3+1/24/d/a^3*tan(1/2*d*x+1/2*c)^3*b^2+1/8/d/a^2*tan(1/2*d
*x+1/2*c)^2*b-1/8/d/a^4*tan(1/2*d*x+1/2*c)^2*b^3+1/16/a/d*tan(1/2*d*x+1/2*c
)-5/8/d/a^3*b^2*tan(1/2*d*x+1/2*c)+1/2/d/a^5*b^4*tan(1/2*d*x+1/2*c)-1/160/d
/a/tan(1/2*d*x+1/2*c)^5+1/32/d/a/tan(1/2*d*x+1/2*c)^3-1/24/d/a^3/tan(1/2*d*
x+1/2*c)^3*b^2-1/16/a/d/tan(1/2*d*x+1/2*c)+5/8/d/a^3/tan(1/2*d*x+1/2*c)*b^2
```

```
-1/2/d/a^5/tan(1/2*d*x+1/2*c)*b^4+1/64/d/a^2*b/tan(1/2*d*x+1/2*c)^4-1/8/d/a
^2*b/tan(1/2*d*x+1/2*c)^2+1/8/d*b^3/a^4/tan(1/2*d*x+1/2*c)^2-3/8/d/a^2*b*ln
(tan(1/2*d*x+1/2*c))+3/2/d/a^4*b^3*ln(tan(1/2*d*x+1/2*c))-1/d/a^6*b^5*ln(ta
n(1/2*d*x+1/2*c))+2/d*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1
/2*c)+2*b)/(a^2-b^2)^(1/2))-4/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4+2/d/a^6*b^6/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 12.18, size = 1082, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^6*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a*d) + (tan(c/2 + (d*x)/2)^2*(b/(32*a^2) + (b*(3/
(32*a) - b^2/(8*a^3))))/a))/d - (tan(c/2 + (d*x)/2)*(b^2/(8*a^3) - 1/(16*a)
+ (2*b*(b/(16*a^2) + (2*b*(3/(32*a) - b^2/(8*a^3))))/a))/a))/d - (tan(c/2 +
(d*x)/2)^3*(1/(32*a) - b^2/(24*a^3)))/d - (b*tan(c/2 + (d*x)/2)^4)/(64*a^2*
d) + (tan(c/2 + (d*x)/2)^2*(a^4 - (4*a^2*b^2)/3) - tan(c/2 + (d*x)/2)^4*(2*
a^4 + 16*b^4 - 20*a^2*b^2) - a^4/5 + tan(c/2 + (d*x)/2)^3*(4*a*b^3 - 4*a^3*
b) + (a^3*b*tan(c/2 + (d*x)/2))/2)/(32*a^5*d*tan(c/2 + (d*x)/2)^5) - (log(t
an(c/2 + (d*x)/2))*((3*a^4*b)/8 + b^5 - (3*a^2*b^3)/2))/(a^6*d) + (b^2*atan
(((b^2*(-(a + b)^3*(a - b)^3)^(1/2)*((tan(c/2 + (d*x)/2)*(3*a^10*b - 32*a^4
*b^7 + 64*a^6*b^5 - 34*a^8*b^3))/(4*a^9) - (16*a^6*b^6 - 28*a^8*b^4 + 11*a^
10*b^2)/(4*a^10) + (b^2*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b
^2))/(4*a^9))*(-(a + b)^3*(a - b)^3)^(1/2))/a^6)*1i)/a^6 - (b^2*(-(a + b)^3
*(a - b)^3)^(1/2)*((16*a^6*b^6 - 28*a^8*b^4 + 11*a^10*b^2)/(4*a^10) - (tan(
c/2 + (d*x)/2)*(3*a^10*b - 32*a^4*b^7 + 64*a^6*b^5 - 34*a^8*b^3))/(4*a^9) +
 (b^2*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(-(a
 + b)^3*(a - b)^3)^(1/2))/a^6)*1i)/a^6)/((8*b^11 - 28*a^2*b^9 + 35*a^4*b^7
- 18*a^6*b^5 + 3*a^8*b^3)/(2*a^10) + (tan(c/2 + (d*x)/2)*(8*b^10 - 26*a^2*b
^8 + 28*a^4*b^6 - 10*a^6*b^4))/(2*a^9) + (b^2*(-(a + b)^3*(a - b)^3)^(1/2)*
```

```
((tan(c/2 + (d*x)/2)*(3*a^10*b - 32*a^4*b^7 + 64*a^6*b^5 - 34*a^8*b^3))/(4*
a^9) - (16*a^6*b^6 - 28*a^8*b^4 + 11*a^10*b^2)/(4*a^10) + (b^2*(2*a^2*b - (
tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(-(a + b)^3*(a - b)^3)
^(1/2))/a^6))/a^6 + (b^2*(-(a + b)^3*(a - b)^3)^(1/2)*((16*a^6*b^6 - 28*a^8
*b^4 + 11*a^10*b^2)/(4*a^10) - (tan(c/2 + (d*x)/2)*(3*a^10*b - 32*a^4*b^7 +
 64*a^6*b^5 - 34*a^8*b^3))/(4*a^9) + (b^2*(2*a^2*b - (tan(c/2 + (d*x)/2)*(2
4*a^12 - 32*a^10*b^2))/(4*a^9))*(-(a + b)^3*(a - b)^3)^(1/2))/a^6))/a^6))*(
-(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^6*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1310 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=212

$$\frac{a^2\left(a^2-b^2\right)^2\sin(c+dx)}{b^7d}-\frac{a\left(a^2-b^2\right)^2\sin^2(c+dx)}{2b^6d}+\frac{\left(a^2-b^2\right)^2\sin^3(c+dx)}{3b^5d}-\frac{a\left(a^2-2b^2\right)\sin^4(c+dx)}{4b^4d}+\frac{\left(a^2-2\right.}{}$$

```
[Out] -a^3*(a^2-b^2)^2*ln(a+b*sin(d*x+c))/b^8/d+a^2*(a^2-b^2)^2*sin(d*x+c)/b^7/d-
1/2*a*(a^2-b^2)^2*sin(d*x+c)^2/b^6/d+1/3*(a^2-b^2)^2*sin(d*x+c)^3/b^5/d-1/4
*a*(a^2-2*b^2)*sin(d*x+c)^4/b^4/d+1/5*(a^2-2*b^2)*sin(d*x+c)^5/b^3/d-1/6*a*
sin(d*x+c)^6/b^2/d+1/7*sin(d*x+c)^7/b/d
```

Rubi [A] time = 0.24, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{\left(a^2-2b^2\right)\sin^5(c+dx)}{5b^3d}-\frac{a\left(a^2-2b^2\right)\sin^4(c+dx)}{4b^4d}+\frac{\left(a^2-b^2\right)^2\sin^3(c+dx)}{3b^5d}-\frac{a\left(a^2-b^2\right)^2\sin^2(c+dx)}{2b^6d}+\frac{a^2\left(a^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a^3*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^8*d)) + (a^2*(a^2 - b^2)^2
*Sin[c + d*x])/(b^7*d) - (a*(a^2 - b^2)^2*Sin[c + d*x]^2)/(2*b^6*d) + ((a^2
 - b^2)^2*Sin[c + d*x]^3)/(3*b^5*d) - (a*(a^2 - 2*b^2)*Sin[c + d*x]^4)/(4*b
^4*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^5)/(5*b^3*d) - (a*Sin[c + d*x]^6)/(6*b^
2*d) + Sin[c + d*x]^7/(7*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3\left(b^2-x^2\right)^2}{b^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\left(b^2-x^2\right)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^8 d}$$

$$= \frac{\text{Subst}\left(\int \left(\left(a^3-ab^2\right)^2 - a\left(a^2-b^2\right)^2 x + \left(a^2-b^2\right)^2 x^2 - a\left(a^2-2b^2\right)x^3 + \left(a^2-\right.\right.\right.}{b^8 d}$$

$$= -\frac{a^3\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{b^8 d} + \frac{a^2\left(a^2-b^2\right)^2\sin(c+dx)}{b^7 d} - \frac{a\left(a^2-b^2\right)^2}{2}$$

Mathematica [A] time = 1.27, size = 180, normalized size = 0.85

$$\frac{420b\left(a^3-ab^2\right)^2\sin(c+dx) - 210ab^2\left(a^2-b^2\right)^2\sin^2(c+dx) + 84b^5\left(a^2-2b^2\right)\sin^5(c+dx) - 105ab^4\left(a^2-2b^2\right)\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-420*a^3*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]] + 420*b*(a^3 - a*b^2)^2*Sin
[c + d*x] - 210*a*b^2*(a^2 - b^2)^2*Sin[c + d*x]^2 + 140*b^3*(a^2 - b^2)^2*
Sin[c + d*x]^3 - 105*a*b^4*(a^2 - 2*b^2)*Sin[c + d*x]^4 + 84*b^5*(a^2 - 2*b
^2)*Sin[c + d*x]^5 - 70*a*b^6*Sin[c + d*x]^6 + 60*b^7*Sin[c + d*x]^7)/(420*
b^8*d)
```

fricas [A] time = 0.68, size = 199, normalized size = 0.94

$$\frac{70\,ab^6\cos(dx+c)^6 - 105\,a^3b^4\cos(dx+c)^4 + 210\left(a^5b^2-a^3b^4\right)\cos(dx+c)^2 - 420\left(a^7-2\,a^5b^2+a^3b^4\right)\log(b\,\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(70*a*b^6*cos(d*x + c)^6 - 105*a^3*b^4*cos(d*x + c)^4 + 210*(a^5*b^2
- a^3*b^4)*cos(d*x + c)^2 - 420*(a^7 - 2*a^5*b^2 + a^3*b^4)*log(b*sin(d*x +
 c) + a) - 4*(15*b^7*cos(d*x + c)^6 - 105*a^6*b + 175*a^4*b^3 - 56*a^2*b^5
- 8*b^7 - 3*(7*a^2*b^5 + b^7)*cos(d*x + c)^4 + (35*a^4*b^3 - 28*a^2*b^5 - 4
*b^7)*cos(d*x + c)^2)*sin(d*x + c))/(b^8*d)
```

giac [A] time = 0.20, size = 261, normalized size = 1.23

$$\frac{60\,b^6 \sin(dx+c)^7 - 70\,ab^5 \sin(dx+c)^6 + 84\,a^2b^4 \sin(dx+c)^5 - 168\,b^6 \sin(dx+c)^5 - 105\,a^3b^3 \sin(dx+c)^4 + 210\,ab^5 \sin(dx+c)^4 + 140\,a^4b^2 \sin(dx+c)^3 - 280\,a^2b^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*((60*b^6*sin(d*x + c)^7 - 70*a*b^5*sin(d*x + c)^6 + 84*a^2*b^4*sin(d*
x + c)^5 - 168*b^6*sin(d*x + c)^5 - 105*a^3*b^3*sin(d*x + c)^4 + 210*a*b^5*
sin(d*x + c)^4 + 140*a^4*b^2*sin(d*x + c)^3 - 280*a^2*b^4*sin(d*x + c)^3 +
140*b^6*sin(d*x + c)^3 - 210*a^5*b*sin(d*x + c)^2 + 420*a^3*b^3*sin(d*x + c
)^2 - 210*a*b^5*sin(d*x + c)^2 + 420*a^6*sin(d*x + c) - 840*a^4*b^2*sin(d*x
 + c) + 420*a^2*b^4*sin(d*x + c))/b^7 - 420*(a^7 - 2*a^5*b^2 + a^3*b^4)*log
(abs(b*sin(d*x + c) + a))/b^8)/d
```

maple [A] time = 0.30, size = 329, normalized size = 1.55

$$\frac{\sin^7(dx+c)}{7bd} - \frac{a\left(\sin^6(dx+c)\right)}{6b^2d} + \frac{\left(\sin^5(dx+c)\right)a^2}{5d\,b^3} - \frac{2\left(\sin^5(dx+c)\right)}{5bd} - \frac{\left(\sin^4(dx+c)\right)a^3}{4d\,b^4} + \frac{a\left(\sin^4(dx+c)\right)}{2b^2d} + \frac{a^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/7*sin(d*x+c)^7/b/d-1/6*a*sin(d*x+c)^6/b^2/d+1/5/d/b^3*sin(d*x+c)^5*a^2-2/
5*sin(d*x+c)^5/b/d-1/4/d/b^4*sin(d*x+c)^4*a^3+1/2*a*sin(d*x+c)^4/b^2/d+1/3/
d/b^5*a^4*sin(d*x+c)^3-2/3/d/b^3*sin(d*x+c)^3*a^2+1/3*sin(d*x+c)^3/b/d-1/2/
d/b^6*sin(d*x+c)^2*a^5+1/d/b^4*sin(d*x+c)^2*a^3-1/2*a*sin(d*x+c)^2/b^2/d+1/
d/b^7*sin(d*x+c)*a^6-2/d/b^5*a^4*sin(d*x+c)+a^2*sin(d*x+c)/b^3/d-1/d*a^7/b^
8*ln(a+b*sin(d*x+c))+2/d*a^5/b^6*ln(a+b*sin(d*x+c))-a^3*ln(a+b*sin(d*x+c))/
b^4/d
```

maxima [A] time = 0.31, size = 205, normalized size = 0.97

$$\frac{\frac{60\,b^6 \sin(dx+c)^7 - 70\,ab^5 \sin(dx+c)^6 + 84\left(a^2b^4 - 2\,b^6\right)\sin(dx+c)^5 - 105\left(a^3b^3 - 2\,ab^5\right)\sin(dx+c)^4 + 140\left(a^4b^2 - 2\,a^2b^4 + b^6\right)\sin(dx+c)^3 - 210\left(a^5b - 2\,a^3b^3 + a\right.}{b^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/420*((60*b^6*sin(d*x + c)^7 - 70*a*b^5*sin(d*x + c)^6 + 84*(a^2*b^4 - 2*b
^6)*sin(d*x + c)^5 - 105*(a^3*b^3 - 2*a*b^5)*sin(d*x + c)^4 + 140*(a^4*b^2
- 2*a^2*b^4 + b^6)*sin(d*x + c)^3 - 210*(a^5*b - 2*a^3*b^3 + a*b^5)*sin(d*x
 + c)^2 + 420*(a^6 - 2*a^4*b^2 + a^2*b^4)*sin(d*x + c))/b^7 - 420*(a^7 - 2*
a^5*b^2 + a^3*b^4)*log(b*sin(d*x + c) + a)/b^8)/d
```

mupad [B] time = 0.13, size = 236, normalized size = 1.11

$$-\frac{\sin(c+dx)^5\left(\frac{2}{5b}-\frac{a^2}{5b^3}\right)-\frac{\sin(c+dx)^7}{7b}-\sin(c+dx)^3\left(\frac{1}{3b}-\frac{a^2\left(\frac{2}{b}-\frac{a^2}{b^3}\right)}{3b^2}\right)+\frac{a\sin(c+dx)^6}{6b^2}+\frac{\ln(a+b\sin(c+dx))\left(a^7-2a^5b^2+a^3b\right.}{b^8}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)^5*(2/(5*b) - a^2/(5*b^3)) - sin(c + d*x)^7/(7*b) - sin(c + d
*x)^3*(1/(3*b) - (a^2*(2/b - a^2/b^3))/(3*b^2)) + (a*sin(c + d*x)^6)/(6*b^2
) + (log(a + b*sin(c + d*x))*(a^7 + a^3*b^4 - 2*a^5*b^2))/b^8 - (a*sin(c +
d*x)^4*(2/b - a^2/b^3))/(4*b) + (a*sin(c + d*x)^2*(1/b - (a^2*(2/b - a^2/b^
3))/b^2))/(2*b) - (a^2*sin(c + d*x)*(1/b - (a^2*(2/b - a^2/b^3))/b^2))/b^2)
/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1311 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{a^2\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{b^7d}-\frac{a\left(a^2-b^2\right)^2\sin(c+dx)}{b^6d}+\frac{\left(a^2-b^2\right)^2\sin^2(c+dx)}{2b^5d}-\frac{a\left(a^2-2b^2\right)\sin^3(c+dx)}{3b^4d}$$

```
[Out] a^2*(a^2-b^2)^2*ln(a+b*sin(d*x+c))/b^7/d-a*(a^2-b^2)^2*sin(d*x+c)/b^6/d+1/2
*(a^2-b^2)^2*sin(d*x+c)^2/b^5/d-1/3*a*(a^2-2*b^2)*sin(d*x+c)^3/b^4/d+1/4*(a
^2-2*b^2)*sin(d*x+c)^4/b^3/d-1/5*a*sin(d*x+c)^5/b^2/d+1/6*sin(d*x+c)^6/b/d
```

Rubi [A] time = 0.21, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{\left(a^2-2b^2\right)\sin^4(c+dx)}{4b^3d}-\frac{a\left(a^2-2b^2\right)\sin^3(c+dx)}{3b^4d}+\frac{\left(a^2-b^2\right)^2\sin^2(c+dx)}{2b^5d}-\frac{a\left(a^2-b^2\right)^2\sin(c+dx)}{b^6d}+\frac{a^2\left(a^2-b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a^2*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^7*d) - (a*(a^2 - b^2)^2*Sin[
c + d*x])/(b^6*d) + ((a^2 - b^2)^2*Sin[c + d*x]^2)/(2*b^5*d) - (a*(a^2 - 2*
b^2)*Sin[c + d*x]^3)/(3*b^4*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^4)/(4*b^3*d) -
 (a*Sin[c + d*x]^5)/(5*b^2*d) + Sin[c + d*x]^6/(6*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(b^2-x^2\right)^2}{b^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(b^2-x^2\right)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(-a\left(a^2-b^2\right)^2 + \left(a^2-b^2\right)^2 x - a\left(a^2-2b^2\right)x^2 + \left(a^2-2b^2\right)x^3 - ax^4\right.\right.}{b^7 d}$$

$$= \frac{a^2\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{b^7 d} - \frac{a\left(a^2-b^2\right)^2\sin(c+dx)}{b^6 d} + \frac{\left(a^2-b^2\right)^2\sin}{2b^5 d}$$

Mathematica [A] time = 0.72, size = 153, normalized size = 0.85

$$\frac{60\left(a^3-ab^2\right)^2\log(a+b\sin(c+dx)) + 30b^2\left(a^2-b^2\right)^2\sin^2(c+dx) - 60ab\left(a^2-b^2\right)^2\sin(c+dx) + 15b^4\left(a^2-2b^2\right.}{60b^7 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (60*(a^3 - a*b^2)^2*Log[a + b*Sin[c + d*x]] - 60*a*b*(a^2 - b^2)^2*Sin[c +
d*x] + 30*b^2*(a^2 - b^2)^2*Sin[c + d*x]^2 - 20*a*b^3*(a^2 - 2*b^2)*Sin[c +
 d*x]^3 + 15*b^4*(a^2 - 2*b^2)*Sin[c + d*x]^4 - 12*a*b^5*Sin[c + d*x]^5 + 1
0*b^6*Sin[c + d*x]^6)/(60*b^7*d)
```

fricas [A] time = 0.77, size = 164, normalized size = 0.91

$$-\frac{10\,b^6\cos(dx+c)^6 - 15\,a^2b^4\cos(dx+c)^4 + 30\left(a^4b^2-a^2b^4\right)\cos(dx+c)^2 - 60\left(a^6-2\,a^4b^2+a^2b^4\right)\log(b\sin(}{60\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^6*cos(d*x + c)^6 - 15*a^2*b^4*cos(d*x + c)^4 + 30*(a^4*b^2 - a^
2*b^4)*cos(d*x + c)^2 - 60*(a^6 - 2*a^4*b^2 + a^2*b^4)*log(b*sin(d*x + c) +
 a) + 4*(3*a*b^5*cos(d*x + c)^4 + 15*a^5*b - 25*a^3*b^3 + 8*a*b^5 - (5*a^3*
b^3 - 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(b^7*d)
```

giac [A] time = 0.18, size = 213, normalized size = 1.18

$$\frac{\frac{10\,b^5\sin(dx+c)^6-12\,ab^4\sin(dx+c)^5+15\,a^2b^3\sin(dx+c)^4-30\,b^5\sin(dx+c)^4-20\,a^3b^2\sin(dx+c)^3+40\,ab^4\sin(dx+c)^3+30\,a^4b\sin(dx+c)^2-60\,a^2b^3\sin(dx}{b^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*((10*b^5*sin(d*x + c)^6 - 12*a*b^4*sin(d*x + c)^5 + 15*a^2*b^3*sin(d*x
 + c)^4 - 30*b^5*sin(d*x + c)^4 - 20*a^3*b^2*sin(d*x + c)^3 + 40*a*b^4*sin(
d*x + c)^3 + 30*a^4*b*sin(d*x + c)^2 - 60*a^2*b^3*sin(d*x + c)^2 + 30*b^5*s
in(d*x + c)^2 - 60*a^5*sin(d*x + c) + 120*a^3*b^2*sin(d*x + c) - 60*a*b^4*s
in(d*x + c))/b^6 + 60*(a^6 - 2*a^4*b^2 + a^2*b^4)*log(abs(b*sin(d*x + c) +
a))/b^7)/d
```

maple [A] time = 0.29, size = 273, normalized size = 1.52

$$\frac{\sin^6(dx+c)}{6bd}-\frac{a\left(\sin^5(dx+c)\right)}{5b^2d}+\frac{\left(\sin^4(dx+c)\right)a^2}{4d\,b^3}-\frac{\sin^4(dx+c)}{2bd}-\frac{\left(\sin^3(dx+c)\right)a^3}{3d\,b^4}+\frac{2a\left(\sin^3(dx+c)\right)}{3b^2d}+\frac{\left(\sin^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/6*sin(d*x+c)^6/b/d-1/5*a*sin(d*x+c)^5/b^2/d+1/4/d/b^3*sin(d*x+c)^4*a^2-1/
2*sin(d*x+c)^4/b/d-1/3/d/b^4*sin(d*x+c)^3*a^3+2/3*a*sin(d*x+c)^3/b^2/d+1/2/
d/b^5*sin(d*x+c)^2*a^4-1/d/b^3*a^2*sin(d*x+c)^2+1/2*sin(d*x+c)^2/b/d-1/d/b^
6*sin(d*x+c)*a^5+2/d/b^4*sin(d*x+c)*a^3-a*sin(d*x+c)/b^2/d+1/d*a^6/b^7*ln(a
+b*sin(d*x+c))-2/d*a^4/b^5*ln(a+b*sin(d*x+c))+1/d/b^3*ln(a+b*sin(d*x+c))*a^
2
```

maxima [A] time = 0.31, size = 172, normalized size = 0.96

$$\frac{\frac{10\,b^5\sin(dx+c)^6-12\,ab^4\sin(dx+c)^5+15\left(a^2b^3-2\,b^5\right)\sin(dx+c)^4-20\left(a^3b^2-2\,ab^4\right)\sin(dx+c)^3+30\left(a^4b-2\,a^2b^3+b^5\right)\sin(dx+c)^2-60\left(a^5-2\,a^3b^2+ab^4\right)\sin}{b^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*((10*b^5*sin(d*x + c)^6 - 12*a*b^4*sin(d*x + c)^5 + 15*(a^2*b^3 - 2*b^
5)*sin(d*x + c)^4 - 20*(a^3*b^2 - 2*a*b^4)*sin(d*x + c)^3 + 30*(a^4*b - 2*a
^2*b^3 + b^5)*sin(d*x + c)^2 - 60*(a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/b
^6 + 60*(a^6 - 2*a^4*b^2 + a^2*b^4)*log(b*sin(d*x + c) + a)/b^7)/d
```

mupad [B] time = 11.78, size = 191, normalized size = 1.06

$$\frac{\sin\left(c+d\,x\right)^2\,\left(\frac{1}{2\,b}-\frac{a^2\,\left(\frac{1}{b}-\frac{a^2}{2\,b^3}\right)}{b^2}\right)-\sin\left(c+d\,x\right)^4\,\left(\frac{1}{2\,b}-\frac{a^2}{4\,b^3}\right)+\frac{\sin(c+d\,x)^6}{6\,b}-\frac{a\sin(c+d\,x)^5}{5\,b^2}+\frac{\ln(a+b\,\sin(c+d\,x))\,\left(a^6-2\,a^4\,b^2+a^2\,b^4\right.}{b^7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^2*(1/(2*b) - (a^2*(1/b - a^2/(2*b^3)))/b^2) - sin(c + d*x)^4*
(1/(2*b) - a^2/(4*b^3)) + sin(c + d*x)^6/(6*b) - (a*sin(c + d*x)^5)/(5*b^2)
 + (log(a + b*sin(c + d*x))*(a^6 + a^2*b^4 - 2*a^4*b^2))/b^7 - (a*sin(c + d
*x)*(1/b - (a^2*(2/b - a^2/b^3))/b^2))/b + (a*sin(c + d*x)^3*(2/b - a^2/b^3
))/(3*b))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1312 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{a\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{b^6d}+\frac{\left(a^2-b^2\right)^2\sin(c+dx)}{b^5d}-\frac{a\left(a^2-2b^2\right)\sin^2(c+dx)}{2b^4d}+\frac{\left(a^2-2b^2\right)\sin^3(c+dx)}{3b^3d}$$

```
[Out] -a*(a^2-b^2)^2*ln(a+b*sin(d*x+c))/b^6/d+(a^2-b^2)^2*sin(d*x+c)/b^5/d-1/2*a*
(a^2-2*b^2)*sin(d*x+c)^2/b^4/d+1/3*(a^2-2*b^2)*sin(d*x+c)^3/b^3/d-1/4*a*sin
(d*x+c)^4/b^2/d+1/5*sin(d*x+c)^5/b/d
```

Rubi [A] time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 772}

$$\frac{\left(a^2-2b^2\right)\sin^3(c+dx)}{3b^3d}-\frac{a\left(a^2-2b^2\right)\sin^2(c+dx)}{2b^4d}+\frac{\left(a^2-b^2\right)^2\sin(c+dx)}{b^5d}-\frac{a\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{b^6d}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^6*d)) + ((a^2 - b^2)^2*Sin[c
 + d*x])/(b^5*d) - (a*(a^2 - 2*b^2)*Sin[c + d*x]^2)/(2*b^4*d) + ((a^2 - 2*b
^2)*Sin[c + d*x]^3)/(3*b^3*d) - (a*Sin[c + d*x]^4)/(4*b^2*d) + Sin[c + d*x]
^5/(5*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x\left(b^2-x^2\right)^2}{b(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x\left(b^2-x^2\right)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2 - a\left(a^2-2b^2\right)x + \left(a^2-2b^2\right)x^2 - ax^3 + x^4 - \frac{a\left(a^2-b^2\right)^2}{a+x}\right) dx,\right.}{b^6 d}$$
$$= -\frac{a\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{b^6 d} + \frac{\left(a^2-b^2\right)^2\sin(c+dx)}{b^5 d} - \frac{a\left(a^2-2b^2\right)\sin}{2b^4 d}$$

Mathematica [A] time = 0.64, size = 128, normalized size = 0.86

$$\frac{-30ab^2\left(a^2-2b^2\right)\sin^2(c+dx) + 60b\left(a^2-b^2\right)^2\sin(c+dx) - 60a\left(a^2-b^2\right)^2\log(a+b\sin(c+dx)) + 20b^3\left(a^2-2\right.}{60b^6 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-60*a*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]] + 60*b*(a^2 - b^2)^2*Sin[c + d
*x] - 30*a*b^2*(a^2 - 2*b^2)*Sin[c + d*x]^2 + 20*b^3*(a^2 - 2*b^2)*Sin[c +
d*x]^3 - 15*a*b^4*Sin[c + d*x]^4 + 12*b^5*Sin[c + d*x]^5)/(60*b^6*d)
```

fricas [A] time = 0.90, size = 142, normalized size = 0.96

$$-\frac{15\,ab^4\cos(dx+c)^4 - 30\left(a^3b^2-ab^4\right)\cos(dx+c)^2 + 60\left(a^5-2\,a^3b^2+ab^4\right)\log(b\sin(dx+c)+a) - 4\left(3\,b^5\cos\right.}{60\,b^6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(15*a*b^4*cos(d*x + c)^4 - 30*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 + 60*(
a^5 - 2*a^3*b^2 + a*b^4)*log(b*sin(d*x + c) + a) - 4*(3*b^5*cos(d*x + c)^4
```

```
+ 15*a^4*b - 25*a^2*b^3 + 8*b^5 - (5*a^2*b^3 - 4*b^5)*cos(d*x + c)^2)*sin(d
*x + c))/(b^6*d)
```

giac [A] time = 0.18, size = 165, normalized size = 1.11

$$\frac{\frac{12\,b^4\sin(dx+c)^5-15\,ab^3\sin(dx+c)^4+20\,a^2b^2\sin(dx+c)^3-40\,b^4\sin(dx+c)^3-30\,a^3b\sin(dx+c)^2+60\,ab^3\sin(dx+c)^2+60\,a^4\sin(dx+c)-120\,a^2b^2\sin(dx+c}{b^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*((12*b^4*sin(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 20*a^2*b^2*sin(d*x
 + c)^3 - 40*b^4*sin(d*x + c)^3 - 30*a^3*b*sin(d*x + c)^2 + 60*a*b^3*sin(d*
x + c)^2 + 60*a^4*sin(d*x + c) - 120*a^2*b^2*sin(d*x + c) + 60*b^4*sin(d*x
+ c))/b^5 - 60*(a^5 - 2*a^3*b^2 + a*b^4)*log(abs(b*sin(d*x + c) + a))/b^6)/
d
```

maple [A] time = 0.22, size = 215, normalized size = 1.45

$$\frac{\sin^5(dx+c)}{5bd}-\frac{a\left(\sin^4(dx+c)\right)}{4b^2d}+\frac{\left(\sin^3(dx+c)\right)a^2}{3d\,b^3}-\frac{2\left(\sin^3(dx+c)\right)}{3bd}-\frac{\left(\sin^2(dx+c)\right)a^3}{2d\,b^4}+\frac{a\left(\sin^2(dx+c)\right)}{b^2d}+\frac{a^4\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/5*sin(d*x+c)^5/b/d-1/4*a*sin(d*x+c)^4/b^2/d+1/3/d/b^3*sin(d*x+c)^3*a^2-2/
3*sin(d*x+c)^3/b/d-1/2/d/b^4*sin(d*x+c)^2*a^3+a*sin(d*x+c)^2/b^2/d+1/d/b^5*
a^4*sin(d*x+c)-2*a^2*sin(d*x+c)/b^3/d+sin(d*x+c)/b/d-1/d*a^5/b^6*ln(a+b*sin
(d*x+c))+2*a^3*ln(a+b*sin(d*x+c))/b^4/d-1/d/b^2*a*ln(a+b*sin(d*x+c))
```

maxima [A] time = 0.31, size = 139, normalized size = 0.94

$$\frac{\frac{12\,b^4\sin(dx+c)^5-15\,ab^3\sin(dx+c)^4+20\left(a^2b^2-2\,b^4\right)\sin(dx+c)^3-30\left(a^3b-2\,ab^3\right)\sin(dx+c)^2+60\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+c)}{b^5}-\frac{60\left(a^5-2\,a^3b^2+ab^4\right)\log}{b^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*((12*b^4*sin(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 20*(a^2*b^2 - 2*b^
4)*sin(d*x + c)^3 - 30*(a^3*b - 2*a*b^3)*sin(d*x + c)^2 + 60*(a^4 - 2*a^2*b
^2 + b^4)*sin(d*x + c))/b^5 - 60*(a^5 - 2*a^3*b^2 + a*b^4)*log(b*sin(d*x +
c) + a)/b^6)/d
```

mupad [B] time = 0.07, size = 150, normalized size = 1.01

$$\frac{\sin\left(c+d\,x\right)\,\left(\frac{1}{b}-\frac{a^2\,\left(\frac{2}{b}-\frac{a^2}{b^3}\right)}{b^2}\right)-{\sin\left(c+d\,x\right)}^3\,\left(\frac{2}{3\,b}-\frac{a^2}{3\,b^3}\right)+\frac{{\sin\left(c+d\,x\right)}^5}{5\,b}-\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(a^5-2\,a^3\,b^2+a\,b^4\right)}{b^6}-\frac{a\,{\sin\left(c+d\,x\right)}^4}{4\,b^2}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)*(1/b - (a^2*(2/b - a^2/b^3))/b^2) - sin(c + d*x)^3*(2/(3*b) -
 a^2/(3*b^3)) + sin(c + d*x)^5/(5*b) - (log(a + b*sin(c + d*x))*(a*b^4 + a^
5 - 2*a^3*b^2))/b^6 - (a*sin(c + d*x)^4)/(4*b^2) + (a*sin(c + d*x)^2*(2/b -
 a^2/b^3))/(2*b))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1313 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=107

$$-\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{ab^4d}+\frac{(a^2-2b^2)\sin(c+dx)}{b^3d}-\frac{a\sin^2(c+dx)}{2b^2d}+\frac{\log(\sin(c+dx))}{ad}+\frac{\sin^3(c+dx)}{3bd}$$

```
[Out] ln(sin(d*x+c))/a/d-(a^2-b^2)^2*ln(a+b*sin(d*x+c))/a/b^4/d+(a^2-2*b^2)*sin(d
*x+c)/b^3/d-1/2*a*sin(d*x+c)^2/b^2/d+1/3*sin(d*x+c)^3/b/d
```

Rubi [A] time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{(a^2-2b^2)\sin(c+dx)}{b^3d}-\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{ab^4d}-\frac{a\sin^2(c+dx)}{2b^2d}+\frac{\log(\sin(c+dx))}{ad}+\frac{\sin^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a*b^4*d)
 + ((a^2 - 2*b^2)*Sin[c + d*x])/(b^3*d) - (a*Sin[c + d*x]^2)/(2*b^2*d) + Si
n[c + d*x]^3/(3*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b\left(b^2-x^2\right)^2}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{\left(a^2-b^2\right)^2}{a(a+x)}\right)dx, x, b\sin(c+dx)\right)}{b^4 d}$$

$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{ab^4 d} + \frac{\left(a^2-2b^2\right)\sin(c+dx)}{b^3 d} -$$

Mathematica [A] time = 0.14, size = 101, normalized size = 0.94

$$\frac{-3a^2b^2\sin^2(c+dx)+6ab\left(a^2-2b^2\right)\sin(c+dx)+6\left(b^4\log(\sin(c+dx))-\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))\right)+2}{6ab^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (6*(b^4*Log[Sin[c + d*x]] - (a^2 - b^2)^2*Log[a + b*Sin[c + d*x]]) + 6*a*b*
(a^2 - 2*b^2)*Sin[c + d*x] - 3*a^2*b^2*Sin[c + d*x]^2 + 2*a*b^3*Sin[c + d*x
]^3)/(6*a*b^4*d)
```

fricas [A] time = 0.96, size = 104, normalized size = 0.97

$$\frac{3\,a^2b^2\cos\left(dx+c\right)^2+6\,b^4\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)-6\left(a^4-2\,a^2b^2+b^4\right)\log\left(b\sin\left(dx+c\right)+a\right)-2\left(ab^3\cos\left(dx+c\right.\right.}{6\,ab^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*b^2*cos(d*x + c)^2 + 6*b^4*log(-1/2*sin(d*x + c)) - 6*(a^4 - 2*a
^2*b^2 + b^4)*log(b*sin(d*x + c) + a) - 2*(a*b^3*cos(d*x + c)^2 - 3*a^3*b +
 5*a*b^3)*sin(d*x + c))/(a*b^4*d)
```

giac [A] time = 0.17, size = 106, normalized size = 0.99

$$\frac{\frac{6\log\left(|\sin(dx+c)|\right)}{a}+\frac{2b^2\sin(dx+c)^3-3ab\sin(dx+c)^2+6a^2\sin(dx+c)-12b^2\sin(dx+c)}{b^3}-\frac{6\left(a^4-2a^2b^2+b^4\right)\log\left(|b\sin(dx+c)+a|\right)}{ab^4}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c)))/a + (2*b^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c
)^2 + 6*a^2*sin(d*x + c) - 12*b^2*sin(d*x + c))/b^3 - 6*(a^4 - 2*a^2*b^2 +
b^4)*log(abs(b*sin(d*x + c) + a))/(a*b^4))/d
```

maple [A] time = 0.42, size = 140, normalized size = 1.31

$$\frac{\sin^3(dx+c)}{3bd}-\frac{a\left(\sin^2(dx+c)\right)}{2b^2d}+\frac{a^2\sin(dx+c)}{b^3d}-\frac{2\sin(dx+c)}{bd}-\frac{a^3\ln(a+b\sin(dx+c))}{b^4d}+\frac{2a\ln(a+b\sin(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/3*sin(d*x+c)^3/b/d-1/2*a*sin(d*x+c)^2/b^2/d+a^2*sin(d*x+c)/b^3/d-2*sin(d*
x+c)/b/d-a^3*ln(a+b*sin(d*x+c))/b^4/d+2/d/b^2*a*ln(a+b*sin(d*x+c))-1/d/a*ln
(a+b*sin(d*x+c))+ln(sin(d*x+c))/a/d
```

maxima [A] time = 0.30, size = 99, normalized size = 0.93

$$\frac{\frac{6\log(\sin(dx+c))}{a}+\frac{2b^2\sin(dx+c)^3-3ab\sin(dx+c)^2+6\left(a^2-2b^2\right)\sin(dx+c)}{b^3}-\frac{6\left(a^4-2a^2b^2+b^4\right)\log(b\sin(dx+c)+a)}{ab^4}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*log(sin(d*x + c))/a + (2*b^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c)^2 +
 6*(a^2 - 2*b^2)*sin(d*x + c))/b^3 - 6*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*
x + c) + a)/(a*b^4))/d
```

mupad [B] time = 12.10, size = 254, normalized size = 2.37

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a^2-2\,b^2\right)}{b^3}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(a^2-2\,b^2\right)}{b^3}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b^2}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{b^2}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(3\,a^2-4\,b^2\right.}{3\,b^3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) + ((2*tan(c/2 + (d*x)/2)*(a^2 - 2*b^2))/b^3 +
 (2*tan(c/2 + (d*x)/2)^5*(a^2 - 2*b^2))/b^3 - (2*a*tan(c/2 + (d*x)/2)^2)/b^
2 - (2*a*tan(c/2 + (d*x)/2)^4)/b^2 + (4*tan(c/2 + (d*x)/2)^3*(3*a^2 - 4*b^2
))/(3*b^3))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 +
 (d*x)/2)^6 + 1)) + (a*log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 - 2*b^2))/(b^4*d)
 - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^2 - b^2)^2)
/(a*b^4*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1314 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=96

$$\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{a\sin(c+dx)}{b^2d}-\frac{\csc(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2bd}$$

```
[Out] -csc(d*x+c)/a/d-b*ln(sin(d*x+c))/a^2/d+(a^2-b^2)^2*ln(a+b*sin(d*x+c))/a^2/b
^3/d-a*sin(d*x+c)/b^2/d+1/2*sin(d*x+c)^2/b/d
```

Rubi [A] time = 0.16, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{a\sin(c+dx)}{b^2d}-\frac{\csc(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) + ((a^2 - b^2)^2*Log[
a + b*Sin[c + d*x]])/(a^2*b^3*d) - (a*Sin[c + d*x])/(b^2*d) + Sin[c + d*x]^
2/(2*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2\left(b^2-x^2\right)^2}{x^2(a+x)}\,dx,x,b\sin(c+dx)\right)}{b^5d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^2(a+x)}\,dx,x,b\sin(c+dx)\right)}{b^3d}$$

$$= \frac{\text{Subst}\left(\int \left(-a+\frac{b^4}{ax^2}-\frac{b^4}{a^2x}+x+\frac{\left(a^2-b^2\right)^2}{a^2(a+x)}\right)dx,x,b\sin(c+dx)\right)}{b^3d}$$

$$= -\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{a\sin(}{b}$$

Mathematica [A] time = 0.18, size = 86, normalized size = 0.90

$$\frac{\frac{2\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^2b^3}-\frac{2b\log(\sin(c+dx))}{a^2}-\frac{2a\sin(c+dx)}{b^2}-\frac{2\csc(c+dx)}{a}+\frac{\sin^2(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*Csc[c + d*x])/a - (2*b*Log[Sin[c + d*x]])/a^2 + (2*(a^2 - b^2)^2*Log[a
 + b*Sin[c + d*x]])/(a^2*b^3) - (2*a*Sin[c + d*x])/b^2 + Sin[c + d*x]^2/b)/
(2*d)
```

fricas [A] time = 0.72, size = 133, normalized size = 1.39

$$\frac{4\,a^3b\cos(dx+c)^2-4\,b^4\log\left(\frac{1}{2}\,\sin(dx+c)\right)\sin(dx+c)-4\,a^3b-4\,ab^3+4\left(a^4-2\,a^2b^2+b^4\right)\log(b\sin(dx+c)}{4\,a^2b^3d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*b*cos(d*x + c)^2 - 4*b^4*log(1/2*sin(d*x + c))*sin(d*x + c) - 4*
a^3*b - 4*a*b^3 + 4*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)*sin(d*x
```

```
 + c) - (2*a^2*b^2*cos(d*x + c)^2 - a^2*b^2)*sin(d*x + c))/(a^2*b^3*d*sin(d
*x + c))
```

giac [A] time = 0.20, size = 105, normalized size = 1.09

$$-\frac{\frac{2\,b\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^2}-\frac{b\sin\left(dx+c\right)^2-2\,a\sin\left(dx+c\right)}{b^2}-\frac{2\,\left(b\sin\left(dx+c\right)-a\right)}{a^2\sin\left(dx+c\right)}-\frac{2\,\left(a^4-2\,a^2b^2+b^4\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^2b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*log(abs(sin(d*x + c)))/a^2 - (b*sin(d*x + c)^2 - 2*a*sin(d*x + c)
)/b^2 - 2*(b*sin(d*x + c) - a)/(a^2*sin(d*x + c)) - 2*(a^4 - 2*a^2*b^2 + b^
4)*log(abs(b*sin(d*x + c) + a))/(a^2*b^3))/d
```

maple [A] time = 0.42, size = 124, normalized size = 1.29

$$\frac{\sin^2\left(dx+c\right)}{2bd}-\frac{a\sin\left(dx+c\right)}{b^2d}+\frac{\ln\left(a+b\sin\left(dx+c\right)\right)a^2}{d\,b^3}-\frac{2\ln\left(a+b\sin\left(dx+c\right)\right)}{bd}+\frac{b\ln\left(a+b\sin\left(dx+c\right)\right)}{a^2d}-\frac{}{da\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2*sin(d*x+c)^2/b/d-a*sin(d*x+c)/b^2/d+1/d/b^3*ln(a+b*sin(d*x+c))*a^2-2*ln
(a+b*sin(d*x+c))/b/d+b*ln(a+b*sin(d*x+c))/a^2/d-1/d/a/sin(d*x+c)-b*ln(sin(d
*x+c))/a^2/d
```

maxima [A] time = 0.30, size = 91, normalized size = 0.95

$$-\frac{\frac{2\,b\log\left(\sin\left(dx+c\right)\right)}{a^2}-\frac{b\sin\left(dx+c\right)^2-2\,a\sin\left(dx+c\right)}{b^2}+\frac{2}{a\sin\left(dx+c\right)}-\frac{2\,\left(a^4-2\,a^2b^2+b^4\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^2b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*log(sin(d*x + c))/a^2 - (b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2
 + 2/(a*sin(d*x + c)) - 2*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/(
a^2*b^3))/d
```

mupad [B] time = 12.03, size = 233, normalized size = 2.43

$$\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(a^2-b^2\right)^2}{a^2\,b^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,\left(a^2-2\,b^2\right)}{b^3\,d}-\frac{b\ \ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^2 - b^2)^2)/(a
^2*b^3*d) - tan(c/2 + (d*x)/2)/(2*a*d) - (log(tan(c/2 + (d*x)/2)^2 + 1)*(a^
2 - 2*b^2))/(b^3*d) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d) - ((2*tan(c/2 + (
d*x)/2)^2*(2*a^2 + b^2))/b^2 + (tan(c/2 + (d*x)/2)^4*(4*a^2 + b^2))/b^2 - (
4*a*tan(c/2 + (d*x)/2)^3)/b + 1)/(d*(2*a*tan(c/2 + (d*x)/2) + 4*a*tan(c/2 +
 (d*x)/2)^3 + 2*a*tan(c/2 + (d*x)/2)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1315 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=105

$$\frac{b\csc(c+dx)}{a^2d}-\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^3b^2d}-\frac{\left(2a^2-b^2\right)\log(\sin(c+dx))}{a^3d}-\frac{\csc^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{bd}$$

```
[Out] b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d-(2*a^2-b^2)*ln(sin(d*x+c))/a^3/d-(a
^2-b^2)^2*ln(a+b*sin(d*x+c))/a^3/b^2/d+sin(d*x+c)/b/d
```

Rubi [A] time = 0.18, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$-\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^3b^2d}-\frac{\left(2a^2-b^2\right)\log(\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((2*a^2 - b^2)*Log[Sin[
c + d*x]])/(a^3*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a^3*b^2*d) +
Sin[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^3\left(b^2-x^2\right)^2}{x^3(a+x)}\,dx,x,b\sin(c+dx)\right)}{b^5d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^3(a+x)}\,dx,x,b\sin(c+dx)\right)}{b^2d}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{b^4}{ax^3}-\frac{b^4}{a^2x^2}+\frac{-2a^2b^2+b^4}{a^3x}-\frac{\left(a^2-b^2\right)^2}{a^3(a+x)}\right)dx,x,b\sin(c+dx)\right)}{b^2d}$$

$$= \frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}-\frac{\left(2a^2-b^2\right)\log(\sin(c+dx))}{a^3d}-\frac{\left(a^2-b^2\right)^2\log(a}{a^3b}$$

Mathematica [A] time = 0.39, size = 97, normalized size = 0.92

$$\frac{\frac{2b\csc(c+dx)}{a^2}+\frac{\frac{2b^2\left(b^2-2a^2\right)\log(\sin(c+dx))-2\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^3}+2b\sin(c+dx)}{b^2}-\frac{\csc^2(c+dx)}{a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*b*Csc[c + d*x])/a^2 - Csc[c + d*x]^2/a + ((2*b^2*(-2*a^2 + b^2)*Log[Sin
[c + d*x]] - 2*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/a^3 + 2*b*Sin[c + d*x
])/b^2)/(2*d)
```

fricas [A] time = 0.68, size = 174, normalized size = 1.66

$$\frac{a^2b^2+2\left(a^4-2a^2b^2+b^4-\left(a^4-2a^2b^2+b^4\right)\cos(dx+c)^2\right)\log(b\sin(dx+c)+a)+2\left(2a^2b^2-b^4-\left(2a^2b^2-b\right.\right.}{2\left(a^3b^2d\cos(dx+c)^2-a^3b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b^2 + 2*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x +
 c)^2)*log(b*sin(d*x + c) + a) + 2*(2*a^2*b^2 - b^4 - (2*a^2*b^2 - b^4)*cos
(d*x + c)^2)*log(-1/2*sin(d*x + c)) + 2*(a^3*b*cos(d*x + c)^2 - a^3*b - a*b
^3)*sin(d*x + c))/(a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)
```

giac [A] time = 0.20, size = 130, normalized size = 1.24

$$\frac{\frac{2\sin(dx+c)}{b} - \frac{2\left(2a^2-b^2\right)\log(|\sin(dx+c)|)}{a^3} - \frac{2\left(a^4-2a^2b^2+b^4\right)\log(|b\sin(dx+c)+a|)}{a^3b^2} + \frac{6a^2\sin(dx+c)^2-3b^2\sin(dx+c)^2+2ab\sin(dx+c)-a^2}{a^3\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*sin(d*x + c)/b - 2*(2*a^2 - b^2)*log(abs(sin(d*x + c)))/a^3 - 2*(a^4
 - 2*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/(a^3*b^2) + (6*a^2*sin(d*x
 + c)^2 - 3*b^2*sin(d*x + c)^2 + 2*a*b*sin(d*x + c) - a^2)/(a^3*sin(d*x + c
)^2))/d
```

maple [A] time = 0.48, size = 140, normalized size = 1.33

$$\frac{\sin(dx+c)}{bd} - \frac{a\ln(a+b\sin(dx+c))}{d\,b^2} + \frac{2\ln(a+b\sin(dx+c))}{da} - \frac{b^2\ln(a+b\sin(dx+c))}{a^3d} - \frac{1}{2da\sin(dx+c)^2} - 2\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] sin(d*x+c)/b/d-1/d/b^2*a*ln(a+b*sin(d*x+c))+2/d/a*ln(a+b*sin(d*x+c))-b^2*ln
(a+b*sin(d*x+c))/a^3/d-1/2/d/a/sin(d*x+c)^2-2*ln(sin(d*x+c))/a/d+b^2*ln(sin
(d*x+c))/a^3/d+1/d/a^2*b/sin(d*x+c)
```

maxima [A] time = 0.31, size = 99, normalized size = 0.94

$$\frac{\frac{2\sin(dx+c)}{b} - \frac{2\left(2a^2-b^2\right)\log(\sin(dx+c))}{a^3} - \frac{2\left(a^4-2a^2b^2+b^4\right)\log(b\sin(dx+c)+a)}{a^3b^2} + \frac{2b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sin(d*x + c)/b - 2*(2*a^2 - b^2)*log(sin(d*x + c))/a^3 - 2*(a^4 - 2*
a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/(a^3*b^2) + (2*b*sin(d*x + c) - a)/(
a^2*sin(d*x + c)^2))/d
```

mupad [B] time = 11.98, size = 238, normalized size = 2.27

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}-\frac{\frac{a}{2}-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(4\,a^2+b^2\right)}{b}}{d\left(4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\right.}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(2*a^2*d) - (a/2 - 2*b*tan(c/2 + (d*x)/2) + (a*tan(c
/2 + (d*x)/2)^2)/2 - (2*tan(c/2 + (d*x)/2)^3*(4*a^2 + b^2))/b)/(d*(4*a^2*ta
n(c/2 + (d*x)/2)^2 + 4*a^2*tan(c/2 + (d*x)/2)^4)) - tan(c/2 + (d*x)/2)^2/(8
*a*d) + (a*log(tan(c/2 + (d*x)/2)^2 + 1))/(b^2*d) - (log(tan(c/2 + (d*x)/2)
)*(2*a^2 - b^2))/(a^3*d) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d
*x)/2)^2)*(a^4 + b^4 - 2*a^2*b^2))/(a^3*b^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1316 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{b\csc^2(c+dx)}{2a^2d}+\frac{b\left(2a^2-b^2\right)\log(\sin(c+dx))}{a^4d}+\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^4bd}+\frac{\left(2a^2-b^2\right)\csc(c+dx)}{a^3d}-\frac{\csc^3(c}{3a}$$

```
[Out] (2*a^2-b^2)*csc(d*x+c)/a^3/d+1/2*b*csc(d*x+c)^2/a^2/d-1/3*csc(d*x+c)^3/a/d+
b*(2*a^2-b^2)*ln(sin(d*x+c))/a^4/d+(a^2-b^2)^2*ln(a+b*sin(d*x+c))/a^4/b/d
```

Rubi [A] time = 0.17, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{\left(2a^2-b^2\right)\csc(c+dx)}{a^3d}+\frac{b\left(2a^2-b^2\right)\log(\sin(c+dx))}{a^4d}+\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^4bd}+\frac{b\csc^2(c+dx)}{2a^2d}-\frac{\csc^3(c}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a^2 - b^2)*Csc[c + d*x])/(a^3*d) + (b*Csc[c + d*x]^2)/(2*a^2*d) - Csc[c
 + d*x]^3/(3*a*d) + (b*(2*a^2 - b^2)*Log[Sin[c + d*x]])/(a^4*d) + ((a^2 - b
^2)^2*Log[a + b*Sin[c + d*x]])/(a^4*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^4(b^2-x^2)^2}{x^4(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^4(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{ax^4} - \frac{b^4}{a^2x^3} + \frac{-2a^2b^2+b^4}{a^3x^2} + \frac{2a^2b^2-b^4}{a^4x} + \frac{(a^2-b^2)^2}{a^4(a+x)}\right)dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{(2a^2-b^2)\csc(c+dx)}{a^3d} + \frac{b\csc^2(c+dx)}{2a^2d} - \frac{\csc^3(c+dx)}{3ad} + \frac{b(2a^2-b^2)\log(\sin(c}{a^4d}$$

Mathematica [A] time = 0.28, size = 110, normalized size = 0.92

$$\frac{-2a^3b\csc^3(c+dx) + 3a^2b^2\csc^2(c+dx) + 6ab\left(2a^2-b^2\right)\csc(c+dx) - 6b^2\left(b^2-2a^2\right)\log(\sin(c+dx)) + 6\left(a^2-b\right.}{6a^4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (6*a*b*(2*a^2 - b^2)*Csc[c + d*x] + 3*a^2*b^2*Csc[c + d*x]^2 - 2*a^3*b*Csc[
c + d*x]^3 - 6*b^2*(-2*a^2 + b^2)*Log[Sin[c + d*x]] + 6*(a^2 - b^2)^2*Log[a
 + b*Sin[c + d*x]])/(6*a^4*b*d)
```

fricas [A] time = 0.89, size = 198, normalized size = 1.65

$$-\frac{3\,a^2b^2\sin(dx+c) + 10\,a^3b - 6\,ab^3 - 6\left(2\,a^3b - ab^3\right)\cos(dx+c)^2 + 6\left(a^4 - 2\,a^2b^2 + b^4 - \left(a^4 - 2\,a^2b^2 + b^4\right)\cos\right.}{6\left(a^4bd\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^2*b^2*sin(d*x + c) + 10*a^3*b - 6*a*b^3 - 6*(2*a^3*b - a*b^3)*cos
(d*x + c)^2 + 6*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x +
```

```
c)^2)*log(b*sin(d*x + c) + a)*sin(d*x + c) + 6*(2*a^2*b^2 - b^4 - (2*a^2*b^
2 - b^4)*cos(d*x + c)^2)*log(1/2*sin(d*x + c))*sin(d*x + c))/((a^4*b*d*cos(
d*x + c)^2 - a^4*b*d)*sin(d*x + c))
```

giac [A] time = 0.22, size = 151, normalized size = 1.26

$$\frac{\frac{6\left(2a^2b-b^3\right)\log(|\sin(dx+c)|)}{a^4}+\frac{6\left(a^4-2a^2b^2+b^4\right)\log(|b\sin(dx+c)+a|)}{a^4b}-\frac{22a^2b\sin(dx+c)^3-11b^3\sin(dx+c)^3-12a^3\sin(dx+c)^2+6ab^2\sin(dx+c)^2-3}{a^4\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(2*a^2*b - b^3)*log(abs(sin(d*x + c)))/a^4 + 6*(a^4 - 2*a^2*b^2 + b^
4)*log(abs(b*sin(d*x + c) + a))/(a^4*b) - (22*a^2*b*sin(d*x + c)^3 - 11*b^3
*sin(d*x + c)^3 - 12*a^3*sin(d*x + c)^2 + 6*a*b^2*sin(d*x + c)^2 - 3*a^2*b*
sin(d*x + c) + 2*a^3)/(a^4*sin(d*x + c)^3))/d
```

maple [A] time = 0.45, size = 163, normalized size = 1.36

$$\frac{\ln(a+b\sin(dx+c))}{bd}-\frac{2b\ln(a+b\sin(dx+c))}{a^2d}+\frac{b^3\ln(a+b\sin(dx+c))}{d\,a^4}-\frac{1}{3da\sin(dx+c)^3}+\frac{2}{da\sin(dx+c)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] ln(a+b*sin(d*x+c))/b/d-2*b*ln(a+b*sin(d*x+c))/a^2/d+1/d/a^4*b^3*ln(a+b*sin(
d*x+c))-1/3/d/a/sin(d*x+c)^3+2/d/a/sin(d*x+c)-1/d/a^3/sin(d*x+c)*b^2+2*b*ln
(sin(d*x+c))/a^2/d-1/d/a^4*b^3*ln(sin(d*x+c))+1/2/d/a^2*b/sin(d*x+c)^2
```

maxima [A] time = 0.30, size = 113, normalized size = 0.94

$$\frac{\frac{6\left(2a^2b-b^3\right)\log(\sin(dx+c))}{a^4}+\frac{6\left(a^4-2a^2b^2+b^4\right)\log(b\sin(dx+c)+a)}{a^4b}+\frac{3ab\sin(dx+c)+6\left(2a^2-b^2\right)\sin(dx+c)^2-2a^2}{a^3\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(2*a^2*b - b^3)*log(sin(d*x + c))/a^4 + 6*(a^4 - 2*a^2*b^2 + b^4)*lo
g(b*sin(d*x + c) + a)/(a^4*b) + (3*a*b*sin(d*x + c) + 6*(2*a^2 - b^2)*sin(d
*x + c)^2 - 2*a^2)/(a^3*sin(d*x + c)^3))/d
```

mupad [B] time = 11.93, size = 227, normalized size = 1.89

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{7}{8\,a}-\frac{b^2}{2\,a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{b\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2\,b-b^3\right)}{a^4\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(7/(8*a) - b^2/(2*a^3)))/d - tan(c/2 + (d*x)/2)^3/(24*a
*d) - log(tan(c/2 + (d*x)/2)^2 + 1)/(b*d) + (log(tan(c/2 + (d*x)/2))*(2*a^2
*b - b^3))/(a^4*d) + (b*tan(c/2 + (d*x)/2)^2)/(8*a^2*d) + (tan(c/2 + (d*x)/
2)^2*(7*a^2 - 4*b^2) - a^2/3 + a*b*tan(c/2 + (d*x)/2))/(8*a^3*d*tan(c/2 + (
d*x)/2)^3) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^2
 - b^2)^2)/(a^4*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1317 $\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{b\csc^3(c+dx)}{3a^2d}+\frac{\left(a^2-b^2\right)^2\log(\sin(c+dx))}{a^5d}-\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^5d}-\frac{b\left(2a^2-b^2\right)\csc(c+dx)}{a^4d}+\frac{\left(2a^2-\right.}{}$$

```
[Out] -b*(2*a^2-b^2)*csc(d*x+c)/a^4/d+1/2*(2*a^2-b^2)*csc(d*x+c)^2/a^3/d+1/3*b*cs
c(d*x+c)^3/a^2/d-1/4*csc(d*x+c)^4/a/d+(a^2-b^2)^2*ln(sin(d*x+c))/a^5/d-(a^2
-b^2)^2*ln(a+b*sin(d*x+c))/a^5/d
```

Rubi [A] time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$\frac{\left(2a^2-b^2\right)\csc^2(c+dx)}{2a^3d}-\frac{b\left(2a^2-b^2\right)\csc(c+dx)}{a^4d}+\frac{\left(a^2-b^2\right)^2\log(\sin(c+dx))}{a^5d}-\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*(2*a^2 - b^2)*Csc[c + d*x])/(a^4*d)) + ((2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^3*d) + (b*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(4*a*d) + ((a^2 -
 b^2)^2*Log[Sin[c + d*x]])/(a^5*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]]
)/(a^5*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^5(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{ax^5} - \frac{b^4}{a^2x^4} + \frac{-2a^2b^2+b^4}{a^3x^3} + \frac{2a^2b^2-b^4}{a^4x^2} + \frac{\left(a^2-b^2\right)^2}{a^5x} - \frac{\left(a^2-b^2\right)^2}{a^5(a+x)}\right) dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\left(2a^2-b^2\right)\csc(c+dx)}{a^4d} + \frac{\left(2a^2-b^2\right)\csc^2(c+dx)}{2a^3d} + \frac{b\csc^3(c+dx)}{3a^2d} - \frac{\csc^4(c+dx)}{4ad} +$$

Mathematica [A] time = 1.04, size = 115, normalized size = 0.78

$$\frac{-3a^4\csc^4(c+dx) + 4a^3b\csc^3(c+dx) + 6a^2\left(2a^2-b^2\right)\csc^2(c+dx) + 12ab\left(b^2-2a^2\right)\csc(c+dx) + 12\left(a^2-b^2\right)^2}{12a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] (12*a*b*(-2*a^2 + b^2)*Csc[c + d*x] + 6*a^2*(2*a^2 - b^2)*Csc[c + d*x]^2 +
4*a^3*b*Csc[c + d*x]^3 - 3*a^4*Csc[c + d*x]^4 + 12*(a^2 - b^2)^2*(Log[Sin[c
 + d*x]] - Log[a + b*Sin[c + d*x]]))/(12*a^5*d)
```

fricas [A] time = 0.71, size = 271, normalized size = 1.83

$$9a^4 - 6a^2b^2 - 6\left(2a^4 - a^2b^2\right)\cos(dx+c)^2 - 12\left(\left(a^4 - 2a^2b^2 + b^4\right)\cos(dx+c)^4 + a^4 - 2a^2b^2 + b^4 - 2\left(a^4 - 2a^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(9*a^4 - 6*a^2*b^2 - 6*(2*a^4 - a^2*b^2)*cos(d*x + c)^2 - 12*((a^4 - 2
*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2
 + b^4)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a) + 12*((a^4 - 2*a^2*b^2 + b^
4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d
*x + c)^2)*log(-1/2*sin(d*x + c)) - 4*(5*a^3*b - 3*a*b^3 - 3*(2*a^3*b - a*b
^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*x +
 c)^2 + a^5*d)
```

giac [A] time = 0.24, size = 201, normalized size = 1.36

$$\frac{\frac{12\left(a^4-2\,a^2b^2+b^4\right)\log(|\sin(dx+c)|)}{a^5}-\frac{12\left(a^4b-2\,a^2b^3+b^5\right)\log(|b\sin(dx+c)+a|)}{a^5b}-\frac{25\,a^4\sin(dx+c)^4-50\,a^2b^2\sin(dx+c)^4+25\,b^4\sin(dx+c)^4+24\,a^3b\sin}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(abs(sin(d*x + c)))/a^5 - 12*(a^4*b - 2
*a^2*b^3 + b^5)*log(abs(b*sin(d*x + c) + a))/(a^5*b) - (25*a^4*sin(d*x + c)
^4 - 50*a^2*b^2*sin(d*x + c)^4 + 25*b^4*sin(d*x + c)^4 + 24*a^3*b*sin(d*x +
 c)^3 - 12*a*b^3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 6*a^2*b^2*sin(d*x
 + c)^2 - 4*a^3*b*sin(d*x + c) + 3*a^4)/(a^5*sin(d*x + c)^4))/d
```

maple [A] time = 0.48, size = 216, normalized size = 1.46

$$-\frac{\ln\left(a+b\sin\left(dx+c\right)\right)}{da}+\frac{2b^2\ln\left(a+b\sin\left(dx+c\right)\right)}{a^3d}-\frac{\ln\left(a+b\sin\left(dx+c\right)\right)b^4}{d\,a^5}-\frac{1}{4da\sin\left(dx+c\right)^4}+\frac{1}{da\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/a*ln(a+b*sin(d*x+c))+2*b^2*ln(a+b*sin(d*x+c))/a^3/d-1/d/a^5*ln(a+b*sin
(d*x+c))*b^4-1/4/d/a/sin(d*x+c)^4+1/d/a/sin(d*x+c)^2-1/2/d/a^3/sin(d*x+c)^2
*b^2+ln(sin(d*x+c))/a/d-2*b^2*ln(sin(d*x+c))/a^3/d+1/d/a^5*ln(sin(d*x+c))*b
^4-2/d/a^2*b/sin(d*x+c)+1/d/a^4*b^3/sin(d*x+c)+1/3/d/a^2*b/sin(d*x+c)^3
```

maxima [A] time = 0.31, size = 139, normalized size = 0.94

$$-\frac{\frac{12\left(a^4-2\,a^2b^2+b^4\right)\log(b\sin(dx+c)+a)}{a^5}-\frac{12\left(a^4-2\,a^2b^2+b^4\right)\log(\sin(dx+c))}{a^5}-\frac{4\,a^2b\sin(dx+c)-12\left(2\,a^2b-b^3\right)\sin(dx+c)^3-3\,a^3+6\left(2\,a^3-ab^2\right)\sin(}{a^4\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/a^5 - 12*(a^4 - 2
*a^2*b^2 + b^4)*log(sin(d*x + c))/a^5 - (4*a^2*b*sin(d*x + c) - 12*(2*a^2*b
 - b^3)*sin(d*x + c)^3 - 3*a^3 + 6*(2*a^3 - a*b^2)*sin(d*x + c)^2)/(a^4*sin
(d*x + c)^4))/d
```

mupad [B] time = 11.83, size = 281, normalized size = 1.90

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{3}{16\,a}-\frac{b^2}{8\,a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{b}{8\,a^2}+\frac{2\,b\left(\frac{3}{8\,a}-\frac{b^2}{4\,a^3}\right)}{a}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a\,b^2-3\,a^3\right)+\mathrm{ta}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(3/(16*a) - b^2/(8*a^3)))/d - tan(c/2 + (d*x)/2)^4/(6
4*a*d) - (tan(c/2 + (d*x)/2)*(b/(8*a^2) + (2*b*(3/(8*a) - b^2/(4*a^3)))/a))
/d - (tan(c/2 + (d*x)/2)^2*(2*a*b^2 - 3*a^3) + tan(c/2 + (d*x)/2)^3*(14*a^2
*b - 8*b^3) + a^3/4 - (2*a^2*b*tan(c/2 + (d*x)/2))/3)/(16*a^4*d*tan(c/2 + (
d*x)/2)^4) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^4
 + b^4 - 2*a^2*b^2))/(a^5*d) + (b*tan(c/2 + (d*x)/2)^3)/(24*a^2*d) + (log(t
an(c/2 + (d*x)/2))*(a^4 + b^4 - 2*a^2*b^2))/(a^5*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1318 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=179

$$\frac{b\csc^4(c+dx)}{4a^2d}-\frac{b\left(a^2-b^2\right)^2\log(\sin(c+dx))}{a^6d}+\frac{b\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^6d}-\frac{\left(a^2-b^2\right)^2\csc(c+dx)}{a^5d}-\frac{b\left(2a^2\right.}{}$$

```
[Out] -(a^2-b^2)^2*csc(d*x+c)/a^5/d-1/2*b*(2*a^2-b^2)*csc(d*x+c)^2/a^4/d+1/3*(2*a
^2-b^2)*csc(d*x+c)^3/a^3/d+1/4*b*csc(d*x+c)^4/a^2/d-1/5*csc(d*x+c)^5/a/d-b*
(a^2-b^2)^2*ln(sin(d*x+c))/a^6/d+b*(a^2-b^2)^2*ln(a+b*sin(d*x+c))/a^6/d
```

Rubi [A] time = 0.20, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{\left(2a^2-b^2\right)\csc^3(c+dx)}{3a^3d}-\frac{b\left(2a^2-b^2\right)\csc^2(c+dx)}{2a^4d}-\frac{\left(a^2-b^2\right)^2\csc(c+dx)}{a^5d}-\frac{b\left(a^2-b^2\right)^2\log(\sin(c+dx))}{a^6d}+\frac{b\left(a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^2*Csc[c + d*x])/(a^5*d)) - (b*(2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + ((2*a^2 - b^2)*Csc[c + d*x]^3)/(3*a^3*d) + (b*Csc[c + d*x]^4)/(
4*a^2*d) - Csc[c + d*x]^5/(5*a*d) - (b*(a^2 - b^2)^2*Log[Sin[c + d*x]])/(a^
6*d) + (b*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a^6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^6\left(b^2-x^2\right)^2}{x^6(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^6(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \left(\frac{b^4}{ax^6} - \frac{b^4}{a^2x^5} + \frac{-2a^2b^2+b^4}{a^3x^4} + \frac{2a^2b^2-b^4}{a^4x^3} + \frac{\left(a^2-b^2\right)^2}{a^5x^2} - \frac{\left(a^2-b^2\right)^2}{a^6x} + \frac{\left(a^2-b^2\right)^2}{a^6(a+x)}\right) dx,\right.}{d}$$

$$= -\frac{\left(a^2-b^2\right)^2\csc(c+dx)}{a^5 d} - \frac{b\left(2a^2-b^2\right)\csc^2(c+dx)}{2a^4 d} + \frac{\left(2a^2-b^2\right)\csc^3(c+dx)}{3a^3 d} +$$

Mathematica [A] time = 6.12, size = 179, normalized size = 1.00

$$\frac{b\csc^4(c+dx)}{4a^2d} - \frac{b\left(a^2-b^2\right)^2\log(\sin(c+dx))}{a^6d} + \frac{b\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^6d} - \frac{\left(a^2-b^2\right)^2\csc(c+dx)}{a^5d} - \frac{b\left(2a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^2*Csc[c + d*x])/(a^5*d)) - (b*(2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + ((2*a^2 - b^2)*Csc[c + d*x]^3)/(3*a^3*d) + (b*Csc[c + d*x]^4)/(
4*a^2*d) - Csc[c + d*x]^5/(5*a*d) - (b*(a^2 - b^2)^2*Log[Sin[c + d*x]])/(a^
6*d) + (b*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a^6*d)
```

fricas [B] time = 1.00, size = 346, normalized size = 1.93

$$-\frac{32a^5 - 100a^3b^2 + 60ab^4 + 60\left(a^5 - 2a^3b^2 + ab^4\right)\cos(dx+c)^4 - 20\left(4a^5 - 11a^3b^2 + 6ab^4\right)\cos(dx+c)^2 - 60}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(32*a^5 - 100*a^3*b^2 + 60*a*b^4 + 60*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d
*x + c)^4 - 20*(4*a^5 - 11*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^2 - 60*(a^4*b -
2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^4 - 2*(a^4*b - 2*a
^2*b^3 + b^5)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a)*sin(d*x + c) + 60*(a^
4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^4 - 2*(a^4*b
 - 2*a^2*b^3 + b^5)*cos(d*x + c)^2)*log(1/2*sin(d*x + c))*sin(d*x + c) + 15
*(3*a^4*b - 2*a^2*b^3 - 2*(2*a^4*b - a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))
/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))
```

giac [A] time = 0.21, size = 251, normalized size = 1.40

$$-\frac{\frac{60\left(a^4b-2a^2b^3+b^5\right)\log(|\sin(dx+c)|)}{a^6}-\frac{60\left(a^4b^2-2a^2b^4+b^6\right)\log(|b\sin(dx+c)+a|)}{a^6b}-\frac{137a^4b\sin(dx+c)^5-274a^2b^3\sin(dx+c)^5+137b^5\sin(dx+c)^5-}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*(a^4*b - 2*a^2*b^3 + b^5)*log(abs(sin(d*x + c)))/a^6 - 60*(a^4*b^
2 - 2*a^2*b^4 + b^6)*log(abs(b*sin(d*x + c) + a))/(a^6*b) - (137*a^4*b*sin(
d*x + c)^5 - 274*a^2*b^3*sin(d*x + c)^5 + 137*b^5*sin(d*x + c)^5 - 60*a^5*s
in(d*x + c)^4 + 120*a^3*b^2*sin(d*x + c)^4 - 60*a*b^4*sin(d*x + c)^4 - 60*a
^4*b*sin(d*x + c)^3 + 30*a^2*b^3*sin(d*x + c)^3 + 40*a^5*sin(d*x + c)^2 - 2
0*a^3*b^2*sin(d*x + c)^2 + 15*a^4*b*sin(d*x + c) - 12*a^5)/(a^6*sin(d*x + c
)^5))/d
```

maple [A] time = 0.49, size = 274, normalized size = 1.53

$$\frac{b\ln\left(a+b\sin\left(dx+c\right)\right)}{a^2d}-\frac{2b^3\ln\left(a+b\sin\left(dx+c\right)\right)}{d\,a^4}+\frac{b^5\ln\left(a+b\sin\left(dx+c\right)\right)}{d\,a^6}-\frac{1}{5da\sin\left(dx+c\right)^5}+\frac{2}{3da\sin\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] b*ln(a+b*sin(d*x+c))/a^2/d-2/d/a^4*b^3*ln(a+b*sin(d*x+c))+1/d/a^6*b^5*ln(a+
b*sin(d*x+c))-1/5/d/a/sin(d*x+c)^5+2/3/d/a/sin(d*x+c)^3-1/3/d/a^3/sin(d*x+c
)^3*b^2-1/d/a/sin(d*x+c)+2/d/a^3/sin(d*x+c)*b^2-1/d/a^5/sin(d*x+c)*b^4-1/d/
a^2*b/sin(d*x+c)^2+1/2/d/a^4*b^3/sin(d*x+c)^2+1/4/d/a^2*b/sin(d*x+c)^4-b*ln
(sin(d*x+c))/a^2/d+2/d/a^4*b^3*ln(sin(d*x+c))-1/d/a^6*b^5*ln(sin(d*x+c))
```

maxima [A] time = 0.31, size = 170, normalized size = 0.95

$$\frac{\frac{60\left(a^4b-2a^2b^3+b^5\right)\log(b\sin(dx+c)+a)}{a^6}-\frac{60\left(a^4b-2a^2b^3+b^5\right)\log(\sin(dx+c))}{a^6}+\frac{15a^3b\sin(dx+c)-60\left(a^4-2a^2b^2+b^4\right)\sin(dx+c)^4-12a^4-30\left(2a^3b-\right.}{a^5\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*(a^4*b - 2*a^2*b^3 + b^5)*log(b*sin(d*x + c) + a)/a^6 - 60*(a^4*b
- 2*a^2*b^3 + b^5)*log(sin(d*x + c))/a^6 + (15*a^3*b*sin(d*x + c) - 60*(a^4
 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 - 12*a^4 - 30*(2*a^3*b - a*b^3)*sin(d*x
+ c)^3 + 20*(2*a^4 - a^2*b^2)*sin(d*x + c)^2)/(a^5*sin(d*x + c)^5))/d
```

mupad [B] time = 11.92, size = 381, normalized size = 2.13

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{b^2}{8\,a^3}-\frac{5}{16\,a}+\frac{2\,b\left(\frac{b}{16\,a^2}+\frac{2\,b\left(\frac{5}{32\,a}-\frac{b^2}{8\,a^3}\right)}{a}\right)}{a}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{b}{32\,a^2}+\frac{b\left(\frac{5}{32\,a}-\frac{b^2}{8\,a^3}\right)}{a}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,a\,d}+\frac{\tan\left(\frac{c}{2}+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^6*(a + b*sin(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(b^2/(8*a^3) - 5/(16*a) + (2*b*(b/(16*a^2) + (2*b*(5/(3
2*a) - b^2/(8*a^3)))/a))/a))/d - (tan(c/2 + (d*x)/2)^2*(b/(32*a^2) + (b*(5/
(32*a) - b^2/(8*a^3)))/a))/d - tan(c/2 + (d*x)/2)^5/(160*a*d) + (tan(c/2 +
(d*x)/2)^3*(5/(96*a) - b^2/(24*a^3)))/d + (log(a + 2*b*tan(c/2 + (d*x)/2) +
 a*tan(c/2 + (d*x)/2)^2)*(a^4*b + b^5 - 2*a^2*b^3))/(a^6*d) + (b*tan(c/2 +
(d*x)/2)^4)/(64*a^2*d) - (log(tan(c/2 + (d*x)/2))*(a^4*b + b^5 - 2*a^2*b^3)
)/(a^6*d) + (tan(c/2 + (d*x)/2)^2*((5*a^4)/3 - (4*a^2*b^2)/3) - a^4/5 - tan
(c/2 + (d*x)/2)^4*(10*a^4 + 16*b^4 - 28*a^2*b^2) + tan(c/2 + (d*x)/2)^3*(4*
a*b^3 - 6*a^3*b) + (a^3*b*tan(c/2 + (d*x)/2))/2)/(32*a^5*d*tan(c/2 + (d*x)/
2)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1319 $\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=212

$$\frac{b\csc^5(c+dx)}{5a^2d}+\frac{b^2\left(a^2-b^2\right)^2\log(\sin(c+dx))}{a^7d}-\frac{b^2\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^7d}+\frac{b\left(a^2-b^2\right)^2\csc(c+dx)}{a^6d}-\frac{(a$$

```
[Out] b*(a^2-b^2)^2*csc(d*x+c)/a^6/d-1/2*(a^2-b^2)^2*csc(d*x+c)^2/a^5/d-1/3*b*(2*
a^2-b^2)*csc(d*x+c)^3/a^4/d+1/4*(2*a^2-b^2)*csc(d*x+c)^4/a^3/d+1/5*b*csc(d*
x+c)^5/a^2/d-1/6*csc(d*x+c)^6/a/d+b^2*(a^2-b^2)^2*ln(sin(d*x+c))/a^7/d-b^2*
(a^2-b^2)^2*ln(a+b*sin(d*x+c))/a^7/d
```

Rubi [A] time = 0.24, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$\frac{\left(2a^2-b^2\right)\csc^4(c+dx)}{4a^3d}-\frac{b\left(2a^2-b^2\right)\csc^3(c+dx)}{3a^4d}-\frac{\left(a^2-b^2\right)^2\csc^2(c+dx)}{2a^5d}+\frac{b\left(a^2-b^2\right)^2\csc(c+dx)}{a^6d}+\frac{b^2\left(a^2-b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*(a^2 - b^2)^2*Csc[c + d*x])/(a^6*d) - ((a^2 - b^2)^2*Csc[c + d*x]^2)/(2*
a^5*d) - (b*(2*a^2 - b^2)*Csc[c + d*x]^3)/(3*a^4*d) + ((2*a^2 - b^2)*Csc[c
+ d*x]^4)/(4*a^3*d) + (b*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6/(6*a*d)
 + (b^2*(a^2 - b^2)^2*Log[Sin[c + d*x]])/(a^7*d) - (b^2*(a^2 - b^2)^2*Log[a
 + b*Sin[c + d*x]])/(a^7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^7\left(b^2-x^2\right)^2}{x^7(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{b^2\,\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^7(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\,\text{Subst}\left(\int \left(\frac{b^4}{ax^7}-\frac{b^4}{a^2x^6}+\frac{-2a^2b^2+b^4}{a^3x^5}+\frac{2a^2b^2-b^4}{a^4x^4}+\frac{\left(a^2-b^2\right)^2}{a^5x^3}-\frac{\left(a^2-b^2\right)^2}{a^6x^2}+\frac{\left(a^2-b^2\right)^2}{a^7x}-\frac{(}{}\right.\right.}{d}$$

$$= \frac{b\left(a^2-b^2\right)^2\csc(c+dx)}{a^6 d}-\frac{\left(a^2-b^2\right)^2\csc^2(c+dx)}{2a^5 d}-\frac{b\left(2a^2-b^2\right)\csc^3(c+dx)}{3a^4 d}+$$

Mathematica [A] time = 2.86, size = 165, normalized size = 0.78

$$\frac{-10a^6\csc^6(c+dx)+12a^5b\csc^5(c+dx)+60\left(b^3-a^2b\right)^2\left(\log(\sin(c+dx))-\log(a+b\sin(c+dx))\right)-30a^2\left(a^2-\right.}{60a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (60*a*b*(a^2 - b^2)^2*Csc[c + d*x] - 30*a^2*(a^2 - b^2)^2*Csc[c + d*x]^2 +
20*a^3*b*(-2*a^2 + b^2)*Csc[c + d*x]^3 + 15*a^4*(2*a^2 - b^2)*Csc[c + d*x]^
4 + 12*a^5*b*Csc[c + d*x]^5 - 10*a^6*Csc[c + d*x]^6 + 60*(-(a^2*b) + b^3)^2
*(Log[Sin[c + d*x]] - Log[a + b*Sin[c + d*x]]))/(60*a^7*d)
```

fricas [B] time = 0.74, size = 464, normalized size = 2.19

$$\frac{10\,a^6-45\,a^4b^2+30\,a^2b^4+30\left(a^6-2\,a^4b^2+a^2b^4\right)\cos\left(dx+c\right)^4-15\left(2\,a^6-7\,a^4b^2+4\,a^2b^4\right)\cos\left(dx+c\right)^2-60\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(10*a^6 - 45*a^4*b^2 + 30*a^2*b^4 + 30*(a^6 - 2*a^4*b^2 + a^2*b^4)*cos
(d*x + c)^4 - 15*(2*a^6 - 7*a^4*b^2 + 4*a^2*b^4)*cos(d*x + c)^2 - 60*((a^4*
b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^6 - a^4*b^2 + 2*a^2*b^4 - b^6 - 3*(a^4*
b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d
*x + c)^2)*log(b*sin(d*x + c) + a) + 60*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*
x + c)^6 - a^4*b^2 + 2*a^2*b^4 - b^6 - 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*
x + c)^4 + 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2)*log(-1/2*sin(d*x +
 c)) - 4*(8*a^5*b - 25*a^3*b^3 + 15*a*b^5 + 15*(a^5*b - 2*a^3*b^3 + a*b^5)*
cos(d*x + c)^4 - 5*(4*a^5*b - 11*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2)*sin(d*x
 + c))/(a^7*d*cos(d*x + c)^6 - 3*a^7*d*cos(d*x + c)^4 + 3*a^7*d*cos(d*x + c
)^2 - a^7*d)
```

giac [A] time = 0.23, size = 301, normalized size = 1.42

$$\frac{\frac{60\left(a^4b^2-2\,a^2b^4+b^6\right)\log(|\sin(dx+c)|)}{a^7}-\frac{60\left(a^4b^3-2\,a^2b^5+b^7\right)\log(|b\sin(dx+c)+a|)}{a^7b}-\frac{147\,a^4b^2\sin(dx+c)^6-294\,a^2b^4\sin(dx+c)^6+147\,b^6\sin(dx+c)^6-}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*(a^4*b^2 - 2*a^2*b^4 + b^6)*log(abs(sin(d*x + c)))/a^7 - 60*(a^4*b
^3 - 2*a^2*b^5 + b^7)*log(abs(b*sin(d*x + c) + a))/(a^7*b) - (147*a^4*b^2*s
in(d*x + c)^6 - 294*a^2*b^4*sin(d*x + c)^6 + 147*b^6*sin(d*x + c)^6 - 60*a^
5*b*sin(d*x + c)^5 + 120*a^3*b^3*sin(d*x + c)^5 - 60*a*b^5*sin(d*x + c)^5 +
 30*a^6*sin(d*x + c)^4 - 60*a^4*b^2*sin(d*x + c)^4 + 30*a^2*b^4*sin(d*x + c
)^4 + 40*a^5*b*sin(d*x + c)^3 - 20*a^3*b^3*sin(d*x + c)^3 - 30*a^6*sin(d*x
+ c)^2 + 15*a^4*b^2*sin(d*x + c)^2 - 12*a^5*b*sin(d*x + c) + 10*a^6)/(a^7*s
in(d*x + c)^6))/d
```

maple [A] time = 0.55, size = 330, normalized size = 1.56

$$-\frac{b^2\ln(a+b\sin(dx+c))}{a^3d}+\frac{2\ln(a+b\sin(dx+c))\,b^4}{d\,a^5}-\frac{b^6\ln(a+b\sin(dx+c))}{d\,a^7}-\frac{1}{6da\sin(dx+c)^6}+\frac{1}{2da\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7/(a+b*sin(d*x+c)),x)
```

```
[Out] -b^2*ln(a+b*sin(d*x+c))/a^3/d+2/d/a^5*ln(a+b*sin(d*x+c))*b^4-1/d/a^7*b^6*ln
(a+b*sin(d*x+c))-1/6/d/a/sin(d*x+c)^6+1/2/d/a/sin(d*x+c)^4-1/4/d/a^3/sin(d*
x+c)^4*b^2-1/2/d/a/sin(d*x+c)^2+1/d/a^3/sin(d*x+c)^2*b^2-1/2/d/a^5/sin(d*x+
c)^2*b^4-2/3/d/a^2*b/sin(d*x+c)^3+1/3/d/a^4*b^3/sin(d*x+c)^3+b^2*ln(sin(d*x
+c))/a^3/d-2/d/a^5*ln(sin(d*x+c))*b^4+1/d/a^7*b^6*ln(sin(d*x+c))+1/5/d/a^2*
```

```
b/sin(d*x+c)^5+1/d/a^2*b/sin(d*x+c)-2/d/a^4*b^3/sin(d*x+c)+1/d/a^6*b^5/sin(
d*x+c)
```

maxima [A] time = 0.32, size = 206, normalized size = 0.97

$$-\frac{\frac{60\left(a^4b^2-2a^2b^4+b^6\right)\log(b\sin(dx+c)+a)}{a^7}-\frac{60\left(a^4b^2-2a^2b^4+b^6\right)\log(\sin(dx+c))}{a^7}-\frac{12a^4b\sin(dx+c)+60\left(a^4b-2a^2b^3+b^5\right)\sin(dx+c)^5-10a^5-30\left(a^5\cdots\right.}{\cdots}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(60*(a^4*b^2 - 2*a^2*b^4 + b^6)*log(b*sin(d*x + c) + a)/a^7 - 60*(a^4
*b^2 - 2*a^2*b^4 + b^6)*log(sin(d*x + c))/a^7 - (12*a^4*b*sin(d*x + c) + 60
*(a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)^5 - 10*a^5 - 30*(a^5 - 2*a^3*b^2 +
a*b^4)*sin(d*x + c)^4 - 20*(2*a^4*b - a^2*b^3)*sin(d*x + c)^3 + 15*(2*a^5 -
 a^3*b^2)*sin(d*x + c)^2)/(a^6*sin(d*x + c)^6))/d
```

mupad [B] time = 12.31, size = 514, normalized size = 2.42

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{1}{64a}-\frac{b^2}{64a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{b}{96a^2}+\frac{2b\left(\frac{1}{16a}-\frac{b^2}{16a^3}\right)}{3a}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{384ad}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{b}{32a^2}-\frac{2b\left(\frac{b^2}{16a^3}\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^7*(a + b*sin(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4*(1/(64*a) - b^2/(64*a^3)))/d - (tan(c/2 + (d*x)/2)^3*
(b/(96*a^2) + (2*b*(1/(16*a) - b^2/(16*a^3)))/(3*a)))/d - tan(c/2 + (d*x)/2
)^6/(384*a*d) + (tan(c/2 + (d*x)/2)*(b/(32*a^2) - (2*b*(b^2/(16*a^3) - 5/(6
4*a) + (2*b*(b/(32*a^2) + (2*b*(1/(16*a) - b^2/(16*a^3)))/a))/a))/a + (2*b*
(1/(16*a) - b^2/(16*a^3)))/a))/d + (tan(c/2 + (d*x)/2)^2*(b^2/(32*a^3) - 5/
(128*a) + (b*(b/(32*a^2) + (2*b*(1/(16*a) - b^2/(16*a^3)))/a))/a))/d + (log
(tan(c/2 + (d*x)/2))*(b^6 - 2*a^2*b^4 + a^4*b^2))/(a^7*d) + (b*tan(c/2 + (d
*x)/2)^5)/(160*a^2*d) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)
```

```
/2)^2)*(b^6 - 2*a^2*b^4 + a^4*b^2))/(a^7*d) - (tan(c/2 + (d*x)/2)^3*((10*a^
4*b)/3 - (8*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^4*(8*a*b^4 + (5*a^5)/2 - 12*a^
3*b^2) - tan(c/2 + (d*x)/2)^5*(20*a^4*b + 32*b^5 - 56*a^2*b^3) - tan(c/2 +
(d*x)/2)^2*(a^5 - a^3*b^2) + a^5/6 - (2*a^4*b*tan(c/2 + (d*x)/2))/5)/(64*a^
6*d*tan(c/2 + (d*x)/2)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1320 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=467

$$-\frac{b\sin^5(c+dx)\cos(c+dx)}{5a^2d}-\frac{\left(28a^4-60a^2b^2+35b^4\right)\sin^4(c+dx)\cos(c+dx)}{140ab^4d}-\frac{a\left(35a^4-77a^2b^2+45b^4\right)\sin^2(c+$$

```
[Out] -1/128*(128*a^8-320*a^6*b^2+240*a^4*b^4-40*a^2*b^6-5*b^8)*x/b^9+2*a^3*(a^2-
b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^9/d-1/105*a*(
105*a^6-245*a^4*b^2+161*a^2*b^4-15*b^6)*cos(d*x+c)/b^8/d+1/128*(64*a^6-144*
a^4*b^2+88*a^2*b^4-5*b^6)*cos(d*x+c)*sin(d*x+c)/b^7/d-1/105*a*(35*a^4-77*a^
2*b^2+45*b^4)*cos(d*x+c)*sin(d*x+c)^2/b^6/d+1/192*(48*a^4-104*a^2*b^2+59*b^
4)*cos(d*x+c)*sin(d*x+c)^3/b^5/d+1/4*cos(d*x+c)*sin(d*x+c)^4/a/d-1/140*(28*
a^4-60*a^2*b^2+35*b^4)*cos(d*x+c)*sin(d*x+c)^4/a/b^4/d-1/5*b*cos(d*x+c)*sin
(d*x+c)^5/a^2/d+1/240*(40*a^4-85*a^2*b^2+48*b^4)*cos(d*x+c)*sin(d*x+c)^5/a^
2/b^3/d-1/7*a*cos(d*x+c)*sin(d*x+c)^6/b^2/d+1/8*cos(d*x+c)*sin(d*x+c)^7/b/d
```

Rubi [A] time = 1.81, antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2896, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{a\left(-245a^4b^2+161a^2b^4+105a^6-15b^6\right)\cos(c+dx)}{105b^8d}+\frac{2a^3\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^9d}+\frac{\left(-85a^2b^2+40a^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((128*a^8 - 320*a^6*b^2 + 240*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*x)/(128*b^9) +
 (2*a^3*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(b^9*d) - (a*(105*a^6 - 245*a^4*b^2 + 161*a^2*b^4 - 15*b^6)*Cos[c + d*x])/
(105*b^8*d) + ((64*a^6 - 144*a^4*b^2 + 88*a^2*b^4 - 5*b^6)*Cos[c + d*x]*Sin
[c + d*x])/(128*b^7*d) - (a*(35*a^4 - 77*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sin
[c + d*x]^2)/(105*b^6*d) + ((48*a^4 - 104*a^2*b^2 + 59*b^4)*Cos[c + d*x]*Si
n[c + d*x]^3)/(192*b^5*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*a*d) - ((28*a^
4 - 60*a^2*b^2 + 35*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(140*a*b^4*d) - (b*Co
s[c + d*x]*Sin[c + d*x]^5)/(5*a^2*d) + ((40*a^4 - 85*a^2*b^2 + 48*b^4)*Cos[
c + d*x]*Sin[c + d*x]^5)/(240*a^2*b^3*d) - (a*Cos[c + d*x]*Sin[c + d*x]^6)/
(7*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^7)/(8*b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\cos(c+dx)\sin^4(c+dx)}{4ad} - \frac{b\cos(c+dx)\sin^5(c+dx)}{5a^2d} - \frac{a\cos(c+dx)\sin^6(c+}{7b^2d}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad} - \frac{b\cos(c+dx)\sin^5(c+dx)}{5a^2d} + \frac{\left(40a^4-85a^2b^2+48\right.}{2}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad} - \frac{\left(28a^4-60a^2b^2+35b^4\right)\cos(c+dx)\sin^4(c+dx)}{140ab^4d}$$

$$= \frac{\left(48a^4-104a^2b^2+59b^4\right)\cos(c+dx)\sin^3(c+dx)}{192b^5d} + \frac{\cos(c+dx)\sin^4(c+dx)}{4ad}$$

$$= -\frac{a\left(35a^4-77a^2b^2+45b^4\right)\cos(c+dx)\sin^2(c+dx)}{105b^6d} + \frac{\left(48a^4-104a^2b^2+59b\right.}{1}$$

$$= \frac{\left(64a^6-144a^4b^2+88a^2b^4-5b^6\right)\cos(c+dx)\sin(c+dx)}{128b^7d} - \frac{a\left(35a^4-77a^2b^2\right.}{}$$

$$= -\frac{a\left(105a^6-245a^4b^2+161a^2b^4-15b^6\right)\cos(c+dx)}{105b^8d} + \frac{\left(64a^6-144a^4b^2+88a\right.}{}$$

$$= -\frac{\left(128a^8-320a^6b^2+240a^4b^4-40a^2b^6-5b^8\right)x}{128b^9} - \frac{a\left(105a^6-245a^4b^2+161\right.}{105b}$$

$$= -\frac{\left(128a^8-320a^6b^2+240a^4b^4-40a^2b^6-5b^8\right)x}{128b^9} - \frac{a\left(105a^6-245a^4b^2+161\right.}{105b}$$

$$= -\frac{\left(128a^8-320a^6b^2+240a^4b^4-40a^2b^6-5b^8\right)x}{128b^9} - \frac{a\left(105a^6-245a^4b^2+161\right.}{105b}$$

$$= -\frac{\left(128a^8-320a^6b^2+240a^4b^4-40a^2b^6-5b^8\right)x}{128b^9} + \frac{2a^3\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+}{}\right.}{b^9d}$$

Mathematica [A] time = 3.32, size = 403, normalized size = 0.86

$$\frac{-107520a^8c-107520a^8dx+26880a^6b^2\sin(2(c+dx))+268800a^6b^2c+268800a^6b^2dx-53760a^4b^4\sin(2(c+}{}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]

[Out] (-107520*a^8*c + 268800*a^6*b^2*c - 201600*a^4*b^4*c + 33600*a^2*b^6*c + 4200*b^8*c - 107520*a^8*d*x + 268800*a^6*b^2*d*x - 201600*a^4*b^4*d*x + 33600*a^2*b^6*d*x + 4200*b^8*d*x + 215040*a^3*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 1680*a*b*(64*a^6 - 144*a^4*b^2 + 88*a^2*b^4 - 5*b^6)*Cos[c + d*x] + 560*(16*a^5*b^3 - 28*a^3*b^5 + 9*a*b^7)*Cos[3*(c + d*x)] - 1344*a^3*b^5*Cos[5*(c + d*x)] + 1680*a*b^7*Cos[5*(c + d*x)] + 240*a*b^7*Cos[7*(c + d*x)] + 26880*a^6*b^2*Sin[2*(c + d*x)] - 53760*a^4*b^4*Sin[2*(c + d*x)] + 25200*a^2*b^6*Sin[2*(c + d*x)] + 1680*b^8*Sin[2*(c + d*x)] - 3360*a^4*b^4*Sin[4*(c + d*x)] + 5040*a^2*b^6*Sin[4*(c + d*x)] - 840*b^8*Sin[4*(c + d*x)] + 560*a^2*b^6*Sin[6*(c + d*x)] - 560*b^8*Sin[6*(c + d*x)] - 105*b^8*Sin[8*(c + d*x)])/(107520*b^9*d)

fricas [A] time = 0.96, size = 706, normalized size = 1.51

$$\left[\frac{1920\, a b^{7} \cos\left(d x + c\right)^{7} - 2688\, a^{3} b^{5} \cos\left(d x + c\right)^{5} + 4480\, \left(a^{5} b^{3} - a^{3} b^{5}\right) \cos\left(d x + c\right)^{3} - 105\, \left(128\, a^{8} - 320\, a^{6} b^{2} + 2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] [1/13440*(1920*a*b^7*cos(d*x + c)^7 - 2688*a^3*b^5*cos(d*x + c)^5 + 4480*(a^5*b^3 - a^3*b^5)*cos(d*x + c)^3 - 105*(128*a^8 - 320*a^6*b^2 + 240*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*d*x + 6720*(a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 13440*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c) - 35*(48*b^8*cos(d*x + c)^7 - 8*(8*a^2*b^6 + b^8)*cos(d*x + c)^5 + 2*(48*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*cos(d*x + c)^3 - 3*(64*a^6*b^2 - 112*a^4*b^4 + 40*a^2*b^6 + 5*b^8)*cos(d*x + c))*sin(d*x + c))/(b^9*d), 1/13440*(1920*a*b^7*cos(d*x + c)^7 - 2688*a^3*b^5*cos(d*x + c)^5 + 4480*(a^5*b^3 - a^3*b^5)*cos(d*x + c)^3 - 105*(128*a^8 - 320*a^6*b^2 + 240*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*d*x - 13440*(a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 13440*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c) - 35*(48*b^8*cos(d*x + c)^7 - 8*(8*a^2*b^6 + b^8)*cos(d*x + c)^5 + 2*(48*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*cos(d*x + c)^3 - 3*(64*a^6*b^2 - 112*a^4*b^4 + 40*a^2*b^6 + 5*b^8)*cos(d*x + c))*sin(d*x + c))/(b^9*d)]

giac [B] time = 0.21, size = 1244, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] -1/13440*(105*(128*a^8 - 320*a^6*b^2 + 240*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*(d*x + c)/b^9 - 26880*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^9) + 2*(6720*a^6*b*tan(1/2*d*x + 1/2*c)^15 - 15120*a^4*b^3*tan(1/2*d*x + 1/2*c)^15 + 9240*a^2*b^5*tan(1/2*d*x + 1/2*c)^15 - 525*b^7*tan(1/2*d*x + 1/2*c)^15 + 13440*a^7*tan(1/2*d*x + 1/2*c)^14 - 40320*a^5*b^2*tan(1/2*d*x + 1/2*c)^14 + 40320*a^3*b^4*tan(1/2*d*x + 1/2*c)^14 - 13440*a*b^6*tan(1/2*d*x + 1/2*c)^14 + 33600*a^6*b*tan(1/2*d*x + 1/2*c)^13 - 62160*a^4*b^3*tan(1/2*d*x + 1/2*c)^13 + 17080*a^2*b^5*tan(1/2*d*x + 1/2*c)^13 + 13895*b^7*tan(1/2*d*x + 1/2*c)^13 + 94080*a^7*tan(1/2*d*x + 1/2*c)^12 - 255360*a^5*b^2*tan(1/2*d*x + 1/2*c)^12 + 201600*a^3*b^4*tan(1/2*d*x + 1/2*c)^12 - 13440*a*b^6*tan(1/2*d*x + 1/2*c)^12 + 60480*a^6*b*tan(1/2*d*x + 1/2*c)^11 - 95760*a^4*b^3*tan(1/2*d*x + 1/2*c)^11 + 31640*a^2*b^5*tan(1/2*d*x + 1/2*c)^11 - 31325*b^7*tan(1/2*d*x + 1/2*c)^11 + 282240*a^7*tan(1/2*d*x + 1/2*c)^10 - 703360*a^5*b^2*tan(1/2*d*x + 1/2*c)^10 + 488320*a^3*b^4*tan(1/2*d*x + 1/2*c)^10 - 67200*a*b^6*tan(1/2*d*x + 1/2*c)^10 + 33600*a^6*b*tan(1/2*d*x + 1/2*c)^9 - 48720*a^4*b^3*tan(1/2*d*x + 1/2*c)^9 + 23800*a^2*b^5*tan(1/2*d*x + 1/2*c)^9 + 61775*b^7*tan(1/2*d*x + 1/2*c)^9 + 470400*a^7*tan(1/2*d*x + 1/2*c)^8 - 1097600*a^5*b^2*tan(1/2*d*x + 1/2*c)^8 + 721280*a^3*b^4*tan(1/2*d*x + 1/2*c)^8 - 67200*a*b^6*tan(1/2*d*x + 1/2*c)^8 - 33600*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 48720*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 23800*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 61775*b^7*tan(1/2*d*x + 1/2*c)^7 + 470400*a^7*tan(1/2*d*x + 1/2*c)^6 - 1052800*a^5*b^2*tan(1/2*d*x + 1/2*c)^6 + 665728*a^3*b^4*tan(1/2*d*x + 1/2*c)^6 - 40320*a*b^6*tan(1/2*d*x + 1/2*c)^6 - 60480*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 95760*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 31640*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 31325*b^7*tan(1/2*d*x + 1/2*c)^5 + 282240*a^7*tan(1/2*d*x + 1/2*c)^4 - 622720*a^5*b^2*tan(1/2*d*x + 1/2*c)^4 + 375424*a^3*b^4*tan(1/2*d*x + 1/2*c)^4 - 40320*a*b^6*tan(1/2*d*x + 1/2*c)^4 - 33600*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 62160*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 17080*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 13895*b^7*tan(1/2*d*x + 1/2*c)^3 + 94080*a^7*tan(1/2*d*x + 1/2*c)^2 - 210560*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 + 124544*a^3*b^4*tan(1/2*d*x + 1/2*c)^2 - 1920*a*b^6*tan(1/2*d*x + 1/2*c)^2 - 6720*a^6*b*tan(1/2*d*x + 1/2*c) + 15120*a^4*b^3*tan(1/2*d*x + 1/2*c) - 9240*a^2*b^5*tan(1/2*d*x + 1/2*c) + 525*b^7*tan(1/2*d*x + 1/2*c) + 13440*a^7 - 31360*a^5*b^2 + 20608*a^3*b^4 - 1920*a*b^6)/((tan(1/2*d*x + 1/2*c)^2 + 1)^8*b^8))/d

maple [B] time = 0.32, size = 2587, normalized size = 5.54

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)

[Out] 5/8/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2+5/64/d/b*arctan(tan(1/2*d*x+1/2*c))+29/4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^9*a^4-14/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^12*a^7+38/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^12*a^5-30/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^12*a^3+11/8/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)*a^2-14/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^2*a^7-2/d*a^3/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-15/4/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^4+5/d/b^7*arctan(tan(1/2*d*x+1/2*c))*a^6-2/d/b^9*arctan(tan(1/2*d*x+1/2*c))*a^8-1765/192/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^9+895/192/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11-397/192/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13+5/64/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15-46/15/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^8*a^3+2/7/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^8*a-2/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^8*a^7+14/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^8*a^5-5/64/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)+397/192/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3-895/192/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5+1765/192/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^7+113/24/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5*a^2-70/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^6*a^7+61/24/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3*a^2+94/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^2*a^5-85/24/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^9*a^2-42/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^10*a^7+314/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^10*a^5-218/3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^10*a^3+10/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^10*a+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^12*a-9/4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)*a^4-278/15/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^2*a^3+6/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^14*a^5-6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^14*a^3+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^14*a+2/7/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^2*a+490/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^8*a^5-70/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^8*a^7-42/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^4*a^7-1486/15/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^6*a^3+6/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^6*a+5/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^7*a^6-29/4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^7*a^4+85/24/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^7*a^2+2/d/b^9*a^9/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-322/3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^8*a^3+10/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^8*a+1/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)*a^6+9/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5*a^6-6/d*a^7/b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d*a^5/b^5/(a^2-b^2)^(1/2)*ar

```
ctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-11/8/d/b^3/(1+tan(1/
2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15*a^2+5/d/b^7/(1+tan(1/2*d*x+1/2*c)^2
)^8*tan(1/2*d*x+1/2*c)^3*a^6-37/4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*
d*x+1/2*c)^3*a^4-5/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^9*a^
6-57/4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5*a^4-2/d/b^8/(1
+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^14*a^7+470/3/d/b^6/(1+tan(1/2*d
*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^6*a^5-61/24/d/b^3/(1+tan(1/2*d*x+1/2*c)^2
)^8*tan(1/2*d*x+1/2*c)^13*a^2-1/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*
x+1/2*c)^15*a^6+9/4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15*
a^4-5/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13*a^6+37/4/d/b^5
/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13*a^4+6/d/b^2/(1+tan(1/2*d*
x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^4*a+278/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^8
*tan(1/2*d*x+1/2*c)^4*a^5-838/15/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d
*x+1/2*c)^4*a^3-9/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11*a^
6+57/4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11*a^4-113/24/d/
b^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11*a^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 15.08, size = 4505, normalized size = 9.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] ((2*(15*a*b^6 - 105*a^7 - 161*a^3*b^4 + 245*a^5*b^2))/(105*b^8) + (2*tan(c/
2 + (d*x)/2)^14*(a*b^6 - a^7 - 3*a^3*b^4 + 3*a^5*b^2))/b^8 + (2*tan(c/2 + (
d*x)/2)^12*(a*b^6 - 7*a^7 - 15*a^3*b^4 + 19*a^5*b^2))/b^8 + (2*tan(c/2 + (d
*x)/2)^10*(15*a*b^6 - 63*a^7 - 109*a^3*b^4 + 157*a^5*b^2))/(3*b^8) + (2*tan
(c/2 + (d*x)/2)^8*(15*a*b^6 - 105*a^7 - 161*a^3*b^4 + 245*a^5*b^2))/(3*b^8)
 + (2*tan(c/2 + (d*x)/2)^4*(45*a*b^6 - 315*a^7 - 419*a^3*b^4 + 695*a^5*b^2)
)/(15*b^8) + (2*tan(c/2 + (d*x)/2)^6*(45*a*b^6 - 525*a^7 - 743*a^3*b^4 + 11
75*a^5*b^2))/(15*b^8) + (2*tan(c/2 + (d*x)/2)^2*(15*a*b^6 - 735*a^7 - 973*a
^3*b^4 + 1645*a^5*b^2))/(105*b^8) + (tan(c/2 + (d*x)/2)*(64*a^6 - 5*b^6 + 8
```

8*a^2*b^4 - 144*a^4*b^2))/(64*b^7) - (tan(c/2 + (d*x)/2)^15*(64*a^6 - 5*b^6 + 88*a^2*b^4 - 144*a^4*b^2))/(64*b^7) + (tan(c/2 + (d*x)/2)^3*(960*a^6 + 397*b^6 + 488*a^2*b^4 - 1776*a^4*b^2))/(192*b^7) - (tan(c/2 + (d*x)/2)^13*(960*a^6 + 397*b^6 + 488*a^2*b^4 - 1776*a^4*b^2))/(192*b^7) + (tan(c/2 + (d*x)/2)^7*(960*a^6 + 1765*b^6 + 680*a^2*b^4 - 1392*a^4*b^2))/(192*b^7) - (tan(c/2 + (d*x)/2)^9*(960*a^6 + 1765*b^6 + 680*a^2*b^4 - 1392*a^4*b^2))/(192*b^7) + (tan(c/2 + (d*x)/2)^5*(1728*a^6 - 895*b^6 + 904*a^2*b^4 - 2736*a^4*b^2))/(192*b^7) - (tan(c/2 + (d*x)/2)^11*(1728*a^6 - 895*b^6 + 904*a^2*b^4 - 2736*a^4*b^2))/(192*b^7))/(d*(8*tan(c/2 + (d*x)/2)^2 + 28*tan(c/2 + (d*x)/2)^4 + 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 + 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1)) - (atan((((((25*a^2*b^24)/512 + (25*a^4*b^22)/32 - (25*a^6*b^20)/16 - (125*a^8*b^18)/4 + 160*a^10*b^16 - 320*a^12*b^14 + 320*a^14*b^12 - 160*a^16*b^10 + 32*a^18*b^8)/b^23 - ((((5*a*b^26)/4 + (35*a^3*b^24)/4 - 38*a^5*b^22 + 44*a^7*b^20 - 16*a^9*b^18)/b^23 - ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(49152*a*b^28 - 32768*a^3*b^26))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9) + (tan(c/2 + (d*x)/2)*(32768*a^4*b^24 - 98304*a^6*b^22 + 98304*a^8*b^20 - 32768*a^10*b^18))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^26 + 775*a^3*b^24 - 2000*a^5*b^22 - 47584*a^7*b^20 + 278144*a^9*b^18 - 655360*a^11*b^16 + 819200*a^13*b^14 - 573440*a^15*b^12 + 212992*a^17*b^10 - 32768*a^19*b^8))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i)*1i)/(128*b^9) + ((((25*a^2*b^24)/512 + (25*a^4*b^22)/32 - (25*a^6*b^20)/16 - (125*a^8*b^18)/4 + 160*a^10*b^16 - 320*a^12*b^14 + 320*a^14*b^12 - 160*a^16*b^10 + 32*a^18*b^8)/b^23 + ((((5*a*b^26)/4 + (35*a^3*b^24)/4 - 38*a^5*b^22 + 44*a^7*b^20 - 16*a^9*b^18)/b^23 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(49152*a*b^28 - 32768*a^3*b^26))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9) + (tan(c/2 + (d*x)/2)*(32768*a^4*b^24 - 98304*a^6*b^22 + 98304*a^8*b^20 - 32768*a^10*b^18))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^26 + 775*a^3*b^24 - 2000*a^5*b^22 - 47584*a^7*b^20 + 278144*a^9*b^18 - 655360*a^11*b^16 + 819200*a^13*b^14 - 573440*a^15*b^12 + 212992*a^17*b^10 - 32768*a^19*b^8))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i)*1i)/(128*b^9))/((32*a^25 - (25*a^5*b^20)/256 + (315*a^7*b^18)/256 + (3205*a^9*b^16)/256 - (39415*a^11*b^14)/256 + (10135*a^13*b^12)/16 - (11217*a^15*b^10)/8 + (3773*a^17*b^8)/2 - (3195*a^19*b^6)/2 + 836*a^21*b^4 - 248*a^23*b^2)/b^23 + (tan(c/2 + (d*x)/2)*(32768*a^26 - 50*a^4*b^22 - 650*a^6*b^20 + 3850*a^8*b^18 + 24850*a^10*b^16 - 254240*a^12*b^14 + 913600*a^14*b^12 - 1834240*a^16*b^10 + 2293760*a^18*b^8 - 1835008*a^20*b^6 + 917504*a^22*b^4 - 262144*a^24*b^2))/(256*b^24) + ((((25*a^2*b^24)/512 + (25*a^4*b^22)/32 - (25*a^6*b^20)/16 - (125*a^8*b^18)/4 + 160*a^10*b^16 - 320*a^12*b^14 + 320*a^14*b^12 - 160*a^16*b^10 + 32*a^18*b^8)/b^23 - ((((5*a*b^26)/4 + (35*a^3*b^24)/4 - 38*a^5*b^22 + 44*a^7*b^20 - 16*a^9*b^18)/b^23 - ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(49152*a*b^28 - 32768*a^3*b^26))/(512*b^24))*(

b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9) + (tan(c/2 + (d*x)/2)*(32768*a^4*b^24 - 98304*a^6*b^22 + 98304*a^8*b^20 - 32768*a^10*b^18))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^26 + 775*a^3*b^24 - 2000*a^5*b^22 - 47584*a^7*b^20 + 278144*a^9*b^18 - 655360*a^11*b^16 + 819200*a^13*b^14 - 573440*a^15*b^12 + 212992*a^17*b^10 - 32768*a^19*b^8))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9) - ((((25*a^2*b^24)/512 + (25*a^4*b^22)/32 - (25*a^6*b^20)/16 - (125*a^8*b^18)/4 + 160*a^10*b^16 - 320*a^12*b^14 + 320*a^14*b^12 - 160*a^16*b^10 + 32*a^18*b^8)/b^23 + ((((5*a*b^26)/4 + (35*a^3*b^24)/4 - 38*a^5*b^22 + 44*a^7*b^20 - 16*a^9*b^18)/b^23 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(49152*a*b^28 - 32768*a^3*b^26))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9) + (tan(c/2 + (d*x)/2)*(32768*a^4*b^24 - 98304*a^6*b^22 + 98304*a^8*b^20 - 32768*a^10*b^18))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^26 + 775*a^3*b^24 - 2000*a^5*b^22 - 47584*a^7*b^20 + 278144*a^9*b^18 - 655360*a^11*b^16 + 819200*a^13*b^14 - 573440*a^15*b^12 + 212992*a^17*b^10 - 32768*a^19*b^8))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9)))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i)*1i)/(64*b^9*d) - (a^3*atan(((a^3*(-(a + b)^5*(a - b)^5)^(1/2)*(((25*a^2*b^24)/512 + (25*a^4*b^22)/32 - (25*a^6*b^20)/16 - (125*a^8*b^18)/4 + 160*a^10*b^16 - 320*a^12*b^14 + 320*a^14*b^12 - 160*a^16*b^10 + 32*a^18*b^8)/b^23 + (tan(c/2 + (d*x)/2)*(50*a*b^26 + 775*a^3*b^24 - 2000*a^5*b^22 - 47584*a^7*b^20 + 278144*a^9*b^18 - 655360*a^11*b^16 + 819200*a^13*b^14 - 573440*a^15*b^12 + 212992*a^17*b^10 - 32768*a^19*b^8))/(512*b^24) + (a^3*(-(a + b)^5*(a - b)^5)^(1/2)*(((5*a*b^26)/4 + (35*a^3*b^24)/4 - 38*a^5*b^22 + 44*a^7*b^20 - 16*a^9*b^18)/b^23 + (tan(c/2 + (d*x)/2)*(32768*a^4*b^24 - 98304*a^6*b^22 + 98304*a^8*b^20 - 32768*a^10*b^18))/(512*b^24) + (a^3*(-(a + b)^5*(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(49152*a*b^28 - 32768*a^3*b^26))/(512*b^24)))/b^9))/b^9)*1i)/b^9 + (a^3*(-(a + b)^5*(a - b)^5)^(1/2)*(((25*a^2*b^24)/512 + (25*a^4*b^22)/32 - (25*a^6*b^20)/16 - (125*a^8*b^18)/4 + 160*a^10*b^16 - 320*a^12*b^14 + 320*a^14*b^12 - 160*a^16*b^10 + 32*a^18*b^8)/b^23 + (tan(c/2 + (d*x)/2)*(50*a*b^26 + 775*a^3*b^24 - 2000*a^5*b^22 - 47584*a^7*b^20 + 278144*a^9*b^18 - 655360*a^11*b^16 + 819200*a^13*b^14 - 573440*a^15*b^12 + 212992*a^17*b^10 - 32768*a^19*b^8))/(512*b^24) - (a^3*(-(a + b)^5*(a - b)^5)^(1/2)*(((5*a*b^26)/4 + (35*a^3*b^24)/4 - 38*a^5*b^22 + 44*a^7*b^20 - 16*a^9*b^18)/b^23 + (tan(c/2 + (d*x)/2)*(32768*a^4*b^24 - 98304*a^6*b^22 + 98304*a^8*b^20 - 32768*a^10*b^18))/(512*b^24) - (a^3*(-(a + b)^5*(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(49152*a*b^28 - 32768*a^3*b^26))/(512*b^24)))/b^9))/b^9)*1i)/b^9)/((32*a^25 - (25*a^5*b^20)/256 + (315*a^7*b^18)/256 + (3205*a^9*b^16)/256 - (39415*a^11*b^14)/256 + (10135*a^13*b^12)/16 - (11217*a^15*b^10)/8 + (3773*a^17*b^8)/2 - (3195*a^19*b^6)/2 + 836*a^21*b^4 - 248*a^23*b^2)/b^23 + (tan(c/2 + (d*x)/2)*(32768*a^26 - 50*a^4*b^22 - 650*a^6*b^20 + 3850*a^8*b^18 + 24850*a^10*b^16 - 254240*a^12*b^14 + 913600*a^14*b^12 - 1834240*a^16*b

```
^10 + 2293760*a^18*b^8 - 1835008*a^20*b^6 + 917504*a^22*b^4 - 262144*a^24*b
^2))/(256*b^24) - (a^3*(-(a + b)^5*(a - b)^5)^(1/2)*(((25*a^2*b^24)/512 + (
25*a^4*b^22)/32 - (25*a^6*b^20)/16 - (125*a^8*b^18)/4 + 160*a^10*b^16 - 320
*a^12*b^14 + 320*a^14*b^12 - 160*a^16*b^10 + 32*a^18*b^8)/b^23 + (tan(c/2 +
 (d*x)/2)*(50*a*b^26 + 775*a^3*b^24 - 2000*a^5*b^22 - 47584*a^7*b^20 + 2781
44*a^9*b^18 - 655360*a^11*b^16 + 819200*a^13*b^14 - 573440*a^15*b^12 + 2129
92*a^17*b^10 - 32768*a^19*b^8))/(512*b^24) + (a^3*(-(a + b)^5*(a - b)^5)^(1
/2)*(((5*a*b^26)/4 + (35*a^3*b^24)/4 - 38*a^5*b^22 + 44*a^7*b^20 - 16*a^9*b
^18)/b^23 + (tan(c/2 + (d*x)/2)*(32768*a^4*b^24 - 98304*a^6*b^22 + 98304*a^
8*b^20 - 32768*a^10*b^18))/(512*b^24) + (a^3*(-(a + b)^5*(a - b)^5)^(1/2)*(
32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(49152*a*b^28 - 32768*a^3*b^26))/(512*b^24
)))/b^9))/b^9))/b^9 + (a^3*(-(a + b)^5*(a - b)^5)^(1/2)*(((25*a^2*b^24)/512
 + (25*a^4*b^22)/32 - (25*a^6*b^20)/16 - (125*a^8*b^18)/4 + 160*a^10*b^16 -
 320*a^12*b^14 + 320*a^14*b^12 - 160*a^16*b^10 + 32*a^18*b^8)/b^23 + (tan(c
/2 + (d*x)/2)*(50*a*b^26 + 775*a^3*b^24 - 2000*a^5*b^22 - 47584*a^7*b^20 +
278144*a^9*b^18 - 655360*a^11*b^16 + 819200*a^13*b^14 - 573440*a^15*b^12 +
212992*a^17*b^10 - 32768*a^19*b^8))/(512*b^24) - (a^3*(-(a + b)^5*(a - b)^5
)^(1/2)*(((5*a*b^26)/4 + (35*a^3*b^24)/4 - 38*a^5*b^22 + 44*a^7*b^20 - 16*a
^9*b^18)/b^23 + (tan(c/2 + (d*x)/2)*(32768*a^4*b^24 - 98304*a^6*b^22 + 9830
4*a^8*b^20 - 32768*a^10*b^18))/(512*b^24) - (a^3*(-(a + b)^5*(a - b)^5)^(1/
2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(49152*a*b^28 - 32768*a^3*b^26))/(512*
b^24)))/b^9))/b^9))/b^9))*(-(a + b)^5*(a - b)^5)^(1/2)*2i)/(b^9*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1321 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=408

$$-\frac{2a^2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^8d}-\frac{b\sin^4(c+dx)\cos(c+dx)}{4a^2d}-\frac{\left(6a^4-13a^2b^2+8b^4\right)\sin^3(c+dx)\cos(c+}{24ab^4d}$$

[Out] 1/16*a*(16*a^6-40*a^4*b^2+30*a^2*b^4-5*b^6)*x/b^8-2*a^2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^8/d+1/105*(105*a^6-245*a^4*b^2+161*a^2*b^4-15*b^6)*cos(d*x+c)/b^7/d-1/16*a*(8*a^4-18*a^2*b^2+11*b^4)*cos(d*x+c)*sin(d*x+c)/b^6/d+1/105*(35*a^4-77*a^2*b^2+45*b^4)*cos(d*x+c)*sin(d*x+c)^2/b^5/d+1/3*cos(d*x+c)*sin(d*x+c)^3/a/d-1/24*(6*a^4-13*a^2*b^2+8*b^4)*cos(d*x+c)*sin(d*x+c)^3/a/b^4/d-1/4*b*cos(d*x+c)*sin(d*x+c)^4/a^2/d+1/140*(28*a^4-60*a^2*b^2+35*b^4)*cos(d*x+c)*sin(d*x+c)^4/a^2/b^3/d-1/6*a*cos(d*x+c)*sin(d*x+c)^5/b^2/d+1/7*cos(d*x+c)*sin(d*x+c)^6/b/d

Rubi [A] time = 1.46, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2896, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{\left(-245a^4b^2+161a^2b^4+105a^6-15b^6\right)\cos(c+dx)}{105b^7d}-\frac{2a^2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^8d}+\frac{\left(-60a^2b^2+28a^4+\right.}{}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]

[Out] (a*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*x)/(16*b^8) - (2*a^2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^8*d) + ((105*a^6 - 245*a^4*b^2 + 161*a^2*b^4 - 15*b^6)*Cos[c + d*x])/(105*b^7*d) - (a*(8*a^4 - 18*a^2*b^2 + 11*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*b^6*d) + ((35*a^4 - 77*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(105*b^5*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(3*a*d) - ((6*a^4 - 13*a^2*b^2 + 8*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/(24*a*b^4*d) - (b*Cos[c + d*x]*Sin[c + d*x]^4)/(4*a^2*d) + ((28*a^4 - 60*a^2*b^2 + 35*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(140*a^2*b^3*d) - (a*Cos[c + d*x]*Sin[c + d*x]^5)/(6*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^6)/(7*b*d)

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\cos(c+dx)\sin^3(c+dx)}{3ad} - \frac{b\cos(c+dx)\sin^4(c+dx)}{4a^2d} - \frac{a\cos(c+dx)\sin^5(c+}{6b^2d}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad} - \frac{b\cos(c+dx)\sin^4(c+dx)}{4a^2d} + \frac{(28a^4-60a^2b^2+35b^4}{14}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad} - \frac{(6a^4-13a^2b^2+8b^4)\cos(c+dx)\sin^3(c+dx)}{24ab^4d} - \frac{b\,c}{}$$

$$= \frac{(35a^4-77a^2b^2+45b^4)\cos(c+dx)\sin^2(c+dx)}{105b^5d} + \frac{\cos(c+dx)\sin^3(c+dx)}{3ad} -$$

$$= -\frac{a(8a^4-18a^2b^2+11b^4)\cos(c+dx)\sin(c+dx)}{16b^6d} + \frac{(35a^4-77a^2b^2+45b^4)\,co}{105b^5}$$

$$= \frac{(105a^6-245a^4b^2+161a^2b^4-15b^6)\cos(c+dx)}{105b^7d} - \frac{a(8a^4-18a^2b^2+11b^4)\cos}{16b^6d}$$

$$= \frac{a(16a^6-40a^4b^2+30a^2b^4-5b^6)x}{16b^8} + \frac{(105a^6-245a^4b^2+161a^2b^4-15b^6)\cos}{105b^7d}$$

$$= \frac{a(16a^6-40a^4b^2+30a^2b^4-5b^6)x}{16b^8} + \frac{(105a^6-245a^4b^2+161a^2b^4-15b^6)\cos}{105b^7d}$$

$$= \frac{a(16a^6-40a^4b^2+30a^2b^4-5b^6)x}{16b^8} + \frac{(105a^6-245a^4b^2+161a^2b^4-15b^6)\cos}{105b^7d}$$

$$= \frac{a(16a^6-40a^4b^2+30a^2b^4-5b^6)x}{16b^8} - \frac{2a^2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^8d} +$$

Mathematica [A] time = 3.07, size = 324, normalized size = 0.79

$$-\frac{-6720a^7c-6720a^7dx+1680a^5b^2\sin(2(c+dx))+16800a^5b^2c+16800a^5b^2dx-3360a^3b^4\sin(2(c+dx))-210}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6720*(-6720*a^7*c + 16800*a^5*b^2*c - 12600*a^3*b^4*c + 2100*a*b^6*c - 6
720*a^7*d*x + 16800*a^5*b^2*d*x - 12600*a^3*b^4*d*x + 2100*a*b^6*d*x + 1344
0*a^2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] +
105*b*(-64*a^6 + 144*a^4*b^2 - 88*a^2*b^4 + 5*b^6)*Cos[c + d*x] + 35*(16*a^
4*b^3 - 28*a^2*b^5 + 9*b^7)*Cos[3*(c + d*x)] - 84*a^2*b^5*Cos[5*(c + d*x)]
+ 105*b^7*Cos[5*(c + d*x)] + 15*b^7*Cos[7*(c + d*x)] + 1680*a^5*b^2*Sin[2*(
c + d*x)] - 3360*a^3*b^4*Sin[2*(c + d*x)] + 1575*a*b^6*Sin[2*(c + d*x)] - 2
10*a^3*b^4*Sin[4*(c + d*x)] + 315*a*b^6*Sin[4*(c + d*x)] + 35*a*b^6*Sin[6*(
c + d*x)])/(b^8*d)
```

fricas [A] time = 0.91, size = 619, normalized size = 1.52

$$\left[-\frac{240\,b^{7}\cos\left(dx+c\right)^{7}-336\,a^{2}b^{5}\cos\left(dx+c\right)^{5}+560\left(a^{4}b^{3}-a^{2}b^{5}\right)\cos\left(dx+c\right)^{3}-105\left(16\,a^{7}-40\,a^{5}b^{2}+30\,a^{3}b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/1680*(240*b^7*cos(d*x + c)^7 - 336*a^2*b^5*cos(d*x + c)^5 + 560*(a^4*b^
3 - a^2*b^5)*cos(d*x + c)^3 - 105*(16*a^7 - 40*a^5*b^2 + 30*a^3*b^4 - 5*a*b
^6)*d*x - 840*(a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^
2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(
d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*si
n(d*x + c) - a^2 - b^2)) - 1680*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)
+ 35*(8*a*b^6*cos(d*x + c)^5 - 2*(6*a^3*b^4 - 5*a*b^6)*cos(d*x + c)^3 + 3*(
8*a^5*b^2 - 14*a^3*b^4 + 5*a*b^6)*cos(d*x + c))*sin(d*x + c))/(b^8*d), -1/1
680*(240*b^7*cos(d*x + c)^7 - 336*a^2*b^5*cos(d*x + c)^5 + 560*(a^4*b^3 - a
^2*b^5)*cos(d*x + c)^3 - 105*(16*a^7 - 40*a^5*b^2 + 30*a^3*b^4 - 5*a*b^6)*d
*x - 1680*(a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 1680*(a^6*b - 2*a^4*b^3 + a^2*b^5
)*cos(d*x + c) + 35*(8*a*b^6*cos(d*x + c)^5 - 2*(6*a^3*b^4 - 5*a*b^6)*cos(d
*x + c)^3 + 3*(8*a^5*b^2 - 14*a^3*b^4 + 5*a*b^6)*cos(d*x + c))*sin(d*x + c)
)/(b^8*d)]
```

giac [B] time = 0.19, size = 863, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1680*(105*(16*a^7 - 40*a^5*b^2 + 30*a^3*b^4 - 5*a*b^6)*(d*x + c)/b^8 - 33
60*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2
```

```
)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2
- b^2)*b^8) + 2*(840*a^5*b*tan(1/2*d*x + 1/2*c)^13 - 1890*a^3*b^3*tan(1/2*d
*x + 1/2*c)^13 + 1155*a*b^5*tan(1/2*d*x + 1/2*c)^13 + 1680*a^6*tan(1/2*d*x
+ 1/2*c)^12 - 5040*a^4*b^2*tan(1/2*d*x + 1/2*c)^12 + 5040*a^2*b^4*tan(1/2*d
*x + 1/2*c)^12 - 1680*b^6*tan(1/2*d*x + 1/2*c)^12 + 3360*a^5*b*tan(1/2*d*x
+ 1/2*c)^11 - 5880*a^3*b^3*tan(1/2*d*x + 1/2*c)^11 + 980*a*b^5*tan(1/2*d*x
+ 1/2*c)^11 + 10080*a^6*tan(1/2*d*x + 1/2*c)^10 - 26880*a^4*b^2*tan(1/2*d*x
 + 1/2*c)^10 + 20160*a^2*b^4*tan(1/2*d*x + 1/2*c)^10 + 4200*a^5*b*tan(1/2*d
*x + 1/2*c)^9 - 6090*a^3*b^3*tan(1/2*d*x + 1/2*c)^9 + 2975*a*b^5*tan(1/2*d*
x + 1/2*c)^9 + 25200*a^6*tan(1/2*d*x + 1/2*c)^8 - 61040*a^4*b^2*tan(1/2*d*x
 + 1/2*c)^8 + 40880*a^2*b^4*tan(1/2*d*x + 1/2*c)^8 - 8400*b^6*tan(1/2*d*x +
 1/2*c)^8 + 33600*a^6*tan(1/2*d*x + 1/2*c)^6 - 76160*a^4*b^2*tan(1/2*d*x +
1/2*c)^6 + 49280*a^2*b^4*tan(1/2*d*x + 1/2*c)^6 - 4200*a^5*b*tan(1/2*d*x +
1/2*c)^5 + 6090*a^3*b^3*tan(1/2*d*x + 1/2*c)^5 - 2975*a*b^5*tan(1/2*d*x + 1
/2*c)^5 + 25200*a^6*tan(1/2*d*x + 1/2*c)^4 - 55440*a^4*b^2*tan(1/2*d*x + 1/
2*c)^4 + 33936*a^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 5040*b^6*tan(1/2*d*x + 1/2*
c)^4 - 3360*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 5880*a^3*b^3*tan(1/2*d*x + 1/2*c
)^3 - 980*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 10080*a^6*tan(1/2*d*x + 1/2*c)^2 -
 22400*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 12992*a^2*b^4*tan(1/2*d*x + 1/2*c)^
2 - 840*a^5*b*tan(1/2*d*x + 1/2*c) + 1890*a^3*b^3*tan(1/2*d*x + 1/2*c) - 11
55*a*b^5*tan(1/2*d*x + 1/2*c) + 1680*a^6 - 3920*a^4*b^2 + 2576*a^2*b^4 - 24
0*b^6)/((tan(1/2*d*x + 1/2*c)^2 + 1)^7*b^7))/d
```

maple [B] time = 0.30, size = 1808, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -5/8/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))-2/7/d/b/(1+tan(1/2*d*x+1/2*c)^2)^7+
202/5/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4*a^2-1/d/b^6/(1+
tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)*a^5+1/d/b^6/(1+tan(1/2*d*x+1/2*c
)^2)^7*tan(1/2*d*x+1/2*c)^13*a^5-9/4/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1
/2*d*x+1/2*c)^13*a^3+146/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2
*c)^8*a^2+40/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6*a^6-272/
3/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6*a^4+176/3/d/b^3/(1+
tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6*a^2-5/d/b^6/(1+tan(1/2*d*x+1/2
*c)^2)^7*tan(1/2*d*x+1/2*c)^5*a^5+29/4/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*tan
(1/2*d*x+1/2*c)^5*a^3-218/3/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/
2*c)^8*a^4-4/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^3*a^5+30/d
/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4*a^6-66/d/b^5/(1+tan(1/
2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4*a^4+9/4/d/b^4/(1+tan(1/2*d*x+1/2*c)^
2)^7*tan(1/2*d*x+1/2*c)*a^3-11/8/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d
*x+1/2*c)*a+2/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12*a^6-6/
```

```
d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12*a^4+6/d/b^3/(1+tan(1
/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12*a^2-7/6/d/b^2/(1+tan(1/2*d*x+1/2*c
)^2)^7*tan(1/2*d*x+1/2*c)^3*a+12/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d
*x+1/2*c)^2*a^6-80/3/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2*
a^4+232/15/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2*a^2+12/d/b
^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^10*a^6+15/4/d/b^4*arctan(t
an(1/2*d*x+1/2*c))*a^3-5/d/b^6*arctan(tan(1/2*d*x+1/2*c))*a^5+46/15/d/b^3/(
1+tan(1/2*d*x+1/2*c)^2)^7*a^2-10/d/b/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x
+1/2*c)^8+2/d/b^8*arctan(tan(1/2*d*x+1/2*c))*a^7-6/d/b/(1+tan(1/2*d*x+1/2*c
)^2)^7*tan(1/2*d*x+1/2*c)^4-2/d/b/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/
2*c)^12+2/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*a^6-14/3/d/b^5/(1+tan(1/2*d*x+1/
2*c)^2)^7*a^4+2/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*
b)/(a^2-b^2)^(1/2))*a^2+4/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*
c)^11*a^5-7/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11*a^3+7/6/
d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11*a+5/d/b^6/(1+tan(1/2
*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^9*a^5-29/4/d/b^4/(1+tan(1/2*d*x+1/2*c)^
2)^7*tan(1/2*d*x+1/2*c)^9*a^3+85/24/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/
2*d*x+1/2*c)^9*a+30/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8*a
^6+7/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^3*a^3-6/d*a^4/b^4/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/
d*a^6/b^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)
^(1/2))-32/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^10*a^4+24/d/
b^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^10*a^2-2/d*a^8/b^8/(a^2-b
^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+11/8/d/b
^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13*a-85/24/d/b^2/(1+tan(1/
2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^5*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 14.46, size = 3797, normalized size = 9.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

[Out] ((2*(105*a^6 - 15*b^6 + 161*a^2*b^4 - 245*a^4*b^2))/(105*b^7) + (tan(c/2 + (d*x)/2)^13*(11*a*b^4 + 8*a^5 - 18*a^3*b^2))/(8*b^6) - (tan(c/2 + (d*x)/2)^3*(7*a*b^4 + 24*a^5 - 42*a^3*b^2))/(6*b^6) + (tan(c/2 + (d*x)/2)^11*(7*a*b^4 + 24*a^5 - 42*a^3*b^2))/(6*b^6) - (tan(c/2 + (d*x)/2)^5*(85*a*b^4 + 120*a^5 - 174*a^3*b^2))/(24*b^6) + (tan(c/2 + (d*x)/2)^9*(85*a*b^4 + 120*a^5 - 174*a^3*b^2))/(24*b^6) + (2*tan(c/2 + (d*x)/2)^12*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/b^7 + (4*tan(c/2 + (d*x)/2)^10*(3*a^6 + 6*a^2*b^4 - 8*a^4*b^2))/b^7 + (8*tan(c/2 + (d*x)/2)^6*(15*a^6 + 22*a^2*b^4 - 34*a^4*b^2))/(3*b^7) + (4*tan(c/2 + (d*x)/2)^2*(45*a^6 + 58*a^2*b^4 - 100*a^4*b^2))/(15*b^7) + (2*tan(c/2 + (d*x)/2)^8*(45*a^6 - 15*b^6 + 73*a^2*b^4 - 109*a^4*b^2))/(3*b^7) + (2*tan(c/2 + (d*x)/2)^4*(75*a^6 - 15*b^6 + 101*a^2*b^4 - 165*a^4*b^2))/(5*b^7) - (a*tan(c/2 + (d*x)/2)*(8*a^4 + 11*b^4 - 18*a^2*b^2))/(8*b^6))/(d*(7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 + 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 + 1)) + (a^2*atan(((a^2*(-(a + b)^5*(a - b)^5)^(1/2)*(((25*a^4*b^19)/8 - (75*a^6*b^17)/2 + (325*a^8*b^15)/2 - 320*a^10*b^13 + 320*a^12*b^11 - 160*a^14*b^9 + 32*a^16*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a^3*b^21 - 881*a^5*b^19 + 4436*a^7*b^17 - 10260*a^9*b^15 + 12800*a^11*b^13 - 8960*a^13*b^11 + 3328*a^15*b^9 - 512*a^17*b^7))/(8*b^21) + (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((10*a^2*b^22 - 38*a^4*b^20 + 44*a^6*b^18 - 16*a^8*b^16)/b^20 + (tan(c/2 + (d*x)/2)*(512*a^3*b^22 - 1536*a^5*b^20 + 1536*a^7*b^18 - 512*a^9*b^16))/(8*b^21) + (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21)))/b^8))/b^8)*1i)/b^8 + (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*(((25*a^4*b^19)/8 - (75*a^6*b^17)/2 + (325*a^8*b^15)/2 - 320*a^10*b^13 + 320*a^12*b^11 - 160*a^14*b^9 + 32*a^16*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a^3*b^21 - 881*a^5*b^19 + 4436*a^7*b^17 - 10260*a^9*b^15 + 12800*a^11*b^13 - 8960*a^13*b^11 + 3328*a^15*b^9 - 512*a^17*b^7))/(8*b^21) - (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((10*a^2*b^22 - 38*a^4*b^20 + 44*a^6*b^18 - 16*a^8*b^16)/b^20 + (tan(c/2 + (d*x)/2)*(512*a^3*b^22 - 1536*a^5*b^20 + 1536*a^7*b^18 - 512*a^9*b^16))/(8*b^21) - (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21)))/b^8))/b^8)*1i)/b^8)/((32*a^22 + (55*a^6*b^16)/4 - (585*a^8*b^14)/4 + (2445*a^10*b^12)/4 - (5511*a^12*b^10)/4 + 1874*a^14*b^8 - 1595*a^16*b^6 + 836*a^18*b^4 - 248*a^20*b^2)/b^20 + (tan(c/2 + (d*x)/2)*(512*a^23 - 50*a^5*b^18 + 750*a^7*b^16 - 4550*a^9*b^14 + 14770*a^11*b^12 - 28880*a^13*b^10 + 35880*a^15*b^8 - 28672*a^17*b^6 + 14336*a^19*b^4 - 4096*a^21*b^2))/(4*b^21) - (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*(((25*a^4*b^19)/8 - (75*a^6*b^17)/2 + (325*a^8*b^15)/2 - 320*a^10*b^13 + 320*a^12*b^11 - 160*a^14*b^9 + 32*a^16*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a^3*b^21 - 881*a^5*b^19 + 4436*a^7*b^17 - 10260*a^9*b^15 + 12800*a^11*b^13 - 8960*a^13*b^11 + 3328*a^15*b^9 - 512*a^17*b^7))/(8*b^21) + (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((10*a^2*b^22 - 38*a^4*b^20 + 44*a^6*b^18 - 16*a^8*b^16)/b^20 + (tan(c/2 + (d*x)/2)*(512*a^3*b^22 - 1536*a^5*b^20 + 1536*a^7*b^18 - 512*a^9*b^16))/(8*b^21) + (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21)))/b^8))/b^8))/b^8 + (a^2*(-(a + b)^

5*(a - b)^5)^(1/2)*(((25*a^4*b^19)/8 - (75*a^6*b^17)/2 + (325*a^8*b^15)/2 - 320*a^10*b^13 + 320*a^12*b^11 - 160*a^14*b^9 + 32*a^16*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a^3*b^21 - 881*a^5*b^19 + 4436*a^7*b^17 - 10260*a^9*b^15 + 12800*a^11*b^13 - 8960*a^13*b^11 + 3328*a^15*b^9 - 512*a^17*b^7))/(8*b^21) - (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((10*a^2*b^22 - 38*a^4*b^20 + 44*a^6*b^18 - 16*a^8*b^16)/b^20 + (tan(c/2 + (d*x)/2)*(512*a^3*b^22 - 1536*a^5*b^20 + 1536*a^7*b^18 - 512*a^9*b^16))/(8*b^21) - (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21)))/b^8))/b^8))/b^8))*(-(a + b)^5*(a - b)^5)^(1/2)*2i)/(b^8*d) + (a*atan(((a*(((25*a^4*b^19)/8 - (75*a^6*b^17)/2 + (325*a^8*b^15)/2 - 320*a^10*b^13 + 320*a^12*b^11 - 160*a^14*b^9 + 32*a^16*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a^3*b^21 - 881*a^5*b^19 + 4436*a^7*b^17 - 10260*a^9*b^15 + 12800*a^11*b^13 - 8960*a^13*b^11 + 3328*a^15*b^9 - 512*a^17*b^7))/(8*b^21) - (a*((10*a^2*b^22 - 38*a^4*b^20 + 44*a^6*b^18 - 16*a^8*b^16)/b^20 + (tan(c/2 + (d*x)/2)*(512*a^3*b^22 - 1536*a^5*b^20 + 1536*a^7*b^18 - 512*a^9*b^16))/(8*b^21) - (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21))*(16*a^6 - 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2)*1i)/(16*b^8))*(16*a^6 - 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2)*1i)/(16*b^8))*(16*a^6 - 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2))/(16*b^8) + (a*(((25*a^4*b^19)/8 - (75*a^6*b^17)/2 + (325*a^8*b^15)/2 - 320*a^10*b^13 + 320*a^12*b^11 - 160*a^14*b^9 + 32*a^16*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a^3*b^21 - 881*a^5*b^19 + 4436*a^7*b^17 - 10260*a^9*b^15 + 12800*a^11*b^13 - 8960*a^13*b^11 + 3328*a^15*b^9 - 512*a^17*b^7))/(8*b^21) + (a*((10*a^2*b^22 - 38*a^4*b^20 + 44*a^6*b^18 - 16*a^8*b^16)/b^20 + (tan(c/2 + (d*x)/2)*(512*a^3*b^22 - 1536*a^5*b^20 + 1536*a^7*b^18 - 512*a^9*b^16))/(8*b^21) + (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21))*(16*a^6 - 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2)*1i)/(16*b^8))*(16*a^6 - 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2)*1i)/(16*b^8))*(16*a^6 - 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2))/(16*b^8))/((32*a^22 + (55*a^6*b^16)/4 - (585*a^8*b^14)/4 + (2445*a^10*b^12)/4 - (5511*a^12*b^10)/4 + 1874*a^14*b^8 - 1595*a^16*b^6 + 836*a^18*b^4 - 248*a^20*b^2)/b^20 + (tan(c/2 + (d*x)/2)*(512*a^23 - 50*a^5*b^18 + 750*a^7*b^16 - 4550*a^9*b^14 + 14770*a^11*b^12 - 28880*a^13*b^10 + 35880*a^15*b^8 - 28672*a^17*b^6 + 14336*a^19*b^4 - 4096*a^21*b^2))/(4*b^21) + (a*(((25*a^4*b^19)/8 - (75*a^6*b^17)/2 + (325*a^8*b^15)/2 - 320*a^10*b^13 + 320*a^12*b^11 - 160*a^14*b^9 + 32*a^16*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a^3*b^21 - 881*a^5*b^19 + 4436*a^7*b^17 - 10260*a^9*b^15 + 12800*a^11*b^13 - 8960*a^13*b^11 + 3328*a^15*b^9 - 512*a^17*b^7))/(8*b^21) - (a*((10*a^2*b^22 - 38*a^4*b^20 + 44*a^6*b^18 - 16*a^8*b^16)/b^20 + (tan(c/2 + (d*x)/2)*(512*a^3*b^22 - 1536*a^5*b^20 + 1536*a^7*b^18 - 512*a^9*b^16))/(8*b^21) - (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21))*(16*a^6 - 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2)*1i)/(16*b^8))*(16*a^6 - 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2)*1i)/(16*b^8))*(16*a^6 - 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2)*1i)/(16*b^8) - (a*(((25*a^4*b^19)/8 - (75*a^6*b^17)/2 + (325*a^8*b^15)/2 - 320*a^10*b^13 + 320*a^12*b^11 - 160*a^14*b^9 + 32*a^16*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a^3*b^21 - 881*a^5*b^19 + 4436*a^7*b^17 - 10260*a^9*b^15 + 12800*a^11*b^13 - 8960*a^13*b^11 + 3328*a^15*b^9 - 512*a^17*b

```
^7))/(8*b^21) + (a*((10*a^2*b^22 - 38*a^4*b^20 + 44*a^6*b^18 - 16*a^8*b^16)
/b^20 + (tan(c/2 + (d*x)/2)*(512*a^3*b^22 - 1536*a^5*b^20 + 1536*a^7*b^18 -
 512*a^9*b^16))/(8*b^21) + (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25
 - 512*a^3*b^23))/(8*b^21))*(16*a^6 - 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2)*1i)/
(16*b^8))*(16*a^6 - 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2)*1i)/(16*b^8))*(16*a^6
- 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2)*1i)/(16*b^8)))*(16*a^6 - 5*b^6 + 30*a^2*
b^4 - 40*a^4*b^2))/(8*b^8*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1322 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=228

$$\frac{2a\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d}+\frac{\cos^3(c+dx)\left(8a\left(a^2-b^2\right)-b\left(6a^2-5b^2\right)\sin(c+dx)\right)}{24b^4d}-\frac{\cos(c+dx)\left(16a\right.}{}$$

```
[Out] -1/16*(16*a^6-40*a^4*b^2+30*a^2*b^4-5*b^6)*x/b^7+2*a*(a^2-b^2)^(5/2)*arctan
((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^7/d-1/30*cos(d*x+c)^5*(6*a-5*b
*sin(d*x+c))/b^2/d+1/24*cos(d*x+c)^3*(8*a*(a^2-b^2)-b*(6*a^2-5*b^2)*sin(d*x
+c))/b^4/d-1/16*cos(d*x+c)*(16*a*(a^2-b^2)^2-b*(8*a^4-14*a^2*b^2+5*b^4)*sin
(d*x+c))/b^6/d
```

Rubi [A] time = 0.52, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2865, 2735, 2660, 618, 204}

$$\frac{2a\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d}+\frac{\cos^3(c+dx)\left(8a\left(a^2-b^2\right)-b\left(6a^2-5b^2\right)\sin(c+dx)\right)}{24b^4d}-\frac{\cos(c+dx)\left(16a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*x)/(16*b^7) + (2*a*(a^2 - b^2)
^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*d) - (Cos[c +
 d*x]^5*(6*a - 5*b*Sin[c + d*x]))/(30*b^2*d) + (Cos[c + d*x]^3*(8*a*(a^2 -
b^2) - b*(6*a^2 - 5*b^2)*Sin[c + d*x]))/(24*b^4*d) - (Cos[c + d*x]*(16*a*(a
^2 - b^2)^2 - b*(8*a^4 - 14*a^2*b^2 + 5*b^4)*Sin[c + d*x]))/(16*b^6*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\int \frac{\cos^4(c+dx)\left(-ab-\left(6a^2-5b^2\right)\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{6b^2}$$

$$= -\frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\cos^3(c+dx)\left(8a\left(a^2-b^2\right)-b\left(6a^2-5b\right.\right.}{24b^4d}$$

$$= -\frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\cos^3(c+dx)\left(8a\left(a^2-b^2\right)-b\left(6a^2-5b\right.\right.}{24b^4d}$$

$$= -\frac{\left(16a^6-40a^4b^2+30a^2b^4-5b^6\right)x}{16b^7} - \frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\cos}{}$$

$$= -\frac{\left(16a^6-40a^4b^2+30a^2b^4-5b^6\right)x}{16b^7} - \frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\cos}{}$$

$$= -\frac{\left(16a^6-40a^4b^2+30a^2b^4-5b^6\right)x}{16b^7} - \frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\cos}{}$$

$$= -\frac{\left(16a^6-40a^4b^2+30a^2b^4-5b^6\right)x}{16b^7} + \frac{2a\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7d} -$$

Mathematica [A] time = 2.26, size = 275, normalized size = 1.21

$$\frac{-960a^6c-960a^6dx+240a^4b^2\sin(2(c+dx))+2400a^4b^2c+2400a^4b^2dx+20\left(4a^3b^3-7ab^5\right)\cos(3(c+dx))-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-960*a^6*c + 2400*a^4*b^2*c - 1800*a^2*b^4*c + 300*b^6*c - 960*a^6*d*x + 2
400*a^4*b^2*d*x - 1800*a^2*b^4*d*x + 300*b^6*d*x + 1920*a*(a^2 - b^2)^(5/2)
*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 120*a*b*(8*a^4 - 18*a^2
*b^2 + 11*b^4)*Cos[c + d*x] + 20*(4*a^3*b^3 - 7*a*b^5)*Cos[3*(c + d*x)] - 1
2*a*b^5*Cos[5*(c + d*x)] + 240*a^4*b^2*Sin[2*(c + d*x)] - 480*a^2*b^4*Sin[2
*(c + d*x)] + 225*b^6*Sin[2*(c + d*x)] - 30*a^2*b^4*Sin[4*(c + d*x)] + 45*b
^6*Sin[4*(c + d*x)] + 5*b^6*Sin[6*(c + d*x)])/(960*b^7*d)
```

fricas [A] time = 1.03, size = 570, normalized size = 2.50

$$\left[-\frac{48\,ab^5\cos\left(dx+c\right)^5-80\left(a^3b^3-ab^5\right)\cos\left(dx+c\right)^3+15\left(16\,a^6-40\,a^4b^2+30\,a^2b^4-5\,b^6\right)dx-120\left(a^5-2\,a^3b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(48*a*b^5*cos(d*x + c)^5 - 80*(a^3*b^3 - a*b^5)*cos(d*x + c)^3 + 15
*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*d*x - 120*(a^5 - 2*a^3*b^2 + a*
b^4)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x +
c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2
 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 240*(a^5*
b - 2*a^3*b^3 + a*b^5)*cos(d*x + c) - 5*(8*b^6*cos(d*x + c)^5 - 2*(6*a^2*b^
4 - 5*b^6)*cos(d*x + c)^3 + 3*(8*a^4*b^2 - 14*a^2*b^4 + 5*b^6)*cos(d*x + c)
)*sin(d*x + c))/(b^7*d), -1/240*(48*a*b^5*cos(d*x + c)^5 - 80*(a^3*b^3 - a*
b^5)*cos(d*x + c)^3 + 15*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*d*x + 2
40*(a^5 - 2*a^3*b^2 + a*b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(
sqrt(a^2 - b^2)*cos(d*x + c))) + 240*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x +
c) - 5*(8*b^6*cos(d*x + c)^5 - 2*(6*a^2*b^4 - 5*b^6)*cos(d*x + c)^3 + 3*(8*
a^4*b^2 - 14*a^2*b^4 + 5*b^6)*cos(d*x + c))*sin(d*x + c))/(b^7*d)]
```

giac [B] time = 0.18, size = 735, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(15*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*(d*x + c)/b^7 - 480*(
a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(
a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)
*b^7) + 2*(120*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 270*a^2*b^3*tan(1/2*d*x + 1/
2*c)^11 + 165*b^5*tan(1/2*d*x + 1/2*c)^11 + 240*a^5*tan(1/2*d*x + 1/2*c)^10
 - 720*a^3*b^2*tan(1/2*d*x + 1/2*c)^10 + 720*a*b^4*tan(1/2*d*x + 1/2*c)^10
+ 360*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 570*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 - 2
5*b^5*tan(1/2*d*x + 1/2*c)^9 + 1200*a^5*tan(1/2*d*x + 1/2*c)^8 - 3120*a^3*b
^2*tan(1/2*d*x + 1/2*c)^8 + 2160*a*b^4*tan(1/2*d*x + 1/2*c)^8 + 240*a^4*b*t
an(1/2*d*x + 1/2*c)^7 - 300*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 450*b^5*tan(1/
2*d*x + 1/2*c)^7 + 2400*a^5*tan(1/2*d*x + 1/2*c)^6 - 5600*a^3*b^2*tan(1/2*d
*x + 1/2*c)^6 + 3680*a*b^4*tan(1/2*d*x + 1/2*c)^6 - 240*a^4*b*tan(1/2*d*x +
 1/2*c)^5 + 300*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 450*b^5*tan(1/2*d*x + 1/2*
```

```
c)^5 + 2400*a^5*tan(1/2*d*x + 1/2*c)^4 - 5280*a^3*b^2*tan(1/2*d*x + 1/2*c)^
4 + 3360*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 360*a^4*b*tan(1/2*d*x + 1/2*c)^3 +
570*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 25*b^5*tan(1/2*d*x + 1/2*c)^3 + 1200*a
^5*tan(1/2*d*x + 1/2*c)^2 - 2640*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 1488*a*b^
4*tan(1/2*d*x + 1/2*c)^2 - 120*a^4*b*tan(1/2*d*x + 1/2*c) + 270*a^2*b^3*tan
(1/2*d*x + 1/2*c) - 165*b^5*tan(1/2*d*x + 1/2*c) + 240*a^5 - 560*a^3*b^2 +
368*a*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*b^6))/d
```

maple [B] time = 0.26, size = 1551, normalized size = 6.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -15/4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2-5/2/d/b^3/(1+tan(1/2*d*x+1/2*c)^
2)^6*tan(1/2*d*x+1/2*c)^5*a^2+3/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*
x+1/2*c)^3*a^4-2/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a^5
+5/8/d/b*arctan(tan(1/2*d*x+1/2*c))-9/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*ta
n(1/2*d*x+1/2*c)*a^2+6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^
10*a^3-20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a^5+44/d/b^
4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a^3+140/3/d/b^4/(1+tan(1/
2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a^3-92/3/d/b^2/(1+tan(1/2*d*x+1/2*c)
^2)^6*tan(1/2*d*x+1/2*c)^6*a+26/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*
x+1/2*c)^8*a^3-18/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8*a+6
/d*a^3/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2
)^(1/2))-2/d*a/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a
^2-b^2)^(1/2))+5/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^4-2/d/b^7*arctan(tan(1/
2*d*x+1/2*c))*a^6-2/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*a^5-11/8/d/b/(1+tan(1/
2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11+5/24/d/b/(1+tan(1/2*d*x+1/2*c)^2)^6
*tan(1/2*d*x+1/2*c)^9-15/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)^7+15/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5-5/24/d/b/(1+ta
n(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+11/8/d/b/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)+14/3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*a^3-46/15/d/b^
2/(1+tan(1/2*d*x+1/2*c)^2)^6*a-19/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/
2*d*x+1/2*c)^3*a^2-10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2
*a^5+2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*a^4-3/d/b^5/(1
+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*a^4+19/4/d/b^3/(1+tan(1/2*d*x
+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*a^2-2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*ta
n(1/2*d*x+1/2*c)^7*a^4+5/2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2
*c)^7*a^2-20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a^5+22/d
/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a^3-62/5/d/b^2/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a-10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)^8*a^5-28/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*
x+1/2*c)^4*a-6/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a+9/4
```

```
/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*a^2+1/d/b^5/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*a^4+2/d*a^7/b^7/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-6/d*a^5/b^5/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/d/b^5/(1+t
an(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*a^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 14.38, size = 3683, normalized size = 16.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] - ((2*(23*a*b^4 + 15*a^5 - 35*a^3*b^2))/(15*b^6) + (4*tan(c/2 + (d*x)/2)^4*
(7*a*b^4 + 5*a^5 - 11*a^3*b^2))/b^6 + (2*tan(c/2 + (d*x)/2)^8*(9*a*b^4 + 5*
a^5 - 13*a^3*b^2))/b^6 + (4*tan(c/2 + (d*x)/2)^6*(23*a*b^4 + 15*a^5 - 35*a^
3*b^2))/(3*b^6) + (2*tan(c/2 + (d*x)/2)^2*(31*a*b^4 + 25*a^5 - 55*a^3*b^2))
/(5*b^6) - (tan(c/2 + (d*x)/2)*(8*a^4 + 11*b^4 - 18*a^2*b^2))/(8*b^5) - (ta
n(c/2 + (d*x)/2)^5*(8*a^4 + 15*b^4 - 10*a^2*b^2))/(4*b^5) + (tan(c/2 + (d*x
)/2)^7*(8*a^4 + 15*b^4 - 10*a^2*b^2))/(4*b^5) + (tan(c/2 + (d*x)/2)^11*(8*a
^4 + 11*b^4 - 18*a^2*b^2))/(8*b^5) + (tan(c/2 + (d*x)/2)^3*(5*b^4 - 72*a^4
+ 114*a^2*b^2))/(24*b^5) - (tan(c/2 + (d*x)/2)^9*(5*b^4 - 72*a^4 + 114*a^2*
b^2))/(24*b^5) + (2*tan(c/2 + (d*x)/2)^10*(3*a*b^4 + a^5 - 3*a^3*b^2))/b^6)
/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/
2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/
2)^12 + 1)) - (atan((((((25*a^2*b^18)/8 - (75*a^4*b^16)/2 + (325*a^6*b^14)/
2 - 320*a^8*b^12 + 320*a^10*b^10 - 160*a^12*b^8 + 32*a^14*b^6)/b^17 - (((10
*a*b^20 - 38*a^3*b^18 + 44*a^5*b^16 - 16*a^7*b^14)/b^17 - ((32*a^2*b^3 + (t
an(c/2 + (d*x)/2)*(768*a*b^22 - 512*a^3*b^20))/(8*b^18))*(a^6*16i - b^6*5i
+ a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(512*a^2*b^20
- 1536*a^4*b^18 + 1536*a^6*b^16 - 512*a^8*b^14))/(8*b^18))*(a^6*16i - b^6*5
i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(50*a*b^20 -
 881*a^3*b^18 + 4436*a^5*b^16 - 10260*a^7*b^14 + 12800*a^9*b^12 - 8960*a^11
```

*b^10 + 3328*a^13*b^8 - 512*a^15*b^6))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i)*1i)/(16*b^7) + ((((25*a^2*b^18)/8 - (75*a^4*b^16)/2 + (325*a^6*b^14)/2 - 320*a^8*b^12 + 320*a^10*b^10 - 160*a^12*b^8 + 32*a^14*b^6)/b^17 + (((10*a*b^20 - 38*a^3*b^18 + 44*a^5*b^16 - 16*a^7*b^14)/b^17 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^22 - 512*a^3*b^20))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(512*a^2*b^20 - 1536*a^4*b^18 + 1536*a^6*b^16 - 512*a^8*b^14))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(50*a*b^20 - 881*a^3*b^18 + 4436*a^5*b^16 - 10260*a^7*b^14 + 12800*a^9*b^12 - 8960*a^11*b^10 + 3328*a^13*b^8 - 512*a^15*b^6))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i)*1i)/(16*b^7))/((32*a^19 + (55*a^3*b^16)/4 - (585*a^5*b^14)/4 + (2445*a^7*b^12)/4 - (5511*a^9*b^10)/4 + 1874*a^11*b^8 - 1595*a^13*b^6 + 836*a^15*b^4 - 248*a^17*b^2)/b^17 + (tan(c/2 + (d*x)/2)*(512*a^20 - 50*a^2*b^18 + 750*a^4*b^16 - 4550*a^6*b^14 + 14770*a^8*b^12 - 28880*a^10*b^10 + 35880*a^12*b^8 - 28672*a^14*b^6 + 14336*a^16*b^4 - 4096*a^18*b^2))/(4*b^18) + ((((25*a^2*b^18)/8 - (75*a^4*b^16)/2 + (325*a^6*b^14)/2 - 320*a^8*b^12 + 320*a^10*b^10 - 160*a^12*b^8 + 32*a^14*b^6)/b^17 - (((10*a*b^20 - 38*a^3*b^18 + 44*a^5*b^16 - 16*a^7*b^14)/b^17 - ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^22 - 512*a^3*b^20))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(512*a^2*b^20 - 1536*a^4*b^18 + 1536*a^6*b^16 - 512*a^8*b^14))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(50*a*b^20 - 881*a^3*b^18 + 4436*a^5*b^16 - 10260*a^7*b^14 + 12800*a^9*b^12 - 8960*a^11*b^10 + 3328*a^13*b^8 - 512*a^15*b^6))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) - ((((25*a^2*b^18)/8 - (75*a^4*b^16)/2 + (325*a^6*b^14)/2 - 320*a^8*b^12 + 320*a^10*b^10 - 160*a^12*b^8 + 32*a^14*b^6)/b^17 + (((10*a*b^20 - 38*a^3*b^18 + 44*a^5*b^16 - 16*a^7*b^14)/b^17 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^22 - 512*a^3*b^20))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(512*a^2*b^20 - 1536*a^4*b^18 + 1536*a^6*b^16 - 512*a^8*b^14))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(50*a*b^20 - 881*a^3*b^18 + 4436*a^5*b^16 - 10260*a^7*b^14 + 12800*a^9*b^12 - 8960*a^11*b^10 + 3328*a^13*b^8 - 512*a^15*b^6))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7)))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i)*1i)/(8*b^7*d) - (a*atan(((a*(-(a + b)^5*(a - b)^5)^(1/2)*(((25*a^2*b^18)/8 - (75*a^4*b^16)/2 + (325*a^6*b^14)/2 - 320*a^8*b^12 + 320*a^10*b^10 - 160*a^12*b^8 + 32*a^14*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(50*a*b^20 - 881*a^3*b^18 + 4436*a^5*b^16 - 10260*a^7*b^14 + 12800*a^9*b^12 - 8960*a^11*b^10 + 3328*a^13*b^8 - 512*a^15*b^6))/(8*b^18) + (a*(-(a + b)^5*(a - b)^5)^(1/2)*((10*a*b^20 - 38*a^3*b^18 + 44*a^5*b^16 - 16*a^7*b^14)/b^17 + (tan(c/2 + (d*x)/2)*(512*a^2*b^20 - 1536*a^4*b^18 + 1536*a^6*b^16 - 512*a^8*b^14))/(8*b^18) + (a*(-(a + b)^5*(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^22 - 512*a^3*b^20))/(8*b^18)))/b^7))/b^7)*1i)/b^7 + (a*(-(a + b)^5*(a - b)^5)^(1/2)*(((25*a^2*b^18)/8 - (75*a^4*b^16)/2 + (325*a^6*b^14)/2 - 320*a^8*b^12 + 320*a^10*b^10 - 160*a^12*b^8 + 32*a^14*b^6)/b^17 +

```
(tan(c/2 + (d*x)/2)*(50*a*b^20 - 881*a^3*b^18 + 4436*a^5*b^16 - 10260*a^7*b
^14 + 12800*a^9*b^12 - 8960*a^11*b^10 + 3328*a^13*b^8 - 512*a^15*b^6))/(8*b
^18) - (a*(-(a + b)^5*(a - b)^5)^(1/2)*((10*a*b^20 - 38*a^3*b^18 + 44*a^5*b
^16 - 16*a^7*b^14)/b^17 + (tan(c/2 + (d*x)/2)*(512*a^2*b^20 - 1536*a^4*b^18
 + 1536*a^6*b^16 - 512*a^8*b^14))/(8*b^18) - (a*(-(a + b)^5*(a - b)^5)^(1/2
)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^22 - 512*a^3*b^20))/(8*b^18)))
/b^7))/b^7)*1i)/b^7)/((32*a^19 + (55*a^3*b^16)/4 - (585*a^5*b^14)/4 + (2445
*a^7*b^12)/4 - (5511*a^9*b^10)/4 + 1874*a^11*b^8 - 1595*a^13*b^6 + 836*a^15
*b^4 - 248*a^17*b^2)/b^17 + (tan(c/2 + (d*x)/2)*(512*a^20 - 50*a^2*b^18 + 7
50*a^4*b^16 - 4550*a^6*b^14 + 14770*a^8*b^12 - 28880*a^10*b^10 + 35880*a^12
*b^8 - 28672*a^14*b^6 + 14336*a^16*b^4 - 4096*a^18*b^2))/(4*b^18) - (a*(-(a
 + b)^5*(a - b)^5)^(1/2)*(((25*a^2*b^18)/8 - (75*a^4*b^16)/2 + (325*a^6*b^1
4)/2 - 320*a^8*b^12 + 320*a^10*b^10 - 160*a^12*b^8 + 32*a^14*b^6)/b^17 + (t
an(c/2 + (d*x)/2)*(50*a*b^20 - 881*a^3*b^18 + 4436*a^5*b^16 - 10260*a^7*b^1
4 + 12800*a^9*b^12 - 8960*a^11*b^10 + 3328*a^13*b^8 - 512*a^15*b^6))/(8*b^1
8) + (a*(-(a + b)^5*(a - b)^5)^(1/2)*((10*a*b^20 - 38*a^3*b^18 + 44*a^5*b^1
6 - 16*a^7*b^14)/b^17 + (tan(c/2 + (d*x)/2)*(512*a^2*b^20 - 1536*a^4*b^18 +
 1536*a^6*b^16 - 512*a^8*b^14))/(8*b^18) + (a*(-(a + b)^5*(a - b)^5)^(1/2)*
(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^22 - 512*a^3*b^20))/(8*b^18)))/b
^7))/b^7))/b^7 + (a*(-(a + b)^5*(a - b)^5)^(1/2)*(((25*a^2*b^18)/8 - (75*a^
4*b^16)/2 + (325*a^6*b^14)/2 - 320*a^8*b^12 + 320*a^10*b^10 - 160*a^12*b^8
+ 32*a^14*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(50*a*b^20 - 881*a^3*b^18 + 4436*
a^5*b^16 - 10260*a^7*b^14 + 12800*a^9*b^12 - 8960*a^11*b^10 + 3328*a^13*b^8
 - 512*a^15*b^6))/(8*b^18) - (a*(-(a + b)^5*(a - b)^5)^(1/2)*((10*a*b^20 -
38*a^3*b^18 + 44*a^5*b^16 - 16*a^7*b^14)/b^17 + (tan(c/2 + (d*x)/2)*(512*a^
2*b^20 - 1536*a^4*b^18 + 1536*a^6*b^16 - 512*a^8*b^14))/(8*b^18) - (a*(-(a
+ b)^5*(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^22 - 512
*a^3*b^20))/(8*b^18)))/b^7))/b^7))/b^7))*(-(a + b)^5*(a - b)^5)^(1/2)*2i)/(
b^7*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1323 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=252

$$\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}-\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{b^4d}+\frac{\left(a^2-3b^2\right)\sin(c+dx)\cos(c+dx)}{2b^3d}-\frac{x\left(a^2-3b^2\right)}{2b^3}$$

```
[Out] -3/8*x/b-1/2*(a^2-3*b^2)*x/b^3-(a^4-3*a^2*b^2+3*b^4)*x/b^5+2*(a^2-b^2)^(5/2
)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/b^5/d-arctanh(cos(d*x+
c))/a/d-a*cos(d*x+c)/b^2/d-a*(a^2-3*b^2)*cos(d*x+c)/b^4/d+1/3*a*cos(d*x+c)^
3/b^2/d+3/8*cos(d*x+c)*sin(d*x+c)/b/d+1/2*(a^2-3*b^2)*cos(d*x+c)*sin(d*x+c)
/b^3/d+1/4*cos(d*x+c)*sin(d*x+c)^3/b/d
```

Rubi [A] time = 0.29, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2897, 3770, 2638, 2635, 8, 2633, 2660, 618, 204}

$$-\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{b^4d}+\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}+\frac{\left(a^2-3b^2\right)\sin(c+dx)\cos(c+dx)}{2b^3d}-\frac{x\left(a^2-3b\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(8*b) - ((a^2 - 3*b^2)*x)/(2*b^3) - ((a^4 - 3*a^2*b^2 + 3*b^4)*x)/b^
5 + (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a*b^5*d) - ArcTanh[Cos[c + d*x]]/(a*d) - (a*Cos[c + d*x])/(b^2*d) - (a*(a^
2 - 3*b^2)*Cos[c + d*x])/(b^4*d) + (a*Cos[c + d*x]^3)/(3*b^2*d) + (3*Cos[c
+ d*x]*Sin[c + d*x])/(8*b*d) + ((a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2
*b^3*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int\left(\frac{-a^4+3a^2b^2-3b^4}{b^5}+\frac{\csc(c+dx)}{a}+\frac{a\left(a^2-3b^2\right)\sin(c+dx)}{b^4}+\frac{\left(-a^2+3b^2\right.}{}\right.$$

$$=-\frac{\left(a^4-3a^2b^2+3b^4\right)x}{b^5}+\frac{\int\csc(c+dx)\,dx}{a}+\frac{a\int\sin^3(c+dx)\,dx}{b^2}-\frac{\int\sin^4(c+}{b}$$

$$=-\frac{\left(a^4-3a^2b^2+3b^4\right)x}{b^5}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{b^4d}+\frac{(a}{}$$

$$=-\frac{\left(a^2-3b^2\right)x}{2b^3}-\frac{\left(a^4-3a^2b^2+3b^4\right)x}{b^5}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{a\cos(c+dx)}{b^2d}$$

$$=-\frac{3x}{8b}-\frac{\left(a^2-3b^2\right)x}{2b^3}-\frac{\left(a^4-3a^2b^2+3b^4\right)x}{b^5}+\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+d\right.}{\sqrt{a^2-b^2}}\right.}{ab^5d}$$

Mathematica [A] time = 0.51, size = 220, normalized size = 0.87

$$-\frac{96a^5c+96a^5dx-24a^3b^2\sin(2(c+dx))-240a^3b^2c-240a^3b^2dx-8a^2b^3\cos(3(c+dx))+24a^2b\left(4a^2-9b^2\right)c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/96*(96*a^5*c - 240*a^3*b^2*c + 180*a*b^4*c + 96*a^5*d*x - 240*a^3*b^2*d*
x + 180*a*b^4*d*x - 192*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]] + 24*a^2*b*(4*a^2 - 9*b^2)*Cos[c + d*x] - 8*a^2*b^3*Cos[3*(
c + d*x)] + 96*b^5*Log[Cos[(c + d*x)/2]] - 96*b^5*Log[Sin[(c + d*x)/2]] - 2
4*a^3*b^2*Sin[2*(c + d*x)] + 48*a*b^4*Sin[2*(c + d*x)] + 3*a*b^4*Sin[4*(c +
 d*x)])/(a*b^5*d)
```

fricas [A] time = 1.24, size = 508, normalized size = 2.02

$$\left[\frac{8\,a^2b^3\cos\left(dx+c\right)^3-12\,b^5\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+12\,b^5\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(8\,a^5-20\,a^3b^2+15\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/24*(8*a^2*b^3*cos(d*x + c)^3 - 12*b^5*log(1/2*cos(d*x + c) + 1/2) + 12*b
^5*log(-1/2*cos(d*x + c) + 1/2) - 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*d*x + 1
2*(a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*co
s(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^
2 - b^2)) - 24*(a^4*b - 2*a^2*b^3)*cos(d*x + c) - 3*(2*a*b^4*cos(d*x + c)^3
 - (4*a^3*b^2 - 7*a*b^4)*cos(d*x + c))*sin(d*x + c))/(a*b^5*d), 1/24*(8*a^2
*b^3*cos(d*x + c)^3 - 12*b^5*log(1/2*cos(d*x + c) + 1/2) + 12*b^5*log(-1/2*
cos(d*x + c) + 1/2) - 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*d*x - 24*(a^4 - 2*a
^2*b^2 + b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)
*cos(d*x + c))) - 24*(a^4*b - 2*a^2*b^3)*cos(d*x + c) - 3*(2*a*b^4*cos(d*x
+ c)^3 - (4*a^3*b^2 - 7*a*b^4)*cos(d*x + c))*sin(d*x + c))/(a*b^5*d)]
```

giac [A] time = 0.23, size = 398, normalized size = 1.58

$$\frac{24\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{3\left(8\,a^{4}-20\,a^{2}b^{2}+15\,b^{4}\right)(dx+c)}{b^{5}}+\frac{48\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,ab^{5}}-\frac{2\left(12\,a^{2}b\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(24*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3*(8*a^4 - 20*a^2*b^2 + 15*b^4)
*(d*x + c)/b^5 + 48*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*a*b^5) - 2*(12*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 27*b^3*ta
n(1/2*d*x + 1/2*c)^7 + 24*a^3*tan(1/2*d*x + 1/2*c)^6 - 72*a*b^2*tan(1/2*d*x
 + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^
5 + 72*a^3*tan(1/2*d*x + 1/2*c)^4 - 168*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a
^2*b*tan(1/2*d*x + 1/2*c)^3 + 3*b^3*tan(1/2*d*x + 1/2*c)^3 + 72*a^3*tan(1/2
*d*x + 1/2*c)^2 - 152*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x +
 1/2*c) + 27*b^3*tan(1/2*d*x + 1/2*c) + 24*a^3 - 56*a*b^2)/((tan(1/2*d*x +
1/2*c)^2 + 1)^4*b^4))/d
```

maple [B] time = 0.45, size = 827, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*a^2+9/4/d/b/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^
4*tan(1/2*d*x+1/2*c)^6*a^3+6/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)^6*a-1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a^2+1/4/d
/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-6/d/b^4/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a^3+14/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan
(1/2*d*x+1/2*c)^4*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3
*a^2-1/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-6/d/b^4/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a^3+38/3/d/b^2/(1+tan(1/2*d*x+1/2
*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*
d*x+1/2*c)*a^2-9/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-2/d/b^
4/(1+tan(1/2*d*x+1/2*c)^2)^4*a^3+14/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*a-2/
d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^4+5/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2
-15/4/d/b*arctan(tan(1/2*d*x+1/2*c))+1/a/d*ln(tan(1/2*d*x+1/2*c))+2/d*a^5/b
^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
-6/d*a^3/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))+6/d*a/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))-2/d*b/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.40, size = 4866, normalized size = 19.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) + ((2*(7*a*b^2 - 3*a^3))/(3*b^4) + (2*tan(c/2
 + (d*x)/2)^6*(3*a*b^2 - a^3))/b^4 + (2*tan(c/2 + (d*x)/2)^4*(7*a*b^2 - 3*a
^3))/b^4 + (2*tan(c/2 + (d*x)/2)^2*(19*a*b^2 - 9*a^3))/(3*b^4) + (tan(c/2 +
 (d*x)/2)*(4*a^2 - 9*b^2))/(4*b^3) + (tan(c/2 + (d*x)/2)^3*(4*a^2 - b^2))/(
4*b^3) - (tan(c/2 + (d*x)/2)^5*(4*a^2 - b^2))/(4*b^3) - (tan(c/2 + (d*x)/2)
^7*(4*a^2 - 9*b^2))/(4*b^3))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)
```

/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (atan((((a^4*
8i + b^4*15i - a^2*b^2*20i)*((840*a*b^14 - 112*a^15 - 3760*a^3*b^12 + (1527
1*a^5*b^10)/2 - (18191*a^7*b^8)/2 + (13697*a^9*b^6)/2 - 3252*a^11*b^4 + 900
*a^13*b^2)/b^11 + (tan(c/2 + (d*x)/2)*(3664*b^20 - 18084*a^2*b^18 + 41125*a
^4*b^16 - 56140*a^6*b^14 + 50084*a^8*b^12 - 29728*a^10*b^10 + 11392*a^12*b^
8 - 2560*a^14*b^6 + 256*a^16*b^4))/(2*b^16) - ((a^4*8i + b^4*15i - a^2*b^2*
20i)*((128*b^16 + 94*a^2*b^14 - (2395*a^4*b^12)/2 + 2068*a^6*b^10 - 1600*a^
8*b^8 + 608*a^10*b^6 - 96*a^12*b^4)/b^11 + (tan(c/2 + (d*x)/2)*(4944*a*b^20
 - 18960*a^3*b^18 + 29985*a^5*b^16 - 24664*a^7*b^14 + 10816*a^9*b^12 - 2240
*a^11*b^10 + 128*a^13*b^8))/(2*b^16) - ((a^4*8i + b^4*15i - a^2*b^2*20i)*((
256*a*b^16 - 452*a^3*b^14 + 340*a^5*b^12 - 112*a^7*b^10)/b^11 - (((128*a^2*
b^16 - 96*a^4*b^14)/b^11 + (tan(c/2 + (d*x)/2)*(1024*a*b^22 - 1088*a^3*b^20
 + 128*a^5*b^18))/(2*b^16))*(a^4*8i + b^4*15i - a^2*b^2*20i))/(8*b^5) + (ta
n(c/2 + (d*x)/2)*(1024*b^22 - 2368*a^2*b^20 + 2432*a^4*b^18 - 1280*a^6*b^16
 + 256*a^8*b^14))/(2*b^16)))/(8*b^5)))/(8*b^5))*1i)/(8*b^5) + ((a^4*8i + b^
4*15i - a^2*b^2*20i)*((840*a*b^14 - 112*a^15 - 3760*a^3*b^12 + (15271*a^5*b
^10)/2 - (18191*a^7*b^8)/2 + (13697*a^9*b^6)/2 - 3252*a^11*b^4 + 900*a^13*b
^2)/b^11 + (tan(c/2 + (d*x)/2)*(3664*b^20 - 18084*a^2*b^18 + 41125*a^4*b^16
 - 56140*a^6*b^14 + 50084*a^8*b^12 - 29728*a^10*b^10 + 11392*a^12*b^8 - 256
0*a^14*b^6 + 256*a^16*b^4))/(2*b^16) + ((a^4*8i + b^4*15i - a^2*b^2*20i)*((
128*b^16 + 94*a^2*b^14 - (2395*a^4*b^12)/2 + 2068*a^6*b^10 - 1600*a^8*b^8 +
 608*a^10*b^6 - 96*a^12*b^4)/b^11 + (tan(c/2 + (d*x)/2)*(4944*a*b^20 - 1896
0*a^3*b^18 + 29985*a^5*b^16 - 24664*a^7*b^14 + 10816*a^9*b^12 - 2240*a^11*b
^10 + 128*a^13*b^8))/(2*b^16) + ((a^4*8i + b^4*15i - a^2*b^2*20i)*((256*a*b
^16 - 452*a^3*b^14 + 340*a^5*b^12 - 112*a^7*b^10)/b^11 + (((128*a^2*b^16 -
96*a^4*b^14)/b^11 + (tan(c/2 + (d*x)/2)*(1024*a*b^22 - 1088*a^3*b^20 + 128*
a^5*b^18))/(2*b^16))*(a^4*8i + b^4*15i - a^2*b^2*20i))/(8*b^5) + (tan(c/2 +
 (d*x)/2)*(1024*b^22 - 2368*a^2*b^20 + 2432*a^4*b^18 - 1280*a^6*b^16 + 256*
a^8*b^14))/(2*b^16)))/(8*b^5)))/(8*b^5))*1i)/(8*b^5))/((224*a^14 - 780*b^14
 + 4445*a^2*b^12 - 10911*a^4*b^10 + 14991*a^6*b^8 - 12481*a^8*b^6 + 6312*a^
10*b^4 - 1800*a^12*b^2)/b^11 + ((a^4*8i + b^4*15i - a^2*b^2*20i)*((840*a*b^
14 - 112*a^15 - 3760*a^3*b^12 + (15271*a^5*b^10)/2 - (18191*a^7*b^8)/2 + (1
3697*a^9*b^6)/2 - 3252*a^11*b^4 + 900*a^13*b^2)/b^11 + (tan(c/2 + (d*x)/2)*
(3664*b^20 - 18084*a^2*b^18 + 41125*a^4*b^16 - 56140*a^6*b^14 + 50084*a^8*b
^12 - 29728*a^10*b^10 + 11392*a^12*b^8 - 2560*a^14*b^6 + 256*a^16*b^4))/(2*
b^16) - ((a^4*8i + b^4*15i - a^2*b^2*20i)*((128*b^16 + 94*a^2*b^14 - (2395*
a^4*b^12)/2 + 2068*a^6*b^10 - 1600*a^8*b^8 + 608*a^10*b^6 - 96*a^12*b^4)/b^
11 + (tan(c/2 + (d*x)/2)*(4944*a*b^20 - 18960*a^3*b^18 + 29985*a^5*b^16 - 2
4664*a^7*b^14 + 10816*a^9*b^12 - 2240*a^11*b^10 + 128*a^13*b^8))/(2*b^16) -
 ((a^4*8i + b^4*15i - a^2*b^2*20i)*((256*a*b^16 - 452*a^3*b^14 + 340*a^5*b^
12 - 112*a^7*b^10)/b^11 - (((128*a^2*b^16 - 96*a^4*b^14)/b^11 + (tan(c/2 +
(d*x)/2)*(1024*a*b^22 - 1088*a^3*b^20 + 128*a^5*b^18))/(2*b^16))*(a^4*8i +
b^4*15i - a^2*b^2*20i))/(8*b^5) + (tan(c/2 + (d*x)/2)*(1024*b^22 - 2368*a^2
*b^20 + 2432*a^4*b^18 - 1280*a^6*b^16 + 256*a^8*b^14))/(2*b^16)))/(8*b^5)))
/(8*b^5)))/(8*b^5) - ((a^4*8i + b^4*15i - a^2*b^2*20i)*((840*a*b^14 - 112*a

^15 - 3760*a^3*b^12 + (15271*a^5*b^10)/2 - (18191*a^7*b^8)/2 + (13697*a^9*b
^6)/2 - 3252*a^11*b^4 + 900*a^13*b^2)/b^11 + (tan(c/2 + (d*x)/2)*(3664*b^20
 - 18084*a^2*b^18 + 41125*a^4*b^16 - 56140*a^6*b^14 + 50084*a^8*b^12 - 2972
8*a^10*b^10 + 11392*a^12*b^8 - 2560*a^14*b^6 + 256*a^16*b^4))/(2*b^16) + ((
a^4*8i + b^4*15i - a^2*b^2*20i)*((128*b^16 + 94*a^2*b^14 - (2395*a^4*b^12)/
2 + 2068*a^6*b^10 - 1600*a^8*b^8 + 608*a^10*b^6 - 96*a^12*b^4)/b^11 + (tan(
c/2 + (d*x)/2)*(4944*a*b^20 - 18960*a^3*b^18 + 29985*a^5*b^16 - 24664*a^7*b
^14 + 10816*a^9*b^12 - 2240*a^11*b^10 + 128*a^13*b^8))/(2*b^16) + ((a^4*8i
+ b^4*15i - a^2*b^2*20i)*((256*a*b^16 - 452*a^3*b^14 + 340*a^5*b^12 - 112*a
^7*b^10)/b^11 + (((128*a^2*b^16 - 96*a^4*b^14)/b^11 + (tan(c/2 + (d*x)/2)*(
1024*a*b^22 - 1088*a^3*b^20 + 128*a^5*b^18))/(2*b^16))*(a^4*8i + b^4*15i -
a^2*b^2*20i))/(8*b^5) + (tan(c/2 + (d*x)/2)*(1024*b^22 - 2368*a^2*b^20 + 24
32*a^4*b^18 - 1280*a^6*b^16 + 256*a^8*b^14))/(2*b^16)))/(8*b^5)))/(8*b^5)))
/(8*b^5) - (tan(c/2 + (d*x)/2)*(4500*a*b^18 - 512*a^19 - 30900*a^3*b^16 + 9
4700*a^5*b^14 - 170260*a^7*b^12 + 198160*a^9*b^10 - 155016*a^11*b^8 + 81600
*a^13*b^6 - 27904*a^15*b^4 + 5632*a^17*b^2))/b^16))*(a^4*8i + b^4*15i - a^2
*b^2*20i)*1i)/(4*b^5*d) - (atan((((-(a + b)^5*(a - b)^5)^(1/2)*((840*a*b^14
 - 112*a^15 - 3760*a^3*b^12 + (15271*a^5*b^10)/2 - (18191*a^7*b^8)/2 + (136
97*a^9*b^6)/2 - 3252*a^11*b^4 + 900*a^13*b^2)/b^11 + (tan(c/2 + (d*x)/2)*(3
664*b^20 - 18084*a^2*b^18 + 41125*a^4*b^16 - 56140*a^6*b^14 + 50084*a^8*b^1
2 - 29728*a^10*b^10 + 11392*a^12*b^8 - 2560*a^14*b^6 + 256*a^16*b^4))/(2*b^
16) + ((-(a + b)^5*(a - b)^5)^(1/2)*((128*b^16 + 94*a^2*b^14 - (2395*a^4*b^
12)/2 + 2068*a^6*b^10 - 1600*a^8*b^8 + 608*a^10*b^6 - 96*a^12*b^4)/b^11 + (
tan(c/2 + (d*x)/2)*(4944*a*b^20 - 18960*a^3*b^18 + 29985*a^5*b^16 - 24664*a
^7*b^14 + 10816*a^9*b^12 - 2240*a^11*b^10 + 128*a^13*b^8))/(2*b^16) + ((-(a
 + b)^5*(a - b)^5)^(1/2)*((256*a*b^16 - 452*a^3*b^14 + 340*a^5*b^12 - 112*a
^7*b^10)/b^11 + (tan(c/2 + (d*x)/2)*(1024*b^22 - 2368*a^2*b^20 + 2432*a^4*b
^18 - 1280*a^6*b^16 + 256*a^8*b^14))/(2*b^16) + (((128*a^2*b^16 - 96*a^4*b^
14)/b^11 + (tan(c/2 + (d*x)/2)*(1024*a*b^22 - 1088*a^3*b^20 + 128*a^5*b^18)
)/(2*b^16))*(-(a + b)^5*(a - b)^5)^(1/2))/(a*b^5)))/(a*b^5)))/(a*b^5))*1i)/
(a*b^5) + ((-(a + b)^5*(a - b)^5)^(1/2)*((840*a*b^14 - 112*a^15 - 3760*a^3*
b^12 + (15271*a^5*b^10)/2 - (18191*a^7*b^8)/2 + (13697*a^9*b^6)/2 - 3252*a^
11*b^4 + 900*a^13*b^2)/b^11 + (tan(c/2 + (d*x)/2)*(3664*b^20 - 18084*a^2*b^
18 + 41125*a^4*b^16 - 56140*a^6*b^14 + 50084*a^8*b^12 - 29728*a^10*b^10 + 1
1392*a^12*b^8 - 2560*a^14*b^6 + 256*a^16*b^4))/(2*b^16) - ((-(a + b)^5*(a -
 b)^5)^(1/2)*((128*b^16 + 94*a^2*b^14 - (2395*a^4*b^12)/2 + 2068*a^6*b^10 -
 1600*a^8*b^8 + 608*a^10*b^6 - 96*a^12*b^4)/b^11 + (tan(c/2 + (d*x)/2)*(494
4*a*b^20 - 18960*a^3*b^18 + 29985*a^5*b^16 - 24664*a^7*b^14 + 10816*a^9*b^1
2 - 2240*a^11*b^10 + 128*a^13*b^8))/(2*b^16) - ((-(a + b)^5*(a - b)^5)^(1/2
)*((256*a*b^16 - 452*a^3*b^14 + 340*a^5*b^12 - 112*a^7*b^10)/b^11 + (tan(c/
2 + (d*x)/2)*(1024*b^22 - 2368*a^2*b^20 + 2432*a^4*b^18 - 1280*a^6*b^16 + 2
56*a^8*b^14))/(2*b^16) - (((128*a^2*b^16 - 96*a^4*b^14)/b^11 + (tan(c/2 + (
d*x)/2)*(1024*a*b^22 - 1088*a^3*b^20 + 128*a^5*b^18))/(2*b^16))*(-(a + b)^5
*(a - b)^5)^(1/2))/(a*b^5)))/(a*b^5)))/(a*b^5))*1i)/(a*b^5))/((224*a^14 - 7
80*b^14 + 4445*a^2*b^12 - 10911*a^4*b^10 + 14991*a^6*b^8 - 12481*a^8*b^6 +

```
6312*a^10*b^4 - 1800*a^12*b^2)/b^11 - (tan(c/2 + (d*x)/2)*(4500*a*b^18 - 51
2*a^19 - 30900*a^3*b^16 + 94700*a^5*b^14 - 170260*a^7*b^12 + 198160*a^9*b^1
0 - 155016*a^11*b^8 + 81600*a^13*b^6 - 27904*a^15*b^4 + 5632*a^17*b^2))/b^1
6 - ((-(a + b)^5*(a - b)^5)^(1/2)*((840*a*b^14 - 112*a^15 - 3760*a^3*b^12 +
 (15271*a^5*b^10)/2 - (18191*a^7*b^8)/2 + (13697*a^9*b^6)/2 - 3252*a^11*b^4
 + 900*a^13*b^2)/b^11 + (tan(c/2 + (d*x)/2)*(3664*b^20 - 18084*a^2*b^18 + 4
1125*a^4*b^16 - 56140*a^6*b^14 + 50084*a^8*b^12 - 29728*a^10*b^10 + 11392*a
^12*b^8 - 2560*a^14*b^6 + 256*a^16*b^4))/(2*b^16) + ((-(a + b)^5*(a - b)^5)
^(1/2)*((128*b^16 + 94*a^2*b^14 - (2395*a^4*b^12)/2 + 2068*a^6*b^10 - 1600*
a^8*b^8 + 608*a^10*b^6 - 96*a^12*b^4)/b^11 + (tan(c/2 + (d*x)/2)*(4944*a*b^
20 - 18960*a^3*b^18 + 29985*a^5*b^16 - 24664*a^7*b^14 + 10816*a^9*b^12 - 22
40*a^11*b^10 + 128*a^13*b^8))/(2*b^16) + ((-(a + b)^5*(a - b)^5)^(1/2)*((25
6*a*b^16 - 452*a^3*b^14 + 340*a^5*b^12 - 112*a^7*b^10)/b^11 + (tan(c/2 + (d
*x)/2)*(1024*b^22 - 2368*a^2*b^20 + 2432*a^4*b^18 - 1280*a^6*b^16 + 256*a^8
*b^14))/(2*b^16) + (((128*a^2*b^16 - 96*a^4*b^14)/b^11 + (tan(c/2 + (d*x)/2
)*(1024*a*b^22 - 1088*a^3*b^20 + 128*a^5*b^18))/(2*b^16))*(-(a + b)^5*(a -
b)^5)^(1/2))/(a*b^5)))/(a*b^5)))/(a*b^5)))/(a*b^5) + ((-(a + b)^5*(a - b)^5
)^(1/2)*((840*a*b^14 - 112*a^15 - 3760*a^3*b^12 + (15271*a^5*b^10)/2 - (181
91*a^7*b^8)/2 + (13697*a^9*b^6)/2 - 3252*a^11*b^4 + 900*a^13*b^2)/b^11 + (t
an(c/2 + (d*x)/2)*(3664*b^20 - 18084*a^2*b^18 + 41125*a^4*b^16 - 56140*a^6*
b^14 + 50084*a^8*b^12 - 29728*a^10*b^10 + 11392*a^12*b^8 - 2560*a^14*b^6 +
256*a^16*b^4))/(2*b^16) - ((-(a + b)^5*(a - b)^5)^(1/2)*((128*b^16 + 94*a^2
*b^14 - (2395*a^4*b^12)/2 + 2068*a^6*b^10 - 1600*a^8*b^8 + 608*a^10*b^6 - 9
6*a^12*b^4)/b^11 + (tan(c/2 + (d*x)/2)*(4944*a*b^20 - 18960*a^3*b^18 + 2998
5*a^5*b^16 - 24664*a^7*b^14 + 10816*a^9*b^12 - 2240*a^11*b^10 + 128*a^13*b^
8))/(2*b^16) - ((-(a + b)^5*(a - b)^5)^(1/2)*((256*a*b^16 - 452*a^3*b^14 +
340*a^5*b^12 - 112*a^7*b^10)/b^11 + (tan(c/2 + (d*x)/2)*(1024*b^22 - 2368*a
^2*b^20 + 2432*a^4*b^18 - 1280*a^6*b^16 + 256*a^8*b^14))/(2*b^16) - (((128*
a^2*b^16 - 96*a^4*b^14)/b^11 + (tan(c/2 + (d*x)/2)*(1024*a*b^22 - 1088*a^3*
b^20 + 128*a^5*b^18))/(2*b^16))*(-(a + b)^5*(a - b)^5)^(1/2))/(a*b^5)))/(a*
b^5)))/(a*b^5)))/(a*b^5)))*(-(a + b)^5*(a - b)^5)^(1/2)*2i)/(a*b^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**6*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.1324 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=183

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^4d}+\frac{ax\left(a^2-3b^2\right)}{b^4}+\frac{\left(a^2-3b^2\right)\cos(c+dx)}{b^3d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{a\sin(c+$$

```
[Out] 1/2*a*x/b^2+a*(a^2-3*b^2)*x/b^4-2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1
/2*c))/(a^2-b^2)^(1/2))/a^2/b^4/d+b*arctanh(cos(d*x+c))/a^2/d+cos(d*x+c)/b/
d+(a^2-3*b^2)*cos(d*x+c)/b^3/d-1/3*cos(d*x+c)^3/b/d-cot(d*x+c)/a/d-1/2*a*co
s(d*x+c)*sin(d*x+c)/b^2/d
```

Rubi [A] time = 0.26, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2897, 3770, 3767, 8, 2638, 2635, 2633, 2660, 618, 204}

$$\frac{\left(a^2-3b^2\right)\cos(c+dx)}{b^3d}-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^4d}+\frac{ax\left(a^2-3b^2\right)}{b^4}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/(2*b^2) + (a*(a^2 - 3*b^2)*x)/b^4 - (2*(a^2 - b^2)^(5/2)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*b^4*d) + (b*ArcTanh[Cos[c + d*x]
])/(a^2*d) + Cos[c + d*x]/(b*d) + ((a^2 - 3*b^2)*Cos[c + d*x])/(b^3*d) - Co
s[c + d*x]^3/(3*b*d) - Cot[c + d*x]/(a*d) - (a*Cos[c + d*x]*Sin[c + d*x])/(
2*b^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int\left(\frac{a^3-3ab^2}{b^4}-\frac{b\csc(c+dx)}{a^2}+\frac{\csc^2(c+dx)}{a}+\frac{\left(-a^2+3b^2\right)\sin(c+dx)}{b^3}+\frac{a}{}\right.
$$

$$= \frac{a\left(a^2-3b^2\right)x}{b^4}+\frac{\int\csc^2(c+dx)\,dx}{a}+\frac{a\int\sin^2(c+dx)\,dx}{b^2}-\frac{\int\sin^3(c+dx)\,dx}{b}$$

$$= \frac{a\left(a^2-3b^2\right)x}{b^4}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\left(a^2-3b^2\right)\cos(c+dx)}{b^3d}-\frac{a\cos(c+$$

$$= \frac{ax}{2b^2}+\frac{a\left(a^2-3b^2\right)x}{b^4}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{bd}+\frac{\left(a^2-3b^2\right)\cos}{b^3d}$$

$$= \frac{ax}{2b^2}+\frac{a\left(a^2-3b^2\right)x}{b^4}-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^4d}+\frac{b\tanh^{-1}(\cos(}{a^2d}$$

Mathematica [A] time = 1.41, size = 208, normalized size = 1.14

$$-\frac{-12a^5c-12a^5dx+3a^3b^2\sin(2(c+dx))+30a^3b^2c+30a^3b^2dx+a^2b^3\cos(3(c+dx))-3a^2b\left(4a^2-9b^2\right)\cos(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/12*(-12*a^5*c + 30*a^3*b^2*c - 12*a^5*d*x + 30*a^3*b^2*d*x + 24*(a^2 - b
^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 3*a^2*b*(4*a^2
 - 9*b^2)*Cos[c + d*x] + a^2*b^3*Cos[3*(c + d*x)] + 6*a*b^4*Cot[(c + d*x)/2
] - 12*b^5*Log[Cos[(c + d*x)/2]] + 12*b^5*Log[Sin[(c + d*x)/2]] + 3*a^3*b^2
*Sin[2*(c + d*x)] - 6*a*b^4*Tan[(c + d*x)/2])/(a^2*b^4*d)
```

fricas [A] time = 1.28, size = 549, normalized size = 3.00

$$\left[\frac{3a^3b^2\cos(dx+c)^3+3b^5\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-3b^5\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^3*b^2*cos(d*x + c)^3 + 3*b^5*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 3*b^5*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(a^4 - 2*a^2*b^2
 + b^4)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
+ c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a
^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x +
 c) - 3*(a^3*b^2 + 2*a*b^4)*cos(d*x + c) - (2*a^2*b^3*cos(d*x + c)^3 - 3*(2
*a^5 - 5*a^3*b^2)*d*x - 6*(a^4*b - 2*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(
a^2*b^4*d*sin(d*x + c)), 1/6*(3*a^3*b^2*cos(d*x + c)^3 + 3*b^5*log(1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) - 3*b^5*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 6*(a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b
)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 3*(a^3*b^2 + 2*a*b^4)*cos(
d*x + c) - (2*a^2*b^3*cos(d*x + c)^3 - 3*(2*a^5 - 5*a^3*b^2)*d*x - 6*(a^4*b
 - 2*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^2*b^4*d*sin(d*x + c))]
```

giac [A] time = 0.19, size = 302, normalized size = 1.65

$$-\frac{\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{3\left(2\,a^3-5\,ab^2\right)(dx+c)}{b^4}-\frac{3\left(2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{12\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)\right.}{\sqrt{a^2-b^2}\,a^2b^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 3*tan(1/2*d*x + 1/2*c)/a - 3
*(2*a^3 - 5*a*b^2)*(d*x + c)/b^4 - 3*(2*b*tan(1/2*d*x + 1/2*c) - a)/(a^2*ta
n(1/2*d*x + 1/2*c)) + 12*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 -
 b^2)))/(sqrt(a^2 - b^2)*a^2*b^4) - 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2
*tan(1/2*d*x + 1/2*c)^4 - 18*b^2*tan(1/2*d*x + 1/2*c)^4 + 12*a^2*tan(1/2*d*
x + 1/2*c)^2 - 24*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) +
 6*a^2 - 14*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^3))/d
```

maple [B] time = 0.44, size = 557, normalized size = 3.04

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{6\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4a^2\left(\tan^2\left(\frac{dx}{2}+\right.\right.}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)+1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x
+1/2*c)^5+2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2-6/d/b
/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4+4/d/b^3/(1+tan(1/2*d*x+1/2
*c)^2)^3*a^2*tan(1/2*d*x+1/2*c)^2-8/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*
d*x+1/2*c)^2-1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)+2/d/b^
3/(1+tan(1/2*d*x+1/2*c)^2)^3*a^2-14/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3+2/d/b^
4*arctan(tan(1/2*d*x+1/2*c))*a^3-5/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))-1/2/a
/d/tan(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tan(1/2*d*x+1/2*c))-2/d*a^4/b^4/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d/b^2/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2-
6/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)
)+2/d*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-
b^2)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 13.10, size = 4892, normalized size = 26.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a*d) - (3*tan(c/2 + (d*x)/2)^4 + (8*tan(c/2 + (d*x)/2
)^3*(2*a*b^2 - a^3))/b^3 + (4*tan(c/2 + (d*x)/2)^5*(3*a*b^2 - a^3))/b^3 + (
tan(c/2 + (d*x)/2)^2*(2*a^2 + 3*b^2))/b^2 - (tan(c/2 + (d*x)/2)^6*(2*a^2 -
b^2))/b^2 + (4*tan(c/2 + (d*x)/2)*(7*a*b^2 - 3*a^3))/(3*b^3) + 1)/(d*(2*a*t
an(c/2 + (d*x)/2) + 6*a*tan(c/2 + (d*x)/2)^3 + 6*a*tan(c/2 + (d*x)/2)^5 + 2
*a*tan(c/2 + (d*x)/2)^7)) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d) - (atan(((
-(a + b)^5*(a - b)^5)^(1/2)*((8*(10*a^2*b^12 - 14*a^14 + 136*a^4*b^10 - 386
*a^6*b^8 + 467*a^8*b^6 - 309*a^10*b^4 + 105*a^12*b^2))/(a^2*b^8) + ((-(a +
```

b)^5*(a - b)^5)^(1/2)*((8*(16*b^15 - 52*a^2*b^13 + 50*a^4*b^11 + 86*a^6*b^9 - 155*a^8*b^7 + 76*a^10*b^5 - 12*a^12*b^3))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(84*a^2*b^18 - 360*a^4*b^16 + 1020*a^6*b^14 - 1264*a^8*b^12 + 661*a^10*b^10 - 140*a^12*b^8 + 8*a^14*b^6))/(a^3*b^12) + ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(32*a^2*b^14 - 64*a^4*b^12 + 50*a^6*b^10 - 14*a^8*b^8))/(a^2*b^8) + (((8*(16*a^4*b^13 - 12*a^6*b^11))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^18 - 68*a^6*b^16 + 8*a^8*b^14))/(a^3*b^12))*(-(a + b)^5*(a - b)^5)^(1/2))/(a^2*b^4) + (8*tan(c/2 + (d*x)/2)*(64*a^2*b^19 - 148*a^4*b^17 + 152*a^6*b^15 - 80*a^8*b^13 + 16*a^10*b^11))/(a^3*b^12)))/(a^2*b^4)))/(a^2*b^4) + (8*tan(c/2 + (d*x)/2)*(4*b^19 - 24*a^2*b^17 + 460*a^4*b^15 - 1300*a^6*b^13 + 1769*a^8*b^11 - 1408*a^10*b^9 + 652*a^12*b^7 - 160*a^14*b^5 + 16*a^16*b^3))/(a^3*b^12))*1i)/(a^2*b^4) + ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(10*a^2*b^12 - 14*a^14 + 136*a^4*b^10 - 386*a^6*b^8 + 467*a^8*b^6 - 309*a^10*b^4 + 105*a^12*b^2))/(a^2*b^8) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(16*b^15 - 52*a^2*b^13 + 50*a^4*b^11 + 86*a^6*b^9 - 155*a^8*b^7 + 76*a^10*b^5 - 12*a^12*b^3))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(84*a^2*b^18 - 360*a^4*b^16 + 1020*a^6*b^14 - 1264*a^8*b^12 + 661*a^10*b^10 - 140*a^12*b^8 + 8*a^14*b^6))/(a^3*b^12) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(32*a^2*b^14 - 64*a^4*b^12 + 50*a^6*b^10 - 14*a^8*b^8))/(a^2*b^8) - (((8*(16*a^4*b^13 - 12*a^6*b^11))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^18 - 68*a^6*b^16 + 8*a^8*b^14))/(a^3*b^12))*(-(a + b)^5*(a - b)^5)^(1/2))/(a^2*b^4) + (8*tan(c/2 + (d*x)/2)*(64*a^2*b^19 - 148*a^4*b^17 + 152*a^6*b^15 - 80*a^8*b^13 + 16*a^10*b^11))/(a^3*b^12)))/(a^2*b^4)))/(a^2*b^4) + (8*tan(c/2 + (d*x)/2)*(4*b^19 - 24*a^2*b^17 + 460*a^4*b^15 - 1300*a^6*b^13 + 1769*a^8*b^11 - 1408*a^10*b^9 + 652*a^12*b^7 - 160*a^14*b^5 + 16*a^16*b^3))/(a^3*b^12))*1i)/(a^2*b^4))/((16*(10*b^13 - 14*a^12*b + 36*a^2*b^11 - 231*a^4*b^9 + 391*a^6*b^7 - 297*a^8*b^5 + 105*a^10*b^3))/(a^2*b^8) + ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(10*a^2*b^12 - 14*a^14 + 136*a^4*b^10 - 386*a^6*b^8 + 467*a^8*b^6 - 309*a^10*b^4 + 105*a^12*b^2))/(a^2*b^8) + ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(16*b^15 - 52*a^2*b^13 + 50*a^4*b^11 + 86*a^6*b^9 - 155*a^8*b^7 + 76*a^10*b^5 - 12*a^12*b^3))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(84*a^2*b^18 - 360*a^4*b^16 + 1020*a^6*b^14 - 1264*a^8*b^12 + 661*a^10*b^10 - 140*a^12*b^8 + 8*a^14*b^6))/(a^3*b^12) + ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(32*a^2*b^14 - 64*a^4*b^12 + 50*a^6*b^10 - 14*a^8*b^8))/(a^2*b^8) + (((8*(16*a^4*b^13 - 12*a^6*b^11))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^18 - 68*a^6*b^16 + 8*a^8*b^14))/(a^3*b^12))*(-(a + b)^5*(a - b)^5)^(1/2))/(a^2*b^4) + (8*tan(c/2 + (d*x)/2)*(64*a^2*b^19 - 148*a^4*b^17 + 152*a^6*b^15 - 80*a^8*b^13 + 16*a^10*b^11))/(a^3*b^12)))/(a^2*b^4)))/(a^2*b^4) + (8*tan(c/2 + (d*x)/2)*(4*b^19 - 24*a^2*b^17 + 460*a^4*b^15 - 1300*a^6*b^13 + 1769*a^8*b^11 - 1408*a^10*b^9 + 652*a^12*b^7 - 160*a^14*b^5 + 16*a^16*b^3))/(a^3*b^12)))/(a^2*b^4) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(10*a^2*b^12 - 14*a^14 + 136*a^4*b^10 - 386*a^6*b^8 + 467*a^8*b^6 - 309*a^10*b^4 + 105*a^12*b^2))/(a^2*b^8) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(16*b^15 - 52*a^2*b^13 + 50*a^4*b^11 + 86*a^6*b^9 - 155*a^8*b^7 + 76*a^10*b^5 - 12*a^12*b^3))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(84*a^2*b^18 - 360*a^4*b^16 + 1020*a^6*b^14 - 1264*a^8*b^12 + 661*a^10*b^10 - 140*a^12*b^8 + 8*a^14

*b^6))/(a^3*b^12) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(32*a^2*b^14 - 64*a^4
*b^12 + 50*a^6*b^10 - 14*a^8*b^8))/(a^2*b^8) - (((8*(16*a^4*b^13 - 12*a^6*b
^11))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^18 - 68*a^6*b^16 + 8*a^8*
b^14))/(a^3*b^12))*(-(a + b)^5*(a - b)^5)^(1/2))/(a^2*b^4) + (8*tan(c/2 + (
d*x)/2)*(64*a^2*b^19 - 148*a^4*b^17 + 152*a^6*b^15 - 80*a^8*b^13 + 16*a^10*
b^11))/(a^3*b^12)))/(a^2*b^4)))/(a^2*b^4) + (8*tan(c/2 + (d*x)/2)*(4*b^19 -
 24*a^2*b^17 + 460*a^4*b^15 - 1300*a^6*b^13 + 1769*a^8*b^11 - 1408*a^10*b^9
 + 652*a^12*b^7 - 160*a^14*b^5 + 16*a^16*b^3))/(a^3*b^12)))/(a^2*b^4) - (16
*tan(c/2 + (d*x)/2)*(32*a^18 - 500*a^4*b^14 + 2500*a^6*b^12 - 5260*a^8*b^10
 + 6036*a^10*b^8 - 4080*a^12*b^6 + 1624*a^14*b^4 - 352*a^16*b^2))/(a^3*b^12
)))*(-(a + b)^5*(a - b)^5)^(1/2)*2i)/(a^2*b^4*d) - (a*atan(((a*(2*a^2 - 5*b
^2)*((8*(10*a^2*b^12 - 14*a^14 + 136*a^4*b^10 - 386*a^6*b^8 + 467*a^8*b^6 -
 309*a^10*b^4 + 105*a^12*b^2))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(4*b^19 -
24*a^2*b^17 + 460*a^4*b^15 - 1300*a^6*b^13 + 1769*a^8*b^11 - 1408*a^10*b^9
+ 652*a^12*b^7 - 160*a^14*b^5 + 16*a^16*b^3))/(a^3*b^12) - (a*(2*a^2 - 5*b^
2)*((8*(16*b^15 - 52*a^2*b^13 + 50*a^4*b^11 + 86*a^6*b^9 - 155*a^8*b^7 + 76
*a^10*b^5 - 12*a^12*b^3))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(84*a^2*b^18 -
360*a^4*b^16 + 1020*a^6*b^14 - 1264*a^8*b^12 + 661*a^10*b^10 - 140*a^12*b^8
 + 8*a^14*b^6))/(a^3*b^12) - (a*(2*a^2 - 5*b^2)*((8*(32*a^2*b^14 - 64*a^4*b
^12 + 50*a^6*b^10 - 14*a^8*b^8))/(a^2*b^8) - (a*((8*(16*a^4*b^13 - 12*a^6*b
^11))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^18 - 68*a^6*b^16 + 8*a^8*
b^14))/(a^3*b^12))*(2*a^2 - 5*b^2)*1i)/(2*b^4) + (8*tan(c/2 + (d*x)/2)*(64*
a^2*b^19 - 148*a^4*b^17 + 152*a^6*b^15 - 80*a^8*b^13 + 16*a^10*b^11))/(a^3*
b^12))*1i)/(2*b^4))*1i)/(2*b^4)))/(2*b^4) + (a*(2*a^2 - 5*b^2)*((8*(10*a^2*
b^12 - 14*a^14 + 136*a^4*b^10 - 386*a^6*b^8 + 467*a^8*b^6 - 309*a^10*b^4 +
105*a^12*b^2))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(4*b^19 - 24*a^2*b^17 + 46
0*a^4*b^15 - 1300*a^6*b^13 + 1769*a^8*b^11 - 1408*a^10*b^9 + 652*a^12*b^7 -
 160*a^14*b^5 + 16*a^16*b^3))/(a^3*b^12) + (a*(2*a^2 - 5*b^2)*((8*(16*b^15
- 52*a^2*b^13 + 50*a^4*b^11 + 86*a^6*b^9 - 155*a^8*b^7 + 76*a^10*b^5 - 12*a
^12*b^3))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(84*a^2*b^18 - 360*a^4*b^16 + 1
020*a^6*b^14 - 1264*a^8*b^12 + 661*a^10*b^10 - 140*a^12*b^8 + 8*a^14*b^6))/
(a^3*b^12) + (a*(2*a^2 - 5*b^2)*((8*(32*a^2*b^14 - 64*a^4*b^12 + 50*a^6*b^1
0 - 14*a^8*b^8))/(a^2*b^8) + (a*((8*(16*a^4*b^13 - 12*a^6*b^11))/(a^2*b^8)
+ (8*tan(c/2 + (d*x)/2)*(64*a^4*b^18 - 68*a^6*b^16 + 8*a^8*b^14))/(a^3*b^12
))*(2*a^2 - 5*b^2)*1i)/(2*b^4) + (8*tan(c/2 + (d*x)/2)*(64*a^2*b^19 - 148*a
^4*b^17 + 152*a^6*b^15 - 80*a^8*b^13 + 16*a^10*b^11))/(a^3*b^12))*1i)/(2*b^
4))*1i)/(2*b^4)))/(2*b^4))/(((16*(10*b^13 - 14*a^12*b + 36*a^2*b^11 - 231*a^
4*b^9 + 391*a^6*b^7 - 297*a^8*b^5 + 105*a^10*b^3))/(a^2*b^8) - (16*tan(c/2
+ (d*x)/2)*(32*a^18 - 500*a^4*b^14 + 2500*a^6*b^12 - 5260*a^8*b^10 + 6036*a
^10*b^8 - 4080*a^12*b^6 + 1624*a^14*b^4 - 352*a^16*b^2))/(a^3*b^12) - (a*(2
*a^2 - 5*b^2)*((8*(10*a^2*b^12 - 14*a^14 + 136*a^4*b^10 - 386*a^6*b^8 + 467
*a^8*b^6 - 309*a^10*b^4 + 105*a^12*b^2))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*
(4*b^19 - 24*a^2*b^17 + 460*a^4*b^15 - 1300*a^6*b^13 + 1769*a^8*b^11 - 1408
*a^10*b^9 + 652*a^12*b^7 - 160*a^14*b^5 + 16*a^16*b^3))/(a^3*b^12) - (a*(2*
a^2 - 5*b^2)*((8*(16*b^15 - 52*a^2*b^13 + 50*a^4*b^11 + 86*a^6*b^9 - 155*a^

```
8*b^7 + 76*a^10*b^5 - 12*a^12*b^3))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(84*a
^2*b^18 - 360*a^4*b^16 + 1020*a^6*b^14 - 1264*a^8*b^12 + 661*a^10*b^10 - 14
0*a^12*b^8 + 8*a^14*b^6))/(a^3*b^12) - (a*(2*a^2 - 5*b^2)*((8*(32*a^2*b^14
- 64*a^4*b^12 + 50*a^6*b^10 - 14*a^8*b^8))/(a^2*b^8) - (a*((8*(16*a^4*b^13
- 12*a^6*b^11))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^18 - 68*a^6*b^1
6 + 8*a^8*b^14))/(a^3*b^12))*(2*a^2 - 5*b^2)*1i)/(2*b^4) + (8*tan(c/2 + (d*
x)/2)*(64*a^2*b^19 - 148*a^4*b^17 + 152*a^6*b^15 - 80*a^8*b^13 + 16*a^10*b^
11))/(a^3*b^12))*1i)/(2*b^4))*1i)/(2*b^4))*1i)/(2*b^4) + (a*(2*a^2 - 5*b^2)
*((8*(10*a^2*b^12 - 14*a^14 + 136*a^4*b^10 - 386*a^6*b^8 + 467*a^8*b^6 - 30
9*a^10*b^4 + 105*a^12*b^2))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(4*b^19 - 24*
a^2*b^17 + 460*a^4*b^15 - 1300*a^6*b^13 + 1769*a^8*b^11 - 1408*a^10*b^9 + 6
52*a^12*b^7 - 160*a^14*b^5 + 16*a^16*b^3))/(a^3*b^12) + (a*(2*a^2 - 5*b^2)*
((8*(16*b^15 - 52*a^2*b^13 + 50*a^4*b^11 + 86*a^6*b^9 - 155*a^8*b^7 + 76*a^
10*b^5 - 12*a^12*b^3))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(84*a^2*b^18 - 360
*a^4*b^16 + 1020*a^6*b^14 - 1264*a^8*b^12 + 661*a^10*b^10 - 140*a^12*b^8 +
8*a^14*b^6))/(a^3*b^12) + (a*(2*a^2 - 5*b^2)*((8*(32*a^2*b^14 - 64*a^4*b^12
 + 50*a^6*b^10 - 14*a^8*b^8))/(a^2*b^8) + (a*((8*(16*a^4*b^13 - 12*a^6*b^11
))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^18 - 68*a^6*b^16 + 8*a^8*b^1
4))/(a^3*b^12))*(2*a^2 - 5*b^2)*1i)/(2*b^4) + (8*tan(c/2 + (d*x)/2)*(64*a^2
*b^19 - 148*a^4*b^17 + 152*a^6*b^15 - 80*a^8*b^13 + 16*a^10*b^11))/(a^3*b^1
2))*1i)/(2*b^4))*1i)/(2*b^4))*1i)/(2*b^4)))*(2*a^2 - 5*b^2))/(b^4*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1325 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=174

$$-\frac{x\left(2a^2-5b^2\right)}{2b^3}+\frac{b\cot(c+dx)}{a^2d}+\frac{\left(5a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}-\frac{a\cos(c}{b^2 d}$$

```
[Out] -1/2*(2*a^2-5*b^2)*x/b^3+2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/
(a^2-b^2)^(1/2))/a^3/b^3/d+1/2*(5*a^2-2*b^2)*arctanh(cos(d*x+c))/a^3/d-a*co
s(d*x+c)/b^2/d+b*cot(d*x+c)/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)/a/d+1/2*cos(d*x
+c)*sin(d*x+c)/b/d
```

Rubi [A] time = 0.38, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2896, 3057, 2660, 618, 204, 3770}

$$\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{\left(5a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{x\left(2a^2-5b^2\right)}{2b^3}+\frac{b\cot(c+dx)}{a^2d}-\frac{a\cos(c+}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((2*a^2 - 5*b^2)*x)/(2*b^3) + (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^3*d) + ((5*a^2 - 2*b^2)*ArcTanh[Cos[c + d
*x]])/(2*a^3*d) - (a*Cos[c + d*x])/(b^2*d) + (b*Cot[c + d*x])/(a^2*d) - (Co
t[c + d*x]*Csc[c + d*x])/(2*a*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{a\cos(c+dx)}{b^2d} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\cos(c+dx)\sin(c}{2bd}$$

$$= -\frac{\left(2a^2-5b^2\right)x}{2b^3} - \frac{a\cos(c+dx)}{b^2d} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + $$

$$= -\frac{\left(2a^2-5b^2\right)x}{2b^3} + \frac{\left(5a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{a\cos(c+dx)}{b^2d} + \frac{b\cot(c}{a^2}$$

$$= -\frac{\left(2a^2-5b^2\right)x}{2b^3} + \frac{\left(5a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{a\cos(c+dx)}{b^2d} + \frac{b\cot(c}{a^2}$$

$$= -\frac{\left(2a^2-5b^2\right)x}{2b^3} + \frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^3d} + \frac{\left(5a^2-2b^2\right)\tanh^{-1}(}{2a^3d}$$

Mathematica [A] time = 5.26, size = 259, normalized size = 1.49

$$\frac{-8a^5c - 8a^5dx - 8a^4b\cos(c+dx) + 2a^3b^2\sin(2(c+dx)) + 20a^3b^2c + 20a^3b^2dx - a^2b^3\csc^2\left(\frac{1}{2}(c+dx)\right) + a^2b^3 s}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-8*a^5*c + 20*a^3*b^2*c - 8*a^5*d*x + 20*a^3*b^2*d*x + 16*(a^2 - b^2)^(5/2
)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 8*a^4*b*Cos[c + d*x] +
 4*a*b^4*Cot[(c + d*x)/2] - a^2*b^3*Csc[(c + d*x)/2]^2 + 20*a^2*b^3*Log[Cos
[(c + d*x)/2]] - 8*b^5*Log[Cos[(c + d*x)/2]] - 20*a^2*b^3*Log[Sin[(c + d*x)
/2]] + 8*b^5*Log[Sin[(c + d*x)/2]] + a^2*b^3*Sec[(c + d*x)/2]^2 + 2*a^3*b^2
*Sin[2*(c + d*x)] - 4*a*b^4*Tan[(c + d*x)/2])/(8*a^3*b^3*d)
```

fricas [B] time = 1.82, size = 770, normalized size = 4.43

$$\left[-\frac{4\,a^4b\cos\left(dx+c\right)^3 + 2\left(2\,a^5-5\,a^3b^2\right)dx\cos\left(dx+c\right)^2 - 2\left(2\,a^5-5\,a^3b^2\right)dx + 2\left(a^4-2\,a^2b^2+b^4-\left(a^4-2\,a^2b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^4*b*cos(d*x + c)^3 + 2*(2*a^5 - 5*a^3*b^2)*d*x*cos(d*x + c)^2 -
2*(2*a^5 - 5*a^3*b^2)*d*x + 2*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b
^4)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2
*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
+ c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^
2)) - 2*(2*a^4*b + a^2*b^3)*cos(d*x + c) + (5*a^2*b^3 - 2*b^5 - (5*a^2*b^3
- 2*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) - (5*a^2*b^3 - 2*b^5 -
 (5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*
b^2*cos(d*x + c)^3 - (a^3*b^2 + 2*a*b^4)*cos(d*x + c))*sin(d*x + c))/(a^3*b
^3*d*cos(d*x + c)^2 - a^3*b^3*d), -1/4*(4*a^4*b*cos(d*x + c)^3 + 2*(2*a^5 -
 5*a^3*b^2)*d*x*cos(d*x + c)^2 - 2*(2*a^5 - 5*a^3*b^2)*d*x - 4*(a^4 - 2*a^2
*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan
(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 2*(2*a^4*b + a^2*b
^3)*cos(d*x + c) + (5*a^2*b^3 - 2*b^5 - (5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)
*log(1/2*cos(d*x + c) + 1/2) - (5*a^2*b^3 - 2*b^5 - (5*a^2*b^3 - 2*b^5)*cos
(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*b^2*cos(d*x + c)^3 - (a^
3*b^2 + 2*a*b^4)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*cos(d*x + c)^2 - a^
3*b^3*d)]
```

giac [B] time = 0.21, size = 431, normalized size = 2.48

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}}-\frac{4\left(2\,a^{2}-5\,b^{2}\right)(dx+c)}{b^{3}}-\frac{4\left(5\,a^{2}-2\,b^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{3}}+\frac{16\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\right.}{\sqrt{a^{2}-b^{2}}\,a^{3}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*((a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/a^2 - 4*(2*a^2 -
 5*b^2)*(d*x + c)/b^3 - 4*(5*a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^
3 + 16*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2
- b^2)*a^3*b^3) + (10*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 - 4*b^4*tan(1/2*d*x +
1/2*c)^6 - 8*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 4*a*b^3*tan(1/2*d*x + 1/2*c)^5
- 16*a^4*tan(1/2*d*x + 1/2*c)^4 + 19*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 8*b^4
*tan(1/2*d*x + 1/2*c)^4 + 8*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 8*a*b^3*tan(1/2*
d*x + 1/2*c)^3 - 16*a^4*tan(1/2*d*x + 1/2*c)^2 + 8*a^2*b^2*tan(1/2*d*x + 1/
2*c)^2 - 4*b^4*tan(1/2*d*x + 1/2*c)^2 + 4*a*b^3*tan(1/2*d*x + 1/2*c) - a^2*
b^2)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2*a^3*b^2))/d
```

maple [B] time = 0.48, size = 483, normalized size = 2.78

$$\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{2d\,a^{2}}-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{2\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/8/a/d*tan(1/2*d*x+1/2*c)^2-1/2/d/a^2*tan(1/2*d*x+1/2*c)*b-1/d/b/(1+tan(1/
2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*t
an(1/2*d*x+1/2*c)^2*a+1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-2
/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*a-2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2+
5/d/b*arctan(tan(1/2*d*x+1/2*c))-1/8/a/d/tan(1/2*d*x+1/2*c)^2-5/2/a/d*ln(ta
n(1/2*d*x+1/2*c))+1/d/a^3*ln(tan(1/2*d*x+1/2*c))*b^2+1/2/d/a^2*b/tan(1/2*d*
x+1/2*c)+2/d*a^3/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b
)/(a^2-b^2)^(1/2))-6/d*a/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2))+6/d*b/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*
x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a^3*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.09, size = 4898, normalized size = 28.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - (a/2 - 2*b*tan(c/2 + (d*x)/2) - (4*tan(c/2 +
 (d*x)/2)^3*(a^2 + b^2))/b + (tan(c/2 + (d*x)/2)^2*(a*b^2 + 8*a^3))/b^2 + (
```

tan(c/2 + (d*x)/2)^4*(a*b^2 + 16*a^3))/(2*b^2) + (2*tan(c/2 + (d*x)/2)^5*(2*a^2 - b^2))/b)/(d*(4*a^2*tan(c/2 + (d*x)/2)^2 + 8*a^2*tan(c/2 + (d*x)/2)^4 + 4*a^2*tan(c/2 + (d*x)/2)^6)) - (b*tan(c/2 + (d*x)/2))/(2*a^2*d) + (atan((((a^2*1i - (b^2*5i)/2)*((4*(20*a^3*b^12 - 28*a^15 + 272*a^5*b^10 - 1272*a^7*b^8 + 1709*a^9*b^6 - 998*a^11*b^4 + 270*a^13*b^2))/(a^6*b^5) + ((a^2*1i - (b^2*5i)/2)*(((a^2*1i - (b^2*5i)/2)*((4*(64*a^5*b^12 - 208*a^7*b^10 + 160*a^9*b^8 - 28*a^11*b^6))/(a^6*b^5) + (((4*(32*a^8*b^10 - 24*a^10*b^8))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(128*a^7*b^14 - 136*a^9*b^12 + 16*a^11*b^10))/(a^6*b^8))*(a^2*1i - (b^2*5i)/2))/b^3 + (4*tan(c/2 + (d*x)/2)*(128*a^4*b^16 - 456*a^6*b^14 + 484*a^8*b^12 - 200*a^10*b^10 + 32*a^12*b^8))/(a^6*b^8)))/b^3 - (4*(184*a^4*b^12 - 32*a^2*b^14 - 360*a^6*b^10 + 78*a^8*b^8 + 240*a^10*b^6 - 152*a^12*b^4 + 24*a^14*b^2))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(8*a^3*b^16 - 160*a^5*b^14 + 1320*a^7*b^12 - 2128*a^9*b^10 + 1242*a^11*b^8 - 280*a^13*b^6 + 16*a^15*b^4))/(a^6*b^8)))/b^3 + (4*tan(c/2 + (d*x)/2)*(8*b^18 - 68*a^2*b^16 + 1040*a^4*b^14 - 3900*a^6*b^12 + 5738*a^8*b^10 - 4201*a^10*b^8 + 1684*a^12*b^6 - 360*a^14*b^4 + 32*a^16*b^2))/(a^6*b^8))*1i)/b^3 + ((a^2*1i - (b^2*5i)/2)*((4*(20*a^3*b^12 - 28*a^15 + 272*a^5*b^10 - 1272*a^7*b^8 + 1709*a^9*b^6 - 998*a^11*b^4 + 270*a^13*b^2))/(a^6*b^5) + ((a^2*1i - (b^2*5i)/2)*(((a^2*1i - (b^2*5i)/2)*((4*(64*a^5*b^12 - 208*a^7*b^10 + 160*a^9*b^8 - 28*a^11*b^6))/(a^6*b^5) - (((4*(32*a^8*b^10 - 24*a^10*b^8))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(128*a^7*b^14 - 136*a^9*b^12 + 16*a^11*b^10))/(a^6*b^8))*(a^2*1i - (b^2*5i)/2))/b^3 + (4*tan(c/2 + (d*x)/2)*(128*a^4*b^16 - 456*a^6*b^14 + 484*a^8*b^12 - 200*a^10*b^10 + 32*a^12*b^8))/(a^6*b^8)))/b^3 + (4*(184*a^4*b^12 - 32*a^2*b^14 - 360*a^6*b^10 + 78*a^8*b^8 + 240*a^10*b^6 - 152*a^12*b^4 + 24*a^14*b^2))/(a^6*b^5) - (4*tan(c/2 + (d*x)/2)*(8*a^3*b^16 - 160*a^5*b^14 + 1320*a^7*b^12 - 2128*a^9*b^10 + 1242*a^11*b^8 - 280*a^13*b^6 + 16*a^15*b^4))/(a^6*b^8)))/b^3 + (4*tan(c/2 + (d*x)/2)*(8*b^18 - 68*a^2*b^16 + 1040*a^4*b^14 - 3900*a^6*b^12 + 5738*a^8*b^10 - 4201*a^10*b^8 + 1684*a^12*b^6 - 360*a^14*b^4 + 32*a^16*b^2))/(a^6*b^8))*1i)/b^3)/(((a^2*1i - (b^2*5i)/2)*((4*(20*a^3*b^12 - 28*a^15 + 272*a^5*b^10 - 1272*a^7*b^8 + 1709*a^9*b^6 - 998*a^11*b^4 + 270*a^13*b^2))/(a^6*b^5) + ((a^2*1i - (b^2*5i)/2)*(((a^2*1i - (b^2*5i)/2)*((4*(64*a^5*b^12 - 208*a^7*b^10 + 160*a^9*b^8 - 28*a^11*b^6))/(a^6*b^5) + (((4*(32*a^8*b^10 - 24*a^10*b^8))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(128*a^7*b^14 - 136*a^9*b^12 + 16*a^11*b^10))/(a^6*b^8))*(a^2*1i - (b^2*5i)/2))/b^3 + (4*tan(c/2 + (d*x)/2)*(128*a^4*b^16 - 456*a^6*b^14 + 484*a^8*b^12 - 200*a^10*b^10 + 32*a^12*b^8))/(a^6*b^8)))/b^3 - (4*(184*a^4*b^12 - 32*a^2*b^14 - 360*a^6*b^10 + 78*a^8*b^8 + 240*a^10*b^6 - 152*a^12*b^4 + 24*a^14*b^2))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(8*a^3*b^16 - 160*a^5*b^14 + 1320*a^7*b^12 - 2128*a^9*b^10 + 1242*a^11*b^8 - 280*a^13*b^6 + 16*a^15*b^4))/(a^6*b^8)))/b^3 + (4*tan(c/2 + (d*x)/2)*(8*b^18 - 68*a^2*b^16 + 1040*a^4*b^14 - 3900*a^6*b^12 + 5738*a^8*b^10 - 4201*a^10*b^8 + 1684*a^12*b^6 - 360*a^14*b^4 + 32*a^16*b^2))/(a^6*b^8)))/b^3 - ((a^2*1i - (b^2*5i)/2)*((4*(20*a^3*b^12 - 28*a^15 + 272*a^5*b^10 - 1272*a^7*b^8 + 1709*a^9*b^6 - 998*a^11*b^4 + 270*a^13*b^2))/(a^6*b^5) + ((a^2*1i - (b^2*5i)/2)*(((a^2*1i - (b^2*5i)/2)*((4*(64*a^5*b^12 - 208*a^7*b^10 + 160*a^9*b^8 - 28*a^11*b^6

))/(a^6*b^5) - (((4*(32*a^8*b^10 - 24*a^10*b^8))/(a^6*b^5) + (4*tan(c/2 + (
d*x)/2)*(128*a^7*b^14 - 136*a^9*b^12 + 16*a^11*b^10))/(a^6*b^8))*(a^2*1i -
(b^2*5i)/2))/b^3 + (4*tan(c/2 + (d*x)/2)*(128*a^4*b^16 - 456*a^6*b^14 + 484
*a^8*b^12 - 200*a^10*b^10 + 32*a^12*b^8))/(a^6*b^8)))/b^3 + (4*(184*a^4*b^1
2 - 32*a^2*b^14 - 360*a^6*b^10 + 78*a^8*b^8 + 240*a^10*b^6 - 152*a^12*b^4 +
24*a^14*b^2))/(a^6*b^5) - (4*tan(c/2 + (d*x)/2)*(8*a^3*b^16 - 160*a^5*b^14
+ 1320*a^7*b^12 - 2128*a^9*b^10 + 1242*a^11*b^8 - 280*a^13*b^6 + 16*a^15*b
^4))/(a^6*b^8)))/b^3 + (4*tan(c/2 + (d*x)/2)*(8*b^18 - 68*a^2*b^16 + 1040*a
^4*b^14 - 3900*a^6*b^12 + 5738*a^8*b^10 - 4201*a^10*b^8 + 1684*a^12*b^6 - 3
60*a^14*b^4 + 32*a^16*b^2))/(a^6*b^8)))/b^3 + (8*(70*a^14 + 20*b^14 + 22*a^
2*b^12 - 642*a^4*b^10 + 1937*a^6*b^8 - 2549*a^8*b^6 + 1695*a^10*b^4 - 553*a
^12*b^2))/(a^6*b^5) - (8*tan(c/2 + (d*x)/2)*(64*a^17 + 700*a^5*b^12 - 3060*
a^7*b^10 + 5412*a^9*b^8 - 4940*a^11*b^6 + 2448*a^13*b^4 - 624*a^15*b^2))/(a
^6*b^8)))*(a^2*1i - (b^2*5i)/2)*2i)/(b^3*d) - (log(tan(c/2 + (d*x)/2))*((5*
a^2)/2 - b^2))/(a^3*d) + (atan((((-(a + b)^5*(a - b)^5)^(1/2)*((4*(20*a^3*b
^12 - 28*a^15 + 272*a^5*b^10 - 1272*a^7*b^8 + 1709*a^9*b^6 - 998*a^11*b^4 +
270*a^13*b^2))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(8*b^18 - 68*a^2*b^16 + 1
040*a^4*b^14 - 3900*a^6*b^12 + 5738*a^8*b^10 - 4201*a^10*b^8 + 1684*a^12*b^
6 - 360*a^14*b^4 + 32*a^16*b^2))/(a^6*b^8) + ((-(a + b)^5*(a - b)^5)^(1/2)*
((4*tan(c/2 + (d*x)/2)*(8*a^3*b^16 - 160*a^5*b^14 + 1320*a^7*b^12 - 2128*a^
9*b^10 + 1242*a^11*b^8 - 280*a^13*b^6 + 16*a^15*b^4))/(a^6*b^8) - (4*(184*a
^4*b^12 - 32*a^2*b^14 - 360*a^6*b^10 + 78*a^8*b^8 + 240*a^10*b^6 - 152*a^12
*b^4 + 24*a^14*b^2))/(a^6*b^5) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(64*a^5*
b^12 - 208*a^7*b^10 + 160*a^9*b^8 - 28*a^11*b^6))/(a^6*b^5) + (((4*(32*a^8*
b^10 - 24*a^10*b^8))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(128*a^7*b^14 - 136*
a^9*b^12 + 16*a^11*b^10))/(a^6*b^8))*(-(a + b)^5*(a - b)^5)^(1/2))/(a^3*b^3
) + (4*tan(c/2 + (d*x)/2)*(128*a^4*b^16 - 456*a^6*b^14 + 484*a^8*b^12 - 200
*a^10*b^10 + 32*a^12*b^8))/(a^6*b^8)))/(a^3*b^3)))/(a^3*b^3))*1i)/(a^3*b^3)
+ ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(20*a^3*b^12 - 28*a^15 + 272*a^5*b^10
- 1272*a^7*b^8 + 1709*a^9*b^6 - 998*a^11*b^4 + 270*a^13*b^2))/(a^6*b^5) + (
4*tan(c/2 + (d*x)/2)*(8*b^18 - 68*a^2*b^16 + 1040*a^4*b^14 - 3900*a^6*b^12
+ 5738*a^8*b^10 - 4201*a^10*b^8 + 1684*a^12*b^6 - 360*a^14*b^4 + 32*a^16*b^
2))/(a^6*b^8) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(184*a^4*b^12 - 32*a^2*b^
14 - 360*a^6*b^10 + 78*a^8*b^8 + 240*a^10*b^6 - 152*a^12*b^4 + 24*a^14*b^2)
)/(a^6*b^5) - (4*tan(c/2 + (d*x)/2)*(8*a^3*b^16 - 160*a^5*b^14 + 1320*a^7*b
^12 - 2128*a^9*b^10 + 1242*a^11*b^8 - 280*a^13*b^6 + 16*a^15*b^4))/(a^6*b^8
) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(64*a^5*b^12 - 208*a^7*b^10 + 160*a^9
*b^8 - 28*a^11*b^6))/(a^6*b^5) - (((4*(32*a^8*b^10 - 24*a^10*b^8))/(a^6*b^5
) + (4*tan(c/2 + (d*x)/2)*(128*a^7*b^14 - 136*a^9*b^12 + 16*a^11*b^10))/(a^
6*b^8))*(-(a + b)^5*(a - b)^5)^(1/2))/(a^3*b^3) + (4*tan(c/2 + (d*x)/2)*(12
8*a^4*b^16 - 456*a^6*b^14 + 484*a^8*b^12 - 200*a^10*b^10 + 32*a^12*b^8))/(a
^6*b^8)))/(a^3*b^3)))/(a^3*b^3))*1i)/(a^3*b^3))/((8*(70*a^14 + 20*b^14 + 22
*a^2*b^12 - 642*a^4*b^10 + 1937*a^6*b^8 - 2549*a^8*b^6 + 1695*a^10*b^4 - 55
3*a^12*b^2))/(a^6*b^5) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(20*a^3*b^12 - 2
8*a^15 + 272*a^5*b^10 - 1272*a^7*b^8 + 1709*a^9*b^6 - 998*a^11*b^4 + 270*a^

```
13*b^2))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(8*b^18 - 68*a^2*b^16 + 1040*a^4
*b^14 - 3900*a^6*b^12 + 5738*a^8*b^10 - 4201*a^10*b^8 + 1684*a^12*b^6 - 360
*a^14*b^4 + 32*a^16*b^2))/(a^6*b^8) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*tan
(c/2 + (d*x)/2)*(8*a^3*b^16 - 160*a^5*b^14 + 1320*a^7*b^12 - 2128*a^9*b^10
+ 1242*a^11*b^8 - 280*a^13*b^6 + 16*a^15*b^4))/(a^6*b^8) - (4*(184*a^4*b^12
 - 32*a^2*b^14 - 360*a^6*b^10 + 78*a^8*b^8 + 240*a^10*b^6 - 152*a^12*b^4 +
24*a^14*b^2))/(a^6*b^5) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(64*a^5*b^12 -
208*a^7*b^10 + 160*a^9*b^8 - 28*a^11*b^6))/(a^6*b^5) + (((4*(32*a^8*b^10 -
24*a^10*b^8))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(128*a^7*b^14 - 136*a^9*b^1
2 + 16*a^11*b^10))/(a^6*b^8))*(-(a + b)^5*(a - b)^5)^(1/2))/(a^3*b^3) + (4*
tan(c/2 + (d*x)/2)*(128*a^4*b^16 - 456*a^6*b^14 + 484*a^8*b^12 - 200*a^10*b
^10 + 32*a^12*b^8))/(a^6*b^8)))/(a^3*b^3)))/(a^3*b^3)))/(a^3*b^3) - ((-(a +
 b)^5*(a - b)^5)^(1/2)*((4*(20*a^3*b^12 - 28*a^15 + 272*a^5*b^10 - 1272*a^7
*b^8 + 1709*a^9*b^6 - 998*a^11*b^4 + 270*a^13*b^2))/(a^6*b^5) + (4*tan(c/2
+ (d*x)/2)*(8*b^18 - 68*a^2*b^16 + 1040*a^4*b^14 - 3900*a^6*b^12 + 5738*a^8
*b^10 - 4201*a^10*b^8 + 1684*a^12*b^6 - 360*a^14*b^4 + 32*a^16*b^2))/(a^6*b
^8) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(184*a^4*b^12 - 32*a^2*b^14 - 360*a
^6*b^10 + 78*a^8*b^8 + 240*a^10*b^6 - 152*a^12*b^4 + 24*a^14*b^2))/(a^6*b^5
) - (4*tan(c/2 + (d*x)/2)*(8*a^3*b^16 - 160*a^5*b^14 + 1320*a^7*b^12 - 2128
*a^9*b^10 + 1242*a^11*b^8 - 280*a^13*b^6 + 16*a^15*b^4))/(a^6*b^8) + ((-(a
+ b)^5*(a - b)^5)^(1/2)*((4*(64*a^5*b^12 - 208*a^7*b^10 + 160*a^9*b^8 - 28*
a^11*b^6))/(a^6*b^5) - (((4*(32*a^8*b^10 - 24*a^10*b^8))/(a^6*b^5) + (4*tan
(c/2 + (d*x)/2)*(128*a^7*b^14 - 136*a^9*b^12 + 16*a^11*b^10))/(a^6*b^8))*(-
(a + b)^5*(a - b)^5)^(1/2))/(a^3*b^3) + (4*tan(c/2 + (d*x)/2)*(128*a^4*b^16
 - 456*a^6*b^14 + 484*a^8*b^12 - 200*a^10*b^10 + 32*a^12*b^8))/(a^6*b^8)))/
(a^3*b^3)))/(a^3*b^3)))/(a^3*b^3) - (8*tan(c/2 + (d*x)/2)*(64*a^17 + 700*a^
5*b^12 - 3060*a^7*b^10 + 5412*a^9*b^8 - 4940*a^11*b^6 + 2448*a^13*b^4 - 624
*a^15*b^2))/(a^6*b^8)))*(-(a + b)^5*(a - b)^5)^(1/2)*2i)/(a^3*b^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1326 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=197

$$\frac{b\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4b^2d}-\frac{b\left(3a^2-b^2\right)\tanh^{-1}(\cos}{a^4d}$$

```
[Out] a*x/b^2-2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/
a^4/b^2/d+1/2*b*arctanh(cos(d*x+c))/a^2/d-b*(3*a^2-b^2)*arctanh(cos(d*x+c))
/a^4/d+cos(d*x+c)/b/d-cot(d*x+c)/a/d+(3*a^2-b^2)*cot(d*x+c)/a^3/d-1/3*cot(d
*x+c)^3/a/d+1/2*b*cot(d*x+c)*csc(d*x+c)/a^2/d
```

Rubi [A] time = 0.28, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2897, 3770, 3767, 8, 3768, 2638, 2660, 618, 204}

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4b^2d}+\frac{\left(3a^2-b^2\right)\cot(c+dx)}{a^3d}-\frac{b\left(3a^2-b^2\right)\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{b\tanh^{-1}(\cos}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/b^2 - (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^4*b^2*d) + (b*ArcTanh[Cos[c + d*x]])/(2*a^2*d) - (b*(3*a^2 - b^2
)*ArcTanh[Cos[c + d*x]])/(a^4*d) + Cos[c + d*x]/(b*d) - Cot[c + d*x]/(a*d)
+ ((3*a^2 - b^2)*Cot[c + d*x])/(a^3*d) - Cot[c + d*x]^3/(3*a*d) + (b*Cot[c
+ d*x]*Csc[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int\left(\frac{a}{b^2} + \frac{\left(3a^2b-b^3\right)\csc(c+dx)}{a^4} + \frac{\left(-3a^2+b^2\right)\csc^2(c+dx)}{a^3} - \frac{b\csc^3(c+dx)}{a^2}\right.$$

$$= \frac{ax}{b^2} + \frac{\int\csc^4(c+dx)\,dx}{a} - \frac{\int\sin(c+dx)\,dx}{b} - \frac{b\int\csc^3(c+dx)\,dx}{a^2} - \frac{\left(a^2-b^2\right)}{}$$

$$= \frac{ax}{b^2} - \frac{b\left(3a^2-b^2\right)\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{bd} + \frac{b\cot(c+dx)\csc(c+}{2a^2d}$$

$$= \frac{ax}{b^2} + \frac{b\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{b\left(3a^2-b^2\right)\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx}{bd}$$

$$= \frac{ax}{b^2} - \frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^2d} + \frac{b\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{b\left(3a^2\right.}{}$$

Mathematica [A] time = 6.20, size = 379, normalized size = 1.92

$$\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} + \frac{\left(5a^2b-2b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\left(2b^3-5a^2b\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(b^2*d) - (2*(a^2 - b^2)^(5/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Co
s[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*b^2*d) + Cos[c
 + d*x]/(b*d) + ((7*a^2*Cos[(c + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*Csc[(c +
 d*x)/2])/(6*a^3*d) + (b*Csc[(c + d*x)/2]^2)/(8*a^2*d) - (Cot[(c + d*x)/2]*
Csc[(c + d*x)/2]^2)/(24*a*d) + ((-5*a^2*b + 2*b^3)*Log[Cos[(c + d*x)/2]])/(
2*a^4*d) + ((5*a^2*b - 2*b^3)*Log[Sin[(c + d*x)/2]])/(2*a^4*d) - (b*Sec[(c
+ d*x)/2]^2)/(8*a^2*d) + (Sec[(c + d*x)/2]*(-7*a^2*Sin[(c + d*x)/2] + 3*b^2
*Sin[(c + d*x)/2]))/(6*a^3*d) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a
*d)
```

fricas [A] time = 1.95, size = 801, normalized size = 4.07

$$\left[\frac{4\left(7a^3b^2-3ab^4\right)\cos(dx+c)^3 - 6\left(a^4-2a^2b^2+b^4-\left(a^4-2a^2b^2+b^4\right)\cos(dx+c)^2\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(7*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^3 - 6*(a^4 - 2*a^2*b^2 + b^4 -
(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)
*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*
x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2))*sin(d*x + c) + 3*(5*a^2*b^3 - 2*b^5 - (5*a^2*b^3 - 2
*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(5*a^2*b
^3 - 2*b^5 - (5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/
2)*sin(d*x + c) - 12*(2*a^3*b^2 - a*b^4)*cos(d*x + c) + 6*(2*a^5*d*x*cos(d*
x + c)^2 + 2*a^4*b*cos(d*x + c)^3 - 2*a^5*d*x - (2*a^4*b + a^2*b^3)*cos(d*x
 + c))*sin(d*x + c))/((a^4*b^2*d*cos(d*x + c)^2 - a^4*b^2*d)*sin(d*x + c)),
 1/12*(4*(7*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^3 - 12*(a^4 - 2*a^2*b^2 + b^4 -
 (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) + 3*(5*a^2*b^3 - 2*
b^5 - (5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d
*x + c) - 3*(5*a^2*b^3 - 2*b^5 - (5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)*log(-1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 12*(2*a^3*b^2 - a*b^4)*cos(d*x + c) +
 6*(2*a^5*d*x*cos(d*x + c)^2 + 2*a^4*b*cos(d*x + c)^3 - 2*a^5*d*x - (2*a^4*
b + a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^4*b^2*d*cos(d*x + c)^2 - a^4*b
^2*d)*sin(d*x + c))]
```

giac [A] time = 0.22, size = 317, normalized size = 1.61

$$\frac{\frac{24\,(dx+c)a}{b^2} + \frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 27\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3} + \frac{48}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b} + \frac{12\left(5\,a^2b-2\,b^3\right)\log\left(\left|\cdots\right.\right.}{a^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*a/b^2 + (a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x
+ 1/2*c)^2 - 27*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/a^3
 + 48/((tan(1/2*d*x + 1/2*c)^2 + 1)*b) + 12*(5*a^2*b - 2*b^3)*log(abs(tan(1
/2*d*x + 1/2*c)))/a^4 - 48*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2
 - b^2)))/(sqrt(a^2 - b^2)*a^4*b^2) - (110*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 4
4*b^3*tan(1/2*d*x + 1/2*c)^3 - 27*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan
(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^4*tan(1/2*d*x
+ 1/2*c)^3))/d
```

maple [B] time = 0.46, size = 442, normalized size = 2.24

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{8d\,a^2}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}+\frac{2}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2a\arctan\left(\tan\left(\frac{dx}{2}\right.\right.}{d\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a^2*tan(1/2*d*x+1/2*c)^2*b-9/8/a/d*tan(
1/2*d*x+1/2*c)+1/2/d/a^3*b^2*tan(1/2*d*x+1/2*c)+2/d/b/(1+tan(1/2*d*x+1/2*c)
^2)+2/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))-1/24/d/a/tan(1/2*d*x+1/2*c)^3+9/8/
a/d/tan(1/2*d*x+1/2*c)-1/2/d/a^3/tan(1/2*d*x+1/2*c)*b^2+1/8/d/a^2*b/tan(1/2
*d*x+1/2*c)^2+5/2/d/a^2*b*ln(tan(1/2*d*x+1/2*c))-1/d/a^4*b^3*ln(tan(1/2*d*x
+1/2*c))-2/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a
^2-b^2)^(1/2))*a^2+6/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2
*b)/(a^2-b^2)^(1/2))-6/d*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*
x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 15.57, size = 4712, normalized size = 23.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] cos(c + d*x)/(32*a*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + (3*sin(
c + d*x))/(32*b*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - (7*cos(3*c
 + 3*d*x))/(96*a*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + sin(2*c +
```

2*d*x)/(32*b*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - sin(3*c + 3*d*x)/(32*b*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - sin(4*c + 4*d*x)/(64*b*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + (b^2*cos(3*c + 3*d*x))/(32*a^3*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - (b^2*cos(c + d*x))/(32*a^3*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + (b*sin(2*c + 2*d*x))/(32*a^2*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + (15*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(64*a^2*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - (3*a*sin(c + d*x)*atan((2*a^5*cos(c/2 + (d*x)/2) - 2*b^5*sin(c/2 + (d*x)/2) + 5*a^2*b^3*sin(c/2 + (d*x)/2))/(2*b^5*cos(c/2 + (d*x)/2) + 2*a^5*sin(c/2 + (d*x)/2) - 5*a^2*b^3*cos(c/2 + (d*x)/2))))/(16*b^2*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - (5*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x))/(64*a^2*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - (3*b^3*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(32*a^4*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + (a*sin(3*c + 3*d*x)*atan((2*a^5*cos(c/2 + (d*x)/2) - 2*b^5*sin(c/2 + (d*x)/2) + 5*a^2*b^3*sin(c/2 + (d*x)/2))/(2*b^5*cos(c/2 + (d*x)/2) + 2*a^5*sin(c/2 + (d*x)/2) - 5*a^2*b^3*cos(c/2 + (d*x)/2))))/(16*b^2*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + (b^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x))/(32*a^4*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + (sin(3*c + 3*d*x)*atan((a^8*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*8i + a^18*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*8i + b^8*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*32i + a*b^7*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*16i - a^7*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*12i - a^17*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*2i - a^3*b^5*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*52i + a^5*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*45i - a^7*b^11*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*2i + a^9*b^9*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*12i - a^11*b^7*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*14i - a^13*b^5*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*12i + a^15*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*15i - a^2*b^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*114i + a^4*b^4*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*121i - a^6*b^2*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*50i + a^2*b^16*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*2i - a^4*b^14*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*17i + a^6*b^12*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b

^4 + 5*a^8*b^2)^(1/2)*60i - a^8*b^10*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*115i + a^10*b^8*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*162i - a^12*b^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*199i + a^14*b^4*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*146i - a^16*b^2*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*50i)/(32*b^23*sin(c/2 + (d*x)/2) + 16*a*b^22*cos(c/2 + (d*x)/2) - 10*a^22*b*sin(c/2 + (d*x)/2) - 172*a^3*b^20*cos(c/2 + (d*x)/2) + 825*a^5*b^18*cos(c/2 + (d*x)/2) - 2334*a^7*b^16*cos(c/2 + (d*x)/2) + 4332*a^9*b^14*cos(c/2 + (d*x)/2) - 5528*a^11*b^12*cos(c/2 + (d*x)/2) + 4906*a^13*b^10*cos(c/2 + (d*x)/2) - 2975*a^15*b^8*cos(c/2 + (d*x)/2) + 1175*a^17*b^6*cos(c/2 + (d*x)/2) - 275*a^19*b^4*cos(c/2 + (d*x)/2) + 30*a^21*b^2*cos(c/2 + (d*x)/2) - 352*a^2*b^21*sin(c/2 + (d*x)/2) + 1734*a^4*b^19*sin(c/2 + (d*x)/2) - 5060*a^6*b^17*sin(c/2 + (d*x)/2) + 9738*a^8*b^15*sin(c/2 + (d*x)/2) - 12976*a^10*b^13*sin(c/2 + (d*x)/2) + 12156*a^12*b^11*sin(c/2 + (d*x)/2) - 7922*a^14*b^9*sin(c/2 + (d*x)/2) + 3470*a^16*b^7*sin(c/2 + (d*x)/2) - 960*a^18*b^5*sin(c/2 + (d*x)/2) + 150*a^20*b^3*sin(c/2 + (d*x)/2)))*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*1i)/(16*a^4*b^2*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - (sin(c + d*x)*atan((a^8*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*8i + a^18*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*8i + b^8*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*32i + a*b^7*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*16i - a^7*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*12i - a^17*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*2i - a^3*b^5*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*52i + a^5*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*45i - a^7*b^11*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*2i + a^9*b^9*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*12i - a^11*b^7*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*14i - a^13*b^5*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*12i + a^15*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*15i - a^2*b^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*114i + a^4*b^4*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*121i - a^6*b^2*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*50i + a^2*b^16*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*2i - a^4*b^14*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*17i + a^6*b^12*sin(c/2 + (d*x)/2)*(b^10 - a^

```
10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*60i - a^8*b^10*
sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a
^8*b^2)^(1/2)*115i + a^10*b^8*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 +
 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*162i - a^12*b^6*sin(c/2 + (d*x)
/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*1
99i + a^14*b^4*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 1
0*a^6*b^4 + 5*a^8*b^2)^(1/2)*146i - a^16*b^2*sin(c/2 + (d*x)/2)*(b^10 - a^1
0 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*50i)/(32*b^23*si
n(c/2 + (d*x)/2) + 16*a*b^22*cos(c/2 + (d*x)/2) - 10*a^22*b*sin(c/2 + (d*x)
/2) - 172*a^3*b^20*cos(c/2 + (d*x)/2) + 825*a^5*b^18*cos(c/2 + (d*x)/2) - 2
334*a^7*b^16*cos(c/2 + (d*x)/2) + 4332*a^9*b^14*cos(c/2 + (d*x)/2) - 5528*a
^11*b^12*cos(c/2 + (d*x)/2) + 4906*a^13*b^10*cos(c/2 + (d*x)/2) - 2975*a^15
*b^8*cos(c/2 + (d*x)/2) + 1175*a^17*b^6*cos(c/2 + (d*x)/2) - 275*a^19*b^4*c
os(c/2 + (d*x)/2) + 30*a^21*b^2*cos(c/2 + (d*x)/2) - 352*a^2*b^21*sin(c/2 +
 (d*x)/2) + 1734*a^4*b^19*sin(c/2 + (d*x)/2) - 5060*a^6*b^17*sin(c/2 + (d*x
)/2) + 9738*a^8*b^15*sin(c/2 + (d*x)/2) - 12976*a^10*b^13*sin(c/2 + (d*x)/2
) + 12156*a^12*b^11*sin(c/2 + (d*x)/2) - 7922*a^14*b^9*sin(c/2 + (d*x)/2) +
 3470*a^16*b^7*sin(c/2 + (d*x)/2) - 960*a^18*b^5*sin(c/2 + (d*x)/2) + 150*a
^20*b^3*sin(c/2 + (d*x)/2)))*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6
*b^4 + 5*a^8*b^2)^(1/2)*3i)/(16*a^4*b^2*d*((3*sin(c + d*x))/32 - sin(3*c +
3*d*x)/32))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1327 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=195

$$\frac{b\cot^3(c+dx)}{3a^2d}+\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5bd}+\frac{b\left(b^2-2a^2\right)\cot(c+dx)}{a^4d}+\frac{\left(7a^2-4b^2\right)\cot(c+dx)\csc(c+\cdots}{8a^3d}$$

```
[Out] -x/b+2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^5
/b/d-1/8*(15*a^4-20*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^5/d+b*(-2*a^2+b^2)
*cot(d*x+c)/a^4/d+1/3*b*cot(d*x+c)^3/a^2/d+1/8*(7*a^2-4*b^2)*cot(d*x+c)*csc
(d*x+c)/a^3/d-1/4*cot(d*x+c)^3*csc(d*x+c)/a/d
```

Rubi [A] time = 0.29, antiderivative size = 275, normalized size of antiderivative = 1.41, number of steps used = 15, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2897, 3770, 3767, 8, 3768, 2660, 618, 204}

$$\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5bd}-\frac{b\left(3a^2-b^2\right)\cot(c+dx)}{a^4d}+\frac{\left(3a^2-b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\left(-3a^2b^2+3a^4\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(x/b) + (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(a^5*b*d) - (3*ArcTanh[Cos[c + d*x]])/(8*a*d) + ((3*a^2 - b^2)*ArcTanh
[Cos[c + d*x]])/(2*a^3*d) - ((3*a^4 - 3*a^2*b^2 + b^4)*ArcTanh[Cos[c + d*x]
])/(a^5*d) + (b*Cot[c + d*x])/(a^2*d) - (b*(3*a^2 - b^2)*Cot[c + d*x])/(a^4
*d) + (b*Cot[c + d*x]^3)/(3*a^2*d) - (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d)
+ ((3*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d) - (Cot[c + d*x]*Csc[c
 + d*x]^3)/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx = \int\left(-\frac{1}{b}+\frac{(3a^4-3a^2b^2+b^4)\csc(c+dx)}{a^5}+\frac{(3a^2b-b^3)\csc^2(c+dx)}{a^4}+\frac{(-3a^2\right.$$

$$= -\frac{x}{b}+\frac{\int\csc^5(c+dx)\,dx}{a}-\frac{b\int\csc^4(c+dx)\,dx}{a^2}+\frac{(a^2-b^2)^3\int\frac{1}{a+b\sin(c+dx)}\,dx}{a^5b}-$$

$$= -\frac{x}{b}-\frac{(3a^4-3a^2b^2+b^4)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{(3a^2-b^2)\cot(c+dx)\csc(c+}{2a^3d}$$

$$= -\frac{x}{b}+\frac{(3a^2-b^2)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{(3a^4-3a^2b^2+b^4)\tanh^{-1}(\cos(c+dx}{a^5d}$$

$$= -\frac{x}{b}+\frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5bd}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{(3a^2-b}{}$$

Mathematica [B] time = 6.19, size = 448, normalized size = 2.30

$$\frac{b\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d}-\frac{b\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d}+\frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(a\sin\left(\frac{1}{2}(c+\right.\right.}{\sqrt{a^2-}}\right.}{a^5bd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((c + d*x)/(b*d)) + (2*(a^2 - b^2)^(5/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(
c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*b*d) + ((-7*a^2*b
*Cos[(c + d*x)/2] + 3*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + (
(9*a^2 - 4*b^2)*Csc[(c + d*x)/2]^2)/(32*a^3*d) + (b*Cot[(c + d*x)/2]*Csc[(c
 + d*x)/2]^2)/(24*a^2*d) - Csc[(c + d*x)/2]^4/(64*a*d) + ((-15*a^4 + 20*a^2
*b^2 - 8*b^4)*Log[Cos[(c + d*x)/2]])/(8*a^5*d) + ((15*a^4 - 20*a^2*b^2 + 8*
b^4)*Log[Sin[(c + d*x)/2]])/(8*a^5*d) + ((-9*a^2 + 4*b^2)*Sec[(c + d*x)/2]^
2)/(32*a^3*d) + Sec[(c + d*x)/2]^4/(64*a*d) + (Sec[(c + d*x)/2]*(7*a^2*b*Si
n[(c + d*x)/2] - 3*b^3*Sin[(c + d*x)/2]))/(6*a^4*d) - (b*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2])/(24*a^2*d)
```

fricas [B] time = 1.51, size = 1034, normalized size = 5.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] [-1/48*(48*a^5*d*x*cos(d*x + c)^4 - 96*a^5*d*x*cos(d*x + c)^2 + 48*a^5*d*x + 6*(9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - 24*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(7*a^4*b - 4*a^2*b^3)*cos(d*x + c) + 3*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) - 3*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*((7*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^3 - 3*(2*a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*x + c)^2 + a^5*b*d), -1/48*(48*a^5*d*x*cos(d*x + c)^4 - 96*a^5*d*x*cos(d*x + c)^2 + 48*a^5*d*x + 6*(9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 + 48*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(7*a^4*b - 4*a^2*b^3)*cos(d*x + c) + 3*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) - 3*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*((7*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^3 - 3*(2*a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*x + c)^2 + a^5*b*d)]

giac [B] time = 0.24, size = 396, normalized size = 2.03

$$-\frac{\frac{192\,(dx+c)}{b} - \frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 8\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 48\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 24\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 216\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 96\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{a^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] -1/192*(192*(d*x + c)/b - (3*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 48*a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 216*a^2*b*tan(1/2*d*x + 1/2*c) - 96*b^3*tan(1/2*d*x + 1/2*c))/a^4 - 24*(15*a^4 - 20*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 - 384*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5*b) + (750*a^4*tan(1/2*d*x + 1/2*c)^4 - 1000*a^2*b^2*tan(1/2*d*x + 1/2*c)

```
^4 + 400*b^4*tan(1/2*d*x + 1/2*c)^4 + 216*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 96
*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 48*a^4*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*b^2*
tan(1/2*d*x + 1/2*c)^2 - 8*a^3*b*tan(1/2*d*x + 1/2*c) + 3*a^4)/(a^5*tan(1/2
*d*x + 1/2*c)^4))/d
```

maple [B] time = 0.49, size = 523, normalized size = 2.68

$$\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64da}-\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^2}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{b^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^3}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8d\,a^2}-\frac{b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^4}-\frac{2\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/64/d/a*tan(1/2*d*x+1/2*c)^4-1/24/d/a^2*b*tan(1/2*d*x+1/2*c)^3-1/4/a/d*tan
(1/2*d*x+1/2*c)^2+1/8/d/a^3*b^2*tan(1/2*d*x+1/2*c)^2+9/8/d/a^2*tan(1/2*d*x+
1/2*c)*b-1/2/d/a^4*b^3*tan(1/2*d*x+1/2*c)-2/d/b*arctan(tan(1/2*d*x+1/2*c))-
1/64/d/a/tan(1/2*d*x+1/2*c)^4+1/4/a/d/tan(1/2*d*x+1/2*c)^2-1/8/d*b^2/a^3/ta
n(1/2*d*x+1/2*c)^2+15/8/a/d*ln(tan(1/2*d*x+1/2*c))-5/2/d/a^3*ln(tan(1/2*d*x
+1/2*c))*b^2+1/d/a^5*ln(tan(1/2*d*x+1/2*c))*b^4+1/24/d/a^2*b/tan(1/2*d*x+1/
2*c)^3-9/8/d/a^2*b/tan(1/2*d*x+1/2*c)+1/2/d*b^3/a^4/tan(1/2*d*x+1/2*c)+2/d*
a/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)
)-6/d*b/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)
^(1/2))+6/d/a^3*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))-2/d/a^5*b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1
/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.40, size = 4334, normalized size = 22.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6/(sin(c + d*x)^5*(a + b*sin(c + d*x))),x)

[Out] tan(c/2 + (d*x)/2)^4/(64*a*d) - (2*atan(-(((((((4*(64*a^9*b^9 - 208*a^11*b^7 + 240*a^13*b^5 - 93*a^15*b^3))/a^12 + (((4*(32*a^14*b^5 - 24*a^16*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^6 - 136*a^15*b^4 + 16*a^17*b^2))/a^12)*1i)/b + (4*tan(c/2 + (d*x)/2)*(128*a^8*b^10 - 456*a^10*b^8 + 604*a^12*b^6 - 335*a^14*b^4 + 62*a^16*b^2))/a^12)*1i)/b - (4*(24*a^16*b - 32*a^4*b^13 + 184*a^6*b^11 - 440*a^8*b^9 + 543*a^10*b^7 - 345*a^12*b^5 + 58*a^14*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(16*a^17 + 8*a^5*b^12 - 40*a^7*b^10 + 100*a^9*b^8 - 148*a^11*b^6 + 252*a^13*b^4 - 180*a^15*b^2))/a^12)*1i)/b - (4*(53*a^15*b + 8*a^5*b^11 - 48*a^7*b^9 + 56*a^9*b^7 + 48*a^11*b^5 - 120*a^13*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(62*a^16 + 8*b^16 - 68*a^2*b^14 + 255*a^4*b^12 - 550*a^6*b^10 + 873*a^8*b^8 - 1096*a^10*b^6 + 929*a^12*b^4 - 410*a^14*b^2))/a^12)/b + ((((4*(24*a^16*b - 32*a^4*b^13 + 184*a^6*b^11 - 440*a^8*b^9 + 543*a^10*b^7 - 345*a^12*b^5 + 58*a^14*b^3))/a^12 + (((4*(64*a^9*b^9 - 208*a^11*b^7 + 240*a^13*b^5 - 93*a^15*b^3))/a^12 - (((4*(32*a^14*b^5 - 24*a^16*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^6 - 136*a^15*b^4 + 16*a^17*b^2))/a^12)*1i)/b + (4*tan(c/2 + (d*x)/2)*(128*a^8*b^10 - 456*a^10*b^8 + 604*a^12*b^6 - 335*a^14*b^4 + 62*a^16*b^2))/a^12)*1i)/b - (4*tan(c/2 + (d*x)/2)*(16*a^17 + 8*a^5*b^12 - 40*a^7*b^10 + 100*a^9*b^8 - 148*a^11*b^6 + 252*a^13*b^4 - 180*a^15*b^2))/a^12)*1i)/b - (4*(53*a^15*b + 8*a^5*b^11 - 48*a^7*b^9 + 56*a^9*b^7 + 48*a^11*b^5 - 120*a^13*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(62*a^16 + 8*b^16 - 68*a^2*b^14 + 255*a^4*b^12 - 550*a^6*b^10 + 873*a^8*b^8 - 1096*a^10*b^6 + 929*a^12*b^4 - 410*a^14*b^2))/a^12)/b)/((((((((4*(64*a^9*b^9 - 208*a^11*b^7 + 240*a^13*b^5 - 93*a^15*b^3))/a^12 + (((4*(32*a^14*b^5 - 24*a^16*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^6 - 136*a^15*b^4 + 16*a^17*b^2))/a^12)*1i)/b + (4*tan(c/2 + (d*x)/2)*(128*a^8*b^10 - 456*a^10*b^8 + 604*a^12*b^6 - 335*a^14*b^4 + 62*a^16*b^2))/a^12)*1i)/b - (4*(24*a^16*b - 32*a^4*b^13 + 184*a^6*b^11 - 440*a^8*b^9 + 543*a^10*b^7 - 345*a^12*b^5 + 58*a^14*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(16*a^17 + 8*a^5*b^12 - 40*a^7*b^10 + 100*a^9*b^8 - 148*a^11*b^6 + 252*a^13*b^4 - 180*a^15*b^2))/a^12)*1i)/b - (4*(53*a^15*b + 8*a^5*b^11 - 48*a^7*b^9 + 56*a^9*b^7 + 48*a^11*b^5 - 120*a^13*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(62*a^16 + 8*b^16 - 68*a^2*b^14 + 255*a^4*b^12 - 550*a^6*b^10 + 873*a^8*b^8 - 1096*a^10*b^6 + 929*a^12*b^4 - 410*a^14*b^2))/a^12)*1i)/b - (((((4*(24*a^16*b - 32*a^4*b^13 + 184*a^6*b^11 - 440*a^8*b^9 + 543*a^10*b^7 - 345*a^12*b^5 + 58*a^14*b^3))/a^12 + (((4*(64*a^9*b^9 - 208*a^11*b^7 + 240*a^13*b^5 - 93*a^15*b^3))/a^12 - (((4*(32*a^14*b^5 - 24*a^16*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^6 - 136*a^15*b^4 + 16*a^17*b^2))/a^12)*1i)/b + (4*tan(c/2 + (d*x)/2)*(128*a^8*b^10 - 456*a^10*b^8 + 604*a^12*b^6 - 335*a^14*b^4 + 62*a^16*b^2))/a^12)*1i)/b - (4*tan(c/2 + (d*x)/2)*(16*a^17 + 8*a^5*b^12 - 40*a^7*b^10 + 100*a^9*b^8 - 148*a^11*b^6 + 252*a^13*b^4 - 180*a^15*b^2))/a^12)*1i)/b - (4*(53*a^15*b + 8*a^5*b^11 - 48*a^7*b^9 + 56*a^9*b^7 + 48*a^11*b^5 - 120*a^13*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(62*a^16 + 8*b^16 - 68*a^2*b^14 + 255*a^4*b^12 - 550*a^6*b^10 + 873*a^8*b^8 - 1096*a^10*b^6 + 929*a^12*b^4 - 410*a^14*b^2))/a^1

2)*1i)/b - (8*(15*a^14*b + 8*b^15 - 68*a^2*b^13 + 223*a^4*b^11 - 366*a^6*b^9 + 305*a^8*b^7 - 97*a^10*b^5 - 20*a^12*b^3))/a^12 + (8*tan(c/2 + (d*x)/2)*(8*a^7*b^8 - 4*a^15 - 20*a^9*b^6 + 12*a^11*b^4 + 4*a^13*b^2))/a^12)))/(b*d) - (tan(c/2 + (d*x)/2)^2*(1/(4*a) - b^2/(8*a^3)))/d + (tan(c/2 + (d*x)/2)*(b/(8*a^2) + (2*b*(1/(2*a) - b^2/(4*a^3)))/a))/d - (tan(c/2 + (d*x)/2)^2*(2*a*b^2 - 4*a^3) + tan(c/2 + (d*x)/2)^3*(18*a^2*b - 8*b^3) + a^3/4 - (2*a^2*b*tan(c/2 + (d*x)/2))/3)/(16*a^4*d*tan(c/2 + (d*x)/2)^4) + (log(tan(c/2 + (d*x)/2))*((15*a^4)/8 + b^4 - (5*a^2*b^2)/2))/(a^5*d) - (b*tan(c/2 + (d*x)/2)^3)/(24*a^2*d) - (atan((((-(a + b)^5*(a - b)^5)^(1/2)*((4*tan(c/2 + (d*x)/2)*(62*a^16 + 8*b^16 - 68*a^2*b^14 + 255*a^4*b^12 - 550*a^6*b^10 + 873*a^8*b^8 - 1096*a^10*b^6 + 929*a^12*b^4 - 410*a^14*b^2))/a^12 - (4*(53*a^15*b + 8*a^5*b^11 - 48*a^7*b^9 + 56*a^9*b^7 + 48*a^11*b^5 - 120*a^13*b^3))/a^12 + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*tan(c/2 + (d*x)/2)*(16*a^17 + 8*a^5*b^12 - 40*a^7*b^10 + 100*a^9*b^8 - 148*a^11*b^6 + 252*a^13*b^4 - 180*a^15*b^2))/a^12 - (4*(24*a^16*b - 32*a^4*b^13 + 184*a^6*b^11 - 440*a^8*b^9 + 543*a^10*b^7 - 345*a^12*b^5 + 58*a^14*b^3))/a^12 + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(64*a^9*b^9 - 208*a^11*b^7 + 240*a^13*b^5 - 93*a^15*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^8*b^10 - 456*a^10*b^8 + 604*a^12*b^6 - 335*a^14*b^4 + 62*a^16*b^2))/a^12 + (((4*(32*a^14*b^5 - 24*a^16*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^6 - 136*a^15*b^4 + 16*a^17*b^2))/a^12)*(-(a + b)^5*(a - b)^5)^(1/2))/(a^5*b)))/(a^5*b)))/(a^5*b))*1i)/(a^5*b) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*tan(c/2 + (d*x)/2)*(62*a^16 + 8*b^16 - 68*a^2*b^14 + 255*a^4*b^12 - 550*a^6*b^10 + 873*a^8*b^8 - 1096*a^10*b^6 + 929*a^12*b^4 - 410*a^14*b^2))/a^12 - (4*(53*a^15*b + 8*a^5*b^11 - 48*a^7*b^9 + 56*a^9*b^7 + 48*a^11*b^5 - 120*a^13*b^3))/a^12 + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(24*a^16*b - 32*a^4*b^13 + 184*a^6*b^11 - 440*a^8*b^9 + 543*a^10*b^7 - 345*a^12*b^5 + 58*a^14*b^3))/a^12 - (4*tan(c/2 + (d*x)/2)*(16*a^17 + 8*a^5*b^12 - 40*a^7*b^10 + 100*a^9*b^8 - 148*a^11*b^6 + 252*a^13*b^4 - 180*a^15*b^2))/a^12 + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(64*a^9*b^9 - 208*a^11*b^7 + 240*a^13*b^5 - 93*a^15*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^8*b^10 - 456*a^10*b^8 + 604*a^12*b^6 - 335*a^14*b^4 + 62*a^16*b^2))/a^12 - (((4*(32*a^14*b^5 - 24*a^16*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^6 - 136*a^15*b^4 + 16*a^17*b^2))/a^12)*(-(a + b)^5*(a - b)^5)^(1/2))/(a^5*b)))/(a^5*b)))/(a^5*b))*1i)/(a^5*b))/((8*(15*a^14*b + 8*b^15 - 68*a^2*b^13 + 223*a^4*b^11 - 366*a^6*b^9 + 305*a^8*b^7 - 97*a^10*b^5 - 20*a^12*b^3))/a^12 - (8*tan(c/2 + (d*x)/2)*(8*a^7*b^8 - 4*a^15 - 20*a^9*b^6 + 12*a^11*b^4 + 4*a^13*b^2))/a^12 - ((-(a + b)^5*(a - b)^5)^(1/2)*((4*tan(c/2 + (d*x)/2)*(62*a^16 + 8*b^16 - 68*a^2*b^14 + 255*a^4*b^12 - 550*a^6*b^10 + 873*a^8*b^8 - 1096*a^10*b^6 + 929*a^12*b^4 - 410*a^14*b^2))/a^12 - (4*(53*a^15*b + 8*a^5*b^11 - 48*a^7*b^9 + 56*a^9*b^7 + 48*a^11*b^5 - 120*a^13*b^3))/a^12 + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*tan(c/2 + (d*x)/2)*(16*a^17 + 8*a^5*b^12 - 40*a^7*b^10 + 100*a^9*b^8 - 148*a^11*b^6 + 252*a^13*b^4 - 180*a^15*b^2))/a^12 - (4*(24*a^16*b - 32*a^4*b^13 + 184*a^6*b^11 - 440*a^8*b^9 + 543*a^10*b^7 - 345*a^12*b^5 + 58*a^14*b^3))/a^12 + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(64*a^9*b^9 - 208*a^11*b^7 + 240*a^13*b^5 - 93*a^15*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^8*b^10 -

```
456*a^10*b^8 + 604*a^12*b^6 - 335*a^14*b^4 + 62*a^16*b^2))/a^12 + (((4*(32*
a^14*b^5 - 24*a^16*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^6 - 136*a
^15*b^4 + 16*a^17*b^2))/a^12)*(-(a + b)^5*(a - b)^5)^(1/2))/(a^5*b)))/(a^5*
b)))/(a^5*b)))/(a^5*b) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*tan(c/2 + (d*x)/
2)*(62*a^16 + 8*b^16 - 68*a^2*b^14 + 255*a^4*b^12 - 550*a^6*b^10 + 873*a^8*
b^8 - 1096*a^10*b^6 + 929*a^12*b^4 - 410*a^14*b^2))/a^12 - (4*(53*a^15*b +
8*a^5*b^11 - 48*a^7*b^9 + 56*a^9*b^7 + 48*a^11*b^5 - 120*a^13*b^3))/a^12 +
((-(a + b)^5*(a - b)^5)^(1/2)*((4*(24*a^16*b - 32*a^4*b^13 + 184*a^6*b^11 -
 440*a^8*b^9 + 543*a^10*b^7 - 345*a^12*b^5 + 58*a^14*b^3))/a^12 - (4*tan(c/
2 + (d*x)/2)*(16*a^17 + 8*a^5*b^12 - 40*a^7*b^10 + 100*a^9*b^8 - 148*a^11*b
^6 + 252*a^13*b^4 - 180*a^15*b^2))/a^12 + ((-(a + b)^5*(a - b)^5)^(1/2)*((4
*(64*a^9*b^9 - 208*a^11*b^7 + 240*a^13*b^5 - 93*a^15*b^3))/a^12 + (4*tan(c/
2 + (d*x)/2)*(128*a^8*b^10 - 456*a^10*b^8 + 604*a^12*b^6 - 335*a^14*b^4 + 6
2*a^16*b^2))/a^12 - (((4*(32*a^14*b^5 - 24*a^16*b^3))/a^12 + (4*tan(c/2 + (
d*x)/2)*(128*a^13*b^6 - 136*a^15*b^4 + 16*a^17*b^2))/a^12)*(-(a + b)^5*(a -
 b)^5)^(1/2))/(a^5*b)))/(a^5*b)))/(a^5*b)))/(a^5*b)))*(-(a + b)^5*(a - b)^5
)^(1/2)*2i)/(a^5*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1328 $\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=241

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{b\left(4b^2-9a^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d}+\frac{\left(11a^2-\right.}{}$$

```
[Out] -2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^6/d+1
/8*b*(15*a^4-20*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^6/d-1/15*(23*a^4-35*a^
2*b^2+15*b^4)*cot(d*x+c)/a^5/d+1/8*b*(-9*a^2+4*b^2)*cot(d*x+c)*csc(d*x+c)/a
^4/d+1/15*(11*a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)^2/a^3/d+1/4*b*cot(d*x+c)*csc
(d*x+c)^3/a^2/d-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d
```

Rubi [A] time = 1.12, antiderivative size = 307, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{\left(-35a^2b^2+23a^4+15b^4\right)\cot(c+dx)}{15a^5d}+\frac{b\left(-20a^2b^2+15a^4+8b^4\right)\tanh^{-1}}{8a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
6*d) + (b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) -
((23*a^4 - 35*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (Cot[c + d*x]*Cs
c[c + d*x])/(b*d) + ((8*a^4 - 9*a^2*b^2 + 4*b^4)*Cot[c + d*x]*Csc[c + d*x])
/(8*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(2*b^2*d) - ((15*a^4 - 22*a^
2*b^2 + 10*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^3*b^2*d) + (b*Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e
 + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^
m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*
m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(
m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x])/Sin[e + f*x]^4,
x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*
x]^2), x] + Simp[(a*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*m*(m
- 1)*Sin[e + f*x]^3), x] - Simp[(b*(m - 4)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1))/(20*a^2*f*Sin[e + f*x]^4), x]) /; FreeQ[{a, b, e, f, m}, x] && N
eQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d} + \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} -$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d} - \frac{\left(15a^4-22a^2b^2+10b^4\right)\cot(c}{30a^3b^2d}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{\left(8a^4-9a^2b^2+4b^4\right)\cot(c+dx)\csc(c+dx)}{8a^4bd} + \frac{a\cot(c+d}{2}$$

$$= -\frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{\left(8a^4-9a^2b^2+4b^4\right)}{8}$$

$$= -\frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{\left(8a^4-9a^2b^2+4b^4\right)}{8}$$

$$= \frac{b\left(15a^4-20a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} -$$

$$= \frac{b\left(15a^4-20a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} -$$

$$= -\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} + \frac{b\left(15a^4-20a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d}$$

Mathematica [B] time = 1.39, size = 504, normalized size = 2.09

$$\frac{736a^5\tan\left(\frac{1}{2}(c+dx)\right) - 3a^5\sin(c+dx)\csc^6\left(\frac{1}{2}(c+dx)\right) + 41a^5\sin(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right) - 656a^5\sin^4\left(\frac{1}{2}(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-1920*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] -
 32*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*Cot[(c + d*x)/2] - 270*a^4*b*Csc[(c +
d*x)/2]^2 + 120*a^2*b^3*Csc[(c + d*x)/2]^2 + 15*a^4*b*Csc[(c + d*x)/2]^4 +
1800*a^4*b*Log[Cos[(c + d*x)/2]] - 2400*a^2*b^3*Log[Cos[(c + d*x)/2]] + 960
*b^5*Log[Cos[(c + d*x)/2]] - 1800*a^4*b*Log[Sin[(c + d*x)/2]] + 2400*a^2*b^
3*Log[Sin[(c + d*x)/2]] - 960*b^5*Log[Sin[(c + d*x)/2]] + 270*a^4*b*Sec[(c
+ d*x)/2]^2 - 120*a^2*b^3*Sec[(c + d*x)/2]^2 - 15*a^4*b*Sec[(c + d*x)/2]^4
- 656*a^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*a^3*b^2*Csc[c + d*x]^3*Si
n[(c + d*x)/2]^4 + 41*a^5*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a^3*b^2*Csc[
(c + d*x)/2]^4*Sin[c + d*x] - 3*a^5*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 736*a
^5*Tan[(c + d*x)/2] - 1120*a^3*b^2*Tan[(c + d*x)/2] + 480*a*b^4*Tan[(c + d*
x)/2] + 6*a^5*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*a^6*d)
```

fricas [B] time = 1.59, size = 1079, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(16*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 1
3*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3 - 120*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x
+ c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*
sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 15
*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x +
 c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 2
0*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d
*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5 - 2*a^3*b^2
 + a*b^4)*cos(d*x + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - (7*a^4*
b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d
*cos(d*x + c)^2 + a^6*d)*sin(d*x + c)), -1/240*(16*(23*a^5 - 35*a^3*b^2 + 1
5*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 13*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3
- 240*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(
a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
 c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 15*(15*a^4*b - 20*a
^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4
*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*
x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5
)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x
```

```
 + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - (7*a^4*b - 4*a^2*b^3)*co
s(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 +
 a^6*d)*sin(d*x + c))]
```

giac [B] time = 0.23, size = 490, normalized size = 2.03

$$\frac{\frac{6 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5-15 a^3 b \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4-70 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3+40 a^2 b^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3+240 a^3 b \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-120 a b^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+660}{a^5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*((6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 70
*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^3*b
*tan(1/2*d*x + 1/2*c)^2 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a^4*tan(1/
2*d*x + 1/2*c) - 1080*a^2*b^2*tan(1/2*d*x + 1/2*c) + 480*b^4*tan(1/2*d*x +
1/2*c))/a^5 - 120*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*d*x + 1/2
*c)))/a^6 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c
)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/
(sqrt(a^2 - b^2)*a^6) + (4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 5480*a^2*b^3*t
an(1/2*d*x + 1/2*c)^5 + 2192*b^5*tan(1/2*d*x + 1/2*c)^5 - 660*a^5*tan(1/2*d
*x + 1/2*c)^4 + 1080*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 480*a*b^4*tan(1/2*d*x
 + 1/2*c)^4 - 240*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x +
1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 40*a^3*b^2*tan(1/2*d*x + 1/2*c)^
2 + 15*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^6*tan(1/2*d*x + 1/2*c)^5))/d
```

maple [B] time = 0.49, size = 629, normalized size = 2.61

$$-\frac{1}{160 d a \tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-\frac{2 \arctan\left(\frac{2 a \tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2 b}{2 \sqrt{a^2-b^2}}\right)}{d \sqrt{a^2-b^2}}+\frac{11 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16 a d}-\frac{11}{16 a d \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{9 b^2 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8 d a^3}+\frac{}{8 d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/160/d/a/tan(1/2*d*x+1/2*c)^5-2/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+11/16/a/d*tan(1/2*d*x+1/2*c)-11/16/a/d/ta
n(1/2*d*x+1/2*c)+1/4/d/a^2*tan(1/2*d*x+1/2*c)^2*b-9/8/d/a^3*b^2*tan(1/2*d*x
+1/2*c)+9/8/d/a^3/tan(1/2*d*x+1/2*c)*b^2-1/8/d/a^4*tan(1/2*d*x+1/2*c)^2*b^3
+1/2/d/a^5*b^4*tan(1/2*d*x+1/2*c)-1/24/d/a^3/tan(1/2*d*x+1/2*c)^3*b^2-1/2/d
```

```
/a^5/tan(1/2*d*x+1/2*c)*b^4+1/64/d/a^2*b/tan(1/2*d*x+1/2*c)^4+1/8/d*b^3/a^4
/tan(1/2*d*x+1/2*c)^2+7/96/d/a/tan(1/2*d*x+1/2*c)^3-15/8/d/a^2*b*ln(tan(1/2
*d*x+1/2*c))-1/4/d/a^2*b/tan(1/2*d*x+1/2*c)^2+5/2/d/a^4*b^3*ln(tan(1/2*d*x+
1/2*c))+1/160/d/a*tan(1/2*d*x+1/2*c)^5-1/d/a^6*b^5*ln(tan(1/2*d*x+1/2*c))-1
/64/d/a^2*tan(1/2*d*x+1/2*c)^4*b+1/24/d/a^3*tan(1/2*d*x+1/2*c)^3*b^2-7/96/d
/a*tan(1/2*d*x+1/2*c)^3+6/d*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/a^6*b^6/(a^2-b^2)^(1/2)*arctan(1/2*(2
*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-6/d/a^4/(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.39, size = 1099, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^6*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a*d) + (tan(c/2 + (d*x)/2)^2*(b/(32*a^2) + (b*(7/
(32*a) - b^2/(8*a^3)))/a))/d - (tan(c/2 + (d*x)/2)*(b^2/(8*a^3) - 11/(16*a)
 + (2*b*(b/(16*a^2) + (2*b*(7/(32*a) - b^2/(8*a^3)))/a))/a))/d - (tan(c/2 +
 (d*x)/2)^3*(7/(96*a) - b^2/(24*a^3)))/d - (b*tan(c/2 + (d*x)/2)^4)/(64*a^2
*d) - (log(tan(c/2 + (d*x)/2))*((15*a^4*b)/8 + b^5 - (5*a^2*b^3)/2))/(a^6*d
) + (tan(c/2 + (d*x)/2)^2*((7*a^4)/3 - (4*a^2*b^2)/3) - a^4/5 - tan(c/2 + (
d*x)/2)^4*(22*a^4 + 16*b^4 - 36*a^2*b^2) + tan(c/2 + (d*x)/2)^3*(4*a*b^3 -
8*a^3*b) + (a^3*b*tan(c/2 + (d*x)/2))/2)/(32*a^5*d*tan(c/2 + (d*x)/2)^5) -
(atan((((-(a + b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b^4 - 3
9*a^10*b^2)/(4*a^10) + ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b
^2))/(4*a^9))*(-(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)*(31*a
^10*b - 32*a^4*b^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9))*1i)/a^6 + ((-(a + b
)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b^4 - 39*a^10*b^2)/(4*a
^10) - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(-
(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)*(31*a^10*b - 32*a^4*b
^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9))*1i)/a^6)/((15*a^10*b - 8*b^11 + 44*
```

```
a^2*b^9 - 99*a^4*b^7 + 113*a^6*b^5 - 65*a^8*b^3)/(2*a^10) + (tan(c/2 + (d*x
)/2)*(16*a^10 - 8*b^10 + 42*a^2*b^8 - 94*a^4*b^6 + 110*a^6*b^4 - 66*a^8*b^2
))/(2*a^9) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b
^4 - 39*a^10*b^2)/(4*a^10) + ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*
a^10*b^2))/(4*a^9))*(-(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)
*(31*a^10*b - 32*a^4*b^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9)))/a^6 + ((-(a
+ b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b^4 - 39*a^10*b^2)/(
4*a^10) - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))
*(-(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)*(31*a^10*b - 32*a^
4*b^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9)))/a^6))*(-(a + b)^5*(a - b)^5)^(1
/2)*2i)/(a^6*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1329 $\int \frac{\cot^6(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=363

$$\frac{b\cot(c+dx)\csc^4(c+dx)}{5a^2d}+\frac{2b\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}+\frac{\left(15a^4-22a^2b^2+10b^4\right)\cot(c+dx)\csc^2(c+dx}{30a^4bd}$$

```
[Out] 2*b*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^7/d+
1/16*(5*a^6-30*a^4*b^2+40*a^2*b^4-16*b^6)*arctanh(cos(d*x+c))/a^7/d+1/15*b*
(23*a^4-35*a^2*b^2+15*b^4)*cot(d*x+c)/a^6/d-1/16*(11*a^4-18*a^2*b^2+8*b^4)*
cot(d*x+c)*csc(d*x+c)/a^5/d-1/2*cot(d*x+c)*csc(d*x+c)^2/b/d+1/30*(15*a^4-22
*a^2*b^2+10*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^4/b/d+1/3*a*cot(d*x+c)*csc(d*x+c
)^3/b^2/d-1/24*(8*a^4-13*a^2*b^2+6*b^4)*cot(d*x+c)*csc(d*x+c)^3/a^3/b^2/d+1
/5*b*cot(d*x+c)*csc(d*x+c)^4/a^2/d-1/6*cot(d*x+c)*csc(d*x+c)^5/a/d
```

Rubi [A] time = 1.48, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}+\frac{b\left(-35a^2b^2+23a^4+15b^4\right)\cot(c+dx)}{15a^6d}+\frac{\left(-30a^4b^2+40a^2b^4+5a^6-16b^6\right)}{16a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^7*d) + ((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*ArcTanh[Cos[c + d*x]])/
(16*a^7*d) + (b*(23*a^4 - 35*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^6*d) - (
(11*a^4 - 18*a^2*b^2 + 8*b^4)*Cot[c + d*x]*Csc[c + d*x])/(16*a^5*d) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(2*b*d) + ((15*a^4 - 22*a^2*b^2 + 10*b^4)*Cot[c +
d*x]*Csc[c + d*x]^2)/(30*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(3*b^2*
d) - ((8*a^4 - 13*a^2*b^2 + 6*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*a^3*b^2
*d) + (b*Cot[c + d*x]*Csc[c + d*x]^4)/(5*a^2*d) - (Cot[c + d*x]*Csc[c + d*x
]^5)/(6*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc^2(c+dx)}{2bd} + \frac{a\cot(c+dx)\csc^3(c+dx)}{3b^2d} + \frac{b\cot(c+dx)\csc^4(c-}{5a^2d}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{2bd} + \frac{a\cot(c+dx)\csc^3(c+dx)}{3b^2d} - \frac{\left(8a^4-13a^2b^2+6b^4\right)}{24}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{2bd} + \frac{\left(15a^4-22a^2b^2+10b^4\right)\cot(c+dx)\csc^2(c+dx)}{30a^4bd}$$

$$= -\frac{\left(11a^4-18a^2b^2+8b^4\right)\cot(c+dx)\csc(c+dx)}{16a^5d} - \frac{\cot(c+dx)\csc^2(c+dx)}{2bd} +$$

$$= \frac{b\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^6d} - \frac{\left(11a^4-18a^2b^2+8b^4\right)\cot(c+dx)\csc(}{16a^5d}$$

$$= \frac{b\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^6d} - \frac{\left(11a^4-18a^2b^2+8b^4\right)\cot(c+dx)\csc(}{16a^5d}$$

$$= \frac{\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)\tanh^{-1}(\cos(c+dx))}{16a^7d} + \frac{b\left(23a^4-35a^2b^2+15b\right.}{15a^6d}$$

$$= \frac{\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)\tanh^{-1}(\cos(c+dx))}{16a^7d} + \frac{b\left(23a^4-35a^2b^2+15b\right.}{15a^6d}$$

$$= \frac{2b\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d} + \frac{\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)\tanh}{16a^7d}$$

Mathematica [A] time = 1.52, size = 356, normalized size = 0.98

$$7680b\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + 240\left(-5a^6+30a^4b^2-40a^2b^4+16b^6\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 240\,(5$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (7680*b*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
+ 240*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*Log[Cos[(c + d*x)/2]] + 24
0*(-5*a^6 + 30*a^4*b^2 - 40*a^2*b^4 + 16*b^6)*Log[Sin[(c + d*x)/2]] + 2*a*C
```

```
ot[c + d*x]*Csc[c + d*x]^5*(-295*a^5 + 570*a^3*b^2 - 360*a*b^4 + 20*(7*a^5
- 42*a^3*b^2 + 24*a*b^4)*Cos[2*(c + d*x)] - 15*(11*a^5 - 18*a^3*b^2 + 8*a*b
^4)*Cos[4*(c + d*x)] + 1168*a^4*b*Sin[c + d*x] - 2320*a^2*b^3*Sin[c + d*x]
+ 1200*b^5*Sin[c + d*x] - 568*a^4*b*Sin[3*(c + d*x)] + 1240*a^2*b^3*Sin[3*(
c + d*x)] - 600*b^5*Sin[3*(c + d*x)] + 184*a^4*b*Sin[5*(c + d*x)] - 280*a^2
*b^3*Sin[5*(c + d*x)] + 120*b^5*Sin[5*(c + d*x)]))/(3840*a^7*d)
```

fricas [B] time = 2.15, size = 1462, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/480*(30*(11*a^6 - 18*a^4*b^2 + 8*a^2*b^4)*cos(d*x + c)^5 - 80*(5*a^6 - 1
2*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c)^3 + 240*((a^4*b - 2*a^2*b^3 + b^5)*cos(
d*x + c)^6 - a^4*b + 2*a^2*b^3 - b^5 - 3*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x
+ c)^4 + 3*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(-
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 30*(5*a^6 - 14*a^4*b^2 + 8*a^2*b^4)*
cos(d*x + c) + 15*((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^
6 - 5*a^6 + 30*a^4*b^2 - 40*a^2*b^4 + 16*b^6 - 3*(5*a^6 - 30*a^4*b^2 + 40*a
^2*b^4 - 16*b^6)*cos(d*x + c)^4 + 3*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b
^6)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) - 15*((5*a^6 - 30*a^4*b^2 +
 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^6 - 5*a^6 + 30*a^4*b^2 - 40*a^2*b^4 + 16
*b^6 - 3*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^4 + 3*(5*a
^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c
) + 1/2) - 32*((23*a^5*b - 35*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^5 - 5*(7*a^5
*b - 13*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^3 + 15*(a^5*b - 2*a^3*b^3 + a*b^5)*
cos(d*x + c))*sin(d*x + c))/(a^7*d*cos(d*x + c)^6 - 3*a^7*d*cos(d*x + c)^4
+ 3*a^7*d*cos(d*x + c)^2 - a^7*d), 1/480*(30*(11*a^6 - 18*a^4*b^2 + 8*a^2*b
^4)*cos(d*x + c)^5 - 80*(5*a^6 - 12*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c)^3 - 4
80*((a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^6 - a^4*b + 2*a^2*b^3 - b^5 - 3*
(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + 3*(a^4*b - 2*a^2*b^3 + b^5)*cos(
d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*c
os(d*x + c))) + 30*(5*a^6 - 14*a^4*b^2 + 8*a^2*b^4)*cos(d*x + c) + 15*((5*a
^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^6 - 5*a^6 + 30*a^4*b^2
- 40*a^2*b^4 + 16*b^6 - 3*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*
x + c)^4 + 3*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^2)*log
(1/2*cos(d*x + c) + 1/2) - 15*((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*c
os(d*x + c)^6 - 5*a^6 + 30*a^4*b^2 - 40*a^2*b^4 + 16*b^6 - 3*(5*a^6 - 30*a^
4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^4 + 3*(5*a^6 - 30*a^4*b^2 + 40*a^
2*b^4 - 16*b^6)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 32*((23*a^5*
b - 35*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^5 - 5*(7*a^5*b - 13*a^3*b^3 + 6*a*b
```

```
^5)*cos(d*x + c)^3 + 15*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*sin(d*x +
 c))/(a^7*d*cos(d*x + c)^6 - 3*a^7*d*cos(d*x + c)^4 + 3*a^7*d*cos(d*x + c)^
2 - a^7*d)]
```

giac [A] time = 0.26, size = 627, normalized size = 1.73

$$\frac{5\,a^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-12\,a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-45\,a^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+30\,a^{3}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+140\,a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-80\,a^{2}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+225\,}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1920*((5*a^5*tan(1/2*d*x + 1/2*c)^6 - 12*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 4
5*a^5*tan(1/2*d*x + 1/2*c)^4 + 30*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 140*a^4*
b*tan(1/2*d*x + 1/2*c)^3 - 80*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 225*a^5*tan(
1/2*d*x + 1/2*c)^2 - 480*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*a*b^4*tan(1/2
*d*x + 1/2*c)^2 - 1320*a^4*b*tan(1/2*d*x + 1/2*c) + 2160*a^2*b^3*tan(1/2*d*
x + 1/2*c) - 960*b^5*tan(1/2*d*x + 1/2*c))/a^6 - 120*(5*a^6 - 30*a^4*b^2 +
40*a^2*b^4 - 16*b^6)*log(abs(tan(1/2*d*x + 1/2*c)))/a^7 + 3840*(a^6*b - 3*a
^4*b^3 + 3*a^2*b^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^7) + (14
70*a^6*tan(1/2*d*x + 1/2*c)^6 - 8820*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 + 11760
*a^2*b^4*tan(1/2*d*x + 1/2*c)^6 - 4704*b^6*tan(1/2*d*x + 1/2*c)^6 + 1320*a^
5*b*tan(1/2*d*x + 1/2*c)^5 - 2160*a^3*b^3*tan(1/2*d*x + 1/2*c)^5 + 960*a*b^
5*tan(1/2*d*x + 1/2*c)^5 - 225*a^6*tan(1/2*d*x + 1/2*c)^4 + 480*a^4*b^2*tan
(1/2*d*x + 1/2*c)^4 - 240*a^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 140*a^5*b*tan(1/
2*d*x + 1/2*c)^3 + 80*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 45*a^6*tan(1/2*d*x +
 1/2*c)^2 - 30*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*a^5*b*tan(1/2*d*x + 1/2*
c) - 5*a^6)/(a^7*tan(1/2*d*x + 1/2*c)^6))/d
```

maple [B] time = 0.51, size = 780, normalized size = 2.15

$$-\frac{b^{2}}{64d\,a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{6}}{d\,a^{7}}+\frac{b}{160d\,a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}+\frac{b^{5}}{2d\,a^{6}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{b^{3}}{24d\,a^{4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/64/d/a^3/tan(1/2*d*x+1/2*c)^4*b^2+1/d/a^7*ln(tan(1/2*d*x+1/2*c))*b^6+1/1
60/d/a^2*b/tan(1/2*d*x+1/2*c)^5+1/2/d*b^5/a^6/tan(1/2*d*x+1/2*c)+1/24/d/a^4
```

```
*b^3/tan(1/2*d*x+1/2*c)^3-1/8/d/a^5/tan(1/2*d*x+1/2*c)^2*b^4-1/160/d/a^2*b*
tan(1/2*d*x+1/2*c)^5+1/64/d/a^3*tan(1/2*d*x+1/2*c)^4*b^2+1/8/d/a^5*tan(1/2*
d*x+1/2*c)^2*b^4-1/24/d/a^4*tan(1/2*d*x+1/2*c)^3*b^3+2/d*b/a/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+15/128/a/d*tan(1
/2*d*x+1/2*c)^2-15/128/a/d/tan(1/2*d*x+1/2*c)^2-6/d/a^3*b^3/(a^2-b^2)^(1/2)
*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-5/16/a/d*ln(tan(1
/2*d*x+1/2*c))+9/8/d/a^4*b^3*tan(1/2*d*x+1/2*c)-7/96/d/a^2*b/tan(1/2*d*x+1/
2*c)^3+1/4/d*b^2/a^3/tan(1/2*d*x+1/2*c)^2-9/8/d*b^3/a^4/tan(1/2*d*x+1/2*c)-
11/16/d/a^2*tan(1/2*d*x+1/2*c)*b+15/8/d/a^3*ln(tan(1/2*d*x+1/2*c))*b^2+11/1
6/d/a^2*b/tan(1/2*d*x+1/2*c)+7/96/d/a^2*b*tan(1/2*d*x+1/2*c)^3-1/4/d/a^3*b^
2*tan(1/2*d*x+1/2*c)^2-5/2/d/a^5*ln(tan(1/2*d*x+1/2*c))*b^4-1/2/d/a^6*b^5*t
an(1/2*d*x+1/2*c)+3/128/d/a/tan(1/2*d*x+1/2*c)^4+6/d/a^5*b^5/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a^7*b^7/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-3/128/
d/a*tan(1/2*d*x+1/2*c)^4+1/384/d/a*tan(1/2*d*x+1/2*c)^6-1/384/d/a/tan(1/2*d
*x+1/2*c)^6
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.34, size = 1289, normalized size = 3.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^7*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^6/(384*a*d) + (tan(c/2 + (d*x)/2)^3*(b/(96*a^2) + (2*b*(
3/(32*a) - b^2/(16*a^3))))/(3*a)))/d - (tan(c/2 + (d*x)/2)^4*(3/(128*a) - b^
2/(64*a^3)))/d - (tan(c/2 + (d*x)/2)*(b/(32*a^2) - (2*b*(b^2/(16*a^3) - 15/
(64*a) + (2*b*(b/(32*a^2) + (2*b*(3/(32*a) - b^2/(16*a^3))))/a))/a))/a + (2*
b*(3/(32*a) - b^2/(16*a^3))))/a))/d - (tan(c/2 + (d*x)/2)^2*(b^2/(32*a^3) -
15/(128*a) + (b*(b/(32*a^2) + (2*b*(3/(32*a) - b^2/(16*a^3))))/a))/a))/d - (
tan(c/2 + (d*x)/2)^3*((14*a^4*b)/3 - (8*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^4*
(8*a*b^4 + (15*a^5)/2 - 16*a^3*b^2) - tan(c/2 + (d*x)/2)^5*(44*a^4*b + 32*b
^5 - 72*a^2*b^3) + a^5/6 - tan(c/2 + (d*x)/2)^2*((3*a^5)/2 - a^3*b^2) - (2*
```

```
a^4*b*tan(c/2 + (d*x)/2))/5)/(64*a^6*d*tan(c/2 + (d*x)/2)^6) - (b*tan(c/2 +
 (d*x)/2)^5)/(160*a^2*d) - (log(tan(c/2 + (d*x)/2))*(5*a^6 - 16*b^6 + 40*a^
2*b^4 - 30*a^4*b^2))/(16*a^7*d) + (b*atan(((b*(-(a + b)^5*(a - b)^5)^(1/2)*
((tan(c/2 + (d*x)/2)*(5*a^13 + 64*a^5*b^8 - 192*a^7*b^6 + 196*a^9*b^4 - 72*
a^11*b^2))/(8*a^11) - (21*a^13*b - 32*a^7*b^7 + 88*a^9*b^5 - 78*a^11*b^3)/(
8*a^12) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^14 - 64*a^12*b^2))/(8*a^1
1))*(-(a + b)^5*(a - b)^5)^(1/2))/a^7)*1i)/a^7 - (b*(-(a + b)^5*(a - b)^5)^
(1/2)*((21*a^13*b - 32*a^7*b^7 + 88*a^9*b^5 - 78*a^11*b^3)/(8*a^12) - (tan(
c/2 + (d*x)/2)*(5*a^13 + 64*a^5*b^8 - 192*a^7*b^6 + 196*a^9*b^4 - 72*a^11*b
^2))/(8*a^11) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^14 - 64*a^12*b^2))/
(8*a^11))*(-(a + b)^5*(a - b)^5)^(1/2))/a^7)*1i)/a^7)/((5*a^12*b + 16*b^13
- 88*a^2*b^11 + 198*a^4*b^9 - 231*a^6*b^7 + 145*a^8*b^5 - 45*a^10*b^3)/(4*a
^12) + (tan(c/2 + (d*x)/2)*(16*b^12 - 84*a^2*b^10 + 178*a^4*b^8 - 190*a^6*b
^6 + 102*a^8*b^4 - 22*a^10*b^2))/(4*a^11) + (b*(-(a + b)^5*(a - b)^5)^(1/2)
*((tan(c/2 + (d*x)/2)*(5*a^13 + 64*a^5*b^8 - 192*a^7*b^6 + 196*a^9*b^4 - 72
*a^11*b^2))/(8*a^11) - (21*a^13*b - 32*a^7*b^7 + 88*a^9*b^5 - 78*a^11*b^3)/
(8*a^12) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^14 - 64*a^12*b^2))/(8*a^
11))*(-(a + b)^5*(a - b)^5)^(1/2))/a^7))/a^7 + (b*(-(a + b)^5*(a - b)^5)^(1
/2)*((21*a^13*b - 32*a^7*b^7 + 88*a^9*b^5 - 78*a^11*b^3)/(8*a^12) - (tan(c/
2 + (d*x)/2)*(5*a^13 + 64*a^5*b^8 - 192*a^7*b^6 + 196*a^9*b^4 - 72*a^11*b^2
))/(8*a^11) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^14 - 64*a^12*b^2))/(8
*a^11))*(-(a + b)^5*(a - b)^5)^(1/2))/a^7))/a^7))*(-(a + b)^5*(a - b)^5)^(1
/2)*2i)/(a^7*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1330 $\int \frac{\cot^6(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=417

$$\frac{b\cot(c+dx)\csc^5(c+dx)}{6a^2d}-\frac{2b^2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}+\frac{\left(8a^4-13a^2b^2+6b^4\right)\cot(c+dx)\csc^3(c+dx)}{24a^4bd}$$

```
[Out] -2*b^2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^8
/d-1/16*b*(5*a^6-30*a^4*b^2+40*a^2*b^4-16*b^6)*arctanh(cos(d*x+c))/a^8/d+1/
105*(15*a^6-161*a^4*b^2+245*a^2*b^4-105*b^6)*cot(d*x+c)/a^7/d+1/16*b*(11*a^
4-18*a^2*b^2+8*b^4)*cot(d*x+c)*csc(d*x+c)/a^6/d-1/105*(45*a^4-77*a^2*b^2+35
*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^5/d-1/3*cot(d*x+c)*csc(d*x+c)^3/b/d+1/24*(8
*a^4-13*a^2*b^2+6*b^4)*cot(d*x+c)*csc(d*x+c)^3/a^4/b/d+1/4*a*cot(d*x+c)*csc
(d*x+c)^4/b^2/d-1/140*(35*a^4-60*a^2*b^2+28*b^4)*cot(d*x+c)*csc(d*x+c)^4/a^
3/b^2/d+1/6*b*cot(d*x+c)*csc(d*x+c)^5/a^2/d-1/7*cot(d*x+c)*csc(d*x+c)^6/a/d
```

Rubi [A] time = 1.83, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b^2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}+\frac{\left(-161a^4b^2+245a^2b^4+15a^6-105b^6\right)\cot(c+dx)}{105a^7d}-\frac{b\left(-30a^4b^2+40a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(a^8*d) - (b*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*ArcTanh[Cos[c + d*
x]])/(16*a^8*d) + ((15*a^6 - 161*a^4*b^2 + 245*a^2*b^4 - 105*b^6)*Cot[c + d
*x])/(105*a^7*d) + (b*(11*a^4 - 18*a^2*b^2 + 8*b^4)*Cot[c + d*x]*Csc[c + d*
x])/(16*a^6*d) - ((45*a^4 - 77*a^2*b^2 + 35*b^4)*Cot[c + d*x]*Csc[c + d*x]^
2)/(105*a^5*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(3*b*d) + ((8*a^4 - 13*a^2*b
^2 + 6*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*a^4*b*d) + (a*Cot[c + d*x]*Csc
[c + d*x]^4)/(4*b^2*d) - ((35*a^4 - 60*a^2*b^2 + 28*b^4)*Cot[c + d*x]*Csc[c
 + d*x]^4)/(140*a^3*b^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^5)/(6*a^2*d) - (C
ot[c + d*x]*Csc[c + d*x]^6)/(7*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc^3(c+dx)}{3bd} + \frac{a\cot(c+dx)\csc^4(c+dx)}{4b^2d} + \frac{b\cot(c+dx)\csc^5(c}{6a^2d}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{3bd} + \frac{a\cot(c+dx)\csc^4(c+dx)}{4b^2d} - \frac{\left(35a^4-60a^2b^2+28\right.}{1}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{3bd} + \frac{\left(8a^4-13a^2b^2+6b^4\right)\cot(c+dx)\csc^3(c+dx)}{24a^4bd} +$$

$$= -\frac{\left(45a^4-77a^2b^2+35b^4\right)\cot(c+dx)\csc^2(c+dx)}{105a^5d} - \frac{\cot(c+dx)\csc^3(c+dx)}{3bd}$$

$$= \frac{b\left(11a^4-18a^2b^2+8b^4\right)\cot(c+dx)\csc(c+dx)}{16a^6d} - \frac{\left(45a^4-77a^2b^2+35b^4\right)\cot}{105a^5d}$$

$$= \frac{\left(15a^6-161a^4b^2+245a^2b^4-105b^6\right)\cot(c+dx)}{105a^7d} + \frac{b\left(11a^4-18a^2b^2+8b^4\right)c}{16a^6d}$$

$$= \frac{\left(15a^6-161a^4b^2+245a^2b^4-105b^6\right)\cot(c+dx)}{105a^7d} + \frac{b\left(11a^4-18a^2b^2+8b^4\right)c}{16a^6d}$$

$$= -\frac{b\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)\tanh^{-1}(\cos(c+dx))}{16a^8d} + \frac{\left(15a^6-161a^4b^2+\right.}{}$$

$$= -\frac{b\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)\tanh^{-1}(\cos(c+dx))}{16a^8d} + \frac{\left(15a^6-161a^4b^2+\right.}{}$$

$$= -\frac{2b^2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d} - \frac{b\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)t}{16a^8d}$$

Mathematica [A] time = 1.99, size = 442, normalized size = 1.06

$$\frac{-107520b^2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)+3360\left(-5a^6b+30a^4b^3-40a^2b^5+16b^7\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-107520*b^2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]] + 3360*(-5*a^6*b + 30*a^4*b^3 - 40*a^2*b^5 + 16*b^7)*Log[Cos[(c + d*x)
/2]] + 3360*b*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*Log[Sin[(c + d*x)/
2]] - 2*a*Cot[c + d*x]*Csc[c + d*x]^6*(1200*a^6 + 8176*a^4*b^2 - 16240*a^2*
b^4 + 8400*b^6 + 8*(225*a^6 - 1519*a^4*b^2 + 3115*a^2*b^4 - 1575*b^6)*Cos[2
*(c + d*x)] + 16*(45*a^6 + 329*a^4*b^2 - 665*a^2*b^4 + 315*b^6)*Cos[4*(c +
d*x)] + 120*a^6*Cos[6*(c + d*x)] - 1288*a^4*b^2*Cos[6*(c + d*x)] + 1960*a^2
*b^4*Cos[6*(c + d*x)] - 840*b^6*Cos[6*(c + d*x)] - 5110*a^5*b*Sin[c + d*x]
+ 13860*a^3*b^3*Sin[c + d*x] - 8400*a*b^5*Sin[c + d*x] + 2135*a^5*b*Sin[3*(
c + d*x)] - 7770*a^3*b^3*Sin[3*(c + d*x)] + 4200*a*b^5*Sin[3*(c + d*x)] - 1
155*a^5*b*Sin[5*(c + d*x)] + 1890*a^3*b^3*Sin[5*(c + d*x)] - 840*a*b^5*Sin[
5*(c + d*x)]))/(53760*a^8*d)
```

fricas [A] time = 1.82, size = 1645, normalized size = 3.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/3360*(32*(15*a^7 - 161*a^5*b^2 + 245*a^3*b^4 - 105*a*b^6)*cos(d*x + c)^7
 + 224*(58*a^5*b^2 - 100*a^3*b^4 + 45*a*b^6)*cos(d*x + c)^5 - 1120*(10*a^5*
b^2 - 19*a^3*b^4 + 9*a*b^6)*cos(d*x + c)^3 + 1680*((a^4*b^2 - 2*a^2*b^4 + b
^6)*cos(d*x + c)^6 - a^4*b^2 + 2*a^2*b^4 - b^6 - 3*(a^4*b^2 - 2*a^2*b^4 + b
^6)*cos(d*x + c)^4 + 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2)*sqrt(-a^
2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) + 105*(5*a^6
*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7 - (5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5
- 16*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*c
os(d*x + c)^4 - 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)
^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 105*(5*a^6*b - 30*a^4*b^3 +
40*a^2*b^5 - 16*b^7 - (5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x
+ c)^6 + 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^4 - 3*
(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^2)*log(-1/2*cos(d
*x + c) + 1/2)*sin(d*x + c) + 3360*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x +
c) - 70*(3*(11*a^6*b - 18*a^4*b^3 + 8*a^2*b^5)*cos(d*x + c)^5 - 8*(5*a^6*b
- 12*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c)^3 + 3*(5*a^6*b - 14*a^4*b^3 + 8*a^2*
b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*d*cos(d*x + c)^6 - 3*a^8*d*cos(d*x +
 c)^4 + 3*a^8*d*cos(d*x + c)^2 - a^8*d)*sin(d*x + c)), 1/3360*(32*(15*a^7 -
 161*a^5*b^2 + 245*a^3*b^4 - 105*a*b^6)*cos(d*x + c)^7 + 224*(58*a^5*b^2 -
100*a^3*b^4 + 45*a*b^6)*cos(d*x + c)^5 - 1120*(10*a^5*b^2 - 19*a^3*b^4 + 9*
a*b^6)*cos(d*x + c)^3 + 3360*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^6 -
a^4*b^2 + 2*a^2*b^4 - b^6 - 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^4 +
3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*si
```

```
n(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) + 105*(5*a^6*b
 - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7 - (5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 -
16*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos
(d*x + c)^4 - 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^2
)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 105*(5*a^6*b - 30*a^4*b^3 + 40
*a^2*b^5 - 16*b^7 - (5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x +
c)^6 + 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^4 - 3*(5
*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^2)*log(-1/2*cos(d*x
 + c) + 1/2)*sin(d*x + c) + 3360*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)
 - 70*(3*(11*a^6*b - 18*a^4*b^3 + 8*a^2*b^5)*cos(d*x + c)^5 - 8*(5*a^6*b -
12*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c)^3 + 3*(5*a^6*b - 14*a^4*b^3 + 8*a^2*b^
5)*cos(d*x + c))*sin(d*x + c))/((a^8*d*cos(d*x + c)^6 - 3*a^8*d*cos(d*x + c
)^4 + 3*a^8*d*cos(d*x + c)^2 - a^8*d)*sin(d*x + c))]
```

giac [A] time = 0.24, size = 776, normalized size = 1.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13440*((15*a^6*tan(1/2*d*x + 1/2*c)^7 - 35*a^5*b*tan(1/2*d*x + 1/2*c)^6 -
 105*a^6*tan(1/2*d*x + 1/2*c)^5 + 84*a^4*b^2*tan(1/2*d*x + 1/2*c)^5 + 315*a
^5*b*tan(1/2*d*x + 1/2*c)^4 - 210*a^3*b^3*tan(1/2*d*x + 1/2*c)^4 + 315*a^6*
tan(1/2*d*x + 1/2*c)^3 - 980*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 560*a^2*b^4*t
an(1/2*d*x + 1/2*c)^3 - 1575*a^5*b*tan(1/2*d*x + 1/2*c)^2 + 3360*a^3*b^3*ta
n(1/2*d*x + 1/2*c)^2 - 1680*a*b^5*tan(1/2*d*x + 1/2*c)^2 - 525*a^6*tan(1/2*
d*x + 1/2*c) + 9240*a^4*b^2*tan(1/2*d*x + 1/2*c) - 15120*a^2*b^4*tan(1/2*d*
x + 1/2*c) + 6720*b^6*tan(1/2*d*x + 1/2*c))/a^7 + 840*(5*a^6*b - 30*a^4*b^3
 + 40*a^2*b^5 - 16*b^7)*log(abs(tan(1/2*d*x + 1/2*c)))/a^8 - 26880*(a^6*b^2
 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) +
arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^8)
 - (10890*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 65340*a^4*b^3*tan(1/2*d*x + 1/2*c)
^7 + 87120*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 34848*b^7*tan(1/2*d*x + 1/2*c)^
7 - 525*a^7*tan(1/2*d*x + 1/2*c)^6 + 9240*a^5*b^2*tan(1/2*d*x + 1/2*c)^6 -
15120*a^3*b^4*tan(1/2*d*x + 1/2*c)^6 + 6720*a*b^6*tan(1/2*d*x + 1/2*c)^6 -
1575*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 3360*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 1
680*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 315*a^7*tan(1/2*d*x + 1/2*c)^4 - 980*a
^5*b^2*tan(1/2*d*x + 1/2*c)^4 + 560*a^3*b^4*tan(1/2*d*x + 1/2*c)^4 + 315*a^
6*b*tan(1/2*d*x + 1/2*c)^3 - 210*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 105*a^7*t
an(1/2*d*x + 1/2*c)^2 + 84*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 - 35*a^6*b*tan(1/
2*d*x + 1/2*c) + 15*a^7)/(a^8*tan(1/2*d*x + 1/2*c)^7))/d
```

maple [B] time = 0.52, size = 952, normalized size = 2.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/128/d/a/tan(1/2*d*x+1/2*c)^5+1/384/d/a^2*b/tan(1/2*d*x+1/2*c)^6-5/128/a/d
*tan(1/2*d*x+1/2*c)+5/128/a/d/tan(1/2*d*x+1/2*c)-15/128/d/a^2*tan(1/2*d*x+1
/2*c)^2*b+11/16/d/a^3*b^2*tan(1/2*d*x+1/2*c)-11/16/d/a^3/tan(1/2*d*x+1/2*c)
*b^2+1/24/d/a^5*tan(1/2*d*x+1/2*c)^3*b^4+1/4/d/a^4*tan(1/2*d*x+1/2*c)^2*b^3
-9/8/d/a^5*b^4*tan(1/2*d*x+1/2*c)+7/96/d/a^3/tan(1/2*d*x+1/2*c)^3*b^2+9/8/d
/a^5/tan(1/2*d*x+1/2*c)*b^4-3/128/d/a^2*b/tan(1/2*d*x+1/2*c)^4-1/4/d*b^3/a^
4/tan(1/2*d*x+1/2*c)^2-3/128/d/a/tan(1/2*d*x+1/2*c)^3+5/16/d/a^2*b*ln(tan(1
/2*d*x+1/2*c))+15/128/d/a^2*b/tan(1/2*d*x+1/2*c)^2-15/8/d/a^4*b^3*ln(tan(1/
2*d*x+1/2*c))-1/128/d/a*tan(1/2*d*x+1/2*c)^5+1/160/d/a^3*tan(1/2*d*x+1/2*c)
^5*b^2-1/64/d/a^4*tan(1/2*d*x+1/2*c)^4*b^3+5/2/d/a^6*b^5*ln(tan(1/2*d*x+1/2
*c))+3/128/d/a^2*tan(1/2*d*x+1/2*c)^4*b-7/96/d/a^3*tan(1/2*d*x+1/2*c)^3*b^2
-1/384/d/a^2*b*tan(1/2*d*x+1/2*c)^6+1/896/d/a*tan(1/2*d*x+1/2*c)^7-1/896/d/
a/tan(1/2*d*x+1/2*c)^7+3/128/d/a*tan(1/2*d*x+1/2*c)^3+1/2/d/a^7*b^6*tan(1/2
*d*x+1/2*c)-1/8/d/a^6*tan(1/2*d*x+1/2*c)^2*b^5-1/d/a^8*b^7*ln(tan(1/2*d*x+1
/2*c))-1/2/d/a^7/tan(1/2*d*x+1/2*c)*b^6+1/8/d/a^6*b^5/tan(1/2*d*x+1/2*c)^2-
1/160/d/a^3/tan(1/2*d*x+1/2*c)^5*b^2-1/24/d/a^5/tan(1/2*d*x+1/2*c)^3*b^4+1/
64/d/a^4*b^3/tan(1/2*d*x+1/2*c)^4-2/d*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2
*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-6/d/a^6*b^6/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/a^8*b^8/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d/a^4/(a^
2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 12.38, size = 1513, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^8*(a + b*sin(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(b^2/(32*a^3) - 5/(128*a) + (2*b*(b/(64*a^2) + (2*b*(5/
(128*a) - b^2/(32*a^3)))/a))/a + (2*b*(b/(64*a^2) - (2*b*(b^2/(32*a^3) - 9/
(128*a) + (2*b*(b/(64*a^2) + (2*b*(5/(128*a) - b^2/(32*a^3)))/a))/a))/a + (
2*b*(5/(128*a) - b^2/(32*a^3)))/a))/a))/d + tan(c/2 + (d*x)/2)^7/(896*a*d)
+ (tan(c/2 + (d*x)/2)^4*(b/(256*a^2) + (b*(5/(128*a) - b^2/(32*a^3)))/(2*a)
))/d - (tan(c/2 + (d*x)/2)^2*(b/(128*a^2) - (b*(b^2/(32*a^3) - 9/(128*a) +
(2*b*(b/(64*a^2) + (2*b*(5/(128*a) - b^2/(32*a^3)))/a))/a))/a + (b*(5/(128*
a) - b^2/(32*a^3)))/a))/d - (tan(c/2 + (d*x)/2)^5*(1/(128*a) - b^2/(160*a^3
)))/d - (tan(c/2 + (d*x)/2)^3*(b^2/(96*a^3) - 3/(128*a) + (2*b*(b/(64*a^2)
+ (2*b*(5/(128*a) - b^2/(32*a^3)))/a))/(3*a)))/d + (tan(c/2 + (d*x)/2)^2*(a
^6 - (4*a^4*b^2)/5) - tan(c/2 + (d*x)/2)^3*(3*a^5*b - 2*a^3*b^3) - a^6/7 -
tan(c/2 + (d*x)/2)^4*(3*a^6 + (16*a^2*b^4)/3 - (28*a^4*b^2)/3) + tan(c/2 +
(d*x)/2)^5*(16*a*b^5 + 15*a^5*b - 32*a^3*b^3) + tan(c/2 + (d*x)/2)^6*(5*a^6
 - 64*b^6 + 144*a^2*b^4 - 88*a^4*b^2) + (a^5*b*tan(c/2 + (d*x)/2))/3)/(128*
a^7*d*tan(c/2 + (d*x)/2)^7) - (b*tan(c/2 + (d*x)/2)^6)/(384*a^2*d) + (log(t
an(c/2 + (d*x)/2))*(5*a^6*b - 16*b^7 + 40*a^2*b^5 - 30*a^4*b^3))/(16*a^8*d)
 - (b^2*atan(((b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((32*a^8*b^8 - 88*a^10*b^6
+ 78*a^12*b^4 - 21*a^14*b^2)/(8*a^14) + (tan(c/2 + (d*x)/2)*(5*a^14*b + 64*
a^6*b^9 - 192*a^8*b^7 + 196*a^10*b^5 - 72*a^12*b^3))/(8*a^13) + (b^2*(2*a^2
*b - (tan(c/2 + (d*x)/2)*(48*a^16 - 64*a^14*b^2))/(8*a^13))*(-(a + b)^5*(a
- b)^5)^(1/2))/a^8)*1i)/a^8 + (b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((32*a^8*b^
8 - 88*a^10*b^6 + 78*a^12*b^4 - 21*a^14*b^2)/(8*a^14) + (tan(c/2 + (d*x)/2)
*(5*a^14*b + 64*a^6*b^9 - 192*a^8*b^7 + 196*a^10*b^5 - 72*a^12*b^3))/(8*a^1
3) - (b^2*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^16 - 64*a^14*b^2))/(8*a^13))
*(-(a + b)^5*(a - b)^5)^(1/2))/a^8)*1i)/a^8)/((16*b^15 - 88*a^2*b^13 + 198*
a^4*b^11 - 231*a^6*b^9 + 145*a^8*b^7 - 45*a^10*b^5 + 5*a^12*b^3)/(4*a^14) +
 (tan(c/2 + (d*x)/2)*(16*b^14 - 84*a^2*b^12 + 178*a^4*b^10 - 190*a^6*b^8 +
102*a^8*b^6 - 22*a^10*b^4))/(4*a^13) + (b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((
32*a^8*b^8 - 88*a^10*b^6 + 78*a^12*b^4 - 21*a^14*b^2)/(8*a^14) + (tan(c/2 +
 (d*x)/2)*(5*a^14*b + 64*a^6*b^9 - 192*a^8*b^7 + 196*a^10*b^5 - 72*a^12*b^3
))/(8*a^13) + (b^2*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^16 - 64*a^14*b^2))/
(8*a^13))*(-(a + b)^5*(a - b)^5)^(1/2))/a^8))/a^8 - (b^2*(-(a + b)^5*(a - b
)^5)^(1/2)*((32*a^8*b^8 - 88*a^10*b^6 + 78*a^12*b^4 - 21*a^14*b^2)/(8*a^14)
 + (tan(c/2 + (d*x)/2)*(5*a^14*b + 64*a^6*b^9 - 192*a^8*b^7 + 196*a^10*b^5
- 72*a^12*b^3))/(8*a^13) - (b^2*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^16 - 6
4*a^14*b^2))/(8*a^13))*(-(a + b)^5*(a - b)^5)^(1/2))/a^8))/a^8))*(-(a + b)^
5*(a - b)^5)^(1/2)*2i)/(a^8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1331 $\int \frac{\cot^6(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=476

$$\frac{b\cot(c+dx)\csc^6(c+dx)}{7a^2d}+\frac{2b^3\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^9d}+\frac{\left(35a^4-60a^2b^2+28b^4\right)\cot(c+dx)\csc^4(c+$$

```
[Out] 2*b^3*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^9/
d+1/128*(5*a^8+40*a^6*b^2-240*a^4*b^4+320*a^2*b^6-128*b^8)*arctanh(cos(d*x+
c))/a^9/d-1/105*b*(15*a^6-161*a^4*b^2+245*a^2*b^4-105*b^6)*cot(d*x+c)/a^8/d
+1/128*(5*a^6-88*a^4*b^2+144*a^2*b^4-64*b^6)*cot(d*x+c)*csc(d*x+c)/a^7/d+1/
105*b*(45*a^4-77*a^2*b^2+35*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^6/d-1/192*(59*a^
4-104*a^2*b^2+48*b^4)*cot(d*x+c)*csc(d*x+c)^3/a^5/d-1/4*cot(d*x+c)*csc(d*x+
c)^4/b/d+1/140*(35*a^4-60*a^2*b^2+28*b^4)*cot(d*x+c)*csc(d*x+c)^4/a^4/b/d+1
/5*a*cot(d*x+c)*csc(d*x+c)^5/b^2/d-1/240*(48*a^4-85*a^2*b^2+40*b^4)*cot(d*x
+c)*csc(d*x+c)^5/a^3/b^2/d+1/7*b*cot(d*x+c)*csc(d*x+c)^6/a^2/d-1/8*cot(d*x+
c)*csc(d*x+c)^7/a/d
```

Rubi [A] time = 2.21, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b^3\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^9d}-\frac{b\left(-161a^4b^2+245a^2b^4+15a^6-105b^6\right)\cot(c+dx)}{105a^8d}+\frac{\left(40a^6b^2-240a^4b^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^3*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^9*d) + ((5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*ArcTa
nh[Cos[c + d*x]])/(128*a^9*d) - (b*(15*a^6 - 161*a^4*b^2 + 245*a^2*b^4 - 10
5*b^6)*Cot[c + d*x])/(105*a^8*d) + ((5*a^6 - 88*a^4*b^2 + 144*a^2*b^4 - 64*
b^6)*Cot[c + d*x]*Csc[c + d*x])/(128*a^7*d) + (b*(45*a^4 - 77*a^2*b^2 + 35*
b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(105*a^6*d) - ((59*a^4 - 104*a^2*b^2 + 48
*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(192*a^5*d) - (Cot[c + d*x]*Csc[c + d*x]
^4)/(4*b*d) + ((35*a^4 - 60*a^2*b^2 + 28*b^4)*Cot[c + d*x]*Csc[c + d*x]^4)/
(140*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^5)/(5*b^2*d) - ((48*a^4 - 85*a
^2*b^2 + 40*b^4)*Cot[c + d*x]*Csc[c + d*x]^5)/(240*a^3*b^2*d) + (b*Cot[c +
d*x]*Csc[c + d*x]^6)/(7*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^7)/(8*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc^4(c+dx)}{4bd} + \frac{a\cot(c+dx)\csc^5(c+dx)}{5b^2d} + \frac{b\cot(c+dx)\csc^6(c+}{7a^2d}$$

$$= -\frac{\cot(c+dx)\csc^4(c+dx)}{4bd} + \frac{a\cot(c+dx)\csc^5(c+dx)}{5b^2d} - \frac{\left(48a^4-85a^2b^2+40b\right.}{24}$$

$$= -\frac{\cot(c+dx)\csc^4(c+dx)}{4bd} + \frac{\left(35a^4-60a^2b^2+28b^4\right)\cot(c+dx)\csc^4(c+dx)}{140a^4bd} +$$

$$= -\frac{\left(59a^4-104a^2b^2+48b^4\right)\cot(c+dx)\csc^3(c+dx)}{192a^5d} - \frac{\cot(c+dx)\csc^4(c+dx)}{4bd}$$

$$= \frac{b\left(45a^4-77a^2b^2+35b^4\right)\cot(c+dx)\csc^2(c+dx)}{105a^6d} - \frac{\left(59a^4-104a^2b^2+48b^4\right)c}{192a}$$

$$= \frac{\left(5a^6-88a^4b^2+144a^2b^4-64b^6\right)\cot(c+dx)\csc(c+dx)}{128a^7d} + \frac{b\left(45a^4-77a^2b^2+\right.}{}$$

$$= -\frac{b\left(15a^6-161a^4b^2+245a^2b^4-105b^6\right)\cot(c+dx)}{105a^8d} + \frac{\left(5a^6-88a^4b^2+144a^2b^4\right.}{}$$

$$= -\frac{b\left(15a^6-161a^4b^2+245a^2b^4-105b^6\right)\cot(c+dx)}{105a^8d} + \frac{\left(5a^6-88a^4b^2+144a^2b^4\right.}{}$$

$$= \frac{\left(5a^8+40a^6b^2-240a^4b^4+320a^2b^6-128b^8\right)\tanh^{-1}(\cos(c+dx))}{128a^9d} - \frac{b\left(15a^6-\right.}{}$$

$$= \frac{\left(5a^8+40a^6b^2-240a^4b^4+320a^2b^6-128b^8\right)\tanh^{-1}(\cos(c+dx))}{128a^9d} - \frac{b\left(15a^6-\right.}{}$$

$$= \frac{2b^3\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^9d} + \frac{\left(5a^8+40a^6b^2-240a^4b^4+320a^2b^6-\right.}{128a^9d}$$

Mathematica [A] time = 3.59, size = 593, normalized size = 1.25

$$\frac{1720320b^3\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) - 6720\left(5a^8+40a^6b^2-240a^4b^4+320a^2b^6-128b^8\right)\log\left(\sin\left(\frac{1}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (1720320*b^3*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]] + 6720*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*Log[
Cos[(c + d*x)/2]] - 6720*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 -
128*b^8)*Log[Sin[(c + d*x)/2]] + a*Csc[c + d*x]^8*(-35*a*(1765*a^6 + 680*a^
4*b^2 - 1392*a^2*b^4 + 960*b^6)*Cos[c + d*x] - 35*(895*a^7 - 904*a^5*b^2 +
2736*a^3*b^4 - 1728*a*b^6)*Cos[3*(c + d*x)] - 13895*a^7*Cos[5*(c + d*x)] -
17080*a^5*b^2*Cos[5*(c + d*x)] + 62160*a^3*b^4*Cos[5*(c + d*x)] - 33600*a*b
^6*Cos[5*(c + d*x)] - 525*a^7*Cos[7*(c + d*x)] + 9240*a^5*b^2*Cos[7*(c + d*
x)] - 15120*a^3*b^4*Cos[7*(c + d*x)] + 6720*a*b^6*Cos[7*(c + d*x)] + 13440*
a^6*b*Sin[2*(c + d*x)] + 88704*a^4*b^3*Sin[2*(c + d*x)] - 174720*a^2*b^5*Si
n[2*(c + d*x)] + 94080*b^7*Sin[2*(c + d*x)] + 13440*a^6*b*Sin[4*(c + d*x)]
- 86912*a^4*b^3*Sin[4*(c + d*x)] + 183680*a^2*b^5*Sin[4*(c + d*x)] - 94080*
b^7*Sin[4*(c + d*x)] + 5760*a^6*b*Sin[6*(c + d*x)] + 42112*a^4*b^3*Sin[6*(c
 + d*x)] - 85120*a^2*b^5*Sin[6*(c + d*x)] + 40320*b^7*Sin[6*(c + d*x)] + 96
0*a^6*b*Sin[8*(c + d*x)] - 10304*a^4*b^3*Sin[8*(c + d*x)] + 15680*a^2*b^5*S
in[8*(c + d*x)] - 6720*b^7*Sin[8*(c + d*x)]))/(860160*a^9*d)
```

fricas [B] time = 3.46, size = 2082, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/26880*(210*(5*a^8 - 88*a^6*b^2 + 144*a^4*b^4 - 64*a^2*b^6)*cos(d*x + c)
^7 + 70*(73*a^8 + 584*a^6*b^2 - 1200*a^4*b^4 + 576*a^2*b^6)*cos(d*x + c)^5
- 70*(55*a^8 + 440*a^6*b^2 - 1104*a^4*b^4 + 576*a^2*b^6)*cos(d*x + c)^3 - 1
3440*((a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^8 + a^4*b^3 - 2*a^2*b^5 + b^
7 - 4*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^6 + 6*(a^4*b^3 - 2*a^2*b^5 +
 b^7)*cos(d*x + c)^4 - 4*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sqrt(-
a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b
^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 210*(5*a^8 + 40*a^6*
b^2 - 112*a^4*b^4 + 64*a^2*b^6)*cos(d*x + c) - 105*((5*a^8 + 40*a^6*b^2 - 2
40*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^8 + 5*a^8 + 40*a^6*b^2 - 2
40*a^4*b^4 + 320*a^2*b^6 - 128*b^8 - 4*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 +
320*a^2*b^6 - 128*b^8)*cos(d*x + c)^6 + 6*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4
 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^4 - 4*(5*a^8 + 40*a^6*b^2 - 240*a^4*
b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) +
105*((5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c
)^8 + 5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8 - 4*(5*a^8 +
 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^6 + 6*(5*a^
8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^4 - 4*(5
```

```
*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^2)*lo
g(-1/2*cos(d*x + c) + 1/2) - 256*((15*a^7*b - 161*a^5*b^3 + 245*a^3*b^5 - 1
05*a*b^7)*cos(d*x + c)^7 + 7*(58*a^5*b^3 - 100*a^3*b^5 + 45*a*b^7)*cos(d*x
+ c)^5 - 35*(10*a^5*b^3 - 19*a^3*b^5 + 9*a*b^7)*cos(d*x + c)^3 + 105*(a^5*b
^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c))*sin(d*x + c))/(a^9*d*cos(d*x + c)^8 -
 4*a^9*d*cos(d*x + c)^6 + 6*a^9*d*cos(d*x + c)^4 - 4*a^9*d*cos(d*x + c)^2 +
 a^9*d), -1/26880*(210*(5*a^8 - 88*a^6*b^2 + 144*a^4*b^4 - 64*a^2*b^6)*cos(
d*x + c)^7 + 70*(73*a^8 + 584*a^6*b^2 - 1200*a^4*b^4 + 576*a^2*b^6)*cos(d*x
 + c)^5 - 70*(55*a^8 + 440*a^6*b^2 - 1104*a^4*b^4 + 576*a^2*b^6)*cos(d*x +
c)^3 + 26880*((a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^8 + a^4*b^3 - 2*a^2*
b^5 + b^7 - 4*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^6 + 6*(a^4*b^3 - 2*a
^2*b^5 + b^7)*cos(d*x + c)^4 - 4*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2
)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c
))) + 210*(5*a^8 + 40*a^6*b^2 - 112*a^4*b^4 + 64*a^2*b^6)*cos(d*x + c) - 10
5*((5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^
8 + 5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8 - 4*(5*a^8 + 4
0*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^6 + 6*(5*a^8
+ 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^4 - 4*(5*a
^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^2)*log(
1/2*cos(d*x + c) + 1/2) + 105*((5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*
b^6 - 128*b^8)*cos(d*x + c)^8 + 5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*
b^6 - 128*b^8 - 4*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8
)*cos(d*x + c)^6 + 6*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*
b^8)*cos(d*x + c)^4 - 4*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 1
28*b^8)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 256*((15*a^7*b - 161
*a^5*b^3 + 245*a^3*b^5 - 105*a*b^7)*cos(d*x + c)^7 + 7*(58*a^5*b^3 - 100*a^
3*b^5 + 45*a*b^7)*cos(d*x + c)^5 - 35*(10*a^5*b^3 - 19*a^3*b^5 + 9*a*b^7)*c
os(d*x + c)^3 + 105*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c))*sin(d*x + c
))/(a^9*d*cos(d*x + c)^8 - 4*a^9*d*cos(d*x + c)^6 + 6*a^9*d*cos(d*x + c)^4
- 4*a^9*d*cos(d*x + c)^2 + a^9*d)]
```

giac [B] time = 0.26, size = 948, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/215040*((105*a^7*tan(1/2*d*x + 1/2*c)^8 - 240*a^6*b*tan(1/2*d*x + 1/2*c)^
7 - 560*a^7*tan(1/2*d*x + 1/2*c)^6 + 560*a^5*b^2*tan(1/2*d*x + 1/2*c)^6 + 1
680*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 1344*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 84
0*a^7*tan(1/2*d*x + 1/2*c)^4 - 5040*a^5*b^2*tan(1/2*d*x + 1/2*c)^4 + 3360*a
^3*b^4*tan(1/2*d*x + 1/2*c)^4 - 5040*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 15680*a
^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 8960*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 1680*
a^7*tan(1/2*d*x + 1/2*c)^2 + 25200*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 - 53760*a
```

```
^3*b^4*tan(1/2*d*x + 1/2*c)^2 + 26880*a*b^6*tan(1/2*d*x + 1/2*c)^2 + 8400*a
^6*b*tan(1/2*d*x + 1/2*c) - 147840*a^4*b^3*tan(1/2*d*x + 1/2*c) + 241920*a^
2*b^5*tan(1/2*d*x + 1/2*c) - 107520*b^7*tan(1/2*d*x + 1/2*c))/a^8 - 1680*(5
*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*log(abs(tan(1/2*d*
x + 1/2*c)))/a^9 + 430080*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*(pi*floor
(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(
a^2 - b^2)))/(sqrt(a^2 - b^2)*a^9) + (22830*a^8*tan(1/2*d*x + 1/2*c)^8 + 18
2640*a^6*b^2*tan(1/2*d*x + 1/2*c)^8 - 1095840*a^4*b^4*tan(1/2*d*x + 1/2*c)^
8 + 1461120*a^2*b^6*tan(1/2*d*x + 1/2*c)^8 - 584448*b^8*tan(1/2*d*x + 1/2*c
)^8 - 8400*a^7*b*tan(1/2*d*x + 1/2*c)^7 + 147840*a^5*b^3*tan(1/2*d*x + 1/2*
c)^7 - 241920*a^3*b^5*tan(1/2*d*x + 1/2*c)^7 + 107520*a*b^7*tan(1/2*d*x + 1
/2*c)^7 - 1680*a^8*tan(1/2*d*x + 1/2*c)^6 - 25200*a^6*b^2*tan(1/2*d*x + 1/2
*c)^6 + 53760*a^4*b^4*tan(1/2*d*x + 1/2*c)^6 - 26880*a^2*b^6*tan(1/2*d*x +
1/2*c)^6 + 5040*a^7*b*tan(1/2*d*x + 1/2*c)^5 - 15680*a^5*b^3*tan(1/2*d*x +
1/2*c)^5 + 8960*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 840*a^8*tan(1/2*d*x + 1/2*
c)^4 + 5040*a^6*b^2*tan(1/2*d*x + 1/2*c)^4 - 3360*a^4*b^4*tan(1/2*d*x + 1/2
*c)^4 - 1680*a^7*b*tan(1/2*d*x + 1/2*c)^3 + 1344*a^5*b^3*tan(1/2*d*x + 1/2*
c)^3 + 560*a^8*tan(1/2*d*x + 1/2*c)^2 - 560*a^6*b^2*tan(1/2*d*x + 1/2*c)^2
+ 240*a^7*b*tan(1/2*d*x + 1/2*c) - 105*a^8)/(a^9*tan(1/2*d*x + 1/2*c)^8))/d
```

maple [B] time = 0.51, size = 1143, normalized size = 2.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9/(a+b*sin(d*x+c)),x)
```

```
[Out] 3/128/d/a^3/tan(1/2*d*x+1/2*c)^4*b^2-5/2/d/a^7*ln(tan(1/2*d*x+1/2*c))*b^6-1
/128/d/a^2*b/tan(1/2*d*x+1/2*c)^5-9/8/d*b^5/a^6/tan(1/2*d*x+1/2*c)-7/96/d/a
^4*b^3/tan(1/2*d*x+1/2*c)^3+1/4/d/a^5/tan(1/2*d*x+1/2*c)^2*b^4+1/128/d/a^2*
b*tan(1/2*d*x+1/2*c)^5-3/128/d/a^3*tan(1/2*d*x+1/2*c)^4*b^2-1/4/d/a^5*tan(1
/2*d*x+1/2*c)^2*b^4+7/96/d/a^4*tan(1/2*d*x+1/2*c)^3*b^3+1/2048/d/a*tan(1/2*
d*x+1/2*c)^8-1/2048/d/a/tan(1/2*d*x+1/2*c)^8+1/128/a/d*tan(1/2*d*x+1/2*c)^2
-1/128/a/d/tan(1/2*d*x+1/2*c)^2+2/d/a^3*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-5/128/a/d*ln(tan(1/2*d*x+1/2*c))-
11/16/d/a^4*b^3*tan(1/2*d*x+1/2*c)+3/128/d/a^2*b/tan(1/2*d*x+1/2*c)^3-15/12
8/d*b^2/a^3/tan(1/2*d*x+1/2*c)^2+11/16/d*b^3/a^4/tan(1/2*d*x+1/2*c)-1/384/d
/a^3/tan(1/2*d*x+1/2*c)^6*b^2-1/64/d/a^5/tan(1/2*d*x+1/2*c)^4*b^4-1/8/d/a^7
/tan(1/2*d*x+1/2*c)^2*b^6+5/128/d/a^2*tan(1/2*d*x+1/2*c)*b-5/16/d/a^3*ln(ta
n(1/2*d*x+1/2*c))*b^2-5/128/d/a^2*b/tan(1/2*d*x+1/2*c)-2/d/a^9*b^9/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-3/128/d/a^
2*b*tan(1/2*d*x+1/2*c)^3+15/128/d/a^3*b^2*tan(1/2*d*x+1/2*c)^2+15/8/d/a^5*l
n(tan(1/2*d*x+1/2*c))*b^4+1/d/a^9*ln(tan(1/2*d*x+1/2*c))*b^8+1/896/d/a^2*b/
tan(1/2*d*x+1/2*c)^7+1/160/d/a^4*b^3/tan(1/2*d*x+1/2*c)^5+1/24/d/a^6*b^5/ta
n(1/2*d*x+1/2*c)^3+1/2/d*b^7/a^8/tan(1/2*d*x+1/2*c)-1/896/d/a^2*b*tan(1/2*d
```

```
*x+1/2*c)^7+1/384/d/a^3*tan(1/2*d*x+1/2*c)^6*b^2-1/160/d/a^4*tan(1/2*d*x+1/
2*c)^5*b^3+1/64/d/a^5*tan(1/2*d*x+1/2*c)^4*b^4-1/24/d/a^6*tan(1/2*d*x+1/2*c
)^3*b^5+1/8/d/a^7*tan(1/2*d*x+1/2*c)^2*b^6-1/2/d/a^8*b^7*tan(1/2*d*x+1/2*c)
+9/8/d/a^6*b^5*tan(1/2*d*x+1/2*c)-1/256/d/a/tan(1/2*d*x+1/2*c)^4-6/d/a^5*b^
5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+
6/d/a^7*b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^
2)^(1/2))+1/256/d/a*tan(1/2*d*x+1/2*c)^4-1/384/d/a*tan(1/2*d*x+1/2*c)^6+1/3
84/d/a/tan(1/2*d*x+1/2*c)^6
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.67, size = 1861, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^9*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^8/(2048*a*d) + (tan(c/2 + (d*x)/2)^5*(b/(640*a^2) + (2*b
*(1/(64*a) - b^2/(64*a^3)))/(5*a)))/d - (tan(c/2 + (d*x)/2)^3*(b/(384*a^2)
- (2*b*(b^2/(64*a^3) - 1/(64*a) + (2*b*(b/(128*a^2) + (2*b*(1/(64*a) - b^2/
(64*a^3)))/a))/a))/(3*a) + (2*b*(1/(64*a) - b^2/(64*a^3)))/(3*a)))/d + (tan
(c/2 + (d*x)/2)^2*(1/(128*a) + b^2/(128*a^3) + (b*(b/(128*a^2) + (2*b*(1/(6
4*a) - b^2/(64*a^3)))/a))/a + (b*(b/(128*a^2) - (2*b*(b^2/(64*a^3) - 1/(64*
a) + (2*b*(b/(128*a^2) + (2*b*(1/(64*a) - b^2/(64*a^3)))/a))/a))/a + (2*b*(
1/(64*a) - b^2/(64*a^3)))/a))/a))/d + (tan(c/2 + (d*x)/2)*(b/(128*a^2) - (2
*b*(b^2/(64*a^3) - 1/(64*a) + (2*b*(b/(128*a^2) + (2*b*(1/(64*a) - b^2/(64*
a^3)))/a))/a))/a - (2*b*(1/(64*a) + b^2/(64*a^3) + (2*b*(b/(128*a^2) + (2*b
*(1/(64*a) - b^2/(64*a^3)))/a))/a + (2*b*(b/(128*a^2) - (2*b*(b^2/(64*a^3)
- 1/(64*a) + (2*b*(b/(128*a^2) + (2*b*(1/(64*a) - b^2/(64*a^3)))/a))/a))/a
+ (2*b*(1/(64*a) - b^2/(64*a^3)))/a))/a))/a + (2*b*(1/(64*a) - b^2/(64*a^3)
))/a))/d - (tan(c/2 + (d*x)/2)^6*(1/(384*a) - b^2/(384*a^3)))/d - (tan(c/2
+ (d*x)/2)^4*(b^2/(256*a^3) - 1/(256*a) + (b*(b/(128*a^2) + (2*b*(1/(64*a)
- b^2/(64*a^3)))/a))/(2*a)))/d - (log(tan(c/2 + (d*x)/2))*(5*a^8 - 128*b^8
+ 320*a^2*b^6 - 240*a^4*b^4 + 40*a^6*b^2))/(128*a^9*d) - (cot(c/2 + (d*x)/2
```

```
)^8*(tan(c/2 + (d*x)/2)^3*(2*a^6*b - (8*a^4*b^3)/5) - tan(c/2 + (d*x)/2)^5*
(6*a^6*b + (32*a^2*b^5)/3 - (56*a^4*b^3)/3) + tan(c/2 + (d*x)/2)^6*(32*a*b^
6 + 2*a^7 - 64*a^3*b^4 + 30*a^5*b^2) + tan(c/2 + (d*x)/2)^7*(10*a^6*b - 128
*b^7 + 288*a^2*b^5 - 176*a^4*b^3) + tan(c/2 + (d*x)/2)^4*(a^7 + 4*a^3*b^4 -
 6*a^5*b^2) + a^7/8 - tan(c/2 + (d*x)/2)^2*((2*a^7)/3 - (2*a^5*b^2)/3) - (2
*a^6*b*tan(c/2 + (d*x)/2))/7))/(256*a^8*d) - (b*tan(c/2 + (d*x)/2)^7)/(896*
a^2*d) + (b^3*atan(((b^3*(-(a + b)^5*(a - b)^5)^(1/2)*((tan(c/2 + (d*x)/2)*
(5*a^17 + 512*a^7*b^10 - 1536*a^9*b^8 + 1568*a^11*b^6 - 576*a^13*b^4 + 30*a
^15*b^2))/(64*a^15) - (5*a^17*b - 256*a^9*b^9 + 704*a^11*b^7 - 624*a^13*b^5
 + 168*a^15*b^3)/(64*a^16) + (b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(384*a^18
- 512*a^16*b^2))/(64*a^15))*(-(a + b)^5*(a - b)^5)^(1/2))/a^9)*1i)/a^9 - (b
^3*(-(a + b)^5*(a - b)^5)^(1/2)*((5*a^17*b - 256*a^9*b^9 + 704*a^11*b^7 - 6
24*a^13*b^5 + 168*a^15*b^3)/(64*a^16) - (tan(c/2 + (d*x)/2)*(5*a^17 + 512*a
^7*b^10 - 1536*a^9*b^8 + 1568*a^11*b^6 - 576*a^13*b^4 + 30*a^15*b^2))/(64*a
^15) + (b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(384*a^18 - 512*a^16*b^2))/(64*a
^15))*(-(a + b)^5*(a - b)^5)^(1/2))/a^9)*1i)/a^9)/((128*b^17 - 704*a^2*b^15
 + 1584*a^4*b^13 - 1848*a^6*b^11 + 1155*a^8*b^9 - 345*a^10*b^7 + 25*a^12*b^
5 + 5*a^14*b^3)/(32*a^16) + (tan(c/2 + (d*x)/2)*(128*b^16 - 672*a^2*b^14 +
1424*a^4*b^12 - 1530*a^6*b^10 + 846*a^8*b^8 - 206*a^10*b^6 + 10*a^12*b^4))/
(32*a^15) + (b^3*(-(a + b)^5*(a - b)^5)^(1/2)*((tan(c/2 + (d*x)/2)*(5*a^17
+ 512*a^7*b^10 - 1536*a^9*b^8 + 1568*a^11*b^6 - 576*a^13*b^4 + 30*a^15*b^2)
)/(64*a^15) - (5*a^17*b - 256*a^9*b^9 + 704*a^11*b^7 - 624*a^13*b^5 + 168*a
^15*b^3)/(64*a^16) + (b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(384*a^18 - 512*a^
16*b^2))/(64*a^15))*(-(a + b)^5*(a - b)^5)^(1/2))/a^9))/a^9 + (b^3*(-(a + b
)^5*(a - b)^5)^(1/2)*((5*a^17*b - 256*a^9*b^9 + 704*a^11*b^7 - 624*a^13*b^5
 + 168*a^15*b^3)/(64*a^16) - (tan(c/2 + (d*x)/2)*(5*a^17 + 512*a^7*b^10 - 1
536*a^9*b^8 + 1568*a^11*b^6 - 576*a^13*b^4 + 30*a^15*b^2))/(64*a^15) + (b^3
*(2*a^2*b - (tan(c/2 + (d*x)/2)*(384*a^18 - 512*a^16*b^2))/(64*a^15))*(-(a
+ b)^5*(a - b)^5)^(1/2))/a^9))/a^9))*(-(a + b)^5*(a - b)^5)^(1/2)*2i)/(a^9*
d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1332 $\int \frac{\sin^2(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{a^3\log(a+b\sin(c+dx))}{b^2d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-\frac{\sin(c+dx)}{bd}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d-1/2*ln(1+sin(d*x+c))/(a-b)/d+a^3*ln(a+b*sin(d
*x+c))/b^2/(a^2-b^2)/d-sin(d*x+c)/b/d
```

Rubi [A] time = 0.18, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 1629}

$$\frac{a^3\log(a+b\sin(c+dx))}{b^2d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
+ (a^3*Log[a + b*Sin[c + d*x]])/(b^2*(a^2 - b^2)*d) - Sin[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^3}{b^3(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{2(a+b)(b-x)}+\frac{a^3}{(a-b)(a+b)(a+x)}-\frac{b^2}{2(a-b)(b+x)}\right)dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d}-\frac{\log(1+\sin(c+dx))}{2(a-b)d}+\frac{a^3\log(a+b\sin(c+dx))}{b^2(a^2-b^2)\,d}-\frac{\text{si}}{}$$

Mathematica [A] time = 0.21, size = 83, normalized size = 0.89

$$-\frac{-\frac{2a^3\log(a+b\sin(c+dx))}{b^2(a^2-b^2)}+\frac{\log(1-\sin(c+dx))}{a+b}+\frac{\log(\sin(c+dx)+1)}{a-b}+\frac{2\sin(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*(Log[1 - Sin[c + d*x]]/(a + b) + Log[1 + Sin[c + d*x]]/(a - b) - (2*a^
3*Log[a + b*Sin[c + d*x]])/(b^2*(a^2 - b^2)) + (2*Sin[c + d*x])/b)/d
```

fricas [A] time = 0.60, size = 100, normalized size = 1.08

$$\frac{2\,a^3\log\left(b\sin\left(dx+c\right)+a\right)-\left(ab^2+b^3\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(ab^2-b^3\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(a^2b-b^3\right.}{2\left(a^2b^2-b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*log(b*sin(d*x + c) + a) - (a*b^2 + b^3)*log(sin(d*x + c) + 1) -
(a*b^2 - b^3)*log(-sin(d*x + c) + 1) - 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*
b^2 - b^4)*d)
```

giac [A] time = 0.21, size = 85, normalized size = 0.91

$$\frac{\frac{2\,a^3\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^2b^2-b^4}-\frac{\log\left(|\sin\left(dx+c\right)+1|\right)}{a-b}-\frac{\log\left(|\sin\left(dx+c\right)-1|\right)}{a+b}-\frac{2\,\sin\left(dx+c\right)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*log(abs(b*sin(d*x + c) + a))/(a^2*b^2 - b^4) - log(abs(sin(d*x +
 c) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b) - 2*sin(d*x + c)/b)/
d
```

maple [A] time = 0.37, size = 95, normalized size = 1.02

$$-\frac{\sin (dx+c)}{bd}-\frac{\ln (\sin (dx+c)-1)}{d(2a+2b)}+\frac{a^3 \ln (a+b\sin (dx+c))}{d\,b^2(a+b)(a-b)}-\frac{\ln (1+\sin (dx+c))}{d(2a-2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -sin(d*x+c)/b/d-1/d/(2*a+2*b)*ln(sin(d*x+c)-1)+1/d/b^2*a^3/(a+b)/(a-b)*ln(a
+b*sin(d*x+c))-1/d/(2*a-2*b)*ln(1+sin(d*x+c))
```

maxima [A] time = 0.31, size = 82, normalized size = 0.88

$$\frac{\frac{2\,a^3 \log(b\sin(dx+c)+a)}{a^2b^2-b^4}-\frac{\log(\sin(dx+c)+1)}{a-b}-\frac{\log(\sin(dx+c)-1)}{a+b}-\frac{2\,\sin(dx+c)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^3*log(b*sin(d*x + c) + a)/(a^2*b^2 - b^4) - log(sin(d*x + c) + 1)/
(a - b) - log(sin(d*x + c) - 1)/(a + b) - 2*sin(d*x + c)/b)/d
```

mupad [B] time = 12.27, size = 134, normalized size = 1.44

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d\,\left(a+b\right)}-\frac{\sin\left(c+d\,x\right)}{b\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{d\,\left(a-b\right)}-\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{b^2\,d}-\frac{a^3\,\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\right.}{d\,\left(b^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] - log(tan(c/2 + (d*x)/2) - 1)/(d*(a + b)) - sin(c + d*x)/(b*d) - log(tan(c/
2 + (d*x)/2) + 1)/(d*(a - b)) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/(b^2*d) -
 (a^3*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(b^4 - a
^2*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

3.1333 $\int \frac{\sin(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{a^2\log(a+b\sin(c+dx))}{bd\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d+1/2*ln(1+sin(d*x+c))/(a-b)/d-a^2*ln(a+b*sin(d
*x+c))/b/(a^2-b^2)/d
```

Rubi [A] time = 0.16, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2837, 12, 1629}

$$-\frac{a^2\log(a+b\sin(c+dx))}{bd\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
- (a^2*Log[a + b*Sin[c + d*x]])/(b*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2}{b^2(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b}{2(a+b)(b-x)} - \frac{a^2}{(a-b)(a+b)(a+x)} + \frac{b}{2(a-b)(b+x)}\right)dx, x, b\sin(c+dx)\right)}{bd}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{a^2\log(a+b\sin(c+dx))}{b\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.07, size = 72, normalized size = 0.90

$$\frac{-2a^2\log(a+b\sin(c+dx)) - b(a-b)\log(1-\sin(c+dx)) + b(a+b)\log(\sin(c+dx)+1)}{2bd(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] (-((a - b)*b*Log[1 - Sin[c + d*x]]) + b*(a + b)*Log[1 + Sin[c + d*x]] - 2*a
^2*Log[a + b*Sin[c + d*x]])/(2*(a - b)*b*(a + b)*d)
```

fricas [A] time = 1.05, size = 74, normalized size = 0.92

$$-\frac{2\,a^2\log\left(b\sin\left(dx+c\right)+a\right) - \left(ab+b^2\right)\log\left(\sin\left(dx+c\right)+1\right) + \left(ab-b^2\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\left(a^2b-b^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(2*a^2*log(b*sin(d*x + c) + a) - (a*b + b^2)*log(sin(d*x + c) + 1) + (
a*b - b^2)*log(-sin(d*x + c) + 1))/((a^2*b - b^3)*d)
```

giac [A] time = 0.21, size = 71, normalized size = 0.89

$$-\frac{\frac{2\,a^2\log(|b\sin(dx+c)+a|)}{a^2b-b^3} - \frac{\log(|\sin(dx+c)+1|)}{a-b} + \frac{\log(|\sin(dx+c)-1|)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x +
c) + 1))/(a - b) + log(abs(sin(d*x + c) - 1))/(a + b))/d
```

maple [A] time = 0.36, size = 81, normalized size = 1.01

$$-\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{d\left(2a+2b\right)}-\frac{a^{2}\ln\left(a+b\sin\left(dx+c\right)\right)}{d\left(a+b\right)\left(a-b\right)b}+\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{d\left(2a-2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(2*a+2*b)*ln(sin(d*x+c)-1)-1/d*a^2/(a+b)/(a-b)/b*ln(a+b*sin(d*x+c))+1/
d/(2*a-2*b)*ln(1+sin(d*x+c))
```

maxima [A] time = 0.38, size = 68, normalized size = 0.85

$$-\frac{\frac{2\,a^{2}\log\left(b\sin\left(dx+c\right)+a\right)}{a^{2}b-b^{3}}-\frac{\log\left(\sin\left(dx+c\right)+1\right)}{a-b}+\frac{\log\left(\sin\left(dx+c\right)-1\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^2*log(b*sin(d*x + c) + a)/(a^2*b - b^3) - log(sin(d*x + c) + 1)/(
a - b) + log(sin(d*x + c) - 1)/(a + b))/d
```

mupad [B] time = 11.94, size = 117, normalized size = 1.46

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{d\,\left(a-b\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d\,\left(a+b\right)}+\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{b\,d}-\frac{a^2\,\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}{b\,d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2) + 1)/(d*(a - b)) - log(tan(c/2 + (d*x)/2) - 1)/(d*(a
 + b)) + log(tan(c/2 + (d*x)/2)^2 + 1)/(b*d) - (a^2*log(a + 2*b*tan(c/2 + (
d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(b*d*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin^{2}\left(c+dx\right)\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.1334 $\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{a\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d-1/2*ln(1+sin(d*x+c))/(a-b)/d+a*ln(a+b*sin(d*x
+c))/(a^2-b^2)/d
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 801}

$$\frac{a\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
+ (a*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)(b-x)} + \frac{a}{(a-b)(a+b)(a+x)} - \frac{1}{2(a-b)(b+x)}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} - \frac{\log(1+\sin(c+dx))}{2(a-b)d} + \frac{a\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.08, size = 87, normalized size = 1.18

$$\frac{a\log(a+b\sin(c+dx)) + (b-a)\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - (a+b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}\right.\right.}{d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a + b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - (a + b)*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] + a*Log[a + b*Sin[c + d*x]])/((a - b)*(a + b)*d)
```

fricas [A] time = 0.68, size = 63, normalized size = 0.85

$$\frac{2\,a\log\left(b\sin\left(dx+c\right)+a\right) - (a+b)\log\left(\sin\left(dx+c\right)+1\right) - (a-b)\log\left(-\sin\left(dx+c\right)+1\right)}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*log(b*sin(d*x + c) + a) - (a + b)*log(sin(d*x + c) + 1) - (a - b)*
log(-sin(d*x + c) + 1))/((a^2 - b^2)*d)
```

giac [A] time = 0.20, size = 71, normalized size = 0.96

$$\frac{\frac{2\,ab\log(|b\sin(dx+c)+a|)}{a^2b-b^3} - \frac{\log(|\sin(dx+c)+1|)}{a-b} - \frac{\log(|\sin(dx+c)-1|)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*b*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x + c
) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b))/d
```

maple [A] time = 0.29, size = 76, normalized size = 1.03

$$-\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{d\left(2a+2b\right)}+\frac{a\ln\left(a+b\sin\left(dx+c\right)\right)}{d\left(a+b\right)\left(a-b\right)}-\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{d\left(2a-2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(2*a+2*b)*ln(sin(d*x+c)-1)+1/d*a/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-1/d/(2
*a-2*b)*ln(1+sin(d*x+c))
```

maxima [A] time = 0.31, size = 65, normalized size = 0.88

$$\frac{\frac{2\,a\log\left(b\sin\left(dx+c\right)+a\right)}{a^{2}-b^{2}}-\frac{\log\left(\sin\left(dx+c\right)+1\right)}{a-b}-\frac{\log\left(\sin\left(dx+c\right)-1\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*log(b*sin(d*x + c) + a)/(a^2 - b^2) - log(sin(d*x + c) + 1)/(a - b
) - log(sin(d*x + c) - 1)/(a + b))/d
```

mupad [B] time = 12.02, size = 91, normalized size = 1.23

$$\frac{a\,\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^2-b^2\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{d\,\left(a-b\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d\,\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] (a*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^2 - b^2)
) - log(tan(c/2 + (d*x)/2) + 1)/(d*(a - b)) - log(tan(c/2 + (d*x)/2) - 1)/(
d*(a + b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(c+dx\right)\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x)

[Out] Integral(sin(c + d*x)*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.1335 $\int \frac{\csc(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{b^2\log(a+b\sin(c+dx))}{ad\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}+\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d+ln(sin(d*x+c))/a/d-1/2*ln(1+sin(d*x+c))/(a-b)
/d+b^2*ln(a+b*sin(d*x+c))/a/(a^2-b^2)/d
```

Rubi [A] time = 0.15, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2837, 12, 894}

$$\frac{b^2\log(a+b\sin(c+dx))}{ad\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) + Log[Sin[c + d*x]]/(a*d) - Log[1 + Si
n[c + d*x]]/(2*(a - b)*d) + (b^2*Log[a + b*Sin[c + d*x]])/(a*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{b}{x(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^2\,\text{Subst}\left(\int \frac{1}{x(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^2\,\text{Subst}\left(\int \left(\frac{1}{2b^2(a+b)(b-x)} + \frac{1}{ab^2x} + \frac{1}{a(a-b)(a+b)(a+x)} - \frac{1}{2(a-b)b^2(b+x)}\right) dx, x, b\sin(c+$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(\sin(c+dx))}{ad} - \frac{\log(1+\sin(c+dx))}{2(a-b)d} + \frac{b^2\log(a+}{a\left(a^2\right.}$$

Mathematica [A] time = 0.11, size = 84, normalized size = 0.90

$$-\frac{-\frac{2b^2\log(a+b\sin(c+dx))}{a(a^2-b^2)} + \frac{\log(1-\sin(c+dx))}{a+b} + \frac{\log(\sin(c+dx)+1)}{a-b} - \frac{2\log(\sin(c+dx))}{a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*(Log[1 - Sin[c + d*x]]/(a + b) - (2*Log[Sin[c + d*x]])/a + Log[1 + Sin
[c + d*x]]/(a - b) - (2*b^2*Log[a + b*Sin[c + d*x]])/(a*(a^2 - b^2)))/d
```

fricas [A] time = 1.03, size = 93, normalized size = 1.00

$$\frac{2\,b^2\log\left(b\sin\left(dx+c\right)+a\right)+2\left(a^2-b^2\right)\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)-\left(a^2+ab\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2-ab\right)\log}{2\left(a^3-ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*log(b*sin(d*x + c) + a) + 2*(a^2 - b^2)*log(-1/2*sin(d*x + c)) -
 (a^2 + a*b)*log(sin(d*x + c) + 1) - (a^2 - a*b)*log(-sin(d*x + c) + 1))/((
a^3 - a*b^2)*d)
```

giac [A] time = 0.21, size = 86, normalized size = 0.92

$$\frac{\frac{2\,b^3\log(|b\sin(dx+c)+a|)}{a^3b-ab^3} - \frac{\log(|\sin(dx+c)+1|)}{a-b} - \frac{\log(|\sin(dx+c)-1|)}{a+b} + \frac{2\,\log(|\sin(dx+c)|)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*log(abs(b*sin(d*x + c) + a))/(a^3*b - a*b^3) - log(abs(sin(d*x +
 c) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b) + 2*log(abs(sin(d*x
+ c)))/a)/d
```

maple [A] time = 0.40, size = 95, normalized size = 1.02

$$-\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{d\left(2a+2b\right)}+\frac{b^2\ln\left(a+b\sin\left(dx+c\right)\right)}{da\left(a+b\right)\left(a-b\right)}+\frac{\ln\left(\sin\left(dx+c\right)\right)}{ad}-\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{d\left(2a-2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(2*a+2*b)*ln(sin(d*x+c)-1)+1/d*b^2/a/(a+b)/(a-b)*ln(a+b*sin(d*x+c))+ln
(sin(d*x+c))/a/d-1/d/(2*a-2*b)*ln(1+sin(d*x+c))
```

maxima [A] time = 0.31, size = 80, normalized size = 0.86

$$\frac{\frac{2\,b^2\log(b\sin(dx+c)+a)}{a^3-ab^2}-\frac{\log(\sin(dx+c)+1)}{a-b}-\frac{\log(\sin(dx+c)-1)}{a+b}+\frac{2\,\log(\sin(dx+c))}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*log(b*sin(d*x + c) + a)/(a^3 - a*b^2) - log(sin(d*x + c) + 1)/(a
 - b) - log(sin(d*x + c) - 1)/(a + b) + 2*log(sin(d*x + c))/a)/d
```

mupad [B] time = 0.17, size = 87, normalized size = 0.94

$$\frac{\ln\left(\sin\left(c+d\,x\right)\right)}{a\,d}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{2\,d\,\left(a-b\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)}{2\,d\,\left(a+b\right)}+\frac{b^2\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{a\,d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(sin(c + d*x))/(a*d) - log(sin(c + d*x) + 1)/(2*d*(a - b)) - log(sin(c +
 d*x) - 1)/(2*d*(a + b)) + (b^2*log(a + b*sin(c + d*x)))/(a*d*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(c+dx\right)\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.1336 $\int \frac{\csc^2(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{b^3\log(a+b\sin(c+dx))}{a^2d\left(a^2-b^2\right)}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-\frac{\csc(c+dx)}{ad}$$

```
[Out] -csc(d*x+c)/a/d-1/2*ln(1-sin(d*x+c))/(a+b)/d-b*ln(sin(d*x+c))/a^2/d+1/2*ln(
1+sin(d*x+c))/(a-b)/d-b^3*ln(a+b*sin(d*x+c))/a^2/(a^2-b^2)/d
```

Rubi [A] time = 0.19, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$-\frac{b^3\log(a+b\sin(c+dx))}{a^2d\left(a^2-b^2\right)}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[1 - Sin[c + d*x]]/(2*(a + b)*d) - (b*Log[Sin[c
+ d*x]])/(a^2*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d) - (b^3*Log[a + b*Sin
[c + d*x]])/(a^2*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{b^2}{x^2(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^3\,\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^3\,\text{Subst}\left(\int \left(\frac{1}{2b^3(a+b)(b-x)}+\frac{1}{ab^2x^2}-\frac{1}{a^2b^2x}-\frac{1}{a^2(a-b)(a+b)(a+x)}-\frac{1}{2b^3(-a+b)(b+x)}\right) dx, x$$

$$= -\frac{\csc(c+dx)}{ad}-\frac{\log(1-\sin(c+dx))}{2(a+b)d}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{\log(1+\sin(c+d}{2(a-b)d}$$

Mathematica [A] time = 0.24, size = 97, normalized size = 0.88

$$\frac{\frac{2b^3\log(a+b\sin(c+dx))}{a^2\left(b^2-a^2\right)}-\frac{2b\log(\sin(c+dx))}{a^2}-\frac{\log(1-\sin(c+dx))}{a+b}+\frac{\log(\sin(c+dx)+1)}{a-b}-\frac{2\csc(c+dx)}{a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*Csc[c + d*x])/a - Log[1 - Sin[c + d*x]]/(a + b) - (2*b*Log[Sin[c + d*x
]])/a^2 + Log[1 + Sin[c + d*x]]/(a - b) + (2*b^3*Log[a + b*Sin[c + d*x]])/(
a^2*(-a^2 + b^2)))/(2*d)
```

fricas [A] time = 0.95, size = 143, normalized size = 1.30

$$-\frac{2\,b^3\log\left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)+2\,a^3-2\,ab^2+2\left(a^2b-b^3\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-\left(a^3+a\right.}{2\left(a^4-a^2b^2\right)d\sin\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*log(b*sin(d*x + c) + a)*sin(d*x + c) + 2*a^3 - 2*a*b^2 + 2*(a^2
*b - b^3)*log(1/2*sin(d*x + c))*sin(d*x + c) - (a^3 + a^2*b)*log(sin(d*x +
```

```
c) + 1)*sin(d*x + c) + (a^3 - a^2*b)*log(-sin(d*x + c) + 1)*sin(d*x + c))/(
(a^4 - a^2*b^2)*d*sin(d*x + c))
```

giac [A] time = 0.21, size = 113, normalized size = 1.03

$$-\frac{\frac{2\,b^{4}\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^{4}b-a^{2}b^{3}}-\frac{\log\left(|\sin\left(dx+c\right)+1|\right)}{a-b}+\frac{\log\left(|\sin\left(dx+c\right)-1|\right)}{a+b}+\frac{2\,b\log\left(|\sin\left(dx+c\right)|\right)}{a^{2}}-\frac{2\,\left(b\sin\left(dx+c\right)-a\right)}{a^{2}\sin\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^4*log(abs(b*sin(d*x + c) + a))/(a^4*b - a^2*b^3) - log(abs(sin(d*
x + c) + 1))/(a - b) + log(abs(sin(d*x + c) - 1))/(a + b) + 2*b*log(abs(sin
(d*x + c)))/a^2 - 2*(b*sin(d*x + c) - a)/(a^2*sin(d*x + c)))/d
```

maple [A] time = 0.42, size = 113, normalized size = 1.03

$$-\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{d\left(2a+2b\right)}-\frac{b^{3}\ln\left(a+b\sin\left(dx+c\right)\right)}{d\,a^{2}\left(a+b\right)\left(a-b\right)}-\frac{1}{da\sin\left(dx+c\right)}-\frac{b\ln\left(\sin\left(dx+c\right)\right)}{a^{2}d}+\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{d\left(2a-2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(2*a+2*b)*ln(sin(d*x+c)-1)-1/d*b^3/a^2/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-
1/d/a/sin(d*x+c)-b*ln(sin(d*x+c))/a^2/d+1/d/(2*a-2*b)*ln(1+sin(d*x+c))
```

maxima [A] time = 0.32, size = 95, normalized size = 0.86

$$-\frac{\frac{2\,b^{3}\log\left(b\sin\left(dx+c\right)+a\right)}{a^{4}-a^{2}b^{2}}-\frac{\log\left(\sin\left(dx+c\right)+1\right)}{a-b}+\frac{\log\left(\sin\left(dx+c\right)-1\right)}{a+b}+\frac{2\,b\log\left(\sin\left(dx+c\right)\right)}{a^{2}}+\frac{2}{a\sin\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3*log(b*sin(d*x + c) + a)/(a^4 - a^2*b^2) - log(sin(d*x + c) + 1)
/(a - b) + log(sin(d*x + c) - 1)/(a + b) + 2*b*log(sin(d*x + c))/a^2 + 2/(a
*sin(d*x + c)))/d
```

mupad [B] time = 11.92, size = 98, normalized size = 0.89

$$-\frac{\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)}{2\,\left(a+b\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{2\,\left(a-b\right)}+\frac{1}{a\,\sin\left(c+d\,x\right)}+\frac{b\,\ln\left(\sin\left(c+d\,x\right)\right)}{a^{2}}+\frac{b^{3}\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{a^{4}-a^{2}\,b^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] -(log(sin(c + d*x) - 1)/(2*(a + b)) - log(sin(c + d*x) + 1)/(2*(a - b)) + 1
/(a*sin(c + d*x)) + (b*log(sin(c + d*x)))/a^2 + (b^3*log(a + b*sin(c + d*x)
))/(a^4 - a^2*b^2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.1337 $\int \frac{\csc^3(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=132

$$\frac{b\csc(c+dx)}{a^2d}+\frac{\left(a^2+b^2\right)\log(\sin(c+dx))}{a^3d}+\frac{b^4\log(a+b\sin(c+dx))}{a^3d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-$$

```
[Out] b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d-1/2*ln(1-sin(d*x+c))/(a+b)/d+(a^2+b
^2)*ln(sin(d*x+c))/a^3/d-1/2*ln(1+sin(d*x+c))/(a-b)/d+b^4*ln(a+b*sin(d*x+c)
)/a^3/(a^2-b^2)/d
```

Rubi [A] time = 0.20, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{b^4\log(a+b\sin(c+dx))}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2+b^2\right)\log(\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - Log[1 - Sin[c + d*x]]/(
2*(a + b)*d) + ((a^2 + b^2)*Log[Sin[c + d*x]])/(a^3*d) - Log[1 + Sin[c + d*
x]]/(2*(a - b)*d) + (b^4*Log[a + b*Sin[c + d*x]])/(a^3*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{b^3}{x^3(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\,\text{Subst}\left(\int \frac{1}{x^3(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\,\text{Subst}\left(\int \left(\frac{1}{2b^4(a+b)(b-x)} + \frac{1}{ab^2x^3} - \frac{1}{a^2b^2x^2} + \frac{a^2+b^2}{a^3b^4x} + \frac{1}{a^3(a-b)(a+b)(a+x)} + \frac{1}{2b^4(-a+b)(b}\right.\right.}{d}$$

$$= \frac{b\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2ad} - \frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\left(a^2+b^2\right)\log(\sin(c+d}{a^3d}$$

Mathematica [A] time = 0.53, size = 132, normalized size = 1.00

$$\frac{b^4\left(\frac{\csc(c+dx)}{a^2b^3} + \frac{\log(a+b\sin(c+dx))}{a^3\left(a^2-b^2\right)} + \frac{\left(a^2+b^2\right)\log(\sin(c+dx))}{a^3b^4} - \frac{\csc^2(c+dx)}{2ab^4} - \frac{\log(1-\sin(c+dx))}{2b^4(a+b)} - \frac{\log(\sin(c+dx)+1)}{2b^4(a-b)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^4*(Csc[c + d*x]/(a^2*b^3) - Csc[c + d*x]^2/(2*a*b^4) - Log[1 - Sin[c + d
*x]]/(2*b^4*(a + b)) + ((a^2 + b^2)*Log[Sin[c + d*x]])/(a^3*b^4) - Log[1 +
Sin[c + d*x]]/(2*(a - b)*b^4) + Log[a + b*Sin[c + d*x]]/(a^3*(a^2 - b^2))))
/d
```

fricas [A] time = 1.24, size = 224, normalized size = 1.70

$$\frac{a^4 - a^2b^2 + 2\left(b^4\cos\left(dx+c\right)^2 - b^4\right)\log\left(b\sin\left(dx+c\right)+a\right) - 2\left(a^4 - b^4 - \left(a^4-b^4\right)\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\sin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^4 - a^2*b^2 + 2*(b^4*cos(d*x + c)^2 - b^4)*log(b*sin(d*x + c) + a) -
 2*(a^4 - b^4 - (a^4 - b^4)*cos(d*x + c)^2)*log(-1/2*sin(d*x + c)) + (a^4 +
 a^3*b - (a^4 + a^3*b)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + (a^4 - a^3*b
 - (a^4 - a^3*b)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^3*b - a*b^3)
*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c)^2 - (a^5 - a^3*b^2)*d)
```

giac [A] time = 0.23, size = 148, normalized size = 1.12

$$\frac{\frac{2\,b^5\log(|b\sin(dx+c)+a|)}{a^5b-a^3b^3}-\frac{\log(|\sin(dx+c)+1|)}{a-b}-\frac{\log(|\sin(dx+c)-1|)}{a+b}+\frac{2\left(a^2+b^2\right)\log(|\sin(dx+c)|)}{a^3}-\frac{3\,a^2\sin(dx+c)^2+3\,b^2\sin(dx+c)^2-2\,ab\sin(dx+}{a^3\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^5*log(abs(b*sin(d*x + c) + a))/(a^5*b - a^3*b^3) - log(abs(sin(d*x
 + c) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b) + 2*(a^2 + b^2)*lo
g(abs(sin(d*x + c)))/a^3 - (3*a^2*sin(d*x + c)^2 + 3*b^2*sin(d*x + c)^2 - 2
*a*b*sin(d*x + c) + a^2)/(a^3*sin(d*x + c)^2))/d
```

maple [A] time = 0.48, size = 144, normalized size = 1.09

$$-\frac{\ln(\sin(dx+c)-1)}{d\,(2a+2b)}+\frac{b^4\ln(a+b\sin(dx+c))}{d\,a^3\,(a+b)\,(a-b)}-\frac{1}{2da\sin(dx+c)^2}+\frac{\ln(\sin(dx+c))}{ad}+\frac{b^2\ln(\sin(dx+c))}{a^3d}+\frac{}{d\,a^2\,\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(2*a+2*b)*ln(sin(d*x+c)-1)+1/d*b^4/a^3/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-
1/2/d/a/sin(d*x+c)^2+ln(sin(d*x+c))/a/d+b^2*ln(sin(d*x+c))/a^3/d+1/d/a^2*b/
sin(d*x+c)-1/d/(2*a-2*b)*ln(1+sin(d*x+c))
```

maxima [A] time = 0.31, size = 114, normalized size = 0.86

$$\frac{\frac{2\,b^4\log(b\sin(dx+c)+a)}{a^5-a^3b^2}-\frac{\log(\sin(dx+c)+1)}{a-b}-\frac{\log(\sin(dx+c)-1)}{a+b}+\frac{2\left(a^2+b^2\right)\log(\sin(dx+c))}{a^3}+\frac{2\,b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^4*log(b*sin(d*x + c) + a)/(a^5 - a^3*b^2) - log(sin(d*x + c) + 1)/
(a - b) - log(sin(d*x + c) - 1)/(a + b) + 2*(a^2 + b^2)*log(sin(d*x + c))/a
^3 + (2*b*sin(d*x + c) - a)/(a^2*sin(d*x + c)^2))/d
```

mupad [B] time = 11.85, size = 125, normalized size = 0.95

$$\frac{\ln\left(\sin\left(c+d\,x\right)\right)\,\left(a^2+b^2\right)}{a^3\,d}-\frac{\frac{1}{2\,a}-\frac{b\,\sin(c+d\,x)}{a^2}}{d\sin\left(c+d\,x\right)^2}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)}{2\,d\,\left(a+b\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{2\,d\,\left(a-b\right)}+\frac{b^4\,\ln\left(a+b\,\sin\left(c+d\right.\right.}{d\,\left(a^5-a^3\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x))*(a^2 + b^2))/(a^3*d) - (1/(2*a) - (b*sin(c + d*x))/a^2)/
(d*sin(c + d*x)^2) - log(sin(c + d*x) - 1)/(2*d*(a + b)) - log(sin(c + d*x)
 + 1)/(2*d*(a - b)) + (b^4*log(a + b*sin(c + d*x)))/(d*(a^5 - a^3*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^3\left(c+dx\right)\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.1338 $\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=268

$$\frac{a\cos(c+dx)}{d\left(a^2-b^2\right)}-\frac{3b\tan(c+dx)}{2d\left(a^2-b^2\right)}+\frac{a\sec(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\sin^2(c+dx)\tan(c+dx)}{2d\left(a^2-b^2\right)}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2bd\left(a^2-b^2\right)}+\frac{3bx}{2\left(a^2-b^2\right)}$$

```
[Out] 3/2*b*x/(a^2-b^2)-1/2*a^2*(2*a^2+b^2)*x/b^3/(a^2-b^2)+2*a^5*arctan((b+a*tan
(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^3/(a^2-b^2)^(3/2)/d+a*cos(d*x+c)/(a^2-b
^2)/d-a^3*cos(d*x+c)/b^2/(a^2-b^2)/d+a*sec(d*x+c)/(a^2-b^2)/d+1/2*a^2*cos(d
*x+c)*sin(d*x+c)/b/(a^2-b^2)/d-3/2*b*tan(d*x+c)/(a^2-b^2)/d+1/2*b*sin(d*x+c
)^2*tan(d*x+c)/(a^2-b^2)/d
```

Rubi [A] time = 0.40, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {2902, 2590, 14, 2591, 288, 321, 203, 2793, 3023, 2735, 2660, 618, 204}

$$-\frac{a^3\cos(c+dx)}{b^2d\left(a^2-b^2\right)}+\frac{a\cos(c+dx)}{d\left(a^2-b^2\right)}+\frac{2a^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\left(a^2-b^2\right)^{3/2}}-\frac{3b\tan(c+dx)}{2d\left(a^2-b^2\right)}+\frac{a\sec(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\sin^2(c+dx)\tan(c+dx)}{2d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*x)/(2*(a^2 - b^2)) - (a^2*(2*a^2 + b^2)*x)/(2*b^3*(a^2 - b^2)) + (2*a^
5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^3*(a^2 - b^2)^(3/2)*
d) + (a*Cos[c + d*x])/((a^2 - b^2)*d) - (a^3*Cos[c + d*x])/(b^2*(a^2 - b^2)
*d) + (a*Sec[c + d*x])/((a^2 - b^2)*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2
*b*(a^2 - b^2)*d) - (3*b*Tan[c + d*x])/(2*(a^2 - b^2)*d) + (b*Sin[c + d*x]^
2*Tan[c + d*x])/(2*(a^2 - b^2)*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sin(c+dx)\tan^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sin^2(c+dx)\tan^2}{a^2-b^2}$$

$$= \frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d} - \frac{a^2\int \frac{a+b\sin(c+dx)-2a\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx}{2b\left(a^2-b^2\right)} - \frac{a\,\mathrm{Subst}\left(\int \frac{1-x}{x^2}\right.}{\left(a^2\right.}$$

$$= -\frac{a^3\cos(c+dx)}{b^2\left(a^2-b^2\right)d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d} + \frac{b\sin^2(c+dx)\tan(c+dx)}{2\left(a^2-b^2\right)d} -$$

$$= -\frac{a^2\left(2a^2+b^2\right)x}{2b^3\left(a^2-b^2\right)} + \frac{a\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{a^3\cos(c+dx)}{b^2\left(a^2-b^2\right)d} + \frac{a\sec(c+dx)}{\left(a^2-b^2\right)d} + \frac{a^2\cos(}{2}$$

$$= \frac{3bx}{2\left(a^2-b^2\right)} - \frac{a^2\left(2a^2+b^2\right)x}{2b^3\left(a^2-b^2\right)} + \frac{a\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{a^3\cos(c+dx)}{b^2\left(a^2-b^2\right)d} + \frac{a\sec(c+dx}{\left(a^2-b^2\right)d}$$

$$= \frac{3bx}{2\left(a^2-b^2\right)} - \frac{a^2\left(2a^2+b^2\right)x}{2b^3\left(a^2-b^2\right)} + \frac{a\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{a^3\cos(c+dx)}{b^2\left(a^2-b^2\right)d} + \frac{a\sec(c+dx}{\left(a^2-b^2\right)d}$$

$$= \frac{3bx}{2\left(a^2-b^2\right)} - \frac{a^2\left(2a^2+b^2\right)x}{2b^3\left(a^2-b^2\right)} + \frac{2a^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}d} + \frac{a\cos(c+dx)}{\left(a^2-b^2\right)d} -$$

Mathematica [A] time = 1.53, size = 221, normalized size = 0.82

$$\frac{\frac{8a^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}} + \frac{4a^4(c+dx)+2a^2b^2(c+dx)-4ab^3-6b^4(c+dx)}{b^5-a^2b^3} - \frac{4a\cos(c+dx)}{b^2} + \frac{4\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{4}{(a-b)\left(\sin\left(\frac{1}{2}\right.\right.}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-4*a*b^3 + 4*a^4*(c + d*x) + 2*a^2*b^2*(c + d*x) - 6*b^4*(c + d*x))/(-(a^
2*b^3) + b^5) + (8*a^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b
^3*(a^2 - b^2)^(3/2)) - (4*a*Cos[c + d*x])/b^2 + (4*Sin[(c + d*x)/2])/((a +
```

```
b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (4*Sin[(c + d*x)/2])/((a - b)*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + Sin[2*(c + d*x)]/b)/(4*d)
```

fricas [A] time = 1.04, size = 521, normalized size = 1.94

$$\left[\frac{\sqrt{-a^2+b^2}\,a^5\cos(dx+c)\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^3b\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^5*cos(d*x + c)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 2*a^3*b^3 - 2*a*b^5 - (2*a^6 - a^4*b^2 - 4*a^2*b^4 + 3*b^6)*d*x*co
s(d*x + c) - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c)^2 - (2*a^2*b^4 - 2*
b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^4*b^3 -
 2*a^2*b^5 + b^7)*d*cos(d*x + c)), -1/2*(2*sqrt(a^2 - b^2)*a^5*arctan(-(a*s
in(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - 2*a^3*b^3 +
 2*a*b^5 + (2*a^6 - a^4*b^2 - 4*a^2*b^4 + 3*b^6)*d*x*cos(d*x + c) + 2*(a^5*
b - 2*a^3*b^3 + a*b^5)*cos(d*x + c)^2 + (2*a^2*b^4 - 2*b^6 - (a^4*b^2 - 2*a
^2*b^4 + b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*
cos(d*x + c))]
```

giac [A] time = 0.22, size = 208, normalized size = 0.78

$$\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^5}{(a^2b^3-b^5)\sqrt{a^2-b^2}}+\frac{4\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}{(a^2-b^2)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{(2a^2+3b^2)(dx+c)}{b^3}-\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\cdots\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\cdots\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*a^5/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) + 4*(b*t
an(1/2*d*x + 1/2*c) - a)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)) - (2*a^
2 + 3*b^2)*(d*x + c)/b^3 - 2*(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x +
1/2*c)^2 - b*tan(1/2*d*x + 1/2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^
2))/d
```

maple [A] time = 0.43, size = 283, normalized size = 1.06

$$-\frac{64}{d\left(64a+64b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{2\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] -64/d/(64*a+64*b)/(tan(1/2*d*x+1/2*c)-1)-1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*t
an(1/2*d*x+1/2*c)^3-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2
*a+1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-2/d/b^2/(1+tan(1/2*d
*x+1/2*c)^2)^2*a-2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2-3/d/b*arctan(tan(1/
2*d*x+1/2*c))+64/d/(64*a-64*b)/(tan(1/2*d*x+1/2*c)+1)+2/d/(a-b)/(a+b)*a^5/b
^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 16.81, size = 2098, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (4*a^5*cos(c + d*x) + (5*a^5)/2 + (3*a^5*cos(2*c + 2*d*x))/2)/(d*cos(c + d*
x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (2*a^8*atan(sin(c/2 + (d*x)/2)/co
s(c/2 + (d*x)/2)))/(b^3*d*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (b*((11*a^
4*sin(c + d*x))/8 + (3*a^4*sin(3*c + 3*d*x))/8 - 3*a^4*cos(c + d*x)*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b
^4 - 2*a^2*b^2)) + (b^4*((3*a)/2 + 2*a*cos(c + d*x) + (a*cos(2*c + 2*d*x))/
2))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (b^5*((9*sin(c +
```

```
d*x))/8 + sin(3*c + 3*d*x)/8 - 3*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2))))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (a
^7*cos(c + d*x) + a^7/2 + (a^7*cos(2*c + 2*d*x))/2)/(b^2*d*cos(c + d*x)*(a^
2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (b^2*(5*a^3*cos(c + d*x) + (7*a^3)/2 +
(3*a^3*cos(2*c + 2*d*x))/2))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2
*b^2)) + ((a^6*sin(c + d*x))/8 + (a^6*sin(3*c + 3*d*x))/8 + 3*a^6*cos(c + d
*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d*cos(c + d*x)*(a^2 - b
^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b^3*((19*a^2*sin(c + d*x))/8 + (3*a^2*sin(3
*c + 3*d*x))/8 - 7*a^2*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2))))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (a^5*atan((a^
12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2)*8i + a^18*s
in(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*8i - b^18*sin(c
/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*18i + a^3*b^15*cos(
c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*42i - a^5*b^13*cos
(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*67i + a^7*b^11*co
s(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*24i + a^9*b^9*co
s(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*45i - a^11*b^7*c
os(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*46i + a^13*b^5*
cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*3i + a^15*b^3*
cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*12i - a^10*b^2
*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2)*12i + a^2*b^1
6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*93i - a^4*b^
14*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*176i + a^6*
b^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*115i + a^
8*b^10*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*66i - a
^10*b^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*133i +
 a^12*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*36i
+ a^14*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*45i
 - a^16*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*36
i - a^11*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2)*4i
- a*b^17*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*9i -
a^17*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*4i)/(18
*b^21*sin(c/2 + (d*x)/2) + 9*a*b^20*cos(c/2 + (d*x)/2) - 60*a^3*b^18*cos(c/
2 + (d*x)/2) + 160*a^5*b^16*cos(c/2 + (d*x)/2) - 200*a^7*b^14*cos(c/2 + (d*
x)/2) + 70*a^9*b^12*cos(c/2 + (d*x)/2) + 116*a^11*b^10*cos(c/2 + (d*x)/2) -
 160*a^13*b^8*cos(c/2 + (d*x)/2) + 80*a^15*b^6*cos(c/2 + (d*x)/2) - 15*a^17
*b^4*cos(c/2 + (d*x)/2) - 120*a^2*b^19*sin(c/2 + (d*x)/2) + 320*a^4*b^17*si
n(c/2 + (d*x)/2) - 400*a^6*b^15*sin(c/2 + (d*x)/2) + 140*a^8*b^13*sin(c/2 +
 (d*x)/2) + 232*a^10*b^11*sin(c/2 + (d*x)/2) - 320*a^12*b^9*sin(c/2 + (d*x)
/2) + 160*a^14*b^7*sin(c/2 + (d*x)/2) - 30*a^16*b^5*sin(c/2 + (d*x)/2)))*(b
^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*2i)/(b^3*d*(a^2 - b^2)*(a^4 + b^4 -
 2*a^2*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1339 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=183

$$\frac{a^2\cos(c+dx)}{bd\left(a^2-b^2\right)}-\frac{b\cos(c+dx)}{d\left(a^2-b^2\right)}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}-\frac{ax}{a^2-b^2}-\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{3/2}}+\frac{a^3x}{b^2\left(a^2-b^2\right)}$$

```
[Out] -a*x/(a^2-b^2)+a^3*x/b^2/(a^2-b^2)-2*a^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a
^2-b^2)^(1/2))/b^2/(a^2-b^2)^(3/2)/d+a^2*cos(d*x+c)/b/(a^2-b^2)/d-b*cos(d*x
+c)/(a^2-b^2)/d-b*sec(d*x+c)/(a^2-b^2)/d+a*tan(d*x+c)/(a^2-b^2)/d
```

Rubi [A] time = 0.27, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2902, 3473, 8, 2590, 14, 2746, 12, 2735, 2660, 618, 204}

$$\frac{a^2\cos(c+dx)}{bd\left(a^2-b^2\right)}-\frac{b\cos(c+dx)}{d\left(a^2-b^2\right)}-\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{3/2}}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}+\frac{a^3x}{b^2\left(a^2-b^2\right)}-\frac{ax}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*x)/(a^2 - b^2)) + (a^3*x)/(b^2*(a^2 - b^2)) - (2*a^4*ArcTan[(b + a*Tan
[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(3/2)*d) + (a^2*Cos[c + d
*x])/(b*(a^2 - b^2)*d) - (b*Cos[c + d*x])/((a^2 - b^2)*d) - (b*Sec[c + d*x]
)/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
```

```
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \tan^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sin(c+dx)\tan^2(c+dx)\,dx}{a^2-b^2}$$

$$= \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d} - \frac{a\int 1\,dx}{a^2-b^2} + \frac{a^2\int \frac{a\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)} + \frac{b\,\text{Subst}\Big(\int$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{a^3\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)} + \frac{b\,\text{Subst}\Big(\int$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^3x}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} - \frac{b\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} + \frac{a\,\mathrm{t}}{(}$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^3x}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} - \frac{b\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} + \frac{a\,\mathrm{t}}{(}$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^3x}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} - \frac{b\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} + \frac{a\,\mathrm{t}}{(}$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^3x}{b^2\left(a^2-b^2\right)} - \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}d} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} - \frac{b\cos}{(a^2}$$

Mathematica [A] time = 1.14, size = 186, normalized size = 1.02

$$-\frac{-\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}}+\frac{-\left(a^3(c+dx)\right)+ab^2(c+dx)+b^3}{b^4-a^2b^2}+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((b^3 - a^3*(c + d*x) + a*b^2*(c + d*x))/(-(a^2*b^2) + b^4) - (2*a^4*ArcTan
[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(3/2)) + Cos[c
 + d*x]/b + Sin[(c + d*x)/2]/((a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
) + Sin[(c + d*x)/2]/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/d
```

fricas [A] time = 0.78, size = 431, normalized size = 2.36

$$\left[\frac{\sqrt{-a^2+b^2}\,a^4\cos(dx+c)\log\left(\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^2b}{2\left(a^4b^2-2a^2b^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^4*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*a^2*b^3 + 2*b^5 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) +
2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^2 + 2*(a^3*b^2 - a*b^4)*sin(d*x +
c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*a^4*arct
an(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2
*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + (a^4*b - 2*a^2*b^
3 + b^5)*cos(d*x + c)^2 + (a^3*b^2 - a*b^4)*sin(d*x + c))/((a^4*b^2 - 2*a^2
*b^4 + b^6)*d*cos(d*x + c))]
```

giac [A] time = 0.21, size = 173, normalized size = 0.95

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^4}{\left(a^2b^2-b^4\right)\sqrt{a^2-b^2}}-\frac{(dx+c)a}{b^2}+\frac{2\left(ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^2-2b^2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-1\right)\left(a^2b-b^3\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a^4/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) - (d*x + c)
*a/b^2 + 2*(a*b*tan(1/2*d*x + 1/2*c)^3 - a^2*tan(1/2*d*x + 1/2*c)^2 + a*b*t
an(1/2*d*x + 1/2*c) + a^2 - 2*b^2)/((tan(1/2*d*x + 1/2*c)^4 - 1)*(a^2*b - b
^3)))/d
```

maple [A] time = 0.37, size = 162, normalized size = 0.89

$$-\frac{32}{d\left(32a+32b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2}{db\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^{2}}-\frac{32}{d\left(32a-32b\right)\left(\tan\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -32/d/(32*a+32*b)/(tan(1/2*d*x+1/2*c)-1)+2/d/b/(1+tan(1/2*d*x+1/2*c)^2)+2/d
/b^2*a*arctan(tan(1/2*d*x+1/2*c))-32/d/(32*a-32*b)/(tan(1/2*d*x+1/2*c)+1)-2
/d/(a-b)/(a+b)*a^4/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2
*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 14.28, size = 1656, normalized size = 9.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (a^5*sin(c + d*x) - 6*a^5*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2)))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (2*a^7*atan
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^2*d*(a^2 - b^2)*(a^4 + b^4 - 2*
a^2*b^2)) + (a^6*cos(c + d*x) + a^6/2 + (a^6*cos(2*c + 2*d*x))/2)/(b*d*cos(
c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b^3*(5*a^2*cos(c + d*x) +
(7*a^2)/2 + (3*a^2*cos(2*c + 2*d*x))/2))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 +
 b^4 - 2*a^2*b^2)) - (b^2*(2*a^3*sin(c + d*x) - 6*a^3*cos(c + d*x)*atan(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^
4 - 2*a^2*b^2)) - (b^5*(2*cos(c + d*x) + cos(2*c + 2*d*x)/2 + 3/2))/(d*cos(
c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b^4*(a*sin(c + d*x) - 2*a*
cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/(d*cos(c + d*x)*
(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (b*(4*a^4*cos(c + d*x) + (5*a^4)/2 +
 (3*a^4*cos(2*c + 2*d*x))/2))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^
2*b^2)) + (a^4*atan(((2*b^14*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*
a^4*b^2)^(1/2) - 2*a^14*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b
^2)^(1/2) - 2*a^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3
/2) - 6*a^3*b^11*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/
2) + 15*a^5*b^9*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2
) - 20*a^7*b^7*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)
 + 15*a^9*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)
- 6*a^11*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) +
 3*a^6*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) - 1
3*a^2*b^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 3
6*a^4*b^10*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 5
6*a^6*b^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 54
*a^8*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 33*
a^10*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 12*
a^12*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a^7
*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + a*b^13*co
s(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a^13*b*cos(c/2
 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))*1i)/((a^4 + b^4 - 2*
a^2*b^2)*(2*b^13*sin(c/2 + (d*x)/2) + a*b^12*cos(c/2 + (d*x)/2) - 6*a^3*b^1
0*cos(c/2 + (d*x)/2) + 15*a^5*b^8*cos(c/2 + (d*x)/2) - 19*a^7*b^6*cos(c/2 +
 (d*x)/2) + 12*a^9*b^4*cos(c/2 + (d*x)/2) - 3*a^11*b^2*cos(c/2 + (d*x)/2) -
 12*a^2*b^11*sin(c/2 + (d*x)/2) + 30*a^4*b^9*sin(c/2 + (d*x)/2) - 38*a^6*b^
7*sin(c/2 + (d*x)/2) + 24*a^8*b^5*sin(c/2 + (d*x)/2) - 6*a^10*b^3*sin(c/2 +
 (d*x)/2))))*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(b^2*d*(a^2 - b^2)*(a^4 + b^4
 - 2*a^2*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+b*sin(d*x+c)),x)
[Out] Integral(sin(c + d*x)**4*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

3.1340 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=133

$$-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)}+\frac{a\sec(c+dx)}{d\left(a^2-b^2\right)}-\frac{a^2x}{b\left(a^2-b^2\right)}+\frac{bx}{a^2-b^2}+\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{3/2}}$$

```
[Out] -a^2*x/b/(a^2-b^2)+b*x/(a^2-b^2)+2*a^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2
-b^2)^(1/2))/b/(a^2-b^2)^(3/2)/d+a*sec(d*x+c)/(a^2-b^2)/d-b*tan(d*x+c)/(a^2
-b^2)/d
```

Rubi [A] time = 0.18, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2902, 2606, 8, 3473, 2735, 2660, 618, 204}

$$\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{3/2}}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)}+\frac{a\sec(c+dx)}{d\left(a^2-b^2\right)}-\frac{a^2x}{b\left(a^2-b^2\right)}+\frac{bx}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a^2*x)/(b*(a^2 - b^2))) + (b*x)/(a^2 - b^2) + (2*a^3*ArcTan[(b + a*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)*d) + (a*Sec[c + d*x])/(
(a^2 - b^2)*d) - (b*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec(c+dx)\tan(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \tan^2(c+dx)\,dx}{a^2-b^2}$$

$$= -\frac{a^2x}{b\left(a^2-b^2\right)} - \frac{b\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{a^3\int \frac{1}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)} + \frac{b\int 1\,dx}{a^2-b^2} + \frac{a\,\text{Subst}(\int 1\,d}{(a^2}$$

$$= -\frac{a^2x}{b\left(a^2-b^2\right)} + \frac{bx}{a^2-b^2} + \frac{a\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{}{a+2b}\right.}{b(}$$

$$= -\frac{a^2x}{b\left(a^2-b^2\right)} + \frac{bx}{a^2-b^2} + \frac{a\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\tan(c+dx)}{\left(a^2-b^2\right)d} - \frac{\left(4a^3\right)\text{Subst}\left(\int \frac{}{-4(a^2}\right.}{}$$

$$= -\frac{a^2x}{b\left(a^2-b^2\right)} + \frac{bx}{a^2-b^2} + \frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} + \frac{a\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\tan(c}{(a^2-}$$

Mathematica [A] time = 0.84, size = 152, normalized size = 1.14

$$\frac{\frac{b(a-b\sin(c+dx))-\left(a^2-b^2\right)(c+dx)\cos(c+dx)}{(a-b)(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2)
 + (-((a^2 - b^2)*(c + d*x)*Cos[c + d*x]) + b*(a - b*Sin[c + d*x]))/((a - b
)*(a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])))/(b*d)
```

fricas [A] time = 0.75, size = 369, normalized size = 2.77

$$\left[\frac{\sqrt{-a^2+b^2}\,a^3\cos(dx+c)\log\left(-\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a}{2\left(a^4b-2a^2b^3+b^5\right)d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^3*cos(d*x + c)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 2*a^3*b - 2*a*b^3 - 2*(a^4 - 2*a^2*b^2 + b^4)*d*x*cos(d*x + c) - 2
*(a^2*b^2 - b^4)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c)),
-(sqrt(a^2 - b^2)*a^3*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x
 + c)))*cos(d*x + c) - a^3*b + a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*d*x*cos(d*x
+ c) + (a^2*b^2 - b^4)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x +
 c))]
```

giac [A] time = 0.29, size = 131, normalized size = 0.98

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^3}{\left(a^2b-b^3\right)\sqrt{a^2-b^2}}-\frac{dx+c}{b}+\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c
) + b)/sqrt(a^2 - b^2)))*a^3/((a^2*b - b^3)*sqrt(a^2 - b^2)) - (d*x + c)/b
+ 2*(b*tan(1/2*d*x + 1/2*c) - a)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1))
)/d
```

maple [A] time = 0.40, size = 138, normalized size = 1.04

$$-\frac{16}{d\left(16a+16b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db}+\frac{16}{d\left(16a-16b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2a^3\arctan\left(\frac{2a\tan}{2}\right.}{d\left(a-b\right)\left(a+b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -16/d/(16*a+16*b)/(tan(1/2*d*x+1/2*c)-1)-2/d/b*arctan(tan(1/2*d*x+1/2*c))+1
6/d/(16*a-16*b)/(tan(1/2*d*x+1/2*c)+1)+2/d/(a-b)/(a+b)*a^3/b/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 14.05, size = 1538, normalized size = 11.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (a^5*cos(c + d*x) + a^5)/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2
)) - (2*a^6*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d*(a^2 - b^2)*(
a^4 + b^4 - 2*a^2*b^2)) - (b*(a^4*sin(c + d*x) - 6*a^4*cos(c + d*x)*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b
^4 - 2*a^2*b^2)) + (b^4*(a + a*cos(c + d*x)))/(d*cos(c + d*x)*(a^2 - b^2)*(
a^4 + b^4 - 2*a^2*b^2)) - (b^2*(2*a^3*cos(c + d*x) + 2*a^3))/(d*cos(c + d*x
)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b^3*(2*a^2*sin(c + d*x) - 6*a^2*c
os(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/(d*cos(c + d*x)*(
a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (b^5*(sin(c + d*x) - 2*cos(c + d*x)*a
tan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/(d*cos(c + d*x)*(a^2 - b^2)*(a
^4 + b^4 - 2*a^2*b^2)) - (a^3*atan(((2*b^14*sin(c/2 + (d*x)/2)*(b^6 - a^6 -
 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 2*a^14*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^
2*b^4 + 3*a^4*b^2)^(1/2) - 2*a^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(3/2) - 6*a^3*b^11*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 +
 3*a^4*b^2)^(1/2) + 15*a^5*b^9*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 +
3*a^4*b^2)^(1/2) - 20*a^7*b^7*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3
*a^4*b^2)^(1/2) + 15*a^9*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*
a^4*b^2)^(1/2) - 6*a^11*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a
^4*b^2)^(1/2) + 3*a^6*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4
*b^2)^(3/2) - 13*a^2*b^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4
*b^2)^(1/2) + 36*a^4*b^10*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4
*b^2)^(1/2) - 56*a^6*b^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*
b^2)^(1/2) + 54*a^8*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b
^2)^(1/2) - 33*a^10*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b
^2)^(1/2) + 12*a^12*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b
^2)^(1/2) + a^7*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3
/2) + a*b^13*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) +
 a^13*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))*1i)/(
(a^4*b + b^5 - 2*a^2*b^3)*(2*b^12*sin(c/2 + (d*x)/2) + a*b^11*cos(c/2 + (d*
```

```
x)/2) - 3*a^11*b*cos(c/2 + (d*x)/2) - 6*a^3*b^9*cos(c/2 + (d*x)/2) + 15*a^5
*b^7*cos(c/2 + (d*x)/2) - 19*a^7*b^5*cos(c/2 + (d*x)/2) + 12*a^9*b^3*cos(c/
2 + (d*x)/2) - 12*a^2*b^10*sin(c/2 + (d*x)/2) + 30*a^4*b^8*sin(c/2 + (d*x)/
2) - 38*a^6*b^6*sin(c/2 + (d*x)/2) + 24*a^8*b^4*sin(c/2 + (d*x)/2) - 6*a^10
*b^2*sin(c/2 + (d*x)/2))))*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(b*d*(a^2 - b^2
)*(a^4 + b^4 - 2*a^2*b^2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1341 $\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

```
[Out] -2*a^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d-b
*sec(d*x+c)/(a^2-b^2)/d+a*tan(d*x+c)/(a^2-b^2)/d
```

Rubi [A] time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2727, 3767, 8, 2606, 2660, 618, 204}

$$-\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (b*Sec[c + d*x])/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec(c+dx)\tan(c+dx)\,dx}{a^2-b^2}$$

$$= -\frac{a\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c+dx))}{(a^2-b^2)\,d} - \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)\,d}$$

$$= -\frac{b\sec(c+dx)}{(a^2-b^2)\,d} + \frac{a\tan(c+dx)}{(a^2-b^2)\,d} + \frac{(4a^2)\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+\right.\right.}{(a^2-b^2)\,d}$$

$$= -\frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} - \frac{b\sec(c+dx)}{(a^2-b^2)\,d} + \frac{a\tan(c+dx)}{(a^2-b^2)\,d}$$

Mathematica [A] time = 0.19, size = 152, normalized size = 1.58

$$\frac{\sqrt{a^2-b^2}\,(a\sin(c+dx)+b\cos(c+dx)-b) - 2a^2\cos(c+dx)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a-b)(a+b)\sqrt{a^2-b^2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqr
t[a^2 - b^2]*(-b + b*Cos[c + d*x] + a*Sin[c + d*x]))/((a - b)*(a + b)*Sqrt[
a^2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]))
```

fricas [A] time = 0.80, size = 305, normalized size = 3.18

$$\left[\frac{\sqrt{-a^2+b^2}\,a^2\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right) - 2a^2b}{2(a^4-2a^2b^2+b^4)d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^2*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 +
 b^4)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*a^2*arctan(-(a*sin(d*x + c) + b)/(s
qrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*si
n(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

giac [A] time = 0.25, size = 107, normalized size = 1.11

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b}{\left(a^2-b^2\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a^2/(a^2 - b^2)^(3/2) + (a*tan(1/2*d*x + 1/2*c) -
 b)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [A] time = 0.38, size = 117, normalized size = 1.22

$$-\frac{8}{d\,(8a+8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{d\,(8a-8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\,(a-b)\,(a+b)\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -8/d/(8*a+8*b)/(tan(1/2*d*x+1/2*c)-1)-8/d/(8*a-8*b)/(tan(1/2*d*x+1/2*c)+1)-
2/d*a^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 11.96, size = 148, normalized size = 1.54

$$\frac{\frac{2\,b}{a^2-b^2}-\frac{2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}-\frac{2\,a^2\,\mathrm{atan}\left(\frac{\frac{a^2\,\left(2\,a^2\,b-2\,b^3\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}+\frac{2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}}{2\,a^2}\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*b)/(a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2))/(a^2 - b^2))/(d*(tan(c/2 + (d
*x)/2)^2 - 1)) - (2*a^2*atan(((a^2*(2*a^2*b - 2*b^3))/((a + b)^(3/2)*(a - b
)^(3/2)) + (2*a^3*tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/2)*(a - b)^(3
/2)))/(2*a^2)))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}{\left(c+dx\right)}\sec^{2}{\left(c+dx\right)}}{a+b\sin{\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

3.1342 $\int \frac{\sec(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{2ab\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{\sec(c+dx)(a-b\sin(c+dx))}{d\left(a^2-b^2\right)}$$

```
[Out] 2*a*b*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d+se
c(d*x+c)*(a-b*sin(d*x+c))/(a^2-b^2)/d
```

Rubi [A] time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2866, 12, 2660, 618, 204}

$$\frac{2ab\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{\sec(c+dx)(a-b\sin(c+dx))}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2)
*d) + (Sec[c + d*x]*(a - b*Sin[c + d*x]))/((a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)d} - \frac{\int \frac{ab}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}$$

$$= \frac{\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)d} + \frac{(ab)\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)d} + \frac{(2ab)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right.}{\left(a^2-b^2\right)d}$$

$$= \frac{\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)d} - \frac{(4ab)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\right.\right.}{\left(a^2-b^2\right)d}$$

$$= \frac{2ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} + \frac{\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.18, size = 151, normalized size = 1.84

$$\frac{\sqrt{a^2-b^2}\,(a(-\cos(c+dx))+a-b\sin(c+dx))+2ab\cos(c+dx)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a-b)(a+b)\sqrt{a^2-b^2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqrt
[a^2 - b^2]*(a - a*Cos[c + d*x] - b*Sin[c + d*x]))/((a - b)*(a + b)*Sqrt[a^
2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]))
```

fricas [A] time = 0.66, size = 308, normalized size = 3.76

$$\left[\frac{\sqrt{-a^2+b^2}\,ab\cos(dx+c)\log\left(-\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^3}{2\left(a^4-2a^2b^2+b^4\right)d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a*b*cos(d*x + c)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 2*a^3 - 2*a*b^2 - 2*(a^2*b - b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2
+ b^4)*d*cos(d*x + c)), -(sqrt(a^2 - b^2)*a*b*arctan(-(a*sin(d*x + c) + b)/
(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^3 + a*b^2 + (a^2*b - b^3)*
sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

giac [A] time = 0.22, size = 106, normalized size = 1.29

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a}{\left(a^2-b^2\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c
) + b)/sqrt(a^2 - b^2)))*a*b/(a^2 - b^2)^(3/2) + (b*tan(1/2*d*x + 1/2*c) -
a)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [A] time = 0.30, size = 116, normalized size = 1.41

$$-\frac{4}{d\left(4a+4b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{d\left(4a-4b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2ab\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\left(a-b\right)\left(a+b\right)\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -4/d/(4*a+4*b)/(tan(1/2*d*x+1/2*c)-1)+4/d/(4*a-4*b)/(tan(1/2*d*x+1/2*c)+1)+
2/d*a*b/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.02, size = 151, normalized size = 1.84

$$\frac{2\,a\,b\,\mathrm{atan}\left(\frac{\frac{a\,b\,\left(2\,a^2\,b-2\,b^3\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}+\frac{2\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}}{2\,a\,b}\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (2*a*b*atan(((a*b*(2*a^2*b - 2*b^3))/((a + b)^(3/2)*(a - b)^(3/2)) + (2*a^2
*b*tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/2)*(a - b)^(3/2)))/(2*a*b))
```

```
/(d*(a + b)^(3/2)*(a - b)^(3/2)) - ((2*a)/(a^2 - b^2) - (2*b*tan(c/2 + (d*x
)/2))/(a^2 - b^2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

3.1343 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{b\sec(c+dx)(b-a\sin(c+dx))}{ad\left(a^2-b^2\right)}+\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{3/2}}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] 2*b^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/(a^2-b^2)^(3/2)/d-
arctanh(cos(d*x+c))/a/d+sec(d*x+c)/a/d+b*sec(d*x+c)*(b-a*sin(d*x+c))/a/(a^2
-b^2)/d
```

Rubi [A] time = 0.23, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2898, 2622, 321, 207, 2696, 12, 2660, 618, 204}

$$\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{3/2}}+\frac{b\sec(c+dx)(b-a\sin(c+dx))}{ad\left(a^2-b^2\right)}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(3/
2)*d) - ArcTanh[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d) + (b*Sec[c + d*x]*
(b - a*Sin[c + d*x]))/(a*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
```

```
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{\csc(c+dx)\sec^2(c+dx)}{a} - \frac{b\sec^2(c+dx)}{a(a+b\sin(c+dx))}\right)dx$$

$$= \frac{\int \csc(c+dx)\sec^2(c+dx)\,dx}{a} - \frac{b\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{b\sec(c+dx)(b-a\sin(c+dx))}{a\left(a^2-b^2\right)d} + \frac{b\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)} + \frac{\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \mathrm{s}\right.}{ad}$$

$$= \frac{\sec(c+dx)}{ad} + \frac{b\sec(c+dx)(b-a\sin(c+dx))}{a\left(a^2-b^2\right)d} + \frac{b^3\int \frac{1}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)} + \frac{\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{b\sec(c+dx)(b-a\sin(c+dx))}{a\left(a^2-b^2\right)d} + \frac{\left(2b^{3}\right.}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{b\sec(c+dx)(b-a\sin(c+dx))}{a\left(a^2-b^2\right)d} - \frac{\left(4b^{3}\right.}{}$$

$$= \frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{3/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{b\sec(c+d}{a}$$

Mathematica [A] time = 0.38, size = 191, normalized size = 1.62

$$\frac{\sqrt{a^2-b^2}\left(a(a-b\sin(c+dx)) - \left(a^2-b^2\right)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right) + 2b^3\cos}{ad(a-b)(a+b)\sqrt{a^2-b^2}\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqrt
[a^2 - b^2]*(-((a^2 - b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c
```

```
 + d*x)/2]])) + a*(a - b*Sin[c + d*x])))/(a*(a - b)*(a + b)*Sqrt[a^2 - b^2]
*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]))
```

fricas [A] time = 1.00, size = 457, normalized size = 3.87

$$\left[\frac{\sqrt{-a^2+b^2}\,b^3\cos(dx+c)\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*b^3*cos(d*x + c)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 2*a^4 - 2*a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)*log(1/2*c
os(d*x + c) + 1/2) + (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)*log(-1/2*cos(d*x
+ c) + 1/2) - 2*(a^3*b - a*b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*
cos(d*x + c)), -1/2*(2*sqrt(a^2 - b^2)*b^3*arctan(-(a*sin(d*x + c) + b)/(sq
rt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - 2*a^4 + 2*a^2*b^2 + (a^4 - 2*a^
2*b^2 + b^4)*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - (a^4 - 2*a^2*b^2 +
b^4)*cos(d*x + c)*log(-1/2*cos(d*x + c) + 1/2) + 2*(a^3*b - a*b^3)*sin(d*x
+ c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c))]
```

giac [A] time = 0.21, size = 135, normalized size = 1.14

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^3}{(a^3-ab^2)\sqrt{a^2-b^2}}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}{(a^2-b^2)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c
) + b)/sqrt(a^2 - b^2)))*b^3/((a^3 - a*b^2)*sqrt(a^2 - b^2)) + log(abs(tan(
1/2*d*x + 1/2*c)))/a + 2*(b*tan(1/2*d*x + 1/2*c) - a)/((a^2 - b^2)*(tan(1/2
*d*x + 1/2*c)^2 - 1)))/d
```

maple [A] time = 0.44, size = 130, normalized size = 1.10

$$-\frac{1}{d\left(a+b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{1}{d\left(a-b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2b^{3}\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{d\left(a-b\right)\left(a+b\right)a\sqrt{a^{2}-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)+1/a/d*ln(tan(1/2*d*x+1/2*c))+1/d/(a-b)/(t
an(1/2*d*x+1/2*c)+1)+2/d*b^3/(a-b)/(a+b)/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.88, size = 659, normalized size = 5.58

$$a^{6}\cos\left(c+d\,x\right)+a^{6}+a^{2}\,b^{4}-2\,a^{4}\,b^{2}+a^{6}\cos\left(c+d\,x\right)\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-b^{6}\cos\left(c+d\,x\right)\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+a^{2}\,b^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] (a^6*cos(c + d*x) + a^6 + a^2*b^4 - 2*a^4*b^2 + a^6*cos(c + d*x)*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - b^6*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/
cos(c/2 + (d*x)/2)) + a^2*b^4*cos(c + d*x) - 2*a^4*b^2*cos(c + d*x) + 2*a^3
*b^3*sin(c + d*x) - a*b^5*sin(c + d*x) - a^5*b*sin(c + d*x) + b^3*cos(c + d
*x)*atan((a^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*
1i + b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*4i -
a^2*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*3i + a
```

```
*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*2i - a^3*
b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*1i)/(a^7*cos
(c/2 + (d*x)/2) - 4*b^7*sin(c/2 + (d*x)/2) - 2*a*b^6*cos(c/2 + (d*x)/2) + 2
*a^6*b*sin(c/2 + (d*x)/2) + 4*a^3*b^4*cos(c/2 + (d*x)/2) - 3*a^5*b^2*cos(c/
2 + (d*x)/2) + 9*a^2*b^5*sin(c/2 + (d*x)/2) - 7*a^4*b^3*sin(c/2 + (d*x)/2))
)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*2i + 3*a^2*b^4*cos(c + d*x)*log
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - 3*a^4*b^2*cos(c + d*x)*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4
- 2*a^2*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

3.1344 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=128

$$\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}-\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{3/2}}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -2*b^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/(a^2-b^2)^(3/2)
/d+b*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a/d-b*sec(d*x+c)/(a^2-b^2)/d+a*ta
n(d*x+c)/(a^2-b^2)/d
```

Rubi [A] time = 0.29, antiderivative size = 150, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2898, 2622, 321, 207, 2620, 14, 2696, 12, 2660, 618, 204}

$$-\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{3/2}}-\frac{b^2\sec(c+dx)(b-a\sin(c+dx))}{a^2d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\tan(c+}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^
(3/2)*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d) - (b*Sec[
c + d*x])/(a^2*d) - (b^2*Sec[c + d*x]*(b - a*Sin[c + d*x]))/(a^2*(a^2 - b^2
)*d) + Tan[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(-\frac{b\csc(c+dx)\sec^2(c+dx)}{a^2} + \frac{\csc^2(c+dx)\sec^2(c+dx)}{a} + \frac{b^2\sec^2(c+d}{a^2(a+b\sin(c+}\right.$$

$$= \frac{\int \csc^2(c+dx)\sec^2(c+dx)\,dx}{a} - \frac{b\int \csc(c+dx)\sec^2(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{\sec^2(c}{a+b\sin(}}{a^2}$$

$$= -\frac{b^2\sec(c+dx)(b-a\sin(c+dx))}{a^2(a^2-b^2)\,d} - \frac{b^2\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{a^2(a^2-b^2)} + \frac{\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x\right.}{ad}$$

$$= -\frac{b\sec(c+dx)}{a^2d} - \frac{b^2\sec(c+dx)(b-a\sin(c+dx))}{a^2(a^2-b^2)\,d} - \frac{b^4\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2(a^2-b^2)} + \frac{\text{Sub}}{}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{b\sec(c+dx)}{a^2d} - \frac{b^2\sec(c+dx)(b-a\sin}{a^2(a^2-b^2)\,d}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{b\sec(c+dx)}{a^2d} - \frac{b^2\sec(c+dx)(b-a\sin}{a^2(a^2-b^2)\,d}$$

$$= -\frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{3/2}\,d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{b\sec(c+}{a^2d}$$

Mathematica [A] time = 1.03, size = 205, normalized size = 1.60

$$\frac{-\frac{4b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{3/2}} - \frac{2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2} + \frac{2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2\sin\left(\frac{1}{2}(c+d\right.}{(a-b)\left(\sin\left(\frac{1}{2}(c+dx)\right)+c\right.}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-4*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)
^(3/2)) - Cot[(c + d*x)/2]/a + (2*b*Log[Cos[(c + d*x)/2]])/a^2 - (2*b*Log[S
in[(c + d*x)/2]])/a^2 + (2*Sin[(c + d*x)/2])/((a + b)*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])) + (2*Sin[(c + d*x)/2])/((a - b)*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])) + Tan[(c + d*x)/2]/a)/(2*d)
```

fricas [B] time = 1.02, size = 582, normalized size = 4.55

$$\left[\frac{\sqrt{-a^2+b^2}\,b^4\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)\sin(dx\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*b^4*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2))*sin(d*x + c) + 2*a^5 - 2*a^3*b^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x
+ c)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*c
os(d*x + c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*(2*a^5 - 3*a^3*b^
2 + a*b^4)*cos(d*x + c)^2 - 2*(a^4*b - a^2*b^3)*sin(d*x + c))/((a^6 - 2*a^4
*b^2 + a^2*b^4)*d*cos(d*x + c)*sin(d*x + c)), 1/2*(2*sqrt(a^2 - b^2)*b^4*ar
ctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c)*sin
(d*x + c) + 2*a^5 - 2*a^3*b^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)*log(
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x +
c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*(2*a^5 - 3*a^3*b^2 + a*b^4
)*cos(d*x + c)^2 - 2*(a^4*b - a^2*b^3)*sin(d*x + c))/((a^6 - 2*a^4*b^2 + a^
2*b^4)*d*cos(d*x + c)*sin(d*x + c))]
```

giac [B] time = 0.21, size = 259, normalized size = 2.02

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^4}{(a^4-a^2b^2)\sqrt{a^2-b^2}}+\frac{6b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{2a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}\cdots\right.}{\cdots}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
 1/2*c) + b)/sqrt(a^2 - b^2)))*b^4/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) + 6*b*
log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 3*tan(1/2*d*x + 1/2*c)/a - (2*a^2*b*ta
n(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x +
1/2*c)^2 + 3*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 10*a^2*b*tan(1/2*d*x + 1/2*c) +
 2*b^3*tan(1/2*d*x + 1/2*c) + 3*a^3 - 3*a*b^2)/((a^4 - a^2*b^2)*(tan(1/2*d*
x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))))/d
```

maple [A] time = 0.44, size = 169, normalized size = 1.32

$$-\frac{1}{d\left(a+b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{1}{2ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{2}}-\frac{1}{d\left(a-b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)+1/2/a/d*tan(1/2*d*x+1/2*c)-1/2/a/d/tan(1/
2*d*x+1/2*c)-1/d/a^2*b*ln(tan(1/2*d*x+1/2*c))-1/d/(a-b)/(tan(1/2*d*x+1/2*c)
+1)-2/d/a^2*b^4/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2
*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.68, size = 778, normalized size = 6.08

$$-\frac{\left(a\,b^6\,1\mathrm{i}-a^7\,\cos\left(2\,c+2\,d\,x\right)\,2\mathrm{i}-a^3\,b^4\,2\mathrm{i}+a^5\,b^2\,1\mathrm{i}+a\,b^6\,\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}-a^6\,b\,\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}-a^2\,b^5\,\mathrm{s}\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] -((a*b^6*1i - a^7*cos(2*c + 2*d*x)*2i - a^3*b^4*2i + a^5*b^2*1i + a*b^6*cos
(2*c + 2*d*x)*1i - a^6*b*sin(2*c + 2*d*x)*1i - a^2*b^5*sin(c + d*x)*2i + a^
4*b^3*sin(c + d*x)*4i - a^3*b^4*cos(2*c + 2*d*x)*4i + a^5*b^2*cos(2*c + 2*d
*x)*5i + b^7*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(2*c + 2*d*x)*1i
 - a^2*b^5*sin(2*c + 2*d*x)*1i + a^4*b^3*sin(2*c + 2*d*x)*2i - a^6*b*sin(c
+ d*x)*2i - a^6*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(2*c + 2*d*
```

```
x)*1i + 2*b^4*sin(2*c + 2*d*x)*atan((a^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*
a^2*b^4 + 3*a^4*b^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b
^4 + 3*a^4*b^2)^(1/2)*4i - a^2*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^
4 + 3*a^4*b^2)^(1/2)*3i + a*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 +
 3*a^4*b^2)^(1/2)*2i - a^3*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*
a^4*b^2)^(1/2)*1i)/(a^7*cos(c/2 + (d*x)/2) - 4*b^7*sin(c/2 + (d*x)/2) - 2*a
*b^6*cos(c/2 + (d*x)/2) + 2*a^6*b*sin(c/2 + (d*x)/2) + 4*a^3*b^4*cos(c/2 +
(d*x)/2) - 3*a^5*b^2*cos(c/2 + (d*x)/2) + 9*a^2*b^5*sin(c/2 + (d*x)/2) - 7*
a^4*b^3*sin(c/2 + (d*x)/2)))*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - a^
2*b^5*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(2*c + 2*d*x)*3i + a^4*
b^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(2*c + 2*d*x)*3i)*1i)/(a^
2*d*sin(2*c + 2*d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

3.1345 $\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=181

$$-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)}+\frac{\left(3a^2-b^2\right)\sec(c+dx)}{2ad\left(a^2-b^2\right)}+\frac{b\cot(c+dx)}{a^2d}-\frac{\left(3a^2+2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{2b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{3/2}}$$

```
[Out] 2*b^5*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/(a^2-b^2)^(3/2)/
d-1/2*(3*a^2+2*b^2)*arctanh(cos(d*x+c))/a^3/d+b*cot(d*x+c)/a^2/d+1/2*(3*a^2
-b^2)*sec(d*x+c)/a/(a^2-b^2)/d-1/2*csc(d*x+c)^2*sec(d*x+c)/a/d-b*tan(d*x+c)
/(a^2-b^2)/d
```

Rubi [A] time = 0.36, antiderivative size = 212, normalized size of antiderivative = 1.17, number of steps used = 17, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2898, 2622, 321, 207, 2620, 14, 288, 2696, 12, 2660, 618, 204}

$$\frac{2b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{3/2}}+\frac{b^2\sec(c+dx)}{a^3d}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b^3\sec(c+dx)(b-a\sin(c+dx))}{a^3d\left(a^2-b^2\right)}-\frac{b\tan(c+}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(
3/2)*d) - (3*ArcTanh[Cos[c + d*x]])/(2*a*d) - (b^2*ArcTanh[Cos[c + d*x]])/(
a^3*d) + (b*Cot[c + d*x])/(a^2*d) + (3*Sec[c + d*x])/(2*a*d) + (b^2*Sec[c +
 d*x])/(a^3*d) - (Csc[c + d*x]^2*Sec[c + d*x])/(2*a*d) + (b^3*Sec[c + d*x]*
(b - a*Sin[c + d*x]))/(a^3*(a^2 - b^2)*d) - (b*Tan[c + d*x])/(a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{b^2\csc(c+dx)\sec^2(c+dx)}{a^3} - \frac{b\csc^2(c+dx)\sec^2(c+dx)}{a^2} + \frac{\csc^3(c+dx)\,\mathrm{s}}{a}\right.$$

$$= \frac{\int \csc^3(c+dx)\sec^2(c+dx)\,dx}{a} - \frac{b\int \csc^2(c+dx)\sec^2(c+dx)\,dx}{a^2} + \frac{b^2\int \csc(}{}$$

$$= \frac{b^3\sec(c+dx)(b-a\sin(c+dx))}{a^3\left(a^2-b^2\right)d} + \frac{b^3\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{a^3\left(a^2-b^2\right)} + \frac{\mathrm{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,d\right.}{ad}$$

$$= \frac{b^2\sec(c+dx)}{a^3 d} - \frac{\csc^2(c+dx)\sec(c+dx)}{2ad} + \frac{b^3\sec(c+dx)(b-a\sin(c+dx))}{a^3\left(a^2-b^2\right)d}$$

$$= -\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3 d} + \frac{b\cot(c+dx)}{a^2 d} + \frac{3\sec(c+dx)}{2ad} + \frac{b^2\sec(c+dx)}{a^3 d} -$$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3 d} + \frac{b\cot(c+dx)}{a^2 d} + \frac{3\sec(c}{2ad}$$

$$= \frac{2b^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}d} - \frac{3\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3 d}$$

Mathematica [A] time = 3.10, size = 261, normalized size = 1.44

$$\frac{-\frac{4b\tan\left(\frac{1}{2}(c+dx)\right)}{a^2} + \frac{4b\cot\left(\frac{1}{2}(c+dx)\right)}{a^2} + \frac{4\left(3a^2+2b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} - \frac{4\left(3a^2+2b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{16b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}} + \cdots}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((16*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)
^(3/2)) + (4*b*Cot[(c + d*x)/2])/a^2 - Csc[(c + d*x)/2]^2/a - (4*(3*a^2 + 2
*b^2)*Log[Cos[(c + d*x)/2]])/a^3 + (4*(3*a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]]
)/a^3 + Sec[(c + d*x)/2]^2/a + (8*Sin[(c + d*x)/2])/((a + b)*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])) - (8*Sin[(c + d*x)/2])/((a - b)*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])) - (4*b*Tan[(c + d*x)/2])/a^2)/(8*d)
```

fricas [B] time = 1.91, size = 878, normalized size = 4.85

$$\left[-\frac{4\,a^6-4\,a^4b^2-2\left(3\,a^6-4\,a^4b^2+a^2b^4\right)\cos\left(dx+c\right)^2-2\left(b^5\cos\left(dx+c\right)^3-b^5\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{(2\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^6 - 4*a^4*b^2 - 2*(3*a^6 - 4*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 -
 2*(b^5*cos(d*x + c)^3 - b^5*cos(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 -
b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*si
n(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*
sin(d*x + c) - a^2 - b^2)) + ((3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x
 + c)^3 - (3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c))*log(1/2*cos(d
*x + c) + 1/2) - ((3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c)^3 - (3
*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1
/2) - 4*(a^5*b - a^3*b^3 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c)^2)*si
n(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)^3 - (a^7 - 2*a^5*b^
2 + a^3*b^4)*d*cos(d*x + c)), -1/4*(4*a^6 - 4*a^4*b^2 - 2*(3*a^6 - 4*a^4*b^
2 + a^2*b^4)*cos(d*x + c)^2 + 4*(b^5*cos(d*x + c)^3 - b^5*cos(d*x + c))*sqr
t(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) +
 ((3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c)^3 - (3*a^6 - 4*a^4*b^2
 - a^2*b^4 + 2*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - ((3*a^6 - 4
*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c)^3 - (3*a^6 - 4*a^4*b^2 - a^2*b^4 +
 2*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 4*(a^5*b - a^3*b^3 - (
2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - 2*a^5*b^
2 + a^3*b^4)*d*cos(d*x + c)^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c))
]
```

giac [A] time = 0.24, size = 245, normalized size = 1.35

$$\frac{\frac{16\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^5}{\left(a^5-a^3b^2\right)\sqrt{a^2-b^2}}+\frac{16\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{4\left(3\,a^2+2\,b^2\right)\log\left(\left|\mathrm{t}\cdots\right.\right.}{a^3}\cdots}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(16*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*b^5/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) + 16*(b
```

```
*tan(1/2*d*x + 1/2*c) - a)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)) + (a*
tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/a^2 + 4*(3*a^2 + 2*b^2)*
log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (18*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^
2*tan(1/2*d*x + 1/2*c)^2 - 4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/(a^3*tan(1/2*d
*x + 1/2*c)^2))/d
```

maple [A] time = 0.50, size = 227, normalized size = 1.25

$$-\frac{1}{d\left(a+b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{2d\,a^2}-\frac{1}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{\ln\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)+1/8/a/d*tan(1/2*d*x+1/2*c)^2-1/2/d/a^2*ta
n(1/2*d*x+1/2*c)*b-1/8/a/d/tan(1/2*d*x+1/2*c)^2+3/2/a/d*ln(tan(1/2*d*x+1/2*
c))+1/d/a^3*ln(tan(1/2*d*x+1/2*c))*b^2+1/2/d/a^2*b/tan(1/2*d*x+1/2*c)+1/d/(
a-b)/(tan(1/2*d*x+1/2*c)+1)+2/d/a^3*b^5/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 15.35, size = 1570, normalized size = 8.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (a^5*((3*b^3*sin(c + d*x))/4 - (5*b^3*sin(3*c + 3*d*x))/4) - a^7*((b*sin(c
+ d*x))/2 - (b*sin(3*c + 3*d*x))/2) + a^2*(b^6/4 + (b^6*cos(2*c + 2*d*x))/4
```

```
 + (3*b^6*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 - (3*b
^6*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/8) - a*((b^
7*sin(c + d*x))/4 + (b^7*sin(3*c + 3*d*x))/4) - a^6*((b^2*cos(c + d*x))/2 +
 b^2/4 - (7*b^2*cos(2*c + 2*d*x))/4 - (b^2*cos(3*c + 3*d*x))/2 + (7*b^2*cos
(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 - (7*b^2*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/8) - a^4*(b^4/4 - (b^4*c
os(c + d*x))/4 + (5*b^4*cos(2*c + 2*d*x))/4 + (b^4*cos(3*c + 3*d*x))/4 - (3
*b^4*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 + (3*b^4*lo
g(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/8) + a^8*(cos(c
+ d*x)/4 - (3*cos(2*c + 2*d*x))/4 - cos(3*c + 3*d*x)/4 + (3*cos(c + d*x)*lo
g(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 - (3*log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/8 + 1/4) - (b^8*cos(c + d*x)*log(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (b^8*log(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2))*cos(3*c + 3*d*x))/4 + a^3*b^5*sin(3*c + 3*d*x) + (b^5*atan((3*a^6
*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 8*b^6*sin(c
/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a^3*b^3*cos(c/2 +
 (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 7*a^4*b^2*sin(c/2 + (
d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 4*a*b^5*cos(c/2 + (d*x)
/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 3*a^5*b*cos(c/2 + (d*x)/2)*
(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^9*cos(c/2 + (d*x)/2)*3i - b^9
*sin(c/2 + (d*x)/2)*8i - a*b^8*cos(c/2 + (d*x)/2)*4i + a^8*b*sin(c/2 + (d*x
)/2)*6i + a^3*b^6*cos(c/2 + (d*x)/2)*5i + a^5*b^4*cos(c/2 + (d*x)/2)*3i - a
^7*b^2*cos(c/2 + (d*x)/2)*7i + a^2*b^7*sin(c/2 + (d*x)/2)*12i + a^4*b^5*sin
(c/2 + (d*x)/2)*4i - a^6*b^3*sin(c/2 + (d*x)/2)*14i))*cos(3*c + 3*d*x)*(-(a
 + b)^3*(a - b)^3)^(1/2)*1i)/2 - (b^5*atan((3*a^6*sin(c/2 + (d*x)/2)*(b^6 -
 a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 8*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 -
 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a^3*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a
^2*b^4 + 3*a^4*b^2)^(1/2) - 7*a^4*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2
*b^4 + 3*a^4*b^2)^(1/2) + 4*a*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
 + 3*a^4*b^2)^(1/2) - 3*a^5*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3
*a^4*b^2)^(1/2))/(a^9*cos(c/2 + (d*x)/2)*3i - b^9*sin(c/2 + (d*x)/2)*8i - a
*b^8*cos(c/2 + (d*x)/2)*4i + a^8*b*sin(c/2 + (d*x)/2)*6i + a^3*b^6*cos(c/2
+ (d*x)/2)*5i + a^5*b^4*cos(c/2 + (d*x)/2)*3i - a^7*b^2*cos(c/2 + (d*x)/2)*
7i + a^2*b^7*sin(c/2 + (d*x)/2)*12i + a^4*b^5*sin(c/2 + (d*x)/2)*4i - a^6*b
^3*sin(c/2 + (d*x)/2)*14i))*cos(c + d*x)*(-(a + b)^3*(a - b)^3)^(1/2)*1i)/2
)/(a^3*d*cos(c + d*x)*sin(c + d*x)^2*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1346 $\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\left(a^2-b^2\right)}-\frac{a^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{(2a+b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(2a-b)\log(\sin(c+a}{4d(a-b)^2}$$

```
[Out] 1/4*(2*a+b)*ln(1-sin(d*x+c))/(a+b)^2/d+1/4*(2*a-b)*ln(1+sin(d*x+c))/(a-b)^2
/d-a^3*ln(a+b*sin(d*x+c))/(a^2-b^2)^2/d+1/2*sec(d*x+c)^2*(a-b*sin(d*x+c))/(
a^2-b^2)/d
```

Rubi [A] time = 0.20, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 801}

$$-\frac{a^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\left(a^2-b^2\right)}+\frac{(2a+b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(2a-b)\log(\sin(c+}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a + b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((2*a - b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^2*d) - (a^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d
) + (Sec[c + d*x]^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2-b^2}-\frac{b^2\left(2a^2-b^2\right)x}{a^2-b^2}}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2b^2d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \left(-\frac{b^2(2a+b)}{2(a+b)^2(b-x)} - \frac{2a^3b^2}{(a-b)^2(a+b)^2(a+x)} + \frac{(2a-b)b^2}{2(a-b)^2(b+x)}\right.\right.}{2b^2d}$$

$$= \frac{(2a+b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(2a-b)\log(1+\sin(c+dx))}{4(a-b)^2d} - \frac{a^3\log(a+b\sin(c+dx)}{\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 0.55, size = 117, normalized size = 0.93

$$\frac{-\frac{4a^3\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2} - \frac{1}{(a+b)(\sin(c+dx)-1)} + \frac{1}{(a-b)(\sin(c+dx)+1)} + \frac{(2a+b)\log(1-\sin(c+dx))}{(a+b)^2} + \frac{(2a-b)\log(\sin(c+dx)+1)}{(a-b)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((2*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^2 + ((2*a - b)*Log[1 + Sin[c + d
*x]])/(a - b)^2 - (4*a^3*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) - 1
/((a + b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])))/(4*d)
```

fricas [A] time = 0.89, size = 157, normalized size = 1.25

$$-\frac{4\,a^3\cos(dx+c)^2\log(b\sin(dx+c)+a) - \left(2\,a^3+3\,a^2b-b^3\right)\cos(dx+c)^2\log(\sin(dx+c)+1) - \left(2\,a^3-3\,a^2b\right.}{4\left(a^4-2\,a^2b^2+b^4\right)d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^3*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - (2*a^3 + 3*a^2*b - b^3
)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^3 - 3*a^2*b + b^3)*cos(d*x +
c)^2*log(-sin(d*x + c) + 1) - 2*a^3 + 2*a*b^2 + 2*(a^2*b - b^3)*sin(d*x + c
))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

giac [A] time = 0.24, size = 177, normalized size = 1.40

$$-\frac{\frac{4\,a^3b\log(|b\sin(dx+c)+a|)}{a^4b-2\,a^2b^3+b^5}-\frac{(2\,a-b)\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2}-\frac{(2\,a+b)\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2}+\frac{2\left(a^3\sin(dx+c)^2-a^2b\sin(dx+c)+b^3\sin(dx+c)-ab^2\right)}{\left(a^4-2\,a^2b^2+b^4\right)\left(\sin(dx+c)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*a^3*b*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - (2*a
 - b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - (2*a + b)*log(abs(si
n(d*x + c) - 1))/(a^2 + 2*a*b + b^2) + 2*(a^3*sin(d*x + c)^2 - a^2*b*sin(d*
x + c) + b^3*sin(d*x + c) - a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2
 - 1)))/d
```

maple [A] time = 0.41, size = 164, normalized size = 1.30

$$-\frac{1}{d\left(4a+4b\right)\left(\sin\left(dx+c\right)-1\right)}+\frac{\ln\left(\sin\left(dx+c\right)-1\right)a}{2d\left(a+b\right)^{2}}+\frac{\ln\left(\sin\left(dx+c\right)-1\right)b}{4d\left(a+b\right)^{2}}-\frac{a^{3}\ln\left(a+b\sin\left(dx+c\right)\right)}{d\left(a+b\right)^{2}\left(a-b\right)^{2}}+\frac{}{d\,(4a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(4*a+4*b)/(sin(d*x+c)-1)+1/2/d/(a+b)^2*ln(sin(d*x+c)-1)*a+1/4/d/(a+b)^
2*ln(sin(d*x+c)-1)*b-1/d*a^3/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))+1/d/(4*a-4*
b)/(1+sin(d*x+c))+1/2*a*ln(1+sin(d*x+c))/(a-b)^2/d-1/4*b*ln(1+sin(d*x+c))/(
a-b)^2/d
```

maxima [A] time = 0.32, size = 142, normalized size = 1.13

$$-\frac{\frac{4\,a^3\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4}-\frac{(2\,a-b)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}-\frac{(2\,a+b)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}-\frac{2\,(b\sin(dx+c)-a)}{\left(a^2-b^2\right)\sin(dx+c)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^3*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - (2*a - b)*log
(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (2*a + b)*log(sin(d*x + c) - 1)/(a
^2 + 2*a*b + b^2) - 2*(b*sin(d*x + c) - a)/((a^2 - b^2)*sin(d*x + c)^2 - a^
2 + b^2))/d
```

mupad [B] time = 12.29, size = 217, normalized size = 1.72

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(2\,a-b\right)}{2\,d\,{\left(a-b\right)}^2}-\frac{\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}-\frac{2\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{a^2-b^2}+\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{a^2-b^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{a^3\,\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\right.\right.}{d\,\left(a^4-2\,a^2\,b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*(2*a - b))/(2*d*(a - b)^2) - ((b*tan(c/2 + (d*
x)/2))/(a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2)^2)/(a^2 - b^2) + (b*tan(c/2 +
(d*x)/2)^3)/(a^2 - b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2
+ 1)) - (a^3*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(
a^4 + b^4 - 2*a^2*b^2)) + (log(tan(c/2 + (d*x)/2) - 1)*(2*a + b))/(2*d*(a +
 b)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1347 $\int \frac{\sec(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{a^2 b\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)}+\frac{a\log(1-\sin(c+dx))}{4d(a+b)^2}-\frac{a\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

```
[Out] 1/4*a*ln(1-sin(d*x+c))/(a+b)^2/d-1/4*a*ln(1+sin(d*x+c))/(a-b)^2/d+a^2*b*ln(
a+b*sin(d*x+c))/(a^2-b^2)^2/d-1/2*sec(d*x+c)^2*(b-a*sin(d*x+c))/(a^2-b^2)/d
```

Rubi [A] time = 0.23, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2837, 12, 1647, 801}

$$\frac{a^2 b\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)}+\frac{a\log(1-\sin(c+dx))}{4d(a+b)^2}-\frac{a\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) - (a*Log[1 + Sin[c + d*x]])/(4*(a
 - b)^2*d) + (a^2*b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) - (Sec[c + d
*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
```

```
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\operatorname{Subst}\left(\int \frac{x^2}{b^2(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{Subst}\left(\int \frac{x^2}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{2d} + \frac{\operatorname{Subst}\left(\int \frac{-\frac{a^2b^2}{a^2-b^2}+\frac{ab^2x}{a^2-b^2}}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2bd}$$

$$= -\frac{\sec^2(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{2d} + \frac{\operatorname{Subst}\left(\int \left(-\frac{ab}{2(a+b)^2(b-x)}+\frac{2a^2b^2}{(a-b)^2(a+b)^2(a+x)}-\right.\right.}{2bd}$$

$$= \frac{a\log(1-\sin(c+dx))}{4(a+b)^2d} - \frac{a\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{a^2b\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2d} - $$

Mathematica [A] time = 0.47, size = 108, normalized size = 0.93

$$-\frac{-\frac{4a^2b\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}+\frac{1}{(a+b)(\sin(c+dx)-1)}+\frac{1}{(a-b)(\sin(c+dx)+1)}-\frac{a\log(1-\sin(c+dx))}{(a+b)^2}+\frac{a\log(\sin(c+dx)+1)}{(a-b)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(-((a*Log[1 - Sin[c + d*x]])/(a + b)^2) + (a*Log[1 + Sin[c + d*x]])/(a
 - b)^2 - (4*a^2*b*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a +
 b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])))/d
```

fricas [A] time = 0.92, size = 154, normalized size = 1.33

$$\frac{4\,a^2b\cos\left(dx+c\right)^2\log\left(b\sin\left(dx+c\right)+a\right)-\left(a^3+2\,a^2b+ab^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+\left(a^3-2\,a^2b\right.}{4\left(a^4-2\,a^2b^2+b^4\right)d\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*b*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - (a^3 + 2*a^2*b + a*b^
2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (a^3 - 2*a^2*b + a*b^2)*cos(d*x +
 c)^2*log(-sin(d*x + c) + 1) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x +
c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

giac [A] time = 0.23, size = 168, normalized size = 1.45

$$\frac{\frac{4\,a^2b^2\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^4b-2\,a^2b^3+b^5}-\frac{a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^2-2\,ab+b^2}+\frac{a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^2+2\,ab+b^2}+\frac{2\left(a^2b\sin\left(dx+c\right)^2-a^3\sin\left(dx+c\right)+ab^2\sin\left(dx+c\right)-b^3\right)}{\left(a^4-2\,a^2b^2+b^4\right)\left(\sin\left(dx+c\right)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(4*a^2*b^2*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - a*l
og(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + a*log(abs(sin(d*x + c) - 1)
)/(a^2 + 2*a*b + b^2) + 2*(a^2*b*sin(d*x + c)^2 - a^3*sin(d*x + c) + a*b^2*
sin(d*x + c) - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)))/d
```

maple [A] time = 0.44, size = 123, normalized size = 1.06

$$-\frac{1}{d\left(4a+4b\right)\left(\sin\left(dx+c\right)-1\right)}+\frac{\ln\left(\sin\left(dx+c\right)-1\right)a}{4d\left(a+b\right)^2}+\frac{a^2b\ln\left(a+b\sin\left(dx+c\right)\right)}{d\left(a+b\right)^2\left(a-b\right)^2}-\frac{1}{d\left(4a-4b\right)\left(1+\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(4*a+4*b)/(sin(d*x+c)-1)+1/4/d/(a+b)^2*ln(sin(d*x+c)-1)*a+1/d*a^2/(a+b
)^2*b/(a-b)^2*ln(a+b*sin(d*x+c))-1/d/(4*a-4*b)/(1+sin(d*x+c))-1/4*a*ln(1+si
n(d*x+c))/(a-b)^2/d
```

maxima [A] time = 0.31, size = 132, normalized size = 1.14

$$\frac{\frac{4\,a^2b\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4}-\frac{a\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{a\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}-\frac{2\,(a\sin(dx+c)-b)}{(a^2-b^2)\sin(dx+c)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^2*b*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - a*log(sin(d*
x + c) + 1)/(a^2 - 2*a*b + b^2) + a*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^
2) - 2*(a*sin(d*x + c) - b)/((a^2 - b^2)*sin(d*x + c)^2 - a^2 + b^2))/d
```

mupad [B] time = 12.22, size = 206, normalized size = 1.78

$$\frac{\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^2-b^2}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^2-b^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{2\,d\,(a-b)^2}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{2\,d\,(a+b)^2}+\frac{a^2\,b\,\ln\left(a\tan\left(\frac{c}{2}+\right.\right.}{d\,\left(a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] ((a*tan(c/2 + (d*x)/2))/(a^2 - b^2) + (a*tan(c/2 + (d*x)/2)^3)/(a^2 - b^2)
- (2*b*tan(c/2 + (d*x)/2)^2)/(a^2 - b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(
c/2 + (d*x)/2)^2 + 1)) - (a*log(tan(c/2 + (d*x)/2) + 1))/(2*d*(a - b)^2) +
(a*log(tan(c/2 + (d*x)/2) - 1))/(2*d*(a + b)^2) + (a^2*b*log(a + 2*b*tan(c/
2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^4 + b^4 - 2*a^2*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

3.1348 $\int \frac{\sec^2(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{ab^2\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\left(a^2-b^2\right)}-\frac{b\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{b\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

```
[Out] -1/4*b*ln(1-sin(d*x+c))/(a+b)^2/d+1/4*b*ln(1+sin(d*x+c))/(a-b)^2/d-a*b^2*ln
(a+b*sin(d*x+c))/(a^2-b^2)^2/d+1/2*sec(d*x+c)^2*(a-b*sin(d*x+c))/(a^2-b^2)/
d
```

Rubi [A] time = 0.17, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2837, 12, 823, 801}

$$-\frac{ab^2\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\left(a^2-b^2\right)}-\frac{b\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{b\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + (b*Log[1 + Sin[c + d*x]])/(4*(
a - b)^2*d) - (a*b^2*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) + (Sec[c +
d*x]^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
```

```
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{x}{b(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\,\text{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2\left(a^2-b^2\right)d} - \frac{\text{Subst}\left(\int \frac{-ab^2+b^2x}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2\left(a^2-b^2\right)d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2\left(a^2-b^2\right)d} - \frac{\text{Subst}\left(\int \left(\frac{b(-a+b)}{2(a+b)(b-x)} + \frac{2ab^2}{(a-b)(a+b)(a+x)} - \frac{b(a+b}{2(a-b)(b}\right.\right.}{2\left(a^2-b^2\right)d}$$

$$= -\frac{b\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{b\log(1+\sin(c+dx))}{4(a-b)^2d} - \frac{ab^2\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2d} +$$

Mathematica [A] time = 0.41, size = 162, normalized size = 1.38

$$\frac{-\frac{4ab^2\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2} + \frac{1}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{1}{(a-b)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.}{(a+b)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^2 + (2*b*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]])/(a - b)^2 - (4*a*b^2*Log[a + b*Sin[c + d*x
]])/(a^2 - b^2)^2 + 1/((a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + 1
/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))/(4*d)
```

fricas [A] time = 0.86, size = 155, normalized size = 1.32

$$-\frac{4\,ab^2\cos\left(dx+c\right)^2\log\left(b\sin\left(dx+c\right)+a\right)-\left(a^2b+2\,ab^2+b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+\left(a^2b-2\,a\right.}{4\left(a^4-2\,a^2b^2+b^4\right)d\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*b^2*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - (a^2*b + 2*a*b^2 + b
^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (a^2*b - 2*a*b^2 + b^3)*cos(d*x
+ c)^2*log(-sin(d*x + c) + 1) - 2*a^3 + 2*a*b^2 + 2*(a^2*b - b^3)*sin(d*x +
 c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

giac [A] time = 0.23, size = 170, normalized size = 1.45

$$-\frac{\frac{4\,ab^3\log\left(|b\sin(dx+c)+a|\right)}{a^4b-2\,a^2b^3+b^5}-\frac{b\log\left(|\sin(dx+c)+1|\right)}{a^2-2\,ab+b^2}+\frac{b\log\left(|\sin(dx+c)-1|\right)}{a^2+2\,ab+b^2}+\frac{2\left(ab^2\sin(dx+c)^2-a^2b\sin(dx+c)+b^3\sin(dx+c)+a^3-2\,ab^2\right)}{\left(a^4-2\,a^2b^2+b^4\right)\left(\sin(dx+c)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*a*b^3*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - b*lo
g(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + b*log(abs(sin(d*x + c) - 1))
/(a^2 + 2*a*b + b^2) + 2*(a*b^2*sin(d*x + c)^2 - a^2*b*sin(d*x + c) + b^3*s
in(d*x + c) + a^3 - 2*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1))
)/d
```

maple [A] time = 0.36, size = 123, normalized size = 1.05

$$-\frac{1}{d\left(4a+4b\right)\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)b}{4d\left(a+b\right)^2}-\frac{a\,b^2\ln\left(a+b\sin\left(dx+c\right)\right)}{d\left(a+b\right)^2\left(a-b\right)^2}+\frac{1}{d\left(4a-4b\right)\left(1+\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(4*a+4*b)/(sin(d*x+c)-1)-1/4/d/(a+b)^2*ln(sin(d*x+c)-1)*b-1/d*a*b^2/(a
+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))+1/d/(4*a-4*b)/(1+sin(d*x+c))+1/4*b*ln(1+si
n(d*x+c))/(a-b)^2/d
```

maxima [A] time = 0.32, size = 132, normalized size = 1.13

$$-\frac{\frac{4\,ab^2\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4}-\frac{b\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{b\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}-\frac{2\,(b\sin(dx+c)-a)}{(a^2-b^2)\sin(dx+c)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*a*b^2*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - b*log(sin(d
*x + c) + 1)/(a^2 - 2*a*b + b^2) + b*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b
^2) - 2*(b*sin(d*x + c) - a)/((a^2 - b^2)*sin(d*x + c)^2 - a^2 + b^2))/d
```

mupad [B] time = 12.11, size = 208, normalized size = 1.78

$$\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{2\,d\,(a-b)^2}-\frac{\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^2-b^2}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^2-b^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{2\,d\,(a+b)^2}-\frac{a\,b^2\,\ln\left(a\tan\left(\frac{c}{2}+\right.\right.}{d\,\left(a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (b*log(tan(c/2 + (d*x)/2) + 1))/(2*d*(a - b)^2) - ((b*tan(c/2 + (d*x)/2))/(
a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2)^2)/(a^2 - b^2) + (b*tan(c/2 + (d*x)/2)
^3)/(a^2 - b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) -
(b*log(tan(c/2 + (d*x)/2) - 1))/(2*d*(a + b)^2) - (a*b^2*log(a + 2*b*tan(c/
2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^4 + b^4 - 2*a^2*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

3.1349 $\int \frac{\csc(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=156

$$-\frac{b^4\log(a+b\sin(c+dx))}{ad\left(a^2-b^2\right)^2}+\frac{1}{4d(a+b)(1-\sin(c+dx))}+\frac{1}{4d(a-b)(\sin(c+dx)+1)}-\frac{(2a+3b)\log(1-\sin(c+dx))}{4d(a+b)^2}$$

```
[Out] -1/4*(2*a+3*b)*ln(1-sin(d*x+c))/(a+b)^2/d+ln(sin(d*x+c))/a/d-1/4*(2*a-3*b)*
ln(1+sin(d*x+c))/(a-b)^2/d-b^4*ln(a+b*sin(d*x+c))/a/(a^2-b^2)^2/d+1/4/(a+b)
/d/(1-sin(d*x+c))+1/4/(a-b)/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.24, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$-\frac{b^4\log(a+b\sin(c+dx))}{ad\left(a^2-b^2\right)^2}+\frac{1}{4d(a+b)(1-\sin(c+dx))}+\frac{1}{4d(a-b)(\sin(c+dx)+1)}-\frac{(2a+3b)\log(1-\sin(c+dx))}{4d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((2*a + 3*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + Log[Sin[c + d*x]]/(a
*d) - ((2*a - 3*b)*Log[1 + Sin[c + d*x]])/(4*(a - b)^2*d) - (b^4*Log[a + b*
Sin[c + d*x]])/(a*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sin[c + d*x])) + 1
/(4*(a - b)*d*(1 + Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\operatorname{Subst}\left(\int \frac{b}{x(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\operatorname{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\operatorname{Subst}\left(\int \left(\frac{1}{4b^3(a+b)(b-x)^2} + \frac{2a+3b}{4b^4(a+b)^2(b-x)} + \frac{1}{ab^4x} - \frac{1}{a(a-b)^2(a+b)^2(a+x)} - \frac{1}{4(a-b)b^3(b+x)^2}\right.\right.}{d}$$

$$= -\frac{(2a+3b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{\log(\sin(c+dx))}{ad} - \frac{(2a-3b)\log(1+\sin(c+}{4(a-b)^2d}$$

Mathematica [A] time = 0.69, size = 151, normalized size = 0.97

$$\frac{b^4\left(-\frac{1}{b^4(a+b)(\sin(c+dx)-1)} + \frac{1}{b^4(a-b)(\sin(c+dx)+1)} - \frac{(2a+3b)\log(1-\sin(c+dx))}{b^4(a+b)^2} + \frac{4\log(\sin(c+dx))}{ab^4} - \frac{(2a-3b)\log(\sin(c+dx)+1)}{b^4(a-b)^2} - \frac{4\log(a+b}{a(a-b)}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^4*(-(((2*a + 3*b)*Log[1 - Sin[c + d*x]])/(b^4*(a + b)^2)) + (4*Log[Sin[c
 + d*x]])/(a*b^4) - ((2*a - 3*b)*Log[1 + Sin[c + d*x]])/((a - b)^2*b^4) - (
4*Log[a + b*Sin[c + d*x]])/(a*(a - b)^2*(a + b)^2) - 1/(b^4*(a + b)*(-1 + S
in[c + d*x])) + 1/((a - b)*b^4*(1 + Sin[c + d*x]))))/(4*d)
```

fricas [A] time = 1.64, size = 213, normalized size = 1.37

$$-\frac{4\,b^4\cos\left(dx+c\right)^2\log\left(b\sin\left(dx+c\right)+a\right) - 2\,a^4 + 2\,a^2b^2 - 4\left(a^4 - 2\,a^2b^2 + b^4\right)\cos\left(dx+c\right)^2\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^4*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - 2*a^4 + 2*a^2*b^2 - 4*
(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2*log(-1/2*sin(d*x + c)) + (2*a^4 + a^
3*b - 4*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (2*a^4 -
a^3*b - 4*a^2*b^2 + 3*a*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(a^3
*b - a*b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2)
```

giac [A] time = 0.23, size = 210, normalized size = 1.35

$$-\frac{\frac{4\,b^5\log(|b\sin(dx+c)+a|)}{a^5b-2\,a^3b^3+ab^5}+\frac{(2\,a-3\,b)\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2}+\frac{(2\,a+3\,b)\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2}-\frac{4\,\log(|\sin(dx+c)|)}{a}-\frac{2\left(a^3\sin(dx+c)^2-2\,ab^2\sin(dx+c)\right.}{\left(a^4-2\,a^2\right.}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*b^5*log(abs(b*sin(d*x + c) + a))/(a^5*b - 2*a^3*b^3 + a*b^5) + (2*a
 - 3*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + (2*a + 3*b)*log(ab
s(sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) - 4*log(abs(sin(d*x + c)))/a - 2*(
a^3*sin(d*x + c)^2 - 2*a*b^2*sin(d*x + c)^2 + a^2*b*sin(d*x + c) - b^3*sin(
d*x + c) - 2*a^3 + 3*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)))
/d
```

maple [A] time = 0.49, size = 181, normalized size = 1.16

$$-\frac{1}{d\left(4a+4b\right)\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)a}{2d\left(a+b\right)^2}-\frac{3\ln\left(\sin\left(dx+c\right)-1\right)b}{4d\left(a+b\right)^2}-\frac{b^4\ln\left(a+b\sin\left(dx+c\right)\right)}{da\left(a+b\right)^2\left(a-b\right)^2}+\frac{\ln(\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(4*a+4*b)/(sin(d*x+c)-1)-1/2/d/(a+b)^2*ln(sin(d*x+c)-1)*a-3/4/d/(a+b)^
2*ln(sin(d*x+c)-1)*b-1/d/a*b^4/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))+ln(sin(d*
x+c))/a/d+1/d/(4*a-4*b)/(1+sin(d*x+c))-1/2*a*ln(1+sin(d*x+c))/(a-b)^2/d+3/4
*b*ln(1+sin(d*x+c))/(a-b)^2/d
```

maxima [A] time = 0.32, size = 156, normalized size = 1.00

$$-\frac{\frac{4\,b^4\log(b\sin(dx+c)+a)}{a^5-2\,a^3b^2+ab^4}+\frac{(2\,a-3\,b)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{(2\,a+3\,b)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}-\frac{2\,(b\sin(dx+c)-a)}{\left(a^2-b^2\right)\sin(dx+c)^2-a^2+b^2}-\frac{4\,\log(\sin(dx+c))}{a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^4*log(b*sin(d*x + c) + a)/(a^5 - 2*a^3*b^2 + a*b^4) + (2*a - 3*b)
*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + (2*a + 3*b)*log(sin(d*x + c) -
 1)/(a^2 + 2*a*b + b^2) - 2*(b*sin(d*x + c) - a)/((a^2 - b^2)*sin(d*x + c)^
2 - a^2 + b^2) - 4*log(sin(d*x + c))/a)/d
```

mupad [B] time = 12.42, size = 170, normalized size = 1.09

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{b}{4\,{\left(a-b\right)}^2}-\frac{1}{2\,\left(a-b\right)}\right)}{d}-\frac{\frac{a}{2\,\left(a^2-b^2\right)}-\frac{b\,\sin\left(c+d\,x\right)}{2\,\left(a^2-b^2\right)}}{d\,\left({\sin\left(c+d\,x\right)}^2-1\right)}+\frac{\ln\left(\sin\left(c+d\,x\right)\right)}{a\,d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{b}{4\,{\left(a+b\right)}^2}+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(b/(4*(a - b)^2) - 1/(2*(a - b))))/d - (a/(2*(a^2 -
b^2)) - (b*sin(c + d*x))/(2*(a^2 - b^2)))/(d*(sin(c + d*x)^2 - 1)) + log(si
n(c + d*x))/(a*d) - (log(sin(c + d*x) - 1)*(b/(4*(a + b)^2) + 1/(2*(a + b))
))/d - (b^4*log(a + b*sin(c + d*x)))/(a*d*(a^2 - b^2)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(c + dx\right)\sec^{3}\left(c + dx\right)}{a + b\sin\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

3.1350 $\int \frac{\csc^2(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=171

$$\frac{b^5\log(a+b\sin(c+dx))}{a^2d\left(a^2-b^2\right)^2}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{1}{4d(a+b)(1-\sin(c+dx))}-\frac{1}{4d(a-b)(\sin(c+dx)+1)}-\frac{(3a+4b)}{}$$

```
[Out] -csc(d*x+c)/a/d-1/4*(3*a+4*b)*ln(1-sin(d*x+c))/(a+b)^2/d-b*ln(sin(d*x+c))/a
^2/d+1/4*(3*a-4*b)*ln(1+sin(d*x+c))/(a-b)^2/d+b^5*ln(a+b*sin(d*x+c))/a^2/(a
^2-b^2)^2/d+1/4/(a+b)/d/(1-sin(d*x+c))-1/4/(a-b)/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.27, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$\frac{b^5\log(a+b\sin(c+dx))}{a^2d\left(a^2-b^2\right)^2}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{1}{4d(a+b)(1-\sin(c+dx))}-\frac{1}{4d(a-b)(\sin(c+dx)+1)}-\frac{(3a+4b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - ((3*a + 4*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d)
 - (b*Log[Sin[c + d*x]])/(a^2*d) + ((3*a - 4*b)*Log[1 + Sin[c + d*x]])/(4*(
a - b)^2*d) + (b^5*Log[a + b*Sin[c + d*x]])/(a^2*(a^2 - b^2)^2*d) + 1/(4*(a
 + b)*d*(1 - Sin[c + d*x])) - 1/(4*(a - b)*d*(1 + Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\,\mathrm{Subst}\left(\int \frac{b^2}{x^2(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^5\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^5\,\mathrm{Subst}\left(\int \left(\frac{1}{4b^4(a+b)(b-x)^2} + \frac{3a+4b}{4b^5(a+b)^2(b-x)} + \frac{1}{ab^4x^2} - \frac{1}{a^2b^4x} + \frac{1}{a^2(a-b)^2(a+b)^2(a+x)} + \cdots\right.\right.}{d}$$

$$= -\frac{\csc(c+dx)}{ad} - \frac{(3a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} - \frac{b\log(\sin(c+dx))}{a^2d} + \frac{(3a-4b\cdots}{\cdots}$$

Mathematica [A] time = 0.78, size = 174, normalized size = 1.02

$$-\frac{\csc(c+dx)(a+b\sin(c+dx))\left(-\frac{4b^5\log(a+b\sin(c+dx))}{a^2(a-b)^2(a+b)^2} + \frac{4b\log(\sin(c+dx))}{a^2} + \frac{1}{(a+b)(\sin(c+dx)-1)} + \frac{1}{(a-b)(\sin(c+dx)+1)} + \frac{(3a+4b\cdots}{\cdots}\right.}{4d(a\csc(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(Csc[c + d*x]*(a + b*Sin[c + d*x])*((4*Csc[c + d*x])/a + ((3*a + 4*b)*
Log[1 - Sin[c + d*x]])/(a + b)^2 + (4*b*Log[Sin[c + d*x]])/a^2 - ((3*a - 4*
b)*Log[1 + Sin[c + d*x]])/(a - b)^2 - (4*b^5*Log[a + b*Sin[c + d*x]])/(a^2*
(a - b)^2*(a + b)^2) + 1/((a + b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Si
n[c + d*x]))))/(d*(b + a*Csc[c + d*x]))
```

fricas [A] time = 1.85, size = 287, normalized size = 1.68

$$\frac{4\,b^5\cos\left(dx+c\right)^2\log\left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right) + 2\,a^5 - 2\,a^3b^2 - 4\left(a^4b - 2\,a^2b^3 + b^5\right)\cos\left(dx+c\right)^2\log\left(\frac{1}{2}\,\mathrm{s}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^5*cos(d*x + c)^2*log(b*sin(d*x + c) + a)*sin(d*x + c) + 2*a^5 - 2*
a^3*b^2 - 4*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^2*log(1/2*sin(d*x + c))*
sin(d*x + c) + (3*a^5 + 2*a^4*b - 5*a^3*b^2 - 4*a^2*b^3)*cos(d*x + c)^2*log
(sin(d*x + c) + 1)*sin(d*x + c) - (3*a^5 - 2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1)*sin(d*x + c) - 2*(3*a^5 - 5*a^3*b^2
+ 2*a*b^4)*cos(d*x + c)^2 - 2*(a^4*b - a^2*b^3)*sin(d*x + c))/((a^6 - 2*a^4
*b^2 + a^2*b^4)*d*cos(d*x + c)^2*sin(d*x + c))
```

giac [A] time = 0.24, size = 279, normalized size = 1.63

$$\frac{\frac{12\,b^6\log(|b\sin(dx+c)+a|)}{a^6b-2\,a^4b^3+a^2b^5}+\frac{3\,(3\,a-4\,b)\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2}-\frac{3\,(3\,a+4\,b)\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2}-\frac{12\,b\log(|\sin(dx+c)|)}{a^2}+\frac{2\left(2\,b^5\sin(dx+c)^3-9\,a^5\sin\right.}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*b^6*log(abs(b*sin(d*x + c) + a))/(a^6*b - 2*a^4*b^3 + a^2*b^5) + 3
*(3*a - 4*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - 3*(3*a + 4*b)
*log(abs(sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) - 12*b*log(abs(sin(d*x + c)
))/a^2 + 2*(2*b^5*sin(d*x + c)^3 - 9*a^5*sin(d*x + c)^2 + 15*a^3*b^2*sin(d*
x + c)^2 - 6*a*b^4*sin(d*x + c)^2 + 3*a^4*b*sin(d*x + c) - 3*a^2*b^3*sin(d*
x + c) - 2*b^5*sin(d*x + c) + 6*a^5 - 12*a^3*b^2 + 6*a*b^4)/((a^6 - 2*a^4*b
^2 + a^2*b^4)*(sin(d*x + c)^3 - sin(d*x + c))))/d
```

maple [A] time = 0.49, size = 199, normalized size = 1.16

$$-\frac{1}{d\,(4a+4b)\,(\sin(dx+c)-1)}-\frac{3\ln(\sin(dx+c)-1)\,a}{4d\,(a+b)^2}-\frac{\ln(\sin(dx+c)-1)\,b}{d\,(a+b)^2}+\frac{b^5\ln(a+b\sin(dx+c))}{d\,a^2\,(a+b)^2\,(a-b)^2}-\frac{}{da\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(4*a+4*b)/(sin(d*x+c)-1)-3/4/d/(a+b)^2*ln(sin(d*x+c)-1)*a-1/d/(a+b)^2*
ln(sin(d*x+c)-1)*b+1/d*b^5/a^2/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))-1/d/a/sin
(d*x+c)-b*ln(sin(d*x+c))/a^2/d-1/d/(4*a-4*b)/(1+sin(d*x+c))+3/4*a*ln(1+sin(
d*x+c))/(a-b)^2/d-b*ln(1+sin(d*x+c))/(a-b)^2/d
```

maxima [A] time = 0.32, size = 200, normalized size = 1.17

$$\frac{\frac{4\,b^5\log(b\sin(dx+c)+a)}{a^6-2\,a^4b^2+a^2b^4}+\frac{(3\,a-4\,b)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}-\frac{(3\,a+4\,b)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}+\frac{2\left(ab\sin(dx+c)-\left(3\,a^2-2\,b^2\right)\sin(dx+c)^2+2\,a^2-2\,b^2\right)}{\left(a^3-ab^2\right)\sin(dx+c)^3-\left(a^3-ab^2\right)\sin(dx+c)}-\frac{4\,b}{}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*b^5*log(b*sin(d*x + c) + a)/(a^6 - 2*a^4*b^2 + a^2*b^4) + (3*a - 4*b
)*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (3*a + 4*b)*log(sin(d*x + c)
- 1)/(a^2 + 2*a*b + b^2) + 2*(a*b*sin(d*x + c) - (3*a^2 - 2*b^2)*sin(d*x +
c)^2 + 2*a^2 - 2*b^2)/((a^3 - a*b^2)*sin(d*x + c)^3 - (a^3 - a*b^2)*sin(d*x
 + c)) - 4*b*log(sin(d*x + c))/a^2)/d
```

mupad [B] time = 12.40, size = 195, normalized size = 1.14

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(3\,a-4\,b\right)}{4\,d\,{\left(a-b\right)}^2}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{b}{4\,{\left(a+b\right)}^2}+\frac{3}{4\,\left(a+b\right)}\right)}{d}-\frac{\frac{1}{a}+\frac{b\,\sin\left(c+d\,x\right)}{2\,\left(a^2-b^2\right)}-\frac{{\sin\left(c+d\,x\right)}^2\,\left(3\,a^2-2\,b^2\right)}{2\,a\,\left(a^2-b^2\right)}}{d\,\left(\sin\left(c+d\,x\right)-{\sin\left(c+d\,x\right)}^3\right)}-\frac{b\,\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(3*a - 4*b))/(4*d*(a - b)^2) - (log(sin(c + d*x) - 1
)*(b/(4*(a + b)^2) + 3/(4*(a + b))))/d - (1/a + (b*sin(c + d*x))/(2*(a^2 -
b^2)) - (sin(c + d*x)^2*(3*a^2 - 2*b^2))/(2*a*(a^2 - b^2)))/(d*(sin(c + d*x
) - sin(c + d*x)^3)) - (b*log(sin(c + d*x)))/(a^2*d) + (b^5*log(a + b*sin(c
 + d*x)))/(a^2*d*(a^2 - b^2)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1351 $\int \frac{\csc^3(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=197

$$\frac{b\csc(c+dx)}{a^2d}+\frac{\left(2a^2+b^2\right)\log(\sin(c+dx))}{a^3d}-\frac{b^6\log(a+b\sin(c+dx))}{a^3d\left(a^2-b^2\right)^2}+\frac{1}{4d(a+b)(1-\sin(c+dx))}+\frac{1}{4d(a-b)(\sin$$

```
[Out] b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d-1/4*(4*a+5*b)*ln(1-sin(d*x+c))/(a+b
)^2/d+(2*a^2+b^2)*ln(sin(d*x+c))/a^3/d-1/4*(4*a-5*b)*ln(1+sin(d*x+c))/(a-b)
^2/d-b^6*ln(a+b*sin(d*x+c))/a^3/(a^2-b^2)^2/d+1/4/(a+b)/d/(1-sin(d*x+c))+1/
4/(a-b)/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.31, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$-\frac{b^6\log(a+b\sin(c+dx))}{a^3d\left(a^2-b^2\right)^2}+\frac{\left(2a^2+b^2\right)\log(\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}+\frac{1}{4d(a+b)(1-\sin(c+dx))}+\frac{1}{4d(a-b)(s$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((4*a + 5*b)*Log[1 - Si
n[c + d*x]])/(4*(a + b)^2*d) + ((2*a^2 + b^2)*Log[Sin[c + d*x]])/(a^3*d) -
((4*a - 5*b)*Log[1 + Sin[c + d*x]])/(4*(a - b)^2*d) - (b^6*Log[a + b*Sin[c
+ d*x]])/(a^3*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sin[c + d*x])) + 1/(4*
(a - b)*d*(1 + Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\,\mathrm{Subst}\left(\int \frac{b^3}{x^3(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^6\,\mathrm{Subst}\left(\int \frac{1}{x^3(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^6\,\mathrm{Subst}\left(\int \left(\frac{1}{4b^5(a+b)(b-x)^2} + \frac{4a+5b}{4b^6(a+b)^2(b-x)} + \frac{1}{ab^4x^3} - \frac{1}{a^2b^4x^2} + \frac{2a^2+b^2}{a^3b^6x} - \frac{1}{a^3(a-b)^2(a+b)\cdots}\right.\right.}{d}$$

$$= \frac{b\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2ad} - \frac{(4a+5b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{\left(2a^2+b^2\right)\log\cdots}{a^3\cdots}$$

Mathematica [A] time = 1.44, size = 168, normalized size = 0.85

$$-\frac{\frac{4b^6\log(a+b\sin(c+dx))}{a^3(a-b)^2(a+b)^2} - \frac{4b\csc(c+dx)}{a^2} - \frac{4\left(2a^2+b^2\right)\log(\sin(c+dx))}{a^3} + \frac{1}{(a+b)(\sin(c+dx)-1)} - \frac{1}{(a-b)(\sin(c+dx)+1)} + \frac{(4a+5b)\log(1-\sin(c+dx))}{(a+b)^2}\cdots}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*((-4*b*Csc[c + d*x])/a^2 + (2*Csc[c + d*x]^2)/a + ((4*a + 5*b)*Log[1 -
 Sin[c + d*x]])/(a + b)^2 - (4*(2*a^2 + b^2)*Log[Sin[c + d*x]])/a^3 + ((4*a
 - 5*b)*Log[1 + Sin[c + d*x]])/(a - b)^2 + (4*b^6*Log[a + b*Sin[c + d*x]])/
(a^3*(a - b)^2*(a + b)^2) + 1/((a + b)*(-1 + Sin[c + d*x])) - 1/((a - b)*(1
 + Sin[c + d*x])))/d
```

fricas [B] time = 3.05, size = 440, normalized size = 2.23

$$-\frac{2a^6 - 2a^4b^2 - 2\left(2a^6 - 3a^4b^2 + a^2b^4\right)\cos(dx+c)^2 + 4\left(b^6\cos(dx+c)^4 - b^6\cos(dx+c)^2\right)\log(b\sin(dx+c)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/4*(2*a^6 - 2*a^4*b^2 - 2*(2*a^6 - 3*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + 4*(b^6*cos(d*x + c)^4 - b^6*cos(d*x + c)^2)*log(b*sin(d*x + c) + a) - 4*((2*a^6 - 3*a^4*b^2 + b^6)*cos(d*x + c)^4 - (2*a^6 - 3*a^4*b^2 + b^6)*cos(d*x + c)^2)*log(-1/2*sin(d*x + c)) + ((4*a^6 + 3*a^5*b - 6*a^4*b^2 - 5*a^3*b^3)*cos(d*x + c)^4 - (4*a^6 + 3*a^5*b - 6*a^4*b^2 - 5*a^3*b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((4*a^6 - 3*a^5*b - 6*a^4*b^2 + 5*a^3*b^3)*cos(d*x + c)^4 - (4*a^6 - 3*a^5*b - 6*a^4*b^2 + 5*a^3*b^3)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^5*b - a^3*b^3 - (3*a^5*b - 5*a^3*b^3 + 2*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)^4 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)^2)

giac [A] time = 0.27, size = 275, normalized size = 1.40

$$-\frac{\frac{4\,b^7\log(|b\sin(dx+c)+a|)}{a^7b-2\,a^5b^3+a^3b^5}+\frac{(4\,a-5\,b)\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2}+\frac{(4\,a+5\,b)\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2}-\frac{2\left(2\,a^3\sin(dx+c)^2-3\,ab^2\sin(dx+c)^2+a^2b\sin(dx+c)-b}{\left(a^4-2\,a^2b^2+b^4\right)\left(\sin(dx+c)^2-1\right.}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] -1/4*(4*b^7*log(abs(b*sin(d*x + c) + a))/(a^7*b - 2*a^5*b^3 + a^3*b^5) + (4*a - 5*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + (4*a + 5*b)*log(abs(sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) - 2*(2*a^3*sin(d*x + c)^2 - 3*a*b^2*sin(d*x + c)^2 + a^2*b*sin(d*x + c) - b^3*sin(d*x + c) - 3*a^3 + 4*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)) - 4*(2*a^2 + b^2)*log(abs(sin(d*x + c)))/a^3 + 2*(6*a^2*sin(d*x + c)^2 + 3*b^2*sin(d*x + c)^2 - 2*a*b*sin(d*x + c) + a^2)/(a^3*sin(d*x + c)^2))/d

maple [A] time = 0.54, size = 231, normalized size = 1.17

$$-\frac{1}{d\left(4a+4b\right)\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)a}{d\left(a+b\right)^{2}}-\frac{5\ln\left(\sin\left(dx+c\right)-1\right)b}{4d\left(a+b\right)^{2}}-\frac{b^{6}\ln\left(a+b\sin\left(dx+c\right)\right)}{d\,a^{3}\left(a+b\right)^{2}\left(a-b\right)^{2}}-\frac{}{2da\,\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^3*sec(d*x+c)^3/(a+b*sin(d*x+c)),x)

[Out] -1/d/(4*a+4*b)/(sin(d*x+c)-1)-1/d/(a+b)^2*ln(sin(d*x+c)-1)*a-5/4/d/(a+b)^2*ln(sin(d*x+c)-1)*b-1/d/a^3*b^6/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))-1/2/d/a/sin(d*x+c)^2+2*ln(sin(d*x+c))/a/d+b^2*ln(sin(d*x+c))/a^3/d+1/d/a^2*b/sin(d*x+c)+1/d/(4*a-4*b)/(1+sin(d*x+c))-a*ln(1+sin(d*x+c))/(a-b)^2/d+5/4*b*ln(1+sin(d*x+c))/(a-b)^2/d

maxima [A] time = 0.32, size = 244, normalized size = 1.24

$$-\frac{\frac{4\,b^6\log(b\sin(dx+c)+a)}{a^7-2\,a^5b^2+a^3b^4}+\frac{(4\,a-5\,b)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{(4\,a+5\,b)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}-\frac{2\left(\left(3\,a^2b-2\,b^3\right)\sin(dx+c)^3+a^3-ab^2-\left(2\,a^3-ab^2\right)\sin(dx+c)^2\cdots\right.}{\left(a^4-a^2b^2\right)\sin(dx+c)^4-\left(a^4-a^2b^2\right)\sin(dx\cdots}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^6*log(b*sin(d*x + c) + a)/(a^7 - 2*a^5*b^2 + a^3*b^4) + (4*a - 5*
b)*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + (4*a + 5*b)*log(sin(d*x + c)
 - 1)/(a^2 + 2*a*b + b^2) - 2*((3*a^2*b - 2*b^3)*sin(d*x + c)^3 + a^3 - a*b
^2 - (2*a^3 - a*b^2)*sin(d*x + c)^2 - 2*(a^2*b - b^3)*sin(d*x + c))/((a^4 -
 a^2*b^2)*sin(d*x + c)^4 - (a^4 - a^2*b^2)*sin(d*x + c)^2) - 4*(2*a^2 + b^2
)*log(sin(d*x + c))/a^3)/d
```

mupad [B] time = 12.48, size = 240, normalized size = 1.22

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\left(\frac{b}{4\,{\left(a-b\right)}^2}-\frac{1}{a-b}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\left(\frac{b}{4\,{\left(a+b\right)}^2}+\frac{1}{a+b}\right)}{d}-\frac{\frac{1}{2\,a}-\frac{b\,\sin(c+d\,x)}{a^2}-\frac{{\sin(c+d\,x)}^2\,\left(2\,a^2-b^2\right)}{2\,a\,\left(a^2-b^2\right)}+\cdots}{d\,\left({\sin\left(c+d\,x\right)}^2-\sin\left(c+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(b/(4*(a - b)^2) - 1/(a - b)))/d - (log(sin(c + d*x)
 - 1)*(b/(4*(a + b)^2) + 1/(a + b)))/d - (1/(2*a) - (b*sin(c + d*x))/a^2 -
(sin(c + d*x)^2*(2*a^2 - b^2))/(2*a*(a^2 - b^2)) + (b*sin(c + d*x)^3*(3*a^2
 - 2*b^2))/(2*a^2*(a^2 - b^2)))/(d*(sin(c + d*x)^2 - sin(c + d*x)^4)) + (lo
g(sin(c + d*x))*(2*a^2 + b^2))/(a^3*d) - (b^6*log(a + b*sin(c + d*x)))/(d*(
a^7 + a^3*b^4 - 2*a^5*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1352 $\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{a\tan^3(c+dx)}{3d(a^2-b^2)}-\frac{b\sec^3(c+dx)}{3d(a^2-b^2)}+\frac{a^2b\sec(c+dx)}{d(a^2-b^2)^2}+\frac{b\sec(c+dx)}{d(a^2-b^2)}+\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{5/2}}-\frac{a^3\tan(c+dx)}{d(a^2-b^2)^2}$$

```
[Out] 2*a^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d+a^
2*b*sec(d*x+c)/(a^2-b^2)^2/d+b*sec(d*x+c)/(a^2-b^2)/d-1/3*b*sec(d*x+c)^3/(a
^2-b^2)/d-a^3*tan(d*x+c)/(a^2-b^2)^2/d+1/3*a*tan(d*x+c)^3/(a^2-b^2)/d
```

Rubi [A] time = 0.23, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2727, 2607, 30, 2606, 3767, 8, 2660, 618, 204}

$$\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{5/2}}+\frac{a\tan^3(c+dx)}{3d(a^2-b^2)}-\frac{a^3\tan(c+dx)}{d(a^2-b^2)^2}-\frac{b\sec^3(c+dx)}{3d(a^2-b^2)}+\frac{a^2b\sec(c+dx)}{d(a^2-b^2)^2}+\frac{b\sec(c+dx)}{d(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) + (a^2*b*Sec[c + d*x])/((a^2 - b^2)^2*d) + (b*Sec[c + d*x])/((a^2 - b^2
)*d) - (b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) - (a^3*Tan[c + d*x])/((a^2 - b^
2)^2*d) + (a*Tan[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec(c+dx)\tan^3(c+dx)}{a^2-b^2}$$

$$= -\frac{a^3\int \sec^2(c+dx)\,dx}{\left(a^2-b^2\right)^2} + \frac{a^4\int \frac{1}{a+b\sin(c+dx)}\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b\right)\int \sec(c+dx)\tan(c+dx)\,dx}{\left(a^2-b^2\right)^2} + \cdots$$

$$= \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a\tan^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a^3\,\mathrm{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{\left(a^2-b^2\right)^2 d}$$

$$= \frac{a^2b\sec(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^3\tan(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{a\tan^3(c+dx)}{3\left(a^2-b^2\right)d}$$

$$= \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{a^2b\sec(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \cdots$$

Mathematica [A] time = 1.42, size = 195, normalized size = 1.10

$$\frac{\frac{48a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{\sec^3(c+dx)\left(8a^3\sin(3(c+dx))+3b\left(11a^2-5b^2\right)\cos(c+dx)+12b\left(b^2-2a^2\right)\cos(2(c+dx))+11a^2b\cos(3(c+dx))-16a^2b+6ab\cdots\right)}{(a-b)^2(a+b)^2}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((48*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
) - (Sec[c + d*x]^3*(-16*a^2*b + 4*b^3 + 3*b*(11*a^2 - 5*b^2)*Cos[c + d*x]
+ 12*b*(-2*a^2 + b^2)*Cos[2*(c + d*x)] + 11*a^2*b*Cos[3*(c + d*x)] - 5*b^3*
Cos[3*(c + d*x)] + 6*a*b^2*Sin[c + d*x] + 8*a^3*Sin[3*(c + d*x)] - 2*a*b^2*
Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(24*d)
```

fricas [A] time = 0.74, size = 476, normalized size = 2.69

$$\left[-\frac{3\sqrt{-a^2+b^2}\,a^4\cos(dx+c)^3\log\left(\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+\cdots}{6\left(a^6-3a\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^4*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 2*a^4*b - 4*a^2*b^3 + 2*b^5 - 6*(2*a^4*b - 3*a^2*b^3 + b^5)*co
s(d*x + c)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*a^3*b^2 + a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3), -1/3*(3*sqrt(a^2 - b^2)*a^4*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2
- b^2)*cos(d*x + c)))*cos(d*x + c)^3 + a^4*b - 2*a^2*b^3 + b^5 - 3*(2*a^4*b
 - 3*a^2*b^3 + b^5)*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*
a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*d*cos(d*x + c)^3)]
```

giac [A] time = 0.27, size = 241, normalized size = 1.36

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^4}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-10a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+1}{\left(a^4-2a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}dx\right.\right.}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*a^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
+ (3*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 10*a^3*t
an(1/2*d*x + 1/2*c)^3 + 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*b*tan(1/2*d
*x + 1/2*c)^2 - 6*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) -
 5*a^2*b + 2*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))
/d
```

maple [A] time = 0.39, size = 269, normalized size = 1.52

$$-\frac{32}{3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(32a+32b)}-\frac{16}{d(32a+32b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{a}{d(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{}{2d(a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -32/3/d/(tan(1/2*d*x+1/2*c)-1)^3/(32*a+32*b)-16/d/(32*a+32*b)/(tan(1/2*d*x+
1/2*c)-1)^2+1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a+1/2/d/(a+b)^2/(tan(1/2*d*x
+1/2*c)-1)*b-32/3/d/(tan(1/2*d*x+1/2*c)+1)^3/(32*a-32*b)+16/d/(32*a-32*b)/(
tan(1/2*d*x+1/2*c)+1)^2+1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*a-1/2/d/(a-b)^2/
(tan(1/2*d*x+1/2*c)+1)*b+2/d*a^4/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 17.43, size = 372, normalized size = 2.10

$$\frac{\frac{2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4-2\,a^2\,b^2+b^4}-\frac{2\,\left(5\,a^2\,b-2\,b^3\right)}{3\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{2\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{a^4-2\,a^2\,b^2+b^4}+\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(2\,a\,b^2-5\,a^3\right)}{3\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2\,b-b^3\right)}{a^4-2\,a^2\,b^2+b^4}-\frac{2\,a^2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{a^4-2\,a^2\,b^2+b^4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*a^3*tan(c/2 + (d*x)/2))/(a^4 + b^4 - 2*a^2*b^2) - (2*(5*a^2*b - 2*b^3))
/(3*(a^4 + b^4 - 2*a^2*b^2)) + (2*a^3*tan(c/2 + (d*x)/2)^5)/(a^4 + b^4 - 2*
a^2*b^2) + (4*tan(c/2 + (d*x)/2)^3*(2*a*b^2 - 5*a^3))/(3*(a^4 + b^4 - 2*a^2
*b^2)) + (4*tan(c/2 + (d*x)/2)^2*(2*a^2*b - b^3))/(a^4 + b^4 - 2*a^2*b^2) -
 (2*a^2*b*tan(c/2 + (d*x)/2)^4)/(a^4 + b^4 - 2*a^2*b^2))/(d*(3*tan(c/2 + (d
*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) + (2*a^4*ata
n(((a^4*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)^(5/2)) + (2*a
^5*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(5/2)
))/(2*a^4)))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1353 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=142

$$-\frac{b\tan^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a\sec^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{2a^3b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}$$

```
[Out] -2*a^3*b*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d
+1/3*a*sec(d*x+c)^3/(a^2-b^2)/d-a^2*sec(d*x+c)*(a-b*sin(d*x+c))/(a^2-b^2)^2
/d-1/3*b*tan(d*x+c)^3/(a^2-b^2)/d
```

Rubi [A] time = 0.24, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2902, 2606, 30, 2607, 2866, 12, 2660, 618, 204}

$$-\frac{2a^3b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{b\tan^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a\sec^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^3*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5
/2)*d) + (a*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]*(a - b*Si
n[c + d*x]))/((a^2 - b^2)^2*d) - (b*Tan[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
```

```
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^3(c+dx)\tan(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sec(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec^2(c+dx)\,\mathrm{t}}{a^2-}$$

$$= -\frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)^2 d} - \frac{a^2\int \frac{ab}{a+b\sin(c+dx)}\,dx}{\left(a^2-b^2\right)^2} + \frac{a\,\mathrm{Subst}\left(\int x^2\,dx, x\right.}{\left(a^2-b^2\right.}$$

$$= \frac{a\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)^2 d} - \frac{b\tan^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{\left(a^3b\right)\int}{\left(a\right.}$$

$$= \frac{a\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)^2 d} - \frac{b\tan^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{\left(2a^3b\right)\mathrm{S}}{}$$

$$= \frac{a\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)^2 d} - \frac{b\tan^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{\left(4a^3b\right)\mathrm{S}}{}$$

$$= -\frac{2a^3b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{a\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)(a-b\sin(c+a}{\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 1.45, size = 184, normalized size = 1.30

$$\frac{\frac{\sec^3(c+dx)\left(-12a^3\cos(2(c+dx))+5a^3\cos(3(c+dx))-4a^3+3a\left(5a^2+b^2\right)\cos(c+dx)+8a^2b\sin(3(c+dx))+ab^2\cos(3(c+dx))-8ab^2+6b^3\sin(c+dx)-2b^3\sin(3\right.}{(a-b)^2(a+b)^2}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-48*a^3*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(
5/2) + (Sec[c + d*x]^3*(-4*a^3 - 8*a*b^2 + 3*a*(5*a^2 + b^2)*Cos[c + d*x] -
```

```
 12*a^3*Cos[2*(c + d*x)] + 5*a^3*Cos[3*(c + d*x)] + a*b^2*Cos[3*(c + d*x)]
+ 6*b^3*Sin[c + d*x] + 8*a^2*b*Sin[3*(c + d*x)] - 2*b^3*Sin[3*(c + d*x)]))/
((a - b)^2*(a + b)^2))/(24*d)
```

fricas [A] time = 0.93, size = 465, normalized size = 3.27

$$\left[-\frac{3\sqrt{-a^2+b^2}\,a^3b\cos(dx+c)^3\log\left(-\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-}{6\left(a^6-3a^4b^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^3*b*cos(d*x + c)^3*log(-((2*a^2 - b^2)*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) +
b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
- a^2 - b^2)) - 2*a^5 + 4*a^3*b^2 - 2*a*b^4 + 6*(a^5 - a^3*b^2)*cos(d*x + c
)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5 - (4*a^4*b - 5*a^2*b^3 + b^5)*cos(d*x + c)
^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1
/3*(3*sqrt(a^2 - b^2)*a^3*b*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*c
os(d*x + c)))*cos(d*x + c)^3 + a^5 - 2*a^3*b^2 + a*b^4 - 3*(a^5 - a^3*b^2)*
cos(d*x + c)^2 - (a^4*b - 2*a^2*b^3 + b^5 - (4*a^4*b - 5*a^2*b^3 + b^5)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3)]
```

giac [A] time = 0.33, size = 227, normalized size = 1.60

$$-\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^3b}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-10a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{\left(a^4-2a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right.}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*a^3*b/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2
)) + (3*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 3*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 10*
a^2*b*tan(1/2*d*x + 1/2*c)^3 + 4*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*tan(1/2
*d*x + 1/2*c)^2 + 3*a^2*b*tan(1/2*d*x + 1/2*c) - 2*a^3 - a*b^2)/((a^4 - 2*a
^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

maple [A] time = 0.42, size = 222, normalized size = 1.56

$$-\frac{16}{3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}(16a+16b)}-\frac{8}{d\,(16a+16b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{a}{2d\,(a+b)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{}{d\,(16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -16/3/d/(tan(1/2*d*x+1/2*c)-1)^3/(16*a+16*b)-8/d/(16*a+16*b)/(tan(1/2*d*x+1
/2*c)-1)^2+1/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a-8/d/(16*a-16*b)/(tan(1/2*
d*x+1/2*c)+1)^2+16/3/d/(tan(1/2*d*x+1/2*c)+1)^3/(16*a-16*b)-1/2/d/(a-b)^2/(
tan(1/2*d*x+1/2*c)+1)*a-2/d*a^3*b/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 17.16, size = 370, normalized size = 2.61

$$\frac{\frac{2\left(2a^{3}+a\,b^{2}\right)}{3\left(a^{4}-2a^{2}b^{2}+b^{4}\right)}-\frac{4a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{a^{4}-2a^{2}b^{2}+b^{4}}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\left(5a^{2}b-2b^{3}\right)}{3\left(a^{4}-2a^{2}b^{2}+b^{4}\right)}-\frac{2a^{2}b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^{4}-2a^{2}b^{2}+b^{4}}+\frac{2a\,b^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{a^{4}-2a^{2}b^{2}+b^{4}}-\frac{2a^{2}b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{a^{4}-2a^{2}b^{2}+b^{4}}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)}-\frac{2a^{3}\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*(a*b^2 + 2*a^3))/(3*(a^4 + b^4 - 2*a^2*b^2)) - (4*a^3*tan(c/2 + (d*x)/2
)^2)/(a^4 + b^4 - 2*a^2*b^2) + (4*tan(c/2 + (d*x)/2)^3*(5*a^2*b - 2*b^3))/(
3*(a^4 + b^4 - 2*a^2*b^2)) - (2*a^2*b*tan(c/2 + (d*x)/2))/(a^4 + b^4 - 2*a^
```

```
2*b^2) + (2*a*b^2*tan(c/2 + (d*x)/2)^4)/(a^4 + b^4 - 2*a^2*b^2) - (2*a^2*b*
tan(c/2 + (d*x)/2)^5)/(a^4 + b^4 - 2*a^2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 -
 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (2*a^3*b*atan(((a^3*
b*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)^(5/2)) + (2*a^4*b*t
an(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(5/2)))/(
2*a^3*b)))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1354 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=165

$$\frac{2a^2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{d\left(a^2-b^2\right)^2}$$

```
[Out] 2*a^2*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/
d-1/3*b*sec(d*x+c)^3/(a^2-b^2)/d+a^2*sec(d*x+c)*(b-a*sin(d*x+c))/(a^2-b^2)^
2/d+a*tan(d*x+c)/(a^2-b^2)/d+1/3*a*tan(d*x+c)^3/(a^2-b^2)/d
```

Rubi [A] time = 0.24, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2902, 3767, 2606, 30, 2696, 12, 2660, 618, 204}

$$\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{2a^2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(
5/2)*d) - (b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) + (a^2*Sec[c + d*x]*(b - a*S
in[c + d*x]))/((a^2 - b^2)^2*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d) + (a*Tan
[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^4(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec^3(c+dx)\tan(c+dx)\,dx}{a^2-b^2}$$

$$= \frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)^2 d} + \frac{a^2\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{\left(a^2-b^2\right)^2} - \frac{a\,\mathrm{Subst}\left(\int \left(1+x^2\right)\right.}{\left(a^2\right.}$$

$$= -\frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)^2 d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{a\tan^3}{3\left(a^2\right.}$$

$$= -\frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)^2 d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{a\tan^3}{3\left(a^2\right.}$$

$$= -\frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)^2 d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{a\tan^3}{3\left(a^2\right.}$$

$$= \frac{2a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} - \frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a^2\sec(c+dx)(b-a\sin(c+\right.}{\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 1.28, size = 200, normalized size = 1.21

$$\frac{\frac{48a^2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{\sec^3(c+dx)\left(-6a^3\sin(c+dx)+2a^3\sin(3(c+dx))+3b\left(5a^2+b^2\right)\cos(c+dx)-12a^2b\cos(2(c+dx))+5a^2b\cos(3(c+dx))-4a\right.}{(a-b)^2(a+b)^2}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((48*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^
(5/2) - (Sec[c + d*x]^3*(-4*a^2*b - 8*b^3 + 3*b*(5*a^2 + b^2)*Cos[c + d*x]
```

```
- 12*a^2*b*Cos[2*(c + d*x)] + 5*a^2*b*Cos[3*(c + d*x)] + b^3*Cos[3*(c + d*x
)] - 6*a^3*Sin[c + d*x] + 12*a*b^2*Sin[c + d*x] + 2*a^3*Sin[3*(c + d*x)] +
4*a*b^2*Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(24*d)
```

fricas [A] time = 0.85, size = 470, normalized size = 2.85

$$\left[-\frac{3\sqrt{-a^2+b^2}\,a^2b^2\cos(dx+c)^3\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+}{6\left(a^6-3a^4b^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^2*b^2*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) +
 b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2)) + 2*a^4*b - 4*a^2*b^3 + 2*b^5 - 6*(a^4*b - a^2*b^3)*cos(d*x
+ c)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - (a^5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c
)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^3),
-1/3*(3*sqrt(a^2 - b^2)*a^2*b^2*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^
2)*cos(d*x + c)))*cos(d*x + c)^3 + a^4*b - 2*a^2*b^3 + b^5 - 3*(a^4*b - a^2
*b^3)*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4 - (a^5 + a^3*b^2 - 2*a*b^4)
*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d
*x + c)^3)]
```

giac [A] time = 0.32, size = 229, normalized size = 1.39

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2b^2}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-4a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+}{(a^4-2a^2b^2+b^4)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*a^2*b^2/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^
2)) + (3*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^4 - 4*a^
3*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b*tan(1/2
*d*x + 1/2*c)^2 + 3*a*b^2*tan(1/2*d*x + 1/2*c) - 2*a^2*b - b^3)/((a^4 - 2*a
^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

maple [A] time = 0.41, size = 224, normalized size = 1.36

$$-\frac{8}{3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}(8a+8b)}-\frac{4}{d\,(8a+8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{b}{2d\,(a+b)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{}{3d\left(\tan\left(\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -8/3/d/(tan(1/2*d*x+1/2*c)-1)^3/(8*a+8*b)-4/d/(8*a+8*b)/(tan(1/2*d*x+1/2*c)
-1)^2-1/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*b-8/3/d/(tan(1/2*d*x+1/2*c)+1)^3
/(8*a-8*b)+4/d/(8*a-8*b)/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/(a-b)^2/(tan(1/2*d*
x+1/2*c)+1)*b+2/d*a^2*b^2/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 17.40, size = 375, normalized size = 2.27

$$\frac{2\,a^2\,b^2\,\mathrm{atan}\left(\frac{\frac{a^2\,b^2\,\left(2\,a^4\,b-4\,a^2\,b^3+2\,b^5\right)}{(a+b)^{5/2}\,(a-b)^{5/2}}+\frac{2\,a^3\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{(a+b)^{5/2}\,(a-b)^{5/2}}}{2\,a^2\,b^2}\right)}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}-\frac{\frac{2\,\left(2\,a^2\,b+b^3\right)}{3\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{2\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{a^4-2\,a^2\,b^2+b^4}+\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^3+a\,b^2\right)}{3\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] (2*a^2*b^2*atan(((a^2*b^2*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a
- b)^(5/2)) + (2*a^3*b^2*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a +
b)^(5/2)*(a - b)^(5/2)))/(2*a^2*b^2)))/(d*(a + b)^(5/2)*(a - b)^(5/2)) - ((
```

```
2*(2*a^2*b + b^3))/(3*(a^4 + b^4 - 2*a^2*b^2)) + (2*b^3*tan(c/2 + (d*x)/2)^
4)/(a^4 + b^4 - 2*a^2*b^2) + (4*tan(c/2 + (d*x)/2)^3*(a*b^2 + 2*a^3))/(3*(a
^4 + b^4 - 2*a^2*b^2)) - (2*a*b^2*tan(c/2 + (d*x)/2))/(a^4 + b^4 - 2*a^2*b^
2) - (4*a^2*b*tan(c/2 + (d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2) - (2*a*b^2*tan(
c/2 + (d*x)/2)^5)/(a^4 + b^4 - 2*a^2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*t
an(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

3.1355 $\int \frac{\sec^3(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=138

$$\frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3d\left(a^2-b^2\right)}-\frac{\sec(c+dx)\left(3ab^2-b\left(a^2+2b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^2}-\frac{2ab^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}$$

```
[Out] -2*a*b^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d
+1/3*sec(d*x+c)^3*(a-b*sin(d*x+c))/(a^2-b^2)/d-1/3*sec(d*x+c)*(3*a*b^2-b*(a
^2+2*b^2)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.22, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2866, 12, 2660, 618, 204}

$$-\frac{2ab^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3d\left(a^2-b^2\right)}-\frac{\sec(c+dx)\left(3ab^2-b\left(a^2+2b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5
/2)*d) + (Sec[c + d*x]^3*(a - b*Sin[c + d*x]))/(3*(a^2 - b^2)*d) - (Sec[c +
 d*x]*(3*a*b^2 - b*(a^2 + 2*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\int \frac{\sec^2(c+dx)\left(-ab+2b^2\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\left(3ab^2-b\left(a^2+2b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\left(3ab^2-b\left(a^2+2b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\left(3ab^2-b\left(a^2+2b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\left(3ab^2-b\left(a^2+2b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{2ab^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)}{}$$

Mathematica [A] time = 1.30, size = 203, normalized size = 1.47

$$\frac{\frac{\sec^3(c+dx)\left(-\frac{1}{2}a^3\cos(3(c+dx))+4a^3-\frac{3}{2}a\left(a^2-7b^2\right)\cos(c+dx)-3a^2b\sin(c+dx)+a^2b\sin(3(c+dx))-6ab^2\cos(2(c+dx))+\frac{7}{2}ab^2\cos(3(c+dx))-10ab^2+6b^3 \text{si}}{(a-b)^2(a+b)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-24*a*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(
5/2) + (Sec[c + d*x]^3*(4*a^3 - 10*a*b^2 - (3*a*(a^2 - 7*b^2)*Cos[c + d*x])
/2 - 6*a*b^2*Cos[2*(c + d*x)] - (a^3*Cos[3*(c + d*x)])/2 + (7*a*b^2*Cos[3*(
c + d*x)])/2 - 3*a^2*b*Sin[c + d*x] + 6*b^3*Sin[c + d*x] + a^2*b*Sin[3*(c +
 d*x)] + 2*b^3*Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(12*d)
```

fricas [A] time = 0.66, size = 469, normalized size = 3.40

$$\left[-\frac{3\sqrt{-a^2+b^2}\,ab^3\cos(dx+c)^3\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-}{6\left(a^6-3a^4b^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a*b^3*cos(d*x + c)^3*log(-((2*a^2 - b^2)*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) +
b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
- a^2 - b^2)) - 2*a^5 + 4*a^3*b^2 - 2*a*b^4 + 6*(a^3*b^2 - a*b^4)*cos(d*x +
 c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)
^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1
/3*(3*sqrt(a^2 - b^2)*a*b^3*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*c
os(d*x + c)))*cos(d*x + c)^3 + a^5 - 2*a^3*b^2 + a*b^4 - 3*(a^3*b^2 - a*b^4
)*cos(d*x + c)^2 - (a^4*b - 2*a^2*b^3 + b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3)]
```

giac [A] time = 0.23, size = 240, normalized size = 1.74

$$-\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab^3}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-4a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{(a^4-2a^2b^2+b^4)\left(\tan\left(\frac{1}{2}dx\right.\right.}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*a*b^3/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2
)) + (3*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/2*c)^4 - 6*a*b^2
*tan(1/2*d*x + 1/2*c)^4 - 4*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*
x + 1/2*c)^3 + 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*b^3*tan(1/2*d*x + 1/2*c)
+ a^3 - 4*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/
d
```

maple [B] time = 0.35, size = 272, normalized size = 1.97

$$-\frac{4}{3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}(4a+4b)}-\frac{2}{d\,(4a+4b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{a}{2d\,(a+b)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{}{d\,(a+b)^{2}\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -4/3/d/(tan(1/2*d*x+1/2*c)-1)^3/(4*a+4*b)-2/d/(4*a+4*b)/(tan(1/2*d*x+1/2*c)
-1)^2-1/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a-1/d/(a+b)^2/(tan(1/2*d*x+1/2*c
)-1)*b-2/d/(4*a-4*b)/(tan(1/2*d*x+1/2*c)+1)^2+4/3/d/(tan(1/2*d*x+1/2*c)+1)^
3/(4*a-4*b)+1/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*a-1/d/(a-b)^2/(tan(1/2*d*x
+1/2*c)+1)*b-2/d*a*b^3/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 17.06, size = 378, normalized size = 2.74

$$\frac{\frac{2\left(4\,a\,b^2-a^3\right)}{3\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{2\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4-2\,a^2\,b^2+b^4}-\frac{2\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^4-2\,a^2\,b^2+b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(2\,a\,b^2-a^3\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(2\,a^2+b^2\right)}{3\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{4\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4-2\,a^2\,b^2+b^4}\quad 2}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*(4*a*b^2 - a^3))/(3*(a^4 + b^4 - 2*a^2*b^2)) - (2*b^3*tan(c/2 + (d*x)/2
))/(a^4 + b^4 - 2*a^2*b^2) - (2*b^3*tan(c/2 + (d*x)/2)^5)/(a^4 + b^4 - 2*a^
```

```
2*b^2) + (2*tan(c/2 + (d*x)/2)^4*(2*a*b^2 - a^3))/(a^4 + b^4 - 2*a^2*b^2) +
 (4*b*tan(c/2 + (d*x)/2)^3*(2*a^2 + b^2))/(3*(a^4 + b^4 - 2*a^2*b^2)) - (4*
a*b^2*tan(c/2 + (d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2))/(d*(3*tan(c/2 + (d*x)/
2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (2*a*b^3*atan(
((a*b^3*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)^(5/2)) + (2*a
^2*b^3*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(
5/2)))/(2*a*b^3)))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

3.1356 $\int \frac{\csc(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{b\sec^3(c+dx)(b-a\sin(c+dx))}{3ad\left(a^2-b^2\right)}-\frac{2b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{5/2}}-\frac{b\sec(c+dx)\left(a\left(2a^2-5b^2\right)\sin(c+dx)+3b^3\right)}{3ad\left(a^2-b^2\right)^2}+$$

```
[Out] -2*b^5*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/(a^2-b^2)^(5/2)/d
-arctanh(cos(d*x+c))/a/d+sec(d*x+c)/a/d+1/3*sec(d*x+c)^3/a/d+1/3*b*sec(d*x+
c)^3*(b-a*sin(d*x+c))/a/(a^2-b^2)/d-1/3*b*sec(d*x+c)*(3*b^3+a*(2*a^2-5*b^2)
*sin(d*x+c))/a/(a^2-b^2)^2/d
```

Rubi [A] time = 0.41, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2898, 2622, 302, 207, 2696, 2866, 12, 2660, 618, 204}

$$-\frac{2b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{5/2}}+\frac{b\sec^3(c+dx)(b-a\sin(c+dx))}{3ad\left(a^2-b^2\right)}-\frac{b\sec(c+dx)\left(a\left(2a^2-5b^2\right)\sin(c+dx)+3b^3\right)}{3ad\left(a^2-b^2\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(5
/2)*d) - ArcTanh[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/
(3*a*d) + (b*Sec[c + d*x]^3*(b - a*Sin[c + d*x]))/(3*a*(a^2 - b^2)*d) - (b*
Sec[c + d*x]*(3*b^3 + a*(2*a^2 - 5*b^2)*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
```

```
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{\csc(c+dx)\sec^4(c+dx)}{a} - \frac{b\sec^4(c+dx)}{a(a+b\sin(c+dx))}\right)dx$$

$$= \frac{\int \csc(c+dx)\sec^4(c+dx)\,dx}{a} - \frac{b\int \frac{\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{b\sec^3(c+dx)(b-a\sin(c+dx))}{3a\left(a^2-b^2\right)d} + \frac{b\int \frac{\sec^2(c+dx)\left(-2a^2+3b^2-2ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3a\left(a^2-b^2\right)} + \text{Subs}$$

$$= \frac{b\sec^3(c+dx)(b-a\sin(c+dx))}{3a\left(a^2-b^2\right)d} - \frac{b\sec(c+dx)\left(3b^3+a\left(2a^2-5b^2\right)\sin(c+dx\right.}{3a\left(a^2-b^2\right)^2 d}$$

$$= \frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} + \frac{b\sec^3(c+dx)(b-a\sin(c+dx))}{3a\left(a^2-b^2\right)d} - \frac{b\sec(c+dx)\left(\right.}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} + \frac{b\sec^3(c+dx)(b-a\sin(}{3a\left(a^2-b^2\right)d}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} + \frac{b\sec^3(c+dx)(b-a\sin(}{3a\left(a^2-b^2\right)d}$$

$$= -\frac{2b^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+d}{3ad}$$

Mathematica [A] time = 4.88, size = 334, normalized size = 1.72

$$\frac{-\frac{24b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}} + \frac{2(7a+10b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{2(10b-7a)\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c\right.\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-24*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^
(5/2)) - (12*Log[Cos[(c + d*x)/2]])/a + (12*Log[Sin[(c + d*x)/2]])/a + 1/((
```

```
a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (2*Sin[(c + d*x)/2])/((a
+ b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (2*(7*a + 10*b)*Sin[(c + d*
x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (2*Sin[(c + d*x)
/2])/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + 1/((a - b)*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^2) + (2*(-7*a + 10*b)*Sin[(c + d*x)/2])/((a
- b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(12*d)
```

fricas [A] time = 2.40, size = 680, normalized size = 3.51

$$\left[-\frac{3\sqrt{-a^2+b^2}\,b^5\cos(dx+c)^3\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*b^5*cos(d*x + c)^3*log(-((2*a^2 - b^2)*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*
cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
a^2 - b^2)) - 2*a^6 + 4*a^4*b^2 - 2*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2) - 3*(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*cos(d*x + c)^3*log(-1/2*cos(d*x + c) + 1/2) - 6*(a^6 - 3*a
^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + (2*a^5*
b - 7*a^3*b^3 + 5*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - 3*a^5*b^2 +
3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^3), 1/6*(6*sqrt(a^2 - b^2)*b^5*arctan(-(a
*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c)^3 + 2*a^6 -
 4*a^4*b^2 + 2*a^2*b^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(d*x + c)
^3*log(1/2*cos(d*x + c) + 1/2) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(
d*x + c)^3*log(-1/2*cos(d*x + c) + 1/2) + 6*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*c
os(d*x + c)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5 + (2*a^5*b - 7*a^3*b^3 + 5*a*b
^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*
cos(d*x + c)^3)]
```

giac [A] time = 0.24, size = 308, normalized size = 1.59

$$-\frac{\frac{6\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^5}{(a^5-2a^3b^2+ab^4)\sqrt{a^2-b^2}}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{2\left(3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6a^3\tan\left(\frac{1}{2}dx+\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3*(6*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*b^5/((a^5 - 2*a^3*b^2 + a*b^4)*sqrt(a^2 - b^2
)) - 3*log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(3*a^2*b*tan(1/2*d*x + 1/2*c)^5
 - 6*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*a^3*tan(1/2*d*x + 1/2*c)^4 + 9*a*b^2*ta
n(1/2*d*x + 1/2*c)^4 - 2*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 8*b^3*tan(1/2*d*x +
 1/2*c)^3 + 6*a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^2
+ 3*a^2*b*tan(1/2*d*x + 1/2*c) - 6*b^3*tan(1/2*d*x + 1/2*c) - 4*a^3 + 7*a*b
^2)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

maple [A] time = 0.49, size = 279, normalized size = 1.44

$$-\frac{1}{3d\left(a+b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{2d\left(a+b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3a}{2d\left(a+b\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2b}{d\left(a+b\right)^{2}\left(\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/3/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)^2-
3/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a-2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*b
+1/a/d*ln(tan(1/2*d*x+1/2*c))-1/2/d/(a-b)/(tan(1/2*d*x+1/2*c)+1)^2+1/3/d/(a
-b)/(tan(1/2*d*x+1/2*c)+1)^3+3/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*a-2/d/(a-
b)^2/(tan(1/2*d*x+1/2*c)+1)*b-2/d*b^5/(a-b)^2/(a+b)^2/a/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 16.20, size = 2162, normalized size = 11.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

[Out] -(a^5*((15*b^5*sin(c + d*x))/4 + (7*b^5*sin(3*c + 3*d*x))/4) - a^7*((9*b^3*sin(c + d*x))/4 + (11*b^3*sin(3*c + 3*d*x))/12) - a^3*((11*b^7*sin(c + d*x))/4 + (17*b^7*sin(3*c + 3*d*x))/12) + a^9*((b*sin(c + d*x))/2 + (b*sin(3*c + 3*d*x))/6) - a^10*(cos(c + d*x) + cos(2*c + 2*d*x)/2 + cos(3*c + 3*d*x)/3 + (3*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/4 + 5/6) + a*((3*b^9*sin(c + d*x))/4 + (5*b^9*sin(3*c + 3*d*x))/12) - a^2*((7*b^8*cos(c + d*x))/4 + (4*b^8)/3 + b^8*cos(2*c + 2*d*x) + (7*b^8*cos(3*c + 3*d*x))/12 + (15*b^8*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (5*b^8*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/4) - a^6*((33*b^4*cos(c + d*x))/4 + (13*b^4)/2 + (9*b^4*cos(2*c + 2*d*x))/2 + (11*b^4*cos(3*c + 3*d*x))/4 + (15*b^4*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (5*b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2) + a^8*((19*b^2*cos(c + d*x))/4 + (23*b^2)/6 + (5*b^2*cos(2*c + 2*d*x))/2 + (19*b^2*cos(3*c + 3*d*x))/12 + (15*b^2*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (5*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/4) + a^4*((25*b^6*cos(c + d*x))/4 + (29*b^6)/6 + (7*b^6*cos(2*c + 2*d*x))/2 + (25*b^6*cos(3*c + 3*d*x))/12 + (15*b^6*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (5*b^6*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2) + (3*b^10*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (b^10*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/4 + (b^5*atan((4*b^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - a^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 2*a*b^5*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + a^5*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 2*a^3*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 5*a^2*b^4*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 4*a^4*b^2*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/(a^11*cos(c/2 + (d*x)/2)*1i - b^11*sin(c/2 + (d*x)/2)*4i - a*b^10*cos(c/2 + (d*x)/2)*2i + a^10*b*sin(c/2 + (d*x)/2)*2i + a^3*b^8*cos(c/2 + (d*x)/2)*7i - a^5*b^6*cos(c/2 + (d*x)/2)*11i + a^7*b^4*cos(c/2 + (d*x)/2)*10i - a^9*b^2*cos(c/2 + (d*x)/2)*5i + a^2*b^9*sin(c/2 + (d*x)/2)*15i - a^4*b^7*sin(c/2 + (d*x)/2)*24i + a^6*b^5*sin(c/2 + (d*x)/2)*21i - a^8*b^3*sin(c/2 + (d*x)/2)*10i))*cos(3*c + 3*d*x)*(-(a + b)^5*(a - b)^5)^(1/2)*1i)/2 + (b^5*atan((4*b^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - a^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 2*a*b^5*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + a^5*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 2*a^3*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 5*a^2*b^4*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 4*a^4*b^2*sin(c/2 + (d*

```
x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)
)/(a^11*cos(c/2 + (d*x)/2)*1i - b^11*sin(c/2 + (d*x)/2)*4i - a*b^10*cos(c/2
 + (d*x)/2)*2i + a^10*b*sin(c/2 + (d*x)/2)*2i + a^3*b^8*cos(c/2 + (d*x)/2)*
7i - a^5*b^6*cos(c/2 + (d*x)/2)*11i + a^7*b^4*cos(c/2 + (d*x)/2)*10i - a^9*
b^2*cos(c/2 + (d*x)/2)*5i + a^2*b^9*sin(c/2 + (d*x)/2)*15i - a^4*b^7*sin(c/
2 + (d*x)/2)*24i + a^6*b^5*sin(c/2 + (d*x)/2)*21i - a^8*b^3*sin(c/2 + (d*x)
/2)*10i))*cos(c + d*x)*(-(a + b)^5*(a - b)^5)^(1/2)*3i)/2)/(a*d*((3*cos(c +
 d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b
^4 - 3*a^4*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1357 $\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=220

$$\frac{b\left(2b^2-a^2\right)\sec(c+dx)}{d\left(a^2-b^2\right)^2}+\frac{b\sec^3(c+dx)(b\sin(c+dx)-a)}{3ad\left(a^2-b^2\right)}+\frac{2b^6\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}+$$

```
[Out] 2*b^6*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/(a^2-b^2)^(5/2)/
d+b*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a/d+b*(-a^2+2*b^2)*sec(d*x+c)/(a^2
-b^2)^2/d+1/3*b*sec(d*x+c)^3*(-a+b*sin(d*x+c))/a/(a^2-b^2)/d+1/3*(6*a^4-10*
a^2*b^2+b^4)*tan(d*x+c)/a/(a^2-b^2)^2/d+1/3*tan(d*x+c)^3/a/d
```

Rubi [A] time = 0.47, antiderivative size = 247, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2898, 2622, 302, 207, 2620, 270, 2696, 2866, 12, 2660, 618, 204}

$$\frac{2b^6\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}-\frac{b^2\sec^3(c+dx)(b-a\sin(c+dx))}{3a^2d\left(a^2-b^2\right)}+\frac{b^2\sec(c+dx)\left(a\left(2a^2-5b^2\right)\sin(c+dx)+3b^3\right)}{3a^2d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^6*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(
5/2)*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d) - (b*Sec[c
 + d*x])/(a^2*d) - (b*Sec[c + d*x]^3)/(3*a^2*d) - (b^2*Sec[c + d*x]^3*(b -
a*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*x]*(3*b^3 + a*(2*a^
2 - 5*b^2)*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*d) + (2*Tan[c + d*x])/(a*d)
+ Tan[c + d*x]^3/(3*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(-\frac{b\csc(c+dx)\sec^4(c+dx)}{a^2} + \frac{\csc^2(c+dx)\sec^4(c+dx)}{a} + \frac{b^2\sec^4(c+d}{a^2(a+b\sin(c+}\right.$$

$$= \frac{\int \csc^2(c+dx)\sec^4(c+dx)\,dx}{a} - \frac{b\int \csc(c+dx)\sec^4(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{\sec^4(c}{a+b\sin(}}{a^2}$$

$$= -\frac{b^2\sec^3(c+dx)(b-a\sin(c+dx))}{3a^2\left(a^2-b^2\right)d} - \frac{b^2\int \frac{\sec^2(c+dx)\left(-2a^2+3b^2-2ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3a^2\left(a^2-b^2\right)} + $$

$$= -\frac{b^2\sec^3(c+dx)(b-a\sin(c+dx))}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sec(c+dx)\left(3b^3+a\left(2a^2-5b^2\right)\sin(c}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{b\sec(c+dx)}{a^2d} - \frac{b\sec^3(c+dx)}{3a^2d} - \frac{b^2\sec^3(c+dx)(b-a\sin(c+}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{b\sec(c+dx)}{a^2d} - \frac{b\sec^3(c+dx)}{3a^2d} - \frac{b^2\sec}{}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{b\sec(c+dx)}{a^2d} - \frac{b\sec^3(c+dx)}{3a^2d} - \frac{b^2\sec}{}$$

$$= \frac{2b^6\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{b\sec(c+}{a^2d}$$

Mathematica [B] time = 6.44, size = 450, normalized size = 2.05

$$\frac{2b^6\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(a\sin\left(\frac{1}{2}(c+dx)\right)+b\cos\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}} - \frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d} + \frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d} + \frac{}{6d(a+b)\left(c\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^6*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/
Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(5/2)*d) - Cot[(c + d*x)/2]/(2*a*d) + (b
```

```
*Log[Cos[(c + d*x)/2]])/(a^2*d) - (b*Log[Sin[(c + d*x)/2]])/(a^2*d) + 1/(12
*(a + b)*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + Sin[(c + d*x)/2]/(6*(
a + b)*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + Sin[(c + d*x)/2]/(6*(a
- b)*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - 1/(12*(a - b)*d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^2) + (10*a*Sin[(c + d*x)/2] - 13*b*Sin[(c + d*
x)/2])/(6*(a - b)^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (10*a*Sin[(c
 + d*x)/2] + 13*b*Sin[(c + d*x)/2])/(6*(a + b)^2*d*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])) + Tan[(c + d*x)/2]/(2*a*d)
```

fricas [A] time = 1.81, size = 831, normalized size = 3.78

$$\left[-\frac{3\sqrt{-a^2+b^2}\,b^6\cos(dx+c)^3\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)\sin\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*b^6*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2))*sin(d*x + c) - 2*a^7 + 4*a^5*b^2 - 2*a^3*b^4 - 3*(a^6*b - 3*a^4*
b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 3*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^3*log(-1/2*cos(d
*x + c) + 1/2)*sin(d*x + c) + 2*(8*a^7 - 22*a^5*b^2 + 17*a^3*b^4 - 3*a*b^6)
*cos(d*x + c)^4 - 2*(4*a^7 - 11*a^5*b^2 + 7*a^3*b^4)*cos(d*x + c)^2 + 2*(a^
6*b - 2*a^4*b^3 + a^2*b^5 + 3*(a^6*b - 3*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^
2)*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^3*
sin(d*x + c)), -1/6*(6*sqrt(a^2 - b^2)*b^6*arctan(-(a*sin(d*x + c) + b)/(sq
rt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c)^3*sin(d*x + c) - 2*a^7 + 4*a^5*b^
2 - 2*a^3*b^4 - 3*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^3*log(
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b
^7)*cos(d*x + c)^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 2*(8*a^7 - 2
2*a^5*b^2 + 17*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^4 - 2*(4*a^7 - 11*a^5*b^2 +
7*a^3*b^4)*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5 + 3*(a^6*b - 3*a
^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4
*b^4 - a^2*b^6)*d*cos(d*x + c)^3*sin(d*x + c))]
```

giac [A] time = 0.23, size = 357, normalized size = 1.62

$$\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^6}{\left(a^6-2a^4b^2+a^2b^4\right)\sqrt{a^2-b^2}}-\frac{6b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{3\left(2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}-\frac{4\left(6a^3\tan\left(\frac{1}{2}\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] 1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*b^6/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(a^2 - b ^2)) - 6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 3*tan(1/2*d*x + 1/2*c)/a + 3*(2*b*tan(1/2*d*x + 1/2*c) - a)/(a^2*tan(1/2*d*x + 1/2*c)) - 4*(6*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 9*b^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 1 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 12*b^3*ta n(1/2*d*x + 1/2*c)^2 + 6*a^3*tan(1/2*d*x + 1/2*c) - 9*a*b^2*tan(1/2*d*x + 1 /2*c) - 4*a^2*b + 7*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d

maple [A] time = 0.48, size = 317, normalized size = 1.44

$$-\frac{2a}{d\left(a+b\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{5b}{2d\left(a+b\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{3d\left(a+b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{2d\left(a+b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2*sec(d*x+c)^4/(a+b*sin(d*x+c)),x)

[Out] -2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a-5/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*b-1/3/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)^2+1/2/a/d*tan(1/2*d*x+1/2*c)-1/2/a/d/tan(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tan(1/2*d*x+1/2*c))-2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*a+5/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*b-1/3/d/(a-b)/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/(a-b)/(tan(1/2*d*x+1/2*c)+1)^2+2/d/a^2*b^6/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more details)Is 4*b^2-4*a^2 positive or negative?

mupad [B] time = 17.38, size = 2317, normalized size = 10.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (a*((3*b^10)/8 + (b^10*cos(2*c + 2*d*x))/2 + (b^10*cos(4*c + 4*d*x))/8) - a
^10*((7*b*sin(c + d*x))/12 + (b*sin(2*c + 2*d*x))/3 + (b*sin(3*c + 3*d*x))/
4 + (b*sin(4*c + 4*d*x))/6 + (b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*
sin(2*c + 2*d*x))/4 + (b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(4*c
 + 4*d*x))/8) - a^6*((17*b^5*sin(c + d*x))/4 + (11*b^5*sin(2*c + 2*d*x))/4
+ (9*b^5*sin(3*c + 3*d*x))/4 + (11*b^5*sin(4*c + 4*d*x))/8 + (5*b^5*log(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(2*c + 2*d*x))/2 + (5*b^5*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(4*c + 4*d*x))/4) - a^2*((5*b^9*sin(c +
 d*x))/6 + (7*b^9*sin(2*c + 2*d*x))/12 + (b^9*sin(3*c + 3*d*x))/2 + (7*b^9*
sin(4*c + 4*d*x))/24 + (5*b^9*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*si
n(2*c + 2*d*x))/4 + (5*b^9*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(4
*c + 4*d*x))/8) + a^8*((31*b^3*sin(c + d*x))/12 + (19*b^3*sin(2*c + 2*d*x))
/12 + (5*b^3*sin(3*c + 3*d*x))/4 + (19*b^3*sin(4*c + 4*d*x))/24 + (5*b^3*lo
g(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(2*c + 2*d*x))/4 + (5*b^3*log(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(4*c + 4*d*x))/8) + a^4*((37*b^7*s
in(c + d*x))/12 + (25*b^7*sin(2*c + 2*d*x))/12 + (7*b^7*sin(3*c + 3*d*x))/4
 + (25*b^7*sin(4*c + 4*d*x))/24 + (5*b^7*log(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*sin(2*c + 2*d*x))/2 + (5*b^7*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2))*sin(4*c + 4*d*x))/4) - a^7*((9*b^4)/8 + 6*b^4*cos(2*c + 2*d*x) + (23*
b^4*cos(4*c + 4*d*x))/8) + a^9*(b^2/4 + (19*b^2*cos(2*c + 2*d*x))/6 + (19*b
^2*cos(4*c + 4*d*x))/12) - a^3*((11*b^8)/8 + (8*b^8*cos(2*c + 2*d*x))/3 + (
23*b^8*cos(4*c + 4*d*x))/24) + a^5*((15*b^6)/8 + (17*b^6*cos(2*c + 2*d*x))/
3 + (59*b^6*cos(4*c + 4*d*x))/24) - a^11*((2*cos(2*c + 2*d*x))/3 + cos(4*c
+ 4*d*x)/3) + (b^11*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(2*c + 2*
d*x))/4 + (b^11*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(4*c + 4*d*x)
)/8 + (b^6*atan((4*b^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4
*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - a^6*sin(c/2 + (d*x)/2)*(b^10 - a^10
- 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 2*a*b^5*cos(c/2
+ (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^
(1/2) + a^5*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10
*a^6*b^4 + 5*a^8*b^2)^(1/2) - 2*a^3*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5
*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 5*a^2*b^4*sin(c/2 +
 (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(
1/2) + 4*a^4*b^2*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 -
 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/(a^11*cos(c/2 + (d*x)/2)*1i - b^11*sin(c/2
+ (d*x)/2)*4i - a*b^10*cos(c/2 + (d*x)/2)*2i + a^10*b*sin(c/2 + (d*x)/2)*2i
 + a^3*b^8*cos(c/2 + (d*x)/2)*7i - a^5*b^6*cos(c/2 + (d*x)/2)*11i + a^7*b^4
```

```
*cos(c/2 + (d*x)/2)*10i - a^9*b^2*cos(c/2 + (d*x)/2)*5i + a^2*b^9*sin(c/2 +
 (d*x)/2)*15i - a^4*b^7*sin(c/2 + (d*x)/2)*24i + a^6*b^5*sin(c/2 + (d*x)/2)
*21i - a^8*b^3*sin(c/2 + (d*x)/2)*10i))*sin(2*c + 2*d*x)*(-(a + b)^5*(a - b
)^5)^(1/2)*1i)/2 + (b^6*atan((4*b^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2
*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - a^6*sin(c/2 + (d*x)/2)*
(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 2*a
*b^5*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4
+ 5*a^8*b^2)^(1/2) + a^5*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10
*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 2*a^3*b^3*cos(c/2 + (d*x)/2)*(b^
10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 5*a^2*
b^4*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 +
 5*a^8*b^2)^(1/2) + 4*a^4*b^2*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 +
 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/(a^11*cos(c/2 + (d*x)/2)*1i -
b^11*sin(c/2 + (d*x)/2)*4i - a*b^10*cos(c/2 + (d*x)/2)*2i + a^10*b*sin(c/2
+ (d*x)/2)*2i + a^3*b^8*cos(c/2 + (d*x)/2)*7i - a^5*b^6*cos(c/2 + (d*x)/2)*
11i + a^7*b^4*cos(c/2 + (d*x)/2)*10i - a^9*b^2*cos(c/2 + (d*x)/2)*5i + a^2*
b^9*sin(c/2 + (d*x)/2)*15i - a^4*b^7*sin(c/2 + (d*x)/2)*24i + a^6*b^5*sin(c
/2 + (d*x)/2)*21i - a^8*b^3*sin(c/2 + (d*x)/2)*10i))*sin(4*c + 4*d*x)*(-(a
+ b)^5*(a - b)^5)^(1/2)*1i)/4)/(a^2*d*sin(c + d*x)*((3*cos(c + d*x))/4 + co
s(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1358 $\int \frac{\csc^3(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=332

$$\frac{b^2\sec^3(c+dx)}{3a^3d}+\frac{b^2\sec(c+dx)}{a^3d}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{b\tan^3(c+dx)}{3a^2d}-\frac{2b\tan(c+dx)}{a^2d}+\frac{b\cot(c+dx)}{a^2d}-\frac{2b^7\tan\ldots}{a\ldots}$$

```
[Out] -2*b^7*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/(a^2-b^2)^(5/2)
/d-5/2*arctanh(cos(d*x+c))/a/d-b^2*arctanh(cos(d*x+c))/a^3/d+b*cot(d*x+c)/a
^2/d+5/2*sec(d*x+c)/a/d+b^2*sec(d*x+c)/a^3/d+5/6*sec(d*x+c)^3/a/d+1/3*b^2*s
ec(d*x+c)^3/a^3/d-1/2*csc(d*x+c)^2*sec(d*x+c)^3/a/d+1/3*b^3*sec(d*x+c)^3*(b
-a*sin(d*x+c))/a^3/(a^2-b^2)/d-1/3*b^3*sec(d*x+c)*(3*b^3+a*(2*a^2-5*b^2)*si
n(d*x+c))/a^3/(a^2-b^2)^2/d-2*b*tan(d*x+c)/a^2/d-1/3*b*tan(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.55, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {2898, 2622, 302, 207, 2620, 270, 288, 2696, 2866, 12, 2660, 618, 204}

$$-\frac{2b^7\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{5/2}}+\frac{b^2\sec^3(c+dx)}{3a^3d}+\frac{b^2\sec(c+dx)}{a^3d}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b^3\sec^3(c+dx)(b-a\sin\ldots}{3a^3d\left(a^2-b^2\right)\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^7*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^
(5/2)*d) - (5*ArcTanh[Cos[c + d*x]])/(2*a*d) - (b^2*ArcTanh[Cos[c + d*x]])/
(a^3*d) + (b*Cot[c + d*x])/(a^2*d) + (5*Sec[c + d*x])/(2*a*d) + (b^2*Sec[c
+ d*x])/(a^3*d) + (5*Sec[c + d*x]^3)/(6*a*d) + (b^2*Sec[c + d*x]^3)/(3*a^3*
d) - (Csc[c + d*x]^2*Sec[c + d*x]^3)/(2*a*d) + (b^3*Sec[c + d*x]^3*(b - a*S
in[c + d*x]))/(3*a^3*(a^2 - b^2)*d) - (b^3*Sec[c + d*x]*(3*b^3 + a*(2*a^2 -
 5*b^2)*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*d) - (2*b*Tan[c + d*x])/(a^2*d)
 - (b*Tan[c + d*x]^3)/(3*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int\Bigg(\frac{b^2\csc(c+dx)\sec^4(c+dx)}{a^3} - \frac{b\csc^2(c+dx)\sec^4(c+dx)}{a^2} + \frac{\csc^3(c+dx)\,\mathrm{se}}{a}$$

$$= \frac{\int \csc^3(c+dx)\sec^4(c+dx)\,dx}{a} - \frac{b\int \csc^2(c+dx)\sec^4(c+dx)\,dx}{a^2} + \frac{b^2\int \csc(c}{}$$

$$= \frac{b^3\sec^3(c+dx)(b-a\sin(c+dx))}{3a^3\left(a^2-b^2\right)d} + \frac{b^3\int \frac{\sec^2(c+dx)\left(-2a^2+3b^2-2ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3a^3\left(a^2-b^2\right)} + \text{Su}$$

$$= -\frac{\csc^2(c+dx)\sec^3(c+dx)}{2ad} + \frac{b^3\sec^3(c+dx)(b-a\sin(c+dx))}{3a^3\left(a^2-b^2\right)d} - \frac{b^3\sec(c+d}{}$$

$$= \frac{b\cot(c+dx)}{a^2d} + \frac{b^2\sec(c+dx)}{a^3d} + \frac{b^2\sec^3(c+dx)}{3a^3d} - \frac{\csc^2(c+dx)\sec^3(c+dx)}{2ad} +$$

$$= -\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b\cot(c+dx)}{a^2d} + \frac{5\sec(c+dx)}{2ad} + \frac{b^2\sec(c+dx)}{a^3d} + \frac{5}{}$$

$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b\cot(c+dx)}{a^2d} + \frac{5\sec(c+}{2ad}$$

$$= -\frac{2b^7\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d} - \frac{5\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}$$

Mathematica [B] time = 6.27, size = 947, normalized size = 2.85

$$16\left(-\frac{\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)\csc(c+dx)(a+b\sin(c+dx))b^7}{8a^3\left(a^2-b^2\right)^{5/2}d(b+a\csc(c+dx))} + \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)(}{32a^2d(b+a\csc(}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] 16*((a*(13*a^2 - 19*b^2)*Csc[c + d*x]*(a + b*Sin[c + d*x]))/(96*(a^2 - b^2)
^2*d*(b + a*Csc[c + d*x])) - (b^7*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)
/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]]*Csc[c + d*x]*(a + b*Sin[c + d*x
]))/(8*a^3*(a^2 - b^2)^(5/2)*d*(b + a*Csc[c + d*x])) + (b*Cot[(c + d*x)/2]*
Csc[c + d*x]*(a + b*Sin[c + d*x]))/(32*a^2*d*(b + a*Csc[c + d*x])) - (Csc[(
c + d*x)/2]^2*Csc[c + d*x]*(a + b*Sin[c + d*x]))/(128*a*d*(b + a*Csc[c + d*
x])) + ((-5*a^2 - 2*b^2)*Csc[c + d*x]*Log[Cos[(c + d*x)/2]]*(a + b*Sin[c +
d*x]))/(32*a^3*d*(b + a*Csc[c + d*x])) + ((5*a^2 + 2*b^2)*Csc[c + d*x]*Log[
Sin[(c + d*x)/2]]*(a + b*Sin[c + d*x]))/(32*a^3*d*(b + a*Csc[c + d*x])) + (
Csc[c + d*x]*Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x]))/(128*a*d*(b + a*Csc[c
 + d*x])) + (Csc[c + d*x]*(a + b*Sin[c + d*x]))/(192*(a + b)*d*(b + a*Csc[c
 + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (Csc[c + d*x]*Sin[(c +
d*x)/2]*(a + b*Sin[c + d*x]))/(96*(a + b)*d*(b + a*Csc[c + d*x])*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^3) - (Csc[c + d*x]*Sin[(c + d*x)/2]*(a + b*Sin[
c + d*x]))/(96*(a - b)*d*(b + a*Csc[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^3) + (Csc[c + d*x]*(a + b*Sin[c + d*x]))/(192*(a - b)*d*(b + a*Csc
[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (Csc[c + d*x]*(-13*a*
Sin[(c + d*x)/2] + 16*b*Sin[(c + d*x)/2])*(a + b*Sin[c + d*x]))/(96*(a - b)
^2*d*(b + a*Csc[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (Csc[c +
 d*x]*(13*a*Sin[(c + d*x)/2] + 16*b*Sin[(c + d*x)/2])*(a + b*Sin[c + d*x]))
/(96*(a + b)^2*d*(b + a*Csc[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
) - (b*Csc[c + d*x]*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2])/(32*a^2*d*(b + a
*Csc[c + d*x])))
```

fricas [A] time = 3.10, size = 1182, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^8 - 8*a^6*b^2 + 4*a^4*b^4 - 6*(5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 -
 a^2*b^6)*cos(d*x + c)^4 + 4*(5*a^8 - 13*a^6*b^2 + 8*a^4*b^4)*cos(d*x + c)^
2 + 6*(b^7*cos(d*x + c)^5 - b^7*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(-((2*a
^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x +
c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2
*a*b*sin(d*x + c) - a^2 - b^2)) + 3*((5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 + a^2*
b^6 - 2*b^8)*cos(d*x + c)^5 - (5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 + a^2*b^6 - 2
*b^8)*cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) - 3*((5*a^8 - 13*a^6*b^2
+ 9*a^4*b^4 + a^2*b^6 - 2*b^8)*cos(d*x + c)^5 - (5*a^8 - 13*a^6*b^2 + 9*a^4
*b^4 + a^2*b^6 - 2*b^8)*cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) - 4*(a
^7*b - 2*a^5*b^3 + a^3*b^5 - (8*a^7*b - 22*a^5*b^3 + 17*a^3*b^5 - 3*a*b^7)*
cos(d*x + c)^4 + (4*a^7*b - 11*a^5*b^3 + 7*a^3*b^5)*cos(d*x + c)^2)*sin(d*x
 + c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^5 - (a^9 - 3
*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^3), -1/12*(4*a^8 - 8*a^6*b^2
```

```
 + 4*a^4*b^4 - 6*(5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 - a^2*b^6)*cos(d*x + c)^4
+ 4*(5*a^8 - 13*a^6*b^2 + 8*a^4*b^4)*cos(d*x + c)^2 - 12*(b^7*cos(d*x + c)^
5 - b^7*cos(d*x + c)^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(
a^2 - b^2)*cos(d*x + c))) + 3*((5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 + a^2*b^6 -
2*b^8)*cos(d*x + c)^5 - (5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 + a^2*b^6 - 2*b^8)*
cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) - 3*((5*a^8 - 13*a^6*b^2 + 9*a^
4*b^4 + a^2*b^6 - 2*b^8)*cos(d*x + c)^5 - (5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 +
 a^2*b^6 - 2*b^8)*cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) - 4*(a^7*b -
 2*a^5*b^3 + a^3*b^5 - (8*a^7*b - 22*a^5*b^3 + 17*a^3*b^5 - 3*a*b^7)*cos(d*
x + c)^4 + (4*a^7*b - 11*a^5*b^3 + 7*a^3*b^5)*cos(d*x + c)^2)*sin(d*x + c))
/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^5 - (a^9 - 3*a^7*b
^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^3)]
```

giac [A] time = 0.24, size = 417, normalized size = 1.26

$$-\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^7}{\left(a^7-2a^5b^2+a^3b^4\right)\sqrt{a^2-b^2}}-\frac{3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^2}-\frac{12\left(5a^2+2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{16\left(6a\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
+ 1/2*c) + b)/sqrt(a^2 - b^2)))*b^7/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 -
 b^2)) - 3*(a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/a^2 - 12*(
5*a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 16*(6*a^2*b*tan(1/2*d*x
 + 1/2*c)^5 - 9*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^3*tan(1/2*d*x + 1/2*c)^4 +
 12*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 14*b^3*
tan(1/2*d*x + 1/2*c)^3 + 12*a^3*tan(1/2*d*x + 1/2*c)^2 - 18*a*b^2*tan(1/2*d
*x + 1/2*c)^2 + 6*a^2*b*tan(1/2*d*x + 1/2*c) - 9*b^3*tan(1/2*d*x + 1/2*c) -
 7*a^3 + 10*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3)
 + 3*(30*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 4*a*b
*tan(1/2*d*x + 1/2*c) + a^2)/(a^3*tan(1/2*d*x + 1/2*c)^2))/d
```

maple [A] time = 0.55, size = 376, normalized size = 1.13

$$-\frac{5a}{2d\left(a+b\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3b}{d\left(a+b\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{3d\left(a+b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2d\left(a+b\right)\left(\tan\left(\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -5/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a-3/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*
b-1/3/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)^2
+1/8/a/d*tan(1/2*d*x+1/2*c)^2-1/2/d/a^2*tan(1/2*d*x+1/2*c)*b-1/8/a/d/tan(1/
2*d*x+1/2*c)^2+5/2/a/d*ln(tan(1/2*d*x+1/2*c))+1/d/a^3*ln(tan(1/2*d*x+1/2*c)
)*b^2+1/2/d/a^2*b/tan(1/2*d*x+1/2*c)+5/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*a
-3/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*b+1/3/d/(a-b)/(tan(1/2*d*x+1/2*c)+1)^3-
1/2/d/(a-b)/(tan(1/2*d*x+1/2*c)+1)^2-2/d/a^3*b^7/(a-b)^2/(a+b)^2/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 18.01, size = 5035, normalized size = 15.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] ((b^9*sin(c + d*x))/24 + (41*b^9*sin(3*c + 3*d*x))/48 + (23*b^9*sin(5*c + 5
*d*x))/48)/(d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4
 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (a*((19*b^8)/48
- (5*b^8*cos(c + d*x))/12 + (17*b^8*cos(2*c + 2*d*x))/12 + (5*b^8*cos(3*c +
 3*d*x))/24 + (11*b^8*cos(4*c + 4*d*x))/16 + (5*b^8*cos(5*c + 5*d*x))/24 -
(5*b^8*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/16 + (5*b^8
*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/32 + (5*b^8*l
og(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x))/32))/(d*sin(c +
 d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(
a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (a^8*((b*sin(3*c + 3*d*x))/6 - (b*sin
(c + d*x))/3 + (b*sin(5*c + 5*d*x))/6))/(d*sin(c + d*x)^2*((3*cos(c + d*x))
/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3
*a^4*b^2)) + ((3*b^10)/16 + (b^10*cos(2*c + 2*d*x))/4 + (b^10*cos(4*c + 4*d
*x))/16 + (5*b^10*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
16 - (5*b^10*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/3
```

```
2 - (5*b^10*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x))/32
)/(a*d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4
- 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - ((b^11*sin(c + d*x))/8
+ (3*b^11*sin(3*c + 3*d*x))/16 + (b^11*sin(5*c + 5*d*x))/16)/(a^2*d*sin(c +
 d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(
a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (a^6*((19*b^3*sin(3*c + 3*d*x))/24 -
(4*b^3*sin(c + d*x))/3 + (19*b^3*sin(5*c + 5*d*x))/24))/(d*sin(c + d*x)^2*(
(3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6
 + 3*a^2*b^4 - 3*a^4*b^2)) + (a^4*((25*b^5*sin(3*c + 3*d*x))/16 - (15*b^5*s
in(c + d*x))/8 + (23*b^5*sin(5*c + 5*d*x))/16))/(d*sin(c + d*x)^2*((3*cos(c
 + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2
*b^4 - 3*a^4*b^2)) - (a^2*((77*b^7*sin(3*c + 3*d*x))/48 - (23*b^7*sin(c + d
*x))/24 + (59*b^7*sin(5*c + 5*d*x))/48))/(d*sin(c + d*x)^2*((3*cos(c + d*x)
)/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 -
3*a^4*b^2)) - (a^9*((5*cos(2*c + 2*d*x))/12 - (7*cos(c + d*x))/24 + (7*cos(
3*c + 3*d*x))/48 + (5*cos(4*c + 4*d*x))/16 + (7*cos(5*c + 5*d*x))/48 - (5*c
os(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/16 + (5*log(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/32 + (5*log(sin(c/2 + (d*
x)/2)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x))/32 - 11/48))/(d*sin(c + d*x)^2*
((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^
6 + 3*a^2*b^4 - 3*a^4*b^2)) - (a^5*((7*b^4*cos(2*c + 2*d*x))/2 - b^4 - (17*
b^4*cos(c + d*x))/8 + (17*b^4*cos(3*c + 3*d*x))/16 + (5*b^4*cos(4*c + 4*d*x
))/2 + (17*b^4*cos(5*c + 5*d*x))/16 - (5*b^4*cos(c + d*x)*log(sin(c/2 + (d*
x)/2)/cos(c/2 + (d*x)/2)))/2 + (5*b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2))*cos(3*c + 3*d*x))/4 + (5*b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
))*cos(5*c + 5*d*x))/4))/(d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c +
3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (a
^3*((19*b^6*cos(2*c + 2*d*x))/6 - b^6/6 - (37*b^6*cos(c + d*x))/24 + (37*b^
6*cos(3*c + 3*d*x))/48 + 2*b^6*cos(4*c + 4*d*x) + (37*b^6*cos(5*c + 5*d*x))
/48 - (15*b^6*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 +
(15*b^6*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/16 + (
15*b^6*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x))/16))/(d
*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^
2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (a^7*((23*b^2*cos(2*c + 2*d*x
))/12 - (41*b^2)/48 - (31*b^2*cos(c + d*x))/24 + (31*b^2*cos(3*c + 3*d*x))/
48 + (23*b^2*cos(4*c + 4*d*x))/16 + (31*b^2*cos(5*c + 5*d*x))/48 - (23*b^2*
cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/16 + (23*b^2*log(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/32 + (23*b^2*log(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x))/32))/(d*sin(c + d*x)
^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 -
 b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (b^12*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*cos(3*c + 3*d*x))/(16*a^3*d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos
(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2
)) + (b^12*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x))/(16
*a^3*d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4
```

- 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (b^12*cos(c + d*x)*log(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(8*a^3*d*sin(c + d*x)^2*((3*cos(c +
 d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b
^4 - 3*a^4*b^2)) - (b^7*atan((8*b^8*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2
*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 5*a^8*sin(c/2 + (d*x)/2
)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 4
*a*b^7*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^
4 + 5*a^8*b^2)^(1/2) + 5*a^7*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8
+ 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + a^3*b^5*cos(c/2 + (d*x)/2)*(
b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 8*a^
5*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4
 + 5*a^8*b^2)^(1/2) - 17*a^4*b^4*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^
8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 18*a^6*b^2*sin(c/2 + (d*x)
/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/
(a^13*cos(c/2 + (d*x)/2)*5i - b^13*sin(c/2 + (d*x)/2)*8i - a*b^12*cos(c/2 +
 (d*x)/2)*4i + a^12*b*sin(c/2 + (d*x)/2)*10i + a^3*b^10*cos(c/2 + (d*x)/2)*
9i + a^5*b^8*cos(c/2 + (d*x)/2)*3i - a^7*b^6*cos(c/2 + (d*x)/2)*30i + a^9*b
^4*cos(c/2 + (d*x)/2)*40i - a^11*b^2*cos(c/2 + (d*x)/2)*23i + a^2*b^11*sin(
c/2 + (d*x)/2)*20i + a^4*b^9*sin(c/2 + (d*x)/2)*2i - a^6*b^7*sin(c/2 + (d*x
)/2)*58i + a^8*b^5*sin(c/2 + (d*x)/2)*80i - a^10*b^3*sin(c/2 + (d*x)/2)*46i
))*cos(c + d*x)*(-(a + b)^5*(a - b)^5)^(1/2)*1i)/(4*a^3*d*sin(c + d*x)^2*((
3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6
+ 3*a^2*b^4 - 3*a^4*b^2)) + (b^7*atan((8*b^8*sin(c/2 + (d*x)/2)*(b^10 - a^1
0 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 5*a^8*sin(c/2
+ (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^
(1/2) + 4*a*b^7*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 -
10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 5*a^7*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5
*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + a^3*b^5*cos(c/2 + (
d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/
2) - 8*a^5*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 1
0*a^6*b^4 + 5*a^8*b^2)^(1/2) - 17*a^4*b^4*sin(c/2 + (d*x)/2)*(b^10 - a^10 -
 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 18*a^6*b^2*sin(c/
2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2
)^(1/2))/(a^13*cos(c/2 + (d*x)/2)*5i - b^13*sin(c/2 + (d*x)/2)*8i - a*b^12*
cos(c/2 + (d*x)/2)*4i + a^12*b*sin(c/2 + (d*x)/2)*10i + a^3*b^10*cos(c/2 +
(d*x)/2)*9i + a^5*b^8*cos(c/2 + (d*x)/2)*3i - a^7*b^6*cos(c/2 + (d*x)/2)*30
i + a^9*b^4*cos(c/2 + (d*x)/2)*40i - a^11*b^2*cos(c/2 + (d*x)/2)*23i + a^2*
b^11*sin(c/2 + (d*x)/2)*20i + a^4*b^9*sin(c/2 + (d*x)/2)*2i - a^6*b^7*sin(c
/2 + (d*x)/2)*58i + a^8*b^5*sin(c/2 + (d*x)/2)*80i - a^10*b^3*sin(c/2 + (d*
x)/2)*46i))*cos(3*c + 3*d*x)*(-(a + b)^5*(a - b)^5)^(1/2)*1i)/(8*a^3*d*sin(
c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2
)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (b^7*atan((8*b^8*sin(c/2 + (d*x)/2
)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 5
*a^8*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4
+ 5*a^8*b^2)^(1/2) + 4*a*b^7*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 +

```
10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 5*a^7*b*cos(c/2 + (d*x)/2)*(b^
10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + a^3*b^
5*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5
*a^8*b^2)^(1/2) - 8*a^5*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 1
0*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 17*a^4*b^4*sin(c/2 + (d*x)/2)*(
b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 18*a
^6*b^2*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^
4 + 5*a^8*b^2)^(1/2))/(a^13*cos(c/2 + (d*x)/2)*5i - b^13*sin(c/2 + (d*x)/2)
*8i - a*b^12*cos(c/2 + (d*x)/2)*4i + a^12*b*sin(c/2 + (d*x)/2)*10i + a^3*b^
10*cos(c/2 + (d*x)/2)*9i + a^5*b^8*cos(c/2 + (d*x)/2)*3i - a^7*b^6*cos(c/2
+ (d*x)/2)*30i + a^9*b^4*cos(c/2 + (d*x)/2)*40i - a^11*b^2*cos(c/2 + (d*x)/
2)*23i + a^2*b^11*sin(c/2 + (d*x)/2)*20i + a^4*b^9*sin(c/2 + (d*x)/2)*2i -
a^6*b^7*sin(c/2 + (d*x)/2)*58i + a^8*b^5*sin(c/2 + (d*x)/2)*80i - a^10*b^3*
sin(c/2 + (d*x)/2)*46i))*cos(5*c + 5*d*x)*(-(a + b)^5*(a - b)^5)^(1/2)*1i)/
(8*a^3*d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^
4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1359 $\int \frac{\sin^3(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=240

$$-\frac{\left(35a^2+57ab+24b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(35a^2-57ab+24b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}-\frac{\sec^4(c+dx)(b-a\sin}{4d\left(a^2-b^2\right)}$$

```
[Out] -1/16*(35*a^2+57*a*b+24*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d+1/16*(35*a^2-57*a*b
+24*b^2)*ln(1+sin(d*x+c))/(a-b)^3/d-a^8*ln(a+b*sin(d*x+c))/b^3/(a^2-b^2)^3/
d+a*sin(d*x+c)/b^2/d-1/2*sin(d*x+c)^2/b/d-1/4*sec(d*x+c)^4*(b-a*sin(d*x+c))
/(a^2-b^2)/d+1/8*sec(d*x+c)^2*(4*b*(4*a^2-3*b^2)-a*(13*a^2-9*b^2)*sin(d*x+c
))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.62, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 12, 1647, 1629}

$$-\frac{a^8\log(a+b\sin(c+dx))}{b^3d\left(a^2-b^2\right)^3}-\frac{\left(35a^2+57ab+24b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(35a^2-57ab+24b^2\right)\log(\sin(c+dx)}{16d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((35*a^2 + 57*a*b + 24*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((35
*a^2 - 57*a*b + 24*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (a^8*Log[
a + b*Sin[c + d*x]])/(b^3*(a^2 - b^2)^3*d) + (a*Sin[c + d*x])/(b^2*d) - Sin
[c + d*x]^2/(2*b*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4*(a^2 - b^2)*
d) + (Sec[c + d*x]^2*(4*b*(4*a^2 - 3*b^2) - a*(13*a^2 - 9*b^2)*Sin[c + d*x]
))/(8*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^8}{b^8(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^8}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^8}{a^2-b^2}+\frac{3ab^8x}{a^2-b^2}-4b^6x^2-4b^4x^4-4b^2x^6}{(a+x)\left(b^2-x^2\right)^2}\,dx\right.}{4b^5 d}$$

$$= \frac{\sec^2(c+dx)\left(4b\left(4a^2-3b^2\right)-a\left(13a^2-9b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} - \frac{\sec^4(c+dx)\left(\frac{b}{a^2\ldots}\right.}{4d}$$

$$= \frac{\sec^2(c+dx)\left(4b\left(4a^2-3b^2\right)-a\left(13a^2-9b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} - \frac{\sec^4(c+dx)\left(\frac{b}{a^2\ldots}\right.}{4d}$$

$$= -\frac{\left(35a^2+57ab+24b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3 d} + \frac{\left(35a^2-57ab+24b^2\right)\log(1+\ldots}{16(a-b)^3 d}$$

Mathematica [A] time = 3.01, size = 212, normalized size = 0.88

$$\frac{-\frac{16a^8\log(a+b\sin(c+dx))}{b^3(a-b)^3(a+b)^3}-\frac{\left(35a^2+57ab+24b^2\right)\log(1-\sin(c+dx))}{(a+b)^3}+\frac{\left(35a^2-57ab+24b^2\right)\log(\sin(c+dx)+1)}{(a-b)^3}+\frac{16a\sin(c+dx)}{b^2}+\frac{13a+11b}{(a+b)^2(\sin(c+dx\ldots}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(((35*a^2 + 57*a*b + 24*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) + ((35*a^2
 - 57*a*b + 24*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 - (16*a^8*Log[a + b*Si
n[c + d*x]])/((a - b)^3*b^3*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2)
+ (13*a + 11*b)/((a + b)^2*(-1 + Sin[c + d*x])) + (16*a*Sin[c + d*x])/b^2 -
 (8*Sin[c + d*x]^2)/b - 1/((a - b)*(1 + Sin[c + d*x])^2) + (13*a - 11*b)/((
a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

fricas [A] time = 1.74, size = 429, normalized size = 1.79

$$-\frac{16\,a^8\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)+4\,a^4b^4-8\,a^2b^6+4\,b^8-8\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)\cos\left(dx+c\right)^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^8*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + 4*a^4*b^4 - 8*a^2*b^
6 + 4*b^8 - 8*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^6 - (35*
a^5*b^3 + 48*a^4*b^4 - 42*a^3*b^5 - 64*a^2*b^6 + 15*a*b^7 + 24*b^8)*cos(d*x
 + c)^4*log(sin(d*x + c) + 1) + (35*a^5*b^3 - 48*a^4*b^4 - 42*a^3*b^5 + 64*
a^2*b^6 + 15*a*b^7 - 24*b^8)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*(a^6
*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^4 - 8*(4*a^4*b^4 - 7*a^2*b
^6 + 3*b^8)*cos(d*x + c)^2 - 2*(2*a^5*b^3 - 4*a^3*b^5 + 2*a*b^7 + 8*(a^7*b
- 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c)^4 - (13*a^5*b^3 - 22*a^3*b^5
+ 9*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7
- b^9)*d*cos(d*x + c)^4)
```

giac [A] time = 0.30, size = 403, normalized size = 1.68

$$-\frac{\frac{16\,a^8\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^6b^3-3\,a^4b^5+3\,a^2b^7-b^9}-\frac{\left(35\,a^2-57\,ab+24\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(35\,a^2+57\,ab+24\,b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{8\left(b\sin\left(dx+c\right)^2-2\,a\sin\left(dx+c\right)\right)}{b^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*a^8*log(abs(b*sin(d*x + c) + a))/(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7
 - b^9) - (35*a^2 - 57*a*b + 24*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^
2*b + 3*a*b^2 - b^3) + (35*a^2 + 57*a*b + 24*b^2)*log(abs(sin(d*x + c) - 1)
)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 8*(b*sin(d*x + c)^2 - 2*a*sin(d*x + c))
/b^2 + 2*(36*a^4*b*sin(d*x + c)^4 - 48*a^2*b^3*sin(d*x + c)^4 + 18*b^5*sin(
d*x + c)^4 - 13*a^5*sin(d*x + c)^3 + 22*a^3*b^2*sin(d*x + c)^3 - 9*a*b^4*si
n(d*x + c)^3 - 56*a^4*b*sin(d*x + c)^2 + 68*a^2*b^3*sin(d*x + c)^2 - 24*b^5
*sin(d*x + c)^2 + 11*a^5*sin(d*x + c) - 18*a^3*b^2*sin(d*x + c) + 7*a*b^4*s
in(d*x + c) + 22*a^4*b - 24*a^2*b^3 + 8*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

maple [A] time = 0.49, size = 338, normalized size = 1.41

$$\frac{1}{2d\left(8a+8b\right)\left(\sin\left(dx+c\right)-1\right)^2}+\frac{13a}{16d\left(a+b\right)^2\left(\sin\left(dx+c\right)-1\right)}+\frac{11b}{16d\left(a+b\right)^2\left(\sin\left(dx+c\right)-1\right)}-\frac{35\ln\left(\sin\left(dx+c\right.\right.}{16d\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^8/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2+13/16/d/(a+b)^2/(sin(d*x+c)-1)*a+11/16/d/(
a+b)^2/(sin(d*x+c)-1)*b-35/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2-57/16/d/(a+b)^
3*ln(sin(d*x+c)-1)*a*b-3/2/d/(a+b)^3*ln(sin(d*x+c)-1)*b^2-1/2*sin(d*x+c)^2/
b/d+a*sin(d*x+c)/b^2/d-1/d/b^3*a^8/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))-1/2/d
/(8*a-8*b)/(1+sin(d*x+c))^2+13/16/d/(a-b)^2/(1+sin(d*x+c))*a-11/16/d/(a-b)^
2/(1+sin(d*x+c))*b+35/16/d/(a-b)^3*ln(1+sin(d*x+c))*a^2-57/16/d/(a-b)^3*ln(
1+sin(d*x+c))*a*b+3/2/d/(a-b)^3*ln(1+sin(d*x+c))*b^2
```

maxima [A] time = 0.38, size = 316, normalized size = 1.32

$$-\frac{\frac{16\,a^{8}\log\left(b\sin\left(dx+c\right)+a\right)}{a^{6}b^{3}-3\,a^{4}b^{5}+3\,a^{2}b^{7}-b^{9}}-\frac{\left(35\,a^{2}-57\,ab+24\,b^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}+\frac{\left(35\,a^{2}+57\,ab+24\,b^{2}\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}-\frac{2\left(\left(13\,a^{3}-9\,ab^{2}\right)\sin\left(dx+c\right)^{3}+14}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sin(}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*a^8*log(b*sin(d*x + c) + a)/(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^
9) - (35*a^2 - 57*a*b + 24*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*
b^2 - b^3) + (35*a^2 + 57*a*b + 24*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*
b + 3*a*b^2 + b^3) - 2*((13*a^3 - 9*a*b^2)*sin(d*x + c)^3 + 14*a^2*b - 10*b
^3 - 4*(4*a^2*b - 3*b^3)*sin(d*x + c)^2 - (11*a^3 - 7*a*b^2)*sin(d*x + c))/
((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 -
2*a^2*b^2 + b^4)*sin(d*x + c)^2) + 8*(b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/
b^2)/d
```

mupad [B] time = 14.19, size = 806, normalized size = 3.36

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{b^2}{4\,{\left(a-b\right)}^3}+\frac{13\,b}{8\,{\left(a-b\right)}^2}+\frac{35}{8\,\left(a-b\right)}\right)}{d}-\frac{\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(-3\,a^4+a^2\,b^2+b^4\right)}{b\,{\left(a^2-b^2\right)}^2}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^4-5\,a^2\,b^2+3\,b^4\right)}{b\,{\left(a^2-b^2\right)}^2}+\tan\left(\frac{c}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*(b^2/(4*(a - b)^3) + (13*b)/(8*(a - b)^2) + 35
/(8*(a - b))))/d - ((4*tan(c/2 + (d*x)/2)^6*(b^4 - 3*a^4 + a^2*b^2))/(b*(a^
2 - b^2)^2) - (2*tan(c/2 + (d*x)/2)^2*(a^4 + 3*b^4 - 5*a^2*b^2))/(b*(a^2 -
b^2)^2) + (tan(c/2 + (d*x)/2)^5*(8*a^5 - 11*a*b^4 + 7*a^3*b^2))/(2*b^2*(a^4
```

```
 + b^4 - 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^7*(8*a^5 - 11*a*b^4 + 7*a^3*b^2)
)/(2*b^2*(a^4 + b^4 - 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^11*(15*a*b^4 + 8*a^
5 - 27*a^3*b^2))/(4*b^2*(a^4 + b^4 - 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^3*(2
5*a*b^4 + 24*a^5 - 45*a^3*b^2))/(4*b^2*(a^4 + b^4 - 2*a^2*b^2)) - (tan(c/2
+ (d*x)/2)^9*(25*a*b^4 + 24*a^5 - 45*a^3*b^2))/(4*b^2*(a^4 + b^4 - 2*a^2*b^
2)) + (4*tan(c/2 + (d*x)/2)^4*(2*a^2 - 3*b^2))/(b*(a^2 - b^2)) + (4*tan(c/2
 + (d*x)/2)^8*(2*a^2 - 3*b^2))/(b*(a^2 - b^2)) - (2*tan(c/2 + (d*x)/2)^10*(
a^4 + 3*b^4 - 5*a^2*b^2))/(b*(a^4 + b^4 - 2*a^2*b^2)) + (a*tan(c/2 + (d*x)/
2)*(8*a^4 + 15*b^4 - 27*a^2*b^2))/(4*b^2*(a^4 + b^4 - 2*a^2*b^2)))/(d*(2*ta
n(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/
2 + (d*x)/2)^8 + 2*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^12 - 1)) - (l
og(tan(c/2 + (d*x)/2) - 1)*(35/(8*(a + b)) - (13*b)/(8*(a + b)^2) + b^2/(4*
(a + b)^3)))/d + (log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 + 3*b^2))/(b^3*d) + (a
^8*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(b^9 - 3*a^
2*b^7 + 3*a^4*b^5 - a^6*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**8/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1360 $\int \frac{\sin^2(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=221

$$-\frac{(24a^2+37ab+15b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{(24a^2-37ab+15b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{\sec^4(c+dx)(a-b\sin}{4d(a^2-b^2)}$$

```
[Out] -1/16*(24*a^2+37*a*b+15*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d-1/16*(24*a^2-37*a*b
+15*b^2)*ln(1+sin(d*x+c))/(a-b)^3/d+a^7*ln(a+b*sin(d*x+c))/b^2/(a^2-b^2)^3/
d-sin(d*x+c)/b/d+1/4*sec(d*x+c)^4*(a-b*sin(d*x+c))/(a^2-b^2)/d-1/8*sec(d*x+
c)^2*(4*a*(3*a^2-2*b^2)-b*(13*a^2-9*b^2)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.54, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 12, 1647, 1629}

$$\frac{a^7\log(a+b\sin(c+dx))}{b^2d(a^2-b^2)^3}-\frac{(24a^2+37ab+15b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{(24a^2-37ab+15b^2)\log(\sin(c+dx)+1}{16d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((24*a^2 + 37*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((24
*a^2 - 37*a*b + 15*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a^7*Log[
a + b*Sin[c + d*x]])/(b^2*(a^2 - b^2)^3*d) - Sin[c + d*x]/(b*d) + (Sec[c +
d*x]^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*d) - (Sec[c + d*x]^2*(4*a*(3*a^
2 - 2*b^2) - b*(13*a^2 - 9*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^7}{b^7(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^7}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{b^2 d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^8}{a^2-b^2}-\frac{b^6\left(4a^2-b^2\right)x}{a^2-b^2}-4b^4x^3-4b^2x^5}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\,\text{si}\right.}{4b^4 d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4a\left(3a^2-2b^2\right)-b\left(13a^2-9b^2\right)\right.}{8\left(a^2-b^2\right)^2 d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4a\left(3a^2-2b^2\right)-b\left(13a^2-9b^2\right)\right.}{8\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(24a^2+37ab+15b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3 d} - \frac{\left(24a^2-37ab+15b^2\right)\log(1+\text{si}}{16(a-b)^3 d}$$

Mathematica [A] time = 2.50, size = 198, normalized size = 0.90

$$\frac{\frac{16a^7\log(a+b\sin(c+dx))}{b^2(a-b)^3(a+b)^3}-\frac{\left(24a^2+37ab+15b^2\right)\log(1-\sin(c+dx))}{(a+b)^3}-\frac{\left(24a^2-37ab+15b^2\right)\log(\sin(c+dx)+1)}{(a-b)^3}+\frac{11a+9b}{(a+b)^2(\sin(c+dx)-1)}+\frac{9b-1}{(a-b)^2(\sin(}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(((24*a^2 + 37*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) - ((24*a^2
 - 37*a*b + 15*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 + (16*a^7*Log[a + b*Si
n[c + d*x]])/((a - b)^3*b^2*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2)
+ (11*a + 9*b)/((a + b)^2*(-1 + Sin[c + d*x])) - (16*Sin[c + d*x])/b + 1/((
a - b)*(1 + Sin[c + d*x])^2) + (-11*a + 9*b)/((a - b)^2*(1 + Sin[c + d*x]))
)/(16*d)
```

fricas [A] time = 1.62, size = 351, normalized size = 1.59

$$16\,a^7\cos(dx+c)^4\log(b\sin(dx+c)+a)+4\,a^5b^2-8\,a^3b^4+4\,ab^6-\Big(24\,a^5b^2+35\,a^4b^3-24\,a^3b^4-42\,a^2b^5+8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^7*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + 4*a^5*b^2 - 8*a^3*b^4
 + 4*a*b^6 - (24*a^5*b^2 + 35*a^4*b^3 - 24*a^3*b^4 - 42*a^2*b^5 + 8*a*b^6 +
 15*b^7)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (24*a^5*b^2 - 35*a^4*b^3 -
24*a^3*b^4 + 42*a^2*b^5 + 8*a*b^6 - 15*b^7)*cos(d*x + c)^4*log(-sin(d*x + c
) + 1) - 8*(3*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^2 - 2*(2*a^4*b^3
- 4*a^2*b^5 + 2*b^7 + 8*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^
4 - (13*a^4*b^3 - 22*a^2*b^5 + 9*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^6*b
^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^4)
```

giac [A] time = 0.31, size = 384, normalized size = 1.74

$$\frac{16\,a^7\log(|b\sin(dx+c)+a|)}{a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8}-\frac{\left(24\,a^2-37\,ab+15\,b^2\right)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(24\,a^2+37\,ab+15\,b^2\right)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{16\,\sin(dx+c)}{b}+\frac{2\left(18\,a^5\sin(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a^7*log(abs(b*sin(d*x + c) + a))/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
- b^8) - (24*a^2 - 37*a*b + 15*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2
```

```
*b + 3*a*b^2 - b^3) - (24*a^2 + 37*a*b + 15*b^2)*log(abs(sin(d*x + c) - 1))
/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 16*sin(d*x + c)/b + 2*(18*a^5*sin(d*x +
c)^4 - 18*a^3*b^2*sin(d*x + c)^4 + 6*a*b^4*sin(d*x + c)^4 - 13*a^4*b*sin(d*
x + c)^3 + 22*a^2*b^3*sin(d*x + c)^3 - 9*b^5*sin(d*x + c)^3 - 24*a^5*sin(d*
x + c)^2 + 16*a^3*b^2*sin(d*x + c)^2 - 4*a*b^4*sin(d*x + c)^2 + 11*a^4*b*si
n(d*x + c) - 18*a^2*b^3*sin(d*x + c) + 7*b^5*sin(d*x + c) + 8*a^5 - 2*a^3*b
^2)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

maple [A] time = 0.49, size = 321, normalized size = 1.45

$$\frac{1}{2d\left(8a+8b\right)\left(\sin\left(dx+c\right)-1\right)^{2}}+\frac{11a}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}+\frac{9b}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{3\ln\left(\sin\left(dx+c\right)\right.}{2d\left(a+b\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^7/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2+11/16/d/(a+b)^2/(sin(d*x+c)-1)*a+9/16/d/(a
+b)^2/(sin(d*x+c)-1)*b-3/2/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2-37/16/d/(a+b)^3*l
n(sin(d*x+c)-1)*a*b-15/16/d/(a+b)^3*ln(sin(d*x+c)-1)*b^2-sin(d*x+c)/b/d+1/d
/b^2*a^7/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/d/(8*a-8*b)/(1+sin(d*x+c))^
2-11/16/d/(a-b)^2/(1+sin(d*x+c))*a+9/16/d/(a-b)^2/(1+sin(d*x+c))*b-3/2/d/(a
-b)^3*ln(1+sin(d*x+c))*a^2+37/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b-15/16/d/(a-
b)^3*ln(1+sin(d*x+c))*b^2
```

maxima [A] time = 0.33, size = 303, normalized size = 1.37

$$\frac{\frac{16\,a^{7}\log\left(b\sin\left(dx+c\right)+a\right)}{a^{6}b^{2}-3\,a^{4}b^{4}+3\,a^{2}b^{6}-b^{8}}-\frac{\left(24\,a^{2}-37\,ab+15\,b^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{\left(24\,a^{2}+37\,ab+15\,b^{2}\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}-\frac{2\left(\left(13\,a^{2}b-9\,b^{3}\right)\sin\left(dx+c\right)^{3}+10\,a^{3}-\cdots\right.}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sin\left(dx+c\right.}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^7*log(b*sin(d*x + c) + a)/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8
) - (24*a^2 - 37*a*b + 15*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - (24*a^2 + 37*a*b + 15*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3) - 2*((13*a^2*b - 9*b^3)*sin(d*x + c)^3 + 10*a^3 - 6*a*b^2
 - 4*(3*a^3 - 2*a*b^2)*sin(d*x + c)^2 - (11*a^2*b - 7*b^3)*sin(d*x + c))/((
a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*
a^2*b^2 + b^4)*sin(d*x + c)^2) - 16*sin(d*x + c)/b)/d
```

mupad [B] time = 14.07, size = 627, normalized size = 2.84

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\left(\frac{b^2}{4\,(a-b)^3}+\frac{11\,b}{8\,(a-b)^2}+\frac{3}{a-b}\right)}{d}-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(3\,a\,b^2-4\,a^3\right)}{\left(a^2-b^2\right)^2}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(a\,b^2-2\,a^3\right)}{\left(a^2-b^2\right)^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(3\,a\right.}{\left(a^2-b^2\right)^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] - (log(tan(c/2 + (d*x)/2) + 1)*(b^2/(4*(a - b)^3) + (11*b)/(8*(a - b)^2) +
3/(a - b)))/d - ((2*tan(c/2 + (d*x)/2)^4*(3*a*b^2 - 4*a^3))/(a^2 - b^2)^2 -
 (2*tan(c/2 + (d*x)/2)^2*(a*b^2 - 2*a^3))/(a^2 - b^2)^2 + (2*tan(c/2 + (d*x
)/2)^6*(3*a*b^2 - 4*a^3))/(a^2 - b^2)^2 - (2*tan(c/2 + (d*x)/2)^8*(a*b^2 -
2*a^3))/(a^4 + b^4 - 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^5*(24*a^4 + 9*b^4 - 2
9*a^2*b^2))/(2*b*(a^2 - b^2)^2) - (2*tan(c/2 + (d*x)/2)^3*(4*a^2 - 5*b^2))/
(b*(a^2 - b^2)) - (2*tan(c/2 + (d*x)/2)^7*(4*a^2 - 5*b^2))/(b*(a^2 - b^2))
+ (tan(c/2 + (d*x)/2)*(8*a^4 + 15*b^4 - 27*a^2*b^2))/(4*b*(a^4 + b^4 - 2*a^
2*b^2)) + (tan(c/2 + (d*x)/2)^9*(8*a^4 + 15*b^4 - 27*a^2*b^2))/(4*b*(a^4 +
b^4 - 2*a^2*b^2)))/(d*(2*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^2 + 2*
tan(c/2 + (d*x)/2)^6 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1))
 - (log(tan(c/2 + (d*x)/2) - 1)*(3/(a + b) - (11*b)/(8*(a + b)^2) + b^2/(4*
(a + b)^3)))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/(b^2*d) - (a^7*log(a + 2
*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(b^8 - 3*a^2*b^6 + 3*a^
4*b^4 - a^6*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**7/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1361 $\int \frac{\sin(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=208

$$-\frac{\left(15a^2+21ab+8b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(15a^2-21ab+8b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}-\frac{\sec^4(c+dx)(b-a\sin(c+}{4d\left(a^2-b^2\right)}$$

```
[Out] -1/16*(15*a^2+21*a*b+8*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d+1/16*(15*a^2-21*a*b+
8*b^2)*ln(1+sin(d*x+c))/(a-b)^3/d-a^6*ln(a+b*sin(d*x+c))/b/(a^2-b^2)^3/d-1/
4*sec(d*x+c)^4*(b-a*sin(d*x+c))/(a^2-b^2)/d+1/8*sec(d*x+c)^2*(4*b*(3*a^2-2*
b^2)-a*(9*a^2-5*b^2)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.51, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2837, 12, 1647, 1629}

$$-\frac{a^6\log(a+b\sin(c+dx))}{bd\left(a^2-b^2\right)^3}-\frac{\left(15a^2+21ab+8b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(15a^2-21ab+8b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((15*a^2 + 21*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((15*
a^2 - 21*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (a^6*Log[a
+ b*Sin[c + d*x]])/(b*(a^2 - b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x
]))/(4*(a^2 - b^2)*d) + (Sec[c + d*x]^2*(4*b*(3*a^2 - 2*b^2) - a*(9*a^2 - 5
*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^6}{b^6(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{bd}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^6}{a^2-b^2}+\frac{3ab^6x}{a^2-b^2}-4b^4x^2-4b^2x^4}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin\right.}{4b^3d}$$

$$= \frac{\sec^2(c+dx)\left(4b\left(3a^2-2b^2\right)-a\left(9a^2-5b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2d} - \frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}\right.}{4d}$$

$$= \frac{\sec^2(c+dx)\left(4b\left(3a^2-2b^2\right)-a\left(9a^2-5b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2d} - \frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}\right.}{4d}$$

$$= -\frac{\left(15a^2+21ab+8b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3d} + \frac{\left(15a^2-21ab+8b^2\right)\log(1+\sin}{16(a-b)^3d}$$

Mathematica [A] time = 1.75, size = 187, normalized size = 0.90

$$-\frac{\frac{16a^6\log(a+b\sin(c+dx))}{b(a-b)^3(a+b)^3}-\frac{(15a^2+21ab+8b^2)\log(1-\sin(c+dx))}{(a+b)^3}+\frac{(15a^2-21ab+8b^2)\log(\sin(c+dx)+1)}{(a-b)^3}+\frac{9a+7b}{(a+b)^2(\sin(c+dx)-1)}+\frac{9a-7b}{(a-b)^2(\sin(c+}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(((15*a^2 + 21*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) + ((15*a^2
- 21*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 - (16*a^6*Log[a + b*Sin[
c + d*x]])/((a - b)^3*b*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (9
*a + 7*b)/((a + b)^2*(-1 + Sin[c + d*x])) - 1/((a - b)*(1 + Sin[c + d*x])^2
) + (9*a - 7*b)/((a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

fricas [A] time = 1.04, size = 303, normalized size = 1.46

$$-\frac{16\,a^6\cos(dx+c)^4\log(b\sin(dx+c)+a)+4\,a^4b^2-8\,a^2b^4+4\,b^6-\left(15\,a^5b+24\,a^4b^2-10\,a^3b^3-24\,a^2b^4+3\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^6*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + 4*a^4*b^2 - 8*a^2*b^
4 + 4*b^6 - (15*a^5*b + 24*a^4*b^2 - 10*a^3*b^3 - 24*a^2*b^4 + 3*a*b^5 + 8*
b^6)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (15*a^5*b - 24*a^4*b^2 - 10*a^3
*b^3 + 24*a^2*b^4 + 3*a*b^5 - 8*b^6)*cos(d*x + c)^4*log(-sin(d*x + c) + 1)
- 8*(3*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 - 2*(2*a^5*b - 4*a^3*b^3
 + 2*a*b^5 - (9*a^5*b - 14*a^3*b^3 + 5*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))
/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)^4)
```

giac [A] time = 0.29, size = 371, normalized size = 1.78

$$-\frac{\frac{16\,a^6\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{(15\,a^2-21\,ab+8\,b^2)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{(15\,a^2+21\,ab+8\,b^2)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(18\,a^4b\sin(dx+c)^4-18\,a^2b^3\sin(\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*a^6*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
 b^7) - (15*a^2 - 21*a*b + 8*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b
 + 3*a*b^2 - b^3) + (15*a^2 + 21*a*b + 8*b^2)*log(abs(sin(d*x + c) - 1))/(a
```

```
^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(18*a^4*b*sin(d*x + c)^4 - 18*a^2*b^3*sin
(d*x + c)^4 + 6*b^5*sin(d*x + c)^4 - 9*a^5*sin(d*x + c)^3 + 14*a^3*b^2*sin(
d*x + c)^3 - 5*a*b^4*sin(d*x + c)^3 - 24*a^4*b*sin(d*x + c)^2 + 16*a^2*b^3*
sin(d*x + c)^2 - 4*b^5*sin(d*x + c)^2 + 7*a^5*sin(d*x + c) - 10*a^3*b^2*sin
(d*x + c) + 3*a*b^4*sin(d*x + c) + 8*a^4*b - 2*a^2*b^3)/((a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

maple [A] time = 0.48, size = 308, normalized size = 1.48

$$\frac{1}{2d\left(8a+8b\right)\left(\sin\left(dx+c\right)-1\right)^{2}}+\frac{9a}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}+\frac{7b}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{15\ln\left(\sin\left(dx+\right.\right.}{16d\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2+9/16/d/(a+b)^2/(sin(d*x+c)-1)*a+7/16/d/(a+
b)^2/(sin(d*x+c)-1)*b-15/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2-21/16/d/(a+b)^3*
ln(sin(d*x+c)-1)*a*b-1/2/d/(a+b)^3*ln(sin(d*x+c)-1)*b^2-1/d*a^6/(a+b)^3/(a-
b)^3/b*ln(a+b*sin(d*x+c))-1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2+9/16/d/(a-b)^2/(
1+sin(d*x+c))*a-7/16/d/(a-b)^2/(1+sin(d*x+c))*b+15/16/d/(a-b)^3*ln(1+sin(d*
x+c))*a^2-21/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b+1/2/d/(a-b)^3*ln(1+sin(d*x+c
))*b^2
```

maxima [A] time = 0.34, size = 289, normalized size = 1.39

$$-\frac{\frac{16\,a^{6}\log\left(b\sin\left(dx+c\right)+a\right)}{a^{6}b-3\,a^{4}b^{3}+3\,a^{2}b^{5}-b^{7}}-\frac{\left(15\,a^{2}-21\,ab+8\,b^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}+\frac{\left(15\,a^{2}+21\,ab+8\,b^{2}\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}-\frac{2\left(\left(9\,a^{3}-5\,ab^{2}\right)\sin\left(dx+c\right)^{3}+10\,a^{2}b\cdots\right.}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sin(dx+c\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*a^6*log(b*sin(d*x + c) + a)/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)
 - (15*a^2 - 21*a*b + 8*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2
 - b^3) + (15*a^2 + 21*a*b + 8*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b +
3*a*b^2 + b^3) - 2*((9*a^3 - 5*a*b^2)*sin(d*x + c)^3 + 10*a^2*b - 6*b^3 - 4
*(3*a^2*b - 2*b^3)*sin(d*x + c)^2 - (7*a^3 - 3*a*b^2)*sin(d*x + c))/((a^4 -
 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b
^2 + b^4)*sin(d*x + c)^2))/d
```

mupad [B] time = 13.38, size = 549, normalized size = 2.64

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{b^2}{4\,{\left(a-b\right)}^3}+\frac{9\,b}{8\,{\left(a-b\right)}^2}+\frac{15}{8\,\left(a-b\right)}\right)}{d}-\frac{\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(11\,a\,b^2-15\,a^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(3\,a\,b^2-7\,a^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(11\cdots\right.}{4\,\left(a^4-2\,a^2\,b\cdots\right.}}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*(b^2/(4*(a - b)^3) + (9*b)/(8*(a - b)^2) + 15/
(8*(a - b))))/d - ((tan(c/2 + (d*x)/2)^3*(11*a*b^2 - 15*a^3))/(4*(a^4 + b^4
 - 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^7*(3*a*b^2 - 7*a^3))/(4*(a^4 + b^4 - 2
*a^2*b^2)) + (tan(c/2 + (d*x)/2)^5*(11*a*b^2 - 15*a^3))/(4*(a^4 + b^4 - 2*a
^2*b^2)) - (2*tan(c/2 + (d*x)/2)^2*(2*a^2*b - b^3))/(a^4 + b^4 - 2*a^2*b^2)
 + (4*tan(c/2 + (d*x)/2)^4*(3*a^2*b - 2*b^3))/(a^4 + b^4 - 2*a^2*b^2) - (2*
tan(c/2 + (d*x)/2)^6*(2*a^2*b - b^3))/(a^4 + b^4 - 2*a^2*b^2) + (a*tan(c/2
+ (d*x)/2)*(7*a^2 - 3*b^2))/(4*(a^4 + b^4 - 2*a^2*b^2)))/(d*(6*tan(c/2 + (d
*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x
)/2)^8 + 1)) + log(tan(c/2 + (d*x)/2)^2 + 1)/(b*d) - (log(tan(c/2 + (d*x)/2
) - 1)*(15/(8*(a + b)) - (9*b)/(8*(a + b)^2) + b^2/(4*(a + b)^3)))/d - (a^6
*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^6*b - b^7
+ 3*a^2*b^5 - 3*a^4*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1362 $\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=204

$$-\frac{(8a^2+9ab+3b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{(8a^2-9ab+3b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{\sec^4(c+dx)(a-b\sin(c+dx$$

```
[Out] -1/16*(8*a^2+9*a*b+3*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d-1/16*(8*a^2-9*a*b+3*b^
2)*ln(1+sin(d*x+c))/(a-b)^3/d+a^5*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d+1/4*sec(
d*x+c)^4*(a-b*sin(d*x+c))/(a^2-b^2)/d-1/8*sec(d*x+c)^2*(4*a*(2*a^2-b^2)-b*(
9*a^2-5*b^2)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.35, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 801}

$$\frac{a^5\log(a+b\sin(c+dx))}{d(a^2-b^2)^3}-\frac{(8a^2+9ab+3b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{(8a^2-9ab+3b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\sec$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((8*a^2
 - 9*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a^5*Log[a + b*
Sin[c + d*x]])/((a^2 - b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4
*(a^2 - b^2)*d) - (Sec[c + d*x]^2*(4*a*(2*a^2 - b^2) - b*(9*a^2 - 5*b^2)*Si
n[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
```

```
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^6}{a^2-b^2}-\frac{b^4\left(4a^2-b^2\right)x}{a^2-b^2}-4b^2x^3}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4b^2d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4a\left(2a^2-b^2\right)-b\left(9a^2-5b^2\right)\sin(c+dx\right.}{8\left(a^2-b^2\right)^2 d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4a\left(2a^2-b^2\right)-b\left(9a^2-5b^2\right)\sin(c+dx\right.}{8\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(8a^2+9ab+3b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3d} - \frac{\left(8a^2-9ab+3b^2\right)\log(1+\sin(c+dx))}{16(a-b)^3d} + \frac{a^5}{\;}$$

Mathematica [A] time = 1.40, size = 184, normalized size = 0.90

$$\frac{\frac{16a^5\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} - \frac{\left(8a^2+9ab+3b^2\right)\log(1-\sin(c+dx))}{(a+b)^3} - \frac{\left(8a^2-9ab+3b^2\right)\log(\sin(c+dx)+1)}{(a-b)^3} + \frac{7a+5b}{(a+b)^2(\sin(c+dx)-1)} + \frac{5b-7a}{(a-b)^2(\sin(c+dx)+1)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) - ((8*a^2 - 9
*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 + (16*a^5*Log[a + b*Sin[c +
d*x]])/((a - b)^3*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (7*a + 5
*b)/((a + b)^2*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])^2) + (-
7*a + 5*b)/((a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

fricas [A] time = 0.95, size = 261, normalized size = 1.28

$$\frac{16\,a^{5}\cos\left(dx+c\right)^{4}\log\left(b\sin\left(dx+c\right)+a\right)-\left(8\,a^{5}+15\,a^{4}b-10\,a^{2}b^{3}+3\,b^{5}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^5*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (8*a^5 + 15*a^4*b - 1
0*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^5 - 15*a^4*b
 + 10*a^2*b^3 - 3*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^5 - 8*a^
3*b^2 + 4*a*b^4 - 8*(2*a^5 - 3*a^3*b^2 + a*b^4)*cos(d*x + c)^2 - 2*(2*a^4*b
 - 4*a^2*b^3 + 2*b^5 - (9*a^4*b - 14*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*sin(d
*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

giac [A] time = 0.31, size = 343, normalized size = 1.68

$$\frac{\frac{16\,a^{5}b\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{6}b-3\,a^{4}b^{3}+3\,a^{2}b^{5}-b^{7}}-\frac{\left(8\,a^{2}-9\,ab+3\,b^{2}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{\left(8\,a^{2}+9\,ab+3\,b^{2}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\frac{2\left(6\,a^{5}\sin\left(dx+c\right)^{4}-9\,a^{4}b\sin\left(dx+c\right)^{3}+}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a^5*b*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5
- b^7) - (8*a^2 - 9*a*b + 3*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3) - (8*a^2 + 9*a*b + 3*b^2)*log(abs(sin(d*x + c) - 1))/(a^3
+ 3*a^2*b + 3*a*b^2 + b^3) + 2*(6*a^5*sin(d*x + c)^4 - 9*a^4*b*sin(d*x + c)
^3 + 14*a^2*b^3*sin(d*x + c)^3 - 5*b^5*sin(d*x + c)^3 - 4*a^5*sin(d*x + c)^
2 - 12*a^3*b^2*sin(d*x + c)^2 + 4*a*b^4*sin(d*x + c)^2 + 7*a^4*b*sin(d*x +
c) - 10*a^2*b^3*sin(d*x + c) + 3*b^5*sin(d*x + c) + 8*a^3*b^2 - 2*a*b^4)/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

maple [A] time = 0.45, size = 304, normalized size = 1.49

$$\frac{1}{2d\left(8a+8b\right)\left(\sin\left(dx+c\right)-1\right)^{2}}+\frac{7a}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}+\frac{5b}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)\right.}{2d\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2+7/16/d/(a+b)^2/(sin(d*x+c)-1)*a+5/16/d/(a+
b)^2/(sin(d*x+c)-1)*b-1/2/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2-9/16/d/(a+b)^3*ln(
sin(d*x+c)-1)*a*b-3/16/d/(a+b)^3*ln(sin(d*x+c)-1)*b^2+1/d*a^5/(a+b)^3/(a-b)
^3*ln(a+b*sin(d*x+c))+1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2-7/16/d/(a-b)^2/(1+si
n(d*x+c))*a+5/16/d/(a-b)^2/(1+sin(d*x+c))*b-1/2/d/(a-b)^3*ln(1+sin(d*x+c))*
a^2+9/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b-3/16/d/(a-b)^3*ln(1+sin(d*x+c))*b^2
```

maxima [A] time = 0.33, size = 288, normalized size = 1.41

$$\frac{\frac{16\,a^5\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\left(8\,a^2-9\,ab+3\,b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(8\,a^2+9\,ab+3\,b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(\left(9\,a^2b-5\,b^3\right)\sin(dx+c)^3+6\,a^3-2\,ab^2-4\,(2\cdots}{\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+c)^4+a^4-2\,a\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^5*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
(8*a^2 - 9*a*b + 3*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^
3) - (8*a^2 + 9*a*b + 3*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2
 + b^3) - 2*((9*a^2*b - 5*b^3)*sin(d*x + c)^3 + 6*a^3 - 2*a*b^2 - 4*(2*a^3
- a*b^2)*sin(d*x + c)^2 - (7*a^2*b - 3*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2
 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*
sin(d*x + c)^2))/d
```

mupad [B] time = 12.54, size = 498, normalized size = 2.44

$$\frac{a^5\,\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{1}{a+b}-\frac{7\,b}{8\,(a+b)^2}+\frac{b^2}{4\,(a+b)^3}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (a^5*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^6 - b^
6 + 3*a^2*b^4 - 3*a^4*b^2)) - (log(tan(c/2 + (d*x)/2) - 1)*(1/(a + b) - (7*
b)/(8*(a + b)^2) + b^2/(4*(a + b)^3)))/d - (log(tan(c/2 + (d*x)/2) + 1)*(b^
2/(4*(a - b)^3) + (7*b)/(8*(a - b)^2) + 1/(a - b)))/d - ((2*a^3*tan(c/2 + (
d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2) + (2*a^3*tan(c/2 + (d*x)/2)^6)/(a^4 + b^
4 - 2*a^2*b^2) + (4*tan(c/2 + (d*x)/2)^4*(a*b^2 - 2*a^3))/(a^4 + b^4 - 2*a^
```

```
2*b^2) - (tan(c/2 + (d*x)/2)^7*(7*a^2*b - 3*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2
)) + (tan(c/2 + (d*x)/2)^3*(15*a^2*b - 11*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2))
 + (tan(c/2 + (d*x)/2)^5*(15*a^2*b - 11*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2)) -
 (b*tan(c/2 + (d*x)/2)*(7*a^2 - 3*b^2))/(4*(a^4 + b^4 - 2*a^2*b^2)))/(d*(6*
tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + ta
n(c/2 + (d*x)/2)^8 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1363 $\int \frac{\sec(c+dx)\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=190

$$-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d\left(a^2-b^2\right)}+\frac{\sec^2(c+dx)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}-\frac{a^4b\log(a+b\sin(c+d}{d\left(a^2-b^2\right)^3}$$

```
[Out] -1/16*a*(3*a+b)*ln(1-sin(d*x+c))/(a+b)^3/d+1/16*a*(3*a-b)*ln(1+sin(d*x+c))/
(a-b)^3/d-a^4*b*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d-1/4*sec(d*x+c)^4*(b-a*sin(
d*x+c))/(a^2-b^2)/d+1/8*sec(d*x+c)^2*(4*b*(2*a^2-b^2)-a*(5*a^2-b^2)*sin(d*x
+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.43, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2837, 12, 1647, 801}

$$-\frac{a^4b\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d\left(a^2-b^2\right)}+\frac{\sec^2(c+dx)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\sin(c+d\right.}{8d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*(3*a + b)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + (a*(3*a - b)*Log[1
+ Sin[c + d*x]])/(16*(a - b)^3*d) - (a^4*b*Log[a + b*Sin[c + d*x]])/((a^2 -
 b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d) + (Sec
[c + d*x]^2*(4*b*(2*a^2 - b^2) - a*(5*a^2 - b^2)*Sin[c + d*x]))/(8*(a^2 - b
^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\mathrm{Subst}\left(\int \frac{x^4}{b^4(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\mathrm{Subst}\left(\int \frac{x^4}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\mathrm{Subst}\left(\int \frac{-\frac{a^2b^4}{a^2-b^2}+\frac{3ab^4x}{a^2-b^2}-4b^2x^2}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+\right.}{4bd}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\sec^2(c+dx)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\right.}{8\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\sec^2(c+dx)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\right.}{8\left(a^2-b^2\right)^2 d}$$

$$= -\frac{a(3a+b)\log(1-\sin(c+dx))}{16(a+b)^3 d} + \frac{a(3a-b)\log(1+\sin(c+dx))}{16(a-b)^3 d} - \frac{a^4 b\log(a}{\left(a^2\right.}$$

Mathematica [A] time = 1.58, size = 169, normalized size = 0.89

$$\frac{-\frac{16a^4b\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3}+\frac{5a+3b}{(a+b)^2(\sin(c+dx)-1)}+\frac{5a-3b}{(a-b)^2(\sin(c+dx)+1)}+\frac{1}{(a+b)(\sin(c+dx)-1)^2}-\frac{1}{(a-b)(\sin(c+dx)+1)^2}-\frac{a(3a+b)\log(1-\sin}{(a+b)^3}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-((a*(3*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^3) + (a*(3*a - b)*Log[1 + Si
n[c + d*x]])/(a - b)^3 - (16*a^4*b*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a +
 b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (5*a + 3*b)/((a + b)^2*(-1 + S
in[c + d*x])) - 1/((a - b)*(1 + Sin[c + d*x])^2) + (5*a - 3*b)/((a - b)^2*(
1 + Sin[c + d*x])))/(16*d)
```

fricas [A] time = 0.97, size = 261, normalized size = 1.37

$$-\frac{16\,a^4b\cos(dx+c)^4\log(b\sin(dx+c)+a)-\left(3\,a^5+8\,a^4b+6\,a^3b^2-ab^4\right)\cos(dx+c)^4\log(\sin(dx+c)+1)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^4*b*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (3*a^5 + 8*a^4*b +
 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (3*a^5 - 8*a^4*b
 + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^4*b - 8*a
^2*b^3 + 4*b^5 - 8*(2*a^4*b - 3*a^2*b^3 + b^5)*cos(d*x + c)^2 - 2*(2*a^5 -
4*a^3*b^2 + 2*a*b^4 - (5*a^5 - 6*a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x +
 c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

giac [A] time = 0.31, size = 333, normalized size = 1.75

$$-\frac{\frac{16\,a^4b^2\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{\left(3\,a^2-ab\right)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(3\,a^2+ab\right)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(6\,a^4b\sin(dx+c)^4-5\,a^5\sin(dx+c)^3+6\,a^3b^2\sin(dx\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*a^4*b^2*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b
^5 - b^7) - (3*a^2 - a*b)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) + (3*a^2 + a*b)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b
^2 + b^3) + 2*(6*a^4*b*sin(d*x + c)^4 - 5*a^5*sin(d*x + c)^3 + 6*a^3*b^2*si
```

```
n(d*x + c)^3 - a*b^4*sin(d*x + c)^3 - 4*a^4*b*sin(d*x + c)^2 - 12*a^2*b^3*s
in(d*x + c)^2 + 4*b^5*sin(d*x + c)^2 + 3*a^5*sin(d*x + c) - 2*a^3*b^2*sin(d
*x + c) - a*b^4*sin(d*x + c) + 8*a^2*b^3 - 2*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

maple [A] time = 0.40, size = 260, normalized size = 1.37

$$\frac{1}{2d\left(8a+8b\right)\left(\sin\left(dx+c\right)-1\right)^{2}}+\frac{5a}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}+\frac{3b}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{3\ln\left(\sin\left(dx+c\right.\right.}{16d\left(a+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2+5/16/d/(a+b)^2/(sin(d*x+c)-1)*a+3/16/d/(a+
b)^2/(sin(d*x+c)-1)*b-3/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2-1/16/d/(a+b)^3*ln
(sin(d*x+c)-1)*a*b-1/d*a^4*b/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))-1/2/d/(8*a-
8*b)/(1+sin(d*x+c))^2+5/16/d/(a-b)^2/(1+sin(d*x+c))*a-3/16/d/(a-b)^2/(1+sin
(d*x+c))*b+3/16/d/(a-b)^3*ln(1+sin(d*x+c))*a^2-1/16/d/(a-b)^3*ln(1+sin(d*x+
c))*a*b
```

maxima [A] time = 0.51, size = 276, normalized size = 1.45

$$-\frac{\frac{16\,a^{4}b\log\left(b\sin\left(dx+c\right)+a\right)}{a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}}-\frac{\left(3\,a^{2}-ab\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}+\frac{\left(3\,a^{2}+ab\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}-\frac{2\left(\left(5\,a^{3}-ab^{2}\right)\sin\left(dx+c\right)^{3}+6\,a^{2}b-2\,b^{3}-4\left(2\,a^{2}b-b^{3}\right)\mathrm{si}\right.}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sin\left(dx+c\right)^{4}+a^{4}-2\,a^{2}b^{2}+b^{4}-2\left(\right.}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*a^4*b*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
 - (3*a^2 - a*b)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (3
*a^2 + a*b)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*((5*a
^3 - a*b^2)*sin(d*x + c)^3 + 6*a^2*b - 2*b^3 - 4*(2*a^2*b - b^3)*sin(d*x +
c)^2 - (3*a^3 + a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^
4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

mupad [B] time = 12.45, size = 507, normalized size = 2.67

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{b^2}{4\,{\left(a-b\right)}^3}+\frac{5\,b}{8\,{\left(a-b\right)}^2}+\frac{3}{8\,\left(a-b\right)}\right)}{d}-\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(3\,a^3+a\,b^2\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(7\,a\,b^2-11\,a^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(7\,a\,b^2\right.}{4\,\left(a^4-2\,a^2\,b^2+\right.}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*(b^2/(4*(a - b)^3) + (5*b)/(8*(a - b)^2) + 3/(
8*(a - b))))/d - ((tan(c/2 + (d*x)/2)^7*(a*b^2 + 3*a^3))/(4*(a^4 + b^4 - 2*
a^2*b^2)) + (tan(c/2 + (d*x)/2)^3*(7*a*b^2 - 11*a^3))/(4*(a^4 + b^4 - 2*a^2
*b^2)) + (tan(c/2 + (d*x)/2)^5*(7*a*b^2 - 11*a^3))/(4*(a^4 + b^4 - 2*a^2*b^
2)) + (4*tan(c/2 + (d*x)/2)^4*(2*a^2*b - b^3))/(a^4 + b^4 - 2*a^2*b^2) - (2
*a^2*b*tan(c/2 + (d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2) - (2*a^2*b*tan(c/2 + (
d*x)/2)^6)/(a^4 + b^4 - 2*a^2*b^2) + (a*tan(c/2 + (d*x)/2)*(3*a^2 + b^2))/(
4*(a^4 + b^4 - 2*a^2*b^2)))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/
2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (log(tan(c/2 +
 (d*x)/2) - 1)*(3/(8*(a + b)) - (5*b)/(8*(a + b)^2) + b^2/(4*(a + b)^3)))/d
 - (a^4*b*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^6
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1364 $\int \frac{\sec^2(c+dx)\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=182

$$\frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4d\left(a^2-b^2\right)}+\frac{a^3b^2\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\sec^2(c+dx)\left(4a^3-b\left(5a^2-b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}+\frac{b(3\ldots}{}$$

```
[Out] 1/16*b*(3*a+b)*ln(1-sin(d*x+c))/(a+b)^3/d-1/16*(3*a-b)*b*ln(1+sin(d*x+c))/(
a-b)^3/d+a^3*b^2*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d+1/4*sec(d*x+c)^4*(a-b*sin
(d*x+c))/(a^2-b^2)/d-1/8*sec(d*x+c)^2*(4*a^3-b*(5*a^2-b^2)*sin(d*x+c))/(a^2
-b^2)^2/d
```

Rubi [A] time = 0.37, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2837, 12, 1647, 823, 801}

$$\frac{a^3b^2\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}+\frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4d\left(a^2-b^2\right)}-\frac{\sec^2(c+dx)\left(4a^3-b\left(5a^2-b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}+\frac{b(3\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*(3*a + b)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((3*a - b)*b*Log[1 +
 Sin[c + d*x]])/(16*(a - b)^3*d) + (a^3*b^2*Log[a + b*Sin[c + d*x]])/((a^2
- b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*d) - (Se
c[c + d*x]^2*(4*a^3 - b*(5*a^2 - b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^3}{b^3(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\,\text{Subst}\left(\int \frac{x^3}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2-b^2}-\frac{b^2\left(4a^2-b^2\right)x}{a^2-b^2}}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx\right.}{4d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(\frac{4a^3}{a^2-b^2}-\frac{b\left(5a^2-b^2\right)\sin(c+dx)}{a^2-b^2}\right)}{8\left(a^2-b^2\right)d} - \text{S}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(\frac{4a^3}{a^2-b^2}-\frac{b\left(5a^2-b^2\right)\sin(c+dx)}{a^2-b^2}\right)}{8\left(a^2-b^2\right)d} - \text{S}$$

$$= \frac{b(3a+b)\log(1-\sin(c+dx))}{16(a+b)^3d} - \frac{(3a-b)b\log(1+\sin(c+dx))}{16(a-b)^3d} + \frac{a^3b^2\log(a}{\left(a^2\right.}$$

Mathematica [A] time = 1.25, size = 166, normalized size = 0.91

$$\frac{\frac{16a^3b^2\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} + \frac{3a+b}{(a+b)^2(\sin(c+dx)-1)} + \frac{b-3a}{(a-b)^2(\sin(c+dx)+1)} + \frac{1}{(a+b)(\sin(c+dx)-1)^2} + \frac{1}{(a-b)(\sin(c+dx)+1)^2} + \frac{b(3a+b)\log(1-\text{si}}{(a+b)^3}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((b*(3*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^3 - ((3*a - b)*b*Log[1 + Sin[c
 + d*x]])/(a - b)^3 + (16*a^3*b^2*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a +
b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (3*a + b)/((a + b)^2*(-1 + Sin[
c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])^2) + (-3*a + b)/((a - b)^2*(1 +
Sin[c + d*x])))/(16*d)
```

fricas [A] time = 1.14, size = 260, normalized size = 1.43

$$\frac{16\,a^3b^2\cos(dx+c)^4\log(b\sin(dx+c)+a) - \left(3\,a^4b+8\,a^3b^2+6\,a^2b^3-b^5\right)\cos(dx+c)^4\log(\sin(dx+c)+1) + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^3*b^2*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (3*a^4*b + 8*a^3*
b^2 + 6*a^2*b^3 - b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (3*a^4*b - 8*
a^3*b^2 + 6*a^2*b^3 - b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^5 -
8*a^3*b^2 + 4*a*b^4 - 8*(a^5 - a^3*b^2)*cos(d*x + c)^2 - 2*(2*a^4*b - 4*a^2
*b^3 + 2*b^5 - (5*a^4*b - 6*a^2*b^3 + b^5)*cos(d*x + c)^2)*sin(d*x + c))/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

giac [A] time = 0.29, size = 326, normalized size = 1.79

$$\frac{\frac{16\,a^3b^3\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7} - \frac{\left(3\,ab-b^2\right)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3} + \frac{\left(3\,ab+b^2\right)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{2\left(6\,a^3b^2\sin(dx+c)^4-5\,a^4b\sin(dx+c)^3+6\,a^2b^3\sin(dx\cdots\right.}{\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a^3*b^3*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^
5 - b^7) - (3*a*b - b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^
2 - b^3) + (3*a*b + b^2)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^
2 + b^3) + 2*(6*a^3*b^2*sin(d*x + c)^4 - 5*a^4*b*sin(d*x + c)^3 + 6*a^2*b^3
*sin(d*x + c)^3 - b^5*sin(d*x + c)^3 + 4*a^5*sin(d*x + c)^2 - 16*a^3*b^2*si
n(d*x + c)^2 + 3*a^4*b*sin(d*x + c) - 2*a^2*b^3*sin(d*x + c) - b^5*sin(d*x
+ c) - 2*a^5 + 6*a^3*b^2 + 2*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(s
in(d*x + c)^2 - 1)^2))/d
```

maple [A] time = 0.42, size = 261, normalized size = 1.43

$$\frac{1}{2d\,(8a+8b)\,(\sin(dx+c)-1)^2} + \frac{3a}{16d\,(a+b)^2\,(\sin(dx+c)-1)} + \frac{b}{16d\,(a+b)^2\,(\sin(dx+c)-1)} + \frac{3\ln(\sin(dx+c)\cdots}{16d\,(a+b)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2+3/16/d/(a+b)^2/(sin(d*x+c)-1)*a+1/16/d/(a+
b)^2/(sin(d*x+c)-1)*b+3/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a*b+1/16/d/(a+b)^3*ln
```

```
(sin(d*x+c)-1)*b^2+1/d*a^3/(a+b)^3*b^2/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/d/(8*
a-8*b)/(1+sin(d*x+c))^2-3/16/d/(a-b)^2/(1+sin(d*x+c))*a+1/16/d/(a-b)^2/(1+s
in(d*x+c))*b-3/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b+1/16/d/(a-b)^3*ln(1+sin(d*
x+c))*b^2
```

maxima [A] time = 0.32, size = 267, normalized size = 1.47

$$\frac{\frac{16\,a^3b^2\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\left(3\,ab-b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(3\,ab+b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(4\,a^3\sin(dx+c)^2-\left(5\,a^2b-b^3\right)\sin(dx+c)^3-2\,a^3-2\,ab^2\right.}{\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+c)^4+a^4-2\,a^2b^2+b^4-2\left(a^4-2\right.}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^3*b^2*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
) - (3*a*b - b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (
3*a*b + b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(4*a
^3*sin(d*x + c)^2 - (5*a^2*b - b^3)*sin(d*x + c)^3 - 2*a^3 - 2*a*b^2 + (3*a
^2*b + b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2
*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

mupad [B] time = 12.45, size = 471, normalized size = 2.59

$$\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(3\,a+b\right)}{8\,d\,{\left(a+b\right)}^3}-\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(11\,a^2\,b-7\,b^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{4\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{a^4-2\,a^2\,b^2+b^4}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(3\,a^2\,b+b^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(11\,a^2\,b\right.}{4\,\left(a^4-2\,a^2\,b^2+b\right.}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (b*log(tan(c/2 + (d*x)/2) - 1)*(3*a + b))/(8*d*(a + b)^3) - ((tan(c/2 + (d*
x)/2)^3*(11*a^2*b - 7*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2)) - (4*a^3*tan(c/2 +
(d*x)/2)^4)/(a^4 + b^4 - 2*a^2*b^2) - (tan(c/2 + (d*x)/2)^7*(3*a^2*b + b^3)
)/(4*(a^4 + b^4 - 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^5*(11*a^2*b - 7*b^3))/(
4*(a^4 + b^4 - 2*a^2*b^2)) + (2*a*b^2*tan(c/2 + (d*x)/2)^2)/(a^4 + b^4 - 2*
a^2*b^2) + (2*a*b^2*tan(c/2 + (d*x)/2)^6)/(a^4 + b^4 - 2*a^2*b^2) - (b*tan(
c/2 + (d*x)/2)*(3*a^2 + b^2))/(4*(a^4 + b^4 - 2*a^2*b^2)))/(d*(6*tan(c/2 +
(d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d
*x)/2)^8 + 1)) - (log(tan(c/2 + (d*x)/2) + 1)*(b^2/(4*(a - b)^3) + (3*b)/(8
*(a - b)^2)))/d + (a^3*b^2*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*
x)/2)^2))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1365 $\int \frac{\sec^3(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=178

$$-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d(a^2-b^2)}+\frac{a\sec^2(c+dx)\left(4ab-\left(a^2+3b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}-\frac{a^2b^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}+-a($$

```
[Out] 1/16*a*(a+3*b)*ln(1-sin(d*x+c))/(a+b)^3/d-1/16*a*(a-3*b)*ln(1+sin(d*x+c))/(
a-b)^3/d-a^2*b^3*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d-1/4*sec(d*x+c)^4*(b-a*sin
(d*x+c))/(a^2-b^2)/d+1/8*a*sec(d*x+c)^2*(4*a*b-(a^2+3*b^2)*sin(d*x+c))/(a^2
-b^2)^2/d
```

Rubi [A] time = 0.38, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2837, 12, 1647, 823, 801}

$$-\frac{a^2b^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d(a^2-b^2)}+\frac{a\sec^2(c+dx)\left(4ab-\left(a^2+3b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}+-a($$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(a + 3*b)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - (a*(a - 3*b)*Log[1 +
 Sin[c + d*x]])/(16*(a - b)^3*d) - (a^2*b^3*Log[a + b*Sin[c + d*x]])/((a^2
- b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d) + (a*
Sec[c + d*x]^2*(4*a*b - (a^2 + 3*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\mathrm{Subst}\left(\int \frac{x^2}{b^2(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^3\,\mathrm{Subst}\left(\int \frac{x^2}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{b\,\mathrm{Subst}\left(\int \frac{-\frac{a^2b^2}{a^2-b^2}+\frac{3ab^2x}{a^2-b^2}}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+d\right.}{4d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{a\sec^2(c+dx)\left(4ab-\left(a^2+3b^2\right)\sin(c+\right.}{8\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{a\sec^2(c+dx)\left(4ab-\left(a^2+3b^2\right)\sin(c+\right.}{8\left(a^2-b^2\right)^2 d}$$

$$= \frac{a(a+3b)\log(1-\sin(c+dx))}{16(a+b)^3 d} - \frac{a(a-3b)\log(1+\sin(c+dx))}{16(a-b)^3 d} - \frac{a^2b^3\log(a}{\left(a^2\right.}$$

Mathematica [A] time = 1.24, size = 163, normalized size = 0.92

$$\frac{-\frac{16a^2b^3\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} + \frac{a-b}{(a+b)^2(\sin(c+dx)-1)} + \frac{a+b}{(a-b)^2(\sin(c+dx)+1)} + \frac{1}{(a+b)(\sin(c+dx)-1)^2} - \frac{1}{(a-b)(\sin(c+dx)+1)^2} + \frac{a(a+3b)\log(1-}{(a+b)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((a*(a + 3*b)*Log[1 - Sin[c + d*x]])/(a + b)^3 - (a*(a - 3*b)*Log[1 + Sin[c
 + d*x]])/(a - b)^3 - (16*a^2*b^3*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a +
b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (a - b)/((a + b)^2*(-1 + Sin[c
+ d*x])) - 1/((a - b)*(1 + Sin[c + d*x])^2) + (a + b)/((a - b)^2*(1 + Sin[c
 + d*x]))))/(16*d)
```

fricas [A] time = 1.21, size = 258, normalized size = 1.45

$$-\frac{16\,a^2b^3\cos(dx+c)^4\log(b\sin(dx+c)+a) + \left(a^5-6\,a^3b^2-8\,a^2b^3-3\,ab^4\right)\cos(dx+c)^4\log(\sin(dx+c)+1)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^2*b^3*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + (a^5 - 6*a^3*b^2
 - 8*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (a^5 - 6*a^3
*b^2 + 8*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^4*b
 - 8*a^2*b^3 + 4*b^5 - 8*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 2*(2*a^5 - 4*a^
3*b^2 + 2*a*b^4 - (a^5 + 2*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))
/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

giac [A] time = 0.28, size = 325, normalized size = 1.83

$$-\frac{\frac{16\,a^2b^4\log\left(|b\sin(dx+c)+a|\right)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}+\frac{\left(a^2-3\,ab\right)\log\left(|\sin(dx+c)+1|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(a^2+3\,ab\right)\log\left(|\sin(dx+c)-1|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(6\,a^2b^3\sin(dx+c)^4-a^5\sin(dx+c)^3-2\,a^3b^2\sin(dx\cdots\right.}{\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*a^2*b^4*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b
^5 - b^7) + (a^2 - 3*a*b)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - (a^2 + 3*a*b)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b
^2 + b^3) + 2*(6*a^2*b^3*sin(d*x + c)^4 - a^5*sin(d*x + c)^3 - 2*a^3*b^2*si
n(d*x + c)^3 + 3*a*b^4*sin(d*x + c)^3 + 4*a^4*b*sin(d*x + c)^2 - 16*a^2*b^3
*sin(d*x + c)^2 - a^5*sin(d*x + c) + 6*a^3*b^2*sin(d*x + c) - 5*a*b^4*sin(d
*x + c) - 2*a^4*b + 6*a^2*b^3 + 2*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*(sin(d*x + c)^2 - 1)^2))/d
```

maple [A] time = 0.41, size = 262, normalized size = 1.47

$$\frac{1}{2d\left(8a+8b\right)\left(\sin\left(dx+c\right)-1\right)^2}+\frac{a}{16d\left(a+b\right)^2\left(\sin\left(dx+c\right)-1\right)}-\frac{b}{16d\left(a+b\right)^2\left(\sin\left(dx+c\right)-1\right)}+\frac{\ln\left(\sin\left(dx+c\right)-\cdots\right.}{16d\left(a+b\right)^3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2+1/16/d/(a+b)^2/(sin(d*x+c)-1)*a-1/16/d/(a+
b)^2/(sin(d*x+c)-1)*b+1/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2+3/16/d/(a+b)^3*ln
(sin(d*x+c)-1)*a*b-1/d*a^2*b^3/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))-1/2/d/(8*
a-8*b)/(1+sin(d*x+c))^2+1/16/d/(a-b)^2/(1+sin(d*x+c))*a+1/16/d/(a-b)^2/(1+s
in(d*x+c))*b-1/16/d/(a-b)^3*ln(1+sin(d*x+c))*a^2+3/16/d/(a-b)^3*ln(1+sin(d*
x+c))*a*b
```

maxima [A] time = 0.33, size = 265, normalized size = 1.49

$$-\frac{\frac{16\,a^2b^3\log\left(b\sin\left(dx+c\right)+a\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{\left(a^2-3\,ab\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(a^2+3\,ab\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(4\,a^2b\sin\left(dx+c\right)^2-\left(a^3+3\,ab^2\right)\sin\left(dx+c\right)^3-2\,a^2b-2\cdots\right.}{\left(a^4-2\,a^2b^2+b^4\right)\sin\left(dx+c\right)^4+a^4-2\,a^2b^2+b^4-2\left(a^4\cdots\right.}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*a^2*b^3*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6) + (a^2 - 3*a*b)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) -
(a^2 + 3*a*b)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(4*
a^2*b*sin(d*x + c)^2 - (a^3 + 3*a*b^2)*sin(d*x + c)^3 - 2*a^2*b - 2*b^3 - (
a^3 - 5*a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4
- 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

mupad [B] time = 12.51, size = 498, normalized size = 2.80

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{b^2}{4\,{\left(a-b\right)}^3}+\frac{b}{8\,{\left(a-b\right)}^2}-\frac{1}{8\,\left(a-b\right)}\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{b}{8\,{\left(a+b\right)}^2}+\frac{1}{8\,\left(a+b\right)}-\frac{b^2}{4\,{\left(a+b\right)}^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{\cdots}}{4\,\left(a^4-2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*(b^2/(4*(a - b)^3) + b/(8*(a - b)^2) - 1/(8*(a
 - b))))/d + (log(tan(c/2 + (d*x)/2) - 1)*(b/(8*(a + b)^2) + 1/(8*(a + b))
- b^2/(4*(a + b)^3)))/d - ((tan(c/2 + (d*x)/2)^3*(3*a*b^2 - 7*a^3))/(4*(a^4
 + b^4 - 2*a^2*b^2)) - (2*b^3*tan(c/2 + (d*x)/2)^6)/(a^4 + b^4 - 2*a^2*b^2)
 - (2*b^3*tan(c/2 + (d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2) + (tan(c/2 + (d*x)/
2)^7*(5*a*b^2 - a^3))/(4*(a^4 + b^4 - 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^5*(
3*a*b^2 - 7*a^3))/(4*(a^4 + b^4 - 2*a^2*b^2)) + (4*a^2*b*tan(c/2 + (d*x)/2)
^4)/(a^4 + b^4 - 2*a^2*b^2) - (a*tan(c/2 + (d*x)/2)*(a^2 - 5*b^2))/(4*(a^4
+ b^4 - 2*a^2*b^2)))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 -
4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (a^2*b^3*log(a + 2*b*
tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3
*a^4*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}{\left (c + dx \right)} \sec^{5}{\left (c + dx \right)}}{a + b \sin{\left (c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

3.1366 $\int \frac{\sec^4(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4d\left(a^2-b^2\right)}-\frac{\sec^2(c+dx)\left(4ab^2-b\left(a^2+3b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}+\frac{ab^4\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{b(a$$

```
[Out] -1/16*b*(a+3*b)*ln(1-sin(d*x+c))/(a+b)^3/d+1/16*(a-3*b)*b*ln(1+sin(d*x+c))/
(a-b)^3/d+a*b^4*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d+1/4*sec(d*x+c)^4*(a-b*sin(
d*x+c))/(a^2-b^2)/d-1/8*sec(d*x+c)^2*(4*a*b^2-b*(a^2+3*b^2)*sin(d*x+c))/(a^
2-b^2)^2/d
```

Rubi [A] time = 0.24, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2837, 12, 823, 801}

$$\frac{ab^4\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}+\frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4d\left(a^2-b^2\right)}-\frac{\sec^2(c+dx)\left(4ab^2-b\left(a^2+3b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}-\frac{b(a$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*(a + 3*b)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((a - 3*b)*b*Log[1
+ Sin[c + d*x]])/(16*(a - b)^3*d) + (a*b^4*Log[a + b*Sin[c + d*x]])/((a^2 -
 b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*d) - (Sec
[c + d*x]^2*(4*a*b^2 - b*(a^2 + 3*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x}{b(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\,\text{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{b^2\,\text{Subst}\left(\int \frac{-ab^2+3b^2x}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4\left(a^2-b^2\right)d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4ab^2-b\left(a^2+3b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4ab^2-b\left(a^2+3b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d}$$

$$= -\frac{b(a+3b)\log(1-\sin(c+dx))}{16(a+b)^3 d} + \frac{(a-3b)b\log(1+\sin(c+dx))}{16(a-b)^3 d} + \frac{ab^4\log(a+}{\left(a^2-\right.}$$

Mathematica [A] time = 0.94, size = 244, normalized size = 1.38

$$\frac{\frac{16ab^4\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^3}+\frac{a+3b}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{a-3b}{(a-b)^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{1}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+d\right.\right.}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*b*(a + 3*b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^3 + (2*(
a - 3*b)*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a - b)^3 + (16*a*b^4*
Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^3 + 1/((a + b)*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^4) + (a + 3*b)/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])^2) + 1/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + (a - 3*b)/((
a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))/(16*d)
```

fricas [A] time = 1.00, size = 255, normalized size = 1.44

$$16\,ab^4\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)+\left(a^4b-6\,a^2b^3-8\,ab^4-3\,b^5\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*b^4*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + (a^4*b - 6*a^2*b^3
- 8*a*b^4 - 3*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (a^4*b - 6*a^2*b^
3 + 8*a*b^4 - 3*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^5 - 8*a^3*
b^2 + 4*a*b^4 - 8*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 - 2*(2*a^4*b - 4*a^2*b^3
 + 2*b^5 - (a^4*b + 2*a^2*b^3 - 3*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

giac [A] time = 0.27, size = 323, normalized size = 1.82

$$\frac{\frac{16\,ab^5\log\left(|b\sin(dx+c)+a|\right)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}+\frac{\left(ab-3\,b^2\right)\log\left(|\sin(dx+c)+1|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(ab+3\,b^2\right)\log\left(|\sin(dx+c)-1|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(6\,ab^4\sin(dx+c)^4-a^4b\sin(dx+c)^3-2\,a^2b^3\sin(dx+\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a*b^5*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5
- b^7) + (a*b - 3*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2
```

```
- b^3) - (a*b + 3*b^2)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2
+ b^3) + 2*(6*a*b^4*sin(d*x + c)^4 - a^4*b*sin(d*x + c)^3 - 2*a^2*b^3*sin(d
*x + c)^3 + 3*b^5*sin(d*x + c)^3 + 4*a^3*b^2*sin(d*x + c)^2 - 16*a*b^4*sin(
d*x + c)^2 - a^4*b*sin(d*x + c) + 6*a^2*b^3*sin(d*x + c) - 5*b^5*sin(d*x +
c) + 2*a^5 - 8*a^3*b^2 + 12*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(si
n(d*x + c)^2 - 1)^2))/d
```

maple [A] time = 0.35, size = 259, normalized size = 1.46

$$\frac{1}{2d\,(8a+8b)\,(\sin(dx+c)-1)^2}-\frac{a}{16d\,(a+b)^2\,(\sin(dx+c)-1)}-\frac{3b}{16d\,(a+b)^2\,(\sin(dx+c)-1)}-\frac{\ln(\sin(dx+c)-}{16d\,(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2-1/16/d/(a+b)^2/(sin(d*x+c)-1)*a-3/16/d/(a+
b)^2/(sin(d*x+c)-1)*b-1/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a*b-3/16/d/(a+b)^3*ln
(sin(d*x+c)-1)*b^2+1/d*a*b^4/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/d/(8*a-
8*b)/(1+sin(d*x+c))^2+1/16/d/(a-b)^2/(1+sin(d*x+c))*a-3/16/d/(a-b)^2/(1+sin
(d*x+c))*b+1/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b-3/16/d/(a-b)^3*ln(1+sin(d*x+
c))*b^2
```

maxima [A] time = 0.32, size = 267, normalized size = 1.51

$$\frac{\frac{16\,ab^4\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{\left(ab-3\,b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(ab+3\,b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(4\,ab^2\sin(dx+c)^2-\left(a^2b+3\,b^3\right)\sin(dx+c)^3+2\,a^3-6\,ab^2-\right.}{\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+c)^4+a^4-2\,a^2b^2+b^4-2\left(a^4-2\,a^2\right.}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a*b^4*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
+ (a*b - 3*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (a*
b + 3*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(4*a*b
^2*sin(d*x + c)^2 - (a^2*b + 3*b^3)*sin(d*x + c)^3 + 2*a^3 - 6*a*b^2 - (a^2
*b - 5*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2
*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

mupad [B] time = 12.40, size = 483, normalized size = 2.73

$$\frac{a\,b^4\,\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a\,b^2-a^3\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(2\,a\,b^2-a^3\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(a^2\,b-5\,b^3\right.}{4\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (a*b^4*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^6 -
b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - ((2*tan(c/2 + (d*x)/2)^2*(2*a*b^2 - a^3))/(
a^4 + b^4 - 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)^6*(2*a*b^2 - a^3))/(a^4 + b^
4 - 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^7*(a^2*b - 5*b^3))/(4*(a^4 + b^4 - 2*a
^2*b^2)) + (tan(c/2 + (d*x)/2)^3*(7*a^2*b - 3*b^3))/(4*(a^4 + b^4 - 2*a^2*b
^2)) + (tan(c/2 + (d*x)/2)^5*(7*a^2*b - 3*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2))
 - (4*a*b^2*tan(c/2 + (d*x)/2)^4)/(a^4 + b^4 - 2*a^2*b^2) + (b*tan(c/2 + (d
*x)/2)*(a^2 - 5*b^2))/(4*(a^4 + b^4 - 2*a^2*b^2)))/(d*(6*tan(c/2 + (d*x)/2)
^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8
 + 1)) - (log(tan(c/2 + (d*x)/2) - 1)*(b/(8*(a + b)^2) + b^2/(4*(a + b)^3))
)/d + (b*log(tan(c/2 + (d*x)/2) + 1)*(a - 3*b))/(8*d*(a - b)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

3.1367 $\int \frac{\csc(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=233

$$-\frac{(8a^2+21ab+15b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{(8a^2-21ab+15b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{b^6\log(a+b\sin(c+dx))}{ad(a^2-b^2)^3}+$$

```
[Out] -1/16*(8*a^2+21*a*b+15*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d+ln(sin(d*x+c))/a/d-1
/16*(8*a^2-21*a*b+15*b^2)*ln(1+sin(d*x+c))/(a-b)^3/d+b^6*ln(a+b*sin(d*x+c))
/a/(a^2-b^2)^3/d+1/16/(a+b)/d/(1-sin(d*x+c))^2+1/16*(5*a+7*b)/(a+b)^2/d/(1-
sin(d*x+c))+1/16/(a-b)/d/(1+sin(d*x+c))^2+1/16*(5*a-7*b)/(a-b)^2/d/(1+sin(d
*x+c))
```

Rubi [A] time = 0.37, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{b^6\log(a+b\sin(c+dx))}{ad(a^2-b^2)^3}-\frac{(8a^2+21ab+15b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{(8a^2-21ab+15b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{1}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + Log[S
in[c + d*x]]/(a*d) - ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sin[c + d*x]])/(16*
(a - b)^3*d) + (b^6*Log[a + b*Sin[c + d*x]])/(a*(a^2 - b^2)^3*d) + 1/(16*(a
 + b)*d*(1 - Sin[c + d*x])^2) + (5*a + 7*b)/(16*(a + b)^2*d*(1 - Sin[c + d*
x])) + 1/(16*(a - b)*d*(1 + Sin[c + d*x])^2) + (5*a - 7*b)/(16*(a - b)^2*d*
(1 + Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b}{x(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^6\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^6\,\text{Subst}\left(\int \left(\frac{1}{8b^4(a+b)(b-x)^3} + \frac{5a+7b}{16b^5(a+b)^2(b-x)^2} + \frac{8a^2+21ab+15b^2}{16b^6(a+b)^3(b-x)} + \frac{1}{ab^6x} + \frac{1}{a(a-b)^3(a+b)^3}\right.\right.}{d}$$
$$= -\frac{\left(8a^2+21ab+15b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3d} + \frac{\log(\sin(c+dx))}{ad} - \frac{\left(8a^2-21ab\right.}{}$$

Mathematica [A] time = 2.89, size = 220, normalized size = 0.94

$$\frac{b^6\left(-\frac{\left(8a^2+21ab+15b^2\right)\log(1-\sin(c+dx))}{b^6(a+b)^3} - \frac{\left(8a^2-21ab+15b^2\right)\log(\sin(c+dx)+1)}{b^6(a-b)^3} + \frac{-5a-7b}{b^6(a+b)^2(\sin(c+dx)-1)} + \frac{5a-7b}{b^6(a-b)^2(\sin(c+dx)+1)} + \frac{}{b^6(a+}\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^6*(-(((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(b^6*(a + b)^3))
+ (16*Log[Sin[c + d*x]])/(a*b^6) - ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sin[c
 + d*x]])/((a - b)^3*b^6) + (16*Log[a + b*Sin[c + d*x]])/(a*(a - b)^3*(a +
b)^3) + 1/(b^6*(a + b)*(-1 + Sin[c + d*x])^2) + (-5*a - 7*b)/(b^6*(a + b)^2
*(-1 + Sin[c + d*x])) + 1/((a - b)*b^6*(1 + Sin[c + d*x])^2) + (5*a - 7*b)/
((a - b)^2*b^6*(1 + Sin[c + d*x]))))/(16*d)
```

fricas [A] time = 4.11, size = 344, normalized size = 1.48

$$16\,b^6\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)+4\,a^6-8\,a^4b^2+4\,a^2b^4+16\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\cos\left(dx+c\right)^4\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^6*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + 4*a^6 - 8*a^4*b^2 + 4
*a^2*b^4 + 16*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(d*x + c)^4*log(-1/2*s
in(d*x + c)) - (8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15
*a*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^6 - 3*a^5*b - 24*a^4*b^
2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) +
1) + 8*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^2 - 2*(2*a^5*b - 4*a^3*b^
3 + 2*a*b^5 + (3*a^5*b - 10*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^2)*sin(d*x + c)
)/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^4)
```

giac [A] time = 0.23, size = 391, normalized size = 1.68

$$\frac{16\,b^7\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^7b-3\,a^5b^3+3\,a^3b^5-ab^7}-\frac{\left(8\,a^2-21\,ab+15\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(8\,a^2+21\,ab+15\,b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{16\,\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a}+\frac{2\left(6\,a^5\sin(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*b^7*log(abs(b*sin(d*x + c) + a))/(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
a*b^7) - (8*a^2 - 21*a*b + 15*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*
b + 3*a*b^2 - b^3) - (8*a^2 + 21*a*b + 15*b^2)*log(abs(sin(d*x + c) - 1))/(
a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 16*log(abs(sin(d*x + c)))/a + 2*(6*a^5*sin
(d*x + c)^4 - 18*a^3*b^2*sin(d*x + c)^4 + 18*a*b^4*sin(d*x + c)^4 + 3*a^4*b
*sin(d*x + c)^3 - 10*a^2*b^3*sin(d*x + c)^3 + 7*b^5*sin(d*x + c)^3 - 16*a^5
*sin(d*x + c)^2 + 48*a^3*b^2*sin(d*x + c)^2 - 44*a*b^4*sin(d*x + c)^2 - 5*a
^4*b*sin(d*x + c) + 14*a^2*b^3*sin(d*x + c) - 9*b^5*sin(d*x + c) + 12*a^5 -
 34*a^3*b^2 + 28*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(sin(d*x + c)^
2 - 1)^2))/d
```

maple [A] time = 0.45, size = 321, normalized size = 1.38

$$\frac{1}{2d\left(8a+8b\right)\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{5a}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{7b}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)-\right.}{2d\left(a+b\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2-5/16/d/(a+b)^2/(sin(d*x+c)-1)*a-7/16/d/(a+
b)^2/(sin(d*x+c)-1)*b-1/2/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2-21/16/d/(a+b)^3*ln
(sin(d*x+c)-1)*a*b-15/16/d/(a+b)^3*ln(sin(d*x+c)-1)*b^2+1/d/a*b^6/(a+b)^3/(
a-b)^3*ln(a+b*sin(d*x+c))+ln(sin(d*x+c))/a/d+1/2/d/(8*a-8*b)/(1+sin(d*x+c))
^2+5/16/d/(a-b)^2/(1+sin(d*x+c))*a-7/16/d/(a-b)^2/(1+sin(d*x+c))*b-1/2/d/(a
-b)^3*ln(1+sin(d*x+c))*a^2+21/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b-15/16/d/(a-
b)^3*ln(1+sin(d*x+c))*b^2
```

maxima [A] time = 0.34, size = 299, normalized size = 1.28

$$\frac{\frac{16\,b^6\log(b\sin(dx+c)+a)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6} - \frac{\left(8\,a^2-21\,ab+15\,b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{\left(8\,a^2+21\,ab+15\,b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{2\left(\left(3\,a^2b-7\,b^3\right)\sin(dx+c)^3+6\,a^3-10\,\cdots\right.}{\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+c)^4+\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*b^6*log(b*sin(d*x + c) + a)/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
- (8*a^2 - 21*a*b + 15*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2
- b^3) - (8*a^2 + 21*a*b + 15*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3
*a*b^2 + b^3) + 2*((3*a^2*b - 7*b^3)*sin(d*x + c)^3 + 6*a^3 - 10*a*b^2 - 4*
(a^3 - 2*a*b^2)*sin(d*x + c)^2 - (5*a^2*b - 9*b^3)*sin(d*x + c))/((a^4 - 2*
a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2
+ b^4)*sin(d*x + c)^2) + 16*log(sin(d*x + c))/a)/d
```

mupad [B] time = 12.43, size = 346, normalized size = 1.48

$$\frac{\ln(\sin(c+d\,x))}{a\,d} - \frac{\ln(\sin(c+d\,x)-1)\left(\frac{5\,b}{16\,(a+b)^2}+\frac{1}{2\,(a+b)}+\frac{b^2}{8\,(a+b)^3}\right)}{d} - \frac{\ln(\sin(c+d\,x)+1)\left(\frac{b^2}{8\,(a-b)^3}-\frac{5\,b}{16\,(a-b)^2}+\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(sin(c + d*x))/(a*d) - (log(sin(c + d*x) - 1)*((5*b)/(16*(a + b)^2) + 1/
(2*(a + b)) + b^2/(8*(a + b)^3)))/d - (log(sin(c + d*x) + 1)*(b^2/(8*(a - b
)^3) - (5*b)/(16*(a - b)^2) + 1/(2*(a - b))))/d - ((5*a*b^2 - 3*a^3)/(4*(a^
4 + b^4 - 2*a^2*b^2)) - (sin(c + d*x)^2*(2*a*b^2 - a^3))/(2*(a^4 + b^4 - 2*
a^2*b^2)) - (sin(c + d*x)^3*(3*a^2*b - 7*b^3))/(8*(a^4 + b^4 - 2*a^2*b^2))
+ (b*sin(c + d*x)*(5*a^2 - 9*b^2))/(8*(a^4 + b^4 - 2*a^2*b^2)))/(d*(cos(c +
 d*x)^2 - sin(c + d*x)^2 + sin(c + d*x)^4)) - (b^6*log(a + b*sin(c + d*x)))
/(d*(a*b^6 - a^7 - 3*a^3*b^4 + 3*a^5*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1368 $\int \frac{\csc^2(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=250

$$-\frac{(15a^2+37ab+24b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{(15a^2-37ab+24b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}-\frac{b^7\log(a+b\sin(c+d}{a^2d\left(a^2-b^2\right)^3}$$

```
[Out] -csc(d*x+c)/a/d-1/16*(15*a^2+37*a*b+24*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d-b*ln
(sin(d*x+c))/a^2/d+1/16*(15*a^2-37*a*b+24*b^2)*ln(1+sin(d*x+c))/(a-b)^3/d-b
^7*ln(a+b*sin(d*x+c))/a^2/(a^2-b^2)^3/d+1/16/(a+b)/d/(1-sin(d*x+c))^2+1/16*
(7*a+9*b)/(a+b)^2/d/(1-sin(d*x+c))-1/16/(a-b)/d/(1+sin(d*x+c))^2+1/16*(-7*a
+9*b)/(a-b)^2/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.40, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$-\frac{b^7\log(a+b\sin(c+dx))}{a^2d\left(a^2-b^2\right)^3}-\frac{(15a^2+37ab+24b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{(15a^2-37ab+24b^2)\log(\sin(c+dx)+}{16d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - ((15*a^2 + 37*a*b + 24*b^2)*Log[1 - Sin[c + d*x]])/
(16*(a + b)^3*d) - (b*Log[Sin[c + d*x]])/(a^2*d) + ((15*a^2 - 37*a*b + 24*b
^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (b^7*Log[a + b*Sin[c + d*x]])
/(a^2*(a^2 - b^2)^3*d) + 1/(16*(a + b)*d*(1 - Sin[c + d*x])^2) + (7*a + 9*b
)/(16*(a + b)^2*d*(1 - Sin[c + d*x])) - 1/(16*(a - b)*d*(1 + Sin[c + d*x])^
2) - (7*a - 9*b)/(16*(a - b)^2*d*(1 + Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b^2}{x^2(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^7\,\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^7\,\text{Subst}\left(\int \left(\frac{1}{8b^5(a+b)(b-x)^3} + \frac{7a+9b}{16b^6(a+b)^2(b-x)^2} + \frac{15a^2+37ab+24b^2}{16b^7(a+b)^3(b-x)} + \frac{1}{ab^6x^2} - \frac{1}{a^2b^6x} - \frac{}{a^2(}\right.\right.}{}$$

$$= -\frac{\csc(c+dx)}{ad} - \frac{\left(15a^2+37ab+24b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3d} - \frac{b\log(\sin(c+dx)}{a^2d}$$

Mathematica [A] time = 6.21, size = 257, normalized size = 1.03

$$\frac{b^7\left(-\frac{\log(\sin(c+dx))}{a^2b^6} - \frac{\left(15a^2+37ab+24b^2\right)\log(1-\sin(c+dx))}{16b^7(a+b)^3} + \frac{\left(15a^2-37ab+24b^2\right)\log(\sin(c+dx)+1)}{16b^7(a-b)^3} - \frac{\log(a+b\sin(c+dx))}{a^2(a-b)^3(a+b)^3} - \frac{\csc(c+dx)}{ab^7} - \frac{}{16b}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^7*(-(Csc[c + d*x]/(a*b^7)) - ((15*a^2 + 37*a*b + 24*b^2)*Log[1 - Sin[c +
 d*x]])/(16*b^7*(a + b)^3) - Log[Sin[c + d*x]]/(a^2*b^6) + ((15*a^2 - 37*a*
b + 24*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*b^7) - Log[a + b*Sin[c + d
*x]]/(a^2*(a - b)^3*(a + b)^3) + 1/(16*b^5*(a + b)*(b - b*Sin[c + d*x])^2)
+ (7*a + 9*b)/(16*b^6*(a + b)^2*(b - b*Sin[c + d*x])) - 1/(16*(a - b)*b^5*(
b + b*Sin[c + d*x])^2) - (7*a - 9*b)/(16*(a - b)^2*b^6*(b + b*Sin[c + d*x])
)))/d
```

fricas [A] time = 4.81, size = 425, normalized size = 1.70

$$-\frac{16\,b^7\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)-4\,a^7+8\,a^5b^2-4\,a^3b^4+16\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*b^7*cos(d*x + c)^4*log(b*sin(d*x + c) + a)*sin(d*x + c) - 4*a^7 +
 8*a^5*b^2 - 4*a^3*b^4 + 16*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x +
 c)^4*log(1/2*sin(d*x + c))*sin(d*x + c) - (15*a^7 + 8*a^6*b - 42*a^5*b^2 -
 24*a^4*b^3 + 35*a^3*b^4 + 24*a^2*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
*sin(d*x + c) + (15*a^7 - 8*a^6*b - 42*a^5*b^2 + 24*a^4*b^3 + 35*a^3*b^4 -
24*a^2*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*(15*a^7
- 42*a^5*b^2 + 35*a^3*b^4 - 8*a*b^6)*cos(d*x + c)^4 - 2*(5*a^7 - 14*a^5*b^2
 + 9*a^3*b^4)*cos(d*x + c)^2 + 4*(a^6*b - 2*a^4*b^3 + a^2*b^5 + 2*(a^6*b -
3*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*
a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^4*sin(d*x + c))
```

giac [A] time = 0.25, size = 418, normalized size = 1.67

$$-\frac{\frac{16\,b^8\log(|b\sin(dx+c)+a|)}{a^8b-3\,a^6b^3+3\,a^4b^5-a^2b^7}-\frac{\left(15\,a^2-37\,ab+24\,b^2\right)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(15\,a^2+37\,ab+24\,b^2\right)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{16\,b\log(|\sin(dx+c)|)}{a^2}+\frac{2\left(6\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*b^8*log(abs(b*sin(d*x + c) + a))/(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 -
 a^2*b^7) - (15*a^2 - 37*a*b + 24*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*
a^2*b + 3*a*b^2 - b^3) + (15*a^2 + 37*a*b + 24*b^2)*log(abs(sin(d*x + c) -
1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 16*b*log(abs(sin(d*x + c)))/a^2 + 2*(
6*a^4*b*sin(d*x + c)^4 - 18*a^2*b^3*sin(d*x + c)^4 + 18*b^5*sin(d*x + c)^4
+ 7*a^5*sin(d*x + c)^3 - 18*a^3*b^2*sin(d*x + c)^3 + 11*a*b^4*sin(d*x + c)^
3 - 16*a^4*b*sin(d*x + c)^2 + 48*a^2*b^3*sin(d*x + c)^2 - 44*b^5*sin(d*x +
c)^2 - 9*a^5*sin(d*x + c) + 22*a^3*b^2*sin(d*x + c) - 13*a*b^4*sin(d*x + c)
 + 12*a^4*b - 34*a^2*b^3 + 28*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(si
n(d*x + c)^2 - 1)^2) - 16*(b*sin(d*x + c) - a)/(a^2*sin(d*x + c)))/d
```

maple [A] time = 0.48, size = 340, normalized size = 1.36

$$\frac{1}{2d\left(8a+8b\right)\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{7a}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{9b}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{15\ln\left(\sin\left(dx+\right.\right.}{16d\left(a+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2-7/16/d/(a+b)^2/(sin(d*x+c)-1)*a-9/16/d/(a+
b)^2/(sin(d*x+c)-1)*b-15/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2-37/16/d/(a+b)^3*
ln(sin(d*x+c)-1)*a*b-3/2/d/(a+b)^3*ln(sin(d*x+c)-1)*b^2-1/d*b^7/a^2/(a+b)^3
/(a-b)^3*ln(a+b*sin(d*x+c))-1/d/a/sin(d*x+c)-b*ln(sin(d*x+c))/a^2/d-1/2/d/(
8*a-8*b)/(1+sin(d*x+c))^2-7/16/d/(a-b)^2/(1+sin(d*x+c))*a+9/16/d/(a-b)^2/(1
+sin(d*x+c))*b+15/16/d/(a-b)^3*ln(1+sin(d*x+c))*a^2-37/16/d/(a-b)^3*ln(1+si
n(d*x+c))*a*b+3/2/d/(a-b)^3*ln(1+sin(d*x+c))*b^2
```

maxima [A] time = 0.35, size = 361, normalized size = 1.44

$$-\frac{\frac{16\,b^7\log(b\sin(dx+c)+a)}{a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6}-\frac{\left(15\,a^2-37\,ab+24\,b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(15\,a^2+37\,ab+24\,b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(\left(15\,a^4-27\,a^2b^2+8\,b^4\right)\sin(dx+c)\right.}{\left(a^5-2\right.}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*b^7*log(b*sin(d*x + c) + a)/(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^
6) - (15*a^2 - 37*a*b + 24*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*
b^2 - b^3) + (15*a^2 + 37*a*b + 24*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*
b + 3*a*b^2 + b^3) + 2*((15*a^4 - 27*a^2*b^2 + 8*b^4)*sin(d*x + c)^4 + 8*a^
4 - 16*a^2*b^2 + 8*b^4 - 4*(a^3*b - 2*a*b^3)*sin(d*x + c)^3 - (25*a^4 - 45*
a^2*b^2 + 16*b^4)*sin(d*x + c)^2 + 2*(3*a^3*b - 5*a*b^3)*sin(d*x + c))/((a^
5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c)^5 - 2*(a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x
 + c)^3 + (a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c)) + 16*b*log(sin(d*x + c))/
a^2)/d
```

mupad [B] time = 12.47, size = 373, normalized size = 1.49

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{b^2}{8\,{\left(a-b\right)}^3}-\frac{7\,b}{16\,{\left(a-b\right)}^2}+\frac{15}{16\,\left(a-b\right)}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{7\,b}{16\,{\left(a+b\right)}^2}+\frac{15}{16\,\left(a+b\right)}+\frac{b^2}{8\,{\left(a+b\right)}^3}\right)}{d}-\frac{\frac{1}{a}-\frac{\sin(c+a}{2\left(a^4\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(b^2/(8*(a - b)^3) - (7*b)/(16*(a - b)^2) + 15/(16*(
a - b))))/d - (log(sin(c + d*x) - 1)*((7*b)/(16*(a + b)^2) + 15/(16*(a + b)
) + b^2/(8*(a + b)^3)))/d - (1/a - (sin(c + d*x)^3*(a^2*b - 2*b^3))/(2*(a^4
 + b^4 - 2*a^2*b^2)) + (sin(c + d*x)*(3*a^2*b - 5*b^3))/(4*(a^4 + b^4 - 2*a
```

```
^2*b^2)) + (sin(c + d*x)^4*(15*a^4 + 8*b^4 - 27*a^2*b^2))/(8*a*(a^4 + b^4 -
 2*a^2*b^2)) - (sin(c + d*x)^2*(25*a^4 + 16*b^4 - 45*a^2*b^2))/(8*a*(a^4 +
b^4 - 2*a^2*b^2)))/(d*(sin(c + d*x) - 2*sin(c + d*x)^3 + sin(c + d*x)^5)) -
 (b*log(sin(c + d*x)))/(a^2*d) - (b^7*log(a + b*sin(c + d*x)))/(d*(a^8 - a^
2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1369 $\int \frac{\csc^3(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=274

$$-\frac{(24a^2+57ab+35b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{(24a^2-57ab+35b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{b\csc(c+dx)}{a^2d}+\frac{(3a^2+b}{}$$

```
[Out] b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d-1/16*(24*a^2+57*a*b+35*b^2)*ln(1-si
n(d*x+c))/(a+b)^3/d+(3*a^2+b^2)*ln(sin(d*x+c))/a^3/d-1/16*(24*a^2-57*a*b+35
*b^2)*ln(1+sin(d*x+c))/(a-b)^3/d+b^8*ln(a+b*sin(d*x+c))/a^3/(a^2-b^2)^3/d+1
/16/(a+b)/d/(1-sin(d*x+c))^2+1/16*(9*a+11*b)/(a+b)^2/d/(1-sin(d*x+c))+1/16/
(a-b)/d/(1+sin(d*x+c))^2+1/16*(9*a-11*b)/(a-b)^2/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.47, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$\frac{b^8\log(a+b\sin(c+dx))}{a^3d\left(a^2-b^2\right)^3}-\frac{(24a^2+57ab+35b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{(3a^2+b^2)\log(\sin(c+dx))}{a^3d}-\frac{(24a^2-57ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((24*a^2 + 57*a*b + 35*
b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((3*a^2 + b^2)*Log[Sin[c + d
*x]])/(a^3*d) - ((24*a^2 - 57*a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*(a -
 b)^3*d) + (b^8*Log[a + b*Sin[c + d*x]])/(a^3*(a^2 - b^2)^3*d) + 1/(16*(a +
 b)*d*(1 - Sin[c + d*x])^2) + (9*a + 11*b)/(16*(a + b)^2*d*(1 - Sin[c + d*x
])) + 1/(16*(a - b)*d*(1 + Sin[c + d*x])^2) + (9*a - 11*b)/(16*(a - b)^2*d*
(1 + Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
```

```
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b^3}{x^3(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^8\,\text{Subst}\left(\int \frac{1}{x^3(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^8\,\text{Subst}\left(\int \left(\frac{1}{8b^6(a+b)(b-x)^3} + \frac{9a+11b}{16b^7(a+b)^2(b-x)^2} + \frac{24a^2+57ab+35b^2}{16b^8(a+b)^3(b-x)} + \frac{1}{ab^6x^3} - \frac{1}{a^2b^6x^2} + \right.\right.}{}$$

$$= \frac{b\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2ad} - \frac{\left(24a^2+57ab+35b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3d} +$$

Mathematica [A] time = 6.26, size = 281, normalized size = 1.03

$$\frac{b^8\left(\frac{\log(a+b\sin(c+dx))}{a^3(a-b)^3(a+b)^3} + \frac{\csc(c+dx)}{a^2b^7} - \frac{\left(24a^2+57ab+35b^2\right)\log(1-\sin(c+dx))}{16b^8(a+b)^3} - \frac{\left(24a^2-57ab+35b^2\right)\log(\sin(c+dx)+1)}{16b^8(a-b)^3} + \frac{\left(3a^2+b^2\right)\log(\sin(c+dx}{a^3b^8}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^8*(Csc[c + d*x]/(a^2*b^7) - Csc[c + d*x]^2/(2*a*b^8) - ((24*a^2 + 57*a*b
 + 35*b^2)*Log[1 - Sin[c + d*x]])/(16*b^8*(a + b)^3) + ((3*a^2 + b^2)*Log[S
in[c + d*x]])/(a^3*b^8) - ((24*a^2 - 57*a*b + 35*b^2)*Log[1 + Sin[c + d*x]]
)/(16*(a - b)^3*b^8) + Log[a + b*Sin[c + d*x]]/(a^3*(a - b)^3*(a + b)^3) +
1/(16*b^6*(a + b)*(b - b*Sin[c + d*x])^2) + (9*a + 11*b)/(16*b^7*(a + b)^2*
(b - b*Sin[c + d*x])) + 1/(16*(a - b)*b^6*(b + b*Sin[c + d*x])^2) + (9*a -
11*b)/(16*(a - b)^2*b^7*(b + b*Sin[c + d*x]))))/d
```

fricas [B] time = 7.88, size = 640, normalized size = 2.34

$$-\frac{4\,a^8-8\,a^6b^2+4\,a^4b^4-8\left(3\,a^8-8\,a^6b^2+6\,a^4b^4-a^2b^6\right)\cos\left(dx+c\right)^4+4\left(3\,a^8-8\,a^6b^2+5\,a^4b^4\right)\cos\left(dx+c\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^8 - 8*a^6*b^2 + 4*a^4*b^4 - 8*(3*a^8 - 8*a^6*b^2 + 6*a^4*b^4 - a
^2*b^6)*cos(d*x + c)^4 + 4*(3*a^8 - 8*a^6*b^2 + 5*a^4*b^4)*cos(d*x + c)^2 -
 16*(b^8*cos(d*x + c)^6 - b^8*cos(d*x + c)^4)*log(b*sin(d*x + c) + a) - 16*
((3*a^8 - 8*a^6*b^2 + 6*a^4*b^4 - b^8)*cos(d*x + c)^6 - (3*a^8 - 8*a^6*b^2
+ 6*a^4*b^4 - b^8)*cos(d*x + c)^4)*log(-1/2*sin(d*x + c)) + ((24*a^8 + 15*a
^7*b - 64*a^6*b^2 - 42*a^5*b^3 + 48*a^4*b^4 + 35*a^3*b^5)*cos(d*x + c)^6 -
(24*a^8 + 15*a^7*b - 64*a^6*b^2 - 42*a^5*b^3 + 48*a^4*b^4 + 35*a^3*b^5)*cos
(d*x + c)^4)*log(sin(d*x + c) + 1) + ((24*a^8 - 15*a^7*b - 64*a^6*b^2 + 42*
a^5*b^3 + 48*a^4*b^4 - 35*a^3*b^5)*cos(d*x + c)^6 - (24*a^8 - 15*a^7*b - 64
*a^6*b^2 + 42*a^5*b^3 + 48*a^4*b^4 - 35*a^3*b^5)*cos(d*x + c)^4)*log(-sin(d
*x + c) + 1) - 2*(2*a^7*b - 4*a^5*b^3 + 2*a^3*b^5 - (15*a^7*b - 42*a^5*b^3
+ 35*a^3*b^5 - 8*a*b^7)*cos(d*x + c)^4 + (5*a^7*b - 14*a^5*b^3 + 9*a^3*b^5)
*cos(d*x + c)^2)*sin(d*x + c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*c
os(d*x + c)^6 - (a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^4)
```

giac [B] time = 0.26, size = 589, normalized size = 2.15

$$\frac{\frac{16\,b^9\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^9b-3\,a^7b^3+3\,a^5b^5-a^3b^7}-\frac{\left(24\,a^2-57\,ab+35\,b^2\right)\log\left(|\sin\left(dx+c\right)+1|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(24\,a^2+57\,ab+35\,b^2\right)\log\left(|\sin\left(dx+c\right)-1|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{16\left(3\,a^2+b^2\right)\log\left(|\sin\left(dx+c\right)|\right)}{a^3}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*b^9*log(abs(b*sin(d*x + c) + a))/(a^9*b - 3*a^7*b^3 + 3*a^5*b^5 -
a^3*b^7) - (24*a^2 - 57*a*b + 35*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a
^2*b + 3*a*b^2 - b^3) - (24*a^2 + 57*a*b + 35*b^2)*log(abs(sin(d*x + c) - 1
))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 16*(3*a^2 + b^2)*log(abs(sin(d*x + c))
)/a^3 + 2*(4*b^8*sin(d*x + c)^6 + 15*a^7*b*sin(d*x + c)^5 - 42*a^5*b^3*sin(
d*x + c)^5 + 35*a^3*b^5*sin(d*x + c)^5 - 8*a*b^7*sin(d*x + c)^5 - 12*a^8*si
n(d*x + c)^4 + 32*a^6*b^2*sin(d*x + c)^4 - 24*a^4*b^4*sin(d*x + c)^4 + 4*a^
2*b^6*sin(d*x + c)^4 - 8*b^8*sin(d*x + c)^4 - 25*a^7*b*sin(d*x + c)^3 + 70*
a^5*b^3*sin(d*x + c)^3 - 61*a^3*b^5*sin(d*x + c)^3 + 16*a*b^7*sin(d*x + c)^
3 + 18*a^8*sin(d*x + c)^2 - 48*a^6*b^2*sin(d*x + c)^2 + 38*a^4*b^4*sin(d*x
+ c)^2 - 8*a^2*b^6*sin(d*x + c)^2 + 4*b^8*sin(d*x + c)^2 + 8*a^7*b*sin(d*x
```

```
+ c) - 24*a^5*b^3*sin(d*x + c) + 24*a^3*b^5*sin(d*x + c) - 8*a*b^7*sin(d*x
+ c) - 4*a^8 + 12*a^6*b^2 - 12*a^4*b^4 + 4*a^2*b^6)/((a^9 - 3*a^7*b^2 + 3*a
^5*b^4 - a^3*b^6)*(sin(d*x + c)^3 - sin(d*x + c))^2))/d
```

maple [A] time = 0.55, size = 371, normalized size = 1.35

$$\frac{1}{2d\left(8a+8b\right)\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{9a}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{11b}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{3\ln\left(\sin\left(dx+c\right.\right.}{2d\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2-9/16/d/(a+b)^2/(sin(d*x+c)-1)*a-11/16/d/(a
+b)^2/(sin(d*x+c)-1)*b-3/2/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2-57/16/d/(a+b)^3*l
n(sin(d*x+c)-1)*a*b-35/16/d/(a+b)^3*ln(sin(d*x+c)-1)*b^2+1/d/a^3*b^8/(a+b)^
3/(a-b)^3*ln(a+b*sin(d*x+c))-1/2/d/a/sin(d*x+c)^2+3*ln(sin(d*x+c))/a/d+b^2*
ln(sin(d*x+c))/a^3/d+1/d/a^2*b/sin(d*x+c)+1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2+
9/16/d/(a-b)^2/(1+sin(d*x+c))*a-11/16/d/(a-b)^2/(1+sin(d*x+c))*b-3/2/d/(a-b
)^3*ln(1+sin(d*x+c))*a^2+57/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b-35/16/d/(a-b)
^3*ln(1+sin(d*x+c))*b^2
```

maxima [A] time = 0.35, size = 422, normalized size = 1.54

$$\frac{\frac{16\,b^{8}\log\left(b\sin\left(dx+c\right)+a\right)}{a^{9}-3\,a^{7}b^{2}+3\,a^{5}b^{4}-a^{3}b^{6}}-\frac{\left(24\,a^{2}-57\,ab+35\,b^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{\left(24\,a^{2}+57\,ab+35\,b^{2}\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\frac{2\left(\left(15\,a^{4}b-27\,a^{2}b^{3}+8\,b^{5}\right)\sin(dx+c\ldots}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*b^8*log(b*sin(d*x + c) + a)/(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6
) - (24*a^2 - 57*a*b + 35*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - (24*a^2 + 57*a*b + 35*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3) + 2*((15*a^4*b - 27*a^2*b^3 + 8*b^5)*sin(d*x + c)^5 - 4*a
^5 + 8*a^3*b^2 - 4*a*b^4 - 4*(3*a^5 - 5*a^3*b^2 + a*b^4)*sin(d*x + c)^4 - (
25*a^4*b - 45*a^2*b^3 + 16*b^5)*sin(d*x + c)^3 + 2*(9*a^5 - 15*a^3*b^2 + 4*
a*b^4)*sin(d*x + c)^2 + 8*(a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c))/((a^6 - 2
*a^4*b^2 + a^2*b^4)*sin(d*x + c)^6 - 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*sin(d*x
+ c)^4 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(d*x + c)^2) + 16*(3*a^2 + b^2)*log
(sin(d*x + c))/a^3)/d
```

mupad [B] time = 12.70, size = 412, normalized size = 1.50

$$\frac{\ln\left(\sin\left(c+d\,x\right)\right)\,\left(3\,a^{2}+b^{2}\right)}{a^{3}\,d}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{b^{2}}{8\,\left(a-b\right)^{3}}-\frac{9\,b}{16\,\left(a-b\right)^{2}}+\frac{3}{2\,\left(a-b\right)}\right)}{d}-\frac{\frac{1}{2\,a}-\frac{b\,\sin\left(c+d\,x\right)}{a^{2}}+\frac{{\sin\left(c+d\,x\right)}^{4}\,\left(3\,a^{4}-\ldots\right)}{2\,a\,\left(a^{4}-2\,a^{2}\ldots\right)}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x))*(3*a^2 + b^2))/(a^3*d) - (log(sin(c + d*x) + 1)*(b^2/(8*
(a - b)^3) - (9*b)/(16*(a - b)^2) + 3/(2*(a - b))))/d - (1/(2*a) - (b*sin(c
 + d*x))/a^2 + (sin(c + d*x)^4*(3*a^4 + b^4 - 5*a^2*b^2))/(2*a*(a^4 + b^4 -
 2*a^2*b^2)) - (sin(c + d*x)^2*(9*a^4 + 4*b^4 - 15*a^2*b^2))/(4*a*(a^4 + b^
4 - 2*a^2*b^2)) - (sin(c + d*x)^5*(15*a^4*b + 8*b^5 - 27*a^2*b^3))/(8*a^2*(
a^4 + b^4 - 2*a^2*b^2)) + (sin(c + d*x)^3*(25*a^4*b + 16*b^5 - 45*a^2*b^3))
/(8*a^2*(a^4 + b^4 - 2*a^2*b^2)))/(d*(sin(c + d*x)^2 - 2*sin(c + d*x)^4 + s
in(c + d*x)^6)) - (log(sin(c + d*x) - 1)*((9*b)/(16*(a + b)^2) + 3/(2*(a +
b)) + b^2/(8*(a + b)^3)))/d + (b^8*log(a + b*sin(c + d*x)))/(d*(a^9 - a^3*b
^6 + 3*a^5*b^4 - 3*a^7*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1370 $\int \frac{\sqrt{g\cos(e+fx)}\sin^4(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=500

$$-\frac{2a^3E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{b^4f\sqrt{\cos(e+fx)}}-\frac{2a^2(g\cos(e+fx))^{3/2}}{3b^3fg}+\frac{a^5g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^5f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{a^5g}{}$$

```
[Out] -2/3*a^2*(g*cos(f*x+e))^(3/2)/b^3/f/g-2/3*(g*cos(f*x+e))^(3/2)/b/f/g+2/7*(g
*cos(f*x+e))^(7/2)/b/f/g^3+2/5*a*(g*cos(f*x+e))^(3/2)*sin(f*x+e)/b^2/f/g+a^
4*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*g^(1/2)/b^(
9/2)/(-a^2+b^2)^(1/4)/f-a^4*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)
^(1/4)/g^(1/2))*g^(1/2)/b^(9/2)/(-a^2+b^2)^(1/4)/f+a^5*g*(cos(1/2*f*x+1/2*e
)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^
2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^5/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e
))^(1/2)+a^5*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(s
in(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^5/f/
(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-2*a^3*(cos(1/2*f*x+1/2*e)^2)^(1/2
)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(
1/2)/b^4/f/cos(f*x+e)^(1/2)-4/5*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+
1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b^2/f/cos
(f*x+e)^(1/2)
```

Rubi [A] time = 1.24, antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {2898, 2640, 2639, 2565, 30, 2568, 14, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{2a^2(g\cos(e+fx))^{3/2}}{3b^3fg}+\frac{a^4\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f\sqrt[4]{b^2-a^2}}-\frac{a^4\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f\sqrt[4]{b^2-a^2}}-\frac{2a^3E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{b^4f\sqrt{\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^4)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^4*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/(b^(9/2)*(-a^2 + b^2)^(1/4)*f) - (a^4*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[
g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)*(-a^2 + b^2)^(1/4)
*f) - (2*a^2*(g*Cos[e + f*x])^(3/2))/(3*b^3*f*g) - (2*(g*Cos[e + f*x])^(3/2
))/(3*b*f*g) + (2*(g*Cos[e + f*x])^(7/2))/(7*b*f*g^3) - (2*a^3*Sqrt[g*Cos[e
 + f*x]]*EllipticE[(e + f*x)/2, 2])/(b^4*f*Sqrt[Cos[e + f*x]]) - (4*a*Sqrt[
```

```
g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*b^2*f*Sqrt[Cos[e + f*x]]) + (
a^5*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)
/2, 2])/(b^5*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^5*g*Sqrt[C
os[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^5
*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (2*a*(g*Cos[e + f*x])^(3/
2)*Sin[e + f*x])/(5*b^2*f*g)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,\sin^4(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(-\frac{a^3\sqrt{g\cos(e+fx)}}{b^4}+\frac{a^2\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{b^3}-\frac{a\sqrt{g\cos(e+fx}}{b^2}\right.$$

$$= -\frac{a^3\int\sqrt{g\cos(e+fx)}\,dx}{b^4}+\frac{a^4\int\frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{b^4}+\frac{a^2\int\sqrt{g\cos(e+fx)}\,\sin(}{b^3}$$

$$= \frac{2a(g\cos(e+fx))^{3/2}\sin(e+fx)}{5b^2fg}-\frac{(2a)\int\sqrt{g\cos(e+fx)}\,dx}{5b^2}-\frac{a^2\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{2a^2(g\cos(e+fx))^{3/2}}{3b^3fg}-\frac{2a^3\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\sqrt{\cos(e+fx)}}+\frac{2a(g\cos(e}{}$$

$$= -\frac{2a^2(g\cos(e+fx))^{3/2}}{3b^3fg}-\frac{2(g\cos(e+fx))^{3/2}}{3bfg}+\frac{2(g\cos(e+fx))^{7/2}}{7bfg^3}-\frac{2a^3\sqrt{}}{}$$

$$= \frac{a^4\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{9/2}\sqrt[4]{-a^2+b^2}\,f}-\frac{a^4\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{9/2}\sqrt[4]{-a^2+b^2}\,f}-\frac{2a^2(g\cos(}{3b^3}$$

Mathematica [C] time = 26.85, size = 816, normalized size = 1.63

$$\frac{\sqrt{g\cos(e+fx)}\left(-\frac{(28a^2+19b^2)\cos(e+fx)}{42b^3}+\frac{\cos(3(e+fx))}{14b}+\frac{a\sin(2(e+fx))}{5b^2}\right)}{f}-\frac{a\sqrt{g\cos(e+fx)}\left(-\frac{(5a^2+2b^2)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\cdots}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^4)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/5*(a*Sqrt[g*Cos[e + f*x]]*((-4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*
AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I
)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*S
qrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1
+ I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Lo
g[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]]
+ I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 -
 Cos[e + f*x]^2]*(a + b*Sin[e + f*x])) - ((5*a^2 + 2*b^2)*(a + b*Sqrt[1 - C
os[e + f*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2
*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)
^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)
] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] -
Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]]
+ b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)
*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 +
 b^2)*(1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(b^3*f*Sqrt[Cos[e + f*x]
]) + (Sqrt[g*Cos[e + f*x]]*(-1/42*((28*a^2 + 19*b^2)*Cos[e + f*x])/b^3 + Co
s[3*(e + f*x)]/(14*b) + (a*Sin[2*(e + f*x)])/(5*b^2)))/f
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\sin\left(fx+e\right)^{4}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^4/(b*sin(f*x + e) + a), x)
```

maple [C] time = 6.60, size = 1674, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 16/7/f/b*cos(1/2*f*x+1/2*e)^6*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-24/7/f/b*c
os(1/2*f*x+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+8/21/f/b*cos(1/2*f*x
+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+8/21/f/b*(2*cos(1/2*f*x+1/2*e)
^2*g-g)^(1/2)-4/3/f/b^3*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(
1/2)*a^2-4/3/f/b^3*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)*a^2+2/f/b^3*a^2*(g*(2
*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+1/2/f*g/b^3*a^4*sum((_R^6-_R^4*g-_R^2*g^2+g
^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2
*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=
RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^
2*g^4))+16/5/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*
g/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+
1/2*e)^5/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)-16/5/f*(g*
(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*g/b^2/(-g*(2*sin(1
/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)^3/(g*(2*cos
(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)+4/5/f*(g*(2*cos(1/2*f*x+1/2*
e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*g/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin
(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))
^(1/2)*cos(1/2*f*x+1/2*e)-2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2
*e)^2)^(1/2)*a^3*g/b^4/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(
1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1
/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e
),2^(1/2))-4/5/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*
a*g/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*
x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*
(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-1/8/
```

```
f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a^3*g/b^6/(-g*(
2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(
2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alpha*(8*(g*(2*_alpha^2*b^2+a^2-2*b
^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2
)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*_alpha
^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)
^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(
g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(g*(2*_alph
a^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)
^2*g)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+
12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2
*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))
*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2
*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2
+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-2*sin(1/2*f*x+1/2*e)
^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\sin\left(fx+e\right)^{4}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^4/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sin\left(e+f\,x\right)}^4\,\sqrt{g\,\cos\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^4*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)^4*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
[Out] Timed out
```

3.1371 $\int \frac{\sqrt{g\cos(e+fx)}\,\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=448

$$\frac{2a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{b^3f\sqrt{\cos(e+fx)}}-\frac{a^4g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{a^4g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-\ldots}}\right.}{b^4f\left(\sqrt{b^2-a^2}+b\right)\sqrt{g}\ldots}$$

```
[Out] 2/3*a*(g*cos(f*x+e))^(3/2)/b^2/f/g-2/5*(g*cos(f*x+e))^(3/2)*sin(f*x+e)/b/f/
g-a^3*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*g^(1/2)
/b^(7/2)/(-a^2+b^2)^(1/4)/f+a^3*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+
b^2)^(1/4)/g^(1/2))*g^(1/2)/b^(7/2)/(-a^2+b^2)^(1/4)/f-a^4*g*(cos(1/2*f*x+1
/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^
2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^4/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f
*x+e))^(1/2)-a^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Elliptic
Pi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^
4/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+2*a^2*(cos(1/2*f*x+1/2*e)^2)^
(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e
))^(1/2)/b^3/f/cos(f*x+e)^(1/2)+4/5*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*
x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b/f/cos
(f*x+e)^(1/2)
```

Rubi [A] time = 0.96, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {2898, 2640, 2639, 2565, 30, 2568, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{a^3\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt[4]{b^2-a^2}}+\frac{a^3\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt[4]{b^2-a^2}}+\frac{2a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{b^3f\sqrt{\cos(e+fx)}}-\frac{a^4g\sqrt{\cos(\ldots}}{b^4f\left(\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a^3*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[g])])/(b^(7/2)*(-a^2 + b^2)^(1/4)*f)) + (a^3*Sqrt[g]*ArcTanh[(Sqrt[b]*Sq
rt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*(-a^2 + b^2)^(1
/4)*f) + (2*a*(g*Cos[e + f*x])^(3/2))/(3*b^2*f*g) + (2*a^2*Sqrt[g*Cos[e + f
*x]]*EllipticE[(e + f*x)/2, 2])/(b^3*f*Sqrt[Cos[e + f*x]]) + (4*Sqrt[g*Cos[
e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*b*f*Sqrt[Cos[e + f*x]]) - (a^4*g*Sq
rt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/
```

```
(b^4*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (a^4*g*Sqrt[Cos[e + f
*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(b + Sq
rt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (2*(g*Cos[e + f*x])^(3/2)*Sin[e +
 f*x])/(5*b*f*g)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
```

```
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
```

```
2, 0])
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(\frac{a^2\sqrt{g\cos(e+fx)}}{b^3} - \frac{a\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{b^2} + \frac{\sqrt{g\cos(e+fx)}\,\text{si}}{b}\right.$$

$$= \frac{a^2\int\sqrt{g\cos(e+fx)}\,dx}{b^3} - \frac{a^3\int\frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{b^3} - \frac{a\int\sqrt{g\cos(e+fx)}\,\sin(e+}{b^2}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}\sin(e+fx)}{5bfg} + \frac{2\int\sqrt{g\cos(e+fx)}\,dx}{5b} + \frac{a\,\text{Subst}\left(\int\sqrt{x}\right.}{}$$

$$= \frac{2a(g\cos(e+fx))^{3/2}}{3b^2fg} + \frac{2a^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{\cos(e+fx)}} - \frac{2(g\cos(e+f}{5}$$

$$= \frac{2a(g\cos(e+fx))^{3/2}}{3b^2fg} + \frac{2a^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{\cos(e+fx)}} + \frac{4\sqrt{g\cos(e+}}{5bf\sqrt{}}$$

$$= -\frac{a^3\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{7/2}\sqrt[4]{-a^2+b^2}\,f} + \frac{a^3\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{7/2}\sqrt[4]{-a^2+b^2}\,f} + \frac{2a(g\cos}{3b}$$

Mathematica [C] time = 26.69, size = 789, normalized size = 1.76

$$\frac{\sqrt{g\cos(e+fx)}\left(\frac{2a\cos(e+fx)}{3b^2} - \frac{\sin(2(e+fx))}{5b}\right)}{f} + \frac{\sqrt{g\cos(e+fx)}\left(-\frac{\left(5a^2+2b^2\right)\sin^2(e+fx)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*((-4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*AppellF
1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[
```

```
e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[
b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*
Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[
-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*C
os[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e
+ f*x]^2]*(a + b*Sin[e + f*x])) - ((5*a^2 + 2*b^2)*(a + b*Sqrt[1 - Cos[e +
f*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*
(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*A
rcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqr
t[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos
[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[C
os[e + f*x]] + b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(
1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(5*b^2*f*Sqrt[Cos[e + f*x]]) +
(Sqrt[g*Cos[e + f*x]]*((2*a*Cos[e + f*x])/(3*b^2) - Sin[2*(e + f*x)]/(5*b))
)/f
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\sin\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

maple [C] time = 7.48, size = 2363, normalized size = 5.27

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 4/3/f*a/b^2*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+4/3/f*a
/b^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-2/f*a/b^2*(g*(2*cos(1/2*f*x+1/2*e)^
2-1))^(1/2)-1/2/f*g*a^3/b^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5
*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^
2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^
2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+16/3/f*(g*(2
*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*x
+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x
+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^5-16/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-
1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+
1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*co
s(1/2*f*x+1/2*e)^5-8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^
(1/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*
f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3-4/f*(g
*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b^3/(-g*(2*sin(1/
2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/
2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e
)^2+1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*a^2+4/3/f*(g*(2*cos(1/2*
f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4
-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2
-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*E
llipticF(cos(1/2*f*x+1/2*e),2^(1/2))-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(
1/2*f*x+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^
2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*
f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticE(cos(1/2*f*x
+1/2*e),2^(1/2))+8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1
/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*
x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3+8/3/f*(g
*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*
f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*
f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)
*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+
1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(s
in(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticF(cos(
1/2*f*x+1/2*e),2^(1/2))*a^2-4/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x
+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/
2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2
*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e)
,2^(1/2))+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b
/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e
)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos
(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-8/3/f*(g*(
2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*
```

```
x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*
x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)-1/4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)
*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b^5/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2
*e)^2-1))^(1/2)*sum((2*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2-sin(1/2*f*x+1/2*e)
^2*a^2-2*b^2*_alpha^2+a^2)/_alpha/(2*_alpha^2-1)*(8*(sin(1/2*f*x+1/2*e)^2)^
(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^
2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha
^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1
)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(
2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-
3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2
*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g
*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)
^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^
(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=Roo
tOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\sin\left(fx+e\right)^3}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^3\,\sqrt{g\,\cos\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^3*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)^3*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1372 $\int \frac{\sqrt{g\cos(e+fx)}\,\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=369

$$\frac{a^2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt[4]{b^2-a^2}}-\frac{a^2\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt[4]{b^2-a^2}}+\frac{a^3g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{a^3g}{}$$

```
[Out] -2/3*(g*cos(f*x+e))^(3/2)/b/f/g+a^2*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a
^2+b^2)^(1/4)/g^(1/2))*g^(1/2)/b^(5/2)/(-a^2+b^2)^(1/4)/f-a^2*arctanh(b^(1/
2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*g^(1/2)/b^(5/2)/(-a^2+b^2
)^(1/4)/f+a^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(
sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^3/f
/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+a^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/
2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)
),2^(1/2))*cos(f*x+e)^(1/2)/b^3/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)
-2*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+
1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b^2/f/cos(f*x+e)^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2898, 2640, 2639, 2565, 30, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{a^2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt[4]{b^2-a^2}}-\frac{a^2\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt[4]{b^2-a^2}}+\frac{a^3g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{a^3g}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^2*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/(b^(5/2)*(-a^2 + b^2)^(1/4)*f) - (a^2*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[
g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*(-a^2 + b^2)^(1/4)
*f) - (2*(g*Cos[e + f*x])^(3/2))/(3*b*f*g) - (2*a*Sqrt[g*Cos[e + f*x]]*Elli
pticE[(e + f*x)/2, 2])/(b^2*f*Sqrt[Cos[e + f*x]]) + (a^3*g*Sqrt[Cos[e + f*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(b - Sqrt
[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^3*g*Sqrt[Cos[e + f*x]]*EllipticP
i[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(b + Sqrt[-a^2 + b^2]
)*f*Sqrt[g*Cos[e + f*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(-\frac{a\sqrt{g\cos(e+fx)}}{b^2}+\frac{\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{b}+\frac{a^2\sqrt{g\cos(e+fx)}}{b^2(a+b\sin(e+fx}\right.$$

$$= -\frac{a\int\sqrt{g\cos(e+fx)}\,dx}{b^2}+\frac{a^2\int\frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{b^2}+\frac{\int\sqrt{g\cos(e+fx)}\,\sin(e+}{b}$$

$$= -\frac{\text{Subst}\left(\int\sqrt{x}\,dx,x,g\cos(e+fx)\right)}{bfg}-\frac{\left(a^3g\right)\int\frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx\right.}}{2b^3}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3bfg}-\frac{2a\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}}+\frac{\left(2a^2g\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3bfg}-\frac{2a\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}}+\frac{a^3g\sqrt{\cos(e+}}{b^3\left(b-\sqrt{-}\right.}$$

$$= \frac{a^2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}\sqrt[4]{-a^2+b^2}\,f}-\frac{a^2\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}\sqrt[4]{-a^2+b^2}\,f}-\frac{2(g\cos(e+}{3bf}$$

Mathematica [C] time = 16.88, size = 372, normalized size = 1.01

$$\frac{\sqrt{g\cos(e+fx)}\left(-\frac{a\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)+3\sqrt{2}\,a\left(a^2-b^2\right)^{3/4}\left(-\log\left(-\sqrt{2}\,\sqrt{b}\,\sqrt[4]{a^2-b^2}\,\sqrt{\text{co}}\right.\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*(-8*b^(3/2)*Cos[e + f*x]^(3/2) - (a*(8*b^(5/2)*Appell
F1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Co
s[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]
*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqr
t[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2
- b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f
```

```
*x]]))*(a + b*Sqrt[Sin[e + f*x]^2]))/((a^2 - b^2)*(a + b*Sin[e + f*x]))))/(
12*b^(5/2)*f*Sqrt[Cos[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\sin\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

maple [C] time = 6.20, size = 924, normalized size = 2.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -4/3/f/b*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-4/3/f/b*(2
*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+2/f/b*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)
+1/2/f*g/b*a^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3
*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-
cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*
a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-2/f*(g*(2*cos(1/2*f*x+1/2
*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g*a/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-si
n(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1)
)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*Elli
pticE(cos(1/2*f*x+1/2*e),2^(1/2))-1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1
/2*f*x+1/2*e)^2)^(1/2)*g*a/b^4/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^
2-1))^(1/2)*sum(1/_alpha*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/
```

```
2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/
2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin
(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1
/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2
)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(
1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2
^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4
-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*
e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*
e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^
2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\sin\left(fx+e\right)^2}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^2\,\sqrt{g\,\cos\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^2*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)^2*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1373 $\int \frac{\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=341

$$-\frac{a^2 g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2 f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{a^2 g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2 f\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}-\frac{a\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g}}{\sqrt{g}}\right.}{b^{3/2}f\sqrt[4]{b^2-}}$$

```
[Out] -a*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*g^(1/2)/b^
(3/2)/(-a^2+b^2)^(1/4)/f+a*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^
(1/4)/g^(1/2))*g^(1/2)/b^(3/2)/(-a^2+b^2)^(1/4)/f-a^2*g*(cos(1/2*f*x+1/2*e)
^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2
)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e)
)^(1/2)-a^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(si
n(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/f/(
b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos
(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/
b/f/cos(f*x+e)^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{a\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt[4]{b^2-a^2}}+\frac{a\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt[4]{b^2-a^2}}-\frac{a^2 g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2 f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{a^2 g}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/(b^(3/2)*(-a^2 + b^2)^(1/4)*f)) + (a*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[g
*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(1/4)*
f) + (2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(b*f*Sqrt[Cos[e + f
*x]]) - (a^2*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
(e + f*x)/2, 2])/(b^2*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (a^2
*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2,
 2])/(b^2*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\sin(e+fx)}{a+b\sin(e+fx)}\,dx = \frac{\int \sqrt{g\cos(e+fx)}\,dx}{b} - \frac{a\int \frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{b}$$

$$= \frac{(a^2g)\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,dx}{2b^2} - \frac{(a^2g)\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}+b\cos\right.}}{2b^2}$$

$$= \frac{2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{\cos(e+fx)}} - \frac{(2ag)\,\text{Subst}\left(\int \frac{x^2}{(a^2-b^2)g^2+b^2x^4}\,dx, x, \sqrt{g\,c\right.}}{f}$$

$$= \frac{2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{\cos(e+fx)}} - \frac{a^2g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(e+f\right.}{b^2\left(b-\sqrt{-a^2+b^2}\right)f\sqrt{g\cos(e+fx}}$$

$$= -\frac{a\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\sqrt[4]{-a^2+b^2}\,f} + \frac{a\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\sqrt[4]{-a^2+b^2}\,f} + \frac{2\sqrt{g\cos(e+}}{bf\sqrt{}}$$

Mathematica [C] time = 19.80, size = 351, normalized size = 1.03

$$-\frac{\sqrt{g\cos(e+fx)}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)+3\sqrt{2}\,a\left(a^2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/12*(Sqrt[g*Cos[e + f*x]]*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e +
f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a
*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2
- b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^
2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Co
s[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2
- b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*(a + b*Sqrt[Sin[e + f*x
]^2]))/(b^(3/2)*(-a^2 + b^2)*f*Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\sin\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

maple [C] time = 6.87, size = 884, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/2/f*g*a*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2
*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(
1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*
g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-4/f*(g*(2*cos(1/2*f*x+1/2*e)^
2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*
x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*
(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticF(co
s(1/2*f*x+1/2*e),2^(1/2))+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2
*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/s
in(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^
2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(
1/2))+1/4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b^3
/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum((-2*sin(1/2*f*
x+1/2*e)^2*_alpha^2*b^2+sin(1/2*f*x+1/2*e)^2*a^2+2*b^2*_alpha^2-a^2)/_alpha
/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1
/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2
*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^
2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b
^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2
```

```
*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*E
llipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootO
f(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\sin\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)\,\sqrt{g\,\cos\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1374 $\int \frac{\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=355

$$-\frac{\sqrt{b}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt[4]{b^2-a^2}}+\frac{\sqrt{b}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt[4]{b^2-a^2}}-\frac{g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{g}{}$$

```
[Out] arctan((g*cos(f*x+e))^(1/2)/g^(1/2))*g^(1/2)/a/f-arctanh((g*cos(f*x+e))^(1/
2)/g^(1/2))*g^(1/2)/a/f-arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4
)/g^(1/2))*b^(1/2)*g^(1/2)/a/(-a^2+b^2)^(1/4)/f+arctanh(b^(1/2)*(g*cos(f*x+
e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*b^(1/2)*g^(1/2)/a/(-a^2+b^2)^(1/4)/f-g*
(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*
e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/f/(b-(-a^2+b^2)^(1/2)
)/(g*cos(f*x+e))^(1/2)-g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*El
lipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1
/2)/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2898, 2565, 329, 298, 203, 206, 2701, 2807, 2805, 205, 208}

$$-\frac{\sqrt{b}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt[4]{b^2-a^2}}+\frac{\sqrt{b}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt[4]{b^2-a^2}}-\frac{g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{g}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g]*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) - (Sqrt[b]*Sqrt[g]*Arc
Tan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2
+ b^2)^(1/4)*f) - (Sqrt[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) + (
Sqrt[b]*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*
Sqrt[g])])/(a*(-a^2 + b^2)^(1/4)*f) - (g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b
)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/((b - Sqrt[-a^2 + b^2])*f*Sqrt[g
*Cos[e + f*x]]) - (g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b
^2]), (e + f*x)/2, 2])/((b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
```

```
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\ \csc(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(\frac{\sqrt{g\cos(e+fx)}\ \csc(e+fx)}{a} - \frac{b\sqrt{g\cos(e+fx)}}{a(a+b\sin(e+fx))}\right)dx$$

$$= \frac{\int\sqrt{g\cos(e+fx)}\ \csc(e+fx)\,dx}{a} - \frac{b\int\frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{a}$$

$$= -\frac{\text{Subst}\left(\int\frac{\sqrt{x}}{1-\frac{x^2}{g^2}}\,dx, x, g\cos(e+fx)\right)}{afg} + \frac{1}{2}g\int\frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-\right.}$$

$$= -\frac{2\,\text{Subst}\left(\int\frac{x^2}{1-\frac{x^4}{g^2}}\,dx, x, \sqrt{g\cos(e+fx)}\right)}{afg} - \frac{(2b^2g)\,\text{Subst}\left(\int\frac{x^2}{(a^2-b^2)g^2+b^2x^4}\,dx\right.}{af}$$

$$= -\frac{g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(e+fx)\middle|2\right)}{\left(b-\sqrt{-a^2+b^2}\right)f\sqrt{g\cos(e+fx)}} - \frac{g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{-a^2+b^2}}\right.}{\left(b+\sqrt{-a^2+b^2}\right)f\sqrt{g\,\text{co}}}$$

$$= \frac{\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{\sqrt{b}\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a\sqrt[4]{-a^2+b^2}\,f} - \frac{\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{g\,\text{c}}}{}\right.}{af}$$

Mathematica [C] time = 14.36, size = 534, normalized size = 1.50

$$\frac{\csc(e+fx)\sqrt{g\cos(e+fx)}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(8ab\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)+3\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]*(8*a*b*AppellF1[3/4, 1/2, 1, 7/4, Cos[e
+ f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*(2*Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]
])/(a^2 - b^2)^(1/4)] - 2*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*a^2*ArcTan[Sqrt[Cos
[e + f*x]]] - 4*b^2*ArcTan[Sqrt[Cos[e + f*x]]] + 2*a^2*Log[1 - Sqrt[Cos[e +
 f*x]]] - 2*b^2*Log[1 - Sqrt[Cos[e + f*x]]] - 2*a^2*Log[1 + Sqrt[Cos[e + f*
```

```
x]]] + 2*b^2*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4
)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]
] + b*Cos[e + f*x]] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2]
 + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))
*(a + b*Sqrt[Sin[e + f*x]^2]))/(12*a*(a^2 - b^2)*f*Sqrt[Cos[e + f*x]]*(b +
a*Csc[e + f*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\csc\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

maple [A] time = 2.94, size = 188, normalized size = 0.53

$$-\frac{\sqrt{g}\,\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}+4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2af}-\frac{\sqrt{g}\,\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}-4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{2af}-\frac{g\ln\left(\frac{2\sqrt{-g}\,\sqrt{}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/2/a/f*g^(1/2)*ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1/2*
e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))-1/2/a/f*g^(1/2)*ln(2/(cos(1/2*f*
x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/
2*e)-g))-1/a/(-g)^(1/2)/f*g*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2
*f*x+1/2*e)^2*g+g)^(1/2)-g))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\csc\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}}{\sin\left(e+f\,x\right)\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/(sin(e + f*x)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(1/2)/(sin(e + f*x)*(a + b*sin(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(e+fx\right)}\,\csc\left(e+fx\right)}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*cos(e + f*x))*csc(e + f*x)/(a + b*sin(e + f*x)), x)
```

3.1375 $\int \frac{\sqrt{g\cos(e+fx)}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=433

$$\frac{bg\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{bg\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}+\frac{b^{3/2}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt[4]{b^2-a^2}}$$

```
[Out] -(g*cos(f*x+e))^(3/2)*csc(f*x+e)/a/f/g-b*arctan((g*cos(f*x+e))^(1/2)/g^(1/2
))*g^(1/2)/a^2/f+b^(3/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/
4)/g^(1/2))*g^(1/2)/a^2/(-a^2+b^2)^(1/4)/f+b*arctanh((g*cos(f*x+e))^(1/2)/g
^(1/2))*g^(1/2)/a^2/f-b^(3/2)*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^
2)^(1/4)/g^(1/2))*g^(1/2)/a^2/(-a^2+b^2)^(1/4)/f+b*g*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(
1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/
2)+b*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f
*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/f/(b+(-a^2+b
^2)^(1/2))/(g*cos(f*x+e))^(1/2)-(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/
2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/a/f/cos(f*x
+e)^(1/2)
```

Rubi [A] time = 0.94, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {2898, 2565, 329, 298, 203, 206, 2570, 2640, 2639, 2701, 2807, 2805, 205, 208}

$$\frac{b^{3/2}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt[4]{b^2-a^2}}-\frac{b^{3/2}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt[4]{b^2-a^2}}+\frac{bg\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((b*Sqrt[g]*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f)) + (b^(3/2)*Sqrt
[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a
^2*(-a^2 + b^2)^(1/4)*f) + (b*Sqrt[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]
)/(a^2*f) - (b^(3/2)*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(1/4)*f) - ((g*Cos[e + f*x])^(3/2
)*Csc[e + f*x])/(a*f*g) - (Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/
(a*f*Sqrt[Cos[e + f*x]]) + (b*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sq
rt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e
```

```
+ f*x]]) + (b*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
 (e + f*x)/2, 2])/(a*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(-\frac{b\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a^2}+\frac{\sqrt{g\cos(e+fx)}\,\csc^2(e+fx)}{a}+\frac{b^2\sqrt{}}{a^2(a+}\right.$$

$$=\frac{\int\sqrt{g\cos(e+fx)}\,\csc^2(e+fx)\,dx}{a}-\frac{b\int\sqrt{g\cos(e+fx)}\,\csc(e+fx)\,dx}{a^2}+$$

$$=-\frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{afg}-\frac{\int\sqrt{g\cos(e+fx)}\,dx}{2a}+\frac{b\,\mathrm{Subst}\left(\int\frac{\sqrt{x}}{1-\frac{x^2}{g^2}}\right.}{a}$$

$$=-\frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{afg}+\frac{(2b)\,\mathrm{Subst}\left(\int\frac{x^2}{1-\frac{x^4}{g^2}}\,dx,x,\sqrt{g\cos(e+fx)}\right.}{a^2fg}$$

$$=-\frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{afg}-\frac{\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{\cos(e+fx)}}+\frac{bg\sqrt{}}{a(}$$

$$=-\frac{b\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}+\frac{b^{3/2}\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{a^2\sqrt[4]{-a^2+b^2}\,f}+\frac{b\sqrt{g}\,\tanh^{-1}}{a}$$

Mathematica [C] time = 27.10, size = 1550, normalized size = 3.58

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((Sqrt[g*Cos[e + f*x]]*Cot[e + f*x])/(a*f)) + (Sqrt[g*Cos[e + f*x]]*((4*a*
(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x
```

```
]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2))
 + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 +
b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2
)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[C
os[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(
-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 +
 b^2)^(1/4))))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) + (5*b*(-1 +
 Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a
^2 - b^2)^(1/4)] - 6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*
Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt
[Cos[e + f*x]]] + 8*a*b*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e +
f*x]]] - 6*b^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x
]]] + 6*b^2*Log[1 + Sqrt[Cos[e + f*x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/
4)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x
]] + b*Cos[e + f*x]] + 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b
^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]
]))/(12*(a^3 - a*b^2)*(1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - ((-1 + C
os[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*
x]*(-42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]
*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a
^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)
] + 84*b^(3/2)*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*Appell
F1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos
[e + f*x]^(3/2) + 48*a*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]^2,
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2 - b
^2)*Log[1 - Sqrt[Cos[e + f*x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[Cos[
e + f*x]]] + 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2]
 - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] -
 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*Sqrt[b]
*(a^3 - a*b^2)*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e +
f*x]))))/(4*a*f*Sqrt[Cos[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\csc\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

maple [C] time = 13.13, size = 1266, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/8*(4*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^
(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*b*(sum(1/_R/
(_R^6*b^2-3*_R^4*b^2*g+8*_R^2*a^2*g^2-5*_R^2*b^2*g^2-b^2*g^3)*ln((-2*sin(1/
2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R)*(_R^6-_R^4
*g-_R^2*g^2+g^3),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z
^4-4*b^2*g^3*_Z^2+b^2*g^4))*(-g)^(1/2)*b^2*g+g^(1/2)*ln(2/(cos(1/2*f*x+1/2*
e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g
))*(-g)^(1/2)+g^(1/2)*ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x
+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)+2*g*ln(2/cos(1/2
*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g)))+(-8*(-g)^(
1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*(sin(1/2*f*x+
1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*
e),2^(1/2))*a*g-(-g)^(1/2)*g^3*sin(1/2*f*x+1/2*e)^4*(2*sin(1/2*f*x+1/2*e)^2
-1)^2/a*sum(1/_alpha*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f
*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+
1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(
1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2
*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b
^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(
1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(4*a^2-3*b^2)
*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_
alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)
^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*
(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2
)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(
```

```
4*_Z^4*b^2-4*_Z^2*b^2+a^2)))*cos(1/2*f*x+1/2*e)-16*(-g)^(1/2)*(-2*sin(1/2*f
*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*a*g*sin(1/2*f*x+1/2*e)^4+16*(-g
)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*a*g*sin(1/
2*f*x+1/2*e)^2-4*(-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2
*g)^(3/2)*a*g)/a^2/(-g)^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)
^2*g)^(3/2)/cos(1/2*f*x+1/2*e)/sin(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^2*
g+g)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\csc\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}}{{\sin\left(e+f\,x\right)}^2\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/(sin(e + f*x)^2*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(1/2)/(sin(e + f*x)^2*(a + b*sin(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(e+fx\right)}\csc^{2}\left(e+fx\right)}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*cos(e + f*x))*csc(e + f*x)**2/(a + b*sin(e + f*x)), x)
```

3.1376 $\int \frac{\sqrt{g\cos(e+fx)}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=544

$$\frac{b^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}-\frac{b^2\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}-\frac{b^2g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{a^2f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{b^2g\sqrt{\cos(e}}{a^2f\left(\right.}$$

```
[Out] b*(g*cos(f*x+e))^(3/2)*csc(f*x+e)/a^2/f/g-1/2*(g*cos(f*x+e))^(3/2)*csc(f*x+
e)^2/a/f/g+1/4*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))*g^(1/2)/a/f+b^2*arctan(
(g*cos(f*x+e))^(1/2)/g^(1/2))*g^(1/2)/a^3/f-b^(5/2)*arctan(b^(1/2)*(g*cos(f
*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*g^(1/2)/a^3/(-a^2+b^2)^(1/4)/f-1/4*a
rctanh((g*cos(f*x+e))^(1/2)/g^(1/2))*g^(1/2)/a/f-b^2*arctanh((g*cos(f*x+e))
^(1/2)/g^(1/2))*g^(1/2)/a^3/f+b^(5/2)*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/
(-a^2+b^2)^(1/4)/g^(1/2))*g^(1/2)/a^3/(-a^2+b^2)^(1/4)/f-b^2*g*(cos(1/2*f*x
+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-
a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a^2/f/(b-(-a^2+b^2)^(1/2))/(g*cos
(f*x+e))^(1/2)-b^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipt
icPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/
a^2/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+b*(cos(1/2*f*x+1/2*e)^2)^(1
/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))
^(1/2)/a^2/f/cos(f*x+e)^(1/2)
```

Rubi [A] time = 1.06, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2898, 2565, 329, 298, 203, 206, 2570, 2640, 2639, 290, 2701, 2807, 2805, 205, 208}

$$-\frac{b^{5/2}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt[4]{b^2-a^2}}+\frac{b^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{b^{5/2}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt[4]{b^2-a^2}}-\frac{b^2\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g\cos}}{}\right.}{a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g]*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) + (b^2*Sqrt[g]*ArcTa
n[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) - (b^(5/2)*Sqrt[g]*ArcTan[(Sqrt[b]
*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2 + b^2)^(1/
4)*f) - (Sqrt[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2*Sqrt
[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (b^(5/2)*Sqrt[g]*ArcTa
nh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2
```

```
 + b^2)^(1/4)*f) + (b*(g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a^2*f*g) - ((g*
Cos[e + f*x])^(3/2)*Csc[e + f*x]^2)/(2*a*f*g) + (b*Sqrt[g*Cos[e + f*x]]*Ell
ipticE[(e + f*x)/2, 2])/(a^2*f*Sqrt[Cos[e + f*x]]) - (b^2*g*Sqrt[Cos[e + f*
x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(b - Sqr
t[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (b^2*g*Sqrt[Cos[e + f*x]]*Elliptic
Pi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(b + Sqrt[-a^2 + b^2
])*f*Sqrt[g*Cos[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\ \csc^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(\frac{b^2\sqrt{g\cos(e+fx)}\ \csc(e+fx)}{a^3} - \frac{b\sqrt{g\cos(e+fx)}\ \csc^2(e+fx)}{a^2} + \frac{\sqrt{g\,\mathrm{c}}}{}\right.$$

$$= \frac{\int\sqrt{g\cos(e+fx)}\ \csc^3(e+fx)\,dx}{a} - \frac{b\int\sqrt{g\cos(e+fx)}\ \csc^2(e+fx)\,dx}{a^2} +$$

$$= \frac{b(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2fg} + \frac{b\int\sqrt{g\cos(e+fx)}\ dx}{2a^2} - \frac{\mathrm{Subst}\left(\int\frac{\sqrt{x}}{\left(1-\frac{x^2}{g^2}\right)}\right.}{}$$

$$= \frac{b(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2fg} - \frac{(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{2afg} - \frac{\mathrm{Subst}\left(\right.}{}$$

$$= \frac{b(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2fg} - \frac{(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{2afg} + \frac{b\sqrt{g\cos}}{}$$

$$= \frac{b^2\sqrt{g}\ \tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f} - \frac{b^{5/2}\sqrt{g}\ \tan^{-1}\left(\frac{\sqrt{b}\ \sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\ \sqrt{g}}\right)}{a^3\sqrt[4]{-a^2+b^2}\ f} - \frac{b^2\sqrt{g}\ \tanh^{-1}}{a^3}$$

$$= \frac{\sqrt{g}\ \tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} + \frac{b^2\sqrt{g}\ \tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f} - \frac{b^{5/2}\sqrt{g}\ \tan^{-1}\left(\frac{\sqrt{b}\ \sqrt{g}}{\sqrt[4]{-}}\right.}{a^3\sqrt[4]{-a^2+b}}$$

Mathematica [C] time = 29.19, size = 1582, normalized size = 2.91

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*((b*Cot[e + f*x])/a^2 - (Cot[e + f*x]*Csc[e + f*x])/(
2*a)))/f - (Sqrt[g*Cos[e + f*x]]*((6*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*(
(a*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
b^2)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1
```

```
+ I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I
)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] -
(1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] +
 Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x
]] + I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/(Sqrt[1 - Cos[e + f
*x]^2]*(b + a*Csc[e + f*x])) - ((-a^2 - 5*b^2)*(-1 + Cos[e + f*x]^2)*(a + b
*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4
)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 6*Sq
rt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*
x]])/(a^2 - b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] + 8*a*b
*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)]*Cos[e + f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*b^2*Log[1 -
Sqrt[Cos[e + f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 6*b^2*Log[1 + Sqr
t[Cos[e + f*x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2]
- Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] +
3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(
a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(12*(a^3 - a*b^2)*(
1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (Sqrt[b]*(-1 + Cos[e + f*x]^2)*
(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*(-42*Sqrt[2]
*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f
*x]])/(a^2 - b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTa
n[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 84*b^(3/2)*
(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*AppellF1[3/4, 1/2, 1,
 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2)
 + 48*a*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2 - b^2)*Log[1 - Sqr
t[Cos[e + f*x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[Cos[e + f*x]]] + 21
*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] - 21*Sqrt[2]*(a^
2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*(a^3 - a*b^2)*(1 - Cos
[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a^2*f*Sqrt[
Cos[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")

[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\csc\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

maple [A] time = 3.25, size = 307, normalized size = 0.56

$$\frac{\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}}{16fa\left(-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}-\frac{\sqrt{g}\,\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}+4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{8fa}+\frac{\sqrt{2\left(\cos^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g-g}}{8fa\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}-\frac{g\ln\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/16/f/a/(-1+cos(1/2*f*x+1/2*e))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/8/f*
g^(1/2)/a*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g
)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))+1/8/f/a/cos(1/2*f*x+1/2*e)^2*(2*cos(1
/2*f*x+1/2*e)^2*g-g)^(1/2)-1/4/f*g/a/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*co
s(1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))-1/16/f/a/(cos(1/2*f*x+1/
2*e)+1)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/8/f*g^(1/2)/a*ln((-4*g*cos(1/
2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*
x+1/2*e)+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\csc\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}}{{\sin\left(e+f\,x\right)}^3\,\left(a+b\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/(sin(e + f*x)^3*(a + b*sin(e + f*x))),x)
[Out] int((g*cos(e + f*x))^(1/2)/(sin(e + f*x)^3*(a + b*sin(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g \cos{\left(e + f x \right)}} \csc^{3}{\left(e + f x \right)}}{a + b \sin{\left(e + f x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
[Out] Integral(sqrt(g*cos(e + f*x))*csc(e + f*x)**3/(a + b*sin(e + f*x)), x)
```

3.1377 $\int \frac{(g\cos(e+fx))^{3/2}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=621

$$-\frac{2a^4g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^5f\sqrt{g\cos(e+fx)}}-\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f}+\frac{2a^2g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3b^3f\sqrt{g\cos(e+fx)}}+\frac{2a^2g\sin(e+\ldots}{\ldots}$$

```
[Out] a^3*(-a^2+b^2)^(1/4)*g^(3/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)
^(1/4)/g^(1/2))/b^(9/2)/f+a^3*(-a^2+b^2)^(1/4)*g^(3/2)*arctanh(b^(1/2)*(g*c
os(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(9/2)/f+2/5*a*(g*cos(f*x+e))^(
5/2)/b^2/f/g-2/7*(g*cos(f*x+e))^(5/2)*sin(f*x+e)/b/f/g-2*a^4*g^2*(cos(1/2*f
*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))
*cos(f*x+e)^(1/2)/b^5/f/(g*cos(f*x+e))^(1/2)+2/3*a^2*g^2*(cos(1/2*f*x+1/2*e
)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x
+e)^(1/2)/b^3/f/(g*cos(f*x+e))^(1/2)+4/21*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/
cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/b
/f/(g*cos(f*x+e))^(1/2)+a^4*(a^2-b^2)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(
1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/
2))*cos(f*x+e)^(1/2)/b^5/f/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2
)+a^4*(a^2-b^2)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipti
cPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b
^5/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-2*a^3*g*(g*cos(f*x+e
))^(1/2)/b^4/f+2/3*a^2*g*sin(f*x+e)*(g*cos(f*x+e))^(1/2)/b^3/f+4/21*g*sin(f
*x+e)*(g*cos(f*x+e))^(1/2)/b/f
```

Rubi [A] time = 1.54, antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.485, Rules used = {2898, 2635, 2642, 2641, 2565, 30, 2568, 2695, 2867, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{a^3g^{3/2}\sqrt[4]{b^2-a^2}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}+\frac{a^3g^{3/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}-\frac{2a^4g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx\ldots\right.}{b^5f\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^3*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)*f) + (a^3*(-a^2 + b^2)^(1/4)*g^(3/2)*Arc
Tanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)
```

```
*f) - (2*a^3*g*Sqrt[g*Cos[e + f*x]])/(b^4*f) + (2*a*(g*Cos[e + f*x])^(5/2))
/(5*b^2*f*g) - (2*a^4*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(b^
5*f*Sqrt[g*Cos[e + f*x]]) + (2*a^2*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*
x)/2, 2])/(3*b^3*f*Sqrt[g*Cos[e + f*x]]) + (4*g^2*Sqrt[Cos[e + f*x]]*Ellipt
icF[(e + f*x)/2, 2])/(21*b*f*Sqrt[g*Cos[e + f*x]]) + (a^4*(a^2 - b^2)*g^2*S
qrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])
/(b^5*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^4*(a^2
- b^2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e +
 f*x)/2, 2])/(b^5*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
+ (2*a^2*g*Sqrt[g*Cos[e + f*x]]*Sin[e + f*x])/(3*b^3*f) + (4*g*Sqrt[g*Cos[e
 + f*x]]*Sin[e + f*x])/(21*b*f) - (2*(g*Cos[e + f*x])^(5/2)*Sin[e + f*x])/(
7*b*f*g)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{a^2(g\cos(e+fx))^{3/2}}{b^3} - \frac{a(g\cos(e+fx))^{3/2}\sin(e+fx)}{b^2} + \frac{(g\cos(e+fx)}{}\right.$$

$$= \frac{a^2\int (g\cos(e+fx))^{3/2}\,dx}{b^3} - \frac{a^3\int \frac{(g\cos(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx}{b^3} - \frac{a\int (g\cos(e+fx))^{3/2}\,\mathrm{s}}{b^2}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a^2g\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{3b^3f} - \frac{2(g\cos(e+f}{}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a(g\cos(e+fx))^{5/2}}{5b^2fg} + \frac{2a^2g\sqrt{g\cos(e+fx)}\,\sin(}{3b^3f}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a(g\cos(e+fx))^{5/2}}{5b^2fg} + \frac{2a^2g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(\right.}{3b^3f\sqrt{g\cos(e+f}}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a(g\cos(e+fx))^{5/2}}{5b^2fg} - \frac{2a^4g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(\right.}{b^5f\sqrt{g\cos(e+f}}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a(g\cos(e+fx))^{5/2}}{5b^2fg} - \frac{2a^4g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(\right.}{b^5f\sqrt{g\cos(e+f}}$$

$$= \frac{a^3\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{9/2}f} + \frac{a^3\sqrt[4]{-a^2+b^2}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{}}{\sqrt[4]{-}}\right.}{b^{9/2}f}$$

Mathematica [C] time = 27.84, size = 1991, normalized size = 3.21

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/420*((g*Cos[e + f*x])^(3/2)*((-2*(70*a^3 - 19*a*b^2)*(a + b*Sqrt[1 - Cos
[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2,
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*
x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^
```

```
2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1,
9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a
^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)
*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] -
Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]
] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e +
 f*x]^2]*(a + b*Sin[e + f*x])) + ((210*a^3 - 21*a*b^2)*(a + b*Sqrt[1 - Cos[
e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^
(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e
 + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[e
 + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e +
f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^
2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1
[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(
2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2
))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-
a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b
^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqr
t[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-
a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(-1 + 2*Cos[e +
f*x]^2)*(a + b*Sin[e + f*x])) - (2*(-98*a^2*b - 40*b^3)*(a + b*Sqrt[1 - Cos
[e + f*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x
]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f
*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2,
1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)
*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[C
os[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a
^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2]
+ Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/
(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2
)*(a + b*Sin[e + f*x]))))/(b^3*f*Cos[e + f*x]^(3/2)) + ((g*Cos[e + f*x])^(3
/2)*Sec[e + f*x]*((a*Cos[2*(e + f*x)])/(5*b^2) + ((28*a^2 + 5*b^2)*Sin[e +
f*x])/(42*b^3) - Sin[3*(e + f*x)]/(14*b)))/f
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

maple [C] time = 10.48, size = 3600, normalized size = 5.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x)
```

```
[Out] 8/5/f*g*a/b^2*cos(1/2*f*x+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-8/5/f
*g*a/b^2*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-8/5/f*g*a/
b^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-2/f*g*a^3/b^4*(g*(2*cos(1/2*f*x+1/2*
e)^2-1))^(1/2)+2/f*g*a/b^2*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+2/f*g^3*a^5
/b^4*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2
-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1
/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^
4-4*b^2*g^3*_Z^2+b^2*g^4))-2/f*g^3*a^3/b^2*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R
^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e
)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*
b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-32/5/f*(g*
(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1/
2*e)^7/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(
1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)+272/15/f*(g*(2*cos(1/2*f*x+1/2
*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1/2*e)^5/(g*(2*cos(1
/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g
)^(1/2)*cos(1/2*f*x+1/2*e)-16/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1
/2*e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1
/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/
```

2*e)-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b^3*
sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2
*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))
*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*a^2+4/3/f*(g
*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1
/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1
/2*f*x+1/2*e)^2*g)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x
+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)+4/f*(g*(2*cos(1/2*f*x+1/2
*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b^3*sin(1/2*f*x+1/2*e)/(g*(2*cos(1
/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g
)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*Ellip
ticE(cos(1/2*f*x+1/2*e),2^(1/2))*a^2-12/5/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*s
in(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e
)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(sin
(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticE(cos(1/2
*f*x+1/2*e),2^(1/2))+64/15/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*
e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(
-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)+
4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b^3/sin(1
/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4
*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin
(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*a^2-4/3/f*(g*(2*c
os(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b/sin(1/2*f*x+1/2*e)
/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*
x+1/2*e)^2*g)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*
e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2
-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b^3/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*
x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/
2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*si
n(1/2*f*x+1/2*e)^2-1)^(1/2)*a^2+12/5/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/
2*f*x+1/2*e)^2)^(1/2)*g^2/b/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1
))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE
(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/
2*e)^2-1)^(1/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(
1/2)*g^2/b^5*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^4*su
m(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*a
rctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2
*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+
a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1
/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2
*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))
^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alp
ha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*s
in(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b^3*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2
*e)^2-1))^(1/2)*a^2*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a

```
^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*
f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/
2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin
(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e
)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*si
n(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alp
ha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/2/f*(g*(2*cos
(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b^5/sin(1/2*f*x+1/2*e)
/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^4*sum(_alpha/(2*_alpha^2-1)*(2^(1/2
)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*
a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alph
a^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*si
n(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2
-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/
2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1
/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2
+a^2))+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/
b^3/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^2*sum(_alpha/
(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/
2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*
f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2
)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^
2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*
e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*El
lipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf
(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^3\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^3*(g*cos(e + f*x))^(3/2))/(a + b*sin(e + f*x)),x)
[Out] int((sin(e + f*x)^3*(g*cos(e + f*x))^(3/2))/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*sin(f*x+e)**3/(a+b*sin(f*x+e)),x)
[Out] Timed out
```

3.1378 $\int \frac{(g\cos(e+fx))^{3/2}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=514

$$\frac{2a^3g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\sqrt{g\cos(e+fx)}}+\frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f}-\frac{a^2g^{3/2}\sqrt[4]{b^2-a^2}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}-\frac{a^2g^{3/2}\sqrt[4]{b^2-}}{}$$

```
[Out] -a^2*(-a^2+b^2)^(1/4)*g^(3/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2
)^(1/4)/g^(1/2))/b^(7/2)/f-a^2*(-a^2+b^2)^(1/4)*g^(3/2)*arctanh(b^(1/2)*(g*
cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(7/2)/f-2/5*(g*cos(f*x+e))^(5
/2)/b/f/g+2*a^3*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipti
cF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/b^4/f/(g*cos(f*x+e))^(1/2)-
2/3*a*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2
*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/b^2/f/(g*cos(f*x+e))^(1/2)-a^3*(a^2-b
^2)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*
f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^4/f/(a^2-b*
(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-a^3*(a^2-b^2)*g^2*(cos(1/2*f*x+1
/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^
2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^4/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/
(g*cos(f*x+e))^(1/2)+2*a^2*g*(g*cos(f*x+e))^(1/2)/b^3/f-2/3*a*g*sin(f*x+e)*
(g*cos(f*x+e))^(1/2)/b^2/f
```

Rubi [A] time = 1.21, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2898, 2635, 2642, 2641, 2565, 30, 2695, 2867, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{a^2g^{3/2}\sqrt[4]{b^2-a^2}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}-\frac{a^2g^{3/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}+\frac{2a^3g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+f\right.}{b^4f\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a^2*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-
a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*f)) - (a^2*(-a^2 + b^2)^(1/4)*g^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7
/2)*f) + (2*a^2*g*Sqrt[g*Cos[e + f*x]])/(b^3*f) - (2*(g*Cos[e + f*x])^(5/2)
)/(5*b*f*g) + (2*a^3*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(b^4
*f*Sqrt[g*Cos[e + f*x]]) - (2*a*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/
```

```
2, 2])/(3*b^2*f*Sqrt[g*Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^2*Sqrt[Cos[e + f
*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(a^2 -
b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^2*Sq
rt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/
(b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (2*a*g*Sqrt
[g*Cos[e + f*x]]*Sin[e + f*x])/(3*b^2*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(-\frac{a(g\cos(e+fx))^{3/2}}{b^2}+\frac{(g\cos(e+fx))^{3/2}\sin(e+fx)}{b}+\frac{a^2(g\cos(e+}{b^2(a+b\sin(e}\right.$$

$$= -\frac{a\int(g\cos(e+fx))^{3/2}\,dx}{b^2}+\frac{a^2\int\frac{(g\cos(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx}{b^2}+\frac{\int(g\cos(e+fx))^{3/2}\,\mathrm{si}}{b}$$

$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f}-\frac{2ag\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{3b^2f}-\frac{\mathrm{Subst}\left(\int x^{3/2}\,dx\right.}{b}$$

$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f}-\frac{2(g\cos(e+fx))^{5/2}}{5bfg}-\frac{2ag\sqrt{g\cos(e+fx)}\,\sin(e+}{3b^2f}$$

$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f}-\frac{2(g\cos(e+fx))^{5/2}}{5bfg}-\frac{2ag^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+\right.}{3b^2f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f}-\frac{2(g\cos(e+fx))^{5/2}}{5bfg}+\frac{2a^3g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+\right.}{b^4f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f}-\frac{2(g\cos(e+fx))^{5/2}}{5bfg}+\frac{2a^3g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+\right.}{b^4f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{a^2\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{7/2}f}-\frac{a^2\sqrt[4]{-a^2+b^2}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt[4]{}}\right.}{b^{7/2}f}$$

Mathematica [C] time = 27.43, size = 1953, normalized size = 3.80

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(-1/5*Cos[2*(e + f*x)]/b - (2*a*Sin[e
+ f*x])/(3*b^2)))/f + ((g*Cos[e + f*x])^(3/2)*((-2*(10*a^2 + 3*b^2)*(a + b*
Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1
- Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2,
```

```
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5
/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e
+ f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcT
an[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[
1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^
2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[
e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[
Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt
[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*x])) + ((30*a^2 - 3*b^2)*(a + b*Sqrt[
1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1
 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2
 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sq
rt[Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*
Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(
a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2,
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e
+ f*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(
-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1
+ I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^
(3/2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(-1 + 2*
Cos[e + f*x]^2)*(a + b*Sin[e + f*x])) + (28*a*b*(a + b*Sqrt[1 - Cos[e + f*x
]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Co
s[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((
-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x
]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (
b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4,
Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 +
b^2*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e
 + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f
*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b
*Sin[e + f*x]))))/(60*b^2*f*Cos[e + f*x]^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

maple [C] time = 8.44, size = 1778, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x)
```

```
[Out] -8/5/f*g/b*cos(1/2*f*x+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+8/5/f*g/
b*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+8/5/f*g/b*(2*cos(
1/2*f*x+1/2*e)^2*g-g)^(1/2)+2/f*g/b^3*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*
a^2-2/f*g/b*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-2/f*g^3/b^3*a^4*sum((_R^4+
_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln
((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R)
,_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^
2+b^2*g^4))+2/f*g^3/b*a^2*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a
^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-co
s(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^
2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+8/3/f*(g*(2*cos(1/2*f*x+1/2
*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*g^2*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1
/2*f*x+1/2*e)^2-1))^(1/2)/b^2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)
^2*g)^(1/2)*cos(1/2*f*x+1/2*e)-4/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*
f*x+1/2*e)^2)^(1/2)*a*g^2*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))
^(1/2)/b^2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2
*f*x+1/2*e)-2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a
^3*g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/b^4/(-2*sin(
1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticF(cos(1/2*f*x+1/2*
```

```
e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)+2
/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*g^2/sin(1/
2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/b^2/(-2*sin(1/2*f*x+1/2*e
)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(
sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)+1/8/f*(g*(2*co
s(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a^5*g^2/sin(1/2*f*x+1/2*e
)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/b^6*sum(1/_alpha/(2*_alpha^2-1)*(2^(
1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/
(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_a
lpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2
*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alph
a^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin
(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*
x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*
b^2+a^2))-1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a
^3*g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/b^4*sum(1/_a
lpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arcta
nh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos
(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-
2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))
+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x
+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/
2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=R
ootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sin\left(e+f\,x\right)}^2\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^2*(g*cos(e + f*x))^(3/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)^2*(g*cos(e + f*x))^(3/2))/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*sin(f*x+e)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1379 $\int \frac{(g\cos(e+fx))^{3/2}\sin(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=426

$$\frac{ag^{3/2}\sqrt[4]{b^2-a^2}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f}+\frac{ag^{3/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f}-\frac{2g^2\left(3a^2-b^2\right)\sqrt{\cos(e+fx)}\,F\left(\right.}{3b^3f\sqrt{g\cos(e+fx)}}$$

```
[Out] a*(-a^2+b^2)^(1/4)*g^(3/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(
1/4)/g^(1/2))/b^(5/2)/f+a*(-a^2+b^2)^(1/4)*g^(3/2)*arctanh(b^(1/2)*(g*cos(f
*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(5/2)/f-2/3*(3*a^2-b^2)*g^2*(cos(1
/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1
/2))*cos(f*x+e)^(1/2)/b^3/f/(g*cos(f*x+e))^(1/2)+a^2*(a^2-b^2)*g^2*(cos(1/2
*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(
b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^3/f/(a^2-b*(b-(-a^2+b^2)^(1
/2)))/(g*cos(f*x+e))^(1/2)+a^2*(a^2-b^2)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/c
os(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^
(1/2))*cos(f*x+e)^(1/2)/b^3/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(
1/2)-2/3*g*(3*a-b*sin(f*x+e))*(g*cos(f*x+e))^(1/2)/b^2/f
```

Rubi [A] time = 0.98, antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{ag^{3/2}\sqrt[4]{b^2-a^2}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f}+\frac{ag^{3/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f}-\frac{2g^2\left(3a^2-b^2\right)\sqrt{\cos(e+fx)}\,F\left(\right.}{3b^3f\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*f) + (a*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTanh
[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*f)
- (2*(3*a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*b^3
*f*Sqrt[g*Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*Elliptic
Pi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 - b*(b - Sqrt[-
a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^2*Sqrt[Cos[e + f*
x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 - b
```

```
*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]
]*(3*a - b*Sin[e + f*x]))/(3*b^2*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sin(e+fx)}{a+b\sin(e+fx)}\,dx = -\frac{2g\sqrt{g\cos(e+fx)}\,(3a-b\sin(e+fx))}{3b^2f} + \frac{\left(2g^2\right)\int \frac{-ab-\frac{1}{2}\left(3a^2-b^2\right)\sin(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}}{3b^2}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}\,(3a-b\sin(e+fx))}{3b^2f} + \frac{\left(a\left(a^2-b^2\right)g^2\right)\int \frac{1}{\sqrt{g\cos(e+fx)}\,(a+}}{b^3}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}\,(3a-b\sin(e+fx))}{3b^2f} + \frac{\left(a^2\sqrt{-a^2+b^2}\,g^2\right)\int \frac{}{\sqrt{g\cos(e+fx)}\,(}}{2b^3}$$

$$= -\frac{2\left(3a^2-b^2\right)g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3b^3f\sqrt{g\cos(e+fx)}} - \frac{2g\sqrt{g\cos(e+fx)}\,(3a-b}{3b^2f}$$

$$= -\frac{2\left(3a^2-b^2\right)g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3b^3f\sqrt{g\cos(e+fx)}} - \frac{a^2\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+}}{b^3\left(b-\sqrt{-a^2+}\right.}$$

$$= \frac{a\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}f} + \frac{a\sqrt[4]{-a^2+b^2}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos}}{\sqrt[4]{-a^2+}}\right.}{b^{5/2}f}$$

Mathematica [C] time = 26.95, size = 1909, normalized size = 4.48

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/6*((g*Cos[e + f*x])^(3/2)*((-2*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)
*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]
^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e
 + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos
[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e
+ f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2
 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2]
```

```
 + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]
]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[
e + f*x])) + (3*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2
- I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2
 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*
ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/
2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2,
 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5
/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e +
 f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - C
os[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b
^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4,
 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f
*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log
[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] +
 I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b
^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e +
f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqr
t[1 - Cos[e + f*x]^2]*(-1 + 2*Cos[e + f*x]^2)*(a + b*Sin[e + f*x])) + (4*b*
(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*S
qrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e
 + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2
, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*
AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 -
(Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] -
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + L
og[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] +
 b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/(
(1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(b*f*Cos[e + f*x]^(3/2)) + (2*
(g*Cos[e + f*x])^(3/2)*Tan[e + f*x])/(3*b*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

maple [C] time = 7.28, size = 2432, normalized size = 5.71

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] -2/f*g*a/b^2*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+2/f*g^3*a^3/b^2*sum((_R^4
+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*l
n((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R
),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z
^2+b^2*g^4))-2/f*g^3*a*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*
g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1
/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g
^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2
-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+
1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)
^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(c
os(1/2*f*x+1/2*e),2^(1/2))+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/
2*e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)
*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(
1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),
2^(1/2))+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/
b/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*
e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-s
in(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))-4/f*(g*(2
*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b/sin(1/2*f*x+1/2*
e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*co
s(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^
2))^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+1/2/f*(g*(2*cos(1/2*f*x+1/2
*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b^3*sin(1/2*f*x+1/2*e)/(g*(2*cos(1
/2*f*x+1/2*e)^2-1))^(1/2)*a^2*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alp
```

```
ha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(
4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*
b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/
2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2
*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)
^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2
/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/2/f
*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b*sin(1/2*f*
x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(_alpha/(2*_alpha^2-1)*(2^
(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)
/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_
alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(
2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alp
ha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-si
n(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f
*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2
*b^2+a^2))-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*
g^2/b^3/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^2*sum(_al
pha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctan
h(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(
1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2
*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+
8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+
1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2
)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=Ro
otOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/
2*f*x+1/2*e)^2)^(1/2)*g^2/b/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1
))^(1/2)*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b
^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^
2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_a
lpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/
2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*
(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1
/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(
1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)\,\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)*(g*cos(e + f*x))^(3/2))/(a + b*sin(e + f*x)),x)
[Out] int((sin(e + f*x)*(g*cos(e + f*x))^(3/2))/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x)
[Out] Timed out
```

3.1380 $\int \frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=439

$$\frac{g^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a\sqrt{b}f}+\frac{g^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a\sqrt{b}f}+\frac{g^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b}\cdots}\right.}{bf\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cdots}}$$

```
[Out] -g^(3/2)*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f-g^(3/2)*arctanh((g*cos(f*
x+e))^(1/2)/g^(1/2))/a/f+(-a^2+b^2)^(1/4)*g^(3/2)*arctan(b^(1/2)*(g*cos(f*x
+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/f/b^(1/2)+(-a^2+b^2)^(1/4)*g^(3/2)*a
rctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/f/b^(1/2)-2
*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+
1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/b/f/(g*cos(f*x+e))^(1/2)+(a^2-b^2)*g^2*(co
s(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),
2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b/f/(a^2-b*(b-(-a^2+b^2)
^(1/2)))/(g*cos(f*x+e))^(1/2)+(a^2-b^2)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/co
s(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(
1/2))*cos(f*x+e)^(1/2)/b/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2
)
```

Rubi [A] time = 1.14, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.516, Rules used = {2898, 2565, 321, 329, 212, 206, 203, 2695, 2867, 2642, 2641, 2702, 2807, 2805, 208, 205}

$$\frac{g^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a\sqrt{b}f}+\frac{g^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a\sqrt{b}f}+\frac{g^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b}\cdots}\right.}{bf\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((g^(3/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f)) + ((-a^2 + b^2)^(1/
4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g
])])/(a*Sqrt[b]*f) - (g^(3/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f)
+ ((-a^2 + b^2)^(1/4)*g^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[g])])/(a*Sqrt[b]*f) - (2*g^2*Sqrt[Cos[e + f*x]]*Elliptic
F[(e + f*x)/2, 2])/(b*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^2*Sqrt[Cos[e
 + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b*(a^2
- b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^2*Sqrt
```

```
[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b
*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{a} - \frac{b(g\cos(e+fx))^{3/2}}{a(a+b\sin(e+fx))}\right) dx$$

$$= \frac{\int (g\cos(e+fx))^{3/2}\csc(e+fx)\,dx}{a} - \frac{b\int \frac{(g\cos(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx}{a}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{af} - \frac{\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{g^2}}\,dx, x, g\cos(e+fx)\right)}{afg} - \frac{g^2\int \frac{b}{\sqrt{g\cos(e}}}{}$$

$$= -\frac{g\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{g^2}\right)}\,dx, x, g\cos(e+fx)\right)}{af} - \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{((-a^2}{}$$

$$= -\frac{(2g)\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{g^2}}\,dx, x, \sqrt{g\cos(e+fx)}\right)}{af} + \frac{\left(\sqrt{-a^2+b^2}\,g^2\right)\int \frac{}{\sqrt{g\cos(e+}}}{2b}$$

$$= -\frac{2g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{g\cos(e+fx)}} - \frac{g^2\,\text{Subst}\left(\int \frac{1}{g-x^2}\,dx, x, \sqrt{g\cos(e+}\right.}{af}$$

$$= -\frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{g^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{2g^2\sqrt{\cos(e+fx}}{bf\sqrt{g\cos}}$$

$$= -\frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} + \frac{\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{a\sqrt{b}\,f} - \frac{g^{3/2}}{}$$

Mathematica [C] time = 6.33, size = 484, normalized size = 1.10

$$\frac{\csc(e+fx)(g\cos(e+fx))^{3/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(8ab^{3/2}\cos^{\frac{5}{2}}(e+fx)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]*(8*a*b^(3/2)*AppellF1[5/4, 1/2, 1, 9/4
, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2) - 5
*(a^2 - b^2)*(2*Sqrt[2]*(a^2 - b^2)^(1/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*Sqrt[2]*(a^2 - b^2)^(1/4)*ArcTan[1 +
(Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*Sqrt[b]*ArcTan[
Sqrt[Cos[e + f*x]]] - 2*Sqrt[b]*Log[1 - Sqrt[Cos[e + f*x]]] + 2*Sqrt[b]*Log
[1 + Sqrt[Cos[e + f*x]]] + Sqrt[2]*(a^2 - b^2)^(1/4)*Log[Sqrt[a^2 - b^2] -
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] - Sq
rt[2]*(a^2 - b^2)^(1/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*(a + b*Sqrt[Sin[e + f*x]^2]))/(
20*a*Sqrt[b]*(a^2 - b^2)*f*Cos[e + f*x]^(3/2)*(b + a*Csc[e + f*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

maple [A] time = 2.94, size = 216, normalized size = 0.49

$$-\frac{g^{\frac{3}{2}}\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}-4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{2af}-\frac{g^{\frac{3}{2}}\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}+4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2af}+\frac{2g\sqrt{-2\left(\sin^{2}\left(\frac{f}{2}\right.\right.}}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/2/a/f*g^(3/2)*ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e
)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))-1/2/a/f*g^(3/2)*ln(2/(-1+cos(1/2*
f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/
2*e)-g))+2/a/f*g*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/a/(-g)^(1/2)/f*g^2*l
n(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\sin\left(e+f\,x\right)\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/(sin(e + f*x)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/(sin(e + f*x)*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1381 $\int \frac{(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=469

$$-\frac{\sqrt{b}\,g^{3/2}\sqrt[4]{b^2-a^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^2f}-\frac{\sqrt{b}\,g^{3/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^2f}-\frac{g^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\,\Gamma}{af\left(a^2-b\left(b-\sqrt{b^2-a\right.\right.}}$$

```
[Out] b*g^(3/2)*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f+b*g^(3/2)*arctanh((g*c
os(f*x+e))^(1/2)/g^(1/2))/a^2/f-(-a^2+b^2)^(1/4)*g^(3/2)*arctan(b^(1/2)*(g*
cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*b^(1/2)/a^2/f-(-a^2+b^2)^(1/4)*
g^(3/2)*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*b^(1
/2)/a^2/f+g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin
(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/a/f/(g*cos(f*x+e))^(1/2)-(a^2-b^2
)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*
x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/f/(a^2-b*(b-(
-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-(a^2-b^2)*g^2*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(
1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x
+e))^(1/2)-g*csc(f*x+e)*(g*cos(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 1.26, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 17, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.515, Rules used = {2898, 2565, 321, 329, 212, 206, 203, 2567, 2642, 2641, 2695, 2867, 2702, 2807, 2805, 208, 205}

$$-\frac{\sqrt{b}\,g^{3/2}\sqrt[4]{b^2-a^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^2f}-\frac{\sqrt{b}\,g^{3/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^2f}-\frac{g^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\,\Gamma}{af\left(a^2-b\left(b-\sqrt{b^2-a\right.\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (b*g^(3/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f) - (Sqrt[b]*(-a^2 +
 b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/
4)*Sqrt[g])])/(a^2*f) + (b*g^(3/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(
a^2*f) - (Sqrt[b]*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*f) - (g*Sqrt[g*Cos[e + f*x]]*C
sc[e + f*x])/(a*f) + (g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(a*
f*Sqrt[g*Cos[e + f*x]]) - ((a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2
*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b*(b - Sqrt[-a^2 + b
```

```
^2]))*f*Sqrt[g*Cos[e + f*x]]) - ((a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*Ellipti
cPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b*(b + Sqrt[-a
^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(-\frac{b(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2}+\frac{(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{a}+\cdots\right.$$

$$=\frac{\int(g\cos(e+fx))^{3/2}\csc^2(e+fx)\,dx}{a}-\frac{b\int(g\cos(e+fx))^{3/2}\csc(e+fx)\,dx}{a^2}\cdots$$

$$=\frac{2bg\sqrt{g\cos(e+fx)}}{a^2f}-\frac{g\sqrt{g\cos(e+fx)}\csc(e+fx)}{af}+\frac{b\,\text{Subst}\left(\int\frac{x^{3/2}}{1-\frac{x^2}{g^2}}dx\cdots\right.}{a^2f\cdots}$$

$$=-\frac{g\sqrt{g\cos(e+fx)}\csc(e+fx)}{af}+\frac{(bg)\,\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x^2}{g^2}\right)}dx,x,g\cos(e+\cdots\right.}{a^2f}$$

$$=-\frac{g\sqrt{g\cos(e+fx)}\csc(e+fx)}{af}-\frac{g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{g\cos(e+fx)}}+\frac{(2bg\cdots}{\cdots}$$

$$=-\frac{g\sqrt{g\cos(e+fx)}\csc(e+fx)}{af}+\frac{g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{g\cos(e+fx)}}+\frac{(bg^2\cdots}{\cdots}$$

$$=\frac{bg^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}+\frac{bg^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}-\frac{g\sqrt{g\cos(e+fx)}\cdots}{af}$$

$$=\frac{bg^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}-\frac{\sqrt{b}\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^2f}+\frac{b\cdots}{\cdots}$$

Mathematica [C] time = 27.11, size = 2099, normalized size = 4.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/4*((g*Cos[e + f*x])^(3/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
```

```
/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)
*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]
^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e
 + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos
[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e
+ f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2
 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2]
 + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]
]))/(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) -
(b*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*
Csc[e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[C
os[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (10*Sqrt[
2]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2
)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*ArcTan[Sqrt[Cos[e + f*x]]])/a
 - (16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (200*b*(a^2 - b^2)*AppellF1[
1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[C
os[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1
, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*Appell
F1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (
-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (10*Lo
g[1 - Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[Cos[e + f*x]]])/a - (5*Sqrt
[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*S
qrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*Sqr
t[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*
Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))))/(20*(
1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (6*b*(-
1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*((5*b*(a^
2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*Ap
pellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
 - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b
^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e +
f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*
x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqr
t[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(3/4)*ArcTan[Sqrt[Cos[e
 + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e + f*x]]] + 2*(a^2 - b^2)
^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] -
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Sq
rt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[
Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^(3/4))))/((1 - Cos[e + f*
```

```
x]^2)*(b + a*Csc[e + f*x]))))/(a*f*Cos[e + f*x]^(3/2)) - ((g*Cos[e + f*x])^
(3/2)*Csc[e + f*x]*Sec[e + f*x])/(a*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

maple [C] time = 13.64, size = 2324, normalized size = 4.96

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/2/f*g^(3/2)*b/a^2*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/
2*e)^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))-2/f*g^3*b*sum((_R^4+_R^2*g)
/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*si
n(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=Roo
tOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g
^4))+2/f*g^3*b^3/a^2*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^
2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2
*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2
-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/f*g^2*b/a^2/(-g)^(1/2)*ln((-2*
g+2*(-g)^(1/2)*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))+1/2/
f*g^(3/2)*b/a^2*ln((-4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e
)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)+1))-1/f*(g*(2*cos(1/2*f*x+1/2*e)^2-
```

1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a*g/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(2*sin(1/2*f*x+1/2*e)^2-1)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a*g/sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a*g/sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a*g/sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(2*sin(1/2*f*x+1/2*e)^2-1)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)+1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/b^2*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a*g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+1/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a*g^2*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a*g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a*g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a*g^2*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\sin\left(e+f\,x\right)^{2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/(sin(e + f*x)^2*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/(sin(e + f*x)^2*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*csc(f*x+e)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1382 $\int \frac{(g\cos(e+fx))^{3/2}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=574

$$-\frac{b^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}-\frac{b^2g^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{bg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right.}{a^2f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}$$

```
[Out] 1/4*g^(3/2)*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f-b^2*g^(3/2)*arctan((g*
cos(f*x+e))^(1/2)/g^(1/2))/a^3/f+b^(3/2)*(-a^2+b^2)^(1/4)*g^(3/2)*arctan(b^
(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^3/f+1/4*g^(3/2)*arct
anh((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f-b^2*g^(3/2)*arctanh((g*cos(f*x+e))^(1
/2)/g^(1/2))/a^3/f+b^(3/2)*(-a^2+b^2)^(1/4)*g^(3/2)*arctanh(b^(1/2)*(g*cos(
f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^3/f-b*g^2*(cos(1/2*f*x+1/2*e)^2)^
(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(
1/2)/a^2/f/(g*cos(f*x+e))^(1/2)+b*(a^2-b^2)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2
)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2))
,2^(1/2))*cos(f*x+e)^(1/2)/a^2/f/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e)
)^(1/2)+b*(a^2-b^2)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ell
ipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/
2)/a^2/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)+b*g*csc(f*x+e)*(
g*cos(f*x+e))^(1/2)/a^2/f-1/2*g*csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 1.37, antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2898, 2565, 321, 329, 212, 206, 203, 2567, 2642, 2641, 288, 2695, 2867, 2702, 2807, 2805, 208, 205}

$$-\frac{b^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{b^{3/2}g^{3/2}\sqrt[4]{b^2-a^2}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f}-\frac{b^2g^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{b^{3/2}g^{3/2}\sqrt[4]{b^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (g^(3/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2*g^(3/2)*ArcTa
n[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (b^(3/2)*(-a^2 + b^2)^(1/4)*g^(3
/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a
^3*f) + (g^(3/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2*g^(3
/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (b^(3/2)*(-a^2 + b^2)^
(1/4)*g^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
```

```
rt[g])])/(a^3*f) + (b*g*Sqrt[g*Cos[e + f*x]]*Csc[e + f*x])/(a^2*f) - (g*Sqr
t[g*Cos[e + f*x]]*Csc[e + f*x]^2)/(2*a*f) - (b*g^2*Sqrt[Cos[e + f*x]]*Ellip
ticF[(e + f*x)/2, 2])/(a^2*f*Sqrt[g*Cos[e + f*x]]) + (b*(a^2 - b^2)*g^2*Sqr
t[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(
a^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (b*(a^2 - b^
2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x
)/2, 2])/(a^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
```

```
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
```

2, 0])

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{b^2(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^3} - \frac{b(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{a^2} +$$

$$= \frac{\int (g\cos(e+fx))^{3/2}\csc^3(e+fx)\,dx}{a} - \frac{b\int (g\cos(e+fx))^{3/2}\csc^2(e+fx)}{a^2}$$

$$= -\frac{2b^2g\sqrt{g\cos(e+fx)}}{a^3f} + \frac{bg\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a^2f} - \frac{\text{Subst}\left(\int \frac{x^{3/}}{\left(1-\frac{x}{g}\right.}\right.}{}$$

$$= \frac{bg\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a^2f} - \frac{g\sqrt{g\cos(e+fx)}\,\csc^2(e+fx)}{2af} + \frac{g\,\text{Sub}}{}$$

$$= \frac{bg\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a^2f} - \frac{g\sqrt{g\cos(e+fx)}\,\csc^2(e+fx)}{2af} + \frac{bg^2\sqrt{}}{}$$

$$= \frac{bg\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a^2f} - \frac{g\sqrt{g\cos(e+fx)}\,\csc^2(e+fx)}{2af} - \frac{bg^2\sqrt{}}{}$$

$$= \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} - \frac{b^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f} + \frac{g^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}}{}\right.}{4af}$$

$$= \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} - \frac{b^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f} + \frac{b^{3/2}\sqrt[4]{-a^2+b^2}\,g}{}$$

Mathematica [C] time = 29.48, size = 2129, normalized size = 3.71

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]

[Out] ((g*Cos[e + f*x])^(3/2)*((-2*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) - (b^2*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*ArcTan[Sqrt[Cos[e + f*x]]])/a - (16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (200*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (10*Log[1 - Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[Cos[e + f*x]]])/a - (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))))/(20*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (2*(-a^2 + 3*b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(3/4)*ArcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e + f*x]]] + 2

```
*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/2)*Log[Sqrt[a
^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e
+ f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^(3/4))))/((1
- Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a^2*f*Cos[e + f*x]^(3/2)) + ((
g*Cos[e + f*x])^(3/2)*((b*Csc[e + f*x])/a^2 - Csc[e + f*x]^2/(2*a))*Sec[e +
 f*x])/f
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

maple [A] time = 3.25, size = 312, normalized size = 0.54

$$\frac{g^{\frac{3}{2}}\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}+4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{8af}+\frac{g\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}}{16fa\left(-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}-\frac{g^{2}\ln\left(\frac{-2g+2\sqrt{-g}\,\sqrt{2\left(\cos^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g-g}}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right.}{4fa\sqrt{-g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/8/f*g^(3/2)/a*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)
^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))+1/16/f*g/a/(-1+cos(1/2*f*x+1/2*
```

e))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/4/f*g^2/a/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))+1/8/f*g^(3/2)/a*ln((-4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)+1))-1/8/f*g/a/cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-1/16/f*g/a/(cos(1/2*f*x+1/2*e)+1)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\sin\left(e+f\,x\right)^{3}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/(sin(e + f*x)^3*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/(sin(e + f*x)^3*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*csc(f*x+e)**3/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1383 $\int \frac{(g\cos(e+fx))^{5/2}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=610

$$-\frac{2a^4g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{b^5f\sqrt{\cos(e+fx)}}-\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f}+\frac{6a^2g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5b^3f\sqrt{\cos(e+fx)}}+\frac{2a^2g}{}$$

```
[Out] -a^3*(-a^2+b^2)^(3/4)*g^(5/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2
)^(1/4)/g^(1/2))/b^(11/2)/f+a^3*(-a^2+b^2)^(3/4)*g^(5/2)*arctanh(b^(1/2)*(g
*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(11/2)/f-2/3*a^3*g*(g*cos(f*
x+e))^(3/2)/b^4/f+2/7*a*(g*cos(f*x+e))^(7/2)/b^2/f/g+2/5*a^2*g*(g*cos(f*x+e
))^(3/2)*sin(f*x+e)/b^3/f+4/45*g*(g*cos(f*x+e))^(3/2)*sin(f*x+e)/b/f-2/9*(g
*cos(f*x+e))^(7/2)*sin(f*x+e)/b/f/g+a^4*(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2
)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^
(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^6/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^
(1/2)+a^4*(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ell
ipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/
2)/b^6/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-2*a^4*g^2*(cos(1/2*f*x+1
/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*
cos(f*x+e))^(1/2)/b^5/f/cos(f*x+e)^(1/2)+6/5*a^2*g^2*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+
e))^(1/2)/b^3/f/cos(f*x+e)^(1/2)+4/15*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(
1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b
/f/cos(f*x+e)^(1/2)
```

Rubi [A] time = 1.36, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.485, Rules used = {2898, 2635, 2640, 2639, 2565, 30, 2568, 2695, 2867, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{a^3g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}f}+\frac{a^3g^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}f}-\frac{2a^4g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^5f\sqrt{\cos(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a^3*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-
a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(11/2)*f)) + (a^3*(-a^2 + b^2)^(3/4)*g^(5/2)
*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(
```

```
11/2)*f) - (2*a^3*g*(g*Cos[e + f*x])^(3/2))/(3*b^4*f) + (2*a*(g*Cos[e + f*x
])^(7/2))/(7*b^2*f*g) - (2*a^4*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)
/2, 2])/(b^5*f*Sqrt[Cos[e + f*x]]) + (6*a^2*g^2*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*b^3*f*Sqrt[Cos[e + f*x]]) + (4*g^2*Sqrt[g*Cos[e + f
*x]]*EllipticE[(e + f*x)/2, 2])/(15*b*f*Sqrt[Cos[e + f*x]]) + (a^4*(a^2 - b
^2)*g^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*
x)/2, 2])/(b^6*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^4*(a^2 -
 b^2)*g^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e +
f*x)/2, 2])/(b^6*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (2*a^2*g*
(g*Cos[e + f*x])^(3/2)*Sin[e + f*x])/(5*b^3*f) + (4*g*(g*Cos[e + f*x])^(3/2
)*Sin[e + f*x])/(45*b*f) - (2*(g*Cos[e + f*x])^(7/2)*Sin[e + f*x])/(9*b*f*g
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(\frac{a^2(g\cos(e+fx))^{5/2}}{b^3} - \frac{a(g\cos(e+fx))^{5/2}\sin(e+fx)}{b^2} + \frac{(g\cos(e+f$$

$$= \frac{a^2\int(g\cos(e+fx))^{5/2}\,dx}{b^3} - \frac{a^3\int\frac{(g\cos(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx}{b^3} - \frac{a\int(g\cos(e+fx))^{5/2}}{b^2}$$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a^2g(g\cos(e+fx))^{3/2}\sin(e+fx)}{5b^3f} - \frac{2(g\cos($$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a(g\cos(e+fx))^{7/2}}{7b^2fg} + \frac{2a^2g(g\cos(e+fx))^{3/2}}{5b^3f}$$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a(g\cos(e+fx))^{7/2}}{7b^2fg} + \frac{6a^2g^2\sqrt{g\cos(e+fx)}}{5b^3f\sqrt{\cos(e}}$$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a(g\cos(e+fx))^{7/2}}{7b^2fg} - \frac{2a^4g^2\sqrt{g\cos(e+fx)}}{b^5f\sqrt{\cos(e}}$$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a(g\cos(e+fx))^{7/2}}{7b^2fg} - \frac{2a^4g^2\sqrt{g\cos(e+fx)}}{b^5f\sqrt{\cos(e}}$$

$$= -\frac{a^3\left(-a^2+b^2\right)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{11/2}f} + \frac{a^3\left(-a^2+b^2\right)^{3/4}g^{5/2}\tanh}{b^{11/2}f}$$

Mathematica [C] time = 27.20, size = 867, normalized size = 1.42

$$\frac{(g\cos(e+fx))^{5/2}\sec^2(e+fx)\left(-\frac{a(28a^2-9b^2)\cos(e+fx)}{42b^4} + \frac{a\cos(3(e+fx))}{14b^2} - \frac{(b^2-18a^2)\sin(2(e+fx))}{90b^3} - \frac{\sin(4(e+fx))}{36b}\right)}{f} - \frac{(g\cos(e}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/15*((g*Cos[e + f*x])^(5/2)*((-2*(6*a^3*b - 2*a*b^3)*(a + b*Sqrt[1 - Cos[
e + f*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*A
rcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcT
an[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[
-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*C
os[e + f*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sq
rt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e
+ f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*x])) - ((15*a^4 - 9*a^2*
b^2 - 2*b^4)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2
, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(
3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Co
s[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e
+ f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*Sin[e
 + f*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x
]))))/(b^4*f*Cos[e + f*x]^(5/2)) + ((g*Cos[e + f*x])^(5/2)*Sec[e + f*x]^2*(
-1/42*(a*(28*a^2 - 9*b^2)*Cos[e + f*x])/b^4 + (a*Cos[3*(e + f*x)])/(14*b^2)
 - ((-18*a^2 + b^2)*Sin[2*(e + f*x)])/(90*b^3) - Sin[4*(e + f*x)]/(36*b)))/
f
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

maple [C] time = 8.45, size = 4548, normalized size = 7.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x)
```

```
[Out] 20/21/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g
*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g
*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(s
in(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)+12/5/f*(g*(2*c
os(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x
+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x
+1/2*e)^2-1))^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*
e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)-20/21/f*(g*(2*cos(1/2*f*x+1/2
*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(
1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^
(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-
2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)-12/5/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/
2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^
2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticE
(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1
/2*e)^2+1)^(1/2)+1216/105/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e
)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*s
in(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^5
-1216/105/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b
/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e
)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^5-40/7/f*(g*(2*co
s(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+
1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+
1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3+40/7/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1
)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x
+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*c
os(1/2*f*x+1/2*e)^3+152/105/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2
*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)
*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)
-152/105/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/
(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)
/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)+64/7/f*(g*(2*cos(1
/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2
*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2
*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^9-64/7/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*s
in(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/
2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(
```

1/2*f*x+1/2*e)^9-576/35/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^7+576/35/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^7-8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3*a^2-8/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)*a^2+8/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)*a^2-16/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^5*a^2+16/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^5*a^2+8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3*a^2+1/4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^7/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum((sin(1/2*f*x+1/2*e)^2*(2*_alpha^2*a^2*b^2-2*_alpha^2*b^4-a^4+a^2*b^2)-2*_alpha^2*a^2*b^2+2*_alpha^2*b^4+a^4-a^2*b^2)/_alpha/(2*_alpha^2-1)*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*a^2+16/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*a^2-16/3/f*(g*(2*cos(1/2*f*x+1/2*e)^

```
2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/
2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1
/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*
cos(1/2*f*x+1/2*e)^2+1)^(1/2)*a^2-4/3/f*g^2*a^3/b^4*(2*cos(1/2*f*x+1/2*e)^2
*g-g)^(1/2)+2/f*g^2*a^3/b^4*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-2/f*g^2*a/
b^2*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-1/2/f*g^3*a^3/b^2*sum((_R^6-_R^4*g
-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*
g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1
/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*
g^3*_Z^2+b^2*g^4))+1/2/f*g^3*a^5/b^4*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b
^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*
x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*
_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+12/
7/f*g^2*a/b^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+16/7/f*g^2*a/b^2*cos(1/2*f
*x+1/2*e)^6*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-24/7/f*g^2*a/b^2*cos(1/2*f*x
+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+12/7/f*g^2*a/b^2*cos(1/2*f*x+1
/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-4/3/f*g^2*a^3/b^4*cos(1/2*f*x+1/
2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)
*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^5/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*
x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*
EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(
1/2*f*x+1/2*e)^2+1)^(1/2)*a^4-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x
+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))
^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticE(co
s(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*
e)^2+1)^(1/2)*a^2-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(
1/2)*g^3/b^5/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1
/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticF(cos(1/2*f*x+1/
2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2
)*a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^3\,\left(g\,\cos\left(e+f\,x\right)\right)^{5/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^3*(g*cos(e + f*x))^(5/2))/(a + b*sin(e + f*x)),x)
[Out] int((sin(e + f*x)^3*(g*cos(e + f*x))^(5/2))/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*sin(f*x+e)**3/(a+b*sin(f*x+e)),x)
[Out] Timed out
```

3.1384 $\int \frac{(g\cos(e+fx))^{5/2}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=501

$$\frac{2a^3g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{b^4f\sqrt{\cos(e+fx)}}+\frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f}+\frac{a^2g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}-\ldots$$

```
[Out] a^2*(-a^2+b^2)^(3/4)*g^(5/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)
^(1/4)/g^(1/2))/b^(9/2)/f-a^2*(-a^2+b^2)^(3/4)*g^(5/2)*arctanh(b^(1/2)*(g*c
os(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(9/2)/f+2/3*a^2*g*(g*cos(f*x+e
))^(3/2)/b^3/f-2/7*(g*cos(f*x+e))^(7/2)/b/f/g-2/5*a*g*(g*cos(f*x+e))^(3/2)*
sin(f*x+e)/b^2/f-a^3*(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x
+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos
(f*x+e)^(1/2)/b^5/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-a^3*(a^2-b^2)
*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x
+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^5/f/(b+(-a^2+b
^2)^(1/2))/(g*cos(f*x+e))^(1/2)+2*a^3*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(
1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b
^4/f/cos(f*x+e)^(1/2)-6/5*a*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/
2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b^2/f/cos(f
*x+e)^(1/2)
```

Rubi [A] time = 1.19, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2898, 2635, 2640, 2639, 2565, 30, 2695, 2867, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{a^2g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}-\frac{a^2g^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}+\frac{2a^3g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\ldots}}{b^4f\sqrt{\cos(e+f\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^2*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)*f) - (a^2*(-a^2 + b^2)^(3/4)*g^(5/2)*Arc
Tanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)
*f) + (2*a^2*g*(g*Cos[e + f*x])^(3/2))/(3*b^3*f) - (2*(g*Cos[e + f*x])^(7/2
))/(7*b*f*g) + (2*a^3*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(
b^4*f*Sqrt[Cos[e + f*x]]) - (6*a*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*
```

```
x)/2, 2])/(5*b^2*f*Sqrt[Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^3*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^5*(b - S
qrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^3*Sqrt[Cos[e
+ f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^5*(b +
 Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (2*a*g*(g*Cos[e + f*x])^(3/2)*
Sin[e + f*x])/(5*b^2*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(-\frac{a(g\cos(e+fx))^{5/2}}{b^2} + \frac{(g\cos(e+fx))^{5/2}\sin(e+fx)}{b} + \frac{a^2(g\cos(e\right.$$

$$= -\frac{a\int (g\cos(e+fx))^{5/2}\,dx}{b^2} + \frac{a^2\int \frac{(g\cos(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx}{b^2} + \frac{\int (g\cos(e+fx))^{5/2}}{b}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f} - \frac{2ag(g\cos(e+fx))^{3/2}\sin(e+fx)}{5b^2f} - \frac{\text{Subst}\left(\int x\right.}{}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f} - \frac{2(g\cos(e+fx))^{7/2}}{7bfg} - \frac{2ag(g\cos(e+fx))^{3/2}\sin}{5b^2f}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f} - \frac{2(g\cos(e+fx))^{7/2}}{7bfg} - \frac{6ag^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}\right.}{5b^2f\sqrt{\cos(e+f}}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f} - \frac{2(g\cos(e+fx))^{7/2}}{7bfg} + \frac{2a^3g^2\sqrt{g\cos(e+fx)}\,E\left(\right.}{b^4f\sqrt{\cos(e+f}}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f} - \frac{2(g\cos(e+fx))^{7/2}}{7bfg} + \frac{2a^3g^2\sqrt{g\cos(e+fx)}\,E\left(\right.}{b^4f\sqrt{\cos(e+f}}$$

$$= \frac{a^2\left(-a^2+b^2\right)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{9/2}f} - \frac{a^2\left(-a^2+b^2\right)^{3/4}g^{5/2}\tanh^{-}}{b^{9/2}f}$$

Mathematica [C] time = 27.12, size = 824, normalized size = 1.64

$$a\left(-\frac{\left(5a^2-3b^2\right)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(8F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)\cos^{\frac{3}{2}}(e+fx)b^{5/2}+3\sqrt{2}\,a\left(a^2-b^2\right)^{3/4}\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)-2\right.\right.}{12b^{3/2}\left(b^2-a^2\right)\left(\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(g*Cos[e + f*x])^(5/2)*((-4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*App
ellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*
Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*S
qrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[S
qrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I
*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Co
s[e + f*x]^2]*(a + b*Sin[e + f*x])) - ((5*a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[
e + f*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Co
s[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3
/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] -
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log
[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b
*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Cos[e + f*x]] + b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 + b^
2)*(1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(5*b^3*f*Cos[e + f*x]^(5/2)
) + ((g*Cos[e + f*x])^(5/2)*Sec[e + f*x]^2*(-1/42*((-28*a^2 + 9*b^2)*Cos[e
+ f*x])/b^3 - Cos[3*(e + f*x)]/(14*b) - (a*Sin[2*(e + f*x)])/(5*b^2)))/f
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

maple [C] time = 6.96, size = 1937, normalized size = 3.87

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x)
```

```
[Out] -16/7/f*g^2/b*cos(1/2*f*x+1/2*e)^6*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+24/7/
f*g^2/b*cos(1/2*f*x+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-12/7/f*g^2/
b*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-12/7/f*g^2/b*(2*c
os(1/2*f*x+1/2*e)^2*g-g)^(1/2)+4/3/f*g^2/b^3*cos(1/2*f*x+1/2*e)^2*(2*cos(1/
2*f*x+1/2*e)^2*g-g)^(1/2)*a^2+4/3/f*g^2/b^3*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1
/2)*a^2-2/f*g^2/b^3*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^2+2/f*g^2/b*(g*(
2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-1/2/f*g^3/b^3*a^4*sum((_R^6-_R^4*g-_R^2*g^
2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln(
(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),
_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2
+b^2*g^4))+1/2/f*g^3/b*a^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*
b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2
*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2
*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+16/5/f*(g*(2*
cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3*a/b^2/(-g*(2*sin(1/
2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/
2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^7-32/5/f*(g*(2*cos(1/2*f*x+1/2*
e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3*a/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-s
in(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1
))^(1/2)*cos(1/2*f*x+1/2*e)^5+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x
+1/2*e)^2)^(1/2)*g^3*a/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2
))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*
x+1/2*e)^3+2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^
3*a^3/b^4/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*
f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2
)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-6
/5/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3*a/b^2/(-
g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(
g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/
2*f*x+1/2*e)^2+1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-4/5/f*(g*(2*c
os(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3*a/b^2/(-g*(2*sin(1/2
*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2
*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)+1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2
-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3*a/b^6/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*
f*x+1/2*e)^2-1))^(1/2)*sum((a^2-b^2)/_alpha*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)
*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2
*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^
2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/
2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_al
pha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4
*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alp
```

```
ha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*s
in(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*
(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)
/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4
*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^2\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^2*(g*cos(e + f*x))^(5/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)^2*(g*cos(e + f*x))^(5/2))/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*sin(f*x+e)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1385 $\int \frac{(g\cos(e+fx))^{5/2}\sin(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=413

$$-\frac{ag^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}+\frac{ag^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}+\frac{a^2g^3\left(a^2-b^2\right)\sqrt{\cos(e+f}}{b^4f\left(b-\sqrt{b^2-\right.}}$$

```
[Out] -a*(-a^2+b^2)^(3/4)*g^(5/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^
(1/4)/g^(1/2))/b^(7/2)/f+a*(-a^2+b^2)^(3/4)*g^(5/2)*arctanh(b^(1/2)*(g*cos(
f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(7/2)/f-2/15*g*(g*cos(f*x+e))^(3/
2)*(5*a-3*b*sin(f*x+e))/b^2/f+a^2*(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2
)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2))
,2^(1/2))*cos(f*x+e)^(1/2)/b^4/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+
a^2*(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticP
i(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^4
/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-2/5*(5*a^2-3*b^2)*g^2*(cos(1/2
*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2
))*(g*cos(f*x+e))^(1/2)/b^3/f/cos(f*x+e)^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2865, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{ag^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}+\frac{ag^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}-\frac{2g^2\left(5a^2-3b^2\right)E\left(\frac{1}{2}(e+\right.}{5b^3f\sqrt{\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*f)) + (a*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcT
anh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*
f) - (2*(5*a^2 - 3*b^2)*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])
/(5*b^3*f*Sqrt[Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*Ell
ipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(b - Sqrt[-a^2
+ b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*E
llipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(b + Sqrt[-a^
```

```
2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (2*g*(g*Cos[e + f*x])^(3/2)*(5*a - 3*b*
Sin[e + f*x]))/(15*b^2*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\sin(e+fx)}{a+b\sin(e+fx)}\,dx = -\frac{2g(g\cos(e+fx))^{3/2}(5a-3b\sin(e+fx))}{15b^2f} + \frac{(2g^2)\int \frac{\sqrt{g\cos(e+fx)}\left(-ab-\frac{1}{2}(5a^2\right.}{a+b\sin(e+f}}{5b^2}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}(5a-3b\sin(e+fx))}{15b^2f} - \frac{((5a^2-3b^2)g^2)\int\sqrt{g\cos(e}}{5b^3}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}(5a-3b\sin(e+fx))}{15b^2f} - \frac{(a^2(a^2-b^2)g^3)\int\frac{}{\sqrt{g\cos(e+fx}}}{2b^4}$$

$$= -\frac{2(5a^2-3b^2)g^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^3f\sqrt{\cos(e+fx)}} - \frac{2g(g\cos(e+fx))^{3/2}(5}{15b^2}$$

$$= -\frac{2(5a^2-3b^2)g^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^3f\sqrt{\cos(e+fx)}} + \frac{a^2(a^2-b^2)g^3\sqrt{\cos(e}}{b^4\left(b-\sqrt{-a^2}\right.}$$

$$= -\frac{a(-a^2+b^2)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{7/2}f} + \frac{a(-a^2+b^2)^{3/4}g^{5/2}\tanh^{-1}\left(\right.}{b^{7/2}f}$$

Mathematica [C] time = 24.81, size = 737, normalized size = 1.78

$$\frac{(g\cos(e+fx))^{5/2}\left(\frac{4a\sin(e+fx)\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(\frac{a\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)}{3(a^2-b^2)}+\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(-\log\left(-(1+i)\sqrt{b}\sqrt[4]{b^2-a^2}\sqrt{\cos(e+fx)}+\sqrt{b}\right.\right.}{}\right)}{b\sqrt{\sin^2(e+f}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((2*Cos[e + f*x]^(3/2)*(-5*a + 3*b*Sin[e + f*x]))/(
3*b^2) + ((5*a^2 - 3*b^2)*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*
x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(
```

```
a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 -
b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)
^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[
e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*(a + b*Sqrt[Sin[e + f*x]^
2]))/(12*b^(7/2)*(-a^2 + b^2)*(a + b*Sin[e + f*x])) + (4*a*((a*AppellF1[3/4
, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f
*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]
*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[-a^2
+ b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e
+ f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x]*(a + b*Sqrt[Sin[e + f*
x]^2]))/(b*Sqrt[Sin[e + f*x]^2]*(a + b*Sin[e + f*x]))))/(5*f*Cos[e + f*x]^(
5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

maple [C] time = 7.89, size = 2612, normalized size = 6.32

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] -4/3/f*g^2*a/b^2*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-4/
3/f*g^2*a/b^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+2/f*g^2*a/b^2*(g*(2*cos(1/
2*f*x+1/2*e)^2-1))^(1/2)+1/2/f*g^3*a^3/b^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(
_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(
1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootO
f(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4
))-1/2/f*g^3*a*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3
*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-
cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*
a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-16/3/f*(g*(2*cos(1/2*f*x+
1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-s
in(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1
))^(1/2)*cos(1/2*f*x+1/2*e)^5+16/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*
f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2)
)^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x
+1/2*e)^5+8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3
/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2
*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3+4/f*(g*(2*cos
(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x
+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x
+1/2*e)^2-1))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*
e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*a^2-16/3/f*(g*(2*cos(1/2*f*x+
1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-s
in(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1
))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)
*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/
2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^
2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticE
(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1
/2*e)^2+1)^(1/2)-8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1
/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*
f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3-8/3/f*
(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(
1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(
1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)-4/f*(g*(2*cos(1/2*f*x+1/2*e)^
2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/
2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1
/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*
cos(1/2*f*x+1/2*e)^2+1)^(1/2)*a^2+16/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(
1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e
)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*Ellipti
cF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x
+1/2*e)^2+1)^(1/2)-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^
(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/
2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticE(cos(1/2*f*x+1/2
```

```
*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)
+8/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*
(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*
(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)+1/4/f*(g*(2*cos(1/2*f*
x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^5/a^2/sin(1/2*f*x+1/2*e)/(g
*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum((sin(1/2*f*x+1/2*e)^2*(2*_alpha^2*a^
2*b^2-2*_alpha^2*b^4-a^4+a^2*b^2)-2*_alpha^2*a^2*b^2+2*_alpha^2*b^4+a^4-a^2
*b^2)/_alpha/(2*_alpha^2-1)*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x
+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^
(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(
sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(co
s(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*
b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*c
os(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2
)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e
)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1
/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1
/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2
*b^2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)\,\left(g\,\cos\left(e+f\,x\right)\right)^{5/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)*(g*cos(e + f*x))^(5/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)*(g*cos(e + f*x))^(5/2))/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1386 $\int \frac{(g\cos(e+fx))^{5/2}\csc(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=425

$$\frac{g^3\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{g^3\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}-\frac{g^5}{\ldots}$$

```
[Out] g^(5/2)*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f-(-a^2+b^2)^(3/4)*g^(5/2)*a
rctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/b^(3/2)/f-g^
(5/2)*arctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f+(-a^2+b^2)^(3/4)*g^(5/2)*ar
ctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/b^(3/2)/f+(a
^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(
1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/f/(b-
(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(
1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(
1/2)-2*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/
2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b/f/cos(f*x+e)^(1/2)
```

Rubi [A] time = 1.15, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.516, Rules used = {2898, 2565, 321, 329, 298, 203, 206, 2695, 2867, 2640, 2639, 2701, 2807, 2805, 205, 208}

$$-\frac{g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{ab^{3/2}f}+\frac{g^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{ab^{3/2}f}+\frac{g^3\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\,\Pi\ldots}{b^2f\left(b-\sqrt{b^2-a^2}\right)\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (g^(5/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) - ((-a^2 + b^2)^(3/4)*
g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])]
)/(a*b^(3/2)*f) - (g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) + (
(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[g])])/(a*b^(3/2)*f) - (2*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE
[(e + f*x)/2, 2])/(b*f*Sqrt[Cos[e + f*x]]) + ((a^2 - b^2)*g^3*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(b - S
qrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^3*Sqrt[Cos[e + f*
```

```
x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(b + Sqr
t[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\csc(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{(g\cos(e+fx))^{5/2}\csc(e+fx)}{a} - \frac{b(g\cos(e+fx))^{5/2}}{a(a+b\sin(e+fx))}\right)dx$$

$$= \frac{\int (g\cos(e+fx))^{5/2}\csc(e+fx)\,dx}{a} - \frac{b\int \frac{(g\cos(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx}{a}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3af} - \frac{\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{g^2}}\,dx, x, g\cos(e+fx)\right)}{afg} - \frac{g^2\int \frac{\sqrt{g\cos}}{}}{}$$

$$= -\frac{g\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{g^2}}\,dx, x, g\cos(e+fx)\right)}{af} - \frac{g^2\int \sqrt{g\cos(e+fx)}\,dx}{b} - \frac{((-a^2}{}$$

$$= -\frac{(2g)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{g^2}}\,dx, x, \sqrt{g\cos(e+fx)}\right)}{af} - \frac{\left(\left(a^2-b^2\right)g^3\right)\int \frac{}{\sqrt{g\cos(e+fx}}}{2b^2}$$

$$= -\frac{2g^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{\cos(e+fx)}} - \frac{g^3\,\text{Subst}\left(\int \frac{1}{g-x^2}\,dx, x, \sqrt{g\cos(e}\right.}{af}$$

$$= \frac{g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{g^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{2g^2\sqrt{g\cos(e+fx}}{bf\sqrt{\cos(}}$$

$$= \frac{g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{\left(-a^2+b^2\right)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{ab^{3/2}f} - \frac{g^{5/2}}{}$$

Mathematica [C] time = 24.46, size = 484, normalized size = 1.14

$$\frac{\csc(e+fx)(g\cos(e+fx))^{5/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(8ab^{5/2}\cos^{\frac{7}{2}}(e+fx)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]*(8*a*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/
4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(7/2) +
7*(a^2 - b^2)*(-2*Sqrt[2]*(a^2 - b^2)^(3/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqr
t[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*(a^2 - b^2)^(3/4)*ArcTan[1
+ (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*b^(3/2)*ArcTa
n[Sqrt[Cos[e + f*x]]] + 2*b^(3/2)*Log[1 - Sqrt[Cos[e + f*x]]] - 2*b^(3/2)*L
og[1 + Sqrt[Cos[e + f*x]]] + Sqrt[2]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2]
- Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] -
Sqrt[2]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*(a + b*Sqrt[Sin[e + f*x]^2]))
/(28*a*b^(3/2)*(a^2 - b^2)*f*Cos[e + f*x]^(5/2)*(b + a*Csc[e + f*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

maple [A] time = 3.17, size = 259, normalized size = 0.61

$$-\frac{g^{\frac{5}{2}}\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}+4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2af}-\frac{g^{\frac{5}{2}}\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}-4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{2af}-\frac{4\sqrt{-2\left(\sin^2\left(\frac{fx}{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x)
```

[Out] -1/2/a/f*g^(5/2)*ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))-1/2/a/f*g^(5/2)*ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))-4/3/a/f*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)*g^2*sin(1/2*f*x+1/2*e)^2+2/3/a/f*g^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/a/(-g)^(1/2)/f*g^3*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)/(b*sin(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{5/2}}{\sin\left(e+f\,x\right)\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(e + f*x))^(5/2)/(sin(e + f*x)*(a + b*sin(e + f*x))),x)

[Out] int((g*cos(e + f*x))^(5/2)/(sin(e + f*x)*(a + b*sin(e + f*x))), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))**(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x)

[Out] Timed out

3.1387 $\int \frac{(g\cos(e+fx))^{5/2}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=462

$$\frac{g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2\sqrt{b}\,f}-\frac{g^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2\sqrt{b}\,f}-\frac{g^3\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\,\Pi\left(\cdots\right.}{abf\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cdots}}$$

```
[Out] -b*g^(5/2)*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f+b*g^(5/2)*arctanh((g*
cos(f*x+e))^(1/2)/g^(1/2))/a^2/f-g*(g*cos(f*x+e))^(3/2)*csc(f*x+e)/a/f+(-a^
2+b^2)^(3/4)*g^(5/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g
^(1/2))/a^2/f/b^(1/2)-(-a^2+b^2)^(3/4)*g^(5/2)*arctanh(b^(1/2)*(g*cos(f*x+e
))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/f/b^(1/2)-(a^2-b^2)*g^3*(cos(1/2*f*x
+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-
a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/b/f/(b-(-a^2+b^2)^(1/2))/(g*cos
(f*x+e))^(1/2)-(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e
)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e
)^(1/2)/a/b/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-g^2*(cos(1/2*f*x+1/
2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*c
os(f*x+e))^(1/2)/a/f/cos(f*x+e)^(1/2)
```

Rubi [A] time = 1.26, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 17, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.515, Rules used = {2898, 2565, 321, 329, 298, 203, 206, 2567, 2640, 2639, 2695, 2867, 2701, 2807, 2805, 205, 208}

$$\frac{g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2\sqrt{b}\,f}-\frac{g^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2\sqrt{b}\,f}-\frac{g^3\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\,\Pi\left(\cdots\right.}{abf\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((b*g^(5/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f)) + ((-a^2 + b^2)
^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[g])])/(a^2*Sqrt[b]*f) + (b*g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]
)/(a^2*f) - ((-a^2 + b^2)^(3/4)*g^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x
]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*Sqrt[b]*f) - (g*(g*Cos[e + f*x])^(3
/2)*Csc[e + f*x])/(a*f) - (g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2,
2])/(a*f*Sqrt[Cos[e + f*x]]) - ((a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*Elliptic
Pi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*b*(b - Sqrt[-a^2 + b^2
```

```
])*f*Sqrt[g*Cos[e + f*x]]) - ((a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*EllipticPi
[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*b*(b + Sqrt[-a^2 + b^2])
*f*Sqrt[g*Cos[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(-\frac{b(g\cos(e+fx))^{5/2}\csc(e+fx)}{a^2}+\frac{(g\cos(e+fx))^{5/2}\csc^2(e+fx)}{a}+\right.$$

$$=\frac{\int(g\cos(e+fx))^{5/2}\csc^2(e+fx)\,dx}{a}-\frac{b\int(g\cos(e+fx))^{5/2}\csc(e+fx)\,dx}{a^2}$$

$$=\frac{2bg(g\cos(e+fx))^{3/2}}{3a^2f}-\frac{g(g\cos(e+fx))^{3/2}\csc(e+fx)}{af}+\frac{b\,\mathrm{Subst}\left(\int\frac{x^{5/2}}{1-\frac{x^2}{g^2}}\right.}{}$$

$$=-\frac{g(g\cos(e+fx))^{3/2}\csc(e+fx)}{af}+\frac{(bg)\,\mathrm{Subst}\left(\int\frac{\sqrt{x}}{1-\frac{x^2}{g^2}}\,dx,x,g\cos(e+fx)\right.}{a^2f}$$

$$=-\frac{g(g\cos(e+fx))^{3/2}\csc(e+fx)}{af}-\frac{3g^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{\cos(e+fx)}}+$$

$$=-\frac{g(g\cos(e+fx))^{3/2}\csc(e+fx)}{af}-\frac{g^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{\cos(e+fx)}}+$$

$$=-\frac{bg^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}+\frac{bg^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}-\frac{g(g\cos(e+fx}{}$$

$$=-\frac{bg^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}+\frac{\left(-a^2+b^2\right)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^2\sqrt{b}\,f}+$$

Mathematica [C] time = 27.28, size = 1556, normalized size = 3.37

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((12*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*AppellF
1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[
```

```
e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[
b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*
Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[
-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*C
os[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/(Sqrt[1 - Cos[e + f*x]^2]*(b
+ a*Csc[e + f*x])) + (5*b*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x
]^2])*Csc[e + f*x]*(6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - (Sqrt[2]
*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 6*Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4
)] + 12*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] + 8*a*b*AppellF1[3/4, 1/2, 1
, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2
) + 6*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*b^2*Log[1 - Sqrt[Cos[e + f*x]]] -
 6*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 6*b^2*Log[1 + Sqrt[Cos[e + f*x]]] - 3*
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^
2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + 3*Sqrt[2]*Sqrt[b]*(a^
2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt
[Cos[e + f*x]] + b*Cos[e + f*x]]))/(12*(a^3 - a*b^2)*(1 - Cos[e + f*x]^2)*(
b + a*Csc[e + f*x])) - ((-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^
2])*Cos[2*(e + f*x)]*Csc[e + f*x]*(-42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b
^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 42
*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[C
os[e + f*x]])/(a^2 - b^2)^(1/4)] + 84*b^(3/2)*(a^2 - b^2)*ArcTan[Sqrt[Cos[e
 + f*x]]] - 56*a*b^(5/2)*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Co
s[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 48*a*b^(5/2)*AppellF1[7/4,
 1/2, 1, 11/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f
*x]^(7/2) + 42*b^(3/2)*(a^2 - b^2)*Log[1 - Sqrt[Cos[e + f*x]]] + 42*b^(3/2)
*(-a^2 + b^2)*Log[1 + Sqrt[Cos[e + f*x]]] + 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2
*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Co
s[e + f*x]] + b*Cos[e + f*x]] - 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*
Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]]
+ b*Cos[e + f*x]]))/(84*Sqrt[b]*(a^3 - a*b^2)*(1 - Cos[e + f*x]^2)*(-1 + 2*
Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a*f*Cos[e + f*x]^(5/2)) - ((g*Co
s[e + f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x])/(a*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

maple [C] time = 14.20, size = 1987, normalized size = 4.30

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/2/f*g^(5/2)*b/a^2*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/
2*e)^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))-1/2/f*g^3*b*sum((_R^6-_R^4*
g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2
*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(
1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2
*g^3*_Z^2+b^2*g^4))+1/2/f*g^3*b^3/a^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*
b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f
*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2
*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1/
f*g^3*b/a^2/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(
1/2))/cos(1/2*f*x+1/2*e))+1/2/f*g^(5/2)*b/a^2*ln((-4*g*cos(1/2*f*x+1/2*e)+2
*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)+1))-1
/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/a/sin(1/2*
f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(2*sin(1/2*f*x+1/2*e)^2-1)*
(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)
+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/a/sin(
1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(2*sin(1/2*f*x+1/2*e)
^2-1)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*Ellipt
icF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)+1/2/f*(g*(2*co
s(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/a/sin(1/2*f*x+1/2*e)^
3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(-2
*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x
+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e
)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/a/sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2
*f*x+1/2*e)^2-1))^(1/2)/(2*sin(1/2*f*x+1/2*e)^2-1)*(-2*sin(1/2*f*x+1/2*e)^4
```

```
*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)-1/8/f*(g*(2*cos(1/2*f*x
+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/a/sin(1/2*f*x+1/2*e)/(g*(2*cos
(1/2*f*x+1/2*e)^2-1))^(1/2)/b^2*sum((-a^2+b^2)/_alpha*(2^(1/2)/(g*(2*_alpha
^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*
cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^
2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*
e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f
*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2
*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a
^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/f*(g*
(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/a*sin(1/2*f*x+1/
2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*
f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)
^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/a/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*
x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/
2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticF(
cos(1/2*f*x+1/2*e),2^(1/2))-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x
+1/2*e)^2)^(1/2)*g^3/a/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1
/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(sin(1/2*f*x+1
/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e
),2^(1/2))+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*
g^3/a*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/cos(1/2*f*x+1
/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{5/2}}{\sin\left(e+f\,x\right)^{2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/(sin(e + f*x)^2*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/(sin(e + f*x)^2*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*csc(f*x+e)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1388 $\int \frac{(g\cos(e+fx))^{5/2}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=557

$$\frac{b^2g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}-\frac{b^2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{g^3\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{a^2f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+$$

```
[Out] -3/4*g^(5/2)*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f+b^2*g^(5/2)*arctan((g
*cos(f*x+e))^(1/2)/g^(1/2))/a^3/f+3/4*g^(5/2)*arctanh((g*cos(f*x+e))^(1/2)/
g^(1/2))/a/f-b^2*g^(5/2)*arctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a^3/f+b*g*(g
*cos(f*x+e))^(3/2)*csc(f*x+e)/a^2/f-1/2*g*(g*cos(f*x+e))^(3/2)*csc(f*x+e)^2
/a/f-(-a^2+b^2)^(3/4)*g^(5/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2
)^(1/4)/g^(1/2))*b^(1/2)/a^3/f+(-a^2+b^2)^(3/4)*g^(5/2)*arctanh(b^(1/2)*(g*
cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*b^(1/2)/a^3/f+(a^2-b^2)*g^3*(co
s(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),
2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a^2/f/(b-(-a^2+b^2)^(1/2
))/(g*cos(f*x+e))^(1/2)+(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*
f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*
cos(f*x+e)^(1/2)/a^2/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+b*g^2*(cos
(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^
(1/2))*(g*cos(f*x+e))^(1/2)/a^2/f/cos(f*x+e)^(1/2)
```

Rubi [A] time = 1.34, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2898, 2565, 321, 329, 298, 203, 206, 2567, 2640, 2639, 288, 2695, 2867, 2701, 2807, 2805, 205, 208}

$$\frac{b^2g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}-\frac{\sqrt{b}\,g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f}-\frac{b^2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{\sqrt{b}\,g^{5/2}\left(b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (-3*g^(5/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) + (b^2*g^(5/2)*Ar
cTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) - (Sqrt[b]*(-a^2 + b^2)^(3/4)*g
^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(a^3*f) + (3*g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2
*g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (Sqrt[b]*(-a^2 +
b^2)^(3/4)*g^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/
```

```
4)*Sqrt[g])])/(a^3*f) + (b*g*(g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a^2*f) -
 (g*(g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^2)/(2*a*f) + (b*g^2*Sqrt[g*Cos[e +
f*x]]*EllipticE[(e + f*x)/2, 2])/(a^2*f*Sqrt[Cos[e + f*x]]) + ((a^2 - b^2)*
g^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2
, 2])/(a^2*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^
3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2,
2])/(a^2*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
```

```
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
```

```
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{b^2(g\cos(e+fx))^{5/2}\csc(e+fx)}{a^3} - \frac{b(g\cos(e+fx))^{5/2}\csc^2(e+fx)}{a^2} + \right.$$

$$= \frac{\int (g\cos(e+fx))^{5/2}\csc^3(e+fx)\,dx}{a} - \frac{b\int (g\cos(e+fx))^{5/2}\csc^2(e+fx)}{a^2}$$

$$= -\frac{2b^2g(g\cos(e+fx))^{3/2}}{3a^3f} + \frac{bg(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2f} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{bg(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2f} - \frac{g(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{2af} + \frac{(3}{}$$

$$= \frac{bg(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2f} - \frac{g(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{2af} + \frac{3b}{}$$

$$= \frac{bg(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2f} - \frac{g(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{2af} + \frac{bg}{}$$

$$= -\frac{3g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} + \frac{b^2g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f} + \frac{3g^{5/2}\tanh^{-1}}{4a}$$

$$= -\frac{3g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} + \frac{b^2g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f} - \frac{\sqrt{b}\left(-a^2+b^2\right.}{}$$

Mathematica [C] time = 29.42, size = 1590, normalized size = 2.85

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/4*((g*Cos[e + f*x])^(5/2)*((6*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*A
ppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)
]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)
*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sq
rt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log
[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] +
 I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/(Sqrt[1 - Cos[e + f*x]^
2]*(b + a*Csc[e + f*x])) - ((3*a^2 - 5*b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sq
rt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*A
rcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 6*Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]]
)/(a^2 - b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] + 8*a*b*Ap
pellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
*Cos[e + f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*b^2*Log[1 - Sqr
t[Cos[e + f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 6*b^2*Log[1 + Sqrt[C
os[e + f*x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] - S
qrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + 3*S
qrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(12*(a^3 - a*b^2)*(1 -
 Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (Sqrt[b]*(-1 + Cos[e + f*x]^2)*(a
+ b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*(-42*Sqrt[2]*(a
^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]
])/(a^2 - b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1
 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 84*b^(3/2)*(a^
2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*AppellF1[3/4, 1/2, 1, 7/
4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) +
48*a*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2 - b^2)*Log[1 - Sqrt[C
os[e + f*x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[Cos[e + f*x]]] + 21*Sq
rt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]
*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] - 21*Sqrt[2]*(a^2 -
 b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*(a^3 - a*b^2)*(1 - Cos[e
+ f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(a^2*f*Cos[e + f*
x]^(5/2)) + ((g*Cos[e + f*x])^(5/2)*((b*Cot[e + f*x])/a^2 - (Cot[e + f*x]*C
sc[e + f*x])/(2*a))*Sec[e + f*x]^2)/f
```

fricas [F] time = 124.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}g^2\cos\left(fx+e\right)^2\csc\left(fx+e\right)^3}{b\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*g^2*cos(f*x + e)^2*csc(f*x + e)^3/(b*sin(f*x
+ e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

maple [A] time = 3.07, size = 318, normalized size = 0.57

$$\frac{3g^{\frac{5}{2}}\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}+4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{8fa}+\frac{g^{2}\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}}{16fa\left(-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}+\frac{3g^{3}\ln\left(\frac{-2g+2\sqrt{-g}\,\sqrt{2\left(\cos^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{4fa\sqrt{-g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x)
```

```
[Out] 3/8/f*g^(5/2)/a*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)
^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))+1/16/f*g^2/a/(-1+cos(1/2*f*x+1/
2*e))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+3/4/f*g^3/a/(-g)^(1/2)*ln((-2*g+2
*(-g)^(1/2)*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))+3/8/f*g
^(5/2)/a*ln((-4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g
)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)+1))+1/8/f*g^2/a/cos(1/2*f*x+1/2*e)^2*(2*co
s(1/2*f*x+1/2*e)^2*g-g)^(1/2)-1/16/f*g^2/a/(cos(1/2*f*x+1/2*e)+1)*(-2*sin(1
/2*f*x+1/2*e)^2*g+g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{5/2}}{\sin\left(e+f\,x\right)^3\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/(sin(e + f*x)^3*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/(sin(e + f*x)^3*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*csc(f*x+e)**3/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1389 $\int \frac{\sin^4(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=509

$$-\frac{2a^3\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\sqrt{g\cos(e+fx)}}-\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg}+\frac{a^5\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^5\sqrt{\text{co}}}{b^4f\left(a^2\right.}$$

```
[Out] 2/5*(g*cos(f*x+e))^(5/2)/b/f/g^3-a^4*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-
a^2+b^2)^(1/4)/g^(1/2))/b^(7/2)/(-a^2+b^2)^(3/4)/f/g^(1/2)-a^4*arctanh(b^(1
/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(7/2)/(-a^2+b^2)^(3/4)
/f/g^(1/2)-2*a^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(
sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/b^4/f/(g*cos(f*x+e))^(1/2)-4/3
*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/
2*e),2^(1/2))*cos(f*x+e)^(1/2)/b^2/f/(g*cos(f*x+e))^(1/2)+a^5*(cos(1/2*f*x+
1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a
^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^4/f/(a^2-b*(b-(-a^2+b^2)^(1/2)))
/(g*cos(f*x+e))^(1/2)+a^5*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*E
llipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(
1/2)/b^4/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-2*a^2*(g*cos(f
*x+e))^(1/2)/b^3/f/g-2*(g*cos(f*x+e))^(1/2)/b/f/g+2/3*a*sin(f*x+e)*(g*cos(f
*x+e))^(1/2)/b^2/f/g
```

Rubi [A] time = 1.51, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2909, 2565, 14, 2568, 2642, 2641, 30, 2867, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg}-\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{2a^3\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^4*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(7/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[g])) - (a^4*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*(-a^2 + b^2)^(3/4)*f*Sq
rt[g]) - (2*a^2*Sqrt[g*Cos[e + f*x]])/(b^3*f*g) - (2*Sqrt[g*Cos[e + f*x]])/
(b*f*g) + (2*(g*Cos[e + f*x])^(5/2))/(5*b*f*g^3) - (2*a^3*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2])/(b^4*f*Sqrt[g*Cos[e + f*x]]) - (4*a*Sqrt[Cos[e
```

```
 + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*b^2*f*Sqrt[g*Cos[e + f*x]]) + (a^5*S
qrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])
/(b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^5*Sqrt[
Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^
4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (2*a*Sqrt[g*Co
s[e + f*x]]*Sin[e + f*x])/(3*b^2*f*g)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sin^3(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{a\int \frac{\sin^3(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{b}$$

$$= -\frac{a\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} + \frac{a^2\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{b^2} - \frac{\text{Subst}\Big(\int \frac{1\cdots}{\cdots}}{\cdots}$$

$$= \frac{2a\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{3b^2fg} + \frac{a^2\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b^3} - \frac{a^3\int \frac{\cdots}{\sqrt{g\cos(e+\cdots}}}{\cdots}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{bfg} + \frac{2(g\cos(e+fx))^{5/2}}{5bfg^3} + \frac{2a\sqrt{g\cos(e+fx)}\,\sin(e\cdots}{3b^2fg}$$

$$= -\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg} - \frac{2\sqrt{g\cos(e+fx)}}{bfg} + \frac{2(g\cos(e+fx))^{5/2}}{5bfg^3} - \frac{4a\cdots}{\cdots}$$

$$= -\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg} - \frac{2\sqrt{g\cos(e+fx)}}{bfg} + \frac{2(g\cos(e+fx))^{5/2}}{5bfg^3} - \frac{2a\cdots}{\cdots}$$

$$= -\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg} - \frac{2\sqrt{g\cos(e+fx)}}{bfg} + \frac{2(g\cos(e+fx))^{5/2}}{5bfg^3} - \frac{2a\cdots}{\cdots}$$

$$= -\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{7/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} - \frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{7/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} - \frac{2a^2\sqrt{g\,\mathrm{co}\cdots}}{b^3\cdots}$$

Mathematica [C] time = 26.92, size = 1953, normalized size = 3.84

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]*(Cos[2*(e + f*x)]/(5*b) + (2*a*Sin[e + f*x])/(3*b^2)))/(f*Sqr
t[g*Cos[e + f*x]]) - (Sqrt[Cos[e + f*x]]*((-2*(10*a^2 - 27*b^2)*(a + b*Sqrt
[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e +
```

f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*x])) + ((30*a^2 + 27*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*((((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(-1 + 2*Cos[e + f*x]^2)*(a + b*Sin[e + f*x])) + (28*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(60*b^2*f*Sqrt[g*Cos[e + f*x]])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^4}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^4/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 6.87, size = 1455, normalized size = 2.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 8/5/f/b*cos(1/2*f*x+1/2*e)^4/g*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-8/5/f/b*c
os(1/2*f*x+1/2*e)^2/g*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-8/5/f/b/g*(2*cos(1
/2*f*x+1/2*e)^2*g-g)^(1/2)-2/f/b^3*a^2/g*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/
2)+2/f*a^4/b^3*g*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*
_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x
+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*
b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-8/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*
sin(1/2*f*x+1/2*e)^2)^(1/2)*a/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1
/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*c
os(1/2*f*x+1/2*e)+4/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)
^(1/2)*a/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1
/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)+2/f*(
g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a^3/b^4/(-g*(2*sin
(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos
```

```
(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*
f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)+4/3/f*(g*(2*cos(1/2*f*
x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4
-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2
-1))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/
2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)-1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin
(1/2*f*x+1/2*e)^2)^(1/2)*a^3/b^6/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/
2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(
1/_alpha/(2*_alpha^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/
2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f
*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(s
in(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(
1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-
2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2
)^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(4*a^2-3*b
^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^
2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2
*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2
*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(
1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=Root
Of(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e
)^2*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sin\left(e+f\,x\right)}^4}{\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)^4/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
[Out] Timed out
```

3.1390 $\int \frac{\sin^3(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=457

$$\frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{g\cos(e+fx)}}-\frac{a^4\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}-\frac{a^4\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\right.}{b^3f\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g}}$$

```
[Out] a^3*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(5/2)/(
-a^2+b^2)^(3/4)/f/g^(1/2)+a^3*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^
2)^(1/4)/g^(1/2))/b^(5/2)/(-a^2+b^2)^(3/4)/f/g^(1/2)+2*a^2*(cos(1/2*f*x+1/2
*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f
*x+e)^(1/2)/b^3/f/(g*cos(f*x+e))^(1/2)+4/3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos
(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/b/f/
(g*cos(f*x+e))^(1/2)-a^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*El
lipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1
/2)/b^3/f/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-a^4*(cos(1/2*f*
x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(
-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^3/f/(a^2-b*(b+(-a^2+b^2)^(1/2)
))/(g*cos(f*x+e))^(1/2)+2*a*(g*cos(f*x+e))^(1/2)/b^2/f/g-2/3*sin(f*x+e)*(g*
cos(f*x+e))^(1/2)/b/f/g
```

Rubi [A] time = 1.18, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {2909, 2568, 2642, 2641, 2565, 30, 2867, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{g\cos(e+fx)}}-\frac{a^4\sqrt{\cos(e+fx)}\,\Pi\left(\frac{}{b-}\right.}{b^3f\left(a^2-b\left(b-\sqrt{b^2-}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a^3*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
b^(5/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) + (a^3*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[
g]) + (2*a*Sqrt[g*Cos[e + f*x]])/(b^2*f*g) + (2*a^2*Sqrt[Cos[e + f*x]]*Elli
pticF[(e + f*x)/2, 2])/(b^3*f*Sqrt[g*Cos[e + f*x]]) + (4*Sqrt[Cos[e + f*x]]
*EllipticF[(e + f*x)/2, 2])/(3*b*f*Sqrt[g*Cos[e + f*x]]) - (a^4*Sqrt[Cos[e
+ f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2
```

```
 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (a^4*Sqrt[Cos[e + f*
x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 - b
*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]]*
Sin[e + f*x])/(3*b*f*g)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
```

```
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
```

```
^2, 0]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{a\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{b}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{3bfg} - \frac{a\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} + \frac{a^2\int \frac{\text{s}}{\sqrt{g\cos(e+}}}{}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{3bfg} + \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{b^3} - \frac{a^3\int}{\sqrt{g\cos(e}}$$

$$= \frac{2a\sqrt{g\cos(e+fx)}}{b^2fg} + \frac{4\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3bf\sqrt{g\cos(e+fx)}} - \frac{2\sqrt{g\cos(e+}}{}$$

$$= \frac{2a\sqrt{g\cos(e+fx)}}{b^2fg} + \frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{g\cos(e+fx)}} + \frac{4\sqrt{\cos(e}}{3bf}$$

$$= \frac{2a\sqrt{g\cos(e+fx)}}{b^2fg} + \frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{g\cos(e+fx)}} + \frac{4\sqrt{\cos(e}}{3bf}$$

$$= \frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} + \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} + \frac{2a\sqrt{g\cos(}}{b^2f}$$

Mathematica [C] time = 26.52, size = 1915, normalized size = 4.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]

[Out] (-2*Cos[e + f*x]*Sin[e + f*x])/(3*b*f*Sqrt[g*Cos[e + f*x]]) + (Sqrt[Cos[e +
 f*x]]*((-2*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1
/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Co
s[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF
1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-
a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 -
 I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^
2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^
(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos
[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a
^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4)
)*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*x])) + (3*a*(a +
 b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*
ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/
2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sq
rt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4))
 + (4*Sqrt[Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^
2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) +
 (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e +
f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e
 + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e +
f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1
+ Cos[e + f*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1
 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b
^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^
2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*
x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*
(-1 + 2*Cos[e + f*x]^2)*(a + b*Sin[e + f*x])) - (8*b*(a + b*Sqrt[1 - Cos[e
+ f*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b
^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2
])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]
^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1,
9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a
^2 + b^2*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[
e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2
```

```
- b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + S
qrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(
a + b*Sin[e + f*x]))))/(6*b*f*Sqrt[g*Cos[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{3}}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^3/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 7.31, size = 2015, normalized size = 4.41

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 2/f*a/b^2/g*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-2/f*a^3/b^2*g*sum((_R^4+_R
^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((
-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_
R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+
b^2*g^4))+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/b/(
-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/
(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*
(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)-4/f*(g*(2*cos
(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/b/(-g*(2*sin(1/2*f*x+1/2*e
)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e
```

)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)
*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*si
n(1/2*f*x+1/2*e)^2)^(1/2)/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^
2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticF
(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/
2*e)^2-1)^(1/2)+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/
2)/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1
/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(
1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)-1/2/f*(
g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/b^3/(-g*(2*sin(1/2
*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2
*f*x+1/2*e)^2-1))^(1/2)*sum(1/(2*_alpha^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-2*b^
2)/b^2)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(
1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*_alpha^
4*b^2-8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*EllipticPi(cos(1/2*f*x+1/2
*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*si
n(1/2*f*x+1/2*e)^2-1)^(1/2)*_alpha^2*b^2+2^(1/2)*a^2*_alpha*arctanh(1/2/(g*
(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*
x+1/2*e)^2*g)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alph
a^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+
1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2
+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(sin(1
/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2)/(g*(2*_alpha^2*b^2+a^2
-2*b^2)/b^2)^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-2*sin(1/2*f*
x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2
-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x
+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*s
um(1/(2*_alpha^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*EllipticPi(
cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(sin(1/2*f*x+1/2*e)
^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*_alpha^4*b^2-8*(g*(2*_alpha^2*b^
2+a^2-2*b^2)/b^2)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^
2)/a^2,2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/
2)*_alpha^2*b^2+2^(1/2)*a^2*_alpha*arctanh(1/2/(g*(2*_alpha^2*b^2+a^2-2*b^2
)/b^2)^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(4*a^
2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/
2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*
x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2
*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin
(1/2*f*x+1/2*e)^2+1))^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2),_alpha
=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+
1/2*e)^2*g)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{3}}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
[Out] integrate(sin(f*x + e)^3/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^{3}}{\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
[Out] int(sin(e + f*x)^3/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
[Out] Timed out
```

3.1391 $\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=380

$$-\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{a^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^3\sqrt{\cos(e}}{b^2f\left(a^2-\right.}$$

```
[Out] -a^2*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(3/2)/
(-a^2+b^2)^(3/4)/f/g^(1/2)-a^2*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b
^2)^(1/4)/g^(1/2))/b^(3/2)/(-a^2+b^2)^(3/4)/f/g^(1/2)-2*a*(cos(1/2*f*x+1/2*
e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*
x+e)^(1/2)/b^2/f/(g*cos(f*x+e))^(1/2)+a^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(
1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/
2))*cos(f*x+e)^(1/2)/b^2/f/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2
)+a^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*
x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/f/(a^2-b*(b
+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-2*(g*cos(f*x+e))^(1/2)/b/f/g
```

Rubi [A] time = 0.93, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.394, Rules used = {2909, 2565, 30, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{a^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^3\sqrt{\cos(e}}{b^2f\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^2*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(3/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[g])) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(3/4)*f*Sq
rt[g]) - (2*Sqrt[g*Cos[e + f*x]])/(b*f*g) - (2*a*Sqrt[Cos[e + f*x]]*Ellipti
cF[(e + f*x)/2, 2])/(b^2*f*Sqrt[g*Cos[e + f*x]]) + (a^3*Sqrt[Cos[e + f*x]]*
EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b*(b
- Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^3*Sqrt[Cos[e + f*x]]*Elli
pticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b*(b + Sq
rt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{a\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{b}$$

$$= -\frac{a\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} + \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{b^2} - \text{Subst}\Big(\int$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{bfg} - \frac{a^3\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,dx}{2b^2\sqrt{-a^2+b^2}} - \frac{a^3\int}{}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{bfg} - \frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{g\cos(e+fx)}} + \frac{(2a^2g)\,\text{Sub}}{}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{bfg} - \frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{g\cos(e+fx)}} + \frac{a^3\sqrt{\cos(e}}{b^2\left(a^2-b\right.}$$

$$= -\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} - \frac{2\sqrt{g\cos(}}{bf}$$

Mathematica [C] time = 25.41, size = 1326, normalized size = 3.49

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*((-2*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)
*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1
/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*
b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
 b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e
 + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))
) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])
/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-
a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*
```

```
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 +
 b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*x]))
- ((a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2
+ b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])
/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1
+ I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)
^(3/4)) + (4*Sqrt[Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e
+ f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 -
b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*C
os[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*
(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]
^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^
2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b
^2*(-1 + Cos[e + f*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^
2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*
x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a
^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos
[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f
*x]^2]*(-1 + 2*Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(2*f*Sqrt[g*Cos[e +
f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^2/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 6.55, size = 855, normalized size = 2.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] -2/f/b/g*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+2/f*a^2/b*g*sum((_R^4+_R^2*g)
/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*si
n(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=Roo
tOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g
^4))+2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a/b^2/(-
g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(
g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/
2*f*x+1/2*e)^2+1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))-1/8/f*(g*(2*c
os(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a/b^4/sin(1/2*f*x+1/2*e)
/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alpha/(2*_alpha^2-1)*(8*(sin(1
/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2
*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/
b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1
/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2
-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1
/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2
*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^
2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-s
in(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2
+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))
^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sin\left(e+f\,x\right)}^2}{\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)^2/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.1392 $\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=352

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{a^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{bf\left(b\sqrt{b^2-a^2}+a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{a^2\sqrt{\cos(e+f\cdots}}{bf\left(a^2-b\left(\sqrt{\cdots}\right.\right.}$$

```
[Out] a*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/(-a^2+b^2)^
(3/4)/f/b^(1/2)/g^(1/2)+a*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(
1/4)/g^(1/2))/(-a^2+b^2)^(3/4)/f/b^(1/2)/g^(1/2)+2*(cos(1/2*f*x+1/2*e)^2)^(
1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1
/2)/b/f/(g*cos(f*x+e))^(1/2)-a^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f
*x+e)^(1/2)/b/f/(a^2-b^2+b*(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-a^2*(cos(
1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*
b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b/f/(a^2-b*(b+(-a^2+b^2)^(
1/2)))/(g*cos(f*x+e))^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{a^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{bf\left(b\sqrt{b^2-a^2}+a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{a^2\sqrt{\cos(e+f\cdots}}{bf\left(a^2-b\left(\sqrt{\cdots}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sq
rt[b]*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) + (a*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*
x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sqrt[b]*(-a^2 + b^2)^(3/4)*f*Sqrt[g])
+ (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(b*f*Sqrt[g*Cos[e + f*x]
]) - (a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e +
f*x)/2, 2])/(b*(a^2 - b^2 + b*Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (
a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2
, 2])/(b*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{b}$$

$$= \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,dx}{2b\sqrt{-a^2+b^2}} + \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}+b\cos\right.}}{2b\sqrt{-a^2+b^2}}$$

$$= \frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{g\cos(e+fx)}} - \frac{(2ag)\,\text{Subst}\left(\int \frac{1}{(a^2-b^2)g^2+b^2x^4}\,dx, x, \right.}{f}$$

$$= \frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{g\cos(e+fx)}} - \frac{a^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(e\right.}{b\left(a^2-b^2+b\sqrt{-a^2+b^2}\right)f\sqrt{g\cos}}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\sqrt{b}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\sqrt{b}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} + \frac{2\sqrt{\cos(e+f}}{bf\sqrt{g\,c}}$$

Mathematica [C] time = 6.20, size = 546, normalized size = 1.55

$$-\frac{2\sqrt{\cos(e+fx)}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(\frac{5b\left(a^2-b^2\right)\sqrt{\sin^2(e+fx)}\sqrt{\cos(e+fx)}F_1\left(\frac{1}{4};\right.}{\left(a^2+b^2\cos^2(e+fx)-b^2\right)\left(2\cos^2(e+fx)\left(2b^2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)+\left(a^2-b^2\right)F_1\left(\frac{5}{4}\right.\right.\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]]*(a + b*Sqrt[Sin[e + f*x]^2])*((a*(-2*ArcTan[1 - (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]
*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[S
qrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*C
os[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*App
ellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
*Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]^2])/((a^2 - b^2 + b^2*Cos[e + f*x]^2)
*(-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2,
```

```
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4
, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2))))/(f
*Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sin(f*x + e)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 6.05, size = 1181, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] -2/f*a*g*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2
*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*
g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)
*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*
f*x+1/2*e)^2)^(1/2)/b*sum(_alpha/(2*_alpha^2-1)*(8*(sin(1/2*f*x+1/2*e)^2)^(
1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2
/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^
3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)
^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2
*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3
)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*
```

```
_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*
(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^
2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(
1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=Root
Of(4*_Z^4*b^2-4*_Z^2*b^2+a^2))/a^2*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2
*e)^2-1))^(1/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(
1/2)/b*sum(_alpha/(2*_alpha^2-1)*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/
2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-
1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_al
pha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*Elliptic
Pi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a
^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)
*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+
2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+
1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*
f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*
f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4
*_Z^2*b^2+a^2))/a^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="ma
xima")
[Out] integrate(sin(f*x + e)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)}{\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
[Out] int(sin(e + f*x)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.1393 $\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=369

$$-\frac{b\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}-\frac{b\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{f\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right.}{af\sqrt{g}\left(b^2-a^2\right)^{3/4}}$$

```
[Out] -arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(1/2)+b^(3/2)*arctan(b^(1/2)*(g
*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/(-a^2+b^2)^(3/4)/f/g^(1/2)-a
rctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(1/2)+b^(3/2)*arctanh(b^(1/2)*(g
*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/(-a^2+b^2)^(3/4)/f/g^(1/2)-b
*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2
*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/f/(a^2-b*(b-(-a^2+b^
2)^(1/2)))/(g*cos(f*x+e))^(1/2)-b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+
1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(
f*x+e)^(1/2)/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)
```

Rubi [A] time = 0.82, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2898, 2565, 329, 212, 206, 203, 2702, 2807, 2805, 208, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{b\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}-\frac{b\sqrt{\cos(e+}}{f\left(a^2-b\right(}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] -(ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*Sqrt[g])) + (b^(3/2)*ArcTan[(Sq
rt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2 + b^2)^
(3/4)*f*Sqrt[g]) - ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*Sqrt[g]) + (b
^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])]
)/(a*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) - (b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)
/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b*(b - Sqrt[-a^2 + b^2]))
*f*Sqrt[g*Cos[e + f*x]]) - (b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt
[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*
Cos[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \int \left(\frac{\csc(e+fx)}{a\sqrt{g\cos(e+fx)}} - \frac{b}{a\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\right)dx$$

$$= \frac{\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{a} - \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{a}$$

$$= \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,dx}{2\sqrt{-a^2+b^2}} + \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}+b\cos\right.}}{2\sqrt{-a^2+b^2}}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{g^2}}\,dx, x, \sqrt{g\cos(e+fx)}\right)}{afg} - \frac{\left(2b^2g\right)\text{Subst}\left(\int \frac{1}{(a^2-b^2)g^2}\right.}{}$$

$$= -\frac{b\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}}; \frac{1}{2}(e+fx)\middle|2\right)}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)f\sqrt{g\cos(e+fx)}} - \frac{b\sqrt{\cos(e+fx)}\,\Pi\left(\frac{}{b-}\right.}{\left(a^2-b\left(b+\sqrt{-a^2+}\right.\right.}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af\sqrt{g}} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{a\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} - \frac{\tanh^{-1}\left(\frac{\sqrt{g\cos}}{\sqrt{}}\right.}{af\sqrt{g}}$$

Mathematica [C] time = 20.62, size = 698, normalized size = 1.89

$$-\frac{2\sqrt{\cos(e+fx)}\left(\cos^2(e+fx)-1\right)\csc(e+fx)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(\frac{}{\sqrt{1-\cos^2(e+fx)}\left(a^2+b^2\left(\cos^2(e+fx)-1\right)\right)\left(5(a^2-\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]]*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2
])*Csc[e + f*x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2
, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e +
f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[
e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x
]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1
```

```
, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*
(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2
]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan
[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2
)^(3/4)*ArcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e
 + f*x]]] + 2*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/
2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x
]] + b*Cos[e + f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b
]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^
(3/4))))/(f*Sqrt[g*Cos[e + f*x]]*(1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(csc(f*x + e)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

maple [A] time = 3.06, size = 186, normalized size = 0.50

$$\frac{\ln\left(\frac{2\sqrt{-g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}-2g}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{a\sqrt{-g}\,f}-\frac{\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}-4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{2a\sqrt{g}\,f}-\frac{\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}+4g\cos\left(\frac{fx}{2}\right.}{-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right.}{2a\sqrt{g}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 1/a/(-g)^(1/2)/f*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)
^2*g+g)^(1/2)-g))-1/2/a/g^(1/2)/f*ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*
sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))-1/2/a/g^(1/2)/f*
ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2
*g*cos(1/2*f*x+1/2*e)-g))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(csc(f*x + e)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)*(g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\sqrt{g\cos\left(e+fx\right)}\,\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/(sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x))), x)
```

3.1394 $\int \frac{\csc^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=448

$$\frac{b^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(b\sqrt{b^2-a^2}+a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{b^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+f}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right.}{a^2f\sqrt{g}\left(b^2-a^2\right)^{3/4}}$$

```
[Out] b*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f/g^(1/2)-b^(5/2)*arctan(b^(1/2)
*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/(-a^2+b^2)^(3/4)/f/g^(1
/2)+b*arctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f/g^(1/2)-b^(5/2)*arctanh(b
^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/(-a^2+b^2)^(3/4)/
f/g^(1/2)+(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2
*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/a/f/(g*cos(f*x+e))^(1/2)+b^2*(cos(1/2
*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(
b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/f/(a^2-b^2+b*(-a^2+b^2)^(1/
2))/(g*cos(f*x+e))^(1/2)+b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e
)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e
)^(1/2)/a/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-csc(f*x+e)*(g
*cos(f*x+e))^(1/2)/a/f/g
```

Rubi [A] time = 0.98, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {2898, 2565, 329, 212, 206, 203, 2570, 2642, 2641, 2702, 2807, 2805, 208, 205}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{b^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(b\sqrt{b^2-a^2}+a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{b^2\sqrt{\cos(}}{af\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (b*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*Sqrt[g]) - (b^(5/2)*ArcTan[
(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2 +
b^2)^(3/4)*f*Sqrt[g]) + (b*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*Sq
rt[g]) - (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4
)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) - (Sqrt[g*Cos[e + f*x]]*Csc
[e + f*x])/(a*f*g) + (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(a*f*Sq
rt[g*Cos[e + f*x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a
^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2 + b*Sqrt[-a^2 + b^2])*f*Sqrt[g*C
```

```
os[e + f*x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b
^2]), (e + f*x)/2, 2])/(a*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e +
 f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \int \left(-\frac{b\csc(e+fx)}{a^2\sqrt{g\cos(e+fx)}} + \frac{\csc^2(e+fx)}{a\sqrt{g\cos(e+fx)}} + \frac{b^2}{a^2\sqrt{g\cos(e+fx)}\,(a\right.$$

$$= \frac{\int \frac{\csc^2(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{a} - \frac{b\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{a^2} + \frac{b^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx)}}{a^2}$$

$$= -\frac{\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{afg} + \frac{\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{2a} - \frac{b^2\int \frac{}{\sqrt{g\cos(e+fx)}}}{2a\sqrt{}}$$

$$= -\frac{\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{afg} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{x^4}{g^2}}\,dx, x, \sqrt{g\cos(e+\right.}{a^2fg}$$

$$= -\frac{\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{afg} + \frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{g\cos(e+fx)}} + \frac{b^2}{a}$$

$$= \frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f\sqrt{g}} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{a^2\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} + \frac{b\tanh^{-1}\left(\frac{\sqrt{g\,c}}{}\right.}{a^2f\sqrt{g}}$$

Mathematica [C] time = 30.00, size = 2093, normalized size = 4.67

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[e + f*x]^2/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]

[Out] -(Cot[e + f*x]/(a*f*Sqrt[g*Cos[e + f*x]])) - (Sqrt[Cos[e + f*x]]*((4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) - (b*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*ArcTan[Sqrt[Cos[e + f*x]]])/a - (16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (200*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (10*Log[1 - Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[Cos[e + f*x]]])/a - (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))))/(20*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (6*b*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(3/4)*ArcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 -

```
Sqrt[Cos[e + f*x]]] + 2*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqr
t[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[C
os[e + f*x]] + b*Cos[e + f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a
^2 - b^2)^(3/4))))/((1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a*f*Sqr
t[g*Cos[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{2}}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(csc(f*x + e)^2/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 13.88, size = 1217, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 1/8*(-4*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*g^(3/2)*(-
g)^(1/2)*a+8*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*g^(3/
2)*(-g)^(1/2)*a*sin(1/2*f*x+1/2*e)^2+(-8*g^(3/2)*(sin(1/2*f*x+1/2*e)^2)^(1/
2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+
1/2*e)^2*g)^(3/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(-g)^(1/2)*a-g^(7/2
)*sin(1/2*f*x+1/2*e)^4*(2*sin(1/2*f*x+1/2*e)^2-1)^2/a*sum(1/_alpha/(2*_alph
a^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(
1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_al
```

```
pha^2*b^2+4*b^2)/a^2,2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)
^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-
4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)
+2^(1/2)*a^2*arctanh(1/2/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-2*sin(1
/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-1
6*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*
_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alph
a^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*
x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x
+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_
Z^2*b^2+a^2))*(-g)^(1/2))*cos(1/2*f*x+1/2*e)+4*cos(1/2*f*x+1/2*e)*sin(1/2*f
*x+1/2*e)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+si
n(1/2*f*x+1/2*e)^2*g)^(3/2)*b*(4*sum(1/(_R^6*b^2-3*_R^4*b^2*g+8*_R^2*a^2*g^
2-5*_R^2*b^2*g^2-b^2*g^3)*_R*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2
*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R)*(_R^2+g),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(1
6*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))*g^(5/2)*(-g)^(1/2)*b^2-
2*g^(3/2)*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)
^(1/2)-g))+ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g
+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)*g+(-g)^(1/2)*ln(2/(cos(1/2*
f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+
1/2*e)-g))*g))/a^2/g^(3/2)/(-g)^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*
x+1/2*e)^2*g)^(3/2)/cos(1/2*f*x+1/2*e)/sin(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1
/2*e)^2*g+g)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(e+f\,x\right)^2\,\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(e+fx\right)}{\sqrt{g\cos\left(e+fx\right)}\,\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
[Out] Integral(csc(e + f*x)**2/(sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x))), x)
```

3.1395 $\int \frac{\csc^3(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=557

$$-\frac{b^2\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}}-\frac{b^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{a^2f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}-\frac{b^3\sqrt{\cos(e+fx)}}{a^2f\left(a^2-b\left(\sqrt{b^2\right.\right.}}$$

```
[Out] -3/4*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(1/2)-b^2*arctan((g*cos(f*x
+e))^(1/2)/g^(1/2))/a^3/f/g^(1/2)+b^(7/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/
2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^3/(-a^2+b^2)^(3/4)/f/g^(1/2)-3/4*arctanh((g*
cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(1/2)-b^2*arctanh((g*cos(f*x+e))^(1/2)/g^(
1/2))/a^3/f/g^(1/2)+b^(7/2)*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)
^(1/4)/g^(1/2))/a^3/(-a^2+b^2)^(3/4)/f/g^(1/2)-b*(cos(1/2*f*x+1/2*e)^2)^(1/
2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2
)/a^2/f/(g*cos(f*x+e))^(1/2)-b^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f
*x+e)^(1/2)/a^2/f/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-b^3*(co
s(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),
2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a^2/f/(a^2-b*(b+(-a^2+b^
2)^(1/2)))/(g*cos(f*x+e))^(1/2)+b*csc(f*x+e)*(g*cos(f*x+e))^(1/2)/a^2/f/g-1
/2*csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/a/f/g
```

Rubi [A] time = 1.03, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2898, 2565, 329, 212, 206, 203, 2570, 2642, 2641, 290, 2702, 2807, 2805, 208, 205}

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{b^2\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}}-\frac{b^3\sqrt{\cdots}}{a^2f\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-3*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f*Sqrt[g]) - (b^2*ArcTan[Sqr
t[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f*Sqrt[g]) + (b^(7/2)*ArcTan[(Sqrt[b]*Sqrt
[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2 + b^2)^(3/4)*f*
Sqrt[g]) - (3*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f*Sqrt[g]) - (b^2
*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f*Sqrt[g]) + (b^(7/2)*ArcTanh[
(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2 +
```

```
b^2)^(3/4)*f*Sqrt[g]) + (b*Sqrt[g*Cos[e + f*x]]*Csc[e + f*x])/(a^2*f*g) - (
Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^2)/(2*a*f*g) - (b*Sqrt[Cos[e + f*x]]*Elli
pticF[(e + f*x)/2, 2])/(a^2*f*Sqrt[g*Cos[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(a^2 - b*
(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x]]*E
llipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(a^2 - b*(b +
 Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \int \left(\frac{b^2\csc(e+fx)}{a^3\sqrt{g\cos(e+fx)}} - \frac{b\csc^2(e+fx)}{a^2\sqrt{g\cos(e+fx)}} + \frac{\csc^3(e+fx)}{a\sqrt{g\cos(e+fx)}} - \cdots\right.$$

$$= \frac{\int \frac{\csc^3(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{a} - \frac{b\int \frac{\csc^2(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{a^2} + \frac{b^2\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{a^3} - \cdots$$

$$= \frac{b\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a^2fg} - \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{2a^2} + \frac{b^3\int \frac{\cdots}{\sqrt{g\cos(e+fx)}\,(\cdots}}{2a^2\cdots} \cdots$$

3 Sub…

$$= \frac{b\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a^2fg} - \frac{\sqrt{g\cos(e+fx)}\,\csc^2(e+fx)}{2afg} - \cdots$$

$$= \frac{b\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a^2fg} - \frac{\sqrt{g\cos(e+fx)}\,\csc^2(e+fx)}{2afg} - \frac{b\sqrt{\cos\cdots}}{\cdots}$$

$$= -\frac{b^2\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{a^3\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} - \frac{b^2\tanh^{-1}\left(\frac{\sqrt{\cdots}}{\cdots}\right.}{a^3f\sqrt{\cdots}}\cdots$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af\sqrt{g}} - \frac{b^2\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos\cdots}}{\sqrt[4]{-a^2+\cdots}}\right.}{a^3\left(-a^2+b^2\right)^{3/4}\cdots}\cdots$$

Mathematica [C] time = 30.85, size = 2129, normalized size = 3.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]*((b*Csc[e + f*x])/a^2 - Csc[e + f*x]^2/(2*a)))/(f*Sqrt[g*Cos[
e + f*x]]) + (Sqrt[Cos[e + f*x]]*((-2*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*
((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2
```

- b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) - (b^2*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*ArcTan[Sqrt[Cos[e + f*x]]])/a - (16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (200*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (10*Log[1 - Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[Cos[e + f*x]]])/a - (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)))/(20*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (2*(3*a^2 + 3*b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(3/4)*ArcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e + f*x]]] + 2*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^(3/4))))/((1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a^2*f*Sqrt[g*Cos[e +

```
f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{3}}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(csc(f*x + e)^3/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

maple [A] time = 3.46, size = 315, normalized size = 0.57

$$-\frac{3\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}+4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{8a\sqrt{g}\,f}+\frac{\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}}{16fag\left(-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}+\frac{3\ln\left(\frac{-2g+2\sqrt{-g}\,\sqrt{2\left(\cos^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g-g}}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{4fa\sqrt{-g}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] -3/8/f/a/g^(1/2)*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e
)^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))+1/16/f/a/g/(-1+cos(1/2*f*x+1/2
*e))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+3/4/f/a/(-g)^(1/2)*ln((-2*g+2*(-g)
^(1/2)*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))-3/8/f/a/g^(1
/2)*ln((-4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/
2)-2*g)/(cos(1/2*f*x+1/2*e)+1))-1/8/f/a/g/cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f
*x+1/2*e)^2*g-g)^(1/2)-1/16/f/a/g/(cos(1/2*f*x+1/2*e)+1)*(-2*sin(1/2*f*x+1/
2*e)^2*g+g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{3}}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(csc(f*x + e)^3/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\sin\left(e+f\,x\right)}^3\,\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)^3*(g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}\left(e+fx\right)}{\sqrt{g\cos\left(e+fx\right)}\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x))), x)
```

3.1396 $\int \frac{\sin^4(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=584

$$\frac{2a^2(g\cos(e+fx))^{3/2}}{3bfg^3\left(a^2-b^2\right)}-\frac{2b(g\cos(e+fx))^{3/2}}{3fg^3\left(a^2-b^2\right)}-\frac{4aE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}-\frac{2b}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{}{f}$$

```
[Out] a^4*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(5/2)/(
-a^2+b^2)^(5/4)/f/g^(3/2)-a^4*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^
2)^(1/4)/g^(1/2))/b^(5/2)/(-a^2+b^2)^(5/4)/f/g^(3/2)+2/3*a^2*(g*cos(f*x+e))
^(3/2)/b/(a^2-b^2)/f/g^3-2/3*b*(g*cos(f*x+e))^(3/2)/(a^2-b^2)/f/g^3-2*b/(a^
2-b^2)/f/g/(g*cos(f*x+e))^(1/2)+2*a*sin(f*x+e)/(a^2-b^2)/f/g/(g*cos(f*x+e))
^(1/2)-a^5*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1
/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^3/(a^2-b
^2)/f/g/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-a^5*(cos(1/2*f*x+1/2*e)^2
)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^
(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^3/(a^2-b^2)/f/g/(b+(-a^2+b^2)^(1/2))/(g*
cos(f*x+e))^(1/2)-4*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellip
ticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/(a^2-b^2)/f/g^2/cos(f
*x+e)^(1/2)+2*a^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE
(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b^2/(a^2-b^2)/f/g^2/cos(f
*x+e)^(1/2)
```

Rubi [A] time = 1.28, antiderivative size = 584, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2902, 2566, 2640, 2639, 2565, 14, 2898, 30, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{2a^2(g\cos(e+fx))^{3/2}}{3bfg^3\left(a^2-b^2\right)}-\frac{2b(g\cos(e+fx))^{3/2}}{3fg^3\left(a^2-b^2\right)}+\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{2a^3E\left(\frac{1}{2}(e+\right.}{b^2fg^2\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a^4*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
b^(5/2)*(-a^2 + b^2)^(5/4)*f*g^(3/2)) - (a^4*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*(-a^2 + b^2)^(5/4)*f*g^(3/
2)) - (2*b)/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a^2*(g*Cos[e + f*x]
)^(3/2))/(3*b*(a^2 - b^2)*f*g^3) - (2*b*(g*Cos[e + f*x])^(3/2))/(3*(a^2 - b
```

```
^2)*f*g^3) - (4*a*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b
^2)*f*g^2*Sqrt[Cos[e + f*x]]) + (2*a^3*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
f*x)/2, 2])/(b^2*(a^2 - b^2)*f*g^2*Sqrt[Cos[e + f*x]]) - (a^5*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 -
 b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) - (a^5*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 -
 b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*Sin[e + f*x])
/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sqrt{g\cos(e+fx)}\,\sin^2(e+f}{a+b\sin(e+fx)}}{\left(a^2-b^2\right)g^2}$$

$$= \frac{2a\sin(e+fx)}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{(2a)\int \sqrt{g\cos(e+fx)}\,dx}{\left(a^2-b^2\right)g^2} - \frac{a^2\int\Big(-}{}$$

$$= \frac{2a\sin(e+fx)}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{a^3\int \sqrt{g\cos(e+fx)}\,dx}{b^2\left(a^2-b^2\right)g^2} - \frac{a^4\int \frac{\sqrt{g\cos}}{a+b\sin}}{b^2\left(a^2-\right.}$$

$$= -\frac{2b}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2b(g\cos(e+fx))^{3/2}}{3\left(a^2-b^2\right)fg^3} - \frac{4a\sqrt{g\cos(e}}{\left(a^2-b^2\right)}$$

$$= -\frac{2b}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2a^2(g\cos(e+fx))^{3/2}}{3b\left(a^2-b^2\right)fg^3} - \frac{2b(g\cos(e}{3\left(a^2-b\right.}$$

$$= -\frac{2b}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2a^2(g\cos(e+fx))^{3/2}}{3b\left(a^2-b^2\right)fg^3} - \frac{2b(g\cos(e}{3\left(a^2-b\right.}$$

$$= \frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{5/2}\left(-a^2+b^2\right)^{5/4}fg^{3/2}} - \frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{5/2}\left(-a^2+b^2\right)^{5/4}fg^{3/2}} - \frac{}{\left(a^2-b^2\right)f}$$

Mathematica [C] time = 26.82, size = 820, normalized size = 1.40

$$\frac{\left(\frac{2\cos(e+fx)}{3b}+\frac{2\sec(e+fx)(a\sin(e+fx)-b)}{a^2-b^2}\right)\cos^2(e+fx)}{f(g\cos(e+fx))^{3/2}} + \frac{a\left(\frac{4ab\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(\frac{aF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)\cos^{\frac{3}{2}}(e+fx)}{3\left(a^2-b^2\right)}+\frac{\left(\frac{1}{8}+\right.}{}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]^2*((2*Cos[e + f*x])/(3*b) + (2*Sec[e + f*x]*(-b + a*Sin[e + f
*x]))/(a^2 - b^2)))/(f*(g*Cos[e + f*x])^(3/2)) + (a*Cos[e + f*x]^(3/2)*((4*
a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[e +
 f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b
^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^
2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 +
 b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sq
rt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a
^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*
x])) - ((a^2 - 2*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*(8*b^(5/2)*AppellF1[
3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e
 + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b
]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b
]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b
^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]
]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[e + f*x]^2)*(a + b*Si
n[e + f*x]))))/((a - b)*b*(a + b)*f*(g*Cos[e + f*x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^4/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 8.63, size = 1990, normalized size = 3.41

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(f*x+e)^4/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)

[Out] 4/3/f/g^2/b*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+4/3/f/g^2/b*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-2/f/g^2/b*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+1/2/f/g^2*b/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/2/f/g/b*a^4/(a-b)/(a+b)*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/2/f/g^2*b/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2/f/g*a^3/(a+b)/(a-b)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/b^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-4/f/g*a/(a+b)/(a-b)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-1/4/f*a^3/(a+b)/(a-b)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/b^4*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alpha*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+4/f/g*a/(a+b)/(a-b)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)+1/8/f*a^3/(a+b)/(a-b)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/b^4*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alpha*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(g*(2*_alpha^2*b^2+a^2-2*b^2

```
)/b^2)^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(4*a^
2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/
2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*
x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2
*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b
^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha
=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^4/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^4}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)^4/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1397 $\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=509

$$-\frac{2a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{bfg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{4bE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{2a}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{}{fg\left(a^2\right.}$$

```
[Out] -a^3*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(3/2)/
(-a^2+b^2)^(5/4)/f/g^(3/2)+a^3*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b
^2)^(1/4)/g^(1/2))/b^(3/2)/(-a^2+b^2)^(5/4)/f/g^(3/2)+2*a/(a^2-b^2)/f/g/(g*
cos(f*x+e))^(1/2)-2*b*sin(f*x+e)/(a^2-b^2)/f/g/(g*cos(f*x+e))^(1/2)+a^4*(co
s(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),
2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/(a^2-b^2)/f/g/(b-(-a
^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+a^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/
2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2)
)*cos(f*x+e)^(1/2)/b^2/(a^2-b^2)/f/g/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1
/2)-2*a^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2
*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b/(a^2-b^2)/f/g^2/cos(f*x+e)^(1/2
)+4*b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x
+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/(a^2-b^2)/f/g^2/cos(f*x+e)^(1/2)
```

Rubi [A] time = 1.07, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {2902, 2565, 30, 2566, 2640, 2639, 2867, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{2a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{bfg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{4bE\left(\frac{1}{2}(e+fx)\middle|2\right)}{fg^2\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^3*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(3/2)*(-a^2 + b^2)^(5/4)*f*g^(3/2))) + (a^3*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(5/4)*f*g^
(3/2)) + (2*a)/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a^2*Sqrt[g*Cos[e
 + f*x]]*EllipticE[(e + f*x)/2, 2])/(b*(a^2 - b^2)*f*g^2*Sqrt[Cos[e + f*x]]
) + (4*b*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^2
*Sqrt[Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-
```

```
a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*S
qrt[g*Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-
a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*S
qrt[g*Cos[e + f*x]]) - (2*b*Sin[e + f*x])/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f
*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
```

```
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
```

```
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sqrt{g\cos(e+fx)}\,\sin(e+}{a+b\sin(e+fx)}}{\left(a^2-b^2\right)g^2}$$

$$= -\frac{2b\sin(e+fx)}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{a^2\int\sqrt{g\cos(e+fx)}\,dx}{b\left(a^2-b^2\right)g^2} + \frac{a^3\int \frac{\sqrt{g}}{a+}}{b\left(a^2\right.}$$

$$= \frac{2a}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2b\sin(e+fx)}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{a^4\int}{}$$

$$= \frac{2a}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2a^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b\left(a^2-b^2\right)fg^2\sqrt{\cos(e+fx)}}$$

$$= \frac{2a}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2a^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b\left(a^2-b^2\right)fg^2\sqrt{\cos(e+fx)}}$$

$$= -\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{5/4}fg^{3/2}} + \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{5/4}fg^{3/2}} + \frac{}{\left(a^2-b\right.}$$

Mathematica [C] time = 26.86, size = 793, normalized size = 1.56

$$\frac{2\cos(e+fx)(a-b\sin(e+fx))}{f\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}}-\frac{\cos^{\frac{3}{2}}(e+fx)\left(-\frac{\left(a^2-2b^2\right)\sin^2(e+fx)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+f\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f*(g*Cos[e + f*x])^(3/2)
) - (Cos[e + f*x]^(3/2)*((4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*Appell
F1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos
[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt
[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]
*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*S
qrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt
[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*
Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e
 + f*x]^2]*(a + b*Sin[e + f*x])) - ((a^2 - 2*b^2)*(a + b*Sqrt[1 - Cos[e + f
*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e +
 f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(
2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*Ar
cTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt
[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[
e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Co
s[e + f*x]] + b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1
 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/((a - b)*(a + b)*f*(g*Cos[e + f*
x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{3}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^3/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 11.94, size = 1613, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/2/f/g^2*a/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2
*f*x+1/2*e)^2*g+g)^(1/2)+1/2/f/g*a^3/(a-b)/(a+b)*sum((_R^6-_R^4*g-_R^2*g^2+
g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-
2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R
=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b
^2*g^4))+1/2/f/g^2*a/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(-2
*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*
f*x+1/2*e)^2)^(1/2)/b/g*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(
1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*
sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2
*e),2^(1/2))-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/
b/g/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/
2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4
-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))-8/f*(g*
(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^2*sin(1/2*f*x+1/
2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/2*e)^2
-1)*cos(1/2*f*x+1/2*e)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(
1/2)+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^2/si
n(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*
f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4
*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+8/f*
(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^2/sin(1/2*f*x
+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/2*e
)^2-1)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x
```

```
+1/2*e)-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^2
/sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin
(1/2*f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*sin(1/2*f*x+1/2
*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))
-1/4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/b^3/g/sin(
1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^2/(a-b)/(a+b)*sum((-2
*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+sin(1/2*f*x+1/2*e)^2*a^2+2*b^2*_alpha^2-
a^2)/_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2
)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*
b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b
^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))
^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(
1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-
1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_
alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{3}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^3/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^3}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)^3/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1398 $\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=453

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}-\frac{2b}{fg\left(a^2-b^2\right)\sqrt{g\cos(e}}$$

```
[Out] a^2*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/(-a^2+b^2
)^(5/4)/f/g^(3/2)/b^(1/2)-a^2*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^
2)^(1/4)/g^(1/2))/(-a^2+b^2)^(5/4)/f/g^(3/2)/b^(1/2)-2*b/(a^2-b^2)/f/g/(g*c
os(f*x+e))^(1/2)+2*a*sin(f*x+e)/(a^2-b^2)/f/g/(g*cos(f*x+e))^(1/2)-a^3*(cos
(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2
*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b/(a^2-b^2)/f/g/(b-(-a^2+
b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-a^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f
*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*c
os(f*x+e)^(1/2)/b/(a^2-b^2)/f/g/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-2
*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/
2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/(a^2-b^2)/f/g^2/cos(f*x+e)^(1/2)
```

Rubi [A] time = 0.88, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.394, Rules used = {2902, 2636, 2640, 2639, 2565, 30, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}-\frac{2b}{fg\left(a^2-b^2\right)\sqrt{g\cos(e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a^2*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
Sqrt[b]*(-a^2 + b^2)^(5/4)*f*g^(3/2)) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sqrt[b]*(-a^2 + b^2)^(5/4)*f*g^(3/
2)) - (2*b)/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a*Sqrt[g*Cos[e + f*
x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^2*Sqrt[Cos[e + f*x]]) - (a^
3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2,
2])/(b*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) - (a^3*
Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2]
```

```
)/(b*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*Si
n[e + f*x])/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
```

```
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{1}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{\left(a^2-b^2\right)g^2}$$

$$= \frac{2a\sin(e+fx)}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{a\int \sqrt{g\cos(e+fx)}\,dx}{\left(a^2-b^2\right)g^2} + \frac{a^3\int \frac{}{\sqrt{g\,\mathrm{co}}}}{}$$

$$= -\frac{2b}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2a\sin(e+fx)}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{(2a}{}$$

$$= -\frac{2b}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2a\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\left(a^2-b^2\right)fg^2\sqrt{\cos(e+fx)}}$$

$$= \frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\sqrt{b}\left(-a^2+b^2\right)^{5/4}fg^{3/2}} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\sqrt{b}\left(-a^2+b^2\right)^{5/4}fg^{3/2}} - \frac{}{\left(a^2-b^2\right)}$$

Mathematica [C] time = 17.01, size = 785, normalized size = 1.73

$$\frac{2\cos(e+fx)(a\sin(e+fx)-b)}{f\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}} - \frac{a\cos^{\frac{3}{2}}(e+fx)\left(-\frac{\sin^2(e+fx)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2}{}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(-b + a*Sin[e + f*x]))/((a^2 - b^2)*f*(g*Cos[e + f*x])^(3/2
)) - (a*Cos[e + f*x]^(3/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*Appe
llF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*C
```

```
os[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sq
rt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[
b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)
*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sq
rt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*
b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos
[e + f*x]^2]*(a + b*Sin[e + f*x])) - ((a + b*Sqrt[1 - Cos[e + f*x]^2])*(8*b
^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 -
 (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sq
rt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2]
- Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] +
Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]]
+ b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*Sqrt[b]*(-a^2 + b^2)*(1 - Cos[e + f
*x]^2)*(a + b*Sin[e + f*x]))))/((a - b)*(a + b)*f*(g*Cos[e + f*x])^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^2/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 8.01, size = 1105, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/2/f/g^2*b/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*
f*x+1/2*e)^2*g+g)^(1/2)-1/2/f/g*b*a^2/(a-b)/(a+b)*sum((_R^6-_R^4*g-_R^2*g^2
+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((
-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_
R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+
b^2*g^4))-1/2/f/g^2*b/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(-
2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-4/f/g*a/(a+b)/(a-b)/sin(1/2*f*x+1/2*e)/(g
*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3-2/f/g*a/(a+b)/(a-b)
/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e
)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos
(1/2*f*x+1/2*e)^2+1)^(1/2)*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)
^2)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+4/f/g*a/(a+b)/(a-b)/sin(1/2
*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)+1/8/f/g
*a/b^2/(a+b)/(a-b)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*
sum(1/_alpha*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1
/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_a
lpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2
*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e
),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)
+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*
e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2
*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x
+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/
2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*si
n(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(g*(
2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^2/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^2}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
[Out] int(sin(e + f*x)^2/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
[Out] Timed out
```

3.1399 $\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=413

$$-\frac{a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{2bE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{2(a-b\,\mathrm{s}}{fg\left(a^2-b^2\right)}$$

```
[Out] -a*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*b^(1/2)/(-
a^2+b^2)^(5/4)/f/g^(3/2)+a*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^
(1/4)/g^(1/2))*b^(1/2)/(-a^2+b^2)^(5/4)/f/g^(3/2)+2*(a-b*sin(f*x+e))/(a^2-b
^2)/f/g/(g*cos(f*x+e))^(1/2)+a^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f
*x+e)^(1/2)/(a^2-b^2)/f/g/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+a^2*(co
s(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),
2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g/(b+(-a^2+b
^2)^(1/2))/(g*cos(f*x+e))^(1/2)+2*b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*
x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/(a^2-b^
2)/f/g^2/cos(f*x+e)^(1/2)
```

Rubi [A] time = 0.95, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{2bE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{2(a-b\,\mathrm{s}}{fg\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/((-a^2 + b^2)^(5/4)*f*g^(3/2))) + (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[g
*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/((-a^2 + b^2)^(5/4)*f*g^(3/2
)) + (2*b*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^
2*Sqrt[Cos[e + f*x]]) + (a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[
-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt
[g*Cos[e + f*x]]) + (a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*C
os[e + f*x]]) + (2*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*
x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{2(a-b\sin(e+fx))}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2\int \frac{\sqrt{g\cos(e+fx)}\left(-ab-\frac{1}{2}b^2\sin(e+fx)\right)}{a+b\sin(e+fx)}\,dx}{\left(a^2-b^2\right)g^2}$$

$$= \frac{2(a-b\sin(e+fx))}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{b\int \sqrt{g\cos(e+fx)}\,dx}{\left(a^2-b^2\right)g^2} + \frac{(ab)\int \frac{\sqrt{g\cos}}{a+b\sin}}{\left(a^2-\right.}$$

$$= \frac{2(a-b\sin(e+fx))}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,d}{2\left(a^2-b^2\right)g}$$

$$= \frac{2b\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\left(a^2-b^2\right)fg^2\sqrt{\cos(e+fx)}} + \frac{2(a-b\sin(e+fx))}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} +$$

$$= \frac{2b\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\left(a^2-b^2\right)fg^2\sqrt{\cos(e+fx)}} + \frac{a^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}}\right.}{\left(a^2-b^2\right)\left(b-\sqrt{-a^2+b^2}\right)fg}$$

$$= -\frac{a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{5/4}fg^{3/2}} + \frac{a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{5/4}fg^{3/2}} + \frac{2b}{}$$

Mathematica [C] time = 16.53, size = 783, normalized size = 1.90

$$\frac{2\cos(e+fx)(a-b\sin(e+fx))}{f\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}} + \frac{b\cos^{\frac{3}{2}}(e+fx)\left(-\frac{\sin^2(e+fx)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos}{}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f*(g*Cos[e + f*x])^(3/2)
) + (b*Cos[e + f*x]^(3/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*Appel
lF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Co
```

```
s[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b
]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqr
t[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b
*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[
e + f*x]^2]*(a + b*Sin[e + f*x])) - ((a + b*Sqrt[1 - Cos[e + f*x]^2])*(8*b^
(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 -
(Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] -
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + L
og[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] +
 b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*Sqrt[b]*(-a^2 + b^2)*(1 - Cos[e + f*
x]^2)*(a + b*Sin[e + f*x]))))/((a - b)*(a + b)*f*(g*Cos[e + f*x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(sin(f*x + e)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 9.04, size = 1938, normalized size = 4.69

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/2/f/g^2*a/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2
*f*x+1/2*e)^2*g+g)^(1/2)+1/2/f/g*a*b^2/(a-b)/(a+b)*sum((_R^6-_R^4*g-_R^2*g^
2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln(
(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),
_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2
+b^2*g^4))+1/2/f/g^2*a/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(
-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-8/f/g*b/(a+b)/(a-b)*sin(1/2*f*x+1/2*e)/(
g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3-4/f/g*b/(a+b)/(a-b
)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*
e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1
/2*f*x+1/2*e)^2)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1
/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)+8/f/g*b/(a+b)/(a-b)/sin(1/
2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3+8/f/
g*b/(a+b)/(a-b)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos
(1/2*f*x+1/2*e)+4/f/g*b/(a+b)/(a-b)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x
+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(
g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*EllipticE(cos(1/2*
f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1
)^(1/2)-8/f/g*b/(a+b)/(a-b)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1
))^(1/2)*cos(1/2*f*x+1/2*e)+1/4/f/g/b/a^2/(a+b)/(a-b)*sin(1/2*f*x+1/2*e)/(g
*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*
x+1/2*e)^2)^(1/2)*sum((2*_alpha^2*b^2-a^2)/_alpha/(2*_alpha^2-1)*(8*(sin(1/
2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*
f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b
^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/
2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-
1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/
2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*
f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2
)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-si
n(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+
a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^
(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/4/f/g/b/a^2/(a+b)/(a-b)/s
in(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum((2*_alpha^2*b^2-
a^2)/_alpha/(2*_alpha^2-1)*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+
1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(
1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(s
in(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos
(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b
^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*co
s(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)
*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)
^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/
2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/
2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*
```

```
b^2+a^2))*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
[Out] integrate(sin(f*x + e)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
[Out] int(sin(e + f*x)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
[Out] Timed out
```

3.1400 $\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=507

$$\frac{2bE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{2b(b-a\sin(e+fx))}{afg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{b^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{fg\left(a^2-b^2\right)\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}$$

```
[Out] arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(3/2)-b^(5/2)*arctan(b^(1/2)*(g*
cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/(-a^2+b^2)^(5/4)/f/g^(3/2)-ar
ctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(3/2)+b^(5/2)*arctanh(b^(1/2)*(g*
cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/(-a^2+b^2)^(5/4)/f/g^(3/2)+2/
a/f/g/(g*cos(f*x+e))^(1/2)+2*b*(b-a*sin(f*x+e))/a/(a^2-b^2)/f/g/(g*cos(f*x+
e))^(1/2)+b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(si
n(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^
2)/f/g/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+b^2*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(
1/2)),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g/(b+(-a^2+b^2)^(1/2))/(g*cos(f
*x+e))^(1/2)+2*b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(
sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/(a^2-b^2)/f/g^2/cos(f*x+e)
^(1/2)
```

Rubi [A] time = 1.36, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.516, Rules used = {2898, 2565, 325, 329, 298, 203, 206, 2696, 2867, 2640, 2639, 2701, 2807, 2805, 205, 208}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{2bE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{2b(b-a\sin}{afg\left(a^2-b^2\right)\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(3/2)) - (b^(5/2)*ArcTan[(Sqrt[
b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2 + b^2)^(5/
4)*f*g^(3/2)) - ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(3/2)) + (b^(5
/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
a*(-a^2 + b^2)^(5/4)*f*g^(3/2)) + 2/(a*f*g*Sqrt[g*Cos[e + f*x]]) + (2*b*Sqr
t[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^2*Sqrt[Cos[e
+ f*x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
```

```
 (e + f*x)/2, 2])/((a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*
x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e
+ f*x)/2, 2])/((a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]])
 + (2*b*(b - a*Sin[e + f*x]))/(a*(a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \int \left(\frac{\csc(e+fx)}{a(g\cos(e+fx))^{3/2}} - \frac{b}{a(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\right) dx$$

$$= \frac{\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a} - \frac{b\int \frac{1}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx}{a}$$

$$= \frac{2b(b-a\sin(e+fx))}{a\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{(2b)\int \frac{\sqrt{g\cos(e+fx)}\left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{1}{2}ab\sin(e+fx\right.}{a+b\sin(e+fx)}}{a\left(a^2-b^2\right)g^2}$$

$$= \frac{2}{afg\sqrt{g\cos(e+fx)}} + \frac{2b(b-a\sin(e+fx))}{a\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{}}{1-}\right.}{}$$

$$= \frac{2}{afg\sqrt{g\cos(e+fx)}} + \frac{2b(b-a\sin(e+fx))}{a\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{2}{afg\sqrt{g\cos(e+fx)}} + \frac{2b\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\left(a^2-b^2\right)fg^2\sqrt{\cos(e+fx)}} + \frac{2b(}{a\left(a^2-\right.}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{3/2}} + \frac{2}{afg\sqrt{g\cos(e+fx)}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{3/2}} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a\left(-a^2+b^2\right)^{5/4}fg^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e}}{\sqrt{g}}\right.}{afg^{3/2}}$$

Mathematica [C] time = 27.98, size = 1587, normalized size = 3.13

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -1/2*(Cos[e + f*x]^(3/2)*((8*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*Appel
lF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Co
```

```
s[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b
]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqr
t[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b
*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/(Sqrt[1 - Cos[e + f*x]^2]*(
b + a*Csc[e + f*x])) - ((-2*a^2 + b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1
- Cos[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan
[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 6*Sqrt[2]*Sq
rt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^
2 - b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] + 8*a*b*AppellF
1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[
e + f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*b^2*Log[1 - Sqrt[Cos
[e + f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 6*b^2*Log[1 + Sqrt[Cos[e
+ f*x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] - Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + 3*Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^
2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(12*(a^3 - a*b^2)*(1 - Cos[
e + f*x]^2)*(b + a*Csc[e + f*x])) - (Sqrt[b]*(-1 + Cos[e + f*x]^2)*(a + b*S
qrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*(-42*Sqrt[2]*(a^2 -
b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a
^2 - b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 + (S
qrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 84*b^(3/2)*(a^2 - b
^2)*ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*AppellF1[3/4, 1/2, 1, 7/4, Co
s[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 48*a*
b^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2 - b^2)*Log[1 - Sqrt[Cos[e
+ f*x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[Cos[e + f*x]]] + 21*Sqrt[2]
*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] - 21*Sqrt[2]*(a^2 - b^2)
^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4
)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*(a^3 - a*b^2)*(1 - Cos[e + f*x
]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/((a - b)*(a + b)*f*(g*
Cos[e + f*x])^(3/2)) + (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f
*(g*Cos[e + f*x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(fx+e)}{(g\cos(fx+e))^{\frac{3}{2}}(b\sin(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(csc(f*x + e)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

maple [A] time = 4.58, size = 425, normalized size = 0.84

$$-\frac{\left(4\ln\left(\frac{2\sqrt{-g}\,\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}-2g}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)g^{\frac{5}{2}}+2\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}+4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)\sqrt{-g}\,g^2+2\ln\left(\frac{2\sqrt{g}\,\sqrt{-2(\sin}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/2*(-(4*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^
(1/2)-g))*g^(5/2)+2*ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1
/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)*g^2+2*ln(2/(cos(1/
2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*
x+1/2*e)-g))*(-g)^(1/2)*g^2)*sin(1/2*f*x+1/2*e)^2+2*ln(2/cos(1/2*f*x+1/2*e)
*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g))*g^(5/2)-4*(-2*sin(1/2*
f*x+1/2*e)^2*g+g)^(1/2)*g^(3/2)*(-g)^(1/2)+ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^
(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(
1/2)*g^2+ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)
^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)*g^2)/g^(7/2)/(-g)^(1/2)/a/(2*s
in(1/2*f*x+1/2*e)^2-1)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(fx+e)}{(g\cos(fx+e))^{\frac{3}{2}}(b\sin(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate(csc(f*x + e)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(e+f\,x\right)\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
[Out] int(1/(sin(e + f*x)*(g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\left(g\cos\left(e+fx\right)\right)^{\frac{3}{2}}\left(a+b\sin\left(e+fx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
[Out] Integral(csc(e + f*x)/((g*cos(e + f*x))**(3/2)*(a + b*sin(e + f*x))), x)
```

3.1401 $\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=627

$$-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{afg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}-\frac{2b^2(b-a\sin(e+fx))}{a^2fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{b^{7/2}\tanh^{-1}\left(\ldots\right.}{a^2fg^{3/2}\left(\ldots\right.}$$

```
[Out] -b*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f/g^(3/2)+b^(7/2)*arctan(b^(1/2
)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/(-a^2+b^2)^(5/4)/f/g^(
3/2)+b*arctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f/g^(3/2)-b^(7/2)*arctanh(
b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/(-a^2+b^2)^(5/4)
/f/g^(3/2)-2*b/a^2/f/g/(g*cos(f*x+e))^(1/2)-csc(f*x+e)/a/f/g/(g*cos(f*x+e))
^(1/2)+3*sin(f*x+e)/a/f/g/(g*cos(f*x+e))^(1/2)-2*b^2*(b-a*sin(f*x+e))/a^2/(
a^2-b^2)/f/g/(g*cos(f*x+e))^(1/2)-b^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*
f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*
cos(f*x+e)^(1/2)/a/(a^2-b^2)/f/g/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-
b^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+
1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/(a^2-b^2)/f/g/(
b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos
(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/
a/f/g^2/cos(f*x+e)^(1/2)-2*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2
*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/a/(a^2-b^2)/
f/g^2/cos(f*x+e)^(1/2)
```

Rubi [A] time = 1.49, antiderivative size = 627, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 18, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2898, 2565, 325, 329, 298, 203, 206, 2570, 2636, 2640, 2639, 2696, 2867, 2701, 2807, 2805, 205, 208}

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{afg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}-\frac{2b^2(b-a\sin\ldots}{a^2fg\left(a^2-b^2\right)\sqrt{g\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((b*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*g^(3/2))) + (b^(7/2)*ArcT
an[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2
 + b^2)^(5/4)*f*g^(3/2)) + (b*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f
*g^(3/2)) - (b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(
```

```
1/4)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(5/4)*f*g^(3/2)) - (2*b)/(a^2*f*g*Sqrt[g*
Cos[e + f*x]]) - Csc[e + f*x]/(a*f*g*Sqrt[g*Cos[e + f*x]]) - (3*Sqrt[g*Cos[
e + f*x]]*EllipticE[(e + f*x)/2, 2])/(a*f*g^2*Sqrt[Cos[e + f*x]]) - (2*b^2*
Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(a*(a^2 - b^2)*f*g^2*Sqrt[C
os[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b
^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos
[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2
]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e
 + f*x]]) + (3*Sin[e + f*x])/(a*f*g*Sqrt[g*Cos[e + f*x]]) - (2*b^2*(b - a*S
in[e + f*x]))/(a^2*(a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
```

```
f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \int\left(-\frac{b\csc(e+fx)}{a^2(g\cos(e+fx))^{3/2}}+\frac{\csc^2(e+fx)}{a(g\cos(e+fx))^{3/2}}+\frac{}{a^2(g\cos(e+fx))}\right.$$

$$=\frac{\int\frac{\csc^2(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a}-\frac{b\int\frac{\csc(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2}+\frac{b^2\int\frac{1}{(g\cos(e+fx))^{3/2}(a+b\sin(}}{a^2}$$

$$=-\frac{\csc(e+fx)}{afg\sqrt{g\cos(e+fx)}}-\frac{2b^2(b-a\sin(e+fx))}{a^2\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}}+\frac{3\int\frac{}{(g\cos(e}}{2}$$

$$=-\frac{2b}{a^2fg\sqrt{g\cos(e+fx)}}-\frac{\csc(e+fx)}{afg\sqrt{g\cos(e+fx)}}+\frac{3\sin(e+fx)}{afg\sqrt{g\cos(e+fx)}}$$

$$=-\frac{2b}{a^2fg\sqrt{g\cos(e+fx)}}-\frac{\csc(e+fx)}{afg\sqrt{g\cos(e+fx)}}+\frac{3\sin(e+fx)}{afg\sqrt{g\cos(e+fx)}}$$

$$=-\frac{2b}{a^2fg\sqrt{g\cos(e+fx)}}-\frac{\csc(e+fx)}{afg\sqrt{g\cos(e+fx)}}-\frac{3\sqrt{g\cos(e+fx)}\,E\Big(}{afg^2\sqrt{\cos(e}}$$

$$=-\frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{3/2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{3/2}}-\frac{2b}{a^2fg\sqrt{g\cos(e+}}$$

$$=-\frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{3/2}}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{a^2\left(-a^2+b^2\right)^{5/4}fg^{3/2}}+\frac{b\tanh^{-1}\Big(\frac{\sqrt{}}{}}{a^2fg}$$

Mathematica [C] time = 28.44, size = 1635, normalized size = 2.61

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -1/4*(Cos[e + f*x]^(3/2)*((-2*(6*a^3 + 2*a*b^2)*(a + b*Sqrt[1 - Cos[e + f*x
]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
-a^2 + b^2)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1
 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 +
((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 +
b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e +
f*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[
e + f*x]] + I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/(Sqrt[1 - Co
s[e + f*x]^2]*(b + a*Csc[e + f*x])) - ((7*a^2*b - 5*b^3)*(-1 + Cos[e + f*x]
^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*Sqrt[b]*(a^2 -
 b^2)^(3/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/
4)] - 6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]
]] + 8*a*b*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*b^
2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 6*b^2*L
og[1 + Sqrt[Cos[e + f*x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a
^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e
+ f*x]] + 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]
*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(12*(a^3
- a*b^2)*(1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - ((-3*a^2*b + b^3)*(-1
 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e
+ f*x]*(-42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*
(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(
1/4)] + 84*b^(3/2)*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*Ap
pellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
*Cos[e + f*x]^(3/2) + 48*a*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]
^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2
 - b^2)*Log[1 - Sqrt[Cos[e + f*x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[
Cos[e + f*x]]] + 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 -
b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x
]] - 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*b^(
3/2)*(a^3 - a*b^2)*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[
e + f*x]))))/(a*(a - b)*(a + b)*f*(g*Cos[e + f*x])^(3/2)) + (Cos[e + f*x]^2
*(-(Cot[e + f*x]/a) + (2*Sec[e + f*x]*(-b + a*Sin[e + f*x]))/(a^2 - b^2)))/
(f*(g*Cos[e + f*x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{2}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(csc(f*x + e)^2/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 17.19, size = 3469, normalized size = 5.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/f/g^(3/2)*b/(2+2^(1/2))/(2^(1/2)-2)/a^2*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(
1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))-1/f/
g^2*b/(2+2^(1/2))/(2^(1/2)-2)/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2*2^(
1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/2/f/g*b^5/(a-b)/(a+b)/a^2*sum((
_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*
g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g
^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*
_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1/f/g*b/a^2/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*
(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))-1/f/g^(3/2)*b/(2+2^
(1/2))/(2^(1/2)-2)/a^2*ln((-4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*
x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)+1))+1/f/g^2*b/(2+2^(1/2))/(2
^(1/2)-2)/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(-2*sin(1/2*f*
x+1/2*e)^2*g+g)^(1/2)-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e
)^2)^(1/2)/g^3*a/cos(1/2*f*x+1/2*e)/sin(1/2*f*x+1/2*e)^5/(2*sin(1/2*f*x+1/2
*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(-2*sin(1/2*f*x+1
/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)
*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^3/a/cos(1/2*f*x+1/2*e)/sin(1/2*f*x+1/2*e)^5/
(2*sin(1/2*f*x+1/2*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)
```

```
*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*b^2-6/f*(g*(2*cos
(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^3*a/cos(1/2*f*x+1/2*e)/s
in(1/2*f*x+1/2*e)/(2*sin(1/2*f*x+1/2*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+
1/2*e)^2-1))^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)
+2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^3/a/cos(1/
2*f*x+1/2*e)/sin(1/2*f*x+1/2*e)/(2*sin(1/2*f*x+1/2*e)^2-1)^2/(a^2-b^2)/(g*(
2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2
*e)^2*g)^(3/2)*b^2+6/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^
(1/2)/g^3*a/cos(1/2*f*x+1/2*e)/sin(1/2*f*x+1/2*e)^3/(2*sin(1/2*f*x+1/2*e)^2
-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(-2*sin(1/2*f*x+1/2*e)
^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)-2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/
2*f*x+1/2*e)^2)^(1/2)/g^3/a/cos(1/2*f*x+1/2*e)/sin(1/2*f*x+1/2*e)^3/(2*sin(
1/2*f*x+1/2*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(-2*si
n(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*b^2-3/f*(g*(2*cos(1/2*f*
x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^3*a/sin(1/2*f*x+1/2*e)^5/(2*sin
(1/2*f*x+1/2*e)^2-1)^(3/2)/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(
-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*EllipticE(cos(1/2*f
*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)+1/f*(g*(2*cos(1/2*f*x+1/2*e
)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^3/a/sin(1/2*f*x+1/2*e)^5/(2*sin(1/2*f*
x+1/2*e)^2-1)^(3/2)/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(-2*sin(
1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*EllipticE(cos(1/2*f*x+1/2*
e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*b^2+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)
^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g/a^3*sin(1/2*f*x+1/2*e)^3/(2*sin(1/2*f*x
+1/2*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alpha*
(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2
*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b
^2+4*b^2)/a^2,2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/
2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alph
a^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2
)*a^2*arctanh(1/2/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-2*sin(1/2*f*x+
1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1
/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^
4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2
-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e
)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2*e)
^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2
+a^2))*b^2-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/
g/a^3*sin(1/2*f*x+1/2*e)/(2*sin(1/2*f*x+1/2*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1
/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alpha*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2
)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*Ellip
ticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*_alpha^3*b^2-
8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*
EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_a
lpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(g*(2*_alpha^2*b^2
+a^2-2*b^2)/b^2)^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(
```

```
1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(
1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*
sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1
/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^
2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1
/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*b^2+1/8/f*(g*(2*cos(1/2*f*x+1
/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g/a^3/sin(1/2*f*x+1/2*e)/(2*sin(1/2*
f*x+1/2*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alp
ha*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)
*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^
2*b^2+4*b^2)/a^2,2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^
(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_a
lpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(
1/2)*a^2*arctanh(1/2/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-2*sin(1/2*f
*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*si
n(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alp
ha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*
a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/
2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2
*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*
b^2+a^2))*b^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(e+f\,x\right)^2\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{\frac{3}{2}}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(csc(e + f*x)**2/((g*cos(e + f*x))**(3/2)*(a + b*sin(e + f*x))), x)
```

3.1402 $\int \frac{\sin^4(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=601

$$\frac{2a^2\sqrt{g\cos(e+fx)}}{bfg^3(a^2-b^2)}-\frac{2b\sqrt{g\cos(e+fx)}}{fg^3(a^2-b^2)}-\frac{4a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2(a^2-b^2)\sqrt{g\cos(e+fx)}}-\frac{2b}{3fg(a^2-b^2)(g\cos(e+fx))^{3/2}}+\frac{}{3fg}$$

```
[Out] -a^4*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(3/2)/
(-a^2+b^2)^(7/4)/f/g^(5/2)-a^4*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b
^2)^(1/4)/g^(1/2))/b^(3/2)/(-a^2+b^2)^(7/4)/f/g^(5/2)-2/3*b/(a^2-b^2)/f/g/(
g*cos(f*x+e))^(3/2)+2/3*a*sin(f*x+e)/(a^2-b^2)/f/g/(g*cos(f*x+e))^(3/2)-4/3
*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/
2*e),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g^2/(g*cos(f*x+e))^(1/2)+2*a^3*(
cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e)
,2^(1/2))*cos(f*x+e)^(1/2)/b^2/(a^2-b^2)/f/g^2/(g*cos(f*x+e))^(1/2)-a^5*(co
s(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),
2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/(a^2-b^2)/f/g^2/(a^2
-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-a^5*(cos(1/2*f*x+1/2*e)^2)^(1
/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2
)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/(a^2-b^2)/f/g^2/(a^2-b*(b+(-a^2+b^2)^(1/2)
))/(g*cos(f*x+e))^(1/2)+2*a^2*(g*cos(f*x+e))^(1/2)/b/(a^2-b^2)/f/g^3-2*b*(g
*cos(f*x+e))^(1/2)/(a^2-b^2)/f/g^3
```

Rubi [A] time = 1.40, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 16, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.485, Rules used = {2902, 2566, 2642, 2641, 2565, 14, 2909, 30, 2867, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{2a^2\sqrt{g\cos(e+fx)}}{bfg^3(a^2-b^2)}-\frac{2b\sqrt{g\cos(e+fx)}}{fg^3(a^2-b^2)}-\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{5/2}(b^2-a^2)^{7/4}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{5/2}(b^2-a^2)^{7/4}}+\frac{2a^3\sqrt{\cos(e+fx)}}{b^2fg^2(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^4*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(3/2)*(-a^2 + b^2)^(7/4)*f*g^(5/2))) - (a^4*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(7/4)*f*g^
(5/2)) - (2*b)/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2)) + (2*a^2*Sqrt[g*C
os[e + f*x]])/(b*(a^2 - b^2)*f*g^3) - (2*b*Sqrt[g*Cos[e + f*x]])/((a^2 - b^
```

```
2)*f*g^3) - (4*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^
2)*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2*a^3*Sqrt[Cos[e + f*x]]*EllipticF[(e + f
*x)/2, 2])/(b^2*(a^2 - b^2)*f*g^2*Sqrt[g*Cos[e + f*x]]) - (a^5*Sqrt[Cos[e +
 f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2
- b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) - (a^5*
Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2]
)/(b^2*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*
x]]) + (2*a*Sin[e + f*x])/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(}}{\left(a^2-b^2\right)g^2}$$

$$= \frac{2a\sin(e+fx)}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{(2a)\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{3\left(a^2-b^2\right)g^2} - \frac{a^2\int \frac{\sin}{\sqrt{g\cos}}}{b\left(a^2-\right.}$$

$$= \frac{2a\sin(e+fx)}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, g\cos(e+f\right.}{b\left(a^2-b^2\right)fg^3}$$

$$= -\frac{2b}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{2a^2\sqrt{g\cos(e+fx)}}{b\left(a^2-b^2\right)fg^3} - \frac{2b\sqrt{g\cos}}{\left(a^2-b\right.}$$

$$= -\frac{2b}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{2a^2\sqrt{g\cos(e+fx)}}{b\left(a^2-b^2\right)fg^3} - \frac{2b\sqrt{g\cos}}{\left(a^2-b\right.}$$

$$= -\frac{2b}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{2a^2\sqrt{g\cos(e+fx)}}{b\left(a^2-b^2\right)fg^3} - \frac{2b\sqrt{g\cos}}{\left(a^2-b\right.}$$

$$= -\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{7/4}fg^{5/2}} - \frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{7/4}fg^{5/2}} - \frac{}{3\left(a^2-b\right.}$$

Mathematica [C] time = 26.84, size = 1958, normalized size = 3.26

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(-b + a*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e + f*x])^(5
/2)) + (Cos[e + f*x]^(5/2)*((-2*(-7*a^2 + 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*
x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Co
s[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(
5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
```

```
2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2
*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Co
s[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^
2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b
]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqr
t[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b
*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2
]*(a + b*Sin[e + f*x])) + ((3*a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])
*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*
Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1
/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-
a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[e + f*x]])/b
- (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[
1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[C
os[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1
, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*Appell
F1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (
-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + ((1/4
- I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(
1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) -
 ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 +
 b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(
3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(-1 + 2*Cos[e + f*x]^2)*(a +
 b*Sin[e + f*x])) - (4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 - b^
2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
 b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*Appell
F1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] +
2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*
Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x
]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)
^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/
4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e +
f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(6*
(a - b)*(a + b)*f*(g*Cos[e + f*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^4/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 11.51, size = 1268, normalized size = 2.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 2/f/g^3/b*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-2/f/g/b*a^4/(a-b)/(a+b)*sum(
(_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g
^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/
2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g
^3*_Z^2+b^2*g^4))-1/12/f/g^3*b/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))^2
*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/12/f/g^3*b*2^(1/2)/(a^2-b^2)/(cos(1/
2*f*x+1/2*e)-1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/12/f/g^3*b/
(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^
(1/2)-1/12/f/g^3*b*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*s
in(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*
x+1/2*e)^2)^(1/2)*a/g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(
1/2)/b^2*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g
*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x
+1/2*e),2^(1/2))+1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^
(1/2)*a^5/g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a-b)
/(a+b)/b^4*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^
2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2
*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(
```

```
2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*
x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1
/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f
*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1)
,2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+1/3/f*(g*(2*cos(1/2*f*
x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a/g^3/sin(1/2*f*x+1/2*e)/(g*(2*co
s(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*cos(1/2*f*x+1/2*e)*(-g*(2*sin(1/2*f*
x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/(cos(1/2*f*x+1/2*e)^2-1/2)^2-2/3/f*
(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a/g^2/sin(1/2*f*x
+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*(sin(1/2*f*x+1/2*e)^
2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(
1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^4/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^4}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)^4/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1403 $\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=528

$$-\frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bfg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{4b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{2a}{3fg\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}}-\frac{}{3fg\left(a^2\right.}$$

```
[Out] 2/3*a/(a^2-b^2)/f/g/(g*cos(f*x+e))^(3/2)-2/3*b*sin(f*x+e)/(a^2-b^2)/f/g/(g*
cos(f*x+e))^(3/2)+a^3*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/
g^(1/2))/(-a^2+b^2)^(7/4)/f/g^(5/2)/b^(1/2)+a^3*arctanh(b^(1/2)*(g*cos(f*x+
e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/(-a^2+b^2)^(7/4)/f/g^(5/2)/b^(1/2)-2*a^
2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2
*e),2^(1/2))*cos(f*x+e)^(1/2)/b/(a^2-b^2)/f/g^2/(g*cos(f*x+e))^(1/2)+4/3*b*
(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e
),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g^2/(g*cos(f*x+e))^(1/2)+a^4*(cos(1
/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b
/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b/(a^2-b^2)/f/g^2/(a^2-b*(b
-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)+a^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/c
os(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^
(1/2))*cos(f*x+e)^(1/2)/b/(a^2-b^2)/f/g^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*c
os(f*x+e))^(1/2)
```

Rubi [A] time = 1.09, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {2902, 2565, 30, 2566, 2642, 2641, 2867, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bfg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{4b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(\right.}{3fg^2\left(a^2-b^2\right)\sqrt{g\,\mathrm{co}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a^3*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
Sqrt[b]*(-a^2 + b^2)^(7/4)*f*g^(5/2)) + (a^3*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sqrt[b]*(-a^2 + b^2)^(7/4)*f*g^(5/
2)) + (2*a)/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2)) - (2*a^2*Sqrt[Cos[e
+ f*x]]*EllipticF[(e + f*x)/2, 2])/(b*(a^2 - b^2)*f*g^2*Sqrt[g*Cos[e + f*x]
]) + (4*b*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^
```

```
2*Sqrt[g*Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqr
t[-a^2 + b^2]), (e + f*x)/2, 2])/(b*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b
^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b
)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b*(a^2 - b^2)*(a^2 - b*(b + Sqr
t[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) - (2*b*Sin[e + f*x])/(3*(a^2 -
b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin}}{\left(a^2-b^2\right)g^2}$$

$$= -\frac{2b\sin(e+fx)}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{b\left(a^2-b^2\right)g^2} + \frac{a^3\int \frac{}{\sqrt{g}}}{}$$

$$= \frac{2a}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{2b\sin(e+fx)}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}}$$

$$= \frac{2a}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right.}{b\left(a^2-b^2\right)fg^2\sqrt{g\cos(e+fx)}}$$

$$= \frac{2a}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right.}{b\left(a^2-b^2\right)fg^2\sqrt{g\cos(e+fx)}}$$

$$= \frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{\sqrt{b}\left(-a^2+b^2\right)^{7/4}fg^{5/2}} + \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{\sqrt{b}\left(-a^2+b^2\right)^{7/4}fg^{5/2}} + \frac{}{3\left(a^2-b\right.}$$

Mathematica [C] time = 23.86, size = 1193, normalized size = 2.26

$$\frac{2\cos(e+fx)(a-b\sin(e+fx))}{3\left(a^2-b^2\right)f(g\cos(e+fx))^{5/2}}-\frac{\cos^{\frac{5}{2}}(e+fx)\left(\frac{4ab\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(\frac{\cdots}{\sqrt{1-\cos^2(e+fx)}\left(5\left(a^2-b^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-\cdots\right.}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e + f*x])^(5/
2)) - (Cos[e + f*x]^(5/2)*((4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a
^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*A
ppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)
] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (
b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e +
 f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e
 + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e +
f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 +
 b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] +
 (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])
)/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e
+ f*x])) - (2*(3*a^2 - 2*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 -
 b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*App
ellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
 + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b
^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e +
f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b
^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^
(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e
 + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/
(3*(a - b)*(a + b)*f*(g*Cos[e + f*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{3}}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^3/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 13.09, size = 2331, normalized size = 4.41

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 2/f/g*a^3/(a-b)/(a+b)*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g
^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/
2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^
2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1/12/f/g^3*a/(a^2-b^2)/(cos(1/2
*f*x+1/2*e)+1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/12/f/g^3*a
*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^
2*g+g)^(1/2)+1/12/f/g^3*a/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))^2*(-2*
sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/12/f/g^3*a*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x
+1/2*e)-1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-8/f*(g*(2*cos(1/2*
f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3*sin(1/2*f*x+1/2*e)/(g*(2*
cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/2*e)^2-1)*cos(1/2
*f*x+1/2*e)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)+4/f*(g
*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3/sin(1/2*f*x+1
/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/2*e)^
2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*
f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+8/f*(g*(2*cos(1
/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3/sin(1/2*f*x+1/2*e)/(g*
(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/2*e)^2-1)*(-2*
```

```
sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)-4/f
*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3/sin(1/2*f*
x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/
2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin
(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+1/2/f*(g*(
2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/b/g^2*sin(1/2*f*x+1/2
*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^2/(a-b)/(a+b)*sum(_alpha/(2*_alp
ha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*
_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2
*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^
(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_
alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)
^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticP
i(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4
*b^2-4*_Z^2*b^2+a^2))-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e
)^2)^(1/2)/b/g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^
2/(a-b)/(a+b)*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b
^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/
2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*
(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f
*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(
1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*
f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1
),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+2/3/f*(g*(2*cos(1/2*f
*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3*sin(1/2*f*x+1/2*e)/(g*(2*c
os(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*cos(1/2*f*x+1/2*e)*(-g*(2*sin(1/2*f
*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/(cos(1/2*f*x+1/2*e)^2-1/2)^2-2/3/f
*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3/sin(1/2*f*
x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*cos(1/2*f*x+1/2*e)*
(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/(cos(1/2*f*x+1/2*e
)^2-1/2)^2-4/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*
b/g^2*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*(si
n(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*
f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1
/2))+4/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^2/
sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*(sin(1/2*
f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/
2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{3}}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^3/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^3}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)^3/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1404 $\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=468

$$-\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2}{3fg\left(a^2-b^2\right)(}$$

```
[Out] -2/3*b/(a^2-b^2)/f/g/(g*cos(f*x+e))^(3/2)+2/3*a*sin(f*x+e)/(a^2-b^2)/f/g/(g
*cos(f*x+e))^(3/2)-a^2*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)
/g^(1/2))*b^(1/2)/(-a^2+b^2)^(7/4)/f/g^(5/2)-a^2*arctanh(b^(1/2)*(g*cos(f*x
+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*b^(1/2)/(-a^2+b^2)^(7/4)/f/g^(5/2)+2/3
*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/
2*e),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g^2/(g*cos(f*x+e))^(1/2)-a^3*(co
s(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),
2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g^2/(a^2-b*(
b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-a^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/
cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2
^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*co
s(f*x+e))^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.394, Rules used = {2902, 2636, 2642, 2641, 2565, 30, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{a^3\sqrt{\cos(e}}{fg^2\left(a^2-b^2\right)(}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[g])])/((-a^2 + b^2)^(7/4)*f*g^(5/2))) - (a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/((-a^2 + b^2)^(7/4)*f*g^
(5/2)) - (2*b)/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2)) + (2*a*Sqrt[Cos[e
 + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^2*Sqrt[g*Cos[e + f*x
]]) - (a^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e +
 f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos
[e + f*x]]) - (a^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2
```

```
]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*g^2*Sq
rt[g*Cos[e + f*x]]) + (2*a*Sin[e + f*x])/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x]
)^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
```

```
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{1}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,(a+b\sin}}{\left(a^2-b^2\right)g^2}$$

$$= \frac{2a\sin(e+fx)}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{3\left(a^2-b^2\right)g^2} - \frac{a^3\int}{}$$

$$= -\frac{2b}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{2a\sin(e+fx)}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/}}$$

$$= -\frac{2b}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\right|}{3\left(a^2-b^2\right)fg^2\sqrt{g\cos(e+fx}}$$

$$= -\frac{a^2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{7/4}fg^{5/2}} - \frac{a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{7/4}fg^{5/2}} -$$

Mathematica [C] time = 23.34, size = 1184, normalized size = 2.53

$$\frac{2\cos(e+fx)(a\sin(e+fx)-b)}{3\left(a^2-b^2\right)f(g\cos(e+fx))^{5/2}} - \frac{a\cos^{\frac{5}{2}}(e+fx)\left(\frac{2b\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(\frac{5b\left(a^2-b^2\right)}{\left(2\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)b^2+\left(a^2-b^2\right)F_1\left(\frac{5}{4};\right.\right.\right.}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(-b + a*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e + f*x])^(5
/2)) - (a*Cos[e + f*x]^(5/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*
```

```
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)
*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]
^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e
 + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos
[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e
+ f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2
 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2]
 + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]
]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[
e + f*x])) + (2*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 - b^2)*Appell
F1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sq
rt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -
1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*
AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) +
(a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] +
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log
[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b
*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))
*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(3*(a - b)*(
a + b)*f*(g*Cos[e + f*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^2/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 10.39, size = 1089, normalized size = 2.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -2/f/g*b*a^2/(a-b)/(a+b)*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^
2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos
(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2
*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/12/f/g^3*b/(a^2-b^2)/(cos(
1/2*f*x+1/2*e)+1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/12/f/g^
3*b*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*
e)^2*g+g)^(1/2)-1/12/f/g^3*b/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))^2*(
-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/12/f/g^3*b*2^(1/2)/(a^2-b^2)/(cos(1/2*
f*x+1/2*e)-1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/8/f*(g*(2*cos
(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a^3/g^2/sin(1/2*f*x+1/2*e)
/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a-b)/(a+b)/b^2*sum(1/_alpha/(2*_alph
a^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_
alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*
e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(
1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_a
lpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^
(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi
(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*
b^2-4*_Z^2*b^2+a^2))+1/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)
^2)^(1/2)*a/g^3/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^
2-b^2)*cos(1/2*f*x+1/2*e)*(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2)
)^(1/2)/(cos(1/2*f*x+1/2*e)^2-1/2)^2-2/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*si
n(1/2*f*x+1/2*e)^2)^(1/2)*a/g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)
^2-1))^(1/2)/(a^2-b^2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^
2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*Ellipti
cF(cos(1/2*f*x+1/2*e),2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^2/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^2}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)^2/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1405 $\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=432

$$-\frac{2b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{a^2b\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{fg^2\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^2b\sqrt{\cos(e+f}}{fg^2\left(a^2-b^2\right)\left(a^2-\right.}$$

```
[Out] a*b^(3/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/(-a
^2+b^2)^(7/4)/f/g^(5/2)+a*b^(3/2)*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^
2+b^2)^(1/4)/g^(1/2))/(-a^2+b^2)^(7/4)/f/g^(5/2)+2/3*(a-b*sin(f*x+e))/(a^2-
b^2)/f/g/(g*cos(f*x+e))^(3/2)-2/3*b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*
x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f
/g^2/(g*cos(f*x+e))^(1/2)+a^2*b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/
2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*
x+e)^(1/2)/(a^2-b^2)/f/g^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2
)+a^2*b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*
f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g
^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)
```

Rubi [A] time = 0.92, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2866, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{2b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{a^2b\sqrt{\cos(e}}{fg^2\left(a^2-b^2\right)\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g
])])/((-a^2 + b^2)^(7/4)*f*g^(5/2)) + (a*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Co
s[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/((-a^2 + b^2)^(7/4)*f*g^(5/2))
- (2*b*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^2*S
qrt[g*Cos[e + f*x]]) + (a^2*b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt
[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]
))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (a^2*b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)
/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-
a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2*(a - b*Sin[e + f*x]))/(3*(a^2
 - b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{2(a-b\sin(e+fx))}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{2\int \frac{-ab+\frac{1}{2}b^2\sin(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{3\left(a^2-b^2\right)g^2}$$

$$= \frac{2(a-b\sin(e+fx))}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{3\left(a^2-b^2\right)g^2} + \frac{(ab)\int \frac{}{\sqrt{g\cos}}}{(a}$$

$$= \frac{2(a-b\sin(e+fx))}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{\left(a^2b\right)\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(\right.}}{2\left(-a^2+b^2\right)^{3/2}g^2}$$

$$= -\frac{2b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3\left(a^2-b^2\right)fg^2\sqrt{g\cos(e+fx)}} + \frac{2(a-b\sin(e+fx))}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/}}$$

$$= -\frac{2b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3\left(a^2-b^2\right)fg^2\sqrt{g\cos(e+fx)}} - \frac{a^2b\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2-}}\right.}{\left(-a^2+b^2\right)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)}$$

$$= \frac{ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{7/4}fg^{5/2}} + \frac{ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{7/4}fg^{5/2}} - \frac{2b\sqrt{c}}{3\left(a\right.}$$

Mathematica [C] time = 23.25, size = 1183, normalized size = 2.74

$$b\left(\frac{2b\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(\frac{5b\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\sqrt{1-\cos^2(e+fx)}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)}{\left(2\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)b^2+\left(a^2-b^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)\right)\cos^2(e+fx)-5\left(a^2-b^2\right)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\cos^2(\right.}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e + f*x])^(5/
2)) + (b*Cos[e + f*x]^(5/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(
```

```
a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2,
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e
+ f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[
e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e +
 f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2
+ b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2]
+ (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]
))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e
 + f*x])) + (2*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 - b^2)*AppellF
1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqr
t[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1
/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*A
ppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (
a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] +
2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[
Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*
Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqr
t[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*
Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(3*(a - b)*(a
 + b)*f*(g*Cos[e + f*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(sin(f*x + e)/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 13.25, size = 2322, normalized size = 5.38

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 2/f/g*a*b^2/(a-b)/(a+b)*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2
*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(
1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*
g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1/12/f/g^3*a/(a^2-b^2)/(cos(1
/2*f*x+1/2*e)+1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/12/f/g^3
*a*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e
)^2*g+g)^(1/2)+1/12/f/g^3*a/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))^2*(-
2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/12/f/g^3*a*2^(1/2)/(a^2-b^2)/(cos(1/2*f
*x+1/2*e)-1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-8/f*(g*(2*cos(1/
2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3*sin(1/2*f*x+1/2*e)/(g*(
2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/2*e)^2-1)*cos(1
/2*f*x+1/2*e)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)+8/f*
(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3/sin(1/2*f*x
+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/2*e
)^2-1)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x
+1/2*e)+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3
/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1
/2*f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*sin(1/2*f*x+1/2*e
)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-4
/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3/sin(1/2*
f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+
1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+s
in(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+1/2/f*(g
*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^2*sin(1/2*f*x+1
/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a-b)/(a+b)*sum(_alpha/(2*_alpha
^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_a
lpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e
)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1
/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_al
pha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(
1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(
cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b
^2-4*_Z^2*b^2+a^2))-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^
```

```
2)^(1/2)*b/g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a-b
)/(a+b)*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^
2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2
*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_al
pha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2
*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(
-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/
2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1
/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+2/3/f*(g*(2*cos(1/2*f*x+1/2
*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2
*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*cos(1/2*f*x+1/2*e)*(-g*(2*sin(1/2*f*x+1/2
*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/(cos(1/2*f*x+1/2*e)^2-1/2)^2-2/3/f*(g*(2
*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3/sin(1/2*f*x+1/2*
e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*cos(1/2*f*x+1/2*e)*(-g*(2
*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/(cos(1/2*f*x+1/2*e)^2-1/
2)^2-4/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^2*
sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*(sin(1/2*
f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/
2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))+4
/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^2/sin(1/
2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*(sin(1/2*f*x+1/
2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4
-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate(sin(f*x + e)/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
[Out] Timed out
```

3.1406 $\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=527

$$-\frac{2b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{2b(b-a\sin(e+fx))}{3afg\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{b^{7/2}\tanh^{-1}\left(\cdots\right.}{afg^{5/2}\left(\cdots\right.}$$

```
[Out] -arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(5/2)+b^(7/2)*arctan(b^(1/2)*(g
*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/(-a^2+b^2)^(7/4)/f/g^(5/2)-a
rctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(5/2)+b^(7/2)*arctanh(b^(1/2)*(g
*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/(-a^2+b^2)^(7/4)/f/g^(5/2)+2
/3/a/f/g/(g*cos(f*x+e))^(3/2)+2/3*b*(b-a*sin(f*x+e))/a/(a^2-b^2)/f/g/(g*cos
(f*x+e))^(3/2)-2/3*b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipt
icF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g^2/(g*cos(f*x
+e))^(1/2)+b^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(s
in(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b
^2)/f/g^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)+b^3*(cos(1/2*f*
x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(
-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g^2/(a^2-b*(b+(-a^2+
b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)
```

Rubi [A] time = 1.31, antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {2898, 2565, 325, 329, 212, 206, 203, 2696, 2867, 2642, 2641, 2702, 2807, 2805, 208, 205}

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{2b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{b^3\sqrt{\cos(e+f\cdots}}{fg^2\left(a^2-b^2\right)\left(a^2-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -(ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(5/2))) + (b^(7/2)*ArcTan[(Sq
rt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2 + b^2)^
(7/4)*f*g^(5/2)) - ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(5/2)) + (b
^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])]
)/(a*(-a^2 + b^2)^(7/4)*f*g^(5/2)) + 2/(3*a*f*g*(g*Cos[e + f*x])^(3/2)) - (
2*b*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^2*Sqrt
[g*Cos[e + f*x]]) + (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2
```

```
 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*
g^2*Sqrt[g*Cos[e + f*x]]) + (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + S
qrt[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b
^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2*b*(b - a*Sin[e + f*x]))/(3*a*(a^2 -
b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \int \left(\frac{\csc(e+fx)}{a(g\cos(e+fx))^{5/2}} - \frac{b}{a(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\right) d$$

$$= \frac{\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a} - \frac{b\int \frac{1}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx}{a}$$

$$= \frac{2b(b-a\sin(e+fx))}{3a\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{(2b)\int \frac{-\frac{a^2}{2}+\frac{3b^2}{2}-\frac{1}{2}ab\sin(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}}{3a\left(a^2-b^2\right)g^2}$$

Subs

$$= \frac{2}{3afg(g\cos(e+fx))^{3/2}} + \frac{2b(b-a\sin(e+fx))}{3a\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} -$$

2 Su

$$= \frac{2}{3afg(g\cos(e+fx))^{3/2}} + \frac{2b(b-a\sin(e+fx))}{3a\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} -$$

$$= \frac{2}{3afg(g\cos(e+fx))^{3/2}} - \frac{2b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3\left(a^2-b^2\right)fg^2\sqrt{g\cos(e+fx)}} + \frac{}{3a\,(}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{5/2}} + \frac{2}{3afg(g\cos(e+f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{5/2}} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a\left(-a^2+b^2\right)^{7/4}fg^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{g\,c}}{}\right.}{afg^{5/}}$$

Mathematica [C] time = 30.57, size = 2136, normalized size = 4.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]^(5/2)*((-8*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 -
b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
+ b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*Appell
F1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2
*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*C
os[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]
^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*
x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]]
)/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a
^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) - (b^2*(-
1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e
 + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e
+ f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (10*Sqrt[2]*(2
*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/
4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*ArcTan[Sqrt[Cos[e + f*x]]])/a - (1
6*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
 b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (200*b*(a^2 - b^2)*AppellF1[1/4,
1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e
+ f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4
, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/
4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2
+ b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (10*Log[1 -
 Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[Cos[e + f*x]]])/a - (5*Sqrt[2]*(
2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[C
os[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*Sqrt[2]*
(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[
Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))))/(20*(1 - C
os[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (2*(6*a^2 -
7*b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*
((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2
- b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f
*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 +
 Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[C
os[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(3/4)*ArcTan[S
qrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e + f*x]]] + 2*(a
^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/2)*Log[Sqrt[a^2
- b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f
```

```
*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1
/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^(3/4))))/((1 - C
os[e + f*x]^2)*(b + a*Csc[e + f*x])))))/(6*(a - b)*(a + b)*f*(g*Cos[e + f*x]
)^(5/2)) + (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e
+ f*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(csc(f*x + e)/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

maple [A] time = 5.24, size = 627, normalized size = 1.19

$$\left(24\ln\left(\frac{2\sqrt{-g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}-2g}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)g^{\frac{7}{2}}-12\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}+4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)\sqrt{-g}\,g^{3}-12\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\cdots}}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/6*((24*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^
(1/2)-g))*g^(7/2)-12*ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+
1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)*g^3-12*ln(2/(cos(
1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*
f*x+1/2*e)-g))*(-g)^(1/2)*g^3)*sin(1/2*f*x+1/2*e)^4+(-24*ln(2/cos(1/2*f*x+1
```

```
/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g))*g^(7/2)+12*ln(2/(
-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos
(1/2*f*x+1/2*e)-g))*(-g)^(1/2)*g^3+12*ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*
(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)*g
^3)*sin(1/2*f*x+1/2*e)^2+6*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*
f*x+1/2*e)^2*g+g)^(1/2)-g))*g^(7/2)+4*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)*(
-g)^(1/2)*g^(5/2)-3*ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1
/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)*g^3-3*ln(2/(cos(1/
2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*
x+1/2*e)-g))*(-g)^(1/2)*g^3)/(-g)^(1/2)/g^(11/2)/a/(4*sin(1/2*f*x+1/2*e)^4-
4*sin(1/2*f*x+1/2*e)^2+1)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate(csc(f*x + e)/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(e+f\,x\right)\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)*(g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1407 $\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=651

$$\frac{2b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3afg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b^2(b-a\sin(e+fx))}{3a^2fg\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}}-\frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{b^{9/2}\tanh^{-1}\left(\right.}{a^2fg^{5/2}\left(\right.}$$

```
[Out] b*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f/g^(5/2)-b^(9/2)*arctan(b^(1/2)
*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/(-a^2+b^2)^(7/4)/f/g^(5
/2)+b*arctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f/g^(5/2)-b^(9/2)*arctanh(b
^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/(-a^2+b^2)^(7/4)/
f/g^(5/2)-2/3*b/a^2/f/g/(g*cos(f*x+e))^(3/2)-csc(f*x+e)/a/f/g/(g*cos(f*x+e)
)^(3/2)+5/3*sin(f*x+e)/a/f/g/(g*cos(f*x+e))^(3/2)-2/3*b^2*(b-a*sin(f*x+e))/
a^2/(a^2-b^2)/f/g/(g*cos(f*x+e))^(3/2)+5/3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos
(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/a/f/
g^2/(g*cos(f*x+e))^(1/2)+2/3*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/a/(a^2-b^2)/f/
g^2/(g*cos(f*x+e))^(1/2)-b^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e
)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e
)^(1/2)/a/(a^2-b^2)/f/g^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)
-b^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x
+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/(a^2-b^2)/f/g^
2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)
```

Rubi [A] time = 1.46, antiderivative size = 651, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 18, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2898, 2565, 325, 329, 212, 206, 203, 2570, 2636, 2642, 2641, 2696, 2867, 2702, 2807, 2805, 208, 205}

$$-\frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{b^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{2b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3afg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{b^4\sqrt{\cos(e}}{afg^2\left(a^2-b^2\right)\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (b*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*g^(5/2)) - (b^(9/2)*ArcTan[
(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2 +
b^2)^(7/4)*f*g^(5/2)) + (b*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*g^
(5/2)) - (b^(9/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4
```

```
)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(7/4)*f*g^(5/2)) - (2*b)/(3*a^2*f*g*(g*Cos[e
 + f*x])^(3/2)) - Csc[e + f*x]/(a*f*g*(g*Cos[e + f*x])^(3/2)) + (5*Sqrt[Cos
[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*a*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2
*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*a*(a^2 - b^2)*f*g^2*S
qrt[g*Cos[e + f*x]]) - (b^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-
a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]
))*f*g^2*Sqrt[g*Cos[e + f*x]]) - (b^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(
b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-
a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (5*Sin[e + f*x])/(3*a*f*g*(g*Cos
[e + f*x])^(3/2)) - (2*b^2*(b - a*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)*f*g*(g*
Cos[e + f*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
```

```
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \int\left(-\frac{b\csc(e+fx)}{a^2(g\cos(e+fx))^{5/2}}+\frac{\csc^2(e+fx)}{a(g\cos(e+fx))^{5/2}}+\frac{}{a^2(g\cos(e+fx}\right.$$

$$=\frac{\int\frac{\csc^2(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a}-\frac{b\int\frac{\csc(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2}+\frac{b^2\int\frac{1}{(g\cos(e+fx))^{5/2}(a+b\sin}}{a^2}$$

$$=-\frac{\csc(e+fx)}{afg(g\cos(e+fx))^{3/2}}-\frac{2b^2(b-a\sin(e+fx))}{3a^2\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}}+\frac{5\int}{}$$

$$=-\frac{2b}{3a^2fg(g\cos(e+fx))^{3/2}}-\frac{\csc(e+fx)}{afg(g\cos(e+fx))^{3/2}}+\frac{5\sin(e+}{3afg(g\cos(e}$$

$$=-\frac{2b}{3a^2fg(g\cos(e+fx))^{3/2}}-\frac{\csc(e+fx)}{afg(g\cos(e+fx))^{3/2}}+\frac{5\sin(e+}{3afg(g\cos(e}$$

$$=-\frac{2b}{3a^2fg(g\cos(e+fx))^{3/2}}-\frac{\csc(e+fx)}{afg(g\cos(e+fx))^{3/2}}+\frac{5\sqrt{\cos(e+f}}{3afg^2\sqrt{g}}$$

$$=\frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{5/2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{5/2}}-\frac{2b}{3a^2fg(g\cos(e+}$$

$$=\frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{5/2}}-\frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^2\left(-a^2+b^2\right)^{7/4}fg^{5/2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{g}}{}\right.}{a^2fg}$$

Mathematica [C] time = 29.96, size = 2183, normalized size = 3.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]^(5/2)*((-2*(10*a^3 - 18*a*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2
])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a
^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e
+ f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-
1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]
*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt
[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a
^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos
[e + f*x]]))/(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e +
f*x])) - ((-5*a^2*b + 3*b^3)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e +
f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1
 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2
 - b^2)^(3/4)) + (10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt
[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*Arc
Tan[Sqrt[Cos[e + f*x]]])/a - (16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^
2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (2
00*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 -
 b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e +
 f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*
x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 +
Cos[e + f*x]^2))) + (10*Log[1 - Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[C
os[e + f*x]]])/a - (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(
a^2 - b^2)^(3/4)) + (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*
Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*
(a^2 - b^2)^(3/4))))/(20*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b +
a*Csc[e + f*x])) - (2*(-7*a^2*b + 9*b^3)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[
1 - Cos[e + f*x]^2])*Csc[e + f*x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/
(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e +
 f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2
, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*Ap
pellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
)*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*
ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt
```

```
[2]*b^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/
4)] + 4*(a^2 - b^2)^(3/4)*ArcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*
Log[1 - Sqrt[Cos[e + f*x]]] + 2*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]
]] - Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4
)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2
] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])
/(8*a*(a^2 - b^2)^(3/4))))/((1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(1
2*a*(a - b)*(a + b)*f*(g*Cos[e + f*x])^(5/2)) + (Cos[e + f*x]^3*(-(Csc[e +
f*x]/a) + (2*Sec[e + f*x]^2*(-b + a*Sin[e + f*x]))/(3*(a^2 - b^2))))/(f*(g*
Cos[e + f*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{2}}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(csc(f*x + e)^2/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

maple [C] time = 19.53, size = 2312, normalized size = 3.55

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 2/f/g^(5/2)*b/(2+2^(1/2))^2/(2^(1/2)-2)^2/a^2*ln((4*g*cos(1/2*f*x+1/2*e)+2*
g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))-2
/f/g*b^5/(a-b)/(a+b)/a^2*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^
2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos
```

(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2
*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/f/g^2*b/a^2/(-g)^(1/2)*ln(
(-2*g+2*(-g)^(1/2)*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))+
1/6/f/g^3*b/(2+2^(1/2))/(2^(1/2)-2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/
2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/6/f/g^3*b*2^(1/2)/(2+2^(1/2))/(
2^(1/2)-2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e
)^2*g+g)^(1/2)+1/6/f/g^3*b/(2+2^(1/2))/(2^(1/2)-2)/(a^2-b^2)/(cos(1/2*f*x+1
/2*e)-1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/6/f/g^3*b*2^(1/2
)/(2+2^(1/2))/(2^(1/2)-2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(-2*si
n(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2/f/g^(5/2)*b/(2+2^(1/2))^2/(2^(1/2)-2)^2/a^2
*ln((-4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-
2*g)/(cos(1/2*f*x+1/2*e)+1))+5/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*
x+1/2*e)^2)^(1/2)*a/g/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3
/2)/(a^2-b^2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*Ellip
ticF(cos(1/2*f*x+1/2*e),2^(1/2))*(2*sin(1/2*f*x+1/2*e)^2-1)^(3/2)*(sin(1/2*
f*x+1/2*e)^2)^(1/2)-1/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)
^(1/2)/a/g/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)/(a^2-b^
2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticF(cos(1/
2*f*x+1/2*e),2^(1/2))*(2*sin(1/2*f*x+1/2*e)^2-1)^(3/2)*(sin(1/2*f*x+1/2*e)^
2)^(1/2)*b^2-10/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/
2)*a/g/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g
)^(3/2)/(a^2-b^2)*sin(1/2*f*x+1/2*e)^5/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)
+2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a/g/cos(1/2*
f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)/(a^2-b^
2)*sin(1/2*f*x+1/2*e)^5/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*b^2+1/8/f*(g*(
2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a^3/g^2/(a^2-b^2)/sin
(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alpha/(2*_alpha
^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1
/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alp
ha^2*b^2+4*b^2)/a^2,2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^
2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4
*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+
2^(1/2)*a^2*arctanh(1/2/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-2*sin(1/
2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16
*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_
alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha
^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x
+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+
1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z
^2*b^2+a^2))*b^2+10/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)
^(1/2)*a/g/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)
^2*g)^(3/2)/(a^2-b^2)*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(
1/2)-2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a/g/cos(
1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)/(a^
2-b^2)*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*b^2-1/2/f*

```
(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a/g/cos(1/2*f*x+1
/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)/(a^2-b^2)*si
n(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+1/2/f*(g*(2*cos(1/2*f
*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a/g/cos(1/2*f*x+1/2*e)/(-2*sin(1
/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)/(a^2-b^2)*sin(1/2*f*x+1/2*e
)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*b^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(e+f\,x\right)^2\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1408 $\int \frac{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=926

$$-\frac{2\sqrt{2}\,a^3\sqrt{g}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}\,d^3}{b^3\sqrt{b-a}\sqrt{a+b}\,f\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,a^3\sqrt{g}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right.\right.}{b^3\sqrt{b-a}\sqrt{a+b}\,f\sqrt{d\sin(e}}$$

```
[Out] -1/2*a^2*d^(5/2)*arctan(-1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*
sin(f*x+e))^(1/2))*g^(1/2)/b^3/f*2^(1/2)-1/8*d^(5/2)*arctan(-1+2^(1/2)*d^(1
/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))*g^(1/2)/b/f*2^(1/2)-
1/2*a^2*d^(5/2)*arctan(1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*si
n(f*x+e))^(1/2))*g^(1/2)/b^3/f*2^(1/2)-1/8*d^(5/2)*arctan(1+2^(1/2)*d^(1/2)
*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))*g^(1/2)/b/f*2^(1/2)-1/4
*a^2*d^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(
1/2)/(d*sin(f*x+e))^(1/2))*g^(1/2)/b^3/f*2^(1/2)-1/16*d^(5/2)*ln(g^(1/2)+co
t(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))
*g^(1/2)/b/f*2^(1/2)+1/4*a^2*d^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)+2^(1/2)*
d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))*g^(1/2)/b^3/f*2^(1/2)+1/
16*d^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/
2)/(d*sin(f*x+e))^(1/2))*g^(1/2)/b/f*2^(1/2)-2*a^3*d^3*EllipticPi((g*cos(f*
x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/
2)*g^(1/2)*sin(f*x+e)^(1/2)/b^3/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(
1/2)+2*a^3*d^3*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2)
,(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/b^3/f/(-a+b)^
(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)-1/2*d^2*(g*cos(f*x+e))^(3/2)*(d*sin(
f*x+e))^(1/2)/b/f/g+a*d^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Ell
ipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)
/b^2/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 1.84, antiderivative size = 926, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {2909, 2568, 2575, 297, 1162, 617, 204, 1165, 628, 2572, 2639, 2906, 2905, 490, 1218}

$$-\frac{2\sqrt{2}\,a^3\sqrt{g}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}\,d^3}{b^3\sqrt{b-a}\sqrt{a+b}\,f\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,a^3\sqrt{g}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right.\right.}{b^3\sqrt{b-a}\sqrt{a+b}\,f\sqrt{d\sin(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2))/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^2*d^(5/2)*Sqrt[g]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqr
t[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^3*f) + (d^(5/2)*Sqrt[g]*ArcTan[1 -
(Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(4*
Sqrt[2]*b*f) - (a^2*d^(5/2)*Sqrt[g]*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[
e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^3*f) - (d^(5/2)*Sqrt
[g]*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e
 + f*x]])])/(4*Sqrt[2]*b*f) - (a^2*d^(5/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Co
t[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/
(2*Sqrt[2]*b^3*f) - (d^(5/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (
Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(8*Sqrt[2]*b*f
) + (a^2*d^(5/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt
[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^3*f) + (d^(5/
2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos
[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(8*Sqrt[2]*b*f) - (2*Sqrt[2]*a^3*d^3*Sqr
t[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(S
qrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^3*Sqrt[-a + b]*
Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a^3*d^3*Sqrt[g]*EllipticPi
[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Si
n[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^3*Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt
[d*Sin[e + f*x]]) - (d^2*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])/(2*b*
f*g) - (a*d^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[
e + f*x]])/(b^2*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
```

```
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx = \frac{d\int \sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}\,dx}{b} - \frac{(ad)\int \frac{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3}}{a+b\sin(e+fx)}}{b}$$

$$= -\frac{d^2(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bfg} - \frac{\left(ad^2\right)\int \sqrt{g\cos(e+fx)}\,\sqrt{d\sin(}}{b^2}$$

$$= -\frac{d^2(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bfg} + \frac{\left(a^2d^3\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{b^3} - \frac{\left(a^3d\right.}{}$$

$$= -\frac{d^2(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bfg} - \frac{ad^2\sqrt{g\cos(e+fx)}\,E\left(e-\frac{\pi}{4}+\right.}{b^2f\sqrt{\sin(2e+}}$$

$$= -\frac{d^2(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bfg} - \frac{ad^2\sqrt{g\cos(e+fx)}\,E\left(e-\frac{\pi}{4}+\right.}{b^2f\sqrt{\sin(2e+}}$$

$$= -\frac{d^{5/2}\sqrt{g}\,\log\left(\sqrt{g}+\sqrt{g}\,\cot(e+fx)-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{8\sqrt{2}\,bf} + \frac{d^{5/2}\sqrt{g}\,\log}{}$$

$$= \frac{d^{5/2}\sqrt{g}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{4\sqrt{2}\,bf} - \frac{d^{5/2}\sqrt{g}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e}}{\sqrt{g}\,\sqrt{d\sin(e+}}\right.}{4\sqrt{2}\,bf}$$

$$= \frac{a^2d^{5/2}\sqrt{g}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^3f} + \frac{d^{5/2}\sqrt{g}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\,co}}{\sqrt{g}\,\sqrt{d\sin(}}\right.}{4\sqrt{2}\,bf}$$

Mathematica [C] time = 27.59, size = 1623, normalized size = 1.75

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2))/(a + b*Sin[e + f*x]
),x]
```

```
[Out] -1/2*(Sqrt[g*Cos[e + f*x]]*Cot[e + f*x]*Csc[e + f*x]*(d*Sin[e + f*x])^(5/2)
)/(b*f) + (Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)*((-2*b*(-(b*AppellF1
```

```
[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a
*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2)
)/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) - (Sqrt[T
an[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*
Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt
[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[
Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^
2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 -
b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*
Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e
 + f*x]^2]))/(12*a*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x
])*(1 + Tan[e + f*x]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Ta
n[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4,
 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^
(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1
+ b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/
2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^
(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcT
an[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)
^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^
2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*
a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3
/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log
[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2
]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2
- b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqr
t[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[
2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[
a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/
2))/Sqrt[1 + Tan[e + f*x]^2])))/(42*a*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f
*x]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))
/(4*b*f*Sqrt[Cos[e + f*x]]*Sin[e + f*x]^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*(d*sin(f*x + e))^(5/2)/(b*sin(f*x + e) + a),
 x)
```

maple [B] time = 1.00, size = 4649, normalized size = 5.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(5/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/4/f*(a-b)*(I*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*b^2+4*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^3-4*(-a^2+b^2)^(1/2)*(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2-(-a^2+b^2)^(1/2)*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2-8*cos(f*x+e)*(-
a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b+4*cos(f*
x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*
EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b-4
*I*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*
```

```
I,1/2*2^(1/2))*a^2-I*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2+4*I*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2-4*(-a^2+b^2)^(1/2)
*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2-(-a^2+b^2)^
(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2+4*(-a^
2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1
/2*2^(1/2))*a^2+4*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(
b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2+2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2),1/2*2^(1/2))*b^2-4*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/
(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2*b+4*(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2*b-4*cos(f*x+e)*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^3+2*sin(f*x
+e)*cos(f*x+e)^2*(-a^2+b^2)^(1/2)*2^(1/2)*b^2-4*(-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^3+4*(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^3-4*cos(f*x+e)^2*(-a
^2+b^2)^(1/2)*2^(1/2)*a*b+4*cos(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*a*b-8*(-a^2
+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b+4*(-a^2+b^2
)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-
```

```
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b-4*I*(-a^2+b^2)^
(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2-I*(-a^
2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2
+4*I*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*
EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(
1/2))*a^2-4*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))*a^2-cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2-4*cos(f*x+e)*(-a^2+b^2)^(1/2)*(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2-cos(f*x+e)*(-
a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b
^2+4*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b
+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2+4*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2+2*cos
(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b^
2-4*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2))
,1/2*2^(1/2))*a^2*b+4*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+
(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2*b+I*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2)*(d*sin(f*x+e))^(5/2)*(g*cos(
f*x+e))^(1/2)/sin(f*x+e)^3/cos(f*x+e)*2^(1/2)*a/b^3/(-a^2+b^2)^(1/2)/(a-b+(
-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*(d*sin(f*x + e))^(5/2)/(b*sin(f*x + e) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(d\,\sin\left(e+f\,x\right)\right)^{5/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(5/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int(((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(5/2))/(a + b*sin(e + f*x)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(5/2)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1409 $\int \frac{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=578

$$\frac{2\sqrt{2}\,a^2d^2\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,a^2d^2\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\ldots\right.\right.}{b^2f\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+\ldots}}$$

```
[Out] 1/2*a*d^(3/2)*arctan(-1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin
(f*x+e))^(1/2))*g^(1/2)/b^2/f*2^(1/2)+1/2*a*d^(3/2)*arctan(1+2^(1/2)*d^(1/2
)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))*g^(1/2)/b^2/f*2^(1/2)+
1/4*a*d^(3/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))^
(1/2)/(d*sin(f*x+e))^(1/2))*g^(1/2)/b^2/f*2^(1/2)-1/4*a*d^(3/2)*ln(g^(1/2)+
cot(f*x+e)*g^(1/2)+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2
))*g^(1/2)/b^2/f*2^(1/2)+2*a^2*d^2*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/
(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e
)^(1/2)/b^2/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)-2*a^2*d^2*Ellip
ticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^
(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/b^2/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d
*sin(f*x+e))^(1/2)-d*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Elliptic
E(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/b/f/
sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 1.13, antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.378, Rules used = {2909, 2572, 2639, 2575, 297, 1162, 617, 204, 1165, 628, 2906, 2905, 490, 1218}

$$\frac{2\sqrt{2}\,a^2d^2\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,a^2d^2\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\ldots\right.\right.}{b^2f\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*d^(3/2)*Sqrt[g]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqr
t[g]*Sqrt[d*Sin[e + f*x]])]])/(Sqrt[2]*b^2*f)) + (a*d^(3/2)*Sqrt[g]*ArcTan[1
 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])]])/
(Sqrt[2]*b^2*f) + (a*d^(3/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (
Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2
*f) - (a*d^(3/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt
[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2*f) + (2*Sqr
```

```
t[2]*a^2*d^2*Sqrt[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*
Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b
^2*Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*a^2*d^2*Sq
rt[g]*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqr
t[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^2*Sqrt[-a + b]*Sq
rt[a + b]*f*Sqrt[d*Sin[e + f*x]]) + (d*Sqrt[g*Cos[e + f*x]]*EllipticE[e - P
i/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(b*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx = \frac{d\int \sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}\,dx}{b} - \frac{(ad)\int \frac{\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,d}{b}$$

$$= -\frac{\left(ad^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{b^2} + \frac{\left(a^2d^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx}{b^2} + \frac{\left(d\sqrt{g}\right.}{}$$

$$= \frac{d\sqrt{g\cos(e+fx)}\,E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin(e+fx)}}{bf\sqrt{\sin(2e+2fx)}} + \frac{\left(2ad^3g\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{d\sqrt{g\cos(e+fx)}\,E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin(e+fx)}}{bf\sqrt{\sin(2e+2fx)}} - \frac{\left(ad^2g\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{d\sqrt{g\cos(e+fx)}\,E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin(e+fx)}}{bf\sqrt{\sin(2e+2fx)}} + \frac{\left(ad^{3/2}\sqrt{g}\right)\text{Subst}}{}$$

$$= \frac{ad^{3/2}\sqrt{g}\,\log\left(\sqrt{g}+\sqrt{g}\,\cot(e+fx)-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}\,b^2f} - \frac{ad^{3/2}\sqrt{g}\,\log}{}$$

$$= -\frac{ad^{3/2}\sqrt{g}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^2f} + \frac{ad^{3/2}\sqrt{g}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g}}{\sqrt{g}\,\sqrt{d\sin}}\right.}{\sqrt{2}\,b^2f}$$

Mathematica [C] time = 18.10, size = 176, normalized size = 0.30

$$\frac{2d\sqrt{d\sin(e+fx)}\,(g\cos(e+fx))^{3/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(bF_1\left(\frac{3}{4};-\frac{3}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-aF_1\left(\frac{3}{4};-\right.\right.}{3fg\left(a^2-b^2\right)\sqrt[4]{\sin^2(e+fx)}\,(a+b\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x]
),x]
```

```
[Out] (2*d*(b*AppellF1[3/4, -3/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)] - a*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
```

```
x]^2)/(-a^2 + b^2)])*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]*(a + b*Sqr
t[Sin[e + f*x]^2]))/(3*(a^2 - b^2)*f*g*(Sin[e + f*x]^2)^(1/4)*(a + b*Sin[e
+ f*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*(d*sin(f*x + e))^(3/2)/(b*sin(f*x + e) + a),
 x)
```

maple [B] time = 0.84, size = 3290, normalized size = 5.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(3/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*(a-b)*(-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)
^(1/2)-a),1/2*2^(1/2))*a*b+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a
/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b-I*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a+(-a^2+b^2)^(1/2)*cos(f*x+e)*E
```

llipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1
/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a-(-a^2+b^2)^(1/2
)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),
1/2*2^(1/2))*a-(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a+(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a+2*(-a^2+b^2)^(1/2
)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b-(-a^2+b^2)^(
1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b+I*(-a^2+b
^2)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a+I*(-
a^2+b^2)^(1/2)*cos(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*a-(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a
^2+b^2)^(1/2)),1/2*2^(1/2))*a-I*(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a-(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2+(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2+(-a^2+b^2)^(1/2)*cos(f
*x+e)^2*2^(1/2)*b-(-a^2+b^2)^(1/2)*cos(f*x+e)*2^(1/2)*b-cos(f*x+e)*(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2+cos(
f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(

```
1/2))*a^2+(-a^2+b^2)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*a-(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a
^2+b^2)^(1/2)-a),1/2*2^(1/2))*a+(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a+2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),1/2*2^(1/2))*b-(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2),1/2*2^(1/2))*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(
1/2)-a),1/2*2^(1/2))*a*b+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b
^2)^(1/2)),1/2*2^(1/2))*a*b)*(d*sin(f*x+e))^(3/2)*(g*cos(f*x+e))^(1/2)/sin(
f*x+e)^2/cos(f*x+e)*2^(1/2)*a/b^2/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(
b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\,\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*(d*sin(f*x + e))^(3/2)/(b*sin(f*x + e) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}\,{\left(d\,\sin\left(e+f\,x\right)\right)}^{3/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int(((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(3/2)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1410 $\int \frac{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=509

$$-\frac{2\sqrt{2}\,ad\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{bf\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,ad\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\cdots\right.\right.}{bf\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+\cdots}}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^
(1/2))*d^(1/2)*g^(1/2)/b/f*2^(1/2)-1/2*arctan(1+2^(1/2)*d^(1/2)*(g*cos(f*x+
e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))*d^(1/2)*g^(1/2)/b/f*2^(1/2)-1/4*ln(
g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+
e))^(1/2))*d^(1/2)*g^(1/2)/b/f*2^(1/2)+1/4*ln(g^(1/2)+cot(f*x+e)*g^(1/2)+2^
(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))*d^(1/2)*g^(1/2)/b/
f*2^(1/2)-2*a*d*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2
),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/b/f/(-a+b)^
(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)+2*a*d*EllipticPi((g*cos(f*x+e))^(1/2
)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*
sin(f*x+e)^(1/2)/b/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {2909, 2575, 297, 1162, 617, 204, 1165, 628, 2906, 2905, 490, 1218}

$$-\frac{2\sqrt{2}\,ad\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{bf\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,ad\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\cdots\right.\right.}{bf\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[d]*Sqrt[g]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]
*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*f) - (Sqrt[d]*Sqrt[g]*ArcTan[1 + (Sqrt[
2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*
b*f) - (Sqrt[d]*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[
d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*f) + (Sqrt[d]*
Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e
+ f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*f) - (2*Sqrt[2]*a*d*Sqrt[g]*El
lipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*
Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b*Sqrt[-a + b]*Sqrt[a +
b]*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a*d*Sqrt[g]*EllipticPi[Sqrt[-a + b]
```

```
/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])]
, -1]*Sqrt[Sin[e + f*x]])/(b*Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]
])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
```

```
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{d\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{b} - \frac{(ad)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx}{b}$$

$$= -\frac{\left(2d^2g\right)\text{Subst}\left(\int \frac{x^2}{g^2+d^2x^4}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{bf} - \frac{\left(ad\sqrt{\sin(e+fx)}\right)\int \frac{}{\sqrt{\sin(}}}{b\sqrt{d\sin(e-}}$$

$$= \frac{(dg)\,\text{Subst}\left(\int \frac{g-dx^2}{g^2+d^2x^4}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{bf} - \frac{(dg)\,\text{Subst}\left(\int \frac{g+dx^2}{g^2+d^2x^4}\,dx, x, \frac{\sqrt{}}{\sqrt{}}\right.}{bf}$$

$$= -\frac{\left(\sqrt{d}\sqrt{g}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{g}}{\sqrt{d}}+2x}{-\frac{g}{d}-\frac{\sqrt{2}\sqrt{g}x}{\sqrt{d}}-x^2}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}\,bf} - \frac{\left(\sqrt{d}\sqrt{g}\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt{d}\sqrt{g}\,\log\left(\sqrt{g}+\sqrt{g}\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}\,bf} + \frac{\sqrt{d}\sqrt{g}\,\log\left(\right.}{}$$

$$= \frac{\sqrt{d}\sqrt{g}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,bf} - \frac{\sqrt{d}\sqrt{g}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+f}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right.}{\sqrt{2}\,bf}$$

Mathematica [C] time = 11.03, size = 178, normalized size = 0.35

$$\frac{2(d\sin(e+fx))^{3/2}(g\cos(e+fx))^{3/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(bF_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-aF_1\left(\frac{3}{4};\frac{1}{4}\right.\right.}{3dfg\left(a^2-b^2\right)\sin^2(e+fx)^{3/4}(a+b\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),
x]
```

```
[Out] (2*(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)] - a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
```

```
2)/(-a^2 + b^2)])*(g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2)*(a + b*Sqrt
[Sin[e + f*x]^2]))/(3*(a^2 - b^2)*d*f*g*(Sin[e + f*x]^2)^(3/4)*(a + b*Sin[e
 + f*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{d\sin\left(fx+e\right)}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a), x
)
```

maple [A] time = 0.86, size = 744, normalized size = 1.46

$$\frac{(a-b)\left(i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{-a^2+b^2}-i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(1/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*(a-b)*(I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-
1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)-I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)+EllipticPi((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^
2)^(1/2)+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*
I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
```

```
(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)+a*Ell
ipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1
/2)),1/2*2^(1/2))+b*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))-EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(
1/2)-a*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^
2+b^2)^(1/2)-a),1/2*2^(1/2))-b*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)))*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)*(g*cos(f*x+e))^(1/2)*s
in(f*x+e)/cos(f*x+e)/(-1+cos(f*x+e))*2^(1/2)*a/b/(-a^2+b^2)^(1/2)/(a-b+(-a^
2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{d\sin\left(fx+e\right)}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}\,\sqrt{d\,\sin\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int(((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sin\left(e+fx\right)}\sqrt{g\cos\left(e+fx\right)}}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(1/2)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
[Out] Integral(sqrt(d*sin(e + f*x))*sqrt(g*cos(e + f*x))/(a + b*sin(e + f*x)), x)
```

3.1411 $\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=208

$$\frac{2\sqrt{2}\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+}}{\sqrt{g}\sqrt{\sin(e+f}}\right.\right.}{f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}$$

```
[Out] 2*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2
)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/f/(-a+b)^(1/2)/(a+b)^(1/2
)/(d*sin(f*x+e))^(1/2)-2*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x
+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/f/(
-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2906, 2905, 490, 1218}

$$\frac{2\sqrt{2}\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+}}{\sqrt{g}\sqrt{\sin(e+f}}\right.\right.}{f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Co
s[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(Sqr
t[-a + b]*Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[g]*Elliptic
Pi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 +
Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt[d
*Sin[e + f*x]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\sqrt{\sin(e+fx)}\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx}{\sqrt{d\sin(e+fx)}}$$

$$= -\frac{\left(4\sqrt{2}\,g\sqrt{\sin(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{\left((a+b)g^2+(a-b)x^4\right)\sqrt{1-\frac{x^4}{g^2}}}\,dx, x, \frac{\sqrt{g\cos(e}}{\sqrt{1+\sin(e}}\right.}{f\sqrt{d\sin(e+fx)}}$$

$$= -\frac{\left(2\sqrt{2}\,g\sqrt{\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a+b}\,g-\sqrt{-a+b}\,x^2\right)\sqrt{1-\frac{x^4}{g^2}}}\,dx, x, \frac{\sqrt{g\cos}}{\sqrt{1+\sin}}\right.}{\sqrt{-a+b}\,f\sqrt{d\sin(e+fx)}}$$

$$= \frac{2\sqrt{2}\,\sqrt{g}\,\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}}{\sqrt{-a+b}\,\sqrt{a+b}\,f\sqrt{d\sin(e+fx)}} - \frac{2}{}$$

Mathematica [A] time = 7.83, size = 182, normalized size = 0.88

$$-\frac{4\sqrt{2}g\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)-1}}\tan^{\frac{3}{2}}\left(\frac{1}{2}(e+fx)\right)\left(-\Pi\left(\frac{a}{\sqrt{b^2-a^2}-b};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}}\right)\middle|-1\right)-\Pi\left(-\frac{a}{b+\sqrt{b^2-}}\right.\right.}{af\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),
x]
```

```
[Out] (-4*Sqrt[2]*g*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(-1 + Cos[e + f*x])]*(El
lipticF[ArcSin[1/Sqrt[Tan[(e + f*x)/2]]], -1] - EllipticPi[a/(-b + Sqrt[-a^
2 + b^2]), ArcSin[1/Sqrt[Tan[(e + f*x)/2]]], -1] - EllipticPi[-(a/(b + Sqrt
[-a^2 + b^2])), ArcSin[1/Sqrt[Tan[(e + f*x)/2]]], -1])*Tan[(e + f*x)/2]^(3/
2))/(a*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))),
 x)
```

maple [B] time = 0.66, size = 590, normalized size = 2.84

$$-\frac{\sqrt{g\cos\left(fx+e\right)}\,\sqrt{-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,(a-b)\left(2\,\mathrm{EllipticF}\left(\sqrt{-\frac{-1+c}{}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/f*(g*cos(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*(a-b)*(2*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2
^(1/2))*(-a^2+b^2)^(1/2)-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)+a*EllipticPi
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/
2*2^(1/2))+b*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a
-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)-a*
EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^
(1/2)-a),1/2*2^(1/2))-b*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)))*sin(f*x+e)^2/(d*sin(f*x+e))^
(1/2)/cos(f*x+e)/(-1+cos(f*x+e))*2^(1/2)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(
1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}}{\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(e+fx\right)}}{\sqrt{d\sin\left(e+fx\right)}\,\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*cos(e + f*x))/(sqrt(d*sin(e + f*x))*(a + b*sin(e + f*x))),
x)
```

3.1412 $\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=320

$$-\frac{2\sqrt{2}\,b\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{adf\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,b\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g}}{\sqrt{g}\,\sqrt{}}\right.\right.}{adf\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+f}}$$

```
[Out] -2*(g*cos(f*x+e))^(3/2)/a/d/f/g/(d*sin(f*x+e))^(1/2)-2*b*EllipticPi((g*cos(
f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(
1/2)*g^(1/2)*sin(f*x+e)^(1/2)/a/d/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))
^(1/2)+2*b*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a
+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/a/d/f/(-a+b)^(1/2
)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)+2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*
Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*
x+e))^(1/2)/a/d^2/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {2910, 2570, 2572, 2639, 2906, 2905, 490, 1218}

$$-\frac{2\sqrt{2}\,b\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{adf\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,b\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g}}{\sqrt{g}\,\sqrt{}}\right.\right.}{adf\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(a*d*f*g*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b*S
qrt[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/
(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*Sqrt[-a + b]*
Sqrt[a + b]*d*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b*Sqrt[g]*EllipticPi[Sqr
t[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
+ f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*Sqrt[-a + b]*Sqrt[a + b]*d*f*Sqrt[d*S
in[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d
*Sin[e + f*x]])/(a*d^2*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
```

```
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}}\,dx}{a} - \frac{b\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx}{ad}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{adfg\sqrt{d\sin(e+fx)}} - \frac{2\int \sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}\,dx}{ad^2} - \cdots$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{adfg\sqrt{d\sin(e+fx)}} + \frac{\left(4\sqrt{2}\,bg\sqrt{\sin(e+fx)}\right)\text{Subst}\left(\int \frac{\cdots}{\left((a+b)g^2+\cdots\right.}\cdots\right)}{adf\sqrt{d\sin(e+\cdots}}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{adfg\sqrt{d\sin(e+fx)}} - \frac{2\sqrt{g\cos(e+fx)}\,E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin\cdots}}{ad^2f\sqrt{\sin(2e+2fx)}}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{adfg\sqrt{d\sin(e+fx)}} - \frac{2\sqrt{2}\,b\sqrt{g}\,\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{1+\sin(e+\cdots}}\cdots\right.\right.}{a\sqrt{-a+b}\,\sqrt{a+b}\,df\sqrt{d\sin\cdots}}$$

Mathematica [C] time = 24.72, size = 1619, normalized size = 5.06

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[g*Cos[e + f*x]]*Sin[e + f*x])/(a*f*(d*Sin[e + f*x])^(
3/2)) + (Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^(3/2)*((4*a*(-(b*AppellF1[3/4, -
1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*Appell
```

```
F1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Co
s[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a
^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) - (b*Sqrt[Tan[e
+ f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[
Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
 + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b
^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^
(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e
 + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*
x]^2]))/(6*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(
1 + Tan[e + f*x]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e
+ f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2
, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2
) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^
2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*A
rcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2
]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4
)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1
 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/
4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]
])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 -
b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3
/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*
Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a
+ Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Ta
n[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2
 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^
2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Ta
n[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b
^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2
- b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/
Sqrt[1 + Tan[e + f*x]^2])))/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]
*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(a*
f*Sqrt[Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/2)
), x)
```

maple [B] time = 0.70, size = 2502, normalized size = 7.82

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/f*(a-b)*(4*cos(f*x+e)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-
a^2+b^2)^(1/2)*a-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2
+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-cos(f*x+e)*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b-cos(f*x+e)*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^
2-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1
/2*2^(1/2))*(-a^2+b^2)^(1/2)*b+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b+cos(f*x+e)*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2-2*cos(f*x+e)*(-(
```

```
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+2*(-a^2+b^2
)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b+4*Elli
pticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*a-(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(
1/2)*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^
(1/2))*a*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/
2*2^(1/2))*b^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2))
,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b
+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(
a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2-2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2),1/2*2^(1/2))*b-2*cos(f*x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*a)*(g*cos(
f*x+e))^(1/2)*sin(f*x+e)/(d*sin(f*x+e))^(3/2)/cos(f*x+e)*2^(1/2)/a/(-a^2+b^
2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\cos\left(e+f\,x\right)}}{{\left(d\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos{\left(e+fx \right)}}}{\left(d\sin{\left(e+fx \right)}\right)^{\frac{3}{2}} \left(a+b\sin{\left(e+fx \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*cos(e + f*x))/((d*sin(e + f*x))**(3/2)*(a + b*sin(e + f*x))
), x)
```

3.1413 $\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=366

$$\frac{2\sqrt{2}\,b^2\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^2d^2f\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,b^2\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos\cdots}}{\sqrt{g}\,\sqrt{\sin\cdots}}\right.\right.}{a^2d^2f\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+f\cdots}}$$

```
[Out] -2/3*(g*cos(f*x+e))^(3/2)/a/d/f/g/(d*sin(f*x+e))^(3/2)+2*b*(g*cos(f*x+e))^(
3/2)/a^2/d^2/f/g/(d*sin(f*x+e))^(1/2)+2*b^2*EllipticPi((g*cos(f*x+e))^(1/2)
/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*
sin(f*x+e)^(1/2)/a^2/d^2/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)-2*
b^2*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/
2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/a^2/d^2/f/(-a+b)^(1/2)/(
a+b)^(1/2)/(d*sin(f*x+e))^(1/2)-2*b*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*P
i+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x
+e))^(1/2)/a^2/d^3/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 1.03, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2910, 2563, 2570, 2572, 2639, 2906, 2905, 490, 1218}

$$\frac{2\sqrt{2}\,b^2\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^2d^2f\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,b^2\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos\cdots}}{\sqrt{g}\,\sqrt{\sin\cdots}}\right.\right.}{a^2d^2f\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+f\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(3*a*d*f*g*(d*Sin[e + f*x])^(3/2)) + (2*b*(g*Co
s[e + f*x])^(3/2))/(a^2*d^2*f*g*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt
[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sq
rt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*Sqrt[-a + b]*S
qrt[a + b]*d^2*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^2*Sqrt[g]*EllipticPi[
Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin
[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*Sqrt[-a + b]*Sqrt[a + b]*d^2*f*S
qrt[d*Sin[e + f*x]]) + (2*b*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x,
2]*Sqrt[d*Sin[e + f*x]])/(a^2*d^3*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
```

```
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}}\,dx}{a} - \frac{b\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx}{ad}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{b^2\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx}{a^2d^2} - \frac{b\int \frac{\sqrt{\ }}{(d}}{}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{a^2d^2fg\sqrt{d\sin(e+fx)}} + \frac{(2b)\int \sqrt{g\cos}}{}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{a^2d^2fg\sqrt{d\sin(e+fx)}} - \frac{\left(4\sqrt{2}\,b^2g\sqrt{\sin}\right.}{}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{a^2d^2fg\sqrt{d\sin(e+fx)}} + \frac{2b\sqrt{g\cos(e+}}{}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{a^2d^2fg\sqrt{d\sin(e+fx)}} + \frac{2\sqrt{2}\,b^2\sqrt{g}\,\Pi\left(\right.}{}$$

Mathematica [C] time = 22.08, size = 1645, normalized size = 4.49

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*((2*b*Cot[e + f*x])/a^2 - (2*Cot[e + f*x]*Csc[e + f*x
])/(3*a))*Sin[e + f*x]^3)/(f*(d*Sin[e + f*x])^(5/2)) - (b*Sqrt[g*Cos[e + f*
x]]*Sin[e + f*x]^(5/2)*((4*a*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^
2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e
 + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sq
rt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x
]^2)^(3/4)*(a + b*Sin[e + f*x])) - (b*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2
)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] +
2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[
-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]
*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x
]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4,
1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*
x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(6*a^2*Cos[e + f*x
]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2))
 + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e
+ f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2,
 (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*Appe
llF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan
[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan
[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4
*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt
[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1
/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan
[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(
1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*
x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[T
an[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e
 + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b
^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(
1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
 f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*
Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b
^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]
*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2
 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2]))
```

```
)/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1
+ Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(a^2*f*Sqrt[Cos[e + f*x]]*(d*
Sin[e + f*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/2)
), x)
```

maple [B] time = 0.73, size = 2672, normalized size = 7.30

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/3/f*(a-b)*(-6*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticF((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*a*b+6*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*El
lipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^2-3*(-a^2+b^2)^(1/2)*cos(f*
x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)
```

),1/2*2^(1/2))*b^2-3*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2+12*(-a^2+b^
2)^(1/2)*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/
2))*a*b+3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+
(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b^2+3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3-3*cos(f*x+e
)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),
1/2*2^(1/2))*a*b^2-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3-6*(-a^2+b^2)^(1/2)*sin(f*x+e
)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b+6*(-a^2+b^2)^(1/2)*si
n(f*x+e)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/
2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^2-3*(-a^2+b^2)^(
1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)
),1/2*2^(1/2))*b^2-3*(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2+12*(-a^2+b^2)^(1/2)*si
n(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b+3*sin(f*x+e)*(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*
b^2+3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2
)),1/2*2^(1/2))*b^3-3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x

```
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+
(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2-3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3+2*(-a^2+b^2)^(1/2)*
cos(f*x+e)^2*2^(1/2)*a^2-6*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*2^(1/2)*a
*b)*(g*cos(f*x+e))^(1/2)*sin(f*x+e)/(d*sin(f*x+e))^(5/2)/cos(f*x+e)*2^(1/2)
/a^2/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}}{{\left(d\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1414 $\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=513

$$-\frac{2\sqrt{2}\,b^3\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^3d^3f\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,b^3\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{}{\sqrt{g}}\right.\right.}{a^3d^3f\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e}}$$

[Out] -2/5*(g*cos(f*x+e))^(3/2)/a/d/f/g/(d*sin(f*x+e))^(5/2)+2/3*b*(g*cos(f*x+e))^(3/2)/a^2/d^2/f/g/(d*sin(f*x+e))^(3/2)-4/5*(g*cos(f*x+e))^(3/2)/a/d^3/f/g/(d*sin(f*x+e))^(1/2)-2*b^2*(g*cos(f*x+e))^(3/2)/a^3/d^3/f/g/(d*sin(f*x+e))^(1/2)-2*b^3*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/a^3/d^3/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)+2*b^3*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/a^3/d^3/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)+4/5*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a/d^4/f/sin(2*f*x+2*e)^(1/2)+2*b^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a^3/d^4/f/sin(2*f*x+2*e)^(1/2)

Rubi [A] time = 1.45, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2910, 2570, 2572, 2639, 2563, 2906, 2905, 490, 1218}

$$-\frac{2b^2(g\cos(e+fx))^{3/2}}{a^3d^3fg\sqrt{d\sin(e+fx)}}-\frac{2b^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{a^3d^4f\sqrt{\sin(2e+2fx)}}-\frac{2\sqrt{2}\,b^3\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{}}{\sqrt{}}\right.}{a^3d^3f\sqrt{b-a}\,\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x])),x]

[Out] (-2*(g*Cos[e + f*x])^(3/2))/(5*a*d*f*g*(d*Sin[e + f*x])^(5/2)) + (2*b*(g*Cos[e + f*x])^(3/2))/(3*a^2*d^2*f*g*(d*Sin[e + f*x])^(3/2)) - (4*(g*Cos[e + f*x])^(3/2))/(5*a*d^3*f*g*Sqrt[d*Sin[e + f*x]]) - (2*b^2*(g*Cos[e + f*x])^(3/2))/(a^3*d^3*f*g*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^3*Sqrt[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^3*Sqrt[-a + b]*Sqrt[a + b]*d^3*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^3*Sqrt[g]*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^3*Sqrt[-a + b]*Sqrt[a + b]*d^3*f*Sqrt[d*Sin[e +

```
f*x]]) - (4*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e
+ f*x]])/(5*a*d^4*f*Sqrt[Sin[2*e + 2*f*x]]) - (2*b^2*Sqrt[g*Cos[e + f*x]]*E
llipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^3*d^4*f*Sqrt[Sin[2*e +
 2*f*x]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}}\,dx}{a} - \frac{b\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx}{ad}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}}\,dx}{5ad^2} + \frac{b^2\int \frac{\sqrt{g\cos}}{(d\sin(e+fx))^{3/2}}}{a^2d}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(g\cos(e}{5ad^3fg\sqrt{d}}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(g\cos(e}{5ad^3fg\sqrt{d}}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(g\cos(e}{5ad^3fg\sqrt{d}}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(g\cos(e}{5ad^3fg\sqrt{d}}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(g\cos(e}{5ad^3fg\sqrt{d}}$$

Mathematica [C] time = 23.61, size = 1726, normalized size = 3.36

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*((-2*(2*a^2*Cos[e + f*x] + 5*b^2*Cos[e + f*x])*Csc[e
+ f*x])/(5*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x])/(3*a^2) - (2*Cot[e + f*x]
*Csc[e + f*x]^2)/(5*a))*Sin[e + f*x]^4)/(f*(d*Sin[e + f*x])^(7/2)) - (Sqrt[
g*Cos[e + f*x]]*Sin[e + f*x]^(7/2)*((-2*(4*a^3 + 10*a*b^2)*(-(b*AppellF1[3/
4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*Ap
```

```
pellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
)*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(
3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((2*a^2*b
+ 10*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a
^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 -
 b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 -
b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqr
t[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e +
 f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2
, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] +
a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*
(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-2*a^2*b - 5*b^3)*Cos[
2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2
])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b
^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4
, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x
]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x
]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]
*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a
^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt
[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sq
rt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (
2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqr
t[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f
*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]
 + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (
Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] -
 Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a +
Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[
e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 -
b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^
(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a^
2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1 + Tan[
e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(5*a^3*f*Sqrt[Cos[e + f*x]]*(d*Sin[
e + f*x])^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/2)
), x)
```

maple [B] time = 0.75, size = 6208, normalized size = 12.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}}{{\left(d\,\sin\left(e+f\,x\right)\right)}^{7/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(7/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(7/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1415 $\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=598

$$\frac{2\sqrt{2}\,b^4\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^4d^4f\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,b^4\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos}}{\sqrt{g}\,\sqrt{\sin}}\right.\right.}{a^4d^4f\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+f}}$$

```
[Out] -2/7*(g*cos(f*x+e))^(3/2)/a/d/f/g/(d*sin(f*x+e))^(7/2)+2/5*b*(g*cos(f*x+e))
^(3/2)/a^2/d^2/f/g/(d*sin(f*x+e))^(5/2)-8/21*(g*cos(f*x+e))^(3/2)/a/d^3/f/g
/(d*sin(f*x+e))^(3/2)-2/3*b^2*(g*cos(f*x+e))^(3/2)/a^3/d^3/f/g/(d*sin(f*x+e
))^(3/2)+4/5*b*(g*cos(f*x+e))^(3/2)/a^2/d^4/f/g/(d*sin(f*x+e))^(1/2)+2*b^3*
(g*cos(f*x+e))^(3/2)/a^4/d^4/f/g/(d*sin(f*x+e))^(1/2)+2*b^4*EllipticPi((g*c
os(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*
2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/a^4/d^4/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(
f*x+e))^(1/2)-2*b^4*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^
(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/a^4/d^4/
f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)-4/5*b*(sin(e+1/4*Pi+f*x)^2)
^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e)
)^(1/2)*(d*sin(f*x+e))^(1/2)/a^2/d^5/f/sin(2*f*x+2*e)^(1/2)-2*b^3*(sin(e+1/
4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(
g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a^4/d^5/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 1.82, antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2910, 2570, 2563, 2572, 2639, 2906, 2905, 490, 1218}

$$\frac{2b^3(g\cos(e+fx))^{3/2}}{a^4d^4fg\sqrt{d\sin(e+fx)}}-\frac{2b^2(g\cos(e+fx))^{3/2}}{3a^3d^3fg(d\sin(e+fx))^{3/2}}+\frac{2b^3E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{a^4d^5f\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{2}\,b^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(7*a*d*f*g*(d*Sin[e + f*x])^(7/2)) + (2*b*(g*Co
s[e + f*x])^(3/2))/(5*a^2*d^2*f*g*(d*Sin[e + f*x])^(5/2)) - (8*(g*Cos[e + f
*x])^(3/2))/(21*a*d^3*f*g*(d*Sin[e + f*x])^(3/2)) - (2*b^2*(g*Cos[e + f*x])
^(3/2))/(3*a^3*d^3*f*g*(d*Sin[e + f*x])^(3/2)) + (4*b*(g*Cos[e + f*x])^(3/2
))/(5*a^2*d^4*f*g*Sqrt[d*Sin[e + f*x]]) + (2*b^3*(g*Cos[e + f*x])^(3/2))/(a
^4*d^4*f*g*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^4*Sqrt[g]*EllipticPi[-(Sqrt
[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
+ f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^4*Sqrt[-a + b]*Sqrt[a + b]*d^4*f*Sqrt
```

```
[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^4*Sqrt[g]*EllipticPi[Sqrt[-a + b]/Sqrt[a +
 b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqr
t[Sin[e + f*x]])/(a^4*Sqrt[-a + b]*Sqrt[a + b]*d^4*f*Sqrt[d*Sin[e + f*x]])
+ (4*b*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x
]])/(5*a^2*d^5*f*Sqrt[Sin[2*e + 2*f*x]]) + (2*b^3*Sqrt[g*Cos[e + f*x]]*Elli
pticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^4*d^5*f*Sqrt[Sin[2*e + 2*
f*x]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{9/2}}\,dx}{a} - \frac{b\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx}{ad}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{4\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}}\,dx}{7ad^2} + \frac{b^2\int \frac{\sqrt{g\,\mathrm{c}}}{(d\sin(e+fx))^{5}}}{a^2}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{8(g\,\mathrm{cc}}{21ad^3fg(}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{8(g\,\mathrm{cc}}{21ad^3fg(}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{8(g\,\mathrm{cc}}{21ad^3fg(}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{8(g\,\mathrm{cc}}{21ad^3fg(}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{8(g\,\mathrm{cc}}{21ad^3fg(}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{8(g\,\mathrm{cc}}{21ad^3fg(}$$

Mathematica [C] time = 22.55, size = 1768, normalized size = 2.96

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*((2*(2*a^2*b*Cos[e + f*x] + 5*b^3*Cos[e + f*x])*Csc[e
 + f*x])/(5*a^4) - (2*(4*a^2*Cos[e + f*x] + 7*b^2*Cos[e + f*x])*Csc[e + f*x
```

```
]^2)/(21*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x]^2)/(5*a^2) - (2*Cot[e + f*x]
*Csc[e + f*x]^3)/(7*a))*Sin[e + f*x]^5)/(f*(d*Sin[e + f*x])^(9/2)) + (b*Sqr
t[g*Cos[e + f*x]]*Sin[e + f*x]^(9/2)*((-2*(4*a^3 + 10*a*b^2)*(-(b*AppellF1[
3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*
AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))
/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((2*a^2*
b + 10*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2
 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + S
qrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e
 + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]
^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x]
+ a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]
]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-2*a^2*b - 5*b^3)*Co
s[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]
^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 +
 b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7
/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f
*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f
*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[
2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/
(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (
Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) -
 (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/S
qrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e +
 f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x
]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1
/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) +
 (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
 - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a
+ Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Ta
n[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2
)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*
a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1 + Ta
n[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(5*a^4*f*Sqrt[Cos[e + f*x]]*(d*Si
n[e + f*x])^(9/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/2)
), x)
```

maple [B] time = 0.85, size = 6593, normalized size = 11.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}}{{\left(d\,\sin\left(e+f\,x\right)\right)}^{9/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(9/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(9/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1416 $\int \frac{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=982

$$\frac{2\sqrt{2}\,a\sqrt{b^2-a^2}\,d^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)g^2}{b^3f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,a\sqrt{b^2-a^2}\,d^{3/2}\sqrt{\cos(e+f}}{}$$

```
[Out] 3/8*d^(3/2)*g^(3/2)*arctan(1-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(
g*cos(f*x+e))^(1/2))/b/f*2^(1/2)+1/2*(a^2-b^2)*d^(3/2)*g^(3/2)*arctan(1-2^(
1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2))/b^3/f*2^(1/
2)-3/8*d^(3/2)*g^(3/2)*arctan(1+2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2
)/(g*cos(f*x+e))^(1/2))/b/f*2^(1/2)-1/2*(a^2-b^2)*d^(3/2)*g^(3/2)*arctan(1+
2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2))/b^3/f*2^
(1/2)-3/16*d^(3/2)*g^(3/2)*ln(d^(1/2)-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/
(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/b/f*2^(1/2)-1/4*(a^2-b^2)*d^(3/2)*
g^(3/2)*ln(d^(1/2)-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2
)+d^(1/2)*tan(f*x+e))/b^3/f*2^(1/2)+3/16*d^(3/2)*g^(3/2)*ln(d^(1/2)+2^(1/2)
*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/b/f*
2^(1/2)+1/4*(a^2-b^2)*d^(3/2)*g^(3/2)*ln(d^(1/2)+2^(1/2)*g^(1/2)*(d*sin(f*x
+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/b^3/f*2^(1/2)+2*a*d^(3/
2)*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(
-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)^(1/2)/b^3/f/(g*cos(
f*x+e))^(1/2)-2*a*d^(3/2)*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+co
s(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x
+e)^(1/2)/b^3/f/(g*cos(f*x+e))^(1/2)+1/2*g*(d*sin(f*x+e))^(3/2)*(g*cos(f*x+
e))^(1/2)/b/f-a*d*g*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/b^2/f-1/2*a*d
^2*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi
+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/b^2/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e
))^(1/2)
```

Rubi [A] time = 1.69, antiderivative size = 982, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 16, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.432, Rules used = {2901, 2838, 2568, 2573, 2641, 2574, 297, 1162, 617, 204, 1165, 628, 2909, 2908, 2907, 1218}

$$\frac{2\sqrt{2}\,a\sqrt{b^2-a^2}\,d^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)g^2}{b^3f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,a\sqrt{b^2-a^2}\,d^{3/2}\sqrt{\cos(e+f}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x]),x]
```

```
[Out] (3*d^(3/2)*g^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[
d]*Sqrt[g*Cos[e + f*x]])])/(4*Sqrt[2]*b*f) + ((a^2 - b^2)*d^(3/2)*g^(3/2)*A
rcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*
x]])])/(Sqrt[2]*b^3*f) - (3*d^(3/2)*g^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[g]*Sqr
t[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(4*Sqrt[2]*b*f) - ((a^2
 - b^2)*d^(3/2)*g^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(
Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b^3*f) + (2*Sqrt[2]*a*Sqrt[-a^2 +
b^2]*d^(3/2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])),
 ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^3*f
*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*a*Sqrt[-a^2 + b^2]*d^(3/2)*g^2*Sqrt[Cos
[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*
x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^3*f*Sqrt[g*Cos[e + f*x]]) -
(3*d^(3/2)*g^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqr
t[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(8*Sqrt[2]*b*f) - ((a^2 - b^2)*d
^(3/2)*g^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*
Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b^3*f) + (3*d^(3/2)*g^(3/
2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]
] + Sqrt[d]*Tan[e + f*x]])/(8*Sqrt[2]*b*f) + ((a^2 - b^2)*d^(3/2)*g^(3/2)*L
og[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] +
Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b^3*f) - (a*d*g*Sqrt[g*Cos[e + f*x]]*Sqrt
[d*Sin[e + f*x]])/(b^2*f) + (g*Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2))
/(2*b*f) + (a*d^2*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/
(2*b^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
```

```
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2901

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/b^2, Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[(g^2*(a^2 - b^2))/b^2, Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])
^n)/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx = \frac{g^2\int \frac{(d\sin(e+fx))^{3/2}(a-b\sin(e+fx))}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{(d\sin(e+fx))^{3}}{\sqrt{g\cos(e+fx)}\,(a+b\sin}}{b^2}$$

$$= \frac{\left(ag^2\right)\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} - \frac{g^2\int \frac{(d\sin(e+fx))^{5/2}}{\sqrt{g\cos(e+fx)}}\,dx}{bd} - \frac{\left(\left(a^2-b^2\right)dg^2\right)\int}{b^3}$$

$$= -\frac{adg\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{b^2f} + \frac{g\sqrt{g\cos(e+fx)}\,(d\sin(e+}{2bf}$$

$$= -\frac{adg\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{b^2f} + \frac{g\sqrt{g\cos(e+fx)}\,(d\sin(e+}{2bf}$$

$$= \frac{2\sqrt{2}\,a\sqrt{-a^2+b^2}\,d^{3/2}g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin}}{\sqrt{d}\,\sqrt{1+c}}\right.\right.}{b^3f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2\sqrt{2}\,a\sqrt{-a^2+b^2}\,d^{3/2}g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin}}{\sqrt{d}\,\sqrt{1+c}}\right.\right.}{b^3f\sqrt{g\cos(e+fx)}}$$

$$= \frac{\left(a^2-b^2\right)d^{3/2}g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b^3f} - \frac{\left(a^2-b^2\right)d^{3/2}g^{3/2}\,\mathrm{ta}}{}$$

$$= \frac{3d^{3/2}g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right)}{4\sqrt{2}\,bf} + \frac{\left(a^2-b^2\right)d^{3/2}g^{3/2}\tan^{-1}\left(1-\right.}{\sqrt{2}\,b^3f}$$

Mathematica [C] time = 28.81, size = 1898, normalized size = 1.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*
x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(d*Sin[e + f*x])^(3/2))/(2*b*f) - ((g*
Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2)*((10*b*(a^2 - b^2)*Sqrt[Cos[e +
f*x]]*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((b*AppellF1[1/4, -3/4, 1, 5/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2])/(-
5*(a^2 - b^2)*AppellF1[1/4, -3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]
^2)/(-a^2 + b^2)] + (4*b^2*AppellF1[5/4, -3/4, 2, 9/4, Cos[e + f*x]^2, (b^2
*Cos[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 1/4, 1, 9/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2) + (a*App
ellF1[1/4, -1/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
)/(5*(a^2 - b^2)*AppellF1[1/4, -1/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, -1/4, 2, 9/4, Cos[e + f*x]^2,
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/4, 1, 9/4
, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2))*Sin[
e + f*x]^(5/2))/((1 - Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))*(a
+ b*Sin[e + f*x])) + (2*a*Sqrt[Sin[e + f*x]]*((Sqrt[a]*(-2*ArcTan[1 - (Sqrt
[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + Log[-a + Sqrt[2]*Sqrt[a]*(
a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] - Log[a
 + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*T
an[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^2)^(3/4)) - (b*AppellF1[5/4, 1/2, 1, 9/4
, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(5/2))/(
5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(Cos[e + f*x]^(5/2)*
(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(1 + Tan[e + f*x]^2)^(3/2)) - (a*Co
s[2*(e + f*x)]*Sqrt[Sin[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]
^2])*(-20*Sqrt[2]*a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] + 20*Sqrt[2]*a*A
rcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] + (10*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*A
rcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b
^2)^(3/4) - (10*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*(a^2 - b^
2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(3/4) + 10*Sqrt[2]*a*Log
[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 10*Sqrt[2]*a*Log[1 + Sqrt
[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (5*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*L
og[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b
^2]*Tan[e + f*x]])/(a^2 - b^2)^(3/4) + (5*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*Log
[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]
*Tan[e + f*x]])/(a^2 - b^2)^(3/4) + 8*b*AppellF1[5/4, 1/2, 1, 9/4, -Tan[e +
 f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(5/2) + (40*b*Sqrt[Tan
```

```
[e + f*x]])/Sqrt[1 + Tan[e + f*x]^2] + (200*a^4*b*AppellF1[1/4, 1/2, 1, 5/4
, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sqrt[Tan[e + f*x]])/(Sqrt
[1 + Tan[e + f*x]^2]*(-5*a^2*AppellF1[1/4, 1/2, 1, 5/4, -Tan[e + f*x]^2, (-
1 + b^2/a^2)*Tan[e + f*x]^2] + 2*(2*(a^2 - b^2)*AppellF1[5/4, 1/2, 2, 9/4,
-Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + a^2*AppellF1[5/4, 3/2, 1,
 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])*Tan[e + f*x]^2)*(-(b
^2*Tan[e + f*x]^2) + a^2*(1 + Tan[e + f*x]^2))))/(10*b^2*Cos[e + f*x]^(5/2
)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Ta
n[e + f*x]^2])))/(4*b*f*Cos[e + f*x]^(3/2)*Sin[e + f*x]^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(d*sin(f*x + e))^(3/2)/(b*sin(f*x + e) + a
), x)
```

maple [B] time = 0.94, size = 2547, normalized size = 2.59

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/4/f*(a-b)*(-I*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1
/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+I*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/
```

```
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-4*I*sin(f*x+e)*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+4*I*
sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2
+b^2)^(1/2)*a^2+4*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*a^2+4*sin(f*x+e)*EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2
*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(
1/2)*a*b-4*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*a^3+4*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2-4*sin(f*x+e)*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+sin(f*x+e)*(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*
b^2+4*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2
)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+4*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+4*sin
(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^
(1/2))*a^3-4*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^
2)^(1/2)),1/2*2^(1/2))*a*b^2-4*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
```

```
2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+sin(f*x+e)*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-4*(-a^2+b^2)
^(1/2)*sin(f*x+e)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b+2*sin
(f*x+e)*cos(f*x+e)^2*(-a^2+b^2)^(1/2)*2^(1/2)*b^2-4*cos(f*x+e)^2*(-a^2+b^2)
^(1/2)*2^(1/2)*a*b-2*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*b^2+4*c
os(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*a*b)*(g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))
^(3/2)/sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^2*2^(1/2)*a/b^3/(-a^2+b^2)^(1/
2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(d*sin(f*x + e))^(3/2)/(b*sin(f*x + e) + a
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(d\,\sin\left(e+f\,x\right)\right)^{3/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1417 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=611

$$-\frac{2\sqrt{2}\sqrt{d}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\sqrt{d}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+f}}{b^2}$$

```
[Out] -1/2*a*g^(3/2)*arctan(1-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos
(f*x+e))^(1/2))*d^(1/2)/b^2/f*2^(1/2)+1/2*a*g^(3/2)*arctan(1+2^(1/2)*g^(1/2
)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2))*d^(1/2)/b^2/f*2^(1/2)+
1/4*a*g^(3/2)*ln(d^(1/2)-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e)
)^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/b^2/f*2^(1/2)-1/4*a*g^(3/2)*ln(d^(1/2)+
2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e
))*d^(1/2)/b^2/f*2^(1/2)-2*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+c
os(f*x+e))^(1/2),-a/(b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*d^(1/2
)*cos(f*x+e)^(1/2)/b^2/f/(g*cos(f*x+e))^(1/2)+2*g^2*EllipticPi((d*sin(f*x+e
))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-
a^2+b^2)^(1/2)*d^(1/2)*cos(f*x+e)^(1/2)/b^2/f/(g*cos(f*x+e))^(1/2)+g*(g*cos
(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/b/f+1/2*d*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/
2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1
/2)/b/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.03, antiderivative size = 611, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {2901, 2838, 2574, 297, 1162, 617, 204, 1165, 628, 2568, 2573, 2641, 2908, 2907, 1218}

$$-\frac{2\sqrt{2}\sqrt{d}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\sqrt{d}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+f}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*Sqrt[d]*g^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqr
t[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b^2*f)) + (a*Sqrt[d]*g^(3/2)*ArcTan[1
 + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/
(Sqrt[2]*b^2*f) - (2*Sqrt[2]*Sqrt[-a^2 + b^2]*Sqrt[d]*g^2*Sqrt[Cos[e + f*x]
]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt
[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^2*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2
]*Sqrt[-a^2 + b^2]*Sqrt[d]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[
```

```
-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])
], -1])/(b^2*f*Sqrt[g*Cos[e + f*x]]) + (a*Sqrt[d]*g^(3/2)*Log[Sqrt[d] - (Sq
rt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e +
f*x]])/(2*Sqrt[2]*b^2*f) - (a*Sqrt[d]*g^(3/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g
]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sq
rt[2]*b^2*f) + (g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(b*f) - (d*g^2
*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(2*b*f*Sqrt[g*Cos[e +
 f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
```

```
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2901

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/b^2, Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[(g^2*(a^2 - b^2))/b^2, Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])
^n)/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{g^2\int \frac{\sqrt{d\sin(e+fx)}\,(a-b\sin(e+fx))}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+f}}{b^2}$$

$$= \frac{\left(ag^2\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} - \frac{g^2\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}}\,dx}{bd} - \frac{\left(\left(a^2-b^2\right)g^2\sqrt{\cos(e+}\right.}{b^2\sqrt{}}$$

$$= \frac{g\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{bf} - \frac{\left(dg^2\right)\int \frac{1}{\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}\,dx}{2b} +$$

$$= -\frac{2\sqrt{2}\,\sqrt{-a^2+b^2}\,\sqrt{d}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+}}{\sqrt{d}\,\sqrt{1+\cos(e}}\right.\right.}{b^2f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2\sqrt{2}\,\sqrt{-a^2+b^2}\,\sqrt{d}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+}}{\sqrt{d}\,\sqrt{1+\cos(e}}\right.\right.}{b^2f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2\sqrt{2}\,\sqrt{-a^2+b^2}\,\sqrt{d}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+}}{\sqrt{d}\,\sqrt{1+\cos(e}}\right.\right.}{b^2f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{a\sqrt{d}\,g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b^2f} + \frac{a\sqrt{d}\,g^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d}}{\sqrt{d}\,\sqrt{g\cos}}\right.}{\sqrt{2}\,b^2f}$$

Mathematica [C] time = 21.41, size = 604, normalized size = 0.99

$$-\frac{(d\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(\frac{2aF_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)}{a^2-b^2}+\frac{5\left(\sin^2(e+fx)\left(a^2-b^2\sin^2(\right.\right.}{b\sin^2(e+fx)^{3/4}\left(a^2-\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]
),x]
```

```
[Out] -1/5*((g*Cos[e + f*x])^(5/2)*(d*Sin[e + f*x])^(3/2)*(a + b*Sqrt[Sin[e + f*x
]^2])*((2*a*AppellF1[5/4, 1/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
```

```
/(-a^2 + b^2)])/(a^2 - b^2) + ((2*a^2 - b^2)*AppellF1[5/4, 3/4, 1, 9/4, Cos
[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])/(-(a^2*b) + b^3) + (5*(-5*
(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)]*(a^2 - 2*b^2 + b^2*Cos[e + f*x]^2) + (-4*b^2*AppellF1[5/4, 3
/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b
^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
 b^2)])*Sin[e + f*x]^2*(a^2 - b^2*Sin[e + f*x]^2)))/(b*(-5*(a^2 - b^2)*Appe
llF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] +
 (4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)] + 3*(-a^2 + b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2
*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(Sin[e + f*x]^2)^(3/4)*(a^2
 - b^2*Sin[e + f*x]^2))))/(d*f*g*(Sin[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x]
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a),
 x)
```

maple [B] time = 0.81, size = 1926, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/f*(a-b)*(I*(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),1/2+1/2*I,1/2*2^(1/2))*a-I*(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a-(-a^2+b^2)^(1/2)*sin(f*x+e)*Ell
ipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1
/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a-(-
a^2+b^2)^(1/2)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*b-(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a-(-a^2+b^2)^(1/2)*s
in(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2
*2^(1/2))*b+(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
1/2*2^(1/2))*b+(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*a+(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a-sin(f*x+e)*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2+sin(f*x+e)*Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2))
,1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^2+sin(
f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^
(1/2))*a^2-sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*
EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^
(1/2)-a),1/2*2^(1/2))*b^2+(-a^2+b^2)^(1/2)*cos(f*x+e)^2*2^(1/2)*b-(-a^2+b^2
)^(1/2)*cos(f*x+e)*2^(1/2)*b)*(g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)/(-1
+cos(f*x+e))/cos(f*x+e)^2*2^(1/2)*a/b^2/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1
```

```
/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{d\,\sin\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1418 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=577

$$\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{ab\sqrt{d}f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\,\Pi\left(-\right.}{ab\sqrt{d}f\sqrt{g\,\mathrm{c}}}$$

```
[Out] 1/2*g^(3/2)*arctan(1-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(f*
x+e))^(1/2))/b/f*2^(1/2)/d^(1/2)-1/2*g^(3/2)*arctan(1+2^(1/2)*g^(1/2)*(d*si
n(f*x+e))^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2))/b/f*2^(1/2)/d^(1/2)-1/4*g^(3/
2)*ln(d^(1/2)-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^(
1/2)*tan(f*x+e))/b/f*2^(1/2)/d^(1/2)+1/4*g^(3/2)*ln(d^(1/2)+2^(1/2)*g^(1/2)
*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/b/f*2^(1/2)/
d^(1/2)+2*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),
-a/(b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)^(1/2)/a/b/f/
d^(1/2)/(g*cos(f*x+e))^(1/2)-2*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/
(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*co
s(f*x+e)^(1/2)/a/b/f/d^(1/2)/(g*cos(f*x+e))^(1/2)-g^2*(sin(e+1/4*Pi+f*x)^2)
^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e
)^(1/2)/a/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {2900, 2838, 2573, 2641, 2574, 297, 1162, 617, 204, 1165, 628, 2908, 2907, 1218}

$$\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{ab\sqrt{d}f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\,\Pi\left(-\right.}{ab\sqrt{d}f\sqrt{g\,\mathrm{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (g^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*
Cos[e + f*x]])])/(Sqrt[2]*b*Sqrt[d]*f) - (g^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[
g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b*Sqrt[d
]*f) + (2*Sqrt[2]*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b
 - Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e
+ f*x]])], -1])/(a*b*Sqrt[d]*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a^2
 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcS
in[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a*b*Sqrt[d
```

```
]*f*Sqrt[g*Cos[e + f*x]]) - (g^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*
Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*S
qrt[d]*f) + (g^(3/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/S
qrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*Sqrt[d]*f) + (g^2
*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a*f*Sqrt[g*Cos[e + f
*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2900

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/(a*b), I
nt[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(b - a*Sin[e + f*x]), x], x]
 + Dist[(g^2*(a^2 - b^2))/(a*b*d), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e +
 f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x
] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LtQ[n, -1] |
| (EqQ[p, 3/2] && EqQ[n, -2^(-1)]))
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{b-a\sin(e+fx)}{\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}\,dx}{ab} + \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+f}}{abd}$$

$$= \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}\,dx}{a} - \frac{g^2\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\,dx}{bd} + \frac{\left(\left(a^2-b^2\right)g^2\sqrt{}\right.}{}$$

$$= -\frac{\left(2g^3\right)\mathrm{Subst}\left(\int \frac{x^2}{d^2+g^2x^4}\,dx, x, \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\right)}{bf} + \frac{\left(2\sqrt{2}\,\left(a^2-b^2\right)\left(1-\frac{}{\sqrt{-}}\right.\right.}{}$$

$$= \frac{2\sqrt{2}\,\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right.\right.}{ab\sqrt{d}\,f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2\sqrt{2}\,\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right.\right.}{ab\sqrt{d}\,f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2\sqrt{2}\,\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right.\right.}{ab\sqrt{d}\,f\sqrt{g\cos(e+fx)}}$$

$$= \frac{g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b\sqrt{d}\,f} - \frac{g^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b\sqrt{d}\,f}$$

Mathematica [C] time = 11.75, size = 178, normalized size = 0.31

$$\frac{2\sqrt{d\sin(e+fx)}\,(g\cos(e+fx))^{5/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(bF_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-aF_1\left(\frac{5}{4};\frac{3}{4},1;\right.\right.}{5dfg\left(a^2-b^2\right)\sqrt[4]{\sin^2(e+fx)}\,(a+b\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (2*(b*AppellF1[5/4, 1/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
 + b^2)] - a*AppellF1[5/4, 3/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
```

```
)/(-a^2 + b^2)])*(g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]]*(a + b*Sqrt[Si
n[e + f*x]^2]))/(5*(a^2 - b^2)*d*f*g*(Sin[e + f*x]^2)^(1/4)*(a + b*Sin[e +
f*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

maple [A] time = 0.63, size = 944, normalized size = 1.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*(a-b)*(I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+EllipticPi
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^
2+b^2)^(1/2)*a+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/
2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+2*EllipticF((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2)
)*(-a^2+b^2)^(1/2)*a-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-a^2*EllipticPi
```

```
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/
2*2^(1/2))+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b
+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*
a-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2
)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b+a^2*EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))-Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-
a),1/2*2^(1/2))*b^2)*sin(f*x+e)^2*(g*cos(f*x+e))^(3/2)*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)/(-1+cos(f*x+e))/cos(f*x+e)^2/(d*sin(f*x+e)
)^(1/2)*2^(1/2)/b/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/
2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(e+fx\right)\right)^{\frac{3}{2}}}{\sqrt{d\sin\left(e+fx\right)}\,\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((g*cos(e + f*x))**(3/2)/(sqrt(d*sin(e + f*x))*(a + b*sin(e + f*x))
), x)
```

3.1419 $\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=321

$$-\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2d^{3/2}f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\,\Pi\left(-\right.}{a^2d^{3/2}f\sqrt{g}}$$

```
[Out] -2*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(
-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)^(1/2)/a^2/d^(3/2)/f
/(g*cos(f*x+e))^(1/2)+2*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(
f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e
)^(1/2)/a^2/d^(3/2)/f/(g*cos(f*x+e))^(1/2)-2*g*(g*cos(f*x+e))^(1/2)/a/d/f/(
d*sin(f*x+e))^(1/2)+b*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Ell
ipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^2/d/f/(g*cos(f*x+e
))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2899, 2563, 2573, 2641, 2908, 2907, 1218}

$$-\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2d^{3/2}f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\,\Pi\left(-\right.}{a^2d^{3/2}f\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqr
t[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]
])], -1])/(a^2*d^(3/2)*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*Sqrt[-a^2 + b^2
]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqr
t[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*d^(3/2)*f*Sq
rt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(a*d*f*Sqrt[d*Sin[e + f*x]
]) - (b*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^2*d*f*S
qrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
```

```
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(}}{a^2d^2}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{adf\sqrt{d\sin(e+fx)}} - \frac{\left(\left(a^2-b^2\right)g^2\sqrt{\cos(e+fx)}\right)\int \frac{\sqrt{d\sin(e+}}{\sqrt{\cos(e+fx)}\,(a+b}}{a^2d^2\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{adf\sqrt{d\sin(e+fx)}} - \frac{bg^2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(2e+2fx)}}{a^2df\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}} - \frac{(2\sqrt{}}{}$$

$$= -\frac{2\sqrt{2}\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+f}}{\sqrt{d}\sqrt{1+\cos(e-}}\right.\right.}{a^2d^{3/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] time = 20.50, size = 1095, normalized size = 3.41

$$-\frac{2\tan(e+fx)(g\cos(e+fx))^{3/2}}{af(d\sin(e+fx))^{3/2}} - \frac{\sin^{\frac{3}{2}}(e+fx)\left(\frac{2a\sqrt{\sin(e+fx)}\left(\frac{\sqrt{a}\left(-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a^2-b^2}\sqrt{\tan(e+fx)}}{\sqrt{a}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a^2-b^2}\sqrt{\tan(e+fx)}}{\sqrt{a}}+\right.\right.}{}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2)*Tan[e + f*x])/(a*f*(d*Sin[e + f*x])^(3/2)) - ((g
*Cos[e + f*x])^(3/2)*Sin[e + f*x]^(3/2)*((-2*b*(a + b*Sqrt[1 - Cos[e + f*x]
^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[
```

```
e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(
5*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*
Cos[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Co
s[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^
2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*S
qrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*Ar
cTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos
[e + f*x]^2)^(1/4))] + Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 +
Cos[e + f*x]^2] - ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(
-1 + Cos[e + f*x]^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqr
t[-1 + Cos[e + f*x]^2] + ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f
*x]])/(-1 + Cos[e + f*x]^2)^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin
[e + f*x]])/((1 - Cos[e + f*x]^2)^(1/4)*(a + b*Sin[e + f*x])) + (2*a*Sqrt[S
in[e + f*x]]*((Sqrt[a]*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
 + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f
*x]])/Sqrt[a]] + Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*
x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(
1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 -
 b^2)^(3/4)) - (b*AppellF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2)
*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt
[1 + Tan[e + f*x]^2]))/(Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e
+ f*x]]*(1 + Tan[e + f*x]^2)^(3/2))))/(a*f*Cos[e + f*x]^(3/2)*(d*Sin[e + f*
x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/2)), x)

maple [B] time = 0.66, size = 2587, normalized size = 8.06

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)

[Out] 1/f*(a-b)*(2*(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b-(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2+2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b-(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(

```
f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a-(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-(-a
^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
Pi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a)
,1/2*2^(1/2))*a-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-
a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a
-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a
/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2-(-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2+2*cos(f*x+e)*2^(1/2)*(-a^2
+b^2)^(1/2)*a)*(g*cos(f*x+e))^(3/2)*sin(f*x+e)/(d*sin(f*x+e))^(3/2)/cos(f*x
+e)^2*2^(1/2)/a/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)
-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{{\left(d\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1420 $\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=435

$$\frac{2bg\sqrt{g\cos(e+fx)}}{a^2d^2f\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,bg^2\sqrt{b^2-a^2}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^3d^{5/2}f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,bg^2\sqrt{b}}{}$$

```
[Out] 2*b*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-
(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)^(1/2)/a^3/d^(5/2)/
f/(g*cos(f*x+e))^(1/2)-2*b*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+c
os(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*
x+e)^(1/2)/a^3/d^(5/2)/f/(g*cos(f*x+e))^(1/2)-2/3*g*(g*cos(f*x+e))^(1/2)/a/
d/f/(d*sin(f*x+e))^(3/2)+2*b*g*(g*cos(f*x+e))^(1/2)/a^2/d^2/f/(d*sin(f*x+e)
)^(1/2)-2/3*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos
(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a/d^2/f/(g*cos(f*x+e))^(1/2)/(
d*sin(f*x+e))^(1/2)+(a^2-b^2)*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+
f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^3/d^2/f/(g
*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.02, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2899, 2570, 2573, 2641, 2563, 2910, 2908, 2907, 1218}

$$-\frac{g^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^3d^2f\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,bg^2\sqrt{b^2-a^2}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d}\sin(e}{\sqrt{d}\,\sqrt{\cos(e}}\right.\right.}{a^3d^{5/2}f\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[2]*b*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sq
rt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x
]])], -1])/(a^3*d^(5/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b*Sqrt[-a^2 +
b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[
Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^3*d^(5/2)*f
*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(3*a*d*f*(d*Sin[e + f*x
])^(3/2)) + (2*b*g*Sqrt[g*Cos[e + f*x]])/(a^2*d^2*f*Sqrt[d*Sin[e + f*x]]) +
 (2*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a*d^2*f*Sqr
t[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]) - ((a^2 - b^2)*g^2*EllipticF[e - Pi
/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^3*d^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d
*Sin[e + f*x]])
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{5/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{1}{\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e}}}{a^2d^2}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{\left(b\left(a^2-b^2\right)g^2\right)}{}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{\left(b\left(a^2-b^2\right)g^2\sqrt{}\right.}{}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{2g^2F\left(e-\frac{\pi}{4}+f\right.}{3ad^2f\sqrt{g\cos(e}}$$

$$= \frac{2\sqrt{2}\,b\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx}}{\sqrt{d}\,\sqrt{1+\cos(e+}}\right.\right.}{a^3d^{5/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] time = 21.51, size = 1138, normalized size = 2.62

$$\frac{(g\cos(e+fx))^{3/2}\left(\frac{2b\csc(e+fx)}{a^2}-\frac{2\csc^2(e+fx)}{3a}\right)\sin^2(e+fx)\tan(e+fx)}{f(d\sin(e+fx))^{5/2}} - \frac{(g\cos(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)\left(-\frac{2\left(a^2-3b^2\right)\left(a\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*((2*b*Csc[e + f*x])/a^2 - (2*Csc[e + f*x]^2)/(3*a))
*Sin[e + f*x]^2*Tan[e + f*x])/(f*(d*Sin[e + f*x])^(5/2)) - ((g*Cos[e + f*x]
)^(3/2)*Sin[e + f*x]^(5/2)*((-2*(a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^
2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e
 + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(5
```

```
*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*C
os[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos
[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2
*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sq
rt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*Arc
Tan[1 + ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[
e + f*x]^2)^(1/4))] + Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + C
os[e + f*x]^2] - ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-
1 + Cos[e + f*x]^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt
[-1 + Cos[e + f*x]^2] + ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*
x]])/(-1 + Cos[e + f*x]^2)^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin[
e + f*x]])/((1 - Cos[e + f*x]^2)^(1/4)*(a + b*Sin[e + f*x])) - (4*a*b*Sqrt[
Sin[e + f*x]]*((Sqrt[a]*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]])/Sqrt[a]] + Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f
*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^
(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2
- b^2)^(3/4)) - (b*AppellF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2
)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqr
t[1 + Tan[e + f*x]^2]))/(Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e
 + f*x]]*(1 + Tan[e + f*x]^2)^(3/2))))/(3*a^2*f*Cos[e + f*x]^(3/2)*(d*Sin[e
 + f*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

maple [B] time = 0.65, size = 3014, normalized size = 6.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/3/f*(a-b)*(3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*a*b+3*(-a^2+b^2)^(1/2)
*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2
)^(1/2)),1/2*2^(1/2))*b^2+3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2*b-3*cos(f*x+e)*sin(f*x
+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2
))*b^3+3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-
a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+3*(-a^2+b^2)^(1/2)*cos(
f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2
)-a),1/2*2^(1/2))*b^2-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2*b+3*cos(f*x+e)*sin(f*x+e)
*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))
*b^3+2*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2)
)*(-a^2+b^2)^(1/2)*a^2-6*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticF((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^2+3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(
```

```
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+3
*(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a
^2+b^2)^(1/2)),1/2*2^(1/2))*b^2+3*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2*b-3*sin(f*x+e)*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3+3*sin(f*x
+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b
^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*(-a^2+b^2)^(1/2)*a*b+3*(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2-3*sin(f*x+e)*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2
*b+3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-
a),1/2*2^(1/2))*b^3+2*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(
1/2))*(-a^2+b^2)^(1/2)*a^2-6*(-a^2+b^2)^(1/2)*sin(f*x+e)*EllipticF((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*b^2-6*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*
2^(1/2)*a*b+2*cos(f*x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*a^2)*(g*cos(f*x+e))^(3/2)
*sin(f*x+e)/cos(f*x+e)^2/(d*sin(f*x+e))^(5/2)*2^(1/2)/a^2/(-a^2+b^2)^(1/2)/
(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\left(d\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1421 $\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=525

$$-\frac{2bg^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{3a^2d^3f\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}+\frac{2bg\sqrt{g\cos(e+fx)}}{3a^2d^2f(d\sin(e+fx))^{3/2}}-\frac{2\sqrt{2}\,b^2g^2\sqrt{b^2-a^2}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2\ldots}}\right.}{a^4d^{7/2}f\sqrt{g\cos(e\ldots}}$$

```
[Out] -2*b^2*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/
(b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)^(1/2)/a^4/d^(7/
2)/f/(g*cos(f*x+e))^(1/2)+2*b^2*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)
/(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*c
os(f*x+e)^(1/2)/a^4/d^(7/2)/f/(g*cos(f*x+e))^(1/2)-2/5*g*(g*cos(f*x+e))^(1/
2)/a/d/f/(d*sin(f*x+e))^(5/2)+2/3*b*g*(g*cos(f*x+e))^(1/2)/a^2/d^2/f/(d*sin
(f*x+e))^(3/2)-8/5*g*(g*cos(f*x+e))^(1/2)/a/d^3/f/(d*sin(f*x+e))^(1/2)+2*(a
^2-b^2)*g*(g*cos(f*x+e))^(1/2)/a^3/d^3/f/(d*sin(f*x+e))^(1/2)+2/3*b*g^2*(si
n(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1
/2))*sin(2*f*x+2*e)^(1/2)/a^2/d^3/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/
2)-b*(a^2-b^2)*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(
cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^4/d^3/f/(g*cos(f*x+e))^(1
/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.35, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2899, 2570, 2563, 2573, 2641, 2910, 2908, 2907, 1218}

$$\frac{bg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^4d^3f\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,b^2g^2\sqrt{b^2-a^2}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d}\sin(\ldots}{\sqrt{d}\,\sqrt{\cos(\ldots}}\right.\right.}{a^4d^{7/2}f\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*b^2*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b -
 Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e +
f*x]])], -1])/(a^4*d^(7/2)*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt[-a
^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), Ar
cSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^4*d^(7
/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(5*a*d*f*(d*Sin[e
+ f*x])^(5/2)) + (2*b*g*Sqrt[g*Cos[e + f*x]])/(3*a^2*d^2*f*(d*Sin[e + f*x])
^(3/2)) - (8*g*Sqrt[g*Cos[e + f*x]])/(5*a*d^3*f*Sqrt[d*Sin[e + f*x]]) + (2*
(a^2 - b^2)*g*Sqrt[g*Cos[e + f*x]])/(a^3*d^3*f*Sqrt[d*Sin[e + f*x]]) - (2*b
```

```
*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*d^3*f*Sqrt
[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]) + (b*(a^2 - b^2)*g^2*EllipticF[e - P
i/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^4*d^3*f*Sqrt[g*Cos[e + f*x]]*Sqrt[
d*Sin[e + f*x]])
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
```

```
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{7/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+}}{a^2d^2}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{\left(2bg^2\right)\int \frac{}{\sqrt{g\,c}}}{}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{8g\sqrt{g\cos(e}}{5ad^3f\sqrt{d\sin(}}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{8g\sqrt{g\cos(e}}{5ad^3f\sqrt{d\sin(}}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{8g\sqrt{g\cos(e}}{5ad^3f\sqrt{d\sin(}}$$

$$= -\frac{2\sqrt{2}\,b^2\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e}}{\sqrt{d}\,\sqrt{1+\cos}}\right.\right.}{a^4d^{7/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] time = 21.56, size = 1165, normalized size = 2.22

$$\frac{b(g\cos(e+fx))^{3/2}\left(-\frac{2\left(a^2-3b^2\right)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\sqrt{\sin(e+fx)}\left(\frac{5a\left(a^2-b^2\right)F_1\left(\frac{1}{4};\right.}{\left(1-\cos^2(e+fx)\right)^{3/4}\left(\left(3\left(a^2-b^2\right)F_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-4b^2F_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4}\right.\right.\right.}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*((2*(a^2 - 5*b^2)*Csc[e + f*x])/(5*a^3) + (2*b*Csc[
e + f*x]^2)/(3*a^2) - (2*Csc[e + f*x]^3)/(5*a))*Sin[e + f*x]^3*Tan[e + f*x]
```

```
)/(f*(d*Sin[e + f*x])^(7/2)) + (b*(g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^(7/2)
*((-2*(a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*Appe
llF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*S
qrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(5*(a^2 - b^2)*AppellF1[1/4,
 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-4*b^2*
AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e +
f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) -
 ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 +
 b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*S
qrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] + Log[
Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] - ((1 + I)*
Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)^(1/4)]
 - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] + ((
1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)
^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin[e + f*x]])/((1 - Cos[e + f
*x]^2)^(1/4)*(a + b*Sin[e + f*x])) - (4*a*b*Sqrt[Sin[e + f*x]]*((Sqrt[a]*(-
2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*Ar
cTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + Log[-a +
 Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan
[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] +
 Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^2)^(3/4)) - (b*AppellF
1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan
[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(C
os[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(1 + Tan[e + f*x]
^2)^(3/2))))/(3*a^3*f*Cos[e + f*x]^(3/2)*(d*Sin[e + f*x])^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/
2)), x)
```

maple [B] time = 0.72, size = 5828, normalized size = 11.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}}{{\left(d\,\sin\left(e+f\,x\right)\right)}^{7/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(7/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(7/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e)),x)
[Out] Timed out
```

3.1422 $\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=688

$$\frac{8bg\sqrt{g\cos(e+fx)}}{5a^2d^4f\sqrt{d\sin(e+fx)}}+\frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}}-\frac{b^2g^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^5d^4f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,b^3g^2\sqrt{b}}{}$$

[Out] 2*b^3*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)^(1/2)/a^5/d^(9/2)/f/(g*cos(f*x+e))^(1/2)-2*b^3*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)^(1/2)/a^5/d^(9/2)/f/(g*cos(f*x+e))^(1/2)-2/7*g*(g*cos(f*x+e))^(1/2)/a/d/f/(d*sin(f*x+e))^(7/2)+2/5*b*g*(g*cos(f*x+e))^(1/2)/a^2/d^2/f/(d*sin(f*x+e))^(5/2)-4/7*g*(g*cos(f*x+e))^(1/2)/a/d^3/f/(d*sin(f*x+e))^(3/2)+2/3*(a^2-b^2)*g*(g*cos(f*x+e))^(1/2)/a^3/d^3/f/(d*sin(f*x+e))^(3/2)+8/5*b*g*(g*cos(f*x+e))^(1/2)/a^2/d^4/f/(d*sin(f*x+e))^(1/2)-2*b*(a^2-b^2)*g*(g*cos(f*x+e))^(1/2)/a^4/d^4/f/(d*sin(f*x+e))^(1/2)-4/7*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a/d^4/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)+2/3*(a^2-b^2)*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^3/d^4/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)+b^2*(a^2-b^2)*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^5/d^4/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)

Rubi [A] time = 1.78, antiderivative size = 688, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2899, 2570, 2573, 2641, 2563, 2910, 2908, 2907, 1218}

$$-\frac{b^2g^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^5d^4f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}-\frac{2g^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{3a^3d^4f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,b^3g^2\sqrt{b^2}}{}$$

Antiderivative was successfully verified.

[In] Int[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x])),x]

[Out] (2*Sqrt[2]*b^3*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^5*d^(9/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b^3*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^5*d^(9/

```
2)*f*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(7*a*d*f*(d*Sin[e +
 f*x])^(7/2)) + (2*b*g*Sqrt[g*Cos[e + f*x]])/(5*a^2*d^2*f*(d*Sin[e + f*x])^
(5/2)) - (4*g*Sqrt[g*Cos[e + f*x]])/(7*a*d^3*f*(d*Sin[e + f*x])^(3/2)) + (2
*(a^2 - b^2)*g*Sqrt[g*Cos[e + f*x]])/(3*a^3*d^3*f*(d*Sin[e + f*x])^(3/2)) +
 (8*b*g*Sqrt[g*Cos[e + f*x]])/(5*a^2*d^4*f*Sqrt[d*Sin[e + f*x]]) - (2*b*(a^
2 - b^2)*g*Sqrt[g*Cos[e + f*x]])/(a^4*d^4*f*Sqrt[d*Sin[e + f*x]]) + (4*g^2*
EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(7*a*d^4*f*Sqrt[g*Cos[
e + f*x]]*Sqrt[d*Sin[e + f*x]]) - (2*(a^2 - b^2)*g^2*EllipticF[e - Pi/4 + f
*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^3*d^4*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin
[e + f*x]]) - (b^2*(a^2 - b^2)*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*
e + 2*f*x]])/(a^5*d^4*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{9/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{}{\sqrt{g\cos(e+fx)}\,(d\sin(}}{a^2d^2}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{\left(4bg^2\right)\int}{}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{4g\sqrt{g\cos}}{7ad^3f(d\sin}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{4g\sqrt{g\cos}}{7ad^3f(d\sin}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{4g\sqrt{g\cos}}{7ad^3f(d\sin}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{4g\sqrt{g\cos}}{7ad^3f(d\sin}$$

$$= \frac{2\sqrt{2}\,b^3\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e}}{\sqrt{d}\,\sqrt{1+\cos}}\right.\right.}{a^5d^{9/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] time = 21.22, size = 1210, normalized size = 1.76

$$\frac{(g\cos(e+fx))^{3/2}\left(-\frac{2\csc^4(e+fx)}{7a}+\frac{2b\csc^3(e+fx)}{5a^2}+\frac{2\left(a^2-7b^2\right)\csc^2(e+fx)}{21a^3}-\frac{2b\left(a^2-5b^2\right)\csc(e+fx)}{5a^4}\right)\sin^4(e+fx)\tan(e+fx)}{f(d\sin(e+fx))^{9/2}} -$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*((-2*b*(a^2 - 5*b^2)*Csc[e + f*x])/(5*a^4) + (2*(a^
2 - 7*b^2)*Csc[e + f*x]^2)/(21*a^3) + (2*b*Csc[e + f*x]^3)/(5*a^2) - (2*Csc
[e + f*x]^4)/(7*a))*Sin[e + f*x]^4*Tan[e + f*x])/(f*(d*Sin[e + f*x])^(9/2))
 - ((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^(9/2)*((-2*(2*a^4 + 7*a^2*b^2 - 21*
b^4)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 3/4,
1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x
]])/((1 - Cos[e + f*x]^2)^(3/4)*(5*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, 3
/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b
^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
 b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*b
*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1
 + Cos[e + f*x]^2)^(1/4))] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x
]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] + Log[Sqrt[-a^2 + b^2
] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] - ((1 + I)*Sqrt[a]*(-a^2 +
 b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)^(1/4)] - Log[Sqrt[-a^
2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] + ((1 + I)*Sqrt[a]*
(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)^(1/4)]))/(Sqrt
[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin[e + f*x]])/((1 - Cos[e + f*x]^2)^(1/4)*(a
 + b*Sin[e + f*x])) + (2*(2*a^3*b - 14*a*b^3)*Sqrt[Sin[e + f*x]]*((Sqrt[a]*
(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*
ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + Log[-a
 + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*T
an[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
 + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^2)^(3/4)) - (b*Appel
lF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*T
an[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/
(Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(1 + Tan[e + f*
x]^2)^(3/2))))/(21*a^4*f*Cos[e + f*x]^(3/2)*(d*Sin[e + f*x])^(9/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/
2)), x)
```

maple [B] time = 0.81, size = 6707, normalized size = 9.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}}{{\left(d\,\sin\left(e+f\,x\right)\right)}^{9/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(9/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(9/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1423 $\int \frac{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=936

$$-\frac{\left(a^2-b^2\right)\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)g^{5/2}}{\sqrt{2}\,b^3f}-\frac{\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)g^{5/2}}{4\sqrt{2}\,bf}+\frac{\left(a^2-b^2\right)\sqrt{d}\,\tan^{-1}\left(\cdots\right.}{\sqrt{2}\cdots}$$

```
[Out] 1/8*g^(5/2)*arctan(-1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f
*x+e))^(1/2))*d^(1/2)/b/f*2^(1/2)+1/2*(a^2-b^2)*g^(5/2)*arctan(-1+2^(1/2)*d
^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))*d^(1/2)/b^3/f*2^(
1/2)+1/8*g^(5/2)*arctan(1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*s
in(f*x+e))^(1/2))*d^(1/2)/b/f*2^(1/2)+1/2*(a^2-b^2)*g^(5/2)*arctan(1+2^(1/2
)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))*d^(1/2)/b^3/f*
2^(1/2)+1/16*g^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f
*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))*d^(1/2)/b/f*2^(1/2)+1/4*(a^2-b^2)*g^(5/2
)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin
(f*x+e))^(1/2))*d^(1/2)/b^3/f*2^(1/2)-1/16*g^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^
(1/2)+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))*d^(1/2)/b/
f*2^(1/2)-1/4*(a^2-b^2)*g^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)+2^(1/2)*d^(1/
2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))*d^(1/2)/b^3/f*2^(1/2)-2*a*d*g
^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)
^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/b^3
/f/(d*sin(f*x+e))^(1/2)+2*a*d*g^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/
2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a
+b)^(1/2)*sin(f*x+e)^(1/2)/b^3/f/(d*sin(f*x+e))^(1/2)+1/2*g*(g*cos(f*x+e))^
(3/2)*(d*sin(f*x+e))^(1/2)/b/f-a*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*
Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*
x+e))^(1/2)/b^2/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 1.60, antiderivative size = 936, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 17, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.460, Rules used = {2901, 2838, 2572, 2639, 2568, 2575, 297, 1162, 617, 204, 1165, 628, 2909, 2906, 2905, 490, 1218}

$$-\frac{\left(a^2-b^2\right)\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)g^{5/2}}{\sqrt{2}\,b^3f}-\frac{\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)g^{5/2}}{4\sqrt{2}\,bf}+\frac{\left(a^2-b^2\right)\sqrt{d}\,\tan^{-1}\left(\cdots\right.}{\sqrt{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -(Sqrt[d]*g^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g
]*Sqrt[d*Sin[e + f*x]])])/(4*Sqrt[2]*b*f) - ((a^2 - b^2)*Sqrt[d]*g^(5/2)*Ar
cTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x
]])])/(Sqrt[2]*b^3*f) + (Sqrt[d]*g^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g
*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(4*Sqrt[2]*b*f) + ((a^2 -
b^2)*Sqrt[d]*g^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqr
t[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^3*f) + (Sqrt[d]*g^(5/2)*Log[Sqrt[g]
 + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin
[e + f*x]]])/(8*Sqrt[2]*b*f) + ((a^2 - b^2)*Sqrt[d]*g^(5/2)*Log[Sqrt[g] + S
qrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e +
 f*x]]])/(2*Sqrt[2]*b^3*f) - (Sqrt[d]*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e +
 f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(8*Sq
rt[2]*b*f) - ((a^2 - b^2)*Sqrt[d]*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x
] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2
]*b^3*f) - (2*Sqrt[2]*a*Sqrt[-a + b]*Sqrt[a + b]*d*g^(5/2)*EllipticPi[-(Sqr
t[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
 + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^3*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[
2]*a*Sqrt[-a + b]*Sqrt[a + b]*d*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b]
, ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[S
in[e + f*x]])/(b^3*f*Sqrt[d*Sin[e + f*x]]) + (g*(g*Cos[e + f*x])^(3/2)*Sqrt
[d*Sin[e + f*x]])/(2*b*f) + (a*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4
+ f*x, 2]*Sqrt[d*Sin[e + f*x]])/(b^2*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
```

```
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2901

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/b^2, Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[(g^2*(a^2 - b^2))/b^2, Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])
^n)/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
```

```
0]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{g^2\int\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}\,(a-b\sin(e+fx))\,dx}{b^2} - \frac{\left(\left(a^2-b^2\right)\right.}{}$$

$$= \frac{\left(ag^2\right)\int\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}\,dx}{b^2} - \frac{g^2\int\sqrt{g\cos(e+fx)}\,(d\sin}{bd}$$

$$= \frac{g(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bf} - \frac{\left(dg^2\right)\int\frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{4b} + \frac{\left(2\left(a^2-\right.\right.}{}$$

$$= \frac{g(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bf} + \frac{ag^2\sqrt{g\cos(e+fx)}\,E\left(e-\frac{\pi}{4}+fx\right.}{b^2f\sqrt{\sin(2e+2f}}$$

$$= \frac{g(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bf} + \frac{ag^2\sqrt{g\cos(e+fx)}\,E\left(e-\frac{\pi}{4}+fx\right.}{b^2f\sqrt{\sin(2e+2f}}$$

$$= \frac{\left(a^2-b^2\right)\sqrt{d}\,g^{5/2}\log\left(\sqrt{g}+\sqrt{g}\,\cot(e+fx)-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}\,b^3f} - \frac{\left(a^2\right.}{}$$

$$= -\frac{\left(a^2-b^2\right)\sqrt{d}\,g^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^3f} + \frac{\left(a^2-b^2\right)\sqrt{d}\,g^{5/2}\tan}{}$$

$$= -\frac{\sqrt{d}\,g^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{4\sqrt{2}\,bf} - \frac{\left(a^2-b^2\right)\sqrt{d}\,g^{5/2}\tan^{-1}\left(1-\right.}{\sqrt{2}\,b^3f}$$

Mathematica [C] time = 27.03, size = 1615, normalized size = 1.73

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]
),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*Sec[e + f*x]*Sqrt[d*Sin[e + f*x]])/(2*b*f) - ((g*Co
s[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]]*((2*b*(-(b*AppellF1[3/4, -1/4, 1, 7/
4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/
4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]
^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/((a^2 - b^2)*(1
 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) - (Sqrt[Tan[e + f*x]]*((3*Sq
rt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])
/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqr
t[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqr
t[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt
[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*Ap
pellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2
]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a*
Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x
]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt
[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[
e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2
+ b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[
2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e +
f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^
2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]
*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a
^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt
[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqr
t[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]
*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[
a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a
^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*S
qrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + S
qrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt
[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e +
f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e
+ f*x]^2])))/(42*a*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e +
 f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(4*b*f*Cos[e + f*x
]^(5/2)*Sqrt[Sin[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
```

```
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a),
 x)
```

maple [B] time = 0.96, size = 6311, normalized size = 6.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{d\,\sin\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(5/2)*(d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int(((g*cos(e + f*x))^(5/2)*(d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1424 $\int \frac{(g\cos(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=572

$$\frac{ag^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^2\sqrt{d}\,f}-\frac{ag^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}+1\right)}{\sqrt{2}\,b^2\sqrt{d}\,f}+\frac{2\sqrt{2}\,g^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\Big(}{b^2f\sqrt{d\sin(}}$$

```
[Out] -1/2*a*g^(5/2)*arctan(-1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*si
n(f*x+e))^(1/2))/b^2/f*2^(1/2)/d^(1/2)-1/2*a*g^(5/2)*arctan(1+2^(1/2)*d^(1/
2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))/b^2/f*2^(1/2)/d^(1/2)
-1/4*a*g^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))
^(1/2)/(d*sin(f*x+e))^(1/2))/b^2/f*2^(1/2)/d^(1/2)+1/4*a*g^(5/2)*ln(g^(1/2)
+cot(f*x+e)*g^(1/2)+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/
2))/b^2/f*2^(1/2)/d^(1/2)+2*g^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)
/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+
b)^(1/2)*sin(f*x+e)^(1/2)/b^2/f/(d*sin(f*x+e))^(1/2)-2*g^(5/2)*EllipticPi((
g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I
)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/b^2/f/(d*sin(f*x+e))^(1
/2)+g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*P
i+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/b/d/f/sin(2*f*x+2
*e)^(1/2)
```

Rubi [A] time = 1.02, antiderivative size = 572, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {2901, 2838, 2575, 297, 1162, 617, 204, 1165, 628, 2572, 2639, 2906, 2905, 490, 1218}

$$\frac{ag^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^2\sqrt{d}\,f}-\frac{ag^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}+1\right)}{\sqrt{2}\,b^2\sqrt{d}\,f}+\frac{2\sqrt{2}\,g^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\Big(}{b^2f\sqrt{d\sin(}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a*g^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[
d*Sin[e + f*x]])])/(Sqrt[2]*b^2*Sqrt[d]*f) - (a*g^(5/2)*ArcTan[1 + (Sqrt[2]
*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^
2*Sqrt[d]*f) - (a*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqr
t[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2*Sqrt[d]*f)
 + (a*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*
Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2*Sqrt[d]*f) + (2*Sqrt[2
```

```
]*Sqrt[-a + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]),
ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin
[e + f*x]])/(b^2*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a +
 b]*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]
]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^2*f*Sqrt[d*
Sin[e + f*x]]) - (g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqr
t[d*Sin[e + f*x]])/(b*d*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2901

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/b^2, Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[(g^2*(a^2 - b^2))/b^2, Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])
^n)/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}\,(a-b\sin(e+fx))}{\sqrt{d\sin(e+fx)}}\,dx}{b^2} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+}}{b^2}$$

$$= \frac{\left(ag^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{b^2} - \frac{g^2\int \sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}\,dx}{bd} - \frac{((}{}$$

$$= -\frac{\left(2adg^3\right)\text{Subst}\left(\int \frac{x^2}{g^2+d^2x^4}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{b^2 f} + \frac{\left(4\sqrt{2}\left(a^2-b^2\right)g^3\sqrt{\text{si}}\right.}{}$$

$$= -\frac{g^2\sqrt{g\cos(e+fx)}\,E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin(e+fx)}}{bdf\sqrt{\sin(2e+2fx)}} + \frac{\left(ag^3\right)\text{Subst}\left(\right.}{}$$

$$= \frac{2\sqrt{2}\sqrt{-a+b}\sqrt{a+b}\,g^{5/2}\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\text{si}}}{b^2 f\sqrt{d\sin(e+fx)}}$$

$$= -\frac{ag^{5/2}\log\left(\sqrt{g}+\sqrt{g}\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}\,b^2\sqrt{d}\,f} + \frac{ag^{5/2}\log\left(\sqrt{\right.}}{}$$

$$= \frac{ag^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^2\sqrt{d}\,f} - \frac{ag^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right.}{\sqrt{2}\,b^2\sqrt{d}\,f}$$

Mathematica [C] time = 25.81, size = 1399, normalized size = 2.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])
),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*Sqrt[Sin[e + f*x]]*((Sqrt[Tan[e + f*x]]*((3*Sqrt[2]
*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt
[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]]
```

```
 - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2
 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF
1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan
[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos
[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2
)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1
+ Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e +
 f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b
^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x
]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*
Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x
]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x
]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 -
 b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^
2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2
- b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Ta
n[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2
]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sq
rt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2
- b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt
[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt
[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt
[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]
*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x
]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f
*x]^2])))/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e +
f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(2*f*Cos[e + f*x]^(
5/2)*Sqrt[d*Sin[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

maple [B] time = 0.69, size = 5224, normalized size = 9.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}}{\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
[Out] Timed out
```

3.1425 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=616

$$-\frac{2\sqrt{2}g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{abdf\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+}{abdf}$$

```
[Out] 1/2*g^(5/2)*arctan(-1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f
*x+e))^(1/2))/b/d^(3/2)/f*2^(1/2)+1/2*g^(5/2)*arctan(1+2^(1/2)*d^(1/2)*(g*c
os(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))/b/d^(3/2)/f*2^(1/2)+1/4*g^(5
/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*s
in(f*x+e))^(1/2))/b/d^(3/2)/f*2^(1/2)-1/4*g^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(
1/2)+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))/b/d^(3/2)/f
*2^(1/2)-2*g*(g*cos(f*x+e))^(3/2)/a/d/f/(d*sin(f*x+e))^(1/2)-2*g^(5/2)*Elli
pticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b
)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/a/b/d/f/(d*sin
(f*x+e))^(1/2)+2*g^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x
+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin
(f*x+e)^(1/2)/a/b/d/f/(d*sin(f*x+e))^(1/2)+2*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2
)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/
2)*(d*sin(f*x+e))^(1/2)/a/d^2/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 1.14, antiderivative size = 616, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.432, Rules used = {2900, 2838, 2570, 2572, 2639, 2575, 297, 1162, 617, 204, 1165, 628, 2906, 2905, 490, 1218}

$$-\frac{2\sqrt{2}g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{abdf\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+}{abdf}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((g^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[
d*Sin[e + f*x]])])/(Sqrt[2]*b*d^(3/2)*f)) + (g^(5/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*d^(
3/2)*f) + (g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sq
rt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*d^(3/2)*f) - (g^(5/
2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x
]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*d^(3/2)*f) - (2*g*(g*Cos[e + f*x])^
```

```
(3/2))/(a*d*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a + b]*g
^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/
(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*b*d*f*Sqrt[d*
Sin[e + f*x]]) + (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[Sqr
t[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
+ f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*b*d*f*Sqrt[d*Sin[e + f*x]]) - (2*g^2*
Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a*
d^2*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2900

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/(a*b), I
nt[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(b - a*Sin[e + f*x]), x], x]
 + Dist[(g^2*(a^2 - b^2))/(a*b*d), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e +
 f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x
] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LtQ[n, -1] |
| (EqQ[p, 3/2] && EqQ[n, -2^(-1)]))
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}\,(b-a\sin(e+fx))}{(d\sin(e+fx))^{3/2}}\,dx}{ab} + \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(}}{abd}$$

$$= \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}}\,dx}{a} - \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{bd} + \frac{\left(\left(a^2-b^2\right)g^2\sqrt{\sin(e+}}{abd}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{adf\sqrt{d\sin(e+fx)}} - \frac{\left(2g^2\right)\int \sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}\,dx}{ad^2}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{adf\sqrt{d\sin(e+fx)}} - \frac{g^3\,\text{Subst}\left(\int \frac{g-dx^2}{g^2+d^2x^4}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{bdf} + \frac{g^3}{}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{adf\sqrt{d\sin(e+fx)}} - \frac{2\sqrt{2}\sqrt{-a+b}\sqrt{a+b}\,g^{5/2}\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\right.}{abdf\sqrt{d\sin(e}}$$

$$= \frac{g^{5/2}\log\left(\sqrt{g}+\sqrt{g}\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}\,bd^{3/2}f} - \frac{g^{5/2}\log\left(\sqrt{g}\right.}{}$$

$$= -\frac{g^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,bd^{3/2}f} + \frac{g^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right.}{\sqrt{2}\,bd^{3/2}f}$$

Mathematica [C] time = 26.65, size = 1611, normalized size = 2.62

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x
]))),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2)*Tan[e + f*x])/(a*f*(d*Sin[e + f*x])^(3/2)) + ((g
*Cos[e + f*x])^(5/2)*Sin[e + f*x]^(3/2)*((2*a*(-(b*AppellF1[3/4, -1/4, 1, 7
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1
```

```
/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x
]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/((a^2 - b^2)*(
1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) - (b*Sqrt[Tan[e + f*x]]*((3
*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x
]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/
Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] -
Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b
*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/
a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(6*
a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e +
 f*x]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*
Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -
Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-
a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[
e + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - S
qrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[
e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]
*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqr
t[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]]
)/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*
Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 -
 Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqr
t[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*S
qrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]]
)/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/
4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) +
(2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
 + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a +
Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[
e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Ta
n[e + f*x]^2])))/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin
[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(a*f*Cos[e + f
*x]^(5/2)*(d*Sin[e + f*x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

maple [B] time = 0.74, size = 5212, normalized size = 8.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}}{{\left(d\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1426 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=359

$$\frac{2bg^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^2d^3f\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{2}g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{}{\sqrt{}}\right.\right.}{a^2d^2f\sqrt{d\sin(e+fx)}}$$

```
[Out] -2/3*g*(g*cos(f*x+e))^(3/2)/a/d/f/(d*sin(f*x+e))^(3/2)+2*b*g*(g*cos(f*x+e))
^(3/2)/a^2/d^2/f/(d*sin(f*x+e))^(1/2)+2*g^(5/2)*EllipticPi((g*cos(f*x+e))^(
1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+
b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/a^2/d^2/f/(d*sin(f*x+e))^(1/2)-2*g^(5
/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/
2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/a^2/d^2
/f/(d*sin(f*x+e))^(1/2)-2*b*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*
x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))
^(1/2)/a^2/d^3/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2899, 2563, 2570, 2572, 2639, 2906, 2905, 490, 1218}

$$\frac{2bg^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^2d^3f\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{2}g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{}{\sqrt{}}\right.\right.}{a^2d^2f\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(3*a*d*f*(d*Sin[e + f*x])^(3/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(a^2*d^2*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*Sqrt[-a
+ b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqr
t[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]]
)/(a^2*d^2*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a + b]*g^
(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqr
t[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*d^2*f*Sqrt[d*Si
n[e + f*x]]) + (2*b*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*S
qrt[d*Sin[e + f*x]])/(a^2*d^3*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
```

```
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
```

```
-3/2] && EqQ[p, 3/2]))
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx}{a^2d^2} -$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{\left(2bg^2\right)\int \sqrt{g\cos}}{}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{\left(4\sqrt{2}\left(a^2-b^2\right)g}{}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{2bg^2\sqrt{g\cos(e+}}{a}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{2\sqrt{2}\sqrt{-a+b}\sqrt{a}}{}$$

Mathematica [C] time = 26.35, size = 1656, normalized size = 4.61

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x
]))),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((2*b*Cot[e + f*x])/a^2 - (2*Cot[e + f*x]*Csc[e + f
*x])/(3*a))*Sin[e + f*x]*Tan[e + f*x]^2)/(f*(d*Sin[e + f*x])^(5/2)) - ((g*C
os[e + f*x])^(5/2)*Sin[e + f*x]^(5/2)*((4*a*b*(-(b*AppellF1[3/4, -1/4, 1, 7
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1
/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x
]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)
*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((a^2 - 2*b^2)*Sqrt[Tan
[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[T
an[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Ta
n[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^
2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Ta
n[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e +
 f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x
])*(1 + Tan[e + f*x]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Ta
n[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4,
 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^
(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1
+ b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/
2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^
(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcT
an[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)
^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^
2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*
a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3
/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log
[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2
]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2
- b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqr
t[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[
2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[
a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/
```

```
2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x
]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(
a^2*f*Cos[e + f*x]^(5/2)*(d*Sin[e + f*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

maple [B] time = 0.68, size = 4668, normalized size = 13.00

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/3/f*(a-b)*(2*(-a^2+b^2)^(1/2)*cos(f*x+e)^2*2^(1/2)*a^2-6*(-a^2+b^2)^(1/2)
*cos(f*x+e)*sin(f*x+e)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b+
12*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),1/2*2^(1/2))*a*b+6*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticF((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
```

-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^2-3*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2*b+3*sin(f*x+e)*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2*b-6
*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2
)*a^2+3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1
/2)),1/2*2^(1/2))*b^3+12*(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2),1/2*2^(1/2))*a*b+3*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*a^2+3*sin(f*x+e
)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))
*(-a^2+b^2)^(1/2)*a^2-3*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2-3*(-a^2+b
^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(
-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2-3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(
1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2*b+3*cos(f*x+
e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
Pi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a)
,1/2*2^(1/2))*a^2*b-6*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+3*cos(f*x+e)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1
/2)*a^2+3*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co

```
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^
(1/2)),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*a^2+3*cos(f*x+e)*EllipticPi
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1
/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)
*a^3-3*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/
2)),1/2*2^(1/2))*sin(f*x+e)*a^3+3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3-3*cos(f*x+e)*sin
(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2
^(1/2))*b^3+6*(-a^2+b^2)^(1/2)*sin(f*x+e)*EllipticF((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*b^2-3*(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2-3*(-a^2+b^2)^(1/2)*sin(
f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^
(1/2))*b^2+3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^
2)^(1/2)),1/2*2^(1/2))*a*b^2-3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2+3*cos(f*x+e)*sin(f*x+e)*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b^
2-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b
^2)^(1/2)-a),1/2*2^(1/2))*a*b^2-6*(-a^2+b^2)^(1/2)*sin(f*x+e)*EllipticF((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b-3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
```

```
*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3-6*(-a^2+b^2)^(1/2)*
cos(f*x+e)*sin(f*x+e)*2^(1/2)*a*b+3*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^3-3*sin(f*x+e)*(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^3)*(g*cos(
f*x+e))^(5/2)*sin(f*x+e)/cos(f*x+e)^3/(d*sin(f*x+e))^(5/2)*2^(1/2)/a^2/(-a^
2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}}{{\left(d\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1427 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=519

$$-\frac{2\sqrt{2}\,bg^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^3d^3f\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,bg^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e}{a^3d^3}$$

```
[Out] -2/5*g*(g*cos(f*x+e))^(3/2)/a/d/f/(d*sin(f*x+e))^(5/2)+2/3*b*g*(g*cos(f*x+e
))^(3/2)/a^2/d^2/f/(d*sin(f*x+e))^(3/2)-4/5*g*(g*cos(f*x+e))^(3/2)/a/d^3/f/
(d*sin(f*x+e))^(1/2)+2*(a^2-b^2)*g*(g*cos(f*x+e))^(3/2)/a^3/d^3/f/(d*sin(f*
x+e))^(1/2)-2*b*g^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+
e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin
(f*x+e)^(1/2)/a^3/d^3/f/(d*sin(f*x+e))^(1/2)+2*b*g^(5/2)*EllipticPi((g*cos(
f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1
/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/a^3/d^3/f/(d*sin(f*x+e))^(1/2
)+4/5*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4
*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a/d^4/f/sin(2*f
*x+2*e)^(1/2)-2*(a^2-b^2)*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)
*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(
1/2)/a^3/d^4/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 1.21, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {2899, 2570, 2572, 2639, 2563, 2910, 2906, 2905, 490, 1218}

$$\frac{2g^2\left(a^2-b^2\right)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{a^3d^4f\sqrt{\sin(2e+2fx)}}+\frac{2g\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}}{a^3d^3f\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,bg^{5/2}\sqrt{b-a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(5*a*d*f*(d*Sin[e + f*x])^(5/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(3*a^2*d^2*f*(d*Sin[e + f*x])^(3/2)) - (4*g*(g*Cos[e +
 f*x])^(3/2))/(5*a*d^3*f*Sqrt[d*Sin[e + f*x]]) + (2*(a^2 - b^2)*g*(g*Cos[e
+ f*x])^(3/2))/(a^3*d^3*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b*Sqrt[-a + b]
*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*
Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a
^3*d^3*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b*Sqrt[-a + b]*Sqrt[a + b]*g^(5
/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[
```

```
g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^3*d^3*f*Sqrt[d*Sin[
e + f*x]]) - (4*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[
d*Sin[e + f*x]])/(5*a*d^4*f*Sqrt[Sin[2*e + 2*f*x]]) + (2*(a^2 - b^2)*g^2*Sq
rt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^3*
d^4*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx}{a^2d^2}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} + \frac{\left(b\left(a^2-b^2\right)\right.}{}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{4g(g\cos(e}{5ad^3f\sqrt{d\sin}}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{4g(g\cos(e}{5ad^3f\sqrt{d\sin}}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{4g(g\cos(e}{5ad^3f\sqrt{d\sin}}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{4g(g\cos(e}{5ad^3f\sqrt{d\sin}}$$

Mathematica [C] time = 24.61, size = 1734, normalized size = 3.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x
]])),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((2*(3*a^2*Cos[e + f*x] - 5*b^2*Cos[e + f*x])*Csc[e
 + f*x])/(5*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x])/(3*a^2) - (2*Cot[e + f*x
]*Csc[e + f*x]^2)/(5*a))*Sin[e + f*x]^2*Tan[e + f*x]^2)/(f*(d*Sin[e + f*x])
^(7/2)) + ((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^(7/2)*((-2*(6*a^3 - 10*a*b^2
)*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x
]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin
[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f
*x])) + ((8*a^2*b - 10*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcT
```

```
an[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1
 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[
2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f
*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[
a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/
4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*
(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*S
qrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-3*a
^2*b + 5*b^3)*Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[
1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e
 + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 +
 b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f
*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2
]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f
*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f
*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2
 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*
a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^
2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[
Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt
[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*
Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a
]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^
2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sq
rt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sq
rt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[
2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f
*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e +
 f*x]^2])))/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e
+ f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(5*a^3*f*Cos[e +
f*x]^(5/2)*(d*Sin[e + f*x])^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/
2)), x)
```

maple [B] time = 0.76, size = 10138, normalized size = 19.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}}{{\left(d\,\sin\left(e+f\,x\right)\right)}^{7/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(7/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(7/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1428 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=612

$$\frac{2\sqrt{2}\,b^2g^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^4d^4f\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,b^2g^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin\cdots}}{a^4\cdots}$$

```
[Out] -2/7*g*(g*cos(f*x+e))^(3/2)/a/d/f/(d*sin(f*x+e))^(7/2)+2/5*b*g*(g*cos(f*x+e
))^(3/2)/a^2/d^2/f/(d*sin(f*x+e))^(5/2)-8/21*g*(g*cos(f*x+e))^(3/2)/a/d^3/f
/(d*sin(f*x+e))^(3/2)+2/3*(a^2-b^2)*g*(g*cos(f*x+e))^(3/2)/a^3/d^3/f/(d*sin
(f*x+e))^(3/2)+4/5*b*g*(g*cos(f*x+e))^(3/2)/a^2/d^4/f/(d*sin(f*x+e))^(1/2)-
2*b*(a^2-b^2)*g*(g*cos(f*x+e))^(3/2)/a^4/d^4/f/(d*sin(f*x+e))^(1/2)+2*b^2*g
^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)
^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/a^4
/d^4/f/(d*sin(f*x+e))^(1/2)-2*b^2*g^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g
^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2
)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/a^4/d^4/f/(d*sin(f*x+e))^(1/2)-4/5*b*g^2*(si
n(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1
/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a^2/d^5/f/sin(2*f*x+2*e)^(1/
2)+2*b*(a^2-b^2)*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Elliptic
E(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a^4/
d^5/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 1.58, antiderivative size = 612, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {2899, 2570, 2563, 2572, 2639, 2910, 2906, 2905, 490, 1218}

$$-\frac{2bg^2\left(a^2-b^2\right)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{a^4d^5f\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{2}\,b^2g^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\Big(-\cdots}{a^4d^4f\sqrt{d\sin(e\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(7*a*d*f*(d*Sin[e + f*x])^(7/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(5*a^2*d^2*f*(d*Sin[e + f*x])^(5/2)) - (8*g*(g*Cos[e +
 f*x])^(3/2))/(21*a*d^3*f*(d*Sin[e + f*x])^(3/2)) + (2*(a^2 - b^2)*g*(g*Cos
[e + f*x])^(3/2))/(3*a^3*d^3*f*(d*Sin[e + f*x])^(3/2)) + (4*b*g*(g*Cos[e +
f*x])^(3/2))/(5*a^2*d^4*f*Sqrt[d*Sin[e + f*x]]) - (2*b*(a^2 - b^2)*g*(g*Cos
[e + f*x])^(3/2))/(a^4*d^4*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt[-a
```

```
 + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sq
rt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]
])/(a^4*d^4*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^2*Sqrt[-a + b]*Sqrt[a +
b]*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]
/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^4*d^4*f*Sqrt
[d*Sin[e + f*x]]) + (4*b*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x,
 2]*Sqrt[d*Sin[e + f*x]])/(5*a^2*d^5*f*Sqrt[Sin[2*e + 2*f*x]]) - (2*b*(a^2
- b^2)*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e +
 f*x]])/(a^4*d^5*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{9/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx}{a^2d^2}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{\left(2bg^2\right)\int \frac{\sqrt{g}}{(d\sin}}{5a^2}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(g\cos(e}{21ad^3f(d\sin(}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(g\cos(e}{21ad^3f(d\sin(}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(g\cos(e}{21ad^3f(d\sin(}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(g\cos(e}{21ad^3f(d\sin(}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(g\cos(e}{21ad^3f(d\sin(}$$

Mathematica [C] time = 23.75, size = 1776, normalized size = 2.90

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((-2*(3*a^2*b*Cos[e + f*x] - 5*b^3*Cos[e + f*x])*Cs
c[e + f*x])/(5*a^4) + (2*(3*a^2*Cos[e + f*x] - 7*b^2*Cos[e + f*x])*Csc[e +
f*x]^2)/(21*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x]^2)/(5*a^2) - (2*Cot[e + f
*x]*Csc[e + f*x]^3)/(7*a))*Sin[e + f*x]^3*Tan[e + f*x]^2)/(f*(d*Sin[e + f*x
])^(9/2)) - (b*(g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^(9/2)*((-2*(6*a^3 - 10*a
```

```
*b^2)*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e +
 f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])
*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e
 + f*x])) + ((8*a^2*b - 10*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*
ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcT
an[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + S
qrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e
 + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + S
qrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1
, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/
2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/
2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((
-3*a^2*b + 5*b^3)*Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*S
qrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -T
an[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a
^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e
 + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sq
rt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e
 + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
 + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt
[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])
/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*S
qrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 -
Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt
[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sq
rt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])
/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (
2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
+ Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + S
qrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e
 + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan
[e + f*x]^2])))/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Si
n[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(5*a^4*f*Cos[
e + f*x]^(5/2)*(d*Sin[e + f*x])^(9/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/
2)), x)
```

maple [B] time = 0.84, size = 10704, normalized size = 17.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}}{{\left(d\,\sin\left(e+f\,x\right)\right)}^{9/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(9/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(9/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1429 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{11/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=822

$$-\frac{2\sqrt{2}\sqrt{b-a}\sqrt{a+b}\,g^{5/2}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}\,b^3}{a^5d^5f\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\sqrt{b-a}\sqrt{a+b}\,g^{5/2}\Pi\left(\frac{\sqrt{b-}}{\sqrt{a-}}\right.}{a^5}$$

```
[Out] -2/9*g*(g*cos(f*x+e))^(3/2)/a/d/f/(d*sin(f*x+e))^(9/2)+2/7*b*g*(g*cos(f*x+e
))^(3/2)/a^2/d^2/f/(d*sin(f*x+e))^(7/2)-4/15*g*(g*cos(f*x+e))^(3/2)/a/d^3/f
/(d*sin(f*x+e))^(5/2)+2/5*(a^2-b^2)*g*(g*cos(f*x+e))^(3/2)/a^3/d^3/f/(d*sin
(f*x+e))^(5/2)+8/21*b*g*(g*cos(f*x+e))^(3/2)/a^2/d^4/f/(d*sin(f*x+e))^(3/2)
-2/3*b*(a^2-b^2)*g*(g*cos(f*x+e))^(3/2)/a^4/d^4/f/(d*sin(f*x+e))^(3/2)-8/15
*g*(g*cos(f*x+e))^(3/2)/a/d^5/f/(d*sin(f*x+e))^(1/2)+4/5*(a^2-b^2)*g*(g*cos
(f*x+e))^(3/2)/a^3/d^5/f/(d*sin(f*x+e))^(1/2)+2*b^2*(a^2-b^2)*g*(g*cos(f*x+
e))^(3/2)/a^5/d^5/f/(d*sin(f*x+e))^(1/2)-2*b^3*g^(5/2)*EllipticPi((g*cos(f*
x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/
2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/a^5/d^5/f/(d*sin(f*x+e))^(1/2)
+2*b^3*g^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2)
,(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1
/2)/a^5/d^5/f/(d*sin(f*x+e))^(1/2)+8/15*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin
(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d
*sin(f*x+e))^(1/2)/a/d^6/f/sin(2*f*x+2*e)^(1/2)-4/5*(a^2-b^2)*g^2*(sin(e+1/
4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(
g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a^3/d^6/f/sin(2*f*x+2*e)^(1/2)-2*b
^2*(a^2-b^2)*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(co
s(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a^5/d^6/
f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 2.14, antiderivative size = 822, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {2899, 2570, 2572, 2639, 2563, 2910, 2906, 2905, 490, 1218}

$$-\frac{2\sqrt{2}\sqrt{b-a}\sqrt{a+b}\,g^{5/2}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}\,b^3}{a^5d^5f\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\sqrt{b-a}\sqrt{a+b}\,g^{5/2}\Pi\left(\frac{\sqrt{b-}}{\sqrt{a-}}\right.}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(11/2)*(a + b*Sin[e + f*x])),x
]
```

```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(9*a*d*f*(d*Sin[e + f*x])^(9/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(7*a^2*d^2*f*(d*Sin[e + f*x])^(7/2)) - (4*g*(g*Cos[e +
 f*x])^(3/2))/(15*a*d^3*f*(d*Sin[e + f*x])^(5/2)) + (2*(a^2 - b^2)*g*(g*Cos
[e + f*x])^(3/2))/(5*a^3*d^3*f*(d*Sin[e + f*x])^(5/2)) + (8*b*g*(g*Cos[e +
f*x])^(3/2))/(21*a^2*d^4*f*(d*Sin[e + f*x])^(3/2)) - (2*b*(a^2 - b^2)*g*(g*
Cos[e + f*x])^(3/2))/(3*a^4*d^4*f*(d*Sin[e + f*x])^(3/2)) - (8*g*(g*Cos[e +
 f*x])^(3/2))/(15*a*d^5*f*Sqrt[d*Sin[e + f*x]]) + (4*(a^2 - b^2)*g*(g*Cos[e
 + f*x])^(3/2))/(5*a^3*d^5*f*Sqrt[d*Sin[e + f*x]]) + (2*b^2*(a^2 - b^2)*g*(
g*Cos[e + f*x])^(3/2))/(a^5*d^5*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^3*Sq
rt[-a + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcS
in[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e +
 f*x]])/(a^5*d^5*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^3*Sqrt[-a + b]*Sqrt
[a + b]*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e +
f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^5*d^5*f
*Sqrt[d*Sin[e + f*x]]) - (8*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f
*x, 2]*Sqrt[d*Sin[e + f*x]])/(15*a*d^6*f*Sqrt[Sin[2*e + 2*f*x]]) + (4*(a^2
- b^2)*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e +
 f*x]])/(5*a^3*d^6*f*Sqrt[Sin[2*e + 2*f*x]]) + (2*b^2*(a^2 - b^2)*g^2*Sqrt[
g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^5*d^6
*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
```

a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n *(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] && IntegersQ[2*m, 2*n]

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{11/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{11/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}}{a^2d^2}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{\left(4bg^2\right)\int}{7}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(g\cos}{15ad^3f(d\sin}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(g\cos}{15ad^3f(d\sin}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(g\cos}{15ad^3f(d\sin}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(g\cos}{15ad^3f(d\sin}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(g\cos}{15ad^3f(d\sin}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(g\cos}{15ad^3f(d\sin}$$

Mathematica [C] time = 25.02, size = 1850, normalized size = 2.25

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(11/2)*(a + b*Sin[e + f*x])),x]

[Out] ((g*Cos[e + f*x])^(5/2)*((2*(2*a^4*Cos[e + f*x] + 9*a^2*b^2*Cos[e + f*x] - 15*b^4*Cos[e + f*x])*Csc[e + f*x])/(15*a^5) - (2*(3*a^2*b*Cos[e + f*x] - 7*b^3*Cos[e + f*x])*Csc[e + f*x]^2)/(21*a^4) + (2*(a^2*Cos[e + f*x] - 3*b^2*Cos[e + f*x])*Csc[e + f*x]^3)/(15*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x]^3)/(7*a^2) - (2*Cot[e + f*x]*Csc[e + f*x]^4)/(9*a))*Sin[e + f*x]^4*Tan[e + f*x]^2)/(f*(d*Sin[e + f*x])^(11/2)) + ((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^(11/2)*((-2*(4*a^5 + 18*a^3*b^2 - 30*a*b^4)*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((2*a^4*b + 24*a^2*b^3 - 30*b^5)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-2*a^4*b - 9*a^2*b^3 + 15*b^5)*Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2

```
- b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))
/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1
 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(15*a^5*f*Cos[e + f*x]^(5/2)
*(d*Sin[e + f*x])^(11/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(11/2)/(a+b*sin(f*x+e)),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(11/2)/(a+b*sin(f*x+e)),x, al
gorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(11
/2)), x)
```

maple [B] time = 0.98, size = 17102, normalized size = 20.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(11/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(11/2)/(a+b*sin(f*x+e)),x, al
gorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(11
/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{5/2}}{\left(d\,\sin\left(e+f\,x\right)\right)^{11/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(11/2)*(a + b*sin(e + f*x))),x
)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(11/2)*(a + b*sin(e + f*x))),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(11/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1430 $\int \frac{(d\sin(e+fx))^{5/2}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=616

$$-\frac{2\sqrt{2}\,a^2d^{5/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,a^2d^{5/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\right.}{b^2f\sqrt{b^2-a^2}\,\sqrt{g\cos}}$$

```
[Out] 1/2*a*d^(5/2)*arctan(1-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(
f*x+e))^(1/2))/b^2/f*2^(1/2)/g^(1/2)-1/2*a*d^(5/2)*arctan(1+2^(1/2)*g^(1/2)
*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2))/b^2/f*2^(1/2)/g^(1/2)-1
/4*a*d^(5/2)*ln(d^(1/2)-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))
^(1/2)+d^(1/2)*tan(f*x+e))/b^2/f*2^(1/2)/g^(1/2)+1/4*a*d^(5/2)*ln(d^(1/2)+2
^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e)
)/b^2/f*2^(1/2)/g^(1/2)-2*a^2*d^(5/2)*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/
2)/(1+cos(f*x+e))^(1/2),-a/(b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)
/b^2/f/(-a^2+b^2)^(1/2)/(g*cos(f*x+e))^(1/2)+2*a^2*d^(5/2)*EllipticPi((d*si
n(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(
1/2)*cos(f*x+e)^(1/2)/b^2/f/(-a^2+b^2)^(1/2)/(g*cos(f*x+e))^(1/2)-d^2*(g*co
s(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/b/f/g-1/2*d^3*(sin(e+1/4*Pi+f*x)^2)^(1
/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(
1/2)/b/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.14, antiderivative size = 616, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.378, Rules used = {2909, 2568, 2573, 2641, 2574, 297, 1162, 617, 204, 1165, 628, 2908, 2907, 1218}

$$-\frac{2\sqrt{2}\,a^2d^{5/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,a^2d^{5/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\right.}{b^2f\sqrt{b^2-a^2}\,\sqrt{g\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(5/2)/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[
g*Cos[e + f*x]])])/(Sqrt[2]*b^2*f*Sqrt[g]) - (a*d^(5/2)*ArcTan[1 + (Sqrt[2]
*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b^
2*f*Sqrt[g]) - (2*Sqrt[2]*a^2*d^(5/2)*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b
- Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e +
 f*x]])], -1])/(b^2*Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*a
^2*d^(5/2)*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSi
```

```
n[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^2*Sqrt[-a
^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) - (a*d^(5/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[
g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*S
qrt[2]*b^2*f*Sqrt[g]) + (a*d^(5/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Si
n[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b^2*f
*Sqrt[g]) - (d^2*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(b*f*g) + (d^3*
EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(2*b*f*Sqrt[g*Cos[e +
f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
```

```
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^{5/2}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{d\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{(ad)\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{b}$$

$$= -\frac{d^2\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{bfg} - \frac{\left(ad^2\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} + \frac{\left(a^2d\right.}{}$$

$$= -\frac{d^2\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{bfg} - \frac{\left(2ad^3g\right)\text{Subst}\left(\int \frac{x^2}{d^2+g^2x^4}\,dx, x\right.}{b^2f}$$

$$= -\frac{d^2\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{bfg} + \frac{d^3F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(2e+}}{2bf\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+}}$$

$$= -\frac{2\sqrt{2}\,a^2d^{5/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right)\right| -}{b^2\sqrt{-a^2+b^2}\,f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2\sqrt{2}\,a^2d^{5/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right)\right| -}{b^2\sqrt{-a^2+b^2}\,f\sqrt{g\cos(e+fx)}}$$

$$= \frac{ad^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b^2f\sqrt{g}} - \frac{ad^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+f}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right.}{\sqrt{2}\,b^2f\sqrt{g}}$$

Mathematica [C] time = 27.08, size = 1318, normalized size = 2.14

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^(5/2)/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x]))
),x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)*((2*Sqrt[Sin[e + f*x]]*((Sqrt[a]
*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2
*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + Log[-
a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*
Tan[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]
```

```
] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^2)^(3/4)) - (b*Appe
llF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*
Tan[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))
/(Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(1 + Tan[e + f
*x]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Sin[e + f*x]]*(b*Tan[e + f*x] + a*Sq
rt[1 + Tan[e + f*x]^2])*(-20*Sqrt[2]*a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]
]] + 20*Sqrt[2]*a*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] + (10*Sqrt[2]*Sqrt
[a]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])
/Sqrt[a]])/(a^2 - b^2)^(3/4) - (10*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*ArcTan[1 +
 (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(3/4)
 + 10*Sqrt[2]*a*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 10*Sqr
t[2]*a*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (5*Sqrt[2]*Sqrt
[a]*(2*a^2 - b^2)*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f
*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(3/4) + (5*Sqrt[2]*Sqrt[a
]*(2*a^2 - b^2)*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]
] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(3/4) + 8*b*AppellF1[5/4, 1/
2, 1, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(5/
2) + (40*b*Sqrt[Tan[e + f*x]])/Sqrt[1 + Tan[e + f*x]^2] + (200*a^4*b*Appell
F1[1/4, 1/2, 1, 5/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sqrt[T
an[e + f*x]])/(Sqrt[1 + Tan[e + f*x]^2]*(-5*a^2*AppellF1[1/4, 1/2, 1, 5/4,
-Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + 2*(2*(a^2 - b^2)*AppellF1
[5/4, 1/2, 2, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + a^2*Ap
pellF1[5/4, 3/2, 1, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])*T
an[e + f*x]^2)*(-(b^2*Tan[e + f*x]^2) + a^2*(1 + Tan[e + f*x]^2))))))/(20*b^
2*Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(-1 + Tan[e +
f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(2*f*Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^
(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin \left(f x+e\right)\right)^{\frac{5}{2}}}{\sqrt{g \cos \left(f x+e\right)} \left(b \sin \left(f x+e\right)+a\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="giac")

[Out] integrate((d*sin(f*x + e))^(5/2)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)
), x)

maple [B] time = 0.89, size = 2344, normalized size = 3.81

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)

[Out] -1/f*(I*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*(-a^2+b^2)^(1/2)*a^2-I*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/
2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+I*sin(f*x+e)*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b-I*sin(f*x+e)*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a
^2+sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*a^2-sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^3+sin(f*x+e)*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2*b-sin(f*x+
e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1
/2)*a^2+sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*(-a^2+b^2)^(1/2)*a*b-sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+

```
1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b-(-a^2+b^2)^(1/2)*si
n(f*x+e)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/
2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b+(-a^2+b^2)^(1/
2)*sin(f*x+e)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*
2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^2+sin(f*x+e
)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))
*(-a^2+b^2)^(1/2)*a^2+sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b
+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^3-sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2*b-cos(f*x+e)^2*(-a^2+b^2)
^(1/2)*2^(1/2)*a*b+cos(f*x+e)^2*2^(1/2)*(-a^2+b^2)^(1/2)*b^2+cos(f*x+e)*(-a
^2+b^2)^(1/2)*2^(1/2)*a*b-cos(f*x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*b^2)*(d*sin(f
*x+e))^(5/2)/(-1+cos(f*x+e))/sin(f*x+e)^2/(g*cos(f*x+e))^(1/2)*2^(1/2)*a/b^
2/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^(5/2)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,\sin\left(e+f\,x\right)\right)}^{5/2}}{\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^(5/2)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((d*sin(e + f*x))^(5/2)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.1431 $\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=508

$$\frac{2\sqrt{2}\,ad^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{bf\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,ad^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\right.}{bf\sqrt{b^2-a^2}\,\sqrt{g\cos(e+}}$$

```
[Out] -1/2*d^(3/2)*arctan(1-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(f
*x+e))^(1/2))/b/f*2^(1/2)/g^(1/2)+1/2*d^(3/2)*arctan(1+2^(1/2)*g^(1/2)*(d*s
in(f*x+e))^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2))/b/f*2^(1/2)/g^(1/2)+1/4*d^(3
/2)*ln(d^(1/2)-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^
(1/2)*tan(f*x+e))/b/f*2^(1/2)/g^(1/2)-1/4*d^(3/2)*ln(d^(1/2)+2^(1/2)*g^(1/2
)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/b/f*2^(1/2)
/g^(1/2)+2*a*d^(3/2)*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))
^(1/2),-a/(b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/b/f/(-a^2+b^2)^(
1/2)/(g*cos(f*x+e))^(1/2)-2*a*d^(3/2)*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/
2)/(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)
/b/f/(-a^2+b^2)^(1/2)/(g*cos(f*x+e))^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {2909, 2574, 297, 1162, 617, 204, 1165, 628, 2908, 2907, 1218}

$$\frac{2\sqrt{2}\,ad^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{bf\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,ad^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\right.}{bf\sqrt{b^2-a^2}\,\sqrt{g\cos(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(3/2)/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[
g*Cos[e + f*x]])])/(Sqrt[2]*b*f*Sqrt[g])) + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b*f*S
qrt[g]) + (2*Sqrt[2]*a*d^(3/2)*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[
-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])
], -1])/(b*Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*a*d^(3/2)*
Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Si
n[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b*Sqrt[-a^2 + b^2]*f*S
qrt[g*Cos[e + f*x]]) + (d^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e
 + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*f*Sqrt
```

```
[g]) - (d^(3/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g
*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*f*Sqrt[g])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{d\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{(ad)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{b}$$

$$= \frac{\left(2d^2g\right)\text{Subst}\left(\int \frac{x^2}{d^2+g^2x^4}\,dx, x, \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\right)}{bf} - \frac{\left(ad\sqrt{\cos(e+fx)}\right)\int -}{b\sqrt{g\cos}}$$

$$= -\frac{d^2\,\text{Subst}\left(\int \frac{d-gx^2}{d^2+g^2x^4}\,dx, x, \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\right)}{bf} + \frac{d^2\,\text{Subst}\left(\int \frac{d+gx^2}{d^2+g^2x^4}\,dx, x,\right.}{bf}$$

$$= \frac{2\sqrt{2}\,ad^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}}; \sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right)\middle| -1\right.}{b\sqrt{-a^2+b^2}\,f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2\sqrt{2}\,ad^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}}; \sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right)\middle| -1\right.}{b\sqrt{-a^2+b^2}\,f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,bf\sqrt{g}} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+fx}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right.}{\sqrt{2}\,bf\sqrt{g}}$$

Mathematica [C] time = 16.45, size = 518, normalized size = 1.02

$$\frac{10\left(a^2-b^2\right)\cot(e+fx)(d\sin(e+fx))^{3/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(\frac{aF_1\left(\right.}{\cos^2(e+fx)\left(\left(b^2-a^2\right)F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-4b\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^(3/2)/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (10*(a^2 - b^2)*Cot[e + f*x]*(d*Sin[e + f*x])^(3/2)*(a + b*Sqrt[Sin[e + f*x
]^2])*((a*AppellF1[1/4, -1/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)])/(5*(a^2 - b^2)*AppellF1[1/4, -1/4, 1, 5/4, Cos[e + f*x]^2, (
b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, -1/4, 2, 9/4, Cos
```

```
[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4
, 3/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e +
f*x]^2) + (b*AppellF1[1/4, -3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)]*Sqrt[Sin[e + f*x]^2])/(-5*(a^2 - b^2)*AppellF1[1/4, -3/4,
1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (4*b^2*AppellF
1[5/4, -3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 3
*(a^2 - b^2)*AppellF1[5/4, 1/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)])*Cos[e + f*x]^2)))/(f*Sqrt[g*Cos[e + f*x]]*(-a + b*Sin[e +
f*x])*(a + b*Sin[e + f*x])^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{g \cos\left(fx+e\right)}\left(b \sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^(3/2)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)
), x)
```

maple [B] time = 0.87, size = 941, normalized size = 1.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*a*(-a^2+b^2)^(1/2)-I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b*(-a^2+b^2)^(1/2)-I*EllipticPi((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a*(-a^2+
```

```
b^2)^(1/2)+I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*b*(-a^2+b^2)^(1/2)-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b*(-a^
2+b^2)^(1/2)-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b*(-a^2+b^2)^(1/2)+EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*
2^(1/2))*(-a^2+b^2)^(1/2)*a+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+a^2*El
lipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(
1/2)),1/2*2^(1/2))-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b-a^2*EllipticPi((-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))+Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)
-a),1/2*2^(1/2))*a*b)*(d*sin(f*x+e))^(3/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)/(-1+cos(f*x+e))/(g*cos(f*x+e))^(1/2)*2^(1/2)*a/b/(-a^2
+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^(3/2)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,\sin\left(e+f\,x\right)\right)}^{3/2}}{\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^(3/2)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((d*sin(e + f*x))^(3/2)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}{\sqrt{g\cos\left(e+fx\right)}\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**(3/2)/(sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x))
), x)
```

3.1432 $\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=209

$$\frac{2\sqrt{2}\,\sqrt{d}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,\sqrt{d}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\right.}{f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+f}}$$

```
[Out] -2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(-a^2
+b^2)^(1/2)),I)*2^(1/2)*d^(1/2)*cos(f*x+e)^(1/2)/f/(-a^2+b^2)^(1/2)/(g*cos(
f*x+e))^(1/2)+2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2
),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*d^(1/2)*cos(f*x+e)^(1/2)/f/(-a^2+b^2)^
(1/2)/(g*cos(f*x+e))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {2908, 2907, 1218}

$$\frac{2\sqrt{2}\,\sqrt{d}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,\sqrt{d}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\right.}{f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*Sqrt[d]*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2]
)), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(Sq
rt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*Sqrt[d]*Sqrt[Cos[e + f*
x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sq
rt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x
]])
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
```

```
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rubi steps

$$\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\sqrt{\cos(e+fx)}\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{\sqrt{g\cos(e+fx)}}$$

$$= \frac{\left(2\sqrt{2}\left(1-\frac{b}{\sqrt{-a^2+b^2}}\right)d\sqrt{\cos(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(\left(b-\sqrt{-a^2+b^2}\right)d+ax^2\right)\sqrt{1-\frac{x^4}{d^2}}}\right.}{f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2\sqrt{2}\sqrt{d}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{\sqrt{-a^2+b^2}\,f\sqrt{g\cos(e+fx)}}$$

Mathematica [A] time = 4.44, size = 166, normalized size = 0.79

$$-\frac{2\sqrt{2}\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}\cot(e+fx)\sqrt{d\sin(e+fx)}\left(\Pi\left(\frac{a}{\sqrt{b^2-a^2}-b};\sin^{-1}\left(\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}\right)\middle|-1\right)-\Pi\left(-\frac{a}{b+\sqrt{b^2-}}\right.\right.}{f\sqrt{b^2-a^2}\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sin[e + f*x]]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),
x]
```

```
[Out] (-2*Sqrt[2]*Cot[e + f*x]*(EllipticPi[a/(-b + Sqrt[-a^2 + b^2]), ArcSin[Sqrt
[Tan[(e + f*x)/2]]], -1] - EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[S
```

```
qrt[Tan[(e + f*x)/2]]], -1])*Sqrt[d*Sin[e + f*x]]*Sqrt[Tan[(e + f*x)/2]])/(
Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x]
)])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(fx+e\right)}}{\sqrt{g \cos\left(fx+e\right)}\left(b \sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e))/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)),
 x)
```

maple [B] time = 0.80, size = 527, normalized size = 2.52

$$-\frac{\sqrt{d \sin\left(fx+e\right)}\left(\operatorname{EllipticPi}\left(\sqrt{-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}, \frac{a}{a-b+\sqrt{-a^2+b^2}}, \frac{\sqrt{2}}{2}\right)\sqrt{-a^2+b^2}+\operatorname{EllipticPi}\left(\sqrt{-\frac{-1+\cos(}{\mathrm{si}}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(d*sin(f*x+e))^(1/2)*(EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)+EllipticPi
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1
/2*2^(1/2))*(-a^2+b^2)^(1/2)+a*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))-b*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))-
```

```
a*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2
)^(1/2)-a),1/2*2^(1/2))+b*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)))*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)/(-1+cos(f*x+e))/(g*cos(f*x+e))^(1/2)*
2^(1/2)*a/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(fx+e\right)}}{\sqrt{g \cos\left(fx+e\right)}\left(b \sin\left(fx+e\right)+a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e))/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,\sin\left(e+f\,x\right)}}{\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^(1/2)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((d*sin(e + f*x))^(1/2)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(e+fx\right)}}{\sqrt{g \cos\left(e+fx\right)}\left(a+b \sin\left(e+fx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*sin(e + f*x))/(sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x))),
x)
```

3.1433 $\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=273

$$\frac{2\sqrt{2}\,b\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a\sqrt{d}\,f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,b\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{d}\,\sqrt{}}\right.\right.}{a\sqrt{d}\,f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx}}$$

```
[Out] 2*b*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(-a^
2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a/f/(-a^2+b^2)^(1/2)/d^(1/2)/(g*c
os(f*x+e))^(1/2)-2*b*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))
^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a/f/(-a^2+b^2)^(
1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2)-(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+
f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a/f/(g*cos(f
*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2910, 2573, 2641, 2908, 2907, 1218}

$$\frac{2\sqrt{2}\,b\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a\sqrt{d}\,f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,b\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{d}\,\sqrt{}}\right.\right.}{a\sqrt{d}\,f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[2]*b*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), Arc
Sin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a*Sqrt[-a
^2 + b^2]*Sqrt[d]*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b*Sqrt[Cos[e + f*x]]
*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[
d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a*Sqrt[-a^2 + b^2]*Sqrt[d]*f*Sqrt[g*Cos[
e + f*x]]) + (EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a*f*Sqr
t[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}\,dx}{a} - \frac{b\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e\ldots}}{ad}$$

$$= -\frac{\left(b\sqrt{\cos(e+fx)}\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{ad\sqrt{g\cos(e+fx)}} + \frac{\sqrt{s\ldots}}{a\sqrt{\ldots}}$$

$$= \frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(2e+2fx)}}{af\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}} - \frac{\left(2\sqrt{2}\,b\left(1-\frac{\ldots}{\sqrt{-a\ldots}}\ldots\right.\right.}{\ldots}$$

$$= \frac{2\sqrt{2}\,b\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e\ldots}}{\sqrt{d}\sqrt{1+\cos\ldots}}\right.\right.}{a\sqrt{-a^2+b^2}\sqrt{d}\,f\sqrt{g\cos(e+fx)}}$$

Mathematica [A] time = 10.10, size = 209, normalized size = 0.77

$$-\frac{4\sqrt{2}\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)-1}}\tan^{\frac{3}{2}}\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{b^2-a^2}\,F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}}\right)\middle|-1\right)+b\left(\Pi\left(\frac{a}{\sqrt{b^2-a^2}-\ldots}\right.\right.\right.}{af\sqrt{b^2-a^2}\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (-4*Sqrt[2]*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(-1 + Cos[e + f*x])]*(Sqrt
[-a^2 + b^2]*EllipticF[ArcSin[1/Sqrt[Tan[(e + f*x)/2]]], -1] + b*(EllipticP
i[a/(-b + Sqrt[-a^2 + b^2]), ArcSin[1/Sqrt[Tan[(e + f*x)/2]]], -1] - Ellipt
icPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[1/Sqrt[Tan[(e + f*x)/2]]], -1]))*T
an[(e + f*x)/2]^(3/2))/(a*Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Si
n[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

maple [B] time = 0.68, size = 631, normalized size = 2.31

$$\frac{\left(2\,\mathrm{EllipticF}\left(\sqrt{-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)a\sqrt{-a^{2}+b^{2}}-2\,\mathrm{EllipticF}\left(\sqrt{-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\sqrt{-a^{2}+b^{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(2*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2)
)*a*(-a^2+b^2)^(1/2)-2*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b-EllipticPi((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2
^(1/2))*b^2+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-
b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-EllipticPi((-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a
*b+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^
2)^(1/2)-a),1/2*2^(1/2))*b^2+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)^2/(d*sin(f*x+e)
)^(1/2)/(-1+cos(f*x+e))/(g*cos(f*x+e))^(1/2)*2^(1/2)/(-a^2+b^2)^(1/2)/(a-b+
(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{g\cos\left(e+fx\right)}\,\sqrt{d\sin\left(e+fx\right)}\,\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))),
x)
```

```
[Out] int(1/((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))),
 x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d\sin\left(e+fx\right)}\,\sqrt{g\cos\left(e+fx\right)}\,\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*sin(e + f*x))*sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x)))
, x)
```

3.1434 $\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=320

$$-\frac{2\sqrt{2}\,b^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2d^{3/2}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,b^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\ldots\right.\right.}{a^2d^{3/2}f\sqrt{b^2-a^2}\sqrt{g\cos(e+\ldots}}$$

```
[Out] -2*b^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(
-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a^2/d^(3/2)/f/(-a^2+b^2)^(1/2)
/(g*cos(f*x+e))^(1/2)+2*b^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(
f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a^2/d^(3/
2)/f/(-a^2+b^2)^(1/2)/(g*cos(f*x+e))^(1/2)-2*(g*cos(f*x+e))^(1/2)/a/d/f/g/(
d*sin(f*x+e))^(1/2)+b*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Ellipti
cF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^2/d/f/(g*cos(f*x+e))^(
1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2910, 2563, 2573, 2641, 2908, 2907, 1218}

$$-\frac{2\sqrt{2}\,b^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2d^{3/2}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,b^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\ldots\right.\right.}{a^2d^{3/2}f\sqrt{b^2-a^2}\sqrt{g\cos(e+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*b^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])),
ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sq
rt[-a^2 + b^2]*d^(3/2)*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt[Cos[e
+ f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]
/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sqrt[-a^2 + b^2]*d^(3/2)*f*Sq
rt[g*Cos[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]])/(a*d*f*g*Sqrt[d*Sin[e + f*x]
]) - (b*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^2*d*f*Sqrt[
g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
```

```
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}}\,dx}{a} - \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e}}}{ad}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{adfg\sqrt{d\sin(e+fx)}} + \frac{b^2\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx)}}{a^2d^2}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{adfg\sqrt{d\sin(e+fx)}} + \frac{\left(b^2\sqrt{\cos(e+fx)}\right)\int \frac{}{\sqrt{\cos(e}}}{a^2d^2\sqrt{g\cos(e}}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{adfg\sqrt{d\sin(e+fx)}} - \frac{bF\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(2e}}{a^2df\sqrt{g\cos(e+fx)}\,\sqrt{d\sin}}$$

$$= -\frac{2\sqrt{2}\,b^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin}}{\sqrt{d}\,\sqrt{1+}}\right.\right.}{a^2\sqrt{-a^2+b^2}\,d^{3/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [A] time = 10.80, size = 225, normalized size = 0.70

$$\frac{2\left(\frac{2\sqrt{2}\,b\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)-1}}\,\tan^{\frac{3}{2}}\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{b^2-a^2}\,F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}}\right)\middle|-1\right)+b\left(\Pi\left(\frac{a}{\sqrt{b^2-a^2}-b};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}}\right)\middle|-1\right)-\Pi\left(-\frac{}{b+\sqrt{}}\right.\right.\right.}{\sqrt{b^2-a^2}}\right.}{a^2df\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x
]))),x]
```

```
[Out] (2*(-(a*Cos[e + f*x]) + (2*Sqrt[2]*b*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(
-1 + Cos[e + f*x])]*(Sqrt[-a^2 + b^2]*EllipticF[ArcSin[1/Sqrt[Tan[(e + f*x)
/2]]], -1] + b*(EllipticPi[a/(-b + Sqrt[-a^2 + b^2]), ArcSin[1/Sqrt[Tan[(e
+ f*x)/2]]], -1] - EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[1/Sqrt[Ta
```

```
n[(e + f*x)/2]]], -1]))*Tan[(e + f*x)/2]^(3/2))/Sqrt[-a^2 + b^2]))/(a^2*d*f
*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

maple [B] time = 0.72, size = 2286, normalized size = 7.14

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(2*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2
^(1/2))*a*b-2*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),1/2*2^(1/2))*b^2+cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2+cos(f*x+e)*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
```

```
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b^2-cos(f*x+e
)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))
*b^3+cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+
(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2+cos(f*x+e)*(-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3+2*(-a^
2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b-2*(-a^2+b^
2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b^2+(-a^2+b^2)^(1/
2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2)
)*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1
/2))*a*b^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2
*2^(1/2))*b^3+(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-
a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(
b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3+2*cos(f*x+e)*2^(1/2)*(-a^2+b^2
)^(1/2)*a^2-2*cos(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*a*b)*sin(f*x+e)/(d*sin(f*
x+e))^(3/2)/(g*cos(f*x+e))^(1/2)*2^(1/2)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(
1/2))/(b+(-a^2+b^2)^(1/2)-a)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{g\,\cos\left(e+f\,x\right)}\,{\left(d\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))),
x)
```

```
[Out] int(1/((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d\sin\left(e+fx\right)\right)^{\frac{3}{2}}\sqrt{g\cos\left(e+fx\right)}\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((d*sin(e + f*x))**(3/2)*sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x
))), x)
```

3.1435 $\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=424

$$\frac{b^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^3d^2f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2b\sqrt{g\cos(e+fx)}}{a^2d^2fg\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,b^3\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^3d^{5/2}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}$$

[Out] 2*b^3*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a^3/d^(5/2)/f/(-a^2+b^2)^(1/2)/(g*cos(f*x+e))^(1/2)-2*b^3*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a^3/d^(5/2)/f/(-a^2+b^2)^(1/2)/(g*cos(f*x+e))^(1/2)-2/3*(g*cos(f*x+e))^(1/2)/a/d/f/g/(d*sin(f*x+e))^(3/2)+2*b*(g*cos(f*x+e))^(1/2)/a^2/d^2/f/g/(d*sin(f*x+e))^(1/2)-2/3*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a/d^2/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)-b^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^3/d^2/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)

Rubi [A] time = 1.16, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {2910, 2570, 2573, 2641, 2563, 2908, 2907, 1218}

$$\frac{b^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^3d^2f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,b^3\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^3d^{5/2}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,b^3}{}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]

[Out] (2*Sqrt[2]*b^3*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^3*Sqrt[-a^2 + b^2]*d^(5/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b^3*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^3*Sqrt[-a^2 + b^2]*d^(5/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]])/(3*a*d*f*g*(d*Sin[e + f*x])^(3/2)) + (2*b*Sqrt[g*Cos[e + f*x]])/(a^2*d^2*f*g*Sqrt[d*Sin[e + f*x]]) + (2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a*d^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]) + (b^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^3*d^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
```

```
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{5/2}}\,dx}{a} - \frac{b\int \frac{}{\sqrt{g\cos(e+fx)}\,(d\sin(e-}}{a}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2\int \frac{1}{\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+f}}}{3ad^2}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b\sqrt{g\cos(e+fx)}}{a^2d^2fg\sqrt{d\sin(e+fx)}}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b\sqrt{g\cos(e+fx)}}{a^2d^2fg\sqrt{d\sin(e+fx)}}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b\sqrt{g\cos(e+fx)}}{a^2d^2fg\sqrt{d\sin(e+fx)}}$$

$$= \frac{2\sqrt{2}\,b^3\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(}}{\sqrt{d}\,\sqrt{1+\cos}}\right.\right.}{a^3\sqrt{-a^2+b^2}\,d^{5/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] time = 20.23, size = 1140, normalized size = 2.69

$$\frac{\cos(e+fx)\left(\frac{2b\csc(e+fx)}{a^2}-\frac{2\csc^2(e+fx)}{3a}\right)\sin^3(e+fx)}{f\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{5/2}}+\frac{\sqrt{\cos(e+fx)}\left(\frac{4ab\sqrt{\sin(e+fx)}\left(\frac{\sqrt{a}\left(-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a^2-b^2}\sqrt{\tan(e+fx)}}{\sqrt{a}}\right)+2\,\mathrm{ta}\dots\right.}{\dots}\right.}{\dots}\right.}{\dots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x
]])),x]
```

```
[Out] (Cos[e + f*x]*((2*b*Csc[e + f*x])/a^2 - (2*Csc[e + f*x]^2)/(3*a))*Sin[e + f
*x]^3)/(f*Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)) + (Sqrt[Cos[e + f*x]
]*Sin[e + f*x]^(5/2)*((-2*(2*a^2 + 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*
((5*a*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(5*(a^
2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)] + (-4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e
 + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e +
 f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1
 + Cos[e + f*x]^2))) - ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[C
os[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*ArcTan[
1 + ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e +
f*x]^2)^(1/4))] + Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e
 + f*x]^2] - ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 +
Cos[e + f*x]^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1
+ Cos[e + f*x]^2] + ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])
/(-1 + Cos[e + f*x]^2)^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin[e +
f*x]])/((1 - Cos[e + f*x]^2)^(1/4)*(a + b*Sin[e + f*x])) + (4*a*b*Sqrt[Sin[
e + f*x]]*((Sqrt[a]*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]
])/Sqrt[a]] + Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
 - Sqrt[a^2 - b^2]*Tan[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^
2)^(3/4)) - (b*AppellF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Ta
n[e + f*x]^2)/a^2]*Tan[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1
+ Tan[e + f*x]^2]))/(Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f
*x]]*(1 + Tan[e + f*x]^2)^(3/2))))/(3*a^2*f*Sqrt[g*Cos[e + f*x]]*(d*Sin[e +
 f*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

maple [B] time = 0.78, size = 2987, normalized size = 7.04

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*(3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*b^3+3*cos(f*x+e)*sin(f*x+e)
*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2
)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*
a*b^3-3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*b^4+3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b
```

^3-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+
b^2)^(1/2)-a),1/2*2^(1/2))*a*b^3+3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^4+4*cos(f*x+e)*s
in(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*
a^3-4*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))
*(-a^2+b^2)^(1/2)*a^2*b+6*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b^2-6*cos(f*x+e)*sin(f*x+e)*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^3+3*sin(f*x+e)*
EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)
^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(
-a^2+b^2)^(1/2)*b^3+3*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b^3-3*sin(f*x+e)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^4+3*sin(f*x+e)*(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^
2)^(1/2)*b^3-3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b
^2)^(1/2)-a),1/2*2^(1/2))*a*b^3+3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^4+4*sin(f*x+e)*(-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^3-4*sin(f*x+e)*(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-s

```
in(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2*b+6*sin(f*x+
e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b^2-6
*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2
)*b^3+6*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*a^2*b-6*cos(f*x+e)*s
in(f*x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*a*b^2-2*(-a^2+b^2)^(1/2)*cos(f*x+e)*2^(1
/2)*a^3+2*cos(f*x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*a^2*b)*sin(f*x+e)/(d*sin(f*x+
e))^(5/2)/(g*cos(f*x+e))^(1/2)*2^(1/2)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/
2))/(b+(-a^2+b^2)^(1/2)-a)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{g\,\cos\left(e+f\,x\right)}\,{\left(d\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))),
x)
```

```
[Out] int(1/((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.1436 $\int \frac{(d\sin(e+fx))^{5/2}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=1064

$$-\frac{2\sqrt{2}\,a^3\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}\,d^3}{b(b-a)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,a^3\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right.}{b(b-a)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e}}$$

```
[Out] 1/2*a^2*d^(5/2)*arctan(-1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*s
in(f*x+e))^(1/2))/b/(a^2-b^2)/f/g^(3/2)*2^(1/2)-1/2*b*d^(5/2)*arctan(-1+2^(
1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))/(a^2-b^2)/f
/g^(3/2)*2^(1/2)+1/2*a^2*d^(5/2)*arctan(1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1
/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))/b/(a^2-b^2)/f/g^(3/2)*2^(1/2)-1/2*b*d^(5/
2)*arctan(1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/
2))/(a^2-b^2)/f/g^(3/2)*2^(1/2)+1/4*a^2*d^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/
2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))/b/(a^2-b^2)/f
/g^(3/2)*2^(1/2)-1/4*b*d^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2
)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))/(a^2-b^2)/f/g^(3/2)*2^(1/2)-1/
4*a^2*d^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^
(1/2)/(d*sin(f*x+e))^(1/2))/b/(a^2-b^2)/f/g^(3/2)*2^(1/2)+1/4*b*d^(5/2)*ln(
g^(1/2)+cot(f*x+e)*g^(1/2)+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+
e))^(1/2))/(a^2-b^2)/f/g^(3/2)*2^(1/2)+2*a*d*(d*sin(f*x+e))^(3/2)/(a^2-b^2)
/f/g/(g*cos(f*x+e))^(1/2)-2*a^3*d^3*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)
/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)
/b/(-a+b)^(3/2)/(a+b)^(3/2)/f/g^(3/2)/(d*sin(f*x+e))^(1/2)+2*a^3*d^3*Ellipt
icPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(
1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/b/(-a+b)^(3/2)/(a+b)^(3/2)/f/g^(3/2)/(d*si
n(f*x+e))^(1/2)-2*b*d^2*(d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/g/(g*cos(f*x+e))^(
1/2)+2*a*d^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+
1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/
g^2/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 1.62, antiderivative size = 1064, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 17, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.460, Rules used = {2902, 2571, 2572, 2639, 2566, 2575, 297, 1162, 617, 204, 1165, 628, 2909, 2906, 2905, 490, 1218}

$$-\frac{2\sqrt{2}\,a^3\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}\,d^3}{b(b-a)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,a^3\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right.}{b(b-a)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(5/2)/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^2*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*S
qrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*(a^2 - b^2)*f*g^(3/2))) + (b*d^(5/2)*ArcT
an[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]]
)])/(Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) + (a^2*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*(a^2
- b^2)*f*g^(3/2)) - (b*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f
*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) + (a
^2*d^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos
[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*(a^2 - b^2)*f*g^(3/2)) - (b
*d^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e
 + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) - (a^2*d
^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e +
 f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*(a^2 - b^2)*f*g^(3/2)) + (b*d^(
5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f
*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) - (2*Sqrt[2]
*a^3*d^3*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]
]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b*(-a + b)^(3
/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a^3*d^3*Elli
pticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[
1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b*(-a + b)^(3/2)*(a + b)^(3/2
)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*b*d^2*Sqrt[d*Sin[e + f*x]])/((a^2 -
b^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*d*(d*Sin[e + f*x])^(3/2))/((a^2 - b^2
)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a*d^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - P
i/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*f*g^2*Sqrt[Sin[2*e + 2*f*x
]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
```

```
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^{5/2}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = -\frac{(bd)\int \frac{(d\sin(e+fx))^{3/2}}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} + \frac{\left(ad^2\right)\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{\left(a^2d^2\right)\int \frac{\sqrt{g\cos}}{\left(a^2\right.}}{}$$

$$= -\frac{2bd^2\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2ad(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{\left(2ad\right.}{}$$

$$= -\frac{2bd^2\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2ad(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{\left(2a^2\right.}{}$$

$$= -\frac{2bd^2\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2ad(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2ad^2}{}$$

$$= -\frac{2bd^2\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2ad(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2ad^2}{}$$

$$= \frac{a^2d^{5/2}\log\left(\sqrt{g}+\sqrt{g}\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}\,b\left(a^2-b^2\right)fg^{3/2}} - \frac{bd^{5/2}\log\left(\right.}{}$$

$$= -\frac{a^2d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b\left(a^2-b^2\right)fg^{3/2}} + \frac{bd^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(}}{\sqrt{g}\sqrt{d\sin(e+}}\right.}{\sqrt{2}\left(a^2-b^2\right)fg^{3/2}}$$

Mathematica [C] time = 80.66, size = 1290, normalized size = 1.21

$$\frac{2\cot(e+fx)\csc(e+fx)(d\sin(e+fx))^{5/2}(a\sin(e+fx)-b)}{\left(a^2-b^2\right)f(g\cos(e+fx))^{3/2}} - \frac{\cos^{\frac{3}{2}}(e+fx)(d\sin(e+fx))^{5/2}\left(-\frac{2\left(3a^2-b^2\right)\left(aF_1\left(\frac{3}{4};\frac{1}{4},1;\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^(5/2)/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] (2*Cot[e + f*x]*Csc[e + f*x]*(d*Sin[e + f*x])^(5/2)*(-b + a*Sin[e + f*x]))/
((a^2 - b^2)*f*(g*Cos[e + f*x])^(3/2)) - (Cos[e + f*x]^(3/2)*(d*Sin[e + f*x
])^(5/2)*((-2*(3*a^2 - b^2)*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2
, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e
+ f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqr
t[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]
^2)^(3/4)*(a + b*Sin[e + f*x])) - (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*T
an[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4
, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]
^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1
 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3
/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + S
qrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)
^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*Arc
Tan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2
)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
 f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a
^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]
*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(
3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Lo
g[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^
2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]
*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2
 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt
[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt
[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3
/2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*
x]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2]))))/
((a - b)*(a + b)*f*(g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^(5/2)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) +
a)), x)
```

maple [B] time = 0.89, size = 4619, normalized size = 4.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(5/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/f*(-I*(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/
2+1/2*I,1/2*2^(1/2))*b^2+I*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^3-I*(-a^2+b^2)^(1/2
)*cos(f*x+e)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*a^2
-(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*a^2+(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^
(1/2))*b^2+I*(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),1/2-1/2*I,1/2*2^(1/2))*b^2+4*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)
```

```
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b-2*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b+2*(-a^2+b^2)^(1/2)*2^(1/2)*a*b-2
*(-a^2+b^2)^(1/2)*sin(f*x+e)*2^(1/2)*b^2-(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2+(-a^2+b^2)^(1/2)*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2+(-a^2+b^2)^(1/2)*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2+(-a^
2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),
1/2*2^(1/2))*a^2-2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*
2^(1/2))*b^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1
/2*2^(1/2))*a^2*b+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2
)-a),1/2*2^(1/2))*a^2*b-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a
-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^3-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a
/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^3+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^3-2*cos(f*x+e)*(-a^2+b^2)^(1/2)*
2^(1/2)*a*b+I*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*
I,1/2*2^(1/2))*b^2-I*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1
/2+1/2*I,1/2*2^(1/2))*b^2+I*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/s
```

in(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2-I*(-a^2+b^2)^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*a^2+4*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b-2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b-cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2+cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2-cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2+cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2+cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2+cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2-2*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b^2-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2*b+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2*b)*(d*sin(f*x+e))^(5/2)*cos(f*x+e)/sin(f*x+e)^3/(g*cos(f*x+e))^(3/2)*2^(1/2)*a/(a+b)/b/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^(5/2)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) +
a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^(5/2)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((d*sin(e + f*x))^(5/2)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(5/2)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1437 $\int \frac{(d\sin(e+fx))^{3/2}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=379

$$\frac{2bdE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{2ad\sqrt{d\sin(e+fx)}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b(d\sin(e+fx))^{3/2}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}$$

```
[Out] -2*b*(d*sin(f*x+e))^(3/2)/(a^2-b^2)/f/g/(g*cos(f*x+e))^(1/2)+2*a^2*d^2*Elli
pticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b
)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/(-a+b)^(3/2)/(a+b)^(3/2)/f/g^(3/2)/(d*s
in(f*x+e))^(1/2)-2*a^2*d^2*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f
*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/(-a+b)^(3
/2)/(a+b)^(3/2)/f/g^(3/2)/(d*sin(f*x+e))^(1/2)+2*a*d*(d*sin(f*x+e))^(1/2)/(
a^2-b^2)/f/g/(g*cos(f*x+e))^(1/2)-2*b*d*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1
/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin
(f*x+e))^(1/2)/(a^2-b^2)/f/g^2/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2902, 2563, 2571, 2572, 2639, 2906, 2905, 490, 1218}

$$\frac{2bdE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{2ad\sqrt{d\sin(e+fx)}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b(d\sin(e+fx))^{3/2}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(3/2)/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[2]*a^2*d^2*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Co
s[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a
 + b)^(3/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*a^2*
d^2*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[
g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b)^(3/2)*(a + b
)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*a*d*Sqrt[d*Sin[e + f*x]])/((a^
2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*b*(d*Sin[e + f*x])^(3/2))/((a^2 - b
^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*b*d*Sqrt[g*Cos[e + f*x]]*EllipticE[e - P
i/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*f*g^2*Sqrt[Sin[2*e + 2*f*x
]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
```

```
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
```

```
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^{3/2}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = -\frac{(bd)\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} + \frac{\left(ad^2\right)\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{a^2-b^2} - \frac{\left(a^2\right.}{}$$

$$= \frac{2ad\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{(2bd)}{}$$

$$= \frac{2ad\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{\left(4\sqrt{2}\right.}{}$$

$$= \frac{2ad\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2bd\sqrt{}}{}$$

$$= \frac{2\sqrt{2}\,a^2d^2\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}}{(-a+b)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+fx)}} - $$

Mathematica [C] time = 23.87, size = 1648, normalized size = 4.35

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^(3/2)/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x
]))),x]
```

```
[Out] (2*Cot[e + f*x]*(d*Sin[e + f*x])^(3/2)*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f
*(g*Cos[e + f*x])^(3/2)) - (Cos[e + f*x]^(3/2)*(d*Sin[e + f*x])^(3/2)*((4*a
*b*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Si
n[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e +
f*x])) + ((a^2 - b^2)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 -
 (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sq
rt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqr
t[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] +
 Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 -
b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Ta
n[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan
[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Si
n[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + (Cos[2*(e +
f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*
b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)
*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2,
1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(7/2)
 + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4
*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*Ar
cTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^
2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
+ f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(
a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[
2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/
(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] +
Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[
e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[
Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]
*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a
^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]
*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x
]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1
/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) +
 (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a^2*Cos[e
```

```
 + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)
*Sqrt[1 + Tan[e + f*x]^2])))/((a - b)*(a + b)*f*(g*Cos[e + f*x])^(3/2)*Sin[
e + f*x]^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^(3/2)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) +
a)), x)
```

maple [B] time = 0.88, size = 2540, normalized size = 6.70

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/f*(-(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a
-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a+2*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+2*(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b-4*(-a^2+b^2)^(1/2)*cos(f*x+e)
```

*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b-(-a^2+b^2)^(1/2)*cos(f*x
+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/
2))*a+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2
)),1/2*2^(1/2))*a^2+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(
-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2-cos(f*x+e)*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b-(-a^2+b^2)^
(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1
/2))*a+2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a
+2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*El
lipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b-4*(-a
^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
E((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b-(-a^2+b^2)^
(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(
1/2))*a+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^
(1/2))*a^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2
*2^(1/2))*a*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
Pi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a)
,1/2*2^(1/2))*a^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli

```
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2
)-a),1/2*2^(1/2))*a*b+2*sin(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*a+2*(-a^2+b^2)^
(1/2)*cos(f*x+e)*2^(1/2)*b-2*(-a^2+b^2)^(1/2)*2^(1/2)*b)*(d*sin(f*x+e))^(3/
2)*cos(f*x+e)/sin(f*x+e)^2/(g*cos(f*x+e))^(3/2)*2^(1/2)*a/(a+b)/(-a^2+b^2)^
(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^(3/2)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) +
a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^(3/2)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((d*sin(e + f*x))^(3/2)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(3/2)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1438 $\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=374

$$-\frac{2aE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{2a(d\sin(e+fx))^{3/2}}{dfg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b\sqrt{d\sin(e+fx)}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+f}}$$

```
[Out] 2*a*(d*sin(f*x+e))^(3/2)/(a^2-b^2)/d/f/g/(g*cos(f*x+e))^(1/2)-2*a*b*d*Ellip
ticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)
^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/(-a+b)^(3/2)/(a+b)^(3/2)/f/g^(3/2)/(d*si
n(f*x+e))^(1/2)+2*a*b*d*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+
e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/(-a+b)^(3/2)
/(a+b)^(3/2)/f/g^(3/2)/(d*sin(f*x+e))^(1/2)-2*b*(d*sin(f*x+e))^(1/2)/(a^2-b
^2)/f/g/(g*cos(f*x+e))^(1/2)+2*a*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f
*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e)
)^(1/2)/(a^2-b^2)/f/g^2/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 0.91, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {2903, 2838, 2563, 2571, 2572, 2639, 2906, 2905, 490, 1218}

$$-\frac{2aE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{2a(d\sin(e+fx))^{3/2}}{dfg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b\sqrt{d\sin(e+fx)}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sin[e + f*x]]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*a*b*d*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos
[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a
+ b)^(3/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a*b*d
*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*
Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b)^(3/2)*(a + b)^(
3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*b*Sqrt[d*Sin[e + f*x]])/((a^2 - b
^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*(d*Sin[e + f*x])^(3/2))/((a^2 - b^2)*d
*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 +
 f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*f*g^2*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
```

```
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2903

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Dist[d/(a^2 - b^
2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1)*(b - a*Sin[e + f*x]), x
], x] + Dist[(a*b*d)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Si
n[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f,
g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n,
0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = -\frac{d\int \frac{b-a\sin(e+fx)}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{a^2-b^2} + \frac{(abd)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx}{\left(a^2-b^2\right)g^2}$$

$$= \frac{a\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{(bd)\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{a^2-b^2} + \frac{\left(abd\sqrt{\sin\right.}}{}$$

$$= -\frac{2b\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2a(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}} - \frac{(2a}{}$$

$$= -\frac{2b\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2a(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}} - \frac{(2}{}$$

$$= -\frac{2\sqrt{2}\,abd\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}}{(-a+b)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+fx)}} +$$

Mathematica [C] time = 23.12, size = 1274, normalized size = 3.41

$$\frac{a\sqrt{d\sin(e+fx)}\left(\frac{4a\left(aF_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-bF_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)\right)\cos^{\frac{3}{2}}(e+fx)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\sin^{\frac{3}{2}}(}{3\left(a^2-b^2\right)\left(1-\cos^2(e+fx)\right)^{3/4}(a+b\sin(e+fx))}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Sin[e + f*x]]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (2*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]]*(-b + a*Sin[e + f*x]))/((a^2 - b^2)*f*
(g*Cos[e + f*x])^(3/2)) + (a*Cos[e + f*x]^(3/2)*Sqrt[d*Sin[e + f*x]]*((4*a*
(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
```

```
 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e
 + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x
])) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan
[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]
^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*A
ppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*
Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[
Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] -
 (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/S
qrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)
^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*Arc
Tan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2
)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
 f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqr
t[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Ta
n[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2
)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a
^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2
- b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt
[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(
a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2
])))/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(
-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/((a - b)*(a + b)*f*(g*Cos[
e + f*x])^(3/2)*Sqrt[Sin[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(fx+e\right)}}{\left(g \cos\left(fx+e\right)\right)^{\frac{3}{2}} \left(b \sin\left(fx+e\right)+a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e))/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)
), x)
```

maple [B] time = 0.89, size = 2536, normalized size = 6.78

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/f*(cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2
)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-2*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-2*(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b+4*cos(f*x+e)*EllipticE((-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*a+cos(f*x+e)*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/
2)*b-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)
),1/2*2^(1/2))*a*b-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-
a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b+cos(f*x+e)*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2+(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1
```

```
/2)*b-2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-
2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b+4*Elli
pticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*a+(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(
1/2)*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(
1/2))*a*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*
2^(1/2))*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),
1/2*2^(1/2))*a*b+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)
-a),1/2*2^(1/2))*b^2-2*cos(f*x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*a-2*sin(f*x+e)*(
-a^2+b^2)^(1/2)*2^(1/2)*b+2*(-a^2+b^2)^(1/2)*2^(1/2)*a)*(d*sin(f*x+e))^(1/2
)*cos(f*x+e)/(g*cos(f*x+e))^(3/2)/sin(f*x+e)*2^(1/2)*a/(a+b)/(-a^2+b^2)^(1/
2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin \left(f x + e\right)}}{\left(g \cos \left(f x + e\right)\right)^{\frac{3}{2}} \left(b \sin \left(f x + e\right) + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e))/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,\sin\left(e+f\,x\right)}}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^(1/2)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((d*sin(e + f*x))^(1/2)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin{\left(e + f x \right)}}}{\left(g \cos{\left(e + f x \right)}\right)^{\frac{3}{2}} \left(a + b \sin{\left(e + f x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(1/2)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(d*sin(e + f*x))/((g*cos(e + f*x))**(3/2)*(a + b*sin(e + f*x))
), x)
```

3.1439 $\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=380

$$-\frac{2b(d\sin(e+fx))^{3/2}}{d^2fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{2bE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{dfg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{2a\sqrt{d\sin(e+fx)}}{dfg\left(a^2-b^2\right)\sqrt{g\cos(e-}}$$

```
[Out] -2*b*(d*sin(f*x+e))^(3/2)/(a^2-b^2)/d^2/f/g/(g*cos(f*x+e))^(1/2)+2*b^2*Elli
pticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b
)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/(-a+b)^(3/2)/(a+b)^(3/2)/f/g^(3/2)/(d*s
in(f*x+e))^(1/2)-2*b^2*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e
))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/(-a+b)^(3/2)/
(a+b)^(3/2)/f/g^(3/2)/(d*sin(f*x+e))^(1/2)+2*a*(d*sin(f*x+e))^(1/2)/(a^2-b^
2)/d/f/g/(g*cos(f*x+e))^(1/2)-2*b*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+
f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e
))^(1/2)/(a^2-b^2)/d/f/g^2/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 0.92, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {2904, 2838, 2563, 2571, 2572, 2639, 2906, 2905, 490, 1218}

$$-\frac{2b(d\sin(e+fx))^{3/2}}{d^2fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{2bE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{dfg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{2a\sqrt{d\sin(e+fx)}}{dfg\left(a^2-b^2\right)\sqrt{g\cos(e-}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[2]*b^2*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e
+ f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b
)^(3/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^2*Elli
pticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[
1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b)^(3/2)*(a + b)^(3/2)*
f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*a*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d
*f*g*Sqrt[g*Cos[e + f*x]]) - (2*b*(d*Sin[e + f*x])^(3/2))/((a^2 - b^2)*d^2*
f*g*Sqrt[g*Cos[e + f*x]]) + (2*b*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 +
f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d*f*g^2*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
```

```
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2904

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 - b^2
), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[b^2/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e + f*x])
^n)/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\int \frac{a-b\sin(e+fx)}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{a^2-b^2} - \frac{b^2\int \frac{\sqrt{g\cos(e+fx}}{\sqrt{d\sin(e+fx)}\,(a+b\sin}}{\left(a^2-b^2\right)g^2}$$

$$= \frac{a\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}}\,dx}{\left(a^2-b^2\right)d}$$

$$= \frac{2a\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin(e+f}{\left(a^2-b^2\right)d^2fg\sqrt{g\cos}}$$

$$= \frac{2a\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin(e+f}{\left(a^2-b^2\right)d^2fg\sqrt{g\cos}}$$

$$= \frac{2\sqrt{2}\,b^2\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\sin}}{(-a+b)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+fx)}}$$

Mathematica [C] time = 21.81, size = 1279, normalized size = 3.37

$$\frac{b\sqrt{\sin(e+fx)}\left(\frac{4a\left(aF_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-bF_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)\right)\cos^{\frac{3}{2}}(e+fx)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\sin^{\frac{3}{2}}(e+}{3\left(a^2-b^2\right)\left(1-\cos^2(e+fx)\right)^{3/4}(a+b\sin(e+fx))}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x
])),x]
```

```
[Out] (2*Cos[e + f*x]*Sin[e + f*x]*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f*(g*Cos[e
+ f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]) + (b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*
x]]*((4*a*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x
```

```
]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]
^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*S
in[e + f*x])) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sq
rt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Ta
n[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^
2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e
+ f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqr
t[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e
+ f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
+ f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(
a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[
2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/
(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - S
qrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[
2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqr
t[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/
(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)
*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2
*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] +
 Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sq
rt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e
+ f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[
e + f*x]^2])))/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e
 + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/((-a + b)*(a + b
)*f*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e
))), x)
```

maple [B] time = 0.72, size = 2559, normalized size = 6.73

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/f*(2*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*
2^(1/2))*a*b+2*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),1/2*2^(1/2))*b^2-4*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2*2^(1/2))*a*b-cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2+cos(f*x+e)*(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b^2+c
os(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*
2^(1/2))*b^3-cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2-cos(f*x+e)*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2-cos(f*x+e)*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^
3+2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b+2*
```

```
(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b^2-4*(-a^
2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b-(-a^2+b^2)
^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(
1/2))*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*
2^(1/2))*a*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2))
,1/2*2^(1/2))*b^3-(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(
b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2-(-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3+2*cos(f*x+e)*(-a^2+b^2)^(1
/2)*2^(1/2)*a*b+2*2^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a^2-2*(-a^2+b^2)^(1/2
)*2^(1/2)*a*b)*cos(f*x+e)/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)*2^(1/2)
/(a+b)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))),
x)
```

```
[Out] int(1/((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d \sin \left(e + f x\right)} \left(g \cos \left(e + f x\right)\right)^{\frac{3}{2}} \left(a + b \sin \left(e + f x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(1/(sqrt(d*sin(e + f*x))*(g*cos(e + f*x))**(3/2)*(a + b*sin(e + f*x
))), x)
```

3.1440 $\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=568

$$\frac{4a(d\sin(e+fx))^{3/2}}{d^3fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{2b^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{ad^2fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}-\frac{4aE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\,\mathrm{s}}}{d^2fg^2\left(a^2-b^2\right)}$$

```
[Out] 4*a*(d*sin(f*x+e))^(3/2)/(a^2-b^2)/d^3/f/g/(g*cos(f*x+e))^(1/2)+2*b^2*(g*co
s(f*x+e))^(3/2)/a/(a^2-b^2)/d/f/g^3/(d*sin(f*x+e))^(1/2)-2*a/(a^2-b^2)/d/f/
g/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)-2*b^3*EllipticPi((g*cos(f*x+e))
^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*si
n(f*x+e)^(1/2)/a/(-a+b)^(3/2)/(a+b)^(3/2)/d/f/g^(3/2)/(d*sin(f*x+e))^(1/2)+
2*b^3*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(
1/2)/(a+b)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/a/(-a+b)^(3/2)/(a+b)^(3/2)/d/f
/g^(3/2)/(d*sin(f*x+e))^(1/2)-2*b*(d*sin(f*x+e))^(1/2)/(a^2-b^2)/d^2/f/g/(g
*cos(f*x+e))^(1/2)+4*a*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Ellipt
icE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/(a
^2-b^2)/d^2/f/g^2/sin(2*f*x+2*e)^(1/2)-2*b^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/si
n(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(
d*sin(f*x+e))^(1/2)/a/(a^2-b^2)/d^2/f/g^2/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 1.41, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2904, 2838, 2570, 2571, 2572, 2639, 2563, 2910, 2906, 2905, 490, 1218}

$$\frac{2b^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{ad^2fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}-\frac{4aE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{d^2fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}-\frac{}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),
x]
```

```
[Out] (-2*a)/((a^2 - b^2)*d*f*g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]) + (2*b
^2*(g*Cos[e + f*x])^(3/2))/(a*(a^2 - b^2)*d*f*g^3*Sqrt[d*Sin[e + f*x]]) - (
2*Sqrt[2]*b^3*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e +
 f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*(-a +
b)^(3/2)*(a + b)^(3/2)*d*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^3*E
llipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sq
rt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*(-a + b)^(3/2)*(a + b)^(
```

```
3/2)*d*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*b*Sqrt[d*Sin[e + f*x]])/((a^2 -
 b^2)*d^2*f*g*Sqrt[g*Cos[e + f*x]]) + (4*a*(d*Sin[e + f*x])^(3/2))/((a^2 -
b^2)*d^3*f*g*Sqrt[g*Cos[e + f*x]]) - (4*a*Sqrt[g*Cos[e + f*x]]*EllipticE[e
- Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d^2*f*g^2*Sqrt[Sin[2*e
+ 2*f*x]]) + (2*b^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[
d*Sin[e + f*x]])/(a*(a^2 - b^2)*d^2*f*g^2*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2904

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 - b^2
), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[b^2/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e + f*x])
^n)/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
```

```
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{a-b\sin(e+fx)}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b^2\int \frac{\sqrt{g\cos(}}{(d\sin(e+fx))^{3/2}(}}{\left(a^2-b^2\right.}$$

$$= \frac{a\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{1}{(g\cos(e+fx))^{3/2}}}{\left(a^2-b\right.}$$

$$= -\frac{2a}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}} + \frac{}{a}$$

$$= -\frac{2a}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}} + \frac{}{a}$$

$$= -\frac{2a}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}} + \frac{}{a}$$

$$= -\frac{2a}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}} + \frac{}{a}$$

Mathematica [C] time = 25.01, size = 1707, normalized size = 3.01

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f
*x])),x]
```

```
[Out] (Cos[e + f*x]^2*Sin[e + f*x]^2*((-2*Cot[e + f*x])/a + (2*Sec[e + f*x]*(-b +
 a*Sin[e + f*x]))/(a^2 - b^2)))/(f*(g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^
```

```
(3/2)) - (Cos[e + f*x]^(3/2)*Sin[e + f*x]^(3/2)*((-2*(4*a^3 - 2*a*b^2)*(-(b
*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b
^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f
*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x]))
+ ((2*a^2*b - 2*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 -
(Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqr
t[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt
[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] +
Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b
^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan
[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[
e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin
[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-2*a^2*b +
b^3)*Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e
 + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2
, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*App
ellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Ta
n[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Ta
n[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (
4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqr
t[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(
1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTa
n[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^
(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f
*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[
Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[
e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 -
b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^
(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
+ f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2
*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 -
b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a
]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^
2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])
))/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(
-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(a*(a - b)*(a + b)*f*(g*Co
s[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, a
```

```
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(
3/2)), x)
```

maple [B] time = 0.72, size = 3104, normalized size = 5.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*(-2*(-a^2+b^2)^(1/2)*2^(1/2)*a^3+4*(-a^2+b^2)^(1/2)*cos(f*x+e)*2^(1/2)*
a^3-2*(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^
(1/2))*a*b^2+4*(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),1/2*2^(1/2))*a*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*
EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)
^(1/2)),1/2*2^(1/2))*a*b^3-cos(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^4+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^4-(-a^2+b^2)^(1/2)*El
lipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1
/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^
3-(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
```

f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3+4*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a^3+4*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b^2+2*sin(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*a^2*b-(-a^2+b^2)^(1/2)*cos(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^3-(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3+4*(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a^3+2*(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b^3-8*(-a^2+b^2)^(1/2)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a^3-cos(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b^3+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b^3-2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b^2-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^4+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^4-2*(-a^2+b^2)^(1/2)*cos(f*x+e)*2^(1/2)*a*b^2+2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b^3-8*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x

```
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a^3-Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a
),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b^3)*
cos(f*x+e)*sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)*2^(1/2)/(a+
b)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(
3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(d\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))),
x)
```

```
[Out] int(1/((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1441 $\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=673

$$\frac{2\sqrt{2}\,b^4\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^2d^2fg^{3/2}(b-a)^{3/2}(a+b)^{3/2}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,b^4\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+f}}{\sqrt{g}\,\sqrt{\sin(e+f}}\right.\right.}{a^2d^2fg^{3/2}(b-a)^{3/2}(a+b)^{3/2}\sqrt{d\sin(e+f}}$$

```
[Out] 2/3*b^2*(g*cos(f*x+e))^(3/2)/a/(a^2-b^2)/d/f/g^3/(d*sin(f*x+e))^(3/2)-2/3*a
/(a^2-b^2)/d/f/g/(d*sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(1/2)-4*b*(d*sin(f*x+e
))^(3/2)/(a^2-b^2)/d^4/f/g/(g*cos(f*x+e))^(1/2)-2*b^3*(g*cos(f*x+e))^(3/2)/
a^2/(a^2-b^2)/d^2/f/g^3/(d*sin(f*x+e))^(1/2)+2*b/(a^2-b^2)/d^2/f/g/(g*cos(f
*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)+2*b^4*EllipticPi((g*cos(f*x+e))^(1/2)/g^(
1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*sin(f*x+e)^(
1/2)/a^2/(-a+b)^(3/2)/(a+b)^(3/2)/d^2/f/g^(3/2)/(d*sin(f*x+e))^(1/2)-2*b^4*
EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(
a+b)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/a^2/(-a+b)^(3/2)/(a+b)^(3/2)/d^2/f/g
^(3/2)/(d*sin(f*x+e))^(1/2)+8/3*a*(d*sin(f*x+e))^(1/2)/(a^2-b^2)/d^3/f/g/(g
*cos(f*x+e))^(1/2)-4*b*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Ellipt
icE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/(a
^2-b^2)/d^3/f/g^2/sin(2*f*x+2*e)^(1/2)+2*b^3*(sin(e+1/4*Pi+f*x)^2)^(1/2)/si
n(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(
d*sin(f*x+e))^(1/2)/a^2/(a^2-b^2)/d^3/f/g^2/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 1.81, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2904, 2838, 2570, 2563, 2571, 2572, 2639, 2910, 2906, 2905, 490, 1218}

$$-\frac{2b^3(g\cos(e+fx))^{3/2}}{a^2d^2fg^3\left(a^2-b^2\right)\sqrt{d\sin(e+fx)}}-\frac{2b^3E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{a^2d^3fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{4bE\left(e+fx-\frac{\pi}{4}\middle|2\right)}{d^3fg^2\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),
x]
```

```
[Out] (-2*a)/(3*(a^2 - b^2)*d*f*g*Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2)) +
(2*b^2*(g*Cos[e + f*x])^(3/2))/(3*a*(a^2 - b^2)*d*f*g^3*(d*Sin[e + f*x])^(3
/2)) + (2*b)/((a^2 - b^2)*d^2*f*g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]
) - (2*b^3*(g*Cos[e + f*x])^(3/2))/(a^2*(a^2 - b^2)*d^2*f*g^3*Sqrt[d*Sin[e
+ f*x]]) + (2*Sqrt[2]*b^4*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sq
```

```
rt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]
])/(a^2*(-a + b)^(3/2)*(a + b)^(3/2)*d^2*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) -
(2*Sqrt[2]*b^4*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f
*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*(-a +
b)^(3/2)*(a + b)^(3/2)*d^2*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (8*a*Sqrt[d*Si
n[e + f*x]])/(3*(a^2 - b^2)*d^3*f*g*Sqrt[g*Cos[e + f*x]]) - (4*b*(d*Sin[e +
 f*x])^(3/2))/((a^2 - b^2)*d^4*f*g*Sqrt[g*Cos[e + f*x]]) + (4*b*Sqrt[g*Cos[
e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d
^3*f*g^2*Sqrt[Sin[2*e + 2*f*x]]) - (2*b^3*Sqrt[g*Cos[e + f*x]]*EllipticE[e
- Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^2*(a^2 - b^2)*d^3*f*g^2*Sqrt[Sin[
2*e + 2*f*x]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
```

```
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2904

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 - b^2
), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[b^2/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e + f*x])
^n)/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
```

```
0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{a-b\sin(e+fx)}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{b^2\int \frac{\sqrt{g\cos(\ldots}}{(d\sin(e+fx))^{5/2}(\ldots}}{\left(a^2-b^2\ldots\right.}$$

$$= \frac{a\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{1}{(g\cos(e+fx))^{3/2}\ldots}}{\left(a^2-\ldots\right.}$$

$$= -\frac{2a}{3\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}} \ldots$$

$$= -\frac{2a}{3\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}} \ldots$$

$$= -\frac{2a}{3\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}} \ldots$$

$$= -\frac{2a}{3\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}} \ldots$$

$$= -\frac{2a}{3\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}} \ldots$$

Mathematica [C] time = 24.37, size = 1727, normalized size = 2.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f
*x])),x]
```

```
[Out] (Cos[e + f*x]^2*Sin[e + f*x]^3*((2*b*Cot[e + f*x])/a^2 - (2*Cot[e + f*x]*Cs
c[e + f*x])/(3*a) + (2*Sec[e + f*x]*(a - b*Sin[e + f*x]))/(a^2 - b^2)))/(f*
(g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(5/2)) - (b*Cos[e + f*x]^(3/2)*Sin[
e + f*x]^(5/2)*((-2*(4*a^3 - 2*a*b^2)*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*
(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Co
s[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((2*a^2*b - 2*b^3)*Sqrt[Tan[e +
 f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[T
an[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
 + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
+ f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^
2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(
1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e
+ f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x
]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(
1 + Tan[e + f*x]^2)^(3/2)) + ((-2*a^2*b + b^3)*Cos[2*(e + f*x)]*Sqrt[Tan[e
+ f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*
AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*
Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e +
 f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*
Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*
ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]
*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqr
t[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]]
)/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*
Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 +
 (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4)
 + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2
*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sq
rt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - S
qrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqr
t[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e +
 f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b
^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(
1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
```

```
f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan
[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a^2*b^2*Cos[e + f*x]^(3/2)
*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan
[e + f*x]^2])))/(a^2*(-a + b)*(a + b)*f*(g*Cos[e + f*x])^(3/2)*(d*Sin[e + f
*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(
5/2)), x)
```

maple [B] time = 0.78, size = 3315, normalized size = 4.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/3/f*(-6*(-a^2+b^2)^(1/2)*2^(1/2)*a^4+3*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(
1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1
/2))*b^4-6*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2*2^(1/2))*b^4+3*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+c
```

os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^4+3*co
s(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*El
lipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1
/2)-a),1/2*2^(1/2))*a*b^4-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b^4+24*sin(f*x+e)*(-a^2+
b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a^3*b-12*sin(f*x
+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b^3-
12*sin(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/
2))*a^3*b+6*sin(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
1/2*2^(1/2))*a*b^3+8*(-a^2+b^2)^(1/2)*cos(f*x+e)^2*2^(1/2)*a^4+6*(-a^2+b^2)
^(1/2)*sin(f*x+e)*2^(1/2)*a^3*b-12*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*2
^(1/2)*a^3*b+6*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*a*b^3+3*sin(f
x+e)(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(
1/2))*b^5-3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2
)^(1/2)),1/2*2^(1/2))*b^5+3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^5-3*cos(f*x+e)*sin(f*x+
e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2)
)*b^5+3*sin(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^4-6*sin(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f
x+e))^(1/2)((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b^4+3*sin(f*x+e)*(-a^2+b^2)^(1/2

```
)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))
*b^4+3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2
)-a),1/2*2^(1/2))*a*b^4-3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/
(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b^4-2*cos(f*x+e)^2*(-a^2+b^2)^(1/2)*2
^(1/2)*a^2*b^2+24*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),1/2*2^(1/2))*a^3*b-12*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/
2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b^3-12*cos(f*x+e)*sin(
f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a^3
*b+6*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),1/2*2^(1/2))*a*b^3)*cos(f*x+e)*sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(d*sin(f
*x+e))^(5/2)*2^(1/2)/(a+b)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2
+b^2)^(1/2)-a)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(
5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(d\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))),
x)
```

```
[Out] int(1/((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1442 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=331

$$\frac{\sqrt{2}\,bg^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2\sqrt{d}\,f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}-\frac{\sqrt{2}\,bg^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{\ldots}}{\sqrt{d}\ldots}\right.\right.}{a^2\sqrt{d}\,f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+f\ldots}}$$

```
[Out] b*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(-
a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a^2/f/(-a^2+b^2)^(1/2)/d^(1/2)/
(g*cos(f*x+e))^(1/2)-b*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f
*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a^2/f/(-a^
2+b^2)^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2)+g*(g*cos(f*x+e))^(1/2)*(d*sin(f*x
+e))^(1/2)/a/d/f/(a+b*sin(f*x+e))-1/2*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e
+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^2/
f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.79, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2887, 2910, 2573, 2641, 2908, 2907, 1218}

$$\frac{\sqrt{2}\,bg^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2\sqrt{d}\,f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}-\frac{\sqrt{2}\,bg^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{\ldots}}{\sqrt{d}\ldots}\right.\right.}{a^2\sqrt{d}\,f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+f\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^2),x]
```

```
[Out] (Sqrt[2]*b*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), A
rcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sqr
t[-a^2 + b^2]*Sqrt[d]*f*Sqrt[g*Cos[e + f*x]]) - (Sqrt[2]*b*g^2*Sqrt[Cos[e +
 f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/
(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sqrt[-a^2 + b^2]*Sqrt[d]*f*Sqr
t[g*Cos[e + f*x]]) + (g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(a*d*f*(
a + b*Sin[e + f*x])) + (g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f
*x]])/(2*a^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2887

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> -Simp[(g*(g*C
os[e + f*x])^(p - 1)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(m + 1))/(a*
d*f*(m + 1)), x] + Dist[(g^2*(2*m + 3))/(2*a*(m + 1)), Int[((g*Cos[e + f*x]
)^(p - 2)*(a + b*Sin[e + f*x])^(m + 1))/Sqrt[d*Sin[e + f*x]], x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && EqQ[m + p +
 1/2, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
```

```
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^2}\,dx = \frac{g\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{adf(a+b\sin(e+fx))} + \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}\,(a+b\sin(}}{2a}$$

$$= \frac{g\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{adf(a+b\sin(e+fx))} + \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}\,dx}{2a^2} - $$

$$= \frac{g\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{adf(a+b\sin(e+fx))} - \frac{\left(bg^2\sqrt{\cos(e+fx)}\right)\int \frac{\sqrt{d\sin(}}{\sqrt{\cos(e+fx)}\,(a}}{2a^2d\sqrt{g\cos(e+fx)}}$$

$$= \frac{g\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{adf(a+b\sin(e+fx))} + \frac{g^2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(2e+2f}}{2a^2f\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+f}}$$

$$= \frac{\sqrt{2}\,bg^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{a^2\sqrt{-a^2+b^2}\,\sqrt{d}\,f\sqrt{g\cos(e+fx)}} - $$

Mathematica [C] time = 16.09, size = 717, normalized size = 2.17

$$\frac{\tan(e+fx)(g\cos(e+fx))^{3/2}}{af\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))} - \frac{\sin(e+fx)(g\cos(e+fx))^{3/2}\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(\frac{}{\left(1-\cos^2(e+fx)\right)^{3/4}\left(a^2+\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])
^2),x]
```

```
[Out] -(((g*Cos[e + f*x])^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2
)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b
^2)]*Sqrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(5*(a^2 - b^2)*AppellF
```

```
1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-
4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Co
s[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^
2))) - ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((
-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*ArcTan[1 + ((1 + I)*Sqr
t[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))]
+ Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] - ((1
 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)^
(1/4)] - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2
] + ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f
*x]^2)^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(a*f*Cos[e + f*
x]^(3/2)*(1 - Cos[e + f*x]^2)^(1/4)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x
]))) + ((g*Cos[e + f*x])^(3/2)*Tan[e + f*x])/(a*f*Sqrt[d*Sin[e + f*x]]*(a +
 b*Sin[e + f*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2/(d*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{2}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2/(d*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e
))), x)
```

maple [B] time = 1.43, size = 3490, normalized size = 10.54

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2/(d*sin(f*x+e))^(1/2),x)

[Out] -1/2/f*(2*(-a^2+b^2)^(1/2)*cos(f*x+e)^2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b+2*(-a^2+b^2)^(1/2)*cos(f*x+e)^2*2^(1/2)*a^2-sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2*b+sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2*b-2*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2-2*cos(f*x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*a^2-sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*a*b-sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b^2-2*cos(f*x+e)^2*(-a^2+b^2)^(1/2)*2^(1/2)*a*b+2*cos(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*a*b+sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b^2-sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2+2*(-a^2+b^2)^(1/2)*sin(f*x+e)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3-(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f

```
*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2-(-a^2+b^2)^(1/2)*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2
+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))
*a*b^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(
1/2))*a*b^2-cos(f*x+e)^2*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*b^3+cos(f*x+e)^2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3-2*(-a^2+b^2)^(1/2)*(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b+(-a^2+b^2)^(1/2)*cos(f*x+e)^2*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/
2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^2+(
-a^2+b^2)^(1/2)*cos(f*x+e)^2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^
2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2-2*(-a^2+b^2)^(1/2)*cos(f*x+e)^2*(-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b^2+cos(f*x+e)^2*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))
*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b^2-cos(f*x+e)^2*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*
b^2)*(g*cos(f*x+e))^(3/2)*sin(f*x+e)/(a+b*sin(f*x+e))/(-1+cos(f*x+e))/(d*si
n(f*x+e))^(1/2)/cos(f*x+e)^2*2^(1/2)/a/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/
2))/(b+(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g \cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b \sin\left(fx+e\right)+a\right)^{2} \sqrt{d \sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2/(d*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^2),
x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e))**2/(d*sin(f*x+e))**(1/2),x
)
```

```
[Out] Timed out
```

3.1443 $\int \sin^2(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=82

$$\frac{3a\tan(c+dx)}{2d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}-\frac{b\cos^3(c+dx)}{3d}+\frac{2b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] -3/2*a*x+2*b*cos(d*x+c)/d-1/3*b*cos(d*x+c)^3/d+b*sec(d*x+c)/d+3/2*a*tan(d*x
+c)/d-1/2*a*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.14, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2591, 288, 321, 203, 2590, 270}

$$\frac{3a\tan(c+dx)}{2d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}-\frac{b\cos^3(c+dx)}{3d}+\frac{2b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*x)/2 + (2*b*Cos[c + d*x])/d - (b*Cos[c + d*x]^3)/(3*d) + (b*Sec[c + d
*x])/d + (3*a*Tan[c + d*x])/(2*d) - (a*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sin^2(c+dx)\tan^2(c+dx)\,dx + b\int \sin^3(c+dx)\tan^2(c+$$

$$= \frac{a\,\mathrm{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d} - \frac{b\,\mathrm{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2}\,dx\right.}{d}$$

$$= -\frac{a\sin^2(c+dx)\tan(c+dx)}{2d} + \frac{(3a)\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(c\right.}{2d}$$

$$= \frac{2b\cos(c+dx)}{d} - \frac{b\cos^3(c+dx)}{3d} + \frac{b\sec(c+dx)}{d} + \frac{3a\tan(c-}{2d}$$

$$= -\frac{3ax}{2} + \frac{2b\cos(c+dx)}{d} - \frac{b\cos^3(c+dx)}{3d} + \frac{b\sec(c+dx)}{d} + \frac{3}{}$$

Mathematica [A]   time = 0.42, size = 82, normalized size = 1.00

$$-\frac{3a(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\tan(c+dx)}{d}+\frac{7b\cos(c+dx)}{4d}-\frac{b\cos(3(c+dx))}{12d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*(c + d*x))/(2*d) + (7*b*Cos[c + d*x])/(4*d) - (b*Cos[3*(c + d*x)])/(1
2*d) + (b*Sec[c + d*x])/d + (a*Sin[2*(c + d*x)])/(4*d) + (a*Tan[c + d*x])/d
```

fricas [A]   time = 0.71, size = 72, normalized size = 0.88

$$-\frac{2\,b\cos\left(dx+c\right)^{4}+9\,adx\cos\left(dx+c\right)-12\,b\cos\left(dx+c\right)^{2}-3\left(a\cos\left(dx+c\right)^{2}+2\,a\right)\sin\left(dx+c\right)-6\,b}{6\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*cos(d*x + c)^4 + 9*a*d*x*cos(d*x + c) - 12*b*cos(d*x + c)^2 - 3*(
a*cos(d*x + c)^2 + 2*a)*sin(d*x + c) - 6*b)/(d*cos(d*x + c))
```

giac [A]   time = 0.21, size = 119, normalized size = 1.45

$$-\frac{9\left(dx+c\right)a+\frac{12\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-10\,b\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)*a + 12*(a*tan(1/2*d*x + 1/2*c) + b)/(tan(1/2*d*x + 1/2*c)
^2 - 1) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^4 - 24*b
*tan(1/2*d*x + 1/2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) - 10*b)/(tan(1/2*d*x + 1
/2*c)^2 + 1)^3)/d
```

maple [A]   time = 0.41, size = 104, normalized size = 1.27

$$\frac{a\left(\frac{\sin^{5}(dx+c)}{\cos(dx+c)}+\left(\sin^{3}\left(dx+c\right)+\frac{3\sin(dx+c)}{2}\right)\cos\left(dx+c\right)-\frac{3dx}{2}-\frac{3c}{2}\right)+b\left(\frac{\sin^{6}(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^{4}\left(dx+c\right)+\frac{4\left(\sin^{2}(dx+\right.}{3}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/
2*d*x-3/2*c)+b*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)
*cos(d*x+c)))
```

maxima [A] time = 0.44, size = 75, normalized size = 0.91

$$-\frac{3\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)a+2\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)b}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*
a + 2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*b)/d
```

mupad [B] time = 18.22, size = 112, normalized size = 1.37

$$-\frac{3\,a\,x}{2}-\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{32\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{16\,b}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + b*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] - (3*a*x)/2 - ((16*b)/3 + 3*a*tan(c/2 + (d*x)/2) + 5*a*tan(c/2 + (d*x)/2)^3
 + 5*a*tan(c/2 + (d*x)/2)^5 + 3*a*tan(c/2 + (d*x)/2)^7 + (32*b*tan(c/2 + (d
*x)/2)^2)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1444 $\int \sin(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=65

$$\frac{a\cos(c+dx)}{d}+\frac{a\sec(c+dx)}{d}+\frac{3b\tan(c+dx)}{2d}-\frac{b\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3bx}{2}$$

```
[Out] -3/2*b*x+a*cos(d*x+c)/d+a*sec(d*x+c)/d+3/2*b*tan(d*x+c)/d-1/2*b*sin(d*x+c)^
2*tan(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2838, 2590, 14, 2591, 288, 321, 203}

$$\frac{a\cos(c+dx)}{d}+\frac{a\sec(c+dx)}{d}+\frac{3b\tan(c+dx)}{2d}-\frac{b\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*b*x)/2 + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (3*b*Tan[c + d*x])/(
2*d) - (b*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sin(c+dx)\tan^2(c+dx)\,dx + b\int \sin^2(c+dx)\tan^2(c+d$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x\right.}{d}$$

$$= -\frac{b\sin^2(c+dx)\tan(c+dx)}{2d} - \frac{a\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, \cos\right.}{d}$$

$$= \frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{3b\tan(c+dx)}{2d} - \frac{b\sin^2(c+d}{}$$

$$= -\frac{3bx}{2} + \frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{3b\tan(c+dx)}{2d} - \frac{b\,\text{s}}{}$$

Mathematica [A] time = 0.19, size = 63, normalized size = 0.97

$$\frac{a\cos(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{3b(c+dx)}{2d}+\frac{b\sin(2(c+dx))}{4d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*b*(c + d*x))/(2*d) + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (b*Sin[2
*(c + d*x)])/(4*d) + (b*Tan[c + d*x])/d
```

fricas [A] time = 0.78, size = 61, normalized size = 0.94

$$-\frac{3\,bdx\cos\left(dx+c\right)-2\,a\cos\left(dx+c\right)^{2}-\left(b\cos\left(dx+c\right)^{2}+2\,b\right)\sin\left(dx+c\right)-2\,a}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(3*b*d*x*cos(d*x + c) - 2*a*cos(d*x + c)^2 - (b*cos(d*x + c)^2 + 2*b)*
sin(d*x + c) - 2*a)/(d*cos(d*x + c))
```

giac [A] time = 0.18, size = 104, normalized size = 1.60

$$-\frac{3\,(dx+c)b+\frac{4\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(3*(d*x + c)*b + 4*(b*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2*d*x + 1/2*c)^
2 - 1) + 2*(b*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1
/2*d*x + 1/2*c) - 2*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.40, size = 94, normalized size = 1.45

$$\frac{a\left(\frac{\sin^{4}(dx+c)}{\cos(dx+c)}+\left(2+\sin^{2}\left(dx+c\right)\right)\cos\left(dx+c\right)\right)+b\left(\frac{\sin^{5}(dx+c)}{\cos(dx+c)}+\left(\sin^{3}\left(dx+c\right)+\frac{3\sin(dx+c)}{2}\right)\cos\left(dx+c\right)-\frac{3dx}{2}-\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+b*(sin(d*x+c)^
5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c))
```

maxima [A] time = 0.42, size = 62, normalized size = 0.95

$$-\frac{\left(3\,dx+3\,c-\frac{\tan\left(dx+c\right)}{\tan\left(dx+c\right)^{2}+1}-2\,\tan\left(dx+c\right)\right)b-2\,a\left(\frac{1}{\cos\left(dx+c\right)}+\cos\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*b
- 2*a*(1/cos(d*x + c) + cos(d*x + c)))/d
```

mupad [B] time = 15.97, size = 98, normalized size = 1.51

$$-\frac{3\,b\,x}{2}-\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,a}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + b*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] - (3*b*x)/2 - (4*a + 3*b*tan(c/2 + (d*x)/2) + 4*a*tan(c/2 + (d*x)/2)^2 + 2*
b*tan(c/2 + (d*x)/2)^3 + 3*b*tan(c/2 + (d*x)/2)^5)/(d*(tan(c/2 + (d*x)/2)^2
 - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)\sin^{3}\left(c+dx\right)\sec^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sin(c + d*x)**3*sec(c + d*x)**2, x)
```

3.1445 $\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{a\tan(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] -a*x+b*cos(d*x+c)/d+b*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2722, 3473, 8, 2590, 14}

$$\frac{a\tan(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx &= \int \left(a\tan^2(c+dx)+b\sin(c+dx)\tan^2(c+dx)\right)\,dx \\
&= a\int \tan^2(c+dx)\,dx+b\int \sin(c+dx)\tan^2(c+dx)\,dx \\
&= \frac{a\tan(c+dx)}{d}-a\int 1\,dx-\frac{b\,\mathrm{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d} \\
&= -ax+\frac{a\tan(c+dx)}{d}-\frac{b\,\mathrm{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx,x,\cos(c+dx)\right)}{d} \\
&= -ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.24

$$
-\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\tan(c+dx)}{d}+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (
a*Tan[c + d*x])/d
```

fricas [A] time = 0.53, size = 47, normalized size = 1.24

$$
-\frac{adx\cos(dx+c)-b\cos(dx+c)^2-a\sin(dx+c)-b}{d\cos(dx+c)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*d*x*cos(d*x + c) - b*cos(d*x + c)^2 - a*sin(d*x + c) - b)/(d*cos(d*x +
c))
```

giac [A] time = 0.17, size = 58, normalized size = 1.53

$$-\frac{(dx+c)a + \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,b\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*a + 2*(a*tan(1/2*d*x + 1/2*c)^3 + a*tan(1/2*d*x + 1/2*c) + 2*b)
/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

maple [A] time = 0.31, size = 59, normalized size = 1.55

$$\frac{a\left(\tan\left(dx+c\right)-dx-c\right)+b\left(\frac{\sin^{4}(dx+c)}{\cos(dx+c)}+\left(2+\sin^{2}\left(dx+c\right)\right)\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(tan(d*x+c)-d*x-c)+b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d
*x+c)))
```

maxima [A] time = 0.43, size = 39, normalized size = 1.03

$$-\frac{(dx+c-\tan\left(dx+c\right))a-b\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a - b*(1/cos(d*x + c) + cos(d*x + c)))/d
```

mupad [B] time = 12.48, size = 55, normalized size = 1.45

$$-a\,x-\frac{2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,b}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + b*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] - a*x - (4*b + 2*a*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c
/2 + (d*x)/2)^4 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin (c + dx))\sin^2 (c + dx)\sec^2 (c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sin(c + d*x)**2*sec(c + d*x)**2, x)
```

3.1446 $\int \sec(c+dx)(a+b\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=27

$$\frac{a\sec(c+dx)}{d}+\frac{b\tan(c+dx)}{d}-bx$$

```
[Out] -b*x+a*sec(d*x+c)/d+b*tan(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2838, 2606, 8, 3473}

$$\frac{a\sec(c+dx)}{d}+\frac{b\tan(c+dx)}{d}-bx$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -(b*x) + (a*Sec[c + d*x])/d + (b*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))\tan(c+dx)\,dx = a\int \sec(c+dx)\tan(c+dx)\,dx + b\int \tan^2(c+dx)\,dx$$
$$= \frac{b\tan(c+dx)}{d} - b\int 1\,dx + \frac{a\,\text{Subst}(\int 1\,dx, x, \sec(c+dx))}{d}$$
$$= -bx + \frac{a\sec(c+dx)}{d} + \frac{b\tan(c+dx)}{d}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.33

$$\frac{a\sec(c+dx)}{d} - \frac{b\tan^{-1}(\tan(c+dx))}{d} + \frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x],x]
[Out] -((b*ArcTan[Tan[c + d*x]])/d) + (a*Sec[c + d*x])/d + (b*Tan[c + d*x])/d
```

fricas [A] time = 0.65, size = 36, normalized size = 1.33

$$-\frac{bdx\cos(dx+c) - b\sin(dx+c) - a}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] -(b*d*x*cos(d*x + c) - b*sin(d*x + c) - a)/(d*cos(d*x + c))
```

giac [A] time = 0.17, size = 43, normalized size = 1.59

$$-\frac{(dx+c)b + \frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] -((d*x + c)*b + 2*(b*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2*d*x + 1/2*c)^2 - 1)
)/d
```

maple [A] time = 0.17, size = 32, normalized size = 1.19

$$\frac{\frac{a}{\cos(dx+c)} + b\left(\tan\left(dx+c\right) - dx - c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x)
[Out] 1/d*(a/cos(d*x+c)+b*(tan(d*x+c)-d*x-c))
```

maxima [A] time = 0.50, size = 32, normalized size = 1.19

$$-\frac{(dx + c - \tan\left(dx+c\right))b - \frac{a}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] -((d*x + c - tan(d*x + c))*b - a/cos(d*x + c))/d
```

mupad [B] time = 11.94, size = 41, normalized size = 1.52

$$-b\,x - \frac{2\,a + 2\,b\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*sin(c + d*x)))/cos(c + d*x)^2,x)
[Out] - b*x - (2*a + 2*b*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin\left(c+dx\right)\right)\sin\left(c+dx\right)\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c)),x)
[Out] Integral((a + b*sin(c + d*x))*sin(c + d*x)*sec(c + d*x)**2, x)
```

3.1447 $\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=36

$$\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d+a*sec(d*x+c)/d+b*tan(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2838, 2622, 321, 207, 3767, 8}

$$\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d + (b*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc(c+dx)\sec^2(c+dx)\,dx + b\int \sec^2(c+dx)\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d} - \frac{b\,\text{Subst}(\int 1\,dx, x, -\text{ta}$$
$$= \frac{a\sec(c+dx)}{d} + \frac{b\tan(c+dx)}{d} + \frac{a\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(\right.}{d}$$
$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\sec(c+dx)}{d} + \frac{b\tan(c+dx)}{d}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.56

$$\frac{a\sec(c+dx)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[(c + d*x)/2]])/d) + (a*Log[Sin[(c + d*x)/2]])/d + (a*Sec[c + d
*x])/d + (b*Tan[c + d*x])/d
```

fricas [A] time = 0.43, size = 65, normalized size = 1.81

$$-\frac{a\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - a\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - 2\,b\sin(dx+c) - 2\,a}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - a*cos(d*x + c)*log(-1/2*
cos(d*x + c) + 1/2) - 2*b*sin(d*x + c) - 2*a)/(d*cos(d*x + c))
```

giac [A] time = 0.20, size = 48, normalized size = 1.33

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(b*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

maple [A] time = 0.42, size = 47, normalized size = 1.31

$$\frac{a}{d\cos\left(dx+c\right)}+\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}+\frac{b\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*a/cos(d*x+c)+1/d*a*ln(csc(d*x+c)-cot(d*x+c))+b*tan(d*x+c)/d
```

maxima [A] time = 0.35, size = 48, normalized size = 1.33

$$\frac{a\left(\frac{2}{\cos\left(dx+c\right)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+2\,b\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(a*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 2
*b*tan(d*x + c))/d
```

mupad [B] time = 11.94, size = 52, normalized size = 1.44

$$\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(cos(c + d*x)^2*sin(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)))/d - (2*a + 2*b*tan(c/2 + (d*x)/2))/(d*(tan(c/2
+ (d*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))\csc(c + dx)\sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*csc(c + d*x)*sec(c + d*x)**2, x)
```

3.1448 $\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=48

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{b\sec(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -b*arctanh(cos(d*x+c))/d-a*cot(d*x+c)/d+b*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2620, 14, 2622, 321, 207}

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{b\sec(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*ArcTanh[Cos[c + d*x]])/d) - (a*Cot[c + d*x])/d + (b*Sec[c + d*x])/d +
(a*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc^2(c+dx)\sec^2(c+dx)\,dx + b\int \csc(c+dx)\sec^2(c+$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x\right.}{d}$$

$$= \frac{b\sec(c+dx)}{d} + \frac{a\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, \tan(c+dx)\right)}{d} + $$

$$= -\frac{b\tanh^{-1}(\cos(c+dx))}{d} - \frac{a\cot(c+dx)}{d} + \frac{b\sec(c+dx)}{d} + $$

Mathematica [A] time = 0.08, size = 68, normalized size = 1.42

$$\frac{a\tan(c+dx)}{d} - \frac{a\cot(c+dx)}{d} + \frac{b\sec(c+dx)}{d} + \frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
[Out] -((a*Cot[c + d*x])/d) - (b*Log[Cos[(c + d*x)/2]])/d + (b*Log[Sin[(c + d*x)/
2]])/d + (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

fricas [A] time = 0.43, size = 96, normalized size = 2.00

$$-\frac{b\cos\left(dx+c\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-b\cos\left(dx+c\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+4\,a\,c}{2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*cos(d*x +
 c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 4*a*cos(d*x + c)^2 - 2*b*si
n(d*x + c) - 2*a)/(d*cos(d*x + c)*sin(d*x + c))
```

giac [B] time = 0.20, size = 103, normalized size = 2.15

$$\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+10\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*b*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a*tan(1/2*d*x + 1/2*c) - (2*b*t
an(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*d*x + 1/2*c)^2 + 10*b*tan(1/2*d*x + 1/
2*c) - 3*a)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

maple [A] time = 0.40, size = 69, normalized size = 1.44

$$\frac{a}{d\sin\left(dx+c\right)\cos\left(dx+c\right)}-\frac{2a\cot\left(dx+c\right)}{d}+\frac{b}{d\cos\left(dx+c\right)}+\frac{b\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*a/sin(d*x+c)/cos(d*x+c)-2*a*cot(d*x+c)/d+1/d*b/cos(d*x+c)+1/d*b*ln(csc(
d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.33, size = 59, normalized size = 1.23

$$\frac{b\left(\frac{2}{\cos\left(dx+c\right)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-2\,a\left(\frac{1}{\tan\left(dx+c\right)}-\tan\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 2
*a*(1/tan(d*x + c) - tan(d*x + c)))/d
```

mupad [B] time = 11.91, size = 92, normalized size = 1.92

$$\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(cos(c + d*x)^2*sin(c + d*x)^2),x)
```

```
[Out] (4*b*tan(c/2 + (d*x)/2) - a + 5*a*tan(c/2 + (d*x)/2)^2)/(d*(2*tan(c/2 + (d*
x)/2) - 2*tan(c/2 + (d*x)/2)^3)) + (a*tan(c/2 + (d*x)/2))/(2*d) + (b*log(ta
n(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1449 $\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=75

$$\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{b\tan(c+dx)}{d}-\frac{b\cot(c+dx)}{d}$$

```
[Out] -3/2*a*arctanh(cos(d*x+c))/d-b*cot(d*x+c)/d+3/2*a*sec(d*x+c)/d-1/2*a*csc(d*
x+c)^2*sec(d*x+c)/d+b*tan(d*x+c)/d
```

Rubi [A] time = 0.14, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2622, 288, 321, 207, 2620, 14}

$$\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{b\tan(c+dx)}{d}-\frac{b\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (b*Cot[c + d*x])/d + (3*a*Sec[c + d*x]
)/(2*d) - (a*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (b*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc^3(c+dx)\sec^2(c+dx)\,dx + b\int \csc^2(c+dx)\sec^2(c$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx\right.}{d}$$

$$= -\frac{a\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{(3a)\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{sec}\right.}{2d}$$

$$= -\frac{b\cot(c+dx)}{d} + \frac{3a\sec(c+dx)}{2d} - \frac{a\csc^2(c+dx)\sec(c+dx}{2d}$$

$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{b\cot(c+dx)}{d} + \frac{3a\sec(c+dx)}{2d}$$

Mathematica [B] time = 0.34, size = 172, normalized size = 2.29

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\sin\left(\frac{1}{2}(\right.}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*Cot[2*(c + d*x)])/d - (a*Csc[(c + d*x)/2]^2)/(8*d) - (3*a*Log[Cos[(c
+ d*x)/2]])/(2*d) + (3*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]
^2)/(8*d) + (a*Sin[(c + d*x)/2])/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
- (a*Sin[(c + d*x)/2])/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.44, size = 128, normalized size = 1.71

$$\frac{6\,a\cos\left(dx+c\right)^{2}-3\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right.\right.}{4\left(d\cos\left(dx+c\right)^{3}-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(6*a*cos(d*x + c)^2 - 3*(a*cos(d*x + c)^3 - a*cos(d*x + c))*log(1/2*cos
(d*x + c) + 1/2) + 3*(a*cos(d*x + c)^3 - a*cos(d*x + c))*log(-1/2*cos(d*x +
 c) + 1/2) + 4*(2*b*cos(d*x + c)^2 - b)*sin(d*x + c) - 4*a)/(d*cos(d*x + c)
^3 - d*cos(d*x + c))
```

giac [A] time = 0.20, size = 116, normalized size = 1.55

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{16\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}-\frac{18\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4}{\tan\left(\frac{1}{2}\,dx+\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 + 12*a*log(abs(tan(1/2*d*x + 1/2*c))) + 4*b*t
an(1/2*d*x + 1/2*c) - 16*(b*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2*d*x + 1/2*c)
^2 - 1) - (18*a*tan(1/2*d*x + 1/2*c)^2 + 4*b*tan(1/2*d*x + 1/2*c) + a)/tan(
1/2*d*x + 1/2*c)^2)/d
```

maple [A] time = 0.46, size = 93, normalized size = 1.24

$$-\frac{a}{2d\sin(dx+c)^2\cos(dx+c)}+\frac{3a}{2d\cos(dx+c)}+\frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{2d}+\frac{b}{d\sin(dx+c)\cos(dx+c)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2/cos(d*x+c)+3/2/d*a/cos(d*x+c)+3/2/d*a*ln(csc(d*x+c)-c
ot(d*x+c))+1/d*b/sin(d*x+c)/cos(d*x+c)-2*b*cot(d*x+c)/d
```

maxima [A] time = 0.33, size = 84, normalized size = 1.12

$$\frac{a\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)-4b\left(\frac{1}{\tan(dx+c)}-\tan(dx+c)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(co
s(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 4*b*(1/tan(d*x + c) - tan(d*x
+ c)))/d
```

mupad [B] time = 11.93, size = 127, normalized size = 1.69

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{-10\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{17\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(cos(c + d*x)^2*sin(c + d*x)^3),x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(2*d) - (a/2 + 2*b*tan(c/2 + (d*x)/2) - (17*a*tan(c/
2 + (d*x)/2)^2)/2 - 10*b*tan(c/2 + (d*x)/2)^3)/(d*(4*tan(c/2 + (d*x)/2)^2 -
 4*tan(c/2 + (d*x)/2)^4)) + (a*tan(c/2 + (d*x)/2)^2)/(8*d) + (3*a*log(tan(c
/2 + (d*x)/2)))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1450 $\int \sin(c+dx)(a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=94

$$\frac{(a^2+2b^2)\cos(c+dx)}{d}+\frac{(a^2+b^2)\sec(c+dx)}{d}+\frac{3ab\tan(c+dx)}{d}-\frac{ab\sin^2(c+dx)\tan(c+dx)}{d}-3abx-\frac{b^2\cos^3(c+}{3d}$$

```
[Out] -3*a*b*x+(a^2+2*b^2)*cos(d*x+c)/d-1/3*b^2*cos(d*x+c)^3/d+(a^2+b^2)*sec(d*x+
c)/d+3*a*b*tan(d*x+c)/d-a*b*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.18, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2911, 2591, 288, 321, 203, 4357, 448}

$$\frac{(a^2+2b^2)\cos(c+dx)}{d}+\frac{(a^2+b^2)\sec(c+dx)}{d}+\frac{3ab\tan(c+dx)}{d}-\frac{ab\sin^2(c+dx)\tan(c+dx)}{d}-3abx-\frac{b^2\cos^3(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -3*a*b*x + ((a^2 + 2*b^2)*Cos[c + d*x])/d - (b^2*Cos[c + d*x]^3)/(3*d) + ((
a^2 + b^2)*Sec[c + d*x])/d + (3*a*b*Tan[c + d*x])/d - (a*b*Sin[c + d*x]^2*T
an[c + d*x])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = (2ab)\int \sin^2(c+dx)\tan^2(c+dx)\,dx + \int \sin(c+dx)\left(a^2+\right.$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(a^2+b^2-b^2x^2)}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{(2ab)\,\text{Sub}}{}$$

$$= -\frac{ab\sin^2(c+dx)\tan(c+dx)}{d} - \frac{\text{Subst}\left(\int \left(-a^2\left(1+\frac{2b^2}{a^2}\right)+\right.\right.}{}$$

$$= \frac{(a^2+2b^2)\cos(c+dx)}{d} - \frac{b^2\cos^3(c+dx)}{3d} + \frac{(a^2+b^2)\sec(c}{d}$$

$$= -3abx + \frac{(a^2+2b^2)\cos(c+dx)}{d} - \frac{b^2\cos^3(c+dx)}{3d} + \frac{(a^2+}{}$$

Mathematica [A] time = 0.43, size = 104, normalized size = 1.11

$$\frac{\sec(c+dx)\left(-24\cos(c+dx)\left(a^2+3ab(c+dx)+b^2\right)+4\left(3a^2+5b^2\right)\cos(2(c+dx))+36a^2+54ab\sin(c+dx)\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (Sec[c + d*x]*(36*a^2 + 45*b^2 - 24*(a^2 + b^2 + 3*a*b*(c + d*x))*Cos[c + d
*x] + 4*(3*a^2 + 5*b^2)*Cos[2*(c + d*x)] - b^2*Cos[4*(c + d*x)] + 54*a*b*Si
n[c + d*x] + 6*a*b*Sin[3*(c + d*x)]))/(24*d)
```

fricas [A] time = 0.43, size = 91, normalized size = 0.97

$$-\frac{b^2\cos(dx+c)^4+9\,abdx\cos(dx+c)-3\left(a^2+2\,b^2\right)\cos(dx+c)^2-3\,a^2-3\,b^2-3\left(ab\cos(dx+c)^2+2\,ab\right)\sin}{3\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(b^2*cos(d*x + c)^4 + 9*a*b*d*x*cos(d*x + c) - 3*(a^2 + 2*b^2)*cos(d*x
 + c)^2 - 3*a^2 - 3*b^2 - 3*(a*b*cos(d*x + c)^2 + 2*a*b)*sin(d*x + c))/(d*c
os(d*x + c))
```

giac [A] time = 0.22, size = 172, normalized size = 1.83

$$-\frac{9(dx+c)ab+\frac{6\left(2ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^2+b^2\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}+\frac{2\left(3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12b^2}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(9*(d*x + c)*a*b + 6*(2*a*b*tan(1/2*d*x + 1/2*c) + a^2 + b^2)/(tan(1/2
*d*x + 1/2*c)^2 - 1) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*tan(1/2*d*x
+ 1/2*c)^4 - 3*b^2*tan(1/2*d*x + 1/2*c)^4 - 6*a^2*tan(1/2*d*x + 1/2*c)^2 -
12*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2 - 5*b^2)
/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.55, size = 147, normalized size = 1.56

$$\frac{a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+2ab\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+2*a*b*(sin(d
*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+
b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)
))
```

maxima [A] time = 0.62, size = 97, normalized size = 1.03

$$-\frac{3\left(3dx+3c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\tan(dx+c)\right)ab+\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\cos(dx+c)\right)b^2-3a^2\left(\frac{1}{\cos(dx+c)}+\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3*(3*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*
a*b + (cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*b^2 - 3*a^2*(1/cos
(d*x + c) + cos(d*x + c)))/d
```

mupad [B] time = 18.49, size = 149, normalized size = 1.59

$$-3\,a\,b\,x-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(8\,a^2+\frac{32\,b^2}{3}\right)+4\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a^2+\frac{16\,b^2}{3}+10\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+10\,a\,b\,\tan\left(\frac{c}{2}+\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + b*sin(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] - 3*a*b*x - (tan(c/2 + (d*x)/2)^2*(8*a^2 + (32*b^2)/3) + 4*a^2*tan(c/2 + (d
*x)/2)^4 + 4*a^2 + (16*b^2)/3 + 10*a*b*tan(c/2 + (d*x)/2)^3 + 10*a*b*tan(c/
2 + (d*x)/2)^5 + 6*a*b*tan(c/2 + (d*x)/2)^7 + 6*a*b*tan(c/2 + (d*x)/2))/(d*
(tan(c/2 + (d*x)/2)^2 - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1451 $\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=94

$$\frac{a^2\tan(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{3b^2\tan(c+dx)}{2d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3b^2x}{2}$$

```
[Out] -a^2*x-3/2*b^2*x+2*a*b*cos(d*x+c)/d+2*a*b*sec(d*x+c)/d+a^2*tan(d*x+c)/d+3/2
*b^2*tan(d*x+c)/d-1/2*b^2*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.14, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 3473, 8, 2590, 14, 2591, 288, 321, 203}

$$\frac{a^2\tan(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{3b^2\tan(c+dx)}{2d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*x) - (3*b^2*x)/2 + (2*a*b*Cos[c + d*x])/d + (2*a*b*Sec[c + d*x])/d +
(a^2*Tan[c + d*x])/d + (3*b^2*Tan[c + d*x])/(2*d) - (b^2*Sin[c + d*x]^2*Tan
[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = \int \left(a^2\tan^2(c+dx)+2ab\sin(c+dx)\tan^2(c+dx)+b^2\sin^2(c+dx)\tan^2\right.$$
$$= a^2\int \tan^2(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan^2(c+dx)\,dx+b^2\int \sin^2($$
$$= \frac{a^2\tan(c+dx)}{d}-a^2\int 1\,dx-\frac{(2ab)\,\mathrm{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d}+ -$$
$$= -a^2x+\frac{a^2\tan(c+dx)}{d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{(2ab)\,\mathrm{Subst}\left(\int \left(-\right.\right.}{}$$
$$= -a^2x+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{3b^2\tan(c}{2d}$$
$$= -a^2x-\frac{3b^2x}{2}+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{3l}{}$$

Mathematica [A] time = 0.43, size = 77, normalized size = 0.82

$$\frac{-4\left(2a^2+3b^2\right)(c+dx)+\left(8a^2+9b^2\right)\tan(c+dx)+b\sec(c+dx)(8a\cos(2(c+dx))+24a+b\sin(3(c+dx)))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (-4*(2*a^2 + 3*b^2)*(c + d*x) + b*Sec[c + d*x]*(24*a + 8*a*Cos[2*(c + d*x)]
 + b*Sin[3*(c + d*x)]) + (8*a^2 + 9*b^2)*Tan[c + d*x])/(8*d)
```

fricas [A] time = 0.42, size = 81, normalized size = 0.86

$$-\frac{\left(2\,a^2+3\,b^2\right)dx\cos\left(dx+c\right)-4\,ab\cos\left(dx+c\right)^2-4\,ab-\left(b^2\cos\left(dx+c\right)^2+2\,a^2+2\,b^2\right)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*((2*a^2 + 3*b^2)*d*x*cos(d*x + c) - 4*a*b*cos(d*x + c)^2 - 4*a*b - (b^
2*cos(d*x + c)^2 + 2*a^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.21, size = 137, normalized size = 1.46

$$-\frac{\left(2\,a^{2}+3\,b^{2}\right)(dx+c)+\frac{4\left(a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}+\frac{2\left(b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((2*a^2 + 3*b^2)*(d*x + c) + 4*(a^2*tan(1/2*d*x + 1/2*c) + b^2*tan(1/2
*d*x + 1/2*c) + 2*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(b^2*tan(1/2*d*x +
1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan(1/2*d*x + 1/2*c) - 4*a*b)
/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.45, size = 116, normalized size = 1.23

$$\frac{a^{2}\left(\tan\left(dx+c\right)-dx-c\right)+2ab\left(\frac{\sin^{4}(dx+c)}{\cos(dx+c)}+\left(2+\sin^{2}\left(dx+c\right)\right)\cos\left(dx+c\right)\right)+b^{2}\left(\frac{\sin^{5}(dx+c)}{\cos(dx+c)}+\left(\sin^{3}\left(dx+c\right)+\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)
*cos(d*x+c))+b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos
(d*x+c)-3/2*d*x-3/2*c))
```

maxima [A] time = 0.43, size = 83, normalized size = 0.88

$$-\frac{2\left(dx+c-\tan\left(dx+c\right)\right)a^{2}+\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^{2}+1}-2\,\tan\left(dx+c\right)\right)b^{2}-4\,ab\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*(2*(d*x + c - tan(d*x + c))*a^2 + (3*d*x + 3*c - tan(d*x + c)/(tan(d*x
 + c)^2 + 1) - 2*tan(d*x + c))*b^2 - 4*a*b*(1/cos(d*x + c) + cos(d*x + c)))
/d
```

mupad [B] time = 17.04, size = 145, normalized size = 1.54

$$\frac{\left(2\,a^2+3\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(4\,a^2+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+8\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\left(2\,a^2+3\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\,a}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + b*sin(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] (8*a*b + tan(c/2 + (d*x)/2)^3*(4*a^2 + 2*b^2) + tan(c/2 + (d*x)/2)^5*(2*a^2
 + 3*b^2) + tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2) + 8*a*b*tan(c/2 + (d*x)/2)^2
)/(d*(tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 +
1)) - x*(a^2 + (3*b^2)/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin (c+dx))^2 \sin ^2(c+dx) \sec ^2(c+dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sin(c + d*x)**2*sec(c + d*x)**2, x)
```

3.1452 $\int \sec(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=42

$$\frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d}-2abx+\frac{2b^2\cos(c+dx)}{d}$$

```
[Out] -2*a*b*x+2*b^2*cos(d*x+c)/d+sec(d*x+c)*(a+b*sin(d*x+c))^2/d
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2861, 12, 2638}

$$\frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d}-2abx+\frac{2b^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -2*a*b*x + (2*b^2*Cos[c + d*x])/d + (Sec[c + d*x]*(a + b*Sin[c + d*x])^2)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] && !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)\,dx = \frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d} - \int 2b(a+b\sin(c+dx))\,dx$$
$$= \frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d} - (2b)\int (a+b\sin(c+dx))\,dx$$
$$= -2abx + \frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d} - \left(2b^2\right)\int \sin(c+dx)$$
$$= -2abx + \frac{2b^2\cos(c+dx)}{d} + \frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d}$$

Mathematica [A] time = 0.31, size = 66, normalized size = 1.57

$$\frac{\sec(c+dx)\left(2a^2+b^2\cos(2(c+dx))+3b^2\right)-2\left(a^2+2ab(c+dx)-2ab\tan(c+dx)+b^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] ((2*a^2 + 3*b^2 + b^2*Cos[2*(c + d*x)])*Sec[c + d*x] - 2*(a^2 + b^2 + 2*a*b
*(c + d*x) - 2*a*b*Tan[c + d*x]))/(2*d)
```

fricas [A] time = 0.45, size = 59, normalized size = 1.40

$$-\frac{2\,abdx\cos\left(dx+c\right)-b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*d*x*cos(d*x + c) - b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)/(d*cos(d*x + c))
```

giac [A] time = 0.17, size = 82, normalized size = 1.95

$$-\frac{2\left((dx+c)ab+\frac{2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2+2\,b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((d*x + c)*a*b + (2*a*b*tan(1/2*d*x + 1/2*c)^3 + a^2*tan(1/2*d*x + 1/2*c
)^2 + 2*a*b*tan(1/2*d*x + 1/2*c) + a^2 + 2*b^2)/(tan(1/2*d*x + 1/2*c)^4 - 1
))/d
```

maple [A] time = 0.31, size = 75, normalized size = 1.79

$$\frac{\frac{a^2}{\cos(dx+c)} + 2ab\left(\tan\left(dx+c\right) - dx - c\right) + b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)} + \left(2+\sin^2\left(dx+c\right)\right)\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2/cos(d*x+c)+2*a*b*(tan(d*x+c)-d*x-c)+b^2*(sin(d*x+c)^4/cos(d*x+c)+(
2+sin(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 0.46, size = 56, normalized size = 1.33

$$-\frac{2\left(dx+c-\tan\left(dx+c\right)\right)ab - b^2\left(\frac{1}{\cos(dx+c)} + \cos\left(dx+c\right)\right) - \frac{a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*(d*x + c - tan(d*x + c))*a*b - b^2*(1/cos(d*x + c) + cos(d*x + c)) - a^
2/cos(d*x + c))/d
```

mupad [B] time = 12.25, size = 81, normalized size = 1.93

$$-\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,b^2}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}-2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*sin(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] - (2*a^2*tan(c/2 + (d*x)/2)^2 + 2*a^2 + 4*b^2 + 4*a*b*tan(c/2 + (d*x)/2)^3
+ 4*a*b*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 1)) - 2*a*b*x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin\left(c + dx\right)\right)^2 \sin\left(c + dx\right)\sec^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
[Out] Integral((a + b*sin(c + d*x))**2*sin(c + d*x)*sec(c + d*x)**2, x)
```

3.1453 $\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=46

$$\frac{(a^2+b^2)\sec(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tan(c+dx)}{d}$$

```
[Out] -a^2*arctanh(cos(d*x+c))/d+(a^2+b^2)*sec(d*x+c)/d+2*a*b*tan(d*x+c)/d
```

Rubi [A] time = 0.17, antiderivative size = 70, normalized size of antiderivative = 1.52, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2911, 3767, 8, 3201, 446, 78, 63, 206}

$$\frac{(a^2+b^2)\sec(c+dx)}{d}-\frac{a^2\sqrt{\cos^2(c+dx)}\sec(c+dx)\tanh^{-1}\left(\sqrt{\cos^2(c+dx)}\right)}{d}+\frac{2ab\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*Sec[c + d*x])/d - (a^2*ArcTanh[Sqrt[Cos[c + d*x]^2]]*Sqrt[Cos[
c + d*x]^2]*Sec[c + d*x])/d + (2*a*b*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3201

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFac
tors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Su
bst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Si
n[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[m/2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \sec^2(c+dx)\,dx + \int \csc(c+dx)\sec^2(c+dx)\left(a^2+\right.$$

$$= -\frac{(2ab)\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d} + \frac{\left(\sqrt{\cos^2(c+dx)}\ \text{se}\right.}{}$$

$$= \frac{2ab\tan(c+dx)}{d} + \frac{\left(\sqrt{\cos^2(c+dx)}\ \sec(c+dx)\right)\text{Subst}\left(\int \frac{}{(1}\right.}{2d}$$

$$= \frac{\left(a^2+b^2\right)\sec(c+dx)}{d} + \frac{2ab\tan(c+dx)}{d} + \frac{\left(a^2\sqrt{\cos^2(c+d\right.}{}$$

$$= \frac{\left(a^2+b^2\right)\sec(c+dx)}{d} + \frac{2ab\tan(c+dx)}{d} - \frac{\left(a^2\sqrt{\cos^2(c+d\right.}{}$$

$$= \frac{\left(a^2+b^2\right)\sec(c+dx)}{d} - \frac{a^2\tanh^{-1}\left(\sqrt{\cos^2(c+dx)}\right)\sqrt{\cos^2(}}{d}$$

Mathematica [A] time = 0.20, size = 58, normalized size = 1.26

$$\frac{\left(a^2+b^2\right)\sec(c+dx) + a\left(a\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right) + 2b\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*Sec[c + d*x] + a*(a*(-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x
)/2]]) + 2*b*Tan[c + d*x]))/d
```

fricas [A] time = 0.45, size = 77, normalized size = 1.67

$$-\frac{a^2\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - a^2\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - 4\,ab\sin(dx+c) - 2\,a^2 -}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - a^2*cos(d*x + c)*log(-
1/2*cos(d*x + c) + 1/2) - 4*a*b*sin(d*x + c) - 2*a^2 - 2*b^2)/(d*cos(d*x +
c))
```

giac [A] time = 0.21, size = 57, normalized size = 1.24

$$\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2+b^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(2*a*b*tan(1/2*d*x + 1/2*c) + a^2 +
 b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

maple [A] time = 0.55, size = 68, normalized size = 1.48

$$\frac{a^2}{d\cos\left(dx+c\right)}+\frac{a^2\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}+\frac{2ab\tan\left(dx+c\right)}{d}+\frac{b^2}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a^2/cos(d*x+c)+1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+2*a*b*tan(d*x+c)/d+1/d
*b^2/cos(d*x+c)
```

maxima [A] time = 0.36, size = 64, normalized size = 1.39

$$\frac{a^2\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+4\,ab\tan\left(dx+c\right)+\frac{2\,b^2}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) +
 4*a*b*tan(d*x + c) + 2*b^2/cos(d*x + c))/d
```

mupad [B] time = 11.85, size = 62, normalized size = 1.35

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a^2+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2\,b^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(cos(c + d*x)^2*sin(c + d*x)),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d - (2*a^2 + 2*b^2 + 4*a*b*tan(c/2 + (d*x)/2)
)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1454 $\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=59

$$\frac{(a^2+b^2)\tan(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -2*a*b*arctanh(cos(d*x+c))/d-a^2*cot(d*x+c)/d+2*a*b*sec(d*x+c)/d+(a^2+b^2)*
tan(d*x+c)/d
```

Rubi [A] time = 0.28, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2911, 2622, 321, 207, 3200, 14}

$$\frac{(a^2+b^2)\tan(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*ArcTanh[Cos[c + d*x]])/d - (a^2*Cot[c + d*x])/d + (2*a*b*Sec[c + d*
x])/d + ((a^2 + b^2)*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3200

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_)*((a_) + (b_.)
*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[
e + f*x], x]}, Dist[ff^(n + 1)/f, Subst[Int[(x^n*(a + (a + b)*ff^2*x^2)^p)/
(1 + ff^2*x^2)^((m + n)/2 + p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{
a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \csc(c+dx)\sec^2(c+dx)\,dx + \int \csc^2(c+dx)\sec^2$$

$$= \frac{\text{Subst}\left(\int \frac{a^2+\left(a^2+b^2\right)x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(2ab)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2ab\sec(c+dx)}{d} + \frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b^2}{a^2}\right)+\frac{a^2}{x^2}\right)dx, x, \tan(c\right.}{d}$$

$$= -\frac{2ab\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cot(c+dx)}{d} + \frac{2ab\sec(c+}{d}$$

Mathematica [A] time = 0.35, size = 102, normalized size = 1.73

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\left(2a^2+b^2\right)\cos(2(c+dx)) - b\left(4a\sin(c+dx) - 2a\sin(2(c+dx))\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*Sec[c + d*x]*((2*a^2 + b^2)*Cos[2*(
c + d*x)] - b*(b + 4*a*Sin[c + d*x] - 2*a*(Log[Cos[(c + d*x)/2]] - Log[Sin[
(c + d*x)/2]])*Sin[2*(c + d*x)])))/d
```

fricas [A] time = 0.45, size = 113, normalized size = 1.92

$$-\frac{ab\cos\left(dx+c\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-ab\cos\left(dx+c\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+(}{d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(a*b*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - a*b*cos(d*x +
 c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (2*a^2 + b^2)*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)/(d*cos(d*x + c)*sin(d*x + c))
```

giac [B] time = 0.21, size = 128, normalized size = 2.17

$$\frac{12\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20\,ab\tan(}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(12*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^2*tan(1/2*d*x + 1/2*c) - (
4*a*b*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1
/2*d*x + 1/2*c)^2 + 20*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/(tan(1/2*d*x + 1/2
*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

maple [A] time = 0.60, size = 90, normalized size = 1.53

$$\frac{a^2}{d\sin\left(dx+c\right)\cos\left(dx+c\right)}-\frac{2a^2\cot\left(dx+c\right)}{d}+\frac{2ab}{d\cos\left(dx+c\right)}+\frac{2ab\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}+\frac{b^2\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a^2/sin(d*x+c)/cos(d*x+c)-2*a^2*cot(d*x+c)/d+2/d*a*b/cos(d*x+c)+2/d*a*b
*ln(csc(d*x+c)-cot(d*x+c))+b^2*tan(d*x+c)/d
```

maxima [A] time = 0.45, size = 71, normalized size = 1.20

$$\frac{ab\left(\frac{2}{\cos(dx+c)} - \log\left(\cos\left(dx+c\right)+1\right) + \log\left(\cos\left(dx+c\right)-1\right)\right) - a^2\left(\frac{1}{\tan(dx+c)} - \tan\left(dx+c\right)\right) + b^2\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] (a*b*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - a^2
*(1/tan(d*x + c) - tan(d*x + c)) + b^2*tan(d*x + c))/d
```

mupad [B] time = 11.87, size = 108, normalized size = 1.83

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(5\,a^2+4\,b^2\right)-a^2+8\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{2\,a\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(cos(c + d*x)^2*sin(c + d*x)^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(5*a^2 + 4*b^2) - a^2 + 8*a*b*tan(c/2 + (d*x)/2))/(d*
(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^3)) + (a^2*tan(c/2 + (d*x)/2))
/(2*d) + (2*a*b*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1455 $\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=100

$$\frac{(3a^2+2b^2)\sec(c+dx)}{2d}-\frac{(3a^2+2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{2ab\tan(c+dx)}{d}-\frac{2ab\cot}{}$$

```
[Out] -1/2*(3*a^2+2*b^2)*arctanh(cos(d*x+c))/d-2*a*b*cot(d*x+c)/d+1/2*(3*a^2+2*b^
2)*sec(d*x+c)/d-1/2*a^2*csc(d*x+c)^2*sec(d*x+c)/d+2*a*b*tan(d*x+c)/d
```

Rubi [A] time = 0.25, antiderivative size = 124, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2911, 2620, 14, 3201, 446, 78, 51, 63, 206}

$$\frac{(3a^2+2b^2)\sec(c+dx)}{2d}-\frac{(3a^2+2b^2)\sqrt{\cos^2(c+dx)}\sec(c+dx)\tanh^{-1}\left(\sqrt{\cos^2(c+dx)}\right)}{2d}-\frac{a^2\csc^2(c+dx)\sec(c+}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cot[c + d*x])/d + ((3*a^2 + 2*b^2)*Sec[c + d*x])/(2*d) - ((3*a^2 +
2*b^2)*ArcTanh[Sqrt[Cos[c + d*x]^2]]*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x])/(2*
d) - (a^2*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (2*a*b*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3201

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFac
tors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Su
bst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Si
```

```
n[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \csc^2(c+dx)\sec^2(c+dx)\,dx + \int \csc^3(c+dx)\sec^2($$

$$= \frac{(2ab)\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} + \frac{\left(\sqrt{\cos^2(c+dx)}\ \text{s}\right.}{}$$

$$= \frac{(2ab)\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, \tan(c+dx)\right)}{d} + \frac{\left(\sqrt{\cos^2(c+d}\right.}{}$$

$$= -\frac{2ab\cot(c+dx)}{d} - \frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{2ab\tan(c}{d}$$

$$= -\frac{2ab\cot(c+dx)}{d} + \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2d} - \frac{a^2\csc^2(c+d}{2}$$

$$= -\frac{2ab\cot(c+dx)}{d} + \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2d} - \frac{a^2\csc^2(c+d}{2}$$

$$= -\frac{2ab\cot(c+dx)}{d} + \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2d} - \frac{\left(3a^2+2b^2\right)\text{t}}{}$$

Mathematica [B] time = 0.48, size = 238, normalized size = 2.38

$$\frac{\csc^4(c+dx)\left(-2\left(3a^2+2b^2\right)\cos(2(c+dx)) - \left(3a^2+2b^2\right)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^4*(2*a^2 + 4*b^2 - 2*(3*a^2 + 2*b^2)*Cos[2*(c + d*x)] + 3*a^2
*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 2*b^2*Cos[3*(c + d*x)]*Log[Cos[(c
 + d*x)/2]] - (3*a^2 + 2*b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2]] - Log[Sin
[(c + d*x)/2]]) - 3*a^2*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 2*b^2*Cos[
3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 8*a*b*Sin[c + d*x] - 8*a*b*Sin[3*(c +
d*x)]))/(2*d*(Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2))
```

fricas [A] time = 0.46, size = 186, normalized size = 1.86

$$\frac{2\left(3\,a^2+2\,b^2\right)\cos\left(dx+c\right)^2-4\,a^2-4\,b^2-\left(\left(3\,a^2+2\,b^2\right)\cos\left(dx+c\right)^3-\left(3\,a^2+2\,b^2\right)\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(\right.\right.}{4\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4*(2*(3*a^2 + 2*b^2)*cos(d*x + c)^2 - 4*a^2 - 4*b^2 - ((3*a^2 + 2*b^2)*co
s(d*x + c)^3 - (3*a^2 + 2*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) +
((3*a^2 + 2*b^2)*cos(d*x + c)^3 - (3*a^2 + 2*b^2)*cos(d*x + c))*log(-1/2*co
s(d*x + c) + 1/2) + 8*(2*a*b*cos(d*x + c)^2 - a*b)*sin(d*x + c))/(d*cos(d*x
 + c)^3 - d*cos(d*x + c))
```

giac [A] time = 0.23, size = 157, normalized size = 1.57

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\left(3\,a^2+2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{16\left(2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2+b^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 + 8*a*b*tan(1/2*d*x + 1/2*c) + 4*(3*a^2 + 2
*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - 16*(2*a*b*tan(1/2*d*x + 1/2*c) + a^2
 + b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (18*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*
b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x
+ 1/2*c)^2)/d
```

maple [A] time = 0.56, size = 140, normalized size = 1.40

$$-\frac{a^2}{2d\sin\left(dx+c\right)^2\cos\left(dx+c\right)}+\frac{3a^2}{2d\cos\left(dx+c\right)}+\frac{3a^2\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{2d}+\frac{2ab}{d\sin\left(dx+c\right)\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2/sin(d*x+c)^2/cos(d*x+c)+3/2/d*a^2/cos(d*x+c)+3/2/d*a^2*ln(csc(d*
x+c)-cot(d*x+c))+2/d*a*b/sin(d*x+c)/cos(d*x+c)-4*a*b*cot(d*x+c)/d+1/d*b^2/c
os(d*x+c)+1/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.45, size = 123, normalized size = 1.23

$$\frac{a^2\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)+2b^2\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right.\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*(a^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(
cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 2*b^2*(2/cos(d*x + c) - log(
cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 8*a*b*(1/tan(d*x + c) - tan(d*
x + c)))/d
```

mupad [B] time = 11.85, size = 148, normalized size = 1.48

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{2}+b^2\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{17\,a^2}{2}+8\,b^2\right)-\frac{a^2}{2}+20\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4\,a\,b}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(cos(c + d*x)^2*sin(c + d*x)^3),x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (log(tan(c/2 + (d*x)/2))*((3*a^2)/2 + b^
2))/d + (tan(c/2 + (d*x)/2)^2*((17*a^2)/2 + 8*b^2) - a^2/2 + 20*a*b*tan(c/2
 + (d*x)/2)^3 - 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 - 4*ta
n(c/2 + (d*x)/2)^4)) + (a*b*tan(c/2 + (d*x)/2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1456 $\int \csc^4(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=104

$$\frac{(a^2+b^2)\tan(c+dx)}{d}-\frac{(2a^2+b^2)\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{3ab\sec(c+dx)}{d}-\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{ab\,c}{}$$

```
[Out] -3*a*b*arctanh(cos(d*x+c))/d-(2*a^2+b^2)*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/
d+3*a*b*sec(d*x+c)/d-a*b*csc(d*x+c)^2*sec(d*x+c)/d+(a^2+b^2)*tan(d*x+c)/d
```

Rubi [A] time = 0.23, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2911, 2622, 288, 321, 207, 3200, 448}

$$\frac{(a^2+b^2)\tan(c+dx)}{d}-\frac{(2a^2+b^2)\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{3ab\sec(c+dx)}{d}-\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{ab\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*a*b*ArcTanh[Cos[c + d*x]])/d - ((2*a^2 + b^2)*Cot[c + d*x])/d - (a^2*Co
t[c + d*x]^3)/(3*d) + (3*a*b*Sec[c + d*x])/d - (a*b*Csc[c + d*x]^2*Sec[c +
d*x])/d + ((a^2 + b^2)*Tan[c + d*x])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3200

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_)*((a_) + (b_.)
*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[
e + f*x], x]}, Dist[ff^(n + 1)/f, Subst[Int[(x^n*(a + (a + b)*ff^2*x^2)^p)/
(1 + ff^2*x^2)^((m + n)/2 + p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{
a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \csc^4(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \csc^3(c+dx)\sec^2(c+dx)\,dx + \int \csc^4(c+dx)\sec$$

$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)\left(a^2+\left(a^2+b^2\right)x^2\right)}{x^4}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(2ab)\,\text{Su}}{}$$

$$= -\frac{ab\csc^2(c+dx)\sec(c+dx)}{d} + \frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b^2}{a^2}\right)+\frac{a^2}{x^4}\right.\right.}{}$$

$$= -\frac{\left(2a^2+b^2\right)\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} + \frac{3ab\sec(c+d}{d}$$

$$= -\frac{3ab\tanh^{-1}(\cos(c+dx))}{d} - \frac{\left(2a^2+b^2\right)\cot(c+dx)}{d} - \frac{a^2\,\mathrm{c}}{}$$

Mathematica [A] time = 0.99, size = 196, normalized size = 1.88

$$\frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-4\left(4a^2+3b^2\right)\cos(2(c+dx))+\left(8a^2+6b^2\right)\cos(4(c+dx))+3b\left(10a\sin(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]^3*(-4*(4*a^2 + 3*b^2)*Cos[2*(c + d*x)]
 + (8*a^2 + 6*b^2)*Cos[4*(c + d*x)] + 3*b*(2*b + 10*a*Sin[c + d*x] - 6*a*(L
og[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[2*(c + d*x)] - 6*a*Sin[3*
(c + d*x)] + 3*a*Log[Cos[(c + d*x)/2]]*Sin[4*(c + d*x)] - 3*a*Log[Sin[(c +
d*x)/2]]*Sin[4*(c + d*x)])))/(192*d*(-1 + Cot[(c + d*x)/2]^2))
```

fricas [A] time = 0.45, size = 192, normalized size = 1.85

$$-\frac{4\left(4\,a^2+3\,b^2\right)\cos\left(dx+c\right)^4-6\left(4\,a^2+3\,b^2\right)\cos\left(dx+c\right)^2+9\left(ab\cos\left(dx+c\right)^3-ab\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/6*(4*(4*a^2 + 3*b^2)*cos(d*x + c)^4 - 6*(4*a^2 + 3*b^2)*cos(d*x + c)^2 +
 9*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2)*sin(
```

```
d*x + c) - 9*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + 6*a^2 + 6*b^2 - 6*(3*a*b*cos(d*x + c)^2 - 2*a*b)*sin(
d*x + c))/((d*cos(d*x + c)^3 - d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.24, size = 204, normalized size = 1.96

$$\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+72ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)+21a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12b^2\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 + 72*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c))) + 21*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2
*d*x + 1/2*c) - 48*(a^2*tan(1/2*d*x + 1/2*c) + b^2*tan(1/2*d*x + 1/2*c) + 2
*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (132*a*b*tan(1/2*d*x + 1/2*c)^3 + 21*a
^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2 + 6*a*b*tan(1/2*d
*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.57, size = 162, normalized size = 1.56

$$-\frac{a^2}{3d\sin(dx+c)^3\cos(dx+c)}+\frac{4a^2}{3d\sin(dx+c)\cos(dx+c)}-\frac{8a^2\cot(dx+c)}{3d}-\frac{ab}{d\sin(dx+c)^2\cos(dx+c)}+\frac{3}{d\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d*a^2/sin(d*x+c)^3/cos(d*x+c)+4/3/d*a^2/sin(d*x+c)/cos(d*x+c)-8/3*a^2*
cot(d*x+c)/d-1/d*a*b/sin(d*x+c)^2/cos(d*x+c)+3/d*a*b/cos(d*x+c)+3/d*a*b*ln(
csc(d*x+c)-cot(d*x+c))+1/d*b^2/sin(d*x+c)/cos(d*x+c)-2*b^2*cot(d*x+c)/d
```

maxima [A] time = 0.38, size = 123, normalized size = 1.18

$$\frac{3ab\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)-6b^2\left(\frac{1}{\tan(dx+c)}-\tan(dx+c)\right)-2a}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/6*(3*a*b*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*lo
g(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 6*b^2*(1/tan(d*x + c) - ta
```

```
n(d*x + c)) - 2*a^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 - 3*tan(d*x + c)
))/d
```

mupad [B] time = 11.86, size = 194, normalized size = 1.87

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{20\,a^2}{3}+4\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(23\,a^2+20\,b^2\right)+\frac{a^2}{3}-34\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(cos(c + d*x)^2*sin(c + d*x)^4),x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (tan(c/2 + (d*x)/2)^2*((20*a^2)/3 + 4*b
^2) - tan(c/2 + (d*x)/2)^4*(23*a^2 + 20*b^2) + a^2/3 - 34*a*b*tan(c/2 + (d*
x)/2)^3 + 2*a*b*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 - 8*tan(c/2
+ (d*x)/2)^5)) + (tan(c/2 + (d*x)/2)*((7*a^2)/8 + b^2/2))/d + (a*b*tan(c/2
+ (d*x)/2)^2)/(4*d) + (3*a*b*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1457 $\int \sin(c+dx)(a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=197

$$\frac{a^3\cos(c+dx)}{d}+\frac{a^3\sec(c+dx)}{d}+\frac{9a^2b\tan(c+dx)}{2d}-\frac{3a^2b\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{9}{2}a^2bx-\frac{ab^2\cos^3(c+dx)}{d}+\frac{6ab^2}{}$$

```
[Out] -9/2*a^2*b*x-15/8*b^3*x+a^3*cos(d*x+c)/d+6*a*b^2*cos(d*x+c)/d-a*b^2*cos(d*x
+c)^3/d+a^3*sec(d*x+c)/d+3*a*b^2*sec(d*x+c)/d+9/2*a^2*b*tan(d*x+c)/d+15/8*b
^3*tan(d*x+c)/d-3/2*a^2*b*sin(d*x+c)^2*tan(d*x+c)/d-5/8*b^3*sin(d*x+c)^2*ta
n(d*x+c)/d-1/4*b^3*sin(d*x+c)^4*tan(d*x+c)/d
```

Rubi [A] time = 0.26, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2912, 2590, 14, 2591, 288, 321, 203, 270}

$$\frac{9a^2b\tan(c+dx)}{2d}-\frac{3a^2b\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{9}{2}a^2bx+\frac{a^3\cos(c+dx)}{d}+\frac{a^3\sec(c+dx)}{d}-\frac{ab^2\cos^3(c+dx)}{d}+\frac{6ab^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (-9*a^2*b*x)/2 - (15*b^3*x)/8 + (a^3*Cos[c + d*x])/d + (6*a*b^2*Cos[c + d*x
])/d - (a*b^2*Cos[c + d*x]^3)/d + (a^3*Sec[c + d*x])/d + (3*a*b^2*Sec[c + d
*x])/d + (9*a^2*b*Tan[c + d*x])/(2*d) + (15*b^3*Tan[c + d*x])/(8*d) - (3*a^
2*b*Sin[c + d*x]^2*Tan[c + d*x])/(2*d) - (5*b^3*Sin[c + d*x]^2*Tan[c + d*x]
)/(8*d) - (b^3*Sin[c + d*x]^4*Tan[c + d*x])/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = \int \left(a^3\sin(c+dx)\tan^2(c+dx)+3a^2b\sin^2(c+dx)\tan^2(c+
$$

$$= a^3\int \sin(c+dx)\tan^2(c+dx)\,dx + \left(3a^2b\right)\int \sin^2(c+dx)\tan
$$

$$= -\frac{a^3\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x}{(1+
$$

$$= -\frac{3a^2b\sin^2(c+dx)\tan(c+dx)}{2d} - \frac{b^3\sin^4(c+dx)\tan(c+dx)}{4d}
$$

$$= \frac{a^3\cos(c+dx)}{d} + \frac{6ab^2\cos(c+dx)}{d} - \frac{ab^2\cos^3(c+dx)}{d} + \frac{a^3\,\text{s}
$$

$$= -\frac{9}{2}a^2bx + \frac{a^3\cos(c+dx)}{d} + \frac{6ab^2\cos(c+dx)}{d} - \frac{ab^2\cos^3(c+}{d}
$$

$$= -\frac{9}{2}a^2bx - \frac{15b^3x}{8} + \frac{a^3\cos(c+dx)}{d} + \frac{6ab^2\cos(c+dx)}{d} - \frac{ab^2}{}
$$

Mathematica [A] time = 0.76, size = 147, normalized size = 0.75

$$\frac{\sec(c+dx)\left(32\left(a^3+5ab^2\right)\cos(2(c+dx))+96a^3-24b\left(12a^2+5b^2\right)(c+dx)\cos(c+dx)+216a^2b\sin(c+dx)+}{6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (Sec[c + d*x]*(96*a^3 + 360*a*b^2 - 24*b*(12*a^2 + 5*b^2)*(c + d*x)*Cos[c +
 d*x] + 32*(a^3 + 5*a*b^2)*Cos[2*(c + d*x)] - 8*a*b^2*Cos[4*(c + d*x)] + 21
6*a^2*b*Sin[c + d*x] + 80*b^3*Sin[c + d*x] + 24*a^2*b*Sin[3*(c + d*x)] + 15
*b^3*Sin[3*(c + d*x)] - b^3*Sin[5*(c + d*x)]))/(64*d)
```

fricas [A] time = 0.44, size = 135, normalized size = 0.69

$$-\frac{8\,ab^2\cos\left(dx+c\right)^4+3\left(12\,a^2b+5\,b^3\right)dx\cos\left(dx+c\right)-8\,a^3-24\,ab^2-8\left(a^3+6\,ab^2\right)\cos\left(dx+c\right)^2+\left(2\,b^3\cos\left(d\right.\right.}{8\,d\cos\left(dx+c\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/8*(8*a*b^2*cos(d*x + c)^4 + 3*(12*a^2*b + 5*b^3)*d*x*cos(d*x + c) - 8*a^
3 - 24*a*b^2 - 8*(a^3 + 6*a*b^2)*cos(d*x + c)^2 + (2*b^3*cos(d*x + c)^4 - 2
4*a^2*b - 8*b^3 - 3*(4*a^2*b + 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(
d*x + c))
```

giac [A] time = 0.24, size = 336, normalized size = 1.71

$$-\frac{3\left(12\,a^2b+5\,b^3\right)(dx+c)+\frac{16\left(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3+3\,ab^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+7\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-8\,a\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(12*a^2*b + 5*b^3)*(d*x + c) + 16*(3*a^2*b*tan(1/2*d*x + 1/2*c) + b
^3*tan(1/2*d*x + 1/2*c) + a^3 + 3*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(
12*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 7*b^3*tan(1/2*d*x + 1/2*c)^7 - 8*a^3*tan(
1/2*d*x + 1/2*c)^6 - 24*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x
 + 1/2*c)^5 + 15*b^3*tan(1/2*d*x + 1/2*c)^5 - 24*a^3*tan(1/2*d*x + 1/2*c)^4
 - 120*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 15*
b^3*tan(1/2*d*x + 1/2*c)^3 - 24*a^3*tan(1/2*d*x + 1/2*c)^2 - 136*a*b^2*tan(
1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c) - 7*b^3*tan(1/2*d*x + 1/
2*c) - 8*a^3 - 40*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.64, size = 214, normalized size = 1.09

$$\frac{a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+3a^2b\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3d}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+3*a^2*b*(sin
(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c
)+3*a*b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(
d*x+c))+b^3*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*si
n(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c))
```

maxima [A] time = 0.48, size = 164, normalized size = 0.83

$$-\frac{12\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)a^2b+8\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\,\cos(dx+c)\right)ab^2+\left(15\,dx+\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/8*(12*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))
*a^2*b + 8*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a*b^2 + (15*d
*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x
 + c)^2 + 1) - 8*tan(d*x + c))*b^3 - 8*a^3*(1/cos(d*x + c) + cos(d*x + c)))
/d
```

mupad [B] time = 16.02, size = 323, normalized size = 1.64

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(9\,a^2\,b+\frac{15\,b^3}{4}\right)+4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\,a\,b^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(12\,a^3+32\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(12\right.}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-3\,\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + b*sin(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(9*a^2*b + (15*b^3)/4) + 4*a^3*tan(c/2 + (d*x)/2)^6 + 1
6*a*b^2 + tan(c/2 + (d*x)/2)^4*(32*a*b^2 + 12*a^3) + tan(c/2 + (d*x)/2)^2*(
48*a*b^2 + 12*a^3) + tan(c/2 + (d*x)/2)^3*(24*a^2*b + 10*b^3) + tan(c/2 + (
d*x)/2)^9*(9*a^2*b + (15*b^3)/4) + tan(c/2 + (d*x)/2)^7*(24*a^2*b + 10*b^3)
 + tan(c/2 + (d*x)/2)^5*(30*a^2*b + (9*b^3)/2) + 4*a^3)/(d*(3*tan(c/2 + (d*
x)/2)^2 + 2*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^6 - 3*tan(c/2 + (d*
x)/2)^8 - tan(c/2 + (d*x)/2)^10 + 1)) - (3*b*atan((3*b*tan(c/2 + (d*x)/2)*(
12*a^2 + 5*b^2))/(36*a^2*b + 15*b^3))*(12*a^2 + 5*b^2))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1458 $\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=146

$$\frac{a^3\tan(c+dx)}{d}+a^3(-x)+\frac{3a^2b\cos(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}+\frac{9ab^2\tan(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-$$

```
[Out] -a^3*x-9/2*a*b^2*x+3*a^2*b*cos(d*x+c)/d+2*b^3*cos(d*x+c)/d-1/3*b^3*cos(d*x+
c)^3/d+3*a^2*b*sec(d*x+c)/d+b^3*sec(d*x+c)/d+a^3*tan(d*x+c)/d+9/2*a*b^2*tan
(d*x+c)/d-3/2*a*b^2*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.21, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2722, 3473, 8, 2590, 14, 2591, 288, 321, 203, 270}

$$\frac{3a^2b\cos(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+a^3(-x)+\frac{9ab^2\tan(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -(a^3*x) - (9*a*b^2*x)/2 + (3*a^2*b*Cos[c + d*x])/d + (2*b^3*Cos[c + d*x])/
d - (b^3*Cos[c + d*x]^3)/(3*d) + (3*a^2*b*Sec[c + d*x])/d + (b^3*Sec[c + d*
x])/d + (a^3*Tan[c + d*x])/d + (9*a*b^2*Tan[c + d*x])/(2*d) - (3*a*b^2*Sin[
c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = \int \left(a^3\tan^2(c+dx)+3a^2b\sin(c+dx)\tan^2(c+dx)+3ab^2\sin^2(c+dx)\right.$$
$$= a^3\int \tan^2(c+dx)\,dx+\left(3a^2b\right)\int \sin(c+dx)\tan^2(c+dx)\,dx+\left(3ab^2\right)$$
$$= \frac{a^3\tan(c+dx)}{d}-a^3\int 1\,dx-\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d}$$
$$= -a^3x+\frac{a^3\tan(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{\left(3a^2b\right)\text{Subst}}{}$$
$$= -a^3x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}+\frac{3a^2b\,\text{s}}{}$$
$$= -a^3x-\frac{9}{2}ab^2x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}$$

Mathematica [A] time = 0.74, size = 113, normalized size = 0.77

$$\frac{3a\left(\left(8a^2+27b^2\right)\tan(c+dx)-4\left(2a^2+9b^2\right)(c+dx)\right)+b\sec(c+dx)\left(4\left(9a^2+5b^2\right)\cos(2(c+dx))+108a^2+9\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (b*Sec[c + d*x]*(108*a^2 + 45*b^2 + 4*(9*a^2 + 5*b^2)*Cos[2*(c + d*x)] - b^
2*Cos[4*(c + d*x)] + 9*a*b*Sin[3*(c + d*x)]) + 3*a*(-4*(2*a^2 + 9*b^2)*(c +
 d*x) + (8*a^2 + 27*b^2)*Tan[c + d*x]))/(24*d)
```

fricas [A] time = 0.43, size = 116, normalized size = 0.79

$$-\frac{2\,b^3\cos\left(dx+c\right)^4+3\left(2\,a^3+9\,ab^2\right)dx\cos\left(dx+c\right)-18\,a^2b-6\,b^3-6\left(3\,a^2b+2\,b^3\right)\cos\left(dx+c\right)^2-3\left(3\,ab^2\,\text{co}\right.}{6\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(2*b^3*cos(d*x + c)^4 + 3*(2*a^3 + 9*a*b^2)*d*x*cos(d*x + c) - 18*a^2*
b - 6*b^3 - 6*(3*a^2*b + 2*b^3)*cos(d*x + c)^2 - 3*(3*a*b^2*cos(d*x + c)^2
+ 2*a^3 + 6*a*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.20, size = 207, normalized size = 1.42

$$-\frac{3\left(2a^3+9ab^2\right)(dx+c)+\frac{12\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3a^2b+b^3\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}+\frac{2\left(9ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-18a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-6b^3\mathrm{t}\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 + 9*a*b^2)*(d*x + c) + 12*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^
2*tan(1/2*d*x + 1/2*c) + 3*a^2*b + b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(9
*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 6*b^3*tan
(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 24*b^3*tan(1/2*d*x
+ 1/2*c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b - 10*b^3)/(tan(1/2*d*x
 + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.55, size = 169, normalized size = 1.16

$$\frac{a^3(\tan(dx+c)-dx-c)+3a^2b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+3a\,b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(tan(d*x+c)-d*x-c)+3*a^2*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^
2)*cos(d*x+c))+3*a*b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c
))*cos(d*x+c)-3/2*d*x-3/2*c)+b^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4
+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 0.47, size = 119, normalized size = 0.82

$$-\frac{6(dx+c-\tan(dx+c))a^3+9\left(3dx+3c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\tan(dx+c)\right)ab^2+2\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\,\mathrm{c}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(6*(d*x + c - tan(d*x + c))*a^3 + 9*(3*d*x + 3*c - tan(d*x + c)/(tan(d
*x + c)^2 + 1) - 2*tan(d*x + c))*a*b^2 + 2*(cos(d*x + c)^3 - 3/cos(d*x + c)
 - 6*cos(d*x + c))*b^3 - 18*a^2*b*(1/cos(d*x + c) + cos(d*x + c)))/d
```

mupad [B] time = 16.66, size = 249, normalized size = 1.71

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(2\,a^3+9\,a\,b^2\right)+12\,a^2\,b+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(2\,a^3+9\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(6\,a^3+15\,a\,b^2\right)+\tan\left(\frac{c}{2}\right.}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + b*sin(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(9*a*b^2 + 2*a^3) + 12*a^2*b + tan(c/2 + (d*x)/2)^7*(9*
a*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^3*(15*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/
2)^5*(15*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/2)^2*(24*a^2*b + (32*b^3)/3) + (1
6*b^3)/3 + 12*a^2*b*tan(c/2 + (d*x)/2)^4)/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*ta
n(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 1)) - (a*atan((a*tan(c/2 + (d*x
)/2)*(2*a^2 + 9*b^2))/(9*a*b^2 + 2*a^3))*(2*a^2 + 9*b^2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1459 $\int \sec(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=75

$$-\frac{3}{2}bx\left(2a^2+b^2\right)+\frac{6ab^2\cos(c+dx)}{d}+\frac{\sec(c+dx)(a+b\sin(c+dx))^3}{d}+\frac{3b^3\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] -3/2*b*(2*a^2+b^2)*x+6*a*b^2*cos(d*x+c)/d+3/2*b^3*cos(d*x+c)*sin(d*x+c)/d+s
ec(d*x+c)*(a+b*sin(d*x+c))^3/d
```

Rubi [A] time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2861, 12, 2644}

$$-\frac{3}{2}bx\left(2a^2+b^2\right)+\frac{6ab^2\cos(c+dx)}{d}+\frac{\sec(c+dx)(a+b\sin(c+dx))^3}{d}+\frac{3b^3\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (-3*b*(2*a^2 + b^2)*x)/2 + (6*a*b^2*Cos[c + d*x])/d + (3*b^3*Cos[c + d*x]*S
in[c + d*x])/(2*d) + (Sec[c + d*x]*(a + b*Sin[c + d*x])^3)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)\,dx = \frac{\sec(c+dx)(a+b\sin(c+dx))^3}{d} - \int 3b(a+b\sin(c+dx))^2\,d$$
$$= \frac{\sec(c+dx)(a+b\sin(c+dx))^3}{d} - (3b)\int (a+b\sin(c+dx))^2$$
$$= -\frac{3}{2}b\left(2a^2+b^2\right)x + \frac{6ab^2\cos(c+dx)}{d} + \frac{3b^3\cos(c+dx)\sin(}{2d}$$

Mathematica [A] time = 0.55, size = 91, normalized size = 1.21

$$\frac{\sec(c+dx)\left(8a^3+12ab^2\cos(2(c+dx))+36ab^2+b^3\sin(3(c+dx))\right)+3b\left(\left(8a^2+3b^2\right)\tan(c+dx)-4\left(2a^2+b\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (Sec[c + d*x]*(8*a^3 + 36*a*b^2 + 12*a*b^2*Cos[2*(c + d*x)] + b^3*Sin[3*(c
+ d*x)]) + 3*b*(-4*(2*a^2 + b^2)*(c + d*x) + (8*a^2 + 3*b^2)*Tan[c + d*x]))
/(8*d)
```

fricas [A] time = 0.42, size = 90, normalized size = 1.20

$$\frac{6\,ab^2\cos\left(dx+c\right)^2-3\left(2\,a^2b+b^3\right)dx\cos\left(dx+c\right)+2\,a^3+6\,ab^2+\left(b^3\cos\left(dx+c\right)^2+6\,a^2b+2\,b^3\right)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*cos(d*x + c)^2 - 3*(2*a^2*b + b^3)*d*x*cos(d*x + c) + 2*a^3 +
6*a*b^2 + (b^3*cos(d*x + c)^2 + 6*a^2*b + 2*b^3)*sin(d*x + c))/(d*cos(d*x +
 c))
```

giac [B] time = 0.20, size = 148, normalized size = 1.97

$$-\frac{3\left(2\,a^2b+b^3\right)(dx+c)+\frac{4\left(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3+3\,ab^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b^3\tan\left(\frac{1}{2}\,d\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(2*a^2*b + b^3)*(d*x + c) + 4*(3*a^2*b*tan(1/2*d*x + 1/2*c) + b^3*t
an(1/2*d*x + 1/2*c) + a^3 + 3*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(b^3*
tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x +
 1/2*c) - 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.45, size = 132, normalized size = 1.76

$$\frac{\frac{a^3}{\cos(dx+c)} + 3a^2b\left(\tan(dx+c) - dx - c\right) + 3a\,b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)} + \left(2+\sin^2(dx+c)\right)\cos(dx+c)\right) + b^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)} + \left(\sin\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3/cos(d*x+c)+3*a^2*b*(tan(d*x+c)-d*x-c)+3*a*b^2*(sin(d*x+c)^4/cos(d*
x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+b^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^
3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c))
```

maxima [A] time = 0.48, size = 99, normalized size = 1.32

$$-\frac{6\,(dx + c - \tan(dx+c))a^2b + \left(3\,dx + 3\,c - \frac{\tan(dx+c)}{\tan(dx+c)^2+1} - 2\,\tan(dx+c)\right)b^3 - 6\,ab^2\left(\frac{1}{\cos(dx+c)} + \cos(dx+c)\right) - \cdots}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*(d*x + c - tan(d*x + c))*a^2*b + (3*d*x + 3*c - tan(d*x + c)/(tan(d
*x + c)^2 + 1) - 2*tan(d*x + c))*b^3 - 6*a*b^2*(1/cos(d*x + c) + cos(d*x +
c)) - 2*a^3/cos(d*x + c))/d
```

mupad [B] time = 16.29, size = 219, normalized size = 2.92

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a^2\,b+3\,b^3\right)+2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+12\,a\,b^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(4\,a^3+12\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(6\,a^2\cdots\right.}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*sin(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(6*a^2*b + 3*b^3) + 2*a^3*tan(c/2 + (d*x)/2)^4 + 12*a*b
^2 + tan(c/2 + (d*x)/2)^2*(12*a*b^2 + 4*a^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*
```

```
b + 3*b^3) + tan(c/2 + (d*x)/2)^3*(12*a^2*b + 2*b^3) + 2*a^3)/(d*(tan(c/2 +
 (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 + 1)) - (3*b*atan
((3*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))/(6*a^2*b + 3*b^3))*(2*a^2 + b^2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^3 \sin(c+dx)\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*sin(c + d*x)*sec(c + d*x)**2, x)
```

3.1460 $\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=78

$$\frac{a^3\sec(c+dx)}{d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\tan(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\tan(c+dx)}{d}-b^3x$$

```
[Out] -b^3*x-a^3*arctanh(cos(d*x+c))/d+a^3*sec(d*x+c)/d+3*a*b^2*sec(d*x+c)/d+3*a^
2*b*tan(d*x+c)/d+b^3*tan(d*x+c)/d
```

Rubi [A] time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2912, 3767, 8, 2622, 321, 207, 2606, 3473}

$$\frac{3a^2b\tan(c+dx)}{d}+\frac{a^3\sec(c+dx)}{d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\tan(c+dx)}{d}-b^3x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(b^3*x) - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Sec[c + d*x])/d + (3*a*b^2*
Sec[c + d*x])/d + (3*a^2*b*Tan[c + d*x])/d + (b^3*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(3a^2b\sec^2(c+dx)+a^3\csc(c+dx)\sec^2(c+dx)+3ab^2\text{se}\right.$$
$$= a^3\int \csc(c+dx)\sec^2(c+dx)\,dx + \left(3a^2b\right)\int \sec^2(c+dx)\,dx$$
$$= \frac{b^3\tan(c+dx)}{d} - b^3\int 1\,dx + \frac{a^3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+\right.}{d}$$
$$= -b^3x + \frac{a^3\sec(c+dx)}{d} + \frac{3ab^2\sec(c+dx)}{d} + \frac{3a^2b\tan(c+dx)}{d}$$
$$= -b^3x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\sec(c+dx)}{d} + \frac{3ab^2\sec(}{d}$$

Mathematica [A] time = 0.30, size = 83, normalized size = 1.06

$$\frac{a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^3\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+b\left(3a^2+b^2\right)\tan(c+dx)+a\left(a^2+3b^2\right)\sec(c+dx)-b^3}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-(b^3*c) - b^3*d*x - a^3*Log[Cos[(c + d*x)/2]] + a^3*Log[Sin[(c + d*x)/2]]
 + a*(a^2 + 3*b^2)*Sec[c + d*x] + b*(3*a^2 + b^2)*Tan[c + d*x])/d
```

fricas [A] time = 0.44, size = 99, normalized size = 1.27

$$-\frac{2\,b^3dx\cos\left(dx+c\right)+a^3\cos\left(dx+c\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-a^3\cos\left(dx+c\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-2\,a^3}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d*x*cos(d*x + c) + a^3*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2)
 - a^3*cos(d*x + c)*log(-1/2*cos(d*x + c) + 1/2) - 2*a^3 - 6*a*b^2 - 2*(3*a
^2*b + b^3)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.21, size = 86, normalized size = 1.10

$$-\frac{(dx+c)b^3 - a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3+3\,ab^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((d*x + c)*b^3 - a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 2*(3*a^2*b*tan(1/2*d
*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c) + a^3 + 3*a*b^2)/(tan(1/2*d*x + 1/2*
c)^2 - 1))/d
```

maple [A] time = 0.60, size = 100, normalized size = 1.28

$$\frac{a^3}{d\cos(dx+c)}+\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{3a^2b\tan(dx+c)}{d}+\frac{3a\,b^2}{d\cos(dx+c)}-b^3x+\frac{b^3\tan(dx+c)}{d}-\frac{b^3c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*a^3/cos(d*x+c)+1/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+3*a^2*b*tan(d*x+c)/d+3
/d*a*b^2/cos(d*x+c)-b^3*x+b^3*tan(d*x+c)/d-1/d*b^3*c
```

maxima [A] time = 0.41, size = 86, normalized size = 1.10

$$-\frac{2\,(dx+c-\tan(dx+c))b^3-a^3\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-6\,a^2b\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c - tan(d*x + c))*b^3 - a^3*(2/cos(d*x + c) - log(cos(d*x +
c) + 1) + log(cos(d*x + c) - 1)) - 6*a^2*b*tan(d*x + c) - 6*a*b^2/cos(d*x +
 c))/d
```

mupad [B] time = 11.93, size = 154, normalized size = 1.97

$$\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a^2\,b+2\,b^3\right)+6\,a\,b^2+2\,a^3}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}+\frac{2\,b^3\,\mathrm{atan}\left(\frac{4\,b^6}{4\,a^3\,b^3+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^6}-\frac{4\,a^3\,b^3\,\tan\left(\frac{c}{2}\right.}{4\,a^3\,b^3+4\,\tan\left(\frac{c}{2}\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)*(6*a^2*b + 2*b^3) + 6
*a*b^2 + 2*a^3)/(d*(tan(c/2 + (d*x)/2)^2 - 1)) + (2*b^3*atan((4*b^6)/(4*a^3
```

```
*b^3 + 4*b^6*tan(c/2 + (d*x)/2)) - (4*a^3*b^3*tan(c/2 + (d*x)/2))/(4*a^3*b^
3 + 4*b^6*tan(c/2 + (d*x)/2))))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1461 $\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=87

$$\frac{a^3\tan(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}$$

```
[Out] -3*a^2*b*arctanh(cos(d*x+c))/d-a^3*cot(d*x+c)/d+3*a^2*b*sec(d*x+c)/d+b^3*se
c(d*x+c)/d+a^3*tan(d*x+c)/d+3*a*b^2*tan(d*x+c)/d
```

Rubi [A] time = 0.20, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2912, 3767, 8, 2622, 321, 207, 2620, 14, 2606}

$$\frac{3a^2b\sec(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3\tan(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^2*b*ArcTanh[Cos[c + d*x]])/d - (a^3*Cot[c + d*x])/d + (3*a^2*b*Sec[c
+ d*x])/d + (b^3*Sec[c + d*x])/d + (a^3*Tan[c + d*x])/d + (3*a*b^2*Tan[c +
d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(3ab^2\sec^2(c+dx)+3a^2b\csc(c+dx)\sec^2(c+dx)+a^3\right.$$
$$= a^3\int \csc^2(c+dx)\sec^2(c+dx)\,dx + \left(3a^2b\right)\int \csc(c+dx)$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} + \frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x}{-1}\right.}{}$$
$$= \frac{3a^2b\sec(c+dx)}{d} + \frac{b^3\sec(c+dx)}{d} + \frac{3ab^2\tan(c+dx)}{d} + $$
$$= -\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^3\cot(c+dx)}{d} + \frac{3a^2b\sec(c}{d}$$

Mathematica [A] time = 0.40, size = 114, normalized size = 1.31

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\left(2a^3+3ab^2\right)\cos(2(c+dx))-2b\left(3a^2+b^2\right)\sin(c+dx)-3ab\left(a\,s\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/4*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*Sec[c + d*x]*((2*a^3 + 3*a*b^2)*Cos
[2*(c + d*x)] - 2*b*(3*a^2 + b^2)*Sin[c + d*x] - 3*a*b*(b + a*(-Log[Cos[(c
+ d*x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[2*(c + d*x)])))/d
```

fricas [A] time = 0.43, size = 131, normalized size = 1.51

$$-\frac{3\,a^2b\cos(dx+c)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-3\,a^2b\cos(dx+c)\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+}{2\,d\cos(dx+c)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/2*(3*a^2*b*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*a^2
*b*cos(d*x + c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*a^3 - 6*a*b^2
 + 2*(2*a^3 + 3*a*b^2)*cos(d*x + c)^2 - 2*(3*a^2*b + b^3)*sin(d*x + c))/(d*
cos(d*x + c)*sin(d*x + c))
```

giac [A] time = 0.22, size = 148, normalized size = 1.70

$$\frac{6\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{2\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+10\,a^2b\tan\left(\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^2*b*log(abs(tan(1/2*d*x + 1/2*c))) + a^3*tan(1/2*d*x + 1/2*c) - (2
*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan
(1/2*d*x + 1/2*c)^2 + 10*a^2*b*tan(1/2*d*x + 1/2*c) + 4*b^3*tan(1/2*d*x + 1
/2*c) - a^3)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

maple [A] time = 0.65, size = 111, normalized size = 1.28

$$\frac{a^3}{d\sin(dx+c)\cos(dx+c)}-\frac{2a^3\cot(dx+c)}{d}+\frac{3a^2b}{d\cos(dx+c)}+\frac{3a^2b\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{3a\,b^2\tan(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*a^3/sin(d*x+c)/cos(d*x+c)-2*a^3*cot(d*x+c)/d+3/d*a^2*b/cos(d*x+c)+3/d*a
^2*b*ln(csc(d*x+c)-cot(d*x+c))+3*a*b^2*tan(d*x+c)/d+1/d*b^3/cos(d*x+c)
```

maxima [A] time = 0.38, size = 90, normalized size = 1.03

$$\frac{3\,a^2b\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-2\,a^3\left(\frac{1}{\tan(dx+c)}-\tan(dx+c)\right)+6\,ab^2\tan(dx+}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/2*(3*a^2*b*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1
)) - 2*a^3*(1/tan(d*x + c) - tan(d*x + c)) + 6*a*b^2*tan(d*x + c) + 2*b^3/c
os(d*x + c))/d
```

mupad [B] time = 11.91, size = 120, normalized size = 1.38

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(12\,a^2\,b+4\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(5\,a^3+12\,a\,b^2\right)-a^3}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+\frac{3\,a^2\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)^2),x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2))/(2*d) + (tan(c/2 + (d*x)/2)*(12*a^2*b + 4*b^3) + t
an(c/2 + (d*x)/2)^2*(12*a*b^2 + 5*a^3) - a^3)/(d*(2*tan(c/2 + (d*x)/2) - 2*
tan(c/2 + (d*x)/2)^3)) + (3*a^2*b*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1462 $\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=132

$$\frac{3a^3\sec(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{3a^2b\tan(c+dx)}{d}-\frac{3a^2b\cot(c+dx)}{d}+\frac{3ab}{}$$

[Out] -3/2*a^3*arctanh(cos(d*x+c))/d-3*a*b^2*arctanh(cos(d*x+c))/d-3*a^2*b*cot(d*x+c)/d+3/2*a^3*sec(d*x+c)/d+3*a*b^2*sec(d*x+c)/d-1/2*a^3*csc(d*x+c)^2*sec(d*x+c)/d+3*a^2*b*tan(d*x+c)/d+b^3*tan(d*x+c)/d

Rubi [A] time = 0.23, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2912, 3767, 8, 2622, 321, 207, 2620, 14, 288}

$$\frac{3a^2b\tan(c+dx)}{d}-\frac{3a^2b\cot(c+dx)}{d}+\frac{3a^3\sec(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{3ab}{}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]

[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*b^2*ArcTanh[Cos[c + d*x]])/d - (3*a^2*b*Cot[c + d*x])/d + (3*a^3*Sec[c + d*x])/(2*d) + (3*a*b^2*Sec[c + d*x])/d - (a^3*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (3*a^2*b*Tan[c + d*x])/d + (b^3*Tan[c + d*x])/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(b^3\sec^2(c+dx)+3ab^2\csc(c+dx)\sec^2(c+dx)+3a^2b\,\mathrm{c}\right.$$
$$= a^3\int\csc^3(c+dx)\sec^2(c+dx)\,dx+\left(3a^2b\right)\int\csc^2(c+dx)\,\mathrm{s}$$
$$= \frac{a^3\,\mathrm{Subst}\left(\int\frac{x^4}{\left(-1+x^2\right)^2}\,dx,x,\sec(c+dx)\right)}{d}+\frac{\left(3a^2b\right)\mathrm{Subst}\left(\int\right.}{}$$
$$= \frac{3ab^2\sec(c+dx)}{d}-\frac{a^3\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{b^3\tan(c+}{d}$$
$$= -\frac{3ab^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^2b\cot(c+dx)}{d}+\frac{3a^3\sec(c}{2d}$$
$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3ab^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a}{}$$

Mathematica [B] time = 0.57, size = 267, normalized size = 2.02

$$\frac{\csc^4(c+dx)\left(-6\left(a^3+2ab^2\right)\cos(2(c+dx))+3a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-3a^3\cos(3(c+dx))\log\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^4*(2*a^3 + 12*a*b^2 - 6*(a^3 + 2*a*b^2)*Cos[2*(c + d*x)] + 3*
a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 6*a*b^2*Cos[3*(c + d*x)]*Log[C
os[(c + d*x)/2]] - 3*a*(a^2 + 2*b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2]] -
Log[Sin[(c + d*x)/2]]) - 3*a^3*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 6*a
*b^2*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 12*a^2*b*Sin[c + d*x] + 6*b^3
*Sin[c + d*x] - 12*a^2*b*Sin[3*(c + d*x)] - 2*b^3*Sin[3*(c + d*x)]))/(2*d*(
Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2))
```

fricas [A] time = 0.44, size = 196, normalized size = 1.48

$$-\frac{4\,a^3+12\,ab^2-6\left(a^3+2\,ab^2\right)\cos\left(dx+c\right)^2+3\left(\left(a^3+2\,ab^2\right)\cos\left(dx+c\right)^3-\left(a^3+2\,ab^2\right)\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/4*(4*a^3 + 12*a*b^2 - 6*(a^3 + 2*a*b^2)*cos(d*x + c)^2 + 3*((a^3 + 2*a*b
^2)*cos(d*x + c)^3 - (a^3 + 2*a*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1
/2) - 3*((a^3 + 2*a*b^2)*cos(d*x + c)^3 - (a^3 + 2*a*b^2)*cos(d*x + c))*log
(-1/2*cos(d*x + c) + 1/2) + 4*(3*a^2*b + b^3 - (6*a^2*b + b^3)*cos(d*x + c)
^2)*sin(d*x + c))/(d*cos(d*x + c)^3 - d*cos(d*x + c))
```

giac [A] time = 0.26, size = 179, normalized size = 1.36

$$\frac{a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 12\,a^2 b \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 12\left(a^3 + 2\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - \frac{16\left(3\,a^2 b \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + b^3 \tan\right.}{\tan\left(\frac{1}{2}\,dx + \right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*b*tan(1/2*d*x + 1/2*c) + 12*(a^3 +
 2*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - 16*(3*a^2*b*tan(1/2*d*x + 1/2*c)
 + b^3*tan(1/2*d*x + 1/2*c) + a^3 + 3*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1) -
 (18*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*
b*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^2)/d
```

maple [A] time = 0.73, size = 161, normalized size = 1.22

$$-\frac{a^3}{2d\sin(dx+c)^2\cos(dx+c)} + \frac{3a^3}{2d\cos(dx+c)} + \frac{3a^3\ln(\csc(dx+c) - \cot(dx+c))}{2d} + \frac{3a^2 b}{d\sin(dx+c)\cos(dx+c)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3/sin(d*x+c)^2/cos(d*x+c)+3/2/d*a^3/cos(d*x+c)+3/2/d*a^3*ln(csc(d*
x+c)-cot(d*x+c))+3/d*a^2*b/sin(d*x+c)/cos(d*x+c)-6*a^2*b*cot(d*x+c)/d+3/d*a
*b^2/cos(d*x+c)+3/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))+b^3*tan(d*x+c)/d
```

maxima [A] time = 0.33, size = 137, normalized size = 1.04

$$\frac{a^3\left(\frac{2\left(3\,\cos(dx+c)^2 - 2\right)}{\cos(dx+c)^3 - \cos(dx+c)} - 3\,\log\left(\cos\left(dx + c\right) + 1\right) + 3\,\log\left(\cos\left(dx + c\right) - 1\right)\right) + 6\,ab^2\left(\frac{2}{\cos(dx+c)} - \log\left(\cos\left(dx + c\right) + \right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*(a^3*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(
cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 6*a*b^2*(2/cos(d*x + c) - lo
g(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 12*a^2*b*(1/tan(d*x + c) - t
an(d*x + c)) + 4*b^3*tan(d*x + c))/d
```

mupad [B] time = 11.91, size = 166, normalized size = 1.26

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^3}{2}+3\,a\,b^2\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{17\,a^3}{2}+24\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(30\,a^2\,b+8\right.}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)^3),x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (log(tan(c/2 + (d*x)/2))*(3*a*b^2 + (3*a
^3)/2))/d + (tan(c/2 + (d*x)/2)^2*(24*a*b^2 + (17*a^3)/2) + tan(c/2 + (d*x)
/2)^3*(30*a^2*b + 8*b^3) - a^3/2 - 6*a^2*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/
2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^4)) + (3*a^2*b*tan(c/2 + (d*x)/2))/(2
*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1463 $\int \csc^4(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=164

$$\frac{a^3\tan(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{2a^3\cot(c+dx)}{d}+\frac{9a^2b\sec(c+dx)}{2d}-\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^2b\csc^2(c+d}{2d}$$

```
[Out] -9/2*a^2*b*arctanh(cos(d*x+c))/d-b^3*arctanh(cos(d*x+c))/d-2*a^3*cot(d*x+c)
/d-3*a*b^2*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d+9/2*a^2*b*sec(d*x+c)/d+b^3*s
ec(d*x+c)/d-3/2*a^2*b*csc(d*x+c)^2*sec(d*x+c)/d+a^3*tan(d*x+c)/d+3*a*b^2*ta
n(d*x+c)/d
```

Rubi [A] time = 0.26, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2912, 2622, 321, 207, 2620, 14, 288, 270}

$$\frac{9a^2b\sec(c+dx)}{2d}-\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^2b\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{a^3\tan(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-9*a^2*b*ArcTanh[Cos[c + d*x]])/(2*d) - (b^3*ArcTanh[Cos[c + d*x]])/d - (2
*a^3*Cot[c + d*x])/d - (3*a*b^2*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d
) + (9*a^2*b*Sec[c + d*x])/(2*d) + (b^3*Sec[c + d*x])/d - (3*a^2*b*Csc[c +
d*x]^2*Sec[c + d*x])/(2*d) + (a^3*Tan[c + d*x])/d + (3*a*b^2*Tan[c + d*x])/
d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rubi steps

$$\int \csc^4(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(b^3\csc(c+dx)\sec^2(c+dx) + 3ab^2\csc^2(c+dx)\sec^2(c\right.$$

$$= a^3\int \csc^4(c+dx)\sec^2(c+dx)\,dx + \left(3a^2b\right)\int \csc^3(c+dx)$$

$$= \frac{a^3\,\mathrm{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4}\,dx, x, \tan(c+dx)\right)}{d} + \frac{\left(3a^2b\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{b^3\sec(c+dx)}{d} - \frac{3a^2b\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{a^3\,\mathrm{Subst}}{}$$

$$= -\frac{b^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{2a^3\cot(c+dx)}{d} - \frac{3ab^2\cot(c+}{d}$$

$$= -\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d} - \frac{b^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{2a}{}$$

Mathematica [A] time = 1.36, size = 287, normalized size = 1.75

$$\frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-8\left(4a^3+9ab^2\right)\cos(2(c+dx)) + 4\left(4a^3+9ab^2\right)\cos(4(c+dx)) + 3b\left(6\left(5a^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]^3*(-8*(4*a^3 + 9*a*b^2)*Cos[2*(c + d*x
)] + 4*(4*a^3 + 9*a*b^2)*Cos[4*(c + d*x)] + 3*b*(12*a*b + 6*(5*a^2 + 2*b^2)
*Sin[c + d*x] - 2*(9*a^2 + 2*b^2)*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x
)/2]])*Sin[2*(c + d*x)] - 18*a^2*Sin[3*(c + d*x)] - 4*b^2*Sin[3*(c + d*x)]
+ 9*a^2*Log[Cos[(c + d*x)/2]]*Sin[4*(c + d*x)] + 2*b^2*Log[Cos[(c + d*x)/2]
]*Sin[4*(c + d*x)] - 9*a^2*Log[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)] - 2*b^2*L
og[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)])))/(384*d*(-1 + Cot[(c + d*x)/2]^2))
```

fricas [A] time = 0.43, size = 252, normalized size = 1.54

$$-\frac{8\left(4\,a^3+9\,ab^2\right)\cos\left(dx+c\right)^4 + 12\,a^3 + 36\,ab^2 - 12\left(4\,a^3+9\,ab^2\right)\cos\left(dx+c\right)^2 + 3\left(\left(9\,a^2b+2\,b^3\right)\cos\left(dx+c\right)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(8*(4*a^3 + 9*a*b^2)*cos(d*x + c)^4 + 12*a^3 + 36*a*b^2 - 12*(4*a^3 +
 9*a*b^2)*cos(d*x + c)^2 + 3*((9*a^2*b + 2*b^3)*cos(d*x + c)^3 - (9*a^2*b +
 2*b^3)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*((9*a^2*
b + 2*b^3)*cos(d*x + c)^3 - (9*a^2*b + 2*b^3)*cos(d*x + c))*log(-1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) + 6*(6*a^2*b + 2*b^3 - (9*a^2*b + 2*b^3)*cos(d*x
 + c)^2)*sin(d*x + c))/((d*cos(d*x + c)^3 - d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.27, size = 245, normalized size = 1.49

$$\frac{a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 9\,a^2 b \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 21\,a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 36\,ab^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 12\left(9\,a^2 b + 2\,b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 21*a^3*
tan(1/2*d*x + 1/2*c) + 36*a*b^2*tan(1/2*d*x + 1/2*c) + 12*(9*a^2*b + 2*b^3)
*log(abs(tan(1/2*d*x + 1/2*c))) - 48*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*ta
n(1/2*d*x + 1/2*c) + 3*a^2*b + b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (198*a^2
*b*tan(1/2*d*x + 1/2*c)^3 + 44*b^3*tan(1/2*d*x + 1/2*c)^3 + 21*a^3*tan(1/2*
d*x + 1/2*c)^2 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 9*a^2*b*tan(1/2*d*x + 1/
2*c) + a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.63, size = 209, normalized size = 1.27

$$-\frac{a^3}{3d\sin(dx+c)^3\cos(dx+c)}+\frac{4a^3}{3d\sin(dx+c)\cos(dx+c)}-\frac{8a^3\cot(dx+c)}{3d}-\frac{3a^2b}{2d\sin(dx+c)^2\cos(dx+c)}+\frac{}{2d\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/3/d*a^3/sin(d*x+c)^3/cos(d*x+c)+4/3/d*a^3/sin(d*x+c)/cos(d*x+c)-8/3*a^3*
cot(d*x+c)/d-3/2/d*a^2*b/sin(d*x+c)^2/cos(d*x+c)+9/2/d*a^2*b/cos(d*x+c)+9/2
/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))+3/d*a*b^2/sin(d*x+c)/cos(d*x+c)-6*a*b^2*
cot(d*x+c)/d+1/d*b^3/cos(d*x+c)+1/d*b^3*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.37, size = 162, normalized size = 0.99

$$\frac{9\,a^2 b\left(\frac{2\left(3\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)} - 3\,\log\left(\cos\left(dx+c\right)+1\right) + 3\,\log\left(\cos\left(dx+c\right)-1\right)\right) + 6\,b^3\left(\frac{2}{\cos(dx+c)} - \log\left(\cos\left(dx+c\right)\right.\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(9*a^2*b*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3
*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 6*b^3*(2/cos(d*x + c) -
 log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 36*a*b^2*(1/tan(d*x + c)
- tan(d*x + c)) - 4*a^3*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 - 3*tan(d*x
+ c)))/d
```

mupad [B] time = 11.89, size = 218, normalized size = 1.33

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{9\,a^2\,b}{2}+b^3\right)}{d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{20\,a^3}{3}+12\,a\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(23\,a^3+60\,a \cdots\right.}{d\,\left(8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right. \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)^4),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))*((9*a^2*b)/2 + b^3))/d + (a^3*tan(c/2 + (d*x)/2)^3
)/(24*d) - (tan(c/2 + (d*x)/2)^2*(12*a*b^2 + (20*a^3)/3) - tan(c/2 + (d*x)/
2)^4*(60*a*b^2 + 23*a^3) - tan(c/2 + (d*x)/2)^3*(51*a^2*b + 16*b^3) + a^3/3
 + 3*a^2*b*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 - 8*tan(c/2 + (d*
x)/2)^5)) + (tan(c/2 + (d*x)/2)*((3*a*b^2)/2 + (7*a^3)/8))/d + (3*a^2*b*tan
(c/2 + (d*x)/2)^2)/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1464 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=222

$$-\frac{2a^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{5/2}}-\frac{a^4\cos(c+dx)}{bd\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{4a^3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{5/2}}+\frac{\text{co}}{2d(a+b)^2}$$

```
[Out] -x/b^2-2*a^5*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^2/(a^2-b^2)
^(5/2)/d+4*a^3*(a^2-2*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))
/b^2/(a^2-b^2)^(5/2)/d+1/2*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))-1/2*cos(d*x+
c)/(a-b)^2/d/(1+sin(d*x+c))-a^4*cos(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.36, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2897, 2648, 2664, 12, 2660, 618, 204}

$$-\frac{2a^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{5/2}}+\frac{4a^3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{5/2}}-\frac{a^4\cos(c+dx)}{bd\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\text{co}}{2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(x/b^2) - (2*a^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a
^2 - b^2)^(5/2)*d) + (4*a^3*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1
 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^4*
Cos[c + d*x])/(b*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(-\frac{1}{b^2} - \frac{1}{2(a+b)^2(-1+\sin(c+dx))} + \frac{1}{2(a-b)^2(1+\sin(c+dx))} + \frac{}{b^2\left(-a^2\right.}\right.$$

$$= -\frac{x}{b^2} + \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2} + \frac{\left(2a^3\left(a^2-2b^2\right)\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^2\left(a^2-b^2\right)^2}$$

$$= -\frac{x}{b^2} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} - \frac{a^4}{b\left(a^2-b^2\right)^2}$$

$$= -\frac{x}{b^2} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} - \frac{a^4}{b\left(a^2-b^2\right)^2}$$

$$= -\frac{x}{b^2} + \frac{4a^3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{}{2}$$

$$= -\frac{x}{b^2} + \frac{4a^3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{}{2}$$

$$= -\frac{x}{b^2} - \frac{2a^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}d} + \frac{4a^3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}d} + \frac{}{2(}$$

Mathematica [A] time = 1.98, size = 236, normalized size = 1.06

$$\frac{-\frac{a^4\cos(c+dx)}{b(a-b)^2(a+b)^2(a+b\sin(c+dx))} - \frac{a^4(c+dx)-2a^2b^2(c+dx)+2ab^3+b^4(c+dx)}{\left(b^3-a^2b\right)^2} + \frac{2a^3\left(a^2-4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}} + \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}\right.\right.}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-((2*a*b^3 + a^4*(c + d*x) - 2*a^2*b^2*(c + d*x) + b^4*(c + d*x))/(-(a^2*b
) + b^3)^2) + (2*a^3*(a^2 - 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
```

```
 - b^2]])/(b^2*(a^2 - b^2)^(5/2)) + Sin[(c + d*x)/2]/((a + b)^2*(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/((a - b)^2*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])) - (a^4*Cos[c + d*x])/((a - b)^2*b*(a + b)^2*(a + b*Si
n[c + d*x])))/d
```

fricas [A] time = 0.51, size = 724, normalized size = 3.26

$$\left[-\frac{2\,a^{4}b^{3}-4\,a^{2}b^{5}+2\,b^{7}+2\left(a^{7}-3\,a^{5}b^{2}+3\,a^{3}b^{4}-ab^{6}\right)dx\cos\left(dx+c\right)+2\left(a^{6}b-b^{7}\right)\cos\left(dx+c\right)^{2}-\left(\left(a^{5}b-4\,a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*(2*a^4*b^3 - 4*a^2*b^5 + 2*b^7 + 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b
^6)*d*x*cos(d*x + c) + 2*(a^6*b - b^7)*cos(d*x + c)^2 - ((a^5*b - 4*a^3*b^3
)*cos(d*x + c)*sin(d*x + c) + (a^6 - 4*a^4*b^2)*cos(d*x + c))*sqrt(-a^2 + b
^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2
*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^5*b^2 - 2*a^3*b^4 + a*
b^6 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*x*cos(d*x + c))*sin(d*x + c))
/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^
7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*cos(d*x + c)), -(a^4*b^3 - 2*a^2*b
^5 + b^7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b
- b^7)*cos(d*x + c)^2 + ((a^5*b - 4*a^3*b^3)*cos(d*x + c)*sin(d*x + c) + (a
^6 - 4*a^4*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/
(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^5*b^2 - 2*a^3*b^4 + a*b^6 - (a^6*b - 3
*a^4*b^3 + 3*a^2*b^5 - b^7)*d*x*cos(d*x + c))*sin(d*x + c))/((a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^7*b^2 - 3*a^5*b^
4 + 3*a^3*b^6 - a*b^8)*d*cos(d*x + c))]
```

giac [A] time = 0.27, size = 264, normalized size = 1.19

$$\frac{\frac{2\left(a^{5}-4\,a^{3}b^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{4}b^{2}-2\,a^{2}b^{4}+b^{6}\right)\sqrt{a^{2}-b^{2}}}-\frac{2\left(2\,a^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+ab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,b\tan\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(a^5 - 4*a^3*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^4*b^2 - 2*a^2*b^4 + b^6)*sqr
t(a^2 - b^2)) - 2*(2*a^3*b*tan(1/2*d*x + 1/2*c)^3 + a*b^3*tan(1/2*d*x + 1/2
*c)^3 + a^4*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x + 1/2*c)^2 - 3*a*b^3
*tan(1/2*d*x + 1/2*c) - a^4 - 2*a^2*b^2)/((a^4*b - 2*a^2*b^3 + b^5)*(a*tan(
1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)
- a)) - (d*x + c)/b^2)/d
```

maple [A] time = 0.55, size = 303, normalized size = 1.36

$$-\frac{1}{d\,(a+b)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^{2}}-\frac{1}{d\,(a-b)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a^{3}}{d\,(a-b)^{2}\,(a+b)^{2}\left(\left(\tan^{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-2/d/b^2*arctan(tan(1/2*d*x+1/2*c))-1/d/
(a-b)^2/(tan(1/2*d*x+1/2*c)+1)-2/d*a^3/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-2/d*a^4/b/(a-b)^2/(a+b)^2/
(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+2/d*a^5/b^2/(a-b)^2/(a+b)
^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
-8/d*a^3/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 19.77, size = 4797, normalized size = 21.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c + d*x)^4/(cos(c + d*x)^2*(a + b*sin(c + d*x))^2),x)

[Out] ((2*tan(c/2 + (d*x)/2)^3*(a*b^2 + 2*a^3))/(a^4 + b^4 - 2*a^2*b^2) + (2*tan(
c/2 + (d*x)/2)^2*(a^4 + 2*b^4))/(b*(a^4 + b^4 - 2*a^2*b^2)) - (2*a^2*(a^2 +
 2*b^2))/(b*(a^2 - b^2)^2) - (6*a*b^2*tan(c/2 + (d*x)/2))/(a^2 - b^2)^2)/(d
*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (d*x)
/2)^3)) - (2*atan((64*a*b^23*tan(c/2 + (d*x)/2))/(64*a*b^23 - 704*a^3*b^21
+ 3520*a^5*b^19 - 9536*a^7*b^17 + 14464*a^9*b^15 - 11072*a^11*b^13 + 1024*a
^13*b^11 + 5440*a^15*b^9 - 4544*a^17*b^7 + 1536*a^19*b^5 - 192*a^21*b^3) -
(704*a^3*b^21*tan(c/2 + (d*x)/2))/(64*a*b^23 - 704*a^3*b^21 + 3520*a^5*b^19
 - 9536*a^7*b^17 + 14464*a^9*b^15 - 11072*a^11*b^13 + 1024*a^13*b^11 + 5440
*a^15*b^9 - 4544*a^17*b^7 + 1536*a^19*b^5 - 192*a^21*b^3) + (3520*a^5*b^19*
tan(c/2 + (d*x)/2))/(64*a*b^23 - 704*a^3*b^21 + 3520*a^5*b^19 - 9536*a^7*b^
17 + 14464*a^9*b^15 - 11072*a^11*b^13 + 1024*a^13*b^11 + 5440*a^15*b^9 - 45
44*a^17*b^7 + 1536*a^19*b^5 - 192*a^21*b^3) - (9536*a^7*b^17*tan(c/2 + (d*x
)/2))/(64*a*b^23 - 704*a^3*b^21 + 3520*a^5*b^19 - 9536*a^7*b^17 + 14464*a^9
*b^15 - 11072*a^11*b^13 + 1024*a^13*b^11 + 5440*a^15*b^9 - 4544*a^17*b^7 +
1536*a^19*b^5 - 192*a^21*b^3) + (14464*a^9*b^15*tan(c/2 + (d*x)/2))/(64*a*b
^23 - 704*a^3*b^21 + 3520*a^5*b^19 - 9536*a^7*b^17 + 14464*a^9*b^15 - 11072
*a^11*b^13 + 1024*a^13*b^11 + 5440*a^15*b^9 - 4544*a^17*b^7 + 1536*a^19*b^5
 - 192*a^21*b^3) - (11072*a^11*b^13*tan(c/2 + (d*x)/2))/(64*a*b^23 - 704*a^
3*b^21 + 3520*a^5*b^19 - 9536*a^7*b^17 + 14464*a^9*b^15 - 11072*a^11*b^13 +
 1024*a^13*b^11 + 5440*a^15*b^9 - 4544*a^17*b^7 + 1536*a^19*b^5 - 192*a^21*
b^3) + (1024*a^13*b^11*tan(c/2 + (d*x)/2))/(64*a*b^23 - 704*a^3*b^21 + 3520
*a^5*b^19 - 9536*a^7*b^17 + 14464*a^9*b^15 - 11072*a^11*b^13 + 1024*a^13*b^
11 + 5440*a^15*b^9 - 4544*a^17*b^7 + 1536*a^19*b^5 - 192*a^21*b^3) + (5440*
a^15*b^9*tan(c/2 + (d*x)/2))/(64*a*b^23 - 704*a^3*b^21 + 3520*a^5*b^19 - 95
36*a^7*b^17 + 14464*a^9*b^15 - 11072*a^11*b^13 + 1024*a^13*b^11 + 5440*a^15
*b^9 - 4544*a^17*b^7 + 1536*a^19*b^5 - 192*a^21*b^3) - (4544*a^17*b^7*tan(c
/2 + (d*x)/2))/(64*a*b^23 - 704*a^3*b^21 + 3520*a^5*b^19 - 9536*a^7*b^17 +
14464*a^9*b^15 - 11072*a^11*b^13 + 1024*a^13*b^11 + 5440*a^15*b^9 - 4544*a^
17*b^7 + 1536*a^19*b^5 - 192*a^21*b^3) + (1536*a^19*b^5*tan(c/2 + (d*x)/2))
/(64*a*b^23 - 704*a^3*b^21 + 3520*a^5*b^19 - 9536*a^7*b^17 + 14464*a^9*b^15
 - 11072*a^11*b^13 + 1024*a^13*b^11 + 5440*a^15*b^9 - 4544*a^17*b^7 + 1536*
a^19*b^5 - 192*a^21*b^3) - (192*a^21*b^3*tan(c/2 + (d*x)/2))/(64*a*b^23 - 7
04*a^3*b^21 + 3520*a^5*b^19 - 9536*a^7*b^17 + 14464*a^9*b^15 - 11072*a^11*b
^13 + 1024*a^13*b^11 + 5440*a^15*b^9 - 4544*a^17*b^7 + 1536*a^19*b^5 - 192*
a^21*b^3)))/(b^2*d) + (a^3*atan(((a^3*(a - 2*b)*(a + 2*b)*(-(a + b)^5*(a -
b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(64*a*b^25 - 672*a^3*b^23 + 3200*a^5*b^21 -
 9632*a^7*b^19 + 20608*a^9*b^17 - 32096*a^11*b^15 + 35776*a^13*b^13 - 27680
*a^15*b^11 + 14272*a^17*b^9 - 4608*a^19*b^7 + 832*a^21*b^5 - 64*a^23*b^3) +
 32*a^2*b^24 - 320*a^4*b^22 + 1440*a^6*b^20 - 3840*a^8*b^18 + 6720*a^10*b^1
6 - 8064*a^12*b^14 + 6720*a^14*b^12 - 3840*a^16*b^10 + 1440*a^18*b^8 - 320*
a^20*b^6 + 32*a^22*b^4 + (a^3*(a - 2*b)*(a + 2*b)*(-(a + b)^5*(a - b)^5)^(1
/2)*(tan(c/2 + (d*x)/2)*(256*a^4*b^24 - 2112*a^6*b^22 + 7680*a^8*b^20 - 161

28*a^10*b^18 + 21504*a^12*b^16 - 18816*a^14*b^14 + 10752*a^16*b^12 - 3840*a
^18*b^10 + 768*a^20*b^8 - 64*a^22*b^6) - 32*a*b^27 + 352*a^3*b^25 - 1632*a^
5*b^23 + 4224*a^7*b^21 - 6720*a^9*b^19 + 6720*a^11*b^17 - 4032*a^13*b^15 +
1152*a^15*b^13 + 96*a^17*b^11 - 160*a^19*b^9 + 32*a^21*b^7 + (a^3*(a - 2*b)
*(a + 2*b)*(-(a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(96*a*b^29 - 10
24*a^3*b^27 + 4960*a^5*b^25 - 14400*a^7*b^23 + 27840*a^9*b^21 - 37632*a^11*
b^19 + 36288*a^13*b^17 - 24960*a^15*b^15 + 12000*a^17*b^13 - 3840*a^19*b^11
 + 736*a^21*b^9 - 64*a^23*b^7) + 32*a^2*b^28 - 320*a^4*b^26 + 1440*a^6*b^24
 - 3840*a^8*b^22 + 6720*a^10*b^20 - 8064*a^12*b^18 + 6720*a^14*b^16 - 3840*
a^16*b^14 + 1440*a^18*b^12 - 320*a^20*b^10 + 32*a^22*b^8))/(b^12 - 5*a^2*b^
10 + 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))/(b^12 - 5*a^2*b^10 +
 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2))*1i)/(b^12 - 5*a^2*b^10 +
10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2) + (a^3*(a - 2*b)*(a + 2*b)*
(-(a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(64*a*b^25 - 672*a^3*b^23
+ 3200*a^5*b^21 - 9632*a^7*b^19 + 20608*a^9*b^17 - 32096*a^11*b^15 + 35776*
a^13*b^13 - 27680*a^15*b^11 + 14272*a^17*b^9 - 4608*a^19*b^7 + 832*a^21*b^5
 - 64*a^23*b^3) + 32*a^2*b^24 - 320*a^4*b^22 + 1440*a^6*b^20 - 3840*a^8*b^1
8 + 6720*a^10*b^16 - 8064*a^12*b^14 + 6720*a^14*b^12 - 3840*a^16*b^10 + 144
0*a^18*b^8 - 320*a^20*b^6 + 32*a^22*b^4 - (a^3*(a - 2*b)*(a + 2*b)*(-(a + b
)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(256*a^4*b^24 - 2112*a^6*b^22 + 76
80*a^8*b^20 - 16128*a^10*b^18 + 21504*a^12*b^16 - 18816*a^14*b^14 + 10752*a
^16*b^12 - 3840*a^18*b^10 + 768*a^20*b^8 - 64*a^22*b^6) - 32*a*b^27 + 352*a
^3*b^25 - 1632*a^5*b^23 + 4224*a^7*b^21 - 6720*a^9*b^19 + 6720*a^11*b^17 -
4032*a^13*b^15 + 1152*a^15*b^13 + 96*a^17*b^11 - 160*a^19*b^9 + 32*a^21*b^7
 - (a^3*(a - 2*b)*(a + 2*b)*(-(a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2
)*(96*a*b^29 - 1024*a^3*b^27 + 4960*a^5*b^25 - 14400*a^7*b^23 + 27840*a^9*b
^21 - 37632*a^11*b^19 + 36288*a^13*b^17 - 24960*a^15*b^15 + 12000*a^17*b^13
 - 3840*a^19*b^11 + 736*a^21*b^9 - 64*a^23*b^7) + 32*a^2*b^28 - 320*a^4*b^2
6 + 1440*a^6*b^24 - 3840*a^8*b^22 + 6720*a^10*b^20 - 8064*a^12*b^18 + 6720*
a^14*b^16 - 3840*a^16*b^14 + 1440*a^18*b^12 - 320*a^20*b^10 + 32*a^22*b^8))
/(b^12 - 5*a^2*b^10 + 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))/(b^
12 - 5*a^2*b^10 + 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2))*1i)/(b^1
2 - 5*a^2*b^10 + 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2))/(2*tan(c/
2 + (d*x)/2)*(256*a^4*b^20 - 2112*a^6*b^18 + 7680*a^8*b^16 - 16128*a^10*b^1
4 + 21504*a^12*b^12 - 18816*a^14*b^10 + 10752*a^16*b^8 - 3840*a^18*b^6 + 76
8*a^20*b^4 - 64*a^22*b^2) + 256*a^5*b^19 - 1088*a^7*b^17 + 1024*a^9*b^15 +
2368*a^11*b^13 - 7040*a^13*b^11 + 7744*a^15*b^9 - 4352*a^17*b^7 + 1216*a^19
*b^5 - 128*a^21*b^3 + (a^3*(a - 2*b)*(a + 2*b)*(-(a + b)^5*(a - b)^5)^(1/2)
*(tan(c/2 + (d*x)/2)*(64*a*b^25 - 672*a^3*b^23 + 3200*a^5*b^21 - 9632*a^7*b
^19 + 20608*a^9*b^17 - 32096*a^11*b^15 + 35776*a^13*b^13 - 27680*a^15*b^11
+ 14272*a^17*b^9 - 4608*a^19*b^7 + 832*a^21*b^5 - 64*a^23*b^3) + 32*a^2*b^2
4 - 320*a^4*b^22 + 1440*a^6*b^20 - 3840*a^8*b^18 + 6720*a^10*b^16 - 8064*a^
12*b^14 + 6720*a^14*b^12 - 3840*a^16*b^10 + 1440*a^18*b^8 - 320*a^20*b^6 +
32*a^22*b^4 + (a^3*(a - 2*b)*(a + 2*b)*(-(a + b)^5*(a - b)^5)^(1/2)*(tan(c/
2 + (d*x)/2)*(256*a^4*b^24 - 2112*a^6*b^22 + 7680*a^8*b^20 - 16128*a^10*b^1

```
8 + 21504*a^12*b^16 - 18816*a^14*b^14 + 10752*a^16*b^12 - 3840*a^18*b^10 +
768*a^20*b^8 - 64*a^22*b^6) - 32*a*b^27 + 352*a^3*b^25 - 1632*a^5*b^23 + 42
24*a^7*b^21 - 6720*a^9*b^19 + 6720*a^11*b^17 - 4032*a^13*b^15 + 1152*a^15*b
^13 + 96*a^17*b^11 - 160*a^19*b^9 + 32*a^21*b^7 + (a^3*(a - 2*b)*(a + 2*b)*
(-(a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(96*a*b^29 - 1024*a^3*b^27
 + 4960*a^5*b^25 - 14400*a^7*b^23 + 27840*a^9*b^21 - 37632*a^11*b^19 + 3628
8*a^13*b^17 - 24960*a^15*b^15 + 12000*a^17*b^13 - 3840*a^19*b^11 + 736*a^21
*b^9 - 64*a^23*b^7) + 32*a^2*b^28 - 320*a^4*b^26 + 1440*a^6*b^24 - 3840*a^8
*b^22 + 6720*a^10*b^20 - 8064*a^12*b^18 + 6720*a^14*b^16 - 3840*a^16*b^14 +
 1440*a^18*b^12 - 320*a^20*b^10 + 32*a^22*b^8))/(b^12 - 5*a^2*b^10 + 10*a^4
*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))/(b^12 - 5*a^2*b^10 + 10*a^4*b^8
 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))/(b^12 - 5*a^2*b^10 + 10*a^4*b^8 - 1
0*a^6*b^6 + 5*a^8*b^4 - a^10*b^2) - (a^3*(a - 2*b)*(a + 2*b)*(-(a + b)^5*(a
 - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(64*a*b^25 - 672*a^3*b^23 + 3200*a^5*b^2
1 - 9632*a^7*b^19 + 20608*a^9*b^17 - 32096*a^11*b^15 + 35776*a^13*b^13 - 27
680*a^15*b^11 + 14272*a^17*b^9 - 4608*a^19*b^7 + 832*a^21*b^5 - 64*a^23*b^3
) + 32*a^2*b^24 - 320*a^4*b^22 + 1440*a^6*b^20 - 3840*a^8*b^18 + 6720*a^10*
b^16 - 8064*a^12*b^14 + 6720*a^14*b^12 - 3840*a^16*b^10 + 1440*a^18*b^8 - 3
20*a^20*b^6 + 32*a^22*b^4 - (a^3*(a - 2*b)*(a + 2*b)*(-(a + b)^5*(a - b)^5)
^(1/2)*(tan(c/2 + (d*x)/2)*(256*a^4*b^24 - 2112*a^6*b^22 + 7680*a^8*b^20 -
16128*a^10*b^18 + 21504*a^12*b^16 - 18816*a^14*b^14 + 10752*a^16*b^12 - 384
0*a^18*b^10 + 768*a^20*b^8 - 64*a^22*b^6) - 32*a*b^27 + 352*a^3*b^25 - 1632
*a^5*b^23 + 4224*a^7*b^21 - 6720*a^9*b^19 + 6720*a^11*b^17 - 4032*a^13*b^15
 + 1152*a^15*b^13 + 96*a^17*b^11 - 160*a^19*b^9 + 32*a^21*b^7 - (a^3*(a - 2
*b)*(a + 2*b)*(-(a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(96*a*b^29 -
 1024*a^3*b^27 + 4960*a^5*b^25 - 14400*a^7*b^23 + 27840*a^9*b^21 - 37632*a^
11*b^19 + 36288*a^13*b^17 - 24960*a^15*b^15 + 12000*a^17*b^13 - 3840*a^19*b
^11 + 736*a^21*b^9 - 64*a^23*b^7) + 32*a^2*b^28 - 320*a^4*b^26 + 1440*a^6*b
^24 - 3840*a^8*b^22 + 6720*a^10*b^20 - 8064*a^12*b^18 + 6720*a^14*b^16 - 38
40*a^16*b^14 + 1440*a^18*b^12 - 320*a^20*b^10 + 32*a^22*b^8))/(b^12 - 5*a^2
*b^10 + 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))/(b^12 - 5*a^2*b^1
0 + 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))/(b^12 - 5*a^2*b^10 +
10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))*(a - 2*b)*(a + 2*b)*(-(a
+ b)^5*(a - b)^5)^(1/2)*2i)/(d*(b^12 - 5*a^2*b^10 + 10*a^4*b^8 - 10*a^6*b^6
 + 5*a^8*b^4 - a^10*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**4*sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

3.1465 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=212

$$-\frac{2a^2\left(a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{5/2}}+\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{5/2}}+\frac{a^3\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\cos}{2d(a+b)^2(}$$

```
[Out] 2*a^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(5/2)/d-
2*a^2*(a^2-3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b/(a^2-b
^2)^(5/2)/d+1/2*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))+1/2*cos(d*x+c)/(a-b)^2/
d/(1+sin(d*x+c))+a^3*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.30, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2897, 2648, 2664, 12, 2660, 618, 204}

$$\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{5/2}}-\frac{2a^2\left(a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{5/2}}+\frac{a^3\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\cos(c}{2d(a+b)^2(1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(5/
2)*d) - (2*a^2*(a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(b*(a^2 - b^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x]
)) + Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) + (a^3*Cos[c + d*x])/(
(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int\left(-\frac{1}{2(a+b)^2(-1+\sin(c+dx))} - \frac{1}{2(a-b)^2(1+\sin(c+dx))} + \frac{a}{b\left(a^2-b^2\right)(a+ \cdots}\right.$$

$$= -\frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2} - \frac{\left(a^2\left(a^2-3b^2\right)\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)^2} + \frac{a^3\int \cdots}{\cdots}$$

$$= \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} + \frac{a^3\cos(c+\cdots}{\left(a^2-b^2\right)^2d(a+b\,\mathrm{s}\cdots}$$

$$= \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} + \frac{a^3\cos(c+\cdots}{\left(a^2-b^2\right)^2d(a+b\,\mathrm{s}\cdots}$$

$$= -\frac{2a^2\left(a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} + \frac{\cdots}{2(a-b\cdots}$$

$$= -\frac{2a^2\left(a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} + \frac{\cdots}{2(a-b\cdots}$$

$$= \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}d} - \frac{2a^2\left(a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}d} + \frac{\mathrm{co}\cdots}{2(a+b)^2d\cdots}$$

Mathematica [A] time = 0.98, size = 162, normalized size = 0.76

$$\frac{\frac{a^3\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))} + \frac{6a^2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{1}{(a-b)^2\left(\sin\left(\frac{1}{2}(c+dx\cdots\right.\right.}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((6*a^2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/
2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
```

```
- 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (a^3*Cos[c + d*x])
/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

fricas [A] time = 0.51, size = 534, normalized size = 2.52

$$\left[\frac{2\,a^5 - 4\,a^3b^2 + 2\,ab^4 + 2\left(a^5 + a^3b^2 - 2\,ab^4\right)\cos\left(dx + c\right)^2 - 3\left(a^2b^2\cos\left(dx + c\right)\sin\left(dx + c\right) + a^3b\cos\left(dx + c\right)\right)}{2\left(\left(a^6b - 3\,a^4b^3 + 3\,a^2b^5 - b^7\right)d\cos\left(dx + c\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(2*a^5 - 4*a^3*b^2 + 2*a*b^4 + 2*(a^5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c
)^2 - 3*(a^2*b^2*cos(d*x + c)*sin(d*x + c) + a^3*b*cos(d*x + c))*sqrt(-a^2
+ b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 +
 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^4*b - 2*a^2*b^3 + b^
5)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(
d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)), (a^5 - 2*
a^3*b^2 + a*b^4 + (a^5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c)^2 - 3*(a^2*b^2*cos
(d*x + c)*sin(d*x + c) + a^3*b*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin
(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^4*b - 2*a^2*b^3 + b^5)*
sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(d*x
 + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c))]
```

giac [A] time = 0.24, size = 222, normalized size = 1.05

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2b}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)\left(a^4-2\,a^2b^2+b^4\right)}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))*a^2*b/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
+ (3*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^2*tan(1/2*d*x + 1/2*c)^2 - a^2*b*
tan(1/2*d*x + 1/2*c) - 2*b^3*tan(1/2*d*x + 1/2*c) - 2*a^3 - a*b^2)/((a*tan(
1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)
- a)*(a^4 - 2*a^2*b^2 + b^4)))/d
```

maple [A] time = 0.53, size = 219, normalized size = 1.03

$$-\frac{1}{d\,(a+b)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{1}{d\,(a-b)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{d\,(a-b)^{2}\,(a+b)^{2}\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)+1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)+2/d*
a^2/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1
/2*d*x+1/2*c)*b+2/d*a^3/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)+6/d*a^2/(a-b)^2/(a+b)^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 15.28, size = 276, normalized size = 1.30

$$\frac{\frac{2\,a\,\left(2\,a^2+b^2\right)}{\left(a^2-b^2\right)^2}-\frac{6\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{\left(a^2-b^2\right)^2}+\frac{2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+2\,b^2\right)}{\left(a^2-b^2\right)^2}-\frac{6\,a\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4-2\,a^2\,b^2+b^4}}{d\,\left(-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}+\frac{6\,a^2\,b\,\mathrm{atan}\left(\frac{\frac{a^2\,b\,\left(2\,a^4\,b-4\,a^2\,b^3+2\,b^5\right)}{2\,(a+b)^{5/2}\,(a-b)^{5/2}}+\frac{a^3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4-\right.}{(a+b)^{5/2}\,(a-b}}{a^2\,b}\right.}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((2*a*(2*a^2 + b^2))/(a^2 - b^2)^2 - (6*a^2*b*tan(c/2 + (d*x)/2)^3)/(a^2 -
b^2)^2 + (2*b*tan(c/2 + (d*x)/2)*(a^2 + 2*b^2))/(a^2 - b^2)^2 - (6*a*b^2*ta
```

```
n(c/2 + (d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2)
 - a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (d*x)/2)^3)) + (6*a^2*b*atan(((a^
2*b*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/(2*(a + b)^(5/2)*(a - b)^(5/2)) + (a^3*b
*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(5/2)))
/(a^2*b)))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1466 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$-\frac{4ab^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{a^2b\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c}$$

```
[Out] -2*a^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d-4
*a*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d+1
/2*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))-1/2*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+
c))-a^2*b*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.31, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2731, 2648, 2664, 12, 2660, 618, 204}

$$-\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{4ab^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{a^2b\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2
)*d) - (4*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b
^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c + d*
x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^2*b*Cos[c + d*x])/((a^2 - b^2)^2
*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int\left(-\frac{1}{2(a+b)^2(-1+\sin(c+dx))}+\frac{1}{2(a-b)^2(1+\sin(c+dx))}-\frac{a^2}{\left(a^2-b^2\right)(a+b\sin\right.$$

$$=\frac{\int\frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2}-\frac{\int\frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2}-\frac{\left(2ab^2\right)\int\frac{1}{a+b\sin(c+dx)}\,dx}{\left(a^2-b^2\right)^2}-\frac{a^2\int\frac{1}{(a+b\sin(c+dx))^2}\,dx}{a^2-b^2}$$

$$=\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))}-\frac{a^2b\cos(c+dx)}{\left(a^2-b^2\right)^2d(a+b\sin(c\right.}$$

$$=\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))}-\frac{a^2b\cos(c+dx)}{\left(a^2-b^2\right)^2d(a+b\sin(c\right.}$$

$$=-\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+\right.}$$

$$=-\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+\right.}$$

$$=-\frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d}-\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+\right.}$$

Mathematica [A] time = 0.91, size = 169, normalized size = 0.84

$$\frac{-\frac{2a\left(a^2+2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{a^2b\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a-b)^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{}{(a+b)^2\left(\cos\right.}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*a*(a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(5/2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c +
```

```
d*x)/2])) + 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - (a^2*b*
Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

fricas [A] time = 0.50, size = 569, normalized size = 2.84

$$\left[-\frac{2\,a^4b-4\,a^2b^3+2\,b^5+2\left(2\,a^4b-a^2b^3-b^5\right)\cos\left(dx+c\right)^2+\left(\left(a^3b+2\,ab^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^4+2\right.\right.}{2\left(\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right)d\,c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*(2*a^4*b - 4*a^2*b^3 + 2*b^5 + 2*(2*a^4*b - a^2*b^3 - b^5)*cos(d*x +
c)^2 + ((a^3*b + 2*a*b^3)*cos(d*x + c)*sin(d*x + c) + (a^4 + 2*a^2*b^2)*cos
(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqr
t(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(
a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*co
s(d*x + c)), -(a^4*b - 2*a^2*b^3 + b^5 + (2*a^4*b - a^2*b^3 - b^5)*cos(d*x
+ c)^2 - ((a^3*b + 2*a*b^3)*cos(d*x + c)*sin(d*x + c) + (a^4 + 2*a^2*b^2)*c
os(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*
cos(d*x + c))) - (a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^
3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*d*cos(d*x + c))]
```

giac [A] time = 0.24, size = 251, normalized size = 1.26

$$-\frac{2\left(\frac{\left(a^3+2\,ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b^3\tan\left(\frac{1}{2}\,dx+\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b\tan\left(\frac{1}{2}\,dx\right.\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((a^3 + 2*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*ta
n(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2
 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a
^2*b*tan(1/2*d*x + 1/2*c)^2 + 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*
x + 1/2*c) - 4*a*b^2*tan(1/2*d*x + 1/2*c) - 3*a^2*b)/((a*tan(1/2*d*x + 1/2*
```

```
c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)*(a^4 - 2*
a^2*b^2 + b^4)))/d
```

maple [A] time = 0.51, size = 282, normalized size = 1.41

$$-\frac{1}{d\,(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{d\,(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,(a-b)^2\,(a+b)^2\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)-2/d*
a/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^2*tan
(1/2*d*x+1/2*c)-2/d*a^2/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)*b-2/d*a^3/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-4/d*a/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 15.96, size = 313, normalized size = 1.56

$$-\frac{\frac{6\,a^2\,b}{{\left(a^2-b^2\right)}^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(4\,a\,b^2-a^3\right)}{{\left(a^2-b^2\right)}^2}-\frac{2\,b\tan{\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\left(a^2+2\,b^2\right)}{a^4-2\,a^2\,b^2+b^4}-\frac{2\,a\tan{\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\left(a^2+2\,b^2\right)}{{\left(a^2-b^2\right)}^2}}{d\left(-a\tan{\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,b\tan{\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\frac{2\,a\,\mathrm{atan}\left(\frac{\frac{a\left(a^2+2\,b^2\right)\left(2\,a^4\,b-4\,a^2\,b^3+2\,b^5\right)}{(a+b)^{5/2}\,(a-b)^{5/2}}+\frac{2\,a^{2}\cdots}{\cdots}}{2\,a^3-\cdots}\right)}{d\,(a+b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] - ((6*a^2*b)/(a^2 - b^2)^2 + (2*tan(c/2 + (d*x)/2)*(4*a*b^2 - a^3))/(a^2 -
b^2)^2 - (2*b*tan(c/2 + (d*x)/2)^2*(a^2 + 2*b^2))/(a^4 + b^4 - 2*a^2*b^2) -
 (2*a*tan(c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a^2 - b^2)^2)/(d*(a + 2*b*tan(c/
2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (d*x)/2)^3)) - (2*a*a
tan(((a*(a^2 + 2*b^2)*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)
^(5/2)) + (2*a^2*tan(c/2 + (d*x)/2)*(a^2 + 2*b^2)*(a^4 + b^4 - 2*a^2*b^2))/
((a + b)^(5/2)*(a - b)^(5/2)))/(4*a*b^2 + 2*a^3))*(a^2 + 2*b^2))/(d*(a + b)
^(5/2)*(a - b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

3.1467 $\int \frac{\sec(c+dx)\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{2b\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{\sec(c+dx)\left(2a^2-3ab\sin(c+dx)+b^2\right)}{d\left(a^2-b^2\right)^2}-\frac{a\sec(c+dx)}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}$$

```
[Out] 2*b*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^
(5/2)/d-a*sec(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))+sec(d*x+c)*(2*a^2+b^2-3*a
*b*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.21, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2864, 2866, 12, 2660, 618, 204}

$$\frac{2b\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{\sec(c+dx)\left(2a^2-3ab\sin(c+dx)+b^2\right)}{d\left(a^2-b^2\right)^2}-\frac{a\sec(c+dx)}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2
- b^2)^(5/2)*d) - (a*Sec[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) + (
Sec[c + d*x]*(2*a^2 + b^2 - 3*a*b*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{a\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^2(c+dx)(b-2a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}$$

$$= -\frac{a\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(2a^2+b^2-3ab\sin(c+dx)\right)}{(a^2-b^2)^2\,d} + \frac{\int}{a}$$

$$= -\frac{a\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(2a^2+b^2-3ab\sin(c+dx)\right)}{(a^2-b^2)^2\,d} + \frac{(b\,(}{}$$

$$= -\frac{a\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(2a^2+b^2-3ab\sin(c+dx)\right)}{(a^2-b^2)^2\,d} + \frac{(2b}{}$$

$$= -\frac{a\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(2a^2+b^2-3ab\sin(c+dx)\right)}{(a^2-b^2)^2\,d} - \frac{(4b}{}$$

$$= \frac{2b\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d} - \frac{a\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\sec(c+a}{}$$

Mathematica [A] time = 0.86, size = 169, normalized size = 1.27

$$\frac{\frac{2b(2a^2+b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{ab^2\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{}{(a-b)^2\left(\sin\left(\frac{1}{2}\right.\right.}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*b*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2
- b^2)^(5/2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])) - 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (a*b^2*C
os[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

fricas [A] time = 0.47, size = 557, normalized size = 4.19

$$\left[\frac{2\,a^5-4\,a^3b^2+2\,ab^4+6\left(a^3b^2-ab^4\right)\cos\left(dx+c\right)^2-\left(\left(2\,a^2b^2+b^4\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(2\,a^3b+ab^3\right)\cos\right.}{2\left(\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right)d\cos\left(d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^5 - 4*a^3*b^2 + 2*a*b^4 + 6*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 - ((
2*a^2*b^2 + b^4)*cos(d*x + c)*sin(d*x + c) + (2*a^3*b + a*b^3)*cos(d*x + c)
)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b
^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^4*b - 2*
a^2*b^3 + b^5)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d
*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)
), (a^5 - 2*a^3*b^2 + a*b^4 + 3*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 - ((2*a^2*
b^2 + b^4)*cos(d*x + c)*sin(d*x + c) + (2*a^3*b + a*b^3)*cos(d*x + c))*sqrt
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) -
(a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b
^7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*c
os(d*x + c))]
```

giac [A] time = 0.22, size = 243, normalized size = 1.83

$$\frac{2\left(\frac{\left(2\,a^2b+b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{2\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)\left(\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((2*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan
(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2
- b^2)) + (2*a^2*b*tan(1/2*d*x + 1/2*c)^3 + b^3*tan(1/2*d*x + 1/2*c)^3 - a^
3*tan(1/2*d*x + 1/2*c)^2 + 4*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*b^3*tan(1/2*d
*x + 1/2*c) - a^3 - 2*a*b^2)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x +
 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)*(a^4 - 2*a^2*b^2 + b^4)))/d
```

maple [B] time = 0.44, size = 280, normalized size = 2.11

$$-\frac{1}{d\left(a+b\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{1}{d\left(a-b\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2b^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a-b\right)^{2}\left(a+b\right)^{2}\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)+1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)+2/d*
b^3/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1
/2*d*x+1/2*c)+2/d*b^2/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x
+1/2*c)*b+a)*a+4/d*a^2/(a-b)^2/(a+b)^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d*b^3/(a-b)^2/(a+b)^2/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 16.32, size = 310, normalized size = 2.33

$$\frac{\frac{6\,b^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\left(a^{2}-b^{2}\right)^{2}}+\frac{2\,a\left(a^{2}+2\,b^{2}\right)}{\left(a^{2}-b^{2}\right)^{2}}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\left(4\,a\,b^{2}-a^{3}\right)}{a^{4}-2\,a^{2}\,b^{2}+b^{4}}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\left(2\,a^{2}+b^{2}\right)}{\left(a^{2}-b^{2}\right)^{2}}}{d\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}+\frac{2\,b\,\mathrm{atan}\left(\frac{\frac{b\left(2\,a^{2}+b^{2}\right)\left(2\,a^{4}\,b-4\,a^{2}\,b^{3}+2\,b^{5}\right)}{(a+b)^{5/2}\,(a-b)^{5/2}}+\frac{2\,a\,b\tan\left(\frac{c}{2}\right.}{}}{4\,a^{2}\,b+2\,b^{3}}\right.}{d\,(a+b)^{5/2}\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((6*b^3*tan(c/2 + (d*x)/2))/(a^2 - b^2)^2 + (2*a*(a^2 + 2*b^2))/(a^2 - b^2)
^2 - (2*tan(c/2 + (d*x)/2)^2*(4*a*b^2 - a^3))/(a^4 + b^4 - 2*a^2*b^2) - (2*
```

```
b*tan(c/2 + (d*x)/2)^3*(2*a^2 + b^2))/(a^2 - b^2)^2)/(d*(a + 2*b*tan(c/2 +
(d*x)/2) - a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (d*x)/2)^3)) + (2*b*atan(
((b*(2*a^2 + b^2)*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)^(5/
2)) + (2*a*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2)*(a^4 + b^4 - 2*a^2*b^2))/((a
+ b)^(5/2)*(a - b)^(5/2)))/(4*a^2*b + 2*b^3))*(2*a^2 + b^2))/(d*(a + b)^(5/
2)*(a - b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

3.1468 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=229

$$\frac{b^4\cos(c+dx)}{ad\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}+\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{\tanh^{-1}(\cos(}{a^2d}$$

```
[Out] 2*b^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d+2*
b^3*(3*a^2-b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/(a^2-b
^2)^(5/2)/d-arctanh(cos(d*x+c))/a^2/d+1/2*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c
))+1/2*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+c))+b^4*cos(d*x+c)/a/(a^2-b^2)^2/d/(
a+b*sin(d*x+c))
```

Rubi [A] time = 0.31, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2897, 3770, 2648, 2664, 12, 2660, 618, 204}

$$\frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}+\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{b^4\cos(c+dx)}{ad\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{\tanh^{-1}(\cos(}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) + (2*b^3*(3*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/(a^2*(a^2 - b^2)^(5/2)*d) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/
(2*(a + b)^2*d*(1 - Sin[c + d*x])) + Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c
 + d*x])) + (b^4*Cos[c + d*x])/(a*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(\frac{\csc(c+dx)}{a^2} - \frac{1}{2(a+b)^2(-1+\sin(c+dx))} - \frac{1}{2(a-b)^2(1+\sin(c+dx))} - \frac{}{a}\right.$$

$$= \frac{\int \csc(c+dx)\,dx}{a^2} - \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2} + \frac{b^3\int \frac{1}{(a+b\sin(c+dx))^2}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+d}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+d}$$

$$= \frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+}{2(a+b)^2d(1-s}$$

$$= \frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+}{2(a+b)^2d(1-s}$$

$$= \frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}d} - \frac{\tanh^{-1}(\mathrm{co}}{a^2}$$

Mathematica [A] time = 2.22, size = 203, normalized size = 0.89

$$\frac{\frac{\frac{ab^4\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2} - \frac{2\left(b^5-4a^2b^3\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+\right.\right.}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*(-4*a^2*b^3 + b^5)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2*(a^2 - b^2)^(5/2)) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] -
```

```
 Sin[(c + d*x)/2])) - 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))
+ (-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]] + (a*b^4*Cos[c + d*x])/((
a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/a^2)/d
```

fricas [B] time = 1.30, size = 957, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^7 - 4*a^5*b^2 + 2*a^3*b^4 + 2*(2*a^5*b^2 - a^3*b^4 - a*b^6)*cos(d
*x + c)^2 + ((4*a^2*b^4 - b^6)*cos(d*x + c)*sin(d*x + c) + (4*a^3*b^3 - a*b
^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x +
c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)
) - ((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)*sin(d*x + c) + (a^7
 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2)
 + ((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)*sin(d*x + c) + (a^7
- 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2)
 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*sin(d*x + c))/((a^8*b - 3*a^6*b^3 + 3*a^
4*b^5 - a^2*b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^9 - 3*a^7*b^2 + 3*a^5*b^4
 - a^3*b^6)*d*cos(d*x + c)), 1/2*(2*a^7 - 4*a^5*b^2 + 2*a^3*b^4 + 2*(2*a^5*
b^2 - a^3*b^4 - a*b^6)*cos(d*x + c)^2 - 2*((4*a^2*b^4 - b^6)*cos(d*x + c)*s
in(d*x + c) + (4*a^3*b^3 - a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*
sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - ((a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 -
a*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + ((a^6*b - 3*a^4*b^3 + 3*
a^2*b^5 - b^7)*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a
*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^6*b - 2*a^4*b^3 + a
^2*b^5)*sin(d*x + c))/((a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*cos(d*x
+ c)*sin(d*x + c) + (a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c))
]
```

giac [A] time = 0.24, size = 314, normalized size = 1.37

$$\frac{\frac{2\left(4a^2b^3-b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-2a^4b^2+a^2b^4\right)\sqrt{a^2-b^2}}+\frac{2\left(2a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^3b^2\tan\left(\frac{1}{2}dx+\right.}{\left(a^6-2a^4b^2+a^2b^4\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(4*a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqr
t(a^2 - b^2)) + 2*(2*a^4*b*tan(1/2*d*x + 1/2*c)^3 + b^5*tan(1/2*d*x + 1/2*c
)^3 - a^5*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + a*b^4
*tan(1/2*d*x + 1/2*c)^2 - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) - b^5*tan(1/2*d*x
+ 1/2*c) - a^5 - a^3*b^2 - a*b^4)/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d
*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a))
 + log(abs(tan(1/2*d*x + 1/2*c)))/a^2)/d
```

maple [A] time = 0.59, size = 304, normalized size = 1.33

$$-\frac{1}{d\,(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}+\frac{1}{d\,(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2b^5\tan}{d\,(a-b)^2\,(a+b)^2\,a^2\left(\left(\tan^2\left(\frac{dx}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c))+1/d/(a-b
)^2/(tan(1/2*d*x+1/2*c)+1)+2/d*b^5/(a-b)^2/(a+b)^2/a^2/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d*b^4/(a-b)^2/(a+b)^2/a/
(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+8/d*b^3/(a-b)^2/(a+b)^2/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d
*b^5/(a-b)^2/(a+b)^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 14.86, size = 2076, normalized size = 9.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)*(a + b*sin(c + d*x))^2),x)

[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) + ((2*(a^4 + b^4 + a^2*b^2))/(a*(a^4 + b^4
- 2*a^2*b^2)) + (2*tan(c/2 + (d*x)/2)*(b^5 + 2*a^2*b^3))/(a^2*(a^4 + b^4 -
2*a^2*b^2)) - (2*tan(c/2 + (d*x)/2)^2*(b^4 - a^4 + 3*a^2*b^2))/(a*(a^4 + b^
4 - 2*a^2*b^2)) - (2*b*tan(c/2 + (d*x)/2)^3*(2*a^4 + b^4))/(a^2*(a^4 + b^4
- 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^4 - 2*
b*tan(c/2 + (d*x)/2)^3)) - (b^3*atan(((b^3*(2*a + b)*(-(a + b)^5*(a - b)^5)
^(1/2)*(2*a - b)*(tan(c/2 + (d*x)/2)*(2*a^16 - 8*a^2*b^14 + 58*a^4*b^12 - 1
60*a^6*b^10 + 222*a^8*b^8 - 168*a^10*b^6 + 70*a^12*b^4 - 16*a^14*b^2) - 2*a
^15*b - 4*a^3*b^13 + 28*a^5*b^11 - 74*a^7*b^9 + 96*a^9*b^7 - 64*a^11*b^5 +
20*a^13*b^3 + (b^3*(2*a + b)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(2*a^17
*b - tan(c/2 + (d*x)/2)*(6*a^18 - 8*a^4*b^14 + 54*a^6*b^12 - 156*a^8*b^10 +
 250*a^10*b^8 - 240*a^12*b^6 + 138*a^14*b^4 - 44*a^16*b^2) + 2*a^5*b^13 - 1
2*a^7*b^11 + 30*a^9*b^9 - 40*a^11*b^7 + 30*a^13*b^5 - 12*a^15*b^3))/(a^12 -
 a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))*1i)/(a^12 -
a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2) - (b^3*(2*a +
b)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(2*a^15*b - tan(c/2 + (d*x)/2)*(2
*a^16 - 8*a^2*b^14 + 58*a^4*b^12 - 160*a^6*b^10 + 222*a^8*b^8 - 168*a^10*b^
6 + 70*a^12*b^4 - 16*a^14*b^2) + 4*a^3*b^13 - 28*a^5*b^11 + 74*a^7*b^9 - 96
*a^9*b^7 + 64*a^11*b^5 - 20*a^13*b^3 + (b^3*(2*a + b)*(-(a + b)^5*(a - b)^5
)^(1/2)*(2*a - b)*(2*a^17*b - tan(c/2 + (d*x)/2)*(6*a^18 - 8*a^4*b^14 + 54*
a^6*b^12 - 156*a^8*b^10 + 250*a^10*b^8 - 240*a^12*b^6 + 138*a^14*b^4 - 44*a
^16*b^2) + 2*a^5*b^13 - 12*a^7*b^11 + 30*a^9*b^9 - 40*a^11*b^7 + 30*a^13*b^
5 - 12*a^15*b^3))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 -
5*a^10*b^2))*1i)/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5
*a^10*b^2))/(4*a*b^13 - 32*a^3*b^11 + 88*a^5*b^9 - 112*a^7*b^7 + 68*a^9*b^5
 - 16*a^11*b^3 + 2*tan(c/2 + (d*x)/2)*(8*a^2*b^12 - 44*a^4*b^10 + 48*a^6*b^
8 + 4*a^8*b^6 - 16*a^10*b^4) + (b^3*(2*a + b)*(-(a + b)^5*(a - b)^5)^(1/2)*
(2*a - b)*(tan(c/2 + (d*x)/2)*(2*a^16 - 8*a^2*b^14 + 58*a^4*b^12 - 160*a^6*
b^10 + 222*a^8*b^8 - 168*a^10*b^6 + 70*a^12*b^4 - 16*a^14*b^2) - 2*a^15*b -
 4*a^3*b^13 + 28*a^5*b^11 - 74*a^7*b^9 + 96*a^9*b^7 - 64*a^11*b^5 + 20*a^13
*b^3 + (b^3*(2*a + b)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(2*a^17*b - ta
n(c/2 + (d*x)/2)*(6*a^18 - 8*a^4*b^14 + 54*a^6*b^12 - 156*a^8*b^10 + 250*a^
10*b^8 - 240*a^12*b^6 + 138*a^14*b^4 - 44*a^16*b^2) + 2*a^5*b^13 - 12*a^7*b
^11 + 30*a^9*b^9 - 40*a^11*b^7 + 30*a^13*b^5 - 12*a^15*b^3))/(a^12 - a^2*b^
10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))/(a^12 - a^2*b^10 +
 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2) + (b^3*(2*a + b)*(-(a +
b)^5*(a - b)^5)^(1/2)*(2*a - b)*(2*a^15*b - tan(c/2 + (d*x)/2)*(2*a^16 - 8*
a^2*b^14 + 58*a^4*b^12 - 160*a^6*b^10 + 222*a^8*b^8 - 168*a^10*b^6 + 70*a^1
2*b^4 - 16*a^14*b^2) + 4*a^3*b^13 - 28*a^5*b^11 + 74*a^7*b^9 - 96*a^9*b^7 +
 64*a^11*b^5 - 20*a^13*b^3 + (b^3*(2*a + b)*(-(a + b)^5*(a - b)^5)^(1/2)*(2
*a - b)*(2*a^17*b - tan(c/2 + (d*x)/2)*(6*a^18 - 8*a^4*b^14 + 54*a^6*b^12 -
 156*a^8*b^10 + 250*a^10*b^8 - 240*a^12*b^6 + 138*a^14*b^4 - 44*a^16*b^2) +
 2*a^5*b^13 - 12*a^7*b^11 + 30*a^9*b^9 - 40*a^11*b^7 + 30*a^13*b^5 - 12*a^1
```

```
5*b^3))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2
)))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))*
(2*a + b)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*2i)/(d*(a^12 - a^2*b^10 +
5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

3.1469 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=248

$$\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{b^5\cos(c+dx)}{a^2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{5/2}}-\frac{\cot(c+dx)}{a^2d}-\frac{4b^4\left(2a^2-\right.}{}$$

```
[Out] -2*b^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/(a^2-b^2)^(5/2)/d
-4*b^4*(2*a^2-b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/(a^
2-b^2)^(5/2)/d+2*b*arctanh(cos(d*x+c))/a^3/d-cot(d*x+c)/a^2/d+1/2*cos(d*x+c
)/(a+b)^2/d/(1-sin(d*x+c))-1/2*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+c))-b^5*cos(
d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.37, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2897, 3770, 3767, 8, 2648, 2664, 12, 2660, 618, 204}

$$-\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{5/2}}-\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{5/2}}-\frac{b^5\cos(c+dx)}{a^2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{2b\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(5
/2)*d) - (4*b^4*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(a^3*(a^2 - b^2)^(5/2)*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[
c + d*x]/(a^2*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c
+ d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (b^5*Cos[c + d*x])/(a^2*(a^2 -
b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int\left(-\frac{2b\csc(c+dx)}{a^3}+\frac{\csc^2(c+dx)}{a^2}-\frac{1}{2(a+b)^2(-1+\sin(c+dx))}+\frac{}{2(a-b)^2\ldots}\right.$$

$$=\frac{\int\csc^2(c+dx)\,dx}{a^2}+\frac{\int\frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2}-\frac{(2b)\int\csc(c+dx)\,dx}{a^3}-\frac{\int\frac{1}{-1+\sin(c+dx)}\,d}{2(a+b)^2}$$

$$=\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c}$$

$$=\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^2d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{}{2(a-}$$

$$=-\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+d}{a^2d}$$

$$=-\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+d}{a^2d}$$

$$=-\frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}d}-\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d}+\frac{2b\tan}{}$$

Mathematica [A] time = 3.25, size = 254, normalized size = 1.02

$$\frac{-\frac{4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}+\frac{4b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}-\frac{2b^5\cos(c+dx)}{a^2(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{a^2}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{a^2}+\frac{4b^4\left(2b^2-5a^2\right)\tan^{-1}\left(\frac{a\tan}{}\right.}{a^3\left(a^2-b^2\right)^{5/}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((4*b^4*(-5*a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^3*(a^2 - b^2)^(5/2)) - Cot[(c + d*x)/2]/a^2 + (4*b*Log[Cos[(c + d*x)/2]]
)/a^3 - (4*b*Log[Sin[(c + d*x)/2]])/a^3 + (2*Sin[(c + d*x)/2])/((a + b)^2*(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2*Sin[(c + d*x)/2])/((a - b)^2*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])) - (2*b^5*Cos[c + d*x])/(a^2*(a - b)^2*(
a + b)^2*(a + b*Sin[c + d*x])) + Tan[(c + d*x)/2]/a^2)/(2*d)
```

fricas [B] time = 1.27, size = 1355, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*(2*a^8 - 4*a^6*b^2 + 2*a^4*b^4 - 2*(2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 - a
^2*b^6)*cos(d*x + c)^2 - ((5*a^2*b^5 - 2*b^7)*cos(d*x + c)^3 - (5*a^3*b^4 -
 2*a*b^6)*cos(d*x + c)*sin(d*x + c) - (5*a^2*b^5 - 2*b^7)*cos(d*x + c))*sqr
t(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
- b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/
(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*((a^6*b^2 - 3*a^
4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^3 - (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
a*b^7)*cos(d*x + c)*sin(d*x + c) - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*
cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 2*((a^6*b^2 - 3*a^4*b^4 + 3*a^2
*b^6 - b^8)*cos(d*x + c)^3 - (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*
x + c)*sin(d*x + c) - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c))
*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^7*b - 2*a^5*b^3 + a^3*b^5 + (2*a^7*b -
 3*a^5*b^3 + 3*a^3*b^5 - 2*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^9*b - 3
*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d*cos(d*x + c)^3 - (a^10 - 3*a^8*b^2 + 3*a^
6*b^4 - a^4*b^6)*d*cos(d*x + c)*sin(d*x + c) - (a^9*b - 3*a^7*b^3 + 3*a^5*b
^5 - a^3*b^7)*d*cos(d*x + c)), -(a^8 - 2*a^6*b^2 + a^4*b^4 - (2*a^8 - 5*a^6
*b^2 + 4*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 - ((5*a^2*b^5 - 2*b^7)*cos(d*x +
 c)^3 - (5*a^3*b^4 - 2*a*b^6)*cos(d*x + c)*sin(d*x + c) - (5*a^2*b^5 - 2*b^
7)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b
```

```
^2)*cos(d*x + c))) - ((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^
3 - (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c)*sin(d*x + c) - (a^
6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1
/2) + ((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^3 - (a^7*b - 3*
a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c)*sin(d*x + c) - (a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - (a^7*b -
 2*a^5*b^3 + a^3*b^5 + (2*a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - 2*a*b^7)*cos(d*x
+ c)^2)*sin(d*x + c))/((a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d*cos(d*x
+ c)^3 - (a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*d*cos(d*x + c)*sin(d*x +
c) - (a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d*cos(d*x + c))]
```

giac [B] time = 0.26, size = 523, normalized size = 2.11

$$-\frac{\frac{20\left(5a^2b^4-2b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^7-2a^5b^2+a^3b^4\right)\sqrt{a^2-b^2}}-\frac{4a^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-8a^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+4ab^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-25a^6\tan}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(20*(5*a^2*b^4 - 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + ar
ctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3
*b^4)*sqrt(a^2 - b^2)) - (4*a^5*b*tan(1/2*d*x + 1/2*c)^5 - 8*a^3*b^3*tan(1/
2*d*x + 1/2*c)^5 + 4*a*b^5*tan(1/2*d*x + 1/2*c)^5 - 25*a^6*tan(1/2*d*x + 1/
2*c)^4 - 2*a^4*b^2*tan(1/2*d*x + 1/2*c)^4 - 21*a^2*b^4*tan(1/2*d*x + 1/2*c)
^4 - 12*b^6*tan(1/2*d*x + 1/2*c)^4 - 10*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 20*a
^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 30*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 20*a^6*ta
n(1/2*d*x + 1/2*c)^2 + 52*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a^2*b^4*tan(1
/2*d*x + 1/2*c)^2 + 12*b^6*tan(1/2*d*x + 1/2*c)^2 + 46*a^5*b*tan(1/2*d*x +
1/2*c) - 12*a^3*b^3*tan(1/2*d*x + 1/2*c) + 26*a*b^5*tan(1/2*d*x + 1/2*c) +
5*a^6 - 10*a^4*b^2 + 5*a^2*b^4)/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x
 + 1/2*c)^5 + 2*b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a*t
an(1/2*d*x + 1/2*c))) + 20*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 5*tan(1/2
*d*x + 1/2*c)/a^2)/d
```

maple [A] time = 0.62, size = 346, normalized size = 1.40

$$-\frac{1}{d\left(a+b\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{1}{2d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}-\frac{1}{d\left(a-b\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)+1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/2/d/a^2/
tan(1/2*d*x+1/2*c)-2/d/a^3*b*ln(tan(1/2*d*x+1/2*c))-1/d/(a-b)^2/(tan(1/2*d*
x+1/2*c)+1)-2/d/a^3*b^6/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-2/d/a^2*b^5/(a-b)^2/(a+b)^2/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-10/d/a*b^4/(a-b)^2/(a+b)^2/(a^2-b^2)^(
1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d/a^3*b^6/(
a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2
-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.33, size = 2151, normalized size = 8.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (a + (2*tan(c/2 + (d*x)/2)*(5*a^4*b + 3*b^5 - 2*a^2*b^3))/(a^4 + b^4 - 2*a^
2*b^2) + (4*tan(c/2 + (d*x)/2)^2*(b^6 - a^6 + 3*a^4*b^2))/(a*(a^2 - b^2)^2)
 - (2*b*tan(c/2 + (d*x)/2)^3*(a^4 + 3*b^4 + 2*a^2*b^2))/(a^4 + b^4 - 2*a^2*
b^2) - (tan(c/2 + (d*x)/2)^4*(5*a^6 + 4*b^6 + a^2*b^4 + 2*a^4*b^2))/(a*(a^4
 + b^4 - 2*a^2*b^2)))/(d*(2*a^3*tan(c/2 + (d*x)/2)^5 - 2*a^3*tan(c/2 + (d*x
)/2) - 4*a^2*b*tan(c/2 + (d*x)/2)^2 + 4*a^2*b*tan(c/2 + (d*x)/2)^4)) + tan(
c/2 + (d*x)/2)/(2*a^2*d) - (2*b*log(tan(c/2 + (d*x)/2)))/(a^3*d) + (b^4*ata
n(((b^4*(5*a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(4
*a^18*b - 16*a^4*b^15 + 104*a^6*b^13 - 272*a^8*b^11 + 372*a^10*b^9 - 288*a^
12*b^7 + 128*a^14*b^5 - 32*a^16*b^3) - 8*a^5*b^14 + 50*a^7*b^12 - 124*a^9*b
^10 + 156*a^11*b^8 - 104*a^13*b^6 + 34*a^15*b^4 - 4*a^17*b^2 + (b^4*(5*a^2
- 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^20*b - tan(c/2 + (d*x)/2)*(6*a^2
1 - 8*a^7*b^14 + 54*a^9*b^12 - 156*a^11*b^10 + 250*a^13*b^8 - 240*a^15*b^6
+ 138*a^17*b^4 - 44*a^19*b^2) + 2*a^8*b^13 - 12*a^10*b^11 + 30*a^12*b^9 - 4
```

```
0*a^14*b^7 + 30*a^16*b^5 - 12*a^18*b^3))/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*
a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))*1i)/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a
^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2) - (b^4*(5*a^2 - 2*b^2)*(-(a + b)^5*(a - b
)^5)^(1/2)*(8*a^5*b^14 - tan(c/2 + (d*x)/2)*(4*a^18*b - 16*a^4*b^15 + 104*a
^6*b^13 - 272*a^8*b^11 + 372*a^10*b^9 - 288*a^12*b^7 + 128*a^14*b^5 - 32*a^
16*b^3) - 50*a^7*b^12 + 124*a^9*b^10 - 156*a^11*b^8 + 104*a^13*b^6 - 34*a^1
5*b^4 + 4*a^17*b^2 + (b^4*(5*a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a
^20*b - tan(c/2 + (d*x)/2)*(6*a^21 - 8*a^7*b^14 + 54*a^9*b^12 - 156*a^11*b^
10 + 250*a^13*b^8 - 240*a^15*b^6 + 138*a^17*b^4 - 44*a^19*b^2) + 2*a^8*b^13
 - 12*a^10*b^11 + 30*a^12*b^9 - 40*a^14*b^7 + 30*a^16*b^5 - 12*a^18*b^3))/(
a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))*1i)/(a
^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))/(16*a^2
*b^15 - 104*a^4*b^13 + 256*a^6*b^11 - 304*a^8*b^9 + 176*a^10*b^7 - 40*a^12*
b^5 - 2*tan(c/2 + (d*x)/2)*(20*a^5*b^12 - 8*a^3*b^14 + 24*a^7*b^10 - 76*a^9
*b^8 + 40*a^11*b^6) + (b^4*(5*a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(ta
n(c/2 + (d*x)/2)*(4*a^18*b - 16*a^4*b^15 + 104*a^6*b^13 - 272*a^8*b^11 + 37
2*a^10*b^9 - 288*a^12*b^7 + 128*a^14*b^5 - 32*a^16*b^3) - 8*a^5*b^14 + 50*a
^7*b^12 - 124*a^9*b^10 + 156*a^11*b^8 - 104*a^13*b^6 + 34*a^15*b^4 - 4*a^17
*b^2 + (b^4*(5*a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^20*b - tan(c/
2 + (d*x)/2)*(6*a^21 - 8*a^7*b^14 + 54*a^9*b^12 - 156*a^11*b^10 + 250*a^13*
b^8 - 240*a^15*b^6 + 138*a^17*b^4 - 44*a^19*b^2) + 2*a^8*b^13 - 12*a^10*b^1
1 + 30*a^12*b^9 - 40*a^14*b^7 + 30*a^16*b^5 - 12*a^18*b^3))/(a^13 - a^3*b^1
0 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))/(a^13 - a^3*b^10 +
5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2) + (b^4*(5*a^2 - 2*b^2)*(-
(a + b)^5*(a - b)^5)^(1/2)*(8*a^5*b^14 - tan(c/2 + (d*x)/2)*(4*a^18*b - 16*
a^4*b^15 + 104*a^6*b^13 - 272*a^8*b^11 + 372*a^10*b^9 - 288*a^12*b^7 + 128*
a^14*b^5 - 32*a^16*b^3) - 50*a^7*b^12 + 124*a^9*b^10 - 156*a^11*b^8 + 104*a
^13*b^6 - 34*a^15*b^4 + 4*a^17*b^2 + (b^4*(5*a^2 - 2*b^2)*(-(a + b)^5*(a -
b)^5)^(1/2)*(2*a^20*b - tan(c/2 + (d*x)/2)*(6*a^21 - 8*a^7*b^14 + 54*a^9*b^
12 - 156*a^11*b^10 + 250*a^13*b^8 - 240*a^15*b^6 + 138*a^17*b^4 - 44*a^19*b
^2) + 2*a^8*b^13 - 12*a^10*b^11 + 30*a^12*b^9 - 40*a^14*b^7 + 30*a^16*b^5 -
 12*a^18*b^3))/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a
^11*b^2)))/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*
b^2)))*(5*a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*2i)/(d*(a^13 - a^3*b^10
 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

3.1470 $\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=295

$$\frac{2b\cot(c+dx)}{a^3d}+\frac{2b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\left(a^2+3b^2\right)\tanh^{-1}(\mathrm{co}}{a^4d}$$

```
[Out] 2*b^5*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/(a^2-b^2)^(5/2)/
d+2*b^5*(5*a^2-3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^4/
(a^2-b^2)^(5/2)/d-1/2*arctanh(cos(d*x+c))/a^2/d-(a^2+3*b^2)*arctanh(cos(d*x
+c))/a^4/d+2*b*cot(d*x+c)/a^3/d-1/2*cot(d*x+c)*csc(d*x+c)/a^2/d+1/2*cos(d*x
+c)/(a+b)^2/d/(1-sin(d*x+c))+1/2*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+c))+b^6*co
s(d*x+c)/a^3/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.39, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2897, 3770, 3767, 8, 3768, 2648, 2664, 12, 2660, 618, 204}

$$\frac{2b^5\left(5a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\left(a^2-b^2\right)^{5/2}}+\frac{2b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}+\frac{b^6\cos(c+dx)}{a^3d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{\left(a^2+3b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(
5/2)*d) + (2*b^5*(5*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^4*(a^2 - b^2)^(5/2)*d) - ArcTanh[Cos[c + d*x]]/(2*a^2*d) - ((a^2
 + 3*b^2)*ArcTanh[Cos[c + d*x]])/(a^4*d) + (2*b*Cot[c + d*x])/(a^3*d) - (Co
t[c + d*x]*Csc[c + d*x])/(2*a^2*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c
 + d*x])) + Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) + (b^6*Cos[c +
d*x])/(a^3*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int\left(\frac{(a^2+3b^2)\csc(c+dx)}{a^4} - \frac{2b\csc^2(c+dx)}{a^3} + \frac{\csc^3(c+dx)}{a^2} - \frac{1}{2(a+b)^2(-1+\right.$$

$$= \frac{\int \csc^3(c+dx)\,dx}{a^2} - \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{(2b)\int \csc^2(c+dx)\,dx}{a^3} - \frac{\int \frac{1}{-1+\sin(c+dx)}}{2(a+b)^2}$$

$$= -\frac{(a^2+3b^2)\tanh^{-1}(\cos(c+dx))}{a^4 d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2 d} + \frac{\cos(c+}{2(a+b)^2 d(1-}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^2 d} - \frac{(a^2+3b^2)\tanh^{-1}(\cos(c+dx))}{a^4 d} + \frac{2b\cot(c+dx)}{a^3 d} -$$

$$= \frac{2b^5(5a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4(a^2-b^2)^{5/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^2 d} - \frac{(a^2+3b^2)\,\mathrm{ta}}{}$$

$$= \frac{2b^5(5a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4(a^2-b^2)^{5/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^2 d} - \frac{(a^2+3b^2)\,\mathrm{ta}}{}$$

$$= \frac{2b^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{5/2}d} + \frac{2b^5(5a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4(a^2-b^2)^{5/2}d} - \frac{\tanh^{-1}}{}$$

Mathematica [A] time = 6.48, size = 356, normalized size = 1.21

$$\frac{b^6\cos(c+dx)}{a^3 d(a-b)^2(a+b)^2(a+b\sin(c+dx))} - \frac{b\tan\left(\frac{1}{2}(c+dx)\right)}{a^3 d} + \frac{b\cot\left(\frac{1}{2}(c+dx)\right)}{a^3 d} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^2 d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (6*b^5*(2*a^2 - b^2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(
c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*(a^2 - b^2)^(5/2)*d) + (b*Cot[(c + d*x
```

```
)/2])/(a^3*d) - Csc[(c + d*x)/2]^2/(8*a^2*d) - (3*(a^2 + 2*b^2)*Log[Cos[(c
+ d*x)/2]])/(2*a^4*d) + (3*(a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]])/(2*a^4*d) +
 Sec[(c + d*x)/2]^2/(8*a^2*d) + Sin[(c + d*x)/2]/((a + b)^2*d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])) - Sin[(c + d*x)/2]/((a - b)^2*d*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])) + (b^6*Cos[c + d*x])/(a^3*(a - b)^2*(a + b)^2*d*(a +
b*Sin[c + d*x])) - (b*Tan[(c + d*x)/2])/(a^3*d)
```

fricas [B] time = 2.30, size = 1844, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/4*(4*a^9 - 8*a^7*b^2 + 4*a^5*b^4 - 4*(4*a^7*b^2 - 8*a^5*b^4 + 7*a^3*b^6
 - 3*a*b^8)*cos(d*x + c)^4 - 6*(a^9 - 5*a^7*b^2 + 7*a^5*b^4 - 5*a^3*b^6 + 2
*a*b^8)*cos(d*x + c)^2 - 6*((2*a^3*b^5 - a*b^7)*cos(d*x + c)^3 - (2*a^3*b^5
 - a*b^7)*cos(d*x + c) + ((2*a^2*b^6 - b^8)*cos(d*x + c)^3 - (2*a^2*b^6 - b
^8)*cos(d*x + c))*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c)
+ b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c
) - a^2 - b^2)) + 3*((a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(
d*x + c)^3 - (a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c)
 + ((a^8*b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c)^3 - (a^8
*b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c))*sin(d*x + c))*l
og(1/2*cos(d*x + c) + 1/2) - 3*((a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*
a*b^8)*cos(d*x + c)^3 - (a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*c
os(d*x + c) + ((a^8*b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x +
c)^3 - (a^8*b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c))*sin(
d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*a^8*b - 4*a^6*b^3 + 2*a^4*b^5
 - (5*a^8*b - 13*a^6*b^3 + 11*a^4*b^5 - 3*a^2*b^7)*cos(d*x + c)^2)*sin(d*x
+ c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^3 - (a^11 -
3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c) + ((a^10*b - 3*a^8*b^3 + 3*
a^6*b^5 - a^4*b^7)*d*cos(d*x + c)^3 - (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4
*b^7)*d*cos(d*x + c))*sin(d*x + c)), -1/4*(4*a^9 - 8*a^7*b^2 + 4*a^5*b^4 -
4*(4*a^7*b^2 - 8*a^5*b^4 + 7*a^3*b^6 - 3*a*b^8)*cos(d*x + c)^4 - 6*(a^9 - 5
*a^7*b^2 + 7*a^5*b^4 - 5*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^2 + 12*((2*a^3*b^5
 - a*b^7)*cos(d*x + c)^3 - (2*a^3*b^5 - a*b^7)*cos(d*x + c) + ((2*a^2*b^6 -
 b^8)*cos(d*x + c)^3 - (2*a^2*b^6 - b^8)*cos(d*x + c))*sin(d*x + c))*sqrt(a
^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 3*
((a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c)^3 - (a^9 -
a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c) + ((a^8*b - a^6*b^3
 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c)^3 - (a^8*b - a^6*b^3 - 3*a^4
*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c))*sin(d*x + c))*log(1/2*cos(d*x + c)
```

```
+ 1/2) - 3*((a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c)^
3 - (a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c) + ((a^8*
b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c)^3 - (a^8*b - a^6*
b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/2*c
os(d*x + c) + 1/2) - 2*(2*a^8*b - 4*a^6*b^3 + 2*a^4*b^5 - (5*a^8*b - 13*a^6
*b^3 + 11*a^4*b^5 - 3*a^2*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^11 - 3*a^9
*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^3 - (a^11 - 3*a^9*b^2 + 3*a^7*b^
4 - a^5*b^6)*d*cos(d*x + c) + ((a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d
*cos(d*x + c)^3 - (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c)
)*sin(d*x + c))]
```

giac [A] time = 0.26, size = 423, normalized size = 1.43

$$\frac{48\left(2\,a^{2}b^{5}-b^{7}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{8}-2\,a^{6}b^{2}+a^{4}b^{4}\right)\sqrt{a^{2}-b^{2}}}+\frac{16\left(2\,a^{6}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+b^{7}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-a^{7}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,a^{5}b^{2}\tan\left(\frac{1}{2}\,dx\right.}{\left(a^{8}-2\,a^{6}b^{2}+a^{4}b^{4}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(48*(2*a^2*b^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4
)*sqrt(a^2 - b^2)) + 16*(2*a^6*b*tan(1/2*d*x + 1/2*c)^3 + b^7*tan(1/2*d*x +
 1/2*c)^3 - a^7*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 +
 a*b^6*tan(1/2*d*x + 1/2*c)^2 - 2*a^4*b^3*tan(1/2*d*x + 1/2*c) - b^7*tan(1/
2*d*x + 1/2*c) - a^7 - a^5*b^2 - a*b^6)/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan
(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)
 - a)) + 12*(a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + (a^2*tan(1/2
*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d*x + 1/2*c))/a^4 - (18*a^2*tan(1/2*d*x + 1
/2*c)^2 + 36*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d*x + 1/2*c) + a^2)
/(a^4*tan(1/2*d*x + 1/2*c)^2))/d
```

maple [A] time = 0.66, size = 404, normalized size = 1.37

$$-\frac{1}{d\left(a+b\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^{2}d}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{d\,a^{3}}-\frac{1}{8a^{2}d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}+\frac{3\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)+1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^3*
tan(1/2*d*x+1/2*c)*b-1/8/a^2/d/tan(1/2*d*x+1/2*c)^2+3/2/d/a^2*ln(tan(1/2*d*
x+1/2*c))+3/d/a^4*ln(tan(1/2*d*x+1/2*c))*b^2+1/d*b/a^3/tan(1/2*d*x+1/2*c)+1
/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)+2/d/a^4*b^7/(a-b)^2/(a+b)^2/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/a^3*b^6/(a-b)^2
/(a+b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+12/d*b^5/(a-b)^2
/(a+b)^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))-6/d/a^4*b^7/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 14.09, size = 2302, normalized size = 7.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^3*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) + ((tan(c/2 + (d*x)/2)^4*(17*a^6 - 32*b^6 +
33*a^2*b^4 - 66*a^4*b^2))/(2*(a^4 + b^4 - 2*a^2*b^2)) - a^2/2 + (8*tan(c/2
+ (d*x)/2)^2*(a^6 + 2*b^6 - 2*a^2*b^4 + 2*a^4*b^2))/(a^4 + b^4 - 2*a^2*b^2)
 + 3*a*b*tan(c/2 + (d*x)/2) - (4*tan(c/2 + (d*x)/2)^5*(5*a^6*b + 2*b^7 + a^
2*b^5 - 2*a^4*b^3))/(a*(a^4 + b^4 - 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^3*(a^
6*b + 8*b^7 + a^2*b^5 + 14*a^4*b^3))/(a*(a^2 - b^2)^2))/(d*(4*a^4*tan(c/2 +
 (d*x)/2)^2 - 4*a^4*tan(c/2 + (d*x)/2)^6 + 8*a^3*b*tan(c/2 + (d*x)/2)^3 - 8
*a^3*b*tan(c/2 + (d*x)/2)^5)) - (b*tan(c/2 + (d*x)/2))/(a^3*d) + (log(tan(c
/2 + (d*x)/2))*(3*a^2 + 6*b^2))/(2*a^4*d) - (b^5*atan(((b^5*(2*a^2 - b^2)*(
-(a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(24*a^22 - 192*a^6*b^16 + 1
152*a^8*b^14 - 2760*a^10*b^12 + 3312*a^12*b^10 - 1944*a^14*b^8 + 288*a^16*b
^6 + 264*a^18*b^4 - 144*a^20*b^2) - 24*a^21*b - 96*a^7*b^15 + 552*a^9*b^13
- 1248*a^11*b^11 + 1368*a^13*b^9 - 672*a^15*b^7 + 24*a^17*b^5 + 96*a^19*b^3
 + (3*b^5*(2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(16*a^23*b - tan(c/2 +
 (d*x)/2)*(48*a^24 - 64*a^10*b^14 + 432*a^12*b^12 - 1248*a^14*b^10 + 2000*a
```

```
^16*b^8 - 1920*a^18*b^6 + 1104*a^20*b^4 - 352*a^22*b^2) + 16*a^11*b^13 - 96
*a^13*b^11 + 240*a^15*b^9 - 320*a^17*b^7 + 240*a^19*b^5 - 96*a^21*b^3))/(a^
14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2))*3i)/(a^
14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2) - (b^5*(
2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(24*a^21*b - tan(c/2 + (d*x)/2)*(
24*a^22 - 192*a^6*b^16 + 1152*a^8*b^14 - 2760*a^10*b^12 + 3312*a^12*b^10 -
1944*a^14*b^8 + 288*a^16*b^6 + 264*a^18*b^4 - 144*a^20*b^2) + 96*a^7*b^15 -
 552*a^9*b^13 + 1248*a^11*b^11 - 1368*a^13*b^9 + 672*a^15*b^7 - 24*a^17*b^5
 - 96*a^19*b^3 + (3*b^5*(2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(16*a^23
*b - tan(c/2 + (d*x)/2)*(48*a^24 - 64*a^10*b^14 + 432*a^12*b^12 - 1248*a^14
*b^10 + 2000*a^16*b^8 - 1920*a^18*b^6 + 1104*a^20*b^4 - 352*a^22*b^2) + 16*
a^11*b^13 - 96*a^13*b^11 + 240*a^15*b^9 - 320*a^17*b^7 + 240*a^19*b^5 - 96*
a^21*b^3))/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12
*b^2))*3i)/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12
*b^2))/(2*tan(c/2 + (d*x)/2)*(144*a^4*b^16 - 576*a^6*b^14 + 864*a^8*b^12 -
864*a^10*b^10 + 720*a^12*b^8 - 288*a^14*b^6) + 288*a^3*b^17 - 1584*a^5*b^15
 + 3168*a^7*b^13 - 2592*a^9*b^11 + 288*a^11*b^9 + 720*a^13*b^7 - 288*a^15*b
^5 + (3*b^5*(2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*
(24*a^22 - 192*a^6*b^16 + 1152*a^8*b^14 - 2760*a^10*b^12 + 3312*a^12*b^10 -
 1944*a^14*b^8 + 288*a^16*b^6 + 264*a^18*b^4 - 144*a^20*b^2) - 24*a^21*b -
96*a^7*b^15 + 552*a^9*b^13 - 1248*a^11*b^11 + 1368*a^13*b^9 - 672*a^15*b^7
+ 24*a^17*b^5 + 96*a^19*b^3 + (3*b^5*(2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(
1/2)*(16*a^23*b - tan(c/2 + (d*x)/2)*(48*a^24 - 64*a^10*b^14 + 432*a^12*b^1
2 - 1248*a^14*b^10 + 2000*a^16*b^8 - 1920*a^18*b^6 + 1104*a^20*b^4 - 352*a^
22*b^2) + 16*a^11*b^13 - 96*a^13*b^11 + 240*a^15*b^9 - 320*a^17*b^7 + 240*a
^19*b^5 - 96*a^21*b^3))/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10
*b^4 - 5*a^12*b^2)))/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^
4 - 5*a^12*b^2) + (3*b^5*(2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(24*a^2
1*b - tan(c/2 + (d*x)/2)*(24*a^22 - 192*a^6*b^16 + 1152*a^8*b^14 - 2760*a^1
0*b^12 + 3312*a^12*b^10 - 1944*a^14*b^8 + 288*a^16*b^6 + 264*a^18*b^4 - 144
*a^20*b^2) + 96*a^7*b^15 - 552*a^9*b^13 + 1248*a^11*b^11 - 1368*a^13*b^9 +
672*a^15*b^7 - 24*a^17*b^5 - 96*a^19*b^3 + (3*b^5*(2*a^2 - b^2)*(-(a + b)^5
*(a - b)^5)^(1/2)*(16*a^23*b - tan(c/2 + (d*x)/2)*(48*a^24 - 64*a^10*b^14 +
 432*a^12*b^12 - 1248*a^14*b^10 + 2000*a^16*b^8 - 1920*a^18*b^6 + 1104*a^20
*b^4 - 352*a^22*b^2) + 16*a^11*b^13 - 96*a^13*b^11 + 240*a^15*b^9 - 320*a^1
7*b^7 + 240*a^19*b^5 - 96*a^21*b^3))/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*
b^6 + 10*a^10*b^4 - 5*a^12*b^2)))/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6
 + 10*a^10*b^4 - 5*a^12*b^2)))*(2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*6
i)/(d*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)
)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1471 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=388

$$-\frac{3a^5\cos(c+dx)}{2bd\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{a^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{7/2}}+\frac{4a^4\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{7/2}}$$

```
[Out] 4*a^4*(a^2-2*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^2/(a^2
-b^2)^(7/2)/d-a^4*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/
2))/b^2/(a^2-b^2)^(7/2)/d-2*a^2*(a^4-3*a^2*b^2+6*b^4)*arctan((b+a*tan(1/2*d
*x+1/2*c))/(a^2-b^2)^(1/2))/b^2/(a^2-b^2)^(7/2)/d+1/2*cos(d*x+c)/(a+b)^3/d/
(1-sin(d*x+c))-1/2*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c))-1/2*a^4*cos(d*x+c)/b
/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^2-3/2*a^5*cos(d*x+c)/b/(a^2-b^2)^3/d/(a+b*s
in(d*x+c))+2*a^3*(a^2-2*b^2)*cos(d*x+c)/b/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.58, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2897, 2648, 2664, 2754, 12, 2660, 618, 204}

$$-\frac{a^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{7/2}}+\frac{4a^4\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{7/2}}-\frac{2a^2\left(-3a^2b^2+a^4+6b^4\right)\tan^{-1}\left(\frac{a}{-}\right)}{b^2d\left(a^2-b^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (4*a^4*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2
*(a^2 - b^2)^(7/2)*d) - (a^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(7/2)*d) - (2*a^2*(a^4 - 3*a^2*b^2 + 6*b
^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(7/2
)*d) + Cos[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a
 - b)^3*d*(1 + Sin[c + d*x])) - (a^4*Cos[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a
+ b*Sin[c + d*x])^2) - (3*a^5*Cos[c + d*x])/(2*b*(a^2 - b^2)^3*d*(a + b*Sin
[c + d*x])) + (2*a^3*(a^2 - 2*b^2)*Cos[c + d*x])/(b*(a^2 - b^2)^3*d*(a + b*
Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
```

```
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(-\frac{1}{2(a+b)^3(-1+\sin(c+dx))}+\frac{1}{2(a-b)^3(1+\sin(c+dx))}+\frac{\quad}{b^2\left(-a^2+b^2\right.}\right.$$

$$=\frac{\int\frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3}-\frac{\int\frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3}+\frac{\left(2a^3\left(a^2-2b^2\right)\right)\int\frac{1}{(a+b\sin(c+dx))^2}\,dx}{b^2\left(a^2-b^2\right)^2}-\frac{a}{}$$

$$=\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))}-\frac{a^4\cos(}{2b\left(a^2-b^2\right)^2d(a}$$

$$=\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))}-\frac{a^4\cos(}{2b\left(a^2-b^2\right)^2d(a}$$

$$=-\frac{2a^2\left(a^4-3a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d}+\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}$$

$$=-\frac{2a^2\left(a^4-3a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d}+\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}$$

$$=\frac{4a^4\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d}-\frac{2a^2\left(a^4-3a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan(}{\sqrt{a^2}}\right.}{b^2\left(a^2-b^2\right)^{7/2}d}$$

$$=\frac{4a^4\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d}-\frac{a^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d}$$

Mathematica [A] time = 3.19, size = 195, normalized size = 0.50

$$\frac{-\frac{6a^2(a^2+4b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}}+\frac{a^3\cos(c+dx)\left((a^2-8b^2)\sin(c+dx)-7ab\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a-b)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-6*a^2*(a^2 + 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])) + 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (a^3*
Cos[c + d*x]*(-7*a*b + (a^2 - 8*b^2)*Sin[c + d*x]))/((a - b)^3*(a + b)^3*(a
 + b*Sin[c + d*x])^2))/(2*d)
```

fricas [A] time = 0.52, size = 906, normalized size = 2.34

$$\left[\frac{4\,a^6b-12\,a^4b^3+12\,a^2b^5-4\,b^7+2\left(11\,a^6b-5\,a^4b^3-8\,a^2b^5+2\,b^7\right)\cos(dx+c)^2+3\left(\left(a^4b^2+4\,a^2b^4\right)\cos(dx+\right.}{4\left(\left(a^8b^2-4\,a^6b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 2*(11*a^6*b - 5*a^4*b^3 -
 8*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + 3*((a^4*b^2 + 4*a^2*b^4)*cos(d*x + c)^
3 - 2*(a^5*b + 4*a^3*b^3)*cos(d*x + c)*sin(d*x + c) - (a^6 + 5*a^4*b^2 + 4*
a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + (a^7 - 11*a^5*b^2 + 4*
a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*
a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5
*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 +
 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)), 1/2*(2*a^6*b -
6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + (11*a^6*b - 5*a^4*b^3 - 8*a^2*b^5 + 2*b^7)*
cos(d*x + c)^2 + 3*((a^4*b^2 + 4*a^2*b^4)*cos(d*x + c)^3 - 2*(a^5*b + 4*a^3
*b^3)*cos(d*x + c)*sin(d*x + c) - (a^6 + 5*a^4*b^2 + 4*a^2*b^4)*cos(d*x + c
))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x +
c))) - (2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + (a^7 - 11*a^5*b^2 + 4*a^3
```

```
*b^4 + 6*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4
*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^
5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*
a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c))]
```

giac [A] time = 0.35, size = 351, normalized size = 0.90

$$-\frac{\frac{3\left(a^4+4a^2b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{2\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^2b-b^3\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a^3\cdots}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(a^4 + 4*a^2*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*
tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6)*sqrt(a^2 - b^2)) + 2*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan(1/2*d*x
+ 1/2*c) - 3*a^2*b - b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*x
 + 1/2*c)^2 - 1)) + (a^5*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*b^2*tan(1/2*d*x + 1
/2*c)^3 + 7*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 14*a^2*b^3*tan(1/2*d*x + 1/2*c)^
2 - a^5*tan(1/2*d*x + 1/2*c) + 22*a^3*b^2*tan(1/2*d*x + 1/2*c) + 7*a^4*b)/(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2
*d*x + 1/2*c) + a)^2))/d
```

maple [A] time = 0.54, size = 590, normalized size = 1.52

$$-\frac{1}{d\left(a+b\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{d\left(a-b\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a^5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a-b\right)^3\left(a+b\right)^3\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)-1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)-1/d*
a^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan
(1/2*d*x+1/2*c)^3-6/d*a^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^2-7/d*a^4/(a-b)^3/(a+b)^3/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b-14/d*a^
2/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1
/2*d*x+1/2*c)^2*b^3+1/d*a^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1
```

```
/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-22/d*a^3/(a-b)^3/(a+b)^3/(tan(1/2*d
*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^2-7/d*a^4/(a
-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b-3/d*a^4
/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a
^2-b^2)^(1/2))-12/d*a^2/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan
(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 18.23, size = 585, normalized size = 1.51

$$\frac{\frac{13a^4b+2a^2b^3}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(a^5+6a^3b^2+8ab^4\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^4b+9a^2b^3+4b^5\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-3a^4+40a^2b^2+8b^4\right)}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{3a^3\,t}{a^6}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-a^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(a^2+4b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(a^2+4b^2\right)+4ab\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] ((13*a^4*b + 2*a^2*b^3)/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (2*tan(c/2 +
(d*x)/2)^3*(8*a*b^4 + a^5 + 6*a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)
 + (2*tan(c/2 + (d*x)/2)^2*(2*a^4*b + 4*b^5 + 9*a^2*b^3))/(a^6 - b^6 + 3*a^
2*b^4 - 3*a^4*b^2) + (a*tan(c/2 + (d*x)/2)*(8*b^4 - 3*a^4 + 40*a^2*b^2))/(a
^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (3*a^3*tan(c/2 + (d*x)/2)^5*(a^2 + 4*b^
2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (9*a^2*b*tan(c/2 + (d*x)/2)^4*(a^
2 + 4*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(d*(a^2*tan(c/2 + (d*x)/2)
^6 - a^2 - tan(c/2 + (d*x)/2)^2*(a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 +
 4*b^2) + 4*a*b*tan(c/2 + (d*x)/2)^5 - 4*a*b*tan(c/2 + (d*x)/2))) - (3*a^2*
atan(((3*a^2*(a^2 + 4*b^2)*(2*a^6*b - 2*b^7 + 6*a^2*b^5 - 6*a^4*b^3))/(2*(a
 + b)^(7/2)*(a - b)^(7/2)) + (3*a^3*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(a^6 -
 b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(a - b)^(7/2)))/(3*a^4 + 12*a
^2*b^2))*(a^2 + 4*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1472 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=366

$$\frac{2ab\left(a^2+3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{a^2\left(a^2-3b^2\right)\cos(c+dx)}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{3a^4\cos(c+dx)}{2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{a^3\left(2a^2\right.}{}$$

```
[Out] -2*a^3*(a^2-3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b/(a^2-
b^2)^(7/2)/d+a^3*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2
))/b/(a^2-b^2)^(7/2)/d+2*a*b*(a^2+3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a
^2-b^2)^(1/2))/(a^2-b^2)^(7/2)/d+1/2*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))+1/
2*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c))+1/2*a^3*cos(d*x+c)/(a^2-b^2)^2/d/(a+b
*sin(d*x+c))^2+3/2*a^4*cos(d*x+c)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))-a^2*(a^2-3
*b^2)*cos(d*x+c)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.49, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2897, 2648, 2664, 2754, 12, 2660, 618, 204}

$$\frac{a^3\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{7/2}}-\frac{2a^3\left(a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{7/2}}+\frac{2ab\left(a^2+3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*a^3*(a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*
(a^2 - b^2)^(7/2)*d) + (a^3*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/(b*(a^2 - b^2)^(7/2)*d) + (2*a*b*(a^2 + 3*b^2)*ArcTan[(b +
 a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]
/(2*(a + b)^3*d*(1 - Sin[c + d*x])) + Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[
c + d*x])) + (a^3*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2)
+ (3*a^4*Cos[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (a^2*(a^2
 - 3*b^2)*Cos[c + d*x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
```

```
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(-\frac{1}{2(a+b)^3(-1+\sin(c+dx))} - \frac{1}{2(a-b)^3(1+\sin(c+dx))} + \frac{a}{b\left(a^2-b^2\right)(a+\ldots}\right.$$

$$= -\frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3} - \frac{\left(a^2\left(a^2-3b^2\right)\right)\int \frac{1}{(a+b\sin(c+dx))^2}\,dx}{b\left(a^2-b^2\right)^2} + \frac{a^3\int\ldots}{\ldots}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} + \frac{a^3\cos(c+\ldots}{2\left(a^2-b^2\right)^2d(a+b\ldots}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} + \frac{a^3\cos(c+\ldots}{2\left(a^2-b^2\right)^2d(a+b\ldots}$$

$$= \frac{2ab\left(a^2+3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} + \frac{\mathrm{c}\ldots}{2(a-b)^3\ldots}$$

$$= \frac{2ab\left(a^2+3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} + \frac{\mathrm{c}\ldots}{2(a-b)^3\ldots}$$

$$= -\frac{2a^3\left(a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{7/2}d} + \frac{2ab\left(a^2+3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}$$

$$= -\frac{2a^3\left(a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{7/2}d} + \frac{a^3\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{7/2}d} + \ldots$$

Mathematica [A] time = 3.51, size = 204, normalized size = 0.56

$$\frac{\frac{6ab\left(3a^2+2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}}+\frac{a^2\cos(c+dx)\left(2a^3+b\left(a^2+6b^2\right)\sin(c+dx)+5ab^2\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((6*a*b*(3*a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])) - 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (a^2
*Cos[c + d*x]*(2*a^3 + 5*a*b^2 + b*(a^2 + 6*b^2)*Sin[c + d*x]))/((a - b)^3*
(a + b)^3*(a + b*Sin[c + d*x])^2))/(2*d)
```

fricas [A] time = 0.53, size = 917, normalized size = 2.51

$$\left[-\frac{4\,a^7-12\,a^5b^2+12\,a^3b^4-4\,ab^6+2\left(2\,a^7+13\,a^5b^2-17\,a^3b^4+2\,ab^6\right)\cos\left(dx+c\right)^2-3\left(\left(3\,a^3b^3+2\,ab^5\right)\cos\left(\right.\right.}{4\left(\left(a^8b^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(4*a^7 - 12*a^5*b^2 + 12*a^3*b^4 - 4*a*b^6 + 2*(2*a^7 + 13*a^5*b^2 -
17*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^2 - 3*((3*a^3*b^3 + 2*a*b^5)*cos(d*x + c
)^3 - 2*(3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)*sin(d*x + c) - (3*a^5*b + 5*a^
3*b^3 + 2*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) +
 b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2)) - 2*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - (a^6*b + 11*a
^4*b^3 - 10*a^2*b^5 - 2*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^
6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b
^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3
*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)), -1/2*
(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + (2*a^7 + 13*a^5*b^2 - 17*a^3*b^4
 + 2*a*b^6)*cos(d*x + c)^2 + 3*((3*a^3*b^3 + 2*a*b^5)*cos(d*x + c)^3 - 2*(3
*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)*sin(d*x + c) - (3*a^5*b + 5*a^3*b^3 + 2*
a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2
 - b^2)*cos(d*x + c))) - (2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - (a^6*b
```

```
+ 11*a^4*b^3 - 10*a^2*b^5 - 2*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2
- 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4
*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^
10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c))]
```

giac [A] time = 0.31, size = 377, normalized size = 1.03

$$\frac{\frac{3\left(3\,a^{3}b+2\,ab^{3}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\sqrt{a^{2}-b^{2}}}+\frac{2\left(3\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^{3}-3\,ab^{2}\right)}{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}+\frac{3\,a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,a}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(3*a^3*b + 2*a*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*sqrt(a^2 - b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x
 + 1/2*c) - a^3 - 3*a*b^2)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*
x + 1/2*c)^2 - 1)) + (3*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 4*a^2*b^3*tan(1/2*d*
x + 1/2*c)^3 + 2*a^5*tan(1/2*d*x + 1/2*c)^2 + 9*a^3*b^2*tan(1/2*d*x + 1/2*c
)^2 + 10*a*b^4*tan(1/2*d*x + 1/2*c)^2 + 5*a^4*b*tan(1/2*d*x + 1/2*c) + 16*a
^2*b^3*tan(1/2*d*x + 1/2*c) + 2*a^5 + 5*a^3*b^2)/((a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2))/d
```

maple [B] time = 0.55, size = 702, normalized size = 1.92

$$-\frac{1}{d\left(a+b\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{1}{d\left(a-b\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{3a^{4}\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{d\left(a-b\right)^{3}\left(a+b\right)^{3}\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)+3/d*
a^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan
(1/2*d*x+1/2*c)^3*b+4/d*a^2/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1
/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^3+2/d*a^5/(a-b)^3/(a+b)^3/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+10/d*a/
(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2
*d*x+1/2*c)^2*b^4+9/d*a^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
```

```
*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^2+5/d*a^4/(a-b)^3/(a+b)^3/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b+16/d*a^2/
(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2
*d*x+1/2*c)*b^3+2/d*a^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)^2+5/d*a^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2*b^2+9/d*a^3/(a-b)^3/(a+b)^3*b/(a^2-b^2)^(1/2)*arctan(1/2*
(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d*a/(a-b)^3/(a+b)^3*b^3/(a^
2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 17.85, size = 583, normalized size = 1.59

$$\frac{3\,a\,b\,\mathrm{atan}\left(\frac{\frac{3\,a\,b\,\left(3\,a^2+2\,b^2\right)\,\left(2\,a^6\,b-6\,a^4\,b^3+6\,a^2\,b^5-2\,b^7\right)}{2\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}+\frac{3\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^2+2\,b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}}{9\,a^3\,b+6\,a\,b^3}\right)\,\left(3\,a^2+2\,b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}-\frac{\frac{4\,a^5+11\,a^3\,b^2}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{d\,\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] (3*a*b*atan(((3*a*b*(3*a^2 + 2*b^2)*(2*a^6*b - 2*b^7 + 6*a^2*b^5 - 6*a^4*b^
3))/(2*(a + b)^(7/2)*(a - b)^(7/2)) + (3*a^2*b*tan(c/2 + (d*x)/2)*(3*a^2 +
2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(a - b)^(7/2)))/
(6*a*b^3 + 9*a^3*b))*(3*a^2 + 2*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2)) - ((4
*a^5 + 11*a^3*b^2)/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (9*tan(c/2 + (d*x)
/2)^4*(2*a*b^4 + 3*a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (2*tan(c
/2 + (d*x)/2)^3*(3*a^4*b + 4*b^5 + 8*a^2*b^3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a
^4*b^2) + (2*tan(c/2 + (d*x)/2)^2*(13*a*b^4 + 2*a^5))/(a^6 - b^6 + 3*a^2*b^
4 - 3*a^4*b^2) + (b*tan(c/2 + (d*x)/2)*(7*a^4 + 38*a^2*b^2))/(a^6 - b^6 + 3
*a^2*b^4 - 3*a^4*b^2) - (3*a^2*b*tan(c/2 + (d*x)/2)^5*(3*a^2 + 2*b^2))/(a^6
```

```
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(d*(a^2*tan(c/2 + (d*x)/2)^6 - a^2 - tan(c
/2 + (d*x)/2)^2*(a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 + 4*b^2) + 4*a*b*
tan(c/2 + (d*x)/2)^5 - 4*a*b*tan(c/2 + (d*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1473 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=350

$$-\frac{a^2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{4a^2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{\quad}{2d}$$

```
[Out] -4*a^2*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)
/d-a^2*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^
2)^(7/2)/d-2*b^2*(3*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2
))/(a^2-b^2)^(7/2)/d+1/2*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))-1/2*cos(d*x+c)
/(a-b)^3/d/(1+sin(d*x+c))-1/2*a^2*b*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c
))^2-3/2*a^3*b*cos(d*x+c)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))-2*a*b^3*cos(d*x+c)
/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.56, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2731, 2648, 2664, 2754, 12, 2660, 618, 204}

$$-\frac{a^2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{4a^2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{\quad}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-4*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^
(7/2)*d) - (a^2*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/((a^2 - b^2)^(7/2)*d) - (2*b^2*(3*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]/(2*(a + b)^3*
d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) - (
a^2*b*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) - (3*a^3*b*C
os[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (2*a*b^3*Cos[c + d*
x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
```

```
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(-\frac{1}{2(a+b)^3(-1+\sin(c+dx))} + \frac{1}{2(a-b)^3(1+\sin(c+dx))} - \frac{a^2}{\left(a^2-b^2\right)(a+b\sin\right.}\right.$$

$$= \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3} - \frac{\left(2ab^2\right)\int \frac{1}{(a+b\sin(c+dx))^2}\,dx}{\left(a^2-b^2\right)^2} - \frac{a^2\int \frac{1}{(a+b\sin(c+dx))}}{a^2-b^2}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} - \frac{a^2b\cos(c+dx}{2\left(a^2-b^2\right)^2d(a+b\sin}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} - \frac{a^2b\cos(c+dx}{2\left(a^2-b^2\right)^2d(a+b\sin}$$

$$= -\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos}{2(a-b)^3d(}$$

$$= -\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos}{2(a-b)^3d(}$$

$$= -\frac{4a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} - \frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\mathrm{c}}{2(a+b)^3}$$

$$= -\frac{4a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} - \frac{a^2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} - \frac{2b^2\left(3a^2+\right.}{}$$

Mathematica [A] time = 3.16, size = 212, normalized size = 0.61

$$\frac{-\frac{2\left(2a^4+11a^2b^2+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}}-\frac{ab\cos(c+dx)\left(4a^3+b\left(3a^2+4b^2\right)\sin(c+dx)+3ab^2\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a-b)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\mathrm{co}\right.}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 + 11*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/(a^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])) + 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
))) - (a*b*Cos[c + d*x]*(4*a^3 + 3*a*b^2 + b*(3*a^2 + 4*b^2)*Sin[c + d*x]))
/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])^2))/(2*d)
```

fricas [A] time = 0.51, size = 934, normalized size = 2.67

$$\left[\frac{4\,a^6b-12\,a^4b^3+12\,a^2b^5-4\,b^7+2\left(8\,a^6b+a^4b^3-11\,a^2b^5+2\,b^7\right)\cos\left(dx+c\right)^2+\left(\left(2\,a^4b^2+11\,a^2b^4+2\,b^6\right)\cos\right.}{4\left(\left(a^8b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 2*(8*a^6*b + a^4*b^3 - 11
*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*cos(d*
x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c) - (
2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log
(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(
d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 -
2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a
^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^
9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c
) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x
 + c)), 1/2*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + (8*a^6*b + a^4*b^3 -
 11*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*cos
(d*x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c)
- (2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^7 - 6*a^
```

```
5*b^2 + 6*a^3*b^4 - 2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c)^
2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*co
s(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(
d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b
^8 + b^10)*d*cos(d*x + c))]
```

giac [A] time = 0.31, size = 384, normalized size = 1.10

$$-\frac{\frac{\left(2\,a^4+11\,a^2b^2+2\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2b-b^3\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{5\,a^3b^2\tan\left(\frac{1}{2}\,dx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((2*a^4 + 11*a^2*b^2 + 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + a
rctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 2*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan
(1/2*d*x + 1/2*c) - 3*a^2*b - b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(ta
n(1/2*d*x + 1/2*c)^2 - 1)) + (5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^4*ta
n(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 11*a^2*b^3*tan(1/2*
d*x + 1/2*c)^2 + 6*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/
2*c) + 10*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b + 3*a^2*b^3)/((a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c)
+ a)^2))/d
```

maple [B] time = 0.56, size = 766, normalized size = 2.19

$$-\frac{1}{d\,(a+b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{d\,(a-b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{5a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{d\,(a-b)^3\,(a+b)^3\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)-1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)-5/d*
a^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan
(1/2*d*x+1/2*c)^3*b^2-2/d/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*a*b^4-4/d*a^4/(a-b)^3/(a+b)^3/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b-11/d*
```

```
a^2/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan
(1/2*d*x+1/2*c)^2*b^3-6/d/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^5-11/d*a^3/(a-b)^3/(a+b)^3/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^2-10/d/(
a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*
d*x+1/2*c)*a*b^4-4/d*a^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2*b-3/d/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)^2*a^2*b^3-2/d*a^4/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-11/d*a^2/(a-b)^3/(a+b)^3/(a^
2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-2
/d/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 18.21, size = 627, normalized size = 1.79

$$\frac{\frac{5\left(2\,a^4\,b+a^2\,b^3\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^5+a^3\,b^2+12\,a\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^4\,b+6\,a^2\,b^3+7\,b^5\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(2\,a^4\,b+11\,a^2\,b^3+2\,b^5\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{a\,\mathrm{t}\cdots}{\cdots}}{d\,\left(a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-a^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^2+4\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(a^2+4\,b^2\right)+4\,a\,b\,\mathrm{ta}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] ((5*(2*a^4*b + a^2*b^3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (2*tan(c/2 +
 (d*x)/2)^3*(12*a*b^4 + 2*a^5 + a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2) + (2*tan(c/2 + (d*x)/2)^2*(2*a^4*b + 7*b^5 + 6*a^2*b^3))/(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2) - (3*tan(c/2 + (d*x)/2)^4*(2*a^4*b + 2*b^5 + 11*a^2*b^
3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (a*tan(c/2 + (d*x)/2)*(18*b^4 - 2
*a^4 + 29*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (a*tan(c/2 + (d*x
```

```
)/2)^5*(2*a^4 + 2*b^4 + 11*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(
d*(a^2*tan(c/2 + (d*x)/2)^6 - a^2 - tan(c/2 + (d*x)/2)^2*(a^2 + 4*b^2) + ta
n(c/2 + (d*x)/2)^4*(a^2 + 4*b^2) + 4*a*b*tan(c/2 + (d*x)/2)^5 - 4*a*b*tan(c
/2 + (d*x)/2))) - (atan((((2*a^4 + 2*b^4 + 11*a^2*b^2)*(2*a^6*b - 2*b^7 + 6
*a^2*b^5 - 6*a^4*b^3))/(2*(a + b)^(7/2)*(a - b)^(7/2)) + (a*tan(c/2 + (d*x)
/2)*(2*a^4 + 2*b^4 + 11*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a +
 b)^(7/2)*(a - b)^(7/2)))/(2*a^4 + 2*b^4 + 11*a^2*b^2))*(2*a^4 + 2*b^4 + 11
*a^2*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

3.1474 $\int \frac{\sec(c+dx)\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=204

$$\frac{3ab\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{\sec(c+dx)\left(3a\left(2a^2+3b^2\right)-b\left(11a^2+4b^2\right)\sin(c+dx)\right)}{2d\left(a^2-b^2\right)^3}-\frac{\left(3a^2+2b\right.}{2d\left(a^2-b^2\right)^2}$$

```
[Out] 3*a*b*(2*a^2+3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b
^2)^(7/2)/d-1/2*a*sec(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^2-1/2*(3*a^2+2*b^
2)*sec(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))+1/2*sec(d*x+c)*(3*a*(2*a^2+3*b
^2)-b*(11*a^2+4*b^2)*sin(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A] time = 0.36, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2864, 2866, 12, 2660, 618, 204}

$$\frac{3ab\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{\sec(c+dx)\left(3a\left(2a^2+3b^2\right)-b\left(11a^2+4b^2\right)\sin(c+dx)\right)}{2d\left(a^2-b^2\right)^3}-\frac{\left(3a^2+2b\right.}{2d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*b*(2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((
a^2 - b^2)^(7/2)*d) - (a*Sec[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]
)^2) - ((3*a^2 + 2*b^2)*Sec[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x
])) + (Sec[c + d*x]*(3*a*(2*a^2 + 3*b^2) - b*(11*a^2 + 4*b^2)*Sin[c + d*x])
)/(2*(a^2 - b^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{a\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)(2b-3a\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{a\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^2(c+}{}}{}$$

$$= -\frac{a\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{\sec(c+}{}$$

$$= -\frac{a\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{\sec(c+}{}$$

$$= -\frac{a\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{\sec(c+}{}$$

$$= -\frac{a\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{\sec(c+}{}$$

$$= \frac{3ab\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} - \frac{a\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{}{2\left(a\right.}$$

Mathematica [A] time = 3.03, size = 206, normalized size = 1.01

$$\frac{\frac{6ab\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + \frac{b^2\cos(c+dx)\left(b\left(5a^2+2b^2\right)\sin(c+dx)+a\left(6a^2+b^2\right)\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((6*a*b*(2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])) - 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (b^2
```

```
*Cos[c + d*x]*(a*(6*a^2 + b^2) + b*(5*a^2 + 2*b^2)*Sin[c + d*x]))/((a - b)^
3*(a + b)^3*(a + b*Sin[c + d*x])^2))/(2*d)
```

fricas [B] time = 0.51, size = 895, normalized size = 4.39

$$\left[-\frac{4\,a^7-12\,a^5b^2+12\,a^3b^4-4\,ab^6+2\left(16\,a^5b^2-17\,a^3b^4+ab^6\right)\cos\left(dx+c\right)^2-3\left(\left(2\,a^3b^3+3\,ab^5\right)\cos\left(dx+c\right)^3\right.}{4\left(\left(a^8b^2-4\,a^6b^4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^7 - 12*a^5*b^2 + 12*a^3*b^4 - 4*a*b^6 + 2*(16*a^5*b^2 - 17*a^3*b
^4 + a*b^6)*cos(d*x + c)^2 - 3*((2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 - 2*(2
*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)*sin(d*x + c) - (2*a^5*b + 5*a^3*b^3 + 3*
a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2)) - 2*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - (11*a^4*b^3 - 7*a^2*b^5
 - 4*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 -
 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*
a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^
4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)), -1/2*(2*a^7 - 6*a^5*b^2
+ 6*a^3*b^4 - 2*a*b^6 + (16*a^5*b^2 - 17*a^3*b^4 + a*b^6)*cos(d*x + c)^2 +
3*((2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 - 2*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x
 + c)*sin(d*x + c) - (2*a^5*b + 5*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*sqrt(a^2
 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a
^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos
(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 +
b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b
^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6
 - 3*a^2*b^8 + b^10)*d*cos(d*x + c))]
```

giac [A] time = 0.32, size = 365, normalized size = 1.79

$$\frac{\frac{3\left(2\,a^3b+3\,ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^3-3\,ab^2\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{7\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

[Out] (3*(2*a^3*b + 3*a*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a *tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c) - a^3 - 3*a*b^2)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*x + 1/2*c)^2 - 1)) + (7*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 13*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 + 2*b^6*tan(1/2*d*x + 1/2*c)^2 + 17*a^3*b^3*tan(1/2*d*x + 1/2*c) + 4*a*b^5*tan(1/2*d*x + 1/2*c) + 6 *a^4*b^2 + a^2*b^4)/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2))/d

maple [B] time = 0.51, size = 643, normalized size = 3.15

$$-\frac{1}{d\left(a+b\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{1}{d\left(a-b\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{7a^{2}\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{3}}{d\left(a-b\right)^{3}\left(a+b\right)^{3}\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)

[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)+7/d* a^2/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan (1/2*d*x+1/2*c)^3*b^3+6/d*a^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan (1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^2+13/d*a/(a-b)^3/(a+b)^3/(tan (1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^4+2/ d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a *tan(1/2*d*x+1/2*c)^2+17/d*a^2/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*ta n(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^3+4/d*b^5/(a-b)^3/(a+b)^3/(tan (1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+6/d*a^3/ (a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^2+1/d *b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a+ 6/d*a^3/(a-b)^3/(a+b)^3*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+9/d*a/(a-b)^3/(a+b)^3*b^3/(a^2-b^2)^(1/2)*arctan(1/ 2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 18.11, size = 624, normalized size = 3.06

$$\frac{3\,a\,b\,\mathrm{atan}\left(\frac{\frac{3\,a\,b\,\left(2\,a^2+3\,b^2\right)\,\left(2\,a^6\,b-6\,a^4\,b^3+6\,a^2\,b^5-2\,b^7\right)}{2\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}+\frac{3\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+3\,b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}}{6\,a^3\,b+9\,a\,b^3}\right)\,\left(2\,a^2+3\,b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}-\frac{\frac{2\,a^5+12\,a^3\,b^2+a\,b^4}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] (3*a*b*atan(((3*a*b*(2*a^2 + 3*b^2)*(2*a^6*b - 2*b^7 + 6*a^2*b^5 - 6*a^4*b^
3))/(2*(a + b)^(7/2)*(a - b)^(7/2)) + (3*a^2*b*tan(c/2 + (d*x)/2)*(2*a^2 +
3*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(a - b)^(7/2)))/
(9*a*b^3 + 6*a^3*b))*(2*a^2 + 3*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2)) - ((a
*b^4 + 2*a^5 + 12*a^3*b^2)/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (2*tan(c/2
 + (d*x)/2)^3*(2*a^4*b + 6*b^5 + 7*a^2*b^3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4
*b^2) - (tan(c/2 + (d*x)/2)^4*(2*b^6 - 2*a^6 + 21*a^2*b^4 + 24*a^4*b^2))/(a
*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (b*tan(c/2 + (d*x)/2)*(2*a^4 + 4*b^
4 + 39*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (2*tan(c/2 + (d*x)/2
)^2*(2*a^6 + b^6 + 14*a^2*b^4 - 2*a^4*b^2))/(a*(a^6 - b^6 + 3*a^2*b^4 - 3*a
^4*b^2)) - (3*a^2*b*tan(c/2 + (d*x)/2)^5*(2*a^2 + 3*b^2))/(a^6 - b^6 + 3*a^
2*b^4 - 3*a^4*b^2))/(d*(a^2*tan(c/2 + (d*x)/2)^6 - a^2 - tan(c/2 + (d*x)/2)
^2*(a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 + 4*b^2) + 4*a*b*tan(c/2 + (d*
x)/2)^5 - 4*a*b*tan(c/2 + (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{\left(c + dx \right)} \sec^{2}{\left(c + dx \right)}}{\left(a + b \sin{\left(c + dx \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

3.1475 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=402

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b^4\left(3a^2-b^2\right)\cos(c+dx)}{a^2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{3b^4\cos(c+dx)}{2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{b^4\cos(c+}{2ad\left(a^2-b^2\right)^2(a+b}$$

```
[Out] 2*b^3*(3*a^2-b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/(a^2-b
^2)^(7/2)/d+b^3*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2)
)/a/(a^2-b^2)^(7/2)/d+2*b^3*(6*a^4-3*a^2*b^2+b^4)*arctan((b+a*tan(1/2*d*x+1
/2*c))/(a^2-b^2)^(1/2))/a^3/(a^2-b^2)^(7/2)/d-arctanh(cos(d*x+c))/a^3/d+1/2
*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))+1/2*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c)
)+1/2*b^4*cos(d*x+c)/a/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^2+3/2*b^4*cos(d*x+c)/
(a^2-b^2)^3/d/(a+b*sin(d*x+c))+b^4*(3*a^2-b^2)*cos(d*x+c)/a^2/(a^2-b^2)^3/d
/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.51, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2897, 3770, 2648, 2664, 2754, 12, 2660, 618, 204}

$$\frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{7/2}}+\frac{b^3\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{7/2}}+\frac{2b^3\left(-3a^2b^2+6a^4+b^4\right)\tan^{-1}\left(\frac{a\tan(}{}\right.}{a^3d\left(a^2-b^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b^3*(3*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(
a^2 - b^2)^(7/2)*d) + (b^3*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sq
rt[a^2 - b^2]])/(a*(a^2 - b^2)^(7/2)*d) + (2*b^3*(6*a^4 - 3*a^2*b^2 + b^4)*
ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(7/2)*d)
 - ArcTanh[Cos[c + d*x]]/(a^3*d) + Cos[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c +
 d*x])) + Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) + (b^4*Cos[c + d*
x])/(2*a*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) + (3*b^4*Cos[c + d*x])/(2*
(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) + (b^4*(3*a^2 - b^2)*Cos[c + d*x])/(a
^2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
```

```
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(\frac{\csc(c+dx)}{a^3} - \frac{1}{2(a+b)^3(-1+\sin(c+dx))} - \frac{1}{2(a-b)^3(1+\sin(c+dx))} - \right.$$

$$= \frac{\int \csc(c+dx)\,dx}{a^3} - \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3} + \frac{b^3\int \frac{1}{(a+b\sin(c+dx))^3}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+$$

$$= \frac{2b^3\left(6a^4-3a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{}{2(a+}$$

$$= \frac{2b^3\left(6a^4-3a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{}{2(a+}$$

$$= \frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{7/2}d} + \frac{2b^3\left(6a^4-3a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\right.}{\sqrt{a^2-}}\right.}{a^3\left(a^2-b^2\right)^{7/2}d}$$

$$= \frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{7/2}d} + \frac{b^3\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{7/2}d} +$$

Mathematica [A] time = 6.59, size = 322, normalized size = 0.80

$$\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3d} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3d} + \frac{9a^2b^4\cos(c+dx)-2b^6\cos(c+dx)}{2a^2d(a-b)^3(a+b)^3(a+b\sin(c+dx))} + \frac{b^3\left(20a^4-7a^2b^2+2b^4\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b^3*(20*a^4 - 7*a^2*b^2 + 2*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)
/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(7/2)*d) - Lo
g[Cos[(c + d*x)/2]]/(a^3*d) + Log[Sin[(c + d*x)/2]]/(a^3*d) + Sin[(c + d*x)
/2]/((a + b)^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - Sin[(c + d*x)/2]/
((a - b)^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (b^4*Cos[c + d*x])/(2
*a*(a - b)^2*(a + b)^2*d*(a + b*Sin[c + d*x])^2) + (9*a^2*b^4*Cos[c + d*x]
- 2*b^6*Cos[c + d*x])/(2*a^2*(a - b)^3*(a + b)^3*d*(a + b*Sin[c + d*x]))
```

fricas [B] time = 3.02, size = 1623, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^10 - 12*a^8*b^2 + 12*a^6*b^4 - 4*a^4*b^6 + 2*(10*a^8*b^2 - 2*a^6
*b^4 - 11*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^2 - ((20*a^4*b^5 - 7*a^2*b^7 +
2*b^9)*cos(d*x + c)^3 - 2*(20*a^5*b^4 - 7*a^3*b^6 + 2*a*b^8)*cos(d*x + c)*s
in(d*x + c) - (20*a^6*b^3 + 13*a^4*b^5 - 5*a^2*b^7 + 2*b^9)*cos(d*x + c))*s
qrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*((a^8*b^2 - 4
*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*
b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*
a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*cos(d*x + c))*log(1/2*c
os(d*x + c) + 1/2) - 2*((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10
)*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*co
s(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2
*b^8 + b^10)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*a^9*b - 6*a^
7*b^3 + 6*a^5*b^5 - 2*a^3*b^7 - (6*a^7*b^3 + 5*a^5*b^5 - 13*a^3*b^7 + 2*a*b
^9)*cos(d*x + c)^2)*sin(d*x + c))/((a^11*b^2 - 4*a^9*b^4 + 6*a^7*b^6 - 4*a^
5*b^8 + a^3*b^10)*d*cos(d*x + c)^3 - 2*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4
*a^6*b^7 + a^4*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^13 - 3*a^11*b^2 + 2*a^
9*b^4 + 2*a^7*b^6 - 3*a^5*b^8 + a^3*b^10)*d*cos(d*x + c)), -1/2*(2*a^10 - 6
*a^8*b^2 + 6*a^6*b^4 - 2*a^4*b^6 + (10*a^8*b^2 - 2*a^6*b^4 - 11*a^4*b^6 + 3
*a^2*b^8)*cos(d*x + c)^2 + ((20*a^4*b^5 - 7*a^2*b^7 + 2*b^9)*cos(d*x + c)^3
 - 2*(20*a^5*b^4 - 7*a^3*b^6 + 2*a*b^8)*cos(d*x + c)*sin(d*x + c) - (20*a^6
*b^3 + 13*a^4*b^5 - 5*a^2*b^7 + 2*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan
(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + ((a^8*b^2 - 4*a^6*
b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 +
 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b
^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*cos(d*x + c))*log(1/2*cos(d*
x + c) + 1/2) - ((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*cos(d
```

```
*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x +
 c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 +
b^10)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - (2*a^9*b - 6*a^7*b^3 + 6
*a^5*b^5 - 2*a^3*b^7 - (6*a^7*b^3 + 5*a^5*b^5 - 13*a^3*b^7 + 2*a*b^9)*cos(d
*x + c)^2)*sin(d*x + c))/((a^11*b^2 - 4*a^9*b^4 + 6*a^7*b^6 - 4*a^5*b^8 + a
^3*b^10)*d*cos(d*x + c)^3 - 2*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4*a^6*b^7
+ a^4*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^13 - 3*a^11*b^2 + 2*a^9*b^4 + 2
*a^7*b^6 - 3*a^5*b^8 + a^3*b^10)*d*cos(d*x + c))]
```

giac [A] time = 0.30, size = 411, normalized size = 1.02

$$\frac{\frac{\left(20\,a^4b^3-7\,a^2b^5+2\,b^7\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^9-3\,a^7b^2+3\,a^5b^4-a^3b^6\right)\sqrt{a^2-b^2}}+\frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^3-3\,ab^2\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{11\,a^3b^5\tan\left(\frac{1}{2}\,dx\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((20*a^4*b^3 - 7*a^2*b^5 + 2*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a)
+ arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^9 - 3*a^7*b^2 +
 3*a^5*b^4 - a^3*b^6)*sqrt(a^2 - b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c) +
b^3*tan(1/2*d*x + 1/2*c) - a^3 - 3*a*b^2)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*(tan(1/2*d*x + 1/2*c)^2 - 1)) + (11*a^3*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*
a*b^7*tan(1/2*d*x + 1/2*c)^3 + 10*a^4*b^4*tan(1/2*d*x + 1/2*c)^2 + 17*a^2*b
^6*tan(1/2*d*x + 1/2*c)^2 - 6*b^8*tan(1/2*d*x + 1/2*c)^2 + 29*a^3*b^5*tan(1
/2*d*x + 1/2*c) - 8*a*b^7*tan(1/2*d*x + 1/2*c) + 10*a^4*b^4 - 3*a^2*b^6)/((
a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(
1/2*d*x + 1/2*c) + a)^2) + log(abs(tan(1/2*d*x + 1/2*c)))/a^3)/d
```

maple [B] time = 0.66, size = 787, normalized size = 1.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c))+1/d/(a-b
)^3/(tan(1/2*d*x+1/2*c)+1)+11/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a
+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-4/d*b^7/(a-b)^3/(a+b)^3/a
^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3
+10/d*a/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2
*tan(1/2*d*x+1/2*c)^2*b^4+17/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+
```

```
2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^2-6/d*b^8/(a-b)^3/(a+b)^3/
a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^
2+29/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a
)^2*tan(1/2*d*x+1/2*c)-8/d*b^7/(a-b)^3/(a+b)^3/a^2/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+10/d*b^4/(a-b)^3/(a+b)^3/(ta
n(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a-3/d*b^6/(a-b)^3/(a+b)^3/
a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+20/d*a/(a-b)^3/(a+b)^
3*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/
2))-7/d*b^5/(a-b)^3/(a+b)^3/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1
/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d*b^7/(a-b)^3/(a+b)^3/a^3/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 17.79, size = 2999, normalized size = 7.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)*(a + b*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) - ((2*a^6 - 3*b^6 + 10*a^2*b^4 + 6*a^4*b^2)
/(a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)*(2*a^6*b - 8
*b^7 + 29*a^2*b^5 + 22*a^4*b^3))/(a^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
- (2*tan(c/2 + (d*x)/2)^3*(2*a^6*b - 2*b^7 + 13*a^2*b^5 + 2*a^4*b^3))/(a^2*
(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)^5*(6*a^6*b - 4*b
^7 + 11*a^2*b^5 + 2*a^4*b^3))/(a^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (
2*tan(c/2 + (d*x)/2)^2*(2*a^8 - 3*b^8 + 10*a^2*b^6 + 8*a^4*b^4 - 2*a^6*b^2)
)/(a^3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)^4*(17*a^2
*b^6 - 6*b^8 - 2*a^8 + 18*a^4*b^4 + 18*a^6*b^2))/(a^3*(a^6 - b^6 + 3*a^2*b^
4 - 3*a^4*b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^6 - a^2 - tan(c/2 + (d*x)/2)^2*
(a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 + 4*b^2) + 4*a*b*tan(c/2 + (d*x)/
2)^5 - 4*a*b*tan(c/2 + (d*x)/2))) + (b^3*atan(((b^3*(-(a + b)^7*(a - b)^7)^
(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c/2 + (d*x)/2)*(2*a^24 + 8*a^4*b^
20 - 76*a^6*b^18 + 346*a^8*b^16 - 938*a^10*b^14 + 1612*a^12*b^12 - 1790*a^1
```

4*b^10 + 1276*a^16*b^8 - 566*a^18*b^6 + 148*a^20*b^4 - 22*a^22*b^2) - 2*a^2
3*b + 4*a^5*b^19 - 37*a^7*b^17 + 164*a^9*b^15 - 433*a^11*b^13 + 722*a^13*b^
11 - 769*a^15*b^9 + 512*a^17*b^7 - 199*a^19*b^5 + 38*a^21*b^3 + (b^3*(-(a +
 b)^7*(a - b)^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c/2 + (d*x)/2)*(
6*a^27 + 8*a^7*b^20 - 78*a^9*b^18 + 342*a^11*b^16 - 888*a^13*b^14 + 1512*a^
15*b^12 - 1764*a^17*b^10 + 1428*a^19*b^8 - 792*a^21*b^6 + 288*a^23*b^4 - 62
*a^25*b^2) - 2*a^26*b + 2*a^8*b^19 - 18*a^10*b^17 + 72*a^12*b^15 - 168*a^14
*b^13 + 252*a^16*b^11 - 252*a^18*b^9 + 168*a^20*b^7 - 72*a^22*b^5 + 18*a^24
*b^3))/(a^17 - a^3*b^14 + 7*a^5*b^12 - 21*a^7*b^10 + 35*a^9*b^8 - 35*a^11*b
^6 + 21*a^13*b^4 - 7*a^15*b^2))*1i)/(a^17 - a^3*b^14 + 7*a^5*b^12 - 21*a^7*
b^10 + 35*a^9*b^8 - 35*a^11*b^6 + 21*a^13*b^4 - 7*a^15*b^2) + (b^3*(-(a + b
)^7*(a - b)^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c/2 + (d*x)/2)*(2*
a^24 + 8*a^4*b^20 - 76*a^6*b^18 + 346*a^8*b^16 - 938*a^10*b^14 + 1612*a^12*
b^12 - 1790*a^14*b^10 + 1276*a^16*b^8 - 566*a^18*b^6 + 148*a^20*b^4 - 22*a^
22*b^2) - 2*a^23*b + 4*a^5*b^19 - 37*a^7*b^17 + 164*a^9*b^15 - 433*a^11*b^1
3 + 722*a^13*b^11 - 769*a^15*b^9 + 512*a^17*b^7 - 199*a^19*b^5 + 38*a^21*b^
3 - (b^3*(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c
/2 + (d*x)/2)*(6*a^27 + 8*a^7*b^20 - 78*a^9*b^18 + 342*a^11*b^16 - 888*a^13
*b^14 + 1512*a^15*b^12 - 1764*a^17*b^10 + 1428*a^19*b^8 - 792*a^21*b^6 + 28
8*a^23*b^4 - 62*a^25*b^2) - 2*a^26*b + 2*a^8*b^19 - 18*a^10*b^17 + 72*a^12*
b^15 - 168*a^14*b^13 + 252*a^16*b^11 - 252*a^18*b^9 + 168*a^20*b^7 - 72*a^2
2*b^5 + 18*a^24*b^3))/(a^17 - a^3*b^14 + 7*a^5*b^12 - 21*a^7*b^10 + 35*a^9*
b^8 - 35*a^11*b^6 + 21*a^13*b^4 - 7*a^15*b^2))*1i)/(a^17 - a^3*b^14 + 7*a^5
*b^12 - 21*a^7*b^10 + 35*a^9*b^8 - 35*a^11*b^6 + 21*a^13*b^4 - 7*a^15*b^2))
/(2*tan(c/2 + (d*x)/2)*(2*a^3*b^18 - 41*a^5*b^16 + 225*a^7*b^14 - 715*a^9*b
^12 + 1135*a^11*b^10 - 792*a^13*b^8 + 146*a^15*b^6 + 40*a^17*b^4) + 4*a^2*b
^19 - 38*a^4*b^17 + 184*a^6*b^15 - 530*a^8*b^13 + 940*a^10*b^11 - 1034*a^12
*b^9 + 688*a^14*b^7 - 254*a^16*b^5 + 40*a^18*b^3 + (b^3*(-(a + b)^7*(a - b)
^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c/2 + (d*x)/2)*(2*a^24 + 8*a^
4*b^20 - 76*a^6*b^18 + 346*a^8*b^16 - 938*a^10*b^14 + 1612*a^12*b^12 - 1790
*a^14*b^10 + 1276*a^16*b^8 - 566*a^18*b^6 + 148*a^20*b^4 - 22*a^22*b^2) - 2
*a^23*b + 4*a^5*b^19 - 37*a^7*b^17 + 164*a^9*b^15 - 433*a^11*b^13 + 722*a^1
3*b^11 - 769*a^15*b^9 + 512*a^17*b^7 - 199*a^19*b^5 + 38*a^21*b^3 + (b^3*(-
(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c/2 + (d*x)/
2)*(6*a^27 + 8*a^7*b^20 - 78*a^9*b^18 + 342*a^11*b^16 - 888*a^13*b^14 + 151
2*a^15*b^12 - 1764*a^17*b^10 + 1428*a^19*b^8 - 792*a^21*b^6 + 288*a^23*b^4
 - 62*a^25*b^2) - 2*a^26*b + 2*a^8*b^19 - 18*a^10*b^17 + 72*a^12*b^15 - 168*
a^14*b^13 + 252*a^16*b^11 - 252*a^18*b^9 + 168*a^20*b^7 - 72*a^22*b^5 + 18*
a^24*b^3))/(a^17 - a^3*b^14 + 7*a^5*b^12 - 21*a^7*b^10 + 35*a^9*b^8 - 35*a^
11*b^6 + 21*a^13*b^4 - 7*a^15*b^2)))/(a^17 - a^3*b^14 + 7*a^5*b^12 - 21*a^7
*b^10 + 35*a^9*b^8 - 35*a^11*b^6 + 21*a^13*b^4 - 7*a^15*b^2) - (b^3*(-(a +
b)^7*(a - b)^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c/2 + (d*x)/2)*(2
*a^24 + 8*a^4*b^20 - 76*a^6*b^18 + 346*a^8*b^16 - 938*a^10*b^14 + 1612*a^12
*b^12 - 1790*a^14*b^10 + 1276*a^16*b^8 - 566*a^18*b^6 + 148*a^20*b^4 - 22*a
^22*b^2) - 2*a^23*b + 4*a^5*b^19 - 37*a^7*b^17 + 164*a^9*b^15 - 433*a^11*b^

```
13 + 722*a^13*b^11 - 769*a^15*b^9 + 512*a^17*b^7 - 199*a^19*b^5 + 38*a^21*b
^3 - (b^3*(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(
c/2 + (d*x)/2)*(6*a^27 + 8*a^7*b^20 - 78*a^9*b^18 + 342*a^11*b^16 - 888*a^1
3*b^14 + 1512*a^15*b^12 - 1764*a^17*b^10 + 1428*a^19*b^8 - 792*a^21*b^6 + 2
88*a^23*b^4 - 62*a^25*b^2) - 2*a^26*b + 2*a^8*b^19 - 18*a^10*b^17 + 72*a^12
*b^15 - 168*a^14*b^13 + 252*a^16*b^11 - 252*a^18*b^9 + 168*a^20*b^7 - 72*a^
22*b^5 + 18*a^24*b^3))/(a^17 - a^3*b^14 + 7*a^5*b^12 - 21*a^7*b^10 + 35*a^9
*b^8 - 35*a^11*b^6 + 21*a^13*b^4 - 7*a^15*b^2)))/(a^17 - a^3*b^14 + 7*a^5*b
^12 - 21*a^7*b^10 + 35*a^9*b^8 - 35*a^11*b^6 + 21*a^13*b^4 - 7*a^15*b^2)))*
(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*2i)/(d*(a^17 -
a^3*b^14 + 7*a^5*b^12 - 21*a^7*b^10 + 35*a^9*b^8 - 35*a^11*b^6 + 21*a^13*b^
4 - 7*a^15*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

3.1476 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=424

$$\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{\cot(c+dx)}{a^3d}-\frac{3b^5\cos(c+dx)}{2ad\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{b^5\cos(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}-\cdots 4$$

```
[Out] -4*b^4*(2*a^2-b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/(a^
2-b^2)^(7/2)/d-b^4*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1
/2))/a^2/(a^2-b^2)^(7/2)/d-2*b^4*(10*a^4-9*a^2*b^2+3*b^4)*arctan((b+a*tan(1
/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^4/(a^2-b^2)^(7/2)/d+3*b*arctanh(cos(d*x+c
))/a^4/d-cot(d*x+c)/a^3/d+1/2*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))-1/2*cos(d
*x+c)/(a-b)^3/d/(1+sin(d*x+c))-1/2*b^5*cos(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*si
n(d*x+c))^2-3/2*b^5*cos(d*x+c)/a/(a^2-b^2)^3/d/(a+b*sin(d*x+c))-2*b^5*(2*a^
2-b^2)*cos(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.59, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2897, 3770, 3767, 8, 2648, 2664, 2754, 12, 2660, 618, 204}

$$-\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{7/2}}-\frac{b^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{7/2}}-\frac{2b^4\left(-9a^2b^2+10a^4+3b^4\right)\tan^{-1}\left(\cdots\right.}{a^4d\left(a^2-b^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-4*b^4*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
2*(a^2 - b^2)^(7/2)*d) - (b^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(7/2)*d) - (2*b^4*(10*a^4 - 9*a^2*b^2 +
 3*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*(a^2 - b^2)^
(7/2)*d) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) - Cot[c + d*x]/(a^3*d) + Cos
[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^3*d*
(1 + Sin[c + d*x])) - (b^5*Cos[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sin[
c + d*x])^2) - (3*b^5*Cos[c + d*x])/(2*a*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x
])) - (2*b^5*(2*a^2 - b^2)*Cos[c + d*x])/(a^3*(a^2 - b^2)^3*d*(a + b*Sin[c
+ d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
```

```
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(-\frac{3b\csc(c+dx)}{a^4} + \frac{\csc^2(c+dx)}{a^3} - \frac{1}{2(a+b)^3(-1+\sin(c+dx))} + \frac{}{2(a-b)^3(}\right.$$

$$= \frac{\int \csc^2(c+dx)\,dx}{a^3} + \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{(3b)\int \csc(c+dx)\,dx}{a^4} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3}$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+}$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\cot(c+dx)}{a^3d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{}{2(a-b)}$$

$$= -\frac{2b^4\left(10a^4-9a^2b^2+3b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{7/2}d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} -$$

$$= -\frac{2b^4\left(10a^4-9a^2b^2+3b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{7/2}d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} -$$

$$= -\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{7/2}d} - \frac{2b^4\left(10a^4-9a^2b^2+3b^4\right)\tan^{-1}\left(\frac{b+a\,\mathrm{ta}}{}\right.}{a^4\left(a^2-b^2\right)^{7/2}d}$$

$$= -\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{7/2}d} - \frac{b^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{7/2}d}$$

Mathematica [A] time = 6.35, size = 379, normalized size = 0.89

$$4\left(-\frac{3b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4a^4d} + \frac{3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{4a^4d} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{8a^3d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{8a^3d} - \frac{b^5\,\mathrm{c}}{8a^2d(a-b)^2(a+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] 4*((-3*b^4*(10*a^4 - 7*a^2*b^2 + 2*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c
+ d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(4*a^4*(a^2 - b^2)^(7/2)
*d) - Cot[(c + d*x)/2]/(8*a^3*d) + (3*b*Log[Cos[(c + d*x)/2]])/(4*a^4*d) -
(3*b*Log[Sin[(c + d*x)/2]])/(4*a^4*d) + Sin[(c + d*x)/2]/(4*(a + b)^3*d*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/(4*(a - b)^3*d*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])) - (b^5*Cos[c + d*x])/(8*a^2*(a - b)^2*(a
+ b)^2*d*(a + b*Sin[c + d*x])^2) + (-11*a^2*b^5*Cos[c + d*x] + 4*b^7*Cos[c
+ d*x])/(8*a^3*(a - b)^3*(a + b)^3*d*(a + b*Sin[c + d*x])) + Tan[(c + d*x)/
2]/(8*a^3*d))
```

<u>fricas</u> [B] time = 2.62, size = 2140, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(4*a^11 - 12*a^9*b^2 + 12*a^7*b^4 - 4*a^5*b^6 + 2*(4*a^9*b^2 - 4*a^7*
b^4 + 17*a^5*b^6 - 23*a^3*b^8 + 6*a*b^10)*cos(d*x + c)^4 - 2*(4*a^11 - 10*a
^9*b^2 + 14*a^7*b^4 + 7*a^5*b^6 - 21*a^3*b^8 + 6*a*b^10)*cos(d*x + c)^2 - 3
*(2*(10*a^5*b^5 - 7*a^3*b^7 + 2*a*b^9)*cos(d*x + c)^3 - 2*(10*a^5*b^5 - 7*a
^3*b^7 + 2*a*b^9)*cos(d*x + c) + ((10*a^4*b^6 - 7*a^2*b^8 + 2*b^10)*cos(d*x
 + c)^3 - (10*a^6*b^4 + 3*a^4*b^6 - 5*a^2*b^8 + 2*b^10)*cos(d*x + c))*sin(d
*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
 + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-
a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(2*(
a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c)^3 - 2*(a
^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c) + ((a^8*b
^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*cos(d*x + c)^3 - (a^10*b - 3
*a^8*b^3 + 2*a^6*b^5 + 2*a^4*b^7 - 3*a^2*b^9 + b^11)*cos(d*x + c))*sin(d*x
+ c))*log(1/2*cos(d*x + c) + 1/2) + 6*(2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 -
 4*a^3*b^8 + a*b^10)*cos(d*x + c)^3 - 2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 -
4*a^3*b^8 + a*b^10)*cos(d*x + c) + ((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^
2*b^9 + b^11)*cos(d*x + c)^3 - (a^10*b - 3*a^8*b^3 + 2*a^6*b^5 + 2*a^4*b^7
- 3*a^2*b^9 + b^11)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2
) - 2*(2*a^10*b - 6*a^8*b^3 + 6*a^6*b^5 - 2*a^4*b^7 + (8*a^10*b - 14*a^8*b^
3 + 28*a^6*b^5 - 31*a^4*b^7 + 9*a^2*b^9)*cos(d*x + c)^2)*sin(d*x + c))/(2*(
a^13*b - 4*a^11*b^3 + 6*a^9*b^5 - 4*a^7*b^7 + a^5*b^9)*d*cos(d*x + c)^3 - 2
*(a^13*b - 4*a^11*b^3 + 6*a^9*b^5 - 4*a^7*b^7 + a^5*b^9)*d*cos(d*x + c) + (
(a^12*b^2 - 4*a^10*b^4 + 6*a^8*b^6 - 4*a^6*b^8 + a^4*b^10)*d*cos(d*x + c)^3
 - (a^14 - 3*a^12*b^2 + 2*a^10*b^4 + 2*a^8*b^6 - 3*a^6*b^8 + a^4*b^10)*d*co
s(d*x + c))*sin(d*x + c)), -1/2*(2*a^11 - 6*a^9*b^2 + 6*a^7*b^4 - 2*a^5*b^6
```

```
 + (4*a^9*b^2 - 4*a^7*b^4 + 17*a^5*b^6 - 23*a^3*b^8 + 6*a*b^10)*cos(d*x + c
)^4 - (4*a^11 - 10*a^9*b^2 + 14*a^7*b^4 + 7*a^5*b^6 - 21*a^3*b^8 + 6*a*b^10
)*cos(d*x + c)^2 - 3*(2*(10*a^5*b^5 - 7*a^3*b^7 + 2*a*b^9)*cos(d*x + c)^3 -
 2*(10*a^5*b^5 - 7*a^3*b^7 + 2*a*b^9)*cos(d*x + c) + ((10*a^4*b^6 - 7*a^2*b
^8 + 2*b^10)*cos(d*x + c)^3 - (10*a^6*b^4 + 3*a^4*b^6 - 5*a^2*b^8 + 2*b^10)
*cos(d*x + c))*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(
sqrt(a^2 - b^2)*cos(d*x + c))) - 3*(2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*
a^3*b^8 + a*b^10)*cos(d*x + c)^3 - 2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a
^3*b^8 + a*b^10)*cos(d*x + c) + ((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b
^9 + b^11)*cos(d*x + c)^3 - (a^10*b - 3*a^8*b^3 + 2*a^6*b^5 + 2*a^4*b^7 - 3
*a^2*b^9 + b^11)*cos(d*x + c))*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) +
3*(2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c)^3
- 2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c) + (
(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*cos(d*x + c)^3 - (a^10
*b - 3*a^8*b^3 + 2*a^6*b^5 + 2*a^4*b^7 - 3*a^2*b^9 + b^11)*cos(d*x + c))*si
n(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - (2*a^10*b - 6*a^8*b^3 + 6*a^6*b^
5 - 2*a^4*b^7 + (8*a^10*b - 14*a^8*b^3 + 28*a^6*b^5 - 31*a^4*b^7 + 9*a^2*b^
9)*cos(d*x + c)^2)*sin(d*x + c))/(2*(a^13*b - 4*a^11*b^3 + 6*a^9*b^5 - 4*a^
7*b^7 + a^5*b^9)*d*cos(d*x + c)^3 - 2*(a^13*b - 4*a^11*b^3 + 6*a^9*b^5 - 4*
a^7*b^7 + a^5*b^9)*d*cos(d*x + c) + ((a^12*b^2 - 4*a^10*b^4 + 6*a^8*b^6 - 4
*a^6*b^8 + a^4*b^10)*d*cos(d*x + c)^3 - (a^14 - 3*a^12*b^2 + 2*a^10*b^4 + 2
*a^8*b^6 - 3*a^6*b^8 + a^4*b^10)*d*cos(d*x + c))*sin(d*x + c))]
```

giac [A] time = 0.30, size = 633, normalized size = 1.49

$$-\frac{\frac{6\left(10\,a^{4}b^{4}-7\,a^{2}b^{6}+2\,b^{8}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{10}-3\,a^{8}b^{2}+3\,a^{6}b^{4}-a^{4}b^{6}\right)\sqrt{a^{2}-b^{2}}}-\frac{2\,a^{6}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6\,a^{4}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6\,a^{2}b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,b}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*(10*a^4*b^4 - 7*a^2*b^6 + 2*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*
sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^10 - 3*a
^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(a^2 - b^2)) - (2*a^6*b*tan(1/2*d*x + 1/2
*c)^3 - 6*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b^5*tan(1/2*d*x + 1/2*c)^3
 - 2*b^7*tan(1/2*d*x + 1/2*c)^3 - 5*a^7*tan(1/2*d*x + 1/2*c)^2 - 9*a^5*b^2*
tan(1/2*d*x + 1/2*c)^2 - 3*a^3*b^4*tan(1/2*d*x + 1/2*c)^2 + a*b^6*tan(1/2*d
*x + 1/2*c)^2 + 10*a^6*b*tan(1/2*d*x + 1/2*c) + 10*a^4*b^3*tan(1/2*d*x + 1/
2*c) - 6*a^2*b^5*tan(1/2*d*x + 1/2*c) + 2*b^7*tan(1/2*d*x + 1/2*c) + a^7 -
3*a^5*b^2 + 3*a^3*b^4 - a*b^6)/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*(t
an(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))) + 2*(13*a^3*b^6*tan(1/2*d*x
+ 1/2*c)^3 - 6*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*b^5*tan(1/2*d*x + 1/2*
```

```
c)^2 + 19*a^2*b^7*tan(1/2*d*x + 1/2*c)^2 - 10*b^9*tan(1/2*d*x + 1/2*c)^2 +
35*a^3*b^6*tan(1/2*d*x + 1/2*c) - 14*a*b^8*tan(1/2*d*x + 1/2*c) + 12*a^4*b^
5 - 5*a^2*b^7)/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*(a*tan(1/2*d*x + 1
/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 6*b*log(abs(tan(1/2*d*x + 1/2*
c)))/a^4 - tan(1/2*d*x + 1/2*c)/a^3)/d
```

maple [B] time = 0.67, size = 829, normalized size = 1.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+1/2/d/a^3*tan(1/2*d*x+1/2*c)-1/2/d/a^3/
tan(1/2*d*x+1/2*c)-3/d/a^4*b*ln(tan(1/2*d*x+1/2*c))-1/d/(a-b)^3/(tan(1/2*d*
x+1/2*c)+1)-13/d/a*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+6/d/a^3*b^8/(a-b)^3/(a+b)^3/(tan(1/2*d
*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-12/d/(a-b)^3
/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/
2*c)^2*b^5-19/d/a^2*b^7/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+10/d/a^4*b^9/(a-b)^3/(a+b)^3/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-35/d/a*b^6
/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/
2*d*x+1/2*c)+14/d/a^3*b^8/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-12/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x
+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+5/d/a^2*b^7/(a-b)^3/(a+b)^3/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-30/d/(a-b)^3/(a+b)^3/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4+21/d/a^
2*b^6/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*
b)/(a^2-b^2)^(1/2))-6/d/a^4*b^8/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*
(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 15.78, size = 3122, normalized size = 7.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - ((tan(c/2 + (d*x)/2)^6*(5*a^8 - 12*b^8 + 25*
a^2*b^6 + 3*a^4*b^4 + 9*a^6*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - a^2
 + (tan(c/2 + (d*x)/2)^4*(9*a^8 - 20*b^8 + 55*a^2*b^6 + 23*a^4*b^4 - 7*a^6*
b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (tan(c/2 + (d*x)/2)^2*(81*a^2*b
^6 - 32*b^8 - 3*a^8 + 7*a^4*b^4 + 37*a^6*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a
^4*b^2) + (2*tan(c/2 + (d*x)/2)*(7*a*b^7 - 8*a^7*b - 18*a^3*b^5 + 4*a^5*b^3
))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (4*tan(c/2 + (d*x)/2)^3*(2*a^8*b -
 5*b^9 + 12*a^2*b^7 + 4*a^4*b^5 + 2*a^6*b^3))/(a*(a^6 - b^6 + 3*a^2*b^4 - 3
*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^5*(4*a^8*b - 10*b^9 + 17*a^2*b^7 + 18*a^
4*b^5 + 16*a^6*b^3))/(a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)))/(d*(2*a^5*tan
(c/2 + (d*x)/2)^7 - tan(c/2 + (d*x)/2)^3*(2*a^5 + 8*a^3*b^2) + tan(c/2 + (d
*x)/2)^5*(2*a^5 + 8*a^3*b^2) - 2*a^5*tan(c/2 + (d*x)/2) - 8*a^4*b*tan(c/2 +
 (d*x)/2)^2 + 8*a^4*b*tan(c/2 + (d*x)/2)^6)) - (3*b*log(tan(c/2 + (d*x)/2))
)/(a^4*d) - (b^4*atan(((b^4*(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + 2*b^4 -
7*a^2*b^2)*(tan(c/2 + (d*x)/2)*(6*a^26*b + 24*a^6*b^21 - 228*a^8*b^19 + 978
*a^10*b^17 - 2454*a^12*b^15 + 3936*a^14*b^13 - 4170*a^16*b^11 + 2928*a^18*b
^9 - 1338*a^20*b^7 + 384*a^22*b^5 - 66*a^24*b^3) + 12*a^7*b^20 - 111*a^9*b^
18 + 462*a^11*b^16 - 1119*a^13*b^14 + 1716*a^15*b^12 - 1707*a^17*b^10 + 108
6*a^19*b^8 - 417*a^21*b^6 + 84*a^23*b^4 - 6*a^25*b^2 - (3*b^4*(-(a + b)^7*(
a - b)^7)^(1/2)*(10*a^4 + 2*b^4 - 7*a^2*b^2)*(tan(c/2 + (d*x)/2)*(6*a^30 +
8*a^10*b^20 - 78*a^12*b^18 + 342*a^14*b^16 - 888*a^16*b^14 + 1512*a^18*b^12
 - 1764*a^20*b^10 + 1428*a^22*b^8 - 792*a^24*b^6 + 288*a^26*b^4 - 62*a^28*b
^2) - 2*a^29*b + 2*a^11*b^19 - 18*a^13*b^17 + 72*a^15*b^15 - 168*a^17*b^13
+ 252*a^19*b^11 - 252*a^21*b^9 + 168*a^23*b^7 - 72*a^25*b^5 + 18*a^27*b^3))
/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6
 + 21*a^14*b^4 - 7*a^16*b^2)))*3i)/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^
8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)) + (b^4*(-(a
 + b)^7*(a - b)^7)^(1/2)*(10*a^4 + 2*b^4 - 7*a^2*b^2)*(tan(c/2 + (d*x)/2)*(
6*a^26*b + 24*a^6*b^21 - 228*a^8*b^19 + 978*a^10*b^17 - 2454*a^12*b^15 + 39
36*a^14*b^13 - 4170*a^16*b^11 + 2928*a^18*b^9 - 1338*a^20*b^7 + 384*a^22*b^
5 - 66*a^24*b^3) + 12*a^7*b^20 - 111*a^9*b^18 + 462*a^11*b^16 - 1119*a^13*b
^14 + 1716*a^15*b^12 - 1707*a^17*b^10 + 1086*a^19*b^8 - 417*a^21*b^6 + 84*a
^23*b^4 - 6*a^25*b^2 + (3*b^4*(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + 2*b^4
- 7*a^2*b^2)*(tan(c/2 + (d*x)/2)*(6*a^30 + 8*a^10*b^20 - 78*a^12*b^18 + 342
*a^14*b^16 - 888*a^16*b^14 + 1512*a^18*b^12 - 1764*a^20*b^10 + 1428*a^22*b^
8 - 792*a^24*b^6 + 288*a^26*b^4 - 62*a^28*b^2) - 2*a^29*b + 2*a^11*b^19 - 1
8*a^13*b^17 + 72*a^15*b^15 - 168*a^17*b^13 + 252*a^19*b^11 - 252*a^21*b^9 +
```

```
 168*a^23*b^7 - 72*a^25*b^5 + 18*a^27*b^3))/(2*(a^18 - a^4*b^14 + 7*a^6*b^1
2 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)))*3
i)/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b
^6 + 21*a^14*b^4 - 7*a^16*b^2)))/(2*tan(c/2 + (d*x)/2)*(18*a^4*b^20 - 189*a
^6*b^18 + 765*a^8*b^16 - 1575*a^10*b^14 + 1575*a^12*b^12 - 468*a^14*b^10 -
306*a^16*b^8 + 180*a^18*b^6) + 36*a^3*b^21 - 342*a^5*b^19 + 1476*a^7*b^17 -
 3690*a^9*b^15 + 5760*a^11*b^13 - 5706*a^13*b^11 + 3492*a^15*b^9 - 1206*a^1
7*b^7 + 180*a^19*b^5 - (3*b^4*(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + 2*b^4
- 7*a^2*b^2)*(tan(c/2 + (d*x)/2)*(6*a^26*b + 24*a^6*b^21 - 228*a^8*b^19 + 9
78*a^10*b^17 - 2454*a^12*b^15 + 3936*a^14*b^13 - 4170*a^16*b^11 + 2928*a^18
*b^9 - 1338*a^20*b^7 + 384*a^22*b^5 - 66*a^24*b^3) + 12*a^7*b^20 - 111*a^9*
b^18 + 462*a^11*b^16 - 1119*a^13*b^14 + 1716*a^15*b^12 - 1707*a^17*b^10 + 1
086*a^19*b^8 - 417*a^21*b^6 + 84*a^23*b^4 - 6*a^25*b^2 - (3*b^4*(-(a + b)^7
*(a - b)^7)^(1/2)*(10*a^4 + 2*b^4 - 7*a^2*b^2)*(tan(c/2 + (d*x)/2)*(6*a^30
+ 8*a^10*b^20 - 78*a^12*b^18 + 342*a^14*b^16 - 888*a^16*b^14 + 1512*a^18*b^
12 - 1764*a^20*b^10 + 1428*a^22*b^8 - 792*a^24*b^6 + 288*a^26*b^4 - 62*a^28
*b^2) - 2*a^29*b + 2*a^11*b^19 - 18*a^13*b^17 + 72*a^15*b^15 - 168*a^17*b^1
3 + 252*a^19*b^11 - 252*a^21*b^9 + 168*a^23*b^7 - 72*a^25*b^5 + 18*a^27*b^3
))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b
^6 + 21*a^14*b^4 - 7*a^16*b^2))))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8
*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)) + (3*b^4*(-(
a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + 2*b^4 - 7*a^2*b^2)*(tan(c/2 + (d*x)/2)*
(6*a^26*b + 24*a^6*b^21 - 228*a^8*b^19 + 978*a^10*b^17 - 2454*a^12*b^15 + 3
936*a^14*b^13 - 4170*a^16*b^11 + 2928*a^18*b^9 - 1338*a^20*b^7 + 384*a^22*b
^5 - 66*a^24*b^3) + 12*a^7*b^20 - 111*a^9*b^18 + 462*a^11*b^16 - 1119*a^13*
b^14 + 1716*a^15*b^12 - 1707*a^17*b^10 + 1086*a^19*b^8 - 417*a^21*b^6 + 84*
a^23*b^4 - 6*a^25*b^2 + (3*b^4*(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + 2*b^4
 - 7*a^2*b^2)*(tan(c/2 + (d*x)/2)*(6*a^30 + 8*a^10*b^20 - 78*a^12*b^18 + 34
2*a^14*b^16 - 888*a^16*b^14 + 1512*a^18*b^12 - 1764*a^20*b^10 + 1428*a^22*b
^8 - 792*a^24*b^6 + 288*a^26*b^4 - 62*a^28*b^2) - 2*a^29*b + 2*a^11*b^19 -
18*a^13*b^17 + 72*a^15*b^15 - 168*a^17*b^13 + 252*a^19*b^11 - 252*a^21*b^9
+ 168*a^23*b^7 - 72*a^25*b^5 + 18*a^27*b^3))/(2*(a^18 - a^4*b^14 + 7*a^6*b^
12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2))))
/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6
 + 21*a^14*b^4 - 7*a^16*b^2))))*(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + 2*b^
4 - 7*a^2*b^2)*3i)/(d*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10
*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1477 $\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=470

$$\frac{3b\cot(c+dx)}{a^4d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{3b^6\cos(c+dx)}{2a^2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{\left(a^2+6b^2\right)\mathrm{ta}}{}$$

```
[Out] 2*b^5*(5*a^2-3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/(a
^2-b^2)^(7/2)/d+b^5*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(
1/2))/a^3/(a^2-b^2)^(7/2)/d+2*b^5*(15*a^4-17*a^2*b^2+6*b^4)*arctan((b+a*tan
(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^5/(a^2-b^2)^(7/2)/d-1/2*arctanh(cos(d*x
+c))/a^3/d-(a^2+6*b^2)*arctanh(cos(d*x+c))/a^5/d+3*b*cot(d*x+c)/a^4/d-1/2*c
ot(d*x+c)*csc(d*x+c)/a^3/d+1/2*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))+1/2*cos(
d*x+c)/(a-b)^3/d/(1+sin(d*x+c))+1/2*b^6*cos(d*x+c)/a^3/(a^2-b^2)^2/d/(a+b*s
in(d*x+c))^2+3/2*b^6*cos(d*x+c)/a^2/(a^2-b^2)^3/d/(a+b*sin(d*x+c))+b^6*(5*a
^2-3*b^2)*cos(d*x+c)/a^4/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.61, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2897, 3770, 3767, 8, 3768, 2648, 2664, 2754, 12, 2660, 618, 204}

$$\frac{b^5\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{7/2}}+\frac{2b^5\left(-17a^2b^2+15a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\left(a^2-b^2\right)^{7/2}}+\frac{2b^5\left(5a^2-3b^2\right)\tan^{-1}}{a^3d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b^5*(5*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^3*(a^2 - b^2)^(7/2)*d) + (b^5*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2]
)/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(7/2)*d) + (2*b^5*(15*a^4 - 17*a^2*b^2
 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5*(a^2 - b^2
)^(7/2)*d) - ArcTanh[Cos[c + d*x]]/(2*a^3*d) - ((a^2 + 6*b^2)*ArcTanh[Cos[c
 + d*x]])/(a^5*d) + (3*b*Cot[c + d*x])/(a^4*d) - (Cot[c + d*x]*Csc[c + d*x]
)/(2*a^3*d) + Cos[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c + d*x])) + Cos[c + d*x
]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) + (b^6*Cos[c + d*x])/(2*a^3*(a^2 - b^2
)^2*d*(a + b*Sin[c + d*x])^2) + (3*b^6*Cos[c + d*x])/(2*a^2*(a^2 - b^2)^3*d
*(a + b*Sin[c + d*x])) + (b^6*(5*a^2 - 3*b^2)*Cos[c + d*x])/(a^4*(a^2 - b^2
)^3*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(\frac{\left(a^2+6b^2\right)\csc(c+dx)}{a^5} - \frac{3b\csc^2(c+dx)}{a^4} + \frac{\csc^3(c+dx)}{a^3} - \frac{1}{2(a+b)^3(-1+s}\right.$$

$$= \frac{\int \csc^3(c+dx)\,dx}{a^3} - \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{(3b)\int \csc^2(c+dx)\,dx}{a^4} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,d}{2(a+b)^3}$$

$$= -\frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5 d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^3 d} + \frac{\cos(c+a}{2(a+b)^3 d(1-si}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^3 d} - \frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5 d} + \frac{3b\cot(c+dx)}{a^4 d} - \frac{co}{}$$

$$= \frac{2b^5\left(15a^4-17a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{7/2} d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^3 d} - \frac{\left(a^2\right.}{}$$

$$= \frac{2b^5\left(15a^4-17a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{7/2} d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^3 d} - \frac{\left(a^2\right.}{}$$

$$= \frac{2b^5\left(5a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2} d} + \frac{2b^5\left(15a^4-17a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a}{}\right.}{a^5\left(a^2-b^2\right)^{7/2} d}$$

$$= \frac{2b^5\left(5a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2} d} + \frac{b^5\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2} d}$$

Mathematica [A] time = 6.78, size = 432, normalized size = 0.92

$$-\frac{3b\tan\left(\frac{1}{2}(c+dx)\right)}{2a^4 d} + \frac{3b\cot\left(\frac{1}{2}(c+dx)\right)}{2a^4 d} + \frac{b^6\cos(c+dx)}{2a^3 d(a-b)^2(a+b)^2(a+b\sin(c+dx))^2} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^3 d} + \frac{\sec^2\left(\frac{1}{2}(c+\right.}{8a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b^5*(14*a^4 - 13*a^2*b^2 + 4*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d
*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*(a^2 - b^2)^(7/2)*d) +
 (3*b*Cot[(c + d*x)/2])/(2*a^4*d) - Csc[(c + d*x)/2]^2/(8*a^3*d) - (3*(a^2
+ 4*b^2)*Log[Cos[(c + d*x)/2]])/(2*a^5*d) + (3*(a^2 + 4*b^2)*Log[Sin[(c + d
*x)/2]])/(2*a^5*d) + Sec[(c + d*x)/2]^2/(8*a^3*d) + Sin[(c + d*x)/2]/((a +
b)^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - Sin[(c + d*x)/2]/((a - b)^3
*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (b^6*Cos[c + d*x])/(2*a^3*(a -
b)^2*(a + b)^2*d*(a + b*Sin[c + d*x])^2) + (13*a^2*b^6*Cos[c + d*x] - 6*b^8
*Cos[c + d*x])/(2*a^4*(a - b)^3*(a + b)^3*d*(a + b*Sin[c + d*x])) - (3*b*Ta
n[(c + d*x)/2])/(2*a^4*d)
```

fricas [B] time = 4.57, size = 2672, normalized size = 5.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(4*a^12 - 12*a^10*b^2 + 12*a^8*b^4 - 4*a^6*b^6 - 2*(21*a^10*b^2 - 56*a
^8*b^4 + 82*a^6*b^6 - 65*a^4*b^8 + 18*a^2*b^10)*cos(d*x + c)^4 - 2*(3*a^12
- 31*a^10*b^2 + 68*a^8*b^4 - 88*a^6*b^6 + 66*a^4*b^8 - 18*a^2*b^10)*cos(d*x
 + c)^2 + 3*((14*a^4*b^7 - 13*a^2*b^9 + 4*b^11)*cos(d*x + c)^5 - (14*a^6*b^
5 + 15*a^4*b^7 - 22*a^2*b^9 + 8*b^11)*cos(d*x + c)^3 + (14*a^6*b^5 + a^4*b^
7 - 9*a^2*b^9 + 4*b^11)*cos(d*x + c) - 2*((14*a^5*b^6 - 13*a^3*b^8 + 4*a*b^
10)*cos(d*x + c)^3 - (14*a^5*b^6 - 13*a^3*b^8 + 4*a*b^10)*cos(d*x + c))*sin
(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqr
t(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 3*(
(a^10*b^2 - 10*a^6*b^6 + 20*a^4*b^8 - 15*a^2*b^10 + 4*b^12)*cos(d*x + c)^5
- (a^12 + 2*a^10*b^2 - 10*a^8*b^4 + 25*a^4*b^8 - 26*a^2*b^10 + 8*b^12)*cos(
d*x + c)^3 + (a^12 + a^10*b^2 - 10*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8 - 11*a^
2*b^10 + 4*b^12)*cos(d*x + c) - 2*((a^11*b - 10*a^7*b^5 + 20*a^5*b^7 - 15*a
^3*b^9 + 4*a*b^11)*cos(d*x + c)^3 - (a^11*b - 10*a^7*b^5 + 20*a^5*b^7 - 15*
a^3*b^9 + 4*a*b^11)*cos(d*x + c))*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2)
 + 3*((a^10*b^2 - 10*a^6*b^6 + 20*a^4*b^8 - 15*a^2*b^10 + 4*b^12)*cos(d*x +
 c)^5 - (a^12 + 2*a^10*b^2 - 10*a^8*b^4 + 25*a^4*b^8 - 26*a^2*b^10 + 8*b^12
)*cos(d*x + c)^3 + (a^12 + a^10*b^2 - 10*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8 -
 11*a^2*b^10 + 4*b^12)*cos(d*x + c) - 2*((a^11*b - 10*a^7*b^5 + 20*a^5*b^7
- 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c)^3 - (a^11*b - 10*a^7*b^5 + 20*a^5*b^7
 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d*x + c)
 + 1/2) - 2*(2*a^11*b - 6*a^9*b^3 + 6*a^7*b^5 - 2*a^5*b^7 + (12*a^9*b^3 - 2
8*a^7*b^5 + 47*a^5*b^7 - 43*a^3*b^9 + 12*a*b^11)*cos(d*x + c)^4 - (6*a^11*b
```

```
 - 10*a^9*b^3 + 2*a^7*b^5 + 29*a^5*b^7 - 39*a^3*b^9 + 12*a*b^11)*cos(d*x +
c)^2)*sin(d*x + c))/((a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b
^10)*d*cos(d*x + c)^5 - (a^15 - 2*a^13*b^2 - 2*a^11*b^4 + 8*a^9*b^6 - 7*a^7
*b^8 + 2*a^5*b^10)*d*cos(d*x + c)^3 + (a^15 - 3*a^13*b^2 + 2*a^11*b^4 + 2*a
^9*b^6 - 3*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) - 2*((a^14*b - 4*a^12*b^3 + 6
*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^3 - (a^14*b - 4*a^12*b^3 +
6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c))*sin(d*x + c)), 1/4*(4*a^1
2 - 12*a^10*b^2 + 12*a^8*b^4 - 4*a^6*b^6 - 2*(21*a^10*b^2 - 56*a^8*b^4 + 82
*a^6*b^6 - 65*a^4*b^8 + 18*a^2*b^10)*cos(d*x + c)^4 - 2*(3*a^12 - 31*a^10*b
^2 + 68*a^8*b^4 - 88*a^6*b^6 + 66*a^4*b^8 - 18*a^2*b^10)*cos(d*x + c)^2 - 6
*((14*a^4*b^7 - 13*a^2*b^9 + 4*b^11)*cos(d*x + c)^5 - (14*a^6*b^5 + 15*a^4*
b^7 - 22*a^2*b^9 + 8*b^11)*cos(d*x + c)^3 + (14*a^6*b^5 + a^4*b^7 - 9*a^2*b
^9 + 4*b^11)*cos(d*x + c) - 2*((14*a^5*b^6 - 13*a^3*b^8 + 4*a*b^10)*cos(d*x
 + c)^3 - (14*a^5*b^6 - 13*a^3*b^8 + 4*a*b^10)*cos(d*x + c))*sin(d*x + c))*
sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))
) - 3*((a^10*b^2 - 10*a^6*b^6 + 20*a^4*b^8 - 15*a^2*b^10 + 4*b^12)*cos(d*x
+ c)^5 - (a^12 + 2*a^10*b^2 - 10*a^8*b^4 + 25*a^4*b^8 - 26*a^2*b^10 + 8*b^1
2)*cos(d*x + c)^3 + (a^12 + a^10*b^2 - 10*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8
- 11*a^2*b^10 + 4*b^12)*cos(d*x + c) - 2*((a^11*b - 10*a^7*b^5 + 20*a^5*b^7
 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c)^3 - (a^11*b - 10*a^7*b^5 + 20*a^5*b^
7 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c))*sin(d*x + c))*log(1/2*cos(d*x + c)
 + 1/2) + 3*((a^10*b^2 - 10*a^6*b^6 + 20*a^4*b^8 - 15*a^2*b^10 + 4*b^12)*co
s(d*x + c)^5 - (a^12 + 2*a^10*b^2 - 10*a^8*b^4 + 25*a^4*b^8 - 26*a^2*b^10 +
 8*b^12)*cos(d*x + c)^3 + (a^12 + a^10*b^2 - 10*a^8*b^4 + 10*a^6*b^6 + 5*a^
4*b^8 - 11*a^2*b^10 + 4*b^12)*cos(d*x + c) - 2*((a^11*b - 10*a^7*b^5 + 20*a
^5*b^7 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c)^3 - (a^11*b - 10*a^7*b^5 + 20*
a^5*b^7 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2) - 2*(2*a^11*b - 6*a^9*b^3 + 6*a^7*b^5 - 2*a^5*b^7 + (12*a^9*
b^3 - 28*a^7*b^5 + 47*a^5*b^7 - 43*a^3*b^9 + 12*a*b^11)*cos(d*x + c)^4 - (6
*a^11*b - 10*a^9*b^3 + 2*a^7*b^5 + 29*a^5*b^7 - 39*a^3*b^9 + 12*a*b^11)*cos
(d*x + c)^2)*sin(d*x + c))/((a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8
+ a^5*b^10)*d*cos(d*x + c)^5 - (a^15 - 2*a^13*b^2 - 2*a^11*b^4 + 8*a^9*b^6
- 7*a^7*b^8 + 2*a^5*b^10)*d*cos(d*x + c)^3 + (a^15 - 3*a^13*b^2 + 2*a^11*b^
4 + 2*a^9*b^6 - 3*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) - 2*((a^14*b - 4*a^12*
b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^3 - (a^14*b - 4*a^12
*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c))*sin(d*x + c))]
```

giac [B] time = 0.37, size = 900, normalized size = 1.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(24*(14*a^4*b^5 - 13*a^2*b^7 + 4*b^9)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
```

```
*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^11 - 3*
a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*sqrt(a^2 - b^2)) + 16*(3*a^2*b*tan(1/2*d*x +
 1/2*c) + b^3*tan(1/2*d*x + 1/2*c) - a^3 - 3*a*b^2)/((a^6 - 3*a^4*b^2 + 3*a
^2*b^4 - b^6)*(tan(1/2*d*x + 1/2*c)^2 - 1)) - (6*a^10*tan(1/2*d*x + 1/2*c)^
6 + 6*a^8*b^2*tan(1/2*d*x + 1/2*c)^6 - 54*a^6*b^4*tan(1/2*d*x + 1/2*c)^6 +
66*a^4*b^6*tan(1/2*d*x + 1/2*c)^6 - 24*a^2*b^8*tan(1/2*d*x + 1/2*c)^6 + 12*
a^9*b*tan(1/2*d*x + 1/2*c)^5 + 60*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 - 252*a^5*
b^5*tan(1/2*d*x + 1/2*c)^5 + 156*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 - 32*a*b^9*
tan(1/2*d*x + 1/2*c)^5 + 13*a^10*tan(1/2*d*x + 1/2*c)^4 - 15*a^8*b^2*tan(1/
2*d*x + 1/2*c)^4 + 63*a^6*b^4*tan(1/2*d*x + 1/2*c)^4 - 341*a^4*b^6*tan(1/2*
d*x + 1/2*c)^4 + 96*a^2*b^8*tan(1/2*d*x + 1/2*c)^4 + 16*b^10*tan(1/2*d*x +
1/2*c)^4 + 4*a^9*b*tan(1/2*d*x + 1/2*c)^3 + 36*a^7*b^3*tan(1/2*d*x + 1/2*c)
^3 - 132*a^5*b^5*tan(1/2*d*x + 1/2*c)^3 - 188*a^3*b^7*tan(1/2*d*x + 1/2*c)^
3 + 112*a*b^9*tan(1/2*d*x + 1/2*c)^3 + 8*a^10*tan(1/2*d*x + 1/2*c)^2 - 44*a
^8*b^2*tan(1/2*d*x + 1/2*c)^2 + 84*a^6*b^4*tan(1/2*d*x + 1/2*c)^2 - 180*a^4
*b^6*tan(1/2*d*x + 1/2*c)^2 + 76*a^2*b^8*tan(1/2*d*x + 1/2*c)^2 - 8*a^9*b*t
an(1/2*d*x + 1/2*c) + 24*a^7*b^3*tan(1/2*d*x + 1/2*c) - 24*a^5*b^5*tan(1/2*
d*x + 1/2*c) + 8*a^3*b^7*tan(1/2*d*x + 1/2*c) + a^10 - 3*a^8*b^2 + 3*a^6*b^
4 - a^4*b^6)/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2
*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))^2) + 12*(a^2 +
 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 + (a^3*tan(1/2*d*x + 1/2*c)^2 -
12*a^2*b*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [B] time = 0.73, size = 897, normalized size = 1.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^
4*tan(1/2*d*x+1/2*c)*b-1/8/d/a^3/tan(1/2*d*x+1/2*c)^2+3/2/d/a^3*ln(tan(1/2*
d*x+1/2*c))+6/d/a^5*ln(tan(1/2*d*x+1/2*c))*b^2+3/2/d*b/a^4/tan(1/2*d*x+1/2*
c)+1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)+15/d*b^7/(a-b)^3/(a+b)^3/a^2/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-8/d*b^9/(a
-b)^3/(a+b)^3/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1
/2*d*x+1/2*c)^3+14/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^2+21/d*b^8/(a-b)^3/(a+b)^3/a^3/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-14/d*b^
10/(a-b)^3/(a+b)^3/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*
tan(1/2*d*x+1/2*c)^2+41/d*b^7/(a-b)^3/(a+b)^3/a^2/(tan(1/2*d*x+1/2*c)^2*a+2
*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-20/d*b^9/(a-b)^3/(a+b)^3/a^4/
(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+14/d
*b^6/(a-b)^3/(a+b)^3/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-
7/d*b^8/(a-b)^3/(a+b)^3/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
```

```
a)^2+42/d*b^5/(a-b)^3/(a+b)^3/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x
+1/2*c)+2*b)/(a^2-b^2)^(1/2))-39/d*b^7/(a-b)^3/(a+b)^3/a^3/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+12/d*b^9/(a-b)^3/(
a+b)^3/a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2
)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 16.82, size = 3266, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^3*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (4*a^2*b*tan(c/2 + (d*x)/2) - a^3/2 + (5*t
an(c/2 + (d*x)/2)^2*(3*a^9 - 20*a*b^8 + 49*a^3*b^6 - 27*a^5*b^4 + 19*a^7*b^
2))/(2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^7*(15*a
^8*b - 16*b^9 + 27*a^2*b^7 + 9*a^4*b^5 - 5*a^6*b^3))/(a^6 - b^6 + 3*a^2*b^4
 - 3*a^4*b^2) - (4*tan(c/2 + (d*x)/2)^5*(5*a^8*b - 18*b^9 + 43*a^2*b^7 - 7*
a^4*b^5 + 7*a^6*b^3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (2*tan(c/2 + (d
*x)/2)^3*(7*a^8*b - 52*b^9 + 115*a^2*b^7 - 27*a^4*b^5 + 47*a^6*b^3))/(a^6 -
 b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (tan(c/2 + (d*x)/2)^4*(33*a^10 - 112*b^10 +
 220*a^2*b^8 + 11*a^4*b^6 + 119*a^6*b^4 - 31*a^8*b^2))/(2*a*(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^6*(17*a^10 + 112*b^10 - 120*a^2
*b^8 - 257*a^4*b^6 + 83*a^6*b^4 - 195*a^8*b^2))/(2*a*(a^6 - b^6 + 3*a^2*b^4
 - 3*a^4*b^2)))/(d*(4*a^6*tan(c/2 + (d*x)/2)^2 - 4*a^6*tan(c/2 + (d*x)/2)^8
 + tan(c/2 + (d*x)/2)^4*(4*a^6 + 16*a^4*b^2) - tan(c/2 + (d*x)/2)^6*(4*a^6
+ 16*a^4*b^2) + 16*a^5*b*tan(c/2 + (d*x)/2)^3 - 16*a^5*b*tan(c/2 + (d*x)/2)
^7)) - (3*b*tan(c/2 + (d*x)/2))/(2*a^4*d) + (log(tan(c/2 + (d*x)/2))*(3*a^2
 + 12*b^2))/(2*a^5*d) + (b^5*atan(((b^5*(-(a + b)^7*(a - b)^7)^(1/2)*(14*a^
4 + 4*b^4 - 13*a^2*b^2)*(tan(c/2 + (d*x)/2)*(24*a^30 + 384*a^8*b^22 - 3552*
a^10*b^20 + 14616*a^12*b^18 - 34872*a^14*b^16 + 52800*a^16*b^14 - 52176*a^1
8*b^12 + 33168*a^20*b^10 - 12576*a^22*b^8 + 2112*a^24*b^6 + 240*a^26*b^4 -
```

168*a^28*b^2) - 24*a^29*b + 192*a^9*b^21 - 1728*a^11*b^19 + 6888*a^13*b^17
- 15816*a^15*b^15 + 22800*a^17*b^13 - 21048*a^19*b^11 + 12048*a^21*b^9 - 37
68*a^23*b^7 + 336*a^25*b^5 + 120*a^27*b^3 - (3*b^5*(-(a + b)^7*(a - b)^7)^(
1/2)*(14*a^4 + 4*b^4 - 13*a^2*b^2)*(tan(c/2 + (d*x)/2)*(48*a^33 + 64*a^13*b
^20 - 624*a^15*b^18 + 2736*a^17*b^16 - 7104*a^19*b^14 + 12096*a^21*b^12 - 1
4112*a^23*b^10 + 11424*a^25*b^8 - 6336*a^27*b^6 + 2304*a^29*b^4 - 496*a^31*
b^2) - 16*a^32*b + 16*a^14*b^19 - 144*a^16*b^17 + 576*a^18*b^15 - 1344*a^20
*b^13 + 2016*a^22*b^11 - 2016*a^24*b^9 + 1344*a^26*b^7 - 576*a^28*b^5 + 144
*a^30*b^3))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 -
35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))*3i)/(2*(a^19 - a^5*b^14 + 7*a^7*b
^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2))
+ (b^5*(-(a + b)^7*(a - b)^7)^(1/2)*(14*a^4 + 4*b^4 - 13*a^2*b^2)*(tan(c/2
+ (d*x)/2)*(24*a^30 + 384*a^8*b^22 - 3552*a^10*b^20 + 14616*a^12*b^18 - 348
72*a^14*b^16 + 52800*a^16*b^14 - 52176*a^18*b^12 + 33168*a^20*b^10 - 12576*
a^22*b^8 + 2112*a^24*b^6 + 240*a^26*b^4 - 168*a^28*b^2) - 24*a^29*b + 192*a
^9*b^21 - 1728*a^11*b^19 + 6888*a^13*b^17 - 15816*a^15*b^15 + 22800*a^17*b^
13 - 21048*a^19*b^11 + 12048*a^21*b^9 - 3768*a^23*b^7 + 336*a^25*b^5 + 120*
a^27*b^3 + (3*b^5*(-(a + b)^7*(a - b)^7)^(1/2)*(14*a^4 + 4*b^4 - 13*a^2*b^2
)*(tan(c/2 + (d*x)/2)*(48*a^33 + 64*a^13*b^20 - 624*a^15*b^18 + 2736*a^17*b
^16 - 7104*a^19*b^14 + 12096*a^21*b^12 - 14112*a^23*b^10 + 11424*a^25*b^8 -
6336*a^27*b^6 + 2304*a^29*b^4 - 496*a^31*b^2) - 16*a^32*b + 16*a^14*b^19 -
144*a^16*b^17 + 576*a^18*b^15 - 1344*a^20*b^13 + 2016*a^22*b^11 - 2016*a^2
4*b^9 + 1344*a^26*b^7 - 576*a^28*b^5 + 144*a^30*b^3))/(2*(a^19 - a^5*b^14 +
7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^1
7*b^2)))*3i)/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 -
35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))/(2*tan(c/2 + (d*x)/2)*(576*a^5*b
^22 - 5040*a^7*b^20 + 18072*a^9*b^18 - 34128*a^11*b^16 + 35640*a^13*b^14 -
20376*a^15*b^12 + 7200*a^17*b^10 - 2952*a^19*b^8 + 1008*a^21*b^6) + 1152*a^
4*b^23 - 10368*a^6*b^21 + 41112*a^8*b^19 - 92448*a^10*b^17 + 126792*a^12*b^
15 - 105552*a^14*b^13 + 48168*a^16*b^11 - 6912*a^18*b^9 - 2952*a^20*b^7 + 1
008*a^22*b^5 - (3*b^5*(-(a + b)^7*(a - b)^7)^(1/2)*(14*a^4 + 4*b^4 - 13*a^2
b^2)(tan(c/2 + (d*x)/2)*(24*a^30 + 384*a^8*b^22 - 3552*a^10*b^20 + 14616*
a^12*b^18 - 34872*a^14*b^16 + 52800*a^16*b^14 - 52176*a^18*b^12 + 33168*a^2
0*b^10 - 12576*a^22*b^8 + 2112*a^24*b^6 + 240*a^26*b^4 - 168*a^28*b^2) - 24
*a^29*b + 192*a^9*b^21 - 1728*a^11*b^19 + 6888*a^13*b^17 - 15816*a^15*b^15
+ 22800*a^17*b^13 - 21048*a^19*b^11 + 12048*a^21*b^9 - 3768*a^23*b^7 + 336*
a^25*b^5 + 120*a^27*b^3 - (3*b^5*(-(a + b)^7*(a - b)^7)^(1/2)*(14*a^4 + 4*b
^4 - 13*a^2*b^2)*(tan(c/2 + (d*x)/2)*(48*a^33 + 64*a^13*b^20 - 624*a^15*b^1
8 + 2736*a^17*b^16 - 7104*a^19*b^14 + 12096*a^21*b^12 - 14112*a^23*b^10 + 1
1424*a^25*b^8 - 6336*a^27*b^6 + 2304*a^29*b^4 - 496*a^31*b^2) - 16*a^32*b +
16*a^14*b^19 - 144*a^16*b^17 + 576*a^18*b^15 - 1344*a^20*b^13 + 2016*a^22*
b^11 - 2016*a^24*b^9 + 1344*a^26*b^7 - 576*a^28*b^5 + 144*a^30*b^3))/(2*(a^
19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a
^15*b^4 - 7*a^17*b^2))))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 3
5*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)) + (3*b^5*(-(a + b)^7*

```
(a - b)^7)^(1/2)*(14*a^4 + 4*b^4 - 13*a^2*b^2)*(tan(c/2 + (d*x)/2)*(24*a^30
 + 384*a^8*b^22 - 3552*a^10*b^20 + 14616*a^12*b^18 - 34872*a^14*b^16 + 5280
0*a^16*b^14 - 52176*a^18*b^12 + 33168*a^20*b^10 - 12576*a^22*b^8 + 2112*a^2
4*b^6 + 240*a^26*b^4 - 168*a^28*b^2) - 24*a^29*b + 192*a^9*b^21 - 1728*a^11
*b^19 + 6888*a^13*b^17 - 15816*a^15*b^15 + 22800*a^17*b^13 - 21048*a^19*b^1
1 + 12048*a^21*b^9 - 3768*a^23*b^7 + 336*a^25*b^5 + 120*a^27*b^3 + (3*b^5*(
-(a + b)^7*(a - b)^7)^(1/2)*(14*a^4 + 4*b^4 - 13*a^2*b^2)*(tan(c/2 + (d*x)/
2)*(48*a^33 + 64*a^13*b^20 - 624*a^15*b^18 + 2736*a^17*b^16 - 7104*a^19*b^1
4 + 12096*a^21*b^12 - 14112*a^23*b^10 + 11424*a^25*b^8 - 6336*a^27*b^6 + 23
04*a^29*b^4 - 496*a^31*b^2) - 16*a^32*b + 16*a^14*b^19 - 144*a^16*b^17 + 57
6*a^18*b^15 - 1344*a^20*b^13 + 2016*a^22*b^11 - 2016*a^24*b^9 + 1344*a^26*b
^7 - 576*a^28*b^5 + 144*a^30*b^3))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^
9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2))))/(2*(a^19
- a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15
*b^4 - 7*a^17*b^2))))*(-(a + b)^7*(a - b)^7)^(1/2)*(14*a^4 + 4*b^4 - 13*a^2
*b^2)*3i)/(d*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35
*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1478 $\int \frac{\sec^2(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=158

$$\frac{\sec(e+fx)\sqrt{d\sin(e+fx)}\,\sqrt{a+b\sin(e+fx)}}{df}-\frac{\sqrt{a+b}\,\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\,\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{d}\,\sqrt{}}{\sqrt{a+b}}\right.\right.}{\sqrt{d}\,f}$$

```
[Out] sec(f*x+e)*(d*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/d/f-EllipticF(d^(1/2
)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2),((-a-b)/(a-b))^(1
/2))*(a+b)^(1/2)*(a*(1-csc(f*x+e))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1
/2)*tan(f*x+e)/f/d^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2888, 2816}

$$\frac{\sec(e+fx)\sqrt{d\sin(e+fx)}\,\sqrt{a+b\sin(e+fx)}}{df}-\frac{\sqrt{a+b}\,\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\,\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{d}\,\sqrt{}}{\sqrt{a+b}}\right.\right.}{\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]])/Sqrt[d*Sin[e + f*x]],x]
```

```
[Out] (Sec[e + f*x]*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])/(d*f) - (Sqrt[
a + b]*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a
- b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt
[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(Sqrt[d]*f)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2888

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(g*Co
s[e + f*x])^(p + 1)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^m)/(d*f*g*(2*
```

```
m + 1)), x] + Dist[(2*a*m)/(g^2*(2*m + 1)), Int[((g*Cos[e + f*x])^(p + 2)*(
a + b*Sin[e + f*x])^(m - 1))/Sqrt[d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, e
, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && EqQ[m + p + 3/2, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx = \frac{\sec(e+fx)\sqrt{d\sin(e+fx)}\,\sqrt{a+b\sin(e+fx)}}{df} + \frac{1}{2}a\int \frac{}{\sqrt{d\sin(e+fx)}}$$

$$= \frac{\sec(e+fx)\sqrt{d\sin(e+fx)}\,\sqrt{a+b\sin(e+fx)}}{df} - \frac{\sqrt{a+b}\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}}{}$$

Mathematica [A] time = 6.23, size = 198, normalized size = 1.25

$$\frac{4a^2\sin^4\left(\frac{1}{4}(2e+2fx-\pi)\right)\sec(e+fx)\sqrt{-\frac{(a+b)\sin(e+fx)(a+b\sin(e+fx))}{a^2(\sin(e+fx)-1)^2}}\sqrt{-\frac{(a+b)\cot^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{a-b}}F\left(\sin^{-1}\left(\sqrt{-\frac{a+b\sin(}{a(\sin(e+}}\right.\right.}{f(a+b)\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]])/Sqrt[d*Sin[e + f*x]],x]
```

```
[Out] (4*a^2*Sqrt[-(((a + b)*Cot[(2*e - Pi + 2*f*x)/4]^2)/(a - b))]*EllipticF[Arc
Sin[Sqrt[-((a + b*Sin[e + f*x])/(a*(-1 + Sin[e + f*x])))]], (2*a)/(a - b)]*
Sec[e + f*x]*Sqrt[-(((a + b)*Sin[e + f*x]*(a + b*Sin[e + f*x]))/(a^2*(-1 +
Sin[e + f*x])^2))]*Sin[(2*e - Pi + 2*f*x)/4]^4 + (a + b)*(a + b*Sin[e + f*x
])*Tan[e + f*x])/((a + b)*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)}\sec\left(fx+e\right)^2}{d\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))*sec(f*x + e)^2/(d*si
n(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sin\left(fx+e\right)+a}\,\sec\left(fx+e\right)^2}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sec(f*x + e)^2/sqrt(d*sin(f*x + e)), x)
```

maple [B] time = 0.72, size = 666, normalized size = 4.22

$$-\frac{\left(\sqrt{-\frac{-\sqrt{-a^2+b^2}\,\sin\left(fx+e\right)-b\sin\left(fx+e\right)+\cos\left(fx+e\right)a-a}{\left(b+\sqrt{-a^2+b^2}\right)\sin\left(fx+e\right)}}\,\sqrt{\frac{\sqrt{-a^2+b^2}\,\sin\left(fx+e\right)-b\sin\left(fx+e\right)+\cos\left(fx+e\right)a-a}{\sqrt{-a^2+b^2}\,\sin\left(fx+e\right)}}\,\sqrt{\frac{a\left(-1+\cos\left(fx+e\right)\right)}{\left(b+\sqrt{-a^2+b^2}\right)\sin\left(fx+e\right)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a
^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)
+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(
-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((-(-(-a^2+b^2)^(1/2)*sin(f*x+e
)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2
^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)*sin(f*x+e)
*(-a^2+b^2)^(1/2)+(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a
-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*
sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*
x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((-(-(-a^2+b^2)^(1/2)
*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(
1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)*
sin(f*x+e)*b-2^(1/2)*cos(f*x+e)*sin(f*x+e)*b-2^(1/2)*cos(f*x+e)*a+2^(1/2)*b
*sin(f*x+e)+2^(1/2)*a)*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)/(d*sin(f*x+e))
^(1/2)/(a+b*sin(f*x+e))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sin\left(fx+e\right)+a}\,\sec\left(fx+e\right)^2}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sec(f*x + e)^2/sqrt(d*sin(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\sin\left(e+f\,x\right)}}{{\cos\left(e+f\,x\right)}^2\,\sqrt{d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)/(cos(e + f*x)^2*(d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/(cos(e + f*x)^2*(d*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sin\left(e+fx\right)}\,\sec^2\left(e+fx\right)}{\sqrt{d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e))**(1/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))*sec(e + f*x)**2/sqrt(d*sin(e + f*x)), x)
```

3.1479 $\int \frac{\sec^2(e+fx)(a+b\sin(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=312

$$\frac{\sec(e+fx)(a\sin(e+fx)+b)\sqrt{a+b\sin(e+fx)}}{f\sqrt{d\sin(e+fx)}}-\frac{(a+b)^{3/2}\tan(e+fx)\sqrt{-\frac{a(\csc(e+fx)-1)}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}F\left(\sin^{-1}\right.}{\sqrt{d}f}$$

```
[Out] sec(f*x+e)*(b+a*sin(f*x+e))*(a+b*sin(f*x+e))^(1/2)/f/(d*sin(f*x+e))^(1/2)-(
a+b)^(3/2)*EllipticF(d^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(d*sin(f*x+
e))^(1/2),((-a-b)/(a-b))^(1/2))*(-a*(-1+csc(f*x+e))/(a+b))^(1/2)*(a*(1+csc(
f*x+e))/(a-b))^(1/2)*tan(f*x+e)/f/d^(1/2)-b*(a+b)*EllipticE(((-b-a*csc(f*x+
e))/(a-b))^(1/2),((-a+b)/(a+b))^(1/2))*(1+sin(f*x+e))*(-a*(-1+csc(f*x+e))/(
a+b))^(1/2)*((b+a*csc(f*x+e))/(-a+b))^(1/2)*tan(f*x+e)/f/(a*(1+csc(f*x+e))/
(a-b))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2)
```

Rubi [F] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^2(e+fx)(a+b\sin(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[e + f*x]^2*(a + b*Sin[e + f*x])^(3/2))/Sqrt[d*Sin[e + f*x]],x]
```

```
[Out] Defer[Int][(Sec[e + f*x]^2*(a + b*Sin[e + f*x])^(3/2))/Sqrt[d*Sin[e + f*x]]
, x]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)(a+b\sin(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}}\,dx=\int \frac{\sec^2(e+fx)(a+b\sin(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}}\,dx$$

Mathematica [A] time = 22.91, size = 602, normalized size = 1.93

$$\frac{\sqrt{\sin(e+fx)}\left(\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)\sqrt{\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{2\tan^2\left(\frac{1}{2}(e+fx)\right)+2}}\sqrt{\frac{a\tan^2\left(\frac{1}{2}(e+fx)\right)+a+2b\tan\left(\frac{1}{2}(e+fx)\right)}{\tan^2\left(\frac{1}{2}(e+fx)\right)+1}}\left(\frac{2\sqrt{b^2-a^2}\left(\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)\sqrt{\ldots}}{\ldots}\right.}{f\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^2*(a + b*Sin[e + f*x])^(3/2))/Sqrt[d*Sin[e + f*x]],
x]
```

```
[Out] (Sqrt[Sin[e + f*x]]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[Tan[(e + f*x)/2]/(2 + 2*T
an[(e + f*x)/2]^2)]*Sqrt[(a + 2*b*Tan[(e + f*x)/2] + a*Tan[(e + f*x)/2]^2)/
(1 + Tan[(e + f*x)/2]^2)]*(-2*b*Tan[(e + f*x)/2]^2 + (2*Sqrt[-a^2 + b^2]*(1
 + Tan[(e + f*x)/2]^2)*Sqrt[(a*(a + 2*b*Tan[(e + f*x)/2] + a*Tan[(e + f*x)/
2]^2))/(a^2 - b^2)]*(-(b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*T
an[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqr
t[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2
+ b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2
])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2]
)]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]]))/(Sqrt[(a*Tan[(e +
 f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*(a + 2*b*Tan[(e + f*x)/2] + a*Tan[(e + f
*x)/2]^2))))/(f*Sqrt[d*Sin[e + f*x]]*(Tan[(e + f*x)/2] + Tan[(e + f*x)/2]^3
)) + ((a + b*Sin[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[d*Sin[e + f*x]])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\sec\left(fx+e\right)^2\sin\left(fx+e\right)+a\sec\left(fx+e\right)^2\right)\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)}}{d\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2*sin(f*x + e) + a*sec(f*x + e)^2)*sqrt(b*sin(f*x
+ e) + a)*sqrt(d*sin(f*x + e))/(d*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{2}}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*sec(f*x + e)^2/sqrt(d*sin(f*x + e)), x
)
```

maple [B] time = 0.61, size = 2377, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(2*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(
-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+
e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b
+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(-(-a^2+b^2)^(1/2)*sin(f*x
+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2
*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)^2*(-a^2+
b^2)^(1/2)*b^2-2*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-
a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*s
in(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x
+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(-(-a^2+b^2)^(1/2)*
sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1
/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)^2
*a^2*b+2*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-
a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e
)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+
(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(-(-a^2+b^2)^(1/2)*sin(f*x+
e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*
2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)^2*b^3-(-(
-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/
2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)
*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(
1/2))/sin(f*x+e))^(1/2)*EllipticF((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x
+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+
```

```
(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)^2*(-a^2+b^2)^(1/2)*a^
2+2*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b
^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos
(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2
+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*
sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/
2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/
2)*b^2-2*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-
a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e
)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+
(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(-(-a^2+b^2)^(1/2)*sin(f*x+
e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*
2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)*a^2*b+2*(
-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(
1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+
e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)
^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f
*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((
b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)*b^3-(-(-(-a^2+b^2)^
(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+
e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+
b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*
x+e))^(1/2)*EllipticF((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+
e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(
1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*a^2+sin(f*x+e)*2
^(1/2)*cos(f*x+e)*a*b^2+2^(1/2)*cos(f*x+e)^2*a^2*b-sin(f*x+e)*2^(1/2)*a^3-s
in(f*x+e)*2^(1/2)*a*b^2+2^(1/2)*cos(f*x+e)*a^2*b-2*2^(1/2)*a^2*b)/cos(f*x+e
)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2)*2^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \sin\left(fx+e\right)+a\right)^{\frac{3}{2}} \sec\left(fx+e\right)^{2}}{\sqrt{d \sin\left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*sec(f*x + e)^2/sqrt(d*sin(f*x + e)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b\,\sin\left(e + f\,x\right)\right)^{3/2}}{\cos\left(e + f\,x\right)^2\,\sqrt{d\,\sin\left(e + f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(3/2)/(cos(e + f*x)^2*(d*sin(e + f*x))^(1/2)),x)
[Out] int((a + b*sin(e + f*x))^(3/2)/(cos(e + f*x)^2*(d*sin(e + f*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e))**(3/2)/(d*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

3.1480 $\int \frac{\sec^4(e+fx)(a+b\sin(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=366

$$\frac{\sec^3(e+fx)\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{5/2}}{3df}+\frac{5a\sec(e+fx)(a\sin(e+fx)+b)\sqrt{a+b\sin(e+fx)}}{6f\sqrt{d\sin(e+fx)}}-\frac{5a(a+b}{}$$

```
[Out] 1/3*sec(f*x+e)^3*(a+b*sin(f*x+e))^(5/2)*(d*sin(f*x+e))^(1/2)/d/f+5/6*a*sec(
f*x+e)*(b+a*sin(f*x+e))*(a+b*sin(f*x+e))^(1/2)/f/(d*sin(f*x+e))^(1/2)-5/6*a
*(a+b)^(3/2)*EllipticF(d^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(d*sin(f*
x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(-a*(-1+csc(f*x+e))/(a+b))^(1/2)*(a*(1+cs
c(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/f/d^(1/2)-5/6*a*b*(a+b)*EllipticE(((-b-a*
csc(f*x+e))/(a-b))^(1/2),((-a+b)/(a+b))^(1/2))*(1+sin(f*x+e))*(-a*(-1+csc(f
*x+e))/(a+b))^(1/2)*((b+a*csc(f*x+e))/(-a+b))^(1/2)*tan(f*x+e)/f/(a*(1+csc(
f*x+e))/(a-b))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2)
```

Rubi [F] time = 0.39, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^4(e+fx)(a+b\sin(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[e + f*x]^4*(a + b*Sin[e + f*x])^(5/2))/Sqrt[d*Sin[e + f*x]],x]
```

```
[Out] (Sec[e + f*x]^3*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(5/2))/(3*d*f) +
(5*a*Defer[Int][(Sec[e + f*x]^2*(a + b*Sin[e + f*x])^(3/2))/Sqrt[d*Sin[e +
f*x]], x])/6
```

Rubi steps

$$\int \frac{\sec^4(e+fx)(a+b\sin(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}}\,dx=\frac{\sec^3(e+fx)\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{5/2}}{3df}+\frac{1}{6}(5a)\int\frac{\sec^2(e+}{}$$

Mathematica [A] time = 22.95, size = 667, normalized size = 1.82

$$\frac{\sin(e+fx)\sqrt{a+b\sin(e+fx)}\left(\frac{1}{3}\sec^3(e+fx)\left(a^2+2ab\sin(e+fx)+b^2\right)+\frac{1}{6}\sec(e+fx)\left(5a^2+5ab\sin(e+f\right.\right.}{f\sqrt{d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]^4*(a + b*Sin[e + f*x])^(5/2))/Sqrt[d*Sin[e + f*x]],
x]
```

```
[Out] (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((Sec[e + f*x]^3*(a^2 + b^2 + 2*a*b*
Sin[e + f*x]))/3 + (Sec[e + f*x]*(5*a^2 - 2*b^2 + 5*a*b*Sin[e + f*x]))/6))/
(f*Sqrt[d*Sin[e + f*x]]) + (5*a*Sqrt[Sin[e + f*x]]*(1 + Tan[(e + f*x)/2]^2)
*Sqrt[Tan[(e + f*x)/2]/(2 + 2*Tan[(e + f*x)/2]^2)]*Sqrt[(a + 2*b*Tan[(e + f
*x)/2] + a*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)]*(-2*b*Tan[(e + f*x
)/2]^2 + (2*Sqrt[-a^2 + b^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a*(a + 2*b*Tan[
(e + f*x)/2] + a*Tan[(e + f*x)/2]^2))/(a^2 - b^2)]*(-(b*EllipticE[ArcSin[Sq
rt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]],
 (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*Ellipt
icF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2
]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e +
f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^
2 + b^2]))]))/(Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*(a + 2*b*
Tan[(e + f*x)/2] + a*Tan[(e + f*x)/2]^2))))/(6*f*Sqrt[d*Sin[e + f*x]]*(Tan[
(e + f*x)/2] + Tan[(e + f*x)/2]^3))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(2\,ab\sec\left(fx+e\right)^4\sin\left(fx+e\right)-\left(b^2\cos\left(fx+e\right)^2-a^2-b^2\right)\sec\left(fx+e\right)^4\right)\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\,s}}{d\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((2*a*b*sec(f*x + e)^4*sin(f*x + e) - (b^2*cos(f*x + e)^2 - a^2 - b
^2)*sec(f*x + e)^4)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))/(d*sin(f*
x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sec\left(fx+e\right)^{4}}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*sec(f*x + e)^4/sqrt(d*sin(f*x + e)), x
)
```

maple [B] time = 0.72, size = 2490, normalized size = 6.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/12/f*(10*cos(f*x+e)^4*(-a^2+b^2)^(1/2)*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b
*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+
b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x
+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*Ellipt
icE((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b
^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(
1/2))^(1/2))*b^2-5*cos(f*x+e)^4*(-a^2+b^2)^(1/2)*(-(-(-a^2+b^2)^(1/2)*sin(f
*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(
((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/
sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)
*EllipticF((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+
(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2
+b^2)^(1/2))^(1/2))*a^2-10*cos(f*x+e)^4*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*s
in(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^
2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e
))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*Elliptic
E((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2
)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/
2))^(1/2))*a^2*b+10*cos(f*x+e)^4*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+
e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2
```

```
)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2
)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(-(
-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2)
)/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/
2))*b^3+10*cos(f*x+e)^3*(-a^2+b^2)^(1/2)*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*
sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b
^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+
e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*Ellipti
cE((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^
2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1
/2))^(1/2))*b^2-5*cos(f*x+e)^3*(-a^2+b^2)^(1/2)*(-(-(-a^2+b^2)^(1/2)*sin(f*
x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*((
(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/s
in(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*
EllipticF((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(
-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+
b^2)^(1/2))^(1/2))*a^2-10*cos(f*x+e)^3*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*si
n(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2
)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e)
)^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE
((-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)
^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2
))^(1/2))*a^2*b+10*cos(f*x+e)^3*(-(-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e
)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)
*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)
*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(-(-
a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(b+(-a^2+b^2)^(1/2))
/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2
))*b^3+5*2^(1/2)*sin(f*x+e)*cos(f*x+e)^3*a*b^2+5*2^(1/2)*cos(f*x+e)^4*a^2*b
-2*2^(1/2)*cos(f*x+e)^4*b^3-5*2^(1/2)*sin(f*x+e)*cos(f*x+e)^2*a^3+2^(1/2)*s
in(f*x+e)*cos(f*x+e)^2*a*b^2+5*2^(1/2)*cos(f*x+e)^3*a^2*b-4*2^(1/2)*cos(f*x
+e)^2*a^2*b+4*2^(1/2)*cos(f*x+e)^2*b^3-2*sin(f*x+e)*2^(1/2)*a^3-6*sin(f*x+e
)*2^(1/2)*a*b^2-6*2^(1/2)*a^2*b-2*2^(1/2)*b^3)/cos(f*x+e)^3/(d*sin(f*x+e))^
(1/2)/(a+b*sin(f*x+e))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sec\left(fx+e\right)^{4}}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*sec(f*x + e)^4/sqrt(d*sin(f*x + e)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\sin\left(e+fx\right)\right)^{5/2}}{\cos\left(e+fx\right)^4\sqrt{d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(5/2)/(cos(e + f*x)^4*(d*sin(e + f*x))^(1/2)),x)
[Out] int((a + b*sin(e + f*x))^(5/2)/(cos(e + f*x)^4*(d*sin(e + f*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

3.1481 $\int \sin^2(c+dx)(a+b\sin(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=155

$$\frac{a\cos^2(c+dx)}{2d}+\frac{a\sec^4(c+dx)}{4d}-\frac{3a\sec^2(c+dx)}{2d}-\frac{3a\log(\cos(c+dx))}{d}-\frac{35b\sin^3(c+dx)}{24d}-\frac{35b\sin(c+dx)}{8d}+\frac{b\sin}{}$$

```
[Out] 35/8*b*arctanh(sin(d*x+c))/d+1/2*a*cos(d*x+c)^2/d-3*a*ln(cos(d*x+c))/d-3/2*
a*sec(d*x+c)^2/d+1/4*a*sec(d*x+c)^4/d-35/8*b*sin(d*x+c)/d-35/24*b*sin(d*x+c
)^3/d-7/8*b*sin(d*x+c)^3*tan(d*x+c)^2/d+1/4*b*sin(d*x+c)^3*tan(d*x+c)^4/d
```

Rubi [A] time = 0.17, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2834, 2590, 266, 43, 2592, 288, 302, 206}

$$\frac{a\cos^2(c+dx)}{2d}+\frac{a\sec^4(c+dx)}{4d}-\frac{3a\sec^2(c+dx)}{2d}-\frac{3a\log(\cos(c+dx))}{d}-\frac{35b\sin^3(c+dx)}{24d}-\frac{35b\sin(c+dx)}{8d}+\frac{b\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (35*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Cos[c + d*x]^2)/(2*d) - (3*a*Log[Co
s[c + d*x]])/d - (3*a*Sec[c + d*x]^2)/(2*d) + (a*Sec[c + d*x]^4)/(4*d) - (3
5*b*Sin[c + d*x])/(8*d) - (35*b*Sin[c + d*x]^3)/(24*d) - (7*b*Sin[c + d*x]^
3*Tan[c + d*x]^2)/(8*d) + (b*Sin[c + d*x]^3*Tan[c + d*x]^4)/(4*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\sin(c+dx))\tan^5(c+dx)\,dx = a\int \sin^2(c+dx)\tan^5(c+dx)\,dx + b\int \sin^3(c+dx)\tan^5(c$$

$$= -\frac{a\,\mathrm{Subst}\left(\int \frac{(1-x^2)^3}{x^5}\,dx, x, \cos(c+dx)\right)}{d} + \frac{b\,\mathrm{Subst}\left(\int \frac{x^8}{(1-x^2)^3}\right.}{}$$

$$= \frac{b\sin^3(c+dx)\tan^4(c+dx)}{4d} - \frac{a\,\mathrm{Subst}\left(\int \frac{(1-x)^3}{x^3}\,dx, x, \cos^2(c\right.}{2d}$$

$$= -\frac{7b\sin^3(c+dx)\tan^2(c+dx)}{8d} + \frac{b\sin^3(c+dx)\tan^4(c+dx)}{4d}$$

$$= \frac{a\cos^2(c+dx)}{2d} - \frac{3a\log(\cos(c+dx))}{d} - \frac{3a\sec^2(c+dx)}{2d} +$$

$$= \frac{a\cos^2(c+dx)}{2d} - \frac{3a\log(\cos(c+dx))}{d} - \frac{3a\sec^2(c+dx)}{2d} +$$

$$= \frac{35b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\cos^2(c+dx)}{2d} - \frac{3a\log(\cos(c+}{d}$$

Mathematica [A] time = 0.60, size = 156, normalized size = 1.01

$$-\frac{a\left(2\sin^2(c+dx)-\sec^4(c+dx)+6\sec^2(c+dx)+12\log(\cos(c+dx))\right)}{4d} - \frac{b\sin^3(c+dx)\tan^4(c+dx)}{3d} - \frac{7b\left(8\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -1/4*(a*(12*Log[Cos[c + d*x]] + 6*Sec[c + d*x]^2 - Sec[c + d*x]^4 + 2*Sin[c
 + d*x]^2))/d - (b*Sin[c + d*x]^3*Tan[c + d*x]^4)/(3*d) - (7*b*(8*Sin[c + d
*x]*Tan[c + d*x]^4 + 5*(6*Sec[c + d*x]^3*Tan[c + d*x] - 8*Sec[c + d*x]*Tan[
c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))))/(24*d
)
```

fricas [A] time = 0.45, size = 149, normalized size = 0.96

$$\frac{24\,a\cos(dx+c)^6 - 3\,(24\,a-35\,b)\cos(dx+c)^4\log(\sin(dx+c)+1) - 3\,(24\,a+35\,b)\cos(dx+c)^4\log(-\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(24*a*cos(d*x + c)^6 - 3*(24*a - 35*b)*cos(d*x + c)^4*log(sin(d*x + c)
 + 1) - 3*(24*a + 35*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 12*a*cos(d*
x + c)^4 - 72*a*cos(d*x + c)^2 + 2*(8*b*cos(d*x + c)^6 - 80*b*cos(d*x + c)^
4 - 39*b*cos(d*x + c)^2 + 6*b)*sin(d*x + c) + 12*a)/(d*cos(d*x + c)^4)
```

giac [A] time = 0.29, size = 135, normalized size = 0.87

$$-\frac{16\, b \sin\left(dx+c\right)^{3} + 24\, a \sin\left(dx+c\right)^{2} + 3\left(24\, a - 35\, b\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right) + 3\left(24\, a + 35\, b\right)\log\left(\left|\sin\left(dx+c\right)\right.\right.}{48\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/48*(16*b*sin(d*x + c)^3 + 24*a*sin(d*x + c)^2 + 3*(24*a - 35*b)*log(abs(
sin(d*x + c) + 1)) + 3*(24*a + 35*b)*log(abs(sin(d*x + c) - 1)) + 144*b*sin
(d*x + c) - 6*(18*a*sin(d*x + c)^4 + 13*b*sin(d*x + c)^3 - 24*a*sin(d*x + c
)^2 - 11*b*sin(d*x + c) + 8*a)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.24, size = 219, normalized size = 1.41

$$\frac{a\left(\sin^{8}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}-\frac{a\left(\sin^{8}\left(dx+c\right)\right)}{2d\cos\left(dx+c\right)^{2}}-\frac{a\left(\sin^{6}\left(dx+c\right)\right)}{2d}-\frac{3a\left(\sin^{4}\left(dx+c\right)\right)}{4d}-\frac{3a\left(\sin^{2}\left(dx+c\right)\right)}{2d}-\frac{3a\ln\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^7*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*a*sin(d*x+c)^8/cos(d*x+c)^2-1/2*a*s
in(d*x+c)^6/d-3/4*a*sin(d*x+c)^4/d-3/2*a*sin(d*x+c)^2/d-3*a*ln(cos(d*x+c))/
d+1/4/d*b*sin(d*x+c)^9/cos(d*x+c)^4-5/8/d*b*sin(d*x+c)^9/cos(d*x+c)^2-5/8*b
*sin(d*x+c)^7/d-7/8*b*sin(d*x+c)^5/d-35/24*b*sin(d*x+c)^3/d-35/8*b*sin(d*x+
c)/d+35/8/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.30, size = 132, normalized size = 0.85

$$-\frac{16\, b \sin\left(dx+c\right)^{3} + 24\, a \sin\left(dx+c\right)^{2} + 3\left(24\, a - 35\, b\right)\log\left(\sin\left(dx+c\right)+1\right) + 3\left(24\, a + 35\, b\right)\log\left(\sin\left(dx+c\right) - \right.}{48\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/48*(16*b*sin(d*x + c)^3 + 24*a*sin(d*x + c)^2 + 3*(24*a - 35*b)*log(sin(
d*x + c) + 1) + 3*(24*a + 35*b)*log(sin(d*x + c) - 1) + 144*b*sin(d*x + c)
- 6*(13*b*sin(d*x + c)^3 + 12*a*sin(d*x + c)^2 - 11*b*sin(d*x + c) - 10*a)/
(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 12.27, size = 346, normalized size = 2.23

$$\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{-\frac{35\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{4}-6\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\frac{35\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{6}+6\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{329\,b\,\mathrm{ta}}{}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^7*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (3*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (6*a*tan(c/2 + (d*x)/2)^4 - 6*a*tan
(c/2 + (d*x)/2)^2 - (35*b*tan(c/2 + (d*x)/2))/4 + 16*a*tan(c/2 + (d*x)/2)^6
 + 16*a*tan(c/2 + (d*x)/2)^8 + 6*a*tan(c/2 + (d*x)/2)^10 - 6*a*tan(c/2 + (d
*x)/2)^12 + (35*b*tan(c/2 + (d*x)/2)^3)/6 + (329*b*tan(c/2 + (d*x)/2)^5)/12
 - 17*b*tan(c/2 + (d*x)/2)^7 + (329*b*tan(c/2 + (d*x)/2)^9)/12 + (35*b*tan(
c/2 + (d*x)/2)^11)/6 - (35*b*tan(c/2 + (d*x)/2)^13)/4)/(d*(tan(c/2 + (d*x)/
2)^2 + 3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^6 - 3*tan(c/2 + (d*x)/
2)^8 + 3*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 - tan(c/2 + (d*x)/2)
^14 - 1)) - (log(tan(c/2 + (d*x)/2) - 1)*(3*a + (35*b)/8))/d - (log(tan(c/2
 + (d*x)/2) + 1)*(3*a - (35*b)/8))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**7*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1482 $\int \sin(c+dx)(a+b\sin(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=135

$$-\frac{15a\sin(c+dx)}{8d}+\frac{a\sin(c+dx)\tan^4(c+dx)}{4d}-\frac{5a\sin(c+dx)\tan^2(c+dx)}{8d}+\frac{15a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\cos^2(c+$$

```
[Out] 15/8*a*arctanh(sin(d*x+c))/d+1/2*b*cos(d*x+c)^2/d-3*b*ln(cos(d*x+c))/d-3/2*
b*sec(d*x+c)^2/d+1/4*b*sec(d*x+c)^4/d-15/8*a*sin(d*x+c)/d-5/8*a*sin(d*x+c)*
tan(d*x+c)^2/d+1/4*a*sin(d*x+c)*tan(d*x+c)^4/d
```

Rubi [A] time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2834, 2592, 288, 321, 206, 2590, 266, 43}

$$-\frac{15a\sin(c+dx)}{8d}+\frac{a\sin(c+dx)\tan^4(c+dx)}{4d}-\frac{5a\sin(c+dx)\tan^2(c+dx)}{8d}+\frac{15a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\cos^2(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (15*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Cos[c + d*x]^2)/(2*d) - (3*b*Log[Co
s[c + d*x]])/d - (3*b*Sec[c + d*x]^2)/(2*d) + (b*Sec[c + d*x]^4)/(4*d) - (1
5*a*Sin[c + d*x])/(8*d) - (5*a*Sin[c + d*x]*Tan[c + d*x]^2)/(8*d) + (a*Sin[
c + d*x]*Tan[c + d*x]^4)/(4*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))\tan^5(c+dx)\,dx = a\int \sin(c+dx)\tan^5(c+dx)\,dx + b\int \sin^2(c+dx)\tan^5(c+d$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \sin(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{(1-x^2)^3}{x^5}\,dx,\right.}{d}$$

$$= \frac{a\sin(c+dx)\tan^4(c+dx)}{4d} - \frac{(5a)\,\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \sin(c\right.}{4d}$$

$$= -\frac{5a\sin(c+dx)\tan^2(c+dx)}{8d} + \frac{a\sin(c+dx)\tan^4(c+dx)}{4d} + $$

$$= \frac{b\cos^2(c+dx)}{2d} - \frac{3b\log(\cos(c+dx))}{d} - \frac{3b\sec^2(c+dx)}{2d} + \frac{b\,\text{se}}{}$$

$$= \frac{15a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b\cos^2(c+dx)}{2d} - \frac{3b\log(\cos(c+d}{d}$$

Mathematica [A] time = 0.36, size = 133, normalized size = 0.99

$$-\frac{a\sin(c+dx)\tan^4(c+dx)}{d} - \frac{5a\left(6\tan(c+dx)\sec^3(c+dx) - 8\tan^3(c+dx)\sec(c+dx) - 3\left(\tanh^{-1}(\sin(c+dx))\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -1/4*(b*(12*Log[Cos[c + d*x]] + 6*Sec[c + d*x]^2 - Sec[c + d*x]^4 + 2*Sin[c
 + d*x]^2))/d - (a*Sin[c + d*x]*Tan[c + d*x]^4)/d - (5*a*(6*Sec[c + d*x]^3*
Tan[c + d*x] - 8*Sec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + S
ec[c + d*x]*Tan[c + d*x])))/(8*d)
```

fricas [A] time = 0.45, size = 138, normalized size = 1.02

$$\frac{8\,b\cos(dx+c)^6 + 3\,(5\,a-8\,b)\cos(dx+c)^4\log(\sin(dx+c)+1) - 3\,(5\,a+8\,b)\cos(dx+c)^4\log(-\sin(dx+c)}{16\,d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*cos(d*x + c)^6 + 3*(5*a - 8*b)*cos(d*x + c)^4*log(sin(d*x + c) +
1) - 3*(5*a + 8*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 4*b*cos(d*x + c)
```

```
^4 - 24*b*cos(d*x + c)^2 - 2*(8*a*cos(d*x + c)^4 + 9*a*cos(d*x + c)^2 - 2*a
)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^4)
```

giac [A] time = 0.28, size = 124, normalized size = 0.92

$$-\frac{8\,b\sin\left(dx+c\right)^{2}-3\,(5\,a-8\,b)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\,(5\,a+8\,b)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+16\,a\sin\left(dx+c\right)-}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(8*b*sin(d*x + c)^2 - 3*(5*a - 8*b)*log(abs(sin(d*x + c) + 1)) + 3*(5
*a + 8*b)*log(abs(sin(d*x + c) - 1)) + 16*a*sin(d*x + c) - 2*(18*b*sin(d*x
+ c)^4 + 9*a*sin(d*x + c)^3 - 24*b*sin(d*x + c)^2 - 7*a*sin(d*x + c) + 8*b)
/(sin(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.24, size = 205, normalized size = 1.52

$$\frac{a\left(\sin^{7}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}-\frac{3a\left(\sin^{7}\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^{2}}-\frac{3a\left(\sin^{5}\left(dx+c\right)\right)}{8d}-\frac{5a\left(\sin^{3}\left(dx+c\right)\right)}{8d}-\frac{15a\sin\left(dx+c\right)}{8d}+\frac{15a\ln\left(\sec\left(dx+\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a*sin(d*x+c)^7/cos(d*x+c)^2-3/8*a*s
in(d*x+c)^5/d-5/8*a*sin(d*x+c)^3/d-15/8*a*sin(d*x+c)/d+15/8/d*a*ln(sec(d*x+
c)+tan(d*x+c))+1/4/d*b*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*b*sin(d*x+c)^8/cos(d
*x+c)^2-1/2*b*sin(d*x+c)^6/d-3/4*b*sin(d*x+c)^4/d-3/2*b*sin(d*x+c)^2/d-3*b*
ln(cos(d*x+c))/d
```

maxima [A] time = 0.31, size = 121, normalized size = 0.90

$$-\frac{8\,b\sin\left(dx+c\right)^{2}-3\,(5\,a-8\,b)\log\left(\sin\left(dx+c\right)+1\right)+3\,(5\,a+8\,b)\log\left(\sin\left(dx+c\right)-1\right)+16\,a\sin\left(dx+c\right)-}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(8*b*sin(d*x + c)^2 - 3*(5*a - 8*b)*log(sin(d*x + c) + 1) + 3*(5*a +
8*b)*log(sin(d*x + c) - 1) + 16*a*sin(d*x + c) - 2*(9*a*sin(d*x + c)^3 + 12
*b*sin(d*x + c)^2 - 7*a*sin(d*x + c) - 10*b)/(sin(d*x + c)^4 - 2*sin(d*x +
c)^2 + 1))/d
```

mupad [B] time = 12.16, size = 304, normalized size = 2.25

$$\frac{3\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{15\,a}{8}+3\,b\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{15\,a}{8}-3\,b\right)}{d}-\frac{\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^6*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (3*b*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (log(tan(c/2 + (d*x)/2) - 1)*((15*a
)/8 + 3*b))/d + (log(tan(c/2 + (d*x)/2) + 1)*((15*a)/8 - 3*b))/d - ((25*a*t
an(c/2 + (d*x)/2)^3)/4 - (15*a*tan(c/2 + (d*x)/2))/4 + (11*a*tan(c/2 + (d*x
)/2)^5)/2 + (11*a*tan(c/2 + (d*x)/2)^7)/2 + (25*a*tan(c/2 + (d*x)/2)^9)/4 -
 (15*a*tan(c/2 + (d*x)/2)^11)/4 - 6*b*tan(c/2 + (d*x)/2)^2 + 12*b*tan(c/2 +
 (d*x)/2)^4 + 4*b*tan(c/2 + (d*x)/2)^6 + 12*b*tan(c/2 + (d*x)/2)^8 - 6*b*ta
n(c/2 + (d*x)/2)^10)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - 4*
tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 2*tan(c/2 + (d*x)/2)^10 - tan
(c/2 + (d*x)/2)^12 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1483 $\int (a + b\sin(c + dx))\tan^5(c + dx)\,dx$

Optimal. Leaf size=116

$$-\frac{(8a+15b)\log(1-\sin(c+dx))}{16d}-\frac{(8a-15b)\log(\sin(c+dx)+1)}{16d}+\frac{\tan^4(c+dx)(a+b\sin(c+dx))}{4d}-\frac{\tan^2(c+dx)}{}$$

```
[Out] -1/16*(8*a+15*b)*ln(1-sin(d*x+c))/d-1/16*(8*a-15*b)*ln(1+sin(d*x+c))/d-15/8
*b*sin(d*x+c)/d-1/8*(4*a+5*b*sin(d*x+c))*tan(d*x+c)^2/d+1/4*(a+b*sin(d*x+c)
)*tan(d*x+c)^4/d
```

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2721, 819, 774, 633, 31}

$$-\frac{(8a+15b)\log(1-\sin(c+dx))}{16d}-\frac{(8a-15b)\log(\sin(c+dx)+1)}{16d}+\frac{\tan^4(c+dx)(a+b\sin(c+dx))}{4d}-\frac{\tan^2(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((8*a + 15*b)*Log[1 - Sin[c + d*x]])/(16*d) - ((8*a - 15*b)*Log[1 + Sin[c
+ d*x]])/(16*d) - (15*b*Sin[c + d*x])/(8*d) - ((4*a + 5*b*Sin[c + d*x])*Tan
[c + d*x]^2)/(8*d) + ((a + b*Sin[c + d*x])*Tan[c + d*x]^4)/(4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+x)}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{(a+b\sin(c+dx))\tan^4(c+dx)}{4d} - \frac{\text{Subst}\left(\int \frac{x^3\left(4ab^2+5b^2x\right)}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+d\right.}{4b^2d}$$

$$= -\frac{(4a+5b\sin(c+dx))\tan^2(c+dx)}{8d} + \frac{(a+b\sin(c+dx))\tan^4(c+dx)}{4d} +$$

$$= -\frac{15b\sin(c+dx)}{8d} - \frac{(4a+5b\sin(c+dx))\tan^2(c+dx)}{8d} + \frac{(a+b\sin(c+dx}{4a}$$

$$= -\frac{15b\sin(c+dx)}{8d} - \frac{(4a+5b\sin(c+dx))\tan^2(c+dx)}{8d} + \frac{(a+b\sin(c+dx}{4a}$$

$$= -\frac{(8a+15b)\log(1-\sin(c+dx))}{16d} - \frac{(8a-15b)\log(1+\sin(c+dx))}{16d} - \frac{15b}{}$$

Mathematica [A] time = 0.32, size = 123, normalized size = 1.06

$$-\frac{a\left(-\tan^4(c+dx)+2\tan^2(c+dx)+4\log(\cos(c+dx))\right)}{4d} - \frac{b\sin(c+dx)\tan^4(c+dx)}{d} - \frac{5b\left(6\tan(c+dx)\sec^3(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((b*Sin[c + d*x]*Tan[c + d*x]^4)/d) - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c +
d*x]^2 - Tan[c + d*x]^4))/(4*d) - (5*b*(6*Sec[c + d*x]^3*Tan[c + d*x] - 8*S
ec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c
+ d*x])))/(8*d)
```

fricas [A] time = 0.44, size = 114, normalized size = 0.98

$$-\frac{(8\,a-15\,b)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)+(8\,a+15\,b)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+16\,a\cos\left(d\right.}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*((8*a - 15*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (8*a + 15*b)*cos
(d*x + c)^4*log(-sin(d*x + c) + 1) + 16*a*cos(d*x + c)^2 + 2*(8*b*cos(d*x +
 c)^4 + 9*b*cos(d*x + c)^2 - 2*b)*sin(d*x + c) - 4*a)/(d*cos(d*x + c)^4)
```

giac [A] time = 0.27, size = 108, normalized size = 0.93

$$-\frac{(8\,a-15\,b)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+(8\,a+15\,b)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+16\,b\sin\left(dx+c\right)-\frac{2\left(6\,a\sin(dx+c)^{4}+9\,b\sin\right.}{(}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*((8*a - 15*b)*log(abs(sin(d*x + c) + 1)) + (8*a + 15*b)*log(abs(sin(d
*x + c) - 1)) + 16*b*sin(d*x + c) - 2*(6*a*sin(d*x + c)^4 + 9*b*sin(d*x + c
)^3 - 4*a*sin(d*x + c)^2 - 7*b*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.24, size = 147, normalized size = 1.27

$$\frac{a\left(\tan^{4}\left(dx+c\right)\right)}{4d}-\frac{a\left(\tan^{2}\left(dx+c\right)\right)}{2d}-\frac{a\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{b\left(\sin^{7}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}-\frac{3b\left(\sin^{7}\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^{2}}-\frac{3b\left(\sin^{5}\left(dx+c\right)\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4*a*tan(d*x+c)^4/d-1/2*a*tan(d*x+c)^2/d-a*ln(cos(d*x+c))/d+1/4/d*b*sin(d*
x+c)^7/cos(d*x+c)^4-3/8/d*b*sin(d*x+c)^7/cos(d*x+c)^2-3/8*b*sin(d*x+c)^5/d-
5/8*b*sin(d*x+c)^3/d-15/8*b*sin(d*x+c)/d+15/8/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.31, size = 108, normalized size = 0.93

$$-\frac{(8a-15b)\log(\sin(dx+c)+1)+(8a+15b)\log(\sin(dx+c)-1)+16b\sin(dx+c)-\frac{2\left(9b\sin(dx+c)^3+8a\sin(dx+\right.}{\sin(dx+c)^4-2\sin}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*((8*a - 15*b)*log(sin(d*x + c) + 1) + (8*a + 15*b)*log(sin(d*x + c) -
 1) + 16*b*sin(d*x + c) - 2*(9*b*sin(d*x + c)^3 + 8*a*sin(d*x + c)^2 - 7*b*
sin(d*x + c) - 6*a)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 12.10, size = 261, normalized size = 2.25

$$\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\left(a-\frac{15b}{8}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\left(a+\frac{15b}{8}\right)}{d}-\frac{\frac{15b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{4}+2a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^5*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (log(tan(c/2 + (d*x)/2) + 1)*(a - (15
*b)/8))/d - (log(tan(c/2 + (d*x)/2) - 1)*(a + (15*b)/8))/d - ((15*b*tan(c/2
 + (d*x)/2))/4 + 2*a*tan(c/2 + (d*x)/2)^2 - 6*a*tan(c/2 + (d*x)/2)^4 - 6*a*
tan(c/2 + (d*x)/2)^6 + 2*a*tan(c/2 + (d*x)/2)^8 - 10*b*tan(c/2 + (d*x)/2)^3
 + (9*b*tan(c/2 + (d*x)/2)^5)/2 - 10*b*tan(c/2 + (d*x)/2)^7 + (15*b*tan(c/2
 + (d*x)/2)^9)/4)/(d*(2*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^2 + 2*t
an(c/2 + (d*x)/2)^6 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1484 $\int \sec(c+dx)(a+b\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan^4(c+dx)}{4d}-\frac{b\tan^2(c+dx)}{2d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d-b*ln(cos(d*x+c))/d-3/8*a*sec(d*x+c)*tan(d*x+c)/
d-1/2*b*tan(d*x+c)^2/d+1/4*a*sec(d*x+c)*tan(d*x+c)^3/d+1/4*b*tan(d*x+c)^4/d
```

Rubi [A] time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2834, 2611, 3770, 3473, 3475}

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan^4(c+dx)}{4d}-\frac{b\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) - (b*Log[Cos[c + d*x]])/d - (3*a*Sec[c +
d*x]*Tan[c + d*x])/(8*d) - (b*Tan[c + d*x]^2)/(2*d) + (a*Sec[c + d*x]*Tan[c
 + d*x]^3)/(4*d) + (b*Tan[c + d*x]^4)/(4*d)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))\tan^4(c+dx)\,dx = a\int \sec(c+dx)\tan^4(c+dx)\,dx + b\int \tan^5(c+dx)\,dx$$
$$= \frac{a\sec(c+dx)\tan^3(c+dx)}{4d} + \frac{b\tan^4(c+dx)}{4d} - \frac{1}{4}(3a)\int \sec(c$$
$$= -\frac{3a\sec(c+dx)\tan(c+dx)}{8d} - \frac{b\tan^2(c+dx)}{2d} + \frac{a\sec(c+dx)}{4}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{b\log(\cos(c+dx))}{d} - \frac{3a\sec(c+dx)}{8d}$$

Mathematica [A] time = 0.31, size = 106, normalized size = 1.03

$$\frac{a\tan^3(c+dx)\sec(c+dx)}{d} - \frac{a\left(6\tan(c+dx)\sec^3(c+dx) - 3\left(\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\sec(c+dx)\right)\right)}{8d} - \frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*Sec[c + d*x]*Tan[c + d*x]^3)/d - (b*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x
]^2 - Tan[c + d*x]^4))/(4*d) - (a*(6*Sec[c + d*x]^3*Tan[c + d*x] - 3*(ArcTa
nh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x])))/(8*d)
```

fricas [A] time = 0.45, size = 104, normalized size = 1.01

$$\frac{(3a-8b)\cos(dx+c)^4\log(\sin(dx+c)+1) - (3a+8b)\cos(dx+c)^4\log(-\sin(dx+c)+1) - 16b\cos(dx+c)}{16d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*((3*a - 8*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*a + 8*b)*cos(d*
x + c)^4*log(-sin(d*x + c) + 1) - 16*b*cos(d*x + c)^2 - 2*(5*a*cos(d*x + c)
^2 - 2*a)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^4)
```

giac [A] time = 0.29, size = 100, normalized size = 0.97

$$\frac{(3a-8b)\log(|\sin(dx+c)+1|)-(3a+8b)\log(|\sin(dx+c)-1|)+\frac{2\left(6b\sin(dx+c)^4+5a\sin(dx+c)^3-4b\sin(dx+c)^2-3a\sin(dx+c)\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*((3*a - 8*b)*log(abs(sin(d*x + c) + 1)) - (3*a + 8*b)*log(abs(sin(d*x
+ c) - 1)) + 2*(6*b*sin(d*x + c)^4 + 5*a*sin(d*x + c)^3 - 4*b*sin(d*x + c)^
2 - 3*a*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.24, size = 133, normalized size = 1.29

$$\frac{a\left(\sin^5(dx+c)\right)}{4d\cos(dx+c)^4}-\frac{a\left(\sin^5(dx+c)\right)}{8d\cos(dx+c)^2}-\frac{a\left(\sin^3(dx+c)\right)}{8d}-\frac{3a\sin(dx+c)}{8d}+\frac{3a\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{b\left(\tan^4(dx+c)\right)}{4d}-\frac{b\left(\tan^2(dx+c)\right)}{2d}-\frac{b\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a*sin(d*x+c)^5/cos(d*x+c)^2-1/8*a*s
in(d*x+c)^3/d-3/8*a*sin(d*x+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/4*b*ta
n(d*x+c)^4/d-1/2*b*tan(d*x+c)^2/d-b*ln(cos(d*x+c))/d
```

maxima [A] time = 0.41, size = 100, normalized size = 0.97

$$\frac{(3a-8b)\log(\sin(dx+c)+1)-(3a+8b)\log(\sin(dx+c)-1)+\frac{2\left(5a\sin(dx+c)^3+8b\sin(dx+c)^2-3a\sin(dx+c)-6b\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*((3*a - 8*b)*log(sin(d*x + c) + 1) - (3*a + 8*b)*log(sin(d*x + c) - 1)
 + 2*(5*a*sin(d*x + c)^3 + 8*b*sin(d*x + c)^2 - 3*a*sin(d*x + c) - 6*b)/(si
n(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 12.03, size = 221, normalized size = 2.15

$$\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-8\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (b*log(tan(c/2 + (d*x)/2)^2 + 1))/d - ((3*a*tan(c/2 + (d*x)/2))/4 - (11*a*t
an(c/2 + (d*x)/2)^3)/4 - (11*a*tan(c/2 + (d*x)/2)^5)/4 + (3*a*tan(c/2 + (d*
x)/2)^7)/4 + 2*b*tan(c/2 + (d*x)/2)^2 - 8*b*tan(c/2 + (d*x)/2)^4 + 2*b*tan(
c/2 + (d*x)/2)^6)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*t
an(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (log(tan(c/2 + (d*x)/2)
- 1)*((3*a)/8 + b))/d + (log(tan(c/2 + (d*x)/2) + 1)*((3*a)/8 - b))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1485 $\int \sec^2(c+dx)(a+b\sin(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=74

$$\frac{a\tan^4(c+dx)}{4d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3b\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] 3/8*b*arctanh(sin(d*x+c))/d-3/8*b*sec(d*x+c)*tan(d*x+c)/d+1/4*b*sec(d*x+c)*
tan(d*x+c)^3/d+1/4*a*tan(d*x+c)^4/d
```

Rubi [A] time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2607, 30, 2611, 3770}

$$\frac{a\tan^4(c+dx)}{4d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3b\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) - (3*b*Sec[c + d*x]*Tan[c + d*x])/(8*d) +
 (b*Sec[c + d*x]*Tan[c + d*x]^3)/(4*d) + (a*Tan[c + d*x]^4)/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))\tan^3(c+dx)\,dx = a\int \sec^2(c+dx)\tan^3(c+dx)\,dx + b\int \sec(c+dx)\tan^4(c+$$
$$= \frac{b\sec(c+dx)\tan^3(c+dx)}{4d} - \frac{1}{4}(3b)\int \sec(c+dx)\tan^2(c+d$$
$$= -\frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec(c+dx)\tan^3(c+dx)}{4d} + $$
$$= \frac{3b\tanh^{-1}(\sin(c+dx))}{8d} - \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec$$

Mathematica [A] time = 0.27, size = 84, normalized size = 1.14

$$\frac{a\tan^4(c+dx)}{4d} + \frac{b\tan^3(c+dx)\sec(c+dx)}{d} - \frac{b\left(6\tan(c+dx)\sec^3(c+dx) - 3\left(\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (b*Sec[c + d*x]*Tan[c + d*x]^3)/d + (a*Tan[c + d*x]^4)/(4*d) - (b*(6*Sec[c
+ d*x]^3*Tan[c + d*x] - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x
]))))/(8*d)
```

fricas [A] time = 0.44, size = 93, normalized size = 1.26

$$\frac{3\,b\cos(dx+c)^4\log(\sin(dx+c)+1) - 3\,b\cos(dx+c)^4\log(-\sin(dx+c)+1) - 8\,a\cos(dx+c)^2 - 2\left(5\,b\cos(\right.}{16\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/16*(3*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*b*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 8*a*cos(d*x + c)^2 - 2*(5*b*cos(d*x + c)^2 - 2*b)*sin(d*x + c) + 4*a)/(d*cos(d*x + c)^4)

giac [A] time = 0.27, size = 81, normalized size = 1.09

$$\frac{3\,b\log\left(|\sin\left(dx+c\right)+1|\right)-3\,b\log\left(|\sin\left(dx+c\right)-1|\right)+\frac{2\left(5\,b\sin(dx+c)^3+4\,a\sin(dx+c)^2-3\,b\sin(dx+c)-2\,a\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] 1/16*(3*b*log(abs(sin(d*x + c) + 1)) - 3*b*log(abs(sin(d*x + c) - 1)) + 2*(5*b*sin(d*x + c)^3 + 4*a*sin(d*x + c)^2 - 3*b*sin(d*x + c) - 2*a)/(sin(d*x + c)^2 - 1)^2)/d

maple [A] time = 0.23, size = 114, normalized size = 1.54

$$\frac{a\left(\sin^4\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4}+\frac{b\left(\sin^5\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4}-\frac{b\left(\sin^5\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^2}-\frac{b\left(\sin^3\left(dx+c\right)\right)}{8d}-\frac{3b\sin\left(dx+c\right)}{8d}+\frac{3b\ln\left(\sec\left(dx+c\right)+\mathrm{t}\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x)

[Out] 1/4/d*a*sin(d*x+c)^4/cos(d*x+c)^4+1/4/d*b*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*b*sin(d*x+c)^5/cos(d*x+c)^2-1/8*b*sin(d*x+c)^3/d-3/8*b*sin(d*x+c)/d+3/8/d*b*ln(sec(d*x+c)+tan(d*x+c))

maxima [A] time = 0.40, size = 89, normalized size = 1.20

$$\frac{3\,b\log\left(\sin\left(dx+c\right)+1\right)-3\,b\log\left(\sin\left(dx+c\right)-1\right)+\frac{2\left(5\,b\sin(dx+c)^3+4\,a\sin(dx+c)^2-3\,b\sin(dx+c)-2\,a\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/16*(3*b*log(sin(d*x + c) + 1) - 3*b*log(sin(d*x + c) - 1) + 2*(5*b*sin(d*x + c)^3 + 4*a*sin(d*x + c)^2 - 3*b*sin(d*x + c) - 2*a)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d

mupad [B] time = 18.08, size = 144, normalized size = 1.95

$$\frac{-\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{11\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{11\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}-\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{3\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (4*a*tan(c/2 + (d*x)/2)^4 - (3*b*tan(c/2 + (d*x)/2))/4 + (11*b*tan(c/2 + (d
*x)/2)^3)/4 + (11*b*tan(c/2 + (d*x)/2)^5)/4 - (3*b*tan(c/2 + (d*x)/2)^7)/4)
/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)
^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*b*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1486 $\int \sec^3(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=74

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan^4(c+dx)}{4d}$$

```
[Out] -1/8*a*arctanh(sin(d*x+c))/d-1/8*a*sec(d*x+c)*tan(d*x+c)/d+1/4*a*sec(d*x+c)
^3*tan(d*x+c)/d+1/4*b*tan(d*x+c)^4/d
```

Rubi [A] time = 0.14, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2834, 2611, 3768, 3770, 2607, 30}

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(8*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b*Tan[c + d*x]^4)/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sec^3(c+dx)\tan^2(c+dx)\,dx + b\int \sec^2(c+dx)\tan^3(c+$$

$$= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{1}{4}a\int \sec^3(c+dx)\,dx + \frac{b\,\mathrm{Subs}}{}$$

$$= -\frac{a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{b}{}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3}{}$$

Mathematica [A] time = 0.02, size = 74, normalized size = 1.00

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)\sec^3(c+dx)}{4d} - \frac{a\tan(c+dx)\sec(c+dx)}{8d} + \frac{b\tan^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -1/8*(a*ArcTanh[Sin[c + d*x]])/d - (a*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b*Tan[c + d*x]^4)/(4*d)
```

fricas [A] time = 0.44, size = 91, normalized size = 1.23

$$-\frac{a\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+8\,b\cos\left(dx+c\right)^{2}+2\left(a\cos\left(dx\right.\right.}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - a*cos(d*x + c)^4*log(-sin(d
*x + c) + 1) + 8*b*cos(d*x + c)^2 + 2*(a*cos(d*x + c)^2 - 2*a)*sin(d*x + c)
 - 4*b)/(d*cos(d*x + c)^4)
```

giac [A] time = 0.37, size = 78, normalized size = 1.05

$$-\frac{a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(a\sin(dx+c)^{3}+4\,b\sin(dx+c)^{2}+a\sin(dx+c)-2\,b\right)}{\left(\sin(dx+c)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(a*log(abs(sin(d*x + c) + 1)) - a*log(abs(sin(d*x + c) - 1)) - 2*(a*s
in(d*x + c)^3 + 4*b*sin(d*x + c)^2 + a*sin(d*x + c) - 2*b)/(sin(d*x + c)^2
- 1)^2)/d
```

maple [A] time = 0.23, size = 100, normalized size = 1.35

$$\frac{a\left(\sin^{3}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}+\frac{a\left(\sin^{3}\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^{2}}+\frac{a\sin\left(dx+c\right)}{8d}-\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{b\left(\sin^{4}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a*sin(d*x+c)^3/cos(d*x+c)^2+1/8*a*s
in(d*x+c)/d-1/8/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*b*sin(d*x+c)^4/cos(d*x+
c)^4
```

maxima [A] time = 0.35, size = 86, normalized size = 1.16

$$-\frac{a\log\left(\sin\left(dx+c\right)+1\right)-a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(a\sin(dx+c)^{3}+4\,b\sin(dx+c)^{2}+a\sin(dx+c)-2\,b\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*(a*sin(d*x + c
)^3 + 4*b*sin(d*x + c)^2 + a*sin(d*x + c) - 2*b)/(sin(d*x + c)^4 - 2*sin(d*
x + c)^2 + 1))/d
```

mupad [B] time = 18.07, size = 144, normalized size = 1.95

$$\frac{\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4}+\frac{7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4}+4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{4}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] ((a*tan(c/2 + (d*x)/2))/4 + (7*a*tan(c/2 + (d*x)/2)^3)/4 + (7*a*tan(c/2 + (
d*x)/2)^5)/4 + (a*tan(c/2 + (d*x)/2)^7)/4 + 4*b*tan(c/2 + (d*x)/2)^4)/(d*(6
*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + t
an(c/2 + (d*x)/2)^8 + 1)) - (a*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1487 $\int \sec^4(c+dx)(a+b\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=74

$$\frac{a\sec^4(c+dx)}{4d}-\frac{b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{b\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] -1/8*b*arctanh(sin(d*x+c))/d+1/4*a*sec(d*x+c)^4/d-1/8*b*sec(d*x+c)*tan(d*x+
c)/d+1/4*b*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2834, 2606, 30, 2611, 3768, 3770}

$$\frac{a\sec^4(c+dx)}{4d}-\frac{b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{b\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -(b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Sec[c + d*x]^4)/(4*d) - (b*Sec[c + d*
x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))\tan(c+dx)\,dx = a\int \sec^4(c+dx)\tan(c+dx)\,dx + b\int \sec^3(c+dx)\tan^2(c+
$$

$$= \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{1}{4}b\int \sec^3(c+dx)\,dx + \frac{a\,\text{Subst}}{}$$

$$= \frac{a\sec^4(c+dx)}{4d} - \frac{b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\,\text{ta}}{4d}$$

$$= -\frac{b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\sec^4(c+dx)}{4d} - \frac{b\sec(c+dx)\tan(}{8d}$$

Mathematica [A] time = 0.02, size = 74, normalized size = 1.00

$$\frac{a\sec^4(c+dx)}{4d} - \frac{b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b\tan(c+dx)\sec^3(c+dx)}{4d} - \frac{b\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -1/8*(b*ArcTanh[Sin[c + d*x]])/d + (a*Sec[c + d*x]^4)/(4*d) - (b*Sec[c + d*
x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

fricas [A] time = 0.42, size = 80, normalized size = 1.08

$$-\frac{b\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-b\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(b\cos\left(dx+c\right)^{2}-2\,b\right)\sin\left(dx\right.}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - b*cos(d*x + c)^4*log(-sin(d
*x + c) + 1) + 2*(b*cos(d*x + c)^2 - 2*b)*sin(d*x + c) - 4*a)/(d*cos(d*x +
c)^4)
```

giac [A] time = 0.23, size = 67, normalized size = 0.91

$$-\frac{b\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-b\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(b\sin\left(dx+c\right)^{3}+b\sin\left(dx+c\right)+2\,a\right)}{\left(\sin\left(dx+c\right)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(b*log(abs(sin(d*x + c) + 1)) - b*log(abs(sin(d*x + c) - 1)) - 2*(b*s
in(d*x + c)^3 + b*sin(d*x + c) + 2*a)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.20, size = 92, normalized size = 1.24

$$\frac{a}{4d\cos\left(dx+c\right)^{4}}+\frac{b\left(\sin^{3}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}+\frac{b\left(\sin^{3}\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^{2}}+\frac{b\sin\left(dx+c\right)}{8d}-\frac{b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/cos(d*x+c)^4+1/4/d*b*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*b*sin(d*x+c)^3
/cos(d*x+c)^2+1/8*b*sin(d*x+c)/d-1/8/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.34, size = 75, normalized size = 1.01

$$-\frac{b\log\left(\sin\left(dx+c\right)+1\right)-b\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(b\sin\left(dx+c\right)^{3}+b\sin\left(dx+c\right)+2\,a\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(b*log(sin(d*x + c) + 1) - b*log(sin(d*x + c) - 1) - 2*(b*sin(d*x + c
)^3 + b*sin(d*x + c) + 2*a)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 18.54, size = 158, normalized size = 2.14

$$\frac{\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\frac{7b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4}+\frac{7b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{4}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}\right.\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] ((b*tan(c/2 + (d*x)/2))/4 + 2*a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/
2)^6 + (7*b*tan(c/2 + (d*x)/2)^3)/4 + (7*b*tan(c/2 + (d*x)/2)^5)/4 + (b*tan
(c/2 + (d*x)/2)^7)/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 -
 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (b*atanh(tan(c/2 + (
d*x)/2)))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1488 $\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=99

$$\frac{a\tan^4(c+dx)}{4d}+\frac{a\tan^2(c+dx)}{d}+\frac{a\log(\tan(c+dx))}{d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+3b\tan$$

```
[Out] 3/8*b*arctanh(sin(d*x+c))/d+a*ln(tan(d*x+c))/d+3/8*b*sec(d*x+c)*tan(d*x+c)/
d+1/4*b*sec(d*x+c)^3*tan(d*x+c)/d+a*tan(d*x+c)^2/d+1/4*a*tan(d*x+c)^4/d
```

Rubi [A] time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2834, 2620, 266, 43, 3768, 3770}

$$\frac{a\tan^4(c+dx)}{4d}+\frac{a\tan^2(c+dx)}{d}+\frac{a\log(\tan(c+dx))}{d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+3b\tan$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Log[Tan[c + d*x]])/d + (3*b*Sec[c +
d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c
 + d*x]^2)/d + (a*Tan[c + d*x]^4)/(4*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc(c+dx)\sec^5(c+dx)\,dx + b\int \sec^5(c+dx)\,dx$$

$$= \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3b)\int \sec^3(c+dx)\,dx + \frac{a\,\text{Sub}}{}$$

$$= \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{8}(3$$

$$= \frac{3b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3}{}$$

$$= \frac{3b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\log(\tan(c+dx))}{d} + \frac{3b\sec(c+dx)}{8d}$$

Mathematica [A] time = 1.26, size = 99, normalized size = 1.00

$$-\frac{a\left(-\sec^4(c+dx)-2\sec^2(c+dx)-4\log(\sin(c+dx))+4\log(\cos(c+dx))\right)}{4d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3b\left(\text{t}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(a*(4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]] - 2*Sec[c + d*x]^2 - Sec
[c + d*x]^4))/d + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*b*(ArcTanh[Sin
[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

fricas [A] time = 0.44, size = 125, normalized size = 1.26

$$\frac{16\,a\cos\left(dx+c\right)^{4}\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-\left(8\,a-3\,b\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-\left(8\,a+3\,b\right)\cos\left(dx+c\right)^{4}}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*cos(d*x + c)^4*log(1/2*sin(d*x + c)) - (8*a - 3*b)*cos(d*x + c)^
4*log(sin(d*x + c) + 1) - (8*a + 3*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1)
 + 8*a*cos(d*x + c)^2 + 2*(3*b*cos(d*x + c)^2 + 2*b)*sin(d*x + c) + 4*a)/(d
*cos(d*x + c)^4)
```

giac [A] time = 0.24, size = 113, normalized size = 1.14

$$-\frac{\left(8\,a-3\,b\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+\left(8\,a+3\,b\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-16\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{2\left(6\,a\sin\left(dx+c\right)^{4}-3\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*((8*a - 3*b)*log(abs(sin(d*x + c) + 1)) + (8*a + 3*b)*log(abs(sin(d*x
 + c) - 1)) - 16*a*log(abs(sin(d*x + c))) - 2*(6*a*sin(d*x + c)^4 - 3*b*sin
(d*x + c)^3 - 16*a*sin(d*x + c)^2 + 5*b*sin(d*x + c) + 12*a)/(sin(d*x + c)^
2 - 1)^2)/d
```

maple [A] time = 0.43, size = 100, normalized size = 1.01

$$\frac{a}{4d\cos\left(dx+c\right)^{4}}+\frac{a}{2d\cos\left(dx+c\right)^{2}}+\frac{a\ln\left(\tan\left(dx+c\right)\right)}{d}+\frac{b\left(\sec^{3}\left(dx+c\right)\right)\tan\left(dx+c\right)}{4d}+\frac{3b\sec\left(dx+c\right)\tan\left(dx+\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/cos(d*x+c)^4+1/2/d*a/cos(d*x+c)^2+a*ln(tan(d*x+c))/d+1/4*b*sec(d*x+
c)^3*tan(d*x+c)/d+3/8*b*sec(d*x+c)*tan(d*x+c)/d+3/8/d*b*ln(sec(d*x+c)+tan(d
*x+c))
```

maxima [A] time = 0.37, size = 109, normalized size = 1.10

$$-\frac{(8\,a-3\,b)\log\left(\sin\left(dx+c\right)+1\right)+(8\,a+3\,b)\log\left(\sin\left(dx+c\right)-1\right)-16\,a\log\left(\sin\left(dx+c\right)\right)+\frac{2\left(3\,b\sin(dx+c)^{3}+4\,a\sin\right.}{\sin(dx+c)^{4}-}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*((8*a - 3*b)*log(sin(d*x + c) + 1) + (8*a + 3*b)*log(sin(d*x + c) - 1
) - 16*a*log(sin(d*x + c)) + 2*(3*b*sin(d*x + c)^3 + 4*a*sin(d*x + c)^2 - 5
*b*sin(d*x + c) - 6*a)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 11.87, size = 116, normalized size = 1.17

$$\frac{-\frac{3\,b\sin(c+d\,x)^{3}}{8}-\frac{a\sin(c+d\,x)^{2}}{2}+\frac{5\,b\,\sin(c+d\,x)}{8}+\frac{3\,a}{4}}{d\,\left(\sin\left(c+d\,x\right)^{4}-2\sin\left(c+d\,x\right)^{2}+1\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{a}{2}-\frac{3\,b}{16}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{a}{2}+\frac{3\,b}{16}\right)}{d}+\frac{a\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)),x)
```

```
[Out] ((3*a)/4 + (5*b*sin(c + d*x))/8 - (a*sin(c + d*x)^2)/2 - (3*b*sin(c + d*x)^
3)/8)/(d*(sin(c + d*x)^4 - 2*sin(c + d*x)^2 + 1)) - (log(sin(c + d*x) + 1)*
(a/2 - (3*b)/16))/d - (log(sin(c + d*x) - 1)*(a/2 + (3*b)/16))/d + (a*log(s
in(c + d*x)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1489 $\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{15a\csc(c+dx)}{8d}+\frac{15a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\csc(c+dx)\sec^4(c+dx)}{4d}+\frac{5a\csc(c+dx)\sec^2(c+dx)}{8d}+\frac{b\tan^4(c+}{4d}$$

[Out] 15/8*a*arctanh(sin(d*x+c))/d-15/8*a*csc(d*x+c)/d+b*ln(tan(d*x+c))/d+5/8*a*csc(d*x+c)*sec(d*x+c)^2/d+1/4*a*csc(d*x+c)*sec(d*x+c)^4/d+b*tan(d*x+c)^2/d+1/4*b*tan(d*x+c)^4/d

Rubi [A] time = 0.14, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2834, 2621, 288, 321, 207, 2620, 266, 43}

$$-\frac{15a\csc(c+dx)}{8d}+\frac{15a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\csc(c+dx)\sec^4(c+dx)}{4d}+\frac{5a\csc(c+dx)\sec^2(c+dx)}{8d}+\frac{b\tan^4(c+}{4d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]

[Out] (15*a*ArcTanh[Sin[c + d*x]])/(8*d) - (15*a*Csc[c + d*x])/(8*d) + (b*Log[Tan[c + d*x]])/d + (5*a*Csc[c + d*x]*Sec[c + d*x]^2)/(8*d) + (a*Csc[c + d*x]*Sec[c + d*x]^4)/(4*d) + (b*Tan[c + d*x]^2)/d + (b*Tan[c + d*x]^4)/(4*d)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc^2(c+dx)\sec^5(c+dx)\,dx + b\int \csc(c+dx)\sec^5(c+$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^3}\,dx, x, \csc(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{\left(1+x^2\right)}{x}\right.}{}$$

$$= \frac{a\csc(c+dx)\sec^4(c+dx)}{4d} - \frac{(5a)\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \text{cs}\right.}{4d}$$

$$= \frac{5a\csc(c+dx)\sec^2(c+dx)}{8d} + \frac{a\csc(c+dx)\sec^4(c+dx)}{4d} -$$

$$= -\frac{15a\csc(c+dx)}{8d} + \frac{b\log(\tan(c+dx))}{d} + \frac{5a\csc(c+dx)\,\text{sec}}{8d}$$

$$= \frac{15a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{15a\csc(c+dx)}{8d} + \frac{b\log(\tan(c+}{d}$$

Mathematica [C] time = 0.39, size = 76, normalized size = 0.66

$$-\frac{a\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\sin^2(c+dx)\right)}{d} - \frac{b\left(-\sec^4(c+dx) - 2\sec^2(c+dx) - 4\log(\sin(c+dx)) + 4\log(\cos(\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[c + d*x]^2])/d) - (b*
(4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]] - 2*Sec[c + d*x]^2 - Sec[c + d*x
]^4))/(4*d)
```

fricas [A] time = 0.44, size = 159, normalized size = 1.38

$$\frac{16\,b\cos\left(dx+c\right)^4\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right) + \left(15\,a-8\,b\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*b*cos(d*x + c)^4*log(1/2*sin(d*x + c))*sin(d*x + c) + (15*a - 8*b)
*cos(d*x + c)^4*log(sin(d*x + c) + 1)*sin(d*x + c) - (15*a + 8*b)*cos(d*x +
```

```
c)^4*log(-sin(d*x + c) + 1)*sin(d*x + c) - 30*a*cos(d*x + c)^4 + 10*a*cos(
d*x + c)^2 + 4*(2*b*cos(d*x + c)^2 + b)*sin(d*x + c) + 4*a)/(d*cos(d*x + c)
^4*sin(d*x + c))
```

giac [A] time = 0.29, size = 134, normalized size = 1.17

$$\frac{(15\,a-8\,b)\log\left(|\sin\left(dx+c\right)+1|\right)-(15\,a+8\,b)\log\left(|\sin\left(dx+c\right)-1|\right)+16\,b\log\left(|\sin\left(dx+c\right)|\right)-\frac{16\,(b\sin(dx+c)+a)}{\sin(dx+c)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*((15*a - 8*b)*log(abs(sin(d*x + c) + 1)) - (15*a + 8*b)*log(abs(sin(d*
x + c) - 1)) + 16*b*log(abs(sin(d*x + c))) - 16*(b*sin(d*x + c) + a)/sin(d*
x + c) + 2*(6*b*sin(d*x + c)^4 - 7*a*sin(d*x + c)^3 - 16*b*sin(d*x + c)^2 +
 9*a*sin(d*x + c) + 12*b)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.30, size = 120, normalized size = 1.04

$$\frac{a}{4d\sin\left(dx+c\right)\cos\left(dx+c\right)^{4}}+\frac{5a}{8d\sin\left(dx+c\right)\cos\left(dx+c\right)^{2}}-\frac{15a}{8d\sin\left(dx+c\right)}+\frac{15a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/sin(d*x+c)/cos(d*x+c)^4+5/8/d*a/sin(d*x+c)/cos(d*x+c)^2-15/8/d*a/si
n(d*x+c)+15/8/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*b/cos(d*x+c)^4+1/2/d*b/co
s(d*x+c)^2+b*ln(tan(d*x+c))/d
```

maxima [A] time = 0.31, size = 126, normalized size = 1.10

$$\frac{(15\,a-8\,b)\log\left(\sin\left(dx+c\right)+1\right)-(15\,a+8\,b)\log\left(\sin\left(dx+c\right)-1\right)+16\,b\log\left(\sin\left(dx+c\right)\right)-\frac{2\left(15\,a\sin(dx+c)^{4}+4\,b\,\mathrm{s}\right.}{\sin(dx\text{-}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*((15*a - 8*b)*log(sin(d*x + c) + 1) - (15*a + 8*b)*log(sin(d*x + c) -
1) + 16*b*log(sin(d*x + c)) - 2*(15*a*sin(d*x + c)^4 + 4*b*sin(d*x + c)^3 -
 25*a*sin(d*x + c)^2 - 6*b*sin(d*x + c) + 8*a)/(sin(d*x + c)^5 - 2*sin(d*x
+ c)^3 + sin(d*x + c)))/d
```

mupad [**B**] time = 11.89, size = 130, normalized size = 1.13

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{15\,a}{16}-\frac{b}{2}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{15\,a}{16}+\frac{b}{2}\right)}{d}+\frac{b\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{\frac{15\,a\,\sin(c+d\,x)^4}{8}+\frac{b\,\sin(c+d\,x)^3}{2}}{d\,\left(\sin\left(c+d\,x\right)^5-2\,\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)^2),x)
```

```
[Out] (log(sin(c + d*x) + 1)*((15*a)/16 - b/2))/d - (log(sin(c + d*x) - 1)*((15*a
)/16 + b/2))/d + (b*log(sin(c + d*x)))/d - (a - (3*b*sin(c + d*x))/4 - (25*
a*sin(c + d*x)^2)/8 + (15*a*sin(c + d*x)^4)/8 + (b*sin(c + d*x)^3)/2)/(d*(s
in(c + d*x) - 2*sin(c + d*x)^3 + sin(c + d*x)^5))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1490 $\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=135

$$\frac{a\tan^4(c+dx)}{4d}+\frac{3a\tan^2(c+dx)}{2d}-\frac{a\cot^2(c+dx)}{2d}+\frac{3a\log(\tan(c+dx))}{d}-\frac{15b\csc(c+dx)}{8d}+\frac{15b\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] 15/8*b*arctanh(sin(d*x+c))/d-1/2*a*cot(d*x+c)^2/d-15/8*b*csc(d*x+c)/d+3*a*l
n(tan(d*x+c))/d+5/8*b*csc(d*x+c)*sec(d*x+c)^2/d+1/4*b*csc(d*x+c)*sec(d*x+c)
^4/d+3/2*a*tan(d*x+c)^2/d+1/4*a*tan(d*x+c)^4/d
```

Rubi [A] time = 0.16, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2834, 2620, 266, 43, 2621, 288, 321, 207}

$$\frac{a\tan^4(c+dx)}{4d}+\frac{3a\tan^2(c+dx)}{2d}-\frac{a\cot^2(c+dx)}{2d}+\frac{3a\log(\tan(c+dx))}{d}-\frac{15b\csc(c+dx)}{8d}+\frac{15b\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (15*b*ArcTanh[Sin[c + d*x]])/(8*d) - (a*Cot[c + d*x]^2)/(2*d) - (15*b*Csc[c
 + d*x])/(8*d) + (3*a*Log[Tan[c + d*x]])/d + (5*b*Csc[c + d*x]*Sec[c + d*x]
^2)/(8*d) + (b*Csc[c + d*x]*Sec[c + d*x]^4)/(4*d) + (3*a*Tan[c + d*x]^2)/(2
*d) + (a*Tan[c + d*x]^4)/(4*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc^3(c+dx)\sec^5(c+dx)\,dx + b\int \csc^2(c+dx)\sec^5(c+$$

$$= \frac{a\,\text{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, \tan(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,d\right.}{d}$$

$$= \frac{b\csc(c+dx)\sec^4(c+dx)}{4d} + \frac{a\,\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, \tan^2(c+\right.}{2d}$$

$$= \frac{5b\csc(c+dx)\sec^2(c+dx)}{8d} + \frac{b\csc(c+dx)\sec^4(c+dx)}{4d} + \frac{a}{\,}$$

$$= -\frac{a\cot^2(c+dx)}{2d} - \frac{15b\csc(c+dx)}{8d} + \frac{3a\log(\tan(c+dx))}{d} + \frac{5}{\,}$$

$$= \frac{15b\tanh^{-1}(\sin(c+dx))}{8d} - \frac{a\cot^2(c+dx)}{2d} - \frac{15b\csc(c+dx)}{8d}$$

Mathematica [C] time = 0.61, size = 86, normalized size = 0.64

$$-\frac{a\left(2\csc^2(c+dx)-\sec^4(c+dx)-4\sec^2(c+dx)-12\log(\sin(c+dx))+12\log(\cos(c+dx))\right)}{4d} - \frac{b\csc(c+dx)\,{}_2F_1}{\,}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[c + d*x]^2])/d) - (a*
(2*Csc[c + d*x]^2 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]] - 4*Sec[c +
 d*x]^2 - Sec[c + d*x]^4))/(4*d)
```

fricas [A] time = 0.46, size = 211, normalized size = 1.56

$$\frac{24\,a\cos(dx+c)^4 - 12\,a\cos(dx+c)^2 + 48\left(a\cos(dx+c)^6 - a\cos(dx+c)^4\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 3\left((8\,a-5\,b)\right.}{\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(24*a*cos(d*x + c)^4 - 12*a*cos(d*x + c)^2 + 48*(a*cos(d*x + c)^6 - a*
cos(d*x + c)^4)*log(1/2*sin(d*x + c)) - 3*((8*a - 5*b)*cos(d*x + c)^6 - (8*
```

```
a - 5*b)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - 3*((8*a + 5*b)*cos(d*x + c
)^6 - (8*a + 5*b)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) + 2*(15*b*cos(d*x
+ c)^4 - 5*b*cos(d*x + c)^2 - 2*b)*sin(d*x + c) - 4*a)/(d*cos(d*x + c)^6 -
d*cos(d*x + c)^4)
```

giac [A] time = 0.32, size = 133, normalized size = 0.99

$$-\frac{3\,(8\,a-5\,b)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\,(8\,a+5\,b)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-48\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+\frac{2\left(15\,b\sin(dx\cdots\right.}{\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(3*(8*a - 5*b)*log(abs(sin(d*x + c) + 1)) + 3*(8*a + 5*b)*log(abs(sin
(d*x + c) - 1)) - 48*a*log(abs(sin(d*x + c))) + 2*(15*b*sin(d*x + c)^5 + 12
*a*sin(d*x + c)^4 - 25*b*sin(d*x + c)^3 - 18*a*sin(d*x + c)^2 + 8*b*sin(d*x
 + c) + 4*a)/(sin(d*x + c)^3 - sin(d*x + c))^2)/d
```

maple [A] time = 0.36, size = 151, normalized size = 1.12

$$\frac{a}{4d\sin\left(dx+c\right)^{2}\cos\left(dx+c\right)^{4}}+\frac{3a}{4d\sin\left(dx+c\right)^{2}\cos\left(dx+c\right)^{2}}-\frac{3a}{2d\sin\left(dx+c\right)^{2}}+\frac{3a\ln\left(\tan\left(dx+c\right)\right)}{d}+\frac{\cdots}{4d\sin\left(dx\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/sin(d*x+c)^2/cos(d*x+c)^4+3/4/d*a/sin(d*x+c)^2/cos(d*x+c)^2-3/2/d*a
/sin(d*x+c)^2+3*a*ln(tan(d*x+c))/d+1/4/d*b/sin(d*x+c)/cos(d*x+c)^4+5/8/d*b/
sin(d*x+c)/cos(d*x+c)^2-15/8/d*b/sin(d*x+c)+15/8/d*b*ln(sec(d*x+c)+tan(d*x+
c))
```

maxima [A] time = 0.33, size = 140, normalized size = 1.04

$$-\frac{3\,(8\,a-5\,b)\log\left(\sin\left(dx+c\right)+1\right)+3\,(8\,a+5\,b)\log\left(\sin\left(dx+c\right)-1\right)-48\,a\log\left(\sin\left(dx+c\right)\right)+\frac{2\left(15\,b\sin(dx+c)^{5}\cdots\right.}{\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(3*(8*a - 5*b)*log(sin(d*x + c) + 1) + 3*(8*a + 5*b)*log(sin(d*x + c)
 - 1) - 48*a*log(sin(d*x + c)) + 2*(15*b*sin(d*x + c)^5 + 12*a*sin(d*x + c)
```

```
^4 - 25*b*sin(d*x + c)^3 - 18*a*sin(d*x + c)^2 + 8*b*sin(d*x + c) + 4*a)/(s
in(d*x + c)^6 - 2*sin(d*x + c)^4 + sin(d*x + c)^2))/d
```

mupad [B] time = 0.11, size = 146, normalized size = 1.08

$$\frac{3\,a\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{3\,a}{2}-\frac{15\,b}{16}\right)}{d}-\frac{\frac{15\,b\,\sin(c+d\,x)^5}{8}+\frac{3\,a\,\sin(c+d\,x)^4}{2}-\frac{25\,b\,\sin(c+d\,x)^3}{8}-\frac{9\,a\,\sin(c+d\,x)^2}{4}+}{d\,\left(\sin\left(c+d\,x\right)^6-2\,\sin\left(c+d\,x\right)^4+\sin\left(c+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)^3),x)
```

```
[Out] (3*a*log(sin(c + d*x)))/d - (log(sin(c + d*x) + 1)*((3*a)/2 - (15*b)/16))/d
 - (a/2 + b*sin(c + d*x) - (9*a*sin(c + d*x)^2)/4 + (3*a*sin(c + d*x)^4)/2
- (25*b*sin(c + d*x)^3)/8 + (15*b*sin(c + d*x)^5)/8)/(d*(sin(c + d*x)^2 - 2
*sin(c + d*x)^4 + sin(c + d*x)^6)) - (log(sin(c + d*x) - 1)*((3*a)/2 + (15*
b)/16))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1491 $\int \csc^4(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=155

$$-\frac{35a\csc^3(c+dx)}{24d}-\frac{35a\csc(c+dx)}{8d}+\frac{35a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\csc^3(c+dx)\sec^4(c+dx)}{4d}+\frac{7a\csc^3(c+dx)\sec}{8d}$$

```
[Out] 35/8*a*arctanh(sin(d*x+c))/d-1/2*b*cot(d*x+c)^2/d-35/8*a*csc(d*x+c)/d-35/24
*a*csc(d*x+c)^3/d+3*b*ln(tan(d*x+c))/d+7/8*a*csc(d*x+c)^3*sec(d*x+c)^2/d+1/
4*a*csc(d*x+c)^3*sec(d*x+c)^4/d+3/2*b*tan(d*x+c)^2/d+1/4*b*tan(d*x+c)^4/d
```

Rubi [A] time = 0.16, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2834, 2621, 288, 302, 207, 2620, 266, 43}

$$-\frac{35a\csc^3(c+dx)}{24d}-\frac{35a\csc(c+dx)}{8d}+\frac{35a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\csc^3(c+dx)\sec^4(c+dx)}{4d}+\frac{7a\csc^3(c+dx)\sec}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (35*a*ArcTanh[Sin[c + d*x]])/(8*d) - (b*Cot[c + d*x]^2)/(2*d) - (35*a*Csc[c
 + d*x])/(8*d) - (35*a*Csc[c + d*x]^3)/(24*d) + (3*b*Log[Tan[c + d*x]])/d +
 (7*a*Csc[c + d*x]^3*Sec[c + d*x]^2)/(8*d) + (a*Csc[c + d*x]^3*Sec[c + d*x]
^4)/(4*d) + (3*b*Tan[c + d*x]^2)/(2*d) + (b*Tan[c + d*x]^4)/(4*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \csc^4(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc^4(c+dx)\sec^5(c+dx)\,dx + b\int \csc^3(c+dx)\sec^5(c$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^8}{\left(-1+x^2\right)^3}\,dx, x, \csc(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{\left(1+x^2\right)}{x^3}\right.}{}$$

$$= \frac{a\csc^3(c+dx)\sec^4(c+dx)}{4d} - \frac{(7a)\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, c\right.}{4d}$$

$$= \frac{7a\csc^3(c+dx)\sec^2(c+dx)}{8d} + \frac{a\csc^3(c+dx)\sec^4(c+dx)}{4d}$$

$$= -\frac{b\cot^2(c+dx)}{2d} + \frac{3b\log(\tan(c+dx))}{d} + \frac{7a\csc^3(c+dx)\text{se}}{8d}$$

$$= -\frac{b\cot^2(c+dx)}{2d} - \frac{35a\csc(c+dx)}{8d} - \frac{35a\csc^3(c+dx)}{24d} + \frac{3b}{}$$

$$= \frac{35a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{b\cot^2(c+dx)}{2d} - \frac{35a\csc(c+dx)}{8d}$$

Mathematica [C] time = 0.82, size = 90, normalized size = 0.58

$$-\frac{a\csc^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\sin^2(c+dx)\right)}{3d} - \frac{b\left(2\csc^2(c+dx) - \sec^4(c+dx) - 4\sec^2(c+dx) - 12\log(\sin(c+\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/3*(a*Csc[c + d*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[c + d*x]^2])/d
- (b*(2*Csc[c + d*x]^2 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]] - 4*Se
c[c + d*x]^2 - Sec[c + d*x]^4))/(4*d)
```

fricas [A] time = 0.47, size = 248, normalized size = 1.60

$$-\frac{210\,a\cos(dx+c)^6 - 280\,a\cos(dx+c)^4 + 42\,a\cos(dx+c)^2 - 144\left(b\cos(dx+c)^6 - b\cos(dx+c)^4\right)\log\left(\frac{1}{2}\,\text{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(210*a*cos(d*x + c)^6 - 280*a*cos(d*x + c)^4 + 42*a*cos(d*x + c)^2 -
144*(b*cos(d*x + c)^6 - b*cos(d*x + c)^4)*log(1/2*sin(d*x + c))*sin(d*x + c
) - 3*((35*a - 24*b)*cos(d*x + c)^6 - (35*a - 24*b)*cos(d*x + c)^4)*log(sin
(d*x + c) + 1)*sin(d*x + c) + 3*((35*a + 24*b)*cos(d*x + c)^6 - (35*a + 24*
b)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1)*sin(d*x + c) - 12*(6*b*cos(d*x +
c)^4 - 3*b*cos(d*x + c)^2 - b)*sin(d*x + c) + 12*a)/((d*cos(d*x + c)^6 - d*
cos(d*x + c)^4)*sin(d*x + c))
```

giac [A] time = 0.30, size = 160, normalized size = 1.03

$$\frac{3\,(35\,a-24\,b)\log\left(|\sin\left(dx+c\right)+1|\right)-3\,(35\,a+24\,b)\log\left(|\sin\left(dx+c\right)-1|\right)+144\,b\log\left(|\sin\left(dx+c\right)|\right)+\frac{6\left(18\,b\sin\right.}{}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/48*(3*(35*a - 24*b)*log(abs(sin(d*x + c) + 1)) - 3*(35*a + 24*b)*log(abs(
sin(d*x + c) - 1)) + 144*b*log(abs(sin(d*x + c))) + 6*(18*b*sin(d*x + c)^4
- 11*a*sin(d*x + c)^3 - 44*b*sin(d*x + c)^2 + 13*a*sin(d*x + c) + 28*b)/(si
n(d*x + c)^2 - 1)^2 - 8*(33*b*sin(d*x + c)^3 + 18*a*sin(d*x + c)^2 + 3*b*si
n(d*x + c) + 2*a)/sin(d*x + c)^3)/d
```

maple [A] time = 0.37, size = 173, normalized size = 1.12

$$\frac{a}{4d\sin\left(dx+c\right)^{3}\cos\left(dx+c\right)^{4}}-\frac{7a}{12d\sin\left(dx+c\right)^{3}\cos\left(dx+c\right)^{2}}+\frac{35a}{24d\sin\left(dx+c\right)\cos\left(dx+c\right)^{2}}-\frac{35a}{8d\sin\left(dx+c\right)}+\frac{35}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/sin(d*x+c)^3/cos(d*x+c)^4-7/12/d*a/sin(d*x+c)^3/cos(d*x+c)^2+35/24/
d*a/sin(d*x+c)/cos(d*x+c)^2-35/8/d*a/sin(d*x+c)+35/8/d*a*ln(sec(d*x+c)+tan(
d*x+c))+1/4/d*b/sin(d*x+c)^2/cos(d*x+c)^4+3/4/d*b/sin(d*x+c)^2/cos(d*x+c)^2
-3/2/d*b/sin(d*x+c)^2+3*b*ln(tan(d*x+c))/d
```

maxima [A] time = 0.36, size = 151, normalized size = 0.97

$$\frac{3\,(35\,a-24\,b)\log\left(\sin\left(dx+c\right)+1\right)-3\,(35\,a+24\,b)\log\left(\sin\left(dx+c\right)-1\right)+144\,b\log\left(\sin\left(dx+c\right)\right)-\frac{2\left(105\,a\sin(dx\right.}{}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

[Out] 1/48*(3*(35*a - 24*b)*log(sin(d*x + c) + 1) - 3*(35*a + 24*b)*log(sin(d*x + c) - 1) + 144*b*log(sin(d*x + c)) - 2*(105*a*sin(d*x + c)^6 + 36*b*sin(d*x + c)^5 - 175*a*sin(d*x + c)^4 - 54*b*sin(d*x + c)^3 + 56*a*sin(d*x + c)^2 + 12*b*sin(d*x + c) + 8*a)/(sin(d*x + c)^7 - 2*sin(d*x + c)^5 + sin(d*x + c)^3))/d

mupad [B] time = 11.91, size = 157, normalized size = 1.01

$$\frac{\ln(\sin(c+dx)+1)\left(\frac{35a}{16}-\frac{3b}{2}\right)}{d}-\frac{\ln(\sin(c+dx)-1)\left(\frac{35a}{16}+\frac{3b}{2}\right)}{d}+\frac{3b\ln(\sin(c+dx))}{d}-\frac{\frac{35a\sin(c+dx)^6}{8}+\frac{3b\sin(c}{2}}{d\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)^4),x)

[Out] (log(sin(c + d*x) + 1)*((35*a)/16 - (3*b)/2))/d - (log(sin(c + d*x) - 1)*((35*a)/16 + (3*b)/2))/d + (3*b*log(sin(c + d*x)))/d - (a/3 + (b*sin(c + d*x))/2 + (7*a*sin(c + d*x)^2)/3 - (175*a*sin(c + d*x)^4)/24 + (35*a*sin(c + d*x)^6)/8 - (9*b*sin(c + d*x)^3)/4 + (3*b*sin(c + d*x)^5)/2)/(d*(sin(c + d*x)^3 - 2*sin(c + d*x)^5 + sin(c + d*x)^7))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**4*sec(d*x+c)**5*(a+b*sin(d*x+c)),x)

[Out] Timed out

3.1492 $\int \sin(c+dx)(a+b\sin(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=189

$$-\frac{(a^2+3b^2)\sin(c+dx)}{d}-\frac{(15a^2+48ab+35b^2)\log(1-\sin(c+dx))}{16d}+\frac{(15a^2-48ab+35b^2)\log(\sin(c+dx)+1)}{16d}$$

```
[Out] -1/16*(15*a^2+48*a*b+35*b^2)*ln(1-sin(d*x+c))/d+1/16*(15*a^2-48*a*b+35*b^2)
*ln(1+sin(d*x+c))/d-(a^2+3*b^2)*sin(d*x+c)/d-a*b*sin(d*x+c)^2/d-1/3*b^2*sin
(d*x+c)^3/d-1/8*sec(d*x+c)^2*(11*b+9*a*sin(d*x+c))*(a+b*sin(d*x+c))/d+1/4*s
ec(d*x+c)^3*(a+b*sin(d*x+c))^2*tan(d*x+c)/d
```

Rubi [A] time = 0.36, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2837, 12, 1645, 1810, 633, 31}

$$-\frac{(a^2+3b^2)\sin(c+dx)}{d}-\frac{(15a^2+48ab+35b^2)\log(1-\sin(c+dx))}{16d}+\frac{(15a^2-48ab+35b^2)\log(\sin(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -((15*a^2 + 48*a*b + 35*b^2)*Log[1 - Sin[c + d*x]])/(16*d) + ((15*a^2 - 48*
a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - ((a^2 + 3*b^2)*Sin[c + d*x])/
d - (a*b*Sin[c + d*x]^2)/d - (b^2*Sin[c + d*x]^3)/(3*d) - (Sec[c + d*x]^2*(
11*b + 9*a*Sin[c + d*x])*(a + b*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a +
 b*Sin[c + d*x])^2*Tan[c + d*x])/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))^2\tan^5(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^6(a+x)^2}{b^6\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6(a+x)^2}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{bd}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{(a+x}{}\right.}{}$$

$$= -\frac{\sec^2(c+dx)(11b+9a\sin(c+dx))(a+b\sin(c+dx))}{8d} + \frac{\sec}{}$$

$$= -\frac{\sec^2(c+dx)(11b+9a\sin(c+dx))(a+b\sin(c+dx))}{8d} + \frac{\sec}{}$$

$$= -\frac{\left(a^2+3b^2\right)\sin(c+dx)}{d} - \frac{ab\sin^2(c+dx)}{d} - \frac{b^2\sin^3(c+dx)}{3d}$$

$$= -\frac{\left(a^2+3b^2\right)\sin(c+dx)}{d} - \frac{ab\sin^2(c+dx)}{d} - \frac{b^2\sin^3(c+dx)}{3d}$$

$$= -\frac{\left(15a^2+48ab+35b^2\right)\log(1-\sin(c+dx))}{16d} + \frac{\left(15a^2-48ab\right.}{}$$

Mathematica [A] time = 1.56, size = 186, normalized size = 0.98

$$\frac{-48\left(a^2+3b^2\right)\sin(c+dx) - 3\left(15a^2+48ab+35b^2\right)\log(1-\sin(c+dx)) + 3\left(15a^2-48ab+35b^2\right)\log(\sin(c+d}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (-3*(15*a^2 + 48*a*b + 35*b^2)*Log[1 - Sin[c + d*x]] + 3*(15*a^2 - 48*a*b +
 35*b^2)*Log[1 + Sin[c + d*x]] + (3*(a + b)^2)/(-1 + Sin[c + d*x])^2 + (3*(
a + b)*(9*a + 13*b))/(-1 + Sin[c + d*x]) - 48*(a^2 + 3*b^2)*Sin[c + d*x] -
48*a*b*Sin[c + d*x]^2 - 16*b^2*Sin[c + d*x]^3 - (3*(a - b)^2)/(1 + Sin[c +
d*x])^2 + (3*(9*a - 13*b)*(a - b))/(1 + Sin[c + d*x]))/(48*d)
```

fricas [A] time = 0.50, size = 198, normalized size = 1.05

$$\frac{48\,ab\cos\left(dx+c\right)^{6}-24\,ab\cos\left(dx+c\right)^{4}+3\left(15\,a^{2}-48\,ab+35\,b^{2}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(15\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/48*(48*a*b*cos(d*x + c)^6 - 24*a*b*cos(d*x + c)^4 + 3*(15*a^2 - 48*a*b +
35*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(15*a^2 + 48*a*b + 35*b^2)
*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 144*a*b*cos(d*x + c)^2 + 24*a*b +
2*(8*b^2*cos(d*x + c)^6 - 8*(3*a^2 + 10*b^2)*cos(d*x + c)^4 - 3*(9*a^2 + 13
*b^2)*cos(d*x + c)^2 + 6*a^2 + 6*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.31, size = 198, normalized size = 1.05

$$-\frac{16\,b^{2}\sin\left(dx+c\right)^{3}+48\,ab\sin\left(dx+c\right)^{2}+48\,a^{2}\sin\left(dx+c\right)+144\,b^{2}\sin\left(dx+c\right)-3\left(15\,a^{2}-48\,ab+35\,b^{2}\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/48*(16*b^2*sin(d*x + c)^3 + 48*a*b*sin(d*x + c)^2 + 48*a^2*sin(d*x + c)
+ 144*b^2*sin(d*x + c) - 3*(15*a^2 - 48*a*b + 35*b^2)*log(abs(sin(d*x + c)
+ 1)) + 3*(15*a^2 + 48*a*b + 35*b^2)*log(abs(sin(d*x + c) - 1)) - 6*(36*a*b
*sin(d*x + c)^4 + 9*a^2*sin(d*x + c)^3 + 13*b^2*sin(d*x + c)^3 - 48*a*b*sin
(d*x + c)^2 - 7*a^2*sin(d*x + c) - 11*b^2*sin(d*x + c) + 16*a*b)/(sin(d*x +
 c)^2 - 1)^2)/d
```

maple [A] time = 0.32, size = 355, normalized size = 1.88

$$\frac{a^{2}\left(\sin^{7}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}-\frac{3a^{2}\left(\sin^{7}\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^{2}}-\frac{3a^{2}\left(\sin^{5}\left(dx+c\right)\right)}{8d}-\frac{5a^{2}\left(\sin^{3}\left(dx+c\right)\right)}{8d}-\frac{15a^{2}\sin\left(dx+c\right)}{8d}+\frac{15a^{2}\ln\left(\sec\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a^2*sin(d*x+c)^7/cos(d*x+c)^2-3/8
*a^2*sin(d*x+c)^5/d-5/8*a^2*sin(d*x+c)^3/d-15/8*a^2*sin(d*x+c)/d+15/8/d*a^2
*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*b*sin(d*x+c)^8/cos(d*x+c)^4-1/d*a*b*sin(
```

```
d*x+c)^8/cos(d*x+c)^2-a*b*sin(d*x+c)^6/d-3/2*a*b*sin(d*x+c)^4/d-3*a*b*sin(d
*x+c)^2/d-6/d*a*b*ln(cos(d*x+c))+1/4/d*b^2*sin(d*x+c)^9/cos(d*x+c)^4-5/8/d*
b^2*sin(d*x+c)^9/cos(d*x+c)^2-5/8*b^2*sin(d*x+c)^7/d-7/8*b^2*sin(d*x+c)^5/d
-35/24*b^2*sin(d*x+c)^3/d-35/8*b^2*sin(d*x+c)/d+35/8/d*b^2*ln(sec(d*x+c)+ta
n(d*x+c))
```

maxima [A] time = 0.30, size = 180, normalized size = 0.95

$$-\frac{16\,b^2\sin\left(dx+c\right)^3+48\,ab\sin\left(dx+c\right)^2-3\left(15\,a^2-48\,ab+35\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(15\,a^2+48\,ab+35\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/48*(16*b^2*sin(d*x + c)^3 + 48*a*b*sin(d*x + c)^2 - 3*(15*a^2 - 48*a*b +
 35*b^2)*log(sin(d*x + c) + 1) + 3*(15*a^2 + 48*a*b + 35*b^2)*log(sin(d*x +
 c) - 1) + 48*(a^2 + 3*b^2)*sin(d*x + c) - 6*(24*a*b*sin(d*x + c)^2 + (9*a^
2 + 13*b^2)*sin(d*x + c)^3 - 20*a*b - (7*a^2 + 11*b^2)*sin(d*x + c))/(sin(d
*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 12.33, size = 433, normalized size = 2.29

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\left(\frac{15\,a^2}{8}-6\,a\,b+\frac{35\,b^2}{8}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\left(\frac{15\,a^2}{8}+6\,a\,b+\frac{35\,b^2}{8}\right)}{d}-\frac{\left(-\frac{15\,a^2}{4}-\frac{35\,b^2}{4}\right)\tan\left(\frac{c}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^6*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*((15*a^2)/8 - 6*a*b + (35*b^2)/8))/d - (log(ta
n(c/2 + (d*x)/2) - 1)*(6*a*b + (15*a^2)/8 + (35*b^2)/8))/d - (tan(c/2 + (d*
x)/2)^7*(11*a^2 - 17*b^2) + tan(c/2 + (d*x)/2)^3*((5*a^2)/2 + (35*b^2)/6) +
 tan(c/2 + (d*x)/2)^11*((5*a^2)/2 + (35*b^2)/6) - tan(c/2 + (d*x)/2)^13*((1
5*a^2)/4 + (35*b^2)/4) + tan(c/2 + (d*x)/2)^5*((47*a^2)/4 + (329*b^2)/12) +
 tan(c/2 + (d*x)/2)^9*((47*a^2)/4 + (329*b^2)/12) - tan(c/2 + (d*x)/2)*((15
*a^2)/4 + (35*b^2)/4) - 12*a*b*tan(c/2 + (d*x)/2)^2 + 12*a*b*tan(c/2 + (d*x
)/2)^4 + 32*a*b*tan(c/2 + (d*x)/2)^6 + 32*a*b*tan(c/2 + (d*x)/2)^8 + 12*a*b
*tan(c/2 + (d*x)/2)^10 - 12*a*b*tan(c/2 + (d*x)/2)^12)/(d*(tan(c/2 + (d*x)/
2)^2 + 3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^6 - 3*tan(c/2 + (d*x)/
2)^8 + 3*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 - tan(c/2 + (d*x)/2)
^14 - 1)) + (6*a*b*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1493 $\int (a+b\sin(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=162

$$-\frac{(4a^2+15ab+12b^2)\log(1-\sin(c+dx))}{8d}+\frac{(15ab-4(a^2+3b^2))\log(\sin(c+dx)+1)}{8d}-\frac{2ab\sin(c+dx)}{d}+\frac{\sec^4(c}{}$$

```
[Out] -1/8*(4*a^2+15*a*b+12*b^2)*ln(1-sin(d*x+c))/d+1/8*(-4*a^2+15*a*b-12*b^2)*ln
(1+sin(d*x+c))/d-2*a*b*sin(d*x+c)/d-1/2*b^2*sin(d*x+c)^2/d+1/4*sec(d*x+c)^4
*(a+b*sin(d*x+c))^2/d-1/4*sec(d*x+c)^2*(a+b*sin(d*x+c))*(4*a+5*b*sin(d*x+c)
)/d
```

Rubi [A] time = 0.27, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2721, 1645, 1810, 633, 31}

$$-\frac{(4a^2+15ab+12b^2)\log(1-\sin(c+dx))}{8d}+\frac{(15ab-4(a^2+3b^2))\log(\sin(c+dx)+1)}{8d}-\frac{2ab\sin(c+dx)}{d}+\frac{\sec^4(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -((4*a^2 + 15*a*b + 12*b^2)*Log[1 - Sin[c + d*x]])/(8*d) + ((15*a*b - 4*(a^
2 + 3*b^2))*Log[1 + Sin[c + d*x]])/(8*d) - (2*a*b*Sin[c + d*x])/d - (b^2*Si
n[c + d*x]^2)/(2*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2)/(4*d) - (Sec[
c + d*x]^2*(a + b*Sin[c + d*x])*(4*a + 5*b*Sin[c + d*x]))/(4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
```

```
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+x)^2}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} + \frac{\text{Subst}\left(\int \frac{(a+x)\left(-2b^6-4ab^4x-4b^4x^2-4ab^2x^3\right.}{\left(b^2-x^2\right)^2}\right.}{4b^2d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx))(4a+}{4d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx))(4a+}{4d}$$

$$= -\frac{2ab\sin(c+dx)}{d} - \frac{b^2\sin^2(c+dx)}{2d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}$$

$$= -\frac{2ab\sin(c+dx)}{d} - \frac{b^2\sin^2(c+dx)}{2d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}$$

$$= -\frac{\left(4a^2+15ab+12b^2\right)\log(1-\sin(c+dx))}{8d} - \frac{\left(4a^2-15ab+12b^2\right)\log(}{8d}$$

Mathematica [A] time = 2.15, size = 164, normalized size = 1.01

$$\frac{-2\left(4a^2+15ab+12b^2\right)\log(1-\sin(c+dx))-2\left(4a^2-15ab+12b^2\right)\log(\sin(c+dx)+1)+\frac{(a-b)^2}{(\sin(c+dx)+1)^2}-\frac{(7a-11b}{\sin(c+a}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (-2*(4*a^2 + 15*a*b + 12*b^2)*Log[1 - Sin[c + d*x]] - 2*(4*a^2 - 15*a*b + 1
2*b^2)*Log[1 + Sin[c + d*x]] + (a + b)^2/(-1 + Sin[c + d*x])^2 + ((a + b)*(
7*a + 11*b))/(-1 + Sin[c + d*x]) - 32*a*b*Sin[c + d*x] - 8*b^2*Sin[c + d*x]
^2 + (a - b)^2/(1 + Sin[c + d*x])^2 - ((7*a - 11*b)*(a - b))/(1 + Sin[c + d
*x]))/(16*d)
```

fricas [A] time = 0.47, size = 178, normalized size = 1.10

$$\frac{4\,b^2\cos(dx+c)^6-2\,b^2\cos(dx+c)^4-\left(4\,a^2-15\,ab+12\,b^2\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-\left(4\,a^2+15\,ab+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(4*b^2*cos(d*x + c)^6 - 2*b^2*cos(d*x + c)^4 - (4*a^2 - 15*a*b + 12*b^2
)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (4*a^2 + 15*a*b + 12*b^2)*cos(d*x
+ c)^4*log(-sin(d*x + c) + 1) - 4*(2*a^2 + 3*b^2)*cos(d*x + c)^2 + 2*a^2 +
2*b^2 - 2*(8*a*b*cos(d*x + c)^4 + 9*a*b*cos(d*x + c)^2 - 2*a*b)*sin(d*x + c
))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.31, size = 175, normalized size = 1.08

$$-\frac{4\,b^2\sin(dx+c)^2+16\,ab\sin(dx+c)+\left(4\,a^2-15\,ab+12\,b^2\right)\log(|\sin(dx+c)+1|)+\left(4\,a^2+15\,ab+12\,b^2\right)\log}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^2*sin(d*x + c)^2 + 16*a*b*sin(d*x + c) + (4*a^2 - 15*a*b + 12*b^2
)*log(abs(sin(d*x + c) + 1)) + (4*a^2 + 15*a*b + 12*b^2)*log(abs(sin(d*x +
c) - 1)) - 2*(3*a^2*sin(d*x + c)^4 + 9*b^2*sin(d*x + c)^4 + 9*a*b*sin(d*x +
 c)^3 - 2*a^2*sin(d*x + c)^2 - 12*b^2*sin(d*x + c)^2 - 7*a*b*sin(d*x + c) +
 4*b^2)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.32, size = 270, normalized size = 1.67

$$\frac{a^2\left(\tan^4(dx+c)\right)}{4d}-\frac{a^2\left(\tan^2(dx+c)\right)}{2d}-\frac{a^2\ln(\cos(dx+c))}{d}+\frac{ab\left(\sin^7(dx+c)\right)}{2d\cos(dx+c)^4}-\frac{3ab\left(\sin^7(dx+c)\right)}{4d\cos(dx+c)^2}-\frac{3ab\left(\sin^5(\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*tan(d*x+c)^4-1/2/d*a^2*tan(d*x+c)^2-1/d*a^2*ln(cos(d*x+c))+1/2/d*
a*b*sin(d*x+c)^7/cos(d*x+c)^4-3/4/d*a*b*sin(d*x+c)^7/cos(d*x+c)^2-3/4*a*b*s
in(d*x+c)^5/d-5/4*a*b*sin(d*x+c)^3/d-15/4*a*b*sin(d*x+c)/d+15/4/d*a*b*ln(se
c(d*x+c)+tan(d*x+c))+1/4/d*b^2*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*b^2*sin(d*x+
c)^8/cos(d*x+c)^2-1/2*b^2*sin(d*x+c)^6/d-3/4*b^2*sin(d*x+c)^4/d-3/2*b^2*sin
(d*x+c)^2/d-3/d*b^2*ln(cos(d*x+c))
```

maxima [A] time = 0.44, size = 157, normalized size = 0.97

$$-\frac{4\,b^2\sin(dx+c)^2+16\,ab\sin(dx+c)+\left(4\,a^2-15\,ab+12\,b^2\right)\log(\sin(dx+c)+1)+\left(4\,a^2+15\,ab+12\,b^2\right)\log}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*(4*b^2*sin(d*x + c)^2 + 16*a*b*sin(d*x + c) + (4*a^2 - 15*a*b + 12*b^2
)*log(sin(d*x + c) + 1) + (4*a^2 + 15*a*b + 12*b^2)*log(sin(d*x + c) - 1) -
 2*(9*a*b*sin(d*x + c)^3 - 7*a*b*sin(d*x + c) + 2*(2*a^2 + 3*b^2)*sin(d*x +
 c)^2 - 3*a^2 - 5*b^2)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 12.22, size = 377, normalized size = 2.33

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\left(a^2+3\,b^2\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\left(a^2-\frac{15\,a\,b}{4}+3\,b^2\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\left(a^2+\frac{15\,a\,b}{4}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^5*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 + 3*b^2))/d - (log(tan(c/2 + (d*x)/2) +
 1)*(a^2 - (15*a*b)/4 + 3*b^2))/d - (log(tan(c/2 + (d*x)/2) - 1)*((15*a*b)/
```

```
4 + a^2 + 3*b^2))/d - (tan(c/2 + (d*x)/2)^4*(4*a^2 + 12*b^2) - tan(c/2 + (d
*x)/2)^10*(2*a^2 + 6*b^2) - tan(c/2 + (d*x)/2)^2*(2*a^2 + 6*b^2) + tan(c/2
+ (d*x)/2)^6*(12*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^8*(4*a^2 + 12*b^2) + (25
*a*b*tan(c/2 + (d*x)/2)^3)/2 + 11*a*b*tan(c/2 + (d*x)/2)^5 + 11*a*b*tan(c/2
 + (d*x)/2)^7 + (25*a*b*tan(c/2 + (d*x)/2)^9)/2 - (15*a*b*tan(c/2 + (d*x)/2
)^11)/2 - (15*a*b*tan(c/2 + (d*x)/2))/2)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c
/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 2*tan(c/2
 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^12 - 1))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1494 $\int \sec(c+dx)(a+b\sin(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=150

$$-\frac{(3a^2+16ab+15b^2)\log(1-\sin(c+dx))}{16d}+\frac{(3a^2-16ab+15b^2)\log(\sin(c+dx)+1)}{16d}-\frac{\sec^2(c+dx)(5a\sin(c+$$

[Out] -1/16*(3*a^2+16*a*b+15*b^2)*ln(1-sin(d*x+c))/d+1/16*(3*a^2-16*a*b+15*b^2)*ln(1+sin(d*x+c))/d-b^2*sin(d*x+c)/d-1/8*sec(d*x+c)^2*(7*b+5*a*sin(d*x+c))*(a+b*sin(d*x+c))/d+1/4*sec(d*x+c)^3*(a+b*sin(d*x+c))^2*tan(d*x+c)/d

Rubi [A] time = 0.28, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2837, 12, 1645, 1810, 633, 31}

$$-\frac{(3a^2+16ab+15b^2)\log(1-\sin(c+dx))}{16d}+\frac{(3a^2-16ab+15b^2)\log(\sin(c+dx)+1)}{16d}-\frac{\sec^2(c+dx)(5a\sin(c+$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]

[Out] -((3*a^2 + 16*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(16*d) + ((3*a^2 - 16*a*b + 15*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - (b^2*Sin[c + d*x])/d - (Sec[c + d*x]^2*(7*b + 5*a*Sin[c + d*x])*(a + b*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*Tan[c + d*x])/(4*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^2\tan^4(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^4(a+x)^2}{b^4\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{x^4(a+x)^2}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{(a\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{\sec^2(c+dx)(7b+5a\sin(c+dx))(a+b\sin(c+dx))}{8d} + \frac{\text{se}\cdots}{\cdots}$$

$$= -\frac{\sec^2(c+dx)(7b+5a\sin(c+dx))(a+b\sin(c+dx))}{8d} + \frac{\text{se}\cdots}{\cdots}$$

$$= -\frac{b^2\sin(c+dx)}{d} - \frac{\sec^2(c+dx)(7b+5a\sin(c+dx))(a+b\,\text{s}\cdots}{8d}$$

$$= -\frac{b^2\sin(c+dx)}{d} - \frac{\sec^2(c+dx)(7b+5a\sin(c+dx))(a+b\,\text{s}\cdots}{8d}$$

$$= -\frac{\left(3a^2+16ab+15b^2\right)\log(1-\sin(c+dx))}{16d} - \frac{\left(16ab-3\left(a^2\cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 1.02, size = 151, normalized size = 1.01

$$\frac{-\left(3a^2+16ab+15b^2\right)\log(1-\sin(c+dx)) + \left(3a^2-16ab+15b^2\right)\log(\sin(c+dx)+1) - \frac{(a-b)^2}{(\sin(c+dx)+1)^2} + \frac{(5a-9b)(a\cdots}{\sin(c+dx)\cdots}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (-((3*a^2 + 16*a*b + 15*b^2)*Log[1 - Sin[c + d*x]]) + (3*a^2 - 16*a*b + 15*
b^2)*Log[1 + Sin[c + d*x]] + (a + b)^2/(-1 + Sin[c + d*x])^2 + ((a + b)*(5*
a + 9*b))/(-1 + Sin[c + d*x]) - 16*b^2*Sin[c + d*x] - (a - b)^2/(1 + Sin[c
+ d*x])^2 + ((5*a - 9*b)*(a - b))/(1 + Sin[c + d*x]))/(16*d)
```

fricas [A] time = 0.47, size = 151, normalized size = 1.01

$$\frac{\left(3\,a^2-16\,ab+15\,b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(3\,a^2+16\,ab+15\,b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)\right.}{16\,d\cos\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/16*((3*a^2 - 16*a*b + 15*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*a
^2 + 16*a*b + 15*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 32*a*b*cos(d*
x + c)^2 + 8*a*b - 2*(8*b^2*cos(d*x + c)^4 + (5*a^2 + 9*b^2)*cos(d*x + c)^2
 - 2*a^2 - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.29, size = 157, normalized size = 1.05

$$-\frac{16\,b^2\sin\left(dx+c\right)-\left(3\,a^2-16\,ab+15\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+\left(3\,a^2+16\,ab+15\,b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(16*b^2*sin(d*x + c) - (3*a^2 - 16*a*b + 15*b^2)*log(abs(sin(d*x + c)
 + 1)) + (3*a^2 + 16*a*b + 15*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(12*a*b*s
in(d*x + c)^4 + 5*a^2*sin(d*x + c)^3 + 9*b^2*sin(d*x + c)^3 - 8*a*b*sin(d*x
 + c)^2 - 3*a^2*sin(d*x + c) - 7*b^2*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/
d
```

maple [A] time = 0.32, size = 262, normalized size = 1.75

$$\frac{a^2\left(\sin^5\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4}-\frac{a^2\left(\sin^5\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^2}-\frac{a^2\left(\sin^3\left(dx+c\right)\right)}{8d}-\frac{3a^2\sin\left(dx+c\right)}{8d}+\frac{3a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a^2*sin(d*x+c)^5/cos(d*x+c)^2-1/8
*a^2*sin(d*x+c)^3/d-3/8*a^2*sin(d*x+c)/d+3/8/d*a^2*ln(sec(d*x+c)+tan(d*x+c)
)+1/2/d*a*b*tan(d*x+c)^4-1/d*a*b*tan(d*x+c)^2-2/d*a*b*ln(cos(d*x+c))+1/4/d*
b^2*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*b^2*sin(d*x+c)^7/cos(d*x+c)^2-3/8*b^2*s
in(d*x+c)^5/d-5/8*b^2*sin(d*x+c)^3/d-15/8*b^2*sin(d*x+c)/d+15/8/d*b^2*ln(se
c(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.33, size = 148, normalized size = 0.99

$$-\frac{16\,b^2\sin\left(dx+c\right)-\left(3\,a^2-16\,ab+15\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(3\,a^2+16\,ab+15\,b^2\right)\log\left(\sin\left(dx+c\right)-1\right)-}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/16*(16*b^2*sin(d*x + c) - (3*a^2 - 16*a*b + 15*b^2)*log(sin(d*x + c) + 1
) + (3*a^2 + 16*a*b + 15*b^2)*log(sin(d*x + c) - 1) - 2*(16*a*b*sin(d*x + c
)^2 + (5*a^2 + 9*b^2)*sin(d*x + c)^3 - 12*a*b - (3*a^2 + 7*b^2)*sin(d*x + c
))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 12.21, size = 332, normalized size = 2.21

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{3\,a^2}{8}-2\,a\,b+\frac{15\,b^2}{8}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{3\,a^2}{8}+2\,a\,b+\frac{15\,b^2}{8}\right)}{d}+\frac{\left(-\frac{3\,a^2}{4}-\frac{15\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*((3*a^2)/8 - 2*a*b + (15*b^2)/8))/d - (log(tan
(c/2 + (d*x)/2) - 1)*(2*a*b + (3*a^2)/8 + (15*b^2)/8))/d + (tan(c/2 + (d*x)
/2)^3*(2*a^2 + 10*b^2) + tan(c/2 + (d*x)/2)^7*(2*a^2 + 10*b^2) + tan(c/2 +
(d*x)/2)^5*((11*a^2)/2 - (9*b^2)/2) - tan(c/2 + (d*x)/2)^9*((3*a^2)/4 + (15
*b^2)/4) - tan(c/2 + (d*x)/2)*((3*a^2)/4 + (15*b^2)/4) - 4*a*b*tan(c/2 + (d
*x)/2)^2 + 12*a*b*tan(c/2 + (d*x)/2)^4 + 12*a*b*tan(c/2 + (d*x)/2)^6 - 4*a*
b*tan(c/2 + (d*x)/2)^8)/(d*(2*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^2
 + 2*tan(c/2 + (d*x)/2)^6 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10
+ 1)) + (2*a*b*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1495 $\int \sec^2(c+dx)(a+b\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=116

$$-\frac{b(3a+4b)\log(1-\sin(c+dx))}{8d}+\frac{b(3a-4b)\log(\sin(c+dx)+1)}{8d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}-\frac{\sec^2(c+dx}{}$$

```
[Out] -1/8*b*(3*a+4*b)*ln(1-sin(d*x+c))/d+1/8*(3*a-4*b)*b*ln(1+sin(d*x+c))/d+1/4*
sec(d*x+c)^4*(a+b*sin(d*x+c))^2/d-1/4*sec(d*x+c)^2*(a+b*sin(d*x+c))*(2*a+3*
b*sin(d*x+c))/d
```

Rubi [A] time = 0.22, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2837, 12, 1645, 633, 31}

$$-\frac{b(3a+4b)\log(1-\sin(c+dx))}{8d}+\frac{b(3a-4b)\log(\sin(c+dx)+1)}{8d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}-\frac{\sec^2(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] -(b*(3*a + 4*b)*Log[1 - Sin[c + d*x]])/(8*d) + ((3*a - 4*b)*b*Log[1 + Sin[c
 + d*x]])/(8*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2)/(4*d) - (Sec[c +
d*x]^2*(a + b*Sin[c + d*x])*(2*a + 3*b*Sin[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^2\tan^3(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^3(a+x)^2}{b^3\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\,\text{Subst}\left(\int \frac{x^3(a+x)^2}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} + \frac{\text{Subst}\left(\int \frac{(a+x)\left(-2b^4-4ab^2\right.}{\left(b^2-x^2\right)}\right.}{}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)(a+b\sin(}{}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)(a+b\sin(}{}$$

$$= -\frac{b(3a+4b)\log(1-\sin(c+dx))}{8d} + \frac{(3a-4b)b\log(1+\sin}{8d}$$

Mathematica [A] time = 0.38, size = 129, normalized size = 1.11

$$\frac{a^2\tan^4(c+dx)}{4d}+\frac{2ab\tan^3(c+dx)\sec(c+dx)}{d}-\frac{ab\left(6\tan(c+dx)\sec^3(c+dx)-3\left(\tanh^{-1}(\sin(c+dx))+\tan(c+\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (2*a*b*Sec[c + d*x]*Tan[c + d*x]^3)/d + (a^2*Tan[c + d*x]^4)/(4*d) - (b^2*(
4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d) - (a*b*(6*S
ec[c + d*x]^3*Tan[c + d*x] - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c
+ d*x])))/(4*d)
```

fricas [A] time = 0.46, size = 127, normalized size = 1.09

$$\frac{\left(3\,ab-4\,b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(3\,ab+4\,b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-4\left(a^2+2\,b^2\right.}{8\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*((3*a*b - 4*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*a*b + 4*b^2)
*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 4*(a^2 + 2*b^2)*cos(d*x + c)^2 + 2
*a^2 + 2*b^2 - 2*(5*a*b*cos(d*x + c)^2 - 2*a*b)*sin(d*x + c))/(d*cos(d*x +
c)^4)
```

giac [A] time = 0.28, size = 130, normalized size = 1.12

$$\frac{\left(3\,ab-4\,b^2\right)\log\left(|\sin\left(dx+c\right)+1|\right)-\left(3\,ab+4\,b^2\right)\log\left(|\sin\left(dx+c\right)-1|\right)+\frac{2\left(3\,b^2\sin(dx+c)^4+5\,ab\sin(dx+c)^3+2\,a^2\sin(dx+c\right.}{\left(\sin(dx+c)^2-\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*((3*a*b - 4*b^2)*log(abs(sin(d*x + c) + 1)) - (3*a*b + 4*b^2)*log(abs(s
in(d*x + c) - 1)) + 2*(3*b^2*sin(d*x + c)^4 + 5*a*b*sin(d*x + c)^3 + 2*a^2*
sin(d*x + c)^2 - 2*b^2*sin(d*x + c)^2 - 3*a*b*sin(d*x + c) - a^2)/(sin(d*x
+ c)^2 - 1)^2)/d
```

maple [A] time = 0.30, size = 168, normalized size = 1.45

$$\frac{a^2\left(\sin^4\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4}+\frac{ab\left(\sin^5\left(dx+c\right)\right)}{2d\cos\left(dx+c\right)^4}-\frac{ab\left(\sin^5\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^2}-\frac{ab\left(\sin^3\left(dx+c\right)\right)}{4d}-\frac{3ab\sin\left(dx+c\right)}{4d}+\frac{3ab\ln\left(\sec\left(dx+\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^4/cos(d*x+c)^4+1/2/d*a*b*sin(d*x+c)^5/cos(d*x+c)^4-1/4
/d*a*b*sin(d*x+c)^5/cos(d*x+c)^2-1/4*a*b*sin(d*x+c)^3/d-3/4*a*b*sin(d*x+c)/
d+3/4/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*b^2*tan(d*x+c)^4-1/2/d*b^2*tan(
d*x+c)^2-1/d*b^2*ln(cos(d*x+c))
```

maxima [A] time = 0.34, size = 123, normalized size = 1.06

$$\frac{\left(3\,ab-4\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(3\,ab+4\,b^2\right)\log\left(\sin\left(dx+c\right)-1\right)+\frac{2\left(5\,ab\sin(dx+c)^3-3\,ab\sin(dx+c)+2\left(a^2+2\,b^2\right)\sin(}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*((3*a*b - 4*b^2)*log(sin(d*x + c) + 1) - (3*a*b + 4*b^2)*log(sin(d*x +
c) - 1) + 2*(5*a*b*sin(d*x + c)^3 - 3*a*b*sin(d*x + c) + 2*(a^2 + 2*b^2)*si
n(d*x + c)^2 - a^2 - 3*b^2)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 12.09, size = 247, normalized size = 2.13

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{3\,a\,b}{4}-b^2\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(b^2+\frac{3\,a\,b}{4}\right)}{d}+\frac{b^2\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{2\,b^2\,\tan\left(\frac{c}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*((3*a*b)/4 - b^2))/d - (log(tan(c/2 + (d*x)/2)
 - 1)*((3*a*b)/4 + b^2))/d + (b^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (2*b^2
*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4*(4*a^2 + 8*b^2) + 2*b^2*tan(c/
2 + (d*x)/2)^6 - (11*a*b*tan(c/2 + (d*x)/2)^3)/2 - (11*a*b*tan(c/2 + (d*x)/
2)^5)/2 + (3*a*b*tan(c/2 + (d*x)/2)^7)/2 + (3*a*b*tan(c/2 + (d*x)/2))/2)/(d
*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6
+ tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1496 $\int \sec^3(c+dx)(a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=93

$$-\frac{(a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{\sec^2(c+dx)\left((a^2+3b^2)\sin(c+dx)+4ab\right)}{8d}+\frac{\tan(c+dx)\sec^3(c+dx)(a+b\sin(c+dx))^2}{4d}$$

```
[Out] -1/8*(a^2-3*b^2)*arctanh(sin(d*x+c))/d-1/8*sec(d*x+c)^2*(4*a*b+(a^2+3*b^2)*
sin(d*x+c))/d+1/4*sec(d*x+c)^3*(a+b*sin(d*x+c))^2*tan(d*x+c)/d
```

Rubi [A] time = 0.19, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2837, 12, 1645, 778, 206}

$$-\frac{(a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{\sec^2(c+dx)\left((a^2+3b^2)\sin(c+dx)+4ab\right)}{8d}+\frac{\tan(c+dx)\sec^3(c+dx)(a+b\sin(c+dx))^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -((a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) - (Sec[c + d*x]^2*(4*a*b + (a^
2 + 3*b^2)*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*Ta
n[c + d*x])/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^2(a+x)^2}{b^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^3\,\text{Subst}\left(\int \frac{x^2(a+x)^2}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)}{4d} + \frac{b\,\text{Subst}\left(\int \ldots\right.}{}$$

$$= -\frac{\sec^2(c+dx)\left(4ab+\left(a^2+3b^2\right)\sin(c+dx)\right)}{8d} + \frac{\sec^3(c+dx\ldots}{}$$

$$= -\frac{\left(a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{\sec^2(c+dx)\left(4ab+\left(a^2\ldots\right.\right.}{8d}$$

Mathematica [A] time = 0.77, size = 85, normalized size = 0.91

$$-\frac{\left(a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))+\frac{1}{4}\sec^4(c+dx)\left(2\sin(c+dx)\left(\left(a^2+5b^2\right)\cos(2(c+dx))-3a^2+b^2\right)+16ab\cos\ldots\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -1/8*((a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]] + (Sec[c + d*x]^4*(16*a*b*Cos[2*(
c + d*x)] + 2*(-3*a^2 + b^2 + (a^2 + 5*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])
)/4)/d
```

fricas [A] time = 0.44, size = 124, normalized size = 1.33

$$-\frac{\left(a^2-3\,b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2-3\,b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+16\,ab\cos\left(d\right.}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/16*((a^2 - 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (a^2 - 3*b^2)*c
os(d*x + c)^4*log(-sin(d*x + c) + 1) + 16*a*b*cos(d*x + c)^2 - 8*a*b + 2*((
a^2 + 5*b^2)*cos(d*x + c)^2 - 2*a^2 - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^
4)
```

giac [A] time = 0.26, size = 124, normalized size = 1.33

$$-\frac{\left(a^2-3\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(a^2-3\,b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(a^2\sin(dx+c)^3+5\,b^2\sin(dx+c)^3+8\,ab\sin(dx+c)^2+\right.}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*((a^2 - 3*b^2)*log(abs(sin(d*x + c) + 1)) - (a^2 - 3*b^2)*log(abs(sin
(d*x + c) - 1)) - 2*(a^2*sin(d*x + c)^3 + 5*b^2*sin(d*x + c)^3 + 8*a*b*sin(
d*x + c)^2 + a^2*sin(d*x + c) - 3*b^2*sin(d*x + c) - 4*a*b)/(sin(d*x + c)^2
 - 1)^2)/d
```

maple [B] time = 0.30, size = 209, normalized size = 2.25

$$\frac{a^2\left(\sin^3\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4}+\frac{a^2\left(\sin^3\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^2}+\frac{a^2\sin\left(dx+c\right)}{8d}-\frac{a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{ab\left(\sin^4\left(dx+c\right)\right)}{2d\cos\left(dx+c\right)^4}+\frac{b}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/8
*a^2*sin(d*x+c)/d-1/8/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*b*sin(d*x+c)^
4/cos(d*x+c)^4+1/4/d*b^2*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*b^2*sin(d*x+c)^5/c
os(d*x+c)^2-1/8*b^2*sin(d*x+c)^3/d-3/8*b^2*sin(d*x+c)/d+3/8/d*b^2*ln(sec(d*
x+c)+tan(d*x+c))
```

maxima [A] time = 0.31, size = 120, normalized size = 1.29

$$-\frac{\left(a^2-3\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2-3\,b^2\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(8\,ab\sin(dx+c)^2+\left(a^2+5\,b^2\right)\sin(dx+c)^3-4\,ab+\left(a^2-3\,b^2\right.\right.}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/16*((a^2 - 3*b^2)*log(sin(d*x + c) + 1) - (a^2 - 3*b^2)*log(sin(d*x + c)
 - 1) - 2*(8*a*b*sin(d*x + c)^2 + (a^2 + 5*b^2)*sin(d*x + c)^3 - 4*a*b + (a
^2 - 3*b^2)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 17.05, size = 191, normalized size = 2.05

$$\frac{\left(\frac{a^2}{4}-\frac{3\,b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{7\,a^2}{4}+\frac{11\,b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(\frac{7\,a^2}{4}+\frac{11\,b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{a^2}{4}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^7*(a^2/4 - (3*b^2)/4) + tan(c/2 + (d*x)/2)^3*((7*a^2)/4
 + (11*b^2)/4) + tan(c/2 + (d*x)/2)^5*((7*a^2)/4 + (11*b^2)/4) + tan(c/2 +
(d*x)/2)*(a^2/4 - (3*b^2)/4) + 8*a*b*tan(c/2 + (d*x)/2)^4)/(d*(6*tan(c/2 +
(d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d
*x)/2)^8 + 1)) - (atanh(tan(c/2 + (d*x)/2))*(a^2/4 - (3*b^2)/4))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1497 $\int \sec^4(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=72

$$-\frac{\sec^2(c+dx)\left(ab\sin(c+dx)+b^2\right)}{4d}-\frac{ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}$$

```
[Out] -1/4*a*b*arctanh(sin(d*x+c))/d+1/4*sec(d*x+c)^4*(a+b*sin(d*x+c))^2/d-1/4*se
c(d*x+c)^2*(b^2+a*b*sin(d*x+c))/d
```

Rubi [A] time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2837, 12, 821, 639, 206}

$$-\frac{\sec^2(c+dx)\left(ab\sin(c+dx)+b^2\right)}{4d}-\frac{ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -(a*b*ArcTanh[Sin[c + d*x]])/(4*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2
)/(4*d) - (Sec[c + d*x]^2*(b^2 + a*b*Sin[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
```

```
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x(a+x)^2}{b\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\,\text{Subst}\left(\int \frac{x(a+x)^2}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{b^2\,\text{Subst}\left(\int \frac{2b^2(a+x)}{\left(b^2-x^2\right)^2}\,dx, x,\right.}{4d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{b^4\,\text{Subst}\left(\int \frac{a+x}{\left(b^2-x^2\right)^2}\,dx, x,\right.}{2d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)\left(b^2+ab\sin(\right.}{4d}$$

$$= -\frac{ab\tanh^{-1}(\sin(c+dx))}{4d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}$$

Mathematica [B] time = 2.94, size = 215, normalized size = 2.99

$$\frac{2a^4b^2\sec^2(c+dx) + ab\left(a^2-b^2\right)^2(\log(1-\sin(c+dx)) - \log(\sin(c+dx)+1)) - 2ab\left(a^2-b^2\right)\tan(c+dx)\sec(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] (a*b*(a^2 - b^2)^2*(Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + 2*a^4*
b^2*Sec[c + d*x]^2 + 2*a^4*(a^2 - b^2)*Sec[c + d*x]^4 + 4*a^3*b*(a^2 - b^2)
*Sec[c + d*x]^3*Tan[c + d*x] + b*(-6*a^4*b + 4*a^2*b^3)*Tan[c + d*x]^2 + 2*
b^4*(-a^2 + b^2)*Tan[c + d*x]^4 - 2*a*b*(a^2 - b^2)*Sec[c + d*x]*Tan[c + d*
x]*(a^2 + b^2 + 2*b^2*Tan[c + d*x]^2))/(8*(a^2 - b^2)^2*d)
```

fricas [A] time = 0.44, size = 104, normalized size = 1.44

$$-\frac{ab\cos(dx+c)^4\log(\sin(dx+c)+1) - ab\cos(dx+c)^4\log(-\sin(dx+c)+1) + 4b^2\cos(dx+c)^2 - 2a^2 - 2}{8d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(a*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - a*b*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 4*b^2*cos(d*x + c)^2 - 2*a^2 - 2*b^2 + 2*(a*b*cos(d*x + c
)^2 - 2*a*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.24, size = 89, normalized size = 1.24

$$-\frac{ab\log(|\sin(dx+c)+1|) - ab\log(|\sin(dx+c)-1|) - \frac{2\left(ab\sin(dx+c)^3 + 2b^2\sin(dx+c)^2 + ab\sin(dx+c) + a^2 - b^2\right)}{\left(\sin(dx+c)^2-1\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(a*b*log(abs(sin(d*x + c) + 1)) - a*b*log(abs(sin(d*x + c) - 1)) - 2*(
a*b*sin(d*x + c)^3 + 2*b^2*sin(d*x + c)^2 + a*b*sin(d*x + c) + a^2 - b^2)/(
sin(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.26, size = 122, normalized size = 1.69

$$\frac{a^2}{4d\cos(dx+c)^4} + \frac{ab\left(\sin^3(dx+c)\right)}{2d\cos(dx+c)^4} + \frac{ab\left(\sin^3(dx+c)\right)}{4d\cos(dx+c)^2} + \frac{ab\sin(dx+c)}{4d} - \frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{4d} + \frac{b^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2/cos(d*x+c)^4+1/2/d*a*b*sin(d*x+c)^3/cos(d*x+c)^4+1/4/d*a*b*sin(d*
x+c)^3/cos(d*x+c)^2+1/4*a*b*sin(d*x+c)/d-1/4/d*a*b*ln(sec(d*x+c)+tan(d*x+c)
)+1/4/d*b^2*sin(d*x+c)^4/cos(d*x+c)^4
```

maxima [A] time = 0.39, size = 97, normalized size = 1.35

$$-\frac{ab\log\left(\sin\left(dx+c\right)+1\right)-ab\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(ab\sin(dx+c)^3+2\,b^2\sin(dx+c)^2+ab\sin(dx+c)+a^2-b^2\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a*b*log(sin(d*x + c) + 1) - a*b*log(sin(d*x + c) - 1) - 2*(a*b*sin(d*
x + c)^3 + 2*b^2*sin(d*x + c)^2 + a*b*sin(d*x + c) + a^2 - b^2)/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 18.87, size = 183, normalized size = 2.54

$$\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+\frac{7\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2}+\frac{7\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+4\,b^2\tan}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (2*a^2*tan(c/2 + (d*x)/2)^2 + 2*a^2*tan(c/2 + (d*x)/2)^6 + 4*b^2*tan(c/2 +
(d*x)/2)^4 + (7*a*b*tan(c/2 + (d*x)/2)^3)/2 + (7*a*b*tan(c/2 + (d*x)/2)^5)/
2 + (a*b*tan(c/2 + (d*x)/2)^7)/2 + (a*b*tan(c/2 + (d*x)/2))/2)/(d*(6*tan(c/
2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2
+ (d*x)/2)^8 + 1)) - (a*b*atanh(tan(c/2 + (d*x)/2)))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1498 $\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$\frac{\sec^4(c+dx)\left(a^2+2ab\sin(c+dx)+b^2\right)}{4d}+\frac{a^2\log(\sin(c+dx))}{d}-\frac{a(4a+3b)\log(1-\sin(c+dx))}{8d}-\frac{a(4a-3b)\log(\ldots}{8\ldots}$$

```
[Out] -1/8*a*(4*a+3*b)*ln(1-sin(d*x+c))/d+a^2*ln(sin(d*x+c))/d-1/8*a*(4*a-3*b)*ln
(1+sin(d*x+c))/d+1/4*a*sec(d*x+c)^2*(2*a+3*b*sin(d*x+c))/d+1/4*sec(d*x+c)^4
*(a^2+b^2+2*a*b*sin(d*x+c))/d
```

Rubi [A] time = 0.22, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2837, 12, 1805, 823, 801}

$$\frac{\sec^4(c+dx)\left(a^2+2ab\sin(c+dx)+b^2\right)}{4d}+\frac{a^2\log(\sin(c+dx))}{d}-\frac{a(4a+3b)\log(1-\sin(c+dx))}{8d}-\frac{a(4a-3b)\log(\ldots}{8\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a*(4*a + 3*b)*Log[1 - Sin[c + d*x]])/(8*d) + (a^2*Log[Sin[c + d*x]])/d -
(a*(4*a - 3*b)*Log[1 + Sin[c + d*x]])/(8*d) + (a*Sec[c + d*x]^2*(2*a + 3*b*
Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^4*(a^2 + b^2 + 2*a*b*Sin[c + d*x]))/(4
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b(a+x)^2}{x\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^6\,\text{Subst}\left(\int \frac{(a+x)^2}{x\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} - \frac{b^4\,\text{Subst}\left(\int \frac{-4a^2-}{x(b^2-x}\right.}{}$$

$$= \frac{a\sec^2(c+dx)(2a+3b\sin(c+dx))}{4d} + \frac{\sec^4(c+dx)\left(a^2+b^2+\right.}{4d}$$

$$= \frac{a\sec^2(c+dx)(2a+3b\sin(c+dx))}{4d} + \frac{\sec^4(c+dx)\left(a^2+b^2+\right.}{4d}$$

$$= -\frac{a(4a+3b)\log(1-\sin(c+dx))}{8d} + \frac{a^2\log(\sin(c+dx))}{d} - \frac{a(4}{}$$

Mathematica [A] time = 0.92, size = 137, normalized size = 1.09

$$\frac{16a^2\log(\sin(c+dx)) - \frac{(a+b)(5a+b)}{\sin(c+dx)-1} + \frac{(a-b)(5a-b)}{\sin(c+dx)+1} + \frac{(a+b)^2}{(\sin(c+dx)-1)^2} + \frac{(a-b)^2}{(\sin(c+dx)+1)^2} - 2a(4a+3b)\log(1-\sin(c+dx))}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*(4*a + 3*b)*Log[1 - Sin[c + d*x]] + 16*a^2*Log[Sin[c + d*x]] - 2*a*(4
*a - 3*b)*Log[1 + Sin[c + d*x]] + (a + b)^2/(-1 + Sin[c + d*x])^2 - ((a + b
)*(5*a + b))/(-1 + Sin[c + d*x]) + (a - b)^2/(1 + Sin[c + d*x])^2 + ((a - b
)*(5*a - b))/(1 + Sin[c + d*x]))/(16*d)
```

fricas [A] time = 0.46, size = 144, normalized size = 1.14

$$\frac{8\,a^2\cos(dx+c)^4\log\left(\frac{1}{2}\sin(dx+c)\right) - \left(4\,a^2-3\,ab\right)\cos(dx+c)^4\log(\sin(dx+c)+1) - \left(4\,a^2+3\,ab\right)\cos(dx}{8\,d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^2*cos(d*x + c)^4*log(1/2*sin(d*x + c)) - (4*a^2 - 3*a*b)*cos(d*x +
 c)^4*log(sin(d*x + c) + 1) - (4*a^2 + 3*a*b)*cos(d*x + c)^4*log(-sin(d*x +
 c) + 1) + 4*a^2*cos(d*x + c)^2 + 2*a^2 + 2*b^2 + 2*(3*a*b*cos(d*x + c)^2 +
 2*a*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.25, size = 134, normalized size = 1.06

$$\frac{8\,a^2\log(|\sin(dx+c)|) - \left(4\,a^2-3\,ab\right)\log(|\sin(dx+c)+1|) - \left(4\,a^2+3\,ab\right)\log(|\sin(dx+c)-1|) + \frac{2\left(3\,a^2\sin(dx\right.}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2*log(abs(sin(d*x + c))) - (4*a^2 - 3*a*b)*log(abs(sin(d*x + c) +
1)) - (4*a^2 + 3*a*b)*log(abs(sin(d*x + c) - 1)) + 2*(3*a^2*sin(d*x + c)^4
- 3*a*b*sin(d*x + c)^3 - 8*a^2*sin(d*x + c)^2 + 5*a*b*sin(d*x + c) + 6*a^2
+ b^2)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.61, size = 125, normalized size = 0.99

$$\frac{a^2}{4d\cos(dx+c)^4} + \frac{a^2}{2d\cos(dx+c)^2} + \frac{a^2\ln(\tan(dx+c))}{d} + \frac{ab\tan(dx+c)\left(\sec^3(dx+c)\right)}{2d} + \frac{3ab\tan(dx+c)\sec(d}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2/cos(d*x+c)^4+1/2/d*a^2/cos(d*x+c)^2+1/d*a^2*ln(tan(d*x+c))+1/2/d*
a*b*tan(d*x+c)*sec(d*x+c)^3+3/4/d*a*b*tan(d*x+c)*sec(d*x+c)+3/4/d*a*b*ln(se
c(d*x+c)+tan(d*x+c))+1/4/d*b^2/cos(d*x+c)^4
```

maxima [A] time = 0.33, size = 130, normalized size = 1.03

$$\frac{8\,a^2\log(\sin(dx+c)) - \left(4\,a^2-3\,ab\right)\log(\sin(dx+c)+1) - \left(4\,a^2+3\,ab\right)\log(\sin(dx+c)-1) - \frac{2\left(3\,ab\sin(dx+c)^3+2\cdots\right.}{\sin(\cdots}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*a^2*log(sin(d*x + c)) - (4*a^2 - 3*a*b)*log(sin(d*x + c) + 1) - (4*a
^2 + 3*a*b)*log(sin(d*x + c) - 1) - 2*(3*a*b*sin(d*x + c)^3 + 2*a^2*sin(d*x
 + c)^2 - 5*a*b*sin(d*x + c) - 3*a^2 - b^2)/(sin(d*x + c)^4 - 2*sin(d*x + c
)^2 + 1))/d
```

mupad [B] time = 0.13, size = 131, normalized size = 1.04

$$\frac{a^2\,\ln(\sin(c+d\,x))}{d}+\frac{-\frac{a^2\sin(c+d\,x)^2}{2}+\frac{3\,a^2}{4}-\frac{3\,a\,b\sin(c+d\,x)^3}{4}+\frac{5\,a\,b\,\sin(c+d\,x)}{4}+\frac{b^2}{4}}{d\,\left(\sin(c+d\,x)^4-2\sin(c+d\,x)^2+1\right)}-\frac{a\,\ln(\sin(c+d\,x)-1)\,(4\,a+3\,b)}{8\,d}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(cos(c + d*x)^5*sin(c + d*x)),x)
```

```
[Out] (a^2*log(sin(c + d*x)))/d + ((3*a^2)/4 + b^2/4 - (a^2*sin(c + d*x)^2)/2 + (
5*a*b*sin(c + d*x))/4 - (3*a*b*sin(c + d*x)^3)/4)/(d*(sin(c + d*x)^4 - 2*si
n(c + d*x)^2 + 1)) - (a*log(sin(c + d*x) - 1)*(4*a + 3*b))/(8*d) - (a*log(s
in(c + d*x) + 1)*(4*a - 3*b))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1499 $\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=168

$$-\frac{\left(15a^2+16ab+3b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{\left(15a^2-16ab+3b^2\right)\log(\sin(c+dx)+1)}{16d}+\frac{b\sec^4(c+dx)\left(\frac{\left(a^2+b^2\right)\sin}{b}\right.}{4d}$$

```
[Out] -a^2*csc(d*x+c)/d-1/16*(15*a^2+16*a*b+3*b^2)*ln(1-sin(d*x+c))/d+2*a*b*ln(si
n(d*x+c))/d+1/16*(15*a^2-16*a*b+3*b^2)*ln(1+sin(d*x+c))/d+1/8*b*sec(d*x+c)^
2*(8*a+(3+7/b^2*a^2)*b*sin(d*x+c))/d+1/4*b*sec(d*x+c)^4*(2*a+(a^2+b^2)*sin(
d*x+c)/b)/d
```

Rubi [A] time = 0.35, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 12, 1805, 1802}

$$-\frac{\left(15a^2+16ab+3b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{\left(15a^2-16ab+3b^2\right)\log(\sin(c+dx)+1)}{16d}+\frac{b\sec^4(c+dx)\left(\frac{\left(a^2+b^2\right)\sin}{b}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) - ((15*a^2 + 16*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])
/(16*d) + (2*a*b*Log[Sin[c + d*x]])/d + ((15*a^2 - 16*a*b + 3*b^2)*Log[1 +
Sin[c + d*x]])/(16*d) + (b*Sec[c + d*x]^2*(8*a + (3 + (7*a^2)/b^2)*b*Sin[c
+ d*x]))/(8*d) + (b*Sec[c + d*x]^4*(2*a + ((a^2 + b^2)*Sin[c + d*x])/b))/(4
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
```

```
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b^2(a+x)^2}{x^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^7\,\text{Subst}\left(\int \frac{(a+x)^2}{x^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\sec^4(c+dx)\left(2a+\frac{\left(a^2+b^2\right)\sin(c+dx)}{b}\right)}{4d} - \frac{b^5\,\text{Subst}\left(\int \frac{-4a^2-8ax\cdots}{x^2\left(b^2\cdots\right.}\right.}{\cdots}$$

$$= \frac{b\sec^2(c+dx)\left(8a+\left(3+\frac{7a^2}{b^2}\right)b\sin(c+dx)\right)}{8d} + \frac{b\sec^4(c+d\cdots}{\cdots}$$

$$= \frac{b\sec^2(c+dx)\left(8a+\left(3+\frac{7a^2}{b^2}\right)b\sin(c+dx)\right)}{8d} + \frac{b\sec^4(c+d\cdots}{\cdots}$$

$$= -\frac{a^2\csc(c+dx)}{d} - \frac{\left(15a^2+16ab+3b^2\right)\log(1-\sin(c+dx))}{16d}\cdots$$

Mathematica [A] time = 2.83, size = 162, normalized size = 0.96

$$-\frac{\left(15a^2+16ab+3b^2\right)\log(1-\sin(c+dx))-\left(15a^2-16ab+3b^2\right)\log(\sin(c+dx)+1)+16a^2\csc(c+dx)+\frac{(a+b}{\sin(}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/16*(16*a^2*Csc[c + d*x] + (15*a^2 + 16*a*b + 3*b^2)*Log[1 - Sin[c + d*x]
] - 32*a*b*Log[Sin[c + d*x]] - (15*a^2 - 16*a*b + 3*b^2)*Log[1 + Sin[c + d*
x]] - (a + b)^2/(-1 + Sin[c + d*x])^2 + ((a + b)*(7*a + 3*b))/(-1 + Sin[c +
 d*x]) + (a - b)^2/(1 + Sin[c + d*x])^2 + ((7*a - 3*b)*(a - b))/(1 + Sin[c
+ d*x]))/d
```

fricas [A] time = 0.47, size = 202, normalized size = 1.20

$$32\,ab\cos\left(dx+c\right)^4\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+\left(15\,a^2-16\,ab+3\,b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/16*(32*a*b*cos(d*x + c)^4*log(1/2*sin(d*x + c))*sin(d*x + c) + (15*a^2 -
16*a*b + 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1)*sin(d*x + c) - (15*a^2
 + 16*a*b + 3*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1)*sin(d*x + c) - 6*(
5*a^2 + b^2)*cos(d*x + c)^4 + 2*(5*a^2 + b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^
2 + 8*(2*a*b*cos(d*x + c)^2 + a*b)*sin(d*x + c))/(d*cos(d*x + c)^4*sin(d*x
+ c))
```

giac [A] time = 0.33, size = 186, normalized size = 1.11

$$\frac{32\,ab\log\left(\left|\sin\left(dx+c\right)\right|\right)+\left(15\,a^2-16\,ab+3\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(15\,a^2+16\,ab+3\,b^2\right)\log\left(\left|\sin\left(dx+c\right.\right.\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(32*a*b*log(abs(sin(d*x + c))) + (15*a^2 - 16*a*b + 3*b^2)*log(abs(sin
(d*x + c) + 1)) - (15*a^2 + 16*a*b + 3*b^2)*log(abs(sin(d*x + c) - 1)) - 16
*(2*a*b*sin(d*x + c) + a^2)/sin(d*x + c) + 2*(12*a*b*sin(d*x + c)^4 - 7*a^2
```

```
*sin(d*x + c)^3 - 3*b^2*sin(d*x + c)^3 - 32*a*b*sin(d*x + c)^2 + 9*a^2*sin(
d*x + c) + 5*b^2*sin(d*x + c) + 24*a*b)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.60, size = 195, normalized size = 1.16

$$\frac{a^2}{4d\sin(dx+c)\cos(dx+c)^4}+\frac{5a^2}{8d\sin(dx+c)\cos(dx+c)^2}-\frac{15a^2}{8d\sin(dx+c)}+\frac{15a^2\ln(\sec(dx+c)+\tan(dx+c))}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2/sin(d*x+c)/cos(d*x+c)^4+5/8/d*a^2/sin(d*x+c)/cos(d*x+c)^2-15/8/d*
a^2/sin(d*x+c)+15/8/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*b/cos(d*x+c)^4+
1/d*a*b/cos(d*x+c)^2+2/d*a*b*ln(tan(d*x+c))+1/4/d*b^2*tan(d*x+c)*sec(d*x+c)
^3+3/8/d*b^2*tan(d*x+c)*sec(d*x+c)+3/8/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.35, size = 163, normalized size = 0.97

$$\frac{32\,ab\log(\sin(dx+c))+\left(15\,a^2-16\,ab+3\,b^2\right)\log(\sin(dx+c)+1)-\left(15\,a^2+16\,ab+3\,b^2\right)\log(\sin(dx+c)-1}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/16*(32*a*b*log(sin(d*x + c)) + (15*a^2 - 16*a*b + 3*b^2)*log(sin(d*x + c)
 + 1) - (15*a^2 + 16*a*b + 3*b^2)*log(sin(d*x + c) - 1) - 2*(8*a*b*sin(d*x
+ c)^3 + 3*(5*a^2 + b^2)*sin(d*x + c)^4 - 12*a*b*sin(d*x + c) - 5*(5*a^2 +
b^2)*sin(d*x + c)^2 + 8*a^2)/(sin(d*x + c)^5 - 2*sin(d*x + c)^3 + sin(d*x +
 c)))/d
```

mupad [B] time = 11.87, size = 169, normalized size = 1.01

$$\frac{\ln(\sin(c+d\,x)+1)\left(\frac{15\,a^2}{16}-a\,b+\frac{3\,b^2}{16}\right)}{d}-\frac{\ln(\sin(c+d\,x)-1)\left(\frac{15\,a^2}{16}+a\,b+\frac{3\,b^2}{16}\right)}{d}-\frac{a^2+\sin(c+d\,x)^4\left(\frac{15\,a^2}{8}+\frac{3\,b^2}{8}\right.}{d\,\left(\sin(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(cos(c + d*x)^5*sin(c + d*x)^2),x)
```

```
[Out] (log(sin(c + d*x) + 1)*((15*a^2)/16 - a*b + (3*b^2)/16))/d - (log(sin(c + d
*x) - 1)*(a*b + (15*a^2)/16 + (3*b^2)/16))/d - (a^2 + sin(c + d*x)^4*((15*a
^2)/8 + (3*b^2)/8) - sin(c + d*x)^2*((25*a^2)/8 + (5*b^2)/8) - (3*a*b*sin(c
```

```
 + d*x))/2 + a*b*sin(c + d*x)^3)/(d*(sin(c + d*x) - 2*sin(c + d*x)^3 + sin(
c + d*x)^5)) + (2*a*b*log(sin(c + d*x)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1500 $\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=185

$$-\frac{(12a^2+15ab+4b^2)\log(1-\sin(c+dx))}{8d}+\frac{(3a^2+b^2)\log(\sin(c+dx))}{d}-\frac{(12a^2-15ab+4b^2)\log(\sin(c+dx)+1)}{8d}$$

```
[Out] -2*a*b*csc(d*x+c)/d-1/2*a^2*csc(d*x+c)^2/d-1/8*(12*a^2+15*a*b+4*b^2)*ln(1-s
in(d*x+c))/d+(3*a^2+b^2)*ln(sin(d*x+c))/d-1/8*(12*a^2-15*a*b+4*b^2)*ln(1+si
n(d*x+c))/d+1/4*sec(d*x+c)^4*(a^2+b^2+2*a*b*sin(d*x+c))/d+1/4*sec(d*x+c)^2*
(4*a^2+2*b^2+7*a*b*sin(d*x+c))/d
```

Rubi [A] time = 0.38, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 12, 1805, 1802}

$$-\frac{(12a^2+15ab+4b^2)\log(1-\sin(c+dx))}{8d}+\frac{(3a^2+b^2)\log(\sin(c+dx))}{d}-\frac{(12a^2-15ab+4b^2)\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - ((12*a^2 + 15*a*b +
4*b^2)*Log[1 - Sin[c + d*x]])/(8*d) + ((3*a^2 + b^2)*Log[Sin[c + d*x]])/d -
 ((12*a^2 - 15*a*b + 4*b^2)*Log[1 + Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*
(a^2 + b^2 + 2*a*b*Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^2*(2*(2*a^2 + b^2)
+ 7*a*b*Sin[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
```

```
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b^3(a+x)^2}{x^3\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^8\,\text{Subst}\left(\int \frac{(a+x)^2}{x^3\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} - \frac{b^6\,\text{Subst}\left(\int \frac{-4a\ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} + \frac{\sec^2(c+dx)\left(2\ldots\right.}{\ldots}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} + \frac{\sec^2(c+dx)\left(2\ldots\right.}{\ldots}$$

$$= -\frac{2ab\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{2d} - \frac{\left(12a^2+15ab+4b^2\right)\ldots}{8\ldots}$$

Mathematica [A] time = 3.66, size = 182, normalized size = 0.98

$$\frac{-2\left(12a^2+15ab+4b^2\right)\log(1-\sin(c+dx))+16\left(3a^2+b^2\right)\log(\sin(c+dx))-2\left(12a^2-15ab+4b^2\right)\log(\sin(c\ldots}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-32*a*b*Csc[c + d*x] - 8*a^2*Csc[c + d*x]^2 - 2*(12*a^2 + 15*a*b + 4*b^2)*
Log[1 - Sin[c + d*x]] + 16*(3*a^2 + b^2)*Log[Sin[c + d*x]] - 2*(12*a^2 - 15
*a*b + 4*b^2)*Log[1 + Sin[c + d*x]] + (a + b)^2/(-1 + Sin[c + d*x])^2 - ((a
 + b)*(9*a + 5*b))/(-1 + Sin[c + d*x]) + (a - b)^2/(1 + Sin[c + d*x])^2 + (
(9*a - 5*b)*(a - b))/(1 + Sin[c + d*x]))/(16*d)
```

fricas [A] time = 0.46, size = 285, normalized size = 1.54

$$\frac{4\left(3\,a^2+b^2\right)\cos\left(dx+c\right)^4-2\left(3\,a^2+b^2\right)\cos\left(dx+c\right)^2-2\,a^2-2\,b^2+8\left(\left(3\,a^2+b^2\right)\cos\left(dx+c\right)^6-\left(3\,a^2+b^2\right)\text{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(4*(3*a^2 + b^2)*cos(d*x + c)^4 - 2*(3*a^2 + b^2)*cos(d*x + c)^2 - 2*a^
2 - 2*b^2 + 8*((3*a^2 + b^2)*cos(d*x + c)^6 - (3*a^2 + b^2)*cos(d*x + c)^4)
*log(1/2*sin(d*x + c)) - ((12*a^2 - 15*a*b + 4*b^2)*cos(d*x + c)^6 - (12*a^
2 - 15*a*b + 4*b^2)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - ((12*a^2 + 15*a
*b + 4*b^2)*cos(d*x + c)^6 - (12*a^2 + 15*a*b + 4*b^2)*cos(d*x + c)^4)*log(
-sin(d*x + c) + 1) + 2*(15*a*b*cos(d*x + c)^4 - 5*a*b*cos(d*x + c)^2 - 2*a*
b)*sin(d*x + c))/(d*cos(d*x + c)^6 - d*cos(d*x + c)^4)
```

giac [A] time = 0.33, size = 190, normalized size = 1.03

$$-\frac{\left(12\,a^2-15\,ab+4\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+\left(12\,a^2+15\,ab+4\,b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-8\left(3\,a^2+b^2\right)\log\left(\right|}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*((12*a^2 - 15*a*b + 4*b^2)*log(abs(sin(d*x + c) + 1)) + (12*a^2 + 15*a
*b + 4*b^2)*log(abs(sin(d*x + c) - 1)) - 8*(3*a^2 + b^2)*log(abs(sin(d*x +
c))) + 2*(15*a*b*sin(d*x + c)^5 + 6*a^2*sin(d*x + c)^4 + 2*b^2*sin(d*x + c)
^4 - 25*a*b*sin(d*x + c)^3 - 9*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 +
8*a*b*sin(d*x + c) + 2*a^2)/(sin(d*x + c)^3 - sin(d*x + c))^2)/d
```

maple [A] time = 0.54, size = 209, normalized size = 1.13

$$\frac{a^2}{4d\sin\left(dx+c\right)^2\cos\left(dx+c\right)^4}+\frac{3a^2}{4d\sin\left(dx+c\right)^2\cos\left(dx+c\right)^2}-\frac{3a^2}{2d\sin\left(dx+c\right)^2}+\frac{3a^2\ln\left(\tan\left(dx+c\right)\right)}{d}+\frac{}{2d\sin\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2/sin(d*x+c)^2/cos(d*x+c)^4+3/4/d*a^2/sin(d*x+c)^2/cos(d*x+c)^2-3/2
/d*a^2/sin(d*x+c)^2+3/d*a^2*ln(tan(d*x+c))+1/2/d*a*b/sin(d*x+c)/cos(d*x+c)^
4+5/4/d*a*b/sin(d*x+c)/cos(d*x+c)^2-15/4/d*a*b/sin(d*x+c)+15/4/d*a*b*ln(sec
(d*x+c)+tan(d*x+c))+1/4/d*b^2/cos(d*x+c)^4+1/2/d*b^2/cos(d*x+c)^2+1/d*b^2*l
n(tan(d*x+c))
```

maxima [A] time = 0.41, size = 183, normalized size = 0.99

$$-\frac{\left(12\,a^{2}-15\,ab+4\,b^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(12\,a^{2}+15\,ab+4\,b^{2}\right)\log\left(\sin\left(dx+c\right)-1\right)-8\left(3\,a^{2}+b^{2}\right)\log\left(s\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*((12*a^2 - 15*a*b + 4*b^2)*log(sin(d*x + c) + 1) + (12*a^2 + 15*a*b +
4*b^2)*log(sin(d*x + c) - 1) - 8*(3*a^2 + b^2)*log(sin(d*x + c)) + 2*(15*a*
b*sin(d*x + c)^5 - 25*a*b*sin(d*x + c)^3 + 2*(3*a^2 + b^2)*sin(d*x + c)^4 +
 8*a*b*sin(d*x + c) - 3*(3*a^2 + b^2)*sin(d*x + c)^2 + 2*a^2)/(sin(d*x + c)
^6 - 2*sin(d*x + c)^4 + sin(d*x + c)^2))/d
```

mupad [B] time = 0.14, size = 194, normalized size = 1.05

$$\frac{\ln\left(\sin\left(c+d\,x\right)\right)\,\left(3\,a^2+b^2\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{3\,a^2}{2}-\frac{15\,a\,b}{8}+\frac{b^2}{2}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{3\,a^2}{2}+\frac{15\,a\,b}{8}+\frac{b^2}{2}\right)}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(cos(c + d*x)^5*sin(c + d*x)^3),x)
```

```
[Out] (log(sin(c + d*x))*(3*a^2 + b^2))/d - (log(sin(c + d*x) + 1)*((3*a^2)/2 - (
15*a*b)/8 + b^2/2))/d - (log(sin(c + d*x) - 1)*((15*a*b)/8 + (3*a^2)/2 + b^
2/2))/d - (a^2/2 + sin(c + d*x)^4*((3*a^2)/2 + b^2/2) - sin(c + d*x)^2*((9*
a^2)/4 + (3*b^2)/4) + 2*a*b*sin(c + d*x) - (25*a*b*sin(c + d*x)^3)/4 + (15*
a*b*sin(c + d*x)^5)/4)/(d*(sin(c + d*x)^2 - 2*sin(c + d*x)^4 + sin(c + d*x)
^6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1501 $\int (a+b\sin(c+dx))^3\tan^5(c+dx)\,dx$

Optimal. Leaf size=202

$$-\frac{b\left(24a^2+35b^2\right)\sin(c+dx)}{8d}-\frac{(a+b)\left(8a^2+37ab+35b^2\right)\log(1-\sin(c+dx))}{16d}-\frac{(a-b)\left(8a^2-37ab+35b^2\right)\log}{16d}$$

```
[Out] -1/16*(a+b)*(8*a^2+37*a*b+35*b^2)*ln(1-sin(d*x+c))/d-1/16*(a-b)*(8*a^2-37*a
*b+35*b^2)*ln(1+sin(d*x+c))/d-1/8*b*(24*a^2+35*b^2)*sin(d*x+c)/d-3/2*a*b^2*
sin(d*x+c)^2/d-1/3*b^3*sin(d*x+c)^3/d+1/4*sec(d*x+c)^4*(a+b*sin(d*x+c))^3/d
-1/8*sec(d*x+c)^2*(a+b*sin(d*x+c))^2*(8*a+11*b*sin(d*x+c))/d
```

Rubi [A] time = 0.34, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2721, 1645, 1629, 633, 31}

$$-\frac{b\left(24a^2+35b^2\right)\sin(c+dx)}{8d}-\frac{(a+b)\left(8a^2+37ab+35b^2\right)\log(1-\sin(c+dx))}{16d}-\frac{(a-b)\left(8a^2-37ab+35b^2\right)\log}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -((a + b)*(8*a^2 + 37*a*b + 35*b^2)*Log[1 - Sin[c + d*x]])/(16*d) - ((a - b
)*(8*a^2 - 37*a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - (b*(24*a^2 + 35
*b^2)*Sin[c + d*x])/(8*d) - (3*a*b^2*Sin[c + d*x]^2)/(2*d) - (b^3*Sin[c + d
*x]^3)/(3*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(4*d) - (Sec[c + d*x
]^2*(a + b*Sin[c + d*x])^2*(8*a + 11*b*Sin[c + d*x]))/(8*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+x)^3}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} + \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(-3b^6-4ab^4x-4b^4x^2-4ab^2x\right.}{\left(b^2-x^2\right)^2}\right.}{4b^2d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2(8a}{8d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2(8a}{8d}$$

$$= -\frac{b\left(24a^2+35b^2\right)\sin(c+dx)}{8d} - \frac{3ab^2\sin^2(c+dx)}{2d} - \frac{b^3\sin^3(c+dx)}{3d} +$$

$$= -\frac{b\left(24a^2+35b^2\right)\sin(c+dx)}{8d} - \frac{3ab^2\sin^2(c+dx)}{2d} - \frac{b^3\sin^3(c+dx)}{3d} +$$

$$= -\frac{(a+b)\left(8a^2+37ab+35b^2\right)\log(1-\sin(c+dx))}{16d} - \frac{(a-b)\left(8a^2-37a\right.}{}$$

Mathematica [A] time = 1.02, size = 199, normalized size = 0.99

$$-\frac{144b\left(a^2+b^2\right)\sin(c+dx)+3\left(8a^2-37ab+35b^2\right)(a-b)\log(\sin(c+dx)+1)+3(a+b)\left(8a^2+37ab+35b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -1/48*(3*(a + b)*(8*a^2 + 37*a*b + 35*b^2)*Log[1 - Sin[c + d*x]] + 3*(a - b
)*(8*a^2 - 37*a*b + 35*b^2)*Log[1 + Sin[c + d*x]] - (3*(a + b)^3)/(-1 + Sin
[c + d*x])^2 - (3*(a + b)^2*(7*a + 13*b))/(-1 + Sin[c + d*x]) + 144*b*(a^2
+ b^2)*Sin[c + d*x] + 72*a*b^2*Sin[c + d*x]^2 + 16*b^3*Sin[c + d*x]^3 - (3*
(a - b)^3)/(1 + Sin[c + d*x])^2 + (3*(7*a - 13*b)*(a - b)^2)/(1 + Sin[c + d
*x]))/d
```

fricas [A] time = 0.49, size = 238, normalized size = 1.18

$$\frac{72\,ab^2\cos(dx+c)^6 - 36\,ab^2\cos(dx+c)^4 - 3\left(8\,a^3-45\,a^2b+72\,ab^2-35\,b^3\right)\cos(dx+c)^4\log(\sin(dx+c)+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/48*(72*a*b^2*cos(d*x + c)^6 - 36*a*b^2*cos(d*x + c)^4 - 3*(8*a^3 - 45*a^2
*b + 72*a*b^2 - 35*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(8*a^3 + 4
5*a^2*b + 72*a*b^2 + 35*b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 12*a^3
 + 36*a*b^2 - 24*(2*a^3 + 9*a*b^2)*cos(d*x + c)^2 + 2*(8*b^3*cos(d*x + c)^6
 - 8*(9*a^2*b + 10*b^3)*cos(d*x + c)^4 + 18*a^2*b + 6*b^3 - 3*(27*a^2*b + 1
3*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.34, size = 251, normalized size = 1.24

$$-\frac{16\,b^3\sin(dx+c)^3+72\,ab^2\sin(dx+c)^2+144\,a^2b\sin(dx+c)+144\,b^3\sin(dx+c)+3\left(8\,a^3-45\,a^2b+72\,ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/48*(16*b^3*sin(d*x + c)^3 + 72*a*b^2*sin(d*x + c)^2 + 144*a^2*b*sin(d*x
+ c) + 144*b^3*sin(d*x + c) + 3*(8*a^3 - 45*a^2*b + 72*a*b^2 - 35*b^3)*log(
abs(sin(d*x + c) + 1)) + 3*(8*a^3 + 45*a^2*b + 72*a*b^2 + 35*b^3)*log(abs(s
in(d*x + c) - 1)) - 6*(6*a^3*sin(d*x + c)^4 + 54*a*b^2*sin(d*x + c)^4 + 27*
a^2*b*sin(d*x + c)^3 + 13*b^3*sin(d*x + c)^3 - 4*a^3*sin(d*x + c)^2 - 72*a*
b^2*sin(d*x + c)^2 - 21*a^2*b*sin(d*x + c) - 11*b^3*sin(d*x + c) + 24*a*b^2
)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [B] time = 0.34, size = 420, normalized size = 2.08

$$\frac{a^3\left(\tan^4(dx+c)\right)}{4d}-\frac{a^3\left(\tan^2(dx+c)\right)}{2d}-\frac{a^3\ln(\cos(dx+c))}{d}+\frac{3a^2b\left(\sin^7(dx+c)\right)}{4d\cos(dx+c)^4}-\frac{9a^2b\left(\sin^7(dx+c)\right)}{8d\cos(dx+c)^2}-\frac{9a^2b\left(\sin\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*tan(d*x+c)^4-1/2/d*a^3*tan(d*x+c)^2-1/d*a^3*ln(cos(d*x+c))+3/4/d*
a^2*b*sin(d*x+c)^7/cos(d*x+c)^4-9/8/d*a^2*b*sin(d*x+c)^7/cos(d*x+c)^2-9/8/d
*a^2*b*sin(d*x+c)^5-15/8/d*a^2*b*sin(d*x+c)^3-45/8*a^2*b*sin(d*x+c)/d+45/8/
d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a*b^2*sin(d*x+c)^8/cos(d*x+c)^4-3/2
/d*a*b^2*sin(d*x+c)^8/cos(d*x+c)^2-3/2/d*a*b^2*sin(d*x+c)^6-9/4/d*a*b^2*sin
(d*x+c)^4-9/2*a*b^2*sin(d*x+c)^2/d-9/d*a*b^2*ln(cos(d*x+c))+1/4/d*b^3*sin(d
*x+c)^9/cos(d*x+c)^4-5/8/d*b^3*sin(d*x+c)^9/cos(d*x+c)^2-5/8/d*b^3*sin(d*x+
```

```
c)^7-7/8/d*sin(d*x+c)^5*b^3-35/24*b^3*sin(d*x+c)^3/d-35/8*b^3*sin(d*x+c)/d+
35/8/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.35, size = 217, normalized size = 1.07

$$-\frac{16\,b^3\sin\left(dx+c\right)^3+72\,ab^2\sin\left(dx+c\right)^2+3\left(8\,a^3-45\,a^2b+72\,ab^2-35\,b^3\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(8\,a^3+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/48*(16*b^3*sin(d*x + c)^3 + 72*a*b^2*sin(d*x + c)^2 + 3*(8*a^3 - 45*a^2*
b + 72*a*b^2 - 35*b^3)*log(sin(d*x + c) + 1) + 3*(8*a^3 + 45*a^2*b + 72*a*b
^2 + 35*b^3)*log(sin(d*x + c) - 1) + 144*(a^2*b + b^3)*sin(d*x + c) - 6*((2
7*a^2*b + 13*b^3)*sin(d*x + c)^3 - 6*a^3 - 30*a*b^2 + 4*(2*a^3 + 9*a*b^2)*s
in(d*x + c)^2 - (21*a^2*b + 11*b^3)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1))/d
```

mupad [B] time = 12.12, size = 512, normalized size = 2.53

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,\left(a^3+9\,a\,b^2\right)}{d}-\frac{\left(-\frac{45\,a^2\,b}{4}-\frac{35\,b^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\left(-2\,a^3-18\,a\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\left(\frac{15\,a\cdots}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^5*(a + b*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(9*a*b^2 + a^3))/d - (tan(c/2 + (d*x)/2)^4*(
18*a*b^2 + 2*a^3) - tan(c/2 + (d*x)/2)^2*(18*a*b^2 + 2*a^3) - tan(c/2 + (d*
x)/2)*((45*a^2*b)/4 + (35*b^3)/4) + tan(c/2 + (d*x)/2)^10*(18*a*b^2 + 2*a^3
) - tan(c/2 + (d*x)/2)^12*(18*a*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^6*(48*a*b
^2 + 16*a^3) + tan(c/2 + (d*x)/2)^8*(48*a*b^2 + 16*a^3) + tan(c/2 + (d*x)/2
)^7*(33*a^2*b - 17*b^3) + tan(c/2 + (d*x)/2)^3*((15*a^2*b)/2 + (35*b^3)/6)
+ tan(c/2 + (d*x)/2)^11*((15*a^2*b)/2 + (35*b^3)/6) - tan(c/2 + (d*x)/2)^13
*((45*a^2*b)/4 + (35*b^3)/4) + tan(c/2 + (d*x)/2)^5*((141*a^2*b)/4 + (329*b
^3)/12) + tan(c/2 + (d*x)/2)^9*((141*a^2*b)/4 + (329*b^3)/12))/(d*(tan(c/2
+ (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^6 - 3*tan(c/2
+ (d*x)/2)^8 + 3*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 - tan(c/2 +
(d*x)/2)^14 - 1)) - (log(tan(c/2 + (d*x)/2) + 1)*(a - b)*(8*a^2 - 37*a*b +
35*b^2))/(8*d) - (log(tan(c/2 + (d*x)/2) - 1)*(a + b)*(37*a*b + 8*a^2 + 35*
b^2))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
[Out] Timed out
```

3.1502 $\int \sec(c+dx)(a+b\sin(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=177

$$-\frac{3(a+b)\left(a^2+7ab+8b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{3(a-b)\left(a^2-7ab+8b^2\right)\log(\sin(c+dx)+1)}{16d}-\frac{29ab^2\sin(c+}{8d}$$

```
[Out] -3/16*(a+b)*(a^2+7*a*b+8*b^2)*ln(1-sin(d*x+c))/d+3/16*(a-b)*(a^2-7*a*b+8*b^
2)*ln(1+sin(d*x+c))/d-29/8*a*b^2*sin(d*x+c)/d-1/2*b^3*sin(d*x+c)^2/d-1/8*se
c(d*x+c)^2*(8*b+5*a*sin(d*x+c))*(a+b*sin(d*x+c))^2/d+1/4*sec(d*x+c)^3*(a+b*
sin(d*x+c))^3*tan(d*x+c)/d
```

Rubi [A] time = 0.37, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2837, 12, 1645, 1629, 633, 31}

$$-\frac{3(a+b)\left(a^2+7ab+8b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{3(a-b)\left(a^2-7ab+8b^2\right)\log(\sin(c+dx)+1)}{16d}-\frac{29ab^2\sin(c+}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (-3*(a + b)*(a^2 + 7*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(16*d) + (3*(a - b
)*(a^2 - 7*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - (29*a*b^2*Sin[c + d
*x])/(8*d) - (b^3*Sin[c + d*x]^2)/(2*d) - (Sec[c + d*x]^2*(8*b + 5*a*Sin[c
+ d*x])*(a + b*Sin[c + d*x])^2)/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c + d*x]
)^3*Tan[c + d*x])/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^3\tan^4(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^4(a+x)^3}{b^4\left(b^2-x^2\right)^3}dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{x^4(a+x)^3}{\left(b^2-x^2\right)^3}dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{(a\cdots}{\cdots}\right)}{\cdots}$$

$$= -\frac{\sec^2(c+dx)(8b+5a\sin(c+dx))(a+b\sin(c+dx))^2}{8d} + \cdots$$

$$= -\frac{\sec^2(c+dx)(8b+5a\sin(c+dx))(a+b\sin(c+dx))^2}{8d} + \cdots$$

$$= -\frac{29ab^2\sin(c+dx)}{8d} - \frac{b^3\sin^2(c+dx)}{2d} - \frac{\sec^2(c+dx)(8b+\cdots}{\cdots}$$

$$= -\frac{29ab^2\sin(c+dx)}{8d} - \frac{b^3\sin^2(c+dx)}{2d} - \frac{\sec^2(c+dx)(8b+\cdots}{\cdots}$$

$$= -\frac{3(a+b)\left(a^2+7ab+8b^2\right)\log(1-\sin(c+dx))}{16d} + \frac{3(a-b)\cdots}{\cdots}$$

Mathematica [A] time = 0.57, size = 174, normalized size = 0.98

$$\frac{3\left(a^2-7ab+8b^2\right)(a-b)\log(\sin(c+dx)+1)-3(a+b)\left(a^2+7ab+8b^2\right)\log(1-\sin(c+dx))-48ab^2\sin(c+\cdots}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (-3*(a + b)*(a^2 + 7*a*b + 8*b^2)*Log[1 - Sin[c + d*x]] + 3*(a - b)*(a^2 -
7*a*b + 8*b^2)*Log[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c + d*x])^2 + ((
a + b)^2*(5*a + 11*b))/(-1 + Sin[c + d*x]) - 48*a*b^2*Sin[c + d*x] - 8*b^3*
Sin[c + d*x]^2 - (a - b)^3/(1 + Sin[c + d*x])^2 + ((5*a - 11*b)*(a - b)^2)/
(1 + Sin[c + d*x]))/(16*d)
```

fricas [A] time = 0.49, size = 208, normalized size = 1.18

$$\frac{8 b^3 \cos(dx+c)^6 - 4 b^3 \cos(dx+c)^4 + 3\left(a^3 - 8 a^2 b + 15 a b^2 - 8 b^3\right) \cos(dx+c)^4 \log(\sin(dx+c)+1) - 3\left(a^3 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(8*b^3*cos(d*x + c)^6 - 4*b^3*cos(d*x + c)^4 + 3*(a^3 - 8*a^2*b + 15*a
*b^2 - 8*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(a^3 + 8*a^2*b + 15*
a*b^2 + 8*b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 12*a^2*b + 4*b^3 - 2
4*(2*a^2*b + b^3)*cos(d*x + c)^2 - 2*(24*a*b^2*cos(d*x + c)^4 - 2*a^3 - 6*a
*b^2 + (5*a^3 + 27*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.32, size = 221, normalized size = 1.25

$$-\frac{8 b^3 \sin(dx+c)^2 + 48 a b^2 \sin(dx+c) - 3\left(a^3 - 8 a^2 b + 15 a b^2 - 8 b^3\right) \log(|\sin(dx+c)+1|) + 3\left(a^3 + 8 a^2 b + 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(8*b^3*sin(d*x + c)^2 + 48*a*b^2*sin(d*x + c) - 3*(a^3 - 8*a^2*b + 15
*a*b^2 - 8*b^3)*log(abs(sin(d*x + c) + 1)) + 3*(a^3 + 8*a^2*b + 15*a*b^2 +
8*b^3)*log(abs(sin(d*x + c) - 1)) - 2*(18*a^2*b*sin(d*x + c)^4 + 18*b^3*sin
(d*x + c)^4 + 5*a^3*sin(d*x + c)^3 + 27*a*b^2*sin(d*x + c)^3 - 12*a^2*b*sin
(d*x + c)^2 - 24*b^3*sin(d*x + c)^2 - 3*a^3*sin(d*x + c) - 21*a*b^2*sin(d*x
 + c) + 8*b^3)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [B] time = 0.33, size = 385, normalized size = 2.18

$$\frac{a^3 \left(\sin^5(dx+c)\right)}{4d\cos(dx+c)^4} - \frac{a^3 \left(\sin^5(dx+c)\right)}{8d\cos(dx+c)^2} - \frac{a^3 \left(\sin^3(dx+c)\right)}{8d} - \frac{3a^3 \sin(dx+c)}{8d} + \frac{3a^3 \ln(\sec(dx+c)+\tan(dx+c))}{8d} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a^3*sin(d*x+c)^5/cos(d*x+c)^2-1/8
*a^3*sin(d*x+c)^3/d-3/8*a^3*sin(d*x+c)/d+3/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c)
)+3/4/d*a^2*b*tan(d*x+c)^4-3/2/d*a^2*b*tan(d*x+c)^2-3/d*a^2*b*ln(cos(d*x+c)
```

```
)+3/4/d*a*b^2*sin(d*x+c)^7/cos(d*x+c)^4-9/8/d*a*b^2*sin(d*x+c)^7/cos(d*x+c)
^2-9/8/d*a*b^2*sin(d*x+c)^5-15/8/d*a*b^2*sin(d*x+c)^3-45/8*a*b^2*sin(d*x+c)
/d+45/8/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*b^3*sin(d*x+c)^8/cos(d*x+c)
^4-1/2/d*b^3*sin(d*x+c)^8/cos(d*x+c)^2-1/2/d*b^3*sin(d*x+c)^6-3/4/d*sin(d*x
+c)^4*b^3-3/2*b^3*sin(d*x+c)^2/d-3/d*b^3*ln(cos(d*x+c))
```

maxima [A] time = 0.32, size = 190, normalized size = 1.07

$$-\frac{8\,b^3\sin\left(dx+c\right)^2+48\,ab^2\sin\left(dx+c\right)-3\left(a^3-8\,a^2b+15\,ab^2-8\,b^3\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(a^3+8\,a^2b+1\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(8*b^3*sin(d*x + c)^2 + 48*a*b^2*sin(d*x + c) - 3*(a^3 - 8*a^2*b + 15
*a*b^2 - 8*b^3)*log(sin(d*x + c) + 1) + 3*(a^3 + 8*a^2*b + 15*a*b^2 + 8*b^3
)*log(sin(d*x + c) - 1) - 2*((5*a^3 + 27*a*b^2)*sin(d*x + c)^3 - 18*a^2*b -
 10*b^3 + 12*(2*a^2*b + b^3)*sin(d*x + c)^2 - 3*(a^3 + 7*a*b^2)*sin(d*x + c
))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 12.11, size = 449, normalized size = 2.54

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\left(3\,a^2\,b+3\,b^3\right)}{d}-\frac{\left(-\frac{3\,a^3}{4}-\frac{45\,a\,b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(-6\,a^2\,b-6\,b^3\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\left(\frac{5\,a^3}{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + b*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(3*a^2*b + 3*b^3))/d - (tan(c/2 + (d*x)/2)^5
*((33*a*b^2)/2 + (15*a^3)/2) - tan(c/2 + (d*x)/2)*((45*a*b^2)/4 + (3*a^3)/4
) + tan(c/2 + (d*x)/2)^7*((33*a*b^2)/2 + (15*a^3)/2) - tan(c/2 + (d*x)/2)^1
1*((45*a*b^2)/4 + (3*a^3)/4) + tan(c/2 + (d*x)/2)^3*((75*a*b^2)/4 + (5*a^3)
/4) + tan(c/2 + (d*x)/2)^9*((75*a*b^2)/4 + (5*a^3)/4) - tan(c/2 + (d*x)/2)^
2*(6*a^2*b + 6*b^3) - tan(c/2 + (d*x)/2)^10*(6*a^2*b + 6*b^3) + tan(c/2 + (
d*x)/2)^4*(12*a^2*b + 12*b^3) + tan(c/2 + (d*x)/2)^8*(12*a^2*b + 12*b^3) +
tan(c/2 + (d*x)/2)^6*(36*a^2*b + 4*b^3))/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c
/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 2*tan(c/2
 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^12 - 1)) - (3*log(tan(c/2 + (d*x)/2) -
1)*(a + b)*(7*a*b + a^2 + 8*b^2))/(8*d) + (3*log(tan(c/2 + (d*x)/2) + 1)*(a
 - b)*(a^2 - 7*a*b + 8*b^2))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1503 $\int \sec^2(c+dx)(a+b\sin(c+dx))^3\tan^3(c+dx)\,dx$

Optimal. Leaf size=142

$$-\frac{3b(a+b)(3a+5b)\log(1-\sin(c+dx))}{16d}+\frac{3b(3a-5b)(a-b)\log(\sin(c+dx)+1)}{16d}+\frac{\sec^4(c+dx)(a+b\sin(c+d$$

[Out] -3/16*b*(a+b)*(3*a+5*b)*ln(1-sin(d*x+c))/d+3/16*(3*a-5*b)*(a-b)*b*ln(1+sin(d*x+c))/d-15/8*b^3*sin(d*x+c)/d+1/4*sec(d*x+c)^4*(a+b*sin(d*x+c))^3/d-1/8*sec(d*x+c)^2*(a+b*sin(d*x+c))^2*(4*a+7*b*sin(d*x+c))/d

Rubi [A] time = 0.30, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2837, 12, 1645, 774, 633, 31}

$$-\frac{3b(a+b)(3a+5b)\log(1-\sin(c+dx))}{16d}+\frac{3b(3a-5b)(a-b)\log(\sin(c+dx)+1)}{16d}+\frac{\sec^4(c+dx)(a+b\sin(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (-3*b*(a + b)*(3*a + 5*b)*Log[1 - Sin[c + d*x]])/(16*d) + (3*(3*a - 5*b)*(a
 - b)*b*Log[1 + Sin[c + d*x]])/(16*d) - (15*b^3*Sin[c + d*x])/(8*d) + (Sec[
c + d*x]^4*(a + b*Sin[c + d*x])^3)/(4*d) - (Sec[c + d*x]^2*(a + b*Sin[c + d
*x])^2*(4*a + 7*b*Sin[c + d*x]))/(8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^3\tan^3(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^3(a+x)^3}{b^3\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\,\text{Subst}\left(\int \frac{x^3(a+x)^3}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} + \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(-3b^4-4ab\right.}{\left(b^2-x^2\right.}\right.}{}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\sec^2(c+dx)(a+b\sin(}{}$$

$$= -\frac{15b^3\sin(c+dx)}{8d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\text{se}}{}$$

$$= -\frac{15b^3\sin(c+dx)}{8d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\text{se}}{}$$

$$= -\frac{3b(a+b)(3a+5b)\log(1-\sin(c+dx))}{16d} + \frac{3(3a-5b)(a-}{}$$

Mathematica [A] time = 0.44, size = 147, normalized size = 1.04

$$\frac{\frac{(a-b)^3}{(\sin(c+dx)+1)^2} - \frac{3(a-3b)(a-b)^2}{\sin(c+dx)+1} + \frac{3(a+b)^2(a+3b)}{\sin(c+dx)-1} + \frac{(a+b)^3}{(\sin(c+dx)-1)^2} + 3b(3a-5b)(a-b)\log(\sin(c+dx)+1) - 3b(a+b)(3a}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (-3*b*(a + b)*(3*a + 5*b)*Log[1 - Sin[c + d*x]] + 3*(3*a - 5*b)*(a - b)*b*L
og[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c + d*x])^2 + (3*(a + b)^2*(a +
3*b))/(-1 + Sin[c + d*x]) - 16*b^3*Sin[c + d*x] + (a - b)^3/(1 + Sin[c + d*
x])^2 - (3*(a - 3*b)*(a - b)^2)/(1 + Sin[c + d*x]))/(16*d)
```

fricas [A] time = 0.47, size = 176, normalized size = 1.24

$$\frac{3\left(3a^2b-8ab^2+5b^3\right)\cos(dx+c)^4\log(\sin(dx+c)+1) - 3\left(3a^2b+8ab^2+5b^3\right)\cos(dx+c)^4\log(-\sin(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(3*(3*a^2*b - 8*a*b^2 + 5*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
3*(3*a^2*b + 8*a*b^2 + 5*b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^3
 + 12*a*b^2 - 8*(a^3 + 6*a*b^2)*cos(d*x + c)^2 - 2*(8*b^3*cos(d*x + c)^4 -
6*a^2*b - 2*b^3 + 3*(5*a^2*b + 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(
d*x + c)^4)
```

giac [A] time = 0.29, size = 188, normalized size = 1.32

$$-\frac{16\,b^3\sin(dx+c)-3\left(3\,a^2b-8\,ab^2+5\,b^3\right)\log(|\sin(dx+c)+1|)+3\left(3\,a^2b+8\,ab^2+5\,b^3\right)\log(|\sin(dx+c)-}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(16*b^3*sin(d*x + c) - 3*(3*a^2*b - 8*a*b^2 + 5*b^3)*log(abs(sin(d*x
+ c) + 1)) + 3*(3*a^2*b + 8*a*b^2 + 5*b^3)*log(abs(sin(d*x + c) - 1)) - 2*(
18*a*b^2*sin(d*x + c)^4 + 15*a^2*b*sin(d*x + c)^3 + 9*b^3*sin(d*x + c)^3 +
4*a^3*sin(d*x + c)^2 - 12*a*b^2*sin(d*x + c)^2 - 9*a^2*b*sin(d*x + c) - 7*b
^3*sin(d*x + c) - 2*a^3)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [B] time = 0.32, size = 297, normalized size = 2.09

$$\frac{a^3\left(\sin^4(dx+c)\right)}{4d\cos(dx+c)^4}+\frac{3a^2b\left(\sin^5(dx+c)\right)}{4d\cos(dx+c)^4}-\frac{3a^2b\left(\sin^5(dx+c)\right)}{8d\cos(dx+c)^2}-\frac{3a^2b\left(\sin^3(dx+c)\right)}{8d}-\frac{9a^2b\sin(dx+c)}{8d}+\frac{9a^2b\ln(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^4/cos(d*x+c)^4+3/4/d*a^2*b*sin(d*x+c)^5/cos(d*x+c)^4-3
/8/d*a^2*b*sin(d*x+c)^5/cos(d*x+c)^2-3/8/d*a^2*b*sin(d*x+c)^3-9/8*a^2*b*sin
(d*x+c)/d+9/8/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a*b^2*tan(d*x+c)^4-3/
2/d*a*b^2*tan(d*x+c)^2-3/d*a*b^2*ln(cos(d*x+c))+1/4/d*b^3*sin(d*x+c)^7/cos(
d*x+c)^4-3/8/d*b^3*sin(d*x+c)^7/cos(d*x+c)^2-3/8/d*sin(d*x+c)^5*b^3-5/8*b^3
*sin(d*x+c)^3/d-15/8*b^3*sin(d*x+c)/d+15/8/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.34, size = 173, normalized size = 1.22

$$-\frac{16\,b^3\sin(dx+c)-3\left(3\,a^2b-8\,ab^2+5\,b^3\right)\log(\sin(dx+c)+1)+3\left(3\,a^2b+8\,ab^2+5\,b^3\right)\log(\sin(dx+c)-1)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(16*b^3*sin(d*x + c) - 3*(3*a^2*b - 8*a*b^2 + 5*b^3)*log(sin(d*x + c)
 + 1) + 3*(3*a^2*b + 8*a*b^2 + 5*b^3)*log(sin(d*x + c) - 1) - 2*(3*(5*a^2*b
 + 3*b^3)*sin(d*x + c)^3 - 2*a^3 - 18*a*b^2 + 4*(a^3 + 6*a*b^2)*sin(d*x + c
)^2 - (9*a^2*b + 7*b^3)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 +
1))/d
```

mupad [B] time = 12.09, size = 356, normalized size = 2.51

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(4\,a^3+18\,a\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{9\,a^2\,b}{4}+\frac{15\,b^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(4\,a^3+18\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-3\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + b*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4*(18*a*b^2 + 4*a^3) - tan(c/2 + (d*x)/2)*((9*a^2*b)/4
+ (15*b^3)/4) + tan(c/2 + (d*x)/2)^6*(18*a*b^2 + 4*a^3) + tan(c/2 + (d*x)/2
)^3*(6*a^2*b + 10*b^3) + tan(c/2 + (d*x)/2)^7*(6*a^2*b + 10*b^3) - tan(c/2
+ (d*x)/2)^9*((9*a^2*b)/4 + (15*b^3)/4) + tan(c/2 + (d*x)/2)^5*((33*a^2*b)/
2 - (9*b^3)/2) - 6*a*b^2*tan(c/2 + (d*x)/2)^2 - 6*a*b^2*tan(c/2 + (d*x)/2)^
8)/(d*(2*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/
2)^6 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (3*a*b^2*log(
tan(c/2 + (d*x)/2)^2 + 1))/d - (3*b*log(tan(c/2 + (d*x)/2) - 1)*(a + b)*(3*
a + 5*b))/(8*d) + (3*b*log(tan(c/2 + (d*x)/2) + 1)*(a - b)*(3*a - 5*b))/(8*
d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1504 $\int \sec^3(c+dx)(a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{(a^3-9ab^2-8b^3)\log(1-\sin(c+dx))}{16d}-\frac{(a^3-9ab^2+8b^3)\log(\sin(c+dx)+1)}{16d}-\frac{\sec^2(c+dx)\left(\left(a^2+4b^2\right)\sin(c+\right.}{8d}$$

```
[Out] 1/16*(a^3-9*a*b^2-8*b^3)*ln(1-sin(d*x+c))/d-1/16*(a^3-9*a*b^2+8*b^3)*ln(1+s
in(d*x+c))/d-1/8*sec(d*x+c)^2*(a+b*sin(d*x+c))*(5*a*b+(a^2+4*b^2)*sin(d*x+c
))/d+1/4*sec(d*x+c)^3*(a+b*sin(d*x+c))^3*tan(d*x+c)/d
```

Rubi [A] time = 0.25, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2837, 12, 1645, 819, 633, 31}

$$\frac{(a^3-9ab^2-8b^3)\log(1-\sin(c+dx))}{16d}-\frac{(a^3-9ab^2+8b^3)\log(\sin(c+dx)+1)}{16d}-\frac{\sec^2(c+dx)\left(\left(a^2+4b^2\right)\sin(c+\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a^3 - 9*a*b^2 - 8*b^3)*Log[1 - Sin[c + d*x]])/(16*d) - ((a^3 - 9*a*b^2 +
8*b^3)*Log[1 + Sin[c + d*x]])/(16*d) - (Sec[c + d*x]^2*(a + b*Sin[c + d*x])
*(5*a*b + (a^2 + 4*b^2)*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c
 + d*x])^3*Tan[c + d*x])/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^2(a+x)^3}{b^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^3\,\text{Subst}\left(\int \frac{x^2(a+x)^3}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)}{4d} + \frac{b\,\text{Subst}\Big(\int \frac{(}{}$$

$$= -\frac{\sec^2(c+dx)(a+b\sin(c+dx))\left(5ab+\left(a^2+4b^2\right)\sin(c+\right.}{8d}$$

$$= -\frac{\sec^2(c+dx)(a+b\sin(c+dx))\left(5ab+\left(a^2+4b^2\right)\sin(c+\right.}{8d}$$

$$= \frac{\left(a^3-9ab^2-8b^3\right)\log(1-\sin(c+dx))}{16d} - \frac{\left(a^3-9ab^2+8b^3\right)}{1}$$

Mathematica [A] time = 0.39, size = 140, normalized size = 0.97

$$\frac{\left(a^3-9ab^2-8b^3\right)\log(1-\sin(c+dx)) - \left(a^3-9ab^2+8b^3\right)\log(\sin(c+dx)+1) - \frac{(a-b)^3}{(\sin(c+dx)+1)^2} + \frac{(a-7b)(a-b)^2}{\sin(c+dx)+1} + \frac{(a+b}{\sin(}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a^3 - 9*a*b^2 - 8*b^3)*Log[1 - Sin[c + d*x]] - (a^3 - 9*a*b^2 + 8*b^3)*Lo
g[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c + d*x])^2 + ((a + b)^2*(a + 7*b
))/(-1 + Sin[c + d*x]) - (a - b)^3/(1 + Sin[c + d*x])^2 + ((a - 7*b)*(a - b
)^2)/(1 + Sin[c + d*x]))/(16*d)
```

fricas [A] time = 0.47, size = 156, normalized size = 1.08

$$-\frac{\left(a^3-9\,ab^2+8\,b^3\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(a^3-9\,ab^2-8\,b^3\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)}{16\,d\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*((a^3 - 9*a*b^2 + 8*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (a^3
- 9*a*b^2 - 8*b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 12*a^2*b - 4*b^3
 + 8*(3*a^2*b + 2*b^3)*cos(d*x + c)^2 - 2*(2*a^3 + 6*a*b^2 - (a^3 + 15*a*b^
2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.27, size = 168, normalized size = 1.17

$$-\frac{\left(a^3-9\,ab^2+8\,b^3\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(a^3-9\,ab^2-8\,b^3\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(6\,b^3\sin\left(dx+c\right)^4+a^3\sin\left(dx+\right.\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*((a^3 - 9*a*b^2 + 8*b^3)*log(abs(sin(d*x + c) + 1)) - (a^3 - 9*a*b^2
- 8*b^3)*log(abs(sin(d*x + c) - 1)) - 2*(6*b^3*sin(d*x + c)^4 + a^3*sin(d*x
 + c)^3 + 15*a*b^2*sin(d*x + c)^3 + 12*a^2*b*sin(d*x + c)^2 - 4*b^3*sin(d*x
 + c)^2 + a^3*sin(d*x + c) - 9*a*b^2*sin(d*x + c) - 6*a^2*b)/(sin(d*x + c)^
2 - 1)^2)/d
```

maple [A] time = 0.32, size = 263, normalized size = 1.83

$$\frac{a^3\left(\sin^3\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4}+\frac{a^3\left(\sin^3\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^2}+\frac{a^3\sin\left(dx+c\right)}{8d}-\frac{a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{3a^2b\left(\sin^4\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a^3*sin(d*x+c)^3/cos(d*x+c)^2+1/8
*a^3*sin(d*x+c)/d-1/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a^2*b*sin(d*x+c
)^4/cos(d*x+c)^4+3/4/d*a*b^2*sin(d*x+c)^5/cos(d*x+c)^4-3/8/d*a*b^2*sin(d*x+
c)^5/cos(d*x+c)^2-3/8/d*a*b^2*sin(d*x+c)^3-9/8*a*b^2*sin(d*x+c)/d+9/8/d*a*b
^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*b^3*tan(d*x+c)^4-1/2/d*b^3*tan(d*x+c)^2-
1/d*b^3*ln(cos(d*x+c))
```

maxima [A] time = 0.33, size = 151, normalized size = 1.05

$$-\frac{\left(a^3-9\,ab^2+8\,b^3\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^3-9\,ab^2-8\,b^3\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(\left(a^3+15\,ab^2\right)\sin\left(dx+c\right)^3-6\,a^2b\right.}{\sin}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*((a^3 - 9*a*b^2 + 8*b^3)*log(sin(d*x + c) + 1) - (a^3 - 9*a*b^2 - 8*b
^3)*log(sin(d*x + c) - 1) - 2*((a^3 + 15*a*b^2)*sin(d*x + c)^3 - 6*a^2*b -
6*b^3 + 4*(3*a^2*b + 2*b^3)*sin(d*x + c)^2 + (a^3 - 9*a*b^2)*sin(d*x + c))/
(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 12.14, size = 299, normalized size = 2.08

$$\frac{b^3 \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\left(-\frac{a^3}{8}+\frac{9\,a\,b^2}{8}+b^3\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\left(\frac{a^3}{8}-\frac{9\,a\,b^2}{8}+b^3\right)}{d}-\frac{\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + b*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (b^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (log(tan(c/2 + (d*x)/2) - 1)*((9*a*
b^2)/8 - a^3/8 + b^3))/d - (log(tan(c/2 + (d*x)/2) + 1)*(a^3/8 - (9*a*b^2)/
8 + b^3))/d - (tan(c/2 + (d*x)/2)*((9*a*b^2)/4 - a^3/4) + 2*b^3*tan(c/2 + (
d*x)/2)^2 + 2*b^3*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^7*((9*a*b^2)/4
- a^3/4) - tan(c/2 + (d*x)/2)^3*((33*a*b^2)/4 + (7*a^3)/4) - tan(c/2 + (d*x
)/2)^5*((33*a*b^2)/4 + (7*a^3)/4) - tan(c/2 + (d*x)/2)^4*(12*a^2*b + 8*b^3)
)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2
)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1505 $\int \sec^4(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=90

$$-\frac{3b\left(a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{3\sec^2(c+dx)(a+b\sin(c+dx))\left(ab\sin(c+dx)+b^2\right)}{8d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d}$$

[Out] -3/8*b*(a^2-b^2)*arctanh(sin(d*x+c))/d+1/4*sec(d*x+c)^4*(a+b*sin(d*x+c))^3/d-3/8*sec(d*x+c)^2*(a+b*sin(d*x+c))*(b^2+a*b*sin(d*x+c))/d

Rubi [A] time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2837, 12, 805, 723, 206}

$$-\frac{3b\left(a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{3\sec^2(c+dx)(a+b\sin(c+dx))\left(ab\sin(c+dx)+b^2\right)}{8d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]

[Out] (-3*b*(a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(4*d) - (3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])*(b^2 + a*b*Sin[c + d*x]))/(8*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x(a+x)^3}{b\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\,\text{Subst}\left(\int \frac{x(a+x)^3}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\left(3b^4\right)\text{Subst}\left(\int \frac{(a+x)^2}{\left(b^2-x^2\right)^2}\,dx \right.}{4d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{3\sec^2(c+dx)(a+b\sin(c}{}$$

$$= -\frac{3b\left(a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\sec^4(c+dx)(a+b\sin(c}{4d}$$

Mathematica [B] time = 1.47, size = 370, normalized size = 4.11

$$\frac{2b\left(a^2-b^2\right)\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^5 + 2a\left(a^2-b^2\right)^2\sec^4(c+dx)(a+b\sin(c+dx))^4 +}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (2*a*(a^2 - b^2)^2*Sec[c + d*x]^4*(a + b*Sin[c + d*x])^4 + 2*b*(a^2 - b^2)*
Sec[c + d*x]^4*(b - a*Sin[c + d*x])*(a + b*Sin[c + d*x])^5 + b*Sec[c + d*x]
^2*(a + b*Sin[c + d*x])^5*(5*a^2*b + b^3 - 3*a*(a^2 + b^2)*Sin[c + d*x]) +
((5*a^4*b + 10*a^2*b^3 + b^5)*(3*((a + b)^4*Log[1 - Sin[c + d*x]] - (a - b)
^4*Log[1 + Sin[c + d*x]]) + 6*b^2*(6*a^2 + b^2)*Sin[c + d*x] + 12*a*b^3*Sin
[c + d*x]^2 + 2*b^4*Sin[c + d*x]^3))/2 - a*b*(a^2 + b^2)*(6*(a + b)^5*Log[1
 - Sin[c + d*x]] - 6*(a - b)^5*Log[1 + Sin[c + d*x]] + 60*a*b^2*(2*a^2 + b^
2)*Sin[c + d*x] + 6*b^3*(10*a^2 + b^2)*Sin[c + d*x]^2 + 20*a*b^4*Sin[c + d*
x]^3 + 3*b^5*Sin[c + d*x]^4))/(8*(a^2 - b^2)^3*d)
```

fricas [A] time = 0.45, size = 143, normalized size = 1.59

$$-\frac{3\left(a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+24\,ab^{2}\cos\ldots}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/16*(3*(a^2*b - b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(a^2*b - b^
3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 24*a*b^2*cos(d*x + c)^2 - 4*a^3
- 12*a*b^2 - 2*(6*a^2*b + 2*b^3 - (3*a^2*b + 5*b^3)*cos(d*x + c)^2)*sin(d*x
 + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.26, size = 142, normalized size = 1.58

$$-\frac{3\left(a^{2}b-b^{3}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-3\left(a^{2}b-b^{3}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(3\,a^{2}b\sin(dx+c)^{3}+5\,b^{3}\sin(dx+c)^{3}+12\,ab^{2}\sin(\ldots\right.}{\left(\sin(\ldots\right.}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(3*(a^2*b - b^3)*log(abs(sin(d*x + c) + 1)) - 3*(a^2*b - b^3)*log(abs
(sin(d*x + c) - 1)) - 2*(3*a^2*b*sin(d*x + c)^3 + 5*b^3*sin(d*x + c)^3 + 12
*a*b^2*sin(d*x + c)^2 + 3*a^2*b*sin(d*x + c) - 3*b^3*sin(d*x + c) + 2*a^3 -
 6*a*b^2)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [B] time = 0.31, size = 231, normalized size = 2.57

$$\frac{a^{3}}{4d\cos\left(dx+c\right)^{4}}+\frac{3a^{2}b\left(\sin^{3}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}+\frac{3a^{2}b\left(\sin^{3}\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^{2}}+\frac{3a^{2}b\sin\left(dx+c\right)}{8d}-\frac{3a^{2}b\ln\left(\sec\left(dx+c\right)+\tan\left(dx\ldots\right.\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^3,x)

[Out] 1/4/d*a^3/cos(d*x+c)^4+3/4/d*a^2*b*sin(d*x+c)^3/cos(d*x+c)^4+3/8/d*a^2*b*sin(d*x+c)^3/cos(d*x+c)^2+3/8*a^2*b*sin(d*x+c)/d-3/8/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a*b^2*sin(d*x+c)^4/cos(d*x+c)^4+1/4/d*b^3*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*b^3*sin(d*x+c)^5/cos(d*x+c)^2-1/8*b^3*sin(d*x+c)^3/d-3/8*b^3*sin(d*x+c)/d+3/8/d*b^3*ln(sec(d*x+c)+tan(d*x+c))

maxima [A] time = 0.38, size = 140, normalized size = 1.56

$$-\frac{3\left(a^2b-b^3\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^2b-b^3\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(12\,ab^2\sin(dx+c)^2+\left(3\,a^2b+5\,b^3\right)\sin(dx+c)^3+2\,a^3-\right.}{\sin(dx+c)^4-2\,\sin(dx+c)^2-}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -1/16*(3*(a^2*b - b^3)*log(sin(d*x + c) + 1) - 3*(a^2*b - b^3)*log(sin(d*x + c) - 1) - 2*(12*a*b^2*sin(d*x + c)^2 + (3*a^2*b + 5*b^3)*sin(d*x + c)^3 + 2*a^3 - 6*a*b^2 + 3*(a^2*b - b^3)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d

mupad [B] time = 16.99, size = 228, normalized size = 2.53

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a^2\,b}{4}-\frac{3\,b^3}{4}\right)+2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(\frac{3\,a^2\,b}{4}-\frac{3\,b^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(c + d*x)*(a + b*sin(c + d*x))^3)/cos(c + d*x)^5,x)

[Out] (tan(c/2 + (d*x)/2)*((3*a^2*b)/4 - (3*b^3)/4) + 2*a^3*tan(c/2 + (d*x)/2)^2 + 2*a^3*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^7*((3*a^2*b)/4 - (3*b^3)/4) + tan(c/2 + (d*x)/2)^3*((21*a^2*b)/4 + (11*b^3)/4) + tan(c/2 + (d*x)/2)^5*((21*a^2*b)/4 + (11*b^3)/4) + 12*a*b^2*tan(c/2 + (d*x)/2)^4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (3*b*atanh(tan(c/2 + (d*x)/2))*(a^2 - b^2))/(4*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c))**3,x)

[Out] Timed out

3.1506 $\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=165

$$\frac{a^3\log(\sin(c+dx))}{d}+\frac{\sec^4(c+dx)\left(b\left(3a^2+b^2\right)\sin(c+dx)+a\left(a^2+3b^2\right)\right)}{4d}-\frac{\left(8a^3+9a^2b-b^3\right)\log(1-\sin(c+d$$

```
[Out] -1/16*(8*a^3+9*a^2*b-b^3)*ln(1-sin(d*x+c))/d+a^3*ln(sin(d*x+c))/d-1/16*(8*a
^3-9*a^2*b+b^3)*ln(1+sin(d*x+c))/d+1/8*sec(d*x+c)^2*(4*a^3+b*(9*a^2-b^2)*si
n(d*x+c))/d+1/4*sec(d*x+c)^4*(a*(a^2+3*b^2)+b*(3*a^2+b^2)*sin(d*x+c))/d
```

Rubi [A] time = 0.25, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2837, 12, 1805, 823, 801}

$$-\frac{\left(9a^2b+8a^3-b^3\right)\log(1-\sin(c+dx))}{16d}-\frac{\left(-9a^2b+8a^3+b^3\right)\log(\sin(c+dx)+1)}{16d}+\frac{\sec^4(c+dx)\left(b\left(3a^2+b^2\right)\mathrm{s}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((8*a^3 + 9*a^2*b - b^3)*Log[1 - Sin[c + d*x]])/(16*d) + (a^3*Log[Sin[c +
d*x]])/d - ((8*a^3 - 9*a^2*b + b^3)*Log[1 + Sin[c + d*x]])/(16*d) + (Sec[c
+ d*x]^2*(4*a^3 + b*(9*a^2 - b^2)*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^4*(a
*(a^2 + 3*b^2) + b*(3*a^2 + b^2)*Sin[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b(a+x)^3}{x\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^6\,\text{Subst}\left(\int \frac{(a+x)^3}{x\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a\left(a^2+3b^2\right)+b\left(3a^2+b^2\right)\sin(c+dx)\right)}{4d} - \frac{b^4\,\text{Su}}{}$$

$$= \frac{\sec^2(c+dx)\left(4a^3+b\left(9a^2-b^2\right)\sin(c+dx)\right)}{8d} + \frac{\sec^4(c+dx)}{}$$

$$= \frac{\sec^2(c+dx)\left(4a^3+b\left(9a^2-b^2\right)\sin(c+dx)\right)}{8d} + \frac{\sec^4(c+dx)}{}$$

$$= -\frac{\left(8a^3+9a^2b-b^3\right)\log(1-\sin(c+dx))}{16d} + \frac{a^3\log(\sin(c+dx)}{d}$$

Mathematica [A] time = 0.57, size = 157, normalized size = 0.95

$$\frac{16a^3 \log(\sin(c+dx)) - \left(8a^3+9a^2b-b^3\right)\log(1-\sin(c+dx)) - \left(8a^3-9a^2b+b^3\right)\log(\sin(c+dx)+1) - \frac{(5a-b)(}{\sin(c+}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-((8*a^3 + 9*a^2*b - b^3)*Log[1 - Sin[c + d*x]]) + 16*a^3*Log[Sin[c + d*x]
] - (8*a^3 - 9*a^2*b + b^3)*Log[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c +
 d*x])^2 - ((5*a - b)*(a + b)^2)/(-1 + Sin[c + d*x]) + (a - b)^3/(1 + Sin[c
 + d*x])^2 + ((a - b)^2*(5*a + b))/(1 + Sin[c + d*x]))/(16*d)
```

fricas [A] time = 0.46, size = 173, normalized size = 1.05

$$16\,a^3\cos(dx+c)^4\log\left(\frac{1}{2}\,\sin(dx+c)\right) - \left(8\,a^3-9\,a^2b+b^3\right)\cos(dx+c)^4\log(\sin(dx+c)+1) - \left(8\,a^3+9\,a^2b-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^3*cos(d*x + c)^4*log(1/2*sin(d*x + c)) - (8*a^3 - 9*a^2*b + b^3)
*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^3 + 9*a^2*b - b^3)*cos(d*x + c
)^4*log(-sin(d*x + c) + 1) + 8*a^3*cos(d*x + c)^2 + 4*a^3 + 12*a*b^2 + 2*(6
*a^2*b + 2*b^3 + (9*a^2*b - b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x +
 c)^4)
```

giac [A] time = 0.35, size = 175, normalized size = 1.06

$$\frac{16\,a^3\log(|\sin(dx+c)|) - \left(8\,a^3-9\,a^2b+b^3\right)\log(|\sin(dx+c)+1|) - \left(8\,a^3+9\,a^2b-b^3\right)\log(|\sin(dx+c)-1|)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/16*(16*a^3*log(abs(sin(d*x + c))) - (8*a^3 - 9*a^2*b + b^3)*log(abs(sin(d
*x + c) + 1)) - (8*a^3 + 9*a^2*b - b^3)*log(abs(sin(d*x + c) - 1)) + 2*(6*a
^3*sin(d*x + c)^4 - 9*a^2*b*sin(d*x + c)^3 + b^3*sin(d*x + c)^3 - 16*a^3*si
n(d*x + c)^2 + 15*a^2*b*sin(d*x + c) + b^3*sin(d*x + c) + 12*a^3 + 6*a*b^2)
/(sin(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.68, size = 216, normalized size = 1.31

$$\frac{a^3}{4d\cos(dx+c)^4}+\frac{a^3}{2d\cos(dx+c)^2}+\frac{a^3\ln(\tan(dx+c))}{d}+\frac{3a^2b\tan(dx+c)\left(\sec^3(dx+c)\right)}{4d}+\frac{9a^2b\tan(dx+c)\sec(}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3/cos(d*x+c)^4+1/2/d*a^3/cos(d*x+c)^2+1/d*a^3*ln(tan(d*x+c))+3/4/d*
a^2*b*tan(d*x+c)*sec(d*x+c)^3+9/8/d*a^2*b*tan(d*x+c)*sec(d*x+c)+9/8/d*a^2*b
*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a*b^2/cos(d*x+c)^4+1/4/d*b^3*sin(d*x+c)^3/
cos(d*x+c)^4+1/8/d*b^3*sin(d*x+c)^3/cos(d*x+c)^2+1/8*b^3*sin(d*x+c)/d-1/8/d
*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.33, size = 160, normalized size = 0.97

$$\frac{16\,a^3\log(\sin(dx+c))-\left(8\,a^3-9\,a^2b+b^3\right)\log(\sin(dx+c)+1)-\left(8\,a^3+9\,a^2b-b^3\right)\log(\sin(dx+c)-1)-\frac{2(}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^3*log(sin(d*x + c)) - (8*a^3 - 9*a^2*b + b^3)*log(sin(d*x + c) +
 1) - (8*a^3 + 9*a^2*b - b^3)*log(sin(d*x + c) - 1) - 2*(4*a^3*sin(d*x + c)
^2 + (9*a^2*b - b^3)*sin(d*x + c)^3 - 6*a^3 - 6*a*b^2 - (15*a^2*b + b^3)*si
n(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 11.91, size = 169, normalized size = 1.02

$$\frac{a^3\,\ln(\sin(c+d\,x))}{d}-\frac{\ln(\sin(c+d\,x)+1)\,\left(\frac{a^3}{2}-\frac{9\,a^2\,b}{16}+\frac{b^3}{16}\right)}{d}-\frac{\ln(\sin(c+d\,x)-1)\,\left(\frac{a^3}{2}+\frac{9\,a^2\,b}{16}-\frac{b^3}{16}\right)}{d}+\frac{\frac{3\,a\,b^2}{4}-\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(cos(c + d*x)^5*sin(c + d*x)),x)
```

```
[Out] (a^3*log(sin(c + d*x)))/d - (log(sin(c + d*x) + 1)*(a^3/2 - (9*a^2*b)/16 +
b^3/16))/d - (log(sin(c + d*x) - 1)*((9*a^2*b)/16 + a^3/2 - b^3/16))/d + ((
3*a*b^2)/4 - sin(c + d*x)^3*((9*a^2*b)/8 - b^3/8) + (3*a^3)/4 + sin(c + d*x
)*((15*a^2*b)/8 + b^3/8) - (a^3*sin(c + d*x)^2)/2)/(d*(sin(c + d*x)^4 - 2*s
in(c + d*x)^2 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1507 $\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=171

$$-\frac{a^3\csc(c+dx)}{d}+\frac{b\sec^4(c+dx)\left(ab\left(\frac{a^2}{b^2}+3\right)\sin(c+dx)+3a^2+b^2\right)}{4d}+\frac{ab\sec^2(c+dx)\left(b\left(\frac{7a^2}{b^2}+9\right)\sin(c+dx)+1\right.}{8d}$$

```
[Out] -a^3*csc(d*x+c)/d-3/16*a*(a+b)*(5*a+3*b)*ln(1-sin(d*x+c))/d+3*a^2*b*ln(sin(
d*x+c))/d+3/16*a*(5*a-3*b)*(a-b)*ln(1+sin(d*x+c))/d+1/4*b*sec(d*x+c)^4*(3*a
^2+b^2+a*(3+1/b^2*a^2)*b*sin(d*x+c))/d+1/8*a*b*sec(d*x+c)^2*(12*a+(9+7/b^2*
a^2)*b*sin(d*x+c))/d
```

Rubi [A] time = 0.37, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 12, 1805, 1802}

$$\frac{b\sec^4(c+dx)\left(ab\left(\frac{a^2}{b^2}+3\right)\sin(c+dx)+3a^2+b^2\right)}{4d}+\frac{ab\sec^2(c+dx)\left(b\left(\frac{7a^2}{b^2}+9\right)\sin(c+dx)+12a\right)}{8d}+\frac{3a^2b\log(\sin}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) - (3*a*(a + b)*(5*a + 3*b)*Log[1 - Sin[c + d*x]])/(
16*d) + (3*a^2*b*Log[Sin[c + d*x]])/d + (3*a*(5*a - 3*b)*(a - b)*Log[1 + Si
n[c + d*x]])/(16*d) + (b*Sec[c + d*x]^4*(3*a^2 + b^2 + a*(3 + a^2/b^2)*b*Si
n[c + d*x]))/(4*d) + (a*b*Sec[c + d*x]^2*(12*a + (9 + (7*a^2)/b^2)*b*Sin[c
+ d*x]))/(8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
```

```
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b^2(a+x)^3}{x^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^7\,\text{Subst}\left(\int \frac{(a+x)^3}{x^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\sec^4(c+dx)\left(3a^2+b^2+a\left(3+\frac{a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} - \frac{b^5\,\text{S}}{}$$

$$= \frac{b\sec^4(c+dx)\left(3a^2+b^2+a\left(3+\frac{a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} + \frac{ab\,\text{s}}{}$$

$$= \frac{b\sec^4(c+dx)\left(3a^2+b^2+a\left(3+\frac{a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} + \frac{ab\,\text{s}}{}$$

$$= -\frac{a^3\csc(c+dx)}{d} - \frac{3a(a+b)(5a+3b)\log(1-\sin(c+dx))}{16d}$$

Mathematica [A] time = 1.45, size = 161, normalized size = 0.94

$$-\frac{16a^3\csc(c+dx) - 48a^2b\log(\sin(c+dx)) + \frac{(a+b)^2(7a+b)}{\sin(c+dx)-1} + \frac{(a-b)^2(7a-b)}{\sin(c+dx)+1} - \frac{(a+b)^3}{(\sin(c+dx)-1)^2} + \frac{(a-b)^3}{(\sin(c+dx)+1)^2} + 3a(a+b)(}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/16*(16*a^3*Csc[c + d*x] + 3*a*(a + b)*(5*a + 3*b)*Log[1 - Sin[c + d*x]]
- 48*a^2*b*Log[Sin[c + d*x]] - 3*a*(5*a - 3*b)*(a - b)*Log[1 + Sin[c + d*x]
] - (a + b)^3/(-1 + Sin[c + d*x])^2 + ((a + b)^2*(7*a + b))/(-1 + Sin[c + d
*x]) + (a - b)^3/(1 + Sin[c + d*x])^2 + ((a - b)^2*(7*a - b))/(1 + Sin[c +
d*x]))/d
```

fricas [A] time = 0.47, size = 226, normalized size = 1.32

$$\frac{48\,a^2b\cos(dx+c)^4\log\left(\frac{1}{2}\,\sin(dx+c)\right)\sin(dx+c)+3\left(5\,a^3-8\,a^2b+3\,ab^2\right)\cos(dx+c)^4\log(\sin(dx+c)+1)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(48*a^2*b*cos(d*x + c)^4*log(1/2*sin(d*x + c))*sin(d*x + c) + 3*(5*a^3
 - 8*a^2*b + 3*a*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1)*sin(d*x + c) - 3
*(5*a^3 + 8*a^2*b + 3*a*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1)*sin(d*x
+ c) - 6*(5*a^3 + 3*a*b^2)*cos(d*x + c)^4 + 4*a^3 + 12*a*b^2 + 2*(5*a^3 + 3
*a*b^2)*cos(d*x + c)^2 + 4*(6*a^2*b*cos(d*x + c)^2 + 3*a^2*b + b^3)*sin(d*x
 + c))/(d*cos(d*x + c)^4*sin(d*x + c))
```

giac [A] time = 0.30, size = 210, normalized size = 1.23

$$\frac{48\,a^2b\log(|\sin(dx+c)|)+3\left(5\,a^3-8\,a^2b+3\,ab^2\right)\log(|\sin(dx+c)+1|)-3\left(5\,a^3+8\,a^2b+3\,ab^2\right)\log(|\sin(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/16*(48*a^2*b*log(abs(sin(d*x + c))) + 3*(5*a^3 - 8*a^2*b + 3*a*b^2)*log(a
bs(sin(d*x + c) + 1)) - 3*(5*a^3 + 8*a^2*b + 3*a*b^2)*log(abs(sin(d*x + c)
- 1)) - 16*(3*a^2*b*sin(d*x + c) + a^3)/sin(d*x + c) + 2*(18*a^2*b*sin(d*x
+ c)^4 - 7*a^3*sin(d*x + c)^3 - 9*a*b^2*sin(d*x + c)^3 - 48*a^2*b*sin(d*x +
 c)^2 + 9*a^3*sin(d*x + c) + 15*a*b^2*sin(d*x + c) + 36*a^2*b + 2*b^3)/(sin
(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.64, size = 221, normalized size = 1.29

$$\frac{a^3}{4d\sin(dx+c)\cos(dx+c)^4}+\frac{5a^3}{8d\sin(dx+c)\cos(dx+c)^2}-\frac{15a^3}{8d\sin(dx+c)}+\frac{15a^3\ln(\sec(dx+c)+\tan(dx+c))}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3/sin(d*x+c)/cos(d*x+c)^4+5/8/d*a^3/sin(d*x+c)/cos(d*x+c)^2-15/8/d*
a^3/sin(d*x+c)+15/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a^2*b/cos(d*x+c)^
4+3/2/d*a^2*b/cos(d*x+c)^2+3/d*a^2*b*ln(tan(d*x+c))+3/4/d*a*b^2*tan(d*x+c)*
sec(d*x+c)^3+9/8/d*a*b^2*tan(d*x+c)*sec(d*x+c)+9/8/d*a*b^2*ln(sec(d*x+c)+ta
n(d*x+c))+1/4/d*b^3/cos(d*x+c)^4
```

maxima [A] time = 0.34, size = 188, normalized size = 1.10

$$\frac{48\,a^2b\log\left(\sin\left(dx+c\right)\right)+3\left(5\,a^3-8\,a^2b+3\,ab^2\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(5\,a^3+8\,a^2b+3\,ab^2\right)\log\left(\sin\left(dx+\right.\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/16*(48*a^2*b*log(sin(d*x + c)) + 3*(5*a^3 - 8*a^2*b + 3*a*b^2)*log(sin(d*
x + c) + 1) - 3*(5*a^3 + 8*a^2*b + 3*a*b^2)*log(sin(d*x + c) - 1) - 2*(12*a
^2*b*sin(d*x + c)^3 + 3*(5*a^3 + 3*a*b^2)*sin(d*x + c)^4 + 8*a^3 - 5*(5*a^3
 + 3*a*b^2)*sin(d*x + c)^2 - 2*(9*a^2*b + b^3)*sin(d*x + c))/(sin(d*x + c)^
5 - 2*sin(d*x + c)^3 + sin(d*x + c)))/d
```

mupad [B] time = 0.13, size = 182, normalized size = 1.06

$$\frac{3\,a^2\,b\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{{\sin\left(c+d\,x\right)}^4\,\left(\frac{15\,a^3}{8}+\frac{9\,a\,b^2}{8}\right)-{\sin\left(c+d\,x\right)}^2\,\left(\frac{25\,a^3}{8}+\frac{15\,a\,b^2}{8}\right)+a^3-\sin\left(c+d\,x\right)\,\left(\frac{9\,a^2\,b}{4}+\right.}{d\,\left({\sin\left(c+d\,x\right)}^5-2\,{\sin\left(c+d\,x\right)}^3+\sin\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(cos(c + d*x)^5*sin(c + d*x)^2),x)
```

```
[Out] (3*a^2*b*log(sin(c + d*x)))/d - (sin(c + d*x)^4*((9*a*b^2)/8 + (15*a^3)/8)
- sin(c + d*x)^2*((15*a*b^2)/8 + (25*a^3)/8) + a^3 - sin(c + d*x)*((9*a^2*b
)/4 + b^3/4) + (3*a^2*b*sin(c + d*x)^3)/2)/(d*(sin(c + d*x) - 2*sin(c + d*x
)^3 + sin(c + d*x)^5)) + (3*a*log(sin(c + d*x) + 1)*(a - b)*(5*a - 3*b))/(1
6*d) - (3*a*log(sin(c + d*x) - 1)*(a + b)*(5*a + 3*b))/(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1508 $\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=221

$$-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3(a+b)\left(8a^2+7ab+b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{3a\left(a^2+b^2\right)\log(\sin(c+dx))}{d}-\frac{3(a-b)\left(8a^2-\right.}{}$$

```
[Out] -3*a^2*b*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d-3/16*(a+b)*(8*a^2+7*a*b+b^2)*l
n(1-sin(d*x+c))/d+3*a*(a^2+b^2)*ln(sin(d*x+c))/d-3/16*(a-b)*(8*a^2-7*a*b+b^
2)*ln(1+sin(d*x+c))/d+1/4*b^2*sec(d*x+c)^4*(a*(3+1/b^2*a^2)+(1+3/b^2*a^2)*b
*sin(d*x+c))/d+1/8*b^2*sec(d*x+c)^2*(4*a*(3+2/b^2*a^2)+3*(1+7/b^2*a^2)*b*si
n(d*x+c))/d
```

Rubi [A] time = 0.43, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 12, 1805, 1802}

$$-\frac{3(a+b)\left(8a^2+7ab+b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{3a\left(a^2+b^2\right)\log(\sin(c+dx))}{d}-\frac{3(a-b)\left(8a^2-7ab+b^2\right)\log(\mathrm{si}}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^2*b*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (3*(a + b)*(8*a^2
+ 7*a*b + b^2)*Log[1 - Sin[c + d*x]])/(16*d) + (3*a*(a^2 + b^2)*Log[Sin[c +
 d*x]])/d - (3*(a - b)*(8*a^2 - 7*a*b + b^2)*Log[1 + Sin[c + d*x]])/(16*d)
+ (b^2*Sec[c + d*x]^4*(a*(3 + a^2/b^2) + (1 + (3*a^2)/b^2)*b*Sin[c + d*x]))
/(4*d) + (b^2*Sec[c + d*x]^2*(4*a*(3 + (2*a^2)/b^2) + 3*(1 + (7*a^2)/b^2)*b
*Sin[c + d*x]))/(8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5\,\mathrm{Subst}\left(\int \frac{b^3(a+x)^3}{x^3\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^8\,\mathrm{Subst}\left(\int \frac{(a+x)^3}{x^3\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\sec^4(c+dx)\left(a\left(3+\frac{a^2}{b^2}\right)+\left(1+\frac{3a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} - \frac{b^6\,\mathrm{S}}{}$$

$$= \frac{b^2\sec^4(c+dx)\left(a\left(3+\frac{a^2}{b^2}\right)+\left(1+\frac{3a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} + \frac{b^2\,\mathrm{s}}{}$$

$$= \frac{b^2\sec^4(c+dx)\left(a\left(3+\frac{a^2}{b^2}\right)+\left(1+\frac{3a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} + \frac{b^2\,\mathrm{s}}{}$$

$$= -\frac{3a^2b\csc(c+dx)}{d} - \frac{a^3\csc^2(c+dx)}{2d} - \frac{3(a+b)\left(8a^2+7ab\right.}{}$$

Mathematica [A] time = 3.10, size = 190, normalized size = 0.86

$$\frac{-8a^3\csc^2(c+dx)+48a\left(a^2+b^2\right)\log(\sin(c+dx))-3(a+b)\left(8a^2+7ab+b^2\right)\log(1-\sin(c+dx))-3(a-b)\left(8\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-48*a^2*b*Csc[c + d*x] - 8*a^3*Csc[c + d*x]^2 - 3*(a + b)*(8*a^2 + 7*a*b +
 b^2)*Log[1 - Sin[c + d*x]] + 48*a*(a^2 + b^2)*Log[Sin[c + d*x]] - 3*(a - b
)*(8*a^2 - 7*a*b + b^2)*Log[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c + d*x
])^2 - (3*(a + b)^2*(3*a + b))/(-1 + Sin[c + d*x]) + (a - b)^3/(1 + Sin[c +
 d*x])^2 + (3*(a - b)^2*(3*a - b))/(1 + Sin[c + d*x]))/(16*d)
```

fricas [A] time = 0.48, size = 337, normalized size = 1.52

$$\frac{24\left(a^3+ab^2\right)\cos\left(dx+c\right)^4-4\,a^3-12\,ab^2-12\left(a^3+ab^2\right)\cos\left(dx+c\right)^2+48\left(\left(a^3+ab^2\right)\cos\left(dx+c\right)^6-\left(a^3+a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(24*(a^3 + a*b^2)*cos(d*x + c)^4 - 4*a^3 - 12*a*b^2 - 12*(a^3 + a*b^2)
*cos(d*x + c)^2 + 48*((a^3 + a*b^2)*cos(d*x + c)^6 - (a^3 + a*b^2)*cos(d*x
+ c)^4)*log(1/2*sin(d*x + c)) - 3*((8*a^3 - 15*a^2*b + 8*a*b^2 - b^3)*cos(d
*x + c)^6 - (8*a^3 - 15*a^2*b + 8*a*b^2 - b^3)*cos(d*x + c)^4)*log(sin(d*x
+ c) + 1) - 3*((8*a^3 + 15*a^2*b + 8*a*b^2 + b^3)*cos(d*x + c)^6 - (8*a^3 +
 15*a^2*b + 8*a*b^2 + b^3)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) + 2*(3*(1
5*a^2*b + b^3)*cos(d*x + c)^4 - 6*a^2*b - 2*b^3 - (15*a^2*b + b^3)*cos(d*x
+ c)^2)*sin(d*x + c))/(d*cos(d*x + c)^6 - d*cos(d*x + c)^4)
```

giac [A] time = 0.35, size = 240, normalized size = 1.09

$$-\frac{3\left(8\,a^3-15\,a^2b+8\,ab^2-b^3\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\left(8\,a^3+15\,a^2b+8\,ab^2+b^3\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(3*(8*a^3 - 15*a^2*b + 8*a*b^2 - b^3)*log(abs(sin(d*x + c) + 1)) + 3*
(8*a^3 + 15*a^2*b + 8*a*b^2 + b^3)*log(abs(sin(d*x + c) - 1)) - 48*(a^3 + a
```

```
*b^2)*log(abs(sin(d*x + c))) + 2*(45*a^2*b*sin(d*x + c)^5 + 3*b^3*sin(d*x +
 c)^5 + 12*a^3*sin(d*x + c)^4 + 12*a*b^2*sin(d*x + c)^4 - 75*a^2*b*sin(d*x
+ c)^3 - 5*b^3*sin(d*x + c)^3 - 18*a^3*sin(d*x + c)^2 - 18*a*b^2*sin(d*x +
c)^2 + 24*a^2*b*sin(d*x + c) + 4*a^3)/(sin(d*x + c)^3 - sin(d*x + c))^2)/d
```

maple [A] time = 0.74, size = 285, normalized size = 1.29

$$\frac{a^3}{4d\sin(dx+c)^2\cos(dx+c)^4}+\frac{3a^3}{4d\sin(dx+c)^2\cos(dx+c)^2}-\frac{3a^3}{2d\sin(dx+c)^2}+\frac{3a^3\ln(\tan(dx+c))}{d}+\frac{}{4d\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3/sin(d*x+c)^2/cos(d*x+c)^4+3/4/d*a^3/sin(d*x+c)^2/cos(d*x+c)^2-3/2
/d*a^3/sin(d*x+c)^2+3/d*a^3*ln(tan(d*x+c))+3/4/d*a^2*b/sin(d*x+c)/cos(d*x+c
)^4+15/8/d*a^2*b/sin(d*x+c)/cos(d*x+c)^2-45/8/d*a^2*b/sin(d*x+c)+45/8/d*a^2
*b*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a*b^2/cos(d*x+c)^4+3/2/d*a*b^2/cos(d*x+c
)^2+3/d*a*b^2*ln(tan(d*x+c))+1/4/d*b^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*b^3*ta
n(d*x+c)*sec(d*x+c)+3/8/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.34, size = 217, normalized size = 0.98

$$-\frac{3\left(8a^3-15a^2b+8ab^2-b^3\right)\log(\sin(dx+c)+1)+3\left(8a^3+15a^2b+8ab^2+b^3\right)\log(\sin(dx+c)-1)-48\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(3*(8*a^3 - 15*a^2*b + 8*a*b^2 - b^3)*log(sin(d*x + c) + 1) + 3*(8*a^
3 + 15*a^2*b + 8*a*b^2 + b^3)*log(sin(d*x + c) - 1) - 48*(a^3 + a*b^2)*log(
sin(d*x + c)) + 2*(3*(15*a^2*b + b^3)*sin(d*x + c)^5 + 12*(a^3 + a*b^2)*sin
(d*x + c)^4 + 24*a^2*b*sin(d*x + c) - 5*(15*a^2*b + b^3)*sin(d*x + c)^3 + 4
*a^3 - 18*(a^3 + a*b^2)*sin(d*x + c)^2)/(sin(d*x + c)^6 - 2*sin(d*x + c)^4
+ sin(d*x + c)^2))/d
```

mupad [B] time = 11.95, size = 221, normalized size = 1.00

$$\frac{3a\ln(\sin(c+dx))\left(a^2+b^2\right)}{d}-\frac{\sin(c+dx)^4\left(\frac{3a^3}{2}+\frac{3ab^2}{2}\right)-\sin(c+dx)^2\left(\frac{9a^3}{4}+\frac{9ab^2}{4}\right)+\sin(c+dx)^5\left(\frac{45a^2b}{8}+\right.}{d\left(\sin(c+dx)^6-2\sin(c+dx)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(cos(c + d*x)^5*sin(c + d*x)^3),x)
```

```
[Out] (3*a*log(sin(c + d*x))*(a^2 + b^2))/d - (sin(c + d*x)^4*((3*a*b^2)/2 + (3*a
^3)/2) - sin(c + d*x)^2*((9*a*b^2)/4 + (9*a^3)/4) + sin(c + d*x)^5*((45*a^2
*b)/8 + (3*b^3)/8) - sin(c + d*x)^3*((75*a^2*b)/8 + (5*b^3)/8) + a^3/2 + 3*
a^2*b*sin(c + d*x))/(d*(sin(c + d*x)^2 - 2*sin(c + d*x)^4 + sin(c + d*x)^6)
) - (3*log(sin(c + d*x) + 1)*(a - b)*(8*a^2 - 7*a*b + b^2))/(16*d) - (3*log
(sin(c + d*x) - 1)*(a + b)*(7*a*b + 8*a^2 + b^2))/(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1509 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=295

$$-\frac{abn\left(a^2(2-n)-b^2(n+2)\right)\sin^{n+2}(c+dx)\,{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{2d(n+2)}-\frac{\left(-\left(a^4\left(n^2-4n+3\right)\right)+6a^2b^2\left(1-n^2\right.\right.}{}$$

```
[Out] -1/8*(6*a^2*b^2*(-n^2+1)-a^4*(n^2-4*n+3)-b^4*(n^2+4*n+3))*hypergeom([1, 1/2
+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^(1+n)/d/(1+n)-1/2*a*b*n*(a^2*(
2-n)-b^2*(2+n))*hypergeom([1, 1+1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(
2+n)/d/(2+n)+1/4*sec(d*x+c)^4*sin(d*x+c)^(1+n)*(a^4+6*a^2*b^2+b^4+4*a*b*(a^
2+b^2)*sin(d*x+c))/d+1/8*sec(d*x+c)^2*sin(d*x+c)^(1+n)*(a^4*(3-n)-6*a^2*b^2
*(1+n)-b^4*(5+n)+4*a*b*(a^2*(2-n)-b^2*(2+n))*sin(d*x+c))/d
```

Rubi [A] time = 0.53, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 1806, 808, 364}

$$-\frac{\left(6a^2b^2\left(1-n^2\right)+a^4\left(-\left(n^2-4n+3\right)\right)-b^4\left(n^2+4n+3\right)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{8d(n+1)}-\frac{abn\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] -((6*a^2*b^2*(1 - n^2) - a^4*(3 - 4*n + n^2) - b^4*(3 + 4*n + n^2))*Hyperge
ometric2F1[1, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(
8*d*(1 + n)) - (a*b*n*(a^2*(2 - n) - b^2*(2 + n))*Hypergeometric2F1[1, (2 +
 n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(2*d*(2 + n)) + (Se
c[c + d*x]^4*Sin[c + d*x]^(1 + n)*(a^4 + 6*a^2*b^2 + b^4 + 4*a*b*(a^2 + b^2
)*Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^2*Sin[c + d*x]^(1 + n)*(a^4*(3 - n)
- 6*a^2*b^2*(1 + n) - b^4*(5 + n) + 4*a*b*(a^2*(2 - n) - b^2*(2 + n))*Sin[c
 + d*x]))/(8*d)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
```

```
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^4\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n (a+x)^4}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^4+6a^2b^2+b^4+4ab\left(a^2+b\right.\right.}{4d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^4+6a^2b^2+b^4+4ab\left(a^2+b\right.\right.}{4d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^4+6a^2b^2+b^4+4ab\left(a^2+b\right.\right.}{4d}$$

$$= -\frac{\left(6a^2b^2\left(1-n^2\right)-a^4\left(3-4n+n^2\right)-b^4\left(3+4n+n^2\right)\right)\ {}_2}{8d(1+n)}$$

Mathematica [A] time = 0.20, size = 164, normalized size = 0.56

$$\frac{\sin^{n+1}(c+dx)\left(6\left(a^2-b^2\right)^2 {}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+2(a-b)^4 {}_2F_1(3,n+1;n+2;-\sin(c+dx))+(3a+5\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((6*(a^2 - b^2)^2*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2
] + (a - b)^3*(3*a + 5*b)*Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]]
 + (3*a - 5*b)*(a + b)^3*Hypergeometric2F1[2, 1 + n, 2 + n, Sin[c + d*x]] +
 2*(a - b)^4*Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]] + 2*(a + b)^
4*Hypergeometric2F1[3, 1 + n, 2 + n, Sin[c + d*x]])*Sin[c + d*x]^(1 + n))/(
16*d*(1 + n))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(4\left(ab^3\cos\left(dx+c\right)^2-a^3b-ab^3\right)\sec\left(dx+c\right)^5\sin\left(dx+c\right)-\left(b^4\cos\left(dx+c\right)^4+a^4+6\,a^2b^2+b^4-2\,\left(\ldots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] integral(-(4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sec(d*x + c)^5*sin(d*x
+ c) - (b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*co
s(d*x + c)^2)*sec(d*x + c)^5)*sin(d*x + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)+a\right)^4\sin\left(dx+c\right)^n\sec\left(dx+c\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

maple [F] time = 5.40, size = 0, normalized size = 0.00

$$\int \left(\sec^5\left(dx+c\right)\right)\left(\sin^n\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x)
```

```
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^4 \sin(dx+c)^n \sec(dx+c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)^n\,(a+b\,\sin(c+d\,x))^4}{\cos(c+d\,x)^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^4)/cos(c + d*x)^5,x)
```

```
[Out] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^4)/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.1510 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=186

$$\frac{a\left(a^2(3-n)-3b^2(n+1)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{4d(n+1)}+\frac{b\left(3a^2(2-n)-b^2(n+2)\right)\sin^{n+2}(c+dx}{4d(n+2)}$$

```
[Out] 1/4*a*(a^2*(3-n)-3*b^2*(1+n))*hypergeom([2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+
c)^2)*sin(d*x+c)^(1+n)/d/(1+n)+1/4*b*(3*a^2*(2-n)-b^2*(2+n))*hypergeom([2,
1+1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)+1/4*sec(d*x+c)^4*
sin(d*x+c)^(1+n)*(a*(a^2+3*b^2)+b*(3*a^2+b^2)*sin(d*x+c))/d
```

Rubi [A] time = 0.30, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 1806, 808, 364}

$$\frac{a\left(a^2(3-n)-3b^2(n+1)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{4d(n+1)}+\frac{b\left(3a^2(2-n)-b^2(n+2)\right)\sin^{n+2}(c+dx}{4d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(a^2*(3 - n) - 3*b^2*(1 + n))*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2,
 Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(4*d*(1 + n)) + (b*(3*a^2*(2 - n) -
b^2*(2 + n))*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin
[c + d*x]^(2 + n))/(4*d*(2 + n)) + (Sec[c + d*x]^4*Sin[c + d*x]^(1 + n)*(a*
(a^2 + 3*b^2) + b*(3*a^2 + b^2)*Sin[c + d*x]))/(4*d)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)^3}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a\left(a^2+3b^2\right)+b\left(3a^2+b^2\right)\sin\right.}{4d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a\left(a^2+3b^2\right)+b\left(3a^2+b^2\right)\sin\right.}{4d}$$

$$= \frac{a\left(a^2(3-n)-3b^2(1+n)\right)\,{}_2F_1\left(2,\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\text{s}}{4d(1+n)}$$

Mathematica [A] time = 0.16, size = 158, normalized size = 0.85

$$\frac{\sin^{n+1}(c+dx)\left(6a(a+b)(a-b)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+2(a-b)^3\,{}_2F_1(3,n+1;n+2;-\sin(c+dx))+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((6*a*(a - b)*(a + b)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sin[c + d*
x]^2] + 3*(a - b)^2*(a + b)*Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x
]] + 3*(a - b)*(a + b)^2*Hypergeometric2F1[2, 1 + n, 2 + n, Sin[c + d*x]] +
 2*(a - b)^3*Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]] + 2*(a + b)^
3*Hypergeometric2F1[3, 1 + n, 2 + n, Sin[c + d*x]])*Sin[c + d*x]^(1 + n))/(
16*d*(1 + n))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

integral $\left(-\left(\left(b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3\right)\sec\left(dx+c\right)^5\sin\left(dx+c\right)+\left(3\,ab^2\cos\left(dx+c\right)^2-a^3-3\,ab^2\right)\sec\left(dx+\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] integral(-((b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sec(d*x + c)^5*sin(d*x + c)
 + (3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2)*sec(d*x + c)^5)*sin(d*x + c)^n,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin\left(dx+c\right)+a)^3\sin\left(dx+c\right)^n\sec\left(dx+c\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

maple [F] time = 5.23, size = 0, normalized size = 0.00

$$\int \left(\sec^5\left(dx+c\right)\right)\left(\sin^n\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x)
```

```
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin\left(dx+c\right)+a)^3\sin\left(dx+c\right)^n\sec\left(dx+c\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)^n\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3}{{\cos\left(c+d\,x\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^3)/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.1511 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=160

$$\frac{\left(a^2(3-n)-b^2(n+1)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{4d(n+1)}+\frac{\sec^4(c+dx)\sin^{n+1}(c+dx)\left(a^2+2ab\sin(c\right.}{4d}$$

```
[Out] 1/4*(a^2*(3-n)-b^2*(1+n))*hypergeom([2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2
)*sin(d*x+c)^(1+n)/d/(1+n)+1/2*a*b*(2-n)*hypergeom([2, 1+1/2*n],[1/2*n+2],s
in(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)+1/4*sec(d*x+c)^4*sin(d*x+c)^(1+n)*(a^
2+b^2+2*a*b*sin(d*x+c))/d
```

Rubi [A] time = 0.26, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 1806, 808, 364}

$$\frac{\left(a^2(3-n)-b^2(n+1)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{4d(n+1)}+\frac{\sec^4(c+dx)\sin^{n+1}(c+dx)\left(a^2+2ab\sin(c\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2*(3 - n) - b^2*(1 + n))*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, Sin
[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(4*d*(1 + n)) + (a*b*(2 - n)*Hypergeomet
ric2F1[2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(2*d*
(2 + n)) + (Sec[c + d*x]^4*Sin[c + d*x]^(1 + n)*(a^2 + b^2 + 2*a*b*Sin[c +
d*x]))/(4*d)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)^2}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} -$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} +$$

$$= \frac{\left(a^2(3-n)-b^2(1+n)\right)\,{}_2F_1\left(2,\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^1}{4d(1+n)}$$

Mathematica [A] time = 0.22, size = 158, normalized size = 0.99

$$\frac{\sin^{n+1}(c+dx)\left(2\left(3a^2-b^2\right)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+2(a-b)^2\,{}_2F_1(3,n+1;n+2;-\sin(c+dx))+(3a+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*(3*a^2 - b^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2
] + (a - b)*(3*a + b)*Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]] + (
3*a - b)*(a + b)*Hypergeometric2F1[2, 1 + n, 2 + n, Sin[c + d*x]] + 2*(a -
b)^2*Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]] + 2*(a + b)^2*Hyperg
eometric2F1[3, 1 + n, 2 + n, Sin[c + d*x]])*Sin[c + d*x]^(1 + n))/(16*d*(1
+ n))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(2\,ab\sec\left(dx+c\right)^{5}\sin\left(dx+c\right)-\left(b^{2}\cos\left(dx+c\right)^{2}-a^{2}-b^{2}\right)\sec\left(dx+c\right)^{5}\right)\sin\left(dx+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral((2*a*b*sec(d*x + c)^5*sin(d*x + c) - (b^2*cos(d*x + c)^2 - a^2 - b
^2)*sec(d*x + c)^5)*sin(d*x + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)+a\right)^{2}\sin\left(dx+c\right)^{n}\sec\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

maple [F] time = 4.76, size = 0, normalized size = 0.00

$$\int\left(\sec^{5}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

```
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)+a\right)^{2}\sin\left(dx+c\right)^{n}\sec\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)^n\,\left(a+b\,\sin(c+d\,x)\right)^2}{\cos(c+d\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1512 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=89

$$\frac{a\sin^{n+1}(c+dx)\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)}+\frac{b\sin^{n+2}(c+dx)\,{}_2F_1\left(3,\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)}$$

```
[Out] a*hypergeom([3, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^(1+n)/d/(1+
n)+b*hypergeom([3, 1+1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n
)
```

Rubi [A] time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 808, 364}

$$\frac{a\sin^{n+1}(c+dx)\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)}+\frac{b\sin^{n+2}(c+dx)\,{}_2F_1\left(3,\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^
(1 + n))/(d*(1 + n)) + (b*Hypergeometric2F1[3, (2 + n)/2, (4 + n)/2, Sin[c
+ d*x]^2]*Sin[c + d*x]^(2 + n))/(d*(2 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)}{\left(b^2-x^2\right)^3}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= \frac{\left(ab^5\right)\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n}{\left(b^2-x^2\right)^3}\,dx,x,b\sin(c+dx)\right)}{d} + \frac{b^6\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{a\,{}_2F_1\left(3,\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)} + \frac{b\,{}_2F_1\left(3,\right.}{}$$

Mathematica [A] time = 0.12, size = 89, normalized size = 1.00

$$\frac{\sin^{n+1}(c+dx)\left(a(n+2)\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+b(n+1)\sin(c+dx)\,{}_2F_1\left(3,\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)\right)}{d(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(a*(2 + n)*Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2,
 Sin[c + d*x]^2] + b*(1 + n)*Hypergeometric2F1[3, (2 + n)/2, (4 + n)/2, Sin
[c + d*x]^2]*Sin[c + d*x]))/(d*(1 + n)*(2 + n))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec(dx+c)^5\sin(dx+c)+a\sec(dx+c)^5\right)\sin(dx+c)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^5*sin(d*x + c) + a*sec(d*x + c)^5)*sin(d*x + c)^n,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)\sin(dx+c)^n\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

maple [F] time = 1.70, size = 0, normalized size = 0.00

$$\int \left(\sec^{5}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)^{n}\sec\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(c+d\,x\right)}^n\,\left(a+b\,\sin\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^n*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
[Out] int((sin(c + d*x)^n*(a + b*sin(c + d*x)))/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c)),x)
[Out] Timed out
```

3.1513 $\int \frac{\sec^5(c+dx)\sin^n(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=360

$$\frac{(3a^2-9ab+8b^2)\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{16d(n+1)(a-b)^3}+\frac{(3a^2+9ab+8b^2)\sin^{n+1}(c+dx)\,{}_2F_1(1,n+$$

```
[Out] 1/16*(3*a^2-9*a*b+8*b^2)*hypergeom([1, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(
1+n)/(a-b)^3/d/(1+n)+1/16*(3*a^2+9*a*b+8*b^2)*hypergeom([1, 1+n],[2+n],sin(
d*x+c))*sin(d*x+c)^(1+n)/(a+b)^3/d/(1+n)-b^6*hypergeom([1, 1+n],[2+n],-b*si
n(d*x+c)/a)*sin(d*x+c)^(1+n)/a/(a^2-b^2)^3/d/(1+n)+1/16*(3*a-5*b)*hypergeom
([2, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(1+n)/(a-b)^2/d/(1+n)+1/16*(3*a+5*b
)*hypergeom([2, 1+n],[2+n],sin(d*x+c))*sin(d*x+c)^(1+n)/(a+b)^2/d/(1+n)+1/8
*hypergeom([3, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(1+n)/(a-b)/d/(1+n)+1/8*h
ypergeom([3, 1+n],[2+n],sin(d*x+c))*sin(d*x+c)^(1+n)/(a+b)/d/(1+n)
```

Rubi [A] time = 0.54, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = $\{2837, 961, 64\}$

$$\frac{(3a^2-9ab+8b^2)\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{16d(n+1)(a-b)^3}+\frac{(3a^2+9ab+8b^2)\sin^{n+1}(c+dx)\,{}_2F_1(1,n+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((3*a^2 - 9*a*b + 8*b^2)*Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*
Sin[c + d*x]^(1 + n))/(16*(a - b)^3*d*(1 + n)) + ((3*a^2 + 9*a*b + 8*b^2)*H
ypergeometric2F1[1, 1 + n, 2 + n, Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(16*(
a + b)^3*d*(1 + n)) - (b^6*Hypergeometric2F1[1, 1 + n, 2 + n, -((b*Sin[c +
d*x])/a)]*Sin[c + d*x]^(1 + n))/(a*(a^2 - b^2)^3*d*(1 + n)) + ((3*a - 5*b)*
Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(16
*(a - b)^2*d*(1 + n)) + ((3*a + 5*b)*Hypergeometric2F1[2, 1 + n, 2 + n, Sin
[c + d*x]]*Sin[c + d*x]^(1 + n))/(16*(a + b)^2*d*(1 + n)) + (Hypergeometric
2F1[3, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(8*(a - b)*d*(1 +
 n)) + (Hypergeometric2F1[3, 1 + n, 2 + n, Sin[c + d*x]]*Sin[c + d*x]^(1 +
n))/(8*(a + b)*d*(1 + n))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
```

```
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)\sin^n(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\mathrm{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^5\,\mathrm{Subst}\left(\int \left(\frac{\left(\frac{x}{b}\right)^n}{8b^3(a+b)(b-x)^3} + \frac{(3a+5b)\left(\frac{x}{b}\right)^n}{16b^4(a+b)^2(b-x)^2} + \frac{\left(3a^2+9ab+8b^2\right)\left(\frac{x}{b}\right)^n}{16b^5(a+b)^3(b-x)} - \frac{\left(\frac{x}{b}\right)^n}{(a-b)^3(a+b)^3(a+x)}\right.\right.}{d}$$

$$= \frac{((3a-5b)b)\,\mathrm{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n}{(b+x)^2}\,dx, x, b\sin(c+dx)\right)}{16(a-b)^2d} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n}{(b+x)^3}\,dx, x, b\,\mathrm{si}\right.}{8(a-b)d}$$

$$= \frac{\left(3a^2-9ab+8b^2\right)\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{16(a-b)^3d(1+n)} + \frac{\left(3a^2+9\right.}{}$$

Mathematica [A] time = 0.46, size = 241, normalized size = 0.67

$$\frac{\sin^{n+1}(c+dx)\left(\frac{\left(3a^2-9ab+8b^2\right){}_2F_1(1,n+1;n+2;-\sin(c+dx))}{(a-b)^3} + \frac{\left(3a^2+9ab+8b^2\right){}_2F_1(1,n+1;n+2;\sin(c+dx))}{(a+b)^3} - \frac{16b^6\,{}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{a(a-b)^3(a+b)^3}\right.}{16d(n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((((3*a^2 - 9*a*b + 8*b^2)*Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]
])/(a - b)^3 + ((3*a^2 + 9*a*b + 8*b^2)*Hypergeometric2F1[1, 1 + n, 2 + n,
Sin[c + d*x]])/(a + b)^3 - (16*b^6*Hypergeometric2F1[1, 1 + n, 2 + n, -((b*
Sin[c + d*x])/a)])/(a*(a - b)^3*(a + b)^3) + ((3*a - 5*b)*Hypergeometric2F1
[2, 1 + n, 2 + n, -Sin[c + d*x]])/(a - b)^2 + ((3*a + 5*b)*Hypergeometric2F
1[2, 1 + n, 2 + n, Sin[c + d*x]])/(a + b)^2 + (2*Hypergeometric2F1[3, 1 + n
, 2 + n, -Sin[c + d*x]])/(a - b) + (2*Hypergeometric2F1[3, 1 + n, 2 + n, Si
n[c + d*x]])/(a + b))*Sin[c + d*x]^(1 + n))/(16*d*(1 + n))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin\left(dx+c\right)^{n}\sec\left(dx+c\right)^{5}}{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^n*sec(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^{n}\sec\left(dx+c\right)^{5}}{b\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*sec(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

maple [F] time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^{5}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)}{a+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^n \sec(dx+c)^5}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^n*sec(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)^n}{\cos(c+d\,x)^5\,\left(a+b\,\sin(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^n/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
[Out] int(sin(c + d*x)^n/(cos(c + d*x)^5*(a + b*sin(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

3.1514 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$

Optimal. Leaf size=487

$$\frac{3\sin^{n+1}(c+dx)(a+b\sin(c+dx))^p\left(\frac{b\sin(c+dx)}{a}+1\right)^{-p}F_1\left(n+1;-p,1;n+2;-\frac{b\sin(c+dx)}{a},-\sin(c+dx)\right)}{16d(n+1)}+\frac{3\sin^{n+}}{}$$

```
[Out] 3/16*AppellF1(1+n,-p,1,2+n,-b*sin(d*x+c)/a,-sin(d*x+c))*sin(d*x+c)^(1+n)*(a
+b*sin(d*x+c))^p/d/(1+n)/((1+b*sin(d*x+c)/a)^p)+3/16*AppellF1(1+n,-p,1,2+n,
-b*sin(d*x+c)/a,sin(d*x+c))*sin(d*x+c)^(1+n)*(a+b*sin(d*x+c))^p/d/(1+n)/((1
+b*sin(d*x+c)/a)^p)+3/16*AppellF1(1+n,-p,2,2+n,-b*sin(d*x+c)/a,-sin(d*x+c))
*sin(d*x+c)^(1+n)*(a+b*sin(d*x+c))^p/d/(1+n)/((1+b*sin(d*x+c)/a)^p)+3/16*Ap
pellF1(1+n,-p,2,2+n,-b*sin(d*x+c)/a,sin(d*x+c))*sin(d*x+c)^(1+n)*(a+b*sin(d
*x+c))^p/d/(1+n)/((1+b*sin(d*x+c)/a)^p)+1/8*AppellF1(1+n,-p,3,2+n,-b*sin(d*
x+c)/a,-sin(d*x+c))*sin(d*x+c)^(1+n)*(a+b*sin(d*x+c))^p/d/(1+n)/((1+b*sin(d
*x+c)/a)^p)+1/8*AppellF1(1+n,-p,3,2+n,-b*sin(d*x+c)/a,sin(d*x+c))*sin(d*x+c
)^(1+n)*(a+b*sin(d*x+c))^p/d/(1+n)/((1+b*sin(d*x+c)/a)^p)
```

Rubi [A] time = 0.57, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2837, 961, 135, 133, 912}

$$\frac{3\sin^{n+1}(c+dx)(a+b\sin(c+dx))^p\left(\frac{b\sin(c+dx)}{a}+1\right)^{-p}F_1\left(n+1;-p,1;n+2;-\frac{b\sin(c+dx)}{a},-\sin(c+dx)\right)}{16d(n+1)}+\frac{3\sin^{n+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p,x]
```

```
[Out] (3*AppellF1[1 + n, -p, 1, 2 + n, -((b*Sin[c + d*x])/a), -Sin[c + d*x]]*Sin[
c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*(1 + n)*(1 + (b*Sin[c + d*x]
)/a)^p) + (3*AppellF1[1 + n, -p, 1, 2 + n, -((b*Sin[c + d*x])/a), Sin[c + d
*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*(1 + n)*(1 + (b*Sin
[c + d*x])/a)^p) + (3*AppellF1[1 + n, -p, 2, 2 + n, -((b*Sin[c + d*x])/a),
-Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*(1 + n)*(
1 + (b*Sin[c + d*x])/a)^p) + (3*AppellF1[1 + n, -p, 2, 2 + n, -((b*Sin[c +
d*x])/a), Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*
(1 + n)*(1 + (b*Sin[c + d*x])/a)^p) + (AppellF1[1 + n, -p, 3, 2 + n, -((b*S
in[c + d*x])/a), -Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p
)/(8*d*(1 + n)*(1 + (b*Sin[c + d*x])/a)^p) + (AppellF1[1 + n, -p, 3, 2 + n,
 -((b*Sin[c + d*x])/a), Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d
*x])^p)/(8*d*(1 + n)*(1 + (b*Sin[c + d*x])/a)^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)^p}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^5\,\text{Subst}\left(\int \left(\frac{\left(\frac{x}{b}\right)^n(a+x)^p}{8b^3(b-x)^3} + \frac{3\left(\frac{x}{b}\right)^n(a+x)^p}{16b^4(b-x)^2} + \frac{\left(\frac{x}{b}\right)^n(a+x)^p}{8b^3(b+x)^3} + \frac{3\left(\frac{x}{b}\right)^n(a+\cdots}{16b^4(b+\cdots}\right.\right.}{d}$$

$$= \frac{(3b)\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)^p}{(b-x)^2}\,dx, x, b\sin(c+dx)\right)}{16d} + \frac{(3b)\,\text{Subst}\cdots}{\cdots}$$

$$= \frac{(3b)\,\text{Subst}\left(\int \left(\frac{\left(\frac{x}{b}\right)^n(a+x)^p}{2b(b-x)} + \frac{\left(\frac{x}{b}\right)^n(a+x)^p}{2b(b+x)}\right)dx, x, b\sin(c+dx)\right)}{8d}$$

$$= \frac{3F_1\left(1+n;-p,2;2+n;-\frac{b\sin(c+dx)}{a},-\sin(c+dx)\right)\sin^{1+n}(\cdots}{16d(1+n)}$$

$$= \frac{3F_1\left(1+n;-p,2;2+n;-\frac{b\sin(c+dx)}{a},-\sin(c+dx)\right)\sin^{1+n}(\cdots}{16d(1+n)}$$

$$= \frac{3F_1\left(1+n;-p,1;2+n;-\frac{b\sin(c+dx)}{a},-\sin(c+dx)\right)\sin^{1+n}(\cdots}{16d(1+n)}$$

Mathematica [F] time = 15.01, size = 0, normalized size = 0.00

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p,x]
[Out] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p, x]
```

fricas [F] time = 1.42, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^p\sin(dx+c)^n\sec(dx+c)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="fricas
")
```

```
[Out] integral((b*sin(d*x + c) + a)^p*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^p \sin(dx+c)^n \sec(dx+c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

maple [F] time = 2.26, size = 0, normalized size = 0.00

$$\int \left(\sec^5(dx+c)\right)\left(\sin^n(dx+c)\right)(a+b\sin(dx+c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x)
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="maxima
")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)^n\,(a+b\,\sin(c+d\,x))^p}{\cos(c+d\,x)^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^p)/cos(c + d*x)^5,x)
[Out] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^p)/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**p,x)
```

```
[Out] Timed out
```

3.1515 $\int \frac{\sec^6(e+fx)(a+b\sin(e+fx))^{9/2}}{\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=502

$$-\frac{3ab\left(b^2-2a^2\right)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{5f\sqrt{d\sin(e+fx)}}-\frac{3a(a+b)^{3/2}\left(5a^2+3ab-4b^2\right)\tan(e+fx)\sqrt{-\frac{a(\csc(e+fx)-1)}{a+b}}\sqrt{\frac{a(\mathrm{cs}}{}}}{20\sqrt{d}f}$$

[Out] 1/5*sec(f*x+e)^5*(a+b*sin(f*x+e))^(9/2)*(d*sin(f*x+e))^(1/2)/d/f-3/5*a*b*(-2*a^2+b^2)*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/f/(d*sin(f*x+e))^(1/2)-3/20*a*sec(f*x+e)^3*(-a*(7*a^2+b^2)+2*b*(-7*a^2+b^2)*sin(f*x+e)+5*a*(a^2-b^2)*sin(f*x+e)^2+(8*a^2*b-4*b^3)*sin(f*x+e)^3)*(d*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/d/f-3/20*a*(a+b)^(3/2)*(5*a^2+3*a*b-4*b^2)*EllipticF(d^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(-a*(-1+csc(f*x+e))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/f/d^(1/2)-3/5*b*(2*a^4-3*a^2*b^2+b^4)*EllipticE(((-b-a*csc(f*x+e))/(a-b))^(1/2),(1-2*a/(a+b))^(1/2))*(-a*(-1+csc(f*x+e))/(a+b))^(1/2)*(d*sin(f*x+e))^(1/2)*(-a*csc(f*x+e)^2*(1+sin(f*x+e))*(a+b*sin(f*x+e))/(a-b)^2)^(1/2)*tan(f*x+e)/d/f/(a+b*sin(f*x+e))^(1/2)

Rubi [F] time = 0.37, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^6(e+fx)(a+b\sin(e+fx))^{9/2}}{\sqrt{d\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

[In] Int[(Sec[e + f*x]^6*(a + b*Sin[e + f*x])^(9/2))/Sqrt[d*Sin[e + f*x]],x]

[Out] (Sec[e + f*x]^5*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(9/2))/(5*d*f) + (9*a*Defer[Int][(Sec[e + f*x]^4*(a + b*Sin[e + f*x])^(7/2))/Sqrt[d*Sin[e + f*x]], x])/10

Rubi steps

$$\int \frac{\sec^6(e+fx)(a+b\sin(e+fx))^{9/2}}{\sqrt{d\sin(e+fx)}}\,dx=\frac{\sec^5(e+fx)\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{9/2}}{5df}+\frac{1}{10}(9a)\int \frac{\sec^4(e-}{}$$

Mathematica [C] time = 9.99, size = 1600, normalized size = 3.19

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^6*(a + b*Sin[e + f*x])^(9/2))/Sqrt[d*Sin[e + f*x]],
x]

[Out] (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((Sec[e + f*x]*(15*a^4 - 15*a^2*b^2
+ 4*b^4 + 24*a^3*b*Sin[e + f*x] - 12*a*b^3*Sin[e + f*x]))/20 + (Sec[e + f*x
]^3*(3*a^4 - 3*a^2*b^2 - 4*b^4 + 9*a^3*b*Sin[e + f*x] - 5*a*b^3*Sin[e + f*x
]))/10 + (Sec[e + f*x]^5*(a^4 + 6*a^2*b^2 + b^4 + 4*a^3*b*Sin[e + f*x] + 4*
a*b^3*Sin[e + f*x]))/5))/(f*Sqrt[d*Sin[e + f*x]]) + (3*a*Sqrt[Sin[e + f*x]]
*((4*a*(5*a^4 - 9*a^2*b^2 + 4*b^4)*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e
+ f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2
]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[
(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqrt[Sin[e + f*x]
]*Sqrt[a + b*Sin[e + f*x]]) + 4*a*(-8*a^3*b + 4*a*b^3)*((Sqrt[((a + b)*Cot[
(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f
*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*S
in[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e
+ f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a
+ b)*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (Sqrt[((a + b)*Cot[(-e
+ Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(-e + Pi/
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f
*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*S
in[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])
/(b*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])) + 2*(8*a^2*b^2 - 4*b^4)*(
(Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(b*Sqrt[Sin[e + f*x]]) + (I*Cos[(-e
 + Pi/2 - f*x)/2]*Csc[e + f*x]*EllipticE[I*ArcSinh[Sin[(-e + Pi/2 - f*x)/2]
/Sqrt[Sin[e + f*x]]], (-2*a)/(-a - b)]*Sqrt[a + b*Sin[e + f*x]])/(b*Sqrt[Co
s[(-e + Pi/2 - f*x)/2]^2*Csc[e + f*x]]*Sqrt[(Csc[e + f*x]*(a + b*Sin[e + f*
x]))/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b
)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[
-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqrt[Sin[e + f*x]]*Sqrt[a
+ b*Sin[e + f*x]]) - (a*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]
*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-
e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/(b*Sqrt[Sin[e + f*x]]*Sqrt[a
 + b*Sin[e + f*x]])))/b)))/(40*f*Sqrt[d*Sin[e + f*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(4\left(ab^3\cos\left(fx+e\right)^2-a^3b-ab^3\right)\sec\left(fx+e\right)^6\sin\left(fx+e\right)-\left(b^4\cos\left(fx+e\right)^4+a^4+6a^2b^2+b^4-\right.\right.}{d\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(4*(a*b^3*cos(f*x + e)^2 - a^3*b - a*b^3)*sec(f*x + e)^6*sin(f*x
+ e) - (b^4*cos(f*x + e)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*co
s(f*x + e)^2)*sec(f*x + e)^6)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
/(d*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{9}{2}}\sec\left(fx+e\right)^6}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(9/2)*sec(f*x + e)^6/sqrt(d*sin(f*x + e)), x
)
```

maple [B] time = 1.10, size = 5578, normalized size = 11.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{9}{2}}\sec\left(fx+e\right)^6}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(9/2)*sec(f*x + e)^6/sqrt(d*sin(f*x + e)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\sin\left(e+f\,x\right)\right)^{9/2}}{{\cos\left(e+f\,x\right)}^6\,\sqrt{d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(9/2)/(cos(e + f*x)^6*(d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int((a + b*sin(e + f*x))^(9/2)/(cos(e + f*x)^6*(d*sin(e + f*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sin(f*x+e))**(9/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.1516 $\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{4/3}\,dx$

Optimal. Leaf size=458

$$-\frac{3(c+d)^2\left(208a^2cd^2-64abd\left(3c^2-5d^2\right)+b^2c\left(54c^2+d^2\right)\right)\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{7}{3};\frac{3}{2};\frac{1}{2}(1-\right.}{1040\sqrt{2}\,d^4f\sqrt{\sin(e+fx)+1}\,\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -9/2080*(64*a*b*c*d-26*a^2*d^2-b^2*(18*c^2-13*d^2))*cos(f*x+e)*(c+d*sin(f*x
+e))^(7/3)/d^3/f-9/208*b*(-2*a*d+3*b*c)*cos(f*x+e)*sin(f*x+e)*(c+d*sin(f*x+
e))^(7/3)/d^2/f+3/16*cos(f*x+e)*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(7/3)/d
/f-3/2080*(c+d)^2*(208*a^2*c*d^2-64*a*b*d*(3*c^2-5*d^2)+b^2*c*(54*c^2+d^2))
*AppellF1(1/2,-7/3,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f
*x+e)*(c+d*sin(f*x+e))^(1/3)/d^4/f/((c+d*sin(f*x+e))/(c+d))^(1/3)*2^(1/2)/(
1+sin(f*x+e))^(1/2)-3/2080*(c-d)*(c+d)^2*(192*a*b*c*d-208*a^2*d^2-b^2*(54*c
^2+91*d^2))*AppellF1(1/2,-4/3,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*
x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/3)/d^4/f/((c+d*sin(f*x+e))/(c+d))^(1/3
)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.24, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2922, 3050, 3033, 3023, 2756, 2665, 139, 138}

$$-\frac{3(c+d)^2\left(208a^2cd^2-64abd\left(3c^2-5d^2\right)+b^2c\left(54c^2+d^2\right)\right)\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{7}{3};\frac{3}{2};\frac{1}{2}(1-\right.}{1040\sqrt{2}\,d^4f\sqrt{\sin(e+fx)+1}\,\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (-9*(64*a*b*c*d - 26*a^2*d^2 - b^2*(18*c^2 - 13*d^2))*Cos[e + f*x]*(c + d*S
in[e + f*x])^(7/3))/(2080*d^3*f) - (9*b*(3*b*c - 2*a*d)*Cos[e + f*x]*Sin[e
+ f*x]*(c + d*Sin[e + f*x])^(7/3))/(208*d^2*f) + (3*Cos[e + f*x]*(a + b*Sin
[e + f*x])^2*(c + d*Sin[e + f*x])^(7/3))/(16*d*f) - (3*(c + d)^2*(208*a^2*c
*d^2 - 64*a*b*d*(3*c^2 - 5*d^2) + b^2*c*(54*c^2 + d^2))*AppellF1[1/2, 1/2,
-7/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*
x]*(c + d*Sin[e + f*x])^(1/3))/(1040*Sqrt[2]*d^4*f*Sqrt[1 + Sin[e + f*x]]*(
(c + d*Sin[e + f*x])/(c + d))^(1/3)) - (3*(c - d)*(c + d)^2*(192*a*b*c*d -
208*a^2*d^2 - b^2*(54*c^2 + 91*d^2))*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sin
[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(1/3))/(1040*Sqrt[2]*d^4*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x]
)/(c + d))^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2922

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*
((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[(a + b*Sin[e
+ f*x])^m*(c + d*Sin[e + f*x])^n*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, c
, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m,
2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
```

```
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{4/3}\,dx = \int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{4/3}\left(1-\mathrm{si}\right.$$

$$= \frac{3\cos(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx}{16df}$$

$$= -\frac{9b(3bc-2ad)\cos(e+fx)\sin(e+fx)(c+d\sin(}{208d^2f}$$

$$= -\frac{9\left(64abcd-26a^2d^2-b^2\left(18c^2-13d^2\right)\right)\cos(e+}{2080d^3f}$$

$$= -\frac{9\left(64abcd-26a^2d^2-b^2\left(18c^2-13d^2\right)\right)\cos(e+}{2080d^3f}$$

$$= -\frac{9\left(64abcd-26a^2d^2-b^2\left(18c^2-13d^2\right)\right)\cos(e+}{2080d^3f}$$

$$= -\frac{9\left(64abcd-26a^2d^2-b^2\left(18c^2-13d^2\right)\right)\cos(e+}{2080d^3f}$$

$$= -\frac{9\left(64abcd-26a^2d^2-b^2\left(18c^2-13d^2\right)\right)\cos(e+}{2080d^3f}$$

Mathematica [B] time = 7.08, size = 3522, normalized size = 7.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(4/3),
x]
```

```
[Out] (513*a*b*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d
)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e
 + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(
1/3))/(455*f) + (81*b^2*c^4*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f
*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*S
qrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c +
d*Sin[e + f*x])^(1/3))/(7280*d^3*f) - (18*a*b*c^3*AppellF1[1/3, 1/2, 1/2, 4
```

/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/
d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e
+ f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/(455*d^2*f) + (54*a^2*c^2*Appe
llF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Si
n[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d
)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/(35*d*f)
+ (5211*b^2*c^2*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c
/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*
Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*
x])^(1/3))/(14560*d*f) + (9*a^2*d*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin
[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e +
f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]
*(c + d*Sin[e + f*x])^(1/3))/(40*f) + (63*b^2*d*AppellF1[1/3, 1/2, 1/2, 4/3
, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)
*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e +
f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/(640*f) + (9*a*b*c^2*((-3*c*Appe
llF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Si
n[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d
)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/d^2 + (3*
AppellF1[4/3, 1/2, 1/2, 7/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c +
d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c
 - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(4/3))/(4*d^
2)))/(65*f) + (81*b^2*c^5*((-3*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[
e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f
*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*
(c + d*Sin[e + f*x])^(1/3))/d^2 + (3*AppellF1[4/3, 1/2, 1/2, 7/3, -((c + d*
Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e
 + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c +
d)]*(c + d*Sin[e + f*x])^(4/3))/(4*d^2)))/(7280*d^3*f) - (18*a*b*c^4*((-3*c
*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c +
 d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(
c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/d^2
+ (3*AppellF1[4/3, 1/2, 1/2, 7/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -(
(c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x
])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(4/3))/
(4*d^2)))/(455*d^2*f) + (3*a^2*c^3*((-3*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((c
 + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*
Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/
(c + d)]*(c + d*Sin[e + f*x])^(1/3))/d^2 + (3*AppellF1[4/3, 1/2, 1/2, 7/3,
-((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d
))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*
x])/(c + d)]*(c + d*Sin[e + f*x])^(4/3))/(4*d^2)))/(70*d*f) - (21*b^2*c^3*(
(-3*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -
((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*
x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))

```
/d^2 + (3*AppellF1[4/3, 1/2, 1/2, 7/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)
), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e
+ f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(4
/3))/(4*d^2)))/(1040*d*f) + (153*a^2*c*d*((-3*c*AppellF1[1/3, 1/2, 1/2, 4/3
, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)
*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e +
f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/d^2 + (3*AppellF1[4/3, 1/2, 1/2,
 7/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 -
c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[
e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(4/3))/(4*d^2)))/(280*f) + (9603*b^
2*c*d*((-3*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)
*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin
[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])
^(1/3))/d^2 + (3*AppellF1[4/3, 1/2, 1/2, 7/3, -((c + d*Sin[e + f*x])/((1 -
c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d
*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f
*x])^(4/3))/(4*d^2)))/(58240*f) + (24*a*b*d^2*((-3*c*AppellF1[1/3, 1/2, 1/2
, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 -
 c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin
[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/d^2 + (3*AppellF1[4/3, 1/2,
 1/2, 7/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((
-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d
*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(4/3))/(4*d^2)))/(91*f) + ((c
+ d*Sin[e + f*x])^(1/3)*((-3*(-216*b^2*c^4 + 768*a*b*c^3*d - 832*a^2*c^2*d^
2 + 332*b^2*c^2*d^2 + 7232*a*b*c*d^3 + 2912*a^2*d^4 + 1729*b^2*d^4)*Cos[e +
 f*x])/(58240*d^3) - (3*(8*b^2*c^2 + 896*a*b*c*d + 416*a^2*d^2 + 117*b^2*d^
2)*Cos[3*(e + f*x)])/(16640*d) + (3*b^2*d*Cos[5*(e + f*x)])/256 + (3*(-18*b
^2*c^3 + 64*a*b*c^2*d + 1144*a^2*c*d^2 + 23*b^2*c*d^2 + 80*a*b*d^3)*Sin[2*(
e + f*x)])/(14560*d^2) - (3*b*(17*b*c + 32*a*d)*Sin[4*(e + f*x)])/1664))/f
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\left(\left(b^2c+2\,abd\right)\cos\left(fx+e\right)^4-\left(2\,abd+\left(a^2+b^2\right)c\right)\cos\left(fx+e\right)^2+\left(b^2d\cos\left(fx+e\right)^4-\left(2\,abc+\left(a^2\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="fricas")
```

```
[Out] integral(-((b^2*c + 2*a*b*d)*cos(f*x + e)^4 - (2*a*b*d + (a^2 + b^2)*c)*cos
(f*x + e)^2 + (b^2*d*cos(f*x + e)^4 - (2*a*b*c + (a^2 + b^2)*d)*cos(f*x + e
)^2)*sin(f*x + e))*(d*sin(f*x + e) + c)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2,
 x)
```

maple [F] time = 3.05, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(a+b\sin\left(fx+e\right)\right)^{2}\left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2,
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos\left(e+f\,x\right)}^2\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(4/3),x)
```

```
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**(4/3),x)
[Out] Timed out
```

3.1517 $\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{4/3}\,dx$

Optimal. Leaf size=341

$$\frac{3(c+d)^2\left(-13acd+6bc^2-10bd^2\right)\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{7}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{65\sqrt{2}\,d^3f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -3/130*(-13*a*d+6*b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(7/3)/d^2/f+3/13*b*cos(f
*x+e)*sin(f*x+e)*(c+d*sin(f*x+e))^(7/3)/d/f+3/130*(c+d)^2*(-13*a*c*d+6*b*c^
2-10*b*d^2)*AppellF1(1/2,-7/3,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*
x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/3)/d^3/f/((c+d*sin(f*x+e))/(c+d))^(1/3
)*2^(1/2)/(1+sin(f*x+e))^(1/2)-3/130*(c-d)*(c+d)^2*(-13*a*d+6*b*c)*AppellF1
(1/2,-4/3,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+
d*sin(f*x+e))^(1/3)/d^3/f/((c+d*sin(f*x+e))/(c+d))^(1/3)*2^(1/2)/(1+sin(f*x
+e))^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2922, 3034, 3023, 2756, 2665, 139, 138}

$$\frac{3(c+d)^2\left(-13acd+6bc^2-10bd^2\right)\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{7}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{65\sqrt{2}\,d^3f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (-3*(6*b*c - 13*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/3))/(130*d^2*f) +
 (3*b*Cos[e + f*x]*Sin[e + f*x]*(c + d*Sin[e + f*x])^(7/3))/(13*d*f) + (3*(
c + d)^2*(6*b*c^2 - 13*a*c*d - 10*b*d^2)*AppellF1[1/2, 1/2, -7/3, 3/2, (1 -
 Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e
 + f*x])^(1/3))/(65*Sqrt[2]*d^3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*
x])/(c + d))^(1/3)) - (3*(c - d)*(c + d)^2*(6*b*c - 13*a*d)*AppellF1[1/2, 1
/2, -4/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^(1/3))/(65*Sqrt[2]*d^3*f*Sqrt[1 + Sin[e + f*x]]
*((c + d*Sin[e + f*x])/(c + d))^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2922

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*
((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[(a + b*Sin[e
+ f*x])^m*(c + d*Sin[e + f*x])^n*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, c
, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m,
2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{4/3}\,dx = \int (a+b\sin(e+fx))(c+d\sin(e+fx))^{4/3}\left(1-\sin^2(\right.$$

$$= \frac{3b\cos(e+fx)\sin(e+fx)(c+d\sin(e+fx))^{7/3}}{13df} + \frac{3}{}$$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{130d^2f} + $$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{130d^2f} + $$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{130d^2f} + $$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{130d^2f} + $$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{130d^2f} + $$

Mathematica [A] time = 5.13, size = 398, normalized size = 1.17

$$3\sec(e+fx)\sqrt[3]{c+d\sin(e+fx)}\left(12\left(d^2-c^2\right)\left(52ac^2d+91ad^3-24bc^3+68bcd^2\right)\sqrt{-\frac{d(\sin(e+fx)-1)}{c+d}}\sqrt{-\frac{d(\sin(e+fx)+1}{c-d}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (3*Sec[e + f*x]*(c + d*Sin[e + f*x])^(1/3)*(12*(-c^2 + d^2)*(-24*b*c^3 + 52
*a*c^2*d + 68*b*c*d^2 + 91*a*d^3)*AppellF1[1/3, 1/2, 1/2, 4/3, (c + d*Sin[e
 + f*x])/(c - d), (c + d*Sin[e + f*x])/(c + d)]*Sqrt[-((d*(-1 + Sin[e + f*x
]))/(c + d))]*Sqrt[-((d*(1 + Sin[e + f*x]))/(c - d))] + 3*(-24*b*c^4 + 52*a
*c^3*d + 84*b*c^2*d^2 + 663*a*c*d^3 + 160*b*d^4)*AppellF1[4/3, 1/2, 1/2, 7/
3, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin[e + f*x])/(c + d)]*Sqrt[-((d*(-
1 + Sin[e + f*x]))/(c + d))]*Sqrt[-((d*(1 + Sin[e + f*x]))/(c - d))]*(c + d
*Sin[e + f*x]) - 4*d^2*Cos[e + f*x]^2*(24*b*c^3 - 52*a*c^2*d + 128*b*c*d^2
+ 91*a*d^3 + 14*d^2*(14*b*c + 13*a*d)*Cos[2*(e + f*x)] - 2*d*(8*b*c^2 + 286
*a*c*d + 45*b*d^2)*Sin[e + f*x] + 70*b*d^3*Sin[3*(e + f*x)])))/(14560*d^4*f
)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(bd\cos\left(fx+e\right)^{4}-(bc+ad)\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-(ac+bd)\cos\left(fx+e\right)^{2}\right)\left(d\sin\left(fx+e\right)+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x, algorithm
="fricas")
```

```
[Out] integral(-(b*d*cos(f*x + e)^4 - (b*c + a*d)*cos(f*x + e)^2*sin(f*x + e) - (
a*c + b*d)*cos(f*x + e)^2)*(d*sin(f*x + e) + c)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x, algorithm
="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2, x
)
```

maple [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(a+b\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(fx + e\right) + a\right)\left(d \sin \left(fx + e\right) + c\right)^{\frac{4}{3}} \cos \left(fx + e\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x, algorithm
="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos \left(e+f\,x\right)}^2\,\left(a+b\,\sin \left(e+f\,x\right)\right)\,{\left(c+d\,\sin \left(e+f\,x\right)\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(4/3),x)
```

```
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

3.1518 $\int \cos^2(e+fx)(c+d\sin(e+fx))^{4/3}\,dx$

Optimal. Leaf size=125

$$\frac{3\cos(e+fx)(c+d\sin(e+fx))^{7/3}F_1\left(\frac{7}{3};-\frac{1}{2},-\frac{1}{2};\frac{10}{3};\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)}{7df\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] 3/7*AppellF1(7/3,-1/2,-1/2,10/3,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+
d))*cos(f*x+e)*(c+d*sin(f*x+e))^(7/3)/d/f/(1+(-c-d*sin(f*x+e))/(c-d))^(1/2)
/(1+(-c-d*sin(f*x+e))/(c+d))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2704, 138}

$$\frac{3\cos(e+fx)(c+d\sin(e+fx))^{7/3}F_1\left(\frac{7}{3};-\frac{1}{2},-\frac{1}{2};\frac{10}{3};\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)}{7df\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (3*AppellF1[7/3, -1/2, -1/2, 10/3, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin
[e + f*x])/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/3))/(7*d*f*Sqrt[1
- (c + d*Sin[e + f*x])/(c - d)]*Sqrt[1 - (c + d*Sin[e + f*x])/(c + d)])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
```

```
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \cos^2(e+fx)(c+d\sin(e+fx))^{4/3}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int(c+dx)^{4/3}\sqrt{-\frac{d}{c-d}-\frac{dx}{c-d}}\sqrt{\frac{d}{c+d}-\frac{dx}{c+d}}\,dx,x,\sin(e+\right.}{f\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= \frac{3F_1\left(\frac{7}{3};-\frac{1}{2},-\frac{1}{2};\frac{10}{3};\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)\cos(e+fx)(c+d\sin(e+f}{7df\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [B] time = 2.13, size = 301, normalized size = 2.41

$$-\frac{3\sec(e+fx)\sqrt[3]{c+d\sin(e+fx)}\left(-3c\left(4c^2+51d^2\right)\sqrt{-\frac{d(\sin(e+fx)-1)}{c+d}}\sqrt{-\frac{d(\sin(e+fx)+1)}{c-d}}(c+d\sin(e+fx))F_1\left(\frac{4}{3};\frac{1}{2},\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (-3*Sec[e + f*x]*(c + d*Sin[e + f*x])^(1/3)*(12*(4*c^4 + 3*c^2*d^2 - 7*d^4)
*AppellF1[1/3, 1/2, 1/2, 4/3, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin[e +
f*x])/(c + d)]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sqrt[-((d*(1 + Sin[
e + f*x]))/(c - d))] - 3*c*(4*c^2 + 51*d^2)*AppellF1[4/3, 1/2, 1/2, 7/3, (c
 + d*Sin[e + f*x])/(c - d), (c + d*Sin[e + f*x])/(c + d)]*Sqrt[-((d*(-1 + S
in[e + f*x]))/(c + d))]*Sqrt[-((d*(1 + Sin[e + f*x]))/(c - d))]*(c + d*Sin[
e + f*x]) + 4*d^2*Cos[e + f*x]^2*(-4*c^2 + 7*d^2 + 14*d^2*Cos[2*(e + f*x)]
- 44*c*d*Sin[e + f*x])))/(1120*d^3*f)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\cos\left(fx+e\right)^2\sin\left(fx+e\right)+c\cos\left(fx+e\right)^2\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e)^2*sin(f*x + e) + c*cos(f*x + e)^2)*(d*sin(f*x + e)
 + c)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2, x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x)
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(c + d*sin(e + f*x))^(4/3),x)
[Out] int(cos(e + f*x)^2*(c + d*sin(e + f*x))^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+d\sin\left(e+fx\right)\right)^{\frac{4}{3}}\cos^{2}\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**(4/3),x)
[Out] Integral((c + d*sin(e + f*x))**(4/3)*cos(e + f*x)**2, x)
```

3.1519 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=38

$$\mathrm{Int}\left(\frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)},x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x)
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x]),x]
```

```
[Out] Defer[Int][(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x])
, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)}\,dx=\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)}\,dx$$

Mathematica [A] time = 62.99, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x]),
x]
```

```
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x]),
 x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2/(b*sin(f*x + e) + a), x
)
```

maple [A] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}}{a+b\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2/(b*sin(f*x + e) + a), x
)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{4/3}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^(4/3))/(a + b*sin(e + f*x)),x)
[Out] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^(4/3))/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**(4/3)/(a+b*sin(f*x+e)),x)
[Out] Timed out
```

3.1520 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x)
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] Defer[Int][(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x])
^2, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2}\,dx = \int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2}\,dx$$

Mathematica [A] time = 46.86, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x])^
2,x]
```

```
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x])^
2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2/(b*sin(f*x + e) + a)^2,
 x)
```

maple [A] time = 5.78, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}}{\left(a+b\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2/(b*sin(f*x + e) + a)^2,
 x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{4/3}}{\left(a+b\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^(4/3))/(a + b*sin(e + f*x))^2,x)
[Out] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^(4/3))/(a + b*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**(4/3)/(a+b*sin(f*x+e))**2,x)
[Out] Timed out
```

3.1521 $\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=36

$$\text{Int}\left(\cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n,x\right)$$

[Out] Unintegrable(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]

[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x]

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+$$

Mathematica [A] time = 7.47, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]

[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x]

fricas [A] time = 2.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [A] time = 0.99, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(a+b\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \cos\left(e+f\,x\right)^{2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)^{m}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.1522 $\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^{4/3}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}(a+b\sin(e+fx))^m,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x)
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^{4/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(4/3)
, x]
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^{4/3}\,dx=\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+f$$

Mathematica [A] time = 36.29, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^{4/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(4/3),
x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(4/3),
 x]
```

fricas [A] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\cos\left(fx+e\right)^2\sin\left(fx+e\right)+c\cos\left(fx+e\right)^2\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{1}{3}}\left(b\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="fricas")
```

```
[Out] integral((d*cos(f*x + e)^2*sin(f*x + e) + c*cos(f*x + e)^2)*(d*sin(f*x + e)
 + c)^(1/3)*(b*sin(f*x + e) + a)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sin \left(f x+e\right)+c\right)^{\frac{4}{3}} \left(b \sin \left(f x+e\right)+a\right)^{m} \cos \left(f x+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*(b*sin(f*x + e) + a)^m*cos(f*x + e)^2,
 x)
```

maple [A] time = 0.75, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(f x+e\right)\right) \left(a+b \sin \left(f x+e\right)\right)^{m} \left(c+d \sin \left(f x+e\right)\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sin \left(f x+e\right)+c\right)^{\frac{4}{3}} \left(b \sin \left(f x+e\right)+a\right)^{m} \cos \left(f x+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*(b*sin(f*x + e) + a)^m*cos(f*x + e)^2,
 x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int {\cos\left(e+f\,x\right)}^2\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(4/3),x)
```

```
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

3.1523 $\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=552

$$-\frac{\sqrt{2}\,(c+d)\cos(e+fx)\left(a^2cd^2\left(n^2+7n+12\right)-2abd(n+4)\left(2c^2-d^2(n+2)\right)+b^2c\left(6c^2-d^2\left(-n^2-n+3\right)\right)\right)(}{d^4f(n+2)(n+3)(n+4)\sqrt{\sin(e}}$$

```
[Out] (2*a^2*d^2*(3+n)-4*a*b*c*d*(4+n)+b^2*(6*c^2-d^2*(3+n)))*cos(f*x+e)*(c+d*sin
(f*x+e))^(1+n)/d^3/f/(2+n)/(3+n)/(4+n)-b*(-2*a*d+3*b*c)*cos(f*x+e)*sin(f*x+
e)*(c+d*sin(f*x+e))^(1+n)/d^2/f/(3+n)/(4+n)+cos(f*x+e)*(a+b*sin(f*x+e))^2*(
c+d*sin(f*x+e))^(1+n)/d/f/(4+n)-(c+d)*(a^2*c*d^2*(n^2+7*n+12)-2*a*b*d*(4+n)
*(2*c^2-d^2*(2+n))+b^2*c*(6*c^2-d^2*(-n^2-n+3)))*AppellF1(1/2,-1-n,1/2,3/2,
d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^
(1/2)/d^4/f/(2+n)/(3+n)/(4+n)/(((c+d*sin(f*x+e))/(c+d))^n)/(1+sin(f*x+e))^(
1/2)-(c^2-d^2)*(4*a*b*c*d*(4+n)-a^2*d^2*(n^2+7*n+12)-b^2*(6*c^2+d^2*(n^2+4*
n+3)))*AppellF1(1/2,-n,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*c
os(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/d^4/f/(2+n)/(3+n)/(4+n)/(((c+d*sin(f*x
+e))/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.51, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2922, 3050, 3033, 3023, 2756, 2665, 139, 138}

$$-\frac{\sqrt{2}\,(c+d)\cos(e+fx)\left(a^2cd^2\left(n^2+7n+12\right)-2abd(n+4)\left(2c^2-d^2(n+2)\right)+b^2\left(6c^3-cd^2\left(-n^2-n+3\right)\right)\right)(}{d^4f(n+2)(n+3)(n+4)\sqrt{\sin(e}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] ((2*a^2*d^2*(3 + n) - 4*a*b*c*d*(4 + n) + b^2*(6*c^2 - d^2*(3 + n)))*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^3*f*(2 + n)*(3 + n)*(4 + n)) - (b*(
3*b*c - 2*a*d)*Cos[e + f*x]*Sin[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^2
*f*(3 + n)*(4 + n)) + (Cos[e + f*x]*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f
*x])^(1 + n))/(d*f*(4 + n)) - (Sqrt[2]*(c + d)*(a^2*c*d^2*(12 + 7*n + n^2)
- 2*a*b*d*(4 + n)*(2*c^2 - d^2*(2 + n)) + b^2*(6*c^3 - c*d^2*(3 - n - n^2))
)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x
]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^4*f*(2 + n)*(3 + n)*(4
 + n)*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (Sqrt[2]*(
c^2 - d^2)*(4*a*b*c*d*(4 + n) - a^2*d^2*(12 + 7*n + n^2) - b^2*(6*c^2 + d^2
*(3 + 4*n + n^2)))*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1
```

```
- Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^4*f*(2 +
n)*(3 + n)*(4 + n)*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2922

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*
((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[(a + b*Sin[e
+ f*x])^m*(c + d*Sin[e + f*x])^n*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, c
, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m,
2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^n\left(1-\sin^2(e\right.$$

$$= \frac{\cos(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{1+n}}{df(4+n)}$$

$$= -\frac{b(3bc-2ad)\cos(e+fx)\sin(e+fx)(c+d\sin(e+}{d^2f(3+n)(4+n)}$$

$$= \frac{\left(2a^2d^2(3+n)-4abcd(4+n)+b^2\left(6c^2-d^2(3+n)\right)\right.}{d^3f(2+n)(3+n)(4-}$$

$$= \frac{\left(2a^2d^2(3+n)-4abcd(4+n)+b^2\left(6c^2-d^2(3+n)\right)\right.}{d^3f(2+n)(3+n)(4-}$$

$$= \frac{\left(2a^2d^2(3+n)-4abcd(4+n)+b^2\left(6c^2-d^2(3+n)\right)\right.}{d^3f(2+n)(3+n)(4-}$$

$$= \frac{\left(2a^2d^2(3+n)-4abcd(4+n)+b^2\left(6c^2-d^2(3+n)\right)\right.}{d^3f(2+n)(3+n)(4-}$$

$$= \frac{\left(2a^2d^2(3+n)-4abcd(4+n)+b^2\left(6c^2-d^2(3+n)\right)\right.}{d^3f(2+n)(3+n)(4-}$$

Mathematica [F] time = 8.85, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n, x]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos\left(fx+e\right)^4-2\,ab\cos\left(fx+e\right)^2\sin\left(fx+e\right)-\left(a^2+b^2\right)\cos\left(fx+e\right)^2\right)\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^4 - 2*a*b*cos(f*x + e)^2*sin(f*x + e) - (a^2 +
b^2)*cos(f*x + e)^2)*(d*sin(f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)+a\right)^{2}\left(d \sin\left(fx+e\right)+c\right)^{n} \cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

maple [F] time = 3.25, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(a+b \sin\left(fx+e\right)\right)^{2}\left(c+d \sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)+a\right)^{2}\left(d \sin\left(fx+e\right)+c\right)^{n} \cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos\left(e+f\,x\right)}^2\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.1524 $\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=375

$$-\frac{\sqrt{2}\,(c+d)\cos(e+fx)\left(acd(n+3)-b\left(2c^2-d^2(n+2)\right)\right)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n-1;\right.}{d^3f(n+2)(n+3)\sqrt{\sin(e+fx)+1}}$$

```
[Out] -(2*b*c-a*d*(3+n))*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d^2/f/(2+n)/(3+n)+b*co
s(f*x+e)*sin(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+n)-(c+d)*(a*c*d*(3+n)-b*(
2*c^2-d^2*(2+n)))*AppellF1(1/2,-1-n,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*
sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/d^3/f/(2+n)/(3+n)/(((c+d*
sin(f*x+e))/(c+d))^n)/(1+sin(f*x+e))^(1/2)-(c^2-d^2)*(2*b*c-a*d*(3+n))*Appe
llF1(1/2,-n,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(
c+d*sin(f*x+e))^n*2^(1/2)/d^3/f/(2+n)/(3+n)/(((c+d*sin(f*x+e))/(c+d))^n)/(1
+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 373, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2922, 3034, 3023, 2756, 2665, 139, 138}

$$\frac{\sqrt{2}\,(c+d)\cos(e+fx)\left(-acd(n+3)+2bc^2-bd^2(n+2)\right)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n-1;\frac{3}{2}\right.}{d^3f(n+2)(n+3)\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -(((2*b*c - a*d*(3 + n))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^2*f*
(2 + n)*(3 + n))) + (b*Cos[e + f*x]*Sin[e + f*x]*(c + d*Sin[e + f*x])^(1 +
n))/(d*f*(3 + n)) + (Sqrt[2]*(c + d)*(2*b*c^2 - b*d^2*(2 + n) - a*c*d*(3 +
n))*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f
*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^3*f*(2 + n)*(3 + n)*
Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (Sqrt[2]*(c^2 -
d^2)*(2*b*c - a*d*(3 + n))*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2
, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^3
*f*(2 + n)*(3 + n)*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
```

```
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2922

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*
((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[(a + b*Sin[e
+ f*x])^m*(c + d*Sin[e + f*x])^n*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, c
, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m,
2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (a+b\sin(e+fx))(c+d\sin(e+fx))^n\left(1-\sin^2(e\right.$$

$$= \frac{b\cos(e+fx)\sin(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+n)} + \frac{\int}{}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+fx))^{1+}}{d^2f(2+n)(3+n)}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+fx))^{1+}}{d^2f(2+n)(3+n)}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+fx))^{1+}}{d^2f(2+n)(3+n)}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+fx))^{1+}}{d^2f(2+n)(3+n)}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+fx))^{1+}}{d^2f(2+n)(3+n)}$$

Mathematica [F] time = 3.57, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)+a\cos\left(fx+e\right)^{2}\right)\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((b*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*(d*sin(f*x + e)
 + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

maple [F] time = 1.19, size = 0, normalized size = 0.00

$$\int\left(\cos^{2}\left(fx+e\right)\right)\left(a+b\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^2\,\left(a+b\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
[Out] Timed out
```

3.1525 $\int \cos^2(e+fx)(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=127

$$\frac{\cos(e+fx)(c+d\sin(e+fx))^{n+1}F_1\left(n+1;-\frac{1}{2},-\frac{1}{2};n+2;\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)}{df(n+1)\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] AppellF1(1+n,-1/2,-1/2,2+n,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*c
os(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(1+n)/(1+(-c-d*sin(f*x+e))/(c-d))^(1/2
)/(1+(-c-d*sin(f*x+e))/(c+d))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2704, 138}

$$\frac{\cos(e+fx)(c+d\sin(e+fx))^{n+1}F_1\left(n+1;-\frac{1}{2},-\frac{1}{2};n+2;\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)}{df(n+1)\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (AppellF1[1 + n, -1/2, -1/2, 2 + n, (c + d*Sin[e + f*x])/(c - d), (c + d*Si
n[e + f*x])/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n
)*Sqrt[1 - (c + d*Sin[e + f*x])/(c - d)]*Sqrt[1 - (c + d*Sin[e + f*x])/(c +
 d)])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
```

```
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \cos^2(e+fx)(c+d\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int (c+dx)^n\sqrt{-\frac{d}{c-d}-\frac{dx}{c-d}}\sqrt{\frac{d}{c+d}-\frac{dx}{c+d}}\,dx,x,\sin(e+ ...}{f\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= \frac{F_1\left(1+n;-\frac{1}{2},-\frac{1}{2};2+n;\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)\cos(e+fx)(c+d\sin ...}{df(1+n)\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n, x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x)
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^2\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(c + d*sin(e + f*x))^n,x)
[Out] int(cos(e + f*x)^2*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n,x)
[Out] Timed out
```

3.1526 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=36

$$\text{Int}\left(\frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)},x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x]),x]
[Out] Defer[Int][(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x]), x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx = \int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx$$

Mathematica [A] time = 3.33, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x]),x]
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x]), x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="fricas")

[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b*sin(f*x + e) + a), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b*sin(f*x + e) + a), x)

maple [A] time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{a+b\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x)

[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b*sin(f*x + e) + a), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(e+f\,x\right)^{2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{n}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + b*sin(e + f*x)),x)
```

```
[Out] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1527 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=36

$$\mathrm{Int}\left(\frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] Defer[Int][(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x])^2,
x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx = \int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$$

Mathematica [A] time = 35.60, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x])^2, x
]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2}{b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b^2*cos(f*x + e)^2 - 2*a*b
*sin(f*x + e) - a^2 - b^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b*sin(f*x + e) + a)^2, x)
```

maple [A] time = 6.54, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+b\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b*sin(f*x + e) + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+b\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + b*sin(e + f*x))^2,x)
[Out] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + b*sin(e + f*x))^2, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+b*sin(f*x+e))**2,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.1528 $\int \cos^7(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=188

$$-\frac{(aB+Ab)\sin^8(c+dx)}{8d}-\frac{(aA-3bB)\sin^7(c+dx)}{7d}+\frac{(aB+Ab)\sin^6(c+dx)}{2d}+\frac{3(aA-bB)\sin^5(c+dx)}{5d}-\frac{3(aB+}{}$$

```
[Out] a*A*sin(d*x+c)/d+1/2*(A*b+B*a)*sin(d*x+c)^2/d-1/3*(3*A*a-B*b)*sin(d*x+c)^3/
d-3/4*(A*b+B*a)*sin(d*x+c)^4/d+3/5*(A*a-B*b)*sin(d*x+c)^5/d+1/2*(A*b+B*a)*s
in(d*x+c)^6/d-1/7*(A*a-3*B*b)*sin(d*x+c)^7/d-1/8*(A*b+B*a)*sin(d*x+c)^8/d-1
/9*b*B*sin(d*x+c)^9/d
```

Rubi [A] time = 0.24, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2837, 772}

$$-\frac{(aB+Ab)\sin^8(c+dx)}{8d}-\frac{(aA-3bB)\sin^7(c+dx)}{7d}+\frac{(aB+Ab)\sin^6(c+dx)}{2d}+\frac{3(aA-bB)\sin^5(c+dx)}{5d}-\frac{3(aB+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) - ((3*a*A - b*B)*
Sin[c + d*x]^3)/(3*d) - (3*(A*b + a*B)*Sin[c + d*x]^4)/(4*d) + (3*(a*A - b*
B)*Sin[c + d*x]^5)/(5*d) + ((A*b + a*B)*Sin[c + d*x]^6)/(2*d) - ((a*A - 3*b
*B)*Sin[c + d*x]^7)/(7*d) - ((A*b + a*B)*Sin[c + d*x]^8)/(8*d) - (b*B*Sin[c
 + d*x]^9)/(9*d)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^3 dx, x, b\sin(c+d\right.}{b^7d}$$

$$= \frac{\text{Subst}\left(\int \left(aAb^6+b^5(Ab+aB)x+b^4(-3aA+bB)x^2\right.\right.}{}$$

$$= \frac{aA\sin(c+dx)}{d}+\frac{(Ab+aB)\sin^2(c+dx)}{2d}-\frac{(3aA-bI}{}$$

Mathematica [A] time = 0.80, size = 151, normalized size = 0.80

$$\frac{\sin(c+dx)\left(-315(aB+Ab)\sin^7(c+dx)-360(aA-3bB)\sin^6(c+dx)+1260(aB+Ab)\sin^5(c+dx)+1512(aA\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(2520*a*A + 1260*(A*b + a*B)*Sin[c + d*x] - 840*(3*a*A - b*B)
*Sin[c + d*x]^2 - 1890*(A*b + a*B)*Sin[c + d*x]^3 + 1512*(a*A - b*B)*Sin[c
+ d*x]^4 + 1260*(A*b + a*B)*Sin[c + d*x]^5 - 360*(a*A - 3*b*B)*Sin[c + d*x]
^6 - 315*(A*b + a*B)*Sin[c + d*x]^7 - 280*b*B*Sin[c + d*x]^8))/(2520*d)
```

fricas [A] time = 0.48, size = 106, normalized size = 0.56

$$-\frac{315\,(Ba+Ab)\cos{(dx+c)}^8+8\left(35\,Bb\cos{(dx+c)}^8-5\,(9\,Aa+Bb)\cos{(dx+c)}^6-6\,(9\,Aa+Bb)\cos{(dx+c)}^4\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2520*(315*(B*a + A*b)*cos(d*x + c)^8 + 8*(35*B*b*cos(d*x + c)^8 - 5*(9*A
*a + B*b)*cos(d*x + c)^6 - 6*(9*A*a + B*b)*cos(d*x + c)^4 - 8*(9*A*a + B*b)
*cos(d*x + c)^2 - 144*A*a - 16*B*b)*sin(d*x + c))/d
```

giac [A] time = 0.31, size = 182, normalized size = 0.97

$$-\frac{Bb\sin{(9\,dx+9\,c)}}{2304\,d}+\frac{7\,Aa\sin{(3\,dx+3\,c)}}{64\,d}-\frac{(Ba+Ab)\cos{(8\,dx+8\,c)}}{1024\,d}-\frac{(Ba+Ab)\cos{(6\,dx+6\,c)}}{128\,d}-\frac{7\,(Ba+Ab)\,c}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2304*B*b*sin(9*d*x + 9*c)/d + 7/64*A*a*sin(3*d*x + 3*c)/d - 1/1024*(B*a
+ A*b)*cos(8*d*x + 8*c)/d - 1/128*(B*a + A*b)*cos(6*d*x + 6*c)/d - 7/256*(B
*a + A*b)*cos(4*d*x + 4*c)/d - 7/128*(B*a + A*b)*cos(2*d*x + 2*c)/d + 1/179
2*(4*A*a - 5*B*b)*sin(7*d*x + 7*c)/d + 1/320*(7*A*a - 2*B*b)*sin(5*d*x + 5*
c)/d + 7/128*(10*A*a + B*b)*sin(d*x + c)/d
```

maple [A] time = 0.46, size = 128, normalized size = 0.68

$$\frac{Bb\left(-\frac{\sin(dx+c)\left(\cos^{8}(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}(dx+c)\right)}{5}+\frac{8\left(\cos^{2}(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)-\frac{Ab\left(\cos^{8}(dx+c)\right)}{8}-\frac{aB\left(\cos^{8}(dx+c)\right)}{8}+\frac{aA\left(\frac{16}{5}+\mathrm{co}\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(B*b*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+
c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/8*A*b*cos(d*x+c)^8-1/8*a*B*cos(d*x+c)^
8+1/7*a*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.35, size = 151, normalized size = 0.80

$$-\frac{280\,Bb\sin\left(dx+c\right)^{9}+315\,(Ba+Ab)\sin\left(dx+c\right)^{8}+360\,(Aa-3\,Bb)\sin\left(dx+c\right)^{7}-1260\,(Ba+Ab)\sin\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2520*(280*B*b*sin(d*x + c)^9 + 315*(B*a + A*b)*sin(d*x + c)^8 + 360*(A*a
 - 3*B*b)*sin(d*x + c)^7 - 1260*(B*a + A*b)*sin(d*x + c)^6 - 1512*(A*a - B*
b)*sin(d*x + c)^5 + 1890*(B*a + A*b)*sin(d*x + c)^4 + 840*(3*A*a - B*b)*sin
(d*x + c)^3 - 2520*A*a*sin(d*x + c) - 1260*(B*a + A*b)*sin(d*x + c)^2)/d
```

mupad [B] time = 0.13, size = 156, normalized size = 0.83

$$-\frac{\frac{B\,b\sin(c+d\,x)^{9}}{9}+\left(\frac{A\,b}{8}+\frac{B\,a}{8}\right)\sin\left(c+d\,x\right)^{8}+\left(\frac{A\,a}{7}-\frac{3\,B\,b}{7}\right)\sin\left(c+d\,x\right)^{7}+\left(-\frac{A\,b}{2}-\frac{B\,a}{2}\right)\sin\left(c+d\,x\right)^{6}+\left(\frac{3\,B\,b}{5}-\frac{3\,A}{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(A + B*sin(c + d*x))*(a + b*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)^3*(A*a - (B*b)/3) - sin(c + d*x)^2*((A*b)/2 + (B*a)/2) - sin
(c + d*x)^6*((A*b)/2 + (B*a)/2) + sin(c + d*x)^4*((3*A*b)/4 + (3*B*a)/4) -
sin(c + d*x)^5*((3*A*a)/5 - (3*B*b)/5) + sin(c + d*x)^7*((A*a)/7 - (3*B*b)/
7) + sin(c + d*x)^8*((A*b)/8 + (B*a)/8) - A*a*sin(c + d*x) + (B*b*sin(c + d
*x)^9)/9)/d
```

sympy [A] time = 15.63, size = 228, normalized size = 1.21

$$\begin{cases} \frac{16Aa\sin^7(c+dx)}{35d} + \frac{8Aa\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2Aa\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{Aa\sin(c+dx)\cos^6(c+dx)}{d} - \frac{Ab\cos^8(c+dx)}{8d} - \frac{Ba\cos^8(c+dx)}{8d} \\ x(A+B\sin(c))(a+b\sin(c))\cos^7(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*A*a*sin(c + d*x)**7/(35*d) + 8*A*a*sin(c + d*x)**5*cos(c + d*
x)**2/(5*d) + 2*A*a*sin(c + d*x)**3*cos(c + d*x)**4/d + A*a*sin(c + d*x)*co
s(c + d*x)**6/d - A*b*cos(c + d*x)**8/(8*d) - B*a*cos(c + d*x)**8/(8*d) + 1
6*B*b*sin(c + d*x)**9/(315*d) + 8*B*b*sin(c + d*x)**7*cos(c + d*x)**2/(35*d
) + 2*B*b*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + B*b*sin(c + d*x)**3*cos(c
 + d*x)**6/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(c))*cos(c)**7, Tr
ue))
```

3.1529 $\int \cos^5(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=143

$$\frac{(aB+Ab)\sin^6(c+dx)}{6d}+\frac{(aA-2bB)\sin^5(c+dx)}{5d}-\frac{(aB+Ab)\sin^4(c+dx)}{2d}-\frac{(2aA-bB)\sin^3(c+dx)}{3d}+\frac{(aB+Ab}{}$$

```
[Out] a*A*sin(d*x+c)/d+1/2*(A*b+B*a)*sin(d*x+c)^2/d-1/3*(2*A*a-B*b)*sin(d*x+c)^3/
d-1/2*(A*b+B*a)*sin(d*x+c)^4/d+1/5*(A*a-2*B*b)*sin(d*x+c)^5/d+1/6*(A*b+B*a)
*sin(d*x+c)^6/d+1/7*b*B*sin(d*x+c)^7/d
```

Rubi [A] time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2837, 772}

$$\frac{(aB+Ab)\sin^6(c+dx)}{6d}+\frac{(aA-2bB)\sin^5(c+dx)}{5d}-\frac{(aB+Ab)\sin^4(c+dx)}{2d}-\frac{(2aA-bB)\sin^3(c+dx)}{3d}+\frac{(aB+Ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) - ((2*a*A - b*B)*
Sin[c + d*x]^3)/(3*d) - ((A*b + a*B)*Sin[c + d*x]^4)/(2*d) + ((a*A - 2*b*B)
*Sin[c + d*x]^5)/(5*d) + ((A*b + a*B)*Sin[c + d*x]^6)/(6*d) + (b*B*Sin[c +
d*x]^7)/(7*d)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^2 dx, x, b\sin(c+d\right.}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(aAb^4+b^3(Ab+aB)x+b^2(-2aA+bB)x^2\right.\right.}{}$$
$$= \frac{aA\sin(c+dx)}{d} + \frac{(Ab+aB)\sin^2(c+dx)}{2d} - \frac{(2aA-bB}{}$$

Mathematica [A] time = 0.29, size = 116, normalized size = 0.81

$$\frac{\sin(c+dx)\left(35(aB+Ab)\sin^5(c+dx)+42(aA-2bB)\sin^4(c+dx)-105(aB+Ab)\sin^3(c+dx)-70(2aA-bB)\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(210*a*A + 105*(A*b + a*B)*Sin[c + d*x] - 70*(2*a*A - b*B)*Si
n[c + d*x]^2 - 105*(A*b + a*B)*Sin[c + d*x]^3 + 42*(a*A - 2*b*B)*Sin[c + d*
x]^4 + 35*(A*b + a*B)*Sin[c + d*x]^5 + 30*b*B*Sin[c + d*x]^6))/(210*d)
```

fricas [A] time = 0.46, size = 88, normalized size = 0.62

$$-\frac{35\,(Ba+Ab)\cos\left(dx+c\right)^6+2\left(15\,Bb\cos\left(dx+c\right)^6-3\,(7\,Aa+Bb)\cos\left(dx+c\right)^4-4\,(7\,Aa+Bb)\cos\left(dx+c\right)^2-\right.}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/210*(35*(B*a + A*b)*cos(d*x + c)^6 + 2*(15*B*b*cos(d*x + c)^6 - 3*(7*A*a
 + B*b)*cos(d*x + c)^4 - 4*(7*A*a + B*b)*cos(d*x + c)^2 - 56*A*a - 8*B*b)*s
in(d*x + c))/d
```

giac [A] time = 0.24, size = 145, normalized size = 1.01

$$-\frac{Bb\sin\left(7\,dx+7\,c\right)}{448\,d}-\frac{(Ba+Ab)\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{(Ba+Ab)\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\,(Ba+Ab)\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{(4\,Aa}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/448*B*b*sin(7*d*x + 7*c)/d - 1/192*(B*a + A*b)*cos(6*d*x + 6*c)/d - 1/32
*(B*a + A*b)*cos(4*d*x + 4*c)/d - 5/64*(B*a + A*b)*cos(2*d*x + 2*c)/d + 1/3
20*(4*A*a - 3*B*b)*sin(5*d*x + 5*c)/d + 1/192*(20*A*a - B*b)*sin(3*d*x + 3*
c)/d + 5/64*(8*A*a + B*b)*sin(d*x + c)/d
```

maple [A] time = 0.46, size = 108, normalized size = 0.76

$$\frac{Bb\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{Ab\left(\cos^6(dx+c)\right)}{6}-\frac{aB\left(\cos^6(dx+c)\right)}{6}+\frac{aA\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2\right.}{}\right.}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(B*b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-1/6*A*b*cos(d*x+c)^6-1/6*a*B*cos(d*x+c)^6+1/5*a*A*(8/3+cos
(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.53, size = 116, normalized size = 0.81

$$\frac{30\,Bb\sin\left(dx+c\right)^7+35\left(Ba+Ab\right)\sin\left(dx+c\right)^6+42\left(Aa-2\,Bb\right)\sin\left(dx+c\right)^5-105\left(Ba+Ab\right)\sin\left(dx+c\right)^4-7}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/210*(30*B*b*sin(d*x + c)^7 + 35*(B*a + A*b)*sin(d*x + c)^6 + 42*(A*a - 2*
B*b)*sin(d*x + c)^5 - 105*(B*a + A*b)*sin(d*x + c)^4 - 70*(2*A*a - B*b)*sin
(d*x + c)^3 + 210*A*a*sin(d*x + c) + 105*(B*a + A*b)*sin(d*x + c)^2)/d
```

mupad [B] time = 12.06, size = 118, normalized size = 0.83

$$\frac{\frac{B\,b\,\sin(c+d\,x)^7}{7}+\left(\frac{A\,b}{6}+\frac{B\,a}{6}\right)\,\sin\left(c+d\,x\right)^6+\left(\frac{A\,a}{5}-\frac{2\,B\,b}{5}\right)\,\sin\left(c+d\,x\right)^5+\left(-\frac{A\,b}{2}-\frac{B\,a}{2}\right)\,\sin\left(c+d\,x\right)^4+\left(\frac{B\,b}{3}-\frac{2\,A\,a}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B*sin(c + d*x))*(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^2*((A*b)/2 + (B*a)/2) - sin(c + d*x)^4*((A*b)/2 + (B*a)/2) -
sin(c + d*x)^3*((2*A*a)/3 - (B*b)/3) + sin(c + d*x)^5*((A*a)/5 - (2*B*b)/5)
 + sin(c + d*x)^6*((A*b)/6 + (B*a)/6) + A*a*sin(c + d*x) + (B*b*sin(c + d*x
)^7)/7)/d
```

sympy [A] time = 5.84, size = 178, normalized size = 1.24

$$\begin{cases} \frac{8Aa\sin^5(c+dx)}{15d} + \frac{4Aa\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{Aa\sin(c+dx)\cos^4(c+dx)}{d} - \frac{Ab\cos^6(c+dx)}{6d} - \frac{Ba\cos^6(c+dx)}{6d} + \frac{8Bb\sin^7(c+dx)}{105d} + \frac{4Bb\sin^5 \\ x(A+B\sin(c))(a+b\sin(c))\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*A*a*sin(c + d*x)**5/(15*d) + 4*A*a*sin(c + d*x)**3*cos(c + d*x
)**2/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**4/d - A*b*cos(c + d*x)**6/(6*d)
 - B*a*cos(c + d*x)**6/(6*d) + 8*B*b*sin(c + d*x)**7/(105*d) + 4*B*b*sin(c
+ d*x)**5*cos(c + d*x)**2/(15*d) + B*b*sin(c + d*x)**3*cos(c + d*x)**4/(3*d
), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(c))*cos(c)**5, True))
```

3.1530 $\int \cos^3(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{(aB+Ab)\sin^4(c+dx)}{4d}-\frac{(aA-bB)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}-\frac{bB\sin^5(c+dx)}{5d}$$

```
[Out] a*A*sin(d*x+c)/d+1/2*(A*b+B*a)*sin(d*x+c)^2/d-1/3*(A*a-B*b)*sin(d*x+c)^3/d-
1/4*(A*b+B*a)*sin(d*x+c)^4/d-1/5*b*B*sin(d*x+c)^5/d
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2837, 772}

$$-\frac{(aB+Ab)\sin^4(c+dx)}{4d}-\frac{(aA-bB)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}-\frac{bB\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) - ((a*A - b*B)*Si
n[c + d*x]^3)/(3*d) - ((A*b + a*B)*Sin[c + d*x]^4)/(4*d) - (b*B*Sin[c + d*x
]^5)/(5*d)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)\,dx, x, b\sin(c+dx\right.}{b^3d}$$

$$= \frac{\text{Subst}\left(\int \left(aAb^2+b(Ab+aB)x-(aA-bB)x^2-\frac{(Ab+}{b}\right.\right.}{b^3d}$$

$$= \frac{aA\sin(c+dx)}{d}+\frac{(Ab+aB)\sin^2(c+dx)}{2d}-\frac{(aA-bB)}{}$$

Mathematica [**A**] time = 0.26, size = 80, normalized size = 0.82

$$\frac{\sin(c+dx)\left(-15(aB+Ab)\sin^3(c+dx)-20(aA-bB)\sin^2(c+dx)+30(aB+Ab)\sin(c+dx)+60aA-12bB\sin\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(60*a*A + 30*(A*b + a*B)*Sin[c + d*x] - 20*(a*A - b*B)*Sin[c
+ d*x]^2 - 15*(A*b + a*B)*Sin[c + d*x]^3 - 12*b*B*Sin[c + d*x]^4))/(60*d)
```

fricas [**A**] time = 0.45, size = 70, normalized size = 0.72

$$-\frac{15(Ba+Ab)\cos(dx+c)^4+4\left(3Bb\cos(dx+c)^4-(5Aa+Bb)\cos(dx+c)^2-10Aa-2Bb\right)\sin(dx+c)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/60*(15*(B*a + A*b)*cos(d*x + c)^4 + 4*(3*B*b*cos(d*x + c)^4 - (5*A*a + B
*b)*cos(d*x + c)^2 - 10*A*a - 2*B*b)*sin(d*x + c))/d
```

giac [**A**] time = 0.20, size = 100, normalized size = 1.03

$$-\frac{12Bb\sin(dx+c)^5+15Ba\sin(dx+c)^4+15Ab\sin(dx+c)^4+20Aa\sin(dx+c)^3-20Bb\sin(dx+c)^3-30B}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/60*(12*B*b*sin(d*x + c)^5 + 15*B*a*sin(d*x + c)^4 + 15*A*b*sin(d*x + c)^
4 + 20*A*a*sin(d*x + c)^3 - 20*B*b*sin(d*x + c)^3 - 30*B*a*sin(d*x + c)^2 -
 30*A*b*sin(d*x + c)^2 - 60*A*a*sin(d*x + c))/d
```

maple [A] time = 0.46, size = 88, normalized size = 0.91

$$\frac{Bb\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{Ab\left(\cos^4(dx+c)\right)}{4}-\frac{aB\left(\cos^4(dx+c)\right)}{4}+\frac{aA\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(B*b*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-1/
4*A*b*cos(d*x+c)^4-1/4*a*B*cos(d*x+c)^4+1/3*a*A*(2+cos(d*x+c)^2)*sin(d*x+c)
)
```

maxima [A] time = 1.13, size = 80, normalized size = 0.82

$$-\frac{12\,Bb\sin\left(dx+c\right)^5+15\,(Ba+Ab)\sin\left(dx+c\right)^4+20\,(Aa-Bb)\sin\left(dx+c\right)^3-60\,Aa\sin\left(dx+c\right)-30\,(Ba+A}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/60*(12*B*b*sin(d*x + c)^5 + 15*(B*a + A*b)*sin(d*x + c)^4 + 20*(A*a - B*
b)*sin(d*x + c)^3 - 60*A*a*sin(d*x + c) - 30*(B*a + A*b)*sin(d*x + c)^2)/d
```

mupad [B] time = 11.99, size = 83, normalized size = 0.86

$$-\frac{\frac{B\,b\sin(c+d\,x)^5}{5}+\left(\frac{A\,b}{4}+\frac{B\,a}{4}\right)\sin\left(c+d\,x\right)^4+\left(\frac{A\,a}{3}-\frac{B\,b}{3}\right)\sin\left(c+d\,x\right)^3+\left(-\frac{A\,b}{2}-\frac{B\,a}{2}\right)\sin\left(c+d\,x\right)^2-A\,a\,\sin\left(c+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*sin(c + d*x))*(a + b*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)^3*((A*a)/3 - (B*b)/3) - sin(c + d*x)^2*((A*b)/2 + (B*a)/2) +
 sin(c + d*x)^4*((A*b)/4 + (B*a)/4) - A*a*sin(c + d*x) + (B*b*sin(c + d*x)^
5)/5)/d
```

sympy [A] time = 1.90, size = 128, normalized size = 1.32

$$\begin{cases} \frac{2Aa\sin^3(c+dx)}{3d}+\frac{Aa\sin(c+dx)\cos^2(c+dx)}{d}-\frac{Ab\cos^4(c+dx)}{4d}-\frac{Ba\cos^4(c+dx)}{4d}+\frac{2Bb\sin^5(c+dx)}{15d}+\frac{Bb\sin^3(c+dx)\cos^2(c+dx)}{3d} & \text{for } d = \\ x\left(A+B\sin(c)\right)\left(a+b\sin(c)\right)\cos^3(c) & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*A*a*sin(c + d*x)**3/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**2/d
 - A*b*cos(c + d*x)**4/(4*d) - B*a*cos(c + d*x)**4/(4*d) + 2*B*b*sin(c + d*
x)**5/(15*d) + B*b*sin(c + d*x)**3*cos(c + d*x)**2/(3*d), Ne(d, 0)), (x*(A
+ B*sin(c))*(a + b*sin(c))*cos(c)**3, True))
```

3.1531 $\int \cos(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{(aB+Ab)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}+\frac{bB\sin^3(c+dx)}{3d}$$

```
[Out] a*A*sin(d*x+c)/d+1/2*(A*b+B*a)*sin(d*x+c)^2/d+1/3*b*B*sin(d*x+c)^3/d
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 43}

$$\frac{(aB+Ab)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}+\frac{bB\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) + (b*B*Sin[c + d*
x]^3)/(3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)\left(A+\frac{Bx}{b}\right)dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(aA+\frac{(Ab+aB)x}{b}+\frac{Bx^2}{b}\right)dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{aA\sin(c+dx)}{d}+\frac{(Ab+aB)\sin^2(c+dx)}{2d}+\frac{bB\sin^3(c-}{3d}$$

Mathematica [**A**] time = 0.08, size = 45, normalized size = 0.87

$$\frac{\sin(c+dx)\left(3(aB+Ab)\sin(c+dx)+6aA+2bB\sin^2(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(6*a*A + 3*(A*b + a*B)*Sin[c + d*x] + 2*b*B*Sin[c + d*x]^2))/
(6*d)
```

fricas [**A**] time = 0.43, size = 51, normalized size = 0.98

$$-\frac{3\,(Ba+Ab)\cos\left(dx+c\right)^2+2\left(Bb\cos\left(dx+c\right)^2-3\,Aa-Bb\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] -1/6*(3*(B*a + A*b)*cos(d*x + c)^2 + 2*(B*b*cos(d*x + c)^2 - 3*A*a - B*b)*s
in(d*x + c))/d
```

giac [**A**] time = 0.16, size = 52, normalized size = 1.00

$$\frac{2\,Bb\sin\left(dx+c\right)^3+3\,Ba\sin\left(dx+c\right)^2+3\,Ab\sin\left(dx+c\right)^2+6\,Aa\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*B*b*sin(d*x + c)^3 + 3*B*a*sin(d*x + c)^2 + 3*A*b*sin(d*x + c)^2 + 6
*A*a*sin(d*x + c))/d
```

maple [A] time = 0.22, size = 44, normalized size = 0.85

$$\frac{\frac{B\left(\sin^{3}(dx+c)\right)b}{3}+\frac{(Ab+aB)\left(\sin^{2}(dx+c)\right)}{2}+A\sin(dx+c)\,a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*B*sin(d*x+c)^3*b+1/2*(A*b+B*a)*sin(d*x+c)^2+A*sin(d*x+c)*a)
```

maxima [A] time = 0.32, size = 45, normalized size = 0.87

$$\frac{2\,Bb\sin(dx+c)^{3}+6\,Aa\sin(dx+c)+3\,(Ba+Ab)\sin(dx+c)^{2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/6*(2*B*b*sin(d*x + c)^3 + 6*A*a*sin(d*x + c) + 3*(B*a + A*b)*sin(d*x + c)
^2)/d
```

mupad [B] time = 12.04, size = 44, normalized size = 0.85

$$\frac{\frac{B\,b\sin(c+d\,x)^{3}}{3}+\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)\sin(c+d\,x)^{2}+A\,a\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*sin(c + d*x))*(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^2*((A*b)/2 + (B*a)/2) + A*a*sin(c + d*x) + (B*b*sin(c + d*x)^
3)/3)/d
```

sympy [A] time = 0.48, size = 75, normalized size = 1.44

$$\begin{cases}\frac{Aa\sin(c+dx)}{d}-\frac{Ab\cos^{2}(c+dx)}{2d}-\frac{Ba\cos^{2}(c+dx)}{2d}+\frac{Bb\sin^{3}(c+dx)}{3d} & \text{for } d\neq 0\\ x\,(A+B\sin(c))\,(a+b\sin(c))\cos(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a*sin(c + d*x)/d - A*b*cos(c + d*x)**2/(2*d) - B*a*cos(c + d*x
)**2/(2*d) + B*b*sin(c + d*x)**3/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b
*sin(c))*cos(c), True))
```

3.1532 $\int \sec(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=64

$$-\frac{(a+b)(A+B)\log(1-\sin(c+dx))}{2d}+\frac{(a-b)(A-B)\log(\sin(c+dx)+1)}{2d}-\frac{bB\sin(c+dx)}{d}$$

```
[Out] -1/2*(a+b)*(A+B)*ln(1-sin(d*x+c))/d+1/2*(a-b)*(A-B)*ln(1+sin(d*x+c))/d-b*B*
sin(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2837, 774, 633, 31}

$$-\frac{(a+b)(A+B)\log(1-\sin(c+dx))}{2d}+\frac{(a-b)(A-B)\log(\sin(c+dx)+1)}{2d}-\frac{bB\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -((a + b)*(A + B)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)*(A - B)*Log[1 + S
in[c + d*x]])/(2*d) - (b*B*Sin[c + d*x])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{b}\right)}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{bB\sin(c+dx)}{d} - \frac{b\,\text{Subst}\left(\int \frac{-aA-bB-\left(A+\frac{aB}{b}\right)x}{b^2-x^2}\,dx, x, b\sin\right.}{d}$$

$$= -\frac{bB\sin(c+dx)}{d} - \frac{((a-b)(A-B))\,\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x\right.}{2d}$$

$$= -\frac{(a+b)(A+B)\log(1-\sin(c+dx))}{2d} + \frac{(a-b)(A-B)}{}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.06

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d} - \frac{aB\log(\cos(c+dx))}{d} - \frac{Ab\log(\cos(c+dx))}{d} - \frac{bB\sin(c+dx)}{d} + \frac{bB\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*ArcTanh[Sin[c + d*x]])/d + (b*B*ArcTanh[Sin[c + d*x]])/d - (A*b*Log[Co
s[c + d*x]])/d - (a*B*Log[Cos[c + d*x]])/d - (b*B*Sin[c + d*x])/d
```

fricas [A] time = 0.47, size = 66, normalized size = 1.03

$$-\frac{2\,Bb\sin(dx+c) - ((A-B)a - (A-B)b)\log(\sin(dx+c)+1) + ((A+B)a + (A+B)b)\log(-\sin(dx+c)+1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] -1/2*(2*B*b*sin(d*x + c) - ((A - B)*a - (A - B)*b)*log(sin(d*x + c) + 1) +
((A + B)*a + (A + B)*b)*log(-sin(d*x + c) + 1))/d
```

giac [A] time = 0.17, size = 67, normalized size = 1.05

$$-\frac{2\,Bb\sin(dx+c)-(Aa-Ba-Ab+Bb)\log(|\sin(dx+c)+1|)+(Aa+Ba+Ab+Bb)\log(|\sin(dx+c)-1|)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*B*b*sin(d*x + c) - (A*a - B*a - A*b + B*b)*log(abs(sin(d*x + c) + 1
)) + (A*a + B*a + A*b + B*b)*log(abs(sin(d*x + c) - 1)))/d
```

maple [A] time = 0.36, size = 83, normalized size = 1.30

$$\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{Ab\ln(\cos(dx+c))}{d}-\frac{bB\sin(dx+c)}{d}+\frac{Bb\ln(\sec(dx+c)+\tan(dx+c))}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*a*A*ln(sec(d*x+c)+tan(d*x+c))-1/d*A*b*ln(cos(d*x+c))-b*B*sin(d*x+c)/d+1
/d*B*b*ln(sec(d*x+c)+tan(d*x+c))-1/d*a*B*ln(cos(d*x+c))
```

maxima [A] time = 0.31, size = 64, normalized size = 1.00

$$-\frac{2\,Bb\sin(dx+c)-((A-B)a-(A-B)b)\log(\sin(dx+c)+1)+((A+B)a+(A+B)b)\log(\sin(dx+c)-1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -1/2*(2*B*b*sin(d*x + c) - ((A - B)*a - (A - B)*b)*log(sin(d*x + c) + 1) +
((A + B)*a + (A + B)*b)*log(sin(d*x + c) - 1))/d
```

mupad [B] time = 0.12, size = 53, normalized size = 0.83

$$-\frac{B\,b\,\sin(c+d\,x)-\frac{\ln(\sin(c+d\,x)+1)\,(A-B)\,(a-b)}{2}+\frac{\ln(\sin(c+d\,x)-1)\,(a+b)\,(A+B)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x)))/cos(c + d*x),x)
```

```
[Out] -(B*b*sin(c + d*x) - (log(sin(c + d*x) + 1)*(A - B)*(a - b))/2 + (log(sin(c
 + d*x) - 1)*(a + b)*(A + B))/2)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sin(c + dx))(a + b\sin(c + dx))\sec(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*(a + b*sin(c + d*x))*sec(c + d*x), x)
```

3.1533 $\int \sec^3(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=59

$$\frac{(aA-bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{2d}$$

```
[Out] 1/2*(A*a-B*b)*arctanh(sin(d*x+c))/d+1/2*sec(d*x+c)^2*(A*b+a*B+(A*a+B*b)*sin
(d*x+c))/d
```

Rubi [A] time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 778, 206}

$$\frac{(aA-bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a*A - b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(A*b + a*B + (a
*A + b*B)*Sin[c + d*x]))/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{b^3\operatorname{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{b}\right)}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{2d} + \frac{(b(a$$

$$= \frac{(aA-bB)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\sec^2(c+dx)(Ab+a$$

Mathematica [A] time = 0.22, size = 54, normalized size = 0.92

$$\frac{(aA-bB)\tanh^{-1}(\sin(c+dx))+\sec^2(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a*A - b*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]^2*(A*b + a*B + (a*A + b*B
)*Sin[c + d*x]))/(2*d)
```

fricas [A] time = 0.46, size = 92, normalized size = 1.56

$$\frac{(Aa-Bb)\cos(dx+c)^2\log(\sin(dx+c)+1)-(Aa-Bb)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,Ba+2\,Ab+2}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/4*((A*a - B*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a - B*b)*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 2*B*a + 2*A*b + 2*(A*a + B*b)*sin(d*x + c)
)/(d*cos(d*x + c)^2)
```

giac [A] time = 0.21, size = 84, normalized size = 1.42

$$\frac{(Aa-Bb)\log(|\sin(dx+c)+1|)-(Aa-Bb)\log(|\sin(dx+c)-1|)-\frac{2\,(Aa\sin(dx+c)+Bb\sin(dx+c)+Ba+Ab)}{\sin(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/4*((A*a - B*b)*log(abs(sin(d*x + c) + 1)) - (A*a - B*b)*log(abs(sin(d*x +
 c) - 1)) - 2*(A*a*sin(d*x + c) + B*b*sin(d*x + c) + B*a + A*b)/(sin(d*x +
c)^2 - 1))/d
```

maple [B] time = 0.55, size = 129, normalized size = 2.19

$$\frac{aA\sec(dx+c)\tan(dx+c)}{2d}+\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{aB}{2d\cos(dx+c)^2}+\frac{Ab}{2d\cos(dx+c)^2}+\frac{Bb\left(\sin^3(\right.}{2d\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/2/d*a*A*sec(d*x+c)*tan(d*x+c)+1/2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a
*B/cos(d*x+c)^2+1/2/d*A*b/cos(d*x+c)^2+1/2/d*B*b*sin(d*x+c)^3/cos(d*x+c)^2+
1/2*b*B*sin(d*x+c)/d-1/2/d*B*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.31, size = 78, normalized size = 1.32

$$\frac{(Aa-Bb)\log(\sin(dx+c)+1)-(Aa-Bb)\log(\sin(dx+c)-1)-\frac{2(Ba+Ab+(Aa+Bb)\sin(dx+c))}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*((A*a - B*b)*log(sin(d*x + c) + 1) - (A*a - B*b)*log(sin(d*x + c) - 1)
- 2*(B*a + A*b + (A*a + B*b)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 0.11, size = 63, normalized size = 1.07

$$\frac{\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(\frac{A\,a}{2}-\frac{B\,b}{2}\right)}{d}-\frac{\frac{A\,b}{2}+\frac{B\,a}{2}+\sin\left(c+d\,x\right)\,\left(\frac{A\,a}{2}+\frac{B\,b}{2}\right)}{d\,\left(\sin\left(c+d\,x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] (atanh(sin(c + d*x))*((A*a)/2 - (B*b)/2))/d - ((A*b)/2 + (B*a)/2 + sin(c +
d*x)*((A*a)/2 + (B*b)/2))/(d*(sin(c + d*x)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sin(c + dx))(a + b\sin(c + dx))\sec^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
[Out] Integral((A + B*sin(c + d*x))*(a + b*sin(c + d*x))*sec(c + d*x)**3, x)
```

3.1534 $\int \sec^5(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=88

$$\frac{(3aA-bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^4(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{4d}+\frac{(3aA-bB)\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] 1/8*(3*A*a-B*b)*arctanh(sin(d*x+c))/d+1/4*sec(d*x+c)^4*(A*b+a*B+(A*a+B*b)*s
in(d*x+c))/d+1/8*(3*A*a-B*b)*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 778, 199, 206}

$$\frac{(3aA-bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^4(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{4d}+\frac{(3aA-bB)\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((3*a*A - b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(A*b + a*B +
(a*A + b*B)*Sin[c + d*x]))/(4*d) + ((3*a*A - b*B)*Sec[c + d*x]*Tan[c + d*x]
)/(8*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
```

```
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{b}\right)}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{4d} + \frac{\left(b^3 \right.}{}$$

$$= \frac{\sec^4(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{4d} + \frac{(3a}{}$$

$$= \frac{(3aA-bB)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\sec^4(c+dx)(Ab+}{}$$

Mathematica [A] time = 0.59, size = 82, normalized size = 0.93

$$\frac{\sec^4(c+dx)\left((bB-3aA)\sin^3(c+dx)+(5aA+bB)\sin(c+dx)+(3aA-bB)\cos^4(c+dx)\tanh^{-1}(\sin(c+dx))+\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^4*(2*(A*b + a*B) + (3*a*A - b*B)*ArcTanh[Sin[c + d*x]]*Cos[c
+ d*x]^4 + (5*a*A + b*B)*Sin[c + d*x] + (-3*a*A + b*B)*Sin[c + d*x]^3))/(8*
d)
```

fricas [A] time = 0.48, size = 114, normalized size = 1.30

$$\frac{(3\,Aa-Bb)\cos(dx+c)^4\log(\sin(dx+c)+1)-(3\,Aa-Bb)\cos(dx+c)^4\log(-\sin(dx+c)+1)+4\,Ba+4\,Ab}{16\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/16*((3*A*a - B*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*A*a - B*b)*co
s(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*B*a + 4*A*b + 2*((3*A*a - B*b)*cos(
d*x + c)^2 + 2*A*a + 2*B*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.23, size = 114, normalized size = 1.30

$$\frac{(3\,Aa-Bb)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-(3\,Aa-Bb)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(3\,Aa\sin(dx+c)^{3}-Bb\sin(dx+c)^{3}-5\,Aa\sin(dx+c)}{\left(\sin(dx+c)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/16*((3*A*a - B*b)*log(abs(sin(d*x + c) + 1)) - (3*A*a - B*b)*log(abs(sin(
d*x + c) - 1)) - 2*(3*A*a*sin(d*x + c)^3 - B*b*sin(d*x + c)^3 - 5*A*a*sin(d
*x + c) - B*b*sin(d*x + c) - 2*B*a - 2*A*b)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [B] time = 0.53, size = 173, normalized size = 1.97

$$\frac{aA\tan\left(dx+c\right)\left(\sec^{3}\left(dx+c\right)\right)}{4d}+\frac{3aA\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{3aA\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{aB}{4d\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*A*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a*A*sec(d*x+c)*tan(d*x+c)+3/8/d*a*A
*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a*B/cos(d*x+c)^4+1/4/d*A*b/cos(d*x+c)^4+1/
4/d*B*b*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*B*b*sin(d*x+c)^3/cos(d*x+c)^2+1/8*b
*B*sin(d*x+c)/d-1/8/d*B*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.32, size = 112, normalized size = 1.27

$$\frac{(3\,Aa-Bb)\log\left(\sin\left(dx+c\right)+1\right)-(3\,Aa-Bb)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left((3\,Aa-Bb)\sin(dx+c)^{3}-2\,Ba-2\,Ab-(5\,Aa+Bb)\sin(}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/16*((3*A*a - B*b)*log(sin(d*x + c) + 1) - (3*A*a - B*b)*log(sin(d*x + c)
- 1) - 2*((3*A*a - B*b)*sin(d*x + c)^3 - 2*B*a - 2*A*b - (5*A*a + B*b)*sin(
d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 0.14, size = 91, normalized size = 1.03

$$\frac{\left(\frac{B\,b}{8}-\frac{3\,A\,a}{8}\right)\,\sin\left(c+d\,x\right)^3+\left(\frac{5\,A\,a}{8}+\frac{B\,b}{8}\right)\,\sin\left(c+d\,x\right)+\frac{A\,b}{4}+\frac{B\,a}{4}}{d\,\left({\sin\left(c+d\,x\right)}^4-2\,{\sin\left(c+d\,x\right)}^2+1\right)}+\frac{\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(\frac{3\,A\,a}{8}-\frac{B\,b}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] ((A*b)/4 + (B*a)/4 + sin(c + d*x)*((5*A*a)/8 + (B*b)/8) - sin(c + d*x)^3*((
3*A*a)/8 - (B*b)/8))/(d*(sin(c + d*x)^4 - 2*sin(c + d*x)^2 + 1)) + (atanh(s
in(c + d*x))*((3*A*a)/8 - (B*b)/8))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1535 $\int \sec^7(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=118

$$\frac{(5aA-bB)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\sec^6(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{6d}+\frac{(5aA-bB)\tan(c+dx)\sec^3}{24d}$$

```
[Out] 1/16*(5*A*a-B*b)*arctanh(sin(d*x+c))/d+1/6*sec(d*x+c)^6*(A*b+a*B+(A*a+B*b)*
sin(d*x+c))/d+1/16*(5*A*a-B*b)*sec(d*x+c)*tan(d*x+c)/d+1/24*(5*A*a-B*b)*sec
(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 778, 199, 206}

$$\frac{(5aA-bB)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\sec^6(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{6d}+\frac{(5aA-bB)\tan(c+dx)\sec^3}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((5*a*A - b*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^6*(A*b + a*B +
 (a*A + b*B)*Sin[c + d*x]))/(6*d) + ((5*a*A - b*B)*Sec[c + d*x]*Tan[c + d*x
])/(16*d) + ((5*a*A - b*B)*Sec[c + d*x]^3*Tan[c + d*x])/(24*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
```

```
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^7(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{b^7\,\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{b}\right)}{\left(b^2-x^2\right)^4}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^6(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{6d} + \frac{\left(b^5\right.}{}$$

$$= \frac{\sec^6(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{6d} + \frac{(5a}{}$$

$$= \frac{\sec^6(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{6d} + \frac{(5a}{}$$

$$= \frac{(5aA-bB)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{\sec^6(c+dx)(Ab+}{}$$

Mathematica [A] time = 0.88, size = 104, normalized size = 0.88

$$-\frac{\sec^6(c+dx)\left((3bB-15aA)\sin^5(c+dx)+8(5aA-bB)\sin^3(c+dx)-3(11aA+bB)\sin(c+dx)-3(5aA-bB)\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/48*(Sec[c + d*x]^6*(-8*(A*b + a*B) - 3*(5*a*A - b*B)*ArcTanh[Sin[c + d*x
]]*Cos[c + d*x]^6 - 3*(11*a*A + b*B)*Sin[c + d*x] + 8*(5*a*A - b*B)*Sin[c +
 d*x]^3 + (-15*a*A + 3*b*B)*Sin[c + d*x]^5))/d
```

fricas [A] time = 0.47, size = 135, normalized size = 1.14

$$\frac{3\,(5\,Aa-Bb)\cos{(dx+c)}^{6}\log{(\sin{(dx+c)}+1)}-3\,(5\,Aa-Bb)\cos{(dx+c)}^{6}\log{(-\sin{(dx+c)}+1)}+16\,Ba+}{96\,d\cos{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/96*(3*(5*A*a - B*b)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(5*A*a - B*b
)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 16*B*a + 16*A*b + 2*(3*(5*A*a - B
*b)*cos(d*x + c)^4 + 2*(5*A*a - B*b)*cos(d*x + c)^2 + 8*A*a + 8*B*b)*sin(d*
x + c))/(d*cos(d*x + c)^6)
```

giac [A] time = 0.26, size = 139, normalized size = 1.18

$$\frac{3\,(5\,Aa-Bb)\log{(|\sin{(dx+c)}+1|)}-3\,(5\,Aa-Bb)\log{(|\sin{(dx+c)}-1|)}-\frac{2\left(15\,Aa\sin(dx+c)^{5}-3\,Bb\sin(dx+c)^{5}-40\,Aa\,\mathrm{s}}{}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/96*(3*(5*A*a - B*b)*log(abs(sin(d*x + c) + 1)) - 3*(5*A*a - B*b)*log(abs(
sin(d*x + c) - 1)) - 2*(15*A*a*sin(d*x + c)^5 - 3*B*b*sin(d*x + c)^5 - 40*A
*a*sin(d*x + c)^3 + 8*B*b*sin(d*x + c)^3 + 33*A*a*sin(d*x + c) + 3*B*b*sin(
d*x + c) + 8*B*a + 8*A*b)/(sin(d*x + c)^2 - 1)^3)/d
```

maple [A] time = 0.56, size = 217, normalized size = 1.84

$$\frac{aA\tan{(dx+c)}\left(\sec^{5}{(dx+c)}\right)}{6d}+\frac{5aA\tan{(dx+c)}\left(\sec^{3}{(dx+c)}\right)}{24d}+\frac{5aA\sec{(dx+c)}\tan{(dx+c)}}{16d}+\frac{5aA\ln{(\sec{(d}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/6/d*a*A*tan(d*x+c)*sec(d*x+c)^5+5/24/d*a*A*tan(d*x+c)*sec(d*x+c)^3+5/16/d
*a*A*sec(d*x+c)*tan(d*x+c)+5/16/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/6/d*a*B/c
os(d*x+c)^6+1/6/d*A*b/cos(d*x+c)^6+1/6/d*B*b*sin(d*x+c)^3/cos(d*x+c)^6+1/8/
d*B*b*sin(d*x+c)^3/cos(d*x+c)^4+1/16/d*B*b*sin(d*x+c)^3/cos(d*x+c)^2+1/16*b
*B*sin(d*x+c)/d-1/16/d*B*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.31, size = 143, normalized size = 1.21

$$\frac{3\,(5\,Aa-Bb)\log\left(\sin\left(dx+c\right)+1\right)-3\,(5\,Aa-Bb)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,(5\,Aa-Bb)\sin(dx+c)^5-8\,(5\,Aa-Bb)\sin(dx+c)^3}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(3*(5*A*a - B*b)*log(sin(d*x + c) + 1) - 3*(5*A*a - B*b)*log(sin(d*x +
 c) - 1) - 2*(3*(5*A*a - B*b)*sin(d*x + c)^5 - 8*(5*A*a - B*b)*sin(d*x + c)
^3 + 8*B*a + 8*A*b + 3*(11*A*a + B*b)*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin
(d*x + c)^4 + 3*sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 12.44, size = 120, normalized size = 1.02

$$\frac{\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\left(\frac{5\,A\,a}{16}-\frac{B\,b}{16}\right)}{d}-\frac{\left(\frac{5\,A\,a}{16}-\frac{B\,b}{16}\right)\sin\left(c+d\,x\right)^5+\left(\frac{B\,b}{6}-\frac{5\,A\,a}{6}\right)\sin\left(c+d\,x\right)^3+\left(\frac{11\,A\,a}{16}+\frac{B\,b}{16}\right)\sin\left(c+\right.}{d\,\left(\sin\left(c+d\,x\right)^6-3\sin\left(c+d\,x\right)^4+3\sin\left(c+d\,x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x)))/cos(c + d*x)^7,x)
```

```
[Out] (atanh(sin(c + d*x))*((5*A*a)/16 - (B*b)/16))/d - ((A*b)/6 + (B*a)/6 + sin(
c + d*x)*((11*A*a)/16 + (B*b)/16) - sin(c + d*x)^3*((5*A*a)/6 - (B*b)/6) +
sin(c + d*x)^5*((5*A*a)/16 - (B*b)/16))/(d*(3*sin(c + d*x)^2 - 3*sin(c + d*
x)^4 + sin(c + d*x)^6 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1536 $\int \cos^7(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=349

$$\frac{3\left(-7a^2B+2aAb+b^2B\right)(a+b\sin(c+dx))^8}{8b^8d}+\frac{\left(a^2-b^2\right)^2\left(-7a^2B+6aAb+b^2B\right)(a+b\sin(c+dx))^4}{4b^8d}-\frac{\left(a^2-b^2\right)}{}$$

```
[Out] -1/3*(a^2-b^2)^3*(A*b-B*a)*(a+b*sin(d*x+c))^3/b^8/d+1/4*(a^2-b^2)^2*(6*A*a*
b-7*B*a^2+B*b^2)*(a+b*sin(d*x+c))^4/b^8/d-3/5*(a^2-b^2)*(5*A*a^2*b-A*b^3-7*
B*a^3+3*B*a*b^2)*(a+b*sin(d*x+c))^5/b^8/d+1/6*(20*A*a^3*b-12*A*a*b^3-35*B*a
^4+30*B*a^2*b^2-3*B*b^4)*(a+b*sin(d*x+c))^6/b^8/d-1/7*(15*A*a^2*b-3*A*b^3-3
5*B*a^3+15*B*a*b^2)*(a+b*sin(d*x+c))^7/b^8/d+3/8*(2*A*a*b-7*B*a^2+B*b^2)*(a
+b*sin(d*x+c))^8/b^8/d-1/9*(A*b-7*B*a)*(a+b*sin(d*x+c))^9/b^8/d-1/10*B*(a+b
*sin(d*x+c))^10/b^8/d
```

Rubi [A] time = 0.39, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$\frac{3\left(-7a^2B+2aAb+b^2B\right)(a+b\sin(c+dx))^8}{8b^8d}-\frac{\left(15a^2Ab-35a^3B+15ab^2B-3Ab^3\right)(a+b\sin(c+dx))^7}{7b^8d}+\frac{\left(20a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -((a^2 - b^2)^3*(A*b - a*B)*(a + b*Sin[c + d*x])^3)/(3*b^8*d) + ((a^2 - b^2
)^2*(6*a*A*b - 7*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^4)/(4*b^8*d) - (3*(a^2
 - b^2)*(5*a^2*A*b - A*b^3 - 7*a^3*B + 3*a*b^2*B)*(a + b*Sin[c + d*x])^5)/(
5*b^8*d) + ((20*a^3*A*b - 12*a*A*b^3 - 35*a^4*B + 30*a^2*b^2*B - 3*b^4*B)*(
a + b*Sin[c + d*x])^6)/(6*b^8*d) - ((15*a^2*A*b - 3*A*b^3 - 35*a^3*B + 15*a
*b^2*B)*(a + b*Sin[c + d*x])^7)/(7*b^8*d) + (3*(2*a*A*b - 7*a^2*B + b^2*B)*
(a + b*Sin[c + d*x])^8)/(8*b^8*d) - ((A*b - 7*a*B)*(a + b*Sin[c + d*x])^9)/
(9*b^8*d) - (B*(a + b*Sin[c + d*x])^10)/(10*b^8*d)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^3\,dx, x, b\sin(c+\right.}{b^7 d}$$

$$= \frac{\text{Subst}\left(\int\left(\frac{\left(-a^2+b^2\right)^3(Ab-aB)(a+x)^2}{b}+\frac{\left(-a^2+b^2\right)^2\left(6aAb-7a^2B+\right.}{b}\right.\right.}{}$$

$$= -\frac{\left(a^2-b^2\right)^3(Ab-aB)(a+b\sin(c+dx))^3}{3b^8 d}+\frac{\left(a^2-b^2\right.}{}$$

Mathematica [A] time = 1.50, size = 295, normalized size = 0.85

$$\frac{2520a^2Ab^8\sin(c+dx)-315b^8\left(a^2B+2aAb-3b^2B\right)\sin^8(c+dx)+360b^8\left(a^2(-A)+6abB+3Ab^2\right)\sin^7(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (-3*a^4*(a^6 - 9*a^4*b^2 + 42*a^2*b^4 - 210*b^6)*B + 2520*a^2*A*b^8*Sin[c +
 d*x] + 1260*a*b^8*(2*A*b + a*B)*Sin[c + d*x]^2 + 840*b^8*(-3*a^2*A + A*b^2
 + 2*a*b*B)*Sin[c + d*x]^3 + 630*b^8*(-6*a*A*b - 3*a^2*B + b^2*B)*Sin[c + d
*x]^4 - 1512*b^8*(-(a^2*A) + A*b^2 + 2*a*b*B)*Sin[c + d*x]^5 + 1260*b^8*(2*
a*A*b + a^2*B - b^2*B)*Sin[c + d*x]^6 + 360*b^8*(-(a^2*A) + 3*A*b^2 + 6*a*b
*B)*Sin[c + d*x]^7 - 315*b^8*(2*a*A*b + a^2*B - 3*b^2*B)*Sin[c + d*x]^8 - 2
80*b^9*(A*b + 2*a*B)*Sin[c + d*x]^9 - 252*b^10*B*Sin[c + d*x]^10)/(2520*b^8
*d)
```

fricas [A] time = 0.51, size = 174, normalized size = 0.50

$$\frac{252\,Bb^2\cos\left(dx+c\right)^{10}-315\left(Ba^2+2\,Aab+Bb^2\right)\cos\left(dx+c\right)^8-8\left(35\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^8-5\left(9\,Aa^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2520*(252*B*b^2*cos(d*x + c)^10 - 315*(B*a^2 + 2*A*a*b + B*b^2)*cos(d*x +
 c)^8 - 8*(35*(2*B*a*b + A*b^2)*cos(d*x + c)^8 - 5*(9*A*a^2 + 2*B*a*b + A*b
^2)*cos(d*x + c)^6 - 6*(9*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^4 - 144*A*a
^2 - 32*B*a*b - 16*A*b^2 - 8*(9*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2)*si
n(d*x + c))/d
```

giac [A] time = 0.50, size = 279, normalized size = 0.80

$$\frac{Bb^2\cos(10\,dx+10\,c)}{5120\,d}+\frac{7\,Aa^2\sin(3\,dx+3\,c)}{64\,d}-\frac{\left(Ba^2+2\,Aab-Bb^2\right)\cos(8\,dx+8\,c)}{1024\,d}-\frac{\left(8\,Ba^2+16\,Aab-Bb^2\right)\mathrm{co}}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/5120*B*b^2*cos(10*d*x + 10*c)/d + 7/64*A*a^2*sin(3*d*x + 3*c)/d - 1/1024*
(B*a^2 + 2*A*a*b - B*b^2)*cos(8*d*x + 8*c)/d - 1/1024*(8*B*a^2 + 16*A*a*b -
 B*b^2)*cos(6*d*x + 6*c)/d - 1/256*(7*B*a^2 + 14*A*a*b + B*b^2)*cos(4*d*x +
 4*c)/d - 7/512*(4*B*a^2 + 8*A*a*b + B*b^2)*cos(2*d*x + 2*c)/d - 1/2304*(2*
B*a*b + A*b^2)*sin(9*d*x + 9*c)/d + 1/1792*(4*A*a^2 - 10*B*a*b - 5*A*b^2)*s
in(7*d*x + 7*c)/d + 1/320*(7*A*a^2 - 4*B*a*b - 2*A*b^2)*sin(5*d*x + 5*c)/d
+ 7/128*(10*A*a^2 + 2*B*a*b + A*b^2)*sin(d*x + c)/d
```

maple [A] time = 0.48, size = 229, normalized size = 0.66

$$\frac{\frac{a^2A\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}-\frac{B\,a^2\left(\cos^8(dx+c)\right)}{8}-\frac{Aab\left(\cos^8(dx+c)\right)}{4}+2Bab\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/7*a^2*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*
x+c)-1/8*B*a^2*cos(d*x+c)^8-1/4*A*a*b*cos(d*x+c)^8+2*B*a*b*(-1/9*sin(d*x+c)
*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*si
n(d*x+c))+A*b^2*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*c
os(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+B*b^2*(-1/10*sin(d*x+c)^2*cos(d*x
+c)^8-1/40*cos(d*x+c)^8))
```

maxima [A] time = 0.31, size = 238, normalized size = 0.68

$$-\frac{252\,Bb^2\sin(dx+c)^{10}+280\left(2\,Bab+Ab^2\right)\sin(dx+c)^9+315\left(Ba^2+2\,Aab-3\,Bb^2\right)\sin(dx+c)^8+360\left(Aa^2\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/2520*(252*B*b^2*sin(d*x + c)^10 + 280*(2*B*a*b + A*b^2)*sin(d*x + c)^9 +
 315*(B*a^2 + 2*A*a*b - 3*B*b^2)*sin(d*x + c)^8 + 360*(A*a^2 - 6*B*a*b - 3*
A*b^2)*sin(d*x + c)^7 - 1260*(B*a^2 + 2*A*a*b - B*b^2)*sin(d*x + c)^6 - 151
2*(A*a^2 - 2*B*a*b - A*b^2)*sin(d*x + c)^5 + 630*(3*B*a^2 + 6*A*a*b - B*b^2
)*sin(d*x + c)^4 - 2520*A*a^2*sin(d*x + c) + 840*(3*A*a^2 - 2*B*a*b - A*b^2
)*sin(d*x + c)^3 - 1260*(B*a^2 + 2*A*a*b)*sin(d*x + c)^2)/d
```

mupad [B] time = 0.18, size = 236, normalized size = 0.68

$$\frac{\sin(c+d\,x)^2\,\left(\frac{B\,a^2}{2}+A\,b\,a\right)-\sin(c+d\,x)^9\,\left(\frac{A\,b^2}{9}+\frac{2\,B\,a\,b}{9}\right)+\sin(c+d\,x)^3\,\left(-A\,a^2+\frac{2\,B\,a\,b}{3}+\frac{A\,b^2}{3}\right)-\sin(c+d\,x)^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^2*((B*a^2)/2 + A*a*b) - sin(c + d*x)^9*((A*b^2)/9 + (2*B*a*b)
/9) + sin(c + d*x)^3*((A*b^2)/3 - A*a^2 + (2*B*a*b)/3) - sin(c + d*x)^5*((3
*A*b^2)/5 - (3*A*a^2)/5 + (6*B*a*b)/5) + sin(c + d*x)^7*((3*A*b^2)/7 - (A*a
^2)/7 + (6*B*a*b)/7) + sin(c + d*x)^6*((B*a^2)/2 - (B*b^2)/2 + A*a*b) - sin
(c + d*x)^4*((3*B*a^2)/4 - (B*b^2)/4 + (3*A*a*b)/2) - sin(c + d*x)^8*((B*a^
2)/8 - (3*B*b^2)/8 + (A*a*b)/4) - (B*b^2*sin(c + d*x)^10)/10 + A*a^2*sin(c
+ d*x))/d
```

sympy [A] time = 25.92, size = 440, normalized size = 1.26

$$\begin{cases} \frac{16Aa^2\sin^7(c+dx)}{35d}+\frac{8Aa^2\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{2Aa^2\sin^3(c+dx)\cos^4(c+dx)}{d}+\frac{Aa^2\sin(c+dx)\cos^6(c+dx)}{d}-\frac{Aab\cos^8(c+dx)}{4d}+\frac{16Ab^2 s}{3} \\ x(A+B\sin(c))(a+b\sin(c))^2\cos^7(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*A*a**2*sin(c + d*x)**7/(35*d) + 8*A*a**2*sin(c + d*x)**5*cos(
c + d*x)**2/(5*d) + 2*A*a**2*sin(c + d*x)**3*cos(c + d*x)**4/d + A*a**2*sin
(c + d*x)*cos(c + d*x)**6/d - A*a*b*cos(c + d*x)**8/(4*d) + 16*A*b**2*sin(c
 + d*x)**9/(315*d) + 8*A*b**2*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 2*A*
b**2*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + A*b**2*sin(c + d*x)**3*cos(c +
 d*x)**6/(3*d) - B*a**2*cos(c + d*x)**8/(8*d) + 32*B*a*b*sin(c + d*x)**9/(3
```

```
15*d) + 16*B*a*b*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 4*B*a*b*sin(c + d
*x)**5*cos(c + d*x)**4/(5*d) + 2*B*a*b*sin(c + d*x)**3*cos(c + d*x)**6/(3*d
) + B*b**2*sin(c + d*x)**10/(40*d) + B*b**2*sin(c + d*x)**8*cos(c + d*x)**2
/(8*d) + B*b**2*sin(c + d*x)**6*cos(c + d*x)**4/(4*d) + B*b**2*sin(c + d*x)
**4*cos(c + d*x)**6/(4*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(c))**2*c
os(c)**7, True))
```

3.1537 $\int \cos^5(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=231

$$-\frac{(-5a^2B+2aAb+b^2B)(a+b\sin(c+dx))^6}{3b^6d}-\frac{(a^2-b^2)(-5a^2B+4aAb+b^2B)(a+b\sin(c+dx))^4}{4b^6d}+\frac{(a^2-b^2)^2(}{}$$

```
[Out] 1/3*(a^2-b^2)^2*(A*b-B*a)*(a+b*sin(d*x+c))^3/b^6/d-1/4*(a^2-b^2)*(4*A*a*b-5
*B*a^2+B*b^2)*(a+b*sin(d*x+c))^4/b^6/d+2/5*(3*A*a^2*b-A*b^3-5*B*a^3+3*B*a*b
^2)*(a+b*sin(d*x+c))^5/b^6/d-1/3*(2*A*a*b-5*B*a^2+B*b^2)*(a+b*sin(d*x+c))^6
/b^6/d+1/7*(A*b-5*B*a)*(a+b*sin(d*x+c))^7/b^6/d+1/8*B*(a+b*sin(d*x+c))^8/b^
6/d
```

Rubi [A] time = 0.26, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$-\frac{(-5a^2B+2aAb+b^2B)(a+b\sin(c+dx))^6}{3b^6d}+\frac{2(3a^2Ab-5a^3B+3ab^2B-Ab^3)(a+b\sin(c+dx))^5}{5b^6d}-\frac{(a^2-b^2)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a^2 - b^2)^2*(A*b - a*B)*(a + b*Sin[c + d*x])^3)/(3*b^6*d) - ((a^2 - b^2)
*(4*a*A*b - 5*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^4)/(4*b^6*d) + (2*(3*a^2*
A*b - A*b^3 - 5*a^3*B + 3*a*b^2*B)*(a + b*Sin[c + d*x])^5)/(5*b^6*d) - ((2*
a*A*b - 5*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^6)/(3*b^6*d) + ((A*b - 5*a*B)
*(a + b*Sin[c + d*x])^7)/(7*b^6*d) + (B*(a + b*Sin[c + d*x])^8)/(8*b^6*d)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^2\,dx, x, b\sin(c$$

$$= \frac{\text{Subst}\left(\int\left(\frac{\left(-a^2+b^2\right)^2(Ab-aB)(a+x)^2}{b}+\frac{\left(-a^2+b^2\right)\left(4aAb-5a^2B-\right.}{b}\right.\right.}{}$$

$$= \frac{\left(a^2-b^2\right)^2(Ab-aB)(a+b\sin(c+dx))^3}{3b^6d} - \frac{\left(a^2-b^2\right.}{}$$

Mathematica [A] time = 0.50, size = 227, normalized size = 0.98

$$\frac{840a^2Ab^6\sin(c+dx)+140b^6\left(a^2B+2aAb-2b^2B\right)\sin^6(c+dx)+168b^6\left(a^2A-4abB-2Ab^2\right)\sin^5(c+dx)+2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^4*(3*a^4 - 28*a^2*b^2 + 210*b^4)*B + 840*a^2*A*b^6*Sin[c + d*x] + 420*a*
b^6*(2*A*b + a*B)*Sin[c + d*x]^2 + 280*b^6*(-2*a^2*A + A*b^2 + 2*a*b*B)*Sin
[c + d*x]^3 + 210*b^6*(-4*a*A*b - 2*a^2*B + b^2*B)*Sin[c + d*x]^4 + 168*b^6
*(a^2*A - 2*A*b^2 - 4*a*b*B)*Sin[c + d*x]^5 + 140*b^6*(2*a*A*b + a^2*B - 2*
b^2*B)*Sin[c + d*x]^6 + 120*b^7*(A*b + 2*a*B)*Sin[c + d*x]^7 + 105*b^8*B*Si
n[c + d*x]^8)/(840*b^6*d)
```

fricas [A] time = 0.49, size = 147, normalized size = 0.64

$$\frac{105\,Bb^2\cos(dx+c)^8-140\left(Ba^2+2\,Aab+Bb^2\right)\cos(dx+c)^6-8\left(15\left(2\,Bab+Ab^2\right)\cos(dx+c)^6-3\left(7\,Aa^2+\right.\right.}{840\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/840*(105*B*b^2*cos(d*x + c)^8 - 140*(B*a^2 + 2*A*a*b + B*b^2)*cos(d*x + c
)^6 - 8*(15*(2*B*a*b + A*b^2)*cos(d*x + c)^6 - 3*(7*A*a^2 + 2*B*a*b + A*b^2
)*cos(d*x + c)^4 - 56*A*a^2 - 16*B*a*b - 8*A*b^2 - 4*(7*A*a^2 + 2*B*a*b + A
*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [A] time = 0.38, size = 231, normalized size = 1.00

$$\frac{Bb^2\cos(8\,dx+8\,c)}{1024\,d}-\frac{\left(2\,Ba^2+4\,Aab-Bb^2\right)\cos(6\,dx+6\,c)}{384\,d}-\frac{\left(8\,Ba^2+16\,Aab+Bb^2\right)\cos(4\,dx+4\,c)}{256\,d}-\frac{\left(10\,Ba^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/1024*B*b^2*cos(8*d*x + 8*c)/d - 1/384*(2*B*a^2 + 4*A*a*b - B*b^2)*cos(6*d
*x + 6*c)/d - 1/256*(8*B*a^2 + 16*A*a*b + B*b^2)*cos(4*d*x + 4*c)/d - 1/128
*(10*B*a^2 + 20*A*a*b + 3*B*b^2)*cos(2*d*x + 2*c)/d - 1/448*(2*B*a*b + A*b^
2)*sin(7*d*x + 7*c)/d + 1/320*(4*A*a^2 - 6*B*a*b - 3*A*b^2)*sin(5*d*x + 5*c
)/d + 1/192*(20*A*a^2 - 2*B*a*b - A*b^2)*sin(3*d*x + 3*c)/d + 5/64*(8*A*a^2
 + 2*B*a*b + A*b^2)*sin(d*x + c)/d
```

maple [A] time = 0.54, size = 199, normalized size = 0.86

$$\frac{\frac{a^2A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5}-\frac{B\,a^2\left(\cos^6(dx+c)\right)}{6}-\frac{Aab\left(\cos^6(dx+c)\right)}{3}+2Bab\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(}{}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/6*B*a^2*cos
(d*x+c)^6-1/3*A*a*b*cos(d*x+c)^6+2*B*a*b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+A*b^2*(-1/7*sin(d*x+c)*cos
(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+B*b^2*(-1/8*
sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6))
```

maxima [A] time = 0.32, size = 184, normalized size = 0.80

$$\frac{105\,Bb^2\sin(dx+c)^8+120\left(2\,Bab+Ab^2\right)\sin(dx+c)^7+140\left(Ba^2+2\,Aab-2\,Bb^2\right)\sin(dx+c)^6+168\left(Aa^2-4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/840*(105*B*b^2*sin(d*x + c)^8 + 120*(2*B*a*b + A*b^2)*sin(d*x + c)^7 + 14
0*(B*a^2 + 2*A*a*b - 2*B*b^2)*sin(d*x + c)^6 + 168*(A*a^2 - 4*B*a*b - 2*A*b
```

```
^2)*sin(d*x + c)^5 - 210*(2*B*a^2 + 4*A*a*b - B*b^2)*sin(d*x + c)^4 + 840*A
*a^2*sin(d*x + c) - 280*(2*A*a^2 - 2*B*a*b - A*b^2)*sin(d*x + c)^3 + 420*(B
*a^2 + 2*A*a*b)*sin(d*x + c)^2)/d
```

mupad [B] time = 0.11, size = 180, normalized size = 0.78

$$\frac{\sin(c+d\,x)^2\,\left(\frac{B\,a^2}{2}+A\,b\,a\right)+\sin(c+d\,x)^7\,\left(\frac{A\,b^2}{7}+\frac{2\,B\,a\,b}{7}\right)+\sin(c+d\,x)^3\,\left(-\frac{2\,A\,a^2}{3}+\frac{2\,B\,a\,b}{3}+\frac{A\,b^2}{3}\right)-\sin(c+d\,x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^2*((B*a^2)/2 + A*a*b) + sin(c + d*x)^7*((A*b^2)/7 + (2*B*a*b)
/7) + sin(c + d*x)^3*((A*b^2)/3 - (2*A*a^2)/3 + (2*B*a*b)/3) - sin(c + d*x)
^5*((2*A*b^2)/5 - (A*a^2)/5 + (4*B*a*b)/5) - sin(c + d*x)^4*((B*a^2)/2 - (B
*b^2)/4 + A*a*b) + sin(c + d*x)^6*((B*a^2)/6 - (B*b^2)/3 + (A*a*b)/3) + (B*
b^2*sin(c + d*x)^8)/8 + A*a^2*sin(c + d*x))/d
```

sympy [A] time = 10.36, size = 335, normalized size = 1.45

$$\begin{cases} \frac{8Aa^2\sin^5(c+dx)}{15d}+\frac{4Aa^2\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{Aa^2\sin(c+dx)\cos^4(c+dx)}{d}-\frac{Aab\cos^6(c+dx)}{3d}+\frac{8Ab^2\sin^7(c+dx)}{105d}+\frac{4Ab^2\sin^5(c+dx)\cos}{15d} \\ x\,(A+B\sin(c))\,(a+b\sin(c))^2\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*A*a**2*sin(c + d*x)**5/(15*d) + 4*A*a**2*sin(c + d*x)**3*cos(c
 + d*x)**2/(3*d) + A*a**2*sin(c + d*x)*cos(c + d*x)**4/d - A*a*b*cos(c + d*
x)**6/(3*d) + 8*A*b**2*sin(c + d*x)**7/(105*d) + 4*A*b**2*sin(c + d*x)**5*c
os(c + d*x)**2/(15*d) + A*b**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - B*a*
*2*cos(c + d*x)**6/(6*d) + 16*B*a*b*sin(c + d*x)**7/(105*d) + 8*B*a*b*sin(c
 + d*x)**5*cos(c + d*x)**2/(15*d) + 2*B*a*b*sin(c + d*x)**3*cos(c + d*x)**4
/(3*d) + B*b**2*sin(c + d*x)**8/(24*d) + B*b**2*sin(c + d*x)**6*cos(c + d*x
)**2/(6*d) + B*b**2*sin(c + d*x)**4*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(A
 + B*sin(c))*(a + b*sin(c))**2*cos(c)**5, True))
```

3.1538 $\int \cos^3(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=132

$$\frac{\left(-3a^2B+2aAb+b^2B\right)(a+b\sin(c+dx))^4}{4b^4d}-\frac{\left(a^2-b^2\right)(Ab-aB)(a+b\sin(c+dx))^3}{3b^4d}-\frac{(Ab-3aB)(a+b\sin(c+d}{5b^4d}$$

```
[Out] -1/3*(a^2-b^2)*(A*b-B*a)*(a+b*sin(d*x+c))^3/b^4/d+1/4*(2*A*a*b-3*B*a^2+B*b^
2)*(a+b*sin(d*x+c))^4/b^4/d-1/5*(A*b-3*B*a)*(a+b*sin(d*x+c))^5/b^4/d-1/6*B*
(a+b*sin(d*x+c))^6/b^4/d
```

Rubi [A] time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$\frac{\left(-3a^2B+2aAb+b^2B\right)(a+b\sin(c+dx))^4}{4b^4d}-\frac{\left(a^2-b^2\right)(Ab-aB)(a+b\sin(c+dx))^3}{3b^4d}-\frac{(Ab-3aB)(a+b\sin(c+d}{5b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -((a^2 - b^2)*(A*b - a*B)*(a + b*Sin[c + d*x])^3)/(3*b^4*d) + ((2*a*A*b - 3
*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^4)/(4*b^4*d) - ((A*b - 3*a*B)*(a + b*S
in[c + d*x])^5)/(5*b^4*d) - (B*(a + b*Sin[c + d*x])^6)/(6*b^4*d)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)\,dx, x, b\sin(c\right.}{b^3d}$$
$$= \frac{\text{Subst}\left(\int\left(\frac{\left(-a^2+b^2\right)(Ab-aB)(a+x)^2}{b}+\frac{\left(2aAb-3a^2B+b^2B\right)(a+x}{b}\right.\right.}{b^3}$$
$$= -\frac{\left(a^2-b^2\right)(Ab-aB)(a+b\sin(c+dx))^3}{3b^4d}+\frac{(2aAb}{}$$

Mathematica [A] time = 0.25, size = 111, normalized size = 0.84

$$\frac{(a+b\sin(c+dx))^3\left(a^3B+3b\left(a^2(-B)+2aAb+5b^2B\right)\sin(c+dx)-2a^2Ab-6b^2(2Ab-aB)\sin^2(c+dx)-5ab\right.}{60b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a + b*Sin[c + d*x])^3*(-2*a^2*A*b + 20*A*b^3 + a^3*B - 5*a*b^2*B + 3*b*(2
*a*A*b - a^2*B + 5*b^2*B)*Sin[c + d*x] - 6*b^2*(2*A*b - a*B)*Sin[c + d*x]^2
 - 10*b^3*B*Sin[c + d*x]^3))/(60*b^4*d)
```

fricas [A] time = 0.47, size = 120, normalized size = 0.91

$$\frac{10\,Bb^2\cos(dx+c)^6-15\left(Ba^2+2\,Aab+Bb^2\right)\cos(dx+c)^4-4\left(3\left(2\,Bab+Ab^2\right)\cos(dx+c)^4-10\,Aa^2-4\,Bab\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/60*(10*B*b^2*cos(d*x + c)^6 - 15*(B*a^2 + 2*A*a*b + B*b^2)*cos(d*x + c)^4
 - 4*(3*(2*B*a*b + A*b^2)*cos(d*x + c)^4 - 10*A*a^2 - 4*B*a*b - 2*A*b^2 - (
5*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [A] time = 0.27, size = 168, normalized size = 1.27

$$-\frac{10\,Bb^2\sin(dx+c)^6+24\,Bab\sin(dx+c)^5+12\,Ab^2\sin(dx+c)^5+15\,Ba^2\sin(dx+c)^4+30\,Aab\sin(dx+c)^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/60*(10*B*b^2*sin(d*x + c)^6 + 24*B*a*b*sin(d*x + c)^5 + 12*A*b^2*sin(d*x
 + c)^5 + 15*B*a^2*sin(d*x + c)^4 + 30*A*a*b*sin(d*x + c)^4 - 15*B*b^2*sin(
d*x + c)^4 + 20*A*a^2*sin(d*x + c)^3 - 40*B*a*b*sin(d*x + c)^3 - 20*A*b^2*s
in(d*x + c)^3 - 30*B*a^2*sin(d*x + c)^2 - 60*A*a*b*sin(d*x + c)^2 - 60*A*a^
2*sin(d*x + c))/d
```

maple [A] time = 0.48, size = 169, normalized size = 1.28

$$\frac{\frac{a^2A(2+\cos^2(dx+c))\sin(dx+c)}{3} - \frac{Ba^2(\cos^4(dx+c))}{4} - \frac{Aab(\cos^4(dx+c))}{2} + 2Bab\left(-\frac{\sin(dx+c)(\cos^4(dx+c))}{5} + \frac{(2+\cos^2(dx+c))\sin(dx+c)}{15}\right) + A}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)-1/4*B*a^2*cos(d*x+c)^4-1/2*A*a*b
*cos(d*x+c)^4+2*B*a*b*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*s
in(d*x+c))+A*b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*
x+c))+B*b^2*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4))
```

maxima [A] time = 0.33, size = 128, normalized size = 0.97

$$-\frac{10\,Bb^2\sin(dx+c)^6 + 12\left(2\,Bab + Ab^2\right)\sin(dx+c)^5 + 15\left(Ba^2 + 2\,Aab - Bb^2\right)\sin(dx+c)^4 - 60\,Aa^2\sin(dx+}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/60*(10*B*b^2*sin(d*x + c)^6 + 12*(2*B*a*b + A*b^2)*sin(d*x + c)^5 + 15*(
B*a^2 + 2*A*a*b - B*b^2)*sin(d*x + c)^4 - 60*A*a^2*sin(d*x + c) + 20*(A*a^2
 - 2*B*a*b - A*b^2)*sin(d*x + c)^3 - 30*(B*a^2 + 2*A*a*b)*sin(d*x + c)^2)/d
```

mupad [B] time = 12.30, size = 127, normalized size = 0.96

$$\frac{\sin(c+d\,x)^2\,\left(\frac{B\,a^2}{2} + A\,b\,a\right) - \sin(c+d\,x)^5\,\left(\frac{A\,b^2}{5} + \frac{2\,B\,a\,b}{5}\right) + \sin(c+d\,x)^3\,\left(-\frac{A\,a^2}{3} + \frac{2\,B\,a\,b}{3} + \frac{A\,b^2}{3}\right) - \sin(c+d\,x)^4}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^2*((B*a^2)/2 + A*a*b) - sin(c + d*x)^5*((A*b^2)/5 + (2*B*a*b)
/5) + sin(c + d*x)^3*((A*b^2)/3 - (A*a^2)/3 + (2*B*a*b)/3) - sin(c + d*x)^4
*((B*a^2)/4 - (B*b^2)/4 + (A*a*b)/2) - (B*b^2*sin(c + d*x)^6)/6 + A*a^2*sin
(c + d*x))/d
```

sympy [A] time = 3.68, size = 228, normalized size = 1.73

$$\begin{cases} \frac{2Aa^2\sin^3(c+dx)}{3d} + \frac{Aa^2\sin(c+dx)\cos^2(c+dx)}{d} - \frac{Aab\cos^4(c+dx)}{2d} + \frac{2Ab^2\sin^5(c+dx)}{15d} + \frac{Ab^2\sin^3(c+dx)\cos^2(c+dx)}{3d} - \frac{Ba^2\cos^4(c+dx)}{4d} + \\ x(A+B\sin(c))(a+b\sin(c))^2\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*A*a**2*sin(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)*cos(c + d*x
)**2/d - A*a*b*cos(c + d*x)**4/(2*d) + 2*A*b**2*sin(c + d*x)**5/(15*d) + A*
b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - B*a**2*cos(c + d*x)**4/(4*d) +
 4*B*a*b*sin(c + d*x)**5/(15*d) + 2*B*a*b*sin(c + d*x)**3*cos(c + d*x)**2/(
3*d) + B*b**2*sin(c + d*x)**6/(12*d) + B*b**2*sin(c + d*x)**4*cos(c + d*x)*
*2/(4*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(c))**2*cos(c)**3, True))
```

3.1539 $\int \cos(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=54

$$\frac{(Ab-aB)(a+b\sin(c+dx))^3}{3b^2d}+\frac{B(a+b\sin(c+dx))^4}{4b^2d}$$

```
[Out] 1/3*(A*b-B*a)*(a+b*sin(d*x+c))^3/b^2/d+1/4*B*(a+b*sin(d*x+c))^4/b^2/d
```

Rubi [A] time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{(Ab-aB)(a+b\sin(c+dx))^3}{3b^2d}+\frac{B(a+b\sin(c+dx))^4}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A*b - a*B)*(a + b*Sin[c + d*x])^3)/(3*b^2*d) + (B*(a + b*Sin[c + d*x])^4)
/(4*b^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(A+\frac{Bx}{b}\right)dx, x, b\sin(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+x)^2}{b}+\frac{B(a+x)^3}{b}\right)dx, x, b\sin(c+dx\right.}{bd}$$

$$= \frac{(Ab-aB)(a+b\sin(c+dx))^3}{3b^2d} + \frac{B(a+b\sin(c+dx)}{4b^2d}$$

Mathematica [A] time = 0.07, size = 41, normalized size = 0.76

$$\frac{(a+b\sin(c+dx))^3(-aB+4Ab+3bB\sin(c+dx))}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a + b*Sin[c + d*x])^3*(4*A*b - a*B + 3*b*B*Sin[c + d*x]))/(12*b^2*d)
```

fricas [A] time = 0.44, size = 92, normalized size = 1.70

$$\frac{3\,Bb^2\cos(dx+c)^4 - 6\left(Ba^2+2\,Aab+Bb^2\right)\cos(dx+c)^2 + 4\left(3\,Aa^2+2\,Bab+Ab^2-\left(2\,Bab+Ab^2\right)\cos(dx+\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*B*b^2*cos(d*x + c)^4 - 6*(B*a^2 + 2*A*a*b + B*b^2)*cos(d*x + c)^2 +
 4*(3*A*a^2 + 2*B*a*b + A*b^2 - (2*B*a*b + A*b^2)*cos(d*x + c)^2)*sin(d*x +
 c))/d
```

giac [A] time = 0.17, size = 86, normalized size = 1.59

$$\frac{3\,Bb^2\sin(dx+c)^4 + 8\,Bab\sin(dx+c)^3 + 4\,Ab^2\sin(dx+c)^3 + 6\,Ba^2\sin(dx+c)^2 + 12\,Aab\sin(dx+c)^2 + 12}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/12*(3*B*b^2*sin(d*x + c)^4 + 8*B*a*b*sin(d*x + c)^3 + 4*A*b^2*sin(d*x + c
)^3 + 6*B*a^2*sin(d*x + c)^2 + 12*A*a*b*sin(d*x + c)^2 + 12*A*a^2*sin(d*x +
 c))/d
```

maple [A] time = 0.24, size = 73, normalized size = 1.35

$$\frac{\frac{B\,b^2\left(\sin^4(dx+c)\right)}{4}+\frac{\left(A\,b^2+2Bab\right)\left(\sin^3(dx+c)\right)}{3}+\frac{\left(2Aab+B\,a^2\right)\left(\sin^2(dx+c)\right)}{2}+a^2A\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/4*B*b^2*sin(d*x+c)^4+1/3*(A*b^2+2*B*a*b)*sin(d*x+c)^3+1/2*(2*A*a*b+B
*a^2)*sin(d*x+c)^2+a^2*A*sin(d*x+c))
```

maxima [A] time = 0.31, size = 74, normalized size = 1.37

$$\frac{3\,Bb^2\sin\left(dx+c\right)^4+12\,Aa^2\sin\left(dx+c\right)+4\left(2\,Bab+Ab^2\right)\sin\left(dx+c\right)^3+6\left(Ba^2+2\,Aab\right)\sin\left(dx+c\right)^2}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(3*B*b^2*sin(d*x + c)^4 + 12*A*a^2*sin(d*x + c) + 4*(2*B*a*b + A*b^2)*
sin(d*x + c)^3 + 6*(B*a^2 + 2*A*a*b)*sin(d*x + c)^2)/d
```

mupad [B] time = 0.07, size = 71, normalized size = 1.31

$$\frac{\sin\left(c+d\,x\right)^2\,\left(\frac{B\,a^2}{2}+A\,b\,a\right)+\sin\left(c+d\,x\right)^3\,\left(\frac{A\,b^2}{3}+\frac{2\,B\,a\,b}{3}\right)+\frac{B\,b^2\,\sin\left(c+d\,x\right)^4}{4}+A\,a^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^2*((B*a^2)/2 + A*a*b) + sin(c + d*x)^3*((A*b^2)/3 + (2*B*a*b)
/3) + (B*b^2*sin(c + d*x)^4)/4 + A*a^2*sin(c + d*x))/d
```

sympy [A] time = 1.02, size = 117, normalized size = 2.17

$$\begin{cases} \frac{Aa^2\sin\left(c+dx\right)}{d}-\frac{Aab\cos^2\left(c+dx\right)}{d}+\frac{Ab^2\sin^3\left(c+dx\right)}{3d}-\frac{Ba^2\cos^2\left(c+dx\right)}{2d}+\frac{2Bab\sin^3\left(c+dx\right)}{3d}+\frac{Bb^2\sin^4\left(c+dx\right)}{4d} & \text{for } d\neq 0\\ x\left(A+B\sin\left(c\right)\right)\left(a+b\sin\left(c\right)\right)^2\cos\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*sin(c + d*x)/d - A*a*b*cos(c + d*x)**2/d + A*b**2*sin(c +
 d*x)**3/(3*d) - B*a**2*cos(c + d*x)**2/(2*d) + 2*B*a*b*sin(c + d*x)**3/(3*
d) + B*b**2*sin(c + d*x)**4/(4*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(
c))**2*cos(c), True))
```

3.1540 $\int \sec(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=94

$$-\frac{b(2aB+Ab)\sin(c+dx)}{d}+\frac{(a-b)^2(A-B)\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2(A+B)\log(1-\sin(c+dx))}{2d}-\frac{b^2B\sin^2(}{2a}$$

```
[Out] -1/2*(a+b)^2*(A+B)*ln(1-sin(d*x+c))/d+1/2*(a-b)^2*(A-B)*ln(1+sin(d*x+c))/d-
b*(A*b+2*B*a)*sin(d*x+c)/d-1/2*b^2*B*sin(d*x+c)^2/d
```

Rubi [A] time = 0.17, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 801, 633, 31}

$$-\frac{b(2aB+Ab)\sin(c+dx)}{d}+\frac{(a-b)^2(A-B)\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2(A+B)\log(1-\sin(c+dx))}{2d}-\frac{b^2B\sin^2(}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -((a + b)^2*(A + B)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)^2*(A - B)*Log[1
 + Sin[c + d*x]])/(2*d) - (b*(A*b + 2*a*B)*Sin[c + d*x])/d - (b^2*B*Sin[c +
 d*x]^2)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{b}\right)}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \left(-A-\frac{2aB}{b}-\frac{Bx}{b}+\frac{b\left(a^2A+Ab^2+2abB\right)+\left(2aAb+a\right.}{b\left(b^2-x^2\right)}\right.\right.}{d}$$

$$= -\frac{b(Ab+2aB)\sin(c+dx)}{d}-\frac{b^2B\sin^2(c+dx)}{2d}+\frac{\text{Sub}}{}$$

$$= -\frac{b(Ab+2aB)\sin(c+dx)}{d}-\frac{b^2B\sin^2(c+dx)}{2d}-\frac{\left((a\right.}{}$$

$$= -\frac{(a+b)^2(A+B)\log(1-\sin(c+dx))}{2d}+\frac{(a-b)^2(A}{}$$

Mathematica [A] time = 0.21, size = 81, normalized size = 0.86

$$-\frac{2b(2aB+Ab)\sin(c+dx)-\left((a-b)^2(A-B)\log(\sin(c+dx)+1)\right)+(a+b)^2(A+B)\log(1-\sin(c+dx))+b^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/2*((a + b)^2*(A + B)*Log[1 - Sin[c + d*x]] - (a - b)^2*(A - B)*Log[1 + S
in[c + d*x]] + 2*b*(A*b + 2*a*B)*Sin[c + d*x] + b^2*B*Sin[c + d*x]^2)/d
```

fricas [A] time = 0.48, size = 111, normalized size = 1.18

$$\frac{Bb^2\cos(dx+c)^2+\left((A-B)a^2-2(A-B)ab+(A-B)b^2\right)\log(\sin(dx+c)+1)-\left((A+B)a^2+2(A+B)ab+\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*(B*b^2*cos(d*x + c)^2 + ((A - B)*a^2 - 2*(A - B)*a*b + (A - B)*b^2)*log
(sin(d*x + c) + 1) - ((A + B)*a^2 + 2*(A + B)*a*b + (A + B)*b^2)*log(-sin(d
*x + c) + 1) - 2*(2*B*a*b + A*b^2)*sin(d*x + c))/d
```

giac [A] time = 0.20, size = 129, normalized size = 1.37

$$-\frac{Bb^2\sin\left(dx+c\right)^2+4\,Bab\sin\left(dx+c\right)+2\,Ab^2\sin\left(dx+c\right)-\left(Aa^2-Ba^2-2\,Aab+2\,Bab+Ab^2-Bb^2\right)\log\left(|\sin\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2*(B*b^2*sin(d*x + c)^2 + 4*B*a*b*sin(d*x + c) + 2*A*b^2*sin(d*x + c) -
(A*a^2 - B*a^2 - 2*A*a*b + 2*B*a*b + A*b^2 - B*b^2)*log(abs(sin(d*x + c) +
1)) + (A*a^2 + B*a^2 + 2*A*a*b + 2*B*a*b + A*b^2 + B*b^2)*log(abs(sin(d*x +
 c) - 1)))/d
```

maple [A] time = 0.38, size = 161, normalized size = 1.71

$$\frac{a^2A\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}-\frac{B\,a^2\ln\left(\cos\left(dx+c\right)\right)}{d}-\frac{2Aab\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{2Bab\ln\left(\sec\left(dx+c\right)+\tan\left(\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))-1/d*B*a^2*ln(cos(d*x+c))-2/d*A*a*b*ln(c
os(d*x+c))+2/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))-2/d*B*a*b*sin(d*x+c)+1/d*A*b
^2*ln(sec(d*x+c)+tan(d*x+c))-1/d*A*b^2*sin(d*x+c)-1/2*b^2*B*sin(d*x+c)^2/d-
1/d*B*b^2*ln(cos(d*x+c))
```

maxima [A] time = 0.34, size = 109, normalized size = 1.16

$$-\frac{Bb^2\sin\left(dx+c\right)^2-\left((A-B)a^2-2\,(A-B)ab+(A-B)b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left((A+B)a^2+2\,(A+B)ab+\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(B*b^2*sin(d*x + c)^2 - ((A - B)*a^2 - 2*(A - B)*a*b + (A - B)*b^2)*lo
g(sin(d*x + c) + 1) + ((A + B)*a^2 + 2*(A + B)*a*b + (A + B)*b^2)*log(sin(d
*x + c) - 1) + 2*(2*B*a*b + A*b^2)*sin(d*x + c))/d
```

mupad [B] time = 12.37, size = 80, normalized size = 0.85

$$-\frac{\sin(c+d\,x)\,\left(A\,b^2+2\,B\,a\,b\right)+\frac{\ln(\sin(c+d\,x)-1)\,{(a+b)}^2\,(A+B)}{2}+\frac{B\,b^2\,{\sin(c+d\,x)}^2}{2}-\frac{\ln(\sin(c+d\,x)+1)\,(A-B)\,{(a-b)}^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2)/cos(c + d*x),x)
```

```
[Out] -(sin(c + d*x)*(A*b^2 + 2*B*a*b) + (log(sin(c + d*x) - 1)*(a + b)^2*(A + B)
)/2 + (B*b^2*sin(c + d*x)^2)/2 - (log(sin(c + d*x) + 1)*(A - B)*(a - b)^2)/
2)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\sin(c+dx))\,(a+b\sin(c+dx))^2\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*(a + b*sin(c + d*x))**2*sec(c + d*x), x)
```

3.1541 $\int \sec^3(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=112

$$-\frac{(a+b)(aA-b(A+2B))\log(1-\sin(c+dx))}{4d}+\frac{(a-b)(aA+b(A-2B))\log(\sin(c+dx)+1)}{4d}+\frac{\sec^2(c+dx)(a+$$

[Out] -1/4*(a+b)*(a*A-b*(A+2*B))*ln(1-sin(d*x+c))/d+1/4*(a-b)*(a*A+b*(A-2*B))*ln(1+sin(d*x+c))/d+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c))*(A*b+a*B+(A*a+B*b)*sin(d* x+c))/d

Rubi [A] time = 0.18, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2837, 819, 633, 31}

$$-\frac{(a+b)(aA-b(A+2B))\log(1-\sin(c+dx))}{4d}+\frac{(a-b)(aA+b(A-2B))\log(\sin(c+dx)+1)}{4d}+\frac{\sec^2(c+dx)(a+$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]

[Out] -((a + b)*(a*A - b*(A + 2*B))*Log[1 - Sin[c + d*x]])/(4*d) + ((a - b)*(a*A + b*(A - 2*B))*Log[1 + Sin[c + d*x]])/(4*d) + (Sec[c + d*x]^2*(a + b*Sin[c + d*x])*(A*b + a*B + (a*A + b*B)*Sin[c + d*x]))/(2*d)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
```

```
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{b}\right)}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))(Ab+aB+(aA+b$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))(Ab+aB+(aA+b$$

$$= -\frac{(a+b)(aA-b(A+2B))\log(1-\sin(c+dx))}{4d} + \frac{(a$$

Mathematica [A] time = 1.53, size = 174, normalized size = 1.55

$$\frac{\left(-6a^3Ab+4aAb^3+2b^4B\right)\tan^2(c+dx)-2a^3(aB-Ab)\sec^2(c+dx)+\left(a^2-b^2\right)((a+b)(aA-b(A+2B))\log(}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a^2 - b^2)*((a + b)*(a*A - b*(A + 2*B))*Log[1 - Sin[c + d*x]] - (a - b)*(
a*A + b*(A - 2*B))*Log[1 + Sin[c + d*x]]) - 2*a^3*(-(A*b) + a*B)*Sec[c + d*
x]^2 - 2*(a^2 - b^2)*(a^2*A + A*b^2 + 2*a*b*B)*Sec[c + d*x]*Tan[c + d*x] +
(-6*a^3*A*b + 4*a*A*b^3 + 2*b^4*B)*Tan[c + d*x]^2)/(4*(-a^2 + b^2)*d)
```

fricas [A] time = 0.48, size = 136, normalized size = 1.21

$$\frac{\left(Aa^2-2\,Bab-(A-2\,B)b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(Aa^2-2\,Bab-(A+2\,B)b^2\right)\cos\left(dx+c\right)^2\log\left(\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/4*((A*a^2 - 2*B*a*b - (A - 2*B)*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1)
 - (A*a^2 - 2*B*a*b - (A + 2*B)*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
+ 2*B*a^2 + 4*A*a*b + 2*B*b^2 + 2*(A*a^2 + 2*B*a*b + A*b^2)*sin(d*x + c))/(
d*cos(d*x + c)^2)
```

giac [A] time = 0.26, size = 146, normalized size = 1.30

$$\frac{\left(Aa^2-2\,Bab-Ab^2+2\,Bb^2\right)\log\left(|\sin\left(dx+c\right)+1|\right)-\left(Aa^2-2\,Bab-Ab^2-2\,Bb^2\right)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\left(Bb^2\right.}{}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/4*((A*a^2 - 2*B*a*b - A*b^2 + 2*B*b^2)*log(abs(sin(d*x + c) + 1)) - (A*a^
2 - 2*B*a*b - A*b^2 - 2*B*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(B*b^2*sin(d*
x + c)^2 + A*a^2*sin(d*x + c) + 2*B*a*b*sin(d*x + c) + A*b^2*sin(d*x + c) +
 B*a^2 + 2*A*a*b)/(sin(d*x + c)^2 - 1))/d
```

maple [B] time = 0.58, size = 231, normalized size = 2.06

$$\frac{a^2A\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{a^2A\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{B\,a^2}{2d\cos\left(dx+c\right)^2}+\frac{Aab}{d\cos\left(dx+c\right)^2}+\frac{Bab\left(\sin^3\right.}{d\cos\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/2/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/2
/d*B*a^2/cos(d*x+c)^2+1/d*A*a*b/cos(d*x+c)^2+1/d*B*a*b*sin(d*x+c)^3/cos(d*x
+c)^2+1/d*B*a*b*sin(d*x+c)-1/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*A*b^2*
sin(d*x+c)^3/cos(d*x+c)^2+1/2/d*A*b^2*sin(d*x+c)-1/2/d*A*b^2*ln(sec(d*x+c)+
tan(d*x+c))+1/2/d*B*b^2*tan(d*x+c)^2+1/d*B*b^2*ln(cos(d*x+c))
```

maxima [A] time = 0.34, size = 122, normalized size = 1.09

$$\frac{\left(Aa^2 - 2\,Bab - (A - 2\,B)b^2\right)\log\left(\sin\left(dx + c\right) + 1\right) - \left(Aa^2 - 2\,Bab - (A + 2\,B)b^2\right)\log\left(\sin\left(dx + c\right) - 1\right) - \frac{2\left(Ba^2+\right.}{}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((A*a^2 - 2*B*a*b - (A - 2*B)*b^2)*log(sin(d*x + c) + 1) - (A*a^2 - 2*B
*a*b - (A + 2*B)*b^2)*log(sin(d*x + c) - 1) - 2*(B*a^2 + 2*A*a*b + B*b^2 +
(A*a^2 + 2*B*a*b + A*b^2)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 12.36, size = 118, normalized size = 1.05

$$\frac{\ln\left(\sin\left(c + d\,x\right) + 1\right)\,\left(a - b\right)\,\left(A\,a + A\,b - 2\,B\,b\right)}{4\,d} - \frac{\sin\left(c + d\,x\right)\,\left(\frac{A\,a^2}{2} + B\,a\,b + \frac{A\,b^2}{2}\right) + \frac{B\,a^2}{2} + \frac{B\,b^2}{2} + A\,a\,b}{d\,\left(\sin\left(c + d\,x\right)^2 - 1\right)} + \frac{\ln\left(\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2)/cos(c + d*x)^3,x)
```

```
[Out] (log(sin(c + d*x) + 1)*(a - b)*(A*a + A*b - 2*B*b))/(4*d) - (sin(c + d*x)*(
(A*a^2)/2 + (A*b^2)/2 + B*a*b) + (B*a^2)/2 + (B*b^2)/2 + A*a*b)/(d*(sin(c +
 d*x)^2 - 1)) + (log(sin(c + d*x) - 1)*(a + b)*(A*b - A*a + 2*B*b))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + B\sin\left(c + dx\right)\right)\left(a + b\sin\left(c + dx\right)\right)^2\sec^3\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*(a + b*sin(c + d*x))**2*sec(c + d*x)**3, x)
```

3.1542 $\int \sec^5(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=122

$$\frac{(3a^2A-2abB-Ab^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^2(c+dx)\left((3a^2A-2abB+Ab^2)\sin(c+dx)+2b(2aA-bB)\right)}{8d}+\frac{\mathrm{se}}{}$$

```
[Out] 1/8*(3*A*a^2-A*b^2-2*B*a*b)*arctanh(sin(d*x+c))/d+1/4*sec(d*x+c)^4*(B+A*sin
(d*x+c))*(a+b*sin(d*x+c))^2/d+1/8*sec(d*x+c)^2*(2*b*(2*A*a-B*b)+(3*A*a^2+A*
b^2-2*B*a*b)*sin(d*x+c))/d
```

Rubi [A] time = 0.16, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2837, 821, 778, 206}

$$\frac{(3a^2A-2abB-Ab^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^2(c+dx)\left((3a^2A-2abB+Ab^2)\sin(c+dx)+2b(2aA-bB)\right)}{8d}+\frac{\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((3*a^2*A - A*b^2 - 2*a*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4
*(B + A*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(4*d) + (Sec[c + d*x]^2*(2*b*
(2*a*A - b*B) + (3*a^2*A + A*b^2 - 2*a*b*B)*Sin[c + d*x]))/(8*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
```

```
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{b}\right)}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(B+A\sin(c+dx))(a+b\sin(c+dx))^2}{4d}$$

$$= \frac{\sec^4(c+dx)(B+A\sin(c+dx))(a+b\sin(c+dx))^2}{4d}$$

$$= \frac{\left(3a^2A-Ab^2-2abB\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\sec^4(c}{}$$

Mathematica [A] time = 1.78, size = 186, normalized size = 1.52

$$\frac{4\left(b^2-a^2\right)\sec^4(c+dx)(a+b\sin(c+dx))^3((bB-aA)\sin(c+dx)-aB+Ab)+\left(-3a^2A+2abB+Ab^2\right)\left(-2\left(a^4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (4*(-a^2 + b^2)*Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3*(A*b - a*B + (-(a*A)
+ b*B)*Sin[c + d*x]) + (-3*a^2*A + A*b^2 + 2*a*b*B)*((a^2 - b^2)^2*(Log[1 -
 Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + 2*a^3*b*Sec[c + d*x]^2 - 2*(a^4 -
 b^4)*Sec[c + d*x]*Tan[c + d*x] + (-6*a^3*b + 4*a*b^3)*Tan[c + d*x]^2))/(16
*(a^2 - b^2)^2*d)
```

fricas [A] time = 0.47, size = 173, normalized size = 1.42

$$\frac{\left(3\,Aa^2-2\,Bab-Ab^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(3\,Aa^2-2\,Bab-Ab^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/16*((3*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3
*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 8*B*b^2*c
os(d*x + c)^2 + 4*B*a^2 + 8*A*a*b + 4*B*b^2 + 2*(2*A*a^2 + 4*B*a*b + 2*A*b^
2 + (3*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x +
c)^4)
```

giac [A] time = 0.27, size = 187, normalized size = 1.53

$$\frac{\left(3\,Aa^2-2\,Bab-Ab^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(3\,Aa^2-2\,Bab-Ab^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(3\,Aa^2\sin(dx+c)^3-2\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/16*((3*A*a^2 - 2*B*a*b - A*b^2)*log(abs(sin(d*x + c) + 1)) - (3*A*a^2 - 2
*B*a*b - A*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(3*A*a^2*sin(d*x + c)^3 - 2*
B*a*b*sin(d*x + c)^3 - A*b^2*sin(d*x + c)^3 - 4*B*b^2*sin(d*x + c)^2 - 5*A*
a^2*sin(d*x + c) - 2*B*a*b*sin(d*x + c) - A*b^2*sin(d*x + c) - 2*B*a^2 - 4*
A*a*b + 2*B*b^2)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [B] time = 0.56, size = 299, normalized size = 2.45

$$\frac{a^2A\tan\left(dx+c\right)\left(\sec^3\left(dx+c\right)\right)}{4d}+\frac{3a^2A\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{3a^2A\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{B\,a}{4d\cos\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a^2*A*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^2*A*sec(d*x+c)*tan(d*x+c)+3/8/d
*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*B*a^2/cos(d*x+c)^4+1/2/d*A*a*b/cos(d
*x+c)^4+1/2/d*B*a*b*sin(d*x+c)^3/cos(d*x+c)^4+1/4/d*B*a*b*sin(d*x+c)^3/cos(
```

```
d*x+c)^2+1/4/d*B*a*b*sin(d*x+c)-1/4/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/4/d
*A*b^2*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*A*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/8/
d*A*b^2*sin(d*x+c)-1/8/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*B*b^2*sin(d*
x+c)^4/cos(d*x+c)^4
```

maxima [A] time = 0.44, size = 171, normalized size = 1.40

$$\frac{\left(3\,Aa^2-2\,Bab-Ab^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(3\,Aa^2-2\,Bab-Ab^2\right)\log\left(\sin\left(dx+c\right)-1\right)+\frac{2\left(4\,Bb^2\sin(dx+c)^2-(3\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/16*((3*A*a^2 - 2*B*a*b - A*b^2)*log(sin(d*x + c) + 1) - (3*A*a^2 - 2*B*a*
b - A*b^2)*log(sin(d*x + c) - 1) + 2*(4*B*b^2*sin(d*x + c)^2 - (3*A*a^2 - 2
*B*a*b - A*b^2)*sin(d*x + c)^3 + 2*B*a^2 + 4*A*a*b - 2*B*b^2 + (5*A*a^2 + 2
*B*a*b + A*b^2)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 12.38, size = 181, normalized size = 1.48

$$\frac{\sin\left(c+d\,x\right)\,\left(\frac{5\,A\,a^2}{8}+\frac{B\,a\,b}{4}+\frac{A\,b^2}{8}\right)+\frac{B\,a^2}{4}-\frac{B\,b^2}{4}+{\sin\left(c+d\,x\right)}^3\,\left(-\frac{3\,A\,a^2}{8}+\frac{B\,a\,b}{4}+\frac{A\,b^2}{8}\right)+\frac{B\,b^2\,{\sin\left(c+d\,x\right)}^2}{2}+\frac{A\,a\,b}{2}}{d\,\left({\sin\left(c+d\,x\right)}^4-2\,{\sin\left(c+d\,x\right)}^2+1\right)}-\frac{\mathrm{at}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (sin(c + d*x)*((5*A*a^2)/8 + (A*b^2)/8 + (B*a*b)/4) + (B*a^2)/4 - (B*b^2)/4
 + sin(c + d*x)^3*((A*b^2)/8 - (3*A*a^2)/8 + (B*a*b)/4) + (B*b^2*sin(c + d*
x)^2)/2 + (A*a*b)/2)/(d*(sin(c + d*x)^4 - 2*sin(c + d*x)^2 + 1)) - (atanh((
4*sin(c + d*x)*((A*b^2)/16 - (3*A*a^2)/16 + (B*a*b)/8))/((A*b^2)/4 - (3*A*a
^2)/4 + (B*a*b)/2))*((A*b^2)/8 - (3*A*a^2)/8 + (B*a*b)/4))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1543 $\int \sec^7(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=160

$$\frac{(5a^2A-2abB-Ab^2)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\sec^4(c+dx)\left((5a^2A-2abB+3Ab^2)\sin(c+dx)+2b(4aA-bB)\right)}{24d}+\cdots$$

```
[Out] 1/16*(5*A*a^2-A*b^2-2*B*a*b)*arctanh(sin(d*x+c))/d+1/6*sec(d*x+c)^6*(B+A*si
n(d*x+c))*(a+b*sin(d*x+c))^2/d+1/24*sec(d*x+c)^4*(2*b*(4*A*a-B*b)+(5*A*a^2+
3*A*b^2-2*B*a*b)*sin(d*x+c))/d+1/16*(5*A*a^2-A*b^2-2*B*a*b)*sec(d*x+c)*tan(
d*x+c)/d
```

Rubi [A] time = 0.21, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2837, 821, 778, 199, 206}

$$\frac{(5a^2A-2abB-Ab^2)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\sec^4(c+dx)\left((5a^2A-2abB+3Ab^2)\sin(c+dx)+2b(4aA-bB)\right)}{24d}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((5*a^2*A - A*b^2 - 2*a*b*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^
6*(B + A*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(6*d) + (Sec[c + d*x]^4*(2*b
*(4*a*A - b*B) + (5*a^2*A + 3*A*b^2 - 2*a*b*B)*Sin[c + d*x]))/(24*d) + ((5*
a^2*A - A*b^2 - 2*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^7(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{b^7\,\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{b}\right)}{\left(b^2-x^2\right)^4}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^6(c+dx)(B+A\sin(c+dx))(a+b\sin(c+dx))^2}{6d}$$

$$= \frac{\sec^6(c+dx)(B+A\sin(c+dx))(a+b\sin(c+dx))^2}{6d}$$

$$= \frac{\sec^6(c+dx)(B+A\sin(c+dx))(a+b\sin(c+dx))^2}{6d}$$

$$= \frac{\left(5a^2A-Ab^2-2abB\right)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{\sec^6(c}{}$$

Mathematica [A] time = 1.54, size = 242, normalized size = 1.51

$$-\frac{3b(-5a^2A+2abB+Ab^2)\left(-2(a^4-b^4)\tan(c+dx)\sec(c+dx)+(4ab^3-6a^3b)\tan^2(c+dx)+2a^3b\sec^2(c+dx)+(a^2-b^2)^2(\log(1-\sin(c+dx))-\log(\sin(c+dx)+1))\right.}{16(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^6*(a + b*Sin[c + d*x])^3*(A*b - a*B + (-(a*A) + b*B)*Sin[c
+ d*x]) + (b*Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3*(3*A*b + (-5*a*A + 2*b*B
)*Sin[c + d*x]))/4 - (3*b*(-5*a^2*A + A*b^2 + 2*a*b*B)*((a^2 - b^2)^2*(Log[
1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + 2*a^3*b*Sec[c + d*x]^2 - 2*(a^
4 - b^4)*Sec[c + d*x]*Tan[c + d*x] + (-6*a^3*b + 4*a*b^3)*Tan[c + d*x]^2))/
(16*(a - b)*(a + b)))/(6*b*(-a^2 + b^2)*d)
```

fricas [A] time = 0.49, size = 203, normalized size = 1.27

$$3\left(5Aa^2-2Bab-Ab^2\right)\cos(dx+c)^6\log(\sin(dx+c)+1)-3\left(5Aa^2-2Bab-Ab^2\right)\cos(dx+c)^6\log(-\sin(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/96*(3*(5*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) -
3*(5*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 24*B*
b^2*cos(d*x + c)^2 + 16*B*a^2 + 32*A*a*b + 16*B*b^2 + 2*(3*(5*A*a^2 - 2*B*a
*b - A*b^2)*cos(d*x + c)^4 + 8*A*a^2 + 16*B*a*b + 8*A*b^2 + 2*(5*A*a^2 - 2*
B*a*b - A*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

giac [A] time = 0.27, size = 229, normalized size = 1.43

$$3\left(5Aa^2-2Bab-Ab^2\right)\log(|\sin(dx+c)+1|)-3\left(5Aa^2-2Bab-Ab^2\right)\log(|\sin(dx+c)-1|)-\frac{2\left(15Aa^2\sin(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/96*(3*(5*A*a^2 - 2*B*a*b - A*b^2)*log(abs(sin(d*x + c) + 1)) - 3*(5*A*a^2
 - 2*B*a*b - A*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(15*A*a^2*sin(d*x + c)^5
```

```
 - 6*B*a*b*sin(d*x + c)^5 - 3*A*b^2*sin(d*x + c)^5 - 40*A*a^2*sin(d*x + c)^
3 + 16*B*a*b*sin(d*x + c)^3 + 8*A*b^2*sin(d*x + c)^3 + 12*B*b^2*sin(d*x + c
)^2 + 33*A*a^2*sin(d*x + c) + 6*B*a*b*sin(d*x + c) + 3*A*b^2*sin(d*x + c) +
 8*B*a^2 + 16*A*a*b - 4*B*b^2)/(sin(d*x + c)^2 - 1)^3)/d
```

maple [B] time = 0.58, size = 396, normalized size = 2.48

$$\frac{a^2 A \tan(dx+c)\left(\sec^5(dx+c)\right)}{6d}+\frac{5a^2 A \tan(dx+c)\left(\sec^3(dx+c)\right)}{24d}+\frac{5a^2 A \sec(dx+c)\tan(dx+c)}{16d}+\frac{5a^2 A \ln(\mathrm{s}\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/6/d*a^2*A*tan(d*x+c)*sec(d*x+c)^5+5/24/d*a^2*A*tan(d*x+c)*sec(d*x+c)^3+5/
16/d*a^2*A*sec(d*x+c)*tan(d*x+c)+5/16/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/6
/d*B*a^2/cos(d*x+c)^6+1/3/d*A*a*b/cos(d*x+c)^6+1/3/d*B*a*b*sin(d*x+c)^3/cos
(d*x+c)^6+1/4/d*B*a*b*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*B*a*b*sin(d*x+c)^3/co
s(d*x+c)^2+1/8/d*B*a*b*sin(d*x+c)-1/8/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/6
/d*A*b^2*sin(d*x+c)^3/cos(d*x+c)^6+1/8/d*A*b^2*sin(d*x+c)^3/cos(d*x+c)^4+1/
16/d*A*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/16/d*A*b^2*sin(d*x+c)-1/16/d*A*b^2*l
n(sec(d*x+c)+tan(d*x+c))+1/6/d*B*b^2*sin(d*x+c)^4/cos(d*x+c)^6+1/12/d*B*b^2
*sin(d*x+c)^4/cos(d*x+c)^4
```

maxima [A] time = 0.45, size = 211, normalized size = 1.32

$$\frac{3\left(5\,Aa^2-2\,Bab-Ab^2\right)\log\left(\sin(dx+c)+1\right)-3\left(5\,Aa^2-2\,Bab-Ab^2\right)\log\left(\sin(dx+c)-1\right)-\frac{2\left(3\left(5\,Aa^2-2\,Bab-\cdots\right.\right.}{\cdots}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/96*(3*(5*A*a^2 - 2*B*a*b - A*b^2)*log(sin(d*x + c) + 1) - 3*(5*A*a^2 - 2*
B*a*b - A*b^2)*log(sin(d*x + c) - 1) - 2*(3*(5*A*a^2 - 2*B*a*b - A*b^2)*sin
(d*x + c)^5 + 12*B*b^2*sin(d*x + c)^2 - 8*(5*A*a^2 - 2*B*a*b - A*b^2)*sin(d
*x + c)^3 + 8*B*a^2 + 16*A*a*b - 4*B*b^2 + 3*(11*A*a^2 + 2*B*a*b + A*b^2)*s
in(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 12.45, size = 220, normalized size = 1.38

$$-\frac{\operatorname{atanh}\left(\frac{4\,\sin(c+d\,x)\left(-\frac{5\,A\,a^2}{32}+\frac{B\,a\,b}{16}+\frac{A\,b^2}{32}\right)}{-\frac{5\,A\,a^2}{8}+\frac{B\,a\,b}{4}+\frac{A\,b^2}{8}}\right)\left(-\frac{5\,A\,a^2}{16}+\frac{B\,a\,b}{8}+\frac{A\,b^2}{16}\right)}{d}-\frac{\sin(c+d\,x)\left(\frac{11\,A\,a^2}{16}+\frac{B\,a\,b}{8}+\frac{A\,b^2}{16}\right)+\frac{B\,a^2}{6}-\frac{B\,b^2}{12}+\cdots}{d\,\left(\mathrm{si}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2)/cos(c + d*x)^7,x)
```

```
[Out] - (atanh((4*sin(c + d*x)*((A*b^2)/32 - (5*A*a^2)/32 + (B*a*b)/16))/((A*b^2)
/8 - (5*A*a^2)/8 + (B*a*b)/4))*((A*b^2)/16 - (5*A*a^2)/16 + (B*a*b)/8))/d -
 (sin(c + d*x)*((11*A*a^2)/16 + (A*b^2)/16 + (B*a*b)/8) + (B*a^2)/6 - (B*b^
2)/12 + sin(c + d*x)^3*((A*b^2)/6 - (5*A*a^2)/6 + (B*a*b)/3) - sin(c + d*x)
^5*((A*b^2)/16 - (5*A*a^2)/16 + (B*a*b)/8) + (B*b^2*sin(c + d*x)^2)/4 + (A*
a*b)/3)/(d*(3*sin(c + d*x)^2 - 3*sin(c + d*x)^4 + sin(c + d*x)^6 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1544 $\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=315

$$-\frac{(a^2-b^2)^3(Ab-aB)\log(a+b\sin(c+dx))}{b^8d}-\frac{(a^2-3b^2)(Ab-aB)\sin^4(c+dx)}{4b^4d}+\frac{(a^2(-B)+aAb+3b^2B)\sin^5(c+dx)}{5b^3d}$$

```
[Out] -(a^2-b^2)^3*(A*b-B*a)*ln(a+b*sin(d*x+c))/b^8/d+(A*a^5*b-3*A*a^3*b^3+3*A*a*
b^5-B*a^6+3*B*a^4*b^2-3*B*a^2*b^4+B*b^6)*sin(d*x+c)/b^7/d-1/2*(a^4-3*a^2*b^
2+3*b^4)*(A*b-B*a)*sin(d*x+c)^2/b^6/d+1/3*(A*a^3*b-3*A*a*b^3-B*a^4+3*B*a^2*
b^2-3*B*b^4)*sin(d*x+c)^3/b^5/d-1/4*(a^2-3*b^2)*(A*b-B*a)*sin(d*x+c)^4/b^4/
d+1/5*(A*a*b-B*a^2+3*B*b^2)*sin(d*x+c)^5/b^3/d-1/6*(A*b-B*a)*sin(d*x+c)^6/b
^2/d-1/7*B*sin(d*x+c)^7/b/d
```

Rubi [A] time = 0.36, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$\frac{(a^2(-B)+aAb+3b^2B)\sin^5(c+dx)}{5b^3d}-\frac{(a^2-3b^2)(Ab-aB)\sin^4(c+dx)}{4b^4d}+\frac{(a^3Ab+3a^2b^2B+a^4(-B)-3aAb^3-\cdots}{3b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^3*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^8*d)) + ((a^5*A*b
- 3*a^3*A*b^3 + 3*a*A*b^5 - a^6*B + 3*a^4*b^2*B - 3*a^2*b^4*B + b^6*B)*Sin[
c + d*x])/(b^7*d) - ((a^4 - 3*a^2*b^2 + 3*b^4)*(A*b - a*B)*Sin[c + d*x]^2)/
(2*b^6*d) + ((a^3*A*b - 3*a*A*b^3 - a^4*B + 3*a^2*b^2*B - 3*b^4*B)*Sin[c +
d*x]^3)/(3*b^5*d) - ((a^2 - 3*b^2)*(A*b - a*B)*Sin[c + d*x]^4)/(4*b^4*d) +
((a*A*b - a^2*B + 3*b^2*B)*Sin[c + d*x]^5)/(5*b^3*d) - ((A*b - a*B)*Sin[c +
 d*x]^6)/(6*b^2*d) - (B*Sin[c + d*x]^7)/(7*b*d)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^3}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^5Ab-3a^3Ab^3+3aAb^5-a^6B+3a^4b^2B-3a^2b^4B+b^6B}{b} - \frac{\left(a^4-3a^2b^2+3b^4\right)(Ab-aB)x}{b}\right.\right.}{}$$

$$= -\frac{\left(a^2-b^2\right)^3(Ab-aB)\log(a+b\sin(c+dx))}{b^8 d} + \frac{\left(a^5Ab-3a^3Ab^3+3aA\right.}{}$$

Mathematica [A] time = 0.87, size = 218, normalized size = 0.69

$$\frac{\frac{(Ab-aB)\left(-30b^2\left(a^2-b^2\right)^2\sin^2(c+dx)-60\left(a^2-b^2\right)^3\log(a+b\sin(c+dx))+15b^4\left(b^2-a^2\right)\cos^4(c+dx)+20ab^3\left(a^2-3b^2\right)\sin^3(c+dx)+60ab\left(a^4-3a^2b^2+3b^4\right)\sin(c\right.}{60b}}{b^7 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((A*b - a*B)*(15*b^4*(-a^2 + b^2)*Cos[c + d*x]^4 + 10*b^6*Cos[c + d*x]^6 -
 60*(a^2 - b^2)^3*Log[a + b*Sin[c + d*x]] + 60*a*b*(a^4 - 3*a^2*b^2 + 3*b^4
)*Sin[c + d*x] - 30*b^2*(a^2 - b^2)^2*Sin[c + d*x]^2 + 20*a*b^3*(a^2 - 3*b^
2)*Sin[c + d*x]^3 + 12*a*b^5*Sin[c + d*x]^5))/(60*b) + (b^6*B*(1225*Sin[c +
 d*x] + 245*Sin[3*(c + d*x)] + 49*Sin[5*(c + d*x)] + 5*Sin[7*(c + d*x)]))/2
240)/(b^7*d)
```

fricas [A] time = 0.55, size = 366, normalized size = 1.16

$$-\frac{70\left(Bab^6-Ab^7\right)\cos(dx+c)^6-105\left(Ba^3b^4-Aa^2b^5-Bab^6+Ab^7\right)\cos(dx+c)^4+210\left(Ba^5b^2-Aa^4b^3-2Ba^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/420*(70*(B*a*b^6 - A*b^7)*cos(d*x + c)^6 - 105*(B*a^3*b^4 - A*a^2*b^5 -
B*a*b^6 + A*b^7)*cos(d*x + c)^4 + 210*(B*a^5*b^2 - A*a^4*b^3 - 2*B*a^3*b^4
+ 2*A*a^2*b^5 + B*a*b^6 - A*b^7)*cos(d*x + c)^2 - 420*(B*a^7 - A*a^6*b - 3*
B*a^5*b^2 + 3*A*a^4*b^3 + 3*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6 + A*b^7)*log(
b*sin(d*x + c) + a) - 4*(15*B*b^7*cos(d*x + c)^6 - 105*B*a^6*b + 105*A*a^5*
b^2 + 280*B*a^4*b^3 - 280*A*a^3*b^4 - 231*B*a^2*b^5 + 231*A*a*b^6 + 48*B*b^
7 - 3*(7*B*a^2*b^5 - 7*A*a*b^6 - 6*B*b^7)*cos(d*x + c)^4 + (35*B*a^4*b^3 -
35*A*a^3*b^4 - 63*B*a^2*b^5 + 63*A*a*b^6 + 24*B*b^7)*cos(d*x + c)^2)*sin(d*
x + c))/(b^8*d)
```

giac [A] time = 0.24, size = 511, normalized size = 1.62

$$-\frac{60\,Bb^6\sin(dx+c)^7-70\,Bab^5\sin(dx+c)^6+70\,Ab^6\sin(dx+c)^6+84\,Ba^2b^4\sin(dx+c)^5-84\,Aab^5\sin(dx+c)^5-252\,Bb^6\sin(dx+c)^5-105\,Ba^3b^3\sin(dx+c)^4+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/420*((60*B*b^6*sin(d*x + c)^7 - 70*B*a*b^5*sin(d*x + c)^6 + 70*A*b^6*sin
(d*x + c)^6 + 84*B*a^2*b^4*sin(d*x + c)^5 - 84*A*a*b^5*sin(d*x + c)^5 - 252
*B*b^6*sin(d*x + c)^5 - 105*B*a^3*b^3*sin(d*x + c)^4 + 105*A*a^2*b^4*sin(d*
x + c)^4 + 315*B*a*b^5*sin(d*x + c)^4 - 315*A*b^6*sin(d*x + c)^4 + 140*B*a^
4*b^2*sin(d*x + c)^3 - 140*A*a^3*b^3*sin(d*x + c)^3 - 420*B*a^2*b^4*sin(d*x
 + c)^3 + 420*A*a*b^5*sin(d*x + c)^3 + 420*B*b^6*sin(d*x + c)^3 - 210*B*a^5
*b*sin(d*x + c)^2 + 210*A*a^4*b^2*sin(d*x + c)^2 + 630*B*a^3*b^3*sin(d*x +
c)^2 - 630*A*a^2*b^4*sin(d*x + c)^2 - 630*B*a*b^5*sin(d*x + c)^2 + 630*A*b^
6*sin(d*x + c)^2 + 420*B*a^6*sin(d*x + c) - 420*A*a^5*b*sin(d*x + c) - 1260
*B*a^4*b^2*sin(d*x + c) + 1260*A*a^3*b^3*sin(d*x + c) + 1260*B*a^2*b^4*sin(
d*x + c) - 1260*A*a*b^5*sin(d*x + c) - 420*B*b^6*sin(d*x + c))/b^7 - 420*(B
*a^7 - A*a^6*b - 3*B*a^5*b^2 + 3*A*a^4*b^3 + 3*B*a^3*b^4 - 3*A*a^2*b^5 - B*
a*b^6 + A*b^7)*log(abs(b*sin(d*x + c) + a))/b^8)/d
```

maple [B] time = 0.40, size = 689, normalized size = 2.19

$$-\frac{3A\left(\sin^2(dx+c)\right)}{2db}-\frac{A\left(\sin^6(dx+c)\right)}{6db}+\frac{3A\sin(dx+c)\,a}{d\,b^2}+\frac{3B\sin(dx+c)\,a^4}{d\,b^5}+\frac{\ln(a+b\sin(dx+c))\,A}{db}-\frac{3B\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] 3/d/b^2*A*sin(d*x+c)*a+3/d/b^5*B*sin(d*x+c)*a^4-3/2/d/b*A*sin(d*x+c)^2+1/d/
b*ln(a+b*sin(d*x+c))*A-1/6/d/b*A*sin(d*x+c)^6+3/4/d/b*A*sin(d*x+c)^4-3/d/b^
3*B*sin(d*x+c)*a^2+1/6/d/b^2*B*sin(d*x+c)^6*a+1/5/d/b^2*A*sin(d*x+c)^5*a-1/
```

```
5/d/b^3*B*sin(d*x+c)^5*a^2-1/4/d/b^3*A*sin(d*x+c)^4*a^2-3/d/b^4*A*sin(d*x+c
)*a^3-3/d/b^6*ln(a+b*sin(d*x+c))*B*a^5+3/d/b^4*ln(a+b*sin(d*x+c))*B*a^3-1/d
/b^2*ln(a+b*sin(d*x+c))*B*a-3/4/d/b^2*B*sin(d*x+c)^4*a+1/3/d/b^4*A*sin(d*x+
c)^3*a^3-1/3/d/b^5*B*sin(d*x+c)^3*a^4-1/d/b^7*ln(a+b*sin(d*x+c))*A*a^6-3/d/
b^3*ln(a+b*sin(d*x+c))*A*a^2-1/2/d/b^5*A*sin(d*x+c)^2*a^4+3/2/d/b^3*A*sin(d
*x+c)^2*a^2+1/2/d/b^6*B*sin(d*x+c)^2*a^5-3/2/d/b^4*B*sin(d*x+c)^2*a^3-1/7*B
*sin(d*x+c)^7/b/d+3/5*B*sin(d*x+c)^5/b/d-B*sin(d*x+c)^3/b/d+B*sin(d*x+c)/b/
d-1/d/b^2*A*sin(d*x+c)^3*a+3/d/b^5*ln(a+b*sin(d*x+c))*A*a^4-1/d/b^7*B*sin(d
*x+c)*a^6+1/d/b^6*A*sin(d*x+c)*a^5+1/d/b^8*ln(a+b*sin(d*x+c))*B*a^7+1/d/b^3
*B*sin(d*x+c)^3*a^2+1/4/d/b^4*B*sin(d*x+c)^4*a^3+3/2/d/b^2*B*sin(d*x+c)^2*a
```

maxima [A] time = 0.44, size = 366, normalized size = 1.16

$$-\frac{\frac{60\,Bb^6\sin(dx+c)^7-70\left(Bab^5-Ab^6\right)\sin(dx+c)^6+84\left(Ba^2b^4-Aab^5-3\,Bb^6\right)\sin(dx+c)^5-105\left(Ba^3b^3-Aa^2b^4-3\,Bab^5+3\,Ab^6\right)\sin(dx+c)^4+140\left(Ba^4b^2-Aa\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/420*((60*B*b^6*sin(d*x + c)^7 - 70*(B*a*b^5 - A*b^6)*sin(d*x + c)^6 + 84
*(B*a^2*b^4 - A*a*b^5 - 3*B*b^6)*sin(d*x + c)^5 - 105*(B*a^3*b^3 - A*a^2*b^
4 - 3*B*a*b^5 + 3*A*b^6)*sin(d*x + c)^4 + 140*(B*a^4*b^2 - A*a^3*b^3 - 3*B*
a^2*b^4 + 3*A*a*b^5 + 3*B*b^6)*sin(d*x + c)^3 - 210*(B*a^5*b - A*a^4*b^2 -
3*B*a^3*b^3 + 3*A*a^2*b^4 + 3*B*a*b^5 - 3*A*b^6)*sin(d*x + c)^2 + 420*(B*a^
6 - A*a^5*b - 3*B*a^4*b^2 + 3*A*a^3*b^3 + 3*B*a^2*b^4 - 3*A*a*b^5 - B*b^6)*
sin(d*x + c))/b^7 - 420*(B*a^7 - A*a^6*b - 3*B*a^5*b^2 + 3*A*a^4*b^3 + 3*B*
a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6 + A*b^7)*log(b*sin(d*x + c) + a)/b^8)/d
```

mupad [B] time = 12.37, size = 435, normalized size = 1.38

$$\frac{\sin\left(c+d\,x\right)^4\,\left(\frac{3\,A}{4\,b}-\frac{a\,\left(\frac{3\,B}{b}+\frac{a\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{b}\right)}{4\,b}\right)}{d}-\frac{\sin\left(c+d\,x\right)^3\,\left(\frac{B}{b}+\frac{a\,\left(\frac{3\,A}{b}-\frac{a\,\left(\frac{3\,B}{b}+\frac{a\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{b}\right)}{b}\right)}{3\,b}\right)}{d}+\frac{\sin\left(c+d\,x\right)\,\left(\frac{B}{b}+\frac{a\,\left(\frac{3\,A}{b}-\frac{a\,\left(\frac{3\,B}{b}+\cdots\right)}{\cdots}\right)}{\cdots}\cdots\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^4*((3*A)/(4*b) - (a*((3*B)/b + (a*(A/b - (B*a)/b^2))/b))/(4*b
)))/d - (sin(c + d*x)^3*(B/b + (a*((3*A)/b - (a*((3*B)/b + (a*(A/b - (B*a)/
b^2))/b))/b))/(3*b)))/d + (sin(c + d*x)*(B/b + (a*((3*A)/b - (a*((3*B)/b +
(a*((3*A)/b - (a*((3*B)/b + (a*(A/b - (B*a)/b^2))/b))/b))/b))/b))/b))/d - (
sin(c + d*x)^6*(A/(6*b) - (B*a)/(6*b^2)))/d - (sin(c + d*x)^2*((3*A)/(2*b)
- (a*((3*B)/b + (a*((3*A)/b - (a*((3*B)/b + (a*(A/b - (B*a)/b^2))/b))/b))/b
))/(2*b)))/d + (sin(c + d*x)^5*((3*B)/(5*b) + (a*(A/b - (B*a)/b^2))/(5*b)))
/d + (log(a + b*sin(c + d*x))*(A*b^7 + B*a^7 - 3*A*a^2*b^5 + 3*A*a^4*b^3 +
3*B*a^3*b^4 - 3*B*a^5*b^2 - A*a^6*b - B*a*b^6))/(b^8*d) - (B*sin(c + d*x)^7
)/(7*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1545 $\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=202

$$\frac{\left(a^2-b^2\right)^2(Ab-aB)\log(a+b\sin(c+dx))}{b^6d}+\frac{\left(a^2-2b^2\right)(Ab-aB)\sin^2(c+dx)}{2b^4d}-\frac{\left(a^2(-B)+aAb+2b^2B\right)\sin^3(c+}{3b^3d}$$

```
[Out] (a^2-b^2)^2*(A*b-B*a)*ln(a+b*sin(d*x+c))/b^6/d-(A*a^3*b-2*A*a*b^3-B*a^4+2*B
*a^2*b^2-B*b^4)*sin(d*x+c)/b^5/d+1/2*(a^2-2*b^2)*(A*b-B*a)*sin(d*x+c)^2/b^4
/d-1/3*(A*a*b-B*a^2+2*B*b^2)*sin(d*x+c)^3/b^3/d+1/4*(A*b-B*a)*sin(d*x+c)^4/
b^2/d+1/5*B*sin(d*x+c)^5/b/d
```

Rubi [A] time = 0.25, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$-\frac{\left(a^2(-B)+aAb+2b^2B\right)\sin^3(c+dx)}{3b^3d}+\frac{\left(a^2-2b^2\right)(Ab-aB)\sin^2(c+dx)}{2b^4d}-\frac{\left(a^3Ab+2a^2b^2B+a^4(-B)-2aAb^3\right.}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((a^2 - b^2)^2*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^6*d) - ((a^3*A*b - 2
*a*A*b^3 - a^4*B + 2*a^2*b^2*B - b^4*B)*Sin[c + d*x])/(b^5*d) + ((a^2 - 2*b
^2)*(A*b - a*B)*Sin[c + d*x]^2)/(2*b^4*d) - ((a*A*b - a^2*B + 2*b^2*B)*Sin[
c + d*x]^3)/(3*b^3*d) + ((A*b - a*B)*Sin[c + d*x]^4)/(4*b^2*d) + (B*Sin[c +
 d*x]^5)/(5*b*d)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^2}{a+x}dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-a^3Ab+2aAb^3+a^4B-2a^2b^2B+b^4B}{b} - \frac{\left(-a^2+2b^2\right)(Ab-aB)x}{b} - \frac{\left(aAb-a^2B+2b^2B\right)}{b}\right.\right.}{b^5 d}$$

$$= \frac{\left(a^2-b^2\right)^2 (Ab-aB)\log(a+b\sin(c+dx))}{b^6 d} - \frac{\left(a^3Ab-2aAb^3-a^4B+2\right.}{b^5}$$

Mathematica [A] time = 0.43, size = 148, normalized size = 0.73

$$\frac{20(Ab-aB)\left(6b^2\left(a^2-b^2\right)\sin^2(c+dx)-12ab\left(a^2-2b^2\right)\sin(c+dx)+12\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))-4ab\right.}{240b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] (20*(A*b - a*B)*(3*b^4*Cos[c + d*x]^4 + 12*(a^2 - b^2)^2*Log[a + b*Sin[c +
d*x]] - 12*a*b*(a^2 - 2*b^2)*Sin[c + d*x] + 6*b^2*(a^2 - b^2)*Sin[c + d*x]^
2 - 4*a*b^3*Sin[c + d*x]^3) + b^5*B*(150*Sin[c + d*x] + 25*Sin[3*(c + d*x)]
 + 3*Sin[5*(c + d*x)]))/(240*b^6*d)
```

fricas [A] time = 0.50, size = 222, normalized size = 1.10

$$-\frac{15\left(Bab^4-Ab^5\right)\cos(dx+c)^4-30\left(Ba^3b^2-Aa^2b^3-Bab^4+Ab^5\right)\cos(dx+c)^2+60\left(Ba^5-Aa^4b-2Ba^3b^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/60*(15*(B*a*b^4 - A*b^5)*cos(d*x + c)^4 - 30*(B*a^3*b^2 - A*a^2*b^3 - B*
a*b^4 + A*b^5)*cos(d*x + c)^2 + 60*(B*a^5 - A*a^4*b - 2*B*a^3*b^2 + 2*A*a^2
*b^3 + B*a*b^4 - A*b^5)*log(b*sin(d*x + c) + a) - 4*(3*B*b^5*cos(d*x + c)^4
 + 15*B*a^4*b - 15*A*a^3*b^2 - 25*B*a^2*b^3 + 25*A*a*b^4 + 8*B*b^5 - (5*B*a
^2*b^3 - 5*A*a*b^4 - 4*B*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(b^6*d)
```

giac [A] time = 0.20, size = 286, normalized size = 1.42

$$\frac{12\,Bb^{4}\sin(dx+c)^{5}-15\,Bab^{3}\sin(dx+c)^{4}+15\,Ab^{4}\sin(dx+c)^{4}+20\,Ba^{2}b^{2}\sin(dx+c)^{3}-20\,Aab^{3}\sin(dx+c)^{3}-40\,Bb^{4}\sin(dx+c)^{3}-30\,Ba^{3}b\sin(dx+c)^{2}+30\,Aa}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/60*((12*B*b^4*sin(d*x + c)^5 - 15*B*a*b^3*sin(d*x + c)^4 + 15*A*b^4*sin(d
*x + c)^4 + 20*B*a^2*b^2*sin(d*x + c)^3 - 20*A*a*b^3*sin(d*x + c)^3 - 40*B*
b^4*sin(d*x + c)^3 - 30*B*a^3*b*sin(d*x + c)^2 + 30*A*a^2*b^2*sin(d*x + c)^
2 + 60*B*a*b^3*sin(d*x + c)^2 - 60*A*b^4*sin(d*x + c)^2 + 60*B*a^4*sin(d*x
+ c) - 60*A*a^3*b*sin(d*x + c) - 120*B*a^2*b^2*sin(d*x + c) + 120*A*a*b^3*s
in(d*x + c) + 60*B*b^4*sin(d*x + c))/b^5 - 60*(B*a^5 - A*a^4*b - 2*B*a^3*b^
2 + 2*A*a^2*b^3 + B*a*b^4 - A*b^5)*log(abs(b*sin(d*x + c) + a))/b^6)/d
```

maple [B] time = 0.40, size = 397, normalized size = 1.97

$$\frac{B\left(\sin^{5}\left(dx+c\right)\right)}{5bd}+\frac{A\left(\sin^{4}\left(dx+c\right)\right)}{4db}-\frac{B\left(\sin^{4}\left(dx+c\right)\right)a}{4d\,b^{2}}-\frac{A\left(\sin^{3}\left(dx+c\right)\right)a}{3d\,b^{2}}+\frac{B\left(\sin^{3}\left(dx+c\right)\right)a^{2}}{3d\,b^{3}}-\frac{2B\left(\sin^{3}\left(dx\right.\right.}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/5*B*sin(d*x+c)^5/b/d+1/4/d/b*A*sin(d*x+c)^4-1/4/d/b^2*B*sin(d*x+c)^4*a-1/
3/d/b^2*A*sin(d*x+c)^3*a+1/3/d/b^3*B*sin(d*x+c)^3*a^2-2/3*B*sin(d*x+c)^3/b/
d+1/2/d/b^3*A*sin(d*x+c)^2*a^2-1/d/b*A*sin(d*x+c)^2-1/2/d/b^4*B*sin(d*x+c)^
2*a^3+1/d/b^2*B*sin(d*x+c)^2*a-1/d/b^4*A*sin(d*x+c)*a^3+2/d/b^2*A*sin(d*x+c
)*a+1/d/b^5*B*sin(d*x+c)*a^4-2/d/b^3*B*sin(d*x+c)*a^2+B*sin(d*x+c)/b/d+1/d/
b^5*ln(a+b*sin(d*x+c))*A*a^4-2/d/b^3*ln(a+b*sin(d*x+c))*A*a^2+1/d/b*ln(a+b*
sin(d*x+c))*A-1/d/b^6*ln(a+b*sin(d*x+c))*B*a^5+2/d/b^4*ln(a+b*sin(d*x+c))*B
*a^3-1/d/b^2*ln(a+b*sin(d*x+c))*B*a
```

maxima [A] time = 0.57, size = 220, normalized size = 1.09

$$\frac{\frac{12\,Bb^{4}\sin(dx+c)^{5}-15\left(Bab^{3}-Ab^{4}\right)\sin(dx+c)^{4}+20\left(Ba^{2}b^{2}-Aab^{3}-2\,Bb^{4}\right)\sin(dx+c)^{3}-30\left(Ba^{3}b-Aa^{2}b^{2}-2\,Bab^{3}+2\,Ab^{4}\right)\sin(dx+c)^{2}+60\left(Ba^{4}-Aa^{3}b-2\right.}{b^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/60*((12*B*b^4*sin(d*x + c)^5 - 15*(B*a*b^3 - A*b^4)*sin(d*x + c)^4 + 20*(
B*a^2*b^2 - A*a*b^3 - 2*B*b^4)*sin(d*x + c)^3 - 30*(B*a^3*b - A*a^2*b^2 - 2
*B*a*b^3 + 2*A*b^4)*sin(d*x + c)^2 + 60*(B*a^4 - A*a^3*b - 2*B*a^2*b^2 + 2*
A*a*b^3 + B*b^4)*sin(d*x + c))/b^5 - 60*(B*a^5 - A*a^4*b - 2*B*a^3*b^2 + 2*
A*a^2*b^3 + B*a*b^4 - A*b^5)*log(b*sin(d*x + c) + a)/b^6)/d
```

mupad [B] time = 0.10, size = 253, normalized size = 1.25

$$\frac{\sin(c+d\,x)^4\left(\frac{A}{4\,b}-\frac{B\,a}{4\,b^2}\right)}{d}-\frac{\sin(c+d\,x)^2\left(\frac{A}{b}-\frac{a\left(\frac{2\,B}{b}+\frac{a\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{b}\right)}{2\,b}\right)}{d}-\frac{\sin(c+d\,x)^3\left(\frac{2\,B}{3\,b}+\frac{a\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{3\,b}\right)}{d}+\frac{\sin(c+d\,x)\left(\frac{B}{b}+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^4*(A/(4*b) - (B*a)/(4*b^2)))/d - (sin(c + d*x)^2*(A/b - (a*((
2*B)/b + (a*(A/b - (B*a)/b^2))/b))/(2*b)))/d - (sin(c + d*x)^3*((2*B)/(3*b)
 + (a*(A/b - (B*a)/b^2))/(3*b)))/d + (sin(c + d*x)*(B/b + (a*((2*A)/b - (a*
((2*B)/b + (a*(A/b - (B*a)/b^2))/b))/b))/b))/d + (log(a + b*sin(c + d*x))*(
A*b^5 - B*a^5 - 2*A*a^2*b^3 + 2*B*a^3*b^2 + A*a^4*b - B*a*b^4))/(b^6*d) + (
B*sin(c + d*x)^5)/(5*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1546 $\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=111

$$-\frac{\left(a^2-b^2\right)(Ab-aB)\log(a+b\sin(c+dx))}{b^4d}+\frac{\left(a^2(-B)+aAb+b^2B\right)\sin(c+dx)}{b^3d}-\frac{(Ab-aB)\sin^2(c+dx)}{2b^2d}-B\sin$$

```
[Out] -(a^2-b^2)*(A*b-B*a)*ln(a+b*sin(d*x+c))/b^4/d+(A*a*b-B*a^2+B*b^2)*sin(d*x+c
)/b^3/d-1/2*(A*b-B*a)*sin(d*x+c)^2/b^2/d-1/3*B*sin(d*x+c)^3/b/d
```

Rubi [A] time = 0.16, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$\frac{\left(a^2(-B)+aAb+b^2B\right)\sin(c+dx)}{b^3d}-\frac{\left(a^2-b^2\right)(Ab-aB)\log(a+b\sin(c+dx))}{b^4d}-\frac{(Ab-aB)\sin^2(c+dx)}{2b^2d}-\frac{B\sin^3(}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^4*d)) + ((a*A*b - a^
2*B + b^2*B)*Sin[c + d*x])/(b^3*d) - ((A*b - a*B)*Sin[c + d*x]^2)/(2*b^2*d)
 - (B*Sin[c + d*x]^3)/(3*b*d)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{aAb-a^2B+b^2B}{b} + \frac{(-Ab+aB)x}{b} - \frac{Bx^2}{b} + \frac{\left(-a^2+b^2\right)(Ab-aB)}{b(a+x)}\right) dx, x, b\sin(c$$
$$= -\frac{\left(a^2-b^2\right)(Ab-aB)\log(a+b\sin(c+dx))}{b^4d} + \frac{\left(aAb-a^2B+b^2B\right)\sin(c}{b^3d}$$

Mathematica [A] time = 0.38, size = 89, normalized size = 0.80

$$\frac{\left(A-\frac{aB}{b}\right)\left(\left(b^2-a^2\right)\log(a+b\sin(c+dx))+ab\sin(c+dx)-\frac{1}{2}b^2\sin^2(c+dx)\right)+\frac{1}{12}b^2B(9\sin(c+dx)+\sin(3(c+}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((A - (a*B)/b)*((-a^2 + b^2)*Log[a + b*Sin[c + d*x]] + a*b*Sin[c + d*x] - (
b^2*Sin[c + d*x]^2)/2) + (b^2*B*(9*Sin[c + d*x] + Sin[3*(c + d*x)]))/12)/(b
^3*d)
```

fricas [A] time = 0.47, size = 112, normalized size = 1.01

$$-\frac{3\left(Bab^2-Ab^3\right)\cos(dx+c)^2-6\left(Ba^3-Aa^2b-Bab^2+Ab^3\right)\log(b\sin(dx+c)+a)-2\left(Bb^3\cos(dx+c)^2-3\right.}{6\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/6*(3*(B*a*b^2 - A*b^3)*cos(d*x + c)^2 - 6*(B*a^3 - A*a^2*b - B*a*b^2 + A
*b^3)*log(b*sin(d*x + c) + a) - 2*(B*b^3*cos(d*x + c)^2 - 3*B*a^2*b + 3*A*a
*b^2 + 2*B*b^3)*sin(d*x + c))/(b^4*d)
```

giac [A] time = 0.20, size = 129, normalized size = 1.16

$$-\frac{\frac{2\,Bb^2\sin(dx+c)^3-3\,Bab\sin(dx+c)^2+3\,Ab^2\sin(dx+c)^2+6\,Ba^2\sin(dx+c)-6\,Aab\sin(dx+c)-6\,Bb^2\sin(dx+c)}{b^3}-\frac{6\left(Ba^3-Aa^2b-Bab^2+Ab^3\right)\log(|b\sin(dx}{b^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*((2*B*b^2*sin(d*x + c)^3 - 3*B*a*b*sin(d*x + c)^2 + 3*A*b^2*sin(d*x +
c)^2 + 6*B*a^2*sin(d*x + c) - 6*A*a*b*sin(d*x + c) - 6*B*b^2*sin(d*x + c))/
b^3 - 6*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*log(abs(b*sin(d*x + c) + a))/b^
4)/d
```

maple [A] time = 0.40, size = 186, normalized size = 1.68

$$-\frac{B\left(\sin^{3}\left(dx+c\right)\right)}{3bd}-\frac{A\left(\sin^{2}\left(dx+c\right)\right)}{2db}+\frac{B\left(\sin^{2}\left(dx+c\right)\right)a}{2d\,b^{2}}+\frac{A\sin\left(dx+c\right)a}{d\,b^{2}}-\frac{B\sin\left(dx+c\right)a^{2}}{d\,b^{3}}+\frac{B\sin\left(dx+c\right)}{bd}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/3*B*sin(d*x+c)^3/b/d-1/2/d/b*A*sin(d*x+c)^2+1/2/d/b^2*B*sin(d*x+c)^2*a+1
/d/b^2*A*sin(d*x+c)*a-1/d/b^3*B*sin(d*x+c)*a^2+B*sin(d*x+c)/b/d-1/d/b^3*ln(
a+b*sin(d*x+c))*A*a^2+1/d/b*ln(a+b*sin(d*x+c))*A+1/d/b^4*ln(a+b*sin(d*x+c))
*B*a^3-1/d/b^2*ln(a+b*sin(d*x+c))*B*a
```

maxima [A] time = 0.51, size = 112, normalized size = 1.01

$$-\frac{\frac{2\,Bb^{2}\sin(dx+c)^{3}-3\left(Bab-Ab^{2}\right)\sin(dx+c)^{2}+6\left(Ba^{2}-Aab-Bb^{2}\right)\sin(dx+c)}{b^{3}}-\frac{6\left(Ba^{3}-Aa^{2}b-Bab^{2}+Ab^{3}\right)\log(b\sin(dx+c)+a)}{b^{4}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/6*((2*B*b^2*sin(d*x + c)^3 - 3*(B*a*b - A*b^2)*sin(d*x + c)^2 + 6*(B*a^2
 - A*a*b - B*b^2)*sin(d*x + c))/b^3 - 6*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)
*log(b*sin(d*x + c) + a)/b^4)/d
```

mupad [B] time = 0.08, size = 122, normalized size = 1.10

$$\frac{\sin\left(c+d\,x\right)\left(\frac{B}{b}+\frac{a\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{b}\right)}{d}-\frac{\sin\left(c+d\,x\right)^2\,\left(\frac{A}{2\,b}-\frac{B\,a}{2\,b^2}\right)}{d}+\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(B\,a^3-A\,a^2\,b-B\,a\,b^2+A\,b^3\right)}{b^4\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)*(B/b + (a*(A/b - (B*a)/b^2))/b))/d - (sin(c + d*x)^2*(A/(2*b)
 - (B*a)/(2*b^2)))/d + (log(a + b*sin(c + d*x))*(A*b^3 + B*a^3 - A*a^2*b -
B*a*b^2))/(b^4*d) - (B*sin(c + d*x)^3)/(3*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1547 $\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=41

$$\frac{(Ab-aB)\log(a+b\sin(c+dx))}{b^2d}+\frac{B\sin(c+dx)}{bd}$$

```
[Out] (A*b-B*a)*ln(a+b*sin(d*x+c))/b^2/d+B*sin(d*x+c)/b/d
```

Rubi [A] time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{(Ab-aB)\log(a+b\sin(c+dx))}{b^2d}+\frac{B\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^2*d) + (B*Sin[c + d*x])/(b*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{a+x}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{B}{b}+\frac{Ab-aB}{b(a+x)}\right)dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{(Ab-aB)\log(a+b\sin(c+dx))}{b^2d} + \frac{B\sin(c+dx)}{bd}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 0.95

$$\frac{\frac{(Ab-aB)\log(a+b\sin(c+dx))}{b}+B\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
[Out] (((A*b - a*B)*Log[a + b*Sin[c + d*x]])/b + B*Sin[c + d*x])/(b*d)
```

fricas [A] time = 0.46, size = 38, normalized size = 0.93

$$\frac{Bb\sin(dx+c)-(Ba-Ab)\log(b\sin(dx+c)+a)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fricas
")
[Out] (B*b*sin(d*x + c) - (B*a - A*b)*log(b*sin(d*x + c) + a))/(b^2*d)
```

giac [A] time = 0.15, size = 41, normalized size = 1.00

$$\frac{\frac{B\sin(dx+c)}{b}-\frac{(Ba-Ab)\log(|b\sin(dx+c)+a|)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] (B*sin(d*x + c)/b - (B*a - A*b)*log(abs(b*sin(d*x + c) + a))/b^2)/d
```

maple [A] time = 0.25, size = 56, normalized size = 1.37

$$\frac{B\sin(dx+c)}{bd}+\frac{\ln(a+b\sin(dx+c))\,A}{db}-\frac{\ln(a+b\sin(dx+c))\,Ba}{d\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] B*sin(d*x+c)/b/d+1/d/b*ln(a+b*sin(d*x+c))*A-1/d/b^2*ln(a+b*sin(d*x+c))*B*a
```

maxima [A] time = 0.52, size = 40, normalized size = 0.98

$$\frac{\frac{B\sin(dx+c)}{b}-\frac{(Ba-Ab)\log(b\sin(dx+c)+a)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] (B*sin(d*x + c)/b - (B*a - A*b)*log(b*sin(d*x + c) + a)/b^2)/d
```

mupad [B] time = 0.06, size = 41, normalized size = 1.00

$$\frac{B\,\sin(c+d\,x)}{b\,d}+\frac{\ln(a+b\,\sin(c+d\,x))\,(A\,b-B\,a)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x)),x)
```

```
[Out] (B*sin(c + d*x))/(b*d) + (log(a + b*sin(c + d*x))*(A*b - B*a))/(b^2*d)
```

sympy [A] time = 0.71, size = 104, normalized size = 2.54

$$\begin{cases} \frac{x(A+B\sin(c))\cos(c)}{a} & \text{for } b=0 \wedge d=0 \\ \frac{\frac{A\sin(c+dx)}{d}-\frac{B\cos^2(c+dx)}{2d}}{a} & \text{for } b=0 \\ \frac{x(A+B\sin(c))\cos(c)}{a+b\sin(c)} & \text{for } d=0 \\ \frac{A\log\left(\frac{a}{b}+\sin(c+dx)\right)}{bd}-\frac{Ba\log\left(\frac{a}{b}+\sin(c+dx)\right)}{b^2d}+\frac{B\sin(c+dx)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*(A + B*sin(c))*cos(c)/a, Eq(b, 0) & Eq(d, 0)), ((A*sin(c + d*x
)/d - B*cos(c + d*x)**2/(2*d))/a, Eq(b, 0)), (x*(A + B*sin(c))*cos(c)/(a +
b*sin(c)), Eq(d, 0)), (A*log(a/b + sin(c + d*x))/(b*d) - B*a*log(a/b + sin(
c + d*x))/(b**2*d) + B*sin(c + d*x)/(b*d), True))
```

3.1548 $\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=90

$$-\frac{(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{(A+B)\log(1-\sin(c+dx))}{2d(a+b)}+\frac{(A-B)\log(\sin(c+dx)+1)}{2d(a-b)}$$

```
[Out] -1/2*(A+B)*ln(1-sin(d*x+c))/(a+b)/d+1/2*(A-B)*ln(1+sin(d*x+c))/(a-b)/d-(A*b
-B*a)*ln(a+b*sin(d*x+c))/(a^2-b^2)/d
```

Rubi [A] time = 0.15, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2837, 801}

$$-\frac{(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{(A+B)\log(1-\sin(c+dx))}{2d(a+b)}+\frac{(A-B)\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((A + B)*Log[1 - Sin[c + d*x]])/(2*(a + b)*d) + ((A - B)*Log[1 + Sin[c + d
*x]])/(2*(a - b)*d) - ((A*b - a*B)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \left(\frac{A+B}{2b(a+b)(b-x)} + \frac{-Ab+aB}{(a-b)b(a+b)(a+x)} + \frac{A-B}{2(a-b)b(b+x)}\right) dx, x, b\sin(c+dx\right.}{d}$$

$$= -\frac{(A+B)\log(1-\sin(c+dx))}{2(a+b)d} + \frac{(A-B)\log(1+\sin(c+dx))}{2(a-b)d} - \frac{(Ab-a}{}$$

Mathematica [A] time = 0.19, size = 99, normalized size = 1.10

$$\frac{\frac{(aB-Ab)\log(a+b\sin(c+dx))+(a+b)(A-B)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a-b} - (A+B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-((A + B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]) + ((a + b)*(A - B)*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (-(A*b) + a*B)*Log[a + b*Sin[c + d*
x]])/(a - b))/((a + b)*d)
```

fricas [A] time = 0.52, size = 88, normalized size = 0.98

$$\frac{2(Ba-Ab)\log(b\sin(dx+c)+a) + ((A-B)a+(A-B)b)\log(\sin(dx+c)+1) - ((A+B)a-(A+B)b)\log(-}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(B*a - A*b)*log(b*sin(d*x + c) + a) + ((A - B)*a + (A - B)*b)*log(si
n(d*x + c) + 1) - ((A + B)*a - (A + B)*b)*log(-sin(d*x + c) + 1))/((a^2 - b
^2)*d)
```

giac [A] time = 0.20, size = 87, normalized size = 0.97

$$\frac{\frac{2\left(Bab-Ab^2\right)\log(|b\sin(dx+c)+a|)}{a^2b-b^3} + \frac{(A-B)\log(|\sin(dx+c)+1|)}{a-b} - \frac{(A+B)\log(|\sin(dx+c)-1|)}{a+b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(B*a*b - A*b^2)*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) + (A - B)
*log(abs(sin(d*x + c) + 1))/(a - b) - (A + B)*log(abs(sin(d*x + c) - 1))/(a
 + b))/d
```

maple [A] time = 0.44, size = 156, normalized size = 1.73

$$-\frac{\ln\left(\sin\left(dx+c\right)-1\right)A}{d\left(2a+2b\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)B}{d\left(2a+2b\right)}-\frac{\ln\left(a+b\sin\left(dx+c\right)\right)Ab}{d\left(a+b\right)\left(a-b\right)}+\frac{\ln\left(a+b\sin\left(dx+c\right)\right)aB}{d\left(a+b\right)\left(a-b\right)}+\frac{\ln\left(1+\mathrm{s}\right.}{d\left(2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(2*a+2*b)*ln(sin(d*x+c)-1)*A-1/d/(2*a+2*b)*ln(sin(d*x+c)-1)*B-1/d/(a+b
)/(a-b)*ln(a+b*sin(d*x+c))*A*b+1/d/(a+b)/(a-b)*ln(a+b*sin(d*x+c))*a*B+1/d/(
2*a-2*b)*ln(1+sin(d*x+c))*A-1/d/(2*a-2*b)*ln(1+sin(d*x+c))*B
```

maxima [A] time = 0.56, size = 79, normalized size = 0.88

$$\frac{\frac{2\,(Ba-Ab)\log(b\sin(dx+c)+a)}{a^2-b^2}+\frac{(A-B)\log(\sin(dx+c)+1)}{a-b}-\frac{(A+B)\log(\sin(dx+c)-1)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/2*(2*(B*a - A*b)*log(b*sin(d*x + c) + a)/(a^2 - b^2) + (A - B)*log(sin(d*
x + c) + 1)/(a - b) - (A + B)*log(sin(d*x + c) - 1)/(a + b))/d
```

mupad [B] time = 0.31, size = 89, normalized size = 0.99

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{A}{2}-\frac{B}{2}\right)}{d\,\left(a-b\right)}-\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(A\,b-B\,a\right)}{d\,\left(a^2-b^2\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{A}{2}+\frac{B}{2}\right)}{d\,\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(A/2 - B/2))/(d*(a - b)) - (log(a + b*sin(c + d*x))*
(A*b - B*a))/(d*(a^2 - b^2)) - (log(sin(c + d*x) - 1)*(A/2 + B/2))/(d*(a +
b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sin(c + dx))\sec(c + dx)}{a + b\sin(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.1549 $\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=159

$$\frac{b^2(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\sec^2(c+dx)(-(aA-bB)\sin(c+dx)-aB+Ab)}{2d\left(a^2-b^2\right)}-\frac{(aA+b(2A+B))\log(1}{4d(a+b)^2}$$

```
[Out] -1/4*(a*A+b*(2*A+B))*ln(1-sin(d*x+c))/(a+b)^2/d+1/4*(a*A-b*(2*A-B))*ln(1+si
n(d*x+c))/(a-b)^2/d+b^2*(A*b-B*a)*ln(a+b*sin(d*x+c))/(a^2-b^2)^2/d-1/2*sec(
d*x+c)^2*(A*b-a*B-(A*a-B*b)*sin(d*x+c))/(a^2-b^2)/d
```

Rubi [A] time = 0.29, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2837, 823, 801}

$$\frac{b^2(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\sec^2(c+dx)(-(aA-bB)\sin(c+dx)-aB+Ab)}{2d\left(a^2-b^2\right)}-\frac{(aA+b(2A+B))\log(1}{4d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*A + b*(2*A + B))*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((a*A - b*(2
*A - B))*Log[1 + Sin[c + d*x]])/(4*(a - b)^2*d) + (b^2*(A*b - a*B)*Log[a +
b*Sin[c + d*x]])/((a^2 - b^2)^2*d) - (Sec[c + d*x]^2*(A*b - a*B - (a*A - b*
B)*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{2\left(a^2-b^2\right)d} - \frac{b\,\text{Subst}\left(\int \frac{-a^2A+2Ab}{(a+}\right.}{}$$

$$= -\frac{\sec^2(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{2\left(a^2-b^2\right)d} - \frac{b\,\text{Subst}\left(\int \left(\frac{(a-b)(-aA}{2b(a+}\right.\right.}{}$$

$$= -\frac{(aA+b(2A+B))\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(aA-b(2A-B))\log(1+\sin}{4(a-b)^2d}$$

Mathematica [A] time = 0.77, size = 197, normalized size = 1.24

$$\frac{\frac{4b^2(Ab-aB)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2} + \frac{A+B}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{B-A}{(a-b)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{2(aA+b(2A+B))\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{(a+b)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*(a*A + b*(2*A + B))*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^
2 + (2*(a*A + b*(-2*A + B))*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a -
b)^2 + (4*b^2*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^2 + (A + B)/
((a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (-A + B)/((a - b)*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^2))/(4*d)
```

fricas [A] time = 0.80, size = 234, normalized size = 1.47

$$\frac{2\,Ba^3 - 2\,Aa^2b - 2\,Bab^2 + 2\,Ab^3 - 4\left(Bab^2 - Ab^3\right)\cos\left(dx+c\right)^2\log\left(b\sin\left(dx+c\right)+a\right) + \left(Aa^3 + Ba^2b - (3\,A - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/4*(2*B*a^3 - 2*A*a^2*b - 2*B*a*b^2 + 2*A*b^3 - 4*(B*a*b^2 - A*b^3)*cos(d*
x + c)^2*log(b*sin(d*x + c) + a) + (A*a^3 + B*a^2*b - (3*A - 2*B)*a*b^2 - (
2*A - B)*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a^3 + B*a^2*b - (3*
A + 2*B)*a*b^2 + (2*A + B)*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(
A*a^3 - B*a^2*b - A*a*b^2 + B*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d
*cos(d*x + c)^2)
```

giac [A] time = 0.32, size = 260, normalized size = 1.64

$$-\frac{\frac{4\left(Bab^3-Ab^4\right)\log\left(\left|b\sin(dx+c)+a\right|\right)}{a^4b-2\,a^2b^3+b^5} + \frac{(Aa+2\,Ab+Bb)\log\left(\left|-\sin(dx+c)+1\right|\right)}{a^2+2\,ab+b^2} - \frac{(Aa-2\,Ab+Bb)\log\left(\left|-\sin(dx+c)-1\right|\right)}{a^2-2\,ab+b^2} + \frac{2\left(Bab^2\sin(dx+c)^2-Ab^3\sin\right.}{}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/4*(4*(B*a*b^3 - A*b^4)*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 +
 b^5) + (A*a + 2*A*b + B*b)*log(abs(-sin(d*x + c) + 1))/(a^2 + 2*a*b + b^2)
 - (A*a - 2*A*b + B*b)*log(abs(-sin(d*x + c) - 1))/(a^2 - 2*a*b + b^2) + 2*
(B*a*b^2*sin(d*x + c)^2 - A*b^3*sin(d*x + c)^2 + A*a^3*sin(d*x + c) - B*a^2
*b*sin(d*x + c) - A*a*b^2*sin(d*x + c) + B*b^3*sin(d*x + c) + B*a^3 - A*a^2
*b - 2*B*a*b^2 + 2*A*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)))/d
```

maple [A] time = 0.50, size = 297, normalized size = 1.87

$$-\frac{A}{d\left(4a+4b\right)\left(\sin\left(dx+c\right)-1\right)}-\frac{B}{d\left(4a+4b\right)\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)aA}{4d\left(a+b\right)^2}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)Ab}{2d\left(a+b\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(4*a+4*b)/(sin(d*x+c)-1)*A-1/d/(4*a+4*b)/(sin(d*x+c)-1)*B-1/4/d/(a+b)^
2*ln(sin(d*x+c)-1)*a*A-1/2/d/(a+b)^2*ln(sin(d*x+c)-1)*A*b-1/4/d/(a+b)^2*ln(
```

```
sin(d*x+c)-1)*B*b+1/d*b^3/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))*A-1/d*b^2/(a+b
)^2/(a-b)^2*ln(a+b*sin(d*x+c))*a*B-1/d/(4*a-4*b)/(1+sin(d*x+c))*A+1/d/(4*a-
4*b)/(1+sin(d*x+c))*B+1/4/d/(a-b)^2*ln(1+sin(d*x+c))*a*A-1/2/d/(a-b)^2*ln(1
+sin(d*x+c))*A*b+1/4/d/(a-b)^2*ln(1+sin(d*x+c))*B*b
```

maxima [A] time = 0.73, size = 175, normalized size = 1.10

$$-\frac{\frac{4\left(Bab^2-Ab^3\right)\log(b\sin(dx+c)+a)}{a^4-2a^2b^2+b^4}-\frac{(Aa-(2A-B)b)\log(\sin(dx+c)+1)}{a^2-2ab+b^2}+\frac{(Aa+(2A+B)b)\log(\sin(dx+c)-1)}{a^2+2ab+b^2}+\frac{2\left(Ba-Ab+(Aa-Bb)\sin(dx+c)\right)}{\left(a^2-b^2\right)\sin(dx+c)^2-a^2+b^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/4*(4*(B*a*b^2 - A*b^3)*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) -
 (A*a - (2*A - B)*b)*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + (A*a + (2*
A + B)*b)*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2) + 2*(B*a - A*b + (A*a -
 B*b)*sin(d*x + c))/((a^2 - b^2)*sin(d*x + c)^2 - a^2 + b^2))/d
```

mupad [B] time = 0.52, size = 197, normalized size = 1.24

$$\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(A\,b^3-B\,a\,b^2\right)}{d\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{\frac{A\,b-B\,a}{2\,\left(a^2-b^2\right)}-\frac{\sin(c+d\,x)\,(A\,a-B\,b)}{2\,\left(a^2-b^2\right)}}{d\cos\left(c+d\,x\right)^2}+\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(A\,a-b\,\left(2\,A-B\right)\right)}{d\,\left(4\,a^2-8\,a\,b+4\,b^2\right)}-\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(a + b*sin(c + d*x))*(A*b^3 - B*a*b^2))/(d*(a^4 + b^4 - 2*a^2*b^2)) - (
(A*b - B*a)/(2*(a^2 - b^2)) - (sin(c + d*x)*(A*a - B*b))/(2*(a^2 - b^2)))/(
d*cos(c + d*x)^2) + (log(sin(c + d*x) + 1)*(A*a - b*(2*A - B)))/(d*(4*a^2 -
 8*a*b + 4*b^2)) - (log(sin(c + d*x) - 1)*(A*a + b*(2*A + B)))/(d*(8*a*b +
4*a^2 + 4*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(A+B\sin(c+dx))\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

3.1550 $\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=263

$$-\frac{\left(3a^2A+ab(9A+B)+b^2(8A+3B)\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(3a^2A-ab(9A-B)+b^2(8A-3B)\right)\log(\sin(c+d}{16d(a-b)^3}$$

```
[Out] -1/16*(3*a^2*A+a*b*(9*A+B)+b^2*(8*A+3*B))*ln(1-sin(d*x+c))/(a+b)^3/d+1/16*(
3*a^2*A+b^2*(8*A-3*B)-a*b*(9*A-B))*ln(1+sin(d*x+c))/(a-b)^3/d-b^4*(A*b-B*a)
*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d-1/4*sec(d*x+c)^4*(A*b-a*B-(A*a-B*b)*sin(d
*x+c))/(a^2-b^2)/d+1/8*sec(d*x+c)^2*(4*b^2*(A*b-B*a)+(3*A*a^3-7*A*a*b^2+B*a
^2*b+3*B*b^3)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.45, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2837, 823, 801}

$$-\frac{b^4(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\left(3a^2A+ab(9A+B)+b^2(8A+3B)\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(3a^2A-ab(9\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((3*a^2*A + a*b*(9*A + B) + b^2*(8*A + 3*B))*Log[1 - Sin[c + d*x]])/(16*(a
 + b)^3*d) + ((3*a^2*A + b^2*(8*A - 3*B) - a*b*(9*A - B))*Log[1 + Sin[c + d
*x]])/(16*(a - b)^3*d) - (b^4*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/((a^2 -
b^2)^3*d) - (Sec[c + d*x]^4*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(4*(a^2
 - b^2)*d) + (Sec[c + d*x]^2*(4*b^2*(A*b - a*B) + (3*a^3*A - 7*a*A*b^2 + a^
2*b*B + 3*b^3*B)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
```

```
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{b^3\,\text{Subst}\left(\int \frac{-3a^2A+4A}{(a}\right.}{}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\sec^2(c+dx)\left(4b^2(A\right.}{}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\sec^2(c+dx)\left(4b^2(A\right.}{}$$

$$= -\frac{\left(3a^2A+ab(9A+B)+b^2(8A+3B)\right)\log(1-\sin(c+dx))}{16(a+b)^3d} + \frac{\left(3a^2A+\right.}{}$$

Mathematica [A] time = 1.31, size = 321, normalized size = 1.22

$$-\frac{2\left(3a^2A+ab(9A+B)+b^2(8A+3B)\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^3} + \frac{2\left(3a^2A+ab(B-9A)+b^2(8A-3B)\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^3} + \frac{16b}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*(3*a^2*A + a*b*(9*A + B) + b^2*(8*A + 3*B))*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]])/(a + b)^3 + (2*(3*a^2*A + b^2*(8*A - 3*B) + a*b*(-9*A + B))
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a - b)^3 + (16*b^4*(A*b - a*B)*
Log[a + b*Sin[c + d*x]])/(-a^2 + b^2)^3 + (A + B)/((a + b)*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^4) + (3*a*A + 5*A*b + a*B + 3*b*B)/((a + b)^2*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])^2) + (-A + B)/((a - b)*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^4) + (-3*a*A + 5*A*b + a*B - 3*b*B)/((a - b)^2*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^2))/(16*d)
```

fricas [A] time = 1.77, size = 413, normalized size = 1.57

$$\frac{4\,Ba^5 - 4\,Aa^4b - 8\,Ba^3b^2 + 8\,Aa^2b^3 + 4\,Bab^4 - 4\,Ab^5 + 16\left(Bab^4 - Ab^5\right)\cos\left(dx + c\right)^4\log\left(b\sin\left(dx + c\right) + a\right) +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/16*(4*B*a^5 - 4*A*a^4*b - 8*B*a^3*b^2 + 8*A*a^2*b^3 + 4*B*a*b^4 - 4*A*b^5
 + 16*(B*a*b^4 - A*b^5)*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + (3*A*a^5 +
 B*a^4*b - 10*A*a^3*b^2 - 6*B*a^2*b^3 + (15*A - 8*B)*a*b^4 + (8*A - 3*B)*b^
5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*A*a^5 + B*a^4*b - 10*A*a^3*b^2
 - 6*B*a^2*b^3 + (15*A + 8*B)*a*b^4 - (8*A + 3*B)*b^5)*cos(d*x + c)^4*log(-
sin(d*x + c) + 1) - 8*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c
)^2 + 2*(2*A*a^5 - 2*B*a^4*b - 4*A*a^3*b^2 + 4*B*a^2*b^3 + 2*A*a*b^4 - 2*B*
b^5 + (3*A*a^5 + B*a^4*b - 10*A*a^3*b^2 + 2*B*a^2*b^3 + 7*A*a*b^4 - 3*B*b^5
)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(
d*x + c)^4)
```

giac [B] time = 0.40, size = 539, normalized size = 2.05

$$\frac{\frac{16\left(Bab^5 - Ab^6\right)\log\left(|b\sin\left(dx + c\right) + a|\right)}{a^6b - 3\,a^4b^3 + 3\,a^2b^5 - b^7} - \frac{\left(3\,Aa^2 + 9\,Aab + Bab + 8\,Ab^2 + 3\,Bb^2\right)\log\left(|-\sin\left(dx + c\right) + 1|\right)}{a^3 + 3\,a^2b + 3\,ab^2 + b^3} + \frac{\left(3\,Aa^2 - 9\,Aab + Bab + 8\,Ab^2 - 3\,Bb^2\right)\log\left(|-\sin\left(dx\right.\right.}{a^3 - 3\,a^2b + 3\,ab^2 - b^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/16*(16*(B*a*b^5 - A*b^6)*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3
+ 3*a^2*b^5 - b^7) - (3*A*a^2 + 9*A*a*b + B*a*b + 8*A*b^2 + 3*B*b^2)*log(ab
s(-sin(d*x + c) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (3*A*a^2 - 9*A*a*b
```

```
+ B*a*b + 8*A*b^2 - 3*B*b^2)*log(abs(-sin(d*x + c) - 1))/(a^3 - 3*a^2*b + 3
*a*b^2 - b^3) + 2*(6*B*a*b^4*sin(d*x + c)^4 - 6*A*b^5*sin(d*x + c)^4 - 3*A*
a^5*sin(d*x + c)^3 - B*a^4*b*sin(d*x + c)^3 + 10*A*a^3*b^2*sin(d*x + c)^3 -
 2*B*a^2*b^3*sin(d*x + c)^3 - 7*A*a*b^4*sin(d*x + c)^3 + 3*B*b^5*sin(d*x +
c)^3 + 4*B*a^3*b^2*sin(d*x + c)^2 - 4*A*a^2*b^3*sin(d*x + c)^2 - 16*B*a*b^4
*sin(d*x + c)^2 + 16*A*b^5*sin(d*x + c)^2 + 5*A*a^5*sin(d*x + c) - B*a^4*b*
sin(d*x + c) - 14*A*a^3*b^2*sin(d*x + c) + 6*B*a^2*b^3*sin(d*x + c) + 9*A*a
*b^4*sin(d*x + c) - 5*B*b^5*sin(d*x + c) + 2*B*a^5 - 2*A*a^4*b - 8*B*a^3*b^
2 + 8*A*a^2*b^3 + 12*B*a*b^4 - 12*A*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*(sin(d*x + c)^2 - 1)^2))/d
```

maple [B] time = 0.51, size = 586, normalized size = 2.23

$$\frac{A}{2d\left(8a+8b\right)\left(\sin\left(dx+c\right)-1\right)^{2}}+\frac{B}{2d\left(8a+8b\right)\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{3aA}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{5Al}{16d\left(a+b\right)^{2}\left(\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2*A+1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2*B-3/16/
d/(a+b)^2/(sin(d*x+c)-1)*a*A-5/16/d/(a+b)^2/(sin(d*x+c)-1)*A*b-1/16/d/(a+b)
^2/(sin(d*x+c)-1)*a*B-3/16/d/(a+b)^2/(sin(d*x+c)-1)*B*b-3/16/d/(a+b)^3*ln(s
in(d*x+c)-1)*a^2*A-9/16/d/(a+b)^3*ln(sin(d*x+c)-1)*A*a*b-1/2/d/(a+b)^3*ln(s
in(d*x+c)-1)*A*b^2-1/16/d/(a+b)^3*ln(sin(d*x+c)-1)*B*a*b-3/16/d/(a+b)^3*ln(
sin(d*x+c)-1)*B*b^2-1/d*b^5/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))*A+1/d*b^4/(a
+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))*a*B-1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2*A+1/2
/d/(8*a-8*b)/(1+sin(d*x+c))^2*B-3/16/d/(a-b)^2/(1+sin(d*x+c))*a*A+5/16/d/(a
-b)^2/(1+sin(d*x+c))*A*b+1/16/d/(a-b)^2/(1+sin(d*x+c))*a*B-3/16/d/(a-b)^2/(
1+sin(d*x+c))*B*b+3/16/d/(a-b)^3*ln(1+sin(d*x+c))*a^2*A-9/16/d/(a-b)^3*ln(1
+sin(d*x+c))*A*a*b+1/2/d/(a-b)^3*ln(1+sin(d*x+c))*A*b^2+1/16/d/(a-b)^3*ln(1
+sin(d*x+c))*B*a*b-3/16/d/(a-b)^3*ln(1+sin(d*x+c))*B*b^2
```

maxima [A] time = 0.38, size = 367, normalized size = 1.40

$$\frac{\frac{16\left(Bab^{4}-Ab^{5}\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}}+\frac{\left(3\,Aa^{2}-\left(9\,A-B\right)ab+\left(8\,A-3\,B\right)b^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{\left(3\,Aa^{2}+\left(9\,A+B\right)ab+\left(8\,A+3\,B\right)b^{2}\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\cdots}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/16*(16*(B*a*b^4 - A*b^5)*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6) + (3*A*a^2 - (9*A - B)*a*b + (8*A - 3*B)*b^2)*log(sin(d*x + c)
```

```
+ 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*A*a^2 + (9*A + B)*a*b + (8*A + 3*
B)*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(2*B*a^3
- 2*A*a^2*b - 6*B*a*b^2 + 6*A*b^3 - (3*A*a^3 + B*a^2*b - 7*A*a*b^2 + 3*B*b^
3)*sin(d*x + c)^3 + 4*(B*a*b^2 - A*b^3)*sin(d*x + c)^2 + (5*A*a^3 - B*a^2*b
 - 9*A*a*b^2 + 5*B*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)
^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

mupad [B] time = 12.95, size = 427, normalized size = 1.62

$$\frac{\frac{B\,a^3-A\,a^2\,b-3\,B\,a\,b^2+3\,A\,b^3}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{\sin(c+d\,x)\,\left(5\,A\,a^3-B\,a^2\,b-9\,A\,a\,b^2+5\,B\,b^3\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{\sin(c+d\,x)^2\,\left(A\,b^3-B\,a\,b^2\right)}{2\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{\sin(c+d\,x)^3\,\left(3\,A\,a^3+B\,a^2\,b-7\,A\,a\,b^2+3\,B}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left(\cos\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)^4-\sin\left(c+d\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] ((3*A*b^3 + B*a^3 - A*a^2*b - 3*B*a*b^2)/(4*(a^4 + b^4 - 2*a^2*b^2)) + (sin
(c + d*x)*(5*A*a^3 + 5*B*b^3 - 9*A*a*b^2 - B*a^2*b))/(8*(a^4 + b^4 - 2*a^2*
b^2)) - (sin(c + d*x)^2*(A*b^3 - B*a*b^2))/(2*(a^4 + b^4 - 2*a^2*b^2)) - (s
in(c + d*x)^3*(3*A*a^3 + 3*B*b^3 - 7*A*a*b^2 + B*a^2*b))/(8*(a^4 + b^4 - 2*
a^2*b^2)))/(d*(cos(c + d*x)^2 - sin(c + d*x)^2 + sin(c + d*x)^4)) - (log(si
n(c + d*x) - 1)*(3*A*a^2 + b^2*(8*A + 3*B) + a*b*(9*A + B)))/(d*(48*a*b^2 +
 48*a^2*b + 16*a^3 + 16*b^3)) - (log(a + b*sin(c + d*x))*(A*b^5 - B*a*b^4))
/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (log(sin(c + d*x) + 1)*(3*A*a^2
+ b^2*(8*A - 3*B) - a*b*(9*A - B)))/(d*(48*a*b^2 - 48*a^2*b + 16*a^3 - 16*b
^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sin(c + dx))\sec^5(c + dx)}{a + b\sin(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*sec(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

3.1551 $\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=383

$$-\frac{\sec^6(c+dx)(-(aA-bB)\sin(c+dx)-aB+Ab)}{6d\left(a^2-b^2\right)}+\frac{b^6(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{\left(5a^3A+a^2b(20A+B)+\right.}{}$$

```
[Out] -1/32*(5*a^3*A+a^2*b*(20*A+B)+a*b^2*(29*A+4*B)+b^3*(16*A+5*B))*ln(1-sin(d*x
+c))/(a+b)^4/d+1/32*(5*a^3*A-b^3*(16*A-5*B)+a*b^2*(29*A-4*B)-a^2*b*(20*A-B)
)*ln(1+sin(d*x+c))/(a-b)^4/d+b^6*(A*b-B*a)*ln(a+b*sin(d*x+c))/(a^2-b^2)^4/d
-1/6*sec(d*x+c)^6*(A*b-a*B-(A*a-B*b)*sin(d*x+c))/(a^2-b^2)/d+1/24*sec(d*x+c
)^4*(6*b^2*(A*b-B*a)+(5*A*a^3-11*A*a*b^2+B*a^2*b+5*B*b^3)*sin(d*x+c))/(a^2-
b^2)^2/d-1/16*sec(d*x+c)^2*(8*b^4*(A*b-B*a)-(5*A*a^5-16*A*a^3*b^2+19*A*a*b^
4+B*a^4*b-4*B*a^2*b^3-5*B*b^5)*sin(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A] time = 0.68, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2837, 823, 801}

$$\frac{b^6(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{\left(a^2b(20A+B)+5a^3A+ab^2(29A+4B)+b^3(16A+5B)\right)\log(1-\sin(c+dx}{32d(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((5*a^3*A + a^2*b*(20*A + B) + a*b^2*(29*A + 4*B) + b^3*(16*A + 5*B))*Log[
1 - Sin[c + d*x]])/(32*(a + b)^4*d) + ((5*a^3*A - b^3*(16*A - 5*B) + a*b^2*
(29*A - 4*B) - a^2*b*(20*A - B))*Log[1 + Sin[c + d*x]])/(32*(a - b)^4*d) +
(b^6*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^4*d) - (Sec[c + d*x]
^6*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(6*(a^2 - b^2)*d) + (Sec[c + d*x
]^4*(6*b^2*(A*b - a*B) + (5*a^3*A - 11*a*A*b^2 + a^2*b*B + 5*b^3*B)*Sin[c +
 d*x]))/(24*(a^2 - b^2)^2*d) - (Sec[c + d*x]^2*(8*b^4*(A*b - a*B) - (5*a^5*
A - 16*a^3*A*b^2 + 19*a*A*b^4 + a^4*b*B - 4*a^2*b^3*B - 5*b^5*B)*Sin[c + d*
x]))/(16*(a^2 - b^2)^3*d)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{b^7\,\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)\left(b^2-x^2\right)^4}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d} - \frac{b^5\,\text{Subst}\left(\int \frac{-5a^2A+$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d} + \frac{\sec^4(c+dx)\left(6b^2(\right.$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d} + \frac{\sec^4(c+dx)\left(6b^2(\right.$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d} + \frac{\sec^4(c+dx)\left(6b^2(\right.$$

$$= -\frac{\left(5a^3A+a^2b(20A+B)+ab^2(29A+4B)+b^3(16A+5B)\right)\log(1-\sin}{32(a+b)^4d}$$

Mathematica [A] time = 2.49, size = 565, normalized size = 1.48

$$\frac{\frac{768b^6(Ab-aB)\log(a+b\sin(c+dx))}{(a^2-b^2)^4}-\frac{48\left(5a^3A+a^2b(20A+B)+ab^2(29A+4B)+b^3(16A+5B)\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^4}+\frac{48\left(5a^3A+a^2b(B-20\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-48*(5*a^3*A + a^2*b*(20*A + B) + a*b^2*(29*A + 4*B) + b^3*(16*A + 5*B))*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^4 + (48*(5*a^3*A + a*b^2*
(29*A - 4*B) + a^2*b*(-20*A + B) + b^3*(-16*A + 5*B))*Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]])/(a - b)^4 + (768*b^6*(A*b - a*B)*Log[a + b*Sin[c + d*x
]])/(a^2 - b^2)^4 + (Sec[c + d*x]^6*(-128*a^4*A*b + 352*a^2*A*b^3 - 368*A*b
^5 + 128*a^5*B - 352*a^3*b^2*B + 368*a*b^4*B - 96*b^2*(a^2 - 3*b^2)*(-(A*b)
 + a*B)*Cos[2*(c + d*x)] - 48*b^4*(A*b - a*B)*Cos[4*(c + d*x)] + 198*a^5*A*
Sin[c + d*x] - 480*a^3*A*b^2*Sin[c + d*x] + 330*a*A*b^4*Sin[c + d*x] - 114*
a^4*b*B*Sin[c + d*x] + 264*a^2*b^3*B*Sin[c + d*x] - 198*b^5*B*Sin[c + d*x]
+ 85*a^5*A*Sin[3*(c + d*x)] - 272*a^3*A*b^2*Sin[3*(c + d*x)] + 259*a*A*b^4*
Sin[3*(c + d*x)] + 17*a^4*b*B*Sin[3*(c + d*x)] - 4*a^2*b^3*B*Sin[3*(c + d*x
)] - 85*b^5*B*Sin[3*(c + d*x)] + 15*a^5*A*Sin[5*(c + d*x)] - 48*a^3*A*b^2*S
in[5*(c + d*x)] + 57*a*A*b^4*Sin[5*(c + d*x)] + 3*a^4*b*B*Sin[5*(c + d*x)]
- 12*a^2*b^3*B*Sin[5*(c + d*x)] - 15*b^5*B*Sin[5*(c + d*x)]))/(a^2 - b^2)^3
)/(768*d)
```

fricas [A] time = 4.32, size = 643, normalized size = 1.68

$$\frac{16\,Ba^7-16\,Aa^6b-48\,Ba^5b^2+48\,Aa^4b^3+48\,Ba^3b^4-48\,Aa^2b^5-16\,Bab^6+16\,Ab^7-96\left(Bab^6-Ab^7\right)\cos\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/96*(16*B*a^7 - 16*A*a^6*b - 48*B*a^5*b^2 + 48*A*a^4*b^3 + 48*B*a^3*b^4 -
48*A*a^2*b^5 - 16*B*a*b^6 + 16*A*b^7 - 96*(B*a*b^6 - A*b^7)*cos(d*x + c)^6*
log(b*sin(d*x + c) + a) + 3*(5*A*a^7 + B*a^6*b - 21*A*a^5*b^2 - 5*B*a^4*b^3
 + 35*A*a^3*b^4 + 15*B*a^2*b^5 - (35*A - 16*B)*a*b^6 - (16*A - 5*B)*b^7)*co
s(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(5*A*a^7 + B*a^6*b - 21*A*a^5*b^2 -
5*B*a^4*b^3 + 35*A*a^3*b^4 + 15*B*a^2*b^5 - (35*A + 16*B)*a*b^6 + (16*A + 5
*B)*b^7)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 48*(B*a^3*b^4 - A*a^2*b^5
- B*a*b^6 + A*b^7)*cos(d*x + c)^4 - 24*(B*a^5*b^2 - A*a^4*b^3 - 2*B*a^3*b^4
 + 2*A*a^2*b^5 + B*a*b^6 - A*b^7)*cos(d*x + c)^2 + 2*(8*A*a^7 - 8*B*a^6*b -
```

```
 24*A*a^5*b^2 + 24*B*a^4*b^3 + 24*A*a^3*b^4 - 24*B*a^2*b^5 - 8*A*a*b^6 + 8*
B*b^7 + 3*(5*A*a^7 + B*a^6*b - 21*A*a^5*b^2 - 5*B*a^4*b^3 + 35*A*a^3*b^4 -
B*a^2*b^5 - 19*A*a*b^6 + 5*B*b^7)*cos(d*x + c)^4 + 2*(5*A*a^7 + B*a^6*b - 2
1*A*a^5*b^2 + 3*B*a^4*b^3 + 27*A*a^3*b^4 - 9*B*a^2*b^5 - 11*A*a*b^6 + 5*B*b
^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6
 + b^8)*d*cos(d*x + c)^6)
```

giac [B] time = 0.32, size = 907, normalized size = 2.37

$$-\frac{\frac{96\left(Bab^7-Ab^8\right)\log(|b\sin(dx+c)+a|)}{a^8b-4a^6b^3+6a^4b^5-4a^2b^7+b^9}+\frac{3\left(5Aa^3+20Aa^2b+Ba^2b+29Aab^2+4Bab^2+16Ab^3+5Bb^3\right)\log(|-\sin(dx+c)+1|)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{3\left(5Aa^3-20Aa^2b+Ba\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/96*(96*(B*a*b^7 - A*b^8)*log(abs(b*sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3
 + 6*a^4*b^5 - 4*a^2*b^7 + b^9) + 3*(5*A*a^3 + 20*A*a^2*b + B*a^2*b + 29*A*
a*b^2 + 4*B*a*b^2 + 16*A*b^3 + 5*B*b^3)*log(abs(-sin(d*x + c) + 1))/(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 3*(5*A*a^3 - 20*A*a^2*b + B*a^2*b +
29*A*a*b^2 - 4*B*a*b^2 - 16*A*b^3 + 5*B*b^3)*log(abs(-sin(d*x + c) - 1))/(a
^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 2*(44*B*a*b^6*sin(d*x + c)^6 -
44*A*b^7*sin(d*x + c)^6 + 15*A*a^7*sin(d*x + c)^5 + 3*B*a^6*b*sin(d*x + c)^
5 - 63*A*a^5*b^2*sin(d*x + c)^5 - 15*B*a^4*b^3*sin(d*x + c)^5 + 105*A*a^3*b
^4*sin(d*x + c)^5 - 3*B*a^2*b^5*sin(d*x + c)^5 - 57*A*a*b^6*sin(d*x + c)^5
+ 15*B*b^7*sin(d*x + c)^5 + 24*B*a^3*b^4*sin(d*x + c)^4 - 24*A*a^2*b^5*sin(
d*x + c)^4 - 156*B*a*b^6*sin(d*x + c)^4 + 156*A*b^7*sin(d*x + c)^4 - 40*A*a
^7*sin(d*x + c)^3 - 8*B*a^6*b*sin(d*x + c)^3 + 168*A*a^5*b^2*sin(d*x + c)^3
 + 24*B*a^4*b^3*sin(d*x + c)^3 - 264*A*a^3*b^4*sin(d*x + c)^3 + 24*B*a^2*b^
5*sin(d*x + c)^3 + 136*A*a*b^6*sin(d*x + c)^3 - 40*B*b^7*sin(d*x + c)^3 + 1
2*B*a^5*b^2*sin(d*x + c)^2 - 12*A*a^4*b^3*sin(d*x + c)^2 - 72*B*a^3*b^4*sin
(d*x + c)^2 + 72*A*a^2*b^5*sin(d*x + c)^2 + 192*B*a*b^6*sin(d*x + c)^2 - 19
2*A*b^7*sin(d*x + c)^2 + 33*A*a^7*sin(d*x + c) - 3*B*a^6*b*sin(d*x + c) - 1
29*A*a^5*b^2*sin(d*x + c) + 15*B*a^4*b^3*sin(d*x + c) + 183*A*a^3*b^4*sin(d
*x + c) - 45*B*a^2*b^5*sin(d*x + c) - 87*A*a*b^6*sin(d*x + c) + 33*B*b^7*si
n(d*x + c) + 8*B*a^7 - 8*A*a^6*b - 36*B*a^5*b^2 + 36*A*a^4*b^3 + 72*B*a^3*b
^4 - 72*A*a^2*b^5 - 88*B*a*b^6 + 88*A*b^7)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 -
4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)^3))/d
```

maple [B] time = 0.50, size = 990, normalized size = 2.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2/d/(a-b)^4*ln(1+sin(d*x+c))*A*b^3+5/32/d/(a-b)^4*ln(1+sin(d*x+c))*B*b^3
-5/32/d/(a+b)^3/(sin(d*x+c)-1)*a^2*A+1/32/d/(a-b)^2/(1+sin(d*x+c))^2*a*B-1/
32/d/(a+b)^4*ln(sin(d*x+c)-1)*B*a^2*b-29/32/d/(a+b)^4*ln(sin(d*x+c)-1)*A*a*
b^2-5/32/d/(a+b)^4*ln(sin(d*x+c)-1)*B*b^3-5/32/d/(a-b)^3/(1+sin(d*x+c))*a^2
*A-11/32/d/(a-b)^3/(1+sin(d*x+c))*A*b^2+1/32/d/(a-b)^3/(1+sin(d*x+c))*B*a^2
-1/8/d/(a+b)^3/(sin(d*x+c)-1)*B*a*b-1/3/d/(16*a+16*b)/(sin(d*x+c)-1)^3*A-1/
3/d/(16*a+16*b)/(sin(d*x+c)-1)^3*B-1/3/d/(16*a-16*b)/(1+sin(d*x+c))^3*A+1/3
/d/(16*a-16*b)/(1+sin(d*x+c))^3*B+3/32/d/(a+b)^2/(sin(d*x+c)-1)^2*A*b-5/32/
d/(a+b)^3/(sin(d*x+c)-1)*B*b^2-7/16/d/(a+b)^3/(sin(d*x+c)-1)*A*a*b+5/32/d/(
a-b)^4*ln(1+sin(d*x+c))*a^3*A-1/d*b^6/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))*a*
B-1/8/d/(a+b)^4*ln(sin(d*x+c)-1)*B*a*b^2+1/32/d/(a-b)^4*ln(1+sin(d*x+c))*B*
a^2*b-5/8/d/(a+b)^4*ln(sin(d*x+c)-1)*A*a^2*b+1/d*b^7/(a+b)^4/(a-b)^4*ln(a+b
*sin(d*x+c))*A-5/8/d/(a-b)^4*ln(1+sin(d*x+c))*A*a^2*b-1/16/d/(a-b)^2/(1+sin
(d*x+c))^2*a*A+3/32/d/(a-b)^2/(1+sin(d*x+c))^2*A*b+7/16/d/(a-b)^3/(1+sin(d*
x+c))*A*a*b+29/32/d/(a-b)^4*ln(1+sin(d*x+c))*A*a*b^2-1/8/d/(a-b)^3/(1+sin(d
*x+c))*B*a*b-1/8/d/(a-b)^4*ln(1+sin(d*x+c))*B*a*b^2+5/32/d/(a-b)^3/(1+sin(d
*x+c))*B*b^2-1/32/d/(a+b)^3/(sin(d*x+c)-1)*B*a^2+1/16/d/(a+b)^2/(sin(d*x+c)
-1)^2*B*b+1/16/d/(a+b)^2/(sin(d*x+c)-1)^2*a*A-5/32/d/(a+b)^4*ln(sin(d*x+c)-
1)*a^3*A+1/32/d/(a+b)^2/(sin(d*x+c)-1)^2*a*B-11/32/d/(a+b)^3/(sin(d*x+c)-1)
*A*b^2-1/16/d/(a-b)^2/(1+sin(d*x+c))^2*B*b-1/2/d/(a+b)^4*ln(sin(d*x+c)-1)*A
*b^3
```

maxima [A] time = 0.45, size = 632, normalized size = 1.65

$$-\frac{\frac{96\left(Bab^6-Ab^7\right)\log(b\sin(dx+c)+a)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{3\left(5Aa^3-(20A-B)a^2b+(29A-4B)ab^2-(16A-5B)b^3\right)\log(\sin(dx+c)+1)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}+\frac{3\left(5Aa^3+(20A+B)a^2b+(29A+4\right.}{a^4+4a^3b+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(96*(B*a*b^6 - A*b^7)*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^2 + 6*a^
4*b^4 - 4*a^2*b^6 + b^8) - 3*(5*A*a^3 - (20*A - B)*a^2*b + (29*A - 4*B)*a*b
^2 - (16*A - 5*B)*b^3)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4
*a*b^3 + b^4) + 3*(5*A*a^3 + (20*A + B)*a^2*b + (29*A + 4*B)*a*b^2 + (16*A
+ 5*B)*b^3)*log(sin(d*x + c) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^
4) + 2*(8*B*a^5 - 8*A*a^4*b - 28*B*a^3*b^2 + 28*A*a^2*b^3 + 44*B*a*b^4 - 44
*A*b^5 + 3*(5*A*a^5 + B*a^4*b - 16*A*a^3*b^2 - 4*B*a^2*b^3 + 19*A*a*b^4 - 5
*B*b^5)*sin(d*x + c)^5 + 24*(B*a*b^4 - A*b^5)*sin(d*x + c)^4 - 8*(5*A*a^5 +
 B*a^4*b - 16*A*a^3*b^2 - 2*B*a^2*b^3 + 17*A*a*b^4 - 5*B*b^5)*sin(d*x + c)^
3 + 12*(B*a^3*b^2 - A*a^2*b^3 - 5*B*a*b^4 + 5*A*b^5)*sin(d*x + c)^2 + 3*(11
*A*a^5 - B*a^4*b - 32*A*a^3*b^2 + 4*B*a^2*b^3 + 29*A*a*b^4 - 11*B*b^5)*sin(
```

```
d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sin(d*x + c)^6 - a^6 + 3*a^4
*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sin(d*x + c)
^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sin(d*x + c)^2))/d
```

mupad [B] time = 13.44, size = 729, normalized size = 1.90

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(5\,A\,a^3+\left(B-20\,A\right)\,a^2\,b+\left(29\,A-4\,B\right)\,a\,b^2+\left(5\,B-16\,A\right)\,b^3\right)}{d\,\left(32\,a^4-128\,a^3\,b+192\,a^2\,b^2-128\,a\,b^3+32\,b^4\right)}-\frac{\frac{-2\,B\,a^5+2\,A\,a^4\,b+7\,B\,a^3\,b^2-7\,A\,\ldots}{12\,\left(a^6-3\,a^4\,b^2+3\,\ldots\right.}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^7*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(5*A*a^3 - b^3*(16*A - 5*B) - a^2*b*(20*A - B) + a*b
^2*(29*A - 4*B)))/(d*(32*a^4 - 128*a^3*b - 128*a*b^3 + 32*b^4 + 192*a^2*b^2
)) - ((11*A*b^5 - 2*B*a^5 - 7*A*a^2*b^3 + 7*B*a^3*b^2 + 2*A*a^4*b - 11*B*a*
b^4)/(12*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (sin(c + d*x)^4*(A*b^5 - B*
a*b^4))/(2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (sin(c + d*x)*(11*A*a^5 -
 11*B*b^5 - 32*A*a^3*b^2 + 4*B*a^2*b^3 + 29*A*a*b^4 - B*a^4*b))/(16*(a^6 -
b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (sin(c + d*x)^2*(5*A*b^5 - A*a^2*b^3 + B*a^
3*b^2 - 5*B*a*b^4))/(4*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (sin(c + d*x)
^3*(5*A*a^5 - 5*B*b^5 - 16*A*a^3*b^2 - 2*B*a^2*b^3 + 17*A*a*b^4 + B*a^4*b))
/(6*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (sin(c + d*x)^5*(5*A*a^5 - 5*B*b
^5 - 16*A*a^3*b^2 - 4*B*a^2*b^3 + 19*A*a*b^4 + B*a^4*b))/(16*(a^6 - b^6 + 3
*a^2*b^4 - 3*a^4*b^2)))/(d*(cos(c + d*x)^2 - 2*sin(c + d*x)^2 + 3*sin(c + d
*x)^4 - sin(c + d*x)^6)) - (log(sin(c + d*x) - 1)*(5*A*a^3 + b^3*(16*A + 5*
B) + a*b^2*(29*A + 4*B) + a^2*b*(20*A + B)))/(d*(128*a*b^3 + 128*a^3*b + 32
*a^4 + 32*b^4 + 192*a^2*b^2)) + (log(a + b*sin(c + d*x))*(A*b^7 - B*a*b^6))
/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.1552 $\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=324

$$\frac{(a^2-b^2)^3(Ab-aB)}{b^8d(a+b\sin(c+dx))}+\frac{(a^2-b^2)^2(-7a^2B+6aAb+b^2B)\log(a+b\sin(c+dx))}{b^8d}+\frac{(-3a^2B+2aAb+3b^2B)\sin^4(c$$

```
[Out] (a^2-b^2)^2*(6*A*a*b-7*B*a^2+B*b^2)*ln(a+b*sin(d*x+c))/b^8/d-(5*A*a^4*b-9*A
*a^2*b^3+3*A*b^5-6*B*a^5+12*B*a^3*b^2-6*B*a*b^4)*sin(d*x+c)/b^7/d+1/2*(4*A*
a^3*b-6*A*a*b^3-5*B*a^4+9*B*a^2*b^2-3*B*b^4)*sin(d*x+c)^2/b^6/d-1/3*(3*A*a^
2*b-3*A*b^3-4*B*a^3+6*B*a*b^2)*sin(d*x+c)^3/b^5/d+1/4*(2*A*a*b-3*B*a^2+3*B*
b^2)*sin(d*x+c)^4/b^4/d-1/5*(A*b-2*B*a)*sin(d*x+c)^5/b^3/d-1/6*B*sin(d*x+c)
^6/b^2/d+(a^2-b^2)^3*(A*b-B*a)/b^8/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.40, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$\frac{(-3a^2B+2aAb+3b^2B)\sin^4(c+dx)}{4b^4d}-\frac{(3a^2Ab-4a^3B+6ab^2B-3Ab^3)\sin^3(c+dx)}{3b^5d}+\frac{(4a^3Ab+9a^2b^2B-5a^4B$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 - b^2)^2*(6*a*A*b - 7*a^2*B + b^2*B)*Log[a + b*Sin[c + d*x]])/(b^8*d)
 - ((5*a^4*A*b - 9*a^2*A*b^3 + 3*A*b^5 - 6*a^5*B + 12*a^3*b^2*B - 6*a*b^4*B
)*Sin[c + d*x])/(b^7*d) + ((4*a^3*A*b - 6*a*A*b^3 - 5*a^4*B + 9*a^2*b^2*B -
 3*b^4*B)*Sin[c + d*x]^2)/(2*b^6*d) - ((3*a^2*A*b - 3*A*b^3 - 4*a^3*B + 6*a
*b^2*B)*Sin[c + d*x]^3)/(3*b^5*d) + ((2*a*A*b - 3*a^2*B + 3*b^2*B)*Sin[c +
d*x]^4)/(4*b^4*d) - ((A*b - 2*a*B)*Sin[c + d*x]^5)/(5*b^3*d) - (B*Sin[c + d
*x]^6)/(6*b^2*d) + ((a^2 - b^2)^3*(A*b - a*B))/(b^8*d*(a + b*Sin[c + d*x]))
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^3}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{-5a^4Ab+9a^2Ab^3-3Ab^5+6a^5B-12a^3b^2B+6ab^4B}{b} - \frac{\left(-4a^3Ab+6aAb^3+5a^4B-9\right.}{b}\right.\right.}{}$$

$$= \frac{\left(a^2-b^2\right)^2\left(6aAb-7a^2B+b^2B\right)\log(a+b\sin(c+dx))}{b^8 d} - \frac{\left(5a^4Ab-9a\right.}{}$$

Mathematica [A] time = 1.63, size = 396, normalized size = 1.22

$$\frac{\frac{6(Ab-aB)\left(-4a^2b^4\sin^4(c+dx)+4\left(a^2-b^2\right)^2\left(15a^2\log(a+b\sin(c+dx))+4a^2-4b^2\right)+b^4\cos^4(c+dx)\left(-a^2+3ab\sin(c+dx)+4b^2\right)+2ab^3\left(5a^2-7b^2\right)\sin^3(c+dx)-\right.}{a+b\sin(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (B*(15*b^4*(-a^2 + b^2)*Cos[c + d*x]^4 + 10*b^6*Cos[c + d*x]^6 - 60*(a^2 -
b^2)^3*Log[a + b*Sin[c + d*x]] + 60*a*b*(a^4 - 3*a^2*b^2 + 3*b^4)*Sin[c + d
*x] - 30*b^2*(a^2 - b^2)^2*Sin[c + d*x]^2 + 20*a*b^3*(a^2 - 3*b^2)*Sin[c +
d*x]^3 + 12*a*b^5*Sin[c + d*x]^5) + (6*(A*b - a*B)*(2*b^6*Cos[c + d*x]^6 +
4*(a^2 - b^2)^2*(4*a^2 - 4*b^2 + 15*a^2*Log[a + b*Sin[c + d*x]]) + 4*a*b*(-
11*a^4 + 18*a^2*b^2 - 4*b^4 + 15*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])*Sin
[c + d*x] - 2*b^2*(15*a^4 - 29*a^2*b^2 + 8*b^4)*Sin[c + d*x]^2 + 2*a*b^3*(5
*a^2 - 7*b^2)*Sin[c + d*x]^3 - 4*a^2*b^4*Sin[c + d*x]^4 + b^4*Cos[c + d*x]^
4*(-a^2 + 4*b^2 + 3*a*b*Sin[c + d*x])))/(a + b*Sin[c + d*x]))/(60*b^8*d)
```

fricas [A] time = 0.55, size = 508, normalized size = 1.57

$$-\frac{480\,Ba^7-480\,Aa^6b-3720\,Ba^5b^2+3360\,Aa^4b^3+5705\,Ba^3b^4-4710\,Aa^2b^5-2402\,Bab^6+1536\,Ab^7+16\left(7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/480*(480*B*a^7 - 480*A*a^6*b - 3720*B*a^5*b^2 + 3360*A*a^4*b^3 + 5705*B*
a^3*b^4 - 4710*A*a^2*b^5 - 2402*B*a*b^6 + 1536*A*b^7 + 16*(7*B*a*b^6 - 6*A*
b^7)*cos(d*x + c)^6 - 8*(35*B*a^3*b^4 - 30*A*a^2*b^5 - 33*B*a*b^6 + 24*A*b^
7)*cos(d*x + c)^4 + 16*(105*B*a^5*b^2 - 90*A*a^4*b^3 - 190*B*a^3*b^4 + 150*
A*a^2*b^5 + 81*B*a*b^6 - 48*A*b^7)*cos(d*x + c)^2 + 480*(7*B*a^7 - 6*A*a^6*
b - 15*B*a^5*b^2 + 12*A*a^4*b^3 + 9*B*a^3*b^4 - 6*A*a^2*b^5 - B*a*b^6 + (7*
B*a^6*b - 6*A*a^5*b^2 - 15*B*a^4*b^3 + 12*A*a^3*b^4 + 9*B*a^2*b^5 - 6*A*a*b
^6 - B*b^7)*sin(d*x + c))*log(b*sin(d*x + c) + a) - (80*B*b^7*cos(d*x + c)^
6 + 2880*B*a^6*b - 2400*A*a^5*b^2 - 5720*B*a^4*b^3 + 4320*A*a^3*b^4 + 2967*
B*a^2*b^5 - 1626*A*a*b^6 - 190*B*b^7 - 24*(7*B*a^2*b^5 - 6*A*a*b^6 - 5*B*b^
7)*cos(d*x + c)^4 + 16*(35*B*a^4*b^3 - 30*A*a^3*b^4 - 54*B*a^2*b^5 + 42*A*a
*b^6 + 15*B*b^7)*cos(d*x + c)^2)*sin(d*x + c))/(b^9*d*sin(d*x + c) + a*b^8*
d)
```

giac [A] time = 0.27, size = 570, normalized size = 1.76

$$-\frac{\frac{60\left(7\,Ba^{6}-6\,Aa^{5}b-15\,Ba^{4}b^{2}+12\,Aa^{3}b^{3}+9\,Ba^{2}b^{4}-6\,Aab^{5}-Bb^{6}\right)\log(|b\sin(dx+c)+a|)}{b^{8}}-\frac{60\left(7\,Ba^{6}b\sin(dx+c)-6\,Aa^{5}b^{2}\sin(dx+c)-15\,Ba^{4}b^{3}\sin(dx+c)\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/60*(60*(7*B*a^6 - 6*A*a^5*b - 15*B*a^4*b^2 + 12*A*a^3*b^3 + 9*B*a^2*b^4
- 6*A*a*b^5 - B*b^6)*log(abs(b*sin(d*x + c) + a))/b^8 - 60*(7*B*a^6*b*sin(d
*x + c) - 6*A*a^5*b^2*sin(d*x + c) - 15*B*a^4*b^3*sin(d*x + c) + 12*A*a^3*b
^4*sin(d*x + c) + 9*B*a^2*b^5*sin(d*x + c) - 6*A*a*b^6*sin(d*x + c) - B*b^7
*sin(d*x + c) + 6*B*a^7 - 5*A*a^6*b - 12*B*a^5*b^2 + 9*A*a^4*b^3 + 6*B*a^3*
b^4 - 3*A*a^2*b^5 - A*b^7)/((b*sin(d*x + c) + a)*b^8) + (10*B*b^10*sin(d*x
+ c)^6 - 24*B*a*b^9*sin(d*x + c)^5 + 12*A*b^10*sin(d*x + c)^5 + 45*B*a^2*b^
8*sin(d*x + c)^4 - 30*A*a*b^9*sin(d*x + c)^4 - 45*B*b^10*sin(d*x + c)^4 - 8
0*B*a^3*b^7*sin(d*x + c)^3 + 60*A*a^2*b^8*sin(d*x + c)^3 + 120*B*a*b^9*sin(
d*x + c)^3 - 60*A*b^10*sin(d*x + c)^3 + 150*B*a^4*b^6*sin(d*x + c)^2 - 120*
A*a^3*b^7*sin(d*x + c)^2 - 270*B*a^2*b^8*sin(d*x + c)^2 + 180*A*a*b^9*sin(d
*x + c)^2 + 90*B*b^10*sin(d*x + c)^2 - 360*B*a^5*b^5*sin(d*x + c) + 300*A*a
^4*b^6*sin(d*x + c) + 720*B*a^3*b^7*sin(d*x + c) - 540*A*a^2*b^8*sin(d*x +
c) - 360*B*a*b^9*sin(d*x + c) + 180*A*b^10*sin(d*x + c))/b^12)/d
```

maple [B] time = 0.70, size = 721, normalized size = 2.23

$$\frac{4B\left(\sin^{3}(dx+c)\right)a^{3}}{3d\,b^{5}}-\frac{3B\left(\sin^{2}(dx+c)\right)}{2b^{2}d}-\frac{A\left(\sin^{5}(dx+c)\right)}{5d\,b^{2}}+\frac{A\left(\sin^{3}(dx+c)\right)}{d\,b^{2}}-\frac{B\left(\sin^{6}(dx+c)\right)}{6b^{2}d}+\frac{3B\left(\sin^{4}(dx+\cdots\right.}{4b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 2/5/d/b^3*B*sin(d*x+c)^5*a-1/5/d/b^2*A*sin(d*x+c)^5+1/d/b^2*A*sin(d*x+c)^3-
3/d/b^2*A*sin(d*x+c)-1/d/b/(a+b*sin(d*x+c))*A+1/2/d/b^3*A*sin(d*x+c)^4*a-3/
4/d/b^4*B*sin(d*x+c)^4*a^2-1/d/b^4*A*sin(d*x+c)^3*a^2+6/d/b^7*B*a^5*sin(d*x
+c)-12/d/b^5*B*a^3*sin(d*x+c)+9/d/b^4*A*a^2*sin(d*x+c)+3/d/b^6/(a+b*sin(d*x
+c))*B*a^5-3/d/b^4/(a+b*sin(d*x+c))*B*a^3+1/d/b^2/(a+b*sin(d*x+c))*a*B-1/d/
b^8/(a+b*sin(d*x+c))*B*a^7-7/d/b^8*ln(a+b*sin(d*x+c))*B*a^6+15/d/b^6*ln(a+b
*sin(d*x+c))*B*a^4-9/d/b^4*ln(a+b*sin(d*x+c))*B*a^2+1/d/b^7/(a+b*sin(d*x+c)
)*A*a^6-3/d/b^5/(a+b*sin(d*x+c))*A*a^4-1/6*B*sin(d*x+c)^6/b^2/d+3/4*B*sin(d
*x+c)^4/b^2/d-3/2*B*sin(d*x+c)^2/b^2/d+3/d/b^3/(a+b*sin(d*x+c))*A*a^2+4/3/d
/b^5*B*sin(d*x+c)^3*a^3-2/d/b^3*B*sin(d*x+c)^3*a+2/d/b^5*A*sin(d*x+c)^2*a^3
-3/d/b^3*A*sin(d*x+c)^2*a-5/2/d/b^6*B*sin(d*x+c)^2*a^4+9/2/d/b^4*B*sin(d*x+
c)^2*a^2-5/d/b^6*A*a^4*sin(d*x+c)+6/d/b^3*B*a*sin(d*x+c)+6/d/b^7*ln(a+b*sin
(d*x+c))*A*a^5-12/d/b^5*ln(a+b*sin(d*x+c))*A*a^3+6/d/b^3*ln(a+b*sin(d*x+c))
*A*a+B*ln(a+b*sin(d*x+c))/b^2/d
```

maxima [A] time = 0.47, size = 377, normalized size = 1.16

$$-\frac{\frac{60\left(Ba^{7}-Aa^{6}b-3\,Ba^{5}b^{2}+3\,Aa^{4}b^{3}+3\,Ba^{3}b^{4}-3\,Aa^{2}b^{5}-Bab^{6}+Ab^{7}\right)}{b^{9}\sin(dx+c)+ab^{8}}+\frac{10\,Bb^{5}\sin(dx+c)^{6}-12\left(2\,Bab^{4}-Ab^{5}\right)\sin(dx+c)^{5}+15\left(3\,Ba^{2}b^{3}-2\,Aab^{4}-3\,Bb^{5}\right)\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/60*(60*(B*a^7 - A*a^6*b - 3*B*a^5*b^2 + 3*A*a^4*b^3 + 3*B*a^3*b^4 - 3*A*
a^2*b^5 - B*a*b^6 + A*b^7)/(b^9*sin(d*x + c) + a*b^8) + (10*B*b^5*sin(d*x +
 c)^6 - 12*(2*B*a*b^4 - A*b^5)*sin(d*x + c)^5 + 15*(3*B*a^2*b^3 - 2*A*a*b^4
 - 3*B*b^5)*sin(d*x + c)^4 - 20*(4*B*a^3*b^2 - 3*A*a^2*b^3 - 6*B*a*b^4 + 3*
A*b^5)*sin(d*x + c)^3 + 30*(5*B*a^4*b - 4*A*a^3*b^2 - 9*B*a^2*b^3 + 6*A*a*b
^4 + 3*B*b^5)*sin(d*x + c)^2 - 60*(6*B*a^5 - 5*A*a^4*b - 12*B*a^3*b^2 + 9*A
*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*sin(d*x + c))/b^7 + 60*(7*B*a^6 - 6*A*a^5*b
 - 15*B*a^4*b^2 + 12*A*a^3*b^3 + 9*B*a^2*b^4 - 6*A*a*b^5 - B*b^6)*log(b*sin
(d*x + c) + a)/b^8)/d
```

mupad [B] time = 12.15, size = 682, normalized size = 2.10

$$\frac{\sin(c+d\,x)^3\left(\frac{A}{b^2}+\frac{a^2\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{3\,b^2}-\frac{2\,a\left(\frac{3\,B}{b^2}+\frac{2\,a\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)}{3\,b}\right)}{d}-\frac{\sin(c+d\,x)^5\left(\frac{A}{5\,b^2}-\frac{2\,B\,a}{5\,b^3}\right)}{d}-\frac{\sin(c+d\,x)^2\left(\frac{3\,B}{2\,b^2}+\frac{a\left(\frac{3\,A}{b^2}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^3*(A/b^2 + (a^2*(A/b^2 - (2*B*a)/b^3))/(3*b^2) - (2*a*((3*B)/
b^2 + (2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/(3*b)))/d - (sin(c + d*
x)^5*(A/(5*b^2) - (2*B*a)/(5*b^3)))/d - (sin(c + d*x)^2*((3*B)/(2*b^2) + (a
*((3*A)/b^2 + (a^2*(A/b^2 - (2*B*a)/b^3))/b^2 - (2*a*((3*B)/b^2 + (2*a*(A/b
^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b))/b + (a^2*((3*B)/b^2 + (2*a*(A/b^2
- (2*B*a)/b^3))/b + (B*a^2)/b^4))/(2*b^2)))/d + (sin(c + d*x)^4*((3*B)/(4*b
^2) + (a*(A/b^2 - (2*B*a)/b^3))/(2*b) + (B*a^2)/(4*b^4)))/d - (sin(c + d*x)
*((3*A)/b^2 - (2*a*((3*B)/b^2 + (2*a*((3*A)/b^2 + (a^2*(A/b^2 - (2*B*a)/b^3
))/b^2 - (2*a*((3*B)/b^2 + (2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b)
)/b + (a^2*((3*B)/b^2 + (2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b^2))
/b + (a^2*((3*A)/b^2 + (a^2*(A/b^2 - (2*B*a)/b^3))/b^2 - (2*a*((3*B)/b^2 +
(2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b))/b^2))/d + (log(a + b*sin(
c + d*x))*(B*b^6 - 7*B*a^6 - 12*A*a^3*b^3 - 9*B*a^2*b^4 + 15*B*a^4*b^2 + 6*
A*a*b^5 + 6*A*a^5*b))/(b^8*d) - (A*b^7 + B*a^7 - 3*A*a^2*b^5 + 3*A*a^4*b^3
+ 3*B*a^3*b^4 - 3*B*a^5*b^2 - A*a^6*b - B*a*b^6)/(b*d*(a*b^7 + b^8*sin(c +
d*x))) - (B*sin(c + d*x)^6)/(6*b^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1553 $\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=206

$$-\frac{(a^2-b^2)^2(Ab-aB)}{b^6d(a+b\sin(c+dx))}-\frac{(a^2-b^2)(-5a^2B+4aAb+b^2B)\log(a+b\sin(c+dx))}{b^6d}-\frac{(-3a^2B+2aAb+2b^2B)\sin^2(c}{2b^4d}$$

```
[Out] -(a^2-b^2)*(4*A*a*b-5*B*a^2+B*b^2)*ln(a+b*sin(d*x+c))/b^6/d+(3*A*a^2*b-2*A*
b^3-4*B*a^3+4*B*a*b^2)*sin(d*x+c)/b^5/d-1/2*(2*A*a*b-3*B*a^2+2*B*b^2)*sin(d
*x+c)^2/b^4/d+1/3*(A*b-2*B*a)*sin(d*x+c)^3/b^3/d+1/4*B*sin(d*x+c)^4/b^2/d-(
a^2-b^2)^2*(A*b-B*a)/b^6/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.27, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$-\frac{(-3a^2B+2aAb+2b^2B)\sin^2(c+dx)}{2b^4d}+\frac{(3a^2Ab-4a^3B+4ab^2B-2Ab^3)\sin(c+dx)}{b^5d}-\frac{(a^2-b^2)^2(Ab-aB)}{b^6d(a+b\sin(c+dx))}-(a$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((a^2 - b^2)*(4*a*A*b - 5*a^2*B + b^2*B)*Log[a + b*Sin[c + d*x]])/(b^6*d)
) + ((3*a^2*A*b - 2*A*b^3 - 4*a^3*B + 4*a*b^2*B)*Sin[c + d*x])/(b^5*d) - ((
2*a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c + d*x]^2)/(2*b^4*d) + ((A*b - 2*a*B)*Sin
[c + d*x]^3)/(3*b^3*d) + (B*Sin[c + d*x]^4)/(4*b^2*d) - ((a^2 - b^2)^2*(A*b
 - a*B))/(b^6*d*(a + b*Sin[c + d*x]))
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)(b^2-x^2)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3a^2Ab-2Ab^3-4a^3B+4ab^2B}{b} + \frac{(-2aAb+3a^2B-2b^2B)x}{b} + \frac{(Ab-2aB)x^2}{b} + \frac{Bx^{\cdot}}{b}\right.\right.}{b^5 d}$$
$$= -\frac{(a^2-b^2)(4aAb-5a^2B+b^2B)\log(a+b\sin(c+dx))}{b^6 d} + \frac{(3a^2Ab-2\ldots}{}$$

Mathematica [A] time = 2.16, size = 234, normalized size = 1.14

$$\frac{4\left(A-\frac{aB}{b}\right)\left(\left(8a^2b-4b^3\right)\sin(c+dx)+\frac{b^4\cos^4(c+dx)-4(a^2-b^2)\left(3a^2\log(a+b\sin(c+dx))+a^2+3ab\sin(c+dx)\log(a+b\sin(c+dx))-b^2\right)}{a+b\sin(c+dx)}-\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (B*(3*b^3*Cos[c + d*x]^4 + (12*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/b - 1
2*a*(a^2 - 2*b^2)*Sin[c + d*x] + 6*b*(a^2 - b^2)*Sin[c + d*x]^2 - 4*a*b^2*S
in[c + d*x]^3) + 4*(A - (a*B)/b)*((8*a^2*b - 4*b^3)*Sin[c + d*x] - 2*a*b^2*
Sin[c + d*x]^2 + (b^4*Cos[c + d*x]^4 - 4*(a^2 - b^2)*(a^2 - b^2 + 3*a^2*Log
[a + b*Sin[c + d*x]] + 3*a*b*Log[a + b*Sin[c + d*x]]*Sin[c + d*x]))/(a + b*
Sin[c + d*x])))/(12*b^5*d)
```

fricas [A] time = 0.54, size = 322, normalized size = 1.56

$$\frac{96\,Ba^5-96\,Aa^4b-504\,Ba^3b^2+432\,Aa^2b^3+383\,Bab^4-256\,Ab^5-8\left(5\,Bab^4-4\,Ab^5\right)\cos\left(dx+c\right)^4+16\left(15\,B\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/96*(96*B*a^5 - 96*A*a^4*b - 504*B*a^3*b^2 + 432*A*a^2*b^3 + 383*B*a*b^4 -
 256*A*b^5 - 8*(5*B*a*b^4 - 4*A*b^5)*cos(d*x + c)^4 + 16*(15*B*a^3*b^2 - 12
*A*a^2*b^3 - 13*B*a*b^4 + 8*A*b^5)*cos(d*x + c)^2 + 96*(5*B*a^5 - 4*A*a^4*b
 - 6*B*a^3*b^2 + 4*A*a^2*b^3 + B*a*b^4 + (5*B*a^4*b - 4*A*a^3*b^2 - 6*B*a^2
```

```
*b^3 + 4*A*a*b^4 + B*b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) + (24*B*b^5
*cos(d*x + c)^4 - 384*B*a^4*b + 288*A*a^3*b^2 + 392*B*a^2*b^3 - 208*A*a*b^4
 - 33*B*b^5 - 16*(5*B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*cos(d*x + c)^2)*sin(d*
x + c))/(b^7*d*sin(d*x + c) + a*b^6*d)
```

giac [A] time = 0.25, size = 328, normalized size = 1.59

$$\frac{\frac{12\left(5\,Ba^4-4\,Aa^3b-6\,Ba^2b^2+4\,Aab^3+Bb^4\right)\log(|b\sin(dx+c)+a|)}{b^6}-\frac{12\left(5\,Ba^4b\sin(dx+c)-4\,Aa^3b^2\sin(dx+c)-6\,Ba^2b^3\sin(dx+c)+4\,Aab^4\sin(dx+c)+Bb^5\sin\right.}{(b\sin(dx+c)+a)b^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/12*(12*(5*B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*log(abs(b*
sin(d*x + c) + a))/b^6 - 12*(5*B*a^4*b*sin(d*x + c) - 4*A*a^3*b^2*sin(d*x +
 c) - 6*B*a^2*b^3*sin(d*x + c) + 4*A*a*b^4*sin(d*x + c) + B*b^5*sin(d*x + c
) + 4*B*a^5 - 3*A*a^4*b - 4*B*a^3*b^2 + 2*A*a^2*b^3 + A*b^5)/((b*sin(d*x +
c) + a)*b^6) + (3*B*b^6*sin(d*x + c)^4 - 8*B*a*b^5*sin(d*x + c)^3 + 4*A*b^6
*sin(d*x + c)^3 + 18*B*a^2*b^4*sin(d*x + c)^2 - 12*A*a*b^5*sin(d*x + c)^2 -
 12*B*b^6*sin(d*x + c)^2 - 48*B*a^3*b^3*sin(d*x + c) + 36*A*a^2*b^4*sin(d*x
 + c) + 48*B*a*b^5*sin(d*x + c) - 24*A*b^6*sin(d*x + c))/b^8)/d
```

maple [B] time = 0.66, size = 422, normalized size = 2.05

$$\frac{B\left(\sin^4(dx+c)\right)}{4b^2d}+\frac{A\left(\sin^3(dx+c)\right)}{3d\,b^2}-\frac{2B\left(\sin^3(dx+c)\right)a}{3d\,b^3}-\frac{A\left(\sin^2(dx+c)\right)a}{d\,b^3}+\frac{3B\left(\sin^2(dx+c)\right)a^2}{2d\,b^4}-\frac{B\left(\sin^2(dx\right.}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4*B*sin(d*x+c)^4/b^2/d+1/3/d/b^2*A*sin(d*x+c)^3-2/3/d/b^3*B*sin(d*x+c)^3*
a-1/d/b^3*A*sin(d*x+c)^2*a+3/2/d/b^4*B*sin(d*x+c)^2*a^2-B*sin(d*x+c)^2/b^2/
d+3/d/b^4*A*a^2*sin(d*x+c)-2/d/b^2*A*sin(d*x+c)-4/d/b^5*B*a^3*sin(d*x+c)+4/
d/b^3*B*a*sin(d*x+c)-4/d/b^5*ln(a+b*sin(d*x+c))*A*a^3+4/d/b^3*ln(a+b*sin(d*
x+c))*A*a+5/d/b^6*ln(a+b*sin(d*x+c))*B*a^4-6/d/b^4*ln(a+b*sin(d*x+c))*B*a^2
+B*ln(a+b*sin(d*x+c))/b^2/d-1/d/b^5/(a+b*sin(d*x+c))*A*a^4+2/d/b^3/(a+b*sin
(d*x+c))*A*a^2-1/d/b/(a+b*sin(d*x+c))*A+1/d/b^6/(a+b*sin(d*x+c))*B*a^5-2/d/
b^4/(a+b*sin(d*x+c))*B*a^3+1/d/b^2/(a+b*sin(d*x+c))*a*B
```

maxima [A] time = 0.39, size = 229, normalized size = 1.11

$$\frac{\frac{12\left(Ba^5-Aa^4b-2\,Ba^3b^2+2\,Aa^2b^3+Bab^4-Ab^5\right)}{b^7\sin(dx+c)+ab^6}+\frac{3\,Bb^3\sin(dx+c)^4-4\left(2\,Bab^2-Ab^3\right)\sin(dx+c)^3+6\left(3\,Ba^2b-2\,Aab^2-2\,Bb^3\right)\sin(dx+c)^2-12\left(4\,Ba^3-3\,Aa\right.}{b^5}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/12*(12*(B*a^5 - A*a^4*b - 2*B*a^3*b^2 + 2*A*a^2*b^3 + B*a*b^4 - A*b^5)/(b
^7*sin(d*x + c) + a*b^6) + (3*B*b^3*sin(d*x + c)^4 - 4*(2*B*a*b^2 - A*b^3)*
sin(d*x + c)^3 + 6*(3*B*a^2*b - 2*A*a*b^2 - 2*B*b^3)*sin(d*x + c)^2 - 12*(4
*B*a^3 - 3*A*a^2*b - 4*B*a*b^2 + 2*A*b^3)*sin(d*x + c))/b^5 + 12*(5*B*a^4 -
 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*log(b*sin(d*x + c) + a)/b^6)/
d
```

mupad [B] time = 0.12, size = 290, normalized size = 1.41

$$\frac{\sin(c+d\,x)^3\left(\frac{A}{3\,b^2}-\frac{2\,B\,a}{3\,b^3}\right)}{d}-\frac{\sin(c+d\,x)\left(\frac{2\,A}{b^2}+\frac{a^2\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b^2}-\frac{2\,a\left(\frac{2\,B}{b^2}+\frac{2\,a\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)}{b}\right)}{d}-\frac{\sin(c+d\,x)^2\left(\frac{B}{b^2}+\frac{a\left(\frac{A}{b^2}-\right.}{b}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^3*(A/(3*b^2) - (2*B*a)/(3*b^3)))/d - (sin(c + d*x)*((2*A)/b^2
 + (a^2*(A/b^2 - (2*B*a)/b^3))/b^2 - (2*a*((2*B)/b^2 + (2*a*(A/b^2 - (2*B*a
)/b^3))/b + (B*a^2)/b^4))/b))/d - (sin(c + d*x)^2*(B/b^2 + (a*(A/b^2 - (2*B
*a)/b^3))/b + (B*a^2)/(2*b^4)))/d + (log(a + b*sin(c + d*x))*(5*B*a^4 + B*b
^4 - 6*B*a^2*b^2 + 4*A*a*b^3 - 4*A*a^3*b))/(b^6*d) - (A*b^5 - B*a^5 - 2*A*a
^2*b^3 + 2*B*a^3*b^2 + A*a^4*b - B*a*b^4)/(b*d*(a*b^5 + b^6*sin(c + d*x)))
+ (B*sin(c + d*x)^4)/(4*b^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1554 $\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=113

$$\frac{(a^2-b^2)(Ab-aB)}{b^4d(a+b\sin(c+dx))}+\frac{(-3a^2B+2aAb+b^2B)\log(a+b\sin(c+dx))}{b^4d}-\frac{(Ab-2aB)\sin(c+dx)}{b^3d}-\frac{B\sin^2(c+dx)}{2b^2d}$$

```
[Out] (2*A*a*b-3*B*a^2+B*b^2)*ln(a+b*sin(d*x+c))/b^4/d-(A*b-2*B*a)*sin(d*x+c)/b^3
/d-1/2*B*sin(d*x+c)^2/b^2/d+(a^2-b^2)*(A*b-B*a)/b^4/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$\frac{(a^2-b^2)(Ab-aB)}{b^4d(a+b\sin(c+dx))}+\frac{(-3a^2B+2aAb+b^2B)\log(a+b\sin(c+dx))}{b^4d}-\frac{(Ab-2aB)\sin(c+dx)}{b^3d}-\frac{B\sin^2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a*A*b - 3*a^2*B + b^2*B)*Log[a + b*Sin[c + d*x]])/(b^4*d) - ((A*b - 2*a
*B)*Sin[c + d*x])/(b^3*d) - (B*Sin[c + d*x]^2)/(2*b^2*d) + ((a^2 - b^2)*(A*
b - a*B))/(b^4*d*(a + b*Sin[c + d*x]))
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)(b^2-x^2)}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-Ab+2aB}{b}-\frac{Bx}{b}+\frac{(-a^2+b^2)(Ab-aB)}{b(a+x)^2}+\frac{2aAb-3a^2B+b^2B}{b(a+x)}\right)dx, x, b\sin(\right.}{b^3d}$$

$$= \frac{(2aAb-3a^2B+b^2B)\log(a+b\sin(c+dx))}{b^4d} - \frac{(Ab-2aB)\sin(c+dx)}{b^3d}$$

Mathematica [A] time = 0.54, size = 111, normalized size = 0.98

$$\frac{\frac{B(b^2-a^2)\log(a+b\sin(c+dx))}{b}+\left(A-\frac{aB}{b}\right)\left(\frac{(a-b)(a+b)}{a+b\sin(c+dx)}+2a\log(a+b\sin(c+dx))-b\sin(c+dx)\right)+aB\sin(c+dx)-\frac{1}{2}}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (((-a^2 + b^2)*B*Log[a + b*Sin[c + d*x]])/b + a*B*Sin[c + d*x] - (b*B*Sin[c
 + d*x]^2)/2 + (A - (a*B)/b)*(2*a*Log[a + b*Sin[c + d*x]] - b*Sin[c + d*x]
+ ((a - b)*(a + b))/(a + b*Sin[c + d*x])))/(b^3*d)
```

fricas [A] time = 0.49, size = 178, normalized size = 1.58

$$-\frac{4\,Ba^3-4\,Aa^2b-11\,Bab^2+8\,Ab^3+2\left(3\,Bab^2-2\,Ab^3\right)\cos(dx+c)^2+4\left(3\,Ba^3-2\,Aa^2b-Bab^2+\left(3\,Ba^2b-2\right.\right.}{4\left(b^5d\sin(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/4*(4*B*a^3 - 4*A*a^2*b - 11*B*a*b^2 + 8*A*b^3 + 2*(3*B*a*b^2 - 2*A*b^3)*
cos(d*x + c)^2 + 4*(3*B*a^3 - 2*A*a^2*b - B*a*b^2 + (3*B*a^2*b - 2*A*a*b^2
- B*b^3)*sin(d*x + c))*log(b*sin(d*x + c) + a) - (2*B*b^3*cos(d*x + c)^2 +
8*B*a^2*b - 4*A*a*b^2 - B*b^3)*sin(d*x + c))/(b^5*d*sin(d*x + c) + a*b^4*d)
```

giac [A] time = 0.23, size = 188, normalized size = 1.66

$$-\frac{\frac{(b\sin(dx+c)+a)^2\left(B-\frac{2(3\,Bab-Ab^2)}{(b\sin(dx+c)+a)b}\right)}{b^4}-\frac{2\left(3\,Ba^2-2\,Aab-Bb^2\right)\log\left(\frac{|b\sin(dx+c)+a|}{(b\sin(dx+c)+a)^2|b|}\right)}{b^4}+\frac{2\left(\frac{Ba^3b^2}{b\sin(dx+c)+a}-\frac{Aa^2b^3}{b\sin(dx+c)+a}-\frac{Bab^4}{b\sin(dx+c)+a}+\frac{Ab^5}{b\sin(dx+c)+a}\right)}{b^6}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*((b*sin(d*x + c) + a)^2*(B - 2*(3*B*a*b - A*b^2)/((b*sin(d*x + c) + a)
*b))/b^4 - 2*(3*B*a^2 - 2*A*a*b - B*b^2)*log(abs(b*sin(d*x + c) + a)/((b*si
n(d*x + c) + a)^2*abs(b)))/b^4 + 2*(B*a^3*b^2/(b*sin(d*x + c) + a) - A*a^2*
b^3/(b*sin(d*x + c) + a) - B*a*b^4/(b*sin(d*x + c) + a) + A*b^5/(b*sin(d*x
+ c) + a))/b^6)/d
```

maple [A] time = 0.66, size = 202, normalized size = 1.79

$$-\frac{B\left(\sin^2(dx+c)\right)}{2b^2d}-\frac{A\sin(dx+c)}{d\,b^2}+\frac{2Ba\sin(dx+c)}{d\,b^3}+\frac{2\ln(a+b\sin(dx+c))\,Aa}{d\,b^3}-\frac{3\ln(a+b\sin(dx+c))\,B\,a^2}{d\,b^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2*B*sin(d*x+c)^2/b^2/d-1/d/b^2*A*sin(d*x+c)+2/d/b^3*B*a*sin(d*x+c)+2/d/b
^3*ln(a+b*sin(d*x+c))*A*a-3/d/b^4*ln(a+b*sin(d*x+c))*B*a^2+B*ln(a+b*sin(d*x
+c))/b^2/d+1/d/b^3/(a+b*sin(d*x+c))*A*a^2-1/d/b/(a+b*sin(d*x+c))*A-1/d/b^4/
(a+b*sin(d*x+c))*B*a^3+1/d/b^2/(a+b*sin(d*x+c))*a*B
```

maxima [A] time = 0.31, size = 118, normalized size = 1.04

$$-\frac{\frac{2\left(Ba^3-Aa^2b-Bab^2+Ab^3\right)}{b^5\sin(dx+c)+ab^4}+\frac{Bb\sin(dx+c)^2-2\,(2\,Ba-Ab)\sin(dx+c)}{b^3}+\frac{2\left(3\,Ba^2-2\,Aab-Bb^2\right)\log(b\sin(dx+c)+a)}{b^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/2*(2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)/(b^5*sin(d*x + c) + a*b^4) + (B
*b*sin(d*x + c)^2 - 2*(2*B*a - A*b)*sin(d*x + c))/b^3 + 2*(3*B*a^2 - 2*A*a*
b - B*b^2)*log(b*sin(d*x + c) + a)/b^4)/d
```

mupad [B] time = 12.10, size = 128, normalized size = 1.13

$$\frac{\ln(a+b\,\sin(c+d\,x))\,\left(-3\,B\,a^2+2\,A\,a\,b+B\,b^2\right)}{b^4\,d}-\frac{B\,a^3-A\,a^2\,b-B\,a\,b^2+A\,b^3}{b\,d\,\left(\sin(c+d\,x)\,b^4+a\,b^3\right)}-\frac{\sin(c+d\,x)\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{d}-\frac{B\sin}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (log(a + b*sin(c + d*x))*(B*b^2 - 3*B*a^2 + 2*A*a*b))/(b^4*d) - (A*b^3 + B*
a^3 - A*a^2*b - B*a*b^2)/(b*d*(a*b^3 + b^4*sin(c + d*x))) - (sin(c + d*x)*(
A/b^2 - (2*B*a)/b^3))/d - (B*sin(c + d*x)^2)/(2*b^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1555 $\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=48

$$\frac{B\log(a+b\sin(c+dx))}{b^2d}-\frac{Ab-aB}{b^2d(a+b\sin(c+dx))}$$

```
[Out] B*ln(a+b*sin(d*x+c))/b^2/d+(-A*b+B*a)/b^2/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{B\log(a+b\sin(c+dx))}{b^2d}-\frac{Ab-aB}{b^2d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (B*Log[a + b*Sin[c + d*x]])/(b^2*d) - (A*b - a*B)/(b^2*d*(a + b*Sin[c + d*x
]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+x)^2}+\frac{B}{b(a+x)}\right)dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{B\log(a+b\sin(c+dx))}{b^2d} - \frac{Ab-aB}{b^2d(a+b\sin(c+dx))}$$

Mathematica [A] time = 0.10, size = 42, normalized size = 0.88

$$\frac{\frac{aB-Ab}{a+b\sin(c+dx)}+B\log(a+b\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (B*Log[a + b*Sin[c + d*x]] + (-(A*b) + a*B)/(a + b*Sin[c + d*x]))/(b^2*d)
```

fricas [A] time = 0.45, size = 54, normalized size = 1.12

$$\frac{Ba-Ab+(Bb\sin(dx+c)+Ba)\log(b\sin(dx+c)+a)}{b^3d\sin(dx+c)+ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] (B*a - A*b + (B*b*sin(d*x + c) + B*a)*log(b*sin(d*x + c) + a))/(b^3*d*sin(d
*x + c) + a*b^2*d)
```

giac [A] time = 0.17, size = 80, normalized size = 1.67

$$-\frac{\frac{B\left(\frac{\log\left(\frac{|b\sin(dx+c)+a|}{(b\sin(dx+c)+a)^2|b|}\right)}{b}-\frac{a}{(b\sin(dx+c)+a)b}\right)}{b}+\frac{A}{(b\sin(dx+c)+a)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] -(B*(log(abs(b*sin(d*x + c) + a)/((b*sin(d*x + c) + a)^2*abs(b)))/b - a/((b
*sin(d*x + c) + a)*b))/b + A/((b*sin(d*x + c) + a)*b))/d
```

maple [A] time = 0.41, size = 63, normalized size = 1.31

$$\frac{B\ln\left(a+b\sin\left(dx+c\right)\right)}{b^{2}d}-\frac{A}{db\left(a+b\sin\left(dx+c\right)\right)}+\frac{aB}{d\,b^{2}\left(a+b\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] B*ln(a+b*sin(d*x+c))/b^2/d-1/d/b/(a+b*sin(d*x+c))*A+1/d/b^2/(a+b*sin(d*x+c)
)*a*B
```

maxima [A] time = 0.46, size = 48, normalized size = 1.00

$$\frac{\frac{Ba-Ab}{b^{3}\sin(dx+c)+ab^{2}}+\frac{B\log(b\sin(dx+c)+a)}{b^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] ((B*a - A*b)/(b^3*sin(d*x + c) + a*b^2) + B*log(b*sin(d*x + c) + a)/b^2)/d
```

mupad [B] time = 0.06, size = 48, normalized size = 1.00

$$\frac{B\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{b^2\,d}-\frac{A\,b-B\,a}{b^2\,d\,\left(a+b\,\sin\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (B*log(a + b*sin(c + d*x)))/(b^2*d) - (A*b - B*a)/(b^2*d*(a + b*sin(c + d*x
)))
```

sympy [A] time = 1.36, size = 178, normalized size = 3.71

$$\begin{cases} \frac{x(A+B\sin(c))\cos(c)}{a^2} & \text{for } b=0 \wedge d=0 \\ \frac{\frac{A\sin(c+dx)}{d} - \frac{B\cos^2(c+dx)}{2d}}{a^2} & \text{for } b=0 \\ \frac{x(A+B\sin(c))\cos(c)}{(a+b\sin(c))^2} & \text{for } d=0 \\ -\frac{Ab}{ab^2d+b^3d\sin(c+dx)} + \frac{Ba\log\left(\frac{a}{b}+\sin(c+dx)\right)}{ab^2d+b^3d\sin(c+dx)} + \frac{Ba}{ab^2d+b^3d\sin(c+dx)} + \frac{Bb\log\left(\frac{a}{b}+\sin(c+dx)\right)\sin(c+dx)}{ab^2d+b^3d\sin(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((x*(A + B*sin(c))*cos(c)/a**2, Eq(b, 0) & Eq(d, 0)), ((A*sin(c +
d*x)/d - B*cos(c + d*x)**2/(2*d))/a**2, Eq(b, 0)), (x*(A + B*sin(c))*cos(c)
/(a + b*sin(c))**2, Eq(d, 0)), (-A*b/(a*b**2*d + b**3*d*sin(c + d*x)) + B*a
*log(a/b + sin(c + d*x))/(a*b**2*d + b**3*d*sin(c + d*x)) + B*a/(a*b**2*d +
 b**3*d*sin(c + d*x)) + B*b*log(a/b + sin(c + d*x))*sin(c + d*x)/(a*b**2*d
+ b**3*d*sin(c + d*x)), True))
```

3.1556 $\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=135

$$\frac{Ab-aB}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\left(a^2(-B)+2aAb-b^2B\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{(A+B)\log(1-\sin(c+dx))}{2d(a+b)^2}+\frac{(A$$

```
[Out] -1/2*(A+B)*ln(1-sin(d*x+c))/(a+b)^2/d+1/2*(A-B)*ln(1+sin(d*x+c))/(a-b)^2/d-
(2*A*a*b-B*a^2-B*b^2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^2/d+(A*b-B*a)/(a^2-b^2)/
d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.19, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2837, 801}

$$\frac{Ab-aB}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\left(a^2(-B)+2aAb-b^2B\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{(A+B)\log(1-\sin(c+dx))}{2d(a+b)^2}+\frac{(A$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((A + B)*Log[1 - Sin[c + d*x]])/(2*(a + b)^2*d) + ((A - B)*Log[1 + Sin[c +
 d*x]])/(2*(a - b)^2*d) - ((2*a*A*b - a^2*B - b^2*B)*Log[a + b*Sin[c + d*x]
])/((a^2 - b^2)^2*d) + (A*b - a*B)/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{b\,\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \left(\frac{A+B}{2b(a+b)^2(b-x)} + \frac{-Ab+aB}{(a-b)b(a+b)(a+x)^2} + \frac{-2aAb+a^2B+b^2B}{(a-b)^2b(a+b)^2(a+x)} + \frac{A-B}{2(a-b)^2b(b+\cdots}\right.\right.}{d}$$

$$= -\frac{(A+B)\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{(A-B)\log(1+\sin(c+dx))}{2(a-b)^2d} - \frac{(2aAb\cdots}{}$$

Mathematica [A] time = 1.37, size = 178, normalized size = 1.32

$$\frac{b\left(A-\frac{aB}{b}\right)\left(\frac{1}{(a^2-b^2)(a+b\sin(c+dx))} - \frac{\log(1-\sin(c+dx))}{2b(a+b)^2} + \frac{\log(\sin(c+dx)+1)}{2b(a-b)^2} - \frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}\right) - \frac{B((b-a)\log(1-\sin(c+dx))+(a+b)\cdots}{2b\cdots}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-1/2*(B*((-a + b)*Log[1 - Sin[c + d*x]] + (a + b)*Log[1 + Sin[c + d*x]] -
2*b*Log[a + b*Sin[c + d*x]]))/(b*(-a + b)*(a + b)) + b*(A - (a*B)/b)*(-1/2*
Log[1 - Sin[c + d*x]]/(b*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*b)
 - (2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a^2 - b^2)*(a
+ b*Sin[c + d*x]))))/d
```

fricas [B] time = 0.71, size = 283, normalized size = 2.10

$$-\frac{2\,Ba^3 - 2\,Aa^2b - 2\,Bab^2 + 2\,Ab^3 - 2\left(Ba^3 - 2\,Aa^2b + Bab^2 + \left(Ba^2b - 2\,Aab^2 + Bb^3\right)\sin(dx+c)\right)\log(b\sin(d\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] -1/2*(2*B*a^3 - 2*A*a^2*b - 2*B*a*b^2 + 2*A*b^3 - 2*(B*a^3 - 2*A*a^2*b + B*
a*b^2 + (B*a^2*b - 2*A*a*b^2 + B*b^3)*sin(d*x + c))*log(b*sin(d*x + c) + a)
 - ((A - B)*a^3 + 2*(A - B)*a^2*b + (A - B)*a*b^2 + ((A - B)*a^2*b + 2*(A -
 B)*a*b^2 + (A - B)*b^3)*sin(d*x + c))*log(sin(d*x + c) + 1) + ((A + B)*a^3
 - 2*(A + B)*a^2*b + (A + B)*a*b^2 + ((A + B)*a^2*b - 2*(A + B)*a*b^2 + (A
```

```
+ B)*b^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/((a^4*b - 2*a^2*b^3 + b^5)*
d*sin(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)
```

giac [A] time = 0.23, size = 205, normalized size = 1.52

$$\frac{\frac{2\left(Ba^2b-2\,Aab^2+Bb^3\right)\log(|b\sin(dx+c)+a|)}{a^4b-2\,a^2b^3+b^5}-\frac{(A+B)\log(|-\sin(dx+c)+1|)}{a^2+2\,ab+b^2}+\frac{(A-B)\log(|-\sin(dx+c)-1|)}{a^2-2\,ab+b^2}-\frac{2\left(Ba^2b\sin(dx+c)-2\,Aab^2\sin(dx+c)+Bb\right.}{\left(a^4-2\,a^2b^2+b^4\right)(b\,s}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] 1/2*(2*(B*a^2*b - 2*A*a*b^2 + B*b^3)*log(abs(b*sin(d*x + c) + a))/(a^4*b -
2*a^2*b^3 + b^5) - (A + B)*log(abs(-sin(d*x + c) + 1))/(a^2 + 2*a*b + b^2)
+ (A - B)*log(abs(-sin(d*x + c) - 1))/(a^2 - 2*a*b + b^2) - 2*(B*a^2*b*sin(
d*x + c) - 2*A*a*b^2*sin(d*x + c) + B*b^3*sin(d*x + c) + 2*B*a^3 - 3*A*a^2*
b + A*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c) + a)))/d
```

maple [A] time = 0.69, size = 240, normalized size = 1.78

$$-\frac{\ln\left(\sin\left(dx+c\right)-1\right)A}{2d\left(a+b\right)^{2}}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)B}{2d\left(a+b\right)^{2}}+\frac{Ab}{d\left(a+b\right)\left(a-b\right)\left(a+b\sin\left(dx+c\right)\right)}-\frac{aB}{d\left(a+b\right)\left(a-b\right)\left(a+b\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d/(a+b)^2*ln(sin(d*x+c)-1)*A-1/2/d/(a+b)^2*ln(sin(d*x+c)-1)*B+1/d/(a+b
)/(a-b)/(a+b*sin(d*x+c))*A*b-1/d/(a+b)/(a-b)/(a+b*sin(d*x+c))*a*B-2/d/(a+b)
^2/(a-b)^2*ln(a+b*sin(d*x+c))*A*a*b+1/d/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))*
B*a^2+1/d/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))*B*b^2+1/2/d/(a-b)^2*ln(1+sin(d
*x+c))*A-1/2/d/(a-b)^2*ln(1+sin(d*x+c))*B
```

maxima [A] time = 0.49, size = 147, normalized size = 1.09

$$\frac{\frac{2\left(Ba^2-2\,Aab+Bb^2\right)\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4}+\frac{(A-B)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}-\frac{(A+B)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}-\frac{2\,(Ba-Ab)}{a^3-ab^2+\left(a^2b-b^3\right)\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*(B*a^2 - 2*A*a*b + B*b^2)*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 +
 b^4) + (A - B)*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (A + B)*log(sin
```

```
(d*x + c) - 1)/(a^2 + 2*a*b + b^2) - 2*(B*a - A*b)/(a^3 - a*b^2 + (a^2*b -
b^3)*sin(d*x + c)))/d
```

mupad [B] time = 0.42, size = 131, normalized size = 0.97

$$\frac{A\,b-B\,a}{d\,\left(a^2-b^2\right)\,\left(a+b\,\sin\left(c+d\,x\right)\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{A}{2}+\frac{B}{2}\right)}{d\,{\left(a+b\right)}^2}+\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(B\,a^2-2\,A\,a\,b+B\,b^2\right)}{d\,{\left(a^2-b^2\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (A*b - B*a)/(d*(a^2 - b^2)*(a + b*sin(c + d*x))) - (log(sin(c + d*x) - 1)*(
A/2 + B/2))/(d*(a + b)^2) + (log(a + b*sin(c + d*x))*(B*a^2 + B*b^2 - 2*A*a
*b))/(d*(a^2 - b^2)^2) + (log(sin(c + d*x) + 1)*(A/2 - B/2))/(d*(a - b)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(A+B\sin{(c+dx)})\sec{(c+dx)}}{(a+b\sin{(c+dx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sin(c + d*x))*sec(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

3.1557 $\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=228

$$-\frac{b\left(a^2A-4abB+3Ab^2\right)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b^2\left(-3a^2B+4aAb-b^2B\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\sec^2(c+dx)(-(aA-bB)\sin\cdots}{2d\left(a^2-b^2\right)(a+b\cdots}$$

```
[Out] -1/4*(A*a+3*A*b+2*B*b)*ln(1-sin(d*x+c))/(a+b)^3/d+1/4*(A*a-3*A*b+2*B*b)*ln(
1+sin(d*x+c))/(a-b)^3/d+b^2*(4*A*a*b-3*B*a^2-B*b^2)*ln(a+b*sin(d*x+c))/(a^2
-b^2)^3/d-1/2*b*(A*a^2+3*A*b^2-4*B*a*b)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))-1/2*
sec(d*x+c)^2*(A*b-a*B-(A*a-B*b)*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.33, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2837, 823, 801}

$$-\frac{b\left(a^2A-4abB+3Ab^2\right)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b^2\left(-3a^2B+4aAb-b^2B\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\sec^2(c+dx)(-(aA-bB)\sin\cdots}{2d\left(a^2-b^2\right)(a+b\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a*A + 3*A*b + 2*b*B)*Log[1 - Sin[c + d*x]])/(4*(a + b)^3*d) + ((a*A - 3*
A*b + 2*b*B)*Log[1 + Sin[c + d*x]])/(4*(a - b)^3*d) + (b^2*(4*a*A*b - 3*a^2
*B - b^2*B)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) - (b*(a^2*A + 3*A*b^
2 - 4*a*b*B))/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (Sec[c + d*x]^2*(A
*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])
)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
```

```
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{b\,\text{Subst}\left(\int \frac{-a^2A+3A}{(}\right.}{}$$

$$= -\frac{\sec^2(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{b\,\text{Subst}\left(\int\left(-\frac{(a-b)(}{2b(}\right.\right.}{}$$

$$= -\frac{(aA+3Ab+2bB)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(aA-3Ab+2bB)\log(1+}{4(a-b)^3d}$$

Mathematica [A] time = 1.72, size = 246, normalized size = 1.08

$$\frac{b\left(a^2A-4abB+3Ab^2\right)\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\log(1-\sin(c+dx))}{2b(a+b)^2}+\frac{\log(\sin(c+dx)+1)}{2b(a-b)^2}-\frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}\right)+\frac{(aA-bB)((a-b)}{}}{2d\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (((a*A - b*B)*((a - b)*Log[1 - Sin[c + d*x]] - (a + b)*Log[1 + Sin[c + d*x]
] + 2*b*Log[a + b*Sin[c + d*x]]))/((a - b)*(a + b)) + (Sec[c + d*x]^2*(A*b
```

```
- a*B + (-(a*A) + b*B)*Sin[c + d*x]))/(a + b*Sin[c + d*x]) + b*(a^2*A + 3*A
*b^2 - 4*a*b*B)*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)^2) + Log[1 + Sin[c +
 d*x]]/(2*(a - b)^2*b) - (2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2
) + 1/((a^2 - b^2)*(a + b*Sin[c + d*x]))))/(2*(-a^2 + b^2)*d)
```

fricas [B] time = 1.35, size = 598, normalized size = 2.62

$$\frac{2\,Ba^5 - 2\,Aa^4b - 4\,Ba^3b^2 + 4\,Aa^2b^3 + 2\,Bab^4 - 2\,Ab^5 - 2\left(Aa^4b - 4\,Ba^3b^2 + 2\,Aa^2b^3 + 4\,Bab^4 - 3\,Ab^5\right)\cos\left(dx + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/4*(2*B*a^5 - 2*A*a^4*b - 4*B*a^3*b^2 + 4*A*a^2*b^3 + 2*B*a*b^4 - 2*A*b^5
- 2*(A*a^4*b - 4*B*a^3*b^2 + 2*A*a^2*b^3 + 4*B*a*b^4 - 3*A*b^5)*cos(d*x + c
)^2 - 4*((3*B*a^2*b^3 - 4*A*a*b^4 + B*b^5)*cos(d*x + c)^2*sin(d*x + c) + (3
*B*a^3*b^2 - 4*A*a^2*b^3 + B*a*b^4)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a)
 + ((A*a^4*b + 2*B*a^3*b^2 - 6*(A - B)*a^2*b^3 - 2*(4*A - 3*B)*a*b^4 - (3*A
 - 2*B)*b^5)*cos(d*x + c)^2*sin(d*x + c) + (A*a^5 + 2*B*a^4*b - 6*(A - B)*a
^3*b^2 - 2*(4*A - 3*B)*a^2*b^3 - (3*A - 2*B)*a*b^4)*cos(d*x + c)^2)*log(sin
(d*x + c) + 1) - ((A*a^4*b + 2*B*a^3*b^2 - 6*(A + B)*a^2*b^3 + 2*(4*A + 3*B
)*a*b^4 - (3*A + 2*B)*b^5)*cos(d*x + c)^2*sin(d*x + c) + (A*a^5 + 2*B*a^4*b
 - 6*(A + B)*a^3*b^2 + 2*(4*A + 3*B)*a^2*b^3 - (3*A + 2*B)*a*b^4)*cos(d*x +
 c)^2)*log(-sin(d*x + c) + 1) + 2*(A*a^5 - B*a^4*b - 2*A*a^3*b^2 + 2*B*a^2*
b^3 + A*a*b^4 - B*b^5)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)
*d*cos(d*x + c)^2*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*co
s(d*x + c)^2)
```

giac [A] time = 0.29, size = 335, normalized size = 1.47

$$-\frac{\frac{4\left(3\,Ba^2b^3 - 4\,Aab^4 + Bb^5\right)\log\left(|b\sin(dx+c)+a|\right)}{a^6b - 3\,a^4b^3 + 3\,a^2b^5 - b^7} - \frac{(Aa - 3\,Ab + 2\,Bb)\log\left(|\sin(dx+c)+1|\right)}{a^3 - 3\,a^2b + 3\,ab^2 - b^3} + \frac{(Aa + 3\,Ab + 2\,Bb)\log\left(|-\sin(dx+c)+1|\right)}{a^3 + 3\,a^2b + 3\,ab^2 + b^3} + \frac{2\left(Aa^2b\sin(dx+\right.}{}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/4*(4*(3*B*a^2*b^3 - 4*A*a*b^4 + B*b^5)*log(abs(b*sin(d*x + c) + a))/(a^6
*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7) - (A*a - 3*A*b + 2*B*b)*log(abs(sin(d*x +
 c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (A*a + 3*A*b + 2*B*b)*log(abs(-
sin(d*x + c) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(A*a^2*b*sin(d*x + c
)^2 - 4*B*a*b^2*sin(d*x + c)^2 + 3*A*b^3*sin(d*x + c)^2 + A*a^3*sin(d*x + c
```

```
) - B*a^2*b*sin(d*x + c) - A*a*b^2*sin(d*x + c) + B*b^3*sin(d*x + c) + B*a^
3 - 2*A*a^2*b + 3*B*a*b^2 - 2*A*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x +
c)^3 + a*sin(d*x + c)^2 - b*sin(d*x + c) - a)))/d
```

maple [A] time = 0.80, size = 388, normalized size = 1.70

$$-\frac{A}{4d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{B}{4d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)aA}{4d\left(a+b\right)^{3}}-\frac{3\ln\left(\sin\left(dx+c\right)-1\right)Ab}{4d\left(a+b\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d/(a+b)^2/(sin(d*x+c)-1)*A-1/4/d/(a+b)^2/(sin(d*x+c)-1)*B-1/4/d/(a+b)^
3*ln(sin(d*x+c)-1)*a*A-3/4/d/(a+b)^3*ln(sin(d*x+c)-1)*A*b-1/2/d/(a+b)^3*ln(
sin(d*x+c)-1)*B*b-1/d*b^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))*A+1/d*b^2/(a+b)^
2/(a-b)^2/(a+b*sin(d*x+c))*a*B+4/d*b^3/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))*A
*a-3/d*b^2/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))*B*a^2-1/d*b^4/(a+b)^3/(a-b)^3
*ln(a+b*sin(d*x+c))*B-1/4/d/(a-b)^2/(1+sin(d*x+c))*A+1/4/d/(a-b)^2/(1+sin(d
*x+c))*B+1/4/d/(a-b)^3*ln(1+sin(d*x+c))*a*A-3/4/d/(a-b)^3*ln(1+sin(d*x+c))*
A*b+1/2/d/(a-b)^3*ln(1+sin(d*x+c))*B*b
```

maxima [A] time = 0.52, size = 346, normalized size = 1.52

$$-\frac{\frac{4\left(3Ba^{2}b^{2}-4Aab^{3}+Bb^{4}\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}}-\frac{\left(Aa-\left(3A-2B\right)b\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}-3a^{2}b+3ab^{2}-b^{3}}+\frac{\left(Aa+\left(3A+2B\right)b\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}+3a^{2}b+3ab^{2}+b^{3}}-\frac{2\left(Ba^{3}-2Aa^{2}b+3B\right.}{a^{5}-2a^{3}b^{2}+ab^{4}-\left(a^{4}\right.}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/4*(4*(3*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(b*sin(d*x + c) + a)/(a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6) - (A*a - (3*A - 2*B)*b)*log(sin(d*x + c) + 1)/(a
^3 - 3*a^2*b + 3*a*b^2 - b^3) + (A*a + (3*A + 2*B)*b)*log(sin(d*x + c) - 1)
/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(B*a^3 - 2*A*a^2*b + 3*B*a*b^2 - 2*A*b
^3 + (A*a^2*b - 4*B*a*b^2 + 3*A*b^3)*sin(d*x + c)^2 + (A*a^3 - B*a^2*b - A*
a*b^2 + B*b^3)*sin(d*x + c))/(a^5 - 2*a^3*b^2 + a*b^4 - (a^4*b - 2*a^2*b^3
+ b^5)*sin(d*x + c)^3 - (a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c)^2 + (a^4*b -
 2*a^2*b^3 + b^5)*sin(d*x + c)))/d
```

mupad [B] time = 12.66, size = 327, normalized size = 1.43

$$\frac{\frac{\sin(c+d\,x)^2\,\left(A\,a^2\,b-4\,B\,a\,b^2+3\,A\,b^3\right)}{2\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{-B\,a^3+2\,A\,a^2\,b-3\,B\,a\,b^2+2\,A\,b^3}{2\,{\left(a^2-b^2\right)}^2}+\frac{\sin(c+d\,x)\,\left(A\,a-B\,b\right)}{2\,\left(a^2-b^2\right)}}{d\,\left(-b\,{\sin\left(c+d\,x\right)}^3-a\,{\sin\left(c+d\,x\right)}^2+b\,\sin\left(c+d\,x\right)+a\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(A\,a+b\,\left(3\,A+\right.\right.}{d\,\left(4\,a^3+12\,a^2\,b+12\,a\,b^2+4\,b^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^3*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((sin(c + d*x)^2*(3*A*b^3 + A*a^2*b - 4*B*a*b^2))/(2*(a^4 + b^4 - 2*a^2*b^2
)) - (2*A*b^3 - B*a^3 + 2*A*a^2*b - 3*B*a*b^2)/(2*(a^2 - b^2)^2) + (sin(c +
 d*x)*(A*a - B*b))/(2*(a^2 - b^2)))/(d*(a + b*sin(c + d*x) - a*sin(c + d*x)
^2 - b*sin(c + d*x)^3)) - (log(sin(c + d*x) - 1)*(A*a + b*(3*A + 2*B)))/(d*
(12*a*b^2 + 12*a^2*b + 4*a^3 + 4*b^3)) + (log(sin(c + d*x) + 1)*(A*a - b*(3
*A - 2*B)))/(d*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3)) - (log(a + b*sin(c +
d*x))*(B*b^4 + 3*B*a^2*b^2 - 4*A*a*b^3))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*
b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sin(c + dx))\sec^3(c + dx)}{(a + b\sin(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sin(c + d*x))*sec(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

3.1558 $\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=372

$$-\frac{\left(3a^2A+2ab(6A+B)+b^2(15A+8B)\right)\log(1-\sin(c+dx))}{16d(a+b)^4}+\frac{\left(3a^2A-2ab(6A-B)+b^2(15A-8B)\right)\log(\sin($$

```
[Out] -1/16*(3*a^2*A+2*a*b*(6*A+B)+b^2*(15*A+8*B))*ln(1-sin(d*x+c))/(a+b)^4/d+1/1
6*(3*a^2*A+b^2*(15*A-8*B)-2*a*b*(6*A-B))*ln(1+sin(d*x+c))/(a-b)^4/d-b^4*(6*
A*a*b-5*B*a^2-B*b^2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^4/d-1/8*b*(3*A*a^4-12*A*a
^2*b^2-15*A*b^4+2*B*a^3*b+22*B*a*b^3)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))-1/4*se
c(d*x+c)^4*(A*b-a*B-(A*a-B*b)*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))+1/8*
sec(d*x+c)^2*(b*(A*a^2+5*A*b^2-6*B*a*b)+(3*A*a^3-9*A*a*b^2+2*B*a^2*b+4*B*b^
3)*sin(d*x+c))/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.56, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2837, 823, 801}

$$-\frac{b\left(-12a^2Ab^2+3a^4A+2a^3bB+22ab^3B-15Ab^4\right)}{8d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{b^4\left(-5a^2B+6aAb-b^2B\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{\left(3a^2A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((3*a^2*A + 2*a*b*(6*A + B) + b^2*(15*A + 8*B))*Log[1 - Sin[c + d*x]])/(16
*(a + b)^4*d) + ((3*a^2*A + b^2*(15*A - 8*B) - 2*a*b*(6*A - B))*Log[1 + Sin
[c + d*x]])/(16*(a - b)^4*d) - (b^4*(6*a*A*b - 5*a^2*B - b^2*B)*Log[a + b*S
in[c + d*x]])/((a^2 - b^2)^4*d) - (b*(3*a^4*A - 12*a^2*A*b^2 - 15*A*b^4 + 2
*a^3*b*B + 22*a*b^3*B))/(8*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (Sec[c +
 d*x]^4*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(4*(a^2 - b^2)*d*(a + b*Sin
[c + d*x])) + (Sec[c + d*x]^2*(b*(a^2*A + 5*A*b^2 - 6*a*b*B) + (3*a^3*A - 9
*a*A*b^2 + 2*a^2*b*B + 4*b^3*B)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d*(a + b*Si
n[c + d*x]))
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{b^3\,\text{Subst}\left(\int \frac{-3a^2A+5\ldots}{(a\ldots}\right.}{\ldots}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^2(c+dx)\left(b\left(a^2A\ldots\right.\right.}{\ldots}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^2(c+dx)\left(b\left(a^2A\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(3a^2A+2ab(6A+B)+b^2(15A+8B)\right)\log(1-\sin(c+dx))}{16(a+b)^4d} + \frac{\left(3a^2A\ldots\right.}{\ldots}$$

Mathematica [A] time = 4.28, size = 370, normalized size = 0.99

$$\frac{\frac{2(b^2-a^2)\sec^4(c+dx)((bB-aA)\sin(c+dx)-aB+Ab)}{a+b\sin(c+dx)}-\frac{(3a^3A+2a^2bB-9aAb^2+4b^3B)((a-b)\log(1-\sin(c+dx))-(a+b)\log(\sin(c+dx)+1)+2b\log(a+b\sin}{(a-b)(a+b)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-(((3*a^3*A - 9*a*A*b^2 + 2*a^2*b*B + 4*b^3*B)*((a - b)*Log[1 - Sin[c + d*
x]] - (a + b)*Log[1 + Sin[c + d*x]] + 2*b*Log[a + b*Sin[c + d*x]]))/((a - b
)*(a + b))) + (2*(-a^2 + b^2)*Sec[c + d*x]^4*(A*b - a*B + (-(a*A) + b*B)*Si
n[c + d*x]))/(a + b*Sin[c + d*x]) + (Sec[c + d*x]^2*(b*(a^2*A + 5*A*b^2 - 6
*a*b*B) + (3*a^3*A - 9*a*A*b^2 + 2*a^2*b*B + 4*b^3*B)*Sin[c + d*x]))/(a + b
*Sin[c + d*x]) + b*(-3*a^4*A + 12*a^2*A*b^2 + 15*A*b^4 - 2*a^3*b*B - 22*a*b
^3*B)*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*
(a - b)^2*b) - (2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a^
2 - b^2)*(a + b*Sin[c + d*x]))))/(8*(a^2 - b^2)^2*d)
```

fricas [B] time = 3.30, size = 881, normalized size = 2.37

$$\frac{4\,Ba^7-4\,Aa^6b-12\,Ba^5b^2+12\,Aa^4b^3+12\,Ba^3b^4-12\,Aa^2b^5-4\,Bab^6+4\,Ab^7-2\left(3\,Aa^6b+2\,Ba^5b^2-15\,Aa^4b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/16*(4*B*a^7 - 4*A*a^6*b - 12*B*a^5*b^2 + 12*A*a^4*b^3 + 12*B*a^3*b^4 - 12
*A*a^2*b^5 - 4*B*a*b^6 + 4*A*b^7 - 2*(3*A*a^6*b + 2*B*a^5*b^2 - 15*A*a^4*b^
3 + 20*B*a^3*b^4 - 3*A*a^2*b^5 - 22*B*a*b^6 + 15*A*b^7)*cos(d*x + c)^4 + 2*
(A*a^6*b - 6*B*a^5*b^2 + 3*A*a^4*b^3 + 12*B*a^3*b^4 - 9*A*a^2*b^5 - 6*B*a*b
^6 + 5*A*b^7)*cos(d*x + c)^2 + 16*((5*B*a^2*b^5 - 6*A*a*b^6 + B*b^7)*cos(d*
x + c)^4*sin(d*x + c) + (5*B*a^3*b^4 - 6*A*a^2*b^5 + B*a*b^6)*cos(d*x + c)^
4)*log(b*sin(d*x + c) + a) + ((3*A*a^6*b + 2*B*a^5*b^2 - 15*A*a^4*b^3 - 20*
B*a^3*b^4 + 5*(9*A - 8*B)*a^2*b^5 + 6*(8*A - 5*B)*a*b^6 + (15*A - 8*B)*b^7)
*cos(d*x + c)^4*sin(d*x + c) + (3*A*a^7 + 2*B*a^6*b - 15*A*a^5*b^2 - 20*B*a
^4*b^3 + 5*(9*A - 8*B)*a^3*b^4 + 6*(8*A - 5*B)*a^2*b^5 + (15*A - 8*B)*a*b^6
)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - ((3*A*a^6*b + 2*B*a^5*b^2 - 15*A*
a^4*b^3 - 20*B*a^3*b^4 + 5*(9*A + 8*B)*a^2*b^5 - 6*(8*A + 5*B)*a*b^6 + (15*
A + 8*B)*b^7)*cos(d*x + c)^4*sin(d*x + c) + (3*A*a^7 + 2*B*a^6*b - 15*A*a^5
*b^2 - 20*B*a^4*b^3 + 5*(9*A + 8*B)*a^3*b^4 - 6*(8*A + 5*B)*a^2*b^5 + (15*A
 + 8*B)*a*b^6)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) + 2*(2*A*a^7 - 2*B*a^
```

```
6*b - 6*A*a^5*b^2 + 6*B*a^4*b^3 + 6*A*a^3*b^4 - 6*B*a^2*b^5 - 2*A*a*b^6 + 2
*B*b^7 + (3*A*a^7 + 2*B*a^6*b - 15*A*a^5*b^2 + 21*A*a^3*b^4 - 6*B*a^2*b^5 -
 9*A*a*b^6 + 4*B*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3 + 6
*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^4*sin(d*x + c) + (a^9 - 4*a^7*b^
2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^4)
```

giac [B] time = 0.34, size = 761, normalized size = 2.05

$$\frac{\frac{16\left(5\,Ba^{2}b^{5}-6\,Aab^{6}+Bb^{7}\right)\log(|b\sin(dx+c)+a|)}{a^{8}b-4\,a^{6}b^{3}+6\,a^{4}b^{5}-4\,a^{2}b^{7}+b^{9}}-\frac{\left(3\,Aa^{2}+12\,Aab+2\,Bab+15\,Ab^{2}+8\,Bb^{2}\right)\log(|-\sin(dx+c)+1|)}{a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}}+\frac{\left(3\,Aa^{2}-12\,Aab+2\,Bab+15\,Ab^{2}-8\,B\right.}{a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/16*(16*(5*B*a^2*b^5 - 6*A*a*b^6 + B*b^7)*log(abs(b*sin(d*x + c) + a))/(a^
8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9) - (3*A*a^2 + 12*A*a*b + 2*B*
a*b + 15*A*b^2 + 8*B*b^2)*log(abs(-sin(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4) + (3*A*a^2 - 12*A*a*b + 2*B*a*b + 15*A*b^2 - 8*B*b^2
)*log(abs(-sin(d*x + c) - 1))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) -
 16*(5*B*a^2*b^5*sin(d*x + c) - 6*A*a*b^6*sin(d*x + c) + B*b^7*sin(d*x + c)
 + 6*B*a^3*b^4 - 7*A*a^2*b^5 + A*b^7)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2
*b^6 + b^8)*(b*sin(d*x + c) + a)) + 2*(30*B*a^2*b^4*sin(d*x + c)^4 - 36*A*a
*b^5*sin(d*x + c)^4 + 6*B*b^6*sin(d*x + c)^4 - 3*A*a^6*sin(d*x + c)^3 - 2*B
*a^5*b*sin(d*x + c)^3 + 15*A*a^4*b^2*sin(d*x + c)^3 - 12*B*a^3*b^3*sin(d*x
+ c)^3 - 5*A*a^2*b^4*sin(d*x + c)^3 + 14*B*a*b^5*sin(d*x + c)^3 - 7*A*b^6*s
in(d*x + c)^3 + 12*B*a^4*b^2*sin(d*x + c)^2 - 16*A*a^3*b^3*sin(d*x + c)^2 -
 68*B*a^2*b^4*sin(d*x + c)^2 + 88*A*a*b^5*sin(d*x + c)^2 - 16*B*b^6*sin(d*x
 + c)^2 + 5*A*a^6*sin(d*x + c) - 2*B*a^5*b*sin(d*x + c) - 17*A*a^4*b^2*sin(
d*x + c) + 20*B*a^3*b^3*sin(d*x + c) + 3*A*a^2*b^4*sin(d*x + c) - 18*B*a*b^
5*sin(d*x + c) + 9*A*b^6*sin(d*x + c) + 2*B*a^6 - 4*A*a^5*b - 14*B*a^4*b^2
+ 24*A*a^3*b^3 + 36*B*a^2*b^4 - 56*A*a*b^5 + 12*B*b^6)/((a^8 - 4*a^6*b^2 +
6*a^4*b^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)^2))/d
```

maple [A] time = 0.82, size = 675, normalized size = 1.81

$$-\frac{\ln\left(\sin\left(dx+c\right)-1\right)Bab}{8d\left(a+b\right)^{4}}+\frac{b^{6}\ln\left(a+b\sin\left(dx+c\right)\right)B}{d\left(a+b\right)^{4}\left(a-b\right)^{4}}+\frac{b^{5}A}{d\left(a+b\right)^{3}\left(a-b\right)^{3}\left(a+b\sin\left(dx+c\right)\right)}-\frac{3\ln\left(1+\sin\left(dx+\right.\right.}{4d\left(a-b\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/8/d/(a+b)^4*ln(sin(d*x+c)-1)*B*a*b+1/d*b^6/(a+b)^4/(a-b)^4*ln(a+b*sin(d*
x+c))*B+1/d*b^5/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))*A-3/4/d/(a-b)^4*ln(1+sin(d
```

```
*x+c))*A*a*b+1/16/d/(a-b)^3/(1+sin(d*x+c))*a*B-5/16/d/(a-b)^3/(1+sin(d*x+c)
)*B*b+3/16/d/(a-b)^4*ln(1+sin(d*x+c))*a^2*A+1/16/d/(a+b)^2/(sin(d*x+c)-1)^2
*A+1/16/d/(a+b)^2/(sin(d*x+c)-1)^2*B-1/16/d/(a-b)^2/(1+sin(d*x+c))^2*A+1/16
/d/(a-b)^2/(1+sin(d*x+c))^2*B-6/d*b^5/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))*A*
a+5/d*b^4/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))*B*a^2-1/d*b^4/(a+b)^3/(a-b)^3/
(a+b*sin(d*x+c))*a*B+1/8/d/(a-b)^4*ln(1+sin(d*x+c))*B*a*b-3/4/d/(a+b)^4*ln(
sin(d*x+c)-1)*A*a*b+7/16/d/(a-b)^3/(1+sin(d*x+c))*A*b+15/16/d/(a-b)^4*ln(1+
sin(d*x+c))*A*b^2-1/2/d/(a-b)^4*ln(1+sin(d*x+c))*B*b^2-3/16/d/(a+b)^3/(sin(
d*x+c)-1)*a*A-7/16/d/(a+b)^3/(sin(d*x+c)-1)*A*b-1/16/d/(a+b)^3/(sin(d*x+c)-
1)*a*B-5/16/d/(a+b)^3/(sin(d*x+c)-1)*B*b-3/16/d/(a+b)^4*ln(sin(d*x+c)-1)*a^
2*A-15/16/d/(a+b)^4*ln(sin(d*x+c)-1)*A*b^2-1/2/d/(a+b)^4*ln(sin(d*x+c)-1)*B
*b^2-3/16/d/(a-b)^3/(1+sin(d*x+c))*a*A
```

maxima [A] time = 0.40, size = 659, normalized size = 1.77

$$\frac{\frac{16\left(5\,Ba^2b^4-6\,Aab^5+Bb^6\right)\log(b\sin(dx+c)+a)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}+\frac{\left(3\,Aa^2-2\,(6\,A-B)ab+(15\,A-8\,B)b^2\right)\log(\sin(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{\left(3\,Aa^2+2\,(6\,A+B)ab+(15\,A+8\,B)b^2\right)\log}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/16*(16*(5*B*a^2*b^4 - 6*A*a*b^5 + B*b^6)*log(b*sin(d*x + c) + a)/(a^8 - 4
*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (3*A*a^2 - 2*(6*A - B)*a*b + (15*
A - 8*B)*b^2)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
b^4) - (3*A*a^2 + 2*(6*A + B)*a*b + (15*A + 8*B)*b^2)*log(sin(d*x + c) - 1)
/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(2*B*a^5 - 4*A*a^4*b - 12*
B*a^3*b^2 + 20*A*a^2*b^3 - 14*B*a*b^4 + 8*A*b^5 - (3*A*a^4*b + 2*B*a^3*b^2
- 12*A*a^2*b^3 + 22*B*a*b^4 - 15*A*b^5)*sin(d*x + c)^4 - (3*A*a^5 + 2*B*a^4
*b - 12*A*a^3*b^2 + 2*B*a^2*b^3 + 9*A*a*b^4 - 4*B*b^5)*sin(d*x + c)^3 + (5*
A*a^4*b + 10*B*a^3*b^2 - 28*A*a^2*b^3 + 38*B*a*b^4 - 25*A*b^5)*sin(d*x + c)
^2 + (5*A*a^5 - 16*A*a^3*b^2 + 6*B*a^2*b^3 + 11*A*a*b^4 - 6*B*b^5)*sin(d*x
+ c))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5
 - b^7)*sin(d*x + c)^5 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)
^4 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)^3 - 2*(a^7 - 3*a^
5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)^2 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5
- b^7)*sin(d*x + c)))/d
```

mupad [B] time = 13.46, size = 615, normalized size = 1.65

$$\frac{\frac{B\,a^5-2\,A\,a^4\,b-6\,B\,a^3\,b^2+10\,A\,a^2\,b^3-7\,B\,a\,b^4+4\,A\,b^5}{4\,\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\sin(c+d\,x)}^4\,\left(3\,A\,a^4\,b+2\,B\,a^3\,b^2-12\,A\,a^2\,b^3+22\,B\,a\,b^4-15\,A\,b^5\right)}{8\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{\sin(c+d\,x)\,\left(5\,A\,a^3-11\,A\,a\,b\right.}{8\,\left(a^4-2\,a^2\,b^2+b^4\right.}}{d\,\left(b\,{\sin\left(c+d\,x\right)}^5+a\,{\sin\left(c+d\,x\right)}^4-2\,b\,{\sin\left(c+d\,x\right)}^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^5*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((4*A*b^5 + B*a^5 + 10*A*a^2*b^3 - 6*B*a^3*b^2 - 2*A*a^4*b - 7*B*a*b^4)/(4*
(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (sin(c + d*x)^4*(2*B*a^3*b^2 - 12*A*
a^2*b^3 - 15*A*b^5 + 3*A*a^4*b + 22*B*a*b^4))/(8*(a^6 - b^6 + 3*a^2*b^4 - 3
*a^4*b^2)) + (sin(c + d*x)*(5*A*a^3 + 6*B*b^3 - 11*A*a*b^2))/(8*(a^4 + b^4
- 2*a^2*b^2)) - (sin(c + d*x)^3*(3*A*a^3 + 4*B*b^3 - 9*A*a*b^2 + 2*B*a^2*b)
)/(8*(a^4 + b^4 - 2*a^2*b^2)) + (sin(c + d*x)^2*(10*B*a^3*b^2 - 28*A*a^2*b^
3 - 25*A*b^5 + 5*A*a^4*b + 38*B*a*b^4))/(8*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b
^2)))/(d*(a + b*sin(c + d*x) - 2*a*sin(c + d*x)^2 + a*sin(c + d*x)^4 - 2*b*
sin(c + d*x)^3 + b*sin(c + d*x)^5)) + (log(a + b*sin(c + d*x))*(B*b^6 + 5*B
*a^2*b^4 - 6*A*a*b^5))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
- (log(sin(c + d*x) - 1)*(3*A*a^2 + b^2*(15*A + 8*B) + a*b*(12*A + 2*B)))/(
d*(64*a*b^3 + 64*a^3*b + 16*a^4 + 16*b^4 + 96*a^2*b^2)) + (log(sin(c + d*x)
 + 1)*(3*A*a^2 + b^2*(15*A - 8*B) - a*b*(12*A - 2*B)))/(d*(16*a^4 - 64*a^3*
b - 64*a*b^3 + 16*b^4 + 96*a^2*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1559 $\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=550

$$-\frac{\sec^6(c+dx)(-(aA-bB)\sin(c+dx)-aB+Ab)}{6d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{b^6\left(-7a^2B+8aAb-b^2B\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^5}-\frac{\left(5a^3A+\right.}{}$$

```
[Out] -1/32*(5*a^3*A+a^2*b*(25*A+2*B)+a*b^2*(47*A+10*B)+b^3*(35*A+16*B))*ln(1-sin
(d*x+c))/(a+b)^5/d+1/32*(5*a^3*A-b^3*(35*A-16*B)+a*b^2*(47*A-10*B)-a^2*(25*
A*b-2*B*b))*ln(1+sin(d*x+c))/(a-b)^5/d+b^6*(8*A*a*b-7*B*a^2-B*b^2)*ln(a+b*s
in(d*x+c))/(a^2-b^2)^5/d-1/16*b*(5*A*a^6-23*A*a^4*b^2+47*A*a^2*b^4+35*A*b^6
+2*B*a^5*b-12*B*a^3*b^3-54*B*a*b^5)/(a^2-b^2)^4/d/(a+b*sin(d*x+c))-1/6*sec(
d*x+c)^6*(A*b-a*B-(A*a-B*b)*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))+1/24*s
ec(d*x+c)^4*(b*(A*a^2+7*A*b^2-8*B*a*b)+(5*A*a^3-13*A*a*b^2+2*B*a^2*b+6*B*b^
3)*sin(d*x+c))/(a^2-b^2)^2/d/(a+b*sin(d*x+c))+1/48*sec(d*x+c)^2*(b*(5*A*a^4
-18*A*a^2*b^2-35*A*b^4+2*B*a^3*b+46*B*a*b^3)+3*(5*A*a^5-18*A*a^3*b^2+29*A*a
*b^4+2*B*a^4*b-10*B*a^2*b^3-8*B*b^5)*sin(d*x+c))/(a^2-b^2)^3/d/(a+b*sin(d*x
+c))
```

Rubi [A] time = 0.93, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2837, 823, 801}

$$-\frac{b\left(-23a^4Ab^2+47a^2Ab^4+5a^6A-12a^3b^3B+2a^5bB-54ab^5B+35Ab^6\right)}{16d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}+\frac{b^6\left(-7a^2B+8aAb-b^2B\right)\log(a+b}{d\left(a^2-b^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((5*a^3*A + a^2*b*(25*A + 2*B) + a*b^2*(47*A + 10*B) + b^3*(35*A + 16*B))*
Log[1 - Sin[c + d*x]])/(32*(a + b)^5*d) + ((5*a^3*A - b^3*(35*A - 16*B) + a
*b^2*(47*A - 10*B) - a^2*(25*A*b - 2*b*B))*Log[1 + Sin[c + d*x]])/(32*(a -
b)^5*d) + (b^6*(8*a*A*b - 7*a^2*B - b^2*B)*Log[a + b*Sin[c + d*x]])/((a^2 -
 b^2)^5*d) - (b*(5*a^6*A - 23*a^4*A*b^2 + 47*a^2*A*b^4 + 35*A*b^6 + 2*a^5*b
*B - 12*a^3*b^3*B - 54*a*b^5*B))/(16*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))
- (Sec[c + d*x]^6*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(6*(a^2 - b^2)*d*
(a + b*Sin[c + d*x])) + (Sec[c + d*x]^4*(b*(a^2*A + 7*A*b^2 - 8*a*b*B) + (5
*a^3*A - 13*a*A*b^2 + 2*a^2*b*B + 6*b^3*B)*Sin[c + d*x]))/(24*(a^2 - b^2)^2
*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^2*(b*(5*a^4*A - 18*a^2*A*b^2 - 35*
A*b^4 + 2*a^3*b*B + 46*a*b^3*B) + 3*(5*a^5*A - 18*a^3*A*b^2 + 29*a*A*b^4 +
2*a^4*b*B - 10*a^2*b^3*B - 8*b^5*B)*Sin[c + d*x]))/(48*(a^2 - b^2)^3*d*(a +
 b*Sin[c + d*x]))
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{b^7\,\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2\left(b^2-x^2\right)^4}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{b^5\,\text{Subst}\left(\int \frac{-5a^2A+\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^4(c+dx)\left(b\left(a^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^4(c+dx)\left(b\left(a^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^4(c+dx)\left(b\left(a^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(5a^3A+a^2b(25A+2B)+ab^2(47A+10B)+b^3(35A+16B)\right)\log(1\cdots}{32(a+b)^5d}$$

Mathematica [A] time = 6.21, size = 766, normalized size = 1.39

$$b^7\left(\frac{\frac{\left(6a\left(5a^5A+2a^4bB-18a^3Ab^2-10a^2b^3B+29aAb^4-8b^5B\right)-3\left(5a^6A+2a^5bB-13a^4Ab^2-8a^3b^3B+11a^2Ab^4+38ab^5B-35Ab^6\right)\right)\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\log(1-\sin(c+dx))}{2b(a+b)^2}+\frac{\log\cdots}{\cdots}\right.}{2b^2\left(b^2-a^2\right)}}{\cdots}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (b^7*(-1/6*(Sec[c + d*x]^6*(-(A*b^2) + a*b*B - b*(-(a*A) + b*B)*Sin[c + d*x
]))/(b^8*(-a^2 + b^2)*(a + b*Sin[c + d*x])) + (-1/4*(Sec[c + d*x]^4*(-6*a*b
^2*(a*A - b*B) - b^2*(-5*a^2*A + 7*A*b^2 - 2*a*b*B) - b*(-6*b^2*(a*A - b*B)
```

```
 - a*(-5*a^2*A + 7*A*b^2 - 2*a*b*B))*Sin[c + d*x]))/(b^6*(-a^2 + b^2)*(a +
b*Sin[c + d*x])) + (-1/2*(Sec[c + d*x]^2*(4*a*b^2*(5*a^3*A - 13*a*A*b^2 + 2
*a^2*b*B + 6*b^3*B) - b^2*(15*a^4*A - 34*a^2*A*b^2 + 35*A*b^4 + 6*a^3*b*B -
 22*a*b^3*B) - b*(4*b^2*(5*a^3*A - 13*a*A*b^2 + 2*a^2*b*B + 6*b^3*B) - a*(1
5*a^4*A - 34*a^2*A*b^2 + 35*A*b^4 + 6*a^3*b*B - 22*a*b^3*B))*Sin[c + d*x]))
/(b^4*(-a^2 + b^2)*(a + b*Sin[c + d*x])) + (-6*(5*a^5*A - 18*a^3*A*b^2 + 29
*a*A*b^4 + 2*a^4*b*B - 10*a^2*b^3*B - 8*b^5*B)*(-1/2*Log[1 - Sin[c + d*x]]/
(b*(a + b)) + Log[1 + Sin[c + d*x]]/(2*(a - b)*b) - Log[a + b*Sin[c + d*x]]
/(a^2 - b^2)) + (6*a*(5*a^5*A - 18*a^3*A*b^2 + 29*a*A*b^4 + 2*a^4*b*B - 10*
a^2*b^3*B - 8*b^5*B) - 3*(5*a^6*A - 13*a^4*A*b^2 + 11*a^2*A*b^4 - 35*A*b^6
+ 2*a^5*b*B - 8*a^3*b^3*B + 38*a*b^5*B))*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a
+ b)^2) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*b) - (2*a*Log[a + b*Sin[c + d*
x]])/((a - b)^2*(a + b)^2) + 1/((a^2 - b^2)*(a + b*Sin[c + d*x]))))/(2*b^2*
(-a^2 + b^2)))/(4*b^2*(-a^2 + b^2)))/(6*b^2*(-a^2 + b^2))))/d
```

<u>**fricas**</u> **[B]** time = 8.22, size = 1244, normalized size = 2.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/96*(16*B*a^9 - 16*A*a^8*b - 64*B*a^7*b^2 + 64*A*a^6*b^3 + 96*B*a^5*b^4 -
96*A*a^4*b^5 - 64*B*a^3*b^6 + 64*A*a^2*b^7 + 16*B*a*b^8 - 16*A*b^9 - 6*(5*A
*a^8*b + 2*B*a^7*b^2 - 28*A*a^6*b^3 - 14*B*a^5*b^4 + 70*A*a^4*b^5 - 42*B*a^
3*b^6 - 12*A*a^2*b^7 + 54*B*a*b^8 - 35*A*b^9)*cos(d*x + c)^6 + 2*(5*A*a^8*b
 + 2*B*a^7*b^2 - 28*A*a^6*b^3 + 42*B*a^5*b^4 + 6*A*a^4*b^5 - 90*B*a^3*b^6 +
 52*A*a^2*b^7 + 46*B*a*b^8 - 35*A*b^9)*cos(d*x + c)^4 + 4*(A*a^8*b - 8*B*a^
7*b^2 + 4*A*a^6*b^3 + 24*B*a^5*b^4 - 18*A*a^4*b^5 - 24*B*a^3*b^6 + 20*A*a^2
*b^7 + 8*B*a*b^8 - 7*A*b^9)*cos(d*x + c)^2 - 96*((7*B*a^2*b^7 - 8*A*a*b^8 +
 B*b^9)*cos(d*x + c)^6*sin(d*x + c) + (7*B*a^3*b^6 - 8*A*a^2*b^7 + B*a*b^8)
*cos(d*x + c)^6)*log(b*sin(d*x + c) + a) + 3*((5*A*a^8*b + 2*B*a^7*b^2 - 28
*A*a^6*b^3 - 14*B*a^5*b^4 + 70*A*a^4*b^5 + 70*B*a^3*b^6 - 28*(5*A - 4*B)*a^
2*b^7 - 2*(64*A - 35*B)*a*b^8 - (35*A - 16*B)*b^9)*cos(d*x + c)^6*sin(d*x +
 c) + (5*A*a^9 + 2*B*a^8*b - 28*A*a^7*b^2 - 14*B*a^6*b^3 + 70*A*a^5*b^4 + 7
0*B*a^4*b^5 - 28*(5*A - 4*B)*a^3*b^6 - 2*(64*A - 35*B)*a^2*b^7 - (35*A - 16
*B)*a*b^8)*cos(d*x + c)^6)*log(sin(d*x + c) + 1) - 3*((5*A*a^8*b + 2*B*a^7*
b^2 - 28*A*a^6*b^3 - 14*B*a^5*b^4 + 70*A*a^4*b^5 + 70*B*a^3*b^6 - 28*(5*A +
 4*B)*a^2*b^7 + 2*(64*A + 35*B)*a*b^8 - (35*A + 16*B)*b^9)*cos(d*x + c)^6*s
in(d*x + c) + (5*A*a^9 + 2*B*a^8*b - 28*A*a^7*b^2 - 14*B*a^6*b^3 + 70*A*a^5
*b^4 + 70*B*a^4*b^5 - 28*(5*A + 4*B)*a^3*b^6 + 2*(64*A + 35*B)*a^2*b^7 - (3
5*A + 16*B)*a*b^8)*cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(8*A*a^9 - 8*
B*a^8*b - 32*A*a^7*b^2 + 32*B*a^6*b^3 + 48*A*a^5*b^4 - 48*B*a^4*b^5 - 32*A*
a^3*b^6 + 32*B*a^2*b^7 + 8*A*a*b^8 - 8*B*b^9 + 3*(5*A*a^9 + 2*B*a^8*b - 28*
```

```
A*a^7*b^2 - 14*B*a^6*b^3 + 70*A*a^5*b^4 + 14*B*a^4*b^5 - 76*A*a^3*b^6 + 6*B
*a^2*b^7 + 29*A*a*b^8 - 8*B*b^9)*cos(d*x + c)^4 + 2*(5*A*a^9 + 2*B*a^8*b -
28*A*a^7*b^2 + 54*A*a^5*b^4 - 12*B*a^4*b^5 - 44*A*a^3*b^6 + 16*B*a^2*b^7 +
13*A*a*b^8 - 6*B*b^9)*cos(d*x + c)^2)*sin(d*x + c))/((a^10*b - 5*a^8*b^3 +
10*a^6*b^5 - 10*a^4*b^7 + 5*a^2*b^9 - b^11)*d*cos(d*x + c)^6*sin(d*x + c) +
 (a^11 - 5*a^9*b^2 + 10*a^7*b^4 - 10*a^5*b^6 + 5*a^3*b^8 - a*b^10)*d*cos(d*
x + c)^6)
```

giac [B] time = 0.43, size = 1185, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/96*(96*(7*B*a^2*b^7 - 8*A*a*b^8 + B*b^9)*log(abs(b*sin(d*x + c) + a))/(a
^10*b - 5*a^8*b^3 + 10*a^6*b^5 - 10*a^4*b^7 + 5*a^2*b^9 - b^11) + 3*(5*A*a^
3 + 25*A*a^2*b + 2*B*a^2*b + 47*A*a*b^2 + 10*B*a*b^2 + 35*A*b^3 + 16*B*b^3)
*log(abs(-sin(d*x + c) + 1))/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a
*b^4 + b^5) - 3*(5*A*a^3 - 25*A*a^2*b + 2*B*a^2*b + 47*A*a*b^2 - 10*B*a*b^2
 - 35*A*b^3 + 16*B*b^3)*log(abs(-sin(d*x + c) - 1))/(a^5 - 5*a^4*b + 10*a^3
*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5) - 96*(7*B*a^2*b^7*sin(d*x + c) - 8*A*a*b
^8*sin(d*x + c) + B*b^9*sin(d*x + c) + 8*B*a^3*b^6 - 9*A*a^2*b^7 + A*b^9)/(
(a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*(b*sin(d*x
+ c) + a)) + 2*(308*B*a^2*b^6*sin(d*x + c)^6 - 352*A*a*b^7*sin(d*x + c)^6 +
 44*B*b^8*sin(d*x + c)^6 + 15*A*a^8*sin(d*x + c)^5 + 6*B*a^7*b*sin(d*x + c)
^5 - 84*A*a^6*b^2*sin(d*x + c)^5 - 42*B*a^5*b^3*sin(d*x + c)^5 + 210*A*a^4*
b^4*sin(d*x + c)^5 - 78*B*a^3*b^5*sin(d*x + c)^5 - 84*A*a^2*b^6*sin(d*x + c
)^5 + 114*B*a*b^7*sin(d*x + c)^5 - 57*A*b^8*sin(d*x + c)^5 + 120*B*a^4*b^4*
sin(d*x + c)^4 - 144*A*a^3*b^5*sin(d*x + c)^4 - 1020*B*a^2*b^6*sin(d*x + c)
^4 + 1200*A*a*b^7*sin(d*x + c)^4 - 156*B*b^8*sin(d*x + c)^4 - 40*A*a^8*sin(
d*x + c)^3 - 16*B*a^7*b*sin(d*x + c)^3 + 224*A*a^6*b^2*sin(d*x + c)^3 + 48*
B*a^5*b^3*sin(d*x + c)^3 - 480*A*a^4*b^4*sin(d*x + c)^3 + 240*B*a^3*b^5*sin
(d*x + c)^3 + 160*A*a^2*b^6*sin(d*x + c)^3 - 272*B*a*b^7*sin(d*x + c)^3 + 1
36*A*b^8*sin(d*x + c)^3 + 36*B*a^6*b^2*sin(d*x + c)^2 - 48*A*a^5*b^3*sin(d*
x + c)^2 - 300*B*a^4*b^4*sin(d*x + c)^2 + 384*A*a^3*b^5*sin(d*x + c)^2 + 11
28*B*a^2*b^6*sin(d*x + c)^2 - 1392*A*a*b^7*sin(d*x + c)^2 + 192*B*b^8*sin(d
*x + c)^2 + 33*A*a^8*sin(d*x + c) - 6*B*a^7*b*sin(d*x + c) - 156*A*a^6*b^2*
sin(d*x + c) + 42*B*a^5*b^3*sin(d*x + c) + 270*A*a^4*b^4*sin(d*x + c) - 210
*B*a^3*b^5*sin(d*x + c) - 60*A*a^2*b^6*sin(d*x + c) + 174*B*a*b^7*sin(d*x +
 c) - 87*A*b^8*sin(d*x + c) + 8*B*a^8 - 16*A*a^7*b - 52*B*a^6*b^2 + 96*A*a^
5*b^3 + 180*B*a^4*b^4 - 288*A*a^3*b^5 - 400*B*a^2*b^6 + 560*A*a*b^7 - 88*B*
b^8)/((a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*(sin(
d*x + c)^2 - 1)^3))/d
```

maple [B] time = 0.86, size = 1080, normalized size = 1.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -19/32/d/(a-b)^4/(1+sin(d*x+c))*A*b^2+1/16/d/(a+b)^3/(sin(d*x+c)-1)^2*a*A+1
/8/d/(a+b)^3/(sin(d*x+c)-1)^2*A*b+1/32/d/(a+b)^3/(sin(d*x+c)-1)^2*a*B+3/32/
d/(a+b)^3/(sin(d*x+c)-1)^2*B*b-5/32/d/(a+b)^4/(sin(d*x+c)-1)*a^2*A-19/32/d/
(a+b)^4/(sin(d*x+c)-1)*A*b^2-1/32/d/(a+b)^4/(sin(d*x+c)-1)*B*a^2-11/32/d/(a
+b)^4/(sin(d*x+c)-1)*B*b^2+1/32/d/(a-b)^4/(1+sin(d*x+c))*B*a^2+11/32/d/(a-b
)^4/(1+sin(d*x+c))*B*b^2+5/32/d/(a-b)^5*ln(1+sin(d*x+c))*a^3*A-35/32/d/(a-b
)^5*ln(1+sin(d*x+c))*b^3*A+1/2/d/(a-b)^5*ln(1+sin(d*x+c))*B*b^3-5/32/d/(a+b
)^5*ln(sin(d*x+c)-1)*a^3*A-35/32/d/(a+b)^5*ln(sin(d*x+c)-1)*b^3*A+9/16/d/(a
-b)^4/(1+sin(d*x+c))*A*a*b+8/d*b^7/(a+b)^5/(a-b)^5*ln(a+b*sin(d*x+c))*A*a-7
/d*b^6/(a+b)^5/(a-b)^5*ln(a+b*sin(d*x+c))*B*a^2+1/d*b^6/(a+b)^4/(a-b)^4/(a+
b*sin(d*x+c))*a*B-1/48/d/(a+b)^2/(sin(d*x+c)-1)^3*A-1/48/d/(a+b)^2/(sin(d*x
+c)-1)^3*B-1/48/d/(a-b)^2/(1+sin(d*x+c))^3*A+1/48/d/(a-b)^2/(1+sin(d*x+c))^
3*B+1/8/d/(a-b)^3/(1+sin(d*x+c))^2*A*b+1/32/d/(a-b)^3/(1+sin(d*x+c))^2*a*B-
3/32/d/(a-b)^3/(1+sin(d*x+c))^2*B*b-5/32/d/(a-b)^4/(1+sin(d*x+c))*a^2*A-1/2
/d/(a+b)^5*ln(sin(d*x+c)-1)*B*b^3-1/16/d/(a-b)^3/(1+sin(d*x+c))^2*a*A-3/16/
d/(a-b)^4/(1+sin(d*x+c))*B*a*b-25/32/d/(a-b)^5*ln(1+sin(d*x+c))*A*a^2*b-3/1
6/d/(a+b)^4/(sin(d*x+c)-1)*B*a*b-25/32/d/(a+b)^5*ln(sin(d*x+c)-1)*A*a^2*b-4
7/32/d/(a+b)^5*ln(sin(d*x+c)-1)*A*a*b^2-1/16/d/(a+b)^5*ln(sin(d*x+c)-1)*B*a
^2*b-9/16/d/(a+b)^4/(sin(d*x+c)-1)*A*a*b+47/32/d/(a-b)^5*ln(1+sin(d*x+c))*A
*a*b^2+1/16/d/(a-b)^5*ln(1+sin(d*x+c))*B*a^2*b-5/16/d/(a-b)^5*ln(1+sin(d*x+
c))*B*a*b^2-5/16/d/(a+b)^5*ln(sin(d*x+c)-1)*B*a*b^2-1/d*b^8/(a+b)^5/(a-b)^5
*ln(a+b*sin(d*x+c))*B-1/d*b^7/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))*A
```

maxima [B] time = 0.40, size = 1083, normalized size = 1.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/96*(96*(7*B*a^2*b^6 - 8*A*a*b^7 + B*b^8)*log(b*sin(d*x + c) + a)/(a^10 -
 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - 3*(5*A*a^3 - (25
*A - 2*B)*a^2*b + (47*A - 10*B)*a*b^2 - (35*A - 16*B)*b^3)*log(sin(d*x + c)
 + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5) + 3*(5*A*a^
3 + (25*A + 2*B)*a^2*b + (47*A + 10*B)*a*b^2 + (35*A + 16*B)*b^3)*log(sin(d
*x + c) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 2*
```

```
(8*B*a^7 - 16*A*a^6*b - 44*B*a^5*b^2 + 80*A*a^4*b^3 + 136*B*a^3*b^4 - 208*A
*a^2*b^5 + 92*B*a*b^6 - 48*A*b^7 + 3*(5*A*a^6*b + 2*B*a^5*b^2 - 23*A*a^4*b^
3 - 12*B*a^3*b^4 + 47*A*a^2*b^5 - 54*B*a*b^6 + 35*A*b^7)*sin(d*x + c)^6 + 3
*(5*A*a^7 + 2*B*a^6*b - 23*A*a^5*b^2 - 12*B*a^4*b^3 + 47*A*a^3*b^4 + 2*B*a^
2*b^5 - 29*A*a*b^6 + 8*B*b^7)*sin(d*x + c)^5 - 8*(5*A*a^6*b + 2*B*a^5*b^2 -
 23*A*a^4*b^3 - 19*B*a^3*b^4 + 55*A*a^2*b^5 - 55*B*a*b^6 + 35*A*b^7)*sin(d*
x + c)^4 - 4*(10*A*a^7 + 4*B*a^6*b - 46*A*a^5*b^2 - 17*B*a^4*b^3 + 86*A*a^3
*b^4 - 2*B*a^2*b^5 - 50*A*a*b^6 + 15*B*b^7)*sin(d*x + c)^3 + 3*(11*A*a^6*b
+ 10*B*a^5*b^2 - 57*A*a^4*b^3 - 76*B*a^3*b^4 + 161*A*a^2*b^5 - 126*B*a*b^6
+ 77*A*b^7)*sin(d*x + c)^2 + (33*A*a^7 + 2*B*a^6*b - 139*A*a^5*b^2 - 8*B*a^
4*b^3 + 227*A*a^3*b^4 - 38*B*a^2*b^5 - 121*A*a*b^6 + 44*B*b^7)*sin(d*x + c)
)/(a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8 - (a^8*b - 4*a^6*b^3 + 6
*a^4*b^5 - 4*a^2*b^7 + b^9)*sin(d*x + c)^7 - (a^9 - 4*a^7*b^2 + 6*a^5*b^4 -
 4*a^3*b^6 + a*b^8)*sin(d*x + c)^6 + 3*(a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a
^2*b^7 + b^9)*sin(d*x + c)^5 + 3*(a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 +
 a*b^8)*sin(d*x + c)^4 - 3*(a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9
)*sin(d*x + c)^3 - 3*(a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*sin(
d*x + c)^2 + (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*sin(d*x + c)
))/d
```

mupad [B] time = 14.38, size = 1024, normalized size = 1.86

$$\frac{\frac{\sin(c+d\,x)\left(33\,A\,a^5+2\,B\,a^4\,b-106\,A\,a^3\,b^2-6\,B\,a^2\,b^3+121\,A\,a\,b^4-44\,B\,b^5\right)}{48\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{\sin(c+d\,x)^5\left(5\,A\,a^5+2\,B\,a^4\,b-18\,A\,a^3\,b^2-10\,B\,a^2\,b^3+29\,A\,a\,b^4-8\,B\,b^5\right)}{16\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^7*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((sin(c + d*x)*(33*A*a^5 - 44*B*b^5 - 106*A*a^3*b^2 - 6*B*a^2*b^3 + 121*A*a
*b^4 + 2*B*a^4*b))/(48*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (sin(c + d*x)
^5*(5*A*a^5 - 8*B*b^5 - 18*A*a^3*b^2 - 10*B*a^2*b^3 + 29*A*a*b^4 + 2*B*a^4*
b))/(16*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (sin(c + d*x)^3*(10*A*a^5 -
15*B*b^5 - 36*A*a^3*b^2 - 13*B*a^2*b^3 + 50*A*a*b^4 + 4*B*a^4*b))/(12*(a^6
- b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (12*A*b^7 - 2*B*a^7 + 52*A*a^2*b^5 - 20*A
*a^4*b^3 - 34*B*a^3*b^4 + 11*B*a^5*b^2 + 4*A*a^6*b - 23*B*a*b^6)/(12*(a^2 -
 b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (sin(c + d*x)^6*(35*A*b^7 + 47
*A*a^2*b^5 - 23*A*a^4*b^3 - 12*B*a^3*b^4 + 2*B*a^5*b^2 + 5*A*a^6*b - 54*B*a
*b^6))/(16*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (sin(c + d*x)
^4*(35*A*b^7 + 55*A*a^2*b^5 - 23*A*a^4*b^3 - 19*B*a^3*b^4 + 2*B*a^5*b^2 + 5
*A*a^6*b - 55*B*a*b^6))/(6*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
 + (sin(c + d*x)^2*(77*A*b^7 + 161*A*a^2*b^5 - 57*A*a^4*b^3 - 76*B*a^3*b^4
+ 10*B*a^5*b^2 + 11*A*a^6*b - 126*B*a*b^6))/(16*(a^2 - b^2)*(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2)))/(d*(a + b*sin(c + d*x) - 3*a*sin(c + d*x)^2 + 3*a*si
```

```
n(c + d*x)^4 - a*sin(c + d*x)^6 - 3*b*sin(c + d*x)^3 + 3*b*sin(c + d*x)^5 -
 b*sin(c + d*x)^7)) - (log(a + b*sin(c + d*x))*(B*b^8 + 7*B*a^2*b^6 - 8*A*a
*b^7))/(d*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2))
- (log(sin(c + d*x) - 1)*(5*A*a^3 + b^3*(35*A + 16*B) + a^2*b*(25*A + 2*B)
+ a*b^2*(47*A + 10*B)))/(d*(160*a*b^4 + 160*a^4*b + 32*a^5 + 32*b^5 + 320*a
^2*b^3 + 320*a^3*b^2)) + (log(sin(c + d*x) + 1)*(5*A*a^3 - b^3*(35*A - 16*B
) - a^2*b*(25*A - 2*B) + a*b^2*(47*A - 10*B)))/(d*(160*a*b^4 - 160*a^4*b +
32*a^5 - 32*b^5 - 320*a^2*b^3 + 320*a^3*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.1560 $\int (g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=40

$$\text{Int}\left((A+B\sin(e+fx))(g\cos(e+fx))^{-m-1}(a+b\sin(e+fx))^m,x\right)$$

```
[Out] Unintegrable((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m)*(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x
]
```

```
[Out] Defer[Int][(g*Cos[e + f*x])^(-1 - m)*(a + b*Sin[e + f*x])^m*(A + B*Sin[e +
f*x]), x]
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A$$

Mathematica [A] time = 5.17, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m)*(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f
*x]),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(-1 - m)*(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f
*x]), x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{-m-1}\left(b\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algo
rithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^(-m - 1)*(b*sin(f*x + e) + a
)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{-m-1}\left(b\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algo
rithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^(-m - 1)*(b*sin(f*x + e) +
a)^m, x)
```

maple [A] time = 2.56, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-m}\left(a+b\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{-m-1}\left(b\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algo
rithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^(-m - 1)*(b*sin(f*x + e) +
a)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^m}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m)/(g*cos(e + f*x))^(m + 1),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m)/(g*cos(e + f*x))^(m + 1),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m)*(a+b*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1561 $\int \frac{(g\cos(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=330

$$\frac{g(g\cos(e+fx))^{p-1}\left(-\frac{d(1-\sin(e+fx))}{c+d\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{d(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{c+d}{c+d\sin(e+fx)},\frac{c-d}{c+d\sin(e+fx)}\right)}{f(1-p)(bc-ad)}-\frac{g(g\,c}{}$$

```
[Out] -g*AppellF1(1-p,1/2-1/2*p,1/2-1/2*p,2-p,(a-b)/(a+b*sin(f*x+e)),(a+b)/(a+b*s
in(f*x+e)))*(g*cos(f*x+e))^(-1+p)*(-b*(1-sin(f*x+e))/(a+b*sin(f*x+e)))^(1/2
-1/2*p)*(b*(1+sin(f*x+e))/(a+b*sin(f*x+e)))^(1/2-1/2*p)/(-a*d+b*c)/f/(1-p)+
g*AppellF1(1-p,1/2-1/2*p,1/2-1/2*p,2-p,(c-d)/(c+d*sin(f*x+e)),(c+d)/(c+d*si
n(f*x+e)))*(g*cos(f*x+e))^(-1+p)*(-d*(1-sin(f*x+e))/(c+d*sin(f*x+e)))^(1/2-
1/2*p)*(d*(1+sin(f*x+e))/(c+d*sin(f*x+e)))^(1/2-1/2*p)/(-a*d+b*c)/f/(1-p)
```

Rubi [A] time = 0.42, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2924, 2703}

$$\frac{g(g\cos(e+fx))^{p-1}\left(-\frac{d(1-\sin(e+fx))}{c+d\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{d(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{c+d}{c+d\sin(e+fx)},\frac{c-d}{c+d\sin(e+fx)}\right)}{f(1-p)(bc-ad)}-\frac{g(g\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((g*AppellF1[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (a + b)/(a + b*Sin[e + f*
x]), (a - b)/(a + b*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((b*(1 - Sin
[e + f*x]))/(a + b*Sin[e + f*x])))^((1 - p)/2)*((b*(1 + Sin[e + f*x]))/(a +
 b*Sin[e + f*x]))^((1 - p)/2))/((b*c - a*d)*f*(1 - p))) + (g*AppellF1[1 - p
, (1 - p)/2, (1 - p)/2, 2 - p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/(c + d
*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((d*(1 - Sin[e + f*x]))/(c + d*
Sin[e + f*x])))^((1 - p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(
(1 - p)/2))/((b*c - a*d)*f*(1 - p))
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
```

```
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 2924

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && Integer
sQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx = \int \left(\frac{b(g\cos(e+fx))^p}{(bc-ad)(a+b\sin(e+fx))} - \frac{d(g\cos(e+fx))^p}{(bc-ad)(c+d\sin(e+fx))}\right) d$$
$$= \frac{b\int \frac{(g\cos(e+fx))^p}{a+b\sin(e+fx)}\,dx}{bc-ad} - \frac{d\int \frac{(g\cos(e+fx))^p}{c+d\sin(e+fx)}\,dx}{bc-ad}$$
$$= -\frac{gF_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)(g\cos(e+f}{(bc-ad)f(1-p)}$$

Mathematica [B] time = 34.72, size = 5085, normalized size = 15.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
[Out] Result too large to show
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(g\cos\left(fx+e\right)\right)^p}{bd\cos\left(fx+e\right)^2-ac-bd-(bc+ad)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(g*cos(f*x + e))^p/(b*d*cos(f*x + e)^2 - a*c - b*d - (b*c + a*d)*
sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^p/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)), x
)
```

maple [F] time = 3.78, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}}{\left(a+b\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^p}{\left(a+b\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.1562 $\int \frac{(g\cos(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=508

$$-\frac{bg(g\cos(e+fx))^{p-1}\left(-\frac{b(1-\sin(e+fx))}{a+b\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(e+fx)+1)}{a+b\sin(e+fx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)}{f(1-p)(bc-ad)^2}+\frac{bg\cdots}{\cdots}$$

```
[Out] -b*g*AppellF1(1-p,1/2-1/2*p,1/2-1/2*p,2-p,(a-b)/(a+b*sin(f*x+e)),(a+b)/(a+b
*sin(f*x+e)))*(g*cos(f*x+e))^(-1+p)*(-b*(1-sin(f*x+e))/(a+b*sin(f*x+e)))^(1
/2-1/2*p)*(b*(1+sin(f*x+e))/(a+b*sin(f*x+e)))^(1/2-1/2*p)/(-a*d+b*c)^2/f/(1
-p)+b*g*AppellF1(1-p,1/2-1/2*p,1/2-1/2*p,2-p,(c-d)/(c+d*sin(f*x+e)),(c+d)/(
c+d*sin(f*x+e)))*(g*cos(f*x+e))^(-1+p)*(-d*(1-sin(f*x+e))/(c+d*sin(f*x+e)))
^(1/2-1/2*p)*(d*(1+sin(f*x+e))/(c+d*sin(f*x+e)))^(1/2-1/2*p)/(-a*d+b*c)^2/f
/(1-p)+g*AppellF1(2-p,1/2-1/2*p,1/2-1/2*p,3-p,(c-d)/(c+d*sin(f*x+e)),(c+d)/
(c+d*sin(f*x+e)))*(g*cos(f*x+e))^(-1+p)*(-d*(1-sin(f*x+e))/(c+d*sin(f*x+e))
)^(1/2-1/2*p)*(d*(1+sin(f*x+e))/(c+d*sin(f*x+e)))^(1/2-1/2*p)/(-a*d+b*c)/f/
(2-p)/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.52, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2924, 2703}

$$-\frac{bg(g\cos(e+fx))^{p-1}\left(-\frac{b(1-\sin(e+fx))}{a+b\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(e+fx)+1)}{a+b\sin(e+fx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)}{f(1-p)(bc-ad)^2}+\frac{bg\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] -((b*g*AppellF1[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (a + b)/(a + b*Sin[e +
f*x]), (a - b)/(a + b*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((b*(1 - S
in[e + f*x]))/(a + b*Sin[e + f*x])))^((1 - p)/2)*((b*(1 + Sin[e + f*x]))/(a
 + b*Sin[e + f*x]))^((1 - p)/2))/((b*c - a*d)^2*f*(1 - p))) + (b*g*AppellF1
[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/
(c + d*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((d*(1 - Sin[e + f*x]))/(
c + d*Sin[e + f*x])))^((1 - p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*
x]))^((1 - p)/2))/((b*c - a*d)^2*f*(1 - p)) + (g*AppellF1[2 - p, (1 - p)/2,
 (1 - p)/2, 3 - p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/(c + d*Sin[e + f*x
])]*(g*Cos[e + f*x])^(-1 + p)*(-((d*(1 - Sin[e + f*x]))/(c + d*Sin[e + f*x]
)))^((1 - p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^((1 - p)/2))/
((b*c - a*d)*f*(2 - p)*(c + d*Sin[e + f*x]))
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 2924

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && Integer
sQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = \int \left(\frac{b^2(g\cos(e+fx))^p}{(bc-ad)^2(a+b\sin(e+fx))} - \frac{d(g\cos(e+fx))^p}{(bc-ad)(c+d\sin(e+fx))^2}\right.$$

$$= \frac{b^2\int \frac{(g\cos(e+fx))^p}{a+b\sin(e+fx)}\,dx}{(bc-ad)^2} - \frac{(bd)\int \frac{(g\cos(e+fx))^p}{c+d\sin(e+fx)}\,dx}{(bc-ad)^2} - \frac{d\int \frac{(g\cos(e+fx))^p}{(c+d\sin(e+fx))^2}}{bc-ad}$$

$$= -\frac{bgF_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)(g\cos(e+}{(bc-ad)^2f(1-p}$$

Mathematica [B] time = 55.25, size = 12568, normalized size = 24.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),
x]
[Out] Result too large to show
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(g\cos\left(fx+e\right)\right)^p}{ac^2+2\,bcd+ad^2-\left(2\,bcd+ad^2\right)\cos\left(fx+e\right)^2-\left(bd^2\cos\left(fx+e\right)^2-bc^2-2\,acd-bd^2\right)\sin\left(fx+e\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p/(a*c^2 + 2*b*c*d + a*d^2 - (2*b*c*d + a*d^2)*co
s(f*x + e)^2 - (b*d^2*cos(f*x + e)^2 - b*c^2 - 2*a*c*d - b*d^2)*sin(f*x + e
)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^p}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^2),
 x)
```

maple [F] time = 3.41, size = 0, normalized size = 0.00

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^p}{\left(a+b\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\cos\left(e+fx\right)\right)^p}{\left(a+b\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^2),x)
```

```
[Out] int((g*cos(e + f*x))^p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.1563 $\int \frac{(g\sec(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=308

$$\frac{\sec(e+fx)(g\sec(e+fx))^p\left(-\frac{d(1-\sin(e+fx))}{c+d\sin(e+fx)}\right)^{\frac{p+1}{2}}\left(\frac{d(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{p+1}{2}}F_1\left(p+1;\frac{p+1}{2},\frac{p+1}{2};p+2;\frac{c+d}{c+d\sin(e+fx)},\frac{c-d}{c+d\sin(e+fx}\right.}{f(p+1)(bc-ad)}$$

```
[Out] -AppellF1(1+p,1/2+1/2*p,1/2+1/2*p,2+p,(a-b)/(a+b*sin(f*x+e)),(a+b)/(a+b*sin
(f*x+e)))*sec(f*x+e)*(g*sec(f*x+e))^p*(-b*(1-sin(f*x+e))/(a+b*sin(f*x+e)))^
(1/2+1/2*p)*(b*(1+sin(f*x+e))/(a+b*sin(f*x+e)))^(1/2+1/2*p)/(-a*d+b*c)/f/(1
+p)+AppellF1(1+p,1/2+1/2*p,1/2+1/2*p,2+p,(c-d)/(c+d*sin(f*x+e)),(c+d)/(c+d*
sin(f*x+e)))*sec(f*x+e)*(g*sec(f*x+e))^p*(-d*(1-sin(f*x+e))/(c+d*sin(f*x+e)
))^(1/2+1/2*p)*(d*(1+sin(f*x+e))/(c+d*sin(f*x+e)))^(1/2+1/2*p)/(-a*d+b*c)/f
/(1+p)
```

Rubi [A] time = 0.62, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2926, 2924, 2703}

$$\frac{\sec(e+fx)(g\sec(e+fx))^p\left(-\frac{d(1-\sin(e+fx))}{c+d\sin(e+fx)}\right)^{\frac{p+1}{2}}\left(\frac{d(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{p+1}{2}}F_1\left(p+1;\frac{p+1}{2},\frac{p+1}{2};p+2;\frac{c+d}{c+d\sin(e+fx)},\frac{c-d}{c+d\sin(e+fx}\right.}{f(p+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((AppellF1[1 + p, (1 + p)/2, (1 + p)/2, 2 + p, (a + b)/(a + b*Sin[e + f*x]
), (a - b)/(a + b*Sin[e + f*x])]*Sec[e + f*x]*(g*Sec[e + f*x])^p*(-((b*(1 -
 Sin[e + f*x]))/(a + b*Sin[e + f*x])))^((1 + p)/2)*((b*(1 + Sin[e + f*x]))/
(a + b*Sin[e + f*x]))^((1 + p)/2))/((b*c - a*d)*f*(1 + p))) + (AppellF1[1 +
 p, (1 + p)/2, (1 + p)/2, 2 + p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/(c +
 d*Sin[e + f*x])]*Sec[e + f*x]*(g*Sec[e + f*x])^p*(-((d*(1 - Sin[e + f*x]))
/(c + d*Sin[e + f*x])))^((1 + p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e +
f*x]))^((1 + p)/2))/((b*c - a*d)*f*(1 + p))
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
```

```
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 2924

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && Integer
sQ[2*m, 2*n]
```

Rule 2926

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]*(g*Sec[e + f*x])^FracPart[p
], Int[((a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(g*Cos[e + f*x])^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx = \left((g\cos(e+fx))^p(g\sec(e+fx))^p\right)\int \frac{(g\cos(e+fx))^{-p}}{(a+b\sin(e+fx))(c+d\sin$$

$$= \left((g\cos(e+fx))^p(g\sec(e+fx))^p\right)\int\left(\frac{b(g\cos(e+fx))^{-p}}{(bc-ad)(a+b\sin(e+fx)}\right.$$

$$= \frac{\left(b(g\cos(e+fx))^p(g\sec(e+fx))^p\right)\int\frac{(g\cos(e+fx))^{-p}}{a+b\sin(e+fx)}\,dx}{bc-ad} - \frac{\left(d(g\cos(\right.}{}$$

$$= -\frac{F_1\left(1+p;\frac{1+p}{2},\frac{1+p}{2};2+p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)\sec(e+fx)(g}{(bc-ad)f(1+}$$

Mathematica [B] time = 30.05, size = 5113, normalized size = 16.60

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
[Out] Result too large to show
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(g\sec\left(fx+e\right)\right)^{p}}{bd\cos\left(fx+e\right)^{2}-ac-bd-(bc+ad)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(g*sec(f*x + e))^p/(b*d*cos(f*x + e)^2 - a*c - b*d - (b*c + a*d)*
sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{p}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*sec(f*x + e))^p/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)), x
)
```

maple [F] time = 4.10, size = 0, normalized size = 0.00

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{p}}{\left(a+b\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] int((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{p}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*sec(f*x + e))^p/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^p}{\left(a+b\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))),x)
```

```
[Out] int((g/cos(e + f*x))^p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g \sec{\left(e + f x \right)}\right)^{p}}{\left(a + b \sin{\left(e + f x \right)}\right) \left(c + d \sin{\left(e + f x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((g*sec(e + f*x))**p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))), x
)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```